EFFICIENT SOLUTIONS FOR THE MULTI-OBJECTIVE WAREHOUSE PROBLEMS

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CERTIFICATE

*I hereby certify that the work which is being presented in the thesis entitled "**Efficient solutions for the multi-objective warehouse problems**" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Mahesh Kumar Sharma.*

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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(AMITA KOHLI)

ABSTRACT

Facility location, also known as location analysis, is a branch of operations research concerning itself with mathematical modeling and solution of problems concerning the placement of facilities in order to minimize transportation costs, avoid placing hazardous materials near housing, outperform competitor's facilities, etc. In a simple facility location problem, a single facility is to be placed, with the only optimization criterion being the minimization of the sum of distances from a given set of point sites. More complex problems considered in this discipline include the placement of multiple facilities, constraints on the locations of facilities, and more complex optimization criteria. Present thesis deals with a real life example of facility location problem which is basically a warehouse location problem which includes assignment of ration shops to the selected number of sites from among the given number of potential sites, for their requirements but with the objectives of minimizing cost for satisfying demands of all ration shops and minimizing the maximum time needed to fulfill the requirements of all the ration shops along with some constraints.

Thesis contains three chapters; first chapter is the introductory in nature and also contains a brief review of literature related to this topic. In second and third chapter, un-capacitated warehouse location problem and capacitated warehouse location problem have been considered respectively. A heuristic approach consisting of a combination of add and drop rules incorporating Tabu search has been used to find the set of efficient solutions.

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CHAPTER-1

The single-objective model was the first to be developed and thus it was received considerably more exposure, been put to more use, and is generally considered to be a relatively high level of refinement. Thus, the implication is simple, well-tested tool is available and we may be inclined to fit the problem to this model despite the assumptions required. But in real life there are many problems with more than one objective for which the multi-objective models are required.

Dantzig's initial concept was centered about the development of a linear programming model but with a single objective. This so set the tone for the development of traditional linear programming that many (if not most) linear programming texts completely ignore even the possibility of more than one objective. Unlike the traditional single objective optimization problem wherein it is settle on optimizing single objective function, there is no single universally accepted approach for solving the multi-objective optimization problems due to usually conflicting nature of objective functions leading to the situation where the optimization of one of these may adversely affect the optimization of others. So in the case of multi-objective optimization problems, there is no need to access the decision maker's utility function that may vary from decision maker to decision maker.

1.1. MULTI-OBJECTIVE OPTIMIZATION AND ALTERNATE APPROACHES

A multi-objective model mainly consists of two absolutely vital aspects; the terminology and concepts that are necessary to describe and understand the process, and the steps employed to formulate the model. These two factors play a much more important part in the understanding and employment of the multi-objective process than was the case in single- objective linear programming.

Multi-objective optimization can be defined as:

"a vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions. Hence, the term "optimize"means finding such a solution which would give the values of all the objective functions acceptable to the designer/decision maker."

This model can be formulated as:

Optimize $f(x) = (f_1(x), f_2(x), ..., f_k(x))$

Subject to

$$g_j(x) \leq, =, \geq b_j \quad , j = 1,2,...,m$$

$$X = (x_1, x_2, ..., x_n)^T \quad , X \geq 0$$

where, $f(x)$ is the objective function to optimize. $f_1(x), f_2(x),..., f_k(x)$ are k number of distinct objective functions subject to m constraints. X is a vector consists of decision variables $x_1, x_2, ..., x_n$.

Unlike single-objective linear programming, there is more than one multiple-objective model (and associated solution technique) from which one may choose. Some of the early investigators who recognized this deficiency in the single-objective approach suggested a rather brute-force approach to circumvent (but not to actually resolve) the problem. They proposed that first list all the problem objectives and pick one of these objectives as the" single objective" in the traditional model and consider all the other objectives to be rigid constraints. Then solve the resultant model and repeat this procedure with another objective as the single objective in the model and at the end pick that solution from all solutions obtained, that appears to "best" satisfy all the objectives. At least there are two drawbacks to this approach, first, it converts the problem into a possibly large number of linear programming problems (i.e., one for each objective) which have to be solved and second and most critical is that the resultant solution as achieved at the end may very well not represent that solution that best satisfies all the objectives (the large the problem, the more likely that this is so). It is rather obvious that something far more systematic is required.

Goal programming model is one of the multi-objective models and is flexible, efficient and easy of use and implementation. This model is also known as *generalized goal programming*. However, the use of the goal programming model does not exclude the consideration of alternative multiple-objective approaches since, as will be shown, the generalized goal programming model may be, through minor changes, extended so as to encompass the facets inherent in the other approaches. Consequently, the choice of the goal programming model for multiple-objective linear programming has been made because:

1. The model development is relatively simple and straightforward.
2. Minor modifications may be employed so as to encompass the alternative approaches (e.g., fuzzy programming, non-dominated or efficient solution methods, weighted objectives, etc.) to the multiple-objective linear programming problem.
3. The method of solution is quite simple and is, in fact, just a refinement to the two-phase simplex method.
4. The goal programming models, and variations thereof, have already found extensive implementations in actual problems since the early 1950s.
5. The model and its assumptions seem consistent with typical real-world problems.

There are large and growing number of approaches in use and/or proposed for the multiple – objective linear model. As mentioned above, goal programming is just one of these. Now consider just a few of the better-known multiple-objective approaches.

There are three primary approaches that form the basis for nearly all the candidate multiple-objective techniques that have been proposed. These are:

1. Weighting or utility methods
2. Ranking or prioritizing methods
3. Efficient solution(or generating)methods

The weighting method refers to those approaches that attempt to express all problem objectives in terms of a single measure. The basic thrust of all such methods is to transform a multiple-objective model into a single-objective model. As such, they are attractive from a strictly computational point of view (e.g., conventional simplex may be used if the model is linear).However, the obvious drawback to such an approach is that associated with actually developing truly credible weights.

The ranking or prioritizing methods try to circumvent the heady problems, rather than attempting to find a numerical weight for each objective, they simply ask that objectives be ranked according to their perceived importance. Most decision makers can do this and, in fact, ranking is a concept that seems inherent to much of decision making. For example, in determining raises, a procedure used by many organizations is to first rank one's employees in order of their work, productivity, value to the company, and so forth. The problem with the ranking approach is how to associate the results of a given solution to the satisfaction of the ranking.

Efficient solution method"avoids" both the problems of finding weights and that of satisfying the ranking. It does this, or at least attempts to, by generating the total set of all *efficient solutions* (or non-dominated solutions, or Pareto optimal solutions).

1.2. EFFICIENT SOLUTIONS OR NON-DOMINATED SOLUTIONS

A set of solutions is said to be efficient if there exists no solution that is superior to it with respect to at least one objective function but is not inferior to it with respect to any of the objective functions.

If x_1 and x_2 are two solutions, then these can have any of two possibilities-one dominates the other or non-dominates the other. In a minimization problem, without the loss of generality, a solution x_1 dominates x_2 iff the following two conditions are satisfied:

$$\forall\ i \in \{1,2,...,N_{obj}\}: f_i(x_1) \le f_i(x_2)$$

$$\exists\ j \in \{1,2,...,N_{obj}\}: f_j(x_1) < f_j(x_2)$$

where, $f(x_1)\, and\, f(x_2)$ are the objective functions.

If any of the above conditions is violated, the solution x_1 does not dominate the solution x_2. If x_1 dominates the solution x_2, x_1 is called the non-dominated solution within the set $\{x_1, x_2\}$ .The solutions that are non-dominated within the entire search space are denoted as Pareto-optimal and constitute the Pareto-optimal set or Pareto-optimal front. From the entire set of efficient (non-dominated) solutions the decision maker can select the solution one believed most attractive.

1.3. HEURISTICS

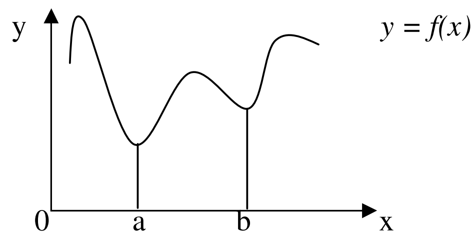
The word "heuristic" originated from the Greek root meaning to discover. In case of optimization of real life problems, the problems are encountered because of the highly complex nature. The regular algorithms are very often ineffective on these problems but there are, however, other approach that may be used to find a solution to models involving integer and or discrete variables. And, in many cases, such approaches have proven capable of providing *acceptable* solutions to truly massive size problems. Thus, the pragmatic or human approach can be considered to solve the problems and hence this shifts the whole paradigm from algorithm-based calculations to the employment of heuristic procedures or heuristic programming.

Heuristics are rules of thumb that are developed through intuition, experience and judgment. In artificial intelligence, a heuristic is a procedure that may lack a proof. It is used when the inter-relationships of the decision variables are not explicitly clear but there is some confidence in understanding the output for certain input. The result of application of heuristics cannot guarantee the optimal solution. A heuristic might help to find solutions which are good, but perhaps not the very best they can be. Obviously the measure of goodness and assessment of a heuristic technique is going to be relative to the domain, and to the specific job that problem solving is going to be applied in that domain. When one or more heuristics are combined with a procedure for deriving a solution from the associated rules, gives a heuristic program. Since the concept is to derive an acceptable solution and not the server-elusive optimal solution, thus the heuristics satisfy certain aspiration criteria as set by the decision maker.

The heuristic approaches are also being used to solve the real world multi-objective problems. These problems always attract us towards the goal programming techniques but of late another approach of the *efficient/non-dominated/Pareto optimal solutions* has gained importance.

1.4. TABU SEARCH

Tabu search is a meta-heuristic approach which is associated with a method of seeking optimal solutions. But since most of the heuristic methods select moves in the neighborhood of the current solution improving the value of the objective function, this leads at times to get stuck at the local optima and terminate the procedure at the arrival of the local optimum instead of the global optimum. The situation is shown in the figure below



Tabu search is used to get out of the local optimum. Hence, *Tabu* search guides local heuristic search procedures to explore the solution space beyond local optimality. The important characteristics of Tabu search are:

1. It allows moves, which even worsen the value of the objective function.
2. It prevents cycling temporarily, i.e., temporarily revisiting solutions examined recently by using Tabu list. Tabu list contains forbidden moves having been selected recently. The Tabu list changes as we move to the next solution whether it is better or worse.
3. It uses incumbent solution which is the best solution obtained so far. The incumbent solution changes to better solution as and when we arrive at it.
4. The procedure terminates when we revisit an already examined solution.

1.5. LITERATURE REVIEW

In mathematical programming, considerable attention has been devoted to the un-capacitated facility location problem (UFLP). Krarup and Pruzan (1983) presented a simple plant location problem. Efroymson and Ray (1966) proposed a formulation of the un-capacitated plant location problem and the use of branch and bound algorithm to solve it. Khumawala(1972) utilized the special structure of UFLP to improve the branch and bound algorithm of Efroymson and Ray(1966). Bilde and Krarup(1977) and Erlenkotter(1978) developed a dual-based branch and bound procedure for the problem and this procedure has been widely accepted as an efficient known procedure. So, one of the main results has been the development of linear programming-based branch and bound algorithms. The standard reference in this area is the algorithm of Erlenkotter(1978), a branch and bound algorithm based on dual descent which is an efficient way to solve the un-capacitated warehouse location problem. Guignard(1977) proposed to strengthen the separable Lagrangian relaxation of UFLP by using Benders inequalities generated during a Lagrangian dual ascent procedure. Galvao and Raggi(1989)proposed a method for solving to optimality un-capacitated location problem. Korkel(1989) showed the exact solution of large-scale simple plant location problem to modify a primal-dual version of Erlenkotter's(1978) exact algorithm. Approximation algorithms for the un-capacitated warehouse location have also been studied heavily. Beasley (1993) proposed lagrangian heuristics for location problems. His results indicated that his proposed approach is robust for solving different location problems. Simao and Thizy(1989) developed a dual simplex algorithm for the canonical representation of UFLP. Conn and Cornuejols (1990) proposed a projection also exploiting a dual formulation. Gao and Robinson (1994) presented a general model and dual-based and branch and bound solution procedure to find optimal solutions for several un-capacitated location problems that include UFLP. They claimed that their proposed solution procedure effectively solves realistically sized UFLP.

Al-Sultan and Al-Fawzan(1999) presented a Tabu search approach to the UFLP. The best neighbor which is not Tabu is selected as the next state if it improves the objective function. The main step is executed for a number of iterations. The algorithm also uses a sophisticated and effective heuristic as the starting point of the Tabu search.

Michel and Hentenryck(2004) presented an simple, yet robust and efficient, Tabu-search algorithm for the un-capacitated warehouse location problem(UWLP). The algorithm finds optimal solutions very quickly and with high frequencies. It also compares favorably with state-of-art genetic algorithms and should be a very valuable addition to the repertoire of tools for un-capacitated warehouse location due to its simplicity and effectiveness.

Ignizio and Cavalier(1994) have considered the problem of selecting upto a fixed number of sites from among a given number of potential sites for locating warehouses at them and clustering customers to the selected sites in such a way that each customer is assigned to a unique selected site. The single objective of this problem was to minimize the sum of distances from each customer to his/her assigned site. Praveena *et al.* (1999) have extended this problem which selects upto a fixed number of sites from among a given number of potential sites for warehouse for clustering ration shops to them subject to several constraints with two objectives. The two objectives are to minimize the total cost and duration of meeting the requirements of all the ration shops from their assigned warehouses at the selected sites. These two objectives are not accorded priorities. One constraint is that each ration shop should be clustered to a unique site, which is selected for locating a warehouse at it but there is no restriction on the number of ration shops to be clustered to a warehouse at the selected site. Other constraint is that the set up cost of the warehouses should not exceed a certain budgetary amount. This problem has been solved by finding the set of efficient solutions of it using heuristic method consisting of a combination of add and drop rules. Prakash *et al.* (2009) has developed a heuristic iterative algorithm incorporating Tabu search to find the set of efficient solutions of this problem.

The capacitated facility location problem (CFLP) has been studied extensively. Many exact algorithms and heuristic methods have been developed to solve it in last 40 years. Because UFLP and CFLP are closely related, many heuristic methods developed for the UFLP are also extended to the CFLP.

Kuehn and Hamburger (1963) developed the first heuristic method for the UFLP, which was later extended to the CFLP by Jacobsen (1983). This heuristic method consists of two phases, the first phase, called ADD, starts with all facilities closed and then the facility that causes the maximum total cost reduction is opened and this phase ends when no more facilities can be opened to further reduce the total cost. The second phase is a local search procedure in which an open facility and a closed facility exchange their status if this exchange reduces the total-cost. Domschke and Drexl(1985) proposed priority rules for the ADD procedure to improve its performance in cases where the facilities have distinct capacities and/or distinct fixed operating costs. Feldman *et al.* (1966) proposed a different strategy for the first phase, named DROP that was also extended to the CFLP by Jacobsen (1983).

Lagrangian relaxation has also been applied to several facility location problems. Cornuejols *et al.* (1991) presented an excellent theoretical analysis of all possible Lagrangian relaxations and the linear programming relaxation for the CFLP. Barahona and Chudak(2001) proposed a Lagrangian heuristic method for the UFLP and the CFLP.

Several exact algorithms based on branch and bound have been proposed. The major differences among these algorithms are in the types of relaxation, the methods of solving the relaxed problem and the strategies to improve the lower bound. Leung and Magnanti(1989) introduced a family of faults and valid inequalities for solving the CFLP with equal capacities. Aardal(1998) proposed new valid inequalities and implemented two branch-and cut algorithms that are tested on small and medium test problems from the literature.

The capacitated warehouse problem consists of the well known transportation problem with the additional feature of affixed charge associated with each warehouse which is put to use. The problem is usually solved as a special type of mixed integer programme, so that relaxation and lower bounding are a vital part of any algorithm. A deeper insight into the relaxation process may eventually lead to more efficient algorithms for the problem. Baker (1982) has shown that the LP relaxation of the capacitated warehouse location problem can incorporate constraints of a much more general nature than those previously described.

Kelly and Khumawala(1982) presented an algorithm for finding a minimal cost warehouse system design wherein individual warehouse have limited capacities and exhibit economies of scale because earlier the mixed integer-linear models that have been used in most analysis of warehouse location problems fail to capture the potential operating efficiencies associated with large scale facilities. The iterative procedure defines and solves a series of conventional transportation problems in order to converge on the optimal system design.

Sun (2008) developed a Tabu search heuristic procedure for the capacitated facility location problem is developed, implemented and computationally tested. A specialized transportation algorithm is developed and employed to exploit the problem structure in solving transportation problems. The performance of the heuristic procedure is tested through computational experiments using test problems from the literature and new test problems randomly generated. It found optimal solutions for almost all the test problems used. As compared to the Lagrangean and the surrogate/Lagrangean heuristic methods, the Tabu search heuristic procedure found much better solutions.

Prakash and Aggarwal(1992) considered the problem of establishing a route between two specified nodes through a network with two objectives-one primary and another secondary is considered. The primary objective is to minimize the total cost of travel and the secondary objective is to minimize the duration of travel.

Prakash and Om (1996) has developed an algorithm to obtain the set of non-dominated solutions of the two-objective problem of determining non-dominated programmes for augmentation of capacities of depots and shipment of buses from them to the starting points of routes along with determining the capacity in reserve and the number of buses to be parked at the respective depots after the augmentation of their capacities is considered with two objectives. One objective is to minimize the present value of the total cost of dead-travelling to be performed by buses between the depots and the starting points of their routes over a planning horizon plus the capital expenditure to be incurred in augmenting capacities of the depots. The other objective is to minimize the maximum distance among the distances traversed by individual buses from depots to the starting points of their respective routes. The two objectives are not accorded priorities.

A detailed discussion about efficient solution can be found in works of Ignizio(1982) and Steuer(1986).

Some algorithms are described for solving plant location problems with non-linear warehousing costs. The heuristic procedures are flexible with respect to the type of warehousing cost structure permitted, and may be used to solve fixed *p*-median location problems as well as problems in which the numbers and locations of warehouses in solution are jointly determined as a trade-off between transportation and fixed and operating plant costs. Computational experience is reported by R. A. Whitaker (1985) on some well known problem sets in which the economies of scale in production are continuously concave. Comparisons with other solution methods indicate that the proposed procedures perform as well as, or better than any currently known on these standard test problems.

Kratica *et al.* (2001) is of special interest as it contains a comprehensive and excellent description of a genetic algorithm and its comparison to Erlenkotter(1978).

In many distribution systems, the location of the distribution facilities and the routing of the vehicles from these facilities are interdependent. The location routing problem (LRP), which combines the facility location and the vehicle routing decisions, is NP-hard. Due to the problem complexity, simultaneous solution methods are limited to heuristics. Tuzun and Burke (1999) presented two-phase Tabu search architecture for the solution of the LRP. First introduced in this paper, the two-phase approach offers a computationally efficient strategy that integrates facility location and routing decisions. This two-phase architecture makes it possible to search the solution space efficiently, thus producing good solutions without excessive computation. An extensive computational study shows that the Tabu search algorithm achieves significant improvement over a recent effective LRP heuristic.

The Tabu search meta-heuristic has been successfully applied to variety of combinatorial optimization problems, but not much research has been reported in using this meta-heuristic to solve the CFLP. The Tabu search heuristic procedure proposed by Grolimund and Ganascia(1997) was applied to the CFLP and limited computational results were reported. However, Tabu search procedures have been developed for more complicated facility location problems by many investigators such as Ferreira Filho and Galvao(1998), Franca *et al.*(1999), Ghosh(2003) and Sun(2006a).

1.6. PRESENT WORK

Now we are going to give the brief explanation of our work. Ignizio and Cavalier(1994) have formulated and solved the problem of selecting upto a fixed number of sites from among a given number of potential sites for locating warehouses at them and clustering customers to the selected sites in such a way that each customer is assigned to a unique selected site. The problem has a single objective to minimize the sum of distances from each customer to his/her assigned site. Praveena *et al.* (1999) have modified and extended this problem to a new situation. The problem selects up to a fixed number of sites from among a given number of potential sites for clustering ration shops to them subject to several constraints with two objectives. One of the constraints is that each ration shop should be clustered to a unique site which is selected for locating a warehouse at it; however there is no restriction on the number of ration shops to be clustered to a warehouse at the selected site. Another constraint is that the setup cost of the warehouses should not exceed a certain budgetary amount. The two objectives are to minimize the total cost and duration of meeting requirements of all the ration shops from their assigned warehouses to the selected sites. The two objectives are not accorded priorities. The set of efficient solutions of this problem has been found through solving using a heuristic algorithm consisting of a combination of add and drop rules. Prakash *et al.* (2009) solved this very problem through solving a sequence of prioritized bi-criterion problems incorporating Tabu search in the heuristic algorithm used for solving the prioritized bi-criterion problems. The incorporation of Tabu search leads to better results than those obtained earlier without its incorporation.

We extended the above problem to new situation in which **(1)** the objective to minimize the total cost, also includes set up cost of warehouse at the site along with the cost of meeting the requirements of all the ration shops. **(2)** The problem becomes capacitated when we put the additional constraints in above problem, which are

a. Each site selected to locate a warehouse must have at least one ration shop assigned to it.
b. Maximum up to a fixed number of ration shops can be assigned to each site selected for locating a warehouse at it.
c. The total number of ration shops that can be given to the selected number of sites, should not be less than total number of ration shops to be clustered.

CHAPTER-2

2.1. INTRODUCTION

Ignizio and Cavalier(1994) have formulated and solved the problem of selecting upto a fixed number of sites from among a given number of potential sites for locating warehouses at them and clustering customers to the selected sites in such a way that each customer is assigned to a unique selected site. The problem has a single objective to minimize the sum of distances from each customer to his/her assigned site. Praveena *et al.* (1999) have modified and extended this problem to a new situation. The problem selects up to a fixed number of sites from among a given number of potential sites for clustering ration shops to them subject to several constraints with two objectives. One of the constraints is that each ration shop should be clustered to a unique site which is selected for locating a warehouse at it: however there is no restriction on the number of ration shops to be clustered to a warehouse at the selected site. Another constraint is that the setup cost of the warehouses should not exceed a certain budgetary amount. The two objectives are to minimize the total cost and duration of meeting requirements of all the ration shops from their assigned warehouses to the selected sites. The two objectives are not accorded priorities. The set of efficient solutions of this problem has been found through solving using a heuristic algorithm consisting of a combination of add and drop rules. Prakash *et al.* (2009) solved this very problem through solving a sequence of prioritized bi-criterion problems incorporating Tabu search in the heuristic algorithm used for solving the prioritized bi-criterion problems. The incorporation of Tabu search leads to better results than those obtained earlier without its incorporation.

We extended the above problem to new situation in which the objective to minimize the total cost, also includes set up cost of warehouse at particular site along with the cost of meeting the requirements of all the ration shops.

2.2. FORMULATION OF UN-CAPACITATED WAREHOUSE LOCATION PROBLEM

Let us consider M ration shops, N potential warehouse sites and K be the maximum number of sites, which can be selected from among the N potential sites for locating warehouses at them. The M ration shops are to be clustered upto K sites in such a way that each shop is assigned to a unique site, which is selected for locating a warehouse. There is no restriction on the number of ration shops to be clustered to a selected site. So this problem is un-capacitated one. Let c_{ij} and t_{ij} $(i=1,2,\ldots,M;\ \ j=1,2,\ldots,N)$ units be the cost and time, respectively of meeting requirements of ration shop i from site j. *Let* C_j units be the cost of setting up a warehouse at the site j and B units be the budgetary amount allocated for setting up warehouses. Let x_{ij} be the variable assuming value 0 or 1 according as ration shop i is not assigned or assigned to site j, and y_j be the variable assuming the value 0 or 1 according as potential site j is not selected or selected for locating a warehouse. Thus, the problem seeks to select upto a fixed number of sites from among a given number of potential sites for locating warehouses at them with two objectives subject to several constraints. The two objectives are not accorded priorities. The objectives are to minimize the total cost and duration of meeting requirements of all ration shops from their assigned warehouses at the selected sites.

The two objectives to be minimized are

$$C = \sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij}\, x_{ij} + \sum_{j=1}^{N} C_j\, y_j \tag{2.1}$$

$$T = \max\{t_{ij},\ x_{ij} = 1 (i = 1, 2, ..., M;\ j = 1, 2, ..., N)\} \tag{2.2}$$

Subject to

$$\sum_{j=1}^{N} y_j \le K \tag{2.3}$$

$$\sum_{j=1}^{N} C_j\, y_j \le B \tag{2.4}$$

$$\sum_{j=1}^{N} x_{ij} = 1\ (i = 1, 2, ..., M) \tag{2.5}$$

$$x_{ij} - y_j \le 0\ (i = 1, 2, ..., M;\ j = 1, 2, ..., N) \tag{2.6}$$

$$x_{ij},\ y_j = 0\ or\ 1\ (i = 1, 2, ..., M;\ j = 1, 2, ..., N) \tag{2.7}$$

The objective functions *C* and *T* given by (2.1) and (2.2) are not accorded any priorities. The constraint (2.3) ensures that upto a maximum of *K* sites can be selected from among the *N* potential warehouse sites and the constraint (2.4) ensures that the total setup cost of warehouses does not exceed the allocated budgetary amount *B*. Constraints (2.5) and (2.6) ensure that each ration shop is assigned to a unique site which is selected for locating a warehouse. It is required to obtain the set of efficient solutions of the problem given by (2.1) to (2.7).

2.3. SOLUTION PROCEDURE

The problem formulated above is a binary integer nonlinear problem because the variables x_{ij} '*s* and y_j*'s* are binary integers and the objective function *T* given by (2.2) is nonlinear. A set of efficient solutions is obtained through an iterative heuristic procedure requiring solutions of bi-criterion problems with prioritized objectives.

Procedure for 1st efficient solution

The first efficient solution (X^1, Y^1) of the problem formulated above by (2.1) to (2.7) is the optimal solution of the problem wherein the total cost *C* and the duration of meeting requirements of the entire ration shops, from their assigned warehouses, are minimized with

1st and 2nd priorities, respectively, subject to the constraint (2.3)-(2.7).The problem yielding 1st efficient solution is designated the 1st prioritized bi-criterion problem of the formulated problem. The heuristic method proposed to solve the 1st bi-criterion problem is a modification and extension of the heuristic method to solve a partial covering problem with a single objective. The proposed method consists of combination of add and drop rules. Start with no sites selected and select the site wherefrom the sum of setup cost of warehouse at that site and total cost of meeting requirements of all the ration shops is minimum without violating the budgetary constraint. In case of tie, select the site wherefrom the maximum time among the times of meeting the requirements of each ration shop is also minimum. And in case, there is a tie again, arbitrarily select a site among the tied ones.

Next, a second site is selected which in combination with the site already selected minimizes the sum of setup cost of warehouse at that site and total cost of meeting requirements of all ration shops without violating the budgetary constraint. Each ration shop is assigned to that site wherefrom the cost of meeting its requirement is minimum, among the selected sites. when tie occur then select that site ,which in combination with the already selected site also minimizes the maximum time among the times of meeting the requirement of each ration shop from its assigned site.

Then select the third site, which in combination with the two sites already selected minimizes the sum of setup cost of warehouse at that site and total cost of meeting the requirements of all the ration shops without violating the budgetary constraint .When tie occurs, the procedure explained above is implemented .The addition of sites can be continued in this way till the desired number of warehouse sites has been selected. Thereafter, drop rule should be used after every add rule. For instance, if warehouses are to be setup at three sites, then following the procedure of add rule select three sites and after the addition of the third site, drop rule must be applied. For this purpose, drop the site that was selected at the earliest in the current combination. Now the add rule is again applied to select a new site following the same procedure explained above. These procedures of combination of add and drop rules are carried out by restricting the choices of sites eligible to be added or drop rules. The $K-1$ most recently added warehouses in the current solution are included in the Tabu list for drop and the site dropped recently is included in the Tabu list for add. The sites in the Tabu list are not considered for the respective operations in the subsequent iterations .The iterative process is continued and various combination of warehouse sites are selected that satisfy not only the constraints (2.3) –(2.7) but also the Tabu lists. The values of both the objective function are also recorded. The combination of warehouse sites, occurring in these iterations, that minimizes the value of objective function given by (2.1), is designed the incumbent solution. The incumbent solution is updated at each iteration. The iterative process stops when we reach the same combination of sites as in the earlier current solution, so as to avoid the recycling of the solutions already examined. The incumbent solution at the end of this process yields the 1st efficient solution (X^1, Y^1) of the formulated problem.

Procedure for 2nd and subsequent efficient solutions

The 2nd efficient solution $(X^2, Y^2)$of the formulated problem is obtained by solving the problem obtained from the 1st prioritized bi-criterion problem after replacing t_{ij}'s corresponding to t_{ij}'s $\geq T(X^1)$ by an arbitrary large positive number M .The problem thus obtained is similar to the 1st prioritized bi-criterion problem and is designated the 2nd prioritized bi-criterion problem. The 2nd efficient solution is obtained in the same manner as follows for the 1st efficient solution by selecting K sites through the use of add rule and then using a combination of add and drop rules and iterating till all the combination of warehouse sites, satisfying the constraints (2.3)-(2.7) and the respective Tabu lists, are explored, remembering that M is an arbitrary large positive number. The incumbent solution at the end of the iterative process yields the 2nd efficient solution (X^2, Y^2) of the formulated problem.

The 3rd and subsequent efficient solutions are obtained in the same way as is done to obtain the 2nd efficient solution. This process of obtaining the efficient solutions is terminated after encountering a prioritized problem whose optimal solution has a variable x_{ij} at level 1 with which is associated the cost M indicating that it is no longer possible to obtain a new efficient solution with lesser duration.

2.4. CASE STUDY

We consider an example of warehouse problem in which there are $N=7$ potential sites, $K=3$ warehouses to be located, $M=5$ ration shops, the budget given for constructing $K=3$ warehouses is $B= 1400$ and assigning numerical values to c_{ij}'s , t_{ij}'s and C_j'S in the problem. In the table below, rows 1-5 correspond to ration shops and columns 1-7 correspond to potential warehouse sites. Upper and lower entries of a cell (i, j) depict the units of costs c_{ij} and time t_{ij} respectively of meeting the requirement of ration shop i from potential site j .On applying the procedure discussed above, obtain the efficient solutions of the numerical problem and then incorporate the tabu search approach to iterate towards the elusive global optimum, while considering the moves which even worsen the values of the objective function.

Ration shops	Warehouse potential sites						
	1	2	3	4	5	6	7
1	40 8	30 2	50 11	120 6	180 10	170 9	150 7
2	70 6	80 6	130 9	130 7	170 8	140 10	20 11
3	80 8	20 9	200 5	70 8	140 11	130 12	60 10
4	60 11	10 6	70 13	80 11	30 9	160 8	210 6
5	90 10	100 13	20 8	60 12	40 6	50 6	220 14
Set up cost(C_j)	100	300	700	800	400	200	500

First Non-dominated Solution

Step 1. Add the entire costs *column wise* and the least of the total costs thus selected.

	Warehouse potential sites						
	1	2	3	4	5	6	7
Sum of costs in single column	340	240	470	460	560	650	660
Set up cost(C_j)	100	300	700	800	400	200	500
Total cost (C)	440	540	1170	1260	960	850	1160
Corresp.Time	11	13	13	12	11	12	14

The solution becomes $y_1 = 1, x_{11} = x_{21} = x_{31} = x_{41} = x_{51} = 1$ and C =*440* units and T=*11* units.

Step 2. Selection of two warehouse potential sites at a time.

	Warehouse potential sites					
	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
Sum of min. costs in two columns	220	270	300	260	300	270
Set up cost(C_j)	400	800	900	500	300	600
Total cost(C)	620	1070	1200	760	600	870
Corresp.Time	10	11	12	9	11	11

The solution at stage is $y_1 = y_6 = 1, x_{11} = x_{21} = x_{31} = x_{41} = x_{56} = 1$ and C=*600* units and T= *11*units.

Step 3. Selection of three warehouse potential sites at a time.

	Warehouse potential sites				
	(1,6,2)	(1,6,3)	(1,6,4)	(1,6,5)	(1,6,7)
Sum of min. costs in three columns	180	270	290	260	230
Set up cost(C_j)	600	1000	1100	700	800
Total cost(C)	780	1270	1390	960	1030
Corresp.Time	9	11	11	9	11

The solution obtained is $y_1 = y_6 = y_2 = 1, x_{12} = x_{21} = x_{32} = x_{42} = x_{56} = 1$ and C=*780* units and T =*9* units. Thus, if three warehouses are to be set up then they should be at sites 1, 6 and 2. Hence, the solution is

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_6, y_2 x_{12}, x_{21}, x_{32} x_{42}, x_{56}	780	9	y_1, y_6, y_2 x_{12}, x_{21}, x_{32} x_{42}, x_{56}	780	9

Step 4. Now perform iterations to improve upon the solution with respect to the first objective function.

Iteration No .1 The locations selected for setting up of warehouses are 1, 6 and 2. Thus, the Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_6, y_2\}$	$\{y_1\}$

Hence, the solutions at this stage are

	Warehouse potential sites			
	(6,2,3)	(6,2,4)	(6,2,5)	(6,2,7)
Sum of min. costs in three columns	160	190	180	130
Set up cost(C_j)	1200	1300	900	1000
Total cost(C)	1360	1490	1080	1130
Corresp.Time	9	9	9	11

Since combination of locations 6, 2 and 5 gives the least cost thus treated as current solution but the incumbent solution still remains the same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_6, y_2, y_5 x_{12}, x_{22}, x_{32} x_{42}, x_{55}	1080	9	y_1, y_6, y_2 x_{12}, x_{21}, x_{32} x_{42}, x_{56}	780	9

Iteration No.2 The Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_2, y_5\}$	$\{y_6\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites		
	(2,5,1)	(2,5,3)	(2,5,7)
Sum of min. costs in three columns	170	160	120
Set up cost(C_j)	800	1400	1200
Total cost(C)	970	1560	1320
Corresp.Time	9	9	11

Here, combination of locations 2, 5 and 1 gives the least cost and hence treated as current solution but the incumbent solution remains same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
x_{42}, x_{55} x_{12}, x_{21}, x_{32} x_{42}, x_{55}	970	9	y_1, y_6, y_2 x_{12}, x_{21}, x_{32} x_{42}, x_{56}	780	9

Iteration No.3 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_5, y_1\}$	$\{y_2\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(5,1,3)	(5,1,4)	(5,1,6)	(5,1,7)
Sum of min. costs in three columns	240	250	260	190
Set up cost(C_j)	1200	1300	700	1000
Total cost(C)	1440	1550	960	1190
Corresp.Time	9	9	9	11

Out of these, combination of locations 5, 1 and 6 is selected for setting up of warehouses and treated as current solution but incumbent solution is still same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_5, y_1, y_6 x_{11}, x_{21}, x_{31} x_{45}, x_{55}	960	9	y_1, y_6, y_2 x_{12}, x_{21}, x_{32} x_{42}, x_{56}	780	9

Iteration No.4 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_1, y_6\}$	$\{y_5\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(1,6,2)	(1,6,3)	(1,6,4)	(1,6,7)
Sum of min. costs in three columns	180	270	290	230
Set up cost(C_j)	600	1000	1100	800
Total cost(C)	780	1270	1390	1030
Corresp.Time	9	11	11	11

The combination of locations 1, 6 and 2 is considered as current solution but the incumbent solution remains the same. Since 1, 6 and 2 is the solution where we started from, so in order to stop the recycling, stop the iteration process. Hence, the 1st non-dominated solution is

Efficient sol.	Variables $x_{ij}'s$ and $y_j's$ at level	Total cost of Meeting req.	Duration of Meeting req.
(X^1,Y^1)	$x_{12}, x_{21}, x_{32}, x_{42}, x_{56}, y_1, y_6, y_2$	$C(X^1)$ =30+70+20 +10+50=780	$T(X^1)$=max {6,6,2,9,6}=9

Step 5. Change $t_{ij}'s$ by M (large +ve no.), $t_{ij}'s \geq 9$

Ration shops	Warehouse potential sites						
	1	2	3	4	5	6	7
1	40 8	30 2	50 M	120 6	180 M	170 M	150 7
2	70 6	80 6	130 M	130 7	170 8	140 M	20 M
3	80 8	20 M	200 5	70 8	140 M	130 M	60 M
4	60 M	10 6	70 M	80 M	30 M	160 8	210 6
5	90 M	100 M	20 8	60 M	40 6	50 6	220 M
Set up cost(C_j)	100	300	700	800	400	200	500

2nd non dominated solution

Step 1. Add the entire costs *column wise* and the least of the total costs thus selected.

	Warehouse potential sites						
	1	2	3	4	5	6	7
Sum of costs in single column	340	240	470	460	560	650	660
Set up cost(C_j)	100	300	700	800	400	200	500
Total cost (C)	440	540	1170	1260	960	850	1160
Corresp.Time	M	M	M	M	M	M	M

The solution becomes $y_1 = 1, x_{11} = x_{21} = x_{31} = x_{41} = x_{51} = 1$ and C =*440* units and T=M units.

Step 2. Selection of two warehouse potential sites at a time.

	Warehouse potential sites					
	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
Sum of min. costs in two columns	280	270	300	260	400	490
Set up cost(C_j)	400	800	900	500	300	600
Total cost(C)	680	1070	1200	760	700	1090
Corresp.Time	M	M	M	M	8	M

The solution at stage is $y_1 = y_6 = 1, x_{11} = x_{21} = x_{31} = x_{46} = x_{56} = 1$ and C=*700* units and T= *8* units.

Step 3. Selection of three warehouse potential sites at a time.

	Warehouse potential sites				
	(1,6,2)	(1,6,3)	(1,6,4)	(1,6,5)	(1,6,7)
Sum of min. costs in three columns	240	370	390	390	400
Set up cost(C_j)	600	1000	1100	700	800
Total cost(C)	840	1370	1490	1090	1200
Corresp.Time	8	8	8	8	8

The solution obtained is $y_1 = y_6 = y_2 = 1, x_{12} = x_{21} = x_{31} = x_{42} = x_{56} = 1$ and C=*840* units and T =*8* units. Thus, if three warehouses are to be set up then they should be at sites 1, 6 and 2. Hence, the solution is

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_6, y_2 x_{12}, x_{21}, x_{31} x_{42}, x_{56}	840	8	y_1, y_6, y_2 x_{12}, x_{21}, x_{31} x_{42}, x_{56}	840	8

Step 4. Now perform iterations to improve upon the solution with respect to the first objective function.

Iteration No.1 The locations selected for setting up of warehouses are 1, 6 and 2. Thus, the Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_6, y_2\}$	$\{y_1\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(6,2,3)	(6,2,4)	(6,2,5)	(6,2,7)
Sum of min. costs in three columns	340	240	180	190
Set up cost(C_j)	1200	1300	900	1000
Total cost(C)	1540	1540	1080	1190
Corresp.Time	8	8	M	M

Since the combinations of locations (6, 2, 3) and (6, 2, 4) give the same current solution, arbitrarily select one of these two combinations. So, select combination of locations 6, 2 and 4.

Iteration No.2 The Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_2, y_4\}$	$\{y_6\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites
	(2,4,1)
Sum of min. costs in three columns	240
Set up cost(C_j)	1200
Total cost(C)	1440
Corresp.Time	M

The combinations of locations (2, 4, 3), (2, 4, 5) and (2, 4, 7) do not satisfy the budgetary constraint and the combination 2, 4 and 1 gives the least cost thus treated as current solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_2, y_4, y_1 x_{12}, x_{21}, x_{34} x_{42}, x_{54}	1440	M	y_1, y_6, y_2 x_{12}, x_{21}, x_{31} x_{42}, x_{56}	840	8

Iteration No.3 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{ y_4, y_1 \}$	$\{ y_2 \}$

Hence, the possible solutions at this stage are

	Warehouse potential sites		
	(4,1,5)	(4,1,6)	(4,1,7)
Sum of min. costs in three columns	250	390	450
Set up cost(C_j)	1300	1100	1400
Total cost(C)	1550	1490	1850
Corresp.Time	M	8	M

The combination of location 4, 1, 3 does not satisfy the budgetary constraint and the combination 4, 1 and 6 is selected for setting up of warehouses.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_4, y_1, y_6 x_{11}, x_{21}, x_{34} x_{46}, x_{56}	1490	8	y_1, y_6, y_2 x_{12}, x_{21}, x_{31} x_{42}, x_{56}	840	8

Iteration No.4 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{ y_1, y_6 \}$	$\{ y_4 \}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(1,6,2)	(1,6,3)	(1,6,5)	(1,6,7)
Sum of min. costs in three columns	240	370	390	400
Set up cost(C_j)	600	1000	700	800
Total cost(C)	840	1370	1090	1200
Corresp.Time	8	8	8	8

Out of these, combination of locations 1, 6 and 2 is selected for setting up of warehouses.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_6, y_2 x_{12}, x_{21}, x_{31} x_{42}, x_{56}	840	8	y_1, y_6, y_2 x_{12}, x_{21}, x_{31} x_{42}, x_{56}	840	8

Since 1, 6 and 2 is the solution where we started from, so in order to stop the recycling, stop the iteration process. Thus, second non-dominated solution is at locations 1, 6 and 2 with $C= 840$ units and $T=8$ units.

Hence, two non-dominated solutions are as shown below.

Efficient sol.	Variables x_{ij}'s and y_j's at level	Total cost of meeting req.	Duration of meeting req.
(X^1,Y^1)	$x_{12}, x_{21}, x_{32}, x_{42}, x_{56}, y_1, y_6, y_2$	$C(X^1)$ =30+70+20 +10+50+600=780	$T(X^1)$= max {6,6,2,9,6}=9
(X^2,Y^2)	$x_{12}, x_{21}, x_{31}, x_{42}, x_{56}, y_1, y_6, y_2$	$C(X^2)$ =30+70+80 +10+50+600=840	$T(X^2)$=max {2,6,8,6,6}=8

Step 6. Change t_{ij}'s by M (large +ve no.), t_{ij}'s $\geq$ 8

Ration shops	Warehouse potential sites						
	1	2	3	4	5	6	7
1	40 M	30 2	50 M	120 6	180 M	170 M	150 7
2	70 6	80 6	130 M	130 7	170 M	140 M	20 M
3	80 M	20 M	200 5	70 M	140 M	130 M	60 M
4	60 M	10 6	70 M	80 M	30 M	160 M	210 6
5	90 M	100 M	20 M	60 M	40 6	50 6	220 M
Set up cost(C_j)	100	300	700	800	400	200	500

3rd non-dominated solution

Step 1. Add the entire costs *column wise* and the least of the total costs thus selected.

	Warehouse potential sites						
	1	2	3	4	5	6	7
Sum of costs in single column	340	240	470	460	560	650	660
Set up cost(C_j)	100	300	700	800	400	200	500
Total cost (C)	440	540	1170	1260	960	850	1160
Corresp.Time	M	M	M	M	M	M	M

The solution becomes $y_1 = 1, x_{11} = x_{21} = x_{31} = x_{41} = x_{51} = 1$ and C =*440* units and T=M units.

Step 2. Selection of two warehouse potential sites at a time.

	Warehouse potential sites					
	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
Sum of min. costs in two columns	220	390	380	260	300	580
Set up cost(C_j)	400	800	900	500	300	600
Total cost(C)	620	1190	1280	760	600	1180
Corresp.Time	M	M	M	M	M	M

The solution at stage is $y_1 = y_6 = 1, x_{11} = x_{21} = x_{31} = x_{41} = x_{56} = 1$ and C=*600* units and T= M units.

Step 3. Selection of three warehouse potential sites at a time.

	Warehouse potential sites				
	(1,6,2)	(1,6,3)	(1,6,4)	(1,6,5)	(1,6,7)
Sum of min. costs in three columns	180	420	370	260	540
Set up cost(C_j)	600	1000	1100	700	800
Total cost(C)	780	1420	1470	960	1340
Corresp.Time	M	M	M	M	M

No improvement of solution is possible. So, there are only two non-dominated solutions.

Efficient sol.	Variables x_{ij}'s and y_j's at level	Total cost of meeting req.	Duration of meeting req.
(X^1,Y^1)	$x_{12}, x_{21}, x_{32}, x_{42}, x_{56}, y_1, y_6, y_2$	$C(X^1)$ =30+70+20 +10+50+600=780	$T(X^1)$= max {6,6,2,9,6}=9
(X^2,Y^2)	$x_{12}, x_{21}, x_{31}, x_{42}, x_{56}, y_1, y_6, y_2$	$C(X^2)$ =30+70+80 +10+50+600=840	$T(X^2)$=max {2,6,8,6,6}=8

If the combination of locations 6, 2 and 3 is selected in iteration 2 for *2nd non-dominated* solution then the following set of solutions is obtained

Efficient sol.	Variables x_{ij}'s and y_j's at level	Total cost of meeting req.	Duration of meeting req.
(X^1,Y^1)	$x_{12}, x_{21}, x_{32}, x_{42}, x_{56}, y_1, y_6, y_2$	$C(X^1)$ =30+70+20 +10+50+600=780	$T(X^1)$= max {6,6,2,9,6}=9
(X^2,Y^2)	$x_{12}, x_{21}, x_{31}, x_{42}, x_{56}, y_1, y_6, y_2$	$C(X^2)$ =30+70+80 +10+50+600=840	$T(X^2)$=max {2,6,8,6,6}=8

2.5. CONCLUSION

This work is an extension of the problem considered by Prakash *et al.* (2009), which represents the un-capacitated warehouse location problem as there is no restriction on number of ration shops to be clustered to a selected site. In this problem setup cost of warehouses has been introduced in the objective function**.** The problem has been solved by solving a sequence of prioritized bi-criterion problems incorporating Tabu search in the heuristic algorithm used for solving the prioritized bi-criterion problems. The incorporation of Tabu search in a heuristic algorithm allows search for a global solution in a wider region compared to the case without incorporating Tabu search, there by increasing the possibility of arriving at the global solution or close to it. Two non-dominated solutions have been obtained for un-capacitated warehouse location problem and the decision maker can select the solution according to his/her priorities.

CHAPTER-3

3.1. INTRODUCTION

Ignizio and Cavalier (1994) have formulated and solved the problem of selecting upto a fixed number of sites from among a given number of potential sites for locating warehouses at them and clustering customers to the selected sites in such a way that each customer is assigned to a unique selected site. The problem has a single objective to minimize the sum of distances from each customer to his/her assigned site. Praveena *et al*. (1999) have modified and extended this problem to a new situation. The problem selects up to a fixed number of sites from among a given number of potential sites for clustering ration shops to them subject to several constraints with two objectives. One of the constraints is that each ration shop should be clustered to a unique site which is selected for locating a warehouse at it: however there is no restriction on the number of ration shops to be clustered to a warehouse at the selected site. Another constraint is that the setup cost of the warehouses should not exceed a certain budgetary amount. The two objectives are to minimize the total cost and duration of meeting requirements of all the ration shops from their assigned warehouses to the selected sites. The two objectives are not accorded priorities. The set of efficient solutions of this problem has been found through solving using a heuristic algorithm consisting of a combination of add and drop rules. Prakash *et al.* (2009) solved this very problem through solving a sequence of prioritized bi-criterion problems incorporating Tabu search in the heuristic algorithm used for solving the prioritized bi-criterion problems. The incorporation of Tabu search leads to better results than those obtained earlier without its incorporation.

We extended the above problem to new situation in which **(1)** the objective to minimize the total cost, also includes set up cost of warehouse at particular site along with the cost of meeting the requirements of all the ration shops. **(2)** The problem becomes capacitated when we put the additional constraints in above problem, which are

I. Each site selected to locate a warehouse must have at least one ration shop assigned to it.
II. Maximum upto a fixed number of ration shops can be assigned to each site selected for locating a warehouse at it.
III. Total number of ration shops that can be given to the selected number of sites selected should not be less than total number of ration shops to be clustered.

3.2. FORMULATION OF CAPACITATED WAREHOUSE LOCATION PROBLEM

Let us consider M ration shops, N potential warehouse sites and K be the maximum number of sites, which can be selected from among the N potential sites for locating warehouses at them. The M ration shops are to be clustered upto K sites in such a way that each shop is assigned to a unique site, which is selected for locating a warehouse. Let c_{ij} and t_{ij} $(i=1,2,\ldots,M;\ j=1,2,\ldots,N)$ units be the cost and time, respectively of meeting requirements of ration shop i from site j. *Let* C_j units be the cost of setting up a warehouse at the site j and B units be the budgetary amount allocated for setting up warehouses. Let aj be the maximum number of ration shops that can be clustered to j^{th} site ($j=1, 2, 3... N$). Let x_{ij} be the variable assuming value 0 or 1 according as ration shop i is not assigned or assigned to site j, and y_j be the variable assuming the value 0 or 1 according as potential site j is not selected or selected for locating a warehouse. Thus, the problem seeks to select up to a fixed number of sites from among a given number of potential sites for locating warehouses at them with two objectives subject to several constraints. The two objectives are not accorded priorities. The objectives are to minimize the total cost and duration of meeting requirements of all ration shops from their assigned warehouses at the selected sites.

The two objectives to be minimized are

$$C = \sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij} x_{ij} + \sum_{j=1}^{N} C_j y_j \tag{3.1}$$

$$T = \max\{t_{ij}, x_{ij} = 1 (i = 1, 2, ..., M; j = 1, 2, ..., N)\} \tag{3.2}$$

Subject to

$$\sum_{j=1}^{N} y_j \le K \tag{3.3}$$

$$\sum_{i=1}^{M} x_{ij} \ge y_j \quad (j = 1, 2, ..., N) \tag{3.4}$$

$$\sum_{j=1}^{N} x_{ij} = 1 \ (i = 1, 2, ..., M) \tag{3.5}$$

$$x_{ij} - y_j \le 0 \ (i = 1, 2, ..., M; j = 1, 2, ..., N) \tag{3.6}$$

$$\sum_{i=1}^{M} x_{ij} \le a_j y_j (j = 1, 2, 3, ..., N) \tag{3.7}$$

$$\sum_{j=1}^{N} C_j y_j \le B \tag{3.8}$$

$$\sum_{j=1}^{N} a_j y_j \ge M , a_j \ge 1 \tag{3.9}$$

$$x_{ij}, y_j = 0 \ or \ 1 \ (i = 1, 2, ..., M; j = 1, 2, ..., N) \tag{3.10}$$

The objective functions C and T given by (3.1) and (3.2) are not accorded any priorities. The constraint (3.3) ensures that upto a maximum of K sites can be selected from among the N potential warehouse sites. Constraint (3.4) ensures that each site selected to locate a warehouse must have at least one ration shop assigned to it. Constraints (3.5) and (3.6) ensure that each ration shop is assigned to a unique site which is selected to locate a warehouse. Constraint (3.7) ensures that upto a maximum of a_j ration shops can be assigned to the site j which is selected to locate a warehouse at it. The constraint (3.8) ensures that the total setup cost of warehouses does not exceed the allocated budgetary amount B. constraint (3.9) ensures that the total capacity of selected sites for locating warehouses at them should not be less than M. It is required to obtain the set of efficient solutions of the problem given by (3.1) to (3.10).

3.3. SOLUTION PROCEDURE

The problem formulated above is a binary integer nonlinear problem because the variables x_{ij} 's and y_j's are binary integers and the objective function T given by (3.2) is nonlinear. A set of efficient solutions is obtained through an iterative heuristic procedure requiring solutions of bi-criterion problems with prioritized objectives.

Procedure for 1st efficient solution

The first efficient solution (X^1, Y^1) of the problem formulated above by (3.1) to (3.10) is the optimal solution of the problem wherein the total cost C and the duration of meeting requirements of the entire ration shops, from their assigned warehouses, are minimized with 1st and 2nd priorities, respectively, subject to the constraint (3.3)-(3.10).The problem yielding 1st efficient solution is designated the 1st prioritized bi-criterion problem of the formulated problem. The heuristic method proposed to solve the 1st bi-criterion problem is a modification and extension of the heuristic method to solve a partial covering problem with a single objective. The proposed method consists of combination of add and drop rules. Start with no sites selected and select the site j wherefrom the sum of setup cost of warehouse at the site j and total cost of meeting requirements of a_j ration shops (but with their cost in increasing order starting with least cost)from the j^{th} site($j=1,2,3,\ldots,N$)is minimum without violating the budgetary constraint. Each ration shop is assigned to that site wherefrom the cost of meeting its requirement is minimum, among the selected sites. In case of tie, select the site wherefrom the maximum time among the times of meeting the requirements of each ration shop is also minimum. And in case, there is a tie again, arbitrarily select a site among the tied ones.

Next, a second site is selected which in combination with the site already selected minimizes the sum of setup cost of warehouses at these sites and total cost of meeting requirements of all ration shops without violating the budgetary constraint. This pair of sites must be selected from amongst those pairs of sites which satisfy the constraint (3.9). Each ration shop is assigned to that site wherefrom the cost of meeting its requirement is minimum, among the selected sites but if capacity of any site from the selected sites have been finished then this site will not be considered for assigning the rest of ration shops. When tie occur then select that site ,which in combination with the already selected site also minimizes the maximum time among the times of meeting the requirement of each ration shop from its assigned site.

Then select the third site, which in combination with the two sites already selected minimizes the sum of setup cost of warehouses at these sites and total cost of meeting the requirements of all the ration shops without violating the budgetary constraint and the constraint (3.9). When tie occurs, the procedure explained above is implemented. If while assigning ration shops to the selected site, any one of the site doesn't even have one ration shop assigned to it then assign all ration shops to the selected number of sites in such a way that increase in cost is minimum for satisfying constraint(3.4). The addition of sites can be continued in this way till the desired number of warehouse sites has been selected. Thereafter, drop rule should be used after every add rule. For instance, if warehouses are to set up at three sites, then following the procedure of add rule select three sites and after the addition of the third site, drop rule must be applied. For this purpose, drop the site that was selected at the earliest in the current combination. Now the add rule is again applied to select a new site following the same procedure explained above. These procedures of combination of add and drop rules are carried out by restricting the choices of sites eligible to be added or drop rules. The *K-1* most recently added warehouses in the current solution are included in the Tabu list for drop and the site dropped recently is included in the Tabu list for add. The sites in the Tabu list are not considered for the respective operations in the subsequent iterations. The iterative process is continued a various combination of warehouse sites are selected that satisfy not only the constraints (3.3) – (3.9) but also the Tabu lists. The values of both the objective function are also recorded. The combination of warehouse sites, occurring in these iterations, that minimizes the value of objective function given by (3.1), is designed the incumbent solution. The incumbent solution is updated at each iteration. The iterative process stops when we reach the solution which has already occurred earlier as the current solution, so as to avoid the recycling of the solutions already examined. The incumbent solution at the end of this process yields the 1st efficient solution $(X^1, Y^1)$of the formulated problem.

Procedure for 2nd and subsequent efficient solutions

The 2nd efficient solution $(X^2, Y^2)$of the formulated problem is obtained by solving the problem obtained from the 1st prioritized bi-criterion problem after replacing t_{ij}'s corresponding to t_{ij}'s $\geq T(X^1)$ by an arbitrary large positive number M .The problem thus obtained is similar to the 1st prioritized bi-criterion problem and is designated the 2nd prioritized bi-criterion problem. The 2nd efficient solution is obtained in the same manner as follows for the 1st efficient solution by selecting *K* sites through the use of add rule and then using a combination of add and drop rules and iterating till all the combination of warehouse sites, satisfying the constraints (3.3)-(3.9) and the respective Tabu lists, are explored, remembering that M is an arbitrary large positive number. The incumbent solution at the end of the iterative process yields the 2nd efficient solution (X^2, Y^2) of the formulated problem.

The 3rd and subsequent efficient solutions are obtained in the same way as is done to obtain the 2nd efficient solution. This process of obtaining the efficient solutions is terminated after encountering a prioritized problem whose optimal solution has a variable x_{ij} at level 1 with which is associated the cost M indicating that it is no longer possible to obtain a new efficient solution with lesser duration.

3.4. CASE STUDY

We consider an example of warehouse problem in which there are $N=7$ potential sites, $K=3$ warehouses to be located, $M=5$ ration shops, the budget given for constructing $K=3$ warehouses is $B= 1400$ and assigning numerical values to a_j's , c_{ij}'s , t_{ij}'s and C_j'S in the problem. In the table below, rows 1-5 correspond to ration shops and columns 1-7 correspond to potential warehouse sites. Upper and lower entries of a cell (i , j) depict the units of costs c_{ij} and time t_{ij} respectively of meeting the requirement of ration shop i from potential site j .On applying the procedure discussed above, obtain the efficient solutions of the numerical problem and then incorporate the Tabu search approach to iterate towards the elusive global optimum, while considering the moves which even worsen the values of the objective function.

Ration shops	Warehouse potential sites						
	1	2	3	4	5	6	7
1	40 8	30 2	50 11	120 6	180 10	170 9	150 7
2	70 6	80 6	130 9	130 7	170 8	140 10	20 11
3	80 8	20 9	200 5	70 8	140 11	130 12	60 10
4	60 11	10 6	70 13	80 11	30 9	160 8	210 6
5	90 10	100 13	20 8	60 12	40 6	50 6	220 14
Set up cost(C_j)	100	300	700	800	400	200	500
Capacity(a_j)	2	2	3	3	2	1	3

First Non-dominated Solution

Step 1. Add the entire costs *column wise* and the least of the total costs thus selected.

	Warehouse potential sites						
	1	2	3	4	5	6	7
Sum of costs in single column	100	30	140	210	70	50	230
Set up cost(C_j)	100	300	700	800	400	200	500
Total cost (C)	200	330	840	1010	470	250	730
Corresp.Time	11	9	13	12	9	6	11

The solution becomes $y_1 = 1, x_{11} = x_{41} = 1$ and $C = 200$ units and $T=11$ units.

Step 2. Selection of two warehouse potential sites at a time.

	Warehouse potential sites		
	(1,3)	(1,4)	(1,7)
Sum of min. costs in two columns	400	320	400
Set up cost(C_j)	800	900	600
Total cost(C)	1200	1220	1000
Corresp.Time	13	12	14

Here, combinations of location (1, 2), (1, 5) and (1, 6) do not satisfy constraint (3.9). The solution at stage is $y_1 = y_7 = 1, x_{11} = x_{27} = x_{37} = x_{41} = x_{57} = 1$ and *C=1000* units and *T= 14* units.

Step 3. Selection of three warehouse potential sites at a time.

	Warehouse potential sites				
	(1,7,2)	(1,7,3)	(1,7,4)	(1,7,5)	(1,7,6)
Sum of min. costs in three columns	220	200	240	190	230
Set up cost(C_j)	900	1300	1400	1000	800
Total cost(C)	1120	1500	1640	1190	1030
Corresp.Time	11	11	11	11	11

The solution obtained is $y_1 = y_7 = y_6 = 1, x_{11} = x_{27} = x_{37} = x_{41} = x_{56} = 1$ and *C=1030* units and *T =11* units. So, if three warehouses are to be set up then they should be at sites 1, 7 and 6. Thus, the solution is

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_7, y_6 x_{11}, x_{27}, x_{37} x_{41}, x_{56}	1030	11	y_1, y_7, y_6 x_{11}, x_{27}, x_{37} x_{41}, x_{56}	1030	11

Step 4. Now perform iterations to improve upon the solution with respect to the first objective function.

Iteration No .1 The locations selected for setting up of warehouses are 1, 6 and 2. Thus, the Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_7, y_6\}$	$\{y_1\}$

Hence, the solutions at this stage are

	Warehouse potential sites		
	(7,6,2)	(7,6,3)	(7,6,5)
Sum of min. costs in three columns	450	250	310
Set up cost(C_j)	1000	1400	1100
Total cost(C)	1450	1650	1410
Corresp.Time	11	13	11

The combination of locations 7, 6 and 4 does not satisfy the budgetary constraint. Since the combination of location 7, 6 and 5 gives the least cost thus treated as current solution. The incumbent solution still remains the same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_7, y_6, y_5 x_{17}, x_{27}, x_{37} x_{45}, x_{56}	1410	11	y_1, y_7, y_6 x_{11}, x_{27}, x_{37} x_{41}, x_{56}	1030	11

Iteration No.2 The Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_6, y_5\}$	$\{y_7\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(6,5,1)	(6,5,2)	(6,5,3)	(6,5,4)
Sum of min. costs in three columns	310	310	360	400
Set up cost(C_j)	700	900	1300	1400
Total cost(C)	1010	1210	1660	1800
Corresp.Time	12	12	12	10

Since combination of locations 6, 5 and 1 gives the least cost thus treated as current solution and here the combination of locations 6, 5 and 1 gives the incumbent solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_6, y_5, y_1 x_{11}, x_{21}, x_{36} x_{45}, x_{55}	1010	12	y_6, y_5, y_1 x_{11}, x_{21}, x_{36} x_{45}, x_{55}	1010	12

Iteration No.3 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_5, y_1\}$	$\{y_6\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(5,1,2)	(5,1,3)	(5,1,4)	(5,1,7)
Sum of min. costs in three columns	190	300	250	190
Set up cost(C_j)	800	1200	1300	1000
Total cost(C)	990	1500	1550	1190
Corresp.Time	9	11	9	11

The combination of locations 5, 1 and 2 is selected for setting up of warehouses. In this case current solution is giving the incumbent solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_5, y_1, y_2 x_{12}, x_{21}, x_{32} x_{45}, x_{55}	990	9	y_5, y_1, y_2 x_{12}, x_{21}, x_{32} x_{45}, x_{55}	990	9

Iteration No.4 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_1, y_2\}$	$\{y_5\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(1,2,3)	(1,2,4)	(1,2,6)	(1,2,7)
Sum of min. costs in three columns	200	240	230	220
Set up cost(C_j)	1100	1200	600	900
Total cost(C)	1300	1440	830	1120
Corresp.Time	11	12	11	11

Here the combination of locations 1, 2 and 6 is giving the current solution as well as the incumbent solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_2, y_6 x_{12}, x_{21}, x_{32} x_{41}, x_{56}	830	11	y_1, y_2, y_6 x_{12}, x_{21}, x_{32} x_{41}, x_{56}	830	11

Iteration No.5 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_2, y_6\}$	$\{y_1\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites		
	(2,6,3)	(2,6,5)	(2,6,7)
Sum of min. costs in three columns	330	310	450
Set up cost(C_j)	1200	900	1000
Total cost(C)	1530	1210	1450
Corresp.Time	13	12	14

The combination of locations 2, 6 and 5 is selected for setting up of warehouses. In this case the incumbent solution remains same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_2, y_6, y_5 x_{12}, x_{22}, x_{36} x_{45}, x_{55}	1210	12	y_1, y_2, y_6 x_{12}, x_{21}, x_{32} x_{41}, x_{56}	830	11

Iteration No.6 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_6, y_5\}$	$\{y_2\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(6,5,1)	(6,5,3)	(6,5,4)	(6,5,7)
Sum of min. costs in three columns	310	360	400	310
Set up cost(C_j)	700	1300	1400	1100
Total cost(C)	1010	1660	1800	1410
Corresp.Time	12	12	10	11

The combination of locations 6, 5and 1 gives the current solution but the incumbent solutions still remains same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_6, y_5, y_1 x_{11}, x_{21}, x_{36} x_{45}, x_{55}	1010	12	y_1, y_2, y_6 x_{12}, x_{21}, x_{32} x_{41}, x_{56}	830	11

Since *C=1010* units, *T=11* units is the solution which has occurred earlier so in order to stop the recycling, stop the iteration process.

So, the 1st non-dominated solution is

Efficient sol.	Variables x_{ij}'s and y_j's at level	Total cost of Meeting req.	Duration of Meeting req.
(X^1,Y^1)	$x_{12}, x_{21}, x_{32}, x_{41}, x_{56}, y_1, y_2, y_6$	$C(X^1)$ =30+70+20+ 60+50+600=830	$T(X^1)$=max {2,6,9,11,6}=11

Step 5. Change t_{ij}'s by M (large +ve no.), t_{ij}'s $\geq$ 11

Ration shops	Warehouse potential sites						
	1	2	3	4	5	6	7
1	40 8	30 2	50 M	120 6	180 10	170 9	150 7
2	70 6	80 6	130 9	130 7	170 8	140 10	20 M
3	80 8	20 9	200 5	70 8	140 M	130 M	60 10
4	60 M	10 6	70 M	80 M	30 9	160 8	210 6
5	90 10	100 M	20 8	60 M	40 6	50 6	220 M
Set up cost(C_j)	100	300	700	800	400	200	500
Capacity(a_j)	2	2	3	3	2	1	3

2^nd^ non-dominated solution

Step 1. Add the entire costs *column wise* and the least of the total costs thus selected.

	Warehouse potential sites						
	1	2	3	4	5	6	7
Sum of costs in single column	110	30	350	320	70	50	420
Set up cost(C_j)	100	330	700	800	400	200	500
Total cost (C)	210	330	1050	1120	470	250	920
Corresp.Time	8	9	9	8	9	6	10

The solution become $y_1 = 1, x_{11} = x_{21} = 1$ and $C = 210$ units and $T=8$ units.

Step 2. Selection of two warehouse potential sites at a time.

	Warehouse potential sites		
	(1,3)	(1,4)	(1,7)
Sum of min. costs in two columns	400	320	600
Set up cost(C_j)	800	900	600
Total cost(C)	1200	1220	1200
Corresp.Time	M	M	M

The combinations of locations (1, 2), (1, 5) and (1, 6) do not satisfy the constraint (3.9). Since the combinations of locations (1, 3) and (1, 4) give the same cost *C* and *T*, arbitrarily select one of these two pairs. So, select the pair (1, 3).

Step 3. Selection of three warehouse potential sites at a time.

	Warehouse potential sites			
	(1,3,2)	(1,3,5)	(1,3,6)	(1,3,7)
Sum of min. costs in three columns	200	360	490	400
Set up cost(C_j)	110	1200	1000	1300
Total cost(C)	1300	1560	1490	1700
Corresp.Time	M	9	8	10

The solution obtained is $y_1 = y_3 = y_6 = 1, x_{11} = x_{21} = x_{33} = x_{46} = x_{53} = 1$ and $C=1490$ units and $T=8$ units. Thus, if three warehouses are to be set up then they should be at sites 1, 3 and 6. Hence, the solution is

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_3, y_6 x_{11}, x_{21}, x_{33} x_{46}, x_{53}	1490	8	y_1, y_3, y_6 x_{11}, x_{21}, x_{33} x_{46}, x_{53}	1490	8

Iteration No.1 The Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_3, y_6\}$	$\{y_1\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites		
	(3,6,2)	(3,6,5)	(3,6,7)
Sum of min. costs in three columns	490	550	520
Set up cost(C_j)	1200	1300	1400
Total cost(C)	1690	1850	1920
Corresp.Time	8	9	10

The combination of locations 3, 6 and 4 does not satisfy the budgetary constraint and the combination 3, 6 and 2 gives the least cost thus treated as current solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_3, y_6, y_2 x_{12}, x_{22}, x_{33} x_{46}, x_{53}	1690	8	y_1, y_3, y_6 x_{11}, x_{21}, x_{33} x_{46}, x_{53}	1490	8

Iteration No.2 The Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_6, y_2\}$	$\{y_3\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(6,2,1)	(6,2,4)	(6,2,5)	(6,2,7)
Sum of min. costs in three columns	370	400	310	550
Set up cost(C_j)	600	1300	900	1000
Total cost(C)	970	1700	1210	1550
Corresp.Time	10	M	M	M

The combination 6, 2 and 1 gives the least cost thus treated as current solution and also as incumbent solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_6, y_2, y_1 x_{12}, x_{21}, x_{32} x_{46}, x_{51}	970	10	y_6, y_2, y_1 x_{12}, x_{21}, x_{32} x_{46}, x_{51}	970	10

Iteration No.3 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_2, y_1\}$	$\{y_6\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(2,1,3)	(2,1,4)	(2,1,5)	(2,1,7)
Sum of min. costs in three columns	200	240	190	420
Set up cost(C_j)	1100	1200	800	900
Total cost(C)	1300	1440	990	1320
Corresp.Time	M	M	9	10

Out of these, combination of locations 2, 1 and 5 is selected for setting up of warehouses.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_2, y_1, y_5 x_{12}, x_{21}, x_{32} x_{45}, x_{55}	990	9	y_6, y_2, y_1 x_{12}, x_{21}, x_{32} x_{46}, x_{51}	970	10

Iteration No.4 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_1, y_5\}$	$\{y_2\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(1,5,3)	(1,5,4)	(1,5,6)	(1,5,7)
Sum of min. costs in three columns	360	250	310	240
Set up cost(C_j)	1200	1300	700	1000
Total cost(C)	1560	1550	1010	1240
Corresp.Time	9	9	M	10

Out of these, combination of locations 1, 5 and 7 is selected for setting up of warehouses and thus treated as the current solution but incumbent solution remains same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_5, y_7 x_{11}, x_{21}, x_{37} x_{45}, x_{55}	1240	10	y_6, y_2, y_1 x_{12}, x_{21}, x_{32} x_{46}, x_{51}	970	10

Iteration No.5 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_5, y_7\}$	$\{y_1\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites	
	(5,7,2)	(5,7,6)
Sum of min. costs in three columns	240	420
Set up cost(C_j)	1200	1100
Total cost(C)	1440	1520
Corresp.Time	10	10

The combinations of locations (5, 7, 3) and (5, 7, 4) do not satisfy the budgetary constraint and the combination 5, 7 and 2 gives the least cost hence treated as current solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_5, y_7, y_2 x_{12}, x_{22}, x_{37} x_{45}, x_{55}	1440	10	y_6, y_2, y_1 x_{12}, x_{21}, x_{32} x_{46}, x_{51}	970	10

Iteration No.6 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_7, y_2\}$	$\{y_5\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites	
	(7,2,1)	(7,2,6)
Sum of min. costs in three columns	420	550
Set up cost(C_j)	900	1000
Total cost(C)	1320	1550
Corresp.Time	10	M

The combinations of locations (7, 2, 3) and (7, 2, 4) do not satisfy the budgetary constraint. The combination of locations 7, 2 and 1 is selected for setting up of warehouses and thus treated as the current solution but incumbent solution remains same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_7, y_2, y_1 x_{12}, x_{21}, x_{32} x_{47}, x_{51}	1320	10	y_6, y_2, y_1 x_{12}, x_{21}, x_{32} x_{46}, x_{51}	970	10

Iteration No.7 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_2, y_1\}$	$\{y_7\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(2,1,3)	(2,1,4)	(2,1,5)	(2,1,6)
Sum of min. costs in three columns	200	240	190	370
Set up cost(C_j)	1100	1200	800	600
Total cost(C)	1300	1440	990	970
Corresp.Time	M	M	9	10

The combination of locations 2, 1 and 6 gives the least cost and hence treated as the current solution but incumbent solution is still same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_2, y_1, y_6 x_{11}, x_{21}, x_{33} x_{46}, x_{51}	970	10	y_6, y_2, y_1 x_{12}, x_{21}, x_{32} x_{46}, x_{51}	970	10

Since *C=970* units, *T=10* units is the solution which has already occurred, so in order to stop the recycling, stop the iteration process. Thus, second non-dominated solution is at locations 6, 2 and 1 with *C=970* units and *T=10* units.

Hence, two non-dominated solutions are as shown below.

Efficient sol.	Variables x_{ij}'s and y_j's at level	Total cost of meeting req.	Duration of meeting req.
(X^1,Y^1)	x_{12}, x_{21}, x_{32}, x_{41}, x_{56}, y_1, y_2, y_6	$C(X^1)$ =30+70+20+ 60+50+600=830	$T(X^1)$=max {2,6,9,11,6}=11
(X^2,Y^2)	x_{12}, x_{21}, x_{32}, x_{46}, x_{51}, y_6, y_2, y_1	$C(X^2)$ =30+70+20 +160+90+600=970	$T(X^2)$=max {2,6,9,8,10}=10

Step 5. Change $t_{ij}'s$ by M (large +ve no.), $t_{ij}'s \geq 10$

Ration shops	Warehouse potential sites						
	1	2	3	4	5	6	7
1	40 8	30 2	50 M	120 6	180 M	170 9	150 7
2	70 6	80 6	130 9	130 7	170 8	140 M	20 M
3	80 8	20 9	200 5	70 8	140 M	130 M	60 M
4	60 M	10 6	70 M	80 M	30 9	160 8	210 6
5	90 M	100 M	20 8	60 M	40 6	50 6	220 M
Set up cost(C_j)	100	300	700	800	400	200	500
Capacity(a_j)	2	2	3	3	2	1	3

3rd non-dominated solution

Step 1. Add the entire costs *column wise* and the least of the total costs thus selected.

	Warehouse potential sites						
	1	2	3	4	5	6	7
Sum of costs in single column	110	30	350	320	70	50	380
Set up cost(C_j)	100	300	700	800	400	200	500
Total cost (C)	210	330	1050	1120	470	250	880
Corresp.Time	8	9	9	8	9	6	M

The solution become $y_1 = 1$, $x_{11} = x_{21} = 1$ and $C = 210$ units and $T=8$ units.

Step 2. Selection of two warehouse potential sites at a time.

	Warehouse potential sites		
	(1,3)	(1,4)	(1,7)
Sum of min. costs in two columns	400	320	600
Set up cost(C_j)	800	900	600
Total cost(C)	1200	1220	1200
Corresp.Time	M	M	M

The combinations of locations (1, 2), (1, 5) and (1, 6) do not satisfy the constraint (3.9). Since combinations of locations (1, 3) and (1, 7) give the same cost *C* and *T*, arbitrarily select one of these two pairs of locations. So select the pair (1, 3).

Step 3. Selection of three warehouse potential sites at a time.

	Warehouse potential sites			
	(1,3,2)	(1,3,5)	(1,3,6)	(1,3,7)
Sum of min. costs in three columns	200	360	490	540
Set up cost(C_j)	1100	1200	1000	1300
Total cost(C)	1300	1560	1490	1840
Corresp.Time	M	9	8	8

The combination of locations 1, 3 and 4 does not satisfy the budgetary constraint. The solution obtained is $y_1 = y_3 = y_6 = 1, x_{11} = x_{21} = x_{33} = x_{46} = x_{53} = 1$ and *C=1490* units and *T =8* units. Thus, if three warehouses are to be set up then they should be at sites 1, 3 and 6. Hence, the solution is

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_3, y_6 x_{11}, x_{21}, x_{33} x_{46}, x_{53}	1490	8	y_1, y_3, y_6 x_{11}, x_{21}, x_{33} x_{46}, x_{53}	1490	8

Step 4.

Iteration No.1 The Tabu Lists are given by

Tabu list for drop	Tabu list for add
$\{y_3, y_6\}$	$\{y_1\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites		
	(3,6,2)	(3,6,5)	(3,6,7)
Sum of min. costs in three columns	490	550	660
Set up cost(C_j)	1200	1300	1400
Total cost(C)	1690	1850	2060
Corresp.Time	8	9	9

The combination of locations 3, 6 and 4 does not satisfy the budgetary constraint and the combination 3, 6 and 2 gives the least cost thus treated as current solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_3, y_6, y_2 x_{12}, x_{22}, x_{33} x_{46}, x_{53}	1690	8	y_1, y_3, y_6 x_{11}, x_{21}, x_{33} x_{46}, x_{53}	1490	8

Iteration No.2 The Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_6, y_2\}$	$\{y_3\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(6,2,1)	(6,2,4)	(6,2,5)	(6,2,7)
Sum of min. costs in three columns	370	400	310	550
Set up cost(C_j)	600	1300	900	1000
Total cost(C)	970	1700	1210	1550
Corresp.Time	M	M	M	M

The combination of locations 6, 2 and 1 gives the least cost and thus treated as the current solution but the incumbent solution remains same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_6, y_2, y_1 x_{12}, x_{21}, x_{32} x_{46}, x_{51}	970	M	y_1, y_3, y_6 x_{11}, x_{21}, x_{33} x_{46}, x_{53}	1490	8

Iteration No.3 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_2, y_1\}$	$\{y_6\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(2,1,3)	(2,1,4)	(2,1,5)	(2,1,7)
Sum of min. costs in three columns	200	240	190	420
Set up cost(C_j)	1100	1200	800	900
Total cost(C)	1300	1440	990	1320
Corresp.Time	M	M	9	M

The combination of locations 2, 1 and 5 is selected for setting up of warehouses. Here the current solution is the incumbent solution also.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_2, y_1, y_5 x_{12}, x_{21}, x_{32} x_{45}, x_{55}	990	9	y_2, y_1, y_5 x_{12}, x_{21}, x_{32} x_{45}, x_{55}	990	9

Iteration No.4 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_1, y_5\}$	$\{y_2\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(1,5,3)	(1,5,4)	(1,5,6)	(1,5,7)
Sum of min. costs in three columns	360	250	310	240
Set up cost(C_j)	1200	1300	700	1000
Total cost(C)	1560	1550	1010	1240
Corresp.Time	9	9	M	M

Out of these, combination of locations 1, 5 and 4 gives the least cost and thus treated as the current solution but the incumbent solution remains same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_5, y_4 x_{11}, x_{21}, x_{34} x_{45}, x_{55}	1550	9	y_2, y_1, y_5 x_{12}, x_{21}, x_{32} x_{45}, x_{55}	990	9

Iteration No.5 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_5, y_4\}$	$\{y_1\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites
	(5,4,6)
Sum of min. costs in three columns	400
Set up cost(C_j)	1400
Total cost(C)	1800
Corresp.Time	9

The combinations of locations (5, 4, 2) and (5, 4, 3) and (5, 4, 7) do not satisfy the budgetary constraint. The combination of locations 5, 4 and 6 is selected for setting up of warehouses and treated as the current solution but the incumbent solution is still same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_5, y_4, y_6 x_{14}, x_{24}, x_{34} x_{45}, x_{56}	1800	9	y_2, y_1, y_5 x_{12}, x_{21}, x_{32} x_{45}, x_{55}	990	9

Iteration No.6 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_4, y_6\}$	$\{y_5\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites	
	(4,6,1)	(4,6,2)
Sum of min. costs in three columns	400	400
Set up cost(C_j)	1100	1300
Total cost(C)	1500	1700
Corresp.Time	M	M

The combinations of locations (4, 6, 3) and (4, 6, 2) do not satisfy the budgetary constraint and the combination 4, 6 and 1 gives the least cost and hence treated as current solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_4, y_6, y_1 x_{11}, x_{21}, x_{34} x_{46}, x_{54}	1500	M	y_2, y_1, y_5 x_{12}, x_{21}, x_{32} x_{45}, x_{55}	990	9

Iteration No.7 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_6, y_1\}$	$\{y_4\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(6,1,2)	(6,1,3)	(6,1,5)	(6,1,7)
Sum of min. costs in three columns	370	490	310	550
Set up cost(C_j)	600	1000	700	800
Total cost(C)	970	1490	1010	1350
Corresp.Time	M	8	M	M

The combination of locations 6, 1 and 3 is selected for setting up of warehouses and thus treated as the current solution but the incumbent solution remains same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_6, y_1, y_3 x_{11}, x_{21}, x_{33} x_{46}, x_{53}	1490	8	y_2, y_1, y_5 x_{12}, x_{21}, x_{32} x_{45}, x_{55}	990	9

Since *C=1490* units, *T=8* units is the solution where we started from, so in order to stop the recycling, stop the iteration process. Thus, third non-dominated solution is at locations 2, 1 and 5 with *C=990* units and *T=9* units.

Hence, three non-dominated solutions are as shown below.

Efficient sol.	Variables x_{ij}'s and y_j's at level	Total cost of meeting req.	Duration of meeting req.
(X^1,Y^1)	$x_{12}, x_{21}, x_{32}, x_{41}, x_{56}, y_1, y_2, y_6$	$C(X^1)$ =30+70+20+ 60+50+600=830	$T(X^1)$=max {2,6,9,11,6}=11
(X^2,Y^2)	$x_{12}, x_{21}, x_{32}, x_{46}, x_{51}, y_2, y_1, y_6$	$C(X^2)$ =30+70+20 +160+90+600=970	$T(X^2)$=max {2,6,9,8,10}=10
(X^3,Y^3)	$x_{12}, x_{21}, x_{32}, x_{45}, x_{55}, y_2, y_1, y_5$	$C(X^3)$ =30+70+20 +30+40+800=990	$T(X^3)$=max {2,6,9,9,6}=9

Step 5. Change t_{ij}'s by M (large +ve no.), t_{ij}'s $\geq$ 9

Ration shops	Warehouse potential sites						
	1	2	3	4	5	6	7
1	40 8	30 2	50 M	120 6	180 M	170 M	150 7
2	70 6	80 6	130 M	130 7	170 8	140 M	20 M
3	80 8	20 M	200 5	70 8	140 M	130 M	60 M
4	60 M	10 6	70 M	80 M	30 M	160 8	210 6
5	90 M	100 M	20 8	60 M	40 6	50 6	220 M
Set up cost(C_j)	100	300	700	800	400	200	500
Capacity(a_j)	2	2	3	3	2	1	3

4^{th} non-dominated solution

Step 1. Add the entire costs *column wise* and the least of the total costs thus selected.

	Warehouse potential sites						
	1	2	3	4	5	6	7
Sum of costs in single column	110	40	270	320	210	50	380
Set up cost(C_j)	100	300	700	800	400	200	500
Total cost (C)	210	340	970	1120	610	250	880
Corresp.Time	8	6	M	8	8	6	M

The solution becomes $y_1 = 1, x_{11} = x_{21} = 1$ and $C = 210$ units and $T=8$ units.

Step 2. Selection of two warehouse potential sites at a time.

	Warehouse potential sites		
	(1,3)	(1,4)	(1,7)
Sum of min. costs in two columns	400	320	600
Set up cost(C_j)	800	900	600
Total cost(C)	1200	1220	1200
Corresp.Time	M	M	M

The combinations of locations (1, 2), (1, 5) and (1, 6) do not satisfy the constraint (3.9). Since combinations of locations (1, 3) and (1, 7) give the same cost *C* and *T*, arbitrarily select one of these two pairs of locations. So, select the pair (1, 3).

Step 3. Selection of three warehouse potential sites at a time.

	Warehouse potential sites			
	(1,3,2)	(1,3,5)	(1,3,6)	(1,3,7)
Sum of min. costs in three columns	210	360	490	540
Set up cost(C_j)	1100	1200	1000	1300
Total cost(C)	1310	1560	1490	1840
Corresp.Time	8	M	8	8

The solution obtained is $y_1 = y_3 = y_2 = 1, x_{12} = x_{21} = x_{31} = x_{42} = x_{53} = 1$ and $C=1310$ units and $T=8$ units. Thus, if three warehouses are to be set up then they should be at sites 1, 3 and 2. Hence, the solution is

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_3, y_2 x_{12}, x_{21}, x_{31} x_{42}, x_{53}	1310	8	y_1, y_3, y_2 x_{12}, x_{21}, x_{31} x_{42}, x_{53}	1310	8

Step 4.

Iteration No.1 The Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_3, y_2\}$	$\{y_1\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites	
	(3,2,5)	(3,2,6)
Sum of min. costs in three columns	360	490
Set up cost(C_j)	1400	1200
Total cost(C)	1760	1690
Corresp.Time	M	8

The combinations of locations (3, 2, 4), (3, 2, 7) do not satisfy the budgetary constraint and the combination 3, 2 and 6 gives the least cost thus treated as current solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_3, y_2, y_6 x_{12}, x_{22}, x_{33} x_{46}, x_{53}	1690	8	y_1, y_3, y_2 x_{12}, x_{21}, x_{31} x_{42}, x_{53}	1310	8

Iteration No.2 The Tabu lists are given by

Tabu list for drop	Tabu list for add
$\{y_2, y_6\}$	$\{y_3\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(2,6,1)	(2,6,4)	(2,6,5)	(2,6,7)
Sum of min. costs in three columns	240	400	310	550
Set up cost(C_j)	600	1300	900	1000
Total cost(C)	840	1700	1210	1550
Corresp.Time	8	M	M	M

The combination of locations 2, 6 and 1 gives the least cost thus treated as current solution and also as incumbent solution.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_2, y_6, y_1 x_{12}, x_{21}, x_{31} x_{42}, x_{56}	840	8	y_2, y_6, y_1 x_{12}, x_{21}, x_{31} x_{42}, x_{56}	840	8

Iteration No.3 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_6, y_1\}$	$\{y_2\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites			
	(6,1,3)	(6,1,4)	(6,1,5)	(6,1,7)
Sum of min. costs in three columns	490	400	310	550
Set up cost(C_j)	1000	1100	700	800
Total cost(C)	1490	1500	1010	1350
Corresp.Time	8	M	M	M

Out of these, combination of locations 6, 1 and 3 is selected for setting up of warehouses and thus treated as the current solution but the incumbent solution remains same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_6, y_1, y_3 x_{11}, x_{21}, x_{33} x_{46}, x_{53}	1490	8	y_2, y_6, y_1 x_{12}, x_{21}, x_{31} x_{42}, x_{56}	840	8

Iteration No.4 The Tabu lists at this stage are

Tabu list for drop	Tabu list for add
$\{y_1, y_3\}$	$\{y_6\}$

Hence, the possible solutions at this stage are

	Warehouse potential sites		
	(1,3,2)	(1,3,5)	(1,3,7)
Sum of min costs in three columns	210	360	540
Set up cost(C_j)	1100	1200	1300
Total cost(C)	1310	1560	1840
Corresp.Time	8	8	8

The combination of locations 1, 3 and 2 gives the least cost and treated as the current solution but the incumbent solution is still same.

Current solution			Incumbent solution		
Variables at level	C	T	Variables at level	C	T
y_1, y_3, y_2 x_{12}, x_{21}, x_{31} x_{42}, x_{53}	1310	8	y_2, y_6, y_1 x_{12}, x_{21}, x_{31} x_{42}, x_{56}	840	8

Since *C=1310* units, *T=8* units is the solution which has already occurred earlier, so in order to stop the recycling, stop the iteration process. Thus, fourth non-dominated solution is at locations 2, 6 and 1 with *C=840* units and *T=8* units, but 4th solution dominates 2nd and 3rd solution as its cost and time is less than the cost and time of 2nd and 3rd solution respectively. So, 2nd and 3rd solution is no longer non-dominated.

Hence, two non-dominated solutions are as shown below.

Efficient sol.	Variables x_{ij}'s and y_j's at level	Total cost of Meeting req.	Duration of Meeting req.
(X^1,Y^1)	$x_{12}, x_{21}, x_{32}, x_{41}, x_{56}, y_1, y_2, y_6$	$C(X^1)$ =30+70+20+ 60+50+600=830	$T(X^1)$=max {2,6,9,11,6}=11
(X^2,Y^2)	$x_{12}, x_{21}, x_{31}, x_{42}, x_{56}, y_2, y_6, y_1$	$C(X^2)$ =30+70+80 +10+50+600=840	$T(X^2)$=max {2,6,8,6,6}=8

If the pair of locations (1, 7) is selected in step 2 for *4th non-dominated* solution then the following set of non-dominated solutions is obtained.

Efficient sol.	Variables x_{ij}'s and y_j's at level	Total cost of meeting req.	Duration of meeting req.
(X^1,Y^1)	$x_{12}, x_{21}, x_{32}, x_{41}, x_{56}, y_1, y_2, y_6$	$C(X^1)$ =30+70+20+ 60+50+600=830	$T(X^1)$=max {2,6,9,11,6}=11
(X^2,Y^2)	$x_{12}, x_{21}, x_{31}, x_{42}, x_{56}, y_6, y_1, y_2$	$C(X^2)$ =30+70+80 +10+50+600=840	$T(X^2)$=max {2,6,8,6,6}=8

If the pair of locations (1, 7) is selected in step 2 for *3rd non-dominated* solution then the following set of non-dominated solutions is obtained.

Efficient sol.	Variables x_{ij}'s and y_j's at level	Total cost of meeting req.	Duration of meeting req.
(X^1,Y^1)	$x_{12}, x_{21}, x_{32}, x_{41}, x_{56}, y_1, y_2, y_6$	$C(X^1)$=30+70+20+ 60+50+600=830	$T(X^1)$=max {2,6,9,11,6}=11
(X^2,Y^2)	$x_{12}, x_{21}, x_{31}, x_{42}, x_{56}, y_6, y_1, y_2$	$C(X^2)$ =30+70+80 +10+50+600=840	$T(X^2)$=max {2,6,8,6,6}=8

If the combination of locations (1, 7) is selected in the step 2 for *2^nd^ non-dominated* solution then the following set of non-dominated solutions is obtained.

Efficient sol.	Variables x_{ij}'s and y_j's at level	Total cost of meeting req.	Duration of meeting req.
(X^1,Y^1)	$x_{12}, x_{21}, x_{32}, x_{41}, x_{56}, y_1, y_2, y_6$	$C(X^1)$=30+70+20+ 60+50+600=830	$T(X^1)$=max {2,6,9,11,6}=11
(X^2,Y^2)	$x_{12}, x_{21}, x_{31}, x_{42}, x_{56}, y_6, y_1, y_2$	$C(X^2)$ =30+70+80 +10+50+600=840	$T(X^2)$=max {2,6,8,6,6}=8

3.5. CONCLUSION

This work is an extension of the problem considered in the chapter two, which represents the capacitated warehouse location problem as there is restriction on number of ration shops to be clustered to a selected site. In this problem setup cost of warehouse has been introduced in the objective function with three additional constraints which ensure that each site selected for locating a warehouse must have at least one ration shop assigned to it, maximum upto a fixed number of ration shops can be assigned to each site selected to locate a warehouse at it and the total number of ration shops that can be given to the selected number of sites should not be less than total number of ration shops to be clustered**.** The problem has been solved by solving a sequence of prioritized bi-criterion problems incorporating Tabu search in the heuristic algorithm used for solving the prioritized bi-criterion problems. The incorporation of Tabu search in a heuristic algorithm allows search for a global solution in a wider region compared to the case without incorporating Tabu search, there by increasing the possibility of arriving at the global solution or close to it. Two non-dominated solutions have been obtained for capacitated warehouse location problem and the decision maker can select the solution according to his/her priorities.

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