

**GRAPHENE BASED 1-D PHOTONIC CRYSTAL WAVEGUIDE
FOR DELAY TUNING**

Dissertation submitted towards the partial fulfillment of requirement for the award of
degree of

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in

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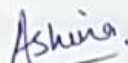
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DECLARATION

I hereby declare that the thesis entitled "Graphene based 1-D Photonic Crystal Waveguide for Delay Tuning" is an authentic record of my own work carried out as the requirement for the award of degree of Masters of Engineering in ECE at Electronics and Communication Department of Thapar University, Patiala under the guidance of Dr. Anil Arora (Assistant Professor), Electronics and Communication Department during the session 2014-2016.

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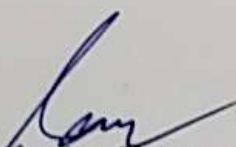

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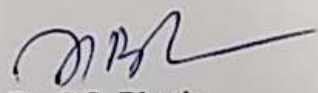
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ABSTRACT

A 1-dimensional (1-D) photonic crystal waveguide based on graphene which is electrically controllable is investigated to achieve slow light having wide bandwidth, low value of group velocity v_g and high value of group index n_g . The photonic crystal (PC) devices have an edge over other materials since they can work on room temperature, on chip integration is highly suitable, and have low dispersion propagation and wider bandwidth. Photonic crystal waveguides (PCW) find applications for tuning the delay as they provide tunable slow light with wide bandwidth

Our proposed structure has a 1-D PCW which is created through a photonic crystal which is otherwise perfect by introduction of a line defect. The bandgap of the PC locates some defect states within it by removal of a central slab of air to obtain an appropriate waveguide design. The light having propagation inside the waveguide has the constraint that it must move with a frequency which is within the crystal's bandgap and it can be made to move along the waveguide. A layer of graphene is inserted in the 1-D PCW to improve the properties of structure further and thus tune the delay by varying the graphene's fermi energy level. Graphene's excellent electrical tuning properties have various advantages and they are taken into consideration for tuning the group delay in a PCW. When graphene is not used in the structure, a value of 54.46ps for group delay is noted which is large enough and that too with a very large applied voltage of 50V, which is not practically realizable for applications involving tuning of delay on chip. Whereas when graphene is used in the design i.e. with core and clad; as the voltage applied to graphene is altered from 1 volt to 4 volts, there is a tuning from 81.1ps to 204.49ps in the group delay value for the structure where core region has graphene on it; and from 50.45ps to 187.6ps where clad region has graphene applied at 1550nm wavelength. Thus, a group delay tuning of 123.39 picoseconds and 137.15 picoseconds is achieved in both the designs respectively which is large enough as compared to 2-D PCW structures. The slow light which is then achieved has many useful applications in signal processing and delay scanning in the optical domain.

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Chapter 1

Introduction

1.1 MOTIVATION

The life of the people and society has immensely changed due to the advancements that have been brought in the semiconductor technology. Even the electrical properties can be controlled now, thus the control on the materials has widened. The revolution of transistor in electronics has begun by the rise of the semiconductor physics that allows the customization of the conducting properties of various materials. In the past few decades, controlling the optical properties of the materials has become the main objective. The scientists prefer photons over electrons for carrying the information nowadays so as to improve the performance of the system and strengthen the development of high density integration. Light has an upper edge over the electrons. It can travel much faster in a dielectric material as compared to the speed of electrons travelling through a metal wire. Light also has the capability to transport massive information in each second. Reduced energy losses are another benefit since the photons or the light particles interact much lesser as compared to those of electrons [1]. There would be a possibility of much technological advancement if it was viable to realize such materials reacting over light in the required frequency range only. This could be done by reflecting them perfectly, or allowing their propagation in some directions only and restraining in some others, or confining them within a mentioned volume [2]. The chief factor leading to all-optical integrated circuits is a new type of optical materials that are referred to as the Photonic Crystals. Photonic crystals make use of a much similar approach for fabrication and other theoretical concepts. The difference lies in the fact that they allow the control over the photons and not on the electrons as in the semiconductor physics. Their potential for interaction with the light over a wavelength scale provides a way towards the photonic integration. The discussed concept behind photonic-crystal materials rises from the very first ideas given by Eli Yablonovitch and S. John. Thus, the desired objective lies in designing such materials that have the ability of influencing the photon's properties just as the ordinary semiconductor crystals affect the electron's properties. The mechanism for controlling the light in photonic crystals is completely different from that of the traditionally used concept of total internal reflection.

The photonic band gap forms a difference which is analogous to that of the electron band gap in semiconductor physics [1].

1.2 PHOTONIC CRYSTALS

A noticeable interest has been found in the photonic crystals in the last few decades. They are called 'photonic' since they work on light and 'crystal' due to their periodical arrangement. When the atoms or molecules are arranged periodically, it is called a Crystal. The sequence defining the repetition of such molecules or atoms in space is called a Crystal Lattice. It is also possible to build some gaps within the crystal's band structure and also forbid some waves to propagate in the lattice. Photonic crystals are the optical analogous of this semiconductor theory. Here, the macroscopic media having contrasting dielectric constants take the place of molecules or atoms and the regular dielectric function replace the periodic potential. Thus the photonic crystals can be defined as periodic dielectric materials whose periodicity is of the order of light's wavelength. The way electron's motion is influenced by the periodic potential in semiconductor physics, similarly the EM (Electro Magnetic) wave's propagation is affected by the periodic dielectric in photonic crystals. Photonic crystals have regularly repeating low as well as high dielectric constant regions. It depends on the wavelength of the photons acting like waves whether they will propagate through the device or not. The wavelengths enabled to pass through can be referred to as modes. The grouping of allowed modes is called Bands. The wavelength bands which are not allowed form Photonic band gaps. The crystal is said to have a complete photonic band gap if for some frequency range, the propagation of EM waves is restricted in a photonic crystal from any source and in any direction. Unless there is a defect in a perfect crystal, the light is not allowed to travel. The type of the gap can control the properties of some localized photonic states that occur due to the defect which disturbs the crystal's periodicity. For example, a micro cavity is a type of point defect, a waveguide is a type of line defect, and a perfect mirror is a type of planar defect. In a periodic dielectric media, the solution of Maxwell's equation explains the concept of light in photonic crystals theoretically [1].

1.2.1 CLASSIFICATION OF PHOTONIC CRYSTALS

The periodic structure's dimensionality can be the basis for classifying the photonic crystals as one-dimensional (1-D), two-dimensional (2-D) and three-dimensional (3-D). The distinct dielectric constants are represented by the contrasting colors. The dielectric material's periodicity along one axis or more is the highlighting feature of the photonic crystal [2].

1. One Dimensional Photonic Crystal

Figure 1.1 shows a multilayer film, which is an example of the easiest and the most convenient photonic crystal possible. The two different colors represent two different materials having distinct dielectric constants. The term "one dimensional" is used as the dielectric function $\epsilon(z)$ is varying along a single direction (z) only [2].

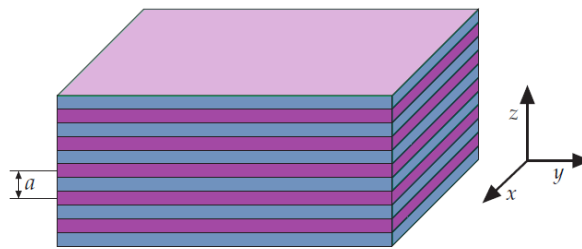


Figure 1.1: Example of a one dimensional photonic crystal [2]

2. Two Dimensional Photonic Crystal

Figure 1.2 represents a photonic crystal in two dimensions. Whereas it has periodicity in two directions, it shows homogeneity in the third direction. A simple example can be dielectric columns arranged in a square lattice. For certain values of the column spacing, there can be a xy-direction photonic band gap in this type of photonic crystal structure[2].

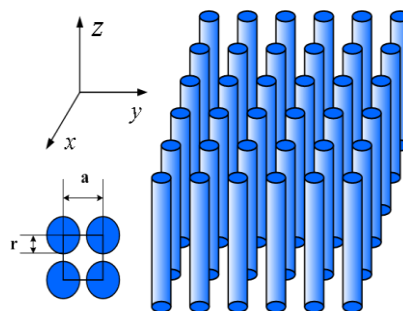


Figure 1.2: Example of a two dimensional photonic crystal [2]

3. Three Dimensional Photonic Crystal

A dielectric structure having periodicity in three different directions is referred to as a three-dimensional photonic crystal. Figure 1.3 represents a 3-D photonic crystal. Examples of such a crystal can be air holes arranged in a diamond lattice, an inverse opal, Yablonovite, rods and holes alternatively arranged in a stack etc.

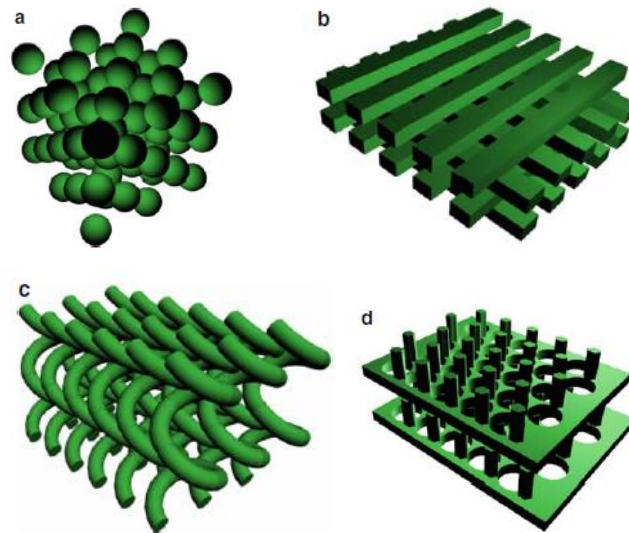


Figure 1.3: Examples of 3-D photonic crystal- FCC, woodpile, spiral lattice, quasi-diamond lattice [3]

In one-dimensional photonic crystals, there is uniformity in structure in two directions while there exists a periodical modulation of the permittivity in the third direction. An example of 1-D PhC can be quoted by the Bragg reflector which finds application in VCSEL (Vertical cavity surface emitting laser) being used as a distributed reflector. Such structures can also be made to use for upgrading the product quality of prisms, lenses, and other components used in optics since they can reduce the surface reflectance and thus find usage as anti reflection coatings. Since 1-D PhCs have a layered structure, only few variations in the periodic structure are possible. It is likely to alter the layer's thickness, the number of layers than can be present in the period or the refractive index of the structure [3].

The generalised information of the nature and properties of photonic crystal can be provided by the photonic crystal's band structure. The various Eigen states of an infinitely periodic structure represents the band structure of PhC. Dispersion contours can be created by slicing the multi dimensional surface of the photonic crystal's band diagram. It is analogous to the fermi band diagram studies in semiconductor physics. The

band diagram is always denoted by a two dimensional plot, irrespective of the photonic crystal's dimensionality. Figure 1.4 shows an example of the 1-D photonic crystal's band structure. The band structure has a physical significance of providing a relationship between the radiation properties and the optical media's properties through which the propagation of radiation occurs [4].

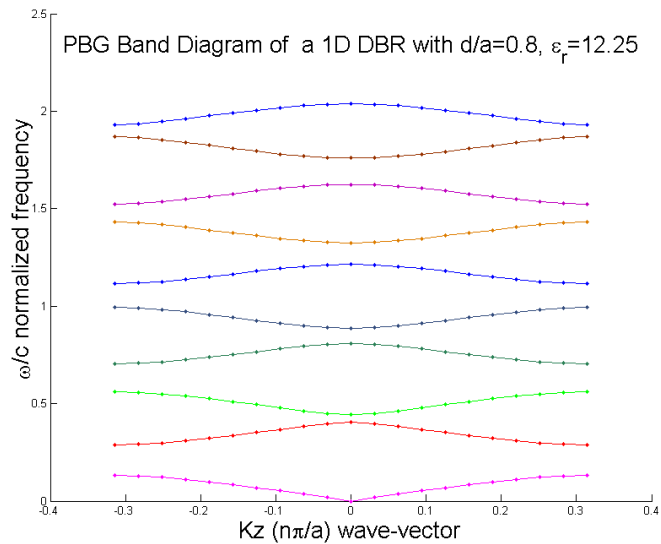


Figure 1.4: The band diagram of a 1-dimensional DBR (Distributed Bragg Reflector) where air is inserted by a dielectric with 12.25 as relative permittivity. The ratio of lattice period and thickness of air core (d/a) is 0.8. It is solved by 101 plane waves.

1.2.2 DEFECTS IN PHOTONIC CRYSTALS

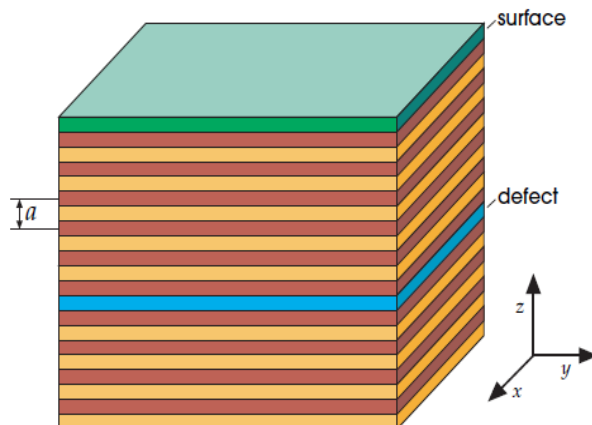


Figure 1.5: Defect states possible in a 1-D Photonic crystal - a) surface state at the crystal's edge represented by green; b) defect state within the crystal's bulk represented by blue [2].

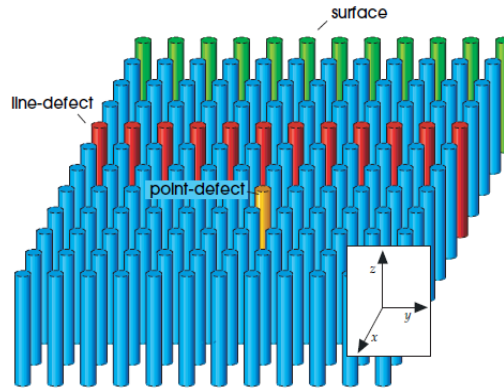


Figure 1.6: Defect states possible in a 2-D photonic crystal - a) point defect formed by disturbance in a single column, b) line defect formed by disturbance in a row, and c) surface defect caused by disturbance in the surface of crystal [2].

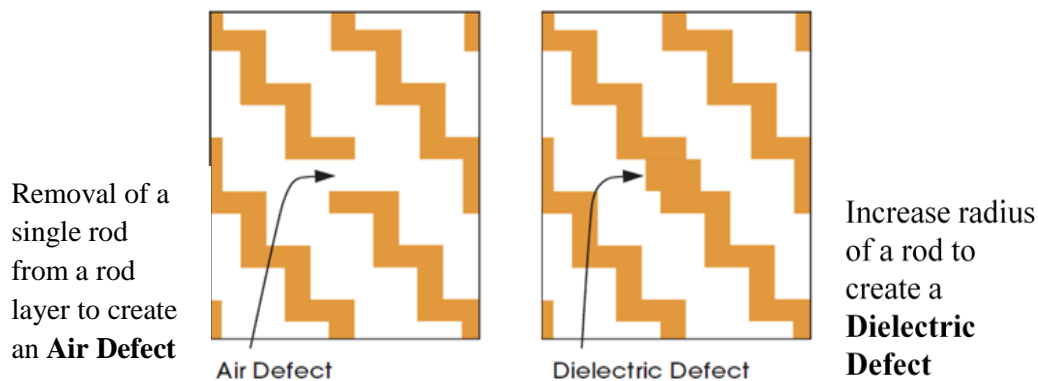


Figure 1.7: A Point defect created by modification of a single rod. Two simple ways to perturb a single lattice are to add extra dielectric material where it does not belong (Dielectric defect), or to remove some that should be there (Air defect) [2].

1.3 PHOTONIC CRYSTAL SLAB

Photonic crystal slab is the simplest design involving the fabrication of a periodic two dimensional PC that has some definite height. The most prominent structure amongst the photonic crystals is the slab which is two dimensionally periodic in plane. The concept of total internal reflection or index guiding confines the light vertically inside the slab in such structures. Here, the guided modes are created by the higher index of the slab that are restricted near by the slab. The periodicity in the structure forms a band gap where there exist no guided modes. The presence of a gap within a slab is determined by two

basic parameters - height and mirror symmetry. The slab must possess an optimum height which is approximately half a wavelength. The gaps should be separately considered in even and odd modes, thus there needs to be mirror symmetry. The modes are confined infirmly with very low value of slab height and the gap is filled by higher order modes with very high value of slab height. Figure 1.8 represents a 1-D photonic crystal slab having alternate air and dielectric bars.

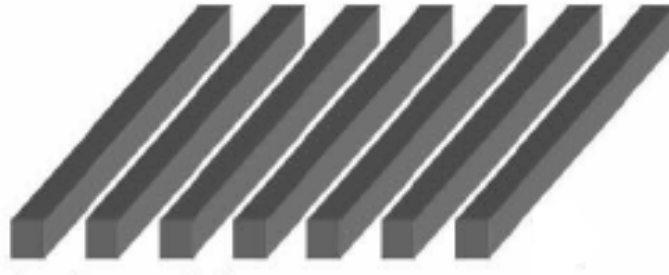


Figure 1.8: Schematic representation of 1-D Photonic crystal slab

Thus, a PC slab that is a $\lambda/2$ -thick high refractive-index dielectric slab (e.g., Si) having alternate bars of air and dielectric is one of the most promising PC structures. This kind of slab has an edge as its fabrication is much easier and simpler compared to higher order dimensionalities photonic crystals. Also, it is easier to manage the integration at chip level. This PC slab has core of high refractive index and cladding of low refractive index material. Thus by index guiding, the light is able to get enclosed in a single direction. It can thus be concluded that the 1-D PC slab possesses much different properties than those of 2-D photonic crystals having finite height.

1.4 PHOTONIC CRYSTAL WAVEGUIDE

Photonic crystals form the main materials in the working of slow light. The photonic bandgap effect is a fundamental property of PhCs, which means prohibiting the wave propagation through the structure of crystal in some directions for some intervals. A guided mode can be attained by deliberately introducing some defects in the crystal structure which propagates in some particular direction within the photonic crystal. Thus, light waves can be trapped by these defects which subdue their propagation in nearby areas. A photonic crystal waveguide is created when these defects are continuously made

in a certain direction. It is possible to propagate the frequencies in a particular direction occupying the bandgap of a photonic crystal. Thus to trap or localize a light, a photonic crystal with a gap within can import a defect. The light can be guided from one position to other if there is an introduction of a line defect. A waveguide is thus created through a photonic crystal which is otherwise perfect. The light having propagation inside the waveguide has the constraint that it must move with a frequency which is within the crystal's bandgap and it can be made to move along the waveguide. The light cannot go anywhere else once it is persuaded to advance within the waveguide [1].

When the periodicity of the photonic crystal structure is disturbed by the introduction of a line defect within it, a photonic crystal waveguide is formed. The defect can be introduced by either altering the diameter of some of the holes, or by removing some of the holes from a row, or by deleting an entire row or more than one rows of holes. The bandgap of the PC must possess some of the defect states. The light gets trapped close to the defect as the light's propagation is not easy at such a frequency. The line defect hence behaves like a waveguide. Some of the illustrations of 2-D linear waveguides are given in Figure 1.9 [5].

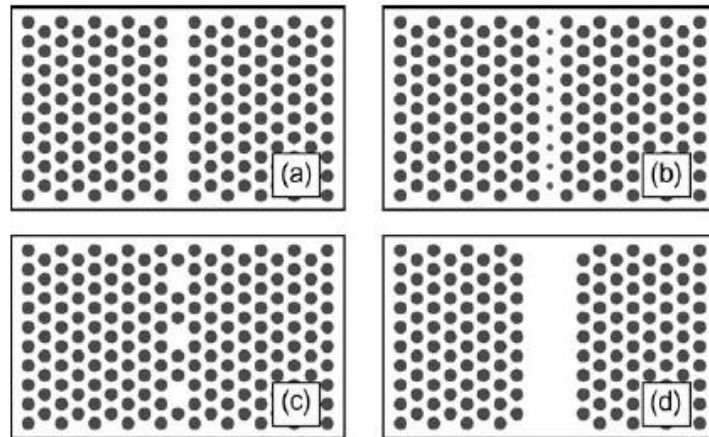


Figure 1.9: Examples of 2-D PC waveguides: a) Waveguide having one row of holes missing i.e. W1 waveguide, b) Waveguide with a row of holes having different radius, c) waveguide with some missing holes in a row, d) Waveguide having three rows of missing holes i.e. W3 waveguide [5].

The defect causes the propagation of light within the photonic crystal. In the vertical side, the propagation occurs through total internal reflection (TIR) and in the lateral side,

it occurs through Bragg reflection. As a result of strong dispersion in the line defect, there is generation of slow light in the proximity of the edge of photonic band [6].

The PC waveguide where only a single bar of dielectric is missing is referred to as W1 waveguide. Figure 1.10 represents a 1-D photonic crystal waveguide structure having a missing bar of dielectric. Various types of waveguides can be formed by similar practice through modifications made to the structure. There is an advantage of the photonic crystal waveguides above the traditionally used dielectric ones. They have the ability to restrict the light on sharp bends also.



Figure 1.10: Representation of a W1 1-D photonic crystal waveguide structure

1.5 SLOW LIGHT

Slow light with an exceptionally low group velocity finds application in buffering and optical data time-domain processing [10]. Slow light takes place as a result of the slowing down of the pulse when it interacts with the medium in which it is propagating. Components with the potential of buffering or delaying information are needed in communication networks but optical/electrical conversion is a hold-up for the data transmission rate increase. The key to the problem are the tunable all-optical delay lines with fine tuning of the delay [11]. By regulating the group velocity and hence the group delay, tunable optical delay lines tend to have usage in buffering the network, time-division multiplexing, optical equalization [12] and optical correlation [13], optical logic gates, optical coherence tomography [14]. Slow light can be achieved in different devices but to have practical applications, it must be obtained over a wide bandwidth. The photonic crystal (PC) devices thus have an edge since they can work on room temperature, on chip integration is highly suitable, and have low dispersion propagation

and wider bandwidth. Photonic crystals (PCs) are of significance for a number of applications.

Photonic crystal waveguides (PCW) find applications for tuning the delay as they provide tunable slow light with wide bandwidth [15]. The alternative for silicon technology is the silicon-on-insulator (SOI) platform which is absolute in photonic crystal waveguides. It is attractive mainly because there is huge difference in the refractive index of silicon and air or in some cases silicon and SiO₂ that provides a large photonic band gap, low losses. The cost of manufacturing is low as well as the yield is quite high since SOI is compatible with silicon technology [16].

By varying the material index, it is possible to widely tune the group index n_g . When the refractive index is varied, there is an associated shift in the band edge. Thus, the group index can be tuned conveniently if this shift is taken into consideration. In the PC waveguide, the guided mode deviates when there is a small variation in refractive index. At large values of frequencies, which are larger than the band edge; there is a sharp change in the group index. The photonic crystal slab's refractive index can be changed by using some common techniques like heating the material and carrier plasma effect. With the use of a heater in a PC waveguide, refractive index variation of about 1% can be achieved with heating which is high enough to tune the group index upto 25-65 range [13]. It is possible to tune the delay line and various methods have been advanced based on the slow light technique to obtain the same. Any dispersion that is there must be tailored to obtain slow light. The data signal carrying the information is highly affected by the group index variation. This causes the signal to delay and such a delay can be taken into control. The ratio of the angular velocity (ω) and the wave number (k) gives the group velocity of light, v_g which is also the inverse of the dispersion (first order) [10]:

$$v_g = \frac{c}{n_g} = \frac{d\omega}{dk} \quad (1.1)$$

The group index, n_g is the ratio of speed of light in vacuum and the group velocity of light, given by [10]:

$$n_g = \frac{c}{v_g} = c \left(\frac{dk}{d\omega} \right) = \frac{d(n\omega)}{d\omega} = n + \omega \frac{dn}{d\omega} \quad (1.2)$$

The group index is a representation of how much the speed is slow as compared to that of the speed of light (c). There is directly proportionate relation between the group index and delay in the pulse propagation [17]. The group delay refers to a delay being caused in the propagation of a slow light pulse. Consider a photonic crystal waveguide having a delay line with length ' l '.

The group delay for such a structure with v_g as the group velocity and n_g as the group index can be given by [10]:

$$\Delta t = \frac{l}{v_g} = \frac{(l \times n_g)}{c} \quad (1.3)$$

The group delay needs to attain a maximum value. There are two ways of doing it- either by increasing the length of the device l , or by decreasing the value of group velocity of light. Length of the device cannot be increased much as the structure might become bulky, thus to increase the group delay the group velocity v_g should be minimized and the group index n_g should be maximized.

Controlling the group velocity of light has found noticeable significance in the past few years as it finds applications in tunable delay lines and electrically tunable delay lines having wide tuning range are of interest at present. Tunable delay lines experimentally noted by the method of varying the propagation path of waveguide are quite cumbersome. Also, the delay resolution is confined to a few nano-seconds and the losses increase with the increasing number of stages. Large bandwidth, large tuning range, continuous tunability and active reconfiguration speed are some of the properties a tunable delay line should possess.

1.6 GRAPHENE

The name "Graphene" given by Hanss-Peter Boehm in 1962 composes of graphite and the suffix -ene. It was in 1987 that graphene was first noticed as a representation of singular layer of graphite sheets being an important aspect of graphite intercalation compounds. Graphene is an allotrope of carbon. It can be represented in the form of a 2-D hexagonal lattice with each vertex having one atom. In addition to carbon, many other

materials like charcoal, graphite, fullerenes and nanotubes of carbon have graphene as the basic structural element. Also, it can be considered as a huge aromatic molecule that is indefinitely massive. This is an extreme case of the flat polycyclic aromatic hydrocarbons family.

Graphene can be defined as firmly packed C atoms in a single flat layer which is in the form of a two dimensional honey comb lattice. Various other materials of different dimensions having graphitic properties use graphene as the basic building block which is represented in Figure 1.11. The 2-D graphene sheet can be wrapped into a 0-dimensional fullerene, or it can form 1-dimensional nano tubes, or it may be form stacks in 3-dimensional graphite. Graphene, which is being reviewed theoretically from around half a century now, represents the different materials based on carbon and their properties. Later, the researchers also made a conclusion that graphene can be a substitute in the quantum electro dynamics as a condensed matter. This further made graphene to grow as a leading material in the photonics world [18].

Recently the demand of graphene, a flat monolayer of carbon atoms which are bonded as sp^2 and are firmly packaged in a honeycomb lattice structure, has increased. Researchers are finding interests in this material as it has some excellent electronic and optical properties. Optical transmittance, large current density, chemically inert, room temperature low resistivity, high mobility, low Johnson noise etc. are some of its distinguished properties which make graphene viable to be replace the existing materials and become the for coming generation's primary material [18].

One single 2-D sheet of hexagonally placed carbon atoms forms the monolayer graphene as shows in Figure 1.12a. The structure having two such layers of graphene is referred to as Bi-layer graphene. When the number of layers is increased to 3-10, it is called few layer graphene (FLG). If the no. of layers is increased further to even more than 10, it forms a thick graphene and these types of structures do not have much use.

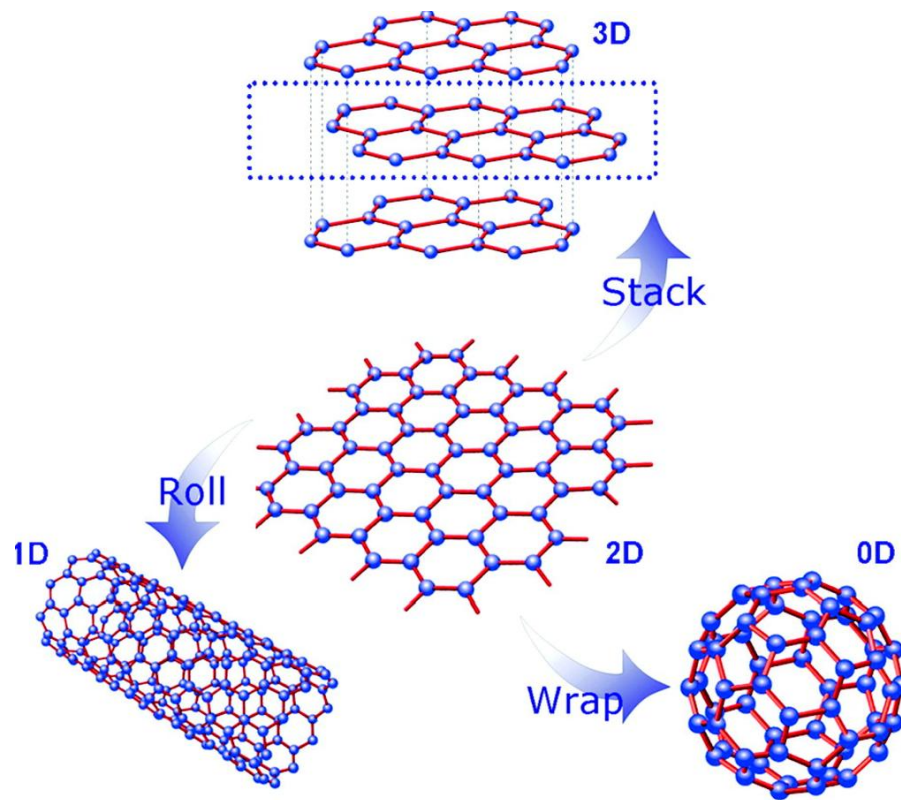


Figure 1.11: Representation of graphene and the different allotropes of carbon - a) 0-dimensional fullerene; b) 1-dimensional nano tube; c) 3-dimensional graphite [19]

There are different ways of stacking C atoms in bi-layer graphene and FLG namely AA stacking, AB stacking, or ABC stacking as shown in Figure 1.12b. Graphene presents a sp^2 bond which is hybridised as it has some π orbitals which are at 90° to the plane and three σ bonds which are all in the plane, represented in Figure 1.12c [20].

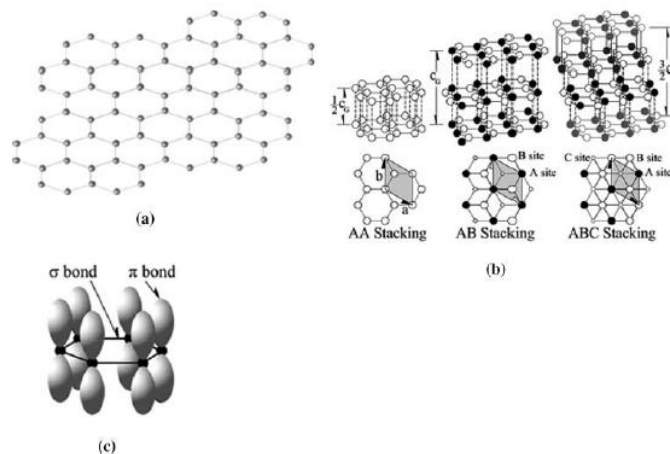


Figure 1.12: a) Representation of graphene's single 2-D sheet with carbon atoms hexagonally arranged, b) Different types of stacking in graphene and some commonly used structures, c) Representation of π bonds which are 90° to the plane and three σ bonds which are in the plane [20].

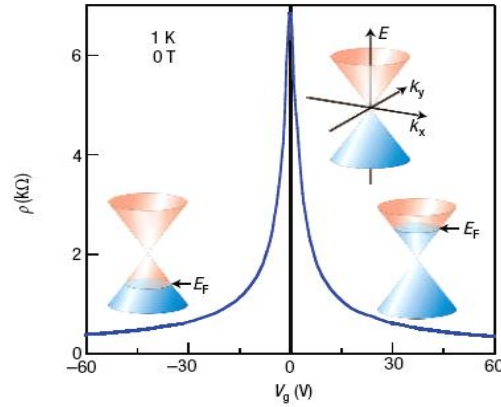


Figure 1.13: Band diagram of a single layer of graphene representing the different positions of Fermi level E_F with change in the voltage applied to gate [20].

1.6.1 TUNING THE DELAY WITH GRAPHENE

There are various applications of the slow light which has an exceptionally low group velocity. It finds usage in buffering and optical data time-domain processing [10]. In various communication systems, there is a requirement for the components that have the potential to delay the information or that can help in buffering. But optical/electrical conversion is a hold-up for the data transmission rate increase. The key to the problem are the tunable all-optical delay lines with fine tuning of the delay [11]. Slow light can be achieved in different devices but to have practical applications, it must be obtained over a wide bandwidth. This gives photonic-crystal (PC) devices an edge due to their ability to integrate on chip, their room temperature operation, low dispersion propagation and wider bandwidth. Photonic crystal waveguides (PCW) find applications for tuning the delay as they provide tunable slow light with wide bandwidth [15].

Electrically tunable delay lines having wide tuning range are of interest at present. But, tunable delay lines experimentally noted by the method of varying the propagation path of waveguide are quite cumbersome. Also, the delay resolution is confined to a few nano-seconds and the losses increase with the increasing number of stages. Large bandwidth, large tuning range, continuous tunability and active reconfiguration speed are some of the properties a tunable delay line should possess.

Graphene has drawn attention in recent years for research interests. The material possesses some excellent electronic and optical properties which makes it viable for

using as the primary material in the upcoming times. The reason for the phenomenal properties of graphene being its uncommon band structure, which can be represented by a pair of Dirac cones. As there is a variation in the applied gate voltage, there is a subsequent change in graphene's density of carrying the charges which further brings change in the Fermi level. Thus, by applying small values of the electric voltage, there is a considerable shift in the Fermi energy due to the low density of states close to the Dirac point [22]. Graphene must be integrated with silicon photonic systems to fully employ its outstanding optical properties for use in optoelectronic applications. Thus, to tune the group delay in a PC waveguide having line-defect, the electrical tuning of graphene must be considered. If graphene has to be used in optoelectronic devices, it is essential to tune graphene's various electrical properties and it can be done in different ways like π -stacking organic molecules [24], applying electric field etc. Since there is no band gap present in graphene, thus it is a semi-metal. This is the distinguishing property of graphene which differentiates it from other materials. FLG creates surface plasmon polariton (SPP) [21] since FLG acts like a semi-metal. Plasmons are the free electrons that are created when a dielectric couples to metal. These plasmons usually oscillate at the optical frequency. Surface plasmons are present at the surface only and when these surface plasmons interact with light, SPPs are formed whose nature is somewhat different as they are a combination of photons and electrons. Thus, the presence of FLG can reduce light's group velocity. Also, through application of electric field, Fermi-level of graphene can be tuned which thus helps in the delay tuning. As the electric field changes, the dielectric constant changes which further changes the refractive index and hence the delay.

A tunable band gap that has been gate controlled up to a voltage of 250 meV is experimentally recorded in bilayer graphene [23]. By electrically gating graphene, it is possible to dynamically tune the magnitude of its dielectric constant in a large range. There are two factors contributing in graphene's total conductivity namely interband and intraband absorptions. The highest level of energy which may be filled up by the electrons is given by the Fermi level or the chemical potential. The valence band consists of only electrons for a pure graphene which is undoped. The chemical potential in this case is absolute zero. There are different ways of varying the chemical potential of graphene. It can be done either by doping through chemicals or by electrostatically doping i.e. applying an electric field between the substrate and graphene. The result is

high tunability of conductivity and thus the optical absorption. As per the approximation of the Kubo formula, the conductivity of graphene is given by:

$$\sigma = \sigma_{\text{inter}} + \sigma_{\text{intra}} \quad (1.4)$$

The intraband and interband components of the conductivity are defined as [25]:

$$\sigma_{\text{intra}} = \frac{2iT e^2}{\pi \hbar^2 (\omega + i\Gamma)} \ln \left[2 \cosh \left(\frac{E_f}{2T} \right) \right] \quad (1.5)$$

$$\sigma_{\text{inter}} = \frac{e^2}{4\hbar} \left[\frac{1}{2} + \frac{1}{\pi} \arctan \left(\frac{\hbar\omega - 2E_f}{2T} \right) - \frac{i}{2\pi} \ln \frac{(\hbar\omega + 2E_f)^2}{(\hbar\omega - 2E_f)^2 + (2T)^2} \right] \quad (1.6)$$

where T being the temperature, Γ is the carrier scattering rate of charge, E_f is the graphene's fermi energy or chemical potential, \hbar and ω are the reduced plank's constant and the angular frequency respectively.

The graphene's fermi energy (E_f) depends on the voltage applied (V) to it and they can be related as [26, 27]:

$$E_f = \hbar V_F \sqrt{\pi \times \left(\frac{C|V|}{q} + n_0 \right)} \quad (1.7)$$

Where C is capacitance per unit area and its value is approximately 20mF/m² when gating is done by ion gel, V_F is graphene's fermi velocity whose value is approx. 10⁶. Here n_0 can be neglected which is the concentration of the carrier.

The optical conductivity of graphene further derives the graphene's dielectric function \mathcal{E} which is a complex unit using the equation [25]:

$$\varepsilon = i\sigma/\varepsilon_0\omega t + 1 \quad (1.8)$$

Where t, ε_0 , σ are the effective thickness of graphene, vacuum permittivity and optical conductivity respectively.

1.6.2 GRAPHENE'S APPLICATIONS IN OPTOELECTRONIC DEVICES

Owing to the extremely large mobility, graphene is known to be the rising alternative to the traditional silicon technology for its usage in digital, very high speed electronic technologies as well as interconnects. Graphene holds various other mechanical,

thermal, optical and electrical properties also in addition to having a high mobility. Graphene finds applications in field emission (FE) displays which are justified from its extra ordinary properties. Graphene is an excellent material for the channels in applications involving radio frequency and high frequency devices. Graphene’s electrical conductivity tuning property is the basis for its usage in sensors, both bio sensors and gas sensors. It is expected that graphene will find potential use as electro optic material including transparent electrodes for LCDs and solar cells. Thus, the phenomenal properties of graphene are leading to new innovations continuously.

1. Graphene based Photonic crystal cavity

A. Majumdar, *et al.* [26] demonstrated that electro optical modulation has the potential if photonic crystals are combined with graphene. If a layer of graphene is electrically gated on the upper side of cavity, a noticeable change is observed in the photonic crystal cavity’s resonance. Ion gel gating is most commonly used for electrical gating purpose. The change in the resonance line width of the cavity and reflectivity of resonance were calculated through experiments. The study showed that the photonic crystal cavity with graphene used can find applications as refractive as well as absorptive modulator which has a low power requirement and higher speed of modulation. The physical size of such a device is also quite small.

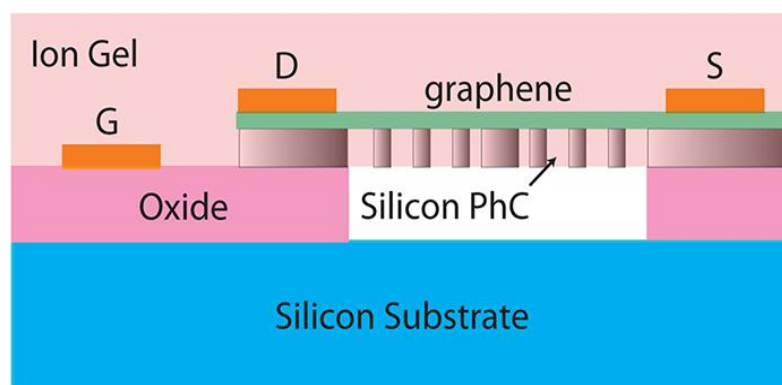


Figure 1.14: Diagrammatic representation of a graphene based photonic crystal cavity. SOI platform is present where the cavity is fabricated. Ion gel gating is there on the three contacts [26].

2. Electrically controlled Graphene Nano cavity

X. Gan *et al.* [29] demonstrated that by connecting graphene to a bus waveguide or a PPC nano cavity, the interaction between light and graphene can be largely intensified. The theoretical description of the graphene based nano cavity was given. Graphene-cavity coupling can be controlled by varying the cavity's intrinsic quality factors or the slab thickness of PPC. It is possible to further intensify graphene's coupling with the light by using graphene on planar photonic crystal nano cavities. A significant improvement was seen in the absorption of the cavity based design. Many other developments were discussed in detail in regard of the device structure proposed, like graphene layer's hot photoluminescence and Raman scattering. An enhancement in the interaction of light and matter is enabled by using graphene in photonic crystal nano-cavity. A transmission attenuation of an approximate 6.2dB was noted for a $70\mu\text{m}$ waveguide length due to the absorption of graphene over a waveguide.

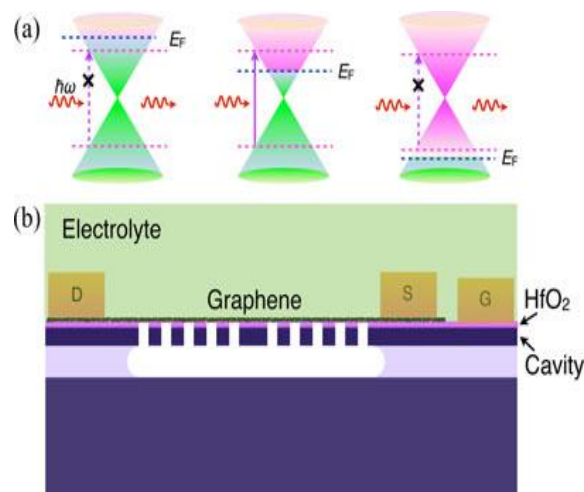


Figure 1.15: a) Graphene's energy band diagram on various levels of doping showing an increment in graphene's transparency, b) Diagrammatic representation of electrically controlled graphene nano cavity. The layer of graphene and silicon are isolated by a layer of HfO_2 . Cavity reflection is regulated by altering the graphene's fermi level [29].

3. Photonic Crystal based on graphene

A different sort of photonic crystal design was proposed by O. Berman *et al.* [30]. This design is created when discs of dielectric and graphene are placed alternatively and periodically in a dielectric material background. Graphene discs form the stack since

they are placed one on top of another and the dielectric discs positioned in between separate them. The photonic band structure as well as transmittance can be calculated for such photonic crystal design. The photonic crystals having their basis on graphene can be used adequately as waveguides or frequency filters at a broad range of temperatures which also includes the room temperature. As low-frequency radiation's absorption is substantially suppressed in the doped graphene, the effects of damping and the skin effect are also suppressed. The dielectric photonic crystals find practice in the optical spectrum region only. So such types of photonic crystals are proposed with the thought that the far infrared spectrum will note some applications for such devices along with their constituting metal elements.

It is possible to further optimize the structure to suit the applicant's requirement. The dielectric layer's thickness between the graphene discs can be varied so as to vary the band structure of the design. The graphene's doping can also be altered as per the need. Also, the material with which the dielectric substrate is made of and the material separating the graphene discs can both be same.

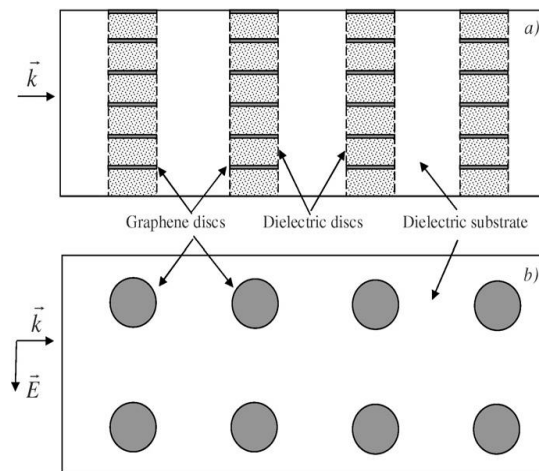


Figure 1.16: a) Front view of the photonic crystal having graphene, b) Top view of the photonic crystal having graphene [30].

4. Graphene based broadband optical modulator

Liu *et al.* [31] demonstrated a waveguide integrated graphene based electro-optic modulator based on the theory of electrostatic gating. By tuning the graphene sheet's

fermi energy level, modulation can be achieved in this structure. A firm electro absorption modulation of about 0.1 dB mm is exhibited by the gigahertz graphene modulator and it works within a wide wavelength range of 1.35 mm-1.6mm under ambient conditions. The device structure has graphene positioned on a doped silicon waveguide that is designed for guiding the light on a wavelength of 1.53 μm . A 7nm thick Al_2O_3 layer separates the waveguide from the graphene. The graphene layer and the silicon waveguide are coupled to two different electrodes. An Au-Pt electrode is used to couple the graphene layer electrically. The Fermi level or chemical potential of graphene is regulated by applying voltage over two contact pads in such a way that graphene's absorptive properties can also be tuned.

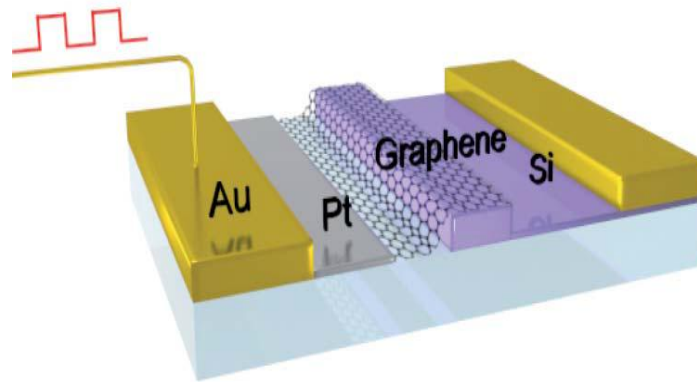


Figure 1.17: Schematic diagram for an electro-optic modulator device based on graphene [31]

Chapter 2

Literature Review

The elementary photonic crystals achievable consist of different materials which have varying dielectric constants that are placed one after the other. In 1887, Lord Rayleigh first represented the various optical properties of the multilayer films. The permittivity modulation occurs periodically only in a single direction for a one-dimensional PC, while the structure is uniform in the two other directions. An example of 1-D PhC can be quoted by the Bragg reflector which finds application in VCSEL (Vertical cavity surface emitting laser) being used as a distributed reflector. The classical way of assessing this system, as established by Lord Rayleigh is by anticipating plane wave propagation within the material and also taking into consideration the multiple refractions and reflections that occur at each interface. A few of the research methodologies that have been developed in the field of slow light, photonic crystals, line-defect waveguides as well as graphene are given below:

S. Yegnanarayanan, *et al.* in 1997 [16] first reported the demonstration for SOI technology based integrated delay line networks used in optics. SOI possesses several advantages over the silica technology e.g. it couples broad area as well as the silicon technology having little cost and high ratio of delay per unit length, it exhibits low losses, and it has compatibility with the circuit technology which is integrated with silicon. It further implies that the manufacturing will have high yield. SOI waveguide's compatibility with CMOS electronics draws attention e.g. it is possible to fabricate a whole beam forming network that consists of thermo optic switches, delay lines and control circuitry on one silicon chip.

S. Johnson, *et al.* in 1999 [4] analyzed the properties of dielectric structures that are periodic in two dimensions. A bandgap is possessed by such structures for propagating in plane. The light is confined by index guiding in the third dimension. The band structure calculation done by various numerical methods in the PC slab has been outlined. Also, the effects of the distinctive backgrounds, the regions below and above the slab are investigated for the different structures.

M. Notomi, et al. in 2001 [6] studied experimentally the line defect photonic crystal slab and showed that its waveguiding characteristics as well as the group velocity dispersion (GVD) are distinctive from that of the traditional waveguides. By regulating the defect width, these characteristics can be tuned resulting in light transmission control through the light path. In interference transmission window, a high value of group velocity is noted that is obtained by altering different parameters of slab like thickness or length of slab. Slow light is thus achieved allowing the guided modes to travel with lower velocity as compared to the light in vacuum.

T. Baba, et al. in 2002 [7] proposed a single line defect PC slab waveguide. The fabrication of the waveguide is done on a SOI platform. Two different types of waveguides with varying photonic crystal slabs were investigated- air bridge type with air claddings and SOI slab type with air and SiO₂ claddings. For air bridge PC slab, propagation having clear light was observed but not for the SOI slab type. An efficient light propagation was seen for the TE type polarisation as well as the TM type polarisation. Minimum propagation loss was noted to be 11dB/mm, which was comparable to the loss for rectangular Si core waveguide. But this value is much smaller than the three line-defect waveguide having SOI slab structure.

S. Noda, et al. in 2002 [17] studied elaborately the 2-D photonic crystals and its different waveguide types. It was further noted that there is a possibility to realize the structures in three dimensions as well with no fabrication complexities. The result of the introducing a light emitter into the 3-D photonic crystal was discussed and it was demonstrated that less number of stacking structures may enhance and suppress the spontaneous emission well. Also, the structure of a single defect cavity was proposed, and the design of a defect with sufficient tunability, a large Q factor, and a wide separation amongst the first and second modes were obtained.

C. Jamois, et al. in 2003 [5] reviewed the properties of silicon based photonic crystals in two dimensions. It was studied that the two-dimensional photonic crystals have a prospect of light guiding in different dimensions. The light can be guided in two and three dimensions through two techniques - index guiding and photonic bandgap guiding. A 3-D confinement of light can be attained in two-dimensional photonic crystals by

combining the line-defect waveguiding with the photonic crystal plane. Also, the losses in the modes of the resonant waveguide were discussed.

N. Moll, *et al.* in 2003 [32] studied butt coupling in a PC waveguide. The study was done for a 2-D PhC and a 3-D PC slab. The guiding medium being optically used in the PC slab is the silicon layer as the fabrication is done on a SOI platform. There is a considerable variation in the two dimensional as well as the slab's transmission spectrum. There is a narrow frequency range for the slab wherein the existence of guided mode is there. The transmission is equivalent to that of two dimensional systems for this range of frequencies. Whereas, there is much lower transmission for other frequency range where only resonance exists for slab system.

X. Hua, *et al.* in 2005 [9] presented a study on the defect modes present in 2-D photonic crystals having a deformed triangular lattice numerically. It was found that by stretching or shrinking the lattice, a change can be obtained on the frequency of the defect modes as well as on their magnetic field distribution. Also, the dipole modes can be separated largely by the stretch and shrink of lattice for realizing single mode excitation in fabrication of photonic crystal lasers.

Y.A. Vlasov, *et al.* in 2005 [33] elaborately discussed the vast amount of decrease in the light's group velocity and also the study causing light pulses to fully stop. The platform used in the silicon fabricated 2-D photonic crystal is SOI. The slow light phenomena find applications in all optical storage as well as all optical switching. An approximate 300 fold reduction in group velocity can be experimentally seen on a silica based ultra compact photonic device. When the PC waveguide is heated using a micro heater, group velocity can be controlled actively and with a high speed.

A.K. Geim, *et al.* in 2007 [18] presented a review article to study rise of graphene as a material used in photonics in the recent years. Graphene consists of a flat monolayer of C atoms which are bonded as sp^2 hybridised bond. It has firmly packed atoms in the form of a two dimensional honey comb lattice. It has excellent electronic and optical properties e.g. the current density is very high, they are chemically inert, optical transmittance, low room temperature resistivity, high mobility, low Johnson noise etc. The fermi level is present at the neutral point of charge called the dirac point at absolute

zero temperature. It must be integrated with silicon photonic systems to fully employ its outstanding optical properties for use in optoelectronic applications. Thus, combination of graphene with nano photonic structures finds incredible potential to obtain electro-optic tunability of light transmission.

V.P. Gusynin, *et al.* in 2007 [27] discussed the properties of graphene exhibiting Dirac-like behaviour. Such properties cause intraband and interband transitions. The longitudinal conductivity is also improved by these properties. There is a constant dependency seen by the spectral weight on the applied gate voltage in case of a graphene based field effect transistor (FET). The hall frequency determines the spectral weight for the case of hall conductivity. This frequency is proportional to the gate voltage applied and differs from the frequency of the cyclotron.

L. Faolain, *et al.* in 2007 [28] explained the propagation of slow light in photonic crystal waveguides and the associated principles and drawbacks too. The linear effects must be increased by the slow light techniques in the PC waveguides. For example, tuning of electro optic as well as thermo optic along with nonlinear effects; gain like Raman amplification, switching; and some effects of parameters like delay lines, conversion of wavelengths are also attainable. Carefully designing the structure leads to wide bandwidth slow light having all-optical functioning e.g. modulators and switches. This is done through formation of a linear and broadband response in order to overpower the limitations of dispersion, restrain the backscattering losses that come across the band-edge and to efficiently inject the light.

T. Baba, *et al.* in 2008 [10] studied the concept of slow light. Slow light with an exceptionally low group velocity finds application in buffering and optical data time-domain processing. Slow light can be achieved in different devices but to have practical applications, it must be obtained over a wide bandwidth. The photonic crystal (PC) devices thus have an edge since they can work on room temperature, on chip integration is highly suitable, and have low dispersion propagation and wider bandwidth. The different factors that should be considered regarding slow light involve dispersion compensation and its wide tunability range. A chirped SOI fabricated photonic crystal waveguide was also proposed and it was evaluated experimentally that a value as high as

57 can be obtained to indicate the capacity of the design to buffer, which is also known as the delay bandwidth product (DBP).

A. E. Willner, *et al.* in 2008 [12] analyzed the degradation effects for slow-light induced signals and the different ways of mitigating them. Various novel slow light based signal processing modules were proposed highlighting several features like variable bit-rate capability, multichannel operation etc. Phase preserving slow light was experimentally demonstrated by delaying 10Gb/s DPSK signals.

F. Wang, *et al.* in 2008 [22] discussed the single layered as well as bilayer graphene showing interesting behaviours of electric transportation. It was demonstrated that the interband transitions are quite strong in these. Also, gating it electrically helps in the adjustment of their optical transitions. Optoelectronics and infrared optics have various applications due to graphene's optical transitions which are strong and dependent on layer, as well as its tunability by electrical gating.

Y. Huang, *et al.* in 2008 [13] analyzed a SOI fabricated 2-D PC waveguide. By measuring the spectrum of transmission, the guided modes and the index guided band gaps were examined. During the fabrication of structure, imperfections were also considered and all characteristics were studied. Short coupling length and high coupling efficiency were also realized successfully by the use of suppressive power reservation technique.

W. Choi, *et al.* in 2010 [20] reviewed the synthesis of graphene, its properties and its applications like field emission, electronics, sensors and energy. The remarkable electrical and optical properties of graphene have grabbed significant attention. It was studied that despite having notable electrical, chemical and opto electric features, and hence the capability to find applications as FET, gas or bio sensors, transparent electrodes in LCDs; synthesising graphene films with the required energy band gap over random substrates is yet to be attained. Graphene has the potential of grabbing more attention than the traditional technologies involving silicon. It can thus provide a large number of upcoming devices whose speed is much higher, are environment friendly, chemically stable and whose functioning is improved.

O. Berman, *et al.* in 2010 [30] proposed a different sort of photonic crystal design. This design is created when discs of dielectric and graphene are placed alternatively and periodically in a dielectric material background. Graphene discs form the stack since they are placed one on top of another and the dielectric discs positioned in between separate them. The photonic band structure as well as transmittance can be calculated for such photonic crystal design. It is possible to further optimize the structure to suit the applicant's requirement. The dielectric layer's thickness between the graphene discs can be varied so as to vary the band structure of the design. The graphene's doping can also be altered as per the need. Also, the material with which the dielectric substrate is made of and the material separating the graphene discs can both be same.

N. Ishikura, *et al.* in 2011 [13] discussed that it is possible to further improve the characteristics of slow light in the photonic crystal waveguide which are achieved by heating. It was experimentally observed that a tunable fractional delay as high as 36 can be attained for picoseconds pulses, which can be further upgraded to 110 by externally compressing the output pulses. Such a device finds application as a delay scanner or correlator wherein the delay scanning resolution can be determined by the fractional delay. Sub- picoseconds pulses with scan frequency upto 2 KHz can be noticed with this correlator. These are many fold faster than the traditional correlators having mechanical scanning devices.

Liu, *et al.* in 2011 [3] explored and presented a demonstration of a waveguide integrated graphene based opto-electrical modulator based over the theory of electrostatic gating. By tuning the graphene sheet's fermi energy level, modulation can be achieved in this structure. The device structure had graphene positioned on a doped silicon waveguide. A 7nm thick Al_2O_3 layer separated the waveguide from the graphene. The graphene layer and the silicon waveguide are coupled to two different electrodes. An Au-Pt electrode is used to couple the graphene layer electrically. The Fermi level or chemical potential of graphene can be regulated by applying voltage over two contact pads in such a way that graphene's absorptive properties can also be tuned.

Y. Francescato, *et al.* in 2013 [25] explored and analyzed the various guided modes that are there in graphene nano ribbon sandwiches. The evolution of strongly confined modes was noticed in which the hybridization intervened by the interlayer spacing has resulted

in the modification of the propagation length. The interaction amongst the sheets of graphene results in the development of edge modes waveguide combined as bonding or non bonding in a single structure. The permittivity as well as the conductivity of graphene was numerically calculated using COMSOL by considering infinitely long ribbons.

A. Majumdar, *et al.* in 2013 [26] demonstrated that the unification of photonics with electronics is caused by the effective conversion of electrical pulse to an optical pulse. The opto electronic modulation requires the combination of photonic crystals with graphene. It was further demonstrated that the cavity resonance can be considerably changed by using a graphene layer which is gated electro statically on the PC cavity. The variation in the cavity's resonance line width and reflectivity of resonance can be calculated through experiments. The study showed that the photonic crystal cavity with graphene used can find applications as refractive as well as absorptive modulator which has a low power requirement and higher speed of modulation. The physical size of such a device is also quite small.

C. Caer, *et al.* in 2013 [36] investigated the ultra fast tuning of delay for a light pulse with low group velocity having a response time which is even less than 10ps. It may be obtained by using two different kinds of slow light. The first is the slow light with its dispersion compensated. The second is slow light having low dispersion. The non linear effects of the controlling signal are increased with low dispersion slow light. The lattice shifted PC waveguide having silicon grows these two types of slow light. The signal pulse spectrum is moved by the control pulse by tuning it dynamically. When there is an overlap between the two pulses in the waveguide, there is a 10ps approx. alteration in the delay.

X. Gan, *et al.* in 2014 [29] demonstrated that by connecting graphene to a bus waveguide or a PPC nano cavity, the interaction between light and graphene can be largely intensified. The theoretical description of the graphene based nano cavity was given. Graphene-cavity coupling can be controlled by varying the cavity's intrinsic quality factors or the slab thickness of PPC. It is possible to further intensify graphene's coupling with the light by using graphene on planar photonic crystal nano cavities. A significant improvement was seen in the absorption of the cavity based design. Many

other developments were discussed in detail in regard of the device structure proposed, like graphene layer's hot photoluminescence and Raman scattering. An enhancement in the interaction of light and matter is enabled by using graphene in photonic crystal nano-cavity. A transmission attenuation of an approximate 6.2dB was noted for a 70 μ m waveguide length due to the absorption of graphene over a waveguide.

H. Zhou, *et al.* in 2014 [35] analyzed four wave mixing of a single layer graphene in PC waveguide with slow light. When graphene is hybridized with silicon slow light, four wave mixing gives a conversion efficiency value of -23 dB and an interaction length of 200 μ m. The calculated results went well with the simulations of non linear coupled mode theory. All optical processing of signal can be efficiently obtained by this idea for chip scale integrated optics.

A. Sharma, *et al.* in 2015 [15] demonstrated that it is possible to attain flat bandwidth slow light in a PC waveguide based on silicon. The normalised delay bandwidth product is high for this slow light while the group velocity dispersion is low. By moving the two rows of holes alongside the waveguide, the waveguide width can be varied which results in high values of NDBP. With the design proposed, a large buffering capacity is obtained with upto 140 bits, thus finding applicability in the slow light buffering applications. The proposed designs are preferable considering the fabrication since there are many challenges to be faced in controlling the hole radius at nanometre scale.

F. Xu, *et al.* in 2015 [34] examined the various tuning characteristics of graphene's complex refractive index which is fabricated using chemical vapour deposition (CVD). The calculations are done at a 1550nm wavelength. The graphene's complex refractive index tunability, which is a function of gate electric voltage matches with that calculated from the Kubo formula.

M. J. Deka, *et al.* in 2016 [24] elaborated that it is important to tune the electrical properties of graphene for its applications in optoelectronic devices. Graphene with various π stacking molecules as well as the electrical properties of graphene were studied. The various π stacking organic molecules that were used included amino naphthol sulphonic acid, ferrocene and hemin. An enhancement in conductivity was presented and it can be associated with the density of π -electron, higher mobility, etc.

Chapter 3

Proposed Methodology and Computational Technique

3.1 METHODOLOGY OF PHOTONIC CRYSTAL MODELLING

An enormous range of models are brought in use for the modelling of optical structures. Firstly, there are a variety of approximate models that are very fast. But, their domain of validity is restricted. Secondly, there are models that involve the precise solving of the Maxwell's equations. There is a single approximation done in this case i.e. definite size of the mesh and the definite no. of terms that are required for the expansion of series. The only limitation in these models is their speed. They are very slow. The method involving the solution of Maxwell's equations has been discussed and explained:

3.1.1 SOLUTION BY MAXWELL'S WAVE EQUATIONS

In 1928, Felix Bloch first pioneered the concept of wave propagation for a three-dimensional periodic media, which was a further extension of the theorem given by G. Floquet in 1883 for one dimension. Bloch established the propagation of waves with no scattering is possible in a medium like this. By multiplying a function involving a periodic envelope with a plane wave, the behaviour of these waves can be studied. It was concluded from the study of quantum mechanics that the only factor resulting in scattering of electrons is imperfections present in a conductor. The periodic ions have no role to play in this. Electromagnetism uses the same approach by considering Maxwell's equation in the form of an Eigen value problem analogous to the Schrodinger's equation. Positive definite and Hermitian are the two properties defining the Eigen operator, thus implying that the Eigen frequencies are also real which further causes perturbation theory relations, variational theorem as well as orthogonality of modes. Light's interaction with matter is explained by the Maxwell's equations. The four equations of Maxwell drive the entire electromagnetism theory. It also explains light's propagation in a photonic crystal. These equations are given by [2]:

$$\nabla \cdot B = 0$$

$$\begin{aligned}
\nabla \cdot \mathbf{D} &= \rho \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}
\end{aligned} \tag{3.1}$$

Where \mathbf{H} = macroscopic magnetic field

\mathbf{E} = macroscopic electric field

\mathbf{B} = magnetic induction field

\mathbf{D} = displacement

\mathbf{J} = current density

ρ = free charge density

In a mixed dielectric medium, the propagation contains a dielectric material's composite regions which are homogeneous. It changes as factor of Cartesian vector but there is no time factored modification in the structure. Since there are no freely available currents or charges, thus $\mathbf{J} = 0$, $\rho = 0$ for the medium where source of light is not there but still propagation of light occurs. The position vector $\vec{\mathbf{r}}$ represents the relative positions for a dielectric material. Consider a material to be linear with permittivity value as positive and real and permeability value as unity. The material is independent of the radiation frequency and is isotropic, lossless and non-magnetic. The expressions for displacement and magnetic induction field in this case can be approximated as:

$$\begin{aligned}
\mathbf{D}(\mathbf{r}) &= \mathcal{E}(\mathbf{r}) \mathcal{E}_0 \mathbf{E}(\mathbf{r}) \\
\mathbf{B}(\mathbf{r}) &= \mu(\mathbf{r}) \mu_0 \mathbf{H}(\mathbf{r})
\end{aligned} \tag{3.2}$$

Where μ_0 = vacuum's permeability whose value is $4\pi \times 10^{-7}$ Henry/m

$\mu(\mathbf{r})$ = relative magnetic permeability

Since the value of $\mu(\mathbf{r})$ is unity for almost all dielectric materials, therefore $\mathbf{B} = \mu_0 \mathbf{H}$. Thus, the refractive index n is the square root of \mathcal{E} which is similar to the Snell's law of optics. In general, $n = \sqrt{\mathcal{E}\mu}$

With all these assumptions, the Maxwell's equations are:

$$\begin{aligned}
\nabla \cdot \mathbf{H}(\mathbf{r}, t) &= 0 \\
\nabla \cdot \mathbf{D} &= \rho \\
\nabla \times \mathbf{E}(\mathbf{r}, t) + \mu_0 \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t} &= 0 \\
\nabla \times \mathbf{H}(\mathbf{r}, t) - \mathcal{E}_0 \mathcal{E}(\mathbf{r}) \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} &= 0
\end{aligned} \tag{3.3}$$

In general, \mathbf{E} and \mathbf{H} are factors of both space and time. Due to linearity of the Maxwell's equations, the spatial dependency as well as the time dependency can be separated. It can be done by forming a harmonic modes set by expanding the fields. Thus, a harmonic mode can be written as product of spatial factor and an exponential which is complex:

$$\begin{aligned}
\mathbf{H}(\mathbf{r}, t) &= \mathbf{H}(\mathbf{r}) \times e^{-j\omega t} \\
\mathbf{E}(\mathbf{r}, t) &= \mathbf{E}(\mathbf{r}) \times e^{-j\omega t}
\end{aligned} \tag{3.4}$$

Mode profile equations for some particular frequency are found by inserting Equation (3.4) into Equation (3.3). Conditions obtained by the divergence equations are:

$$\begin{aligned}
\nabla \cdot \mathbf{H}(\mathbf{r}) &= 0 \\
\nabla \cdot [\mathcal{E}(\mathbf{r}) \mathbf{E}(\mathbf{r})] &= 0
\end{aligned} \tag{3.5}$$

The medium having magnetic fields and displacement do not have any point sources or sinks. This is the physical interpretation of these equations. The transverse EM waves form the configuration of fields. The two curl equations providing a relation between $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ are:

$$\begin{aligned}
\nabla \times \mathbf{E}(\mathbf{r}) - i\mu_0\omega \mathbf{H}(\mathbf{r}) &= 0 \\
\nabla \times \mathbf{H}(\mathbf{r}) + i\omega \mathcal{E}(\mathbf{r}) \mathcal{E}_0 \mathbf{E}(\mathbf{r}) &= 0
\end{aligned} \tag{3.6}$$

It is possible to separate these equations. First, divide the second equation of (3.6) by $\mathcal{E}(\mathbf{r})$ and then take its curl. Then, use the first equation of (3.6) to remove $\mathbf{E}(\mathbf{r})$. The speed of light in vacuum is obtained by combining the two constants μ_0 and \mathcal{E}_0 i.e. $c = 1/\sqrt{\mathcal{E}_0\mu_0}$.

The resulting equation which is entirely in $\mathbf{H}(\mathbf{r})$ is the **Master Equation** given as:

$$\nabla \times \left(\frac{1}{\mathcal{E}(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right) = \left(\frac{\omega}{c} \right)^2 \times \mathbf{H}(\mathbf{r}) \tag{3.7}$$

This Master equation provides all the information about $\mathbf{H}(\mathbf{r})$. The modes $\mathbf{H}(\mathbf{r})$ are found using the Master equation for a given structure. Also the corresponding frequencies are calculated by taking the transversal condition into consideration. $\mathbf{E}(\mathbf{r})$ is then recovered by using the second equation of (3.6):

$$\mathbf{E}(\mathbf{r}) = i\omega \mathcal{E}(\mathbf{r}) \mathcal{E}_0 (\nabla \times \mathbf{H}(\mathbf{r})) \quad (3.8)$$

As the divergence of curl is always zero, thus \mathbf{E} must satisfy $\nabla \cdot (\mathcal{E}\mathbf{E}) = 0$ which is the requirement of transversality. Thus, we are required to put only a single constraint of transversality rather than two. For mathematical convenience, the problem is formulated in $\mathbf{H}(\mathbf{r})$ only and not on $\mathbf{E}(\mathbf{r})$. Using the first equation of (3.6), \mathbf{H} can be found from \mathbf{E} :

$$\mathbf{H}(\mathbf{r}) = -i\omega\mu_0 (\nabla \times \mathbf{E}(\mathbf{r})) \quad (3.9)$$

3.1.2 EIGEN VALUE PROBLEM

The Master Equation (3.7) which is entirely in $\mathbf{H}(\mathbf{r})$ is the heart of Maxwell's equation for a harmonic mode in case of a mixed dielectric media. If $\mathbf{H}(\mathbf{r})$ is an electromagnetic mode which is allowed, and a series of operations are performed over $\mathbf{H}(\mathbf{r})$, the outcome is still the original function of $\mathbf{H}(\mathbf{r})$ which is multiplied by a constant. This is called as an Eigen value problem. When an operation is performed on a function and if the outcome is the constant times function itself, it is known as an Eigen Vector or Eigen function of the operator. The constant by which the function is multiplied is known as Eigen value. Thus, in equation (3.7), $\mathbf{H}(\mathbf{r})$ is the Eigen vector and $(\omega/c)^2$ the Eigen value. Here, $\mathbf{H}(\mathbf{r})$ being the Eigen vector is the harmonic mode's spatial or field pattern. Also, the Eigen value $(\omega/c)^2$ is in square proportion with these modes's frequency [2]. The Π operator $\nabla \times \left(\frac{1}{\mathcal{E}(\mathbf{r})} \nabla \times \right)$ has linear properties. It is also a Hermitian operator which signifies that it has real Eigen value and orthogonal Eigen function. Equivalent frequencies of two different harmonic modes do not necessarily means orthogonality but it means they are degenerate. After determining the Eigen function as well as Eigen value of the Master Equation, it is possible to scale them till the dielectric constant is constant.

3.2 METHOD OF PLANE WAVE EXPANSION

A solution for the Eigen value problem is found numerically using a software package from MIT known as MPB (MIT's Photonic Band). It can be used to calculate the band structure, Eigen values, and various other parameters of the PC waveguide and other structures. This method makes use of the frequency domain and by the method of plane wave expansion (PWE), the Eigen modes are computed [38]. The PWE method in electromagnetic refers to the use of a computational technique involving the Eigen value problem which is being used for solving the equations of Maxwell. A periodic dielectric structure is taken into consideration by MPB and the Eigen modes are calculated for such structure. These are the EM waves propagating with some specified frequency inside the device. For some primitive lattice vectors, a PC is equivalent to a dielectric function that is periodic in nature. In this case, a distinct Eigen problem having Hermitian property is given by the Bloch theorem for every Bloch wave vector in a primitive cell lattice. The primitive cell has a fixed domain and discrete Eigen value if there is a periodicity in structure in all the directions. The Eigen values are functions of k periodically and thus form discrete bands when they are plotted against k in the band structure. Because of this periodicity, only those Eigen solutions need to be calculated for k which is there in reciprocal lattice's primitive cell. In other words, wave vectors close to $k = 0$ origin are only considered and this region forms the first Brillouin zone. If the crystal has some additional symmetry like the mirror planes, the first Brillouin zone might be redundant. An irreducible Brillouin zone may be obtained by eliminating such redundancy. On solving the Maxwell's equations, the range of frequencies where there exist no solutions of propagation for any real value of k , and are enclosed by propagating states below and above the gap is referred to as a complete photonic bandgap. Incomplete band gaps are also there that exist only over a small subset of all the possible polarization, wave vectors and symmetries. A bandgap is formed by any kind of periodic dielectric variation in a single dimension. A small variation causes the formation of a small gap. To attain a complete photonic bandgap, some additional obstacles need to be overcome. There is no overlapping of these gaps in frequency despite having a photonic band gap in one-dimension at each k -point of the crystal. For overlapping, the gaps between the bands must be sufficiently high. Same periodicity in different directions also leads to large band gaps. Therefore, the hexagonal lattices have the largest band gaps in two dimensions and the largest band gap in three dimensions is for fcc lattices.

Chapter 4

Graphene's Electrical Control and Group Delay calculation in Graphene based 1-D PCW

In the structure proposed, variation is made in the graphene's electrical voltage. This provides a corresponding change in the graphene's dielectric constant ' ϵ ' which further causes the group velocity v_g of graphene to change. This gives a varying group delay Δt of slow light. Hence the delay is tuned in a 1-D PCW using graphene in it.

4.1 PROPOSED DEVICE DESIGN

A PC slab with SOI fabricated platform having alternating rods of silicon ($n=3.48$) and air ($n=1$) is considered. By removing the central rod and replacing it with a rod of graphene ($n=1.8$), defect is formed. It is possible to vertically confine the light in such a structure by the concept of total internal reflection i.e. TIR. The index contrast creates a vertical confinement and when it is combined with the in plane confinement, 3-D confinement of light within a photonic crystal slab waveguide can be attained.

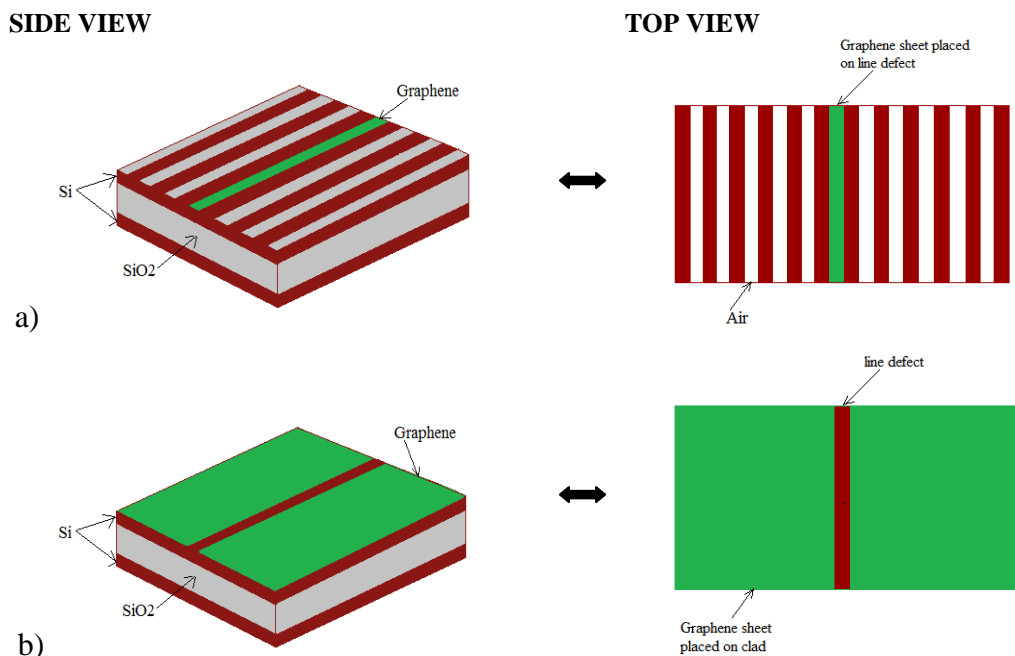


Figure 4.1: a) Side view and top view of a 1-D PCW with graphene positioned on the core, b) Side view and top view of a 1-D PCW with graphene positioned on the clad.

The lattice period that is being chosen is 290 nm. The elementary 1-D PhC has a SOI fabricated platform. On the silicon substrate, there is a deposition of SiO₂ layer. A Si layer is kept on top it where etching of air is done. A necessary contrast of dielectric is thus obtained.

Two structures have been designed - first where the core region has graphene layer deposited and the second where the cladding region has graphene layer. Figure 4.1 represents the design of such a graphene based structure which is SOI fabricated. As previously discussed, graphene is a flat monolayer of carbon atoms which are bonded as sp² and are firmly packaged in a honeycomb lattice structure, and by application of an electric field it is possible to efficiently control its properties. Thus, a graphene based PCW has been proposed for the light transmission through the waveguide to be controlled by using the electric voltage.

Since the thickness of graphene layer is very less, it will take enormous amount of time in computations during simulations. Thus, it is replaced by a layer with an equivalent dielectric constant which makes the computations easier and the simulations become faster. It is therefore possible in a graphene based PCW to realise the photonic crystal's band structure and other profiles for studying the guided light.

4.2 GRAPHENE'S EFFECT ON THE CHARACTERISTICS OF LIGHT

The plane wave expansion method is most suitable for evaluating the properties of slow light in PCW and it thus helps in the numerical analysis of the proposed structure's properties. By selecting relevant supercell, the slab equivalent index method is used. The refractive index of graphene and silicon used at 1550nm were 1.8 and 3.48 respectively. A very low group velocity can be achieved in the vector range of $0.3 \leq k_x \leq 0.5$. Thus, this range is best suited for the calculations.

The narrow region acquires a strong confinement of light optically which occurs when the line-defect perturbs the periodicity of air-slabs. As the defect is surrounded by a periodic structure, it results in the guiding of light within the region having defect due to the light scattering cancellation.

Figure 4.2 represents a plot of group velocity variation versus the wavelength in PCW structure when graphene is absent. The plot is made for the defect guided mode.

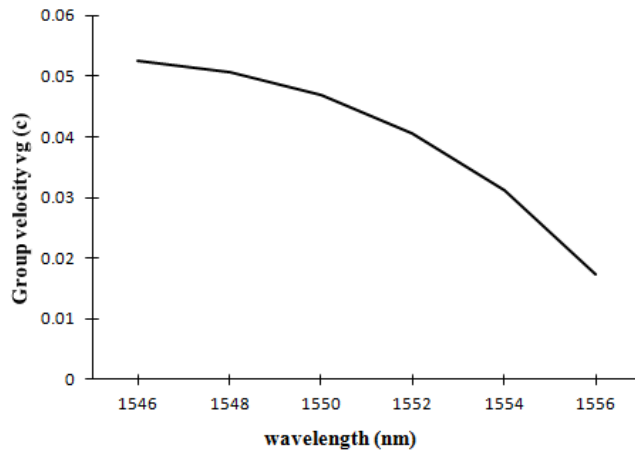
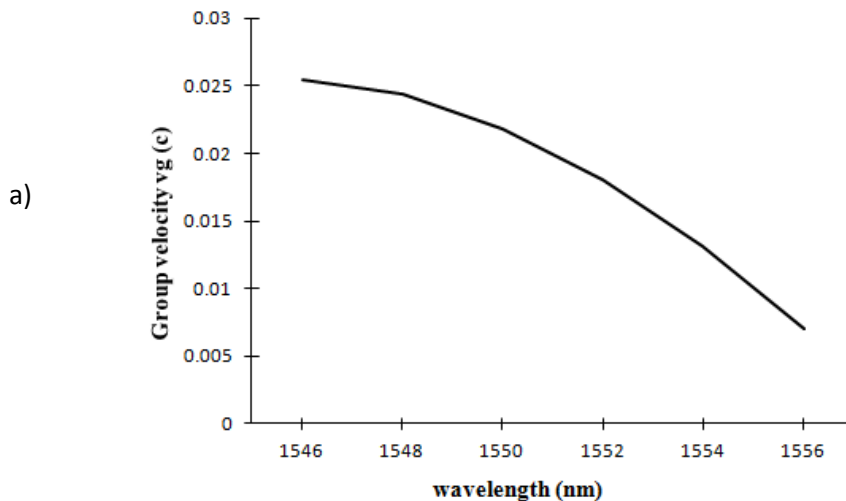


Figure 4.2: Plot of group velocity (v_g) of the defect guided mode versus the wavelength in nm when graphene is not used. The group velocity is in the units of light's speed in vacuum (c).

Figure 4.3 shows a plot of the group velocity variation for the PCW when the core region has graphene used and when the cladding region has graphene used. The calculations are done for the guided mode. The group velocity is taken in the units of c and wavelength is in nm.



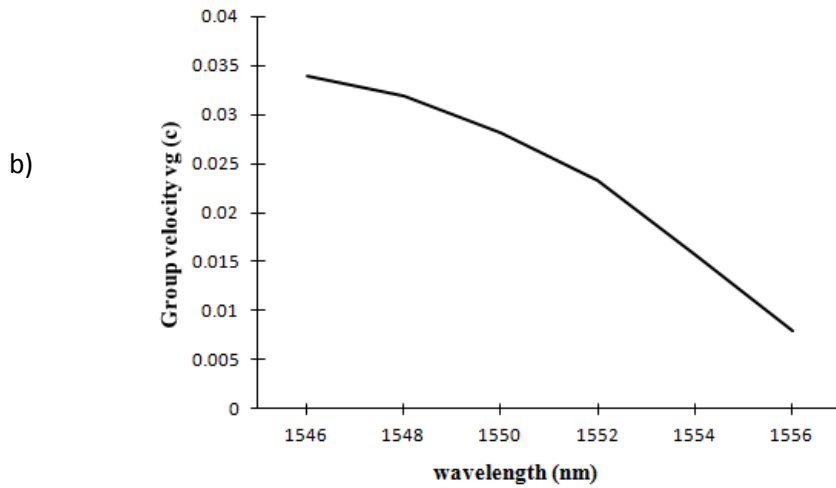


Figure 4.3: a) Plot of group velocity versus wavelength when core region has graphene used, b) Plot of group velocity versus wavelength when cladding region has graphene used.

Figure 4.4 represents the plot for comparison of varying group velocity with wavelength inside the PCW structure with no graphene used and when core/clad region has graphene applied to it.

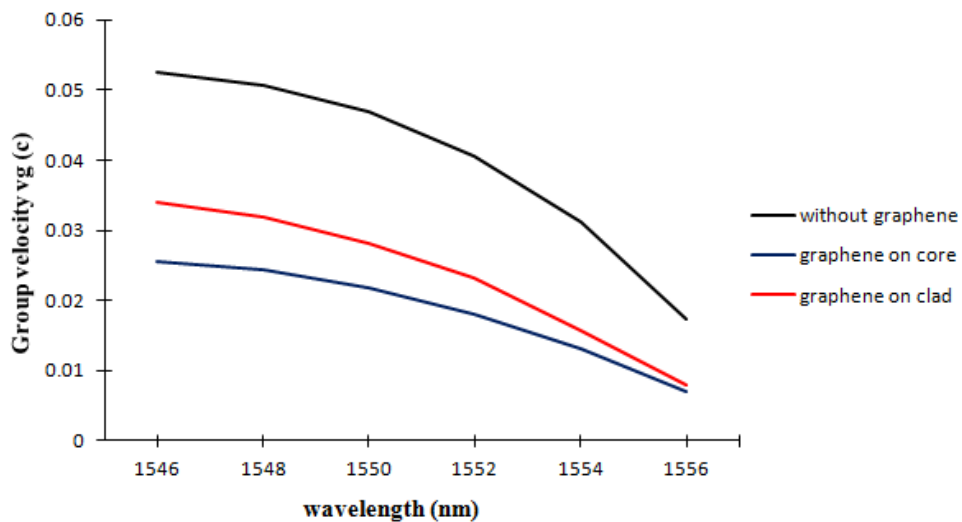


Figure 4.4: Plot representing the varying group velocity with wavelength with no graphene and when core/clad region has graphene applied to it.

When graphene is not used, the value of group velocity obtained at the wavelength of 1550nm is 0.0468c. When the core region has graphene applied to it, the value of group velocity is 0.02182c and when the clad region uses graphene, the value of group velocity obtained is 0.0282c at the concerned wavelength of 1550nm.

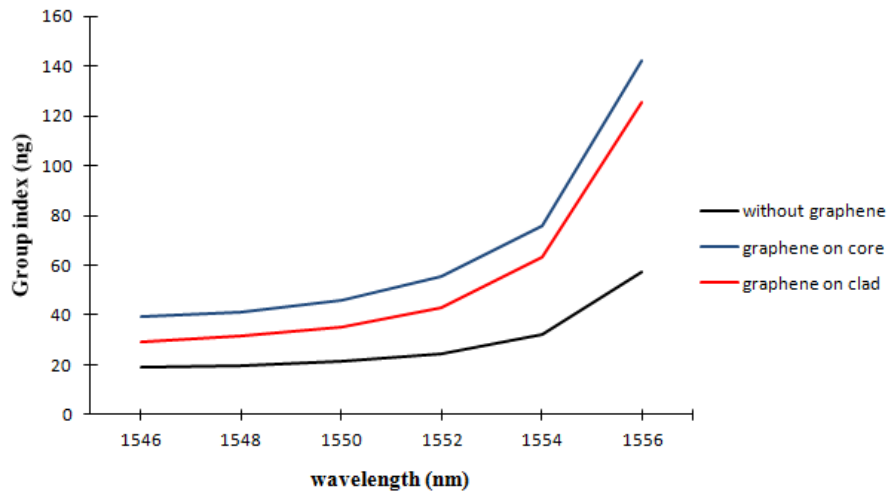


Figure 4.5: Plot of group index (ng) versus wavelength when graphene is not used and when the core/clad region has graphene. This demonstrates slow light enhancement.

Few layer graphene (FLG) creates surface plasmon polariton (SPP) on the graphene-silicon interface since FLG acts like a semi-metal. The mode which is being guided optically in the PCW is modified by the SPP. The guided mode's group index is raised by its interaction with SPP. This reduces the group velocity of light and thus slows down the light furthermore as shown in the plot of Figure 4.5. The value of group velocity decreases further when graphene is used in the structure in comparison to the absence of graphene at 1550nm wavelength. The reason this particular wavelength is of importance because this is the wavelength used for telecommunication purposes. With decrease in group velocity v_g , the group delay increases as the two are inversely proportional. This has advantages in various fields involving tuning of delay.

4.3 GRAPHENE'S ELECTRICAL CONTROL

In the structure proposed, variation is made in the graphene's electrical voltage. The optical properties of graphene sheet vary i.e. graphene's fermi level changes with the change in applied voltage. This provides a corresponding change in the graphene's dielectric constant ' ϵ ' which further causes the group velocity v_g of graphene to change. This gives a varying group delay Δt of slow light. Hence the delay is tuned in a 1-D PCW using graphene in it.

The gating which is electrically done varies the imaginary and real constituents of the dielectric constant of graphene. They are given by the equation [27]:

$$\mathcal{E}'_{\mathbf{g}}(E_{\mathbf{f}}) = 1 + \frac{e^2}{8\pi E \epsilon_0 d} \ln \frac{(E + 2|E_{\mathbf{f}}|)^2 + \Gamma^2}{(E - 2|E_{\mathbf{f}}|)^2 + \Gamma^2} - \frac{e^2}{\pi \epsilon_0 d} \frac{|E_{\mathbf{f}}|}{E^2 + \left(\frac{1}{\tau}\right)^2} \quad (4.1)$$

$$\begin{aligned} \mathcal{E}''_{\mathbf{g}}(E_{\mathbf{f}}) = & \frac{e^2}{4E \epsilon_0 d} \left[1 + \frac{1}{\pi} \left\{ \tan^{-1} \frac{E - 2|E_{\mathbf{f}}|}{\Gamma} - \tan^{-1} \frac{E + 2|E_{\mathbf{f}}|}{\Gamma} \right\} \right] \\ & + \frac{e^2}{\pi \tau E \epsilon_0 d} \frac{|E_{\mathbf{f}}|}{E^2 + \left(\frac{1}{\tau}\right)^2} \end{aligned} \quad (4.2)$$

Here, Γ is the charge carrier scattering rate whose value is 110meV. The scattering rate of carrier $1/\tau$ may be neglected as it does not affect the dielectric constant much. Since the waveguide operates on a narrow bandwidth, the dielectric constant is calculated on a single wavelength of 1550nm only. The graphene's fermi energy ($E_{\mathbf{f}}$) depends on the voltage applied (V) to it and they can be related as [26, 27]:

$$E_{\mathbf{f}} = \hbar V_{\mathbf{f}} \sqrt{\pi \times \left(\frac{C|V|}{q} + n_0 \right)} \quad (4.3)$$

Where C is capacitance per unit area and its value is approximately 20mF/m² when gating is done by ion gel, $V_{\mathbf{f}}$ is graphene's fermi velocity whose value is approx. 10⁶. Here n_0 can be neglected which is the concentration of the carrier.

4.4 TUNING OF DELAY

It is possible to express the complex refractive index tunability in terms of electric voltage applied to gate based on the Kubo formula. Firstly, the group velocity variation is calculated on applying electric voltage for the case when graphene is absent in the PCW structure. There is a vast amount of variation in the group velocity but with the disadvantage that a very large electric voltage is required. Thus, we can tune the delay widely but an enormously large electric voltage needs to be applied. It is not practically realizable for applications involving the delay tuning on chip. Figure 4.6 shows a plot depicting the group velocity variation as the electric voltage applied is varied.

Figure 4.6 shows that as the applied electric voltage in the PCW structure increases, there is a corresponding decrease in the group velocity thus slowing down the light.

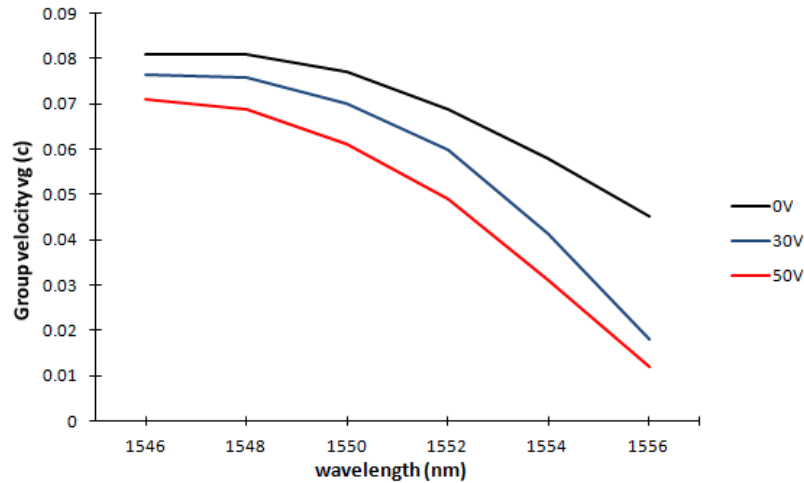


Figure 4.6: Plot of group velocity variation versus wavelength when the voltage applied is varied for the case when no graphene is used.

Thus, with reduction in the group velocity, the group delay increases consequently. When no electric voltage is applied, a value of 0.07716c is noted for the group velocity and a value of 43.23 picoseconds is attained for the group delay at a wavelength of 1550nm. When a voltage of 30 volts is applied, a value of 0.0701c is noted for the group velocity and thus 47.55 picoseconds is the group delay noted. While when a voltage of 50 volts is applied on the structure, a value of 0.0612c is noted for group velocity and 54.46 picoseconds is the group delay observed at 1550nm. Once the group velocity has been calculated, we can calculate the group delay according to the Equation 1.2. The length of the delay line L is taken to be 1mm for calculations to obtain better optical results [39].

Figure 4.7 shows a plot representing the variation of delay in picoseconds with the voltage applied at 1550nm wavelength which is the telecommunication wavelength, for the case when graphene is not present in the PCW structure. For our wavelength of interest, there is tuning from 0.07716c to 0.0612c in the group velocity value by application of voltage as high as 50 volts. Consequently, there is tuning of group delay with values changing from 43.23ps to 54.46ps. Thus, as voltage increases, value of group delay also increases. We can conclude from this that there is a requirement of vast amount of electric voltage to obtain such high values of group delay. This is not practically realizable for applications involving the tuning of delay on chip.

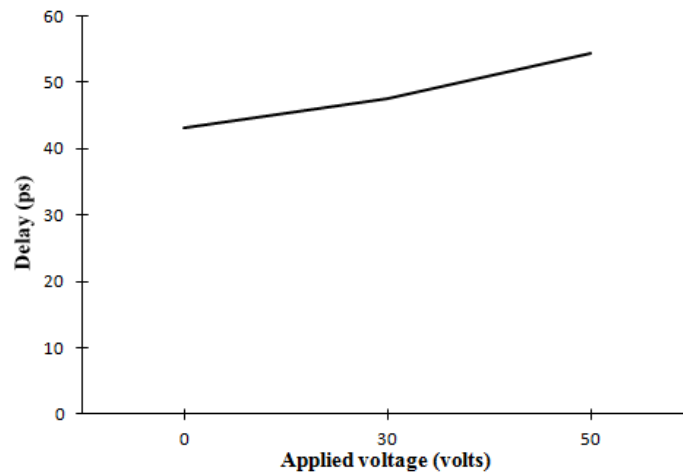


Figure 4.7: Variation of delay (in picoseconds) for the photonic crystal waveguide when no graphene is used with the applied voltage at 1550nm wavelength. This represents that as the voltage increases, the group delay increases.

Now consider the structure of PCW that involves graphene. Firstly the structure where core region or line defect region has graphene placed on it is analyzed. The variation of group velocity with the change in electric voltage applied on the PCW is determined. A large variation in the group velocity can be noticed with less requirement of voltage. Thus, by using only a very small amount of voltage or electric field, the delay can be tuned widely which is feasible for applications involving tuning the delay on chip.

Figure 4.8 depicts the group velocity variation by changing the applied electric voltage in the PCW structure. As shown by the figure, there is a decrease in the group velocity with increase in the applied electric field. Hence, the group delay increases since the group velocity and group delay are related by an inverse relation.

In the absence of any electric voltage, a value of $0.0511c$ is noted for the group velocity and $65.23ps$ as the group delay value at $1550nm$. When a voltage of 2 volts is applied, a value of $0.0285c$ is reported for the group velocity and $116.95ps$ for the group delay. Whereas when a voltage of 4 volts is applied, a value of $0.0163c$ is noted for group velocity and $204.29ps$ for the group delay at $1550nm$.

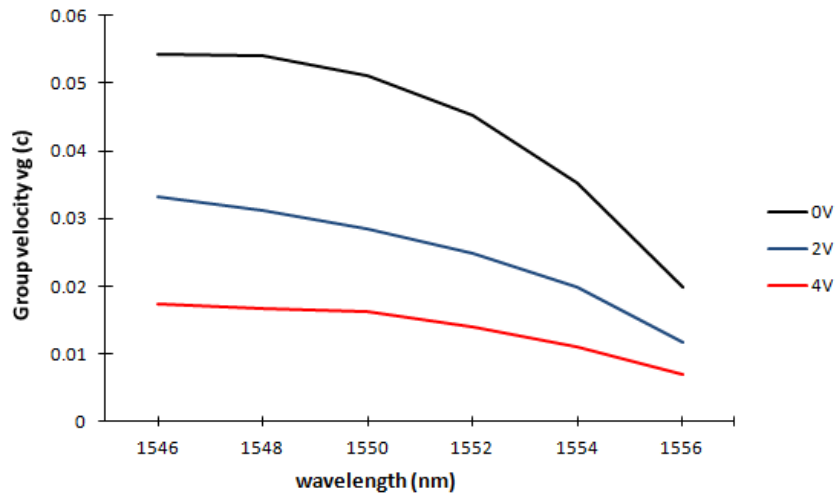


Figure 4.8: Plot of group velocity variation versus wavelength when the voltage applied is varied for the case when core region has graphene placed on it.

Figure 4.9 represents a plot of variation in the group delay with the applied voltage at 1550nm for the case when the core region in PCW structure has graphene sheet placed on it.

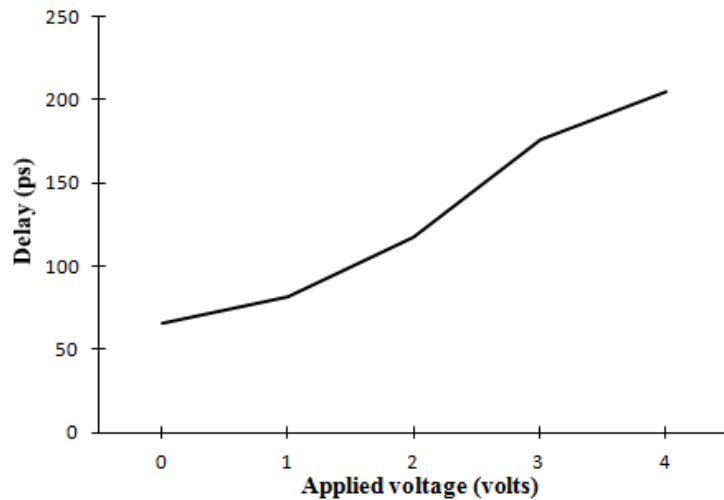


Figure 4.9: Plot of delay variation (in picoseconds) with the applied voltage at 1550nm for the case when the core region has graphene placed on it.

By applying small amount of electric voltage, group delay having very high values can be obtained. This is feasible for applications involving the tuning on delay on chip. For the interested wavelength, there is tuning from 0.0511c to 0.0163c in the value of group velocity by application of voltage as small as 4 volts. Consequently, there is tuning in the group delay for values changing from 65.23ps to 204.49ps in PCW. Thus, the delay is widely tuned with 139.26ps tuning range for a design like this.

Now we calculate group velocity variation for another device design. A structure where the cladding region has graphene placed on it is now analyzed. Again the application of small amount of voltage results in high values of group velocity variation. Thus by applying an electric voltage which is very less, delay can be tuned widely and it is feasible for applications involving the tuning of delay on chip.

Figure 4.10 depicts the plot of group velocity variation by changing the applied electric voltage in the PCW structure. As shown in the figure, there is a decrease in the values of group velocity with increase in the electric voltage. The group thus subsequently increases due to the inverse relation between group delay and group velocity.

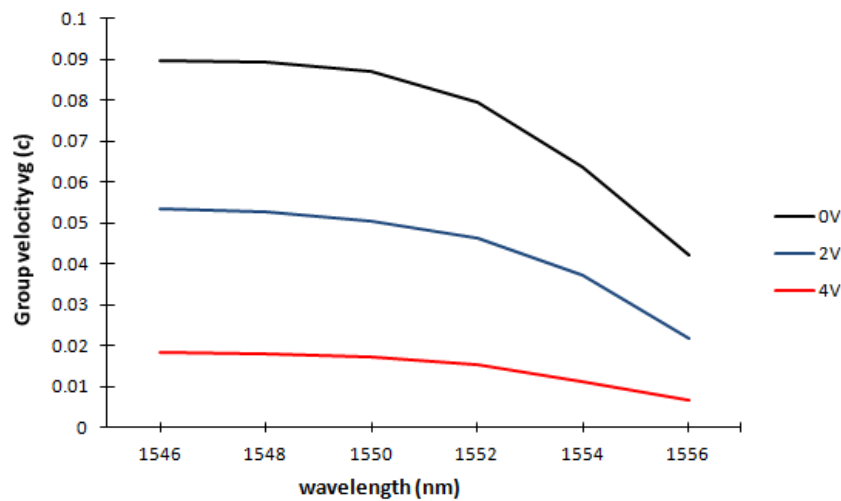


Figure 4.10: Plot of group velocity variation versus wavelength when the voltage applied is varied for the case when clad region has graphene placed on it.

As shown by the figure, there is a decrease in the group velocity with increase in the applied electric field. Hence, the group delay increases. In the absence of any electric voltage, a value of 0.08703c is noted for the group velocity and 38.3ps as the group delay value at 1550nm. When a voltage of 2 volts is applied, a value of 0.04354c is reported for the group velocity and 76.55ps for the group delay. Whereas when a voltage of 4 volts is applied, a value of 0.01776c is noted for group velocity and 187.6ps for the group delay at 1550nm.

Figure 4.11 represents a plot of variation in the group delay with the applied voltage at 1550nm for the case when the clad region in PCW structure has graphene sheet placed on it.

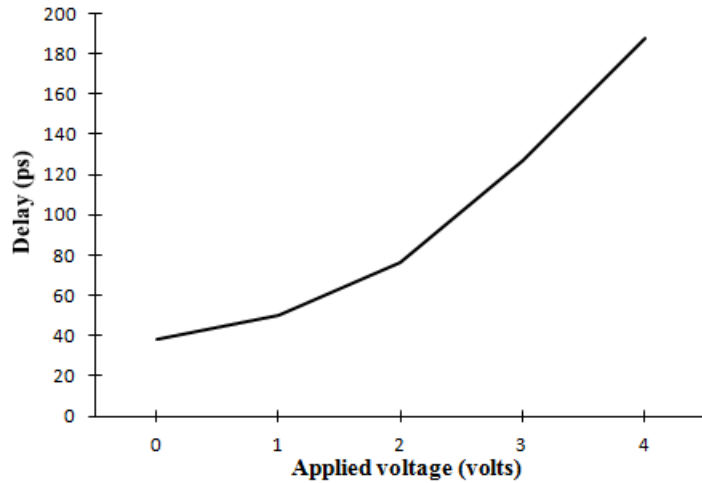


Figure 4.11: Plot of delay variation (in picoseconds) with the applied voltage at 1550nm for the case when the clad region has graphene placed on it.

By applying small amount of electric voltage, group delay having very high values can be obtained. This is feasible for applications involving the tuning on delay on chip. For the interested wavelength, there is tuning from 0.08703c to 0.01776c in the value of group velocity by application of voltage as small as 4 volts. Consequently, there is tuning in the group delay for values changing from 38.3ps to 187.6ps in PCW. Thus, the delay is widely tuned with 149.3ps tuning range for a design like this.

Figure 4.12 shows the comparison plot of the delay variation for a 1-D PCW on application of an electric voltage on graphene when it is placed on core as well as on cladding region.

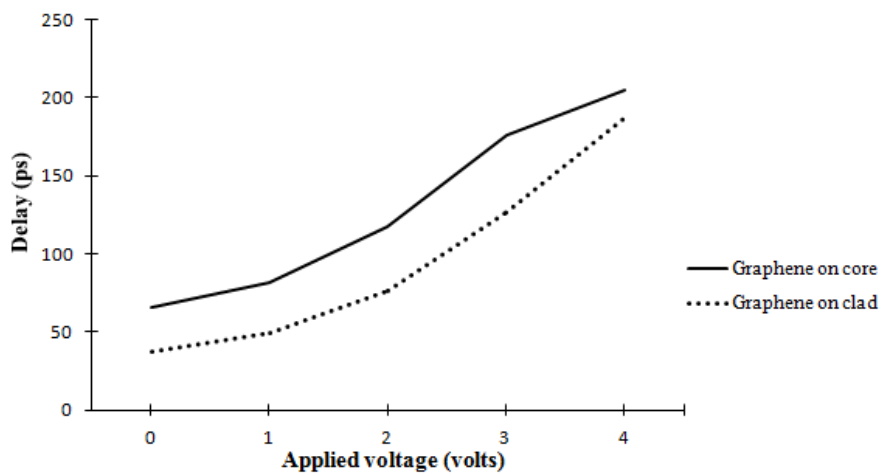


Figure 4.12: Comparison plot of the delay variation with applied voltage at 1550nm with core and clad region having graphene placed on it.

It is possible that the delay is extensively tuned in a PCW through the application of very low electric voltage on graphene. When graphene is not present in the structure, a group delay value as high as 54.46 picoseconds is noted in the PCW structure. But a large amount of voltage needs to be applied to obtain high group delay values. It is not practically realizable for applications involving the tuning of delay on chip.

Contradictory to this, we can use graphene in the PCW structure to obtain group delay which is comparably large but requires only a small electric voltage. This is possible since we can tune the graphene's fermi level remarkably by applying comparatively small electrical field. Thus, for the applications which require less power; PCW structure involving graphene is highly suitable. Furthermore, it is noted that the value of group velocity decreases while the value of group delay increases when graphene is used in the structure, thus slowing down the light. As seen in Figure 4.12, a value of 204.49ps for group delay is noted when the core has graphene layer over it. For the interested wavelength, there is tuning from $0.0511c$ to $0.0163c$ in the value of group velocity by application of voltage as small as 4 volts. Thus, there occurs a tuning from 65.23ps to 204.49ps for group delay in the device where core i.e. line-defect has graphene applied on it. It can be noted that group delay increases with the increase in voltage. For the device design where cladding region has graphene sheet placed over it, a value of 187.6ps for group delay is observed which is comparably smaller than that of the structure for core. There is tuning from $0.08703c$ to $0.01776c$ in the group velocity value by application of small voltage (4volts). There occurs a tuning from 38.3ps to 187.6ps for group delay in the device where the cladding region has graphene sheet placed on it. Therefore, for the structure with core having the graphene sheet on it, a value of 139.26ps as the tuning range is obtained which is wide enough while value of 149.3ps is obtained for the structure where clad region has graphene applied on it. Hence, group delay which is tunable electrically by using low values of electric power can be obtained in a PCW using graphene. Such a model is likely for applications involving tuning of delay on chip.

Chapter 5

Conclusion and Future Scope

5.1 CONCLUSION

A 1-dimensional (1-D) photonic crystal waveguide based on graphene which is electrically controllable is investigated to achieve slow light having wide bandwidth, low value of group velocity v_g and high value of group index n_g . The aim here is to attain tuning of optical delay with application of electric field on graphene in a PCW structure. A straightforward technique is taken into consideration for delay tuning in a PCW structure without having many complications. Firstly, analyze the PCW structure with the absence of graphene and the values of the group velocities with changing electric voltages are noted. When graphene is not used in the structure, a value of 54.46ps for group delay is noted which is large enough and that too with a very large applied voltage of 50V, which is not practically realizable for applications involving tuning of delay on chip. Then graphene is added in the structure and it is analyzed again. The slow light effect is enhanced by reducing the group velocity of light on using few layer graphene (FLG) in the PCW structure. FLG creates surface plasmon polariton (SPP) since FLG acts like a semi-metal. The guided mode's group index is thus enhanced. By using low electrical voltage, graphene's fermi level can be tuned remarkably. The graphene's refractive index is thus varied and consequently the dielectric constant varies. The result of this is that the light is further slowed down. Therefore, the use of graphene in the structure not only tunes the delay electrically with reduced power, but it raises the group delay also. Two design structures have been proposed - first where core region or the line defect has graphene sheet placed on it; second where clad region has graphene placed on it. When the voltage used on graphene is altered from 1 volt to upto 4 volts, there is a tuning from 81.1ps to 204.49ps in the group delay value for the structure where core region has graphene on it; and from 50.45ps to 187.6ps where clad region has graphene applied at 1550nm wavelength. Thus, a group delay tuning of 123.39 picoseconds and 137.15 picoseconds is achieved in both the designs respectively which is large enough as compared to 2-D PCW structures. A conclusion is thus formed that while keeping up a small physical footprint, graphene based 1-D PCW can be valuable in low power delay tuning applications.

5.2 FUTURE SCOPE

The slow light can be further slowed down by optimizing the proposed device structure of PCW based on graphene. Different optimizations can be done in the structure like changing the dimensions of the slab or the layer of graphene, or by varying the lattice period. The values of the group delay will be improved which depends on the upgraded device structure and it helps in further enhancing the delay tuning range advantageous for applications involving tuning of delay. It is also possible to improve the delay resolution. Improving the optical delay is of use only if tuning is done with continuity at a bandwidth range which is wide enough. Also, it must be done through the use of devices uncomplicated and can be managed easily and efficient from cost point of view. It is expected that graphene will replace the traditional technologies based on silicon and will become more captivating in future. It may come up with electrical devices that are more chemically stable, have higher speed, are environmental friendly, and function in a better way. So, graphene's integration with architectures having nano photonics such as photonic crystal waveguides can lead to the formation of compact sized, ultrafast, and efficient devices of graphene which have an excellent potential for optical communication on chip.

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