

Economic Load Dispatch Using Modified Particle Swarm Optimization

A dissertation

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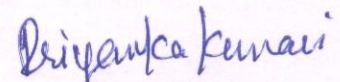
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CERTIFICATE

I hereby certify that the work which is being presented in dissertation entitled, "Economic load dispatch using modified particle Swarm Optimization", in partial fulfillment of the requirements for the award of degree of **Master of Engineering in Power System and Electric drives** at Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of **Mr. Nitin Narang**, Assistant Professor (EIED). The matter embodied in this dissertation has not been submitted for the award of any other degree to any other university.

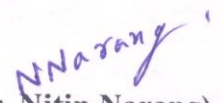
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ABSTRACT

Modified particle swarm optimization technique is presented in this thesis work to solve economic load dispatch (ELD) problem taking into account the functional constraints of power system. The theory of modified PSO involves modification in the velocity of each particle in such a manner that every particle in the swarm try to improve its velocity and position based on an α -factor which helps it to achieve the feasible solution easily and a β -factor in order to exploit the solution to converge it to improved results. Thus, the modified velocity equation follows a better convergence by providing a balance between exploring the searching area and exploiting those searched areas. It is applied on ELD problem to obtain optimum solution within feasible functional limits, satisfying the load demand at the same time. In order to determine the optimization performance of the proposed modified PSO based approach, it is applied on two standard test problems. The optimization strategy proposed in this work is easy to implement and provide an improved exploration and exploitation strategy.

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1.1 Overview

Electric energy is the most flexible form of energy which is used to operate various domestic appliances in households and electrical equipments in large factories. Electricity demand is rising day by day following such a highly power dependent society. These demands can be fulfilled either by utilizing electrical energy in a more efficient and way or by producing energy in most economical manner. Generating the exact amount of power as demanded meeting all the losses is crucial and follows a complex procedure. Economic load dispatch (ELD) can be utilized to figure and to schedule the required quantity of power to be generated among the various generating units in the system [1]. The operation of the various generating systems in an economical manner is always considered an important factor in power industry while considering various diversified form of constraints regarding to power generation, power demand and operational limits of the generating units.

Most of the industrial practices known so far consider a quadratic function of real power output generated for the operating fuel cost of generators and valve point loading effect if considered in case of thermal power plants calls for discontinuity in the optimization problem. Thus, ELD problem exhibit non-linear and non-convex features due to the presence of valve point loading effects. Classical optimization techniques include continuously differentiable functions which are difficult to handle and at times, does not converge to optimum solution [2]. The evolutionary computational techniques can handle such non-convex and non-differential objective functions very easily and provide a feasibly optimal solution in a very interval of time. Many meta-heuristic algorithms have the ability to efficiently solve non-continuous, non-linear and non-convex optimization problems as such techniques do not impose any requirements on the given problem. Similar to these evolutionary algorithms, randomly followed artificial intelligence techniques such as swarm intelligence, for instance say particle swarm optimization (PSO) is also applied to many complicated optimization fields use of which not only avoids the problem related to coding of the structure and repetitive decoding as seen in case of GA but also lead to reduction in

time and memory while deciding parameters, maximum number of iterations and size of population [3].

1.2 Literature Review

Optimization techniques can be classified into three categories involving classical techniques, stochastic method and hybrid approach. A classical technique is dependent on the selection of parameter and thus appropriate selection of parameter value is necessary for satisfactory operation. Lung *et al.* [1] have presented ELD using direct search method. Liang *et al.* [2] proposed a dynamic programming (DP) technique used to solve economic dispatch including transmission losses. But these classical methods are totally based on assumption values and do not follow converging solution for inappropriate values selected.

Classical approaches were replaced by stochastic techniques which are efficient and flexible than conventional approaches because of their ability to obtain satisfactory solution with well defined handling of constraints. Many techniques have been employed to minimize total generating cost in electric power system which includes genetic algorithm (GA) [3-5], bacteria foraging method [6], differential evolution (DE) [7], artificial immune system (AIS) [8], gravitational searching algorithm [9], artificial intelligence (AI) techniques, evolutionary programming (EP) etc. Similar to EP approaches, behavioral intelligence technique namely gravitational search intelligence has been applied in various optimization fields. Hong-da liu *et al.* [3] suggested immune GA for solving ELD. Chukiat Worasuchep *et al.* [4] introduced real-coded GA to solve scheduling of generating unit problem. But all the above discussed methods suffer from the problem of memory requirement. S.H. Ling *et al.* [5] developed improved GA for obtaining the solution of standard ELD problem considering valve-point loading. J. Hazra *et al.* [6] using bacteria foraging optimization method to solve ELD. Nidul Sinha *et al.* [7] discussed the solution of ELD problem using non-dominated sorting DE algorithm. B.Vanaja *et al.* [8] presented AIS algorithm for carrying out the optimization of constrained ELD problem with valve-point loading effect and it has been tested by changing the size of population for obtaining the fast convergence, robustness and improved efficiency. T. Kumano *et al.* [9] solved ELD optimization problem considering valve point loading effect and transmission losses using gravitational searching algorithm. Jun Wang [10] presented a modified mutative scale chaotic optimization approach for solving ELD

optimization problem in which a descriptive load map is entitled to ensure the perfect satisfaction of constraints. H. T. Jadhav [11] investigated plant growth simulation algorithm for the scheduling of various diversified wind farms and coal based units by using waybill probability distribution function considering various constraints like valve Kron's power loss, ramp rates, point loading effect, and prohibited zones. N. A. Rahmat *et al.* [12] suggested differential evolution ant colony optimization to solve ELD problem. Sunny Orike [13] investigated the solution of power flow problem optimally for ELD in case of power generating industry using modified EA combined with hill-climbing and smart mutation. Following a smart mutation operator and benchmark instances involving operational limits, ramp rates, prohibited operating zones and power balancing constraints. Sunny Orike *et al.* [14] proposed a novel EA with a smart mutation operator method to solve ELD problem that set its focus mutation towards genes contributing regarding limit violation and cost minimization. Sk Md Ali Bulbull *et al.* [15] presented a modified structure of GSA to solve nonlinear ELD optimization problem using quasi-oppositional GSA employing population initialization based on opposition and accelerate the convergence velocity of conventional GSA using generation jumping and numerical results demonstrates improved performance of the proposed scheme both based on quality of the solution obtained and the numerical efficiency.

Shaik Affijulla *et al.* [16] solved ELD problem considering valve point loading using gravitational search algorithm. Dieu Ngoc Vo *et al.* [17] solved ELD problem containing piecewise quadratic cost functions using a continuous neural network having its energy function corresponding to augmented Lagrangian function i.e. enhanced augmented Lagrange Hopfield network for the purpose of determining fuel type in two phases. R.K. Swain *et al.* [18] suggested gravitational search algorithm based on mass interaction and law of gravity to solve highly constrained ELD problems. Manisha Sharma *et al.* [19] explored and compared the performance of a variety of DE strategies improved with time-varying mutation for solving the reserve constrained multi-area economic dispatch problem. Qun Niu *et al.* [20] proposed a hybrid harmony search namely arithmetic crossover operation incorporated with opposition based learning algorithm to enhance the diversity of solutions obtained in static ELD with valve point loading effects, ELD with multiple fuel cells, ELD with restrictive operating zones, dynamic ELD and combined heat and power economic

dispatch (CHPED).

Particle swarm optimization proves to be effective for solving various types of problems. G.Pranava *et al.* [21] proposed PSO to solve ELD problem. Yamille del Valle *et al.* [22] explained basic structure of PSO with its different form. Gary G. Yen *et al.* [23] proposed information exchanging method between distinct swarms of PSO. Jun Sun *et al.* [24] investigated the searching ability of a particle based on quantum behaved PSO. Abolfazl Zaraki *et al.* [25] suggested a reliable based on evolutionary approach, PSO to solve the constraint ELD problem considering the fuel cost equation as piecewise quadratic function for each generation units and the transmission losses expressed using B-coefficient. K. Vaisakh *et al.* [26] presented integration of bacterial foraging optimization algorithm, PSO and DE to evolve bacterial foraging PSO-DE algorithm for solving economic dispatch problem considering security constraints eliminating the difficulty of stagnation in solution using PSO and DE operators and fine tuning of the solution is possible through DE operator. Farheen *et al.* [27] presented PSO algorithm to solve dynamic ELD satisfying all constraints including generator constraints, valve point effect, transmission losses and ramp rate limits. B. Mohammadi-Ivatloo *et al.* [28] proposed time varying acceleration coefficients method to solve ELD problem which is considered to be non-linear and non-convex in nature. Jong-Bae *et al.* [29] suggested new PSO framework approach to solve ELD problems considering with smooth cost functions as well as with non-smooth cost functions with satisfactory satisfaction of equality and inequality constraints within a realistic confiscation time. Ahmed Y. Saber *et al.* [30] suggested modification of particle swarm optimization technique to solve ELD problem which prove to be reliable and accurate method to track a constantly varying solution with an suitable fitness function that lead to its convergence very quickly and benchmark data is used to determine the effectiveness of the suggested method. Ahmed Y. Saber *et al.* [31] proposed a technique for effectively solving ELD problems using a novel modified bacterial foraging technique (BFT) where particle's best biased velocity is included to the random velocity of BFT in order to minimize the randomness in movement. Xin MA *et al.* [32] investigated the solution of dynamic load economic dispatch problems in power systems using an improved PSO approach considering constraints like prohibited operation zones and ramp rate limits including the loss criterion where an improved strategy is utilized constraint handling that guarantee the feasible solution employed with penalty functions

resulting in high accuracy, fast speed and better convergence. A. Jaini *et al.* [33] suggested PSO algorithm to solve standards IEEE 26-bus RTS and the results proved that the suggested approach has the merit of attaining feasible solution for solution of given problems. Yongqiang Wang *et al.* [34] proposed a hybrid technique to solve ELD problem using PSO combined simulated annealing method in which SA algorithm help PSO in jumping out of the local minima and obtain a feasible converging solution employed with feasibility-based rule for the satisfactory handling of the constraints. Shaik Affijulla *et al.* [35] applied PSO technique to solve large scale ELD problem including valve point loading, tested on standard test problems and results has shown that the proposed method can handle large scale optimization problems with a great potential in obtaining feasible solution in comparatively a short interval of time. Ahmed Y. Saber *et al.* [36] suggested hybridized technique to exactly provide a balance in local and global search to solve ELD problem taking into consideration non-smooth cost functions and all realistic complex constraints using particle swarm differential evolution optimization technique combining basic structures of PSO and DE strategies and was tested for standard ELD problems to indicate the improved overall performance of the suggested PSDEO strategy. N. Rugthaicharoencheep *et al.* [37] investigated ELD solution using PSO approach and the performance considering standard test systems is demonstrated which shows that developed methodology is better in results when compared to other stochastic algorithms. T. Mabu Subhani [38] presented PSO algorithm with an automation strategy in which time varying acceleration coefficients (TVAC) are included for controlling local and global searching more effectively to avoid premature convergence of the solution using time varying acceleration coefficients based PSO in order to carry out the solution of ELD problem considering non-convex cost functions and the results shows the better performance of the proposed technique. Nagendra Singh [39] suggested the solution of ELD optimization problem considering emission as constraint using moderate-random PSO (MRPSO) which is combination of a novel PSO with a moderate random search strategy and results shows the validation of the suggested MRPSO technique. Farheen M.A. *et al.* [40] presented the solution to basic dynamic ELD optimization problem using PSO algorithm developed through the integration of a basic social system to search a feasible operating schedule for each of the generator units considering various system constraints such as transmission losses, valve point effect,

generator constraints, and ramp rate limits. Koustav Dasgupta *et al.* [41] considered four modified structures of basic PSO i.e. PSO, PSO with constriction factor approach (PSOCFA), PSO with inertia weight factor approach (PSOIWA) and PSO with constriction factor and inertia weight factor approach (PSOCFIWA) algorithms to solve ELD problem obtaining the power output and feasible minimal cost in a very short duration of time that shows successful implementation of these evolutionary algorithms. Yu Zhu *et al.* [42] presented a hybridized structured algorithm to solve basic ELD problem following PSO approach employed with chaotic searching (CPSO) to avoid trapping in the local optimum and obtaining improved performance and the results verified the validity of the proposed technique. Chao-Lung Chiang [43] suggested the solution of power ELD problem of units considering valve-point effect using an improved GA with multiplier updating. Koustav Dasgupta *et al.* [44] investigated the performance of four evolutionary algorithms i.e. PSO, PSO with inertia weight factor approach, PSO with constriction factor and inertia weight factor approach and PSO with constriction factor approach algorithms to ELD.

1.3 Objective of the Work

In this thesis work, modified PSO technique is implemented to solve ELD solution to obtain the best converging solution. During the iteration, acceleration constant is varied to improve the searching capability of PSO. Thus, optimal solution of the problem is obtained assuring constraint satisfaction. Results thus obtained using modified particle swarm optimization is converging in nature.

1.4 Organization of Dissertation

The thesis work titled as “Economic load dispatch using modified particle swarm optimization” has been summarized in six chapters. Chapter one provides a comprehensive introduction, literature review and objective of the work. Chapter two highlights the formulation of ELD. Chapter three includes the basic structure of modified PSO, its algorithm, advantages and disadvantages. Chapter four includes the implementation of modified PSO to ELD problem. Chapter five covers the results section and chapter six concluded the thesis work and suggested the scope for future work followed by the reference section.

2.1 Introduction

In power system, ELD in general terms deals with the minimization of the total generation cost while meeting the equality and inequality constraints imposed by the power generating units [4]. Thus, ELD attempts to schedule the generation with capability of minimizing the total operating cost and operate under unit operating limits [5]. The operating cost can be minimized by employing any optimization technique for solving ELD.

The economic load scheduling problem can be formulated as an objective to minimize the operating cost in power systems with all constraints satisfied in a reliable way. In handling such a dispatch problem, several constraint handling control strategies are required [6]. Many techniques have been suggested to schedule generation to various power units. These methods require adjusting some control variables and thus allocating the required power to each of the generating units throughout the system optimally [7]. The generation cost subjected to various constraints including equality and inequality constraints makes the load dispatch problem non-linear and constrained optimization problem [8].

2.2 Problem Formulation

The economic load dispatch problem includes a cost function employed with various power balance and inequality constraints. The cost function considered and constraints which are taken into account for the purpose of formulating the ELD problem are explained as follows [9]:

2.2.1 Objective Function

The problem in the suggested case involves thermal units. Cost function for each of the generating unit is considered to be quadratic function and can be expressed as:

$$F = \sum_{i=1}^G (a_i P_i^2 + b_i P_i + c_i + |d_i \sin \{e_i (P_i^{\min} - P_i)\}|) \quad (2.1)$$

$$\text{OF}(P) = F(P) + \text{Error} \quad (2.2)$$

$$\min(\text{OF}(P))$$

where

a_i, b_i, c_i, d, e are the cost coefficients.

$F(P)$ = Fuel cost function of thermal units.

P_i = Real generated power of i^{th} unit.

G = Total number of generating unit.

2.2.2 Equality constraints

For power balance, equality constraints should be satisfied as expressed in Eq. (2.3) [8]. The equality constraints indicate the power equation where total generation should be equal to total demand plus total line loss. Equality constraint for power unit can be expressed as:

$$\sum_{i=1}^G P_i = P_D + P_L \quad (2.3)$$

2.2.3 Inequality constraints

Minimum and maximum operational limits for satisfactory working of all the units represent the inequality constraints which are totally based on their operational curves and output values [10]. Thus, the generation values from each of the units must lie within their respective working limits. Operational limits for power generating units are:

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad (2.4)$$

2.2.4 Active power transmission line loss

Power transmission loss is an important factor and can be represented by using Kron's loss formula expressed as follows [11]:

$$P_L = B_{00} + \sum_{i=1}^G \sum_{j=1}^G P_i B_{ij} P_j + \sum_{i=1}^G B_{i0} P_i \quad (2.5)$$

3.1 Introduction

Most of the engineering optimization problems in power system are complex in nature having nonlinear characteristics employed with equality and inequality constraints. Many stochastic techniques have been implemented successfully to solve the optimization problems over the past decade [12]. Some of the popular stochastic optimization techniques include GA, PSO, EP, DE etc. Out of them, particle swarm optimization method being a salient optimizer and can be used in variety of nonlinear and complex engineering optimization problems [13]. Kennedy and Eberhart in 1995 motivated by social activities of individuals such as schooling of fishes and bird flocking for the first time introduced particle swarm optimization technique. Particle swarm optimization technique involves a number of particles forming the swarm moving around a specified searching area where particles forming the swarm are allowed to move in the search space and acquiring a best position based on their personal best values and the global best value of the system considered and obtaining global best solution [14]. During the optimization procedure, each particle makes a track of position following its coordinates in the pre-specified searching area [15].

Particle swarm optimization is basically a population-based searching process which searches in parallel and does not leading to elimination of any of the solution. It is a kind of searching method which traces its progression based on the evolving movement of birds flocking or fish schooling search for food [16]. It initializes particles randomly into the specified search space and these particles gather information through their corresponding positions [17]. The modification in position of each particle in the swarm is based on their own personal best experience and their neighbor's experiences. The inertia weight introduced at the velocity updating step is responsible for the momentum of each particle. It was introduced by Shi and Eberhart and it works by weighing the involvement of the previous velocity for the purpose of eliminating the requirement for velocity clamping [18]. Value of w decides the particle velocity updating and thus whole system direction is based on its value. With w having a value greater than 0, divergent behavior of the system result due to increase in system's velocity and thus particles fail to acquire the best position. With w being

less than 0, deceleration of particles takes place which result in slow convergence to the best solution. Static inertia values or adaptive values may lead to convergent solution [19].

Particle swarm optimization has numerous key features that prove to be very reliable in different applications where conventional optimization techniques might fail as compared to other evolutionary optimization algorithms [20]. Following are some of the advantages of PSO over other conventional algorithms:

- It makes use of basic logical and mathematical functions which are easy to handle and operate with a sequence of procedure [5].
- It has fewer working parameters which are easy to operate [21].
- It utilizes real valued operational integers which eliminates the requirement of converting binary form to real coded structure as in classical GA [16].
- It is better in terms of time utilized for computation and memory storage requirement [22].

3.1.1 Disadvantages of PSO

Particle swarm optimization prove to be very effective for solving many complex type of power system problems very quickly and in a reliable manner but it gets trapped at local minima. As a result, it becomes monotonous to obtain the global best value for such a complicated non-convex system with multiple local minima [23]. Following are some of the disadvantages of PSO:

- The velocities of the particle are random and thus it is very important to suitably identify their value. With an inappropriate value selection, it may suddenly increase or decrease the searching speed that may result in fly off the particle out of the searching region or slow movement to acquire a suitable position in space [24].
- It may convergence prematurely due to heavier constraints in PSO or handling problems with multiple local optima [25].
- The performance of the PSO algorithm may deteriorate with a lengthy problem considered. Thus, it may not lead to optimal solution due to lack of balance between exploration and exploitation [26].

For the solution to this problem, modified PSO includes an α -factor and β -factor to obtain a stable point easily and ensure convergence [27]. The objective of these two factors is to carry out the optimization process in a well balance manner throughout the iteration.

3.2 Variations in PSO

Hybrid of Genetic Algorithm and PSO (GA-PSO) [22,23] combines the advantages of both the techniques involving a natural selection mechanism of GA and swarm intelligence approach of PSO for changing the current searching space successively and thus, jump from one searched area to another following the selection mechanism. This result in increasing the speed of convergence for obtaining an optimal solution. Evolutionary PSO [28] is basically a hybridized structure of EP and PSO incorporating a selection procedure and self-adapting properties for the selection of its parameters. Particle Swarm Differential Evolution Optimization is a new hybridized optimization technique comprising of the conventional PSO approach and DE algorithm considering improved structure of both the methods to enhance improved balancing between exploration and exploitation where DE explores and PSO exploits the searching area. This method results in a more efficient way of improving the convergence speed with a better quality of optimal solution. A modified mutative scale chaotic optimization algorithm [29] involves the concept of novel load map after following the chaotic search for ensuring the fitness of the load variables into the optimal region. Afterwards, mutative scale chaotic sequence arrangements are changed into load variables using these load maps for cost function calculation. It uses stochastic random searching strategy instead of a gradient search and does not require any derivative criterion. This approach is easy with fewer numbers of parameters and simple implementation. Gaussian PSO [30] eliminates the problem of parameter dependence as in case of PSO for guiding the particles and assigning them a particular direction. In this approach, the acceleration constant is replaced by randomly defined numbers using Gaussian distribution and the inertia constant is eliminated. Stretching PSO [31] is basically a modified structure involving PSO algorithm that is based on finding all global minima values thus, eliminating the disadvantage of conventional PSO of getting trapped into a particular value and stagnating the solution. Cooperative PSO [32] employs cooperative behavior using multiple swarms in order to

significantly improve the performance of the original PSO algorithm leading to cooperative optimization of different components of the solution vector.

3.3 Overview to Modified PSO

The theory of modified particle swarm optimization approach involves modification in the velocity of each particle in swarm following its local best and global best position at the closing stages of every iteration in a well defined manner. Every particle in the swarm try to improve its velocity and position based on the distance between its local best value and current position using an α -factor which helps it to achieve the optimal solution easily and the distance between its current position and global best solution using an β -factor in order to exploit the solution to converge it to better results. Thus, the modified velocity equation follows a better convergence by providing a balance between exploring the searching area and exploiting those searched areas. Modified Particle swarm optimization is a modern heuristic approach being effectively utilized to solve nonlinear engineering optimization problems. The position for these particles can be updated using their current velocity and the distance between respective local best and global best value. Following procedural steps are followed for the purpose of implementing modified PSO approach:

Step1 Swarm initialization

Each particle in the swarm can be initialized randomly within their operational working limits. The velocity and position initialized values follow a uniform random number approach lying between its maximum and minimum values [33]. Velocity and position can be represented as follows:

$$\vec{V}_i = \vec{V}_{i1}, \vec{V}_{i2}, \dots, \vec{V}_{in} \quad (3.1)$$

$$\vec{P}_i = \vec{P}_{i1}, \vec{P}_{i2}, \dots, \vec{P}_{in} \quad (3.2)$$

The full swarm can be expressed as a matrix:

$$\text{swarm} = \begin{bmatrix} P_{1,1} & P_{1,2} & \dots & P_{1,G} \\ P_{2,1} & P_{2,2} & \dots & P_{2,G} \\ \dots & \dots & \dots & \dots \\ P_{PR,1} & P_{PR,2} & \dots & P_{PR,G} \end{bmatrix} \quad (3.3)$$

where G represent number of generators and PR is the number of particles in the swarm.

The velocity and position values are randomly initialized following the equation as given indicated:

$$V_{i,j} = V_j^{\min} + \text{rand}() \times (V_j^{\max} - V_j^{\min}) \quad (j = 1,2,3 \dots \dots G; i = 1,2,3 \dots \dots PR) \quad (3.4)$$

$$P_{i,j} = P_j^{\min} + \text{rand}() \times (P_j^{\max} - P_j^{\min}) \quad (3.5)$$

All the initialized values are made to fall within the pre-specified operational limits of generating unit as follows [34]:

$$\bar{V}_{ij}^{\min} \leq \bar{V}_{ij} \leq \bar{V}_{ij}^{\max} \quad (3.6)$$

$$\bar{P}_{ij}^{\min} \leq \bar{P}_{ij} \leq \bar{P}_{ij}^{\max} \quad (3.7)$$

Step2 Velocity updating

The velocities of each of the particles in the swarm of a modified PSO can be updated using α -factor and β -factor using the following equation:

$$\bar{V}_{ij}^{\text{new}} = w \times \bar{v}_{ij} + \alpha \times C_1 \times \text{rand}() \times (\bar{P}_{ij}^{\text{best}} - \bar{P}_{ij}) + \beta \times C_2 \times \text{rand}() \times (\bar{G}_i^{\text{best}} - \bar{P}_{ij}) \quad (3.8)$$

Such that $\alpha + \beta = 1$ providing dominant nature of exploration initially while at later stages, exploitation follows. Thus, α -factor provides local searching while β -factor provide global exploitation of the obtained results. It can be expressed as:

$$\alpha = \alpha^{\max} - (\alpha^{\max} - \alpha^{\min}) * IT/IT^{\max} \quad (3.9)$$

where

α^{\max} initial α -factor value, IT^{\max} maximum iteration number, IT iteration number, α^{\min} final value of α -factor, C_1 is a positive coefficient known as coefficient of the self-cognition component, C_2 is also a positive constant term, called as coefficient of the social component,

\bar{v}_{ij} represents the velocity of each particle in the swarm, $\text{rand}()$ and $\text{rand}()$ are the random numbers distributed uniformly in the interval [0,1], and w is the inertia weight and can be expressed by the following expression [35]:

$$w = w^{\max} - (w^{\max} - w^{\min}) * IT/IT^{\max} \quad (3.10)$$

where w^{\max} initial weight and w^{\min} final weight.

Step3 Position updating

Each particle's positions are updated between successive iterations using the updated velocity term as explained above according to the following equation:

$$\bar{P}_{ij}^{\text{new}} = \bar{P}_{ij} + \bar{V}_{ij}^{\text{new}} \quad (i = 1, 2, \dots, PR; j = 1, 2, \dots, G) \quad (3.11)$$

Step4 Memory updating

The local best and global best values are modified using the following equation:

$$\bar{P}_i^{\text{best}} = \bar{P}_i \quad \text{if} \quad F(\bar{P}_i) < F(\bar{P}_i^{\text{best}}) \quad (3.12)$$

$$\bar{G}^{\text{best}} = \bar{P}_i \quad \text{if} \quad F(\bar{P}_i) < F(\bar{G}^{\text{best}}) \quad (3.13)$$

where $F(\bar{P}_i)$ is the value of objective function at i th individual's position and $F(\bar{G}^{\text{best}})$ is the value of objective function of global best individual's position [36].

Step5 Termination criteria examination

The algorithm is repeated until pre-specified terminating rules are satisfied.

4.1 Introduction

The ELD problem involves generation scheduling with minimized cost and all other constraints satisfied. In general, power system tends to deal with such objectives to achieve economic operations [37]. Economic load dispatch problem with quadratic nature of cost function is incorporated with some of the equality and inequality constraints which are required to be handled in an organized manner [38]. Modified particle swarm optimization is an algorithm used to solve such load dispatch problems. But the main problem with such a technique is its unconstrained nature. A constraint handling method should be incorporated in the modified PSO technique for solving the power generation optimization application for solving ELD problem [39]. Modified PSO can be employed to the given problem by allocating optimally the power generation within its working limits and thus satisfying the total demand for the system.

4.2 Formulation of Objective Function

The formulation of ELD problem attempts to minimize cost with all the constraints satisfied. It can be done by formulating the given problem being implemented by modified PSO which may include an objective function containing a cost function component and error values [40]. The cost function corresponding to fossil fuel employed for each of the generator is expressed as a quadratic expression [41]. Thus, the total cost of fuel in terms of power output can be represented as:

$$F = \sum_{i=1}^G (a_i P_i^2 + b_i P_i + c_i + |d_i \sin \{e_i (P_i^{\min} - P_i)\}|) \quad (4.1)$$

4.2.1 Equality and inequality constraint handling:

Out of all the units, one of the generating units is selected randomly as a dependent operating unit in order to satisfy the heat balance constraint [42]. The power corresponding to this depended generating unit is calculated as follows:

$$P_1 = \{P_D - \sum_{i=1, i \neq dp}^G P_i \quad ; P_l = 0 \quad (4.2)$$

The dependent generating unit can be set at its respective limiting values in case it violates the respective constraint as indicated in the equation:

$$P_i = \begin{cases} P_i^{min} ; & P_i < P_i^{min} \\ P_i^{max} ; & P_i > P_i^{max} \\ 0 ; & P_i^{min} \leq P_i \leq P_i^{max} \end{cases} \quad (i = 1, 2 \dots G, i \neq dp)$$

(4.3)

If the solution lies outside the limits, corresponding error can be computed as follows:

$$E(P) = \begin{cases} (P_{dp} - P_{dp}^{max})^2 ; & P_{dp} > P_{dp}^{max} \\ (P_{dp}^{min} - P_{dp})^2 ; & P_{dp} < P_{dp}^{min} \end{cases}$$

(4.4)

Thus, for violation of constraint regarding the power of dependent unit, inequality constraint handling technique as explained in equation above is incorporated. In case it violates the limit then error term is calculated and a penalty term is incorporated to the objective function after restricting the value of the dependent generator within the limits [43].

4.2.2 Objective function:

In order to judge the modified particle swarm optimization technique effectiveness on ELD problem, the objective function is evaluated for each of the generating units [26]. It is evaluated by simulating the cost function and penalty and the main objective is to minimize this objective function with satisfactory handling of constraints which expressed in the following equation:

$$OF(P) = F(P) + r \times \text{error}$$

(4.5)

where error = E(P) and 'r' represent penalty parameter.

The procedure of mapping the technique to the given problem directly affects its feasibility and performance. Thus following steps are followed while implementing the modified PSO algorithm to the ELD problem [18]:

Step1 Parameter setting

The parameters for the modified PSO technique must be selected correctly since it is a parameter dependent approach. These parameters may include cognitive coefficients, swarm size, social coefficient, Inertial Weight, constriction factor, operational limits of the generating units and maximum number of iterations.

Step2 Generation of initial population

The population is initialized randomly within the restrictive maximum and minimum limits for each of the generating units such that whole of the population thus formed must be satisfying all the constraints [18].

Step3 Treatment of constraints

Constraint must be satisfied throughout the optimization process and it can be obtained by following the independent equality and inequality constraint handling techniques as explained before in equation (2.3) and (2.4).

Step4 Objective function evaluation

Objective function for each of the generating units is formulated by simulating cost function and penalty value and thus objective is to minimize this function as explained in equation (4.5).

Step5 Updating the best local values for each generating unit

The local best values can be updated by comparing the modified values with the previous local best ones [19]. In every iteration previous values and the updated values are compared for each of the units to obtain local best value. In a similar manner, the global best value can be updated by comparing the previously obtained global best to the minimum value

out of the local best values. At each iteration, all obtained values are compared and resulting value forms the global best solution.

Step6 Checking the updated population for bound violations

It is possible that constraints may get violated during the updating process. Thus, each of the generating unit is checked for the restrictive values and in case found violated, it made to follow the procedure as indicated in equation (4.2) to (4.4).

Step7 Stopping criterion

The above procedural steps are followed in the provided sequence until maximum number of iteration is reached where the optimization process terminates and obtained global best value forms the minimum cost and the local best values corresponding to each of the generating units represents the required solution.

5.1 Introduction

The proposed thesis work involves obtaining the solution of ELD problem using modified PSO. The suggested method has been tested on given test system. The considered test system consists of thermal units. Input data is provided in Appendix A.

5.2 Case Study

The following cases have been studied: case study 1 for thirteen generating units and case study 2 for fifteen generating units.

5.2.1 Case Study 1

In this test system, thirteen units for generation are considered in which PSO algorithm has been employed to solve the ELD problem. Power demand for test system 1 is $P_D=1800$ MW [43]. Comparison indicating overall system cost achieved is given in table 5.1.

5.2.2 Case Study 2

Thirteen units for generation are considered in this case study to solve the ELD problem. Power demand for test system 2 is $P_D = 2630$ MW [43]. Comparison indicating overall system cost achieved is given in table 5.2.

5.3 Simulation Results

Proposed approach has been employed to obtain the solution of ELD problem in this thesis work. For the purpose of obtaining optimal solution having a stable nature, number of trials is carried out for obtaining the value of parameters of PSO that include acceleration coefficients (C_1, C_2) selected to be 2 each, maximum and minimum value of inertia weight are $w^{\max}=0.9$ and $w^{\min}=0.4$, maximum and minimum value of α -factor are $\alpha^{\max}=1$ and $\alpha^{\min}=0.4$ respectively and maximum value of iteration (ITmax) is 800.

In order to determine the effectiveness of PSO over the problem of economic load dispatch, two case studies has been carried out and obtained results are compared with results

using other conventional techniques on the same standard test problem. Comparison indicating overall system cost achieved by PSO technique is given in table 5.2.

Table 5.1 Comparison of operating cost in case study 1

Control variables	CGA_MU[43]	IGA_MU[43]	Modified PSO
P₁(MW)	448.799	628.3151	179.4281
P₂(MW)	302.5353	148.1027	229.0516
P₃(MW)	299.1993	224.2713	157.3579
P₄(MW)	109.8666	109.8617	159.5676
P₅(MW)	60	109.8637	156.8495
P₆(MW)	109.8666	109.8643	109.3413
P₇(MW)	109.8666	109.855	159.5125
P₈(MW)	60	109.8662	157.7454
P₉(MW)	109.8666	60	155.4088
P₁₀(MW)	40	40	74.58614
P₁₁(MW)	40	40	80.97797
P₁₂(MW)	55	55	90.24188
P₁₃(MW)	55	55	89.93122
Cost(\$)	17975.3437	17963.9848	17985.69

Note: CGA_MU: Conventional GA with multiplier updating; IGA_MU: Improved GA with multiplier updating.

Table 5.2 Comparison of operating cost in case study 2

Control variables	PSOCFA[44]	PSOIWA[44]	PSOCFIWA[44]	Modified PSO
P₁(MW)	455	455	425.4942	285.428
P₂(MW)	455	455	444.613	304.3447
P₃(MW)	130	130	124.1373	130
P₄(MW)	130	130	126.9129	130
P₅(MW)	284.2286	277.2810	340.4991	378.72
P₆(MW)	460	460	413.1213	402.1822
P₇(MW)	465	465	441.4832	465
P₈(MW)	60	60	78.0288	185.8915
P₉(MW)	25.0695	25	36.8862	69.36272
P₁₀(MW)	25.0010	25.3315	38.5051	58.76132
P₁₁(MW)	35.7478	37.9364	34.4401	78.23976
P₁₂(MW)	49.9522	54.4432	28.9741	46.8193
sP₁₃(MW)	25.0002	25.0078	28.8596	49.52972
P₁₄(MW)	15	15	53.0448	23.93864
P₁₅(MW)	15.0006	15	15.0003	21.78197
Cost(\$)	32257	32257	32398	32571.06

Note: PSOCFA: PSO constriction factor approach; PSOIWA: PSO inertia weight factor approach; PSOFIWA: PSO constriction factor and inertia weight factor approach.

6.1 Conclusion

A heuristic algorithm i.e. modified PSO for optimizing the ELD problem has been proposed in this work. The proposed method results in exploitation of the optimal solution and thus searches efficiently. Obtained numerical results indicate effectiveness of the proposed algorithm for solving the ELD problem.

6.2 Scope for future work

The method proposed can be extended in many forms for improving the results in present case and are identified as follows:

- Modified particle swarm optimization technique can be hybridized using other stochastic approaches to improve the performance of proposed algorithm.
- The proposed techniques can be applied to solve other power system generation scheduling problems like reactive power, hydrothermal and combined heat and power problems.

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APPENDIX-A

Appendix –A.1 System data for case study 1

Unit	P_{\min} (MW)	P_{\max} (MW)	a_i (\$/MW ²)	b_i (\$/MW)	c_i (\$)	d_i (\$)	e_i (rad/MW)
1	0	680	0.00028	8.1	550	300	.035
2	0	360	0.00056	8.1	309	200	.042
3	0	360	0.00056	8.1	309	200	.042
4	60	180	0.00324	7.74	240	150	.063
5	60	180	0.00324	7.74	240	150	.063
6	60	180	0.00324	7.74	240	150	.063
7	60	180	0.00324	7.74	240	150	.063
8	60	180	0.00324	7.74	240	150	.063
9	60	180	0.00324	7.74	240	150	.063
10	40	120	0.00028	8.6	126	100	.084
11	40	120	0.00028	8.6	126	100	.084
12	55	120	0.00028	8.6	126	100	.084
13	55	120	0.00028	8.6	126	100	.084

Appendix –A.2 System data for case study 2

Unit	P_{\min} (MW)	P_{\max} (MW)	a_i (\$/MW ²)	b_i (\$/MW)	c_i (\$)
1	150	455	0.0003	10.1	671
2	150	455	0.0002	10.2	574
3	20	130	0.0011	8.8	374
4	20	130	0.0011	8.8	374
5	150	470	0.0002	10.4	461
6	135	460	0.0003	10.1	630
7	135	465	0.0004	9.8	548
8	60	300	0.0003	11.2	227
9	25	162	0.0008	11.2	173
10	25	160	0.0012	10.7	175
11	20	80	0.0036	10.2	186
12	20	80	0.0055	9.9	230
13	25	85	0.0004	13.1	225
14	15	55	0.0019	12.1	309
15	15	55	0.0044	12.4	323