# FUZZY INVENTORY MODELS FOR CRISP AND FUZZY PRODUCTION QUANTITY WITH TRAPEZOIDAL FUZZY NUMBERS

***Thesis Submitted in partial fulfillment of the requirements for***
***The award of the degree of***
***Masters of Science***
***In***
**Mathematics and Computing**

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**DEDICATED**

**TO**

**GOD, MY PARENTS**

**AND**

**SUPERVISOR**

# CERTIFICATE

*I hereby certify that the work which is being presented in the thesis entitled* " **Fuzzy Inventory Models for Crisp and Fuzzy Production Quantity with Trapezoidal Fuzzy Numbers**" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Mahesh Kumar Sharma.

*The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.*

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*This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.*

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Neha Aggarwal
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# ABSTRACT

The main objective of inventory management is to get satisfactory levels of customer service while keeping inventory costs within reasonable bounds and that is why the subject of inventory control is a major consideration in economic situations. Inventory management and control in manufacturing environments involve various types of data, which may be deterministic or probalistic, however in real life situations in inventory management the data is uncertain and imprecise. Conventional inventory models that include uncertainty are based on the concept of randomness and on probability theory. However, in certain situations uncertainties are due to fuzziness and in these cases the fuzzy set theory is applicable only.

In this study inventory models in fuzzy environment have been considered and the solution are obtained.

The present thesis consists of five chapters.

**Chapter one** is introductory in which basic inventory models and literature related to the work have been discussed.

The fuzzy production inventory models with fuzzy parameters for crisp and fuzzy production quantity involves constant rate demand with uniform order replenishment have been considered by Hsieh [6] and Extension of the lagrangean method used to solve the problem. In **chapter two** this problem has been reviewed in detail with instantaneous and uniform replenishment separately and in place of Extension of the lagrangean method, KT conditions has been used to solve inequality constraints problem for uniform and instantaneous replenishment.

In **chapter three**, the fuzzy shortage cost has been introduced in existing fuzzy production inventory model with fuzzy parameters for crisp production quantity involve constant rate demand with instantaneous replenishment and solution has been obtained.

In **chapter Four**, A fully fuzzy production inventory models with price breaks involving uniform and instantaneous replenishment is developed and the technique used in chapter two and three has been used to find the solution of the problem.

In **chapter Five**, A Multi-item fuzzy production inventory model with fuzzy parameters for crisp production quantity involving uniform and instantaneous replenishment has been considered and solution is obtained.

# Contents

# Chapter 1

# INTRODUCTION

To ensure smooth functioning of a business or a company it is always better to maintain a reasonable inventory of goods. A company dealing with physical products including manufacturers, wholesaler and retailer needs to maintain the inventory. Manufactures needs inventories of the materials to make their products, however wholesalers and retailers need to maintain inventories of goods to be available for costumers to purchase. Most managers do not like inventories because they understand that the money is placed in a drawer, assets tied up in investments that are not producing any return and, in fact, incurring a borrowing cost and the costs for the care of the stored material and are subject to spoilage and obsolescence. The main objective of inventory management is to get satisfactory levels of customer service while keeping inventory costs within reasonable bounds and that is why the subject of inventory control is a major consideration in economic situations. The principal factor which affect the result is the nature of demand. The demand may be deterministic or probabilistic, however in real life situation demand is probabilistic but in same situations the uncertainties are due to fuzzyness and fuzzy set theory may be acceptable. Complexity of inventory problem it is almost impossible to develop a generalized model that covers all possible situations. Regardless of the method for the solution of any inventory model always two questions arises that how much and when to order.

In this chapter the background of the basic inventory models(deterministic) have been discussed.

## 1.1 GENERAL INVENTORY MODEL:

An inventory model is based on placing and receiving the order periodically. It may be periodic review in which the new orders are placed in the starting of each period.on the other hand, it may also be a continuous review in which a new order is placed when a inventory level drops to a certain level. To answer that how much to order? and when to order is to minimize the total inventory cost which includes

**Purchasing cost**-Based on the price per unit of the item or may be depends upon the offered discount which depends on the size of the order.

**Setup cost**-It is the fixed costs incurred when an order is placed. This costs is independent on the size of the order.

**Holding cost**-It represents the costs which is used to maintaining the inventory. Costs include interest, insurance, taxes, deterioration, spoilage, breakage, and warehousing costs etc.

**Shortage cost**-This cost incurred when the amount of the commodity required exceeds the available stock. These costs can include the opportunity cost of not making a sale, loss of customer good will and late charges. Shortage costs are sometimes difficult to measure, and they may be subjectively estimated.

Costs involving in inventory are shown by Figure 1.1

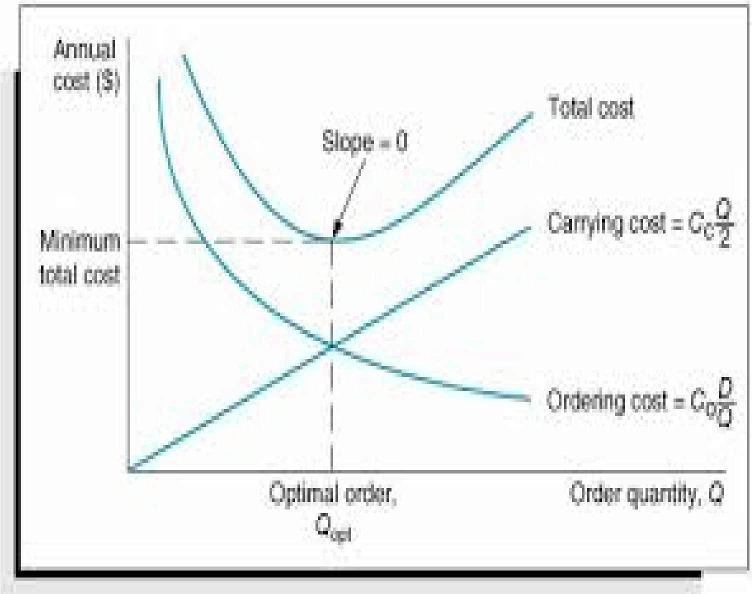

Figure 1.1: Inventory costs

There is usually lag between when an order is placed and when it arrives in inventory. The amount of time between the placement of an order and its receipt is referred to as the lead time. The inventory level at which the order is placed is called reorder point. The time between consecutive replenishment of inventory is referred to as a cycle.

### 1.1.1 Economic Order Quantity(EOQ) Model

The classical or basic EOQ models are based on the assumptions that there is continuous, constant and known demand rate. Planned shortages are not allowed in it and the order quantity to replenish inventory arrives all at one point in time when it is desired.

The inventory follows the following pattern given in Figure 1.2

If $T$, $D$, $Q$ and $A$ are the setup cost, demand rate, order quantity and holding cost respectively, then The total cost per unit time is obtained from the following components.

setup cost per cycle=$T$

$$\text{Holding cost per cycle}=\frac{AQ^2}{2D}$$

Therefore,

$$\text{So, Total cost per unit time(TCU)}=\frac{TD}{Q}+\frac{AQ}{2}$$

### 1.1.2 EOQ Model with Shortage

In addition with basic EOQ model planned shortages are allowed in this model. When a shortage occurs, the affected customers will wait for the product to become available again. This causes a variety of headache including dealing with unhappy customers and having extra record keeping to arrange for filling the demand later. When the order quantity arrives to replenish inventory their backorders are filled immediately. However, there are situations where permitting limited planned shortage make sense from a managerial perspective. The most important is that the customers generally are able and willing to accept a reasonable delay in filling their order if need be. If the

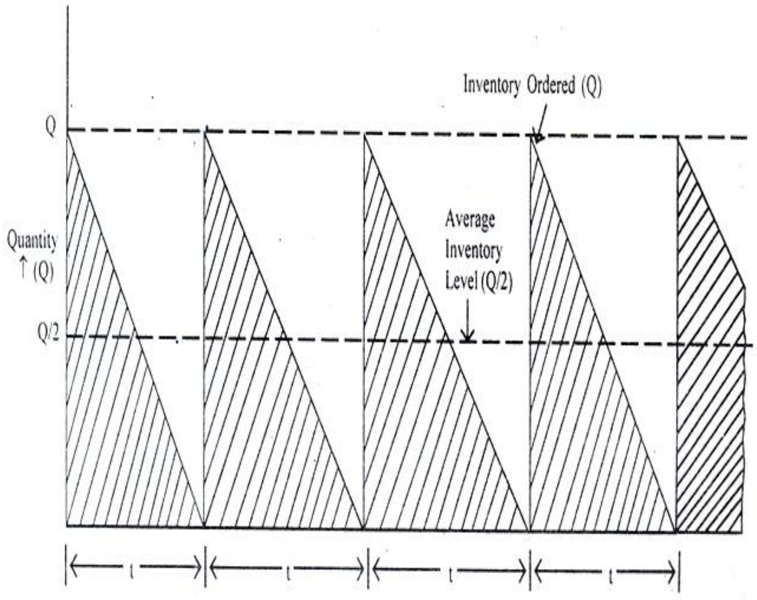

Figure 1.2: A basic EOQ model

cost of holding inventory is high relative to these shortage costs,then lowering the average inventory level by permitting occasional brief shortages may be a sound business decision.

The inventory follows the following pattern given in Figure 1.4

Total cost per unit cycle=setup cost per unit cycle + holding cost per unit cycle +

shortage cost per unit cycle.

$$\text{So, Total cost per unit item (TCU(y))}=\frac{TD}{Q}+DC+\frac{S^2A}{2Q}+\frac{PM^2}{2Q}$$

where, $M$- inventory level just after a batch of $Q$ units is added to inventory, $P$- shortage cost per unit short per unit of time short, $S$=$Q-M$=shortage in inventory just before a batch of $Q$ units is added, C= purchasing cost.

### 1.1.3 EOQ Model With Price Break

In some of the cases basic EOQ models fails when there is discount offered when purchases are made in large quantities. When a large quantities of items are ordered then certain manufactures

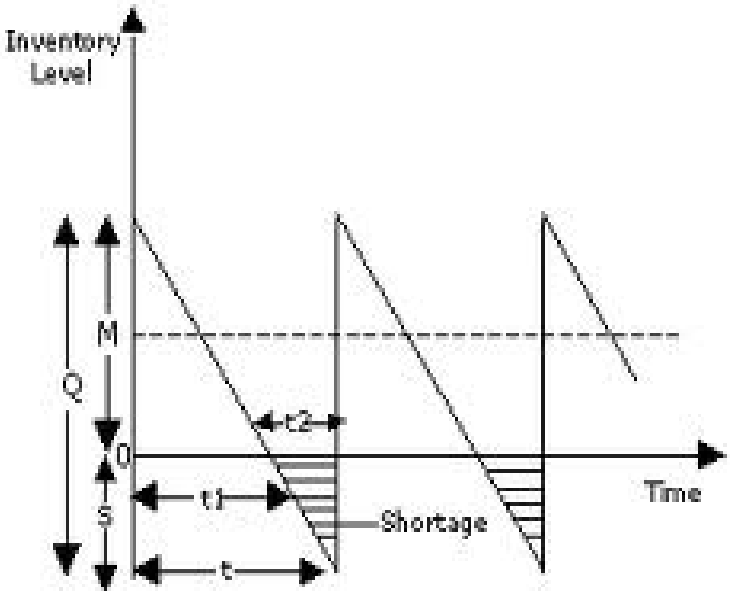

Figure 1.3: EOQ with shortage

offers reduced rate for items. It may appears that the inventory holding cost may increases if large quantities of items are ordered. But if the discount offered is so attractive than it even outweighs the holding cost, the probably the orders at levels other than EOQ would be economical.
The inventory follows the following pattern given in Figure 1.4

The EOQ model when the quantity discounts are available can be formulated as below. The total cost per unit item includes purchasing cost per unit item, holding cost per unit item.

$$\text{Total cost per unit item(TC(Q))=} \begin{cases} TC_0(Q) = DC_0 + \frac{TD}{Q} + \frac{AQ}{2}\ , Q \leq X_1 \\ TC_1(Q) = DC_1 + \frac{TD}{Q} + \frac{AQ}{2}\ , X_1 < Q < X_2 \\ TC_2(Q) = DC_2 + \frac{TD}{Q} + \frac{AQ}{2}\ , Q \geq X_2 \end{cases}$$

where

$$\text{purchasing cost c=} \begin{cases} c_0, \text{ if } Q \leq X_1 \\ \\ c_1, \text{ if } X_1 < Q < X_2 \\ \\ c_2, \text{ if } Q \geq X_2 \end{cases}$$

Where, $X_1$ and $X_2$ are limits.

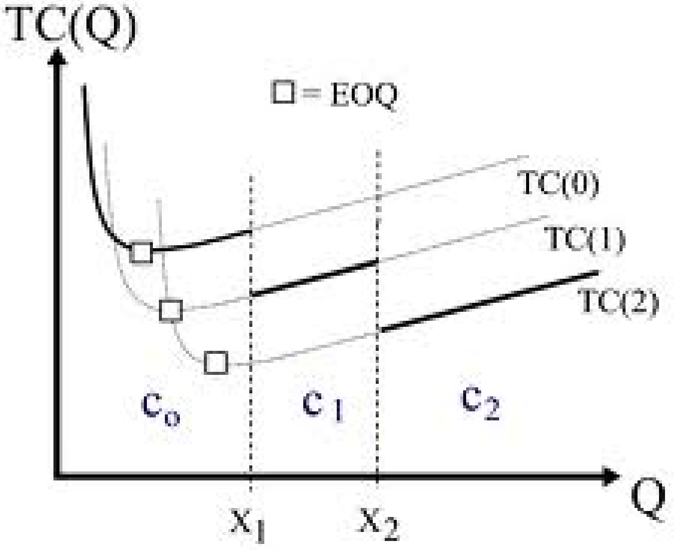

Figure 1.4: EOQ with price breaks

### 1.1.4 Multi Item EOQ Model With Storage Limitation

The inventory model presented in this section is based on same assumption as basic EOQ model in that we are attempting to determine how much we should order and when the order should be placed. This model deals with more than one item the only difference is that the items are competing for a limited storage space. For this model, total cost as a function of the production lot size is expressed in the form of holding cost and set up cost. Then we attempt to find the production lot size that minimize the total cost. This inventory model follow the same pattern in Figure 1.2

The mathematical model representing the inventory situation is given as,

$$Min\ TCU(Q_1, Q_2, Q_3, ......., Q_n) = \sum_{i=1}^{n}(\frac{T_i D_i}{Q_i} + \frac{A_i Q_i}{2})$$

such that

$$\sum_{i=1}^{n} s_i Q_i \leq S$$

$Q_i > 0$, i=1,2,.....,n

where, $D_i$ is demand rate, $T_i$ is setup cost, $A_i$ is holding cost per unit time, $Q_i$ is order quantity, $s_i$ is storage area requirement per inventory unit and $S$ is maximum available storage area for all n items.

In inventory models where there is uncertainties in the data due to fuzziness and this case fuzzy set theory is applicable. Here we briefly discuss the fuzzy definitions and arithmetic operations.

## 1.2 BASIC DEFINITIONS IN FUZZY THEORY

**Definition 1.2.1 [2]**

Let $X$ be a universal set.Then, the set of ordered pairs $\widetilde{A}$=$\{(x, \mu_{\widetilde{A}}(x))$:$x \in X\}$ is called a fuzzy set in $X$ where $\mu_{\widetilde{A}} : X \to [0, 1]$ is called membership function.

**Definition 1.2.2 [2]**

A fuzzy number is an extension of a regular number. It does not refer to a single number but rather to a connected set of possible values where each possible values lies between 0 and 1.

**Definition 1.2.3 [2]**

A fuzzy set $\widetilde{A}$ in $X$ is said to be normal if $\exists x \in X$ such that $\mu_{\widetilde{A}}(x) = 1$

**Definition 1.2.4 [2]**

A fuzzy set $\widetilde{A}$ in $X$ is said to be convex fuzzy set if and only if

$$\mu_{\widetilde{A}}[\lambda x_1 + (1 - \lambda)x_2] \geq \text{minimum}\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}, \forall x_1, x_2 \in X \ and \ \lambda \in [0, 1]$$

**Definition 1.2.5 [2]**

Let $\widetilde{A}$ be a fuzzy set in $X$ and $\lambda \in [0, 1]$ be a real number.Then the set

$$A^{\lambda} = \{x \in X : \mu_{\widetilde{A}} \geq \lambda\}$$

is called $\lambda$ - level set or $\lambda$ - cut of $\widetilde{A}$

**Definition 1.2.6 [2]**

An interval of confidence is a closed and bounded set of real numbers $[a, b]$ =$\{x : a \leq x \leq b \ \forall a, b \in R\}$

**Definition 1.2.7 [2]**

A convex normalized fuzzy set $\widetilde{A}$=$\{(x, \mu_{\widetilde{A}}(x))$:$x \in R\}$ on real line $R$ is called a fuzzy number if and only if $\mu_{\widetilde{A}}(x)$ is piecewise continuous in $R$.

**Definition 1.2.8 [2]**

Let $f : X \to Y$ be a mapping from a set $X$ to a set $Y$. Then extension principle is defined the fuzzy set $\widetilde{B}$ in $Y$ induced by fuzzy set $\widetilde{A}$ in $X$ through $f$ as follows

$$\widetilde{B} = \{(y, \mu_{\widetilde{B}}(x))|y = f(x); x \in X\}$$

$$\mu_{\widetilde{B}}(y) = \mu_{\widetilde{f(A)}}(y) = \begin{cases} sup_{y=f(x)}\mu_{\widetilde{A}}(x); f^{-1}(y) \neq \phi \\ 0 \qquad\qquad\quad ;f^{-1}(y) = \phi \end{cases} \tag{1.1}$$

**Definition 1.2.9 [2]**

A fuzzy number $\widetilde{A}$ defined on the universal set of real numbers $R$ denoted as $\widetilde{A}$=$(a, b, c, d)$ is said to be trapezoidal fuzzy number if its membership function $\mu_{\widetilde{A}}(x)$ is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} (\frac{x-a}{b-a}) \ ; \ a \leq x < b \\ 1 \qquad ;b \leq x \leq c \\ (\frac{x-d}{c-d}); c < x \leq d \\ 0 \qquad \text{;othewise} \end{cases} \tag{1.2}$$

**Definition 1.2.10 [2]**

Let $\widetilde{A}$=$(a, b, c, d)$ be a trapezoidal fuzzy number. Then, its $\lambda - cut \ A^{\lambda}$ is defined as follows:

$$A^{\lambda} = [a + (b - a)\lambda, d - (d - c)\lambda] \ , 0 \leq \lambda \leq 1$$

**Definition 1.2.11[3]**

A ranking function $\Re : F(R) \rightarrow R$, where $F(R)$ set of fuzzy numbers defined on set of real numbers, maps each fuzzy number into a real number, where a natural order exists.

Let $\widetilde{A} = (a_1, b_1, c_1, d_1) \ and \ \widetilde{B} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers.Then,

(i) $\widetilde{A} \prec \widetilde{B}$ if and only if $\Re(\widetilde{A}) < \Re(\widetilde{B})$

(ii) $\widetilde{A} \preceq \widetilde{B}$ if and only if $\Re(\widetilde{A}) \leq \Re(\widetilde{B})$

(iii) $\widetilde{A} \approx \widetilde{B}$ if and only if $\Re(\widetilde{A}) = \Re(\widetilde{B})$

where, $\widetilde{A} = \frac{(a_1+b_1+c_1+d_1)}{4} \ and \ \widetilde{B} = \frac{(a_2+b_2+c_2+d_2)}{4}$

**Definition 1.2.12** [4]

A trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be zero trapezoidal fuzzy number if and only if $\Re(\widetilde{A}) = 0$.

**Definition 1.2.13** [4]

A trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be non-negative trapezoidal fuzzy number if and only if $\Re(\widetilde{A}) \geq 0$.

**Definition 1.2.14** [4]

A trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be positive trapezoidal fuzzy number if and only if $\Re(\widetilde{A}) > 0$.

### 1.2.1 Fuzzy Arithmetic operations

In this section, some arithmetic operations [2] between two trapezoidal fuzzy numbers defined on the universal set of real numbers $R$ are presented.

Let $\widetilde{A} = (a_1, b_1, c_1, d_1) \ and \ \widetilde{B} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers.Then,

(i) $\widetilde{A} \oplus \widetilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$

(ii) $\widetilde{A} \ominus \widetilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$

(iii) $\widetilde{A} \otimes \widetilde{B} \simeq (a, b, c, d)$

where

$a$=Minimum $\{a_1a_2, a_1d_2, a_2d_1, d_1d_2\}$ , $b$=Minimum $\{b_1b_2, b_1c_2, c_1b_2, c_1c_2\}$

$c$=Maximum $\{b_1b_2, b_1c_2, c_1b_2, c_1c_2\}$ , $d$=Maximum $\{a_1a_2, a_1d_2, a_2d_1, d_1d_2\}$

(iv)

$$\gamma A = \begin{cases} (\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1), \gamma \geq 0 \\ (\gamma d_1, \gamma c_1, \gamma b_1, \gamma a_1), \gamma < 0 \end{cases} \tag{1.3}$$

**Remark 1.1**

Let { $\widetilde{a_i}$: i=1,2,....,n } be a set of trapezoidal fuzzy numbers.If $\Re(\widetilde{a_k}) \leq \Re(\widetilde{a_i})$, $\forall$ i then, the fuzzy number $\widetilde{a_k}$ is the Minimum { $\widetilde{a_i}$: i=1,2,....,n } and if $\Re(\widetilde{a_k}) \geq \Re(\widetilde{a_i})$, $\forall$ i then, the fuzzy number $\widetilde{a_k}$ is the Maximum { $\widetilde{a_i}$: i=1,2,....,n } .

## 1.3 LITERATURE REVIEW

Zadeh [5] introduced the basic properties and implications of concept of fuzzy set theory and later this concept has been applied in inventory control systems to model behavior more realistically. Later on Bellman and Zadeh [8] used the fuzzy theory to the decision making problem. Zimmerman [9] introduced the concept to solve multi objective linear programming problem and fuzzy set theory has been entered into the inventory control systems. Park [10] developed the concept to examine the inventory problem with fuzzy inventory cost using arithmetic operations of extension principle under fuzzy environment.

chang [12] presented a membership function of the fuzzy total cost of production inventory model, and used Extension Principle and centroid method to obtain an estimate of total cost, and to obtain the economic production quantity. Chen and Hsieh (1988) defined a Graded Mean Integration Representation of generalized fuzzy numbers.

Hsieh [6] developed how a fuzzy production inventory models with fuzzy parameters for for crisp production quantity and for fuzzy production quantity. They used the concept of Graded Mean Integration Representation method to defuzzifing fuzzy total production inventory cost. Extension of the Lagrangean method has been used for solving inequality constraint problem. It has been concluded that the optimal solution obtained from the model can be specified to meet the classical inventory models and these fuzzy production inventory models are executable and useful for real world. Later on, Optimization of fuzzy inventory models is developed by Hsieh [13] with fuzzy demand and fuzzy lead time. In this paper, lead time and demand is taken as a trapezoidal fuzzy number. To obtained the optimal ordered quantity, function principle and graded mean integration representation method has been used.

chen et al. [11] discussed the backorder fuzzy inventory model under the fuzzy environment of fuzzy demand, fuzzy order cost, fuzzy inventory cost, and fuzzy backorder cost. In this paper to calculate the total fuzzy inventory cost, Function principle has been used and significance of function principle over extension principle has been concluded. They also apply the median rule to find the optimal economic order quantity and shortage quantity.

Dey and Chakraborty [20] presented a fuzzy random continuous review system, with the annual customer demand assumed to be a uniformly distributed continuous fuzzy random variable. In this model, beside the reorder point and the production lot size, the setup cost and the 'out of control' probability for a production process has been assumed to be control parameters. In this paper, a methodology has been proposed to minimize the cost.

In real world, the single period inventory models plays an important role in assisting the decision maker to determine the optimal quantity to order. Due to lack of historical data, the demand has to be subjectively determined in many cases. Kao et al. [19] constructed a single period inventory model for cases of fuzzy demand. In this paper, a method for ranking fuzzy number is used to find the optimal order quantity in terms of the cost. The methodology of this paper can also be applied to construct other inventory model with fuzzy demand. This paper concentrates on possibilistic situations, in that demands are described by subjectively determined membership functions. Also Yager's method for ranking fuzzy numbers has been applied to calculate the quantity with smallest fuzzy cost.

Xiaobin et al. [17] explained the concept of fuzzy economic order quantity inventory models without backordering. In this model, the order cost of each cost and the stock cost of each unit quantity is taken as independent fuzzy variables. Based on an expected value criterion or a credibility criterion a fuzzy expected value model and a fuzzy dependent chance programming models are developed by them. They designed a particle swarm optimization algorithm which is based on fuzzy simulation.

A new method for fuzzy risk analysis is given by chen at el. [18] which is based on ranking generalized fuzzy numbers with different heights and different spreads. Firsly, they presented a new method for ranking generalized fuzzy numbers which considered the heights, the spreads and the defuzzified values. Then, based on the proposed method for ranking generalized fuzzy numbers, they proposed a fuzzy risk analysis algorithm to deal with fuzzy risk analysis problems.

Syed and Aziz [16] developed a fuzzy inventory model with price break in which cost function of both buffer and seller is represented by fuzzy membership function. They obtained the optimum result by reformulating the problem as fuzzy linear programming.

xu et al. [21] developed a fuzzy random multi-objective models about inventory problems. They designed a method of solving solution sets of fuzzy random multi-objective programming problems. These are applied to numerical problems in which all inventory costs, purchasing and selling prices in the objectives and constraints are assumed to be fuzzy random variables in nature, and then the impreciseness of fuzzy random variables in the above objectives and constraints are transformed into fuzzy variables which are similar trapezoidal fuzzy numbers. They claimed that, no inventory model has been formulated in such environments i.e. a fuzzy random atmosphere and very few research papers are available for the solution of fuzzy multi-objective non-linear problems by the non-linear programming method.

Kasthuri et al. [15] presented Multi item fuzzy inventory model with shortage. This model involved two constraints limited storage space and production cost. They used Karush-Khun-Tucker conditions for solving the problem and unit cost demand has been taken in fuzzy environment.

Li et al. [22] considered the single-period inventory problem in the presence of uncertainties. In this paper there are two types of uncertainties on which inventory is defined. One is arising from randomness which can be incorporated through a probability distribution and the other from fuzziness which can be characterized by fuzzy numbers, are considered. They developed two models, in one the demand is probabilistic while the cost components are fuzzy and in the other the costs are deterministic but the demand is fuzzy. In each, the objective is maximization of profit which is fuzzy and optimization is achieved through fuzzy ordering of fuzzy numbers with respect to their total integral values.They showed that the first model reduces to the classical newsboy problem, and therefore an optimal solution is easily available. In second model, they showed that the objective function is concave and hence characterization of the optimal solution has been presented from which one can readily compute an optimal solution.

# Chapter 2

# Fuzzy Production Inventory Model for Crisp and Fuzzy Production Quantity

## 2.1 Introduction

Hsieh [6] considered fuzzy production inventory models with fuzzy parameters for crisp and fuzzy production quantity involves constant rate demand with uniform order replenishment and no shortage is allowed. Graded mean integration representation method for trapezoidal fuzzy number is used for defuzzify the total inventory cost and extension of lagrangean method for solving inequality constraints problem.

In this chapter the models considered by Hsieh [6] have been reviewed in details for instantaneous and uniform replenishment separately and in place of Extension of lagrangean method, KT conditions has been used to solve inequality constraints problem for both uniform and instantaneous replenishment.

## 2.2 The fuzzy production inventory model for crisp production quantity with instantaneous replenishment using trapezoidal fuzzy numbers

In this section, we use the following variables in order to simplify the treatment of the fuzzy production inventory models:

$\widetilde{D}$ fuzzy yearly demand,

$\widetilde{A}$ fuzzy inventory cost (dollars/item year),

$\widetilde{T}$ fuzzy setup cost,

$\widetilde{Q_P}$ fuzzy production quantity,

$Q_P$ crisp production quantity.

The fuzzy production inventory model with fuzzy parameters for crisp production quantity $Q_P$ as follows:

In this model, the fuzzy annual inventory cost $TCU_1$ and fuzzy annual setup cost $TCU_2$ is

$TCU_1 = \widetilde{A} \otimes \frac{Q_P}{2}$ and $TCU_1 = \widetilde{T} \otimes (\widetilde{D} \oslash Q_P)$

Then, the fuzzy total production inventory cost is

$$\widetilde{C_1}=TCU_1 + TCU_2=\widetilde{T} \otimes (\widetilde{D} \oslash Q_P) \oplus \widetilde{A} \otimes \frac{Q_P}{2} \tag{2.1}$$

where $\oslash$, $\otimes$, $\ominus$ and $\oplus$ are the fuzzy arithmetical operations under Function Principle.

let $\widetilde{A}$=$(a_1, a_2, a_3, a_4)$, $\widetilde{D} = (d_1, d_2, d_3, d_4)$ and $\widetilde{T} = (t_1, t_2, t_3, t_4)$ are non-negative trapezoidal fuzzy numbers.

Then, the fuzzy total production inventory cost given in formula (2.1) becomes

$$\widetilde{C_1} = \left[\frac{t_1 d_1}{Q_P} + \frac{a_1 Q_P}{2}, \frac{t_2 d_2}{Q_P} + \frac{a_2 Q_P}{2}, \frac{t_3 d_3}{Q_P} + \frac{a_3 Q_P}{2}, \frac{t_4 d_4}{Q_P} + \frac{a_4 Q_P}{2}\right]$$

The fuzzy total production inventory cost is defuzzified by using graded mean integration technique given by the formula in equation (2.2) [6]

$$\mathrm{P}(\widetilde{B}\ )=\frac{b_1+2b_2+2b_3+b_4}{6}. \tag{2.2}$$

where, $\widetilde{B}=(b_1,b_2,b_3,b_4)$ is trapezoidal fuzzy number.

Then the total fuzzy production cost becomes

$$P(\widetilde{C_1})=\frac{1}{6}\left[\frac{t_1d_1}{Q_P}+\frac{a_1Q_P}{2}+\frac{2t_2d_2}{Q_P}+\frac{2a_2Q_P}{2}+\frac{2t_3d_3}{Q_P}+\frac{2a_3Q_P}{2}+\frac{t_4d_4}{Q_P}+\frac{a_4Q_P}{2}\right].$$

To get optimal production quantity $Q_P^*$

$$\frac{\partial}{\partial Q_P}(P(\widetilde{C_1}))=\frac{1}{6}\left[-\frac{1}{Q_P^2}(t_1d_1+2t_2d_2+2t_3d_3+t_4d_4)+\frac{1}{2}(a_1+2a_2+2a_3+a_4)\right]=0$$

And the optimal production quantity is

$$Q_P^*=\sqrt{\frac{2(t_1d_1+2t_2d_2+2t_3d_3+t_4d_4)}{a_1+2a_2+2a_3+a_4}} \tag{2.3}$$

## 2.3 The fuzzy production inventory model for fuzzy production quantity with instantaneous replenishment using trapezoidal fuzzy numbers

In this section, the fuzzy production inventory models by changing the crisp production quantity in Section 2.2 into fuzzy production quantity.

Suppose fuzzy production quantity $\widetilde{Q_P}$ be a trapezoidal fuzzy number, $\widetilde{Q_P}=(q_{P_1},q_{P_2},q_{P_3},q_{P_4})$ with $0<q_{P_1}\le q_{P_2}\le q_{P_3}\le q_{P_4}$.Then the fuzzy annual inventory cost $TCU_1$ and fuzzy annual setup cost $TCU_2$ is

$TCU_1=\widetilde{A}\otimes\frac{\widetilde{Q_P}}{2}$ and $TCU_2=\widetilde{T}\otimes(\widetilde{D}\oslash\widetilde{Q_P})$ and the total fuzzy production inventory cost is

$$\widetilde{C_2}=\left[\frac{t_1d_1}{q_{P_4}}+\frac{a_1q_{p_1}}{2},\frac{t_2d_2}{q_{P_3}}+\frac{a_2q_{p_2}}{2},\frac{t_3d_3}{q_{P_2}}+\frac{a_3q_{p_3}}{2},\frac{t_4d_4}{q_{P_1}}+\frac{a_4q_{p_4}}{2}\right] \tag{2.4}$$

The fuzzy total production inventory cost can be defuzzified by using the formula (2.2) and is given by

$$P(\widetilde{C_2}) = \frac{1}{6}\left[\frac{t_1 d_1}{q_{P_4}} + \frac{a_1 q_{P_1}}{2} + \frac{2t_2 d_2}{q_{P_3}} + \frac{2a_2 q_{P_2}}{2} + \frac{2t_3 d_3}{q_{P_2}} + \frac{2a_3 q_{P_3}}{2} + \frac{t_4 d_4}{q_{P_1}} + \frac{a_4 q_{P_4}}{2}\right]. \tag{2.5}$$

with constraints $0 < q_{P_1} \leq q_{P_2} \leq q_{P_3} \leq q_{P_4}$

These inequality conditions $0 < q_{P_1} \leq q_{P_2} \leq q_{P_3} \leq q_{P_4}$ can also be converted into the following inequality without changing the meaning of (2.5)

$$q_{P_1} - q_{P_2} \leq 0, \quad q_{P_2} - q_{P_3} \leq 0, \quad q_{P_3} - q_{P_4} \leq 0$$

To find the optimal solution of the problem KT conditions have been used. So, the problem can be written as

$$\text{Minimize } P(\widetilde{C_2}) = \frac{1}{6}\left[\frac{t_1 d_1}{q_{P_4}} + \frac{a_1 q_{P_1}}{2} + \frac{2t_2 d_2}{q_{P_3}} + \frac{2a_2 q_{P_2}}{2} + \frac{2t_3 d_3}{q_{P_2}} + \frac{2a_3 q_{P_3}}{2} + \frac{t_4 d_4}{q_{P_1}} + \frac{a_4 q_{P_4}}{2}\right]. \tag{2.6}$$

$$\begin{aligned} \text{s.t. } & q_{P_1} - q_{P_2} \leq 0 \\ & q_{P_2} - q_{P_3} \leq 0 \\ & q_{P_3} - q_{P_4} \leq 0 \\ & \text{where } q_{P_1} > 0 \end{aligned}$$

Then we have the lagrangean function as

$$L(q_{P_1}, q_{P_2}, q_{P_3}, q_{P_4}, \lambda_1, \lambda_2, \lambda_3) = P(\widetilde{C_2}) + \lambda_1(q_{P_1} - q_{P_2}) + \lambda_2(q_{P_2} - q_{P_3}) + \lambda_3(q_{P_3} - q_{P_4})$$

where, $\lambda_1$, $\lambda_2$, $\lambda_3$ are Lagrangean Multiplier.

Applying KT conditions on Lagrangean Function the following equations have been obtained

$$\frac{\partial}{\partial q_{P_1}}(L) = \frac{1}{6}\left[\frac{a_1}{2} - \frac{t_4 d_4}{q_{P_1}^2}\right] + \lambda_1 \geq 0$$

$$\frac{\partial}{\partial q_{P_2}}(L) = \frac{1}{6}\left[\frac{2a_2}{2} - \frac{2t_3 d_3}{q_{P_2}^2}\right] - \lambda_1 + \lambda_2 \geq 0$$

$$\frac{\partial}{\partial q_{P_3}}(L) = \frac{1}{6}\left[\frac{2a_3}{2} - \frac{2t_2 d_2}{q_{P_3}^2}\right] - \lambda_2 + \lambda_3 \geq 0$$

$$\frac{\partial}{\partial q_{P_4}}(L) = \frac{1}{6}\left[\frac{a_4}{2} - \frac{t_1 d_1}{q_{P_4}^2}\right] - \lambda_3 \geq 0$$

$$q_{P_1}\left[\frac{1}{6}\left(\frac{a_1}{2} - \frac{t_4 d_4}{q_{P_1}^2}\right) + \lambda_1\right] = 0,$$

$$q_{P_2}\left[\frac{1}{6}\left(\frac{2a_2}{2} - \frac{2t_3 d_3}{q_{P_2}^2}\right) - \lambda_1 + \lambda_2\right] = 0,$$

$$q_{P_3}[\frac{1}{6}\left(\frac{2a_3}{2} - \frac{2t_2 d_2}{q_{P_3}^2}\right) - \lambda_2 + \lambda_3\Bigg] = 0,$$

$$q_{P_4}\left[\frac{1}{6}\left(\frac{a_4}{2} - \frac{t_1 d_1}{q_{P_4}^2}\right) - \lambda_3\right] = 0$$

$$\frac{\partial}{\partial \lambda_1}(L) = q_{p_1} - q_{p_2} \leq 0$$

$$\frac{\partial}{\partial \lambda_2}(L) = q_{p_2} - q_{p_3} \leq 0$$

$$\frac{\partial}{\partial \lambda_3}(L) = q_{p_3} - q_{p_4} \leq 0$$

$$\lambda_1(q_{p_1} - q_{p_2}) = 0$$

$$\lambda_2(q_{p_2} - q_{p_3}) = 0$$

$$\lambda_3(q_{p_3} - q_{p_4}) = 0$$

Solving the above equations we get,

$$q_{P_1} = q_{P_2} = q_{P_3} = q_{P_4} = \sqrt{\frac{2(t_1 d_1 + 2t_2 d_2 + 2t_3 d_3 + t_4 d_4)}{a_1 + 2a_2 + 2a_3 + a_4}}.$$

$$\lambda_1 = \frac{1}{6}\left[\frac{t_4 d_4}{q_{P_1}^2} - \frac{a_1}{2}\right]$$

$$\lambda_2 = \frac{1}{6}\left[\frac{t_4 d_4}{q_{P_1}^2} - \frac{a_1}{2} + \frac{2t_3 d_3}{q_{P_2}^2} - \frac{2a_2}{2}\right]$$

$$\lambda_3 = \frac{1}{6}\left[\frac{a_4}{2} - \frac{t_1 d_1}{q_{P_1}^2}\right]$$

Since the equation (2.6) is a convex nonlinear programming so the local optimum will be a global optimum which satisfies all conditions.

Let $q_{P_1} = q_{P_2} = q_{P_3} = q_{P_4} = q_P$. Then the optimal fuzzy production quantity is $\widetilde{Q_P^*} = (q_P^*, q_P^*, q_P^*, q_P^*)$, where

$$q_P^* = \sqrt{\frac{2(t_1d_1 + 2t_2d_2 + 2t_3d_3 + t_4d_4)}{a_1 + 2a_2 + 2a_3 + a_4}}. \tag{2.7}$$

## 2.4 Numerical Example

Let the fuzzy parameters are trapezoidal fuzzy number

Fuzzy demand=$\widetilde{D}$=(9000, 9500, 10500, 11000)

Fuzzy setupcost=$\widetilde{T}$=(95, 100, 100, 105)

Fuzzy holding cost=$\widetilde{A}$=(0.475, 0.5, 0.5, 0.525)

Now using formula (2.3) and (2.7), the crisp and fuzzy production quantity

$Q_P^*$= $\widetilde{Q_P^*}$=(1868.135, 1868.135, 1868.135, 1868.135 ).

## 2.5 The fuzzy basic production inventory model for crisp production quantity with uniform replenishment using trapezoidal fuzzy numbers

In this section we use the following variables in order to simplify the treatment of the fuzzy production inventory models:

| | |
|---|---|
| $\widetilde{D}$ | fuzzy yearly demand, |
| $\widetilde{A}$ | fuzzy inventory cost (dollars/item year), |
| $\widetilde{T}$ | fuzzy setup cost, |
| $\widetilde{Q_P}$ | fuzzy production quantity, |
| $Q_P$ | crisp production quantity. |

$\widetilde{P}$ fuzzy daily production rate,

$\widetilde{R}$ fuzzy daily demand rate,

The fuzzy production inventory model with fuzzy parameters for crisp production quantity $Q_P$ as follows:

In this model, the fuzzy annual inventory cost $TCU_1$ and fuzzy annual setup cost $TCU_2$ is

$TCU_1 = \widetilde{A} \otimes \frac{Q_P}{2} \otimes [1 - (\widetilde{R} \oslash \widetilde{P})]$ and $TCU_2 = \widetilde{T} \otimes (\widetilde{D} \oslash Q_P)$

Then, the fuzzy total production inventory cost is

$$\widetilde{C_1} = TCU_1 + TCU_2 = \widetilde{T} \otimes (\widetilde{D} \oslash Q_P) \oplus \widetilde{A} \otimes \frac{Q_P}{2} \otimes \left[1 - (\widetilde{R} \oslash \widetilde{P})\right] \tag{2.8}$$

where $\oslash$, $\otimes$, $\ominus$ and $\oplus$ are the fuzzy arithmetical operations under Function Principle.

let $\widetilde{A}=(a_1, a_2, a_3, a_4)$, $\widetilde{R}=(r_1, r_2, r_3, r_4)$, $\widetilde{P}=(p_1, p_2, p_3, p_4)$, $\widetilde{D} = (d_1, d_2, d_3, d_4)$ and $\widetilde{T} = (t_1, t_2, t_3, t_4)$ are non-negative trapezoidal fuzzy numbers.

Then, the fuzzy total production inventory cost by given in formula (2.8) becomes

$$\widetilde{C_1} = \left[\frac{t_1 d_1}{Q_P} + \frac{a_1 Q_P}{2}\left(1 - \frac{r_4}{p_1}\right), \frac{t_2 d_2}{Q_P} + \frac{a_2 Q_P}{2}\left(1 - \frac{r_3}{p_2}\right), \frac{t_3 d_3}{Q_P} + \frac{a_3 Q_P}{2}\left(1 - \frac{r_2}{p_3}\right), \frac{t_4 d_4}{Q_P} + \frac{a_4 Q_P}{2}\left(1 - \frac{r_1}{p_4}\right)\right]$$

Then, the fuzzy total production inventory cost is defuzzified by using formula (2.2). The result is

$$\begin{aligned} P(\widetilde{C_1}) = \frac{1}{6}\Bigg[&\frac{t_1 d_1}{Q_P} + \frac{a_1 Q_P}{2}\left(1 - \frac{r_4}{p_1}\right) + \frac{2t_2 d_2}{Q_P} + \frac{2a_2 Q_P}{2}\left(1 - \frac{r_3}{p_2}\right) + \\ &\frac{2t_3 d_3}{Q_P} + \frac{2a_3 Q_P}{2}\left(1 - \frac{r_2}{p_3}\right) + \frac{t_4 d_4}{Q_P} + \frac{a_4 Q_P}{2}\left(1 - \frac{r_1}{p_4}\right)\Bigg]. \end{aligned}$$

To get optimal production quantity $Q_P^*$

$$\begin{aligned} \frac{\partial}{\partial Q_P}(P(\widetilde{C_1})) = \frac{1}{6}\left[-\frac{1}{Q_P^2}(t_1 d_1 + 2t_2 d_2 + 2t_3 d_3 + t_4 d_4)\right] + \frac{1}{2}\Bigg[&a_1\left(1 - \frac{r_4}{p_1}\right) + \\ &2a_2\left(1 - \frac{r_3}{p_2}\right) + 2a_3\left(1 - \frac{r_2}{p_3}\right) + a_4\left(1 - \frac{r_1}{p_4}\right)\Bigg] = 0 \end{aligned}$$

And the optimal production quantity is

$$Q_P^* = \sqrt{\frac{2(t_1d_1 + 2t_2d_2 + 2t_3d_3 + t_4d_4)}{a_1(1-\frac{r_4}{p_1}) + 2a_2(1-\frac{r_3}{p_2}) + 2a_3(1-\frac{r_2}{p_3}) + a_4(1-\frac{r_1}{p_4})}} \tag{2.9}$$

## 2.6 The fuzzy production inventory model for fuzzy production quantity with uniform replenishment using trapezoidal fuzzy numbers

In this section, the fuzzy production inventory models by changing the crisp production quantity in Section 2.2 into fuzzy production quantity.

Suppose fuzzy production quantity $\widetilde{Q_P}$ be a trapezoidal fuzzy number, $\widetilde{Q_P} = (q_{P_1}, q_{P_2}, q_{P_3}, q_{P_4})$ with $0 < q_{P_1} \leq q_{P_2} \leq q_{P_3} \leq q_{P_4}$. Then the fuzzy annual inventory cost $TCU_1$ and fuzzy annual setup cost $TCU_2$ is

$TCU_1 = \widetilde{A} \otimes \frac{\widetilde{Q_P}}{2} \otimes \left[1 - \left(\widetilde{R} \oslash \widetilde{P}\right)\right]$ and $TCU_2 = \widetilde{T} \otimes (\widetilde{D} \oslash \widetilde{Q_P})$

Then, we get the fuzzy total production inventory cost

$$\begin{aligned}\widetilde{C_2} = \Bigg[&\frac{t_1d_1}{q_{P_4}} + \frac{a_1q_{p_1}}{2}\left(1 - \frac{r_4}{p_1}\right), \frac{t_2d_2}{q_{P_3}} + \frac{a_2q_{p_2}}{2}\left(1 - \frac{r_3}{p_2}\right),\\ &\frac{t_3d_3}{q_{P_2}} + \frac{a_3q_{p_3}}{2}\left(1 - \frac{r_2}{p_3}\right), \frac{t_4d_4}{q_{P_1}} + \frac{a_4q_{p_4}}{2}\left(1 - \frac{r_1}{p_4}\right)\Bigg]\end{aligned} \tag{2.10}$$

The fuzzy total production inventory cost can be defuzzified by using equation (2.2) and is given by

$$\begin{aligned}P(\widetilde{C_2}) = \frac{1}{6}\Bigg[&\frac{t_1d_1}{q_{P_4}} + \frac{a_1q_{P_1}}{2}\left(1 - \frac{r_4}{p_1}\right) + \frac{2t_2d_2}{q_{P_3}} + \frac{2a_2q_{P_2}}{2}\left(1 - \frac{r_3}{p_2}\right) +\\ &\frac{2t_3d_3}{q_{P_2}} + \frac{2a_3q_{P_3}}{2}\left(1 - \frac{r_2}{p_3}\right) + \frac{t_4d_4}{q_{P_1}} + \frac{a_4q_{P_4}}{2}\left(1 - \frac{r_1}{p_4}\right)\Bigg].\end{aligned} \tag{2.11}$$

with constraints $0 < q_{P_1} \leq q_{P_2} \leq q_{P_3} \leq q_{P_4}$

These inequality conditions $0 < q_{P_1} \leq q_{P_2} \leq q_{P_3} \leq q_{P_4}$ can also be converted into the following inequality without changing the meaning of (2.5)

$$q_{P_1} - q_{P_2} \leq 0, \quad q_{P_2} - q_{P_3} \leq 0, \quad q_{P_3} - q_{P_4} \leq 0$$

To find the optimal solution of the problem KT conditions have been used. So, the problem can be written as

$$Minimize P(\widetilde{C_2}) = \frac{1}{6}\Bigg[\frac{t_1d_1}{q_{P_4}} + \frac{a_1q_{P_1}}{2}\Bigg(1-\frac{r_4}{p_1}\Bigg)+\frac{2t_2d_2}{q_{P_3}} + \frac{2a_2q_{P_2}}{2}\Bigg(1-\frac{r_3}{p_2}\Bigg)+\frac{2t_3d_3}{q_{P_2}} \tag{2.12}$$

$$+\frac{2a_3q_{P_3}}{2}\Bigg(1-\frac{r_2}{p_3}\Bigg)+\frac{t_4d_4}{q_{P_1}} + \frac{a_4q_{P_4}}{2}\Bigg(1-\frac{r_1}{p_4}\Bigg)\Bigg]. \tag{2.13}$$

$$\text{s.t. } q_{P_1} - q_{P_2} \leq 0$$

$$q_{P_2} - q_{P_3} \leq 0$$

$$q_{P_3} - q_{P_4} \leq 0$$

$$\text{with } q_{P_1} > 0$$

Then we have the lagrangean function as

$$L(q_{P_1}, q_{P_2}, q_{P_3}, q_{P_4}, \lambda_1, \lambda_2, \lambda_3) = P(\widetilde{C_2}) + \lambda_1(q_{P_1} - q_{P_2}) + \lambda_2(q_{P_2} - q_{P_3}) + \lambda_3(q_{P_3} - q_{P_4})$$

Where, $\lambda_1$, $\lambda_2$, $\lambda_3$ are Lagrangean Multiplier.

Applying KT conditions on Lagrangean Function we get the following equations

$$\frac{\partial}{\partial q_{P_1}}(L) = \frac{1}{6}\Bigg[\frac{a_1}{2}\Bigg(1-\frac{r_4}{p_1}\Bigg)-\frac{t_4d_4}{q_{P_1}^2}\Bigg]+\lambda_1 \geq 0$$

$$\frac{\partial}{\partial q_{P_2}}(L) = \frac{1}{6}\Bigg[\frac{2a_2}{2}\Bigg(1-\frac{r_3}{p_2}\Bigg)-\frac{2t_3d_3}{q_{P_2}^2}\Bigg]-\lambda_1 + \lambda_2 \geq 0$$

$$\frac{\partial}{\partial q_{P_3}}(L) = \frac{1}{6}\Bigg[\frac{2a_3}{2}\Bigg(1-\frac{r_2}{p_3}\Bigg)-\frac{2t_2d_2}{q_{P_3}^2}\Bigg]-\lambda_2 + \lambda_3 \geq 0$$

$$\frac{\partial}{\partial q_{P_4}}(L) = \frac{1}{6}\Bigg[\frac{a_4}{2}\Bigg(1-\frac{r_1}{p_4}\Bigg)-\frac{t_1d_1}{q_{P_4}^2}\Bigg]-\lambda_3 \geq 0$$

$$q_{p_1}\Bigg[\frac{1}{6}\Bigg(\frac{a_1}{2}\Bigg(1-\frac{r_4}{p_1}\Bigg)-\frac{t_4d_4}{q_{P_1}^2}\Bigg)+\lambda_1\Bigg] = 0,$$

$$q_{p_2}\left[\frac{1}{6}\left(\frac{2a_2}{2}\left(1-\frac{r_3}{p_2}\right)-\frac{2t_3d_3}{q_{P_2}^2}\right)-\lambda_1+\lambda_2\right]=0,$$

$$q_{p_3}\left[\frac{1}{6}\left(\frac{2a_3}{2}\left(1-\frac{r_2}{p_3}\right)-\frac{2t_2d_2}{q_{P_3}^2}\right)-\lambda_2+\lambda_3\right]=0,$$

$$q_{p_4}\left[\frac{1}{6}\left(\frac{a_4}{2}\left(1-\frac{r_1}{p_4}\right)-\frac{t_1d_1}{q_{P_4}^2}\right)-\lambda_3\right]=0$$

$$\frac{\partial}{\partial\lambda_1}(L)=q_{p_1}-q_{p_2}\le 0$$

$$\frac{\partial}{\partial\lambda_2}(L)=q_{p_2}-q_{p_3}\le 0$$

$$\frac{\partial}{\partial\lambda_3}(L)=q_{p_3}-q_{p_4}\le 0$$

$$\lambda_1(q_{p_1}-q_{p_2})=0$$

$$\lambda_2(q_{p_2}-q_{p_3})=0$$

$$\lambda_3(q_{p_3}-q_{p_4})=0$$

Solving the above equations we get,

$$q_{P_1}=q_{P_2}=q_{P_3}=q_{P_4}=\sqrt{\frac{2(t_1d_1+2t_2d_2+2t_3d_3+t_4d_4)}{a_1(1-\frac{r_4}{p_1})+2a_2(1-\frac{r_3}{p_2})+2a_3(1-\frac{r_2}{p_3})+a_4(1-\frac{r_1}{p_4})}}.$$

$$\lambda_1=\frac{1}{6}\left[\frac{t_4d_4}{q_{P_1}^2}-\frac{a_1}{2}\left(1-\frac{r_4}{p_1}\right)\right] \tag{2.14}$$

$$\lambda_2=\frac{1}{6}\left[\frac{t_4d_4}{q_{P_1}^2}-\frac{a_1}{2}\left(1-\frac{r_4}{p_1}\right)+\frac{2t_3d_3}{q_{P_2}^2}-\frac{2a_2}{2}\left(1-\frac{r_3}{p_2}\right)\right] \tag{2.15}$$

$$\lambda_3=\frac{1}{6}\left[\frac{a_4}{2}\left(1-\frac{r_1}{p_4}\right)-\frac{t_1d_1}{q_{P_1}^2}\right] \tag{2.16}$$

Since the equation (2.12) is a convex nonlinear programming so the local optimum will be a global optimum which satisfies all the conditions.

Let $q_{P_1}=q_{P_2}=q_{P_3}=q_{P_4}=q_P$. Then the optimal fuzzy production quantity is $\widetilde{Q_P^*}=(q_P^*,q_P^*,q_P^*,q_P^*)$, where

$$q_P^* = \sqrt{\frac{2(t_1d_1 + 2t_2d_2 + 2t_3d_3 + t_4d_4)}{a_1(1 - \frac{r_4}{p_1}) + 2a_2(1 - \frac{r_3}{p_2}) + 2a_3(1 - \frac{r_2}{p_3}) + a_4(1 - \frac{r_1}{p_4})}}.$$

## 2.7 Numerical Example

Let the fuzzy parameters are trapezoidal fuzzy.

Fuzzy demand=$\widetilde{D}$=(9000, 9500, 10500, 11000)

Fuzzy setupcost=$\widetilde{T}$=(95, 100, 100, 105)

Fuzzy holding cost=$\widetilde{A}$=(0.475, 0.5, 0.5, 0.525)

Fuzzy daily demand=$\widetilde{R}$=(57, 60, 60, 63)

Fuzzy daily production=$\widetilde{P}$=(72, 76, 84, 88)

Now using formula (2.9) and (2.6), the crisp and fuzzy production quantity

$Q_P^*$=$\widetilde{Q_P^*}$=(4028.77, 4028.77, 4028.77, 4028.77).

## 2.8 Conclusion

In this chapter the models considered by Hsieh [6] have been reviewed in details for instantaneous and uniform replenishment and in place of Extension of Lagrangean method, KT conditions have been used and same solution has been obtained. And the inequality constraints problem for both uniform and instantaneous replenishment has been solved in single step. So, it is concluded that it is easier to apply the KT conditions in place of Extension of Lagrangean method.

# Chapter 3

# Fuzzy Production Inventory Model With Shortage

## 3.1 Introduction

Due to the fact that the inventory models considered in literature that the each cost assumed with certainty and there is no ambiguity in them. In spite of this fact the uncertainty of shortage cost is widely admitted. Keeping this in view the fuzzy shortage cost has been introduced in fuzzy production inventory model with fuzzy parameters for crisp production quantity involve constant rate demand with instantaneous replenishment and trapezoidal fuzzy number has been used for fuzzy parameters and solution is obtained.

## 3.2 The fuzzy production inventory model for crisp production quantity with instantaneous replenishment with shortage using trapezoidal fuzzy numbers

In this section we use the following variables in order to simplify the treatment of the fuzzy production inventory models:

$\widetilde{D}$ fuzzy yearly demand,

$\widetilde{A}$ fuzzy inventory cost (dollars/item year),

$\widetilde{T}$ fuzzy setup cost,

$Q_P$ crisp production quantity.

$S$ crisp inventory level just after Q unit is added to the inventory,

$\widetilde{p}$ fuzzy shortage cost per unit short per unit of time short ,

$\widetilde{C}$ fuzzy purchasing cost.

In this model, the fuzzy total production inventory cost is

$$\begin{aligned} C_1 &= \widetilde{T} \otimes (\widetilde{D} \oslash Q_P) \oplus \widetilde{D} \otimes \widetilde{C} \\ &\oplus \frac{S^2 \otimes \widetilde{A}}{2Q_P} \oplus \frac{(Q_P - S)^2 \widetilde{p}}{2Q_P} \end{aligned} \tag{3.1}$$

where $\oslash$, $\otimes$, $\ominus$ and $\oplus$ are the fuzzy arithmetical operations under Function Principle.

Let $\widetilde{A}$=$(a_1, a_2, a_3, a_4)$, $\widetilde{D} = (d_1, d_2, d_3, d_4)$, $\widetilde{T} = (t_1, t_2, t_3, t_4)$, $\widetilde{p} = (p_1, p_2, p_3, p_4)$ and $\widetilde{C} = (c_1, c_2, c_3, c_4)$, are non-negative trapezoidal fuzzy numbers.

So, the total fuzzy production inventory cost by using formula (3.1) is

$$\begin{aligned} \widetilde{C_1} = (t_1, t_2, t_3, t_4) \otimes \left(\frac{d_1}{Q_P}, \frac{d_2}{Q_P}, \frac{d_3}{Q_P}, \frac{d_4}{Q_P}\right) \oplus (d_1c_1, d_2c_2, d_3c_3, d_4c_4) \oplus \left(\frac{S^2a_1}{2Q_P}, \frac{S^2a_2}{2Q_P}, \frac{S^2a_3}{2Q_P}, \frac{S^2a_4}{2Q_P}\right) \\ \oplus (\frac{(Q_P - S)^2 p_1}{2Q_P}, \frac{(Q_P - S)^2 p_2}{2Q_P}, \frac{(Q_P - S)^2 p_3}{2Q_P}, \frac{(Q_P - S)^2 p_4}{2Q_P}) \end{aligned}$$

$$\begin{aligned} \Rightarrow \widetilde{C_1} = \Big[ &\frac{t_1d_1}{Q_P} + d_1c_1 + \frac{S^2a_1}{2Q_P} + \frac{(Q_P - S)^2 p_1}{2Q_P}, \frac{t_2d_2}{Q_P} + d_2c_2 + \frac{S^2a_2}{2Q_P} + \frac{(Q_P - S)^2 p_2}{2Q_P}, \\ &\frac{t_3d_3}{Q_P} + d_3c_3 + \frac{S^2a_3}{2Q_P} + \frac{(Q_P - S)^2 p_3}{2Q_P}, \frac{t_4d_4}{Q_P} + d_4c_4 + \frac{S^2a_4}{2Q_P} + \frac{(Q_P - S)^2 p_4}{2Q_P} \Big] \end{aligned}$$

To defuzzify the fuzzy total production inventory cost the formula (2.2) is used and the cost becomes

$$\begin{aligned} P(\widetilde{C_1}) = \frac{1}{6} \Big[ &\frac{t_1d_1}{Q_P} + d_1c_1 + \frac{S^2a_1}{2Q_P} + \frac{(Q_P - S)^2 p_1}{2Q_P} + \frac{2t_2d_2}{Q_P} + 2d_2c_2 + \frac{2S^2a_2}{2Q_P} + \frac{2(Q_P - S)^2 p_2}{2Q_P} + \\ &\frac{2t_3d_3}{Q_P} + 2d_3c_3 + \frac{2S^2a_3}{2Q_P} + \frac{2(Q_P - S)^2 p_3}{2Q_P} + \frac{t_4d_4}{Q_P} + d_4c_4 + \frac{S^2a_4}{2Q_P} + \frac{(Q_P - S)^2 p_4}{2Q_P} \Big] \end{aligned}$$

To get the optimal production quantity $Q_P^*$ and $S^*$, $P(\widetilde{C_1})$ is to be minimized. To find the minimization of $P(\widetilde{C_1})$, the derivative of $P(\widetilde{C_1})$ with respect to $Q_P$ and $S$ are zero.

$$\frac{\partial}{\partial Q_P}(P(\widetilde{C_1})) = \frac{1}{6}\Bigg[-\frac{1}{Q_P^2}(t_1d_1 + 2t_2d_2 + 2t_3d_3 + t_4d_4) -$$

$$\frac{S^2}{2Q_P^2}(a_1 + 2a_2 + 2a_3 + a_4) + \left(\frac{p_1}{2} + \frac{2p_2}{2} + \frac{2p_3}{2} + \frac{p_4}{2}\right)\left(\frac{Q_P^2 - S^2}{Q_P^2}\right)\Bigg] = 0$$

$$\frac{\partial}{\partial S}(P(\widetilde{C_1})) = \frac{S}{Q_P}(a_1 + 2a_2 + 2a_3 + a_4) + \left(\frac{S - Q_P}{Q_P}\right)(p_1 + 2p_2 + 2p_3 + p_4) = 0$$

Then, solving both equations we find the optimal production quantity $Q_P^*$ and $S^*$ as

$$Q_P^* = \sqrt{\frac{2(t_1d_1 + 2t_2d_2 + 2t_3d_3 + t_4d_4)(p_1 + 2p_2 + 2p_3 + p_4 + a_1 + 2a_2 + 2a_3 + a4)}{(p_1 + 2p_2 + 2p_3 + p_4)(a_1 + 2a_2 + 2a_3 + a_4)}} \tag{3.2}$$

$$S^* = \sqrt{\frac{2(t_1d_1 + 2t_2d_2 + 2t_3d_3 + t_4d_4)(p_1 + 2p_2 + 2p_3 + p_4)}{(p_1 + 2p_2 + 2p_3 + p_4 + a_1 + 2a_2 + 2a_3 + a_4)(a_1 + 2a_2 + 2a_3 + a_4)}} \tag{3.3}$$

## 3.3 Numerical Example

Let the fuzzy parameters are in terms of trapezoidal fuzzy number and are as follows.

fuzzy yearly demand, $\widetilde{D}$=(80, 90, 100, 140)

fuzzy inventory cost (dollars/item year), $\widetilde{A}$=(0.30, 0.40, 0.40, 0.50)

fuzzy setup cost, $\widetilde{T}$=(900, 1000, 1000, 1100)

fuzzy shortage cost per unit short per unit of time short, $\widetilde{p}$=(0.8, 1, 1, 1.2)

Now, using equations (3.2) and (3.3) and using above values we get, $Q_P^* = 840.832$ and $S^* = 600.595$.

If we convert all the trapezoidal fuzzy numbers into crisp numbers using formula (2.2) we get $D$=100, $A$=0.40, $T$=1000 and $p$=1.

Then, in classical inventory model

$$Q_P^* = \sqrt{\frac{2TD}{A}}\sqrt{\frac{p+A}{p}} = 836.66 \text{ and } S^* = \sqrt{\frac{2TD}{A}}\sqrt{\frac{p}{p+A}} = 597.614$$

which is similar to the fuzzy optimal production inventory obtained above with trapezoidal fuzzy parameters.

## 3.4 Conclusion

In this chapter, fuzzy shortage cost is allowed is in fuzzy inventory model for crisp production quantity used in chapter 2 and the new model has been developed. It is concluded that in fuzzy environment we can also discuss the production inventory problem when shortage is allowed.

Furthermore, when all fuzzy parameters are the crisp real numbers, that is $\widetilde{D} = (d_1, d_2, d_3, d_4) = (D, D, D, D)$, $\widetilde{p} = (p_1, p_2, p_3, p_4) = (p, p, p, p)$, $\widetilde{A} = (a_1, a_2, a_3, a_4) = (A, A, A, A)$ and $\widetilde{T} = (t_1, t_2, t_3, t_4) = (T, T, T, T)$.Then, $Q_P^*$ and $S^*$ becomes
$Q_P^* = \sqrt{\frac{2TD}{A}}\sqrt{\frac{p+A}{p}}$ and $S^* = \sqrt{\frac{2TD}{A}}\sqrt{\frac{p}{p+A}}$ which is similar to the result in classical inventory problem and the similar results are obtained for numerical example (3.3) for the classical inventory model.

# Chapter 4

# Fuzzy Production Inventory Model with Price Breaks

## 4.1 Introduction

Price Break models are used where the price of inventory varies with the order size. In these models the economic order quantity is calculated for each possible price and compared to the amount of inventory that will be available at that price. Where the amount of inventory desired is available at that price this option can be considered by the organization.

In this chapter, the fuzzy production inventory models with fuzzy parameters for fuzzy production quantity with price breaks involving uniform and instantaneous replenishment is considered and the same technique which was used in chapter two and three has been used to find the solution of the problem.

## 4.2 The fuzzy production inventory model with price breaks with instantaneous replenishment using trapezoidal fuzzy numbers

we use the following variables in order to simplify the treatment of the fuzzy production inventory models:

$\widetilde{D}$ fuzzy yearly demand,

$\widetilde{A}$ fuzzy inventory cost (dollars/item year),

$\widetilde{T}$ fuzzy setup cost,

$\widetilde{Q_P}$ fuzzy production quantity,

$\widetilde{C}$ purchasing cost.

$$\widetilde{C} = \begin{cases} \widetilde{C_a} \; if \; \widetilde{Q_P} \leq \widetilde{q} \\ \widetilde{C_b} \; if \; \widetilde{Q_P} > \widetilde{q} \end{cases} \tag{4.1}$$

For this model, the total cost per unit time with fuzzy parameters is

$$TCU(\widetilde{Q_P}) = \begin{cases} TCU_1(\widetilde{Q_P}) = \widetilde{D} \otimes \widetilde{C_a} \oplus \widetilde{T} \otimes \left( \widetilde{D} \oslash \widetilde{Q_P} \right) \oplus \widetilde{A} \otimes \frac{\widetilde{Q_P}}{2}; \; \widetilde{Q_P} \leq \widetilde{q} \\ TCU_2(\widetilde{Q_P}) = \widetilde{D} \otimes \widetilde{C_b} \oplus \widetilde{T} \otimes \left( \widetilde{D} \oslash \widetilde{Q_P} \right) \oplus \widetilde{A} \otimes \frac{\widetilde{Q_P}}{2}; \; \widetilde{Q_P} > \widetilde{q} \end{cases} \tag{4.2}$$

The steps for determining $\widetilde{Q_P}^*$ are

**Step 1:** we first Determine $\widetilde{Q_m}$ by using formula (2.7) obtained in chapter 1. If $\widetilde{q}$ is in zone I that is $(0, \widetilde{Q_m})$ then $\widetilde{Q_P}^* = \widetilde{Q_m}$. Otherwise go to step 2.

**Step 2:** Now, we determine $\widetilde{Q}$ from the equation

$TCU_2(\widetilde{Q}) = TCU_1(\widetilde{Q_m})$.

using formula (4.2) in above equation we get,

$$\widetilde{D} \otimes \widetilde{C_b} \; \oplus \; \widetilde{T} \otimes \left( \widetilde{D} \oslash \widetilde{Q} \right) \oplus \widetilde{A} \otimes \frac{\widetilde{Q}}{2} = \widetilde{D} \otimes \widetilde{C_a} \; \oplus \; \widetilde{T} \otimes \left( \widetilde{D} \oslash \widetilde{Q_m} \right) \oplus \widetilde{A} \otimes \frac{\widetilde{Q_m}}{2}$$

Here, we suppose $\widetilde{A} = (a_1, a_2, a_3, a_4)$, $\widetilde{D} = (d_1, d_2, d_3, d_4)$, $\widetilde{T} = (t_1, t_2, t_3, t_4)$, $\widetilde{C_a} = (c_{a_1}, c_{a_2}, c_{a_3}, c_{a_4})$ and $\widetilde{C_b} = (c_{b_1}, c_{b_2}, c_{b_3}, c_{b_4})$ are non-negative trapezoidal fuzzy numbers. Then using these in above equation and comparing on both side we get,

$$d_1 c_{b_1} + \frac{t_1 d_1}{Q_4} + \frac{a_1 Q_1}{2} = d_1 c_{a_1} + \frac{t_1 d_1}{Q_{m_4}} + \frac{a_1 Q_{m_1}}{2} \tag{4.3}$$

$$d_2 c_{b_2} + \frac{t_2 d_2}{Q_3} + \frac{a_2 Q_2}{2} = d_2 c_{a_2} + \frac{t_2 d_2}{Q_{m_3}} + \frac{a_2 Q_{m_2}}{2} \tag{4.4}$$

$$d_3c_{b_3} + \frac{t_3d_3}{Q_2} + \frac{a_3Q_3}{2} = d_3c_{a_3} + \frac{t_3d_3}{Q_{m_2}} + \frac{a_3Q_{m_3}}{2} \tag{4.5}$$

$$d_4c_{b_4} + \frac{t_4d_4}{Q_1} + \frac{a_4Q_4}{2} = d_4c_{a_4} + \frac{t_4d_4}{Q_{m_1}} + \frac{a_4Q_{m_4}}{2} \tag{4.6}$$

Further solving equations (4.3), (4.4), (4.5) and (4.6) we get,

$$\Rightarrow \; d_1(c_{a_1} - c_{b_1}) + t_1d_1\left(\frac{1}{Q_{m_4}} - \frac{1}{Q_4}\right) + \frac{a_1}{2}(Q_{m_1} - Q_1) = 0 \tag{4.7}$$

$$\Rightarrow \; d_2(c_{a_2} - c_{b_2}) + t_2d_2\left(\frac{1}{Q_{m_3}} - \frac{1}{Q_3}\right) + \frac{a_2}{2}(Q_{m_2} - Q_2) = 0 \tag{4.8}$$

$$\Rightarrow \; d_3(c_{a_3} - c_{b_3}) + t_3d_3\left(\frac{1}{Q_{m_2}} - \frac{1}{Q_2}\right) + \frac{a_3}{2}(Q_{m_3} - Q_3) = 0 \tag{4.9}$$

$$\Rightarrow \; d_4(c_{a_4} - c_{b_4}) + t_4d_4\left(\frac{1}{Q_{m_1}} - \frac{1}{Q_1}\right) + \frac{a_4}{2}(Q_{m_4} - Q_4) = 0 \tag{4.10}$$

Using equations (4.7), (4.8), (4.9) and (4.10) and solving we can determine the value of $\widetilde{Q} = (Q_1, Q_2, Q_3, Q_4)$ from which zones II can be defined that is $(\widetilde{Q_m}, \widetilde{Q})$ and III that is $(\widetilde{Q}, \infty)$. If $\widetilde{q}$ is in zone II, $\widetilde{Q_P}^* = \widetilde{q}$. Otherwise, $\widetilde{q}$ is in zone III, and $\widetilde{Q_P}^* = \widetilde{Q_m}$ which can be obtained by using the formula (2.7) in chapter 2.

## 4.3 Numerical Example

Let the fuzzy parameters are in terms of trapezoidal fuzzy numbers and are as follows.

fuzzy demand, $\widetilde{D}$= (186, 186.5, 187.5, 191)

fuzzy inventory cost, $\widetilde{A}$=(0.01, 0.02, 0.02, 0.03)

fuzzy setup cost, $\widetilde{T}$=(18, 19, 20, 24)

fuzzy purchasing cost, $\widetilde{C_a}$=(2, 3, 3, 4)

fuzzy purchasing cost, $\widetilde{C_b}$=(2, 2.5, 2.5, 3)

fuzzy limit, $\widetilde{q}$=(800, 900, 1000, 1400).

Now according to step 1 $\widetilde{Q_m} = (Q_{m_1}, Q_{m_2}, Q_{m_3}, Q_{m_4})$=(612.63, 612.63, 612.63, 612.63) since, $\widetilde{Q_m} < \widetilde{q}$ we go to step 2,

According to step 2, using the equations (4.7), (4.8), (4.9) and (4.10) and solving we get the value of $\widetilde{Q} = (Q_1, Q_2, Q_3, Q_4)$=(1656.582, 10482.457, 10563.926, 13660.257).

zone I=$[0, \widetilde{Q_m}]$=[(0, 0, 0, 0) , (612.63, 612.63, 612.63, 612.63)]

zone II=$[\widetilde{Q_m}, \widetilde{Q}]$=[(612.63, 612.63, 612.63, 612.63) , (1656.582, 10482.457, 10563.926, 13660.257) ]

zone III=$[\widetilde{Q}, \infty]$=[(1656.582, 10482.457, 10563.926, 13660.257) , $(\infty, \infty, \infty, \infty)$]

Now, $\widetilde{q}$ lies in II zone. so, we get $\widetilde{Q_P}^* = \widetilde{q}$=(800, 900, 1000, 1400).

If we convert all the trapezoidal fuzzy numbers into crisp numbers using formula (2.2) we get $D$=187.5, $A$=0.02, $T$=20, $C_a$=3, $C_b$=3 and $q$=1000 and $Q_P^*$=1000

In classical inventory model the optimal production inventory $Q$= 1000 [1] which is same as $Q_P^*$.

## 4.4 The fuzzy production inventory model with price breaks with uniform replenishment using trapezoidal fuzzy numbers

we use the following variables in order to simplify the treatment of the fuzzy production inventory models:

$\widetilde{D}$ fuzzy yearly demand,

$\widetilde{A}$ fuzzy inventory cost (dollars/item year),

$\widetilde{T}$ fuzzy setup cost,

$\widetilde{Q_P}$ fuzzy production quantity,

$\widetilde{P}$ fuzzy daily production rate,

$\widetilde{R}$ fuzzy daily demand rate,

$\widetilde{C}$ purchasing cost.

$$\widetilde{C} = \begin{cases} \widetilde{C_a} \ if \ \widetilde{Q_P} \leq q \\ \widetilde{C_b} \ if \ \widetilde{Q_P} > q \end{cases}$$

For this model, the total cost per unit time with fuzzy parameters is

$$TCU(\widetilde{Q_P}) = \begin{cases} TCU_1(\widetilde{Q_P}) = \widetilde{D} \otimes \widetilde{C_a} \oplus \widetilde{T} \otimes \left(\widetilde{D} \oslash \widetilde{Q_P}\right) \oplus \widetilde{A} \otimes \frac{\widetilde{Q_P}}{2} \otimes \left[1 - \widetilde{R} \oslash \widetilde{P}\right]; \ \widetilde{Q_P} \leq q \\ TCU_2(\widetilde{Q_P}) = \widetilde{D} \otimes \widetilde{C_b} \oplus \widetilde{T} \otimes \left(\widetilde{D} \oslash \widetilde{Q_P}\right) \oplus \widetilde{A} \otimes \frac{\widetilde{Q_P}}{2} \otimes \left[1 - \widetilde{R} \oslash \widetilde{P}\right]; \ \widetilde{Q_P} > q \end{cases} \tag{4.11}$$

The steps for determining $\widetilde{Q_P}^*$ are

**Step 1:** we first Determine $\widetilde{Q_m}$ by using formula (2.7) obtained in chapter 1. If q is in zone I that is $(0, \widetilde{Q_m})$, then $\widetilde{Q_P}^* = \widetilde{Q_m}$. Otherwise go to step 2.

**Step 2:** Now, we determine $\widetilde{Q}$ from the equation

$TCU_2(\widetilde{Q}) = TCU_1(\widetilde{Q_m})$.

using formula (4.11) in above equation we get,

$$\widetilde{D} \otimes \widetilde{C_b} \ \oplus \ \widetilde{T} \otimes \left(\widetilde{D} \oslash \widetilde{Q}\right) \ \oplus \ \widetilde{A} \otimes \frac{\widetilde{Q}}{2} \otimes \left[1 - \widetilde{R} \oslash \widetilde{P}\right] =$$
$$\widetilde{D} \otimes \widetilde{C_a} \ \oplus \ \widetilde{T} \otimes \left(\widetilde{D} \oslash \widetilde{Q_m}\right) \ \oplus \ \widetilde{A} \otimes \frac{\widetilde{Q_m}}{2} \otimes \left[1 - \widetilde{R} \oslash \widetilde{P}\right]$$

Here, we suppose $\widetilde{A} = (a_1, a_2, a_3, a_4)$, $\widetilde{D} = (d_1, d_2, d_3, d_4)$, $\widetilde{T} = (t_1, t_2, t_3, t_4)$, $\widetilde{R} = (r_1, r_2, r_3, r_4)$, $\widetilde{P} = (p_1, p_2, p_3, p_4)$ , $\widetilde{C_a} = (c_{a_1}, c_{a_2}, c_{a_3}, c_{a_4})$ and $\widetilde{C_b} = (c_{b_1}, c_{b_2}, c_{b_3}, c_{b_4})$ are non-negative trapezoidal fuzzy numbers. Then using these in above equation and comparing on both side we get,

$$d_1 c_{b_1} + \frac{t_1 d_1}{Q_4} + \frac{a_1 Q_1}{2}\left(1 - \frac{r_4}{p_1}\right) = d_1 c_{a_1} + \frac{t_1 d_1}{Q_{m_4}} + \frac{a_1 Q_{m_1}}{2}\left(1 - \frac{r_4}{p_1}\right) \tag{4.12}$$

$$d_2 c_{b_2} + \frac{t_2 d_2}{Q_3} + \frac{a_2 Q_2}{2}\left(1 - \frac{r_3}{p_2}\right) = d_2 c_{a_2} + \frac{t_2 d_2}{Q_{m_3}} + \frac{a_2 Q_{m_2}}{2}\left(1 - \frac{r_3}{p_2}\right) \tag{4.13}$$

$$d_3 c_{b_3} + \frac{t_3 d_3}{Q_2} + \frac{a_3 Q_3}{2}\left(1 - \frac{r_2}{p_3}\right) = d_3 c_{a_3} + \frac{t_3 d_3}{Q_{m_2}} + \frac{a_3 Q_{m_3}}{2}\left(1 - \frac{r_2}{p_3}\right) \tag{4.14}$$

$$d_4 c_{b_4} + \frac{t_4 d_4}{Q_1} + \frac{a_4 Q_4}{2}\left(1 - \frac{r_1}{p_4}\right) = d_4 c_{a_4} + \frac{t_4 d_4}{Q_{m_1}} + \frac{a_4 Q_{m_4}}{2}\left(1 - \frac{r_1}{p_4}\right) \tag{4.15}$$

Further solving equations (4.12), (4.13), (4.14) and (4.15) we get the result follows:

$$\Rightarrow\ d_1(c_{a_1}-c_{b_1})+t_1d_1\left(\frac{1}{Q_{m_4}}-\frac{1}{Q_4}\right)+\frac{a_1}{2}\left(1-\frac{r_4}{p_1}\right)(Q_{m_1}-Q_1)=0 \tag{4.16}$$

$$\Rightarrow\ d_2(c_{a_2}-c_{b_2})+t_2d_2\left(\frac{1}{Q_{m_3}}-\frac{1}{Q_3}\right)+\frac{a_2}{2}\left(1-\frac{r_3}{p_2}\right)(Q_{m_2}-Q_2)=0 \tag{4.17}$$

$$\Rightarrow\ d_3(c_{a_3}-c_{b_3})+t_3d_3\left(\frac{1}{Q_{m_2}}-\frac{1}{Q_2}\right)+\frac{a_3}{2}\left(1-\frac{r_2}{p_3}\right)(Q_{m_3}-Q_3)=0 \tag{4.18}$$

$$\Rightarrow\ d_4(c_{a_4}-c_{b_4})+t_4d_4\left(\frac{1}{Q_{m_1}}-\frac{1}{Q_1}\right)+\frac{a_4}{2}\left(1-\frac{r_1}{p_4}\right)(Q_{m_4}-Q_4)=0 \tag{4.19}$$

Using equations (4.16), (4.17), (4.18) and (4.19) and solving we can determine the value of $\widetilde{Q}=(Q_1,Q_2,Q_3,Q_4)$ from which the zones II can be defined as that is $(\widetilde{Q_m},\widetilde{Q})$ and III, that is $(\widetilde{Q},\infty)$. If $\widetilde{q}$ is in zone II, $\widetilde{Q_P}^*=\widetilde{q}$. Otherwise, $\widetilde{q}$ is in zone III, and $\widetilde{Q_P}^*=\widetilde{Q_m}$ which can be obtained by formula (2.7) in chapter 2.

## 4.5 Numerical Example

Let the fuzzy parameters are in terms of trapezoidal fuzzy number and are as follows.

fuzzy yearly demand, $\widetilde{D}$= (20, 30, 30, 40)

fuzzy inventory cost, $\widetilde{A}$=(0.04, 0.05, 0.05, 0.06)

fuzzy setup cost, $\widetilde{T}$=(80, 90, 100, 140)

fuzzy daily production rate, $\widetilde{P}$=(0.8, 1, 1, 1.2)

fuzzy daily demand rate, $\widetilde{R}$=(0.07, 0.08, 0.08, 0.09)

fuzzy purchasing cost, $\widetilde{C_a}$=(9, 10, 10, 11)

fuzzy purchasing cost, $\widetilde{C_b}$=(7, 8, 8, 9)

fuzzy limit, $\widetilde{q}$=(400, 500, 500, 600).

Now according to step 1 using the formula (2.7) and using the above given parameters we get the value of $\widetilde{Q_m}=(Q_{m_1},Q_{m_2},Q_{m_3},Q_{m_4})$=(367.128, 367.128, 367.128, 367.128)

since, $\widetilde{Q_m}<\widetilde{q}$ we go to step 2, According to step 2, using the equations (4.7), (4.8), (4.9) and (4.10) and solving we get the value of $\widetilde{Q} = (Q_1, Q_2, Q_3, Q_4)$=(2807.418, 3259.896, 3291.118, 3697.835).

zone I=[0, $\widetilde{Q_m}$]=[(0, 0, 0, 0) , (367.128, 367.128, 367.128, 367.128)]

zone II=[$\widetilde{Q_m}$, $\widetilde{Q}$]=[(367.128, 367.128, 367.128, 367.128) , (2807.418, 3259.896, 3291.118, 3697.835)]

zone III=[$\widetilde{Q}$, $\infty$]=[(2807.418, 3259.896, 3291.118, 3697.835) , $(\infty, \infty, \infty, \infty)$]

Now, $\widetilde{q}$ lies in II zone. so, we get $\widetilde{Q_P}^* = \widetilde{q}$=(400, 500, 500, 600).

## 4.6 Conclusion

In this chapter, Inventory model with price breaks is presented with trapezoidal fuzzy number in both the cases, for instantaneous and uniform replenishment. The numerical example considered in this chapter gives the similar results as the classical inventory model if trapezoidal fuzzy number are converted into crisp number. So, proposed inventory model can be useful in real world.

# Chapter 5

# Fuzzy Production Inventory Model for Multi-item with Storage Limitation

## 5.1 Introduction

Classical inventory models generally deal with a single-item. But in real world situations, a single-item inventory seldom occurs and multi-item inventory is common. In a multi-item inventory system, the companies or the retailers are required to maximize/minimize two or more objectives simultaneously over a given set of decision variables. In this chapter, the same technique which was used in chapter three has been used on the Multi-item fuzzy production inventory model with fuzzy parameters for crisp production quantity involving uniform and instantaneous replenishment.

## 5.2 The fuzzy production Multi-item inventory model with instantaneous replenishment using trapezoidal fuzzy numbers

we use the following variables in order to simplify the treatment of the fuzzy production inventory models:

$\widetilde{D_i}$ fuzzy demand,

$\widetilde{A_i}$ fuzzy inventory cost (dollars/item year),

$\widetilde{T_i}$ fuzzy setup cost,

$Q_{P_i}$ order quantity,

$\widetilde{s_i}$ fuzzy storage area requirement,

$\widetilde{S}$ fuzzy maximum available storage area for all n item.

For this model, the mathematical model representing the inventory situation is given as

$$\text{Minimize } TCU(Q_{P_1}, Q_{P_2}, ........., Q_{P_n}) = \sum_{i=1}^{n}\left(\widetilde{T_i} \otimes (\widetilde{D_i} \oslash Q_{P_i}) + \widetilde{A_i} \otimes \tfrac{Q_{P_i}}{2}\right)$$

subject to

$$\sum_{i=1}^{n}(\widetilde{s_i}Q_{P_i}) \leq \widetilde{S}$$

$$Q_{P_i} > 0; \text{ i=1, 2,....., n}$$

To find the optimal order quantity the following procedure has been used[1] :

**Step 1:** Compute the unconstrained optimal values of the order quantities by using formula (2.6).

**Step 2:** We check if the unconstrained optimal value $Q^*_{P_i}$ satisfy the storage constraint. If it does, stop; $Q^*_{P_i}$ , i=1,2,....,n are optimal. Otherwise, go to step 3.

**Step 3:** The storage constraint must be satisfied in equation form. Then we use lagrange multipliers method to determine the constrained optimal values of the order quantities.

Now, the Lagrangean function is formulated as

$$L(\lambda, Q_{P_1}, Q_{P_2}, ......., Q_{P_n}) = TCU(Q_{P_1}, Q_{P_2}, ......., Q_{P_n}) - \lambda\left(\sum_{i=1}^{n}(\widetilde{s_i}Q_{P_i}) - \widetilde{S}\right)$$

$$L(\lambda, Q_{P_1}, Q_{P_2}, ......., Q_{P_n}) = \sum_{i=1}^{n}\left(\widetilde{T_i} \otimes (\widetilde{D_i} \oslash Q_{P_i}) + \widetilde{A_i} \otimes \frac{Q_{P_i}}{2}\right) - \lambda\left(\sum_{i=1}^{n}(\widetilde{s_i}Q_{P_i}) - \widetilde{S}\right) \tag{5.1}$$

where $\lambda(< 0)$ is the Lagrange Multiplier and $\oslash$, $\otimes$, $\ominus$ and $\oplus$ are the fuzzy arithmetical operations under Function Principle.

Here, we suppose $\widetilde{A_i}=(a_{i_1}, a_{i_2}, a_{i_3}, a_{i_4})$, $\widetilde{D_i} = (d_{i_1}, d_{i_2}, d_{i_3}, d_{i_4})$, $\widetilde{s_i} = (s_{i_1}, s_{i_2}, s_{i_3}, s_{i_4})$, $\widetilde{S} = (S_1, S_2, S_3, S_4)$

and $\widetilde{T_i} = (t_{i_1}, t_{i_2}, t_{i_3}, t_{i_4})$ are non-negative trapezoidal fuzzy numbers. Then we solve for the equation (5.1) and using formula (2.2) for defuzzification we get the result as

$$L = \frac{1}{6}\Bigg[\frac{t_{i_1}d_{i_1}}{Q_{P_i}} + \frac{a_{i_1}Q_{P_i}}{2} + \frac{2t_{i_2}d_{i_2}}{Q_{P_i}} + \frac{2a_{i_2}Q_{P_i}}{2} + \frac{2t_{i_3}d_{i_3}}{Q_{P_i}} + \frac{2a_{i_3}Q_{P_i}}{2} + \frac{t_{i_4}d_{i_4}}{Q_{P_i}} + \frac{a_{i_4}Q_{P_i}}{2} - \lambda\bigg(\sum_{i=1}^{n}(Q_{P_i}s_{i_1} + 2Q_{P_i}s_{i_2} + 2Q_{P_i}s_{i_3} + Q_{P_i}s_{i_4}) - (S_1 + 2S_2 + 2S_3 + S_4)\bigg)\Bigg]$$

The optimal values of $Q^*_{P_i}$ and $\lambda$ are determined from the conditions $\frac{\partial}{\partial Q_{P_i}}(L)$ =0 and $\frac{\partial}{\partial \lambda}(L)$=0

The derivative of L with $\lambda$ is

$$\frac{\partial}{\partial \lambda}(L) = -\sum_{i=1}^{n}(\widetilde{s_i}Q_{P_i}) + \widetilde{S}$$

Let $\frac{\partial}{\partial \lambda}(L)$=0, then we get,

$$\sum_{i=1}^{n}(\widetilde{s_i}Q_{P_i}) - \widetilde{S} = 0 \tag{5.2}$$

The equation (5.2) shows that the storage constraint must satisfied in equation form at the optimum. The derivative of L with $Q_{P_i}$ is

$$\frac{\partial}{\partial Q_{P_i}}(L) = \frac{1}{6}\left[-\frac{1}{Q^2_{P_i}}(t_{i_1}d_{i_1} + 2t_{i_2}d_{i_2} + 2t_{i_3}d_{i_3} + t_{i_4}d_{i_4})\right] + \frac{1}{2}\left[(a_{i_1} + 2a_{i_2} + 2a_{i_3} + a_{i_4}) - \lambda\sum_{i=1}^{n}(s_{i_1} + 2s_{i_2} + 2s_{i_3} + s_{i_4})\right]$$

Let $\frac{\partial}{\partial Q_{P_i}}(L)$=0, we find the optimal production quantity $Q^*_{P_i}$

$$Q^*_{P_i} = \sqrt{\frac{2(t_{i_1}d_{i_1} + 2t_{i_2}d_{i_2} + 2t_{i_3}d_{i_3} + t_{i_4}d_{i_4})}{[a_{i_1} + 2a_{i_2} + 2a_{i_3} + a_{i_4}] - 2\lambda^*(s_{i_1} + 2s_{i_2} + 2s_{i_3} + s_{i_4})}} \tag{5.3}$$

## 5.3 Numerical Example

The parameters values for three items are given in table 5.1

Table 5.1: Parameter

| items | T | D | A | $s_i$ |
|---|---|---|---|---|
| 1 | 10 | 2 | 0.3 | 1 |
| 2 | 5 | 4 | 0.1 | 1 |
| 2 | 15 | 4 | 0.2 | 1 |

In this case the optimal order quantities are $Q_1^*$=6.35 units, $Q_2^*$=7.11 units, $Q_3^*$=11.6 units [1].

Table 5.2: Fuzzy Parameter

| items | $\widetilde{T}$ | $\widetilde{D}$ | $\widetilde{A}$ | $\widetilde{s_i}$ |
|---|---|---|---|---|
| 1 | (9, 10, 10, 11) | (1, 2, 2, 3) | (0.2, 0.3, 0.3, 0.4) | (0.8, 1, 1, 1.2) |
| 2 | (4, 5, 5, 6) | (3, 4, 4, 5) | (0.08, 0.1, 0.1, 0.12) | (0.8, 1, 1, 1.2) |
| 3 | (14, 15, 15, 16) | (3, 4, 4, 5) | (0.1, 0.2, 0.2, 0.3) | (0.8, 1, 1, 1.2) |

Total available space $\widetilde{S}$=(24, 25, 25, 26)

For the first item, using the formula (5.3) we have the value of $Q_{P_1}$ as

$$Q_{P_1} = \sqrt{\frac{230}{1.8 - 12\lambda}} \tag{5.4}$$

For the second item, using the formula (5.3) we have the value of $Q_{P_2}$ as

$$Q_{P_2} = \sqrt{\frac{244}{0.6 - 12\lambda}} \tag{5.5}$$

For the third item, using the formula (5.3) we have the value of $Q_{P_3}$ as

$$Q_{P_3} = \sqrt{\frac{724}{1.2 - 12\lambda}} \tag{5.6}$$

Using the values of table 5.2 and equations (5.4), (5.5) and (5.6) we have table 5.3 The last column of table 5.3 shows that equation (5.2) is satisfied somewhere in range $-0.3 > \lambda > -0.4$. Then by approximation $\lambda^* \approx$ -0.33, which gives

$Q_{P_1}^* \approx$ 6.25 units, $Q_{P_2}^* \approx$ 7.05 units and $Q_{P_3}^* \approx$ 11.5 units units which is same as the optimal order quantity obtained above for classical inventory problem.

Table 5.3: Resulting table

| $\lambda$ | $Q_{P_1}$ | $Q_{P_2}$ | $Q_{P_3}$ | $\sum_{i=1}^{n}(\widetilde{s}_i Q_{P_i}) - \widetilde{S} = 0$ |
|---|---|---|---|---|
| 0 | 11.304 | 20.166 | 24.563 | 31.033 |
| -0.1 | 8.756 | 11.643 | 17.369 | 12.435 |
| -0.2 | 7.4 | 9.018 | 14.181 | 5.6 |
| -0.3 | 6.526 | 7.622 | 12.281 | 1.0956 |
| -0.4 | 5.903 | 6.722 | 10.985 | -1.39 |

## 5.4 The fuzzy production Multi-item inventory model with uniform replenishment using trapezoidal fuzzy numbers

In this section we use the following variables in order to simplify the treatment of the fuzzy production inventory models:

$\widetilde{D_i}$ fuzzy demand,

$\widetilde{A_i}$ fuzzy inventory cost (dollars/item year),

$\widetilde{T_i}$ fuzzy setup cost,

$Q_{P_i}$ order quantity,

$\widetilde{s_i}$ fuzzy storage area requirement,

$\widetilde{R_i}$ fuzzy daily demand,

$\widetilde{P_i}$ fuzzy daily production,

$\widetilde{S}$ fuzzy maximum available storage area for all n item.

For this model, the mathematical model representing the inventory situation is given as

$$\text{Minimize } TCU(Q_{P_1}, Q_{P_2}, ........., Q_{P_n}) = \sum_{i=1}^{n}\left(\widetilde{T_i} \otimes (\widetilde{D_i} \oslash Q_{P_i}) + \widetilde{A_i} \otimes \tfrac{Q_{P_i}}{2} \otimes \left(1 - (\widetilde{R_i} \oslash \widetilde{P_i})\right)\right)$$

subject to

$$\sum_{i=1}^{n}(\widetilde{s_i} Q_{P_i}) \leq \widetilde{S}$$

$$Q_{P_i} > 0; \text{ i=1, 2,...., n}$$

The following steps we have used for finding optimal order quantity:

**Step 1:** Compute the unconstrained optimal values of the order quantities by using formula (2.6).

**Step 2:** We check if the unconstrained optimal value $Q^*_{P_i}$ satisfy the storage constraint.If it does stop; $Q^*_{P_i}$ , i=1, 2,...., n are optimal. Othewise, go to step 3

**Step 3:** The storage constraint must be satisfied in equation form. Then we use lagrange multiplier method to determine the constrained optimal values of the order quantities.

Now, the Lagrangean function is formulated as

$$L(\lambda, Q_{P_1}, Q_{P_2}, ......., Q_{P_n}) = TCU(Q_{P_1}, Q_{P_2}, ......., Q_{P_n}) - \lambda\left(\sum_{i=1}^{n}(\widetilde{s_i} Q_{P_i}) - \widetilde{S}\right)$$

$$L(\lambda, Q_{P_1}, Q_{P_2}, ......., Q_{P_n}) = \sum_{i=1}^{n}\left(\widetilde{T_i} \otimes (\widetilde{D_i} \oslash Q_{P_i}) + \widetilde{A_i} \otimes \frac{Q_{P_i}}{2} \otimes \left(1 - (\widetilde{R_i} \oslash \widetilde{P_i})\right)\right) - \lambda\left(\sum_{i=1}^{n}(\widetilde{s_i} Q_{P_i}) - \widetilde{S}\right) \tag{5.7}$$

where $\lambda (< 0)$ is Lagrange multiplier and $\oslash$, $\otimes$, $\ominus$ and $\oplus$ are the fuzzy arithmetical operations under Function Principle.

Here, we suppose $\widetilde{A_i}=(a_{i_1}, a_{i_2}, a_{i_3}, a_{i_4})$, $\widetilde{D_i} = (d_{i_1}, d_{i_2}, d_{i_3}, d_{i_4})$, $\widetilde{T_i} = (t_{i_1}, t_{i_2}, t_{i_3}, t_{i_4})$, $\widetilde{s_i} = (s_{i_1}, s_{i_2}, s_{i_3}, s_{i_4})$, $\widetilde{P_i}=(p_{i_1}, p_{i_2}, p_{i_3}, p_{i_4})$, $\widetilde{R_i}=(r_{i_1}, r_{i_2}, r_{i_3}, r_{i_4})$ are non-negative trapezoidal fuzzy numbers.Then we solve for the equation (5.7) and using formula (2.2) for defuzzification we get the result as

$$\begin{aligned} L = \frac{1}{6}\Bigg[&\frac{t_{i_1} d_{i_1}}{Q_{P_i}} + \frac{a_{i_1} Q_{P_i}}{2}\left(1 - \frac{r_{i_4}}{p_{i_1}}\right) + \frac{2t_{i_2} d_{i_2}}{Q_{P_i}} + \frac{2a_{i_2} Q_{P_i}}{2}\left(1 - \frac{r_{i_3}}{p_{i_2}}\right) + \\ &\frac{2t_{i_3} d_{i_3}}{Q_{P_i}} + \frac{2a_{i_3} Q_{P_i}}{2}\left(1 - \frac{r_{i_2}}{p_{i_3}}\right) + \frac{t_{i_4} d_{i_4}}{Q_{P_i}} + \frac{a_{i_4} Q_{P_i}}{2}\left(1 - \frac{r_{i_1}}{p_{i_4}}\right) - \\ &\lambda\left(\sum_{i=1}^{n}(Q_{P_i} s_{i_1} + 2Q_{P_i} s_{i_2} + 2Q_{P_i} s_{i_3} + Q_{P_i} s_{i_4}) - (S_1 + 2S_2 + 2S_3 + S_4)\right)\Bigg] \end{aligned}$$

The optimal values of $Q^*_{P_i}$ and $\lambda$ are determined from the conditions $\frac{\partial}{\partial Q_{P_i}}(L)$ =0 and $\frac{\partial}{\partial \lambda}(L)$=0

The derivative of L with $\lambda$ is

$$\frac{\partial}{\partial \lambda}(L) = -\sum_{i=1}^{n}(\widetilde{s}_i Q_{P_i}) + \widetilde{S}$$

Let $\frac{\partial}{\partial \lambda}(L)$=0, then we get,

$$\sum_{i=1}^{n}(\widetilde{s}_i Q_{P_i}) - \widetilde{S} = 0 \tag{5.8}$$

The equation (5.8) shows that the storage constraint must satisfied in equation form at the optimum. The derivative of L with $Q^*_{P_i}$ is

$$\frac{\partial}{\partial Q_{P_i}}(L) = \frac{1}{6}\left[-\frac{1}{Q^2_{P_i}}(t_{i_1}d_{i_1} + 2t_{i_2}d_{i_2} + 2t_{i_3}d_{i_3} + t_{i_4}d_{i_4})\right] + \frac{1}{2}\left[a_{i_1}\left(1 - \frac{r_{i4}}{p_{i_1}}\right) + 2a_{i_2}\left(1 - \frac{r_{i_3}}{p_{i_2}}\right) + 2a_{i3}\left(1 - \frac{r_{i2}}{p_{i3}}\right) + a_{i_4}\left(1 - \frac{r_{i_1}}{p_{i_4}}\right) - \lambda(s_{i_1} + 2s_{i_2} + 2s_{i_3} + s_{i_4})\right]$$

Let $\frac{\partial}{\partial Q_{P_i}}(L)$=0, we find the optimal production quantity $Q^*_{P_i}$ as

$$Q^*_{P_i} = \sqrt{\frac{2(t_{i_1}d_{i_1} + 2t_{i_2}d_{i_2} + 2t_{i_3}d_{i_3} + t_{i_4}d_{i_4})}{[a_{i_1}(1 - \frac{r_{i_4}}{p_{i_1}}) + 2a_{i_2}(1 - \frac{r_{i_3}}{p_{i_2}}) + 2a_{i_3}(1 - \frac{r_{i_2}}{p_{i_3}}) + a_{i_4}(1 - \frac{r_{i_1}}{p_{i_4}})] - 2\lambda^*(s_{i_1} + 2s_{i_2} + 2s_{i_3} + s_{i_4})}} \tag{5.9}$$

## 5.5 Numerical Example

The parameters values for three items are given in table 5.4

Table 5.4: Parameter

| items | T | D | A | $s_i$ | R | P |
|---|---|---|---|---|---|---|
| 1 | 10 | 2 | 0.3 | 1 | 0.5 | 1 |
| 2 | 5 | 4 | 0.1 | 1 | 0.3 | 1 |
| 2 | 15 | 4 | 0.2 | 1 | 0.3 | 1 |

Total available space $\widetilde{S}$=(24, 25, 25, 26)

Table 5.5: Fuzzy Parameter

| items | $\widetilde{T}$ | $\widetilde{D}$ | $\widetilde{A}$ | $\widetilde{s_i}$ | $\widetilde{R}$ | $\widetilde{P}$ |
|---|---|---|---|---|---|---|
| 1 | (9,10,10,11) | (1,2,2,3) | (0.2,0.3,0.3,0.4) | (0.8,1,1,1.2) | (0.4,0.5,0.5,0.6 ) | (0.8,1,1,1.2) |
| 2 | (4,5,5,6) | (3,4,4,5) | (0.08,0.1,0.1,0.12) | (0.8,1,1,1.2) | (0.2,0.3,0.3,0.4 ) | (0.8,1,1,1.2) |
| 3 | (14,15,15,16) | (3,4,4,5) | (0.1,0.2,0.2,0.3) | (0.8,1,1,1.2) | (0.2,0.3,0.3,0.4 ) | (0.8,1,1,1.2) |

For the first item, using the formula (5.9) we have the value of $Q_{P_1}$ as

$$Q_{P_1} = \sqrt{\frac{244}{0.917 - 12\lambda}} \tag{5.10}$$

For the second item, using the formula (5.9) we have the value of $Q_{P_2}$ as

$$Q_{P_2} = \sqrt{\frac{244}{0.42 - 12\lambda}} \tag{5.11}$$

For the third item, using the formula (5.9) we have the value of $Q_{P_3}$ as

$$Q_{P_3} = \sqrt{\frac{724}{0.86 - 12\lambda}} \tag{5.12}$$

Using the values of table 5.5 and equations (5.10), (5.11) and (5.12) we have table 5.6

Table 5.6: Resulting table

| $\lambda$ | $Q_{P_1}$ | $Q_{P_2}$ | $Q_{P_3}$ | $\sum_{i=0}^{n}(\widetilde{s_i} Q_{P_i}) - \widetilde{S} = 0$ |
|---|---|---|---|---|
| 0 | 16.312 | 24.103 | 29.015 | 44.43 |
| -0.1 | 10.736 | 12.273 | 18.747 | 16.756 |
| -0.2 | 8.577 | 9.302 | 14.903 | 7.782 |
| -0.3 | 7.35 | 7.791 | 12.741 | 1.953 |
| -0.4 | 6.533 | 6.837 | 11.31 | -0.6533 |

The last column of table 5.6 shows that equation (5.8) is satisfied somewhere in range $-0.3 > \lambda > -0.4$. Then by approximation $\lambda^* \approx$ -0.33, which gives

$Q^*_{P_1} \approx 6.937$ units, $Q^*_{P_2} \approx 7.012$ units and $Q^*_{P_3} \approx 11.93$.

## 5.6 Conclusion

In this chapter, Multi-item inventory model is presented with trapezoidal fuzzy number in both the cases, for instantaneous and for uniform replenishment and result is compared with classical inventory model.

It is concluded that, when all fuzzy parameters are the crisp real numbers, that is $\widetilde{A_i}=(a_{i_1}, a_{i_2}, a_{i_3}, a_{i_4})=$

$(A_i, A_i, A_i, A_i)$, $\widetilde{D_i} = (d_{i_1}, d_{i_2}, d_{i_3}, d_{i_4}) = (D_i, D_i, D_i, D_i)$, $\widetilde{T_i} = (t_{i_1}, t_{i_2}, t_{i_3}, t_{i_4}) = (T_i, T_i, T_i, T_i)$, $\widetilde{s_i} = (s_{i_1}, s_{i_2}, s_{i_3}, s_{i_4}) = (s_i, s_i, s_i, s_i)$, $\widetilde{P_i}=(p_{i_1}, p_{i_2}, p_{i_3}, p_{i_4}) = (P_i, P_i, P_i, P_i)$, $\widetilde{R_i}=(r_{i_1}, r_{i_2}, r_{i_3}, r_{i_4}) = (R_i, R_i, R_i, R_i)$ ,the optimal production quantity that is $Q^*_P$ becomes

$Q^*_P = \sqrt{\frac{2T_iD_i}{A_i(1-\frac{R_i}{P_i})-2\lambda s_i}}$ which is similar to the result in classical inventory problem for multi item. Numerical example (5.3) considered in this chapter gives the similar results as the classical inventory model when all the trapezoidal fuzzy numbers are converted in crisp numbers.

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