ECONOMIC ORDERING POLICY OF DETERIORATED ITEM FOR VENDOR AND BUYER WITH VARIABLE DEMAND

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Submitted by

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Certificate

This is to certify that the thesis entitled **"Economic Ordering Policy Of Deteriorated Item For Vendor And Buyer With Variable Demand"**, being presented in partial fulfillment of the requirements for the award of the degree of Master of Science in the School of Mathematics, Thapar University, Patiala, is a bonafide work carried out under the supervision of **Mahesh Kumar sharma**.

That no part of this thesis has been submitted for the award of any other degree.

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Abstract

In a production-inventory system, the manufacturer produces the items at a rate, dispatches the order quantities to the customers in specific intervals and stores the excess inventory for subsequent deliveries. In real situations often inventory manager have to hold thousand of items in an inventory. But these single item inventory model are developed for stationary and non stationary demand. As in a competitive market, the aim of the vendor and the buyer is to maximize their profit and in general an integrated policy is required by the vendor. In real life, decay and deterioration occur in some of the products, such as fruits, milk products, medicine and vegetables. Various models have been developed in literature for deteriorating item with constant demand. Later on it has been observed that in real life demand of the item depends on either selling price or time. The present work divided into 3 Chapters.

Chapter 1, is introductory in nature in which some inventory models have been discussed and literature related to the topic has been also discussed. In Chapter 2, the "Economic ordering policy of deteriorated items for vendor and buyer: an integrated approach" given by Yang and Wee(2000) has been reviewed in details. In Chapter 3, the inventory model and solution methodology given by Yang and Wee(2000) with variable demand rate is considered and numerical results are obtained.

Towards the end, references of various publications cited in the present dissertation have been reported.

Contents

Chapter 1

Introduction

Inventory is the stock of materials and finished goods that a manufacturer or seller keeps to cater to fluctuations in unanticipated demand from the consumer end. For example wholesalers and retailers need to maintain inventories of goods to be available for customers to purchase.

Inventory model helps an organization in determining the economic order quantity, the frequency of ordering and to keep goods or services flowing to the customer without delay. Inventory modeling deals with determining the level of commodity that business or organization must maintain to ensure smooth operation. The basis for the decision is a model that balances the cost of capital resulting from holding too much inventory against the penalty cost resulting from inventory shortage. The principal factor affecting the solution is the nature of the demand which can be deterministic or probabilistic however in real life situation demand is probabilistic. We have different methods for different models but regardless of methodology an inventory model always seeks two basic results that how much and when to order. In this chapter some basic inventory models have been discussed.

1.1 General Inventory Models

Inventory problems consists of placing and receiving orders of given sizes at some intervals. From this point, an inventory policy always answers two questions:

- How much to order?
- When to order?

To answer these questions the total inventory cost which includes purchasing cost, setup cost, holding cost and shortage cost is to be minimized.

1.1.1 Classic Economic Order Quantity(EOQ) Model

Classical inventory model concern with single-item model. The objective is to determine economic order quantity, y which minimizes the total cost of an inventory system when demand is constant with instantaneous order replenishment and no shortage is allowed. The model developed under following assumptions:

1. This model deals with single item.
2. The demand rate is known and constant.
3. Quantity discounts are not available.
4. The ordering cost is constant.
5. Shortages are not allowed and lead time is known and is constant.
6. The inventory holding cost per inventory unit per time is known and constant.

$$y = \text{Order quantity(number of units)}$$

$$D = \text{Demand rate(units per unit time)}$$

$$t_0 = \text{Ordering cycle length(time units)}$$

The inventory levels follows the pattern as shown in Fig. 1.1

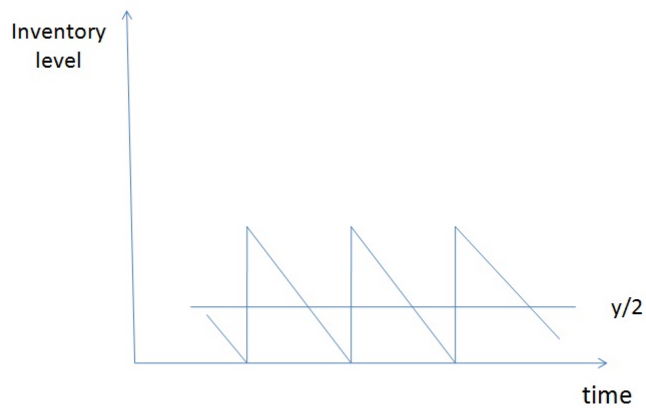

Figure 1.1: Inventory pattern in the classic EOQ model

In above Fig. 1.1 order of size y units is placed and received instantaneously when the inventory reaches zero. The ordering cycle for this pattern is

$$t_0 = \frac{y}{D} \text{ time units}$$

The resulting average inventory level is given as $\frac{y}{2}$ units. The model requires two cost parameters with K as a setup cost of an item(dollar per order) and h is the holding cost(dollars per inventory unit per unit time).

The total cost per unit time(TCU) is

$$\begin{aligned} \text{TCU}(y) &= \text{setup cost per unit time + Holding cost per unit time} \\ &= \frac{K}{\left(\frac{y}{D}\right)} + h\frac{y}{2} \end{aligned}$$

The optimum solution is determined by minimizing TCU(y) with respect to y. Assuming y is continuous, a necessary condition for finding the optimal value of y is

$$\frac{dTCU(y)}{dy} = -\frac{KD}{y^2} + \frac{h}{2} = 0$$

The condition is also sufficient because TCU(y) is convex.

The solution of the equation yields the EOQ, y^* as

$$y^* = \sqrt{\frac{2KD}{h}}$$

Thus, the optimum inventory policy for the proposed model is summarized as order $y^* = \sqrt{\frac{2KD}{h}}$ units every $t_0^* = \frac{y^*}{D}$ time units.

In real situation it is not possible that new orders received instantly. But, a positive lead time L, may occur between the placement and the receipt of an order. In case of lead time, reorder point occurs when inventory level drops to LD units.

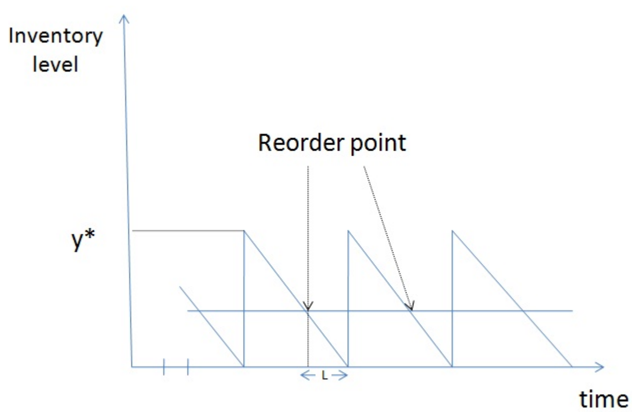

Figure 1.2: Reorder point in the classic EOQ model

In above Fig.1.2 L is the lead time. It is assumed that the lead time L is less than the cycle length t_0^*, which is not in general. To account for other situations, the effective lead time is defined as $L_e = L - nt_0^*$, where n is the largest integer not exceeding $\frac{L}{t_0^*}$.

Inventory policy says that order the quantity y^* whenever the inventory level drops to $L_e D$ units.

1.1.2 EOQ With Price Breaks

In this model items may be purchased at a discount if the size of the order y exceeds a given limit q the unit purchasing price c is given as

$$c = \begin{cases} c_1, & y \leq q \\ c_2, & y > q, \end{cases}$$

where, $c_1 > c_2$

Hence,

$$Purchasing \;\; cost \;\; per \;\; unit \;\; time = \begin{cases} \frac{c_1 y}{t_0} = Dc_1, & y \leq q \\ \frac{c_2 y}{t_0} = Dc_2, & y > q \end{cases} \tag{1.1}$$

Total cost per unit time is

$$TCU(y) = \begin{cases} TCU_1(y) = Dc_1 + \frac{KD}{y} + \frac{hy}{2}, & y \leq q \\ TCU_2(y) = Dc_2 + \frac{KD}{y} + \frac{hy}{2}, & y > q, \end{cases} \tag{1.2}$$

The functions TCU_1 and TCU_2 are represented below. Minima occurs at

$$y_m = \sqrt{\frac{2KD}{h}}$$

The cost function TCU(y) starts on the left with TCU_1 and drops to TCU_2

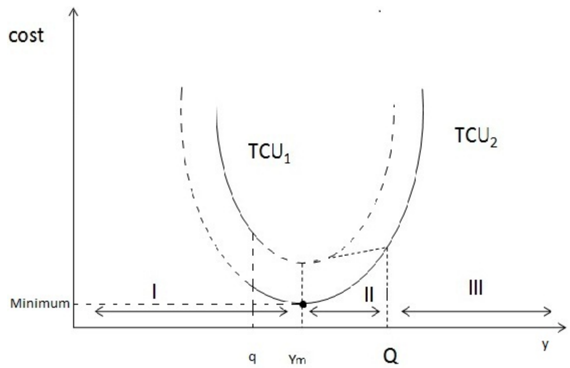

Figure 1.3: Inventory cost function with price breaks

at the price break point q which lies in zone I. Optimum value y^* depends where q, lies with zones I, II and III given in Fig. 1.3 as given below by (0,y_m), (y_m,Q) and (Q,∞), respectively. The value of Q($> y_m$) is determined from

$$TCU_2(Q) = TCU_1(Y_m)$$

$$c_2D + \frac{KD}{Q} + \frac{hQ}{2} = TCU_1(y_m)$$

which simplifies to

$$Q^2 + \left(\frac{2(c_2D - TCU_1(y_m))}{h}\right)Q + \frac{2KD}{h} = 0 \tag{1.3}$$

Fig 1.3 shows the optimum quantity y^* is

$$y^* = \begin{cases} y_m, & \text{if } q \text{ is in zones I or III} \\ q, & \text{if } q \text{ is in zone II,} \end{cases}$$

Steps for determining y^* are

Step1. $y^* = y_m$, if q is in zone I or zone III. Otherwise go to step2.

Step2. Determine $Q(> y_m)$ from equation (1.3). If q is in zone II, $y^*=q$.

1.2 A Single-Vendor And Multiple-Buyers Production–Inventory Policy For A Deteriorating Item(Yang And Wee(2002))

Yang and Wee(2002) considered a single-vendor, multi-buyers production–inventory policy for a deteriorating item with a constant production rate and demand rate. A mathematical model incorporating the costs of both the vendor and the buyers is developed and results shows that the integrated policy results in an impressive cost reduction when it is compared with the independent decisions made by the vendor and the buyers. It has been shown that the heuristic approach is better for multiple retailers and results in low error.

1.2.1 Fundamental Assumptions and Notations

Yang and Wee(2002) considered a model on the basis of following assumptions and notations:

1.2.2 Assumptions

1. A single item with constant deteriorating rate of the on-hand inventory is considered.

2. Single-vendor, multi-buyers with one item is assumed.

3. Shortage is not allowed.

4. A stationary policy where the buyers order the same lot size is assumed.

5. There is no replacement or repair of deteriorated units.

6. Holding cost applies to good units only.

7. The production rate is finite and is greater than the sum of all the buyer's demand.

1.2.3 Notations

p = Production rate.

d_i = Demand rate per year for buyer i, $i = 1, 2.....N$.

$I_{v1}(t_1)$ = Inventory level for vendor when t_1 is between 0 and T_1.

$I_{v2}(t_2)$ = Inventory level for vendor when t_2 is between 0 and T_2.

T = Time length of each cycle, where T=$T_1 + T_2$.

T_1 = The length of production time in each production cycle T

T_2 = The length of non-production time in each production cycle T.

θ = The deterioration rate.

n_i = Number of deliveries per order.

$I_{mv}(t)$ = Maximum inventory level of the vendor.

$I_{mi}(t)$ = Maximum inventory level of the buyer i.

$I_{bi}(t)$ = Inventory level for buyer i when t is between 0 and $\frac{T}{ni}$

P_v = Unit production cost for vendor.

P_b = Unit price for the buyer.

C_{sv} = Setup cost for the vendor per production cycle.

C_{sb} = The setup or ordering cost per order for buyer.

F_v = The holding cost per dollar per year for vendor.

F_b = The holding cost per dollar per year for buyer.

K_{0b} = The incoming control cost per delivery for the buyer.

K_{0v} = The transportation charge per delivery for the vendor.

BC = The total cost function for the buyer who buys goods from the vendor.

VC = The total cost function for the vendor.

TC = The integrated total cost function including BC and VC.

Inventory level depicted in Fig 1.4:

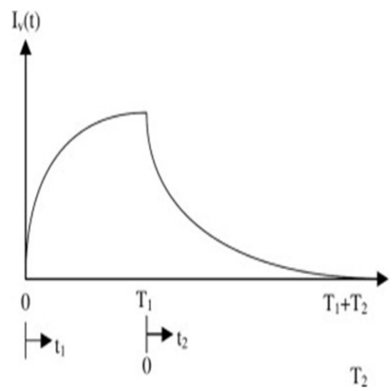

Figure 1.4: Production–inventory system for the vendor.

$$\frac{dI_{v1}(t_1)}{dt_1} + \theta I_{v1}(t_1) = p - \sum_{i=1}^{N} d_i \quad ; \qquad 0 \le t_1 \le T_1, \tag{1.4}$$

$$\frac{dI_{v2}(t_2)}{dt_2} + \theta I_{v2}(t_2) = - \sum_{i=1}^{N} d_i \quad ; \qquad 0 \le t_2 \le T_2, \tag{1.5}$$

$$\frac{dI_{bi}(t)}{dt} + \theta I_{bi}(t) = -d_i \quad ; \qquad 0 \le t \le \frac{T}{n_i} \tag{1.6}$$

$$i = 1, 2, ...N$$

We consider simple case means single buyer, N=1. From Spiegel(1960) and by using various boundary conditions $I_{v1}(0)$=0, $I_{v2}(T_2)$=0 and $I_{bi}(\frac{T}{n_i})$=0, the solution of the above differential equation is,

$$I_{v1}(t_1) = \frac{p - \sum_{i=1}^{N} d_i}{\theta}\big(1 - \exp(-\theta t_1)\big) \quad ; \qquad 0 \le t_1 \le T_1 \tag{1.7}$$

$$I_{v2}(t_2) = \frac{\sum_{i=1}^{N} d_i}{\theta}\left[\frac{\exp(\theta T_2) - \exp(\theta t_2)}{\exp(\theta t_2)}\right] \quad ; \quad 0 \leq t_2 \leq T_2 \tag{1.8}$$

and the value of the buyer is,

$$I_{bi}(t) = \frac{d_i}{\theta}\left[\frac{\exp(\frac{\theta T}{n_i}) - \exp(\theta t)}{\exp(\theta t)}\right] \quad ; \quad 0 \leq t \leq \frac{T}{n_i} \tag{1.9}$$
$$i = 1, 2...., N,$$

As θ cannot be zero so it is to be assume as a very small value θ=0.001 representing a negligible deterioration rate. From equations (1.7) and (1.8) and Fig. 1.4, the maximum inventory level of the vendor when $I_{mv} = I_{v2}(0)$,

$$I_{mv} = \frac{\sum_{i=1}^{N} d_i}{\theta}\Big[\exp\big(\theta T_2\big) - 1\Big] \tag{1.10}$$

Similarly from equation (1.9) and Fig. 1.5, the maximum inventory level

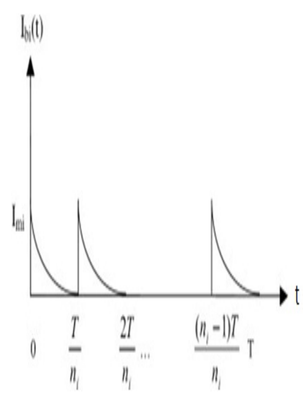

Figure 1.5: Inventory system for buyer i.

of the buyer when $I_{mi} = I_{bi}(0)$ is,

$$I_{mi} = \frac{d_i}{\theta}\left[\exp\left(\frac{\theta T}{n_i}\right) - 1\right] \quad ; \qquad i = 1, 2, ...N \tag{1.11}$$

with the boundary condition, $I_{v1}(T_1) = I_{v2}(0)$ following expression is obtained,

$$\begin{aligned}&\left(p - \sum_{i=1}^{N} d_i\right)\left[1 - \exp(-\theta T_1)\right] \\ &= \left(\sum_{i=1}^{N} d_i\right)\left[\exp(\theta T_2) - 1\right]\end{aligned} \tag{1.12}$$

By using taylor's expansion and assumption on $\theta \ll 1$, and from Misra(1975) length of production time is given by,

$$T_1 \approx \frac{\sum_{i=1}^{N} d_i}{p - \sum_{i=1}^{N} d_i} T_2\left(1 + \frac{1}{2}\theta T_2\right) \tag{1.13}$$

Relation between production and non production cycle is,

$$T = T_1 + T_2 \tag{1.14}$$

$$T = \frac{\sum_{i=1}^{N} d_i}{p - \sum_{i=1}^{N} d_i} T_2\left(1 + \frac{1}{2}\theta T_2\right) + T_2 \tag{1.15}$$

From equations (1.14) and (1.15) time length of each cycle is obtained,

$$T \approx \frac{T_2}{p - \sum_{i=1}^{N} d_i}\left(p + \frac{1}{2}\sum_{i=1}^{N} d_i \theta T_2\right) \tag{1.16}$$

Holding cost for the buyers and vendor are as follows:

$$\begin{aligned}
HC_b &= \frac{p_b F_b}{T}\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} I_{bi}(t)dt \\
HC_b &= \frac{p_b F_b}{T}\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} \left(\frac{d_i}{\theta}\left[\frac{\exp(\frac{\theta T}{n_i})-\exp(\theta t)}{\exp(\theta t)}\right]\right)dt \\
HC_b &= \frac{p_b F_b}{2n_i^2}\sum_{i=1}^{N} n_i \left[1+\frac{\theta T^2}{3n_i}\right] \qquad (1.17)
\end{aligned}$$

and

$$\begin{aligned}
HC_v &= \frac{p_v F_v}{T}\left[\int_0^{T_1} I_{v1}(t_1)dt_1 + \int_0^{T_2} I_{v2}(t_2)dt_2 - \sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}} I_{bi}(t)dt\right] \\
HC_v &= \frac{p_v F_v}{T}\left[\int_0^{T_1}\left(\frac{p-\sum_{i=1}^{N} d_i}{\theta}\left(1-\exp(-\theta t_1)\right)\right)dt_1\right. \\
&+ \int_0^{T_2}\left(\frac{\sum_{i=1}^{N} d_i}{\theta}\left[\frac{\exp(\theta T_2)-\exp(\theta t_2)}{\exp(\theta t_2)}\right]\right)dt_2 \\
&- \left.\sum_{i=1}^{N} n_i \int_0^{\frac{T}{n_i}}\left(\frac{d_i}{\theta}\left[\frac{\exp(\frac{\theta T}{n_i})-\exp(\theta t)}{\exp(\theta t)}\right]\right)dt\right] \\
HC_v &= \frac{p_v F_v}{T}\left[\frac{p-\sum_{i=1}^{N} d_i T_1^2}{2}\left(1-\frac{\theta T_1}{3}\right)+\frac{\sum_{i=1}^{N} d_i T_2^2}{2}\right. \\
&\left.\left(1+\frac{\theta T_2}{3}\right)-\frac{T^2\sum_{i=1}^{N} n_i d_i}{2\sum_{i=1}^{N} n_i^2}\left(1+\frac{\theta T^3}{6\sum_{i=1}^{N} n_i^3}\right)\right] \qquad (1.18)
\end{aligned}$$

Annual deteriorated costs for all the buyers and vendor are,

$$\begin{aligned} DC_b &= \sum_{i=1}^{N}\left(I_{mi} - \frac{Td_i}{n_i}\right)\frac{n_i p_b}{T} \\ DC_b &= \sum_{i=1}^{N}\left(\frac{d_i}{\theta}\left[\exp\left(\frac{\theta T}{n_i}\right) - 1\right] - \frac{Td_i}{n_i}\right)\frac{n_i p_b}{T} \qquad (1.19) \\ DC_v &= \frac{p_v}{T}\left(pT_1 - \sum_{i=1}^{N} n_i I_{mi}\right) \\ DC_v &= \frac{p_v}{T}\left(pT_1 - \sum_{i=1}^{N} n_i\left(\frac{d_i}{\theta}\left[\exp\left(\frac{\theta T}{n_i}\right) - 1\right]\right)\right) \qquad (1.20) \end{aligned}$$

Setup costs per year for both are,

$$SC_b = \sum_{i=1}^{N}\frac{n_i C_{sb}}{T} \qquad (1.21)$$

$$SC_v = \frac{C_{sv}}{T}, \qquad (1.22)$$

Total buyers and vendor cost is the sum of various costs, from equations (1.17), (1.19) and (1.21), total cost of the buyer is,

$$\begin{aligned} BC &= \frac{p_b F_b}{2n_i^2}\sum_{i=1}^{N} n_i\left[1 + \frac{\theta T^2}{3n_i}\right] + \sum_{i=1}^{N}\left(\frac{d_i}{\theta}\left[\exp\left(\frac{\theta T}{n_i}\right)\right.\right. \\ &- \left.\left. 1\right] - \frac{Td_i}{n_i}\right)\frac{n_i p_b}{T} + \sum_{i=1}^{N}\frac{n_i C_{sb}}{T} \qquad (1.23) \\ VC &= HC_v + DC_v + SC_v \\ VC &= \frac{p_v F_v}{T}\left[\frac{p - \sum_{i=1}^{N} d_i T_1^2}{2}\left(1 - \frac{\theta T_1}{3}\right) + \frac{\sum_{i=1}^{N} d_i T_2^2}{2}\right. \qquad (1.24) \end{aligned}$$

$$\left(1+\frac{\theta T_2}{3}\right)-\frac{T^2\sum\limits_{i=1}^{N}n_id_i}{2\sum\limits_{i=1}^{N}n_i^2}\left(1+\frac{\theta T^3}{6\sum\limits_{i=1}^{N}n_i^3}\right)\Bigg]$$

$$+\quad\frac{p_v}{T}\left(pT_1-\sum_{i=1}^{N}n_i\left(\frac{d_i}{\theta}\left[\exp\left(\frac{\theta T}{n_i}\right)-1\right]\right)\right)+\frac{C_{sv}}{T}$$

And finally, the integrated cost of the buyers and the vendor is sum of total costs of both, for a fixed value n_i, where $i=1,2....,N$, from equations (1.23) and (1.24) total cost is,

$$\begin{aligned}
TC \quad = \quad & \frac{p_bF_b}{T}\sum_{i=1}^{N}n_i\int\limits_{0}^{\frac{T}{n_i}}\left(\frac{d_i}{\theta}\left[\frac{\exp(\frac{\theta T}{n_i})-\exp(\theta t)}{\exp(\theta t)}\right]\right)dt \\
+ \quad & \sum_{i=1}^{N}\left(\frac{d_i}{\theta}\left[\exp\left(\frac{\theta T}{n_i}\right)-1\right]-\frac{Td_i}{n_i}\right)\frac{n_ip_b}{T}+\sum_{i=1}^{N}\frac{n_iC_{sb}}{T} \\
+ \quad & \frac{p_vF_v}{T}\Bigg[\int\limits_{0}^{T_1}\left(\frac{p-\sum\limits_{i=1}^{N}d_i}{\theta}\Big(1-\exp(-\theta t_1)\Big)\right)dt_1 \\
+ \quad & \int\limits_{0}^{T_2}\left(\frac{\sum\limits_{i=1}^{N}d_i}{\theta}\left[\frac{\exp(\theta T_2)-\exp(\theta t_2)}{\exp(\theta t_2)}\right]\right)dt_2 \\
- \quad & \sum_{i=1}^{N}n_i\int\limits_{0}^{\frac{T}{n_i}}\left(\frac{d_i}{\theta}\left[\frac{\exp(\frac{\theta T}{n_i})-\exp(\theta t)}{\exp(\theta t)}\right]\right)dt\Bigg] \\
+ \quad & \frac{p_v}{T}\left(pT_1-\sum_{i=1}^{N}n_i\left(\frac{d_i}{\theta}\left[\exp\left(\frac{\theta T}{n_i}\right)-1\right]\right)\right) \\
+ \quad & \frac{C_{sv}}{T}
\end{aligned} \tag{1.25}$$

1.3 Literature Review

The term inventory refers to the goods or materials used by a firm for the purpose of production and sale. Maintaining optimum investment in inventory and applying effective control system so as to minimize the total inventory cost is the basic concerned of inventory theory. Inventory management is important as it enables to address two important issues first is the firm has to maintain adequate inventory for smooth production. Second is to minimize the investment in inventory to enhance profit. Many models of single item or multi item had been developed based on different assumptions like partial backlogging, constant lead time, constant deterioration rate and static or variable demand. An implicit assumption in most inventory control models is that when there is a shortage, either complete back-ordering or complete lost sale is assumed. Wee(1993) formulated a model in which an economic production policy for deteriorating items with partial back-ordering. They developed an economic production plan for deteriorating items with partial back-ordering of the items. Two numerical examples are used to illustrate the theory and it has been shown that the policy of this model leads to lower cost. Later on Wee(1995) used a modified model in which he considered a replenishment policy of deteriorating product where demand declines exponentially over a fixed time horizon. In this model deterioration is assumed to be a constant fraction of the total on-hand inventory and complete backordering of demand is assumed. Ghare and Schrader(1963) is the first who analysed the decaying inventory problems. He developed a simple economic order quantity model with a constant rate of decay. Later on Covert and Philip(1973) extended the model presented by Ghare and Schrader(1963) and obtained model for variable rate of deterioration by assuming two parameter weibull distribution. Weibull distribution is used to represent the distribution of the time to deterioration. In this model EOQ formula is derived for constant demand, instantaneous delivery and no shortages, a computer program is developed for the numerical solution. It has been verified that the solution gives minimum total cost per unit. The theoretical derivation was shown to reduce to the model presented by Ghare and Schrader(1963). Further philip(2007) use three parameter weibull distribution to represent the time to deterioration. The solutions to two numerical examples are compared to Covert and Philip(1973) solution. Misra(1975) later on considered both constant and variable demand results in approximate expression for production lot size model with no backlogging. In this model a numerical method has been suggested for the varying rate and for the constant rate of deterioration an approximate expression has been derived for the production lot size. To show the impact of deterioration a numerical example is solved. Furthermore, many models has been developed by considering different situations like in Misra(1979) finds optimal inventory management under inflation. The terms "multi-echelon" or "multi-level" refers supply with such supply chains, when an item moves through more than one step before reaching the final customer. Inventories exist throughout the supply chain in various forms for various reasons. Keeping this in mind, many researchers have studied all aspects of multi echelon inventory since the use of multiple suppliers, in a majority of cases, reduces the overall inventory and distribution system costs. Ganeshan(1999) developed a model for single product with constant unit price, N_r identical retailers and n suppliers with one distribution center. This gives an exact representation of the retailer and warehouse inventory level and an approximation to the demand process at the warehouse. Graves(1985) presented a multi echelon inventory in which problem is to obtain the inventory level for a repairable items with replenishment allowed. Law and Wee(2006) consider a single-buyer, single-vendor supply chain which produces and delivers a product of which the raw material is livestock. The manufacturer buys young livestock and grows them, the mature livestock is then used to make food. This food is delivered to the buyer in batches. In their research, Law and Wee(2006) consider the time value of money by discounting the cost with a specific rate, ultimately minimizing the total cost of the system. Taking a discounted cash flow approach, Lo et al.(2007) model a production-inventory system which consists of one buyer and one manufacturer. Similar to the previous research works, Lo et al.(2007) aim to minimize the total cost of the system. Lee(1987) developed a continuous-review multi-echelon model for repairable items when emergency lateral trans shipments between identical bases are allowed. In this lateral trans-shipments between bases is commonly practiced to provide improved service support for products. Approximation are derived and tested for the expected level of backorders and the quantity of emergency lateral tran shipments in cases where fairly high service levels are required. These approximations are used to determine the optimal stocking levels in such a system and a procedure is developed to find the these stocking levels. Ray et al.(1998) developed an order level inventory model by assuming that the demand rate is stock-dependent and two separate warehouses are used. In this when stock level exceeds the capacity of the own warehouse(OW), a rented warehouse(RW) is used. The stock is transferred periodically in bulk from RW to OW and there is an associated transportation cost. An algorithm is developed to obtain the optimal solution of the model. A numerical example is given to show the effects of changes in the system parameters on the decision variables. Janseen(1999) developed a model in which he considered two suppliers with deterministic lead time. Supply agreements are made with the main supplier to deliver a fixed quantity Q, every review period. In this model, Inventory level evaluated at each review period and the replenishment orders from the second supplier will arrive after given lead times. This is a combination of push and pull system if we are using more than one source one must know how to divide the purchase quantity. An algorithm is defined to determine the value of S, Q and R where R is the length of review period for which the total relevant costs are minimized. Further Yang and Wee(2002) developed a model for single vendor and multiple buyers production inventory policy for deteriorating items by taking constant demand rate and production rate. They also considered a different method which is used when number of buyers are less than two but when it is more than two then they use heuristic approach as considered by Yang and Wee(2000) and then sensitivity analysis is conducted to show the effect of parameters on total cost. It is also based on the view that integrated approach results in joint total cost reduction for both the buyer and the vendor. Later on Ghiami and williams(2015) developed a two echelon production inventory model for deteriorating items with constant demand and production rate. In this model, they considered physical inventory and echelon stock of the vendor during production and non production time. The same approach heuristic approach has been used to solve the model as considered by Yang and Wee(2002). Results of Ghiami and williams(2015) were compared with the Yang and Wee(2002) and It has been observed that in Yang and Wee(2002) the production rate must be higher than the demand rate and they dropped a part of production time due to huge surplus. Model given by Ghiami and williams(2015) results in better optimal solution than yang and Wee(2002) as there is relaxation of huge surplus. Rau et al.(2003) develop a single-supplier, single-manufacturer, single buyer model. Similar to Yang and Wee(2002), Rau et al.(2003) implicitly assume that the production rate is significantly larger than the demand,therefore they drop part of the manufacturer's production period. In most of cases demand of the item is considered as a constant but in realistic situation this will change according to time. So to show the effect of time on demand a model is developed by Ouyang et al.(2005) for deteriorating with demand declining exponentially and shortages are allowed and partially backlogged. They considered the variable rate for backlogging which is inversely proportional to waiting time for the next replenishment with constant rate of deterioration to find the optimal solution of the problem and also verified that the total cost function is convex. Then sensitivity analysis is performed to show how total cost depends on different parameter. In a realistic inventory system, some inventory parameters are breakable, some probabilistic, and others imprecise. Vats(2014) considered a model in which the demand is dependent as a quadratic function $a + bt + ct^2$ with backlogging depends on length of replenishment and shortages are not allowed. Here 'a' is fix fraction of demand, 'b' and 'c' are that fraction of demand which is vary with time. Analytical solution has been obtained for minimizing the cost. This model is useful for those industries where demand rate is depending upon the time and holding cost is constant. Further work has been done where demand is considered as a exponential function or linearly depending on time with different assumptions.

Chapter 2

Economic Ordering Policy Of Deteriorated Item For Vendor And Buyer : An Integrated Approach(Yang and Wee(2000))

2.1 Introduction

A supply chain is a network that typically comprises suppliers, producers, distributors, and retailers. In a competitive market environment, a buyer has the privilege to decide on the number of deliveries when an order is made. The optimal number of deliveries favored by the buyers may not be the most economical for the vendor. If the number of deliveries is decided in cooperation with the vendor, the overall integrated cost can be minimized. A major issue in supply chain management research is supply chain coordination. To overcome this situation the integration approach has been researched for years.In this work they developed an economic ordering policy of deteriorating items in which production and demand rate are constant with some assumptions. They proved that the integrated approach is better than the independent decision approach through sensitivity analysis and results in an impressive reduction in cost of both. A heuristic approach is developed to find the optimal solution and numerical examples are presented to illustrate the results of the proposed models. In this chapter a research paper entitled "Economic ordering policy of deteriorated item for vendor and buyer : an integrated approach" by Yang and Wee(2000) has been reviewed in detail.

2.1.1 Fundamental Assumptions And Notations

Yang and Wee(2000), considered a model on the basis of following assumption:

2.1.2 Assumptions

1. The production rate is constant.

2. Demand rate is constant.

3. Shortage is not allowed.

4. A single item with a constant rate of deterioration is considered.

5. Deterioration of the units is considered only after they have been received into inventory.

6. There is no replacement or repair of deteriorated units.

7. Carrying cost applied to good units only.

8. Single producer and single distributor are considered.

9. There are multiple deliveries per order.

10. There is only one production cycle.

2.1.3 Notations

p = Production rate.

d = Consumer's demand rate.

$I_{(t_1)}$ = Inventory level that changes with time t_1 during production period.

$I_{(t_2)}$ = Inventory level that changes with time during non-production period.

T_1 = The production period in each cycle.

T_2 = The non-production period in each cycle.

T = Time length of cycle.

θ = The deterioration rate.

n = Number of deliveries per order.

$I_{pc}(t)$ = Inventory level of the vendor.

$I_b(t)$ = Inventory level of the buyer.

C_{ob} = The ordering cost of the buyer, per order.

C_{sv} = Setup cost for the vendor, per production cycle.

C_{cb} = Inventory carrying cost for the buyer, per time and per unit.

C_{cv} = Inventory carrying cost for the vendor, per time per unit.

C_b = Cost of deteriorated unit for the buyer.

C_v = Cost of deteriorated unit for the vendor.

K_{0b} = The incoming control cost per delivery for the buyer.

K_{0v} = The transportation charge per delivery for the vendor.

TC_b = The total cost function for the buyer who buys goods from the vendor.

TC_v = The total cost function for the vendor.

TC = The integrated total cost function including TC_b and TC_v.

2.2 Formulation Of The Model

The objective of the model is to determine the optimum profit for items with constant demand and constant rate of deterioration. The differential equations $I_{pc}(t)$ which describes the inventory level for a vendor with consumer demand are as follows:

$$\frac{dI_1(t_1)}{dt_1} = (p-d) - \theta I_1(t_1) \quad ; \qquad 0 \le t_1 \le T_1 \qquad (2.1)$$

$$\frac{dI_2(t_2)}{dt_2} = -d - \theta I_2(t_2) \quad ; \qquad 0 \le t_2 \le T_2 \qquad (2.2)$$

Using Spiegel (1960), the obtained boundary conditions are condition as $I_2(0) = I_2(T_2)$, the solutions of the above differential equations are:

$$I_1(t_1) \exp \theta(t_1) = (p-d) \int \exp(\theta t_1) dt_1 + c$$

on solving it and by using boundary condition we have,

$$I_1(t_1) = \frac{(p-d)}{\theta}\Big(1 - \exp\big(-\theta t_1\big)\Big) \quad ; \qquad 0 \le t_1 \le T_1 \qquad (2.3)$$

$$I_2(t_2) = \frac{-d}{\theta}\bigg(1 - \exp\Big(\theta\big(T_2 - t_2\big)\Big)\bigg) \quad ; \qquad 0 \le t_1 \le T_2 \qquad (2.4)$$

the inventory level depicted in Fig. 2.1, together with boundary condition

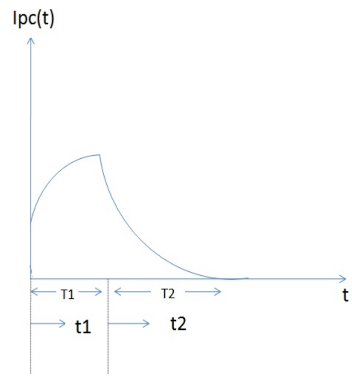

Figure 2.1: $I_{pc}(t)$ versus t.

$I_1(T_1) = I_2(0)$,

$$(p-d)\Big(1-\exp(-\theta T_1)\Big) = -d\big(1-\exp(\theta T_2)\big) \tag{2.5}$$

Value of θ assumed to be very small $\theta \ll 1$, as deterioration rate cannot be zero by using taylor's expansion following expression obtained,

$$\begin{aligned}(p-d)T_1\left(1-\left(1-\theta T_1+\frac{\theta {T_1}^2}{2}-\frac{\theta {T_1}^3}{6}\right)\right) &= -d\left(1-\left(1+\theta T_2\right.\right.\\ &+ \left.\left.\frac{\theta {T_2}^2}{2}+\frac{\theta {T_2}^3}{6}\right)\right)\end{aligned}$$

on solving this and neglecting higher terms,

$$(p-d)T_1\left(1-\frac{1}{2}\theta T_1\right) = dT_2\left(1+\frac{1}{2}\theta T_2\right) \tag{2.6}$$

From Misra (1975), and from equation (2.6) production period of each cycle is,

$$T_1 \approx \frac{d}{p-d}T_2\left(1+\frac{1}{2}\theta T_2\right) \tag{2.7}$$

Total length of time cycle obtained as,

$$T = T_1 + T_2 \tag{2.8}$$

$$T = \frac{d}{p-d} T_2 \left(1 + \frac{1}{2}\theta T_2\right) + T_2 \tag{2.9}$$

On solving equations (2.8) and (2.9), final expression for total length of the time cycle is,

$$T \approx \frac{T_2}{p-d}\left(p + \frac{1}{2} d\theta T_2\right) \tag{2.10}$$

Inventory function for a buyer for n deliveries per order is given by:

$$\frac{d}{dt} I_b(t) = -d - \theta I_b(t) \tag{2.11}$$

On solving above equation inventory level of the buyer is,

$$I_{b(t)} = \frac{d}{\theta}\left(\exp\left(\theta\left(\frac{T}{n} - t\right)\right) - 1\right) \quad ; \qquad 0 \leq t \leq \frac{T}{n} \tag{2.12}$$

Now the maximum inventory $I_b(t)$ of the buyer at $t = 0$ one get,

$$I_b(t = 0) = \frac{d}{\theta}\left(\exp\left(\frac{\theta T}{n}\right) - 1\right) \tag{2.13}$$

Fig 2.2 shows inventory level $I_b(t)$. By the assumption $\theta \ll 1$ and taylor's series expansion, the total cost function for the buyer TC_b is given as follows

$$TC_b(t) = \frac{1}{T}(C_{ob} + nK_{0b}) + C_{cb}\frac{n}{T}\int_0^{\frac{T}{n}} I_b(t)dt + C_b\frac{n}{T}\left(I_b(0) - \frac{T}{n}(d)\right)$$

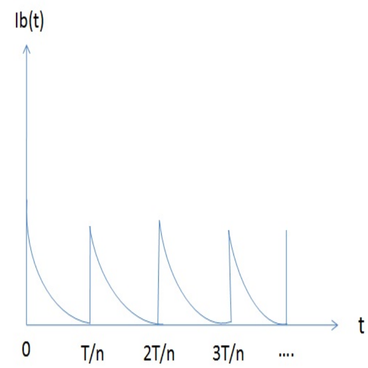

Figure 2.2: The buyer's inventory function $I_b(t)$ versus t.

From equations (2.12) and (2.13) total cost for the buyer is given as,

$$\begin{aligned}
TC_b(t) &= \frac{1}{T}\big(C_{ob}+nK_{0b}\big)+C_{cb}\frac{n}{T}\int_0^{\frac{T}{n}}\frac{d}{\theta}\Bigg(\exp\Bigg(\theta\Big(\frac{T}{n}-t\Big)\Bigg)-1\Bigg)dt \\
&+ C_b\frac{n}{T}\Bigg(\frac{d}{\theta}\Bigg(\exp\Big(\frac{\theta T}{n}\Big)-1\Bigg)-\frac{T}{n}d\Bigg) \\
&= \frac{1}{T}\big(C_{ob}+nK_{0b}\big)+C_{cb}\frac{n}{T}\frac{d}{\theta}\frac{1}{\theta}\Bigg(\Bigg(-1+\exp\frac{\theta T}{n}\Bigg) \\
&\quad -\frac{T}{n}\Bigg)+C_b\frac{n}{T}\Bigg(\frac{d}{\theta}\Bigg(\exp\Big(\frac{\theta T}{n}\Big)-1\Bigg)-\frac{T}{n}d\Bigg) \\
&= \frac{1}{T}\big(C_{ob}+nK_{0b}\big)+C_{cb}\frac{n}{T}\frac{d}{\theta}\Bigg(\frac{1}{\theta}\Big(-1+1+\frac{\theta T}{n}+\frac{\theta^2T^2}{2n^2}+\frac{\theta^3T^3}{6n}.........\Big)\Bigg) \\
&+ C_b\frac{n}{T}+C_b\frac{n}{T}\frac{d}{\theta}\Bigg(\Bigg(\Bigg(1+\frac{\theta T}{n}+\frac{\theta^2T^2}{2n^2}+\frac{\theta^3T^3}{6n}.........\Bigg)-1\Bigg)-\frac{Td}{n}\Bigg) \\
&= \frac{1}{T}\big(C_{ob}+nK_{0b}\big)+C_{cb}\frac{n}{T}\frac{dT^2}{2n^2}\Bigg(1+\frac{\theta T}{3n}+\frac{\theta^2T^2}{12n^2}+.........\Bigg) \\
&+ C_b\frac{n}{T}+C_b\frac{n}{T}\frac{d}{\theta}\Bigg(\frac{\theta^2T^2}{2n^2}+\frac{\theta^3T^3}{6n^3}+.....\Bigg)
\end{aligned}$$

$$TC_b(t) \approx \frac{1}{T}\big(C_{ob}+nK_{0b}\big)+\frac{C_{cb}Td}{2n}\Bigg(1+\frac{\theta T}{3n}\Bigg)+\frac{C_bTd\theta}{2n} \qquad (2.14)$$

now total cost function for the vendor is TC_v as follows,

$$TC_v = \frac{1}{T}(C_{sv}+nK_{0v})+C_{cv}\frac{1}{T}\Big(\int_0^{T_1} I_1(t_1)dt_1+\int_0^{T_2} I_2(t_2)dt_2 - n\int_0^{\frac{T}{n}} I_b(t)dt\Big)+C_v\frac{1}{T}\Big(pT_1-dT-n\Big(I_d(0)-\Big(\frac{T}{n}\Big)d\Big)\Big)$$

from equations (2.3), (2.4), (2.12) and (2.13), following expression is obtained for the total cost of vendor,

$$= \frac{1}{T}(C_{sv}+nK_{0v})+\frac{C_{cv}}{T}\Big(\int_0^{T_1}\Big(\frac{p-d}{\theta}(1-\exp(-\theta t_1))\Big)dt_1 + \int_0^{T_2}\Big(\frac{-d}{\theta}\Big(1-\exp\big(\theta(T_2-t_2)\big)\Big)dt_2 - n\int_0^{\frac{T}{n}}\Big(\frac{d}{\theta}\Big(\exp\Big(\theta\Big(\frac{T}{n}-t\Big)\Big) - 1\Big)\Big)dt\Big)\Big)+C_v\frac{1}{T}\Big(pT_1-dT-n\Big(\frac{d}{\theta}\Big(\exp\Big(\frac{\theta T}{n}\Big)-1\Big)-\Big(\frac{T}{n}\Big)d\Big)\Big)$$

again by the same assumption on θ,

$$TC_v \approx \frac{1}{T}\big(C_{sv}+nK_{0v}\big)+\frac{C_{cv}}{T}\Big[\frac{(p-d){T_1}^2}{2}(1-\frac{\theta T_1}{3})+\frac{d{T_2}^2}{2}(1+\frac{\theta T_2}{3}) -\frac{dT^2}{2n}(1-\frac{\theta T}{3n})\Big]+\frac{C_v}{T}\Big(pT_1-Td-\frac{d\theta T^2}{2n}\Big) \quad (2.15)$$

Total cost function TC for the vendor and the buyer is the sum of cost function of buyer and vendor as follows $TC = TC_b + TC_v$ By using equations

(2.7), (2.9), (2.14) and (2.15) total cost obtained as,

$$\begin{aligned} &= \frac{1}{T}(C_{sv} + nK_{0v}) + \frac{C_{cv}}{T}\left[\frac{(p-d){T_1}^2}{2}\left(1 - \frac{\theta T_1}{3}\right)\right. \\ &+ \left.\frac{d{T_2}^2}{2}\left(1 + \frac{\theta T_2}{3}\right) - \frac{dT^2}{2n}\left(1 - \frac{\theta T}{3n}\right)\right] + \frac{C_v}{T}\left(pT_1 - dT - \frac{d\theta T^2}{2n}\right) \\ &+ \frac{1}{T}(C_{ob} + nK_{0b}) + \frac{C_{cb}Td}{2n}\left(1 + \frac{\theta T}{3n}\right) + \frac{C_bTd\theta}{2n} \end{aligned} \tag{2.16}$$

2.3 Solution Procedure

The aim is to minimize the total cost of both the buyer and the vendor. For that, the value of n is to be find to minimize the total cost. As number of deliveries per order is a discrete variable, so it is easy to find the value of n by the following method.

1. For a range of n-values, find the partial derivative of TC from equation (2.16) with respect to T_2 and equate it to zero. For each find the minimizing T_2 value by $T_2(n^*)$.

2. Obtain the optimal value of n, denoted by n, such that $TC(T_2(n^* - 1), n^* - 1) \geq$TC$(T_2(n^*), n^*)$

$$\leq TC(T_2(n+1), n+1) \tag{2.17}$$

3. By above condition, find the optimal value of T_1 and T from equation (2.7) and (2.9).

4. Find production quantity, pT_1.

5. Find the delivery quantity $\frac{d}{\theta}(\exp \frac{\theta T}{n} - 1)$ from equation (2.13).

2.4 Example

The preceding theory can be illustrated by considering the numerical example. The capacity of production is 2000000 units per year the demand rate is 500000 units per year. The rate of deterioration is 0.1 per year. The other related factors are as follows: ordering cost 2000 per order, production set-up cost 100000 per cycle, the quality-control cost per delivery is 500, the transportation charge for vendor per delivery is 1000, carrying costs per unit per year for buyer and vendor are 60 and 40, respectively. Deteriorated costs per unit for buyer and vendor are 600 and 400, respectively. In above example, $p = 2 \times 10^5$ units per year

$d = 5 \times 10^4$ units per year

$\theta = 0.1$ per year

$C_{ob} = 2000$ per order

$C_{sb} = 1000000$ per cycle

$K_{0b} = 500$ per delivery

$K_{0v} = 1000$ per delivery

$C_{cb} = 60$ per unit per year

$C_{cv} = 40$ per unit per year

$C_b = 600$ per unit

$C_v = 400$ per unit

Using the solution procedure given above the example has been solved and results are given in table (2.1) to (2.9). The result shows that the buyer follows the integrated policy and agrees on 7 deliveries instead of his original optimal value 25 and will incur an increased cost of \$174803. The vendor on the other hand will have a cost saving of 6.85% here vendor is the winner, so vendor will offer some incentive for the buyer to accept the integrated policy of seven deliveries. To develop a win-win situation for both the buyer and the vendor. The vendor provide discount up to a certain percentage of his extra benefit due to integrated approach.

2.4.1 Sensitivity Analysis

From this integration approach the optimal values of n and $TC(n)$ for constant parameters,we will take a set of these parameters as $S = \{(c_{ob}, C_{sv}),$ $(K_{0b}, K_{0v}), (C_{cb}, C_{cv}), (C_b, C_v), \theta, p, d)\}$ denoted by n^* and $TC(n^*)$,respectively. If the decision is from buyer's point of view then optimal value is denoted by $n^{\#}$ and $TC(n^{\#})$. Then consider the changes in optimal values by increasing or decreasing the parameters by 10%.

Table 2.1: optimal solution of n

n	$T_2(10^{-4})$	$T_1(10^{-4})$	$T(10^{-4})$	$TC_b \times 10^3$	$TC_v \times 10^3$	$TC \times 10^3$
1	501	167	668	2046.84	1048.9	3095.7
2	570	191	761	1182.07	1577.6	2759.7
6	649	217	866	491.00	2071.9	2562.9
7*	659	220	879	439.47	2119.9	2559.3*
8	667	223	890	401.41	2159.1	2560.5
15	712	238	950	290.04	2330.9	2600.9
20	737	247	984	269.58	2412.8	2682.4
24	756	253	1009	264.89	2469.4	2734.2
25#	761	255	1015	264.66#	2482.7	2747.4#
26	765	256	1021	264.72	2495.8	2760.5
30	783	262	1045	267.19	2546.1	2813.3

* integrated optimal solution on n which minimize TC.

buyer's optimal solution of n which minimizes TC_b

$PICR$, percentage of integrated cost reduction=$[TC(n^{\#})$-$TC(n^*)]/TC(n^{\#})$=6.85%

Table 2.2: The vendor and buyer's costs

Cost items	$n^* = 25$	$n^{\#} = 7$	cost $
Ordering cost ofTC_b	19696	22751	+3055
IQC cost of TC_b	123100	39815	-83285
Deteriorated cost of TC_b	60935	188451	+127516
Carrying cost of TC_b	60935	188451	+127516
TC_b	264665	439468	+174803
Setup cost of TC_v	984797	1137568	-152771
delivery cost of TC_v	246199	79630	-166569
Carrying cost of TC_v	722242	534631	-187611
Deteriorated cost of TC_v	529476	368029	-161447
TC_v	2482713	2119858	-362855
TC	2747377	2559325	-188052

+ represent the increment and - represent the decrement in the cost.

2.4.2 Comment On The Sensitivity Analysis

The observation drawn from analysis is as follows.

1. The range of $PICR$(percentage of integrated cost reduction)varies from 6.03 to 7.68% and the average value is $\sim$6.8

2. The values of $PICR$ and n^* changes more frequently by changing the parameter of subset p, d,(K_{0b}, K_{0v}) and (C_{ob}, C_{sv}), and is less sensitive to the parameters of subset (C_{cb}, C_{cv}),(C_b, C_v) and θ.

3. The value of $PICR$ is directly proportional,whereas n^* is inversely proportional to the subset of parameter (K_{ob}, K_{0v}).

4. The value of $PICR$ and n^* decreases as the value of (C_{ob}, C_{sv}) increases.

5. production rate is also inversely proportional to the values of n^* and $PICR$.

6. Demand rate increases as the value of n and $PICR$ increases.

Table 2.3: Sensitivity analysis where C_b and C_v are changed by 10%

C_b	420	480	540	600	660	720	780
C_v	280	320	360	400	440	480	520
n^*	7	7	7	7	7	7	7
TCn^*	2386569	2445506	2503061	2559325	2614383	2668308	2721167
$n^\#$	25	25	25	25	25	25	26
$TCn^\#$	2564170	2626653	2687690	2747377	2805799	2863034	2933101
$PICR$	6.93%	6.90%	6.87%	6.84%	6.82%	6.80%	7.23%

Table 2.4: Sensitivity analysis where C_{cb} and C_{cv} are changed by 10%

C_{cb}	42	48	54	60	66	72	78
C_{cv}	28	32	36	40	44	48	52
n^*	7	7	7	7	7	7	7
TCn^*	2332306	2410362	2485963	2559325	2630637	2700063	2767743
$n^\#$	26	26	25	25	25	25	26
$TCn^\#$	2513320	2598350	2667907	2747377	2824606	2899773	2973035
$PICR$	7.20%	7.23%	6.82%	6.84%	6.87%	6.89%	6.91%

Table 2.5: Sensitivity analysis where θ is changed by 10%

θ	0.07	0.08	0.09	0.10	0.11	0.12	0.13
n^*	7	7	7	7	7	7	7
TCn^*	2386418	2445428	2503032	2559325	2614395	2668316	2721159
$n^\#$	25	25	25	25	25	25	26
$TCn^\#$	2563988	2626561	2687658	2747377	2805809	2863032	2933070
$PICR$	6.93%	6.90%	6.87%	6.84%	6.82%	6.80%	7.20%

where * Integrated optimal solution on n which minimize TC.

$^\#$ The optimal buyer's solution which minimizes cost of buyer.

Table 2.6: Sensitivity analysis when $K_{0b} and K_{0v}$ are changed by 10%

K_{0b}	350	400	450	500	550	600	650
K_{0v}	700	800	900	1000	1100	1200	1300
n^*	9	8	8	7	7	7	6
TCn^*	2519402	2533371	2546956	2559325	2571242	2583104	2593915
$n^\#$	30	28	26	25	24	23	23
$TCn^\#$	2681008	2704403	2722090	2747377	2769686	2789099	2822454
$PICR$	6.03%	6.43%	6.43%	6.84%	7.16%	7.39%	8.10%

Table 2.7: Sensitivity analysis when $C_{ob} and C_{ov}$ are changed by 10%

C_{ob}	1400	1600	1800	2000	2200	2400	2600
C_{ov}	70000	80000	90000	100000	110000	120000	130000
n^*	6	6	7	7	8	8	8
TCn^*	2181211	2315451	2440537	2559325	2672572	2780159	2883735
$n^\#$	22	23	24	25	26	27	28
$TCn^\#$	2381503	2509587	2631253	2747377	2858652	2965638	3068796
$PICR$	8.41%	7.74%	7.25%	6.84%	6.51%	6.25%	6.03%

Table 2.8: Sensitivity analysis when p is changed by 10%

$p(10^6)$	1.4	1.6	1.8	2.0	2.2	2.4	2.6
n^*	8	8	7	7	7	7	7
TCn^*	2320483	2420716	2498089	2559325	2609359	2651011	2686224
$n^\#$	29	27	26	25	25	24	24
$TCn^\#$	2518945	2612098	2689288	2747377	2805152	2839423	2879749
$PICR$	7.88%	7.33%	7.11%	6.84%	6.98%	6.64%	6.72%

Table 2.9: Sensitivity analysis when d is changed by 10%

$p(10^6)$	350	400	450	500	550	600	650
n^*	7	7	7	7	8	8	8
TCn^*	2279348	2387565	2480216	2559325	2622886	2682438	2728104
$n^\#$	23	24	25	25	26	27	28
$TCn^\#$	2433956	2558477	2666693	2747377	2828032	2896949	2955000
$PICR$	6.35%	6.68%	6.99%	6.84%	7.25%	7.40%	7.68%

In above tables, $PICR$ is percentage of integrated cost reduction=$[TC(n^{\#})$-$TC(n^{*})]/TC(n^{\#})$, $\{\}$, base column.

2.5 Conclusion

In this paper, it has been shown that optimal policy using integrated approach reduces the total joint cost for both the vendor and the buyer through Sensitivity analysis. As from above table we have seen that the buyer's cost is higher, therefore an incentive in the form of cost reduction or discount for the buyer to unite and to make situation realistic.

Chapter 3

Economic Ordering Policy Of Deteriorated Item For Vendor And Buyer With Demand Depending On Time

3.1 Formulation And Solution Methodology

Most inventory models considered to managing its inventory policy to minimize its own cost or maximize its own profit when demand and various factors are constant. This one-sided-optimal-strategy is not suitable for global markets. In real situation, demand rate of any product is always in dynamic state. This variation is due to time or price of the items in any inventory. In this Chapter "Economic ordering policy of deteriorated item for vendor and buyer : an integrated approach" with variable demand rate is considered and other all assumption and notation are same as used by

Yang and Wee(2000). The objective of the model is to determine the optimum profit for items with variable demand and other factors. The demand function is depending on the time of the production and non production cycle. Demand function is represented as follows for different period:
$d(t) = a + bt + ct^2$ for the buyer where a is the initial rate of demand, b and c are constants.
The differential equations which describes the inventory level at time t_1 and t_2 are given by:

$$\frac{dI_1(t_1)}{dt_1} = (p - (a + bt_1 + ct_1^2)) - \theta I_1(t_1) \quad ; 0 \leq t_1 \leq T_1 \qquad (3.1)$$

$$\frac{dI_2(t_2)}{dt_2} = -(a + bt_2 + ct_2^2) - \theta I_2(t_2) \quad ; 0 \leq t_2 \leq T_2 \qquad (3.2)$$

using Spiegel (1960), obtained boundary condition are

$I_1(0) = I_2(T_2)$, solution of the above equations are as follows:

$$I_1(t_1) = \frac{1}{\theta}(p - a - bt_1 - ct_1^2) + \frac{1}{\theta^2}\left(b + 2t_1c - \frac{2c}{\theta}\right) + \left(\frac{-p}{\theta} + \frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^3}\right)\exp -\theta t_1 \quad ; \quad 0 \leq t_1 \leq T_1 \qquad (3.3)$$

$$I_2(t_2) = -\frac{a}{\theta} - \frac{bt_2}{\theta} + \frac{b}{\theta^2} - \frac{ct_2^2}{\theta} + \frac{2ct_2}{\theta^2} - \frac{2c}{\theta^3} + \left(a + bT_2 + cT_2^2\right)\frac{\exp\theta\big(T_2 - t_2\big)}{\theta} + \left(-b - 2T_2c + \frac{2c}{\theta}\right)\frac{\exp\theta\big(T_2 - t_2\big)}{\theta^2} \quad ; \quad 0 \leq t_2 \leq T_2 \qquad (3.4)$$

Then again by boundary conditions $I_1(T_1) = I_2(0)$

$$\frac{1}{\theta}\big(p - a - bT_1 - cT_1^2\big) + \frac{1}{\theta^2}(b + 2T_1c - \frac{2c}{\theta}) + (\frac{-p}{\theta} + \frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^3})\exp -\theta T_1 = -\frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} + \big(a + bT_2 + cT_2^2\big)\frac{\exp\theta(T_2 - t_2)}{\theta} + \big(-b - 2T_2c + \frac{2c}{\theta}\big)\frac{\exp\theta T_2}{\theta^2} \quad ; \quad 0 \leq t_2 \leq T_2 \qquad (3.5)$$

By using taylor's expansion and by using Yang and Wee(2000) assumption we have,

$$T_1\Bigg((p-a)\Bigg(1-\frac{\theta T_1}{2}\Bigg)-\frac{bT_1}{2}\Bigg)=T_2\Bigg(a\Bigg(1+\frac{\theta T_2}{2}\Bigg)+\frac{bT_2}{2}(1+\theta T_2)+\frac{c\theta T_2^3}{2}\Bigg) \tag{3.6}$$

From Misra(1975) we have,

$$T_1\approx T_2\Bigg(1+\frac{1}{p-a-\frac{bT_1}{2}}\Bigg(a(1+\frac{\theta T_1}{2})+\frac{bT_2}{2}(1+\theta T_2)+\frac{c\theta T_2^3}{2}\Bigg)\Bigg) \tag{3.7}$$

From the relation

$T=T_1+T_2$

By using above relation we have,

$$T\approx\frac{T_2}{(p-a-\frac{bT_1}{2})}\Bigg(p+\frac{1}{2}\Bigg(aT_2\theta+bT_2+b\frac{T_2}{\theta}+\frac{cT_2^2}{\theta}\Bigg)-2bT_1\Bigg) \tag{3.8}$$

the differential equation for the buyer with n deliveries per order are as follows:

$$\begin{aligned}\frac{I_b(t)}{dt} &= -(a+bt+ct^2)-\theta I_b(t)\\ I_b(t) &= \frac{1}{\theta}\Bigg(\Bigg(-a-b\Bigg(t-\frac{1}{\theta}\Bigg)-c\Bigg(t^2-\frac{2tc}{\theta}+\frac{2}{\theta^2}\Bigg)\Bigg)\Bigg)\\ &+ \frac{\exp\theta(\frac{T}{n}-t)}{\theta}\Bigg(a+b\Bigg(\frac{T}{n}-\frac{1}{\theta}\Bigg)+\frac{c}{\theta}\Bigg(\frac{T^2}{n^2}-\frac{2T}{n}+\frac{2}{\theta}\Bigg)\Bigg)\quad;\qquad 0\le t\le\frac{T}{n}\end{aligned} \tag{3.9}$$

Maximum inventory of the buyer is at $t=0$ given by,

$$\begin{aligned}I_b(t=0) &= \frac{1}{\theta}\Bigg(-a+\frac{b}{\theta}\Bigg)-\frac{2c}{\theta^3}+\frac{\exp\theta\Big(\frac{T}{n}\Big)}{\theta}\Bigg(a+b\Bigg(\frac{T}{n}-\frac{1}{\theta}\Bigg)\\ &+ \frac{c}{\theta}\Bigg(\frac{T^2}{n^2}-\frac{2T}{n}+\frac{2}{\theta}\Bigg)\Bigg)\end{aligned} \tag{3.10}$$

from taylor's series expansion and assumption made in Chapter 2, $\theta \ll 1$.

Total cost of the buyer is

$$\begin{aligned} TC_b(t) &= \frac{1}{T}\big(C_{ob} + nK_{0b}\big) + C_{cb}\frac{n}{T}\int_0^{\frac{T}{n}} I_b(t)dt \\ &+ C_b\frac{n}{T}\bigg(I_b(0) - \frac{T}{n}(a + bt + ct^2)\bigg) \end{aligned}$$

using equations (3.9) and (3.10), we have

$$\begin{aligned} &= \frac{1}{T}\big(C_{ob} + nK_{0b}\big) + C_{cb}\frac{n}{T}\int_0^{\frac{T}{n}} \bigg(\frac{1}{\theta}\bigg(\bigg(-a - b\bigg(t - \frac{1}{\theta}\bigg) - c\big(t^2 - \frac{2tc}{\theta} + \frac{2}{\theta^2}\big)\bigg)\bigg) \\ &+ \frac{\exp\theta\bigg(\frac{T}{n} - t\bigg)}{\theta}\bigg(a + b\bigg(\frac{T}{n} - \frac{1}{\theta}\bigg) + \frac{c}{\theta}\bigg(\frac{T^2}{n^2} - \frac{2T}{n} + \frac{2}{\theta}\bigg)\bigg)\bigg)dt \\ &+ C_b\frac{n}{T}\bigg(\bigg(\frac{1}{\theta}(-a + \frac{b}{\theta}) - \frac{2c}{\theta^3} + \frac{\exp\theta\bigg(\frac{T}{n}\bigg)}{\theta}\bigg(a + b(\frac{T}{n} - \frac{1}{\theta}) \\ &+ \frac{c}{\theta}\bigg(\frac{T^2}{n^2} - \frac{2T}{n} + \frac{2}{\theta}\bigg)\bigg)\bigg) - \frac{T}{n}(a + bt + ct^2)\bigg) \end{aligned}$$

on solving above equation and by using taylor's expansion we get,

$$\begin{aligned} TC_b(t) \approx\ & \frac{1}{T}(C_{ob} + nK_{0b}) + \frac{C_{cb}n}{\theta T}\bigg(-\frac{aT}{n} - \frac{bT^2}{2n^2} + \frac{bT}{n\theta} - \frac{cT^3}{3n^3} + \frac{c^2T^2}{n^2\theta} - \frac{2cT}{n\theta^2}\bigg) + \frac{C_{cb}n}{\theta^2 T}\bigg(-a \\ -\ & b\bigg(\frac{T}{n} - \frac{1}{\theta}\bigg) - \frac{c}{\theta}\bigg(\frac{T^2}{n^2} - \frac{2T}{n} + \frac{2}{\theta}\bigg)\bigg) - \frac{C_{cb}n}{T\theta^2}\bigg(-a - b(\frac{T}{n} - \frac{1}{\theta}) \\ & \bigg(1 + \frac{\theta T}{n} + \frac{\theta^2T^2}{n^2}\bigg) - \frac{c}{\theta}\bigg(\frac{T^2}{n^2} - \frac{2T}{n} + \frac{2}{\theta}\bigg)\bigg)\bigg(1 + \frac{\theta T}{n} + \frac{\theta^2T^2}{2n^2} + \\ & \frac{\theta^3T^3}{6n^3}\bigg) + \frac{C_bn}{T}\bigg(-\frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} + \frac{1}{\theta}\bigg(1 + \frac{\theta T}{n} + \frac{\theta^2T^2}{2n^2}\bigg) \\ & \bigg(a + b\bigg(\frac{T}{n} - \frac{1}{\theta}\bigg) + \frac{c}{\theta}\bigg(\frac{T^2}{n^2} - \frac{2T}{n} + \frac{2}{\theta}\bigg)\bigg) - \frac{T}{n}(a + bt + ct^2)\bigg) \end{aligned} \tag{3.11}$$

The total cost function TC_v for the vendor is

$$
\begin{aligned}
TC_v &= \frac{1}{T}(C_{sv}+nK_{0v}) + C_{cv}\frac{1}{T}\Bigg(\int_0^{T_1} I_1(t_1)dt_1 + \int_0^{T_2} I_2(t_2)dt_2 - n\int_0^{\frac{T}{n}} I_b(t)dt\Bigg)\\
&+ C_v\frac{1}{T}\Bigg(pT_1 - (a+bT_1+T_1^2)T - n(I_d(0) - \Big(\frac{T}{n}\Big)(a+bT_2+cT_2^2))\Bigg)\\
&= \frac{1}{T}(C_{sv}+nK_{0v}) + C_{cv}\frac{1}{T}\Bigg(\int_0^{T_1}\Bigg(\frac{1}{\theta}(p-a-bt_1-ct_1^2) + \frac{1}{\theta^2}\Big(b+2t_1c-\frac{2c}{\theta}\Big)\\
&+ \Big(\frac{-p}{\theta}+\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^3}\Big)\exp -\theta t_1\Bigg)dt_1 + \int_0^{T_2}\Bigg(-\frac{a}{\theta}-\frac{bt_2}{\theta}+\frac{b}{\theta^2}-\frac{ct_2^2}{\theta}\\
&+ \frac{2ct_2}{\theta^2}-\frac{2c}{\theta^3}+\big(a+bT_2+cT_2^2\big)\frac{\exp\theta\big(T_2-t_2\big)}{\theta}+(-b-2T_2c+\frac{2c}{\theta})\frac{\exp\theta\big(T_2-t_2\big)}{\theta^2}\Bigg)dt_2\\
&- n\int_0^{\frac{T}{n}}\Bigg(\frac{1}{\theta}\Bigg(\Bigg(-a-\Big(t-\frac{1}{\theta}\Big)-c\Big(t^2-\frac{2tc}{\theta}+\frac{2}{\theta^2}\Big)\Bigg)\Bigg)+\frac{\exp\theta(\frac{T}{n}-t)}{\theta}\\
&\quad\Bigg(a+b\Big(\frac{T}{n}-\frac{1}{\theta}\Big)+\frac{c}{\theta}\Big(\frac{T^2}{n^2}-\frac{2T}{n}+\frac{2}{\theta}\Big)\Bigg)\Bigg)dt\Bigg)+C_v\frac{1}{T}\Bigg(pT_1-(a+bT_1+T_1^2)T\\
&- n\Bigg(\Bigg(\frac{1}{\theta}\Big(-a+\frac{b}{\theta}\Big)-\frac{2c}{\theta^3}+\frac{\exp\theta(\frac{T}{n})}{\theta}\Big(a+b(\frac{T}{n}-\frac{1}{\theta})+\frac{c}{\theta}\Big(\frac{T^2}{n^2}-\frac{2T}{n}+\frac{2}{\theta}\Big)\Big)\Bigg)\\
&- (\frac{T}{n})\big(a+bT_2+cT_2^2\big)\Bigg)\Bigg)\\
&\approx \frac{1}{T}\big(C_{sv}+nK_{0v}\big)+\frac{C_{cv}}{T}\Bigg(\frac{1}{\theta}\Bigg(\Big(pT_1-aT_1-\frac{bT_1^2}{2}-\frac{cT_1^3}{3}\Big)+\frac{1}{\theta}\Big(bT_1^2c-\frac{2c}{\theta}\Big)\\
&- \Big(1-\theta T_1+\frac{\theta^2T_1^2}{2}-\frac{\theta^3T_1^3}{6}\Big)\Big(-\frac{p}{\theta}+\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\Big)\Bigg)+\frac{1}{\theta^2}\Big(-p-a-\frac{b}{\theta}+\frac{2c}{\theta}\Big)\\
&+ \Big(-\frac{aT_2}{\theta}-\frac{bT_2^2}{2\theta}+\frac{bT_2}{\theta^2}-\frac{cT_2^3}{3\theta}+\frac{cT_2^2}{\theta^2}-\frac{2cT_2}{\theta^3}\Big)-\big(a+bT_2+cT_2^2\big)\frac{1}{\theta^2}\\
&- \frac{1}{\theta^3}\Big(-b-2T_2c+\frac{2c}{\theta}\Big)+\frac{1}{\theta^2}\Bigg(\Big(1+\theta T_2+\frac{\theta^2T_2^2}{2}+\frac{\theta^3T_2^3}{6}\Big)\big(a+bT_2+cT_2^2\big)\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
+ & \frac{1}{\theta^3}\Bigg(\Big(-b-2T_2c+\frac{2c}{\theta}\Big)\Big(1+\theta T_2+\frac{\theta^2T_2^2}{2}+\frac{\theta^3T_2^3}{6}\Big)\Bigg)-n\Bigg(\frac{1}{\theta}\Bigg(-\frac{aT}{n} \\
- & \frac{bT^2}{2n^2}+\frac{bT}{n\theta}-\frac{cT^3}{3n^3}+\frac{T^2c^2}{n^2\theta}-\frac{2cT}{n\theta^2}\Bigg)-\frac{1}{\theta^2}\Bigg(\frac{c}{\theta}\Big(\frac{T^2}{n^2}-\frac{2T}{n}+\frac{2}{\theta}\Big)\Bigg) \\
+ & a+b\Big(\frac{T}{n}-\frac{1}{\theta}\Big)\Bigg)+\frac{1}{\theta^2}\Big(1+\frac{\theta T}{n}+\frac{\theta^2T^2}{2n^2}+\frac{\theta^3T^3}{6n^3}\Big)\Bigg(\frac{c}{\theta}(\frac{T^2}{n^2}-\frac{2T}{n} \\
+ & \frac{2}{\theta})+a+b\Big(\frac{T}{n}-\frac{1}{\theta}\Big)\Bigg)\Bigg)\Bigg)+\frac{c_v}{T}\Bigg(pT_1-(a+bt_1+ct_1^2)T-n\Bigg(\frac{1}{\theta}(-a+\frac{b}{\theta}) \\
- & \frac{2c}{\theta^3}+\frac{1}{\theta}\Big(1+\frac{\theta T}{n}+\frac{\theta^2T^2}{2n^2}\Big)\Bigg(a+b\Big(\frac{T}{n}-\frac{1}{\theta}\Big)+\frac{c}{\theta}\Big(\frac{T^2}{n^2} \\
- & \frac{2T}{n}+\frac{2}{\theta}\Big)\Bigg)-\frac{T}{n}(a+bt_2+ct_2^2)\Bigg)\Bigg) \qquad (3.12)
\end{aligned}
$$

Total cost function TC is the sum of the total cost of buyer and vendor. From equations (3.7), (3.8), (3.12) and (3.13), integrated cost can be written as a function of T_2, T_1 and n as follows, TC=TC_b+TC_v

$$
\begin{aligned}
= & \frac{1}{T}(C_{ob}+nK_{0b})+\frac{C_{cb}n}{\theta T}\Big(-\frac{aT}{n}-\frac{bT^2}{2n^2}+\frac{bT}{n\theta}-\frac{cT^3}{3n^3}+\frac{c^2T^2}{n^2\theta}-\frac{2cT}{n\theta^2}\Big)+\frac{C_{cb}n}{\theta^2 T}\Bigg(-a \\
- & b(\frac{T}{n}-\frac{1}{\theta})-\frac{c}{\theta}\Big(\frac{T^2}{n^2}-\frac{2T}{n}+\frac{2}{\theta}\Big)\Bigg)-\frac{C_{cb}n}{T\theta^2}\Bigg(-a-b\Big(\frac{T}{n}-\frac{1}{\theta}\Big)\Big(1+\frac{\theta T}{n}+\frac{\theta^2T^2}{n^2}\Big) \\
- & \frac{c}{\theta}\Big(\frac{T^2}{n^2}-\frac{2T}{n}+\frac{2}{\theta}\Big)\Bigg)\Big(1+\frac{\theta T}{n}+\frac{\theta^2T^2}{2n^2}+\frac{\theta^3T^3}{6n^3}\Big)+\frac{C_bn}{T}\Bigg(-\frac{a}{\theta}+\frac{b}{\theta^2}-\frac{2c}{\theta^3}+\frac{1}{\theta} \\
& \Big(1+\frac{\theta T}{n}+\frac{\theta^2T^2}{2n^2}\Big)\Bigg(a+b(\frac{T}{n}-\frac{1}{\theta})+\frac{c}{\theta}\Big(\frac{T^2}{n^2}-\frac{2T}{n}+\frac{2}{\theta}\Big)\Bigg)-\frac{T}{n}(a+bt+ct^2)\Bigg) \\
+ & \frac{1}{T}(C_{sv}+nK_{0v})+\frac{C_{cv}}{T}\Bigg(\frac{1}{\theta}\Bigg(\Big(pT_1-aT_1-\frac{bT_1^2}{2}-\frac{cT_1^3}{3}\Big)+\frac{1}{\theta}\Big(bT_1^2c-\frac{2c}{\theta}\Big)-\Big(1-\theta T_1 \\
+ & \frac{\theta^2T_1^2}{2}-\frac{\theta^3T_1^3}{6}\Big)\Big(-\frac{p}{\theta}+\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\Big)\Bigg)+\frac{1}{\theta^2}\Big(-p-a-\frac{b}{\theta}+\frac{2c}{\theta}\Big)+\Bigg(-\frac{aT_2}{\theta} \\
- & \frac{bT_2^2}{2\theta}+\frac{bT_2}{\theta^2}-\frac{cT_2^3}{3\theta}+\frac{cT_2^2}{\theta^2}-\frac{2cT_2}{\theta^3}\Bigg)-(a+bT_2+cT_2^2)\frac{1}{\theta^2}-\frac{1}{\theta^3}\Big(-b-2T_2c+\frac{2c}{\theta}\Big) \\
+ & \frac{1}{\theta^2}\Bigg(\Big(1+\theta T_2+\frac{\theta^2T_2^2}{2}+\frac{\theta^3T_2^3}{6}\Big)(a+bT_2+cT_2^2)\Bigg)+\frac{1}{\theta^3}\Bigg((-b-2T_2c+\frac{2c}{\theta})\Big(1+\theta T_2+\frac{\theta^2T_2^2}{2}
\end{aligned}
$$

$$
\begin{aligned}
+ \quad & \frac{\theta^3 T_2^3}{6}\Bigg)\Bigg) - n\Bigg(\frac{1}{\theta}\Bigg(- \frac{aT}{n} - \frac{bT^2}{2n^2} + \frac{bT}{n\theta} - \frac{cT^3}{3n^3} + \frac{T^2c^2}{n^2\theta} - \frac{2cT}{n\theta^2}\Bigg) \\
- \quad & \frac{1}{\theta^2}\Bigg(\frac{c}{\theta}\Bigg(\frac{T^2}{n^2} - \frac{2T}{n} + \frac{2}{\theta}\Bigg) + a + b\Bigg(\frac{T}{n} - \frac{1}{\theta}\Bigg)\Bigg) + \frac{1}{\theta^2}(1 + \frac{\theta T}{n} + \frac{\theta^2 T^2}{2n^2} \\
+ \quad & \frac{\theta^3 T^3}{6n^3})\Bigg(\frac{c}{\theta}(\frac{T^2}{n^2} - \frac{2T}{n} + \frac{2}{\theta}) + a + b\Bigg(\frac{T}{n} - \frac{1}{\theta}\Bigg)\Bigg)\Bigg)\Bigg) + \frac{c_v}{T}\Bigg(pT_1 - \\
& (a + bt_1 + ct_1^2)T - n\Bigg(\frac{1}{\theta}\Bigg(- a + \frac{b}{\theta}\Bigg) - \frac{2c}{\theta^3} + \frac{1}{\theta}\Bigg(1 + \frac{\theta T}{n} + \frac{\theta^2 T^2}{2n^2}\Bigg)\Bigg(a + \\
& b\Bigg(\frac{T}{n} - \frac{1}{\theta}\Bigg) + \frac{c}{\theta}\Bigg(\frac{T^2}{n^2} - \frac{2T}{n} + \frac{2}{\theta}\Bigg)\Bigg) - \frac{T}{n}(a + bt_2 + ct_2^2)\Bigg)\Bigg) \qquad (3.13)
\end{aligned}
$$

3.2 Numerical Example

The preceding theory can be illustrated by considering the numerical example. The capacity of production is 2000000 units per year the demand rate is 500000 units per year. The rate of deterioration is 0.1 per year. The other related factors are as follows: ordering cost 2000 per order, production set-up cost 100000 per cycle, the quality-control cost per delivery is 500, the transportation charge for vendor per delivery is 1000, carrying costs per unit per year for buyer and vendor are 60 and 40, respectively. Deteriorated costs per unit for buyer and vendor are 600 and 400, respectively. The example used in Chapter 2 has been considered here with demand as a function of time $d(t) = a + bt + ct^2$ Where 'a' is the initial rate of demand, 'b' and 'c' are the rates with which the demand rate increases or decreases is considered and the solution procedure given by Yang and Wee(2000) is used to find the solution of this model. The results are given in Table (3.1) to (3.4) which shows that $PICR$ vary from 1.2% to 10.02% for different demand rates.

Table 3.1: optimal solution of n with $a = 500000, b = 0, c = 0$

n	$T_2(10^{-4})$	$T_1(10^{-4})$	$T(10^{-4})$	$TC_b \times 10^3$	$TC_v \times 10^3$	$TC \times 10^3$
1	501	167	668	2046.84	1048.9	3095.7
2	570	191	761	1182.07	1577.6	2759.7
6	649	217	866	491.00	2071.9	2562.9
7^*	659	220	879	439.47	2119.9	2559.3^*
8	667	223	890	401.41	2159.1	2560.5
15	712	238	950	290.04	2330.9	2600.9
20	737	247	984	269.58	2412.8	2682.4
24	756	253	1009	264.89	2469.4	2734.2
$25^{\#}$	761	255	1015	$264.66^{\#}$	2482.7	$2747.4^{\#}$
26	765	256	1021	264.72	2495.8	2760.5
30	783	262	1045	267.19	2546.1	2813.3

* integrated optimal solution on n which minimize TC.
$^{\#}$ buyer's optimal solution of n which minimizes TC_b.
$PICR$, percentage of integrated cost reduction=$[TC(n^{\#})\text{-}TC(n^*)]/TC(n^{\#}) = 6.85\%$.

Table 3.2: optimal solution of n with $a = 500000, b = -0.1, c = -0.01$

n	T_2	T_1	$T(10^{-3})$	$TC_b \times 10^3$	$TC_v \times 10^3$	$TC \times 10^3$
1	0.05266	0.017600	702	2298.346	9536.592	3252.00
2^*	0.06298	0.021060	840	1570.876	14766.40	3047.52^*
5	0.07929	0.026538	0105	1425.661	19586.76	3202.13
$7^{\#}$	0.08717	0.029182	0116	$1271.73^{\#}$	21154.28	$3387.16^{\#}$
15	0.11180	0.037476	0149	1586.89	25319.18	4118.70
25	0.13585	0.045590	0181	1982.68	29268.67	4909.55
26	0.13800	0.046320	0184	2019.50	29626.99	4982.20
30	0.14633	0.049132	0194	2168.65	40116.87	6180.34

* integrated optimal solution on n which minimize TC.
$^{\#}$ buyer's optimal solution of n which minimizes TC_b.
$PICR$, percentage of integrated cost reduction=$[TC(n^{\#})\text{-}TC(n^*)]/TC(n^{\#}) = 10.02\%$.

Table 3.3: optimal solution of n with $a = 500000, b = -0.01, c = -0.1$

n	T_2	T_1	T	$TC_b \times 10^6$	$TC_v \times 10^5$	$TC \times 10^3$
1^*	0.071584516	0.023946991	0.09553143	4.016228305	3.997765463	4416.004851^*
$2^\#$	0.099837204	0.03344519	0.13328240	$3.645356543^\#$	11.82750612	$4416.54202^\#$
3	0.1214431296	0.04072685	0.16216998	3.651070318	16.84031509	5335.1018
4	0.1396571220	0.046877442	0.18653456	3.751729180	20.7519477	5826.8588
5	0.1557115513	0.052307952	0.20801950	3.885756327	24.0522082	6290.9771
6	0.1702329501	0.057227304	0.22746025	4.032504756	26.9573909	6728.2438
7	0.1835921871	0.0617591637	0.24535135	4.183438552	29.5805418	7141.4927
15	0.2671922182	0.090253933	0.35744615	5.324777815	45.4093986	9865.7177
20	0.3080110646	0.1042515347	0.41226260	5.949256914	52.94774745	11244.0317
25	0.3439597944	0.1166250701	0.46058486	6.517931817	59.54096138	12472.0279
26	0.3507013047	0.1189502911	0.46965160	6.626146746	60.77388481	12703.5352
30	0.3764439476	0.1278431493	0.50428710	7.043457722	65.46995278	13590.4532

* integrated optimal solution on n which minimize TC.

$^\#$ buyer's optimal solution of n which minimizes TC_b.

$PICR$, percentage of integrated cost reduction=$[TC(n^\#)$-$TC(n^*)]/TC(n^\#) = 1.2\%$.

Table 3.4: optimal solution of n with $a = 500000, b = 0.1, c = -0.1$

n	T_2	T_1	T	$TC_b \times 10^6$	$TC_v \times 10^6$	$TC \times 10^6$
1^*	0.071584436	0.02394688437	0.09553132	4.016207924	3.991660514	4.415373976^*
$2^\#$	0.099837077	0.03344515004	0.13328223	$3.645304273^\#$	1.182158638	$4.827462911^\#$
3	0.1214429668	0.04072679598	0.16216976	3.651084416	1.683381755	5.334466171
5	0.1557113317	0.05230787839	0.20801921	3.885721994	2.404588497	6.290310491
7	0.1835919218	0.06175907496	0.24535100	4.183434488	2.957413881	7.140848369
8	0.1960303172	0.06598390519	0.26201423	4.334691962	3.198407689	7.533099651
10	0.2187695359	0.07372084873	0.29249038	4.63200245	3.632646732	8.264649187
15	0.2671918178	0.09025379947	0.35744562	5.324892282	4.5401915016	9.865083798
20	0.3080105986	0.1042513783	0.41226198	5.948976273	5.294274906	11.24325118
25	0.3439592709	0.1212322663	0.47854542	6.315884490	6.164796218	12.48068070
30	0.3769433729	0.1280159010	0.50495927	7.035672823	6.554202453	13.58987528

* integrated optimal solution on n which minimize TC.

$^\#$ buyer's optimal solution of n which minimizes TC_b.

$PICR$, percentage of integrated cost reduction=$[TC(n^\#)$-$TC(n^*)]/TC(n^\#) = 8.5\%$.

3.3 Conclusion

In this work, a mathematical model for deteriorating items with constant production and quadratic demand rate is considered. The results obtained for different values of demand rate coefficients. It can be easily verified that if we set $b, c = 0$ in demand rate function we obtained the usual expressions given by yang and Wee(2000). The numerical results also have been verified with yang and Wee(2000) and are given in Table (3.1). Increment in 'b' results in retarded growth in demand whereas change in 'c' corresponds to rapid change in demand and the results are given in Table (3.2), (3.3) and (3.4). The model is very practical for the industries in which the demand rate is depending upon the time and holding cost is constant.

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