# EFFECT OF DISC THICKNESS GRADIENT AND THERMAL GRADIENT ON THE CREEP RESPONSE IN FGM ROTATING DISC

**A thesis submitted**

**in fulfilment of the requirements**

**for the award of the degree**

**of**

**MASTER OF ENGINEERING**
**IN**
**CAD/CAM Engineering**

by

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# CERTIFICATE

This is to Certify that the work in this thesis report entitled **"EFFECT OF DISC THICKNESS GRADIENT AND THERMAL GRADIENT ON THE CREEP RESPONSE IN FGM ROTATING DISC"** submitted in partial fulfilment of requirement for the award of **Master of Engineering Degree** in **CAD/CAM Engineering** in Mechanical Engineering Department of Thapar University, Patiala, is an authentic record of work carried out by me under the supervision and guidance of **Mr. Kishore Khanna**, Assistant Professor, Mechanical Engineering Department, Thapar University, Patiala.

This matter embodied in this report has not been submitted in part or full to any other university or institute for the award of any degree.

**DATE** JULY 15, 2016

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This is to certify that the above declaration made by the candidate is correct to the best of my knowledge and belief.

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# ABSTRACT

In the present thesis work, the analysis of creep stresses and strain rates has been carried out in a functionally graded disc with linearly varying thickness, rotating at constant angular velocity and subjected to thermal gradient by using Tresca criterion in the disc. For the purpose of investigation, five discs made of silicon carbide particles ($SiC_p$) in a matrix of pure aluminum with different thickness profiles are considered. The density and reinforcement content is varying along the radial direction. The variable thickness disc with higher disc thickness gradient undergoes lower strain rates. Further the analysis has been carried out on the same disc to analyze the effect of thermal gradient. The disc with minimum creep strain rates is considered as the best disc.

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# CHAPTER 1
# INTRODUCTION

## 1.1 COMPOSITE MATERIALS

Modern age is the age of invention and the study of materials. This is the area which needs continuous research to make the life of human better and more comfortable. All the big structures constructed by the humans are impossible to make without the materials. Nowadays we need materials which have unusual properties which we cannot find in conventional metals, alloys, polymer and ceramic materials. Generally strong materials are dense in nature and if we increase the strength then it leads to reduction in impact strength of the material [6]. This need has encouraged the scientists to develop a new low cost material which can follow severe and contradictory property requirements which cannot satisfy by the normal materials.

To full fill this ambition humans find out a new material named as composite material. Today we live in a world where we have high–tech machines, aircrafts, racing cars which are getting better and better because we are able to produce new materials which are stronger, lighter and able to handle heavy loads. The progression of technology is the reason which helps us to continuously enhance the properties of the materials. The need of enhancement of properties encouraged from requirement of increasing the efficiency and life of the component. The composite materials have revealed great promise in this side. From the start of the civilization, the composite materials have always been the part of human life. The Chinese wall is one of the best examples of the ancient composite structures which have a great strength and durability. There are some problems like corrosion and high temperature which can shrink life, decrease its strength and increase the creep rate of the materials [7]. So a lot of composite materials have been made with enhanced high temperature properties. But with the increase in expectation in the modern age, the researchers are continuously increasing the temperature limit of the composite by using different materials in different compositions.

A composite material is a material produced using two or more constituent materials with altogether diverse physical or chemical properties that when consolidated produce a material with qualities not the same as the individual parts[6]. Composite material is likewise called a composition material or abbreviated to composite. Composites are multifunctional material that give attributes which are not achievable by any discrete material. The composite material behaviour depends upon the properties of reinforced material, matrix phase material and interaction between these two materials in the composite. The parameter considered during selection of composite are strength, corrosion, behaviour at high temperature, weight, damping properties and toughness.

## 1.2 CLASSIFICATION OF COMPOSITE MATERIALS

Composites are grouped into following types:-

### 1.2.1 Classification based on geometry of reinforcement

Its sub types are:-

(a) Particulate Composites.

(b) Fiber Composites.

(c) Flake Composites.

**(a) Particulate Composites**:-If the matrix is fortify by suspending the particles in the pure matrix material then it is called particulate composite. They are generally isotropic since the particles are dispersed at random [12]. Particles have very shot dimensions as compared to fibers. In the majority of cases particles are basically utilized as filer to decrease the expense of material without degrading the characteristics of material. The choice of molecule lattice affiliation depends upon the properties required.Particulate composites have advantages like enhanced strength, expanded working temperature and oxidation resistance etc.

e.g.:- machining of copper alloy is easier when lead is added in it.

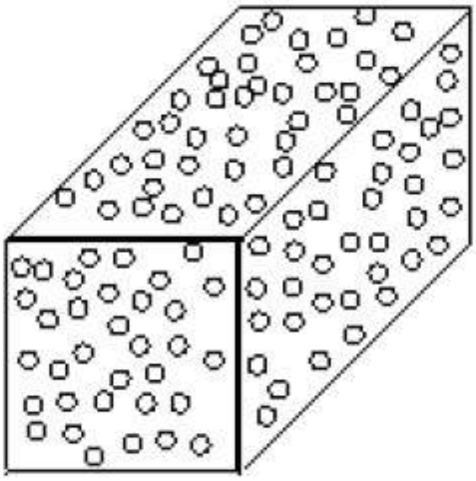
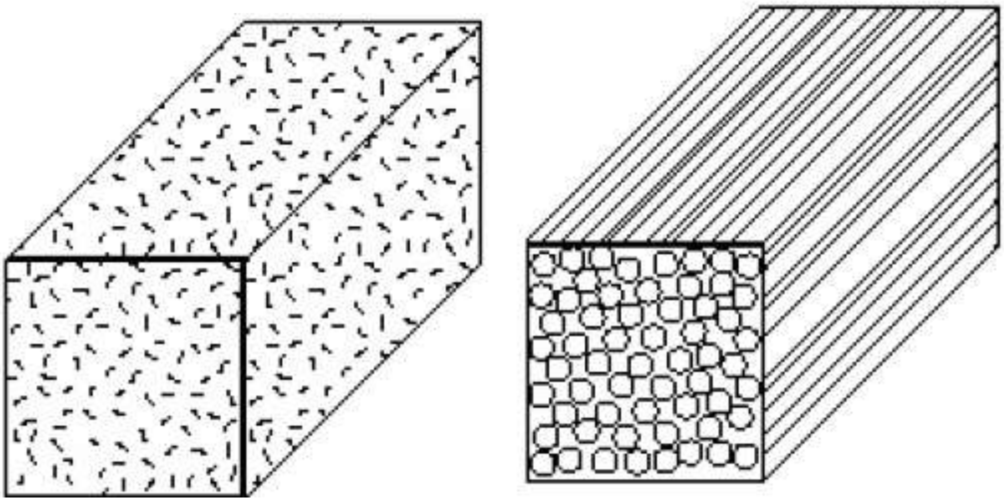


**Figure 1.1 Particulate composite [33]**

**(b) Fiber Composites**:**-** These are made out of matrix strengthened by short or long fibers. The fibers which are used in matrix can be continuous or discontinuous in nature [7]. Fibers are usually anisotropic. The collection of fibers and their orientation permit us to change the mechanical properties of composites from strongly anisotropic to isotropic.



**(a)** **(b)**

**Figure 1.2 Fiber composite (a) Short fiber (b) Continuous fiber [33]**

**(c)Flake Composites**:**-**If the matrix is strengthened with flakes such as glass, mica, silica, silver etc. then it is called flake composite. Flake composite provide high out of plane flexural modulus, large strength and less cost but the flakes can't be oriented easily which is the biggest disadvantage of flake composite[12]. Due to this reason, a small number of components are available for use.

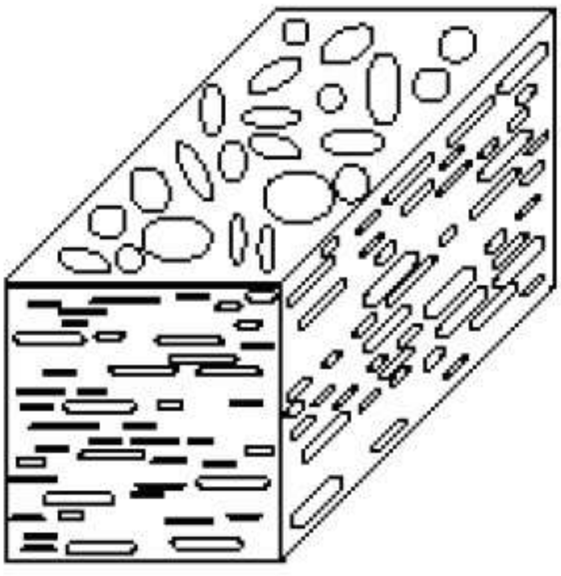


**Figure 1.3 Flake composite [33]**

### 1.2.2 Classification based on type of matrix

(a) Polymer Matrix Composites

(b) Metal Matrix Composites

(c) Ceramic Matrix Composites

(d) Carbon Matrix Composites

**(a) Polymer Matrix Composite**:**-** Polymer matrix composites are the most widely recognized composites and are also called as fiber reinforced polymers. These materials utilize a polymer-based resin as the matrix and a variety of fibers like glass and carbon as the reinforcement. Matrix materials are thermosetting and thermoplastic. On weight for-weight premises, Graphite/Epoxy composites are around five times robust than steel. They are normally utilised because of their low cost, high strength and easy manufacturing. Primary downsides of polymer matrix composites (PMCs) contain elevated coefficient of thermal and moisture expansion, low working temperature and low elastic properties in specific directions[6]. However, it has great Insulating property and high chemical resistance. The innovation of polymer composites has been headed to large extent towards aerospace military applications.

**(b) Metal Matrix Composite**:-Metal matrix composite is made of metal matrix like aluminium, titanium and magnesium etc. with reinforcement of fibers, particulates or whiskers like carbon, silicon carbide etc. These composites can be utilised at elevated service temperatures than their base metal counterparts. The reinforcement in metal matrix composites can enhance thermal and electrical conductivities, specific stiffness, creep resistance, abrasion resistance, specific strength and dimensional stability [6]. MMCs are providing broad applications in parts which are exposed to elevated temperatures because of their high strength, improved creep characteristics and great stiffness at elevated temperature. Metal matrix composites are widely used in different areas such as defence, aerospace devices, shipbuilding and automotive.

**(c) Ceramic Matrix Composite**:- Ceramic matrix composite is a material consisting of a ceramic matrix like alumina reinforced with a ceramic (oxides, carbides) dispersed phase. Here we only use those fibers which have the ability to remain stable at temperature more than 1000°C [33].The major advantages of CMC are high service temperature, less density, chemical inertness and large strength and hardness. CMC display better high temperature creep behaviour and thermal shock resistance. Brittleness is the main disadvantage of CMC. CMCs are used mainly in those applications where temperature is very high and where we can't able to use MMCs and PMCs.

**(d) Carbon Matrix Composite**:-It is a special type of composite material which is made of carbon matrix toughened by carbon fibers. These composites are very vital to make structures which are light in weight. But these structures are comparatively costly. These composites are able to keep their properties up to 2000°C and this range can be exceed by using protecting coating[7].These composites are 20 times stronger than graphite fibers. Its advantages are capability to withstand high temperatures, good tensile and compressive strength, and low creep at high temperature, high thermal conductivity and high coefficient of friction, low density, high fatigue resistance. High cost, low shear strength and susceptibility to oxidations at high temperature are the disadvantages of this composite. Carbon materials react with oxygen at temperatures above 900ºF (482ºC).

## 1.3 FUNCTIONALLY GRADED MATERIAL (FGM)

Several engineering applications include a severe thermo-mechanical loading which means there is simultaneous presence of mechanical loadings and high temperature in these applications. The conventional materials like metals/ceramics alone at thermal gradient and high temperature can't able to survive[22].Pure metals are used very less because of increasing demand of conflicting properties which is not possible to acquire in pure materials. For example, in nature there is no material which is both hard and ductile. Thus, a new idea emerged named as Functionally Graded Materials (FGM) and which steer us to the advancement of better heat resistant materials although keeping the expense under control. The functionally graded material concept was introduced and developed in Japan in 1984[16]. FGM is of advanced material class where the contents of two or more materials are varied about some position coordinates, which permit them to give unique performance [12].FGM is a composite which is heterogeneous in nature whose properties change continuously with the varying dimension. Ceramic phase in the FGM provide thermal barrier effects and shield the metal from oxidation and corrosion. The metallic phase in FGM provides strength and toughness to the material. With the advancement of preparing techniques, Scientists are able to mix the ceramic and metal in a continuously changing volume fraction to create FGM with desirable properties. There are some interface problems like residual stresses in material which initiate because of variation in coefficient of thermal expansion is removed.FGM is mainly manufactured to work in high temperature environment like light weight, temperature resistant materials for aerospace vehicles [31].Teeth, bones etc are named as natural functionally graded material.FGM is used in many engineering devices like steam and gas turbine rotors, clutch plates, turbo generator gears and brake discs.

Other than admirable mechanical and thermal properties, FGMs additionally have the benefit of optimum utilization of costly dispersoids because in FGM the volume of dispersoids changes from higher temperature region to lower temperature region. The utilization of FGM aids in enhancing the capability of structural components which can accomplish by customising the microstructure and appropriate control of stresses, undesirable features and Deformation like internal residual stresses. In various applications, the materials have to work for longer time under steady loads in different temperature conditions such as turbine blades, furnace parts, filaments in vacuum tubes *etc* which may guide to persistent deformation and may seriously impair its helpfulness. Such deformations which are dependent of time can develop large and result in breakage of the component without further increment of load. The FGM fall in the category of that material which can handle these problems. Due to oil crisis, Engineers are trying to make lightweight components to reduce the overall weight of vehicle to accomplish better fuel economy [15]. Aluminium or its alloys are used to reduce the weight of engineering components to save power and fuel, which increased their utilization in aerospace and automobile industries. But at elevated temperature, the creep rate of aluminium and its alloys increase very fast. So, the components prepared of MMCs like aluminium or its alloys matrix toughened with ceramic like fibers, particles or whiskers are observed to be extremely valuable.

Experimental studies relating to uniaxial creep have shown that the creep rate in aluminium and its alloy toughened with ceramic such as silicon carbide is decreased by big magnitude when compared with aluminium or its alloys [6]. These composites are able to achieve considerable enhancement in stiffness and specific strength .In comparison to other ceramic reinforcement, aluminium oxide (Al2O3) and silicon carbide (SiC) are mostly used because of their encouraging combinations of density, cost and property enhancement potential. The selection of silicon carbide as fortification of aluminium or its alloy is mainly decide because of its excellent physical properties, accessibility and low cost. Silicon carbide is accessible in the form of particles or whiskers and fibers. By optimizing variables such as geometry, size and volume of reinforcement in composites, one can produce cost effective components with improved performance. A cost effective product with improved performance can be made by upgrading variables like size, geometry and content of reinforcement.

Most of the disc applications are involved with creep. So, Researchers are constantly keen on controlling the creep deformation to improve its service life. In this manner, the studies related to creep help the researchers to optimise the dispersion of reinforcement in the rotating disk to control the creep response. Any enhancement in the disc design can lead us to better service life and efficiency of device having rotating disc. Further, an exact investigation of creep in rotating disc will help us understanding of materials and working parameters which are responsible for stresses and strain in the disc under thermo mechanical loadings[7]. Recently, a lot of research is happening in the field of FGM, researchers engaged in developing process of FGMs are lacking a united framework to choose a best possible mix of material ingredient and composition profiles. The problem studied in this thesis may provide some help to the researchers of rotating disc to create a reliable design.

### 1.3.1 Applications of FGMs in different fields

**(a) Automotive:-** Automobile parts like combustion chamber, shock absorbers, Racing car brakes and leaf springs for suspension etc are made by functionally graded materials because these parts needs high strength and resistance to work as a system in the car.

**(b) Medicine**:- FGM is widely used in dental and bone replacement side of medical field.

**(c) Aerospace**:- FGM is put in a special class of composites because it can withstand high thermal gradient. This property makes it appropriate for constructing space components.

**(d) Defence**:**-**FGM is used in bullet proof vests and armour plates as penetration resistant materials.

**(e) Energy**:-FGM act as protecting coating and thermal barrier on turbine blades of gas turbine which is a energy conversion device.

**(f) Other areas**:-FGMs are also used in other applications like nuclear reactor components, heat exchangers and fire retardant doors etc.

## 1.4 CREEP

Creep is time dependent deformation process in which a solid material under the effect of mechanical stresses deforms slowly and permanently. This deformation occurs because of high level of stresses exposure for long time which is still less than yield strength of material [6]. Research shows that the increase in temperature decreases the strength of material. When the material is in contact to heat for long term then it increases the creep rate. Creep rate keep on increasing as the temperature reach near to melting point. Creep mainly increases because of movement of atoms at elevated temperature. The materials with good thermal and mechanical properties are able to control the movement of these atoms. Deformation rate is the function of exposure temperature, material properties, exposure time and load applied on the material. Sometimes the deformation turns out to be so large that the material can not able to perform its function any longer. The properties of material are independent of time at room temperature, but material behaves like viscoelastic at high temperature. For instance creep on the blades of turbine can bend the blades which can cause the contact of blade with casing resulting to failure of turbine blades [31]. Creep is generally a big concern to metallurgist when estimating components that work under large stresses or high temperature. Failure mode may or may not occur in the creep mechanism but it will definitely decrease the component life. Creep in some components creates good effect like moderate creep reduces tensile stresses in concrete which can otherwise create crack in the concrete in future. Creep deformation is not all of a sudden deformation process like brittle fracture. It is a time dependent deformation process where strains build because of long time exposure of stress. The temperature range varies in different materials for the occurrence of creep deformation. The impact of creep deformation becomes recognizable at roughly 40-45% of melting point for ceramics and 30% of melting point of metals. When the temperature approach to melting point then any material can creep virtually [12]. But creep can also occur at relatively low temperature in some materials. We can measure the dimensional change that happen at high temperature by using creep test and rupture test to measure the impact of long time load handling capacity. These tests give the data to better comprehend the creep deformation and to make a better product with increased life.

### 1.4.1 Creep stages

Creep is divided into three stages:-

1. Primary Creep (Transiting Creep)

2. Secondary Creep (Steady State Creep)

3. Tertiary Creep

**(1)Primary Creep**:- In the initial stage or primary creep, the strain rate is relatively high, but decreases with increasing time. This is because of work hardening. Stage1 represents primary creep.

**(2)Secondary Creep**:- It has a relatively uniform rate. After primary creep, the strain rate eventually reaches a minimum and becomes near constant. The equilibrium between annealing (thermal softening) and work hardening is the reason for uniform creep rate [6]. This stage is known as steady-state creep or secondary. This is the stage where we want to reduce the creep of disc. . Stage2 represents secondary creep.

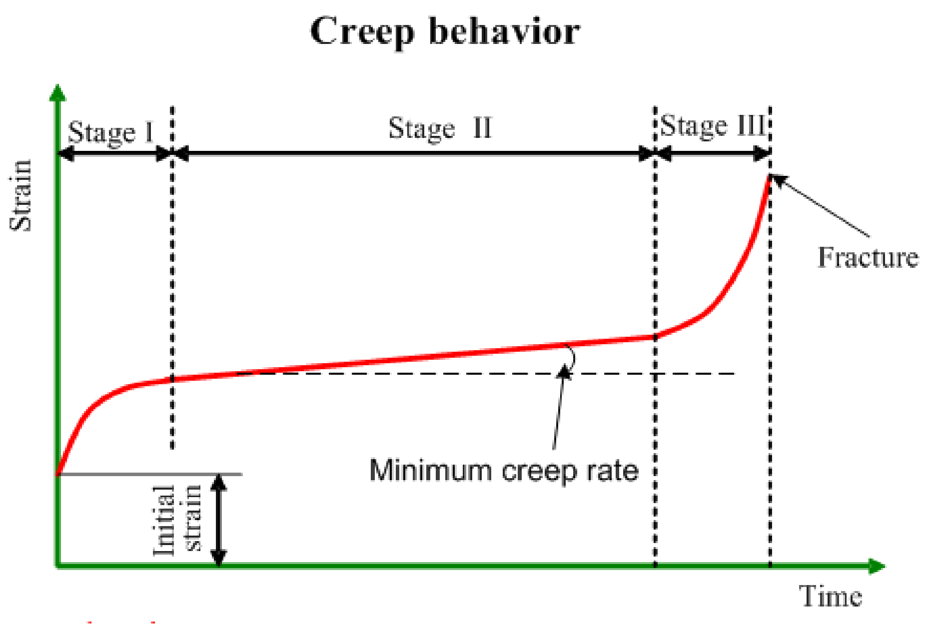



**Figure 1.4 Strain time curve for creep test [34]**

**(3) Tertiary Creep:** It is an accelerated creep rate and terminates when the material breaks or ruptures. It is linked with both necking and development of grain boundary voids. Stage3 represents tertiary creep.

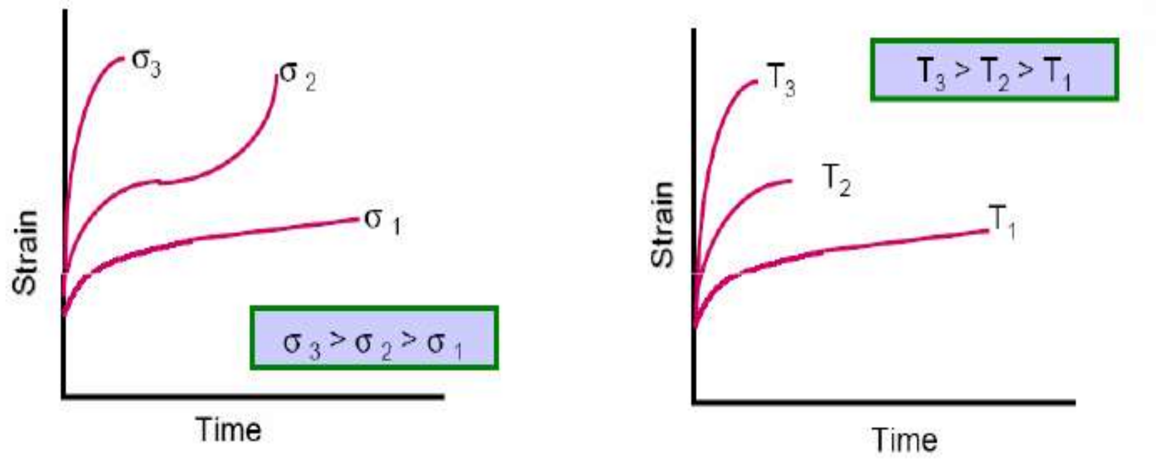



**Figure 1.5 Effect of stress and temperature [7]**

The strength of material declines with increase in temperature. The mobility of atom increase rapidly with temperature. It also causes the deformation of grain boundary at high temperature. The tensile property of most of metal is independent of time at room temperature, but at high temperature the strength become very depend on strain rate and the time of exposure .The number of metals in these condition behaves like a viscoelastic material[12]. The metal which is subjected to steady tensile load at high temperature will creep and experience a time dependent change in length. A strong time dependence of strength becomes important in different material at different temperature. What is high temperature for one may not be for another material. Creep is performance based behaviour since it is not an intrinsic material response. Creep is highly dependent on the environment include temperature and ambient conditions.

## 1.5 ROTATING DISC

In engineering, Rotating discs are getting a lot of attention because they are used in wide range of instruments which can either be mechanical and electronic instruments. Rotating disc gives a region of research because of their immense usage in rotating machinery like compressors, gas turbine rotors, pumps, compressors, flywheels, computer disc drives, automotive braking systems, ship propellers and turbo generators[12].In majority of the applications, Disc operates under high temperature and at the same time subjected to elevated stresses cause by disc rotation [14]. The disc material experiences creep deformations because of rigorous thermal and mechanical loadings which created big effect to disc performance. For example, there is always a probability in turbine rotor that heat from the outside surface transfer to the shaft and which badly affects efficiency and functioning of rotor [18].A detailed study of creep mechanisms and creep characteristics is necessary, if we want to use a composite disc under elevated temperature and high stresses. Qualitative information relating to creep rate is additional key for predicting the functional life of rotating disc working under given loading condition. For an effective design and better material, it is important to study the strains and stresses of discs which are under different boundary conditions. These boundary conditions vary with the way of attachment of disc with the shaft.

The availability of such information helps to modify the processing parameters of the disc material, tailor the disc design or change the composition of disc material which can improve the creep performance of disc. The designers working with Rotating discs offer an area of research and study due to their vast utilization in rotating machinery [9] because in most of the rotating machinery the disc has to rotate under elevated temperature and is simultaneously subjected to high stresses caused by disc rotation at high speed and because of mechanical and thermal loadings, the material of disc undergoes creep deformations which affect the performance of the system. We are able to reduce creep deformation by varying thickness, properties of disc by using functionally graded material and by doing this we are able to reduce the cost and increase efficiency of disc.

# CHAPTER 2

# LITERATURE REVIEW

Several investigators have carried out creep analysis of a rotating disc which is discussed as under:

**Singh *et al.* (2001)** studied the creep behaviour of two type of discs which were made of same Al-Sic functionally graded material but one had variation of particle distribution linearly in the radial direction and other had same quantity of particle distribution uniformly. The main goal was to save weight and to decrease the creep response. Creep response was calculated by using Norton's law in the disc made of 6061 aluminium alloy containing silicon carbide particles. The tangential stress found to be increased in inner radius because of high density occurred due to high particle content but got compensated by decreasing creep parameters which happened due to high particle content. This effect created a big decrease in creep rate in this region. The lower particle content in outer radius overcome the effect of increased creep parameters which leads to big decrease in tangential stress and reduces the creep rate. So, on comparing author found out that linear particle distributed discs had less radial and tangential creep rates than the disc with uniform particle distribution.

**Singh and Ray (2003)** investigated the creep analysis of rotating disc made of Al-Sic functionally graded materials which was isotropic in nature. Author studied a linear decreasing particle distribution disc which decrease from inner radius to outer radius. Norton's law was used to describe the creep behaviour of the disc. The variation of creep parameters of material which change because of varying composition in the radial direction of disc were calculated by using the method of regression fit in the accessible experimental data. Author found that the isotropic rotating disc with linear particle distribution was better than the same disc which had uniform particle distribution. The radial and tangential creep rates were less by a big magnitude in disc with linear decreasing distribution of particle than uniform particle distribution disc.

**Singh and Ray (2003)** investigated the creep behaviour of anisotropic functionally graded material rotating disc which occurred by doing the preferential alignment of whiskers and short fibers. New criteria were used to describe plasticity which at certain limits changes to Hoffman isotropic and Hill anisotropic yield criterion. The new yield criterion stresses and strain rates were compared with the stresses and strain rates calculated by hill anisotropic criteria. Author found change in yield stress in tension and compression that occur because of residual stresses present in disc. Experimental results of many studies are used to find the material parameters which characterize difference in yield stresses and anisotropy. It was concluded that occurrence of residual stress in anisotropic disc increases the tangential stress which increases the creep rate and made the radial strain compressive at outer and inner radius but create tensile at middle region of disc. So this area needs a more attention in future to control these types of problems.

**Gupta *et al.* (2004)** studied functionally graded material (FGM) disc prepared by particles of silicon carbide in pure aluminium matrix which was isotropic in nature. Sherby's law was used to describe the creep behaviour. Author studied linear particle distribution and thermal gradient in the disc and found out that strain rates are considerably lower in linear particle distribution disc as compared to uniform particle distribution isotropic disc. Linear particle distribution was used to make the strain rates more uniform. It was also concluded that the disc having thermal gradient decrease the strain rates by a big magnitude to that of uniform average temperature disc. So the disc with thermal gradient and linear particle distribution was concluded as the best disc in this paper.

**Callioglu *et al.* (2004)** investigated orthotropic rotating discs with few angular velocities to see the separation of the plastic region and tried to find radial and circumferential stress component to increase angular velocity. Author found out that elastic and plastic stress components had nearly the same values where the plastic region just started. The plastic yielding occurred first at the inner surface where residual stress was maximum. The circumferential stress component was lowest at the outer surface but highest at the inner surface. The radial stress component was always tension and zero along the radial section. The magnitude of circumferential stress component was found to be higher than that of radial stress component. The magnitude of residual stress was higher at inner surface than that of outer surface. The residual stress component of radial stress was always compressive and zero along radial section where as in circumferential stress it was at compression and tension at outer and inner surface. So the magnitude of the plastic flow was found to be highest at inner surface. By using analytical method author found that radial displacement increases at both inner and outer surfaces.

**Bayat *et al.* (2007)** investigated steady state thermal loading and axisymmetric bending in rotating disc made of functionally graded material. The disc material properties were supposed to be graded in the path of thickness by using power law distribution. Author employed von Karman and Mindlin plate theories for first order shear deformation. They developed new equilibrium equations for large and small deflections. They found a precise solution for displacement field by utilizing small deflection theory. In large deflection theory, Solutions were acquired in series form. Mechanical responses were compared with homogeneous versus FG discs as well as small verses large deflection. It was observed that mechanical response in functionally graded disc can be less than homogeneous disc for accurate values of grading index of mechanical properties. Author found that $w_{max}/h$ close to 0.5 for homogeneous disc give large errors with small deflection theory. So, functionally graded disc was considered as compared to full ceramic disc for using small deflection theory.

**Singh (2008)** investigated the creep behaviour of Al-Sic whisker composite disc which was anisotropic in nature. Nortons's power law was used to describe the creep behaviour of the rotating disc. The calculated results of Strain rate and stresses of anisotropic disc were compared with the results of isotropic disc. It was concluded that anisotropic disc with anisotropy parameter less than 1 had tensile radial strain rate in the middle region of the disc where as other disc had compressive radial strain rate in that region. The magnitude of tensile strain rate in tangential direction reduced by five orders, when anisotropy parameter was decreased from 1.3 to 0.7 but the variation of tensile strain rate remained same. Author found that anisotropy presents significant change in strain rates, where as resulting stress was increased by a very small amount.

**Bayat *et al.* (2008)** investigated elastic solutions for variable thickness functionally graded rotating disc which was axisymmetric in nature. The two power law distributions were used for disc thickness profile and material properties. Semi analytical and analytical solutions for hollow disc under free-free and fixed-free conditions were calculated by using power law for thickness profile function and material properties .Only semi analytical solution was presented for solid disc .Author investigated the effects of geometry and grading index on the displacement and stresses of the disc. It was found that FGM disc with hyperbolic or parabolic convergent thickness profile had less displacements and stresses when compared with uniform thickness disc. The maximum radial stress of parabolic thickness profile solid functionally graded disc was not at middle such as uniform thickness disc. Author concluded that hyperbolic convergent or parabolic concave thickness profile disc was better than one with the uniform thickness.

**Rattan *et al.* (2010)** studied creep behaviour of particle reinforced functionally graded material axisymmetric rotating disc which is isotropic in nature. Author compared rotating disc which had non linear particle distribution in radial direction with the discs which had equal amount of particle but distributed either uniformly or linearly in the radial direction. Sherry's model was used to study the creep behaviour of Al-Sic particulate composite disc. The creep parameters change along the radial direction because of varying composition. Regression fit of accessible experimental data was used to calculate this variation. Author concluded that creep behaviour can be controlled by appropriate distribution of particles because strain was minimum during parabolic distribution of particles. So parabolic profile of non linear FGM disc is better than linear variation and uniform particle distribution disc.

**Deepak *et al*. (2010)** investigated creep behaviour of functionally graded material rotating disc which had linear varying thickness. The disc was made of silicon carbide particles in pure aluminium matrix and threshold stress based law was used to describe the creep behaviour of the disc by considering stress exponent 5. The density and creep constants were varied in radial direction because of linearly decrease of silicon carbide content from inner radius to outer radius of disc. It was found that radial stress increased all over the disc with the increase in particle gradient in the disc where as effective and tangential stresses reduced at the outer radius but increased at the inner radius. The strain rates in uniform distribution composite disc were very high as compared to rotating disc which had gradient in reinforcement. They concluded that FGM discs with gradient in reinforcement were better than rotating disc with uniform distribution of particles.

**Callioglu *et al.* (2011)** studied stress analysis of functionally graded material annular disc. Power law function was used where density and elasticity modulus was supposed to be varying in radial direction of disc but Poisson's ratio of the material remains constant. Gradient parameter (n) was selected between 0 and 1.The disc with gradient parameter equal to zero was called isotropic homogeneous material. Numerical and analytical solutions were used to investigate radial and tangential stress and displacements at different gradient parameters with varying density and elasticity modulus. Least radial stress and homogeneous tangential stress was achieved along the radial direction of disc at n=1, when compared with n=0 which is homogeneous isotropic material disc. Rotating capability of disc was increased nearly two times by reducing tangential stress which was happening in FGM disc. So it was concluded that functionally graded material disc can handle higher angular rotations than homogeneous isotropic disc.

**Loghman *et al.* (2011)** investigated creep stress analysis which was time dependent in Al-SiC composite disc by using Mendelson's method. All thermal and mechanical properties were varying in radial direction according to volume percentage of silicon carbide reinforcement except Poisson's ratio. Sherby's model was used to describe the creep behaviour of Al–SiC disc by using Pandey's experimental results. Strains, strain displacements and stresses were calculated by using equilibrium equations. Successive elastic solution method was used to calculate history of deformations and stresses. It was concluded that stresses did not reduced considerably by uniform SiC reinforcement. However, composite disc of aluminium with 40% SiC at outer surface and 0% at inner surface had most uniform and minimum circumferential stresses and effective stresses. It was concluded that creep strains, stresses and displacement were varying with time at declining rate so that the disc approaches to steady state condition after approximately 50 years.

**Callioglu *et al.* (2011)** studied stress analysis of rotating annular discs which were subjected to internal pressure. Discs were under different temperature distributions like uniform temperature ($T_0$), linearly decreasing ($T_a$) and increasing ($T_b$) temperature in the radial direction. Power law functions and infinitesimal deformation elasticity theory were used to get the results. Results demonstrated that tangential stress decrease at outer and increase at inner surface when $T_0$ and $T_b$

temperature increased. If $T_a$ was increased then tangential stress rise at outer surface and decrease at inner surface of disc. Radial stresses were decreased gradually with rising $T_a$ and $T_0$ temperatures and increased with rising $T_b$ temperatures in the radial direction. The radial displacement was highest in disc having uniform temperature ($T_0$) as compared to the discs having other temperature distributions. The radial displacement was lower at inner surface than that of outer surface at all temperature distributions except in uniform temperature case. It was concluded after comparison between literature values of displacement and stresses and disc value at uniform temperature that they were consistent with each other.

**Kursun *et al.* (2011)** investigated elastic stress analysis of functionally graded annular discs which was under linearly reducing temperature distribution and uniform pressures at inner radius. Power law functions considered that thermal expansion coefficient and elasticity modulus were varied radially in the disc. Plane stress assumption was used to calculate analytical solutions of elastic stress and then compared with matlab program results prepared by author.

**Garg *et al.* (2013)** investigated creep behaviour of linearly varying thickness rotating disc under linear thermal gradient. The content of silicon carbide particles were varied non linearly in pure aluminium matrix. The variation of strain rates and stresses during disc rotation were calculated by imposing different thermal gradient temperature and also at constant temperature calculated by averaging the given thermal gradient. It was found that radial stresses were increased all over the disc where as tangential stresses reduced at outer radius and increases at inner radius when disc operate at linear thermal gradient. It was concluded that strain rates were decreased considerably in thermal gradient in comparison to functionally graded material disc working under constant average temperature.

**Hosseini *et al*. [2013]** investigated semi analytical solution of functionally graded axisymmetric discs. Thermoelastic creep behaviour was studied for variable thickness disc in the time domain. Governing differential equations were derived and then solved analytically as reduced linear algebraic equations. This method was used to avoid restrictions and simplifications which were normally needed in other creep calculation techniques. The structural and thermal properties of base metal were supposed to be function of temperature. It was found that ignoring these properties dependency on temperature caused nearly 200% errors in creep results. Thermal loadings and centrifugal force impact on strain rate was calculated at different profiles of disc as well as at different boundary conditions. The results were then compared with available literature to verify the results.

**Garg *et.al.* (2013)** investigated the creep analysis of linear varying thickness rotating disc. The silicon carbide particles were distributed non-linearly in pure aluminium matrix in radial direction of disc. The threshold stress law was used to describe the creep behaviour with stress exponent of 5. The strain rates and stresses were calculated in the disc and then compared with the disc having uniform allocation of reinforcement. The inhomogeneity in radial as well as tangential stress was increased by raising the particle gradient but tangential and radial strain inhomogeneity was reduced by significant amount. It was found that creep response in non linear distribution disc was considerably better then uniform distribution composite disc. So it was concluded that the extent of distortion and magnitude of strain rates can be reduced considerably by increasing the gradient in reinforcement.

**Gupta and Singh (2014)** studied the effect of anisotropy on creep behaviour of Al-SiC functionally graded rotating disc having hyperbolic thickness. Silicon carbide content in pure aluminium matrix was supposed to decrease linearly from inner to outer radius of rotating disc. Sherby's law was used to study the creep behaviour of FGM disc under stress due to rotation. Stress exponent of 8 was used to calculate creep response in threshold stress. It was found that anisotropy created a significant effect on creep response of FGM disc. The disc with both the anisotropy constant greater than 1 was called as the best disc. This disc had minimum tangential and radial strain when compared with other isotropic and anisotropic discs. Author concluded that anisotropy should be considered for safe design of functionally graded material rotating disc having hyperbolic thickness.

**Khanna *et al.* (2015)** investigated the creep behaviour of Al-SiC functionally graded discs which had different reinforcement gradients of silicon carbide and Thickness profiles. The rotating discs were considered to show creep behaviour according to tersca criterion and threshold stress based law. The equilibrium equation was solved along creep equations to estimate the strain rates and stresses of the disc. It was calculated that with the rise in thickness gradient of disc, the tangential stress reduced over whole radius where as radial stress increased at outer radius but decreased at inner radius. When silicon carbide gradient was increased in the disc, the tangential stress was decreased at outer radius and increased at outer radius whereas radial stress was increased over the entire disc in a considerable amount. There was considerable reduction and uniform strain rates throughout the disc because of increase in reinforcement gradient or thickness gradient. So it was concluded that the disc with high reinforcement and thickness gradients gets less distortions.

**Dai *et al.* (2015)** investigated stress distribution and displacement of functionally graded rotating disc rotating at different angular speed. The mass density and modulus of elasticity were considered to be related with volume fraction and change in radial direction. The motion equations were simplified to ordinary differential equation and then Runge-Kutta method was used to solve them. After that shooting method was used to solve whole problem. Numerical calculations explained that geometric shape, angular speeds and functionally graded index created considerable effect on mechanical behavior of rotating disc. These three parameters were able to reduce the displacements and stresses by a big amount and needs more research to increase the life of the disc.

**Deepak *et al.* (2015)** studied steady state creep of variable thickness functionally graded Al-SiCp disc by developing a mathematical model. The silicon carbide content was considered to vary from inner radius to outer radius. Stress exponent of 5 was used in threshold stress law to calculate the creep behaviour of disc. The strain rates and stresses were calculated for three same type of FGM disc but with three dissimilar thickness profiles which were hyperbolic varying thickness, linearly varying thickness and constant thickness profiles. It was found that hyperbolic varying thickness disc had least strain rates and stresses in comparison to constant or linear thickness disc. The radial and tangential strain rates of linear and hyperbolic discs were less by three orders in magnitude in comparison to constant thickness disc. It was concluded that hyperbolic and linear thickness discs possesses uniform radial strain rates, so there was less chances of failure in the disc.

**Singh and Sahni (2016)** investigated elastic plastic stresses of a functionally graded annular disc which was subjected to variable pressure and thickness. Young modulus was considered to be varying in the radial direction whereas poison ratio was considered as constant. A considerable increase in tangential and radial stress was seen at high internal pressure. Hoop stress was always greater than radial stress which did not allow the disc to move out and leads the disc to fail. Internal pressure was reason of maximum stress at internal radii. Inverse behaviour was found when disc was under external pressure which leads to maximum stress at outer radius. Author found that the resisting stress was decreased with the rise in geometric parameters m and n. Author found that displacement was maximum at outer radius when internal pressure was less than outer pressure and vice versa.

**Dai *et al.* (2016)** studied semi analytical approach to calculate stress and displacements in functionally graded hollow disc having angular acceleration and varying temperature. Material properties were assumed to change along radial coordinate. Material properties also related with volume fraction of every material. Coefficient of thermal expansion and modulus of elasticity were supposed to be dependent on temperature where as Poisson's ratio was considered as constant. It was found from numerical calculations that stress and displacement were increased with the rise of functionally grading index. The $r_0/r_i$ ratio and angular speed had the capability to increase the stress and displacement of disc where as enlarged temperature gradient had increased circumferential and radial stress while reduced the radial displacement. So it was concluded that these four parameters should be keep in mind while manufacturing an optimum rotating disc.

# CHAPTER 3

# RESEARCH PROBLEM

## 3.1 PROBLEM FORMULATION

The literature discussed so far reveals that most of the work on rotating disc has been done by using uniform composite. But functionally graded material can increase the life of the rotating disc. In most of the work, the considered disc has constant thickness and operates at constant temperature. But a little work reported for functionally graded rotating disc having lineally varying thickness and operates under linearly changing temperature. Most of the researches have used Von-mises yield criterion in their analysis. But the literature also reports that Tresca criterion has the ability to deal with more complex problem like thermal effects, acceleration effects etc. In a study, it is reported that analytical results by Tresca criterion are much closer to experimental results when compared with the results obtained by using Von-Mises criterion. So it is decided to analyse an FGM rotating disc made of Aluminium and reinforced with linearly varying silicon carbide particles.The rotating FGM disc will have variable thickness which decreases linearly from inner radius to outer radius of the disc. This FGM disc will also be subjected to a linear thermal gradient and yields according to Tresca criterion to estimate creep stresses and strain rates. Mathematical investigation will accomplish by using proper boundary conditions on the given FGM rotating disc.

## 3.2 METHODOLOGY

To achieve the accurate results of above problem, following steps are necessary to follow:-

I. Development of creep constitutive equations.
II. Solving the equilibrium equation along with the constitutive equations of FGM rotating disc under imposed boundary conditions.
III. Generate a software code for estimating creep stresses and strain rates in the rotating FGM disc of variable thickness.
IV. Validation of the analysis carried out and software code with published results.
V. Analysing the estimated results to draw important conclusion.

# CHAPTER 4

# MATHEMATICAL MODELLING

In the present chapter, a mathematical formulation is carried out to obtain the creep behaviour of rotating disc made of Al matrix reinforced with silicon carbide particles. The thickness and reinforcement of disc is decreasing linearly from inner radius to outer radius. The volume of the variable disc is kept equal to the uniform thickness disc. Disc is assumed to yield according to the Tresca criterion. The rotating FGM disc of variable thickness is assumed to operate at constant angular velocity (ω) and linear thermal gradient. The mathematical model is used to study the effect of varying the disc thickness profile and by imposing different kinds of linear thermal gradients.

(1) Linearly changing thickness profile.
(2) Linearly varying temperature profile.

## 4.1 DISC PROFILE

This study consider a composite disc rotating at constant angular velocity (ω) of 15000rpm with inner radius ($r_i$) =31.75mm and outer radius ($r_o$) =152.4mm [13].The disc is considered to have linearly varying thickness and linearly varying $SiC_p$ content.

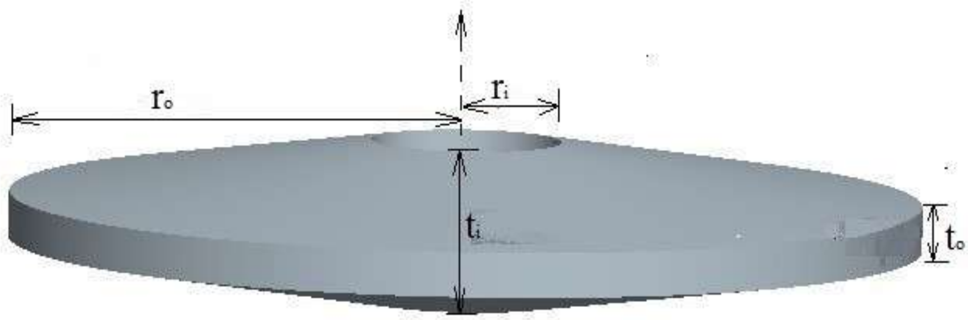



**Figure 4.1 Schematic showing disc geometry**

The disc thickness t(r) is assumed to vary linearly along the radius [4] in the following form,

$$t(r) = t_o + 2c(r_o - r) \qquad (1)$$

Where, t(r) is the thickness of disc at any radius r

$c = \left[\frac{t_o - t_i}{2(r_o - r_i)}\right]$ is a constant, $t_0$ and $t_i$ are disc thickness at inner and outer disc radius.

## 4.2 DISTRIBUTION OF REINFORCEMENT

The reinforcement content ($SiC_p$) in the FGM disc is linearly decreasing from inner radius to outer radius [31], as:-

$$V(r) = V_{max} - \left(\frac{r - r_i}{r_o - r_i}\right)(V_{max} - V_{min}) \qquad (2)$$

Where

$V_{min}$ and $V_{max}$ are minimum and maximum volume of silicon carbide particles reinforcement at inner and outer disc radii and V(r) is the SiCp reinforcement at any disc radius r.

The density of disc is calculated by using rule of mixture. So the density ρ(r) of disc at any radius r [31] is given as:-

$$\rho(r) = \frac{[100 - V(r)]\rho_m + V(r)\rho_d}{100}$$

$$\rho(r) = \rho_m + \frac{(\rho_d - \rho_m)V(r)}{100} \qquad (3)$$

Where $\rho_d$ =3210 kg/m$^3$ and $\rho_m$=2698.9 kg/m$^3$ are the densities of $SiC_p$ reinforcement and pure Al matrix, respectively and ρ(r) is the density of disc at any radius r.

## 4.3 DISTRIBUTION OF TEMPERATURE

Temperature profile is considered to vary linearly from inner radius to outer radius [18] as given by,

$$T(r) = T_a + \left(\frac{r_o - r}{r_o - r_i}\right)(T_b - T_a)$$

Where

$T_b$ and $T_a$ are temperature at outer and inner radius of the disc, and T(r) is the temperature at any disc radius r.

## 4.4 CREEP LAW

Threshold stress based law for steady state creep behaviour of disc [13] is shown as follows:-

$$\dot{\varepsilon} = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n \qquad (4)$$

Where

$\dot{\varepsilon}$ =effective strain rate, $\bar{\sigma}$ =Effective stress, $\sigma_0(r)$= threshold stress

$M(r) = \frac{1}{E}\left(A' \exp\frac{-Q}{RT}\right)^{\frac{1}{n}}$ are the creep parameters.

Where

$A'$=Structure dependent parameter.

n =True Stress Exponent, Q=True activation energy.

E=Temperature dependent young's modulus, R= Gas constant.

The value of true stress exponent is taken as 5.

The value of creep parameter M(r) and threshold stress σo(r) actually depends on the particle size (P=1.7μm), particle content V(r) and the operating temperature T(r) which can be explained by these equations[4]:-

$$M(r) = 0.0288 - \left(\frac{0.0088}{P}\right) - \left(\frac{14.0267}{T(r)}\right) + \left(\frac{0.0322}{V(r)}\right) \qquad (5)$$

$$\sigma_0(r) = -(0.084 \times P) - (0.023 \times T(r)) + (1.185 \times V(r)) + 22.207 \qquad (6)$$

The analysis of FGM disc is based on certain assumptions given below:-

i. Material of disc is locally isotropic and incompressible.
ii. Disc is considered under steady state condition of stress.

iii. Elastic deformations are neglected because they are small when compared with creep deformations.

iv. Disc thickness is small compared to diameter because of which axial stress is considered as zero.

Generalised creep constitutive equations under plane stress conditions [13] are given by,

$$\dot{\varepsilon}_r = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}[2\sigma_r(r) - \sigma_\theta(r)]$$

$$\dot{\varepsilon}_\theta = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}[2\sigma_\theta(r) - \sigma_r(r)] \qquad (7)$$

$$\dot{\varepsilon}_z = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}[-\sigma_r(r) - \sigma_\theta(r)]$$

Where

$\dot{\varepsilon}_z, \dot{\varepsilon}_r$ and $\dot{\varepsilon}_\theta$ are strain rates along axial, radial and tangential directions as given by the subscripts.

$\sigma_\theta$ and $\sigma_r$ are tangential and radial stresses in the disc.

The yielding of disc material is considered to be according Tresca criterion. In disc $\sigma_\theta > \sigma_r > \sigma_z$, So effective stress under biaxial state of stress [13] is written as:-

$$\bar{\sigma} = \sigma_\theta \qquad (8)$$

Putting $\bar{\sigma}$ and $\dot{\bar{\varepsilon}}$ values from equation (8) and (4) in first eqn of set of eqn (7), one gets,

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n \frac{[2\sigma_r(r) - \sigma_\theta(r)]}{2\sigma_\theta(r)}$$

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n \frac{[2x(r) - 1]}{2} \qquad (9)$$

Where

$x(r) = \frac{\sigma_r(r)}{\sigma_\theta(r)}$ = Ratio of radial and tangential stresses.

$\dot{u_r} = \frac{du}{dt}$ = radial deformation rate and u is radial deformation in disc.

Similarly, second equation of set of eqn (7) is written as,

$$\dot{\varepsilon_\theta} = \frac{\dot{u_r}}{r} = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n \frac{[2\sigma_\theta(r) - \sigma_r(r)]}{2\sigma_\theta(r)}$$

$$\dot{\varepsilon_\theta} = \frac{\dot{u_r}}{r} = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n \frac{[2 - x(r)]}{2} \qquad (10)$$

Divide equation (9) by equation (10) and then integrate the equation between limits $r_i$ to r,one gets,

$$\int_{r_i}^{r} \frac{d\dot{u_r}}{\dot{u_r}} = \int_{r_i}^{r} \frac{[2x(r) - 1]dr}{[2 - x(r)]r}$$

$$\int_{r_i}^{r} \frac{d\dot{u_r}}{\dot{u_r}} = \int_{r_i}^{r} \frac{\Phi(r)dr}{r}$$

$$\log\left(\frac{\dot{u_r}}{\dot{u_{r_i}}}\right) = \int_{r_i}^{r} \frac{\Phi(r)dr}{r}$$

$$\dot{u_r} = \dot{u_{r_i}} \times \exp\left[\int_{r_i}^{r} \frac{\Phi(r)}{r}\right] dr \qquad (11)$$

$\dot{u_{r_i}}$= Radial Deformation rate at inner radius

$$\Phi(r) = \frac{[2x(r) - 1]}{[2 - x(r)]}$$

Putting the value of $\dot{u_r}$ from equation (11) in equation (10) one get,

$$\frac{\dot{u_{r_i}} \times \exp\left[\int_{r_i}^{r} \frac{\Phi(r)}{r}\right] dr}{r} = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n \times \frac{[2 - x(r)]}{2}$$

$$\bar{\sigma} = \left(\frac{\dot{u_{r_i}} \times \exp\left[\int_{r_i}^{r} \frac{\Phi(r)}{r}\right] dr}{r \times M(r)[2 - x(r)]}\right)^{1/n} + \sigma_0(r)$$

$$\sigma_\theta(r) = \left(\dot{u}_{r_1}\right)^{1/n} \times \frac{\Psi_1(r)}{M(r)} + \sigma_0(r) \qquad (12)$$

Where

$$\bar{\sigma} = \sigma_\theta(r)\,,\ \Psi_1(r) = \Psi_2(r)^{1/n} \text{ and } \Psi_2(r) = \frac{2\times\exp\left[\int_{r_i}^{r}\frac{\Phi(r)}{r}\right]dr}{r[2-x(r)]}$$

The force equilibrium equation for variable thickness functionally graded material disc at angular velocity ω [13] is written as:-

$$\frac{d}{dr}[rt(r)\sigma_0(r)] - t(r)\sigma_\theta + \rho(r)\omega^2 r^2 t(r) = 0 \qquad (13)$$

The disc is assumed to operate under free-free boundary condition [13], which is given as,

$\sigma_r(r)$ =0 at $r=r_i$ and $\sigma_r(r)$ =0 at $r=r_o$ (14)

Substituting eqn (1) in the equilibrium eqn (13) and then Integrating the resulting equation between limits of $r_i$ to $r_o$ by using boundary equations of equation (14), we get:-

$$\int_{r_i}^{r_o} t(r)\sigma_\theta(r)\,dr = \omega^2 \int_{r_i}^{r_o} \rho(r) r^2 t(r)$$

$$\int_{r_i}^{r_o} t(r)\sigma_\theta(r)\,dr = \omega^2 \int_{r_i}^{r_o} \rho_m + \frac{(\rho_d - \rho_m)}{100}\left(V_{max} - \frac{(r-r_i)}{(r_o - r_i)}(V_{max} - V_{min})\right) r^2 (t_o + 2c(r - r_i))$$

$$\int_{r_i}^{r_o} t(r)\sigma_\theta(r)\,dr = \omega^2 \left[\int_{r_i}^{r_o} \left(A_\rho - B_\rho r\right)(Mr^2 - Nr^3)\right]$$

$$\int_{r_i}^{r_o} t(r)\sigma_\theta(r)\,dr = \omega^2 \left[A_\rho M \int_{r_i}^{r_o} r^2 - A_\rho N \int_{r_i}^{r_o} r^3 - B_\rho M \int_{r_i}^{r_o} r^3 + B_\rho N \int_{r_i}^{r_o} r^4\right]$$

$$\int_{r_i}^{r_o} t(r)\sigma_\theta(r)\,d = \omega^2\left[\frac{A_\rho M}{3}(r_o{}^3 - r_i{}^3) - \frac{A_\rho N}{4}(r_o{}^4 - r_i{}^4) - \frac{B_\rho M}{4}(r_o{}^4 - r_i{}^4) + \frac{B_\rho N}{5}(r_o{}^5 - r_i{}^5)\right] \quad (15)$$

Where,

$M = (t_o + 2r_o c)$, $N = 2c$

$$A_\rho = \rho_m + \frac{(\rho_d - \rho_m)}{100}\left[V_{max} + \frac{r_i(V_{max} - V_{min})}{(r_o - r_i)}\right]$$

$$B_\rho = \frac{(\rho_d - \rho_m)}{100} \times \frac{(V_{max} - V_{min})}{(r_o - r_i)}$$

The average tangential stress ($\sigma_\theta(avg)$) in the FGM disc [13] is given by:-

$$\sigma_\theta(avg) = \frac{\int_{r_i}^{r_o} t(r)\,\sigma_\theta(r)dr}{\int_{r_i}^{r_o} t(r)dr} \quad (16)$$

Putting equation (15) and (1) in equation (16), we get:-

$$\sigma_\theta(avg) = \frac{\omega^2\left[\frac{A_\rho M}{3}(r_0{}^3 - r_i{}^3) - \frac{A_\rho N}{4}(r_o{}^4 - r_i{}^4) - \frac{B_\rho M}{4}(r_o{}^4 - r_i{}^4) + \frac{B_\rho N}{5}(r_o{}^5 - r_i{}^5)\right]}{M(r_o - r_i) - \frac{N}{2}(r_o{}^2 - r_i{}^2)} \quad (17)$$

Integrate the equation (13) between limits $r_i$ to r and also using the boundary conditions of equation (14), one gets:-

$$\sigma_r(r) = \frac{\left[\int_{r_i}^{r_o} t(r)\sigma_\theta(r)dr - \int_{r_i}^{r_o} \rho(r)\omega^2 r^2 t(r)dr\right]}{r \times t(r)} \quad (18)$$

Multiplying equation (12) by t(r) dr and integrating between limits $r_i$ to $r_o$, we get:-

$$\int_{r_i}^{r_o} \sigma_\theta(r) \times t(r)dr = \int_{r_i}^{r_o} \frac{(\dot{u}_{r_1})^{1/n} \times \Psi_1(r) \times t(r)}{M(r)}dr + \int_{r_i}^{r_o} \sigma_0(r) \times t(r)dr$$

$$\sigma_\theta(avg) \int_{r_i}^{r_o} t(r)dr = (\dot{u}_{r_1})^{1/n} \int_{r_i}^{r_o} \left(\frac{t(r)\Psi_1(r)}{M(r)}\right)dr + \int_{r_i}^{r_o} \sigma_0(r)t(r)dr$$

$$\left(\dot{u_{r_1}}\right)^{1/n} = \frac{\sigma_\theta(avg)\int_{r_i}^{r_o} t(r)dr - \int_{r_i}^{r_o} \sigma_0(r)t(r)dr}{\int_{r_i}^{r_o}\left(\frac{t(r)\Psi_1(r)}{M(r)}\right)dr} \tag{19}$$

Where

$$\sigma_\theta(avg) = \frac{\int_{r_i}^{r_o} \sigma_\theta(r)\times t(r)dr}{\int_{r_i}^{r_o} t(r)dr}$$

Putting the value of $\left(\dot{u_{r_1}}\right)^{1/n}$ in equation (12), we get:-

$$\sigma_\theta(r) = \frac{[\sigma_\theta(avg)\int_{r_i}^{r_o} t(r)dr - \int_{r_i}^{r_o} \sigma_0(r)t(r)dr]\Psi_1(r)}{M(r)\int_{r_i}^{r_o}\left(\frac{t(r)\Psi_1(r)}{M(r)}\right)dr} + \sigma_0(r) \tag{20}$$

After knowing the values of radial stress and tangential stress, radial strain rate and tangential strain rate is estimated from equations (9) and (10).

# CHAPTER 5

# VALIDATION

It is essential to verify the correctness of developed computer code which is created according to mathematical model before explaining the results of current work. The stresses and strain rates are estimated for same functionally graded disc at same boundary conditions with the help of computer code and then compared the results obtained with that of published results (khanna *et al.*(2015)).

## 5.1 DATA FOR VALIDATION

Inner radius of disc ($r_i$) =31.75, Outer radius of disc ($r_o$) =152.4mm.

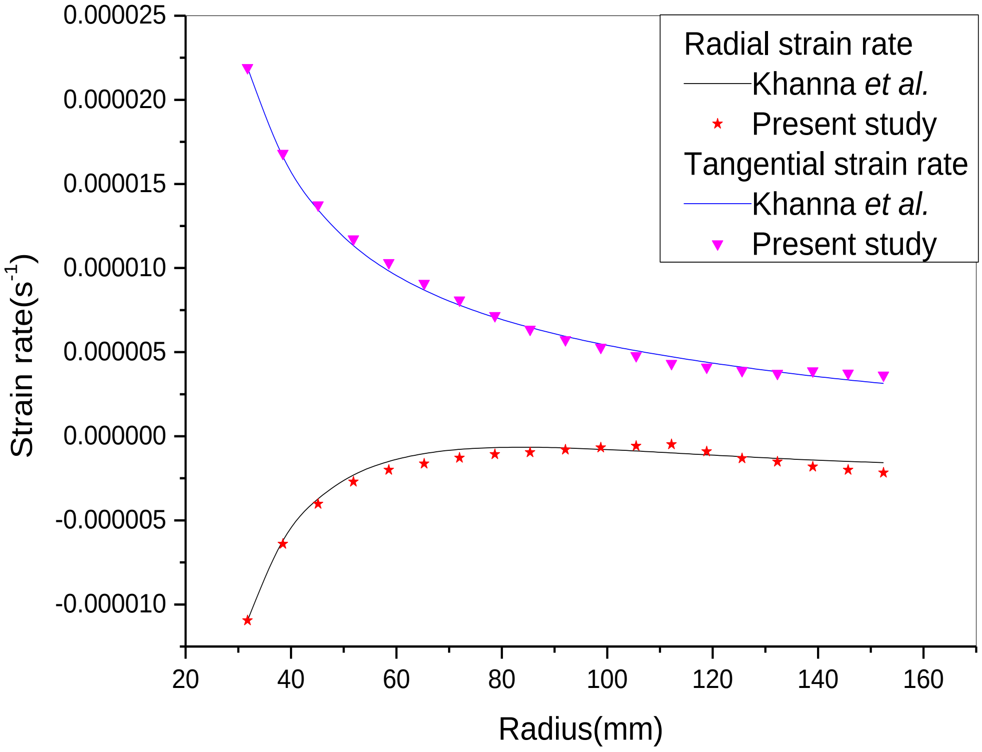



**Figure 5.1 Comparison of strain rates (present study) with published results.**

The variation of radial and tangential strain rate along the radius has been showed in Figure 5.1, which shows a good agreement between the two i.e. the results obtained from the present analysis and the published results. So, this validates the present analysis.

# CHAPTER 6

# RESULTS AND DISCUSSION

The main aim of present research is to estimate the effect of thickness variation and thermal gradient on the stresses and creep strain rates of the functionally graded material rotating disc. The following section explains the effects with the help of obtained results.

## 6.1 EFFECT OF THICKNESS GRADIENT

To analyze the effect of variation of thickness on the creep stresses and strain rates, one disc of constant thickness and four discs of higher inner and lower outer thickness are considered which are varying linearly. It is important to mention that the volume is kept same for all the discs and the volume fraction of $SiC_p$ in the matrix of Al is kept same (20%) for all the discs. The five discs are as follows:-

**Disc-1:** having constant thickness (t=25.4) throughout the disc.

**Disc-2:** having inner thickness ($t_i$ =30mm) and outer thickness ($t_o$ =22.45mm).

**Disc-3:** having inner thickness ($t_i$ =35mm) and outer thickness ($t_o$ =19.24mm).

**Disc-4:** having inner thickness ($t_i$ =40mm) and outer thickness ($t_o$ =16.03mm).

**Disc-5:** having inner thickness ($t_i$ =45mm) and outer thickness ($t_o$ =12.83mm).

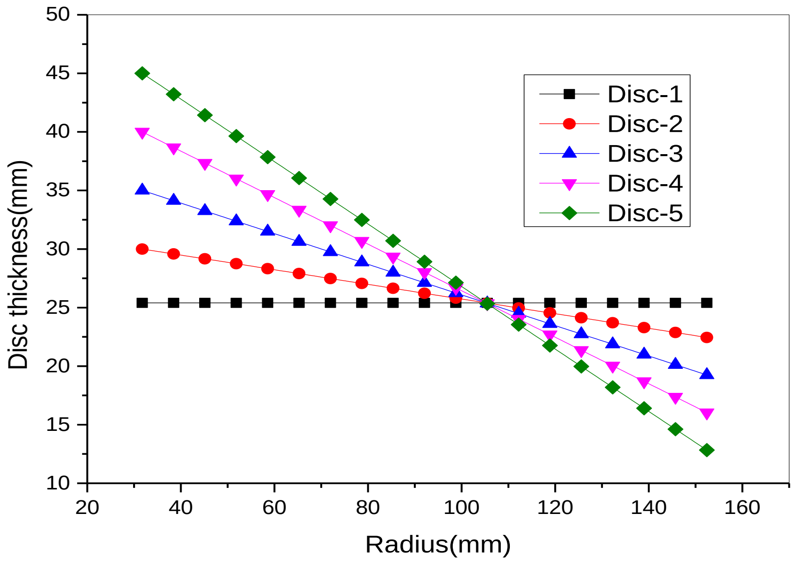



**Figure 6.1 Variation of thickness in the disc**

The stresses and strain rates in varying thickness discs are estimated and compared with constant thickness disc.

The variation of radial stress along the radius is as shown in figure 6.2 which depicts that the magnitude of radial stress keeps on decreasing with the increase in thickness gradient in the discs. The radial stress in all the five discs follows the same profile where as only the magnitude changes. The value of radial stress at inner and outer radius remain zero in all the five discs due to boundary conditions but maximum value of radial stress is achieved somewhere at radius of 65mm in all the five discs. Here Disc-5 which has highest value of thickness gradient from inner to outer radius, exhibits lowest radial stress throughout the disc. The highest value of radial stress in the Disc-5 is estimated as 25.9 MPa. The maximum reduction noticed in the radial stress is 6.09MPa, is between the Disc5 and Disc1 which is a uniform thickness disc.

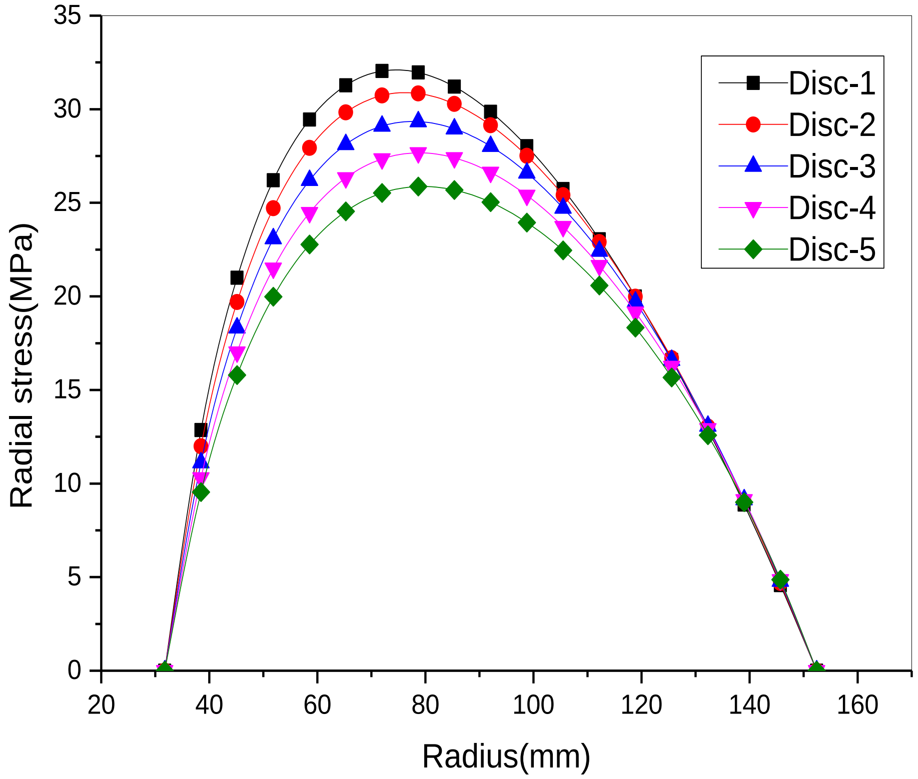



**Figure 6.2 Effect of thickness gradient on radial stress in the FGM disc**

The figure 6.3 shows the distribution of tangential stress in all the five discs. All the discs follows the same decreasing profile where highest value of tangential stress is at the inner radius and then stress keep on decreasing up to outer radius, But the magnitude of tangential stress in Disc-5 is lowest in all the discs. The tangential stress decreases significantly in all the discs with increasing radius. On increasing the disc thickness gradient, the tangential stress reduces throughout in all the discs in comparison the disc having uniform thickness. Although the reduction in the tangential stress with increase in disc thickness gradient is more near the inner radius in comparison to the reduction near the outer radius. The maximum reduction noticed is 20.63MPa and 13.44MPa at the inner and outer radius respectively between Disc-5 and Disc-1.

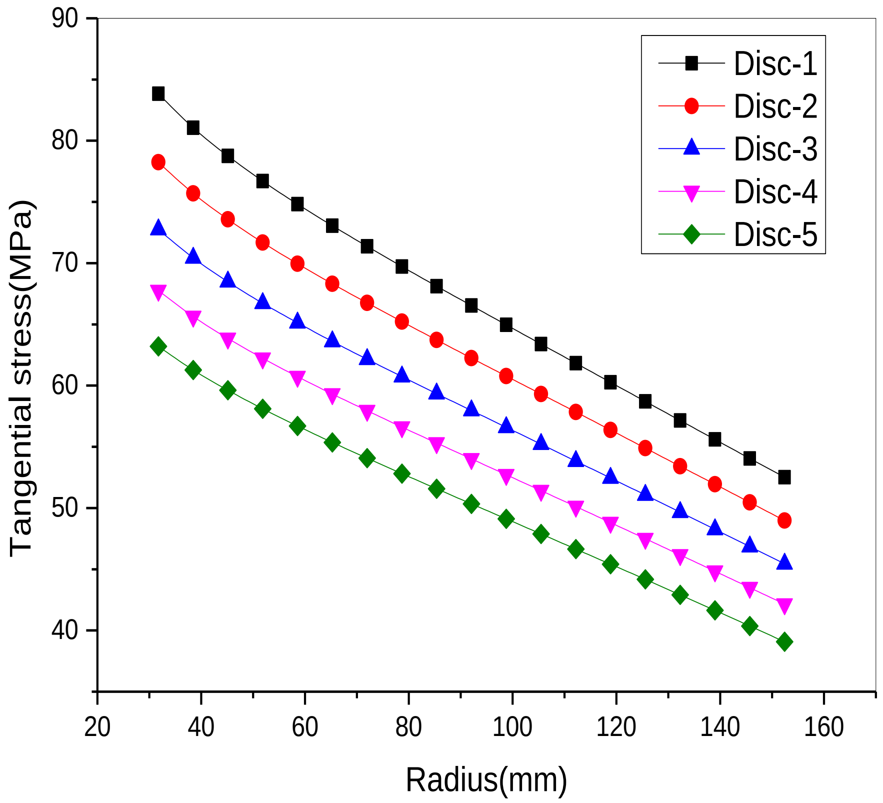



**Figure 6.3 Effect of thickness gradient on tangential stress in the FGM disc**

The radial strain rate depends on the creep parameter, threshold stress, radial and tangential stresses. Figure 6.4 depicts the variation of radial strain rate along the radius of FGM disc with the variation in disc thickness gradient. The radial strain rate decreases drastically with the increasing radius in the uniform thickness disc. As the disc thickness varies from inner to the outer disc radius, it reduces at the inner radius with increase in disc thickness gradient. However a slight increase is also observed after the mid radius. The variation in the radial strain rates reduces with increase in disc thickness gradient. Minimum radial strain rate is observed in the Disc-5.These results show that the maximum difference observed in the radial strain rate is $7.38\times10^{-6}s^{-1}$ in the Disc-5 and Disc-1. Disc-5 undergoes minimum radial strain rate and with very small variation in the strain rate from inner to the outer radius.

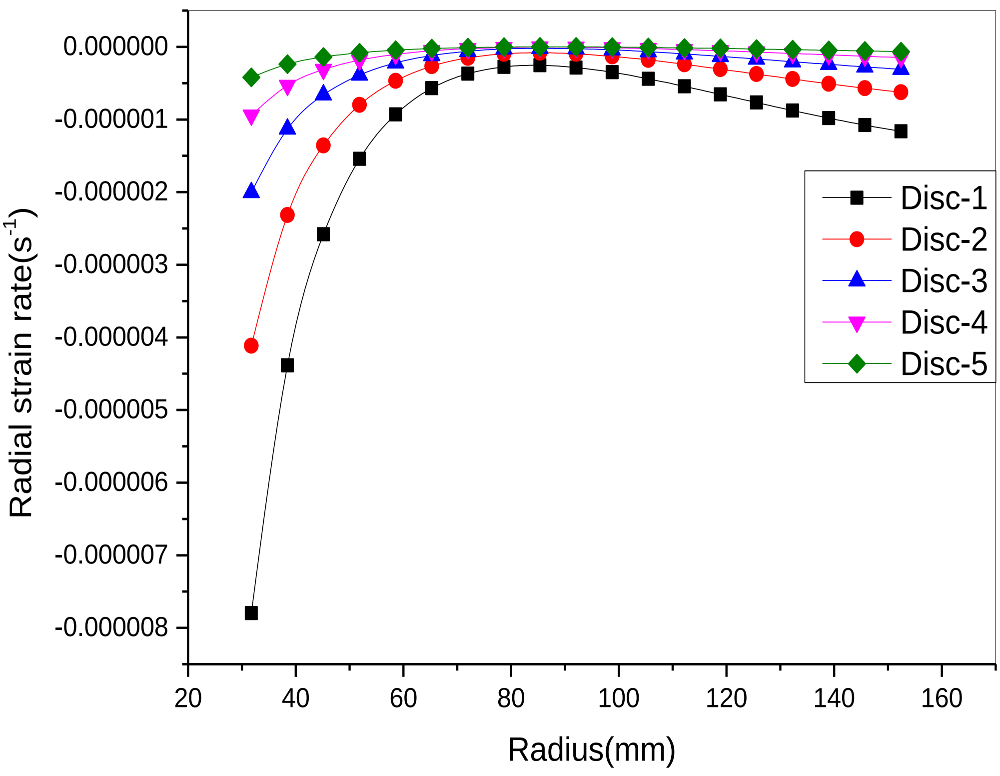



**Figure 6.4 Effect of thickness gradient on radial strain rate in the FGM disc**

The tangential strain rate also depends on creep parameter, Threshold, radial and Tangential stress of the FGM disc. Figure 6.5 depicts the variation of tangential strain rate in all the five discs with the variation in disc thickness gradient. All the discs follow same decreasing trend from inner radius to outer radius. But Tangential strain rate of uniform thickness disc decreases drastically with the increasing radius. On increasing the disc thickness gradient, the tangential strain rate reduces throughout in all the discs in comparison to constant thickness disc. With the increase in thickness gradient, the reduction in tangential strain rate is more near the inner radius in comparison to the reduction near the outer radius. The maximum reduction noticed in tangential strain rate is $1.47\times10^{-5}s^{-1}$ is between Disc-5 and Disc-1 which is uniform thickness disc.

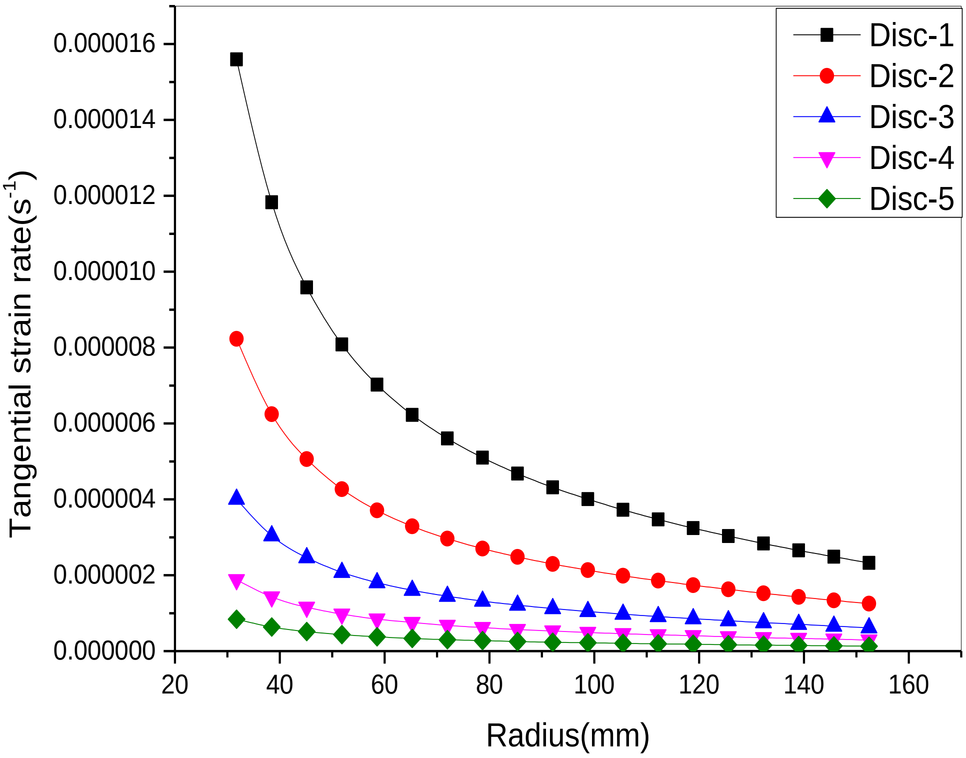



**Figure 6.5 Effect of thickness gradient on tangential strain rate in FGM disc**

The Figure 6.6 shows the distribution of axial strain rate in all the five discs which have linearly varying thickness profile. All the discs follows the same profile where highest value of axial strain rate is at the inner radius and then strain rate keeps on decreasing up to outer radius, But the magnitude of axial strain rate in Disc-5 is lowest in all the discs. There is severe decrease in axial strain rate from inner radius to outer radius in uniform thickness disc. The axial strain rate reduces at inner radius with the increase in thickness gradient in all the five discs. The variation in axial strain rate reduces with increase in thickness gradient. The maximum difference noticed in axial strain rate is $7.38\times10^{-6}s^{-1}$ between inner radius of Disc-5 and Disc-1. Disc-5 undergoes minimum axial strain and with very small variation in the strain rate from inner radius to the outer radius.

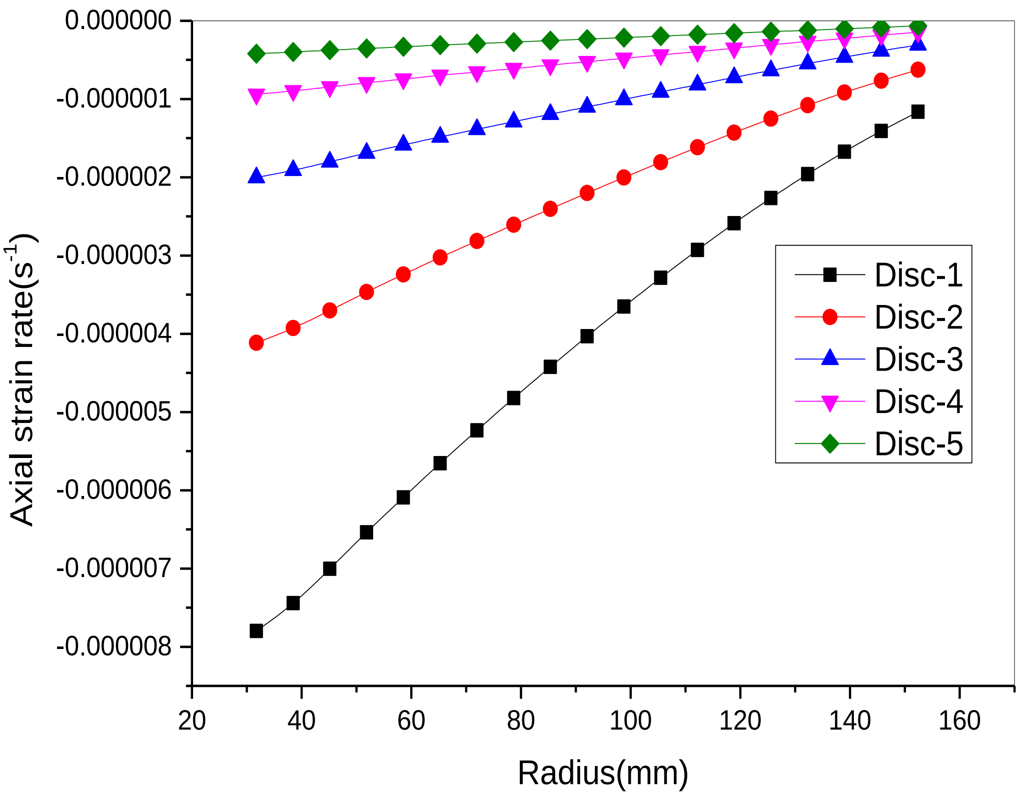



**Figure 6.6 Effect of thickness gradient on axial strain rate in the FGM disc**

The value of creep parameter depends on three terms which are particle size, particle content and operating temperature of disc. In all the considered discs particle size and operating temperature is taken as constant but particle content varies linearly from inner radius to outer radius. The increase in particle content at inner radius decreases the value of creep parameter but with the linearly decrease in particle content along the radius, the creep parameter keep on increasing and reach at maximum value at outer radius. Figure 6.7 depicts the effect of varying the thickness profile on the creep parameter in the FGM disc. The creep parameter value at inner radius is same in all five discs but all five discs follow different slopes towards outer radius. On increasing the disc thickness gradient, the creep parameter increases throughout in all the discs in comparison to constant thickness disc. Disc-5 has highest value of creep parameter along the radius because it has highest thickness gradient. The creep parameter value in all discs at inner radius is $2.35\times10^{-3}$MPa but the maximum difference of creep parameter is $1.8\times10^{-4}$MPa at outer radius between Disc-1 and Disc-5.

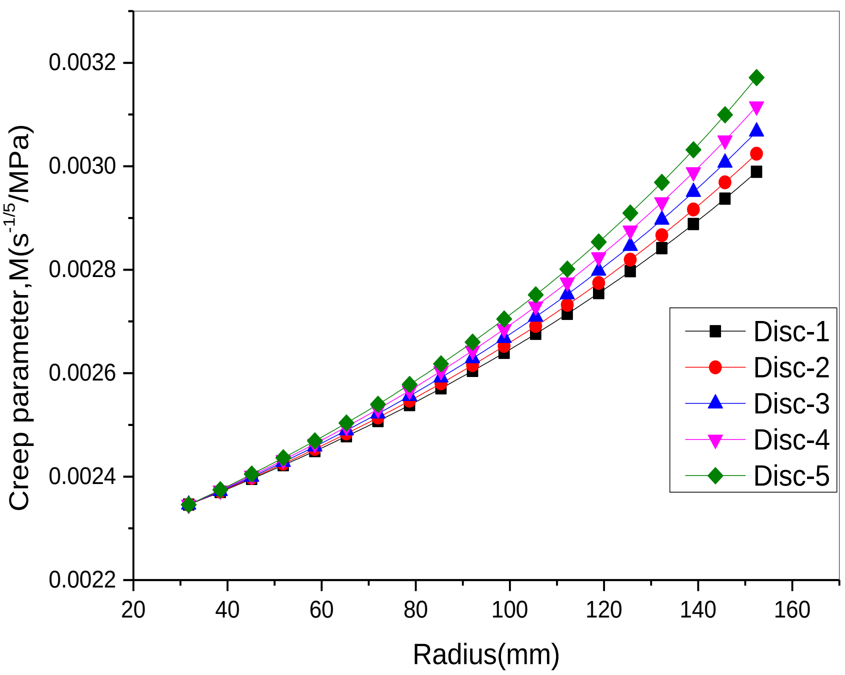



**Figure 6.7 Effect of thickness gradient on creep parameter in the FGM disc**

Threshold stress also depends on particle size, operating temperature and particle content. The particle content is varying linearly from inner to outer radius. The threshold stress is maximum at inner radius and then keeps on decreasing along the radius. The reason of this decrease in threshold stress is linearly decreasing particle content. Figure 6.8 depicts the variation of threshold stress along the radius with the variation in disc thickness gradient. All the five discs followed the same profile but there is change in magnitude of threshold stress along the radius. The threshold stress decrease drastically from inner radius to outer radius of disc in uniform thickness disc. This variation of threshold stress keeps on increasing along the radius with the increase in thickness gradient. The value of threshold stress in all the discs at inner radius is 37.2MPa.The maximum difference in threshold stress is between Disc-1 and Disc-5 at outer radius and which is 1.7MPa.

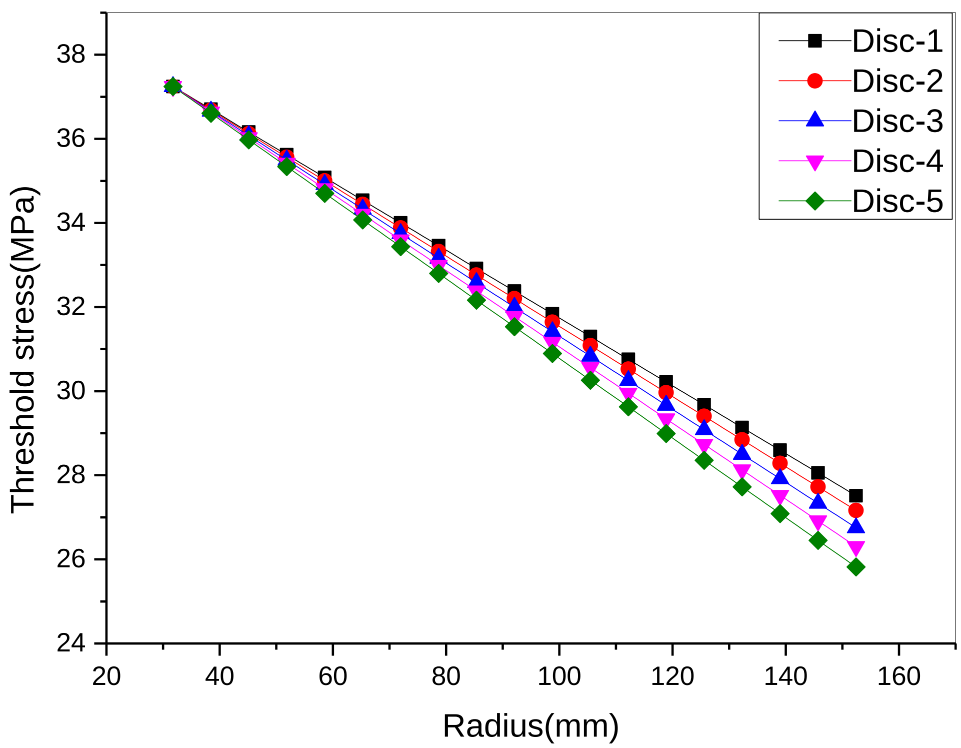



**Figure 6.8 Effect of thickness gradient on threshold stress in the FGM disc**

After discussing the effect of thickness gradient, it is concluded that Disc-5 is the best disc in comparison to remaining four discs because Disc-5 has lowest radial and tangential stress and it also has uniform radial, tangential and axial strain rate.

## 6.2 EFFECT OF THERMAL GRADIENT

In the previous section, it has been observed that the disc with highest disc thickness gradient, exhibits best creep performance. So, it has been decided to observe the effect of variation in temperature on the same disc. The inner thickness of this disc is 45mm and outer thickness is 12.826mm which is varying linearly. The results of FGM disc with constant temperature are taken as reference to compare the same disc at different thermal gradient. The temperature is varying linearly throughout the disc.

The three different temperature profiles which are imposed on the FGM rotating disc of variable thickness are given in the table :

**Table 6.1: Description of thermal gradient in the FGM disc**

| Disc Notation | Temperature at inner radius ($T_i$) | Temperature at outer radius ($T_0$) | Thermal Gradient ($TG=T_o-T_i$) |
|---|---|---|---|
| DTG-1 | 623 | 623 | 0 |
| DTG-2 | 598 | 648 | 50 |
| DTG-3 | 573 | 673 | 100 |

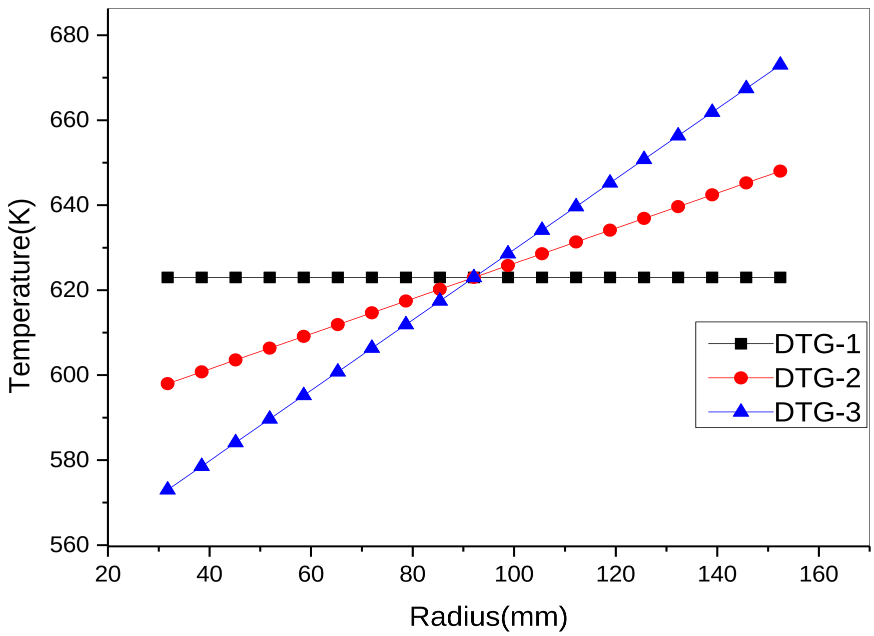



**Figure 6.9 Variation of temperature in the FGM disc**

The stresses and strain rates are estimated in the FGM disc under the effect of three different temperature profiles shown in figure 6.9 and are compared to investigate the effect of imposing thermal gradient.

The present section explains the effect of linear thermal gradient on radial stress in functionally graded material rotating disc. Figure 6.10 depicts that the profile of radial stress remain same under imposed boundary condition but the magnitude keeps on increasing with the increase in thermal gradient from 0 to 100K.The maximum value of radial stress is observed somewhere at radius of 70mm in all the discs. The disc with maximum thermal gradient exhibits highest radial stress throughout the disc. The highest value of radial stress is estimated as 32.23MPa in disc DTG-3. The maximum increase noticed in radial stress is 6.33MPa between DTG-3 disc and the disc that operates at uniform temperature i.e. DTG-1.

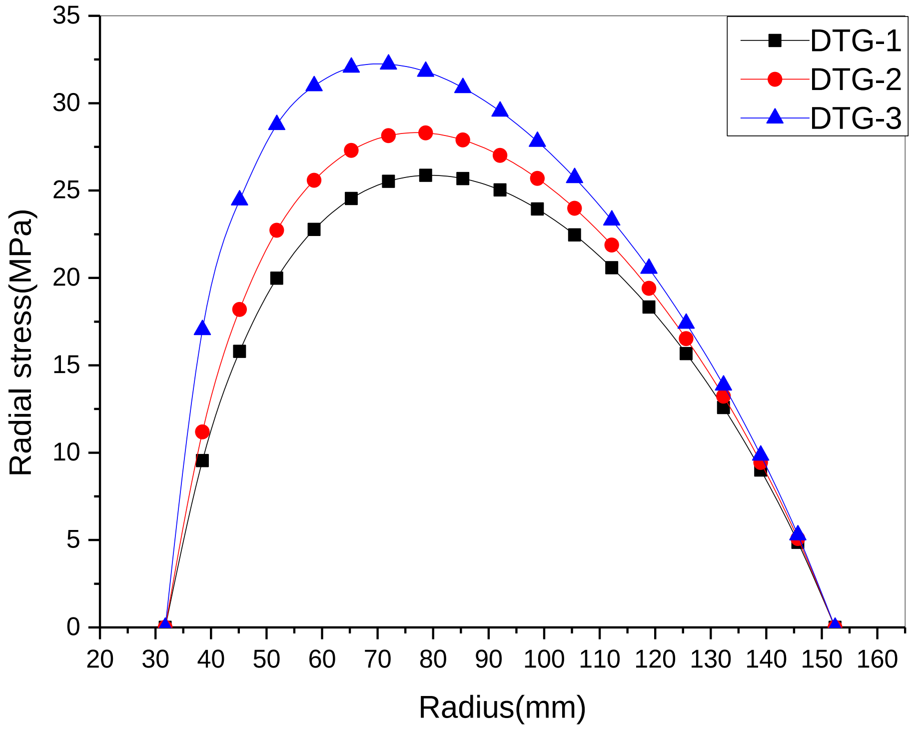



**Figure 6.10 Effect of thermal gradient on radial stress in the FGM disc**

Figure 6.11 explains the effect of thermal gradient on the magnitude of tangential stress in the FGM disc. The tangential stress decreases with increasing radius in constant temperature disc. As the temperature at inner radius decreases, the tangential stress increases by a large amount at inner radius which decreases drastically along the radius of disc with increase in temperature from inner to the outer disc radius. Tangential stress in thermal gradient discs decreases significantly at the outer radius in comparison to constant temperature disc. The highest value of tangential stress is 123.76MPa at inner radius in DTG-3 disc. The maximum difference noticed in tangential stress is 55.55MPa and 9.61MPa at inner and outer radius between DTG-3 disc and constant temperature disc i.e. DTG-1.Although the decrease in tangential stress is less at outer disc radius in comparison to the increase at the inner radius.

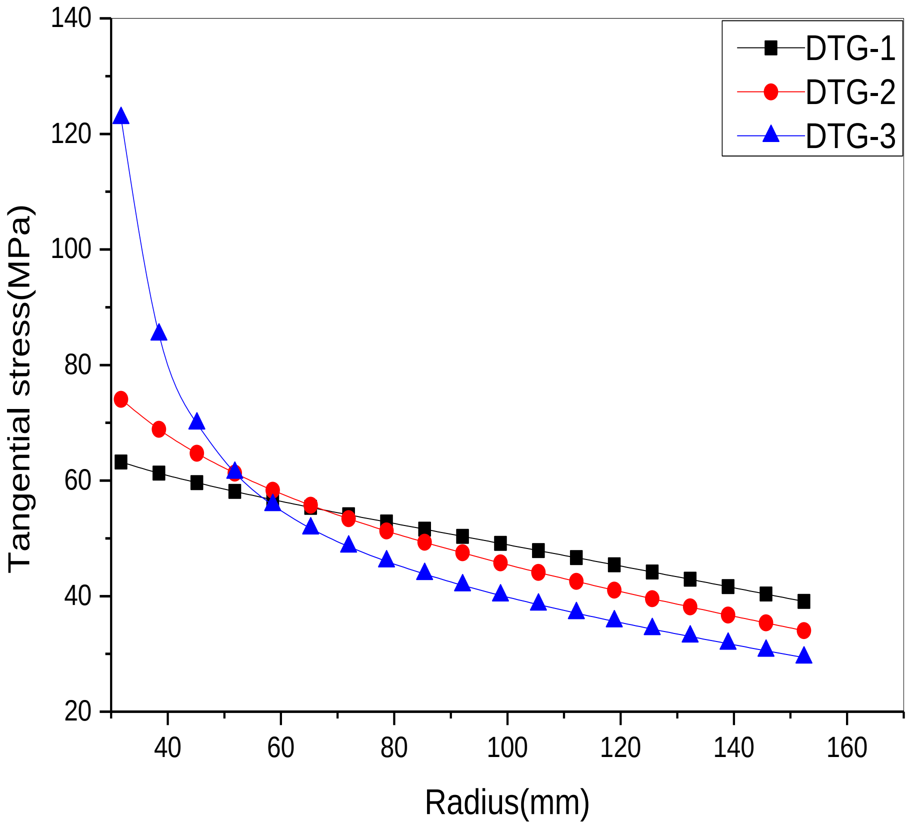



**Figure 6.11 Effect of thermal gradient on tangential stress in the FGM disc**

The radial strain rate depends on creep parameter, Threshold stress, radial and tangential stress. Figure 6.12 depicts the variation in radial strain rate in FGM disc under the effect of thermal gradient. The radial strain rate decreases drastically with the increase in radius in uniform thickness disc. Radial strain rate decreases at the inner radius of FGM discs with the increase in thickness gradient However a slight increase is also observed after mid radius. Minimum radial strain rate is observed in the disc DTG-3. The variation of radial strain rate decreases with the increase in thermal gradient. The minimum value of radial strain rate is $-4.19\times 10^{-7}s^{-1}$ at inner radius in the disc DTG-3 in comparison to the disc DTG-1 having thermal gradient of 100K. The maximum reduction in radial strain rate is $4.038\times10^{-7}s^{-1}$ at inner radius between constant temperature disc and DTG-3 thermal gradient disc.

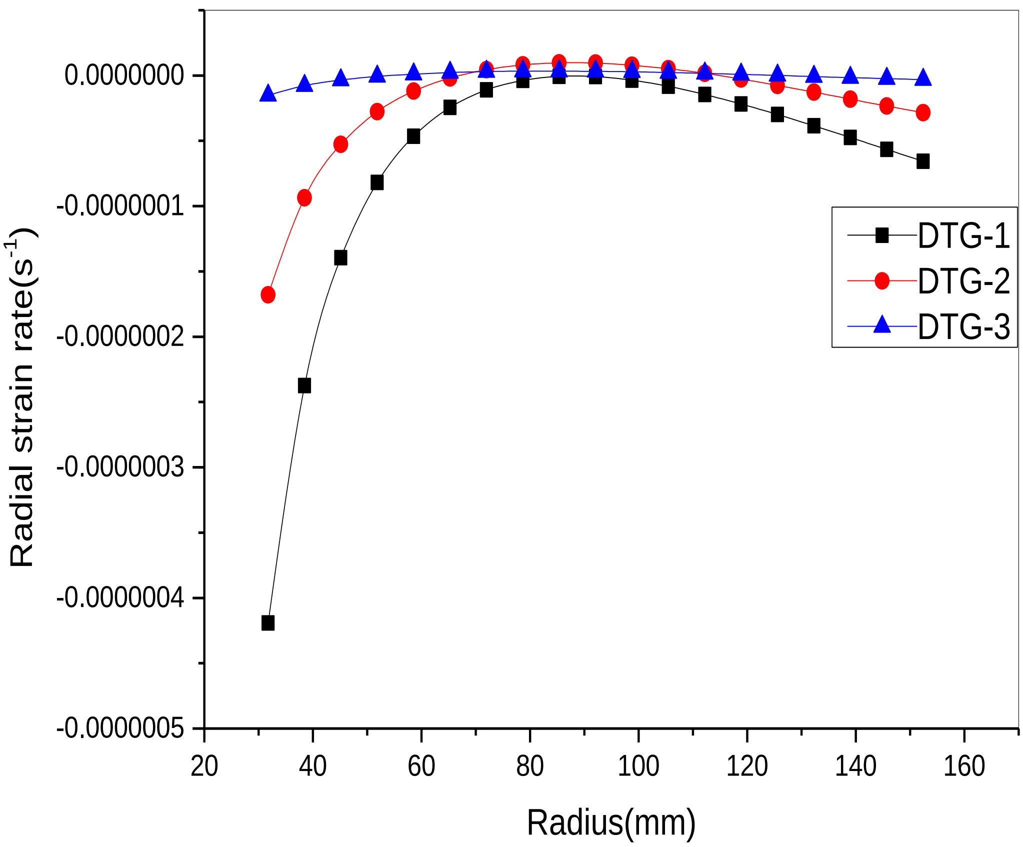



**Figure 6.12 Effect of thermal gradient on radial strain rate in the FGM disc**

Figure 6.13 depicts the effect of thermal gradient on tangential strain rate in the FGM disc. Tangential strain rate reduces severely from inner radius to outer radius in the constant temperature disc. But under the influence of thermal gradient, the tangential strain rate decreases at inner radius in all the FGM discs. The tangential strain rate of thermal gradient discs always remain less than constant thickness disc from inner radius to outer radius of disc. The variation in tangential strain rate from inner radius to outer radius decreases with the increase in thermal gradient. The minimum value of tangential strain rate is $1.31\times10^{-7}s^{-1}$ at outer radius of disc DTG-3. The maximum difference observed in tangential strain rate is $8.08\times10^{-7}s^{-1}$ at inner radius between disc DTG-3 and constant temperature disc i.e. DTG-1.

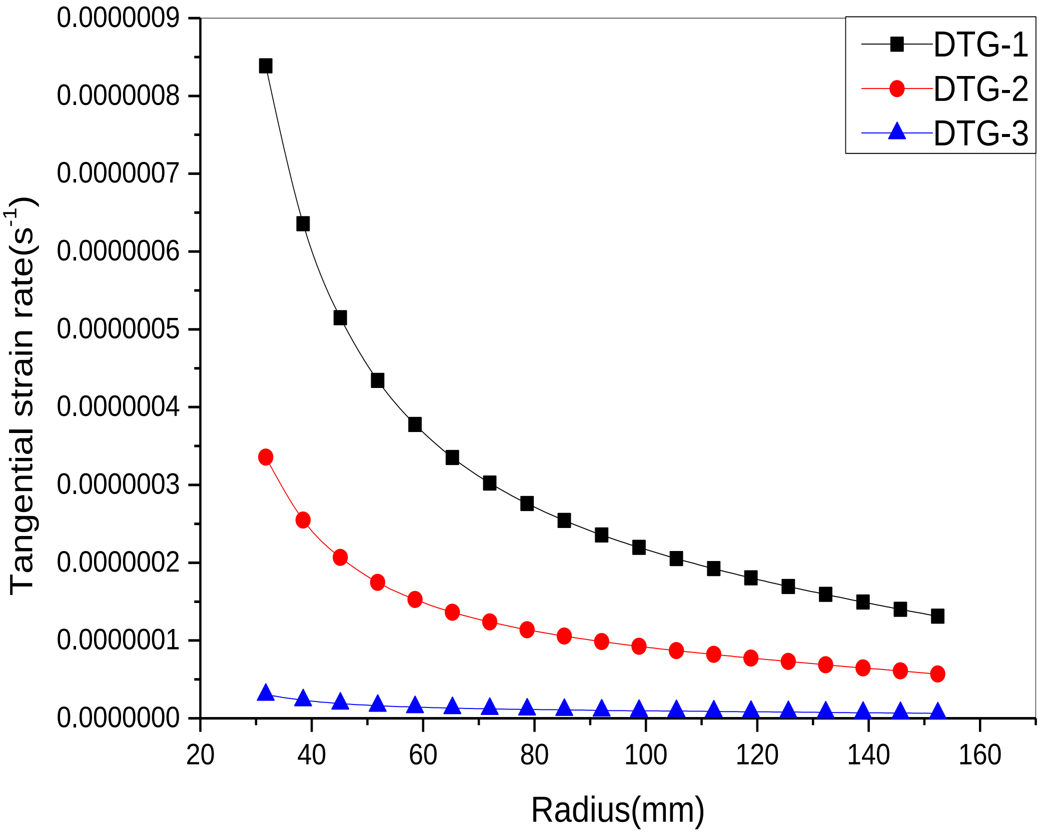



**Figure 6.13 Effect of thermal gradient on tangential strain rate in FGM disc**

Figure 6.14 shows the effect of thermal gradient on axial strain rate in the considered FGM discs. A significant decrease in axial strain rate along the radius in the constant temperature disc has been observed. Axial strain rate depends on radial and tangential stress and creep parameter according to its mathematical formulation. As the disc temperature increases from inner radius to outer radius in the disc, Axial strain rate reduces at inner radius of the discs. The minimum value of axial strain rate is $-6.56\times10^{-8}s^{-1}$ at outer radius in the disc DTG-3 having a thermal gradient of 100K. The maximum reduction noticed in axial strain rate is $2.51\times10^{-7}s^{-1}$ at inner radius between constant temperature disc i.e. DTG-1 and disc DTG-3 under the effect of thermal gradient.

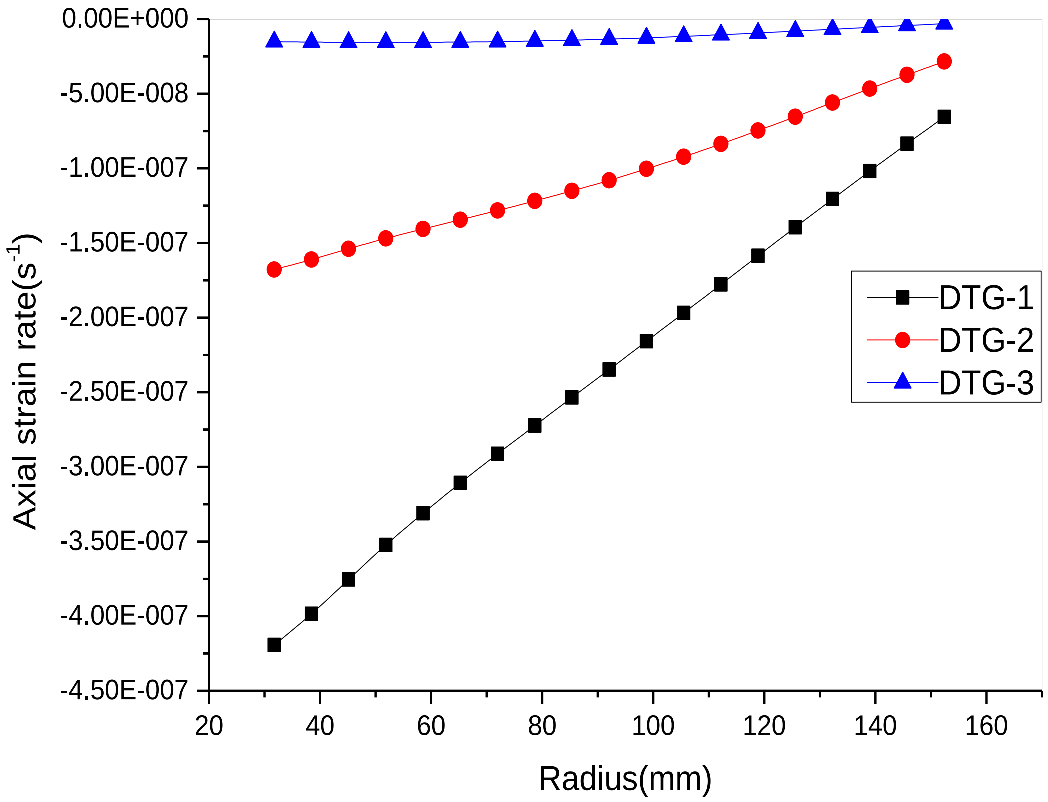



**Figure 6.14 Effect of thermal gradient on axial strain rate in the FGM disc**

Figure 6.15 depicts the variation of creep parameter because of thermal gradient. Creep parameter actually depends upon particle size, particle content and operating temperature of FGM disc. In this case, all the considered discs have same particle size and particle content but only difference between them is linearly increasing temperature distribution from inner radius to outer radius of disc which are compared with constant temperature disc. When the temperature at inner radius of disc is less than constant temperature disc then it reduces the value of creep parameter because of its place in mathematical formulation. But as the temperature start increasing from inner radius to outer radius, the value of creep parameter also increases and even exceeds the creep parameter value of constant temperature disc. Due to this reason, Disc with highest thermal gradient shows minimum creep parameter value at inner radius and maximum value at outer radius of disc as shown in Figure 6.15.The maximum difference in creep parameter is between constant temperature disc i.e. DTG-1 and DTG-3 and which is $9.5\times10^{-4}$ at inner radius and $8.7\times10^{-4}$ at outer radius of disc.

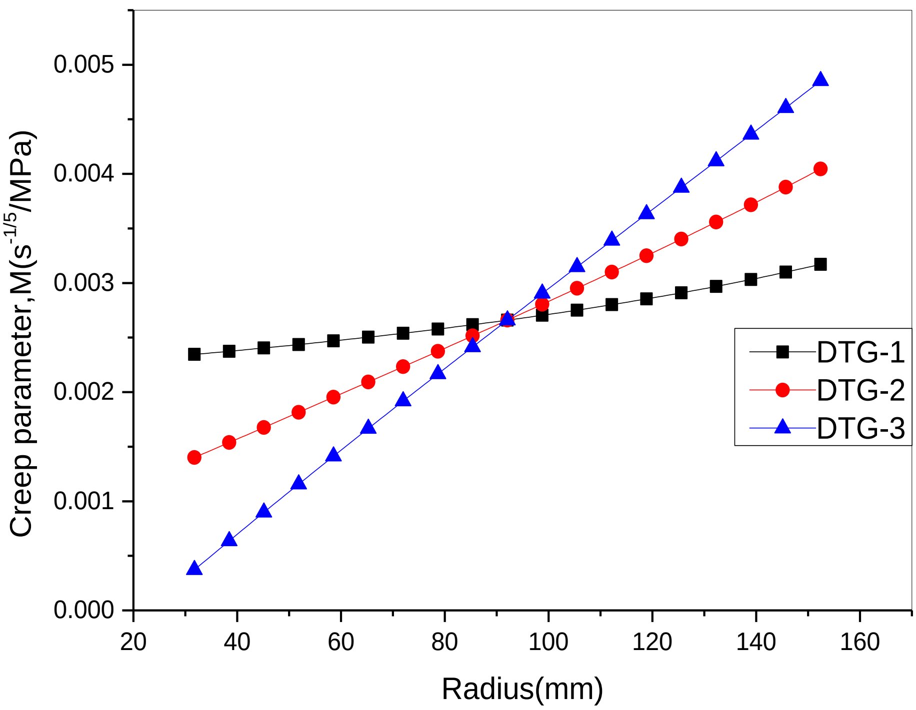



**Figure 6.15 Effect of thermal gradient on creep parameter in the FGM disc**

The variation in threshold stress because of thermal gradient is completely opposite to creep parameter as shown in figure 6.16. Threshold stress also depends on particle size, particle content and operating temperature. But due to its mathematical formulation, Temperature less than constant temperature disc leads to increase in magnitude of threshold stress at inner radius of disc. With the linearly increasing temperature distribution from inner radius to outer radius, the value of threshold stress starts decreasing and even provides threshold stress less than constant temperature disc at outer radius of disc. The reason of this decrease is higher temperature than constant temperature disc at outer radius of disc. The maximum difference in magnitude of threshold stress is 0.6MPa at inner radius and outer radius between constant temperature disc i.e. DTG-1 and disc DTG-3 under the effect of thermal gradient.

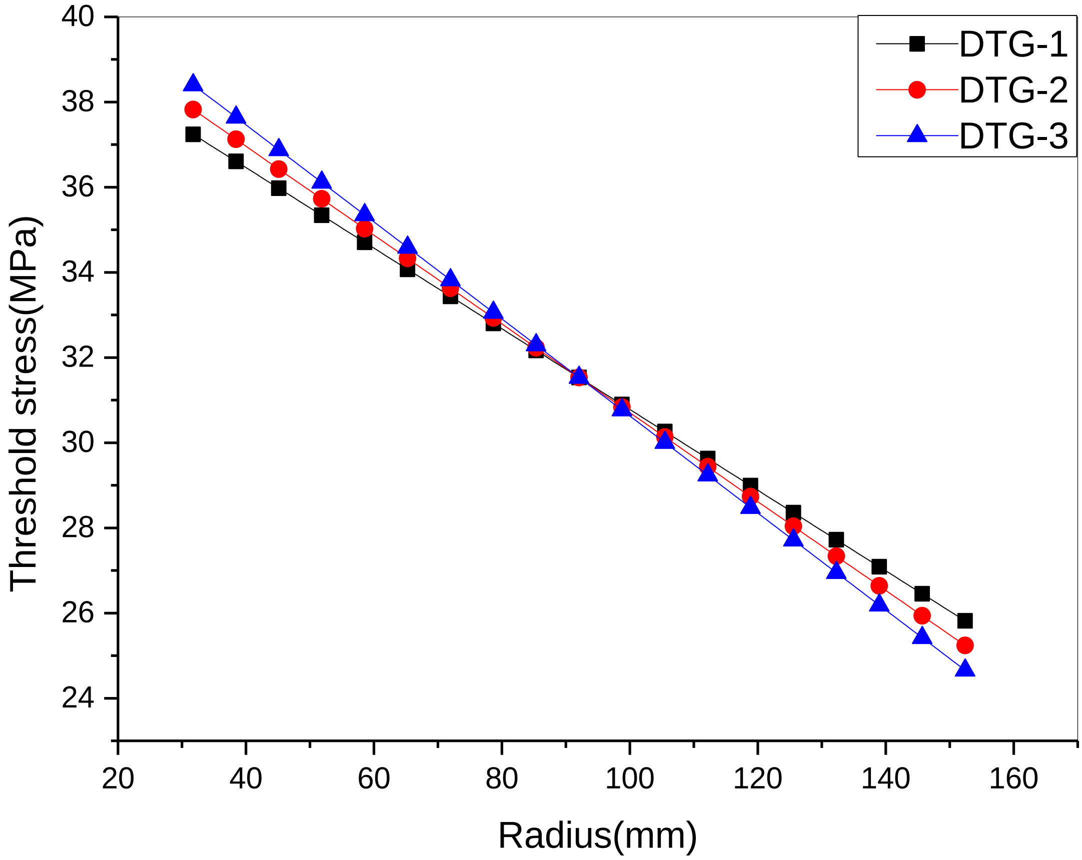



**Figure 6.16 Effect of thermal gradient on threshold stress in the FGM disc**

# CHAPTER 7
# CONCLUSIONS

From previous results and discussion, some observations in the present research work can be explained as follows:-

(i) The magnitude of radial and tangential stress decreases as the thickness decreases along the radial direction from inner radius to the outer radius.

(ii) The radial, tangential and axial strain rates are noticed to be significantly lower and uniform with the increase in disc thickness gradient.

(iii) With the increase in thermal gradient along the radius, the magnitude of radial stress and tangential stress increases at inner radius and decreases at outer radius of FGM rotating disc.

(iv) Radial, tangential and axial strain rate reduce and become more uniform with the increase in thermal gradient.

## 7.1 FUTURE SCOPE OF WORK

The present work may be further extended to in following directions:

- The creep analysis of the disc which is made of some other composite material.
- Analysis can be carried out for other volume fraction of $SiC_p$ in the matrix of Aluminium.
- Different disc thickness profiles can also be considered.
- Reverse variation of temperature can be considered.

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