A Thesis
On
Low Energy Properties of Baryons In Phenomenological Models

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Submitted by

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CERTIFICATE

This is to certify that the report entitled "**Low Energy Properties of Baryons in Phenomenological Models**" submitted by **Ms. Vani Gilhotra (301004018**) of M.Sc (Physics), Thapar University, Patiala was carried out by her under my Supervision. I certified that the matter embodied in this report is of candidate's own record and not submitted to any other university in any part or full form for the award of such degree.

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ABSTRACT

Low energy properties of Baryon octet are analyzed here by using various approaches. We take nucleons in addition to other baryon octets for determining fundamental properties. Here the baryon is considered to be made up of the sea and valence part where sea is assumed to have various fock states in addition to the three valence quarks. Here the sea comes with the different contents contributing to the spin 0,1,2 depending whether scalar, vector and tensor sea. Here we have a statistical approach to study the behavior of nucleons in contributing to these properties and hence the different approaches where the similar framework has been used is studied for the respective properties. We have used a statistical model in which a nucleon is taken as an ensemble of quark-gluon Fock states and where a probability to find a Fock state is decided by the principle of balance or principle of detailed balance. Within this framework, various models have been discussed for these properties and their domain of validity is check for calculating these parameter to find the effects of the sea for them.

Our aim in the thesis is to find out various approaches to calculate these fundamental properties of the baryon octets and to analyze the difference in various approached based on the dynamics of the sea contributions. We calculate the coefficient coming from this sea in terms of α & β. We see the effect of sea contribution on different models just by considering the sea or by suppressing it. Hence we get the different values of α & β for both the approaches which show the importance of these contributions coming from various fock states and hence put a contain on the values of alpha and beta. The results for these properties baryon showing the different values in different models. For betterment for our analysis we have also taken few other theoretical models and properties calculated in them and then we have shown a comparison between these models. Some shows higher relevancy to the experimental values while some shows less. Thus we tabulate the different results validating these models.

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Greek Alphabets

Alpha	Α	Α
Beta	Β	Β
Gamma	Γ	Γ
Delta	Δ	Δ
Epsilon	Ε	Ε
Zeta	Ξ	Ζ
Eta	Η	Η
Theta	Θ	Θ
Iota	Ί	Ι
Lamda	Λ	Λ
Mu	μ	Μ
Pi	Π	Π
Rho	Ρ	Ρ
Sigma	Σ	Σ
Tau	Τ	Τ
Phi	Φ	Φ
Chi	Χ	Χ
Psi	Φ	Ψ

Table I. Greek alphabets

Baryons ($J^p=1/2^+$)

BARYONS	**QUARK CONTENT**	**CHARGE**	**MASS(MeV)**
P	uud	+1	938.280
N	udd	0	939.573
Λ	uds	0	1115.6
Σ^+	uus	+1	1189.4
Σ^-	dds	-1	1197.3
Σ^0	uds	0	1192.5
Ξ^0	uus	0	1314.9
Ξ^-	dss	-1	1321.3
Λ_c^+	udc	+1	2281

Table.II Baryons octet

Baryons ($J^p=3/2^+$)

BARYONS	QUARK CONTENTS	CHARGE	MASS(Mev)
Δ^{++}	uuu	+2	1232
Δ^{+}	uud	+1	1232
Δ^{0}	udd	0	1232
Δ^{-}	ddd	-1	1232
Σ^{+}	uus	+1	1382.8
Σ^{0}	uds	0	1383.7
Σ^{-}	dds	-1	1387.2
Ξ^{0}	uss	0	1531.80
Ξ^{-}	dss	-1	1535
Ω^{-}	sss	-1	1672

Table. III Baryons decouplet

Leptons

LEPTONS	CHARGE	MASS
Electron (e^-)	-1	.511
Muon (μ^-)	-1	105.6
Tau (τ^-)	-1	1777
Electron neutrino(υ	0	<.000003
Muon neutrino(υ_μ)	0	<0.19
Tau neutrino(υ_τ)	0	<18.2

Table. IV Leptons

Mesons

PARTICLES	SYMBOL	QUARK CONTENTS	MASS(MeV)
Charged pion	Π^+	ud	139.6
Neutral pion	Π^0	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	135.0
Charged kaon	K^+	$u\bar{s}$	493.7

Neutral kaon	K^0	$d\bar{s}$	497.7
Eta	H	$\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$	547.8
Eta prime	η'	$\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{3}}$	957.6
Negative pion	Π^-	$\bar{u}d$	139.6
Negative kaon	K^-	$\bar{u}s$	493.7
Neutral kaon bar	$\bar{k}^0$	$\bar{d}s$	497.7

Table. V Meson Nonet

Natural Units	**Ћ=C=1**
Mass Mc^2/c^2	GeV
Length ħc/Mc^2	GeV^{-1}=0.1975 fm
Time ħc/Mc^3	Gev^{-1}=6.59*10^{-25} s

Introduction

Subatomic particles, also known as elementary particle, Subatomic particles include electrons, the negatively charged and they include the heavier building blocks of the small but very dense nucleus of the atom, the positively charged protons and the electrically neutral neutrons. For instance Protons and neutrons, are themselves made up of elementary particles called quarks, and the electron is only one member that includes the muon and the neutrino. More-unusual subatomic particles—such as the positron, that is the antimatter of electron have been detected in cosmic-ray interactions in the Earth's atmosphere. The field of elementary particles has dramatically extended with the construction of powerful particle accelerators to study high-energy collisions of electrons, protons, and other particles with matter.

As particles collide at high energy, the collision energy becomes available for the creation of subatomic particles such as mesons and hyperons. The study of subatomic particles has been transformed by the discovery that the actions of forces are due to the exchange of "force" particles such as photons and gluons. More than 200 subatomic particles have been detected most of them highly unstable. As a result of collisions produced in cosmic-ray reactions or particle-accelerator experiments. Theoretical and experimental research in particle physics, has given scientists a clearer understanding of the nature of matter and energy and of the origin of the universe. The current understanding of the state of particle physics is expended within a conceptual framework known as the Standard Model. The Standard Model provides a classification scheme for all the known subatomic particles based on theoretical descriptions of the basic forces of matter.

High Energy Physics

High-energy physics is the study of the smallest components of matter and how they interact. These fundamental particles are the stuff from which our entire world is made. Invisible to the naked eye so tiny that the most powerful electron microscope can't see them these particles, and only these, compose our homes, the cells of our bodies, and the warm rays of light on a summer's day. Some are not even particles of matter: they're bundles of pure energy. In studying them, high-energy physicists seek to learn what our world is made of, how it is put together, and how it works. Since molecules & atoms are basic elements of familiar substances that we can see & feel, we have to look within atoms in order to learn about "elementary" subatomic particles and to understand the nature of our universe. The science of this study is called particle physics, elementary particle physics or sometime high energy physics (HEP) [1], because it involves energies of hundreds of giga electron volts or more. The phenomena of antimatter and annihilation are very important to particle physicists, who often produce and collide particles and antiparticles in the laboratory, and use the resulting “mini-explosions” as a tool for concentrating a lot of energy in a very small region. Out of these bursts of energy, many new particles may be produced. In order to explore the possible existence of new phenomena, physicists try to produce them in higher and higher energy collisions, hence the name High Energy Physics. Elementary particles can detected and created during energetic collisions of other particles [2].

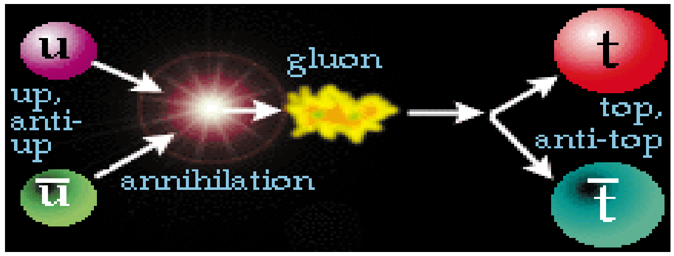


Fig 1.1 Collisions of particles

1.1) Motivation

The driving motivation behind particle physics experiments is the desire to uncover the true nature of fundamental forces and particles. The standard model is believed to be an effective theory, which has a deeper underlying theory reachable in the next generation of experiments. In the electroweak sector, where great successes of the past decades have predicted and verified the unification of the electromagnetic and weak nuclear forces, precision measurements at the CERN large electron positron collider (LEP) and the Fermilab proton-antiproton collider demand that there be either a light Higgs particle with a mass less than about 200 GeV and concern with flavor also, a decade of increasingly precise measurements of the properties of heavy quarks has shown remarkable agreement with the standard model predictions, and now moving a steps into an era of precise investigations of the neutrino sector. The demonstration by the Japanese neutrinos flavors oscillate but that their masses are likely much smaller than those of the other elementary particles suggests that there are critical phenomena in particle physics which cannot be explained by the standard model. This physics also have a significant role in cosmology. There are some hints that this new physics might be accessible to upcoming experiments. In the strong field, the theory of quantum chromodynamics (QCD) has been successfully used to predict the behavior of quarks and gluons at high energies observed at the HERA collider at DESY Hamburg and many more theoretically and experimentally challenging which requires further investigation.

1.2) Fundamental Particles and Interactions

The central nucleus contains protons and neutrons which in turn contains quarks. Quarks constitute all hadrons (baryons and mesons)--i.e., all particles that interact by means of the strong force. The objects which carry color force between quarks and hence glue them into hadrons are called gluons. Unlike photons, gluons can interact with each other. As a consequences the force increases with the distance, and quarks are confined inside hadrons. So that free individual quarks have not been observed.

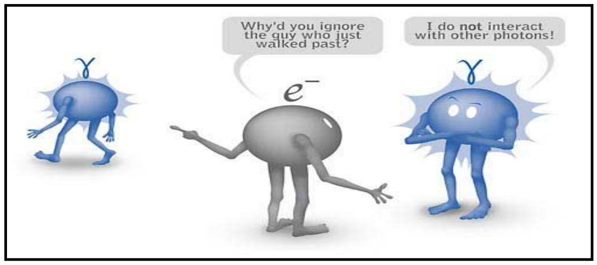


Fig 1.2: Interaction of Photons with Electron

The set of particles also includes their antiparticles. Initially quarks are “up”,”down”,”strange” upto 1963.In 1974 a new particle was unexpectedly discovered at SLAC(Stanford linear accelerator center).it was given the dual name J/Psi that shows the bound state of quark-antiquarks pair so the new fourth quark called “charm ” quark. This quark scheme was extended to its present state of six quarks by the addition of new pair. The six quark prediction was fulfilled when in 1977 a new heavy meson called the upsilon was discovered in fermilab and later shown to be a bound state of bottom and antibottom quark pair. so the six quarks are:-Up, Down, Strange, Charm, Bottom, Top and these are denoted by: ‘u’, ‘d’, ‘s’, ‘c’, ‘b’, ‘t’ respectively and also six leptons are Electron, Muon, Tau, Electron neutrino, Muon neutrino, Tau neutrino. Leptons are also a group of subatomic particles. Hadrons are made up of 3 quarks and are further subdivided according to the spin. Hadrons having half spin(1/2,3/2) are called fermions or baryons and having integral spin(1,2,3) are called bosons or mesons so these quarks combine to form heavier particles called Baryons, quarks and anti-quarks combine to form Mesons.

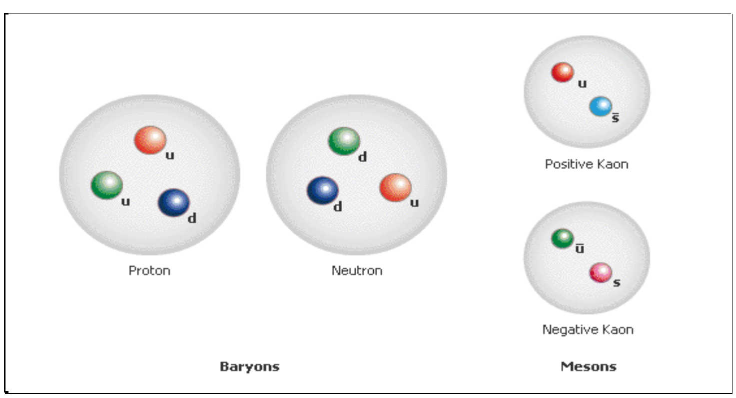


Fig 1.3: Baryons and Mesons

Quarks and leptons are building blocks of matter. The universe, which we know exists because the fundamental particles interact. These interactions include attractive and repulsive forces, decay, and annihilation. Four fundamental interactions between particles are Gravitational interaction, Electromagnetic interaction, Strong interaction, Weak interaction and all forces in the world can be attributed to these four interactions.

1.2.1) Gravitational forces

In subatomic world in which we are dealing with very small masses, gravity is weak as to be negligible. So this force is not included in the standard model of particle physics, that describes the role of fundamental particles and interactions between them. Graviton is the carrier particle for gravitational force.

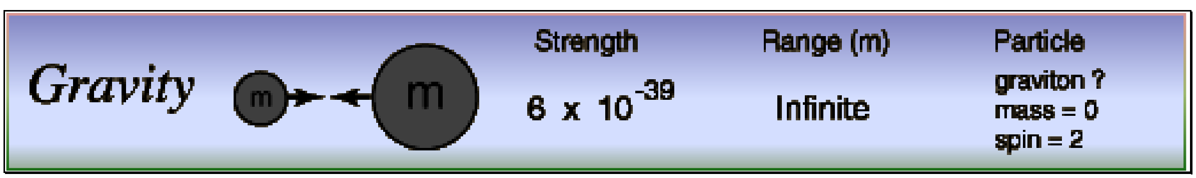


Fig 1.4: Properties of gravitational force

1.2.2) Electromagnetic force

Electromagnetism combines both the effect of electrical charge and magnetism, as different aspects of a single force.

Fig 1.5: Properties of electromagnetic force

Its strength is 100 times weaker than the strong interaction. photons (γ) are the exchange particles of this force. It has a infinite range and obey inverse square law. Electromagnetic interactions are also responsible for electric and magnetic field formation around electric charges and electric currents. All these phenomena are electromagnetic waves and differ only in wavelength. In the quantum field theory, any changing electromagnetic fields or electromagnetic waves can be described in terms of photons. When there are many photons involved, the effects are equally correct given by the earlier non-quantum theory, namely Maxwell's equations. Theory of electromagnetic interaction is quantum electrodynamics (QED).QED is the generalization of quantum mechanics to include the special relativity (where the particle travel almost closed to the velocity of light). It is a field theory of electromagnetic field force. Thus it is applied to all electromagnetic phenomenon associated with the fundamental charge particle.

1.2.3) Strong force

A force which can hold the nucleus together against the forces of repulsion of proton is strong indeed. The carrier particles of strong force are called gluons, because they glues the quarks together. This force acts between the quarks only. It acts over a very short distance. Coupling constant (α) $=g_s^2/4\pi<=1$.Gluon is a mass less like a photon, but it has a color charge. Gluons only interact with the objects which having a color charge, they can interact among themselves.

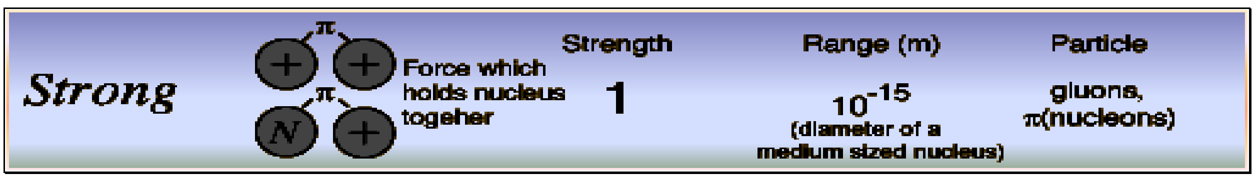


Fig 1.6: Properties of strong force

Beyond a hadron separation of about 1.7 femto meters (10^{-15} meters), the strong nuclear force becomes negligibly small. Quantum Chromodynamics (QCD) is the theory of strong interaction. It deals with quarks, gluons, & their interaction & is part of the standard model of particle physics [3]. Two peculiar properties of QCD: Asymptotic freedom and Confinement. Confinement means force between quarks does not diminish as they are separated because of this it would take an infinite amount of energy to separate two quarks however they bound into hadrons. Asymptotic means that in very high energy reactions, quarks and gluons interact very weakly means nearly free, when the exchange momentum is very big. Confinement is dominant in low-energy scales but as energy increases, asymptotic freedom becomes dominant.

1.2.4) Weak force

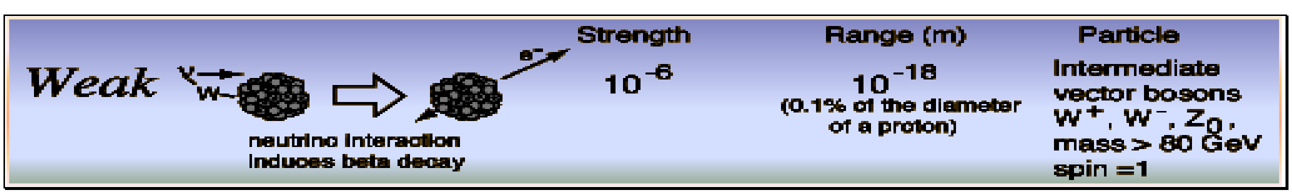


Fig 1.7: Properties of weak force

The weak interaction acts between both quarks and leptons. It is mediated by intermediate vector bosons w+, w- and z. The charge of weak interaction is called weak charge. Since the W's have a charge of +1 and –1 they cause a "charge-changing" interaction. That is when they are emitted or absorbed, to conserve charge, the "emitting" or "absorbing" particle changes charge by +1 or –1 unit. It's a short range force. Coupling strength (α) $=\varepsilon^2/4\pi = 1/137$. It accounts for certain type of radioactivity that are classed together as beta decay. The neutrino has no strong or electromagnetic interaction and feels only the weak force. That is why neutrinos are so elusive that they can pass through the entire earth without interacting.

1.3) Standard Model

Standard model is renormalisable field theory. It comes in 1978.It describes the universe in terms of *Matter* (fermions) and *Force* (bosons).The Fundamental particle interactions described by the Standard Model are the electromagnetic, weak and strong nuclear forces[4].The Standard Model describes role of fundamental particles and their interactions between them. S.M. using 17 fundamental particles, all of which are fermions or bosons: 6 quarks (fermions), 6 leptons (fermions), 4 force-carrying particles (bosons), which are physical manifestations of the forces through which particles interact. The graviton is not included. Attempts to include gravity in the Standard Model have failed. and the hypothetical Higgs boson. Unlike the force-carrying particles, the matter particles have associated antimatter particles, such as the antielectron called positron and antiquarks.

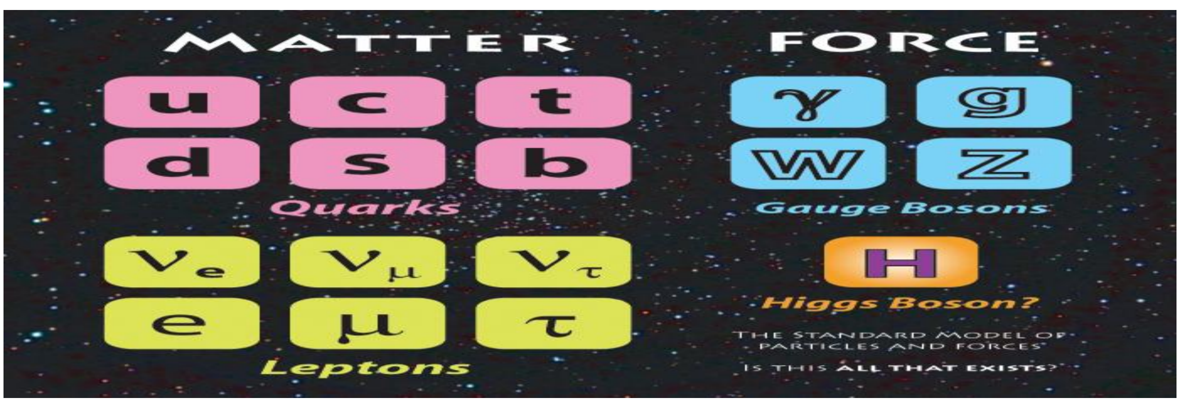


Fig 1.8: Quarks and Leptons

- **Limitations of standard model**

1. Does not attempt to explain gravitational interactions.

2. A fraction (1/4) of the matter in the Universe is cold, dark matter which is not SM particles.

3. There is no explanation given in the SM, why the weak interactions should be short range mediated by massive gauge bosons while the EM and strong forces are mediated by mass less gauge bosons.

4. The total number of free parameters is 19, which is too large.

5. Although, the model unifies the strong, weak and the electromagnetic interactions, the SM does not complete this unification because the strengths of these interactions are not related by the model.

6. There is no explanation given in the SM, why the weak interactions should be short range mediated by massive gauge bosons while the EM and strong forces are mediated by mass less gauge bosons.

- **Challenges to standard model**

1. There is some experimental evidence consistent with neutrinos having mass, which the Standard Model does not allow. To accommodate such findings, the Standard Model can be modified.

2. There is one elementary particle predicted by the Standard Model that has yet to be observed: the Higgs boson.

- **Beyond Standard Model**

Theories called the " Grand Unification theory" have been proposed to unify the electroweak force with the strong force. Unfortunately we still are not able to unite gravitation into unified model. If in future we have a quantum theory of gravitation, we may unite gravitation also with the other forces. The unification is accomplish under an $SU(3) \otimes SU(2) \otimes U(1)$.

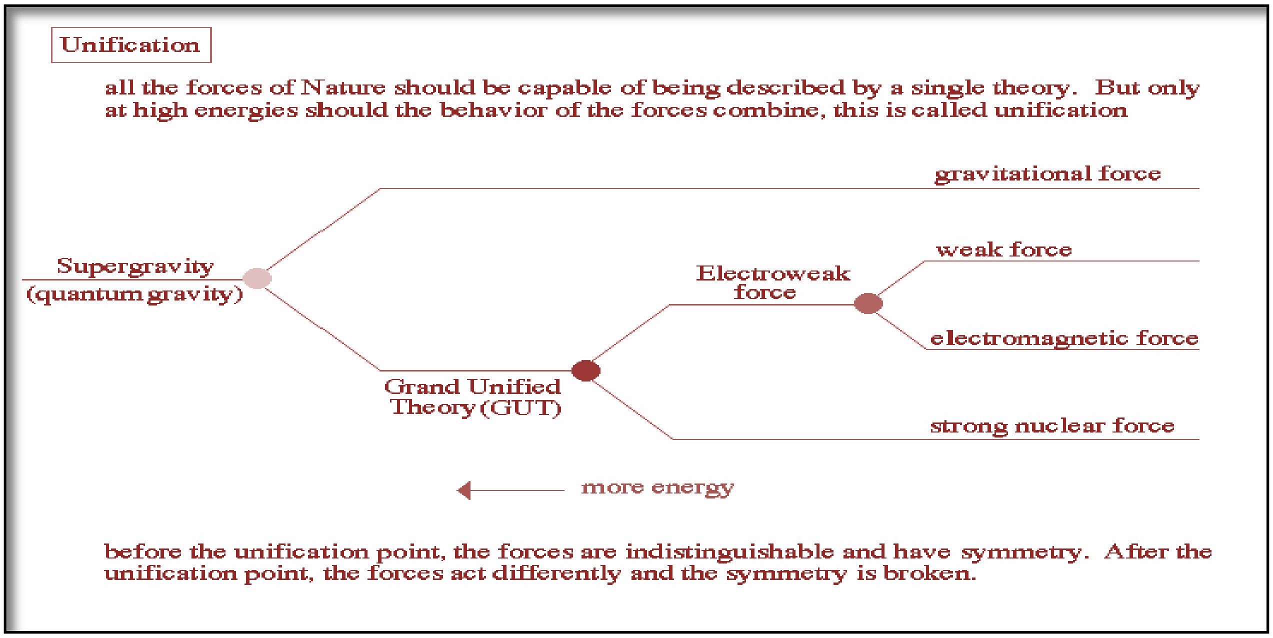


Fig 1.9: Beyond Standard Model

1.4) Groups and Symmetries

In physics, a symmetry of a physical system is a physical or mathematical feature of the system that is "preserved" under some change. The standard model of physics is based on a combination of three internal symmetries, U(1) x SU(2) x SU(3). SU(2) group determines the isospin invariance and SU(3) is a eight fold way theory of Gell-Mann and Ne'eman [5]. An understanding of these symmetries requires that we know something about the mathematical behavior of symmetrical groups. A group is a set of elements having closure, associative, identity and inverse properties. The unitary group U(n), is the set of n x n unitary matrices :$UU^\dagger = U^\dagger U=1$.The group of n x n unitary matrices with a unit determinant is called the special unitary group, SU(n).

SU(2) Symmetry

The Special Unitary group of dimension 2, SU(2) is group of matrices. There are three isospin- 1/2 matrices so the three Pauli spin matrices generate the group SU(2) [6]. The group structure of isospin symmetry is very similar to that of usual spin. The elements(matrices) of SU(2) group have their determinant= +1. The generators of this must be hermitian and traceless matrices of order n and maximum number of traceless hermitian matrices of order n are n^2- 1. The isospin generator T_i satisfy the lie algebra of SU(2) as:

$$[T_i, Tj]=i\varepsilon_{i,\,j,\,k}T_k$$

Where indices ranges from 1 to 3. The proton & neutron from an isospin double (p, n) means that

$$T_3|p\rangle = 1/2|p\rangle,\ T_3|n\rangle = -1/2|n\rangle$$

Strong interactions Hamiltonian H_s has the property:

$$[T_i, H_s]=0 \qquad i=1, 2, 3$$

SU(3) Symmetry

A symmetry provides the kind of grouping of hadrons that is observed in nature. The mathematical symmetry is SU(3), which stands for "special unitary group in 3-D". Gell-Mann and Ne'eman feels that the basic subgroups of SU(3) contain either 8 or 10 members and the observed hadrons can be grouped together in 8s or 10s in the same way generators of SU(3) in terms of eight gell-mann matrices λ^a

$$T^a = 1/2\lambda^a$$

The generator T^a of SU(3) satisfies the commutation relation:

$$[T^a, T^b]=T^aT^b-T^bT^a=i\Sigma^8{}_{c=1}f^{abc}T^c$$

Where f^{abc} is the structure constant of SU(3) are real numbers. The Eightfold Way classification is named after the following fact. If we take three flavors of quarks, then the quarks lie in the fundamental representation, 3 (called the triplet) of flavor SU (3). The antiquarks lie in the complex conjugate representation 3. The nine states (nonet) made out of a pair can be decomposed into 1 (called the singlet), and the 8 (called the octet). The notation for this decomposition is

$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}.$$

The antisymmetric wave function is obtained by making it fully antisymmetric in color and symmetric in flavor, spin and space put together. With three flavors, the decomposition in flavor is

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_M \oplus \mathbf{8}_M \oplus \mathbf{1}_A$$

SU(3) flavor symmetry is not an exact symmetry because masses of u,d & s quark are not same & so the quark flavor are distinguishable. SU(3) color group is an exact symmetry of standard model, which accounts for the strong interaction of quarks & gluons.

1.5) Experimental Setups

There are many experimental setups in India and abroad also, related with particle physics which are helpful in producing the remarkable and unpredictable results. some are below:

a) ATLAS
b) HERA

c) LHCb
d) EMC

ATLAS: The ATLAS experiment is one of the two general-purpose detectors at the LHC at CERN, the European Laboratory for Particle Physics. The LHC is designed to accelerate intense beams consisting of thousands of bunches, each containing up to 10^{11} protons, to an energy of 7 TeV about 7 times more energy than the present world record. The protons will collide in the heart of the detectors, allowing their constituent partons to annihilate and liberate up to 14 TeV per collision for the creation of new particles. Bunches will collide 40 million times every second, giving a luminosity of 10^{34} $cm^{-2}s^{-1}$. This unprecedented combination of centre-of-mass energy and luminosity will allow the LHC to produce previously undiscovered particles with masses of a TeV (such as the Higgs) and more in sufficient numbers to ensure their discovery, probing the phase space crucial to understanding how electroweak symmetry is broken and mass is generated. This, in turn, is expected to contribute to an understanding of the connection of gravity to the three forces already described by the standard model. Among other outstanding issues, which the LHC is designed to address, are the nature of dark matter and the preponderance of matter over anti-matter in the composition of our universe.

HERA: We are continuing to use electron scattering to learn about quarks and the particles they form. One of the laboratories at the forefront of this research is DESY (Deutsches Elektronen-Synchrotron) in Hamburg, where electrons and protons collide in HERA (Hadron Electron Ring Accelerator). In HERA, electrons are accelerated in a ring in one direction, the protons in another ring, in the opposite direction. The two accelerator rings occupy the same tunnel which forms a circle 6.3km in circumference. The rings intersect at four points around the tunnel, where the electrons and protons collide head on. There are two main experiments for studying deep inelastic scattering, called H1 and ZEUS

Fig 1.10: The pictorial view of HERA- the proton ring with electron ring below.

LHCb: The Large Hadron Collider beauty (LHCb) experiment at CERN observed two new excited states of the Λ_b beauty baryon. The Standard Model of particle physics predicts the existence of these new states and the first time they have been confirmed in an experiment. Baryons are subatomic particles whose mass is equal to or greater than that of a proton. Like protons and neutrons, the Λ_b beauty baryon is composed of three quarks. In Λ_b these are up, down and beauty quarks.

LHCb physicists found the signals for the Λ_b particlesin a sample of about 60 trillion proton—proton collisions which were delivered by the LHC operating at a centre-of-mass energy of 7 TeV in 2011. They measured the masses of the new excited states as 5912 MeV/c2 and 5920 MeV/c2 respectively. which are five times greater than the mass of a proton or neutron.

EMC: The European Muon Collaboration conducted experiments at CERN related with high energy physics. The EMC experiment took a high energy muon beam produced by colliding high energy protons of a solid target at CERN accelerator and selected muon energies so that the spin of the muon pointed along its momentum [7]. So in 1983, it discovered that nucleons inside a nucleus have a different distribution of momentum among their component quarks. This is the original so-called "EMC Effect". The muon beam is directed on to a polarized ammonia target in which the protons in the ammonia have their own spins aligned either parallel or anti-parallel to the muon beam direction. After one year of collecting data, the EMC experimentalists analyzed a few million events and extracted the proton spin structure function. It was thought that when the muons interacted and transferred a large energy to the proton, the proton would break up and the fraction of spin carried by the broken pieces would be less than by the entire proton. So the spin dependence would get smaller at higher energies. This surprising effect was that the EMC experiment found in 1987 that only a small part of the proton spin is carried by quarks, and that the strange quark sea is probably polarized are sometimes called as proton spin crisis.

1.6) Deep Inelastic Scattering

The clarity for the presence of quarks inside the proton is given by deep inelastic scattering. The proton is composed of point like, spin-1/2 particles - quarks; this is the quark hypothesis. At sufficiently high energies it should be possible to model inelastic electron scattering off a proton as the sum of several elastic scatterings with free quarks. Electron-proton scattering at these energies is called deep inelastic scattering as seen in figure.

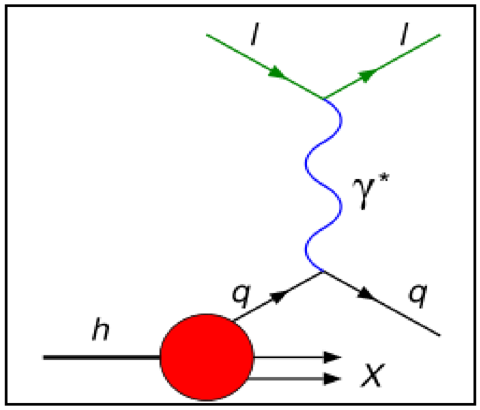


Fig 1.11: Deep Inelastic scattering

At high energies, proton size is larger as compared to the wavelength associated with the electron. So the electron can probes deep within the proton. High energies tends to alter the proton so that the new particles can be produced. Deep inelastic scattering can be seen in two different ways: as inelastic scattering off a proton because it has constituents inside, or as elastic scattering from one of the constituents inside, avoiding the whole proton. The measurements of polarized structure functions of proton in the deep inelastic scattering (DIS) experiments [8,9,10] showing that the valence quarks of the proton carry only about 30% of its spin, many interesting facts have been seen regarding the polarized structure functions of the nucleon as well as the quark distribution functions in these experiments. DIS experiments have given good deal of information about the quark distribution functions.

References

[1] Nari Mistry, A Brief Introduction To Particle Physics, Laboratory for elementary particle physics, Cornell University.

[2] Edward Eric Johnson, Measurements of charm meson production in 10.5 GeV electron-positron annihilation.

[3] R.Machleidt & D.R Entem, chiral symmetry & Nucleon-Nucleon interaction (2011).

[4] John Ellis, Standard model of particle physics (2006).

[5] Ta-pei chung and Ling fong Li, Guage Theory of elementary particles 90 (1984).

[6] Javier M.G. Duarte, MA 02142 (2009).

[7] Rene Donaldson & Bill Kirk, Vol.27 (1997).

[8]J.Ashman et al, EMC Collaboration phys. Lett.206B, 364 (1998); nucl.phys.B328,(1989).

[9]B.Adeva.et al, SMC Collaboration, phy.Lett.302B, 533 (1993); P.Adams et al, phys.Rev.D56, 5330 (1997).

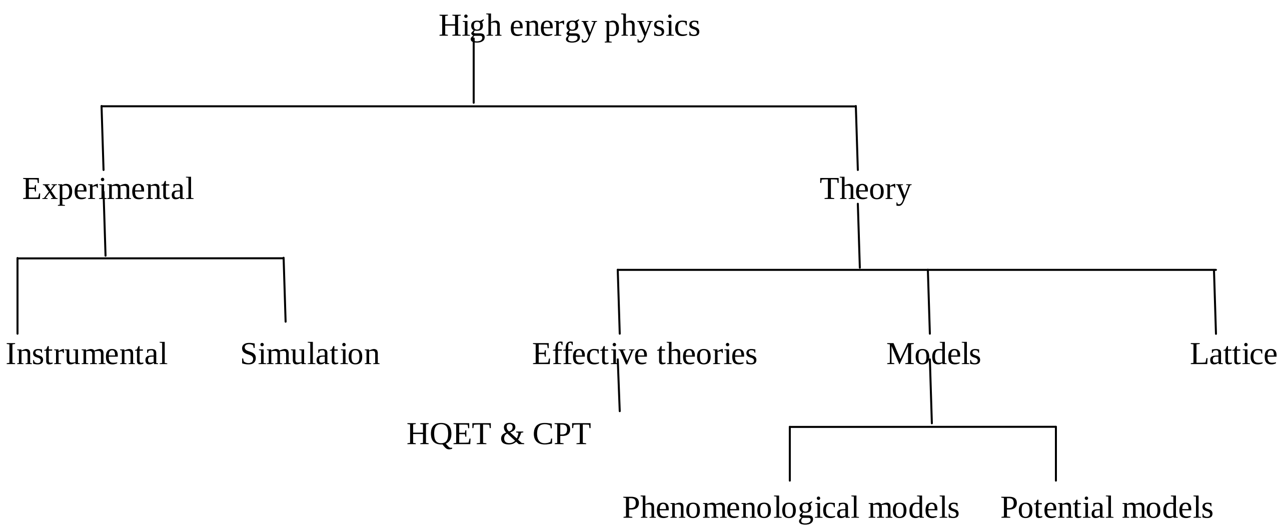
[10] P.L. Anthony et al. [E142 Collaboration], Phys. Rev. Lett.71,959 (1993); K.Abe et al., Phys.Rev.Lett.75,391(1995).

Chapter 2

Various Models

2.1) Theoretical and Experimental aspects of studying particle physics

Particle physics phenomenology is the part of theoretical particle physics that deals with the application of theory to high-energy particle physics experiments. Phenomenology means the experimental consequences of models: how their new particles could be searched for, how the model parameters could be measured, and how the model could be distinguished from other, competing model.



Here we are discussing only about phenomenological models some are: Simple Quark Model, Chiral Quark Model, Statistical Model and Non-Relativistic model. There are various hadronic models in high energy physics whose remarkable contribution in different parametres results produces a good results. To study the low energy properties of nucleons use models. We are discussing here only four different models.

2.2) Phenomenological Models

- Simple Quark Model
- Chiral Quark Model

- Statistical Model
- Non-Relativistic Quark Model

The mainly Simple Quark Model and Statistical Model uses the same wavefunction.

$$\langle \phi_{1/2}^{(\uparrow)} | \hat{o} | \Phi_{1/2}^{(\uparrow)} \rangle = \frac{1}{N^2} \left[a\sum_i \begin{bmatrix} \langle o_f^i \rangle^{\lambda\lambda} \langle \sigma_z^i \rangle^{\lambda\uparrow\lambda\uparrow} + \langle o_f^i \rangle^{\rho\rho} \langle \sigma_z^i \rangle^{\rho\uparrow\rho\uparrow} + 2\langle o_f^i \rangle^{\lambda\rho} \langle \sigma_z^i \rangle^{\lambda\uparrow\rho\uparrow} \\ + b\sum_i \left(\langle o_f^i \rangle^{\lambda\lambda} + \langle o_f^i \rangle^{\rho\rho} \right) \left(< \sigma_z^i >^{\lambda\uparrow\lambda\uparrow} + < \sigma_z^i >^{\rho\uparrow\rho\uparrow} \right) + \end{bmatrix} \right.$$

$$c\sum_i [< o_f^i >^{\lambda\lambda} < \sigma_z^i >^{\rho\uparrow\rho\uparrow} + < o_f^i >^{\rho\rho} < \sigma_z^i >^{\lambda\uparrow\lambda\uparrow} - 2 < o_f^i >^{\lambda\rho} < \sigma_z^i >^{\lambda\uparrow\rho\uparrow}]$$
$$+ d[\sum_i < o_f^i >^{\lambda\lambda} + \sum_i < o_f^i >^{\rho\rho}] + e[\sum_i (< o_f^i >^{\rho\rho} - < o_f^i >^{\lambda\lambda}) < \sigma_z^i >^{\lambda\uparrow 3/2\uparrow}$$
$$\left. + 2\sum_i < o_f^i >^{\lambda\rho} < \sigma_z^i >^{\rho\uparrow 3/2\uparrow}] \right]$$

When we apply this operator $\hat{o}$, to get the eigenvalues in terms of certain properties that can explain phenomenon at low energies. These properties are based on SU(3) flavor and SU(2) spin space. For the same purpose, to get the whole idea about these properties we have calculated some of the quantum numbers like charge, spin and isospin, charge square using their respective operators. Some of the solved examples are shown below in appendix.

2.2.1) Simple Quark Model

Quark model is a classification scheme for hadrons in terms of their valence quarks-quarks and antiquarks which give rise to the quantum number of hadrons. These quantum number are of two kinds-one set comes Poincare symmetry- $\mathbf{J^{pc}}$ where J is a total angular momentum, p is symmetry and c is symmetry respectively. Other set comes flavor quantum number such as isospin, strangeness, charm and so on. Quark model is follow up to eight-fold way classification scheme.

Quark model asserts that:

1. Every baryon is composed of three quarks (antibaryons composed of antiquarks) [1].
2. Every meson composed of quark and antiquarks.

Quark	Charge	Baryon number	Strangeness
Up	+ (2/3)e	1/3	0
Down	-(1/3)e	1/3	0

Strange	-(1/3)e	1/3	-1
Charm	+(2/3)e	1/3	0
Bottom	-(1/3)e	1/3	0
Top	+(2/3)e	1/3	0

Table. VI Properties of Quarks

Eight fold way:

In the eight-fold symmetry the mesons and baryons were arranged into various geometrical patterns on the basis of their charge and strangeness.

- The baryons were fit into a hexagonal array with two particles at the centre. This is called BARYON OCTET.

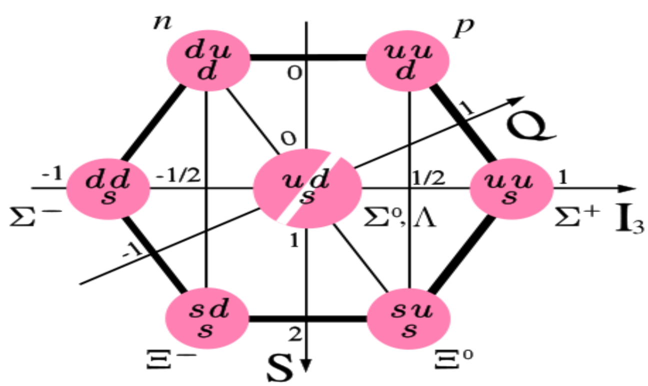


Fig 2.1: Baryon octet

- The eight mesons fill the similar hexagonal pattern called MESON OCTET.

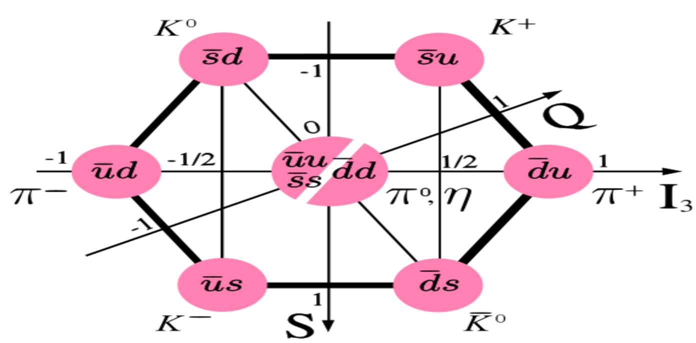


Fig 2.2: Meson octet

Quark model requires mesons with Q=0 & S=0,such a particle had already been found experimentally-the ή.the 8-fold way has been classified the ή as a singlet.according to quark model it properly belongs with the eight mesons to form meson nonet.

In 1961, after the discovery of ‘hypercharge’, the strangeness was replaced by hypercharge , which is equal to S for mesons and S+1 for baryons.

For heavier baryons, a triangular array incorporating 10 baryons was introduced called the BARYON DECOUPLET.

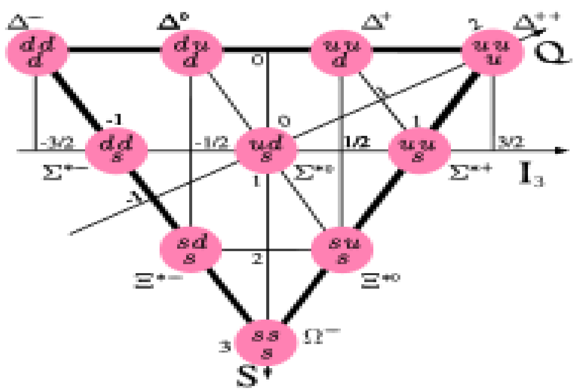


Fig 2.3: Baryon decouplet

- **Limitations of Quark Model**

No combination of the three quarks (u, d, s) can form a baryon with S=0 Q=-2.And a meson with S= -3 Q=+2. These particles were called exotic particles. But were not found even after long experimental searches.

- Free quarks doesn’t exists. i.e. quarks are confined in hadrons.
- Pauli principle violated in (D^{++}= (uuu)).

In 1964, this dilemma was ruled out by Greenberg by formulating that quarks have different color .i.e. $\Delta^{++} = u_R\, u_G\, u_B$.

- How many different types of quarks exist? (6?)

These additions to quark model gives that total number of quarks becomes six with their corresponding anti-quarks.

- **More Additions in Quark Model**

Quark	**Date**	**Mass [GeV/c^2]**
Up, Down	1968 1964	~ 0.005 ~0.010
Strange	1947	~0.200
Charm	1974	~1.500
Bottom	1977	~4.500
Top	1995	~175.0

Table. VII Origin of quarks

2.2.2) Chiral Quark Model

These models are designed to give us insight into some aspects of hadron structure & spectroscopy. A Chiral quark model should be investigated that relies essentially on those degrees of freedom that are brought about by the spontaneous breaking of chiral symmetry, namely constituent quarks and Goldstone bosons. Goldstone bosons are bosons that appear necessarily in models exhibiting spontaneous breakdown of continuous symmetries. These spin-less bosons correspond to the spontaneously broken internal symmetry generators, and are characterized by the quantum numbers of these. Goldstone bosons are infraparticles. An infraparticle is an electrically charged particle and its surrounding cloud of soft photons. That is, it is a dressed particle rather than a bare particle. The χQM as formulated by Manohar and Georgi find a good deal of attention as it not only provides a valuable description of depolarization of valence quarks through the emission of a Goldstone boson [2] which causes a modification in the flavor content along it also able to account for the $\bar{u} - \bar{d}$ asymmetry existence of significant strange quark content $\bar{s}$ in the nucleon , various quark flavor contributions to the proton spin, baryon magnetic moments etc. They are done by including the parameters of the χQM without configuration mixing are fixed by incorporating the latest data pertaining to $\bar{u}$ - $\bar{d}$ asymmetry and the spin polarization functions, [3] in the case of octet magnetic moments the results not only show improvement over the nonrelativistic quark model results but also give a nonzero value for the right hand side of the Coleman Glashow sum rule, usually zero in most of the models. So in the case of the octet, the predictions of the χQM with the generalized Cheng-Li mechanism show remarkable improvements in general when the effects of configuration mixing and ‘‘mass adjustments’’ due to confinement are included. It has

been shown that invoking configuration mixing, having its origin in spin-spin forces, in the χQM (referred to as χQMgcm) with SU(3) and U(1) symmetry breakings improves the predictions of χQM regarding the spin and quark distribution functions [4].
The basic process in the χQM is the emission of a GB by a constituent quark which further splits into a $q\bar{q}$ pair, for example,

$$q_{\pm} \rightarrow GB^{0} + q_{\mp}{}' \rightarrow (q\overline{q'}) + q'$$

where $(q\bar{q}')+q'$ constitute the ''quark sea'' The effective Lagrangian describing interaction between quarks and a nonet of GBs, consisting of the octet and a singlet, can be expressed as

$$L = g_8 \bar{q} \phi_q ,$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} , \qquad \phi = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} + \xi\frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} + \xi\frac{\eta'}{\sqrt{3}} & \alpha K^0 \\ \alpha K^- & \alpha \overline{K^0} & -\beta\frac{2\eta}{\sqrt{6}} + \xi\frac{\eta'}{\sqrt{3}} \end{bmatrix}$$

where $\xi = g_1/g_8$ and g_1 and g_8 are the coupling constants for the singlet and octet GBs, respectively. SU(3) symmetry breaking is introduced by considering $M_s > M_{u,d}$ as well as by considering the masses of GBs to be non-degenerate ($M_{K,\eta} > M_\pi$) , whereas the axial U(1) breaking is introduced by $M_{\eta'} > M_{K,\eta}$ .The parameter $a(=|g_8|^2)$ denotes the transition probability of chiral fluctuation of the splittings $u(d) \rightarrow d(u)+\pi^{+(-)}$ whereas $\alpha^2 a$, $\beta^2 a$, and $\xi^2 a$, respectively, denote the probabilities of transitions of $u(d) \rightarrow s+K^{-(0)}$, $u(d,s) \rightarrow u(d,s)+\eta$, and $u(d,s) \rightarrow u(d,s)+\eta'$.

2.2.3) Statistical Model

Hadrons are composite particles its composition is one of the central issues of hadronic physics and can be handled in two terms, i.e., in terms of quark-gluon degrees of freedom or meson-baryon degrees of freedom.[5]The proton is the simple in which the three colors of QCD neutralize into a colorless bound state, still its difficult how to describe the proton in terms of its fundamental quark and gluon degrees of freedom from basic principles. The structure of the proton is complicated due to the non-perturbative and relativistic nature of the quark and gluon in the protons and also for the sea quarks contribution in it.[6].

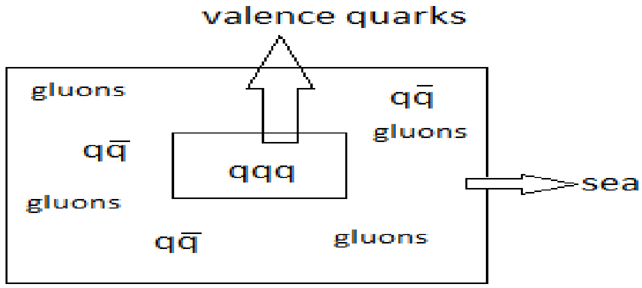


Fig 2.4: Valence and Sea contribution

In the statistical approach, the nucleon is treated as a collection of massless quarks, antiquarks and gluons in thermal equilibrium within a finite size volume. A statistical model consider a nucleon as an ensemble of quark-gluon Fock states. This model, uses the principle of balance that every Fock state should be balanced with all of the nearby Fock states or using the principle of detailed balance that any two nearby Fock states should be balanced with each other. It has been seen that the model gives an excellent details of the light flavor sea asymmetry (i.e., $u \neq d$) without any parameter. In this article, this model can be used to calculate the light quark spin content of nucleons, the ratio of their magnetic moments, the semi-leptonic decay constant of neutron, and the ratio of SU(3) reduced matrix elements for the axial current.

2.2.4) Non-Relativistic Quark Model

A very simple non-relativistic potential model pioneer by Zweig and Gell-mann and further developed by Dalitz and Greenberg and many others to make it successful. Inspite of simplicity, this model is successfully analysis the baryon spectrum. NRQM are based on the assumption that baryons are composed of three massive constituents quarks bound in a confining potential[7]. One of the success of this model is that it gives a simple description of anomalous magnetic moment of proton and neutron together with those of the hyperons. It can also explain most of the observed baryon spectrum. The basic hypothesis of NRQM is the existence of constituents quarks as quasi particle degree of freedom inside hadron. CQ share with the fundamental QCD quarks their conserved charges and quantum numbers: electric charge, baryon number, color and flavor. A constituents quark may be viewed as an object in which a “bare” valence quark is dressed by clouds of quarks-antiquarks pairs and gluons.

2.3) Low Energy Properties in phenomenological models

- Spin distribution
- Magnetic moment
- G_A / G_v
- F/D

Spin Distribution:

Its a essential parameter for classification of particles or intrinsic angular momentum. In fact, the spin of a planet is the sum of the spins and the orbital angular momenta of all its elementary particles. So are the spins of other composite objects such as atoms and protons (which are made of quarks). Half-integer spin fermions are constrained by the Pauli Exclusion Principle whereas particles having integer spin bosons are not. The spin classification of particles determines the nature of the energy distribution in a collection of the particles. Particles of integer spin obey Bose-Einstein statistics, whereas those of half-integer spin behave according to Fermi-Dirac statistics. spin is basically something about symmetry:-

- The multiparticle wave function is antisymmetric under exchange of identical fermions because fermions have half spin particles.
- The multiparticle wave function is symmetric under exchange of identical bosons because bosons have integral spin particles.

The spin "crisis" in high-energy physics began in 1988. As it has been learned since from accelerator-based experiments with polarized protons at lower energies, quark spin only accounts for one quarter of the proton's spin. The remaining three-quarters of a proton's spin may be carried by particles called gluons, which bind quarks inside the proton, or by something called orbital angular momentum, or by both.

In this below diagram representing the head-on collision of a quark (red ball) from one proton (orange ball) with a gluon (green ball) from another proton with opposite spin, spin is represented by the blue arrows circling the protons and the quark.

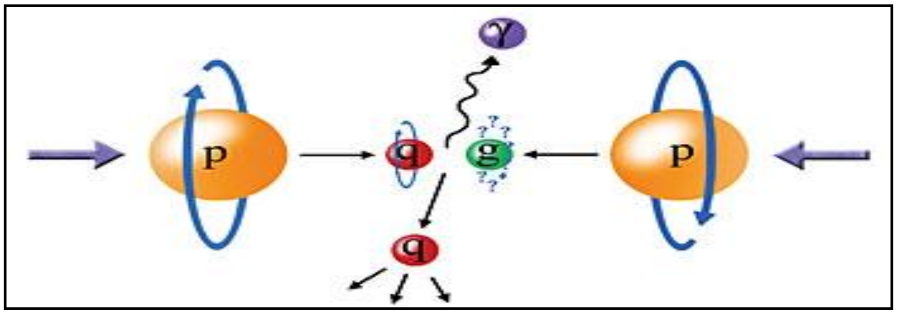


Fig 2.5: Collision of quark and gluon

Magnetic Moment

The electromagnetic properties like magnetic moment gives the valuable information regarding the internal structure of baryons. Magnetic moment of baryon is simply a sum of moments of three constituents quarks: $\mu = \mu_1 + \mu_2 + \mu_3$

It depends on quark flavor and spin configuration because three quarks carry different magnetic moments and spin that determines the relative orientations of three dipoles. The magnetic dipole moment of spin ½ particles of charge q and mass m is:

$$\mu = \frac{q}{mc} s$$

$$\mu_{iz}\left|\uparrow\right\rangle = \mu_i S_z \left|\uparrow\right\rangle = \frac{q}{mc} S_z \left|\uparrow\right\rangle = \frac{q}{mc}\frac{\hbar}{2}\left|\uparrow\right\rangle$$

As for mesons have no spin so zero magnetic moment. The magnetic moment of a given baryon receives contributions from the valence quarks, "quark sea" and orbital angular momentum of the "quark sea" following Cheng and Li [7,8] and is expressed as

$$\mu(B)_{total} = \mu(\mathrm{B})_{val} + \mu(\mathrm{B})_{Sea} + \mu(\mathrm{B})_{orbit,},$$

where $\mu(B)_{Val}$ and $\mu(B)_{sea}$ represent the contributions of the valence quarks and the "quark sea" to the magnetic moments due to spin polarizations. The term $\mu(B)_{orbit}$ corresponds to the orbital angular momentum contribution of the "quark sea". In terms of quark magnetic moments and spin polarizations, the valence, sea and orbital contributions can be defined as

$$\mu(\mathrm{B})_{val} = \sum_{q=u,d,s} \Delta q_{val}\mu_q \qquad \mu(\mathrm{B})_{sea} = \sum_{q=u,d,s} \Delta q_{sea}\mu_q$$

$$\mu(\mathrm{B})_{orbit} = \sum_{q=u,d,s,c} q_{val}\mu(q_+ \rightarrow q_-)$$

where μ_q = $e_q/2M_q$,(q = u, d, s, c) is the quark magnetic moment, $\mu = q_+ \rightarrow q_-$ is the orbital moment for any chiral fluctuation, eq and Mq are the electric charge and the mass respectively for the quark q.

G_A/G_V :

According to standard model the charge weak current is left handed means it is an equal admixture of axial vector(A) and polar vector(v) currents of quarks and leptons. Vector current give rise to Fermi β-transitions with couplings constant G_v and having a spin parity selection rule is $\Delta I = 0$. Axial current give gammow-teller β-transitions with coupling constant G_A and having selection rule is $\Delta I=0, \pm 1$

$$\lambda = G_A/G_V$$

Determination of λ and thus G_V and G_A can each be determined in both sign and magnitude from neutron decay alone. F and D are the weak matrix elements in beta decay

REFERENCES

[1] David Griffiths , Introduction to elementary particles, 38-39 (1942).

[2] H.Dahiya and M.Gupta ,Chiral quark model and nuclear spin (2008).

[3] H.Dahiya and M.Gupta, phys.rev, D 67 ,114015 (2003).

[4] H.Dahiya and M.Gupta ,phys.rev ,D66 (2003) .

[5] Lijing.Shao ,Yong-jun.Zhang , Bo-Qiang.Ma , Phys. Lett, B 686 , 136(2010).

[6] Bin.Zhang and Yong-Jun.Zhang (2010).

[7] A.W. Thomas, The Structure Of Nucleon (2001).

Fundamental Properties of Baryon Octet In various Approaches

3.1) Framework for Various Approaches

The composition of nucleons, in terms of fundamental quarks and gluons degrees of freedom have been modeled variously to account for its observed properties.[1] It is important to calculate as many nucleonic parameters as possible in these models to check their merits and their domains of validity. Proton is the simple system which have the three colors of QCD which neutralize into a colorless bound state. In many models we consider only valence quarks of proton which contributes to spin and mass of nucleon. The structure of the proton is rather complicated due to the non-perturbative and relativistic nature of the quark and gluon in the protons. The proton is also complicated due to the presence of sea quarks. Since proton and neutron are fermions, they should obey Fermi statistic and hence both valence and sea should combine to give the proton an antisymmetric wave function.

Many possible attempts have been made to describe the origin of the nucleon sea and its antiquark asymmetry. Sea part can be generated by gluon splitting into $\bar{u}u$ and $\bar{d}d$ pairs . In order to understand the sea flavor asymmetry of the proton is from a "statistical balance model" The sea quark-antiquark $\bar{u}u$ and $\bar{d}d$ pairs can be produced by gluon splitting with equal probabilities , the processes of the annihilation of the antiquarks with their quark into gluons are not flavor symmetric because of net excess of u quarks over d quarks . Thus the $\bar{u}$ quarks have a larger probability to annihilate with the u quarks than that of the $\bar{d}$ quarks, and this brings an excess of $\bar{d}$ over $\bar{u}$ inside the proton. In a statistical model, a nucleon is taken as an ensemble of quark-gluon Fock states [2][3]. This model, using the principle of detailed balance that every Fock state should be balanced with all of the nearby Fock states, or using the principle of detailed balance that any two nearby Fock states should be balanced with each other[3], the probability of finding every Fock state of the proton accounting upto ≈ 90% of the total Fock state has been obtained. We construct such Fock states of a nucleon which have definite color and spin quantum numbers. Now , Take the proton as an ensemble of a complete set of quark-gluon Fock states, and these different Fock states can be written as:

$$|\psi 1\rangle = |uud\rangle = |\{uud\}, \{0, 0, 0\}\rangle \tag{1}$$
$$|\psi 2\rangle = |uudg\rangle = |\{uud\}, \{0, 0, 1\}\rangle \tag{2}$$
$$|\psi 3\rangle = |uud\bar{u}u\rangle = |\{uud\}, \{1, 0, 0\}\rangle \tag{3}$$
$$\ldots \tag{4}$$

$$|\psi^n\rangle = |uud\ \bar{u}u \ldots \bar{u}u\ \bar{d}d \ldots \bar{d}d\ g \ldots g\rangle = |\{uud\},\{i,j,k\}\rangle \qquad (5)$$

where {uud} are the valence quarks of the proton , i is the number of quark antiquark ($\bar{u}u$ pairs), j is the number of quark -antiquark ($\bar{d}d$ pairs), and k is the number of gluons. Then the density operator of the ensemble is

$$\hat{\rho} = \rho_{0,0,0}|uud\rangle\langle uud| + \rho_{0,0,1}|uudg\rangle\langle uudg| + \ldots \qquad (6)$$

$$= \sum \rho_{i,j,k}|\{uud\},\{i,j,k\}\rangle\langle\{uud\},\{i,j,k\}|, \qquad (7)$$

where ρi,j,k is the probability of finding a proton in the state |{uud}, {i, j, k}> .Theses probability can help to find the alpha and beta parameter and should satisfy the normalization condition,

$$\sum_{i,j,k} \rho_{i,j,k} = 1 \qquad (8)$$

$\rho_{i,j,k}$ can be calculated by using the principle of detailed balance without any parameter as shown below from which values of α & β are calculated. Then on this basis we analyze the value of low energy properties in the context of the different models. Statistical and simple quark model used the same basic idea but in their approach, the values of parameters are different.

Here the total probability for the occupancy of various fock states is based on the principle of detailed balance and there by we can get the generalized term for the probability. The principle of detailed balance means that any two states balance each other when changes occur between them. So the number of events that changed from state A to state B ($n_{A\longrightarrow B}$) equals the number of events that changed from state B to state A

($n_{B\longrightarrow A}$).

$$\text{i.e.} \quad n_{A\longrightarrow B} = n_{B\longrightarrow A}. \qquad (9)$$

Here shows some cases by using these ways probabilities can be calculated:

(1) The first process involving the creation or annihilation of gluon, only q ⇔ qg will be considered while g ⇔ gg neglected. So we get

$$|uud\rangle \underset{1\times 3}{\overset{3}{\rightleftharpoons}} |uudg\rangle, \qquad (10)$$

$$|uudg\rangle \underset{2\times 3}{\rightleftharpoons} |uudgg\rangle, \tag{11}$$

$$|uudgg\rangle \underset{3\times 3}{\overset{3}{\rightleftharpoons}} |uudggg\rangle, \tag{12}$$

And so on in general we get

$$|\{q\},\{i,j,k-1\}\rangle \underset{(3+2i+2j)k}{\overset{3+2i+2j}{\rightleftharpoons}} |\{q\},\{i,j,k\}\rangle \tag{13}$$

In general we get formula:

$$\frac{\rho_{i,j,k}}{\rho_{i,j,k-1}} = \frac{1}{k} \tag{14}$$

$$\frac{\rho_{i,j,k}}{\rho_{i,j,0}} = \frac{1}{k!} \tag{15}$$

2)Now consider g ⇔ $\bar{q}q$, it gives as shown :

$$|uudg\rangle \underset{1\times 3}{\overset{3}{\rightleftharpoons}} |uud\bar{u}u\rangle, \tag{16}$$

$$|uudg\rangle \underset{1\times 2}{\overset{1}{\rightleftharpoons}} |uud\bar{d}d\rangle \tag{17}$$

So on…….

$$|\{q\},\{i-1, j, 1\}\rangle \underset{i(i+2)}{\overset{1}{\rightleftharpoons}} |\{q\},\{i,j,0\}\rangle , \tag{18}$$

$$|\{q\},\{i, j-1,1\}> \underset{j(j+1)}{\overset{1}{\rightleftharpoons}} |\{q\},\{i,j,0\}>, \qquad (19)$$

In, General we get formula ,

$$\frac{\rho_{i,j,0}}{\rho_{i-1,j,1}} = \frac{1}{i(i+2)} \qquad (20)$$

$$\frac{\rho_{i,j,0}}{\rho_{i,j-1,1}} = \frac{1}{j(j+1)} \qquad (21)$$

Using relation $\rho_{i,j,1} = \rho_{i,j,0}$ from formula (15), we get

$$\frac{\rho_{i,j,0}}{\rho_{0,0,0}} = \frac{1}{i!(i+2)!\,j!(j+1)!} \qquad (22)$$

By combining (15) & (22) we get the general formula,

$$\frac{\rho_{i,j,k}}{\rho_{0,0,0}} = \frac{2}{i!(i+2)!\,j!(j+1)!k!} \qquad (23)$$

Now check whether the including of g ⇔ gg can change the above calculation or not by analysis on two kinds of similar relation:

3) Detailed balance involving q ⇔ qg and g ⇔ gg gives the relation

$$|uud> \underset{1\times 3}{\overset{3}{\rightleftharpoons}} |uudg>, \qquad (24)$$

$$|uudg> \underset{2\times 3+1}{\overset{3+1}{\rightleftharpoons}} |uudgg>, \qquad (25)$$

$$|uudgg> \underset{3\times 3+3}{\overset{3+2}{\rightleftharpoons}} |uudggg>, \tag{26}$$

....

So on

Generally,

$$|\{q\},\{i,j,k-1\}> \underset{(3+2i+2j)k+c^2_k}{\overset{3+2i+2j+k-1}{\rightleftharpoons}} |\{q\},\{i,j,k\}> \tag{27}$$

Where $c^2_k=k(k-1)/2$,Generally we get a formula:

$$\frac{\rho_{i,j,k}}{\rho_{i,j,k-1}} = \frac{(3+2i+2j+2k-1)}{(3+2i+2j)+k(k-1)/2} \tag{28}$$

Detailed balance involving g ⇔q̄q has no effect by the process g ⇔gg so the Eq. (28) remains unchanged. So shows that detailed balance involving g ⇔ gg has no effect on above conclusion general formula remains unchanged.

As seen further, similar basic idea can be using for calculating the low energy properties in context of another model by Song & Gupta. It considering the different way to calculate these properties by applying constraints as a four or three parameter fit.

Hadrons (Baryons & Mesons) are color-singlet combinations of three quarks or quark-antiquark pairs in the appropriate flavor and spin combinations.

The three (valence) quark wave function of the baryon can be written as:

$$\Psi = \Phi\,(|\phi>|\chi>|\varphi>)\,(|\xi>)$$

Φ, χ, φ, ξ denote flavor, spin, color, and space-time q wave functions. The wave function Flavor, spin-color part should be total anti-symmetric because the wave function ξ be total symmetric part. Sea is supposed to be flavorless and in s- wave, spin of sea part are chosen 0,1,2 and color can be $1_c, 8_c, \overline{10}_c$ The possible combinations of q^3 and sea wave functions, which can give a spin 1/2, flavor octet, color singlet state, for a flavor octet baryons are:

$\Phi_1^{1/2}H_0G_0$, $\Phi_8^{1/2}H_0G_8$, $\Phi_{10}^{1/2}H_0G_{10}$, $\Phi_1^{1/2}H_1G_1$, $\Phi_8^{1/2}H_1G_8$, $\Phi_{10}^{1/2}H_1G_{10}$,
$\Phi_8^{3/2}H_1G_8$, $\Phi_8^{3/2}H_1G_8$

Total flavor-spin-color wave function of a spin up baryons which consists of three valence quarks and a sea component can be given:

$$|\Phi_{1/2}^{(\uparrow)}> = \frac{1}{N} \Big[\Phi_1^{(1/2\uparrow)} .H_0.G_1 + a_8\Phi_8^{(1/2\uparrow)}.H_0 .G_8 + a_{10}\Phi_{10}^{(1/2\uparrow)} . H_0 .G_{10}$$
$$+ b_1(\Phi_1^{(1/2)} \otimes H_1)^{\uparrow} . G_1 + b_8(\Phi_8^{(1/2)} \otimes H_1)^{\uparrow} .G_8 + b_{10}(\Phi_{10}^{(1/2)} \otimes H_1)^{\uparrow} . G_{10}$$
$$+ c_8 (\Phi_8^{(3/2)} \otimes H_1)^{\uparrow} . G_8 + d_8(\Phi_8^{(3/2)} \otimes H_2)\uparrow . G_8 \Big]$$

Where

$$N^2 = 1+ a_8^{(2)} + a_{10}^{(2)} + b_1^{(2)} + b_8^{(2)} + b_{10}^{(2)} + c_8^{(2)} + d_8^{(2)}$$

These are seven correction terms first three terms come from scalar sea and next three terms come from vector sea and c_8, d_8 tensor sea . $\hat{O}$ is any operator which depends on the flavor of ith quark and σ^i_z is the spin projection operator of ith quark . $\hat{O}$ is like a $\hat{O} = \Sigma_i \hat{O}^i_f \sigma^i_z$. The suitable operators are switched for flavor and spin so as to get a simplified relation for each of parameter in the wave function.

Some notations are used for this operator are listed below:

1. $< O^i_f >^{\lambda\lambda} \equiv < \phi^{\lambda} | O^i_f |\phi^{\lambda} >, < \sigma^i_z >^{\lambda\uparrow\lambda\uparrow} \equiv < \chi^{\lambda\uparrow} | \sigma^i_z | \chi^{\lambda\uparrow} >$
2. $< O^i_f >^{\rho\rho} \equiv < \phi^{\rho} | O^i_f |\phi^{\rho} >, < \sigma^i_z >^{\rho\uparrow\rho\uparrow} \equiv < \chi^{\rho\uparrow} | \sigma^i_z | \chi^{\rho\uparrow} >$
3. For magnetic moments, $O^i_f = e^i/2m_i$ (i= u,d, s)

The baryonic magnetic moments can be expressed in terms of quark magnetic moments (μ_u, μ_d, μ_s) and two parametres α &β as shown:

$\mu_p = 3(\mu_u\alpha - \mu_d\beta)$, $\mu_n = 3(\mu_d\alpha - \mu_u\beta)$, $\mu_\Lambda = 1/2 (\alpha - 4\beta)(\mu_u +\mu_d) + (2\alpha +\beta)\mu_s$,

$\mu_{\Sigma+} = 3(\mu_u\alpha - \mu_s\beta)$, $\mu_{\Sigma^-} = 3(\mu_d\alpha - \mu_s\beta)$, $\mu_{\Sigma}^0 = 1/2(\mu_{\Sigma}^+ + \mu_{\Sigma}^-)$,

$\mu_{\Xi}^0 = 3(\mu_s\alpha - \mu_u\beta)$, $\mu_{\Xi}^- = 3(\mu_s\alpha - \mu_d\beta)$

Where $\mu_q = e/2m_q$ (q = u,d,s)

$$\alpha = 1/N^2(4/9) (2a +2b +3d + \sqrt{2}e)$$

$$\beta = 1/N^2 (1/9) (2a -4b - 6c -6d+ 4\sqrt{2}e)$$

Seven parametres contribute in the wave function through the combinations given by α & β. The Physical reason for that is α & β are connected with number of spin-up and spin-down quarks in the spin-up proton.

If , $\Delta q \equiv n(q_\uparrow) - n(q_\downarrow) + n(\bar{q}_\uparrow) - n(\bar{q}_\downarrow)$, q= u,d,s then $\Delta u = 3\alpha$ & $\Delta d = -3\beta$.

3.2) Experimental Value v/s Theoretical

For baryons spin distribution, flavors and spin may be combined in an appropriate flavor-spin SU(6) symmetry ,in which three basic quark states are $u^{\uparrow}$,$u^{\downarrow}$, $d^{\uparrow}$,$d^{\downarrow}$, $s^{\uparrow}$ and $s^{\downarrow}$. The simplifies wave function for proton

$$|p^{\uparrow}> = 1/\sqrt{6}\ (2|u^{\uparrow}u^{\uparrow}d^{\downarrow}> - |\ u^{\uparrow}\ u^{\downarrow}\ d^{\uparrow}> - |\ u^{\downarrow}\ u^{\uparrow}\ d^{\uparrow}>)$$

From this wave function, we can count the number of quark flavors with spin parallel and anti parallel to the total spin of the proton. The result is

$$n_{u\uparrow}(p) = 5/3\ ,\ n_{u\downarrow}(P)= 1/3\ ,\ n_{d\uparrow}(P) =1/3\ ,\ n_{d\downarrow}(P) = 2/3$$

These numbers are the Eigen values of the quark number operator with respect to helicity and sum up to two u quark and one d quark. From the differences, which are exactly the expectation values of twice the quark spin operator, one obtains the contribution by each of the quark flavors to the proton helicity . The differences are according to the non- Relativistic Quark Model:

$$\Delta u^{p} =4/3\ ,\ \Delta d^{p}= -1/3\ ,\ \Delta s^{p}= 0$$

Sum of these differences is in NQM :

$$\Delta\Sigma^{p} = \Delta u^{p} + \Delta d^{p} + \Delta s^{p} = 1$$

In 1988 the European muon collaboration (EMC)[4,5] announced that it had measured the fraction of the proton spin carried by the quarks $\Delta\Sigma \approx 0$, also $\Delta s \neq 0$.this was the beginning of what was to be called the “nucleon spin crisis”. A lot of research has been going on and the present experimental results may be summarized as[6].

$$\Delta u = 0.83 \pm 0.03,$$

$$\Delta d= -0.43 \pm 0.03,$$

$$\Delta s= -0.10 \pm 0.03,$$

$$\Delta\Sigma= 0.31 \pm 0.07.$$

If $\Delta\Sigma = 1$ means that the spin is carried by the quark alone. If $\Delta\Sigma<1$, then the remainder spin could be built by something else.

Further as according to simple quark model, Song and Gupta [7] computed the low energy properties of nucleons by evaluating α & β from the fits obtained using the experimental data of magnetic moment with suppressed scalar & tensor sea. Even otherwise these values of α & β can be calculated by choice of basic parametres a_8, a_{10}, b_8, b_{10} etc. However song and gupta suppressed some of the contributions just to get the good fit to reproduce the experimental values. It consider the four parameter fit and three parameter fit to put constraints on values of α & β. We have used their values of α & β to find the spin distribution function, as per the definition below :

By taking $\alpha = 0.3415$, $\beta = 0.0775$ and putting the value in below formula's:

$I_1^p = 1/6(4\alpha - \beta)$ and $I_1^n = 1/6(\alpha - 4\beta)$ we get the values as 0.2147 & 0.00525 respectively.

However the statistical model by J.P Singh & A. Upadhyay[1] treated the proton as an ensemble of quark gluon fock states by using the principle of detailed balance that every fock state should be balanced. It consider the sub processes as $q \Leftrightarrow q$, $g \Leftrightarrow gg$ and $g \Leftrightarrow q\bar{q}$. By calculating the probability associated with different fock states coming from the scalar, vector and tensor sea, α & β have been determined. Thus using probabilities ratio, different coefficients are calculated where all coefficients are expressed in terms of α & β. Hence the value for I_1^p, I_1, ΔI_1^p and ΔI_1^n, F/D, ratio of magnetic moment have been shown in table.

We can calculate the magnetic moment also as shown below. In the NQM, the magnetic moments of octet baryons can be written as [8]

$$\mu(p) = \Delta u\mu_u + \Delta d\mu_d + \Delta s\mu_s ,$$

$$\mu(n) = \Delta u\mu_d + \Delta d\mu_u + \Delta s\mu_s ,$$

$$\mu(\Sigma^+) = \Delta u\mu_u + \Delta d\mu_s + \Delta s\mu_d ,$$

$$\mu(\Sigma^-) = \Delta u\mu_d + \Delta d\mu_s + \Delta s\mu_u ,$$

$$\mu(\Xi^0) = \Delta u\mu_s + \Delta d\mu_u + \Delta s\mu_d ,$$

$$\mu(\Xi^-) = \Delta u\mu_s + \Delta d\mu_d + \Delta s\mu_u ,$$

$$\mu(\Lambda) = \mu_s ,$$

$$\mu(\Sigma^0) = \tfrac{1}{2}(\mu(\Sigma^+) + \mu(\Sigma^-)) .$$

In first six formulas the factors $\Delta u = 4/3$, $\Delta d = -1/3$, $\Delta s = 0$ are the differences and μ_u, μ_d, μ_s are the quark magnetic moments. As assumption of isospin symmetry $m_u = m_d$, one has $\mu_u = -2\mu_d$. By using the magnetic moment of proton and neutron are :

$$\mu(p) = -3\mu_d \text{ and } \mu(d) = 2\mu_d \text{, and ratio between them is } \mu(p)/\mu(n) = -1.5 ,$$

Which is close to the experimental value of -1.46.

The experimentally measured magnetic moments of octet baryons are shown in table below. Data have been obtained from Ref.[9] and are given in units of nuclear magneton, μ_N

Particle	Magnetic Moment
P	2.79 ± 0.00
N	-1.91± 0.00
Λ	-0.61± 0.01
Σ^0	-
Σ^+	2.46 ±0.02
Σ^-	-1.16±0.03
Ξ^0	-1.25±0.02
Ξ^-	-0.65±0.01

Table.VIII Magnetic moments of baryon octet

References

[1] J. P. Singh, A. Upadhyay, Phys. G, **30**, 881, 2004.

[2] Y. J Zhang, B. Zhang and B.-Q. Ma, Phys. Lett. B523, 260, (2002).

[3]. Y.J. Zhang, W.Z. Deng and B.-Q. Ma, Phys. Rev. D 65, 114005 (2002).

[4] J. Ashman et al., European Muon Collaboration, Phys. Lett. B 206, 364 (1988).

[5] J. Ashman et al., European Muon Collaboration, Nucl. Phys. B328, 1(1989).

[6] J. Ellis and M. Karliner, Phys. Lett. B 341, 397 (1995).

[7] X. Song, V. Gupta, Phys. Rev. D. **49**, 2211, 1994.

[8] D. Griffiths, Introduction to Elementary Particles (Wiley, New York,(1987).

[9] R.M. Barnett et al., Particle Data Group, Review of Particle Physics, Phys. Rev. D 54, 1 (1999)

Results and Conclusions

Nucleon is a microscopic system, and its behavior at low energies is determined by perturbative QCD. Hence model plays an important roles in studying nucleonic properties. Quantum chromo dynamics allows the nucleon to have virtual quark antiquark pair and gluon into the sea in addition to the three valence quarks. This type of sea plays an important and crucial role in determining various properties. This type of sea plays an important and crucial role in determining various properties. Here we have a statistical approach to study the behavior of nucleons in contributing to these properties and hence the different approaches where the similar framework has been used is studied for the respective properties. We have used a statistical model in which a nucleon is taken as an ensemble of quark-gluon Fock states and where a probability to find a Fock state is decided by the principle of balance or principle of detailed balance. These quarks and gluons have to be understood as 'intrinsic' partons of the nucleon. The total flavor-spin-color wave function of a spin-up nucleon has been decomposed in a three-quark core and a sea with definite spin and color quantum numbers and the respective expansion coefficients have been determined. The sea is taken to be flavorless but with angular momentum and color quantum numbers which, when combined with the corresponding quantum numbers of three-quark core makes nucleon spin 1/2 and colorless system respectively.

Within this framework, various models have been discussed for these properties and their domain of validity is check for calculating these parameter to find the effects of the sea for them. Here we took four models in general and two in specific for our analysis. Simple quark model and the statistical model follow the same kind of approach and for enhances analysis other model and their result are discussed.

Properties	**Theoretical value**		**Experimental**
	Statistical Model	**SQM**	
I_1^p	0.1677	0.2147	0.136
I_1^n	0.0291	0.00525	-0.030
ΔI_1^p	0.03080	-	-
ΔI_1^n	0.04059	-	-
$\frac{\mu_p}{\mu_n}$	-1.4050	-1.46	-1.4600
F/D	0.60330	0.6878	0.57500
G_A/G_v	1.24328	1.66	1.2670

Table. IX Low Energy Properties In Different Models

Distribution of spin among the valence quarks and the sea quarks for the proton and neutron are denoted by I_1^p & I_1^n and magnetic moments for proton and neutron is denoted by μ_p &μ_n respectively.

Here it is important to note that QCD based model are useful in study the baryonic properties. Specially the low energy properties can be studied in the phenomenological model where baryon has been taken to be combination of various condensates of quark in addition to the gluons with limited numbers. Based on their multiplicities, the probability for finding these fock states have been calculated which finally comes in terms of the coefficients that shows individual contribution coming from scalar, vector and tensor sea. Here we have show the contribution from the sum of all the sea in the statistical model and have compared them with the simple quark model where the sea is supposed to be suppressed. We conclude here that these sea contribution here plays an important role in studying these parameters and their inclusion is important for the study. Suppression of sea helps in making the data better or we can say close to the experimental values for the spin distribution in neutron as compared to proton. F/D ratio is deviating from the experimental data by 4% while SQM is showing an error of 19%. Spin distribution for proton is more accurate in statistical with a divergence of 23% as compared to 57% divergence of SQM. The magnetic moment ration seems to be accurate in both statistical model as well as simple quark model. The errors for statistical model is 3.7% and for simple quark model it is 0%. The G_A/G_V ratio is more accurate in statistical model as compared to SQM. Error in statistical model is 1% and in SQM is 31%.

Hence we conclude that for most of the QCD phenomenon the low energy properties need to take sea part in addition to the valence quarks to get the better accuracy and the high experimental match. Sea plays and important role in determining these parameters and they are consider to be useful for at least low energy scale upto $1GeV^2$.

Solved Appendix

(i) Spin projection operator

- For $< \sigma_z^{(1)} >^{\lambda\uparrow\lambda\uparrow} = < \sigma_z^{(2)} >^{\lambda\uparrow\lambda\uparrow} = 2/3$; $<\sigma_z^{(3)} >^{\lambda\uparrow\lambda\uparrow} = -1/3$ (1)

By taking the mixed-symmetric wave function for "proton". Where "λ" denotes the symmetric and "σ" is spin projection operator.

$\lambda^{\uparrow} = 1/\sqrt{6}\,|\,(\uparrow\downarrow + \uparrow\downarrow)\uparrow - 2\uparrow\uparrow\downarrow >$ By applying it on above equation (1) firstly on 1st then for 2nd and 3rd as shown below :

$1/\sqrt{6}\,|\,(\uparrow\downarrow + \uparrow\downarrow)\uparrow - 2\uparrow\uparrow\downarrow > \;|< \sigma_z^{(1)} >^{\lambda\uparrow\lambda\uparrow}|\; 1/\sqrt{6}\,|\,(\uparrow\downarrow + \uparrow\downarrow)\uparrow - 2\uparrow\uparrow\downarrow >$

$1/6(1-1+4(1)) = 2/3$

Similarly for 2nd function we get 2/3.

For 3rd function :

$1/\sqrt{6}\,|\,(\uparrow\downarrow + \uparrow\downarrow)\uparrow - 2\uparrow\uparrow\downarrow > \;|< \sigma_z^{(3)} >^{\lambda\uparrow\lambda\uparrow}|\; 1/\sqrt{6}\,|\,(\uparrow\downarrow + \uparrow\downarrow)\uparrow - 2\uparrow\uparrow\downarrow >$

$1/6(1+1+4(-1)) = -1/3.$

- For $< \sigma_z^{(1)} >^{\rho\uparrow\rho\uparrow} = < \sigma_z^{(2)} >^{\rho\uparrow\rho\uparrow} = 0$; $<\sigma_z^{(3)} >^{\rho\uparrow\rho\uparrow} = 1$ (2)

By taking the mixed antisymmetric wave function for "p". Where "ρ" denotes the anti -symmetric and "σ" is spin projection operator.

$\rho^{\uparrow} = 1/\sqrt{2}\,|(\uparrow\downarrow - \downarrow\uparrow)\uparrow >$ By applying it on above equation (2) firstly on 1st then for 2nd and 3rd as shown below :

$<1/\sqrt{2}\,|(\uparrow\downarrow - \downarrow\uparrow)\uparrow > |< \sigma_z^{(1)} >^{\rho\uparrow\rho\uparrow}| 1/\sqrt{2}\,|(\uparrow\downarrow - \downarrow\uparrow)\uparrow >$

$1/2(1+ (-1)) = 0$

Similarly for 2nd function, we get 0.

For 3rd function:

$$<1/\sqrt{2}\ |(\uparrow\downarrow - \downarrow\uparrow)\uparrow > |< \sigma_z^{(3)} >^{\rho\uparrow\rho\uparrow}|1/\sqrt{2}\ |(\uparrow\downarrow - \downarrow\uparrow)\uparrow >$$

$$1/2(1+1) = 1.$$

- $$< \sigma_z^{(1)} >^{\lambda\uparrow\rho\uparrow} = - < \sigma_z^{(2)} >^{\lambda\uparrow\rho\uparrow} = 1/\sqrt{3}\ ; \qquad < \sigma_z^{(3)} >^{\lambda\uparrow\rho\uparrow} = 0 \quad (3)$$

By taking the mixed-antisymmetric & mixed-symmetric wave function for "p". Where "ρ" denotes the anti-symmetric and "λ" denotes the symmetric. "σ" is spin projection operator.

$\rho^{\uparrow} = 1/\sqrt{2}\ |(\uparrow\downarrow - \downarrow\uparrow)\uparrow >$ & $\lambda^{\uparrow} = 1/\sqrt{6}\ |(\uparrow\downarrow + \uparrow\downarrow)\uparrow - 2\uparrow\uparrow\downarrow >$ By applying it on above equation (3) firstly on 1st then for 2nd and 3rd operator as shown below :

$$<1/\sqrt{6}\ |(\uparrow\downarrow + \uparrow\downarrow)\uparrow - 2\uparrow\uparrow\downarrow > |< \sigma_z^{(1)} >^{\lambda\uparrow\rho\uparrow}|1/\sqrt{2}\ |(\uparrow\downarrow - \downarrow\uparrow)\uparrow >$$

$$1/\sqrt{12}(1+0-2(1)) = 1/\sqrt{3}.$$

Similarly for $- < \sigma_z^{(2)} >^{\lambda\uparrow\rho\uparrow} = 1/\sqrt{3}$.

For 3rd operator:

$$<1/\sqrt{6}\ |(\uparrow\downarrow + \uparrow\downarrow)\uparrow - 2\uparrow\uparrow\downarrow > |< \sigma_z^{(3)} >^{\lambda\uparrow\rho\uparrow}|1/\sqrt{2}\ |(\uparrow\downarrow - \downarrow\uparrow)\uparrow >$$

$$1/\sqrt{12}(1-1) = 0.$$

Matrix elements in (1) & (2) satisfy

$$\Sigma^{3}_{i=1} < \sigma_z^{(i)} >^{\lambda\uparrow\lambda\uparrow} = \Sigma^{3}_{i=1} < \sigma_z^{(i)} >^{\rho\uparrow\rho\uparrow} = 1 \quad (4)$$

In addition to that,

- $$< \sigma_z^{(1)} >^{\lambda\uparrow 3/2\uparrow} = < \sigma_z^{(2)} >^{\lambda\uparrow 3/2\uparrow} = -\sqrt{2}/3\ ; \qquad < \sigma_z^{(3)} >^{\lambda\uparrow 3/2\uparrow} = 2\sqrt{2}/3 \quad (5)$$

By taking the mixed-symmetric wave function for "p" . Where "λ" denotes the symmetric. "σ" is spin projection operator.

$\lambda^{\uparrow} =|1/\sqrt{6}\ |\ (\uparrow\downarrow + \uparrow\downarrow)\uparrow-2\uparrow\uparrow\downarrow >$ and for state 3/2, $|1/\sqrt{3}(\uparrow\downarrow\uparrow +\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow)>$ by applying it on equation (5) firstly on 1st then 2nd & 3rd as shown below:

$|1/\sqrt{6}\ |\ (\uparrow\downarrow + \uparrow\downarrow)\uparrow-2\uparrow\uparrow\downarrow >\ \sigma_z^{(1)} >^{\lambda\uparrow 3/2\uparrow} |1/\sqrt{3}(\uparrow\downarrow\uparrow +\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow) >$

$1/3\sqrt{2}(1\text{-}1\text{-}2(1)) = \sqrt{2}/3.$

Similarly for $< \sigma_z^{(2)} >^{\lambda\uparrow 3/2\uparrow} >$ we get , $\sqrt{2}/3$.

For 3rd operator:

$|1/\sqrt{6}\ |\ (\uparrow\downarrow + \uparrow\downarrow)\uparrow-2\uparrow\uparrow\downarrow >\ \sigma_z^{(3)} >^{\lambda\uparrow 3/2\uparrow} |1/\sqrt{3}(\uparrow\downarrow\uparrow +\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow) >$

$1/3\sqrt{2}(1+1-2(-1)) = 2\sqrt{2}/3.$

(ii) Isospin Projection operator

For the Σ^+ - hyperon,

- $< I_3^{(1)} >_{\Sigma+}^{\lambda\lambda} = < I_3^{(2)} >_{\Sigma+}^{\lambda\lambda} = 5/12$; $< I_3^{(3)} >_{\Sigma+}^{\lambda\lambda} = 1/6$ (6)

By taking the mixed-symmetric wave function for "Σ^+" . "λ" denotes the symmetric. "I_3" is iso-spin projection operator.

Where $\Sigma^+ = 1/\sqrt{6}[(\,us+su)u-2uus]$, By applying it on equation (6) firstly on 1st then 2nd & 3rd as shown below:

$<1/\sqrt{6}((\,us+su)u-2uus)|\ I_3^{(1)} >_{\Sigma+}^{\lambda\lambda} |1/\sqrt{6}((\,us+su)u-2uus)>$

$1/6[(1/2+(0) +4(1/2)] = 5/12.$

Similarly for $< I_3^{(2)} >_{\Sigma+}^{\lambda\lambda}$, we get 5/12.

For 3rd operator:

$<1/\sqrt{6}((\,us+su)u-2uus)|\ I_3^{(3)} >_{\Sigma+}^{\lambda\lambda} |1/\sqrt{6}((\,us+su)u-2uus)>$

$1/6[(1/2) +(1/2) +4(0)] = 1/6.$

- $< I_3^{(1)} >_{\Sigma+}^{\rho\rho} = < I_3^{(2)} >_{\Sigma+}^{\rho\rho} = 1/4$; $< I_3^{(3)} >_{\Sigma+}^{\rho\rho} = 1/2$ (7)

By taking the Mixed-anti symmetric wave function for "Σ^+" . "ρ" denotes the anti-symmetric. "I_3" is Iso- spin projection operator.

$\Sigma^+ = 1/\sqrt{2}[(us-su)u]$, By applying it on equation (7) firstly on 1st then 2nd & 3rd as shown below:

$<1/\sqrt{2}((us-su)u)| \; I_3^{(1)} >_{\Sigma+}^{\rho\rho} |1/\sqrt{2}((us-su)u)>$

$1/2[(1/2)+(0)] = 1/4.$

Similarly for the $< I_3^{(2)} >_{\Sigma+}^{\rho\rho}$, we get 1/4.

For the 3rd operator:

$<1/\sqrt{2}((us-su)u)| \; I_3^{(3)} >_{\Sigma+}^{\rho\rho} |1/\sqrt{2}((us-su)u)>$

$1/2[(1/2) +(1/2)] = 1/2.$

- $< I_3^{(1)} >_{\Sigma+}^{\lambda\rho} = - < I_3^{(2)} >_{\Sigma+}^{\lambda\rho} = 1/4\sqrt{3}$; $< I_3^{(3)} >_{\Sigma+}^{\lambda\rho} = 0$ (8)

By taking the Mixed-anti symmetric or mixed symmetric wave function for "Σ^+" . "ρ" denotes the anti-symmetric & "λ" denotes the symmetric. "I_3" is Iso- spin projection operator.

$<1/\sqrt{6}((us+su)u-2uus)| \; I_3^{(1)} >_{\Sigma+}^{\lambda\rho}|1/\sqrt{2}((us-su)u)>$

$1/\sqrt{12}[(1/2)+(0)] = 1/4\sqrt{3}.$

Similarly for the $< I_3^{(2)} >_{\Sigma+}^{\lambda\rho}$ we get, $1/4\sqrt{3}$.

For the 3rd operator:

$1/\sqrt{6}((us+su)u-2uus)| \; I_3^{(3)} >_{\Sigma+}^{\lambda\rho}|1/\sqrt{2}((us-su)u)>$

$1/\sqrt{12}[(1/2)-(1/2)] = 0.$

All this is similar for Σ^- hyperon, the matrix elements reverse their signs. We can also solve it for other baryon octets by taking their wave-functions.

(iii) **Charge Operator With Symmetric Breaking Effect**

For the Proton,

- $< e^{(1)}{}_{m/m1} >_p^{\lambda\lambda} = < e^{(2)}{}_{m/m2} >_p^{\lambda\lambda} = 1/2; \qquad < e^{(3)}{}_{m/m3} >_p^{\lambda\lambda} = 0$ (9)

By taking the mixed symmetric wave function for "proton" . "λ" denotes the symmetric. "e" is charge projection operator.

Where, P = 1/√6((ud+du)u-2uud)). By applying it on equation (9), firstly on 1st then 2nd & 3rd ,as shown below:

$<1/\sqrt{6}((ud+du)u-2uud))| < e^{(1)}{}_{m/m1} >_p^{\lambda\lambda}|1/\sqrt{6}((ud+du)u-2uud)) >$

$1/6[(2/3)+(-1/3)+4(2/3)] = 1/2.$

Similarly for the $< e^{(2)}{}_{m/m2} >_p^{\lambda\lambda}$, we get, 1/2.

For the 3rd operator:

$<1/\sqrt{6}((ud+du)u-2uud))| < e^{(3)}{}_{m/m1} >_p^{\lambda\lambda}|1/\sqrt{6}((ud+du)u-2uud)) >$

$1/6[(2/3)+(2/3)+4(-1/3)] = 0.$

- $< e^{(1)}{}_{m/m1} >_p^{\rho\rho} = < e^{(2)}{}_{m/m2} >_p^{\rho\rho} = 1/6; \qquad < e^{(3)}{}_{m/m3} >_p^{\rho\rho} = 2/3$ (10)

By taking the mixed anti-symmetric wave function for "proton" . "ρ" denotes the anti-symmetric. "e" is charge projection operator.

Where, P = 1/√2(ud-du)u , By applying it on equation (10), firstly solved for 1st then 2nd & 3rd as shown below :

$<1/\sqrt{2}(ud-du)u|< e^{(1)}{}_{m/m1} >_p^{\rho\rho}|1/\sqrt{2}(ud-du)u>$

$1/2[(2/3)+(-1/3)] = 1/6.$

Similarly for the $< e^{(2)}{}_{m/m2} >_p{}^{\rho\rho}$ we get, 1/6.

For the 3rd function:

$$<1/\sqrt{2}(ud-du)u|< e^{(3)}{}_{m/m1} >_p{}^{\rho\rho}|1/\sqrt{2}(ud-du)u>$$

$$1/2[(2/3)+(2/3)] = 2/3.$$

➢ $< e^{(1)}{}_{m/m1} >_p{}^{\lambda\rho} = < e^{(2)}{}_{m/m2} >_p{}^{\lambda\rho} = 1/2\sqrt{3};$ $< e^{(3)}{}_{m/m3} >_p{}^{\lambda\rho} = 0$ (11)

By taking the mixed anti-symmetric & mixed symmetric wave function for "proton" . "ρ" denotes the anti-symmetric& "λ" denotes the symmetric. "e" is charge projection operator.

$$<1/\sqrt{6}((ud+du)u-2uud))| < e^{(1)}{}_{m/m1} >_p{}^{\lambda\rho}|1/\sqrt{2}(ud-du)u>$$

$$1/\sqrt{12}[(2/3)-(-1/3)] = 1/2\sqrt{3}.$$

Similarly for the $< e^{(2)}{}_{m/m2} >_p{}^{\lambda\rho}$ we get, $1/2\sqrt{3}$.

For the 3rd function:

$$<1/\sqrt{6}((ud+du)u-2uud))| < e^{(3)}{}_{m/m1} >_p{}^{\lambda\rho}|1/\sqrt{2}(ud-du)u>$$

$$1/\sqrt{12}[(2/3)-(2/3)] = 0.$$

Where $m = m_u = m_d$, so cancels out. For the Neutron the matrix elements in (9) reverse their signs. But in (10) first two matrix elements do not change and 3rd one becomes -1/3.

(iv) **Charge Square Operator**

For the Neutron,

➢ $< e^{(1)\wedge 2} >_n^{\lambda\lambda} = < e^{(2)\wedge 2} >_n^{\lambda\lambda} = 1/6;$ $< e^{(3)\wedge 2} >_p^{\lambda\lambda} = 1/3$ (12)

By taking the mixed symmetric wave function for "neutron" . "λ" denotes the symmetric. "e^2" is charge square projection operator.

N = -1/√6((ud+du)d-2ddu), By applying it on equation (12),firstly solved for 1st then for 2nd & 3rd function as shown below:

$< -1/\sqrt{6}((ud+du)d-2ddu| < e^{(1)\wedge 2} >_n^{\lambda\lambda} |-1/\sqrt{6}((ud+du)d-2ddu>$

$$1/6[(2/3)^2+(-1/3)^2+4(-1/3)^2] = 1/6.$$

Similarly for the $|< e^{(2)\wedge 2} >_n^{\lambda\lambda}$ we get, 1/6.

For the 3rd function:

$$< -1/\sqrt{6}((ud+du)d-2ddu| < e^{(3)\wedge 2} >_n^{\lambda\lambda} |-1/\sqrt{6}((ud+du)d-2ddu>$$

$$1/6[(-1/3)^2+(-1/3)^2+4(2/3)^2] = 1/3.$$

➢ $< e^{(1)\wedge 2} >_n^{\rho\rho} = < e^{(2)\wedge 2} >_n^{\rho\rho} = 5/18;$ $< e^{(3)\wedge 2} >_p^{\rho\rho} = 1/9$ (13)

By taking the anti-mixed symmetric wave function for "neutron" . "ρ" denotes the anti-symmetric. "e^2" is charge square projection operator.

N= 1/√2(ud-du)d, By applying it on equation(13),firstly solved for 1st then for 2nd & 3rd function, as shown below:

$< 1/\sqrt{2}(ud-du)d| < e^{(1)\wedge 2} >_n^{\rho\rho} |1/\sqrt{2}(ud-du)d>$

$$1/2[(2/3)^2+(-1/3)^2] = 5/18.$$

Similarly for the $< e^{(2)\wedge 2} >_n^{\rho\rho}$ we get, 5/18.

For the 3rd function:

$< 1/\sqrt{2}(ud-du)d|< e^{(3)\wedge 2} >_n^{\rho\rho}|1/\sqrt{2}(ud-du)d >$

$1/2[(-1/3)^2+(-1/3)^2] = 1/9.$

- $< e^{(1)\wedge 2} >_n^{\lambda\rho} = < e^{(2)\wedge 2} >_n^{\lambda\rho} = 1/6\sqrt{3};$ $< e^{(3)\wedge 2} >_p^{\lambda\rho} = 0$ (14)

By taking the mixed symmetric & anti-mixed symmetric wave function for "neutron" . "ρ" denotes the anti-symmetric & "λ" denotes the symmetric. "e^2" is charge square projection operator.

$< -1/\sqrt{6}((ud+du)d-2ddu|< e^{(1)\wedge 2} >_n^{\lambda\rho}|1/\sqrt{2}(ud-du)d>$

$1/\sqrt{12}[(2/3)^2-(-1/3)^2] = 1/6\sqrt{3}.$

Similarly for the $< e^{(2)\wedge 2} >_n^{\lambda\rho}$ we get, $1/6\sqrt{3}$.

For the 3rd function :

$< -1/\sqrt{6}((ud+du)d-2ddu|< e^{(3)\wedge 2} >_n^{\lambda\rho}|1/\sqrt{2}(ud-du)d>$

$1/\sqrt{12}[(-1/3)^2-(-1/3)^2] = 0.$