STABILIZATION OF ROTARY INVERTED PENDULUM USING DIFFERENT SLIDING MODE CONTROL TECHNIQUES: AN EXPERIMENTAL STUDY

A Thesis for fulfillment of the requirements for the Degree

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in

Electronic Instrumentation & Control Engineering

submitted by
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DECLARATION

I hereby certify that the work which is presented in this dissertation entitled, "**Stabilization of Rotary Inverted Pendulum Using Different Sliding Mode Control Techniques: An Experimental Study**", in complete fulfilment of the requirements for the award of the degree of Master of Engineering in Electronic Instrumentation and Control Engineering, submitted to Electrical & Instrumentation Engineering Department of TIET, Patiala is as authentic record of my own work carried under the supervision of Dr. Vikram Chopra. It refers others researcher's work which is duly listed in the reference section. The matter contained in this dissertation has not been submitted, neither in part nor in full to any other degree to any other university or institute except as reported in the text and references.

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It is certified that the above statement made by the student is correct to the best of my knowledge and belief.

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Date: 29/06/2018

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LIST OF ABBREVIATIONS

IFAC	International Federation of Automatic Control
RIP	Rotary Inverted Pendulum
LQR	Linear Quadratic Regulator
PID	Proportional Integral Derivative
SMC	Sliding Mode Controller
FLC	Fuzzy Logic Controller
ISMC	Integral Sliding Mode Controller
FSMC	Fuzzy Sliding Mode Controller
VTOL	Vertical Take-Off and Landing Aircraft
LQG	Linear Quadratic Gaussian Control
ANN	Artificial Neural Network
ANFIS	Adaptive Neural Fuzzy Inference System
GA	Genetic Algorithm
PSO	Particle Swarm Optimization
MIMO	Multi Input Multi Output
FISMC	Fuzzy Integral Sliding Mode Control
DOF	Degree of Freedom
FSF	Full State Feedback
PD	Proportional Derivative
SISO	Single Input Single Output
ASCFNN	Adaptive Self-Constructing Fuzzy Neural Network
DE	Differential Evolution
ACO	Ant Colony Optimization
KE	Kinetic Energy
PE	Potential Energy
DAQ	Data Acquisition
DC	Direct Current
PWM	Pulse Width Modulation
VSCS	Variable Structure Control Systems

LPF	Low Pass Filter
EA	Evolutionary Algorithm

ABSTRACT

In last few decades, inverted pendulum has been considered as one of the benchmark problem in control theory. The system is highly nonlinear, underactuated, multivariable, open loop unstable and non-minimum phase. Different type of inverted pendulums is found in the literature. These are: linear inverted pendulum, rotary inverted pendulum, spherical inverted pendulum, double inverted pendulum, and mobile wheeled pendulum etc. The inverted pendulum control has been classified into three categories i.e. the swing control, the stabilization control and the tracking control. This work presents the stabilization of rotary inverted pendulum (RIP). Stabilization means keeping the inverted pendulum in the vertical upright position by applying suitable control effort on the rotary base. The conventional control techniques do not perform well in the presence of uncertainties or disturbances. Hence there is a need of robust control techniques for stabilizing of such an unstable system. The stabilization of RIP is done using different sliding mode control (SMC) techniques. These control techniques include SMC, integral sliding mode control (ISMC) and fuzzy integral sliding mode control (FISMC). SMC has two phases: reaching phase and sliding phase and it shows robustness in the presence of disturbance during sliding phase. In case of SMC the reaching phase is noise prone. This limitation of SMC can be eliminated by the ISMC technique. Both SMC and ISMC techniques are affected by the chattering phenomena which has adverse effect on the mechanical system. Therefore, FISMC technique is used to minimize the chattering effect. These control techniques have been implemented using MATLAB simulink and their results are compared with the conventional linear quadratic regulator (LQR) controller (without and with disturbance) in terms of settling rise time (t_r) and time (t_s). Simulation results show that the performance of LQR deteriorate with disturbance while SMC, ISMC and FISMC techniques gives better performance in the presence of disturbance. These control techniques are further validated on the real-time RIP system called QUANSER QUBE-SERVO.

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CHAPTER 1: INTRODUCTION

1.1 Introduction of Pendulum:

Inverted pendulum system is one of the most interesting problem in control theory. In year 1990, a bunch of control problems that show resemblance to real-world control problems were defined by the International Federation of Automatic Control (IFAC) theory committee to validate conventional and modern control schemes, called benchmark problem [1]. Control of an inverted pendulum is one of these problem that is defined by the IFAC committee.

The real-world applications of inverted pendulum system are: control of an underactuated system [2], the stabilization of rocket, missile and ship cabin, stabilization of mobile wheeled vehicle like Segway [3], hoverboard [4], unicycle, stabilization of vertical take-off and landing (VTOL) aircraft [5], stabilization of humanoid robot which is combination of simple pendulum and inverted pendulum [6], design of earthquake resistant building [7], etc. Some of these are shown from Fig.1.1 to Fig. 1.8.

Fig.1.1: Rocket

Fig.1.2: Missile

Fig.1.3: Stabilization of Ship Cabin

Fig.1.4: Segway

Fig.1.5: Hoverboard

Fig.1.6: VTOL Aircraft

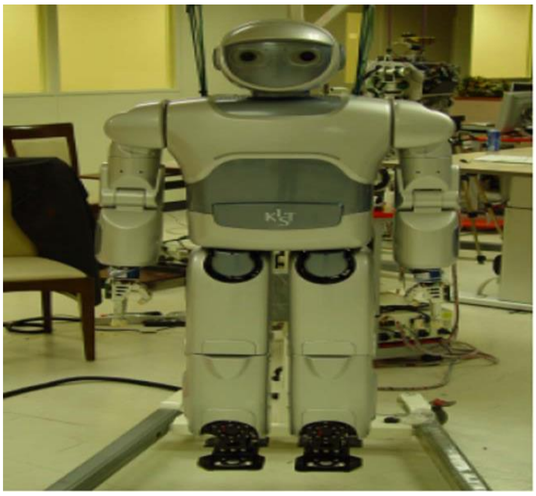

Fig.1.7: Humanoid Robot

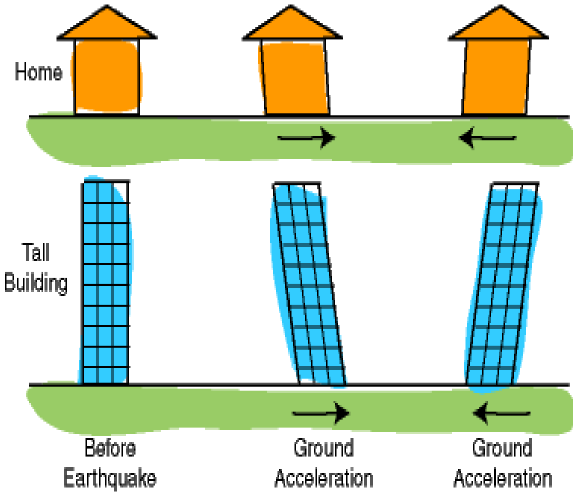

Fig.1.8: Earthquake Resistant Building

There are many types of inverted pendulum found in the literature like, linear inverted pendulum (x-inverted pendulum) [8], rotary inverted pendulum (RIP) [9], spherical inverted pendulum (x-y inverted pendulum) [8], x-z inverted pendulum [5], double inverted pendulum [10], wheeled inverted pendulum [11], and recently x-y-z inverted pendulum.

Three types of control problems related to inverted pendulum systems are found in literature. These are:

(i) Stabilization Control Problem: - In this problem, pendulum is stabilized at vertically upright position which is against gravity so, this position of pendulum is unstable in nature. It is a linear problem [12].

(ii) Swing-Up Control Problem: - In this problem, pendulum is controlled to swing up from hanging stable equilibrium position to vertically upright unstable position. It is a highly nonlinear problem [13,14].

(iii) Tracking Control Problem: - In this problem, cart or rotary arm is made to follow the desired trajectory while stabilizing the pendulum at vertically upright position. It can be linear or nonlinear problem [15].

The RIP system was first introduced by K. Furuta (in 1992) [16] and hence it is also called as Futura Pendulum. RIP is a highly non-linear, open-loop unstable, non-minimum phase, multivariable and underactuated system [17]. This system consists of a pendulum hinged to a rotary arm as shown in Fig.1.9.

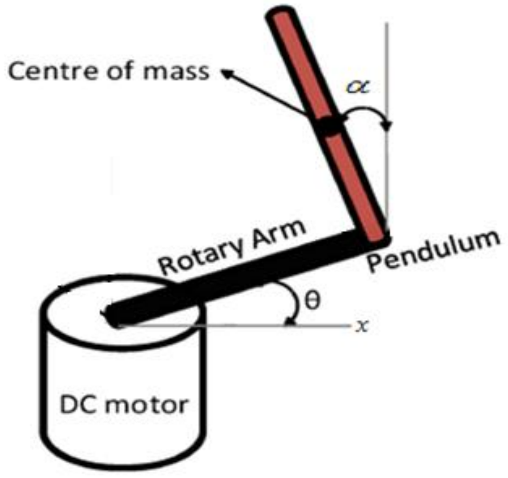

Fig.1.9: Rotary Inverted Pendulum

Fig.1.10 Quanser QUBE-SERVO (USB Interface)

Here θ represent the rotary arm position angle and α represent the pendulum arm position angle. The rotary arm is controlled by an actuator and pendulum is indirectly controlled. This system has advantage of infinite track length unlike linear inverted pendulum systems.

The controller design for pendulum arm stabilization is quite difficult task because of the physical limitations of rotary arm position angle, pendulum arm position angle and actuator that applied to motor. In this work, the stabilization of RIP system is achieved using different sliding mode techniques. These control techniques are implemented on the MATLAB simulink and are further tested on the QUANSER experiment setup called QUBE-SERVO shown in Fig.1.10.

1.2 Scope of Thesis:

In this thesis, stabilization control of RIP is achieved using different sliding mode control techniques. These control techniques include sliding mode control (SMC), integral sliding mode control (ISMC) and fuzzy integral sliding mode control (FISMC). Thereafter, these control techniques are validated on the simulink and experimental setup of RIP which is provided by QUANSER and the robustness of these control techniques is verified by comparing them to linear quadratic regulator (LQR) control technique without and with disturbance environment.

The work in thesis is organized below:

- Firstly, the dynamic mathematical model of RIP has been analyzed and is developed in the MATLAB simulink.
- Secondly, the conventional control technique i.e. LQR is implemented on the simulink model of RIP system.
- Thereafter, robust control techniques i.e. SMC, ISMC and FISMC are implemented on the simulink model of RIP system.
- Finally, these control techniques are compared and validated on the experimental setup of RIP system which is QUANSER QUBE-SERVO setup.

1.3 Organization of the thesis:

The thesis is organized as follow:

- Chapter 2 describes the literature survey of inverted pendulum and its control techniques. In this chapter, the control techniques are distinguished in three groups: conventional control techniques, robust control techniques and intelligent control techniques. The literature survey of inverted pendulum has been done on the basis of these control techniques.
- Chapter 3 explains the dynamic mathematical modeling of RIP system with the help of Euler-Lagrangian method. The non-linear model of RIP is achieved by this method. Thereafter, the non-linear model is linearized around the equilibrium point which is unstable vertical position. Then, the state space representation of the RIP system has been obtained.

- Chapter 4 describes the control techniques used to stabilize the pendulum arm of RIP system in vertical upright position. These control techniques include SMC, ISMC and FISMC are discussed.
- Chapter 5 described the simulation and experimental implementation of these control techniques. In this chapter, the simulation models of these control techniques are shown. Thereafter, the simulation and experimental results are shown.
- Chapter 6 gives the conclusion and future scope of the thesis.

CHAPTER 2: LITERATURE SURVEY

2.1 Introduction: -

Since 1950, the inverted pendulum is a benchmark [1] for teaching and studying different types of control strategies [18]. Boubaker, [19] described variety of controllers available for the linear pendulum system in a survey article. Many researchers worked on the stabilization, swing-up and tracking control of inverted pendulum. Different control techniques had been developed to stabilize the inverted pendulum in vertical upright position which is unstable equilibrium position. However, most of the controllers were tested in the simulation environment only and not demonstrated on real-time experimental setups. The results on simulation are quite different from the real-time experimental results. In simulation, physical restrictions are ignored such as constrained cart length, friction effect, damping effect etc. In this study, literature survey has been done on the basis of control techniques i.e. conventional, robust and intelligent control techniques.

2.2 Brief History of Inverted Pendulum: -

The inverted pendulum is a standard robotics system which is used to study different types of control techniques by researchers and scholars in control, physics and mechanics. It is one of the simple and unstable mechanical system. Roberge in 1960 proposed a controller to inverted-pendulum system at M.I.T. in his dissertation, "The Mechanical Seal" [20]. Higdon and Cannon in 1963 at Standford, described the multiple independent inverted pendula. Schaefer and Cannon in 1966 discussed jointed and flexible inverted pendulum systems. In classes of physics and classic mechanics, stabilization of inverted pendulum by a vertically oscillating base is one of the favourite examples. In 1976, Mori *et al.* proposed the swing-up control of an inverted pendulum system. Then, in 1991, Futura *et al.* proposed the RIP system as an alternative to the linear inverted pendulum system [20].

In this survey, the control techniques are divided into three categories:

a. Conventional control techniques
b. Robust control techniques
c. Intelligent control techniques

2.3 Conventional Control Techniques: -

The basic conventional control techniques include proportional-integral-derivative (PID) control [21], pole placement control, LQR control [22], bang-bang (On-Off) control.

2.3.1 PID Control Technique: -

The PID control technique is the combination of proportional, derivative and integral term. In this technique, all the three terms contribute and reduce the error to minimum level. The individual proportional term, reduces the error of the system but not completely (2% error is present). The integral term is used to reduces the error completely but to do so it increases the number of oscillations. To remove this problem, the derivative term is introduced, that remove the error completely and decreases percentage overshoot. The output voltage of PID controller is defined as [23]:

$$V_d = K_d \dot{e}(t) + K_p e(t) + K_i \int e(t)dt \tag{2.1}$$

where K_p, K_d and K_i are the gains of PID controller. The gains are tuned by various methods like Ziegler's Nicholas method, trial and error, Cohen-coon, etc. In PID control technique, proportional gain term result depends on the current error in system, integral gain term results depend on sum of previous errors and derivative gain term result depends on rate of change of error [24]. When any uncertainty occurs, the PID controller performance may deteriorate.

2.3.2 Bang-Bang Controller: -

It is also called as on-off controller. It works on the principle of switching between two states for controlling a process. Its principle is just same as relay system. Bang-Bang controller is easily coded and tune it. The main use of this controller is in minimum time problem. The main drawback of bang-bang controller is intolerable accuracy level and complex calculation. [25].

2.3.3 Linear Quadratic Regulator (LQR) Controller: -

LQR control technique is the optimal conventional control technique which depend on the state feedback law. In this technique, the poles of the system are placed at the desired location by using Ackerman's formulae. The gain of the LQR controller is observed by using algebraic Riccati equation and depends on state weight matrix 'Q', input weight matrix 'R', input matrix 'B' and system state matrix 'A'. The state weight matrix 'Q' and input weight matrix 'R' is tuned to assign the weight according to the performance of all the states and input to the system. The 'MATLAB' code is used to find the gain of controller is '*lqr*' The main drawback of this control technique is to determine all the state of the system, so that the weight of all the states are assign accordingly [22].

2.3.4 Pole Placement Controller: -

This control technique depends on the state feedback law. In this technique, the poles of the system are placed at the desired location. The gains of controller are calculated by the 'MATLAB' code '*place*'. The two dominant poles on the left half of the s-plane are calculated on the basis of desired overshoot and settling time of the system shown below and other poles of the system are chosen far away from the left half of s-plane that they do not affect the response of the system [17].

$$\text{Overshoot,} \qquad M_p = e^{\frac{-\Pi\xi}{\sqrt{1-\xi^2}}} \tag{2.2}$$

$$\text{Settling time,} \qquad t_s = \frac{4}{\xi\omega_n} \tag{2.3}$$

For the desired closed loop system characteristics equation given by:

$$(s^2 + 2\xi\omega_n s + \omega_n^2) = 0 \tag{2.4}$$

The dominant poles are determined by solving the equation (2.4). The stability of the system is defined by the poles placed on the left half side of s-plane.

Literature survey has been done below on the basis of conventional controllers: -

- Akhtaruzzaman and Shafie [24] proposed the different type of controllers for stabilizing the RIP in upright position. They proposed a two degree of freedom (DOF) PID controller, full state feedback (FSF) and LQR control techniques. In this study, FSF and LQR control scheme were used for both swing-up as well as stabilization control of inverted pendulum. In this study, the performance of two DOF PID could be improved by tuning the controller parameters. The results showed that both LQR and FSF controllers were robust to the parameter variations and LQR controller was more robust and reliable than FSF controller. The LQR controller was also faster than the FSF controller.
- Wang [8] proposed the PID controller's simulation studies of inverted pendulum i.e. x, x-y and x-z inverted pendulum. The control design of PID controller for inverted pendulum system step by step. It gave a new solution to three types of inverted pendulum. The proposed scheme gave good dynamic performance and robustness to external disturbances.
- Ghosh *et.al* [26] proposed a two loop PID control scheme to stabilize the inverted pendulum arm in upright direction via pole placement technique. In this study, the closed loop dominant poles to be placed at the desired locations were obtained from an LQR control scheme. The

proposed PID controller was robust than the LQR controller and gave the better responses due to additional integral term.

- Nath and Mitra [27] proposed a counter based and pole placement controllers for stabilization of RIP system. A mode controller was used to change the control action which is either swing-up and stabilization controllers. In this study, With the help of counter-based swing-up controller the pendulum arm was placed in vertical position within 4.5 second. This type of controller is also implement with microcontroller because they have inbuilt counter. Pole placement controller with integrator eliminated the error in rotary arm position and the error band was within 0.5%.
- Avelar and Valenzuela [28] proposed a new composite scheme where the total control action was constituted the sum of feedback-linearization-based controller and an energy-based compensation. In this study, the objective was the tracking of a desired periodic trajectory, while the pendulum arm was stabilized at the upward position. The new proposed algorithm showed the better performance in both arm trajectory and pendulum regulation. The proposed controller was showed the good tracking the desired signal and the stabilize the pendulum position.
- Kathpal and Singla [17] proposed the stabilization control techniques like pole placement, proportional derivative (PD) and LQR control. They used these control techniques on the real-time RIP hardware and compared their results with the simulink based model. Among these three controllers, LQR gave better results among other control techniques.
- Roshdy *et al.* [29] proposed the pole placement technique that is using pole separation factor to stabilize the real inverted pendulum. In this paper, they used the FSF controller for stabilization of pendulum arm. First two dominant poles were selected and then it is used the separation factor between selected dominant poles and the other poles to assured that their effect on system performance was neglected. Then, feedback gain matrix was calculated with the help of Ackermann's formula. According to paper, the proposed controller handled the external disturbances very well. The proposed controller showed the excellent performance, 1.5 sec settling time and low overshoot. When the external disturbance observed, the pendulum was stabilized within 2 second.
- Seman *et al.* [30] proposed the swinging up control techniques and model predictive control technique to stabilize the pendulum in upright position. In this paper, the authors mentioned two swing-up control methods. First method depends upon the comparison among the total energy of the system with the energy stored in its stabilize unstable position. The second

method was related to the use of exponentiation operation over the pendulum position. They tested the control techniques on the real-time RIP setup.

- Wanli *et al.* [31] proposed the control method to stabilize the pendulum arm based on Kalman filter. According to paper, earlier the effect of measurement noise and output noise of the sensor were not considered and because of this, strong random jitter occurred which effects the control accuracy and stability of inverted pendulum. So, the Kalman filter was used to eliminate the noise effect. The Kalman filter can predict the signal disturbed by the noise.
- Sukontanakarn and Parnichkun [32] proposed the energy-based PD controller for the swing-up and LQR controller to stabilize the RIP arm in upright position. In this study, both controllers tested on the real-time setup of the RIP and compared with the computer simulation.
- Yusuf and Magaji [33] proposed the optimized PID controller to stabilize the inverted pendulum. The parameters of PID were optimized by genetic algorithm (GA). The proposed optimized controller was compared with the conventional PID controller whose gains were manually tuned. The comparison was done on the basis of settling time and percentage overshoot. The percentage overshoot and settling time of the optimized PID controller was 3% and 5.02 seconds respectively whereas 5% and 70 seconds for the conventional PID controller.

2.4 Robust Control Techniques: -

The robust control techniques are the techniques which do not allow external disturbances to affect the system performance. Some robust control techniques such as SMC [34], ISMC, fuzzy sliding mode control (FSMC) [35], FISMC, H-infinity controller, linear quadratic gaussian (LQG) control [36] technique, etc.

2.4.1 Sliding Mode Control (SMC) Technique: -

The SMC is a control technique which is robust against the matched uncertainties, parametric uncertainties and external disturbance. In this technique, the sliding surface has to be designed and there after a control law is defined which take the nonlinear system onto the designed sliding surface which is linear in nature. The SMC control technique is affected by the chattering phenomena (high frequency switching) which is not good for the mechanical system. The SMC has two phases: reaching phase, sliding phase. The SMC shows robustness when it comes to sliding phase. The SMC controller affected by noise, when it is in the reaching phase. This control is widely used in the swing-up and stabilization control of RIP [34].

2.4.2 Integral Sliding Mode Control (ISMC) Technique: -

The ISMC control technique is the improved version of SMC. In this technique, the reachability phase is eliminated which is noise prone phase. In this technique, control law is divided into two parts. First part of control law is nominal control which is defined by LQR, pole-placement control technique, PID control technique etc. Second control law part is the discontinuous control which is integral sliding mode is added to it. The discontinuous control part provides robustness to parametric uncertainties but it is influenced by chattering phenomena [37].

2.4.3 Fuzzy Sliding Mode Control (FSMC) And Fuzzy Integral Sliding Mode Control (FISMC):-

The FSMC and FISMC control techniques are used to remove the high frequency switching around the sliding surface called chattering phenomena. The chattering mainly introduced due to the discontinuous control. So, in this technique the discontinuous control is replaced by the smooth control technique provided by fuzzy logic control (FLC). The inputs of the FLC are sliding surface and rate of change of sliding surface and output is control law which drive the system [9].

2.4.4 H-Infinity Control Technique: -

The H-infinity control technique is the most robust control technique. Generally, the desired condition of any system is that it has proper and stable transfer closed loop transfer. So, these two conditions satisfy when the system in the H-infinity space. In this space, the cost function is optimized and select the best transfer function in the space which satisfy the above two conditions. In H-infinity control, comparison of transfer function is done on the basis of their H-infinity norm. The H-infinity norm of the transfer function is defined as:

$$\|G\|_{\infty} = \sup_{\omega} |G(jw)| \tag{2.5}$$

So, in H-infinity control technique, the objective is to minimize the H-infinity norm which is shown in equation (2.5) [38].

2.4.5 Linear Quadratic Gaussian (LQG): -

The LQG control technique is the optimal control technique and it is robust against the disturbance (additive white gaussian noise). In this control technique, the Kalman filter which is act as an estimator is introduced with the LQR control technique. The Kalman filter and LQR controller is designed and computed individually [38].

The literature survey based on the Robust control technique is shown below: -

- Khanesar *et al.* [39] proposed the FSMC control technique to control RIP system. They combined the fuzzy system with the classical SMC controller to estimate the signal of the SMC controller. The disadvantage of SMC controller was difficult to design of equivalent control signal. Fuzzy neural network was employed to add adaption to the classical SMC controller. The stability of the controller and the adaption algorithm were controlled by a proper lyapunov function.

- Hoang and Wongsaisuwan [40] proposed a robust controller designed using linear matrix inequalities to stabilize the RIP system. They proposed the state feedback H_∞ and an output feedback H_∞ controller. The performance index was minimized when two sinusoidal disturbances were injected to the system.
- Yang *et al.* [41] proposed the swing up control method for an inverted pendulum with fixed cart length. The basic fundamental method for swing up is based on the energy methods. The authors used the new lyapunov function to control the swing-up of the inverted pendulum. The new lyapunov function is summation of square of the pendulum energy and the weighted square of the cart's velocity. In this study, design parameters were reduced. Instead of four or more parameters to tuned, there were only two design parameters to be tuned.
- Anvar *et al.* [42] proposed the sliding mode-state feedback control scheme for stabilization of RIP. In this study, the authors combined the genetic based state feedback control and SMC. The disadvantage of SMC like chattering phenomena was eliminated by switched to GA based state feedback control which is a smooth control law. The proposed control scheme consumed less control signal energy when compared to SMC controller.
- Wang [5] proposed SMC for stabilization and tracking control of the x-z inverted pendulum. The performance of SMC scheme was compared with the conventional PID control scheme. Good control performance was achieved with the help of SMC.
- Balamurugan *et al.* [9] proposed the FSMC controller with low pass filter (LPF) to reduce the chattering effect which presents in the conventional SMC controller. In this paper FSMC controller was tested on the QUANSER real time setup of the RIP. According to paper FSMC controller completely removed the chattering but when it is applied to the real time hardware setup, the chattering was not completely removed. To eliminate this drawback, the FSMC controller is used with the LPF. In this paper, the design of SMC and FSMC was restricted for linear operating region.
- Sirisha and Junghare [43] proposed the four types of controllers to stabilize the RIP arm and showed a comparative study of these four controllers. The four controllers used were fuzzy logic, H_∞, LQR and PID controllers. In simulation results the FLC controller gave better result in term of percentage peak overshoot and rise time among all four controllers. In test bed result,

the LQR controller gave the betters result in term of percentage peak overshoot and rise time among PID and FLC controller.

- Bhavsar and Kumar [37] proposed the ISMC to track the trajectory of the linear inverted pendulum. ISMC techniques is the advance version of conventional SMC technique. ISMC avoids the reaching phase which is present in the conventional SMC. In this paper, ISMC control technique was compare with LQR control technique with and without disturbance. According to the paper ISMC showed the better results in presence of uncertainties.
- Yu *et al.* [44] proposed the robust control technique to stabilize the RIP arm. In this paper, standard H_∞ robust control technique was used for stabilization purpose. They apply fixed-order H_∞ controller to simplify the control structure. The standard H_∞ control technique results were compared with the H_∞ fixed-order control technique
- Dastranj *et al*. [45] proposed the FSMC and GA to control the inverted pendulum. They combined both FSMC and GA technique to make the robust control to stabilize the pendulum arm

2.5 Intelligent Control Techniques: -

The intelligent control techniques are FLC, artificial neural network (ANN), adaptive neuro-fuzzy inference system (ANFIS) etc.

2.5.1 Fuzzy Logic Control (FLC) Technique: -

The FLC is a control technique which deals with the concept of vagueness means it is not expressed as ‘true’ or ‘false’, it is expressed as ‘partial true’ or ‘partial false’. The FLC inputs are the error of system and rate of change of error of the system. The FLC needs the expert knowledge of the system rather than the mathematical model. This technique based on the IF-THEN rule-base structure. It has fast computational speed and very popular in inverted pendulum control problems. The drawbacks of this technique are, it is difficult to tune because it requires complete knowledge about the system and difficult to make the rule-base for MIMO (multi input multi output) systems [22].

2.5.2 Artificial Neural Network (ANN) Control Technique: -

ANN is the intelligent control technique inspired by biological neural network. In this technique, generally three layers are present: input layer, activation function, output layer. The synaptic weights are assigned to each input and these weights are trained by some learning techniques [46].

2.5.3 Adaptive Neuro-Fuzzy Inference System (ANFIS) Control Technique: -

ANFIS is the hybrid control technique which is a combination of FLC and ANN. There are total five-layer present in this structure. The Takagi-Sugeno fuzzy inference system is used in the ANFIS structure. In this technique, two optimization algorithms are used: least square estimate algorithm and gradient decent algorithm. The ANFIS structure layers are: -

a) Input layer
b) Membership layer
c) Rule-base layer
d) Normalization layer
e) Output layer

The basic structure of ANFIS controller are shown in Fig.2.1 [47].

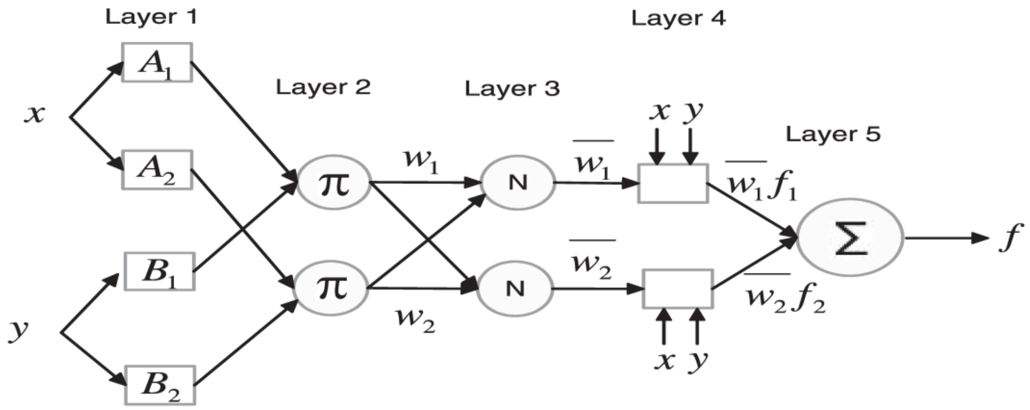

Fig.2.1: Basic Structure of ANFIS Controller

The literature survey based on intelligent control technique is shown below: -

- Chiu and Chen [48] proposed a FLC to swing up pendulum from downward position to upright position and then balanced at the set point. They proposed a FLC based on a microprocessor platform. In FLC, precise mathematical model is not required. Thus, it is simpler than the conventional control schemes. The whole controller is designed around an 8051-microcontroller.
- Rahmanian *et al.* [49] proposed an off-line and on-line fuzzy backstepping controllers for stabilization of RIP. In this, an adaptive fuzzy controller was developed with the help of backstepping technique for a class of single input single output (SISO) nonlinear systems. When the backstepping scheme and a neuro-fuzzy system or a fuzzy neural net was combined, the performance of controller was improved.

- Hassanzadeh and Mobayen [50] proposed a binary GA based input-output feedback linearization controller for stabilization of RIP. The proposed method efficiently gave better performance in terms of rise time, settling time and overshoot.
- Chiu *et al.* [51] proposed the model free intelligent controller to control wheeled inverted pendulums. In this method, without knowing about the model information, controlling of angle and position of the wheeled inverted pendulum is done.
- Dwivedi *et al.* [52] proposed the fractional order controller for stabilization of unstable equilibrium point of RIP. The focus of this study was to minimize the deviation in rotational arm. The parameters of the fractional controller were tuned by a simple tuning method based on the frequency response is used. Parameters were also tuned by dynamic particle swarm optimization (dPSO).
- Oh *et al.* [53] proposed the optimized cascade fuzzy controller based on differential evolution (DE) algorithm to stabilize the RIP. The author concluded that a DE algorithm was simpler than other optimization algorithms like GA, particle swarm optimization (PSO), tabu search etc. and its convergence rate to the best value was best among all. In this paper, fuzzy PD controller in cascade form was used to stabilize the pendulum arm and optimize the fuzzy PD cascade controller by the GA and DE algorithm. The simulation results of DE algorithm with GA were compared and it was found that GA optimized fuzzy PD cascade controller results in large overshoot
- Liu *et al.* [54] proposed FLC control technique to stabilize the inverted pendulum in real-time environment. The author first implemented FLC scheme in the MATLAB simulink environment and then tested this control scheme on real-time hardware setup of inverted pendulum. In this paper, x-inverted pendulum was used. To remove the complexity of the system and real-time requirement, MATLAB based hardware in loop simulation was used. The FLC control scheme stabilize the pendulum arm within one second in both software and hardware-based setup.
- Rahimi *et al.* [55] proposed a controller based on PSO algorithm to stabilize the pendulum arm of RIP system. In this paper, the authors implemented a PID controller with the state-feedback controller and optimized the performance of the state feedback controller using PSO algorithm. PID controller was used to swing-up the pendulum and the state feedback controller was used to stabilize the pendulum in upright position. The result showed that PSO algorithm used with pole placement gave the better response and it could be used for other systems and control techniques.

- Lu *et al.* [56] proposed an adaptive self-constructing fuzzy neural network (ASCFNN) controller for tracking control of real-time hardware setup of inverted pendulum system. The ASCFNN identifier has structure and parameter learning to get the approximate performance. To determine the fuzzy rules eliminated/generated or not, the Mahalanobis distance (M-distance) method in the structure learning was used.
- Sunil and Manju [57] proposed a FLC controller to stabilize the QUANSER RIP. According to the paper, the pendulum angle, velocity and arm velocity were stabilized at zero within 0.5 sec.
- Melba and Marimuthu [58] proposed an intelligent hybrid controller for swing-up and stabilization of RIP system. The PD position control was used to swing-up the pendulum to and fro motion. When the pendulum reached under certain limit then the stabilization controller which is FLC is operated. The LQR control technique was also used for the stabilization purpose.
- Kharola [59] proposed PID based ANFIS and fuzzy control technique to stabilize the inverted pendulum on the plane inclined at an angle of 10^o from the horizontal plane. The author used PID controller whose gains were found by the trial and error method. Then the dataset generated by the PID controller-based simulation result was used to train the ANFIS controller. The fuzzy controller was also used to stabilize the pendulum arm. The ANFIS controller gave the better results than the remaining two controllers. The error in stabilization after training an ANFIS controller is 0.0052838. The pendulum was stabilized by the PID, fuzzy and ANFIS control techniques within 10 sec, 7.6 sec and 7.5 sec respectively.
- Yurkovich and Widjaja [60] proposed a fuzzy controller to stabilize the inverted pendulum system. In this paper the authors used the three features of fuzzy control in the design of controller i.e. direct, auto-tuning and supervisory. The fuzzy supervisory feature was used in the energy pumping strategy for swing-up the pendulum. The direct feature of fuzzy control was the generalized technique of LQR-based linear control technique, it is used in the balancing of inverted pendulum. The auto-tune feature of fuzzy control was used with the LQR design when the model uncertainties affect the performance of controller. This new technique was developed to reject the model uncertainty, online vary its resolution. All these control techniques were validated on the real-time hardware setup of inverted pendulum system.
- Saidi and Allad [61] proposed an optimization technique to optimized the FLC parameters to stabilize the inverted pendulum. The GA optimization technique was used to optimized the structure and gains of FLC.

- Kuo *et al.* [62] proposed the FLC to stabilize RIP system. In this paper, the parameters of FLC were optimized by GA. The GA optimized the input membership function of FLC. The proposed method was effective to search the optimal membership function instead of trial and error method and gave better results than FLC without GA.
- Ibtissem and Nouredine [63] proposed the PD controller to stabilize the x-inverted pendulum in vertical upright position. In this paper, the gains of PD controller were optimized by the DE algorithm. The multi-objective function had to be minimize in order to get the optimum gains of PD controller. The proposed method gave better response then conventional PD controller.
- Hassanzadeh and Mobayen [64] proposed the control techniques based on evolutionary algorithm (EA) to stabilize the RIP. The EA consists GA, ant colony optimization (ACO) and PSO. The proposed schemes were validated on the experimental setup. According to authors ACO method was better than other two techniques in term of convergence efficiency. Also, ACO technique was slower than PSO but faster than GA.
- Huang and Huang [65] proposed an intelligent control technique including grey prediction model and PD controller to control the inverted pendulum. In this study, they used two different grey model controllers for swing-up the pendulum and stabilize it at upright position. This control technique was also validated on the experimental setup. The proposed control technique showed robustness against the disturbance.

CHAPTER 3: MATHEMATICAL MODELING OF ROTARY INVERTED PENDULUM

This chapter introduces the dynamic mathematical modeling of RIP system with the help of Euler-Lagrange approach. There are two different mathematical modeling techniques present in the literature: Newton-Euler and Euler-Lagrange. The Newton-Euler approach depends upon Newton's second law of motion and Euler-Lagrange approach depends upon total energy present in the system. In this work, for mathematical modeling of RIP system, Euler-Lagrange method is used.

The generalized Lagrange equation is:

$$L(q,\dot{q},t) = k(q,\dot{q},t) - p(q,t) \tag{3.1}$$

where, L is the Lagrangian operator, q is the vector of generalized coordinates, k is the kinetic energy (KE) of a system and p is the potential energy (PE) of a system.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = f_i \forall i = 1,2,...,n \tag{3.2}$$

where, f_i is the i^{th} generalized force acting on the system and *R* is the Rayleigh's dissipation function.

3.1 Dynamics of RIP System: -

A schematic representation of RIP system is shown in Fig.3.1. The equations of motion are computed using Euler-Lagrange method. The two generalized coordinates chosen in this case are θ and α, which represent the angular displacement of the rotary arm and pendulum arm from their reference point respectively. To derive the equations, pendulum is considered as a lumped mass at its center of mass. It consists of a rotary arm on which a pendulum is pivoted. The rotary arm is directly controlled by motor while pendulum is indirectly controlled, thus making it an underactuated system. Here, L_r represents length of rotary arm, L_p represents length of the pendulum, theta (θ) represents rotary arm angle which is taken positive in counter-clockwise direction and alpha (α) represent pendulum angle from vertically upright pendulum position and is taken positive in counter clockwise direction as seen from outside of system.

The servo should turn in the counter clockwise direction when the control voltage is positive i.e. $V_m > 0$. The generalized coordinates of the system are θ and α. The parameters of the RIP are given in Table 3.1.

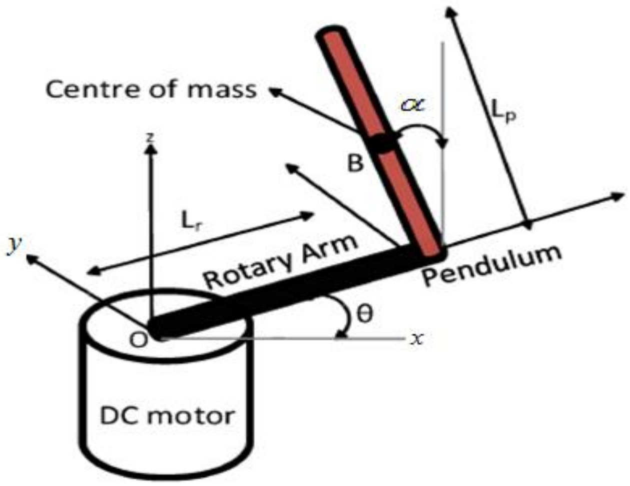

Fig.3.1: Schematic Diagram of Rotary Inverted Pendulum

Table 3.1: Parameters of RIP System

Symbol	**Description**	**Units**	**Values**
m_p	Mass of pendulum	Kg	0.024
J_p	Pendulum inertia	Kgm^2	3.3×10^{-5}
L_p	Length of pendulum	m	0.129
B_p	Pendulum damping coefficient	Nms/rad	0.0005
J_r	Rotary arm inertia	Kgm^2	5.7×10^{-5}
L_r	Length of rotary arm	m	0.085
B_r	Viscous damping coefficient	Nms/rad	0.0015

3.2 Mathematical Modeling of RIP System: -

In this section, dynamic modeling of RIP is performed by using analytical approach. In this approach, the Euler-Lagrange method is used to develop the equations of motion.

Total KE of the system is given by summation of kinetic energy of the pendulum and the rotary arm, which is given as:

$$KE = KE_{arm} + KE_{pend} \tag{3.3}$$

$$= \frac{1}{2}(m_p L_r^2 + J_r + m_p \frac{L_p^2}{4}\sin^2\alpha)\dot{\theta}^2 + \frac{1}{2}(m_p \frac{L_p^2}{4} + J_p)\dot{\alpha}^2$$

$$-\frac{1}{2} m_p L_r L_p \dot{\theta}\dot{\alpha}\cos\alpha \tag{3.4}$$

In this system only, pendulum possesses PE. The total PE of the system is given as:

$$PE = PE_{arm} + PE_{pend} \tag{3.5}$$

$$= 0 + (-m_p g(\frac{L_p}{2} - \frac{L_p}{2}\cos\alpha)) \tag{3.6}$$

Due to two generalized coordinates, there are two equations of motion of the system. Therefore, Lagrange's equation of the system is given as:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}}) - \frac{\partial L}{\partial \theta} + \frac{\partial R}{\partial \dot{\theta}} = \tau \tag{3.7}$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\alpha}}) - \frac{\partial L}{\partial \alpha} + \frac{\partial R}{\partial \dot{\alpha}} = 0 \tag{3.8}$$

where, R is the Rayleigh's dissipation function which is used to drive damping force is given as:

$$R = \frac{1}{2} B_r \dot{\theta}^2 + \frac{1}{2} B_p \dot{\alpha}^2 \tag{3.9}$$

Here, L = K - P and τ represents torque applied by motor at rotary arm. After solving the equations and linearizing them about operating point ($\alpha = 0$), the following equations of motion are obtained:

$$(m_p L_r^{\ 2} + J_r)\ddot{\theta} - \frac{1}{2} m_p L_p L_r \ddot{\alpha} + B_r \dot{\theta} = \tau \tag{3.10}$$

$$-\frac{1}{2} m_p L_p L_r \ddot{\theta} + (J_p + \frac{1}{4} m_p L_p^{\ 2})\ddot{\alpha} - \frac{1}{2} m_p L_p g \alpha + B_p \dot{\alpha} = 0 \tag{3.11}$$

Solving the acceleration term yields:

$$\ddot{\theta} = \frac{1}{J_t}(-(J_p + \frac{1}{4} m_p L_p^{\ 2}) B_r \dot{\theta} - \frac{1}{2} m_p L_p L_r B_p \dot{\alpha} + \frac{1}{4} m_p^{\ 2} L_p^{\ 2} L_r g \alpha + (J_p + \frac{1}{4} m_p L_p^{\ 2})\tau) \tag{3.12}$$

$$\ddot{\alpha} = \frac{1}{J_t}(\frac{1}{2} m_p L_p L_r B_r \dot{\theta} - (J_r + m_p L_r^{\ 2}) B_p \dot{\alpha} + \frac{1}{2} m_p L_p g (J_r + m_p L_r^{\ 2})\alpha - \frac{1}{2} m_p L_p L_r \tau) \tag{3.13}$$

where, $$J_t = J_p m_p L_r^{\ 2} + J_r J_p + \frac{1}{4} J_r m_p L_p^{\ 2} \tag{3.14}$$

τ represent the torque generated by a direct-drive rotary servo system. The dynamics of the actuator is described by the following equation:

$$\tau = K_t\left(\frac{V_m - K_m\dot{\theta}}{R_m}\right) \tag{3.15}$$

where, the actuator parameters are described in Table 3.2

Table 3.2: Parameters of Actuator

Symbol	**Description**	**Units**	**Value**
R_m	Terminal Resistance	Ω	8.4
K_m	Motor back EMF constant	Vs/rad	0.042
K_t	Torque constant	Nm/A	0.042
L_m	Rotor inductance	mH	0.85

3.3 State-Space Representation of RIP System: -

The matrix differential equation can be represented in a state-space form as:

$$\dot{x} = Ax + Bu \tag{3.16}$$
$$y = Cx + Du \tag{3.17}$$

where,

$x = \begin{bmatrix}\theta & \alpha & \dot{\theta} & \dot{\alpha}\end{bmatrix}$, is called as the state vector, u is known as the input vector and A, B, C and

D are called as state-weighing coefficient matrices. The derived mathematical model of the RIP system can be represented in state-state form as:

$$\begin{bmatrix}\dot{\theta}\\ \dot{\alpha}\\ \ddot{\theta}\\ \ddot{\alpha}\end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \dfrac{m_p^2 g L_p^2 L_r}{4J_t} & \dfrac{-B_r}{J_t}\left(m_p\dfrac{L_p^2}{4} + J_p\right) - \dfrac{K_t K_m}{J_t R_m}\left(J_p + m_p\dfrac{L_p^2}{4}\right) & \dfrac{-m_p L_r L_p B_p}{2J_t}\\ 0 & \dfrac{m_p L_p g (J_r + m_p L_r^2)}{2J_t} & \dfrac{-B_r m_p L_p L_r}{2J_t} - \dfrac{K_t K_m m_p L_p L_r}{2J_t R_m} & \dfrac{-(J_r + m_p L_r^2)B_p}{J_t} \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ 0 \\ \frac{K_t}{J_t R_m}(J_p + \frac{m_p L_p^2}{4}) \\ \frac{K_t}{J_t R_m}(\frac{m_p L_p L_r}{2}) \end{bmatrix} [V_m] \tag{3.18}$$

Incorporating the values of parameters given in Table 3.1 and Table 3.2, the state-space representation can be expressed as:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 149.2751 & -2.0886 & 0 \\ 0 & 261.6091 & -2.0643 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 49.7275 \\ 49.1493 \end{bmatrix} [V_m] \tag{3.19}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [V_m] \tag{3.20}$$

From state space equation, derive transfer function: -

$$P_1 = \frac{\theta(s)}{V_m(s)} = \frac{49.73s^2 - 5672}{s^4 + 2.089s^3 - 261.6s^2 - 238.2s} \tag{3.21}$$

$$P_2 = \frac{\alpha(s)}{V_m(s)} = \frac{49.15s^2 - 5.495\times10^{-14}s + 7.938\times10^{-30}}{s^4 + 2.089s^3 - 261.6s^2 - 238.2s} \tag{3.22}$$

The step response and root-locus plot of rotary arm position (θ) and pendulum arm position (α) are shown in Fig.3.2 to Fig.3.5 respectively.

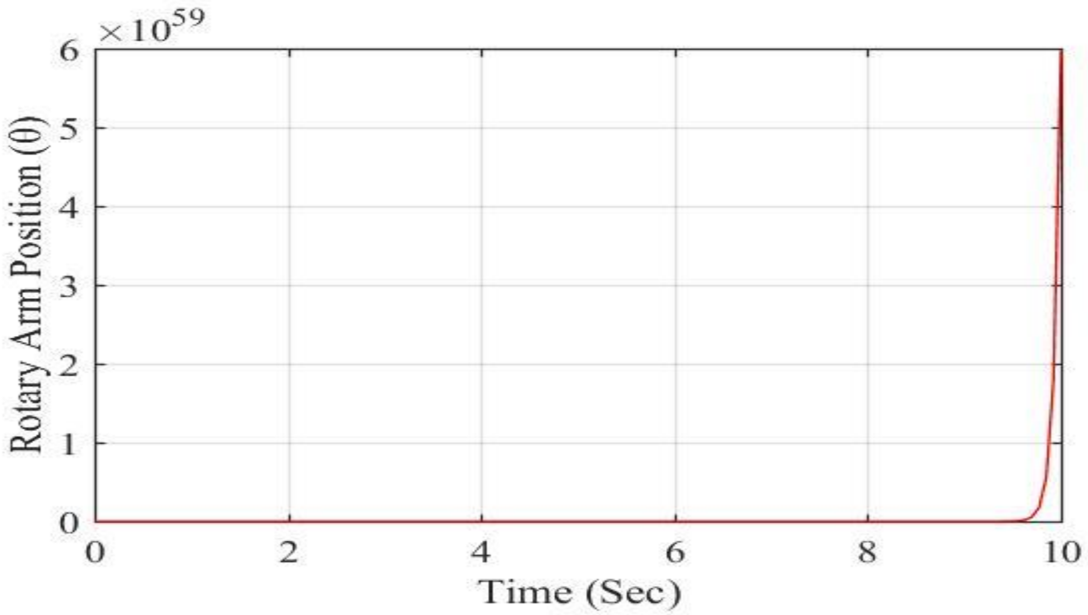

Fig.3.2: Step Response of Rotary Arm Position (θ)

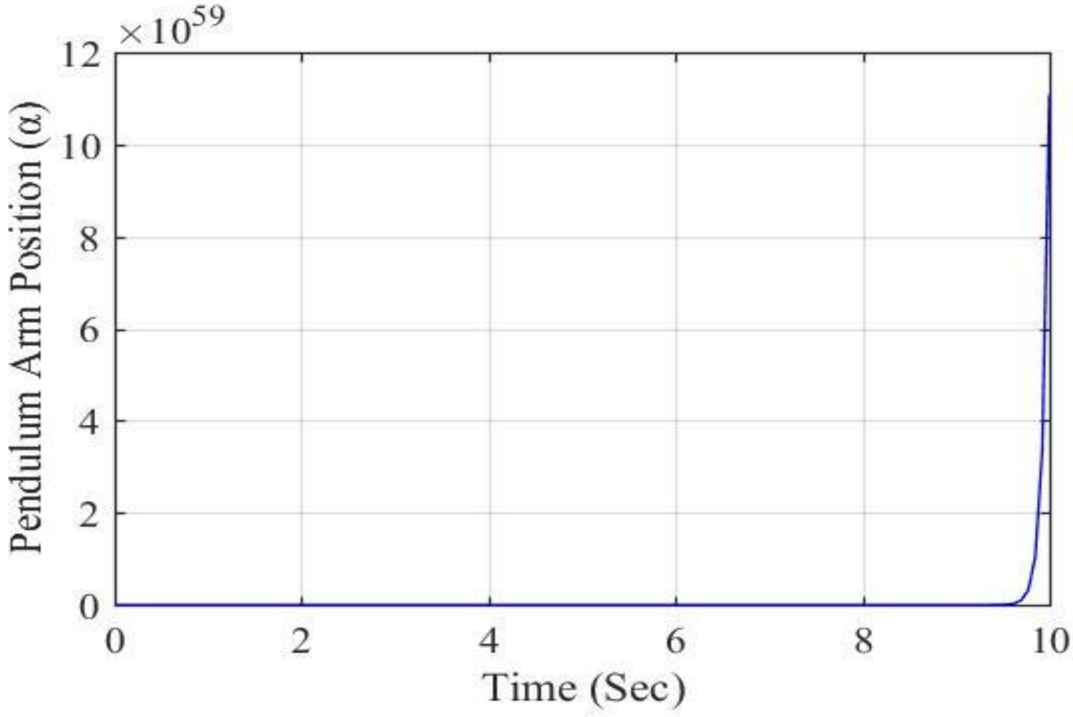

Fig.3.3: Step Response of Pendulum Arm Position (α)

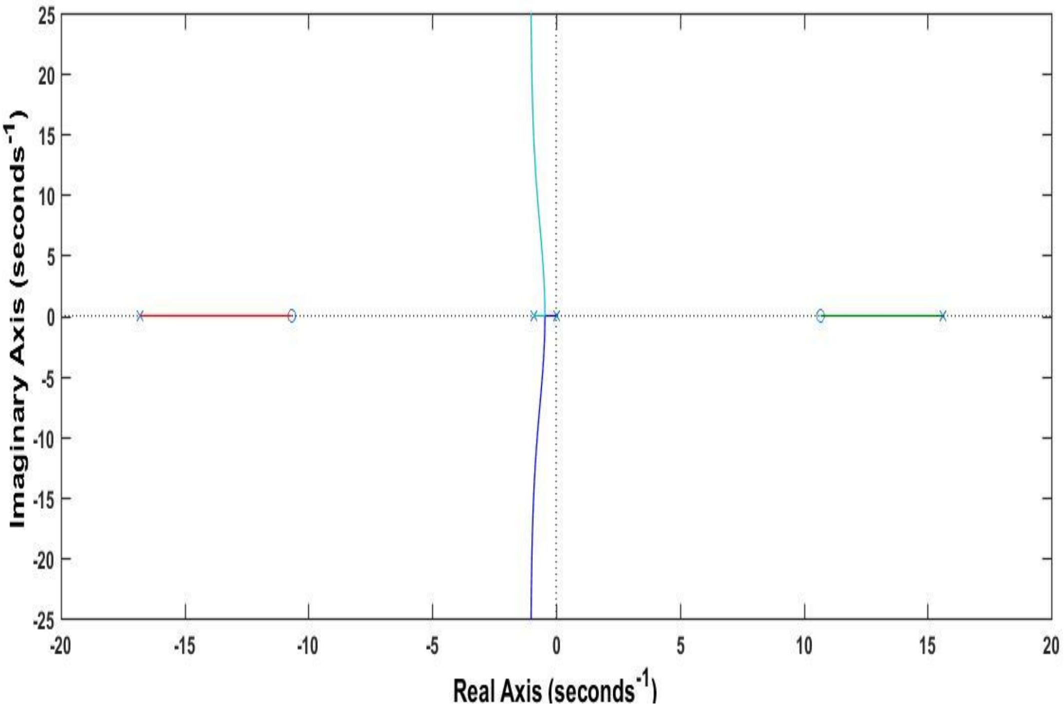

Fig.3.4: Root-Locus Plot of Rotary Arm Position (θ)

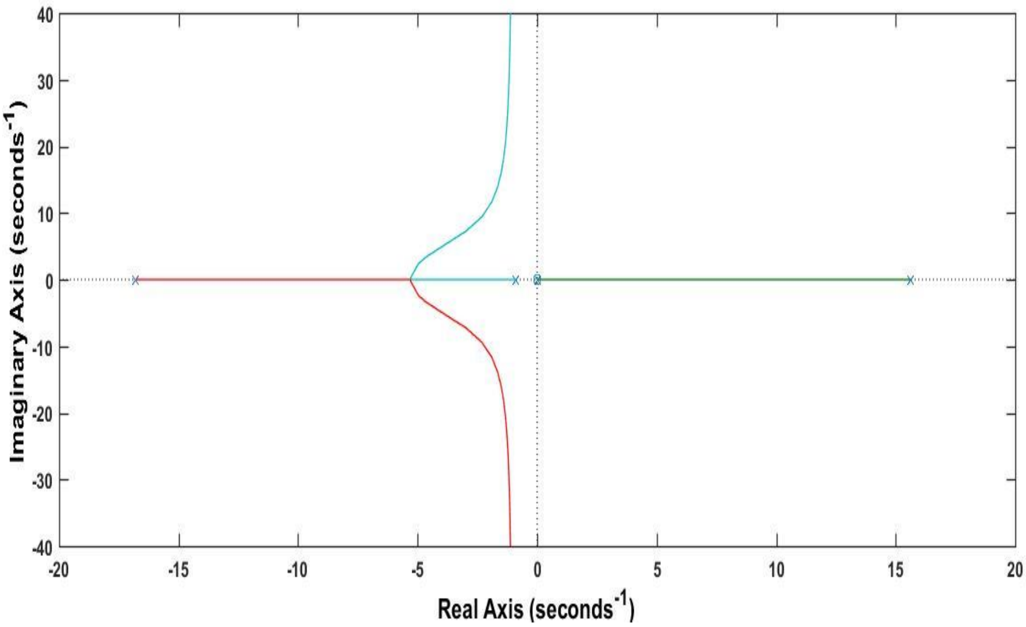

Fig.3.5: Root-Locus Plot of Pendulum Arm Position (α)

The step response and root-locus plot show that the RIP system is open-loop unstable system (one pole present in the right half of s-plane).

3.4 Experimental Setup of RIP System: -

The real time hardware setup of RIP is provided by QUANSER which is called QUBE-SERVO setup. The QUBE-SERVO [66] setup is of three types: the USB interface, Direct I/O interface and NI myRIO interface. In this study, the USB interface QUBE-SERVO is used which is shown in Fig.3.6.

Fig.3.6: Experimental Setup of Quanser QUBE-SERVO (USB Interface)

The USB interface QUBE-SERVO setup has an inbuilt power amplifier and data acquisition (DAQ) device. The QUANSER QUBE-SERVO setup has direct drive 18 V brushed direct current (DC) motor and two encoders mounted on the rotary arm and pendulum arm which is used to measure the angle position of rotary arm and pendulum arm. The encoder output is 2048 counts per revolution in quadrature mode (512 lines per revolution). The inbuilt DAQ device includes two 16-bit encoder channels with quadrature decoding and two pulse width modulated (PWM) output channels. It also includes the PWM voltage-controlled power amplifier which provide 2A of peak current and 0.5A of continuous current (based on thermal current rating of the DC motor). The output voltage range to the load is ±10V. It has 1×12-bit analog output.

CHAPTER 4: SLIDING MODE CONTROL TECHNIQUES

4.1 Introduction: -

This chapter discusses different sliding mode control techniques. These techniques include:

a. Sliding Mode Control (SMC)

b. Integral Sliding Mode control (ISMC)

c. Fuzzy Integral Sliding Mode Control (FISMC)

4.1.1 SMC (Sliding Mode Control) Technique: -

SMC technique is a robust control technique. It gives the better result in presence of external disturbance and matched uncertainties. It is a special case of the variable structure system [9]. Variable structure control systems (VSCS) are a class of systems where the control law, as a function of system state, is changed according to some predefined rules like relay system. In SMC control scheme, first step is to design a sliding (switching) surface and then established the control law which will force the system state on to the sliding surface. Sliding surface is the set of points for which the switching function is zero. In SMC control scheme, there are two phases. First phase is the reaching phase which is before the occurrence of sliding mode and the next phase is sliding phase. When system in the reaching phase then it is affected by external disturbance even matched ones. The system shows the robustness against the external disturbance and parameter variations when the system is in the sliding phase. The necessary condition which ensures that the system should always be on the sliding surface is reachability condition and is given as:

$$s\dot{s} < 0 \tag{4.1}$$

The reachability condition is derived by using lyapunov theorem. Lyapunov theorem is used to find the stability of the nonlinear systems. The lyapunov function is defined as,

$$v = \frac{1}{2}s^2 \tag{4.2}$$

where, s is the sliding surface

The stability of sliding motion depends on the choice of the sliding surface design matrix. The control law defined for SMC control scheme is divided into two part, one is linear part and other is non-linear part. Non-linear part contains the discontinuous control and this component is responsible for the system attain the sliding surface and linear part contain the nominal equivalent control which help to maintain the sliding or to ensure the system kept on the sliding surface. The idea of sliding mode is shown in Fig.4.1.

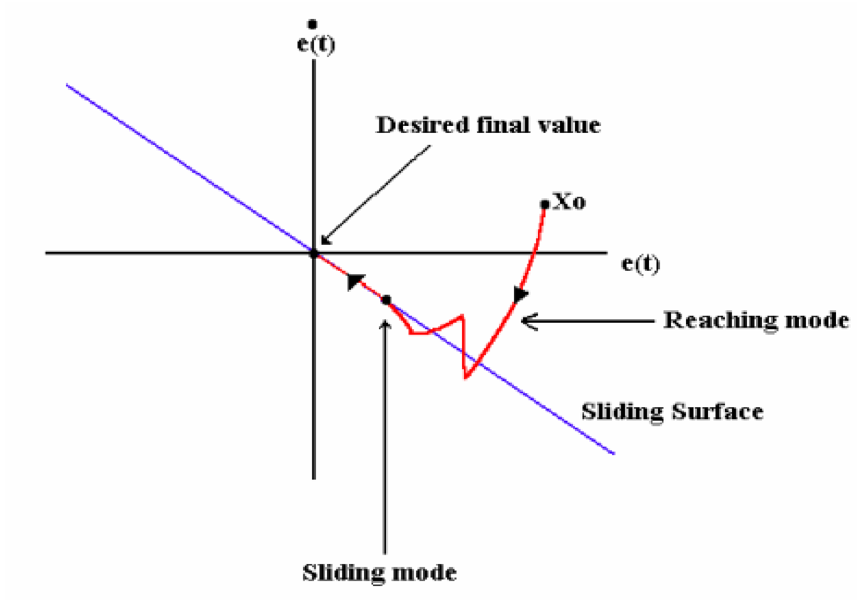

Fig.4.1: Idea of Sliding Mode

There are four classical reaching law which is used to take system on to sliding surface [35]: -

i. Constant rate reaching law: -

$$\dot{s} = -\varepsilon \operatorname{sgn}(s), \qquad \varepsilon > 0 \tag{4.3}$$

where, ε is the constant rate and *s* is the sliding surface

ii. Exponential reaching law: -

$$\dot{s} = -\varepsilon \operatorname{sgn}(s) - ks, \qquad \varepsilon > 0, k > 0 \tag{4.4}$$

where, $\dot{s} = -ks$ is exponential term and its solution is $s=s(0)e^{-kt}$

iii. Power rate reaching law: -

$$\dot{s} = -k|s|^{\alpha} \operatorname{sgn}(s), \qquad k > 0, 1 > \alpha > 0 \tag{4.5}$$

where, k and α represents constant gain term

iv. General reaching law: -

$$\dot{s} = -\varepsilon \operatorname{sgn}(s) - f(s), \qquad \varepsilon > 0 \tag{4.6}$$

where, $f(0) = 0$ and $sf(s) > 0$ when $s \neq 0$

The advantage of SMC control scheme is, it reduces the order of system, convert the nonlinear model into the linear one, high degree of robustness against external disturbance, parameter invariance and matched uncertainties. In this scheme, system performance specification depends on the design of sliding surface.

The disadvantage of this control scheme is that, it is affected by the chattering phenomena.

The design approach using SMC has two steps:

a. Design of sliding surface in the state space
b. Derive the control law that drive the system towards sliding surface

Let the LTI system is:

$$\dot{x} = Ax + Bu \tag{4.7}$$

where, *A* represent the system state matrix and *B* represent the input matrix

The sliding surface is defined in the state space as:

$$s = G \times x_e \tag{4.8}$$

where, *G* is a [1×4] design matrix and x_e is the error state vector. If x_d is the desired state vector

and *x* is the actual state vector then:

$$x_e = x - x_d \tag{4.9}$$

Constant rate reaching law which guarantees finite time reaching is:

$$\dot{s} = (-Msign(s))(1 + |k \times x_e|) \tag{4.10}$$

where, $M > 0$

Differentiating equation (4.8):

$$\dot{s} = G \times \dot{x}_e \tag{4.11}$$

Substituting for $\dot{x}_e$ in equation (4.11), we get:

$$\dot{s} = G\dot{x} - G\dot{x}_d \tag{4.12}$$

Putting the value of $\dot{x}$ from equation (4.7):

$$\dot{s} = GAx + GBu - G\dot{x}_d \tag{4.13}$$

Compare equation (4.10) and (4.13), we get:

$$(-Msign(s))(1 + |kx_e|) = GAx + GBu - G\dot{x}_d \tag{4.14}$$

Finally, the control law become:

$$u = -(GB)^{-1}[GAx + (Msign(s))(1 + |kx_e|)] \tag{4.15}$$

The sliding mode design matrix which is denoted by 'G' is formulated by the Ackermann's formulae [67]. With the help of Ackermann's formulae, a control law for linear state-feedback system with desired eigen value is obtained. In the same way, designing of SMC with linear discontinuity surface, Ackermann's formula is useful when it has linear sliding mode equation and completely depends on its coefficient of surface equation [67].

The desired eigenvalues $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \ldots\ldots, \lambda_n$ of the LTI system given in equation (4.16) of the matrix (A-Bk) is commissioned using the Ackermann's formula.

$$u = -Gx \tag{4.16}$$

$$G = e^T P(\lambda) \tag{4.17}$$

where,

$$e^T = (0,......0,1)(B, AB, ...A^{n-1}B)^{-1}$$

$$P(\lambda) = (A - \lambda_1 I)(A - \lambda_2 I).........(A - \lambda_{n-1} I)(A - \lambda_n I)$$

'I' is the identity matrix.

Also, G×B=1

4.1.2 ISMC (Integral Sliding Mode Control) Technique: -

ISMC control technique is the advancement of the conventional SMC control technique. This control technique is used to eliminate the reaching phase which is non-robust or noise prone in nature. This means the sliding mode is enforce from the beginning of the system and system show the robustness and compensate the matched uncertainties and external disturbances throughout the system response. In this control scheme, the nominal plant already exists which provides the nominal control. Then, a discontinuous controller is added to the nominal state feedback controller to ensure the nominal performance is maintained and system is insensitive to external disturbance. The nominal control can be of any type like LQR, pole placement, PID control etc.

In this control law is divided into two parts, nominal control as well as discontinuous control

$$u = u_{nom} + u_{dis} \tag{4.18}$$

The integral mode sliding surface is defined as:

$$s = P\{\int_0^t (x(\tau) - x_d(\tau) d\tau)\} \tag{4.19}$$

where, $x(t)$ is the actual trajectory, $x_d(t)$ is the desired trajectory given by nominal control in the absence of uncertainty and disturbances. P is the gain value of sliding surface or projection matrix and it is tuned to left inverse of the matrix B. The projection matrix P is defined as:

$$P = B^{+} = (B^{T}B)^{-1}B \tag{4.20}$$

So, the sliding surface is:

$$s = P(x(t) - x(0) - \int_{0}^{t} (Ax(\tau) + Bu_o(\tau))d\tau) \tag{4.21}$$

Finally, the control law is defined as:

$$u = -ke(t) - Msign(s) \tag{4.22}$$

4.1.3 FISMC (Fuzzy Integral Sliding Mode Control) Technique: -

The SMC control technique and ISMC control technique are generally affected by the chattering phenomena. The chattering phenomena shown in Fig.4.2 is defined as the high frequency switching with some amplitude like relay system around the sliding surface. The chattering phenomena is not good as it may damage the moving mechanical parts results in high heat losses in power circuit and low control accuracy.

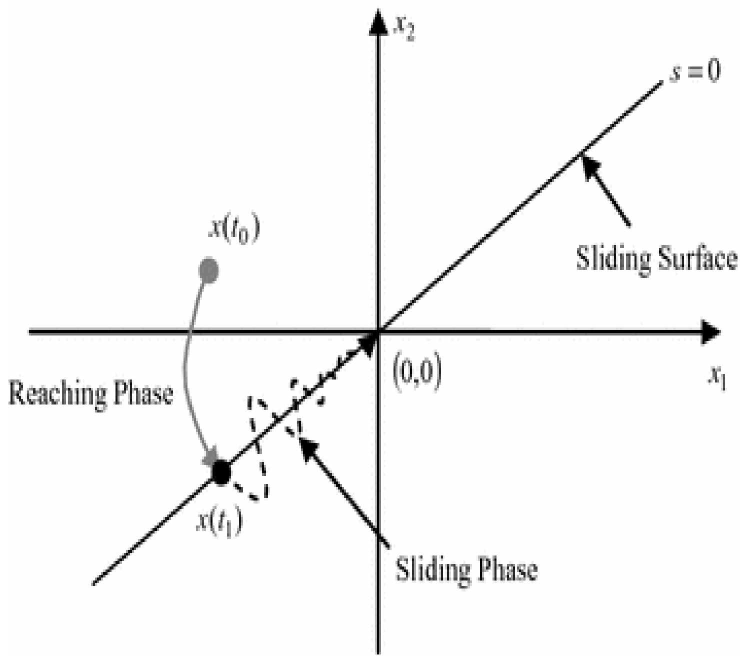

Fig.4.2: The Chattering Phenomena

The chattering is introduced because of two reasons [68]:

(i) It occurs due to fast system dynamics which is not considered into account in the ideal model.

(ii) It occurs due to the digital controller with finite sampling rate.

The amplitude of chattering is directly proportional to the amplitude of discontinuous control. Naturally, the chattering is removed by enhancing the switching frequency but it is not convenient always [68]. So, to avoid the chattering phenomena, many other techniques are presented in the literature such as: saturation function used instead of signum function, use of FLC technique instead of signum function to give the smooth control, use of high order observer, using LPF etc. In this study, the FLC technique is used in place of signum function which provide the discontinuous control in ISMC as shown in equation (4.22).

FLC technique works on the IF-THEN rule base. The rule base of fuzzy inference system (FIS) depends on the expert knowledge instead of mathematical model. FLC uses vague linguistic form of variables to define quantities which avoids discontinuous and provide the smooth control law which eliminate chattering.

The basic structure of fuzzy logic controller is shown in Fig.4.3.

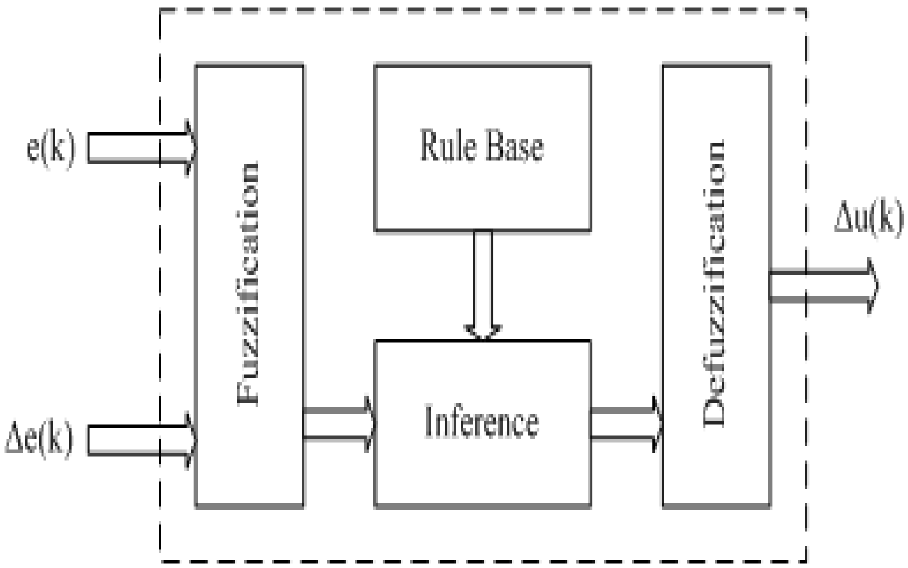

Fig.4.3: Basic Structure of Fuzzy Logic Controller

The inputs provided to the FIS is sliding surface (s) and rate of change in sliding surface ($\dot{s}$) and output is control input (u). Now the final control law become:

$$u = -ke(t) - M \times F(s, \dot{s}) \tag{4.23}$$

In this study, the universe of discourse of both inputs is divided into five linguistic variables i.e. negative big-NB, negative small-NS, zero-Z, positive small-PS, positive big-PB. The membership function is chosen for define inputs and output is triangular shape. The range of universe of discourse for 's' is -0.1 to 0.1 and for '$\dot{s}$' is -5 to 5. The range for output is -10 to 10. The Fuzzy ISMC rule-based matrix is shown in Table 4.1.

Table 4.1: Fuzzy ISMC Rule-Based Matrix

$\dot{s}$ / s	NB	NS	Z	PS	PB
NB	PB	PB	PB	PS	Z
NS	PB	PB	PS	Z	NS
Z	PB	PS	Z	NS	NB
PS	PS	Z	NS	NB	NB
PB	Z	NS	NB	NB	NB

The fuzzy membership function of input variable s and $\dot{s}$ are shown in Fig.4.4 & Fig.4.5 respectively.

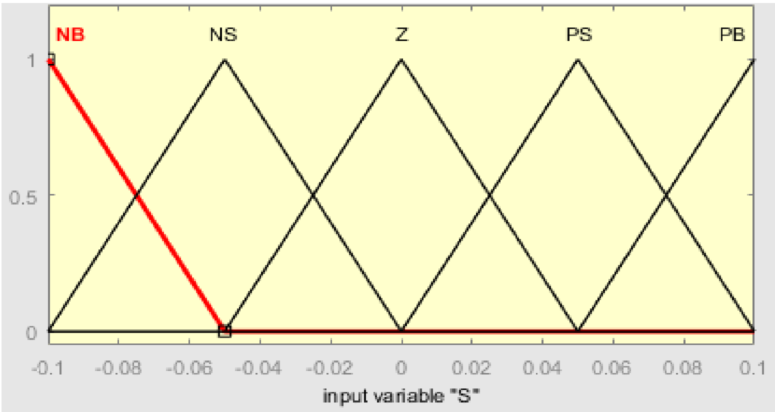

Fig.4.4: Fuzzy Membership Function of Input Variable s

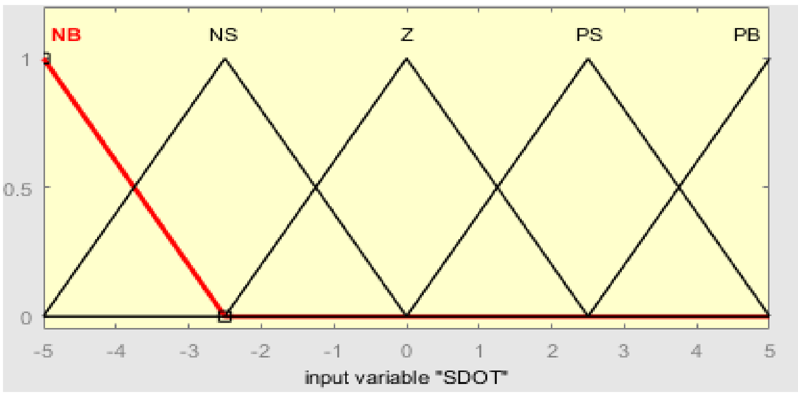

Fig.4.5: Fuzzy Membership Function of Input Variable $\dot{s}$

The membership function of output variable u is shown in Fig.4.7.

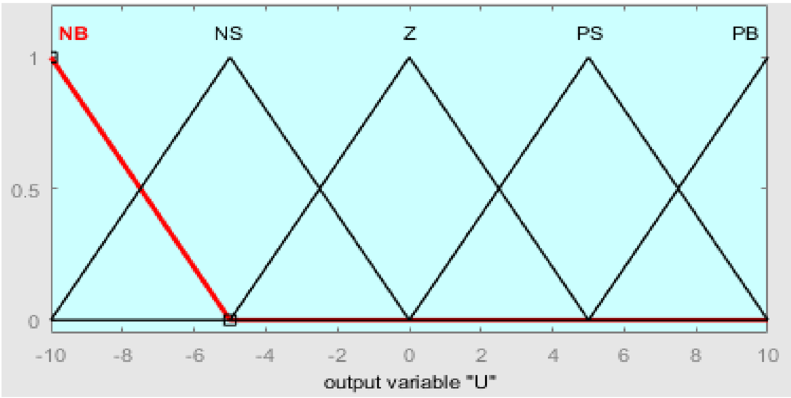

Fig.4.6: Fuzzy Membership Function of Output Variable *u*

The centroid method is used for defuzzification.

CHAPTER 5: SIMULATION AND HARDWARE RESULTS

5.1 Introduction: -

In this chapter, different sliding mode control techniques are implemented for the stabilization of RIP using MATLAB simulink and their results are compared with conventional LQR controller. After that these control techniques are validated on the QUANSER QUBE-SERVO experimental setup.

5.2 Simulink Implementation of Conventional Controller: -

5.2.1 Linear Quadratic Regulator (LQR) Based Simulink Model: -

The simulink model of LQR controller is shown in Fig.5.1. In this RIP system model, there are total 4 states i.e. θ, α, $\dot{\theta}$, $\dot{\alpha}$. All these states are fed back and then multiply with the appropriate gain matrix which is derived from the following MATLAB command:

K= *lqr* (A, B, Q, R)

where, Q is the state weight matrix and R is the control weight matrix. In this model there are total 4 states therefore Q will be 4×4 matrix and there is only one control input is present so, R will be scalar.

The following values of Q and R matrix are taken to find the appropriate gain matrix K:

$$Q = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 20 \end{bmatrix} \qquad R = [1]$$

These values are found using trial and error method.

Then, finally the gain matrix K is calculated and is given as:

K= [-2.2361,53.8740, -1.7400,6.5269]

From this gain matrix K, find the eigen value

$$|sI-(A-BK)|=0 \tag{5.1}$$

Eigen values represent the closed loop poles of the system. These values are:

$$s=-2.2042,-226.50,-3.8269\pm 3.2805i \tag{5.2}$$

In this model, reference angle 30^0 for theta and constant 0 for alpha

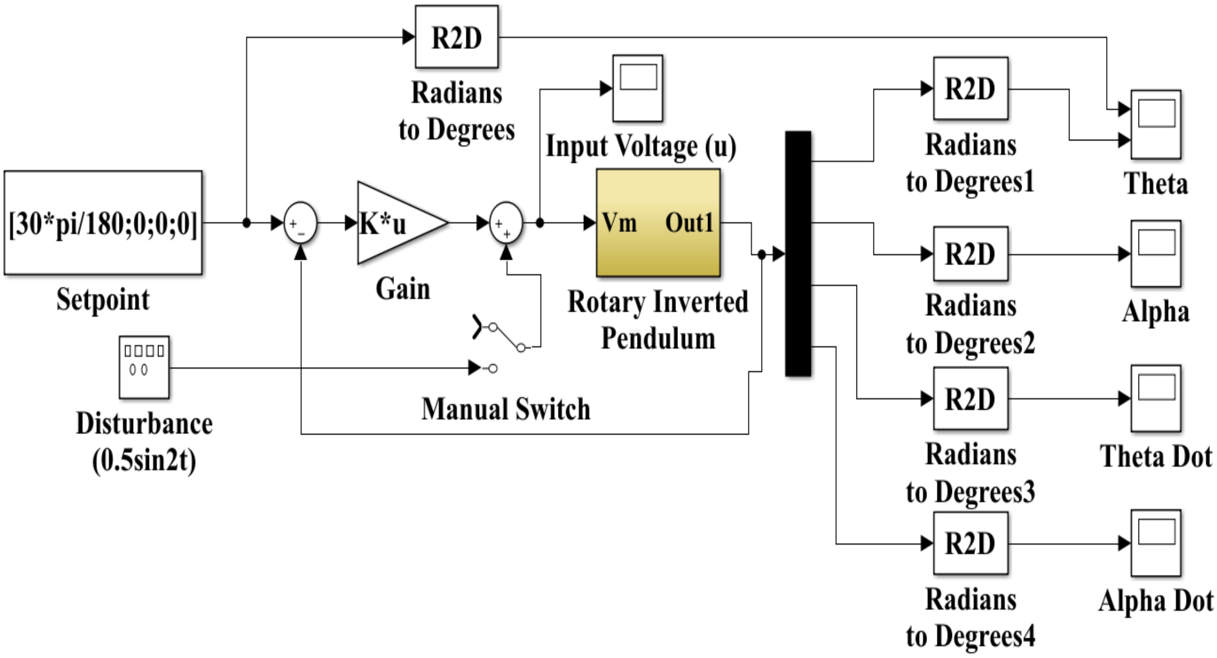

Fig.5.1: LQR Based Simulink Model

A sinusoidal disturbance (noise) of amplitude 0.5 is added to the input channel of the RIP system and the performance of control techniques is tested in this noisy environment.

5.3 Simulink Implementation of Sliding Mode Control Techniques

5.3.1 Sliding Mode Control (SMC) Based Simulink Model: -

In the model, reference is 30^0 for theta and constant 0 for alpha. The sliding mode design matrix G and the gains used in the simulink based model given in the Fig.5.2 is given below: -

$$G= [-0.0094\ 0.2415\ -0.0076\ 0.0280], \qquad M=0.95$$

The value of design matrix is found by using Ackermann's formula shown in equation (4.16) and equation (4.17).

The value of desired eigen variables chosen in this model are

$$\lambda_1= 3.2,\ \lambda_2= 3.8,\ \lambda_3= 4.4$$

The eigen values of the system are found by the trial and error method. The rise time of rotary arm position (θ) and overshoot of pendulum arm position (α) depends on the desired eigen values.

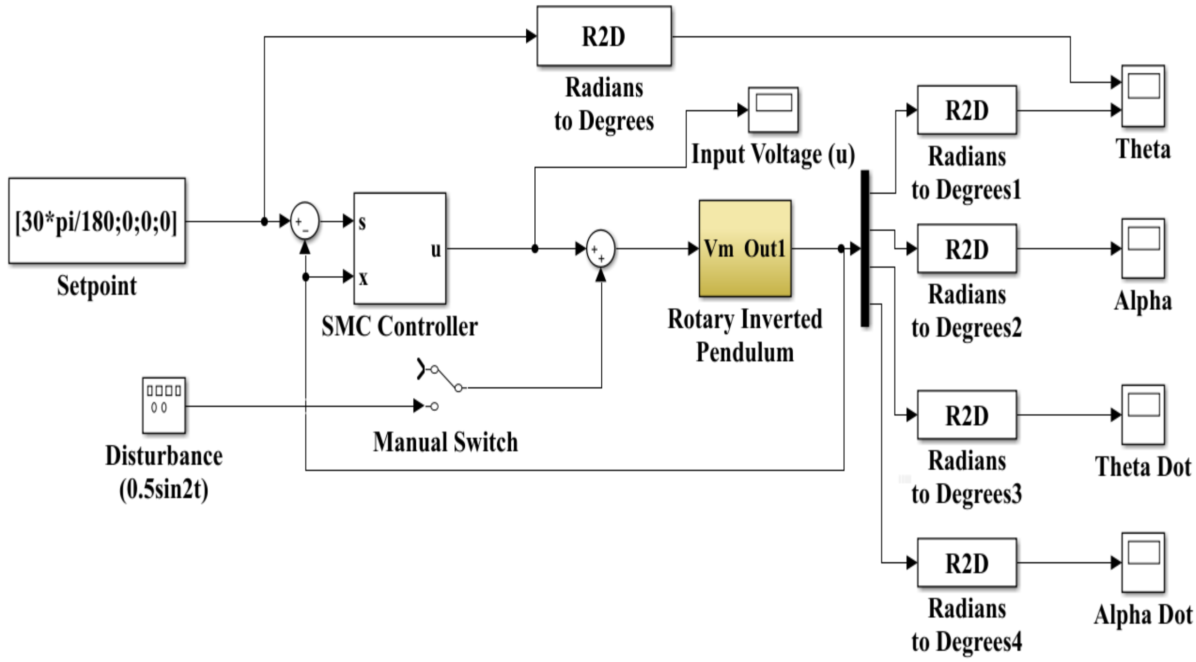

Fig.5.2: SMC Based Simulink Model

5.3.2 Integral Sliding Mode Control (SMC) Based Simulink Model: -

The simulink model of ISMC shown in Fig.5.3. The nominal controller used in the ISMC technique is LQR.

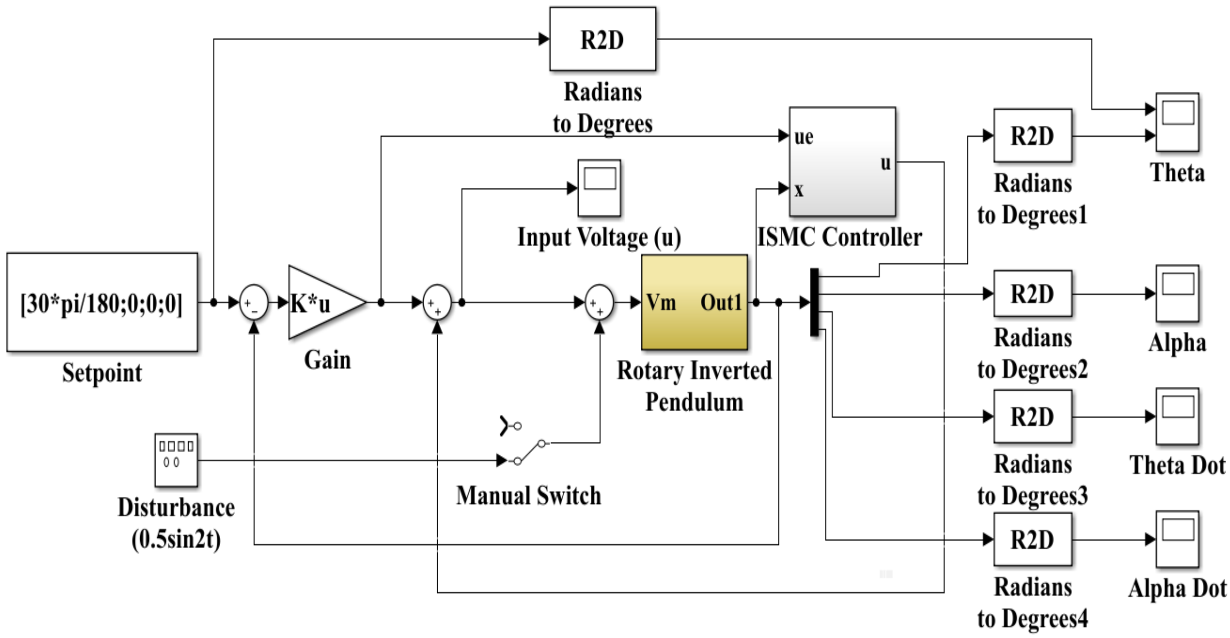

Fig.5.3: ISMC Based Simulink Model

The weight state matrix, Q and weight input matrix, R are defined as:

$$Q = \text{diag}\,[12,1,1,18] \text{ and } R = [1]$$

The value of state feedback gain matrix, K is given as:

K= [-3.4641, 58.1521, -2.0848, 6.6765]

The value of projection matrix, P and gain, M is given below as:

P= [0, 0, 0.0102, 0.0101], M= -2.2

So, final control law for ISMC is:

$$u(t) = -Ke(t) - (-2.2)sign(s(x,t)) \tag{5.3}$$

After implementation, the amplitude of chattering is quite high which is not good for the mechanical system. So, to remove this problem, first order LPF given in equation (5.4) has been introduced. LPF reduced the chattering but does not remove it completely.

$$T/F = \frac{1/\tau}{s + \frac{1}{\tau}} \tag{5.4}$$

where, τ is the Time constant of system. In this model, choose time constant, τ = 0.1 sec.

5.3.3 Fuzzy Integral Sliding Mode Control (FISMC) Based Simulink Model: -

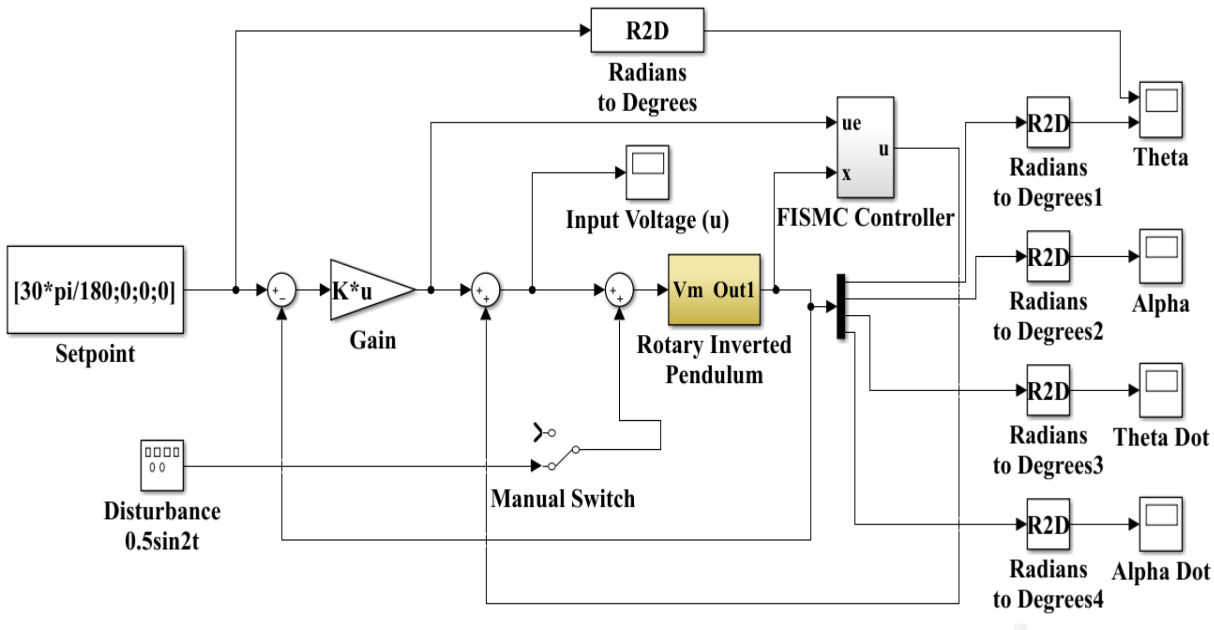

Fig.5.4: FISMC Based Simulink Model

In this model, instead of signum function which is a hard non-linear function given in equation (5.3), FLC technique is used to provide the smooth control law. The simulink model of FISMC is shown in Fig.5.4. The value of gain M is 2.3. All other parameter values are similar to the ISMC technique. So, the final control law of FISMC is:

$$u = -ke(t) - 2.3 \times F(s, \dot{s}) \tag{5.5}$$

5.4 Simulation-Based Results of Conventional Control Scheme (LQR): -

5.4.1 LQR without Disturbance: -

Here reference angle is 30^0 for theta and constant 0 for alpha. The simulation results for LQR without disturbance are shown from Fig.5.5 to Fig.5.7 respectively.

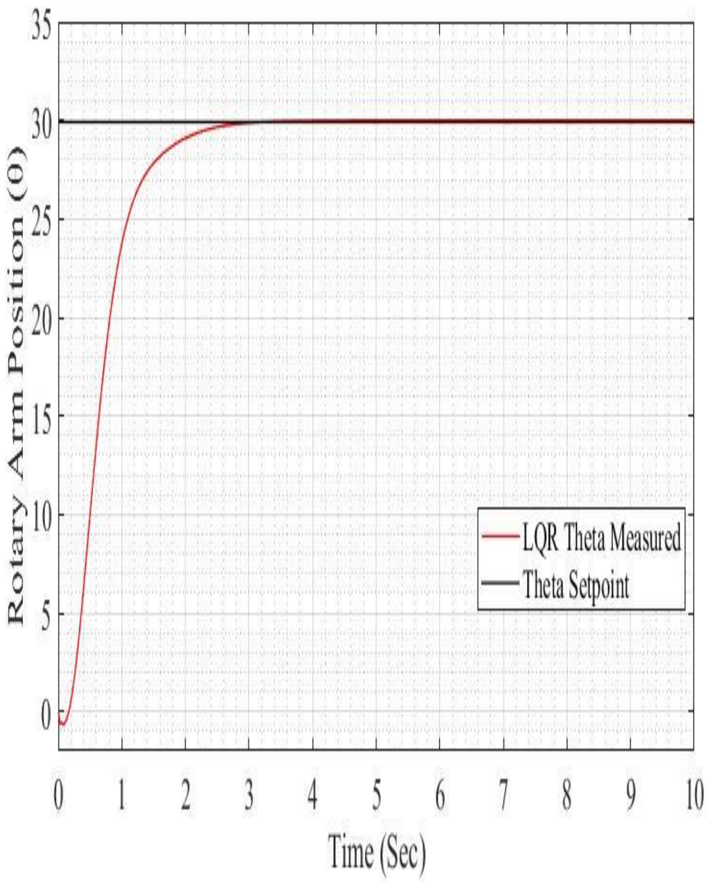

Fig.5.5: Theta Vs Time without disturbance

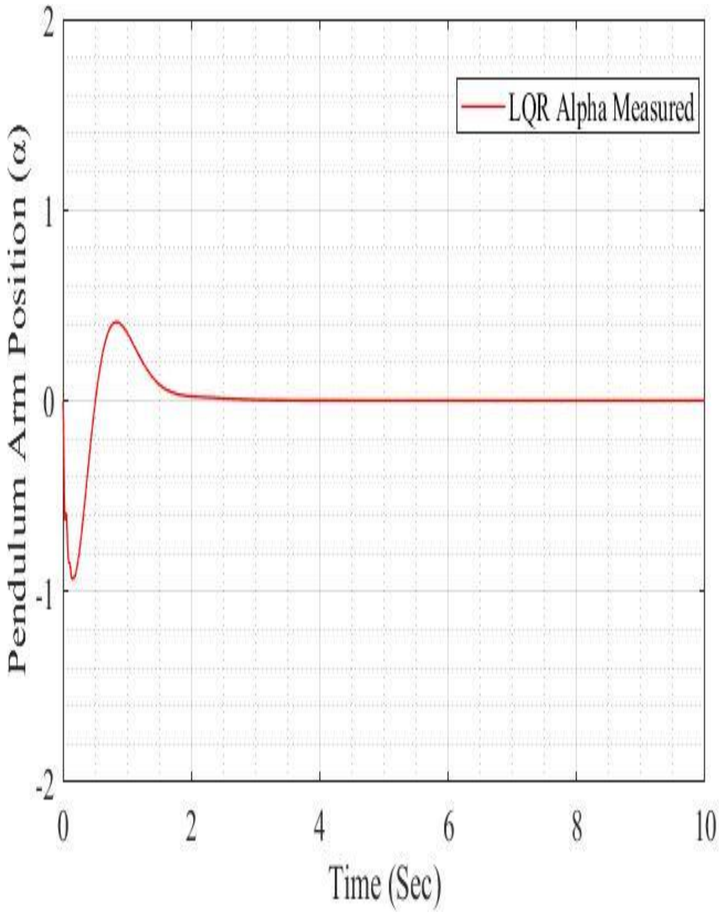

Fig.5.6: Alpha Vs Time without disturbance

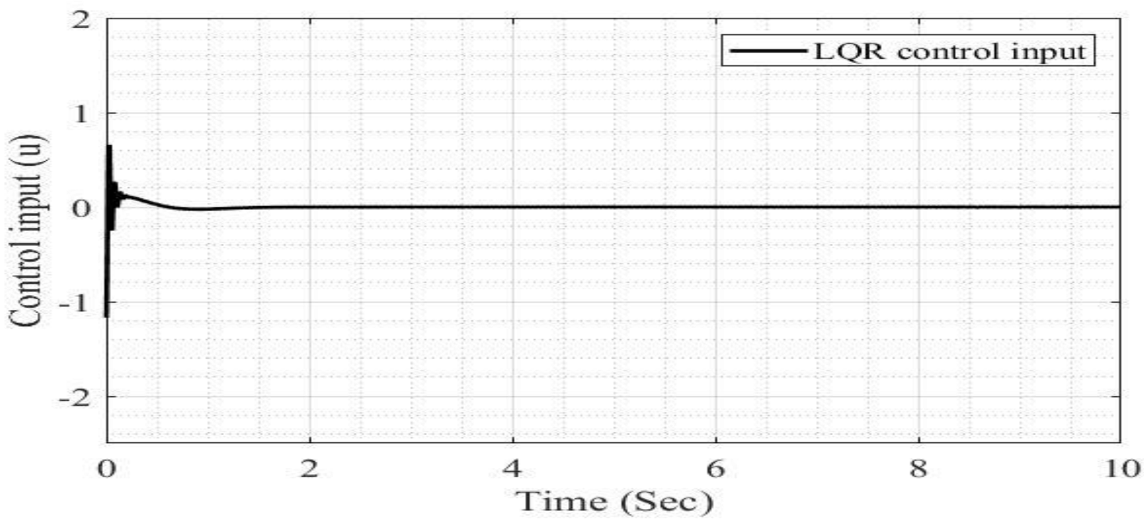

Fig.5.7: Control Effort Vs Time without disturbance

5.4.2 LQR with Disturbance: -

The sinusoidal disturbance is introduced to the input channel of the system and is given as:

$$d = 0.5 \sin 2t \tag{5.6}$$

The simulation results for LQR with disturbance are shown from Fig.5.8 to Fig.5.10 respectively.

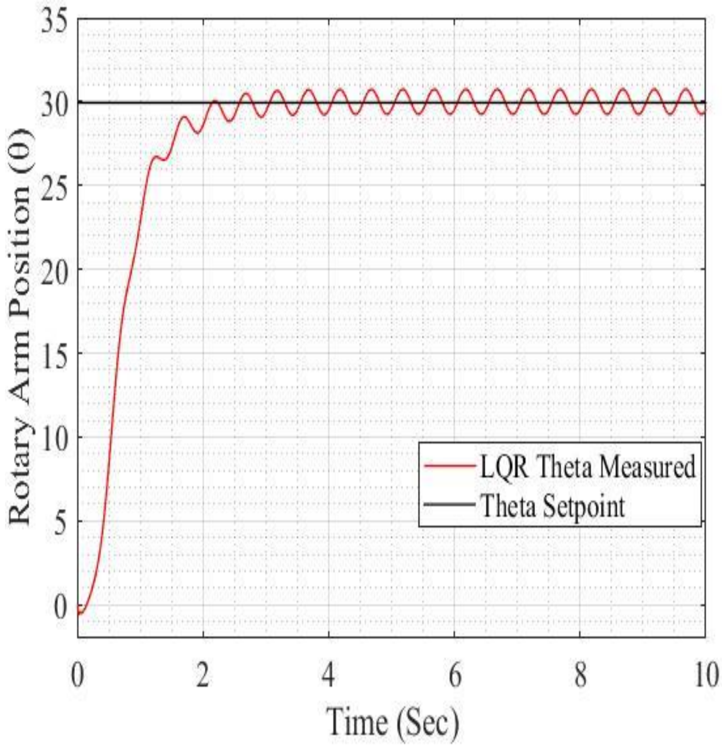

Fig.5.8: Theta Vs Time with disturbance

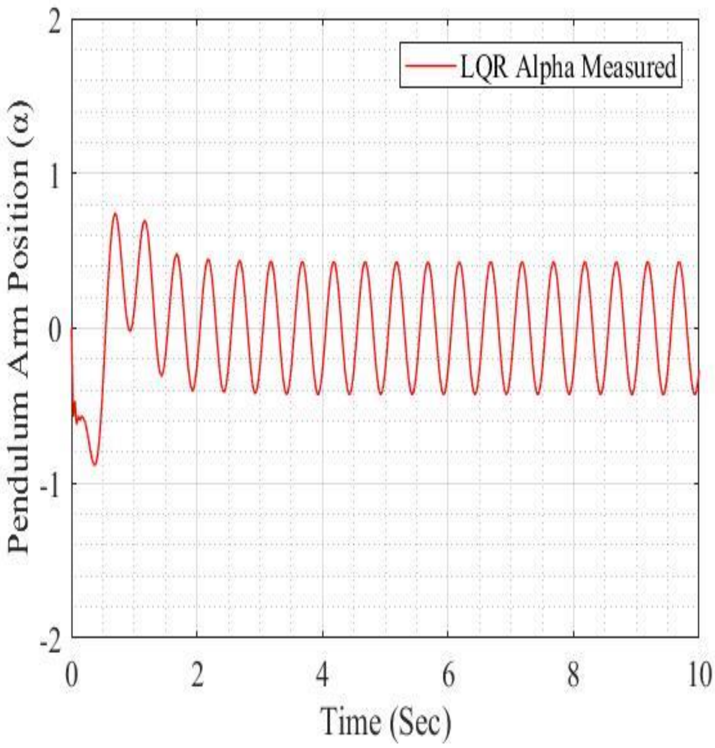

Fig.5.9: Alpha Vs Time with disturbance

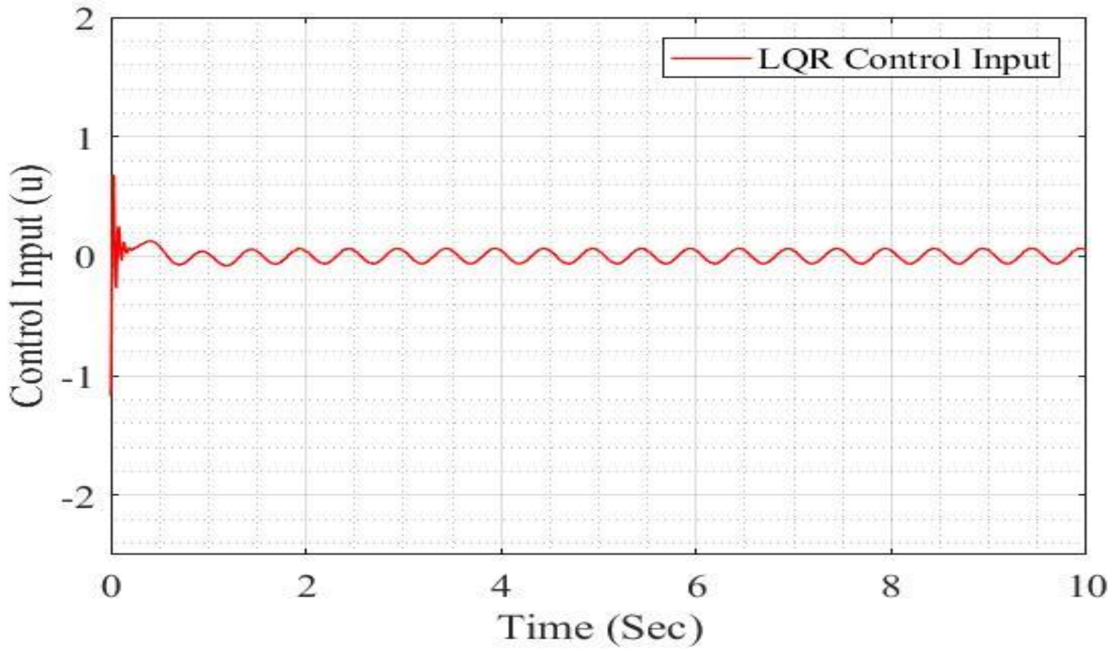

Fig.5.10: Control Effort Vs Time with disturbance

5.5 Simulation-Based Result of Different Sliding Mode Control Schemes: -

5.5.1 SMC without Disturbance: -

The simulation results for SMC without disturbance are shown from Fig.5.11 to Fig.5.13.

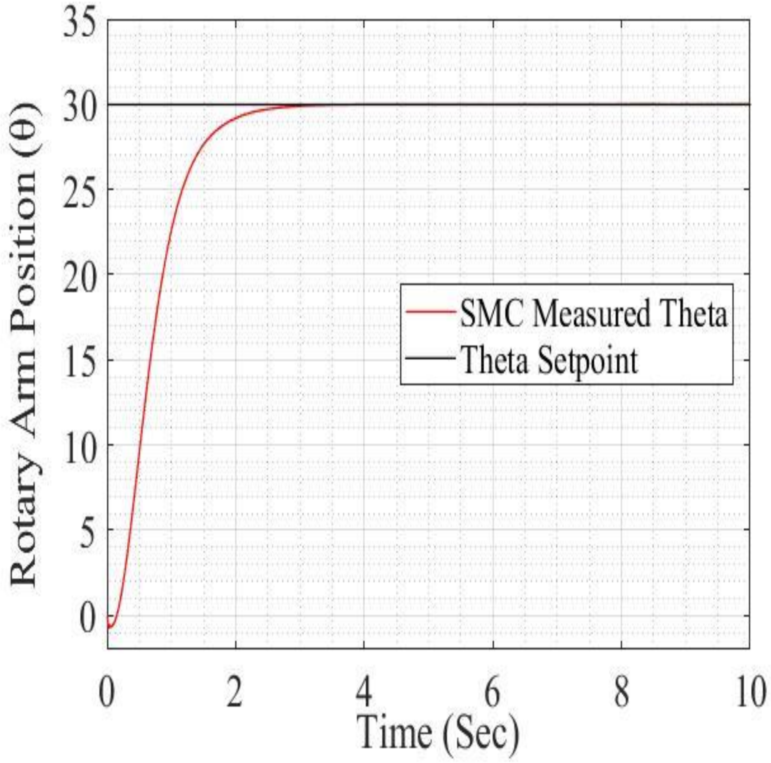

Fig.5.11: Theta Vs Time without disturbance

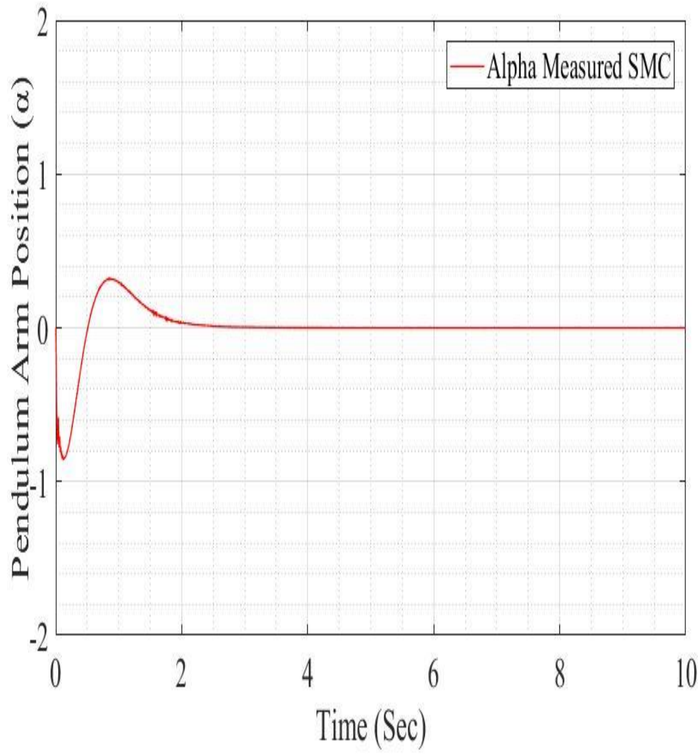

Fig.5.12: Alpha Vs Time without disturbance

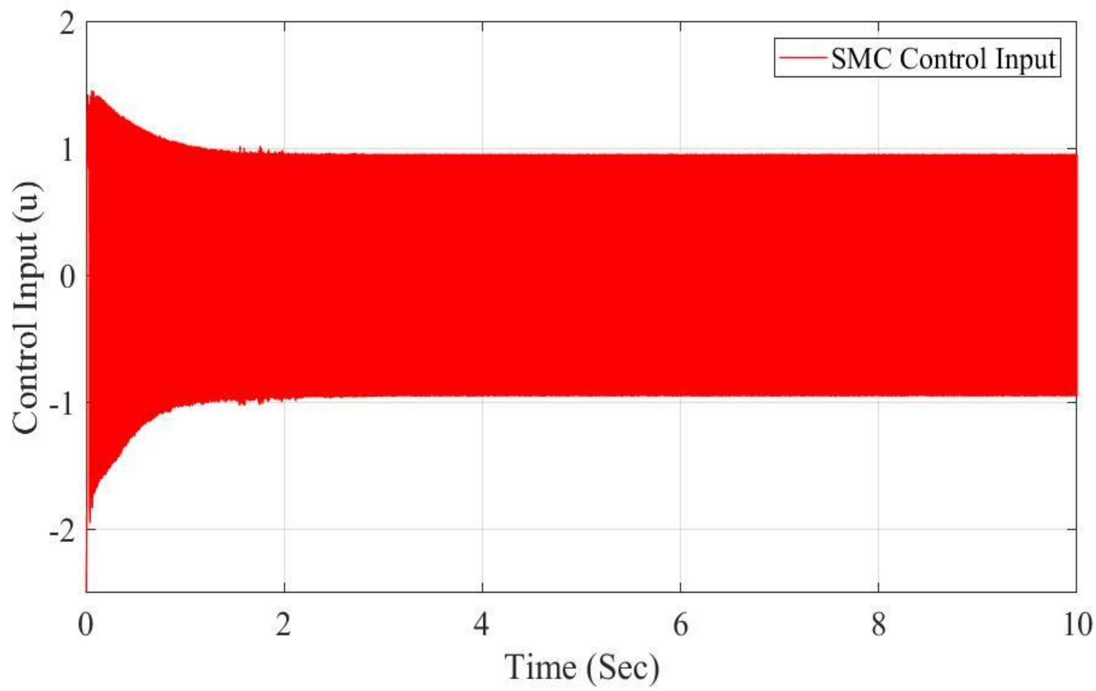

Fig.5.13: Control Effort Vs Time without disturbance

5.5.2 SMC with Disturbance: -

Simulation results for SMC with disturbance shown from Fig.5.14 to Fig.5.16.

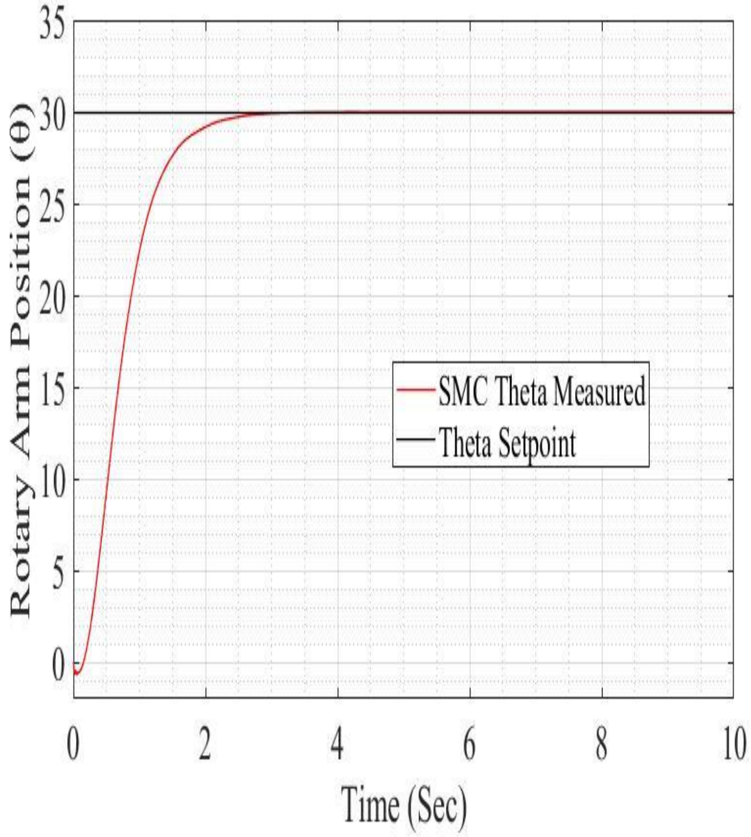

Fig.5.14: Theta Vs Time with disturbance

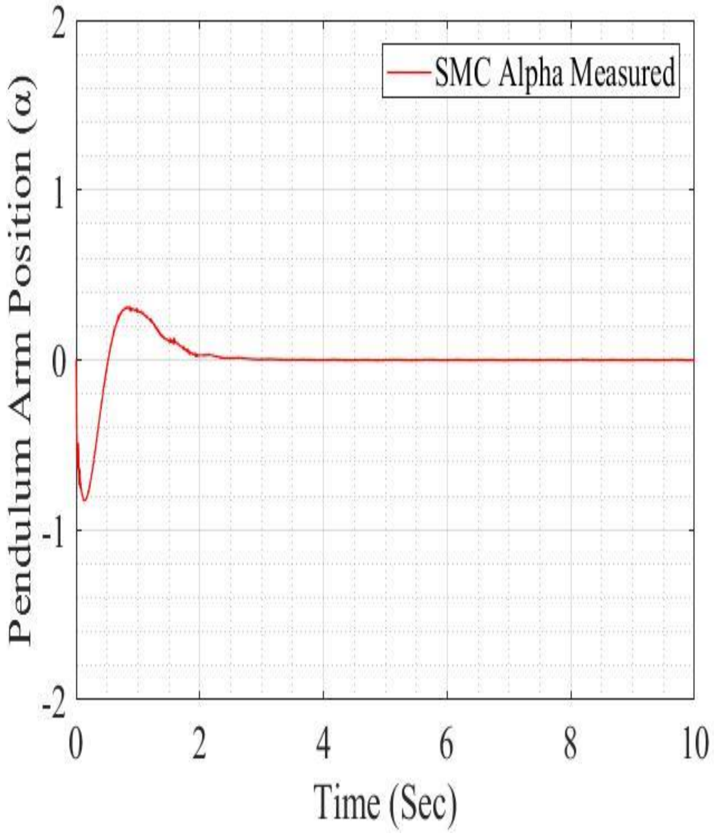

Fig.5.15: Alpha Vs Time with disturbance

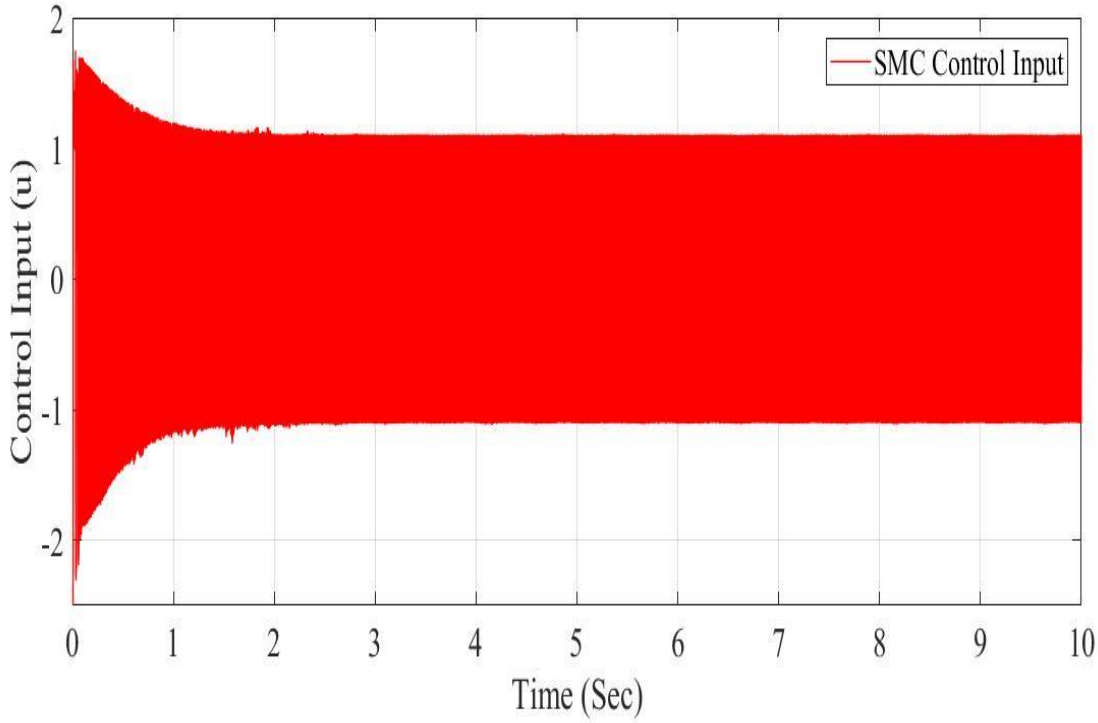

Fig.5.16: Control Effort Vs Time with disturbance

5.5.3 ISMC without Disturbance: -

The simulation results for ISMC without disturbance are shown from Fig.5.17 to Fig.5.19.

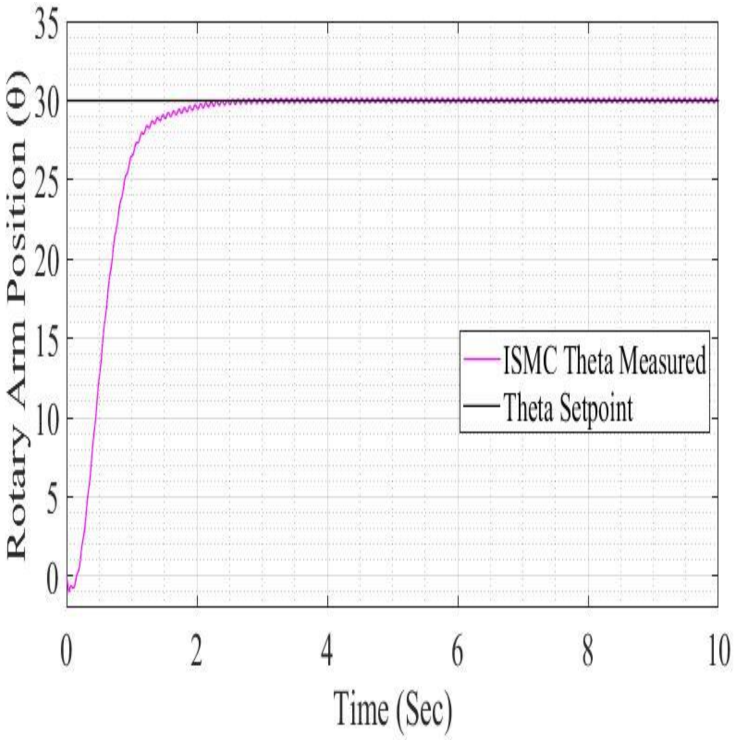

Fig.5.17: Theta Vs Time without disturbance

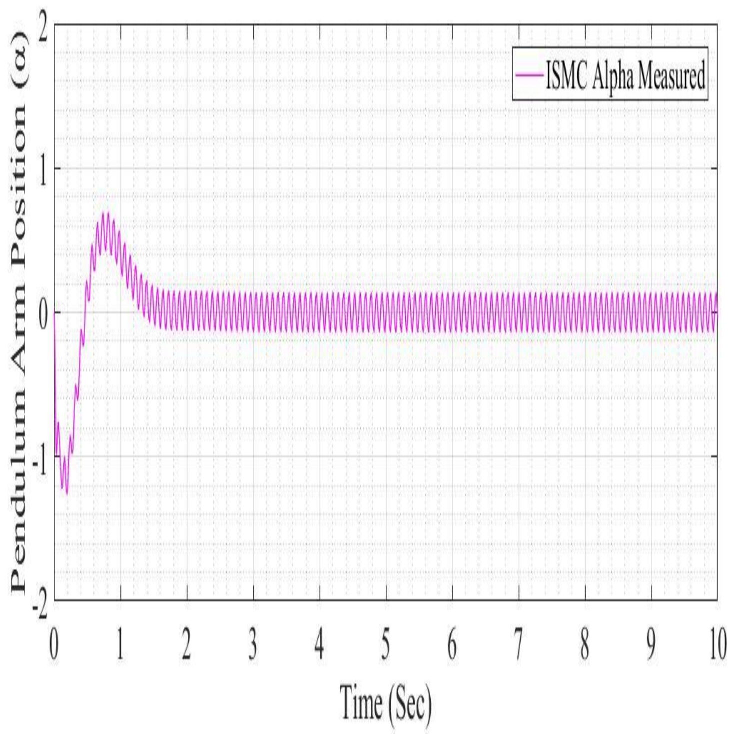

Fig.5.18: Alpha Vs Time without disturbance

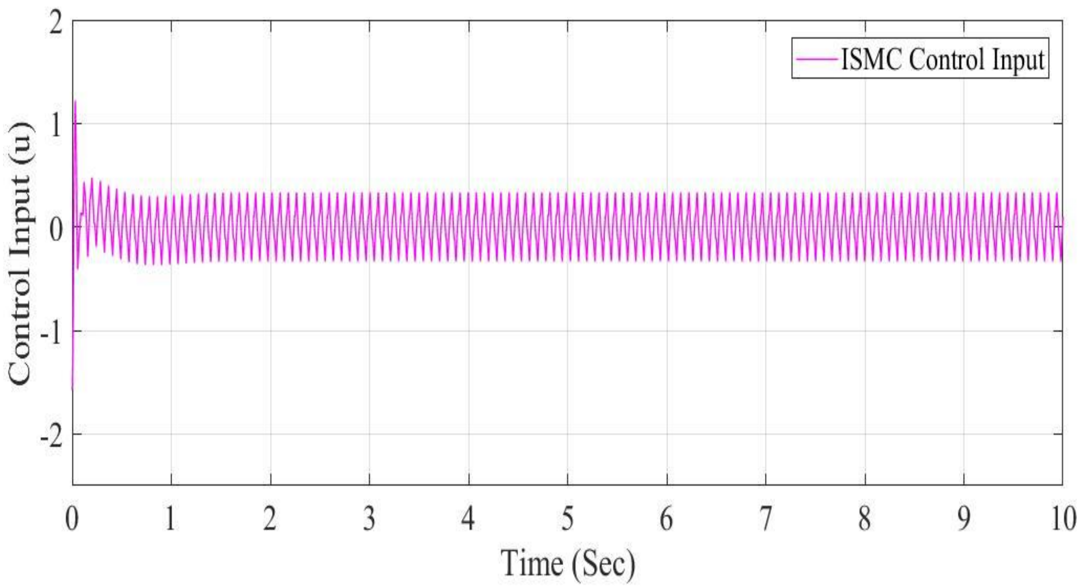

Fig.5.19: Control Effort Vs Time without disturbance

The response shows that the ISMC technique is affected by the chattering due to the discontinuous control (signum function) law given in equation (4.22).

5.5.4 ISMC with Disturbance: -

The simulation results of ISMC with disturbance are shown from Fig.5.20 to Fig.5.22.

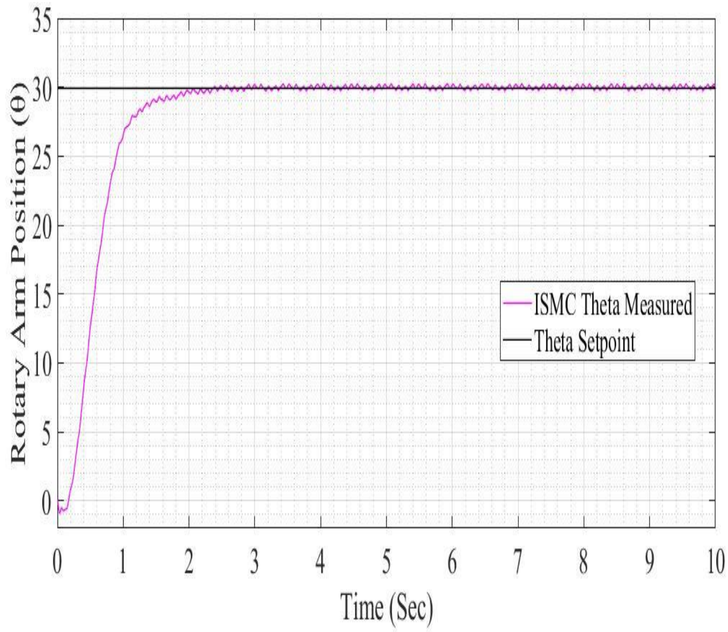

Fig.5.20: Theta Vs Time with disturbance

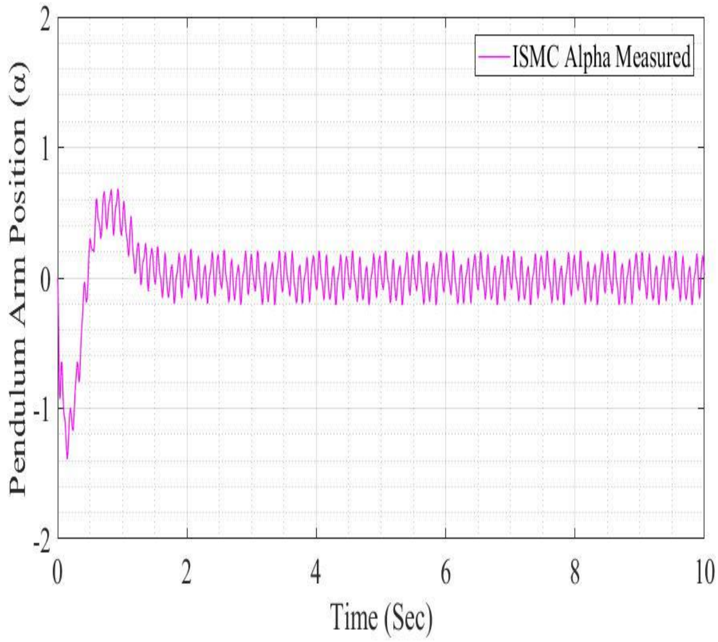

Fig.5.21: Alpha Vs Time with disturbance

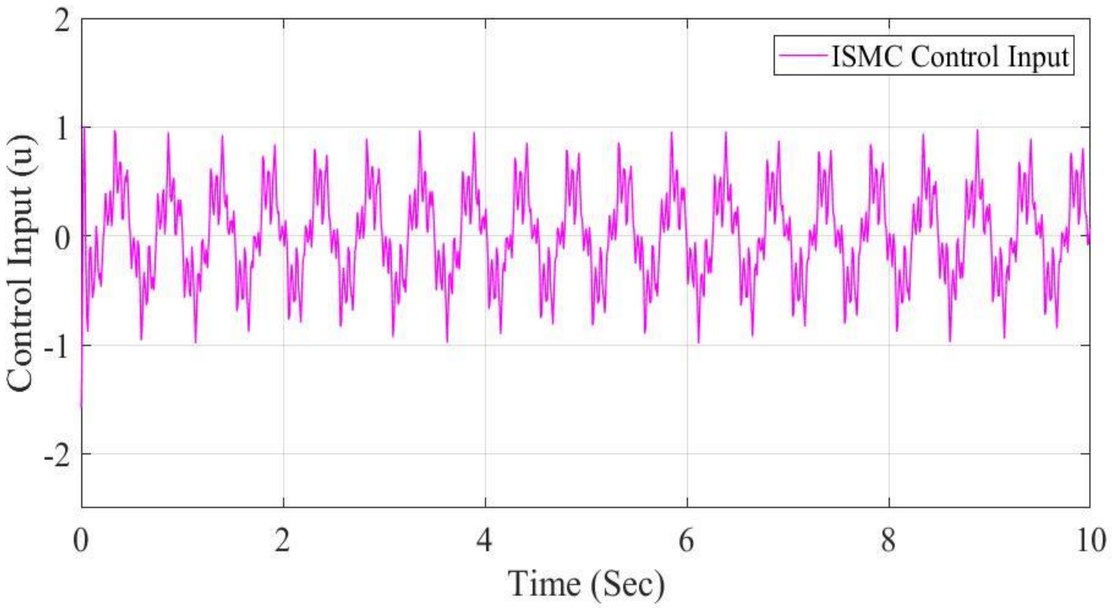

Fig.5.22: Control Effort Vs Time with disturbance

5.5.5 FISMC without Disturbance: -

The simulation results of FISMC without disturbance are shown from Fig.5.23 to Fig.5.25.

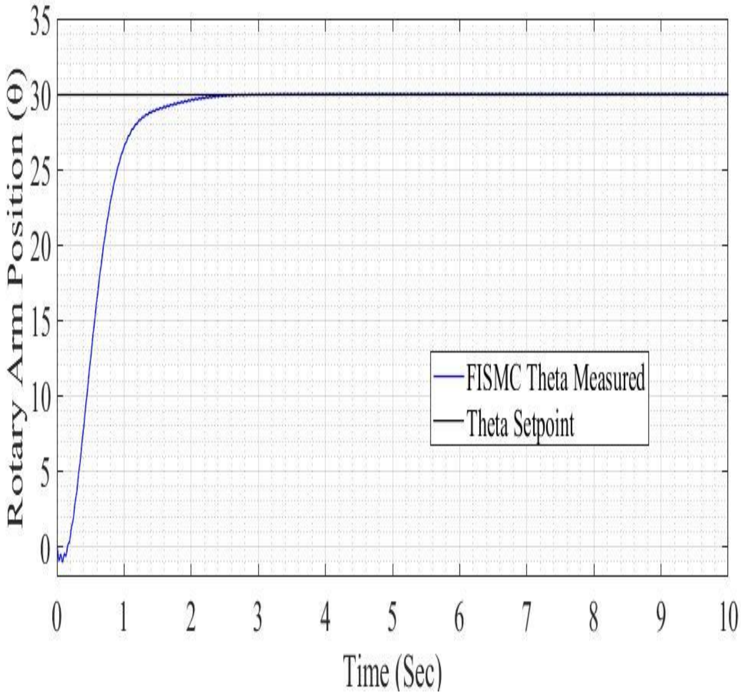

Fig.5.23: Theta Vs Time without disturbance

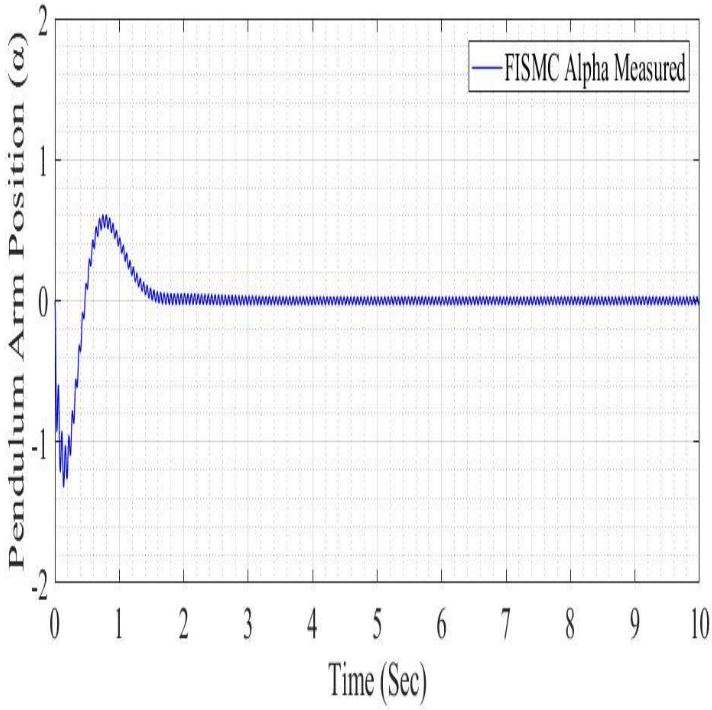

Fig.5.24: Alpha Vs Time without disturbance

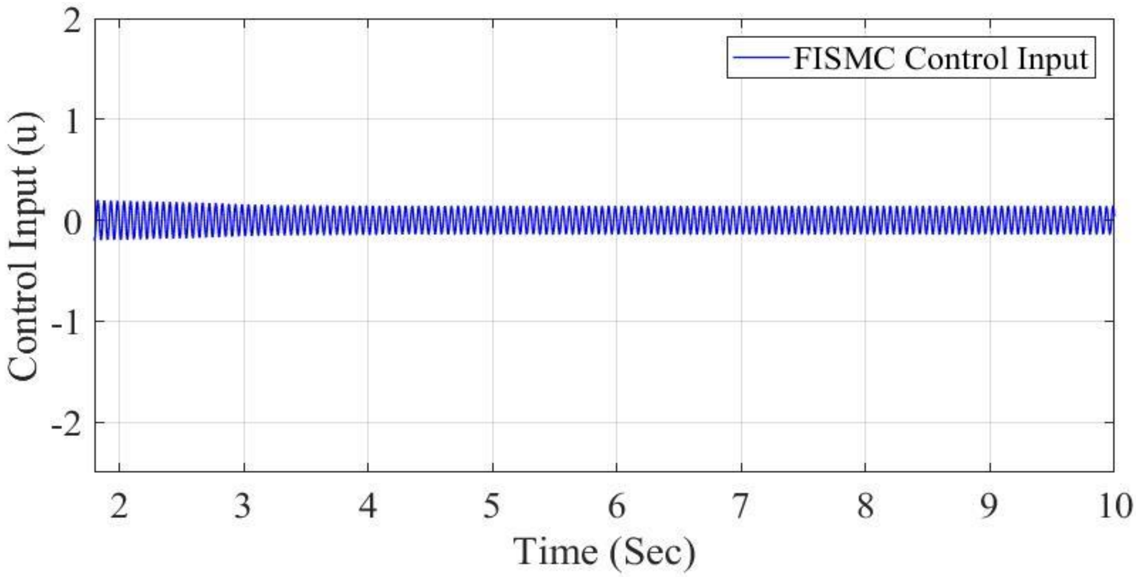

Fig.5.25: Control Effort Vs Time without disturbance

5.5.6 FISMC with Disturbance: -

The simulation results of FISMC with disturbance are shown from Fig.5.26 to Fig.5.28.

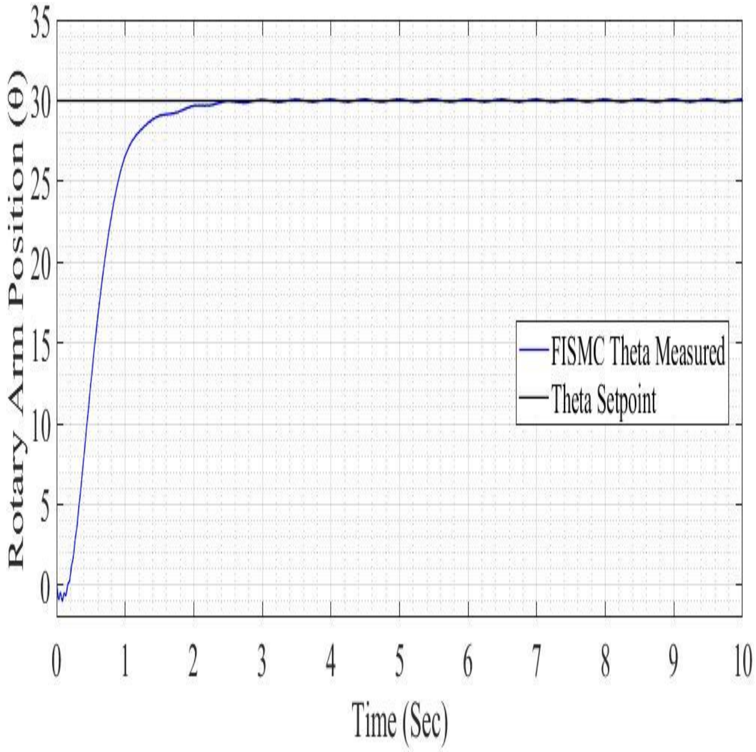

Fig.5.26: Theta Vs Time with disturbance

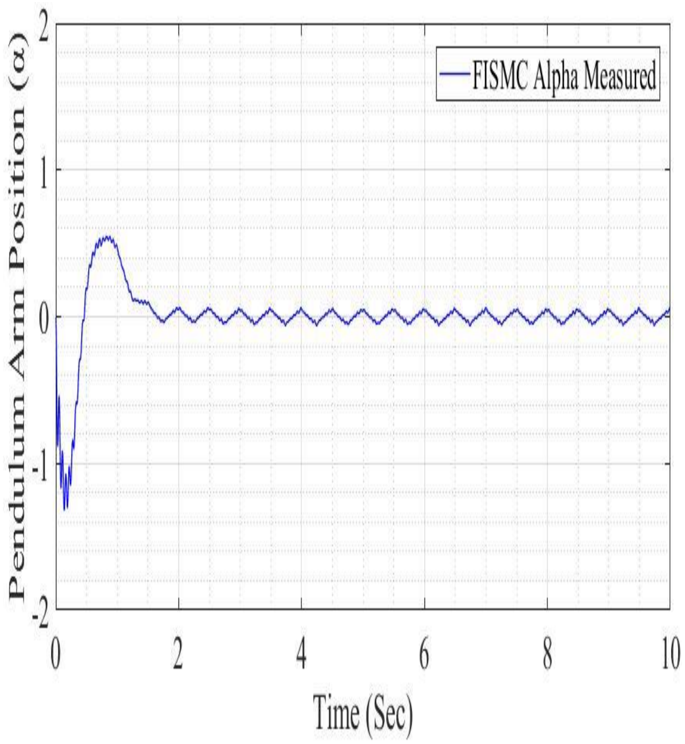

Fig.5.27: Alpha Vs Time with disturbance

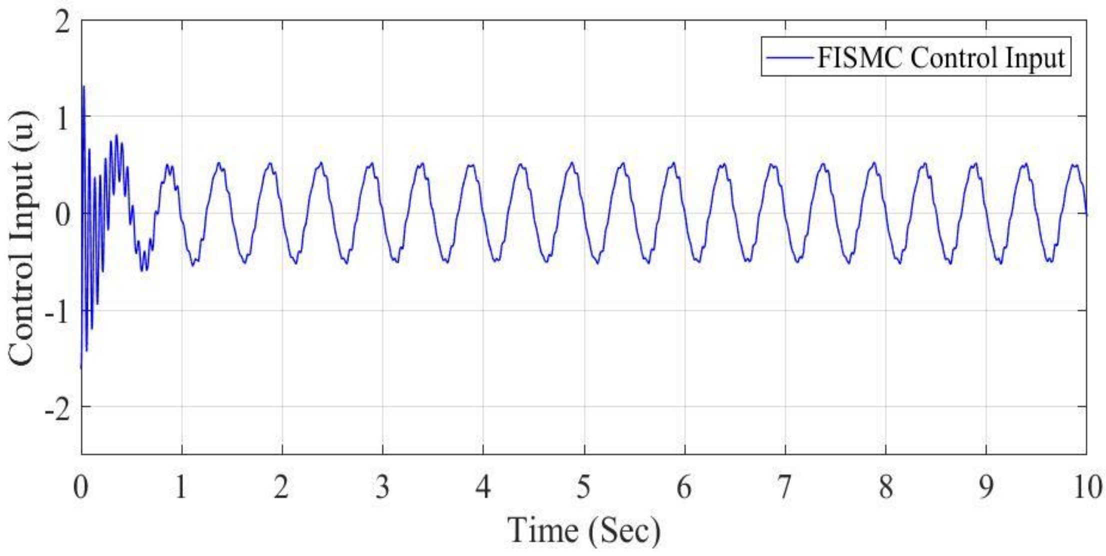

Fig.5.28: Control Effort Vs Time with disturbance

The above responses show that performance of different SMC techniques is not affected by the disturbance while the performance of LQR control technique affected by the disturbance. ISMC response shows that chattering phenomena which is removed by FISMC control technique shown in Fig.5.24.

5.6 Comparison Among Different Control Techniques: -

5.6.1 Comparison of Rotary Arm Position (θ) without Disturbance: -

A comparison among the LQR, SMC, ISMC and FISMC without disturbance is shown in Fig.5.29.

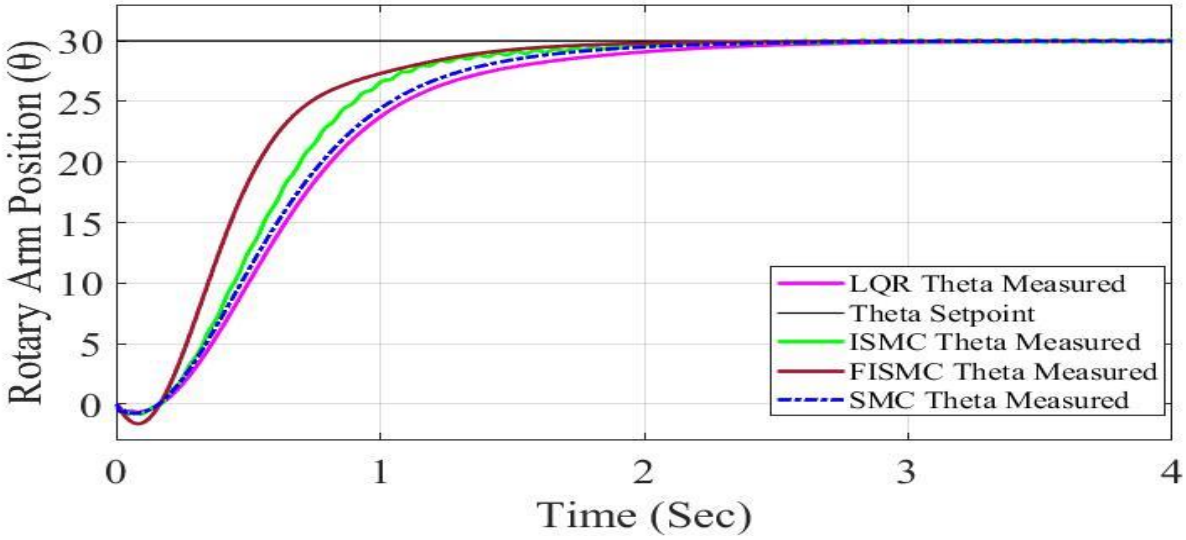

Fig.5.29: Theta Comparison of Conventional and Robust Control Techniques without Disturbance

5.6.2 <u>Comparison of Rotary Arm Position (θ) with Disturbance</u>: -

A comparison among the LQR, SMC, ISMC and FISMC with disturbance is shown in Fig.5.30.

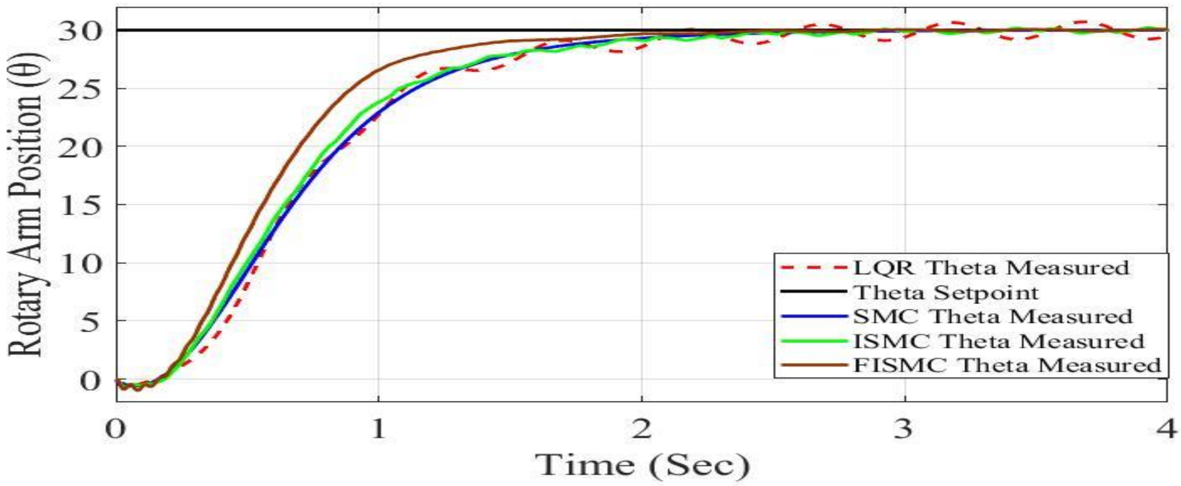

Fig.5.30: Theta Comparison of Conventional and Robust Control Techniques with Disturbance

The response shown in Fig.5.29 shows that the FISMC gives better response in terms of settling time (t_s) among all controllers. Fig.5.30 shows that the LQR control technique is affected due to the presence of disturbance whereas different SMC techniques show their robustness even in the presence of disturbance.

5.6.3 <u>Comparison of Pendulum Arm Position (α) without Disturbance: -</u>

A comparison among the LQR, SMC, ISMC and FISMC without disturbance is shown in Fig.5.31.

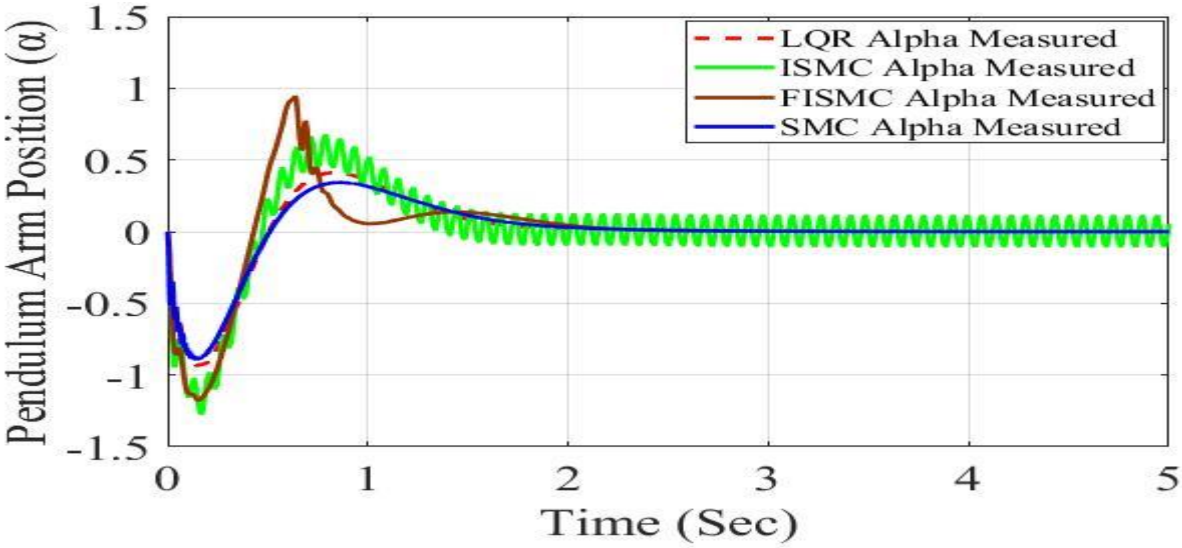

Fig.5.31: Alpha Comparison of Conventional and Robust Control Techniques without Disturbance

5.6.4 Comparison of Pendulum Arm Position (α) with Disturbance: -

A comparison among the LQR, SMC, ISMC and FISMC with disturbance is shown in Fig.5.32.

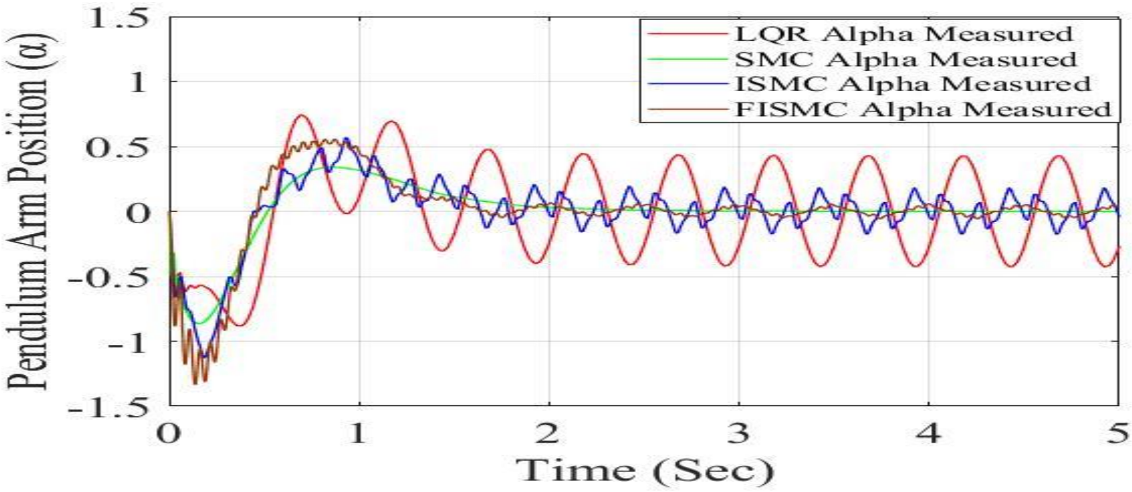

Fig.5.32: Alpha Comparison of Conventional and Robust Control Techniques with Disturbance

Fig.5.31 gives the comparison of alpha angle and response shows that SMC has stabilized the pendulum arm with less overshoot and undershoot whereas the ISMC has some chattering issue which is removed by the FISMC.

5.7 Hardware Implementation of Various Control Schemes: -

The real-time hardware setup module provided by QUANSER is shown in Fig.5.33. After applying various control techniques, the pendulum arm of the real-time hardware system is successfully stabilized in the vertical upright position (unstable equilibrium position) as shown in Fig.5.34.

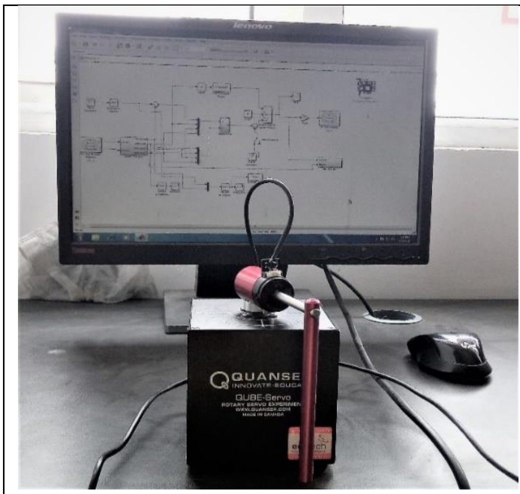

Fig. 5.33: Real-Time Setup of RIP System

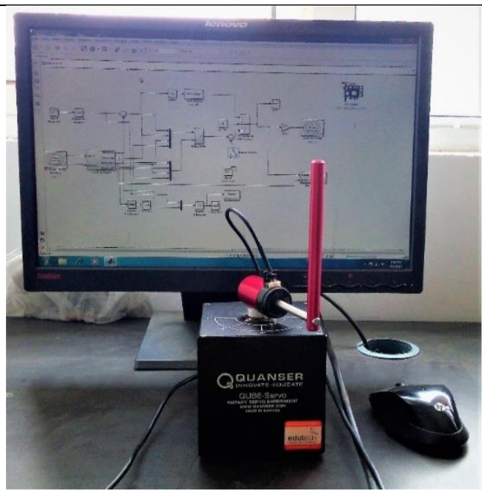

Fig.5.34: Pendulum Arm Stabilization

5.7.1 LQR without Disturbance: -

In the hardware setup, a square wave of 30^0 amplitude is given as reference for theta (Rotary arm position) and 0^0 is given as reference for alpha (Pendulum arm position). The experimental results of LQR without disturbance are shown from Fig.5.35 to Fig.5.37.

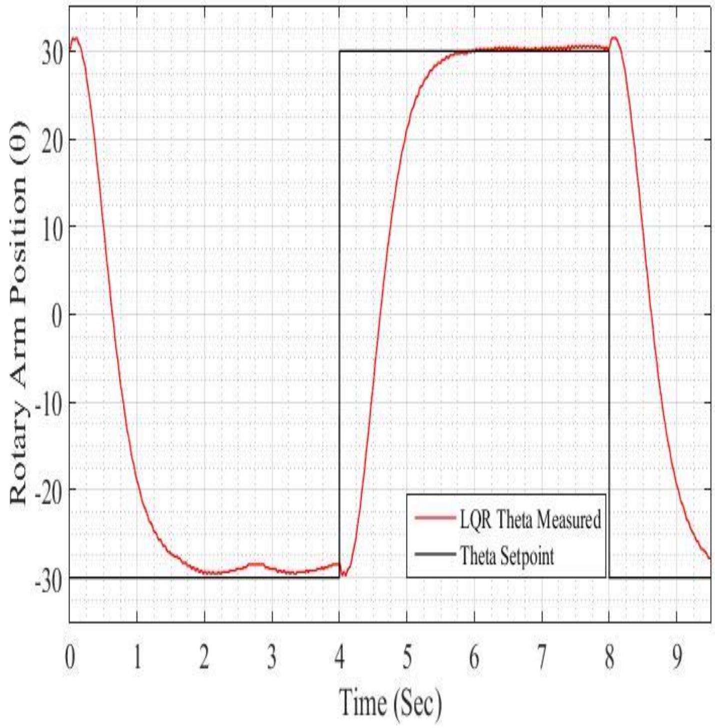

Fig.5.35: Theta Vs Time without disturbance

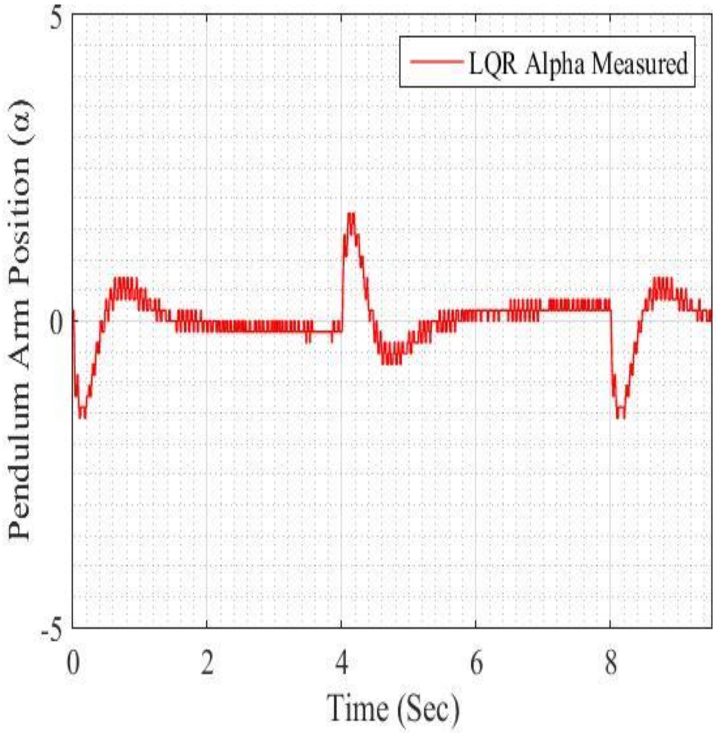

Fig.5.36: Alpha Vs Time without disturbance

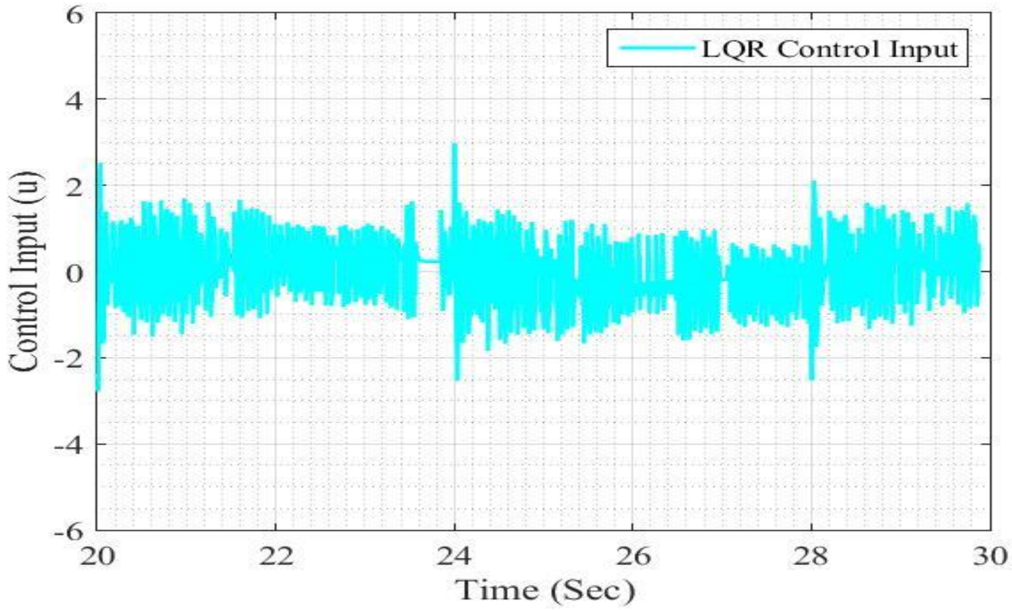

Fig.5.37: Control Effort Vs Time without disturbance

5.7.2 LQR with Disturbance: -

The sinusoidal disturbance introduce to the input channel of the system is given as,

$$d = 0.5\sin 3.1t \tag{5.7}$$

The experimental results of LQR with disturbance are shown from Fig.5.38 to Fig.5.40.

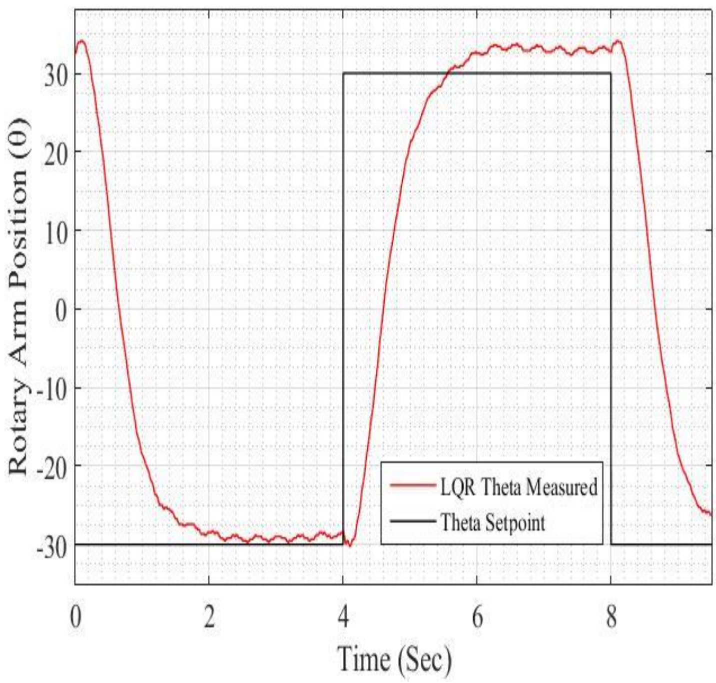

Fig.5.38: Theta Vs Time with disturbance

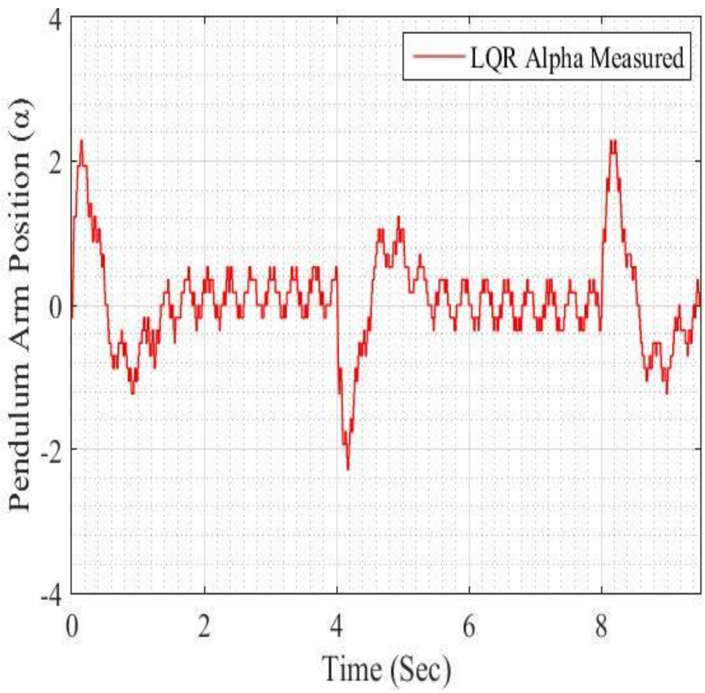

Fig.5.39: Alpha Vs Time with disturbance

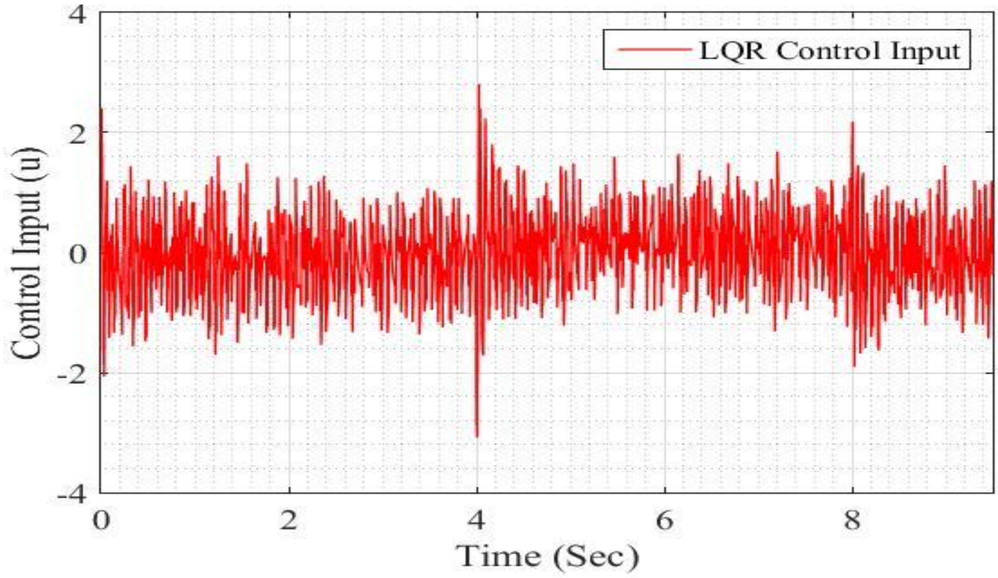

Fig.5.40: Control Effort Vs Time with disturbance

5.8 Hardware Implementation of Sliding Mode Control Techniques: -

5.8.1 SMC without Disturbance : -

The experimental results of SMC without disturbance are shown from Fig.5.41 to Fig.5.43.

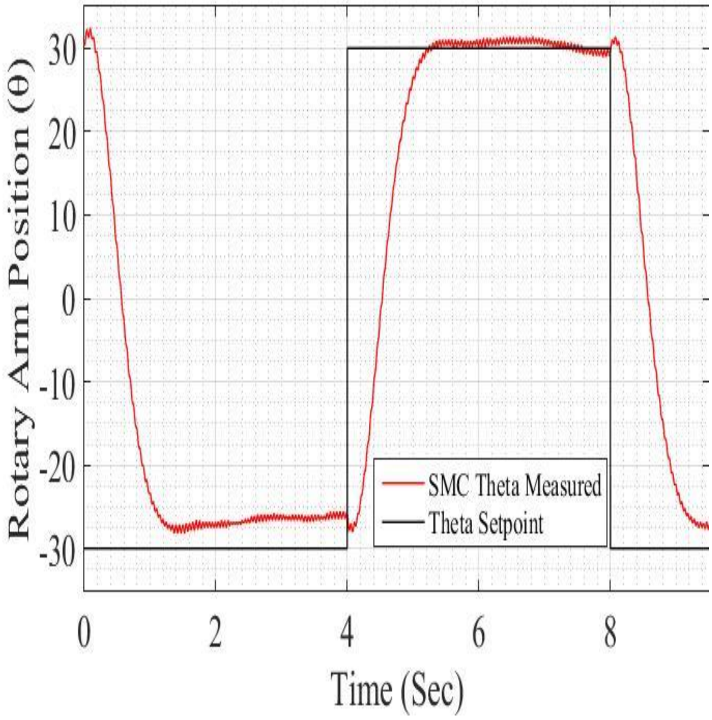

Fig.5.41: Theta Vs Time without disturbance

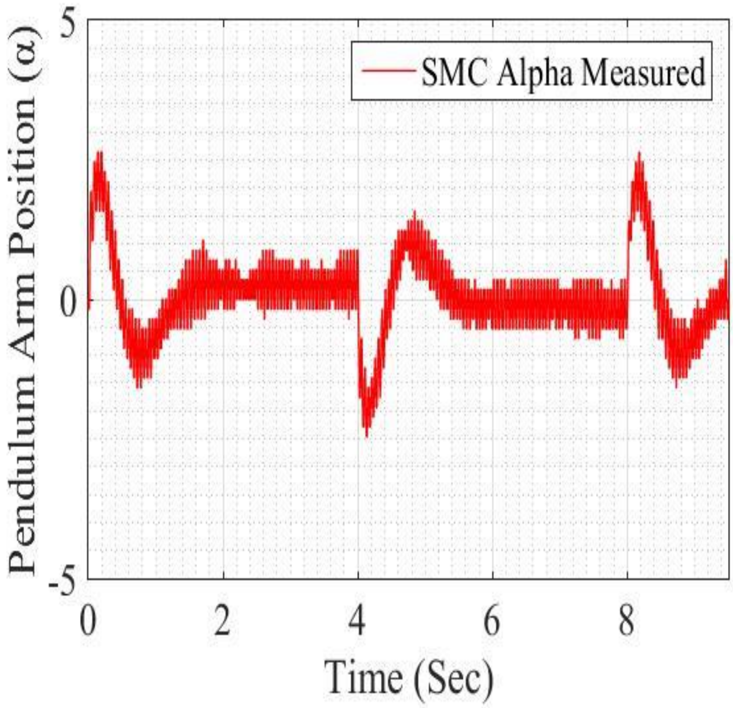

Fig.5.42: Alpha Vs Time without Disturbance

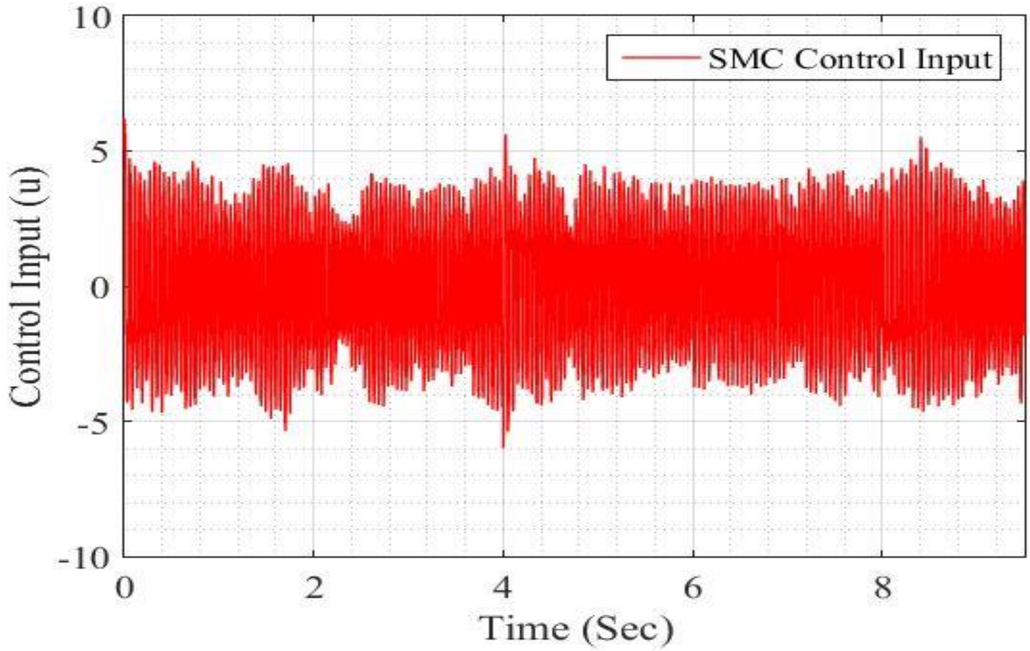

Fig.5.43: Control Effort Vs Time without Disturbance

5.8.2 SMC with Disturbance: -

The experimental results of SMC with disturbance are shown from Fig.5.44 to Fig.5.46.

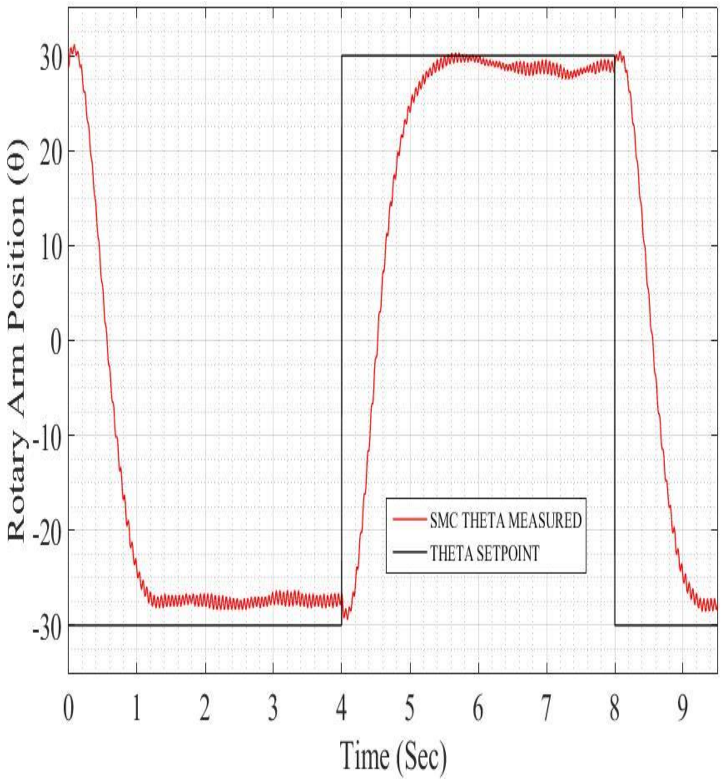

Fig.5.44: Theta Vs Time with disturbance

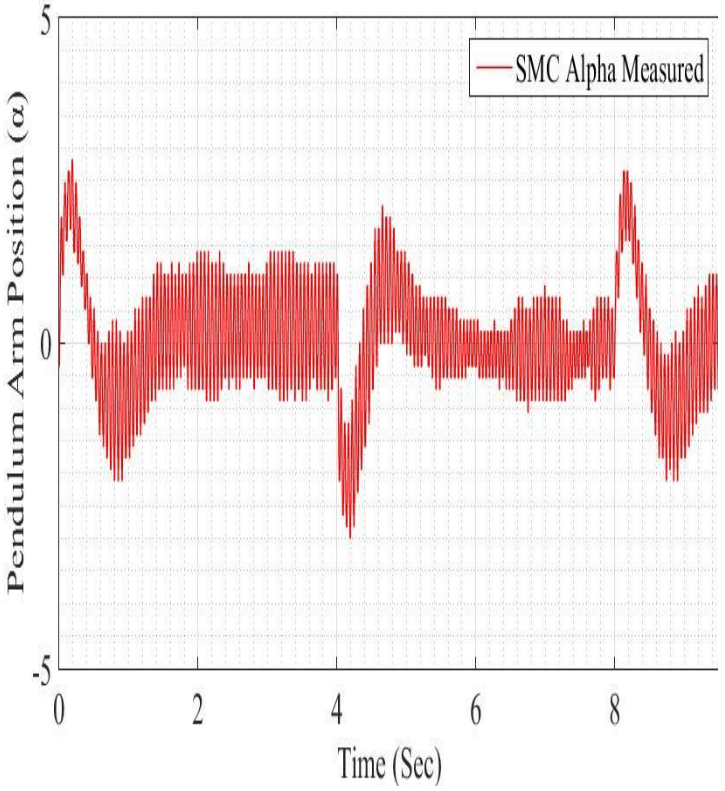

Fig.5.45: Alpha Vs Time with disturbance

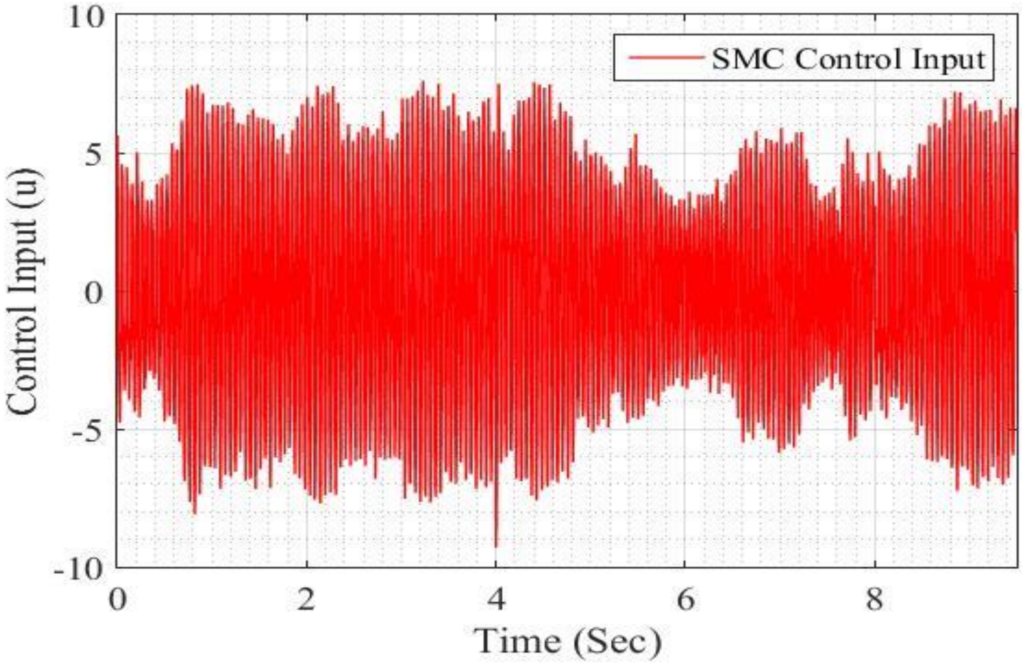

Fig.5.46: Control Effort Vs Time with disturbance

5.8.3 ISMC without Disturbance: -

The experimental results of ISMC without disturbance are shown from Fig.5.47 to Fig.5.49.

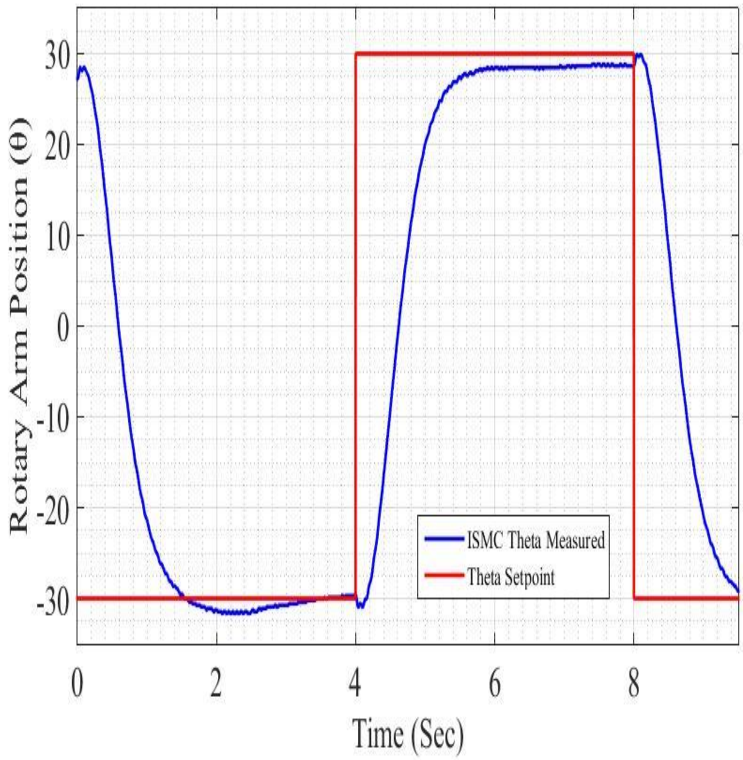

Fig.5.47: Theta Vs Time without disturbance

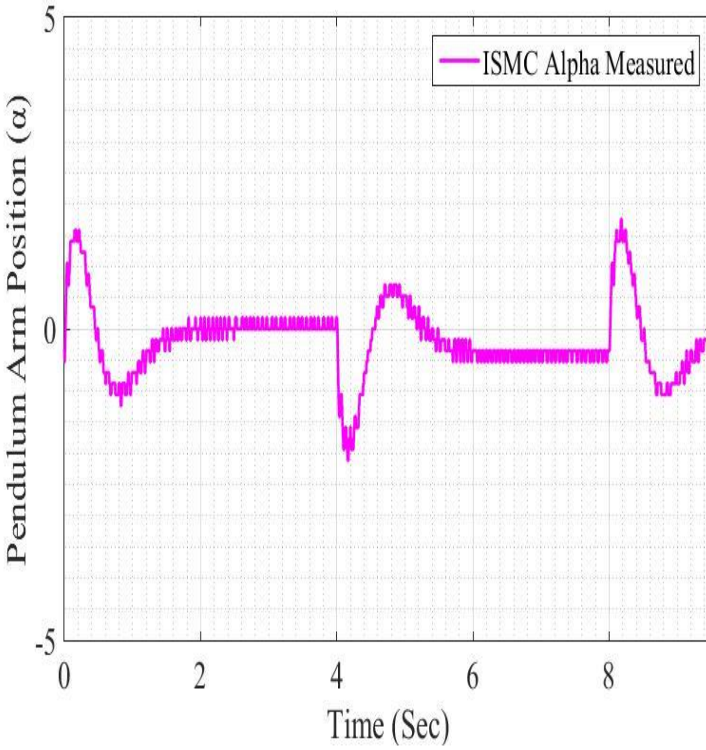

Fig.5.48: Alpha Vs Time without disturbance

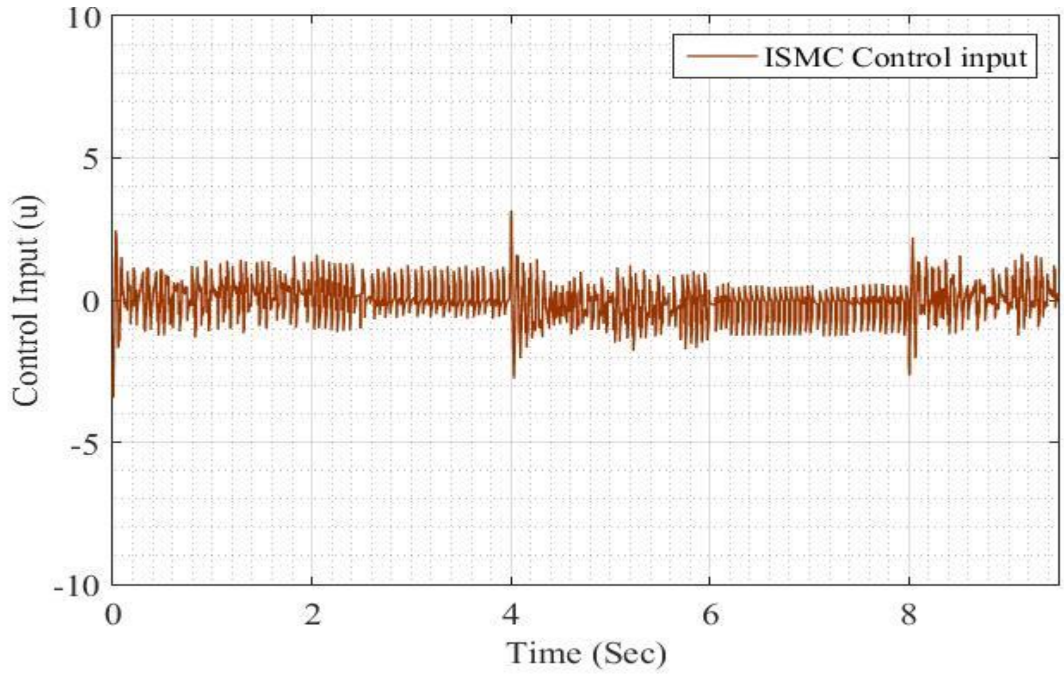

Fig.5.49: Control Effort Vs Time without disturbance

5.8.4 ISMC with Disturbance: -

The experimental results of ISMC with disturbance are shown from Fig.5.50 to Fig.5.52.

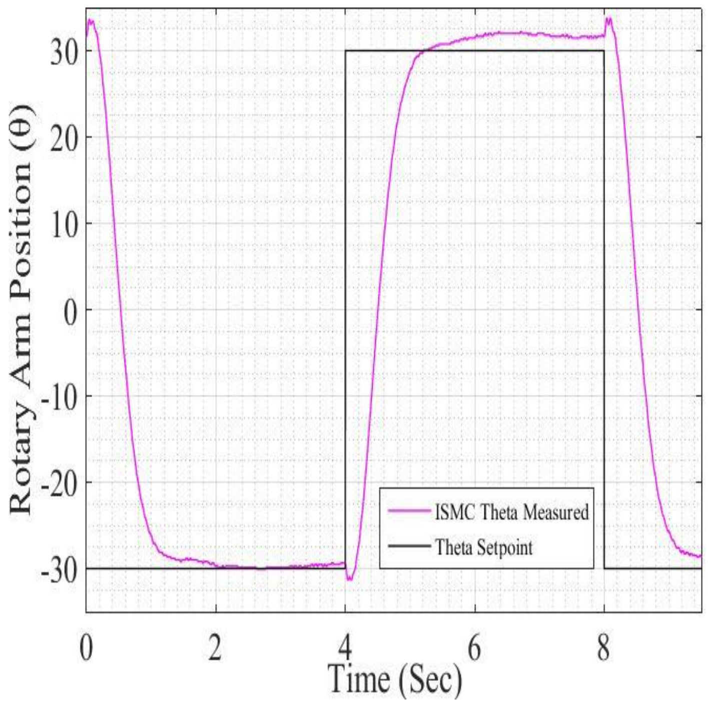

Fig.5.50 Theta Vs Time with disturbance

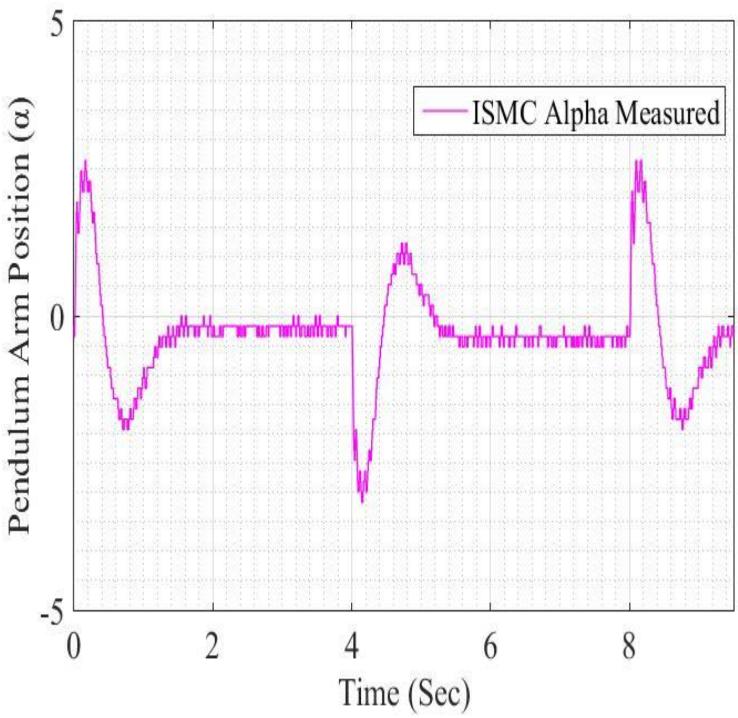

Fig.5.51: Alpha Vs Time with disturbance

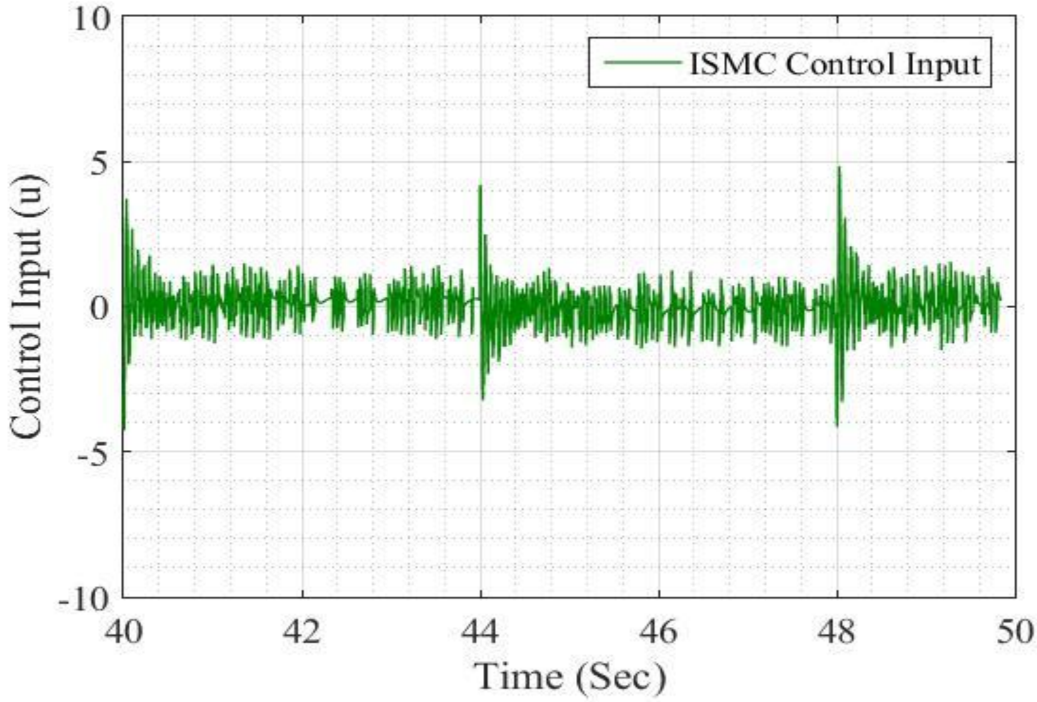

Fig.5.52: Control Effort Vs Time with disturbance

5.8.5 FISMC without Disturbance: -

The experimental results of FISMC without disturbance are shown from Fig.5.53 to Fig.5.55.

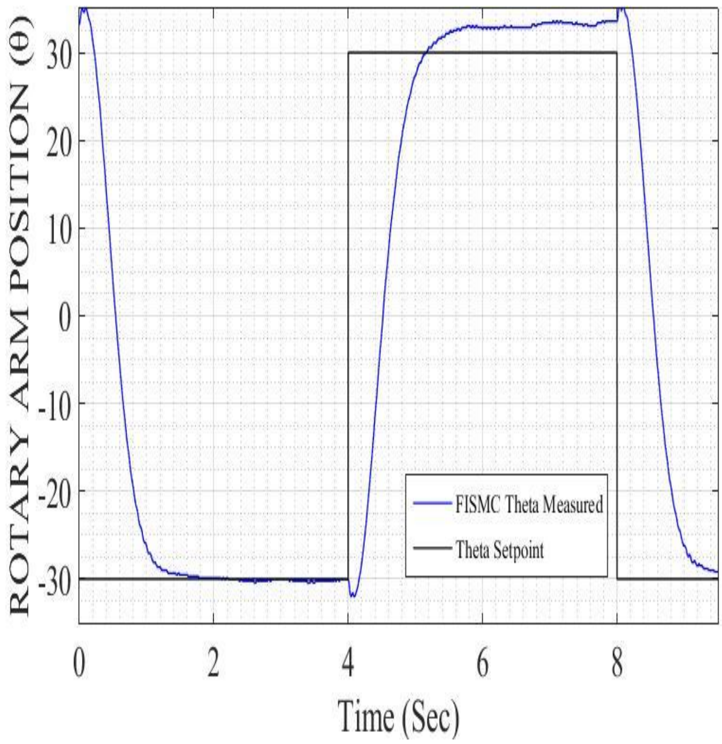

Fig.5.53: Theta Vs Time without disturbance

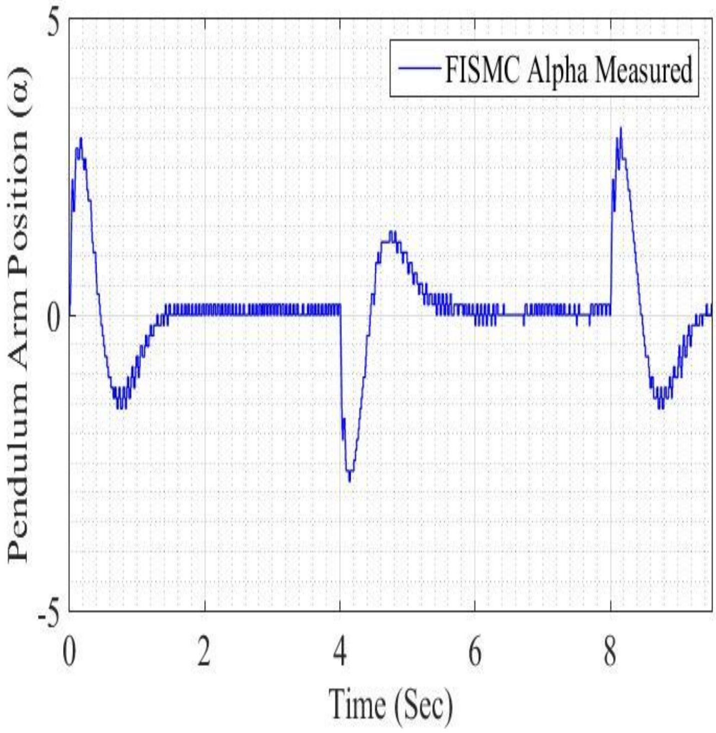

Fig.5.54: Alpha Vs Time without disturbance

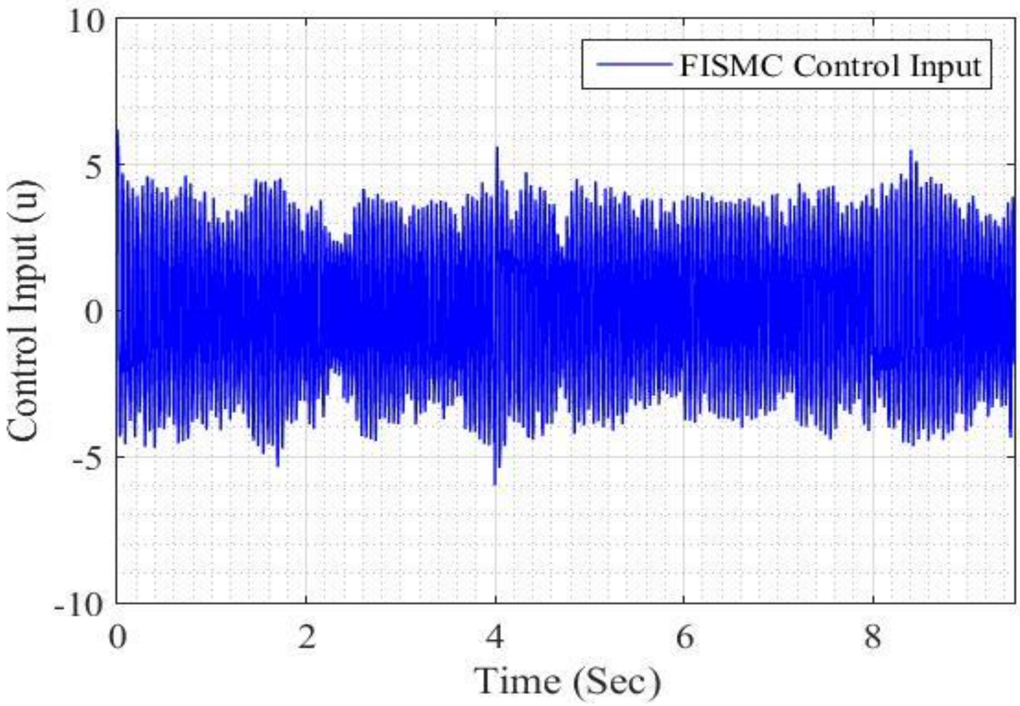

Fig.5.55: Control Effort Vs Time without disturbance

5.8.6 FISMC with Disturbance: -

The experimental results of FISMC with disturbance are shown from Fig.5.56 to Fig.5.58.

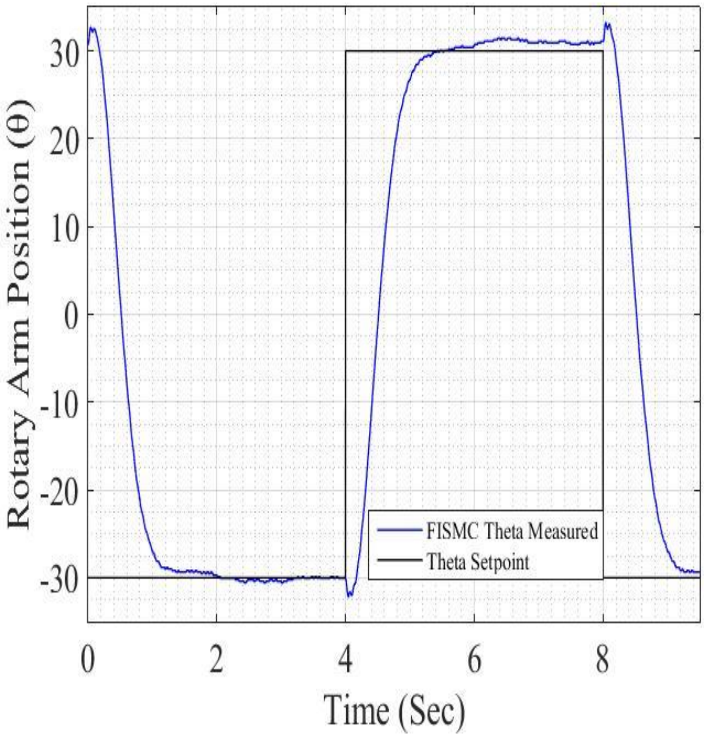

Fig.5.56: Theta Vs Time with disturbance

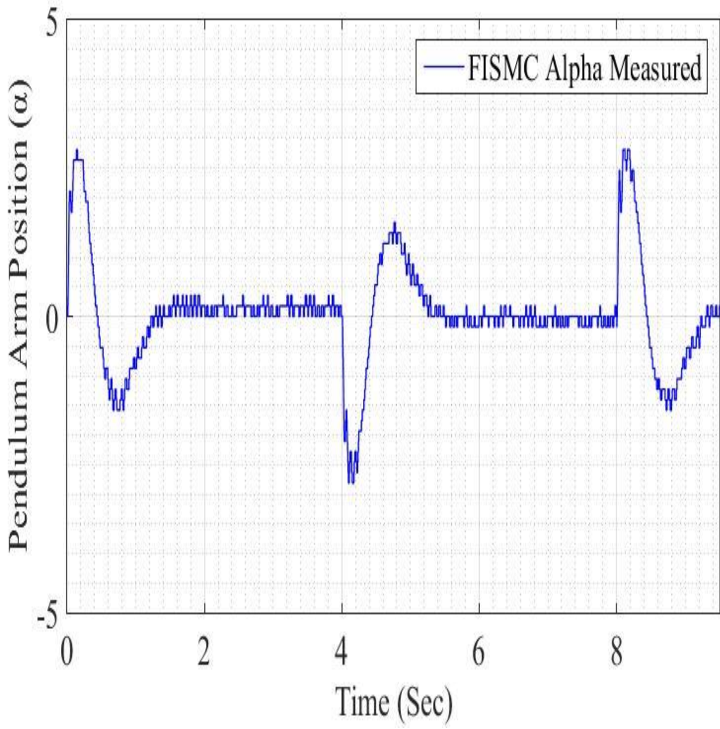

Fig.5.57: Alpha Vs Time with disturbance

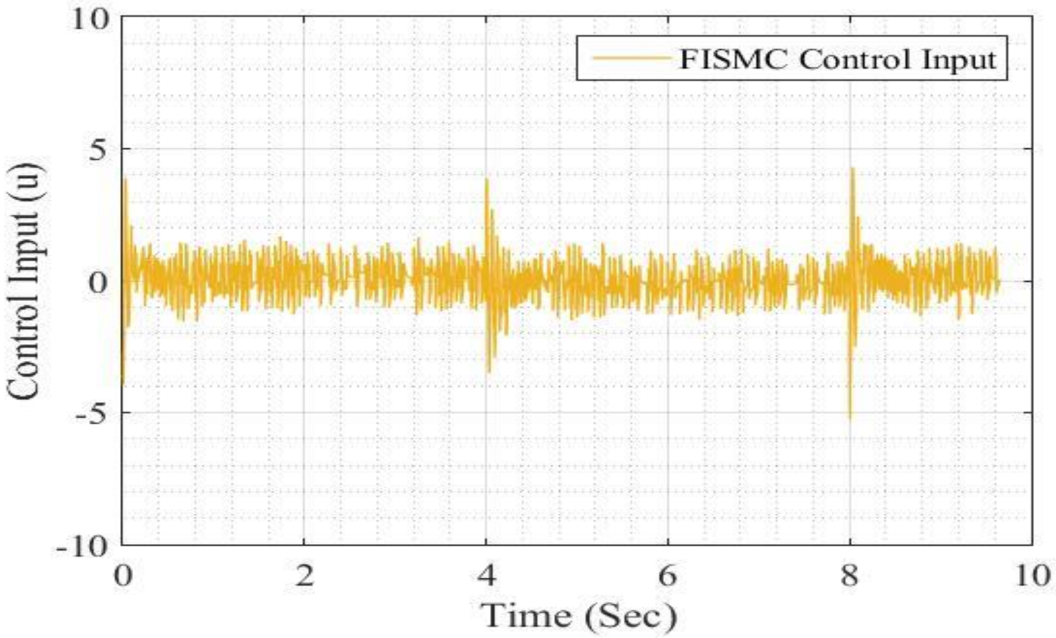

Fig.5.58: Control Effort Vs Time with disturbance

The time-domain parameters i.e. rise time (t_r) and settling time (t_s) of different control techniques are shown in the Table 5.1 and Table 5.2 respectively.

Table 5.1: Time-Domain Parameters of Conventional and Robust Control Techniques (Simulation)

Control Technique	Rise Time (t_r) (sec)	Settling Time (t_s) (sec)
LQR	1.013	2.5
SMC	1.093	2.3
ISMC	0.764	1.8
FISMC	0.767	1.7

Table 5.2: Time-Domain Parameters of Conventional and Robust Control Techniques (Experimental)

Control Technique	Rise Time (t_r) (sec)	Settling Time (t_s) (sec)
LQR	0.8	1.75
SMC	0.7	1.25
ISMC	0.89	1.22
FISMC	0.88	1.21

The control effort of these control techniques with and without disturbance are shown in Table 5.3 and Table 5.4 respectively.

Table 5.3: Control Effort of Conventional and Robust Control Techniques (Simulation)

Control Technique	Control Effort without Disturbance (Volt)		Control Effort with Disturbance (Volt)	
	Minimum	Maximum	Minimum	Maximum
LQR	-1.1708	0.6530	-1.1708	0.6823
SMC	-3.0952	1.6996	-3.0952	1.7491
ISMC	-1.5778	1.2208	-1.5778	1.0078
FISMC	-0.1998	0.1948	-1.6140	1.3151

The comparison among different control techniques show that the FISMC control technique settle the rotary arm within 1.7 sec which is fastest among other control techniques. The rise time (t_r) of ISMC and FISMC control techniques are quite similar and better than other two techniques.

Table 5.4: Control Effort of Conventional and Robust Control Techniques (Experimental)

Control Technique	Control Effort without Disturbance (Volt)		Control Effort with Disturbance (Volt)	
	Minimum	Maximum	Minimum	Maximum
LQR	-2.8	3.05	-3.1	2.8
SMC	-6	6.2	-9.3	7.5
ISMC	-3.46	3.15	-4.3	4.85
FISMC	-6	6.2	-5.25	4.3

In simulation results, control effort of SMC control technique is large among other control techniques. The amplitude of control effort is increased when come in experimental environment.

CHAPTER 6: CONCLUSION AND FUTURE SCOPE

6.1 Conclusion: -

Inverted pendulum has been considered as one of the benchmark problem in control theory. The system is highly nonlinear, underactuated, multivariable, open loop unstable and non-minimum phase. In this work different robust control techniques were used for the stabilization of RIP. These techniques were based on SMC, ISMC and FISMC. SMC has two phases: reaching phase and sliding phase and it shows robustness in the presence of disturbance during sliding phase. In case of SMC the reaching phase is noise prone. This limitation of SMC can be eliminated by the ISMC technique. Both SMC and ISMC techniques are affected by the chattering phenomena which has adverse effect on the mechanical system. Therefore, FISMC technique is used to minimize the chattering effect. The robustness of the given control schemes was checked in the presence of sinusoidal external disturbance. These control techniques have been implemented using MATLAB Simulink and their results are compared with the conventional linear quadratic regulator (LQR) controller (without and with disturbance) in terms of settling rise time (t_r) and time (t_s). Simulation results show that the performance of LQR deteriorates with disturbance while SMC, ISMC and FISMC techniques give better performance in the presence of disturbance. The comparison of LQR and sliding mode control techniques has been done in terms of settling time (t_s) and results show that % improvement in terms of settling time of SMC, ISMC and FISMC control technique is 8%, 27.97% and 32% respectively in simulation and 28.57%, 30.29% and 30.86% respectively in experimental with respect to LQR control technique. These control techniques are further validated on the real-time RIP system called QUANSER QUBE-SERVO.

6.2 Future Scope of Thesis: -

The following work can be done in future: -

- During mathematical modeling of RIP, many variables such as actuator delay, static friction, viscous coefficient of rotary and pendulum arm, actuator wire dynamics etc. were neglected. Therefore, in future, these variables can be considered to derive the mathematical model.
- Different types of noises like white noise, Gaussian noise, band-limited noise etc. can be added to input channel of system and robustness of the controller can be checked.

- The state weight matrix Q and input weight matrix R will be optimized with the different optimization algorithm like genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO), evolutionary algorithm (EA) etc.
- The robustness of the control techniques can be tested by varying the weight of the pendulum arm.

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