SOME ALGORITHMS FOR SOLVING TRANSPORTATION PROBLEMS IN FUZZY ENVIRONMENT

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CANDIDATE'S DECLARATION

I hereby certify that the work being presented in the thesis entitled "Some Algorithms for Solving Transportation Problems in Fuzzy Environment" in fulfillment of the requirements for the award of degree of Doctor of Philosophy, submitted in the School of Mathematics and Computer Applications of Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Amit Kumar, Assistant Professor, School of Mathematics and Computer Applications, Thapar University, Patiala.

The matter embodied in this thesis has not been submitted by me to any other university or institute for the award of any other degree

Amarpreet Kaur 19/6/2012

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

19/06/2012

Dr. Amit Kumar

DEDICATED

TO

MY PARENTS

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Abstract

In this thesis, the shortcomings and limitations of some existing methods for solving fully fuzzy transportation problems, fully fuzzy transshipment problems and fully fuzzy solid transportation problems are pointed out and new methods are proposed to overcome the shortcomings and limitations of existing methods.

The chapter wise summary of the thesis is as follows:

Chapter 1

In this chapter, a brief review of the work done in the area of single-objective fuzzy transportation problems, fuzzy transshipment problems, fuzzy solid transportation problems and fuzzy solid transshipment problems are presented.

Chapter 2

To the best of my knowledge, only the methods [14, 66, 134, 161, 162, 191, 192], are proposed in the literature to find the fuzzy optimal solution of fully fuzzy transportation problems. In this chapter, the shortcomings of all these existing methods are pointed out and to overcome these shortcomings two new methods are proposed for solving fully fuzzy transportation problems. The advantages of the proposed methods over existing methods are discussed. To illustrate the proposed methods, an existing fully fuzzy transportation problem is solved. Also, to show the application of proposed methods in real life problems the fuzzy optimal solution of an

existing real life fuzzy transportation problem is obtained by using the proposed methods.

Chapter 3

In this chapter, the limitations of the methods, proposed in Chapter 2, are pointed out. To overcome these limitations new methods are proposed for solving fully fuzzy transportation problems by modifying the methods, proposed in Chapter 2. The advantages of the proposed methods over the methods, proposed in Chapter 2, are discussed. To illustrate the proposed methods, a fully fuzzy transportation problem is solved.

Chapter 4

To the best of my knowledge, only the method [70] is proposed in the literature to find the fuzzy optimal solution of fully fuzzy transshipment problems. In this chapter, limitations of this method are pointed out and to overcome these limitations, two new methods are proposed for solving fully fuzzy transshipment problems. The advantages of the proposed methods over existing method [70] and over the methods, proposed in previous chapters, are discussed. To illustrate the proposed methods a fully fuzzy transshipment problem is solved. Also, to show the application of the proposed methods in real life problems an existing real life fully fuzzy transshipment problem is solved by the proposed methods.

Chapter 5

To the best of my knowledge, only the method [132] is proposed in the literature to find the fuzzy optimal solution of fully fuzzy solid transportation problems. In this chapter, the shortcomings of this method are pointed out and to overcome these shortcomings, two new methods are proposed for solving fully fuzzy solid transportation problems. The advantages of the proposed methods over the existing method [132] and over the methods, proposed in Chapter 2 and Chapter 3, are discussed. To illustrate the proposed methods an existing fully fuzzy solid transportation problem is solved. Also, to show the application of the proposed methods in real life problem an existing real life fuzzy solid transportation problem is solved by the proposed methods.

Chapter 6

The fully fuzzy transshipment problems are obtained by introducing the intermediate nodes in fully fuzzy transportation problems and the fully fuzzy solid transportation problems are obtained by introducing the additional conveyances in fully fuzzy transportation problems. But, in real life problems both the intermediate nodes and additional conveyances are used simultaneously. So, in this chapter, by combining the concept of fully fuzzy solid transportation problems and fully fuzzy transshipment problems, new type of problems, named as fully fuzzy solid transshipment problems, its fuzzy linear programming formulation and two new methods for finding its fuzzy optimal solution, are proposed. The advantages of the proposed methods over the methods, proposed in previous chapters and over the existing method [70], are discussed. To illustrate the methods, proposed in this chapter, a fully fuzzy solid transshipment problem is solved.

Chapter 7

Finally, in this chapter, based on the present study, conclusions are drawn and future work have been suggested.

List of Research Papers

1. **A. Kaur**, A. Kumar, A new method for solving fuzzy transportation problems using ranking function, Applied Mathematical Modelling, 35 (2011) 5652-5661.

2. A. Kumar, **A. Kaur**, Methods for solving unbalanced fuzzy transportation problems, Operational Research-An International Journal, 12 (2012) DOI 10.10 07/s12351-010-0101-3.

3. **A. Kaur**, A. Kumar, A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers, Applied Soft Computing, 12 (2012) 1201-1213.

4. A. Kumar, **A. Kaur**, A. Gupta, Fuzzy linear programming approach for solving fuzzy transportation problems with transshipment, Journal of Mathematical Modelling and Algorithms, 10 (2011) 163-180.

5. A. Kumar, **A. Kaur**, Application of classical transportation methods to find the fuzzy optimal solution of fuzzy transportation problems, Fuzzy Information and Engineering, 3 (2011) 81-99.

6. A. Kumar, **A. Kaur**, Application of linear programming for solving fuzzy transportation problems, Journal of Applied Mathematics and Informatics, 29 (2011) 831-846.

7. A. Kumar, **A. Kaur**, Optimal solution of fuzzy transportation problems based on fuzzy linear programming formulation, Journal of Advanced Research in Applied Mathematics, 2 (2010) 70-84.

8. A. Kumar, **A. Kaur**, A. Gupta, New methods for solving fuzzy transportation problems with some additional transshipments, Australian Society for Operations Research, 30 (2011) 42-61.

9. A. Kumar, **A. Kaur**, M. Kaur, Fuzzy optimal solution of fuzzy transportation problems with transshipments, Lecture Notes in Computer Science, 6743 (2011) 167-170.

10. A. Kumar, **A. Kaur**, Methods for solving fully fuzzy transportation problems based on classical transportation methods, International Journal of Operations Research and Information Systems, 2 (2011) 52-71.

11. A. Kumar, **A. Kaur**, Application of classical transportation methods for solving fuzzy transportation problems, Journal of Transportation Systems Engineering and Information Technology, 11 (2011) 68-80.

12. A. Kumar, **A. Kaur**, A. Gupta, Method for solving a special type of fuzzy transportation problems, The Journal of Fuzzy Mathematics, 20 (2012) 119-132.

13. **A. Kaur**, A. Kumar, M. Kaur, Fuzzy solid transshipment problems and its fuzzy optimal solution, Applied and Computational Mathematics (Accepted).

14. A. Kumar, **A. Kaur**, Exact fuzzy optimal solution of unbalanced fuzzy solid transportation problems, Information Sciences (Communicated).

15. A. Kumar, **A. Kaur**, Optimal way of selecting cities and conveyances for supplying coal in uncertain environment, Optimization Letters (Communicated).

16. **A. Kaur**, A. Kumar, Fuzzy optimal solution of fully fuzzy transshipment problems, Communications in Mathematical Sciences (Communicated).

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Chapter 1

Introduction

In today's highly competitive market, the pressure on organizations to find better ways to send the products to the customers in a cost-effective manner, becomes more challenging. Transportation models [86] provide a powerful framework to meet this challenge.

To find the optimal solution of transportation problems it is assumed that the direct route between a source node and a destination node is a minimum-cost route. However, in real life transportation problems there may exist some nodes, called intermediate nodes, at which the product may be stored in case of excess of the available product and later on the product may be supplied from these intermediate nodes to the destinations. Such types of transportation problems are known as transshipment problems.

Also, to find the optimal solution of transportation problems it is assumed that same type of conveyances are used to transport the product from sources to destinations. However, in real life problems different types of conveyances are used for transporting the product from sources to destinations e.g., in many industrial problems, a homogeneous product is delivered from a source to a destination by different conveyances such as trucks, cargo flights, trains, ships etc. For such a case, the transportation problem turns into the solid transportation problem.

In conventional transportation problems, transshipment problems and solid transportation problems it is assumed that decision maker is sure about the precise values of transportation cost, availability and demand of the product. In the real world applications all the parameters of the transportation problems may not be known precisely due to uncontrollable factors.

To deal quantitatively with imprecise information, the concepts and techniques of probability could be employed. However, probability distributions require either a priori predictable regularity or a posteriori frequency distribution to construct. Moreover, the premise that imprecision can be equated with randomness is still questionable.

As an alternative, uncertain values can be represented by membership functions of the fuzzy set theory [219]. The main advantages of methodologies based on fuzzy theory are that they do not require prior predictable regularities or posterior frequency distributions and they can deal with imprecise input information containing feelings and emotions quantified based on the decision-makers subjective judgment. Due to the same reason several authors have represented some or all the parameters of transportation problems, transshipment problems and solid transportation problems by fuzzy numbers and proposed different methods for solving these problems.

The existing methods for solving fuzzy transportation problems, depending upon the fuzziness of decision variables, can be broadly divided into two groups. In the first group, the decision variables are assumed as real numbers i.e., in an uncertain environment, a crisp decision is to be made to meet some decision criteria. In the second group, decision variables are assumed as fuzzy numbers. Since, on assuming the decision variables as fuzzy numbers instead of a single optimal solution, a set of possible optimal solutions are obtained and an appropriate solution can be chosen from the set of possible solutions i.e., fuzzy optimal solution provides a range of flexibility to the decision maker. So, in the literature [7, 15, 27, 28, 29, 44, 76, 84, 101, 116, 143, 156, 198, 199, 200] it is pointed out, that it is better to assume the decision variables as fuzzy numbers.

1.1 Literature review

In this section, a brief review of the work done in the area of single-objective fuzzy transportation problems, fuzzy transshipment problems and fuzzy solid transportation problems are presented.

Oheigeartaigh [157] proposed an algorithm to find the crisp optimal solution of such fuzzy transportation problems in which the availabilities and demands are represented by triangular fuzzy numbers. Chanas et al. [33] presented a fuzzy linear programming model to find the crisp optimal solution of fuzzy transportation problems with crisp cost coefficients, fuzzy availability and fuzzy demand values. Ishii et al. [90] considered a fuzzy version of the transportation problem by introducing two kinds of membership functions which characterize fuzzy supply and fuzzy demands to determine a crisp optimal flow that maximize the smallest value of all membership functions under the constraints that the total transportation cost must not exceed a certain upper limit.

Tada et al. [196] generalized the existing fuzzy transportation problem [90] into an integer fuzzy transportation problem by adding an additional integral constraint of flow and proposed a method for solving integer fuzzy transportation problems. Chanas et al. [32] formulated the fuzzy transportation problems in three different situations and proposed a method for finding the crisp optimal solution of fuzzy transportation problems. Tada and Ishii [195] pointed out that in the existing fuzzy transportation problem [196] the budget constraint is not considered due to which there may exist more than one crisp optimal solution with the same objective value and considered another generalized problem by adding the budget constraint in the existing fuzzy transportation problem [196] and modified the existing method [196] for solving these types of transportation problems. Chanas and Kuchta [34] proposed a method to find the crisp optimal solution of such fuzzy transportation problems in which all the cost parameters are represented by LR type fuzzy numbers.

Jimenez and Verdegay [98] pointed out that there is no solution method to solve the parametric solid transportation problem and proposed a genetic algorithm based solution method to find crisp optimal solution of such fuzzy solid transportation problems in which all the parameters except cost parameters are represented by trapezoidal fuzzy numbers. Chanas and Kuchta [35] proposed a method to find the integer crisp optimal solution of such fuzzy transportation problems in which availability and demand are represented by fuzzy numbers. Jimenez and Verdegay [99] proposed the methods for finding the crisp optimal solution of two kinds of fuzzy solid transportation problem i.e., the availabilities, demands and conveyance capacities are interval numbers and fuzzy numbers, respectively. Jimenez and Verdegay [100] designed an evolutionary algorithm based parametric approach to find the crisp optimal solution of such fuzzy solid transportation problems in which all the parameters except cost parameters are represented by trapezoidal fuzzy numbers. Kikuchi [110] represented all the parameters of transportation problem by triangular fuzzy numbers and used the fuzzy linear programming approach to find the set of values such that the smallest membership grade among them is maximized. Ahlatcioglu et al. [3] proposed an algorithm for finding the crisp optimal solution of fuzzy transportation problem by converting all the fuzzy parameters into intervals. Saad and Abbas [182] discussed the solution algorithm for solving the transportation problem in fuzzy environment.

Liu and Kao [134] proposed a method, based on extension principle, to find the fuzzy optimal solution of such fuzzy transportation problems in which all the parameters are represented by trapezoidal fuzzy numbers. Liu and Kao [135] proposed a method for find the crisp optimal solution of such fuzzy transshipment problems in which the cost parameters are represented by trapezoidal fuzzy numbers. Chiang [38] proposed a method to find the crisp optimal solution of fuzzy transportation problems with fuzzy demand and fuzzy availability. Liang et al. [127] proposed an interactive possibilistic linear programming approach for finding the crisp optimal solution of transportation planning decision problems with imprecise cost, availabilities and demand. Gani and Samuel [64] proposed an algorithm for finding the fuzzy optimal solution of such fuzzy transportation problems in which availabilities and demands are represented by triangular fuzzy numbers.

Liu [132] extended the existing method [134] to find the fuzzy optimal solution of such fuzzy solid transportation problems in which all the parameters are represented by trapezoidal fuzzy numbers. Gupta and Mehlawat [79] proposed a method to find the crisp optimal solution of such fuzzy transportation problems in which availability and demand are represented by (λ, ρ) interval-valued fuzzy numbers and used it to select a new type of coal for a steel manufacturing unit. Smimou [189] proposed a method to find the crisp optimal solution of a fuzzy transshipment model with fuzzy costs and transshipment model with fuzzy supply and fuzzy demand. Gani et al. [67] proposed an algorithm for finding the fuzzy initial basic feasible solution of such fuzzy transportation problems in which cost, availabilities and demands are represented by triangular fuzzy numbers.

Das and Baruah [41] proposed vogel's approximation method to find fuzzy initial basic feasible solution and modi method to find the fuzzy optimal solution of such fuzzy transportation problems in which all the parameters are represented by triangular fuzzy numbers. Ghatee and Hashemi [70] proposed a method to find the fuzzy optimal solution of such balanced fuzzy transshipment problems in which all the parameters are represented by LR type fuzzy numbers. Ghatee and Hashemi [71, 73] and Ghatee et al. [74] applied the existing method [70] for solving real life problems. Li et al. [121] proposed a method, based on goal programming, to find the crisp optimal solution of such fuzzy transportation problems in which cost parameters are represented by fuzzy numbers. Jana and Roy [95] proposed a method for solving fuzzy linear programming problems with fuzzy variables and used it to find the fuzzy optimal solution of such fuzzy transportation problems in which availability and demand are represented by triangular fuzzy numbers. Lin [128] used genetic algorithm for finding the crisp optimal solution of fuzzy transportation problems with fuzzy coefficients. Stephen Dinagar and Palanivel [191] proposed fuzzy modified distribution method to find the fuzzy optimal solution of such fuzzy transportation problems in which all the parameters are represented by trapezoidal fuzzy numbers. Stephen Dinagar and Palanivel [192] proposed fuzzy vogel's approximation method for finding initial fuzzy solution of such fuzzy transportation problems in which all the parameters are represented by trapezoidal fuzzy numbers and proposed fuzzy modified distribution method to find the fuzzy optimal solution from the obtained initial fuzzy solution.

Pandian and Natarajan [161] proposed a new method, namely fuzzy zero point method, for finding fuzzy optimal solution of such fuzzy transportation problems in which all the parameters are represented by trapezoidal fuzzy numbers. Pandian and Natarajan [162] proposed a new method based on fuzzy zero point method for finding more-for-less fuzzy optimal solution for a such fuzzy transportation problems with mixed constraints in which all the parameters are represented by trapezoidal fuzzy numbers. De and Yadav [43] modified the existing method [110] by using trapezoidal fuzzy numbers instead of triangular fuzzy numbers. Lin [129] pointed out that differential evolution has received increasing attention owing to its simplicity and effectiveness in solving numerical optimization problems and introduced differential evolution to find the crisp optimal solution of fuzzy transportation problems with fuzzy coefficients. Guzel [80] proposed a method to find the crisp optimal solution of such fuzzy transportation problems in which all the parameters are represented by triangular fuzzy numbers.

Dutta and Murthy [51] proposed a method to find the crisp optimal solution of such fuzzy transportation problems with additional impurity restrictions in which cost parameters are represented by fuzzy numbers. Basirzadeh [14] used classical algorithms for finding the fuzzy optimal solution of fully fuzzy transportation problems by transforming the fuzzy parameters into crisp parameters. Samuel and

Venkatachalapathy [185] proposed a method namely, modified vogel's approximation method, for finding the fuzzy optimal solution of fully fuzzy transportation problems. Gani et al. [66] used Arsham-Khan's simplex algorithm [12] to find the fuzzy optimal solution of such fuzzy transportation problems in which all the parameters are represented by trapezoidal fuzzy numbers. Kumar Murugesan [117] proposed modified revised simplex method to find the fuzzy optimal solution of such fuzzy transportation problems in which availability and demand are represented by triangular fuzzy numbers. Gani et al. [61] proposed a method to find the fuzzy optimal solution of such fuzzy transshipment problems with mixed constraints in which all the parameters are represented by triangular fuzzy numbers.

After reviewing the literature, it can be concluded that very few methods are proposed in the literature for solving such fuzzy transportation problems, fuzzy transshipment problems and fuzzy solid transportation problems in which atleast one parameter of each type (atleast one of the cost parameters, atleast one of the availability parameters, atleast one of the demand parameters) and all the decision variables are represented by fuzzy numbers.

Since, in the literature [7, 8, 28, 44, 84, 116, 143] such linear programming problems in which atleast one parameter of each type as well as all the decision variables are represented by fuzzy numbers are named as fully fuzzy linear programming problems. So, on the same direction such fuzzy transportation problems, fuzzy transshipment problems and fuzzy solid transportation problems in which atleast one of the cost parameters, atleast one of the availability parameters, atleast one of the demand parameters and all the decision variables are represented by fuzzy numbers may be named as fully fuzzy transportation problems, fully fuzzy transshipment problems and fully fuzzy solid transportation problems.

In this thesis, the shortcomings and limitations of all the existing methods for finding the fuzzy optimal solution of single-objective fully fuzzy transportation problems, fully fuzzy transshipment problems and fully fuzzy solid transportation problems are pointed out. To overcome these shortcomings and limitations, new methods are proposed. Also, by combining the concept of fully fuzzy solid transportation problems and fully fuzzy transshipment problems, new type of problems, named as fully fuzzy solid transshipment problems, its fuzzy linear programming formulation and methods for finding its fuzzy optimal solution are proposed.

1.2 Organization of the thesis

The chapter wise summary of the thesis is as follows:

Chapter 2

To the best of my knowledge, only the methods [14, 66, 134, 161, 162, 191, 192], are proposed in the literature to find the fuzzy optimal solution of fully fuzzy transportation problems. In this chapter, the shortcomings of all these existing methods are pointed out and to overcome these shortcomings two new methods are proposed for solving fully fuzzy transportation problems. The advantages of the proposed methods over existing methods are discussed. To illustrate the proposed methods, an existing fully fuzzy transportation problem is solved. Also, to show the application of proposed methods in real life problems the fuzzy optimal solution of an existing real life fuzzy transportation problem is obtained by using the proposed methods.

Chapter 3

In this chapter, the limitations of the methods, proposed in Chapter 2, are pointed out. To overcome these limitations new methods are proposed for solving fully fuzzy transportation problems by modifying the methods, proposed in Chapter 2. The advantages of the proposed methods over the methods, proposed in Chapter 2, are discussed. To illustrate the proposed methods, a fully fuzzy transportation problem is solved.

Chapter 4

To the best of my knowledge, only the method [70] is proposed in the literature to find the fuzzy optimal solution of fully fuzzy transshipment problems. In this chapter, limitations of this method are pointed out and to overcome these limitations, two new methods are proposed for solving fully fuzzy transshipment problems. The advantages of the proposed methods over existing method [70] and over the methods, proposed in previous chapters, are discussed. To illustrate the proposed methods a fully fuzzy transshipment problem is solved. Also, to show the application of the proposed methods in real life problems an existing real life fully fuzzy transshipment problem is solved by the proposed methods.

Chapter 5

To the best of my knowledge, only the method [132] is proposed in the literature to find the fuzzy optimal solution of fully fuzzy solid transportation problems. In this chapter, the shortcomings of this method are pointed out and to overcome these shortcomings, two new methods are proposed for solving fully fuzzy solid transportation problems. The advantages of the proposed methods over the existing method [132] and over the methods, proposed in Chapter 2 and Chapter 3, are discussed. To illustrate the proposed methods an existing fully fuzzy solid transportation problem is solved. Also, to show the application of the proposed methods in real life problem an existing real life fuzzy solid transportation problem is solved by the proposed methods.

Chapter 6

The fully fuzzy transshipment problems are obtained by introducing the intermediate nodes in fully fuzzy transportation problems and the fully fuzzy solid transportation problems are obtained by introducing the additional conveyances in fully fuzzy transportation problems. But, in real life problems both the intermediate nodes and additional conveyances are used simultaneously. So, in this chapter, by combining the concept of fully fuzzy solid transportation problems and fully fuzzy transshipment problems, new type of problems, named as fully fuzzy solid transshipment problems, its fuzzy linear programming formulation and two new methods for finding its fuzzy optimal solution, are proposed. The advantages of the proposed methods over the methods, proposed in previous chapters and over the existing method [70], are discussed. To illustrate the methods, proposed in this chapter, a fully fuzzy solid transshipment problem is solved.

Chapter 7

Finally, in this chapter, based on the present study, conclusions are drawn and future work have been suggested.

Chapter 2

New Methods For Solving Fully Fuzzy Transportation Problems With Trapezoidal Fuzzy Parameters

To the best of my knowledge, only the methods [14, 66, 134, 161, 162, 191, 192], are proposed in the literature to find the fuzzy optimal solution of fully fuzzy transportation problems. In this chapter, the shortcomings of all these existing methods are pointed out and to overcome these shortcomings two new methods are proposed for solving fully fuzzy transportation problems. The advantages of the proposed methods over existing methods are discussed. To illustrate the proposed methods, an existing fully fuzzy transportation problem is solved. Also, to show the application of proposed methods in real life problems the fuzzy optimal solution of an existing real life fuzzy transportation problem is obtained by using the proposed methods.

The contents of this chapter are published in *Operational Research : An International Journal* 11 (2011) DOI 10.1007/s12351-010-0101-3 which are generalization of the contents of my published papers *Applied Mathematical Modelling* 35 (2011) 5652-5661 and *Applied Soft Computing* 12 (2012) 1201-1213.

2.1 Preliminaries

In this section, some basic definitions and arithmetic operations of trapezoidal fuzzy numbers are presented [104].

2.1.1 Basic definitions

In this section, some basic definitions are presented.

Definition 2.1 Let X be a classical set of objects. Then, the set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$, where $\mu_{\tilde{A}} : X \to [0,1]$, is called a fuzzy set in X. The evaluation function $\mu_{\tilde{A}}(x)$ is called the membership function.

Definition 2.2 Let $\tilde{A}$ be a fuzzy set in X and $\lambda \in [0,1]$ be a real number. Then, the classical set $A^{\lambda} = \{x \in X : \mu_{\tilde{A}}(x) \geq \lambda\}$ is called a λ-cut of $\tilde{A}$.

Definition 2.3 A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ is called a normalized fuzzy set if and only if $\underset{x \in X}{\text{Supremum}}\{\mu_{\tilde{A}}(x)\} = 1$.

Definition 2.4 A fuzzy set $\tilde{A}$ is called a convex fuzzy set if and only if

$\mu_{\tilde{A}}(\alpha x_1 + (1-\alpha)x_2) \geq \text{Minimum}\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\},\ \forall\ x_1, x_2 \in X,\ \alpha \in [0,1]$.

Definition 2.5 A convex normalized fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in \mathbb{R}\}$ on the real line $\mathbb{R}$ is called a fuzzy number if and only if $\mu_{\tilde{A}}(x)$ is piecewise continuous in $\mathbb{R}$.

Definition 2.6 A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0,\ \forall\ x < 0$.

Definition 2.7 A fuzzy number $\tilde{A}$ defined on the universal set of real numbers $\mathbb{R}$, denoted as $\tilde{A} = (a, b, c)$, is said to be a triangular fuzzy number if its membership function, $\mu_{\tilde{A}}(x)$, is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{(x-a)}{(b-a)} & , \quad a \leq x < b \\ 1 & , \quad x = b \\ \frac{(x-c)}{(b-c)} & , \quad b < x \leq c \\ 0 & , \quad \text{otherwise} \end{cases}$$

Definition 2.8 A fuzzy number $\tilde{A}$ defined on the universal set of real numbers $\mathbb{R}$, denoted as $\tilde{A} = (a, b, c, d)$, is said to be a trapezoidal fuzzy number if its membership function, $\mu_{\tilde{A}}(x)$, is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{(x-a)}{(b-a)} & , \quad a \leq x < b \\ 1 & , \quad b \leq x \leq c \\ \frac{(x-d)}{(c-d)} & , \quad c < x \leq d \\ 0 & , \quad \text{otherwise} \end{cases}$$

Definition 2.9 Let $\tilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number. Then, its λ-cut A^{λ} is defined as follows:

$$A^{\lambda} = [a + (b - a)\lambda, d - (d - c)\lambda], \; 0 \leq \lambda \leq 1$$

Definition 2.10 A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be non-negative trapezoidal fuzzy number if and only if $a \geq 0$.

Definition 2.11 A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be zero trapezoidal fuzzy number if and only if $a = 0, b = 0, c = 0$ and $d = 0$.

Definition 2.12 Two trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ are said to be equal i.e., $\tilde{A}_1 = \tilde{A}_2$ if and only if $a_1 = a_2, b_1 = b_2, c_1 = c_2$ and $d_1 = d_2$.

2.1.2 Arithmetic operations

In this section, some arithmetic operations between two trapezoidal fuzzy numbers, defined on universal set of real numbers $\mathbb{R}$, are presented.

(i) Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then, $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$.

(ii) Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two non-negative trapezoidal fuzzy numbers. Then, $\tilde{A}_1 \otimes \tilde{A}_2 \simeq (a_1a_2, b_1b_2, c_1c_2, d_1d_2)$

(iii) Let $\tilde{A} = (a, b, c, d)$ be trapezoidal fuzzy number. Then,

$$\gamma\tilde{A} = \begin{cases} (\gamma a, \gamma b, \gamma c, \gamma d) & \gamma \geq 0 \\ (\gamma d, \gamma c, \gamma b, \gamma a) & \gamma \leq 0 \end{cases}$$

Remark 2.1 If $b = c$ then a trapezoidal fuzzy number (a, b, c, d) is said to be triangular fuzzy number and is denoted as (a, b, b, d) or (a, b, d) or (a, c, d).

2.2 Fuzzy linear programming formulation of balanced fully fuzzy transportation problems

The fuzzy linear programming formulation of balanced fully fuzzy transportation problem can be written as [134]:

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij})$$

subject to

$$\begin{aligned} &\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, && i = 1, 2, 3, ..., m \\ &\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, && j = 1, 2, 3, ..., n \\ &\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j \\ &\tilde{x}_{ij} \text{ is a non-negative fuzzy number} \end{aligned} \qquad (P_{2.1})$$

where, m : total number of sources

n : total number of destinations

$\tilde{a}_i$: the fuzzy availability of the product at i^{th} source (S_i)

$\tilde{b}_j$: the fuzzy demand of the product at j^{th} destination (D_j)

$\tilde{c}_{ij}$: the fuzzy cost for transporting one unit quantity of the product from i^{th} source (S_i) to j^{th} destination (D_j)

$\tilde{x}_{ij}$: the fuzzy quantity of the product that should be transported from

i^{th} source (S_i) to j^{th} destination (D_j) (or fuzzy decision variables) to minimize the total fuzzy transportation cost

$\sum_{i=1}^{m} \tilde{a}_i$: total fuzzy availability of the product

$\sum_{j=1}^{n} \tilde{b}_j$: total fuzzy demand of the product

$\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$: total fuzzy transportation cost

Remark 2.2 If $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$ then the fully fuzzy transportation problem is said to be balanced fully fuzzy transportation problem otherwise it is called unbalanced fully fuzzy transportation problem.

2.3 Existing methods

To the best of my knowledge, only the methods [14, 66, 134, 161, 162, 191, 192], are proposed in the literature to find the fuzzy optimal solution of fully fuzzy transportation problems.

In the existing method [134] all the parameters are represented by non-negative trapezoidal fuzzy numbers. While, in the existing methods [14, 66, 161, 162, 191, 192] either the cost parameters or the obtained fuzzy optimal solution (the fuzzy quantity of the product that should be transported from different sources to different destinations to minimize the total fuzzy transportation cost) are not non-negative fuzzy numbers and hence the obtained minimum total fuzzy transportation cost is not a non-negative fuzzy number.

Since, in the real life problems there is no physical meaning of negative cost and the negative quantity of the product. So, it is not genuine to apply the existing methods [14, 66, 161, 162, 191, 192] for solving fully fuzzy transportation problems.

In this section, only the existing method [134] for solving fully fuzzy transportation problems, in which all the parameters are represented by non-negative trapezoidal fuzzy numbers, is discussed.

2.3.1 Liu and Kao method

Liu and Kao [134] proposed a new method for solving such fully fuzzy transportation problems with inequality and equality constraints in which the parameters are either represented by triangular fuzzy numbers or trapezoidal fuzzy numbers. In this section, the existing method [134] for solving fully fuzzy transportation problems is presented.

2.3.1.1 Fully fuzzy transportation problems with inequality constraints

The fuzzy optimal solution of fully fuzzy transportation problems with inequality constraints having m sources and n destinations can be obtained by using the following steps of the existing method [134]:

Step 1 Find the α-cuts $[(c_{ij})^L_\alpha, (c_{ij})^U_\alpha]$, $[(a_i)^L_\alpha, (a_i)^U_\alpha]$, $[(b_j)^L_\alpha, (b_j)^U_\alpha]$ of $\tilde{c}_{ij}$, $\tilde{a}_i$ and $\tilde{b}_j$ respectively.

Step 2 Check that $\sum\limits_{i=1}^{m}(a_i)^L_{\alpha=0} \geq \sum\limits_{j=1}^{n}(b_j)^U_{\alpha=0}$ or not.

Case (i) If $\sum\limits_{i=1}^{m}(a_i)^L_{\alpha=0} \geq \sum\limits_{j=1}^{n}(b_j)^U_{\alpha=0}$ then the chosen problem is feasible and Go to Step 3.

Case (ii) If $\sum\limits_{i=1}^{m}(a_i)^L_{\alpha=0} < \sum\limits_{j=1}^{n}(b_j)^U_{\alpha=0}$ then the chosen problem is infeasible.

Step 3 Solve the problem $(P_{2.2})$ to find the left end point $(x_{ij})^L_\alpha$ of α-cut of fuzzy decision variable $\tilde{x}_{ij}$ and the left end point (Z^L_α) of α-cut of minimum total fuzzy transportation cost $\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij})$ corresponding to different values of $\alpha \in [0,1]$.

$$Z_\alpha^L = \underset{\substack{(c_{ij})_\alpha^L \le c_{ij} \le (c_{ij})_\alpha^U \\ (a_i)_\alpha^L \le a_i \le (a_i)_\alpha^U \\ (b_j)_\alpha^L \le b_j \le (b_j)_\alpha^U \\ \forall \ i,\ j}}{\text{Minimize}} \begin{cases} \text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{subject to} \\ \sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1,2,...,m \\ \sum_{i=1}^{m} x_{ij} \ge b_j, \quad j = 1,2,...,n \\ x_{ij} \ge 0 \quad \forall \ i,j \end{cases} \qquad (P_{2.2})$$

Step 4 Solve the problem $(P_{2.3})$ to find the right end point $(x_{ij})_\alpha^U$ of α-cut of fuzzy decision variable $\tilde{x}_{ij}$ and the right end point (Z_α^U) of α-cut of minimum total fuzzy transportation cost $\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij})$ corresponding to different values of $\alpha \in [0,1]$.

$$Z_\alpha^U = \underset{\substack{(c_{ij})_\alpha^L \le c_{ij} \le (c_{ij})_\alpha^U \\ (a_i)_\alpha^L \le a_i \le (a_i)_\alpha^U \\ (b_j)_\alpha^L \le b_j \le (b_j)_\alpha^U \\ \forall \ i,\ j}}{\text{Maximize}} \begin{cases} \text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{subject to} \\ \sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1,2,...,m \\ \sum_{i=1}^{m} x_{ij} \ge b_j, \quad j = 1,2,...,n \\ x_{ij} \ge 0 \quad \forall \ i,j \end{cases} \qquad (P_{2.3})$$

Step 5 Use the values of $(x_{ij})_\alpha^L$, $(x_{ij})_\alpha^U$, Z_α^L and Z_α^U, obtained from Step 3 and Step 4, to find the α-cuts $[(x_{ij})_\alpha^L,\ (x_{ij})_\alpha^U]$ and $[Z_\alpha^L,\ Z_\alpha^U]$ corresponding to optimal fuzzy quantity of the product $\tilde{x}_{ij}$ and minimum total fuzzy transportation cost $\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij})$.

2.3.1.2 Fully fuzzy transportation problems with equality constraints

The fuzzy optimal solution of fully fuzzy transportation problems with equality constraints having m sources and n destinations can be obtained by using the following steps of the existing method [134]:

Step 1 Use Step 1 and Step 2 of the existing method [134], discussed in Section 2.3.1.1, to check the feasibility of the chosen fully fuzzy transportation problem.

Step 2 If the chosen fully fuzzy transportation problem is feasible then solve the problem $(P_{2.4})$ to find the left end point $(x_{ij})^L_\alpha$ of α-cut of fuzzy decision variable $\tilde{x}_{ij}$ and the left end point (Z^L_α) of α-cut of minimum total fuzzy transportation cost $\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij})$ corresponding to different values of $\alpha \in [0,1]$.

$$Z^L_\alpha = \underset{\substack{(c_{ij})^L_\alpha \le c_{ij} \le (c_{ij})^U_\alpha \\ (a_i)^L_\alpha \le a_i \le (a_i)^U_\alpha \\ (b_j)^L_\alpha \le b_j \le (b_j)^U_\alpha \\ \forall \;\; i,\; j}}{\text{Minimize}} \left\{ \begin{array}{lll} \text{Minimize} & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} & \\ \text{subject to} & & \\ \sum_{j=1}^{n} x_{ij} = a_i, & i = 1,2,...,m & \\ \sum_{i=1}^{m} x_{ij} = b_j, & j = 1,2,...,n & \\ x_{ij} \ge 0 & \forall \;\; i,j & \end{array} \right. \qquad (P_{2.4})$$

Step 3 Solve the problem $(P_{2.5})$ to find the right end point $(x_{ij})^U_\alpha$ of α- cut of fuzzy decision variable $\tilde{x}_{ij}$ and the right end point (Z^U_α) of α-cut of minimum total fuzzy transportation cost $\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij})$ corresponding to different values of $\alpha \in [0,1]$.

$$Z^U_\alpha = \underset{\substack{(c_{ij})^L_\alpha \le c_{ij} \le (c_{ij})^U_\alpha \\ (a_i)^L_\alpha \le a_i \le (a_i)^U_\alpha \\ (b_j)^L_\alpha \le b_j \le (b_j)^U_\alpha \\ \forall \;\; i,\; j}}{\text{Maximize}} \left\{ \begin{array}{lll} \text{Minimize} & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} & \\ \text{subject to} & & \\ \sum_{j=1}^{n} x_{ij} = s_i, & i = 1,2,...,m & \\ \sum_{i=1}^{m} x_{ij} = d_j, & j = 1,2,...,n & \\ x_{ij} \ge 0 & \forall \;\; i,j & \end{array} \right. \qquad (P_{2.5})$$

Step 4 Use the values of $(x_{ij})^L_\alpha$, $(x_{ij})^U_\alpha$, Z^L_α and Z^U_α, obtained from Step 2 and Step 3, to find the α-cuts of $[(x_{ij})^L_\alpha, (x_{ij})^U_\alpha]$ and $[Z^L_\alpha, Z^U_\alpha]$ corresponding to optimal fuzzy quantity of the product $\tilde{x}_{ij}$ and and minimum total fuzzy transportation cost $\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij})$.

2.4 Shortcomings of existing methods

In this section, the shortcomings of the existing methods [14, 66, 134, 161, 162, 191, 192] for solving fully fuzzy transportation problems are pointed out:

(1) To show the shortcomings of the existing methods [102, 164], Liu and Kao [134] solved the fuzzy transportation problem, presented in Example 2.1, by using the existing methods [102, 164].

Example 2.1 [134, pp. 663] Consider a fuzzy transportation problem with inequality constraints having two sources and two destinations. Availabilities, demands and the costs are as follows:

Availabilities: $\tilde{a}_1 = (2, 3, 5)$, $a_2 = 5$.

Demands: $b_1 = 4$, $\tilde{b}_2 = (1, 3, 6)$.

Costs: $c_{11} = 1$, $c_{12} = 3, c_{21} = 7, c_{22} = 2$.

According to Liu and Kao [134], the optimal solution of fuzzy transportation problem, presented in Example 2.1, is $Z_0^L = 18$ and $Z_0^U = 17$, where Z_0^L and Z_0^U are the values of left and right end points of $\alpha-$cut $[Z_\alpha^L, Z_\alpha^U]$ of minimum total fuzzy transportation cost $\tilde{Z}$ at $\alpha = 0$. Liu and Kao [134] pointed out that $Z_0^L > Z_0^U$ which contradicts the existing result $Z_\alpha^U \geq Z_\alpha^L \ \forall \ \alpha \in [0, 1]$.

To overcome the shortcomings of the existing methods [102, 164], Liu and Kao [134], proposed a new method for solving fully fuzzy transportation problems but there are the following shortcomings in the existing method [134]:

(a) Liu and Kao [134] solved the fully fuzzy transportation problem, presented in Example 2.2, to illustrate their method.

Example 2.2 [134, pp. 671] Consider a fully fuzzy transportation problem with equality constraints having two sources and three destinations. Fuzzy availabilities, fuzzy demands and the costs are as follows:

Fuzzy availabilities: $\tilde{a}_1 = (70, 90, 100)$, $\tilde{a}_2 = (40, 60, 70, 80)$.

Fuzzy demands: $\tilde{b}_1 = (30, 40, 50, 70)$, $\tilde{b}_2 = (20, 30, 40, 50), \tilde{b}_3 = (40, 50, 80)$.

Costs: $c_{11} = 10$, $c_{12} = 50, c_{13} = 80, \tilde{c}_{21} = (60, 70, 80, 90), c_{22} = 60, c_{23} = 20$.

Liu and Kao [134] claimed that on solving the fully fuzzy transportation problem, presented in Example 2.2, at $\alpha = 0$, the optimal solution $x_{11}^L = 50$ and $x_{11}^U = 30$ is obtained. Since, $x_{11}^L > x_{11}^U$ so the same shortcoming, pointed out by Liu and Kao [134], in the existing methods [102, 164], is also occurring in the method proposed by Liu and Kao [134].

(b) Liu and Kao [134] claimed that on solving the fully fuzzy transportation problem, presented in Example 2.2, values of Z_α^L and Z_α^U for $\alpha = 0.9$ are 3680 and for $\alpha = 1$ the solution is infeasible. Liu and Kao [134] also claimed that, since $Z_\alpha^L = Z_\alpha^U = 3680$ for $\alpha = 0.9$ and for higher values of α the obtained solution is infeasible. So, the obtained fuzzy optimal solution will be triangular fuzzy number with maximum membership degree 0.9. However, there may also exist a fully fuzzy transportation problem in which $Z_\alpha^L \neq Z_\alpha^U$ for $\alpha = 0.9$ but solution is infeasible for $\alpha = 1$. In such a situation, there will exist infinite values of α between 0.9 and 1 but in such a case it is neither possible to find the maximum degree of membership nor to find the transportation cost corresponding to which the maximum degree of membership will exist i.e., in such a case the obtained solution can be represented by an interval but not a triangular fuzzy number.

(2) Stephen Dinagar and Palanivel [191] proposed a method for solving fully fuzzy transportation problems and solved the fully fuzzy transportation problem,

presented in Example 2.3, to illustrate the proposed method.

Example 2.3 [191, pp. 69] Consider a fully fuzzy transportation problem having three sources and four destinations. Fuzzy availabilities, fuzzy demands and the fuzzy costs are as follows:

Fuzzy availabilities: $\tilde{a}_1 = (0, 2, 4, 6)$, $\tilde{a}_2 = (2, 4, 9, 13)$, $\tilde{a}_3 = (2, 4, 6, 8)$.

Fuzzy demands: $\tilde{b}_1 = (1, 3, 5, 7)$, $\tilde{b}_2 = (0, 2, 4, 6)$, $\tilde{b}_3 = (1, 3, 5, 7)$, $\tilde{b}_4 = (1, 3, 5, 7)$.

Fuzzy costs: $\tilde{c}_{11} = (-2, 0, 2, 8)$, $\tilde{c}_{12} = (-2, 0, 2, 8), \tilde{c}_{13} = (-2, 0, 2, 8), \tilde{c}_{14} = (-1, 0, 1, 4), \tilde{c}_{21} = (4, 8, 12, 16), \tilde{c}_{22} = (4, 7, 9, 12), \tilde{c}_{23} = (2, 4, 6, 8), \tilde{c}_{24} = (1, 3, 5, 7)$, $\tilde{c}_{31} = (2, 4, 9, 13), \tilde{c}_{32} = (0, 6, 8, 10), \tilde{c}_{33} = (0, 6, 8, 10), \tilde{c}_{34} = (4, 7, 9, 12)$.

It is obvious that there exist negative part in the trapezoidal fuzzy numbers $\tilde{c}_{11}, \tilde{c}_{12}, \tilde{c}_{13}$ and $\tilde{c}_{14}$, representing the fuzzy costs for transporting one unit quantity of the product from first source to all destinations, which depicts that the transportation cost can be negative.

Also, Stephen Dinagar and Palanivel [191] claimed that on solving the fully fuzzy transportation problem, presented in Example 2.3, the following results are obtained:

(a) The fuzzy optimal quantity of the product that should be transported from second source to third destination, third source to first destination and third source to third destination are $(-5, -1, 6, 12)$, $(-5, -1, 3, 7)$, and $(-11, -3, 6, 12)$ respectively.

(b) The minimum total fuzzy transportation cost is $(-122, -2, 139, 257)$.

It is clear that there exist negative part in all the obtained trapezoidal fuzzy numbers. Similarly, there exist negative part in the results obtained by using the existing methods [14, 66, 161, 162, 192].

Since, in the real life problems there is no physical meaning of negative cost and the negative quantity of the product. So, it is not genuine to apply the existing methods [14, 66, 161, 162, 191, 192] for solving fully fuzzy transportation problems.

Also, the fuzzy optimal solution, obtained by using the existing methods [14, 66, 161, 162, 191, 192], does not satisfy the constraints exactly i.e., on putting the values of fuzzy decision variables, obtained by using the existing methods, in the left hand side of the constraints the value at right hand side are not obtained e.g., Stephen Dinagar and Palanivel [191] claimed that on solving the fully fuzzy transportation problem, presented in Example 2.3, the obtained fuzzy optimal quantity of the product that should be transported from second source to third and fourth destinations are $(-5, -1, 6, 12)$ and (1,3,5,7) respectively. It is obvious that the sum of these quantities is not exactly equal to the the fuzzy availability $\tilde{a}_2 = (2, 4, 9, 13)$ of the product at second source i.e., the constraint $\sum_{j=1}^{4} \tilde{x}_{2j} = \tilde{a}_2$ is not exactly satisfying.

2.5 Proposed methods

In this section, to overcome all the shortcomings of the existing methods [14, 66, 134, 161, 162, 191, 192], discussed in Section 2.4, two new methods (based on fuzzy linear programming formulation and based on tabular representation) are proposed for solving such fully fuzzy transportation problems in which all the parameters are represented by trapezoidal fuzzy numbers. Also, the advantages of the proposed methods over existing methods [14, 66, 134, 161, 162, 191, 192] are discussed.

2.5.1 Method based on fuzzy linear programming formulation

In this section, a new method, based on the fuzzy linear programming formulation of the fully fuzzy transportation problems, is proposed to find the fuzzy optimal solution of fully fuzzy transportation problems.

The steps of the proposed method are as follow:

Step 1 Find the total fuzzy availability $\sum_{i=1}^{m} \tilde{a}_i$ and the total fuzzy demand $\sum_{j=1}^{n} \tilde{b}_j$. Let $\sum_{i=1}^{m} \tilde{a}_i = (a, b, c, d)$ and $\sum_{j=1}^{n} \tilde{b}_j = (a', b', c', d')$. Use Definition 2.12 to examine that the problem is balanced or not, i.e., $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$ or $\sum_{i=1}^{m} \tilde{a}_i \neq \sum_{j=1}^{n} \tilde{b}_j$.

Case (i) If the problem is balanced i.e., $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$, then Go to Step 2.

Case (ii) If $\sum_{i=1}^{m} \tilde{a}_i \neq \sum_{j=1}^{n} \tilde{b}_j$ then convert the unbalanced problem into balanced problem as follows:

Case (a) If $a \leq a'$, $b - a \leq b' - a'$, $c - b \leq c' - b'$ and $d - c \leq d' - c'$ then introduce a dummy source with fuzzy availability $(a' - a, b' - b, c' - c, d' - d)$. Assume the fuzzy cost for transporting one unit quantity of the product from the introduced dummy source to all destinations as zero trapezoidal fuzzy number. Go to Step 2.

Case (b) If $a \geq a'$, $b - a \geq b' - a'$, $c - b \geq c' - b'$ and $d - c \geq d' - c'$ then introduce a dummy destination with fuzzy demand $(a - a', b - b', c - c', d - d')$. Assume the fuzzy cost for transporting one unit quantity of the product from all sources to the introduced dummy destination as zero trapezoidal fuzzy number. Go to Step 2.

Case (c) If neither Case (a) nor Case (b) is satisfied then introduce a dummy source with fuzzy availability $\Big($maximum $\{0, a' - a\}$, maximum $\{0, (a' - a)\}$ + maximum $\{0, (b' - a') - (b - a)\}$, maximum $\{0, a' - a\}$ + maximum $\{0, (b' - a') - (b - a)\}$ +

maximum $\{0, (c' - b') - (c - b)\}$, maximum $\{0, (a' - a)\}$ + maximum $\{0, (b' - a') - (b - a)\}$ + maximum $\{0, (c' - b') - (c - b)\}$ + maximum $\{0, (d' - c') - (d - c)\}\big)$ and a dummy destination with fuzzy demand (maximum $\{0, (a - a')\}$, maximum $\{0, (a - a')\}$ + maximum $\{0, (b - a) - (b' - a')\}$, maximum $\{0, (a - a')\}$ + maximum $\{0, (b - a) - (b' - a')\}$ + maximum $\{0, (c - b) - (c' - b')\}$, maximum $\{0, (a - a')\}$ + maximum $\{0, (b - a) - (b' - a')\}$ + maximum $\{0, (c - b) - (c' - b')\}$ + maximum $\{0, (d - c) - (d' - c')\}$). Assume the fuzzy cost for transporting one unit quantity of the product from the introduced dummy source to all destinations and from all sources to the introduced dummy destination as zero trapezoidal fuzzy number. Go to Step 2.

Step 2 Formulate the balanced fully fuzzy transportation problem, obtained in Step 1, into the fuzzy linear programming problem $(P_{2.1})$.

Step 3 Assuming $\tilde{c}_{ij} = (a'_{ij}, b'_{ij}, c'_{ij}, d'_{ij})$, $\tilde{a}_i = (a_i, b_i, c_i, d_i)$, $\tilde{b}_j = (a'_j, b'_j, c'_j, d'_j)$ and $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$, the fuzzy linear programming problem $(P_{2.1})$, can be written as:

$$\text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\Big((a'_{ij}, b'_{ij}, c'_{ij}, d'_{ij}) \otimes (a_{ij}, b_{ij}, c_{ij}, d_{ij})\Big)$$

subject to

$$\sum_{j=1}^{n}(a_{ij}, b_{ij}, c_{ij}, d_{ij}) = (a_i, b_i, c_i, d_i), \qquad i = 1, 2, 3, ..., m$$

$$\sum_{i=1}^{m}(a_{ij}, b_{ij}, c_{ij}, d_{ij}) = (a'_j, b'_j, c'_j, d'_j), \qquad j = 1, 2, 3, ..., n$$

$(a_{ij}, b_{ij}, c_{ij}, d_{ij})$ is a non-negative trapezoidal fuzzy number.

Step 4 Using the arithmetic operations, defined in Section 2.1.2, the fuzzy linear programming problem, obtained in Step 3, can be written as:

$$\text{Minimize} \quad \Big(\sum_{i=1}^{m}\sum_{j=1}^{n} a'_{ij}a_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} b'_{ij}b_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} c'_{ij}c_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} d'_{ij}d_{ij}\Big)$$

subject to

$$\Big(\sum_{j=1}^{n} a_{ij}, \sum_{j=1}^{n} b_{ij}, \sum_{j=1}^{n} c_{ij}, \sum_{j=1}^{n} d_{ij}\Big) = (a_i, b_i, c_i, d_i), \quad i = 1, 2, 3, ..., m$$
$$\Big(\sum_{i=1}^{m} a_{ij}, \sum_{i=1}^{m} b_{ij}, \sum_{i=1}^{m} c_{ij}, \sum_{i=1}^{m} d_{ij}\Big) = (a'_j, b'_j, c'_j, d'_j), \quad j = 1, 2, 3, ..., n$$

$(a_{ij}, b_{ij}, c_{ij}, d_{ij})$ is a non-negative trapezoidal fuzzy number.

Step 5 Using Definition 2.10 and Definition 2.12, the fuzzy linear programming problem, obtained in Step 4, can be converted into fuzzy linear programming problem $(P_{2.6})$:

$$\text{Minimize} \ \Big(\sum_{i=1}^{m}\sum_{j=1}^{n} a'_{ij}a_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} b'_{ij}b_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} c'_{ij}c_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} d'_{ij}d_{ij}\Big)$$

subject to

$$\begin{aligned}
&\sum_{j=1}^{n} a_{ij} = a_i \ , && i = 1, 2, 3, ..., m \\
&\sum_{j=1}^{n} b_{ij} = b_i, && i = 1, 2, 3, ..., m \\
&\sum_{j=1}^{n} c_{ij} = c_i, && i = 1, 2, 3, ..., m \\
&\sum_{j=1}^{n} d_{ij} = d_i, && i = 1, 2, 3, ..., m \qquad (P_{2.6}) \\
&\sum_{i=1}^{m} a_{ij} = a'_j, && j = 1, 2, 3, ..., n \\
&\sum_{i=1}^{m} b_{ij} = b'_j, && j = 1, 2, 3, ..., n \\
&\sum_{i=1}^{m} c_{ij} = c'_j, && j = 1, 2, 3, ..., n \\
&\sum_{i=1}^{m} d_{ij} = d'_j, && j = 1, 2, 3, ..., n
\end{aligned}$$

$$a_{ij}, b_{ij} - a_{ij}, c_{ij} - b_{ij}, d_{ij} - c_{ij} \geq 0 \qquad \forall \ i = 1, 2, ..., m; j = 1, 2, ..., n$$

Step 6 Suppose the fuzzy linear programming problem $(P_{2.6})$ has h basic feasible solutions and $\{a^w_{ij}, b^w_{ij}, c^w_{ij}, d^w_{ij}\}$ be the w^{th} basic feasible solution then the aim is to find the feasible solution with the smallest objective value i.e., $\underset{1\leq w\leq h}{\text{minimum}} \Big\{\Big(\sum_{i=1}^{m}\sum_{j=1}^{n} a'_{ij}a^w_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} b'_{ij}b^w_{ij}, \ \sum_{i=1}^{m}\sum_{j=1}^{n} c'_{ij}c^w_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} d'_{ij}d^w_{ij}\Big)\Big\}$.

Although, till now there is no unique way to compare fuzzy numbers but several authors [30, 38, 53, 58, 66, 109, 116, 135, 146, 147, 148, 153, 161, 162, 163, 191, 192] have used the concept that if $\underset{1\leq w\leq h}{\text{minimum}}\Big\{\Re\Big(\sum_{i=1}^{m}\sum_{j=1}^{n}a'_{ij}a^{w}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}b'_{ij}b^{w}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}c'_{ij}c^{w}_{ij},\sum_{i=1}^{m}\sum_{j=1}^{n}d'_{ij}d^{w}_{ij}\Big)\Big\}$ is $\Re\Big(\sum_{i=1}^{m}\sum_{j=1}^{n}a'_{ij}a^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}b'_{ij}b^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}c'_{ij}c^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}d'_{ij}d^{\eta}_{ij}\Big)$ then $\underset{1\leq w\leq h}{\text{minimum}}\Big\{\Big(\sum_{i=1}^{m}\sum_{j=1}^{n}a'_{ij}a^{w}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}b'_{ij}b^{w}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}c'_{ij}c^{w}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}d'_{ij}d^{w}_{ij}\Big)\Big\}$ will also be $\Big(\sum_{i=1}^{m}\sum_{j=1}^{n}a'_{ij}a^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}b'_{ij}b^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}c'_{ij}c^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}d'_{ij}d^{\eta}_{ij}\Big)$, where, $\Re\Big(\sum_{i=1}^{m}\sum_{j=1}^{n}a'_{ij}a^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}b'_{ij}b^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}c'_{ij}c^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}d'_{ij}d^{\eta}_{ij}\Big) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}a'_{ij}a^{\eta}_{ij}+\sum_{i=1}^{m}\sum_{j=1}^{n}b'_{ij}b^{\eta}_{ij}+\sum_{i=1}^{m}\sum_{j=1}^{n}c'_{ij}c^{\eta}_{ij}+\sum_{i=1}^{m}\sum_{j=1}^{n}d'_{ij}d^{\eta}_{ij}}{4}$ represents the Liou and Wang ranking index [131] of a trapezoidal fuzzy number $(\sum_{i=1}^{m}\sum_{j=1}^{n}a'_{ij}a^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}b'_{ij}b^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}c'_{ij}c^{\eta}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n}d'_{ij}d^{\eta}_{ij})$.

In other words, according to existing methods [30, 38, 53, 58, 66, 109, 116, 135, 146, 147, 148, 153, 161, 162, 163, 191, 192] the fuzzy optimal solution of the fuzzy linear programming problem ($P_{2.6}$) can be obtained by solving the following crisp linear programming problem:

$$\text{Minimize}\left(\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}a'_{ij}a_{ij}+\sum_{i=1}^{m}\sum_{j=1}^{n}b'_{ij}b_{ij}+\sum_{i=1}^{m}\sum_{j=1}^{n}c'_{ij}c_{ij}+\sum_{i=1}^{m}\sum_{j=1}^{n}d'_{ij}d_{ij}}{4}\right)$$

subject to

constraints of the fuzzy linear programming problem ($P_{2.6}$).

Step 7 Solve the crisp linear programming problem, obtained in Step 6, to find the optimal solution $\{a_{ij}, b_{ij}, c_{ij}, d_{ij}\}$.

Step 8 Find the fuzzy optimal solution $\{\tilde{x}_{ij}\}$ by putting the values of $a_{ij}, b_{ij}, c_{ij}, d_{ij}$ in $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$.

Step 9 Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ij}$ in $\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}\otimes\tilde{x}_{ij})$.

Remark 2.3 If $\sum_{i=1}^{m}\tilde{a}_i \neq \sum_{j=1}^{n}\tilde{b}_j$ i.e., $(a, b, c, d) \neq (a', b', c', d')$ and neither Case (a) $a \leq a'$, $b-a \leq b'-a'$, $c-b \leq c'-b'$, $d-c \leq d'-c'$ nor Case (b) $a \geq a'$, $b-a \geq b'-a'$, $c-b \geq c'-b'$, $d-c \geq d'-c'$ is satisfied then there may exist infinite non-negative trapezoidal fuzzy numbers (a_1, b_1, c_1, d_1) and (a'_1, b'_1, c'_1, d'_1) such that $(a, b, c, d) \oplus (a_1, b_1, c_1, d_1) = (a', b', c', d') \oplus (a'_1, b'_1, c'_1, d'_1)$ but the aim is to be find the trapezoidal fuzzy numbers (a_1, b_1, c_1, d_1) and (a'_1, b'_1, c'_1, d'_1) which satisfies the following characteristics:

(i) (a_1, b_1, c_1, d_1) and (a'_1, b'_1, c'_1, d'_1) are non-negative trapezoidal fuzzy numbers

(ii) $(a, b, c, d) \oplus (a_1, b_1, c_1, d_1) = (a', b', c', d') \oplus (a'_1, b'_1, c'_1, d'_1)$

(iii) If there exist non-negative trapezoidal fuzzy numbers (x, y, z, w) and (x', y', z', w') such that $(a, b, c, d) \oplus (x, y, z, w) = (a', b', c', d') \oplus (x', y', z', w')$ then $\Re(x, y, z, w) \geq \Re(a_1, b_1, c_1, d_1)$ and $\Re(x', y', z', w') \geq \Re(a'_1, b'_1, c'_1, d'_1)$.

2.5.2 Method based on tabular representation

In this section, a new method, based on the tabular representation of the fully fuzzy transportation problems, is proposed to find the fuzzy optimal solution of fully fuzzy transportation problems.

The steps of the proposed method are as follows:

Step 1 Use Step 1 of the method, proposed in Section 2.5.1, to obtain a balanced fully fuzzy transportation problem.

Step 2 Represent the balanced fully fuzzy transportation problem, obtained in Step 1, into tabular form as shown in Table 2.1.

Table 2.1 Tabular representation of balanced fully fuzzy transportation problem

Destinations→ Sources↓	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	Availability ($\tilde{a}_i$)
S_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\cdots$	$\tilde{c}_{1j}$	$\cdots$	$\tilde{c}_{1n}$	$\tilde{a}_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	$\tilde{c}_{i1}$	$\tilde{c}_{i2}$	$\cdots$	$\tilde{c}_{ij}$	$\cdots$	$\tilde{c}_{in}$	$\tilde{a}_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	$\cdots$	$\tilde{c}_{mj}$	$\cdots$	$\tilde{c}_{mn}$	$\tilde{a}_m$
Demand ($\tilde{b}_j$)	$\tilde{b}_1$	$\tilde{b}_2$	$\cdots$	$\tilde{b}_j$	$\cdots$	$\tilde{b}_n$	$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$

Step 3 Split Table 2.1 into four crisp transportation tables i.e., Table 2.2, Table 2.3, Table 2.4 and Table 2.5 respectively.

Table 2.2 Tabular representation of first crisp transportation problem

Destinations→ Sources↓	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	a_i
S_1	λ_{11}	λ_{12}	$\cdots$	λ_{1j}	$\cdots$	λ_{1n}	a_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	λ_{i1}	λ_{i2}	$\cdots$	λ_{ij}	$\cdots$	λ_{in}	a_i
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	λ_{m1}	λ_{m2}	$\cdots$	λ_{mj}	$\cdots$	λ_{mn}	a_m
a'_j	a'_1	a'_2	$\cdots$	a'_j	$\cdots$	a'_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

where, $\lambda_{ij} = \frac{(a'_{ij}+b'_{ij}+c'_{ij}+d'_{ij})}{4}, \quad i = 1, 2, 3, ..., m$ and $j = 1, 2, 3, ..., n$

Table 2.3 Tabular representation of second crisp transportation problem

Destinations→ Sources↓	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	$b_i - a_i$
S_1	ρ_{11}	ρ_{12}	$\cdots$	ρ_{1j}	$\cdots$	ρ_{1n}	$b_1 - a_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	ρ_{i1}	ρ_{i2}	$\cdots$	ρ_{ij}	$\cdots$	ρ_{in}	$b_i - a_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	ρ_{m1}	ρ_{m2}	$\cdots$	ρ_{mj}	$\cdots$	ρ_{mn}	$b_m - a_m$
$b'_j - a'_j$	$b'_1 - a'_1$	$b'_2 - a'_2$	$\cdots$	$b'_j - a'_j$	$\cdots$	$b'_n - a'_n$	$\sum_{i=1}^{m}(b_i - a_i) = \sum_{j=1}^{n}(b'_j - a'_j)$

where, $\rho_{ij} = \frac{(b'_{ij}+c'_{ij}+d'_{ij})}{4}, \quad i = 1, 2, 3, ..., m$ and $j = 1, 2, 3, ..., n$

Table 2.4 Tabular representation of third crisp transportation problem

Destinations→ Sources↓	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	$c_i - b_i$
S_1	δ_{11}	δ_{12}	$\cdots$	δ_{1j}	$\cdots$	δ_{1n}	$c_1 - b_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	δ_{i1}	δ_{i2}	$\cdots$	δ_{ij}	$\cdots$	δ_{in}	$c_i - b_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	δ_{m1}	δ_{m2}	$\cdots$	δ_{mj}	$\cdots$	δ_{mn}	$c_m - b_m$
$c'_j - b'_j$	$c'_1 - b'_1$	$c'_2 - b'_2$	$\cdots$	$c'_j - b'_j$	$\cdots$	$c'_n - b'_n$	$\sum_{i=1}^{m}(c_i - b_i) = \sum_{j=1}^{n}(c'_j - b'_j)$

where, $\delta_{ij} = \frac{(c'_{ij}+d'_{ij})}{4}, \quad i = 1, 2, 3, ..., m$ and $j = 1, 2, 3, ..., n$

Table 2.5 Tabular representation of fourth crisp transportation problem

Destinations→ Sources↓	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	$d_i - c_i$
S_1	ξ_{11}	ξ_{12}	$\cdots$	ξ_{1j}	$\cdots$	ξ_{1n}	$d_1 - c_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	ξ_{i1}	ξ_{i2}	$\cdots$	ξ_{ij}	$\cdots$	ξ_{in}	$d_i - c_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	ξ_{m1}	ξ_{m2}	$\cdots$	ξ_{mj}	$\cdots$	ξ_{mn}	$d_m - c_m$
$d'_j - c'_j$	$d'_1 - c'_1$	$d'_2 - c'_2$	$\cdots$	$d'_j - c'_j$	$\cdots$	$d'_n - c'_n$	$\sum_{i=1}^{m}(d_i - c_i) = \sum_{j=1}^{n}(d'_j - c'_j)$

where, $\xi_{ij} = \frac{d'_{ij}}{4}$, $\quad i = 1, 2, 3, ..., m$ and $j = 1, 2, 3, ..., n$

Step 4 Solve the crisp transportation problems, shown by Table 2.2, Table 2.3, Table 2.4 and Table 2.5, to find the optimal solution $\{a_{ij}\}$, $\{b_{ij} - a_{ij}\}$, $\{c_{ij} - b_{ij}\}$ and $\{d_{ij} - c_{ij}\}$ respectively.

Step 5 Solve the equations, obtained in Step 4, to find the values of a_{ij}, b_{ij}, c_{ij} and d_{ij}.

Step 6 Find the fuzzy optimal solution $\{\tilde{x}_{ij}\}$ by putting the values of $a_{ij}, b_{ij}, c_{ij}, d_{ij}$ in $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$.

Step 7 Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ij}$ in $\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij})$.

Remark 2.4 Since, in the transportation problems negative parameters has no physical meaning. So, in the proposed methods all the parameters can be assumed as non-negative trapezoidal fuzzy numbers.

2.5.3 Advantages of the proposed methods over existing methods

In this section, the advantages of the proposed method over existing methods [14, 66, 134, 161, 162, 191, 192] are discussed:

(1) The advantages of the proposed methods over existing method [134] can be explained as follows:

(a) Since, in the proposed methods the restrictions $b-a \geq 0, c-b \geq 0$ and $d-c \geq 0$ are used and due to these restrictions the restriction, $d \geq a$ (or $x^U \geq x^L$) will always be satisfied. So, by using the proposed methods the shortcoming of the existing method [134], pointed out in 1(a) of Section 2.4, is resolved.

(b) Since, on solving the fully fuzzy transportation problems by using the proposed methods the obtained minimum total fuzzy transportation cost will always be a fuzzy number. So, by using the proposed methods the shortcoming of the existing method [134], pointed out in 1(b) of Section 2.4, is resolved.

(2) In Section 2.4, it is pointed out that on solving the fully fuzzy transportation problems by using the existing methods [14, 66, 161, 162, 191, 192] negative minimum total fuzzy transportation cost and negative quantity of the product are obtained which is not appropriate according to real life situations.

Also, it is pointed out that fuzzy optimal solution, obtained by using the existing methods does not satisfy the constraints exactly.

Since, in the proposed method all the cost parameters are represented by non-negative trapezoidal fuzzy numbers and also the restrictions $a \geq 0$ is used. So, on solving the fully fuzzy transportation problems by using the proposed methods the obtained minimum total fuzzy transportation cost and fuzzy quantity of the product will be either zero or non-negative.

Also, the fuzzy optimal solution, obtained by using the proposed methods, will always satisfy all the constraints exactly.

Hence, on applying the proposed methods all the shortcomings of the existing methods [14, 66, 134, 161, 162, 191, 192], pointed out in Section 2.4, are resolved.

2.6 Illustrative example

In this section, to illustrate the proposed methods, the existing fully fuzzy transportation problem [134], presented in Example 2.2, is solved by the proposed methods.

2.6.1 Fuzzy optimal solution using the method based on fuzzy linear programming formulation

Using the proposed method, based on fuzzy linear programming formulation, the fuzzy optimal solution of the fully fuzzy transportation problem, presented in Example 2.2, can be obtained as follows:

Step 1 Total fuzzy availability = $(110, 150, 160, 180)$ and total fuzzy demand = $(90, 120, 140, 200)$. Since, total fuzzy availability $\neq$ total fuzzy demand, so it is an unbalanced fully fuzzy transportation problem.

Now as described in the proposed method (using Case (c) of Step 1 of the proposed method, discussed in Section 2.5.1), the unbalanced fully fuzzy transportation problem can be converted into a balanced fully fuzzy transportation problem by introducing a dummy source S_3 with fuzzy availability $(0, 0, 10, 50)$ and a dummy destination D_4 with fuzzy demand $(20, 30, 30, 30)$.

Step 2 Assuming the fuzzy cost transporting for one unit quantity of the product from dummy source S_3 to all destinations and from all sources to dummy destination D_4 as zero trapezoidal fuzzy number i.e., $\tilde{c}_{14} = \tilde{c}_{24} = \tilde{c}_{31} = \tilde{c}_{32} = \tilde{c}_{33} = \tilde{c}_{34} = (0, 0, 0, 0)$, the balanced fully fuzzy transportation problem, obtained in Step 1, can be formulated into the following fuzzy linear programming problem:

Minimize $\Big((10, 10, 10, 10) \otimes \tilde{x}_{11} \oplus (50, 50, 50, 50) \otimes \tilde{x}_{12} \oplus (80, 80, 80, 80) \otimes \tilde{x}_{13} \oplus$

$(0,0,0,0) \otimes \tilde{x}_{14} \oplus (60,70,80,90) \otimes \tilde{x}_{21} \oplus (60,60,60,60) \otimes \tilde{x}_{22} \oplus (20,20,20,20) \otimes \tilde{x}_{23} \oplus$

$(0,0,0,0) \otimes \tilde{x}_{24} \oplus (0,0,0,0) \otimes \tilde{x}_{31} \oplus (0,0,0,0) \otimes \tilde{x}_{32} \oplus (0,0,0,0) \otimes \tilde{x}_{33} \oplus (0,0,0,0) \otimes \tilde{x}_{34}\Big)$

subject to

$$\tilde{x}_{11} \oplus \tilde{x}_{12} \oplus \tilde{x}_{13} \oplus \tilde{x}_{14} = (70,90,90,100)$$

$$\tilde{x}_{21} \oplus \tilde{x}_{22} \oplus \tilde{x}_{23} \oplus \tilde{x}_{24} = (40,60,70,80)$$

$$\tilde{x}_{31} \oplus \tilde{x}_{32} \oplus \tilde{x}_{33} \oplus \tilde{x}_{34} = (0,0,10,50)$$

$$\tilde{x}_{11} \oplus \tilde{x}_{21} \oplus \tilde{x}_{31} = (30,40,50,70)$$

$$\tilde{x}_{12} \oplus \tilde{x}_{22} \oplus \tilde{x}_{32} = (20,30,40,50)$$

$$\tilde{x}_{13} \oplus \tilde{x}_{23} \oplus \tilde{x}_{33} = (40,50,50,80)$$

$$\tilde{x}_{14} \oplus \tilde{x}_{24} \oplus \tilde{x}_{34} = (20,30,30,30)$$

$\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34}$ are non-negative trapezoidal fuzzy numbers.

Step 3 Assuming $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ and using arithmetic operations, defined in Section 2.1.2, the fuzzy linear programming problem, obtained in Step 2, can be converted into the following fuzzy linear programming problem:

Minimize $\Big(10a_{11} + 50a_{12} + 80a_{13} + 60a_{21} + 60a_{22} + 20a_{23},\ 10b_{11} + 50b_{12} + 80b_{13} + 70b_{21} + 60b_{22} + 20b_{23},\ 10c_{11} + 50c_{12} + 80c_{13} + 80c_{21} + 60c_{22} + 20c_{23},\ 10d_{11} + 50d_{12} + 80d_{13} + 90d_{21} + 60d_{22} + 20d_{23}\Big)$

subject to

$$\Big(\sum_{j=1}^{4} a_{1j}, \sum_{j=1}^{4} b_{1j}, \sum_{j=1}^{4} c_{1j}, \sum_{j=1}^{4} d_{1j}\Big) = (70,90,90,100)$$
$$\Big(\sum_{j=1}^{4} a_{2j}, \sum_{j=1}^{4} b_{2j}, \sum_{j=1}^{4} c_{2j}, \sum_{j=1}^{4} d_{2j}\Big) = (40,60,70,80)$$
$$\Big(\sum_{j=1}^{4} a_{3j}, \sum_{j=1}^{4} b_{3j}, \sum_{j=1}^{4} c_{3j}, \sum_{j=1}^{4} d_{3j}\Big) = (0,0,10,50)$$
$$\Big(\sum_{i=1}^{3} a_{i1}, \sum_{i=1}^{3} b_{i1}, \sum_{i=1}^{3} c_{i1}, \sum_{i=1}^{3} d_{i1}\Big) = (30,40,50,70)$$

$$\left(\sum_{i=1}^{3} a_{i2}, \sum_{i=1}^{3} b_{i2}, \sum_{i=1}^{3} c_{i2}, \sum_{i=1}^{3} d_{i2}\right) = (20, 30, 40, 50)$$
$$\left(\sum_{i=1}^{3} a_{i3}, \sum_{i=1}^{3} b_{i3}, \sum_{i=1}^{3} c_{i3}, \sum_{i=1}^{3} d_{i3}\right) = (40, 50, 50, 80)$$
$$\left(\sum_{i=1}^{3} a_{i4}, \sum_{i=1}^{3} b_{i4}, \sum_{i=1}^{3} c_{i4}, \sum_{i=1}^{3} d_{i4}\right) = (20, 30, 30, 30)$$

$(a_{ij}, b_{ij}, c_{ij}, d_{ij})$ is a non-negative trapezoidal fuzzy number.

Step 4 Using Definition 2.10 and Definition 2.12, the fuzzy linear programming problem, obtained in Step 3, can be converted into the fuzzy linear programming problem ($P_{2.7}$):

Minimize $\Big(10a_{11} + 50a_{12} + 80a_{13} + 60a_{21} + 60a_{22} + 20a_{23},\ 10b_{11} + 50b_{12} + 80b_{13} + 70b_{21} + 60b_{22} + 20b_{23},\ 10c_{11} + 50c_{12} + 80c_{13} + 80c_{21} + 60c_{22} + 20c_{23},\ 10d_{11} + 50d_{12} + 80d_{13} + 90d_{21} + 60d_{22} + 20d_{23}\Big)$

subject to

$$\begin{aligned}
&\sum_{j=1}^{4} a_{1j} = 70\ , \quad \sum_{j=1}^{4} b_{1j} = 90\ , \quad \sum_{j=1}^{4} c_{1j} = 90\ , \quad \sum_{j=1}^{4} d_{1j} = 100\\
&\sum_{j=1}^{4} a_{2j} = 40\ , \quad \sum_{j=1}^{4} b_{2j} = 60\ , \quad \sum_{j=1}^{4} c_{2j} = 70\ , \quad \sum_{j=1}^{4} d_{2j} = 80\\
&\sum_{j=1}^{4} a_{3j} = 0\ , \quad \sum_{j=1}^{4} b_{3j} = 0\ , \quad \sum_{j=1}^{4} c_{3j} = 10\ , \quad \sum_{j=1}^{4} d_{3j} = 50\\
&\sum_{i=1}^{3} a_{i1} = 30\ , \quad \sum_{i=1}^{3} b_{i1} = 40\ , \quad \sum_{i=1}^{3} c_{i1} = 50\ , \quad \sum_{i=1}^{3} d_{i1} = 70 \qquad (P_{2.7})\\
&\sum_{i=1}^{3} a_{i2} = 20\ , \quad \sum_{i=1}^{3} b_{i2} = 30\ , \quad \sum_{i=1}^{3} c_{i2} = 40\ , \quad \sum_{i=1}^{3} d_{i2} = 50\\
&\sum_{i=1}^{3} a_{i3} = 40\ , \quad \sum_{i=1}^{3} b_{i3} = 50\ , \quad \sum_{i=1}^{3} c_{i3} = 50\ , \quad \sum_{i=1}^{3} d_{i3} = 80\\
&\sum_{i=1}^{3} a_{i4} = 20\ , \quad \sum_{i=1}^{3} b_{i4} = 30\ , \quad \sum_{i=1}^{3} c_{i4} = 30\ , \quad \sum_{i=1}^{3} d_{i4} = 30\\
&a_{ij}, b_{ij} - a_{ij},\ c_{ij} - b_{ij},\ d_{ij} - c_{ij} \geq 0\ \forall\ i = 1, 2, 3; j = 1, 2, 3, 4
\end{aligned}$$

Step 5 Using Step 6 of the method, proposed in Section 2.5.1, the fuzzy optimal solution of the fuzzy linear programming problem ($P_{2.7}$), can be obtained by solving the following crisp linear programming problem:

Minimize $\Big(\frac{1}{4}(10a_{11} + 10b_{11} + 10c_{11} + 10d_{11} + 50a_{12} + 50b_{12} + 50c_{12} + 50d_{12} + 80a_{13} +$

$80b_{13}+80c_{13}+80d_{13}+60a_{21}+70b_{21}+80c_{21}+90d_{21}+60a_{22}+60b_{22}+60c_{22}+60d_{22}+ 20a_{23}+20b_{23}+20c_{23}+20d_{23})\Big)$

subject to

constraints of the problem $(P_{2.7})$

Step 6 The optimal solution of the crisp linear programming problem, obtained in Step 5, is $a_{11}=30, b_{11}=40, c_{11}=40, d_{11}=40, a_{12}=20, b_{12}=30, c_{12}=30, d_{12}=40, a_{13}=0, b_{13}=0, c_{13}=0, d_{13}=0, a_{14}=20, b_{14}=20, c_{14}=20, d_{14}=20, a_{21}=0, b_{21}=0, c_{21}=0, d_{21}=0, a_{22}=0, b_{22}=0, c_{22}=10, d_{22}=10, a_{23}=40, b_{23}=50, c_{23}=50, d_{23}=60, a_{24}=0, b_{24}=10, c_{24}=10, d_{24}=10, a_{31}=0, b_{31}=0, c_{31}=10, d_{31}=30, a_{32}=0, b_{32}=0, c_{32}=0, d_{32}=0, a_{33}=0, b_{33}=0, c_{33}=0, d_{33}=20, a_{34}=0, b_{34}=0, c_{34}=0, d_{34}=0.$

Step 7 Putting the values of a_{ij}, b_{ij}, c_{ij} and d_{ij}, obtained from Step 6, in $\tilde{x}_{ij}=(a_{ij}, b_{ij}, c_{ij}, d_{ij})$, the fuzzy optimal solution is $\tilde{x}_{11}=(30,40,40,40)$ $\tilde{x}_{12}=(20,30,30,40)$, $\tilde{x}_{13}=(0,0,0,0)$, $\tilde{x}_{14}=(20,20,20,20)$, $\tilde{x}_{21}=(0,0,0,0)$, $\tilde{x}_{22}=(0,0,10,10)$, $\tilde{x}_{23}=(40,50,50,60)$, $\tilde{x}_{24}=(0,10,10,10)$, $\tilde{x}_{31}=(0,0,10,30)$, $\tilde{x}_{32}=(0,0,0,0)$, $\tilde{x}_{33}=(0,0,0,20)$, $\tilde{x}_{34}=(0,0,0,0)$.

Step 8 Putting the values of $\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34}$ in $\Big((10,10,10,10)\otimes\tilde{x}_{11}\oplus(50,50,50,50)\otimes\tilde{x}_{12}\oplus(80,80,80,80)\otimes\tilde{x}_{13}\oplus(0,0,0,0)\otimes\tilde{x}_{14}\oplus(60,70,80,90)\otimes\tilde{x}_{21}\oplus(60,60,60,60)\otimes\tilde{x}_{22}\oplus(20,20,20,20)\otimes\tilde{x}_{23}\oplus(0,0,0,0)\otimes\tilde{x}_{24}\oplus(0,0,0,0)\otimes\tilde{x}_{31}\oplus(0,0,0,0)\otimes\tilde{x}_{32}\oplus(0,0,0,0)\otimes\tilde{x}_{33}\oplus(0,0,0,0)\otimes\tilde{x}_{34}\Big)$ the obtained minimum total fuzzy transportation cost is $(2100, 2900, 3500, 4200)$.

2.6.2 Fuzzy optimal solution using the method based on tabular representation

Using the proposed method, based on tabular representation, the fuzzy optimal solution of the fully fuzzy transportation problem, chosen in Example 2.2, can be obtained as follows:

Step 1 The balanced fully fuzzy transportation problem, obtained from Step 1 of Section 2.6.1, can be represented by Table 2.6.

Table 2.6 Tabular representation of balanced fully fuzzy transportation problem

	D_1	D_2	D_3	D_4	$\tilde{a}_i$
S_1	(10,10,10,10)	(50,50,50,50)	(80,80,80,80)	(0,0,0,0)	(70,90,90,100)
S_2	(60,70,80,90)	(60,60,60,60)	(20,20,20,20)	(0,0,0,0)	(40,60,70,80)
S_3	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,10,50)
$\tilde{b}_j$	(30,40,50,70)	(20,30,40,50)	(40,50,50,80)	(20,30,30,30)	$\sum_{i=1}^{3} \tilde{a}_i = \sum_{j=1}^{4} \tilde{b}_j$

Step 2 Using Step 3 of the method, proposed in Section 2.5.2, Table 2.6 can be split into four crisp transportation tables i.e., Table 2.7, Table 2.8, Table 2.9, Table 2.10.

Table 2.7 Tabular representation of first crisp transportation problem

	D_1	D_2	D_3	D_4	a_i
S_1	10	50	80	0	70
S_2	75	60	20	0	40
S_3	0	0	0	0	0
a'_j	30	20	40	20	

Table 2.8 Tabular representation of second crisp transportation problem

	D_1	D_2	D_3	D_4	$b_i - a_i$
S_1	7.5	37.5	60	0	20
S_2	60	45	15	0	20
S_3	0	0	0	0	0
$b'_j - a'_j$	10	10	10	10	

Table 2.9 Tabular representation of third crisp transportation problem

	D_1	D_2	D_3	D_4	$c_i - b_i$
S_1	5	25	40	0	0
S_2	42.5	30	10	0	10
S_3	0	0	0	0	10
$c'_j - b'_j$	10	10	0	0	

Table 2.10 Tabular representation of fourth crisp transportation problem

	D_1	D_2	D_3	D_4	$d_i - c_i$
S_1	2.5	22.5	30	0	10
S_2	22.5	15	5	0	10
S_3	0	0	0	0	40
$d'_j - c'_j$	20	10	30	0	

Step 3 The optimal solution of crisp transportation problems, shown by Table 2.7; Table 2.8; Table 2.9 and Table 2.10, are $a_{11} = 30, a_{12} = 20, a_{13} = 0, a_{14} = 20, a_{21} = 0, a_{22} = 0, a_{23} = 40, a_{24} = 0, a_{31} = 0, a_{32} = 0, a_{33} = 0, a_{34} = 0; b_{11} - a_{11} =$

$10, b_{12} - a_{12} = 10, b_{13} - a_{13} = 0, b_{14} - a_{14} = 0, b_{21} - a_{21} = 0, b_{22} - a_{22} = 0, b_{23} - a_{23} = 10, b_{24} - a_{24} = 10, b_{31} - a_{31} = 0, b_{32} - a_{32} = 0, b_{33} - a_{33} = 0, b_{34} - a_{34} = 0; c_{11} - b_{11} = 0, c_{12} - b_{12} = 0, c_{13} - b_{13} = 0, c_{14} - b_{14} = 0, c_{21} - b_{21} = 0, c_{22} - b_{22} = 10, c_{23} - b_{23} = 0, c_{24} - b_{24} = 0, c_{31} - b_{31} = 0, c_{32} - b_{32} = 10, c_{33} - b_{33} = 0, c_{34} - b_{34} = 0$ and $d_{11} - c_{11} = 10, d_{12} - c_{12} = 10, d_{13} - c_{13} = 0, d_{14} - c_{14} = 0, d_{21} - c_{21} = 0, d_{22} - c_{22} = 0, d_{23} - c_{23} = 10, d_{24} - c_{24} = 0, d_{31} - c_{31} = 20, d_{32} - c_{32} = 0, d_{33} - c_{33} = 20, d_{34} - c_{34} = 0$ respectively.

Step 4 On solving the equations, obtained in Step 3, the values of a_{ij}, b_{ij}, c_{ij} and d_{ij} are $a_{11} = 30, b_{11} = 40, c_{11} = 40, d_{11} = 40, a_{12} = 20, b_{12} = 30, c_{12} = 30, d_{12} = 40, a_{13} = 0, b_{13} = 0, c_{13} = 0, d_{13} = 0, a_{14} = 20, b_{14} = 20, c_{14} = 20, d_{14} = 20, a_{21} = 0, b_{21} = 0, c_{21} = 0, d_{21} = 0, a_{22} = 0, b_{22} = 0, c_{22} = 10, d_{22} = 10, a_{23} = 40, b_{23} = 50, c_{23} = 50, d_{23} = 60, a_{24} = 0, b_{24} = 10, c_{24} = 10, d_{24} = 10, a_{31} = 0, b_{31} = 0, c_{31} = 10, d_{31} = 30, a_{32} = 0, b_{32} = 0, c_{32} = 0, d_{32} = 0, a_{33} = 0, b_{33} = 0, c_{33} = 0, d_{33} = 20, a_{34} = 0, b_{34} = 0, c_{34} = 0, d_{34} = 0$.

Step 5 Putting the values of a_{ij}, b_{ij}, c_{ij} and d_{ij} in $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$, the fuzzy optimal solution is $\tilde{x}_{11} = (30, 40, 40, 40)$ $\tilde{x}_{12} = (20, 30, 30, 40)$, $\tilde{x}_{13} = (0, 0, 0, 0)$, $\tilde{x}_{14} = (20, 20, 20, 20)$, $\tilde{x}_{21} = (0, 0, 0, 0)$, $\tilde{x}_{22} = (0, 0, 10, 10)$, $\tilde{x}_{23} = (40, 50, 50, 60)$, $\tilde{x}_{24} = (0, 10, 10, 10)$, $\tilde{x}_{31} = (0, 0, 10, 30)$, $\tilde{x}_{32} = (0, 0, 0, 0)$, $\tilde{x}_{33} = (0, 0, 0, 20)$, $\tilde{x}_{34} = (0, 0, 0, 0)$.

Step 6 Putting the values of $\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34}$ in $((10, 10, 10, 10) \otimes \tilde{x}_{11} \oplus (50, 50, 50, 50) \otimes \tilde{x}_{12} \oplus (80, 80, 80, 80) \otimes \tilde{x}_{13} \oplus (0, 0, 0, 0) \otimes \tilde{x}_{14} \oplus (60, 70, 80, 90) \otimes \tilde{x}_{21} \oplus (60, 60, 60, 60) \otimes \tilde{x}_{22} \oplus (20, 20, 20, 20) \otimes \tilde{x}_{23} \oplus (0, 0, 0, 0) \otimes \tilde{x}_{24} \oplus (0, 0, 0, 0) \otimes \tilde{x}_{31} \oplus (0, 0, 0, 0) \otimes \tilde{x}_{32} \oplus (0, 0, 0, 0) \otimes \tilde{x}_{33} \oplus (0, 0, 0, 0) \otimes \tilde{x}_{34})$ the minimum total fuzzy transportation cost is $(2100, 2900, 3500, 4200)$.

2.6.3 Physical interpretation of the results

In this section, the minimum total fuzzy transportation cost, obtained by using the proposed methods, is physically interpreted. Similarly, the obtained fuzzy optimal solution can also be physically interpreted.

Using the proposed methods the minimum total fuzzy transportation cost is $(2100, 2900, 3500, 4200)$, which can be physically interpreted as follows:

(1) The least amount of minimum total transportation cost is 2100.

(2) The most possible amount of minimum total transportation cost lies between 2900 and 3500.

(3) The greatest amount of minimum total transportation cost is 4200 i.e., the minimum total transportation cost will always be greater than 2100 and less than 4200 and maximum chances are that the minimum total transportation cost will lie between 2900 and 3500.

The variation in minimum total transportation cost with respect to chances are shown in Figure 2.1.

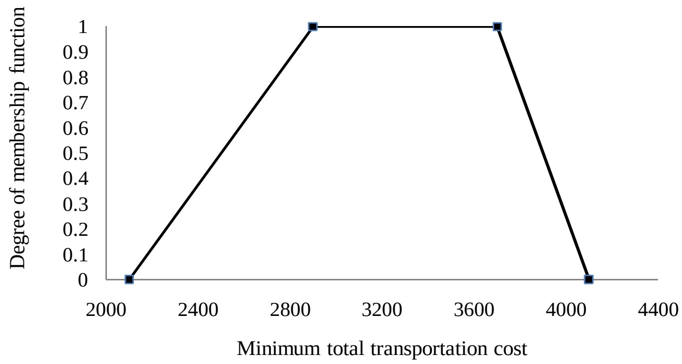

Figure 2.1. Membership function of trapezoidal fuzzy number representing the minimum total fuzzy transportation cost

2.7 Case study

Liang [127] proposed a method to find the crisp optimal solution of such fuzzy transportation problems in which all the parameters are represented by triangular fuzzy numbers and used it to find the crisp optimal solution of a real life fuzzy transportation problem described in Section 2.7.1.

However, in Chapter 1, it is pointed out that it is better to find the fuzzy optimal solution as compared to crisp optimal solution. So, in this section, the fuzzy optimal solution of the same real life problem are obtained with the help of proposed methods.

2.7.1 Description of problem

Dali Company is the leading producer of soft drinks and low-temperature foods in Taiwan. Currently, Dali plans to develop the South-East Asian market and broaden the visibility of Dali products in the Chinese market. Notably, following the entry of Taiwan to the World Trade Organization, Dali plans to seek strategic alliance with prominent international companies, and introduced international bread to lighten the embedded future impact. In the domestic soft drinks market, Dali produces tea beverages to meet demand from four distribution centers in Taichung, Chiayi, Kaohsiung, and Taipei, with production being based at three plants in Changhua, Touliu, and Hsinchu. According to the preliminary environmental information, Table 2.11 summarizes the potential availability of the product at these three plants, the forecast demand from the four distribution centers, and the unit transportation cost for each route used by Dali for the upcoming season.

The environmental coefficients and related parameters are generally imprecise numbers with triangular possibility distributions over the planning horizon due to incomplete or unobtainable information. For example, the available supply of the Changhua plant is (7.2, 8, 8.8) thousand dozen bottles, the forecast demand of the Taichung distribution center is (6.2, 7, 7.8) thousand dozen bottles and the transportation cost per dozen bottles from Changhua to Taichung is ($\$8, \$10, \$10.8$).

Due to transportation costs being a major expense, the management of Dali is initiating a study to reduce these costs as much as possible.

Table 2.11. Summarized data in the Dali case (in U.S. dollar)

Source (i)	Destination (j)				Supply(000 dozen bottles)
	Taichung (1)	Chiayi (2)	Kaohsiung (3)	Taipei (4)	
Changhua (1)	($\$8, \$10, \$10.8$)	($\$20.4, \$22, \$24$)	($\$8, \$10, \$10.6$)	($\$18.8, \$20, \$22$)	($7.2, 8, 8.8$)
Touliu (2)	($\$14, \$15, \$16$)	($\$18.2, \$20, \$22$)	($\$10, \$12, \$13$)	($\$6, \$8, \$8.8$)	($12, 14, 16$)
Hsinchu (3)	($\$18.4, \$20, \$21$)	($\$9.6, \$12, \$13$)	($\$7.8, \$10, \$10.8$)	($\$14, \$15, \$16$)	($10.2, 12, 13.8$)
Demand(000 dozen bottles)	($6.2, 7, 7.8$)	($8.6, 10, 11.4$)	($6.5, 8, 9.5$)	($7.8, 9, 10.2$)	

2.7.2 Results

On solving the real life problem, shown by Table 2.11, by using the proposed methods the obtained fuzzy optimal solution $\{\tilde{x}_{ij}\}$, representing the fuzzy quantity of the soft drinks that should be transported from i^{th} source j^{th} destination to minimize the total fuzzy transportation cost, is $\tilde{x}_{11} = (6.2, 7, 7.8)$, $\tilde{x}_{13} = (1, 1, 1)$, $\tilde{x}_{23} = (3.9, 4.7, 5.5)$, $\tilde{x}_{24} = (7.8, 9, 10.2)$, $\tilde{x}_{25} = (.3, .3, .3)$, $\tilde{x}_{32} = (8.6, 9.7, 10.8)$, $\tilde{x}_{33} = (1.6, 2.3, 3)$, $\tilde{x}_{42} = (0, .3, .6)$ and the minimum total fuzzy transportation cost is ($\$238.44, \$347.8, \$428.9$).

Remark 2.5 Since, the real life problem, chosen in Section 2.7.1, is an unbalanced problem. So, to find the solution of the problem a dummy source (4) and a dummy destination (5) is introduced. In the results, presented in Section 2.7.2, $\tilde{x}_{25}$ and $\tilde{x}_{42}$ represents the fuzzy quantity of the product that should be transported from source (2) to dummy destination (5) and from dummy source (4) to destination (2) respectively.

2.7.3 Physical interpretation of the results

In this section, the minimum total fuzzy transportation cost, obtained by using the proposed methods, is physically interpreted. Similarly, the obtained fuzzy optimal solution can also be physically interpreted.

Using the proposed methods the minimum total fuzzy transportation cost is (\$238.44, \$347.8, \$428.9), which can be physically interpreted as follows:

(1) The least amount of minimum total transportation cost is \$238.44.

(2) The most possible amount of minimum total transportation cost is \$347.8.

(3) The greatest amount of minimum total transportation cost is \$428.9 i.e., the minimum total transportation cost will always be greater than \$238.44 and less than \$428.9 and maximum chances are that the minimum total transportation cost will be \$347.8.

The variation in minimum total transportation cost with respect to chances are shown in Figure 2.2.

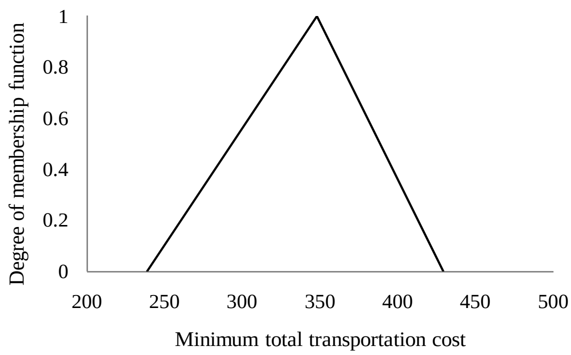

Figure 2.2. Membership function of triangular fuzzy number representing the minimum total fuzzy transportation cost

2.8 Conclusions

On solving the fully fuzzy transportation problems by applying the proposed methods all the shortcomings, occurring due to the existing methods, are resolved. So, it can be concluded that it is better to use the proposed methods as compared to the existing methods.

Chapter 3

New Methods For Solving Fully Fuzzy Transportation Problems With LR Flat Fuzzy Parameters

In this chapter, the limitations of the methods, proposed in Chapter 2, are pointed out. To overcome these limitations new methods are proposed for solving fully fuzzy transportation problems by modifying the methods, proposed in Chapter 2. The advantages of the proposed methods over the methods, proposed in Chapter 2, are discussed. To illustrate the proposed methods, a fully fuzzy transportation problem is solved.

3.1 Preliminaries

In the literature [49, 225] it is pointed out that the computational efforts required to solve a fuzzy linear programming problem can be reduced, if the decision maker can express his subjective impression using LR flat fuzzy numbers. All kinds of crisp numbers, triangular and trapezoidal fuzzy numbers are LR flat fuzzy numbers.

In this section, some basic definitions and arithmetic operations of LR flat fuzzy numbers are presented.

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3.1.1 Basic definitions

In this section, some basic definitions are presented.

Definition 3.1 [49] A function $L : [0, \infty) \to [0, 1]$ (or $R : [0, \infty) \to [0, 1]$) is said to be reference function of fuzzy number if and only if

(i) $L(0) = 1$ (or $R(0) = 1$)

(ii) L (or R) is non-increasing on $[0, \infty)$.

Definition 3.2 [49] A fuzzy number $\widetilde{A}$ defined on universal set of real numbers $\mathbb{R}$, denoted as $(m, n, \alpha, \beta)_{LR}$, is said to be an LR flat fuzzy number if its membership function $\mu_{\widetilde{A}}(x)$ is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} L(\frac{m-x}{\alpha}), & x \leq m,\ \alpha > 0 \\ R(\frac{x-n}{\beta}), & x \geq n,\ \beta > 0 \\ 1, & m \leq x \leq n \end{cases}$$

Definition 3.3 [49] Let $\widetilde{A} = (m, n, \alpha, \beta)_{LR}$ be an LR flat fuzzy number and λ be a real number in the interval $[0, 1]$. Then, the classical set $A_\lambda = \{x \in X : \mu_{\widetilde{A}}(x) \geq \lambda\} = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$, is said to be λ-cut of $\widetilde{A}$.

Definition 3.4 [49] An LR flat fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be zero LR flat fuzzy number if and only if $m = 0, n = 0, \alpha = 0$ and $\beta = 0$.

Definition 3.5 [49] Two LR flat fuzzy numbers $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ are said to be equal i.e., $\tilde{A}_1 = \tilde{A}_2$ if and only if $m_1 = m_2, n_1 = n_2, \alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$.

Definition 3.6 [44] An LR flat fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be non-negative LR flat fuzzy number if and only if $m - \alpha \geq 0$.

Definition 3.7 [44] An LR flat fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be negative (positive) LR flat fuzzy number if and only if $n + \beta < 0$ $(m - \alpha > 0)$.

Remark 3.1 If $m = n$ then an LR flat fuzzy number $(m, n, \alpha, \beta)_{LR}$ is said to be an LR fuzzy number and is denoted as $(m, m, \alpha, \beta)_{LR}$ or $(n, n, \alpha, \beta)_{LR}$ or $(m, \alpha, \beta)_{LR}$ or $(n, \alpha, \beta)_{LR}$.

Remark 3.2 If $m = n$ and $L(x) = R(x) =$ maximum $\{0, 1 - x\}$ then an LR flat fuzzy number $(m, n, \alpha, \beta)_{LR}$ is said to be a triangular fuzzy number and is denoted as (a, b, c) where, $a = m - \alpha, b = m(\text{or } n), c = m + \beta(\text{or } n + \beta)$.

Remark 3.3 If $m \neq n$ but $L(x) = R(x) =$ maximum $\{0, 1 - x\}$ then an LR flat fuzzy number $(m, n, \alpha, \beta)_{LR}$ is said to be a trapezoidal fuzzy number and is denoted as (a, b, c, d) where, $a = m - \alpha, b = m, c = n, d = n + \beta$.

3.1.2 Arithmetic operations

In this section, addition and multiplication operations of LR flat fuzzy numbers are presented [49]:

(i) Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be two LR flat fuzzy numbers. Then, $\tilde{A}_1 \oplus \tilde{A}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)_{LR}$

(ii) Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be two non-negative LR flat fuzzy numbers. Then,

$$\tilde{A}_1 \otimes \tilde{A}_2 \simeq (m_1 m_2, n_1 n_2, m_1 m_2 - (m_1 - \alpha_1)(m_2 - \alpha_2), (n_1 + \beta_1)(n_2 + \beta_2) - n_1 n_2)_{LR}$$

3.2 Limitations of the methods proposed in previous chapter

The methods, proposed in Chapter 2, can be used only for solving such fully fuzzy transportation problems in which the parameters are either represented by triangular fuzzy numbers or trapezoidal fuzzy numbers. However, the same methods can not be used for solving such fully fuzzy transportation problems in which the parameters are represented by LR fuzzy numbers or LR flat fuzzy numbers e.g., the fully fuzzy transportation problem, chosen in Example 3.1, can not be solved by the methods proposed in Chapter 2.

Example 3.1 A company has two sources S_1 and S_2 and three destinations D_1, D_2 and D_3 ; the fuzzy cost for transporting one unit quantity of the product from i^{th} source to j^{th} destination is $\tilde{c}_{ij}$

where, $[\tilde{c}_{ij}]_{2\times 3} = \begin{pmatrix} (20,30,10,10)_{LR} & (60,70,10,20)_{LR} & (90,110,10,10)_{LR} \\ (70,80,10,10)_{LR} & (80,100,10,20)_{LR} & (30,50,10,10)_{LR} \end{pmatrix}$. The fuzzy availability of the product at first, second sources are $(90,90,20,10)_{LR}$ and $(60,70,20,10)_{LR}$ and the fuzzy demand of the product at first, second and third destinations are $(40,50,10,20)_{LR}$, $(30,40,10,10)_{LR}$ and $(50,50,10,30)_{LR}$ respectively, where, $L(x) = R(x) =$ maximum $\{0, 1 - x^4\}$. The owner of the company want to determine the fuzzy quantity of the product that should be transported from each of the sources to each destination so that the total fuzzy transportation cost is minimum.

3.3 Proposed methods

In this section, to overcome the limitations of the methods, proposed in Chapter 2, two new methods are proposed for solving such fully fuzzy transportation problems in which all the parameters are represented by LR flat fuzzy numbers. Also, the advantages of the proposed methods over the methods, proposed in Chapter 2, are discussed.

3.3.1 Method based on fuzzy linear programming formulation

In this section, a new method, based on the fuzzy linear programming formulation of the fully fuzzy transportation problems, is proposed to find the fuzzy optimal solution of such fully fuzzy transportation problems in which the parameters are represented by LR flat fuzzy numbers.

The steps of the proposed method are as follows:

Step 1 Find the total fuzzy availability $\sum_{i=1}^{m} \tilde{a}_i$ and the total fuzzy demand $\sum_{j=1}^{n} \tilde{b}_j$. Let $\sum_{i=1}^{m} \tilde{a}_i = (m, n, \alpha, \beta)_{LR}$ and $\sum_{j=1}^{n} \tilde{b}_j = (m', n', \alpha', \beta')_{LR}$. Use Definition 3.5 to examine that the problem is balanced or not, i.e., $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$ or $\sum_{i=1}^{m} \tilde{a}_i \neq \sum_{j=1}^{n} \tilde{b}_j$.

Case (i) If the problem is balanced, i.e., $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$, then Go to Step 2.

Case (ii) If $\sum_{i=1}^{m} \tilde{a}_i \neq \sum_{j=1}^{n} \tilde{b}_j$ then convert the unbalanced problem into balanced problem as follows:

Case (a) If $m - \alpha \leq m' - \alpha'$, $\alpha \leq \alpha'$, $n - m \leq n' - m'$, and $\beta \leq \beta'$ then introduce a dummy source with fuzzy availability $(m' - m, n' - n, \alpha' - \alpha, \beta' - \beta)_{LR}$. Assume the fuzzy cost for transporting one unit quantity of the product from the introduced dummy source to all destinations as zero LR flat fuzzy number. Go to Step 2.

Case (b) If $m-\alpha \geq m'-\alpha'$, $\alpha \geq \alpha'$, $n-m \geq n'-m'$, and $\beta \geq \beta'$ then introduce a dummy destination with fuzzy demand $(m-m', n-n', \alpha-\alpha', \beta-\beta')_{LR}$. Assume the fuzzy cost for transporting one unit quantity of the product from all sources to the introduced dummy destination as zero LR flat fuzzy number. Go to Step 2.

Case (c) If neither Case (a) nor Case (b) is satisfied then introduce a dummy source with fuzzy availability (maximum $\{0, (m'-\alpha')-(m-\alpha)\}$ + maximum $\{0, (\alpha'-\alpha)\}$, maximum $\{0, (m'-\alpha')-(m-\alpha)\}$ + maximum $\{0, (\alpha'-\alpha)\}$ + maximum $\{0, (n'-m')-(n-m)\}$, maximum $\{0, (\alpha'-\alpha)\}$, maximum $\{0, (\beta'-\beta)\})_{LR}$ and dummy destination with fuzzy demand (maximum $\{0, (m-\alpha)-(m'-\alpha')\}$ + maximum $\{0, (\alpha-\alpha')\}$, maximum $\{0, (m-\alpha)-(m'-\alpha')\}$ + maximum $\{0, (\alpha-\alpha')\}$ + maximum $\{0, (n-m)-(n'-m')\}$, maximum $\{0, (\alpha-\alpha')\}$, maximum $\{0, (\beta-\beta')\})_{LR}$. Assume the fuzzy cost for transporting one unit quantity of the product from the introduced dummy source to all destinations and from all sources to the introduced dummy destination as zero LR flat fuzzy number. Go to Step 2.

Step 2 Formulate the balanced fully fuzzy transportation problem, obtained in Step 1, into the fuzzy linear programming problem $(P_{2.1})$.

Step 3 Assuming $\tilde{c}_{ij} = (m'_{ij}, n'_{ij}, \alpha'_{ij}, \beta'_{ij})_{LR}$, $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$, $\tilde{a}_i = (m_i, n_i, \alpha_i, \beta_i)_{LR}$ and $\tilde{b}_j = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR}$, the fuzzy linear programming problem, obtained in Step 2, can be written as:

$$\text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\Big((m'_{ij}, n'_{ij}, \alpha'_{ij}, \beta'_{ij})_{LR} \otimes (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}\Big)$$

subject to

$$\sum_{j=1}^{n}((m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}) = (m_i, n_i, \alpha_i, \beta_i)_{LR}, \qquad i = 1, 2, 3, ..., m$$

$$\sum_{i=1}^{m}((m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}) = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR} \qquad j = 1, 2, 3, ..., n$$

$(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$ is a non-negative LR flat fuzzy number.

Step 4 Using the arithmetic operations, defined in Section 3.1.2 and assuming $\sum_{i=1}^{m}\sum_{j=1}^{n}\Big((m'_{ij}, n'_{ij}, \alpha'_{ij}, \beta'_{ij})_{LR} \otimes (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}\Big) = (m_0, n_0, \alpha_0, \beta_0)_{LR}$, the fuzzy linear programming problem, obtained in Step 3, can be written as:

Minimize $(m_0, n_0, \alpha_0, \beta_0)_{LR}$

subject to

$$\Big(\sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} n_{ij}, \sum_{j=1}^{n} \alpha_{ij}, \sum_{j=1}^{n} \beta_{ij}\Big)_{LR} = (m_i, n_i, \alpha_i, \beta_i)_{LR}, \qquad i = 1, 2, 3, ..., m$$
$$\Big(\sum_{i=1}^{m} m_{ij}, \sum_{i=1}^{m} n_{ij}, \sum_{i=1}^{m} \alpha_{ij}, \sum_{i=1}^{m} \beta_{ij}\Big)_{LR} = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR}, \qquad j = 1, 2, 3, ..., n$$

$(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$ is a non-negative LR flat fuzzy number.

Step 5 Using Definition 3.5 and Definition 3.6, the fuzzy linear programming problem, obtained in Step 4, can be converted into the fuzzy linear programming problem $(P_{3.1})$:

Minimize $(m_0, n_0, \alpha_0, \beta_0)_{LR}$

subject to

$$\begin{aligned}
&\sum_{j=1}^{n} m_{ij} = m_i\,, && i = 1, 2, 3, ..., m \\
&\sum_{j=1}^{n} n_{ij} = n_i, && i = 1, 2, 3, ..., m \\
&\sum_{j=1}^{n} \alpha_{ij} = \alpha_i, && i = 1, 2, 3, ..., m \\
&\sum_{j=1}^{n} \beta_{ij} = \beta_i, && i = 1, 2, 3, ..., m \qquad\qquad (P_{3.1}) \\
&\sum_{i=1}^{m} m_{ij} = m'_j, && j = 1, 2, 3, ..., n \\
&\sum_{i=1}^{m} n_{ij} = n'_j, && j = 1, 2, 3, ..., n \\
&\sum_{i=1}^{m} \alpha_{ij} = \alpha'_j, && j = 1, 2, 3, ..., n \\
&\sum_{i=1}^{m} \beta_{ij} = \beta'_j, && j = 1, 2, 3, ..., n
\end{aligned}$$
$$m_{ij} - \alpha_{ij}, n_{ij} - m_{ij}, \alpha_{ij}, \beta_{ij} \geq 0 \qquad \forall\ i, j$$

Step 6 As discussed in Step 6 of the method, proposed in Section 2.5.1 of Chapter 2, the fuzzy optimal solution of the fuzzy linear programming problem $(P_{3.1})$ can be obtained by solving the following crisp linear programming problem:

Minimize $\Re(m_0, n_0, \alpha_0, \beta_0)_{LR}$

subject to

constraints of the problem $(P_{3.1})$

Step 7 Using the existing formula [131], $\Re(m_0, n_0, \alpha_0, \beta_0)_{LR} = \frac{1}{2}(\int_0^1 (m_0 - \alpha_0 L^{-1}(\lambda))\, d\lambda + \int_0^1 (n_0 + \beta_0 R^{-1}(\lambda))\, d\lambda)$, the crisp linear programming problem, obtained in Step 6, can be converted into the following crisp linear programming problem:

Minimize $\frac{1}{2}(\int_0^1 (m_0 - \alpha_0 L^{-1}(\lambda))\ d\lambda + \int_0^1 (n_0 + \beta_0 R^{-1}(\lambda))\ d\lambda)$

subject to

constraints of the problem $(P_{3.1})$

Step 8 Solve the crisp linear programming problem, obtained from Step 7, to find the optimal solution $\{m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}\}$.

Step 9 Find the fuzzy optimal solution $\{\tilde{x}_{ij}\}$ by putting the values of $m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}$ in $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$.

Step 10 Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ij}$ in $\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij})$.

Remark 3.4 Let $\tilde{A} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$ be an LR flat fuzzy number with $L(x) = R(x) =$ maximum $\{0, 1 - x^4\}$. Then,

$\Re(\tilde{A}) = \frac{1}{2}((m_{ij} + n_{ij}) - \alpha_{ij} \int_0^1 L^{-1}(\lambda) d\lambda + \beta_{ij} \int_0^1 R^{-1}(\lambda) d\lambda) = \frac{1}{10}(5m_{ij} + 5n_{ij} + 4\beta_{ij} - 4\alpha_{ij})$

3.3.2 Method based on tabular representation

In this section, a new method, based on tabular representation of the fully fuzzy transportation problems, is proposed to find the fuzzy optimal solution of such fully fuzzy transportation problems in which the parameters are represented by LR flat fuzzy numbers.

The steps of the proposed method are as follows:

Step 1 Use Step 1 of the method, proposed in Section 3.3.1, to obtain a balanced fully fuzzy transportation problem.

Step 2 Represent the balanced fully fuzzy transportation problem, obtained from Step 1, into tabular form as shown by Table 3.1.

Table 3.1 Tabular representation of balanced fully fuzzy transportation problem

Destinations→ Sources↓	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	Availability ($\tilde{a}_i$)
S_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\cdots$	$\tilde{c}_{1j}$	$\cdots$	$\tilde{c}_{1n}$	$\tilde{a}_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	$\tilde{c}_{i1}$	$\tilde{c}_{i2}$	$\cdots$	$\tilde{c}_{ij}$	$\cdots$	$\tilde{c}_{in}$	$\tilde{a}_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	$\cdots$	$\tilde{c}_{mj}$	$\cdots$	$\tilde{c}_{mn}$	$\tilde{a}_m$
Demand($\tilde{b}_j$)	$\tilde{b}_1$	$\tilde{b}_2$	$\cdots$	$\tilde{b}_j$	$\cdots$	$\tilde{b}_n$	$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$

Step 3 Split Table 3.1 into four crisp transportation tables i.e., Table 3.2, Table 3.3, Table 3.4 and Table 3.5.

Table 3.2 Tabular representation of first crisp transportation problem

Destinations→ Sources↓	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	$m_i - \alpha_i$
S_1	η_{11}	η_{12}	$\cdots$	η_{1j}	$\cdots$	η_{1n}	$m_1 - \alpha_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	η_{i1}	η_{i2}	$\cdots$	η_{ij}	$\cdots$	η_{in}	$m_i - \alpha_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	η_{m1}	η_{m2}	$\cdots$	η_{mj}	$\cdots$	η_{mn}	$m_m - \alpha_m$
$m'_j - \alpha'_j$	$m'_1 - \alpha'_1$	$m'_2 - \alpha'_2$	$\cdots$	$m'_j - \alpha'_j$	$\cdots$	$m'_n - \alpha'_n$	$\sum_{i=1}^{m}(m_i - \alpha_i) = \sum_{j=1}^{n}(m'_j - \alpha'_j)$

where, $\eta_{ij} = \frac{1}{2}((m'_{ij} + n'_{ij}) - \alpha'_{ij}\int_0^1 L^{-1}(\lambda)d\lambda + \beta'_{ij}\int_0^1 R^{-1}(\lambda)d\lambda), \quad i = 1, 2, 3, ..., m$ and $j = 1, 2, 3, ..., n$

Table 3.3 Tabular representation of second crisp transportation problem

Destinations→ Sources↓	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	α_i
S_1	ρ_{11}	ρ_{12}	$\cdots$	ρ_{1j}	$\cdots$	ρ_{1n}	α_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	ρ_{i1}	ρ_{i2}	$\cdots$	ρ_{ij}	$\cdots$	ρ_{in}	α_i
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	ρ_{m1}	ρ_{m2}	$\cdots$	ρ_{mj}	$\cdots$	ρ_{mn}	α_m
α'_i	α'_1	α'_2	$\cdots$	α'_j	$\cdots$	α'_n	$\sum_{i=1}^{m} \alpha_i = \sum_{j=1}^{n} \alpha'_j$

where, $\rho_{ij} = \frac{1}{2}((m'_{ij} + n'_{ij}) - m'_{ij} \int_0^1 L^{-1}(\lambda)d\lambda + \beta'_{ij} \int_0^1 R^{-1}(\lambda)d\lambda), \quad i = 1, 2, 3, ..., m$ and $j = 1, 2, 3, ..., n$

Table 3.4 Tabular representation of third crisp transportation problem

Destinations→ Sources↓	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	$n'_i - m'_i$
S_1	δ_{11}	δ_{12}	$\cdots$	δ_{1j}	$\cdots$	δ_{1n}	$n_1 - m_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	δ_{i1}	δ_{i2}	$\cdots$	δ_{ij}	$\cdots$	δ_{in}	$n_i - m_i$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	δ_{m1}	δ_{m2}	$\cdots$	δ_{mj}	$\cdots$	δ_{mn}	$n_m - m_m$
$n'_j - m'_j$	$n'_1 - m'_1$	$n'_2 - m'_2$	$\cdots$	$n'_j - m'_j$	$\cdots$	$n'_n - m'_n$	$\sum_{i=1}^{m} (n_i - m_i) = \sum_{j=1}^{n} (n'_j - m'_j)$

where, $\delta_{ij} = \frac{1}{2}(n'_{ij} + \beta'_{ij} \int_0^1 R^{-1}(\lambda)d\lambda), \quad i = 1, 2, 3, ..., m$ and $j = 1, 2, 3, ..., n$

Table 3.5 Tabular representation of fourth crisp transportation problem

Destinations→ Sources↓	D_1	D_2	$\cdots$	D_j	$\cdots$	D_n	β'_i
S_1	ξ_{11}	ξ_{12}	$\cdots$	ξ_{1j}	$\cdots$	ξ_{1n}	β_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	ξ_{i1}	ξ_{i2}	$\cdots$	ξ_{ij}	$\cdots$	ξ_{in}	β_i
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	ξ_{m1}	ξ_{m2}	$\cdots$	ξ_{mj}	$\cdots$	ξ_{mn}	β_m
β'_j	β'_1	β'_2	$\cdots$	β'_j	$\cdots$	β'_n	$\sum_{i=1}^{m} \beta_i = \sum_{j=1}^{n} \beta'_j$

where, $\xi_{ij} = \frac{1}{2}((n'_{ij} + \beta'_{ij}) \int_0^1 R^{-1}(\lambda)d\lambda), \quad i = 1, 2, 3, ..., m$ and $j = 1, 2, 3, ..., n$

Step 4 Solve the crisp transportation problems, shown by Table 3.2, Table 3.3, Table 3.4 and Table 3.5, to find the optimal solution $\{m_{ij} - \alpha_{ij}\}$; $\{\alpha_{ij}\}$; $\{n_{ij} - m_{ij}\}$ and $\{\beta_{ij}\}$ respectively.

Step 5 Solve the equations, obtained in Step 4, to find the values of $m_{ij}, n_{ij}, \alpha_{ij}$

and β_{ij}.

Step 6 Find the fuzzy optimal solution $\{\tilde{x}_{ij}\}$ by putting the values of $m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}$ in $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$.

Step 7 Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ij}$ in $\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij})$.

Remark 3.5 If in the proposed methods all the parameters are represented by such LR flat fuzzy numbers in which $L(x) = R(x) =$ maximum $\{0, 1 - x\}$ then the methods, proposed in this chapter, will be same as the methods proposed in Chapter 2.

Remark 3.6 Let $\tilde{A} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$ be an LR flat fuzzy number with $L(x) = R(x) =$ maximum $\{0, 1 - x^4\}$. Then,

$\eta_{ij} = \frac{1}{2}((m_{ij}+n_{ij}) - \alpha_{ij}\int_0^1 L^{-1}(\lambda)d\lambda + \beta_{ij}\int_0^1 R^{-1}(\lambda)d\lambda) = \frac{1}{10}(5m_{ij}+5n_{ij}+4\beta_{ij}-4\alpha_{ij})$,

$\rho_{ij} = \frac{1}{2}((m_{ij} + n_{ij}) - m_{ij}\int_0^1 L^{-1}(\lambda)d\lambda + \beta_{ij}\int_0^1 R^{-1}(\lambda)d\lambda) = \frac{1}{10}(m_{ij} + 5n_{ij} + 4\beta_{ij})$,

$\delta_{ij} = \frac{1}{2}(n_{ij} + \beta_{ij}\int_0^1 R^{-1}(\lambda)d\lambda) = \frac{1}{10}(5n_{ij} + 4\beta_{ij})$ and

$\xi_{ij} = \frac{1}{2}((n_{ij} + \beta_{ij})\int_0^1 R^{-1}(\lambda)d\lambda) = \frac{4}{10}(n_{ij} + \beta_{ij})$.

3.3.3 Advantages of the proposed methods

The methods, proposed in Chapter 2, can be used only to find the fuzzy optimal solution of such fully fuzzy transportation problems in which the parameters are either represented by triangular fuzzy numbers or trapezoidal fuzzy numbers but can not be used for solving such fully fuzzy transportation problems in which parameters are either represented by LR fuzzy numbers or LR flat fuzzy numbers.

Since, triangular fuzzy numbers, trapezoidal fuzzy numbers and LR fuzzy numbers are special type of LR flat fuzzy numbers. So, the methods, proposed in this chapter, can also be used for solving such fully fuzzy transportation problems in which the parameters are represented by triangular fuzzy numbers or trapezoidal fuzzy numbers or LR fuzzy numbers.

3.4 Illustrative example

In this section, to illustrate the proposed methods, the fully fuzzy transportation problem, chosen in Example 3.1, is solved by the proposed methods.

3.4.1 Fuzzy optimal solution using the method based on fuzzy linear programming formulation

Using the proposed method, based on fuzzy linear programming formulation, the fuzzy optimal solution of the fully fuzzy transportation problem, chosen in Example 3.1, can be obtained as follows:

Step 1 Total fuzzy availability $= (150, 160, 40, 20)_{LR}$ and total fuzzy demand $= (120, 140, 30, 60)_{LR}$. Since, total fuzzy availability $\neq$ total fuzzy demand, so it is an unbalanced fully fuzzy transportation problem.

Now as described in the proposed method (using Case (c) of Step 1 of the proposed method, discussed in Section 3.3.1), the chosen unbalanced fully fuzzy transportation problem can be converted into a balanced fully fuzzy transportation problem by introducing a dummy source S_3 with fuzzy availability $(0, 10, 0, 40)_{LR}$ and a dummy destination D_4 with fuzzy demand $(30, 30, 10, 0)_{LR}$

Step 2 Assuming the fuzzy cost for transporting one unit quantity of the product from dummy source S_3 to all destinations and from all sources to dummy destination D_4, as zero LR flat fuzzy number i.e., $\tilde{c}_{14} = \tilde{c}_{24} = \tilde{c}_{31} = \tilde{c}_{32} = \tilde{c}_{33} = \tilde{c}_{34} = (0, 0, 0, 0)_{LR}$, the balanced fully fuzzy transportation problem, obtained in Step 1, can be formulated into the following fuzzy linear programming problem:

Minimize $((20,30,10,10)_{LR}\otimes\tilde{x}_{11}\oplus(60,70,10,20)_{LR}\otimes\tilde{x}_{12}\oplus(90,110,10,10)_{LR}\otimes\tilde{x}_{13}\oplus$ $(0,0,0,0)_{LR}\otimes\tilde{x}_{14}\oplus(70,80,10,10)_{LR}\otimes\tilde{x}_{21}\oplus(80,100,10,20)_{LR}\otimes\tilde{x}_{22}\oplus(30,50,10,10)_{LR}\otimes$ $\tilde{x}_{23}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{24}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{31}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{32}\oplus(0,0,0,0)_{LR}\otimes$ $\tilde{x}_{33}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{34})$

subject to

$$\tilde{x}_{11}\oplus\tilde{x}_{12}\oplus\tilde{x}_{13}\oplus\tilde{x}_{14}=(90,90,20,10)_{LR}$$
$$\tilde{x}_{21}\oplus\tilde{x}_{22}\oplus\tilde{x}_{23}\oplus\tilde{x}_{24}=(60,70,20,10)_{LR}$$
$$\tilde{x}_{31}\oplus\tilde{x}_{32}\oplus\tilde{x}_{33}\oplus\tilde{x}_{34}=(0,10,0,40)_{LR}$$
$$\tilde{x}_{11}\oplus\tilde{x}_{21}\oplus\tilde{x}_{31}=(40,50,10,20)_{LR}$$
$$\tilde{x}_{12}\oplus\tilde{x}_{22}\oplus\tilde{x}_{32}=(30,40,10,10)_{LR}$$
$$\tilde{x}_{13}\oplus\tilde{x}_{23}\oplus\tilde{x}_{33}=(50,50,10,30)_{LR}$$
$$\tilde{x}_{14}\oplus\tilde{x}_{24}\oplus\tilde{x}_{34}=(30,30,10,0)_{LR}$$

$\tilde{x}_{11},\tilde{x}_{12},\tilde{x}_{13},\tilde{x}_{14},\tilde{x}_{21},\tilde{x}_{22},\tilde{x}_{23},\tilde{x}_{24},\tilde{x}_{31},\tilde{x}_{32},\tilde{x}_{33},\tilde{x}_{34}$ are non-negative LR flat fuzzy numbers

Step 3 Using Step 4 to Step 7, of the method, proposed in Section 3.3.1 and Remark 3.4, the fuzzy linear programming problem, obtained in Step 2, can be converted into the following crisp linear programming problem:

Minimize $\frac{1}{10}(60m_{11}+190n_{11}-40\alpha_{11}+160\beta_{11}+260m_{12}+430n_{12}-200\alpha_{12}+360\beta_{12}+$ $410m_{13}+590n_{13}-320\alpha_{13}+480\beta_{13}+310m_{21}+440n_{21}-240\alpha_{21}+360\beta_{21}+360m_{22}+$ $580n_{22}-280\alpha_{22}+480\beta_{22}+110m_{23}+290n_{23}-80\alpha_{23}+240\beta_{23})$

subject to

$$\sum_{j=1}^{4}m_{1j}=90\ ,\quad \sum_{j=1}^{4}n_{1j}=90\ ,\quad \sum_{j=1}^{4}\alpha_{1j}=20\ ,\quad \sum_{j=1}^{4}\beta_{1j}=10$$
$$\sum_{j=1}^{4}m_{2j}=60\ ,\quad \sum_{j=1}^{4}n_{2j}=70\ ,\quad \sum_{j=1}^{4}\alpha_{2j}=20\ ,\quad \sum_{j=1}^{4}\beta_{2j}=10$$

$$\sum_{j=1}^{4} m_{3j} = 0\ , \quad \sum_{j=1}^{4} n_{3j} = 10\ , \quad \sum_{j=1}^{4} \alpha_{3j} = 0\ , \quad \sum_{j=1}^{4} \beta_{3j} = 40$$
$$\sum_{i=1}^{3} m_{i1} = 40\ , \quad \sum_{i=1}^{3} n_{i1} = 50\ , \quad \sum_{i=1}^{3} \alpha_{i1} = 10\ , \quad \sum_{i=1}^{3} \beta_{i1} = 20$$
$$\sum_{i=1}^{3} m_{i2} = 30\ , \quad \sum_{i=1}^{3} n_{i2} = 40\ , \quad \sum_{i=1}^{3} \alpha_{i2} = 10\ , \quad \sum_{i=1}^{3} \beta_{i2} = 10$$
$$\sum_{i=1}^{3} m_{i3} = 50\ , \quad \sum_{i=1}^{3} n_{i3} = 50\ , \quad \sum_{i=1}^{3} \alpha_{i3} = 10\ , \quad \sum_{i=1}^{3} \beta_{i3} = 30$$
$$\sum_{i=1}^{3} m_{i4} = 30\ , \quad \sum_{i=1}^{3} n_{i4} = 30\ , \quad \sum_{i=1}^{3} \alpha_{i4} = 10\ , \quad \sum_{i=1}^{3} \beta_{i4} = 0$$
$$m_{ij} - \alpha_{ij},\ n_{ij} - m_{ij},\ \alpha_{ij}, \beta_{ij} \geq 0\ \forall\ i = 1,2,3; j = 1,2,3,4$$

Step 4 The optimal solution of the crisp linear programming problem, obtained from Step 3, is $m_{11} = 40, n_{11} = 40, \alpha_{11} = 10, \beta_{11} = 10, m_{12} = 20, n_{12} = 20, \alpha_{12} = 0, \beta_{12} = 0, m_{13} = 25, n_{13} = 25, \alpha_{13} = 10, \beta_{13} = 0, m_{14} = 5, n_{14} = 5, \alpha_{14} = 0, \beta_{14} = 0, m_{21} = 0, n_{21} = 10, \alpha_{21} = 0, \beta_{21} = 10, m_{22} = 10, n_{22} = 10, \alpha_{22} = 10, \beta_{22} = 0, m_{23} = 25, n_{23} = 25, \alpha_{23} = 0, \beta_{23} = 0, m_{24} = 25, n_{24} = 25, \alpha_{24} = 10, \beta_{24} = 0, m_{31} = 0, n_{31} = 0, \alpha_{31} = 0, \beta_{31} = 0, m_{32} = 0, n_{32} = 10, \alpha_{32} = 0, \beta_{32} = 10, m_{33} = 0, n_{33} = 0, \alpha_{33} = 0, \beta_{33} = 30, m_{34} = 0, n_{34} = 0, \alpha_{34} = 0, \beta_{34} = 0$.

Step 5 Putting the values of $m_{ij}, n_{ij}, \alpha_{ij}$ and β_{ij} in $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$, the fuzzy optimal solution is $\tilde{x}_{11} = (40, 40, 10, 10)_{LR}$ $\tilde{x}_{12} = (20, 20, 0, 0)_{LR}$, $\tilde{x}_{13} = (25, 25, 10, 0)_{LR}$, $\tilde{x}_{14} = (5, 5, 0, 0)_{LR}$, $\tilde{x}_{21} = (0, 10, 0, 10)_{LR}$, $\tilde{x}_{22} = (10, 10, 10, 0)_{LR}$, $\tilde{x}_{23} = (25, 25, 0, 0)_{LR}$, $\tilde{x}_{24} = (25, 25, 10, 0)_{LR}$, $\tilde{x}_{31} = (0, 0, 0, 0)_{LR}$, $\tilde{x}_{32} = (0, 10, 0, 10)_{LR}$, $\tilde{x}_{33} = (0, 0, 0, 30)_{LR}$, $\tilde{x}_{34} = (0, 0, 0, 0)_{LR}$

Step 6 Putting the values of $\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34}$ in $((20, 30, 10, 10)_{LR} \otimes \tilde{x}_{11} \oplus (60, 70, 10, 20)_{LR} \otimes \tilde{x}_{12} \oplus (90, 110, 10, 10)_{LR} \otimes \tilde{x}_{13} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{14} \oplus (70, 80, 10, 10)_{LR} \otimes \tilde{x}_{21} \oplus (80, 100, 10, 20)_{LR} \otimes \tilde{x}_{22} \oplus (30, 50, 10, 10)_{LR} \otimes \tilde{x}_{23} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{24} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{31} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{32} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{33} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{34})$ the minimum total fuzzy transportation cost is $(5800, 8400, 2800, 2900)_{LR}$.

3.4.2 Fuzzy optimal solution using the method based on tabular representation

Using the proposed method, based on tabular representation, the fuzzy optimal solution of the fully fuzzy transportation problem, chosen in Example 3.1, can be obtained as follows:

Step 1 The balanced fully fuzzy transportation problem, obtained from Step 1 of Section 3.4.1, can be represented by Table 3.6.

Table 3.6 Tabular representation of balanced fully fuzzy transportation problem

	D_1	D_2	D_3	D_4	$\tilde{a}_i$
S_1	$(20,30,10,10)_{LR}$	$(60,70,10,20)_{LR}$	$(90,110,10,10)_{LR}$	$(0,0,0,0)_{LR}$	$(90,90,20,10)_{LR}$
S_2	$(70,80,10,10)_{LR}$	$(80,100,10,20)_{LR}$	$(30,50,10,10)_{LR}$	$(0,0,0,0)_{LR}$	$(60,70,20,10)_{LR}$
S_3	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,10,0,40)_{LR}$
$\tilde{b}_j$	$(40,50,10,20)_{LR}$	$(30,40,10,10)_{LR}$	$(50,50,10,30)_{LR}$	$(30,30,10,0)_{LR}$	$\sum_{i=1}^{3} \tilde{a}_i = \sum_{j=1}^{4} \tilde{b}_j$

Step 2 Using Step 3 of the method, proposed in Section 3.3.2 and Remark 3.6, Table 3.6 can be split into four crisp transportation tables i.e., Table 3.7, Table 3.8, Table 3.9, and Table 3.10.

Table 3.7 Tabular representation of first crisp transportation problem

	D_1	D_2	D_3	D_4	$m_i - \alpha_i$
S_1	25	69	100	0	70
S_2	75	94	40	0	40
S_3	0	0	0	0	0
$m'_j - \alpha'_j$	30	20	40	20	

Table 3.8 Tabular representation of second crisp transportation problem

	D_1	D_2	D_3	D_4	α_i
S_1	21	49	68	0	20
S_2	51	66	32	0	20
S_3	0	0	0	0	0
α'_j	10	10	10	10	

Table 3.9 Tabular representation of third crisp transportation problem

	D_1	D_2	D_3	D_4	$n_i - m_i$
S_1	19	43	59	0	0
S_2	44	58	29	0	10
S_3	0	0	0	0	10
$n'_j - m'_j$	10	10	0	0	

Table 3.10 Tabular representation of fourth crisp transportation problem

	D_1	D_2	D_3	D_4	β_i
S_1	16	36	48	0	10
S_2	36	48	24	0	10
S_3	0	0	0	0	40
β'_j	20	10	30	0	

Step 3 The optimal solution of crisp transportation problems, shown by Table 3.7, Table 3.8, Table 3.9 and Table 3.10, are $m_{11} - \alpha_{11} = 30, m_{12} - \alpha_{12} = 20, m_{13} - \alpha_{13} = 15, m_{14} - \alpha_{14} = 5, m_{21} - \alpha_{21} = 0, m_{22} - \alpha_{22} = 0, m_{23} - \alpha_{23} = 25, m_{24} - \alpha_{24} =$

$15, m_{31} - \alpha_{31} = 0, m_{32} - \alpha_{32} = 0, m_{33} - \alpha_{33} = 0, m_{34} - \alpha_{34} = 0; \alpha_{11} = 10, \alpha_{12} = 0, \alpha_{13} = 10, \alpha_{14} = 0, \alpha_{21} = 0, \alpha_{22} = 10, \alpha_{23} = 0, \alpha_{24} = 10, \alpha_{31} = 0, \alpha_{32} = 0, \alpha_{33} = 0, \alpha_{34} = 0; n_{11} - m_{11} = 0, n_{12} - m_{12} = 0, n_{13} - m_{13} = 0, n_{14} - m_{14} = 0, n_{21} - m_{21} = 10, n_{22} - m_{22} = 0, n_{23} - m_{23} = 0, n_{24} - m_{24} = 0, n_{31} - m_{31} = 0, n_{32} - m_{32} = 10, n_{33} - m_{33} = 0, n_{34} - m_{34} = 0$ and $\beta_{11} = 10, \beta_{12} = 0, \beta_{13} = 0, \beta_{14} = 0, \beta_{21} = 10, \beta_{22} = 0, \beta_{23} = 0, \beta_{24} = 0, \beta_{31} = 0, \beta_{32} = 10, \beta_{33} = 30, \beta_{34} = 0$ respectively.

Step 4 On solving the equations, obtained from Step 3, the values of $m_{ij}, n_{ij}, \alpha_{ij}$ and β_{ij} are $m_{11} = 40, n_{11} = 40, \alpha_{11} = 10, \beta_{11} = 10, m_{12} = 20, n_{12} = 20, \alpha_{12} = 0, \beta_{12} = 0, m_{13} = 25, n_{13} = 25, \alpha_{13} = 10, \beta_{13} = 0, m_{14} = 5, n_{14} = 5, \alpha_{14} = 0, \beta_{14} = 0, m_{21} = 0, n_{21} = 10, \alpha_{21} = 0, \beta_{21} = 10, m_{22} = 10, n_{22} = 10, \alpha_{22} = 10, \beta_{22} = 0, m_{23} = 25, n_{23} = 25, \alpha_{23} = 0, \beta_{23} = 0, m_{24} = 25, n_{24} = 25, \alpha_{24} = 10, \beta_{24} = 0, m_{31} = 0, n_{31} = 0, \alpha_{31} = 0, \beta_{31} = 0, m_{32} = 0, n_{32} = 10, \alpha_{32} = 0, \beta_{32} = 10, m_{33} = 0, n_{33} = 0, \alpha_{33} = 0, \beta_{33} = 30, m_{34} = 0, n_{34} = 0, \alpha_{34} = 0, \beta_{34} = 0.$

Step 5 Putting the values of $m_{ij}, n_{ij}, \alpha_{ij}$ and β_{ij} in $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$, the fuzzy optimal solution is $\tilde{x}_{11} = (40, 40, 10, 10)_{LR}$ $\tilde{x}_{12} = (20, 20, 0, 0)_{LR}$, $\tilde{x}_{13} = (25, 25, 10, 0)_{LR}$, $\tilde{x}_{14} = (5, 5, 0, 0)_{LR}$, $\tilde{x}_{21} = (0, 10, 0, 10)_{LR}$, $\tilde{x}_{22} = (10, 10, 10, 0)_{LR}$, $\tilde{x}_{23} = (25, 25, 0, 0)_{LR}$, $\tilde{x}_{24} = (25, 25, 10, 0)_{LR}$, $\tilde{x}_{31} = (0, 0, 0, 0)_{LR}$, $\tilde{x}_{32} = (0, 10, 0, 10)_{LR}$, $\tilde{x}_{33} = (0, 0, 0, 30)_{LR}$, $\tilde{x}_{34} = (0, 0, 0, 0)_{LR}$

Step 6 Putting the values of $\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34}$ in $((20, 30, 10, 10)_{LR} \otimes \tilde{x}_{11} \oplus (60, 70, 10, 20)_{LR} \otimes \tilde{x}_{12} \oplus (90, 110, 10, 10)_{LR} \otimes \tilde{x}_{13} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{14} \oplus (70, 80, 10, 10)_{LR} \otimes \tilde{x}_{21} \oplus (80, 100, 10, 20)_{LR} \otimes \tilde{x}_{22} \oplus (30, 50, 10, 10)_{LR} \otimes \tilde{x}_{23} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{24} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{31} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{32} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{33} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{34})$ the minimum total fuzzy transportation cost is $(5800, 8400, 2800, 2900)_{LR}$.

3.4.3 Physical interpretation of the results

In this section, the minimum total fuzzy transportation cost, obtained by using the proposed methods, is physically interpreted. Similarly, the obtained fuzzy optimal solution can also be physically interpreted.

Using the proposed methods the minimum total fuzzy transportation cost is $(5800, 8400, 2800, 2900)_{LR}$, which can be physically interpreted as follows:

(1) The least amount of minimum total transportation cost is 3000.

(2) The most possible amount of minimum total transportation cost lies between is 5800 and 8400.

(3) The greatest amount of minimum total transportation cost is 11300. i.e., the minimum total transportation cost will always be greater than 3000 and less than 11300 and maximum chances are that the minimum total transportation cost will lie between 5800 and 8400.

The variation in cost with respect to chances are shown in Figure 3.1.

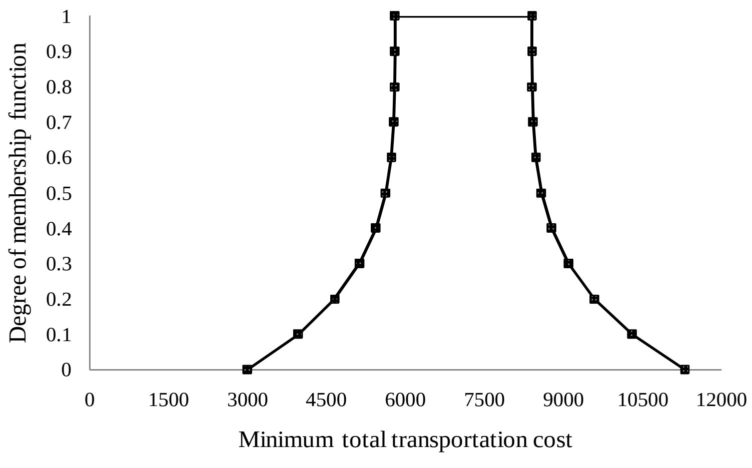

Figure 3.1. Membership function of *LR* flat fuzzy number representing the minimum total fuzzy transportation cost

3.5 Comparative study

The results obtained by the methods, proposed in this chapter and by the methods proposed in Chapter 2, are compared in Table 3.11.

Table 3.11 Results obtained by using proposed methods

Example	Minimum total fuzzy transportation cost	
	Methods proposed in Chapter 2	Methods proposed in this chapter
2.2	$(2100, 2900, 3500, 4200)$	$(2900, 3500, 800, 700)_{LR}$
3.1	Not applicable	$(5800, 8400, 2800, 2900)_{LR}$

The results, shown in Table 3.11, can be explained as follows:

(1) The methods, proposed in Chapter 2, can be used only for solving such fully fuzzy transportation problems in which the parameters are either represented by triangular fuzzy numbers or trapezoidal fuzzy numbers. Since, in the fully fuzzy transportation problem, chosen in Example 2.2, the parameters are represented by trapezoidal fuzzy numbers. So, this problem can be solved by the methods, proposed in Chapter 2. However, in the fully fuzzy transportation problem, chosen in Example 3.1, the parameters are represented by such LR flat fuzzy numbers which are neither triangular fuzzy numbers nor trapezoidal fuzzy numbers. So, the fully fuzzy transportation problem, chosen in Example 3.1, can not be solved by using the methods, proposed in Chapter 2.

(2) The methods, proposed in this chapter, can be used for solving such fully fuzzy transportation problems in which the parameters are represented by LR flat fuzzy numbers and as discussed in Remark 3.3, the trapezoidal fuzzy numbers are special type of LR flat fuzzy numbers so both the fully fuzzy transportation problems, chosen in Example 2.2 and Example 3.1, can be solved by using the methods, proposed in this chapter.

3.6 Conclusions

On the basis of the comparison of the results, it can be concluded that the problems which can be solved by using the methods proposed in Chapter 2, can also be solved by the methods proposed in this chapter. However, there exist several problems which can only be solved by the methods, proposed in this chapter, but not by the methods, proposed in Chapter 2. Hence, it is better to use the method, proposed in this chapter as compared to the methods proposed in Chapter 2.

Chapter 4

New Methods For Solving Fully Fuzzy Transshipment Problems With LR Flat Fuzzy Parameters

To the best of my knowledge, only the method [70] is proposed in the literature to find the fuzzy optimal solution of fully fuzzy transshipment problems. In this chapter, limitations of this method are pointed out and to overcome these limitations, two new methods are proposed for solving fully fuzzy transshipment problems. The advantages of the proposed methods over existing method [70] and over the methods, proposed in previous chapters, are discussed. To illustrate the proposed methods a fully fuzzy transshipment problem is solved. Also, to show the application of the proposed methods in real life problems an existing real life fully fuzzy transshipment problem is solved by the proposed methods.

4.1 Fuzzy linear programming formulation of balanced fully fuzzy transshipment problems

In the existing method [70] the fuzzy linear programming formulation ($P_{4.1}$) of balanced fully fuzzy transshipment problems is used to find the fuzzy optimal solution of fully fuzzy transshipment problems.

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The nodes which are used to formulate ($P_{4.1}$), are categorized as follows:

Purely source node: A node S is said to be a purely source node if there exist atleast one node S' such that the product may be supplied from S to S' but there does not exist any node S'' such that product may be supplied from S'' to S. The set of all such nodes may be represented by N_{PS}.

Purely destination node: A node D is said to be a purely destination node if there does not exist any node D' such that the product may be supplied from D to D' but there exist atleast one node D'' such that product may be supplied from D'' to D. The set of all such nodes may be represented by N_{PD}.

Intermediate node: The following nodes in the network are said to be intermediate nodes:

(i) A node S at which some quantity of the product is available to transship at other nodes and also there exist some nodes such that some quantity of the product is supplying from that nodes to node S. All such nodes are said to be source nodes and the set of all such nodes may be represented by N_S.

(ii) A node D at which some quantity of the product is required and also there exist some nodes such that the product is supplying from node D to that nodes. All such nodes are said to be destination nodes and the set of all such nodes may be represented by N_D.

(iii) A node T at which neither any quantity of the product is available to transship at other nodes nor any quantity of the product is required but there exist some nodes such that some quantity of the product is supplying from that nodes to node T and the same quantity of the product is supplying from T to some other nodes. All such nodes are said to be transition nodes and the set of all such nodes may be represented by N_T.

Let us consider a balanced fully fuzzy transshipment problem having m purely source nodes N_{PS_i}, $i = 1$ to m, t purely destination nodes N_{PD_j}, $j = 1$ to t and $(l+q+r)$ intermediate nodes. Let out of $(l+q+r)$ intermediate nodes, l intermediate nodes N_{S_i}, $i = 1$ to l are working as source nodes, q intermediate nodes N_{D_j}, $j = 1$ to q are working as destination nodes and remaining r intermediate nodes N_{T_i}, $i = 1$ to r are working as transition nodes. Let the availability of the product at i^{th} purely source node N_{PS_i} and at i^{th} source node N_{S_i} be $\tilde{a}_i$ and $\tilde{a}_{m+i}$ respectively and the demand of the product at j^{th} purely destination node N_{PD_j} and at j^{th} destination node N_{D_j} be $\tilde{b}_j$ and $\tilde{b}_{t+j}$ respectively.

Let $\tilde{c}_{ij}$ be the fuzzy cost for transporting one unit quantity of the product from i^{th} node to j^{th} node and $\tilde{x}_{ij}$ be the fuzzy quantity of the product that should be transported from i^{th} node to j^{th} node to minimize the total fuzzy transportation cost. Then, fuzzy linear programming formulation of balanced fully fuzzy transshipment problem can be written as [70]:

$$\text{Minimize} \sum_{(i,j)\in A} (\tilde{c}_{ij} \otimes \tilde{x}_{ij})$$

subject to

$$\begin{aligned}
&\sum_{j:(i,j)\in A} \tilde{x}_{ij} = \tilde{a}_i \ , && i = 1 \text{ to } m \\
&\sum_{j:(i,j)\in A} \tilde{x}_{ij} \ominus_H \sum_{j:(j,i)\in A} \tilde{x}_{ji} \ = \ \tilde{a}_{m+i} \ , && i = 1 \text{ to } l \\
&\sum_{i:(i,j)\in A} \tilde{x}_{ij} = \tilde{b}_j \ , && j = 1 \text{ to } t \qquad (P_{4.1}) \\
&\sum_{i:(i,j)\in A} \tilde{x}_{ij} \ominus_H \sum_{i:(j,i)\in A} \tilde{x}_{ji} \ = \ \tilde{b}_{t+j} \ , && j = 1 \text{ to } q \\
&\sum_{j:(i,j)\in A} \tilde{x}_{ij} = \sum_{j:(j,i)\in A} \tilde{x}_{ji} \ , && i = 1 \text{ to } r
\end{aligned}$$

$\tilde{x}_{ij}$ is a non-negative fuzzy number $\forall \ (i,j) \in A$

where, A is the set of arcs (i,j) joining node i to node j.

Remark 4.1 [70] Let $\tilde{A}_1 = (m_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{A}_2 = (m_2, \alpha_2, \beta_2)_{LR}$ be two LR fuzzy numbers such that $\alpha_1 \geq \alpha_2$ and $\beta_1 \geq \beta_2$. Then, $\tilde{A}_1 \ominus_H \tilde{A}_2 = (m_1 - m_2, \alpha_1 - \alpha_2, \beta_1 - \beta_2)_{LR}$.

Remark 4.2 [73] Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be two LR flat fuzzy numbers such that $\alpha_1 \geq \alpha_2$ and $\beta_1 \geq \beta_2$. Then, $\tilde{A}_1 \ominus_H \tilde{A}_2 = (m_1 - m_2, n_1 - n_2, \alpha_1 - \alpha_2, \beta_1 - \beta_2)_{LR}$.

Remark 4.3 If $\sum_{i=1}^{m} \tilde{a}_i \oplus \sum_{i=1}^{l} \tilde{a}_{m+i} = \sum_{j=1}^{t} \tilde{b}_j \oplus \sum_{j=1}^{q} \tilde{b}_{t+j}$ then the fully fuzzy transshipment problem is said to be balanced fully fuzzy transshipment problem otherwise it is called unbalanced fully fuzzy transshipment problem.

4.2 Ghatee and Hashemi method

Ghatee and Hashemi [70] proposed a method to find the fuzzy optimal solution of balanced fully fuzzy transshipment problems. Ghatee and Hashemi [71, 73] and Ghatee et al. [74] applied the existing method [70] for solving real life problems.

The steps of the existing method [70] are as follows:

Step 1 Assuming $\tilde{c}_{ij} = (m'_{ij}, \alpha'_{ij}, \beta'_{ij})_{LR}$, $\tilde{x}_{ij} = (m_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$, $\tilde{a}_i = (m_i, \alpha_i, \beta_i)_{LR}$, $\tilde{a}_{m+i} = (m_{m+i}, \alpha_{m+i}, \beta_{m+i})_{LR}$, $\tilde{b}_j = (m'_j, \alpha'_j, \beta'_j)_{LR}$ and $\tilde{b}_{t+j} = (m_{t+j}, \alpha_{t+j}, \beta_{t+j})_{LR}$, the fuzzy linear programming problem $(P_{4.1})$ can be written as:

Minimize $\sum_{(i,j)\in A} \Big((m'_{ij}, \alpha'_{ij}, \beta'_{ij})_{LR} \otimes (m_{ij}, \alpha_{ij}, \beta_{ij})_{LR} \Big)$

subject to

$$\sum_{j:(i,j)\in A} (m_{ij}, \alpha_{ij}, \beta_{ij})_{LR} = (m_i, \alpha_i, \beta_i)_{LR}\ , \qquad i = 1 \text{ to } m$$

$$\sum_{j:(i,j)\in A} (m_{ij}, \alpha_{ij}, \beta_{ij})_{LR} \ominus_H \sum_{j:(j,i)\in A} (m_{ji}, \alpha_{ji}, \beta_{ji})_{LR} = (m_{m+i}, \alpha_{m+i}, \beta_{m+i})_{LR}, \ i = 1 \text{ to } l$$

$$\sum_{i:(i,j)\in A} (m_{ij}, \alpha_{ij}, \beta_{ij})_{LR} = (m'_j, \alpha'_j, \beta'_j)_{LR}\ , \qquad j = 1 \text{ to } t$$

$$\sum_{i:(i,j)\in A}(m_{ij},\alpha_{ij},\beta_{ij})_{LR}\ominus_H\sum_{i:(j,i)\in A}(m_{ji},\alpha_{ji},\beta_{ji})_{LR}=(m_{t+j},\alpha_{t+j},\beta_{t+j})_{LR},\quad j=1\text{ to }q$$

$$\sum_{j:(i,j)\in A}(m_{ij},\alpha_{ij},\beta_{ij})_{LR}=\sum_{j:(j,i)\in A}(m_{ji},\alpha_{ji},\beta_{ji})_{LR}\ ,\qquad i=1\text{ to }r$$

$(m_{ij},\alpha_{ij},\beta_{ij})_{LR}$ is a non-negative LR fuzzy number $\forall\ (i,j)\in A$

Step 2 Using the arithmetic operations, defined in Section 3.1.2 and assuming $\sum_{(i,j)\in A}\big((m'_{ij},\alpha'_{ij},\beta'_{ij})_{LR}\otimes(m_{ij},\alpha_{ij},\beta_{ij})_{LR}\big)=(m_0,\alpha_0,\beta_0)_{LR}$, the fuzzy linear programming problem, obtained in Step 1, can be written as:

Minimize $(m_0,\alpha_0,\beta_0)_{LR}$

subject to

$$\Big(\sum_{j:(i,j)\in A}m_{ij},\sum_{j:(i,j)\in A}\alpha_{ij},\sum_{j:(i,j)\in A}\beta_{ij}\Big)_{LR}=(m_i,\alpha_i,\beta_i)_{LR},\qquad i=1\text{ to }m$$

$$\Big(\sum_{j:(i,j)\in A}m_{ij},\sum_{j:(i,j)\in A}\alpha_{ij},\sum_{j:(i,j)\in A}\beta_{ij}\Big)_{LR}\ominus_H\Big(\sum_{j:(j,i)\in A}m_{ji},\sum_{j:(j,i)\in A}\alpha_{ji},\sum_{j:(j,i)\in A}\beta_{ji}\Big)_{LR}$$
$$=(m_{m+i},\alpha_{m+i},\beta_{m+i})_{LR},\qquad i=1\text{ to }l$$

$$\Big(\sum_{i:(i,j)\in A}m_{ij},\sum_{i:(i,j)\in A}\alpha_{ij},\sum_{i:(i,j)\in A}\beta_{ij}\Big)_{LR}=(m'_j,\alpha'_j,\beta'_j)_{LR}\ ,\qquad j=1\text{ to }t$$

$$\Big(\sum_{i:(i,j)\in A}m_{ij},\sum_{i:(i,j)\in A}\alpha_{ij},\sum_{i:(i,j)\in A}\beta_{ij}\Big)_{LR}\ominus_H\Big(\sum_{i:(j,i)\in A}m_{ji},\sum_{i:(j,i)\in A}\alpha_{ji},\sum_{i:(j,i)\in A}\beta_{ji}\Big)_{LR}$$
$$=(m_{t+j},\alpha_{t+j},\beta_{t+j})_{LR},\qquad j=1\text{ to }q$$

$$\Big(\sum_{j:(i,j)\in A}m_{ij},\sum_{j:(i,j)\in A}\alpha_{ij},\sum_{j:(i,j)\in A}\beta_{ij}\Big)_{LR}=\Big(\sum_{j:(j,i)\in A}m_{ji},\sum_{j:(j,i)\in A}\alpha_{ji},\sum_{j:(j,i)\in A}\beta_{ji}\Big)_{LR}\ ,$$
$$i=1\text{ to }r$$

$(m_{ij},\alpha_{ij},\beta_{ij})_{LR}$ is a non-negative LR fuzzy number $\forall\ (i,j)\in A$

Step 3 Using Definition 3.5, Definition 3.6 and Remark 4.1, the fuzzy linear programming problem, obtained in Step 2, can be converted into the fuzzy linear programming problem ($P_{4.2}$):

Minimize $(m_0,\alpha_0,\beta_0)_{LR}$

subject to

$$\sum_{j:(i,j)\in A}m_{ij}=m_i\ ,\qquad i=1\text{ to }m$$

$$\sum_{j:(i,j)\in A} \alpha_{ij} = \alpha_i \ , \qquad i = 1 \text{ to } m$$
$$\sum_{j:(i,j)\in A} \beta_{ij} = \beta_i \ , \qquad i = 1 \text{ to } m$$
$$\sum_{j:(i,j)\in A} m_{ij} - \sum_{j:(j,i)\in A} m_{ji} = m_{m+i} \ , \qquad i = 1 \text{ to } l$$
$$\sum_{j:(i,j)\in A} \alpha_{ij} - \sum_{j:(j,i)\in A} \alpha_{ji} = \alpha_{m+i} \ , \qquad i = 1 \text{ to } l$$
$$\sum_{j:(i,j)\in A} \beta_{ij} - \sum_{j:(j,i)\in A} \beta_{ji} = \beta_{m+i} \ , \qquad i = 1 \text{ to } l$$
$$\sum_{i:(i,j)\in A} m_{ij} = m'_j \ , \qquad j = 1 \text{ to } t$$
$$\sum_{i:(i,j)\in A} \alpha_{ij} = \alpha'_j \ , \qquad j = 1 \text{ to } t \qquad (P_{4.2})$$
$$\sum_{i:(i,j)\in A} \beta_{ij} = \beta'_j \ , \qquad j = 1 \text{ to } t$$
$$\sum_{i:(i,j)\in A} m_{ij} - \sum_{i:(j,i)\in A} m_{ji} = m_{t+j} \ , \qquad j = 1 \text{ to } q$$
$$\sum_{i:(i,j)\in A} \alpha_{ij} - \sum_{i:(j,i)\in A} \alpha_{ji} = \alpha_{t+j} \ , \qquad j = 1 \text{ to } q$$
$$\sum_{i:(i,j)\in A} \beta_{ij} - \sum_{i:(j,i)\in A} \beta_{ji} = \beta_{t+j} \ , \qquad j = 1 \text{ to } q$$
$$\sum_{j:(i,j)\in A} m_{ij} = \sum_{j:(j,i)\in A} m_{ji} \ , \qquad i = 1 \text{ to } r$$
$$\sum_{j:(i,j)\in A} \alpha_{ij} = \sum_{j:(j,i)\in A} \alpha_{ji} \ , \qquad i = 1 \text{ to } r$$
$$\sum_{j:(i,j)\in A} \beta_{ij} = \sum_{j:(j,i)\in A} \beta_{ji} \qquad i = 1 \text{ to } r$$
$$m_{ij} - \alpha_{ij}, \alpha_{ij}, \beta_{ij} \geq 0 \ \forall \ (i,j) \in A$$

Step 4 The fuzzy optimal solution of fuzzy linear programming $(P_{4.2})$ can be obtained by solving the following crisp linear programming problem:

Minimize $(km_0 + l\alpha_0 + r\beta_0)$

subject to

constraints of the problem $(P_{4.2})$

where, $k = q_1\vartheta^{n_1}, l = q_2\vartheta^{n_2}$ and $r = q_3\vartheta^{n_3}$, $q_1, q_2, q_3 \in \mathbb{Q}^+$, $n_1 \neq n_2 \neq n_3$ are non-negative integers and ϑ is a non-algebraic positive real number .

Step 5 Solve the crisp linear programming problem, obtained in Step 4, to find the optimal solution $\{m_{ij}, \alpha_{ij}, \beta_{ij}\}$.

Step 6 Put the values of $m_{ij}, \alpha_{ij}, \beta_{ij}$ in $\tilde{x}_{ij} = (m_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$ to find the fuzzy optimal solution $\{\tilde{x}_{ij}\}$.

Step 7 Put the values of $\tilde{x}_{ij}$, obtained from Step 6, in $\sum\limits_{(i,j)\in A} (\tilde{c}_{ij} \otimes \tilde{x}_{ij})$, to find the minimum total fuzzy transportation cost.

Remark 4.4 In the existing method [70] the following existing multiplication [215], which is an approximation of the multiplication, presented in Section 3.1.2, is used:

Let $\tilde{A}_1 = (m, \alpha, \beta)_{LR}$ and $\tilde{A}_2 = (m_1, \alpha_1, \beta_1)_{LR}$ be two non-negative LR fuzzy numbers. Then,

$\tilde{A}_1 \otimes \tilde{A}_2 \simeq (mm_1, m\alpha_1 + m_1\alpha - \kappa_1\alpha\alpha_1, m\beta_1 + m_1\beta + \kappa_2\beta\beta_1)_{LR}$

where, $\kappa_1 = \frac{\int_0^1 [L^{-1}(t)]^3 dt}{\int_0^1 [L^{-1}(t)]^2 dt}$ and $\kappa_2 = \frac{\int_0^1 [R^{-1}(t)]^3 dt}{\int_0^1 [R^{-1}(t)]^2 dt}$.

4.3 Limitations of Ghatee and Hashemi method and the methods proposed in previous chapters

In this section, the limitations of the existing method [70] and the methods, proposed in previous chapters, are pointed out:

(1) The existing method [70] is proposed for solving balanced fully fuzzy transshipment problems. Since, the balanced fully fuzzy transportation problems are special type of fully fuzzy transshipment problems so, the existing method [70] can also be used to find the fuzzy optimal solution of balanced fully fuzzy transportation problems. However, the existing method [70] can neither be used to find the fuzzy optimal solution of unbalanced fully fuzzy transportation problems nor to find the fuzzy optimal solution of unbalanced fully fuzzy transshipment problems e.g., the balanced fully fuzzy transportation problem, chosen in Example 4.1, can be solved by the existing method [70] but the

unbalanced fully fuzzy transportation problems, chosen in Example 2.1 and Example 3.1, can not be solved by using the existing method [70]. Similarly, the existing balanced fully fuzzy transshipment problem [71, Example 3.5, pp. 2498] can be solved by the existing method [70] but the unbalanced fully fuzzy transshipment problem, chosen in Example 4.2, can not be solved by using the existing method [70].

Example 4.1 A company has two sources S_1 and S_2 and three destinations D_1, D_2 and D_3 ; the fuzzy cost for transporting one unit quantity of the product from i^{th} source to j^{th} destination is $\tilde{c}_{ij}$

where, $[\tilde{c}_{ij}]_{2\times 3} = \begin{pmatrix} (20,30,10,10)_{LR} & (60,70,10,20)_{LR} & (90,110,10,10)_{LR} \\ (70,80,10,10)_{LR} & (80,100,10,20)_{LR} & (30,50,10,10)_{LR} \end{pmatrix}$. The fuzzy availability of the product at first and second sources are $(90,100,20,30)_{LR}$ and $(60,70,20,30)_{LR}$ and the fuzzy demand of the product at first, second and third destinations are $(40,50,10,20)_{LR}$, $(30,40,10,10)_{LR}$ and $(80,80,20,30)_{LR}$ respectively, where, $L(x) = R(x) =$ maximum $\{0, 1-x\}$. The owner of the company want to determine the fuzzy quantity of the product that should be transported from each of the sources to each destination so that the total fuzzy transportation cost is minimum.

Example 4.2 Consider a network with five nodes, shown in Figure 4.1, including one purely source node (1), one source node (2), one transition node (3) and two purely destination nodes (4 and 5). The fuzzy cost for transporting one unit quantity of the product from one node to another node is shown on the respective arcs. The fuzzy availability and fuzzy demand are represented by the following LR flat fuzzy numbers.

Fuzzy availability: $\tilde{a}_1 = (40,40,10,20)_{LR}$, $\tilde{a}_2 = (30,40,10,30)_{LR}$

Fuzzy demand: $\tilde{b}_4 = (20, 30, 10, 10)_{LR}$, $\tilde{b}_5 = (60, 70, 30, 10)_{LR}$

where, $L(x) = R(x) =$ maximum $\{0, 1 - x^4\}$. Find the fuzzy optimum shipping schedule.

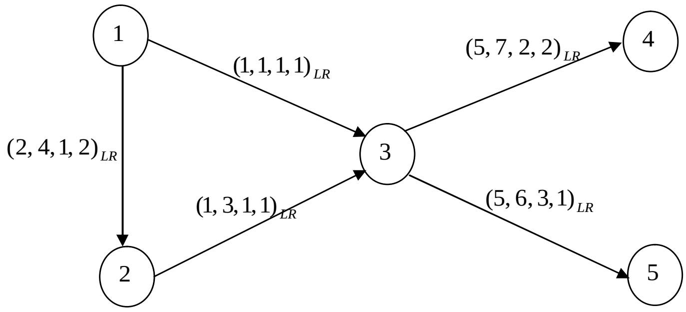

Figure 4.1. A network representing different shipping routes

(2) The methods, proposed in previous chapters, can be used only to find the fuzzy optimal solution of fully fuzzy transportation problems. Since, the fully fuzzy transshipment problems are generalization of fully fuzzy transportation problems so, the methods, proposed in previous chapters, can not be used for solving fully fuzzy transshipment problems.

4.4 Proposed methods

In this section, to overcome the limitations of the existing method [70] and the methods of previous chapters (discussed in Section 4.3), two new methods are proposed to find the fuzzy optimal solution of such fully fuzzy transshipment problems in which all the parameters are represented by LR flat fuzzy numbers. Also, the advantages of the proposed methods over the existing method [70] and over the methods, proposed in previous chapters, are discussed.

4.4.1 Proposed method based on fuzzy linear programming formulation

In this section, a new method, based on fuzzy linear programming formulation, is proposed to find the fuzzy optimal solution of such fully fuzzy transshipment problems in which all the parameters are represented by LR flat fuzzy numbers.

The steps of the proposed method are as follows:

Step 1 Find the total fuzzy availability $\sum_{i=1}^{m} \tilde{a}_i \oplus \sum_{i=1}^{l} \tilde{a}_{m+i}$ and the total fuzzy demand $\sum_{j=1}^{t} \tilde{b}_j \oplus \sum_{j=1}^{q} \tilde{b}_{t+j}$. Let $\sum_{i=1}^{m} \tilde{a}_i \oplus \sum_{i=1}^{l} \tilde{a}_{m+i} = (m, n, \alpha, \beta)_{LR}$ and $\sum_{j=1}^{t} \tilde{b}_j \oplus \sum_{j=1}^{q} \tilde{b}_{t+j} = (m', n', \alpha', \beta')_{LR}$. Use Definition 3.5 to examine that the problem is balanced or not, i.e., $\sum_{i=1}^{m} \tilde{a}_i \oplus \sum_{i=1}^{l} \tilde{a}_{m+i} = \sum_{j=1}^{t} \tilde{b}_j \oplus \sum_{j=1}^{q} \tilde{b}_{t+j}$ or $\sum_{i=1}^{m} \tilde{a}_i \oplus \sum_{i=1}^{l} \tilde{a}_{m+i} \neq \sum_{j=1}^{t} \tilde{b}_j \oplus \sum_{j=1}^{q} \tilde{b}_{t+j}$.

Case (1) If the problem is balanced, i.e., $\sum_{i=1}^{m} \tilde{a}_i \oplus \sum_{i=1}^{l} \tilde{a}_{m+i} = \sum_{j=1}^{t} \tilde{b}_j \oplus \sum_{j=1}^{q} \tilde{b}_{t+j}$, then Go to Step 2.

Case (2) If $\sum_{i=1}^{m} \tilde{a}_i \oplus \sum_{i=1}^{l} \tilde{a}_{m+i} \neq \sum_{j=1}^{t} \tilde{b}_j \oplus \sum_{j=1}^{q} \tilde{b}_{t+j}$ then convert the unbalanced problem into balanced problem as follows:

Case (2a) If $m - \alpha \leq m' - \alpha'$, $\alpha \leq \alpha'$, $n - m \leq n' - m'$, and $\beta \leq \beta'$ then introduce a dummy purely source node with fuzzy availability $(m' - m, n' - n, \alpha' - \alpha, \beta' - \beta)_{LR}$. Assume the fuzzy cost for transporting one unit quantity of the product from the introduced dummy purely source node to all purely destination nodes and all intermediate nodes as zero LR flat fuzzy number. Go to Step 2.

Case (2b) If $m - \alpha \geq m' - \alpha'$, $\alpha \geq \alpha'$, $n - m \geq n' - m'$, and $\beta \geq \beta'$ then introduce a dummy purely destination node with fuzzy demand $(m - m', n - n', \alpha - \alpha', \beta - \beta')_{LR}$. Assume the fuzzy cost for transporting one unit quantity of the product from all purely source nodes and intermediate nodes to the introduced dummy purely destination node as zero LR flat fuzzy number. Go to Step 2.

Case (2c) If neither Case (a) nor Case (b) is satisfied then introduce a dummy purely source node with fuzzy availability (maximum $\{0,(m'-\alpha')-(m-\alpha)\}$ + maximum $\{0,(\alpha'-\alpha)\}$, maximum $\{0,(m'-\alpha')-(m-\alpha)\}$ + maximum $\{0,(\alpha'-\alpha)\}$ + maximum $\{0,(n'-m')-(n-m)\}$, maximum $\{0,(\alpha'-\alpha)\}$, maximum $\{0,(\beta'-\beta)\})_{LR}$ and also introduce a dummy purely destination node with fuzzy demand (maximum $\{0,(m-\alpha)-(m'-\alpha')\}$ + maximum $\{0,(\alpha-\alpha')\}$, maximum $\{0,(m-\alpha)-(m'-\alpha')\}$ + maximum $\{0,(\alpha-\alpha')\}$ + maximum $\{0,(n-m)-(n'-m')\}$, maximum $\{0,(\alpha-\alpha')\}$, maximum $\{0,(\beta-\beta')\})_{LR}$. Assume the fuzzy cost for transporting one unit quantity of the product from the introduced dummy purely source node to all intermediate nodes, existing purely destination nodes and introduced dummy purely destination node as zero LR flat fuzzy number. Similarly, assume the fuzzy cost for transporting one unit quantity of the product from all intermediate nodes, existing purely source nodes and introduced dummy purely source node to the introduced dummy purely destination node as zero LR flat fuzzy number. Go to Step 2

Step 2 Formulate the balanced fully fuzzy transshipment problem, obtained in Step 1, into the fuzzy linear programming problem $(P_{4.1})$.

Step 3 Assuming $\tilde{c}_{ij}=(m'_{ij},n'_{ij},\alpha'_{ij},\beta'_{ij})_{LR}$, $\tilde{x}_{ij}=(m_{ij},n_{ij},\alpha_{ij},\beta_{ij})_{LR}$, $\tilde{a}_i=(m_i,$ $n_i,\alpha_i,\beta_i)_{LR}$, $\tilde{a}_{m+i}=(m_{m+i},n_{m+i},\alpha_{m+i},\beta_{m+i})_{LR}$, $\tilde{b}_j=(m'_j,n'_j,\alpha'_j,\beta'_j)_{LR}$ and $\tilde{b}_{t+j}=$ $(m_{t+j},n_{t+j},\alpha_{t+j},\beta_{t+j})_{LR}$, the fuzzy linear programming problem $(P_{4.1})$, can be written as:

Minimize $\sum\limits_{(i,j)\in A}\Big((m'_{ij},n'_{ij},\alpha'_{ij},\beta'_{ij})_{LR}\otimes(m_{ij},n_{ij},\alpha_{ij},\beta_{ij})_{LR}\Big)$

subject to

$$\sum_{j:(i,j)\in A}(m_{ij},n_{ij},\alpha_{ij},\beta_{ij})_{LR}=(m_i,n_i,\alpha_i,\beta_i)_{LR}\ , \qquad i=1 \text{ to } m$$

$$\sum_{j:(i,j)\in A}(m_{ij},n_{ij},\alpha_{ij},\beta_{ij})_{LR}\ominus_H\sum_{j:(j,i)\in A}(m_{ji},n_{ji},\alpha_{ji},\beta_{ji})_{LR}=(m_{m+i},n_{m+i},\alpha_{m+i},\beta_{m+i})_{LR},$$

$$i = 1 \text{ to } l$$

$$\sum_{i:(i,j)\in A} (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR} = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR}\ , \qquad j = 1 \text{ to } t$$

$$\sum_{i:(i,j)\in A} (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR} \ominus_H \sum_{i:(j,i)\in A} (m_{ji}, n_{ji}, \alpha_{ji}, \beta_{ji})_{LR} = (m_{t+j}, n_{t+j}, \alpha_{t+j}, \beta_{t+j})_{LR}, \quad j = 1 \text{ to } q$$

$$\sum_{j:(i,j)\in A} (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR} = \sum_{j:(j,i)\in A} (m_{ji}, n_{ji}, \alpha_{ji}, \beta_{ji})_{LR}\ , \qquad i = 1 \text{ to } r$$

$(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$ is a non-negative LR flat fuzzy number $\forall\ (i, j) \in A$

Step 4 Using the arithmetic operations of LR flat fuzzy numbers, defined in Section 3.1.2 of Chapter 3 and assuming $\sum_{(i,j)\in A} \Big(m'_{ij}, n'_{ij}, \alpha'_{ij}, \beta'_{ij})_{LR} \otimes (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR} \Big) = (m_0, n_0, \alpha_0, \beta_0)_{LR}$, the fuzzy linear programming problem, obtained in Step 3, can be written as:

Minimize $(m_0, n_0, \alpha_0, \beta_0)_{LR}$

subject to

$$\Big(\sum_{j:(i,j)\in A} m_{ij}, \sum_{j:(i,j)\in A} n_{ij}, \sum_{j:(i,j)\in A} \alpha_{ij}, \sum_{j:(i,j)\in A} \beta_{ij}\Big)_{LR} = (m_i, n_i, \alpha_i, \beta_i)_{LR}, \qquad i = 1 \text{ to } m$$

$$\Big(\sum_{j:(i,j)\in A} m_{ij}, \sum_{j:(i,j)\in A} n_{ij}, \sum_{j:(i,j)\in A} \alpha_{ij}, \sum_{j:(i,j)\in A} \beta_{ij}\Big)_{LR} \ominus_H \Big(\sum_{j:(j,i)\in A} m_{ji}, \sum_{j:(j,i)\in A} n_{ji}, \sum_{j:(j,i)\in A} \alpha_{ji}, \sum_{j:(j,i)\in A} \beta_{ji}\Big)_{LR} = (m_{m+i}, n_{m+i}, \alpha_{m+i}, \beta_{m+i})_{LR}, \quad i = 1 \text{ to } l$$

$$\Big(\sum_{i:(i,j)\in A} m_{ij}, \sum_{i:(i,j)\in A} n_{ij}, \sum_{i:(i,j)\in A} \alpha_{ij}, \sum_{i:(i,j)\in A} \beta_{ij}\Big)_{LR} = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR}\ , \qquad j = 1 \text{ to } t$$

$$\Big(\sum_{i:(i,j)\in A} m_{ij}, \sum_{i:(i,j)\in A} n_{ij}, \sum_{i:(i,j)\in A} \alpha_{ij}, \sum_{i:(i,j)\in A} \beta_{ij}\Big)_{LR} \ominus_H \Big(\sum_{i:(j,i)\in A} m_{ji}, \sum_{i:(j,i)\in A} n_{ji}, \sum_{i:(j,i)\in A} \alpha_{ji}, \sum_{i:(j,i)\in A} \beta_{ji}\Big)_{LR} = (m_{t+j}, n_{t+j}, \alpha_{t+j}, \beta_{t+j})_{LR}, \quad j = 1 \text{ to } q$$

$$\Big(\sum_{j:(i,j)\in A} m_{ij}, \sum_{j:(i,j)\in A} n_{ij}, \sum_{j:(i,j)\in A} \alpha_{ij}, \sum_{j:(i,j)\in A} \beta_{ij}\Big)_{LR} = \Big(\sum_{j:(j,i)\in A} m_{ji}, \sum_{j:(j,i)\in A} n_{ji}, \sum_{j:(j,i)\in A} \alpha_{ji}, \sum_{j:(j,i)\in A} \beta_{ji}\Big)_{LR}, \qquad i = 1 \text{ to } r$$

$(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$ is a non-negative LR flat fuzzy number $\forall\ (i, j) \in A$

Step 5 Using Definition 3.5, Definition 3.6 and Remark 4.2, the fuzzy linear programming problem, obtained in Step 4, can be converted into the fuzzy linear programming problem $(P_{4.3})$:

Minimize $(m_0, n_0, \alpha_0, \beta_0)_{LR}$

subject to

$$
\begin{array}{lll}
\sum\limits_{j:(i,j)\in A} m_{ij} = m_i \ , & i = 1 \text{ to } m & \\
\sum\limits_{j:(i,j)\in A} n_{ij} = n_i \ , & i = 1 \text{ to } m & \\
\sum\limits_{j:(i,j)\in A} \alpha_{ij} = \alpha_i \ , & i = 1 \text{ to } m & \\
\sum\limits_{j:(i,j)\in A} \beta_{ij} = \beta_i \ , & i = 1 \text{ to } m & \\
\sum\limits_{j:(i,j)\in A} m_{ij} - \sum\limits_{j:(j,i)\in A} m_{ji} = m_{m+i} \ , & i = 1 \text{ to } l & \\
\sum\limits_{j:(i,j)\in A} n_{ij} - \sum\limits_{j:(j,i)\in A} n_{ji} = n_{m+i} \ , & i = 1 \text{ to } l & \\
\sum\limits_{j:(i,j)\in A} \alpha_{ij} - \sum\limits_{j:(j,i)\in A} \alpha_{ji} = \alpha_{m+i} \ , & i = 1 \text{ to } l & \\
\sum\limits_{j:(i,j)\in A} \beta_{ij} - \sum\limits_{j:(j,i)\in A} \beta_{ji} = \beta_{m+i} \ , & i = 1 \text{ to } l & \\
\sum\limits_{i:(i,j)\in A} m_{ij} = m'_j \ , & j = 1 \text{ to } t & \\
\sum\limits_{i:(i,j)\in A} n_{ij} = n'_j \ , & j = 1 \text{ to } t & (P_{4.3}) \\
\sum\limits_{i:(i,j)\in A} \alpha_{ij} = \alpha'_j \ , & j = 1 \text{ to } t & \\
\sum\limits_{i:(i,j)\in A} \beta_{ij} = \beta'_j \ , & j = 1 \text{ to } t & \\
\sum\limits_{i:(i,j)\in A} m_{ij} - \sum\limits_{i:(j,i)\in A} m_{ji} = m_{t+j} \ , & j = 1 \text{ to } q & \\
\sum\limits_{i:(i,j)\in A} n_{ij} - \sum\limits_{i:(j,i)\in A} n_{ji} = n_{t+j} \ , & j = 1 \text{ to } q & \\
\sum\limits_{i:(i,j)\in A} \alpha_{ij} - \sum\limits_{i:(j,i)\in A} \alpha_{ji} = \alpha_{t+j} \ , & j = 1 \text{ to } q & \\
\sum\limits_{i:(i,j)\in A} \beta_{ij} - \sum\limits_{i:(j,i)\in A} \beta_{ji} = \beta_{t+j} \ , & j = 1 \text{ to } q & \\
\sum\limits_{j:(i,j)\in A} m_{ij} = \sum\limits_{j:(j,i)\in A} m_{ji} \ , & i = 1 \text{ to } r & \\
\sum\limits_{j:(i,j)\in A} n_{ij} = \sum\limits_{j:(j,i)\in A} n_{ji} \ , & i = 1 \text{ to } r & \\
\sum\limits_{j:(i,j)\in A} \alpha_{ij} = \sum\limits_{j:(j,i)\in A} \alpha_{ji} \ , & i = 1 \text{ to } r & \\
\sum\limits_{j:(i,j)\in A} \beta_{ij} = \sum\limits_{j:(j,i)\in A} \beta_{ji} & i = 1 \text{ to } r & \\
m_{ij} - \alpha_{ij}, n_{ij} - m_{ij}, \alpha_{ij}, \beta_{ij} \geq 0 \ \forall \ (i,j) \in A & &
\end{array}
$$

Step 6 As discussed in Step 6 of the method, proposed in Section 2.5.1 of Chapter

2, the fuzzy optimal solution of fuzzy linear programming problem ($P_{4.3}$), can be obtained by solving the following crisp linear programming problem:

Minimize $\Re(m_0, n_0, \alpha_0, \beta_0)_{LR}$

subject to

constraints of the problem ($P_{4.3}$)

Step 7 Using the existing formula [131], $\Re(m_0, n_0, \alpha_0, \beta_0)_{LR} = \frac{1}{2}(\int_0^1 (m_0 - \alpha_0 L^{-1}(\lambda))\ d\lambda + \int_0^1 (n_0 + \beta_0 R^{-1}(\lambda))\ d\lambda)$, the crisp linear programming problem, obtained in Step 6, can be written as:

Minimize $\frac{1}{2}(\int_0^1 (m_0 - \alpha_0 L^{-1}(\lambda))\ d\lambda + \int_0^1 (n_0 + \beta_0 R^{-1}(\lambda))\ d\lambda)$

subject to

constraints of the problem ($P_{4.3}$)

Step 8 Solve the crisp linear programming problem, obtained in Step 7, to find the optimal solution $\{m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}\}$.

Step 9 Put the values of $m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}$ in $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$ to find the fuzzy optimal solution $\{\tilde{x}_{ij}\}$.

Step 10 Put the values of $\tilde{x}_{ij}$, obtained from Step 9, in $\sum\limits_{(i,j)\in A} (\tilde{c}_{ij} \otimes \tilde{x}_{ij})$, to find the minimum total fuzzy transportation cost.

Remark 4.5 Since, the method, used in existing method [70], for comparing fuzzy numbers is applicable only for comparing LR fuzzy numbers so, this method can not be used for comparing LR flat fuzzy numbers. While, Liou and Wang ranking approach [131] can be used for comparing both LR fuzzy numbers and LR flat fuzzy numbers so, in Step 6 of the proposed method Liou and Wang ranking approach [131] is used for comparing LR flat fuzzy numbers.

4.4.2 Proposed method based on tabular representation

In this section, a new method, based on tabular representation, is proposed to find the fuzzy optimal solution of such fully fuzzy transshipment problems in which all the parameters are represented by LR flat fuzzy numbers.

The steps of the proposed method are as follows:

Step 1 Use Step 1 of the method, proposed in Section 4.4.1, to obtain a balanced fully fuzzy transshipment problem.

Step 2 Represent the balanced fully fuzzy transshipment problem, obtained from Step 1, into a tabular form as shown by Table 4.1.

Step 3 Convert the balanced fully fuzzy transshipment problem, represented by Table 4.1, into balanced fully fuzzy transportation problem as follows:

Add an amount of fuzzy buffer stock $\tilde{P} = \sum_{i=1}^{m} \tilde{a}_i \oplus \sum_{i=1}^{l} \tilde{a}_{m+i}$ (or$\sum_{j=1}^{t} \tilde{b}_j \oplus \sum_{j=1}^{q} \tilde{b}_{t+j}$) $= (P_1, P_2, \alpha_p, \beta_p)_{LR}$ in the fuzzy availability and fuzzy demand corresponding to intermediate nodes of Table 4.1. Let after adding the fuzzy buffer stock $\tilde{P}$, the new fuzzy availabilities corresponding to the i^{th} intermediate nodes N_{S_i}, N_{T_i} and N_{D_i} are $\tilde{a}'_{m+i} = \tilde{a}_{m+i} \oplus \tilde{P} = (m''_{m+i}, n''_{m+i}, \alpha''_{m+i}, \beta''_{m+i})_{LR}$, $\tilde{P}$ and $\tilde{P}$ respectively and the new fuzzy demands corresponding to the j^{th} intermediate nodes N_{D_j}, N_{T_j} and N_{S_j} are $\tilde{b}'_{t+j} = \tilde{b}_{t+j} \oplus \tilde{P} = (m''_{t+j}, n''_{t+j}, \alpha''_{t+j}, \beta''_{t+j})_{LR}$, $\tilde{P}$ and $\tilde{P}$ respectively and Table 4.1 is converted into Table 4.2.

Step 4 Split Table 4.2 into four crisp transportation tables i.e., Table 4.3, Table 4.4, Table 4.5 and Table 4.6 respectively. The cost for transporting one unit quantity of the product from i^{th} node to j^{th} node in Table 4.3, Table 4.4, Table 4.5 and Table 4.6 are represented by $\eta_{ij}, \rho_{ij}, \delta_{ij}$ and ξ_{ij} respectively.

where,

$\eta_{ij} = \frac{1}{2}((m'_{ij}+n'_{ij})-\alpha'_{ij}\int_0^1 L^{-1}(\lambda)d\lambda+\beta'_{ij}\int_0^1 R^{-1}(\lambda)d\lambda),\qquad i=1,2,...,m+l+q+r,$

$j=1,2,...,t+q+r+l$

$\rho_{ij} = \frac{1}{2}((m'_{ij}+n'_{ij})-m'_{ij}\int_0^1 L^{-1}(\lambda)d\lambda+\beta'_{ij}\int_0^1 R^{-1}(\lambda)d\lambda),\qquad i=1,2,...,m+l+q+r\ ,$

$j=1,2,...,t+q+r+l$

$\delta_{ij} = \frac{1}{2}(n'_{ij}+\beta'_{ij}\int_0^1 R^{-1}(\lambda)d\lambda),\qquad i=1,2,...,m+l+q+r,\ \ j=1,2,...,t+q+r+l$

$\xi_{ij} = \frac{1}{2}((n'_{ij}+\beta'_{ij})\int_0^1 R^{-1}(\lambda)d\lambda),\qquad i=1,2,...,m+l+q+r\ ,\ \ j=1,2,...,t+q+r+l$

Step 5 Solve the crisp transportation problems, shown by Table 4.3, Table 4.4, Table 4.5 and Table 4.6, to find the optimal solution $\{m_{ij}-\alpha_{ij}\}$; $\{\alpha_{ij}\}$; $\{n_{ij}-m_{ij}\}$ and $\{\beta_{ij}\}$ respectively.

Step 6 Solve the equations, obtained in Step 5, to find the values of $m_{ij}, n_{ij}, \alpha_{ij}$ and β_{ij}.

Step 7 Find the fuzzy optimal solution $\{\tilde{x}_{ij}\}$ by putting the values of $m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}$ in $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$.

Step 8 Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ij}$, obtained from Step 7, in $\sum\limits_{(i,j)\in A}(\tilde{c}_{ij}\otimes\tilde{x}_{ij})$.

Table 4.1 Tabular representation of balanced fully fuzzy transshipment problem

	N_{PD_1}	$\cdots$	N_{PD_t}	N_{D_1}	$\cdots$	N_{D_q}	N_{T_1}	$\cdots$	N_{T_r}	N_{S_1}	$\cdots$	N_{S_l}	
N_{PS_1}	$\tilde{c}_{11}$	$\cdots$	$\tilde{c}_{1t}$	$\tilde{c}_{1(t+1)}$	$\cdots$	$\tilde{c}_{1(t+q)}$	$\tilde{c}_{1(t+q+1)}$	$\cdots$	$\tilde{c}_{1(t+q+r)}$	$\tilde{c}_{1(t+q+r+1)}$	$\cdots$	$\tilde{c}_{1(t+q+r+l)}$	$\tilde{a}_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{PS_m}	$\tilde{c}_{m1}$	$\cdots$	$\tilde{c}_{mt}$	$\tilde{c}_{m(t+1)}$	$\cdots$	$\tilde{c}_{m(t+q)}$	$\tilde{c}_{m(t+q+1)}$	$\cdots$	$\tilde{c}_{m(t+q+r)}$	$\tilde{c}_{m(t+q+1)}$	$\cdots$	$\tilde{c}_{m(t+q+r+l)}$	$\tilde{a}_m$
N_{S_1}	$\tilde{c}_{(m+1)1}$	$\vdots$	$\tilde{c}_{(m+1)t}$	$\tilde{c}_{(m+1)(t+1)}$	$\cdots$	$\tilde{c}_{(m+1)(t+q)}$	$\tilde{c}_{(m+1)(t+q+1)}$	$\cdots$	$\tilde{c}_{(m+1)(t+q+r)}$	$\tilde{c}_{(m+1)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+1)(t+q+r+l)}$	$\tilde{a}_{m+1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{S_l}	$\tilde{c}_{(m+l)1}$	$\vdots$	$\tilde{c}_{(m+l)t}$	$\tilde{c}_{(m+l)(t+1)}$	$\cdots$	$\tilde{c}_{(m+l)(t+q)}$	$\tilde{c}_{(m+l)(t+q+1)}$	$\cdots$	$\tilde{c}_{(m+l)(t+q+r)}$	$\tilde{c}_{(m+l)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+l)(t+q+r+l)}$	$\tilde{a}_{m+l}$
N_{D_1}	$\tilde{c}_{(m+l+1)1}$	$\cdots$	$\tilde{c}_{(m+l+1)t}$	$\tilde{c}_{(m+l+1)(t+1)}$	$\cdots$	$\tilde{c}_{(m+l+1)(t+q)}$	$\tilde{c}_{(m+l+1)(t+q+1)}$	$\cdots$	$\tilde{c}_{(m+l+1)(t+q+r)}$	$\tilde{c}_{(m+l+1)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+l+1)(t+q+r+l)}$	–
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	–
N_{D_q}	$\tilde{c}_{(m+l+q)1}$	$\cdots$	$\tilde{c}_{(m+l+q)t}$	$\tilde{c}_{(m+l+q)(t+1)}$	$\cdots$	$\tilde{c}_{(m+l+q)(t+q)}$	$\tilde{c}_{(m+l+q)(t+q+1)}$	$\cdots$	$\tilde{c}_{(m+l+q)(t+q+r)}$	$\tilde{c}_{(m+l+q)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+l+q)(t+q+r+l)}$	–
N_{T_1}	$\tilde{c}_{(m+l+q+1)1}$	$\cdots$	$\tilde{c}_{(m+l+q+1)t}$	$\tilde{c}_{(m+l+q+1)(t+1)}$	$\cdots$	$\tilde{c}_{(m+l+q+1)(t+q)}$	$\tilde{c}_{(m+l+q+1)(t+q+1)}$	$\cdots$	$\tilde{c}_{(m+l+q+1)(t+q+r)}$	$\tilde{c}_{(m+l+q+1)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+l+q+1)(t+q+r+l)}$	–
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	–
N_{T_r}	$\tilde{c}_{(m+l+q+r)1}$	$\cdots$	$\tilde{c}_{(m+l+q+r)t}$	$\tilde{c}_{(m+l+q+r)(t+1)}$	$\cdots$	$\tilde{c}_{(m+l+q+r)(t+q)}$	$\tilde{c}_{(m+l+q+r)(t+q+r)}$	$\cdots$	$\tilde{c}_{(m+l+q+r)(t+q+1)}$	$\tilde{c}_{(m+l+q+r)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+l+q+r)(t+q+r+l)}$	–
	$\tilde{b}_1$	$\cdots$	$\tilde{b}_t$	$\tilde{b}_{t+1}$	$\cdots$	$\tilde{b}_{t+q}$	–	–	–	–	–	–	

$$\tilde{c}_{ij} = \begin{cases} (0,0,0,0)_{LR} & \text{; If the product is supplying from some intermediate node to same intermediate node} \\ & \text{or} \\ & \text{If the product is supplying from some dummy purely source node to any intermediate node or} \\ & \text{to any purely destination node} \\ & \text{or} \\ & \text{If the product is supplying from any purely source node to dummy purely destination node} \\ & \text{or} \\ & \text{If the product is supplying from any intermediate node to dummy purely destination node} \\ (M,M,0,0)_{LR} & \text{; If the product can not be directly supplied from } i^{th} \text{ node to } j^{th} \text{ node} \\ & \text{or If the product can not be supplied from } i^{th} \text{ node to } j^{th} \text{ node} \\ (m'_{ij}, n'_{ij}, \alpha'_{ij}, \beta'_{ij})_{LR} & \text{; If the product can be directly supplied from } i^{th} \text{ node to } j^{th} \text{ node} \end{cases}$$

Table 4.2 Tabular representation of balanced fully fuzzy transportation problem obtained by adding fuzzy buffer stock

	N_{PD_1}	$\cdots$	N_{PD_t}	N_{D_1}	$\cdots$	N_{D_q}	N_{T_1}	$\cdots$	N_{T_r}	N_{S_1}	$\cdots$	N_{S_l}	
N_{PS_1}	$\tilde{c}_{11}$	$\cdots$	$\tilde{c}_{1t}$	$\tilde{c}_{1(t+1)}$	$\cdots$	$\tilde{c}_{1(t+q)}$	$\tilde{c}_{1(t+q+1)}$	$\cdots$	$\tilde{c}_{1(t+q+r)}$	$\tilde{c}_{1(t+q+r+1)}$	$\cdots$	$\tilde{c}_{1(t+q+r+l)}$	$\tilde{a}_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{PS_m}	$\tilde{c}_{m1}$	$\cdots$	$\tilde{c}_{mt}$	$\tilde{c}_{m(t+1)}$	$\cdots$	$\tilde{c}_{m(t+q)}$	$\tilde{c}_{m(t+q+1)}$	$\cdots$	$\tilde{c}_{m(t+q+r)}$	$\tilde{c}_{m(t+q+1)}$	$\cdots$	$\tilde{c}_{m(t+q+r+l)}$	$\tilde{a}_m$
N_{S_1}	$\tilde{c}_{(m+1)1}$	$\vdots$	$\tilde{c}_{(m+1)t}$	$\tilde{c}_{(m+1)(t+1)}$	$\cdots$	$\tilde{c}_{(m+1)(t+q)}$	$\tilde{c}_{(m+1)(t+q+1)}$	$\cdots$	$\tilde{c}_{(m+1)(t+q+r)}$	$\tilde{c}_{(m+1)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+1)(t+q+r+l)}$	$\tilde{a}'_{m+1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{S_l}	$\tilde{c}_{(m+l)1}$	$\vdots$	$\tilde{c}_{(m+l)t}$	$\tilde{c}_{(m+l)(t+1)}$	$\cdots$	$\tilde{c}_{(m+l)(t+q)}$	$\tilde{c}_{(m+l)(t+q+1)}$	$\cdots$	$\tilde{c}_{(m+l)(t+q+r)}$	$\tilde{c}_{(m+l)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+l)(t+q+r+l)}$	$\tilde{a}'_{m+l}$
N_{D_1}	$\tilde{c}_{(m+l+1)1}$	$\cdots$	$\tilde{c}_{(m+l+1)t}$	$\tilde{c}_{(m+l+1)(t+1)}$	$\cdots$	$\tilde{c}_{(m+l+1)(t+q)}$	$\tilde{c}_{(m+l+1)(t+q+1)}$	$\cdots$	$\tilde{c}_{(m+l+1)(t+q+r)}$	$\tilde{c}_{(m+l+1)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+l+1)(t+q+r+l)}$	$\widetilde{P}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{D_q}	$\tilde{c}_{(m+l+q)1}$	$\cdots$	$\tilde{c}_{(m+l+q)t}$	$\tilde{c}_{(m+l+q)(t+1)}$	$\cdots$	$\tilde{c}_{(m+l+q)(t+q)}$	$\tilde{c}_{(m+l+q)(t+q+1)}$	$\cdots$	$\tilde{c}_{(m+l+q)(t+q+r)}$	$\tilde{c}_{(m+l+q)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+l+q)(t+q+r+l)}$	$\widetilde{P}$
N_{T_1}	$\tilde{c}_{(m+l+q+1)1}$	$\cdots$	$\tilde{c}_{(m+l+q+1)t}$	$\tilde{c}_{(m+l+q+1)(t+1)}$	$\cdots$	$\tilde{c}_{(m+l+q+1)(t+q)}$	$\tilde{c}_{(m+l+q+1)(t+q+1)}$	$\cdots$	$\tilde{c}_{(m+l+q+1)(t+q+r)}$	$\tilde{c}_{(m+l+q+1)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+l+q+1)(t+q+r+l)}$	$\widetilde{P}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{T_r}	$\tilde{c}_{(m+l+q+r)1}$	$\cdots$	$\tilde{c}_{(m+l+q+r)t}$	$\tilde{c}_{(m+l+q+r)(t+1)}$	$\cdots$	$\tilde{c}_{(m+l+q+r)(t+q)}$	$\tilde{c}_{(m+l+q+r)(t+q+r)}$	$\cdots$	$\tilde{c}_{(m+l+q+r)(t+q+1)}$	$\tilde{c}_{(m+l+q+r)(t+q+r+1)}$	$\cdots$	$\tilde{c}_{(m+l+q+r)(t+q+r+l)}$	$\widetilde{P}$
	$\widetilde{b}_1$	$\cdots$	$\widetilde{b}_t$	$\widetilde{b}'_{t+1}$	$\cdots$	$\widetilde{b}'_{t+q}$	$\widetilde{P}$	$\cdots$	$\widetilde{P}$	$\widetilde{P}$	$\cdots$	$\widetilde{P}$	

Table 4.3 Tabular representation of first crisp transportation table

	N_{PD_1}	$\cdots$	N_{PD_t}	N_{D_1}	$\cdots$	N_{D_q}	N_{T_1}	$\cdots$	N_{T_r}	N_{S_1}	$\cdots$	N_{S_l}	
N_{PS_1}	η_{11}	$\cdots$	η_{1t}	$\eta_{1(t+1)}$	$\cdots$	$\eta_{1(t+q)}$	$\eta_{1(t+q+1)}$	$\cdots$	$\eta_{1(t+q+r)}$	$\eta_{1(t+q+r+1)}$	$\cdots$	$\eta_{1(t+q+r+l)}$	$m_1-\alpha_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{PS_m}	η_{m1}	$\cdots$	η_{mt}	$\eta_{m(t+1)}$	$\cdots$	$\eta_{m(t+q)}$	$\eta_{m(t+q+1)}$	$\cdots$	$\eta_{m(t+q+r)}$	$\eta_{m(t+q+1)}$	$\cdots$	$\eta_{m(t+q+r+l)}$	$m_m-\alpha_m$
N_{S_1}	$\eta_{(m+1)1}$	$\vdots$	$\eta_{(m+1)t}$	$\eta_{(m+1)(t+1)}$	$\cdots$	$\eta_{(m+1)(t+q)}$	$\eta_{(m+1)(t+q+1)}$	$\cdots$	$\eta_{(m+1)(t+q+r)}$	$\eta_{(m+1)(t+q+r+1)}$	$\cdots$	$\eta_{(m+1)(t+q+r+l)}$	$m''_{m+1}-\alpha''_{m+1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{S_l}	$\eta_{(m+l)1}$	$\vdots$	$\eta_{(m+l)t}$	$\eta_{(m+l)(t+1)}$	$\cdots$	$\eta_{(m+l)(t+q)}$	$\eta_{(m+l)(t+q+1)}$	$\cdots$	$\eta_{(m+l)(t+q+r)}$	$\eta_{(m+l)(t+q+r+1)}$	$\cdots$	$\eta_{(m+l)(t+q+r+l)}$	$m''_{m+l}-\alpha''_{m+l}$
N_{D_1}	$\eta_{(m+l+1)1}$	$\cdots$	$\eta_{(m+l+1)t}$	$\eta_{(m+l+1)(t+1)}$	$\cdots$	$\eta_{(m+l+1)(t+q)}$	$\eta_{(m+l+1)(t+q+1)}$	$\cdots$	$\eta_{(m+l+1)(t+q+r)}$	$\eta_{(m+l+1)(t+q+r+1)}$	$\cdots$	$\eta_{(m+l+1)(t+q+r+l)}$	$P_1-\alpha_p$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{D_q}	$\eta_{(m+l+q)1}$	$\cdots$	$\eta_{(m+l+q)t}$	$\eta_{(m+l+q)(t+1)}$	$\cdots$	$\eta_{(m+l+q)(t+q)}$	$\eta_{(m+l+q)(t+q+1)}$	$\cdots$	$\eta_{(m+l+q)(t+q+r)}$	$\eta_{(m+l+q)(t+q+r+1)}$	$\cdots$	$\eta_{(m+l+q)(t+q+r+l)}$	$P_1-\alpha_p$
N_{T_1}	$\eta_{(m+l+q+1)1}$	$\cdots$	$\eta_{(m+l+q+1)t}$	$\eta_{(m+l+q+1)(t+1)}$	$\cdots$	$\eta_{(m+l+q+1)(t+q)}$	$\eta_{(m+l+q+1)(t+q+1)}$	$\cdots$	$\eta_{(m+l+q+1)(t+q+r)}$	$\eta_{(m+l+q+1)(t+q+r+1)}$	$\cdots$	$\eta_{(m+l+q+1)(t+q+r+l)}$	$P_1-\alpha_p$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{T_r}	$\eta_{(m+l+q+r)1}$	$\cdots$	$\eta_{(m+l+q+r)t}$	$\eta_{(m+l+q+r)(t+1)}$	$\cdots$	$\eta_{(m+l+q+r)(t+q)}$	$\eta_{(m+l+q+r)(t+q+r)}$	$\cdots$	$\eta_{(m+l+q+r)(t+q+1)}$	$\eta_{(m+l+q+r)(t+q+r+1)}$	$\cdots$	$\eta_{(m+l+q+r)(t+q+r+l)}$	$P_1-\alpha_p$
	$m'_1-\alpha'_1$	$\cdots$	$m'_t-\alpha'_t$	$m''_{t+1}-\alpha''_{t+1}$	$\cdots$	$m''_{t+q}-\alpha''_{t+q}$	$P_1-\alpha_p$	$\cdots$	$P_1-\alpha_p$	$P_1-\alpha_p$	$\cdots$	$P_1-\alpha_p$	

Table 4.4 Tabular representation of second crisp transportation table

	N_{PD_1}	$\cdots$	N_{PD_t}	N_{D_1}	$\cdots$	N_{D_q}	N_{T_1}	$\cdots$	N_{T_r}	N_{S_1}	$\cdots$	N_{S_l}	
N_{PS_1}	ρ_{11}	$\cdots$	ρ_{1t}	$\rho_{1(t+1)}$	$\cdots$	$\rho_{1(t+q)}$	$\rho_{1(t+q+1)}$	$\cdots$	$\rho_{1(t+q+r)}$	$\rho_{1(t+q+r+1)}$	$\cdots$	$\rho_{1(t+q+r+l)}$	α_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{PS_m}	ρ_{m1}	$\cdots$	ρ_{mt}	$\rho_{m(t+1)}$	$\cdots$	$\rho_{m(t+q)}$	$\rho_{m(t+q+1)}$	$\cdots$	$\rho_{m(t+q+r)}$	$\rho_{m(t+q+1)}$	$\cdots$	$\rho_{m(t+q+r+l)}$	α_m
N_{S_1}	$\rho_{(m+1)1}$	$\vdots$	$\rho_{(m+1)t}$	$\rho_{(m+1)(t+1)}$	$\cdots$	$\rho_{(m+1)(t+q)}$	$\rho_{(m+1)(t+q+1)}$	$\cdots$	$\rho_{(m+1)(t+q+r)}$	$\rho_{(m+1)(t+q+r+1)}$	$\cdots$	$\rho_{(m+1)(t+q+r+l)}$	α''_{m+1}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{S_l}	$\rho_{(m+l)1}$	$\vdots$	$\rho_{(m+l)t}$	$\rho_{(m+l)(t+1)}$	$\cdots$	$\rho_{(m+l)(t+q)}$	$\rho_{(m+l)(t+q+1)}$	$\cdots$	$\rho_{(m+l)(t+q+r)}$	$\rho_{(m+l)(t+q+r+1)}$	$\cdots$	$\rho_{(m+l)(t+q+r+l)}$	α''_{m+l}
N_{D_1}	$\rho_{(m+l+1)1}$	$\cdots$	$\rho_{(m+l+1)t}$	$\rho_{(m+l+1)(t+1)}$	$\cdots$	$\rho_{(m+l+1)(t+q)}$	$\rho_{(m+l+1)(t+q+1)}$	$\cdots$	$\rho_{(m+l+1)(t+q+r)}$	$\rho_{(m+l+1)(t+q+r+1)}$	$\cdots$	$\rho_{(m+l+1)(t+q+r+l)}$	α_p
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{D_q}	$\rho_{(m+l+q)1}$	$\cdots$	$\rho_{(m+l+q)t}$	$\rho_{(m+l+q)(t+1)}$	$\cdots$	$\rho_{(m+l+q)(t+q)}$	$\rho_{(m+l+q)(t+q+1)}$	$\cdots$	$\rho_{(m+l+q)(t+q+r)}$	$\rho_{(m+l+q)(t+q+r+1)}$	$\cdots$	$\rho_{(m+l+q)(t+q+r+l)}$	α_p
N_{T_1}	$\rho_{(m+l+q+1)1}$	$\cdots$	$\rho_{(m+l+q+1)t}$	$\rho_{(m+l+q+1)(t+1)}$	$\cdots$	$\rho_{(m+l+q+1)(t+q)}$	$\rho_{(m+l+q+1)(t+q+1)}$	$\cdots$	$\rho_{(m+l+q+1)(t+q+r)}$	$\rho_{(m+l+q+1)(t+q+r+1)}$	$\cdots$	$\rho_{(m+l+q+1)(t+q+r+l)}$	α_p
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{T_r}	$\rho_{(m+l+q+r)1}$	$\cdots$	$\rho_{(m+l+q+r)t}$	$\rho_{(m+l+q+r)(t+1)}$	$\cdots$	$\rho_{(m+l+q+r)(t+q)}$	$\rho_{(m+l+q+r)(t+q+r)}$	$\cdots$	$\rho_{(m+l+q+r)(t+q+1)}$	$\rho_{(m+l+q+r)(t+q+r+1)}$	$\cdots$	$\rho_{(m+l+q+r)(t+q+r+l)}$	α_p
	α'_1	$\cdots$	α'_t	α''_{t+1}	$\cdots$	α''_{t+q}	α_p	$\cdots$	α_p	α_p	$\cdots$	α_p	

Table 4.5 Tabular representation of third crisp transportation table

	N_{PD_1}	$\cdots$	N_{PD_t}	N_{D_1}	$\cdots$	N_{D_q}	N_{T_1}	$\cdots$	N_{T_r}	N_{S_1}	$\cdots$	N_{S_l}	
N_{PS_1}	δ_{11}	$\cdots$	δ_{1t}	$\delta_{1(t+1)}$	$\cdots$	$\delta_{1(t+q)}$	$\delta_{1(t+q+1)}$	$\cdots$	$\delta_{1(t+q+r)}$	$\delta_{1(t+q+r+1)}$	$\cdots$	$\delta_{1(t+q+r+l)}$	$n_1 - m_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{PS_m}	δ_{m1}	$\cdots$	δ_{mt}	$\delta_{m(t+1)}$	$\cdots$	$\delta_{m(t+q)}$	$\delta_{m(t+q+1)}$	$\cdots$	$\delta_{m(t+q+r)}$	$\delta_{m(t+q+1)}$	$\cdots$	$\delta_{m(t+q+r+l)}$	$n_m - m_m$
N_{S_1}	$\delta_{(m+1)1}$	$\vdots$	$\delta_{(m+1)t}$	$\delta_{(m+1)(t+1)}$	$\cdots$	$\delta_{(m+1)(t+q)}$	$\delta_{(m+1)(t+q+1)}$	$\cdots$	$\delta_{(m+1)(t+q+r)}$	$\delta_{(m+1)(t+q+r+1)}$	$\cdots$	$\delta_{(m+1)(t+q+r+l)}$	$n''_{m+1} - m''_{m+1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{S_l}	$\delta_{(m+l)1}$	$\vdots$	$\delta_{(m+l)t}$	$\delta_{(m+l)(t+1)}$	$\cdots$	$\delta_{(m+l)(t+q)}$	$\delta_{(m+l)(t+q+1)}$	$\cdots$	$\delta_{(m+l)(t+q+r)}$	$\delta_{(m+l)(t+q+r+1)}$	$\cdots$	$\delta_{(m+l)(t+q+r+l)}$	$n''_{m+l} - m''_{m+l}$
N_{D_1}	$\delta_{(m+l+1)1}$	$\cdots$	$\delta_{(m+l+1)t}$	$\delta_{(m+l+1)(t+1)}$	$\cdots$	$\delta_{(m+l+1)(t+q)}$	$\delta_{(m+l+1)(t+q+1)}$	$\cdots$	$\delta_{(m+l+1)(t+q+r)}$	$\delta_{(m+l+1)(t+q+r+1)}$	$\cdots$	$\delta_{(m+l+1)(t+q+r+l)}$	$P_2 - P_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{D_q}	$\delta_{(m+l+q)1}$	$\cdots$	$\delta_{(m+l+q)t}$	$\delta_{(m+l+q)(t+1)}$	$\cdots$	$\delta_{(m+l+q)(t+q)}$	$\delta_{(m+l+q)(t+q+1)}$	$\cdots$	$\delta_{(m+l+q)(t+q+r)}$	$\delta_{(m+l+q)(t+q+r+1)}$	$\cdots$	$\delta_{(m+l+q)(t+q+r+l)}$	$P_2 - P_1$
N_{T_1}	$\delta_{(m+l+q+1)1}$	$\cdots$	$\delta_{(m+l+q+1)t}$	$\delta_{(m+l+q+1)(t+1)}$	$\cdots$	$\delta_{(m+l+q+1)(t+q)}$	$\delta_{(m+l+q+1)(t+q+1)}$	$\cdots$	$\delta_{(m+l+q+1)(t+q+r)}$	$\delta_{(m+l+q+1)(t+q+r+1)}$	$\cdots$	$\delta_{(m+l+q+1)(t+q+r+l)}$	$P_2 - P_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{T_r}	$\delta_{(m+l+q+r)1}$	$\cdots$	$\delta_{(m+l+q+r)t}$	$\delta_{(m+l+q+r)(t+1)}$	$\cdots$	$\delta_{(m+l+q+r)(t+q)}$	$\delta_{(m+l+q+r)(t+q+r)}$	$\cdots$	$\delta_{(m+l+q+r)(t+q+1)}$	$\delta_{(m+l+q+r)(t+q+r+1)}$	$\cdots$	$\delta_{(m+l+q+r)(t+q+r+l)}$	$P_2 - P_1$
	$n'_1 - m'_1$	$\cdots$	$n'_t - m'_t$	$n''_{t+1} - m''_{t+1}$	$\cdots$	$n''_{t+q} - m''_{t+q}$	$P_2 - P_1$	$\cdots$	$P_2 - P_1$	$P_2 - P_1$	$\cdots$	$P_2 - P_1$	

Table 4.6 Tabular representation of fourth crisp transportation table

	N_{PD_1}	$\cdots$	N_{PD_t}	N_{D_1}	$\cdots$	N_{D_q}	N_{T_1}	$\cdots$	N_{T_r}	N_{S_1}	$\cdots$	N_{S_l}	
N_{PS_1}	ξ_{11}	$\cdots$	ξ_{1t}	$\xi_{1(t+1)}$	$\cdots$	$\xi_{1(t+q)}$	$\xi_{1(t+q+1)}$	$\cdots$	$\xi_{1(t+q+r)}$	$\xi_{1(t+q+r+1)}$	$\cdots$	$\xi_{1(t+q+r+l)}$	β_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{PS_m}	ξ_{m1}	$\cdots$	ξ_{mt}	$\xi_{m(t+1)}$	$\cdots$	$\xi_{m(t+q)}$	$\xi_{m(t+q+1)}$	$\cdots$	$\xi_{m(t+q+r)}$	$\xi_{m(t+q+1)}$	$\cdots$	$\xi_{m(t+q+r+l)}$	β_m
N_{S_1}	$\xi_{(m+1)1}$	$\vdots$	$\xi_{(m+1)t}$	$\xi_{(m+1)(t+1)}$	$\cdots$	$\xi_{(m+1)(t+q)}$	$\xi_{(m+1)(t+q+1)}$	$\cdots$	$\xi_{(m+1)(t+q+r)}$	$\xi_{(m+1)(t+q+r+1)}$	$\cdots$	$\xi_{(m+1)(t+q+r+l)}$	β''_{m+1}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{S_l}	$\xi_{(m+l)1}$	$\vdots$	$\xi_{(m+l)t}$	$\xi_{(m+l)(t+1)}$	$\cdots$	$\xi_{(m+l)(t+q)}$	$\xi_{(m+l)(t+q+1)}$	$\cdots$	$\xi_{(m+l)(t+q+r)}$	$\xi_{(m+l)(t+q+r+1)}$	$\cdots$	$\xi_{(m+l)(t+q+r+l)}$	β''_{m+l}
N_{D_1}	$\xi_{(m+l+1)1}$	$\cdots$	$\xi_{(m+l+1)t}$	$\xi_{(m+l+1)(t+1)}$	$\cdots$	$\xi_{(m+l+1)(t+q)}$	$\xi_{(m+l+1)(t+q+1)}$	$\cdots$	$\xi_{(m+l+1)(t+q+r)}$	$\xi_{(m+l+1)(t+q+r+1)}$	$\cdots$	$\xi_{(m+l+1)(t+q+r+l)}$	β_p
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{D_q}	$\xi_{(m+l+q)1}$	$\cdots$	$\xi_{(m+l+q)t}$	$\xi_{(m+l+q)(t+1)}$	$\cdots$	$\xi_{(m+l+q)(t+q)}$	$\xi_{(m+l+q)(t+q+1)}$	$\cdots$	$\xi_{(m+l+q)(t+q+r)}$	$\xi_{(m+l+q)(t+q+r+1)}$	$\cdots$	$\xi_{(m+l+q)(t+q+r+l)}$	β_p
N_{T_1}	$\xi_{(m+l+q+1)1}$	$\cdots$	$\xi_{(m+l+q+1)t}$	$\xi_{(m+l+q+1)(t+1)}$	$\cdots$	$\xi_{(m+l+q+1)(t+q)}$	$\xi_{(m+l+q+1)(t+q+1)}$	$\cdots$	$\xi_{(m+l+q+1)(t+q+r)}$	$\xi_{(m+l+q+1)(t+q+r+1)}$	$\cdots$	$\xi_{(m+l+q+1)(t+q+r+l)}$	β_p
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_{T_r}	$\xi_{(m+l+q+r)1}$	$\cdots$	$\xi_{(m+l+q+r)t}$	$\xi_{(m+l+q+r)(t+1)}$	$\cdots$	$\xi_{(m+l+q+r)(t+q)}$	$\xi_{(m+l+q+r)(t+q+r)}$	$\cdots$	$\xi_{(m+l+q+r)(t+q+1)}$	$\xi_{(m+l+q+r)(t+q+r+1)}$	$\cdots$	$\xi_{(m+l+q+r)(t+q+r+l)}$	β_p
	β'_1	$\cdots$	β'_t	β''_{t+1}	$\cdots$	β''_{t+q}	β_p	$\cdots$	β_p	β_p	$\cdots$	β_p	

4.4.3 Advantages of the proposed methods over existing method

In this section, the advantages of the methods, proposed in this chapter, over the existing method [70] and the methods, proposed in previous chapters, are discussed.

(1) The existing method [70] can be used for solving balanced fully fuzzy transshipment problems and balanced fully fuzzy transportation problems. But, the existing method [70] can neither be used for solving unbalanced fully fuzzy transportation problems nor for solving unbalanced fully fuzzy transshipment problems. While, the methods, proposed in this chapter, can be used for solving balanced and unbalanced fully fuzzy transportation problems as well as balanced and unbalanced fully fuzzy transshipment problems.

(2) The methods, proposed in previous chapters, can be used to find the fuzzy optimal solution of fully fuzzy transportation problems but can not be used for solving fully fuzzy transshipment problems. Since, fully fuzzy transportation problems are special type of fully fuzzy transshipment problems so, the methods, proposed in this chapter, can also be used to find the fuzzy optimal solution of fully fuzzy transportation problems.

4.5 Illustrative example

In this section, the fully fuzzy transshipment problem, chosen in Example 4.2, is solved by the proposed methods.

4.5.1 Fuzzy optimal solution of the chosen problem using proposed method based on fuzzy linear programming formulation

The fuzzy optimal solution of the fully fuzzy transshipment problem, chosen in Example 4.2, by using the method, proposed in Section 4.4.1, can be obtained as follows:

Step 1 Total fuzzy availability $= (70, 80, 20, 50)_{LR}$ and total fuzzy demand $= (80, 100, 40, 20)_{LR}$. Since total fuzzy availability $\neq$ total fuzzy demand, so it is an unbalanced fully fuzzy transshipment problem.

Now as described in the proposed method (using Case (c) of Step 1 of the proposed method discussed in Section 4.4.1), the unbalanced fuzzy transshipment problem can be converted into a balanced fully fuzzy transshipment problem by introducing a dummy purely source node 6 with fuzzy availability $(20, 30, 20, 0)_{LR}$ and a dummy purely destination node 7 with fuzzy demand $(10, 10, 0, 30)_{LR}$.

Assume the fuzzy cost for transporting one unit quantity of the product from the introduced dummy purely source node (6) to transition node (3), source node (2), existing purely destination nodes (4 and 5) and introduced dummy purely destination node 7 as zero LR flat fuzzy number. Similarly, assume the fuzzy cost for transporting one unit quantity of the product from source node (2), transition node (3), existing purely source node 1 and introduced dummy purely source node 6 to the introduced dummy purely destination node (7) as zero LR flat fuzzy number i.e., $\tilde{c}_{17} = \tilde{c}_{27} = \tilde{c}_{37} = \tilde{c}_{62} = \tilde{c}_{63} = \tilde{c}_{64} = \tilde{c}_{65} = \tilde{c}_{67} = (0, 0, 0, 0)_{LR}$.

Step 2 The balanced fully fuzzy transshipment problem, obtained from Step 1, can be formulated into the following fuzzy linear programming problem:

Minimize $((2,4,1,2)_{LR}\otimes\tilde{x}_{12}\oplus(1,1,1,1)_{LR}\otimes\tilde{x}_{13}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{17}\oplus(1,3,1,1)_{LR}\otimes \tilde{x}_{23}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{27}\oplus(5,7,2,2)_{LR}\otimes\tilde{x}_{34}\oplus(5,6,3,1)_{LR}\otimes\tilde{x}_{35}\oplus(0,0,0,0)_{LR}\otimes \tilde{x}_{37}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{62}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{63}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{64}\oplus(0,0,0,0)_{LR}\otimes \tilde{x}_{65}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{67})$

subject to

$$\tilde{x}_{12}\oplus\tilde{x}_{13}\oplus\tilde{x}_{17}=(40,40,10,20)_{LR}$$

$$(\tilde{x}_{23}\oplus\tilde{x}_{27})\ominus_H(\tilde{x}_{12}\oplus\tilde{x}_{62})=(30,40,10,30)_{LR}$$

$$\tilde{x}_{62}\oplus\tilde{x}_{63}\oplus\tilde{x}_{64}\oplus\tilde{x}_{65}\oplus\tilde{x}_{67}=(20,30,20,0)_{LR}$$

$$\tilde{x}_{34}\oplus\tilde{x}_{35}\oplus\tilde{x}_{37}=\tilde{x}_{13}\oplus\tilde{x}_{23}\oplus\tilde{x}_{63}$$

$$\tilde{x}_{34}\oplus\tilde{x}_{64}=(20,30,10,10)_{LR}$$

$$\tilde{x}_{35}\oplus\tilde{x}_{65}=(60,70,30,10)_{LR}$$

$$\tilde{x}_{17}\oplus\tilde{x}_{27}\oplus\tilde{x}_{37}\oplus\tilde{x}_{67}=(10,10,0,30)_{LR}$$

$\tilde{x}_{12},\tilde{x}_{13},\tilde{x}_{17},\tilde{x}_{23},\tilde{x}_{27},\tilde{x}_{34},\tilde{x}_{35},\tilde{x}_{37},\tilde{x}_{62},\tilde{x}_{63},\tilde{x}_{64},\tilde{x}_{65},\tilde{x}_{67}$ are non-negative LR flat fuzzy numbers.

Step 3 Using Step 3 to Step 7, of the method, proposed in Section 4.4.1, the fuzzy linear programming problem, obtained in Step 2, can be converted into the following crisp linear programming problem:

Minimize $(\frac{1}{10}(6m_{12}+28n_{12}-4\alpha_{12}+24\beta_{12}+m_{13}+9n_{13}+8\beta_{13}+m_{23}+19n_{23}+ 16\beta_{23}+17m_{34}+43n_{34}-12\alpha_{34}+36\beta_{34}+13m_{35}+34n_{35}-8\alpha_{35}+28\beta_{35}))$

subject to

$m_{12}+m_{13}+m_{17}=40;\quad m_{23}+m_{27}-m_{12}-m_{62}=30;\quad m_{62}+m_{63}+m_{64}+m_{65}+m_{67}=20;$

$n_{12}+n_{13}+n_{17}=40\,;\quad n_{23}+n_{27}-n_{12}-n_{62}=40\,;\quad n_{62}+n_{63}+n_{64}+n_{65}+n_{67}=30;$

$\alpha_{12}+\alpha_{13}+\alpha_{17}=10\,;\quad \alpha_{23}+\alpha_{27}-\alpha_{12}-\alpha_{62}=10\,;\quad \alpha_{62}+\alpha_{63}+\alpha_{64}+\alpha_{65}+\alpha_{67}=20;$

$\beta_{12}+\beta_{13}+\beta_{17}=20\,;\quad \beta_{23}+\beta_{27}-\beta_{12}-\beta_{62}=30;\quad \beta_{62}+\beta_{63}+\beta_{64}+\beta_{65}+\beta_{67}=0;$

$m_{34}+m_{35}+m_{37}=m_{13}+m_{23}+m_{63}$; $m_{34}+m_{64}=20$; $m_{35}+m_{65}=60$;

$n_{34}+n_{35}+n_{37}=n_{13}+n_{23}+n_{63}$; $n_{34}+n_{64}=30$; $n_{35}+n_{65}=70$;

$\alpha_{34}+\alpha_{35}+\alpha_{37}=\alpha_{13}+\alpha_{23}+\alpha_{63}$; $\alpha_{34}+\alpha_{64}=10$; $\alpha_{35}+\alpha_{65}=30$;

$\beta_{34}+\beta_{35}+\beta_{37}=\beta_{13}+\beta_{23}+\beta_{63}$; $\beta_{34}+\beta_{64}=10$; $\beta_{35}+\beta_{65}=10$;

$m_{17}+m_{27}+m_{37}+m_{67}=10$; $n_{17}+n_{27}+n_{37}+n_{67}=10$;

$\alpha_{17}+\alpha_{27}+\alpha_{37}+\alpha_{67}=0$; $\beta_{17}+\beta_{27}+\beta_{37}+\beta_{67}=30$;

$m_{12}-\alpha_{12},\ n_{12}-m_{12},\ m_{13}-\alpha_{13},\ n_{13}-m_{13},\ m_{23}-\alpha_{23},\ n_{23}-m_{23},\ m_{27}-\alpha_{27},$ $n_{27}-m_{27},\ m_{34}-\alpha_{34},\ n_{34}-m_{34},\ m_{35}-\alpha_{35},\ n_{35}-m_{35},\ m_{62}-\beta_{62},\ n_{62}-m_{62},$ $m_{63}-\alpha_{63}, n_{63}-m_{63}, m_{64}-\alpha_{64}, n_{64}-m_{64}, m_{65}-\alpha_{65}, n_{65}-m_{65}, m_{67}-\alpha_{67}, n_{67}-$ $m_{67}, m_{12}, n_{12}, \alpha_{12}, \beta_{12}, m_{13}, n_{13}, \alpha_{13}, \beta_{13}, m_{23}, n_{23}, \alpha_{23}, \beta_{23}, m_{27}, n_{27}, \alpha_{27}, \beta_{27}, m_{34}, n_{34},$ $\alpha_{34}, \beta_{34}, m_{35}, n_{35}, \alpha_{35}, \beta_{35}, m_{62}, n_{62}, \alpha_{62}, \beta_{62}, m_{63}, n_{63}, \alpha_{63}, \beta_{63}, m_{64}, n_{64}, \alpha_{64}, \beta_{64}, m_{65},$ $n_{65}, \alpha_{65}, \beta_{65}, m_{67}, n_{67}, \alpha_{67}, \beta_{67} \ \geq \ 0$

Step 4 The optimal solution of the crisp linear programming problem, obtained in Step 3, is $m_{13}=40, n_{13}=40, \alpha_{13}=10, \beta_{13}=20, m_{23}=20, n_{23}=30, \alpha_{23}=10, \beta_{23}=0, m_{27}=10, n_{27}=10, \alpha_{27}=0, \beta_{27}=30, m_{34}=10, n_{34}=10, \alpha_{34}=0, \beta_{34}=10, m_{35}=50, n_{35}=60, \alpha_{35}=20, \beta_{35}=10, m_{64}=10, n_{64}=20, \alpha_{64}=10, \beta_{64}=0, m_{65}=10, n_{65}=10, \alpha_{65}=10, \beta_{65}=0$ and the remaining values of $m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}$ are zero.

Step 5 Putting the values of $m_{ij}, n_{ij}, \alpha_{ij}$ and β_{ij} in $\tilde{x}_{ij}=(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})$, the fuzzy optimal solution is $\tilde{x}_{13}=(40,40,10,20)_{LR}$, $\tilde{x}_{23}=(20,30,10,0)_{LR}$, $\tilde{x}_{27}=(10,10,0,30)_{LR}$, $\tilde{x}_{34}=(10,10,0,10)_{LR}$, $\tilde{x}_{35}=(50,60,20,10)_{LR}$, $\tilde{x}_{64}=(10,20,10,0)_{LR}$, $\tilde{x}_{65}=(10,10,10,0)_{LR}$ and remaining values of $\tilde{x}_{ij}$ are $(0,0,0,0)_{LR}$

Step 6 Putting the values of $\tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{17}, \tilde{x}_{23}, \tilde{x}_{27}, \tilde{x}_{34}, \tilde{x}_{35}, \tilde{x}_{37}, \tilde{x}_{62}, \tilde{x}_{63}, \tilde{x}_{64}, \tilde{x}_{65}, \tilde{x}_{67}$ in $((2,4,1,2)_{LR} \otimes \tilde{x}_{12} \oplus (1,1,1,1)_{LR} \otimes \tilde{x}_{13} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{17} \oplus (1,3,1,1)_{LR} \otimes$

$\tilde{x}_{23} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{27} \oplus (5,7,2,2)_{LR} \otimes \tilde{x}_{34} \oplus (5,6,3,1)_{LR} \otimes \tilde{x}_{35} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{37} \oplus (0,0,0,0) \otimes \tilde{x}_{62} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{63} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{64} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{65} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{67})$, the minimum total fuzzy transportation cost is $(360,560,270,350)_{LR}$.

4.5.2 Fuzzy optimal solution of the chosen problem using proposed method based on tabular representation

The fuzzy optimal solution of the fully fuzzy transshipment problem, chosen in Example 4.2, by using the method, proposed in Section 4.4.2, can be obtained as follows:

Step 1 Use Step 1 of the method, discussed in Section 4.5.1, to obtain a balanced fully fuzzy transshipment problem.

Step 2 Using Step 2 of the method, proposed in Section 4.4.2, the balanced fully fuzzy transshipment problem, obtained from Step 1, can be represented into a tabular form as shown in Table 4.7.

Table 4.7 Tabular representation of balanced fully fuzzy transshipment problem

	2	3	4	5	7	
1	$(2,4,1,2)_{LR}$	$(1,1,1,1)_{LR}$	$(M,M,0,0)_{LR}$	$(M,M,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(40,40,10,20)_{LR}$
2	$(0,0,0,0)_{LR}$	$(1,3,1,1)_{LR}$	$(M,M,0,0)_{LR}$	$(M,M,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(30,40,10,30)_{LR}$
3	$(M,M,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(5,7,2,2)_{LR}$	$(5,6,3,1)_{LR}$	$(0,0,0,0)_{LR}$	–
6	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(20,30,20,0)_{LR}$
	–	–	$(20,30,10,10)_{LR}$	$(60,70,30,10)_{LR}$	$(10,10,0,30)_{LR}$	

Step 3 Using Step 3 of the method, proposed in Section 4.4.2, add an amount of fuzzy buffer stock $\tilde{P} = \sum \tilde{a}_i = \sum \tilde{b}_j = (90,110,40,50)_{LR}$ in the fuzzy availability and fuzzy demand corresponding to each intermediate node of Table 4.7. After adding the fuzzy buffer stock $\tilde{P}$, the Table 4.7 is converted into Table 4.8.

Table 4.8 Tabular representation of fully fuzzy transportation problem obtained by adding fuzzy buffer stock

	2	3	4	5	7	
1	$(2,4,1,2)_{LR}$	$(1,1,1,1)_{LR}$	$(M,M,0,0)_{LR}$	$(M,M,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(40,40,10,20)_{LR}$
2	$(0,0,0,0)_{LR}$	$(1,3,1,1)_{LR}$	$(M,M,0,0)_{LR}$	$(M,M,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(120,150,50,80)_{LR}$
3	$(M,M,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(5,7,2,2)_{LR}$	$(5,6,3,1)_{LR}$	$(0,0,0,0)_{LR}$	$(90,110,40,50)_{LR}$
6	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(20,30,20,0)_{LR}$
	$(90,110,40,50)_{LR}$	$(90,110,40,50)_{LR}$	$(20,30,10,10)_{LR}$	$(60,70,30,10)_{LR}$	$(10,10,0,30)_{LR}$	

Step 4 Using Step 4 of the method, proposed in Section 4.4.2, Table 4.8 can be split into four crisp transportation tables i.e., Table 4.9, Table 4.10, Table 4.11, and Table 4.12.

Table 4.9 Tabular representation of first crisp transportation problem

	2	3	4	5	7	
1	3.4	1	M	M	0	30
2	0	2	M	M	0	70
3	M	0	6	4.7	0	50
6	0	0	0	0	0	0
	50	50	10	30	10	

Table 4.10 Tabular representation of second crisp transportation problem

	2	3	4	5	7	
1	3	1	$0.6M$	$0.6M$	0	10
2	0	2	$0.6M$	$0.6M$	0	50
3	$0.6M$	0	4.8	3.9	0	40
6	0	0	0	0	0	20
	40	40	10	30	0	

Table 4.11 Tabular representation of third crisp transportation problem

	2	3	4	5	7	
1	2.8	.9	$0.5M$	$0.5M$	0	0
2	0	1.9	$0.5M$	$0.5M$	0	30
3	$0.5M$	0	4.3	3.4	0	20
6	0	0	0	0	0	10
	20	20	10	10	0	

Table 4.12 Tabular representation of fourth crisp transportation problem

	2	3	4	5	7	
1	2.4	.8	$0.4M$	$0.4M$	0	20
2	0	1.6	$0.4M$	$0.4M$	0	80
3	$0.4M$	0	3.6	2.8	0	50
6	0	0	0	0	0	0
	50	50	10	10	30	

Step 5 The optimal solution of crisp transportation problems, shown by Table 4.9; Table 4.10; Table 4.11 and Table 4.12, are $m_{13}-\alpha_{13}=30, m_{22}-\alpha_{22}=50, m_{23}-\alpha_{23}=10, m_{27}-\alpha_{27}=10, m_{33}-\alpha_{33}=10, m_{34}-\alpha_{34}=10, m_{35}-\alpha_{35}=50, m_{64}-\alpha_{64}=0, m_{65}-\alpha_{65}=0; \alpha_{13}=10, \alpha_{22}=50, \alpha_{23}=10, \alpha_{33}=20, \alpha_{35}=20, \alpha_{64}=10, \alpha_{65}=10; n_{13}-m_{13}=0, n_{22}-m_{22}=20, n_{23}-m_{23}=10, n_{27}-m_{27}=0, n_{33}-m_{33}=10, n_{34}-m_{34}=0, n_{35}-m_{35}=10, n_{64}-m_{64}=10, n_{65}-m_{65}=0$ and $\beta_{13}=20, \beta_{22}=50, \beta_{27}=30, \beta_{33}=30, \beta_{34}=10, \beta_{35}=10$ respectively.

Step 6 On solving the equations, obtained in Step 5, the values of $m_{ij}, n_{ij}, \alpha_{ij}$ and β_{ij} are $m_{13}=40, n_{13}=40, \alpha_{13}=10, \beta_{13}=20, m_{22}=90, n_{22}=110, \alpha_{22}=40, \beta_{22}=$

$50, m_{23} = 20, n_{23} = 30, \alpha_{23} = 10, \beta_{23} = 0, m_{27} = 10, n_{27} = 10, \alpha_{27} = 0, \beta_{27} = 30, m_{33} = 30, n_{33} = 40, \alpha_{33} = 20, \beta_{33} = 30, m_{34} = 10, n_{34} = 10, \alpha_{34} = 0, \beta_{34} = 10, m_{35} = 50, n_{35} = 60, \alpha_{35} = 20, \beta_{35} = 10, m_{64} = 10, n_{64} = 20, \alpha_{64} = 10, \beta_{64} = 0, m_{65} = 10, n_{65} = 10, \alpha_{65} = 10, \beta_{65} = 0$ and the remaining values of $m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}$ are zero.

Step 7 Putting the values of $m_{ij}, n_{ij}, \alpha_{ij}$ and β_{ij} in $\tilde{x}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$, the fuzzy optimal solution is $\tilde{x}_{13} = (40, 40, 10, 20)_{LR}$, $\tilde{x}_{22} = (90, 110, 40, 50)_{LR}$, $\tilde{x}_{23} = (20, 30, 10, 0)_{LR}$, $\tilde{x}_{27} = (10, 10, 0, 30)_{LR}$, $\tilde{x}_{33} = (30, 40, 20, 30)_{LR}$, $\tilde{x}_{34} = (10, 10, 0, 10)_{LR}$, $\tilde{x}_{35} = (50, 60, 20, 10)_{LR}$, $\tilde{x}_{64} = (10, 20, 10, 0)_{LR}$, $\tilde{x}_{65} = (10, 10, 10, 0)_{LR}$ and remaining values of $\tilde{x}_{ij}$ are $(0, 0, 0, 0)_{LR}$.

Step 8 Putting the values of $\tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{15}, \tilde{x}_{17}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{25}, \tilde{x}_{27}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34}, \tilde{x}_{35}, \tilde{x}_{37}, \tilde{x}_{62}, \tilde{x}_{63}, \tilde{x}_{64}, \tilde{x}_{65}, \tilde{x}_{67}$ in $((2, 4, 1, 2)_{LR} \otimes \tilde{x}_{12} \oplus (1, 1, 1, 1)_{LR} \otimes \tilde{x}_{13} \oplus (M, M, 0, 0)_{LR} \otimes \tilde{x}_{14} \oplus (M, M, 0, 0)_{LR} \otimes \tilde{x}_{15} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{17} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{22} \oplus (1, 3, 1, 1)_{LR} \otimes \tilde{x}_{23} \oplus (M, M, 0, 0)_{LR} \otimes \tilde{x}_{24} \oplus (M, M, 0, 0)_{LR} \otimes \tilde{x}_{25} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{27} \oplus (M, M, 0, 0)_{LR} \otimes \tilde{x}_{32} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{33} \oplus (5, 7, 2, 2)_{LR} \otimes \tilde{x}_{34} \oplus (5, 6, 3, 1)_{LR} \otimes \tilde{x}_{35} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{37} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{62} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{63} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{64} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{65} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{67})$, the minimum total fuzzy transportation cost is $(360, 560, 270, 350)_{LR}$.

4.5.3 Physical interpretation of the results

In this section, the minimum total fuzzy transportation cost, obtained by using the proposed methods, is physically interpreted. Similarly, the obtained fuzzy optimal solution can also be physically interpreted.

Using both the proposed methods the minimum total fuzzy transportation cost is $(360, 560, 270, 350)_{LR}$, which can be physically interpreted as follows:

(1) The least amount of minimum total transportation cost is 90 units.

(2) The most possible amount of minimum total transportation cost lies between 360 units and 560 units.

(3) The greatest amount of minimum total transportation cost is 910 units. i.e., the minimum total transportation cost will always be greater than 90 units and less than 910 units and maximum chances are that the minimum total transportation cost will lie between 360 units and 560 units.

The variation in minimum total transportation cost with respect to chances are shown in Figure 4.2.

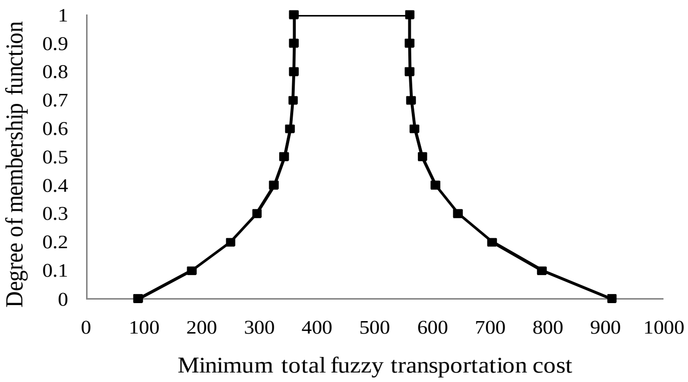

Figure 4.2. Membership function of *LR* flat fuzzy number representing the minimum total fuzzy transportation cost

4.6 Comparative study

The results obtained by the existing method [70] and by the methods proposed in this chapter and previous chapters, are compared in Table 4.13.

Table 4.13 Results obtained by existing and proposed methods

Example	Minimum total fuzzy transportation cost			
	Existing method [70]	Methods proposed in Chapter 2	Methods proposed in Chapter 3	Methods proposed in this chapter
2.2	Not applicable	(2100, 2900, 3700, 4100)	$(2900, 3700, 800, 400)_{LR}$	$(2900, 3700, 800, 400)_{LR}$
3.1	Not applicable	Not applicable	$(5800, 8400, 2800, 2900)_{LR}$	$(5800, 8400, 2800, 2900)_{LR}$
4.1	$(4100, 6600, 2000, 2600)_{LR}$	(2100, 4100, 6600, 9200)	$(4100, 6600, 2000, 2600)_{LR}$	$(4100, 6600, 2000, 2600)_{LR}$
3.5 [71, pp. 2498]	$(1924000, 1903300, 7299800)_{LR}$	Not applicable	Not applicable	$(1924000, 1903300, 7299800)_{LR}$
4.2	Not applicable	Not applicable	Not applicable	$(360, 560, 270, 350)_{LR}$

The results, shown in Table 4.13, can be explained as follows:

(1) The existing method [70] can be used only for solving balanced fully fuzzy transportation problems and balanced fully fuzzy transshipment problems. Since, the existing fully fuzzy transshipment problem [71, Example 3.5, pp. 2498] and the fully fuzzy transportation problem, chosen in Example 4.1, are balanced problems. So, these problems can be solved by the existing method [70]. But, the problem, chosen in Example 2.2, Example 3.1 and Example 4.2, are unbalanced problems so the existing method [70] can not be used for solving these problems.

(2) The methods, proposed in Chapter 2, can be used only for solving such balanced and unbalanced fully fuzzy transportation problems in which all the parameters are either represented by triangular fuzzy numbers or trapezoidal fuzzy numbers. Since, in the fully fuzzy transportation problem, chosen in Example 2.2 and Example 4.1, all the parameters are represented by trapezoidal fuzzy numbers so, this problem can be solved by the method, proposed in Chapter 2. But, in the fully fuzzy transportation problem, chosen in Example 3.1, the parameters are represented by LR flat fuzzy numbers so, this problem can not be solved by the methods, proposed in Chapter 2.

Also, the problems, chosen in Example 3.5 and Example 4.2, are fully fuzzy transshipment problems which is an extension of fully fuzzy transportation problem so, these problems can not be solved by the methods, proposed in Chapter 2.

(3) The methods, proposed in Chapter 3, can be used to solve such balanced and unbalanced fully fuzzy transportation problems in which all the parameters

are represented by LR flat fuzzy numbers. Since, in the problems, chosen in Example 2.2, Example 3.1 and Example 4.1, all the parameters are represented by LR flat fuzzy numbers so, all these problems can be solved by the methods, proposed in Chapter 3.

Also, the existing problem [71, Example 3.5, pp. 2498] and the problem, chosen in Example 4.2, are fully fuzzy transshipment problems which are extension of fully fuzzy transportation problems so, these problems can not be solved by the methods, proposed in Chapter 3.

(4) The methods, proposed in this chapter, can be used to solve balanced as well as unbalanced fully fuzzy transshipment problems. Since, the fully fuzzy transportation problems are also a special type of fully fuzzy transshipment problems so the proposed method can also be used for solving fully fuzzy transportation problems i.e., all the chosen problems can be solved by the methods, proposed in this chapter.

Remark 4.6 Since, in the existing methods [70, 71] and proposed methods different type of multiplication is used in the objective function so the different fuzzy optimal values are obtained by using the existing and proposed methods. But, to compare the results of existing method [71] and proposed methods same type of multiplicative operation is used for solving fully fuzzy transportation problems and fully fuzzy transshipment problems.

4.7 Case study

Ghatee and Hashemi [73, Definition 5.2, pp. 804] have claimed that if $\tilde{a}$ and $\tilde{b}$ and are two non-negative fuzzy numbers such that $\tilde{a} \neq \tilde{b}$. Then, $\tilde{a} \neq \tilde{b}$ can be converted into $\tilde{a} = \tilde{b}$ by the following manner:

Find $\tilde{e} = \tilde{a} \ominus \tilde{b}$ and check that $\tilde{e}$ is negative or positive.

Case (i) If $\tilde{e}$ is positive then $\tilde{b} \oplus \tilde{e} = \tilde{a}$.

Case (ii) If $\tilde{e}$ is negative then $\tilde{a} \oplus \tilde{e'} = \tilde{b}$, where, $\tilde{e'} = \ominus_H \tilde{e}$.

However, it is not always possible to convert $\tilde{a} \neq \tilde{b}$ into $\tilde{a} = \tilde{b}$ by using the described method due to the following reasons:

If $\tilde{a}$ and $\tilde{b}$ are two non-negative fuzzy numbers such that $\tilde{a} \neq \tilde{b}$ then $\tilde{e} = \tilde{a} \ominus \tilde{b}$ may be neither negative nor positive. i.e., neither $\tilde{b} \oplus \tilde{e} = \tilde{a}$ nor $\tilde{a} \oplus \tilde{e'} = \tilde{b}$. e.g., in the existing real life fully fuzzy transshipment problem [73], described in Section 4.7.1, total fuzzy availability $\tilde{a} = (1580, 49, 100)$ is not equal to the total fuzzy demand $\tilde{b} = (1498.9, 64, 59)$ so it is an unbalanced fully fuzzy transshipment problem. However, $\tilde{e} = \tilde{a} \ominus \tilde{b} = (1580 - 1498.9, 49 + 59, 100 + 64) = (81.1, 108, 164)$ is neither negative nor positive fuzzy number and neither $\tilde{b} \oplus \tilde{e} = \tilde{a}$ nor $\tilde{a} \oplus \tilde{e'} = \tilde{b}$.

Since, Ghatee and Hashemi [73] have used the fuzzy linear programming formulation $(P_{4.1})$, to obtain the fuzzy optimal solution of this unbalanced real life problem without converting it into the balanced fully fuzzy transshipment problem. So, the results of this real life problem, obtained by Ghatee and Hashemi [73], are not genuine.

In this section, the methods, proposed in this chapter, are used to find the fuzzy optimal solution of this real life problem.

4.7.1 Description of problem

A simplified network of petroleum industry distribution system Iran, shown in Figure 4.3, which is operated to transport crude oil from production units and import terminals to refineries, export terminals and storage tanks, and from their to destinations with minimal cost is depicted in Figure 4.4.

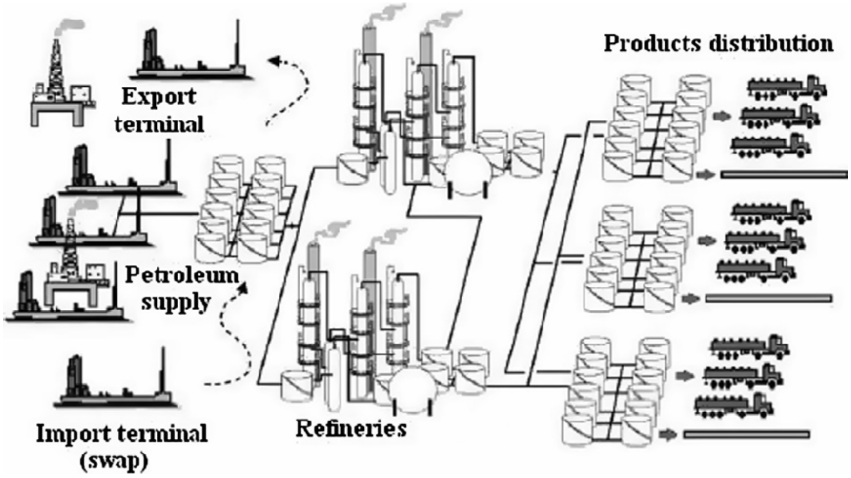

Figure 4.3. General petroleum supply chain, including the suppliers and demanders of petroleum.

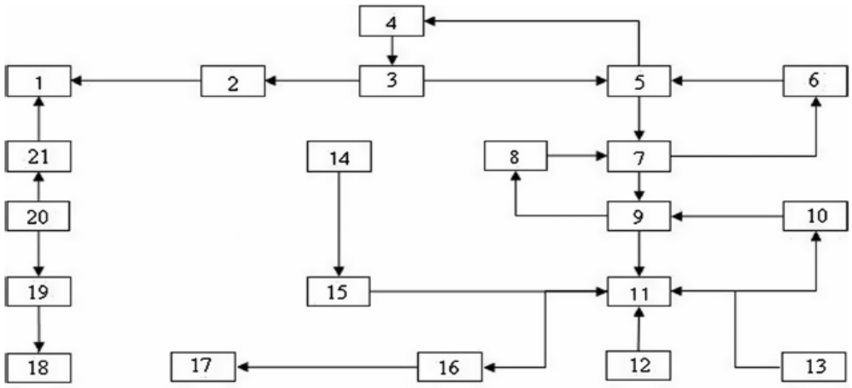

Figure 4.4. A simple diagram of a pilot in Iranian petroleum industry.

The fuzzy availability of the crude oil at different sources and destinations are shown in Table 4.14 and the fuzzy cost for transporting one unit quantity of the crude oil from different sources to different destinations are shown in Table 4.15

with $L(x) = R(x) =$ maximum $\{0, 1 - x\}$.

Table 4.14 The fuzzy availability and fuzzy demand

Node	Fuzzy availability	Fuzzy demand
1	-	$(588, 10, 5)_{LR}$
2	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$
3	$(200, 10, 10)_{LR}$	-
4	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$
5	$(80, 4, 7)_{LR}$	-
6	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$
7	$(220, 4, 12)_{LR}$	-
8	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$
9	$(150, 5, 9)_{LR}$	-
10	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$
11	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$
12	$(220, 5, 16)_{LR}$	-
13	$(70, 2, 5)_{LR}$	-
14	$(50, 3, 5)_{LR}$	-
15	$(70, 1, 4)_{LR}$	-
16	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$
17	-	$(400, 17, 20)_{LR}$
18	-	$(158.9, 3, 15)_{LR}$
19	$(0, 0, 0)_{LR}$	(0,0,0)
20	$(520, 15, 32)_{LR}$	-
21	-	$(352, 34, 19)_{LR}$

Table 4.15 The fuzzy transportation cost

Node	Node	Fuzzy transportation cost
2	1	$(4550, 220.68, 1419.6)_{LR}$
3	2	$(4550, 22.75, 1656.2)_{LR}$
3	5	$(5000, 160, 940)_{LR}$
5	4	$(10000, 595, 2360)_{LR}$
4	3	$(10000, 830, 440)_{LR}$
5	7	$(10000, 525, 80)_{LR}$
7	6	$(10000, 610, 960)_{LR}$
6	5	$(10000, 150, 2840)_{LR}$
7	9	$(10000, 440, 920)_{LR}$
9	8	$(2500, 76.25, 940)_{LR}$
8	7	$(10000, 625, 2280)_{LR}$
9	11	$(10000, 590, 1920)_{LR}$
11	10	$(10000, 430, 1040)_{LR}$
10	9	$(10000, 515, 960)_{LR}$
13	11	$(10000, 770, 240)_{LR}$
14	15	$(10000, 895, 400)_{LR}$
15	11	$(2000, 131, 368)_{LR}$
11	16	$(5000, 127.5, 1460)_{LR}$
16	17	$(5000, 227.5, 1520)_{LR}$
12	11	$(5000, 167.5, 1520)_{LR}$
20	19	$(1600, 89.6, 96)_{LR}$
20	21	$(2500, 148.75, 700)_{LR}$
21	1	$(1600, 103.2, 89.6)_{LR}$
19	18	$(30000, 585, 7800)_{LR}$

4.7.2 Results

The fuzzy quantity of crude oil that should be transported from one node to another node, obtained by using the proposed methods, are shown in Table 4.16.

Table 4.16 The fuzzy optimal flow between each couple of node in the simplified pilot in Iran

Node ⟶ Node	Fuzzy flow	Node ⟶ Node	Fuzzy flow
2 ⟶ 1	$(557, 10, 5)_{LR}$	11 ⟶ 16	$(400, 17, 20)_{LR}$
3 ⟶ 2	$(557, 10, 5)_{LR}$	12 ⟶ 11	$(220, 5, 16)_{LR}$
3 ⟶ 23	$(0, 0, 5)_{LR}$	13 ⟶ 11	$(70, 2, 0)_{LR}$
4 ⟶ 3	$(357, 0, 0)_{LR}$	13 ⟶ 23	$(0, 0, 5)_{LR}$
5 ⟶ 21	$(4, 4, 0)_{LR}$	14 ⟶ 15	$(3, 3, 0)_{LR}$
5 ⟶ 4	$(357, 0, 0)_{LR}$	14 ⟶ 23	$(47, 0, 3)_{LR}$
5 ⟶ 23	$(0, 0, 7)_{LR}$	14 ⟶ 18	$(0, 0, 2)_{LR}$
6 ⟶ 5	$(281, 0, 0)_{LR}$	15 ⟶ 11	$(73, 4, 4)_{LR}$
7 ⟶ 9	$(1, 1, 0)_{LR}$	16 ⟶ 17	$(400, 17, 20)_{LR}$
7 ⟶ 6	$(281, 0, 0)_{LR}$	19 ⟶ 18	$(156, 0, 13)_{LR}$
7 ⟶ 21	$(3, 3, 0)_{LR}$	20 ⟶ 19	$(156, 0, 13)_{LR}$
7 ⟶ 23	$(0, 0, 12)_{LR}$	20 ⟶ 21	$(364, 15, 19)_{LR}$
8 ⟶ 7	$(65, 0, 0)_{LR}$	21 ⟶ 1	$(31, 0, 0)_{LR}$
9 ⟶ 8	$(65, 0, 0)_{LR}$	22 ⟶ 18	$(3, 3, 0)_{LR}$
9 ⟶ 11	$(37, 6, 0)_{LR}$	22 ⟶ 21	$(12, 12, 0)_{LR}$
9 ⟶ 23	$(49, 0, 9)_{LR}$		

4.7.3 Discussion

Since, it is obvious from Figure 4.4 that node 5, node 7 and node 14 are not connected to node 21, node 21 and node 18 respectively. So, in the fuzzy optimal solution the fuzzy quantity of the crude oil that should be transported from node 5, node 7 and node 14 to node 21, node 21 and node 18 respectively should be zero fuzzy number. However, it is obvious from the results, shown in Table 4.16, that these quantities are not zero fuzzy number. So, the obtained fuzzy optimal solution of the chosen real life problem is a pseudo fuzzy optimal solution.

Since, the fuzzy cost for transporting one unit quantity of the crude oil from node 5, node 7 and node 14 to node 21, node 21 and node 18 respectively are not given in the existing data [73]. So, assuming this cost as M the obtained minimum total fuzzy transportation cost is $(29896400 + 7M, 26984213.51 + 7M, 9121057.2 + 2M)_{LR}$.

Remark 4.7 Since the chosen real life problem is an unbalanced problem so to find the fuzzy optimal solution of the chosen problem a purely dummy source node (22) and a purely dummy destination node (23) is introduced. In the results, presented in Section 4.7.2, $i \longrightarrow 23$, where, $i = 3, 5, 7, 9, 14$ represents the fuzzy quantity of the crude oil that should be transported from i^{th} node to purely destination node (23). Similarly, $22 \longrightarrow j$, where, $j = 18, 21$ represents the fuzzy quantity of the crude oil that should be transported from purely dummy source node 22 to j^{th} node.

Remark 4.8 If the product can not be supplied from i^{th} node to j^{th} node then in the optimal solution of a transshipment problem the optimal quantity of the product (x_{ij}) that should be transported from i^{th} node to j^{th} node should be zero. However, if there exist any two nodes i and j such that the product can not be supplied from i^{th} node to j^{th} node but in the optimal quantity of the product (x_{ij}) is non-zero then such an optimal solution is called pseudo optimal solution e.g., on solving the network with six nodes, shown in Figure 4.5, including three purely source node and three purely destination nodes with cost, availability and demand are as follows:

Cost: $c_{14} = 2, c_{15} = 2, c_{16} = 3, c_{24} = 4, c_{25} = 1, c_{26} = 2, c_{34} = 1, c_{35} = 3$

Availability: $a_1 = 10,\ a_2 = 15, a_3 = 40$ Demand: $b_1 = 20,\ b_2 = 15, b_3 = 30$

The obtained optimal solution is $x_{16} = 10,\ x_{26} = 15,\ x_{34} = 20,\ x_{26} = 15,\ x_{36} = 5$. Since, $x_{36} \neq 0$, so the obtained optimal solution is pseudo optimal solution.

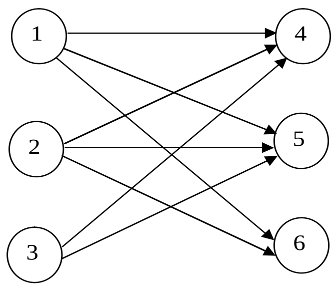

Figure 4.5. A network representing different shipping routes

4.8 Conclusions

On the basis of the comparison of the results, it can be concluded that all the problems which can be solved by using the existing method [70] and the methods, proposed in the previous chapters, can also be solved by the methods proposed in this chapter. However, there exist several problems which can be solved by the methods, proposed in this chapter, but neither can be solved by using any of the existing method [70] nor by the methods proposed in previous chapters. Hence, it is better to use the methods, proposed in this chapter as compared to the existing method [70] and the methods, proposed in previous chapters.

Chapter 5

New Methods For Solving Fully Fuzzy Solid Transportation Problems With LR Flat Fuzzy Parameters

To the best of my knowledge, only the method [132] is proposed in the literature to find the fuzzy optimal solution of fully fuzzy solid transportation problems. In this chapter, the shortcomings of this method are pointed out and to overcome these shortcomings, two new methods are proposed for solving fully fuzzy solid transportation problems. The advantages of the proposed methods over the existing method [132] and over the methods, proposed in Chapter 2 and Chapter 3, are discussed. To illustrate the proposed methods an existing fully fuzzy solid transportation problem is solved. Also, to show the application of the proposed methods in real life problem an existing real life fuzzy solid transportation problem is solved by the proposed methods.

The contents of this chapter is communicated for publication in *Optimization Letters.*

5.1 Fuzzy linear programming formulation of balanced fully fuzzy solid transportation problems

The fuzzy linear programming formulation of a balanced fully fuzzy solid transportation problem having p sources, q destinations and r conveyances with fuzzy availability $(\tilde{a}_i)$ of the product at i^{th} source, fuzzy demand $(\tilde{b}_j)$ of the product at j^{th} destination, fuzzy capacity $(\tilde{e}_k)$ of the k^{th} conveyance i.e., the maximum quantity of the product which can be carried by the k^{th} conveyance and the fuzzy cost for transporting one unit quantity of the product from i^{th} source to j^{th} destination by means of the k^{th} conveyance $(\tilde{c}_{ijk})$ can be written as [132]:

$$\text{Minimize} \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{r}(\tilde{c}_{ijk} \otimes \tilde{x}_{ijk})$$

subject to

$$\begin{aligned}
&\sum_{j=1}^{q}\sum_{k=1}^{r}\tilde{x}_{ijk} = \tilde{a}_i, && i = 1,2,3,...,p,\\
&\sum_{i=1}^{p}\sum_{k=1}^{r}\tilde{x}_{ijk} = \tilde{b}_j, && j = 1,2,3,...,q, \qquad (P_{5.1})\\
&\sum_{i=1}^{p}\sum_{j=1}^{q}\tilde{x}_{ijk} = \tilde{e}_k, && k = 1,2,3,...,r,\\
&\sum_{i=1}^{p}\tilde{a}_i = \sum_{j=1}^{q}\tilde{b}_j = \sum_{k=1}^{r}\tilde{e}_k
\end{aligned}$$

$\tilde{x}_{ijk}$ is a non-negative fuzzy number.

Remark 5.1 If $\sum_{i=1}^{p}\tilde{a}_i = \sum_{j=1}^{q}\tilde{b}_j = \sum_{k=1}^{r}\tilde{e}_k$ then the fully fuzzy solid transportation problem is said to be balanced fully fuzzy solid transportation problem, otherwise it is called unbalanced fully fuzzy solid transportation problem.

5.2 Liu method

Liu [132] proposed a new method for solving such fully fuzzy solid transportation problems having p sources, q destinations and r conveyances in which the parameters are either represented by triangular fuzzy numbers or trapezoidal fuzzy numbers.

The steps of the existing method [132] are as follows:

Step 1 Find the α-cuts $[(c_{ijk})^L_\alpha, (c_{ijk})^U_\alpha]$, $[(a_i)^L_\alpha, (a_i)^U_\alpha]$, $[(b_j)^L_\alpha, (b_j)^U_\alpha]$, $[(e_k)^L_\alpha, (e_k)^U_\alpha]$ of $\tilde{c}_{ijk}$, $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{e}_k$ respectively.

Step 2 Check that $\sum\limits_{i=1}^{p}(a_i)^L_{\alpha=0} \geq \sum\limits_{j=1}^{q}(b_j)^U_{\alpha=0}$ and $\sum\limits_{k=1}^{r}(e_k)^U_{\alpha=0} \geq \sum\limits_{j=1}^{q}(b_j)^L_{\alpha=0}$ or not.

Case (i) If $\sum\limits_{i=1}^{p}(a_i)^L_{\alpha=0} \geq \sum\limits_{j=1}^{q}(b_j)^U_{\alpha=0}$ and $\sum\limits_{k=1}^{r}(e_k)^U_{\alpha=0} \geq \sum\limits_{j=1}^{q}(b_j)^L_{\alpha=0}$ then the chosen problem is feasible and go to Step 3.

Case (ii) If $\sum\limits_{i=1}^{p}(a_i)^L_{\alpha=0} < \sum\limits_{j=1}^{q}(b_j)^U_{\alpha=0}$ or $\sum\limits_{k=1}^{r}(e_k)^U_{\alpha=0} < \sum\limits_{j=1}^{q}(b_j)^L_{\alpha=0}$ then the chosen problem is infeasible.

Step 3 Solve the problem $(P_{5.2})$ to find the left end point $(x_{ijk})^L_\alpha$ of α-cut of fuzzy decision variable $\tilde{x}_{ijk}$ and the left end point (Z^L_α) of α-cut of minimum total fuzzy transportation cost $\tilde{Z} = \sum\limits_{i=1}^{p}\sum\limits_{j=1}^{q}\sum\limits_{k=1}^{r}(\tilde{c}_{ijk} \otimes \tilde{x}_{ijk})$ corresponding to different values of $\alpha \in [0, 1]$.

$$Z^L_\alpha = \underset{\substack{(c_{ijk})^L_\alpha \leq c_{ijk} \leq (c_{ijk})^U_\alpha \\ (a_i)^L_\alpha \leq a_i \leq (a_i)^U_\alpha \\ (b_j)^L_\alpha \leq b_j \leq (b_j)^U_\alpha \\ (e_k)^L_\alpha \leq e_k \leq (e_k)^U_\alpha \\ \forall \;\; i,\, j,\, k}}{\text{Minimize}} \left\{ \begin{array}{ll} \text{Minimize } \sum\limits_{i=1}^{p}\sum\limits_{j=1}^{q}\sum\limits_{k=1}^{r} c_{ijk}x_{ijk} & \\ \text{subject to} & \\ \sum\limits_{j=1}^{q}\sum\limits_{k=1}^{r} x_{ijk} \leq a_i, & i = 1, 2, ..., p \\ \sum\limits_{i=1}^{p}\sum\limits_{k=1}^{r} x_{ijk} \geq b_j, & j = 1, 2, ..., q \\ \sum\limits_{i=1}^{p}\sum\limits_{j=1}^{q} x_{ijk} \leq e_k, & k = 1, 2, ..., r \\ x_{ijk} \geq 0 & \forall \;\; i, j, k \end{array} \right. \qquad (P_{5.2})$$

Step 4 Solve the problem $(P_{5.3})$ to find the right end point $(x_{ijk})^U_\alpha$ of α-cut of fuzzy decision variable $\tilde{x}_{ijk}$ and the right end point (Z^U_α) of α- cut of minimum total fuzzy transportation cost $\tilde{Z} = \sum\limits_{i=1}^{p}\sum\limits_{j=1}^{q}\sum\limits_{k=1}^{r}(\tilde{c}_{ijk} \otimes \tilde{x}_{ijk})$ corresponding to different values of $\alpha \in [0, 1]$.

$$Z_\alpha^U = \underset{\substack{(c_{ijk})_\alpha^L \le c_{ijk} \le (c_{ijk})_\alpha^U \\ (a_i)_\alpha^L \le a_i \le (a_i)_\alpha^U \\ (b_j)_\alpha^L \le b_j \le (b_j)_\alpha^U \\ (e_k)_\alpha^L \le e_k \le (e_k)_\alpha^U \\ \forall \;\; i,\; j,\; k}}{\text{Maximize}} \begin{cases} \text{Minimize } \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{r} c_{ijk}x_{ijk} \\ \text{subject to} \\ \sum_{j=1}^{q}\sum_{k=1}^{r} x_{ijk} \le a_i, \qquad i = 1,2,...,p \\ \sum_{i=1}^{p}\sum_{k=1}^{r} x_{ijk} \ge b_j, \qquad j = 1,2,...,q \\ \sum_{i=1}^{p}\sum_{j=1}^{q} x_{ijk} \le e_k, \qquad k = 1,2,...,r \\ x_{ijk} \ge 0 \qquad\qquad \forall \;\; i,j,k \end{cases} \qquad (P_{5.3})$$

Step 5 Use the values of $(x_{ijk})_\alpha^L$, $(x_{ijk})_\alpha^U$, Z_α^L and Z_α^U, obtained from Step 3 and Step 4, to find the α-cuts $[(x_{ijk})_\alpha^L,\ (x_{ijk})_\alpha^U]$ and $[Z_\alpha^L,\ Z_\alpha^U]$ corresponding to optimal fuzzy quantity of the product $\tilde{x}_{ijk}$ and minimum total fuzzy transportation cost $\tilde{Z} = \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{r}(\tilde{c}_{ijk} \otimes \tilde{x}_{ijk})$.

5.3 Shortcomings of Liu method

In this section, the shortcomings of the existing method [132] are pointed out. Liu [132] solved the fully fuzzy solid transportation problem, presented in Example 5.1, to illustrate his method.

Example 5.1 [132, pp. 937] Consider a fully fuzzy solid transportation problem having two sources, three destinations and two conveyances. Fuzzy availabilities, fuzzy demands, fuzzy capacities and the costs with $L(x) = R(x) =$ maximum $\{0, 1-x\}$ are as follows:

Fuzzy availabilities: $\tilde{a}_1 = (80, 100, 10, 20)_{LR}$, $\tilde{a}_2 = (70, 70, 10, 20)_{LR}$.

Fuzzy demands: $\tilde{b}_1 = (30, 40, 20, 10)_{LR}$, $\tilde{b}_2 = (50, 50, 10, 10)_{LR}$, $\tilde{b}_3 = (40, 60, 10, 10)_{LR}$.

Fuzzy capacities: $\tilde{e}_1 = (80, 80, 10, 20)_{LR}$, $\tilde{e}_2 = (70, 70, 10, 20)_{LR}$.

Costs: $\tilde{c}_{111} = (30, 30, 10, 10)_{LR}$, $c_{112} = 70, c_{121} = 60, c_{122} = 60, c_{131} = 50, c_{132} = 30, \tilde{c}_{211} = (20, 20, 10, 10)_{LR}, c_{212} = 40, c_{221} = 20, c_{222} = 50, c_{231} = 40, c_{232} = 50$.

Liu [132] claimed that on solving the fully fuzzy solid transportation problem, presented in Example 5.1, for $\alpha = 0$, the following optimal solution is obtained $x^L_{122} = 40$ and $x^U_{122} = 10$. Since, $x^L_{122} > x^U_{122}$ so the same shortcoming, pointed out by Liu and Kao [134] in the existing methods [102, 164], which is already discussed in Chapter 2, is also occurring in the existing method [132].

5.4 Limitations of the methods proposed in previous chapters

Since, fully fuzzy solid transportation problems are generalization of fully fuzzy transportation problems and there is no link between fully fuzzy transshipment problems and fully fuzzy solid transportation problems. So, the methods for solving fully fuzzy transshipment problems can not be used for solving fully fuzzy solid transportation problems i.e., neither the existing method [70] nor the methods, proposed in previous chapters, can be used for solving fully fuzzy solid transportation problems.

5.5 Proposed methods

In this section, to overcome the shortcomings of the existing method [132], discussed in Section 5.3, and to overcome the limitations of the methods proposed in Chapter 2 and Chapter 3, discussed in Section 5.4, two new methods are proposed to find the fuzzy optimal solution of such fully fuzzy solid transportation problems in which all the parameters are represented by LR flat fuzzy numbers. Also, the advantages of the proposed methods over the existing method [132] and over the methods, proposed in Chapter 2 and Chapter 3, are discussed.

5.5.1 Proposed method based on fuzzy linear programming formulation

In this section, a new method, based on fuzzy linear programming formulation, is proposed to find the fuzzy optimal solution of such fully fuzzy solid transportation problems in which all the parameters are represented by LR flat fuzzy numbers.

The steps of the proposed method are as follows:

Step 1 Find $\sum\limits_{i=1}^{p}\tilde{a}_i$, $\sum\limits_{j=1}^{q}\tilde{b}_j$ and $\sum\limits_{k=1}^{r}\tilde{e}_k$. Let $\sum\limits_{i=1}^{p}\tilde{a}_i=(m,n,\alpha,\beta)_{LR}$, $\sum\limits_{j=1}^{q}\tilde{b}_j=(m',n',\alpha',\beta')_{LR}$ and $\sum\limits_{k=1}^{r}\tilde{e}_k=(m'',n'',\alpha'',\beta'')_{LR}$. Use Definition 3.5 to examine that the problem is balanced or unbalanced.

Case (1) If the problem is balanced, i.e., $\sum\limits_{i=1}^{p}\tilde{a}_i=\sum\limits_{j=1}^{q}\tilde{b}_j=\sum\limits_{k=1}^{r}\tilde{e}_k$, then Go to Step 4.

Case (2) If the problem is unbalanced i.e., $\sum\limits_{i=1}^{p}\tilde{a}_i=\sum\limits_{j=1}^{q}\tilde{b}_j\neq\sum\limits_{k=1}^{r}\tilde{e}_k$ or $\sum\limits_{i=1}^{p}\tilde{a}_i\neq\sum\limits_{j=1}^{q}\tilde{b}_j=\sum\limits_{k=1}^{r}\tilde{e}_k$ or $\sum\limits_{i=1}^{p}\tilde{a}_i=\sum\limits_{k=1}^{r}\tilde{e}_k\neq\sum\limits_{j=1}^{q}\tilde{b}_j$ or $\sum\limits_{i=1}^{p}\tilde{a}_i\neq\sum\limits_{j=1}^{q}\tilde{b}_j\neq\sum\limits_{k=1}^{r}\tilde{e}_k$ then Go to Step 2.

Step 2 Check that $\sum\limits_{i=1}^{p}\tilde{a}_i=\sum\limits_{j=1}^{q}\tilde{b}_j$ or $\sum\limits_{i=1}^{p}\tilde{a}_i\neq\sum\limits_{j=1}^{q}\tilde{b}_j$.

Case (1) If $\sum\limits_{i=1}^{p}\tilde{a}_i=\sum\limits_{j=1}^{q}\tilde{b}_j$ then Go to Step 3.

Case (2) If $\sum\limits_{i=1}^{p}\tilde{a}_i\neq\sum\limits_{j=1}^{q}\tilde{b}_j$ then convert $\sum\limits_{i=1}^{p}\tilde{a}_i\neq\sum\limits_{j=1}^{q}\tilde{b}_j$ into $\sum\limits_{i=1}^{u}\tilde{a}_i=\sum\limits_{j=1}^{t}\tilde{b}_j$, where, $u=p$ or $p+1$ and $t=q$ or $q+1$ by using one of the following cases:

Case (2a) If $m-\alpha\leq m'-\alpha'$, $\alpha\leq\alpha'$, $n-m\leq n'-m'$ and $\beta\leq\beta'$ then introduce a dummy source S_{p+1} with fuzzy availability $\tilde{a}_{p+1}=(m'-m,n'-n,\alpha'-\alpha,\beta'-\beta)_{LR}$ so that $\sum\limits_{i=1}^{p+1}\tilde{a}_i=\sum\limits_{j=1}^{q}\tilde{b}_j$. Go to Step 3.

Case (2b) If $m-\alpha\geq m'-\alpha'$, $\alpha\geq\alpha'$, $n-m\geq n'-m'$ and $\beta\geq\beta'$ then introduce a dummy destination D_{q+1} with fuzzy demand $\tilde{b}_{q+1}=(m-m',n-n',\alpha-\alpha',\beta-\beta')_{LR}$ so that $\sum\limits_{i=1}^{p}\tilde{a}_i=\sum\limits_{j=1}^{q+1}\tilde{b}_j$. Go to Step 3.

Case (2c) If neither Case (2a) nor Case (2b) is satisfied then introduce a dummy source S_{p+1} with fuzzy availability $\tilde{a}_{p+1}=$ (maximum $\{0,(m'-\alpha')-(m-\alpha)\}$ + maximum $\{0,(\alpha'-\alpha)\}$, maximum $\{0,(m'-\alpha')-(m-\alpha)\}$ + maximum $\{0,(\alpha'-\alpha)\}$ + maximum $\{0,(n'-m')-(n-m)\}$, maximum $\{0,(\alpha'-\alpha)\}$, maximum $\{0,(\beta'-\beta)\})_{LR}$ and a dummy destination D_{q+1} with fuzzy demand $\tilde{b}_{q+1}=$ (maximum $\{0,(m-\alpha)-(m'-\alpha')\}$ + maximum $\{0,(\alpha-\alpha')\}$, maximum $\{0,(m-\alpha)-(m'-\alpha')\}$ + maximum $\{0,(\alpha-\alpha')\}$ + maximum $\{0,(n-m)-(n'-m')\}$, maximum $\{0,(\alpha-\alpha')\}$, maximum $\{0,(\beta-\beta')\})_{LR}$ so that $\sum_{i=1}^{p+1}\tilde{a}_i=\sum_{j=1}^{q+1}\tilde{b}_j$. Go to Step 3.

Step 3 Using Step 2, $\sum_{i=1}^{u}\tilde{a}_i=\sum_{j=1}^{t}\tilde{b}_j$, where, $u=p$ or $p+1$ and $t=q$ or $q+1$.

Let $\sum_{i=1}^{u}\tilde{a}_i=\sum_{j=1}^{t}\tilde{b}_j=(m_1,n_1,\alpha_1,\beta_1)_{LR}$ and $\sum_{k=1}^{r}\tilde{e}_k=(m'',n'',\alpha'',\beta'')_{LR}$

Now check $\sum_{i=1}^{u}\tilde{a}_i=\sum_{j=1}^{t}\tilde{b}_j=\sum_{k=1}^{r}\tilde{e}_k$ or $\sum_{i=1}^{u}\tilde{a}_i=\sum_{j=1}^{t}\tilde{b}_j\neq\sum_{k=1}^{r}\tilde{e}_k$

Case (1) If $\sum_{i=1}^{u}\tilde{a}_i=\sum_{j=1}^{t}\tilde{b}_j=\sum_{k=1}^{r}\tilde{e}_k$, then Go to Step 4.

Case (2) If $\sum_{i=1}^{u}\tilde{a}_i=\sum_{j=1}^{t}\tilde{b}_j\neq\sum_{k=1}^{r}\tilde{e}_k$ then convert $\sum_{i=1}^{u}\tilde{a}_i=\sum_{j=1}^{t}\tilde{b}_j\neq\sum_{k=1}^{r}\tilde{e}_k$ into $\sum_{i=1}^{m}\tilde{a}_i=\sum_{j=1}^{n}\tilde{b}_j=\sum_{k=1}^{l}\tilde{e}_k$, where, $m=p$ or $p+1$, $n=q$ or $q+1$ and $l=r$ or $r+1$ by using one of the following cases:

Case (2a) If $m_1-\alpha_1\leq m''-\alpha''$, $\alpha_1\leq\alpha''$, $n_1-m_1\leq n''-m''$ and $\beta_1\leq\beta''$ then check that in Step 2 a dummy source S_{p+1} with fuzzy availability $\tilde{a}_{p+1}$ is introduced or not and also check that a dummy destination D_{q+1} with fuzzy demand $\tilde{b}_{q+1}$ is introduced or not.

Case (i) If both the dummy source S_{p+1} and dummy destination D_{q+1} are introduced then increase both the fuzzy availability $\tilde{a}_{p+1}$ of the already introduced dummy source S_{p+1} and the fuzzy demand $\tilde{b}_{q+1}$ of the already introduced dummy destination D_{q+1} by the same fuzzy quantity $\tilde{a}=(m''-m_1,n''-n_1,\alpha''-\alpha_1,\beta''-\beta_1)_{LR}$ i.e., replace fuzzy availability $\tilde{a}_{p+1}$ and fuzzy demand $\tilde{b}_{q+1}$ of the already introduced dummy source S_{p+1} and destinations D_{q+1} by $\tilde{a}_{p+1} \oplus \tilde{a}$ and $\tilde{b}_{q+1} \oplus \tilde{a}$ so that $\sum_{i=1}^{p+1} \tilde{a}_i = \sum_{j=1}^{q+1} \tilde{b}_j = \sum_{k=1}^{r} \tilde{e}_k$. Go to Step 4.

Case (ii) If a dummy source S_{p+1} with fuzzy availability $\tilde{a}_{p+1}$ is introduced but no dummy destination D_{q+1} with fuzzy demand $\tilde{b}_{q+1}$ is introduced then increase the fuzzy availability $\tilde{a}_{p+1}$ of the already introduced dummy source S_{p+1} by the fuzzy quantity $\tilde{a} = (m'' - m_1, n'' - n_1, \alpha'' - \alpha_1, \beta'' - \beta_1)_{LR}$ i.e., replace fuzzy availability $\tilde{a}_{p+1}$ of the already introduced dummy source S_{p+1} by $\tilde{a}_{p+1} \oplus \tilde{a}$ and also introduce a dummy destination with fuzzy demand $\tilde{b}_{q+1} = \tilde{a} = (m'' - m_1, n'' - n_1, \alpha'' - \alpha_1, \beta'' - \beta_1)_{LR}$ so that $\sum_{i=1}^{p+1} \tilde{a}_i = \sum_{j=1}^{q+1} \tilde{b}_j = \sum_{k=1}^{r} \tilde{e}_k$. Go to Step 4.

Case (iii) If a dummy destination D_{q+1} with fuzzy demand $\tilde{b}_{q+1}$ is introduced but no dummy source S_{p+1} with fuzzy availability $\tilde{a}_{p+1}$ is introduced then increase the fuzzy demand $\tilde{b}_{q+1}$ of the already introduced dummy destination D_{q+1} by the fuzzy quantity $\tilde{a} = (m'' - m_1, n'' - n_1, \alpha'' - \alpha_1, \beta'' - \beta_1)_{LR}$ i.e., replace fuzzy demand $\tilde{b}_{q+1}$ of the already introduced dummy destination D_{q+1} by $\tilde{b}_{q+1} \oplus \tilde{a}$ and also introduce a dummy source S_{p+1} with fuzzy availability $\tilde{a}_{p+1} = \tilde{a} = (m'' - m_1, n'' - n_1, \alpha'' - \alpha_1, \beta'' - \beta_1)_{LR}$ so that $\sum_{i=1}^{p+1} \tilde{a}_i = \sum_{j=1}^{q+1} \tilde{b}_j = \sum_{k=1}^{r} \tilde{e}_k$. Go to Step 4.

Case (2b) If $m_1 - \alpha_1 \geq m'' - \alpha''$, $\alpha_1 \geq \alpha''$, $n_1 - m_1 \geq n'' - m''$, and $\beta_1 \geq \beta''$ then introduce a dummy conveyance E_{r+1} with fuzzy capacity $\tilde{e}_{r+1} = (m_1 - m'', n_1 - n'', \alpha_1 - \alpha'', \beta_1 - \beta'')_{LR}$ so that $\sum_{i=1}^{u} \tilde{a}_i = \sum_{j=1}^{t} \tilde{b}_j = \sum_{k=1}^{r+1} \tilde{e}_k$. Go to Step 4.

Case (2c) If neither Case (2a) nor Case (2b) is satisfied then check that in Step 2 a dummy source S_{p+1} with fuzzy availability $\tilde{a}_{p+1}$ is introduced or not and also check that a dummy destination D_{q+1} with fuzzy demand $\tilde{b}_{q+1}$ is introduced or not.

Case (i) If both the dummy source S_{p+1} and dummy destination D_{q+1} are introduced then increase both the fuzzy availability $\tilde{a}_{p+1}$ of the already introduced dummy source S_{p+1} and the fuzzy demand of the already introduced dummy destination D_{q+1} by the same fuzzy quantity $\tilde{a}$ =(maximum $\{0,(m''-\alpha'')-(m_1-\alpha_1)\}$ + maximum $\{0,(\alpha''-\alpha_1)\}$, maximum $\{0,(m''-\alpha'')-(m_1-\alpha_1)\}$ + maximum $\{0,(\alpha''-\alpha_1)\}$ + maximum $\{0,(n''-m'')-(n_1-m_1)\}$, maximum $\{0,(\alpha''-\alpha_1)\}$, maximum $\{0,(\beta''-\beta_1)\})_{LR}$ i.e., replace fuzzy availability $\tilde{a}_{p+1}$ and fuzzy demand $\tilde{b}_{q+1}$ of the already introduced dummy source S_{p+1} and destinations D_{q+1} by $\tilde{a}_{p+1}\oplus\tilde{a}$ and $\tilde{b}_{q+1}\oplus\tilde{a}$ respectively. Also, introduce a dummy conveyance E_{r+1} with fuzzy capacity $\tilde{e}_{r+1}$ = (maximum $\{0,(m_1-\alpha_1)-(m''-\alpha'')\}$ + maximum $\{0,(\alpha_1-\alpha'')\}$, maximum $\{0,(m_1-\alpha_1)-(m''-\alpha'')\}$ + maximum $\{0,(\alpha_1-\alpha'')\}$ + maximum $\{0,(n_1-m_1)-(n''-m'')\}$, maximum $\{0,(\alpha_1-\alpha'')\}$, maximum $\{0,(\beta_1-\beta'')\})_{LR}$ so that $\sum_{i=1}^{p+1}\tilde{a}_i=\sum_{j=1}^{q+1}\tilde{b}_j=\sum_{k=1}^{r+1}\tilde{e}_k$. Go to Step 4.

Case (ii) If a dummy source S_{p+1} with fuzzy availability $\tilde{a}_{p+1}$ is introduced but no dummy destination D_{q+1} with fuzzy demand $\tilde{b}_{q+1}$ is introduced then increase the fuzzy availability $\tilde{a}_{p+1}$ of the already introduced dummy source S_{p+1} by the fuzzy quantity $\tilde{a}$ = (maximum $\{0,(m''-\alpha'')-(m_1-\alpha_1)\}$ + maximum $\{0,(\alpha''-\alpha_1)\}$, maximum $\{0,(m''-\alpha'')-(m_1-\alpha_1)\}$ + maximum $\{0,(\alpha''-\alpha_1)\}$ + maximum $\{0,(n''-m'')-(n_1-m_1)\}$, maximum $\{0,(\alpha''-\alpha_1)\}$, maximum $\{0,(\beta''-\beta_1)\})_{LR}$ i.e., replace fuzzy availability $\tilde{a}_{p+1}$ of the already introduced dummy source S_{p+1} by $\tilde{a}_{p+1}\oplus\tilde{a}$ and also introduce a dummy destination with fuzzy demand $\tilde{b}_{q+1}=\tilde{a}=$ (maximum $\{0,(m''-\alpha'')-(m_1-\alpha_1)\}$ + maximum $\{0,(\alpha''-\alpha_1)\}$, maximum $\{0,(m''-\alpha'')-(m_1-\alpha_1)\}$ + maximum $\{0,(\alpha''-\alpha_1)\}$ + maximum $\{0,(n''-m'')-(n_1-m_1)\}$, maximum $\{0,(\alpha''-\alpha_1)\}$, maximum $\{0,(\beta''-\beta_1)\})_{LR}$. Also, introduce a dummy conveyance E_{r+1} with fuzzy capacity $\tilde{e}_{r+1}$ = (maximum $\{0,(m_1-\alpha_1)-(m''-\alpha'')\}$ + maximum $\{0,(\alpha_1-\alpha'')\}$, maximum $\{0,(m_1-\alpha_1)-(m''-\alpha'')\}$ + maximum

$\{0, (\alpha_1 - \alpha'')\}$ + maximum $\{0, (n_1 - m_1) - (n'' - m'')\}$, maximum $\{0, (\alpha_1 - \alpha'')\}$, maximum $\{0, (\beta_1 - \beta'')\})_{LR}$ so that $\sum_{i=1}^{p+1} \tilde{a}_i = \sum_{j=1}^{q+1} \tilde{b}_j = \sum_{k=1}^{r+1} \tilde{e}_k$. Go to Step 4.

Case (iii) If a dummy destination D_{q+1} with fuzzy demand $\tilde{b}_{q+1}$ is introduced but no dummy source S_{p+1} with fuzzy availability $\tilde{a}_{p+1}$ is introduced then increase the fuzzy demand $\tilde{b}_{q+1}$ of the already introduced dummy destination D_{q+1} by the fuzzy quantity $\tilde{a}$ = (maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$ + maximum $\{0, (n'' - m'') - (n_1 - m_1)\}$, maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (\beta'' - \beta_1)\})_{LR}$ i.e., replace fuzzy demand $\tilde{b}_{q+1}$ of the already introduced dummy destination D_{q+1} by $\tilde{b}_{q+1} \oplus \tilde{a}$ and also introduce a dummy source S_{p+1} with fuzzy availability $\tilde{a}_{p+1} = \tilde{a}$ = (maximum (maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$ + maximum $\{0, (n'' - m'') - (n_1 - m_1)\}$, maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (\beta'' - \beta_1)\})_{LR}$. Also, introduce a dummy conveyance E_{r+1} with fuzzy capacity $\tilde{e}_{r+1}$ = (maximum $\{0, (m_1 - \alpha_1) - (m'' - \alpha'')\}$ + maximum $\{0, (\alpha_1 - \alpha'')\}$, maximum $\{0, (m_1 - \alpha_1) - (m'' - \alpha'')\}$ + maximum $\{0, (\alpha_1 - \alpha'')\}$ + maximum $\{0, (n_1 - m_1) - (n'' - m'')\}$, maximum $\{0, (\alpha_1 - \alpha'')\}$, maximum $\{0, (\beta_1 - \beta'')\})_{LR}$ so that $\sum_{i=1}^{p+1} \tilde{a}_i = \sum_{j=1}^{q+1} \tilde{b}_j = \sum_{k=1}^{r+1} \tilde{e}_k$. Go to Step 4.

Step 4 The balanced fully fuzzy solid transportation problem, obtained by using Step 1 to Step 3, can be formulated into the fully fuzzy linear programming problem ($P_{5.1}$) by assuming the following fuzzy transportation costs as zero LR flat fuzzy numbers:

(i) If it is required to add any dummy source then assume the fuzzy cost for transporting one unit quantity of the product from the introduced dummy source to all destinations by all conveyances as zero LR flat fuzzy number.

(ii) If it is required to add any dummy destination then assume the fuzzy cost for transporting one unit quantity of the product from all sources to the introduced dummy destination by all conveyances as zero LR flat fuzzy number.

(iii) If it is required to add any dummy conveyance then assume the fuzzy cost for transporting one unit quantity of the product from the all sources to all destinations by introduced dummy conveyance as zero LR flat fuzzy number.

Step 5 Assuming, $\tilde{c}_{ijk} = (m'_{ijk}, n'_{ijk}, \alpha'_{ijk}, \beta'_{ijk})_{LR}$, $\tilde{a}_i = (m_i, n_i, \alpha_i, \beta_i)_{LR}$, $\tilde{b}_j = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR}$, $\tilde{e}_k = (m''_k, n''_k, \alpha''_k, \beta''_k)_{LR}$ and $\tilde{x}_{ijk} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$, the fuzzy linear programming problem $(P_{5.1})$, can be written as:

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\Big((m'_{ijk}, n'_{ijk}, \alpha'_{ijk}, \beta'_{ijk})_{LR} \otimes (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}\Big)$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l}(m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR} = (m_i, n_i, \alpha_i, \beta_i)_{LR}, \quad i = 1, 2, 3, ..., m$$
$$\sum_{i=1}^{m}\sum_{k=1}^{l}(m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR} = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR}, \quad j = 1, 2, 3, ..., n$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n}(m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR} = (m''_k, n''_k, \alpha''_k, \beta''_k)_{LR}, \quad k = 1, 2, 3, ..., l$$

$(m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$ is a non-negative LR flat fuzzy number.

Step 6 Using the arithmetic operations, defined in Section 3.1.2 and assuming $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\Big((m'_{ijk}, n'_{ijk}, \alpha'_{ijk}, \beta'_{ijk})_{LR} \otimes (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}\Big) = (m_0, n_0, \alpha_0, \beta_0)_{LR}$, the fuzzy linear programming problem, obtained in Step 5, can be written as:

Minimize $(m_0, n_0, \alpha_0, \beta_0)_{LR}$

subject to

$$(\sum_{j=1}^{n}\sum_{k=1}^{l} m_{ijk}, \sum_{j=1}^{n}\sum_{k=1}^{l} n_{ijk}, \sum_{j=1}^{n}\sum_{k=1}^{l} \alpha_{ijk}, \sum_{j=1}^{n}\sum_{k=1}^{l} \beta_{ijk})_{LR} = (m_i, n_i, \alpha_i, \beta_i)_{LR}, \; i = 1, 2, 3, ..., m$$
$$(\sum_{i=1}^{m}\sum_{k=1}^{l} m_{ijk}, \sum_{i=1}^{m}\sum_{k=1}^{l} n_{ijk}, \sum_{i=1}^{m}\sum_{k=1}^{l} \alpha_{ijk}, \sum_{i=1}^{m}\sum_{k=1}^{l} \beta_{ijk})_{LR} = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR}, \; j = 1, 2, 3, ..., n$$

$(\sum_{i=1}^{m}\sum_{j=1}^{n} m_{ijk}, \sum_{i=1}^{m}\sum_{j=1}^{n} n_{ijk}, \sum_{i=1}^{m}\sum_{j=1}^{n} \alpha_{ijk}, \sum_{i=1}^{m}\sum_{j=1}^{n} \beta_{ijk})_{LR} = (m''_k, n''_k, \alpha''_k, \beta''_k)_{LR}, \;\; k = 1, 2, 3, ..., l$

$(m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$ is a non-negative LR flat fuzzy number.

Step 7 Using Definition 3.5 and Definition 3.6, the fuzzy linear programming problem, obtained in Step 6, can be converted into the fuzzy linear programming problem $(P_{5.4})$:

Minimize $(m_0, n_0, \alpha_0, \beta_0)_{LR}$

subject to

$$\begin{aligned}
&\sum_{j=1}^{n}\sum_{k=1}^{l} m_{ijk} = m_i\,, && i = 1, 2, 3, ..., m\\
&\sum_{j=1}^{n}\sum_{k=1}^{l} n_{ijk} = n_i, && i = 1, 2, 3, ..., m\\
&\sum_{j=1}^{n}\sum_{k=1}^{l} \alpha_{ijk} = \alpha_i, && i = 1, 2, 3, ..., m\\
&\sum_{j=1}^{n}\sum_{k=1}^{l} \beta_{ijk} = \beta_i, && i = 1, 2, 3, ..., m\\
&\sum_{i=1}^{m}\sum_{k=1}^{l} m_{ijk} = m'_j, && j = 1, 2, 3, ..., n\\
&\sum_{i=1}^{m}\sum_{k=1}^{l} n_{ijk} = n'_j, && j = 1, 2, 3, ..., n\\
&\sum_{i=1}^{m}\sum_{k=1}^{l} \alpha_{ijk} = \alpha'_j, && j = 1, 2, 3, ..., n \qquad (P_{5.4})\\
&\sum_{i=1}^{m}\sum_{k=1}^{l} \beta_{ijk} = \beta'_j, && j = 1, 2, 3, ..., n\\
&\sum_{i=1}^{m}\sum_{j=1}^{n} m_{ijk} = m''_k, && k = 1, 2, 3, ..., l\\
&\sum_{i=1}^{m}\sum_{j=1}^{n} n_{ijk} = n''_k, && k = 1, 2, 3, ..., l\\
&\sum_{i=1}^{m}\sum_{j=1}^{n} \alpha_{ijk} = \alpha''_k, && k = 1, 2, 3, ..., l\\
&\sum_{i=1}^{m}\sum_{j=1}^{n} \beta_{ijk} = \beta''_k, && k = 1, 2, 3, ..., l\\
&m_{ijk} - \alpha_{ijk}, n_{ijk} - m_{ijk}, \alpha_{ijk}, \beta_{ijk} \geq 0 && \forall \;\; i, j, k
\end{aligned}$$

Step 8 As discussed in Step 6, of the method, proposed in Section 2.5.1 of Chapter 2, the fuzzy optimal solution of fuzzy linear programming problem $(P_{5.4})$, can be obtained by solving the following crisp linear programming problem:

Minimize $\Re(m_0, n_0, \alpha_0, \beta_0)_{LR}$

subject to

constraints of the problem $(P_{5.4})$

Step 9 Using the existing formula [131], $\Re(m_0, n_0, \alpha_0, \beta_0)_{LR} = \frac{1}{2}(\int_0^1 (m_0 - \alpha_0 L^{-1}(\lambda))\, d\lambda + \int_0^1 (n_0 + \beta_0 R^{-1}(\lambda))\, d\lambda)$, the crisp linear programming problem, obtained in Step 8, can be written as:

Minimize $\frac{1}{2}(\int_0^1 (m_0 - \alpha_0 L^{-1}(\lambda))\, d\lambda + \int_0^1 (n_0 + \beta_0 R^{-1}(\lambda))\, d\lambda)$

subject to

constraints of the problem $(P_{5.4})$

Step 10 Solve the crisp linear programming problem, obtained in Step 9, to find the optimal solution $\{m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk}\}$.

Step 11 Find the fuzzy optimal solution $\{\tilde{x}_{ijk}\}$ by putting the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}$, β_{ijk} in $\tilde{x}_{ijk} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$.

Step 12 Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ijk}$ in $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}(\tilde{c}_{ijk} \otimes \tilde{x}_{ijk})$.

Remark 5.2 In Step 1 of the proposed method, to convert an unbalanced fuzzy solid transportation problem into balanced fuzzy solid transportation problem, it is checked $\sum_{i=1}^{p}\tilde{a}_i \neq \sum_{j=1}^{q}\tilde{b}_j$ or $\sum_{i=1}^{p}\tilde{a}_i = \sum_{j=1}^{q}\tilde{b}_j$. One can start with $\sum_{i=1}^{p}\tilde{a}_i$ and $\sum_{k=1}^{r}\tilde{e}_k$ or $\sum_{j=1}^{q}\tilde{b}_j$ and $\sum_{k=1}^{r}\tilde{e}_k$ also. The method and the solution will remain same.

Remark 5.3 Let $\tilde{A} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$ be an LR flat fuzzy number with $L(x) = R(x) = \text{maximum}\ \{0, 1 - x\}$. Then,

$\Re(\tilde{A}) = \frac{1}{2}((m_{ijk} + n_{ijk}) - \alpha_{ijk}\int_0^1 L^{-1}(\lambda)d\lambda + \beta_{ijk}\int_0^1 R^{-1}(\lambda)d\lambda) = \frac{1}{4}(2m_{ijk} + 2n_{ijk} + \beta_{ijk} - \alpha_{ijk})$

5.5.2 Proposed method based on tabular representation

In this section, a new method, based on tabular representation of the fully fuzzy solid transportation problems, is proposed to find the fuzzy optimal solution of such fully fuzzy solid transportation problems in which the parameters are represented by LR flat fuzzy numbers.

The steps of the proposed method are as follows:

Step 1 Use Step 1 to Step 3 of the method, proposed in Section 5.5.1, to obtain a balanced fully fuzzy solid transportation problem.

Step 2 Represent the balanced fully fuzzy solid transportation problem, obtained from Step 1, into tabular form as shown by Table 5.1.

Step 3 Split Table 5.1 into four crisp transportation tables i.e., Table 5.2, Table 5.3, Table 5.4 and Table 5.5. The cost for transporting one unit quantity of the product from i^{th} source to j^{th} destination by means of k^{th} conveyance in Table 5.2, Table 5.3, Table 5.4 and Table 5.5 are represented by $\eta_{ijk}, \rho_{ijk}, \delta_{ijk}$ and ξ_{ijk} respectively.

where,

$\eta_{ijk} = \frac{1}{2}((m'_{ijk} + n'_{ijk}) - \alpha'_{ijk} \int_0^1 L^{-1}(\lambda)d\lambda + \beta'_{ijk} \int_0^1 R^{-1}(\lambda)d\lambda),$

$$i = 1, 2, ..., m \ \ , j = 1, 2, ..., n \text{ and } \ k = 1, 2, ..., l$$

$\rho_{ijk} = \frac{1}{2}((m'_{ijk} + n'_{ijk}) - m'_{ijk} \int_0^1 L^{-1}(\lambda)d\lambda + \beta'_{ijk} \int_0^1 R^{-1}(\lambda)d\lambda),$

$$i = 1, 2, ..., m \ \ , j = 1, 2, ..., n \text{ and } \ k = 1, 2, ..., l$$

$\delta_{ijk} = \frac{1}{2}(n'_{ijk} + \beta'_{ijk} \int_0^1 R^{-1}(\lambda)d\lambda),$

$$i = 1, 2, ..., m \ \ , j = 1, 2, ..., n \text{ and } \ k = 1, 2, ..., l$$

$\xi_{ijk} = \frac{1}{2}((n'_{ijk} + \beta'_{ijk}) \int_0^1 R^{-1}(\lambda)d\lambda),$

$$i = 1, 2, ..., m \ \ , j = 1, 2, ..., n \text{ and } \ k = 1, 2, ..., l$$

Table 5.1 Tabular representation of balanced fully fuzzy solid transportation problem

																											Capacity ($\tilde{e}_k$)
Conveyance	E_1						E_1						$\cdots$	E_1						$\cdots$	E_1						$\tilde{e}_1$
		E_2						E_2					$\cdots$		E_2					$\cdots$		E_2					$\tilde{e}_2$
			$\cdots$						$\cdots$				$\cdots$			$\cdots$				$\cdots$			$\cdots$				$\vdots$
				E_k						E_k			$\cdots$				E_k			$\cdots$				E_k			$\tilde{e}_k$
					$\cdots$						$\cdots$		$\cdots$					$\cdots$		$\cdots$					$\cdots$		$\vdots$
						E_l						E_l	$\cdots$						E_l	$\cdots$						E_l	$\tilde{e}_l$
Destinations → Sources↓	D_1						D_2							D_j							D_n						Availability $\tilde{a}_i$
S_1	$\tilde{c}_{111}$	$\tilde{c}_{112}$	$\cdots$	$\tilde{c}_{11k}$	$\cdots$	$\tilde{c}_{11l}$	$\tilde{c}_{121}$	$\tilde{c}_{122}$	$\cdots$	$\tilde{c}_{12k}$	$\cdots$	$\tilde{c}_{12l}$	$\cdots$	$\tilde{c}_{1j1}$	$\tilde{c}_{1j2}$	$\cdots$	$\tilde{c}_{1jk}$	$\cdots$	$\tilde{c}_{1jl}$	$\cdots$	$\tilde{c}_{1n1}$	$\tilde{c}_{1n2}$	$\cdots$	$\tilde{c}_{1nk}$	$\cdots$	$\tilde{c}_{1nl}$	$\tilde{a}_1$
S_2	$\tilde{c}_{211}$	$\tilde{c}_{212}$	$\cdots$	$\tilde{c}_{21k}$	$\cdots$	$\tilde{c}_{21l}$	$\tilde{c}_{221}$	$\tilde{c}_{222}$	$\cdots$	$\tilde{c}_{22k}$	$\cdots$	$\tilde{c}_{22l}$	$\cdots$	$\tilde{c}_{2j1}$	$\tilde{c}_{2j2}$	$\cdots$	$\tilde{c}_{2jk}$	$\cdots$	$\tilde{c}_{2jl}$	$\cdots$	$\tilde{c}_{2n1}$	$\tilde{c}_{2n2}$	$\cdots$	$\tilde{c}_{2nk}$	$\cdots$	$\tilde{c}_{2nl}$	$\tilde{a}_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	$\tilde{c}_{i11}$	$\tilde{c}_{i12}$	$\cdots$	$\tilde{c}_{i1k}$	$\cdots$	$\tilde{c}_{i1l}$	$\tilde{c}_{i21}$	$\tilde{c}_{i22}$	$\cdots$	$\tilde{c}_{i2k}$	$\cdots$	$\tilde{c}_{i2l}$	$\cdots$	$\tilde{c}_{ij1}$	$\tilde{c}_{ij2}$	$\cdots$	$\tilde{c}_{ijk}$	$\cdots$	$\tilde{c}_{ijl}$	$\cdots$	$\tilde{c}_{in1}$	$\tilde{c}_{in2}$	$\cdots$	$\tilde{c}_{ink}$	$\cdots$	$\tilde{c}_{inl}$	$\tilde{a}_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$\tilde{c}_{m11}$	$\tilde{c}_{m12}$	$\cdots$	$\tilde{c}_{m1k}$	$\cdots$	$\tilde{c}_{m1l}$	$\tilde{c}_{m21}$	$\tilde{c}_{m22}$	$\cdots$	$\tilde{c}_{m2k}$	$\cdots$	$\tilde{c}_{m2l}$	$\cdots$	$\tilde{c}_{mj1}$	$\tilde{c}_{mj2}$	$\cdots$	$\tilde{c}_{mjk}$	$\cdots$	$\tilde{c}_{mjl}$	$\cdots$	$\tilde{c}_{mn1}$	$\tilde{c}_{mn2}$	$\cdots$	$\tilde{c}_{mnk}$	$\cdots$	$\tilde{c}_{mnl}$	$\tilde{a}_m$
Demand ($\tilde{b}_j$)	$\tilde{b}_1$						$\tilde{b}_2$						$\cdots$	$\tilde{b}_j$						$\cdots$	$\tilde{b}_n$						

$$\tilde{c}_{ijk} = \begin{cases} (0,0,0,0)_{LR}; & \text{If } i^{th} \text{ source node is dummy source node} \\ (0,0,0,0)_{LR}; & \text{If } j^{th} \text{ destination node is dummy destination node} \\ (0,0,0,0)_{LR}; & \text{If } k^{th} \text{ conveyance is dummy conveyance} \\ (m'_{ijk}, n'_{ijk}, \alpha'_{ijk}, \beta'_{ijk})_{LR}; & \text{otherwise} \end{cases}$$

Table 5.2 Tabular representation of first crisp solid transportation problem

																											Capacity
Conveyance	E_1						E_1						$\cdots$	E_1						$\cdots$	E_1						$m''_1-\alpha''_1$
		E_2						E_2					$\cdots$		E_2					$\cdots$		E_2					$m''_2-\alpha''_2$
			$\cdots$						$\cdots$				$\cdots$			$\cdots$				$\cdots$			$\cdots$				$\vdots$
				E_k						E_k			$\cdots$				E_k			$\cdots$				E_k			$m''_k-\alpha''_k$
					$\cdots$						$\cdots$		$\cdots$					$\cdots$		$\cdots$					$\cdots$		$\vdots$
						E_l						E_l	$\cdots$						E_l	$\cdots$						E_l	$m''_l-\alpha''_l$
Destinations → Sources↓	D_1						D_2							D_j							D_n						Availability
S_1	η_{111}	η_{112}	$\cdots$	η_{11k}	$\cdots$	η_{11l}	η_{121}	η_{122}	$\cdots$	η_{12k}	$\cdots$	η_{12l}	$\cdots$	η_{1j1}	η_{1j2}	$\cdots$	η_{1jk}	$\cdots$	η_{1jl}	$\cdots$	η_{1n1}	η_{1n2}	$\cdots$	η_{1nk}	$\cdots$	η_{1nl}	$m_1-\alpha_1$
S_2	η_{211}	η_{212}	$\cdots$	η_{21k}	$\cdots$	η_{21l}	η_{221}	η_{222}	$\cdots$	η_{22k}	$\cdots$	η_{22l}	$\cdots$	η_{2j1}	η_{2j2}	$\cdots$	η_{2jk}	$\cdots$	η_{2jl}	$\cdots$	η_{2n1}	η_{2n2}	$\cdots$	η_{2nk}	$\cdots$	η_{2nl}	$m_2-\alpha_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	η_{i11}	η_{i12}	$\cdots$	η_{i1k}	$\cdots$	η_{i1l}	η_{i21}	η_{i22}	$\cdots$	η_{i2k}	$\cdots$	η_{i2l}	$\cdots$	η_{ij1}	η_{ij2}	$\cdots$	η_{ijk}	$\cdots$	η_{ijl}	$\cdots$	η_{in1}	η_{in2}	$\cdots$	η_{ink}	$\cdots$	η_{inl}	$m_i-\alpha_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	η_{m11}	η_{m12}	$\cdots$	η_{m1k}	$\cdots$	η_{m1l}	η_{m21}	η_{m22}	$\cdots$	η_{m2k}	$\cdots$	η_{m2l}	$\cdots$	η_{mj1}	η_{mj2}	$\cdots$	η_{mjk}	$\cdots$	η_{mjl}	$\cdots$	η_{mn1}	η_{mn2}	$\cdots$	η_{mnk}	$\cdots$	η_{mnl}	$m_m-\alpha_m$
Demand	$m'_1-\alpha'_1$						$m'_2-\alpha'_2$						$\cdots$	$m'_j-\alpha'_j$						$\cdots$	$m'_n-\alpha'_n$						

Table 5.3 Tabular representation of second crisp solid transportation problem

																											Capacity
Conveyance	E_1						E_1						$\cdots$	E_1						$\cdots$	E_1						α''_1
		E_2						E_2					$\cdots$		E_2					$\cdots$		E_2					α''_2
			$\cdots$						$\cdots$				$\cdots$			$\cdots$				$\cdots$			$\cdots$				$\vdots$
				E_k						E_k			$\cdots$				E_k			$\cdots$				E_k			α''_k
					$\cdots$						$\cdots$		$\cdots$					$\cdots$		$\cdots$					$\cdots$		$\vdots$
						E_l						E_l	$\cdots$						E_l	$\cdots$						E_l	α''_l
Destinations → Sources↓	D_1						D_2							D_j							D_n						Availability
S_1	ρ_{111}	ρ_{112}	$\cdots$	ρ_{11k}	$\cdots$	ρ_{11l}	ρ_{121}	ρ_{122}	$\cdots$	ρ_{12k}	$\cdots$	ρ_{12l}	$\cdots$	ρ_{1j1}	ρ_{1j2}	$\cdots$	ρ_{1jk}	$\cdots$	ρ_{1jl}	$\cdots$	ρ_{1n1}	ρ_{1n2}	$\cdots$	ρ_{1nk}	$\cdots$	ρ_{1nl}	α_1
S_2	ρ_{211}	ρ_{212}	$\cdots$	ρ_{21k}	$\cdots$	ρ_{21l}	ρ_{221}	ρ_{222}	$\cdots$	ρ_{22k}	$\cdots$	ρ_{22l}	$\cdots$	ρ_{2j1}	ρ_{2j2}	$\cdots$	ρ_{2jk}	$\cdots$	ρ_{2jl}	$\cdots$	ρ_{2n1}	ρ_{2n2}	$\cdots$	ρ_{2nk}	$\cdots$	ρ_{2nl}	α_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	ρ_{i11}	ρ_{i12}	$\cdots$	ρ_{i1k}	$\cdots$	ρ_{i1l}	ρ_{i21}	ρ_{i22}	$\cdots$	ρ_{i2k}	$\cdots$	ρ_{i2l}	$\cdots$	ρ_{ij1}	ρ_{ij2}	$\cdots$	ρ_{ijk}	$\cdots$	ρ_{ijl}	$\cdots$	ρ_{in1}	ρ_{in2}	$\cdots$	ρ_{ink}	$\cdots$	ρ_{inl}	α_i
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	ρ_{m11}	ρ_{m12}	$\cdots$	ρ_{m1k}	$\cdots$	ρ_{m1l}	ρ_{m21}	ρ_{m22}	$\cdots$	ρ_{m2k}	$\cdots$	ρ_{m2l}	$\cdots$	ρ_{mj1}	ρ_{mj2}	$\cdots$	ρ_{mjk}	$\cdots$	ρ_{mjl}	$\cdots$	ρ_{mn1}	ρ_{mn2}	$\cdots$	ρ_{mnk}	$\cdots$	ρ_{mnl}	α_m
Demand	α'_1						α'_2						$\cdots$	α'_j						$\cdots$	α'_n						

Table 5.4 Tabular representation of third crisp solid transportation problem

																											Capacity
Conveyance	E_1						E_1						$\cdots$	E_1						$\cdots$	E_1						$n''_1 - m''_1$
		E_2						E_2					$\cdots$		E_2					$\cdots$		E_2					$n''_2 - m''_2$
			$\cdots$						$\cdots$				$\cdots$			$\cdots$				$\cdots$			$\cdots$				$\vdots$
				E_k						E_k			$\cdots$				E_k			$\cdots$				E_k			$n''_k - m''_k$
					$\cdots$						$\cdots$		$\cdots$					$\cdots$		$\cdots$					$\cdots$		$\vdots$
						E_l						E_l	$\cdots$						E_l	$\cdots$						E_l	$n''_l - m''_l$
Destinations → Sources↓	D_1						D_2							D_j							D_n						Availability
S_1	δ_{111}	δ_{112}	$\cdots$	δ_{11k}	$\cdots$	δ_{11l}	δ_{121}	δ_{122}	$\cdots$	δ_{12k}	$\cdots$	δ_{12l}		δ_{1j1}	δ_{1j2}	$\cdots$	δ_{1jk}	$\cdots$	δ_{1jl}		δ_{1n1}	δ_{1n2}	$\cdots$	δ_{1nk}	$\cdots$	δ_{1nl}	$n_1 - m_1$
S_2	δ_{211}	δ_{212}	$\cdots$	δ_{21k}	$\cdots$	δ_{21l}	δ_{221}	δ_{222}	$\cdots$	δ_{22k}	$\cdots$	δ_{22l}		δ_{2j1}	δ_{2j2}	$\cdots$	δ_{2jk}	$\cdots$	δ_{2jl}		δ_{2n1}	δ_{2n2}	$\cdots$	δ_{2nk}	$\cdots$	δ_{2nl}	$n_2 - m_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	δ_{i11}	δ_{i12}	$\cdots$	δ_{i1k}	$\cdots$	δ_{i1l}	δ_{i21}	δ_{i22}	$\cdots$	δ_{i2k}	$\cdots$	δ_{i2l}		δ_{ij1}	δ_{ij2}	$\cdots$	δ_{ijk}	$\cdots$	δ_{ijl}		δ_{in1}	δ_{in2}	$\cdots$	δ_{ink}	$\cdots$	δ_{inl}	$n_i - m_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	δ_{m11}	δ_{m12}	$\cdots$	δ_{m1k}	$\cdots$	δ_{m1l}	δ_{m21}	δ_{m22}	$\cdots$	δ_{m2k}	$\cdots$	δ_{m2l}		δ_{mj1}	δ_{mj2}	$\cdots$	δ_{mjk}	$\cdots$	δ_{mjl}		δ_{mn1}	δ_{mn2}	$\cdots$	δ_{mnk}	$\cdots$	δ_{mnl}	$n_m - m_m$
Demand	$n'_1 - m'_1$						$n'_2 - n'_2$						$\cdots$	$n'_j - n'_j$						$\cdots$	$n'_n - n'_n$						

Table 5.5 Tabular representation of fourth crisp solid transportation problem

																											Capacity
Conveyance	E_1						E_1						$\cdots$	E_1						$\cdots$	E_1						β''_1
		E_2						E_2					$\cdots$		E_2					$\cdots$		E_2					β''_2
			$\cdots$						$\cdots$				$\cdots$			$\cdots$				$\cdots$			$\cdots$				$\vdots$
				E_k						E_k			$\cdots$				E_k			$\cdots$				E_k			β''_k
					$\cdots$						$\cdots$		$\cdots$					$\cdots$		$\cdots$					$\cdots$		$\vdots$
						E_l						E_l	$\cdots$						E_l	$\cdots$						E_l	β''_l
Destinations → Sources↓	D_1						D_2							D_j							D_n						Availability
S_1	ξ_{111}	ξ_{112}	$\cdots$	ξ_{11k}	$\cdots$	ξ_{11l}	ξ_{121}	ξ_{122}	$\cdots$	ξ_{12k}	$\cdots$	ξ_{12l}		ξ_{1j1}	ξ_{1j2}	$\cdots$	ξ_{1jk}	$\cdots$	ξ_{1jl}		ξ_{1n1}	ξ_{1n2}	$\cdots$	ξ_{1nk}	$\cdots$	ξ_{1nl}	β_1
S_2	ξ_{211}	ξ_{212}	$\cdots$	ξ_{21k}	$\cdots$	ξ_{21l}	ξ_{221}	ξ_{222}	$\cdots$	ξ_{22k}	$\cdots$	ξ_{22l}		ξ_{2j1}	ξ_{2j2}	$\cdots$	ξ_{2jk}	$\cdots$	ξ_{2jl}		ξ_{2n1}	ξ_{2n2}	$\cdots$	ξ_{2nk}	$\cdots$	ξ_{2nl}	β_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	ξ_{i11}	ξ_{i12}	$\cdots$	ξ_{i1k}	$\cdots$	ξ_{i1l}	ξ_{i21}	ξ_{i22}	$\cdots$	ξ_{i2k}	$\cdots$	ξ_{i2l}		ξ_{ij1}	ξ_{ij2}	$\cdots$	ξ_{ijk}	$\cdots$	ξ_{ijl}		ξ_{in1}	ξ_{in2}	$\cdots$	ξ_{ink}	$\cdots$	ξ_{inl}	β_i
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	ξ_{m11}	ξ_{m12}	$\cdots$	ξ_{m1k}	$\cdots$	ξ_{m1l}	ξ_{m21}	ξ_{m22}	$\cdots$	ξ_{m2k}	$\cdots$	ξ_{m2l}		ξ_{mj1}	ξ_{mj2}	$\cdots$	ξ_{mjk}	$\cdots$	ξ_{mjl}		ξ_{mn1}	ξ_{mn2}	$\cdots$	ξ_{mnk}	$\cdots$	ξ_{mnl}	β_m
Demand	β'_1						β'_2						$\cdots$	β'_j						$\cdots$	β'_n						

Step 4 Solve the crisp solid transportation problems [83], shown by Table 5.2, Table 5.3, Table 5.4 and Table 5.5, to find the optimal solution $\{m_{ijk}-\alpha_{ijk}\}$; $\{\alpha_{ijk}\}$; $\{n_{ijk}-m_{ijk}\}$ and $\{\beta_{ijk}\}$ respectively.

Step 5 Solve the equations, obtained in Step 4, to find the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}$ and β_{ijk}.

Step 6 Find the fuzzy optimal solution $\{\tilde{x}_{ijk}\}$ by putting the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}$, β_{ijk} in $\tilde{x}_{ijk} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$.

Step 7 Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ijk}$ in $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}(\tilde{c}_{ijk} \otimes \tilde{x}_{ijk})$.

Remark 5.4 Let $\tilde{A} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$ be an LR flat fuzzy number with $L(x) = R(x) = \text{maximum } \{0, 1-x\}$. Then,

$\eta_{ijk} = \frac{1}{2}((m_{ijk} + n_{ijk}) - \alpha_{ijk}\int_0^1 L^{-1}(\lambda)d\lambda + \beta_{ijk}\int_0^1 R^{-1}(\lambda)d\lambda)$

$\quad = \frac{1}{4}(2m_{ijk} + 2n_{ijk} + \beta_{ijk} - \alpha_{ijk})$,

$\rho_{ijk} = \frac{1}{2}((m_{ijk}+n_{ijk}) - m_{ijk}\int_0^1 L^{-1}(\lambda)d\lambda + \beta_{ijk}\int_0^1 R^{-1}(\lambda)d\lambda) = \frac{1}{4}(m_{ijk}+2n_{ijk}+\beta_{ijk})$,

$\delta_{ijk} = \frac{1}{2}(n_{ijk} + \beta_{ijk}\int_0^1 R^{-1}(\lambda)d\lambda) = \frac{1}{4}(2n_{ijk} + \beta_{ijk})$ and

$\xi_{ijk} = \frac{1}{2}((n_{ijk} + \beta_{ijk})\int_0^1 R^{-1}(\lambda)d\lambda) = \frac{1}{4}(n_{ijk} + \beta_{ijk})$.

5.5.3 Advantages of the proposed methods

In this section, the advantages of the methods, proposed in this chapter, over the existing method [132] and over the methods, proposed in Chapter 2 and Chapter 3, are discussed.

(1) Since, in the proposed methods the restrictions $b-a \geq 0, c-b \geq 0$ and $d-c \geq 0$ are used and due to these restrictions, the restriction $d \geq a$ (or $x^U \geq x^L$) will always be automatically satisfied. So, by using the proposed methods the shortcomings of the existing methods [132], pointed out in Section 5.3, are

resolved.

(2) The methods, proposed in Chapter 2 and Chapter 3, can be used to find the fuzzy optimal solution of fully fuzzy transportation problems but can not be used for solving fully fuzzy solid transportation problems. Since, fully fuzzy transportation problems are special type of fully fuzzy solid transportation problems so, the methods, proposed in this chapter, can also be used to find the fuzzy optimal solution of fully fuzzy transportation problems.

5.6 Illustrative example

In this section, to illustrate the proposed methods, the existing fully fuzzy solid transportation problem, presented in Example 5.1, is solved by the proposed methods.

5.6.1 Fuzzy optimal solution of the chosen problem using the proposed method based on fuzzy linear programming formulation

Using the proposed method, based on fuzzy linear programming formulation, the fuzzy optimal solution of the fully fuzzy solid transportation problem, chosen in Example 5.1, can be obtained as follows:

Step 1 Using values of $\tilde{a}_1, \tilde{a}_2, \tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{e}_1$ and $\tilde{e}_2$, total fuzzy availability $\sum\limits_{i=1}^{2} \tilde{a}_i = (150, 170, 20, 40)_{LR}$, total fuzzy demand $\sum\limits_{j=1}^{3} \tilde{b}_j = (120, 150, 40, 30)_{LR}$ and total fuzzy capacity $\sum\limits_{k=1}^{2} \tilde{e}_k = (150, 150, 20, 40)_{LR}$. Since $\sum\limits_{i=1}^{2} \tilde{a}_i \neq \sum\limits_{j=1}^{3} \tilde{b}_j \neq \sum\limits_{k=1}^{2} \tilde{e}_k$, so it is an unbalanced fully fuzzy solid transportation problem.

Step 2 Comparing $\sum\limits_{i=1}^{2} \tilde{a}_i = (150, 170, 20, 40)_{LR}$ by $\sum\limits_{i=1}^{p} \tilde{a}_i = (m, n, \alpha, \beta)_{LR}$ and $\sum\limits_{j=1}^{3} \tilde{b}_j = (120, 150, 40, 30)_{LR}$ by $\sum\limits_{j=1}^{q} \tilde{b}_j = (m', n', \alpha', \beta')_{LR}$ the values of $m, n, \alpha, \beta, m', n', \alpha'$

and β' are 150,170,20,40,120,150,40 and 30 respectively.

Since, $\sum_{i=1}^{2} \tilde{a}_i \neq \sum_{j=1}^{3} \tilde{b}_j$ and neither the condition $m - \alpha \leq m' - \alpha'$, $\alpha \leq \alpha'$, $n - m \leq n' - m'$, $\beta \leq \beta'$ nor the condition $m - \alpha \geq m' - \alpha'$, $\alpha \geq \alpha'$, $n - m \geq n' - m'$, $\beta \geq \beta'$ is satisfying. So, as described in Step 2 (Case (2c)) of the proposed method, there is need to introduce a dummy source S_3 with fuzzy availability $\tilde{a}_3 = (20, 30, 20, 0)_{LR}$ and a dummy destination D_4 with fuzzy demand $\tilde{b}_4 = (50, 50, 0, 10)_{LR}$ so that $\sum_{i=1}^{3} \tilde{a}_i = \sum_{j=1}^{4} \tilde{b}_j$.

Step 3 Using Step 2, $\sum_{i=1}^{3} \tilde{a}_i = \sum_{j=1}^{4} \tilde{b}_j = (170, 200, 40, 40)_{LR}$. Since, $\sum_{i=1}^{3} \tilde{a}_i = \sum_{j=1}^{4} \tilde{b}_j \neq \sum_{k=1}^{2} \tilde{e}_k$ so Go to Step 3 of the proposed method.

Comparing $\sum_{i=1}^{3} \tilde{a}_i = \sum_{j=1}^{4} \tilde{b}_j = (170, 200, 40, 40)_{LR}$ by $\sum_{i=1}^{3} \tilde{a}_i = \sum_{j=1}^{4} \tilde{b}_j = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\sum_{k=1}^{2} \tilde{e}_k = (150, 150, 20, 40)_{LR}$ by $\sum_{k=1}^{r} \tilde{e}_k = (m'', n'', \alpha'', \beta'')_{LR}$ the values of $m_1, n_1, \alpha_1, \beta_1, m'', n'', \alpha''$ and β'' are 170, 200, 40, 40, 150, 150, 20 and 40 respectively.

Since, the condition $m_1 \geq m''$, $m_1 - \alpha_1 \geq m'' - \alpha''$, $\alpha_1 \geq \alpha''$, $n_1 - m_1 \geq n'' - m''$ is satisfying so as described in Step 3 (Case (2b)) of the proposed method there is need to introduce a dummy conveyance E_3 with fuzzy capacity $\tilde{e}_3 = (20, 50, 20, 0)_{LR}$ so that $\sum_{i=1}^{3} \tilde{a}_i = \sum_{j=1}^{4} \tilde{b}_j = \sum_{k=1}^{3} \tilde{e}_k$.

Step 4 Since, a dummy source S_3 with fuzzy availability $\tilde{a}_3$, a dummy destination D_4 with fuzzy demand $\tilde{b}_4$ and a dummy conveyance E_3 with fuzzy capacity $\tilde{e}_3$ are introduced. So, as described in Step 4 of the proposed method, by assuming $\tilde{c}_{3jk} = \tilde{c}_{i4k} = \tilde{c}_{ij3} = (0, 0, 0, 0)_{LR}$ $\forall$ $i = 1, 2, 3;$ $j = 1, 2, 3, 4;$ $k = 1, 2, 3$, the fuzzy linear programming formulation of balanced fully fuzzy solid transportation problem, obtained from Step 3, can be written as:

Minimize($(30, 30, 10, 10)_{LR} \otimes \tilde{x}_{111} \oplus (70, 70, 0, 0)_{LR} \otimes \tilde{x}_{112} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{113} \oplus$

$(60,60,0,0)_{LR}\otimes\tilde{x}_{121}\oplus(60,60,0,0)_{LR}\otimes\tilde{x}_{122}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{123}\oplus(50,50,0,0)_{LR}\otimes$
$\tilde{x}_{131}\oplus(30,30,0,0)_{LR}\otimes\tilde{x}_{132}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{133}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{141}\oplus(0,0,0,0)_{LR}\otimes$
$\tilde{x}_{142}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{143}\oplus(20,20,10,10)_{LR}\otimes\tilde{x}_{211}\oplus(40,40,0,0)_{LR}\otimes\tilde{x}_{212}\oplus(0,0,0,0)_{LR}\otimes$
$\tilde{x}_{213}\oplus(20,20,0,0)_{LR}\otimes\tilde{x}_{221}\oplus(50,50,0,0)_{LR}\otimes\tilde{x}_{222}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{223}\oplus(40,40,0,0)_{LR}\otimes$
$\tilde{x}_{231}\oplus(50,50,0,0)_{LR}\otimes\tilde{x}_{232}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{233}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{241}\oplus(0,0,0,0)_{LR}\otimes$
$\tilde{x}_{242}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{243}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{311}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{312}\oplus(0,0,0,0)_{LR}\otimes$
$\tilde{x}_{313}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{321}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{322}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{323}\oplus(0,0,0,0)_{LR}\otimes$
$\tilde{x}_{331}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{332}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{333}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{341}\oplus(0,0,0,0)_{LR}\otimes$
$\tilde{x}_{342}\oplus(0,0,0,0)_{LR}\otimes\tilde{x}_{343})$

subject to

$$\begin{array}{ll}
\sum_{j=1}^{4}\sum_{k=1}^{3}\tilde{x}_{1jk}=(80,100,10,20)_{LR}; & \sum_{j=1}^{4}\sum_{k=1}^{3}\tilde{x}_{2jk}=(70,70,10,20)_{LR}\\
\sum_{j=1}^{4}\sum_{k=1}^{3}\tilde{x}_{3jk}=(20,30,20,0)_{LR}; & \sum_{i=1}^{3}\sum_{k=1}^{3}\tilde{x}_{i1k}=(30,40,20,10)_{LR}\\
\sum_{i=1}^{3}\sum_{k=1}^{3}\tilde{x}_{i2k}=(50,50,10,10)_{LR}; & \sum_{i=1}^{3}\sum_{k=1}^{3}\tilde{x}_{i3k}=(40,60,10,10)_{LR}\\
\sum_{i=1}^{3}\sum_{k=1}^{3}\tilde{x}_{i4k}=(50,50,0,10)_{LR}; & \sum_{i=1}^{3}\sum_{j=1}^{4}\tilde{x}_{ij1}=(80,80,10,20)_{LR}\\
\sum_{i=1}^{3}\sum_{j=1}^{4}\tilde{x}_{ij2}=(70,70,10,20)_{LR}; & \sum_{i=1}^{3}\sum_{j=1}^{4}\tilde{x}_{ij3}=(20,50,20,0)_{LR}
\end{array}$$

$\tilde{x}_{ijk}$ are non-negative trapezoidal fuzzy numbers $\forall\, i=1,2,3;\ j=1,2,3,4;\ k=1,2,3$

Step 5 Using Step 7 to Step 9 of the method, proposed in Section 5.5.1, the fuzzy linear programming problem, obtained in Step 4, can be converted into the following crisp linear programming problem:

Minimize $(\frac{1}{4}(50m_{111}+70n_{111}-20\alpha_{111}+40\beta_{111}+140m_{112}+140n_{112}-70\alpha_{112}+70\beta_{112}+$
$120m_{121}+120n_{121}-60\alpha_{121}+60\beta_{121}+120m_{122}+120n_{122}-60\alpha_{122}+60\beta_{122}+100m_{131}+$
$100n_{131}-50\alpha_{131}+50\beta_{131}+60m_{132}+60n_{132}-30\alpha_{132}+30\beta_{132}+30m_{211}+50n_{211}-$
$10\alpha_{211}+30\beta_{211}+80m_{212}+80n_{212}-40\alpha_{212}+40\beta_{212}+40m_{221}+40n_{221}-20\alpha_{221}+$
$20\beta_{221}+100m_{222}+100n_{222}-50\alpha_{222}+50\beta_{222}+80m_{231}+80n_{231}-40\alpha_{231}+40\beta_{231}+$

$100m_{232}+100n_{232}-50\alpha_{232}+50\beta_{232}))$

subject to

$$\sum_{j=1}^{4}\sum_{k=1}^{3}m_{1jk}=80; \quad \sum_{j=1}^{4}\sum_{k=1}^{3}n_{1jk}=100; \quad \sum_{j=1}^{4}\sum_{k=1}^{3}\alpha_{1jk}=10; \quad \sum_{j=1}^{4}\sum_{k=1}^{3}\beta_{1jk}=20$$
$$\sum_{j=1}^{4}\sum_{k=1}^{3}m_{2jk}=70; \quad \sum_{j=1}^{4}\sum_{k=1}^{3}n_{2jk}=70; \quad \sum_{j=1}^{4}\sum_{k=1}^{3}\alpha_{2jk}=10; \quad \sum_{j=1}^{4}\sum_{k=1}^{3}\beta_{2jk}=20$$
$$\sum_{j=1}^{4}\sum_{k=1}^{3}m_{3jk}=20; \quad \sum_{j=1}^{4}\sum_{k=1}^{3}n_{3jk}=30; \quad \sum_{j=1}^{4}\sum_{k=1}^{3}\alpha_{3jk}=20; \quad \sum_{j=1}^{4}\sum_{k=1}^{3}\beta_{3jk}=0$$
$$\sum_{i=1}^{3}\sum_{k=1}^{3}m_{i1k}=30; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}n_{i1k}=40; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}\alpha_{i1k}=20; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}\beta_{i1k}=10$$
$$\sum_{i=1}^{3}\sum_{k=1}^{3}m_{i2k}=50; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}n_{i2k}=50; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}\alpha_{i2k}=10; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}\beta_{i2k}=10$$
$$\sum_{i=1}^{3}\sum_{k=1}^{3}m_{i3k}=40; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}n_{i3k}=60; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}\alpha_{i3k}=10; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}\beta_{i3k}=10$$
$$\sum_{i=1}^{3}\sum_{k=1}^{3}m_{i4k}=50; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}n_{i4k}=50; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}\alpha_{i4k}=0; \quad \sum_{i=1}^{3}\sum_{k=1}^{3}\beta_{i4k}=10$$
$$\sum_{i=1}^{3}\sum_{j=1}^{4}m_{ij1}=80; \quad \sum_{i=1}^{3}\sum_{j=1}^{4}n_{ij1}=80; \quad \sum_{i=1}^{3}\sum_{j=1}^{4}\alpha_{ij1}=10; \quad \sum_{i=1}^{3}\sum_{j=1}^{4}\beta_{ij1}=20$$
$$\sum_{i=1}^{3}\sum_{j=1}^{4}m_{ij2}=70; \quad \sum_{i=1}^{3}\sum_{j=1}^{4}n_{ij2}=70; \quad \sum_{i=1}^{3}\sum_{j=1}^{4}\alpha_{ij2}=10; \quad \sum_{i=1}^{3}\sum_{j=1}^{4}\beta_{ij2}=20$$
$$\sum_{i=1}^{3}\sum_{j=1}^{4}m_{ij3}=20; \quad \sum_{i=1}^{3}\sum_{j=1}^{4}n_{ij3}=50; \quad \sum_{i=1}^{3}\sum_{j=1}^{4}\alpha_{ij3}=20; \quad \sum_{i=1}^{3}\sum_{j=1}^{4}\beta_{ij3}=0$$
$$m_{ijk}-\alpha_{ijk},\ n_{ijk}-m_{ijk},\ \alpha_{ijk},\ \beta_{ijk}\geq 0 \quad \forall\ i=1,2,3;\ j=1,2,3,4;\ k=1,2,3$$

Step 6 The optimal solution of the crisp linear programming problem, obtained in Step 5, is $m_{113}=10, n_{113}=10, \alpha_{113}=10, m_{132}=30, n_{132}=30, \beta_{132}=10, n_{133}=20, m_{141}=10, n_{141}=10, m_{142}=30, n_{142}=30, \beta_{142}=10, m_{211}=10, n_{211}=10, \beta_{211}=10, m_{221}=40, n_{221}=40, \beta_{221}=10, m_{241}=10, n_{241}=10, m_{213}=10, n_{213}=10, \alpha_{213}=10, m_{321}=10, n_{321}=10, \alpha_{321}=10, n_{313}=10, m_{332}=10, n_{332}=10, \alpha_{332}=10$ and remaining values of $m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk}$ are zero respectively.

Step 7 Putting the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}$ and β_{ijk} in $\tilde{x}_{ijk}=(m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$, the fuzzy optimal solution is $\tilde{x}_{113}=(10,10,10,0)_{LR}$, $\tilde{x}_{132}=(30,30,0,10)_{LR}$, $\tilde{x}_{133}=(0,20,0,0)_{LR}$, $\tilde{x}_{141}=(10,10,0,0)_{LR}$, $\tilde{x}_{142}=(30,30,0,10)_{LR}$, $\tilde{x}_{211}=(10,10,0,10)_{LR}$,

$\tilde{x}_{221} = (40, 40, 0, 10)_{LR}$, $\tilde{x}_{241} = (10, 10, 0, 0)_{LR}$, $\tilde{x}_{213} = (10, 10, 10, 0)_{LR}$, $\tilde{x}_{321} = (10, 10, 10, 0)_{LR}$, $\tilde{x}_{313} = (0, 10, 0, 0)_{LR}$, $\tilde{x}_{332} = (10, 10, 10, 0)_{LR}$ and remaining values of $\tilde{x}_{ijk}$ are zero respectively.

Step 8 Putting the values of $\tilde{x}_{111}, \tilde{x}_{112}, \tilde{x}_{113}, \tilde{x}_{121}, \tilde{x}_{122}, \tilde{x}_{123}, \tilde{x}_{131}, \tilde{x}_{132}, \tilde{x}_{133}, \tilde{x}_{141}, \tilde{x}_{142}, \tilde{x}_{143}, \tilde{x}_{211}, \tilde{x}_{212}, \tilde{x}_{213}, \tilde{x}_{221}, \tilde{x}_{222}, \tilde{x}_{223}, \tilde{x}_{231}, \tilde{x}_{232}, \tilde{x}_{233}, \tilde{x}_{241}, \tilde{x}_{242}, \tilde{x}_{243}, \tilde{x}_{311}, \tilde{x}_{312}, \tilde{x}_{313}, \tilde{x}_{321}, \tilde{x}_{322}, \tilde{x}_{323}, \tilde{x}_{331}, \tilde{x}_{332}, \tilde{x}_{333}, \tilde{x}_{341}, \tilde{x}_{342}, \tilde{x}_{343}$ in

$((30, 30, 10, 10)_{LR} \otimes \tilde{x}_{111} \oplus (70, 70, 0, 0)_{LR} \otimes \tilde{x}_{112} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{113} \oplus (60, 60, 0, 0)_{LR} \otimes \tilde{x}_{121} \oplus (60, 60, 0, 0)_{LR} \otimes \tilde{x}_{122} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{123} \oplus (50, 50, 0, 0)_{LR} \otimes \tilde{x}_{131} \oplus (30, 30, 0, 0)_{LR} \otimes \tilde{x}_{132} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{133} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{141} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{142} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{143} \oplus (20, 20, 10, 10)_{LR} \otimes \tilde{x}_{211} \oplus (40, 40, 0, 0)_{LR} \otimes \tilde{x}_{212} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{213} \oplus (20, 20, 0, 0)_{LR} \otimes \tilde{x}_{221} \oplus (50, 50, 0, 0)_{LR} \otimes \tilde{x}_{222} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{223} \oplus (40, 40, 0, 0)_{LR} \otimes \tilde{x}_{231} \oplus (50, 50, 0, 0)_{LR} \otimes \tilde{x}_{232} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{233} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{241} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{242} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{243} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{311} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{312} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{313} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{321} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{322} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{323} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{331} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{332} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{333} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{341} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{342} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{343})$

the minimum total fuzzy transportation cost is $(1900, 1900, 100, 900)_{LR}$.

5.6.2 Fuzzy optimal solution of the chosen problem using the proposed method based on tabular representation

Using the proposed method, based on tabular representation, the fuzzy optimal solution of the fully fuzzy solid transportation problem, chosen in Example 5.1, can be obtained as follows:

Step 1 The balanced fully fuzzy solid transportation problem, obtained from Step 1 to Step 3 of Section 5.6.1, can be represented by Table 5.6.

Table 5.6 Tabular representation of balanced fully fuzzy solid transportation problem

													Capacity
	E_1			E_1			E_1			E_1			$(80, 80, 10, 20)_{LR}$
		E_2			E_2			E_2			E_2		$(70, 70, 10, 20)_{LR}$
			E_3			E_3			E_3			E_3	$(20, 50, 20, 0)_{LR}$
	D_1			D_2			D_3			D_4			
S_1	$(30, 30, 10, 10)_{LR}$	$(70, 70, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(60, 60, 0, 0)_{LR}$	$(60, 60, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(50, 50, 0, 0)_{LR}$	$(30, 30, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(80, 100, 10, 20)_{LR}$
S_2	$(20, 20, 10, 10)_{LR}$	$(40, 40, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(20, 20, 0, 0)_{LR}$	$(50, 50, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(40, 40, 0, 0)_{LR}$	$(50, 50, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(70, 70, 10, 20)_{LR}$
S_3	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(0, 0, 0, 0)_{LR}$	$(20, 30, 20, 0)_{LR}$
	$(30, 40, 20, 10)_{LR}$			$(50, 50, 10, 10)_{LR}$			$(40, 60, 10, 10)_{LR}$			$(50, 50, 0, 10)_{LR}$			

Table 5.7 Tabular representation of first crisp solid transportation problem

													Capacity$(m''_{ijk} - \alpha''_{ijk})$
	E_1			E_1			E_1			E_1			70
		E_2			E_2			E_2			E_2		60
			E_3			E_3			E_3			E_3	0
	D_1			D_2			D_3			D_4			$(m_i - \alpha_i)$
S_1	30	70	0	60	60	0	50	30	0	0	0	0	70
S_2	20	40	0	20	50	0	40	50	0	0	0	0	60
S_3	0	0	0	0	0	0	0	0	0	0	0	0	0
$(m'_j - \alpha'_j)$	10			40			30			50			

Table 5.8 Tabular representation of second crisp solid transportation problem

													Capacity(α''_{ijk})
	E_1			E_1			E_1			E_1			10
		E_2			E_2			E_2			E_2		10
			E_3			E_3			E_3			E_3	20
	D_1			D_2			D_3			D_4			(α_i)
S_1	25	52.5	0	45	45	0	37.5	22.5	0	0	0	0	10
S_2	17.5	30	0	15	37.5	0	30	37.5	0	0	0	0	10
S_3	0	0	0	0	0	0	0	0	0	0	0	0	20
(α'_j)	20			10			10			0			

Table 5.9 Tabular representation of third crisp solid transportation problem

													Capacity $(n''_{ijk} - m''_{ijk})$
	E_1			E_1			E_1			E_1			0
		E_2			E_2			E_2			E_2		0
			E_3			E_3			E_3			E_3	30
	D_1			D_2			D_3			D_4			$(n_i - m_i)$
S_1	17.5	35	0	30	30	0	25	15	0	0	0	0	20
S_2	12.5	20	0	10	25	0	20	25	0	0	0	0	0
S_3	0	0	0	0	0	0	0	0	0	0	0	0	10
$(n'_j - m'_j)$	10			0			20			0			

Table 5.10 Tabular representation of fourth crisp solid transportation problem

													Capacity (β''_{ijk})
	E_1			E_1			E_1			E_1			20
		E_2			E_2			E_2			E_2		20
			E_3			E_3			E_3			E_3	0
	D_1			D_2			D_3			D_4			(β_i)
S_1	10	17.5	0	15	15	0	12.5	7.5	0	0	0	0	20
S_2	7.5	10	0	5	12.5	0	10	12.5	0	0	0	0	20
S_3	0	0	0	0	0	0	0	0	0	0	0	0	0
(β'_{ijk})	10			10			10			10			

Step 2 Using Step 3 of the method, proposed in Section 5.5.2, Table 5.6 can be split into four crisp solid transportation tables i.e., Table 5.7, Table 5.8, Table 5.9 and Table 5.10.

Step 3 The optimal solution of crisp solid transportation problems, shown by Table 5.7, Table 5.8, Table 5.9 and Table 5.10 are $m_{113} - \alpha_{113} = 0, m_{132} - \alpha_{132} = 30, m_{133} - \alpha_{133} = 0, m_{141} - \alpha_{141} = 10, m_{142} - \alpha_{142} = 30, m_{211} - \alpha_{211} = 10, m_{221} - \alpha_{221} = 40, m_{242} - \alpha_{242} = 10, m_{213} - \alpha_{213} = 0, m_{321} - \alpha_{321} = 0, m_{313} - \alpha_{313} = 0, m_{332} - \alpha_{332} = 0; \alpha_{113} = 10, \alpha_{132} = 0, \alpha_{133} = 0, \alpha_{141} = 0, \alpha_{142} = 0, \alpha_{211} = 0, \alpha_{221} = 0, \alpha_{241} = 0, \alpha_{213} = 10, \alpha_{321} = 10, \alpha_{313} = 0, \alpha_{332} = 10; n_{113} - m_{113} = 0, n_{132} - m_{132} = 0, n_{133} - m_{133} = 20, n_{141} - m_{141} = 0, n_{142} - m_{142} = 0, n_{211} - m_{211} = 0, n_{221} - m_{221} = 0, n_{242} - m_{242} = 0, n_{213} - m_{213} = 0, n_{321} - m_{321} = 0, n_{313} - m_{313} = 10, n_{332} - m_{332} = 0$ and $\beta_{132} = 10, \beta_{142} = 10, \beta_{211} = 10, \beta_{221} = 10$ respectively.

Step 4 On solving the equations, obtained from Step 3, the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}$ and β_{ijk} are $m_{113} = 10, n_{113} = 10, \alpha_{113} = 10, \beta_{113} = 0,$ $m_{132} = 30, n_{132} = 30, \alpha_{132} = 0, \beta_{132} = 10,$ $m_{133} = 0, n_{133} = 20, \alpha_{133} = 0, \beta_{133} = 0,$ $m_{141} = 10, n_{141} = 10, \alpha_{141} = 0, \beta_{141} = 0,$ $m_{142} = 30, n_{142} = 30, \alpha_{142} = 0, \beta_{142} = 10,$ $m_{211} = 10, n_{211} = 10, \alpha_{211} = 0, \beta_{211} = 10,$ $m_{221} = 40, n_{221} = 40, \alpha_{221} = 0, \beta_{221} = 10,$ $m_{241} = 10, n_{241} = 10, \alpha_{241} = 0, \beta_{241} = 0,$ $m_{213} = 10, n_{213} = 10, \alpha_{213} = 10, \beta_{213} = 0,$ $m_{321} = 10, n_{321} = 10, \alpha_{321} = 10, \beta_{321} = 0,$ $m_{313} = 0, n_{313} = 10, \alpha_{313} = 0, \beta_{313} = 0,$ $m_{332} = 10, n_{332} = 10, \alpha_{332} = 10, \beta_{332} = 0$ and remaining values of $m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk}$ are zero.

Step 5 Putting the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}$ and β_{ijk} in $\tilde{x}_{ijk} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$, the fuzzy optimal solution is $\tilde{x}_{113} = (10, 10, 10, 0)_{LR}$, $\tilde{x}_{132} = (30, 30, 0, 10)_{LR}$, $\tilde{x}_{133} = (0, 20, 0, 0)_{LR}$, $\tilde{x}_{141} = (10, 10, 0, 0)_{LR}$, $\tilde{x}_{142} = (30, 30, 0, 10)_{LR}$, $\tilde{x}_{211} = (10, 10, 0, 10)_{LR}$, $\tilde{x}_{221} = (40, 40, 0, 10)_{LR}$, $\tilde{x}_{241} = (10, 10, 0, 0)_{LR}$, $\tilde{x}_{213} = (10, 10, 10, 0)_{LR}$, $\tilde{x}_{321} =$

$(10, 10, 10, 0)_{LR}$, $\tilde{x}_{313} = (0, 10, 0, 0)_{LR}$, $\tilde{x}_{332} = (10, 10, 10, 0)_{LR}$ and remaining values of $\tilde{x}_{ijk}$ are zero.

Step 6 Putting the values of $\tilde{x}_{111}, \tilde{x}_{112}, \tilde{x}_{113}, \tilde{x}_{121}, \tilde{x}_{122}, \tilde{x}_{123}, \tilde{x}_{131}, \tilde{x}_{132}, \tilde{x}_{133}, \tilde{x}_{141}, \tilde{x}_{142}, \tilde{x}_{143}, \tilde{x}_{211}, \tilde{x}_{212}, \tilde{x}_{213}, \tilde{x}_{221}, \tilde{x}_{222}, \tilde{x}_{223}, \tilde{x}_{231}, \tilde{x}_{232}, \tilde{x}_{233}, \tilde{x}_{241}, \tilde{x}_{242}, \tilde{x}_{243}, \tilde{x}_{311}, \tilde{x}_{312}, \tilde{x}_{313}, \tilde{x}_{321}, \tilde{x}_{322}, \tilde{x}_{323}, \tilde{x}_{331}, \tilde{x}_{332}, \tilde{x}_{333}, \tilde{x}_{341}, \tilde{x}_{342}, \tilde{x}_{343}$ in

$((30, 30, 10, 10)_{LR} \otimes \tilde{x}_{111} \oplus (70, 70, 0, 0)_{LR} \otimes \tilde{x}_{112} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{113} \oplus (60, 60, 0, 0)_{LR} \otimes \tilde{x}_{121} \oplus (60, 60, 0, 0)_{LR} \otimes \tilde{x}_{122} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{123} \oplus (50, 50, 0, 0)_{LR} \otimes \tilde{x}_{131} \oplus (30, 30, 0, 0)_{LR} \otimes \tilde{x}_{132} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{133} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{141} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{142} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{143} \oplus (20, 20, 10, 10)_{LR} \otimes \tilde{x}_{211} \oplus (40, 40, 0, 0)_{LR} \otimes \tilde{x}_{212} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{213} \oplus (20, 20, 0, 0)_{LR} \otimes \tilde{x}_{221} \oplus (50, 50, 0, 0)_{LR} \otimes \tilde{x}_{222} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{223} \oplus (40, 40, 0, 0)_{LR} \otimes \tilde{x}_{231} \oplus (50, 50, 0, 0)_{LR} \otimes \tilde{x}_{232} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{233} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{241} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{242} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{243} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{311} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{312} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{313} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{321} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{322} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{323} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{331} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{332} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{333} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{341} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{342} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{343})$

the minimum total fuzzy transportation cost is $(1900, 1900, 100, 900)_{LR}$.

5.6.3 Physical interpretation of the results

In this section, the minimum total fuzzy transportation cost, obtained by using the proposed methods, is physically interpreted. Similarly, the obtained fuzzy optimal solution can also be physically interpreted.

Using the proposed methods the minimum total fuzzy transportation cost is $(1900, 1900, 100, 900)_{LR}$, which can be physically interpreted as follows:

(1) The least amount of minimum total transportation cost is 1800.

(2) The most possible amount of minimum total transportation cost is 1900.

(3) The greatest amount of minimum total transportation cost is 2800. i.e., the

minimum total transportation cost will be always greater than 1800 and less than 2800 and maximum chances are that minimum total transportation cost will be 1900.

The variation in minimum total transportation cost with respect to chances are shown in Figure 5.1.

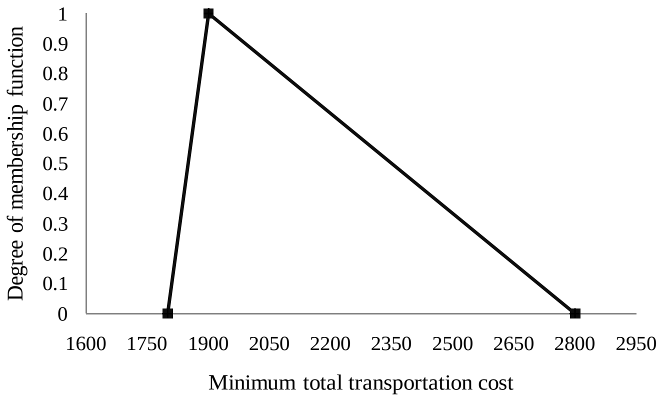

Figure 5.1. Membership function of LR flat fuzzy number representing the minimum total fuzzy transportation cost

5.7 Comparative study

The results obtained by the methods, proposed in this chapter and by the methods proposed in Chapter 2 and Chapter 3, are compared in Table 5.11.

Table 5.11 Results obtained by using proposed methods

Example	Minimum total fuzzy transportation cost		
	Methods proposed in Chapter 2	Methods proposed in Chapter 3	Methods proposed in this chapter
2.2	$(2100, 2900, 3500, 4200)$	$(2100, 2900, 3500, 4200)$	$(2100, 2900, 3500, 4200)$
3.1	Not applicable	$(5800, 8400, 2800, 2900)_{LR}$	$(5800, 8400, 2800, 2900)_{LR}$
5.1	Not applicable	Not applicable	$(1900, 1900, 100, 900)_{LR}$

The results, shown in Table 5.11, can be explained as follows:

(1) The methods, proposed in Chapter 2, can be used only for solving such fully fuzzy transportation in which either all the parameters are represented by triangular fuzzy numbers or by trapezoidal fuzzy numbers. Similarly, the methods, proposed in Chapter 3, can be used for solving such fully fuzzy

transportation problems in which all the parameters are represented by LR flat fuzzy numbers. Since, in the fully fuzzy transportation problem, chosen in Example 3.1, all the parameters are represented by LR flat fuzzy numbers so the problem, chosen in Example 3.1, can not be solved by the method proposed in Chapter 2 but the same problem can be solved by the method proposed in Chapter 3.

Since, fully fuzzy solid transportation problems are the generalization of fully fuzzy transportation problems so, fully fuzzy solid transportation problem, chosen in Example 5.1, can not be solved by the methods proposed in Chapter 2 and Chapter 3.

(2) Since, fully fuzzy transportation problems are special type of fully fuzzy solid transportation problems so, the methods, proposed in this chapter, can also be used for solving fully fuzzy transportation problems i.e., the method, proposed in this chapter, can be used to find the fuzzy optimal solution of all the chosen problems.

5.8 Case study

Yang and Liu [217] proposed a method to find the crisp optimal solution of fully fuzzy fixed charge solid transportation problems and used it to find the crisp optimal solution of a real life fully fuzzy fixed charge solid transportation problem described in Section 5.8.1.

However, in Chapter 1, it is pointed out that it is better to find the fuzzy optimal solution as compared to crisp optimal solution. So, the fuzzy optimal solution of the same real life problem, assuming that there is no fixed charge, are obtained with the help of proposed methods.

5.8.1 Description of problem

As we know, coal is a kind of crucial energy source in the development of economy and society. Accordingly, how to transport the coal from mines to the different areas economically is also an important issue in the coal transportation. For the convenience of description, the problem can be summarized as follows. Suppose that there are two coal mines to supply the coal for two cities. During the process of transportation, two kinds of conveyances are available to be selected, i.e., train and cargo ship. Now, the task for the decision-maker is to make the transportation plan for the next month. At the beginning of this task, the decision maker needs to obtain the basic data, such as supply capacity, demand, transportation capacity, transportation cost of unit product, and so on. In fact, since the transportation plan is made in advance, we generally cannot get these data exactly. For this condition, the usual way is to obtain the fuzzy data by means of experience evaluation or expert advice. In this example, the notations $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{e}_k$ are employed to denote the availability, demand and transportation capacity, respectively. The corresponding fuzzy data, with $L(x) = R(x) = \text{maximum } \{0, 1-x\}$ are listed as follows:

$\tilde{a}_1 = (28, 29, 3, 2)_{LR}$, $\tilde{a}_2 = (20, 23, 4, 2)_{LR}$, $\tilde{a}_3 = (34, 36, 2, 2)_{LR}$, $\tilde{a}_4 = (30, 32, 2, 2)_{LR}$,

$\tilde{b}_1 = (13, 14, 1, 4)_{LR}$, $\tilde{b}_2 = (23, 26, 3, 1)_{LR}$, $\tilde{b}_3 = (21, 23, 2, 1)_{LR}$, $\tilde{b}_4 = (27, 29, 2, 2)_{LR}$,

$\tilde{e}_1 = (45, 50, 6, 5)_{LR}$, $\tilde{e}_2 = (65, 70, 5, 5)_{LR}$

For the same reason, the transportation cost of unit amount in advance can not be obtained accurately, and it can also be treated as a fuzzy variable by experience or expert advice. For this example, the transportation cost of unit amount is listed in Tables 5.12 and 5.13.

Table 5.12 The direct cost by Train

Cities→ Mines↓	1	2	3	4
1	$(5,8,2,2)_{LR}$	$(8,9,1,1)_{LR}$	$(17,19,2,1)_{LR}$	$(15,17,2,2)_{LR}$
2	$(7,8,2,1)_{LR}$	$(9,10,3,1)_{LR}$	$(4,6,1,1)_{LR}$	$(20,23,4,2)_{LR}$
3	$(8,9,3,1)_{LR}$	$(15,17,2,2)_{LR}$	$(6,8,3,1)_{LR}$	$(8,11,1,2)_{LR}$
4	$(19,21,1,3)_{LR}$	$(12,13,3,2)_{LR}$	$(10,11,3,3)_{LR}$	$(10,13,1,2)_{LR}$

Table 5.13 The direct cost by Cargo Ship

Cities→ Mines↓	1	2	3	4
1	$(9,12,2,1)_{LR}$	$(9,10,4,2)_{LR}$	$(12,13,3,2)_{LR}$	$(25,26,5,2)_{LR}$
2	$(12,14,2,1)_{LR}$	$(14,16,2,2)_{LR}$	$(17,18,6,2)_{LR}$	$(5,8,2,2)_{LR}$
3	$(10,12,2,2)_{LR}$	$(23,25,3,2)_{LR}$	$(25,27,2,2)_{LR}$	$(8,10,2,2)_{LR}$
4	$(8,9,2,1)_{LR}$	$(28,30,2,2)_{LR}$	$(32,33,2,2)_{LR}$	$(32,38,2,2)_{LR}$

Now, the aim is to find how much quantity of the product should be transported from which coal mine to which city by which conveyance so that the the total fuzzy transportation cost is minimum.

5.8.2 Results

On solving the chosen real life problem by using the proposed methods, the obtained fuzzy optimal solution $\{\tilde{x}_{ijk}\}$, representing the fuzzy quantity of the coal, that should be transported from i^{th} coal mine to j^{th} city by k^{th} conveyance to minimize the total fuzzy transportation cost and the fuzzy optimal value, representing the minimum total fuzzy transportation cost, are $\tilde{x}_{111} = (7,7,0,0)_{LR}$, $\tilde{x}_{121} = (16,17,3,0)_{LR}$, $\tilde{x}_{231} = (2,3,2,0)_{LR}$, $\tilde{x}_{331} = (19,20,0,0)_{LR}$, $\tilde{x}_{122} = (5,5,0,0)_{LR}$, $\tilde{x}_{242} = (18,20,2,0)_{LR}$, $\tilde{x}_{342} = (9,9,0,0)_{LR}$, $\tilde{x}_{412} = (6,7,1,0)_{LR}$, $\tilde{x}_{352} = (5,6,1,0)_{LR}$, $\tilde{x}_{423} = (2,2,0,0)_{LR}$, $\tilde{x}_{452} = (22,23,1,0)_{LR}$, $\tilde{x}_{351} = (1,1,1,0)_{LR}$, $\tilde{x}_{521} = (0,2,0,0)_{LR}$, $\tilde{x}_{152} = (0,0,0,2)_{LR}$, $\tilde{x}_{213} = (0,0,0,1)_{LR}$, $\tilde{x}_{252} = (0,0,0,1)_{LR}$, $\tilde{x}_{343} = (0,0,0,2)_{LR}$, $\tilde{x}_{451} = (0,0,0,2)_{LR}$, $\tilde{x}_{511} = (0,0,0,3)_{LR}$, $\tilde{x}_{522} = (0,0,0,1)_{LR}$, $\tilde{x}_{532} = (0,0,0,1)_{LR}$

and $(540, 750, 214, 129)_{LR}$ respectively.

Remark 5.5 Since, the chosen real life problem is an unbalanced problem so, to find the solution of this problem a dummy source (5), a dummy destination (5) and dummy conveyance (3) is introduced. In the results, presented in Section 5.8.2, $\tilde{x}_{5jk}$, $\tilde{x}_{i5k}$ and $\tilde{x}_{ij3}$ represents the fuzzy quantity of the product that should be transported from dummy source (5) to j^{th} destination by means of k^{th} conveyance, i^{th} source to dummy destination (5) by means of k^{th} conveyance and i^{th} source to j^{th} destination by means of dummy conveyance (3) respectively.

5.8.3 Physical interpretation of the results

In this section, the minimum total fuzzy transportation cost, obtained by using the proposed method, is physically interpreted. Similarly, the obtained fuzzy optimal solution can also be physically interpreted.

Using the proposed method the minimum total fuzzy transportation cost is $(540, 750, 214, 129)_{LR}$, which can be physically interpreted as follows:

(1) The least amount of minimum total transportation cost is 326 units.

(2) The most possible amount of minimum total transportation cost lies between 540 units and 750 units.

(3) The greatest amount of minimum total transportation cost is 879 units. i.e., the minimum total transportation cost will be always greater than 326 units and less than 879 units and maximum chances are that the minimum total transportation cost will lie between 540 units and 750 units.

The variation in minimum total transportation cost with respect to chances are shown in Figure 5.2.

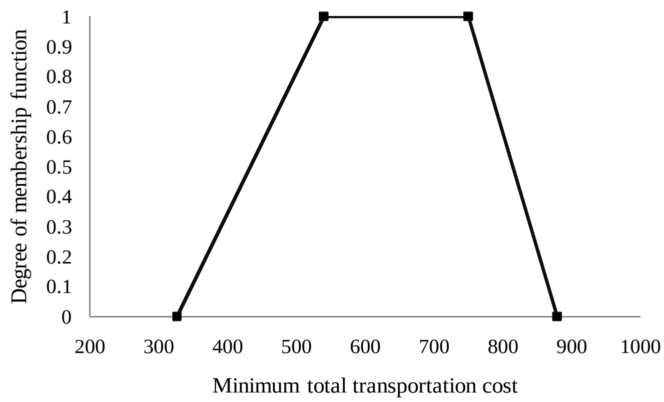

Figure 5.2. Membership function of *LR* flat fuzzy number representing the minimum total fuzzy transportation cost

5.9 Conclusions

On the basis of the comparison of the results, it can be concluded that on solving the fuzzy solid transportation problems by using the proposed methods all the shortcomings, occurring in the results, due to applying the existing method [132] are resolved and all the problems which can be solved by using the methods, proposed in the Chapter 2 and Chapter 3, can also be solved by the methods proposed in this chapter. Also, there can exist several problems which can not be solved by using the methods proposed in Chapter 2 and Chapter 3 but can be solved by the methods proposed in this chapter. Hence, it is better to use the methods proposed in this chapter as compared to the existing method [132] and the methods proposed in Chapter 2 and Chapter 3.

Chapter 6

New Methods For Solving Fully Fuzzy Solid Transshipment Problems With LR Flat Fuzzy Parameters

The fully fuzzy transshipment problems are obtained by introducing the intermediate nodes in fully fuzzy transportation problems and the fully fuzzy solid transportation problems are obtained by introducing the additional conveyances in fully fuzzy transportation problems. But, in real life problems both the intermediate nodes and additional conveyances are used simultaneously. So, in this chapter, by combining the concept of fully fuzzy solid transportation problems and fully fuzzy transshipment problems, new type of problems, named as fully fuzzy solid transshipment problems, its fuzzy linear programming formulation and two new methods for finding its fuzzy optimal solution, are proposed. The advantages of the proposed methods over the methods, proposed in previous chapters and over the existing method [70], are discussed. To illustrate the methods, proposed in this chapter, a fully fuzzy solid transshipment problem is solved.

Some part of this chapter is accepted for publication in *Applied and Computational Mathematics* and remaining part is communicated for publication in *Applied Soft Computing.*

6.1 Proposed fuzzy linear programming formulation of balanced fully fuzzy solid transshipment problems

In this section, fuzzy linear programming formulation of balanced fully fuzzy solid transshipment problems is proposed.

Let $\tilde{a}_i$ and $\tilde{a}'_i$ be the fuzzy availability of the product at i^{th} purely source node and at i^{th} source node, $\tilde{b}_j$ and $\tilde{b}'_j$ be the fuzzy demand of the product at j^{th} purely destination node and at the j^{th} destination node, $\tilde{e}_k$ be fuzzy capacity of the k^{th} conveyance (the maximum fuzzy quantity of the product which can be carried by the k^{th} conveyance), $\tilde{c}_{ijk}$ be the fuzzy cost for transporting one unit quantity of the product from i^{th} source to j^{th} destination by means of the k^{th} conveyance and $\tilde{x}_{ijk}$ be the fuzzy quantity of the product that should be transported from i^{th} node to j^{th} node by means of the k^{th} conveyance to minimize the total fuzzy transportation cost. Then, the fuzzy linear programming formulation of a balanced fully fuzzy solid transshipment problem can be written as:

$$\text{Minimize} \sum_{(i,j)\in A} \sum_{k\in S_C} (\tilde{c}_{ijk} \otimes \tilde{x}_{ijk})$$

subject to

$$\begin{aligned}
&\sum_{j:(i,j)\in A} \sum_{k\in S_C} \tilde{x}_{ijk} = \tilde{a}_i, && i \in N_{PS} \\
&\sum_{j:(i,j)\in A} \sum_{k\in S_C} \tilde{x}_{ijk} \ominus_H \sum_{j:(j,i)\in A} \sum_{k\in S_C} \tilde{x}_{jik} = \tilde{a}'_i, && i \in N_S \\
&\sum_{i:(i,j)\in A} \sum_{k\in S_C} \tilde{x}_{ijk} = \tilde{b}_j, && j \in N_{PD} \\
&\sum_{i:(i,j)\in A} \sum_{k\in S_C} \tilde{x}_{ijk} \ominus_H \sum_{i:(j,i)\in A} \sum_{k\in S_C} \tilde{x}_{jik} = \tilde{b}'_j, && j \in N_D \qquad (P_{6.1}) \\
&\sum_{j:(i,j)\in A} \sum_{k\in S_C} \tilde{x}_{ijk} = \sum_{j:(j,i)\in A} \sum_{k\in S_C} \tilde{x}_{jik}, && i \in N_T \\
&\sum_{i:(i,j)\in A} \tilde{x}_{ijk} = \tilde{e}_k, && k \in S_C
\end{aligned}$$

$\tilde{x}_{ijk}$ is a non-negative LR flat fuzzy number $\forall\ (i,j) \in A,\ k \in S_C$

where, A is set of arcs (i, j) joining node i and node j and S_C is the set of all available conveyances.

Remark 6.1 If $\sum\limits_{i \in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i \in N_S} \tilde{a}'_i = \sum\limits_{j \in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j \in N_D} \tilde{b}'_j = \sum\limits_{k \in S_C} \tilde{e}_k$ then the fully fuzzy solid transshipment problem is said to be balanced fully fuzzy solid transshipment problem otherwise it is said to be unbalanced fully fuzzy solid transshipment problem.

6.2 Limitations of the existing method and methods proposed in previous chapters

Since, fully fuzzy solid transshipment problems are the generalization of fully fuzzy transportation problems, fully fuzzy solid transportation problems and fully fuzzy transshipment problems. So, the methods, proposed for solving these problems, can not be used for solving fully fuzzy solid transshipment problems i.e., neither the existing method [70] nor the methods, proposed in previous chapters, can be used for solving fully fuzzy solid transshipment problems e.g., the fully fuzzy solid transshipment problem, chosen in Example 6.1, can neither be solved by using the existing method [70] nor by using any of the methods proposed in previous chapters.

Example 6.1 Consider a network with three nodes, shown in Figure 6.1, including one purely source node (2), one source node (1) and one purely destination node (3). The fuzzy cost $\tilde{c}_{ijk}$, fuzzy availability $\tilde{a}_i$, fuzzy demand $\tilde{b}_j$ and the fuzzy capacity $\tilde{e}_k$ are represented by the following LR flat fuzzy numbers:

Fuzzy costs: $\tilde{c}_{131}=(8, 10, 2, 2)_{LR}$, $\tilde{c}_{132}=(4, 8, 3, 2)_{LR}$, $\tilde{c}_{211}=(8, 10, 4, 4)_{LR}$,

$\tilde{c}_{212}=(6, 8, 4, 4)_{LR}$, $\tilde{c}_{231}=(9, 12, 6, 3)_{LR}$, $\tilde{c}_{232}=(3, 6, 2, 3)_{LR}$

Fuzzy availability: $\tilde{a}_1 = (60, 80, 20, 20)_{LR}$, $\tilde{a}_2 = (50, 70, 20, 20)_{LR}$

Fuzzy demand: $\tilde{b}_3 = (50, 80, 30, 50)_{LR}$

Fuzzy capacity: $\tilde{e}_1 = (60, 80, 20, 10)_{LR}$, $\tilde{e}_2 = (50, 70, 20, 40)_{LR}$

where, $L(x) = R(x) =$ maximum $\{0, 1 - x\}$. Find the fuzzy optimum shipping schedule.

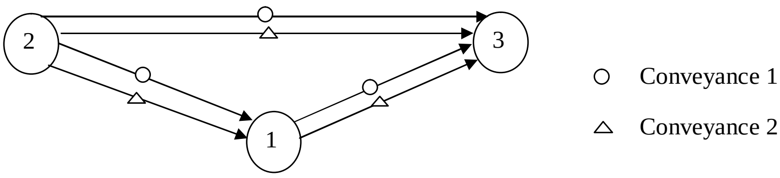

Figure 6.1. Network representing fully fuzzy solid transshipment problem

6.3 Proposed methods

In this section, to overcome the limitations of the existing method [70] and methods of previous chapters, discussed in Section 6.2, two new methods are proposed to find the fuzzy optimal solution of such fully fuzzy solid transshipment problems in which all the parameters are represented by LR flat fuzzy numbers. Also, the advantages of the proposed methods over the existing method [70] and over the methods, proposed in previous chapters, are discussed.

6.3.1 Proposed method based on fuzzy linear programming formulation

In this section, a new method, based on fuzzy linear programming formulation, is proposed to find the fuzzy optimal solution of such fully fuzzy solid transshipment problems in which all the parameters are represented by LR flat fuzzy numbers.

The steps of the proposed method are as follows:

Step 1 Find $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i$, $\sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$ and $\sum\limits_{k\in S_C} \tilde{e}_k$.

Let $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i = (m, n, \alpha, \beta)_{LR}$, $\sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j = (m', n', \alpha', \beta')_{LR}$ and $\sum\limits_{k\in S_C} \tilde{e}_k = (m'', n'', \alpha'', \beta'')_{LR}$. Use Definition 3.5 to examine that the problem is balanced or unbalanced.

Case (1) If the problem is balanced i.e., $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i = \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j = \sum\limits_{k\in S_C} \tilde{e}_k$, then Go to Step 4.

Case (2) If the problem is unbalanced i.e., $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i = \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j \neq \sum\limits_{k\in S_C} \tilde{e}_k$ or $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i \neq \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j = \sum\limits_{k\in S_C} \tilde{e}_k$ or $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i = \sum\limits_{k\in S_C} \tilde{e}_k \neq \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$ or $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i \neq \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j \neq \sum\limits_{k\in S_C} \tilde{e}_k$ then Go to Step 2.

Step 2 Check that $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i = \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$ or $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i \neq \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$.

Case (1) If $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i = \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$ then Go to Step 3.

Case (2) If $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i \neq \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$ then convert $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i \neq \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$ into $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i = \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$ as follows:

Case (2a) If $m-\alpha \leq m'-\alpha'$, $\alpha \leq \alpha'$, $n-m \leq n'-m'$ and $\beta \leq \beta'$ then introduce a dummy purely source node with fuzzy availability $(m'-m, n'-n, \alpha'-\alpha, \beta'-\beta)_{LR}$ so that $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i = \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$. Go to Step 3.

Case (2b) If $m-\alpha \geq m'-\alpha'$, $\alpha \geq \alpha'$, $n-m \geq n'-m'$ and $\beta \geq \beta'$ then introduce a dummy purely destination node with fuzzy demand $(m-m', n-n', \alpha-\alpha', \beta-\beta')_{LR}$ so that $\sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i = \sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$. Go to Step 3.

Case (2c) If neither Case (2a) nor Case (2b) is satisfied then introduce a dummy purely source node with fuzzy availability (maximum $\{0, (m'-\alpha')-(m-\alpha)\}$ + maximum $\{0, (\alpha'-\alpha)\}$, maximum $\{0, (m'-\alpha')-(m-\alpha)\}$ + maximum $\{0, (\alpha'-\alpha)\}$ +

maximum $\{0,(n'-m')-(n-m)\}$, maximum $\{0,(\alpha'-\alpha)\}$, maximum $\{0,(\beta'-\beta)\})_{LR}$ and a dummy purely destination node with fuzzy demand (maximum $\{0,(m-\alpha)-(m'-\alpha')\}$ + maximum $\{0,(\alpha-\alpha')\}$, maximum $\{0,(m-\alpha)-(m'-\alpha')\}$ + maximum $\{0,(\alpha-\alpha')\}$ + maximum $\{0,(n-m)-(n'-m')\}$, maximum $\{0,(\alpha-\alpha')\}$, maximum $\{0,(\beta-\beta')\})_{LR}$ so that $\sum_{i\in N_{PS}}\tilde{a}_i \oplus \sum_{i\in N_S}\tilde{a}'_i = \sum_{j\in N_{PD}}\tilde{b}_j \oplus \sum_{j\in N_D}\tilde{b}'_j$. Go to Step 3.

Step 3 Using Step 2, $\sum_{i\in N_{PS}}\tilde{a}_i \oplus \sum_{i\in N_S}\tilde{a}'_i = \sum_{j\in N_{PD}}\tilde{b}_j \oplus \sum_{j\in N_D}\tilde{b}'_j$.

Let $\sum_{i\in N_{PS}}\tilde{a}_i \oplus \sum_{i\in N_S}\tilde{a}'_i = \sum_{j\in N_{PD}}\tilde{b}_j \oplus \sum_{j\in N_D}\tilde{b}'_j = (m_1,n_1,\alpha_1,\beta_1)_{LR}$ and $\sum_{k\in S_C}\tilde{e}_k = (m'',n'',\alpha'',\beta'')_{LR}$

Now check $\sum_{i\in N_{PS}}\tilde{a}_i \oplus \sum_{i\in N_S}\tilde{a}'_i = \sum_{j\in N_{PD}}\tilde{b}_j \oplus \sum_{j\in N_D}\tilde{b}'_j = \sum_{k\in S_C}\tilde{e}_k$ or $\sum_{i\in N_{PS}}\tilde{a}_i \oplus \sum_{i\in N_S}\tilde{a}'_i = \sum_{j\in N_{PD}}\tilde{b}_j \oplus \sum_{j\in N_D}\tilde{b}'_j \neq \sum_{k\in S_C}\tilde{e}_k$

Case (1) If $\sum_{i\in N_{PS}}\tilde{a}_i \oplus \sum_{i\in N_S}\tilde{a}'_i = \sum_{j\in N_{PD}}\tilde{b}_j \oplus \sum_{j\in N_D}\tilde{b}'_j = \sum_{k\in S_C}\tilde{e}_k$ then Go to Step 4.

Case (2) If $\sum_{i\in N_{PS}}\tilde{a}_i \oplus \sum_{i\in N_S}\tilde{a}'_i = \sum_{j\in N_{PD}}\tilde{b}_j \oplus \sum_{j\in N_D}\tilde{b}'_j \neq \sum_{k\in S_C}\tilde{e}_k$ then convert $\sum_{i\in N_{PS}}\tilde{a}_i \oplus \sum_{i\in N_S}\tilde{a}'_i = \sum_{j\in N_{PD}}\tilde{b}_j \oplus \sum_{j\in N_D}\tilde{b}'_j \neq \sum_{k\in S_C}\tilde{e}_k$ into $\sum_{i\in N_{PS}}\tilde{a}_i \oplus \sum_{i\in N_S}\tilde{a}'_i = \sum_{j\in N_{PD}}\tilde{b}_j \oplus \sum_{j\in N_D}\tilde{b}'_j = \sum_{k\in S_C}\tilde{e}_k$ as follows:

Case (2a) If $m_1-\alpha_1 \leq m''-\alpha''$, $\alpha_1 \leq \alpha''$, $n_1-m_1 \leq n''-m''$ and $\beta_1 \leq \beta''$ then check that in Step 2 a dummy purely source node is introduced or not and also check that a dummy purely destination node is introduced or not.

Case (i) If both the dummy purely source node and dummy purely destination node are introduced then increase both the fuzzy availability of the already introduced dummy purely source node and the fuzzy demand of the already introduced dummy purely destination node by the same fuzzy quantity $(m''-m_1,n''-n_1,\alpha''-\alpha_1,\beta''-\beta_1)_{LR}$ so that $\sum_{i\in N_{PS}}\tilde{a}_i \oplus \sum_{i\in N_S}\tilde{a}'_i = \sum_{j\in N_{PD}}\tilde{b}_j \oplus \sum_{j\in N_D}\tilde{b}'_j = \sum_{k\in S_C}\tilde{e}_k$. Go to Step 4.

Case (ii) If a dummy purely source node is introduced but no dummy purely destination node is introduced then increase the fuzzy availability of the already introduced dummy purely source node by the fuzzy quantity $(m''-m_1,n''-n_1,\alpha''-$

$\alpha_1, \beta'' - \beta_1)_{LR}$ and also introduce a dummy purely destination node with fuzzy demand $(m'' - m_1, n'' - n_1, \alpha'' - \alpha_1, \beta'' - \beta_1)_{LR}$ so that $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = \sum_{j \in N_{PD}} \tilde{b}_j \oplus \sum_{j \in N_D} \tilde{b}'_j = \sum_{k \in S_C} \tilde{e}_k$. Go to Step 4.

Case (iii) If a dummy purely destination node is introduced but no dummy purely source node is introduced then increase the fuzzy demand of the already introduced dummy purely destination node by the fuzzy quantity $(m'' - m_1, n'' - n_1, \alpha'' - \alpha_1, \beta'' - \beta_1)_{LR}$ and also introduce a dummy purely source node with fuzzy availability $(m'' - m_1, n'' - n_1, \alpha'' - \alpha_1, \beta'' - \beta_1)_{LR}$ so that $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = \sum_{j \in N_{PD}} \tilde{b}_j \oplus \sum_{j \in N_D} \tilde{b}'_j = \sum_{k \in S_C} \tilde{e}_k$. Go to Step 4.

Case (2b) If $m_1 - \alpha_1 \geq m'' - \alpha''$, $\alpha_1 \geq \alpha''$, $n_1 - m_1 \geq n'' - m''$ and $\beta_1 \geq \beta''$ then introduce a dummy conveyance with fuzzy capacity $(m_1 - m'', n_1 - n'', \alpha_1 - \alpha'', \beta_1 - \beta'')_{LR}$ so that $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = \sum_{j \in N_{PD}} \tilde{b}_j \oplus \sum_{j \in N_D} \tilde{b}'_j = \sum_{k \in S_C} \tilde{e}_k$. Go to Step 4.

Case (2c) If neither Case (2a) nor Case (2b) is satisfied then check that in Step 2 a dummy purely source node is introduced or not and also check that a dummy purely destination node is introduced or not.

Case (i) If both the dummy purely source node and dummy purely destination node are introduced then increase both the fuzzy availability of the already introduced dummy purely source node and the fuzzy demand of the already introduced dummy purely destination node by the same fuzzy quantity (maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$ + maximum $\{0, (n'' - m'') - (n_1 - m_1)\}$, maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (\beta'' - \beta_1)\})_{LR}$ and also introduce a dummy purely conveyance with fuzzy capacity (maximum $\{0, (m_1 - \alpha_1) - (m'' - \alpha'')\}$ + maximum $\{0, (\alpha_1 - \alpha'')\}$, maximum $\{0, (m_1 - \alpha_1) - (m'' - \alpha'')\}$ + maximum $\{0, (\alpha_1 - \alpha'')\}$ + maximum

$\{0, (n_1 - m_1) - (n'' - m'')\}$, maximum $\{0, (\alpha_1 - \alpha'')\}$, maximum $\{0, (\beta_1 - \beta'')\})_{LR}$ so that $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = \sum_{j \in N_{PD}} \tilde{b}_j \oplus \sum_{j \in N_D} \tilde{b}'_j = \sum_{k \in S_C} \tilde{e}_k$. Go to Step 4.

Case (ii) If a dummy purely source node is introduced but no dummy purely destination node is introduced then increase the fuzzy availability of the already introduced dummy purely source node by the fuzzy quantity (maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$ + maximum $\{0, (n'' - m'') - (n_1 - m_1)\}$, maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (\beta'' - \beta_1)\})_{LR}$ and also introduce a dummy purely destination node with fuzzy demand (maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$ + maximum $\{0, (n'' - m'') - (n_1 - m_1)\}$, maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (\beta'' - \beta_1)\})_{LR}$. Also, introduce a dummy conveyance with fuzzy capacity (maximum $\{0, (m_1 - \alpha_1) - (m'' - \alpha'')\}$ + maximum $\{0, (\alpha_1 - \alpha'')\}$, maximum $\{0, (m_1 - \alpha_1) - (m'' - \alpha'')\}$ + maximum $\{0, (\alpha_1 - \alpha'')\}$ + maximum $\{0, (n_1 - m_1) - (n'' - m'')\}$, maximum $\{0, (\alpha_1 - \alpha'')\}$, maximum $\{0, (\beta_1 - \beta'')\})_{LR}$ so that $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = \sum_{j \in N_{PD}} \tilde{b}_j \oplus \sum_{j \in N_D} \tilde{b}'_j = \sum_{k \in S_C} \tilde{e}_k$. Go to Step 4.

Case (iii) If a dummy purely destination node is introduced but no dummy purely source node is introduced then increase the fuzzy demand of the already introduced dummy purely destination node by the fuzzy quantity (maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$ + maximum $\{0, (n'' - m'') - (n_1 - m_1)\}$, maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (\beta'' - \beta_1)\})_{LR}$ and also introduce a dummy purely source node with fuzzy availability (maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (m'' - \alpha'') - (m_1 - \alpha_1)\}$ + maximum $\{0, (\alpha'' - \alpha_1)\}$ + maximum

$\{0, (n'' - m'') - (n_1 - m_1)\}$, maximum $\{0, (\alpha'' - \alpha_1)\}$, maximum $\{0, (\beta'' - \beta_1)\})_{LR}$. Also, introduce a dummy conveyance with fuzzy capacity (maximum $\{0, (m_1 - \alpha_1) - (m'' - \alpha'')\}$ + maximum $\{0, (\alpha_1 - \alpha'')\}$, maximum $\{0, (m_1 - \alpha_1) - (m'' - \alpha'')\}$ + maximum $\{0, (\alpha_1 - \alpha'')\}$ + maximum $\{0, (n_1 - m_1) - (n'' - m'')\}$, maximum $\{0, (\alpha_1 - \alpha'')\}$, maximum $\{0, (\beta_1 - \beta'')\})_{LR}$ so that $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = \sum_{j \in N_{PD}} \tilde{b}_j \oplus \sum_{j \in N_D} \tilde{b}'_j = \sum_{k \in S_C} \tilde{e}_k$. Go to Step 4.

Step 4 The balanced fully fuzzy solid transshipment problem, obtained by using Step 1 to Step 3, can be formulated into the fuzzy linear programming problem ($P_{6.1}$) by assuming the following fuzzy costs as zero LR flat fuzzy numbers:

(i) If it is required to add any dummy purely source node then assume the fuzzy cost for transporting one unit quantity of the product from the introduced dummy purely source node to all purely destination nodes and all intermediate nodes by any conveyance as zero LR flat fuzzy number.

(ii) If it is required to add any dummy purely destination node then assume the fuzzy cost for transporting one unit quantity of the product from all purely source nodes and intermediate nodes to the introduced dummy purely destination node by any conveyance as zero LR flat fuzzy number.

(iii) If it is required to add any dummy conveyance then assume the fuzzy cost for transporting one unit quantity of the product from all purely source nodes and intermediate nodes to all intermediate nodes and any purely destination nodes by introduced dummy conveyance as zero LR flat fuzzy number.

Step 5 Assuming $\tilde{c}_{ijk} = (m'_{ijk}, n'_{ijk}, \alpha'_{ijk}, \beta'_{ijk})_{LR}$, $\tilde{x}_{ijk} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$, $\tilde{a}_i = (m_i, n_i, \alpha_i, \beta_i)_{LR}$, $\tilde{a}'_i = (m'_i, n'_i, \alpha'_i, \beta'_i)_{LR}$, $\tilde{b}_j = (m_j, n_j, \alpha_j, \beta_j)_{LR}$, $\tilde{b}'_j = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR}$ and $\tilde{e}_k = (m''_k, n''_k, \alpha''_k, \beta''_k)_{LR}$, the fuzzy linear programming problem ($P_{6.1}$) can be written as:

Minimize $\sum_{(i,j)\in A}\sum_{k\in S_C}\Big((m'_{ijk},n'_{ijk},\alpha'_{ijk},\beta'_{ijk})_{LR}\otimes(m_{ijk},n_{ijk},\alpha_{ijk},\beta_{ijk})_{LR}\Big)$

subject to

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C}(m_{ijk},n_{ijk},\alpha_{ijk},\beta_{ijk})_{LR}=(m_i,n_i,\alpha_i,\beta_i)_{LR}\ , \qquad i\in N_{PS}$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C}(m_{ijk},n_{ijk},\alpha_{ijk},\beta_{ijk})_{LR}\ominus_H\sum_{j:(j,i)\in A}\sum_{k\in S_C}(m_{jik},n_{jik},\alpha_{jik},\beta_{jik})_{LR}=(m'_i,n'_i,\alpha'_i,\beta'_i)_{LR}\ , \qquad i\in N_S$$

$$\sum_{i:(i,j)\in A}\sum_{k\in S_C}(m_{ijk},n_{ijk},\alpha_{ijk},\beta_{ijk})_{LR}=(m_j,n_j,\alpha_j,\beta_j)_{LR}\ , \qquad j\in N_{PD}$$

$$\sum_{i:(i,j)\in A}\sum_{k\in S_C}(m_{ijk},n_{ijk},\alpha_{ijk},\beta_{ijk})_{LR}\ominus_H\sum_{i:(j,i)\in A}\sum_{k\in S_C}(m_{jik},n_{jik},\alpha_{jik},\beta_{jik})_{LR}=(m'_j,n'_j,\alpha'_j,\beta'_j)_{LR}\ , \qquad j\in N_D$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C}(m_{ijk},n_{ijk},\alpha_{ijk},\beta_{ijk})_{LR}=\sum_{j:(j,i)\in A}\sum_{k\in S_C}(m_{jik},n_{jik},\alpha_{jik},\beta_{jik})_{LR},\quad i\in N_T$$

$$\sum_{i:(i,j)\in A}(m_{ijk},n_{ijk},\alpha_{ijk},\beta_{ijk})_{LR}=(m''_k,n''_k,\alpha''_k,\beta''_k)_{LR}\ , \qquad k\in S_C$$

$(m_{ijk},n_{ijk},\alpha_{ijk},\beta_{ijk})_{LR}$ is non-negative LR flat fuzzy number $\forall\ (i,j)\in A,\ k\in S_C$.

Step 6 Using the arithmetic operations of LR flat fuzzy numbers, defined in Section 3.1.2, and assuming $\sum_{(i,j)\in A}\sum_{k\in S_C}(\tilde{c}_{ijk}\otimes\tilde{x}_{ijk})=(m_0,n_0,\alpha_0,\beta_0)_{LR}$, the fuzzy linear programming problem, obtained in Step 5, can be written as:

Minimize $(m_0,n_0,\alpha_0,\beta_0)_{LR}$

subject to

$$\Big(\sum_{j:(i,j)\in A}\sum_{k\in S_C}m_{ijk},\sum_{j:(i,j)\in A}\sum_{k\in S_C}n_{ijk},\sum_{j:(i,j)\in A}\sum_{k\in S_C}\alpha_{ijk},\sum_{j:(i,j)\in A}\sum_{k\in S_C}\beta_{ijk}\Big)_{LR}=(m_i,n_i,\alpha_i,\beta_i)_{LR}\ , \qquad i\in N_{PS}$$

$$\Big(\sum_{j:(i,j)\in A}\sum_{k\in S_C}m_{ijk},\sum_{j:(i,j)\in A}\sum_{k\in S_C}n_{ijk},\sum_{j:(i,j)\in A}\sum_{k\in S_C}\alpha_{ijk},\sum_{j:(i,j)\in A}\sum_{k\in S_C}\beta_{ijk}\Big)_{LR}\ominus_H\Big(\sum_{j:(j,i)\in A}\sum_{k\in S_C}m_{jik},\sum_{j:(j,i)\in A}\sum_{k\in S_C}n_{jik},\sum_{j:(j,i)\in A}\sum_{k\in S_C}\alpha_{jik},\sum_{j:(j,i)\in A}\sum_{k\in S_C}\beta_{jik}\Big)_{LR}=(m'_i,n'_i,\alpha'_i,\beta'_i)_{LR} \qquad i\in N_S$$

$$\Big(\sum_{i:(i,j)\in A}\sum_{k\in S_C}m_{ijk},\sum_{i:(i,j)\in A}\sum_{k\in S_C}n_{ijk},\sum_{i:(i,j)\in A}\sum_{k\in S_C}\alpha_{ijk},\sum_{i:(i,j)\in A}\sum_{k\in S_C}\beta_{ijk}\Big)_{LR}=(m_j,n_j,\alpha_j,\beta_j)_{LR}\ ,$$

$j \in N_{PD}$

$(\sum_{i:(i,j)\in A}\sum_{k\in S_C} m_{ijk}, \sum_{i:(i,j)\in A}\sum_{k\in S_C} n_{ijk}, \sum_{i:(i,j)\in A}\sum_{k\in S_C} \alpha_{ijk}, \sum_{i:(i,j)\in A}\sum_{k\in S_C} \beta_{ijk})_{LR} \ominus_H (\sum_{i:(j,i)\in A}\sum_{k\in S_C} m_{jik}, \sum_{i:(j,i)\in A}\sum_{k\in S_C} n_{jik}, \sum_{i:(j,i)\in A}\sum_{k\in S_C} \alpha_{jik}, \sum_{i:(j,i)\in A}\sum_{k\in S_C} \beta_{jik})_{LR} = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR}$, $j \in N_D$

$(\sum_{j:(i,j)\in A}\sum_{k\in S_C} m_{ijk}, \sum_{j:(i,j)\in A}\sum_{k\in S_C} n_{ijk}, \sum_{j:(i,j)\in A}\sum_{k\in S_C} \alpha_{ijk}, \sum_{j:(i,j)\in A}\sum_{k\in S_C} \beta_{ijk})_{LR} = (\sum_{j:(j,i)\in A}\sum_{k\in S_C} m_{jik}, \sum_{j:(j,i)\in A}\sum_{k\in S_C} n_{jik}, \sum_{j:(j,i)\in A}\sum_{k\in S_C} \alpha_{jik}, \sum_{j:(j,i)\in A}\sum_{k\in S_C} \beta_{jik})_{LR},$ $i \in N_T$

$(\sum_{i:(i,j)\in A} m_{ijk}, \sum_{i:(i,j)\in A} n_{ijk}, \sum_{i:(i,j)\in A} \alpha_{ijk}, \sum_{i:(i,j)\in A} \beta_{ijk})_{LR} = (m''_k, n''_k, \alpha''_k, \beta''_k)_{LR}$, $k \in S_C$

$(m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$ is a non-negative LR flat fuzzy number $\forall\ (i,j) \in A,\ k \in S_C$

Step 7 Using Definition 3.5, Definition 3.6 and Remark 4.1, the fuzzy linear programming problem, obtained in Step 6, can be converted into the fuzzy linear programming problem ($P_{6.2}$):

Minimize $(m_0, n_0, \alpha_0, \beta_0)_{LR}$

subject to

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} m_{ijk} = m_i\ , \qquad i \in N_{PS}$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} n_{ijk} = n_i\ , \qquad i \in N_{PS}$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \alpha_{ijk} = \alpha_i\ , \qquad i \in N_{PS}$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \beta_{ijk} = \beta_i\ , \qquad i \in N_{PS}$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} m_{ijk} - \sum_{j:(j,i)\in A}\sum_{k\in S_C} m_{jik} = m'_i\ , \qquad i \in N_S$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} n_{ijk} - \sum_{j:(j,i)\in A}\sum_{k\in S_C} n_{jik} = n'_i\ , \qquad i \in N_S$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \alpha_{ijk} - \sum_{j:(j,i)\in A}\sum_{k\in S_C} \alpha_{jik} = \alpha'_i\ , \qquad i \in N_S$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \beta_{ijk} - \sum_{j:(j,i)\in A}\sum_{k\in S_C} \beta_{jik} = \beta'_i\ , \qquad i \in N_S$$
$$\sum_{i:(i,j)\in A}\sum_{k\in S_C} m_{ijk} = m_j\ , \qquad j \in N_{PD}$$
$$\sum_{i:(i,j)\in A}\sum_{k\in S_C} n_{ijk} = n_j\ , \qquad j \in N_{PD}$$
$$\sum_{i:(i,j)\in A}\sum_{k\in S_C} \alpha_{ijk} = \alpha_j\ , \qquad j \in N_{PD}$$

$$\sum_{i:(i,j)\in A}\sum_{k\in S_C}\beta_{ijk}=\beta_j\ , \qquad j\in N_{PD} \qquad (P_{6.2})$$
$$\sum_{i:(i,j)\in A}\sum_{k\in S_C} m_{ijk}-\sum_{i:(j,i)\in A}\sum_{k\in S_C} m_{jik}=m'_j\ , \qquad j\in N_D$$
$$\sum_{i:(i,j)\in A}\sum_{k\in S_C} n_{ijk}-\sum_{i:(j,i)\in A}\sum_{k\in S_C} n_{jik}=n'_j\ , \qquad j\in N_D$$
$$\sum_{i:(i,j)\in A}\sum_{k\in S_C} \alpha_{ijk}-\sum_{i:(j,i)\in A}\sum_{k\in S_C} \alpha_{jik}=\alpha'_j\ , \qquad j\in N_D$$
$$\sum_{i:(i,j)\in A}\sum_{k\in S_C} \beta_{ijk}-\sum_{i:(j,i)\in A}\sum_{k\in S_C} \beta_{jik}=\beta'_j\ , \qquad j\in N_D$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} m_{ijk}=\sum_{j:(j,i)\in A}\sum_{k\in S_C} m_{jik}\ , \qquad i\in N_T$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} n_{ijk}=\sum_{j:(j,i)\in A}\sum_{k\in S_C} n_{jik}\ , \qquad i\in N_T$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \alpha_{ijk}=\sum_{j:(j,i)\in A}\sum_{k\in S_C} \alpha_{jik}\ , \qquad i\in N_T$$
$$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \beta_{ijk}=\sum_{j:(j,i)\in A}\sum_{k\in S_C} \beta_{jik}, \qquad i\in N_T$$
$$\sum_{i:(i,j)\in A} m_{ijk}=m''_k\ , \qquad k\in S_C$$
$$\sum_{i:(i,j)\in A} n_{ijk}=n''_k\ , \qquad k\in S_C$$
$$\sum_{i:(i,j)\in A} \alpha_{ijk}=\alpha''_k\ , \qquad k\in S_C$$
$$\sum_{i:(i,j)\in A} \beta_{ijk}=\beta''_k\ , \qquad k\in S_C$$
$$m_{ijk}-\alpha_{ijk},\ n_{ijk}-m_{ijk},\ \alpha_{ijk},\beta_{ijk}\geq 0\ \forall\ (i,j)\in A,\ k\in S_C$$

Step 8 As discussed in Step 6 of the method, proposed in Section 2.5.1 of Chapter 2, the fuzzy optimal solution of the fuzzy linear programming problem $(P_{6.2})$, can be obtained by solving the following crisp linear programming problem:

Minimize $\Re((m_0,n_0,\alpha_0,\beta_0)_{LR})$

subject to

constraints of the problem $(P_{6.2})$

Step 9 Using the existing formula [131], $\Re(m_0,n_0,\alpha_0,\beta_0)_{LR}=\frac{1}{2}(\int_0^1(m_0-\alpha_0L^{-1}(\lambda))\,d\lambda+\int_0^1(n_0+\beta_0R^{-1}(\lambda))\,d\lambda)$, the crisp linear programming problem, obtained in Step 8, can be written as:

Minimize $\frac{1}{2}(\int_0^1(m_0-\alpha_0L^{-1}(\lambda))\ d\lambda+\int_0^1(n_0+\beta_0R^{-1}(\lambda))\ d\lambda)$

subject to

constraints of the problem $(P_{6.2})$

Step 10 Solve the crisp linear programming problem, obtained in Step 9, to find the optimal solution $\{m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk}\}$.

Step 11 Put the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk}$ in $\tilde{x}_{ijk} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$ to find the fuzzy optimal solution $\{\tilde{x}_{ijk}\}$.

Step 12 Put the values of $\tilde{x}_{ijk}$, obtained from Step 11, in $\sum\limits_{(i,j)\in A} \sum\limits_{k\in S_C} (\tilde{c}_{ijk} \otimes \tilde{x}_{ijk})$, to find the minimum total fuzzy transportation cost.

6.3.2 Proposed method based on tabular representation

In this section, a new method, based on tabular representation, is proposed to find the fuzzy optimal solution of such fully fuzzy solid transshipment problems in which the parameters are represented by LR flat fuzzy numbers.

The steps of the proposed method are as follows:

Step 1 Use Step 1 to Step 3 of the method, proposed in Section 6.3.1, to obtain a balanced fully fuzzy solid transshipment problem.

Step 2 Convert the balanced fully fuzzy solid transshipment problem, obtained in Step 1, into balanced fully fuzzy solid transportation problem as follows:

Increase the fuzzy availability and fuzzy demand corresponding to intermediate nodes by the fuzzy quantity $\tilde{P} = \sum\limits_{i\in N_{PS}} \tilde{a}_i \oplus \sum\limits_{i\in N_S} \tilde{a}'_i$ (or $\sum\limits_{j\in N_{PD}} \tilde{b}_j \oplus \sum\limits_{j\in N_D} \tilde{b}'_j$ or $\sum\limits_{k\in S_C} \tilde{e}_k$). Also, check that in Step 1 a dummy conveyance is introduced or not.

Case (i) If a dummy conveyance is already introduced then increase the fuzzy capacity of the already introduced dummy conveyance by the fuzzy quantity $= K\tilde{P}$ where, $K =$ number of intermediate nodes.

Case (ii) If no dummy conveyance is introduced then introduce a dummy conveyance with fuzzy capacity $= K\tilde{P}$ where, $K =$ number of intermediate nodes.

Step 3 Let in the balanced fully fuzzy solid transportation problem, obtained from Step 2, the number of purely source nodes, source nodes, transshipment nodes, purely destination nodes, destination nodes and conveyances be m, l, r, t, q and s respectively. Also, let the fuzzy availability of the product at i^{th} purely source node (N_{PS_i}), fuzzy availability of the product at i^{th} source node (N_{S_i}), fuzzy demand of the product at j^{th} purely destination node (N_{PD_j}), fuzzy demand of the product at j^{th} destination node (N_{D_j}), the fuzzy capacity of the k^{th} conveyance (E_k) and the fuzzy cost for transporting one unit quantity of the product from i^{th} source to j^{th} destination by means of the k^{th} conveyance be denoted by $\tilde{a}_i$, $\tilde{a'}_{m+i}$, $\tilde{b}_j$, $\tilde{b'}_{t+j}$, $\tilde{e}_k$ and $\tilde{c}_{ijk}$ respectively. Then, it can be represented by Table 6.1.

Step 4 Split Table 6.1 into four crisp solid transportation tables i.e., Table 6.2, Table 6.3, Table 6.4 and Table 6.5 respectively. The cost for transporting one unit quantity of the product from i^{th} node to j^{th} node by means of k^{th} conveyance in Table 6.2, Table 6.3, Table 6.4 and Table 6.5 are represented by $\eta_{ijk}, \rho_{ijk}, \delta_{ijk}$ and ξ_{ijk} respectively.

where,

$$\eta_{ijk} = \frac{1}{2}((m'_{ijk} + n'_{ijk}) - \alpha'_{ijk}\int_0^1 L^{-1}(\lambda)d\lambda + \beta'_{ijk}\int_0^1 R^{-1}(\lambda)d\lambda),$$

$$i = 1, 2, ..., m+l+q+r,\ j = 1, 2, ..., t+q+r+l \text{ and } \ k = 1, 2, ..., s$$

$$\rho_{ijk} = \frac{1}{2}((m'_{ijk} + n'_{ijk}) - m'_{ijk}\int_0^1 L^{-1}(\lambda)d\lambda + \beta'_{ijk}\int_0^1 R^{-1}(\lambda)d\lambda),$$

$$i = 1, 2, ..., m+l+q+r,\ j = 1, 2, ..., t+q+r+l \text{ and } \ k = 1, 2, ..., s$$

$$\delta_{ijk} = \frac{1}{2}(n'_{ijk} + \beta'_{ijk}\int_0^1 R^{-1}(\lambda)d\lambda),$$

$$i = 1, 2, ..., m+l+q+r,\ j = 1, 2, ..., t+q+r+l \text{ and } \ k = 1, 2, ..., s$$

$\xi_{ijk} = \frac{1}{2}((n'_{ijk} + \beta'_{ijk}) \int_0^1 R^{-1}(\lambda)d\lambda)$,

$i = 1, 2, ..., m+l+q+r,\ j = 1, 2, ..., t+q+r+l$ and $k = 1, 2, ..., s$

Step 5 Solve the crisp solid transportation problems shown by Table 6.2, Table 6.3, Table 6.4 and Table 6.5 to find the optimal solution $\{m_{ijk} - \alpha_{ijk}\}$; $\{\alpha_{ijk}\}$; $\{n_{ijk} - m_{ijk}\}$ and $\{\beta_{ijk}\}$ respectively.

Step 6 Solve the equations, obtained in Step 5, to find the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}$ and β_{ijk}.

Step 7 Find the fuzzy optimal solution $\{\tilde{x}_{ijk}\}$ by putting the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk}$ in $\tilde{x}_{ijk} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$.

Step 8 Find the minimum total fuzzy transportation cost by putting the values of $\tilde{x}_{ijk}$ in $\sum_i \sum_j \sum_k (\tilde{c}_{ijk} \otimes \tilde{x}_{ijk})\ \ \forall\, i = 1, 2, ..., m+l+q+r,\ j = 1, 2, ..., t+q+r+l,\ k = 1, 2, ..., s$.

Remark 6.2 Let $\tilde{A} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$ be an LR flat fuzzy number with $L(x) = R(x) =$ maximum $\{0, 1 - x\}$. Then,

$\eta_{ijk} = \frac{1}{2}((m_{ijk} + n_{ijk}) - \alpha_{ijk} \int_0^1 L^{-1}(\lambda)d\lambda + \beta_{ijk} \int_0^1 R^{-1}(\lambda)d\lambda)$

$= \frac{1}{4}(2m_{ijk} + 2n_{ijk} + \beta_{ijk} - \alpha_{ijk})$,

$\rho_{ijk} = \frac{1}{2}((m_{ijk}+n_{ijk})-m_{ijk} \int_0^1 L^{-1}(\lambda)d\lambda+\beta_{ijk} \int_0^1 R^{-1}(\lambda)d\lambda) = \frac{1}{4}(m_{ijk}+2n_{ijk}+\beta_{ijk})$,

$\delta_{ijk} = \frac{1}{2}(n_{ijk} + \beta_{ijk} \int_0^1 R^{-1}(\lambda)d\lambda) = \frac{1}{4}(2n_{ijk} + \beta_{ijk})$ and

$\xi_{ijk} = \frac{1}{2}((n_{ijk} + \beta_{ijk}) \int_0^1 R^{-1}(\lambda)d\lambda) = \frac{1}{4}(n_{ijk} + \beta_{ijk})$.

Table 6.1 Tabular representation of balanced fully fuzzy solid transportation problem obtained by adding fuzzy buffer stock ($\tilde{P}$)

Conveyance	E_1			⋯	E_1			E_1			⋯
		⋯		⋯		⋯			⋯		⋯
			E_s	⋯			E_s			E_s	⋯
Destinations → Sources↓	N_{PD_1}			⋯	N_{PD_t}			N_{D_1}			⋯
N_{PS_1}	$\tilde{c}_{111}$	⋯	$\tilde{c}_{11s}$	⋯	$\tilde{c}_{1t1}$	⋯	$\tilde{c}_{1ts}$	$\tilde{c}_{1(t+1)1}$	⋯	$\tilde{c}_{1(t+1)s}$	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{PS_m}	$\tilde{c}_{m11}$	⋯	$\tilde{c}_{m1s}$	⋯	$\tilde{c}_{mt1}$	⋯	$\tilde{c}_{mts}$	$\tilde{c}_{m(t+1)1}$	⋯	$\tilde{c}_{m(t+1)s}$	⋯
N_{S_1}	$\tilde{c}_{(m+1)11}$	⋯	$\tilde{c}_{(m+1)1s}$	⋯	$\tilde{c}_{(m+1)t1}$	⋯	$\tilde{c}_{(m+1)ts}$	$\tilde{c}_{(m+1)(t+1)1}$	⋯	$\tilde{c}_{(m+1)(t+1)s}$	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{S_l}	$\tilde{c}_{(m+l)11}$	⋯	$\tilde{c}_{(m+l)1s}$	⋯	$\tilde{c}_{(m+l)t1}$	⋯	$\tilde{c}_{(m+l)ts}$	$\tilde{c}_{(m+l)(t+1)1}$	⋯	$\tilde{c}_{(m+l)(t+1)s}$	⋯
N_{D_1}	$\tilde{c}_{(m+l+1)11}$	⋯	$\tilde{c}_{(m+l+1)1s}$	⋯	$\tilde{c}_{(m+l+1)t1}$	⋯	$\tilde{c}_{(m+l+1)ts}$	$\tilde{c}_{(m+l+1)(t+1)1}$	⋯	$\tilde{c}_{(m+l+1)(t+1)s}$	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{D_q}	$\tilde{c}_{(m+l+q)11}$	⋯	$\tilde{c}_{(m+l+q)1s}$	⋯	$\tilde{c}_{(m+l+q)t1}$	⋯	$\tilde{c}_{(m+l+q)ts}$	$\tilde{c}_{(m+l+q)(t+1)1}$	⋯	$\tilde{c}_{(m+l+q)(t+1)s}$	⋯
N_{T_1}	$\tilde{c}_{(m+l+q+1)11}$	⋯	$\tilde{c}_{(m+l+q+1)1s}$	⋯	$\tilde{c}_{(m+l+q+1)t1}$	⋯	$\tilde{c}_{(m+l+q+1)ts}$	$\tilde{c}_{(m+l+q+1)(t+1)1}$	⋯	$\tilde{c}_{(m+l+q+1)(t+1)s}$	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{T_r}	$\tilde{c}_{(m+l+q+r)11}$	⋯	$\tilde{c}_{(m+l+q+r)1s}$	⋯	$\tilde{c}_{(m+l+q+r)t1}$	⋯	$\tilde{c}_{(m+l+q+r)ts}$	$\tilde{c}_{(m+l+q+r)(t+1)1}$	⋯	$\tilde{c}_{(m+l+q+r)(t+1)s}$	⋯
Demand	$\tilde{b}_1$			⋯	$\tilde{b}_t$			$\tilde{b}'_{t+1}$			⋯

Conveyance	E_1			E_1			⋯	E_1			⋯
		⋯			⋯		⋯		⋯		⋯
			E_s			E_s	⋯			E_s	⋯
Destinations → Sources↓	N_{D_q}			N_{T_1}			⋯	N_{T_r}			⋯
N_{PS_1}	$\tilde{c}_{1(t+q)1}$	⋯	$\tilde{c}_{1(t+q)s}$	$\tilde{c}_{1(t+q+1)1}$	⋯	$\tilde{c}_{1(t+q+1)s}$	⋯	$\tilde{c}_{1(t+q+r)1}$	⋯	$\tilde{c}_{1(t+q+r)s}$	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{PS_m}	$\tilde{c}_{m(t+q)1}$	⋯	$\tilde{c}_{m(t+q)s}$	$\tilde{c}_{m(t+q+1)1}$	⋯	$\tilde{c}_{m(t+q+1)s}$	⋯	$\tilde{c}_{m(t+q+r)1}$	⋯	$\tilde{c}_{m(t+q+r)s}$	⋯
N_{S_1}	$\tilde{c}_{(m+1)(t+q)1}$	⋯	$\tilde{c}_{(m+1)(t+q)s}$	$\tilde{c}_{(m+1)(t+q+1)1}$	⋯	$\tilde{c}_{(m+1)(t+q+1)s}$	⋯	$\tilde{c}_{(m+1)(t+q+r)1}$	⋯	$\tilde{c}_{(m+1)(t+q+r)s}$	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{S_l}	$\tilde{c}_{(m+l)(t+q)1}$	⋯	$\tilde{c}_{(m+l)(t+q)s}$	$\tilde{c}_{(m+l)(t+q+1)1}$	⋯	$\tilde{c}_{(m+l)(t+q+1)s}$	⋯	$\tilde{c}_{(m+l)(t+q+r)1}$	⋯	$\tilde{c}_{(m+l)(t+q+r)s}$	⋯
N_{D_1}	$\tilde{c}_{(m+l+1)(t+q)1}$	⋯	$\tilde{c}_{(m+l+1)(t+q)s}$	$\tilde{c}_{(m+l+1)(t+q+1)1}$	⋯	$\tilde{c}_{(m+l+1)(t+q+1)s}$	⋯	$\tilde{c}_{(m+l+1)(t+q+r)1}$	⋯	$\tilde{c}_{(m+l+1)(t+q+r)s}$	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{D_q}	$\tilde{c}_{(m+l+q)(t+q)1}$	⋯	$\tilde{c}_{(m+l+q)(t+q)s}$	$\tilde{c}_{(m+l+q)(t+q+1)1}$	⋯	$\tilde{c}_{(m+l+q)(t+q+1)s}$	⋯	$\tilde{c}_{(m+l+q)(t+q+r)1}$	⋯	$\tilde{c}_{(m+l+q)(t+q+r)s}$	⋯
N_{T_1}	$\tilde{c}_{(m+l+q+1)(t+q)1}$	⋯	$\tilde{c}_{(m+l+q+1)(t+q)s}$	$\tilde{c}_{(m+l+q+1)(t+q+1)1}$	⋯	$\tilde{c}_{(m+l+q+1)(t+q+1)s}$	⋯	$\tilde{c}_{(m+l+q+1)(t+q+r)1}$	⋯	$\tilde{c}_{(m+l+q+1)(t+q+r)s}$	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{T_r}	$\tilde{c}_{(m+l+q+r)(t+q)1}$	⋯	$\tilde{c}_{(m+l+q+r)(t+q)s}$	$\tilde{c}_{(m+l+q+r)(t+q+1)1}$	⋯	$\tilde{c}_{(m+l+q+r)(t+q+1)s}$	⋯	$\tilde{c}_{(m+l+q+r)(t+q+r)1}$	⋯	$\tilde{c}_{(m+l+q+r)(t+q+r)s}$	⋯
Demand	$\tilde{b}'_{t+q}$			$\tilde{P}$			⋯	$\tilde{P}$			⋯

								Capacity
Conveyance	E_1			⋯	$\tilde{E}_1$			$\tilde{e}_1$
		⋯		⋯		⋯		⋮
			E_s	⋯			E_s	$\tilde{e}_s$
Destinations → Sources↓	N_{S_1}			⋯	N_{S_l}			Availability
N_{PS_1}	$\tilde{c}_{1(t+q+r+1)1}$	⋯	$\tilde{c}_{1(t+q+r+1)s}$	⋯	$\tilde{c}_{1(t+q+r+l)1}$	⋯	$\tilde{c}_{1(t+q+r+l)s}$	$\tilde{a}_l$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{PS_m}	$\tilde{c}_{m(t+q+r+1)1}$	⋯	$\tilde{c}_{m(t+q+r+1)s}$	⋯	$\tilde{c}_{m(t+q+r+l)1}$	⋯	$\tilde{c}_{m(t+q+r+l)s}$	$\tilde{a}_m$
N_{S_1}	$\tilde{c}_{(m+1)(t+q+r+1)1}$	⋯	$\tilde{c}_{(m+1)(t+q+r+1)s}$	⋯	$\tilde{c}_{(m+1)(t+q+r+l)1}$	⋯	$\tilde{c}_{(m+1)(t+q+r+l)s}$	$\tilde{a}'_{m+1}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{S_l}	$\tilde{c}_{(m+l)(t+q+r+1)1}$	⋯	$\tilde{c}_{(m+l)(t+q+r+1)s}$	⋯	$\tilde{c}_{(m+l)(t+q+r+l)1}$	⋯	$\tilde{c}_{(m+l)(t+q+r+l)s}$	$\tilde{a}'_{m+l}$
N_{D_1}	$\tilde{c}_{(m+l+1)(t+q+r+1)1}$	⋯	$\tilde{c}_{(m+l+1)(t+q+r+1)s}$	⋯	$\tilde{c}_{(m+l+1)(t+q+r+l)1}$	⋯	$\tilde{c}_{(m+l+1)(t+q+r+l)s}$	$\tilde{P}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{D_q}	$\tilde{c}_{(m+l+q)(t+q+r+1)1}$	⋯	$\tilde{c}_{(m+l+q)(t+q+r+1)s}$	⋯	$\tilde{c}_{(m+l+q)(t+q+r+l)1}$	⋯	$\tilde{c}_{(m+l+q)(t+q+r+l)s}$	$\tilde{P}$
N_{T_1}	$\tilde{c}_{(m+l+q+1)(t+q+r+1)1}$	⋯	$\tilde{c}_{(m+l+q+1)(t+q+r+1)s}$	⋯	$\tilde{c}_{(m+l+q+1)(t+q+r+l)1}$	⋯	$\tilde{c}_{(m+l+q+1)(t+q+r+l)s}$	$\tilde{P}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{T_r}	$\tilde{c}_{(m+l+q+r)(t+q+r+1)1}$	⋯	$\tilde{c}_{(m+l+q+r)(t+q+r+1)s}$	⋯	$\tilde{c}_{(m+l+q+r)(t+q+r+l)1}$	⋯	$\tilde{c}_{(m+l+q+r)(t+q+r+l)s}$	$\tilde{P}$
Demand	$\tilde{P}$			⋯	$\tilde{P}$			

$$\tilde{c}_{ijk} = (m'_{ijk}, n'_{ijk}, \alpha'_{ijk}, \beta'_{ijk})_{LR} = \begin{cases} (0,0,0,0)_{LR} & \text{; If the product is supplying from some intermediate node to same intermediate node by dummy conveyance} \\ & \text{or If the product is supplying from some dummy purely source node to any intermediate node or to any purely} \\ & \text{destination node by any conveyance} \\ & \text{or If the product is supplying from any purely source node to purely dummy destination node by any conveyance} \\ & \text{or If the product is supplying from any intermediate node to purely dummy destination node by any conveyance} \\ & \text{If the product is supplying from some purely source node to any purely destination node or to any intermediate node} \\ & \text{by dummy conveyance} \\ & \text{or If the product is supplying from any intermediate node to any purely destination node or to any intermediate node} \\ & \text{by dummy conveyance} \\ (M,M,0,0)_{LR} & \text{; If the product is supplying from some intermediate node to same intermediate node by such a conveyance which is} \\ & \text{not a dummy conveyance} \\ & \text{or If the product can not be directly supplied from } i^{th} \text{ node to } j^{th} \text{ node} \\ & \text{or If the product can not be supplied from } i^{th} \text{ node to } j^{th} \text{ node} \\ (m''_{ijk}, n''_{ijk}, \alpha''_{ijk}, \beta''_{ijk})_{LR} & \text{; If the product can be directly supplied from } i^{th} \text{ node to } j^{th} \text{ node} \end{cases}$$

$\tilde{a}_i = (m_i, n_i, \alpha_i, \beta_i)_{LR}$; $\tilde{a}'_{m+i} = (m''_{m+i}, n''_{m+i}, \alpha''_{m+i}, \beta''_{m+i})_{LR}$; $\tilde{b}_j = (m'_j, n'_j, \alpha'_j, \beta'_j)_{LR}$; $\tilde{b}'_{t+j} = (m''_{t+j}, n''_{t+j}, \alpha''_{t+j}, \beta''_{t+j})_{LR}$ and $\tilde{P} = (P_1, P_2, \alpha_p, \beta_p)_{LR}$

Table 6.2 Tabular representation of first crisp solid transportation problem

																													Capacity
Conveyance	E_1			⋯	E_1			E_1			⋯	E_1			E_1			⋯	E_1			E_1			⋯	E_1			$m'''_1-\alpha'''_1$
		⋯		⋯		⋯			⋯		⋯		⋯			⋯		⋯		⋯			⋯		⋯		⋯		⋮
			E_s	⋯			E_s			E_s	⋯			E_s			E_s	⋯			E_s			E_s	⋯			E_s	$m'''_s-\alpha'''_s$
Destinations → Sources↓	N_{PD_1}			⋯	N_{PD_t}			N_{D_1}			⋯	N_{D_q}			N_{T_1}			⋯	N_{T_r}			N_{S_1}			⋯	N_{S_l}			Availability
N_{PS_1}	η_{111}	⋯	η_{11s}	⋯	η_{1t1}	⋯	η_{1ts}	$\eta_{1(t+1)1}$	⋯	$\eta_{1(t+1)s}$	⋯	$\eta_{1(t+q)1}$	⋯	$\eta_{1(t+q)s}$	$\eta_{1(t+q+1)1}$	⋯	$\eta_{1(t+q+1)s}$	⋯	$\eta_{1(t+q+r)1}$	⋯	$\eta_{1(t+q+r)s}$	$\eta_{1(t+q+r+1)1}$	⋯	$\eta_{1(t+q+r+1)s}$	⋯	$\eta_{1(t+q+r+l)1}$	⋯	$\eta_{1(t+q+r+l)s}$	$m_1-\alpha_1$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{PS_m}	η_{m11}	⋯	η_{m1s}	⋯	η_{mt1}	⋯	η_{mts}	$\eta_{m(t+1)1}$	⋯	$\eta_{m(t+1)s}$	⋯	$\eta_{m(t+q)1}$	⋯	$\eta_{m(t+q)s}$	$\eta_{m(t+q+1)1}$	⋯	$\eta_{m(t+q+1)s}$	⋯	$\eta_{m(t+q+r)1}$	⋯	$\eta_{m(t+q+r)s}$	$\eta_{m(t+q+r+1)1}$	⋯	$\eta_{m(t+q+r+1)s}$	⋯	$\eta_{m(t+q+r+l)1}$	⋯	$\eta_{m(t+q+r+l)s}$	$m_m-\alpha_m$
N_{S_1}	$\eta_{(m+1)11}$	⋯	$\eta_{(m+1)1s}$	⋯	$\eta_{(m+1)t1}$	⋯	$\eta_{(m+1)ts}$	$\eta_{(m+1)(t+1)1}$	⋯	$\eta_{(m+1)(t+1)s}$	⋯	$\eta_{(m+1)(t+q)1}$	⋯	$\eta_{(m+1)(t+q)s}$	$\eta_{(m+1)(t+q+1)1}$	⋯	$\eta_{(m+1)(t+q+1)s}$	⋯	$\eta_{(m+1)(t+q+r)1}$	⋯	$\eta_{(m+1)(t+q+r)s}$	$\eta_{(m+1)(t+q+r+1)1}$	⋯	$\eta_{(m+1)(t+q+r+1)s}$	⋯	$\eta_{(m+1)(t+q+r+l)1}$	⋯	$\eta_{(m+1)(t+q+r+l)s}$	$m''_{m+1}-\alpha''_{m+1}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{S_l}	$\eta_{(m+l)11}$	⋯	$\eta_{(m+l)1s}$	⋯	$\eta_{(m+l)t1}$	⋯	$\eta_{(m+l)ts}$	$\eta_{(m+l)(t+1)1}$	⋯	$\eta_{(m+l)(t+1)s}$	⋯	$\eta_{(m+l)(t+q)1}$	⋯	$\eta_{(m+l)(t+q)s}$	$\eta_{(m+l)(t+q+1)1}$	⋯	$\eta_{(m+l)(t+q+1)s}$	⋯	$\eta_{(m+l)(t+q+r)1}$	⋯	$\eta_{(m+l)(t+q+r)s}$	$\eta_{(m+l)(t+q+r+1)1}$	⋯	$\eta_{(m+l)(t+q+r+1)s}$	⋯	$\eta_{(m+l)(t+q+r+l)1}$	⋯	$\eta_{(m+l)(t+q+r+l)s}$	$m''_{m+l}-\alpha''_{m+l}$
N_{D_1}	$\eta_{(m+l+1)11}$	⋯	$\eta_{(m+l+1)1s}$	⋯	$\eta_{(m+l+1)t1}$	⋯	$\eta_{(m+l+1)ts}$	$\eta_{(m+l+1)(t+1)1}$	⋯	$\eta_{(m+l+1)(t+1)s}$	⋯	$\eta_{(m+l+1)(t+q)1}$	⋯	$\eta_{(m+l+1)(t+q)s}$	$\eta_{(m+l+1)(t+q+1)1}$	⋯	$\eta_{(m+l+1)(t+q+1)s}$	⋯	$\eta_{(m+l+1)(t+q+r)1}$	⋯	$\eta_{(m+l+1)(t+q+r)s}$	$\eta_{(m+l+1)(t+q+r+1)1}$	⋯	$\eta_{(m+l+1)(t+q+r+1)s}$	⋯	$\eta_{(m+l+1)(t+q+r+l)1}$	⋯	$\eta_{(m+l+1)(t+q+r+l)s}$	$P_1-\alpha_p$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{D_q}	$\eta_{(m+l+q)11}$	⋯	$\eta_{(m+l+q)1s}$	⋯	$\eta_{(m+l+q)t1}$	⋯	$\eta_{(m+l+q)ts}$	$\eta_{(m+l+q)(t+1)1}$	⋯	$\eta_{(m+l+q)(t+1)s}$	⋯	$\eta_{(m+l+q)(t+q)1}$	⋯	$\eta_{(m+l+q)(t+q)s}$	$\eta_{(m+l+q)(t+q+1)1}$	⋯	$\eta_{(m+l+q)(t+q+1)s}$	⋯	$\eta_{(m+l+q)(t+q+r)1}$	⋯	$\eta_{(m+l+q)(t+q+r)s}$	$\eta_{(m+l+q)(t+q+r+1)1}$	⋯	$\eta_{(m+l+q)(t+q+r+1)s}$	⋯	$\eta_{(m+l+q)(t+q+r+l)1}$	⋯	$\eta_{(m+l+q)(t+q+r+l)s}$	$P_1-\alpha_p$
N_{T_1}	$\eta_{(m+l+q+1)11}$	⋯	$\eta_{(m+l+q+1)1s}$	⋯	$\eta_{(m+l+q+1)t1}$	⋯	$\eta_{(m+l+q+1)ts}$	$\eta_{(m+l+q+1)(t+1)1}$	⋯	$\eta_{(m+l+q+1)(t+1)s}$	⋯	$\eta_{(m+l+q+1)(t+q)1}$	⋯	$\eta_{(m+l+q+1)(t+q)s}$	$\eta_{(m+l+q+1)(t+q+1)1}$	⋯	$\eta_{(m+l+q+1)(t+q+1)s}$	⋯	$\eta_{(m+l+q+1)(t+q+r)1}$	⋯	$\eta_{(m+l+q+1)(t+q+r)s}$	$\eta_{(m+l+q+1)(t+q+r+1)1}$	⋯	$\eta_{(m+l+q+1)(t+q+r+1)s}$	⋯	$\eta_{(m+l+q+1)(t+q+r+l)1}$	⋯	$\eta_{(m+l+q+1)(t+q+r+l)s}$	$P_1-\alpha_p$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{T_r}	$\eta_{(m+l+q+r)11}$	⋯	$\eta_{(m+l+q+r)1s}$	⋯	$\eta_{(m+l+q+r)t1}$	⋯	$\eta_{(m+l+q+r)ts}$	$\eta_{(m+l+q+r)(t+1)1}$	⋯	$\eta_{(m+l+q+r)(t+1)s}$	⋯	$\eta_{(m+l+q+r)(t+q)1}$	⋯	$\eta_{(m+l+q+r)(t+q)s}$	$\eta_{(m+l+q+r)(t+q+1)1}$	⋯	$\eta_{(m+l+q+r)(t+q+1)s}$	⋯	$\eta_{(m+l+q+r)(t+q+r)1}$	⋯	$\eta_{(m+l+q+r)(t+q+r)s}$	$\eta_{(m+l+q+r)(t+q+r+1)1}$	⋯	$\eta_{(m+l+q+r)(t+q+r+1)s}$	⋯	$\eta_{(m+l+q+r)(t+q+r+l)1}$	⋯	$\eta_{(m+l+q+r)(t+q+r+l)s}$	$P_1-\alpha_p$
Demand	$m'_1-\alpha'_1$			⋯	$m'_t-\alpha'_t$			$m''_{t+1}-\alpha''_{t+1}$			⋯	$m''_{t+q}-\alpha''_{t+q}$			$P_1-\alpha_p$			⋯	$P_1-\alpha_p$			$P_1-\alpha_p$			⋯	$P_1-\alpha_p$			

Table 6.3 Tabular representation of second crisp solid transportation problem

																													Capacity
Conveyance	E_1			⋯	E_1			E_1			⋯	E_1			E_1			⋯	E_1			E_1			⋯	E_1			α'''_1
		⋯		⋯		⋯			⋯		⋯		⋯			⋯		⋯		⋯			⋯		⋯		⋯		⋮
			E_s	⋯			E_s			E_s	⋯			E_s			E_s	⋯			E_s			E_s	⋯			E_s	α'''_s
Destinations → Sources↓	N_{PD_1}			⋯	N_{PD_t}			N_{D_1}			⋯	N_{D_q}			N_{T_1}			⋯	N_{T_r}			N_{S_1}			⋯	N_{S_l}			Availability
N_{PS_1}	ρ_{111}	⋯	ρ_{11s}	⋯	ρ_{1t1}	⋯	ρ_{1ts}	$\rho_{1(t+1)1}$	⋯	$\rho_{1(t+1)s}$	⋯	$\rho_{1(t+q)1}$	⋯	$\rho_{1(t+q)s}$	$\rho_{1(t+q+1)1}$	⋯	$\rho_{1(t+q+1)s}$	⋯	$\rho_{1(t+q+r)1}$	⋯	$\rho_{1(t+q+r)s}$	$\rho_{1(t+q+r+1)1}$	⋯	$\rho_{1(t+q+r+1)s}$	⋯	$\rho_{1(t+q+r+l)1}$	⋯	$\rho_{1(t+q+r+l)s}$	α_1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{PS_m}	ρ_{m11}	⋯	ρ_{m1s}	⋯	ρ_{mt1}	⋯	ρ_{mts}	$\rho_{m(t+1)1}$	⋯	$\rho_{m(t+1)s}$	⋯	$\rho_{m(t+q)1}$	⋯	$\rho_{m(t+q)s}$	$\rho_{m(t+q+1)1}$	⋯	$\rho_{m(t+q+1)s}$	⋯	$\rho_{m(t+q+r)1}$	⋯	$\rho_{m(t+q+r)s}$	$\rho_{m(t+q+r+1)1}$	⋯	$\rho_{m(t+q+r+1)s}$	⋯	$\rho_{m(t+q+r+l)1}$	⋯	$\rho_{m(t+q+r+l)s}$	α_m
N_{S_1}	$\rho_{(m+1)11}$	⋯	$\rho_{(m+1)1s}$	⋯	$\rho_{(m+1)t1}$	⋯	$\rho_{(m+1)ts}$	$\rho_{(m+1)(t+1)1}$	⋯	$\rho_{(m+1)(t+1)s}$	⋯	$\rho_{(m+1)(t+q)1}$	⋯	$\rho_{(m+1)(t+q)s}$	$\rho_{(m+1)(t+q+1)1}$	⋯	$\rho_{(m+1)(t+q+1)s}$	⋯	$\rho_{(m+1)(t+q+r)1}$	⋯	$\rho_{(m+1)(t+q+r)s}$	$\rho_{(m+1)(t+q+r+1)1}$	⋯	$\rho_{(m+1)(t+q+r+1)s}$	⋯	$\rho_{(m+1)(t+q+r+l)1}$	⋯	$\rho_{(m+1)(t+q+r+l)s}$	α''_{m+1}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{S_l}	$\rho_{(m+l)11}$	⋯	$\rho_{(m+l)1s}$	⋯	$\rho_{(m+l)t1}$	⋯	$\rho_{(m+l)ts}$	$\rho_{(m+l)(t+1)1}$	⋯	$\rho_{(m+l)(t+1)s}$	⋯	$\rho_{(m+l)(t+q)1}$	⋯	$\rho_{(m+l)(t+q)s}$	$\rho_{(m+l)(t+q+1)1}$	⋯	$\rho_{(m+l)(t+q+1)s}$	⋯	$\rho_{(m+l)(t+q+r)1}$	⋯	$\rho_{(m+l)(t+q+r)s}$	$\rho_{(m+l)(t+q+r+1)1}$	⋯	$\rho_{(m+l)(t+q+r+1)s}$	⋯	$\rho_{(m+l)(t+q+r+l)1}$	⋯	$\rho_{(m+l)(t+q+r+l)s}$	α''_{m+l}
N_{D_1}	$\rho_{(m+l+1)11}$	⋯	$\rho_{(m+l+1)1s}$	⋯	$\rho_{(m+l+1)t1}$	⋯	$\rho_{(m+l+1)ts}$	$\rho_{(m+l+1)(t+1)1}$	⋯	$\rho_{(m+l+1)(t+1)s}$	⋯	$\rho_{(m+l+1)(t+q)1}$	⋯	$\rho_{(m+l+1)(t+q)s}$	$\rho_{(m+l+1)(t+q+1)1}$	⋯	$\rho_{(m+l+1)(t+q+1)s}$	⋯	$\rho_{(m+l+1)(t+q+r)1}$	⋯	$\rho_{(m+l+1)(t+q+r)s}$	$\rho_{(m+l+1)(t+q+r+1)1}$	⋯	$\rho_{(m+l+1)(t+q+r+1)s}$	⋯	$\rho_{(m+l+1)(t+q+r+l)1}$	⋯	$\rho_{(m+l+1)(t+q+r+l)s}$	α_p
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{D_q}	$\rho_{(m+l+q)11}$	⋯	$\rho_{(m+l+q)1s}$	⋯	$\rho_{(m+l+q)t1}$	⋯	$\rho_{(m+l+q)ts}$	$\rho_{(m+l+q)(t+1)1}$	⋯	$\rho_{(m+l+q)(t+1)s}$	⋯	$\rho_{(m+l+q)(t+q)1}$	⋯	$\rho_{(m+l+q)(t+q)s}$	$\rho_{(m+l+q)(t+q+1)1}$	⋯	$\rho_{(m+l+q)(t+q+1)s}$	⋯	$\rho_{(m+l+q)(t+q+r)1}$	⋯	$\rho_{(m+l+q)(t+q+r)s}$	$\rho_{(m+l+q)(t+q+r+1)1}$	⋯	$\rho_{(m+l+q)(t+q+r+1)s}$	⋯	$\rho_{(m+l+q)(t+q+r+l)1}$	⋯	$\rho_{(m+l+q)(t+q+r+l)s}$	α_p
N_{T_1}	$\rho_{(m+l+q+1)11}$	⋯	$\rho_{(m+l+q+1)1s}$	⋯	$\rho_{(m+l+q+1)t1}$	⋯	$\rho_{(m+l+q+1)ts}$	$\rho_{(m+l+q+1)(t+1)1}$	⋯	$\rho_{(m+l+q+1)(t+1)s}$	⋯	$\rho_{(m+l+q+1)(t+q)1}$	⋯	$\rho_{(m+l+q+1)(t+q)s}$	$\rho_{(m+l+q+1)(t+q+1)1}$	⋯	$\rho_{(m+l+q+1)(t+q+1)s}$	⋯	$\rho_{(m+l+q+1)(t+q+r)1}$	⋯	$\rho_{(m+l+q+1)(t+q+r)s}$	$\rho_{(m+l+q+1)(t+q+r+1)1}$	⋯	$\rho_{(m+l+q+1)(t+q+r+1)s}$	⋯	$\rho_{(m+l+q+1)(t+q+r+l)1}$	⋯	$\rho_{(m+l+q+1)(t+q+r+l)s}$	α_p
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{T_r}	$\rho_{(m+l+q+r)11}$	⋯	$\rho_{(m+l+q+r)1s}$	⋯	$\rho_{(m+l+q+r)t1}$	⋯	$\rho_{(m+l+q+r)ts}$	$\rho_{(m+l+q+r)(t+1)1}$	⋯	$\rho_{(m+l+q+r)(t+1)s}$	⋯	$\rho_{(m+l+q+r)(t+q)1}$	⋯	$\rho_{(m+l+q+r)(t+q)s}$	$\rho_{(m+l+q+r)(t+q+1)1}$	⋯	$\rho_{(m+l+q+r)(t+q+1)s}$	⋯	$\rho_{(m+l+q+r)(t+q+r)1}$	⋯	$\rho_{(m+l+q+r)(t+q+r)s}$	$\rho_{(m+l+q+r)(t+q+r+1)1}$	⋯	$\rho_{(m+l+q+r)(t+q+r+1)s}$	⋯	$\rho_{(m+l+q+r)(t+q+r+l)1}$	⋯	$\rho_{(m+l+q+r)(t+q+r+l)s}$	α_p
Demand	α'_1			⋯	α'_t			α''_{t+1}			⋯	α''_{t+q}			α_p			⋯	α_p			α_p			⋯	α_p			

Table 6.4 Tabular representation of third crisp solid transportation problem

																													Capacity
Conveyance	E_1			⋯	E_1			E_1			⋯	E_1			E_1			⋯	E_1			E_1			⋯	E_1			$n'''_1 - m'''_1$
		⋯		⋯		⋯			⋯		⋯		⋯			⋯		⋯		⋯			⋯		⋯		⋯		⋮
			E_s	⋯			E_s			E_s	⋯			E_s			E_s	⋯			E_s			E_s	⋯			E_s	$n'''_s - m'''_s$
Destinations → Sources↓	N_{PD_1}			⋯	N_{PD_t}			N_{D_1}			⋯	N_{D_q}			N_{T_1}			⋯	N_{T_r}			N_{S_1}			⋯	N_{S_l}			Availability
N_{PS_1}	δ_{111}	⋯	δ_{11s}	⋯	δ_{1t1}	⋯	δ_{1ts}	$\delta_{1(t+1)1}$	⋯	$\delta_{1(t+1)s}$	⋯	$\delta_{1(t+q)1}$	⋯	$\delta_{1(t+q)s}$	$\delta_{1(t+q+1)1}$	⋯	$\delta_{1(t+q+1)s}$	⋯	$\delta_{1(t+q+r)1}$	⋯	$\delta_{1(t+q+r)s}$	$\delta_{1(t+q+r+1)1}$	⋯	$\delta_{1(t+q+r+1)s}$	⋯	$\delta_{1(t+q+r+l)1}$	⋯	$\delta_{1(t+q+r+l)s}$	$n_1 - m_1$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{PS_m}	δ_{m11}	⋯	δ_{m1s}	⋯	δ_{mt1}	⋯	δ_{mts}	$\delta_{m(t+1)1}$	⋯	$\delta_{m(t+1)s}$	⋯	$\delta_{m(t+q)1}$	⋯	$\delta_{m(t+q)s}$	$\delta_{m(t+q+1)1}$	⋯	$\delta_{m(t+q+1)s}$	⋯	$\delta_{m(t+q+r)1}$	⋯	$\delta_{m(t+q+r)s}$	$\delta_{m(t+q+r+1)1}$	⋯	$\delta_{m(t+q+r+1)s}$	⋯	$\delta_{m(t+q+r+l)1}$	⋯	$\delta_{m(t+q+r+l)s}$	$n_m - m_m$
N_{S_1}	$\delta_{(m+1)11}$	⋯	$\delta_{(m+1)1s}$	⋯	$\delta_{(m+1)t1}$	⋯	$\delta_{(m+1)ts}$	$\delta_{(m+1)(t+1)1}$	⋯	$\delta_{(m+1)(t+1)s}$	⋯	$\delta_{(m+1)(t+q)1}$	⋯	$\delta_{(m+1)(t+q)s}$	$\delta_{(m+1)(t+q+1)1}$	⋯	$\delta_{(m+1)(t+q+1)s}$	⋯	$\delta_{(m+1)(t+q+r)1}$	⋯	$\delta_{(m+1)(t+q+r)s}$	$\delta_{(m+1)(t+q+r+1)1}$	⋯	$\delta_{(m+1)(t+q+r+1)s}$	⋯	$\delta_{(m+1)(t+q+r+l)1}$	⋯	$\delta_{(m+1)(t+q+r+l)s}$	$n''_{m+1} - m''_{m+1}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{S_l}	$\delta_{(m+l)11}$	⋯	$\delta_{(m+l)1s}$	⋯	$\delta_{(m+l)t1}$	⋯	$\delta_{(m+l)ts}$	$\delta_{(m+l)(t+1)1}$	⋯	$\delta_{(m+l)(t+1)s}$	⋯	$\delta_{(m+l)(t+q)1}$	⋯	$\delta_{(m+l)(t+q)s}$	$\delta_{(m+l)(t+q+1)1}$	⋯	$\delta_{(m+l)(t+q+1)s}$	⋯	$\delta_{(m+l)(t+q+r)1}$	⋯	$\delta_{(m+l)(t+q+r)s}$	$\delta_{(m+l)(t+q+r+1)1}$	⋯	$\delta_{(m+l)(t+q+r+1)s}$	⋯	$\delta_{(m+l)(t+q+r+l)1}$	⋯	$\delta_{(m+l)(t+q+r+l)s}$	$n''_{m+l} - m''_{m+l}$
N_{D_1}	$\delta_{(m+l+1)11}$	⋯	$\delta_{(m+l+1)1s}$	⋯	$\delta_{(m+l+1)t1}$	⋯	$\delta_{(m+l+1)ts}$	$\delta_{(m+l+1)(t+1)1}$	⋯	$\delta_{(m+l+1)(t+1)s}$	⋯	$\delta_{(m+l+1)(t+q)1}$	⋯	$\delta_{(m+l+1)(t+q)s}$	$\delta_{(m+l+1)(t+q+1)1}$	⋯	$\delta_{(m+l+1)(t+q+1)s}$	⋯	$\delta_{(m+l+1)(t+q+r)1}$	⋯	$\delta_{(m+l+1)(t+q+r)s}$	$\delta_{(m+l+1)(t+q+r+1)1}$	⋯	$\delta_{(m+l+1)(t+q+r+1)s}$	⋯	$\delta_{(m+l+1)(t+q+r+l)1}$	⋯	$\delta_{(m+l+1)(t+q+r+l)s}$	$P_2 - P_1$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{D_q}	$\delta_{(m+l+q)11}$	⋯	$\delta_{(m+l+q)1s}$	⋯	$\delta_{(m+l+q)t1}$	⋯	$\delta_{(m+l+q)ts}$	$\delta_{(m+l+q)(t+1)1}$	⋯	$\delta_{(m+l+q)(t+1)s}$	⋯	$\delta_{(m+l+q)(t+q)1}$	⋯	$\delta_{(m+l+q)(t+q)s}$	$\delta_{(m+l+q)(t+q+1)1}$	⋯	$\delta_{(m+l+q)(t+q+1)s}$	⋯	$\delta_{(m+l+q)(t+q+r)1}$	⋯	$\delta_{(m+l+q)(t+q+r)s}$	$\delta_{(m+l+q)(t+q+r+1)1}$	⋯	$\delta_{(m+l+q)(t+q+r+1)s}$	⋯	$\delta_{(m+l+q)(t+q+r+l)1}$	⋯	$\delta_{(m+l+q)(t+q+r+l)s}$	$P_2 - P_1$
N_{T_1}	$\delta_{(m+l+q+1)11}$	⋯	$\delta_{(m+l+q+1)1s}$	⋯	$\delta_{(m+l+q+1)t1}$	⋯	$\delta_{(m+l+q+1)ts}$	$\delta_{(m+l+q+1)(t+1)1}$	⋯	$\delta_{(m+l+q+1)(t+1)s}$	⋯	$\delta_{(m+l+q+1)(t+q)1}$	⋯	$\delta_{(m+l+q+1)(t+q)s}$	$\delta_{(m+l+q+1)(t+q+1)1}$	⋯	$\delta_{(m+l+q+1)(t+q+1)s}$	⋯	$\delta_{(m+l+q+1)(t+q+r)1}$	⋯	$\delta_{(m+l+q+1)(t+q+r)s}$	$\delta_{(m+l+q+1)(t+q+r+1)1}$	⋯	$\delta_{(m+l+q+1)(t+q+r+1)s}$	⋯	$\delta_{(m+l+q+1)(t+q+r+l)1}$	⋯	$\delta_{(m+l+q+1)(t+q+r+l)s}$	$P_2 - P_1$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{T_r}	$\delta_{(m+l+q+r)11}$	⋯	$\delta_{(m+l+q+r)1s}$	⋯	$\delta_{(m+l+q+r)t1}$	⋯	$\delta_{(m+l+q+r)ts}$	$\delta_{(m+l+q+r)(t+1)1}$	⋯	$\delta_{(m+l+q+r)(t+1)s}$	⋯	$\delta_{(m+l+q+r)(t+q)1}$	⋯	$\delta_{(m+l+q+r)(t+q)s}$	$\delta_{(m+l+q+r)(t+q+1)1}$	⋯	$\delta_{(m+l+q+r)(t+q+1)s}$	⋯	$\delta_{(m+l+q+r)(t+q+r)1}$	⋯	$\delta_{(m+l+q+r)(t+q+r)s}$	$\delta_{(m+l+q+r)(t+q+r+1)1}$	⋯	$\delta_{(m+l+q+r)(t+q+r+1)s}$	⋯	$\delta_{(m+l+q+r)(t+q+r+l)1}$	⋯	$\delta_{(m+l+q+r)(t+q+r+l)s}$	$P_2 - P_1$
Demand	$n'_1 - m'_1$			⋯	$n'_t - m'_t$			$n''_{t+1} - m''_{t+1}$			⋯	$n''_{t+q} - m''_{t+q}$			$P_2 - P_1$			⋯	$P_2 - P_1$			$P_2 - P_1$			⋯	$P_2 - P_1$			

Table 6.5 Tabular representation of fourth crisp solid transportation problem

																													Capacity
Conveyance	E_1			⋯	E_1			E_1			⋯	E_1			E_1			⋯	E_1			E_1			⋯	E_1			β'''_1
		⋯		⋯		⋯			⋯		⋯		⋯			⋯		⋯		⋯			⋯		⋯		⋯		⋮
			E_s	⋯			E_s			E_s	⋯			E_s			E_s	⋯			E_s			E_s	⋯			E_s	β'''_s
Destinations → Sources↓	N_{PD_1}			⋯	N_{PD_t}			N_{D_1}			⋯	N_{D_q}			N_{T_1}			⋯	N_{T_r}			N_{S_1}			⋯	N_{S_l}			Availability
N_{PS_1}	ξ_{111}	⋯	ξ_{11s}	⋯	ξ_{1t1}	⋯	ξ_{1ts}	$\xi_{1(t+1)1}$	⋯	$\xi_{1(t+1)s}$	⋯	$\xi_{1(t+q)1}$	⋯	$\xi_{1(t+q)s}$	$\xi_{1(t+q+1)1}$	⋯	$\xi_{1(t+q+1)s}$	⋯	$\xi_{1(t+q+r)1}$	⋯	$\xi_{1(t+q+r)s}$	$\xi_{1(t+q+r+1)1}$	⋯	$\xi_{1(t+q+r+1)s}$	⋯	$\xi_{1(t+q+r+l)1}$	⋯	$\xi_{1(t+q+r+l)s}$	β_1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{PS_m}	ξ_{m11}	⋯	ξ_{m1s}	⋯	ξ_{mt1}	⋯	ξ_{mts}	$\xi_{m(t+1)1}$	⋯	$\xi_{m(t+1)s}$	⋯	$\xi_{m(t+q)1}$	⋯	$\xi_{m(t+q)s}$	$\xi_{m(t+q+1)1}$	⋯	$\xi_{m(t+q+1)s}$	⋯	$\xi_{m(t+q+r)1}$	⋯	$\xi_{m(t+q+r)s}$	$\xi_{m(t+q+r+1)1}$	⋯	$\xi_{m(t+q+r+1)s}$	⋯	$\xi_{m(t+q+r+l)1}$	⋯	$\xi_{m(t+q+r+l)s}$	β_m
N_{S_1}	$\xi_{(m+1)11}$	⋯	$\xi_{(m+1)1s}$	⋯	$\xi_{(m+1)t1}$	⋯	$\xi_{(m+1)ts}$	$\xi_{(m+1)(t+1)1}$	⋯	$\xi_{(m+1)(t+1)s}$	⋯	$\xi_{(m+1)(t+q)1}$	⋯	$\xi_{(m+1)(t+q)s}$	$\xi_{(m+1)(t+q+1)1}$	⋯	$\xi_{(m+1)(t+q+1)s}$	⋯	$\xi_{(m+1)(t+q+r)1}$	⋯	$\xi_{(m+1)(t+q+r)s}$	$\xi_{(m+1)(t+q+r+1)1}$	⋯	$\xi_{(m+1)(t+q+r+1)s}$	⋯	$\xi_{(m+1)(t+q+r+l)1}$	⋯	$\xi_{(m+1)(t+q+r+l)s}$	β''_{m+1}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{S_l}	$\xi_{(m+l)11}$	⋯	$\xi_{(m+l)1s}$	⋯	$\xi_{(m+l)t1}$	⋯	$\xi_{(m+l)ts}$	$\xi_{(m+l)(t+1)1}$	⋯	$\xi_{(m+l)(t+1)s}$	⋯	$\xi_{(m+l)(t+q)1}$	⋯	$\xi_{(m+l)(t+q)s}$	$\xi_{(m+l)(t+q+1)1}$	⋯	$\xi_{(m+l)(t+q+1)s}$	⋯	$\xi_{(m+l)(t+q+r)1}$	⋯	$\xi_{(m+l)(t+q+r)s}$	$\xi_{(m+l)(t+q+r+1)1}$	⋯	$\xi_{(m+l)(t+q+r+1)s}$	⋯	$\xi_{(m+l)(t+q+r+l)1}$	⋯	$\xi_{(m+l)(t+q+r+l)s}$	β''_{m+l}
N_{D_1}	$\xi_{(m+l+1)11}$	⋯	$\xi_{(m+l+1)1s}$	⋯	$\xi_{(m+l+1)t1}$	⋯	$\xi_{(m+l+1)ts}$	$\xi_{(m+l+1)(t+1)1}$	⋯	$\xi_{(m+l+1)(t+1)s}$	⋯	$\xi_{(m+l+1)(t+q)1}$	⋯	$\xi_{(m+l+1)(t+q)s}$	$\xi_{(m+l+1)(t+q+1)1}$	⋯	$\xi_{(m+l+1)(t+q+1)s}$	⋯	$\xi_{(m+l+1)(t+q+r)1}$	⋯	$\xi_{(m+l+1)(t+q+r)s}$	$\xi_{(m+l+1)(t+q+r+1)1}$	⋯	$\xi_{(m+l+1)(t+q+r+1)s}$	⋯	$\xi_{(m+l+1)(t+q+r+l)1}$	⋯	$\xi_{(m+l+1)(t+q+r+l)s}$	β_p
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{D_q}	$\xi_{(m+l+q)11}$	⋯	$\xi_{(m+l+q)1s}$	⋯	$\xi_{(m+l+q)t1}$	⋯	$\xi_{(m+l+q)ts}$	$\xi_{(m+l+q)(t+1)1}$	⋯	$\xi_{(m+l+q)(t+1)s}$	⋯	$\xi_{(m+l+q)(t+q)1}$	⋯	$\xi_{(m+l+q)(t+q)s}$	$\xi_{(m+l+q)(t+q+1)1}$	⋯	$\xi_{(m+l+q)(t+q+1)s}$	⋯	$\xi_{(m+l+q)(t+q+r)1}$	⋯	$\xi_{(m+l+q)(t+q+r)s}$	$\xi_{(m+l+q)(t+q+r+1)1}$	⋯	$\xi_{(m+l+q)(t+q+r+1)s}$	⋯	$\xi_{(m+l+q)(t+q+r+l)1}$	⋯	$\xi_{(m+l+q)(t+q+r+l)s}$	β_p
N_{T_1}	$\xi_{(m+l+q+1)11}$	⋯	$\xi_{(m+l+q+1)1s}$	⋯	$\xi_{(m+l+q+1)t1}$	⋯	$\xi_{(m+l+q+1)ts}$	$\xi_{(m+l+q+1)(t+1)1}$	⋯	$\xi_{(m+l+q+1)(t+1)s}$	⋯	$\xi_{(m+l+q+1)(t+q)1}$	⋯	$\xi_{(m+l+q+1)(t+q)s}$	$\xi_{(m+l+q+1)(t+q+1)1}$	⋯	$\xi_{(m+l+q+1)(t+q+1)s}$	⋯	$\xi_{(m+l+q+1)(t+q+r)1}$	⋯	$\xi_{(m+l+q+1)(t+q+r)s}$	$\xi_{(m+l+q+1)(t+q+r+1)1}$	⋯	$\xi_{(m+l+q+1)(t+q+r+1)s}$	⋯	$\xi_{(m+l+q+1)(t+q+r+l)1}$	⋯	$\xi_{(m+l+q+1)(t+q+r+l)s}$	β_p
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N_{T_r}	$\xi_{(m+l+q+r)11}$	⋯	$\xi_{(m+l+q+r)1s}$	⋯	$\xi_{(m+l+q+r)t1}$	⋯	$\xi_{(m+l+q+r)ts}$	$\xi_{(m+l+q+r)(t+1)1}$	⋯	$\xi_{(m+l+q+r)(t+1)s}$	⋯	$\xi_{(m+l+q+r)(t+q)1}$	⋯	$\xi_{(m+l+q+r)(t+q)s}$	$\xi_{(m+l+q+r)(t+q+1)1}$	⋯	$\xi_{(m+l+q+r)(t+q+1)s}$	⋯	$\xi_{(m+l+q+r)(t+q+r)1}$	⋯	$\xi_{(m+l+q+r)(t+q+r)s}$	$\xi_{(m+l+q+r)(t+q+r+1)1}$	⋯	$\xi_{(m+l+q+r)(t+q+r+1)s}$	⋯	$\xi_{(m+l+q+r)(t+q+r+l)1}$	⋯	$\xi_{(m+l+q+r)(t+q+r+l)s}$	β_p
Demand	β'_1			⋯	β'_t			β''_{t+1}			⋯	β''_{t+q}			β_p			⋯	β_p			β_p			⋯	β_p			

6.3.3 Advantages of the proposed methods over existing methods

Since, the advantages of the methods, proposed in Chapter 4 and Chapter 5, over the methods proposed in Chapter 2, Chapter 3 and over the existing methods [70, 132] are already discussed in Chapter 4 and Chapter 5 respectively. So, in this section, only the advantages of the methods, proposed in this chapter, over the methods, proposed in Chapter 4 and Chapter 5, are discussed.

(1) The methods, proposed in Chapter 4, can be used to find the fuzzy optimal solution of fully fuzzy transportation problems and fully fuzzy transshipment problems but these methods can neither be used for solving fully fuzzy solid transportation problems nor for solving fully fuzzy solid transshipment problems. Since, fully fuzzy transportation problems, fully fuzzy transshipment problems and fully fuzzy solid transportation problems are special type of fully fuzzy solid transshipment problems so, the methods, proposed in this chapter, can be used for finding the fuzzy optimal solution of all these problems.

(2) The methods, proposed in Chapter 5, can be used to find the fuzzy optimal solution of fully fuzzy transportation problems and fully fuzzy solid transportation problems but these methods can neither be used for solving fully fuzzy transshipment problems nor for solving fully fuzzy solid transshipment problems. Since, fully fuzzy transportation problems, fully fuzzy transshipment problems and fully fuzzy solid transportation problems are special type of fully fuzzy solid transshipment problems so, the methods, proposed in this chapter, can be used for finding the fuzzy optimal solution of all these problems.

6.4 Illustrative example

In this section, the fuzzy optimal solution of the fully fuzzy solid transshipment problem, chosen in Example 6.1, is obtained by using the proposed methods.

6.4.1 Fuzzy optimal solution of the chosen problem using the proposed method based on fuzzy linear programming formulation

Using the proposed method, based on fuzzy linear programming formulation, the fuzzy optimal solution of the fully fuzzy solid transshipment problem, chosen in Example 6.1, can be obtained as follows:

Step 1 Total fuzzy availability $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = (110, 150, 40, 40)_{LR}$, total fuzzy demand $\sum_{j \in N_D} \tilde{b}_j \oplus \sum_{j \in N_{PD}} \tilde{b}_j = (50, 80, 30, 50)_{LR}$ and total fuzzy capacity $\sum_{k \in S_C} \tilde{e}_k = (110, 150, 40, 50)_{LR}$. Since $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i \neq \sum_{j \in N_D} \tilde{b}_j \oplus \sum_{j \in N_{PD}} \tilde{b}_j \neq \sum_{k \in S_C} \tilde{e}_k$, so it is an unbalanced fully fuzzy solid transshipment problem.

Step 2 Comparing $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = (110, 150, 40, 40)_{LR}$ by $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = (m, n, \alpha, \beta)_{LR}$ and $\sum_{j \in N_{PD}} \tilde{b}_j \oplus \sum_{j \in N_D} \tilde{b'}_j = (50, 80, 30, 50)_{LR}$ by $\sum_{j \in N_{PD}} \tilde{b}_j \oplus \sum_{j \in N_D} \tilde{b}'_j = (m', n', \alpha', \beta')_{LR}$ the values of $m, n, \alpha, \beta, m', n', \alpha'$ and β' are 110, 150, 40, 40, 50, 80, 30 and 50 respectively.

Since, $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i \neq \sum_{j \in N_{PD}} \tilde{b}_j \oplus \sum_{j \in N_D} \tilde{b'}_j$ and neither the condition $m - \alpha \leq m' - \alpha'$, $\alpha \leq \alpha'$, $n - m \leq n' - m'$, $\beta \leq \beta'$ nor the condition $m - \alpha \geq m' - \alpha'$, $\alpha \geq \alpha'$, $n - m \geq n' - m'$, and $\beta \geq \beta'$ is satisfying so, as described in Step 2 (Case (2c)) of the proposed method, there is need to introduce a dummy purely source node 4 with fuzzy availability $\tilde{a}_4 = (0, 0, 0, 10)_{LR}$ and a dummy purely destination node 5 with fuzzy demand $\tilde{b}_5 = (60, 70, 10, 0)_{LR}$ so that $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = \sum_{j \in N_D} \tilde{b}_j \oplus \sum_{j \in N_{PD}} \tilde{b}_j$.

Step 3 Since, $\sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = \sum_{j \in N_D} \tilde{b}_j \oplus \sum_{j \in N_{PD}} \tilde{b}_j = (110, 150, 40, 50)_{LR} = \sum_{k \in S_C} \tilde{e}_k$, so the fully fuzzy solid transshipment problem, obtained in Step 2, is a balanced fully fuzzy solid transshipment problem.

Step 4 Since, a dummy purely source node (4) and a dummy purely destination node (5) are introduced. So, as described in Step 4 of the proposed method, by assuming $\tilde{c}_{4jk} = \tilde{c}_{i5k} = (0, 0, 0, 0)_{LR}\ \forall\ i = 1, 2, 4;\ j = 3, 5;\ k = 1, 2$, the fuzzy linear programming formulation of the balanced fully fuzzy solid transshipment problem, obtained from Step 3, can be written as:

Minimize $((8, 10, 2, 2)_{LR} \otimes \tilde{x}_{131} \oplus (4, 8, 3, 2)_{LR} \otimes \tilde{x}_{132} \oplus (8, 10, 4, 4)_{LR} \otimes \tilde{x}_{211} \oplus (6, 8, 4, 4)_{LR} \otimes \tilde{x}_{212} \oplus (9, 12, 6, 3)_{LR} \otimes \tilde{x}_{231} \oplus (3, 6, 2, 3)_{LR} \otimes \tilde{x}_{232} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{411} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{412} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{431} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{432} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{251} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{252} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{451} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{452} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{151} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{152})$

subject to

$$\sum_{k=1}^{2} (\tilde{x}_{13k} \oplus \tilde{x}_{15k}) \ominus_H \sum_{k=1}^{2} (\tilde{x}_{41k} \oplus \tilde{x}_{21k}) = (60, 80, 20, 20)_{LR}$$
$$\sum_{k=1}^{2} (\tilde{x}_{21k} \oplus \tilde{x}_{23k} \oplus \tilde{x}_{25k}) = (50, 70, 20, 20)_{LR}$$
$$\sum_{k=1}^{2} (\tilde{x}_{13k} \oplus \tilde{x}_{23k} \oplus \tilde{x}_{43k}) = (50, 80, 30, 50)_{LR}$$
$$\sum_{k=1}^{2} (\tilde{x}_{41k} \oplus \tilde{x}_{43k} \oplus \tilde{x}_{45k}) = (0, 0, 0, 10)_{LR}$$
$$\sum_{k=1}^{2} (\tilde{x}_{15k} \oplus \tilde{x}_{25k} \oplus \tilde{x}_{45k}) = (60, 70, 10, 0)_{LR}$$
$$\tilde{x}_{131} \oplus \tilde{x}_{151} \oplus \tilde{x}_{211} \oplus \tilde{x}_{231} \oplus \tilde{x}_{251} \oplus \tilde{x}_{411} \oplus \tilde{x}_{431} \oplus \tilde{x}_{451} = (60, 80, 20, 10)_{LR}$$
$$\tilde{x}_{132} \oplus \tilde{x}_{152} \oplus \tilde{x}_{212} \oplus \tilde{x}_{232} \oplus \tilde{x}_{252} \oplus \tilde{x}_{412} \oplus \tilde{x}_{432} \oplus \tilde{x}_{452} = (50, 70, 20, 40)_{LR}$$

$\tilde{x}_{ijk}$ are non-negative LR flat fuzzy numbers $\forall\ i = 1, 2, 4;\ j = 3, 5\ ;\ k = 1, 2$.

Step 5 Using Step 6 to Step 9 of the method, proposed in Section 6.3.1, the fuzzy linear programming problem, obtained in Step 5, can be converted into the following crisp linear programming problem:

Minimize $(\frac{1}{4}(14m_{131} - 6\alpha_{131} + 22n_{131} + 12\beta_{131} + 5m_{132} - \alpha_{132} + 18n_{132} + 10\beta_{132} +$

$12m_{211} - 4\alpha_{211} + 24n_{211} + 14\beta_{211} + 8m_{212} - 2\alpha_{212} + 20n_{212} + 12\beta_{212} + 12m_{231} - 3\alpha_{231} +$

$27n_{231} + 15\beta_{231} + 4m_{232} - 1\alpha_{232} + 15n_{232} + 9\beta_{232}))$

subject to

$$\sum_{k=1}^{2}(m_{13k} + m_{15k}) - \sum_{k=1}^{2}(m_{41k} + m_{21k}) = 60$$
$$\sum_{k=1}^{2}(n_{13k} + n_{15k}) - \sum_{k=1}^{2}(n_{41k} + n_{21k}) = 80$$
$$\sum_{k=1}^{2}(\alpha_{13k} + \alpha_{15k}) - \sum_{k=1}^{2}(\alpha_{41k} + \alpha_{21k}) = 20$$
$$\sum_{k=1}^{2}(\beta_{13k} + \beta_{15k}) - \sum_{k=1}^{2}(\beta_{41k} + \beta_{21k}) = 20$$
$$\sum_{k=1}^{2}(m_{21k} + m_{23k} + m_{25k}) = 50$$
$$\sum_{k=1}^{2}(n_{21k} + n_{23k} + n_{25k}) = 70$$
$$\sum_{k=1}^{2}(\alpha_{21k} + \alpha_{23k} + \alpha_{25k}) = 20$$
$$\sum_{k=1}^{2}(\beta_{21k} + \beta_{23k} + \beta_{25k}) = 20$$
$$\sum_{k=1}^{2}(m_{13k} + m_{23k} + m_{43k}) = 50$$
$$\sum_{k=1}^{2}(n_{13k} + n_{23k} + n_{43k}) = 80$$
$$\sum_{k=1}^{2}(\alpha_{13k} + \alpha_{23k} + \alpha_{43k}) = 30$$
$$\sum_{k=1}^{2}(\beta_{13k} + \beta_{23k} + \beta_{43k}) = 50$$
$$\sum_{k=1}^{2}(m_{41k} + m_{43k} + m_{45k}) = 0$$
$$\sum_{k=1}^{2}(n_{41k} + n_{43k} + n_{45k}) = 0$$
$$\sum_{k=1}^{2}(\alpha_{41k} + \alpha_{43k} + \alpha_{45k}) = 0$$
$$\sum_{k=1}^{2}(\beta_{41k} + \beta_{43k} + \beta_{45k}) = 10$$
$$\sum_{k=1}^{2}(m_{15k} + m_{25k} + m_{45k}) = 60$$
$$\sum_{k=1}^{2}(n_{15k} + n_{25k} + n_{45k}) = 70$$

$\sum_{k=1}^{2}(\alpha_{15k}+\alpha_{25k}+\alpha_{45k})=10$

$\sum_{k=1}^{2}(\beta_{15k}+\beta_{25k}+\beta_{45k})=0$

$m_{131}+m_{151}+m_{211}+m_{231}+m_{251}+m_{411}+m_{431}+m_{451}=60$

$n_{131}+n_{151}+n_{211}+n_{231}+n_{251}+n_{411}+n_{431}+n_{451}=80$

$\alpha_{131}+\alpha_{151}+\alpha_{211}+\alpha_{231}+\alpha_{251}+\alpha_{411}+\alpha_{431}+\alpha_{451}=20$

$\beta_{131}+\beta_{151}+\beta_{211}+\beta_{231}+\beta_{251}+\beta_{411}+\beta_{431}+\beta_{451}=10$

$m_{132}+m_{152}+m_{212}+m_{232}+m_{252}+m_{412}+m_{432}+m_{452}=50$

$n_{132}+n_{152}+n_{212}+n_{232}+n_{252}+n_{412}+n_{432}+n_{452}=70$

$\alpha_{132}+\alpha_{152}+\alpha_{212}+\alpha_{232}+\alpha_{252}+\alpha_{412}+\alpha_{432}+\alpha_{452}=20$

$\beta_{132}+\beta_{152}+\beta_{212}+\beta_{232}+\beta_{252}+\beta_{412}+\beta_{432}+\beta_{452}=40$

$m_{ijk}-\alpha_{ijk}, n_{ijk}-m_{ijk}, \alpha_{ijk}, \beta_{ijk} \geq\ 0\ \forall\ i=1,2,4\ ;\ j=3,5\ ;\ k=1,2.$

Step 6 The optimal solution of the crisp linear programming problem, obtained from Step 5, is $m_{131}=10, n_{131}=20, \alpha_{131}=10, \beta_{131}=0, m_{132}=0, n_{132}=0, \alpha_{132}=0, \beta_{132}=20, m_{232}=40, n_{232}=60, \alpha_{232}=20, \beta_{232}=20, m_{151}=40, n_{151}=50, \alpha_{151}=10, \beta_{151}=0, m_{152}=10, n_{152}=10, \alpha_{152}=0, \beta_{152}=0, m_{251}=10, n_{251}=10, \alpha_{251}=0, \beta_{251}=0, m_{431}=0, n_{431}=0, \alpha_{431}=0, \beta_{431}=10$ and the remaining values of $m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk}$ are zero.

Step 7 Putting the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}$ and β_{ijk} in $\tilde{x}_{ij}=(m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$, the fuzzy optimal solution is $\tilde{x}_{131}=(10,20,10,0)_{LR}$, $\tilde{x}_{132}=(0,0,0,20)_{LR}$, $\tilde{x}_{232}=(40,60,20,20)_{LR}$, $\tilde{x}_{151}=(40,50,10,0)_{LR}$, $\tilde{x}_{251}=(10,10,0,0)_{LR}$, $\tilde{x}_{431}=(0,0,0,10)_{LR}$, $\tilde{x}_{152}=(10,10,0,0)_{LR}$ and remaining values of $\tilde{x}_{ijk}=(0,0,0,0)_{LR}$.

Step 8 Putting the values of $\tilde{x}_{131}, \tilde{x}_{1321}, \tilde{x}_{211}, \tilde{x}_{212}, \tilde{x}_{231}, \tilde{x}_{232}, \tilde{x}_{411}, \tilde{x}_{412}, \tilde{x}_{431}, \tilde{x}_{432}, \tilde{x}_{451}, \tilde{x}_{452}, \tilde{x}_{251}, \tilde{x}_{252}, \tilde{x}_{151}, \tilde{x}_{152}$ in $((8,10,2,2)_{LR}\otimes\tilde{x}_{131}\oplus(4,8,3,2)_{LR}\otimes\tilde{x}_{132}\oplus(8,10,4,4)_{LR}\otimes$

$\tilde{x}_{211} \oplus (6,8,4,4)_{LR} \otimes \tilde{x}_{212} \oplus (9,12,6,3)_{LR} \otimes \tilde{x}_{231} \oplus (3,6,2,3)_{LR} \otimes \tilde{x}_{232} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{411} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{412} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{431} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{432} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{251} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{252} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{451} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{452} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{151} \oplus (0,0,0,0)_{LR} \otimes \tilde{x}_{152})$, the minimum total fuzzy transportation cost is $(200, 560, 180, 600)_{LR}$.

6.4.2 Fuzzy optimal solution of the chosen problem using the proposed method based on tabular representation

Using the proposed method, based on tabular representation, the fuzzy optimal solution of the fully fuzzy solid transshipment problem, chosen in Example 6.1, can be obtained as follows:

Step 1 The balanced fully fuzzy solid transshipment problem, obtained from Step 1 to Step 3 of Section 6.4.1, can be represented by Table 6.6.

Table 6.6 Tabular representation of balanced fully fuzzy solid transshipment problem

							Capacity
	E_1		E_1		E_1		$(60,80,20,10)_{LR}$
		E_2		E_2		E_2	$(50,70,20,40)_{LR}$
	1		3		5		Availability
1	$(M,M,0,0)_{LR}$	$(M,M,0,0)_{LR}$	$(8,10,2,2)_{LR}$	$(4,8,3,2)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(60,80,20,20)_{LR}$
2	$(8,10,4,4)_{LR}$	$(6,8,4,4)_{LR}$	$(9,12,6,3)_{LR}$	$(3,6,2,3)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(50,70,20,20)_{LR}$
4	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,10)_{LR}$
	–		$(50,80,30,50)_{LR}$		$(60,70,10,0)_{LR}$		

Step 2 Using Step 2 of the method, proposed in Section 6.3.2, add an amount of fuzzy buffer stock $\tilde{P} = \sum_{i \in N_{PS}} \tilde{a}_i \oplus \sum_{i \in N_S} \tilde{a}'_i = \sum_{j \in N_{PD}} \tilde{b}_j \oplus \sum_{j \in N_D} \tilde{b}'_j = \sum_{k \in S_C} \tilde{e}_k = (110,150,40,50)_{LR}$ in the fuzzy availability and fuzzy demand corresponding to each intermediate node and also introduce a dummy conveyance E_3 with fuzzy capacity $\tilde{e}_3 = (110,150,40,50)_{LR}$. After adding the fuzzy buffer stock $\tilde{P}$ and introducing dummy conveyance E_3, the Table 6.6 is converted into Table 6.7.

Table 6.7 Tabular representation of balanced fuzzy solid transportation problem after adding fuzzy buffer stock

										Capacity
	E_1			E_1			E_1			$(60,80,20,10)_{LR}$
		E_2			E_2			E_2		$(50,70,20,40)_{LR}$
			E_3			E_3			E_3	$(110,150,40,50)_{LR}$
	1			3			5			Availability
1	$(M,M,0,0)_{LR}$	$(M,M,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(8,10,2,2)_{LR}$	$(4,8,3,2)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(170,230,60,70)_{LR}$
2	$(8,10,4,4)_{LR}$	$(6,8,4,4)_{LR}$	$(0,0,0,0)_{LR}$	$(9,12,6,3)_{LR}$	$(3,6,2,3)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(50,70,20,20)_{LR}$
4	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,0)_{LR}$	$(0,0,0,10)_{LR}$
	$(110,150,40,50)_{LR}$			$(50,80,30,50)_{LR}$			$(60,70,10,0)_{LR}$			

Step 3 Using Step 4 of the method, proposed in Section 6.3.2, Table 6.7 can be split into four crisp solid transportation tables i.e., Table 6.8, Table 6.9, Table 6.10 and Table 6.11.

Table 6.8 Tabular representation of first crisp solid transportation problem

										Capacity
	E_1			E_1			E_1			40
		E_2			E_2			E_2		30
			E_3			E_3			E_3	70
	1			3			5			Availability
1	M	M	0	9	5.75	0	0	0	0	110
2	9	7	0	9.75	4.75	0	0	0	0	30
4	0	0	0	0	0	0	0	0	0	0
	70			20			50			

Table 6.9 Tabular representation of second crisp solid transportation problem

										Capacity
	E_1			E_1			E_1			20
		E_2			E_2			E_2		20
			E_3			E_3			E_3	40
	1			3			5			Availability
1	$\frac{3M}{4}$	$\frac{3M}{4}$	0	7.5	5.5	0	0	0	0	60
2	8	6.5	0	9	4.5	0	0	0	0	20
4	0	0	0	0	0	0	0	0	0	0
	40			30			10			

Table 6.10 Tabular representation of third crisp solid transportation problem

										Capacity
	E_1			E_1			E_1			20
		E_2			E_2			E_2		20
			E_3			E_3			E_3	40
	1			3			5			Availability
1	$\frac{M}{2}$	$\frac{M}{2}$	0	5.5	4.5	0	0	0	0	60
2	6	5	0	6.75	3.75	0	0	0	0	20
4	0	0	0	0	0	0	0	0	0	0
	40			30			10			

Table 6.11 Tabular representation of fourth crisp solid transportation problem

										Capacity
	E_1			E_1			E_1			10
		E_2			E_2			E_2		40
			E_3			E_3			E_3	50
	1			3			5			Availability
1	$\frac{M}{4}$	$\frac{M}{4}$	0	3	2.5	0	0	0	0	70
2	3.5	3	0	3.75	2.25	0	0	0	0	20
4	0	0	0	0	0	0	0	0	0	10
	50			50			0			

Step 4 The optimal solution of crisp solid transportation problems, shown by Table 6.8, Table 6.9, Table 6.10 and Table 6.11, are $m_{113} - \alpha_{113} = 70, m_{151} - \alpha_{151} = 40, m_{232} - \alpha_{232} = 20, m_{252} - \alpha_{252} = 10; \alpha_{113} = 40, \alpha_{131} = 10, \alpha_{151} = 10, \alpha_{232} = 20; n_{113} - m_{113} = 40, n_{131} - m_{131} = 10, n_{151} - m_{151} = 10, n_{232} - m_{232} = 20$ and $\beta_{113} = 40, \beta_{131} = 10, \beta_{151} = 10, \beta_{232} = 20,\ \beta_{411} = 10$ respectively.

Step 5 On solving the equations, obtained from Step 4, the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}$ and β_{ijk} are $m_{113} = 110, n_{113} = 150, \alpha_{113} = 40, \beta_{113} = 40, m_{151} = 50, n_{151} = 60, \alpha_{151} = 10, \beta_{151} = 0, m_{232} = 40, n_{232} = 60, \alpha_{232} = 20, \beta_{232} = 20, m_{252} = 10, n_{252} = 10, \alpha_{252} = 0, \beta_{252} = 0, m_{131} = 10, n_{131} = 20, \alpha_{131} = 10, \beta_{131} = 0, m_{132} = 0, n_{132} = 0, \alpha_{132} = 0, \beta_{132} = 20, m_{133} = 0, n_{133} = 0, \alpha_{133} = 0, \beta_{133} = 10, m_{411} = 0, n_{411} =$

$0, \alpha_{411} = 0, \beta_{411} = 10$ and the remaining values of $m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk}$ are zero.

Step 6 Putting the values of $m_{ijk}, n_{ijk}, \alpha_{ijk}$ and β_{ijk} in $\tilde{x}_{ijk} = (m_{ijk}, n_{ijk}, \alpha_{ijk}, \beta_{ijk})_{LR}$, the fuzzy optimal solution is $\tilde{x}_{113} = (110, 150, 40, 40)_{LR}$, $\tilde{x}_{151} = (50, 60, 10, 0)_{LR}$, $\tilde{x}_{232} = (40, 60, 20, 20)_{LR}$, $\tilde{x}_{252} = (10, 10, 0, 0)_{LR}$, $\tilde{x}_{131} = (10, 20, 10, 0)_{LR}$, $\tilde{x}_{132} = (0, 0, 0, 20)_{LR}$, $\tilde{x}_{133} = (0, 0, 0, 10)_{LR}$, $\tilde{x}_{411} = (0, 0, 0, 10)_{LR}$ and remaining values of $\tilde{x}_{ijk}$ are zero.

Step 7 Putting the values of $\tilde{x}_{111}, \tilde{x}_{112}, \tilde{x}_{113}, \tilde{x}_{131}, \tilde{x}_{132}, \tilde{x}_{133}, \tilde{x}_{151}, \tilde{x}_{152}, \tilde{x}_{153}, \tilde{x}_{211}, \tilde{x}_{212}, \tilde{x}_{213}, \tilde{x}_{231}, \tilde{x}_{232}, \tilde{x}_{233}, \tilde{x}_{251}, \tilde{x}_{252}, \tilde{x}_{253}, \tilde{x}_{411}, \tilde{x}_{412}, \tilde{x}_{413}, \tilde{x}_{431}, \tilde{x}_{432}, \tilde{x}_{433}, \tilde{x}_{451}, \tilde{x}_{452}, \tilde{x}_{453}$ in $((M, M, 0, 0)_{LR} \otimes \tilde{x}_{111} \oplus (M, M, 0, 0)_{LR} \otimes \tilde{x}_{112} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{113} \oplus (8, 10, 2, 2)_{LR} \otimes \tilde{x}_{131} \oplus (4, 8, 3, 2)_{LR} \otimes \tilde{x}_{132} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{133} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{151} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{152} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{153} \oplus (8, 10, 4, 4)_{LR} \otimes \tilde{x}_{211} \oplus (6, 8, 4, 4)_{LR} \otimes \tilde{x}_{212} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{213} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{251} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{252} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{253} \oplus (9, 12, 6, 3)_{LR} \otimes \tilde{x}_{231} \oplus (3, 6, 2, 3)_{LR} \otimes \tilde{x}_{232} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{411} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{412} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{413} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{431} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{432} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{433} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{451} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{452} \oplus (0, 0, 0, 0)_{LR} \otimes \tilde{x}_{453})$, the minimum total fuzzy transportation cost is $(200, 560, 180, 600)_{LR}$.

6.4.3 Physical interpretation of the results

In this section, the minimum total fuzzy transportation cost, obtained by using the proposed methods, is physically interpreted. Similarly, the obtained fuzzy optimal solution can also be physically interpreted.

Using the proposed method the minimum total fuzzy transportation cost is $(200, 560, 180, 600)_{LR}$, which can be physically interpreted as follows:

(1) The least amount of minimum total transportation cost is 20 units.

(2) The most possible amount of minimum total transportation cost lies between

200 units and 560 units.

(3) The greatest amount of minimum total transportation cost is 1160 units i.e., the minimum total transportation cost will be always greater than 20 units and less than 1160 units and maximum chances are that the minimum total transportation cost will lie between 200 units and 560 units.

The variation in minimum total transportation cost with respect to chances are shown in Figure 6.2.

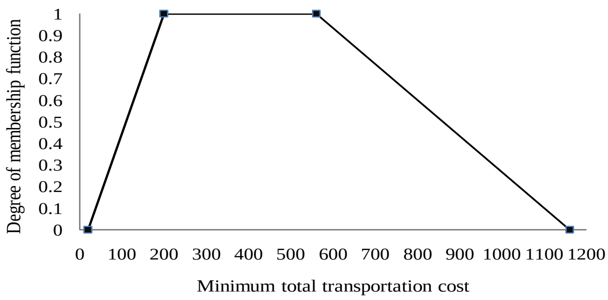

Figure 6.2. Membership function of *LR* flat fuzzy number representing the minimum total fuzzy transportation cost

6.5 Comparative study

The results obtained by the existing method [70] and by the methods proposed in this chapter and previous chapters, are compared in Table 6.12.

Table 6.12 Results obtained by using existing method and proposed methods

Example	Minimum total fuzzy transportation cost					
	Existing method [70]	Methods proposed in Chapter 2	Methods proposed in Chapter 3	Methods proposed in Chapter 4	Methods proposed in Chapter 5	Methods proposed in this chapter
2.2	Not applicable	(2100, 2900, 3500, 4200)	(2100, 2900, 3500, 4200)	(2100, 2900, 3500, 4200)	(2100, 2900, 3500, 4200)	(2100, 2900, 3500, 4200)
3.1	Not applicable	Not applicable	$(5800, 8400, 2800, 2900)_{LR}$	$(5800, 8400, 2800, 2900)_{LR}$	$(5800, 8400, 2800, 2900)_{LR}$	$(5800, 8400, 2800, 2900)_{LR}$
3.5 [71]	$(1924000, 1903300, 7299800)_{LR}$	Not applicable	Not applicable	$(1924000, 1903300, 7299800)_{LR}$	Not applicable	$(1924000, 1903300, 7299800)_{LR}$
4.1	$(4100, 6600, 2000, 2600)_{LR}$	(2100, 4100, 6600, 9200)	$(4100, 6600, 2000, 2600)_{LR}$	$(4100, 6600, 2000, 2600)_{LR}$	$(4100, 6600, 2000, 2600)_{LR}$	$(4100, 6600, 2000, 2600)_{LR}$
4.2	Not applicable	Not applicable	Not applicable	$(360, 560, 270, 350)_{LR}$	Not applicable	$(360, 560, 270, 350)_{LR}$
5.1	Not applicable	Not applicable	Not applicable	Not applicable	$(1900, 1900, 100, 900)_{LR}$	$(1900, 1900, 100, 900)_{LR}$
6.1	Not applicable	Not applicable	Not applicable	Not applicable	Not applicable	$(200, 560, 180, 600)_{LR}$

The results, shown in Table 6.12, can be explained as follows:

(1) The existing method [70] is proposed for solving balanced fully fuzzy transshipment problems. Since, the balanced fully fuzzy transportation problems are special type of balanced fully fuzzy transshipment problems so the existing method can also be used for solving these type of problems. Since, the existing fully fuzzy transshipment problem [71, Example 3.5, pp. 2498] and the fully fuzzy transportation problem, chosen in Example 4.1, are balanced problems so these problems can be solved by using the existing method [70]. However, the fully fuzzy transportation problems, chosen in Example 2.2, Example 3.1 and the fully fuzzy transshipment problem, chosen in Example 4.2, are unbalanced problems so these problems can not be solved by using the existing method [70].

There is no link between fully fuzzy transshipment problems and fully fuzzy solid transportation problems. Also, fully fuzzy solid transshipment problems are the generalization of fully fuzzy transshipment problems. So, neither the fully fuzzy solid transportation problem, chosen in Example 5.1, nor the fully fuzzy solid transshipment problem, chosen in Example 6.1, can be solved by the existing method [70].

(2) The methods, proposed in Chapter 2, can be used only for solving such fully fuzzy transportation problems in which all the parameters are either represented by triangular fuzzy numbers or by trapezoidal fuzzy numbers. Similarly, the methods, proposed in Chapter 3, can be used for solving such fully fuzzy transportation problems in which all the parameters are represented by LR flat fuzzy numbers. Since, in the fully fuzzy transportation problem, chosen in Example 3.1, all the parameters are represented by LR flat fuzzy numbers

so the problem, chosen in Example 3.1, can not be solved by the methods proposed in Chapter 2 but the same problem can be solved by the methods proposed in Chapter 3.

Since, fully fuzzy transshipment problems, fully fuzzy solid transportation problems and fully fuzzy solid transshipment problems are the generalization of fully fuzzy transportation problems. So, the existing fully fuzzy transshipment problem [71, Example 3.5, pp. 2498], fully fuzzy transshipment problem, chosen in Example 4.2, fully fuzzy solid transportation problem, chosen in Example 5.1 and fully fuzzy solid transshipment problem, chosen in Example 6.1, can not be solved by the methods proposed in Chapter 2 and Chapter 3.

(3) Since the methods, proposed in Chapter 4, can be used for solving such balanced and unbalanced fully fuzzy transshipment problems in which all the parameters are represented by LR flat fuzzy numbers and fully fuzzy transportation problems are also special type of fully fuzzy transhipment problems so the fully fuzzy transportation problems, chosen in Example 2.2 and Example 3.1, the existing fully fuzzy transshipment problem [71, Example 3.5, pp. 2498] and fully fuzzy transshipment problem, chosen in Example 4.2, can be solved by the methods proposed in Chapter 4.

There is no link between fully fuzzy transshipment problems and fully fuzzy solid transportation problems. Also, fully fuzzy solid transshipment problems are the generalization of fully fuzzy transshipment problems. So neither the fully fuzzy solid transportation problem, chosen in Example 5.1, nor the fully fuzzy solid transshipment problem, chosen in Example 6.1, can be solved by the methods proposed in Chapter 4.

(4) Since, the methods proposed in Chapter 5, can be used for solving such balanced and unbalanced fully fuzzy solid transportation problems in which all the parameters are represented by LR flat fuzzy numbers and fully fuzzy transportation problems are special type of fully fuzzy solid transportation problems so the fully fuzzy transportation problems, chosen in Example 2.2, Example 3.1, Example 4.1 and the fully fuzzy solid transportation problem, chosen in Example 5.1, can be solved by the methods proposed in Chapter 5.

There is no link between fully fuzzy solid transportation problems and fully fuzzy transshipment problems. Also, the fully fuzzy solid transshipment problems are the generalization of fully fuzzy solid transportation problems. So the existing fully fuzzy transshipment problem [71, Example 3.5, pp. 2498], fully fuzzy transshipment problem, chosen in Example 4.2, and fully fuzzy solid transshipment problem, chosen in Example 6.1 can not be solved by the method proposed in Chapter 5.

(5) Since, fully fuzzy transportation problems, fully fuzzy transshipment problems and fully fuzzy solid transportation problems are special types of fully fuzzy solid transshipment problems. So, the methods, proposed in this chapter, for solving fully fuzzy solid transshipment problems, can be used for solving all these problems. Due to the same reason the method, proposed in this chapter, can be used to find the fuzzy optimal solution of all the chosen problems.

6.6 Conclusions

On the basis of the comparison of the results, it can be concluded that all the problems which can be solved by using the existing method [70] and the methods, proposed in the previous chapters, can also be solved by the method proposed in this chapter. However, there exist several problems which can be solved by the methods, proposed in this chapter, but can neither be solved by using any of the existing method nor by the methods proposed in previous chapters. Hence, it is better to use the methods proposed in this chapter as compared to the existing method [70] and the methods proposed in previous chapters.

Chapter 7

Conclusions And Future Scope

On the basis of the work, proposed in the thesis, the following conclusions can be drawn:

(1) It is better to use the methods, proposed in Chapter 2, as compared to the existing methods for solving such fully fuzzy transportation problems in which parameters are either represented by triangular fuzzy numbers or trapezoidal fuzzy numbers.

(2) Neither the methods, proposed in Chapter 2, nor any existing method can be used to find the optimal solution of such fully fuzzy transportation problems in which the parameters are represented by LR flat fuzzy numbers. However, the fuzzy optimal solution of all similar type of problems, the problems which can be solved by the existing methods and the methods, proposed in Chapter 2, can be obtained by using the method proposed in Chapter 3.

(3) The existing method [70] can be used to find the fuzzy optimal solution of balanced fully fuzzy transportation problems and balanced fully fuzzy transshipment problems. But, the existing method [70] can not be used to find the fuzzy optimal solution of similar type of unbalanced problems.

However, the fuzzy optimal solution of all these problems can be obtained by using the methods proposed in Chapter 4.

(4) It is better to use the methods, proposed in Chapter 5, as compared to the existing method [132] for solving fully fuzzy solid transportation problems.

(5) Neither, the existing methods nor the methods, proposed in Chapters 2, Chapter 3, Chapter 4 and Chapter 5 can be used to find the fuzzy optimal solution of fully fuzzy solid transshipment problems. However, the fuzzy optimal solution of these problems can be obtained by using the methods proposed in Chapter 6.

The methods, proposed in Chapter 6, can be used to find the fuzzy optimal solution of single-objective fully fuzzy transportation problems, transshipment problems, solid transportation problems and solid transshipment problems. However, these methods can not be used to find the fuzzy optimal solution of similar type of multi-objective problems. In future, it may be tried to extend the methods, proposed in Chapter 6, for solving multi-objective problems.

Also, in the future the existing method [217] which is used to find the crisp optimal solution of single-objective fuzzy solid fixed charge transportation problems may be modified to find the fuzzy optimal solution of single-objective fuzzy solid fixed charge transportation problems.

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