STUDY AND ANALYSIS OF COOPERATIVE DIVERSITY COMMUNICATION SYSTEMS UNDER VARIOUS FADING CHANNEL ENVIRONMENTS

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DECLARATION

I hereby declare that the work, which is being presented in the dissertation, entitled as **"Study and Analysis of Cooperative Diversity Communication Systems Under Various Fading Channel Environments"**, submitted by me in the partial fulfillment for the award of degree of Master of Engineering in Electronics and Communication submitted at Electronics and Communication Engineering Department, Thapar University, Patiala, is an authentic record of my own work carried out under the esteemed guidance of **Dr. Hem Dutt Joshi** (Assistant Professor), Electronics and Communication Engineering Department and refers the work of other researchers which is duly listed in the reference section.

The matter presented in this dissertation has not been submitted in any other University/Institute for the award of degree.

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ABSTRACT

The propagation of a radio wave through the wireless medium is affected by a host of impairments. Fading is one such phenomenon which has been an area of significant research for the evaluation of the performance of a digital communication system. Based on the nature of the radio propagation medium, a variety of statistical modelsare used to characterize the fading of a wireless signal.

Diversity is one of the prime techniques to deal with the severity induced in a signal transmitted over a fading channel. The goal for a system designer is to arrive at a unified set of mathematical models characterized by diversity combining schemes.

Spatial diversity is a well known method to generate independent communication paths between the sender and the receiver through the use of multiple antennas at the transmitter and/or receiver. A recent method to achieve spatial diversity is the cooperative diversity communication. It makes use of the mobile users in a network to relay the original source signal to the destination which creates a virtual form of multiple-input and multiple-output antenna array system.

In a cooperative communication system, it is possible to deter the effects of shadowing and deep fading both of which severely affect the quality of signal transmitted in a wireless network. The mobile user which acts as the relay node processes the original signal received from the source and retransmits it to the destination. The resultant signal at the receiver is analyzed using different parameters to bring improvement in the overall performance of the communication system.

The objective of this dissertation is to present a holistic analysis of the performance of a cooperative communication system under different fading channel conditions. The performance evaluation is done analytically as well as through Monte Carlo simulation to achieve performance curves for a variety of modulation schemes over a range of signal-to-noise ratios.

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LIST OF ACRONYMS

RMS	Root Mean Square
SNR	Signal-to-Noise Ratio
PDF	Probability Distribution Function
MGF	Moment Generating Function
CDF	Cumulative Distribution Function
BEP	Bit Error Probability
AWGN	Additive White Gaussian Noise
LOS	Line-of-Sight
SC	Selection Combining
MRC	Maximal Ratio Combining
EGC	Equal Gain Combining
MIMO	Multiple Input Multiple Output
AF	Amplify and Forward
DF	Decode and Forward
BPSK	Binary Phase Shift Keying
QPSK	Quadrature Phase Shift Keying
SER	Symbol Error Rate
ASER	Average Symbol Error Rate
M-PSK	M-ary Phase Shift Keying
QAM	Quadrature Amplitude Modulation
SAF	Selection Amplify and Forward
M-ASK	M-ary Amplitude Shift Keying
SEP	Symbol Error Probability
DBPSK	Differential Binary Phase Shift Keying
TWDP	Two Wave With Diffuse Power
PSD	Power Spectral Density
CSI	Channel State Information

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CHAPTER 1
INTRODUCTION

1.1 RADIO PROPAGATION

With the ever increasing use of the wireless technologies in the current times for the purpose of communication between devices/hosts, there is no doubt that the prime consideration in the design of a communication system employing one or the other of wireless technology is to ensure that the system performs with a reasonably good margin over a wireless channel that is affected by a variety of impairments. In this scenario, fading is one of the severe impairments that affects the quality and reliability of a signal transmitted over a mobile radio channel [1].

The phenomenon of multipath fading channels has been a challenge for the communication system designers for nearly as long as 40 years and owing to the dependence on the wireless technologies in the present communication intensive era, the significance to counter fading is ever-increasing. All the means that have been developed by the present day communication engineer for the up to date wireless applications are also capable of dealing with the remote and less complicated communication scenarios employed in the past for the mobile radio environment [1].

1.2. FADING

The transmitted signal which propagates from the base station to the receiver in a mobile communication system experiences three phenomena of propagation-reflection, diffraction and scattering [2]. These different propagated signals are because of the presence of several manmade and natural obstructions such as tall buildings, forests, hills etc. This mechanism results in multipath propagation of the signal which gives a resultant signal at the receiver that varies in the amplitude, frequency and phase characteristics. The multipath versions of the signal may combine constructively or destructively at the receiver to give a randomly varying resultant signal [2]. The rapid variations in the receiver signal give rise to the phenomenon of fading. Fading is, therefore, a result of the interference between the different multipath versions of the transmitted signal, also referred to as the multipath waves.

The intensity distribution, the relative time of propagation, and the bandwidth of these multipath waves affects the nature of variations in the resultant signal at the receiver.

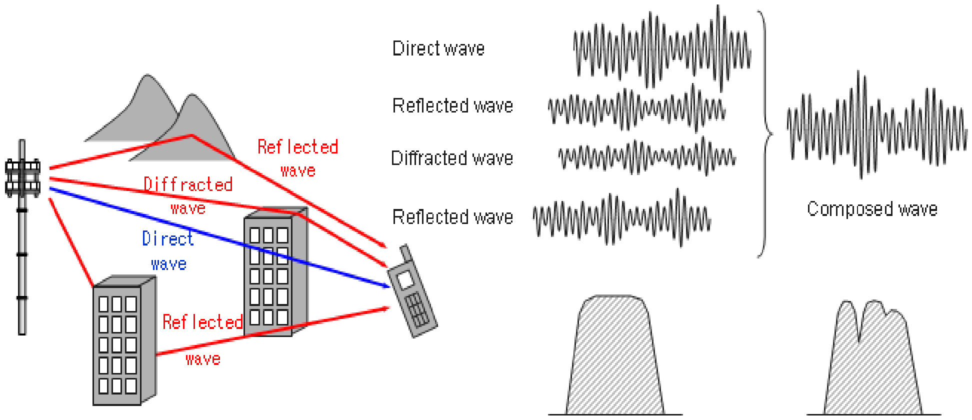

Figure 1.1 Propagation of wave in mobile radio environment [3]

When a signal is received over a fading channel, there are fluctuations in the envelope and phase of the signal over time. For the coherent modulation schemes, the phase fluctuations caused by fading can adversely affect the performance unless appropriate measures are taken to nullify these effects at the receiver [2]. Thus, in the analysis of such modulation schemes it is assumed that the phase variations introduced by fading are reasonably corrected to achieve the ideal coherent demodulation at the receiver. While in the case of non coherent modulation scheme, the phase information is not required at the receiver and thus the phase fluctuations due to fading do not affect the system performance. Therefore, the performance analysis of both the coherent and non coherent modulation techniques requires only the knowledge of the fading envelope statistics [4].

1.2.1 TYPES OF FADING

Depending on the parameters of the channel and the signal there are different kinds of fading encountered in the wireless communication system [2].

1.2.1.1 LARGE SCALE FADING

The fading, resulting in the variations in the average strength of the received signal over a large distance between the transmitter and the receiver (of the magnitude of the order of kilometers), is known as large scale fading. The presence of hills, buildings, forests and other structures in the environment cause shadowing of the signal which results in the variation of the strength of the received signal. The technique to compensate the effects of large scale fading is known as macroscopic diversity which is employed over considerable distances comparable to the distance between one base station and another [2].

1.2.1.2 SMALL SCALE FADING

Small scale fading is one in which the resultant received signal experiences variations in the amplitude and phase when the receiver moves over smaller distances (of the order of a few wavelengths) or smaller time intervals (of the magnitude of seconds). Over the small distances of mobility, the received signal results from the constructive or destructive combining of the multipath signals. The small scale fading can be combated with the use of a diversity technique known as microscopic diversity. Microscopic diversity makes use of the rapidly changing characteristics of the received signal [2].

1.2.2 PARAMETERS OF SMALL SCALE FADING

1.2.2.1 DOPPLER SPREAD

When a mobile receiver moves over a certain distance, there are changes in the frequency of the carrier signal at the receiver caused by differences in the path lengths of the resultant signal at the receiver. Doppler Spread is, therefore, a term that gives the quantity of shift in the frequency of the received signal [2].

The Doppler Spread f_d is given by [7]

$$f_d = \frac{v}{\lambda} \cos \alpha \tag{1.1}$$

where, α is the angle between the direction in which the receiver moves and the direction of the incoming signal, v is the velocity with which the receiver moves and λ is the wavelength of the signal received at the receiver [2].

1.2.2.2 COHERENCE TIME

Coherence time is defined as the time duration over which the signals received at different times at the receiver have identical channel characteristics. In the duration of the coherence time, the impulse response of the channel remains invariant [2]. The coherence time T_c is given as an approximation by the following expression [2]

$$T_c \approx \frac{9}{16\pi f_m} \tag{1.2}$$

where, f_m is maximum Doppler Spread.

1.2.2.3 RMS TIME DELAY SPREAD

The RMS (Root Mean Square) delay spread is a parameter that characterizes the time dispersion of a channel. It is measured as the square root of the second central moment of the power delay profile of a channel. It also serves as a measure for the maximum achievable data rate without causing inter-symbol interference [2]. The RMS delay spread is given as [2]

$$\tau_{rms} = \sqrt{\frac{1}{P_T}\sum_{i=1}^{N} P_i \tau_i^2 - \tau_o^2} \tag{1.3}$$

where,

$$\tau_o = \frac{1}{P_T}\sum_{i=1}^{N} P_i \tau_i \tag{1.4}$$

is the mean delay spread.

1.2.2.3 COHERENCE BANDWIDTH

The coherence bandwidth is defined as the range of frequencies for which the channel allows all the spectral components to pass with approximately equal linear phase and channel gain [2]. The coherence bandwidth is given by [2]

$$B_C \approx \frac{1}{50\sigma_\tau} \tag{1.5}$$

where, σ_τ is the root mean square (RMS) delay spread.

1.2.3 CLASSIFICATION OF SMALL-SCALE FADING

There are different types of small-scale fading that affect the signals transmitted from a source which depend on different parameters of the signal (such as bandwidth, symbol period etc.) and that of the channel (such as Doppler spread, RMS delay spread) [2].

1.2.3.1 ON THE BASIS OF MULTIPATH TIME DELAY SPREAD

- **FLAT FADING**

When the spectral properties of a signal transmitted from the source are maintained over a bandwidth which is more than the bandwidth of the signal, the signal is said to undergo flat fading. There are, however, variations in the signal because of the variations that take place in the channel because of multipath propagation [2].

A signal experiences flat fading if the bandwidth of the transmitted signal B_s is smaller than the coherence bandwidth B_c of the channel i.e. $B_s \ll B_c$ and the symbol period T_s is greater than the RMS delay spread i.e. $T_s \gg \sigma_\tau$. A flat fading channel reduces the SNR (Signal-to-Noise ratio) [2].

- **FREQUENCY SELECTIVE FADING**

When the gain and phase response of the channel remain constant over a bandwidth which is lesser than the bandwidth of the transmitted signal, the received signal undergoes frequency selective fading [2].

Mathematically, frequency selective fading takes place when $B_s \gg B_c$ and $T_s \ll \sigma_\tau$, where B_s is the bandwidth of the transmitted signal, T_s is the symbol period, σ_τ is the RMS delay spread, B_c is the coherence bandwidth of the channel [2].

1.2.3.2 ON THE BASIS OF DOPPLER SPREAD

- **FAST FADING**

When the impulse response of a channel varies quite faster than the changes in the characteristics of the transmitted signal, the fading encountered by the transmitted signal is fast fading i.e. $T_s > T_c$ and $B_s < B_d$, where B_d is the Doppler shift [2].

- **SLOW FADING**

When the variations in the impulse response of the channel are quite slower than the variations in the transmitted signal, the signal encounters slow fading i.e. $T_s < T_c$ and $B_s > B_d$ [2].

1.3 SYSTEM PERFORMANCE MEASURES

The emphasis of this thesis work is on the word performance which serves the purpose of this introductory chapter that shall give a fair amount of knowledge of the several performance measures that are necessary to evaluate in a practical wireless communication system. At the same time, it is equally important to know the analytical methods using which these system parameters can be evaluated [4].

1.3.1 AVERAGE SIGNAL-TO-NOISE RATIO

The signal-to-noise ratio (SNR) is the most common performance measure in the evaluation of a digital communication system. The calculation of the average SNR is done at the output of the receiver and thus forms a part of the data detection process. This is one of the easiest performance measures to evaluate and gives a fairly good idea of the overall reliability of the system [4]. In a communication system affected by fading the term used is average SNR where average implies the statistical averaging over the PDF (Probability Distribution Function) of the fading channel model. To express this mathematically, if γ is the instantaneous SNR calculated at the output of the receiver (where γ is a random variable), thus the average SNR is given as [4]

$$\bar{\gamma} \triangleq \int_0^\infty \gamma p_\gamma(\gamma) d\gamma \tag{1.6}$$

where, $p_\gamma(\gamma)$ is the PDF of γ.

To follow the methodology of unified evaluation of the performance of a system, the term called moment generating function (MGF) is introduced. The MGF for the instantaneous SNR γ is given as [4]

$$M_\gamma(s) = \int_0^\infty p_\gamma(\gamma) e^{s\gamma} d\gamma \tag{1.7}$$

The first derivative of (1.7) with respect to s and substituting s=0 in the results gives the expression [4]

$$\bar{\gamma} = \left. \frac{dM_\gamma(s)}{ds} \right|_{s=0} \tag{1.8}$$

Thus, the calculation of the MGF of γ can be used in the evaluation of the average SNR by differentiation of the MGF as seen in (1.8).

1.3.1 OUTAGE PROBABILITY

The outage probability (P_{out}) is the second performance characteristic of a diversity communication system that models fading channels [4]. It is defined as the probability that the instantaneous value of the probability of error exceeds a certain value, or the probability that the SNR γ at the output of the receiver falls below a certain threshold value γ_{th}. P_{out} is given as [4]

$$P_{out} = \int_0^{\gamma_{th}} p_\gamma(\gamma)\, d\gamma. \tag{1.9}$$

P_{out} is the cumulative distribution function (CDF) of the instantaneous SNR calculated at the value of $\gamma = \gamma_{th}$.

The relation between the PDF $p_\gamma(\gamma)$ and the CDF $P_\gamma(\gamma)$ is [4]

$$\frac{dP_\gamma(\gamma)}{d\gamma} = p_\gamma(\gamma)$$

(1.10)

and $P_\gamma(\gamma)$= 0 at $\gamma = 0$, therefore, the relation between their Laplace Transforms is [4]

$$\hat{P}_\gamma(s) = \frac{\hat{p}_\gamma(s)}{s}.$$

. (1.11)

Also, $\hat{p}_\gamma(s) = M_\gamma(-s)$, hence P_{out} can also be given as [4]

$$P_{out} = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{M_\gamma(-s)}{s} e^{s\gamma_{th}}\, ds$$

(1.12)

where, σ lies in the region of convergence in the complex s-plane.

1.3 AVERAGE BIT ERROR PROBABILITY

The average bit error probability is the third performance measure and is usually the most difficult to compute in a communication system. At the same, is the BEP (Bit Error Probability) that gives a fair evaluation of the performance of a communication system and hence it becomes important to formulate less difficult methods for its calculation [4]. The main reason for the tedious calculations in the evaluation of the average BEP is the nature of the conditional BEP (depending on the type of fading) which varies nonlinearly with the instantaneous SNR. The nature of this nonlinearity depends on the type of modulation or detection technique used in the system. As a solution to this scenario, the MGF based approach is most commonly used to simplify the performance analysis of a communication system [4].

Mathematically, the average BEP is expressed as [4]

$$P_b(e) \triangleq \int_0^\infty P_b(E|\gamma)p_\gamma(\gamma)d\gamma$$

(1.13)

where, $P_b(E|\gamma)$ is the conditional BEP(on the fading) and $p_\gamma(\gamma)$ is the probability distribution function of the instantaneous SNR γ.

1.4 STATISTICAL MODEL OF FADING CHANNELS

There are different types of accurate yet simple statistical models to characterize the various fading channels depending on the nature of the propagation environment and the communication system scenario [2]. A qualitative description of the main fading channel models is presented in this section.

When fading affects a communication system, the amplitude of the received carrier signal is modulated by the parameter called fading amplitude α and the PDF $p_\alpha(\alpha)$ of the fading model which depends on the nature of the propagation environment. The fading amplitude α is a random variable whose mean square value is $\Omega = \overline{\alpha^2}$. The signal passes through the fading channel after which it is affected by additive white Gaussian noise (AWGN) at the receiver. The AWGN is characterized by the power spectral density N_o (one sided) measured in the units of W/Hz. Also the communication system model assumes that statistically the AWGN does not depend on the fading amplitude α. The instantaneous signal power at the receiver is modulated by α^2 [4].

For a given symbol, the instantaneous SNR γ is given by [4]

$$\gamma = \alpha^2 \frac{E_s}{N_o} \tag{1.14}$$

and the average SNR $\bar{\gamma}$ is given by [4]

$$\bar{\gamma} = \Omega \frac{E_s}{N_o} \tag{1.15}$$

where, E_s is the energy of one symbol.

To obtain the PDF $p_\gamma(\gamma)$ of γ from the PDF $p_\alpha(\alpha)$, a change of variables is introduced in the mathematical expression of the fading PDF $p_\alpha(\alpha)$ to give the relation [4]

$$p_\gamma(\gamma) = \frac{p_\alpha\left(\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}\right)}{2\sqrt{\frac{\gamma\bar{\gamma}}{\Omega}}}. \tag{1.16}$$

For a given fading PDF, another important parameter called as the amount of fading (AF), also called as the fading figure, is given as [4]

$$AF = \frac{\mathrm{var}(\alpha^2)}{(E[\alpha^2])^2} = \frac{E[(\alpha^2-\Omega)^2]}{\Omega^2} = \frac{E[\gamma^2]-(E[\gamma])^2}{(E[\gamma])^2} \tag{1.17}$$

where, E[.] is the statistical average and var(.) is the variance.

A signal undergoes multipath fading when the random signal components which are delayed, scattered, diffracted or reflected combine constructively or destructively at the receiver. Based on the nature of the propagation medium, there are different types of multipath fading models to statistically describe the behavior of the fluctuations in the received carrier signal [2].

1.4.1 RAYLEIGH FADING MODEL

The Rayleigh fading model is used to describe the multipath fading in which there is no direct line-of-sight(LOS) signal component. The distribution for the fading amplitude α is given as [4]

$$p_\alpha(\alpha) = \frac{2\alpha}{\Omega}\exp\left(-\frac{\alpha^2}{\Omega}\right), \alpha \geq 0 \tag{1.18}$$

and the distribution for the instantaneous SNR γ is given as [36]

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \gamma \geq 0. \tag{1.19}$$

The MGF for this distribution is given by [4]

$$M_{\gamma}(s) = (1 - s\bar{\gamma})^{-1}. \tag{1.20}$$

This model is also used in the modeling of the propagation of the reflected and refracted paths through the ionosphere and the troposphere and to the ship-to-ship radio channel links [4].

1.4.2 NAKAGAMI-q (HOYT) MODEL

The fading PDF $p_{\alpha}(\alpha)$ for the Nakagami-q (Hoyt) Model is given by [4]

$$p_{\alpha}(\alpha) = \frac{(1+q^2)\alpha}{q\Omega} \exp\left(-\frac{(1+q^2)^2\alpha^2}{4q^2\Omega}\right) I_o\left(\frac{(1-q^4)\alpha^2}{4q^2\Omega}\right), \alpha \geq 0 \tag{1.21}$$

where, $I_0(.)$ represents the zeroth-order modified Bessel function of the first kind, and q represents the Nakagami-q fading parameter such that $0 \leq q \leq 1$ [4].

The PDF $p_{\gamma}(\gamma)$ is given by [4]

$$p_{\gamma}(\gamma) = \frac{(1+q^2)}{2q\bar{\gamma}} \exp\left(-\frac{(1+q^2)^2\gamma}{4q^2\bar{\gamma}}\right) I_o\left(\frac{(1-q^4)\gamma}{4q^2\bar{\gamma}}\right), \gamma \geq 0 \tag{1.22}$$

The MGF for this distribution is given by [4]

$$M_{\gamma}(s) = \left[1 - 2s\bar{\gamma} + \frac{(2s\bar{\gamma})^2 q^2}{(1+q^2)^2}\right]^{-\frac{1}{2}}. \tag{1.23}$$

The Nakagami-q distribution type of fading is experienced on the satellite links with strong ionospheric scintillation [4].

1.4.3 NAKAGAMI-n (RICE) MODEL

This distribution is used to model a propagation environment which has a strong direct LOS component and other random weak signal components. The fading PDF $p_{\alpha}(\alpha)$ is given by [4]

$$p_\alpha(\alpha) = \frac{2(1+n^2)e^{-n^2}\alpha}{\Omega}\exp\left[-\frac{(1+n^2)\alpha}{\Omega}\right]I_o\left(2n\alpha\sqrt{\frac{1+n^2}{\Omega}}\right), \alpha \geq 0 \tag{1.24}$$

where, n represents the Nakagami-n fading parameter such that $0 \leq n \leq \infty$.

The PDF for the instantaneous SNR is given as [4]

$$p_\gamma(\gamma) = \frac{(1+n^2)e^{-n^2}}{\bar{\gamma}}\exp\left[-\frac{(1+n^2)\gamma}{\bar{\gamma}}\right]I_o\left(2n\sqrt{\frac{(1+n^2)\gamma}{\bar{\gamma}}}\right). \tag{1.25}$$

The MGF for the Nakagami-n(Rice) model is given by [4]

$$M_\gamma(s) = \frac{(1+n^2)}{(1+n^2)-\bar{\gamma}s}\exp\left[-\frac{n^2\bar{\gamma}s}{(1+n^2)-\bar{\gamma}s}\right]. \tag{1.26}$$

This type of fading is experienced in the LOS paths of the microcellular suburban and urban land-mobile communication, pico-cellular indoor environments and factory radio propagation environments [4].

1.4.4 NAKAGAMI-m MODEL

The fading PDF for the Nakagami-m model is given by [4]

$$p_\alpha(\alpha) = \frac{2m^m\alpha^{2m-1}}{\Omega^m\Gamma(m)}\exp\left(-\frac{m\alpha^2}{\Omega}\right), \alpha \geq 0 \tag{1.27}$$

where, m is defined as the Nakagami-m fading parameter such that $\frac{1}{2} \leq m \leq \infty$. The PDF $p_\alpha(\alpha)$ for the Nakagami-m distribution is essentially a central chi-square distribution. The PDF for the instantaneous SNR γ per symbol is given by [4]

$$p_\gamma(\gamma) = \frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\Gamma(m)}\exp\left(-\frac{m\gamma}{\bar{\gamma}}\right), \gamma \geq 0. \tag{1.28}$$

The MGF for the Nakagami-m distribution is given by [4]

$$M_\gamma(s) = (1 - \frac{s\bar{\gamma}}{m})^{-m} \tag{1.29}$$

The Nakagami-m distribution spans all the other fading distributions with different values of the fading parameter m. The Nakagami-m distribution can be used to obtain one-sided Gaussian distribution using $m = \frac{1}{2}$ and the Rayleigh fading distribution using $m = 1$. When the value approaches $+\infty$, the Nakagami-m fading channel model approaches a non-fading AWGN communication channel model [4].

The Nakagami-m distribution is used to model the land to mobile, indoor to mobile multipath propagation environment and scintillating ionospheric propagation environment [4].

1.5 DIVERSITY

Diversity is a powerful method to create two or more communication channel paths between the sender and the receiver to improve the reliability of the message signal. Diversity-combining is employed to combat the effects of fading caused by the channel through the use of independent fading paths for the signal [5].

The principle of diversity-combining relies on the fact that the signal transmitted over the several paths will not be affected simultaneously by deep fades. The independent signal paths are combined in a way that the resultant signal is less affected by fading. When two transmit antennas are placed sufficiently apart from each other, it is improbable that the paths offered by the two antennas both experience the deep fades simultaneously [5].

The diversity method which combats the effect of multipath fading is known as micro-diversity. The diversity method that combats the shadowing due to buildings and objects is known as macro-diversity [2].

1.5.1 TYPES OF DIVERSITY

On the basis of the property or mechanism which is used to achieve diversity there are different types of diversity techniques.

- **FREQUENCY DIVERSITY**

When multiple frequencies are used at the same time to transmit a signal to overcome the undesired effects of multipath fading, the diversity technique is called as frequency diversity. Hence, the use of multiple frequencies creates different and uncorrelated fading characteristics [2].

- **TIME DIVERSITY**

When the same information signal is transmitted multiple times, the diversity technique is known as time diversity. Time diversity can also be achieved by addition of a redundant error correcting code in which the error bursts are spread in time by the use of bit-interleaving. Time diversity is used mainly to combat the variations offered by time-varying channel [2].

- **SPATIAL DIVERSITY**

Spatial diversity makes use of multiple antennas at the transmitter or the receiver to achieve independent paths to combat fading in a wireless communication system. The elements of the antenna array are spatially separated from one another. To obtain transmit spatial diversity, the transmit power is divided among the multiple antennas in the array. The separation between the antennas should be such that the fading amplitudes for each path are nearly independent of each other [2].

1.5.2 DIVERSITY COMBINING TECHNIQUES

The independent fading paths created by the use of multiple antennas are combined at the receiver to give a resultant signal which is passed for detection to the demodulator. There are different types of diversity combining techniques which are of different complexity [5].

1.5.2.1 SELECTION COMBINING

In the selection combining (SC) technique, the output of the combiner equals the output of the branch that gives the highest SNR. This technique requires only one receiver that is switched into the antenna branch with the highest SNR and thus only one branch is used actively. A dedicated receiver may be required at each antenna branch is the system is required to transmit signals continuously such that the SNRs on each antenna branch are continuously monitored. This technique does not require co-phasing of the branches because the output of only one branch is used [5].

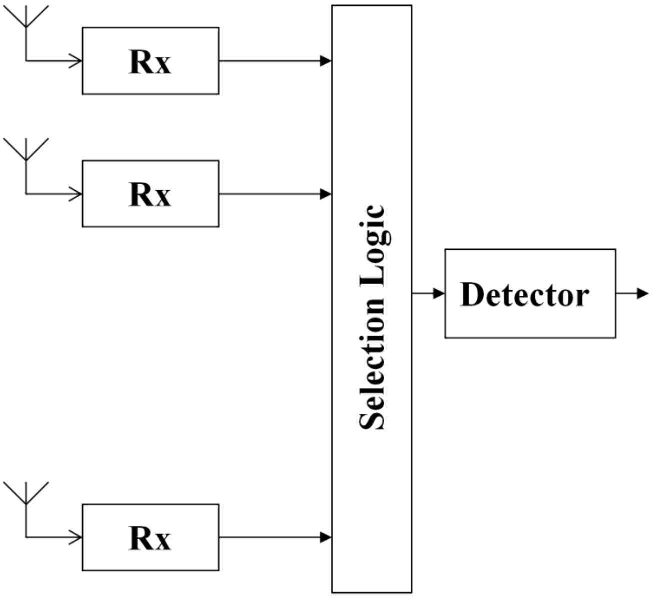

Figure 1.2 Selection Combining Diversity [3]

1.5.2.2 MAXIMAL RATIO COMBINING

In maximal ratio combining (MRC), the resultant signal at the output is the weighted sum of the signal on all the branches. In an M-branch MRC scheme, the gain α_i for each branch is non-zero where$1 \leq i \leq M$. The signals on the different branches are co-phased, therefore, for the i^{th} branch, $\alpha_i = \alpha_i e^{-j\theta_i}$, where θ_i is the phase for the signal on the i^{th} branch [5]. Therefore, the envelope of the output at the combiner is $\mathrm{r} = \sum_{\mathrm{i}=1}^{\mathrm{M}} \mathrm{a}_{\mathrm{i}} \mathrm{r}_{\mathrm{i}}$.

The SNR of the output of the combiner is equal to the sum of the SNRs at each branch. The performance of MRC scheme is considerably better than the SC scheme. At high values of SNR, the diversity order achieved with MRC equals M and thus MRC is said to achieve full diversity order [5].

A general system model for the MRC diversity scheme is depicted in figure 1.2. There are L-receivers at the destination with each receiver having one receiving antenna. The outputs generated by the receivers are processed using a switching logic unit to generate a resultant output which is passed through the detector for demodulation.

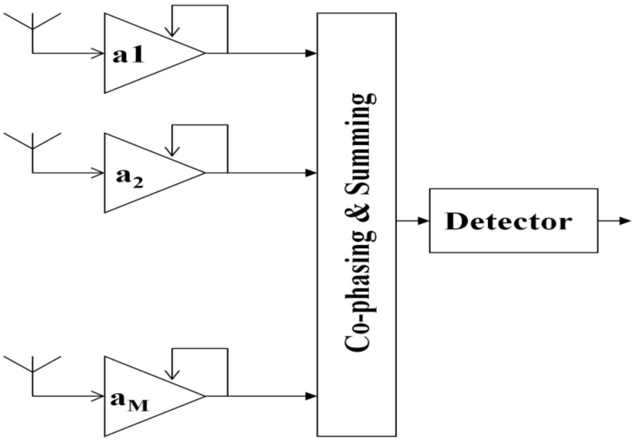

Figure 1.3 Maximal Ratio Diversity Combining [3]

1.5.2.3 EQUAL GAIN COMBINING

In equal gain combining (EGC) technique, signals on each branch are co-phased and are combined at the output with identical weighting such that $\alpha_i = e^{-\theta_i}$. EGC is less complex in comparison to MRC. In terms of performance, EGC is nearly identical to MRC. EGC is a relatively simpler technique because unlike MRC, it does not require the knowledge of the SNR (varying with time) for each branch as this is difficult to compute [5].

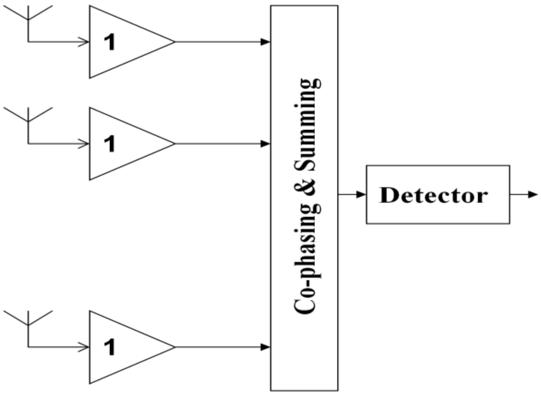

Figure 1.4 Equal Gain Combining Diversity [3]

1.6 COOPERATIVE DIVERSITY

In the transmit diversity scheme, a number of antennas are required at the transmitter to create independent fading paths to transmit the signal [6]. But in case of wireless devices there is one antenna at the device terminal because of limitation in the size or the complexity of hardware. Thus, a new method called cooperative diversity communication was introduced in which multiple users in a mobile radio environment share their antennas in order to achieve transmit diversity. The cooperative diversity communication can be viewed as a virtual transmit diversity scheme with one device using its own antenna as well as the antenna of other devices in the network. Because of its inherent advantages, cooperative diversity is a burgeoning field in wireless network communication systems [6].

1.6.1 MIMO SYSTEMS

In wireless communication systems, the demands to achieve capacity are ever increasing because of the number of applications such as cellular mobile devices, internet and a host of multimedia services. The radio frequency spectrum is however limited and thus to improve the capacity there is a need to improve the spectral efficiency. Significant gains in performance and capacity can be achieved by use of multiple antennas at the transmitter or the receiver. This can be achieved by using MIMO systems. The use of multiple antennas at the transmitter (multiple inputs) and multiple antennas at the receiver (multiple outputs) is known as a MIMO System (multiple input and multiple output system) [7].

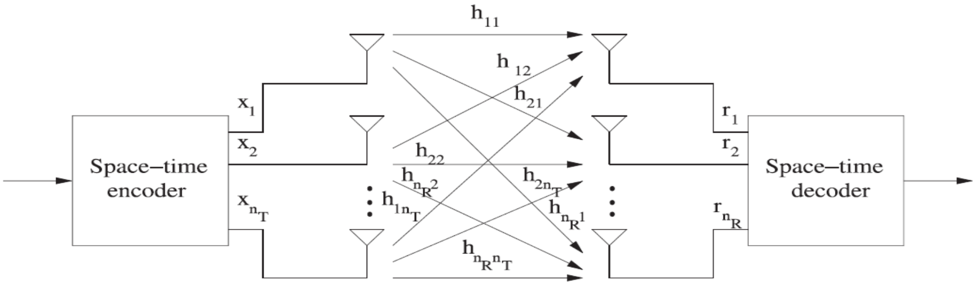

Figure 1.5 MIMO System Model [7]

The figure 1.5 shows the system model for a MIMO system (point-to-point) with n_T transmitter antennas and n_R receiver antennas. The fading coefficients designated by h from one transmitting antenna to the receiving antennas are statistically independent of each other and are determined by the type of fading channel model. This system ensures high data rates because of the use of multiple antennas for one user. It provides high speed channel links with improved reliability. There is however increased complexity in MIMO systems because of the presence of a number of antennas [7].

The MIMO systems are being incorporated into the wireless technologies because of their many advantages such as enhanced spectral efficiency, high data rates and minimized probability of error [7]. However, in certain communication scenarios, a MIMO system might not be practical owing to limitations in size, hardware, cost or the inability of a wireless agent to allow multiple antennas at its source terminal. Examples of such a scenario are the mobile handsets (limitation in size) or the wireless sensor network nodes (limitation in the size and power). Thus, cooperative communication systems offer an alternative to MIMO systems as they can tap the benefits of the MIMO systems in a virtual manner [6].

1.6.2 COOPERATIVE DIVERSITY MODEL

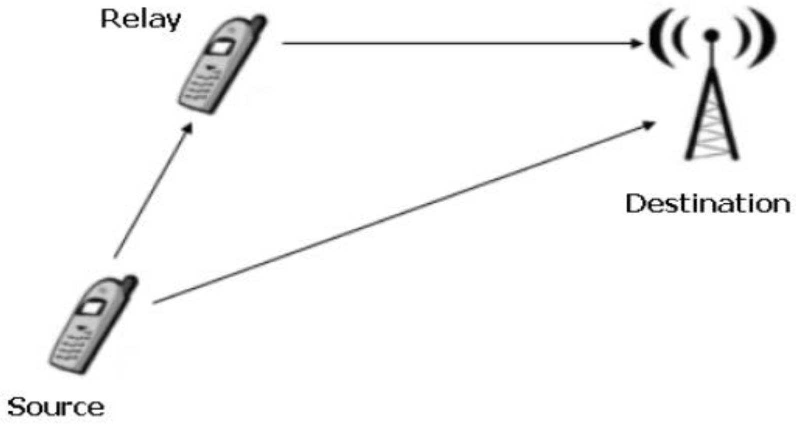

Figure 1.6 Cooperative Diversity Model [8]

The figure 1.6 illustrates a basic cooperative communication system model. It shows two mobile subscribers- one acting as the source and the other acting as the relay. The source transmits the original message signal to the receiver as well as to the relay in the first phase of transmission. In the second phase of transmission, the relay forwards the message signal sent by the source to the receiver based on a particular cooperative signaling protocol [8].

There are three channel links in the model. The relay acts as an inter-user channel that forwards the message transmitted by the source. The channel links between the users and the receivers are statistically independent fading channel paths. The receiver thus obtains two copies of the original message signal. The resultant received signal is obtained after combining the message signals with the use of an appropriate combining technique. Thus, because of the presence of statistically independent fading paths, spatial diversity is achieved in a virtual manner [8].

1.6.3 COOPERATIVE RELAYING PROTOCOLS

There are mainly two types of cooperative communication relaying protocols- Amplify and Forward (AF) and the Decode and Forward (DF).

1.6.3.1 AMPLIFY AND FORWARD

In the AF relaying protocol, the signal received at the relay from the source is a noisy form of the original source signal. The relay amplifies this received signal and forwards it to the destination. The figure 1.7 shows the transmission of the message signal from the source to the destination using the AF relaying method.

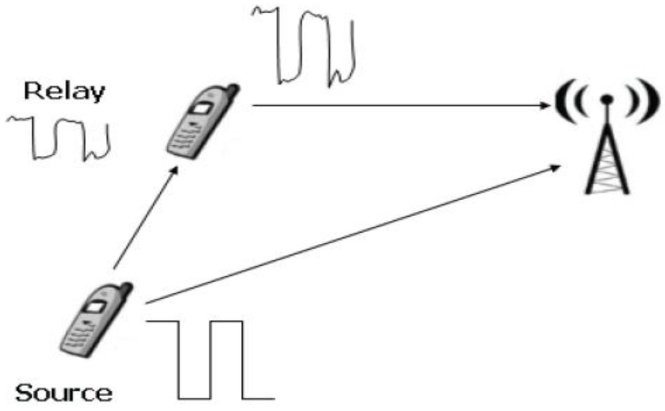

Figure 1.7 AF Relaying Protocol [8]

In the first time-slot, the source sends the message signal to all the relays and the destination node. In the second time-slot, the relay amplifies the signal which is received from the source in the first time-slot. At the end of the second time-slot, the destination processes the copies of the message signal received directly from the source as well as from the relays in the network [8].

The Amplify and Forward protocol is a less complicated method to achieve cooperative diversity.

1.6.3.2 DECODE AND FORWARD

In the DF relaying protocol, the signal received at the relay is a copy of the original signal transmitted by the source affected by noise. The relay decodes this received signal and then re-encodes the signal to send it to the destination terminal. The figure 1.8 shows the transmission of the message signal from the source to the destination using the DF relaying method [8].

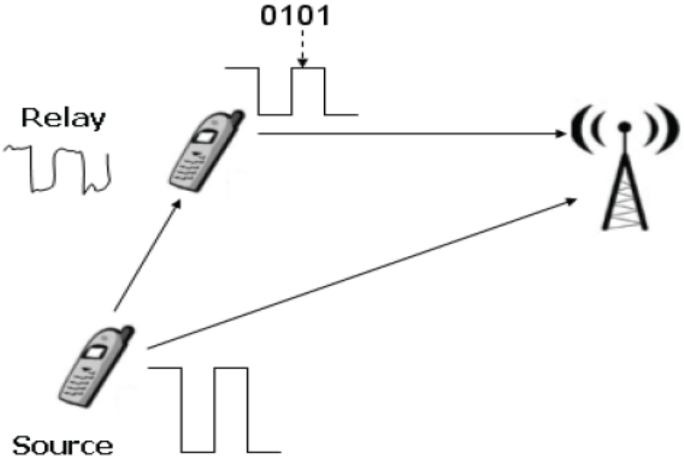

Figure 1.8 DF Relaying Protocol [8]

The algorithm of the DF protocol can be understood in two time-slots. In the first time-slot, the source sends the original message signal which is listened by all the relay nodes in the system as well as the receiver node. After this, all those relays that can successfully decode the message signal, re-encode the received message signal and the message is ready to be sent to the destination node. At the end of the second time slot, the destination is able to decode the message. No message is transmitted by the source in the second time slot [8].

CHAPTER 2
LITERATURE REVIEW

This chapter gives an overview of the methods available in literature for the analysis of the performance of the cooperative diversity communication system.

The performance analysis of a cooperative diversity system relies on the understanding of the different channel statistical models as well as the performance parameters for the system. To transmit information through several mobile stations that are located spatially apart from each other, the antennas are distributed at different locations in the network.

Cooperative diversity communication system is basically a system in which two terminals (source and destination) communicate via a third terminal known as the relay node. The idea of this strategy was born from the concept of bent-pipe satellites in which the spacecraft primarily relays the uplink carrier with the downlink carrier which was introduced by R.M. Galiardi in the year 1988 [9]. This idea is also applied in the use of fixed microwave channel links which enables a larger coverage without the increase in the power of the transmitter. Over the years, this concept gained popularity to achieve diversity in collaborative wireless communication systems.

The capacity of the mobile users is limited because within the time duration of a call, there are severe fluctuations in the signal attenuation levels, which makes it necessary to employ some form of diversity scheme. In 1998, Sendonaris *et.al* [10] presented a new kind of diversity scheme in which the diversity gains were achieved through the cooperation of the mobile users. The results illustrated that the technique of user cooperation leads to an increase in the capacity of the system despite the noise present on the inter-user channel. It was also shown that cooperation improves the overall robustness of the system in which the achievable data rates are less prone to the variations in the channel.

The improved data rates achieved with cooperation thus translate into reduced transmitter powers for the users and this in turn helps to improve the battery life.

Another major benefit of this newly introduced scheme was the overall increase in the cell coverage for a cellular system.

In 2000, Laneman *et.al* [11] developed an energy-efficient method for transmission of signals over wireless networks that employ spatial diversity through the coordinated and collaborated transmission by the use of several radios to incorporate a kind of spatial diversity. The authors used transmission of messages by a single-user and explored the several possibilities for the appropriate strategy that must be used by the relaying radio. The relaying strategies examined were Decode and Forward (DF) and Amplify and Forward (AF) schemes.

The receivers developed for both the strategies, were based either on the maximum-likelihood criterion or the maximum SNR criterion. The bit error probability rates were compared through the analysis and simulation of the cooperative diversity model. A comparison between the single-hop and the multi-hop routing systems was done. It was shown that the use of antenna sharing through the AF and the DF protocols achieved better diversity gains when compared with the single hop and the multi-hop transmission systems.

It was observed that despite the amplification of the noise in the proposed AF scheme, this scheme was more energy-efficient than the DF scheme. The simulation was done for the different transmission protocols for the un-coded BPSK modulation scheme at different relay locations. The analytical bounds obtained were found to give nearly the same performance as achieved with the simulation. The diversity transmission with the use of AF scheme gave better results in terms of performance when compared with a decoding relay.

Laneman *et.al* [12] continued this research further in 2001 by introducing two different models of an energy-efficient cooperative communication protocol which was capable of mitigating the effects of the multipath propagation in a wireless communication network. The protocols developed were suited for the ad-hoc wireless networks and the peer-to-peer wireless communication networks in which each radio uses a single antenna. The protocols were found to give advantages such as reduction in the battery drain and improvement in the network performance in terms of parameters such as capacity.

The performance achieved with the two different protocols namely Amplify and Forward and Hybrid Decode and Forward were compared with the direct transmission scheme and the transmit diversity scheme by evaluating the outage probability for each case for large values of average SNR. Each of the relaying mechanism was found to achieve full diversity order. The relative performances of the two protocols were found to depend on the network architecture parameters.

In 2002, Hasna *et.al* [13] investigated the performance of communication systems using non-regenerative relays with Rayleigh flat fading channels. This model was studied using closed form expressions obtained for the statistical analysis of the harmonic mean of two exponential variables that were independent of each other. The results obtained through the analysis were found to depend on the relay gain and were accordingly given as the exact or the tight lower analytical bounds.

Specifically, the outage probability was studied as the performance parameter. In addition to this, for the given system with binary differential phase shift keying modulation scheme, the expressions for the outage capacity and the bit error rate were derived. The comparison between the regenerative and the non-regenerative systems had shown that the former system performs better for the low average SNR values. The two systems however were found to give nearly the same performance at high average SNR values.

The study that followed this research was based on analyzing the performance of the multiple relay networks under different fading environments. Anghel *et.al* [14] presented an analysis of the distributed cooperative diversity system using K amplifying relays with Rayleigh fading environment to calculate the average symbol error rate (SER) for the proposed system. The formula derived for the average SER illustrated that the proposed cooperative diversity system was capable of overcoming the severe degradation in the signal-to-noise ratio (SNR) due to Rayleigh fading in the channels. The cooperative diversity network achieved full diversity order with the use of simple bounds on the error probability rate.

It is also important to note that the modulation scheme acts as an important parameter for the performance evaluation. For the above proposed system, the error probability analysis was done for M-PSK transmissions. The maximum-likelihood performance was achieved when the incoming signals from all the K+1 branches were combined using the MRC combining technique. Peculiarly, however, it was found that the bandwidth efficiency achieved with the use of K relays was K+1 times lower than that achieved with the single-hop transmissions. It was presented that the efficient bandwidth utilization could be achieved by the use of space-time encoder across each relay.

The results obtained so far encouraged the researchers to formulate more generalized formulas for the performance analysis. By this time, it had been known that the cooperative diversity scheme achieves dramatic gains in the fading environments through the use of virtual antenna arrays. The all-participate relay was further followed by the study in the field of selection relay strategies that selected the best relay out of all the available relays instead of using all the relays for participation.

In 2006, Blestas *et.al* [15] proposed a novel cooperative diversity model that provided diversity gains proportional to the order of the number of relays in the network. In this scheme, a best relay was selected out of all the M available relays in the system. The selected best relay was then used for cooperating signals between the source terminal and the destination terminal. The best relay was selected using the measurements of the instant values of the channel conditions without the requirement of the topology information. Also, there was no need for the relay nodes to communicate with each other explicitly.

The objective of the selection of the best relay was found to depend on the statistical nature of the wireless channel and the authors introduced a methodology to evaluate the system performance in any kind of wireless channel model. The proposed scheme could achieve the same diversity-multiplexing tradeoff which was achieved by the more complicated protocols. This simplified technique was proposed as a method for implementation in the future 4G wireless communication systems because of the enhanced reliability, flexibility and spectral efficiency.

Zhao *et.al* [16] presented an analysis of the selection AF relay system which was shown to achieve the same diversity order as that achieved with the all-participate relay system. It was shown that the outage probability of the selection relaying was lower than the all-participate relaying technique. The authors presented an asymptotic analysis with the average symbol error rates as the performance parameter and the performance was compared with the all-participate relay scheme.

In Selection Amplify and Forward (SAF) relay system, the node that provides the maximum value of the SNR at the receiver is chosen as the relay for the transmission. The comparison of the SER for S-AF networks with two relays was done for the 4-QAM and 16-QAM modulation schemes for the theoretical and the simulation results. The SER comparison for the all-participate relay and the SAF system was done using the 4-QAM modulation scheme. The results proved that the SER for the SAF system were better in comparison to the all-participate relay for both the theoretical approximation and the simulation analysis.

In 2008, Ikki *et.al* [17] investigated the performance of a cooperative diversity system employing the best relay selection scheme in a Rayleigh fading channel environment. The authors presented a model which used the Amplify and Forward (AF) cooperative diversity communication to transmit the message from the source node to the destination node through the use of M relaying nodes.

Generally, in a cooperative diversity system, all the relay nodes present in the network contribute in relaying the message signal to the destination through the use of orthogonal channels with the use of carriers, time slots or codes to mitigate the effects of co-channel interference. The authors had presented the idea that for a cooperative diversity system with M- relay nodes, M+1channels are required in total out of which one channel is used for the direct link and the rest of the M channels are used for the indirect relay links. This also implied that the number of channels in the cooperative diversity network increased linearly with the increase in the number of relays.

The scheme proposed by the authors investigated the performance of the system with the selection of the best relay out of all the available relays. This reduced the number of channels in the system from M+1 to 2 irrespective of the number of relays. The best relay was the one that could achieve the maximum SNR at the destination terminal. The benefit of the best relay selection scheme, apart from using lesser number of resources, was the attainment of a full diversity order.

The closed form expressions for the SER and the outage probability were derived in the form of tight lower bounds. An approximate value of the probability density function (PDF) of the SNR of the signal transmitted from the source to the destination via the relay was used in the analysis. A landmark closed form expression was derived for the Moment Generating Function (MGF) for the total SNR at the receiver.

The MGF is a powerful mathematical tool that can be effectively used to evaluate the values of other performance parameters such as the outage probability, the average SER, the amount of fading, the average SNR as well as other moments of the SNR. The asymptotic analysis presented had shown that the diversity order depends on the number of relays.

The analyses for selection relaying scheme have so far focused on the high SNR values. However, the practical communication systems operate at much lower values of the SNR than the high SNR values considered for analytical evaluation. Fan *et.al* [18] presented the analysis of the selection relaying system for a reasonably good range of SNR in a cooperative communication system. The authors provided the exact expressions for the outage performance and the capacity bounds. The analytical results derived were shown to corroborate the simulation results.

The asymptotic results for the different performance parameters were also found to depend on the mathematical approach followed in the analysis. For instance, the three possible ways to do the analysis could be on the basis of one of the three performance parameters namely- the probability density function (PDF), the moment generating function (MGF) and the cumulative distribution function (CDF).

In 2009, Torabi *et.al* [19] investigated the performance of a cooperative diversity communication system employing the best relay selection method in a Rayleigh fading environment. The mathematical expressions for the PDF, the MGF and the CDF of the average SNR at the destination were derived. The expressions for these parameters were used to derive the average SER in a closed form along with the average gain for the end-to-end SNR achieved with the use of the best relay selection technique. The comprehensive study carried out by the authors highlighted the main benefits of using the best relay selection scheme for different scenarios.

The investigation gave important results for the average SER for the modulation schemes employing BPSK, M-PSK along with the M-QAM modulation schemes. The results for the average SER using the BPSK modulation were presented with the number of relays taking the values 1, 2 and 4. It was shown that for a given SNR value, the SER reduced significantly with the increase in the number of relays. Similar results were presented for the 8-PSK, 4-QAM and 64-QAM modulation schemes.

The best relay selection scheme in the cooperative diversity system met with the challenge that the selected relay may or may not be available for the active transmission of the message from the source terminal to the destination terminal. This propelled the research to devise a scheme which selects the second best relay if the first best relay was not ready for transmission. Thus, in 2010, Ikki *et.al* [20] investigated the performance of a cooperative diversity communication system using the N^{th} best relay selection technique.

The system proposed two cooperative diversity systems-one with the adaptive Decode-and-Forward (DF) and the other with the Amplify-and-Forward (AF) based on the N^{th} best relay selection technique. The closed-form mathematical expressions were derived for three performance parameters- the average SEP (Symbol Error Probability), the asymptotic probability of error and the outage probability.

The mathematical approach followed for the performance evaluation first found the expression for the PDF of the average SNR of the signal sent through the relay to the destination in a closed form. This expression was used to derive the MGF of the total SNR at the receiver terminal. It was shown that the MGF is a powerful tool to devise a unified framework for the calculation of different performance metrics. The fading environment considered followed the Rayleigh fading model. It was shown that the diversity order achieved with the Nth best relay selection scheme was (M-N+2) where M represents the total number of relays in the network.

Jeongtae *et.al* [21] presented an analysis of the opportunistic AF (OAF) cooperative diversity system with Rayleigh fading channels using the N^{th} best relay selection method. The authors used the MGF based approach to compute the approximate value of the average SER (symbol error rate) of the system.

The transmission of signals for multimedia services over wireless transmission medium requires a large bandwidth. Since in the scenario of cooperative diversity, the performance of the system also depends on the nature of the modulation technique employed, therefore, it becomes necessary to employ a modulation technique that can meet the high bandwidth demand adequately. Thus, the investigations laid an emphasis on the QAM modulation scheme because it is bandwidth efficient.

Kim *et.al* [22] investigated the performance of the DF cooperative communication model for BPSK signaling method in Nakagami-m fading environment for different values of the Nakagami fading parameter m. The authors derived the expression for the bit error probability in the exact form.

Mallik *et.al* [23] presented the analysis of a cooperative diversity system using a single relay with DF relaying protocol in a Rayleigh fading environment. The diversity combining scheme employed was a scaled SNR based selection combining technique. This new form of technique was based on incorporating a deterministic scaling factor to select the source-to-relay link for transmission. The authors presented an analytical expression in the closed-form for the bit error probability for BPSK modulation scheme.

In 2012, Dixit *et.al* [24] expressed the average symbol error rate (ASER) analytically in the form of a lower bound for a cooperative diversity communication system employing Amplify and Forward (AF) relaying protocol in Rayleigh fading environment using the best relay selection scheme with rectangular quadrature amplitude modulation scheme of the general order. The idea behind the use of the rectangular quadrature amplitude modulation lies in the fact that this is a general modulation technique whose results can be conveniently applied to other modulation techniques such as square quadrature amplitude modulation (QAM), orthogonal BFSK (binary frequency shift keying), BPSK (binary phase shift keying), QPSK (quadrature phase shift keying) and M-ASK (amplitude shift keying) techniques.

It was shown that the numerical results obtained with the derived lower bound of the ASER when compared with the simulation results obtained with Monte Carlo simulation, led to the observation that the derived lower bound is more suitable when the number of relays is less. The results obtained numerically using the lower bound were plotted for a range of SNR values and the effect of the system parameters on the performance was depicted precisely. The diversity combining technique employed was MRC.

In a regular cooperative communication network, irrespective of the conditions of the channels, the relays always transmit the message signal from the source to the destination which results in the inefficient utilization of the channels. A solution to this is to use incremental relaying technique in which the relay processes the message signal sent by the source only when the channel conditions are bad.

The incremental relaying technique makes use of the feedback sent from the destination node, i.e. if there is negative feedback from the destination about the message sent over the direct channel from the source, only then the relay forwards a processed version of the message signal to the destination. Ikki *et.al* [25] investigated the performance of the incremental relaying technique in the un-coded form in Rayleigh fading environment (with independent non-identical links). The authors derived novel expressions in the closed form for the BER, the outage probability and the achievable rates. It was shown that the incremental relaying technique achieved the maximum amount of diversity in comparison with the conventional cooperative diversity network.

The diversity combining scheme is yet another factor that plays an important role in the overall performance of the system. In place of MRC, the performance analysis of the cooperative diversity communication system could also use the simpler SC technique. Selvaraj *et.al* [26] investigated the SC technique in a cooperative diversity system to express the end-to-end SEP (symbol error probability) for MPSK (M-ary phase shift keying) with a new mathematical approach known as the paired-error approach. The authors presented a new selection combining technique that was shown to offer considerable gains in the SNR when compared with the conventional SC technique.

The performances of the selection combining scheme and the maximal ratio combining in a cooperative diversity scheme were compared in the work of Avendi *et.al* [27]. The authors proposed a SC scheme with the differential AF relaying protocol in slow Rayleigh fading environment. In SC combining technique, the link is selected on the basis of the maximum absolute value of the decision statistic for the non-coherent detection of the symbols transmitted by the source. Therefore, unlike MRC, the SC technique does not require the channel information at the destination terminal. The exact analytical expressions for the BER for SC scheme had shown that the performance of the SC technique closely matches that of the MRC scheme.

Co-channel interference is a challenge in the cooperative communication technique. Aissa *et.al* [28] performed the analysis of a two-way AF relay system, with Rayleigh fading channels in the presence of co-channel interferers. The authors derived the expressions for the CDF of the signal-to-interference-plus-noise ratio, also known as the SINR, the outage probability as well as the probability of error. The study also presented an analysis of the data rates achievable in the presence of co-channel interference.

The channel estimation is another area of research in the cooperative communication systems. Seyfi *et.al* [29], presented a comprehensive analysis of the performance of a selection relaying system in which the channel estimation is not perfectly known with AF relaying protocol in frequency and flat fading links. The expressions for the average SER and the outage probability in the closed form were derived.

In certain types of propagation environment, the resultant signal at the receiver is made up of two different, strong multipath signals which are specular in nature. This environment is statistically modeled using TWDP (Two wave with diffuse power) fading statistical model. This model typically finds application in the wireless sensor networks. A comprehensive evaluation of the TWDP fading environment was presented by Yao *et.al* [30] for the cooperative relaying system using DF protocol with MRC diversity combining at the receiver. The authors derived the outage probability of the system in the form of a lower bound.

Correlation among the several channel links in a cooperative diversity communication system is yet another area of research. Swaminathan *et.al* [31] proposed a cooperative diversity scheme with a single-relay in a relatively realistic scenario in which correlation was assumed among the different channel links namely- the source to destination (SD), the source to relay (SR), and the relay to destination (RD). A closed form expression in a generic form was derived for the symbol error probability for the MPSK modulation technique in a Nakagami-m fading environment having correlated channel with decode and forward relaying protocol. The mathematical approach used was the paired error approach. The derived results were verified using the results obtained with the Monte Carlo simulation.

This work was continued further by Chen *et.al* [32] for the Nakagami-m fading environment with correlated links using AF relaying protocol. The exact analytical expression for the MGF was derived for different values of the Nakagami-m fading parameter assuming that the correlated links each have the same value of the

Nakagami-m fading parameter. The effect of correlation among the channel links was shown for the fixed gain relaying and the variable gain relaying. Roy [32] *et.al* proposed a comprehensive system based on the correlation between the SD and the RD links with DF relaying protocol for the MPSK modulation scheme using the paired error approach. The authors derived the end-to-end probability of error for each symbol transmitted by the source expressed in the closed form.

Ramabadran *et.al* [33] presented the analysis of correlated links by assuming correlation between the SD, the RD and the SR links for MPSK signaling and the DBPSK (Differential BPSK) signaling using DF cooperative diversity system with SC combining technique. The channel links were slow and flat Nakagami-m fading links. The exact expressions of the SEP were derived in the closed form for the two modulation schemes. It was shown that the performance of the SR-RD correlated links was better than the conventional uncorrelated channel system. It was also presented that there is a slight improvement in the performance of the outage probability at low values of the SNR for the SR-RD channel correlation when compared with the uncorrelated channel system.

The fading environment has a significant impact in the overall system performance. Chang *et.al* [34] presented an analysis of the AF cooperative diversity communication system with the N^{th} best relay selection technique in a Ricean fading environment with independent and non-identical channel links. The authors derived an analytical expression for the average symbol error rate. Ricean fading is observed in wireless mesh and microcellular mobile communication systems where the LOS (line-of-sight) component is predominant. It was shown that the diversity gain for the proposed system was 'R-N+2', where R represents the total number of relays in the system. The accuracy of the derived result was verified against the simulation results.

The cooperative communication systems offer a promising technology that can be implemented in industrial applications. In the cluttered industrial settings, the wireless communications applications are faced with the challenges to offer utmost reliability for monitoring of factory and other control applications.

Masood *et.al* [35] investigated the cooperative relaying system in such fading environments so as to reduce the probability of outage. The authors presented an experimental setup based analysis for the devices compatible with the IEEE 802.15.4

analyzed in an industrial production setting. It was shown with the use of different relay update mechanism that the relaying protocol method gives better performance over the conventional system in which time diversity is used for retransmission of the packets that are not able to reach the destination successfully. The implementation of the system had shown that the overall performance depends on the protocol and the network metrics.

Most recently, in 2015 Beaulieu *et.al* [36] presented an analysis of the DP cooperative system with BPSK modulation technique in Rayleigh fading environment to give the expression for the average BEP (bit error probability) in the exact form. The derivation of the BEP in the closed-form was derived on the basis of the calculation of the characteristic functions evaluated for the decision statistics.

Kanatas *et.al* [2] investigated the analysis of rectangular QAM modulation scheme for the K-fading environment (which is a statistical model for the composite fading channels).

The authors presented the analytical expression for the average symbol error probability (SEP) considering the rectangular quadrature amplitude modulation (QAM) signalling modelled statistically by the generalized-K (also known as the (K_G)) distribution. The analysis makes use of a fast converging mathematical series representation (infinite in nature) to calculate the average of the product of two Gaussian-Q mathematical functions over K_G fading links. Numerically computed results have been validated by comparing with the simulated results.

The holistic elaboration of the area of cooperative diversity networks as presented by the literature review in this chapter forms the motivation for the work presented in the subsequent chapters.

CHAPTER 3
PERFORMANCE ANALYSIS OF DIVERSITY SYSTEMS

This chapter presents an analysis of the error probability performance in a flat fading environment using a diversity combining receiver.

3.1 AWGN CHANNEL

The modulated signal for an AWGN channel gives the signal [5]

$$s(t) = \Re\{u(t)e^{j2\pi f_c t}\}, \tag{3.1}$$

where, u(t) is the original transmitted signal, f_c is the carrier frequency and $\Re\{.\}$ represents the real part of a signal.

The received signal, denoted by r(t), is expressed as [5]

$$r(t) = s(t) + n(t), \tag{3.2}$$

where, n(t) is the additive white Gaussian random noise with zero mean and PSD (power spectral density) as $N_0/2$.

The signal power-to-noise power ratio (SNR) is defined as the ratio of the power P_r at the receiver to the noise power which lies within the bandwidth of the signal transmitted by the source. The power at the receiver is determined by the power transmitted by the source, the propagation path loss and effects of fading. The noise power is determined by the bandwidth of the original transmitted signal and the spectral characteristics of $n(t)$ [5].

If B is the bandwidth of the signal u(t) (where, u(t) is the complex envelope of the signal s(t)), then the bandwidth for the transmitted signal (s(t)) is found to be 2B [5].

The total power of the noise within the defined bandwidth of 2B is [27]

$$N = N_0/2 \times 2B = N_0B. \tag{3.3}$$

The SNR at the receiver is expressed as [5]

$$SNR = \frac{P_r}{N_o B}, \tag{3.4}$$

where, P_r is the received power and B is the bandwidth of the signal u(t).

The SNR is usually given as a function of the energy of the signal for one bit, denoted by E_b and the energy of one symbol, denoted by E_s[5].

Therefore, the SNR is expressed mathematically as [5]

$$SNR = \frac{P_r}{N_o B} = \frac{E_s}{N_o B T_s} = \frac{E_b}{N_o B T_b}, \tag{3.5}$$

where, T_s is the time duration for one symbol and T_b is the time duration for one bit.

The SNR per symbol γ_s and the SNR per bit γ_b is expressed as [21] $\gamma_s = E_s/N_o$ and $\gamma_b = E_b/N_o$ respectively.

For an M-ary signaling scheme [6],

$$\gamma_b \approx \frac{\gamma_s}{\log_2 M}, \text{and } E_b \approx \frac{E_s}{\log_2 M}. \tag{3.6}$$

The approximate values of the symbol error probability $P_s(\gamma_s)$ and the bit error probability $P_b(\gamma_b)$ for different modulation schemes in the AWGN channel are given in table 3.1.

MODULATION	$\mathbf{P_s(\gamma_s)}$	$\mathbf{P_b(\gamma_b)}$
BFSK	$Q(\sqrt{\gamma_b})$	$Q(\sqrt{\gamma_b})$
BPSK	$Q(\sqrt{2\gamma_b})$	$Q(\sqrt{2\gamma_b})$
QPSK,4-QAM	$2Q(\sqrt{\gamma_s})$	$Q(\sqrt{2\gamma_b})$
MPAM	$\frac{2(M-1)}{M}Q\left(\sqrt{\frac{6\bar{\gamma}_s}{M^2-1}}\right)$	$\frac{2(M-1)}{M\log_2 M}Q\left(\sqrt{\frac{6\overline{\gamma_b}(\log_2 M)}{M^2-1}}\right)$
MPSK	$2Q(\sqrt{2\gamma_s}\sin(\pi/M))$	$\frac{2}{\log_2 M}Q(\sqrt{2\gamma_b \log_2 M}\sin(\pi/M))$
RECTANGULAR M-QAM	$\frac{4(\sqrt{M}-1)}{\sqrt{M}}Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M-1}}\right)$	$\frac{4(\sqrt{M}-1)}{\sqrt{M}\log_2 M}Q\left(\sqrt{\frac{3\overline{\gamma_b}\log_2 M}{M-1}}\right)$
NON RECTANGULAR M-QAM	$4Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M-1}}\right)$	$\frac{4}{\log_2 M}Q\left(\sqrt{\frac{3\bar{\gamma}_s \log_2 M}{M-1}}\right)$

Table 3.1 Approximate symbol error probability$P_s(\gamma_s)$ and bit error probability $P_b(\gamma_b)$ for coherent modulation schemes [5]

3.2 BER OF BPSK IN RAYLEIGH FADING CHANNEL

This section derives the bit error probability for a signal modulated using BPSK and transmitted over a Rayleigh fading channel. Consider h as the Rayleigh fading coefficient on a channel. The effective value of the ratio of the bit energy to the noise power is $|h|^2\frac{E_b}{N_0}$, where, E_b denoted the bit energy and N_0 denotes the noise power [4].

The expression for the conditional probability of error on a fading channel with Rayleigh fading coefficient h is [4]

$$P_{b|h} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{|h|^2\frac{E_b}{N_0}}\right) = \frac{1}{2}\operatorname{erfc}(\sqrt{\gamma}).$$

(3.7)

where $\gamma = |h|^2 \frac{E_b}{N_0}$ is the value of the instantaneous SNR(a random variable).

The expression for the bit error probability is found by the integration of the conditional bit error probability expressed in (3.7) over the PDF(Probability Distribution Function) of the Rayleigh fading envelope, given by [4]

$$P_b = \int_0^{\infty} \frac{1}{2} \mathrm{erfc}\left(\sqrt{\gamma}\right) p(\gamma) d\gamma \tag{3.8}$$

where, the PDF $p(\gamma)$ is given by [4]

$$p(\gamma) = \frac{1}{\gamma} e^{-\frac{\gamma}{\bar{\gamma}}}, \gamma \geq 0 \text{ and } \bar{\gamma} = \frac{E_b}{N_0}. \tag{3.9}$$

To solve the equation (3.8), an expression for the integral $\int \mathrm{erfc}(x) e^{-\frac{x}{a}} dx$ is derived in the subsequent steps.

In general, the complementary error function is given by [4]

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt. \tag{3.10}$$

and the error function erf(x) is given by [4]

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^{x} e^{-t^2} dt. \tag{3.11}$$

Differentiate (3.10) with respect to x to get

$$\frac{d}{dx} \mathrm{erf}(x) = -\frac{2}{\sqrt{\pi}} e^{-x^2}. \tag{3.12}$$

The definitions expressed in (3.10), (3.11) and (3.12) are given to evaluate the expression for the erfc(x) given by [4]

$$\int \mathrm{erfc}(x)dx = x\,\mathrm{erfc}(x) - \frac{1}{\sqrt{\pi}}e^{-x^2}. \tag{3.13}$$

Differentiation of the $\mathrm{erfc}(\sqrt{x})$ and $\mathrm{erf}(\sqrt{x})$ with respect to x gives [4]

$$\frac{d}{dx}\mathrm{erfc}(\sqrt{x}) = -\frac{1}{\sqrt{\pi}}e^{-x}x^{-\frac{1}{2}} \tag{3.14}$$

and

$$\frac{d}{dx}\mathrm{erf}(\sqrt{x}) = \frac{1}{\sqrt{\pi}}e^{-x}x^{-\frac{1}{2}}. \tag{3.15}$$

Substitute (3.14) and (3.15) in (3.13) and rearrange the terms to obtain the expression given by [4]

$$\int \mathrm{erfc}(x)e^{-\frac{x}{a}}dx = -a\,\mathrm{erfc}(\sqrt{x})e^{-\frac{x}{a}} - a\sqrt{\frac{a}{a+1}}\,\mathrm{erf}\left(\sqrt{\frac{a+1}{a}}\sqrt{x}\right). \tag{3.16}$$

Substitute (3.16) in (3.8) to solve the expression for the bit error probability P_b given by [4]

$$P_b = \frac{1}{2\bar{\gamma}}\int_0^{\infty} \mathrm{erfc}(\sqrt{\gamma})e^{-\frac{\gamma}{\bar{\gamma}}}\,d\gamma \tag{3.17}$$

P_b in (3.17) is expressed using (3.16) as

$$P_b = \frac{1}{2\bar{\gamma}}\left[\bar{\gamma}\,\mathrm{erfc}\left(\sqrt{\gamma}\right)e^{-\frac{\gamma}{\bar{\gamma}}} + \bar{\gamma}\sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+1}}\,\mathrm{erf}\left(\sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+1}}\sqrt{\gamma}\right)\right]_0^{\infty}.$$

(3.18)

Solve the integral in (3.18) to obtain [4]

$$P_b = \frac{1}{2\bar{\gamma}}\left(1 - \sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+1}}\right).$$

(3.19)

Use the value of $\bar{\gamma} = \frac{E_b}{N_0}$, where, E_b is the bit energy and N_0 is the noise power to give the final expression for P_b expressed as [4]

$$P_b = \frac{1}{2\bar{\gamma}}\left(1 - \sqrt{\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}+1}}\right).$$

(3.20)

Therefore, the expression derived in (3.20) gives the probability of bit error for a BPSK modulated signal over a Rayleigh fading channel.

3.3 BER OF BPSK IN RAYLEIGH FADING CHANNEL WITH MRC DIVERSITY

This section derives the bit error probability for a signal modulated using BPSK and transmitted over a Rayleigh fading channel with MRC diversity combining at the receiver.

Consider a system that transmits equally likely BPSK modulated symbols with an MRC diversity receiver with L branches.

For the MRC combiner, the total SNR γ_t at the output is given by [4]

$$\gamma_t = \sum_{l=1}^{L} \gamma_l .$$

(3.21)

The conditional bit error rate is given by [4]

$$P_b(E/\{\gamma_t\}_1^L) = Q(\sqrt{2a\gamma_t}),$$

(3.22)

where a=1 for BPSK modulation, Q(.) is the Q function.

The bit error probability is found by the integration of the conditional bit error rate given in [4] over the PDF $p_{\gamma_t}(\gamma_t)$ of the total SNR γ_t given by [4]

$$P_b(E) = \int_0^{\infty} Q\left(\sqrt{2a\gamma_t}\right) p_{\gamma_t}(\gamma_t) d\gamma_t .$$

(3.23)

In the MRC diversity system, there are L i.i.d (independent and identically distributed) Rayleigh fading links. The PDF $p_{\gamma_l}(\gamma_l)$ for the lth path is given by [4]

$$p_{\gamma_l}(\gamma_l) = \left(\frac{1}{\bar{\gamma}}\right) e^{-\frac{\gamma_l}{\bar{\gamma}}},$$

(3.24)

where, γ_l is the SNR for the Lth branch.

At the receiver, the bit error probability $P_b(E)$ is given by [4]

$$P_b(E) = \int_0^{\infty}\int_0^{\infty} \ldots \int_0^{\infty} P_b\left(\{\gamma_l\}_{l=1}^{L}\right) \prod_{l=1}^{L} p_{\gamma_l}(\gamma_l) d\gamma_1 d\gamma_2 \ldots d\gamma_L .$$

(3.25)

The value of the Q-function used in (3.23) is given by [4]

$$Q\left(\sqrt{2a\gamma_l}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(\frac{-a\gamma_l}{\sin^2\theta}\right) d\theta.$$

(3.26)

Use (3.22) and (3.26) in (3.25) to obtain the expression for $P_b(E)$ in the form [4]

$$P_b(E) = \int_0^{\infty}\int_0^{\infty} \dots \int_0^{\infty} \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{l=1}^{L} \exp\left(\frac{-a\gamma_l}{\sin^2\phi}\right) p_{\gamma_l}(\gamma_l) d\gamma_l d\gamma_2 \dots d\gamma_L$$

(3.27)

Express (3.27) in the form of the moment generating function $M_{\gamma_l}\left(\frac{-a}{\sin^2\phi}\right)$ as [4]

$$P_b(E) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{l=1}^{L} M_{\gamma_l}\left(\frac{-a}{\sin^2\phi}\right) d\phi$$

(3.28)

The product term in (3.28) used for the L-branches is rearranged to obtain the expression [4]

$$P_b(E) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(M_{\gamma}\left(\frac{-a}{\sin^2\phi}\right)\right)^L d\phi$$

(3.29)

The MGF for the Rayleigh fading channel is given by [4]

$$M_{\gamma}(s) = \frac{1}{(1 - s\gamma)}$$

(3.30)

This MGF can be expressed in an alternative form with the argument $\frac{-a}{\sin^2\phi}$ [4].

Use the definition of $M_\gamma(s)$ in (3.30) to express the term $M_\gamma\left(\frac{-a}{\sin^2\phi}\right)$ used in the equation (3.29) to obtain [4]

$$M_\gamma\left(\frac{-a}{\sin^2\phi}\right) = \left(\frac{\sin^2\phi}{\sin^2\phi + a\bar{\gamma}}\right)$$

(3.31)

Substitute the value of $M_\gamma\left(\frac{-a}{\sin^2\phi}\right)$ from (3.31) in (3.28) to obtain the expression for $P_b(E)$ given by [4]

$$P_b(E) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\phi}{\sin^2\phi + a\bar{\gamma}}\right)^L d\phi.$$

(3.32)

Equation (3.32) is solved using the special integrals in the form of summation [4] to give [4]

$$P_b(E) = \frac{1}{2} - \frac{1}{\pi}\sqrt{\frac{a\bar{\gamma}}{1 + a\bar{\gamma}}}\left[\frac{\pi}{2}\sum_{j=0}^{L-1}\binom{2j}{j}\frac{1}{(4(1 + a\bar{\gamma}))^k}\right]$$

(3.33)

The expression in (3.33) gives the probability of error for Rayleigh fading channels with L-order MRC diversity scheme [4].

With the use of similar mathematical approach, expressions can be derived for different modulation schemes and fading environments in MRC diversity combining method.

CHAPTER 4
PERFORMANCE ANALYSIS OF COOPERATIVE DIVERSITY SYSTEMS

4.1 SYSTEM MODEL

This chapter presents a comprehensive analysis of AF cooperative diversity systems with Rayleigh fading channels for different modulation techniques.

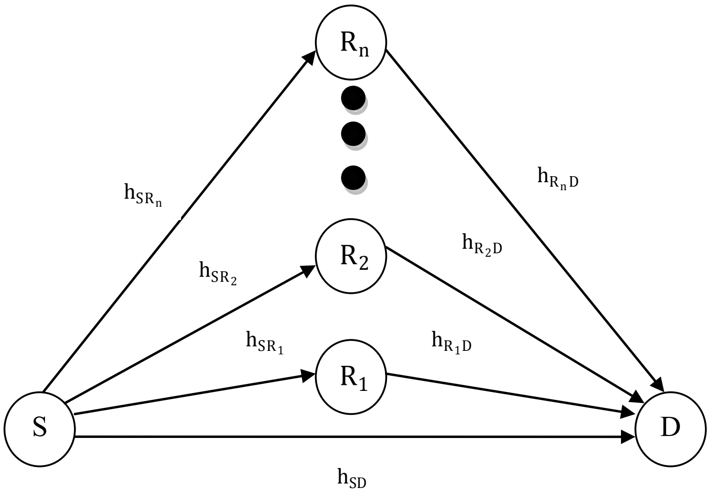

Figure 4.1 System Model

The proposed system as illustrated in figure 4.1 is a cooperative diversity system with the source S, the relays R_1, R_2 .. and R_N, and the destination D with flat Rayleigh fading links. The message from the source S is propagated through the direct channel between S and D as well as through one indirect channel consisting out of one best relay out of all the relays present in the network. There is one transmit antenna on S, D and the relays R_n each (where$1 \leq n \leq N$). The direct link is designated as $S \rightarrow D$ link and the indirect two-hop AF relay links are designated as $S \rightarrow R_n \rightarrow D$ (where$1 \leq n \leq N$). We assume that the CSI (channel state information) is known at the receiver terminal.

In the first phase of transmission, the source sends the message signal x to the relay nodes and the destination node. The signal obtained at the receiver through the i-th relay node is given by [19]

$$y_{SR_i} = h_{SR_i}\, x + n_{SR_i}, \tag{4.1}$$

and the signal obtained at the destination terminal is given by [19]

$$y_{SD} = h_{SD}\, x + n_{SD}, \tag{4.2}$$

where, h_{SR_i} and h_{SD} are the respective gains for the channel between the source node and the i-th relay terminal and the channel between the source node and the destination node. The complex AWGN noise at the i-th relay node is represented by $n_{SR_i} \sim \mathcal{CN}(0, N_o)$ and at the destination node is represented by $n_{SD} \sim \mathcal{CN}(0, N_o)$, where N_o represents the variance of noise [19].

In the second phase of user cooperation, the i-th relay node amplifies the received signal which is forwarded to the destination node over the h_{R_iD} link. The signal received at the destination node through the relay link is given as [19]

$$y_{R_iD} = G_i h_{R_iD} y_{SR_i} + n_{R_iD}, \tag{4.3}$$

where, h_{R_iD} is gain on the channel between the i-th relay node and the destination node and $n_{R_iD} \sim \mathcal{CN}(0, N_o)$ is the complex AWGN. The gain for the i-th relay node is given by [19]

$$G_i = \frac{E_s}{(E_s |h_{SR_i}|^2 + N_o)}, \tag{4.4}$$

where, E_s is the average energy of each symbol.

The signals received at the destination node are combined with MRC diversity combining technique. The instantaneous value of the end-to-end SNR at the destination node is written as [18]

$$\gamma_D = \gamma_{SD} + \sum_{i=1}^{N} \frac{\gamma_{SR_i}\gamma_{R_iD}}{\gamma_{SR_i} + \gamma_{R_iD}}, \tag{4.5}$$

where,

$$\gamma_{SR_i} = \left|h_{SR_i}\right|^2 (E_s/N_o), \tag{4.6}$$

$$\gamma_{R_iD} = \left|h_{R_iD}\right|^2 (E_s/N_o), \tag{4.7}$$

and γ_{SR_i}, γ_{R_iD} represent the instantaneous value of the SNR for the $S \rightarrow R_i$ and the $R_i \rightarrow D$ channel links respectively.

The instantaneous value of the SNR measured at the $S \rightarrow D$ link is given by [11]

$$\gamma_{SD} = |h_{SD}|^2 (E_s/N_o). \tag{4.8}$$

In this study, the Rayleigh fading channels have been considered. The PDF and the CDF for the SNR over the channels can be expressed respectively as [19]

$$f_{\gamma_v}(\gamma) = \frac{1}{\bar{\gamma}_v} e^{-\frac{\gamma}{\bar{\gamma}_v}}, \tag{4.9}$$

$$F_{\gamma_v}(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}_v}}, \tag{4.10}$$

where, $v \in \{SR_i, R_iD, SD\}$ represents the index for the link.

The analysis is simplified with the use of a simple upper bound for the second term in the equation for the SNR expressed in the equivalent form. This upper bound is given by [14]

$$\gamma_i = \min\left(\gamma_{SR_i}, \gamma_{R_iD}\right) \geq \frac{\gamma_{SR_i}\gamma_{R_iD}}{\gamma_{SR_i} + \gamma_{R_iD}}.$$

(4.11)

Therefore, the end-to-end SNR (equivalent) can be expressed in the form of an upper bound given by [19]

$$\gamma_D \leq \gamma_{up} = \gamma_{SD} + \sum_{i=1}^{N} \gamma_i$$

. (4.12)

The SNR expressed in (4.12) in the form of an upper bound is more convenient for the performance evaluation of the system and is found to be fairly accurate at the SNR values for the medium and the high range.

4.2 NO-RELAY SELECTION MECHANISM

This section presents the performance analysis of the considered system for the simple case in which there is no selection of the best relay [19].

4.2.1 EVALUATION OF MGF, CDF AND PDF OF END-TO-END SIGNAL TO NOISE RATIO

To determine the PDF and the CDF of the end-to-end signal-to-noise ratio γ_{up}, the first step is to calculate the MGF of γ_{up}. The next step is to take the inverse Laplace transform of the obtained MGF to obtain the PDF [19].

The analytical framework presented in this section assumes that the SNRs γ_{SD} and γ_i are independent of each other. The MGF of γ_{up} is given by [19]

$$M_{\gamma_{up}}(s) = M_{\gamma_{SD}}(s) \prod_{i=1}^{N} M_{\gamma_i}(s)$$

(4.13)

where, $M_{\gamma_{SD}}(s)$ and $M_{\gamma_i}(s)$ are the MGF of the SNRs γ_{SD} and γ_i respectively.

The MGF is defined by [19]

$$M_\gamma(s) = E\{e^{-s\gamma}\} = \int_0^\infty e^{-s\gamma} f_\gamma(\gamma) d\gamma. \tag{4.14}$$

For the S → D channel, the MGF $M_{\gamma_{SD}}(s)$ is given by [19]

$$M_{\gamma_{SD}}(s) = (1 + \bar{\gamma}_{SD}\, s)^{-1} \tag{4.15}$$

where, $\bar{\gamma}_{SD} = \bar{\gamma}_0 = E_s/N_o$.

To evaluate the MGF for the $S \rightarrow R_i \rightarrow D$ channel, the PDF for the channel is obtained from the corresponding CDF. When γ_{SR_i} and γ_{R_iD} are independent of each other, the CDF for the SNR $\gamma_i = \min(\gamma_{SR_i}, \gamma_{R_iD})$ is given by [39]

$$F_{\gamma_i}(\gamma) = 1 - \Pr\left(\gamma_{SR_i} > \gamma\right) \Pr\left(\gamma_{R_iD} > \gamma\right) \tag{4.16}$$

$$= 1 - [1 - \Pr\left(\gamma_{SR_i} \leq \gamma\right)][1 - \Pr\left(\gamma_{R_iD} \leq \gamma\right)] \tag{4.17}$$

$$= 1 - \left[1 - F_{\gamma_{SR_i}}(\gamma)\right]\left[1 - F_{\gamma_{R_iD}}(\gamma)\right], \tag{4.18}$$

where, $F_{\gamma_{SR_i}}(\gamma)$ and $F_{\gamma_{R_iD}}(\gamma)$ are the CDF for the SNR values for the $S \rightarrow R_i$ and the $R_i \rightarrow D$ channel links respectively for the i-th relay node.

Put the value of CDF from (4.10) in (4.18) with the use of the proper link index to obtain the CDF $F_{\gamma_i}(\gamma)$ as [19]

$$F_{\gamma_i}(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}_{C_i}}} \tag{4.19}$$

where, $\bar{\gamma}_{C_i} = \frac{\bar{\gamma}_{SR_i} \bar{\gamma}_{R_iD}}{\bar{\gamma}_{SR_i} + \bar{\gamma}_{R_iD}}$.

Since the channels are assumed to experience flat Rayleigh fading with independent and identical links, and $\overline{\gamma}_{SR_i} = \overline{\gamma}_{R_iD} = \overline{\gamma}_0$, hence, the average SNRs are related as [19]

$$\overline{\gamma}_{C_i} = \overline{\gamma}_C = 0.5\overline{\gamma}_0. \tag{4.20}$$

The PDF $f_{\gamma_i}(\gamma)$ for the SNR γ_i is obtained by the differentiation of the CDF $F_{\gamma_i}(\gamma)$ (expressed in (4.19)) with respect to γ and is given as [19]

$$f_{\gamma_i}(\gamma) = \frac{1}{\overline{\gamma}_{C_i}} e^{-\frac{\gamma}{\overline{\gamma}_{C_i}}}. \tag{4.21}$$

The MGF of the SNR γ_i is given by [19]

$$M_{\gamma_i}(s) = (1 + \overline{\gamma}_{C_i} s)^{-1}. \tag{4.22}$$

Use the equation (4.15) and (4.22) in the equation (4.13) to obtain the MGF of the SNR γ_{up} expressed by [19]

$$M_{\gamma_{up}}(s) = (1 + \overline{\gamma}_{SD} s)^{-1} \prod_{i=1}^{N} (1 + \overline{\gamma}_{C_i} s)^{-1}. \tag{4.23}$$

With the use of the Rayleigh i.i.d fading channels (for which $\overline{\gamma}_{SR_i} = \overline{\gamma}_{R_iD} = \overline{\gamma}_0$ and $\overline{\gamma}_{C_i} = 0.5\overline{\gamma}_0$), the MGF $M_{\gamma_{up}}(s)$ is expressed as [19]

$$M_{\gamma_{up}}(s) = (1 + \overline{\gamma}_0 s)^{-1} (1 + 0.5\overline{\gamma}_0 s)^{-N}. \tag{4.24}$$

4.3 BEST RELAY SELECTION MECHANISM

This section presents the performance analysis of the considered system for the case of the best relay selection mechanism.

The method by which the best relay (designated as b) is selected out of the N relays in the network is expressed as [24]

$$b = \mathrm{argmax}_{i\in R}\{\gamma_i\},$$

(4.25)

where, $R = \{1,2,\dots N\}$ and

$$\gamma_i = \min(\gamma_{SR_i}, \gamma_{R_iD})$$

(4.26)

where, γ_i denotes the instantaneous value of the end-to-end SNR of the $S \rightarrow R_i \rightarrow D$ link expressed as an upper bound for the relay denoted by R_i.

The total signal-to-noise ratio (SNR) γ_t expressed as an upper bound at the destination terminal is [24]

$$\gamma_t \leq \gamma_{SD} + \max_{n\in R}\left\{\min(\gamma_{SR_i}, \gamma_{R_iD})\right\}.$$

(4.27)

4.3.1 EVALUATION OF MGF, CDF AND PDF OF END-TO-END SIGNAL TO NOISE RATIO

For the case of i.i.d channels ($\bar{\gamma}_{C_i} = \bar{\gamma}_C = 0.5\bar{\gamma}_0$), the CDF given in (4.18) can be expressed as [19]

$$F_{\gamma_b}(\gamma) = [F_{\gamma_b}(\gamma)]^N = \left[1 - e^{-\frac{\gamma}{\bar{\gamma}_C}}\right]^N .$$

(4.28)

The PDF $f_{\gamma_b}(\gamma)$ for the SNR γ_b is obtained with the differentiation of the CDF $F_{\gamma_b}(\gamma)$ with respect to the random variable γ, and can be given by [19]

$$f_{\gamma_b}(\gamma) = N f_{\gamma_i}(\gamma)\left[F_{\gamma_i}(\gamma)\right]^{N-1}$$

(4.29)

$$f_{\gamma_b}(\gamma) = \frac{N}{\bar{\gamma}_C} e^{-\frac{\gamma}{\bar{\gamma}_C}} \left[1 - e^{-\frac{\gamma}{\bar{\gamma}_C}}\right]^{N-1}.$$

(4.30)

Express equation (4.14) using equation (4.30) and use the property of binomial expansion. This gives the MGF $M_{\gamma_b}(s)$ as [19]

$$M_{\gamma_b}(s) = \sum_{n=1}^{N} \binom{N}{n} \frac{n(-1)^{n-1}}{(n + \bar{\gamma}_C s)},$$

(4.31)

where, $\binom{N}{n} = N!/[n!(N-n)!]$ denotes the binomial coefficient.

The MGF $M_{\gamma_{up}}(s)$ for the end-to-end SNR γ_{up} is expressed as [19]

$$M_{\gamma_{up}}(s) = M_{\gamma_{SD}}(s) M_{\gamma_b}(s).$$

(4.32)

Use the equations (4.15) and (4.31), to obtain (23) in the form [19]

$$M_{\gamma_{up}}(s) = \sum_{n=1}^{N} \binom{N}{n} \frac{n(-1)^{n-1}}{(n + \bar{\gamma}_C s)(1 + \bar{\gamma}_{SD} s)}$$

(4.33)

The PDF $f_{\gamma_{up}}(\gamma)$ for the SNR γ_{up} is given by [19]

$$f_{\gamma_{up}}(\gamma) = \mathcal{L}^{-1}\left\{M_{\gamma_{up}}(s)\right\} = \int_0^{\infty} e^{sx} M_{\gamma_{up}}(s) ds,$$

(4.34)

where, $\mathcal{L}^{-1}\{.\}$ is the operator for the inverse Laplace Transform operation.

Now, substitute the value of MGF from (4.33) in (4.34) to obtain [19]

$$f_{\gamma_{up}}(\gamma) = \sum_{n=1}^{N} \binom{N}{n} \frac{n(-1)^{n-1}}{(\bar{\gamma}_C - n\bar{\gamma}_{SD})} \left[e^{-n\frac{\gamma}{\bar{\gamma}_C}} - e^{-\frac{\gamma}{\bar{\gamma}_{SD}}}\right].$$

(4.35)

The CDF $F_{\gamma_{up}}(\gamma)$ is found by the integration of the PDF in (4.35) with respect to the random variable γ and is given by [19]

$$F_{\gamma_{up}}(\gamma) = 1 + \sum_{n=1}^{N} \binom{N}{n} \frac{n(-1)^{n-1}}{\left(\bar{\gamma}_C - n\bar{\gamma}_{SD}\right)} \left[\bar{\gamma}_{SD}\, e^{-\frac{\gamma}{\bar{\gamma}_{SD}}} - \frac{\bar{\gamma}_C}{n} e^{-n\frac{\gamma}{\bar{\gamma}_C}}\right]. \tag{4.36}$$

4.3.2 EVALUATION OF AVERAGE END-TO-END SNR GAIN

The average value of the SNR gain in the relay selection cooperative diversity system considered in the proposed system is defined as [19]

$$Gain_{SNR} = \frac{\overline{\gamma_{up}}}{\overline{\gamma_{up,1}}}, \tag{4.37}$$

where, $\overline{\gamma_{up,1}}$ is the average signal-to-noise-ratio of a one relay cooperative diversity system, and $\overline{\gamma_{up}}$ is the average signal-to-noise-ratio of a relay selection cooperative diversity system.

In case of i.i.d channels, the SNR $\overline{\gamma_{up}}$ is given by [19]

$$\overline{\gamma_{up}} = \bar{\gamma}_{C_1} + 0.5\bar{\gamma}_0 = 1.5\bar{\gamma}_0. \tag{4.38}$$

The SNR $\overline{\gamma_{up}}$ can be expressed as [19]

$$\overline{\gamma_{up}} = \int_0^{\infty} \gamma_{up}\, f_{\gamma_{up}}\left(\gamma_{up}\right) d\gamma_{up}. \tag{4.39}$$

Use the equation (4.35) in (4.39) and solve the integral to obtain the expression for the average SNR $\overline{\gamma_{up}}$ as [19]

$$\overline{\gamma_{up}} = \sum_{n=1}^{N} \frac{\bar{\gamma}_C}{n} + \bar{\gamma}_{SD}. \tag{4.40}$$

Since it is known that $\bar{\gamma}_C = 0.5\bar{\gamma}_0$ and $\bar{\gamma}_{SD} = \bar{\gamma}_0$, can be expressed as [19]

$$\overline{\gamma_{up}} = \left[1 + \sum_{n=1}^{N} \frac{1}{2n}\right] \bar{\gamma}_0. \tag{4.41}$$

Use (4.37) to obtain average SNR gain in the form [19]

$$Gain_{SNR} = \frac{2}{3}\left[1 + \sum_{n=1}^{N} \frac{1}{2n}\right]. \tag{4.42}$$

4.3.3 EVALUATION OF OUTAGE PROBABILITY

For the given system, consider the outage probability P_{out} as the probability that the average SNR (end-to-end) is below a certain threshold α.

P_{out} is given as [19]

$$P_{out} = \int_0^{\alpha} f_{\gamma_b}(\gamma) d\gamma = F_{\gamma_{up}}(\alpha) \tag{4.43}$$

Use the expression in (4.36), to obtain P_{out} in the form [19]

$$P_{out} = 1 + \sum_{n=1}^{N} \binom{N}{n} \frac{n(-1)^{n-1}}{(\bar{\gamma}_C - n\bar{\gamma}_{SD})} \left[\bar{\gamma}_{SD}\, e^{-\frac{\alpha}{\bar{\gamma}_{SD}}} - \frac{\bar{\gamma}_C}{n} e^{-n\frac{\alpha}{\bar{\gamma}_C}}\right]. \tag{4.44}$$

4.3.4 EVALUATION OF AVERAGE ASER FOR BPSK MODULATION

The average SER ($\overline{SER}$) is found by integration of the instantaneous SEP (symbol error probability) over the PDF $f_{\gamma_{up}}(\gamma)$ evaluated at the destination node and is represented by $SER(\gamma)$, is expressed as [19]

$$\overline{SER} = \int_0^{\infty} SER(\gamma) f_{\gamma_{up}}(\gamma) d\gamma. \tag{4.45}$$

With the use of the approach based on the MGF [37], the average SER can be obtained for the M-PSK and the M-QAM signaling techniques. Since in the analysis of the given system we make use of the MGF for the SNR γ_{up} expressed as an upper bound, hence, the expression for the average SER is basically a lower bound expression.

For binary signaling methods, the average SER is expressed as [40]

$$\overline{\mathrm{SER}} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_{\gamma_{up}}\left(\frac{g}{\sin^2\theta}\right) d\theta. \tag{4.46}$$

where g=1 for the BPSK modulation method while g=0.5 in the case of the orthogonal BFSK scheme and the MGF $M_{\gamma_{up}}$ is as expressed in (4.33).

Substitute (4.33) in (4.46), and rearrange the terms to obtain the average SER as [19]

$$\overline{\mathrm{SER}} = \sum_{n=1}^{N}\binom{N}{n}(-1)^{n-1} \times \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{\sin^2\theta + c_1}\right)\left(\frac{\sin^2\theta}{\sin^2\theta + c_2}\right) d\theta \tag{4.47}$$

where, $c_1 = g\bar{\gamma}_C/n = 0.5g\bar{\gamma}_0/n$ and $c_2 = g\bar{\gamma}_{SD} = 0.g\bar{\gamma}_0$.

Use the method of partial fraction expansion, and rearrange the terms to obtain the average symbol error rate for the BPSK modulation scheme in the form of the expression [19]

$$\overline{\mathrm{SER}} = \sum_{n=1}^{N}\binom{N}{n}\frac{(-1)^{n-1}}{(1-2n)} \times \left[I_1\left(\frac{\bar{\gamma}_0}{2n}\right) - 2nI_1(\bar{\gamma}_0)\right] \tag{4.48}$$

expressed in the closed-form, where, $I_1(c)$ is given by [4, (eq.5A.9)]

$$I_1(c) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right) d\theta = \frac{1}{2}\left(1 - \sqrt{\frac{c}{1+c}}\right). \tag{4.49}$$

4.3.5 EVALUATION OF AVERAGE ASER FOR MPSK MODULATION

The average symbol error rate for the M-PSK signaling method is given by [24]

$$\overline{\mathrm{SER}} = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\int_0^{\frac{\pi}{2}} M_{\gamma_{up}}\left(\frac{g_{PSK}}{\sin^2\theta}\right) d\theta \tag{4.50}$$

where, $g_{PSK} = \sin^2(\pi/M)$.

Express (4.33) in the form of (4.50) and use the steps followed from (4.47)-(4.48) and rearrange the terms, the average symbol error rate is expressed as [12]

$$\overline{\mathrm{SER}} = \sum_{n=1}^{N}\binom{N}{n}\frac{(-1)^{n-1}}{(1-2n)} \times \left[I_2\left(\frac{\bar{\gamma}_0}{2n}\sin^2\left(\frac{\pi}{M}\right)\right) - 2nI_2(\bar{\gamma}_0\sin^2\left(\frac{\pi}{M}\right))\right]. \tag{4.51}$$

in which $I_2(.)$ is given as [4,(eq.5A.15)]

$$I_2(c) = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right) d\theta$$

$$= \frac{M-1}{M} \times \left\{1 - \sqrt{\frac{c}{1+c}}\left(\frac{M}{(M-1)\pi}\right) \times \left[\frac{\pi}{2} + \tan^{-1}\left(\sqrt{\frac{c}{1+c}}\cot\left(\frac{\pi}{M}\right)\right)\right]\right\} \tag{4.52}$$

4.3.6 EVALUATION OF AVERAGE ASER FOR M-QAM MODULATION

The average symbol error rate for the M-QAM modulation scheme is given by [18]

$$\overline{\mathrm{SER}} = \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}} M_{\gamma_{up}}\left(\frac{g_{QAM}}{\sin^2\theta}\right) d\theta - \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)^2\int_0^{\frac{\pi}{4}} M_{\gamma_{up}}\left(\frac{g_{QAM}}{\sin^2\theta}\right) d\theta \tag{4.53}$$

where, $g_{QAM} = 1.5/(M-1)$.

Substitute (4.33) into the equation (4.53) and rearrange the equations from the equation (4.47) to the equation (4.48), the average symbol error rate is expressed as [18]

$$\overline{\mathrm{SER}} = 4\sum_{n=1}^{N}\binom{N}{n}\frac{(-1)^{n-1}}{(1-2n)}$$

$$\times\left\{\left(1-\frac{1}{\sqrt{M}}\right)\left[I_1\left(\frac{0.75\bar{\gamma}_0}{n(M-1)}\right)-2nI_1\left(\frac{1.5\bar{\gamma}_0}{(M-1)}\right)\right]\right.$$

$$\left.-\left(1-\frac{1}{\sqrt{M}}\right)^2\left[I_3\left(\frac{0.75\bar{\gamma}_0}{n(M-1)}\right)-2nI_3\left(\frac{1.5\bar{\gamma}_0}{(M-1)}\right)\right]\right\}$$

(4.54)

where,

$$I_3(c) = \frac{1}{\pi}\int_0^{\frac{\pi}{4}}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)d\theta = \frac{1}{4}\left\{1-\sqrt{\frac{c}{1+c}}\left[\frac{4}{\pi}\tan^{-1}\left(\sqrt{\frac{c+1}{c}}\right)\right]\right\}.$$

(4.55)

4.4 Nth BEST RELAY SELECTION SCHEME

In the Nth best relay selection scheme, the system selects the next best relay out of all the available relays if the first best relay is not present, where N is the number of the relay which is selected to transmit the source signal to the destination node.

4.4.1 EVALUATION OF MGF AND PDF OF END-TO-END SNR

In the first time slot of the transmission, the signal received at the destination node is expressed as [20]

$$y_{S,D}(t) = h_{S,D}\sqrt{E_S}x(t) + n_{S,D}\ (t)$$

(4.56)

and the signal received at the i-th relay node from the source is given by [20]

$$y_{S,R_i}(t) = h_{S,R_i}\sqrt{E_S}x(t) + n_{S,R_i}\ (t),$$

(4.57)

where, E_S is the energy of the transmitted signal, $x(t)$ is the signal transmitted by the source with energy as one unit and the AWGN noise terms are $n_{S,D}\ (t)$ and $n_{S,R_i}\ (t)$.

In the second time slot of the transmission, the relay(Nth best) processes the signal received by it from the source and gives a new signal $x_r(t)$and forwards it to the destination.

The signal received at the destination node from the relay is expressed as [20]

$$y_{R_{sel},D}(t) = h_{R_{sel},D}\sqrt{E_S}x_r(t) + n_{R_{sel},D}(t) \tag{4.58}$$

where, $n_{R_{sel},D}$ represents the AWGN noise for the $R_i \rightarrow D$ channel.

The signal x_r transmitted by the relay is obtained after the amplification of $y_{R_{sel},D}$ and is written as [20]

$$x_r = G \times y_{R_{sel},D}, \tag{4.59}$$

where, G denotes the gain factor expressed as [20]

$$G = \sqrt{E_S/\left(E_S h_{R_{sel},D}{}^2 + N_0\right)}. \tag{4.60}$$

It can be observed that the gain factor depends on the channel fading coefficient $h_{R_{sel},D}$ and the relay selected for transmission computes the value of $h_{R_{sel},D}$ with a precise accuracy.

At the destination node, the total SNR is expressed in the exact form as [20]

$$\gamma_{AF} = \gamma_{SD} + N^{th} \max_{i=1,\ldots,M}\left[\frac{\gamma_{SR_i}\gamma_{R_iD}}{\gamma_{SR_i} + \gamma_{R_iD} + 1}\right] \tag{4.61}$$

$$\leq \gamma_{SD} + N^{th} \max_{i=1,\ldots,M}\left[\min(\gamma_{SR_i}, \gamma_{R_iD})\right] \tag{4.62}$$

$$= \gamma_{SD} + N^{th} \max_{i=1,\ldots,M}[\gamma_i] = \gamma_{SD} + \gamma_b = \gamma_{ub}. \tag{4.63}$$

where, N^{th} $\max\{\gamma_i\}$ represents the selection of the N^{th} maximum value of γ_i, $\gamma_b = \min(\gamma_i)$, M is the number of relays in the network[20] and

$$\gamma_b = N^{th} \max_{i=1,\ldots,M} \left[\min(\gamma_{SR_i}, \gamma_{R_iD})\right]. \tag{4.64}$$

The system analysis is therefore based on the SNR γ_{ub} because the upper bound SNR is fairly accurate for performance evaluation [20].

To find expression for the PDF for the SNR γ_i is found mathematically in terms of the average SNR values given as [20]

$$\bar{\gamma}_{SR_i} = \mathbf{E}(|h_{SR_i}|^2)\,(E_s/N_o), \tag{4.65}$$

$$\bar{\gamma}_{R_iD} = \mathbf{E}(|h_{R_iD}|^2)\,(E_s/N_o) \tag{4.66}$$

and the PDF is given by [20]

$$f_{\gamma_i}(\gamma) = \frac{1}{\bar{\gamma}_i}\exp(-\gamma/\gamma_i) \tag{4.67}$$

where, the SNR $\bar{\gamma}_i$ is given by [18]

$$\bar{\gamma}_i = \overline{\gamma_{S,R_i}}\,\overline{\gamma_{R_iD}}/(\overline{\gamma_{S,R_i}} + \overline{\gamma_{R_iD}}). \tag{4.68}$$

Because the SNRs γ_b and $\gamma_{S,D}$ are independent of each other, the MGF $M_{\gamma_{ub}}(s)$ for the N^{th} best relay is given by [20]

$$M_{\gamma_{ub}}(s) = M_{\gamma_{S,D}}(s)M_{\gamma_b}(s) \tag{4.69}$$

where, $M_{\gamma_{S,D}}(s)$ is the MGF of the SNR $\gamma_{S,D}$ and $M_{\gamma_b}{}^{N}(s)$ is the MGF of the SNR γ_b.

The MGF is given by [20]

$$\mathrm{M_X(s) = \mathbf{E}(\exp(-sX))}$$

(4.70)

where, **E(.)** is the expectation operator.

For the given Rayleigh fading i.i.d links, the MGF $\mathrm{M}_{\gamma_{\mathrm{S,D}}}(\mathrm{s})$ is given by [20]

$$\mathrm{M}_{\gamma_{\mathrm{S,D}}}(\mathrm{s}) = 1/(1 + \mathrm{s}\,\bar{\gamma}_{\mathrm{SD}}),$$

(4.71)

where,

$$\bar{\gamma}_{\mathrm{SD}} = \mathbf{E}\left(\mathrm{h_{SD}}^{\mathbf{2}}\right)(\mathrm{E_s/N_o}).$$

(4.72)

Since the system considers the i.i.d channel links, therefore, $\overline{\gamma_{\mathrm{S,R_i}}} = \overline{\gamma_{\mathrm{R_iD}}} = \bar{\gamma}, \forall \mathrm{i}, \mathrm{i} = 1,2 \ldots \mathrm{M}$, and $\bar{\gamma_{\mathrm{i}}} = 0.5\bar{\gamma}$ [20].

The PDF $\mathrm{f}_{\gamma_\mathrm{b}}{}^{\mathrm{N}}(\gamma)$or the SNR γ_b is expressed as [20]

$$\mathrm{f}_{\gamma_\mathrm{b}}{}^{\mathrm{N}}(\gamma) = \mathrm{M}\binom{\mathrm{M}-1}{\mathrm{N}-1}\frac{1}{\bar{\gamma}/2} \times \sum_{\mathrm{k}=0}^{\mathrm{M-N}}(-1)^{\mathrm{k}}\binom{\mathrm{M}-\mathrm{N}}{\mathrm{k}}\exp\left(\frac{-\mathrm{x}(\mathrm{k}+\mathrm{N})}{\bar{\gamma}/2}\right)$$

(4.73)

The MGF $\mathrm{M}_{\gamma_\mathrm{b}}{}^{\mathrm{N}}(\mathrm{s})$ for the SNR γ_b is expressed as [20]

$$\mathrm{M}_{\gamma_\mathrm{b}}{}^{\mathrm{N}}(\mathrm{s}) = \mathrm{M}\binom{\mathrm{M}-1}{\mathrm{N}-1} \times \sum_{\mathrm{k}=0}^{\mathrm{M-N}}(-1)^{\mathrm{k}}\frac{\binom{\mathrm{M-N}}{\mathrm{k}}}{\mathrm{k}+\mathrm{N}+\bar{\gamma_\mathrm{s}}/2}$$

(4.74)

Use (4.73) and (4.74) in equation (4.69) to obtain the MGF $\mathrm{M}_{\gamma_{\mathrm{ub}}}{}^{\mathrm{N}}(\mathrm{s})$ as [20]

$$\mathrm{M}_{\gamma_{\mathrm{ub}}}{}^{\mathrm{N}}(\mathrm{s}) = \mathrm{M}\binom{\mathrm{M}-1}{\mathrm{N}-1}\sum_{\mathrm{k}=0}^{\mathrm{M-N}}(-1)^{\mathrm{k}}\binom{\mathrm{M}-\mathrm{N}}{\mathrm{k}}\left[\frac{(-1)^{\mathrm{k}}\binom{\mathrm{M-N}}{\mathrm{k}}}{\bar{\gamma}/2-(\mathrm{k}+\mathrm{N})\bar{\gamma}_{\mathrm{S,D}}}\right]$$

(4.75)

The method of partial fraction is used to express (4.75) in the form given by [20]

$$M_{\gamma_{ub}}{}^{N}(s) = M\binom{M-1}{N-1}\sum_{k=0}^{M-N}(-1)^k\binom{M-N}{k}\left[\frac{\alpha}{1+s\overline{\gamma}_{S,D}} + \frac{\beta}{k+N+\frac{\overline{\gamma}}{2}s}\right],$$

(4.76)

where,

$$\alpha = \frac{\overline{\gamma}_{S,D}}{(k+N)\overline{\gamma}_{S,D} - \overline{\gamma}/2}$$

(4.77)

and

$$\beta = -\frac{\overline{\gamma}}{(k+N)\overline{\gamma}_{S,D} - \overline{\gamma}/2}.$$

(4.78)

The MGF $M_{\gamma_{ub}}(s)$ is expressed by the use of the inverse Laplace transform of (15), the PDF $f_{\gamma_{ub}}{}^{N}(s)$ is expressed as [20]

$$f_{\gamma_{ub}}{}^{N}(s) = M\binom{M-1}{N-1}\sum_{k=0}^{M-N}(-1)^k\binom{M-N}{k}\left[\frac{(-1)^k\binom{M-N}{k}}{\overline{\gamma}/2-(k+N)\overline{\gamma}_{S,D}}\right]\left(\exp\left(-\frac{x(k+N)}{\overline{\gamma}/2}\right) - \exp\left(\frac{-x}{\overline{\gamma}_{S,D}}\right)\right).$$

(4.79)

4.4.2 EXPRESSION FOR ERROR PROBABILITY

It is known that the average probability of error in a fading environment with slow, flat fading channels is calculated using the conditional probability of error in the AWGN channel, $P_s^N(e|\gamma)$, in which γ is the random variable to denote the instantaneous value of the SNR [20].

In general, the error probability is given by [20],

$$P_s^N(e) = \int_0^{\infty} P_s^N(e|\gamma)\, f_\gamma(\gamma)d\gamma,$$

(4.80)

where, $f_\gamma(\gamma)$ is the PDF for the SNR (instantaneous) at the output of the diversity combiner.

For various constellations (with gray mapping of symbols, mostly used in the practical communication systems), $P_s^N(e|\gamma)$ is given by [20]

$$P_s^N(e|\gamma) = a\,\mathrm{erfc}\left(\sqrt{b\gamma}\right), \tag{4.81}$$

a and b are constants that depend on the type of modulation scheme.

Therefore, $P_s^N(e)$ is expressed as [20]

$$P_s^N(e) = a\int_0^\infty \mathrm{erfc}(\sqrt{b\gamma})\, f_\gamma(\gamma) d\gamma.$$

The probability of error for the proposed AF system for the i.i.d relay channels is given by [20]

$$\begin{aligned} P_s^N(e) = {} & aM\binom{M-1}{N-1}\sum_{k=0}^{M-N}(-1)^k\binom{M-N}{k}\frac{1}{(k+N)} \\ & \times\left(1-\frac{\bar{\gamma}/(2k+2N)}{\bar{\gamma}/(2k+2N)-\bar{\gamma}_{S,D}}\sqrt{\frac{b\bar{\gamma}/(2k+2N)}{1+b\bar{\gamma}/(2k+2N)}}\right. \\ & \left.+\frac{\bar{\gamma}_{S,D}}{\bar{\gamma}/(2k+2N)-\bar{\gamma}_{S,D}}\sqrt{\frac{b\bar{\gamma}_{S,D}}{b\bar{\gamma}_{S,D}+1}}\right). \end{aligned} \tag{4.81}$$

4.5 SER FOR RECTANGULAR QAM MODULATION

The aim of this section is to present an analysis for a cooperative diversity system which selects one relay out of all the available relays in a Rayleigh fading environment for a rectangular QAM scheme of the general order with the Amplify and Forward relaying protocol. The analysis is based on the derivation of the average SER (symbol error rate) for the proposed system [24]. The numerically computed values for the expression obtained for the average SER in the form of a lower bound-* shall is compared against the values obtained with the results of the Monte Carlo simulation of the system model. The numerical values of the computed ASER are plotted and the ASER is investigated as a function of the parameters of the communication system [24].

The cooperative diversity system considered for the proposed system model assumed the AF cooperative scheme in which the relay amplifies the original signal transmitted by the source. This scheme has lower complexity in comparison to the DF scheme [24].

The QAM modulation scheme is a generic signaling method that encompasses other modulation techniques such as square QAM, orthogonal BFSK, BPSK, QPSK and the M-ary ASK (Amplitude Shift Keying). Therefore, QAM is of significant interest in the performance analysis of a communication system based on cooperative diversity [24].

The ASER obtained for the proposed system model is basically a general expression from which the values of the ASER for square QAM can be conveniently obtained.

The method by which the best relay (designated as b) is selected out of the N relays in the network is [24]

$$b = \mathrm{argmax}_{n \in R}\{\gamma_n\}, \tag{4.83}$$

where, $R = \{1,2, \ldots N\}$ and

$$\gamma_n = \min(\gamma_{SR_n}, \gamma_{R_nD}) \tag{4.84}$$

where, γ_n denotes the instantaneous value of the end-to-end SNR of the $S \rightarrow R_n \rightarrow D$ link expressed as an upper bound for the relay denoted by R_n.

The total signal-to-noise ratio (SNR) γ_t expressed as an upper bound at the destination terminal is [24]

$$\gamma_t \leq \gamma_{SD} + \max_{n \in R}\{\min(\gamma_{SR_n}, \gamma_{R_nD})\}, \tag{4.85}$$

where, $\gamma_{SR_n} = |h_{SR_n}|^2 \frac{E_s}{N_o}$, $\gamma_{R_nD} = |h_{R_nD}|^2 \frac{E_s}{N_o}$ and $\gamma_{SD} = |h_{SD}|^2 \frac{E_s}{N_o}$ are defined as the instantaneous values of the SNRs for the $S \rightarrow R_n$, the $R_n \rightarrow D$ and the $S \rightarrow D$ channels, respectively and h_{SR_n}, h_{R_nD} and h_{SD} are the link gains (statistically distributed according to Rayleigh fading distribution) for the $S \to R_n$, the $R_n \to D$ and the $S \to D$ channels, respectively. E_s represents the average energy of each symbol and N_o represents the variance of AWGN.

4.5.1 DERIVATION OF ASER

It is well known that the ASER for a rectangular QAM modulation technique in a communication system is given by the general formula [24]

$$P_s(e) = \int_0^\infty P_s(e|\gamma_t)\, f_\gamma(\gamma_t) d\gamma_t \tag{4.86}$$

where, $f_\gamma(\gamma_t)$ represents the PDF (Probability Density Function) of the SNR γ_t as expressed in (4.85) and $P_s(e|\gamma_t)$ denotes the conditional symbol error rate for the rectangular M QAM modulation technique in which $M = M_I \times M_Q$.

$P_s(e|\gamma_t)$ is given as [24]

$$P_s(e|\gamma_t) = 2pQ(a\sqrt{\gamma_t}) + 2qQ(b\sqrt{\gamma_t}) - 4pqQ(a\sqrt{\gamma_t})Q(b\sqrt{\gamma_t}) \tag{4.87}$$

where, Q(x) is the Q-function expressed as [24]

$$Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-t^2/2}\, dt, \tag{4.88}$$

p is the modulation parameter given by [24]

$$p = 1 - \frac{1}{M_I}, \tag{4.89}$$

q is the modulation parameter given by [24]

$$q = 1 - \frac{1}{M_Q}, \tag{4.90}$$

a is the modulation parameter given by [24]

$$a = \sqrt{\frac{6}{(M_I{}^2 - 1) + (M_Q{}^2 - 1)\beta^2}},$$

(4.91)

$b = \beta a$ and $\beta = d_Q/d_I$, where d_Q and d_I are respective decisions distances for the quadrature-phase and the in-phase components. Also, the average signal energy for the in-phase component is $(M_I{}^2 - 1)/(M_Q{}^2 - 1)\,\beta^2$ times the average signal energy for the quadrature-phase component [24].

Substitute the equation (4.87) in the equation (4.86), and rearrange the terms to obtain [24]

$$P_s(e) = 2p\int_0^{\infty} Q\left(a\sqrt{\gamma_t}\right)f_\gamma\left(\gamma_t\right)d\gamma_t + 2q\int_0^{\infty} Q\left(b\sqrt{\gamma_t}\right)f_\gamma\left(\gamma_t\right)d\gamma_t - 4pq\int_0^{\infty} Q\left(a\sqrt{\gamma_t}\right)Q\left(b\sqrt{\gamma_t}\right)f_\gamma\left(\gamma_t\right)d\gamma_t\,.$$

(4.92)

The alternative representation of the Gaussian Q-functions is given as [4, (4.2)]

$$Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta\,, x \geq 0.$$

(4.93)

The product of the two Gaussian Q functions is given as [4]

$$Q(x)Q(y) = \frac{1}{2\pi}\int_0^{\frac{\pi}{2}-\arctan(y/x)} \exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta + \frac{1}{2\pi}\int_0^{\arctan(y/x)} \exp\left(-\frac{y^2}{2\sin^2\theta}\right)d\theta.$$

(4.94)

Using the equations (4.93) and (4.94) in (4.92), $P_s(e)$ is expressed as [4]

$$P_s(e) = 2pI_1 + 2qI_2 - 4pqI_3$$

(4.95)

where,

$$I_1 = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_{\gamma_t}\left(\frac{a^2}{2\sin^2\theta}\right)d\theta,$$

(4.96)

$$I_2 = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_{\gamma_t}\left(\frac{b^2}{2\sin^2\theta}\right)d\theta$$

(4.97)

$$I_3 = \underbrace{\frac{1}{2\pi}\int_0^{\frac{\pi}{2}-\arctan(b/a)} M_{\gamma_t}\left(\frac{a^2}{2\sin^2\theta}\right)d\theta}_{Y_1}$$

$$+ \underbrace{\frac{1}{2\pi}\int_0^{\arctan(b/a)} M_{\gamma_t}\left(\frac{b^2}{2\sin^2\theta}\right)d\theta}_{Y_2}$$

(4.98)

where, M_{γ_t} is the MGF of the SNR γ_t (γ_t is a random variable).

For the independent and the identical relay channel links, $M_{\gamma_t}(s)$ is given by [17,(24)]

$$M_{\gamma_t}(s) = \sum_{n=1}^{N}\binom{N}{n}\frac{n(-1)^{n-1}}{(n+\bar{\gamma}_C s)(1+\bar{\gamma}_{SD} s)}.$$

(4.99)

where, $\bar{\gamma}_{SD} = \bar{\gamma}_0$ and $\bar{\gamma}_C = \frac{\bar{\gamma}_{SR_n}\bar{\gamma}_{SR_n}}{\bar{\gamma}_{SR_n}+\bar{\gamma}_{R_nD}}$.

To calculate the exact value of the expression given by (4.95), the integrals I_1, I_2 and I_3 are calculated as given in appendix A in (A.1), (A.2) and (A.3) respectively [24].

Substitute the solutions of I_1, I_2 and I_3 in (4.95), the expression for the average ASER is given by [24]

$$
\begin{aligned}
\mathrm{P_s(e)} = \sum_{n=1}^{N} \binom{N}{n} \frac{(-1)^{n-1}}{(1-2n)} \Bigg[& 2p \left\{ J_1\left(\frac{a^2\bar{\gamma}_0}{4n}\right) - 2nJ_1\left(\frac{a^2\bar{\gamma}_0}{2}\right) \right\} \\
& + 2q \left\{ J_1\left(\frac{b^2\bar{\gamma}_0}{4n}\right) - 2nJ_1\left(\frac{b^2\bar{\gamma}_0}{2}\right) \right\} \\
& - 2pq \Bigg\{ J_2\left(a, b, \frac{a^2\bar{\gamma}_0}{4n}\right) + J_3\left(a, b, \frac{b^2\bar{\gamma}_0}{4n}\right) - 2nJ_2\left(a, b, \frac{a^2\bar{\gamma}_0}{2}\right) \\
& - 2nJ_3\left(a, b, \frac{b^2\bar{\gamma}_0}{2}\right) \Bigg\} \Bigg].
\end{aligned}
\tag{4.100}
$$

where, $J_1(.)$, $J_2(.,.,.)$ and $J_3(.,.,.)$ are given in the equations (A.3), (A.6) and (A.7) respectively.

The expression of the average ASER as derived in (4.100) is applicable for the case of square M-QAM for which $M_I = M_Q = \sqrt{M}$ and $\beta = 1$ [24].

This chapter has provided a comprehensive insight into the performance evaluation of cooperative diversity systems for different modulation techniques.

The subsequent chapter expresses the results obtained analytically for different fading channel scenarios for a cooperative diversity communication system.

CHAPTER 5
RESULTS AND DISCUSSION

This chapter presents the results obtained with the performance analysis of the cooperative diversity system for the system model given in chapter 4. The results presented in the subsequent sections are obtained using analytical expressions as well as Monte Carlo simulation for the Bit Error Rate/Symbol Error Rate for different modulation schemes over a range of SNR values.

5.1 BER IN AWGN CHANNEL

The figure 5.1 is the Bit Error Probability curve for BPSK scheme over AWGN channel.

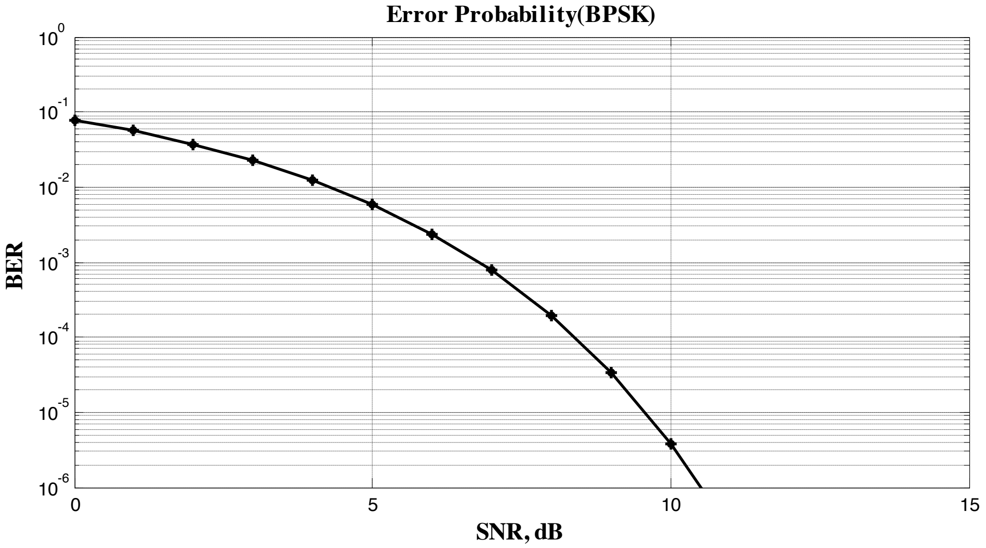

Figure 5.1 Simulated BER Of BPSK In AWGN Channel

In AWGN channel, the noise present in the channel is additive in nature and severely degrades the quality of the signal. The BER improves with the increase in the SNR. For instance, at SNR value of 5 dB, the BER is 5.954×10^{-3} and at SNR value of 10 dB, the BER is 3.872×10^{-6}.

5.2 BER FOR Nth BEST RELAY SELECTION

This section presents the analytical and simulation results for a cooperative diversity model employing BPSK modulation technique for the Nth best relay selection scheme with M relays in the network. The BER is found to depend on the values of M and N [20].

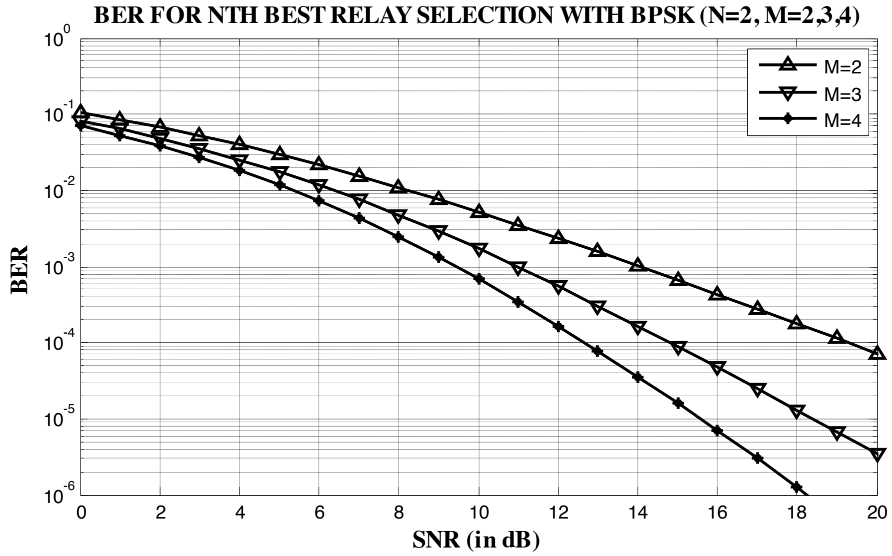

Figure 5.2 BER of BPSK with Nth Best Relay Selection, M=2,3,4 and N=2

The results presented in figure 5.2 give the analytical BER for different number of relays in the network when N is selected as 2, where, N is the order of the relay which is selected as the best relay.

For instance at SNR value of 10 dB, the BER values for M=2 is 5.217×10^{-3}, M=3 is 1.693×10^{-3} and for M=4 is 6.806×10^{-4}. Therefore, for same value of SNR, the BER is least for highest value of M (where, M is the number of relays in the network) and for the given case, the BER is least for N=2 and M=4.

The relative difference between the BER for a given SNR for different values of M increases with the increase in the SNR.

For instance, at SNR value of 18 dB, the BER value for M=2 is 1.766×10^{-4}, M=3 is 1.288×10^{-5} and M=4 is 1.278×10^{-6}. Thus, BER the relative gap in BERs is more for higher values of SNR.

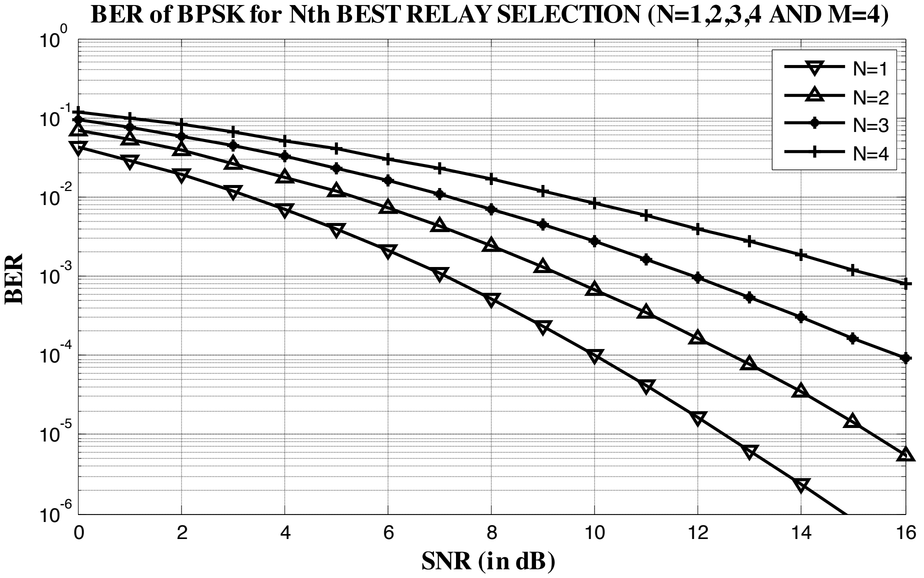

Figure 5.3 BER of BPSK with Nth Best Relay Selection (N=1,2,3,4 and M=4)

The results shown in figure 5.3 are given for a cooperative diversity system with 4 relays in the network for Rayleigh fading links using BPSK modulation technique for analytical ASER. Different cases are being presented for values of N=1,2,3 and 4, where, N denotes the order of the relay which is being selected as the best relay.

For the given value of M=4, the BER is least for N=1 and maximum for N=4. For instance at SNR value of 8 dB, the BER for N=1 is 5.149×10^{-4}, for N=2 is 2.401×10^{-4}, for N=3 is 7.004×10^{-3} and for N=4 is 1.653×10^{-2}.

For the given cooperative diversity system using the Nth best relay selection scheme, the diversity order is equal to M-N+2. The increase in diversity order is linear with the increase in the value of M (the number of branches) [20].

The diversity order also depends on the value of N, also known as the order of the relay. As the order of the relay increases, the diversity order decreases. If the objective of a system designer is to achieve an optimum value of the diversity order, then a fair combination of the values of M and N must be selected. The results presented for the system in this section can be conveniently used to select the desired BER performance for the cooperative diversity system by selection of the value of M and N [20].

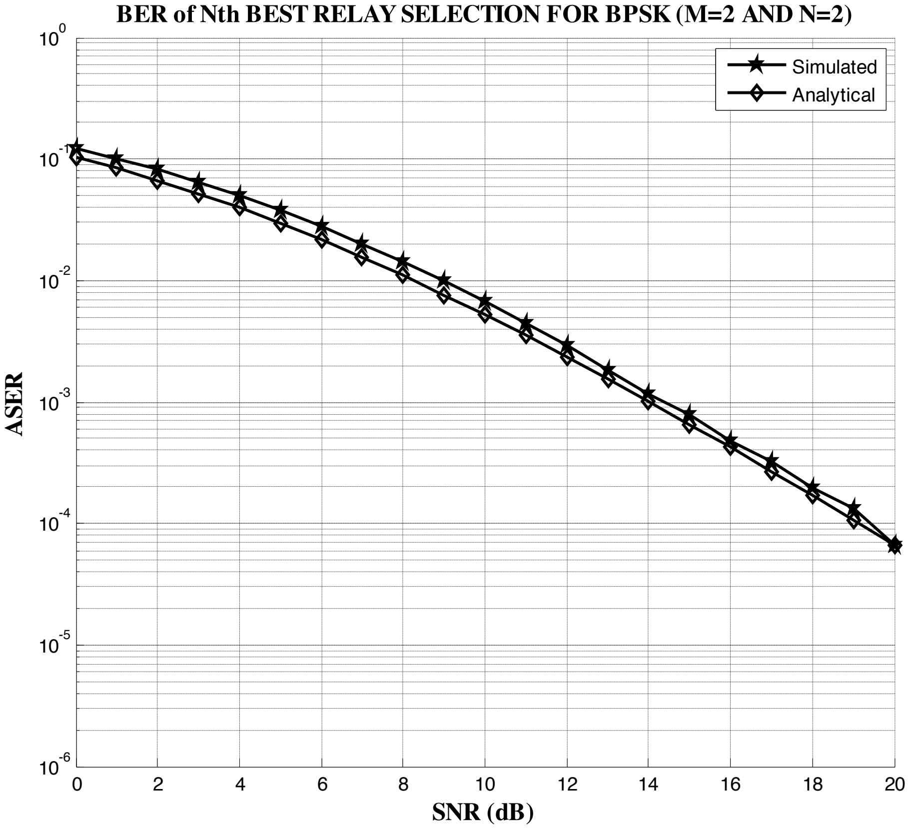

Figure 5.4 BER of Nth Best Relay Selection for BPSK (M=2 and N=2)

The figure 5.4 depicts the average ASER curve for BPSK modulation technique used in a cooperative diversity system for both the analytical results as well as the results obtained with Monte Carlo Simulation with M=2 and N=2. It can be observed from the figure that for a given value of SNR, the BER is lesser for the analytical bound as compared to the simulation results. For instance at the SNR value of 10 dB, the BER for analytical bound is 5.211×10^{-3} and with simulation result is 6.71×10^{-3}.

5.3 ASER FOR RECTANGULAR QAM MODULATION

The lower bound of the average SER obtained in (4.20) is compared with the exact simulation values obtained using the Monte Carlo simulation for the cooperative diversity communication model. The results are plotted as a function of $\frac{E_s}{N_o}$ measured in decibels.

For the case of 4×2 QAM as shown in figure 5.5, with β=1 and N=2 and 4, there is an increase in the difference in the values of exact ASER and the lower bound ASER with the increase in the value of N. However, at high SNRs of about 25 dB, the relative gap reduces. Therefore, the derived lower bound serves as a fair approximation when N is small. Also, ASER improves with the increase in N because of the corresponding improvement in the diversity gain [24]. In addition to this, for SNR values greater than 10 dB, the difference between the exact and the lower bound ASER reduces much more than the values of SNR below 10 dB.

$\frac{E_s}{N_0}$ in dB	N	Exact ASER	Lower Bound	Relative Difference (%)
0	1	0.57869	0.5411	6.49398
0	2	0.55813	0.50653	9.24516
0	4	0.53541	0.47008	12.2019
14	1	0.03792	0.03028	20.1477
14	2	0.02049	0.01333	34.9439
14	4	0.00971	0.00454	53.2441
25	1	3.384×10^{-4}	3.07×10^{-4}	9.26051
25	2	2.43×10^{-5}	1.86×10^{-5}	23.2169
25	4	3.2×10^{-7}	2.042×10^{-7}	36.1778

Table 5.1 Relative difference between the values of exact ASER for 4 × 2 QAM and β=1 [24]

For the case of 4 × 2 QAM, the values of the relative differences in the exact results obtained with simulation and the analytical results obtained using the derived ASER as presented in table 5.1 shows the closeness of the simulated results and the obtained ASER expression [24].

For the same value of N, ASER is found to depend on the value of β. For instance, for the case of 8×4 QAM, as presented in figure 5.12, the ASER is least for β=1. In case of 16×2 QAM, ASER is least when β=2 and N=1 or 4. For a given N and β=1, ASER performance of the 8×4 QAM is found to be better than the ASER performance for the 16×2 QAM over the range of SNR values considered.

However, with an increase in the value of β, ASER for the 8×4 QAM is observed falls more rapidly than the case of 16×2 QAM (presented in the figure 5.7). To understand this, it is required to observe the definition of β, which is basically the ratio of two decision distances i.e $\beta = d_Q/d_I$, where d_Q and d_I are respective decisions distances for the quadrature-phase and the in-phase components. Also, β depends on the values of M_I and M_Q. Therefore, for a given QAM constellation with size M (where, $M = M_I \times M_Q$), ASER values may differ with change in the values of β, M_I and M_Q.

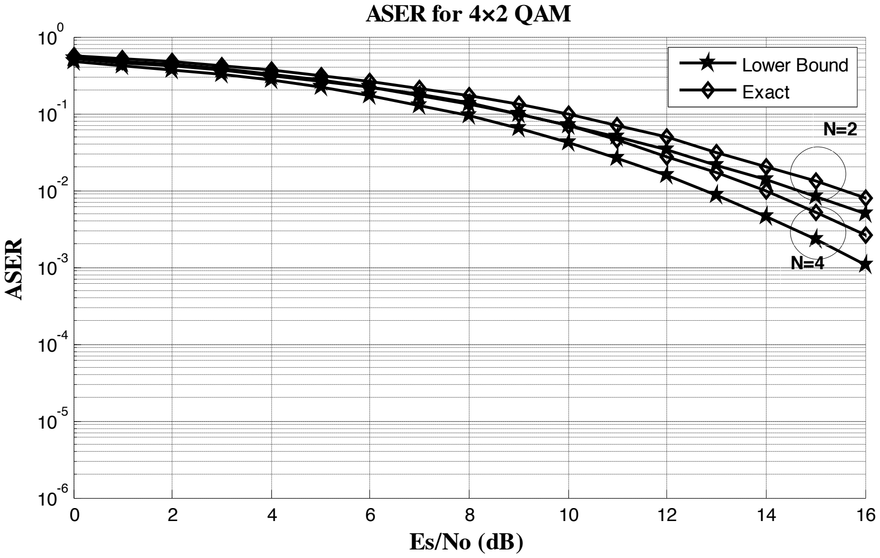

Figure 5.5 ASER for 4×2 QAM

When a given value of β is considered, the ASER depends on the absolute difference between the values of M_I and M_Q. Those constellations which have large difference between M_I and M_Q show poor ASER performance as compared to the constellations with smaller difference between M_I and M_Q. This can be understood from the dependence of β on the decision distances in the sense that for more number of symbols in one quadrant, the symbols will be more close to each other resulting in an increase in the ASER. Therefore, the constellation with smallest difference between M_I and M_Q and β=1 gives the least value of ASER.

For instance, in the case of 32-QAM modulation scheme, the 8 × 4 QAM constellation with β=1 has reduced SER in comparison with the 16 × 2 QAM constellation irrespective of the value of β. Similarly, for 64 QAM modulation technique, the best ASER performance is achieved with the 8 × 8 QAM constellation with β=1. In addition to this, for any QAM modulation order, ASER performance differs when β changes to $\beta' = \frac{1}{\beta}$. For any M-QAM constellation, the ASER performance with $\beta \geq 1$ is better in comparison to any case of β' [24].

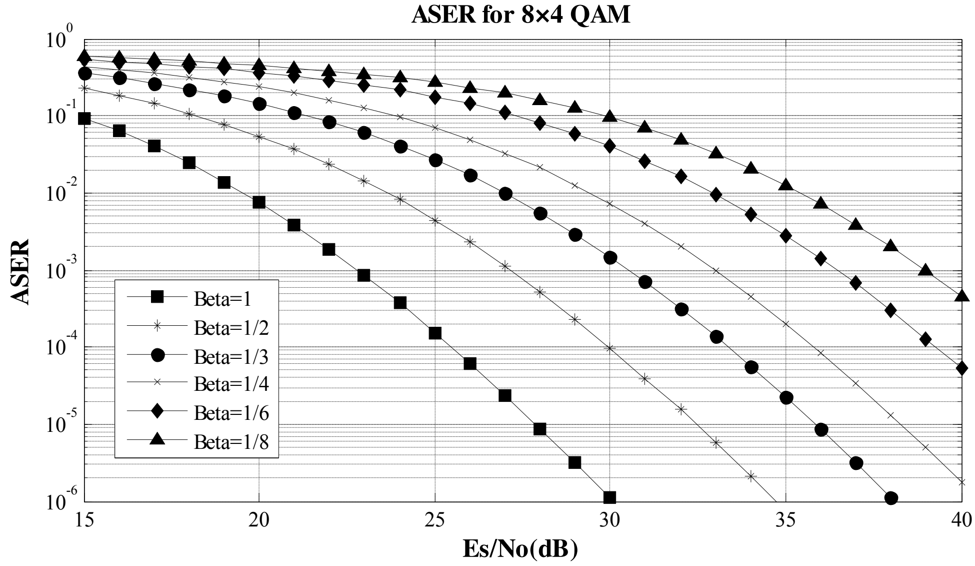

Figure 5.6 ASER for 8×4 QAM

The performance curve for the 8×4 QAM Modulation scheme presented in figure 5.6 is given for $\frac{1}{8} \leq \beta \leq 1$. At the SNR value of 30 dB, the ASER is least for the value of $\beta = 1$. The worst performance for this SNR value is observed for $\beta = \frac{1}{8}$.

At the lowest SNR value of 15 dB, the difference in the values achieved in the performance curves for $\frac{1}{8} \leq \beta \leq 1$ is relatively lesser than the the difference in the values achieved for SNR values above 30 dB.

The performance curve for the 16×2 QAM Modulation scheme presented in figure 5.7 is given for $\frac{1}{8} \leq \beta \leq 1$. At the SNR value of 35 dB, the ASER is least for the value of $\beta = 1$. The worst performance for this SNR value is observed for $\beta = \frac{1}{8}$.

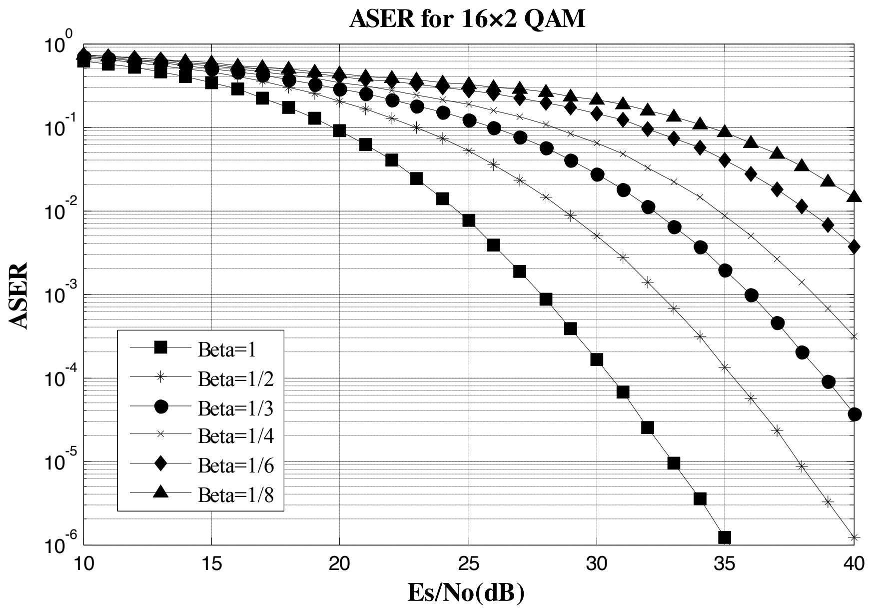

Figure 5.7 ASER for 16×2 QAM

It can be observed from the ASER values obtained in the analytical and the simulation results that the derived ASER depends on the number of relays in the network, the size of the constellation, and the value of β [24].

All the results presented in this chapter have verified the analytical expressions obtained through the performance evaluation of the cooperative diversity system model given in chapter 4.

CHAPTER 6
CONCLUSION AND FUTURE SCOPE

This chapter concludes the dissertation which has compiled a comprehensive evaluation of a cooperative diversity communication system model under various fading environments and different system parameters.

The values of the various performance parameters have been verified with the use of Monte Carlo simulation and analytical evaluation. This achieves the objective to show that cooperative diversity offers a promising method to deal with the severe fluctuations induced in a wireless fading environment apart from achieving higher data rates to meet the demands of the current wireless applications. The cooperative communication offers reasonable reliability for use in wireless sensor networks and ad-hoc networks to overcome the limitations in terms of size and the transmitter power.

A basic cooperative diversity system model is a powerful technique that can find applications in diverse areas of research in communication engineering. The analysis presented in the dissertation can be applied to other fading scenarios with different channel parameters. It is also possible to extend the aforementioned analysis to other diversity techniques such as Generalized Selection Combining and Equal Gain Combining and the results can be conveniently compared with the results obtained using MRC combining technique.

As a scope for future research, it is worthwhile to include the effect of the frequency offset in the system model presented in the dissertation. Frequency offset in a signal occurs due to the variation in the path lengths when a mobile subscriber moves over a certain distance in the wireless network.

It is suggested that the inclusion of frequency offset in the performance evaluation will provide a more realistic evaluation of the practical communication systems that are based on mobile wireless communication.

APPENDIX A

A.1 SOLUTIONS FOR THE INTEGRALS I_1 AND I_2

Use the partial fraction method in (4.17), the expression obtained for $M_{\gamma_t}(s)$ is given by [24]

$$M_{\gamma_t}(s) = \sum_{n=1}^{N} \binom{N}{n} \frac{(-1)^{n-1}}{(1-2n)} \left[\frac{1}{\left(1 + \frac{\bar{\gamma}_0}{2n} s\right)} - \frac{2n}{(1 + \bar{\gamma}_0 s)} \right]. \tag{A.1}$$

Substitute the equation (A.1) in the equation (4.14), I_1 is given by [24]

$$I_1 = \sum_{n=1}^{N} \binom{N}{n} \frac{(-1)^{n-1}}{(1-2n)} \left[J_1\left(\frac{a^2 \bar{\gamma}_0}{4n}\right) - 2n J_1\left(\frac{a^2 \bar{\gamma}_0}{2}\right) \right] \tag{A.2}$$

where, $J_1(.)$ is given by [4,(5A.9)]

$$J_1(c) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\theta}{\sin^2\theta + c} \right) d\theta = \frac{1}{2}\left(1 - \sqrt{\frac{c}{1+c}}\right). \tag{A.3}$$

Similarly, to obtain the expression for I_2, substitute (A.1) in (4.15) to obtain [24]

$$I_2 = \sum_{n=1}^{N} \binom{N}{n} \frac{(-1)^{n-1}}{(1-2n)} \left[J_1\left(\frac{b^2 \bar{\gamma}_0}{4n}\right) - 2n J_1\left(\frac{b^2 \bar{\gamma}_0}{2}\right) \right] \tag{A.4}$$

B. SOLUTION FOR THE INTEGRAL I_3

Substitute (A.1) in (4.16) and integrate to obtain the expression for γ_1 as [24]

$$\Upsilon_1 = \frac{1}{2}\sum_{n=1}^{N}\binom{N}{n}\frac{(-1)^{n-1}}{(1-2n)} \times \left[J_2\left(a, b, \frac{a^2\bar{\gamma}_0}{4n}\right) - 2nJ_2\left(a, b, \frac{a^2\bar{\gamma}_0}{2}\right)\right]$$

(A.5)

where, $J_2(.,.,.,.)$ is expressed as [24]

$$J_2(a, b, c) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}-\arctan(b/a)}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)d\theta$$

$$= \frac{1}{2} - \frac{1}{\pi}\arctan\left(\frac{b}{a}\right) - \frac{1}{\pi}\sqrt{\frac{c}{1+c}}\arctan\left(\frac{a}{b}\sqrt{\frac{1+c}{c}}\right)$$

(A.6)

The value of Υ_2 is given by [24]

$$\Upsilon_2 = \frac{1}{2}\sum_{n=1}^{N}\binom{N}{n}\frac{(-1)^{n-1}}{(1-2n)} \times \left[J_3\left(a, b, \frac{b^2\bar{\gamma}_0}{4n}\right) - 2nJ_3\left(a, b, \frac{b^2\bar{\gamma}_0}{2}\right)\right]$$

(A.7)

where, $J_3(.,.,.,.)$ is given by [24]

$$J_3(a, b, c) = \frac{1}{\pi}\int_0^{\arctan(b/a)}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)d\theta$$

$$= \frac{1}{\pi}\arctan\left(\frac{b}{a}\right) - \frac{1}{\pi}\sqrt{\frac{c}{1+c}}\arctan\left(\frac{b}{a}\sqrt{\frac{1+c}{c}}\right)$$

(A.7)

Therefore, the integral I_3 is given by [24]

$$I_3 = \frac{1}{2}\sum_{n=1}^{N}\binom{N}{n}\frac{(-1)^{n-1}}{(1-2n)}$$

$$\times\left[J_2\left(a, b, \frac{a^2\bar{\gamma}_0}{4n}\right) + J_3\left(a, b, \frac{b^2\bar{\gamma}_0}{4n}\right) - 2nJ_2\left(a, b, \frac{a^2\bar{\gamma}_0}{2}\right)\right.$$

$$\left. - 2nJ_3\left(a, b, \frac{b^2\bar{\gamma}_0}{2}\right)\right]$$

(A.8)

REFERENCES

[1] J. G. Proakis, *Digital Communications,* 4th edition. McGraw-Hill, 2001.

[2] T. S. Rappaport, Wireless Communications: Principles and Practice. Upper Saddle River, NJ: Prentice-Hall, 1996.

[3] B. Sklar, “Rayleigh Fading Channels in Mobile Digital Communication Systems I. Characterization,” IEEE Commun. Mag., vol. 35, no. 7, pp. 90–100, Jul. 1997.

[4] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd edition. Wiley, 2005.

[5] A. Goldsmith, *Wireless Communications*, Cambridge, U.K.: Cambridge Univ. Press, 2005.

[6] S.S. Ikki and S. Aissa, “Performance analysis of two-way amplify-and-forward relaying in the presence of co-channel interferences”, *IEEE Trans. Commun.,* vol. 60, no. 4, pp. 933-939, Apr. 2012.

[7] B. Vucetic and J. Yuan, Space-Time Coding. Chichester, U.K.: Wiley, 2003.

[8] A. Mahmood, “Cooperative diversity in wireless networks”, Journal of Engineering xScience and Technology Review, vol. 3, no. 1, pp. 184-187, 2010.

[9] R.M. Galiardi, Introduction to Communications Engineering. New York: Wiley, 1988.

[10] A. Sendonaris, E. Erkip, and B. Aazhang, “Increasing uplink capacity via user cooperation diversity,” in *Proc. IEEE Int. Symp. Information Theory (ISIT’98),* Cambridge, MA, p. 156, Aug. 1998.

[11] J.N. Laneman and G.W. Wornell, “Energy efficient antenna sharing and relaying for wireless networks,” in *Proc. IEEE Wireless Communication and Networking Conf. (WCNC’00),* Chicago, IL, pp. 7-12, Oct.2000.

[12] J.N. Laneman, G.W. Wornell, and D.N.C. Tse, “An efficient protocol for realizing cooperative diversity in wireless networks,” in *Proc. IEEE Int. Symp. Communications, (ISC’10),* Taiwan, R.O.C., pp. 105.1-105.4, Nov. 2001.

[13] M.O. Hasna and M.S. Alouini, “Performance analysis of two-hop Relayed transmission over Rayeleigh-fading channels,” in *Proc. IEEE Vehicular Technology Conf. (VTC’02)*, Vancouver, BC, Canada, pp. 1992-1996, Sep. 2002.

[14] P.A. Anghel and M. Kaveh, “Exact symbol error probability of a cooperative network in a Rayleigh-fading environment,” *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1416-1421, Sep. 2004.

[15] A. Blestas, A. Khisti, D. P. Reed, and A. Lippman, “A simple cooperative diversity method based on network path selection,” *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659-672, Mar. 2006.

[16] Y. Zhao, R. Adve, and T.J. Lim, “Symbol error rate of selection amplify and forward relay systems,” *IEEE Commun. Lett.,* vol. 10, no. 11, pp. 757-759, Nov. 2006.

[17] S.S. Ikki and M.H. Ahmed, “Performance of multiple-relay cooperative diversity systems with best relay selection over Rayleigh fading channels,” *Eurasip J. Advances in Signal Process.*, vol. 2008, Jan. 2008.

[18] A. Adinoyi, Y. Fan, H. Yanikomeroglu, H. Poor, and F. Al-Shaalan, “Performance of selection relaying and cooperative diversity,” *Wireless Communications, IEEE Transactions on*, vol. 8, no. 12, pp. 5790–5795, 2009.

[19] M. Torabi, W. Ajib, and D. Haccoun, “Performance analysis of amplify and-forward cooperative networks with relay selection over Rayleigh fading channels,” in Proc. 2009 IEEE VTC – Spring, pp. 1–5.

[20] S.S. Ikki and M.H. Ahmed, “On the performance of cooperative-diversity networks with the Nth best-relay selection scheme,” *IEEE Trans. Commun.,* vol. 58, no. 11, pp. 3062-3069, Nov. 2010.

[21] K. Kyunbyoung, S. Jeongtae and W. Choongchae, “Another closed-form expression of average error rate for the Nth best relay selection AF relaying over Rayleigh fading channels”, *Proc. 2011 IEEE International Conference on ICT Convergence, (ICTC)*, pp. 106-109, Sep. 2011.

[22] Y. G. Kim and N. C. Beaulieu, “Exact closed-form solutions for the BEP of decode-and-forward cooperative systems in Nakagami- m fading channels,” *IEEE Trans. Commun.*, vol. 59, no. 9, pp. 2355–2361, Sep. 2011.

[23] Y. G. Kim and N. C. Beaulieu, “Exact closed-form solutions for the BEP of decode-and-forward cooperative systems in Nakagami- m fading channels,” *IEEE Trans. Commun.*, vol. 59, no. 9, pp. 2355–2361, Sep. 2011.

[24] D. Dixit and P.R. Sahu, “ Symbol error rate of rectangular QAM with best-relay selection in cooperative systems over Rayleigh fading channels”, *IEEE Commun. Lett.*,vol. 16, no. 4, pp. 466-469, Apr. 2012.

[25] A. M. and S.S. Ikki, “Performance analysis of incremental-relaying cooperative-diversity networks over rayleigh fading channels communications,” *IET*, vol. 5, pp. 337–349, Feb. 2011.

[26] R. Swaminathan, R. Roy and M. D. Selvaraj, “Performance analysis of triple correlated selection combining for cooperative diversity systems,” *proc. IEEE ICC*, pp. 5483–5488, 2013.

[27] M. R. Avendi and Ha H. Nguyen, “Selection combining for differential amplify-and-forward relaying over Rayleigh-fading channels,” *IEEE Signal Process. Lett.*, vol. 20, no. 3, pp. 277–280, Mar. 2013.

[28] S.S. Ikki and S. Aissa, “Performance analysis of two-way amplify-and-forward relaying in the presence of co-channel interferences”, *IEEE Trans. Commun.,* vol. 60, no. 4, pp. 933-939, Apr. 2012.

[29] M. Seyfi, S. Muhaidat and J. Liang, “Amplify-and-Forward Selection Cooperation over Rayleigh Fading Channels with Imperfect CSI”, *Wireless Communications, IEEE Transactions on,* vol. 11, no. 1, pp. 199-209, Jan. 2012.

[30] L. Yao, W. Xiaoxiang and Y. Nan, "Outage Probability of Cooperative Relay Networks in Two-Wave with Diffuse Power Fading Channels", *IEEE Trans. Commun.,* vol. 60, no. 1, pp. 42-47, Jan. 2012.

[31] R. Swaminathan, M. D. Selvaraj and R. Roy, "Performance analysis of double correlated selection combining for cooperative diversity systems," *proc. IEEE NCC*, pp. 1–5, 2013.

[32] Y. Chen, R. Shi and M. Long, "Performance analysis of amplify-and-forward relaying with correlated links," *Vehicular Technology, IEEE Transactions on,* vol. 62, no. 5, pp. 2344–2349, June 2013.

[32] R. Swaminathan, M. D. Selvaraj and R. Roy, "Performance analysis of double correlated selection combining for cooperative diversity systems," *proc. IEEE NCC*, pp. 1–5, 2013.

[33] S. Ramabadran, "On the error and outage performance of decode and forward cooperative selection diversity system with correlated links", *IEEE Trans. Veh. Tech.*, no. 99, pp. 1, Sep. 2014.

[34] S.I Chu, H.C Chang and H.P Lee, "Asymptotic performance of amplify-and-forward cooperative diversity networks with the N^{th} best relay over Rician fading channels", *Transactions on Emerging Telecommunications Technologies*, vol. 23, no. 4, pp. 327-333, Jun. 2012.

[35] N. Marchenko, W. Masood, T. Andre and G. Brandner, "An experimental study of selective cooperative relaying in industrial wireless sensor networks", *IEEE Trans. Industrial Informatics*, vol. 10, no. 3, pp. 1806-1816, Jun. 2014.

[36] N.C. Beaulieu and Y.G. Kim, " Exact BEP of Decode-and-Forward Cooperative Systems with Multiple Relays in Rayleigh Fading Channels", *IEEE Trans. Veh. Tech.*, vol. 64, no. 2, pp. 823-828, Feb. 2015.

[37] P.S. Bithas, G.P. Efthymoglou and A.G. Kanatas, "SEP of rectangular QAM in composite fading channels", *AEU International Journal of Electronics and Communictions,* vol. 69, no. 1, pp. 246-252, Jan. 2015.

[38] M.D.Selvaraj and R.K.Mallik, “Single-relay cooperative diversity with scaled selection combining,” *IEEE Trans. Commun.*, vol. 59, no.3, pp. 3086-3094, Mar. 2011.

[39] A. Papoulis, Probability, Random Variables, and Stochastic Process. McGraw Hill, 1991.

[40] M.S. Alouini and M. Simon, “An MGF-based performance analysis of generalized selection combining over Rayleigh fading channels,” *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 401–415, Mar. 2000.