Online Handwritten Signature Verification based on FIR Filters using Discrete Fractional Cosine Transform

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Submitted by

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Certificate

I, hereby declare that the work, which is being presented in the dissertation, entitled "**Online Handwritten Signature Verification based on FIR Filters using Discrete Fractional Cosine Transform**" in partial fulfillment of the requirements for the award of degree of Master of Engineering in Wireless Communications submitted in Electronics and Communication Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the guidance of **Dr. Kulbir Singh (Associate Professor) and Dr. Neeru Jindal (Assistant Professor)**, Electronics and Communication Department and refers other research's work which are duly listed in reference section. The matter presented in this dissertation has not been submitted in any other University/Institute for the award of degree.

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It is certified that the above statement made by the student is correct to the best of my knowledge and belief.

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Abstract

Biometrics is being widely implemented in today's society to deal with the security requirement issues. A biometric system can either do identification or verification task. In identification the biometric system can accept or reject identity of a person whereas verification authenticates the person's claimed identity from database samples. Biometric verification technology can be divided into two branches: physiological verification and behavioural verification. Physiological recognition includes speech, fingerprint, iris and retinal recognition etc. Physiological recognition is used to authenticate or identify an individual whereas Behavioural recognition examines the mannerisms of an individual for example keyboard typing recognition, signature verification etc., these verification techniques analyse and recognize how an individual signs his signature or uses a keyboard etc.

Signature verification is an intriguing intellectual challenge with many practical applications. It is one of the customary ways to verify person in many countries. However, it is known that signatures signed by same person are never precisely the same. It is widely and commonly accepted practice for authentication of an individual. Whereas off-line signature verification contributes very less to accurate identification, on-line signature verification has been successfully implemented in recent researches to achieve 80%-98% of accuracy. Various approaches have been used to implement biometric signature recognition some of which are dynamic time warping (DTW), Bayesian Learning, Hidden Markov model (HMM), Neural Networks, Support Vector machine (SVM) etc.

In literature, many techniques have developed given to extract online signature features. The best field of signature verification is still under developing phase, many methodologies are yet to be explored. One such less explored methodology is based on Fractional Transform. Fractional Transform is generalization of classical transforms. This transform has an additional parameter which gives an extra degree of freedom. This field is still in development.

An algorithm for online handwritten signature verification which is based on discrete fractional Cosine Transform (DFrCT) is used. The Experiments were performed on SVC

2004 and SUSIG databases. In this system ten features of tested signature are extracted. Then system is realized with the help of five FIR filters. After this feature vectors are calculated. Euclidean norm of difference between the feature vectors of test signatures and reference signatures is calculated. This Euclidean norm is compared with threshold to classify the signatures as genuine or forgery. The equal error rate (EER) is calculated to compare the efficiency of this method with the existing methods. It has been observed that the results of proposed DFrCT with ten features are better than discrete cosine transform (DCT) results.

Contents

List of Abbreviations

EER	Equal Error Rate
FAR	False Acceptance Rate
FRR	False Rejection Rate
DTW	Dynamic Time Warping
DCT	Discrete Cosine Transform
HMM	Hidden Markov Model
PCs	Personal Computers
FrFT	Fractional Fourier Transform
DFrFT	Discrete Fractional Fourier Transform
PCA	Principal Component Analysis
FT	Fourier Transform
CT	Cosine Transform
ST	Sine Transform
DFrCT	Discrete Fractional Cosine Transform
FIR	Finite Impulse Response
DFrST	Discrete Fractional Sine Transform
SVM	Support Vector Machine
PDA	Personal Digital Assistant
ANN	Artificial Neural Network
AFT	Affine Fourier Transform
DAFT	Discrete Affine Transform
NN	Neural Networks

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Chapter 1

Introduction

1.1 Preamble

Humans use various characteristics to recognize each other for ages. Recognition is done by voice when someone speaks and is done by face during meeting. In computer system authentication traditionally was based on magnetic chip card or PIN passwords. However these things can be lost or stolen. One may forget the password. For the achievement of more reliable verification systems something should be used that really characterises the person. Biometrics offer verification identification methods which are based on physiological or behavioural characteristics [1]. Physiological characteristics are finger print voice sample, iris detection etc. Behavioural characteristics are signature verification system, gaint, Pattern recognition systems etc. as given in Figure 1.1.

Biometrics system can be in two different modes: Verification and Identification. Verification occurs if the user is already registered in the in the system. In this case the recorded biometrics data of user is compared with the obtained biometric data. In identification case is different. In this obtained biometric data of user is matched against all recorded biometric data. The process of registration of user with system is called as enrollment. Several behavioural or physiological characteristics of humans can be used in biometrics system [2].

- **Universality:** All individuals should possess this behavioural or physiological characteristic**.**
- **Distinctiveness:** There must be sufficient difference in form of characteristic for two persons.
- **Permanence:** This behavioural or physiological characteristic must amply invariant over a very long span of time.
- **Collectability:** This behavioural or physiological characteristic needs to be quantitatively measured.
- **Performance:** This means speed and accuracy. The environment and operational factors should not have any effect on the accuracy and speed.

- **Acceptability:** It denotes the amount of people which are in favour to accept the use of this behavioural or physiological characteristic.

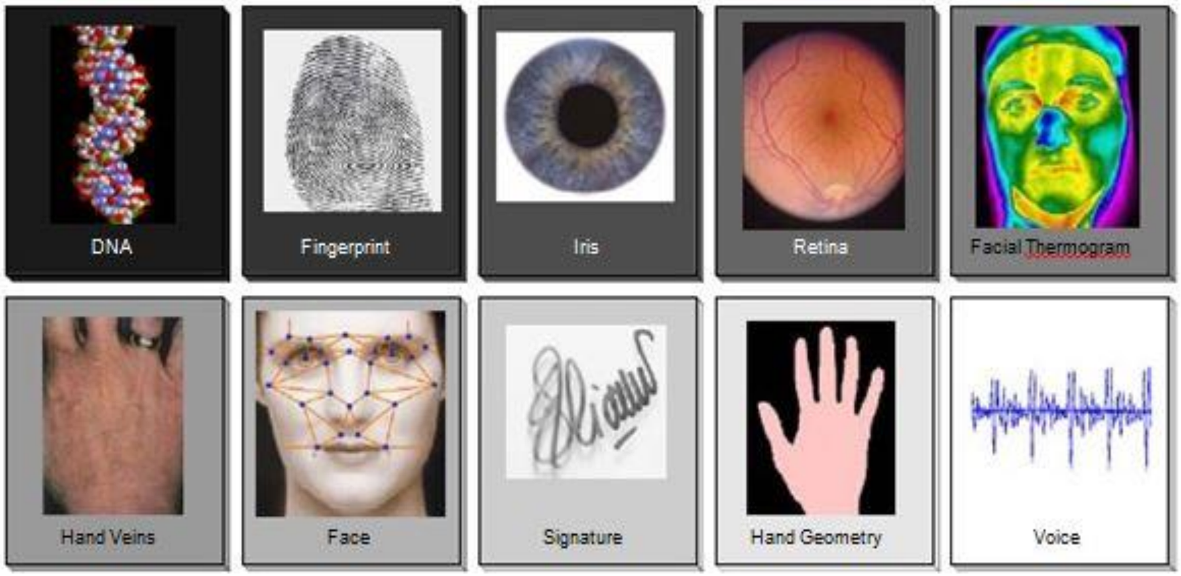


Figure 1.1: Different Biometrics Recognition Systems

- **Harmless:** This behavioural or physiological characteristic must not have any harm effect on the users.
- **Reducible and Comparable:** It must be easy to convert this behavioural or physiological characteristic to a simple format. This characteristic must be capable of being compared digitally to others [2].

Biometrics systems based on verification methods of individuals are more accurate and reliable. Mostly, the the physiological characteristics are not affected by illness or stress. Systems with nearly 100 % match can be built. Some variability has to be allowed in the biometric system. The variability is known as threshold. If it is high then the security level is high, but if it is low then the security level is low.

Biometric systems can be operated in two different modes. First is verification mode. Verification of an identity occurs when an individual privileges to be already registered with the system. In verification mode, the feature vectors collected from the person are matched to the person's already stored feature vectors in the system. In Figure 1.2 block diagram of verification is given. Second mode is identification. Identification (which is also called search) arises, if identity of the user is not known. In this mode the user's obtained biometric data is tested with all the biometric data in the database, because user can be anywhere in the database or it is also possible that user does not exist in the system.

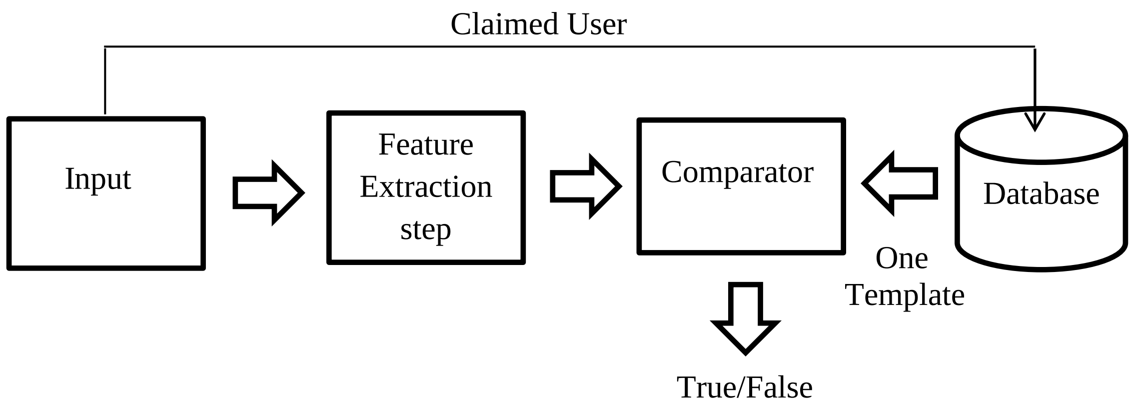


Figure 1.2: Model for Verification System [3]

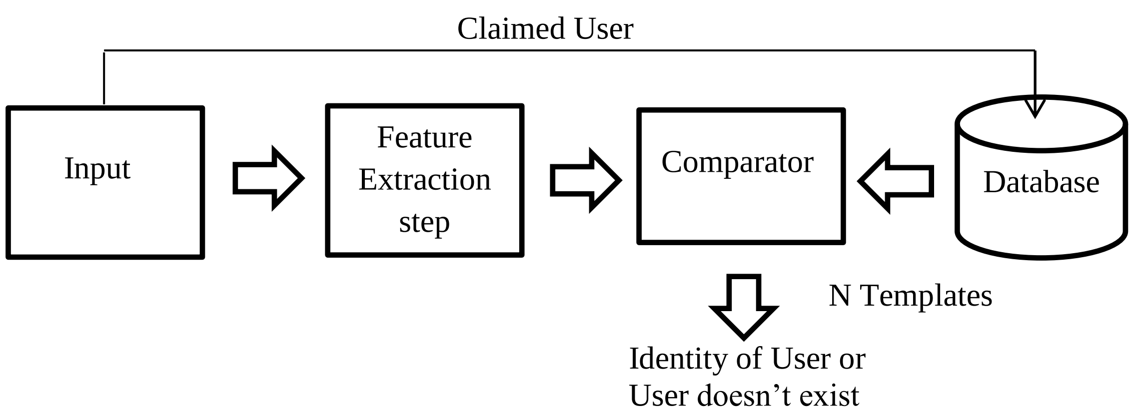


Figure 1.3: Model for Identification System [3]

1.1.1 Advantages of a Biometrics System

Fundamental methods that are used in verification mechanisms:

- Something a user knows. It refers to PINs and passwords.
- Something a user has. This refers to tokens and smart cards.
- Something a user is. It refers to physiological characteristics of the user.

Increased Security: Biometric systems offer a greater security than classical verification methods. A password is crammed by a single user. Password is never written down. PINs and passwords can be guessed easily. One may stole tokens easily. Many users may use obvious words or numbers for their password, because of this an unauthorized person can access the account. Password-based systems provide various cryptographic methods to stop attacks, particularly password hashing. On the other side, biometric data can never be guessed or stolen. In systems, where the biometric authentication releases passwords (leveraging the existing username-password infrastructure), the user can create more complex passwords and longer passwords than would be feasible without biometrics.

Increased Accountability: There is a large increase in awareness of security issues in last two decades. The utilization of biometrics offers high security environment to computers facilities. Biometrics provides a high degree of certainty than what was accessed and at what time.

Increased Convenience: Passwords are easily forgotten so these are kept simple. As users are have to manage a number of passwords, the passwords can be forgotten easily. If a user chooses a universal password then it reduces security further. Cards and tokens can be forgotten very easily. But one can reduce the risk by attaching the cards and tokens to key chains. Biometrics can provide more advantage than systems which are based on passwords and on keeping possession of an authentication token. In a personal computer environment, a person accesses multiple resources. In that environment biometrics can be used to simplify the authentication process. Biometrics can replace passwords. This thing will automatically reduce the burden on the person and the administration.

1.1.2 Disadvantages of a Biometrics System

Change of Voice: Voice of human doesn't remain same for all ages. Also the voice of a person changes who is suffering from flu or throat infection. There must not be too much noise in the environment, because this technique can't work efficiently in much noisy environment. So this technique of authentication is not efficient in all situations.

Change of Finger Print: Generally finger prints of the persons changes who work in chemical industries. So these industries should not utilize the finger print verification.

Change of Iris: The eyes of persons change who are suffering from diabetes. So this thing results errors [4].

In spite of these disadvantages, biometric systems are used to complete current verification requirements. Biometric systems provide very high level of security. Biometric system is very convenient and comfortable for everyone. These systems have already been utilized in many different situations.

1.2 Handwritten Signature as Biometrics

1.2.1 Terms and concepts

As automatized signature verification is still in development, some terms in the field may have ambiguous interpretations. To avoid miscomprehension, throughout this thesis following interpretations of basic terms is used.

Signature: A person's name written as a form of identification in authorizing a document.

Signer: The creator of a handwritten signature.

Genuine signature: A signature originating from a genuine signer.

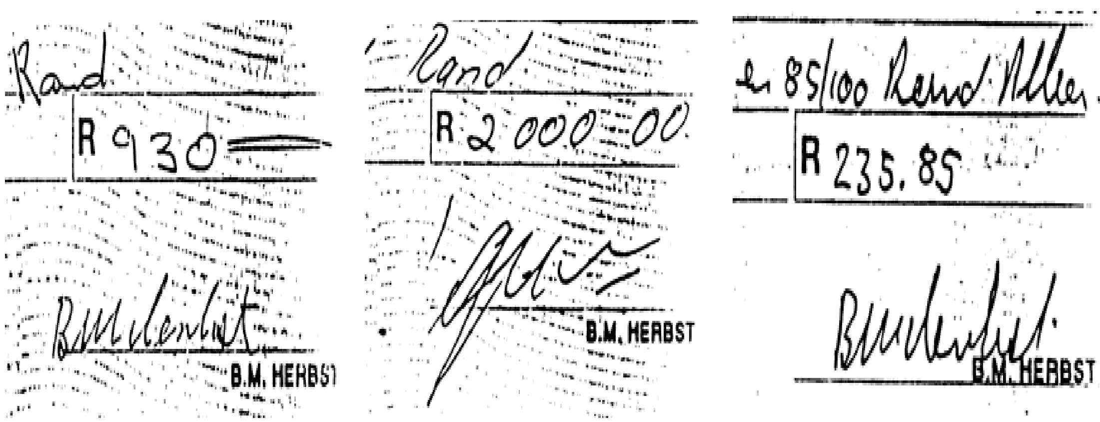
Forged signature (forgery): A signature which imitates the signature of a given person but does not originate from him/her.

Reference signature: Stored signatures of a user.

Test signature: Signature to be tested, which is classified as either genuine or forgery

Many biometric methods are tested for identity verification. The handwritten signatures have been widely utilized for a very long span of time for verification of an identity amongst all possible biometrics techniques. It is used in transactions in banks, transactions of commerce, payments of credit cards and in almost all types of licit documents. If one consider all different types of biometric methods, the handwritten signature is the most preferred biometric method for the different situations. ID documents of a person already contain image of the person. The error rates are not ample in verifying handwritten signature images for broadly deployment. In this, the user is authenticated signing activity of the individual. Signing activity is measured in this offline mode. This activity contains information like pressure applied by person with the pen on tablet, in addition to the viewable appearance of the handwritten signatures. The handwritten signatures have high licit value. Despite this, the handwritten signature can be affected by physical and emotional conditions. So this technique shows a significant variability. This variability must be taken into consideration in the process of verification. In handwritten signature verification system the main motive is to find three types of forgeries. These three types of forgeries are connected to intra and inter-personal variability. Random forgery is the first type of forgery. This forgery is represented by a handwritten signature sample that belongs to other writer [5]. The second forgery is called simple forgery. It is represented by a handwritten signature sample, which have same shape of the authenticated user's name. The last type of forgery is skilled forgery. This forgery is represented by appropriate information of the authenticated person's signature. Forgeries are given in Figure 1.5.

Figure 1.4: Forgery in Cheques



(a) (b) (c)

Figure 1.5: (a) Genuine Signature (b) Random Forgery (c) Skilled Forgery [5]

1.2.2 Modes of Verification

Handwritten Signature Verification Systems are categorized into two modes: offline mode (also called as static) and online (also called as dynamic). The main difference between these modes lies in the information acquisition [6].

1.2.2.1 Offline Mode of Verification

Offline signature verification system is always a challenging system. In an offline signature mode of verification system, a handwritten signature of a person is captured as an image. This image presents a personal style of handwriting of human. Consequently, an offline signature verification system has to cope with a significant number of errors and uncertainties in the recovered data. These difficulties are not present in the online mode. The general approaches to offline handwritten signature verification make use of static features of the handwriting, which treats the complete signature as a single entity. These techniques involve the analysis and comparison of image projections, gradient features, geometric features shadow-code descriptors, transform features, and moment features etc.

1.2.2.2 Online Mode of Verification

Online mode of signature verification system uses a special hardware, a digitizing tablet and a pressure sensitive pen, which records the movements of pen during writing. In addition to shape of signatures, the dynamics features of signatures are also captured in online signatures, Dynamic features are not present in the 2-D representation of the signature and so online written signatures are difficult to forge [7].

1.2.3 Advantages of Signature Verification

According to view of adaption in the market place, handwritten signature verification offers three advantages over other biometrics methods.

- It is a widely socially accepted authentication method. It is already in use in bank transactions and credit card transactions.
- A Handwritten signature is very useful for new generation personal digital assistants (PDAs) and portable computers which use online handwriting as the input for verification.

- A handwritten signature can be changed by the user a number of times, similarly to a password. But change of retina patterns and finger prints is not possible.

So, handwritten signature verification has become a good choice for verification. It is not used only in electronic transactions, but also for other industries.

1.2.4 Applications of Signature Verification

There are numerous numbers of applications of signature verification. The range of these applications is from governmental use to commercial level. A few numbers of applications are discussed below:

- **Commercial Transactions Security:** Nowadays handwritten signature verification can be used for commercial use. This method of verification can be used for identification on ATMs during transactions. It can also be used in package delivery companies. The international courier service UPS has been using handwritten signature verification for many years for verification.
- **Secure computer system authentication:** Combination of handwritten signature verification system and fingerprint authentication system can be used to get a very higher level security in a sensitive area like logging on to PCs. One can also use combination of a password and handwritten signature verification system.
- **Cheque Authentication:** Banks have been using handwritten signature verification system for cheque authentication for many years. Even experts on forgeries can make mistakes while verifying a handwritten signature. Offline handwritten signature verification system can be utilized for cheque verification in banks.
- **Forensic Applications:** Handwritten signature verification techniques have been used for forensic applications.

1.3 Model for Signature Verification System

Model for handwritten signature verification system is given in Figure 1.6. During enrollment stage (registration), system input comprise of input handwritten signatures produced by the user who is going to be enrolled. The input data is first preprocessed. After that features are extracted from the preprocessed data. The obtained data is stored in a database with a unique identity (ID). A threshold on the matching score is calculated from the users training data.

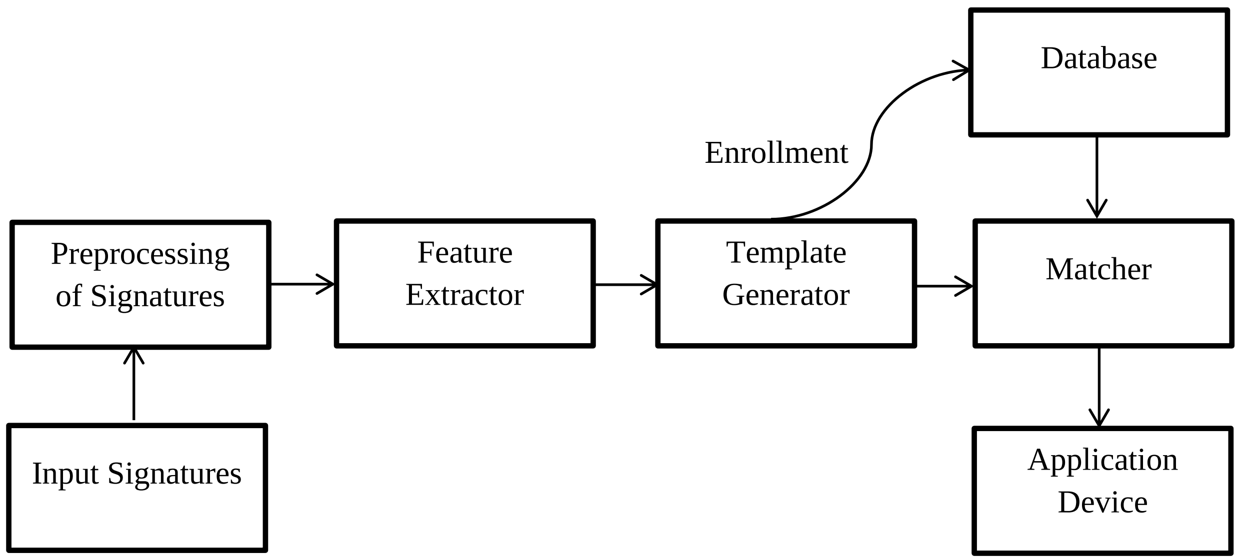


Figure 1.6: Model for Signature Verification System [8]

For authentication, a test signature is fed to the system along with the claimed identity. Same preprocessing and feature extraction methods are used. The handwritten signature is then compared to the average of signatures that are stored in database. Difference between the testing signature and average of stored signatures is calculated. Then resulting difference value is compared with pre-calculated threshold. After comparison the signature is as accepted or rejected.

1.3.1 Acquisition

In offline mode, the acquisition step transforms a number of sheets to a group of digital images. A single sheet contains one or more handwritten signatures. The two ways to acquire online signature are scanning of signed paper sheets and with digital tablets. In offline mode only static features are recorded.

1.3.2 Preprocessing

This step is a continuity of image transformations. This step creates the best possible input for next step, which is feature extraction step. In on-line signature verification systems, the obtained signatures are of different sizes and there are fluctuations in writing. So in this preprocessing remove these fluctuations and make all signatures of same size. In the off-line signature verification system it is mandatory to eliminate the introduced noise, which was introduced during the acquisition step [8].

1.3.3 Feature extraction

Features can be categorized as static and dynamic features. First type features, static features are of two types- global and local features. In case of global characteristics, whole signature is concerned. On the other hand, local features are confined only to a small portion of signature. A large number of features of a handwritten signature is related to vertical projection length or the trace length etc. These parameters have distinct measurement values for different instances of same signature, but, they have a constant relative value. The following parameters are calculated in this manner [9].

- Area of signature: Area of signature is related pixels of the signature image. It is equal to the number of pixels in image.
- Height-to-width ratio: This parameter is obtained by dividing a handwritten signature's height by its width. Signatures of one user may vary in height and width. Signatures of one user have approximately same height to width ratio.
- Maximum vertical histogram: In this parameter the horizontal histograms of signature are calculated for each row. After that the row with the highest value is chosen as maximum horizontal histogram.
- Maximum horizontal histogram: Similarly for each column, vertical histograms are calculated. After that the column which have highest value, is elected as maximum vertical histogram.
- Signature's edge point numbers: Edge point of a signature is the pixel which has only one neighbour.

- Signature's local maxima numbers: In this parameter the number of local maxima of the vertical and horizontal histogram is calculated.

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1.3.3.1 Local Features

Local features are limited only to a small portion of signature.

Table 1.1: Local Features

Signature Length of Signature	Initial direction	Pen elevation of signature
Horizontal position of signature	Total velocity of signature	Acceleration X direction
Vertical position of signature	Velocity in X direction	Acceleration Y direction
Normal pressure of signature	Velocity in Y direction	Log radius of curvature of signal
Path tangent angle of signature	Total acceleration of signature	Pen azimuth of signature

1.3.3.2 Dynamic Feature Extraction

The dynamic features are more difficult to replicate. The most widely used dynamic features are: velocity in X or Y direction, absolute and relative speed between two crucial points, force, acceleration in X or Y direction, position pressure. The list of dynamic features is given below:

- Signing time: In this feature the total time taken to sign the signature is recorded. Signing time can be calculated by calculating the number of coordinates, which were recorded during the signing process.
- Maximum velocity: Velocity is found between two consecutive points of coordinates. Maximum of all velocities is selected.
- Minimum velocity: Minimum of all velocities is selected.
- Number of pen ups: In this feature, the number of times when pen leaves the screen is calculated at the time of signing process. For every time the pen is up a ";" is recorded. The total number of recorded ";" is called the number of pen-ups.
- Duration of V_x : The total time taken by the pen in moving from left to right is recoded by this feature. It is calculated by summing up the number of times the pen is going from left to right between two consecutive coordinates of signatures.

- Duration of V_y: The total time taken by the pen in moving from down to up is recoded by this feature. This time is calculated by summing up the number of times the pen is going from down to up between two consecutive coordinates of signatures.

1.3.4 Learning

This step makes use of the extracted features to calculate various defined parameters. Standard deviation, mean etc. are calculated for each feature. The obtained values are used as feature vector. These feature vectors are stored in the database against the unique identification number of user. To provide higher accuracy, the number of learning samples should be more.

1.3.5 Classification

In this step the test signatures are compared with the reference signatures. Feature vectors for each user are calculated. These feature vectors are matched against the already stored feature vectors of reference signatures. The resulting difference is compared with the threshold. After that the signatures are classified as genuine and forgery.

1.3.6 Fractional Transforms

The Fourier transform is widely used in signal processing. The fractional Fourier transform is representation of frequency domain and time domain. The FrFT is a generalized form of the ordinary Fourier transform (FT). It was introduced 75 years ago, but it has been widely used in signal processing, quantum mechanics and optics only in the last two decades. The Fourier Transform (FT) is widely used in signal processing. Fourier transform is widely used in many engineering applications. The Fractional Fourier Transform was proposed in by Victor Namias in1980 [10]. It was established in same year that other one can fractional other transforms too. McBride and Keer analyzed its mathematical definition in 1987 [11]. FrFT has become powerful tool for signal processing in a very less time.

1.4 Dissertation Structure

This dissertation consists of 6 chapters, which are organized as below:

Chapter 1: Introduction, firstly concept of biometrics and type of biometrics are given, after that signature verification has been discussed. Two mode of signature verification and general model for verification have also been discussed.

Chapter 2: Literature review, in this chapter, the work which has been done regarding designing of various methods for online/offline handwritten signature verification has been studied. Comparison is also done between the different methods in this chapter and based on that gaps in study are found.

Chapter 3: Online signature verification system, this chapter discusses the phases of handwritten signature verification system. Various methods for phases involved in verification have been discussed.

Chapter 4: Fractional transforms, this chapter studies the fractional transforms i.e. fractional Fourier and fractional Cosine transform. Applications of fractional transforms are also given in this chapter.

Chapter 5: Signature verification System using discrete fractional cosine transform (DFrCT) is discussed. It is compared with the existing methods and observed that proposed method gives better performance in terms of performance in terms of parameters EER, FA, FR.

Chapter 6: Conclusion, in this chapter whole work has been concluded; also future scope has been discussed on the basis of observations of experiments.

Chapter 2

Literature Survey

2.1 Introduction

The work which has been done regarding Online/Offline signature verification systems and Fractional transform is overviewed in this chapter. In the literature survey different areas of work regarding the topic, in which further research has to be done are discussed. At the end of chapter objective for dissertation has been given.

2.2 Online Signature Verification

A.K.Jain *et al.* [2] made a discussion regarding on-line handwritten signature verification. A digitized tablet was utilized to acquire both information of writing i.e. both dynamic and spatial information. Extraction of several features is done after preprocessing the signature. To authenticate a writer comparison is made out between the input signature and stored feature vectors of the writer. In order to measure the correlation between signatures to be tested and the reference signatures, string matching is used. After this, the obtained correlation value was matched with a threshold. In this paper, FRR was obtained as 2.8 % and FAR was obtained as 1.6%.

P. Thumwarin *et al.* [12] discussed a technique of online signature verification system. In this method FIR filters were used. First, the DCT of the characteristics was calculated in order to minimize the fluctuation. After that features are extracted.. After this three FIR subsystems were introduced in the system. Feature vectors are obtained using the impulse response of the subsystems. The difference between impulse responses was used to check whether the signature is original or not.

SaeidRashidi *et al.* [13] proposed an method which used discrete cosine transform. This method is applied to nineteen time signals such as position, angles of pen and pressure. Two databases, SVC2004 and SUSIG were used to carry out the experiments. Best performing signals were obtained using a simple mean and variance based methodology. Different classifiers were used to test proposed system for skilled forgery. For SVC 2004, equal error rate was 5.07 %. For SUSIG equal error rate was 4.33%.

Edson J. R. Justin *et al.* [14] discussed different forgery types for signature verification. In this paper HMM (Hidden Markov Model) frame work was used. The experiments conducted in this paper concluded that the results regarding EER(Equal Error Rate) for simple and random forgery signatures were too much close. Simple forgery representation of principal fraudulent case as a real application is shown by the result.

F L. Liu *et al.* [15] discussed an approach to identify handwritten signatures using variance. In this paper variance and Dynamic Time Warping algorithm were combined. These algorithms were combined to calculate the interclass distance and intraclass distance. This paper was very simple and efficient. A conclusion was drawn in this paper that the signatures of real forged ones shows much difference than the real ones.

S.Emerich *et al.* [16] used method which was based on TESPAR DZ for online handwritten signature system. At initial stage, decomposition of pen position parameter of handwritten signature into multiscale signals is done. In this paper TESPAR DZ based method was used. A fixed dimension feature vector was obtained for each analysed time. The models were tested with Support Vector Machines after training.

D. Muramatsu *et al.* [17] discussed a which is based on sequential Monte Carlo. In this web camera was used for data input. For the tracking of pin tip sequential Monte Carlo was used. In this several distances were computed from online signature. In the end a fusion model was used to compute the final score by combining distances. A private database was used to conduct preliminary experiments.

Q.Z.Wu *et al.* [18] proposed a method which was based on correlation recording of logarithmic spectrum for online signature verification. Extraction of principle component of logarithmic spectrum is done in order to compute the correlation between the signature to be tested and the reference signature. After this, the obtained logarithmic spectrum was compared with threshold to check the authenticity.

A. Zimmer *et al.* [19] discussed a new hybrid handwritten signature verification system. The segmentation of scanned offline data was done on the basis of acquired online data, which is obtained using a digitized tablet. In this paper a learning technique which is self-adjustable is used to emphasis the feature extraction process to determine the foci of attention. Processing of both local and global primitives was done and then decision about authenticity of specimen was checked by similarity measurements.

Takahiro Yoshida *et al.* [20] discussed the DCT spectrum distance of genuine signatures after DP matching becomes tremendously small, also DCT spectra in low frequency domain had essential individuality. In this paper 20 Japanese persons signatures were used for experiments. 30 DCT spectra of Y position was calculated, which was used for verification of individuals with EER (Equal Error Rate) of 1.7%. The EER obtained in this paper was similar to the results obtained by the signature verification system in which whole series data of pressure and XY position was used.

MohitArora *et al.* [21] discussed a discrete fractional cosine transformation (DFrCT) based methodology for online signature verification. Various features of hand written signatures were extracted using six characteristics of the hand-written signature. Three FIR systems were characterized for realization of system for the signature verification. Euclidean norm is calculated by impulse response of the FIR system. Feature vectors are calculated by the impulse responses of FIR systems. Euclidean norm of difference between the features of the signature to be tested and the reference signature is calculated. After that this Euclidean norm is matched with the threshold. After this the signature is declared as accepted or rejected. The efficiency of proposed method is checked by the equal error rate (EER). DFrCT obtain superior results than discrete cosine transform (DCT). In this paper SVC2004 signature database was used for handwritten signature verification experiment.

R.K.Bharathi *et al.* [22] discussed a method in which the transform based approach discrete cosine transform (DCT) was combined with dimensionality reduction technique (2D2FLD). This technique was used for off-line signature verification. Features were extracted with the use of DCT in this paper. Then these features were fed into 2D2FLD for reduction. This thing represented the signature sample with optimal set of features. Features collected from all the samples in the dataset which included both genuine and skilled forge samples form the knowledge base. The performance of this method was examined using support vector machine (SVM) with 2 set of experimentations.

Ritesh C. Sonawane *et al.* [23] discussed a method in which various features of dynamic signature were described by consideration of spatial and time domain characteristics. In this paper there was an individual identification strokes by locating the points where pen tip pressure decreases, rapid change in pen angle and decrease in pen velocity.

Canggih Puspo Wibowo *et al.* [24] discussed a method which used forward and backward variance. In this paper it was shown that the difference between K-L coefficients of the backward and forward variances between reference and test signature was computed for the recognition of signatures. Experiments of this paper were carried out on MCYT-100 database.

J.F. Velez *et al.* [25] discussed a comparison of different algorithms for offline signature verification system. For segmentation snakes were applied to the images. In this paper many hybrid snake algorithms were introduced for practical demand of signature verification system. Fuzziness was introduced for the extraction of features. In this paper four hybrid snake based approaches were compared.

Roushanak Rahmat *et al*. [27] discussed a method for online signature authentication which used Singular Value Decomposition (SVD). The proposed method comprise of four steps: 1) Input phase and preprocessing of input 2) Feature extraction step 3) Comparison step 4) Result The handwritten signature verification system's performance was checked with reduced-sensor. In this paper F-value for each sensor was utilized for the selection of the most important sensors. The proposed method of this paper investigated with a number of genuine and false signatures. It offers a secure environment for applications like banking transactions etc.

H. Feng *et al.* [28] discussed a new warping technique. This warping technique is used for the functional approach. The broadly used warping technique is Dynamic time warping (DTW). It was originally used in speech recognition. Now this technique has also been utilized in the area of signature authentication. The technique which was proposed in this paper is known as extreme points warping (EPW). This warping technique was proved more superior in the area of signature authentication system.

S. Shirato *et al.* [29] discussed a method for online handwritten signature authentication. This method is camera-based. From a camera time-series images were obtained while a handwritten signature was being written. A particle filter (Sequential Monte Carlo) was used to achieve the online signature data. Data is achieved by tracking the pen tip from the images. The proposed system has an advantage that special devices such as tablets were not necessary. Different camera positions had an effect on verification accuracy because the signature shapes obtained by tracking pen tip changes with change in camera position.

M. T. Ibrahim *et al.* [30] discussed a method in which the use of trajectories in isolation is introduced. First, velocity and pressure profiles are decomposed into two partitions. Underlying horizontal and vertical were obtained in next step. The whole process was a process decomposing signature trajectories exploited the inter-feature dependencies. Each partition was verified separately which depended upon the velocity and pressure information. It was possible to extract eight features which were discriminating. After that the most stable discriminating feature among them was used. Principal Component Analysis (PCA) in this paper had been discussed to make the handwritten signatures rotation invariant. Obtained results demonstrated that the proposed method is superior than other methods.

C. T. Yuen *et al.* [31] proposed a method in this paper used dynamic parameters of the signature: velocity and position, pen pressure. The system was proposed to read, process and verify the signatures from the SUSIG online database. Firstly, the testing and reference samples were normalized, re-sampled and smoothed through pre-processing step. In verification stage, the difference between testing and reference signatures was calculated. This was based on the proposed threshold standard deviation method. A probabilistic acceptance model had been designed.

2.3 Offline Signature Verification

Saba Mushtaq *et al*. [32] discussed more reliable biometric feature signature verification. This paper presented a survey of handwritten signature verification systems. This paper also gave comparison of the various methods used for handwritten signature verification.

Othman O-khalifa *et al.* [34] discussed offline signature verification schemes which considered as a highly secured technique to recognize the genuine person's identity. It proposed a offline signature verification technique using Artificial Neural Network (ANN) method. It also explained the fundamental characteristics of offline signature verification processes also it highlighted the comparison among various offline signature verification approaches.

A.Hamadene *et al.* [35] discussed a new feature generation method for two different approaches of offline handwritten signature verification which were writer independent and writer dependent HSV. This method used contour let transform and cooccurance matrix conjointly. CEDAR dataset was used to conduct experiments. SVM (Support Vector Machines) was used to conduct classification.

M. E. Karslıgil *et al.* [36] discussed an offline handwritten signature verification system. This system used the global features as well as directional features and signature's grid features. For classification and verification, Support Vector Machine (SVM) was used. In this paper obtained classification ratio was 0.95. Verification of signatures represented a multiclass problem. In this paper, SVM's one-against-all method was used. In this paper, there was a comparison between the SVM method Artificial Neural Network‘s (ANN) back propagation method.

I. S. I. Abuhaiba *et al.* [37] discussed a handwritten signature verification method which was based on the binary pixel intensities of raw. A graph matching problem is utilized for authentication problem. This method verified genuine and forgery signatures which were made by five subjects. In this method at the point of equal FR and skilled forger FAR for random forgery became very small. This was the positive property of algorithm which was discussed this paper. The verification time was in two seconds range for the normalization size of 32 × 64 pixels.

Yazan M. Al-Omari *et al.* [38] discussed the verification system for offline signatures. The method of this paper was based on the State-of-the-Art of offline signatures. Due to its of its need in daily life routines, this verification technique had more attraction in last years, as this mode is used for verification of bank checks. Different types of forgeries of handwritten signature were also discussed. In this paper, types of feature, popular methods used for features extraction are also discussed.

I. Guler *et al.* [39] discussed a method that is based on global features. In this different signature shapes and dynamics of signature were produced. In this paper, the discussed method did not pay any attention to the size and thickness of it. The obtained results of this paper had shown the perfectness of proposed algorithm. At first, preprocessing of signatures was done to remove noise. In next step, the signature was studied to extract its features. These features were store in database for the comparison. In next step, dynamic time warping algorithm was used to classify a signature as a forgery or an original one.

K. Huang *et al.* [40] discussed a method of statistical mode for both pixel distribution and structural layout description. The directional frontier feature of the signature was used as a structural descriptor. The verification algorithm which was based on the geometric handwriting feature was used to attain the signatures which were very similar to the reference samples The proposed algorithm verified the correlation between the input and reference signatures in to detect skilled forgeries.

Srikanta Pal *et al*. [41] discussed a method which combined two techniques SURF and Gabor filter. In this paper classification of 50 pairs of original and forged signatures was done. GPDS database was used in this paper. The proposed method could be used even when a part of handwritten signature was missing. The dimension of feature vector was 2628. In future the dimension would be reduced by using PCA or some other feature selection methods.

2.4 Fractional Transform

S. C. Pei *et al*. [42] explored the continuous fractional Fourier transform (FrFT). The fractional Fourier transform is a rotation of signal in time-frequency plane. In this paper a discrete version of fractional Fourier transform was calculated. The obtained results were no same as in case of continuous case. It provided similar transform as those of continuous fractional Fourier transform. It has become powerful tool for signal processing

L. B. Almeida [43] introduced FrFT and number of its properties. Some new results like interpretation as the rotation in time frequency plane and the FrFT‘s relationship with the time-frequency representation like ambiguity function, Wigner distribution, the short time Fourier transform and its spectrogram. In this paper some examples of FrFT of simple signals had been given. This paper also explained the example of its applications which showed how the usage of FrFT allowed a analysis of swept frequency filters that same to classical analysis of shift-invariant filter with the Fourier transform. This paper presented The extension of classic Fourier transform which was designated as fractional Fourier transform is presented in this paper. The transform depends upon the parameter alpha.

J. J. Ding *et al*. [44] discussed a new type of DFrFT. The new discussed DFrFT was unitary, revocable and adaptable. DFrFT had been studied in many types. Its performance was same as the continuous fractional Fourier transform. FFT can be used to calculate it. Continuous affine Fourier transform (AFT) is generalization of continuous FrFT . It is called canonical transform. DFRCT was extended to into the discrete affine Fourier transform (DAFT) in this paper.

T. Alieva *et al.* [45] discussed that Fractional cosine and sine transforms were additive on the index. Both reserve the same relationships with the FrFT as the FT had with ordinary sine transforms and cosine transform (ST, CT). The key features of the fractional cosine transform (FrCT) and fractional sine transforms (FrST) were computed. There were distinct methods for the fractionalization of transforms like the CT, the ST, and the FT. Fractional CT and ST were also measured in relative to the fractional in this paper.

S.C Pei *et al.* [20] discussed Discrete Fractional Cosine Transform (DFrCT) and Discrete Fractional Sine Transform (DFrST). Eigen decomposition DCT (Discrete Cosine Transform) and DST (Discrete Sine Transform) kernels were used to define DFrCT and DFrST. The same procedure was followed by the Discrete Fractional Fourier Transform. In paper Eigen vector and eigen value relationships between DFrFT, DFrST and DFrCT were established.

Table 2.1: Comparison Table

Year	Authors	Technique	Method	Results
1996	T. Matsuura *et.al.*	Online	FIR filter design	FAR was found to be 2-12% and FRR was found to be 0-2%.
1997	C. Schmidt *et al.*	Online	Establishment of Personalized Templates	Right acceptance rate *78*% and right rejection rate of 100% were achieved respectively.
1998	Q. Z. Wu *et.al.*	Online	Based on logarithmic spectrum	The FRR and FAR were 1.4 and 2.8%, respectively
2000	E. J. R.Justino *et.al.*	Offline	Hidden Markov Model	Mean error was approximately .88%

2001	Y. Mizukami *et.al.*	Off line	Extracted Displacement Function	The proposed system achieved error rates as low as 24.9%,
2002	A. K. Jain *et el*.	Online	String Matching	FRR of 2.8% and FAR of 1.6% were obtained.
2002	K. Huang *et al.*	Offline	Statistical model	The obtained FRR and FRR were 6.3% and 8.2%.
2003	A. Zimmer *et al.*	Offline/Online	Segmentation of signature	EER of about 5% when reference signatures are 5 and 1% when reference signatures are 10, are achieved.
2003	B. Fang *et al.*	Offline	Tracking of stroke and feature positions	In this FAR, FRR and EER were obtained 23.3%, 23.5% and 23.4% respectively
2003	H. Feng*et al.*	Online	Extreme points warping technique	There was an improvement by a factor of 1.3.over using DTW.
2005	M. E. Karslıgil *et al.*	Offline	Support vector machine	The EER of 5% reach achieved.
2007	D. Muramatsu *et al.*	Online	Effectiveness of Pen Pressure, Altitude and Azimuth Features	EER of 3.61% with user-dependent thresho-ld parameter, and for global threshold parameter it was 10.15

2007	I. S. I. Abuhaiba	Offline	Graph matching	For skilled forgery obtained EER was 26.7 %. For random it was 5.6% .
2008	Guler *et al.*	Offline	Dynamic time warping	EER of 25.1% was achieved for skilled forgery and 5.5% was achieved for random forgeries.
2009	L. Liu *et al.*	Online	Combined variance with dynamic time warping	The resulted FRR is 15.8% and FAR is 0.85%.
2009	B. Schafer *et al.*	Offline	Euclidean distance	EER of 15.9 was Achieved.
2009	D. Muramatsu *et al.*	Online	Sequential Monte Carlo	An EER of 3.2% was achieved.
2010	T. Matsuura *et al.*	Online	Based on fir	An EER of 7.04% was achieved
2010	S. Emerich *et al.*	Online	Wavelet transforms technique**.**	The result was a success rate of 92.37%
2010	S. Shirato *et al.*	Online	Sequential Monte Carlo	An EER of 6% was achieved
2010	L. Nanni *et al.*	Online	Dynamic Time Warping, a Hidden Markov Model	An EER of 4.51% was achieved

2010	M. T. Ibrahim *et al.*	Online	Velocity and pressure based partitions.	An EER of 3.4% was Achieved
2011	C. T. Yuen *et al.*	Online	Probabilistic Model	FRR of 14.8% and FAR of 2.64%
2014	M. Arora *et al.*	Online	DFrCT	EER of 5% was achieved
2014	R. K Bharathi	Online	DCT and SVM	FAR 7.84%, FRR 9.36% was achieved

2.5 Gaps in Study

Many methodologies for signature verification system have been developed till date. There are online verification schemes such as extreme points warping technique, as Sequential Monte Carlo, Wavelet transforms technique etc. and offline verification schemes such as Hidden Markov Model, Graph matching, Euclidean distance etc. But this field of signature verification is still under development phase. Many new methodologies are yet to be explored. One such unexplored methodology is Discrete Fractional Cosine Transform. More features can be added to reduce EER.

2.6 Objectives

On the basis of literature review of existing signature verification systems, gaps were identified, following are the objectives of this study:

- To study the existing online signature verification techniques.
- To develop a technique based on DFrCT with more features.
- To compare the technique based on DCT and DFrCT methods with that of DFrCT with more features.

Chapter 3

Online Signature Verification System

3.1 Introduction

Handwritten signature verification is a behavioural biometric. It is one of the widely used biometrics. The verification system can be functioned in modes:

Static: In static mode, signers sign their signatures on paper. After that signatures are digitized by an optical scanner or a camera. After that their shape is studied for recognition. This system is also known as "off-line system".

Dynamic: In dynamic mode, digitizing tablet is used for acquisition. These tablets attain the signatures in real time as shown in Figure 3.1. Acquisition in this mode can also be done with the help of stylus-operated PDAs. This mode is also called as "on-line system". In dynamic mode the obtained data comprise of the following information [6].

Dynamic information comprise of the following information:

- Spatial coordinate x(t)
- Spatial coordinate y(t)
- Pressure p(t)
- Azimuth az(t)
- Altitude al(t)
- Pen up/down

One must not confuse dynamic signature devices with electronic handwritten signature capture systems. Electronic handwritten signature capture systems are used to capture a graphic image of the handwritten signature. Online signature verification system comprised of three main states: data acquisition (which means input) and preprocessing for fluctuation deduction, feature extraction (feature vectors are calculated) and verification. In enrolment phase, the input signatures are processed. After processing that the features are extracted, then feature vectors are stored into the database.

Figure 3.1: Digitizing Tablet

3.2 Data Acquisition and Management

Handwritten signature verification system can be splited into two types based on acquisition method: online systems and offline systems as shown in Figure 3.2. Data acquisition in offline system is done after the writing process. In offline case, the signature is expressed as a gray level image $G(x, y)$ $0 \leq x \leq X$, $0 \leq y \leq Y$. $G(x, y)$ means the gray level at the x, y position of the image. On the other hand, in dynamic systems acquisition devices, tablets are used which generate electronic signals which represent signature during the signing process. The handwritten signature can be expressed as a sequence $\{G(n)\}_{n=0,1...N}$, where $G(n)$ is the sampled signal value at time $n\Delta t$ of the signing process, where Δt is the sampling period [47].

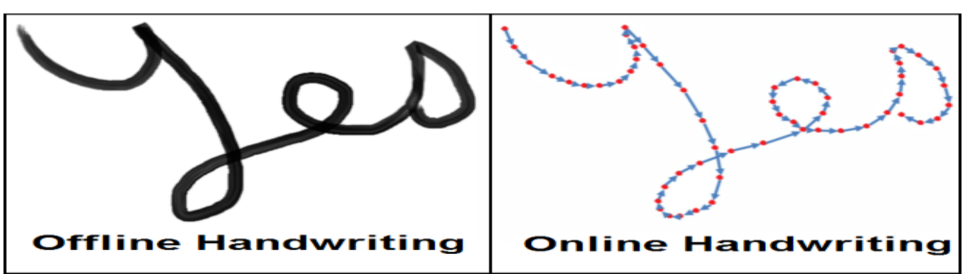


Figure 3.2: (a) Offline Signature (b) Online Signature

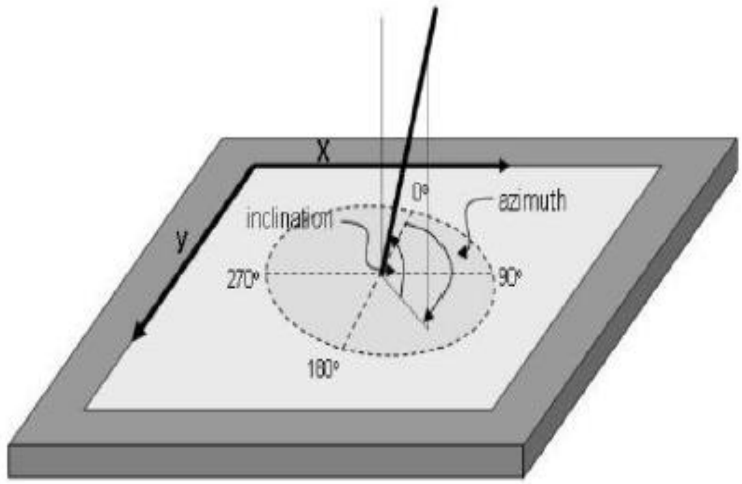


Figure 3.3: Electrical Signals Acquired by Digital Tablets

3.3 Preprocessing

This step is an important step because it strong affects all the other steps of handwritten signature verification. This step is called preprocessing. Signatures written by the same person differ from each other due to location, size and pen movement. So preprocessing is done to remove these fluctuations.

3.4 Feature Extraction

Features used for handwritten signature verification are of two types: functions or parameters. The handwritten signatures in function features are represented in terms of a time function. The value function feature establishes the feature set. Signatures are represented as a vector of elements in parameter features. A value of a feature is represented by each vector. Parameters can be categorized into two main types: global and local. The whole signature is concerned global parameters. Total time duration of a signature, number of components, number of pen lifts, global orientation of the signature, coefficients obtained by mathematical transforms are some examples of global parameters etc.

In Table 3.1, function features which are commonly used in signature verification system, are given. Broadly used functions for for online signature verification are position, velocity, and acceleration. Acquisition devices directly records the position function, whereas velocity and acceleration functions numerically derived from position the acquisition device and can be captured by acquisition device directly. The most frequently used functions are pressure and function. Distinctive devices have been established to record these functions. Pressure information has been used in signature verification to take advantage of inter-feature dependencies. To boost the performance in handwritten online signature verification systems, direction of pen inclination and pen movement also have also been used. Pen trajectory functions have been extracted from offline signature verification systems.

Table 3.1: Function Features [47]

Functions Category	Category
Position	Online
Velocity	Online
Force	Online
Pressure	Online
Acceleration	Online
Pen inclination	Online
Direction of pen movement	Online

The most consistent features are position, pen inclination and velocity in online handwritten signature verification. These are consistent features only when a distance-based consistency model is used. Table 3.2 gives the various signals that can be used in online signature verification system.

3.5 Verification

In the verification process, the verification of the user take place by matching the features against the stored feature vectors, which were stored during the enrolment (registration) stage. The result of this process is a single response (accepted or rejected). This step states the verification of the test signature.

Table 3.2: List of Features [13]

	Feature Description		Feature Description
1	Coordinate $x(t)$	2	Coordinate $y(t)$
3	Absolute position, $ap(t)=\sqrt{x^2+y^2}$	4	Velocity in x, $v_x(t)$
5	Velocity in y, $v_y(t)$	6	Absolute velocity, $v(t)=\sqrt{v(t)_x^2+v(t)_y^2}$
7	Velocity in $ap(t)$, $v_{ap}(t)$	8	Velocity vector's angle, $\theta_v=tg^{-1}\frac{v_y(t)}{v_x(t)}$
9	Acceleration in x, $a_x(t)$	10	Acceleration in y , $a_y(t)$
11	Absolute acceleration, $a(t)=\sqrt{a(t)_x^2+a(t)_y^2}$	12	Tangential acceleration, $a_{ta}(t)=\frac{v_x(t)a_x(t)+v_y(t)a_y(t)}{v(t)}$
13	Centripetal acceleration $a_c(t)=\frac{v_x(t)a_y(t)-v_y(t)a_x(t)}{v(t)}$	14	Acceleration of $r(t)$, $a_r(t)$
15	Acceleration vector's angle $\theta_a=tg^{-1}\frac{a_y(t)}{a_x(t)}$	16	Pressure, $p(t)$
17	Azimuth, $az(t)$	18	Altitude angle, $al(t)$
19	Curvature, $c(t)=\log\frac{v_x(t)a_y(t)-v_y(t)a_x(t)}{v^3(t)}$		

Chapter 4

Fractional Transforms

4.1 Introduction

In recent years, fractional transforms have been examined broadly in engineering science and applications. The fractional integral and derivative are described by many mathematicians. Some physical problems are also solved with the help of fractional transforms. The fractional Fourier transform is widely used in the signal processing. Real-world data like dust in the air, neuron networks in the body, clouds and coastline are recoded with the help of fractional dimension. The fractional dimension is extensively used in pattern classification and recognition. Non-Gaussian signals are analysed by the fractional lower order moment. Rotation of signals in the time–frequency plane corresponds to FrFT. It is a special case of the linear canonical transform. FrFT provides a tool to analyze the mixed time and frequency components of signals. The FrFT is a generalized form the ordinary Fourier transform. It has an order parameter α. If value of free parameter α is equal to π/2 then FrFT is equal to ordinary FT. The ordinary FT is extensively used in distinct fields like signal processing, communications and control systems. Fractional Fourier transform also find applications in these fields. The fractional Fourier transform is related to the group of time-frequency representations. This transform broadly utilized by the signal processing area. A plane with two orthogonal axes is normally used in time–frequency representation. Time and frequency are the two axes [43]. Assume a signal $x(t)$. The signal $x(t)$ is to be characterized along the time axis and $X(f)$ is its ordinary Fourier transform which is to be characterised along the frequency axis. F, which is the Fourier transform operator can be taken as alteration in depiction of the signal. Alteration means clockwise turning of the axis by an angle of π/2.

4.2 Fractional Fourier Transform (FrFT)

FrFT, which is a generalization of conventional FT. FrFT, is very affluent theory. It is affluent not in theory only but is more flexible in application too. FrFT is also cheaper in implementation. FrFT is a very influential tool to analyze the signals which are time varying. In any area where frequency domain is used, potential for implementation and generalization by using fractional Fourier transform also exists in that area.

FT of a function is a linear differential. The FrFT computes this differential operator by making it depend on a parameter α. Mathematically, α th order FrFT is the α th power of FT operator [45].

The FrFT $F^{\alpha}(\mu)$ of a function $f(x)$ is defined as

$$F^{\alpha}(\mu) = R_F^{\alpha}\left[f(x)\right](u) \tag{4.1}$$

$$F^{\alpha}(\mu) = \frac{1}{2}\int_{-\infty}^{\infty} k_{\alpha}(x,\mu) f(x) e^{-\frac{j\mu x}{\sin\alpha}} dx \tag{4.2}$$

where α the rotation is angle and $k_{\alpha}(x,\mu)$ is the kernel. Kernel is given by

$$k_{\alpha}(x,\mu) = \frac{e^{j\frac{1}{2}\alpha}}{n} e^{i\theta}\left[\frac{1}{2} j\left(x^2+\mu^2\right)\cot\alpha\right] \tag{4.3}$$

For $\alpha = \frac{1}{2}\pi$ kernel is equal to 1. When $\alpha = 0$, we have normal FT i.e identity transformation $F^{0}(x) = f(x)$. For $k_{\alpha+\pi}(x,\mu) = k_{\alpha}(\mu,x)$ and $F^{\alpha+\pi}(\mu) = F^{\alpha}(-\mu)$. Also $k_{-\alpha}(x,\mu) = k_{\alpha}^{*}(x,\mu)$.

4.3 Fractional Cosine Transform

FrCT is defined as

$$F_c^{\alpha} = R_c^{\alpha}\left[f(x)\right](u) \tag{4.4}$$

$$F_c^{\alpha} = R_c^{\alpha}\left[f(x)+f(x)\right](u) \tag{4.5}$$

$$F_c^{\alpha}(\mu) = F^{\alpha}(\mu) + F^{\alpha}(-\mu) \tag{4.6}$$

$$F_c^{\alpha}(\mu) = \sqrt{\frac{2}{\pi}}\int_{0}^{\infty} k_{\alpha}(x,\mu)\, f(x)\cos\left(\frac{\mu x}{\sin\alpha}\right) dx \tag{4.7}$$

It reduces to normal CT for $\alpha = \frac{1}{2}\pi$. In order to show in a different approach, the link among the FrFT of a causal, function which is one sided and FrCT of this function, the kernels T^{α} of the fractional transform can be written by

$$T_F^{\alpha} : \left(\frac{1}{\sqrt{2\pi}}\right) k_{\alpha}(x,\mu) e^{\left(-\frac{j\mu x}{\sin\alpha}\right)} \quad (4.8)$$

$$T_c^{\alpha} : \left(\frac{1}{\sqrt{2\pi}}\right) k_{\alpha}(x,\mu) \cos\left(\frac{\mu x}{\sin\alpha}\right) \quad (4.9)$$

T_c^{α} is related to the even part of T_F^{α}. [45]

4.4 Properties

Many properties of the FrFT can be immediately translated to the FrCT [45]. In particular, fractional transforms holds the *additive property* for the angle α. It is given by

$$T^{\alpha_1} T^{\alpha_2} \left[f(x) \right](\mu) = T^{\alpha_1+\alpha_2} \left[f(x) \right](\mu) \quad (4.10)$$

Scaling property for FrCT is given by

$$T_c^{\alpha} \left[f(\lambda x) \right](u) = C T_c^{\beta} \left[f(x) \right]\left(\frac{\mu \sin\beta}{\lambda \sin\alpha}\right) \quad (4.11)$$

Shifting property for FrCT is given by

$$T^{\alpha} \left[f(x - x_0) \right] \mu = e\left[-jx_0 \sin\alpha \left(\mu - \frac{1}{2} x_0 \cos\alpha\right) \right] \times T^{\alpha} \left[f(x) \right](\mu - x_0 \cos\alpha) \quad (4.12)$$

FrCT is *periodic* with period π

$$T_c^{\alpha+\pi} \left[f(x) \right](\mu) = T_c^{\alpha} \left[f(x) \right](\mu) \quad (4.13)$$

4.5 Applications of Fractional Transform

- FrFT has many applications in optics, wave and beam propagation, phase-space tomography and wave field reconstruction.
- FrFT is used for analyze of time-frequency and space-frequency distributions.

- In signal processing applications this transform is basically used for signal reconstruction, filtering, signal synthesis. It is widely used for signal detectors, image recovery, beam forming, matched filtering and pattern recognition.
- Fractional transforms are widely used in image processing.

Chapter 5

Signature Verification System using DFrCT and FIR Filters

5.1 Implemented Technique

An online handwritten signature verification method is presented in this dissertation, which is based on DFrCT. In this thesis previously proposed techniques based on DCT and DFrCT has been taken as reference. DFrCT is used to extract the features as shown in Figure 5.1. The proposed method is based on five FIR subsystems. In the first subsystem, link between the vertical and horizontal parts of the barycenter trajectory is characterized. DFrCT of horizontal component of barycenter trajectory is used as input of FIR filter and DFrCT of vertical component of barycenter trajectory is used as output of finite impulse response (FIR) filter. In the second subsystem, relation between the change of direction of barycenter and velocity is characterized. DFrCT of velocity is used as input of FIR filter and DFrCT of change of direction of barycenter is used as output of FIR filter. In the third subsystem, relation between the displacement and areal velocity is characterized.

DFrCT of displacement and DFrCT of areal velocity are used like input and output of FIR filter. In the fourth subsystem, relation between the pressure and absolute position is characterized. DFrCT of pressure is used as input of FIR filter and DFrCT of absolute position is used as output of FIR filter. In the fifth subsystem, relation between the altitude and azimuth position is characterized. DFrCT of altitude is used as input of FIR filter and DFrCT of azimuth is used as output of FIR filter.

5.2 Overview of the System

System proposed for online handwritten signature verification system shown in Figure 5.1 consists of following steps:

- **Input signature**: The SVC 2004 and SUSIG databases are used for experimentation. In this database, 40 signatures are provided for each user, out of which 20 signatures are forgery and 20 signatures are genuine.

- **Preprocessing process**: In order to remove the fluctuations, firstly signature is being preprocessed. Normalizing of size, location is calculated to make signatures of a standard size. Trajectory of barycenter is find to overcome the fluctuation related to pen movement.
- **Features extraction**: After preprocessing step, ten features namely horizontal pen point movement, vertical pen point movement, areal velocity, displacement, pressure, absolute position, azimuth, altitude, magnitude of velocity and change of angle trajectory of barycenter have been extracted. DFrCT of these features are calculated to define five FIR systems to characterize a given signature.
- **Database**: Feature vectors are calculated after the calculation of impulse responses of five FIR subsystems. These feature vectors are stored against an identity in the database for future purpose.
- **Signature verification**: In another step, impulse responses of FIR systems are used. All five impulse response are combined together to form a feature vector. To verify a given signature, difference between feature vectors of signature to be tested and reference signature is measured. After this, Euclidean norm of the difference is calculated and is matched with the defined threshold value. The given signature is genuine if the Euclidean norm of the difference is lower than the threshold value.

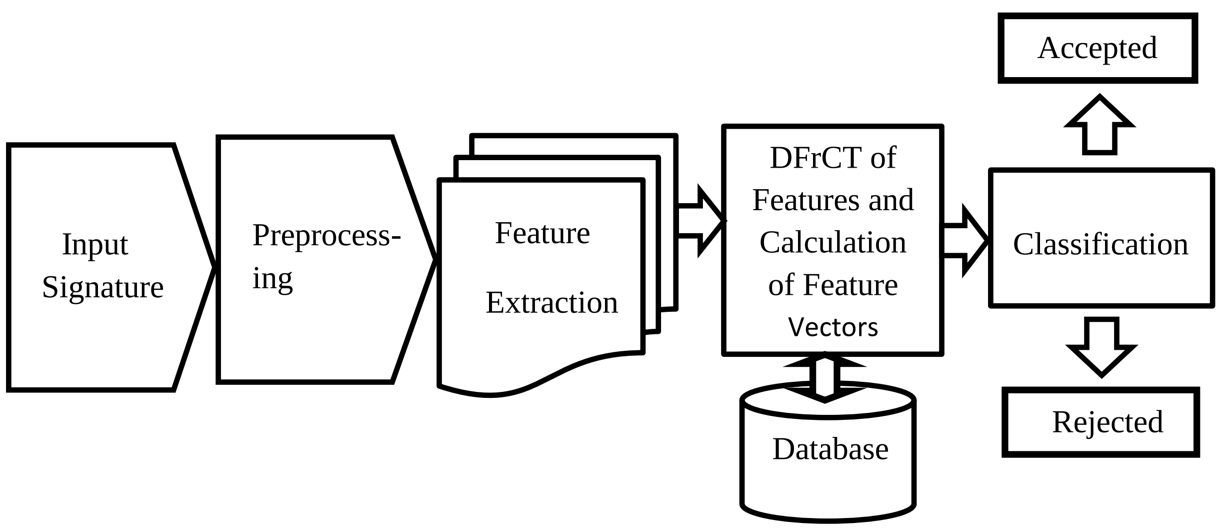


Figure 5.1: Overview of the System

5.3 SVC 2004 Database

The SVC2004 database was constructed in 2003. This database was developed to help partakers to create and test methods in order to take part in first signature verification contest (SVC2004). This database was recorded by Wacom Intuos 2 A6 tablet. This database comprise of x and y position, 2 angles (inclination and azimuth), pressure. In this database, for two different sessions every user wrote 20 genuine handwritten signatures. For privacy reasons, the users were told not to use their real signatures. The users were advised to design their new signature for the database. For each user 20 forgery signatures were recorded. The forgers were given the chance to watch the genuine signatures carefully to be reproduced in order to capture forgery signatures. A software application was used in this process.

5.4 SUSIG Database

SUSIG is a new online signature database. This database used for developing or testing signature verification algorithms. There are two parts of this database, one part was collected using pressure sensitive tablets (one without LCD and one with LCD display). 100 people contributed to each part. So this database consists of a total of 3000 genuine signatures. It contains 2000 skilled forgery handwritten signatures also. There are two parts of database: Visual and Blind subcorpora. Signatures in the Visual subcorpus were captured with the help of a pressure sensitive tablet having LCD display such that a user could see his/her signature while writing. In Blind subcorpus, no visual feedback was used.

5.5 Preprocessing Process

The horizontal component $x(t_n)$ and vertical component $y(t_n)$ of the signatures are collected with the help of a graphical tablet at time t_n. Preprocessing is done to remove the fluctuations, because signatures signed by same person can never be same. The steps of preprocessing are as follows

5.5.1 Size Normalization

This step is used to remove the fluctuations related to the size of signatures. This step results in a standard size of signatures.

$$w(t_n) = \frac{w(t_n) - w_{\min}}{w_{\max} - w_{\min}}, \ (w = x, y) \tag{5.1}$$

$$w_{\min} = \min(w(t_n)), \quad w_{\max} = \max(w(t_n))$$

5.5.2 Location Normalization

This step is used to remove the fluctuations related to the location of signatures. The standard size of signatures results in this step

$$c_w = \frac{1}{N} \sum_{n=0}^{N-1} w(t_n), \tag{5.2}$$

$$\tilde{w} = \hat{w}(t_n) - c_z, \tag{5.3}$$

where c_w is the center point of signature. Example of normalized signature is shown in Figure 5.3.

5.5.3 Barycenter Trajectory

In signing process, there are fluctuations in pen movement. Two adjacent pen-point positions are used in this step.

$$b_x(t_n) = \frac{x(t_n) + x(t_{(n+1)})}{3} \tag{5.4}$$

$$b_y(t_n) = \frac{y(t_n) + y(t_{(n+1)})}{3} \tag{5.5}$$

$b_x(t_n)$ and $b_y(t_n)$ are barycenter trajectories of horizontal and vertical components.

5.6 Feature Extraction

Features of tested signatures are extracted as given below :

- Horizontal component of barycenter trajectory.
- Vertical component of barycenter trajectory.
- Areal velocity which is equal to area swept in unit time.

$$av(t_n) = \frac{1}{2} \begin{vmatrix} b_{x-1}(t_{n-1}) & b_{y-1}(t_{n-1}) \\ b_x(t_n) & b_y(t_n) \end{vmatrix} \tag{5.6}$$

- Displacement $d(t_n)$ which is equal to distance from barycenter trajectory to center of signature.

$$d(t_n) = \sqrt{b_x(t_n)^2 + b_y(t_n)^2} \tag{5.7}$$

- Magnitude of velocity $v(t_n)$ is given by

$$v(t_n) = \sqrt{(s_x(t_n))^2 + (s_y(t_n))^2} \tag{5.8}$$

$$s_x(t_n) = b_x(t_{n+1}) - b_x(t_n) \tag{5.9}$$

$$s_y(t_n) = b_y(t_{n+1}) - b_y(t_n) \tag{5.10}$$

- Change of direction of barycenter trajectory

$$\theta(t_n) = \tan^{-1}\left(\frac{b_y(t_{n+1}) - b_y(t_n)}{b_x(t_{n+1}) - b_x(t_n)}\right) \tag{5.11}$$

- Pressure $p(t)$ which is equal to force per unit area.

- Absolute position which is equal to

$$ap(t_n) = \sqrt{x^2(t_n)^2 + y^2(t_n)^2} \tag{5.12}$$

- The altitude which is equal to the distance appears to be above the horizon.it is denoted by $al(t)$

- The azimuth $az(t_n)$ which is equal to the angular distance along the horizon to the location of sign. Features are shown in Figure 5.5.

5.7 Discrete Fractional Cosine Transform

The fractional Fourier transform is the generalized form of Fourier transform. It can be explained as revolving by some angle in time-frequency plane. In this all digital implementations are based on approximation of continuous version. This approximations depend on the number of samples, more number of samples leads to more successful approximation. DFrCT is a generalized form of discrete cosine tranform. $X_\alpha(z)$ as the FrCT of $x(t)$. Discretization of FrCT can be defined by the following method. The FrCT is defined as,

$$C_\alpha(p) = X(p) + X(-p) \tag{5.13}$$

$$C_\alpha(p) = 2\sqrt{\frac{1-\cot\alpha}{2\pi}} \times \int_{-\infty}^{\infty} e^{\frac{j\cot\alpha\left(p^2+t^2\right)}{2}} \cos\left(pt\csc\alpha\right)x(t)\,dt \tag{5.14}$$

The samples $l_\alpha(p)$ can be written as

$$Y_\alpha^c(m) = \sum_{L=0}^{L-1} F_\alpha^c(m,l)\,y(l) \tag{5.15}$$

$$m = 0,1...M-1$$

$$l = 0,1...L-1$$

Where $p = m\Delta p$, $t = l\Delta t$, $y(l)$ represents the input sample. The kernel $F_\alpha^c(m,l)$ can be defined as [44]

$$F_\alpha^c(m,l) = 2\sqrt{\frac{1-j\cot\alpha}{2\pi}}\,2\Delta t e^{\frac{i\cot\alpha\left(m^2\Delta p^2+l^2\Delta t^2\right)}{2}} \times \cos\left(ml\Delta t\Delta u\csc\alpha\right) \tag{5.16}$$

Conventional DCT-I is given by

$$C_{L+1} = \sqrt{\frac{2}{L}} \times k_m k_l \cos\left(\frac{ml\pi}{L}\right) \tag{5.17}$$

To get the same kernel, $\alpha = \frac{\pi}{2}$

Sample must be such that $\Delta t\Delta p = \frac{s\pi\sin\alpha}{L}$ where s is $\operatorname{sgn}(\sin(\alpha))$

$$y(n) = \frac{(2\Delta t)^2}{2\pi|\sin\alpha|}\sum_{k=0}^{L-1} y(k)\,e^{\frac{\cot\alpha\left(k^2-l^2\right)\Delta t^2}{2}}\sum_{m=0}^{M}\cos\left(\frac{sml\pi}{L}\right)\times\cos\left(\frac{smk\pi}{L}\right) \tag{5.18}$$

In order to reduce RHS, $F_\alpha^c(m,l)$ must be normalized, then kernel becomes

$$F_\alpha^c(m,l) = k_m k_l\sqrt{\frac{2(1-j\cot\alpha)|\sin\alpha|}{L}} \tag{5.19}$$

where $k_{m,l} = \begin{cases} \sqrt{\frac{1}{2}} & for\, m,l=0 \\ 1 & Otherwise \end{cases}$

5.8 Signature Verification

After preprocessing features are calculated. DFrCT is used to convert features into frequency domain which is used in calculation of feature vectors. Feature vectors are used for classification of signatures as genuine or fake.

Five subsystems are introduced in the identification system. In the first subsystem, link among the vertical part of the barycenter trajectory and horizontal part of the barycenter trajectory is characterized. DFrCT of horizontal component of barycenter trajectory is used as input and DFrCT of vertical component of barycenter trajectory is used as output of finite impulse response (FIR) filter. The optimal impulse response (h) can be found by decreasing least square error. M is order of filter.

$$\Lambda h = c \tag{5.20}$$

where $\Lambda = \delta_{d,m}$ (5.21)

$$\delta_{d,m} = \sum_{n=0}^{N} A_{b_x}(n-m) A_{b_x}(n-d) \tag{5.22}$$

$$(d, m = 0, ..., M)$$

where A_{b_x} and A_{b_y} are the autocorrelation functions of b_x an b_y

$$C = [c_0 c_1, ..., c_M]^T \tag{5.23}$$

$$c_d = \sum_{n=0}^{N} A_{b_y} A_{b_x}(n-d) \tag{5.24}$$

If Λ is non-singular, then the optimal impulse response h is given by

$$h = \Lambda^{-1} C \tag{5.25}$$

In the first subsystem, link among the vertical part of barycenter trajectory and horizontal part of the barycenter trajectory is characterized. DFrCT of horizontal component of barycenter is used as input and DFrCT of vertical component of barycenter trajectory is used as output of FIR filter respectively

$$F_y(k)=\sum_{m=0}^{M} h_1(m) F_x(k-m) \tag{5.26}$$

In the second subsystem, relation between the change of direction of barycenter and velocity is characterized. DFrCT of velocity and change of direction of barycenter are used like input and output of FIR filter.

$$F_v(k)=\sum_{m=0}^{M} h_2(m) F_\theta(k-m) \tag{5.27}$$

In the third subsystem, relation between the displacement and areal velocity is characterized. DFrCT of displacement and DFrCT of areal velocity are used like input and output of FIR filter.

$$F_{av}(k)=\sum_{m=0}^{M} h_3(m) F_d(k-m) \tag{5.28}$$

In the fourth subsystem, relation between the pressure and absolute position is characterized. DFrCT of pressure and DFrCT of absolute position are used like input and output of FIR filter.

$$F_{ap}(k)=\sum_{m=0}^{M} h_4(m) F_p(k-m) \tag{5.29}$$

In the fifth subsystem, relation between the altitude and azimuth position is characterized. DFrCT of altitude and DFrCT of azimuth are used like input and output of FIR filter.

$$F_{al}(k)=\sum_{m=0}^{M} h_5(m) F_{az}(k-m) \tag{5.30}$$

The algorithm for signature verification is given follow:

1. The feature vectors are defined as

$$h_1^{'}=\left[h_1(0),\, h_1(1),\ldots,\, h_1(m)\right] \tag{5.31}$$

$$h_2^{'}=\left[h_2(0),\, h_2(1),\ldots,\, h_2(m)\right] \tag{5.32}$$

$$h_3^{'}=\left[h_3(0),\, h_3(1),\ldots,\, h_3(m)\right] \tag{5.33}$$

$$h_4^{'}=\left[h_4(0),\, h_4(1),\ldots,\, h_4(m)\right] \tag{5.34}$$

$$h_5^{'}=\left[h_5(0), h_5(1),\ldots, h_5(m)\right] \tag{5.35}$$

2. Join the feature vectors

$$h' = \left\lfloor h_1' \, h_2' \, h_3' \, h_4' \, h_5' \right\rfloor \tag{5.36}$$

3. The signature is confirmed as

$$\begin{array}{ll} \text{True} & \text{if } \left\| h^{(ref)} - h \right\| < \eta \\ \text{False} & \text{Otherwise} \end{array} \tag{5.37}$$

where η is threshold.

5.9 Simulation Results

Simulated results are discussed in this section. Various extracted features and characteristics corresponding to a given signature are shown graphically here.

Normalization of Size and Location

Figure 5.2 shows the original signatures of a user, it can be seen from the figure that all signatures from same user are different, i.e. the fluctuations due to size and location are removed in this step. This step is used to remove the fluctuations related to the size of signatures. This step results in a standard size of signatures.

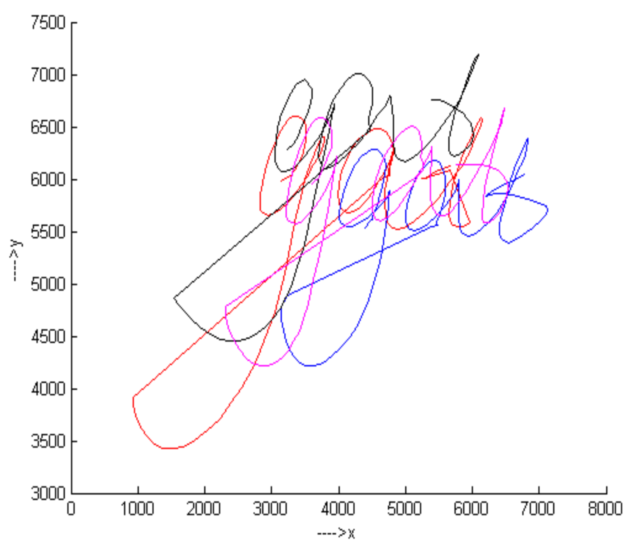


Figure 5.2: Original Signatures

Figure 5.3 shows the signatures after normalization of size and location. It can be seen that the after normalization of signatures their size and location is standardized. We can see that the fluctuation related to size and location is removed.

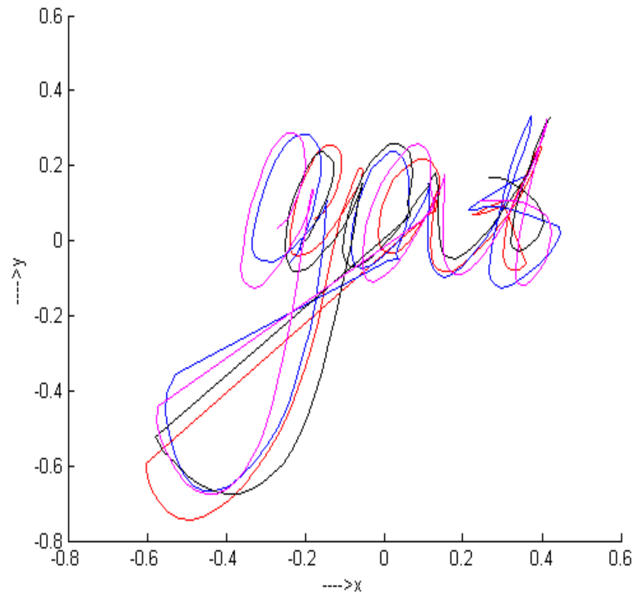


Figure 5.3: After Normalization of Size and Location

Barycenter Trajectory

This step is used to reduce fluctuations related to pen movement.

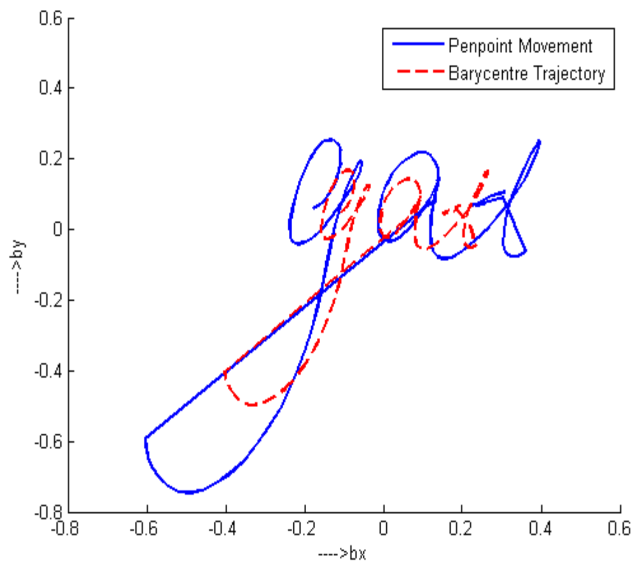
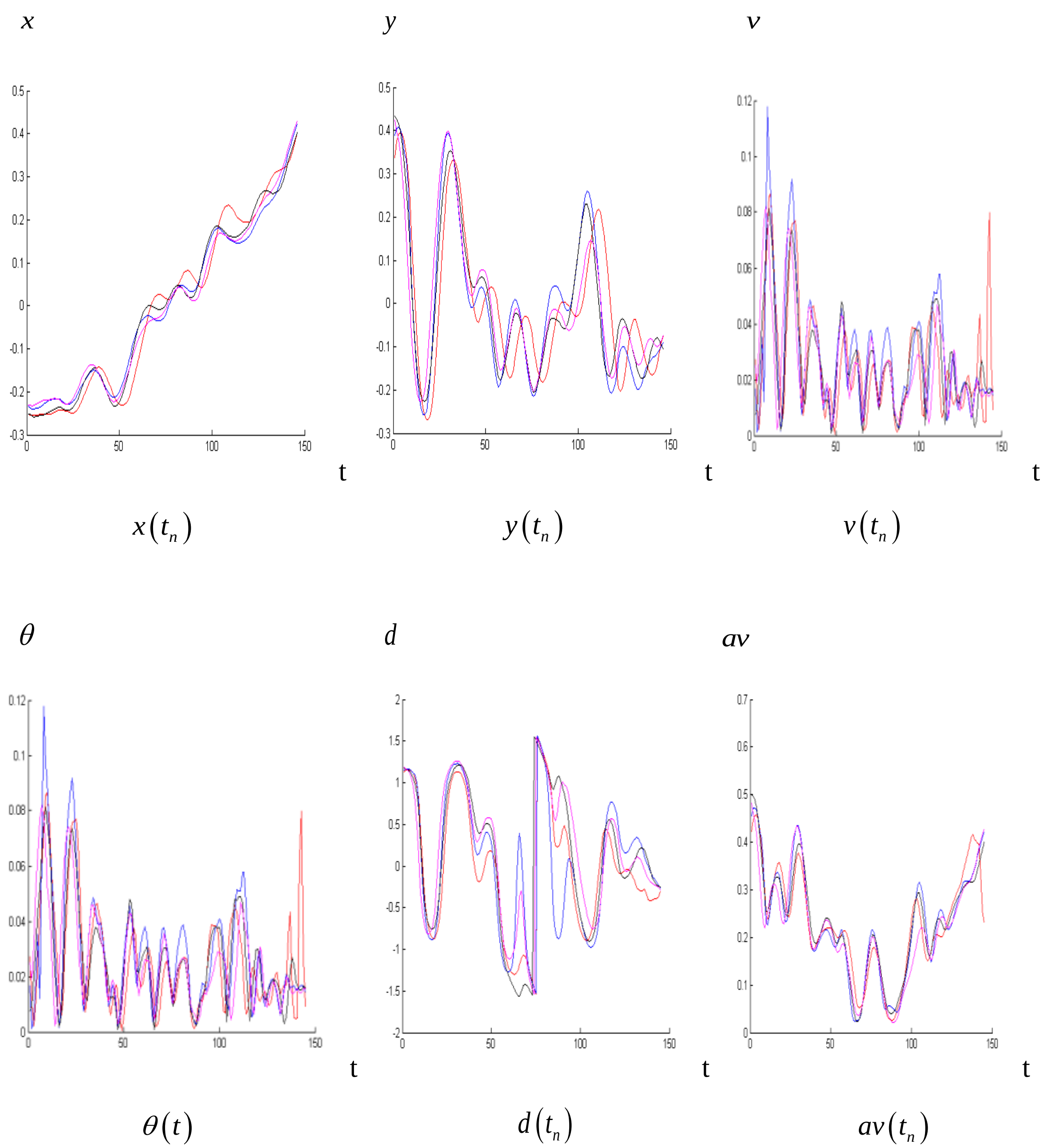


Figure 5.4: Trajectories of Barycenter of Signature and Pen Point Position

Features of Signatures

Ten features extracted from given signature are given in Figure



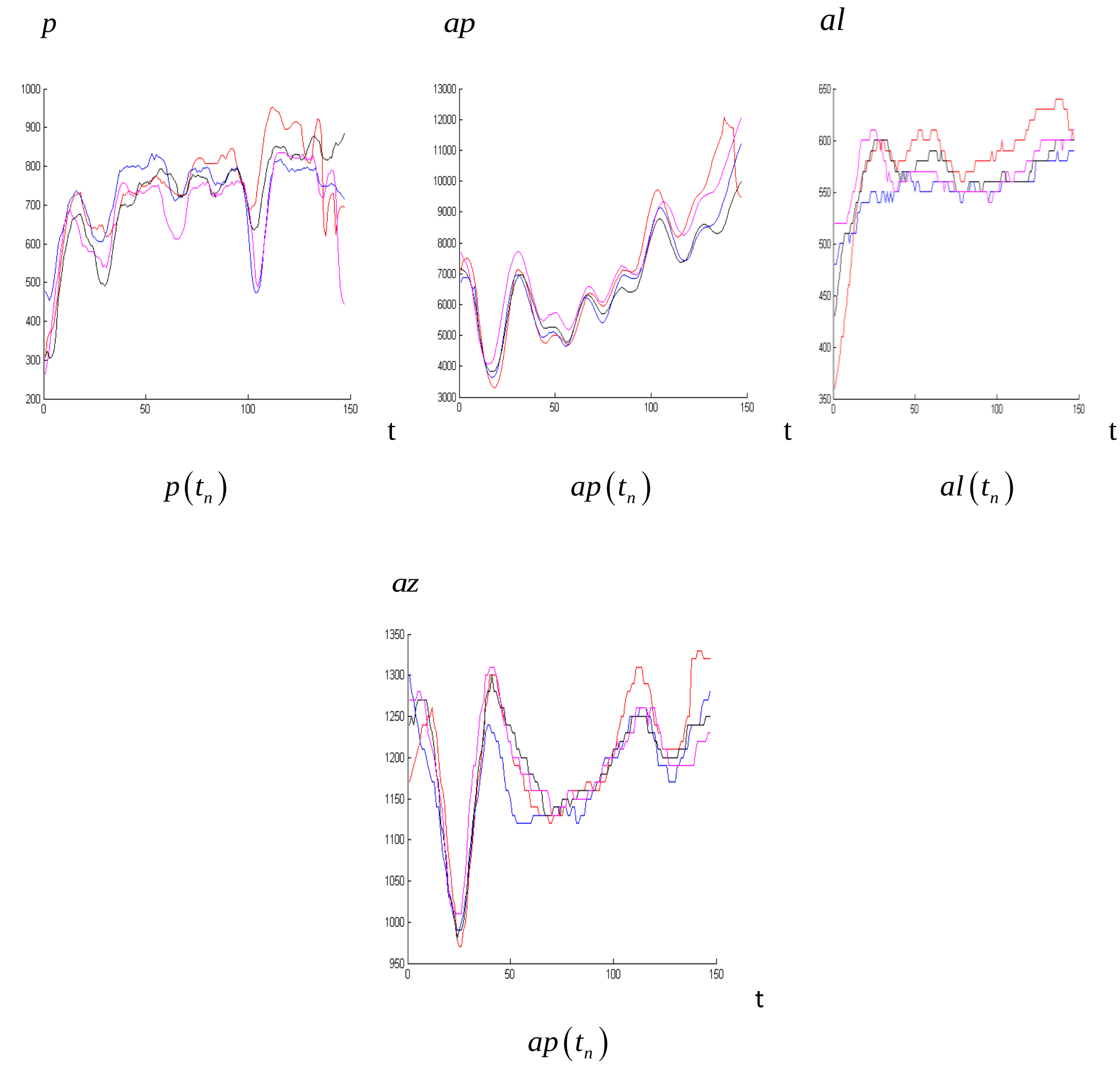


Figure 5.5: Characteristics of Tested Signature

DFrCT of Features of Signatures

DFrCT of extracted features for genuine and forgery signatures are shown in Figure 5.6 & Figure 5.7. A comparison between features of genuine and forgery signatures are shown here. DFrCT $F(m)$ of each feature is calculated for optimized value of fractional order (α). Optimum value can be obtained by varying fractional order between 0 and 1until a minimum Equal Error Rate (EER) is achieved. DFrCT for genuine and fake signatures are given in Figure 5.6 and Figure 5.7 respectively. X axis of figure is equal to M. Y axis shows the value of coefficients. It is calculated for four signatures and is plotted on same graph. Variation of

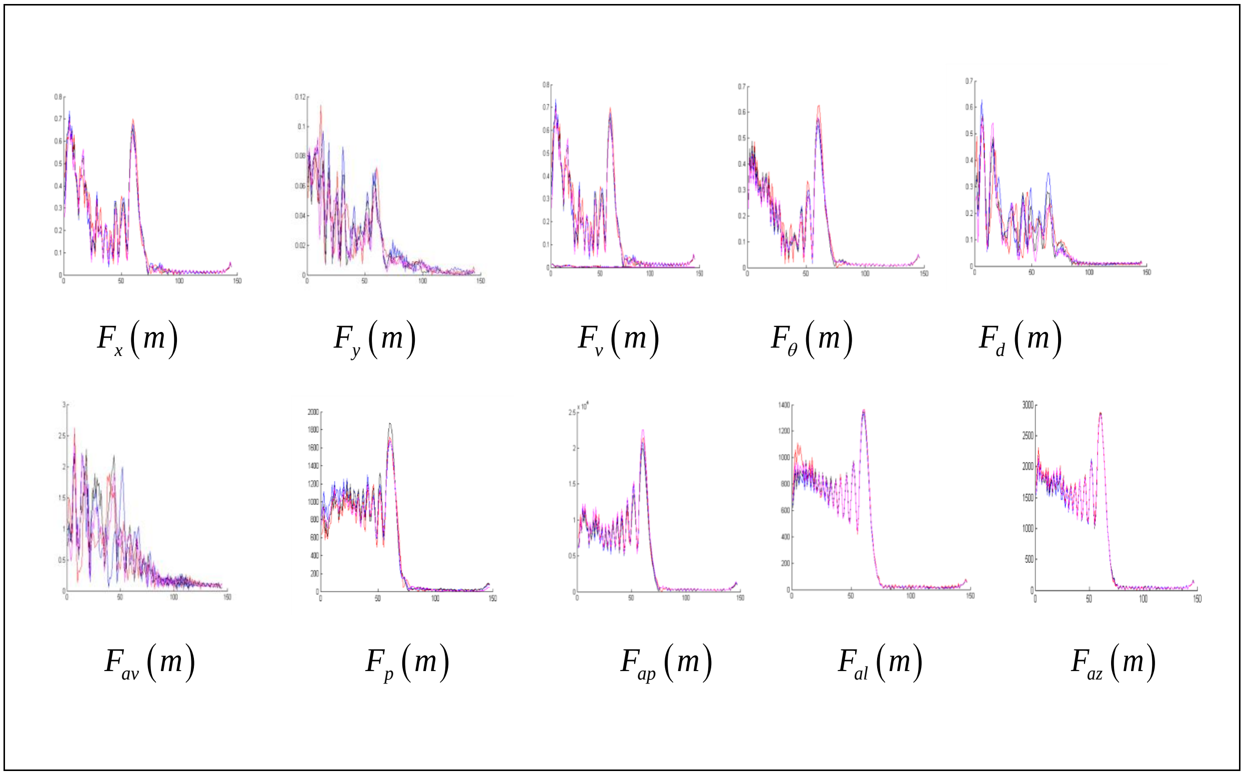


Figure 5.6: DFrCT of Features for Genuine Signatures

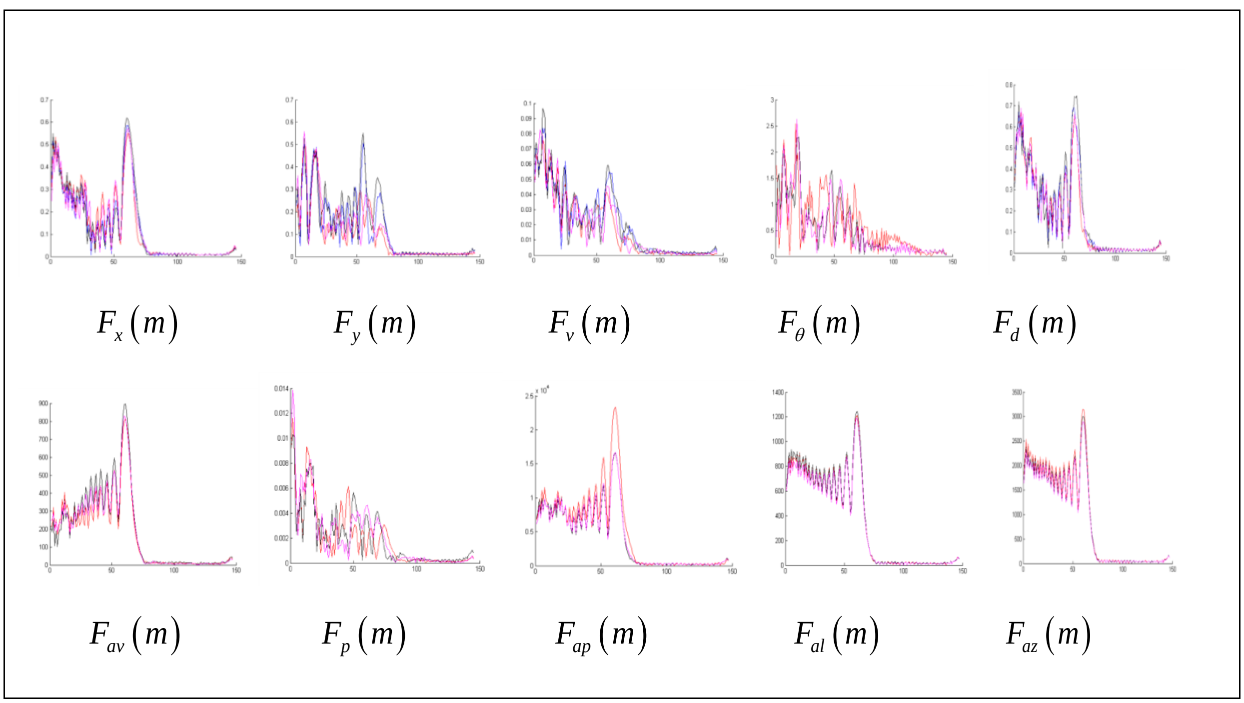
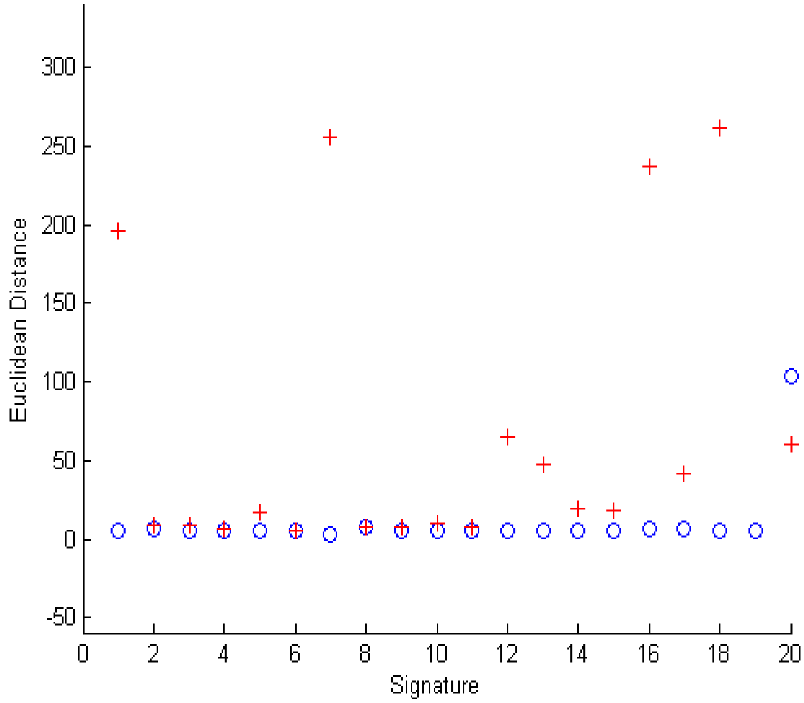


Figure 5.7: DFrCT of Features of Fake Signatures

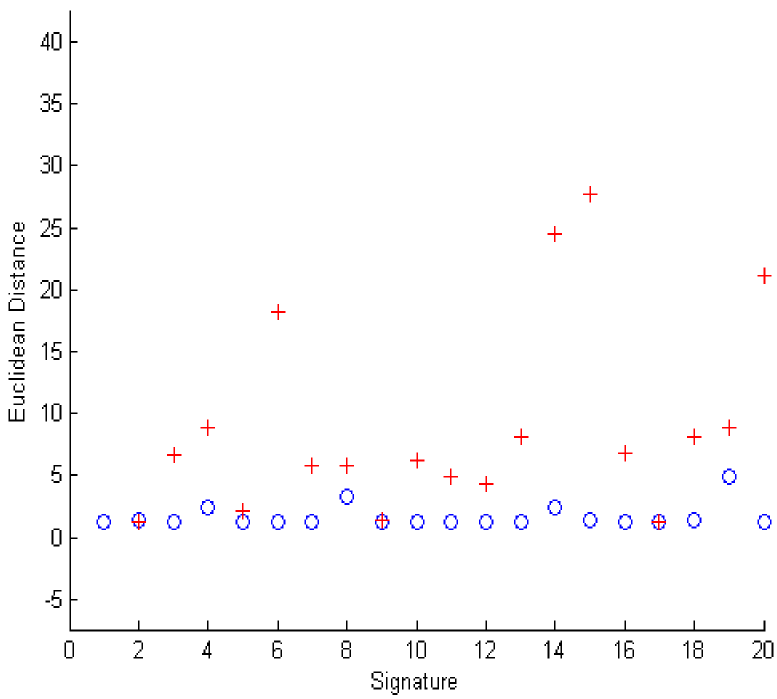
From Figure 5.6 and Figure 5.7 it can be seen that for more features the variation is less because of skilled forgery. So it is difficult to detect skilled forgery. This thing results in EER. Variation can also be seen in the signatures written by the same person. So we have calculated the average of the features for more accurate results. DFrCT is different in each feature. In some features the variation is less because of skilled forgery. Variation in DFrCT for pressure and areal velocity is more than variation of other features.

Experimental Results of Verification system

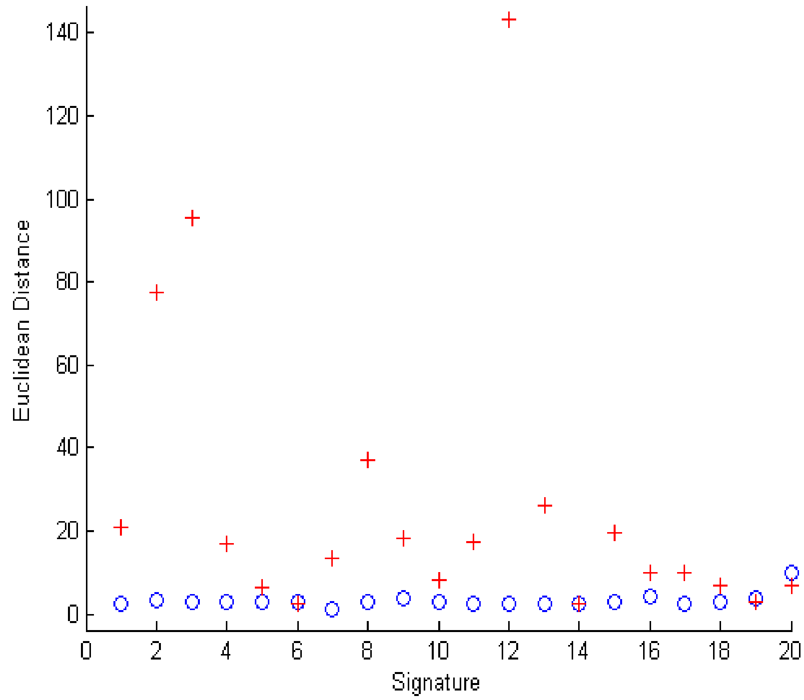
The efficiency of authentication system is indicated by terms; the first is false accept rate (FAR) and the second one is false reject rate (FRR). FA occurs when the verification system accepts an invalid sign. A FR occurs when a valid signer is rejected. The two errors ratios FAR and FRR are correlated directly. Other rate is inversely affected when there is variation in one of the error percentage. EER is the most common alternate to check the efficiency of an authentication technique. FAR and FRR are equal at EER. The SVC2004 and SUSIG database was used in the experiment. In SVC 2004 database, twenty genuine signatures and twenty forgery signatures are given for each user. In the experiment, five users were selected from the database randomly. So, in total there were two hundred signatures for this experiment. In SUSIG database, there are eight genuine signatures and eight forgery signatures for each user. Out of which five signatures are chosen for each user. Euclidean norm of difference between feature vectors of test and reference signatures is calculated for verification. This is compared with the threshold. If the difference is less than the threshold then the signatures are classified as accepted else rejected. The signature verification system is trained by 5 genuine signatures corresponding to each user. In this thesis a comparison has been made between results obtained by the method using DCT for feature extraction to that of one using DFrCT for feature extractions for handwritten signature verification. The opted evaluation criterion is Equal Error Rate (EER). While using DFrCT, the optimized value of α (free parameter) for each user can be achieved by varying its value in between 0 to 1. Algorithm is repeated for all values of α until a minimum EER is achieved. This is more time consuming as compare to DCT but its performance is better than DCT. Figure 5.7 show some examples of the plots of Euclidean norm of difference between feature vectors of genuine signatures and forge signatures using DFrCT for feature extraction.



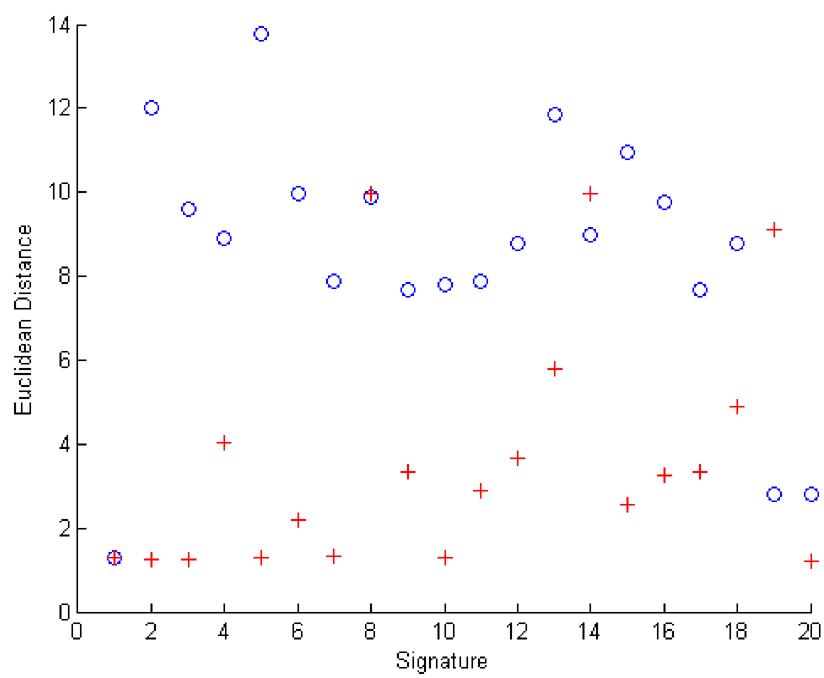
α =0.75



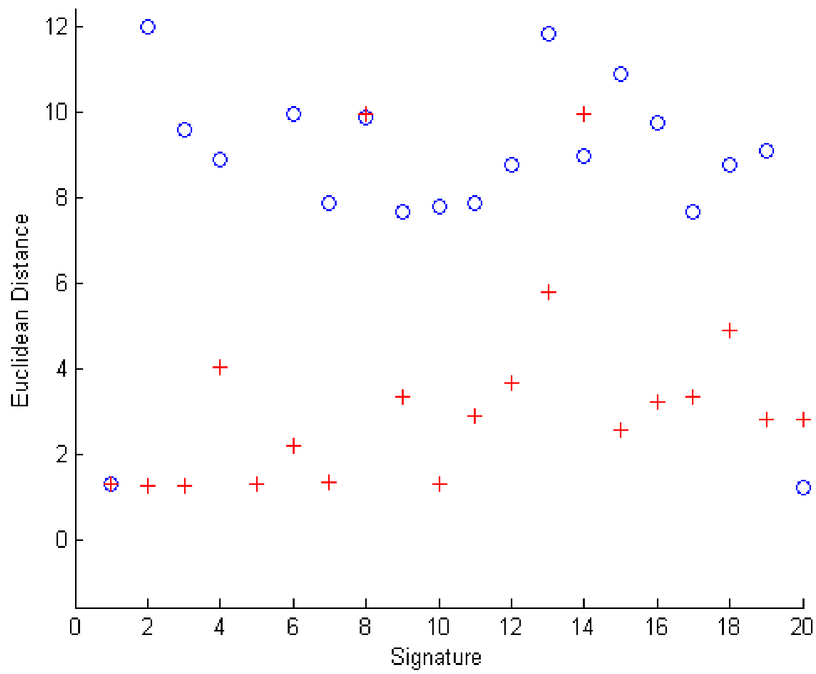
α = 0.33



$\alpha = 0.19$



$\alpha = 0.95$



α= 0.51

Figure 5.8: Euclidean Distance plot for Genuine and Forgery Signatures

5.10 Experimental Results of Verification System

5.10.1 Analysis of System using DCT Method

Table 5.1 shows the Euclidean norm of difference between feature vectors of test signature and reference signature.

Table 5.1: Euclidean Norm Of Difference Of Joined Feature Vectors Of Reference And Tested Signature using DCT (Genui.= Genuine, Forge.= Forgery, Optimi.= Optimized)

Sign	User 1		User 2		User 3		User 4		User 5	
	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.
1	7.239	15.37	2.237	29.22	13.89	33.78	13.91	27.14	29.22	29.22
2	8.927	17.33	3.444	28.98	14.59	32.41	13.29	34.54	19.27	31.94
3	13.92	25.31	13.72	27.13	17.64	33.64	12.22	19.63	18.49	33.54

4	**19.21**	19.81	32.74	32.31	23.47	27.42	11.32	32.19	21.31	25.54
5	8.303	27.99	37.91	35.81	21.74	33.34	14.64	21.71	**31.65**	37.40
6	3.212	26.32	13.98	29.58	20.41	37.23	17.29	36.14	29.27	39.24
7	11.71	13.79	23.91	35.36	18.47	28.82	24.20	37.21	33.27	36.30
8	17.29	27.12	**29.42**	33.55	13.29	47.80	14.27	31.39	23.41	30.67
9	12.11	20.41	11.27	17.73	3.194	31.42	27.54	38.41	27.14	27.59
10	14.29	22.29	8.977	29.88	19.33	37.20	33.22	39.22	32.65	38.41
11	12.97	25.99	12.28	31.35	33.38	28.41	14.52	43.99	27.19	39.77
12	15.33	27.17	14.33	45.97	**30.93**	29.93	19.65	33.97	23.26	60.17
13	17.99	28.79	15.97	37.79	20.74	30.99	**33.25**	35.25	24.31	100.6
14	18.91	13.28	15.34	32.21	24.77	31.54	37.24	38.61	26.21	45.70
15	17.27	27.39	18.92	38.63	23.19	29.28	16.52	30.29	23.20	39.56
16	12.18	20.33	7.744	30.58	30.46	24.33	17.76	37.98	20.92	32.21
17	14.41	29.42	19.27	29.43	23.23	45.32	19.21	47.23	21.92	38.45
18	22.92	21.42	27.81	33.72	25.92	37.42	17.22	33.35	27.77	30.97
19	18.29	30.22	12.97	42.61	30.73	41.55	18.32	32.72	21.91	32.14
20	23.71	19.12	9.814	31.83	24.73	32.27	23.92	42.21	19.79	37.44

Table 5.2: Total number of False Accepted (FA) and Rejected (FR) signatures for each user

User 1			User 2			User 3			User 4			User 5			Total	
η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR	FA	FR
19.21	4	2	29.42	4	3	30.93	5	1	33.25	5	1	31.65	4	1	22	8

Table 5.3: False Accept Rate (FAR) and False Reject Rate (FRR) and Equal Error Rate (EER)

Total		FAR %	FRR %	EER %
FA	FR			
22	8	11	4	7.5

5.10.2 Analysis of System using DFrCT Method (SVC 2004 Database)

In this section results are calculated using DFrCT. In this section SVC 2004 database is used for calculations.

Table 5.4 gives Euclidean norm for fake and genuine signatures. For each user 20 genuine and 20 forgery signatures are taken. Optimum value of fractional order is calculated for each user. A threshold is calculated for each user for minimum FAR and FRR. In Table 5.5 FAR, FRR and ERR is calculated. In Table 5.6 obtained results are compared with previous results. It can be seen that the proposed system is superior than the previous results.

Table 5.4: Euclidean Norm of Difference of Joined Feature Vectors Of Reference and Tested Signature using DFrCT (Genui.=Genuine, Forge.=Forgery, Optimi.=Optimized).

Sign	User 1 Optimi. α=0.75		User 2 Optimi. α=0.33		User 3 Optimi. α=0.19		User 4 Optimi. α=0.62		User 4 Optimi. α=0.51	
	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.
1	5.533	195.7	1.183	46.41	2.589	21.05	6.618	24.49	1.276	1.279
2	6.084	8.250	1.379	1.262	3.457	77.43	6.301	7.937	12.01	1.250
3	5.552	8.998	1.191	6.624	2.701	95.46	6.571	7.556	9.586	1.266
4	5.552	6.654	2.418	8.847	2.668	17.04	**7.567**	320.1	8.899	4..044
5	5.532	17.26	1.222	2.101	2.700	6.404	6.532	42.48	13.78	1.272
6	5.529	5.730	1.263	18.25	2.663	2.545	6.717	147.0	9.987	2.200
7	3.346	255.9	1.257	5.762	1.087	13.42	1.380	6.623	7.897	1.345
8	**7.393**	7.833	3.277	5.818	2.781	37.06	6.596	32.02	9.884	9.987

9	5.678	7.897	1.263	1.343	3.862	18.16	3.952	183.4	7.672	3.345
10	5.556	9.456	1.287	6.251	2.812	8.059	6.572	9.225	7.789	1.289
11	5.544	7.795	1.233	4.909	2.450	17.32	6.667	15.11	7.895	2.899
12	5.730	64.42	1.264	4.300	2.470	143.2	7.116	8.116	8.789	3.678
13	5.530	47.94	1.267	8.182	2.589	25.95	7.778	3.456	11.87	5.789
14	5.533	19.64	2.453	24.53	2.596	2.582	7.223	8.561	**8.990**	9.983
15	5.703	18.12	1.372	27.78	2.678	19.70	6.667	8.562	10.93	2.567
16	5.915	236.6	1.256	6.782	**4.229**	9.789	6.572	8.540	9.776	3.234
17	5.974	42.04	1.263	1.224	2.345	9.890	6.291	8.564	7.675	3.334
18	5.532	261.2	1.329	8.144	2.789	6.896	7.492	9.376	8.801`	4.895
19	5.533	373.2	**4.892**	8.814	3.765	2.674	7.343	9.180	9.126	2.789
20	103.7	60.58	1.221	21.18	9.888	6.606	7.323	20.96	1.209	2.790

Table 5.5: Total number of False Accepted (FA) and Rejected (FR) Signatures for Each User

User 1 Optimi. α=0.75			User 2 Optimi. α=0.33			User 3 Optimi. α=0.19			User 4 Optimi. α=0.62			User 5 Optimi. α=0.51			Total	
η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR	FA	FR
7.393	1	1	4.892	4	0	4.229	3	1	7.567	1	1	8.990	2	1	11	4

Table 5.6: False Accept Rate (FAR), False Reject Rate (FRR) and Comparison

FAR (%)	FRR (%)	Skilled Forgery (Presented Work) EER (%)	Matsuura *et al.* EER (%) [21]	Mohit *et al.* EER (%) [12]
5.50	2.00	3.75	7.47	5.00

5.10.3 Analysis of System using DFrCT (SUSIG Database)

In this section, SUSIG database is used.

Table 5.7: Euclidean Distance for User 1, User 2, User 3, User 4 and User 5

Sign	User 1 Optimi. α=0.27		User 2 Optimi. α=0.41		User 3 Optimi. α=0.71		User 4 Optimi. α=0.89		User 5 Optimi. α=0.57	
	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.
1	5.342	7.421	3.321	9.984	**8.412**	10.12	2.899	6.221	3.985	8.972
2	7.892	9.819	**8.984**	7.193	11.21	11.13	2.021	9.892	7.792	11.92
3	8.714	10.21	6.248	8.991	2.131	10.02	9.411	8.181	8.192	9.551
4	2.421	11.44	4.431	13.12	7.012	13.27	5.203	10.91	7.224	9.422
5	**8.925**	8.978	4.998	10.59	2.277	11.14	**8.881**	10.12	**8.912**	12.72

Table 5.8: Euclidean Distance for User 6, User 7, User 8, User 9 and User 10

Sign	User 6 Optimi. α=0.76		User 7 Optimi. α=0.29		User 8 Optimi. α=0.93		User 9 Optimi. α=0.81		User 10 Optimi. α=0.15	
	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.
1	**8.411**	9.224	**8.812**	10.12	2.231	9.959	7.131	13.92	2.124	9.421
2	2.812	8.401	9.214	14.23	2.544	8.221	12.14	17.99	3.299	10.05
3	9.219	9.212	7.212	9..230	3.422	4.778	9.712	21.22	3.992	11.87
4	5.217	7.712	7.729	10.22	**4.778**	5.531	2.227	22.61	**8.384**	12.27
5	7.521	8.525	8.811	12.31	2.789	10.94	**17.29**	17.37	7.129	11.41

Table 5.9: Euclidean Distance for User 11, User 12, User 13, User 14 and User 15

Sign	User 11 Optimi. α=0.11		User 12 Optimi. α=0.69		User 13 Optimi. α=0.24		User 14 Optimi. α=0.65		User 15 Optimi. α=0.15	
	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.
1	2.659	5.782	3.727	9.121	1.625	9.474	3.121	12.66	2.371	6.122
2	3.288	7.266	4.422	15.12	2.118	8.259	**8.816**	16.50	2.495	13.12
3	2.785	8.118	3.671	21.22	**6.044**	9.258	6.526	9.927	5.226	14.13

4	**6.123**	7.250	4.001	12.99	2.002	10.27	6.881	10.21	**6.296**	13.21
5	2.479	4.279	**7.122**	9.998	4.641	10.22	5.291	20.13	4.227	10.21

Table 5.10: Euclidean Distance for User 16, User 17, User 18, User 19 and User 20

Sign	User 16 Optimi. α=0.19		User 17 Optimi. α=0.57		User 18 Optimi. α=0.21		User 19 Optimi. α=0.53		User 20 Optimi. α=0.23	
	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.	Genu.	Forg.
1	**13.92**	12.13	3.221	7.721	**6.925**	111.6	7.291	12.19	5.316	13.96
2	6.527	13.95	2.420	6.621	6.731	7.213	6.733	15.53	2.622	15.23
3	9.212	17.90	5.678	7.219	5.299	6.232	5.213	10.22	3.906	8.362
4	13.97	18.80	**6.225**	9.112	3.883	10.93	5.921	11.27	7.197	9.259
5	6.596	28.52	6.225	10.22	7.322	19.89	**8.911**	13.29	**7.329**	21.00

Table 5.11: Total number of False Accepted (FA) and Rejected (FR) Signatures for Users 1 to 5

User 1 Optimi. α=0.27			User 2 Optimi. α=0.41			User 3 Optimi. α=0.71			User 4 Optimi. α=0.89			User 5 Optimi. α=0.57		
η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR
8.925	1	0	8.984	1	0	8.412	0	0	8.881	1	1	8.912	0	0

Table 5.12: Total number of False Accepted (FA) and Rejected (FR) Signatures for Users 6 to 10

User 6 Optimi. α=0.76			User 7 Optimi. α=0.29			User 8 Optimi. α=0.93			User 9 Optimi. α=0.81			User 10 Optimi. α=0.15		
η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR
8.411	1	1	8.812	0	1	4.778	1	0	17.29	1	0	8.384	0	0

Table 5.13: Total number of False Accepted (FA) and Rejected (FR) Signatures for Users 11 to 15

User 11 Optimi. α=0.11			User 12 Optimi. α=0.69			User 13 Optimi. α=0.24			User 14 Optimi. α=0.65			User 15 Optimi. α=0.15		
η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR
6.123	1	0	7.122	0	0	6.044	0	0	8.816	0	0	6.296	1	0

Table 5.14: Total number of False Accepted (FA) and Rejected (FR) Signatures for Users 16 to 20

User 16 Optimi. α=0.19			User 17 Optimi. α=0.57			User 18 Optimi. α=0.21			User 19 Optimi. α=0.53			User 20 Optimi. α=0.23		
η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR	η	FA	FR
13.92	1	1	6.225	1	0	6.925	1	0	8.911	0	0	7.329	1	1

Table 5.15: Total number of False Accepted (FA) and Rejected (FR) Signatures of All Users and EER

Total		FAR %	FRR %	EER %
FA	FR			
13	4	6.5	2	4.25

Table 5.16: Obtained EER for SVC 2004 and SUSIG Databases

Result	
EER for SVC 2004 Database	EER for SUSIG Database
3.75 %	4.25 %

Chapter 6

Conclusion and Future Scope

6.1 Conclusion

Nowadays with developing progress in identification/verification applications have drastically increase the demand for new generation ID documents that include additional biometric information of users. This additional information provides more accurate user recognition.

Handwritten Signatures can be categorized as offline signature when only the image of signature is given. Signatures can be categorized as online signature when certain dynamic parameters such as velocity of signature, movement with respect to time, acceleration, altitude, azimuth, pressure etc. are also recorded which cannot be seen by eyes. ID documents already contain image of the person’s signatures, so the trend is shifting towards online handwritten signatures. In order to be in synchronism worldwide, the method of features extraction should be standardized worldwide, this is a challenging part. Various methods have been proposed for features extraction till now. This system is still at developing stage, as many more methodologies are expected to be proposed. One such effort has been put up by using DFrCT in this study to extract features for a given handwritten signature. Use of DFrCT which according to literature was not much explored has been discussed in this study. DFrCT is a powerful tool for signal processing. It has a free parameter, which is called its fractional order. One can optimize this free parameter to get better results. Optimization is done for every user. So, feature extraction has given far better results than the existing methodologies in literature based on DCT and DFrCT with less features. An EER (equal error rate) of only 3.75% is achieved. In DCT EER of 7.5%was achieved. A significant difference between the error rates proves that DFrCT is better tool than DCT.

6.2 Future Scope

In this study ten features are used. In order to increase efficiency, PCA (principle component analysis) can be done in future.

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