

RELIABILITY ANALYSIS OF POWER SYSTEMS USING FUZZY SETS AND THEIR EXTENSIONS

A THESIS SUBMITTED TO

THAPAR UNIVERSITY
PATIALA, INDIA

FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

IN

MATHEMATICS

BY

Manjit Verma

(Registration No. 900911008)



School of Mathematics and Computer Applications

Thapar University, Patiala-147004

Punjab-India

DEDICATED

TO

THE ALMIGHTY

&

MY FAMILY

CANDIDATE'S DECLARATION

I hereby certify that the work being presented in the thesis entitled "**Reliability Analysis of Power Systems Using Fuzzy Sets and their Extensions**" in fulfillment of the requirements for the award of degree of Doctor of Philosophy, submitted in the School of Mathematics and Computer Applications of Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Amit Kumar, Assistant Professor, School of Mathematics and Computer Applications, Thapar University, Patiala and Prof. Yaduvir Singh, Professor and Director, NOIDA Institute of Engineering and Technology, Greater Noida.

The matter embodied in the thesis has not been submitted by me or any other university or institute for the award of other degree.


(Manjit Verma)
(Regn. No. 900911008)

This is to certify that the above statement made by candidate is correct to best of my knowledge.


Dr. Amit Kumar
Assistant Professor
School of Mathematics and
Computer Applications
Thapar University, Patiala


Prof. Yaduvir Singh
Director
NOIDA Institute of Engineering
and Technology, Greater Noida

ACKNOWLEDGEMENTS

First of all, I bow my head in reverence to the Almighty God for giving me an opportunity and strength to complete this task. It is my privilege to express my sincere regards and gratitude to my supervisors Dr. Amit Kumar and Dr. Yaduvir Singh for their expert guidance, valuable suggestions, advice, continuous encouragement and support throughout the course of my research work.

I want to acknowledge the innocent blessings of Mehar. My supervisor, Dr. Amit Kumar, believes that Mehar is an angel for him and without Mehar's blessings it was not possible to think of the ideas presented in this thesis. Mehar is a lovely daughter of Parmpreet Kaur (Research Scholar under the supervision of Dr. Amit Kumar).

I am highly obliged to Prof. S.S. Bhatia, Head, School of Mathematics and Computer Applications for his encouraging attitude, keen personal interest and constant inspiration in guiding me during my work.

I am sincerely thankful to the members of the Doctoral Committee for their valuable guidance and help in completion of my study.

I express my gratitude to Dr. P.K. Bajpai, Dean of Research and Sponsored Projects, for his constant encouragement that was of great importance in the completion of the thesis.

My thanks are also due to Dr. Abhijit Mukherjee, Director, Thapar University, Patiala.

I convey my thanks to Dr. Rajesh Gupta and Mrs. Usha Gupta for their time to time help and moral support. I am also grateful to my friends Puspinder Singh, Anila Gupta, Neha Bhatia, Meenakshi, Priya Shahi, Rashmi Sachdeva for their timely help and the moral support they provided me during my research work.

My friends Amit Kumar, Nischint Sharma, Ch. V. Rukmini, Sourav Goyal, Praveen Kumar, Gourav Mittal and Ashutosh Tiwari have been always there whenever I needed them for help and advice.

I feel a deep sense of gratitude for my parents who formed part of my initial vision and inculcated in me the real value of education which really matters in life. I would like to give my special thanks to my brother and sister-in-law for their constant encouragement that was of great importance in the completion of the thesis.

Patiala

July 20, 2012



(Manjit Verma)

In this thesis, a new type of fuzzy fault tree, named as non-normal p -norm trapezoidal fuzzy number based fuzzy fault tree and an approach, based on the proposed fuzzy fault tree, for evaluating the fuzzy reliability of industrial systems is proposed. Also, new approaches, based on Petri net, for evaluating some important vague reliability parameters of industrial systems as well as a new approach for evaluating the fuzzy reliability of such industrial systems in which failure probability of all the components are represent by different types of fuzzy numbers are proposed. To illustrate the proposed approach, based on non-normal p -norm trapezoidal fuzzy number based fuzzy fault tree, the fuzzy reliability of an electrical substation and to illustrate the approaches, based on Petri net, the vague reliability parameters of a compressor system and a gas turbine are evaluated.

Chapter 1

Chapter 1 is introductory in nature. This Chapter gives brief review of the work done in the area of fuzzy reliability analysis.

Chapter 2

In order to handle the uncertain situation and insufficient information, Tanaka et al. [92] represented the failure probabilities of basic events of fault tree by trapezoidal fuzzy numbers and named the obtained fault tree as fuzzy fault tree. Chen and Tang [23] extended the trapezoidal fuzzy number to non-normal p -norm trapezoidal fuzzy number to handle the uncertainty in more intelligent manner. In this Chapter, non-normal p -norm trapezoidal fuzzy number based fuzzy fault tree approach is proposed by modifying the existing

trapezoidal fuzzy number based fuzzy fault tree approach [92]. The proposed approach is illustrated by evaluating the fuzzy reliability of a 66/11 kV substation.

Chapter 3

Liu and Chiou [68] discussed the advantages of Petri net over fault tree and proposed a approach, based on Petri net, for evaluating some crisp reliability parameters. Knezevic and Odoom [56] discussed the advantage of fuzzy sets over crisp sets and proposed a approach, based on Petri net, by modifying the existing approach [68] for evaluating some fuzzy reliability parameters. On the same direction, in this Chapter, the advantage of vague sets over fuzzy sets are discussed and a new approach, based on Petri net, by modifying the existing approach [56] is proposed for evaluating some vague reliability parameters. Also, the proposed approach is used to evaluate the vague reliability parameters of a compressor system.

Chapter 4

Chen et al. [29] extended the concept of trapezoidal fuzzy number [52] by trapezoidal fuzzy number with different left and right height and pointed out that it is better to represent the data with the proposed trapezoidal fuzzy number as compared to the existing trapezoidal fuzzy number [52]. On the same direction in this Chapter, the concept of trapezoidal vague set [25] is extended by trapezoidal vague set with different left and right height. Also, the approach proposed in Chapter 3, is modified on the basis of proposed trapezoidal vague set. The modified approach is illustrated by evaluating the vague reliability of a gas turbine.

Chapter 5

Kumar et al. [62] proposed T_{ω} -based vague fault tree approach for evaluating the fuzzy reliability of industrial systems and shown that it is better to use the proposed approach

as compared to the existing T_ω -based fuzzy fault tree approach [48]. On the same direction, in this Chapter, T_ω -based vague Petri net approach is proposed for evaluating the vague reliability of industrial systems and shown that it is better to use T_ω -based vague Petri net approach as compared to the approach, proposed in previous Chapter. The proposed approach is illustrated by evaluating the vague reliability of a gas turbine.

Chapter 6

Chang and Cheng [19] pointed out that there is no approach in the literature for finding the arithmetic operations between different types of vague sets and proposed a approach for the same. Chang and Cheng [19] also used the proposed approach for evaluating the vague reliability of a Printed Circuit Board Assembly (PCBA). In this Chapter, the shortcomings of the approach, proposed by Chang and Cheng [19], are pointed out and to resolve the shortcomings of existing approach [19], a new approach is proposed for finding the arithmetic operations between different types of vague sets. Also, it is pointed out that due to the shortcoming of the existing approach [19], the vague reliability of PCBA, obtained by Chang and Cheng [19], is not exact and the proposed approach is used to evaluate the exact vague reliability of PCBA.

LIST OF PUBLICATIONS

- [1]. **Verma, M.,** Kumar, A. and Singh, Y. and Allahviranloo, T., Application of non-normal p -norm trapezoidal fuzzy number in reliability evaluation of electrical substations. **Neural Computing and Applications (Springer)** DOI 10.1007/s00521-012-0949-7 (SCI).
- [2]. **Verma, M.,** Kumar, A. and Singh, Y. Vague reliability assessment of combustion system using Petri nets and vague lambda-tau methodology, **Engineering Computations (Accepted)** (SCI).
- [3]. **Verma, M.,** Kumar, A. and Singh, Y. Electrical substations reliability evaluation using fault tree approach based on generalized fuzzy number. **Eksploracja I Niezawodnosc-Maintenance and Reliability (Accepted)** (SCI).
- [4]. **Verma, M.,** Kumar, A. and Singh, Y. Vague modelling for risk and reliability analysis of compressor system, **Concurrent Engineering: Research and Applications**, DOI: 10.1177/1063293X11435178, 2012 (SCI).
- [5]. **Verma, M.,** Kumar, A. and Singh, Y. Fuzzy fault tree approach for analyzing the fuzzy reliability of a gas power plant, **International Journal of Reliability and Safety (Accepted)**.
- [6]. **Verma, M.,** Kumar, A. and Lata, S. Fuzzy reliability analysis of a gas power plant using trapezoidal vague sets, **International Journal of Performability Engineering (Accepted)**.
- [7]. **Verma, M.** and Kumar, A. Fuzzy reliability analysis of a gas power plant using T_{ω} -based arithmetic operations on LR type flat vague set, **Electroenergetics - Electrotechnics - Electromechanics +Control**, 2(1): 32- 44, 2011.

- [8]. **Verma, M.,** Kumar, A. and Singh, Y., Power system reliability evaluation using fault tree approach based on generalized fuzzy number, **Journal of Fuzzy Set Valued Analysis (Accepted).**
- [9]. **Verma, M.,** Kumar, A. and Singh, Y. A new approach for analysis the fuzzy reliability using *LR* type interval valued flat vague set, **International Journal of Systems Assurance Engineering and Management (Accepted).**
- [10]. **Verma, M.** and Kumar, A. Performance analysis of 66/11 kV substation using Petri nets and vague lambda tau methodology, **Kuwait Journal of Science and Engineering (Submitted after Revision).**
- [11]. **Verma, M.** and Kumar, A. A note on “A novel general approach to evaluating the PCBA for components with different membership function”, **Applied Soft Computing (Communicated).**
- [12]. **Verma, M.,** Kumar, A. and Singh, Y. A novel general approach to evaluating the reliability of gas turbine system, **Engineering Applications of Artificial Intelligence (Communicated).**
- [13]. **Verma, M.,** Kumar, A. and Singh, Y. Application of the weakest t-norm (T_{ω}) based vague lambda-tau methodology for reliability analysis of gas turbine system, **International Journal of Systems Science (Communicated).**

CONTENTS

Acknowledgements.....	i
Abstract.....	iii
List of Publications	vi
List of Figures	xii
List of Tables	xiv
CHAPTER 1 Introduction	1
1.1 Literature Survey	2
1.2 Organization of The Thesis.....	5
CHAPTER 2 Non-normal P -norm Trapezoidal Fuzzy Number Based Fuzzy Fault Tree Approach for Evaluating the Fuzzy Reliability of Industrial systems	8
2.1 Fault tree Analysis	9
2.2 Existing Trapezoidal Fuzzy Number Based Fuzzy Fault Tree Analysis	11
2.2.1 Basic Definitions.....	12
2.2.2 Arithmetic Operations Between Two Trapezoidal Fuzzy Numbers.....	14
2.2.3 Existing Approach	15
2.3 Proposed Non-Normal P -norm Trapezoidal Fuzzy Number Based Fuzzy Fault Tree Approach	16
2.3.1 Basic Definitions.....	16

2.3.2 Arithmetic Operations Between Two Non-Normal P -Norm Trapezoidal Fuzzy Numbers	17
2.4 Proposed Approach.....	18
2.4.1 Fuzzy Reliability Evaluation of 66/11 kV Substation	19
2.4.2 Comparative Study.....	25
2.5 Conclusions.....	27
CHAPTER 3 A Petri Net based Approach with Trapezoidal Vague Sets for Evaluating the Vague Reliability Parameters of Industrial Systems	28
3.1 A Brief Review of Petri Net.....	28
3.2 Existing Approach	31
3.3 Vague Set Theory	33
3.4 Proposed Approach.....	35
3.5 Evaluation of Vague Reliability Parameters of a Compressor System.....	39
3.5.1 Comparative Study.....	46
3.6 Conclusions.....	48
CHAPTER 4 A Petri Net based Approach with a new type of Trapezoidal Vague Set for Evaluating the Vague Reliability Parameters of Industrial Systems	49
4.1 Proposed Trapezoidal vague Set with Different Left and Right Heights	49

4.2 Proposed Approach.....	53
4.3 Evaluation of Vague Reliability Parameters of a Gas turbine	59
4.4 Comparative Study.....	67
4.5 Conclusions.....	68
 CHAPTER 5 A Petri Net based Approach with T_{ω} -based Arithmetic Operations for Evaluating the Vague Reliability Parameters of Industrial Systems	 69
5.1 T_{ω} - Based Arithmetic Operations Between Two Vague Sets	69
5.1.1 Illustrative Example	72
5.2 Proposed Approach.....	75
5.3 Evaluation of Vague Reliability Parameters of a Gas Turbine.....	75
5.4 Comparative Study.....	79
5.5 Conclusions.....	80
 CHAPTER 6 Modified Approach for Evaluating the Vague Reliability of PCBA with Different Membership Function	 81
6.1 Shortcomings of Existing Approach.....	81
6.1.1 Existing Approach	82
6.1.2 Existing Approach for finding the Arithmetic Operations Between Triangular Vague Sets.....	83

6.1.3 Illustrative Example	84
6.1.4 Shortcomings of Obtained Results.....	85
6.2 Proposed Approach.....	86
6.3 Advantage of Proposed Approach	87
6.3.1 Proposed Approach for Finding the Arithmetic Operations Between Triangular Vague Sets.....	87
6.3.2 Illustrative Example	88
6.4 Exact Vague Reliability of PCBA	89
6.4.1 Fault Tree of PCBA	90
6.4.2 Failure Probabilities of Basic Events	92
6.4.3 Exact Interval of Confidence	103
6.4.4 Vague Reliability Evaluation.....	112
6.5 Conclusions.....	114
CHAPTER 7 Future Directions	115
Bibliography	116

LIST OF FIGURES

Figure 2.1: Triangular fuzzy number	13
Figure 2.2: Trapezoidal fuzzy number	14
Figure 2.3: Generalized trapezoidal fuzzy number	17
Figure 2.4: A single line diagram of 66/11 kV substation	20
Figure 2.5: Fault tree of a chosen substation	21
Figure 2.6: Reliability of substation obtained by using different approaches	26
Figure 3.1: Basic fault tree logic gates along their representation in Petri net model	30
Figure 3.2: The fault tree model	31
Figure 3.3: The Petri net transformation of Figure 3.2	31
Figure 3.4: Triangular vague set	35
Figure 3.5: Fault tree of compressor system	40
Figure 3.6: Petri net model of compressor system	40
Figure 3.7: Vague reliability parameters of compressor system	45
Figure 4.1: Trapezoidal fuzzy sets with different left and right heights	50
Figure 4.2: Trapezoidal vague set	51
Figure 4.3: Trapezoidal vague set with different left and right heights	52

Figure 4.4: Fault tree of gas turbine.....	60
Figure 4.5: Petri net model of gas turbine.....	60
Figure 4.6: Vague reliability parameters of the gas turbine.....	66
Figure 5.1: Triangular vague set	74
Figure 5.2: Vague reliability parameters of the gas turbine.....	77
Figure 5.3: Reliability of gas turbine over the time interval.....	79
Figure 6.1: Fault tree of PCBA.....	90
Figure 6.2: Vague set representing of vague reliability of PCBA	114

LIST OF TABLES

Table 2.1: Intermediate events of chosen substation	21
Table 2.2: Basic events of chosen substation.....	22
Table 2.3: Failure Probabilities of basic events.....	23
Table 2.4: Results obtained by using the existing approaches and proposed approach	25
Table 3.1: Reliability parameters for repairable system	31
Table 3.2: Expressions for evaluating failure rate (λ_s) and repair rate (τ_s) of the system ...	32
Table 3.3: Intermediate events of compressor system	41
Table 3.4: Basic events of compressor system	41
Table 3.5: Crisp failures rates and repair time	42
Table 3.6: Left end point and right end point of interval of confidence corresponding to truth membership function of vague reliability parameters	43
Table 3.7: Left end point and right end point of interval of confidence corresponding to complement of membership function of vague reliability parameters	44
Table 3.8: Results obtained by using existing approach [56] and proposed approach	46
Table 4.1: Intermediate events of the chosen system	61
Table 4.2: Basic events of the chosen system.....	61

Table 4.3: Crisp failures rates and repair rate	63
Table 4.4: Left end point and right end point of interval of confidence corresponding to truth membership function of vague reliability parameters	64
Table 4.5: Left end point and right end point of interval of confidence corresponding to complement of false membership function of vague reliability parameters.....	65
Table 5.1: Comparison of the results obtained by using existing and proposed approaches corresponding to different membership values.....	78
Table 6.1: The description of the intermediate events.....	91
Table 6.2: The description of the basic events.....	91
Table 6.3: Vague sets representing the failure probability of basic events	92
Table 6.4: Interval of confidence corresponding to truth membership function of $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \dots, \tilde{S}$	104
Table 6.5: Interval of confidence of corresponding to complement of false membership function of $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \dots, \tilde{S}$	108
Table 6.6: Interval of confidence corresponding to truth membership function of vague reliability of PCBA	113
Table 6.7: Interval of confidence corresponding to complement of false membership function of vague reliability of PCBA	113

CHAPTER 1

INTRODUCTION

The power systems are usually large, complex and in many ways nonlinear systems. They include subsystems and components such as generators, switching substations, power lines and loads. The evaluation of the overall system reliability is extremely complex as it is necessary to include detailed modeling of both generation and transmission facilities and their auxiliary elements. A failure of components or subsystems can result in a failure of power delivery to specific loads or in certain cases, a full blackout of the power system. Therefore, power system reliability analysis [8] is an important issue both on the academic research and practice.

Fault tree models are often used to determine the reliability of large integrated systems whose failure occurs due to failure of components in the systems. Fault tree is a logical diagram from which the probability of the top event can be determined with known failure probabilities of components in the system.

In general, the failure probabilities of the components are considered as crisp. These crisp values are determined from past occurrences which are very uncertain, since the system's environment keep changing. Moreover, failures of components which have never failed before must often be considered. Therefore, some degree of uncertainty exists in such a determination. In order to handle the insufficient information, the fuzzy approach is used to evaluate system reliability.

1.1 LITERATURE SURVEY

Fault tree analysis is a powerful diagnosis technique and is widely used for demonstrating the root causes of undesired events in system failure. The concept of fault tree analysis [99] was developed by Bell telephone laboratory in 1961. It is widely used in many fields such as nuclear reactors [10], chemical [22] and aviation industries [20].

Singer [86] presented a fuzzy set approach [101] based fault tree for reliability analysis in which the relative frequencies of the basic events are considered as fuzzy numbers.

Cai et al. [12] pointed out that there are generally two assumptions in the conventional reliability theory, given as follows:

- (i) Binary state assumptions: the system is precisely defined as functioning or failing.
- (ii) Probability assumptions: the system behaviour is fully characterized in the context of probability measures.

Due to inaccuracy and uncertainty in data, the estimation of precise values of probability becomes very difficult in many systems. Therefore, Cai et al. [13] presented the following two assumptions:

- (i) Fuzzy-state assumption: the meaning of the system failure can't be precisely defined in a reasonable way. At any time the system may be in one of the following two states: fuzzy success state or fuzzy failure state.
- (ii) Possibility assumption: the system behaviour can be fully characterized in the context of possibility measures.

Cai et al. [14] has given the following three forms of “fuzzy reliability theories”

- (i) PROFUST reliability theory, based on the PRObability assumption and FUZZY-STate assumption.
- (ii) POSBIST reliability theory, based on the POSSibility assumption and BINARY STate assumption.
- (iii) POSFUST reliability theory, based on the POSSibility assumption and the FUZZY-STate assumption.

Chen [26] proposed a new approach for analysing the fuzzy system reliability using fuzzy number arithmetic operations. Pan and Yun [77] used the fuzzy set theory for modelling the fuzzy system structure and proposed the new procedure to calculate the system reliability. Chanda and Bhattacharjee [16] used fuzzy fault tree for reliability analysis of an optimally planned transmission system. Chang et al. [22] developed a novel approach by incorporating digraph models, fault trees and fuzzy inference mechanisms in a unified framework to relieve the online computation load.

Verma et al. [94] optimized the reactive power control variables by using fuzzy linear programming and by considering the voltage collapse phenomena in the existing framework of Bulk Power System Reliability Evaluation. Huang et al. [49] presented a model of posbist fault tree analysis of coherent systems and used the same model for quantitative analysis of a real-life safety system. Bai and Asgarpour [5] described a fuzzy-based analytical approach and a fuzzy-based Monte Carlo simulation technique to obtain a possibility distribution of reliability indices for substations. Konstandinidou et al. [58] analysed the human reliability using fuzzy classification system and calculate the probability of erroneous actions according to Cognitive Reliability and Error Analysis Approach in specific contexts. Volkanovski et al. [96] used the fault tree analysis approach for power system reliability analysis.

Chen [25] presented the arithmetic operations between vague sets and also proposed a new approach for analysing the fuzzy system reliability based on vague sets. Shu et al. [85] proposed an algorithm to evaluate the fault interval of the system components using vague fault tree analysis and applied this approach to the failure analysis problem of Printed Circuit Board Assembly (PCBA). Kumar et al. [61] presented fuzzy reliability analysis of marine power plant using vague set theory, where the reliability of components of the system is represented by trapezoidal vague sets. Chang and Cheng [19] obtained fault interval and reliability interval of the PCBA with different membership function using fault-tree analysis. Taheri and Zarei [91] investigated bayesian system reliability assessment in vague environment.

Hura and Atwood [50] proposed the use of Petri nets to represent fault trees. Kumar and Aggarwal [65] used both Petri nets and fault tree approaches for software reliability analysis. Liu and Chiou [68] discussed the advantages of Petri net over fault tree and proposed an approach, based on Petri net, for evaluating some crisp reliability parameters. Knezevic and Odoom [56] proposed new methodology which uses Petri nets instead of the fault tree methodology and solves for reliability indices utilizing fuzzy lambda-tau approach. Sharma et al. [83] computing various performance measures, namely reliability, availability, mean time between failures, expected number of failures, failure rate and repair time, for any industrial system using Petri net and fuzzy lambda-tau methodology.

In this thesis, a new type of fuzzy fault tree, named as non-normal p -norm trapezoidal fuzzy number based fuzzy fault tree and an approach, based on the proposed fuzzy fault tree, for evaluating the fuzzy reliability of industrial systems is proposed. Also, new approaches, based on Petri net, for evaluating some important vague reliability parameters of industrial systems as well as a new approach for evaluating the fuzzy reliability

of such industrial systems in which failure probability of all the components are represent by different types of fuzzy numbers are proposed. To illustrate the proposed approach, based on non-normal p -norm trapezoidal fuzzy number based fuzzy fault tree, the fuzzy reliability of an electrical substation and to illustrate the approaches, based on Petri net, the vague reliability parameters of a compressor system and a gas turbine are evaluated.

1.2 ORGANIZATION OF THE THESIS

The chapter wise summary of the thesis is as follows:

Chapter 2

In order to handle the uncertain situation and insufficient information, Tanaka et al. [92] represented the failure probabilities of basic events of fault tree by trapezoidal fuzzy numbers and named the obtained fault tree as fuzzy fault tree. Chen and Tang [23] extended the trapezoidal fuzzy number to non-normal p -norm trapezoidal fuzzy number to handle the uncertainty in more intelligent manner. In this Chapter, non-normal p -norm trapezoidal fuzzy number based fuzzy fault tree approach is proposed by modifying the existing trapezoidal fuzzy number based fuzzy fault tree approach [92]. The proposed approach is illustrated by evaluating the fuzzy reliability of a 66/11 kV substation.

Chapter 3

Liu and Chiou [68] discussed the advantages of Petri net over fault tree and proposed an approach, based on Petri net, for evaluating some crisp reliability parameters. Knezevic and Odoom [56] discussed the advantage of fuzzy sets over crisp sets and proposed a approach, based on Petri net, by modifying the existing approach [68] for evaluating some fuzzy reliability parameters. On the same direction, in this Chapter, the advantage of vague sets over fuzzy sets are discussed and a new approach, based on Petri net, by modifying the existing approach [56] is proposed for evaluating some vague reliability parameters. Also, the

proposed approach is used to evaluate the vague reliability parameters of a compressor system.

Chapter 4

Chen et al. [29] extended the concept of trapezoidal fuzzy number [52] by trapezoidal fuzzy number with different left and right height and pointed out that it is better to represent the data with the proposed trapezoidal fuzzy number as compared to the existing trapezoidal fuzzy number [52]. On the same direction in this Chapter, the concept of trapezoidal vague set [25] is extended by trapezoidal vague set with different left and right height. Also, the approach proposed in Chapter 3, is modified on the basis of proposed trapezoidal vague set. The modified approach is illustrated by evaluating the vague reliability of a gas turbine.

Chapter 5

Kumar et al. [62] proposed T_ω -based vague fault tree approach for evaluating the fuzzy reliability of industrial systems and shown that it is better to use the proposed approach as compared to the existing T_ω -based fuzzy fault tree approach [48]. On the same direction, in this Chapter, T_ω -based vague Petri net approach is proposed for evaluating the vague reliability of industrial systems and shown that it is better to use T_ω -based vague Petri net approach as compared to the approach, proposed in previous Chapter. The proposed approach is illustrated by evaluating the vague reliability of a gas turbine.

Chapter 6

Chang and Cheng [19] pointed out that there is no approach in the literature for finding the arithmetic operations between different types of vague sets and proposed a approach for the same. Chang and Cheng [19] also used the proposed approach for evaluating

the vague reliability of a Printed Circuit Board Assembly (PCBA). In this Chapter, the shortcomings of the approach, proposed by Chang and Cheng [19], are pointed out and to resolve the shortcomings of existing approach [19], a new approach is proposed for finding the arithmetic operations between different types of vague sets. Also, it is pointed out that due to the shortcoming of the existing approach [19], the vague reliability of PCBA, obtained by Chang and Cheng [19], is not exact and the proposed approach is used to evaluate the exact vague reliability of PCBA.

CHAPTER 2

NON-NORMAL P -NORM TRAPEZOIDAL FUZZY NUMBER BASED FUZZY FAULT TREE APPROACH FOR EVALUATING THE FUZZY RELIABILITY OF INDUSTRIAL SYSTEMS^{*}

Fault tree analysis is a powerful diagnosis technique and is used widely for demonstrating the root causes of undesired event in system failure and logical functional relationship among components, manufacturing processes and subsystems of a system. In conventional fault-tree analysis, it is assumed that exact probabilities of events are given. In addition, a lot of data of failures should be collected before Fault tree analysis; otherwise, some important events may be omitted during the fault-tree construction, which may lead to incorrect results. However, it is often difficult to obtain past failure data because the environmental change of process and Fault tree analysis is made at early design stage, especially, there is often need to use new components that have not failure data. Furthermore, in real system/process, the information is inaccurate and supposed to linguistic representation, the estimation of precise values of possibility becomes very difficult in many cases.

^{*} The contents of this Chapter, published in *Neural Computing and Applications*, DOI 10.1007/s00521-012-0949-7, (2012), are the generalisation of the contents of the paper accepted for publication in *Eksploatacja I Niezawodnosc-Maintenance and Reliability*.

In order to handle the uncertain situation and insufficient information, Tanaka et al. [92] represented the failure probabilities of basic events of fault tree by trapezoidal fuzzy numbers and named the obtained fault tree as fuzzy fault tree. Chen and Tang [23] extended the trapezoidal fuzzy number to non-normal p -norm trapezoidal fuzzy number to handle the uncertainty in more intelligent manner. In this Chapter, non-normal p -norm trapezoidal fuzzy number based fuzzy fault tree approach is proposed by modifying the existing trapezoidal fuzzy number based fuzzy fault tree approach [92]. The proposed approach is illustrated by evaluating the fuzzy reliability of a 66/11 kV substation.

2.1 FAULT TREE ANALYSIS

In this section, a brief review of the fault tree analysis is presented [85].

Fault tree analysis is a powerful diagnosis technique and is used widely for demonstrating the root causes of undesired event in system failure and logical functional relationship among components, manufacturing processes and subsystems of a system. Fault tree analysis is now widely used in many fields, such as in the semiconductor industry, man-machine system, flexible manufacturing system, software-implemented shutdown system of the nuclear power plants, and chemical and aviation industries. Fault tree analysis is suitable for the manufacturing system/process fault diagnosis because it has the following characteristics:

- (i) Fault tree analysis can be used deeply to analyze a specific fault level by level. It uses clear graphics to vividly give visibility to fault dependencies between the part/component faults and the manufacturing system/process fault.

- (ii) The fault tree can clearly point out aspects of the system/process relevant to significant failures. It can also be seen whether a part/component fault will cause a system/process fault, what the effect is, how great the effect is, and its mechanism.
- (iii) A fault tree is a clear illustration for those management and maintenance personnel who have never participated in the system/process design and trial-manufacture, which will greatly shorten the training time of the maintenance personnel, and therefore cut down the expense for personnel training.
- (iv) The qualitative analysis of fault trees a system/ process may make the designers with an insight into the system/ process behavior, so as to be able to find out the weak links in the design scheme, and take corrective measures.

It is important to understand that a fault tree is not a model of all possible system failures or all possible causes for system failure. A fault tree is tailored to its top event that corresponds to some particular system failure mode, and the fault tree thus includes only those faults that contribute to this top event. Moreover, these faults are not exhaustive; they cover only the faults that are assessed to be realistic by the analyst. It is also important to point out that a fault tree is not itself a quantitative model. It is a qualitative model that can be evaluated quantitatively. This qualitative aspect, of course, is true of virtually all varieties of system models. The fact that a fault tree is a particularly convenient model to quantify does not change the qualitative nature of the model itself. Both qualitative and quantitative evaluations can be performed on a fault tree. The fault tree itself is a qualitative assessment of the events and relationships that lead to the top event.

Some important symbols that are used to construct the fault tree are as follows:

 Top Event:

An undesired state of a system caused by an events occurring within the system.

Top event is represented by rectangle.



Intermediate Event:

An intermediate event is a fault event which occurs from a combination of other events via logic gates. Intermediate event is also represented by rectangle.



Basic Event:

The circle describes a basic initiating fault event that requires no further development. In other words, it signifies that the appropriate limit of resolution has been reached.



Undeveloped Event:

An event which is not further developed either because it is of insufficient consequence or because information is unavailable.



OR Gate:

Output fault occurs if at least one of the input faults occurs.



AND Gate:

Output fault occurs if all of the input faults occur.

2.2 EXISTING TRAPEZOIDAL FUZZY NUMBER BASED FUZZY FAULT TREE ANALYSIS

Tanaka et al. [92] represented the failure probabilities of basic events of fault tree by trapezoidal fuzzy numbers and named the obtained fault tree as fuzzy fault tree.

In this section, some basic concept of fuzzy sets theory [101] and the existing approach [92], based on fuzzy fault tree analysis, for evaluating the fuzzy reliability of industrial systems is presented.

2.2.1 BASIC DEFINITIONS

In this section, some basic definitions are presented [52].

Definition 2.1 The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range $[0,1]$ i.e. $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned values indicate the membership grade of the element in the set A . The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2 A fuzzy number $\tilde{A}$ is described as a subset of real line whose membership function is a continuous mapping from real line to a closed interval $[0,1]$, which has the following characteristics:

- (i) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, c] \cup [d, \infty)$.
- (ii) Is strictly increasing on $[c, a]$ and strictly decreasing on $[b, d]$.
- (iii) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [a, b]$.

Eventually, it can be $c = -\infty$ or $c = a$ or $a = b$ or $b = d$ or $d = \infty$.

Definition 2.3 The α -cut, A_α of fuzzy set $\tilde{A}$ in the universe of discourse X is defined by

$$A_\alpha = \{x_i \mid \mu_{\tilde{A}}(x_i) \geq \alpha, x_i \in X\}$$

where, $\alpha \in [0,1]$.

Definition 2.4 A fuzzy number $\tilde{A}=(a,b,c)$ is said to be a triangular fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\frac{x-a}{b-a}\right), & a \leq x < b \\ 1, & x = b \\ \left(\frac{c-x}{c-b}\right), & b < x \leq c \\ 0, & \text{otherwise} \end{cases}$$

where, $a,b,c \in \Re$ and $a < b < c$. The triangular fuzzy number $\tilde{A}$, is shown in Figure 2.1.

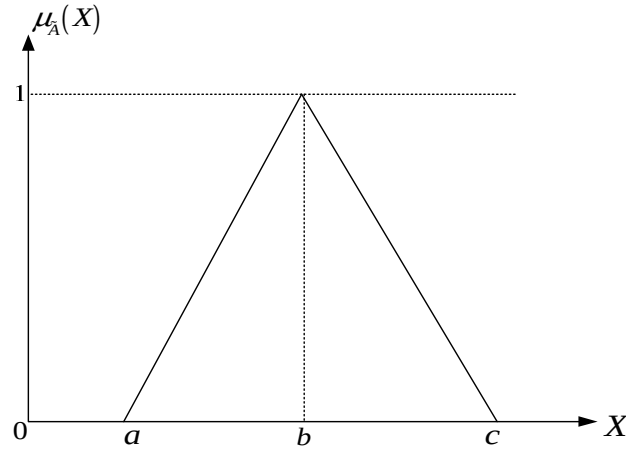


Figure 2.1: Triangular fuzzy number

Definition 2.5 A fuzzy number $\tilde{A}=(a,b,c,d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \leq x < b, \\ 1, & b < x \leq c, \\ (d-x)/(d-c), & c < x \leq d, \\ 0, & \text{otherwise} \end{cases}$$

where, $a, b, c, d, x \in \Re$ and $a < b < c < d$. The trapezoidal fuzzy number $\tilde{A}$, is shown in Figure 2.2

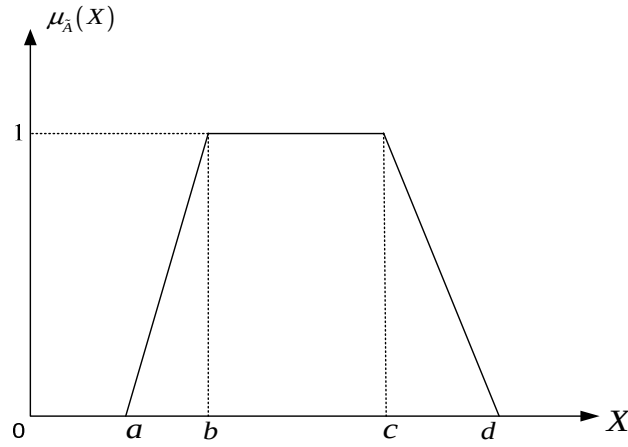


Figure 2.2: Trapezoidal fuzzy number

2.2.2 ARITHMETIC OPERATIONS BETWEEN TWO TRAPEZOIDAL FUZZY NUMBERS

In this section, some arithmetic operations between two trapezoidal fuzzy numbers are presented.

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers.

Then,

- (i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$
- (ii) $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$

$$(iii) \quad \tilde{A}_1 \otimes \tilde{A}_2 = (a, b, c, d)$$

where $a = \text{minimum}(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$, $b = \text{minimum}(b_1c_2, b_2c_1, c_1b_2, c_1c_2)$,

$c = \text{maximum}(b_1c_2, b_2c_1, c_1b_2, c_1c_2)$, $d = \text{maximum}(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$

2.2.3 EXISTING APPROACH

In this section, the existing approach [92], based on fuzzy fault tree, for evaluating the fuzzy reliability of industrial systems is presented [92].

Step 1: Construct a fault-tree model for the chosen industrial system

Step 2: Use Boolean algebra and fault tree model, obtained in Step 1, to find an expression of failure of system in terms of failure of basic events.

Step 3: Use the absorption law of Boolean algebra to remove the repetition of basic events from the expression obtained in Step 2.

Step 4: Use the expression, obtained in Step 3, for finding an expression for evaluating the failure probability of the system.

Step 5: Construct trapezoidal fuzzy number for representing the failure probability of each basic event.

Step 6: Replace the failure probability of each basic event of the expression, obtained from Step 4, from the corresponding trapezoidal fuzzy number constructed in Step 5.

Step 7: Use the arithmetic of trapezoidal fuzzy numbers to obtain a trapezoidal fuzzy number representing the probability of failure of system.

Step 8: Subtract the trapezoidal fuzzy number, obtained from Step 7, from 1 to obtain a trapezoidal fuzzy number representing the fuzzy reliability of the chosen industrial system.

2.3 PROPOSED NON-NORMAL P -NORM TRAPEZOIDAL FUZZY NUMBER BASED FUZZY FAULT TREE APPROACH

In the literature [23], it is pointed out that it is better to use non-normal p -norm trapezoidal fuzzy number instead of trapezoidal fuzzy number to handle the uncertainty in more effective manner. Due to this reason, in this section, a non-normal p -norm trapezoidal fuzzy number based fuzzy fault tree approach is proposed by modifying the existing trapezoidal fuzzy number based fuzzy fault tree approach [92], presented in Section 2.2.

2.3.1 BASIC DEFINITIONS

In this section, some basic definitions are presented [23].

Definition 2.6 A fuzzy number $\tilde{A} = (a, b, c, d; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w \frac{(x-a)}{(b-a)} & , \quad a \leq x < b \\ w, & b \leq x \leq c \\ w \frac{(x-d)}{(c-d)} & , \quad c < x \leq d \end{cases}$$

where, $a, b, c, d, x \in \Re$ and $a < b < c < d$. The generalized trapezoidal fuzzy number $\tilde{A}$, is shown in Figure 2.3.

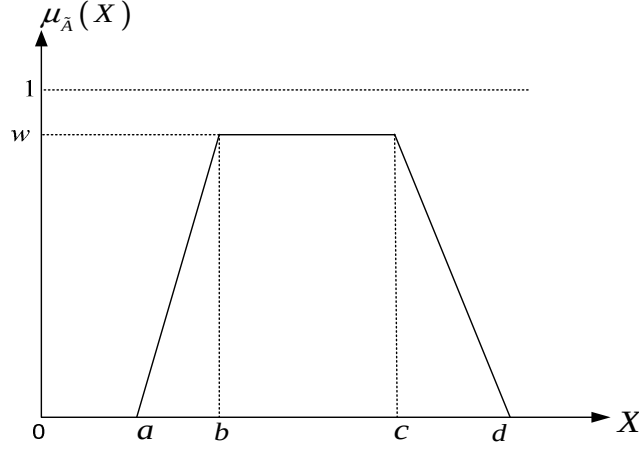


Figure 2.3: Generalized trapezoidal fuzzy number

Definition 2.7 A fuzzy number $\tilde{A} = (a, b, c, d; w)_p$ is said to be a non-normal p -norm trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w \left(1 - \left(\frac{x-b}{a-b} \right)^p \right)^{\frac{1}{p}}, & a \leq x < b \\ w & , b \leq x < c \\ w \left(1 - \left(\frac{x-c}{d-c} \right)^p \right)^{\frac{1}{p}}, & c \leq x < d \end{cases}$$

where, p is a positive integer.

2.3.2 ARITHMETIC OPERATIONS BETWEEN TWO NON-NORMAL p -NORM TRAPEZOIDAL FUZZY NUMBERS

In this section, arithmetic operations between two non-normal p -norm trapezoidal fuzzy numbers are presented.

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)_p$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)_p$ be two non-normal p -norm trapezoidal fuzzy numbers. Then,

$$(i) \quad \tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(w_1, w_2))_p$$

$$(ii) \quad \tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \min(w_1, w_2))_p$$

$$(iii) \quad \tilde{A}_1 \otimes \tilde{A}_2 = (a, b, c, d; \min(w_1, w_2))_p$$

where $a = \text{minimum}(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2)$, $b = \text{minimum}(b_1 c_2, b_2 c_1, c_1 b_2, c_1 c_2)$,

$c = \text{maximum}(b_1 c_2, b_2 c_1, c_1 b_2, c_1 c_2)$, $d = \text{maximum}(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2)$

2.4 PROPOSED APPROACH

In this section, a new approach for evaluating the fuzzy reliability of industrial systems is proposed by modifying the existing approach [92].

Step 1: Use Step 1 to Step 4 of the existing approach, presented in Section 2.2.3 to obtain the expression for evaluating the failure probability of the system.

Step 2: Construct a non-normal p -norm trapezoidal fuzzy number for representing the failure probability of each basic event.

Step 3: Replace the failure probability of each basic event of the expression, obtained from Step 1, from the corresponding non-normal p -norm trapezoidal fuzzy numbers constructed in Step 2.

Step 4: Use the arithmetic of non-normal p -norm trapezoidal fuzzy numbers to obtain a non-normal p -norm trapezoidal fuzzy number representing the probability of failure of system.

Step 5: Subtract the non-normal p -norm trapezoidal fuzzy number, obtained from Step 4, from 1 to obtain a non-normal p -norm trapezoidal fuzzy number, representing the fuzzy reliability of the chosen industrial system.

2.4.1 FUZZY RELIABILITY EVALUATION OF 66/11 kV SUBSTATION

Substations serve as source of energy supply for the local areas of distribution in which these are located. The main function of substations is to receive energy transmitted at high voltage from the generating stations, reduce the voltage to a value appropriate for local distribution and provide facilities for switching. Substations have some additional functions. Substations provide points where safety devices may be installed to disconnect equipment or circuit in the event of fault voltage on the outgoing distribution feeders can be regulated at a substation. A substation is a convenient place for installing synchronous condensers at the end of the transmission line for the purpose of improving power factor and make measurements to check the operation of the various parts of the power system. Street lightning equipment as well as switching controls for street lights can be installed in a substation.

A single line diagram of 66/11 kV substation is shown in Figure 2.4. This system has one power supply source, this source supply power to two systems (1 and 2). Each of them has a transformers T/F 1 and T/F 2; the primary breakers, CB1 and CB2; secondary breakers, CB3 and CB4; and over-current protections. These systems (1 and 2) are further connected

with two feeders (I1 and I2). The two feeder systems are inter-connected by the bus-coupler CB5. This bus-coupler is normally closed and open only to isolate both systems (I1 and I2).

This system has six loads (L1 to L6); L1 to L3 are connected to feeder I1 and L4 to L6 are connected to feeder I2, each of which equipped with a breaker system.

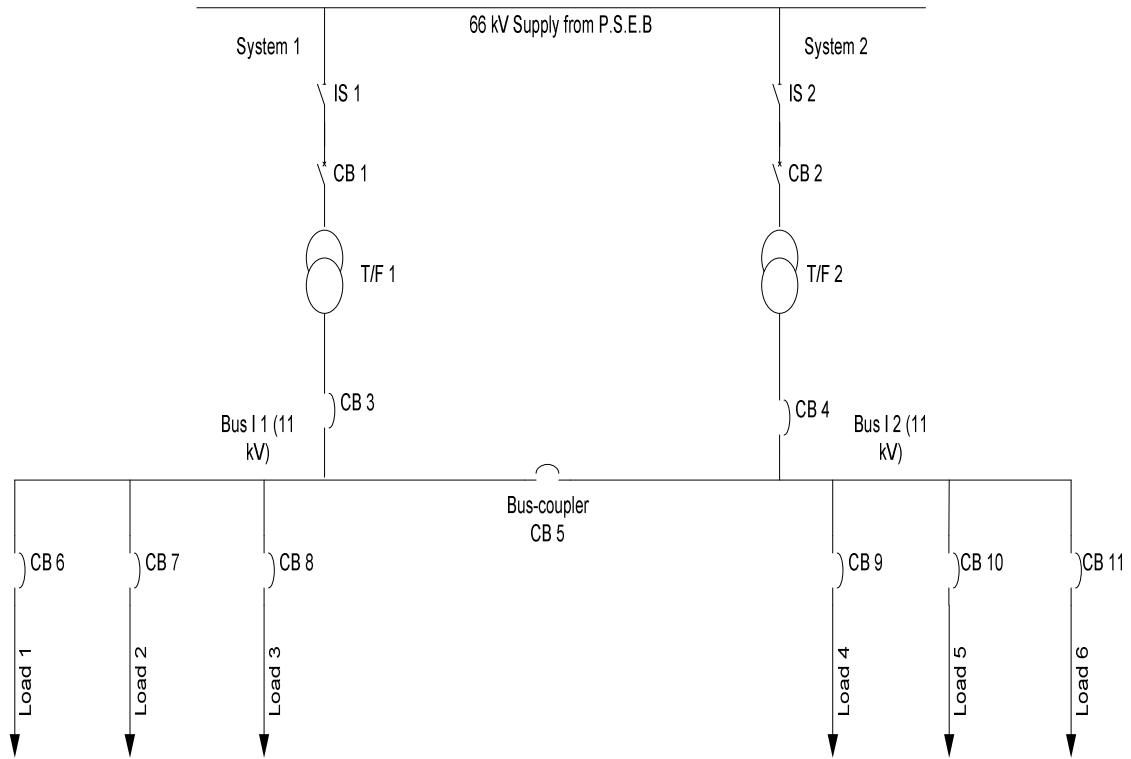


Figure 2.4: A single line diagram of 66/11 kV substation

In this section, to illustrate the proposed approach the fuzzy reliability of the substation, shown in Figure 2.4, is evaluated.

Step 1: On the basis of a deep study of the substation and its components, its fault tree, depicted in Figure 2.5, is constructed. The description of the symbols used in fault tree is listed in Table 2.1 and Table 2.2.

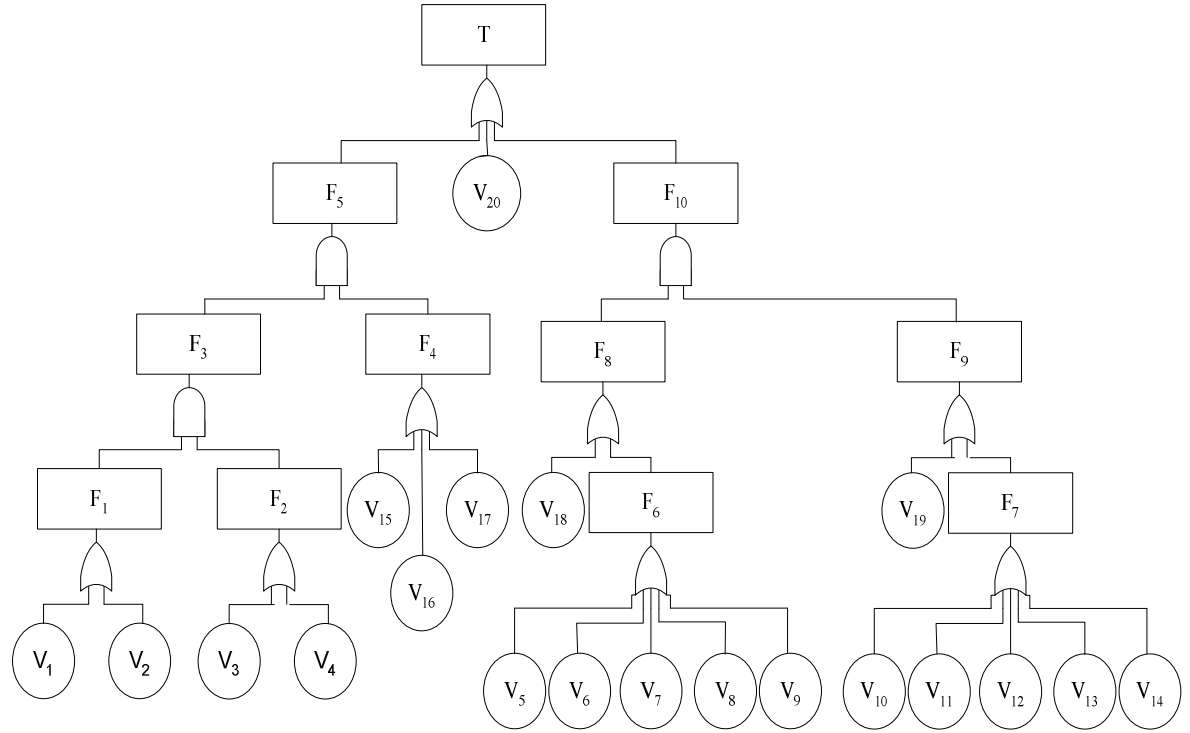


Figure 2.5: Fault tree of a chosen substation

Table 2.1: Intermediate events of chosen substation

Code	Fault
T	All Loads L1 to L6 power loss
F_1	Bus I1 fault
F_2	Bus I2 fault
F_3	Bus I1 and Bus I2 fault
F_4	CB5 fails to open

F_5	Bus I1 and Bus I2 faults is not isolated
F_6	T/F 1 outage
F_7	T/F 2 outage
F_8	T/F1 system unavailable
F_9	T/F2 system unavailable
F_{10}	System 1 and System 2 power loss

Table 2.2: Basic events of chosen substation

Code	Fault	Code	Fault
V_1	Bus I1 short	V_{11}	CB 1 faulty open
V_2	System I1 outage	V_{12}	T/F 1 sec. feeder short
V_3	Bus I2 short	V_{13}	T/F 1 prim. feeder short
V_4	System I2 outage	V_{14}	T/F 1 winding short
V_5	CB 3 faulty open	V_{15}	Relay fails to open CB5
V_6	CB 1 faulty open	V_{16}	Human error mis-calibration
V_7	T/F 1 sec. feeder short	V_{17}	CB5 fails to open
V_8	T/F 1 prim. feeder short	V_{18}	System 1 outage
V_9	T/F 1 winding short	V_{19}	System 1 outage
V_{10}	CB 3 faulty open	V_{20}	Bus-coupler CB5 short

Step 2: Using the Boolean algebra and fault tree, shown in Figure 2.5, the following expression is obtained for failure of substation in terms of failure of basic events.

$$T = \left\{ \begin{aligned} & (V_1 \cap V_{15} \cap V_3) \cup (V_1 \cap V_{15} \cap V_4) \cup (V_1 \cap V_{16} \cap V_3) \cup (V_1 \cap V_{16} \cap V_4) \cup (V_1 \cap V_{17} \cap V_3) \cup \\ & (V_1 \cap V_{17} \cap V_4) \cup (V_{10} \cap V_{18}) \cup (V_{10} \cap V_5) \cup (V_{10} \cap V_6) \cup (V_{10} \cap V_7) \cup (V_{10} \cap V_8) \cup (V_{10} \\ & \cap V_9) \cup (V_{11} \cap V_{18}) \cup (V_{11} \cap V_5) \cup (V_{11} \cap V_6) \cup (V_{11} \cap V_7) \cup (V_{11} \cap V_8) \cup (V_{11} \cap V_9) \cup (V_{12} \\ & \cap V_{18}) \cup (V_{12} \cap V_5) \cup (V_{12} \cap V_6) \cup (V_{12} \cap V_7) \cup (V_{12} \cap V_8) \cup (V_{12} \cap V_9) \cup (V_{13} \cap V_{18}) \cup (\\ & V_{13} \cap V_5) \cup (V_{13} \cap V_6) \cup (V_{13} \cap V_7) \cup (V_{13} \cap V_8) \cup (V_{13} \cap V_9) \cup (V_{14} \cap V_{18}) \cup (V_{14} \cap V_5) \cup (\\ & V_{14} \cap V_6) \cup (V_{14} \cap V_7) \cup (V_{14} \cap V_8) \cup (V_{14} \cap V_9) \cup (V_{15} \cap V_2 \cap V_3) \cup (V_{15} \cap V_2 \cap V_4) \cup (V_{16} \\ & \cap V_2 \cap V_3) \cup (V_{16} \cap V_2 \cap V_4) \cup (V_{17} \cap V_2 \cap V_3) \cup (V_{17} \cap V_2 \cap V_4) \cup (V_{18} \cap V_{19}) \cup (V_{19} \cap \\ & V_5) \cup (V_{19} \cap V_6) \cup (V_{19} \cap V_7) \cup (V_{19} \cap V_8) \cup (V_{19} \cap V_9) \cup V_{20} \end{aligned} \right\}$$

Step 3: Using the expression, obtained in Step 2, the following expression is obtained for evaluating the failure probability of substation:

$$\begin{aligned} T = & (1 - (1 - (V_1 V_{15} V_3)) \times (1 - (V_1 V_{15} V_4)) \times (1 - (V_1 V_{16} V_3)) \times (1 - (V_1 V_{16} V_4)) \times (1 - (V_1 V_{17} V_3)) \\ & \times (1 - (V_1 V_{17} V_4)) \times (1 - (V_{10} V_{18})) \times (1 - (V_{10} V_5)) \times (1 - (V_{10} V_6)) \times (1 - (V_{10} V_7)) \times (1 - (V_{10} V_8)) \times (1 - (V_{10} V_9)) \\ & \times (1 - (V_{11} V_{18})) \times (1 - (V_{11} V_5)) \times (1 - (V_{11} V_6)) \times (1 - (V_{11} V_7)) \times (1 - (V_{11} V_8)) \times (1 - (V_{11} V_9)) \times (1 - (V_{12} V_{18})) \\ & \times (1 - (V_{12} V_5)) \times (1 - (V_{12} V_6)) \times (1 - (V_{12} V_7)) \times (1 - (V_{12} V_8)) \times (1 - (V_{12} V_9)) \times (1 - (V_{13} V_{18})) \times (1 - (V_{13} V_5)) \\ & \times (1 - (V_{13} V_6)) \times (1 - (V_{13} V_7)) \times (1 - (V_{13} V_8)) \times (1 - (V_{13} V_9)) \times (1 - (V_{14} V_{18})) \times (1 - V_{14}) \times (1 - V_5) \\ & \times (1 - (V_{14} V_6)) \times (1 - (V_{14} V_7)) \times (1 - (V_{14} V_8)) \times (1 - (V_{14} V_9)) \times (1 - (V_{15} V_2 V_3)) \times (1 - (V_{15} V_2 V_4)) \times (1 - (V_{16} \\ & V_2 V_3)) \times (1 - (V_{16} V_2 V_4)) \times (1 - (V_{17} V_2 V_3)) \times (1 - (V_{17} V_2 V_4)) \times (1 - (V_{18} V_{19})) \times (1 - (V_{19} V_5)) \times (1 - (V_{19} V_6)) \\ & \times (1 - (V_{19} V_7)) \times (1 - (V_{19} V_8)) \times (1 - (V_{19} V_9)) \times (1 - V_{20})) \end{aligned}$$

Step 4: The non-normal p -norm trapezoidal fuzzy numbers, assumed for representing the failure probabilities of basic events, are shown in Table 2.3.

Table 2.3: Failure probabilities of basic events

Non-normal p -norm trapezoidal fuzzy number	a_i	b_i	c_i	d_i	w_i
$\tilde{V}_1$	0.018	0.019	0.020	0.021	0.9
$\tilde{V}_2$	0.008	0.009	0.010	0.011	0.75

$\tilde{V}_3$	0.028	0.029	0.030	0.031	0.77
$\tilde{V}_4$	0.003	0.004	0.005	0.006	0.89
$\tilde{V}_5$	0.001	0.002	0.003	0.004	0.91
$\tilde{V}_6$	0.043	0.044	0.045	0.046	0.95
$\tilde{V}_7$	0.003	0.004	0.005	0.006	0.85
$\tilde{V}_8$	0.088	0.089	0.090	0.091	0.93
$\tilde{V}_9$	0.048	0.049	0.050	0.051	0.87
$\tilde{V}_{10}$	0.007	0.008	0.009	0.010	0.89
$\tilde{V}_{11}$	0.198	0.199	0.200	0.201	0.9
$\tilde{V}_{12}$	0.125	0.126	0.127	0.128	0.76
$\tilde{V}_{13}$	0.042	0.043	0.044	0.045	0.94
$\tilde{V}_{14}$	0.028	0.029	0.030	0.031	0.77
$\tilde{V}_{15}$	0.068	0.069	0.070	0.071	0.70
$\tilde{V}_{16}$	0.078	0.079	0.080	0.081	0.79
$\tilde{V}_{17}$	0.088	0.089	0.090	0.091	0.84
$\tilde{V}_{18}$	0.025	0.026	0.027	0.028	0.82
$\tilde{V}_{19}$	0.003	0.004	0.005	0.006	0.96
$\tilde{V}_{20}$	0.068	0.069	0.070	0.071	0.76

Step 5: Replacing the failure probability of basic events by the corresponding non-normal p -norm trapezoidal fuzzy numbers, the non-normal p -norm trapezoidal fuzzy number $(0.0803, 0.0836, 0.0868, 0.0902; 0.7)_p$, representing the failure probability of substation, is obtained.

Step 6: Subtracting the non-normal p -norm trapezoidal fuzzy number, obtained from Step 5, the non-normal p -norm trapezoidal fuzzy number $(0.9098, 0.9132, 0.9164, 0.9197; 0.7)_p$, representing the fuzzy reliability of substation is obtained.

2.4.2 COMPARATIVE STUDY

The interval of confidence for crisp reliability of substation obtained by using the existing approach [38] by assuming the crisp probability of failure of i^{th} basic event V_i as $\frac{b_i + c_i}{2}$, the interval of confidence for fuzzy reliability of substation obtained by using the existing approach [5] by assuming the fuzzy probability of failure of i^{th} basic event V_i as $(a_i, b_i, c_i, d_i; 1)$ and the interval of confidence for fuzzy reliability, obtained in Section 2.4.1, are listed in Table 2.4. Also, the graphical representation of these results is shown in Figure 2.6.

Table 2.4: Results obtained by using the existing approaches and proposed approach

α	Existing approach [38]	Existing approach [5]				Proposed Approach			
		a	b	c	d	a	b	c	d
1.0	0.9148	0.9132	0.9132	0.9164	0.9164	--	--	--	--
0.9	0.9148	0.9129	0.9132	0.9164	0.9167	--	--	--	--
0.8	0.9148	0.9125	0.9132	0.9164	0.9171	--	--	--	--

0.7	0.9148	0.9122	0.9132	0.9164	0.9174	0.9132	0.9132	0.9164	0.9164
0.6	0.9148	0.9118	0.9132	0.9164	0.9177	0.9114	0.9132	0.9164	0.9181
0.5	0.9148	0.9115	0.9132	0.9164	0.9181	0.9108	0.9132	0.9164	0.9187
0.4	0.9148	0.9112	0.9132	0.9164	0.9184	0.9104	0.9132	0.9164	0.9191
0.3	0.9148	0.9108	0.9132	0.9164	0.9187	0.9101	0.9132	0.9164	0.9194
0.2	0.9148	0.9105	0.9132	0.9164	0.9190	0.9099	0.9132	0.9164	0.9196
0.1	0.9148	0.9101	0.9132	0.9164	0.9194	0.9098	0.9132	0.9164	0.9197
0	0.9148	0.9098	0.9132	0.9164	0.9197	0.9098	0.9132	0.9164	0.9197

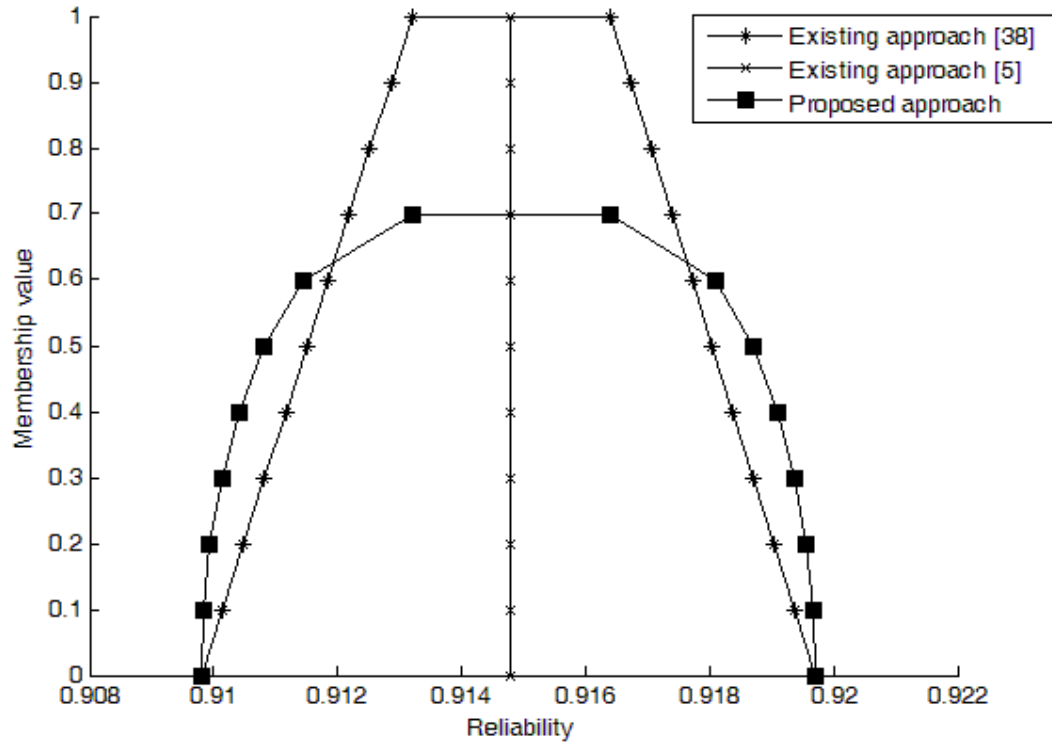


Figure 2.6: Reliability of substation obtained by using different approaches

- (i) Using the existing approach [38], the interval of confidence for reliability of the substation corresponding to all values of α is 0.9148, i.e. degree of acceptance is 1 for the statement that the reliability of substation will be 0.9148 and degree of rejection about the same statement is 0. Since, in real life problems there also exist rejection with acceptance. So, the results obtained by using existing approach [38] are not appropriate according to real life situations.
- (ii) On the basis of the results, obtained by using existing approach [5], shown in Table 2.4, it can be concluded that the reliability of the substation will lie between 0.9098 to 0.9197. The degree of acceptance and degree of rejection about the statement that the reliability of substation will lie between 0.9132 to 0.9164 are 1 and 0 respectively. Since, in real life situations it rarely happens that maximum degree of acceptance is 1. So, the results obtained by using existing approach [5] are not appropriate according to real life situation.
- (iii) On the basis of the results, obtained by using proposed approach, shown in Table 2.4, it can be concluded that the reliability of substation will lie between 0.9098 to 0.9197. The degree of acceptance and degree of rejection about the statement that the reliability of substation will lie between 0.9132 to 0.9164 are 0.7 and 0.3 respectively. Since, the maximum degree of acceptance. So, the results obtained by using proposed approach are appropriate according to real life situations.

2.5 CONCLUSIONS

On the basis of the presented study, it can be concluded that the results obtained by using the existing approaches [5,38] are not appropriate according to real life situations. While the results obtained by using the proposed approach are appropriate according to real life situations. Therefore, it is better to use the proposed approach as compared to existing approaches [5,38] for evaluating the fuzzy reliability of industrial systems.

CHAPTER 3

A PETRI NET BASED APPROACH WITH TRAPEZOIDAL VAGUE SETS FOR EVALUATING THE VAGUE RELIABILITY PARAMETERS OF INDUSTRIAL SYSTEMS[†]

Liu and Chiou [68] discussed the advantages of Petri net over fault tree and proposed an approach, based on Petri net, for evaluating some crisp reliability parameters. Knezevic and Odoom [56] discussed the advantages of fuzzy sets over crisp sets and proposed an approach, based on Petri net, by modifying the existing approach [68] for evaluating some fuzzy reliability parameters. On the same direction, in this Chapter, the advantages of vague sets over fuzzy sets are discussed and a new approach, based on Petri net, by modifying the existing approach [56] is proposed for evaluating some vague reliability parameters. Also, the proposed approach is used to evaluate the vague reliability parameters of a compressor system.

3.1 A BRIEF REVIEW OF PETRI NET

The fault tree analysis approach assumes that the causes of events are random and statistically independent but certain common causes can lead to correlations in event probabilities which violate the independence assumptions and could exaggerate the likelihood of an event fault. State space based approaches like Markov model overcome this drawback,

[†] The contents of this Chapter are published in *Concurrent Engineering: Research and Applications*, DOI: 10.1177/1063293X11435178, 2012.

but it requires the solution to be computed over a set of equations that increases exponentially with the number of components. To overcome above said limitations, Petri net [68] can be used instead of fault tree analysis and Markov model since it allows us to describe large state spaces by a restricted number of model primitives (places, transitions and tokens). Unlike the fault tree, the Petri net allows efficient simultaneous generation of minimal cut (smallest set of basic events, which if they all occur will result in the top event occurring) and minimal path sets (smallest number of events, which if they do not occur, will result in the non-occurrence of the top event).

The basic symbols of Petri nets include:

○ : *Place*, drawn as a circle, denotes event.

— : *Immediate transition*, drawn as a thin bar, denotes event transfer with no delay time.

— : *Timed transition*, drawn as a thick bar, denotes event transfer with a period of delay time.

↗ : *Arc*, drawn as an arrow, between places and transitions.

● : *Token*, drawn as a dot, contained in places, denotes the data.

⊖ : *Inhibitor arc*, drawn as a line with a circle end, between places and transitions.

The transition is said to *fire* if input places satisfy an enabled condition. Transitions firing will remove one token from each of its input places and put one token into all of its output places. In Petri net there are two types of input places for the transitions; namely, specified type and conditional type. The former one has single output arc whereas the latter has multiples. Tokens in the specified type places have only one outgoing destination, i.e. if

the input place(s) holds a token then the transition fires and gives the output place(s) a token. However, tokens in the conditional type place have more than one outgoing paths that may lead the different situations. Owing to the variety of logical relations that can be represented with Petri net, it is a powerful tool for modelling system. It can be used not only for simulation, reliability analysis and failure monitoring, but also for dynamic behaviour observation. The AND/OR logic gates operations of Petri net model corresponding to the fault tree AND/OR logic gates is shown in Figure 3.1 and the Petri net transformation of the fault tree model, shown in Figure 3.2, is shown in Figure 3.3.

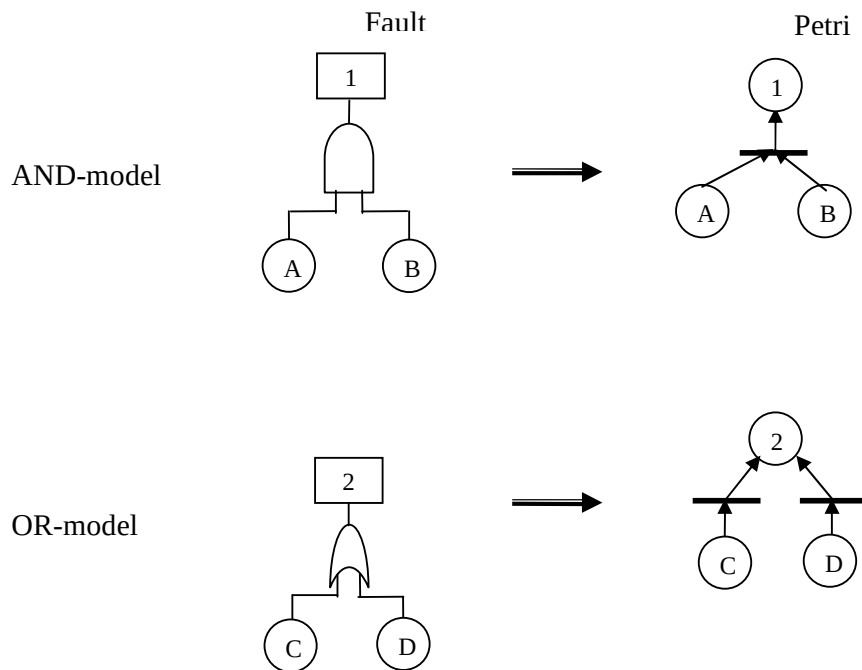


Figure 3.1: Basic fault tree logic gates along their representation in Petri net model

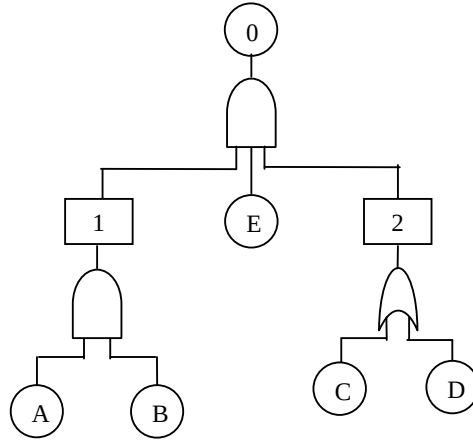


Figure 3.2: The fault tree model

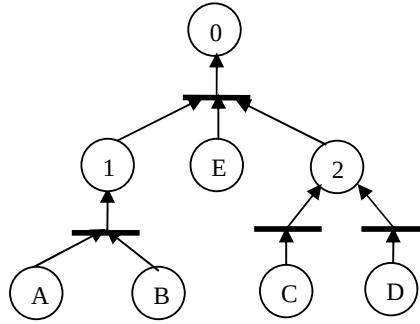


Figure 3.3: The Petri net transformation of Figure 3.2

3.2 EXISTING APPROACH

Liu and Chiou [68] proposed the following approach for evaluating the crisp reliability parameters shown in Table 3.1.

Table 3.1: Reliability parameters for repairable system

Parameters	Expression
Mean time to failure	$MTTF_s = \frac{1}{\lambda_s}$

Mean time to repair	$MTTR_s = \frac{1}{\mu_s} = \tau_s$
Mean time between failure	$MTBF_s = MTTF_s + MTTR_s$
Availability	$A_s(t) = \frac{\mu_s}{\lambda_s + \mu_s} + \frac{\lambda_s}{\lambda_s + \mu_s} e^{-(\lambda_s + \mu_s)t}$
Reliability	$R_s(t) = e^{-\lambda_s t}$
Expected number of failures	$W_s(0,t) = \frac{\lambda_s \mu_s}{\lambda_s + \mu_s} t + \frac{\lambda_s^2}{(\lambda_s + \mu_s)^2} [1 - e^{-(\lambda_s + \mu_s)t}]$

Step 1: Construct a Petri net model for the chosen industrial system.

Step 2: Use the Petri net model and expressions, presented in Table 3.2, for obtaining the expression for evaluating failure rate (λ_s) and repair rate (τ_s) of the chosen system.

Table 3.2: Expressions for evaluating failure rate (λ_s) and repair rate (τ_s) of the system

Gate	λ_{AND}	τ_{AND}	λ_{OR}	τ_{OR}
Conventional expression (n -Inputs)	$\prod_j \lambda_j \left[\sum_{i=1}^n \prod_{i \neq j} \tau_i \right]$	$\frac{\prod_{j=1}^n \tau_j}{\sum_{j=1}^n \left[\prod_{i=1, i \neq j}^n \tau_i \right]}$	$\sum_{j=1}^n \lambda_j$	$\frac{\sum_{j=1}^n \lambda_j \tau_j}{\sum_{j=1}^n \lambda_j}$

Step 3: Put the failure rates (λ_j) and repair rate (τ_j) of basic events in the expression, obtained in Step 2, for evaluating the failure rate (λ_s) and repair rate (τ_s) of the system.

Step 4: Find the crisp reliability parameters by putting the failure rate (λ_s) and repair rate (τ_s) , obtained from Step 4, in the expression shown in Table 3.1.

3.3 VAGUE SET THEORY

In conventional reliability analysis the failure probabilities of the components of a system are treated as exact values. It is often difficult to obtain data for failure probabilities under changing environmental conditions. Hence, fuzzy sets [101] are used to analyze the fuzzy system reliability.

In real life, a person may assume that an object belongs to a set to certain degree, but it is possible that he is not sure about it. In other words, there may be hesitation or uncertainty about the membership degree of an object belonging to a set. In fuzzy set theory, there is no means to incorporate the hesitation in the membership degree. A possible solution of such problems is to use vague sets [42].

Definition 3.1 A vague set $\tilde{V} = \{x, \mu_{\tilde{V}}(x), 1 - \nu_{\tilde{V}}(x) \mid x \in X\}$ on the universal set X is characterized by a truth membership function $\mu_{\tilde{V}}, \mu_{\tilde{V}} : X \rightarrow [0,1]$ and a false membership function $\nu_{\tilde{V}}, \nu_{\tilde{V}} : X \rightarrow [0,1]$. The values $\mu_{\tilde{V}}(x)$ and $\nu_{\tilde{V}}(x)$ represents the degree of truth membership and degree of false membership of x and always satisfies the condition $\mu_{\tilde{V}}(x) + \nu_{\tilde{V}}(x) \leq 1 \quad \forall \quad x \in X$. The value $1 - \mu_{\tilde{V}}(x) - \nu_{\tilde{V}}(x)$ represents the degree of hesitation of $x \in X$.

For example, let the membership values of a vague set $\tilde{V}$ on the universal set X corresponding to $x \in X$ are $\mu_{\tilde{V}}(x) = 0.5$, $1 - \nu_{\tilde{V}}(x) = 0.7$ and $\nu_{\tilde{V}}(x) = 0.3$. Then, x belong to vague set $\tilde{V}$ with accept evidence is 0.5 and decline evidence is 0.3. If x is the vote result

from ten people, it implies that five people vote in favour, three persons vote against, and two persons is abstention. The hesitation of x can be described by differential value of $1 - \nu_{\tilde{V}}(x) - \mu_{\tilde{V}}(x)$. If the differential value is small, then the value of x is more certain and if the differential value is high, then the value of x is more uncertain. The crisp sets and fuzzy sets are the special cases of vague sets. If $1 - \nu_{\tilde{V}}(x) = \mu_{\tilde{V}}(x)$, then vague set $\tilde{V}$ regresses to a fuzzy set and if, $1 - \nu_{\tilde{V}}(x) = \mu_{\tilde{V}}(x) = 1$ or $1 - \nu_{\tilde{V}}(x) = \mu_{\tilde{V}}(x) = 0$, then vague set $\tilde{V}$ regresses to a crisp set. Therefore, vague sets can be used to describe real life in more intelligent manner than crisp sets and fuzzy sets.

Definition 3.2 A vague set $\tilde{V} = \langle [(a, b, c); \mu, \nu] \rangle$ is said to be a triangular vague set if

$$\mu_{\tilde{V}}(x) = \begin{cases} \mu \left(\frac{x-a}{b-a} \right), & a \leq x < b \\ \mu, & x = b \\ \mu \left(\frac{c-x}{c-b} \right), & b < x \leq c \\ 0, & \text{otherwise} \end{cases}$$

and,

$$1 - \nu_{\tilde{V}}(x) = \begin{cases} \nu \left(\frac{x-a}{b-a} \right), & a \leq x < b \\ \nu, & x = b \\ \nu \left(\frac{c-x}{c-b} \right), & b < x \leq c \\ 0, & \text{otherwise} \end{cases}$$

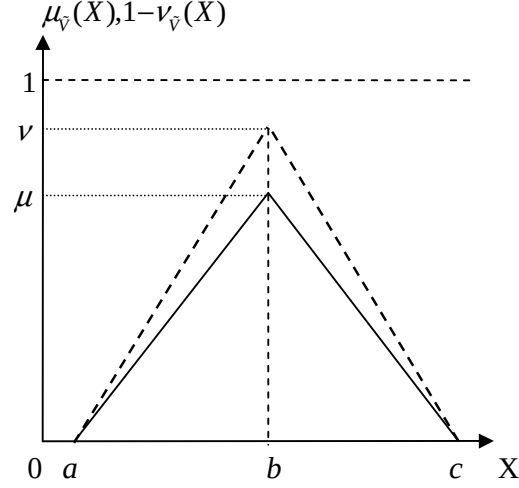


Figure 3.4: Triangular vague set

3.4 PROPOSED APPROACH

In this section, a new approach, by modifying the existing approach, presented in Section 3.2, is proposed for evaluating the interval of confidence corresponding to truth membership function and complement of false membership function of vague reliability parameters.

The steps of proposed approach are as follows:

Step 1: Use Step 1 to Step 3 of the existing approach, presented in Section 3.2, to find the expressions for evaluating failure rate (λ_s) and repair rate (τ_s) of the chosen system.

Step 2: Construct the triangular vague sets $\tilde{\lambda}_j = \langle [(\lambda_{j1}, \lambda_{j2}, \lambda_{j3}); \mu_j, \nu_j] \rangle$ and $\tilde{\tau}_j = \langle [(\tau_{j1}, \tau_{j2}, \tau_{j3}); \mu'_j, \nu'_j] \rangle$ to represent the uncertain failure rate (λ_j) and uncertain repair rate (τ_j) of the j^{th} basic event.

Step 3: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1,

$$\text{by } \left\{ \left(\lambda_{j2} - \lambda_{j1} \right) \frac{\alpha}{\mu} + \lambda_{j1} \right\}, \quad \mu = \text{minimum}(\mu_j), \quad \alpha \in [0, \mu] \quad \text{and} \quad \left\{ \left(\tau_{j2} - \tau_{j1} \right) \frac{\alpha}{\mu'} + \tau_{j1} \right\},$$

$\mu' = \text{minimum}(\mu'_j), \quad \alpha \in [0, \mu']$ respectively and replace λ_j and τ_j , presented in the

denominator of the expression, obtained in Step 1, by $\left\{ - \left(\lambda_{j3} - \lambda_{j2} \right) \frac{\alpha}{\mu} + \lambda_{j3} \right\},$

$$\mu = \text{minimum}(\mu_j), \quad \alpha \in [0, \mu] \quad \text{and} \quad \left\{ - \left(\tau_{j3} - \tau_{j2} \right) \frac{\alpha}{\mu'} + \tau_{j3} \right\}, \quad \mu' = \text{minimum}(\mu'_j), \quad \alpha \in [0, \mu']$$

respectively, in the expression, obtained in Step 1, for evaluating the value of left end point of

interval of confidence corresponding to truth membership function of vague failure rate $(\tilde{\lambda}_s)$

and vague repair rate $(\tilde{\tau}_s)$ of the chosen system.

Step 4: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1,

$$\text{by } \left\{ - \left(\lambda_{j3} - \lambda_{j2} \right) \frac{\alpha}{\mu} + \lambda_{j3} \right\}, \quad \mu = \text{minimum}(\mu_j), \quad \alpha \in [0, \mu] \quad \text{and} \quad \left\{ - \left(\tau_{j3} - \tau_{j2} \right) \frac{\alpha}{\mu'} + \tau_{j3} \right\},$$

$\mu' = \text{minimum}(\mu'_j), \quad \alpha \in [0, \mu']$ respectively and replace λ_j and τ_j , presented in the

denominator of the expression, obtained in Step 1, by $\left\{ \left(\lambda_{j2} - \lambda_{j1} \right) \frac{\alpha}{\mu} + \lambda_{j1} \right\},$

$$\mu = \text{minimum}(\mu_j), \quad \alpha \in [0, \mu] \quad \text{and} \quad \left\{ \left(\tau_{j2} - \tau_{j1} \right) \frac{\alpha}{\mu'} + \tau_{j1} \right\}, \quad \mu' = \text{minimum}(\mu'_j), \quad \alpha \in [0, \mu']$$

respectively, in the expression, obtained in Step 1, for evaluating the value of right end point

of interval of confidence corresponding to truth membership function of vague failure rate

$(\tilde{\lambda}_s)$ and vague repair rate $(\tilde{\tau}_s)$ of the chosen system.

Step 5: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1,

$$\text{by } \left\{ (\lambda_{j2} - \lambda_{j1}) \frac{\alpha}{\nu} + \lambda_{j1} \right\}, \quad \nu = \text{minimum}(\nu_j), \quad \alpha \in [0, \nu], \quad \text{and} \quad \left\{ (\tau_{j2} - \tau_{j1}) \frac{\alpha}{\nu'} + \tau_{j1} \right\},$$

$\nu' = \text{minimum}(\nu'_j), \alpha \in [0, \nu']$ respectively and replace λ_j and τ_j , presented in the

denominator of the expression, obtained in Step 1, by $\left\{ -(\lambda_{j3} - \lambda_{j2}) \frac{\alpha}{\nu} + \lambda_{j3} \right\},$

$$\nu = \text{minimum}(\nu_j), \quad \alpha \in [0, \nu] \quad \text{and} \quad \left\{ -(\tau_{j3} - \tau_{j2}) \frac{\alpha}{\nu'} + \tau_{j3} \right\}, \quad \nu' = \text{minimum}(\nu'_j), \quad \alpha \in [0, \nu']$$

respectively, in the expression, obtained in Step 1, for evaluating the value of left end point of interval of confidence to corresponding to complement of false membership function of vague failure rate $(\tilde{\lambda}_s)$ and vague repair rate $(\tilde{\tau}_s)$ of the chosen system.

Step 6: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1,

$$\text{by } \left\{ -(\lambda_{j3} - \lambda_{j2}) \frac{\alpha}{\nu} + \lambda_{j3} \right\}, \quad \nu = \text{minimum}(\nu_j), \quad \alpha \in [0, \nu] \quad \text{and} \quad \left\{ -(\tau_{j3} - \tau_{j2}) \frac{\alpha}{\nu'} + \tau_{j3} \right\},$$

$\nu' = \text{minimum}(\nu'_j), \alpha \in [0, \nu']$ respectively and replace λ_j and τ_j , presented in the

denominator of the expression, obtained in Step 1, by $\left\{ (\lambda_{j2} - \lambda_{j1}) \frac{\alpha}{\nu} + \lambda_{j1} \right\},$

$$\nu = \text{minimum}(\nu_j), \quad \alpha \in [0, \nu] \quad \text{and} \quad \left\{ (\tau_{j2} - \tau_{j1}) \frac{\alpha}{\nu'} + \tau_{j1} \right\}, \quad \nu' = \text{minimum}(\nu'_j), \quad \alpha \in [0, \nu']$$

respectively, in the expression, obtained in Step 1, for evaluating the value of right end point of interval of confidence to corresponding to complement of false membership function of vague failure rate $(\tilde{\lambda}_s)$ and vague repair rate $(\tilde{\tau}_s)$ of the chosen system.

Step 7: Replace λ_s and τ_s presented in numerator of the expressions, presented in Table 3.1

by the left end point of interval of confidence corresponding to truth membership function,

obtained from Step 3 to Step 6 and replace λ_s and τ_s , presented in denominator of the expressions, presented in Table 3.1 by the right end point of interval of confidence corresponding to truth membership function, obtained from Step 3 to Step 6, to evaluate the left end point of interval of confidence corresponding truth membership function of vague reliability parameters.

Step 8: Replace λ_s and τ_s presented in numerator of the expressions, presented in Table 3.1 by the right end point of interval of confidence corresponding to truth membership function, obtained from Step 3 to Step 6 and replace λ_s and τ_s , presented in denominator of the expressions, presented in Table 3.1 by the left end point of interval of confidence corresponding to truth membership function, obtained from Step 3 to Step 6, to evaluate the right end point of interval of confidence corresponding truth membership function of vague reliability parameters.

Step 9: Replace λ_s and τ_s presented in numerator of the expressions, presented in Table 3.1 by the left end point of interval of confidence corresponding to complement of false membership function, obtained from Step 3 to Step 6 and replace λ_s and τ_s , presented in denominator of the expressions, presented in Table 3.1 by the right end point of interval of confidence corresponding to complement of false membership function, obtained from Step 3 to Step 6, to evaluate the left end point of interval of confidence corresponding to complement of false membership function of vague reliability parameters.

Step 10: Replace λ_s and τ_s presented in numerator of the expressions, presented in Table 3.1 by the right end point of interval of confidence corresponding to complement of false membership function, obtained from Step 3 to Step 6 and replace λ_s and τ_s , presented in denominator of the expressions, presented in Table 3.1 by the left end point of interval of

confidence corresponding to complement of false membership function, obtained from Step 3 to Step 6, to evaluate the right end point of interval of confidence corresponding to complement of false membership function of vague reliability parameters.

3.5 EVALUATION OF VAGUE RELIABILITY PARAMETERS OF A COMPRESSOR SYSTEM

The compressor system is generally used to compress the atmospheric air/fluid to a higher pressure by reducing its volume. The working principle of a compressor can vary according to the application for which it can be used. For example, bottle filling plants, hospitals, internal combustion engines, pressurized aircraft and pipeline transport application, etc. The compressor system is main part of the gas power plant and its components reliability affects the overall system reliability.

In this section, to show the application of proposed approach, a Petri net model of a compressor system is constructed with the help of fault tree model of the same system and with the help of proposed approach, the interval of confidence corresponding to truth membership functions as well as complement of false membership function of vague reliability parameters of compressor system are evaluated.

Step 1: On the basis of a deep study of the compressor system and its components, its fault tree, depicted in Figure 3.5, is constructed. Also, using the constructed fault tree the Petri net model, depicted in Figure 3.6, is obtained. The description of the symbols used in fault tree as well as in the Petri net model is listed in Table 3.3 and Table 3.4.

.

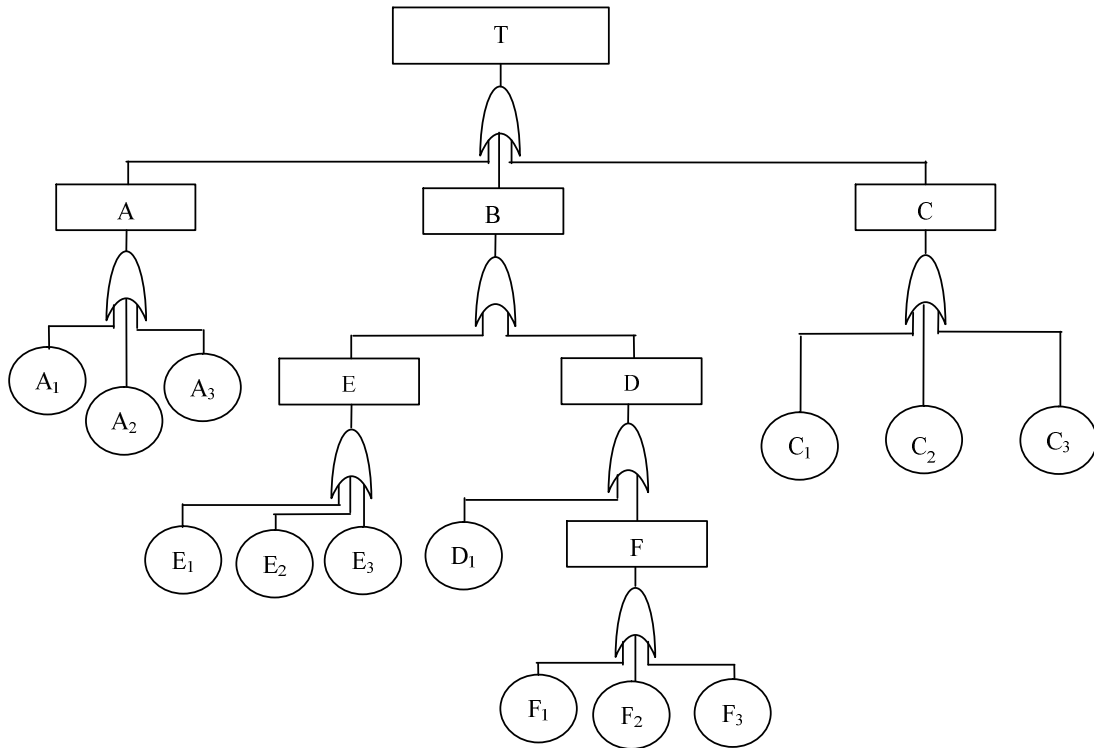


Figure 3.5: Fault tree of compressor system

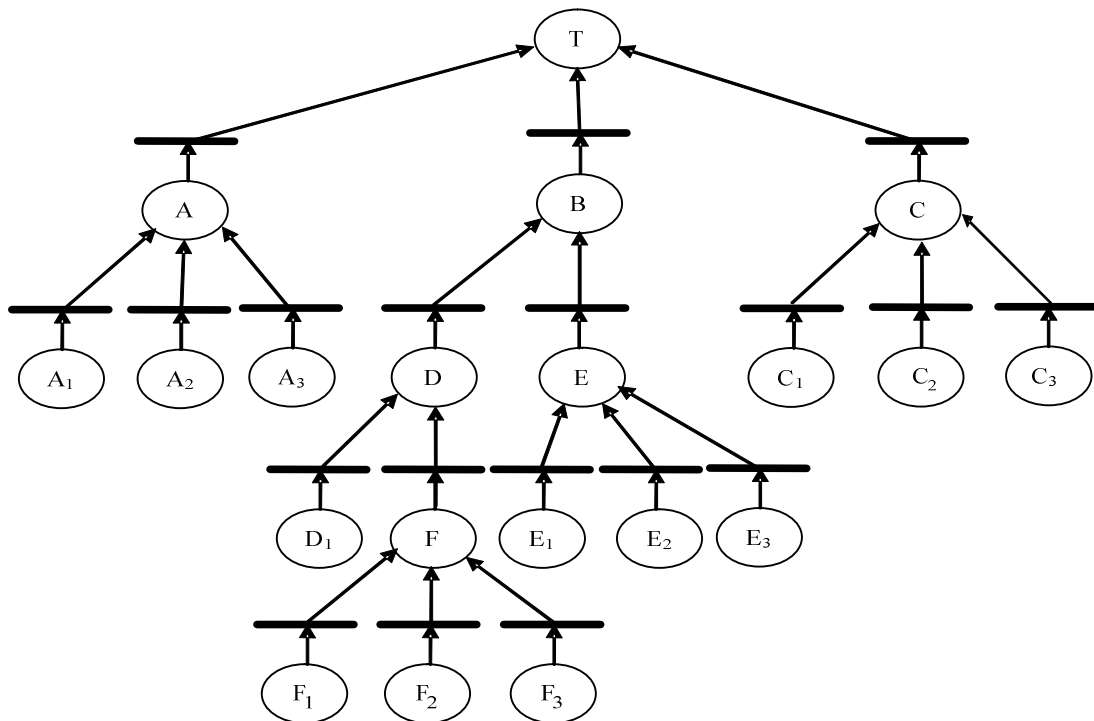


Figure 3.6: Petri net model of compressor system

Table 3.3: Intermediate events of compressor system

Code	Fault
T	Failure of total compressor system
A	Failure of blade system
B	Failure of journal bearing
C	Failure of trust bearing
D	Failure of lubrication system
E	Failure of casing system
F	Less lube oil supply from reservoir

Table 3.4: Basic events of compressor system

Code	Fault	Code	Fault
A ₁	Rings of the blade system got broken	E ₁	Stationary blades of the casing system got broken
A ₂	Inlet guide vane of the blades System got broken	E ₂	Casing of the casing system damaged
A ₃	Shaft of the blade system starts vibrate	E ₃	Pad of the casing system got ruptured
C ₁	Housing of the trust bearing got broken	F ₁	Pressure gauge got corrupted
C ₂	Shoes of the trust bearing got broken	F ₂	Failure of heating system
C ₃	Filler ring of the trust bearing got broken	F ₃	Failure of temperature sensor (RTD)

D ₁	Failure of cooling system of the lubrication system
----------------	--

Step 2: Using the Petri net model, shown in Figure 3.6, the following expression are obtained for evaluating the failure rate (λ_s) and repair rate (τ_s) of compressor system.

$$\lambda_s = \sum \lambda_j \text{ and } \tau_s = \frac{\sum \lambda_j \tau_j}{\sum \lambda_j}$$

where, λ_j and τ_j are failure rates and repair rates of basic events of compressor system.

Step 3: To handle the uncertainties and imprecision in the reliability data (assuming mission time $t=1$ year), the crisp failure rate (λ_j) and crisp repair rate (τ_j) of basic events, presented in the Table 3.5, are converted into triangular vague sets

$$\tilde{\lambda}_j = \left\langle \left[(\lambda_{j1}, \lambda_{j2}, \lambda_{j3}); \mu_j, \nu_j \right] \right\rangle \quad \text{and} \quad \tilde{\tau}_j = \left\langle \left[(\tau_{j1}, \tau_{j2}, \tau_{j3}); \mu'_j, \nu'_j \right] \right\rangle \quad \text{where,}$$

$$\lambda_{j1} = \lambda_j - 0.15 \times \lambda_j, \lambda_{j3} = \lambda_j + 0.15 \times \lambda_j, \mu_j = \mu'_j = 0.6, \nu_j = \nu'_j = 0.8.$$

Table 3.5: Crisp failures rates and repair time

$\lambda_{A_1} = 0.003, \lambda_{A_2} = 0.005, \lambda_{A_3} = 0.0013, \lambda_{C_1} = \lambda_{A_1}, \lambda_{C_2} = 0.0051, \lambda_{C_3} = 0.0015, \lambda_{D_1} = \lambda_{A_1}, \lambda_{E_1} = \lambda_{A_1}, \lambda_{E_2}$
$= \lambda_{A_1}, \lambda_{E_3} = \lambda_{C_3}, \lambda_{F_1} = 0.002, \lambda_{F_2} = 0.007, \lambda_{F_3} = \lambda_{A_1}.$
$\tau_{A_1} = 10, \tau_{A_2} = 10, \tau_{A_3} = 2, \tau_{C_1} = \tau_{A_1}, \tau_{C_2} = 3, \tau_{C_3} = 4, \tau_{D_1} = 10, \tau_{E_1} = \tau_{A_1}, \tau_{E_2} = \tau_{A_1}, \tau_{E_3} = 3, \tau_{F_1} = 4, \tau_{F_2}$
$= 10, \tau_{F_3} = \tau_{A_1}.$

Table 3.6: Left end point and right end point of interval of confidence corresponding to truth membership function of vague reliability parameters

α	Left end point						Right end point					
	λ_s	τ_s	R_s	A_s	ENOF	MTBF	λ_s	τ_s	R_s	A_s	ENOF	MTBF
0	0.0081	4.0499	0.9892	0.9897	0.0080	95.7762	0.0109	10.0295	0.9920	0.9929	0.0108	134.1298
0.1	0.0083	4.3870	0.9894	0.9899	0.0083	98.1516	0.0107	9.3240	0.9917	0.9926	0.0106	129.8785
0.2	0.0085	4.7467	0.9896	0.9902	0.0085	100.6424	0.0104	8.6666	0.9915	0.9923	0.0104	125.8724
0.3	0.0088	5.1307	0.9899	0.9905	0.0087	103.2565	0.0102	8.0534	0.9913	0.9921	0.0101	122.0915
0.4	0.0090	5.5407	0.9901	0.9907	0.0090	106.0028	0.0100	7.4810	0.9910	0.9918	0.0099	118.5181
0.5	0.0092	5.9785	0.9903	0.9910	0.0092	108.8909	0.0097	6.9462	0.9908	0.9915	0.0097	115.1362
0.6	0.0095	6.4462	0.9906	0.9913	0.0094	111.9314	0.0095	6.4462	0.9906	0.9913	0.0094	111.9314
0.7	--	--	--	--	--	--	--	--	--	--	--	--
0.8	--	--	--	--	--	--	--	--	--	--	--	--
0.9	--	--	--	--	--	--	--	--	--	--	--	--
1.0	--	--	--	--	--	--	--	--	--	--	--	--

Table 3.7: Left end point and right end point of interval of confidence corresponding to complement of false membership function of vague reliability parameters

α	Left end point						Right end point					
	λ_s	τ_s	R_s	A_s	ENOF	MTBF	λ_s	τ_s	R_s	A_s	ENOF	MTBF
0	0.0081	4.0499	0.9892	0.9897	0.0080	95.7762	0.0109	10.0295	0.9920	0.9929	0.0108	134.1298
0.1	0.0082	4.3007	0.9893	0.9899	0.0082	97.5473	0.0107	9.4957	0.9918	0.9927	0.0107	130.9175
0.2	0.0084	4.5639	0.9895	0.9901	0.0084	99.3821	0.0105	8.9895	0.9916	0.9925	0.0105	127.8461
0.3	0.0086	4.8404	0.9897	0.9903	0.0086	101.2840	0.0104	8.5093	0.9914	0.9923	0.0103	124.9068
0.4	0.0088	5.1307	0.9899	0.9905	0.0087	103.2565	0.0102	8.0534	0.9913	0.9921	0.0101	122.0915
0.5	0.0089	5.4356	0.9900	0.9907	0.0089	105.3033	0.0100	7.6204	0.9911	0.9919	0.0100	119.3929
0.6	0.0091	5.7560	0.9902	0.9909	0.0091	107.4285	0.0098	7.2091	0.9909	0.9917	0.0098	116.8041
0.7	0.0093	6.0925	0.9904	0.9911	0.0093	109.6363	0.0097	6.8180	0.9907	0.9915	0.0096	114.3189
0.8	0.0095	6.4462	0.9906	0.9913	0.0094	111.9314	0.0095	6.4462	0.9906	0.9913	0.0094	111.9314
0.9	--	--	--	--	--	--	--	--	--	--	--	--
1.0	--	--	--	--	--	--	--	--	--	--	--	--

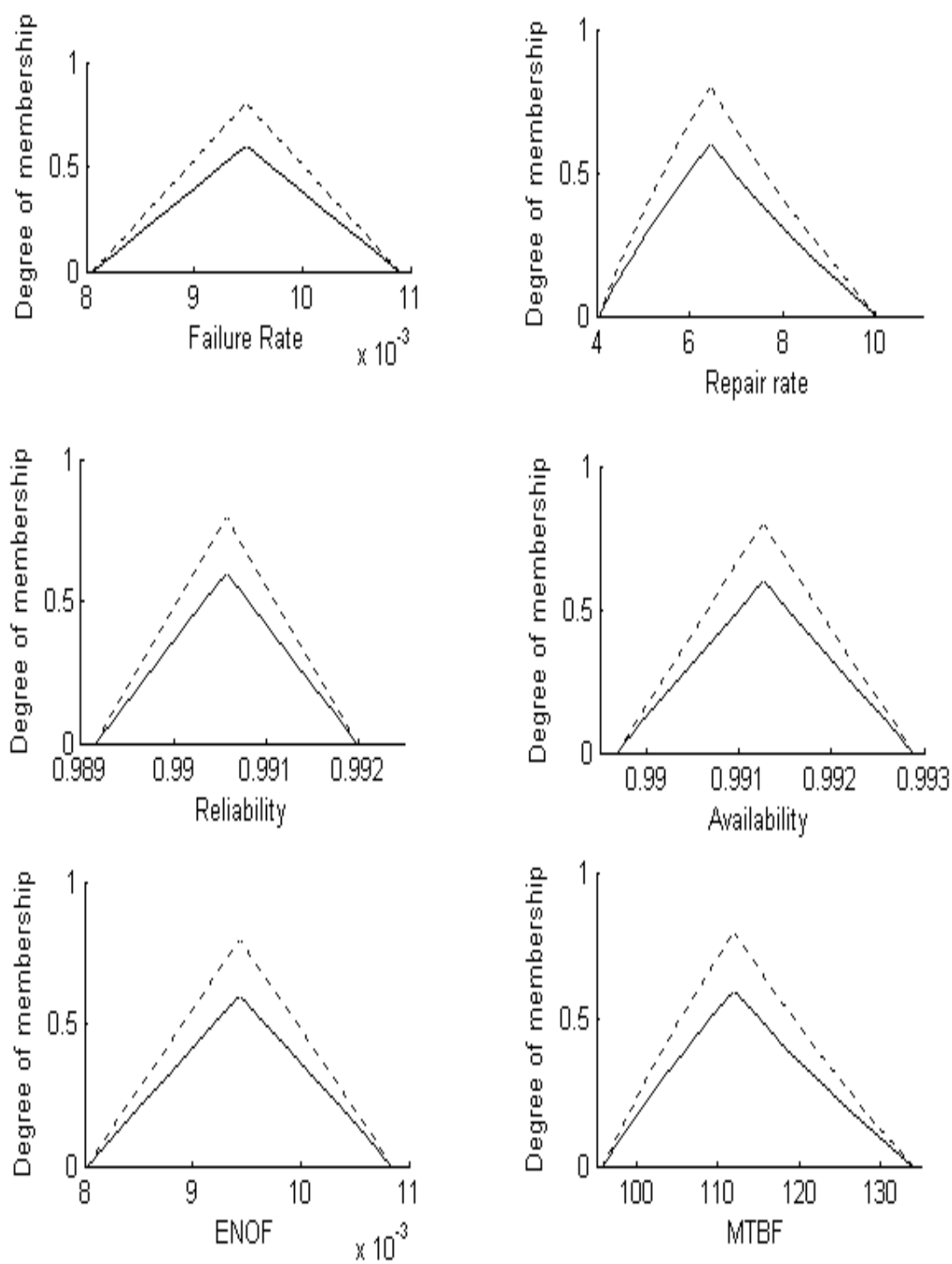


Figure 3.7: Vague reliability parameters of compressor system

Step 4: The left end point and the right end point of the interval of confidence corresponding to truth membership function as well as complement of false membership function of vague failure rate, vague repair rate as well as other vague reliability parameters of the compressor system obtained by using the data obtained from Step 3 as well as Step 3 to Step 10 of the proposed approach are listed in Table 3.6 and Table 3.7 respectively.

Step 5: The vague sets, representing vague reliability parameters obtained by using the left end point and right end point of interval of confidence, shown in Table 3.6 and Table 3.7 respectively, are depicted in Figure 3.7.

3.5.1 COMPARATIVE STUDY

The interval of confidence for fuzzy reliability of compressor system obtained by using the existing approach [56] with triangular fuzzy number (constructed with 15% left and right spread in the data of Table 3.5) and the interval of confidence for vague reliability, obtained in Section 3.5.2, are listed in Table 3.8.

Table 3.8: Results obtained by using existing approach [56] and proposed approach

α	Existing approach [56]			Proposed approach				
	a	b	c	a	a'	b	c'	c
0	0.9892	0.9906	0.9920	0.9892	0.9892	0.9906	0.9920	0.9920
0.1	0.9893	0.9906	0.9919	0.9894	0.9893	0.9906	0.9918	0.9917
0.2	0.9895	0.9906	0.9917	0.9896	0.9895	0.9906	0.9916	0.9915
0.3	0.9896	0.9906	0.9916	0.9899	0.9897	0.9906	0.9914	0.9913
0.4	0.9898	0.9906	0.9914	0.9901	0.9899	0.9906	0.9913	0.9910
0.5	0.9899	0.9906	0.9913	0.9903	0.9900	0.9906	0.9911	0.9908

0.6	0.9900	0.9906	0.9912	0.9906	0.9902	0.9906	0.9909	0.9906
0.7	0.9902	0.9906	0.9910	--	0.9904	0.9906	0.9907	--
0.8	0.9903	0.9906	0.9909	--	0.9906	0.9906	0.9906	--
0.9	0.9905	0.9906	0.9907	--	--	--	--	--
1.0	0.9906	0.9906	0.9906	--	--	--	--	--

- (i) On the basis of the results, obtained by using existing approach [56], shown in Table 3.8, it can be concluded that the reliability of compressor system will lie between 0.9892 to 0.9920 and the degree of acceptance, degree of rejection and degree of hesitation about the statement that the reliability of compressor system will be 0.9906 are 1, 0 and 0 respectively. Also, it can be concluded that, if the degree of acceptance is α for the statement that the reliability of compressor system is r , where $r \in [0.9892, 0.9920]$ and $r \neq 0.9906$, then for the same statement the degree of rejection and degree of hesitation will be $1-\alpha$ and 0 respectively i.e for all the values of reliability obtained by using existing approach [56], the degree of hesitation is zero. Since, in real life situations there also exist hesitation with acceptance and rejection. So, the results obtained by using existing approach [56] are not appropriate according to real life situations.
- (ii) On the basis of the results, obtained by using proposed approach, shown in Table 3.8,, it can be concluded that the reliability of compressor system will lie between 0.9892 to 0.9920 and the degree of acceptance, degree of rejection, and degree of hesitation about the statement that the reliability of compressor system will be 0.9906 are 0.6, 0.2 and 0.2 respectively. Also, it can be concluded that, if the degree of acceptance, degree of rejection, and degree of hesitation for the statement that the reliability of

compressor system is r , are α and β respectively, where $r \in [0.9892, 0.9920]$ and $r \neq 0.9906$, then the degree of hesitation will be β and $1 - (\alpha - \beta)$, i.e. for all the values of reliability obtained by using proposed approach, there exist degree of hesitation, acceptance and rejection. Since, in real life situation there always exists hesitation, acceptance and rejection for the same statement. So, the results obtained by using proposed approach are appropriate according to real life situations.

3.6 CONCLUSIONS

On the basis of the presented study, it can be concluded that the results obtained by using the existing approach [56] are not appropriate according to real life situations. While, the results obtained by using the proposed approach are appropriate according to real life situations. Therefore, it is better to use the proposed approach as compared to existing approach [56] for evaluating the reliability parameters of industrial systems.

CHAPTER 4

A PETRI NET BASED APPROACH WITH A NEW TYPE OF TRAPEZOIDAL VAGUE SET FOR EVALUATING THE VAGUE RELIABILITY PARAMETERS OF INDUSTRIAL SYSTEMS[‡]

Chen et al. [29] extended the concept of trapezoidal fuzzy number [52] by trapezoidal fuzzy number with different left and right height and pointed out that it is better to represent the data with the proposed trapezoidal fuzzy number as compared to the existing trapezoidal fuzzy number [52]. On the same direction in this Chapter, the concept of trapezoidal vague set [25] is extended by trapezoidal vague set with different left and right height. Also, the approach, proposed in Chapter 3, is modified on the basis of proposed trapezoidal vague set. The modified approach is illustrated by evaluating the vague reliability of a gas turbine.

4.1 PROPOSED TRAPEZOIDAL VAGUE SET WITH DIFFERENT LEFT AND RIGHT HEIGHTS

Chen et al. [29] proposed the definition of trapezoidal fuzzy set with different left and right heights, presented in Definition 4.1, by modifying the definition of trapezoidal fuzzy sets with equal left and right heights, presented in Definition 2.5. On the same direction in this section, the definition of trapezoidal vague set with different left and right heights, presented in Definition 4.3, is proposed by modifying the trapezoidal vague set with equal heights, presented in Definition 4.2.

[‡] The contents of this Chapter are communicated in *Engineering Applications of Artificial Intelligence*.

Definition 4.1 [29] A fuzzy set $\tilde{V} = \langle [a, b, c, d]; \mu_L, \mu_R \rangle$ is said to be a trapezoidal fuzzy set with different left and right heights if:

$$\mu_{\tilde{V}}(x) = \begin{cases} \mu_L \left(\frac{x-a}{b-a} \right), & a \leq x < b \\ \mu_L + \frac{(x-b)}{(c-b)} (\mu_R - \mu_L), & b \leq x < c \\ \mu_R \left(\frac{d-x}{d-c} \right), & c \leq x < d \\ 0, & \text{otherwise} \end{cases}$$

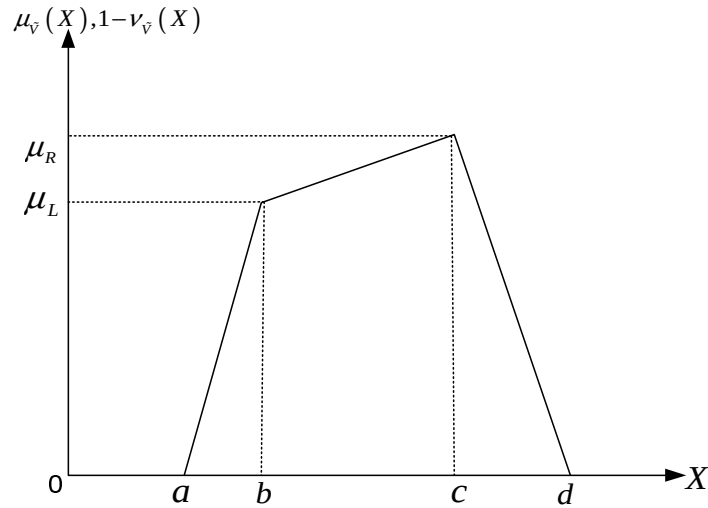


Figure 4.1: Trapezoidal fuzzy sets with different left and right heights

Definition 4.2 [25] A vague set $\tilde{V} = \langle [a, b, c, d]; \mu, \nu \rangle$ is said to be a trapezoidal vague set if :

$$\mu_{\tilde{v}}(x) = \begin{cases} \mu\left(\frac{x-a}{b-a}\right), & a \leq x < b \\ \mu, & b \leq x < c \\ \mu\left(\frac{d-x}{d-c}\right), & c \leq x < d \\ 0, & \text{otherwise} \end{cases}$$

and,

$$1-\nu_{\tilde{v}}(x) = \begin{cases} \nu\left(\frac{x-a}{b-a}\right), & a \leq x < b \\ \nu, & b \leq x < c \\ \nu\left(\frac{d-x}{d-c}\right), & c \leq x < d \\ 0, & \text{otherwise} \end{cases}$$

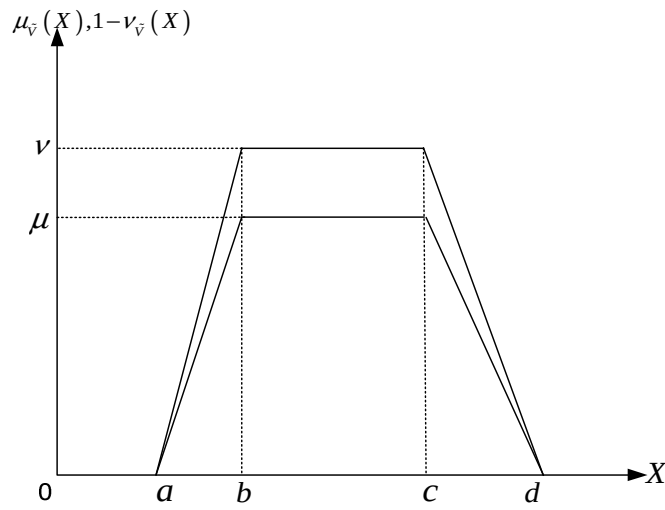


Figure 4.2: Trapezoidal vague set

Definition 4.3 A vague set $\tilde{V} = \langle [(a, b, c, d); \mu_L, \mu_R, \nu_L, \nu_R] \rangle$ is said to be a trapezoidal

vague set with different left and right heights if:

$$\mu_{\tilde{V}}(x) = \begin{cases} \mu_L \left(\frac{x-a}{b-a} \right), & a \leq x < b \\ \mu_L + \frac{(x-b)}{(c-b)} (\mu_R - \mu_L), & b \leq x < c \\ \mu_R \left(\frac{d-x}{d-c} \right), & c \leq x < d \\ 0, & \text{otherwise} \end{cases}$$

and,

$$\nu_{\tilde{V}}(x) = \begin{cases} \nu_L \left(\frac{x-a}{b-a} \right), & a \leq x < b \\ \nu_L + \frac{(x-b)}{(c-b)} (\nu_R - \nu_L), & b \leq x < c \\ \nu_R \left(\frac{d-x}{d-c} \right), & c \leq x < d \\ 0, & \text{otherwise} \end{cases}$$

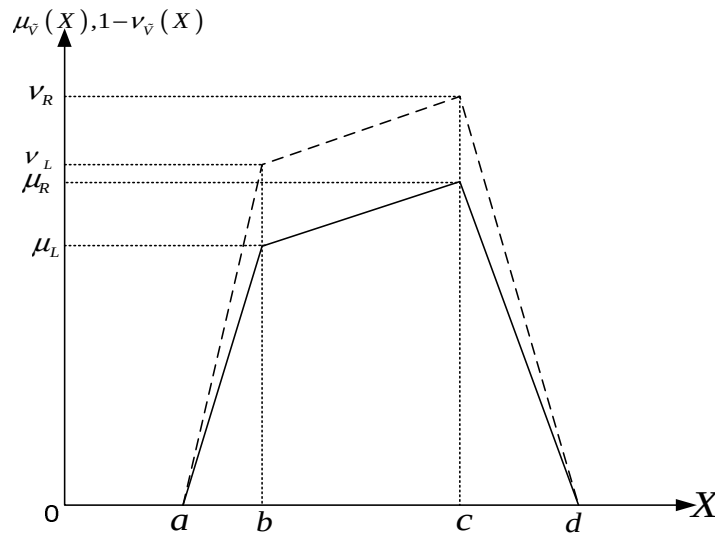


Figure 4.3: Trapezoidal vague set with different left and right heights

4.2 PROPOSED APPROACH

In this section, a new approach, by modifying the approach, proposed in Section 3.4, is proposed for evaluating the interval of confidence corresponding to truth membership function and complement of false membership function of vague reliability parameters.

The steps of proposed approach are as follows:

Step 1: Use Step 1 of the approach, proposed in Section 3.4, to find the expressions for evaluating failure rate (λ_s) and repair rate (τ_s) of the chosen system.

Step 2: Construct the trapezoidal vague sets with different left and right heights $\tilde{\lambda}_j = \left\langle \left[(\lambda_{j1}, \lambda_{j2}, \lambda_{j3}, \lambda_{j4}); \mu_{jL}, \mu_{jR}, \nu_{jL}, \nu_{jR} \right] \right\rangle$, $\tilde{\tau}_j = \left\langle \left[(\tau_{j1}, \tau_{j2}, \tau_{j3}, \tau_{j4}); \mu'_{jL}, \mu'_{jR}, \nu'_{jL}, \nu'_{jR} \right] \right\rangle$ to represent the uncertain failure rate (λ_j) and repair rate (τ_j) .

Step 3: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1,

by $\left\{ (\lambda_{j2} - \lambda_{j1}) \frac{\alpha}{\mu_L} + \lambda_{j1} \right\}$, $\mu_L = \text{minimum}(\mu_{jL})$, $\alpha \in [0, \mu_L]$ and $\left\{ (\tau_{j2} - \tau_{j1}) \frac{\beta}{\mu'_L} + \tau_{j1} \right\}$,

$\mu'_L = \text{minimum}(\mu'_{jL})$, $\beta \in [0, \mu'_L]$ respectively and replace λ_j and τ_j , presented in the

denominator of the expression, obtained in Step 1, by $\left\{ -(\lambda_{j4} - \lambda_{j3}) \frac{\alpha}{\mu_L} + \lambda_{j4} \right\}$,

$\mu_L = \text{minimum}(\mu_{jL})$, $\alpha \in [0, \mu_L]$ and $\left\{ -(\tau_{j4} - \tau_{j3}) \frac{\beta}{\mu'_L} + \tau_{j4} \right\}$, $\mu'_L = \text{minimum}(\mu'_{jL})$,

$\beta \in [0, \mu'_L]$ respectively, in the expression, obtained in Step 1, for evaluating the value of left end point of interval of confidence corresponding to truth membership function of vague failure rate (λ_j) and vague repair rate (τ_j) of the chosen system.

Step 4: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1

by $\left\{ -(\lambda_{j4} - \lambda_{j3}) \frac{\alpha}{\mu_L} + \lambda_{j4} \right\}$, $\mu_L = \text{minimum}(\mu_{jL})$, $\alpha \in [0, \mu_L]$ and $\left\{ -(\tau_{j4} - \tau_{j3}) \frac{\beta}{\mu'_L} + \tau_{j4} \right\}$,

$\mu'_L = \text{minimum}(\mu'_{jL})$, $\beta \in [0, \mu'_L]$ respectively and replace λ_j and τ_j , presented in the

denominator of the expression, obtained in Step 1, by $\left\{ (\lambda_{j2} - \lambda_{j1}) \frac{\alpha}{\mu_L} + \lambda_{j1} \right\}$,

$\mu_L = \text{minimum}(\mu_{jL})$, $\alpha \in [0, \mu_L]$ and $\left\{ (\tau_{j2} - \tau_{j1}) \frac{\beta}{\mu'_L} + \tau_{j1} \right\}$, $\mu'_L = \text{minimum}(\mu'_{jL})$,

$\beta \in [0, \mu'_L]$ respectively, in the expression, obtained in Step 1, for evaluating the value of

right end point of interval of confidence corresponding to truth membership function of

vague failure rate $(\tilde{\lambda}_s)$ and vague repair rate $(\tilde{\tau}_s)$ of the chosen system.

Step 5: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1,

by $\left\{ (\lambda_{j3} - \lambda_{j2}) \frac{(\alpha - \mu_L)}{(\mu_R - \mu_L)} + \lambda_{j2} \right\}$, $\mu_L = \text{minimum}(\mu_{jL})$, $\mu_R = \text{minimum}(\mu_{jR})$, $\alpha \in [\mu_L, \mu_R]$

and $\left\{ (\tau_{j3} - \tau_{j2}) \frac{(\beta - \mu'_L)}{(\mu'_R - \mu'_L)} + \tau_{j2} \right\}$, $\mu'_L = \text{minimum}(\mu'_{jL})$, $\mu'_R = \text{minimum}(\mu'_{jR})$, $\beta \in [\mu'_L, \mu'_R]$

respectively and replace λ_j and τ_j , presented in the denominator of the expression, obtained

in Step 1, by $\left\{ -(\lambda_{j4} - \lambda_{j3}) \frac{\alpha}{\mu_R} + \lambda_{j4} \right\}$, $\mu_R = \text{minimum}(\mu_{jR})$, $\alpha \in [0, \mu_R]$ and

$\left\{ -(\tau_{j4} - \tau_{j3}) \frac{\beta}{\mu'_R} + \tau_{j4} \right\}$, $\mu'_R = \text{minimum}(\mu'_{jR})$, $\beta \in [0, \mu'_R]$ respectively, in the expression,

obtained in Step 1, for evaluating the value of left end point of interval of confidence

corresponding to truth membership function of vague failure rate $(\tilde{\lambda}_s)$ and vague repair rate $(\tilde{\tau}_s)$ of the chosen system.

Step 6: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1

by $\left\{ -(\lambda_{j4} - \lambda_{j3}) \frac{\alpha}{\mu_R} + \lambda_{j4} \right\}$, $\mu_R = \text{minimum}(\mu_{jR})$, $\alpha \in [0, \mu_R]$, and $\left\{ -(\tau_{j4} - \tau_{j3}) \frac{\beta}{\delta_R} + \tau_{j4} \right\}$,

$\mu'_R = \text{minimum}(\mu'_{jR})$, $\beta \in [0, \mu'_R]$ respectively and replace λ_j and τ_j , presented in the

denominator of the expression, obtained in Step 1, by $\left\{ (\lambda_{j3} - \lambda_{j2}) \frac{(\alpha - \mu_L)}{(\mu_R - \mu_L)} + \lambda_{j2} \right\}$,

$\mu_L = \text{minimum}(\mu_{jL})$, $\mu_R = \text{minimum}(\mu_{jR})$, $\alpha \in [\mu_L, \mu_R]$ and $\left\{ (\tau_{j3} - \tau_{j2}) \frac{(\beta - \mu'_L)}{(\mu'_R - \mu'_L)} + \tau_{j2} \right\}$,

$\mu'_L = \text{minimum}(\mu'_{jL})$, $\mu'_R = \text{minimum}(\mu'_{jR})$, $\beta \in [\mu'_L, \mu'_R]$ respectively, in the expression,

obtained in Step 1, for evaluating the value of right end point of interval of confidence

corresponding to truth membership function of vague failure rate $(\tilde{\lambda}_s)$ and vague repair rate

$(\tilde{\tau}_s)$ of the chosen system.

Step 7: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1,

by $\left\{ (\lambda_{j2} - \lambda_{j1}) \frac{\alpha}{\nu_L} + \lambda_{j1} \right\}$, $\nu_L = \text{minimum}(\nu_{jL})$, $\alpha \in [0, \nu_L]$ and $\left\{ (\tau_{j2} - \tau_{j1}) \frac{\beta}{\nu'_L} + \tau_{j1} \right\}$,

$\nu'_L = \text{minimum}(\nu'_{jL})$, $\beta \in [0, \nu'_L]$ respectively and replace λ_j and τ_j , presented in the

denominator of the expression, obtained in Step 1, by $\left\{ -(\lambda_{j4} - \lambda_{j3}) \frac{\alpha}{\mu_L} + \lambda_{j4} \right\}$,

$\nu_L = \text{minimum}(\nu_{jL})$, $\alpha \in [0, \nu_L]$ and $\left\{ -(\tau_{j4} - \tau_{j3}) \frac{\beta}{\nu'_L} + \tau_{j4} \right\}$, $\nu'_L = \text{minimum}(\nu'_{jL})$,

$\beta \in [0, \nu'_L]$ respectively, in the expression, obtained in Step 1, for evaluating the value of left end point of interval of confidence corresponding to complement of false membership function of vague failure rate $(\tilde{\lambda}_s)$ and vague repair rate $(\tilde{\tau}_s)$ of the chosen system.

Step 8: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1

by $\left\{ -(\lambda_{j4} - \lambda_{j3}) \frac{\alpha}{\nu_L} + \lambda_{j4} \right\}$, $\nu_L = \text{minimum}(\nu_{jL})$, $\alpha \in [0, \nu_L]$, $\alpha \in [0, \mu_L]$ and

$\left\{ -(\tau_{j4} - \tau_{j3}) \frac{\beta}{\nu'_L} + \tau_{j4} \right\}$, $\nu'_L = \text{minimum}(\nu'_{jL})$, $\beta \in [0, \nu'_L]$ respectively and replace λ_j and τ_j ,

presented in the denominator of the expression, obtained in Step 1, by $\left\{ (\lambda_{j2} - \lambda_{j1}) \frac{\alpha}{\nu_L} + \lambda_{j1} \right\}$,

$\nu_L = \text{minimum}(\nu_{jL})$, $\alpha \in [0, \nu_L]$, $\alpha \in [0, \mu_L]$ and $\left\{ (\tau_{j2} - \tau_{j1}) \frac{\beta}{\nu'_L} + \tau_{j1} \right\}$, $\nu'_L = \text{minimum}(\nu'_{jL})$,

$\beta \in [0, \nu'_L]$ respectively, in the expression, obtained in Step 1, for evaluating the value of right end point of interval of confidence corresponding to complement of false membership function of vague failure rate $(\tilde{\lambda}_s)$ and vague repair rate $(\tilde{\tau}_s)$ of the chosen system.

Step 9: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1,

by $\left\{ (\lambda_{j3} - \lambda_{j2}) \frac{(\alpha - \nu_L)}{(\nu_R - \nu_L)} + \lambda_{j2} \right\}$, $\nu_L = \text{minimum}(\nu_{jL})$, $\nu_R = \text{minimum}(\nu_{jR})$, $\alpha \in [\nu_L, \nu_R]$ and

$\left\{ (\tau_{j3} - \tau_{j2}) \frac{(\beta - \nu'_L)}{(\nu'_R - \nu'_L)} + \tau_{j2} \right\}$, $\nu'_L = \text{minimum}(\nu'_{jL})$, $\nu'_R = \text{minimum}(\nu'_{jR})$, $\beta \in [\nu'_L, \nu'_R]$

respectively and replace λ_j and τ_j , presented in the denominator of the expression, obtained

in Step 1, by $\left\{ -(\lambda_{j4} - \lambda_{j3}) \frac{\alpha}{\mu_R} + \lambda_{j4} \right\}$, $\nu_R = \text{minimum}(\nu_{jR})$, $\alpha \in [0, \nu_R]$ and

$$\left\{ -(\tau_{j4} - \tau_{j3}) \frac{\beta}{\nu'_R} + \tau_{j4} \right\}, \quad \nu'_R = \text{minimum}(\nu'_{jR}), \quad \beta \in [0, \nu'_R] \quad \text{respectively, in the expression,}$$

obtained in Step 1, for evaluating the value of left end point of interval of confidence corresponding to complement of false membership function of vague failure rate $(\tilde{\lambda}_s)$ and vague repair rate $(\tilde{\tau}_s)$ of the chosen system.

Step 10: Replace λ_j and τ_j presented in the numerator of the expression, obtained in Step 1

$$\text{by } \left\{ -(\lambda_{j4} - \lambda_{j3}) \frac{\alpha}{\mu_R} + \lambda_{j4} \right\}, \quad \nu_R = \text{minimum}(\nu_{jR}), \quad \alpha \in [0, \nu_R] \quad \text{and} \quad \left\{ -(\tau_{j4} - \tau_{j3}) \frac{\beta}{\delta_R} + \tau_{j4} \right\},$$

$\nu'_R = \text{minimum}(\nu'_{jR}), \quad \beta \in [0, \nu'_R]$ respectively and replace λ_j and τ_j , presented in the

denominator of the expression, obtained in Step 1, by $\left\{ (\lambda_{j3} - \lambda_{j2}) \frac{(\alpha - \nu_L)}{(\nu_R - \nu_L)} + \lambda_{j2} \right\},$

$$\nu_L = \text{minimum}(\nu_{jL}), \quad \nu_R = \text{minimum}(\nu_{jR}), \quad \alpha \in [\nu_L, \nu_R] \quad \text{and} \quad \left\{ (\tau_{j3} - \tau_{j2}) \frac{(\beta - \nu'_L)}{(\nu'_R - \nu'_L)} + \tau_{j2} \right\},$$

$\nu'_L = \text{minimum}(\nu'_{jL}), \quad \nu'_R = \text{minimum}(\nu'_{jR}), \quad \beta \in [\nu'_L, \nu'_R]$ respectively, in the expression,

obtained in Step 1, for evaluating the value of right end point of interval of confidence corresponding to complement of false membership function of vague failure rate $(\tilde{\lambda}_s)$ and vague repair rate $(\tilde{\tau}_s)$ of the chosen system.

Step 11: Replace λ_s and τ_s presented in numerator of the expressions, presented in Table 3.1

by the left end point of interval of confidence corresponding to truth membership function, obtained from Step 2 to Step 10 and replace λ_s and τ_s , presented in denominator of the expressions, presented in Table 3.1 by the right end point of interval of confidence corresponding to truth membership function, obtained from Step 2 to Step 10, to evaluate the

left end point of interval of confidence corresponding to truth membership function of vague reliability parameters.

Step 12: Replace λ_s and τ_s presented in numerator of the expressions, presented in Table 3.1 by the right end point of interval of confidence corresponding to truth membership function, obtained from Step 2 to Step 10 and replace λ_s and τ_s , presented in denominator of the expressions, presented in Table 3.1 by the left end point of interval of confidence corresponding to truth membership function, obtained from Step 2 to Step 10, to evaluate the right end point of interval of confidence of corresponding to truth membership function of vague reliability parameters.

Step 13: Replace λ_s and τ_s presented in numerator of the expressions, presented in Table 3.1 by the left end point of interval of confidence corresponding to complement of false membership function, obtained from Step 2 to Step 10 and replace λ_s and τ_s , presented in denominator of the expressions, presented in Table 3.1 by the right end point of interval of confidence corresponding to complement of false membership function, obtained from Step 2 to Step 10, to evaluate the left end point of interval of confidence corresponding to complement of false membership function of vague reliability parameters.

Step 14: Replace λ_s and τ_s presented in numerator of the expressions, presented in Table 3.1 by the right end point of interval of confidence corresponding to complement of false membership function, obtained from Step 2 to Step 10, and replace λ_s and τ_s , presented in denominator of the expressions, presented in Table 3.1 by the left end point of interval of confidence corresponding to complement of false membership function, obtained from Step 2 to Step 10, to evaluate the right end point of interval of confidence corresponding to complement of false membership function of vague reliability parameters.

4.3 EVALUATION OF VAGUE RELIABILITY PARAMETERS OF A GAS TURBINE

In this section, to show the application of proposed approach, a Petri net model of a gas turbine is constructed with the help of fault tree of the same system and with the help of proposed approach, the interval of confidence corresponding to truth membership function as well as complement of false membership function of vague reliability parameters of gas turbine are evaluated.

Step 1: On the basis of a deep study of the gas turbine and its components, its fault tree, depicted in Figure 4.4, is constructed. Also, using the constructed fault tree, the Petri net, depicted in Figure 4.5, is obtained. The description of the symbols used in fault tree as well as in the Petri net is listed in Table 4.1 and

Table 4.2.

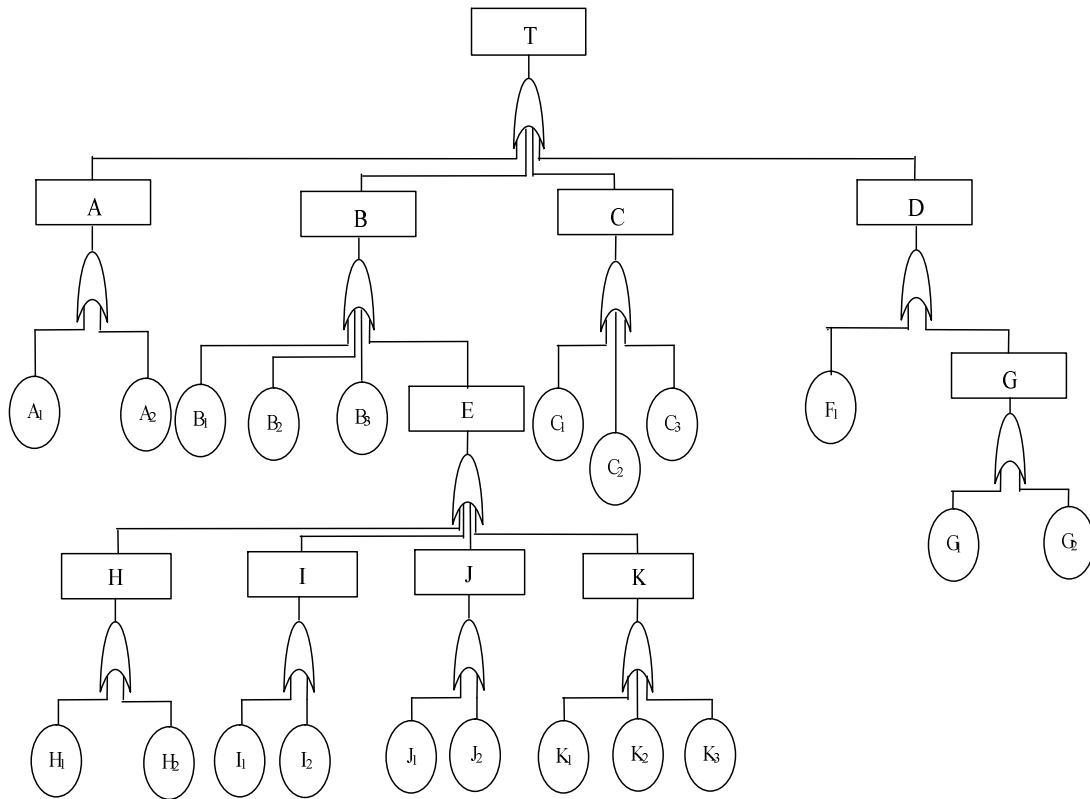


Figure 4.4: Fault tree of gas turbine

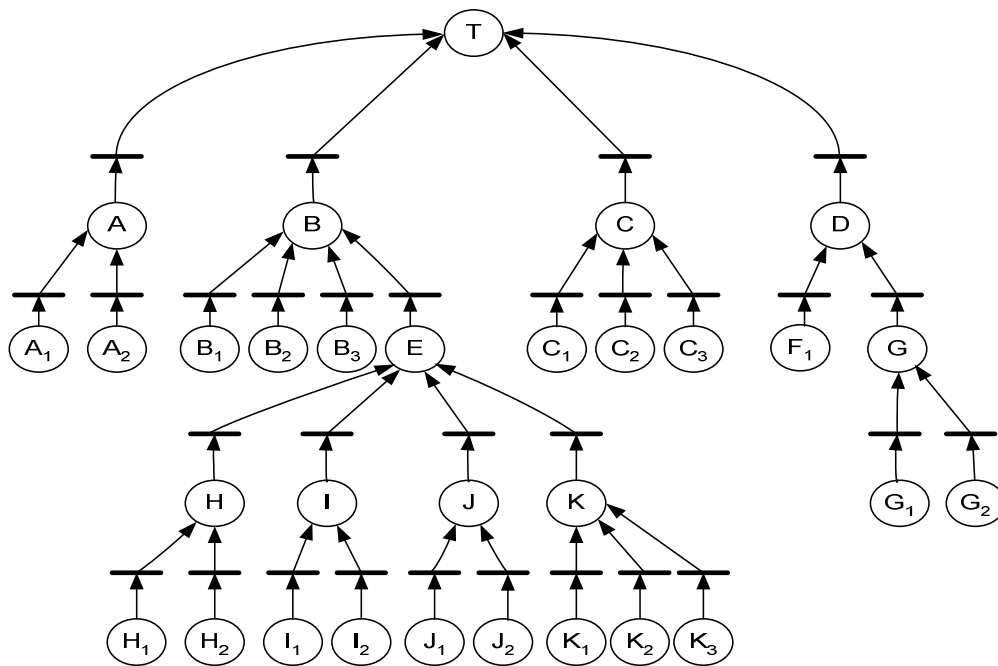


Figure 4.5: Petri net model of gas turbine

Table 4.1: Intermediate events of the chosen system

Code	Fault	Code	Fault
T	Failure of total turbine system	G	Failure of exhaust connection
A	Failure of vane system turbine	H	Failure of pumping system
B	Failure of radial bearings of the turbine	I	Failure of filtering system
C	Failure of cylinder of the turbine	J	Failure of cooling system
D	Failure of exhaust system of the turbine	K	Less lube oil supply from reservoir
E	Failure of Lubrication system		

Table 4.2: Basic events of the chosen system

Code	Fault	Code	Fault
A ₁	Vanes got damaged	G ₂	Exhaust pipes of the exhaust system got damaged
A ₂	Pins of the vane system got broken	H ₁	Pump failure
B ₁	Failure of pads of the radial bearing.	H ₂	Pipes rupture
B ₂	Failure of thermocouple	I ₁	Filter clogged
B ₃	Shell of the radial bearing system got damaged.	I ₂	Failure of Differential pressure gauge
C ₁	Blade rings of the turbine cylinder system got broken	J ₁	Failure of control system
C ₂	Blade pins of the turbine cylinder system	J ₂	Failure of bypass valve

	got broken		
C ₃	Blade of the turbine cylinder system got broken	K ₁	Pressure gauge got corrupted.
F ₁	Casing of the exhaust cylinder got damaged	K ₂	Failure of heating system.
G ₁	Static seal of the exhaust system got damaged.	K ₃	Failure of temperature sensor.

Step 2: Using the Petri net, shown in Figure 4.5, the following expressions are obtained for evaluating the failure rate (λ_s) and repair rate (τ_s) of gas turbine.

$$\lambda_s = \sum \lambda_j \quad \text{and} \quad \tau_s = \frac{\sum \lambda_j \tau_j}{\sum \lambda_j}$$

where, λ_j and τ_j are failure rates and repair rates of basic events of gas turbine.

Step 3: To handle the uncertainties and imprecision in the reliability data (assuming mission time $t = 1$ year), the crisp value of (λ_j) and (τ_j), presented, in the

Table 4.3 are converted into trapezoidal vague sets with different left and right height

$$\tilde{\lambda}_j = \left\langle \left[(\lambda_{j1}, \lambda_{j2}, \lambda_{j3}, \lambda_{j4}); \mu_{jL}, \mu_{jR}, \nu_{jL}, \nu_{jR} \right] \right\rangle, \quad \tilde{\tau}_j = \left\langle \left[(\tau_{j1}, \tau_{j2}, \tau_{j3}, \tau_{j4}); \mu'_{jL}, \mu'_{jR}, \nu'_{jL}, \nu'_{jR} \right] \right\rangle$$

where, $\lambda_{j1} = \lambda_j - 0.3 \times \lambda_j$, $\lambda_{j2} = \lambda_j - 0.15 \times \lambda_j$, $\lambda_{j3} = \lambda_j + 0.15 \times \lambda_j$, $\lambda_{j4} = \lambda_j + 0.3 \times \lambda_j$

$\tau_{j1} = \tau_j - 0.3 \times \tau_j$, $\tau_{j2} = \tau_j - 0.15 \times \tau_j$, $\tau_{j3} = \tau_j + 0.15 \times \tau_j$, $\tau_{j4} = \tau_j + 0.3 \times \tau_j$, $\mu_{jL} = \mu'_{jL} = 0.5$,

$\mu_{jR} = \mu'_{jR} = 0.6$, $\nu_{jL} = \nu'_{jL} = 0.7$, $\nu_{jR} = \nu'_{jR} = 0.8$.

Table 4.3: Crisp failures rates and repair rate

$\lambda_{A_1} = 0.003, \lambda_{A_2} = 0.005, \lambda_{B_1} = 0.0013, \lambda_{B_2} = \lambda_{B_1}, \lambda_{B_3} = 0.0051, \lambda_{C_1} = 0.0015, \lambda_{C_2} = \lambda_{A_1}, \lambda_{C_3} = \lambda_{B_1}, \lambda_{F_1}$ $= \lambda_{B_1}, \lambda_{G_1} = \lambda_{C_1}, \lambda_{G_2} = 0.002, \lambda_{H_1} = 0.007, \lambda_{H_2} = \lambda_{A_1}, \lambda_{I_1} = 0.001, \lambda_{I_2} = \lambda_{B_1}, \lambda_{J_1} = \lambda_{C_1}, \lambda_{J_2} = \lambda_{G_2}, \lambda_{K_1} = \lambda_{I_1}$ $, \lambda_{K_2} = \lambda_{A_1}, \lambda_{K_3} = \lambda_{I_1}.$
$\tau_{A_1} = 10, \tau_{A_2} = 10, \tau_{B_1} = 2, \tau_{B_2} = \tau_{B_1}, \tau_{B_3} = 3, \tau_{C_1} = 4, \tau_{C_2} = 10, \tau_{C_3} = \tau_{A_1}, \tau_{F_1} = \tau_{B_1}, \tau_{G_1} = 3, \tau_{G_2} = 4, \tau_{H_1}$ $= 10, \tau_{H_2} = \tau_{A_1}, \tau_{I_1} = 2, \tau_{I_2} = \tau_{B_1}, \tau_{J_1} = 3, \tau_{J_2} = 4, \tau_{K_1} = 10, \tau_{K_2} = \tau_{A_1}, \tau_{K_3} = 2.$

Table 4.4: Left end point and right end point of interval of confidence corresponding to truth membership function of vague reliability parameters

α	Left end point						Right end point					
	λ_s	τ_s	R_s	A_s	ENOF	MTBF	λ_s	τ_s	R_s	A_s	ENOF	MTBF
0	0.0127	2.2535	0.9766	0.9774	0.0127	44.4957	0.0237	14.4340	0.9873	0.9898	0.0234	92.8838
0.1	0.0133	2.4988	0.9770	0.9779	0.0132	45.5693	0.0232	13.3136	0.9868	0.9891	0.0230	88.5395
0.2	0.0138	2.7626	0.9775	0.9784	0.0138	46.6945	0.0228	12.2915	0.9863	0.9885	0.0225	84.5479
0.3	0.0144	3.0459	0.9779	0.9789	0.0143	47.8744	0.0223	11.3565	0.9857	0.9878	0.0221	80.8690
0.4	0.0149	3.3500	0.9784	0.9794	0.0148	49.1124	0.0219	10.4990	0.9852	0.9872	0.0216	77.4683
0.5	0.0155	3.6762	0.9788	0.9799	0.0154	50.4123	0.0214	9.7108	0.9846	0.9865	0.0212	74.3165
0.6	0.0209	6.8754	0.9793	0.9807	0.0207	54.6274	0.0209	6.8754	0.9793	0.9807	0.0207	54.6274
0.7	--	--	--	--	--	--	--	--	--	--	--	--
0.8	--	--	--	--	--	--	--	--	--	--	--	--
0.9	--	--	--	--	--	--	--	--	--	--	--	--
1	--	--	--	--	--	--	--	--	--	--	--	--

Table 4.5: Left end point and right end point of interval of confidence corresponding to complement of false membership function of vague reliability parameters

α	Left end point						Right end point					
	λ_s	τ_s	R_s	A_s	ENOF	MTBF	λ_s	τ_s	R_s	A_s	ENOF	MTBF
0	0.0127	2.2535	0.9766	0.9774	0.0127	44.4957	0.0237	14.4340	0.9873	0.9898	0.0234	92.8838
0.1	0.0131	2.4286	0.9769	0.9778	0.0131	45.2890	0.0233	13.6042	0.9869	0.9893	0.0231	89.7238
0.2	0.0135	2.6132	0.9773	0.9781	0.0134	46.1102	0.0230	12.8279	0.9866	0.9888	0.0227	86.7518
0.3	0.0139	2.8079	0.9776	0.9785	0.0138	46.9605	0.0226	12.1007	0.9862	0.9884	0.0224	83.9519
0.4	0.0143	3.0129	0.9779	0.9789	0.0142	47.8414	0.0223	11.4184	0.9858	0.9879	0.0221	81.3101
0.5	0.0147	3.2290	0.9783	0.9792	0.0146	48.7542	0.0220	10.7776	0.9854	0.9874	0.0217	78.8137
0.6	0.0151	3.4564	0.9786	0.9796	0.0150	49.7005	0.0216	10.1750	0.9850	0.9870	0.0214	76.4516
0.7	0.0155	3.6959	0.9789	0.9800	0.0154	50.6818	0.0213	9.6078	0.9846	0.9865	0.0211	74.2135
0.8	0.0209	6.8754	0.9793	0.9807	0.0207	54.6274	0.0209	6.8754	0.9793	0.9807	0.0207	54.6274
0.9	--	--	--	--	--	--	--	--	--	--	--	--
1.0	--	--	--	--	--	--	--	--	--	--	--	--

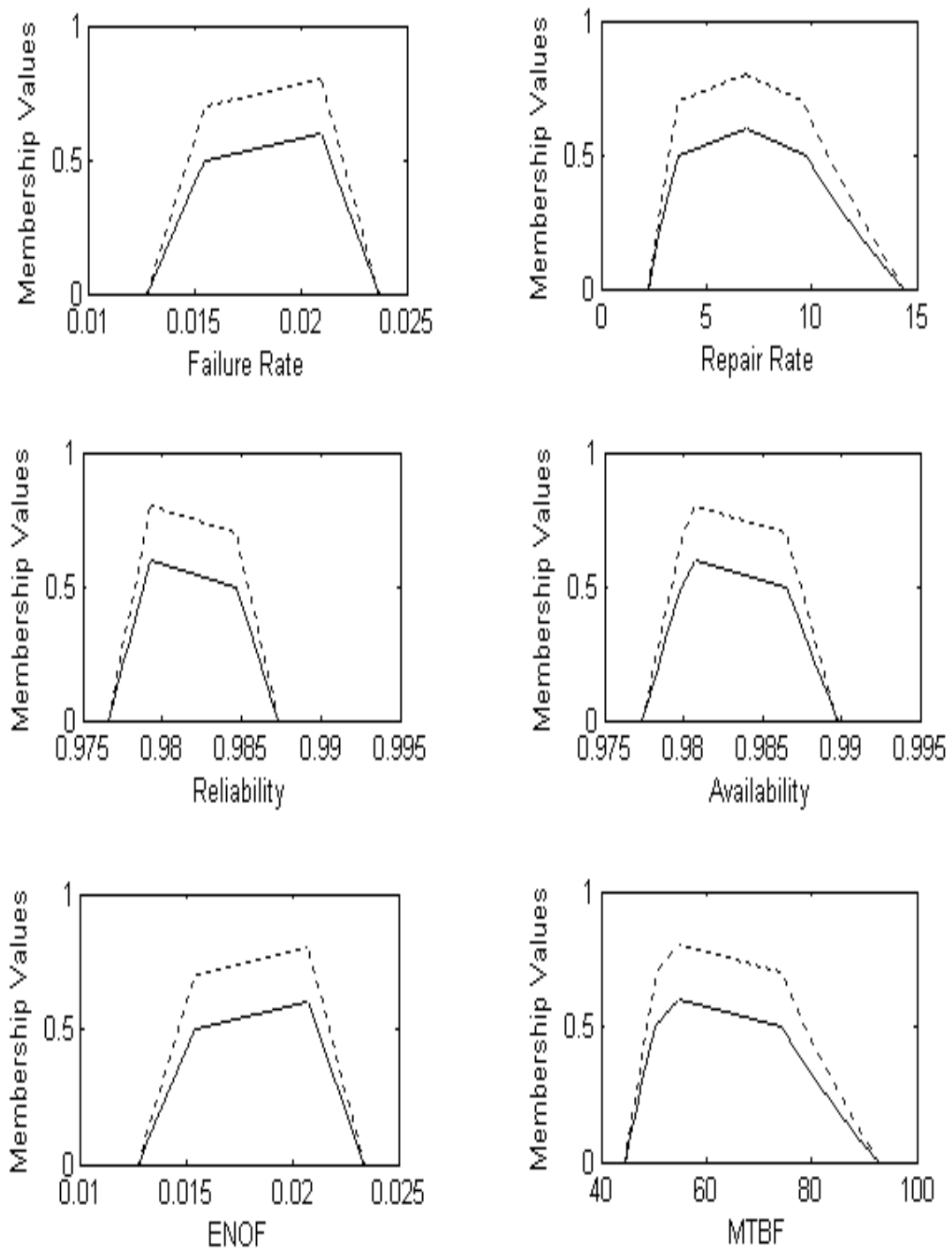


Figure 4.6: Vague reliability parameters of the gas turbine

Step 4: The left end point and the right end point of the interval of confidence corresponding to truth membership function as well as complement of false membership function of vague failure rate, vague repair rate as well as other vague reliability parameters of the compressor system obtained by using the data obtained from Step 3 as well as Step 3 to Step 14 of the proposed approach, are listed in Table 4.4 and Table 4.5 respectively.

Step 5: The vague sets, representing vague reliability parameters obtained by using the left end point and right end point of interval of confidence, shown in Table 4.4 and Table 4.5 respectively, are depicted in Figure 4.6.

4.4 COMPARATIVE STUDY

To show the advantage of the approach, proposed in this chapter, over the approach, proposed in Chapter 3, the trapezoidal vague set with equal heights, representing the failure probabilities of basic events, are constructed by assuming $\mu_{jL} = \mu_{jR}$ and $\nu_{jL} = \nu_{jR}$ in the vague set with different left and right height, obtained in Step 3 of Section 4.3 and then the approach, proposed in Chapter 3, is modified to find the interval of confidence, corresponding to truth membership function and complement of false membership function of vague reliability parameters.

On the basis of the obtained results, it is concluded that on applying the approach, proposed in Chapter 3, degree of acceptance, degree of rejection and degree of hesitation is same corresponding to all the values of reliability lying between 0.9793 to 0.9852 respectively. While, on applying the approach, proposed in this chapter, degree of

acceptance, degree of rejection and degree of hesitation are not same corresponding to different values of reliability.

Since, in real life problem it rarely occurs that degree of acceptance, degree of rejection and degree of hesitation will be same corresponding to different values of reliability. So, the results obtained by approach, proposed in Chapter 3, is not appropriate according to real life situations. While the results obtained by using the approach, proposed in this chapter, are appropriate according to real life situations.

4.5 CONCLUSIONS

On the basis of the presented study, it can be concluded that the results obtained by using the approach, proposed in Chapter 3, are not closer to real life situations. While the results obtained by using the proposed approach are closer to real life situation. Therefore, it is better to use the approach, proposed in this chapter, as compared to approach, proposed in Chapter 3, for evaluating the reliability parameters of industrial systems.

CHAPTER 5

A PETRI NET BASED APPROACH WITH T_ω -BASED ARITHMETIC OPERATIONS FOR EVALUATING THE VAGUE RELIABILITY PARAMETERS OF INDUSTRIAL SYSTEMS[§]

Kumar et al. [62] proposed T_ω -based vague fault tree approach for evaluating the fuzzy reliability of industrial systems and shown that it is better to use the proposed approach as compared to the existing T_ω -based fuzzy fault tree approach [48]. On the same direction, in this Chapter, T_ω -based vague Petri net approach is proposed for evaluating some vague reliability parameters of industrial systems and shown that it is better to use approach, proposed in this Chapter, as compared to the approach, proposed in Chapter 3. The proposed approach is illustrated by evaluating the vague reliability parameters of a gas turbine.

5.1 T_ω - BASED ARITHMETIC OPERATIONS BETWEEN TWO VAGUE SETS

Hong and Do [48] shown that, T_ω -based multiplication preserve the shape of fuzzy numbers. Also, T_ω -based arithmetic operations provide control on fuzzy spread during calculations in uncertain environment.

[§] The contents of this Chapter are communicated in *Journal of Intelligent and Fuzzy Systems*

In this section, T_ω -based arithmetic operations between two triangular vague sets are presented [62].

Let $T = T_\omega$ be the weakest t -norm and $\tilde{V}_1 = \langle [(a_1, b_1, c_1); \mu_1, \nu_1] \rangle$ and $\tilde{V}_2 = \langle [(a_2, b_2, c_2); \mu_2, \nu_2] \rangle$ be two triangular vague sets. Then, the T_ω -based arithmetic operations between $\tilde{V}_1$ and $\tilde{V}_2$ are defined as follows:

$$(i) \quad \tilde{V}_1 \oplus_T \tilde{V}_2 = \langle [(b_1 + b_2, \max(b_1 - a_1, b_2 - a_2), \max(c_1 - b_1, c_2 - b_2)); \min(\mu_1, \mu_2), \min(\nu_1, \nu_2)] \rangle \quad (5.1)$$

$$(ii) \quad \tilde{V}_1 \ominus_T \tilde{V}_2 = \langle [(b_1 - b_2, \max(b_1 - a_1, c_2 - b_2), \max(c_1 - b_1, b_2 - a_2)); \min(\mu_1, \mu_2), \min(\nu_1, \nu_2)] \rangle \quad (5.2)$$

$$(iii) \quad \tilde{V}_1 \otimes_T \tilde{V}_2 = \begin{cases} \langle [(b_1 b_2, \max((b_1 - a_1)b_2, (b_2 - a_2)b_1), \max((c_1 - b_1)b_2, (c_2 - b_2)b_1)); \min(\mu_1, \mu_2), \min(\nu_1, \nu_2)] \rangle, & \text{for } b_1, b_2 > 0; \\ \langle [(b_1 b_2, \max(-(c_1 - b_1)b_2, -(c_2 - b_2)b_1), \max(-(b_1 - a_1)b_2, -(b_1 - a_1)b_1)); \min(\mu_1, \mu_2), \min(\nu_1, \nu_2)] \rangle, & \text{for } b_1, b_2 < 0, \\ \langle [(b_1 b_2, \max((b_1 - a_1)b_2, -(c_2 - b_2)b_1), \max((c_1 - b_1)b_2, -(b_2 - a_2)b_1)); \min(\mu_1, \mu_2), \min(\nu_1, \nu_2)] \rangle, & \text{for } b_1 < 0, b_2 > 0. \\ \langle [(0, (b_1 - a_1)b_2, (c_1 - b_1)b_2); \min(\mu_1, \mu_2), \min(\nu_1, \nu_2)] \rangle, & \text{for } b_1 = 0, b_2 > 0. \\ \langle [(0, -(c_1 - b_1)b_2, -(b_1 - a_1)b_2); \min(\mu_1, \mu_2), \min(\nu_1, \nu_2)] \rangle, & \text{for } b_1 = 0, b_2 < 0. \\ \langle [(0, 0, 0); \min(\mu_1, \mu_2), \min(\nu_1, \nu_2)] \rangle, & \text{for } b_1 = 0, b_2 = 0. \end{cases} \quad (5.3)$$

(iv) $\tilde{V}_1 \otimes_T \tilde{V}_2$

$$\begin{aligned}
& \left\langle \left[\left(b_1 / b_2, \max \left((c_2 - b_2) b_1 / b_2^2, (b_1 - a_1) b_2 / b_2^2 \right), \max \left((b_2 - a_2) b_1 / b_2^2, (c_1 - b_1) b_2 / b_2^2 \right) \right); \right. \right. \\
& \quad \left. \left. \min (\mu_1, \mu_2), \min (\nu_1, \nu_2) \right] \right\rangle \text{ for } b_1, b_2 > 0; \\
& \left\langle \left[\left(b_1 / b_2, \max \left(-(c_1 - b_1) b_2 / b_2^2, -(b_2 - a_2) b_1 / b_2^2 \right), \max \left(-(b_1 - a_1) b_2 / b_2^2, \right. \right. \right. \\
& \quad \left. \left. \left. -(c_2 - b_2) b_1 / b_2^2 \right) \right); \min (\mu_1, \mu_2), \min (\nu_1, \nu_2) \right] \right\rangle \text{ for } b_1, b_2 < 0; \\
& = \left\langle \left[\left(b_1 / b_2, \max \left((b_1 - a_1) b_2 / b_2^2, -(b_2 - a_2) b_1 / b_2^2 \right), \max \left((c_1 - b_1) b_2 / b_2^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -(c_2 - b_2) b_1 / b_2^2 \right) \right); \min (\mu_1, \mu_2), \min (\nu_1, \nu_2) \right] \right\rangle \text{ for } b_1 < 0, b_2 > 0; \\
& \left\langle \left[\left(0, (b_1 - a_1) b_2 / b_2^2, (c_1 - b_1) b_2 / b_2^2 \right); \min (\mu_1, \mu_2), \min (\nu_1, \nu_2) \right] \right\rangle, \text{ for } b_1 = 0, b_2 > 0; \\
& \left\langle \left[\left(0, -(c_1 - b_1) b_2 / b_2^2, -(b_2 - a_2) b_2 / b_2^2 \right); \min (\mu_1, \mu_2), \min (\nu_1, \nu_2) \right] \right\rangle, \text{ for } b_1 = 0, b_2 < 0; \\
& \left\langle \left[\left(0, 0, 0 \right); \min (\mu_1, \mu_2), \min (\nu_1, \nu_2) \right] \right\rangle, \text{ for } b_1 = 0, b_2 = 0;
\end{aligned}
\tag{5.4}$$

In above arithmetic the term $(b_1 - a_1) \times (b_2 - a_2)$ is neglected by assuming that the values of $(b_1 - a_1)$ and $(b_2 - a_2)$ are small as compared to b_1 and b_2 . But, if spreads are not small compared with mean values, then

$$\begin{aligned}
\text{(v)} \quad \tilde{V}_1 \otimes_T \tilde{V}_2 &= \left\langle \left[(b_1 b_2, \max \left((b_1 - a_1) b_2, (b_2 - a_2) b_1 \right) - (b_1 - a_1)(b_2 - a_2), \right. \right. \\
& \quad \left. \left. \max \left((c_1 - b_1) b_2, (c_1 - b_1) b_1 \right) + (c_1 - b_1)(c_1 - b_1); \min (\mu_1, \mu_2), \min (\nu_1, \nu_2) \right] \right\rangle \\
& \text{ for } b_1, b_2 > 0
\end{aligned}
\tag{5.5}$$

$$\begin{aligned}
\text{(vi)} \quad \tilde{V}_1 \otimes_T \tilde{V}_2 &= \left\langle \left[\left(b_1 / b_2, \max \left((c_2 - b_2) b_1 / (b_2^2 + b_2 (c_2 - b_2)), (b_1 - a_1) b_2 / (b_2^2 + b_2 (c_2 - b_2)) \right); \right. \right. \\
& \quad \left. \left. \max \left((b_2 - a_2) b_1 / (b_2^2 + b_2 (b_2 - a_2)), (c_1 - b_1) b_2 / (b_2^2 + b_2 (b_2 - a_2)) \right); \right. \right. \\
& \quad \left. \left. \min (\mu_1, \mu_2), \min (\nu_1, \nu_2) \right] \right\rangle
\end{aligned}$$

$$\min(\mu_1, \mu_2), \min(\nu_1, \nu_2)]\rangle \text{ for } b_1, b_2 > 0 \quad (5.6)$$

5.1.1 ILLUSTRATIVE EXAMPLE

In this section, with the help of numerical example, the advantage of T_ω -based arithmetic operations is presented.

Let $\tilde{A} = \langle [(5, 8, 13); 0.5, 0.7] \rangle$ and $\tilde{B} = \langle [(1, 4, 6); 0.5, 0.7] \rangle$ be two triangular vague sets. Then,

Case I Using existing arithmetic [25]:

$$\begin{aligned} \text{(i)} \quad \tilde{A} \oplus \tilde{B} &= \langle [(5, 8, 13); 0.5, 0.7] \rangle \oplus \langle [(1, 4, 6); 0.5, 0.7] \rangle \\ &= \langle [(6, 12, 19); 0.5, 0.7] \rangle. \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \tilde{A} \ominus \tilde{B} &= \langle [(5, 8, 13); 0.5, 0.7] \rangle \ominus \langle [(1, 4, 6); 0.5, 0.7] \rangle \\ &= \langle [(-1, 4, 12); 0.5, 0.7] \rangle \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \tilde{A} \otimes \tilde{B} &= \langle [(5, 8, 13); 0.5, 0.7] \rangle \otimes \langle [(1, 4, 6); 0.5, 0.7] \rangle \\ &= \langle [(5, 32, 78); 0.5, 0.7] \rangle. \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad \tilde{A} \oslash \tilde{B} &= \langle [(5, 8, 13); 0.5, 0.7] \rangle \oslash \langle [(1, 4, 6); 0.5, 0.7] \rangle \\ &= \langle [(0.84, 2, 13); 0.5, 0.7] \rangle. \end{aligned}$$

Case II Using T_ω -based arithmetic [62]:

$$\text{(i)} \quad \tilde{A} \oplus_r \tilde{B} = \langle [(5, 8, 13); (0.5, 0.7)] \rangle \oplus_r \langle [(1, 4, 6); (0.5, 0.7)] \rangle$$

$$=\langle [(9,12,17);0.5,0.7]\rangle$$

$$(ii) \quad \tilde{A} \Theta_T \tilde{B} = \langle [(5,8,13);0.5,0.7]\rangle \Theta_T \langle [(1,4,6);0.5,0.7]\rangle$$

$$=\langle [(1,4,9);0.5,0.7]\rangle$$

$$(iii) \quad \tilde{A} \otimes_T \tilde{B} = \langle [(5,8,13);0.5,0.7]\rangle \otimes_T \langle [(1,4,6);0.5,0.7]\rangle$$

$$=\langle [(17,32,62);0.5,0.7]\rangle$$

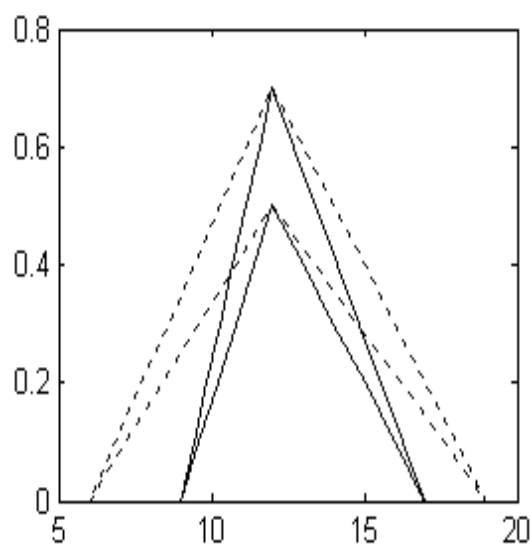
$$(iv) \quad \tilde{A} \oslash_T \tilde{B} = \langle [(5,8,13);0.5,0.7]\rangle \oslash_T \langle [(1,4,6);0.5,0.7]\rangle$$

$$=\langle [(1.34,2,6);0.5,0.7]\rangle$$

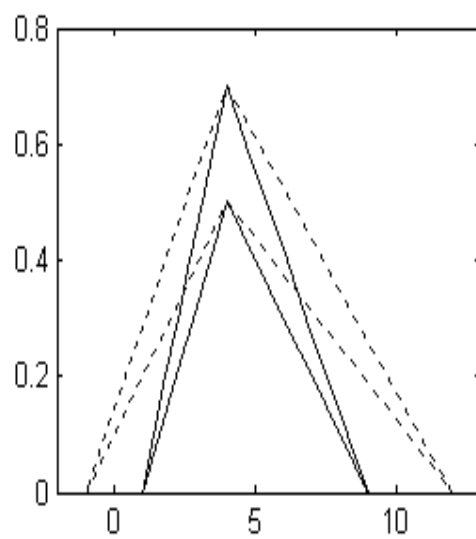
The results, obtained in Case I and Case II are shown in Figure 5.1. It is obvious from the result that the spread of the vague set is increasing in case of traditional arithmetic operations [25] but in T_ω -based operation [62] the spread is controlled.

.....Using existing arithmetic [25]

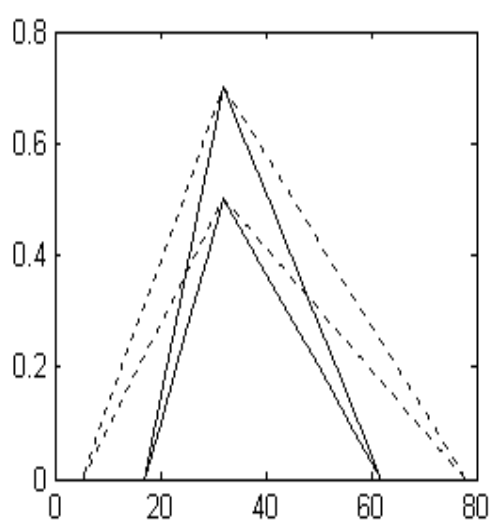
— Using T_ω -based vague arithmetic [62]



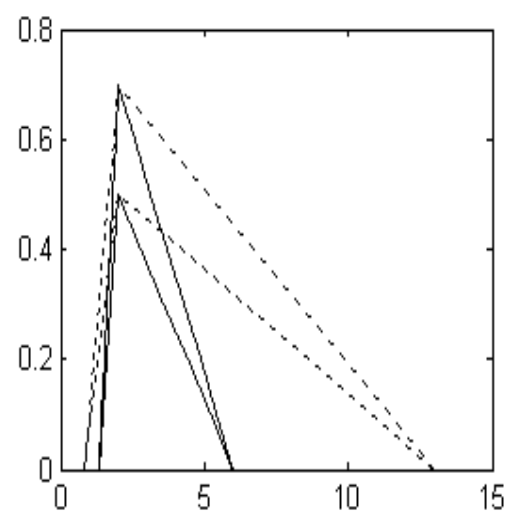
(a) Addition



(b) Subtraction



(c) Multiplication



(d) Division

Figure 5.1: Triangular vague set

5.2 PROPOSED APPROACH

In this section, a new approach, by modifying the existing approach, presented in Section 3.2, is proposed for evaluating the vague sets representing of vague reliability parameters.

The steps of proposed approach are as follows:

Step 1: Use Step 1 to Step 3 of the existing approach, presented in Section 3.2, to find the expressions for evaluating failure rate λ_s and repair rate τ_s of the chosen system.

Step 2: Construct the vague sets $\tilde{\lambda}_j = \langle [(\lambda_{j1}, \lambda_{j2}, \lambda_{j3}); \mu_j, \nu_j] \rangle$ and $\tilde{\tau}_j = \langle [(\tau_{j1}, \tau_{j2}, \tau_{j3}); \mu'_j, \nu'_j] \rangle$ to represent the uncertain failure rate (λ_s) and uncertain repair rate (τ_s) of the j^{th} basic event.

Step 3: Replace λ_j and τ_j presented in the expression, obtained in Step 1, by $\tilde{\lambda}_j$ and $\tilde{\tau}_j$ respectively and use T_ω -based arithmetic operations, presented in Section 5.1, to calculate $\tilde{\lambda}_s$ and $\tilde{\tau}_s$.

Step 4: Replace λ_s and τ_s presented in the expressions, presented in Table 3.2, by $\tilde{\lambda}_s$ and $\tilde{\tau}_s$ obtained from Step 2 and use T_ω -based arithmetic operations, presented in Section 5.1, to calculate the vague sets representing the vague reliability parameters.

5.3 EVALUATION OF VAGUE RELIABILITY PARAMETERS OF A GAS TURBINE

In this section, to show the application of proposed approach, vague reliability parameters of gas turbine are evaluated.

Step 1: Using Step 1 of the proposed approach, presented in Section 5.2, the expressions for evaluating failure rate (λ) and repair rate (τ) of the basic events of the gas turbine are:

$$\lambda_s = \sum \lambda_j \text{ and } \tau_s = \frac{\sum \lambda_j \tau_j}{\sum \lambda_j}$$

where, λ_j and τ_j are failure rates and repair rates of basic events of gas turbine.

Step 2: Using Step 2 of the proposed approach, presented in Section 5.2, the expressions for evaluating vague failure rate ($\tilde{\lambda}$) and vague repair rate ($\tilde{\tau}$) of the basic events of the gas turbine are:

$$\tilde{\lambda}_s = \sum \tilde{\lambda}_j \text{ and } \tilde{\tau}_s = \frac{\sum \tilde{\lambda}_j \tilde{\tau}_j}{\sum \tilde{\lambda}_j}$$

where, $\tilde{\lambda}_j$ and $\tilde{\tau}_j$ are vague failure rates and vague repair rates of basic events of gas turbine.

Step 3: Using Step 3, of the proposed approach, presented in Section 5.2, the vague sets representing the vague reliability parameters are:

$$\tilde{\lambda}_s = \langle [(0.01776, 0.01821, 0.01866); 0.5, 0.8] \rangle$$

$$\tilde{\tau}_s = \langle [(5.73, 5.97, 6.22); 0.5, 0.8] \rangle$$

$$\tilde{R}_s = \langle [(0.98151, 0.98195, 0.98239); 0.5, 0.8] \rangle$$

$$\tilde{A}_s = \langle [(0.98151, 0.98195, 0.98239); 0.5, 0.8] \rangle$$

$$\text{ENOF}_s = \langle [(0.01761, 0.01805, 0.01849); 0.5, 0.8] \rangle$$

$$MTBF_s = \langle [(59.32, 60.89, 62.53); 0.5, 0.8] \rangle$$

The triangular vague sets, representing the vague reliability parameters, obtained by using the approach, proposed in Chapter 3 and the approach, proposed in this chapter, are shown in Figure 5.2 and the variation in reliability with respect to time is shown in Figure 5.3.

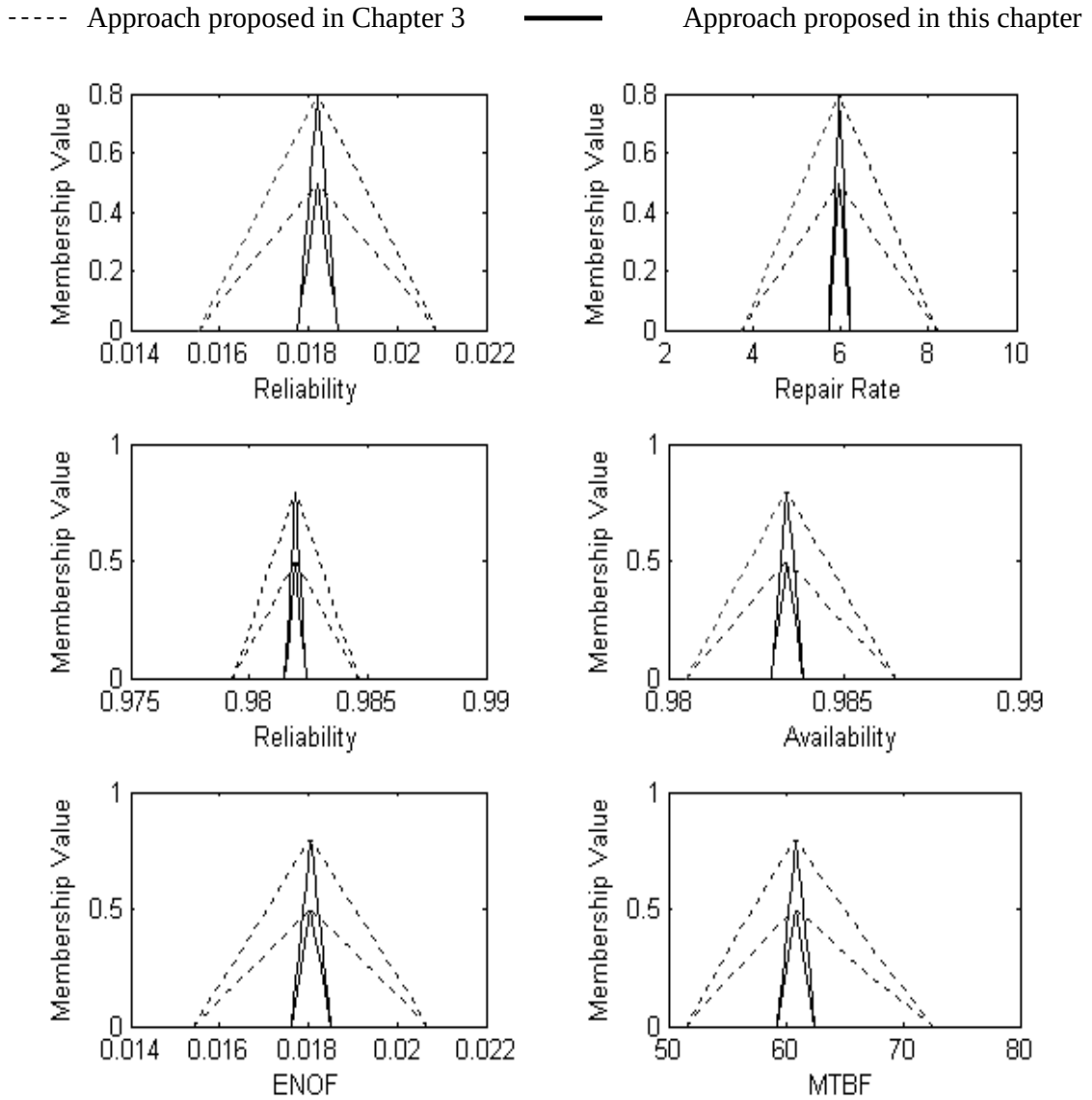


Figure 5.2: Vague reliability parameters of the gas turbine

Table 5.1: Comparison of the results obtained by using existing and proposed approaches corresponding to different membership values

α	Proposed in Chapter 3					Proposed in this chapter				
	a	a'	b	c'	c	a	a'	b	c'	c
0	0.97930	0.97930	0.98195	0.98460	0.98460	0.98151	0.98151	0.98195	0.98239	0.98239
0.1	0.97983	0.97963	0.98195	0.98427	0.98407	0.98160	0.98156	0.98195	0.98234	0.98230
0.2	0.98036	0.97996	0.98195	0.98394	0.98354	0.98168	0.98162	0.98195	0.98228	0.98221
0.3	0.98089	0.98029	0.98195	0.98361	0.98301	0.98177	0.98167	0.98195	0.98223	0.98213
0.4	0.98142	0.98063	0.98195	0.98328	0.98248	0.98186	0.98173	0.98195	0.98217	0.98204
0.5	0.98195	0.98096	0.98195	0.98294	0.98195	0.98195	0.98178	0.98195	0.98212	0.98195
0.6	--	0.98129	0.98195	0.98261	--	0.9793	0.98184	0.98195	0.98206	--
0.7	--	0.98162	0.98195	0.98228	--	--	0.98189	0.98195	0.98201	--
0.8	--	0.98195	0.98195	0.98195	--	--	0.98195	0.98195	0.98195	--
0.9	--	--	--	--	--	--	--	--	--	--
1.0	--	--	--	--	--	--	--	--	--	--

----- Approach proposed in Chapter 3 ——— Approach proposed in this chapter

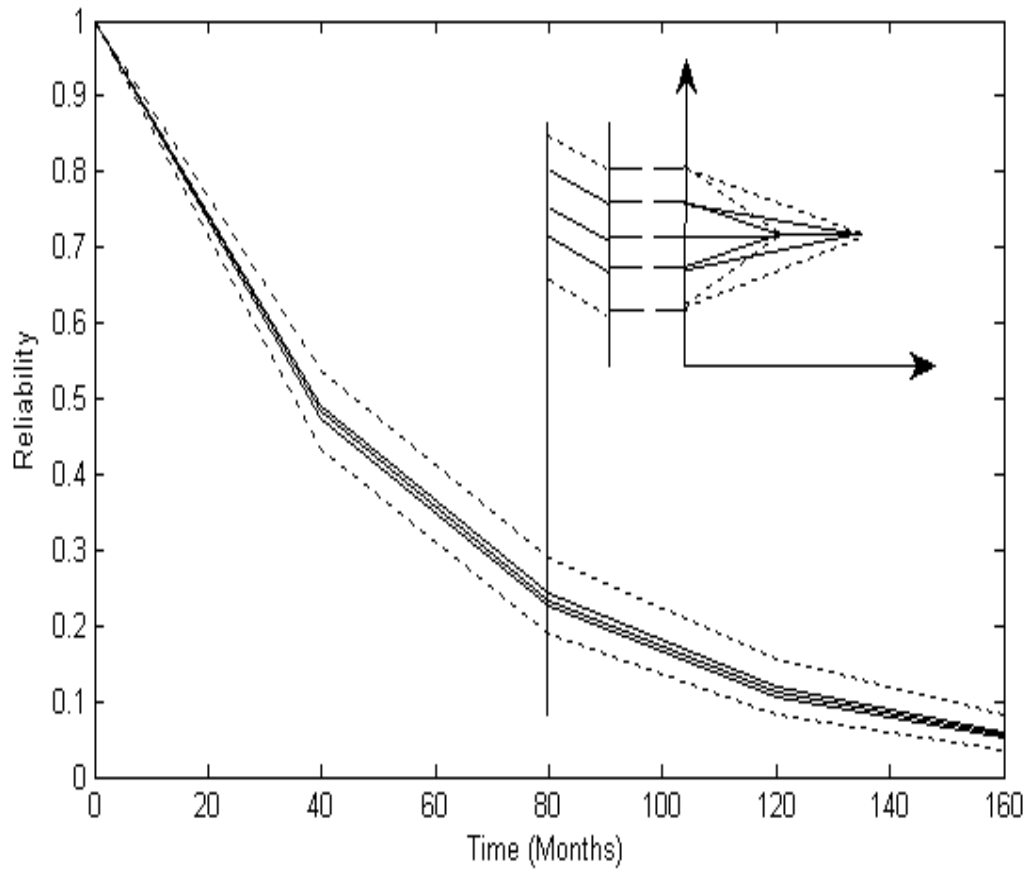


Figure 5.3: Reliability of gas turbine over the time interval.

5.4 COMPARATIVE STUDY

The interval of confidence for the vague reliability of gas turbine, obtained by using the approach, proposed in Chapter 3, and the by using approach, proposed in this Chapter, are shown in Table 5.1.

It is obvious from the results shown in Table 5.1, Figure 5.2, and Figure 5.3, that for the same value of α the length of interval of confidence of the vague reliability parameters, obtained by using the approach proposed in Chapter 3, is larger than the length of interval of

confidence of the vague reliability parameters, obtained by using the approach proposed in this Chapter, i.e., in the results, obtained by using the approach, proposed in this Chapter, the vagueness about the reliability is less than in the results obtained by using the approach proposed in Chapter 3.

5.5 CONCLUSIONS

On the basis of the presented study, it can be concluded it is better to use the approach, proposed in this chapter, as compared to approach, proposed in Chapter 3, for evaluating the vague reliability parameters of industrial systems.

CHAPTER 6

MODIFIED APPROACH FOR EVALUATING THE VAGUE RELIABILITY OF PCBA WITH DIFFERENT MEMBERSHIP FUNCTION^{**}

Chang and Cheng [19] pointed out that there is no approach in the literature for finding the arithmetic operations between different types of vague sets and proposed a approach for the same. Chang and Cheng [19] also used the proposed approach for evaluating the vague reliability of a Printed Circuit Board Assembly (PCBA). In this Chapter, the shortcomings of the approach, proposed by Chang and Cheng [19], are pointed out and to resolve the shortcomings of existing approach [19], a new approach is proposed for finding the arithmetic operations between different types of vague sets. Also, it is pointed out that due to the shortcomings of the existing approach [19], the vague reliability of PCBA, obtained by Chang and Cheng [19], is not exact and the proposed approach is used to evaluate the exact vague reliability of PCBA.

6.1 SHORTCOMINGS OF EXISTING APPROACH

Chang and Cheng [19] pointed out that there is no approach in the literature for finding the arithmetic operation between different types of vague sets and proposed a approach for the same.

^{**} The contents of this Chapter are communicated in *Applied Soft Computing*.

In this section, the existing approach [19] is presented and with the help of a numerical example, it is shown that, it is not genuine to use the existing approach [19] for finding the arithmetic operations between different types of vague sets.

6.1.1 EXISTING APPROACH

Let $\tilde{V}_1 = \left\langle \left[(a_1, b_1, c_1; \mu_1) \right], \left[(a'_1, b_1, c'_1; \nu_1) \right] \right\rangle_{L_1 R_1}$ and

$\tilde{V}_2 = \left\langle \left[(a_2, b_2, c_2; \mu_2) \right], \left[(a'_2, b_2, c'_2; \nu_2) \right] \right\rangle_{L_2 R_2}$ be two different types of LR type vague sets [62].

Then, using the existing approach [19], the value of $\tilde{V}_1 * \tilde{V}_2$, where ‘*’ represents four arithmetic operations: addition, subtraction, multiplication and division can be obtained as follows:

Step 1: Calculate the interval of confidence $\left[b_1 - \frac{(b_1 - a_1)}{\mu_1} L_1^{-1}(\alpha), b_1 + \frac{(c_1 - b_1)}{\mu_1} R_1^{-1}(\alpha) \right]$ and

$\left[b_2 - \frac{(b_2 - a_2)}{\mu_2} L_2^{-1}(\alpha), b_2 + \frac{(c_2 - b_2)}{\mu_2} R_2^{-1}(\alpha) \right]$ corresponding to truth membership function of

vague sets $\tilde{V}_1$ and $\tilde{V}_2$ respectively, where $\alpha \in [0, \text{minimum}\{\mu_1, \mu_2\}]$.

Step 2: Calculate the interval of confidence $\left[b_1 - \frac{(b_1 - a'_1)}{\nu_1} L_1^{-1}(\alpha), b_1 + \frac{(c'_1 - b_1)}{\nu_1} R_1^{-1}(\alpha) \right]$ and

$\left[b_2 - \frac{(b_2 - a'_2)}{\nu_2} L_2^{-1}(\alpha), b_2 + \frac{(c'_2 - b_2)}{\nu_2} R_2^{-1}(\alpha) \right]$ corresponding to complement of false

membership function of vague sets $\tilde{V}_1$ and $\tilde{V}_2$ respectively, where $\alpha \in [0, \text{minimum}\{\nu_1, \nu_2\}]$.

Step 3: Calculate the interval of confidence

$\left[b_1 - \frac{(b_1 - a_1)}{\mu_1} L_1^{-1}(\alpha), b_1 + \frac{(c_1 - b_1)}{\mu_1} R_1^{-1}(\alpha) \right] * \left[b_2 - \frac{(b_2 - a_2)}{\mu_2} L_2^{-1}(\alpha), b_2 + \frac{(c_2 - b_2)}{\mu_2} R_2^{-1}(\alpha) \right]$

corresponding to truth membership function of $\tilde{V}_1 * \tilde{V}_2$, where $\alpha \in [0, \min\{\mu_1, \mu_2\}]$.

Step 4: Calculate the interval of confidence

$$\left[b_1 - \frac{(b_1 - a'_1)}{\nu_1} L_1^{-1}(\alpha), b_1 + \frac{(c'_1 - b_1)}{\nu_1} R_1^{-1}(\alpha) \right] * \left[b_2 - \frac{(b_2 - a'_2)}{\nu_2} L_2^{-1}(\alpha), b_2 + \frac{(c'_2 - b_2)}{\nu_2} R_2^{-1}(\alpha) \right]$$

corresponding to complement of false membership function of $\tilde{V}_1 * \tilde{V}_2$, where $\alpha \in [0, \text{minimum}\{\nu_1, \nu_2\}]$.

6.1.2 EXISTING APPROACH FOR FINDING THE ARITHMETIC OPERATIONS BETWEEN TRIANGULAR VAGUE SETS

In this section, the existing approach [19] is illustrated with the help of two triangular vague sets.

$$\text{Let } \tilde{V}_1 = \langle [(a_1, b_1, c_1; \mu_1)], [(a'_1, b_1, c'_1; \nu_1)] \rangle \text{ and } \tilde{V}_2 = \langle [(a_2, b_2, c_2; \mu_2)], [(a'_2, b_2, c'_2; \nu_2)] \rangle$$

be two triangular vague sets. Then, using the existing approach [19], the value of $\tilde{V}_1 * \tilde{V}_2$, where ‘*’ represents four arithmetic operations: addition, subtraction, multiplication and division can be obtained as follows:

Step 1: Calculate the interval of confidence $\left[a_1 + (b_1 - a_1) \frac{\alpha}{\mu_1}, c_1 - (c_1 - b_1) \frac{\alpha}{\mu_1} \right]$ and

$$\left[a_2 + (b_2 - a_2) \frac{\alpha}{\mu_2}, c_2 - (c_2 - b_2) \frac{\alpha}{\mu_2} \right] \text{ corresponding to truth membership function of vague}$$

sets $\tilde{V}_1$ and $\tilde{V}_2$ respectively, where $\alpha \in [0, \text{minimum}\{\mu_1, \mu_2\}]$.

Step 2: Calculate the interval of confidence $\left[a'_1 + (b_1 - a'_1) \frac{\alpha}{\nu_1}, c'_1 - (c'_1 - b_1) \frac{\alpha}{\nu_1} \right]$ and

$$\left[a'_2 + (b_2 - a'_2) \frac{\alpha}{\nu_2}, c'_2 - (c'_2 - b_2) \frac{\alpha}{\nu_2} \right] \text{ corresponding to complement of false membership}$$

function of vague sets $\tilde{V}_1$ and $\tilde{V}_2$ respectively, where $\alpha \in [0, \text{minimum}\{\nu_1, \nu_2\}]$.

Step 3: Calculate the interval of confidence

$$\left[a_1 + (b_1 - a_1) \frac{\alpha}{\mu_1}, c_1 - (c_1 - b_1) \frac{\alpha}{\mu_1} \right] * \left[a_2 + (b_2 - a_2) \frac{\alpha}{\mu_2}, c_2 - (c_2 - b_2) \frac{\alpha}{\mu_2} \right] \text{ corresponding to truth}$$

membership function of $\tilde{V}_1 * \tilde{V}_2$, where $\alpha \in [0, \text{minimum}\{\mu_1, \mu_2\}]$.

Step 4: Calculate the interval of confidence

$$\left[a'_1 + (b_1 - a'_1) \frac{\alpha}{\nu_1}, c'_1 - (c'_1 - b_1) \frac{\alpha}{\nu_1} \right] * \left[a'_2 + (b_2 - a'_2) \frac{\alpha}{\nu_2}, c'_2 - (c'_2 - b_2) \frac{\alpha}{\nu_2} \right] \text{ corresponding to}$$

complement of false membership function of $\tilde{V}_1 * \tilde{V}_2$, where $\alpha \in [0, \text{minimum}\{\nu_1, \nu_2\}]$.

6.1.3 ILLUSTRATIVE EXAMPLE

If $\tilde{V}_1 = \langle [(3, 5, 7; 0.6)], [(1, 5, 11; 0.8)] \rangle$ and $\tilde{V}_2 = \langle [(4, 7, 9; 0.7)], [(2, 7, 10; 0.9)] \rangle$ are two triangular vague sets, then using the existing approach, presented in Section 6.1.2, the interval of confidence corresponding to truth membership function and complement of false membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ at $\alpha = 0.6$ can be obtained as follows:

Step 1: Using Step 1 of the existing approach, presented in Section 6.1.2, the interval of confidence corresponding to truth membership function of $\tilde{V}_1$ and $\tilde{V}_2$ at $\alpha = 0.6$ are $[5, 5]$ and $[6.5714, 7.2857]$ respectively.

Step 2: Using Step 2 of the existing approach, presented in Section 6.1.2, the interval of confidence corresponding to complement of false membership function of $\tilde{V}_1$ and $\tilde{V}_2$ at $\alpha = 0.6$ are $[4, 6.5]$ and $[5.3333, 8]$ respectively.

Step 3: Using Step 3 of the existing approach, presented in Section 6.1.2, the interval confidence corresponding to truth membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ at $\alpha = 0.6$ is $[11.5714, 12.2857]$.

Step 4: Using Step 4 of the existing approach, presented in Section 6.1.2, the interval of confidence corresponding to complement of false membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ at $\alpha = 0.6$ is $[9.3333, 14.5]$.

6.1.4 SHORTCOMINGS OF OBTAINED RESULTS

In this section, the interval of confidence corresponding to truth membership function as well as complement of false membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ at $\alpha = 0.6$ are obtained by using the existing approach [17] and shown that the result obtained by using the existing approach [17] is not same as the result obtained by using the existing approach [19].

Step 1: Using existing arithmetic operations between triangular vague sets [17], the value of $\tilde{V}_1 \oplus \tilde{V}_2$ is $\langle [(7, 12, 16; 0.6)], [(3, 12, 21; 0.8)] \rangle$.

Step 2: The interval of confidence corresponding to truth membership function and complement of false membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ are

$$\left[7 + \frac{5\alpha}{0.6}, 16 - \frac{4\alpha}{0.6} \right] \text{ and } \left[3 + \frac{9\alpha}{0.8}, 21 - \frac{9\alpha}{0.8} \right] \text{ respectively.}$$

Step 3: The interval of confidence corresponding to truth membership function and complement of false membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ at $\alpha = 0.6$ are $[12, 12]$ and $[9.75, 14.25]$ respectively.

It is obvious from the results, obtained in Section 6.1.3 and Section 6.1.4, that the interval of confidence corresponding to truth membership function and complement of false membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ at $\alpha = 0.6$ obtained by using the existing approach [19] is not same as obtained by using existing approach [17].

6.2 PROPOSED APPROACH

In this section, to overcome the shortcoming of the existing approach, discussed in Section 6.1.4, a new approach by modifying existing approach [19] is proposed for finding the arithmetic operations between different types of vague sets.

The steps of the proposed approach are as follows:

Step 1: Calculate the interval of confidence $\left[b_1 - \frac{(b_1 - a_1)}{\mu_N} L_1^{-1}(\alpha), b_1 + \frac{(c_1 - b_1)}{\mu_N} R_1^{-1}(\alpha) \right]$ and

$\left[b_2 - \frac{(b_2 - a_2)}{\mu_N} L_2^{-1}(\alpha), b_2 + \frac{(c_2 - b_2)}{\mu_N} R_2^{-1}(\alpha) \right]$ corresponding to truth membership function of

vague sets $\tilde{V}_1$ and $\tilde{V}_2$ respectively, where $\mu_N = \text{minimum}\{\mu_1, \mu_2\}$, $\alpha \in [0, \mu_N]$

Step 2: Calculate the interval of confidence $\left[b_1 - \frac{(b_1 - a'_1)}{\nu_N} L_1^{-1}(\alpha), b_1 + \frac{(c'_1 - b_1)}{\nu_N} R_1^{-1}(\alpha) \right]$ and

$\left[b_2 - \frac{(b_2 - a'_2)}{\nu_N} L_2^{-1}(\alpha), b_2 + \frac{(c'_2 - b_2)}{\nu_N} R_2^{-1}(\alpha) \right]$ corresponding to complement of false

membership function of vague sets $\tilde{V}_1$ and $\tilde{V}_2$ respectively, where $\nu_N = \text{minimum}\{\nu_1, \nu_2\}$,

$\alpha \in [0, \nu_N]$.

Step 3: Calculate the interval of confidence

$$\left[b_1 - \frac{(b_1 - a_1)}{\mu_N} L_1^{-1}(\alpha), b_1 + \frac{(c_1 - b_1)}{\mu_N} R_1^{-1}(\alpha) \right] * \left[b_2 - \frac{(b_2 - a_2)}{\mu_N} L_2^{-1}(\alpha), b_2 + \frac{(c_2 - b_2)}{\mu_N} R_2^{-1}(\alpha) \right]$$

corresponding to truth membership function of $\tilde{V}_1 * \tilde{V}_2$, where $\mu_N = \min\{\mu_1, \mu_2\}$ and $\forall \alpha \in [0, \mu_N]$.

Step 4: Calculate the interval of confidence

$$\left[b_1 - \frac{(b_1 - a'_1)}{\nu_N} L_1^{-1}(\alpha), b_1 + \frac{(c'_1 - b_1)}{\nu_N} R_1^{-1}(\alpha) \right] * \left[b_2 - \frac{(b_2 - a'_2)}{\nu_N} L_2^{-1}(\alpha), b_2 + \frac{(c'_2 - b_2)}{\nu_N} R_2^{-1}(\alpha) \right]$$

corresponding complement of false membership function of $\tilde{V}_1 * \tilde{V}_2$, where $\nu_N = \min\{\nu_1, \nu_2\}$, $\alpha \in [0, \nu_N]$.

6.3 ADVANTAGE OF PROPOSED APPROACH

The main advantage of the proposed approach over existing approach [19] is that on using the proposed approach for finding the arithmetic operation between different types of vague sets all the shortcomings, occurring in the results, due to applying the existing approach [19] are resolved.

6.3.1 PROPOSED APPROACH FOR FINDING THE ARITHMETIC OPERATIONS BETWEEN TRIANGULAR VAGUE SETS

In this section, with help of proposed approach, a new approach is proposed for finding the arithmetic operations between triangular vague sets.

The steps of the proposed approach are as follows:

Step 1: Calculate the interval of confidence $\left[a_1 + (b_1 - a_1) \frac{\alpha}{\mu_N}, c_1 - (c_1 - b_1) \frac{\alpha}{\mu_N} \right]$ and

$\left[a_2 + (b_2 - a_2) \frac{\alpha}{\mu_N}, c_2 - (c_2 - b_2) \frac{\alpha}{\mu_N} \right]$ corresponding to truth membership function of vague

sets $\tilde{V}_1$ and $\tilde{V}_2$ respectively, where $\mu_N = \min\{\mu_1, \mu_2\}$, $\alpha \in [0, \mu_N]$

Step 2: Calculate the interval of confidence $\left[a'_1 + (b_1 - a'_1) \frac{\alpha}{\nu_N}, c'_1 - (c'_1 - b_1) \frac{\alpha}{\nu_N} \right]$ and

$\left[a'_2 + (b_2 - a'_2) \frac{\alpha}{\nu_N}, c'_2 - (c'_2 - b_2) \frac{\alpha}{\nu_N} \right]$ corresponding to complement of false membership

function of vague sets $\tilde{V}_1$ and $\tilde{V}_2$ respectively, where $\nu_N = \text{minimum}\{\nu_1, \nu_2\}$, $\alpha \in [0, \nu_N]$.

Step 3: Calculate the interval of confidence

$\left[a_1 + (b_1 - a_1) \frac{\alpha}{\mu_N}, c_1 - (c_1 - b_1) \frac{\alpha}{\mu_N} \right] * \left[a_2 + (b_2 - a_2) \frac{\alpha}{\mu_N}, c_2 - (c_2 - b_2) \frac{\alpha}{\mu_N} \right]$ corresponding to

truth membership function of $\tilde{V}_1 * \tilde{V}_2$, where $\mu_N = \text{minimum}\{\mu_1, \mu_2\}$ and $\forall \alpha \in [0, \mu_N]$.

Step 4: Calculate the interval of confidence

$\left[a'_1 + (b_1 - a'_1) \frac{\alpha}{\nu_N}, c'_1 - (c'_1 - b_1) \frac{\alpha}{\nu_N} \right] * \left[a'_2 + (b_2 - a'_2) \frac{\alpha}{\nu_N}, c'_2 - (c'_2 - b_2) \frac{\alpha}{\nu_N} \right]$ corresponding

complement of false membership function of $\tilde{V}_1 * \tilde{V}_2$, where $\nu_N = \text{minimum}\{\nu_1, \nu_2\}$,

$\alpha \in [0, \nu_N]$.

6.3.2 ILLUSTRATIVE EXAMPLE

To show the advantage of the proposed approach, the interval of confidence corresponding to truth membership function and complement of false membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ at $\alpha = 0.6$, for which the results obtained by using existing approach [19] are not appropriate, are obtained by the approach, proposed in Section 6.3.1, and shown that all shortcomings occurring in the result due to applying the existing approach [19], is resolved.

Using the proposed approach the interval of confidence corresponding to truth membership function and complement of false membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ at $\alpha = 0.6$ can be obtained as follows:

Step 1: Using Step 1 of the approach, proposed in Section 6.3.1, the interval of confidence corresponding to truth membership function of $\tilde{V}_1$ and $\tilde{V}_2$ at $\alpha=0.6$ are $[5,5]$ and $[7,7]$ respectively.

Step 2: Using Step 2 of the approach, proposed in Section 6.3.1, the interval of confidence corresponding to complement of false membership function of $\tilde{V}_1$ and $\tilde{V}_2$ at $\alpha=0.6$ are $[4,6.5]$ and $[5.75,7.75]$ respectively.

Step 3: Using Step 3 of the approach, proposed in Section 6.3.1, the interval of confidence corresponding to truth membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ at $\alpha=0.6$ is $[12,12]$.

Step 4: Using Step 4 of the approach, proposed in Section 6.3.1, the interval of confidence corresponding to complement of false membership function of $\tilde{V}_1 \oplus \tilde{V}_2$ at $\alpha=0.6$ is $[9.75,14.25]$.

6.4 EXACT VAGUE RELIABILITY OF PCBA

Chang and Cheng [19] used the existing approach, presented in Section 6.1.1, for evaluating the vague reliability of PCBA. However, due to shortcomings of the existing approach, discussed in Section 6.1.4, the vague reliability, obtained by Chang and Cheng [19], is not exact.

In this section, with the help of proposed approach the exact vague reliability of PCBA is evaluated.

6.4.1 FAULT TREE OF PCBA

The fault tree of PCBA, constructed by Chang and Cheng [19], is shown in Figure 6.1. Also, the description of intermediate-events and basic events, provided by Chang and Cheng [19], are presented in Table 6.1 and Table 6.2 respectively.

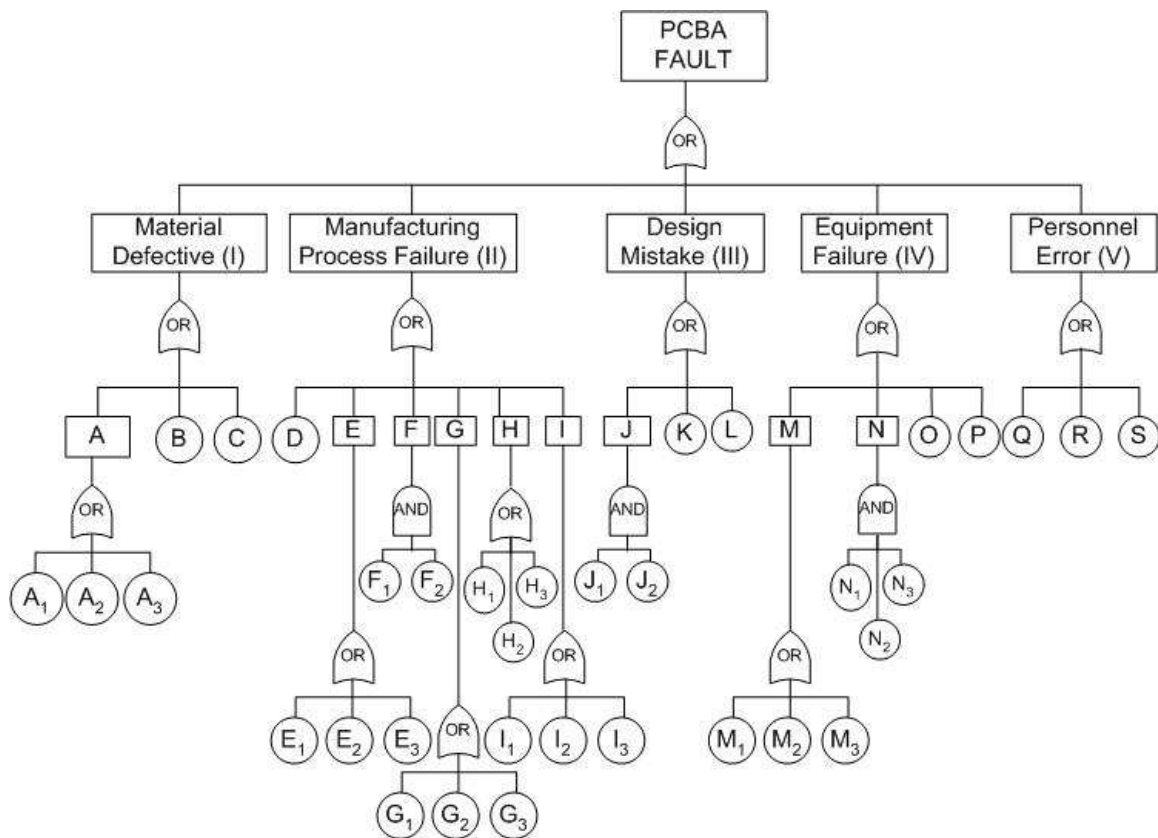


Figure 6.1: Fault tree of PCBA

Table 6.1: The description of the intermediate events.

Code	Fault	Code	Fault	Code	Fault
A	IC lead bend	G	Inappropriate solder paster	J	PCB pad disequilibrium
E	Electrostatic discharge (ESD)	H	Lack of uniformity solder	M	Inaccuracy
F	Improper fixture	I	Printing halftone	N	Optics deviation

Table 6.2: The description of the basic events

Code	Fault	Code	Fault	Code	Fault	Code	Fault
A ₁	Insufficient IQC sampling	F ₁	Lower technology	I ₂	Half-ton bad management	N ₁	Tolerance of brightness prob.
A ₂	Mixed up failure ICs from vendors	F ₂	Tolerance mistake	I ₃	Clean solvent problem	N ₂	Bad preventive maintenance
A ₃	Bad handling operation	G ₁	Without building up standard	J ₁	Lower technology	N ₃	Lower technology
B	Improper electrode oxide	G ₂	Deficient execution	J ₂	Without building up standard	O	Oven Δt problem
C	Improper electroplate	G ₃	Refrigerator capability problem	K	Component interval problem	P	Feeder without pre-maintenance
D	Excessive temperature profile	H ₁	Squeezing speed problem	L	Matching failure	Q	Insufficient training
E ₁	Insufficient skill	H ₂	Squeezing pressure problem	M ₁	Short of preventive maintenance	R	Incorrect set up equipment
E ₂	Improper ground connection	H ₃	Printer Machine Horizontal problem	M ₂	Inadequate machine capability	S	Improper handling
E ₃	Inferior ESD technology	I ₁	Inferior technology	M ₃	Lower technology		

6.4.2 FAILURE PROBABILITIES OF BASIC EVENTS

The vague sets, used by Chang and Cheng [19] for representing the probability of failure of basic events, are listed in Table 6.3.

Table 6.3: Vague sets representing the failure probability of basic events

Vague sets	a_i	a'_i	b_i	c'_i	c_i	μ	ν
$\tilde{A}_1$	0.00058	0.00070	0.0011	0.00124	0.00151	0.7	0.7
$\tilde{A}_2$	0.00072	0.00096	0.0011	0.00134	0.00149	0.9	0.9
$\tilde{A}_3$	0.00289	0.00411	0.00473	0.00513	0.00557	0.9	0.9
$\tilde{B}$	0.00089	0.00111	0.00145	0.00178	0.00192	0.9	0.95
$\tilde{C}$	0.00079	0.00079	0.00099	0.00119	0.00119	1.0	1.0
$\tilde{D}$	0.00067	0.00136	0.00165	0.00194	0.00263	0.8	0.85
$\tilde{E}_1$	0.00092	0.00092	0.0022	0.00348	0.00348	1.0	1.0
$\tilde{E}_2$	0.00048	0.00048	0.00077	0.00106	0.00106	1.0	1.0
$\tilde{E}_3$	0.00005	0.00023	0.00055	0.00087	0.00105	0.6	0.75
$\tilde{F}_1$	0.00068	0.001	0.0011	0.00162	0.00182	0.75	0.85
$\tilde{F}_2$	0.00149	0.00185	0.0022	0.00259	0.00299	0.8	0.9
$\tilde{G}_1$	0.00027	0.0005	0.00061	0.00079	0.00086	0.85	0.9
$\tilde{G}_2$	0.00237	0.00254	0.0032	0.00395	0.00461	0.7	0.8
$\tilde{G}_3$	0.00052	0.00096	0.0013	0.00171	0.00207	0.8	0.95
$\tilde{H}_1$	0.00004	0.00027	0.00055	0.00083	0.00106	0.6	0.75
$\tilde{H}_2$	0.00195	0.00195	0.00297	0.00399	0.00399	1.0	1.0
$\tilde{H}_3$	0.00082	0.00091	0.0011	0.00129	0.00138	0.6	0.8
$\tilde{I}_1$	0.00161	0.00161	0.0033	0.00499	0.00499	1.0	1.0
$\tilde{I}_2$	0.00172	0.00172	0.0033	0.00488	0.00488	1.0	1.0
$\tilde{I}_3$	0.0012	0.0012	0.00242	0.00364	0.00364	1.0	1.0
$\tilde{J}_1$	0.00188	0.00441	0.00462	0.00495	0.00551	0.95	0.95
$\tilde{J}_2$	0.00188	0.00194	0.00264	0.00285	0.00291	1.0	1.0
$\tilde{K}$	0.00022	0.00057	0.00077	0.00097	0.00132	0.75	0.9
$\tilde{L}$	0.00083	0.00106	0.00121	0.00168	0.00205	1.0	1.0
$\tilde{M}_1$	0.00259	0.00461	0.00583	0.00592	0.00617	0.7	0.85

$\tilde{M}_2$	0.00018	0.0002	0.00022	0.00028	0.00045	0.65	0.8
$\tilde{M}_3$	0.00102	0.00119	0.00143	0.00161	0.00193	0.8	0.9
$\tilde{N}_1$	0.00191	0.00197	0.00231	0.00258	0.00273	0.9	0.9
$\tilde{N}_2$	0.00086	0.00104	0.00132	0.00139	0.00142	0.9	0.9
$\tilde{N}_3$	0.00117	0.00145	0.00176	0.00188	0.00251	1.0	1.0
$\tilde{O}$	0.00033	0.00088	0.00121	0.00154	0.00209	0.8	0.9
$\tilde{P}$	0.00211	0.00258	0.00282	0.00306	0.00327	0.7	0.8
$\tilde{Q}$	0.00331	0.00331	0.0044	0.00549	0.00549	1.0	1.0
$\tilde{R}$	0.00116	0.0018	0.00187	0.00194	0.00258	0.65	0.7
$\tilde{S}$	0.00307	0.00307	0.00418	0.00529	0.00529	1.0	1.0

The truth membership function and complement of false membership function, used by Chang and Cheng [19] for vague sets presented in Table 6.3, are as follows:

Basic event A_1 is a triangular type

$$\mu_{\tilde{A}_1}(X_{A_1}) = \begin{cases} 1750X_{A_1} - 1.225, & 0.00070 \leq X_{A_1} \leq 0.00110, \\ -5000X_{A_1} + 6.2, & 0.00110 \leq X_{A_1} \leq 0.00124, \end{cases}$$

$$1 - \nu_{\tilde{A}_1}(X_{A_1}) = \begin{cases} \frac{17500}{13}X_{A_1} - \frac{203}{260}, & 0.00058 \leq X_{A_1} \leq 0.00110, \\ -\frac{70000}{41}X_{A_1} + \frac{1057}{410}, & 0.00110 \leq X_{A_1} \leq 0.00151, \end{cases}$$

Basic event A_2 is a triangular type

$$\mu_{\tilde{A}_2}(X_{A_2}) = \begin{cases} \frac{45000}{7}X_{A_2} - \frac{216}{35}, & 0.00096 \leq X_{A_2} \leq 0.00110, \\ -3750X_{A_2} + \frac{201}{40}, & 0.00110 \leq X_{A_2} \leq 0.00134, \end{cases}$$

$$1 - \nu_{\bar{A}_2}(X_{A_2}) = \begin{cases} \frac{45000}{19}X_{A_2} - \frac{162}{95}, & 0.00072 \leq X_{A_2} \leq 0.00110, \\ \frac{-30000}{13}X_{A_2} + \frac{447}{130}, & 0.00110 \leq X_{A_2} \leq 0.00149, \end{cases}$$

Basic event A_3 is a triangular type

$$\mu_{\bar{A}_3}(X_{A_3}) = \begin{cases} \frac{45000}{31}X_{A_3} - \frac{3699}{620}, & 0.00411 \leq X_{A_3} \leq 0.00473, \\ -2250X_{A_3} + \frac{4617}{400}, & 0.00473 \leq X_{A_3} \leq 0.00513, \end{cases}$$

$$1 - \nu_{\bar{A}_3}(X_{A_3}) = \begin{cases} \frac{11250}{23}X_{A_3} - \frac{2601}{1840}, & 0.00289 \leq X_{A_3} \leq 0.00473, \\ \frac{-22500}{21}X_{A_3} + \frac{5013}{840}, & 0.00473 \leq X_{A_3} \leq 0.00557, \end{cases}$$

Basic event B is a trapezoidal type

$$\mu_{\bar{B}}(X_B) = \begin{cases} 10000X_B - 11.1, & 0.00111 \leq X_B \leq 0.00120, \\ 0.9, & 0.00120 \leq X_B \leq 0.00170, \\ -11250X_B + \frac{801}{40}, & 0.00170 \leq X_B \leq 0.00178, \end{cases}$$

$$1 - \nu_{\bar{B}}(X_B) = \begin{cases} \frac{95000}{31}X_B - \frac{1691}{620}, & 0.00089 \leq X_B \leq 0.00120, \\ 0.95, & 0.00120 \leq X_B \leq 0.00170, \\ -\frac{47500}{11}X_B + \frac{456}{55}, & 0.00170 \leq X_B \leq 0.00192, \end{cases}$$

Basic event C is a normal type

$$\mu_{\bar{C}}(X_C) = \exp\left(-240000000(X_C - 0.00099)^2\right), \quad 0.00079 \leq X_C \leq 0.00119,$$

$$1 - \nu_{\bar{C}}(X_C) = \exp\left(-240000000(X_C - 0.00099)^2\right), \quad 0.00079 \leq X_C \leq 0.00119,$$

Basic event D is a sharp gamma type

$$\mu_{\bar{D}}(X_D) = \begin{cases} \exp(-98000(X_D - 0.00165)), & 0.00136 \leq X_D \leq 0.00165, \\ \exp(-98000(X_D - 0.00165)), & 0.00165 \leq X_D \leq 0.00194, \end{cases}$$

$$1 - \nu_{\bar{D}}(X_D) = \begin{cases} \exp(-35000(X_D - 0.00165)), & 0.00136 \leq X_D \leq 0.00165, \\ \exp(-35000(X_D - 0.00165)), & 0.00165 \leq X_D \leq 0.00263, \end{cases}$$

Basic event E_1 is a normal type

$$\mu_{\bar{E}_1}(X_{E_1}) = \exp(-8000000(X_{E_1} - 0.0022)^2), \quad 0.00092 \leq X_{E_1} \leq 0.00348,$$

$$1 - \nu_{\bar{E}_1}(X_{E_1}) = \exp(-8000000(X_{E_1} - 0.0022)^2), \quad 0.00092 \leq X_{E_1} \leq 0.00348,$$

Basic event E_2 is a normal type

$$\mu_{\bar{E}_2}(X_{E_2}) = \exp(-60000000(X_{E_2} - 0.00077)^2), \quad 0.000477 \leq X_{E_2} \leq 0.00106,$$

$$1 - \nu_{\bar{E}_2}(X_{E_2}) = \exp(-60000000(X_{E_2} - 0.00077)^2), \quad 0.000477 \leq X_{E_2} \leq 0.00106,$$

Basic event E_3 is a normal type

$$\mu_{\bar{E}_3}(X_{E_3}) = \begin{cases} \exp(-105000(X_{E_3} - 0.00055)), & 0.00023 \leq X_{E_3} \leq 0.00055, \\ \exp(-105000(X_{E_3} - 0.00055)), & 0.00055 \leq X_{E_3} \leq 0.00087, \end{cases}$$

$$1 - \nu_{\bar{E}_3}(X_{E_3}) = \begin{cases} \exp(-60000(X_{E_3} - 0.00055)), & 0.00005 \leq X_{E_3} \leq 0.00055, \\ \exp(-60000(X_{E_3} - 0.00055)), & 0.00055 \leq X_{E_3} \leq 0.00105, \end{cases}$$

Basic event F_1 is a triangular type

$$\mu_{\hat{F}_1}(X_{F_1}) = \begin{cases} 7500X_{F_1} - 7.5, & 0.0010 \leq X_{F_1} \leq 0.0011, \\ -\frac{18750}{13}X_{F_1} + \frac{243}{104}, & 0.0011 \leq X_{F_1} \leq 0.00162, \end{cases}$$

$$1 - \nu_{\hat{F}_1}(X_{F_1}) = \begin{cases} 850000X_{F_1} - \frac{11611}{8340}, & 0.000683 \leq X_{F_1} \leq 0.0011, \\ -\frac{10625}{9}X_{F_1} + \frac{1547}{720}, & 0.0011 \leq X_{F_1} \leq 0.00182, \end{cases}$$

Basic event F_2 is a triangular type

$$\mu_{\hat{F}_2}(X_{F_2}) = \begin{cases} \frac{1600}{7}X_{F_2} - \frac{148}{35}, & 0.00185 \leq X_{F_2} \leq 0.0022, \\ -\frac{80000}{39}X_{F_2} + \frac{1036}{195}, & 0.0022 \leq X_{F_2} \leq 0.00259, \end{cases}$$

$$1 - \nu_{\hat{F}_2}(X_{F_2}) = \begin{cases} \frac{90000}{71}X_{F_2} - \frac{1341}{710}, & 0.00149 \leq X_{F_2} \leq 0.0022, \\ -\frac{90000}{79}X_{F_2} + \frac{2691}{790}, & 0.0022 \leq X_{F_2} \leq 0.00299, \end{cases}$$

Basic event G_1 is a trapezoidal type

$$\mu_{\hat{G}_1}(X_{G_1}) = \begin{cases} \frac{106250}{3}X_{G_1} - \frac{527}{30}, & 0.000496 \leq X_{G_1} \leq 0.00052, \\ 0.85, & 0.00052 \leq X_{G_1} \leq 0.00070, \\ -\frac{85000}{9}X_{G_1} + \frac{1343}{180}, & 0.00070 \leq X_{G_1} \leq 0.00079, \end{cases}$$

$$1 - \nu_{\hat{G}_1}(X_{G_1}) = \begin{cases} \frac{900000}{247}X_{G_1} - \frac{2457}{2470}, & 0.000273 \leq X_{G_1} \leq 0.00052, \\ 0.9, & 0.00052 \leq X_{G_1} \leq 0.00070, \\ -5625X_{G_1} + \frac{387}{80}, & 0.00070 \leq X_{G_1} \leq 0.00086, \end{cases}$$

Basic event G_2 is a trapezoidal type

$$\mu_{\tilde{G}_2}(X_{G_2}) = \begin{cases} \frac{35000}{23}X_{G_2} - \frac{889}{230}, & 0.00254 \leq X_{G_2} \leq 0.00300, \\ 0.7, & 0.00300 \leq X_{G_2} \leq 0.00340, \\ -\frac{14000}{9}X_{G_2} + \frac{1343}{180}, & 0.00340 \leq X_{G_2} \leq 0.00461, \end{cases}$$

$$1 - \nu_{\tilde{G}_2}(X_{G_2}) = \begin{cases} \frac{80000}{63}X_{G_2} - \frac{316}{105}, & 0.00237 \leq X_{G_2} \leq 0.00300, \\ 0.8, & 0.00300 \leq X_{G_2} \leq 0.00340, \\ -\frac{80000}{121}X_{G_2} + \frac{1844}{605}, & 0.00340 \leq X_{G_2} \leq 0.00461, \end{cases}$$

Basic event G_3 is a trapezoidal type

$$\mu_{\tilde{G}_3}(X_{G_3}) = \begin{cases} 20000X_{G_3} - \frac{96}{5}, & 0.00096 \leq X_{G_3} \leq 0.0010, \\ 0.8, & 0.0010 \leq X_{G_3} \leq 0.0016, \\ -\frac{80000}{11}X_{G_3} + \frac{684}{55}, & 0.0016 \leq X_{G_3} \leq 0.00171, \end{cases}$$

$$1 - \nu_{\tilde{G}_3}(X_{G_3}) = \begin{cases} \frac{950000}{483}X_{G_3} - \frac{9823}{9660}, & 0.000517 \leq X_{G_3} \leq 0.0010, \\ 0.95, & 0.0010 \leq X_{G_3} \leq 0.0016, \\ -\frac{95000}{47}X_{G_3} + \frac{3933}{940}, & 0.0016 \leq X_{G_3} \leq 0.00207, \end{cases}$$

Basic event H_1 is a sharp gamma type

$$\mu_{\tilde{H}_1}(X_{H_1}) = \begin{cases} \exp(-103000(X_{H_1} - 0.00055)), & 0.00027 \leq X_{H_1} \leq 0.00055, \\ \exp(-103000(X_{H_1} - 0.00055)), & 0.0005 \leq X_{H_1} \leq 0.00083, \end{cases}$$

$$1 - \nu_{\tilde{H}_1}(X_{H_1}) = \begin{cases} \exp(-60000(X_{H_1} - 0.00055)), & 0.00004 \leq X_{H_1} \leq 0.00055, \\ \exp(-60000(X_{H_1} - 0.00055)), & 0.0005 \leq X_{H_1} \leq 0.00106, \end{cases}$$

Basic event H_2 is a normal type

$$\mu_{\tilde{H}_2}(X_{H_2}) = \exp\left(-3600000(X_{H_2} - 0.00297)^2\right), \quad 0.00195 \leq X_{H_2} \leq 0.00399,$$

$$1 - \nu_{\tilde{H}_2}(X_{H_2}) = \exp\left(-3600000(X_{H_2} - 0.00297)^2\right), \quad 0.00195 \leq X_{H_2} \leq 0.00399,$$

Basic event H_3 is a sharp gamma type

$$\mu_{\tilde{H}_3}(X_{H_3}) = \begin{cases} \exp(-110000(X_{H_3} - 0.0011)), & 0.00091 \leq X_{H_3} \leq 0.00110, \\ \exp(-110000(X_{H_3} - 0.0011)), & 0.0011 \leq X_{H_3} \leq 0.00129, \end{cases}$$

$$1 - \nu_{\tilde{H}_3}(X_{H_3}) = \begin{cases} \exp(-50000(X_{H_3} - 0.0011)), & 0.00082 \leq X_{H_3} \leq 0.00110, \\ \exp(-50000(X_{H_3} - 0.0011)), & 0.0011 \leq X_{H_3} \leq 0.00138, \end{cases}$$

Basic event I_1 is a normal type

$$\mu_{\tilde{I}_1}(X_{I_1}) = \exp\left(-5000000(X_{I_1} - 0.0033)^2\right), \quad 0.00161 \leq X_{I_1} \leq 0.0033,$$

$$1 - \nu_{\tilde{I}_1}(X_{I_1}) = \exp\left(-5000000(X_{I_1} - 0.0033)^2\right), \quad 0.00161 \leq X_{I_1} \leq 0.00499,$$

Basic event I_2 is a normal type

$$\mu_{\tilde{I}_2}(X_{I_2}) = \exp\left(-4000000(X_{I_2} - 0.0033)^2\right), \quad 0.00172 \leq X_{I_2} \leq 0.00488,$$

$$1 - \nu_{\tilde{I}_2}(X_{I_2}) = \exp\left(-4000000(X_{I_2} - 0.0033)^2\right), \quad 0.00172 \leq X_{I_2} \leq 0.00488,$$

Basic event I_3 is a normal type

$$\mu_{\tilde{I}_3}(X_{I_3}) = \exp\left(-9000000(X_{I_3} - 0.00242)^2\right), \quad 0.00120 \leq X_{I_3} \leq 0.00364,$$

$$1 - \nu_{\bar{I}_3}(X_{I_3}) = \exp\left(-9000000(X_{I_3} - 0.00242)^2\right), \quad 0.00120 \leq X_{I_3} \leq 0.00364,$$

Basic event J_1 is a triangular type

$$\mu_{J_1}(X_{J_1}) = \begin{cases} \frac{95000}{21}X_{J_1} - \frac{8379}{420}, & 0.00441 \leq X_{J_1} \leq 0.00462, \\ -\frac{95000}{33}X_{J_1} + \frac{57}{4}, & 0.00462 \leq X_{J_1} \leq 0.00495, \end{cases}$$

$$1 - \nu_{J_1}(X_{J_1}) = \begin{cases} \frac{47500}{137}X_{J_1} - \frac{893}{1370}, & 0.00188 \leq X_{J_1} \leq 0.00462, \\ -\frac{95000}{33}X_{J_1} + \frac{10469}{1780}, & 0.00462 \leq X_{J_1} \leq 0.00551, \end{cases}$$

Basic event J_2 is a triangular type

$$\mu_{J_2}(X_{J_2}) = \begin{cases} \frac{10000}{7}X_{J_2} - \frac{97}{35}, & 0.00194 \leq X_{J_2} \leq 0.00264, \\ -\frac{100000}{21}X_{J_2} + \frac{95}{7}, & 0.00264 \leq X_{J_2} \leq 0.00285, \end{cases}$$

$$1 - \nu_{J_2}(X_{J_2}) = \begin{cases} \frac{25000}{19}X_{J_2} - \frac{47}{19}, & 0.00188 \leq X_{J_2} \leq 0.00264, \\ -\frac{100000}{21}X_{J_2} + \frac{97}{9}, & 0.00264 \leq X_{J_2} \leq 0.00291, \end{cases}$$

Basic event K is a sharp gamma type

$$\mu_{\bar{K}}(X_K) = \begin{cases} \exp(-70000(X_K - 0.00077)), & 0.00057 \leq X_K \leq 0.00077, \\ \exp(-70000(X_K - 0.00077)), & 0.00077 \leq X_K \leq 0.00097, \end{cases}$$

$$1 - \nu_{\bar{K}}(X_K) = \begin{cases} \exp(-30000(X_K - 0.00077)), & 0.00022 \leq X_K \leq 0.00077, \\ \exp(-30000(X_K - 0.00077)), & 0.00077 \leq X_K \leq 0.00132, \end{cases}$$

Basic event L is a triangular type

$$\mu_{\tilde{L}}(X_L) = \begin{cases} \frac{200000}{3}X_L - \frac{106}{15}, & 0.00106 \leq X_L \leq 0.00121, \\ -\frac{100000}{47}X_L + \frac{168}{47}, & 0.00121 \leq X_L \leq 0.00168, \end{cases}$$

$$1 - \nu_{\tilde{L}}(X_L) = \begin{cases} \frac{50000}{19}X_L - \frac{83}{38}, & 0.00083 \leq X_L \leq 0.00121, \\ -\frac{25000}{21}X_L + \frac{205}{84}, & 0.00121 \leq X_L \leq 0.00205, \end{cases}$$

Basic event M_1 is a triangular type

$$\mu_{\tilde{M}_1}(X_{M_1}) = \begin{cases} \frac{35000}{61}X_{M_1} - \frac{3227}{1220}, & 0.00461 \leq X_{M_1} \leq 0.00583, \\ -\frac{70000}{9}X_{M_1} + \frac{2072}{45}, & 0.00583 \leq X_{M_1} \leq 0.00592, \end{cases}$$

$$1 - \nu_{\tilde{M}_1}(X_{M_1}) = \begin{cases} \frac{21250}{81}X_{M_1} - \frac{4403}{6480}, & 0.00259 \leq X_{M_1} \leq 0.00583, \\ -\frac{42500}{17}X_{M_1} + \frac{10489}{680}, & 0.00583 \leq X_{M_1} \leq 0.00617, \end{cases}$$

Basic event M_2 is a triangular type

$$\mu_{\tilde{M}_2}(X_{M_2}) = \begin{cases} \frac{650000}{17}X_{M_2} - \frac{2639}{340}, & 0.000203 \leq X_{M_2} \leq 0.00022, \\ -\frac{32500}{3}X_{M_2} + \frac{91}{30}, & 0.00022 \leq X_{M_2} \leq 0.00028, \end{cases}$$

$$1 - \nu_{\tilde{M}_2}(X_{M_2}) = \begin{cases} \frac{200000}{9}X_{M_2} - \frac{184}{45}, & 0.000184 \leq X_{M_2} \leq 0.00022, \\ -\frac{80000}{23}X_{M_2} + \frac{36}{23}, & 0.00022 \leq X_{M_2} \leq 0.00045, \end{cases}$$

Basic event M_3 is a triangular type

$$\mu_{\tilde{M}_3}(X_{M_3}) = \begin{cases} \frac{10000}{3}X_{M_3} - \frac{119}{30}, & 0.00119 \leq X_{M_3} \leq 0.00143, \\ -\frac{40000}{9}X_{M_3} + \frac{322}{45}, & 0.00143 \leq X_{M_3} \leq 0.00161, \end{cases}$$

$$1-\nu_{\tilde{M}_3}(X_{M_3})=\begin{cases}\frac{90000}{41}X_{M_3}-\frac{459}{205}, 0.00102\leq X_{M_3}\leq 0.00143, \\ -1800X_{M_3}+\frac{1737}{500}, 0.00143\leq X_{M_3}\leq 0.00193,\end{cases}$$

Basic event N_1 is a triangular type

$$\mu_{\tilde{N}_1}(X_{N_1})=\begin{cases}\frac{45000}{17}X_{N_1}-\frac{1773}{340}, 0.00197\leq X_{N_1}\leq 0.00231, \\ -\frac{10000}{3}X_{N_1}+\frac{129}{15}, 0.00231\leq X_{N_1}\leq 0.00258,\end{cases}$$

$$1-\nu_{\tilde{N}_1}(X_{N_1})=\begin{cases}2250X_{N_1}-\frac{1719}{400}, 0.00191\leq X_{N_1}\leq 0.00231, \\ -\frac{15000}{7}X_{N_1}+\frac{819}{140}, 0.00231\leq X_{N_1}\leq 0.00273,\end{cases}$$

Basic event N_2 is a triangular type

$$\mu_{\tilde{N}_2}(X_{N_2})=\begin{cases}\frac{22500}{7}X_{N_2}-\frac{117}{35}, 0.00104\leq X_{N_2}\leq 0.00132, \\ -\frac{90000}{7}X_{N_2}+\frac{1251}{70}, 0.00132\leq X_{N_2}\leq 0.00139,\end{cases}$$

$$1-\nu_{\tilde{N}_2}(X_{N_2})=\begin{cases}\frac{450000}{229}X_{N_2}-\frac{3879}{2290}, 0.00862\leq X_{N_2}\leq 0.00132, \\ -9000X_{N_2}+\frac{639}{50}, 0.00132\leq X_{N_2}\leq 0.00142,\end{cases}$$

Basic event N_3 is a triangular type

$$\mu_{\tilde{N}_3}(X_{N_3})=\begin{cases}\frac{100000}{31}X_{N_3}-\frac{145}{31}, 0.00145\leq X_{N_3}\leq 0.00176, \\ -\frac{25000}{3}X_{N_3}+\frac{47}{3}, 0.00176\leq X_{N_3}\leq 0.00188,\end{cases}$$

$$1-\nu_{\tilde{N}_3}(X_{N_3})=\begin{cases}\frac{100000}{59}X_{N_3}-\frac{117}{59}, 0.00117\leq X_{N_3}\leq 0.00176, \\ -\frac{4000}{3}X_{N_3}+\frac{251}{75}, 0.00176\leq X_{N_3}\leq 0.00188,\end{cases}$$

Basic event O is a sharp gamma type

$$\mu_{\bar{O}}(X_o) = \begin{cases} \exp(-50000(X_o - 0.00121)), & 0.00088 \leq X_o \leq 0.00121, \\ \exp(-50000(X_o - 0.00121)), & 0.00121 \leq X_o \leq 0.00154, \end{cases}$$

$$1 - \nu_{\bar{O}}(X_o) = \begin{cases} \exp(-30000(X_o - 0.00121)), & 0.00033 \leq X_o \leq 0.00121, \\ \exp(-30000(X_o - 0.00121)), & 0.00121 \leq X_o \leq 0.00209, \end{cases}$$

Basic event P is a trapezoidal type

$$\mu_{\bar{P}}(X_p) = \begin{cases} \frac{17500}{3}X_p - \frac{903}{60}, & 0.00258 \leq X_p \leq 0.00270, \\ 0.7, & 0.00270 \leq X_p \leq 0.00294, \\ -\frac{17500}{3}X_p + \frac{1071}{60}, & 0.00294 \leq X_p \leq 0.00306, \end{cases}$$

$$1 - \nu_{\bar{P}}(X_p) = \begin{cases} \frac{80000}{59}X_p - \frac{844}{295}, & 0.00211 \leq X_p \leq 0.00270, \\ 0.8, & 0.00270 \leq X_p \leq 0.00294, \\ -\frac{80000}{33}X_p + \frac{436}{55}, & 0.00294 \leq X_p \leq 0.00306, \end{cases}$$

Basic event Q is a normal type

$$\mu_{\bar{Q}}(X_q) = \exp(-8000000(X_q - 0.0044)^2), \quad 0.00331 \leq X_q \leq 0.00594,$$

$$1 - \nu_{\bar{Q}}(X_q) = \exp(-8000000(X_q - 0.0044)^2), \quad 0.00331 \leq X_q \leq 0.00594,$$

Basic event R is a sharp gamma type

$$\mu_{\bar{R}}(X_r) = \begin{cases} \exp(-100000(X_r - 0.00187)), & 0.00108 \leq X_r \leq 0.00187, \\ \exp(-100000(X_r - 0.00187)), & 0.00187 \leq X_r \leq 0.00194, \end{cases}$$

$$1-\nu_{\bar{R}}(X_R)=\begin{cases}\exp(-800000(X_R-0.00187)), & 0.00116 \leq X_R \leq 0.00187, \\ \exp(-800000(X_R-0.00187)), & 0.00187 \leq X_R \leq 0.00258, \end{cases}$$

Basic event S is a normal type

$$\mu_{\bar{S}}(X_S)=\exp(-5000000(X_S-0.00418)^2), \quad 0.00307 \leq X_S \leq 0.00529,$$

$$1-\nu_{\bar{S}}(X_S)=\exp(-5000000(X_S-0.00418)^2), \quad 0.00307 \leq X_S \leq 0.00529,$$

6.4.3 EXACT INTERVAL OF CONFIDENCE

Since, it is not genuine to use the existing approach [19] for evaluating the interval of confidence. So, in this section the interval of confidence corresponding of truth membership function as well as complement of false membership function of the vague sets, representing the probability of failure of basic events, at different values of α are obtained by using proposed approach.

6.4.3.1 INTERVAL OF CONFIDENCE CORRESPONDING TO TRUTH MEMBERSHIP FUNCTION

The interval of confidence corresponding to truth membership function of different vague sets at different values of α , obtained by using proposed approach, is shown in Table 6.4.

.

Table 6.4: Interval of confidence corresponding to truth membership function of $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \dots, \tilde{S}$.

α	A_1^α	A_2^α	A_3^α	B^α	C^α	D^α	E_1^α
0.0	[0.00070, 0.00124]	[0.00096, 0.00134]	[0.00411, 0.00513]	[0.00089, 0.00192]	[0.00079, 0.00119]	[0.00136, 0.00194]	[0.00092, 0.00348]
0.1	[0.00076, 0.00121]	[0.00098, 0.00130]	[0.00421, 0.00506]	[0.00093, 0.00188]	[0.00090, 0.00107]	[0.001631, 0.001668]	[0.00172, 0.00267]
0.2	[0.00083, 0.00119]	[0.00100, 0.00126]	[0.00431, 0.00499]	[0.00098, 0.00185]	[0.00092, 0.00105]	[0.001638, 0.001661]	[0.00182, 0.00257]
0.3	[0.00090, 0.00117]	[0.00103, 0.00122]	[0.00442, 0.00493]	[0.00103, 0.00181]	[0.00093, 0.00104]	[0.001642, 0.001657]	[0.00190, 0.00249]
0.4	[0.00096, 0.00114]	[0.00105, 0.00118]	[0.00452, 0.00486]	[0.00108, 0.00178]	[0.00094, 0.00103]	[0.001645, 0.001654]	[0.00197, 0.00242]
0.5	[0.00103, 0.00112]	[0.00107, 0.00114]	[0.00462, 0.00479]	[0.00113, 0.00174]	[0.00096, 0.00101]	[0.001648, 0.001651]	[0.00204, 0.00235]
0.6	[0.00110, 0.00110]	[0.00110, 0.00110]	[0.00473, 0.00473]	[0.00118, 0.00171]	[0.00099, 0.00099]	[0.00165, 0.00165]	[0.0022, 0.0022]
0.7	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.8	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.9	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
1.0	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
α	E_2^α	E_3^α	F_1^α	F_2^α	G_1^α	G_2^α	G_3^α
0.0	[0.00048, 0.00106]	[0.00023, 0.00087]	[0.00100, 0.00162]	[0.00185, 0.00259]	[0.0005; 0.00079]	[0.00254, 0.00395]	[0.00096, 0.00171]
0.1	[0.00059, 0.00094]	[0.000532, 0.000567]	[0.00101, 0.00153]	[0.00190, 0.00252]	[0.000503, 0.000775]	[0.00261, 0.00385]	[0.00096, 0.00169]
0.2	[0.00063, 0.00090]	[0.000539, 0.000560]	[0.00103, 0.00144]	[0.00196, 0.00246]	[0.000506; 0.000760]	[0.00269, 0.00376]	[0.00097, 0.00167]

0.3	[0.00066, 0.00087]	[0.000543, 0.000556]	[0.00104, 0.00135]	[0.00202, 0.00239]	[0.00051; 0.000745]	[0.00277, 0.00367]	[0.00098, 0.00165]
0.4	[0.00068, 0.00085]	[0.000546, 0.000553]	[0.00106, 0.00126]	[0.00208, 0.00233]	[0.000513; 0.00073]	[0.00284, 0.00358]	[0.00098, 0.00163]
0.5	[0.00071, 0.00082]	[0.000548, 0.000551]	[0.00107, 0.00117]	[0.00214, 0.00226]	[0.000516; 0.000715]	[0.00292, 0.00349]	[0.00099, 0.00161]
0.6	[0.00077, 0.00077]	[0.00055, 0.00055]	[0.00109, 0.00109]	[0.00220, 0.00220]	[0.00052; 0.0007]	[0.003, 0.0034]	[0.001, 0.0016]
0.7	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.8	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.9	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
1.0	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
α	H_1^α	H_2^α	H_3^α	I_1^α	I_2^α	I_3^α	J_1^α
0.0	[0.00027, 0.00083]	[0.00195, 0.00399]	[0.00091, 0.00129]	[0.00161, 0.00499]	[0.00172, 0.00488]	[0.00120, 0.00364]	[0.00441, 0.00495]
0.1	[0.000532, 0.000567]	[0.00226, 0.00367]	[0.001083, 0.001116]	[0.00270, 0.00389]	[0.00263, 0.00396]	[0.00197, 0.00286]	[0.00444, 0.00489]
0.2	[0.000539, 0.000560]	[0.00241, 0.00352]	[0.001090, 0.001109]	[0.00283, 0.00376]	[0.00277, 0.00382]	[0.00207, 0.00276]	[0.00448, 0.00484]
0.3	[0.000543, 0.000556]	[0.00253, 0.00340]	[0.001093, 0.001106]	[0.00292, 0.00367]	[0.00288, 0.00371]	[0.00214, 0.00269]	[0.00451, 0.00478]
0.4	[0.000546, 0.000553]	[0.00263, 0.00330]	[0.001096, 0.001103]	[0.00301, 0.00358]	[0.00298, 0.00361]	[0.00220, 0.00263]	[0.00455, 0.00478]
0.5	[0.000548, 0.000551]	[0.00274, 0.00319]	[0.001098, 0.001101]	[0.00310, 0.00349]	[0.00308, 0.00351]	[0.00227, 0.00256]	[0.00458, 0.00467]
0.6	[0.00055, 0.00055]	[0.00297, 0.00297]	[0.0011, 0.0011]	[0.0033, 0.0033]	[0.0033, 0.0033]	[0.00242, 0.00242]	[0.00462, 0.00462]
0.7	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.8	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]

0.9	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
1.0	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
α	J_2^α	K^α	L^α	M_1^α	M_2^α	M_3^α	N_1^α
0.0	[0.00194, 0.00285]	[0.00057, 0.00097]	[0.00106, 0.00168]	[0.00461, 0.00593]	[0.00020, 0.00028]	[0.00119, 0.00161]	[0.00197, 0.00258]
0.1	[0.00205, 0.00281]	[0.000744, 0.000795]	[0.00108, 0.00160]	[0.00481, 0.00591]	[0.00021, 0.00027]	[0.00123, 0.00158]	[0.00202, 0.00253]
0.2	[0.00217, 0.00278]	[0.000754, 0.000785]	[0.00111, 0.00152]	[0.00501, 0.00589]	[0.00021, 0.00026]	[0.00127, 0.00155]	[0.00208, 0.00249]
0.3	[0.00229, 0.00274]	[0.000760, 0.000779]	[0.00113, 0.00144]	[0.00522, 0.00588]	[0.00021, 0.00025]	[0.00131, 0.00152]	[0.00214, 0.00244]
0.4	[0.00240, 0.00271]	[0.000764, 0.000775]	[0.00116, 0.00136]	[0.00542, 0.00586]	[0.00021, 0.00024]	[0.00135, 0.00149]	[0.00219, 0.00240]
0.5	[0.00252, 0.00267]	[0.000767, 0.000772]	[0.00118, 0.00128]	[0.00562, 0.00584]	[0.00022, 0.00023]	[0.00139, 0.00146]	[0.00225, 0.002355]
0.6	[0.00264, 0.00264]	[0.00077, 0.00077]	[0.00121, 0.00121]	[0.00583, 0.00583]	[0.00022, 0.00022]	[0.00143, 0.00143]	[0.00231, 0.00231]
0.7	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.8	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.9	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
1.0	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
α	N_2^α	N_3^α	O^α	P^α	Q^α	R^α	S^α
0.0	[0.00104, 0.00139]	[0.00145, 0.00188]	[0.00088, 0.00154]	[0.00258, 0.00306]	[0.00331, 0.00549]	[0.00180, 0.00194]	[0.00307, 0.00529]
0.1	[0.00108, 0.00137]	[0.00150, 0.00186]	[0.001174, 0.001245]	[0.00260, 0.00304]	[0.00392, 0.00487]	[0.00185, 0.00189]	[0.00358, 0.00477]

0.2	[0.00113, 0.00136]	[0.00155, 0.00184]	[0.001188,0.001231]	[0.00262, 0.00302]	[0.00402, 0.00477]	[0.00185, 0.00189]	[0.00371, 0.00464]
0.3	[0.00118, 0.00135]	[0.00160, 0.00182]	[0.001196,0.001223]	[0.00264, 0.00300]	[0.00410, 0.00469]	[0.00186, 0.00188]	[0.00380, 0.00455]
0.4	[0.00122, 0.00134]	[0.00165, 0.00180]	[0.001201,0.001218]	[0.00266, 0.00298]	[0.00417, 0.00462]	[0.00186, 0.00188]	[0.00389, 0.00446]
0.5	[0.00127, 0.00133]	[0.00170, 0.00178]	[0.001206,0.001213]	[0.00268, 0.00296]	[0.00424, 0.00455]	[0.00186, 0.00188]	[0.00398, 0.00437]
0.6	[0.00132, 0.00132]	[0.00176, 0.00176]	[0.00121, 0.001221]	[0.0027, 0.00294]	[0.0044, 0.0044]	[0.00186, 0.00188]	[0.00418, 0.00418]
0.7	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.8	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.9	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
1.0	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]

Table 6.5: Interval of confidence of corresponding to complement of false membership function of $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \dots, \tilde{S}$.

α	A_1^α	A_2^α	A_3^α	B^α	C^α	D^α	E_1^α
0.0	[0.00058, 0.00151]	[0.00072, 0.00149]	[0.00289, 0.00557]	[0.00089, 0.00192]	[0.00079, 0.00119]	[0.00067, 0.00263]	[0.00093, 0.00348]
0.1	[0.00065, 0.00145]	[0.00077, 0.00143]	[0.00315, 0.00545]	[0.00093, 0.00189]	[0.00089, 0.00108]	[0.001630, 0.001669]	[0.00170, 0.00269]
0.2	[0.00072, 0.00139]	[0.00082, 0.00137]	[0.00350, 0.00533]	[0.00097, 0.00186]	[0.00091, 0.00106]	[0.001637, 0.001662]	[0.00180, 0.002595]
0.3	[0.00080, 0.00133]	[0.00088, 0.00132]	[0.00367, 0.00521]	[0.00101, 0.00183]	[0.00093, 0.00104]	[0.001641, 0.001658]	[0.00187, 0.00252]
0.4	[0.00087, 0.00127]	[0.00093, 0.00126]	[0.00394, 0.00509]	[0.00105, 0.0018]	[0.00094, 0.00103]	[0.001644, 0.001655]	[0.00193, 0.00246]
0.5	[0.00095, 0.00121]	[0.00099, 0.00121]	[0.00420, 0.00497]	[0.00109, 0.00177]	[0.00095, 0.00102]	[0.001646, 0.001653]	[0.00199, 0.00240]
0.6	[0.00102, 0.00115]	[0.00104, 0.00115]	[0.00446, 0.00485]	[0.00113, 0.00174]	[0.00096, 0.00101]	[0.001648, 0.001651]	[0.00206, 0.00233]
0.7	[0.00110, 0.00110]	[0.00110, 0.00110]	[0.00473, 0.00473]	[0.00118, 0.00171]	[0.00099, 0.00099]	[0.00165, 0.00165]	[0.0022, 0.0022]
0.8	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.9	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
1.0	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
α	E_2^α	E_3^α	F_1^α	F_2^α	G_1^α	G_2^α	G_3^α
0.0	[0.00048, 0.00106]	[0.00195, 0.00399]	[0.00082, 0.00138]	[0.00161, 0.00499]	[0.000273; 0.00086]	[0.000237, 0.00461]	[0.000517, 0.00207]
0.1	[0.00058, 0.00095]	[0.000531, 0.000568]	[0.00086, 0.00134]	[0.00185, 0.00474]	[0.000308; 0.000837]	[0.000246, 0.00443]	[0.000586, 0.00200]
0.2	[0.00062, 0.00091]	[0.000538, 0.000561]	[0.0009, 0.0013]	[0.00209, 0.00450]	[0.000343; 0.000814]	[0.000255, 0.004264]	[0.000655, 0.00193]

0.3	[0.00065, 0.00088]	[0.000541, 0.000558]	[0.00094, 0.00126]	[0.00233, 0.00426]	[0.000378; 0.000791]	[0.000264, 0.00409]	[0.000724, 0.00186]
0.4	[0.00067, 0.00086]	[0.000544, 0.000555]	[0.00098, 0.00122]	[0.00257, 0.00402]	[0.000414; 0.000768]	[0.000273, 0.00391]	[0.000793, 0.00180]
0.5	[0.00069, 0.00084]	[0.000546, 0.000553]	[0.00102, 0.00118]	[0.00281, 0.00378]	[0.000449; 0.000745]	[0.000282, 0.00374]	[0.000862, 0.00173]
0.6	[0.00071, 0.00082]	[0.000548, 0.000551]	[0.00106, 0.00114]	[0.00305, 0.00354]	[0.000484; 0.000722]	[0.000291, 0.00357]	[0.000931, 0.00166]
0.7	[0.00077, 0.00077]	[0.00055, 0.00055]	[0.0011, 0.0011]	[0.0033, 0.0033]	[0.00052; 0.0007]	[0.0003, 0.0034]	[0.001, 0.0016]
0.8	[None, none]	[0.00272, 0.00322]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.9	[None, none]	[0.00280, 0.00314]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
1.0	[None, none]	[0.00297, 0.00297]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
α	H_1^α	H_2^α	H_3^α	I_1^α	I_2^α	I_3^α	J_1^α
0.0	[0.00004, 0.00106]	[0.00195, 0.00399]	[0.00082, 0.00138]	[0.00161, 0.00499]	[0.00172, 0.00488]	[0.00120, 0.00364]	[0.00188, 0.00551]
0.1	[0.000517, 0.000582]	[0.00223, 0.00370]	[0.001061, 0.001138]	[0.00267, 0.00392]	[0.00267, 0.00399]	[0.00195, 0.00288]	[0.00227, 0.00538]
0.2	[0.000529, 0.000570]	[0.00238, 0.00355]	[0.001074, 0.001125]	[0.00279, 0.00380]	[0.00279, 0.00385]	[0.00204, 0.00279]	[0.00266, 0.00525]
0.3	[0.000535, 0.000564]	[0.00248, 0.00345]	[0.001083, 0.001116]	[0.00288, 0.00371]	[0.00288, 0.00376]	[0.00211, 0.00272]	[0.00305, 0.00512]
0.4	[0.000540, 0.000559]	[0.00257, 0.00336]	[0.001088, 0.001111]	[0.00296, 0.00363]	[0.00296, 0.00367]	[0.00217, 0.00266]	[0.00344, 0.00500]
0.5	[0.000544, 0.000555]	[0.00266, 0.00327]	[0.001093, 0.001106]	[0.00304, 0.00355]	[0.00304, 0.00359]	[0.00222, 0.00261]	[0.00383, 0.00487]
0.6	[0.000547, 0.000552]	[0.00276, 0.00317]	[0.001096, 0.001103]	[0.00312, 0.00347]	[0.00312, 0.00349]	[0.00228, 0.00255]	[0.00422, 0.00474]
0.7	[0.00055, 0.00055]	[0.00297, 0.00297]	[0.0011, 0.0011]	[0.0033, 0.0033]	[0.0033, 0.0033]	[0.00242, 0.00242]	[0.00462, 0.00462]
0.8	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]

0.9	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
1.0	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
α	J_2^α	K^α	L^α	M_1^α	M_2^α	M_3^α	N_1^α
0.0	[0.00188, 0.00291]	[0.00022, 0.00132]	[0.00083, 0.00205]	[0.00259, 0.00617]	[0.00018, 0.00045]	[0.00102, 0.00193]	[0.00191, 0.00273]
0.1	[0.00198, 0.00287]	[0.000705, 0.000834]	[0.00088, 0.00193]	[0.00302, 0.00609]	[0.000185, 0.000417]	[0.00107, 0.00185]	[0.00196, 0.00267]
0.2	[0.00209, 0.00283]	[0.000728, 0.000811]	[0.00093, 0.00181]	[0.00346, 0.00601]	[0.000191, 0.000384]	[0.00113, 0.00178]	[0.00202, 0.00261]
0.3	[0.00220, 0.00279]	[0.000741, 0.000798]	[0.00099, 0.00169]	[0.00389, 0.00594]	[0.000197, 0.000351]	[0.00119, 0.00171]	[0.00208, 0.00255]
0.4	[0.00231, 0.00275]	[0.000751, 0.000788]	[0.00104, 0.00157]	[0.00433, 0.00586]	[0.000202, 0.000318]	[0.00125, 0.00164]	[0.00213, 0.00249]
0.5	[0.00242, 0.00271]	[0.000758, 0.000781]	[0.00110, 0.00145]	[0.00476, 0.00579]	[0.000208, 0.000285]	[0.00131, 0.00157]	[0.00219, 0.00243]
0.6	[0.00253, 0.00267]	[0.000764, 0.000775]	[0.00115, 0.00133]	[0.00520, 0.00571]	[0.000214, 0.000252]	[0.00137, 0.00150]	[0.00225, 0.00237]
0.7	[0.00264, 0.00264]	[0.00077, 0.00077]	[0.00121, 0.00121]	[0.00564, 0.00564]	[0.00022, 0.00022]	[0.00143, 0.00143]	[0.00231, 0.00231]
0.8	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.9	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
1.0	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
α	N_2^α	N_3^α	O^α	P^α	Q^α	R^α	S^α
0.0	[0.00086, 0.00142]	[0.00117, 0.00251]	[0.00033, 0.00209]	[0.00211, 0.00327]	[0.00331, 0.00549]	[0.00116, 0.00258]	[0.00307, 0.00529]
0.1	[0.00092, 0.00140]	[0.00125, 0.00240]	[0.00114, 0.00127]	[0.00219, 0.00322]	[0.00390, 0.00489]	[0.001845, 0.001894]	[0.00355, 0.00480]

0.2	[0.00099, 0.00139]	[0.00133, 0.00229]	[0.00116, 0.00125]	[0.00227, 0.00317]	[0.00400, 0.00479]	[0.001854, 0.001885]	[0.00367, 0.00468]
0.3	[0.00105, 0.00137]	[0.00142, 0.00218]	[0.00118, 0.00123]	[0.00236, 0.00312]	[0.00407, 0.00472]	[0.001859, 0.001880]	[0.00376, 0.00459]
0.4	[0.00112, 0.00136]	[0.00150, 0.00208]	[0.00119, 0.00122]	[0.00244, 0.00308]	[0.00413, 0.00466]	[0.001863, 0.001876]	[0.00384, 0.00451]
0.5	[0.00118, 0.00134]	[0.00159, 0.00197]	[0.00119, 0.00122]	[0.00253, 0.00303]	[0.00419, 0.00460]	[0.001865, 0.001874]	[0.00392, 0.00443]
0.6	[0.00125, 0.00133]	[0.00167, 0.00186]	[0.00120, 0.00121]	[0.00261, 0.00298]	[0.00426, 0.00453]	[0.001868, 0.001871]	[0.00400, 0.00435]
0.7	[0.00132, 0.00132]	[0.00176, 0.00176]	[0.00121, 0.00121]	[0.0027, 0.00294]	[0.0044, 0.0044]	[0.00187, 0.00187]	[0.00418, 0.00418]
0.8	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
0.9	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]
1.0	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]	[None, none]

6.4.3.2 INTERVAL OF CONFIDENCE CORRESPONDING TO COMPLEMENT OF FALSE MEMBERSHIP FUNCTION

The interval of confidence corresponding to complement of false membership function of different vague sets at different values of α , obtained by using proposed approach, is shown in Table 6.5.

6.4.4 VAGUE RELIABILITY EVALUATION

The exact interval of confidence corresponding to truth membership function and complement of false membership function of the vague reliability at different value of α , obtained by solving equation 6.1, is shown in Table 6.6 and Table 6.7 respectively, and the vague set, representing the exact vague reliability, is shown in Figure 6.2.

$$q_{PCBA}^{\alpha} = 1 - \left((q_I^{\alpha}) (q_{II}^{\alpha}) (q_{III}^{\alpha}) (q_{IV}^{\alpha}) (q_V^{\alpha}) \right) \quad (6.1)$$

where,

$$q_I^{\alpha} = (1 - A_1^{\alpha}) (1 - A_2^{\alpha}) (1 - A_3^{\alpha}) (1 - B^{\alpha}) (1 - C^{\alpha})$$

$$q_{II}^{\alpha} = (1 - D^{\alpha}) (1 - E_1^{\alpha}) (1 - E_2^{\alpha}) (1 - E_3^{\alpha}) (1 - F_1^{\alpha} F_2^{\alpha}) (1 - G_1^{\alpha}) (1 - G_2^{\alpha}) (1 - G_3^{\alpha}) (1 - H_1^{\alpha}) (1 - H_2^{\alpha}) (1 - H_3^{\alpha}) (1 - I_1^{\alpha}) (1 - I_2^{\alpha}) (1 - I_3^{\alpha})$$

$$q_{III}^{\alpha} = (1 - J_1^{\alpha} J_2^{\alpha}) (1 - K^{\alpha}) (1 - L^{\alpha})$$

$$q_{IV}^{\alpha} = (1 - M_1^{\alpha}) (1 - M_2^{\alpha}) (1 - M_3^{\alpha}) (1 - N_1^{\alpha} N_2^{\alpha} N_3^{\alpha}) (1 - O^{\alpha}) (1 - P^{\alpha})$$

$$q_V^{\alpha} = (1 - Q^{\alpha}) (1 - R^{\alpha}) (1 - S^{\alpha})$$

Table 6.6: Interval of confidence corresponding to truth membership function of vague reliability of PCBA

α	Interval of confidence
0.0	[0.0536, 0.0716]
0.1	[0.0609, 0.0735]
0.2	[0.0627, 0.0749]
0.3	[0.0644, 0.0761]
0.4	[0.0658, 0.0775]
0.5	[0.0674, 0.0791]
0.6	[0.0697, 0.0861]
0.7	[None, none]
0.8	[None, none]
0.9	[None, none]
1.0	[None, none]

Table 6.7: Interval of confidence corresponding to complement of false membership function of vague reliability of PCBA

α	Interval of confidence
0.0	[0.0423, 0.0724]
0.1	[0.0513, 0.0747]
0.2	[0.0543, 0.0766]
0.3	[0.0569, 0.0784]
0.4	[0.0594, 0.0802]
0.5	[0.0619, 0.0821]
0.6	[0.0645, 0.0843]

0.7	[0.0677, 0.0968]
0.8	[None, none]
0.9	[None, none]
1.0	[None, none]

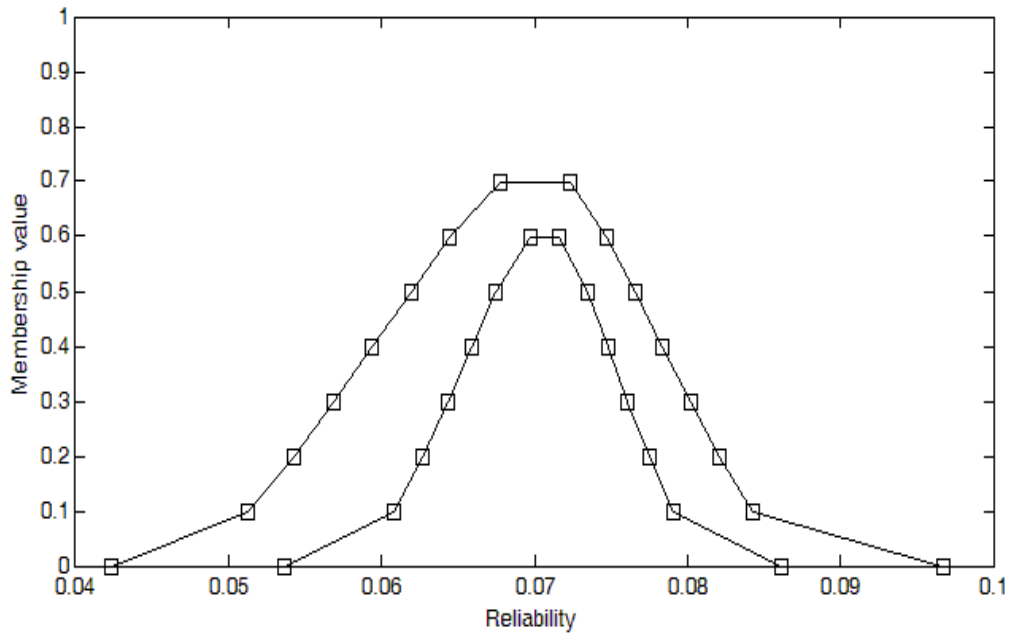


Figure 6.2: Vague set representing of vague reliability of PCBA

6.5 CONCLUSIONS

On the basis of the presented study, it can be concluded that it is better to use the proposed approach as compared to the existing approach [19] for evaluating the vague reliability of such industrial systems in which the probabilities of failure are represented by different types of vague sets.

CHAPTER 7

FUTURE DIRECTIONS

The following may be treated as future directions:

- (i) In the Chapter 6, a new approach is proposed to find the arithmetic operations between different types of vague sets and used it, to evaluate the vague reliability of PCBA. In future, with the help of approach, proposed in Chapter 6, the approach proposed in Chapter 3, may be modified to find the vague reliability parameters of such industrial systems in which the failure probabilities of all the components are represented by different types of vague sets.

- (ii) To the best of our knowledge, in the literature, T_ω -based arithmetic operations are defined only for same types of vague sets and no one have defined these operations for different types of vague sets. In future, it may be tried to develop an approach to find the T_ω -based arithmetic operations for different types of vague sets and with the help of proposed approach, the approach proposed in Chapter 6 can be further modified.

BIBLIOGRAPHY

- [1]. Aggarwal, K K. "A fast algorithm for the performance index of a telecommunication network." *IEEE Transactions on Reliability* 37 (1988): 65-69.
- [2]. Aggarwal, K K, K B Misra, and J S Gupta. "A fast algorithm far reliability evaluation." *IEEE Transactions on Reliability* R-24 (1975): 83-85.
- [3]. Ang, A H S, and W H Tang. *Probability Concepts in Engineering Planning and Design*. New York: John Wiley and Sons, 1984.
- [4]. Arora, Navneet, and D Kumar. "Availability analysis of steam and power generation systems in the thermal power plant." *Microelectronics Reliability* 37, no. 5 (1997): 795-800.
- [5]. Bai, X, and S Asgarpoor. "A fuzzy-based approach to substation reliability evaluation." *Electric Power Systems Research* 69, no. 2-3 (2004): 197-204.
- [6]. Baumik, S. "Failure of turbine rotor blisk of an aircraft engine." *Engineering Failure Analysis* 9, no. 3 (2002): 287-301.
- [7]. Bian, Y H, and H W Huang. "Fuzzy fault tree analysis of deep foundation pit supporting system's failure probability." *Proceedings of the First Sino-Japan Seminar for the Graduate Student in Civil Engineering*. Shanghai, China, 2004. 109-113.
- [8]. Billinton, Roy, and R N Allan. *Reliability Evaluation of Engineering Systems: Concepts and Techniques*. London.: Plenum Press, 1983.
- [9]. Black, and Veatch. *Power Plant Engineering* . New York: Chapman & Hall, An International Thomson Publishing Company, 1996.
- [10]. Burgazzi, L. "Evaluation of uncertainties related to passive systems performance." *Nuclear Engineering and Design* 230, no. 1-3 (2004): 93-106.
- [11]. Bustince, H, and P Burillo. "Vague sets are intuitionistic fuzzy sets." *Fuzzy Sets and System* 79 (1996): 403-405.

- [12]. Cai, K Y, C Y Wen, and M L Zhang. "Fuzzy reliability modeling of gracefully degradable computing systems." *Reliability Engineering and System Safety* 33, no. 1 (1991): 141-157.
- [13]. Cai, K Y, C Y Wen, and M L Zhang. "Fuzzy variables as a basis for a theory of fuzzy reliability in the possibility context." *Fuzzy Sets and Systems* 42, no. 2 (1991): 145-172.
- [14]. Cai, K Y, C Y Wen, and M L Zhang. "Posbist reliability behavior of typical systems with two types of failure." *Fuzzy Sets and Systems* 43, no. 1 (1991): 17-32.
- [15]. Chanda, R S, and P K Bhattachacjee. "A reliability approach to transmission expansion planning using minimal cut theory." *Electric Power Systems Research* 33 (1995): 111-117.
- [16]. Chanda, R S, and P K Bhattacharjee. "A reliability approach to transmission expansion planning using fuzzy fault-tree model." *Electric Power Systems Research* 45, no. 2 (1998): 101-108.
- [17]. Chang, J R, K H Chang, S H Liao, and C H Cheng. "The reliability of general vague fault tree analysis on weapon systems fault diagnosis." *Soft Computing* 10 (2006): 531-542.
- [18]. Chang, J, Y Yun, C Choi, and J Kim. "Failure analysis of gas turbine buckets." *Engineering Failure Analysis* 10, no. 5 (2003): 559-567.
- [19]. Chang, K H, and C H Cheng. "A novel general approach to evaluating the PCBA for components with different membership function." *Applied Soft Computing* 9, no. 2 (2009): 1044-1056.
- [20]. Chang, K H, C H Cheng, and Y C Chang. "Reliability assessment of an aircraft propulsion system using IFS and OWA tree." *Engineering Optimization* 40 (2008): 907-921.

- [21]. Chang, P T, P F Pai, K P Lin, and M S Wu. "Applying fuzzy arithmetic to system dynamics for the customer-producer-employment model." *International Journal of Systems Science* 37 (2006): 673-698.
- [22]. Chang, S Y, C R Lin, and C T Chang. "A fuzzy diagnosis approach using dynamic fault trees." *Chemical Engineering Science* 57, no. 15 (2002): 2971-2985.
- [23]. Chen, C C, and H C Tang. "Ranking non-normal p- norm trapezoidal fuzzy numbers with integral value." *Computers and Mathematics with Applications* 56 (2008): 2340-2346.
- [24]. Chen, S J, and S M Chen. "Fuzzy risk analysis based on similarity measure of generalized fuzzy numbers." *IEEE Transactions on Fuzzy Systems* 11, no. 1 (2003): 45-56.
- [25]. Chen, S M. "Analyzing fuzzy system reliability using vague set theory." *International Journal of Applied Science and Engineering* 1 (2003): 82-88.
- [26]. Chen, S M. "Fuzzy system reliability analysis using fuzzy number arithmetic operations." *Fuzzy Sets and Systems* 64, no. 1 (1994): 31-38.
- [27]. Chen, S M. "Measures of similarity between vague sets." *Fuzzy Sets and Systems* 74 (1995): 217-223.
- [28]. Chen, S M. "New method for fuzzy system reliability analysis." *Cybernetics and Systems: An International Journal* 27 (1996): 385- 401.
- [29]. Chen, S M, A Munif, G S Chen, H C Liu, and B C Kuo. "Fuzzy risk analysis based on ranking generalized fuzzy numbers with different left heights and right heights." *Expert Systems with Applications* 39, no. 7 (2012): 6320-6334.
- [30]. Cheng, C H, and D L Mon. "Fuzzy system reliability analysis by interval of confidence." *Fuzzy Sets and Systems* 56 (1993): 29-35.

- [31]. Cheng, S R, B Lin, B M Hsu, and M H Shu. "Fault-tree analysis for liquefied natural gas terminal emergency shutdown system." *Expert Systems with Applications* 36 (2009): 11918-11824.
- [32]. Chevalier, P B, and L M Wein. "Inspection for circuit board assembly." *Management Science* 43, no. 9 (1997): 1198–1213.
- [33]. Chowdhury, S G, and K B Misra. "Evaluation of fuzzy reliability of a non-series parallel network." *Microelectronics Reliability* 32 (1992): 1-4.
- [34]. Deshpande, Ashok. "Fuzzy fault tree analysis: revisited." *International Journal of Systems Assurance Engineering and Management* 2, no. 1 (2011): 3-13.
- [35]. Dhillon, B S. *Robot Reliability and Safety*. New York: Springer-Verlag, 1991.
- [36]. Dhillon, B S, and S N Rayapati. "A method to evaluate reliability of three-state device networks." *Microelectronics Reliability* 26 (1986): 525-554.
- [37]. Diwekar, U M. *Introduction to Applied Optimization*. Cambridge: Springer-Verlag, 2008.
- [38]. Ebeling, C E. *An Introduction to Reliability and Maintainability Engineering*. New Delhi: Tata McGraw-Hill Publishing Company Limited, 2000.
- [39]. Ejlali, A, and S G Miremadi. "FPGA-based Monte Carlo simulation for fault tree analysis." *Microelectronics Reliability* 44, no. 6 (2004): 1017-1028.
- [40]. Ferdous, R, F Khan, B. Veitch, and P R Amyotte. " Methodology for computer aided fuzzy fault tree analysis." *Process Safety and Environmental Protection* 87, no. 4 (2009): 217-226.
- [41]. Furuta, H, and N Shiraishi. "Fuzzy importance in fault tree analysis." *Fuzzy Sets and Systems* 12 (1984): 205-213.
- [42]. Gau, W L, and D J Buehrer. "Vague sets." *IEEE Transactions on Systems, Man and Cybernetics* 23 (1993): 610-614.

- [43]. Goodman, Gerrit V R. "An assessment of coal mine escape way reliability using fault tree analysis." *Mining Science and Technology* 7 (1988): 205-215.
- [44]. Guimarees, A C F, and N F F Ebecken. "Fuzzy FTA: a fuzzy fault tree system for uncertainty analysis." *Annals of Nuclear Energy* 26 (1999): 523-532.
- [45]. Gupta, P, S Verma, and M K Mehlawat. "A membership function approach for cost-reliability trade-off of cots selection in fuzzy environment." *International Journal of Reliability, Qquality and Safety Engineering* 18, no. 6 (2011): 573-595.
- [46]. Gupta, S, and J Bhattacharya. "Reliability analysis of a conveyor system using hybrid data." *Quality and Reliability Engineering International* 23 (2007): 867–882.
- [47]. Hong, D H. " Fuzzy measures for a correlation coefficient of fuzzy numbers under Tw (the weakest t-norm)-based fuzzy arithmetic operations." *Information Sciences* 176 (2006): 150-160.
- [48]. Hong, D H, and H Y Do. "Fuzzy system reliability analysis by the use of Tw (the weakest t-norm) on fuzzy number arithmetic operations." *Fuzzy Sets and Systems* 90, no. 3 (1997): 307-316.
- [49]. Huang, H Z, X Tong, and M J Zuo. "Posbist fault tree analysis of coherent systems." *Reliability Engineering and System Safety* 84, no. 2 (2004): 141-148.
- [50]. Hura, G S, and J W Atwood. "The use of Petri nets to analyze coherent fault trees." *IEEE Transactions on Reliability* 375 (1988): 469-474.
- [51]. Joseph, L A, J T Watt, and N Wigglesworth. "Modeling and analysis of a new product development in electronics sub-assembly manufacturing." *Proceedings of the Winter Simulation Conference*. 1990. 899-903.
- [52]. Kaufmann, A, and M M Gupta. *Introduction to Fuzzy Arithmetics: Theory and Applications*. New York: Van Nostrand Reinhold, 1985.

- [53]. Keller, A Z, and C K Zaitri. "Further application of fuzzy logic to reliability assessment safety analysis." *Microelectronics Reliability* 29 (1989): 399-404.
- [54]. Khan, F I, and R Sadiq. "Risk-based prioritization of air pollution monitoring using fuzzy synthetic evaluation technique." *Environmental Monitoring and Assessment* 105 (2005): 261-283.
- [55]. Klir, G J, and B Yuan. *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. New Delhi: Prentice-Hall of India Pvt. Ltd., 1995.
- [56]. Knezevic, J, and E R Odoom. "Reliability modelling of repairable systems using Petri nets and fuzzy Lambda-Tau methodology." *Reliability Engineering and System Safety* 73, no. 1 (2001): 1-17.
- [57]. Kolesarova, A. "Triangular norm-based addition of linear fuzzy numbers." *Tatra Mountains Mathematical Publications* 6 (1995): 75-81.
- [58]. Konstandinidou, M, Z Nivolianitou, C Kiranoudis, and N Markatos. " A fuzzy modeling application of CREAM methodology for human reliability analysis." *Reliability Engineering & System Safety* 91, no. 6 (2006): 706-716.
- [59]. Kseibat, D S, G S Mittal, and O A Basir. "Predicting safety and quality of thermally processed canned foods using a neural network." *Transactions of the Institute of Measurement and Control* 26 (2004): 55-68.
- [60]. Kumar, A, P Singh, P Kaur, and A Kaur. "A new approach for ranking non-normal p-norm trapezoidal fuzzy numbers." *Computers and Mathematics with Applications* 61 (2011): 881–887.
- [61]. Kumar, A, S P Yadav, and S Kumar. "Fuzzy reliability of a marine power plant using vague set theory." *Proceeding International Conference on Reliability and Safety Engineering*. 2005. 281- 292.

- [62]. Kumar, A, S P Yadav, and S Kumar. "New approach for electric robot fuzzy reliability analysis." *International Journal of Performability Engineering* 3 (2007): 157-166.
- [63]. Kumar, A, S P Yadav, and S. Kumar. "Fuzzy system reliability analysis using Tw (the weakest t-norm) based arithmetic operations on LR type interval valued vague sets." *International Journal of Quality and Reliability Management* 24 (2007): 71–82.
- [64]. Kumar, E V, S K Chaturvedi, and A W Deshpande. "Failure probability estimation using fuzzy fault tree analysis (FFTA) with PDM data in process plants." *International Journal of Performability Engineering* 4 (2008): 271-284.
- [65]. Kumar, V, and K K Aggarwal. "Petri net modeling and reliability evaluation of distributed processing systems." *Reliability Engineering and System Safety* 41 (1993): 167-176.
- [66]. Li, D F. "A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to MADM problems." *Computers and Mathematics with Applications* 60 (2010): 1557-1570.
- [67]. Li, W, J Zhou, J Lu, and W Yan. "Incorporating a combined fuzzy and probabilistic load model in power system reliability assessment." *IEEE Transactions on Power Systems* 22 (2007): 1386-1388.
- [68]. Liu, T S, and S B Chiou. "The application of petri nets to failure analysis." *Reliability Engineering and System Safety* 57 (1997): 129-142.
- [69]. Lv, B, et al. "Gender consistency and difference in healthy adults revealed by cortical thickness." *NeuroImage* 53 (2010): 373-382.
- [70]. Lv, B, et al. "Structural and functional deficits in human amblyopia." *Neuroscience Letters* 437 (2008): 5-9.
- [71]. Mehta, V K, and Rohit Mehta. *Principles of Power System*. S.Chand, 2005.

- [72]. Mesiar, R. "Shape preserving additions of fuzzy intervals." *Fuzzy Sets and Systems* 86 (1997): 73-78.
- [73]. Misra, K B, and G G Weber. "A new method for fuzzy fault tree analysis." *Microelectronics Reliability* 29 (1989): 195-216.
- [74]. Misra, K B, and G G Weber. "Use of fuzzy set theory for level1 studies in probabilistic risk assessment." *Fuzzy Sets and Systems* 37 (1990): 139-160.
- [75]. Mon, D L, and C H Cheng. "Fuzzy system reliability analysis for components with different membership functions." *Fuzzy Sets and Systems* 64 (1994): 145-157.
- [76]. Onisawa, T. "An application of fuzzy concepts to modeling of reliability analysis." *Fuzzy Sets and Systems* 37 (1990): 267-287.
- [77]. Pan, H S, and W Y Yun. "Fault tree analysis with fuzzy gates." *Computers and Industrial Engineering* 33, no. 3-4 (1997): 569-572.
- [78]. Park, K.S. "Fuzzy apportionment of system reliability." *IEEE Transactions on Reliability* 36 (1987): 129-132.
- [79]. Park, M, Y Hwang, Y Choi, and T Kim. "Analysis of a J69-T-25 engine turbine blade fracture." *Engineering Failure Analysis* 9, no. 5 (2002): 593-601.
- [80]. Petch, M, D Das, and A Ramakrishnan. "The IEEE standards of reliability program and reliability prediction methods for electronic equipment." *Microelectronics Reliability* 42 (2002): 1259-1266.
- [81]. Petri, C A. *Communication with automata*. Griffs (NY): Rome Air Development Center: Technical Report (English) RADC-TR-65-377, 1966.
- [82]. Sabat, S, L d S Coelho, and A Abraham. "MESFET DC Model Parameter Extraction Using Quantum Particle Swarm Optimization." *Microelectronics Reliability* 49, no. 6 (2009): 660-666.

- [83]. Sharma, R K, D Kumar, and P Kumar. "Fuzzy modeling of system behavior for risk and reliability analysis." *International Journal of Systems Science* 39, no. 6 (2008): 563 -581.
- [84]. Sharma, T K, M Pant, and A Abraham. "Dichotomous Search in ABC and its Application in Parameter Estimation of Software Reliability Growth Models Salamanca." *Third World Congress on Nature and Biologically Inspired Computing* . 2011.
- [85]. Shu, H M, H C Cheng, and R J Chang. "Using intuitionistic fuzzy sets for fault tree analysis on printed circuit board assembly." *Microelectronics Reliability* 46 (2006): 2139-2148.
- [86]. Singer, D. "A fuzzy set approach to fault tree and reliability analysis." *Fuzzy Sets and Systems* 34, no. 2 (1990): 145-155.
- [87]. Smith, D J. *Reliability and Maintainability Risk*. Oxford: Butterworth Heinemann, 2002.
- [88]. Srinath, L.S. *Reliability engineering, 3rd ed*. East-west Press Pvt. Ltd., 2003.
- [89]. Stamatelatos, Michael, and William Vesely. "Fault Tree Handbook with Aerospace Applications." 2002.
- [90]. Suresh, P V, A K Babar, and V V Raj. "Uncertainty in fault tree analysis: a fuzzy approach." *Fuzzy Sets and Systems* 83 (1996): 135-141.
- [91]. Taheri, S M, and R Zarei. "Bayesian system reliability assessment under the vague environment." *Applied Soft Computing* 11, no. 2 (2011): 1614-1622.
- [92]. Tanaka, H, L T Fan, F S Lai, and K Toguchi. "Fault-tree analysis by fuzzy probability." *IEEE Transactions on Reliability* 32 (1983): 150–163.

- [93]. Verma, A K, A Srividya, and B C Deka. "Composite system reliability assessment using fuzzy linear programming." *Electric Power Systems Research* 73 (2005): 143-149.
- [94]. Verma, A K, A Srividya, and H M Ravi Kumar. "A framework using uncertainties in the composite power system reliability evaluation." *Electric Power Components and Systems* 30 (2002): 679-691.
- [95]. Verma, A K, A Srividya, and R S Prabhu Gaonkar. *Fuzzy-Reliability Engineering: Concepts and Applications*. New Delhi: Narosa Publishing House, 2007.
- [96]. Volkanovski, A, M Cepin, and B Mavko. "Application of the fault tree analysis for assessment of power system reliability." *Reliability Engineering and System Safety* 94, no. 6 (2009): 1116-1127.
- [97]. Wang, H, and H Pham. *Reliability and Optimal Maintenance*. London: Springer, 2006.
- [98]. Wang, J. "On the parameterized OWA operators for fuzzy MCDM based on vague set theory." *Fuzzy Optimization and Decision Making* 5 (2006): 5-20.
- [99]. Watson, H A. *Launch Control Safety*. Murray Hill, NJ: Bell Labs, 1961.
- [100]. Zadeh, L A. "Fuzzy set as a basis for a theory of possibility." *Fuzzy Sets and Systems* 1 (1978): 3-28.
- [101]. Zadeh, L A. "Fuzzy sets." *Information and Control* 8, no. 3 (1965): 338-353.
- [102]. Zhang, Q, S X Yang, G S Mittal, and S Yi. "Prediction of performance indices and optimal parameters of rough rice drying using neural networks." *Biosystems Engineering* 83, no.3 (2002): 281-290.
- [103]. Zimmermann, H J. *Fuzzy Set Theory And Its Applications*, 2nd ed. Allied Publisher Ltd., 1996.

- [104]. Zimmermann, H J. *Fuzzy Set Theory And Its Applications*, 3rd ed. London: KluwerAcademic Publishers, 1996.
- [105]. Zou, Q, J Zhou, C Zhou, L Song, J Guo, and Y Liu. "The practical research on flood risk analysis based on IIOSM and fuzzy α -cut technique." *Applied Mathematical Modelling* 36, no. 7 (2012): 3271-3282.