Impact parameter dependence of elliptical flow at intermediate energy

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Shitu Raheja

Roll no. - 301004015

Under

the guidance of

Dr. Suneel Kumar

School of physics and Material Science

Thapar University

Patiala – 147004 (PUNJAB)

INDIA

Dedicated

To

My family

CERTIFICATE

This is to certify that Miss. Shitu Raheja, Roll No. 301004015 has worked out on this dissertation as partial fulfillment for the award of degree of MASTER OF SCIENCE in PHYSICS. I certify that matter embodied in this dissertation is of candidate's own record and not submitted to other university in any part or full form for the award of such degree.

SKumar 29/6/12

Dr. Suneel Kumar
Supervisor
SPMS, Thapar University
Patiala

Countersigned by:

Dr. Kulvir Singh
Head
SPMS, Thapar University
Patiala

Dr. S.K Mohapatra
Dean of Academic affairs
Thapar University
Patiala

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Shitu Raheja

(Shitu Raheja)

Roll number: 301004015

Date: 29-6-2012

ABSTRACT

Here we deal with theoretical study of elliptical flow in heavy ion collisions at intermediate energy 100 MeV/nucleon over different impact prarmeter ranges and results are also compared with the experimental finding of INDRA and ALADIN collabration. Simulation of heavy ions is done by using IQMD model and fragments are constructed using MST method. A systematic study is carried out for reactions ($^{40}_{20}Ca$ + $^{40}_{20}Ca$ and $^{124}_{47}Ag$ + $^{124}_{47}Ag$) by using the soft equation of state.

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CHAPTER 1

Introduction

Nuclear physics is branch of physics that studies the interaction of atomic nuclei, nucleon-nucleon scattering and structure of nucleus. Nucleus came into the picture in 1911 discovered by Rutherford and his co-workers. Rutherford established that 99.95 present mass of atom resided into the nucleus. At that time, main aspects of nuclear physics to study structure of nucleus by proton, neutron, deuteron, alpha particles etc., which are generally produced, either in some nuclear reaction or obtained from some radioactive decay of substances. Device which is used to accelerate these particle with increased energy is called accelerator [1]. With the passage of time these accelerators are able to accelerate the heavy ions. The first heavy ion (which has mass number greater than the alpha particle) collisions at modestly relativistic conditions were undertaken at the Lawerence Berkelely National Laboratory, LBNL, at Berkeley, USA, and at the Joint Institute for Nuclear Research, JINR, in Dubna, USSR. The energy scale at the level of 1-2 GeV per nucleon attained which is used to compress the nuclear matter. On the physical point of view nuclear physics is differentiated into three parts on the basis of their energy-:

1) Low energy nuclear physics ($E < 10$ MeV/nucleon)
2) Intermediate energy nuclear physics (10 MeV/nucleon $\leq E \leq$ 2 GeV/nucleon)
3) High energy nuclear physics ($E > 2$ GeV/nucleon)

1.1) LOW ENERGY NUCLEAR PHYSICS -:

Aim of low energy nuclear physics heavy ion collision is to look for low density phenomena. The low energy heavy-ion reactions give unique possibility to look for the nuclear interactions, fusion-fission, cluster radioactivity, formation of super heavy nuclei, and possibilities of synthesis of super heavy elements, halo nuclei etc [2,3,4]. Low energy nuclear physics research concerns itself with understanding the structure and stability of the nuclei in the world around us as well as the reactions which formed them in the cosmos. The reaction cross section at low energies consists of three categories i.e., the fusion, quasi-elastic and deep inelastic scattering. All these processes depend on the projectile-target combinations, on the bombarding energy of projectile and angular momentum. Due to large efforts in low energies heavy ion collisions, the understanding of different phenomena is quite rich [5]. Let's see what happen at low energy nuclear physics.

Two nuclei are approaching each other with energy E < 10 MeV/nucleon as shown in fig1.1

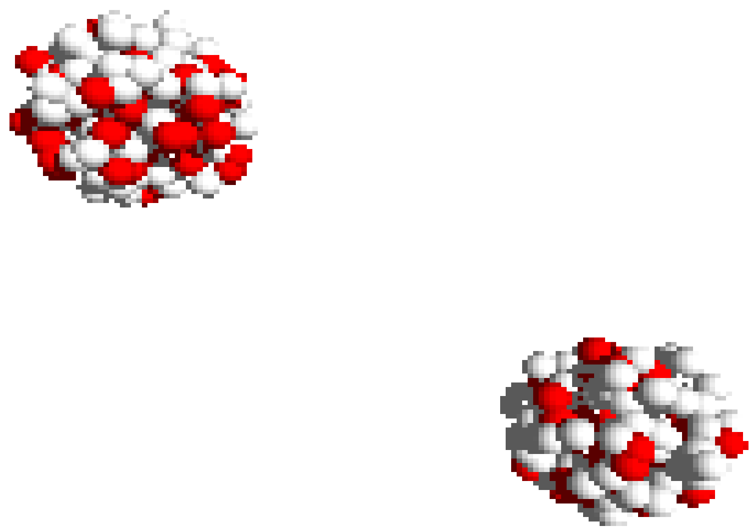

Fig1.1 Initialized projectile and target nuclei.

. At low energy colliding nuclei can not compress each other; their density is equal to their normal nuclear density.

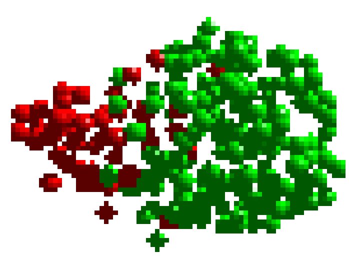

Fig1.2 Collision at low energy (E < 10 MeV/nucleon).

So there available free space is very small due to this 98% of collision are blocked the whole dynamics at low energies is due to the mean field. Because at low incident energies the nucleon-nucleon collisions are negligible and mean field is maximum.

INTERMEDIATE ENERGY NUCLEAR PHYSICS-:

The energy range start around 10MeV/nucleon to 2 GeV/nucleon beam energy. At this energy range, density is equal to 2-3 times more than the normal nuclear density as shown in fig1.3. Mean field and cascade picture both are important at intermediate energy[6].

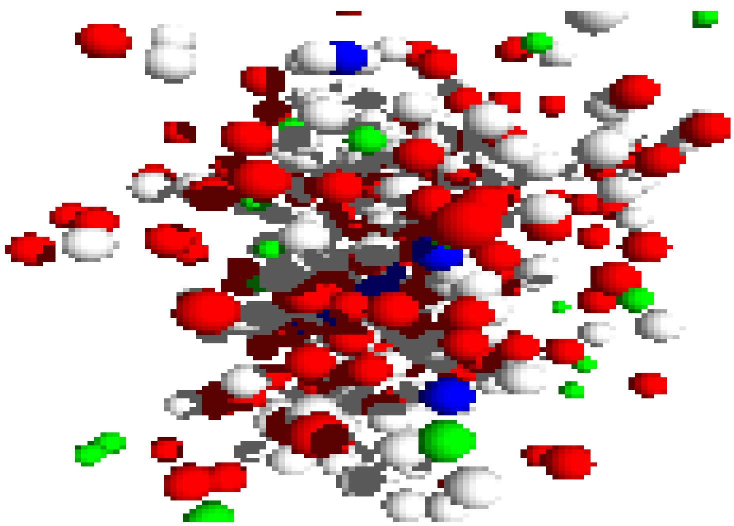

Fig1.3 Collision at intermediate energy (10 MeV/nucleon $\leq$ E $\leq$ 2 GeV/nucleon).

At this energy range, there is production of hadrons (pions, kenos, neutrons and protons). The compressibility and other basic properties of the nuclear equation of state (EOS) and nuclear interactions can be tested. The results in this energy range have astrophysical relevance to neutron stars and supernova explosions. This is the area where the research is most developed and real quantitative questions on the nuclear incompressibility, in medium nuclear cross sections, momentum dependence of the nucleon-nucleon interaction, etc., are studied. Collective processes are well established both experimentally and theoretically, such as different collective flow patterns. The most dominant is the collective flow which is used as a tool to extract the EOS and transport properties of nuclear matter.

HIGH ENERGY NUCLEAR PHYSICS -:

A highly dense and hot region represents the quark gluon plasma. When the temperature is between 0 MeV to 10 MeV and density lie between 4 to 6 w. r. t normal density. At this energy due to presence of large free space, only 4% of attempted collision are blocked and dynamics as termed as cascade approach. Collisions at this energy produce quark gluon plasma.

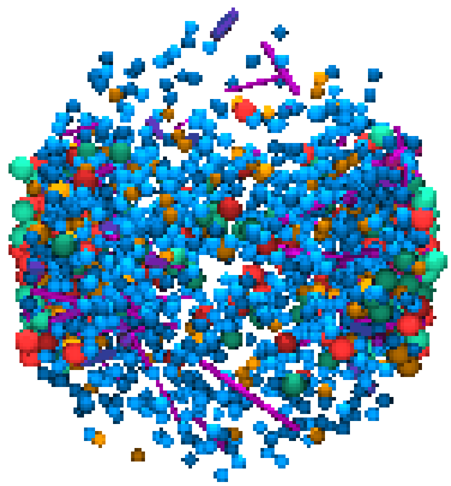

Fig 1.4 Collision at high energy (E > 2 GeV/nucleon)

1.2) NUCLEAR PHASE OF MATTER -:

Now the question is that why we study the nuclear matter produced from relativistic heavy ion physics or from high energy. The answer is that we need this information to understand the early history of our universe, and to understand high-density objects, called "neutron stars" in our present-day universe. At even higher temperatures and densities, nucleons themselves can undergo a phase transition. We can view each nucleon as a "bag" containing quarks and gluons. These quarks and gluons can move relatively freely inside their own bag, but "bag" theory says that they cannot escape from the bag–they are "confined." For this reason, we have never been able to detect individual free quarks or gluons. However, if we are able to produce an extremely dense gas of hadrons (mainly pions and nucleons), then their bags can overlap. This overlap let the quarks and gluons from different bags to mix freely and travel across the entire nuclear volume. We call this state a "quark-gluon plasma". From theoretical calculations, we also expect the phase transition to a quark-gluon plasma to be of first order, with a phase coexistence region.

Major research efforts at BNL (Brookhaven National Laboratory) in New York are directed toward establishing the conditions for creating this phase transition and observing its signatures. These high energy densities, sufficient to produce QGP, may be reached in laboratory in heavy ion reactions already at relatively modest energies, in the laboratory projectile energy. When these conditions are created, there will be a large number of quarks and gluons in the reaction zone. This is the only possibility to produce unbound quarks and gluons in laboratory in a small volume (10–100 fm for a short time, 2–10 fm/c).

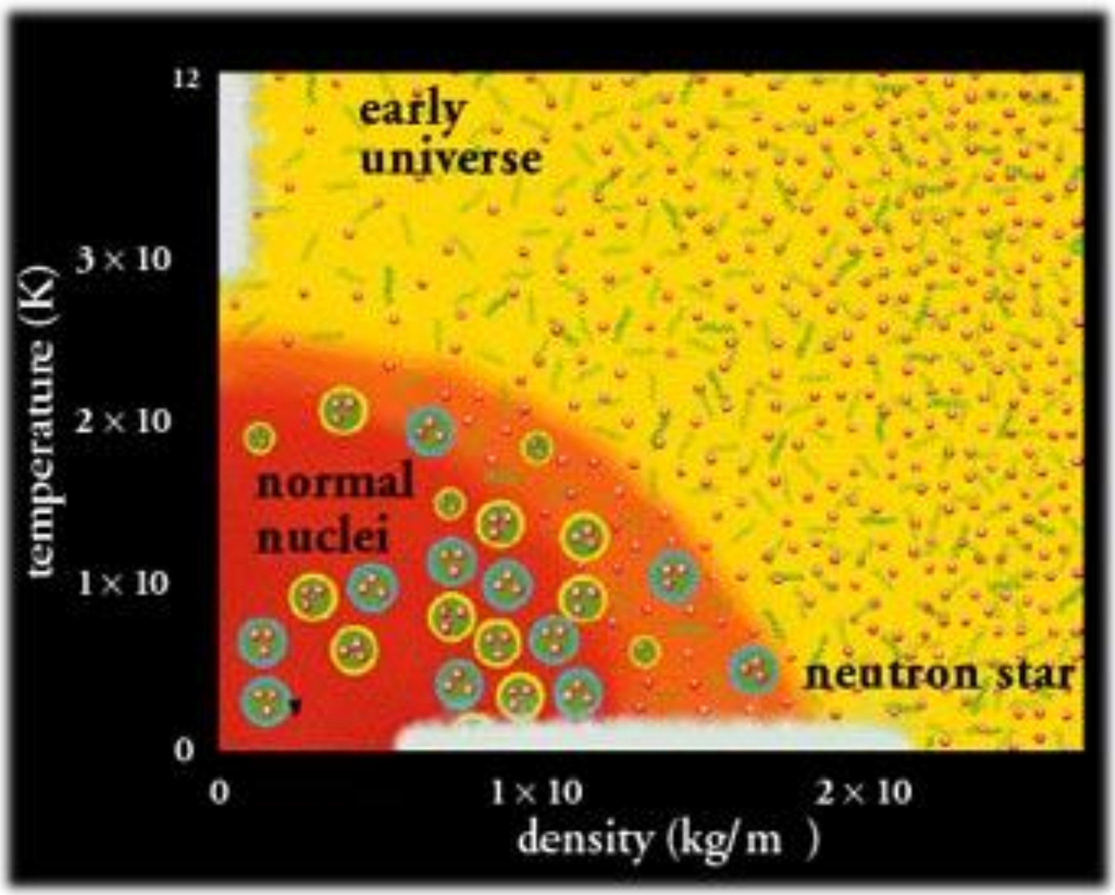

Fig 1.5 Phase diagram of nuclear matter.

Even if quark gluon plasma (QGP) is not formed in a given reaction it is very important to know the behaviour of the matter under high densities and temperatures, which can be reached in heavy ion reactions. This in formation can be obtained through nuclear equation of state.

1.3) NUCLEAR EQUATION OF STATE -: Nuclear equation of state give as the information about how much we can compress the nuclear matter at given energy. Nuclear matter has an energy minimum about 0.15 and 0.17 particles per cubic fermi. We can compress the nuclear matter by colliding two ions, it means we add the compressional energy into the system. Nuclear equation of state tells us that which compressional energy corresponds to which density.

To achieve parametrization one uses the so-called Skyrme-ansatz and uses different parameter sets corresponding to different compressibility values ranging from about 200 MeV (less repulsive) a so-called soft EOS up to about 400 MeV (more repulsive) a so-called *hard* EOS. The corresponding compressional energy curves as shown in fig 1.6

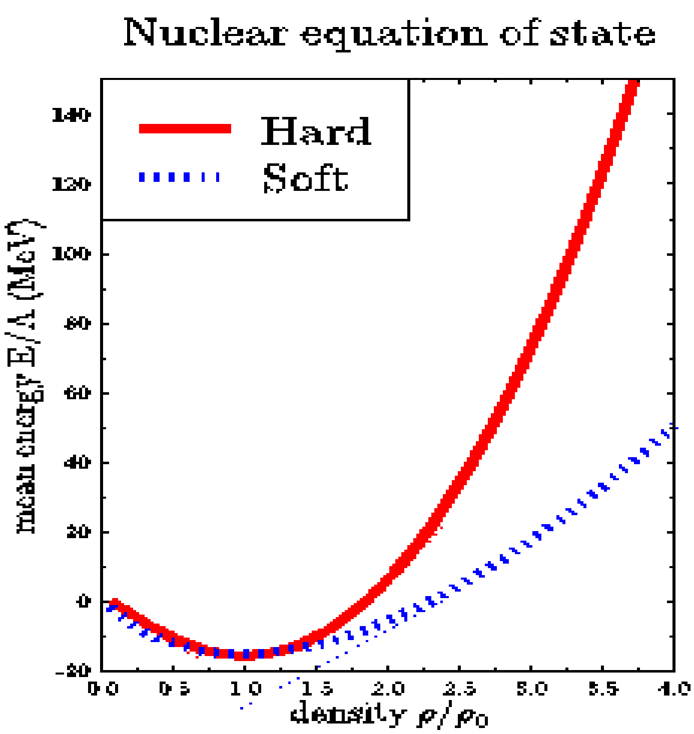

Fig 1.6 The density dependence of compression energy per nucleon. The soft and hard interaction are shown by dash and solid lines.

Heavy ion collisions allow to search for large number of observables which may be used as indicator of the properties of matter under extreme conditions. Frequently these observables are related to quantitative description of collective effects like the bounce -off of cold spectator matter in the reaction plane and squeeze – out of hot and compressed participant matter perpendicular to the reaction plane as well as production of secondary particles.

The most impressive results of high energy heavy ion research so far are the new, collective phenomena discovered in these reactions. The hot and compressed nuclear matter behaves like a compressible fluid (not like a dilute gas) and fluid dynamical effects are observed in these reactions.

1.4) COLLECTIVE FLOW-: The collective flow is measure of transverse motion imparted to particles and fragments during the collision of two nuclei. There are different type of flow and defined by the different n-th harmonic coefficient [7] -:

$$\frac{dN}{d\emptyset} = p_o(1+2v_1\cos\emptyset + \cos 2\emptyset \text{ ....}) \qquad (1.1)$$

Where $\emptyset$ is the angle between the transverse momentum of the particle and the reaction plane. Z-axis is defined as along the beam axis and Y-axis is defined perpendicular to the beam direction.

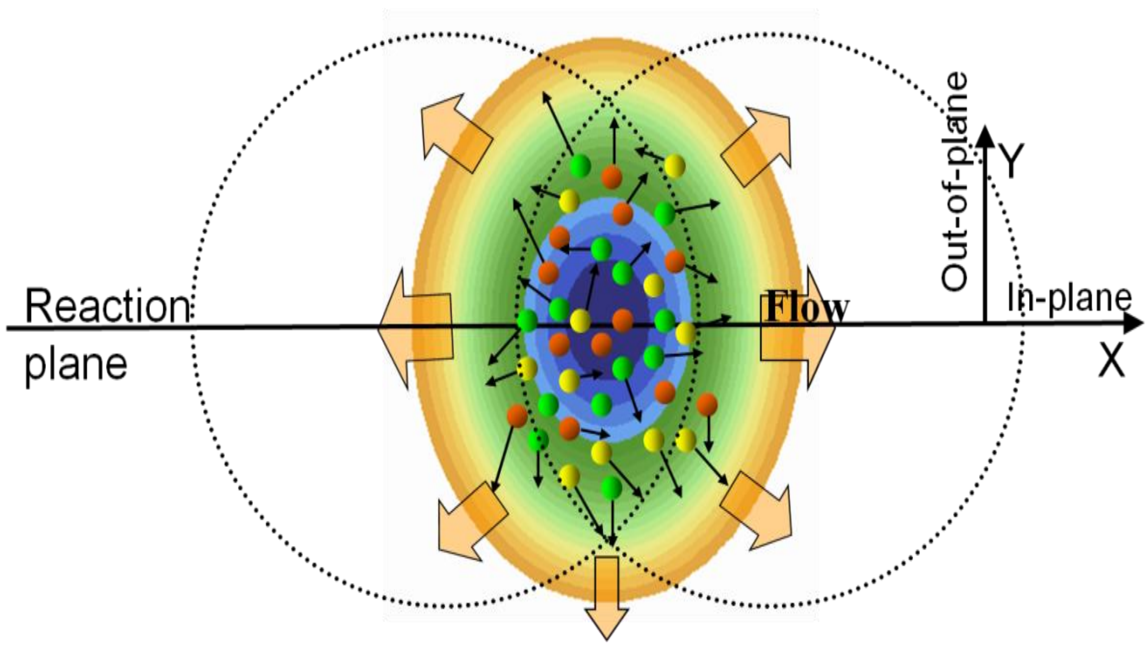

Fig 1.7 The direction of in plane and out of plane emission.

There are different type of flows which are described here.

1.5) Radial flow-: describes the azimuthal symmetric flow observed in the central collision in which the particle are emitted near the beam axis.

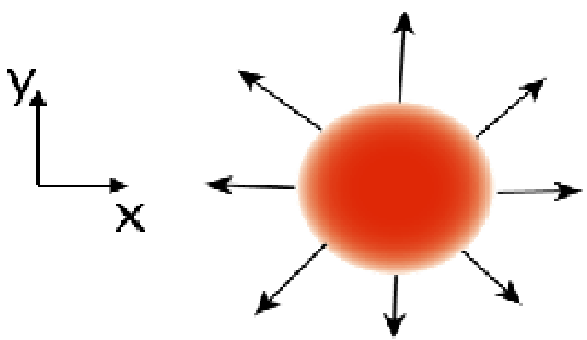

Fig1.8 Radial flow observed in the central collision.

1.4.2) Directed flow-: Also called in plane flow refers to the azimuthal asymmetric emission of particles with in the reaction plane.

$$v_1 = \frac{P_x}{P_t} \quad (1.2)$$

The first harmonic coefficient represent the directed flow.

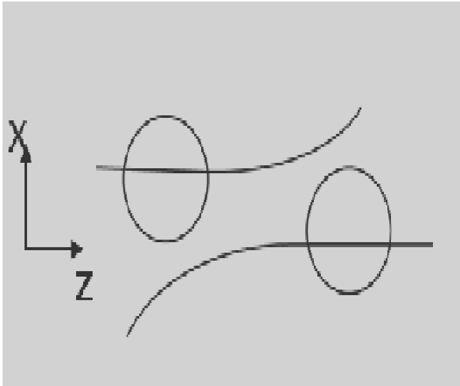

Fig 1.9 Directed flow observed in non central collision.

1.4.3) Elliptical flow-: Describe the azimuthal asymmetric emission pattern in which particle found to be preferentially emitted perpendicular to reaction plane.

$$v_2 = \frac{P_x^2 - P_y^2}{P_x^2 + P_y^2} \qquad (1.3)$$

While v_2 measure the eccentricity of the particle distribution in the momentum space represent the elliptical flow. Second harmonic coefficient represents the elliptical flow.

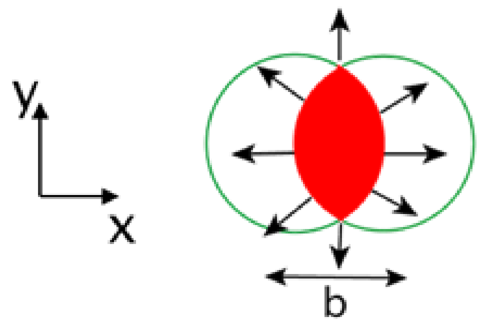

Fig1.10 elliptical flow observed in non central collision.

1.4.4) Triangular flow -: Third coefficient of harmonic coefficient is defined as triangular flow.

$$v_3 = \frac{P_x^2 - 3P_x P_y^2}{P_t^3} \qquad (1.4)$$

where P_x and P_y are the projections of particle transverse momentum parallel and perpendicular to the reaction plane. Triangular flow[8] is a sensitive probe of viscosity of viscosity. Viscous effects the drive energy and impact parameter. More central collision have less fluctuation, and hence smaller triangularity.

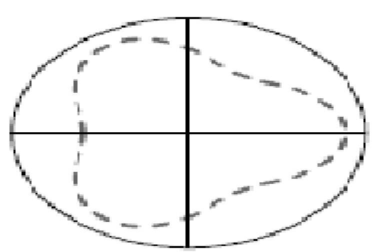

Fig1.11 Triangular flow

1.4.5) Hexadecouple flow -: v_4 represent the 4th momentum anisotropy namely the hexadecouple flow.

$$v_4 = \frac{P_x^4 - 6\, P_x^2 P_y^2 + P_y^4}{P_t^4} \qquad (1.5)$$

Where p_x and p_y are the projections of particle transverse momentum parallel and perpendicular to the reaction plane.

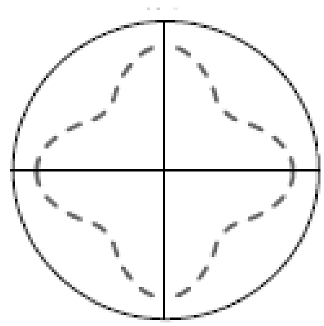

Fig 1.12 Hexadecouple flow

To study these flows, there are many theoretical models available in literature. We shall review them briefly.

1.5) REVIEW OF THEORETICAL MODELS-:

These observables (multi fragmentation, collective flow and stopping) are studied by theoretical models. Two type of theoretical models are available in the literature -:

i) Statistical models ii) Dynamical models

Statistical models used to study the final break up, neglecting the dynamics of reactions. On the other hand, dynamical models are used to study these reactions when two nuclei are well defined to final state where the matter is well separated and cold. The statistical models give the better description only of the final stage of a reaction. Since no dynamical information is possible. The study of dynamics of reaction including correlation and fluctuation is possible by using the dynamical models.

Due to lack of free space at low incident energies, 98% of the attempted collisions are blocked. The whole dynamics at low energy is governed by mean field or by the mutual two or three body interactions. On the contrary, the availability of large phase space at relativistic energies makes the Pauli blocking quite small and hence the dynamics of the reaction in governed by the cascade picture. On the other hand both the mean field and the cascade picture emerge at intermediate energies. These dynamical model are therefore, are capable of studying the detailed reaction phenomena with many body features. Moreover, isospin picture also come into the existence in this energy range. Isospin is the factor that differentiate neutrons from protons inside the nucleus on the basis of charges.

The dynamical approaches such as the Time Dependent Hartree Fock (TDHF)[9,10] or semi classical version called Vlasov equation are suitable at low incident energies where the nucleon nucleon collisions are negligible. Next model is the Extended Time Dependent Hartree Fock (ETDHF)[11] theory in which nucleon nucleon collisions are included. Then the BUU model was developed to study the large deviation problems of low, intermediate and relativistic heavy ion collisions. For intermediate energy region, where the correlation and fluctuation among the nucleons can be preserved. Classical Molecular Dynamics Model [12] and Quantum Molecular Dynamics Models are capable of predicting the the fragment production. With the development of the radioactive ion – beam physics, several rather isospin–dependent, but mostly semi – transport model such as Isospin dependent Boltzmann-Uehling-Uhlenback (IBUU) [13] and Isospin Quantum Molecular Dynamical Model (IQMD)[19] have been successfully developed in the recent years to describe nuclear reaction induced by the neutron-rich nuclei at intermediate energies.

1.6) EXPERIMENTAL REVIEW OF ELLIPTICAL FLOW -:

As discussed earlier, elliptical flow can be classified in Radial, Directed, elliptical, Triangular and Hexadecouple flow while elliptical flow is yet fully explored in nuclear science community at intermediate and high energy physics. The elliptical flow was first introduce by H. Sorge[14].

The review of experimental finding on different collaborations is presented as follow -:

Experimentally, out of plane emission, termed as squeeze out was first observed in 1989 by two competing collaborators. The Diogene collaboration at Saturne Synchrotron at Saclay observed few peaks in azimuthal distribution at mid rapidity at 800 MeV/nucleon. At BEVALAC in Berkeley, the plastic Bell/Well group observed the out of plane emission in $Au^{197} + Au^{197}$ collisions at 400 Mev/nucleon[15]. The BEVALAC group also characterize the sequeeze out as a function of projectile energy, mass as well as rapidity dependence by using a novel ratio of out of plane/in plane emission [16].

Other collaboration INDRA at GSI and ALADIN at GANIL emphasis the elliptical flow for different kind of fragments at intermediate energy heavy ion collisions. The results of a flow analysis for the set of reactions of $^{124,129}Xe$ projectiles and $^{112,124}Sn$ targets at incident energies 100 and 150 A MeV studied by INDRA detector at GSI[17]. The dependence on centrality and on p_t of the elliptic flow are determined for isotopically selected reaction products with $Z \leq 3$. The v_1 and $v_2$follow expected trends but isotopic effects are small. Also the transition from in plane to out of plane emission was first observed using Zn^{64} + Ni^{58} at GANIL facility by NAUTILUS collaboration in 1994. The energy of transition increase with impact parameter. Moreover, ALADIN collaboration observed the out of plane emission in $^{197}_{79}Au + ^{197}_{79}Au$ collision at 100 MeV/nucleon, where the out of plane emission is seen foe central collision, while peripheral collision clearly show in plane emission.

CHAPTER 2

Methodology

2.1) QUANTUM MOLECULAR DYNAMICS MODEL -:

The quantum molecular dynamics (QMD)[18] model is based on event by event method. Here each event is independent of each other. In contrast to BUU model, no averaging is done over the various events and hence, the correlations among the nucleons can be preserved. When we compare the result of TDHF and Vlasov equation, one find the quite similarity among these results.

The simulation model QMD needs three steps to describe the heavy ion collisions-:

1) Initialisation
2) Propagation
3) Collisions

1) **Initialisation** -: First one has to generate the nuclei. This procedure is called as initialisation. These nuclei are produced by applying some constrains on the ground state properties like radii and binding energy etc.
2) **Propagation** -: These nucleons, then propagate under the influence of surrounding mean field using the Hamilton's equation of motion with a given Hamiltonian.
   This is called as propagation.
3) **Collision** -: Finally, nucleon are bound to collide if they come close to each other in the coordinate space. The nucleon change their momenta respecting the Pauli principle. This part is called nucleon-nucleon collision.

Quantum Molecular Dynamics (QMD) model describes the N-body correlation, nuclear equation of state and many important quantum features, namely the Pauli principle, stochastic scattering as well as particle production. Now, at intermediate energies, there is charge independence of nuclear forces. In QMD model nucleon nucleon cross section (i.e. isospin independent) is same. To differentiate the nucleon state, we have isospin quantum number. This was one of the motivations for the development of **isospin dependent quantum molecular dynamics (IQMD)**[19] model. Isospin is treated explicitly through symmetry potential (to achieve the corrected distribution of protons and neutrons in the nucleus) and isospin dependent cross section.

The IQMD model treats the different charge states of nucleon, deltas and pions explicitly. The IQMD model has been used successfully for the analysis of large number of observables (collective flow, multi fragmentation, stopping) from low to relativistic energies.

2.2) ISOSPIN – DEPENDENT QUANTUM MOLECOULAR DYNAMICS MODEL-:

The best suited method to study the isospin effects is the isospin dependent quantum molecular dynamics model. It has basically two advantages -:

(1) Many-body process, in particular, the formation of complex fragments is treated.

(2)The model allows an event – by – event analysis of heavy ion reactions.

This model also includes three important steps: First, one has to generate the nuclei. This procedure is called as initialization. Then propagate under the influence of surrounding mean field. This is termed as propagation. Finally, nucleons are bound to collide if they come too close to each other. This part is dubbed as collisions. The elastic and inelastic cross-sections for proton-proton, neutron-neutron as well as proton-neutron are supposed to be affected in the presence of isospin. In the following, we shall discuss all of these steps in detail.

2.2.1) Initialisation -:

In this model the nucleons are represented by the Gaussian-shaped density distributions.

$$\varphi(r,p,t) = \frac{1}{\pi^2 \hbar^2} \; e^{\frac{-(r-r_i)^2}{2L}} e^{\frac{-(p-p_i)^2 . 2L}{\hbar^2}} \qquad (2.1)$$

Here the centroids of the Gaussian in the nucleus are randomly distributed in a phase space sphere ($r \leq R$ and $P \leq P_F$) with radius $R = 1.12A^{1/3}$ fermi in accordance with the liquid drop model. Each nucleon occupies a volume of h^3, so that phase space is uniformly filled. The Fermi momentum depends on the ground state density. For $\rho_0 = 0.17fm^{-3}$ the value of P_F is 268 MeV/c. As a result binding energy is low as compared to the Weizsacker mass formula i.e there is reduced binding energy per nucleon as compared to Weizsacker mass formula. Hence, the initialized nuclei are less stable against spurious particle evaporation as compared to QMD model. Finally, it should be noted that IQMD performs a Lorentz contraction of the nucleus coordinate distribution which is not present in QMD and also important for high energies. Gaussian width is regarded as a description of the interaction range of a particle. Its influence disappears for infinite nuclear matter, whereas, for finite systems it

plays an important role. It is denoted by L. The interaction range parameter L influences the interaction density for finite system. In IQMD, the Gaussian width can be used as an optional input parameter. The system dependence of parameter L in IQMD has been introduced in order to obtain maximum stability of the nucleonic density profiles. As an example, for Au+Au a value of L= 8.66fm^2 is chosen, for Ca+ Ca and lighter system L= 4.33fm^2.

2.2.2) Propagation-:

The successfully initialized nuclei are then boosted towards each other with a proper center of mass velocity using relativistic kinematics. The nucleons of target and projectile interact via two and three body Skyrme forces and the Yukawa interactions. The isospin degree of freedom is treated explicitly by employing a symmetry potential and explicit Coulomb forces between the protons of colliding target and projectile. All this provide us a correct distribution of protons and neutrons within the nucleus.

The Hamilton equations of motion for the propagation of hadrons are:

$$\frac{dr}{dt} = \frac{d<H>}{dp_i} \quad ; \quad \frac{dp}{dt} = - \frac{d<H>}{dr_i} \tag{2.2}$$

2.2.3) POTENTIAL USED IN IQMD -:

The IQMD-model offers rather stable density distributions and good energy conservation, however for the price of nucleon evaporation and improper binding energies (E_{Bind}~ 4-5MeV/nucleon for heavy nuclei instead of 8 MeV/nucleon). The Hamiltonian function <H> contains all the possible interactions among the particles of the system. The expectation value of the Hamiltonian is i.e a total Hamiltonian function with a kinetic energy T and potential energy V is given by

$$H = <\mathbf{T}> + <\mathbf{V}>$$

The potential in above equation's is the sum of the following specific elementary potential

$$V^{ij}(\mathrm{r} - r') = V^{ij}_{Skr} + V^{ij}_{Yuk} + V^{ij}_{Coul} + V^{ij}_{mdi} + V^{ij}_{sym}$$

$$= (t_1\delta(r' - \mathrm{r}) + t_2\delta(r' - \mathrm{r})\rho^{\Upsilon-1}(\frac{r'+r}{2}))$$

$$+ t_3 \frac{exp(|r' - r|/)}{(|r' - r|/)} + \frac{Z_i Z_j e^2}{|r' - r|}$$

$$+ t_4 \mathrm{In}^2(t_5 (p_i' - p)^2 + 1)\delta(r' - \mathrm{r})$$

$$+ t_6 \rho_o T_i^3 T_j^3 \delta(r_i' - r_j). \qquad (2.3)$$

2.2.4) Collisions

Two particles collide if their minimum distance d, i.e. the minimum relative distance of the centroids of the Gaussian during their motion, in their CM frame fulfil the requirement :

$$|\mathrm{r_i} - \mathrm{r_j}| \leq \sqrt{\frac{\sigma_{tot}}{\pi}} \qquad (2.4)$$

Where the cross-section is assumed to be the free cross section of the regarded collision type (N − N, N − Δ...). The total cross-section is the sum of the elastic cross -section and all inelastic cross-sections. The main processes include:

Elastic processes -: N + N→N + N

N+Δ→ N + Δ

Δ + Δ→ Δ + Δ

Inelastic processes -: N+N→N+Δ

N+Δ→N+N (2.5)

In addition to this the Pauli blocking of baryons is also taken into account by checking the phase space densities in the final state. The final phase space fractions and Pi , Pj which are already occupied by other nucleons, are determined for each of the scattering baryons. For each collision the phase space densities in the final state are checked in order to assure that the final distribution in phase space is in agreement with the Pauli principle.

2.3) METHODS OF CLUSTERIZATION

2.3.1) Minimum spanning tree method (MST)

In MST [20], two nucleons share the same fragment if their centroids are closer than a distance d_{min},

$$|r_i - r_j| \leq d_{min} \qquad (2.6)$$

Where r_i and r_j are the spatial positions of both nucleons. The minimum distance d_{min} has been used as a free parameter which varies between 2 fm - 4 fm. This approach (being a spatial distance approach) cannot detect different fragments which are almost overlapping and therefore, will give a single big fragment during the early stage of the reaction where density is quite high and the interactions among the nucleons are still active. In other words, the simple coordinate space approach cannot address the question of time scale of the fragments. To study the time of fragment formation, one needs to derive a method which should be able to detect the overlapping fragments.

CHAPTER 3

Elliptical flow

The information about the nature of equation of state is still one of the burning topic of present day nuclear physics research in general and heavy ion collisions in particular. Significant progress has been made in determining the nuclear equation of state from heavy ion reactions. Multifragmentation, elliptical flow and stopping are important phenomena at intermediate energy in heavy ion collisions played a fascinating role in exploring the various aspects of nuclear dynamics such as nuclear equation of state [21]. Among the different observables collective flow play the special status. This is due to sensitive response to model ingredients that define the equation of state. A lot of effort has been made in studying the collective flow in heavy ion collisions. Here first we study the basic properties of a reaction. Scaled density $<\rho/\rho_0>$ as a function of impact parameter has been studied here.

We simulate reactions of ${}^{40}_{20}Ca + {}^{40}_{20}Ca$ and ${}^{124}_{47}Ag + {}^{124}_{47}Ag$ at incident energy 100 MeV/nucleon. The relativistic effects do not play a significant role at this incident energy and the sub threshold particle production is negligible. The phase space generated by IQMD has been analyzed using the minimum spanning tree method (MST) [20]. Here we also use the MSTM method in which we imply the momentum cut of order of Fermi momentum. The entire calculations are done at t =200 fm/c.

3.1) IMPACT PARAMETER DEPENDENCE OF DENSITY -:

The final state of nuclear matter is closely related to density of collision[18]. We define the average density as

$$<\rho(r_i)> = \sum_i \frac{1}{(2\pi L)^{3/2}} e^{-(r_i-r_j)^2/2L} \qquad (3.1)$$

The above density definition shows the number of nucleons in the vicinity of each nucleon. Fig.3.1 shows that scaled density as the function of impact parameter. The final stage density is different at different impact parameter in addition to it is different for compressed zone and freeze out zone. When two nuclei collide with each other, in the compressed stage, density is lower for higher impact parameter range but this trend changes for freeze out stage or for final stage.

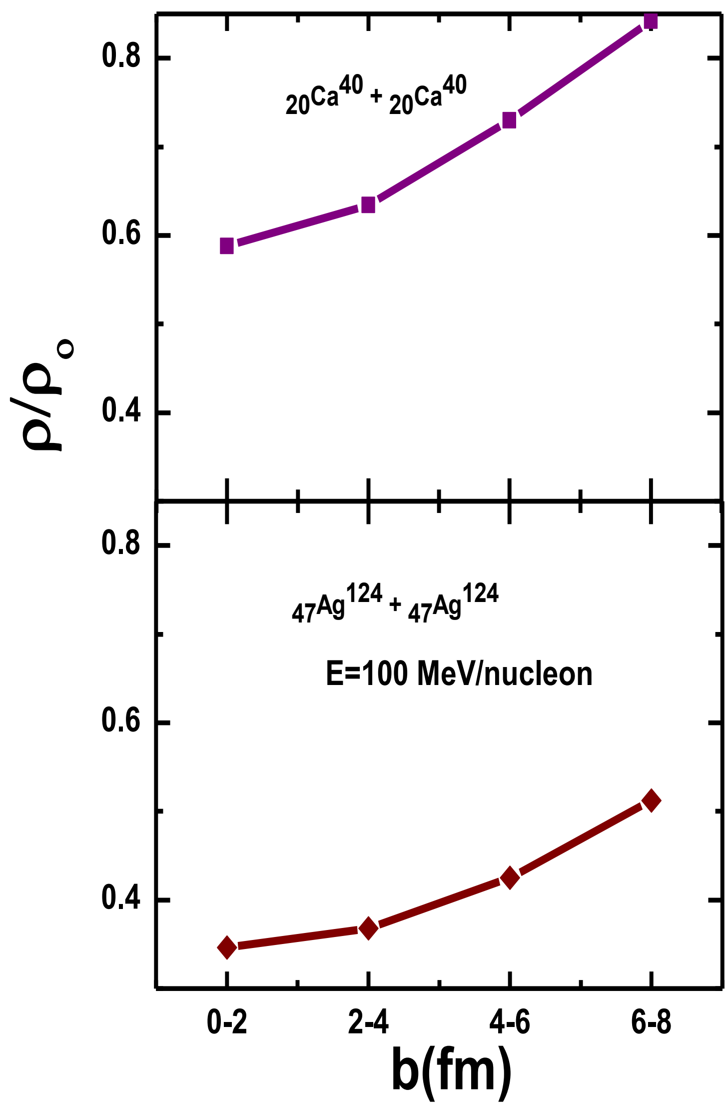

Fig 3.1 Upper panel represents density as fuction of impact parameter at different ranges for ${}^{40}_{20}Ca$+ ${}^{40}_{20}Ca$ and lower pnel for ${}^{124}_{47}Ag$ + ${}^{124}_{47}Ag$.

This is due to the fact that at higher impact parameter range, it preserves most of the initial correlations or in simple language; it cannot breaks nuclear matter into many fragments, hence the density in close to normal nuclear matter density. Naturally, the final stage density depends strongly on the masses of colliding nuclei and maximum impact parameter(b_{max}). When we collide the heavier nuclei, then it break into the heavier fragments hence the final stage density is larger for heavier nuclei as comparison to lighter nuclei, here scaled density is larger for the lighter nuclei because lighter nuclei ($^{40}_{20}Ca$ + $^{40}_{20}Ca$) achieve the b_{max} earlier than heavier nuclei ($^{124}_{47}Ag$ + $^{124}_{47}Ag$).

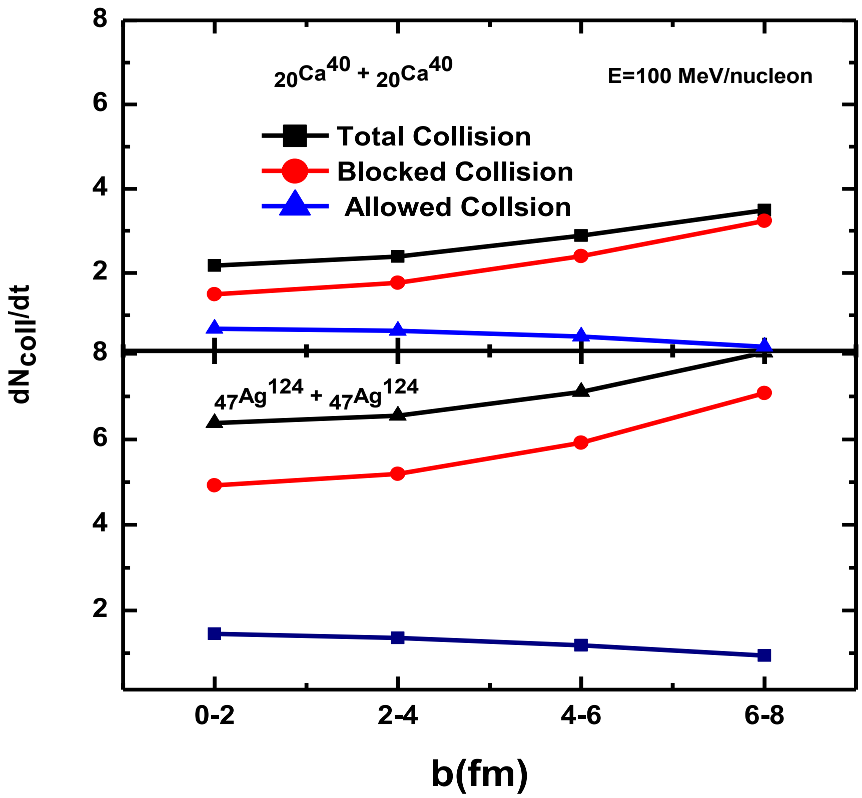

Fig3.2 Rate of collisions as a function of impact parameter $^{40}_{20}Ca + ^{40}_{20}Ca$ (upper panel) & $^{124}_{47}Ag$ + $^{124}_{47}Ag$ (lower panel).

3.2) GEOMETRY DEPENDENCE OF COLLISION DYNAMICS-:

In fig.3.2, rate of collisions is displayed under the same condition as density profile. Generally collision rate depends on the participant zone and the incident energy. For two systems Ca + Ca and Ag + Ag, we have fixed the incident energy i.e. 100 MeV/nucleon and played with the participant zone by changing the geometry of the collision. At the peripheral geometries, the participant zone decreases and hence the decrease is observed in the allowed collisions and increase observed in the number of blocked collisions. Total collision is the sum of allowed and blocked collisions which increase with the increase of impact parameter. Also the numbers of collisions depend on the mass of nuclei. For lighter nuclei, number of collisions is less in comparison to heavier nuclei as a result collisions increase with increase of mass number.

Now we shall discuss the elliptical flow, which play important role to know about nuclear equation of state.

3.3) ELLIPTICAL FLOW -:

Collective flow can be studied via in plane flow and out of plane flow. Here $\emptyset$ is the azimuthal angle between the transverse momentum of the particle and the reaction plane. Elliptical flow[18] is defined by second order Fourier coefficient $< cos2\emptyset >$ [7].

$$< \cos 2\emptyset > = < v_2 > = < \frac{P_x^2 - P_y^2}{P_x^2 + P_y^2} > \qquad (3.2)$$

The positive value of elliptical flow reflects in plane emission, whereas, out of plane is reflected by its negative value. The reason for the anisotropic flow is the orthogonal asymmetry in the configuration space and re-scattering. The directed flow is reported to diminish at higher incident energy due to the large beam rapidity. Therefore, the elliptical flow is much more suited at this incident energy. The elliptical flow describes the eccentricity of an ellipse like distribution.

Flow in X-Z direction representing the in plane flow or sideward flow and flow in X – Y direction representing the out of plane flow or squeeze out flow, while indicating separately both of emission(in plane and out of plane) in fig3.3.

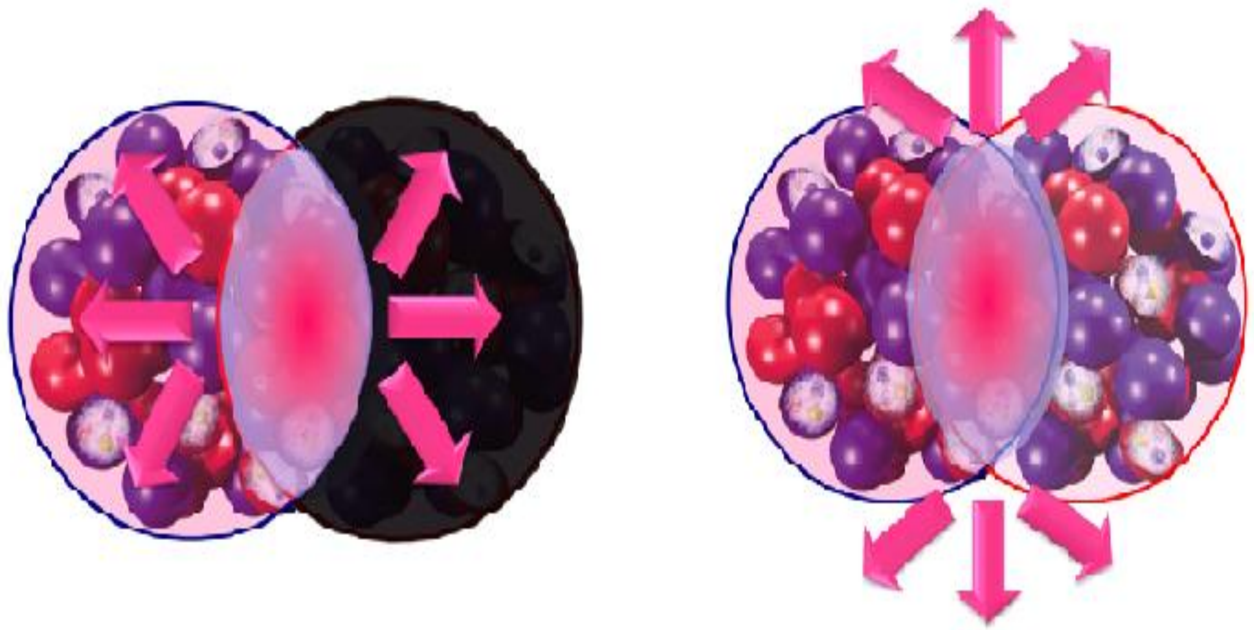

Fig.3.3 a) In plane flow ($v_2 > 0$) b) Out of plane flow($v_2 < 0$)

Elliptical flow is an interesting phenomena which can occur in non central collisions, has been observed in the intermediate energy range. The two factors are responsible for out of plane emission.

1 The pressure builds up in the compression stage compared to energy density.

2 The passage time for the removal of projectile and target.

The first concept of elliptical flow was given by H. Stocker [22]. He measured the angular distribution of protons and provides further evidence for predominant sideward emission of fragments from high energy heavy ion collisions through fluid dynamical model in central collisions. The reason for the out of plane flow was compressed matter which emit the particles in the perpendicular directions. Out of plane flow is also expressed as sideward flow where directed flow is also known as rotational flow.

3.4) TRANSVERSE MOMENTUM DEPENDENCE OF ELLIPTICAL FLOW -:

In fig.3.4 and fig.3.5, the free nucleons and different fragment i.e. light mass fragments (LMF's) ($2 \leq A \geq 4$) and intermediate mass fragments ($5 \leq A \geq A_{tot}/6$) are selected to study the elliptical flow analysis. We perform a complete systematic study using the reactions Ca + Ca (lighter nuclei) and Ag + Ag (heavier nuclei) at incident energy 100 MeV/nucleon. We have simulated these reactions at different impact parameter range. A Gaussian type behavior is observed in all the cases as observed by Colona and Toro et al[23]. One sees that elliptical flow is positive in the whole range of p_t. The collective rotation is one of the main mechanism to induce the positive elliptical flow. It is also evident from the figure that peak of Gaussian shift towards the lower value of p_t for IMF, Obviously, IMF have lower momentum in comparison to LMF and free nucleon. With the increase of impact parameter range, participant zone decreases as a result, elliptical flow increases. With the increase of composite mass of the system, elliptical flow increases. Also at some impact parameter, flow vanish.

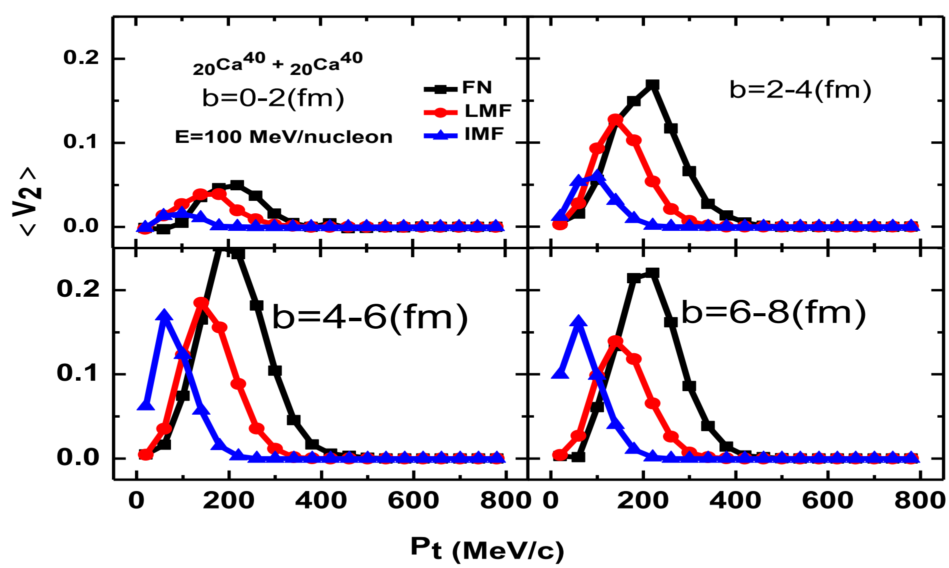

Fig.3.4 The transverse momentum dependence of elliptical flow at different impact parameter range for ${}^{40}_{20}Ca + {}^{40}_{20}Ca$ at 100MeV/nucleon.

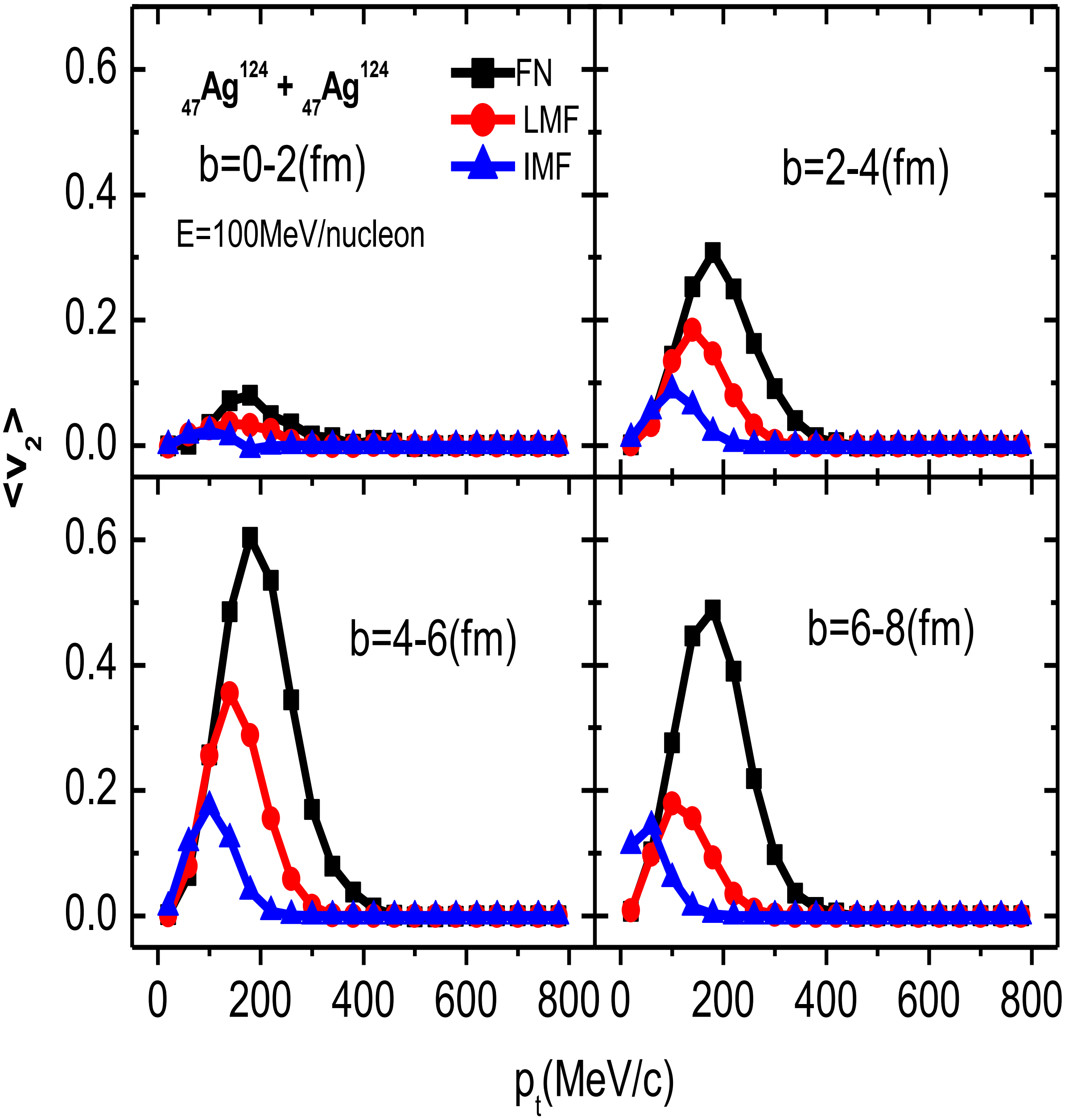

Fig.3.5 The transverse momentum dependence of elliptical flow at different impact parameter range for $^{124}_{47}Ag + ^{124}_{47}Ag$ at 100 MeV/nucleon.

3.5) IMPACT PARAMETER DEPENDENCE ON ELLIPTICAL FLOW -:

The value of elliptical flow become more positive and less negative with increase of impact parameter at E=100 MeV/nucleon. With the increase of impact parameter, participant zone decreases as a result elliptical flow increases. This is indicating the dominance of in plane flow with increase of impact parameter as matter is less squeezed. Elliptical flow also depends on the composite mass of the system. With the increase of the composite mass of the system elliptical flow increases. The value of elliptical flow is higher for the system Ag + Ag having the less positive value and more negative value. These observations are consistent with the experimental finding and other theoretical work.

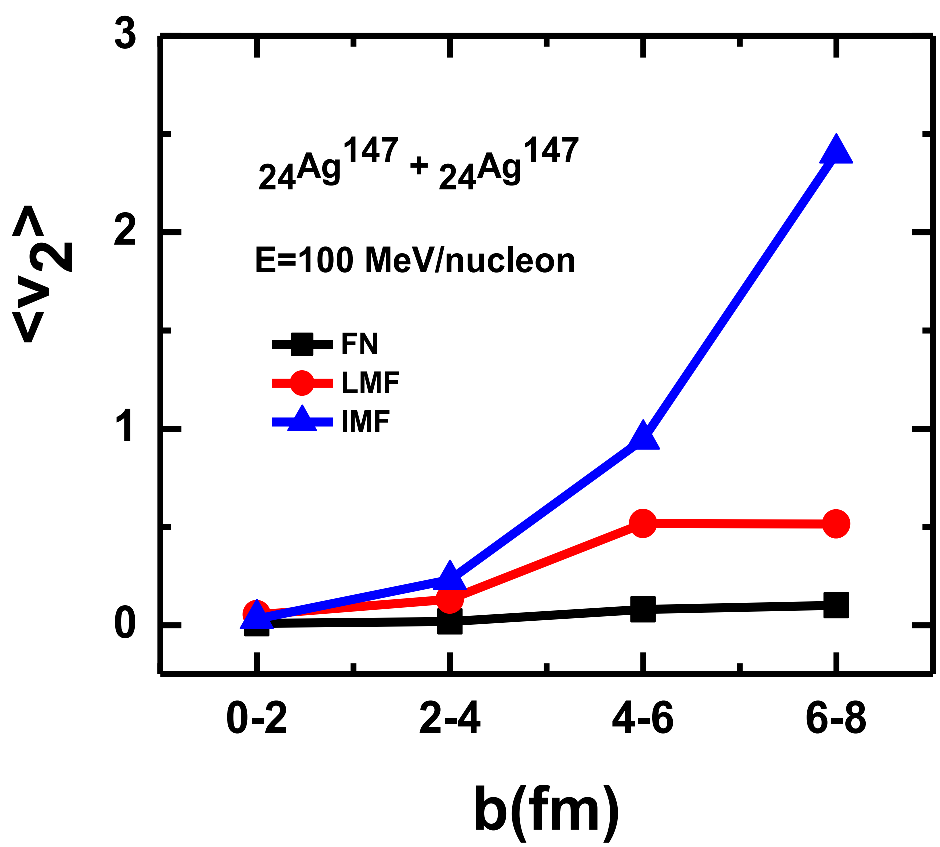

Fig.3.6 Impact parameter dependence of elliptical flow summed over the entire rapidity range at incident energy 100 MeV/nucleon.

3.7) COMPARISION WITH EXPERIMENTAL DATA -:

To, further strengthen our interpretation of results, we compare our result (impact parameter) dependence of v_2 for the reaction of $^{124}_{47}Ag$ + $^{124}_{47}Ag$ for p, He and Li using the same conditions with experimental data extracted by INDRA and ALADIN collaboration[17]. We find that elliptical flow become more positive with increase in impact parameter. With the increase of impact parameter, lesser nucleons participate in the collision process leading to enhanced elliptical flow. Production of free nucleon and lighter fragments decrease with increase of impact parameter. One can see from the fig.3.7, the corresponding value of $< v_2 >$ for fragment decreases. We have attempted to compare all results with experimental data because the mass of experimentally collided nuclei are very close of Ag + Ag,

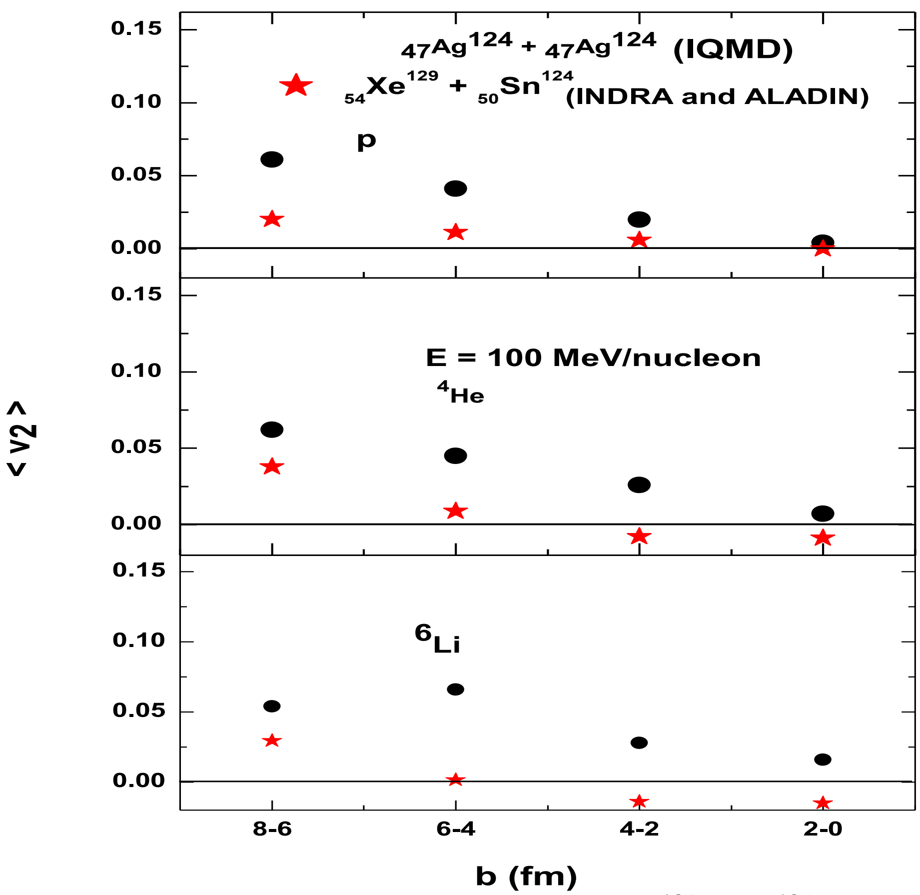

Fig. 3.7 Comparison of theoretical result (simulated for the reaction $^{124}_{47}Ag$ + $^{124}_{47}Ag$ using IQMD model for p, He, Li) with experimental data extracted by INDRA and ALADIN collaboration for reaction $^{129}_{54}Xe$ + $^{124}_{51}Sn$.

therefore as we increase the impact parameter, azimuthal anisotropy become large, hence further increase in impact parameter. The theoretical observation are consistent with the experimental data obtained for p, He and Li .

3.6) DISAPPEARANCE OF ELLIPTICAL FLOW (TRANSITION ENERGY) -: A useful feature of elliptical flow is the existence of transition between the two forms of observables. The transition energy in that, at which we have isotropic angular distribution or elliptical flow disappear [24]. At energy below the transition energy, emission is primarily in the reaction plane (rotational flow) above the transition energy, a maximum emission in the direction perpendicular to the reaction plane (known as squeeze out) because the compressed matter in the interaction region can escape the particle in the perpendicular direction. In the other words, one can say that the azimuthal emission pattern is primarily out of plane. The transition energy increases with the

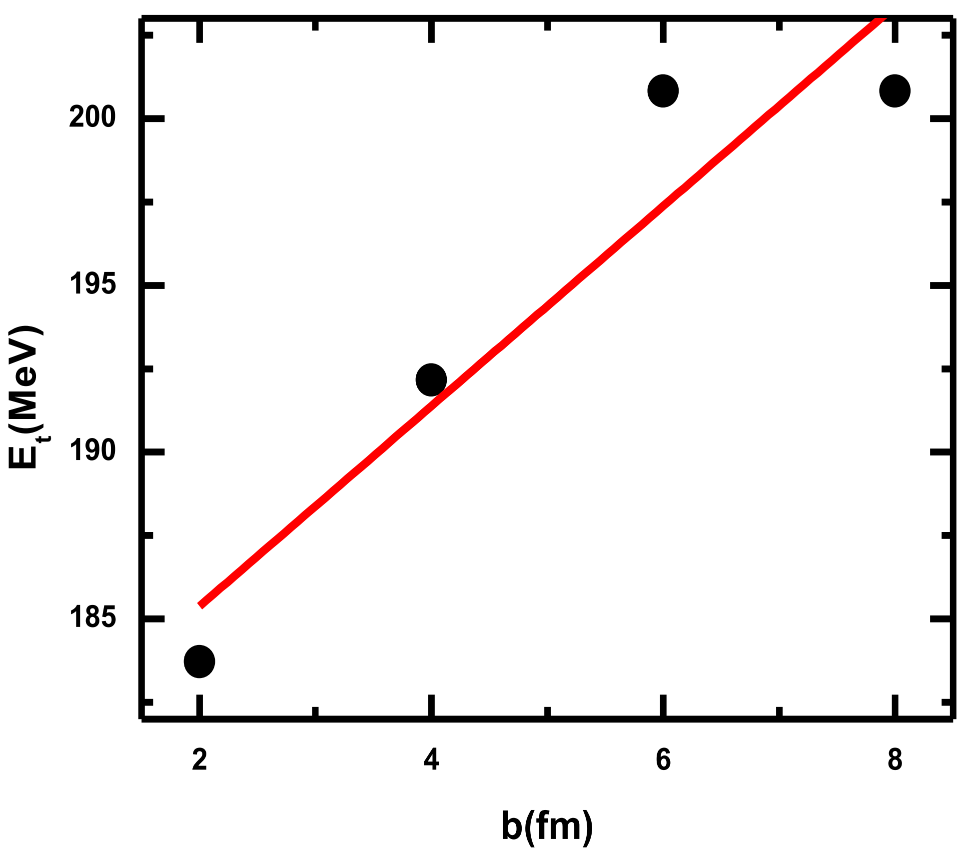

Fig.3.8 Transition energy at different impact parameter for ${}^{124}_{47}Ag$ + ${}^{124}_{47}Ag$.

increase of impact parameter. With the increase of impact parameter v_2 increases which means collective expansion process based on the nucleon – nucleon scattering starts predominating over the mean field[25], which contributing to the formation of rotating compound system, become less important which lead to higher transition energy.

3.7) CONCLUSION -:

In this chapter, We have discussed transverse momentum dependence of elliptical flow at different impact parameter range which increases with the increase of impact parameter A Gaussian type behavior is observed for transverse momentum dependence of v_2. Also, with the increase of composite mass of the system, elliptical flow increases. To further strengthen our interpretation of results, we compare our theoretical result with experimental results extracted by INDRA and ALADIN collaboration. This proves that with increase of impact parameter elliptical flow increases. Then disappearance of elliptical flow at different impact parameter range has been discussed.

REFERENCES -:

[1] Nuclear Physics, 8th edition, D. C. Tayal, Himalaya Publishing House(2011).

[2] L. C. Vaz, J. M. Alexander and G. R. Satchler, Phys. Rep. **69,** 373(1981); M. Beckerman, Rep. Prog. Phys. **51**, 1047 (1988).

[3] K. E. Zyromski et al., Phys. Rev. **C 55**, R562 (1997).

[4] C. Ngo, B. Tamain, M. Breiner, R. J. Lombard, D. Mas, and H. H. Deubler, Nucl. Phys**. A 252**, 237 (1975); H. Ngo and C.h. Ngo, Nucl. Phys. **A 348**, 140(1980); K. C. Panda and T. Patra, J. Phys. **G 14**, 1489 (1988).

[5] R. K. Puri *et al*, Eur. *Phys. J.* **A23**, 429(2005).

[6]. H. Stocker and W. Greiner, Phys. Rep. **137**, 277(1986).

[7] S. Voloshin and Y.Zhang Z. Phys. C**70**, 665(1996).

[8] YAN Ting Zhi, MA yu-Gang, FANG De-Quing, Chinese physics Lett., **24**, 338(2007).

[9] H. Stocker and W. Greiner, Phys. Rep **137**, 277(1986),

[10] A. K Kerman and S. E Konnie, Ann. of Phys. **100**, 332(1976).

[11] E Surand and C. Gregorie, and B. Tamain, Prog. Part. Nucl. Phys. **23**, 375(1989).

[12] L. Wilets , Y. Yariv, R. Chestnut, Nucl. Phys. **A 301**, 359(1978); A. Vicetini, G. Jacucci and V. R. Pandharipande, Phys. Rev. C **31**, 1783 (1985).

[13] B. A. Li et al. Phys. Rev. C52, R1746(1995).

[14] H. Sorge, Phys. Rev. Lett. **78**, 2309 (1997); J. Y. Ollitrault, Phys. Rev. **D 46**, 229 (1992).

[15]H.H. Gutbroad et al., Phys. Lett. B **216**, 267 (1989).

[16] H.H. Gutbroad et al., Phys. C **42**, 640 (1990).

[17] J. Lukasik et al., Proc. of INPC **2**, 513 (2007).

[18] J. Aichelin Phys. Rep. **202**, 233 (1991).

[19] C. Hartnack, Rajiv K Puri, J. Aichelin, J Konopka, S. A. Bass, H. Stocker, W. Greiner, Eur Phys. J. A **1**, 151(1998).

[20] J. Singh, S. Kumar and R. K. Puri, *Phys. Rev.* **C62**, 044617 (2000); S Kumar and R K Puri, Phys**.** Rev. **C 58**, 1618 (1998); Y K Vermani, S Goyal and R K Puri, Phys. Rev. **C 79**, 064613 (2009).

[21] P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002); H. Stocker and W. Greiner. Phys. Rep. **137**, 277 (1986); W. Reisdorf and H. G. Ritter, Ann. Rev. Nucl. Sci. **47**,

633 (1997); C. Hartnack and J. Aichelin, Phys. Rev**.** **C 49**, 2801 (1994); S. Kumar and R. K. Puri, Phys. Rev. **C 78**, 064602 (2008);

[22] H. Stocker et al. Phys. Rev. **C 25**, 1873 (1982).

[23] M. Di. Toro, S. J. Yennello, and B. A. Li, Eur. Phys. **16**, 2676 (2007).

[24] S. Kumar, S. Kumar and R. K. Puri, Phys. Rev. C 81, 014611(2010)

[25] Y. Zhang and Z. Li, Phys. Rev. **C 74**, 014602 (2006).