AN EFFICIENT HEURISTIC ALGORITHM FOR MULTI- OBJECTIVE FIRE STATION LOCATION PROBLEM

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By

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CERTIFICATE

I hereby certify that the work present in the thesis entitled. "**An Efficient Heuristic Algorithm For Multi-Objective Fire Station Location problem**" which is being presented for the award of the degree of Master of Science, School of Mathematics and Computer Science Application, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Mahesh Kumar Sharma .

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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ABSTRACT

In this work, a real-life problem of locating a fixed number of fire station at a potential location sites, and allocating the areas to them is considered. In this prioritized bi-criteria problem, one of the constraint is that each area should be allocated to a unique fire station site selected for locating a fire station at it accordance with the capacity of each fire station site. The maximum number of areas that can be allocated to a fire station site is also fixed. The two objective functions are to minimize the total operating cost and the duration of covering the areas from the locations identified for setting up the fire stations at the least setup cost. Each potential location has a capacity to cover up to a fixed number of areas. A heuristic algorithm is proposed to find the set of efficient solution of this problem.

The present thesis consists of three chapters. Chapter one is introducing in nature in which multi-objective optimization has been described and brief survey of the literature to the topic has been discussed. In the second chapter an algorithms for multi-objective fire station location problem given by Singh A. (2010) have been reviewed. In chapter three, a heuristic algorithm has been developed for multi-objective fire station location problem which is the combination of an algorithm to convert bi-objective problem to its equivalent single objective problem given by Prakash el. al. (2008) and an algorithm reviewed in chapter second.

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CHAPTER 1
INTRODUCTION

Optimization is an activity that aims at finding the best (i.e., optimal) solution to a problem. For optimization to be meaningful there must be an objective function to be optimized and there must exist more than one feasible solution, i.e., a solution which does not violate the constraints. The term optimization does not apply, usually, when the number of solutions permits the best to be chosen by inspection, using an appropriate criterion. One distinguishes single objective and multi objective optimization. In the first case, the objective is scalar-valued (it can be measured by a single number); in the second, the objective is vector-valued (its value is expressed by an n-tuple of numbers).

The single objective model was first to be developed and thus it was received considerably more exposure, been put to more use, and is generally considered to be relatively high level of refinement. Thus the implication is simple, well-tested tool is available and we may be inclined to fit the problem to this model despite the assumptions required. But in real life there are many problems to this model despite the assumption required. But in the real life there are many problems with more than one objective for which the multi objective models are required.

Dantzig's initial concept was concerned about the development of the linear programming model but with single objective. This so set the tone for development of traditional linear programming that many (if not most) linear programming texts completely ignore even the possibility of more than one objective. Unlike the traditional single objective problem wherein it is settle on optimizing single objective function, there is no single universally accepted approach for solving multi-objective optimization problems due to usually conflicting nature of objective functions leading to the situation where the optimization of one of these may adversely affect the optimization of others. So in case of multi-objective optimization problem, there is no need to access the decision maker's utility function that may vary to decision makers to decision makers

1.1 MULTI-OBJECTIVE OPTIMIZATION

Multi-objective optimization is also known as Multi-criterion or Multi-attribute optimization is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. Multi –objective optimization can be found in various fields: product and process design, finance, aircraft design, the oil and gas design, automobile design, or wherever optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives. Maximizing profit and minimizing the cost of a product, maximization performance and minimizing fuel consumption of vehicle, minimizing weight and maximizing the strength of a particular component are the example of Multi objective optimization problem.

For nontrivial multi-objective problems, one cannot identify a single solution that simultaneously optimizes each objective. While searching for solutions, one reaches points such that, when attempting to improve an objective further, other objectives suffer as a result. A tentative solution is called non-dominated, Pareto optimal, or Pareto efficient if it cannot be eliminated from consideration by replacing it with another solution which improves an objective without worsening another one. Finding such non-dominated solutions, and quantifying the trade-offs in satisfying the different objectives, is the goal when setting up and solving a multi objective optimization problem.

A number of techniques have been developed, some are to simplified; other invokes lot of assumptions ; quit a number of possess mathematically rigor but tend to unfortunately lose individual or organizations preference or values. It is more flexible approaching the complex problem, and it is absorb the existing single objective methodology as a special case.

A general constraint optimization problem is defined as a problem of optimizing (minimization or maximizing) of a function $f(X)$ subject to constraints $g_1(X) \leq v_1, g_2(X) \leq v_2, \ldots, g_k(X) \leq v_m$ where X is a vector of non negative real numbers. When there are k objectives, Multi-objective optimization problem can be formulated in the following form.

$$
\begin{array}{lll}
\text{Maximize} & Z_i = f_i(x) & i = 1, \ldots, k \\
\text{Subject to} & g_i(x) \le v_i & i = 1, \ldots, m \\
 & x \ge 0 &
\end{array}
$$

It is always possible to express the objective function in their “Maximize” form since a minimization problem can always be transferred to a maximization problem by proper sign manipulation. Likewise for the constraints, “greater than” and “equal” always convertible to their equivalent “less than”.

1.2 CONCEPT OF OPTIMAL AND EFFICIENT SOLUTIONS

1.2.1 Optimal Solution

An optimal solution in the classical sense is one which attains the maximum value of all the objectives simultaneously. The solution x^* is optimal to the problem defined if and only if $x^* \in S$ and $f_l(x*) \ge f_l(x)$ for all l and for all $x \in S$, where S is the feasible reason.

In general, there is no optimal solution to a multi-objective problem. Therefore, optimality replaced by the concept of “satisfying” or the best compromise solution, which depends on the decision makers preferences with respect to the object. Optimality is not an illusion only when the objectives are non-conflicting. Therefore, one must be satisfied with obtaining efficient solutions in multi-objective problem.

1.2.2 Efficient or Non-Dominated solutions:

A set of solutions is said to be efficient if there exists no solution that is superior to it with respect to at least one objective function but is not inferior to it with respect to any of the objective functions.

If x_1 and x_2 are two solutions, then these can have any of two possibilities-one dominates the other or non-dominates the other. In a minimization problem, without the loss of generality, a solution x_1 dominates x_2 iff the following two conditions are satisfied:

$\forall\, i \in \{1, 2\ ,...,\ N_{obj}\}: f_i(x_1) \le f_i(x_2)$

$\forall\, i \in \{1, 2 ,...,\ N_{obj}\}: f_l(x_1) < f_l(x_2)$

Where $f(x_1)$ and $f(x_2)$ are the objective functions.

If any of the above conditions is violated, the solution x_1 does not dominate the solution x_2. If x_1 dominates the solution x_2 is called the non-dominated solution with in the set { x_1, x_2 }.

The solutions that are non-dominated with in the entire search space are denoted as Pareto-optimal and constitute the Pareto-optimal set or Pareto-optimal front. From the entire set off efficient (non-dominated) solutions the decision maker can select the solution one believed most attractive.

1.3 HEURISTICS

The word "heuristic" originated from a Greek root meaning to discover. In the case of optimization of real life problems, the problems are encountered because of the highly complex nature. The regular algorithms are very often ineffective on these problems but there are, however, other approach that may be used to find a solution to model i9nvolving integer or discrete variable. And in many cases, such approaches have proven capable of providing acceptable solutions to truly massive size problem. Thus, the pragmatic or human approach can be considered to solve the problems and hence this shifted the whole paradigm from algorithm based calculations to the employment of heuristic procedure or heuristic programming.

Heuristics are rule of thumbs that are develop through intuition, experience and judgment. In artificial intelligence, a heuristic is a procedure that may lack a proof. It is used when the inter-relationship between variables is not explicitly clear but there is some confidence in understanding the output for certain input. The result of application of heuristic cannot guarantee the optimal solution. A heuristic might help to find the solutions, which are good, but perhaps the not very best they can be. Obviously the measures of goodness and assessments of a heuristic technique is going to be relative to the domain. When one or more heuristic are combined with the procedure for driving a solution from the associated rules, given a heuristic program. Since the concept is to drive an acceptable solution and not the server-elusive optimal solution, thus the heuristic satisfied a certain aspiration criterion as set by the decision maker.

The heuristic approach is also being used to solve the real world multi-objective problem. These problems always attract us goal programming techniques but of late another approach of the efficient/non-dominated/Pareto optimal solutions has gained importance.

1.4 FIRE STATION LOCATION PROBLEM

Fire service is conceived of as organized public service having the primary objective of preventing fires from occurring and reducing the loss of life and property due to fires. In this thesis the fire station location problem of selecting a fixed no of sites, from among given number of potential fire station sites for clustering the areas to them subject to several constraints is considered. One of the constraint is that each area should be allocated to a unique fire station site selected for locating a fire station at it accordance with the capacity of each fire station site. The maximum number of areas that can be allocated to a fire station site, is also fixed. The objective functions of the problem is to minimize the total operating cost to cover the areas from the selected sites.

1.5 FORMULATION OF THE PROBLEM

Suppose there are M areas, N potential fire station locations, K is the number of sites where the fire stations are to be located. The M areas are to be clustered to K sites in such a way that each area is assigned to a unique site which is selected for the task. Each selected potential location may cater a maximum of L areas. Let C_{ij} $(i = 1,2, \ldots, M;\ j = 1,2,\ldots,N)$ units be the cost of covering the area i from locate on j. Let C_j be the setup cost of a fire station at the site j. Let x_{ij} be the decision variable assuming value 0 or 1 according as area i is not clustered or clustered to site j, and y_j be the variable assuming value 0 or 1 according as potential site is not selected or selected for locating a fire station at it. Let C denote the total cost of meeting requirements of all the areas from their assigned fire stations. The mathematical formulation of this problem is as follows.

$$C = \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij}\, x_{ij} \tag{1.1}$$

Constraints of the problem are

$$\sum_{j=1}^{N} y_j = K \tag{1.2}$$

$$\sum_{i=1}^{M} x_{ij} y_{ij} \leq L \ (j = 1,2,\ldots,N) \tag{1.3}$$

$$\sum_{j=1}^{N} x_{ij} = 1 (i = 1,2,\ldots,M) \tag{1.4}$$

$$x_{ij} - y_j \leq 0 \ (i = 1,2,\ldots,M; \ j = 1,2,\ldots,N) \tag{1.5}$$

$$x_{ij}, y_j = 0 \, or \, 1 (i = 1,2,\ldots,M; \ j = 1,2,\ldots,N) \tag{1.6}$$

The constraint (1.2) ensure that K sites are selected for locating fire stations at them, whereas constraint (1.3) ensure that each of the selected potential locations can serve at the most L areas. The constraints (1.4) and (1.5) ensure that each area is assigned to a unique site selected for locating a fire station.

1.6 MULTI-OBJECTIVE FIRE STATION LOCATION PROBLEM

The multi-objective optimization model is set to solve the optimization problem simultaneously associated with several objectives. Normally, existing multi-objective optimization models use a minimization of the total cost objective as one of their objectives. The other objectives may concern about quantity of goods delivered, underused capacity, energy consumption, total delivery time, etc. In this work the multi-objective fire station location problem with two objectives is considered. One objective is to minimize the total operating cost and other is to minimize the duration to cover the areas from the selected fire station sites. The mathematical formulation of this problem is as follows. The two objective functions which are sought to be minimized are:

$$C \;=\; \sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij}\, x_{ij} \tag{1.1}$$

$$T \;=\; \max\left\{ t_{ij} x_{ij} : i = 1,2,..., M;\ j = 1,2,..., N \right\} \tag{1.2}$$

Constraints of the problem are same as from (1.2) - (1.6) given in section (1.5). Note that the above problem is formulated with two objective functions given by Eqns. (1.1) and (1.2). C and T denote the total cost and the total duration respectively of meeting requirements of all the areas from their assigned fire stations. c_{ij} and t_{ij} $(i = 1, 2, ..., M;\ j = 1, 2, ..., N)$ units be the cost and time respectively of covering the area i from location j

1.7 LITERATURE SURVEY

Present work deal with multi-objective optimization problem. Multi-objective differs from single objective problem in the sense that the former problem has more than one objective whereas latter problem have only one objective. There are many approaches to solve multi objective optimization problem whereas there is universally accepted single objective approach seeking to optimize single objective function. The various approaches for solving the multi-objective optimization problem are lexicographic/prioritized and Pareto optimal/ efficient/ non-dominating solution approach. A discussion about them can be found in the work of zeenlay(1974), Prakash(1981), igzino(1982), sharma and Prakash(1986), Steuer (1986), Prakash, Aggarwal and shah(1988), Prakash and Pradeep (1991),shah and Prakash (1992), Prakash,Balaji and Tuneja (1999), Prakash, Kumar, Prasad and Gupta (2008), Taha (2008). The first two approaches reduce the multi-objective optimization to a single objective optimization problem while the last two approached do not alter the nature of the problem.

There are two main approaches for solving the multi-objective optimization problem-(a) analytic (b) heuristic. The analytic approach yields an exact solution of the problem whereas the heuristic approach yield a satisfying solution which can also be at time an exact solution. There are many real-life problems which either cannot be solved or even if can be solved through analytic approach, the amount of time and labor spent on solving them through analytic approach renders it unsuitable. In such cases, heuristic comes to our rescue. The heuristic approach does not require specialized knowledge of the subject. It is based on intuition, experience and judgment, thereby making it easy to apply and has wider applicability. A discussion about heuristic approach used for solving problems can be found in the works of Ignizio (1982), Glover (1989, 1990), Reeves (1993), Ignizio and Cavalier (1994), Osman and Kelly (1996), Prakash, Natrajan and Roy (1999), Deb (2001), Prakash and Gupta (2006), Prakash, Madhusudan and Kunal (2007), Prakash, Tuli and Sharma (2009), Prakash and Tuli (2010).

Facility location also known as location analysis, is a branch of Operational Research concerning with mathematical modeling an solution of problems of facilities so as to minimize transportation cost, avoid placing hazardous material near residential areas, outperform competitor's facilities, etc.

A number of case studies describing the application of facility location models to the strategic design of real-life have been discussed in past, showing the growing awareness and importance that practitioners are devoting to this area. The rapid evolution of computer and communications technology has made it possible the optimization of facility location in real word production-distribution system. In some cases, however the problem size and complexity along with the management's wish to obtain "good" solutions in reasonable time have driven researchers to develop heuristic procedure. This work concern with real-life application of the location of emergency facilities or services that are related to the public sector.

Hoggs (1968) have developed a technique which minimizes the total number of fire application journey time to fire for any given number of fire station, thereby generating a set of solutions giving the best combination of r station sites from a set of solutions, thereby generating a set of solutions giving the best combination of r station sites from a set of n alternative sites, where r varies from n -1 to 1.Kolesar and Blum (1973) have developed an inverse square root function

For the relation between average response distance and the number of locations at which response units are stationed in a region. The square-root response distance model is combined with response distance-response time relation to find the optimal resource allocations given the resource constraints and response time standards.

Plane and Hedrick (1997) had setup a hierarchical objectives function for the set-covering problem which simultaneously minimize the number of fire station and the maximization of the number of fire stations within the minimum total number of stations, thereby reducing the annual costs for the fire companies in Denver. Brueckner(1981) has shown that the fire protection exhibits substantial publicness which suggest that increasing the community's population should not greatly reduce the level of fire protection, holding the suppression capacity fixed, thereby providing the fire services at a lower per capita cost in a larger community.

Sanli and AI-Tamini (1990) have reviewed the principles affecting the performance of services systems with regard to spatial relationships and communication patterns within cities and in particular the principles related to location and allocating of fire services. Badri, Mortagy and Alsayed (1998) have presented a multiple criteria modeling approach, through integer goal programming, to the fire –station location problem that involves conflicting objectives incorporating both the travel distances from stations to demand sites.

Ignizio and Cavalier (1994) have consider the problem of selecting up to fixed number of sites from among given number of potential sites for locating warehouses at them and clustering customer to the selected sites in such a way that each customer is assigned to a unique selected site. The single objective of this problem was to minimize the sum of distances from each customer to his/her assigned site. Praveena et al. (1999) have extended this problem which selects up to fixed number of sites from among given number of potential sites for locating warehouses at them and clustering ration shop to them subject to several constraints with two objectives. The two objectives is to minimize the total cost and duration of meeting the requirements of all the ration shops from their assigned warehouse at their selected sites. These two objectives are not accorded priorities. One constraint is that each ration shop should be clustered to a unique site, which is selected to locate a warehouse at it but there is no restriction on number of ration shops to be clustered to a warehouse at the selected sites. Other constraint is that the setup cost of the warehouse should not exceed a certain budgetary amount. This problem has been solved by finding the set of efficient solutions of it using heuristic method consisting of a combination of add and drop rules. Prakash et al. (2009) has developed a heuristic iterative algorithm incorporating Tabu search to find the set of efficient solutions.

Liu, Huang and Chandramouli (2006) present an approach to suitably situating new fire stations, considering multiple objective of maximizing the coverage of the routes, achieving a reasonable distance between fire stations and maximizing the areas that can be served by fire station within 6 minutes, using GIS and ANT algorithm. Guo and Fu (2007) have reviewed how the fire situation in china has become relatively stable in recent years and have also discussed the ongoing research in the areas of theory of fire dynamics, evaluation techniques for fire risk and fire and resource services of urban areas, modernization of the fire department apparatus and equipment, human behavior in fire among others.

Beraldi and Burni (2009) have formulated and solved a probabilistic model for determining the optimal location of facilities in congested emergency systems. Iannoni, Morabito and saydam (2009) have developed a method that can be used to make decisions regarding the optimal location of the embulance on the highways and the coverage areas of the embulance in order to minimize user response time or remedy an imbalance in ambulance workloads within the system.

Prakash et. al.(2008) have considered the same problem as that by Prakash and Ram (1995) with the alternate that the objective are not assigned priorities. For obtaining the set of non-dominating solutions of altered BTP, developed two algorithms each requiring a sequence of single objective bulk transportation problem to be solved. Sharma M.K., Singh S. , Gupta G.(2011) proposed a heuristic algorithm for multi-objective bulk transportation problem by firstly converting it into an equivalent single objective problem by the procedure provided by Prakash et.al.(2008) using preemptive priority factors and then a heuristic algorithm is proposed to solve this problem. Singh A. (2010) has consider the problem of locating a fixed number of fire station at potential location sites, and allocating the areas to them , has been modeled as a bi-objective problem. A heuristic procedure based on the cost penalties and the adaptive search technique has been proposed to solve the problem.

1.8 PRESENT WORK

The present work deal with multi-objective optimization problems which have a bearing on real-life. We come across these problems in everyday life. In this work, the general and complex real-life problem of locating a fixed number of fire station at a potential location sites, and allocating the areas to them is considered.

The present thesis consists of three chapters. Chapter 1, is introducing in nature in which multi-objective optimization has been described and brief survey of the literature to the topic has been discussed. In the chapter 2, a heuristic algorithm for multi-objective fire station location problem given by Singh A. (2010) has been reviewed. In chapter 3, a multi-objective fire station location problem considered in chapter 2 has been converted into a single objective problem and a heuristic procedure is developed to find its efficient solutions.

CHAPTER 2
MULTI-OBJECTIVE FIRE STATION LOCATION PROBLEM

2.1 INTRODUCTION

The problem of selecting a fixed no of sites, from a given number of potential fire station sites for clustering the areas to them subject to several constraints and two objectives is considered. One of the constraints is that each area should be allocated to a unique fire station site selected for locating a fire station at it accordance with the capacity of each fire station site. The maximum number of areas that can be allocated to a fire station site is also fixed. The two objective functions are to minimize the total operating cost and the duration to cover the areas from the selected sites. A heuristic algorithm given by Singh, A. (2010) to find the set of efficient solution of this problem has been reviewed in this chapter.

2.2 FORMULATION OF PROBLEM

Suppose there are M and, N potential fire station locations, K is the number of sites where the fire stations are to be located. The M areas are to be clustered to K sites in such a way that each area is assigned to a unique site which is selected for the task. Each selected potential location may cater a maximum of L areas. Let c_{ij} and t_{ij} $(i = 1, 2, ..., M;\ j = 1, 2, ..., N)$ units be the cost and time respectively of covering the area i from location j. Let C_j be the setup cost of a fire station at the site j. Let x_{ij} be the decision variable assuming value 0 or 1 according as area is not clustered or clustered to site , and be the variable assuming value 0 or 1 according as potential site is not selected or selected for locating a fire station at it. Let C and T denote the total cost and the total duration respectively of meeting requirements of all the areas from their assigned fire stations. The mathematical formulation of this problem is as follows. The two objective functions which are sought to be minimized are:

$$C = \sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij}\, x_{ij} \tag{2.1}$$

$$T = \max\left\{t_{ij}\, x_{ij} : i = 1,2,..., M;\ j = 1,2,..., N\right\} \tag{2.2}$$

Constraints of the problem are:

$$\sum_{j=1}^{N} y_j = K \tag{2.3}$$

$$\sum_{i=1}^{M} x_{ij}\, y_{ij} \le L\ (j = 1,2,\ldots,N) \tag{2.4}$$

$$\sum_{j=1}^{N} x_{ij} = 1 (i = 1,2,\ldots,M) \tag{2.5}$$

$$x_{ij} - y_j \le 0\ (i = 1,2,\ldots,M;\ j = 1,2,\ldots,N) \tag{2.6}$$

$$x_{ij}, y_j = 0\, or\, 1 (i = 1,2,\ldots,M;\ j = 1,2,\ldots,N) \tag{2.7}$$

Note that the objective functions given by Eqns. (2.1) and (2.2) are not accorded any Priorities. The constraint (2.3) ensure that K sites are selected for locating fire stations at them, whereas constraint (2.4) ensure that each of the selected potential locations can serve at the most L areas. The constraints (2.5) and (2.6) ensure that each area is assigned to a unique site selected for locating a fire station.

For the purpose of listing the efficient solutions, a solution $(\overline{X}^{(1)}, \overline{Y}^{(1)})$ shall be called the 1^{st} Efficient solution if it is the optimal solution of the problem with minimization of C and T as the first and second prioritized objectives respectively. A solution $(\overline{X}^{(2)}, \overline{Y}^{(2)})$ shall be called the 2^{nd} efficient solution if no efficient solution $(\overline{X}, \overline{Y})$ of the problem exists satisfying the conditions: (1) $C(\overline{X}^{(1)}) < C(\overline{X}) < C(\overline{X}^{(2)})$ and (2) $T(\overline{X}^{(1)}) > T(\overline{X}) > T(\overline{X}^{(2)})$. The 3^{rd} and subsequent efficient solutions are defined in the same way as is done for the 2^{nd} efficient solution.

2.3 SOLUTION PROCEDURE (Singh, A. 2010)

The problem formulated above is a binary integer nonlinear problem because the variables $x_{ij}s$ and $y_j s$ are binary integers and the objective function T given by 2.1 is nonlinear. The prioritized bi-criteria problems are solved as such i.e. without being linearized. Procedure to obtain 1^{st}, 2^{nd} and subsequent efficient solutions are explained below.

2.3.1 Procedure to Obtained 1^{st} Efficient Solution

Note that the 1^{st} efficient solution $(\overline{X}^{(1)}, \overline{Y}^{(1)})$ of the problem formulated above given by Eqns. (2.1)-(2.7) is the optimal solution of the problem wherein the total cost C and the duration T of covering the areas from their assigned fire stations, are minimized with the first and second priorities respectively, subject to constraints(2.2)-(2.7). The problem yielding the 1^{st} efficient solution is designated as the 1^{st} prioritized bi-criterion problem. The procedure to obtain the 1^{st} efficient solution is given below.

Step I: Identify all the ${}^{N}C_{K}$ combinations of the potential locations.

Step II: Rank the combinations in accordance with their total setup cost. The least setup cost given the first rank.

Step III: Starting from first ranked combination of the potential fire station locations, calculated the cost penalty for each of the selected potential location. The cost penalty is the minimum penalty that might be incurred in case the potential location is not associated with an area which corresponds to least cost for the said location, i.e. the cost penalty CP_j for the location j is the positive difference between the least and second least cost associated with j^{th} location, i.e. $CP_j = c_{2j} - c_{1j}$, where c_{1j} and c_{2j} are the least and second least costs of the j^{th} destination. If the least cost occurs more than once in a column j, then the corresponding penalty $CP_j = 0$.

Step IV: Select the j^{th} destination with largest cost penalty CP_j. Make an allocation at the least cost cell of the selected column j. In case of tie of largest penalty between columns, select the column which has the least cost lower than the least of the others. In case of tie on the least cost count as well, select the column which has least amount of time associated with the least cost. In case both the least cost and the associated time are same for the tied column, then it is an arbitrary choice for the decision maker. After allocation the area, drop it for further considerations.

Step V: Repeat the step 3 till M -1 areas have been allocated to the selected potential locations Keeping in view their capacities. The last area is to be allocated following the least cost rule and not on the basis of cost penalty. The area will be allocated to the least cost cell of the available locations, i.e. the potential location with available capacity. When all the areas have been allocated to unique selected potential fire station location, the first efficient solution $(\overline{X}^{(1)}, \overline{Y}^{(1)})$ has been found.

2.3.2 Procedure to Obtain 2^{nd} and Subsequent Efficient Solutions

The 2^{nd} efficient solution $(\overline{X}^{(2)}, \overline{Y}^{(2)})$ of the formulated problem is obtained by solving the problem obtained from the $1^{st,}$ prioritized bi-criterion problem after dropping all those cells (i, j) corresponding to $t_{ij} \geq T(\overline{X}^{(1)})$. The problem, thus obtained, is similar to the $1^{st,}$ prioritized bi-criterion problem and is designated as 2^{nd} prioritized bi-criterion problem. The 2^{nd} prioritized bi-criterion problem is solved adopting the method for solving $1^{st,}$ prioritized bi-criterion problem. The 3^{rd} and subsequent efficient solutions are obtained in the same way as done to obtained the 2^{nd} efficient solution. This process of obtaining the efficient solution is terminated after encountering a prioritized bi-criterion problem where it is impossible to allocate at least one area to one of the selected potential fire station location because of the unavailability of cost sells satisfying time constraints

The above mentioned procedures to obtain the efficient solutions should be implemented for all the ${}^{N}C_{K}$ combination of the potential fire station locations to obtain different set of efficient solution for all the combinations. Thereafter all the solutions should be compared to generate a final set of efficient set of solutions out of which the decision maker may choose the one depending on his/her priority of the decision parameters.

2.4 NUMERICAL PROBLEM

Table 2.1: Tableau representation of the problem

Areas	Fire Station Potential Sites				
	1	2	3	4	5
1	40 2	70 6	80 8	60 11	90 10
2	30 8	80 3	20 9	10 6	100 13
3	50 11	130 9	200 3	70 13	20 8
4	120 6	130 7	70 8	80 4	60 12
5	180 10	170 8	140 11	30 9	40 2
6	170 9	140 10	130 12	160 8	50 6
7	150 7	20 11	60 10	210 4	220 14
Setup Cost (C_{ij})	1,00,000	8,00,000	7,00,000	3,00,000	4,00,000

Now starts the procedure explained in section 2.3 by applying it to obtained the set of efficient solutions of the numerical problem presented in Table 2.1considering $M = 7, N = 5, K = L = 3$ and assigning numerical values to all the $c_{ij}s, t_{ij}s, C_j s$ in the problem formulated in section 2.2.

In Table 2.1, rows 1-7 correspond to areas and columns 1-5 correspond to potential fire station locations. Upper and lower entries of a cell (i, j) depicts the units of costs c_{ij} and time t_{ij} respectively of covering area i from the fire station location j.

2.4.1 Procedure to Obtained 1^{st} Efficient Solution of the Numerical Problem

Step I: Since three fire stations are to be located, thus the combinations will be $^5C_3$=10 in number. The procedure starts by considering the combination that has least setup cost. In the numerical problem being considered here, the combination of 3 potential fire station locations with the lest setup cost is 1, 4, 5 with a total set up cost as 8, 00,000 units.

Table-2.2

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	40 2	60 11	90 10
2	30 8	10 6	100 13
3	50 11	70 13	20 8
4	120 6	80 4	60 12
5	180 10	30 9	40 2
6	170 9	160 8	50 6
7	150 7	210 4	220 14
Setup Cost (C_{ij})	1,00,000	3,00,000	4,00,000

Step II: Calculate the cost penalties of each selected sites, i.e. the positive difference of the least and second least costs of selected locations. The penalties are 10, 20,and 20 respectively for the location 1,4 and 5. Since there is a tie in the largest penalty for location 1,4 and 5 is given the first priority since the least cost for location 4 is lower than that for the location 5. Thus the first allocation is made in the cell (2, 4) i.e. area 2 is clustered to location 4.

Table-2.3

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	40 2	60 11	90 10
2	30 8	10 6 ⇦	100 13
3	50 11	70 13	20 8
4	120 6	80 4	60 12
5	180 10	30 9	40 2
6	170 9	160 8	50 6
7	150 7	210 4	220 14

Since area 2 is allocated to unique potential site 4, drop this area for further consideration. Thus updated table is given below in Table.

Table-2.4

Areas	Fire Station Potential Sites		
	1	**4**	**5**
1	40 2	60 11	90 10
3	50 11	70 13	20 8
4	120 6	80 4	60 12
5	180 10	30 9	40 2
6	170 9	160 8	50 6
7	150 7	210 4	220 14

Step III: Recalculate the cost penalties for the selected potential locations. The new penalties are 10, 30, and 20 for the location 1, 4 and 5 respectively. Since the largest penalty again correspond to the location 4, thus the allocation is made in the next least cost cell for location 4. i.e. (5, 4)

Table-2.5

Areas	Fire Station Potential Sites		
	1	**4**	**5**
1	40 2	60 11	90 10
3	50 11	70 13	20 8
4	120 6	80 4	60 12

5	180 10	30 9	40 2
6	170 9	160 8	50 6
7	150 7	210 4	220 14

Hence area 5 is allocated to the location 4. Drop this area for further consideration. Thus updated table is given below.

Table-2.6

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	40 2	60 11	90 10
3	50 11	70 13	20 8
4	120 6	80 4	60 12
6	170 9	160 8	50 6
7	150 7	210 4	220 14

Step IV: Recalculate the cost penalties for the selected potential locations. The new penalties are 10, 10, and 30 for the location 1, 4 and 5 respectively. Since the largest penalty is correspond to the location 5, thus the allocation is made in the least cost cell for location 5, i.e. (3, 5).

Table-2.7

Areas	Fire Station Potential Sites		
	1	**4**	**5**
1	40 2	60 11	90 10
3	50 11	70 13	20 ⇦ 8
4	120 6	80 4	60 12
6	170 9	160 8	50 6
7	150 7	210 4	220 14

Since area 3 is allocated to a unique potential site 5, drop this area for further consideration. Thus updated table is given below.

Table-2.8

Areas	Fire Station Potential Sites		
	1	**4**	**5**
1	40 2	60 11	90 10
4	120 6	80 4	60 12
6	170 9	160 8	50 6
7	150 7	210 4	220 14

Step V: Recalculate the cost penalties for the selected potential locations. The new penalties are 80, 20, and 10 for the location 1, 4 and 5 respectively. Since the largest penalty is correspond to the location 1, thus the allocation is made in the least cost cell for location 1, i.e in the cell (1, 1).

Table-2.9

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	40 2	60 11	90 10
4	120 6	80 4	60 12
6	170 9	160 8	50 6
7	150 7	210 4	220 14

Since area 1 is allocated to a unique potential site 1. Drop this area for further consideration. Thus updated table is given below.

Table-2.10

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
4	120 6	80 4	60 12
6	170 9	160 8	50 6
7	150 7	210 4	220 14

Step VI: Recalculate the cost penalties for the selected potential locations. The new penalties are 30, 80, and 10 for the location 1,4 and 5 respectively. Since the largest penalty is correspond to the location 4, thus the allocation is made in the least cost cell for location 4, i.e in the cell (4, 4).

Table-2.11

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
4	120 6	80 ⇦ 4	60 12
6	170 9	160 8	50 6
7	150 7	210 4	220 14

Since area 4 is allocated to a unique location 4. Drop this area for further consideration. Thus updated table is given below.

Table-2.12

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
6	170 9	160 8	50 6
7	150 7	210 4	220 14

Step VII: Recalculate the cost penalties for the selected potential locations. The new penalties are 20, 50, and 170 for the location 1, 4 and 5 respectively. Since the largest penalty is correspond to the location 5, thus the allocation is made in the least cost cell for location 5, i.e in the cell (6, 5).

Table-2.13

Areas	Fire Station Potential Sites		
	1	**4**	**5**
6	170 9	160 8	50 ⇦ 6
7	150 7	210 4	220 14

Since area 6 is allocated to the location 5. Drop this area for further consideration. Thus updated table is given below.

Table2.14

Areas	Fire Station Potential Sites		
	1	**4**	**5**
7	150 7	210 4	220 14

Step VIII: The last area is to be allocated following the least cost rule and not on the basis of cost penalty. The area will be allocated to the least cost cell of the available locations, i.e. the potential location with available capacity. Thus allocation is made in the cell (7, 1), i.e. area 7 is allocated to the location 1.

Table-2.15

Areas	Fire Station Potential Sites		
	1	**4**	**5**
7	150 ⇦ 7	210 4	220 14

Thus the allocation of the areas to the location 1, 4 and 5 keeping in view their capacity is given below in Table 2.16.

Table 2.16: Allocation of the areas to the locations 1, 4, 5

Areas	**Fire Station Potential Sites**
1	1
2	4
3	5
4	4
5	4
6	5
7	1

Thus the 1^{st} **Efficient Solution** has been obtained wherein the cost and time to cover all areas from the selected potential location is given in the Table 2.17.

Table 2.17: 1^{st} **Efficient Solution**

Cost to cover all areas	380 units
Time to cover all areas	9 units
Total set up Cost	8,00,000

2.4.2 Procedure to Obtained 2^{nd} Efficient Solution of the Numerical Problem

The 2^{nd} Efficient Solution of the problem for the same combination of the selected potential locations i.e. 1,4 and 5 is obtained by dropping the cell (i, j) wherein the time $t_{ij} \geq T(\overline{X}^{(1)})$ =9 units. The updated table of the problem for the 2^{nd} **Efficient Solution** is given in Table 2.18.

Table-2.18: Table for 2^{nd} Efficient Solution

Areas	Fire Station Potential Sites				
	1	2	3	4	5
1	10 2	70 6	80 8	- -	- -
2	30 8	80 3	20 9	10 6	- -
3	- -	130 9	200 3	- -	20 8
4	120 6	130 7	70 8	80 4	- -
5	- -	170 8	140 11	- -	40 2
6	- -	140 10	130 12	160 8	50 6
7	150 7	20 11	60 10	210 4	- -

Step I: Table corresponds to location 1, 4 and 5 for the 2^{nd} Efficient solution is given below in Table 2.19.

Table-2.19

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	40 2	- -	- -
2	30 8	10 6	- -
3	- -	- -	20 8
4	120	80	-

	6	4	-
5	- -	- -	40 2
6	- -	160 8	50 6
7	150 7	210 4	- -
Setup Cost (C_{ij})	1,00,000	3,00,000	4,00,000

Step II: Calculate the cost penalties for the selected potential locations. The new penalties are 10, 70, and 20 for the location 1, 4 and 5 respectively. Since the largest penalty is correspond to the location 4, thus the allocation is made in the least cost cell for location 4, i.e in the cell (2, 4).

Table-2.20

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	40 2	- -	- -
2	30 8	10 ⇐ 6	- -
3	- -	- -	20 8
4	120 6	80 4	- -
5	- -	- -	40 2
6	- -	160 8	50 6
7	150 7	210 4	- -

Hence area 2 is allocated to the location 4.Drop this area for further consideration. Thus updated table is given below.

Table-2.21

Areas	Fire Station Potential Sites		
	1	**4**	**5**
1	40 2	- -	- -
3	- -	- -	20 8
4	120 6	80 4	- -
5	- -	- -	40 2
6	- -	160 8	50 6
7	150 7	210 4	- -

Step III: Recalculate the cost penalties for the selected potential locations. The new penalties are 80, 80, and 20 for the location 1,4 and 5 respectively. Since there is a tie in the largest penalty for location 1 and 4. Location 1 is given the first priority since the least cost for location 1 is lower than that for the location 4. Thus the allocation is made in the cell (1, 1) i.e. area 1 is clustered to location 1.

Table-2.22

Areas	Fire Station Potential Sites		
	1	**4**	**5**
1	40 ⇐ 2	- -	- -

3	- -	- -	20 8
4	120 6	80 4	- -
5	- -	- -	40 2
6	- -	160 8	50 6
7	150 7	210 4	- -

Hence area 1 is allocated to a unique potential location site 1. Drop this area for further consideration. Thus updated table is given below.

Table2.23

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
3	- -	- -	20 8
4	120 6	80 4	- -
5	- -	- -	40 2
6	- -	160 8	50 6
7	150 7	210 4	- -

Step IV: Recalculate the cost penalties for the selected potential locations. The new penalties are 30, 80, and 20 for the location 1,4 and 5 respectively. Since the largest penalty is correspond to the location 4, thus the allocation is made in the least cost cell for location 4, i.e in the cell (4, 4).

Table-2.24

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
3	- -	- -	20 8
4	120 6	80 4 ⇦	- -
5	- -	- -	40 2
6	- -	160 8	50 6
7	150 7	210 4	- -

Hence area 4 is allocated to a unique potential location site 4. Drop this area for further consideration. Thus updated table is given below

Table -2.25

Areas	Fire Station Potential Sites		
	1	4	5
3	- -	- -	20 8
5	- -	- -	40 2
6	- -	160 8	50 6
7	150 7	210 4	- -

Step V: In Table (2.25), it can be observed that there is only one choice for allocating the areas 3 and 5; since in each of these two cases, two potential locations have been dropped. Thus areas 3 and 5 are allocated to location 5and 5 respectively.

Table-2.26

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
3	- -	- -	20 8
5	- -	- -	40 2
6	- -	160 8	50 6
7	150 7	210 4	- -

Hence area 3 and 5 is allocated to the same location 5 .Drop these areas for further consideration. Thus updated table is given below

Table-2.27

Areas	**Fire Station Potential Sites**		
	1	4	5
6	- -	160 8	50 6
7	150 7	210 4	- -

Step VI: In Table 2.27, it can be observed that, allocation for location 7 to be made in the least cost cell (7, 1) i.e. area 7 is allocated to location 1.

Table-2.28

Areas	Fire Station Potential Sites		
	1	4	5
6	- -	160 8	50 6
7	150 7 ⇐	210 4	- -

Hence area 7 is allocated to a unique location 1. Drop this area for further consideration. Thus updated table is given below

. **Table-2.29**

Areas	Fire Station Potential Sites		
	1	**4**	**5**
6	- -	160 8	50 6

Step VII: The last area is to be allocated following the least cost rule and not on the basis of cost penalty. The area will be allocated to the least cost cell of the available locations, i.e. the potential location with available capacity. Thus allocation is made in the cell (6, 5), i.e. area 6 is allocated to the location 5.

Step VII: After applying whole procedure, 2^{nd} efficient solution has been obtained. Thus allocation made for 2^{nd} Efficient Solution are given in Table 2.30.

Table 2.30: Allocation of the areas to the locations 1, 4, 5

Areas	**Fire Station Potential Sites**
1	1
2	4
3	5
4	4
5	5
6	5
7	1

.Thus the 2^{nd} **Efficient Solution** has been obtained wherein the cost and time to cover all areas from the selected potential location is given in the Table 2.31.

Table-2.31: 2^{nd} Efficient Solution

Cost to cover all areas	**390 units**
Time to cover all areas	**8 units**
Total set up Cost	**8,00,000**

2.4.3 Procedure to Obtained 3^{rd} Efficient Solution of the Numerical Problem

The 3^{rd} Efficient Solution of the problem for the same combination of the selected potential locations i.e. 1,4 and 5 is obtained by dropping the cell (i, j) wherein the time $t_{ij} \geq T(\overline{X}^{(1)})$ =8 units. The updated table of the problem is given in Table 2.32.

Table-2.32: Table for 3^{rd} Efficient Solution

Areas	Fire Station Potential Sites				
	1	2	3	4	5
1	10 2	70 6	80 8	- -	- -
2	- -	80 3	20 9	10 6	- -
3	- -	130 9	200 3	- -	- -
4	120 6	130 7	70 8	80 4	- -
5	- -	170 8	140 11	- -	40 2
6	- -	140 10	130 12	- -	50 6
7	150 7	20 11	60 10	210 4	- -

From Table 2.32, it can be observed that 3rd efficient solution for the location 1, 4 and 5, to the numerical problem is not possible since 3 cannot be allocated to any of the location. Thus two efficient solutions obtained using the combination 1, 4 and 5 of the proposed problem is given below in Table 2.33.

Table 2.33: Efficient Solutions obtained with Locations 1, 4 and 5

	Cost	**Duration**	**Setup Cost**
1^{st} **Efficient Solution**	380	9	8,00,000
2^{nd} **Efficient Solution**	390	8	8,00,000

There are in total ten numbers of combinations the remaining are considered on the basis of the least setup cost of the selected sites. Thereafter the same procedure that is applied in above Step I to VI given in section (2.3) is followed to generate a set of efficient solutions of the problem. So the next selected fire station location is 1, 3 and 4 with next least setup cost 11, 00,000 and the efficient solutions are given in Table 2.34.

Table 2.34: Efficient Solutions obtained with Locations 1, 3 and 4

	Cost	**Duration**	**Setup Cost**
1st **Efficient Solution**	410	12	11,00,000
2nd **Efficient Solution**	470	11	11,00,000
3rd **Efficient Solution**	590	10	11,00,000
4th **Efficient Solution**	670	9	11,00,000

After applying the same procedure on the remaining combination of the potential fire station locations, the possible efficient solutions are given in Tables 2.35 – 2.42.

Table 2.35: Efficient Solutions obtained with Locations 1, 3 and 5

	Cost	**Duration**	**Setup Cost**
1st **Efficient Solution**	350	10	12,00,000
2nd **Efficient Solution**	390	9	12,00,000
3rd **Efficient Solution**	400	8	12,00,000

Table 2.36: Efficient Solutions obtained with Locations 1, 2 and 4

	Cost	**Duration**	**Setup Cost**
1st **Efficient Solution**	400	11	12,00,000
2nd **Efficient Solution**	580	10	12,00,000

3rd **Efficient Solution**	610	19	12,00,000

Table 2.37: Efficient Solutions obtained with Locations 1, 2 and 5

	Cost	**Duration**	**Setup Cost**
1st **Efficient Solution**	410	11	13,00,000
2nd **Efficient Solution**	580	8	13,00,000

Table 2.38: Efficient Solutions obtained with Locations 3, 4 and 5

	Cost	**Duration**	**Setup Cost**
1st **Efficient Solution**	320	11	14,00,000
2nd **Efficient Solution**	460	10	14,00,000
3rd **Efficient Solution**	480	8	14,00,000

Table 2.39: Efficient Solutions obtained with Locations 2, 4 and 5

	Cost	**Duration**	**Setup Cost**
1st **Efficient Solution**	280	11	15,00,000
2nd **Efficient Solution**	480	8	15,00,000

Table 2.40: Efficient Solutions obtained with Locations 1, 2 and 3

	Cost	**Duration**	**Setup Cost**
1st **Efficient Solution**	510	11	16,00,000
2nd **Efficient Solution**	720	10	16,00,000
3rd **Efficient Solution**	750	9	16,00,000

Table 2.41: Efficient Solutions obtained with Locations 2, 3 and 4

	Cost	**Duration**	**Setup Cost**
1^{st} **Efficient Solution**	410	13	18,00,000
2^{nd} **Efficient Solution**	470	12	18,00,000
3^{rd} **Efficient Solution**	500	11	18,00,000
4^{th} **Efficient Solution**	530	10	18,00,000
5^{th} **Efficient Solution**	690	9	18,00,000
6^{th} **Efficient Solution**	890	8	18,00,000

Table 2.42: Efficient Solutions obtained with Locations 2, 3 and 5

	Cost	**Duration**	**Setup Cost**
1^{st} **Efficient Solution**	290	11	19,00,000
2^{nd} **Efficient Solution**	330	10	19,00,000

2.5 Set of Efficient Solution of the Numerical Problem:

After comparing all the efficient solutions shown in Table 2.33-2.42 for all the combinations of the potential fire station locations, the final efficient solutions in the increasing order of cost of operation to cover the areas from the selected sites, is given below in Table 2.43.

Table 2.43 Set of Efficient Solutions of the Numerical Problem

	Cost	**Duration**	**Combination of Locations**	**Setup Cost**
1^{st} **Efficient Solution**	280	11	2, 4, 5	15,00,000
2^{nd} **Efficient Solution**	330	10	2, 3, 5	19,00,000
3^{rd} **Efficient Solution**	380	9	1, 4, 5	8,00,000
4^{th} **Efficient Solution**	390	8	1, 3, 5	11,00,000

2.5 Conclusion

In this chapter, a real-life problem of locating a fixed number of fire stations at potential location sites, and allocating the areas to them, has been modeled as a biobjective problem. A heuristic procedure based on the cost penalties and the adaptive search techniques has been developed to solve the problem. Application of the heuristic procedure yields four solutions are shown in Table 2.43. The choice of the final solution depends on the priorities of the decision maker which, in this case, may be the civic administration and the priorities of the civic administrations may differ depending upon their respective budget.

CHAPTER 3

AN EFFICIENT HEURESTIC ALGORITHM FOR MULTIOBJECTIVE FIRE STATION LOCATION PROBLEM

3.1 AN EFFICIENT HEURESTIC ALGORITHM FOR MULTIOBJECTIVE FIRE STATION LOCATION PROBLEM

In this chapter the multi-objective fire station location problem (2.1 to 2.7) has been considered and a procedure is proposed to find the set of efficient solutions. The prioritized bi-criterion problem is converted into an equivalent single objective problem by using preemptive priority factors given by Prakash et. al. (2008) and then a heuristic algorithm is proposed to find the set of efficient solutions of this Problem.

3.1.1 Procedure to Convert Bi-Objective Problem into an Equivalent Single Objective Problem

There are many approaches to solve for solving multi-objective optimization problem whereas there is universally acceptable approach seeking to optimize the single objective function.
The solution of the prioritized bi-criterion problem is the solution with minimization of C Provided by Eq. (2.1) as first priority objective and that of T provided by Eq. (2.2) as second priority objective. So for obtaining the efficient solution, the prioritized bi-criteria problem with prioritized objectives is solved. For this purpose, we reduce it to an equivalent single –objective problem.

From ready reference, we explain the procedure provided by Parakash et. al. (2008) to convert the prioritized bi-criterion problem (2.2) into an equivalent single –objective problem. This is accomplished as follow. First, the $\{t_{ij} : i = 1,2,..., M,\ j = 1,2,..., N\}$ is petitioned into subset $L_k\ (k = 1,..., q)$ in the following way. Each of the subset L_k's consists of the t_{ij}'s having the same numerical value. L_1 consist of the t_{ij}'s having the largest numerical value, L_2 consist of the t_{ij}'s having the next largest numerical value, and so on.. Finally, L_q consist of the t_{ij}'s having the smallest numerical value. After this preemptive priority factor $M_0, M_1, ..., M_q$ are assigned to C provided by Eq. (2.1), $\sum_{L_1} x_{ij}, ..., \sum_{L_q} x_{ij}$ respectively. Here $\sum_{L_k} x_{ij}$ is the sum of x_{ij}'s corresponding to t_{ij}'s belonging to L_k. All the priority factors M_k's are fixed positive real numbers and such that the expression $\sum_{k=0}^{q} \alpha_k M_k$ where α_k's are real numbers which can be negative or zero or positive has the same sign as the nonzero α_k with the smallest subscript in it irrespective of the values of the other α_k's. This implies that $M_0, M_1, ..., M_q$ the such that $M_0 >> M_1 >> ... >> M_q$. The symbol >> indicates that quantity on left hand side is arbitrary large compared to that on right hand side. Having done this, the prioritized bi-criteria problem with minimization of C as first priority objective and that of T as second priority objective is reduced to an equivalent single –objective problem seeking to determine x_{ij}'s which minimize

$$Z = M_0 \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij} x_{ij} + \sum_{k=1}^{q} M_k \sum_{L_k} x_{ij} \tag{3.1}$$

Subject to given constraints.

Table 3.1: Tableau representation of the problem given by Eq. (3.1)

Areas (A_i)	Fire Station Potential Sites (S_j)				
	S_1	S_2	$\cdots$	$\cdots$	S_N
A_1	$c_{11}M_0 + M_k$	$c_{11}M_0 + M_k$	$\cdots$	$\cdots$	$c_{11}M_0 + M_k$
A_2	$c_{11}M_0 + M_k$	$c_{11}M_0 + M_k$	$\cdots$	$\cdots$	$c_{11}M_0 + M_k$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\cdots$	$\vdots$
A_M	$c_{11}M_0 + M_k$	$c_{11}M_0 + M_k$	$\cdots$	$\cdots$	$c_{11}M_0 + M_k$
Setup Cost (C_j)	C_1	C_2	$\cdots$	$\cdots$	(C_N)

3.1.2 Procedure to Obtained 1^{st} Efficient Solution

Note that the procedure to obtain the 1^{st} efficient solution $(\overline{X}^{(1)}, \overline{Y}^{(1)})$ of the problem formulated above given by Eq. (3.1) is given below.

Step I: Identify all the ${}^{N}C_{K}$ combinations of the potential locations.

Step II: Rank the combinations in accordance with their total setup cost. The least setup cost given the first rank.

Step III: Starting from first ranked combination of the potential fire station locations and then obtain row reduce matrix (the matrix which contains at least one zero in each row) of the selected potential locations.

Step IV: In the row reduce matrix, if the number of rows that contain zero corresponding to selected potential sites, are greater than the available capacity of these potential sites, then there is a need of blockage. Block that cell for which cost difference of this site with other selected sites is minimum to cover that area. And again find a row reduce matrix.

Step V: Calculate cost penalty corresponding to each row (corresponding to those cells which contains zero) i.e. positive difference between the least cost corresponding to rows and column.

S**tep VI:** Select that cell for which penalty is largest and make an allocation to that cell. In the case of tie of the largest penalty, then it is an arbitrary choice for decision maker.

Step VII: If the area is allocated to a unique potential site, drop it for further consideration.

Step VIII: Repeat the same procedure till all areas have been allocated to the selected potential locations keeping in view their capacities. When all the areas have been allocated to unique selected potential location, the first efficient solution $(\overline{X}^{(1)}, \overline{Y}^{(1)})$ has been found.

3.1.3 Procedure to Obtain 2^{nd} and Subsequent Efficient Solutions

The 2^{nd} efficient solution $(\overline{X}^{(2)}, \overline{Y}^{(2)})$ of the formulated problem is obtained by solving the problem obtained from the 1^{st} proposed problem given by Table (3.1), after dropping all those cells (i, j) corresponding to $t_{ij} \geq T(\overline{X}^{(1)})$. The problem, thus obtained, is similar to the $1^{st,}$ proposed problem. The 2^{nd} problem is solved by adopting the method as used for solving $1^{st,}$

proposed problem. The 3^{rd} and subsequent efficient solutions are obtained in the same way as done to obtained the 2^{nd} efficient solution.

3.2. NUMERICAL PROBLEM

Table 3.2: Tableau representation of the problem

Areas	Fire Station Potential Sites				
	1	2	3	4	5
1	10 2	70 6	80 8	60 11	90 10
2	30 8	80 3	20 9	10 6	100 13
3	50 11	130 9	200 3	70 13	20 8
4	120 6	130 7	70 8	80 4	60 12
5	180 10	170 8	140 11	30 9	40 2
6	170 9	140 10	130 12	160 8	50 6
7	150 7	20 11	60 10	210 4	220 14
Setup Cost (C_{ij})	1,00,000	8,00,000	7,00,000	3,00,000	4,00,000

Now starts the procedure explained in section (3.1)by applying it to obtained the set of efficient solutions of the numerical problem presented in Table 3.2 considering $M = 7, N = 5, K = L = 3$ and assigning numerical values to all the $c_{ij}s, t_{ij}s, C_j s$ in the problem formulated in section 2.2. In Table 3.2, rows 1-7 correspond to areas and columns 1-5 correspond to potential fire station locations. Upper and lower entries of a cell (i, j) depicts the units of costs c_{ij} and time t_{ij} respectively of covering area i from the fire station location j.

Now apply the procedure explained in section (3.1.1) to obtain an equivalent single objective problem of the Numerical Problem given by the following equation.

$$
\begin{aligned}
Z = M_0(&40x_{11} + 70x_{12} + 80x_{13} + 60x_{14} + 90x_{15} + 30x_{21} + 80x_{22} + 20x_{23} + 10x_{24} + 100x_{25} + 50x_{31} \\
&+ 130x_{32} + 200x_{33} + 70x_{34} + 20x_{35} + 120x_{41} + 130x_{42} + 70x_{43} + 80x_{44} + 60x_{45} + 180x_{51} \\
&+ 170x_{52} + 140x_{53} + 30x_{54} + 40x_{55} + 170x_{61} + 140x_{62} + 130x_{63} + 160x_{64} + 50x_{65} + 150x_{71} \\
&+ 20x_{72} + 60x_{73} + 210x_{74} + 220x_{75}) + M_1\{x_{75}\} + M_2\{x_{25}, x_{34}\} + M_3\{x_{45}, x_{63}\} + \\
&M_4\{x_{14}, x_{31}, x_{53}, x_{72}\} + M_5\{x_{15}, x_{51}, x_{62}, x_{73}\} + M_6\{x_{23}, x_{32}, x_{54}, x_{61}\} + \\
&M_7\{x_{13}, x_{21}, x_{45}, x_{52}, x_{64}, x_{35}\} + M_8\{x_{42}, x_{71}\} + M_9\{x_{12}, x_{24}, x_{41}, x_{65}\} + M_{10}\{x_{44}, x_{74}\} + \\
&M_{11}\{x_{22}, x_{33}\} + M_{12}\{x_{11}, x_{55}\}
\end{aligned}
$$

Table3.3: Tableau representation of the problem

Areas	Fire Station Potential Sites				
	1	2	3	4	5
1	$40M_0 + M_{12}$	$70M_0 + M_9$	$80M_0 + M_7$	$60M_0 + M_4$	$90M_0 + M_5$
2	$30M_0 + M_7$	$80M_0 + M_{11}$	$20M_0 + M_6$	$10M_0 + M_9$	$100M_0 + M_2$
3	$50M_0 + M_4$	$130M_0 + M_6$	$200M_0 + M_{11}$	$70M_0 + M_2$	$20M_0 + M_7$
4	$120M_0 + M_9$	$130M_0 + M_8$	$70M_0 + M_7$	$80M_0 + M_{10}$	$60M_0 + M_3$
5	$180M_0 + M_5$	$170M_0 + M_7$	$140M_0 + M_4$	$30M_0 + M_6$	$40M_0 + M_{12}$
6	$170M_0 + M_6$	$140M_0 + M_5$	$130M_0 + M_3$	$160M_0 + M_7$	$50M_0 + M_9$
7	$150M_0 + M_8$	$20M_0 + M_4$	$60M_0 + M_5$	$210M_0 + M_{10}$	$220M_0 + M_1$
Setup Cost (C_{ij})	1,00,000	8,00,000	7,00,000	3,00,000	4,00,000

Now apply the procedure provided in section (3.1.2) and in section (3.1.3) to obtain the efficient solution of the problem obtained in Table 3.3.

3.2.1: Procedure to Obtain the 1^{st} Efficient Solution of the Numerical Problem

Step I: Since three fire stations are to be located, thus the combinations of the locations will be ${}^{5}C_{3}$=10 in numbers.

Step II: Start the procedure by considering the combination that has least setup cost. In the numerical problem being considered here, the combination of 3 potential fire station locations with the least setup cost is 1, 4 and 5 with a total set up cost as 8, 00,000 units. Thus three selected sites are given below.

Table-3.4

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	$40M_0 + M_{12}$	$60M_0 + M_4$	$90M_0 + M_5$
2	$30M_0 + M_7$	$10M_0 + M_9$	$100M_0 + M_2$
3	$50M_0 + M_4$	$70M_0 + M_2$	$20M_0 + M_7$
4	$120M_0 + M_9$	$80M_0 + M_{10}$	$60M_0 + M_3$
5	$180M_0 + M_5$	$30M_0 + M_6$	$40M_0 + M_{12}$
6	$170M_0 + M_6$	$160M_0 + M_7$	$50M_0 + M_9$
7	$150M_0 + M_8$	$210M_0 + M_{10}$	$220M_0 + M_1$
Setup Cost (C_{ij})	1,00,000	3,00,000	4,00,000

Step III: Obtain a Row reduce matrix (i.e. every row contains at least one zero) of is given matrix by Table 3.4 with 3 fire station locations. Thus Row reduce matrix is given below.

Table -3.5

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	0	$20M_0 + M_4 - M_{12}$	$50M_0 + M_5 - M_{12}$
2	$20M_0 + M_7 - M_9$	0	$90M_0 + M_2 - M_9$
3	$30M_0 + M_4 - M_7$	$50M_0 + M_2 - M_7$	0
4	$60M_0 + M_9 - M_3$	$20M_0 + M_{10} - M_3$	0
5	$150M_0 + M_5 - M_6$	0	$10M_0 + M_{12} - M_6$
6	$120M_0 + M_6 - M_9$	$110M_0 + M_7 - M_9$	0
7	0	$60M_0 + M_{10} - M_8$	$70M_0 + M_1 - M_8$

Step IV: Since number of rows that contain zero corresponding to each site is not greater than the capacity of these fire station sites, there is no need to any blockage.

Step V: Calculate cost penalties (i.e. corresponding to those cells which contains zero) i.e. positive difference between the least cost corresponding to rows and columns. In this case penalties corresponding to the cell (1,1), (2, 4), (3,5), (4,5), (5,4), (6,5), (7,1) are $(20M_0 + M_4 - M_{12})$,$(20M_0 + M_7 - M_9)$,$(30M_0 + M_4 - M_7)$, $(20M_0 + M_{10} - M_3)$, $(10M_0 + M_{12} - M_6)$, $(110M_0 + M_7 - M_9)$,$(60M_0 + M_{10} - M_8)$ respectively.

Step VI: Since largest penalty is corresponding to cell (6, 5) i.e. area 6 is allocated to unique location 5. Make an allocation to this cell. i.e. (6.5).

Step VII: Drop this area 6 because it is allocated to a unique site, and thus updated table after dropping this area is given below.

Table -3.6

Areas	Fire Station Potential Sites		
	1	**4**	**5**
1	0	$20M_0 + M_4 - M_{12}$	$50M_0 + M_5 - M_{12}$
2	$20M_0 + M_7 - M_9$	0	$90M_0 + M_2 - M_9$
3	$30M_0 + M_4 - M_7$	$50M_0 + M_2 - M_7$	0
4	$60M_0 + M_9 - M_3$	$20M_0 + M_{10} - M_3$	0
5	$150M_0 + M_5 - M_6$	0	$10M_0 + M_{12} - M_6$
7	0	$60M_0 + M_{10} - M_8$	$70M_0 + M_1 - M_8$

Repeat the procedure i.e. (step 3-step 7) given in section (3.1.2) until all the areas have been allocated to a unique site.

Thus in this case i.e. in Table (3.6), penalties corresponding to the cell (1,1), (2,4), (3,5),(4,5), (5,4),(7,1)are($20M_0 + M_4 - M_{12}$),($20M_0 + M_7 - M_9$),($30M_0 + M_4 - M_7$),($20M_0 + M_{10} - M_3$), ($10M_0 + M_{12} - M_6$), ($60M_0 + M_{10} - M_8$) respectively. Since largest penalty is corresponding to the cell (7, 1). Make an allocation to cell (7, 1).Drop the area 7 because it is allocated to site 1. New updated table is given below.

Table-3.7

Areas	Fire Station Potential Sites		
	1	**4**	**5**
1	0	$20M_0 + M_4 - M_{12}$	$50M_0 + M_5 - M_{12}$
2	$20M_0 + M_7 - M_9$	0	$90M_0 + M_2 - M_9$
3	$30M_0 + M_4 - M_7$	$50M_0 + M_2 - M_7$	0
4	$60M_0 + M_9 - M_3$	$20M_0 + M_{10} - M_3$	0
5	$150M_0 + M_5 - M_6$	0	$10M_0 + M_{12} - M_6$

In this case, calculated penalties are $(M_1 + M_{12} - M_9 - M_{12})$, $(20M_0 + M_7 - M_9)$, $(30M_0 + M_4 - M_7)$, $(20M_0 + M_{10} - M_3)$, $(10M_0 + M_{12} - M_6)$, corresponding to cells cell (1,1), (2,4), (3,5),(4,5), (5,4), respectively. Since largest penalty is corresponding to the cell (3, 5). Make an allocation to cell (3, 5).Drop the area 3 because it is allocated to site 5. New updated table is given below.

Table -3.8

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	0	$20M_0 + M_4 - M_{12}$	$50M_0 + M_5 - M_{12}$
2	$20M_0 + M_7 - M_9$	0	$90M_0 + M_2 - M_9$
4	$60M_0 + M_9 - M_3$	$20M_0 + M_{10} - M_3$	0
5	$150M_0 + M_5 - M_6$	0	$10M_0 + M_{12} - M_6$

From this Table (3.8), penalties corresponding to the cell (1, 1), (2, 4), (4, 5), (5, 4), are $(M_1 + M_{12} - M_9 - M_{12})$,$(20M_0 + M_7 - M_9)$,$(10M_1 + M_6 + M_{10} - M_3 - M_{12})$, $10M_0 + M_{12} - M_6$), respectively. Since largest penalty is corresponding to the cell (2, 4). Make an allocation to cell (2, 4).Drop the area 2 because it is allocated to site 4. New updated table is given below in table 3.9.

Table -3.9

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	0	$20M_0 + M_4 - M_{12}$	$50M_0 + M_5 - M_{12}$
4	$60M_0 + M_9 - M_3$	$20M_0 + M_{10} - M_3$	0
5	$150M_0 + M_5 - M_6$	0	$10M_0 + M_{12} - M_6$

From Table (3.9), penalties corresponding to the cell (1, 1), (4, 5), (5, 4),are $(40M_0 + M_{12} + M_9 - M_4 - M_3)$,$(10M_0 + M_6 + M_{10} - M_3 - M_{12})$,$10M_0 + M_6 + M_{10} - M_3 - M_{12}$ respectively. Since largest penalty is corresponding to the cell (1, 1). Make an allocation to cell (1, 1).Drop the area 1 because it is allocated to site 1. New updated table is given below.

Table -3.10

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
4	$60M_0 + M_9 - M_3$	$20M_0 + M_{10} - M_3$	0
5	$150M_0 + M_5 - M_6$	0	$10M_0 + M_{12} - M_6$

Penalties in this case corresponding to the cell (4, 5), (5, 4), are $(10M_0 + M_6 + M_{10} - M_3 - M_{12})$, $(10M_0 + M_6 + M_{10} - M_3 - M_{12})$, respectively. Since penalties corresponding to the cell (4, 5), (5,4) are same we can choose any of the cell i.e. take cell (5, 4). Make an allocation to cell (5, 4). Drop the area 5 because it is allocated to site 4. New updated table is given below in Table (3.11).

Table- 3.11

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
4	$60M_0 + M_9 - M_3$	$20M_0 + M_{10} - M_3$	0

Now we simply make an allocation to the cell (4, 5). Since all the areas have been allocated to unique site, so allocation of the areas to the selected site for the 1^{st} efficient solution is given below.

Table 3.12: Allocation of the areas to the locations 1, 4, 5

Areas	**Fire Station Potential Sites**
1	1
2	4
3	5
4	5
5	4
6	5
7	1

Now all the areas have been clustered to unique a site. Thus the 1^{st} **Efficient Solution** has been obtained wherein the cost and time to cover all areas from the selected potential location is given in the Table 3.13.

Table 3.13: 1^{st} Efficient Solution

Cost to cover all areas	**360 units**
Time to cover all areas	**12 units**
Total set up Cost	**8,00,000**

3.2.2 2^{nd} Efficient Solution of the Numerical Problem:

The second efficient solution of the problem for the same combination of the selected potential locations i.e. 1, 4 and 5 is obtained by dropping the cell (i, j) wherein the time $t_{ij} \geq T(\overline{X}^{(1)})=12$ units. Thus updated table of the problem is given in Table 3.14.

Table- 3.14

Areas	Fire Station Potential Sites		
	1	**4**	**5**
1	$40M_0 + M_{12}$	$60M_0 + M_4$	$90M_0 + M_5$
2	$30M_0 + M_7$	$10M_0 + M_9$	–
3	$50M_0 + M_4$	–	$20M_0 + M_7$
4	$120M_0 + M_9$	$80M_0 + M_{10}$	–
5	$180M_0 + M_5$	$30M_0 + M_6$	$40M_0 + M_{12}$
6	$170M_0 + M_6$	$160M_0 + M_7$	$50M_0 + M_9$
7	$150M_0 + M_8$	$210M_0 + M_{10}$	–
Setup Cost (C_{ij})	1,00,000	3,00,000	4,00,000

Applying the same procedure i.e. (step 3-step 7) given in section (3.1.2) until all the areas have been allocated to a unique site, to the problem provided in Table 3.14. After applying same procedure as we have done to find the 1^{st} efficient solution, we will get the 2^{nd} efficient solution for which the variable $x_{ij}'s$ at level 1 are x_{11}, x_{24}, x_{35}, x_{44}, x_{54}, x_{65}, x_{71}; the total cost and duration to cover all the areas from the selected sites $C(\bar{x}^{(2)})$ = 40+10+20+80+30+50+150= 380 and $T(\bar{x}^{(2)})$ =max{2,6,8,4,9,6,7}= 9.unit respectively. Thus the 2^{nd} Efficient Solution of the problem is given below in Table 3.16. Allocation of the areas to the selected location is given below in Table 3.15.

Table 3.15: Allocation of the areas to the locations 1, 4, 5

Areas	**Fire Station Potential Sites**
1	1
2	4
3	5
4	4
5	4
6	5
7	1

Table 3.16: 2^{nd} Efficient Solution

Cost to cover all areas	**380 units**
Time to cover all areas	**9 units**
Total set up Cost	**8,00,000**

3.2.3: 3^{rd} Efficient Solution of the Numerical Problem:

The 3^{rd} efficient solution of the problem for the same combination of the selected potential locations i.e. 1, 4 and 5 is obtained by dropping the cell (i, j) wherein the time $t_{ij} \geq T(\overline{X}^{(1)})$ =9. Thus updated table of the problem is given below.

Table 3.17

Areas	Fire Station Potential Sites		
	1	4	5
1	$40M_0 + M_{12}$	–	–
2	$30M_0 + M_7$	$10M_0 + M_9$	–
3	–	–	$20M_0 + M_7$
4	$120M_0 + M_9$	$80M_0 + M_{10}$	–
5	$180M_0 + M_5$	–	$40M_0 + M_{12}$
6	–	$160M_0 + M_7$	$50M_0 + M_9$
7	$150M_0 + M_8$	$210M_0 + M_{10}$	–
Setup Cost (C_{ij})	1,00,000	3,00,000	4,00,000

Applying the same procedure as have done to find the 1^{st} and 2^{nd} efficient solution**,** we will get the 3^{rd} efficient solution for which the variable x_{ij}'s at level 1 are x_{11}, x_{24}, x_{35}, x_{44}, x_{55}, x_{65}, x_{71}; the total cost and duration to cover all the areas from the selected sites are $C(\bar{x}^{(3)})$=40+10+20+80+40+50+150= 390 and $T(\bar{x}^{(3)})$ =max{2,6,8,4,2,6,7}= 8.unit respectively. Allocation of the areas to the selected location for 3^{rd} Efficient Solution is given below.

Table 3.18: Allocation of the areas to the locations 1, 4, 5

Areas	**Fire Station Potential Sites**
1	1
2	4
3	5
4	4
5	5
6	5
7	1

Table 3.19: 3^{rd} Efficient Solution

Cost to cover all areas	**390 units**
Time to cover all areas	**8 units**
Total set up Cost	**8,00,000**

3.2.3: 4^{th} Efficient Solution of the Numerical Problem:

The 4^{th} efficient solution of the problem for the same combination of the selected potential locations i.e. 1, 4 and 5 is obtained by dropping the cell (i, j) wherein the time $t_{ij} \geq T(\overline{X}^{(1)}) = 8$ Thus updated table of the problem for obtaining 4^{th} efficient solution is given below.

Table- 3.20

Areas	**Fire Station Potential Sites**		
	1	**4**	**5**
1	$40M_0 + M_{12}$	–	–
2	–	$10M_0 + M_9$	–
3	–	–	–
4	$120M_0 + M_9$	$80M_0 + M_{10}$	–
5	–	–	$40M_0 + M_{12}$
6	–	–	$50M_0 + M_9$
7	$150M_0 + M_8$	$210M_0 + M_{10}$	–
Setup Cost (C_{ij})	1,00,000	3,00,000	4,00,000

It can be observed from Table 3.20, that a solution to the numerical problem with the selected location sites 1, 4 and 5 with $t_{ij} < 8$ units is not possible since the area 3 cannot be allocated to any of the location. Thus considering the combination1, 4 and 5 of potential fire station sites, it is not possible to find 4^{th} efficient solution. Thus the three efficient solutions obtained using the combination 1, 4 and 5 of the potential fire station locations is given in Table 3.21

Table 3.21: Efficient Solutions obtained with Locations 1, 4 and 5

	Cost	**Duration**	**Setup Cost**
1^{st} **Efficient Solution**	360	12	8,00,000
2^{nd} **Efficient Solution**	380	9	8,00,000
3^{rd} **Efficient Solution**	390	8	8,00,000

Since there are total of ${}^5C_3$ combinations, in this case 10 in number, the remaining are considered on the basis of the least setup cost. Then the same procedure as applied above i.e. step (3-7) given in section (3.1.2-3.13) is followed to generate a set of efficient solutions for all the combinations of the problem.

For instance, the next combination of the potential fire station locations considered is location 1, 3 and 4, with next least set-up cost (11,00,000), and the efficient solution corresponding to these sites is given in Table 3.22.

Table 3.22: Efficient Solutions obtained with Locations 1, 3 and 4

	Cost	**Duration**	**Setup Cost**
1^{st} **Efficient Solution**	390	12	11,00,000
2^{nd} **Efficient Solution**	420	11	11,00,000
3^{rd} **Efficient Solution**	570	10	11,00,000

Similarly, the remaining combinations of the potential fire station sites are considered and after applying the same procedure, the possible efficient solutions for other remaining combinations are given in table (3.23 – 3.30).

Table 3.23: Efficient Solutions obtained with Locations 1, 3 and 5

	Cost	**Duration**	**Setup Cost**
1^{st} **Efficient Solution**	300	10	12,00,000
2^{nd} **Efficient Solution**	390	9	12,00,000
3^{rd} **Efficient Solution**	400	8	12,00,000

Table 3.24: Efficient Solutions obtained with Locations 1, 2 and 4

	Cost	**Duration**	**Setup Cost**
1^{st} **Efficient Solution**	370	11	12,00,000
2^{nd} **Efficient Solution**	580	10	12,00,000
3^{rd} **Efficient Solution**	610	9	12,00,000

Table 3.25: Efficient Solutions obtained with Locations 1, 2 and 5

	Cost	**Duration**	**Setup Cost**
1^{st} **Efficient Solution**	290	12	13,00,000
2^{nd} **Efficient Solution**	320	11	13,00,000
3^{rd} **Efficient Solution**	460	8	13,00,000

Table 3.26: Efficient Solutions obtained with Locations 3, 4 and 5

	Cost	Duration	Setup Cost
1st **Efficient Solution**	290	12	14,00,000
2nd **Efficient Solution**	300	11	14,00,000
3rd **Efficient Solution**	320	10	14,00,000
4th **Efficient Solution**	470	9	14,00,000
5th **Efficient Solution**	480	8	14,00,000

Table 3.27: Efficient Solutions obtained with Locations 2, 4 and 5

	Cost	Duration	Setup Cost
1st **Efficient Solution**	250	12	15,00,000
2nd **Efficient Solution**	280	11	15,00,000
3rd **Efficient Solution**	480	8	15,00,000

Table 3.28: Efficient Solutions obtained with Locations 1, 2 and 3

	Cost	Duration	Setup Cost
1st **Efficient Solution**	480	11	16,00,000
2nd **Efficient Solution**	630	10	16,00,000
3rd **Efficient Solution**	750	9	16,00,000

Table 3.29: Efficient Solutions obtained with Locations 2, 3 and 4

	Cost	Duration	Setup Cost
1st Efficient Solution	400	13	18,00,000
2nd Efficient Solution	450	12	18,00,000
3rd Efficient Solution	460	11	18,00,000
4th Efficient Solution	510	10	18,00,000
5th Efficient Solution	690	9	18,00,000
6th Efficient Solution	890	8	18,00,000

Table 3.30: Efficient Solutions obtained with Locations 2, 3 and 5

	Cost	Duration	Setup Cost
1st Efficient Solution	290	11	19,00,000
2nd Efficient Solution	330	10	19,00,000

3.3: Set of Efficient Solution of the Numerical Problem:

After comparing all the efficient solutions shown in Table 3.21-3.30 for all the combinations of the potential fire station locations, the final efficient solutions in the increasing order of cost of operation to cover the areas from the selected sites, is given below in Table 3.31.

Table 3.31: Set of Efficient Solutions of the Numerical Problem

	Cost	Duration	Combination of Locations	Setup Cost
1st Efficient Solution	250	12	2, 4, 5	15,00,000
2nd Efficient Solution	280	11	2, 4, 5	15,00,000
3rd Efficient Solution	300	10	1, 3, 5	12,00,000
4th Efficient Solution	380	9	1, 4, 5	8,00,000
5th Efficient Solution	390	8	1, 4, 5	8,00,000

3.4 CONCLUSION

In this work, a real-life bi-criterion problem of locating a fixed number of fire station at a potential location sites, and allocating the areas to them is considered. The prioritized bi-criterion problem is converted into an equivalent single objective problem by using preemptive priority factors and then a heuristic algorithm is proposed to find the set of efficient solutions of this Problem. Application of the heuristic procedure given by Singh, Amrinder. (2010) yields four efficient solutions are shown in Table 2.43.However proposed algorithm, yields five efficient solutions are shown in Table 3.31, in which three efficient solutions are same and two solutions are better as obtained by Singh, Amrinder (2010).

The procedure applied here is dynamic in nature. It can be used to other real-life problems of locating the ambulances in the cities and on the highways, police patrol vehicles and other emergency services that are related to public sector.

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