A THESIS

On

Temperature Dependence of Nuclear Potentials and its Relevance

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Dedicated to:

God who gave me life

And

My Lovable Parents who made it worth living

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CONTENTS

List of figures

Abstract

In reference to the recent developments in the field of nuclear structure and reaction dynamics in low energy range, it has been observed that a large number of experiments are performed at the extreme conditions of temperature, energy, angular momentum etc.. Many such experiments are being planned in order to establish the dynamical behaviour of exotic nuclear systems. In order to complement these advancements in nuclear experimentations, it becomes essential and important to develop the theoretical aspects of related phenomena. One of the major requirement in this front is the availability of realistic nuclear potentials and their proper and accurate temperature dependence. In order to meet this challenging task the nuclear theory group at Thapar University Patiala and Punjab University Chandigarh, have done a very useful effort to establish temperature dependence on nuclear potential. In this effort the bulk constant α(0) and proton-neutron asymmetry constant a_a of Seeger et.al. are refitted at T=0 for all isotopes of periodic table upto Z=118. In this thesis, I have reviewed the nuclear mass formulae and their possible temperature dependence. For the purpose, a chronological development of nuclear potentials is presented in a systematic manner. The need and requirement of temperature dependence on these nuclear potentials is discussed in detail. An effort is made to find out the possible reasons behind refitting of selected set of Seeger's constants.

It is important to note that these refitted constants can be used in order to understand the dynamics of excited nuclear systems. Actually the aim of refitted these constants is simply to include the T-dependence on experimental binding energies, which could play a very crucial role in order to understand the dynamical behaviour of exotic nuclear systems. Beside this, these temperature dependent potentials can be utilized to make meaningful predictions for future activities of experimental nuclear physics.

CHAPTER -1

1. INTRODUCTION

Nuclear physics deals with the properties of nuclei along understanding of these nuclear properties in terms of the interactions between the constituents. The basic questions facing nuclear physics today range over the extremes of physical phenomena from the microscopic to the macroscopic discriptions. Thus the microscopic properties of nuclei have their origins in the basic interactions of elementary particles, quarks and gluons, while the macroscopic evolution of the universe is directly related to the interactions between nuclei.

The experimental techniques have improved with time and lot of experimental data related to different nuclear phenomena has become available. In the last about three decades, reactions induced by heavy ions have become the principal tool in nuclear physics research. It has led to phenomenological studies of atomic nuclei, which resulted in better and better understanding of related concepts. A significant amount of information is available regarding synthesis of new heavy and super heavy nuclei, study of the radioactive systems, chemical properties of new nuclei and their isotopes, production of nuclei far from the beta-stability line, formation and decay of exotic nuclear systems etc.

Presently a lot of work is being done on the dynamics of nuclear system at extreme composition, excitation energy, angular momentum, nuclear deformation, etc. In brief the study of nucleus at higher excitation energies carries lot of information, which in turn could be extremely useful for the better understanding of nuclear properties.

The fragmentation potential consists of the macroscopic liquid drop energy V_{LDM} besides coulomb potential, proximity potential and rotational energy. Therefore, in order to investigate the dynamics of a excited nuclear system, the knowledge of temperature dependence of macroscopic part of nuclear potential becomes essential. This temperature dependent liquid drop potential can be calculated as suggested by Davidson et. al. [1]. The predictions of this model find its basis in

the well-established semi-empirical mass formula of Seeger [2]. Although Seeger's formula was quite successful in explaining the available literature of its time (1961) but our knowledge regarding the understanding of nuclear properties has grown, consequently the constants used in this model need major revival in order to meet the present day requirements. So there is need to modify the mass formulas.

In order to achieve this tedious task of obtaining temperature dependent potentials, I plan to discuss different mass formulas and their modifications in chapter 2. However, before knowing about the mass formulae, we should know the relation between mass formula and binding energies of nucleus.

Binding energy is the mechanical energy required to disassemble a composite nuclear system into separate parts. A bound system has a lower potential energy than its constituent parts; this is what keeps the system together. The usual convention is that this corresponds to a positive binding energy.

In general, binding energy represents the mechanical work which must be done in acting against the forces which hold an object together, while disassembling the object into component parts separated by sufficient distance that the further separation requires negligible additional work.

At the atomic level, the atomic binding energy of the atom derives from electromagnetic interaction and is the energy required to disassemble an atom into free electrons and a nucleus. Electron binding energy is a measure of the energy required to free electrons from their atomic orbits. Nuclear binding energy is derived from the strong nuclear force and is the energy required to disassemble a nucleus into free unbound neutrons and protons, strictly so that the relative distances of the particles from each other are infinite (essentially far enough so that the strong nuclear force and electromagetic forces can no longer cause the particles to interact).

In astrophysics, gravitational binding energy of a celestial body is the energy required to disassemble it into space debris (dust and gas). This quantity is not to be confused with the gravitational potential energy, which is the energy required to separate two bodies, such as a celestial body and a satellite, to infinite

distance, keeping each intact (the latter energy is lower).In bound systems, if the binding energy is removed from the system, it must be subtracted from the mass of the unbound system, simply because this energy has mass, and if subtracted from the system at the time it is bound, will result in removal of mass from the system. System mass is not conserved in this process because the system is not closed during the binding process.

1.1 Mass defect:-

Because a bound system is at a lower energy level than its unbound constituents, its mass must be less than the total mass of its unbound constituents. For systems with low binding energies, this "lost" mass after binding may be fractionally small. For systems with high binding energies, however, the missing mass may be an easily measurable fraction.

Since all forms of energy have mass, the question of where the missing mass of the binding energy goes, is of interest. The answer is that this mass is lost from a system which is not closed. It transforms to heat, light, higher energy states of the nucleus/atom or other forms of energy, but these types of energy also have mass, and it is necessary that they be removed from the system before its mass may decrease. The "mass defect" from binding energy is therefore removed mass that corresponds with removed energy, according to Einstein's equation $E=mc^2$. Once the system cools to normal temperatures and returns to ground states in terms of energy levels, there is less mass remaining in the system than there was when it first combined and was at high energy. Mass measurements are almost always made at low temperatures with systems in ground states, and this difference between the mass of a system and the sum of the masses of its isolated parts is called a mass deficit. Thus, if binding energy mass is transformed into heat, the system must be cooled (the heat removed) before the mass-deficit appears in the cooled system. In that case, the removed heat represents exactly the mass "deficit", and the heat itself retains the mass which was lost.

As an illustration, consider two objects attracting each other in space through their gravitational field. The attraction force accelerates the objects and they gain some speed toward each other converting the potential (gravity) energy into kinetic (movement) energy. When either the particles 1) pass through each other without interaction or 2) elastically repel during the collision, the gained kinetic energy (related to speed), starts to revert into potential form driving the collided particles apart. The deacelerating particles will return to the initial distance or stop and repeat the collision (oscillation takes place). This shows that the system, which loses no energy, does not combine (bind) into a solid object, parts of which oscillate at short distances. Therefore, in order to bind the particles, the kinetic energy gained due to the attraction must be dissipated (by resistive force). Complex objects in collision ordinarily undergo inelastic collision, transforming some kinetic energy into internal energy (heat content, which is atomic movement), which is further radiated in the form of photons -- the light and heat. Once the energy to escape the gravity is dissipated in the collision, the parts will oscillate at closer, possibly atomic, distance, thus looking like one solid object. This lost energy, necessary to overcome the potential barrier in order to separate the objects, is the binding energy. If this binding energy were retained in the system as heat, its mass would not decrease. However, binding energy lost from the system (as heat radiation) would itself have mass, and directly represent of the "mass deficit" of the cold, bound system. Closely analogous considerations apply in chemical and nuclear considerations. Exothermic chemical reactions in closed systems do not change mass, but (in theory) become less massive once the heat of reaction is removed. This mass change is too small to measure with standard equipment. In nuclear reactions, however, the fraction of mass that may be removed as light or heat, i.e., binding energy, is often a much larger fraction of the system mass. It may thus be measured directly as a mass difference between rest masses of reactants and products. This is because nuclear forces are comparatively stronger than Coulombic forces associated with the interactions between electrons and protons that generate heat in chemical reactions.

In nuclear reactions, the energy that must be radiated or otherwise removed as binding energy may be in the form of electromagnetic waves, such as gamma radiation, or as heat. Again, however, no mass-deficit can appear until this radiation has been emitted and is no longer part of the system. The energy given off during either nuclear fusion or nuclear fission is the difference between the binding energies of the fuel and the fusion or fission products. In practice, this energy may also be calculated from the substantial mass differences between the fuel and products, once the evolved heat and radiation have been removed.

1.1.1Mass Defect calculations:-

It is observed experimentally that the mass of the nucleus is smaller than the sum of the masses of the constituent nucleons in free states. This difference is called mass defect.

The difference between the actual mass of the nucleus and the sum of masses of the constituent nucleons is called mass defect i.e.

Mass Defect, (Δm) = M - A

where, M = actual mass of the nucleus and A = mass number

When the nucleons are grouped together to form a nucleus, they lose a small amount mass i.e. There is mass defect. This mass defect is converted into energy according to the relation $E = mc^2$. This energy holds the nucleons together and is known as binding energy. In fact, mass defect is a measure of the binding energy of the nucleus. The greater the mass defect, the greater is the binding energy of the nucleus.

1.1.2 Illustration:

The deutron (i.e. an isotope of hydrogen) nucleus contains only one neutron and one proton. Let us compare the mass of deutron nucleus to that of its constituent nucleons.

Neutron Mass = 1.008665 a.m.u, Hydrogen Mass = 1.007825 a.m.u

Therefore, Neutron Mass + Hydrogen Mass = 2.016490 a.m.u

Deutron Mass = 2.014103 a.m.u

Mass Defect, (Δm) = 2.014103 – 2.016490 = -0.002387 a.m.u

Thus, the mass of deuterium nucleus is less than the sum of the constituent nucleons by 0.002387 a.m.u. This difference in the mass from the actual mass is called the mass defect. This difference in mass is not due to inaccurate measurements of masses but is a real difference to be accounted. When a neutron and a proton come together to form deutron, a small part of their mass, namely 0.002387 a.m.u (equivalent to 2.22 MeV of energy) is radiated from the newly formed nucleus. The two particles now have a binding energy of 2.22 MeV. In other words, in order to break the deutron nucleus into its constituent nucleons, an energy of 2.22 MeV is required.

The release of energy results in the stability of the nucleus. The energy released in the formation of a nucleus from its constituent nucleons is called the binding energy of the nucleus.

If ΔM a.m.u is the mass defect, then the binding energy of the nucleus in MeV will be given by ΔM x 931.494 MeV, because,

1 a.m.u = 931.494 MeV

This relation between mass units and energy units may be derived from $E = \Delta M c^2$ using the proper units and unit conversions. The greater the mass defect, the greater is the binding energy of the nucleus. The binding energy when divided by the number of nucleons gives the mean binding energy per nucleons. The binding energy per nucleons is a measure of the stability of the nucleus. The greater the binding energy per nucleon, the more stable is the nucleus.

1.2 Nucleur binding energy

Practice: Binding energy for atoms

The measured mass deficits of isotopes are always listed as mass deficits of the neutral atoms of that isotope, mostly in MeV. As a consequence, the listed mass deficits are not a measure for the stability or binding energy of isolated nuclei, but for the whole atoms. This has very practical reasons, because it is very hard to totally ionize heavy elements, i.e. it is always difficult to strip off all electrons.

This practice is useful for other reasons, too: Stripping all the electrons from a heavy unstable nucleus (thus producing a bare nucleus) will change the lifetime of the nucleus, indicating that the nucleus cannot be treated independently (Experiments at the heavy ion accelerator GSI). This is also evident from phenomena like electron capture. Theoretically, in orbital models of heavy atoms, the electron orbits partially inside the nucleus (it doesn't orbit in a strict sense, but has a non-vanishing probability of being located inside the nucleus).

Of course, a nuclear decay happens to the nucleus, meaning that properties ascribed to the nucleus will change in the event. But for the following considerations and examples, you should keep in mind that "mass deficit" as a measure for "binding energy", and as listed in nuclear data tables, means "mass deficit of the neutral atom" and is a measure for stability of the whole atom.

Specific quantitative example: a deuteron

A deuteron (the nucleus of a deuterium atom, with no electron) consists of one proton and one neutron.

The binding energy of deutron can be calculated as:-

0.002387 u × 931.494 MeV/u = 2.224 MeV.

Thus, expressed in another way, the binding energy is [0.002387/2.01649] x 100% = about 0.1184% of the total energy corresponding to the mass. This corresponds to 1.07×10^{14} J/kg = 107 TJ/kg. This energy must be removed in the form of a gamma ray for the system to de-excite and allow the deuteron to remain bound, and the system mass to decrease to the value of a deuteron.

Another example:-α particle

For the alpha particle Δm= 0.0304 u which gives a binding energy of 28.3 MeV.

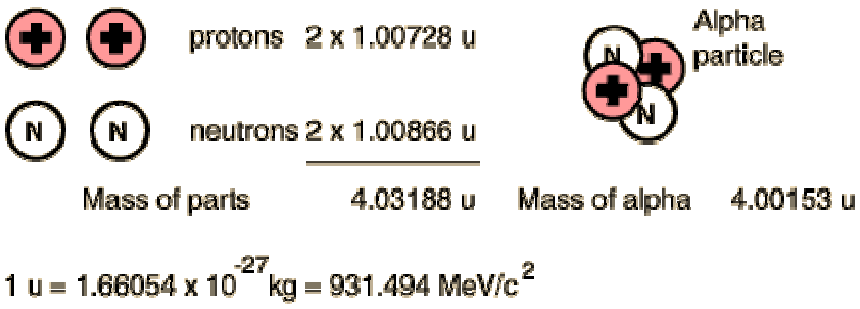


Figure. 1.1: α-particle ($^4He_2$ nucleus) representation

The enormity of the nuclear binding energy can perhaps be better appreciated by comparing it to the binding energy of an electron in an atom. The comparison of the alpha particle binding energy with the binding energy of the electron in a hydrogen atom is shown below. The nuclear binding energies are on the order of a million times greater than the electron binding energies of atoms.

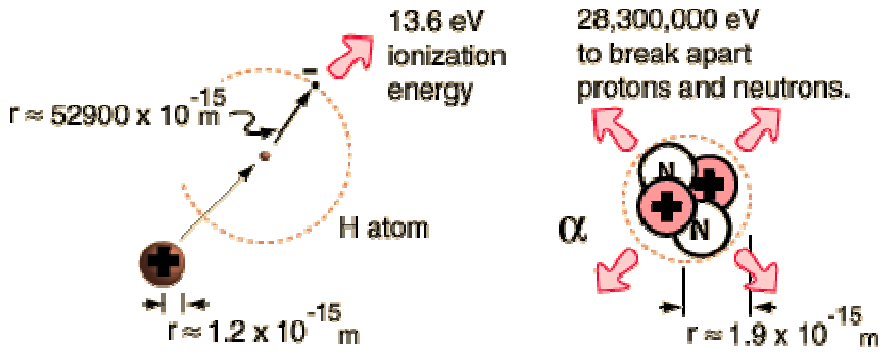


Figure 1.2 Comparison of atomic and nuclear scales and binding energy

Nuclear binding energy curve:-

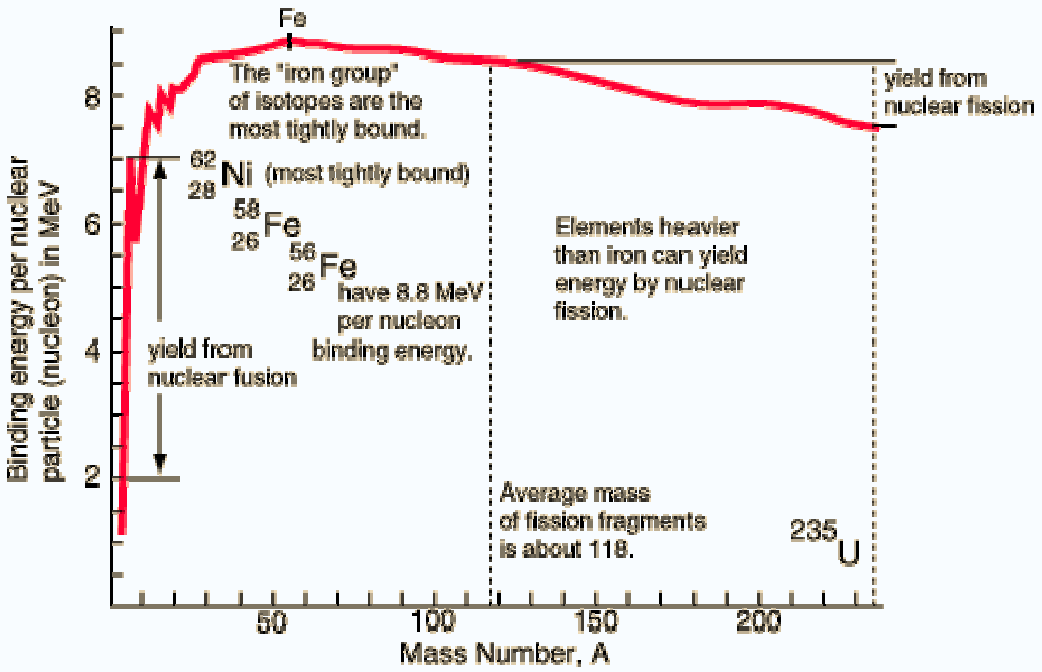


Figure 1.3 Binding energy per nucleon as a function of mass number.

In the periodic table of elements, the series of light elements from hydrogen up to sodium is observed to exhibit generally increasing binding energy per nucleon as the atomic mass increases. This increase is generated by increasing forces per nucleon in the nucleus, as each additional nucleon is attracted by all of the other nucleons, and thus more tightly bound to the average field in the nucleus.

The region of increasing binding energy is followed by a region of relative stability (saturation) in the sequence from magnesium through xenon. In this region, the nucleus has become large enough that nuclear forces no longer completely extend efficiently across its width. Attractive nuclear forces in this region, as atomic mass increases, are nearly balanced by repellent electromagnetic forces between protons, as atomic number increases. Finally, in elements heavier than xenon, there is a decrease in binding energy per nucleon as atomic number increases. In this region of nuclear size, electromagnetic repulsive forces are beginning to gain against the strong nuclear force.

At the peak of binding energy, nickel-62 is the most tightly-bound nucleus, followed by iron-58 and iron-56. (This is the basic reason why iron and nickel are very common metals in planetary cores, since they are produced profusely as end products in supernovae and in the final stages of silicon burning in Stars). Iron-56 is more common than nickel isotopes because its unstable progenitor nickel-56 is copiously made by staged build-up of 14 helium nuclei inside supernovas, and it then decays to iron-56 within a few weeks.

The existence of a maximum in binding energy in medium-sized nuclei is a consequence of the trade-off in the effects of two opposing forces which have different range characteristics. The attractive nuclear force (strong nuclear force), which binds protons and neutrons equally to each other, has a limited range due to a rapid exponential decrease in this force with distance. However, the repelling electromagnetic force, which acts between protons to force nuclei apart, falls off with distance much more slowly (as the inverse square of distance). For nuclei larger than about four nucleons in diameter, the additional repelling force of additional protons more than offsets any binding energy which results between further added nucleons as a result of additional strong force interactions; such nuclei become less and less tightly bound as their size increases, though most of them are still stable. Finally, nuclei containing more than 209 nucleons (larger than about 6 nucleons in diameter) are all too large to be stable, and are subject to spontaneous decay to smaller nuclei.

Nuclear fusion produces energy by combining the very lightest elements into more tightly-bound elements (such as hydrogen into helium), and nuclear fission produces energy by splitting the heaviest elements (such as uranium and plutonium) into more tightly-bound elements (such as barium and krypton). Both processes produce energy, because middle-sized nuclei are the most tightly bound of all.

1.3 Semiempirical formula:-

For a nucleus with A nucleons including Z protons, a semiempirical formula for the binding energy per nucleon (E/A) is:

$$E/A = a - b/\sqrt[3]{A} - c(Z/\sqrt[3]{A^2})^2 - d(N-Z)^2/\sqrt[3]{A^{3+3}} \pm e/\sqrt[4]{A^{3+4}}$$

where the binding energy is in MeV for the following numerical values of the constants: a = 14.0; b = 13.0; c = 0.585; d = 19.3; e = 33.

Most terms in this formula can be explained by the liquid drop model of the nucleus, which treats the nucleus as a drop of uniform, incompressible fluid, whose radius can be derived from its density.

- The first term $a$is called the saturation contribution and ensures that the binding energy (B.E.) per nucleon is the same for all nuclei, to a first approximation.
- The term $-b/A^{1/3}$is a surface tension effect and is proportional to the number of nucleons that are situated on the nuclear surface. It is largest for light nuclei.
- The term $-cZ^2/A^{4/3}$is the Coulomb electrostatic repulsion. This becomes more important as Z increases.
- The symmetry correction term $-d(N-Z)^2/A^2$takes into account Pauli's exclusion principle. In the absence of other effects the most stable

arrangement (i.e. one with lowest energy) has equal numbers of protons and neutrons.

- The pairing term $\pm e/A^{7/4}$ is + for even-even nuclei and − for odd-odd nuclei. This too is a result of Pauli's exclusion principle, together with the protons and neutrons having spin 1/2.

Table 1 The following table gives the binding energy per nucleon in MeV for selected isotopes.:-

	Formula	Measured
^{27}Al	8.42	8.33
^{63}Cu	8.75	8.75
^{98}Mo	8.62	8.63
^{195}Pt	7.87	7.92
^{238}U	7.56	7.58

1.3.1 Measuring the binding energy:-

As seen in the example of deuterium, nuclear binding energies are large enough that they may be easily measured as fractional mass deficits, according to the equivalence of mass and energy. The atomic binding energy is simply the amount of energy (and mass) released, when a collection of free nucleons are joined together to form a nucleus.

Nuclear binding energy can be easily computed from the easily measurable difference in mass of a nucleus, and the sum of the masses of the number of free neutrons and protons that make up the nucleus. Once this mass difference, called the mass defect or mass deficiency, is known, Einstein's mass-energy equivalence formula can be used to compute the binding energy of any nucleus.

(As a historical note, early nuclear physicists used to refer to computing this value as a "packing fraction" calculation.)

For example, the atomic mass unit (1 u) is defined to be 1/12 of the mass of a ^{12}C atom—but the atomic mass of a ^{1}H atom (which is a proton plus electron) is 1.007825 u, so each nucleon in ^{12}C has lost, on average, about 0.8% of its mass in the form of binding energy.

1.4 Nuclear units :-

Nuclear energies are very high compared to atomic processes, and need larger units

1electron volt= 1eV=1.6×10^{-19} joules

1MeV=10^6 eV, 1GeV=10^9 eV, 1TeV=10^{12} eV

However, the sizes are quite small and need smaller units:
Atomic sizes are on the order of $1A^0 = 10^{-10}$ m
Nuclear sizes are on the order of femtometers which in the nuclear context are usually called fermis: 1fm= 10^{-15} m.Nuclear masses are measured in terms of atomic mass units with the carbon-12 nucleus defined as having a mass of exactly 12 amu. It is also common practice to quote the rest mass energy $E=m_0c^2$ as if it were the mass. The conversion to amu is:

$$1\,u = 1.66054 \times 10^{-27}\,kg = 931.494\,MeV$$

In the third chapter we will discuss about the temperature dependent binding energies. The study of nucleus at higher excitation energies carries lot of information, which in turn could be extremely useful for the better understanding of nuclear properties at extreme conditions.

References

[1] N. J. Davidson, S. S. Hsiao, J. Markram, H. G. Miller and Y. Tzeng, Nucl. Phys. A 570, 61c (1994).

[2] P. A. Seeger. Nucl. Phys., 25, 1 (1961).

[3] Fewell, M. P. (1995). "The atomic nuclide with the highest mean binding energy". American Journal of Physics 63 (7): 653–658-doi:10.1119/1.17828. http://adsabs.harvard.edu/abs/1995AmJPh..63..653F.

[4] Binding energy- wikipedia the free encyclopedia.

CHAPTER-2

MUCLEAR MASS FORMULAS AND THEIR MODIFICATIONS

2.1 Current status of nuclear mass formulae and their predictability:

The study of nuclear mass formulae has a long history since Weizsacker [1] or Bethe and Bacher [2] proposed a formula based on the liquid-drop model. Figure 2.1 shows the chronological table of mass formulae accompanied by that of nuclear models. Three epoch-making stages can be seen in Fig. 2.1. The first is when Levy [3] Cameron [4] Zeldes [5] and others proposed to take account of so-called shell effects and in succession Garvey and Kelson [6] proposed a mass relation, the second is when Myers and Swiatecki [7] proposed a formula with consideration of shell and deformation effects, and the third is when Strutinsky [8] proposed an extremely smart prescription to calculate shell energies from a nuclear single-particle potential like the Nilsson potential. Each stage corresponds to the proposal of major nuclear model, that is, the shell model, the collective model, or the Nilsson model. These are the Euro-American trends. Meanwhile, in the orient world, Yamada and his collaborators [9] independently started the study on this subject mainly from the standpoint of empirical shell terms. After those stages, many authors have struggled to construct more improved formulae, which were converged into the 1976 and 1988 mass-Olympics [10,11]. At the present stage it should of course be said that the main theme of the study on mass formulae is not only to give more precise mass values of known nuclides but also to predict reliable masses of unknown nuclides. Most of the unknown nuclides lie in the region far from stability or in the super-heavy region. In order to accomplish this theme it is most important to treat both shell and deformation effects as consistently as possible. There are three prominent groups who have been studying mass formulae along this line: P. Moller and his collaborators [12], J. M. Pearson and his collaborators [13]

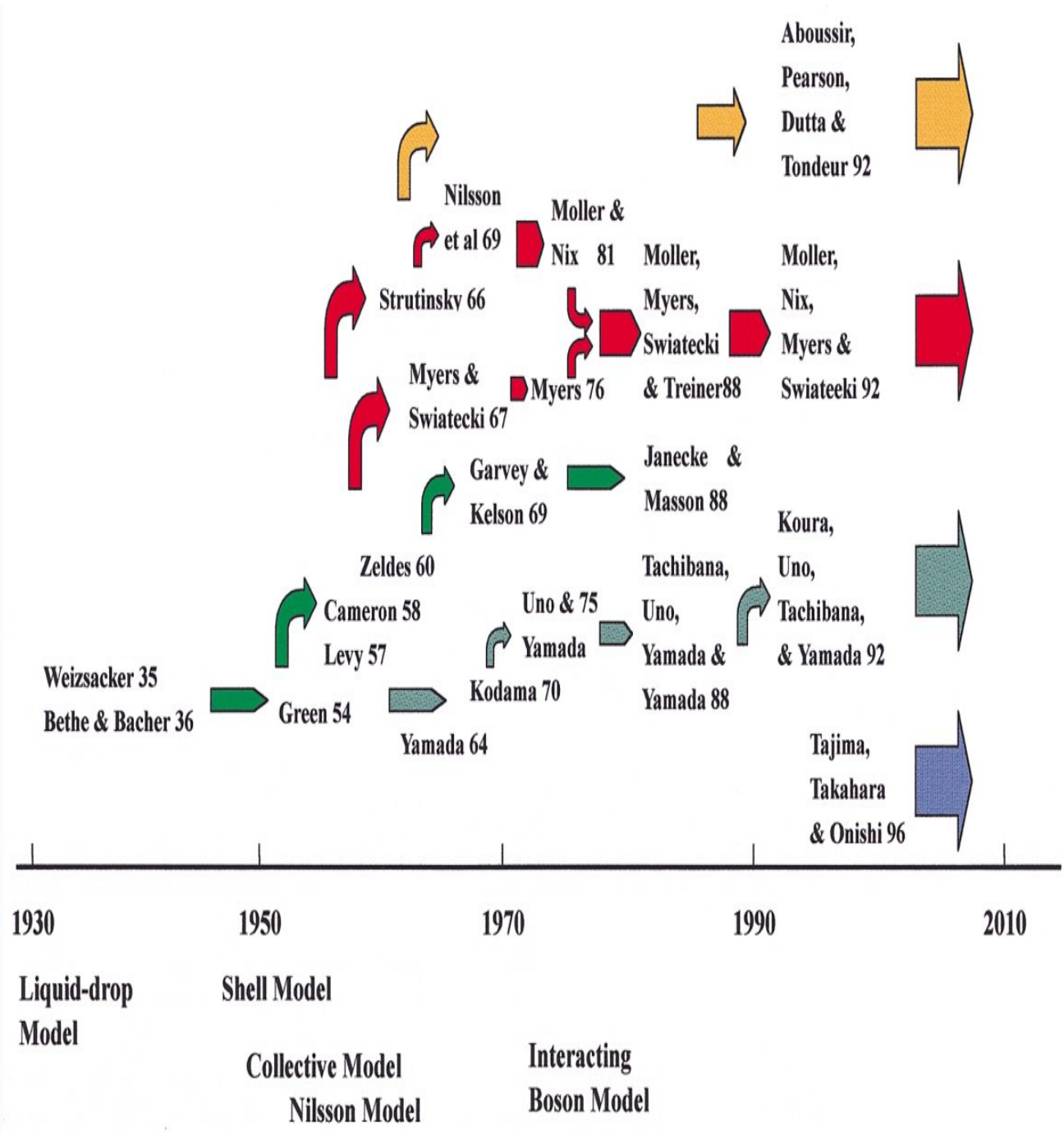


Figure 2.1: - The chronological table of mass formulae accompanied by that of nuclear models.

In recent years nuclear masses have realistically been treated in the framework of microscopic theories like the deformed Hartree-Fock method, the relativistic or non-relativistic mean field theories and so forth.

2.2 OVERVIEW OF THE THEORETICAL FRAMEWORK

2.2.1 P. Moller and his collaborators:-

The first group has long followed the golden road of conventional Nilsson-Strutinsky method. Their mass formula has two parts, a macroscopic part and a microscopic (or shell) part. The former is a smooth function of proton number Z and neutron number N, which is derived from the finite-range droplet model. The latter represents the individuality of each nucleus, and is usually called shell part in a broad sense. As for this part they start from a deformed single-particle potential of folded-Yukawa type. In order to obtain shell energies
they apply the conventional Strutinsky averaging prescription to this potential. They further add a Lipkin-Nogami type pairing energy. Their formula is often called FRDM formula, which is the abbreviation of the Finite-Range Droplet Model.

2.2.2 J. M. Pearson and his collaborators:-

The second group has adopted a semi-classical approximation to the Hartree-Fock method that includes full Strutinsky shell corrections. They extend Thomas-Fermi calculation to be available for a Skyrme-type Hartree-Fock force and a δ-function pairing force incorporating a full Strutinsky integral. Their treatment of shell effects is essentially the Strutinsky method except that they do not divide masses into two parts. Their formula is often called ETFSI formula, which is the abbreviation of the Extended Thomas-Fermi and Strutinsky Integral.

2.2.3 H. Koura and his collaborators:-

The third group has adopted another new approach. Here is given somewhat detailed explanation of this method. Formula has also two parts, the gross part and the shell part. The gross part is a smooth function of Z and N based on the liquid-drop model including an average pairing energy. This is essentially of the

same form as that of formula [16]. As for the shell part, we apply a new method. We start from a spherical single-particle potential, which has been well adjusted to about 150 single-particle levels of nuclei in the neighborhood of 15 double-magic or magic-sub magic nuclei ranging from ^{4}He to ^{208}Pb[17]. The central part of the potential is an extension of the Woods-Saxon potential as follows: -

$$V_{\rm cent}(r) = V_0 \frac{1}{\{1+\exp[(r-R_{\rm v})/a_{\rm v}]\}^{a_{\rm v}/\kappa}} \cdot \left\{1 + V_{\rm c} \frac{1}{1+\exp[(r-R_{\rm v})/a_{\rm v}]}\right\}.$$

2.2.4 Weizsacker (1935) [1]:-

In nuclear physics, the semi-empirical mass formula (SEMF), sometimes also called Weizsäcker's formula, is a formula used to approximate the mass and various other properties of an atomic nucleus. As the name suggests, it is partially based on theory and partly on empirical measurements; the theory is based on the liquid drop model, and can account for most of the terms in the formula, and gives rough estimates for the values of the coefficients. It was first formulated in 1935 by German physicist Carl Friedrich von Weizsäcker, and although refinements have been made to the coefficients over the years, the form of the formula remains the same today.

The liquid drop model and its analysis:-

The liquid drop model is a model in nuclear physics which treats the nucleus as a drop of incompressible nuclear fluid, first proposed by George Gamow and developed by Niels Bohr and John Archibald Wheeler. The fluid is made of nucleons (protons and neutrons), which are held together by the strong nuclear force.This is a crude model that does not explain all the properties of nuclei, but

does explain the spherical shape of most nuclei. It also helps to predict the binding energy of the nucleus.

Mathematical analysis of the theory delivers an equation which attempts to predict the binding energy of a nucleus in terms of the numbers of protons and neutrons it contains. This equation has five terms comparising of cohesive binding of all the nucleons by the strong nuclear force, the electrostatic mutual repulsion of the protons, a surface energy term, an asymmetry term (derivable from the protons and neutrons occupying independent quantum momentum states) and a pairing term (partly derivable from the protons and neutrons occupying independent quantum spin states).

If we consider the sum of these five types of energies, then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus:

Volume energy:- When an assembly of nucleons of the same size is packed together into the smallest volume, each interior nucleon has a certain number of other nucleons in contact with it. So, this nuclear energy is proportional to the volume.

Surface energy:- A nucleon at the surface of a nucleus interacts with fewer other nucleons than one in the interior of the nucleus and hence its binding energy is less. This surface energy term takes that into account and is therefore negative and is proportional to the surface area.

Coulomb Energy:- The electric repulsion between each pair of protons in a nucleus contributes toward decreasing its binding energy.

Asymmetry energy (also called Pauli Energy). An energy associated with the Pauli exclusion principle. If it wasn't for the Coulomb energy, the most stable form of nuclear matter would have N=Z, since unequal values of N and Z imply filling

higher energy levels for one type of particle, while leaving lower energy levels vacant for the other type.

Pairing energy:- An energy which is a correction term that arises from the tendency of proton pairs and neutron pairs to occur. An even number of particles is more stable than an odd number.

The formula:-

In the following formulae, let A be the total number of nucleons, Z the number of protons, and N the number of neutrons.

The mass of an atomic nucleus is given by

$$m = Zm_p + Nm_n - \frac{E_B}{c^2}$$

where m_p and m_n are the rest mass of a proton and a neutron, respectively, and E_B is the binding energy of the nucleus. The semi-empirical mass formula states that the binding energy will take the following form:

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z)$$

2.2.5 GREEN (1954)[24]:-

In the privious study Green and Enguler found an adjustment of constants which brought the Weiszacker Semiempirical equation into good agreement with experimental nuclear masses. It was noted that the energy constants which accomplish this were substantially larger than those appearing earlier in the litrature. In view of the lack of the understanding of nuclear forces,the significance of the increased nuclear energy constants is obscure. However, the increased couloumb energy constants(a_3) has a simple interpretation if one assumes that the charge in a nucleus may be characterized by a simple charge

distribution. In particular, if simple charge distribution And the radius formula $R= r_0A^{1/3}$ are accepted,

$$a_3=3e^2/5r_0,$$

Accordingly, the larger coulomb energy constant (0.750 mMu) corresponds to a smaller coulomb radius constant (1.2369 in units of $10^{-13\ cm}$). It has been pointed out by Bitter and Feshbach that this new constant is in good agreement with the radius constant obtained recently from μ-mesonic x-ray studies, electron scattering studies, and isotope shift studies.

Green and Engler halted their interative adjustment process when they found a convenient rounded set of energy constants (a_1=16.720, a_2=18.500,a_3=0.750 and a_4=100, all in mMu) which reduced the discrepancy between the Weiszsacker formula and the mass data to the order of magnitude of uncertainty caused by shell effects. In view of the importance of the question of the coulomb radius, it seems worthwhile to attempt to refine the determination of the coulomb radius constant based upon nuclear masses by the use of an objective criterion for best fit and to establish the precision of such a determination.

2.2.6 PHILIP A. SEEGER (1961) [18]:-

An improved expression for atomic masses as a function of position in the NZ-plane has been sought for the purpose of studying in detail the process of nucleosynthesis by rapid neutron capture. For this purpose, it must be possible to extrapolate the mass law to atoms with high neutron excess, but there exits no means of directly testing the law in this region of the NZ-plane. Criteria for testing the law in the valley of beta stability are listed in next section.

The familiar weizsacker semiempirical mass law consists of terms whose forms follow from the statistical model of the nucleus, each multiplied by a coefficient to be determined empirically. The mass excess of the atom with Z proton and N neutrons was taken to be

$$\Delta M_w(Z,A)=NM_n+ZM_H-A-\alpha A+\beta I^2/A+\gamma A^{2/3}+(3e^2/5R_0)Z^2/A^{1/2}, \qquad (2.1)$$

Where M_n and M_H are the neutron and hydrogen atom masses, A=N+Z, and I=N-Z. Modifications of this original form will be discussed.

The free coefficients, such as α, β, and γ in eq .(2.1) and similar quantities appearing in modified forms of the mass law, are found from least squares analysis by fitting either to the mass excess , and since A varies over a wide range , the results here reported were determined by fitting to the mass excess. In many treatments of the problem, a limited number of masses for the atoms on the line of beta stability have been used. In this study, however, all stable odd mass atoms were used, all data being weighted inversely as the first power of the estimated error the principal source was the table of masses omitted from that table but estimated in the earlier work of Huizenga were corrected to the later value of pb208 by addition of 1.123 mMU and used with the quoted error doubled. Thus a total of 488 odd mass atoms with A ≥19 were used to determine a mass law for odd mass atoms. All computations were made on an IBM 704 at Los Alamos Scientific Laboratory.

Criteria for testing the mass laws:-

Three criteria for testing the mass laws have been considered. First, the errors §I of the masses calculated by the law compared to the input data were plotted, and the standard deviation calculated by

$$\sigma = (n/n-k)^{1/2}(\Sigma wi\ §_i^2/\Sigma w_i)^{1/2}, \qquad (2.2)$$

Where n is the number of input masses, k the number of free coefficients, and w_i the weight of the input datum. This is a measure of the ability of the law to fit known masses; it is to be made as small as possible. A standard deviation σ≤1 mMU is considered to be satisfactory.

As another test, called the “interpolation” test, the 488 odd-mass atoms were divided at random into two groups. This was done by ascertaining whether the last bit in the binary expression for the mass excess was even or odd. The first group, consisting of 232 masses, was used to determine the coefficients of a mass law of the same form as the law in question, and then that law was used to “predict” the masses of the 256 atoms in the second group. The standard

deviation σ1of the first group was calculated by eq. (2.2) ,and that of the second group by

$$\sigma_2 = (\Sigma wi\ \S_i^2/\Sigma w_i)^{1/2} \qquad (2.3)$$

The ratio $(\sigma_2/\sigma_1)_{int}$ is taken as a measure of the ability of the particular form of mass law to interpolate; σ1 should be approximately the same as σ, and the ratio (σ_2/σ_1) int should be less than, say, 1.05.

For our purposes the most significant criterion was a third test called the 'extrapolation 'test. The 488 odd mass atoms were again devided into two groups: the first containing those atoms on the line of beta stability plus those on the neutron poor side of the valley of beta stability, the second containing atoms only from the neutron rich side of beta stability. The first group had 282 atoms and the second had 206. The standard deviations σ_1 and σ_2 and the ratio $(\sigma_2/\sigma_1)_{ext}$ were calculated as in the interpolation test. Once again σ_1 should be similar to σ, and the ratio $(\sigma_2/\sigma_1)_{ext}$ should be close to unity. It is difficult to estimate an acceptable value for this ratio, but we might hope to make $(\sigma_2/\sigma_1)^2_{ext} \leq 2$ or $(\sigma_2/\sigma_1)^2_{ext} \leq 1.4$.

Standard form of mass law:-

For our standard form of the mass law, the symmetry term was modified to include |I| as well as I^2 as suggested by Wigner and the coefficient was modified to include composition dependence of the surface effects. The factor 2 in 2 |I| was suggested by E.E.Salpeter.following mozer, the coulomb term was modified to trapezoidal charge distribution and exchange effects. The coefficients $3e^2/5R_0=0.8076$ MeV/c^2 and $7/24(tR_0)^2=2.29$ were determined using $R_0=1.07$ fm, t=2.8 fm from scattering experiments with high-energy electrons. Then in MeV/c^2 on the O^{16} scale the mass excess becomes

$$\Delta M_{stan}(Z, A)=8.367N+7.5848Z-\alpha A+(\beta-1/2/A/^{1/2})(I^2+2|I|)/A+ A^{2/3}$$
$$+0.8076Z^2/A^{1/2}(1-0.7636/Z^{2/3}-2.29/A^{2/3}), \qquad (2.4)$$

The coefficients as determined by least squares are

$$\alpha = 16.11 \text{ MeV}/c^2,\ \gamma = 20.21 \text{ MeV}/c^2, \qquad (2.4a)$$

β =20.65 MeV/c^2 , η =48.00MeV/c^2.

Standard deviation: σ =2.61 MeV/c^2

Interpolation: σ_1 =2.87 MeV/c^2 $(\sigma_2/\sigma_1)^2_{int}$ =0.84

Extrapolation: σ_1 = 2.19 MeV/c^2 $(\sigma_2/\sigma_1)^2_{ext}$ =1.75

It will be noted that standard form fails in the two most crucial test: the standard deviation and the extrapolation ratio are too large.

Additional terms in the mass law:-

In the attempt to find a mass law which would extrapolate more successfully than the standard form (2.4) , a function S(N,Z), representing the increase in nuclear binding energy both at closed shells due to magic numbers of N or Z and between closed shells due to the deformation of the nucleus, was subtracted from ΔM_{stan} to give M_0 ,the mass excess for odd- mass atoms. Thus $M_0 = M_{stan} - S(N,Z)$. We may now write

$$S(N,Z)=k_s c_0+k_d(c_1-c_2^2), \qquad (2.5)$$

Where k_s and k_d are shell and deformation coefficients which may vary from shell to shell, c_0 (N, Z) is the functional representation of the excess binding energy due to shell effects, and the extra energy due to deformation is approximated by $(c_{1\eta} - c_2\eta^2)$. In this last expression = $\Delta R/R$ is the spheroid eccentricity, $c_{1\eta}$ is the excess number binding energy due to deformation and $c_2\eta^2$ is the change in coulomb and surface energies in deforming a uniformly charged sphere into a spheroid. By the usual argument that the nucleus take that deformation η_0 such that the deformation energy is a maximum.

$d/d\eta(c_{1\eta} - c_2\eta^2)|_0 = c_1 - 2c_2\eta_0 = 0$

$\eta_0 = c_1/2c_2$, $c_1\eta_0 - c_2\eta_0^2 = c_1^2/4c_2$.

The coefficient c_2 can be expressed as: -

$C_2 = 8A^{2/3}(1-0.005Z^2/A)$ MeV/c^2.

In this study, c_2 has been considered constant over the range of A included within any one shell of N or Z. The function $c_{1\eta}$ can be calculated as a sum over all nucleon outside closed shell. Each such nucleon is in a spheroidal rather than spherical potential and the excess binding energy can be calculated by the methods of rainwater or nilsson . It is to be expected that c1 is zero at a nucleus for which both Z and N are magic, and has a maximum near the middle of each shell of Z or N this behavior makes it possible to approximate c_1 with sine functions. The term C_0 must be a maximum at closed shells and reach a minimum near the middle of shell. It has therefore been assumed that $C_0=K-C_1$ where K is a constant.

With these assumptions, (2.5) becomes

$$S=k_sK-k_sC_1+k_dC_1^2/4C_2, \qquad (2.6)$$

Where k_sK, k_s and $k_d/4C_2$ are constants within any given shell.

For the nucleus with N neutrons and Z protons, let N' and Z' be the fractional occupations of the last shells,

$$N'=(N-N_f)/(N_{f+1}-N_f)\ , \quad Z'=(Z-Z_k)/(Z_{k+1}-Z_K) \qquad (2.7)$$

Where the magic closed shell number are:-

N_f, Z_k=8,20,50,82,126,184,

And

$N_f \le N<N_{f+1}$, $Z_k \le Z <Z_{k+1}$.

Consider eq.(2.6) if C_1 is expanded in a Fourier sine series,

$C_1=A_1\sin N'\Omega+A_2\sin 2N'\Omega +B_1 \sin Z'\Omega +B_2 \sin 2Z'\Omega +$……..

Hence the square of the sine has same qualitative properties as the sine itself, that is, going to zero at magic numbers and has its maximum halfway in between, such terms in C_1^2 add nothing essentially new to our approximation for S which is not already included in the linear terms in C_1. The cross product in C_1^2, such as (sin N'Ω)(sin Z'Ω), are unlike any terms in C_1 itself, and must be included in S. previous attempts to include shall structure have used S=f(N)+g(Z), which neglect cross product terms. We feel it is essential to include these terms in order to have some representation of the effects of the proton

shell structure on the neutron shell structure and vice versa. Thus, the final approximate form adopted for S was

$$S_{jk}(N',Z')=\zeta_j \sin N'\Omega+\zeta_k \sin Z'\Omega+v_j \sin 2N'\Omega+v_k \sin 2Z'\Omega+(\varphi_{j+}\varphi_k)(\sin\Omega)(\sin Z'\Omega)+X \quad (2.8)$$

Where indices j and k refer to the shells of N and Z respectively, and the ζ_j, v_j, φ_j and X are constants approximately related to the constants in eq.(2.6). Note that S_{jk} is symmetric in N and Z, so that it is charge independent . also note that the constant term X does not vary from shell to shell, in order to make S_{jk} a continuous function. Then the mas law for odd mass atom is

$$\Delta M_0 (Z,A)=\Delta M_{stan}(Z,A)- S_{jk}(N',Z') \quad (2.9)$$

The coefficients α, β, γ and η as determined by least squares are, in MeV/C^2.

$$\alpha=17.06,\ \beta=33.61,\ \gamma=25.00,\ \eta=59.54 \quad (2.9a)$$

Table 2 :-The coefficients ζ, v, φ and X(in MeV/C^2) :-

≤Z or N<	Z	N	Φ	X
8,20	4.008	-0.428	-1.389	13.51
20,50	-0.508	2.331	-0.463	13.51
50,82	-7.636	0.496	1.950	13.51
82,126	-15.63	-2.284	10.67	13.51
126,184	-27.59	2.660	26.99	13.51

The coefficients ζ, v, φ and X as determined by least square are given in above table 2. Other forms of S were tried, but only the function S_{jk} as given in eq. (2.8) gave this improved result for the extrapolation test. Thus we have σ<1 MeV/C^2, $(\sigma_2/\sigma_1)_{int=1}$ and $(\sigma_2/\sigma_1)_{ext}$ within the proposed criteria .

TABLE OF MASSES:-

The mass law (9) with least square determined coefficients yield the masses of odd mass atoms directly. In order to calculate masses of even-mass atoms, it was necessary to consider the pairing energy term $\S_A$ given by

$$\Delta M(Z,A) = \Delta M_0(Z,A) + \begin{cases} +\tfrac{1}{2}\,\S_A \text{ for odd atoms} \\ +0 \text{ for odd mass atoms,} \\ -1/2\,\S_A \text{ for even mass atoms} \end{cases} \qquad (2.10)$$

The function $\S_A$ was estimated by calculating ΔM_0 from eq. (2.9) for all known even-mass atoms. For each even value of A, the difference between ΔM_0 and the experimental mass excess averaged over Z was plotted as $\pm\tfrac{1}{2}\S_A$. A table of mass excesses (on the O^{16} scale) and binding energy of 7479 atoms was calculated using eq. (2.10). It is hoped that by simultaneously determining the S_{jk} function and the standard mass law terms, the influence for shell effects in the determination of α, β, γ and η has been reduced. A comparison of the coefficients of the standard and form (2.4a) with those of the modified form (2.9a) shows that all of the major coefficients α, β, γ and η are increased when S_{jk} is included.

2.2.7 JAMES WING and PETER FONG (1964)[19]:-

In the past thirty years, more than twenty mathematical expressions have been formulated to account for the systematics of the nuclidic masses, binding energies, and nucleon separation energies[19]. These expressions, commonly called nuclidic mass equations, represent the mass or the equivalents as a function of the proton number Z and the neutron number N of the nuclide. Some of these expressions are quite inconvenient to use because they involve many complicated functions and adjustable constants. The physical significance of some of the complicated functions is also uncertain. Although many of the quations describe fairly well the general features of the nuclidic mass surface(a three-dimensional plot of nuclidic masses versus N and Z), systematic deviations from the experimental data are observed in every mass equation hither to published [19]. Most of these systematic deviations originate from the inadequate

treatment of the nuclear shell effects and the isobaric mass variation. We have developed a relatively simple semiempirical expression to account for the nuclidic mass systematics starting from an equation based on the liquid-drop model [20,21], with the shell effects included in a correction term. The numerical values of the constants in the mass equation are evaluated by least-squares fitting of the experimental nuclidic masses[22,23] based on the scale of C^{12} = 12.000000 mass units.

FORMULATION OF THE MASS EQUATION:-

The basic form of our nuclidic mass equation is:-

$$M(Z,A)=M_A + ½ B_A (Z-Z_A)^2 + P_A – S(N,Z), \quad (2.11)$$

Where M is the mass excess (nuclidic mass minus mass number), M_A the mass excess of the stable nuclide ($Z=Z_A$) for

mass number A, B_A a measure of the curvature of the isobaric mass section, Z_A the charge (not necessarily an integer) of the most stable isobar, P_A the pairing energy due to the even-odd variation, and S the shell correction term. M_A is assumed to be a parabolic function of A, and the coefficients were evaluated in a first approximation by a least –squares fit of the experimental masses of the stable, odd-A nuclides not containing closed-shell configurations:

$$M_A=0.0089794A^2-2.0717A+33.448 \text{ MeV}. \quad (2.12)$$

For odd-A nuclides, it can be shown that

$$B_A=M(Z,A)-2M(Z+1,A)+M(Z+2,A). \quad (2.13)$$

With the experimental masses of odd-A nuclides inserted into eq.(2.12), we calculated the values of B_A on the basis of which the following expression was obtained as the first approximation of B_A:

$$B_A'=4.68-(86.32/A^{1/2})+(550/A) \text{ MeV}. \quad (2.14)$$

The expression for Z_A was taken from Green's work [24]; his Z_A seems sufficiently satisfatory as a continuous approximation and is adopted without change:

$$Z_A= (A+0.003A^2)/(2+0.01A). \quad (2.15)$$

The pairing energy term was obtained by comparison of eq. (2.11), with eqs. (2.13),(214) and (2.15) inserted, with the experimental mass data, disregarding the nuclides with the closed shell configurations:

$$P_A=11.51\ \delta/A^{1/2}\ \text{MeV}, \qquad (2.16)$$

Where δ equals 0 for odd-A, -1 for even-Z-even-A, and +1 for odd-Z-even-A nuclides. Wapstra [25] suggested a bell-shaped correction curve for the shell effect. We tentatively assumed the shell effect term in our mass equation to take the form of the sum of a series of cauchy distribution functions in terms of N and Z, with their maxima located at the magic numbers:

$$S'(N,Z)=\sum_I(a_ib_i^2)/((N-N_i^*)^2+b_i^2)+\sum_j(a_jb_j^2)/((Z-Z_j^*)^2+bj^2) \qquad (2.17)$$

Where N_i^* and Z_j^* are respectively the magic numbers of neutrons and protons in the closed-shell configurations, a is the maximum magnitude of the shell correction in the ith neutron or jth proton shell and b is the half-width at (½)a. in the neighborhood of a shell edge, only one term in the sum is large; other terms from other shells are small. The advantage of this form of shell correction is that the mass equation remains a continous function (except for the pairing energy) from one shell region to another. It soon becomes apparent that the correction curve of eq.(2.17), which is symmetric with respect to the shell edge cannot fit experimental data on both sides of the shell edge. On the other hand, each side may be fitted by a cauchy distribution curve with a width different from that of the other side. Thus we assumed different values of the width $b_\pm$ for N (or Z) greater and smaller than the magic number. The shell correction term thus has the following form:

$$S(N,Z)=\sum_I(a_ib_{i\pm}^2)/((N-N_i^*)^2+b_{i\pm}^2)+\sum_j(a_jb_{j\pm}^2)/((Z-Z_j^*)^2+bj_\pm^2) \qquad (2.18)$$

Where $b_{i\pm}$ and $b_{j\pm}$ are used when $(N-N_i^*)><0$ and $(Z-Z_j^*)><0$, respectively. In spite of the discontinuity of b at the shell edge, the equation remains continuous to the first order derivative. The values of the constants in eq. (2.18) were

evaluated by a variable metric method for minimization [26], the input data being the values of S obtained from the combination of eqs. (2.11),(2.12) and (2.14)to (2.16) and substitution of experimental masses.

We then applied the variable metric minimization method for the improvement of the B_A expression, using the experimental masses and all the previous equations and constants except eqs. (12), (13), and (16). We obtained the following expression for B_A and doscarded eq. (13):

$$B_A = 3.258-(60.22/A^{1/2})+ ((431.6)/A) \text{ MeV}. \qquad (2.19)$$

A similar iteration was performed for M_A. However, the values of the coefficient so obtained were essentially identical with those in eq. (2.12) and therefore eq. (2.12) is used for M_A without change.

we have also studied the possibility of developing a mass formula by correcting Z_A for the shell effects. We know that the charge of the beta- stable isotope is not a smooth function of A, but shows fluctuations due to shells [21].

The resulting mass equation, reffered to as the alternate equation to avoid confusion, is not as simple and as successful as the present one and is listed only for reference. In addition to the shell correction for Z_A , the terms S remains indispensable and again appears to be assymetric with respect to the shell edges. The present mass equation includes all shell corrections in the assynetric S term and thus is simpler; the empirical variation of the charge (integral) of the beta-stable isotope is thus a manifestation of the S term.

DISCUSSION:-

The nuclidic masses predicted by the present equation are compared with the experimental data by plotting the differences. No large systematic deviations are observed in the mass region of A=60 to 220. for A>230, there is a wide spread of almost 5 MeV of differences; the neutron-rich nuclides have positive deviations and the neutron-deficient nuclides have negative deviations. This wide spread of

differences is a result of inadequate treatment of the isobaric mass variation in this region (B_A and Z_A small compared with the experimental data) .

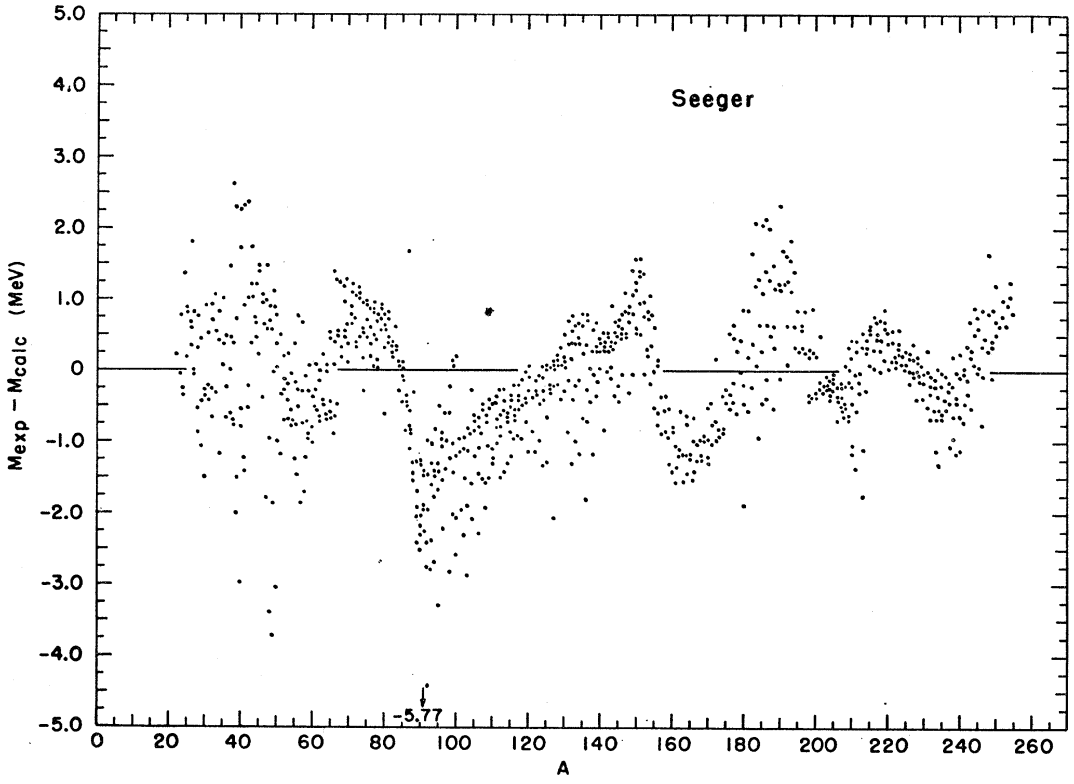


Figure 2.2 :- Deviation of Seeger's mass equation from the experimental values of nuclidic masses.

Improvement of the present mass equation could be made with additional adjustable constants especially for the B_A and Z_A expressions. However, this was not done because we did not want to add adjustable constants into our present equation.

For comparison, we have also plotted the differences between the experimental and the calculated (masses or binding energies) values, for the mass equations of Cameron[4], Seeger[18], Levy[3], Baker[26], and Green[24]. In the mass region of A>201, Cameron and Baker used, for the evaluation of their constants, the experimental masses [27] which are about one MeV lower than those we used [22] we have, therefore, allowed for a one-MeV correction in the calculations using the equations of Cameron and Baker in this mass region .

It has long recognized that a liquid-drop model mass formula, such as Fermi's, deviates from the experimental results by a uniform shift plus systematic fluctuations related to shells[21]. The former may be eliminated by proper changes of the parameters of the formula, but the latter can not be expressed in terms of simple functions of N and Z. We are thus led to expect a nuclidic mass formula consisting of two parts: a smoothly varying part similar to the liquid-drop model formula plus a rapidly varying shell correction term. How to formulate the shell correction is thus the central problem. Levy[3] and Green[24] tried to fit the experimental data between the shells with smooth functions, this being obviously the simplest approach bacause the variation of mass between the shells is the least drastic. The fitted smooth functions expectedly lead to large deviations near the closed shells. Yet the most undesirable feature of this approach is that the formulas are discontinuous over the shell boundaries. Levy divided the mass surface into sections by the magic number lines.

The mass surface in each section is approximated by a quadratic surface, the parameters of which change from section to section. Besides large deviations at the shell edge there is also a constant deviation of about 1.5 to 2.0 MeV for the heavy elements region. Green's shell correction term consists of a set of parabolic curves with vertices located midway between two magic numbers. The discontinuity at the shell edge is considerable.The other alternative is to correct the shell effects in the neighborhood of the shell edges; this approach is mathematically more difficult but physically more reasonable. Seeger[18] expressed the shell effects by a series of sine functions of N and Z with cross product terms. The systematic oscillation of his deviations (Figure.2.2) is probably a result of the symmetric nature of the sine functions with respect to the shell edges.

Since this leaves the asymmetric correction over the shell edges as the only alternative, we adopted it. From the point of view of nuclear structure we have no reason to expect a nucleus with extra nucleons outside a closed shell to behave exactly the same as one an equal number of unfilled levels (holes). The nuclear level spacing bahaves differently for these two types of nuclides. So does the

nuclear mass which fixes the position of the ground level. Apart from the shell correction term, the rest is empirically fitted into a smooth formula in the present approach. The formula is similar to the liquid-drop model formula in its quadratic dependence on Z, though the expressions of M_A, B_A, and Z_A are purely empirical. Cameron [4] empirically determined the combined effects of the shell and pairing interactions for each value of N and Z. There is a notable systematic deviation in the rare-earth region. This may be due to the fact that very few and perhaps poor experimental data were available in this mass region for the evaluation of Cameron's constants. The complication of his formula is brought out by the fact more than 200 parameters are involved. Many of the deviations in Cameron's, Seeger's, and Levy's mass equations become very large for nuclidic far away from the beta stability line, indicating inadequate treatment for the isobaric mass sequences. Baker[26] expressed the binding energies of nuclides by polynomial functions of neutron excess and mass number. No shell effect was included in his binding energy formula. The spread of his deviations is wider than those we have examined so far, except for the heavy elements region. These deviations are obviously related to the shell effects which are not adequately accounted for.

Table 3 lists the frequency distributions of deviations (absolute difference between the experimental and calculated values) found in the nuclidic mass equations which we have considered. The number of the adjustable constants used in a given mass equation is placed in parentheses. The present mass equation has the fewest deviations greater than 2.0 MeV, and, next to Cameron's, has the highest parcent (and number) of deviations smaller than 0.5 MeV.

Table 3. Frequency distributions of deviations.

Mass region	Deviation (MeV)	Cameron[a]	Seeger	Levy	Baker[a]	Ref. [19]	The alternate equation
		(232)	(25)	(81)	(63)	(34)	(47)
$40 \leqslant A \leqslant 70$	<0.5	89	45	16	66	76	57
	0.5–1.0	26	51	8	38	35	38
	1.1–2.0	8	23	5	19	13	27
	>2.0	3	7	6	2	2	4
$70 < A \leqslant 115$	<0.5	146	50	132	118	137	93
	0.5–1.0	44	75	40	70	65	76
	1.1–2.0	11	53	16	15	6	33
	>2.0	2	25	6	6	1	7
$115 < A \leqslant 162$	<0.5	96	101	127	72	92	103
	0.5–1.0	70	68	55	63	78	65
	1.1–2.0	27	23	8	45	21	23
	>2.0	0	1	3	13	2	2
$162 < A \leqslant 208$	<0.5	29	65	40	24	107	46
	0.5–1.0	21	43	33	28	37	60
	1.1–2.0	38	33	46	54	1	34
	>2.0	57	4	14	39	0	5
$208 < A \leqslant 255$	<0.5	158	106	1	154	68	50
	0.5–1.0	11	50	12	12	65	65
	1.1–2.0	0	13	133	2	30	44
	>2.0	0	0	6	1	6	9
$40 \leqslant A \leqslant 255$	<0.5	518	367	316	434	480	349
	0.5–1.0	172	287	148	211	280	304
	1.1–2.0	84	145	208	135	71	161
	>2.0	62	37	35	61	11	27

No deviation in the present mass formula is larger than 3.10 MeV, whereas all the other mass equations we have examined so far have one or more deviations larger than 4.0 MeV. In the mass region of A=60 to 220, our calculated values are in good agreement with the experimental data. However, in the heavy mass region (A 210),

Cameron's and Baker's predictions are much better. Finally, it may be mentioned that the present mass equation has only 34 adjustable constants while all the others, except Seeger's, require more than 40 adjustable constants.

The nuclidic mass excesses, neutron and proton binding energies, alpha-particle binding energies, and total beta-decay energies predicted by the present mass equation for Z=13 to 110 and A=22 to 315 are tabulated in an Argonne National Laboratory report.[28]

2.2.8 G. Audi and A.H. Wapstra (2003) [25]:-

The description of the general procedures and policies are given in Part I [33]of this series of two papers, where the input data used in the evaluation are presented. In this paper we give tables and graphs derived from the evaluation of the input data in Part I [33]. Firstly, we present the table of atomic masses

expressed as mass excesses in energy units, together with the binding energy per nucleon, the beta-decay energy and the full atomic mass in mass units. This work has been undertaken with the encouragement of the IUPAP Commission on Symbols, Units, Nomenclature, Atomic Masses and Fundamental Constants (SUN-AMCO). The second table is the table of influences on primary nuclides (Table II). For each of the primary nuclides entering this evaluation, we give the three main data and their influences on the mass of this nuclide.

Thirdly, we give a table for values and their estimated precision for the separation energies and reaction energies for twelve carefully selected combinations of nuclides. This selection, together with the ß -decay energies above, yields all differences in masses between any pair of nuclei differing at most by 2 units in Z and N. A method is indicated in which many more reaction energy values can be derived from the present table.

The following series of graphs are then presented: two-neutron separation energies and a-decay energies as a function of neutron number, two-proton separation energies as a function of proton number and double ß -decay energies as a function of mass number which are considered as the most illustrative ones for the systematic trends.

The atomic mass table:-

As in our previous work AME'93 [29-32],[4] and AME'95 [33], the tables presented in this work give atomic masses and derived quantities. With very few exceptions, experimental data on masses of nuclei refer to "atomic" masses or to masses of singly ionized atoms. In this last case the ionization energy is generally (much) smaller than the error on the mass, and, for the small number of very precise mass measurements, corrections for the first -and second-ionization potentials could be applied without much loss of accuracy. The same is true for the electron mass Me involved, see Table 4 in Part I. This is the reason for the decision to present, in our evaluations, atomic rather than nuclear masses.

Nuclear masses can be calculated from atomic ones by using the formula:

$$M_N(A,Z) = M_A(A,Z) - Z \times M_e + B_e(Z) \tag{2.19}$$

Nowadays, several mass measurements are made on fully or almost fully ionized particles. Then, a correction must be made for the total binding energy of all removed electrons Be(Z). They can be found in the table for calculated total atomic binding energy of all electrons of Huang et al. [34]. Unfortunately, the precision of the calculated values Be(Z)is not clear; this quantity (up to 760 keV for 92U) cannot be measured easily. Very probably, its precision for 92U is rather better than the 2 keV accuracy with which the mass of, e.g., 238U is known. A simple formula, approximating the results of [34], is given in the review of Lunney, Pearson and Thibault [35]:

$$B_{el}(Z) = 14.4381\, Z^{2.39} + 1.55468 \times 10^{-6} Z^{5.35}\ \text{eV} \tag{2.20}$$

The atomic masses are given in mass units and the derived quantities in energy units. For the atomic mass unit we use the “unified atomic mass unit,” symbol “u”, defined as 1/12 of the atomic mass of one 12C atom in its electronic and nuclear ground states and in its rest coordinate system. In our work energy values are expressed as electron-volt, using the maintained volt V90. we no longer give values for the binding energies, $ZM_H + NM_n - M$, as we used to in earlier tables. Otherwise than before, its error equals that in the value of the mass excess, which makes its use unnecessary. We now give instead the binding energy per nucleon, which is of educational interest, connected to the Aston curve and the maximum stability around the ‘iron-peak’ of importance in astrophysics.

Due to the drastic increase in the precision of the mass values of the very light nuclei, the printing format of the mass table is not adequate. Table 4 gives, for the most precise among them, values of mass excesses and atomic masses. Conversion of the errors from μu to keV were obtained by:

$$\sigma^2_{M_{keV}} = (\sigma_{M_u} \times u)^2 + (M_u \times \sigma_u)^2 \tag{2.21}$$

where M_u is the mass excess in μ_u, and su the error of u expressed in eV_{90}. The part dependent on Mu is only important for very few nuclides

Table 4:- The most precisely known massess[25]

	Mass excess (keV_{90})		Atomic mass (μu)	
^{1}n	8 071.317 10	0.000 53	1 008 664.915 74	0.000 56
^{1}H	7 288.970 50	0.000 11	1 007 825.032 07	0.000 10
^{2}H	13 135.721 58	0.000 35	2 014 101.777 85	0.000 36
^{3}H	14 949.806 00	0.002 31	3 016 049.277 67	0.002 47
^{3}He	14 931.214 75	0.002 42	3 016 029.319 14	0.002 60
^{4}He	2 424.915 65	0.000 06	4 002 603.254 15	0.000 06
^{13}C	3 125.011 29	0.000 91	13 003 354.837 78	0.000 98
^{14}C	3 019.893 05	0.003 80	14 003 241.988 70	0.004 08
^{14}N	2 863.417 04	0.000 58	14 003 074.004 78	0.000 62
^{15}N	101.438 05	0.000 70	15 000 108.898 23	0.000 75
^{16}O	− 4 737.001 41	0.000 16	15 994 914.619 56	0.000 16
^{20}Ne	− 7 041.931 31	0.001 79	19 992 440.175 42	0.001 92
^{23}Na	− 9 529.853 58	0.002 73	22 989 769.280 87	0.002 93
^{28}Si	− 21 492.796 78	0.001 81	27 976 926.532 46	0.001 94
^{40}Ar	− 35 039.896 02	0.002 68	39 962 383.122 51	0.002 86

References

[1] C. F. von Weizs¨acker: Z. Phys. 96, 431 (1935).

[2] H. A. Bethe and R. F. Bacher: Rev. Mod. Phys. 8, 82 (1936).

[3] H. B. Levy: Phys. Rev. 106, 1265 (1957).

[4] A. G. W. Cameron: Can. J. Phys. 35, 1021 (1957).

[5] N. Zeldes: Nucl. Phys. 7, 27 (1958).

[6] G. T. Garvey, W. J. Gerace, R. L. Jaffe, I. Talmi, and I. Kelson: Rev Mod. Phys. 41, S1 (1969).

[7] W. D. Myers and W. J. Swiatecki: Nucl. Phys. 81, 1 (1966).

[8] V. M. Strutinsky: Yad. Fiz. 3, 614 (1966); Nucl. Phys. A 95, 420 (1967).

[9] M. Yamada: Prog. Theor. Phys. 32, 512 (1964); T. Kodama: Prog Theor. Phys. 45, 1112 (1970); M. Uno and M. Yamada: Prog. Theor Phys. 53, 987 (1975).

[10] S. Maripuu: 1975 Mass Predictions; At. Data Nucl. Data Tables 17, 411 (1976).

[11] P. Haustein: 1986-1987 Atomic Mass Predictions; At. Data Nucl Data Tables 39, 185 (1988).

[12] P. M¨oller, J. R. Nix, W. D. Myers, and W. J. Swiatecki: At. Dat Nucl. Data Tables 59, 185 (1995).

[13] Y. Aboussir, J. M. Pearson, A. K. Dutta, and F. Tondeur: At. Data Nucl. Data Tables 61, 127 (1995).

[14] H. Koura, M. Uno, T. Tachibana, and M. Yamada: AIP Conf.Proc.455, 114 (1998).

[15] N. Tajima, S. Takahara, and N. Onishi: Nucl. Phys. A 603, 23 (1996).

[16] T. Tachibana, M. Uno, M. Yamada, and S. Yamada: At. Data Nucl. Data Tables 39, 251 (1988).

[17] H. Koura and M. Yamada: Nucl. Phys., to be published.

[18] P. A. Seeger. Nucl. Phys., 25, 1 (1961).

[19] J. Wing, Atomic Energy Commission Report, ANL-6814,1964(unpublished).

[20] N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939).

[21] P. Fong, Phys. Rev. 102, 434 (1956).
[22] L. A. Konig, J. H. E. Mattauch, and A.H . Wapstra, Nucl. Phys 31, 28 (1962).
[23] V. A. Kravtsov, Nucl. Phys. 41, 330 (1963).
[24] A. E. S. Green, Nuclear Physics (McGraw-Hill Book Company, Inc, New York, 1955).
[25] A. H. Wapstra, Physica 18, 83 (1952).
[26] G. A. Baker, Phys. Rev. 112,954 (1958).
[27] J. R. Huizenga, Physica 21, 410 (1955).
[28] J. Wing and J. D. Varley, Atomic Energy Commission Report, ANL-6886, 1964 (unpublished).
[29] G. Audi and A.H. Wapstra, Nucl. Phys. A 565 (1993) 1;
http://csnwww.in2p3.fr/AMDC/masstables/Ame1993/
[30] G. Audi and A.H. Wapstra, Nucl. Phys. A 565 (1993) 66.
[31] C. Borcea, G. Audi, A.H. Wapstra and P. Favaron, Nucl. Phys. A 565 (1993) 158. 193.
[32] G. Audi and A.H. Wapstra and M. Dedieu, Nucl. Phys. A 565 (1993) 193.
[33] G. Audi and A.H. Wapstra, Nucl. Phys. A 595 (1995) 409;
http://csnwww.in2p3.fr/AMDC/masstables/Ame1995/
[34] K.-N. Huang, M. Aoyagi, M.H. Chen, B. Crasemann and H. Mark, At. Nucl. Data Tables 18 (1976) 243.
[35] D. Lunney, J.M. Pearson and C. Thibault, Rev. Mod. Phys. 75 (2003) 1021.
[36] The AME2003 files in the electronic distribution and complementary documents can be retrieved from the Atomic Mass Data Center (AMDC) through the Web:
http://csnwww.in2p3.fr/amdc/
[37] Nuclear Structure Reference (NSR): a computer file of indexed references maintained by NNDC, Brookhaven National Laboratory; http://www2.nndc.bnl.gov/nsr/

CHAPTER-3

TEMPERATURE DEPENDENCE OF NUCLEAR POTENTIALS

Presently a lot of work is being done on the dynamics of nuclear system at extreme composition, excitation energy, angular momentum, nuclear deformation, etc. in brief the study of nucleus at higher excitation energies carries lot of information, which in turn could be extremely useful for the better understanding of nuclear properties.

The fragmentation potential consists of the macroscopic liquid drop energy V_{LDM} besides coulomb potential, proximity potential and rotational energy. Therefore, in order to investigate the dynamics of a excited nuclear system, the knowledge of temperature dependence of macroscopic part of nuclear potential becomes essential. This temperature dependent liquid drop potential can be calculated as suggested by Davidson et al. [3.1]. The predictions of this model find its basis in the well-established semi-empirical mass formula of Seeger [3.2].

3.1 Seeger's liquid drop energy and its temperature dependence:-

In nuclei the nuclear force between the nucleons is short-ranged which leads to saturation of the binding energy, E(A,Z), per nucleon. Empirically it is known that for the stable isotopes

$$\frac{E(A,Z)}{A} \approx \alpha \tag{3.1}$$

where α= 8 MeV and A ≥20. This fundamental gross property of nuclei is well accounted for by the semi-empirical mass formula which provides a simple parametrization of the binding energy per nucleon for all known nuclei. Qualitatively it is also consistent with the simple Fermi gas model prediction .

$$\frac{\mathcal{E}(A)}{A} = \frac{3}{5}\epsilon_f \tag{3.2}$$

Where E is the ground state energy and ε_f is the Fermi energy, which is constant as long as the particle density remains constant. It is interesting to note that at low temperatures ($T < T_f$) the excitation energy in the Fermi gas model is given by

$$\begin{aligned}\frac{\mathcal{E}(A,T)}{A} &= \frac{3}{5}\epsilon_f + \frac{\pi^2}{4}\frac{T^2}{\epsilon_f} \quad \text{or} \\ &= a + bT^2\end{aligned} \tag{3.3}$$

Where a and b are constant again as long as the particle density remains constant. Although the Fermi gas model may be an oversimplified model, nonetheless it underscores the relevance of independent particle (or quasi-particle) methods in nuclear structure physics. Mean field methods have been used throughout the periodic table, both at zero temperature and at finite but low temperatures, and typically yield T^2 behaviour of the energy density. This suggests that perhaps at low but finite temperatures a simple scaling relation might also exist for the excitation energy, E (A, T), of finite nuclei.

In order to test the validity of equation (3.3) we have made use of a finite temperature extension of the semi-empirical mass formula. The following form for the temperature dependent binding energy has been assumed

$$\begin{aligned}E(A,Z,T) = {} & \overbrace{\alpha(T)A}^{1} + \overbrace{\beta(T)A^{\frac{2}{3}}}^{2} + \overbrace{(\gamma(T) - \frac{\eta(T)}{A^{\frac{1}{3}}})(\frac{4t_\zeta^2 + 4|t_\zeta|}{A})}^{3} \\ & + \underbrace{\kappa(T)\frac{Z^2}{A^{\frac{1}{3}}}(1 - \frac{0.7636}{Z^{\frac{2}{3}}} - \frac{2.29\,\kappa^2(T)}{(0.8076)^2 A^{\frac{2}{3}}})}_{4} + \underbrace{\delta(T)f(A,Z)A^{-\frac{3}{4}}}_{5}\end{aligned} \tag{3.4}$$

Where A=N+Z, t_ζ =(1/2)(Z-N) and f(A,Z)= (-1,0,+1) for (even-even, even-odd-odd-odd) nuclei. Here 1 is the volume energy, 2 is the surface energy, 3 is the

asymmetry energy, 4 is the Coulomb energy and 5 is the pairing energy contribution to temperature dependent binding energy. In this parametrization the temperature dependence of the contributions to the Coulomb energy term, which arise from exchange and surface effects, was ignored. Also no attempt has been made to include shell effects in the finite temperature expression.

The excitation energy for particle is given by:-

$$\frac{\mathcal{E}(A,Z,T)}{A} = \frac{E(A,Z,T) - E(A,Z,0)}{A}.$$

(3.5)

At T = 0 the coefficients are given by α(0) = -16.11 MeV, β(0) = 20.21 MeV, γ(0) = 20.65 MeV, η(0) = 48.00 MeV, and κ(0) = 0.8076 MeV obtained from a fit to the experimental nuclear ground state energies of 488 odd mass nuclei. The T = 0 coefficient for the pairing term is taken as § (0) = 33.0 MeV

To obtain the temperature dependence of the coefficients, the available experimental information about the excited states of nuclei throughout the periodic table was used to determine the partition function of each neucleus in the canonical ensemble

$$\mathcal{Z}(A,Z,T) = \sum_{i}^{n} g_i \exp(-\beta E_i) + \int_{E_n}^{E_{max}} dE\ g_{A,Z}(E) \exp(-\beta E)$$

(3.6)

where g_i = 2ji + 1 is the spin degeneracy factor and E_i the excitation energy of the ith state of the nucleus, and β = 1/T . The sum in the first term of equation (3.6) runs over the experimentally measured (discrete) excited states.

Since the experimentally known spectrum in most cases is only sufficient to allow the accurate determination of Z for very low temperatures (T <<1 MeV), it is necessary to supplement the experimentally known spectrum with an appropriate approximation to the continuum g_A, Z(E). For sufficiently large energies, the usual

Fermi gas expression for the total density of states (i.e including the spin degeneracy) is used:

$$g_{A,Z}(E) = \frac{\sqrt{\pi}}{12} \frac{\exp\left(2\sqrt{a_{A,Z}U}\right)}{a_{A,Z}^{1/4} U^{5/4}}. \tag{3.7}$$

Here $a_{A,Z}$ is the level density parameter and U = E -P(N)-P(Z), where P(N) and P(Z) are the pairing corrections for neutron number N and proton number Z respectively .

This parametrization is obtained by means of a saddle point approximation, and is probably only valid up to T = 6 MeV. However, in the region up to T=4 MeV, this parametrization is probably acceptable.

At lower energies, a suitable fit to the nuclear energy level density can be obtained to the form

$$g_{A,Z}(E) = \frac{\sqrt{2\pi}\sigma}{\tau} \exp\left(E - E_0\right)/\tau, \tag{3.8}$$

with σ the spin-dependence parameter. Values for the parameters $a_{A,Z,\delta}$,E_0 and σ, as well as the respective regions where (3.7) and (3.8) should be used, for a large number of nuclei can be found in [9].

The nuclei used to determine the aforementioned coefficients can therefore be three groups:

1). Nuclei where sufficient discrete states are known to allow the use of the discrete spectrum at low energies and the Fermi gas expression (3.7) at higher energies.

2). Nuclei where the discrete spectrum does not extend high enough for (3.7) to be valid. For these nuclei, the discrete spectrum is used for low excitation energies, followed by the exponential form (3.8) for intermediate energies and finally the Fermi gas expression (3.7) at high energies.

3). Nuclei where very little of the discrete spectrum is known. In these cases, (3.8) is used for the low- and intermediate- excitation portions of the spectra and (3.7) for the highly excited part.

All three groups are spread across the whole periodic table. The lower bound E_n on the integral in (3.6) is taken to be the energy at which (3.7) should become

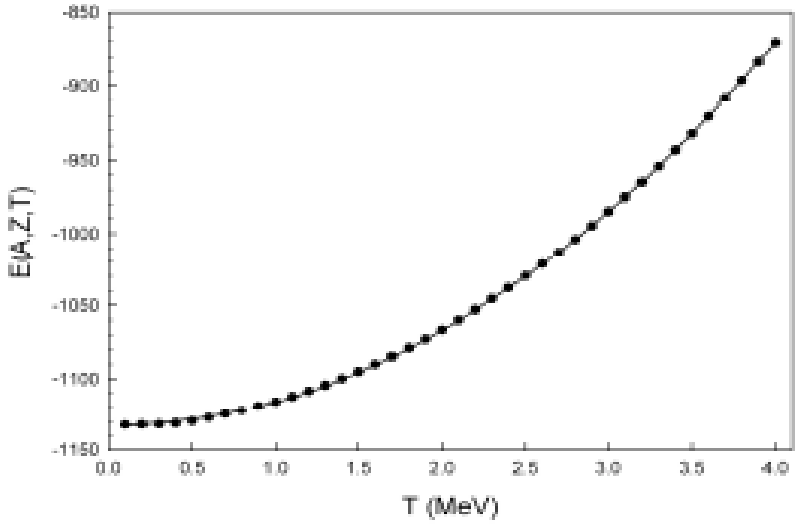


Figure.3.1: Quadratic fit of the calculated binding energy of Xe (A=130) using equation (3.4).

valid .The fit to the Xe data is representative of the fits obtained for other nuclei (From [9]). For the first case above, 80% of the largest discrete energy level for the second case above, and zero for the third case. For temperatures up to T = 4 MeV, the upper bound E_{max} =3 GeV was used. Increasing E_{max} has no effect on the numerical results in this temperature range.

The coefficients in the mass formula have been determined by a least squares fit of (3.5) to the ensemble average of the excitation energy

$$\mathcal{E}(A,Z,T) = -\frac{\partial}{\partial\beta}\ln\mathcal{Z}(A,Z,T) \tag{3.9}$$

determined from a total of 313 nuclei in the mass region 22 ≤ A ≤ 250 for temperatures T ≤ 4 MeV.

The temperature dependence can be applied as suggested in [1]. The recently developed Dynamical Cluster Decay Model (DCM)[5],[6],[8],[10]-[13] is used to study heavy ion reactions at extreme conditions of temperature, deformation and orientation etc. It is relevant to mention here that this model has been applied successfully to study the decay of compound nuclei formed in light (^{48}Cr*, ^{56}Ni*), intermediate (^{116}Ba*) and heavy (^{246}Bk*),(^{202}pb*) mass region [6], [8], [10]-[14],[44]. It is well established that macroscopic liquid drop energy V_{LDM} of the reaction partners or decaying fragments play important role in the interaction potential besides coulomb potential, proximity potential and rotational energy etc. In recent calculations [14], it is shown that the complete mass spectra in decay of excited ^{56}Ni* or the intermediate mass fragments, IMFs in the decay of excited ^{116}Ba* have their origin in the macroscopic liquid drop energy (the shell effects are almost zero at the excitation energies involved). Therefore, V_{LDM} certainly plays a significant role in the decay process of a nuclear system. In order to investigate the dynamics of a excited nuclear systems, the knowledge of temperature dependence of macroscopic part of nuclear potential becomes essential. This temperature dependent liquid drop potential can be calculated as suggested by Davidson et. al. [1]. The predictions of this model find its basis in the well-established semi- empirical mass formula of Seeger [2].

3.1.1 METHODOLOGY

The binding energy B of a nucleus at temperature T has been defined, within the Strutinsky renormalization procedure, as the sum of liquid drop energy VLDM (T) and shell correction δU (T) i.e

$$B(T)=V_{LDM}(T)+\delta U_{exp}(T^2/T_0^2) \qquad (3.10)$$

The T dependent liquid drop part of the binding energy V_{LDM} (T) or E_b is from Davidson et al.[1], based on the semi-empirical mass formula of Seeger [2], as

$$E_b(A,Z,T) = \alpha(T)A + \beta(T)A^{2/3} + (\gamma(T) - \frac{\eta(T)}{A^{1/3}})(\frac{4t_\zeta^2 + 4|t_\zeta|}{A}) + \frac{Z^2}{R(T)A^{1/3}}(1 - \frac{0.7636}{Z^{2/3}} - \frac{2.29}{[R(T)\ A^{1/3}]^2}) + \delta(T)\frac{f(A,Z)}{A^{3/4}}, \quad (3.11)$$

where

$I = a_a (Z - N), \quad a_a = 1.0,$

and, respectively, for even-even, even-odd, and odd-odd nuclei,

$f(Z,A) = (-1, 0, 1).$

α(T), β(T), γ(T), η(T) and δ(T)

For T = 0, Seeger [2] obtained the constants, by fitting all even-even nuclei and 488 odd-A nuclei available at that time, as

α(0) = -16.11MeV, β(0) = 20.21MeV,

γ (0) = 20.65MeV, η (0) = 48.00MeV,

with the pairing energy term

δ(0) = 33.0 MeV,

from Ref. [17]. Evidently, these constants need to be refitted since a large amount of data has become available [15], [16] particularly for neutron-rich nuclei. Gupta and collaborators [5], [6], [8] found that the measured binding energies could be fitted within 1 to 1.5 MeV by changing the bulk constant α(0) and introducing a proton, neutron asymmetry constant a_a. These constants are fitted for 1 ≤ Z≤ 118.

The T-dependent constants in Eq. (3.11) were obtained numerically by Davidson et al. [1] for the available experimental information on excited states of 313 nuclei in the mass region 22 ≤ A ≤ 250 by determining the partition function Z(A,Z, T) of each nucleus in the canonical ensemble and making a least squares fit of the excitation energy.

$E_{ex}(A,Z\ T) = V_{LDM}(A, Z, T) - V_{LDM}(A, Z, 0)$

to the ensemble average

$$E_{ex}(A, Z, T) = T^2 (d/dT)[\ln Z(A, Z, T)]$$

The α(T), β(T), γ (T), η(T) and δ(T) thus obtained are given in Fig.3.1 of Ref.[18] for T≤4 MeV, extrapolated linearly for higher temperatures. For the bulk constant α(T), instead, an empirically fitted expression to a Fermi gas model is used, as

$$\alpha(T) = \alpha(0) + (T^2/15)$$

Also, the δ(T) is constrained to be positive definite at all temperatures, with δ (T >2MeV) =0. Finally, the analytical form used for $R_0(T)$ is

$$R_0(T) = 1.07(1 + 0.01T).$$

3.2 Shell corrections and their temperature dependence

3.2.1 Role of shell structure

Effects of shell structure are important for all nuclei. Their role for the heaviest ones is, however, essential as these nuclei would not exit without these effects [35]. Figure 3.2. [34] illustrates shell effects in masses of heaviest nuclei. This is the difference between the experimental value of mass and those calculated within a model, which does not contain shell structure (e.g., liquid drop model).

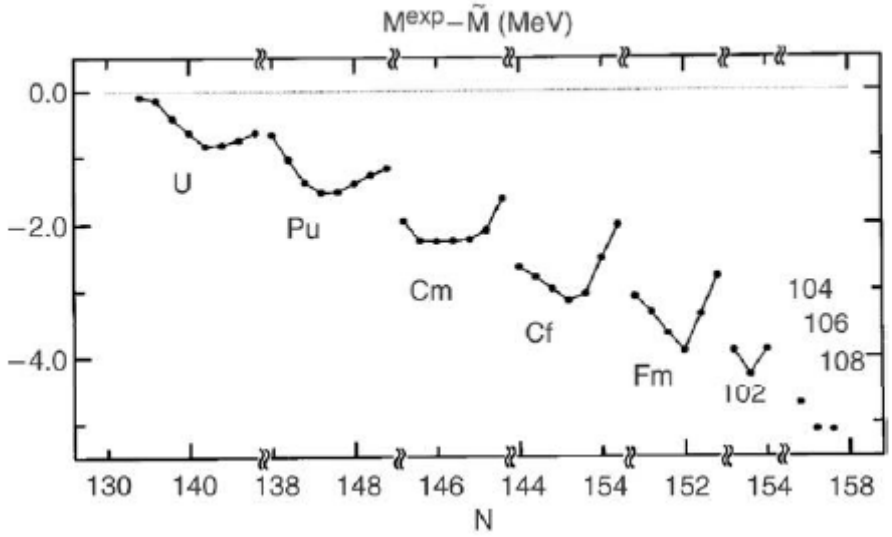


Figure 3.2. shell effects in the masses of nuclei [62].

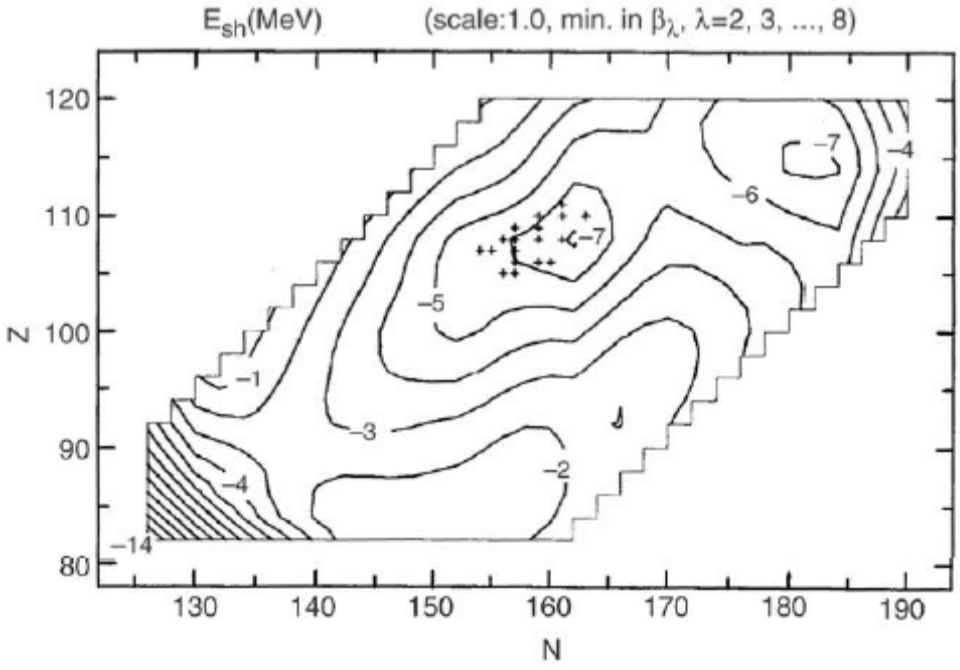


Figure 3.3 Contour map of the ground-state shell correction energy $E_{sh.}$. Crosses denote the heaviest nuclei which were synthesized at the time, when the figure was plotted [37].

One can see that the shell effect, $M^{exp} - M$, is negative (i.e. stabilizes the nuclei) and decreases (increases in its absolute value) with increasing atomic number Z, down to about -5 MeV for the heaviest even-even nuclei ^{260}Sg (Z=106) and ^{264}Hs (Z=108), known at the time when the figure was drawn. The effect of the known deformed shell at the neutron number N=152 is clearly seen in the results for Cf, Fm and No (Z=102) isotopes. The experimental values M exp, available at the time when the picture was done, have been taken from [34,36]

Similarly clear and large shell effects were found in other important properties of heaviest nuclei, such as α-decay energy Q_α and half-life T_α or height of fission barrier B_f [35]. An especially large effect, up to 15 orders of magnitude, appears in the spontaneous-fission half-life T_{sf} [35].

A first orientation to in which regions of the nuclear chart the largest effects may be expected is obtained from calculations of these effects. Figure 3.3 [37] shows a contour map of shell correction E_{sh} to the ground-state energy (or mass) of

nuclei in a large region of nuclides with proton number Z=82-120 and neutron number N= 126-190. One can see that E_{sh} has three minima in this region.

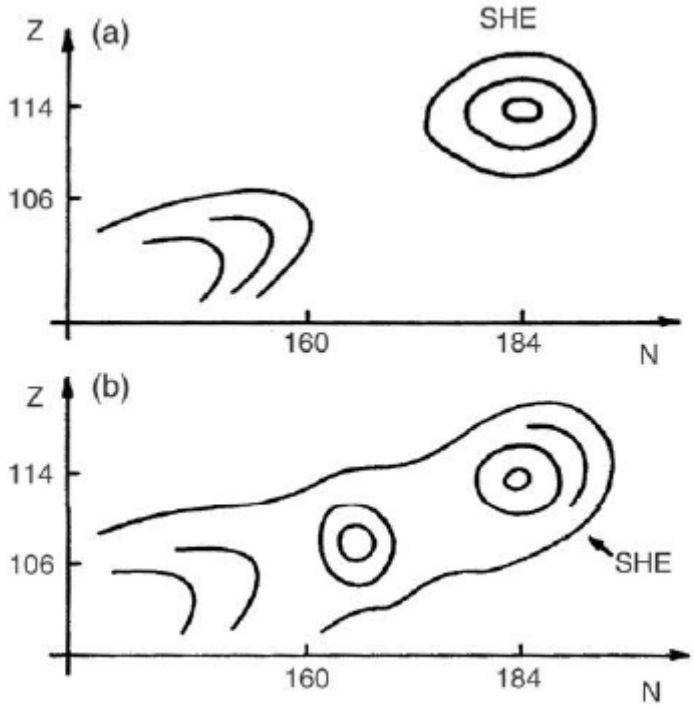


Figure.3.4. Regions of relatively long- lived heavy nuclei as believed earlier (a), and expected presently (b) [38].

The first one, which is the deepest (E_{sh} = -14.3 MeV) is obtained for the doubly magic spherical nucleus ^{208}Pb. The second one (E_{sh} = -7.2 MeV) appears at the nucleus $^{270}_{162}108$, which is predicted [65,66] to be a doubly magic deformed nucleus. The third minimum, with the same depth (E_{sh} = -7.2 MeV) as that of the second one, is obtained for the nucleus $^{296}_{182}114$, which is close to the nucleus $^{298}_{184}114$ predicted [39-41] to be a doubly magic spherical nucleus, the next one to the last experimentally known double-magic 208 Pb,. Beside these three minima, there appears a rather wide plateau around the nucleus 252 Fm, which although having a smaller (in absolute value) shell correction (E_{sh} = -5.2 MeV) than the nucleus 270 108 (270 Hs), may be considered as a nucleus with closed deformed subshells [42,43]

One can also see in Fig 3.3 that some of the already synthesized nuclei profit by 6-7 MeV in their binding energy from the shell correction. Without this profit they could not exist, as discussed in [35]

The appearance of the region of nuclei around the second minimum (deformed super heavy nuclei) constitutes probably the main change in our view of stability of heaviest nuclei in the whole history of SHE. Before this, it was believed for a long time that spherical superheavy nuclei, predicted to be situated around the third minimum, would constitute an island, separated from the usual peninsula of relatively long-lived nuclei by an “ocean” of full instability. After the appearance of deformed super heavy nuclei. This is qualitatively illustrated in fig .3.4 taken from [38].

3.2.2 Shell corrections by Strutinsky method

For the shell corrections δU in Eq. (10), since there is no microscopic shell model known that gives the shell corrections for light nuclei, we use the empirical formula of Myers and Swiatecki [19]. For spherical shapes,

$$\delta U = C \left[\frac{F(N) + F(Z)}{(A/2)^{2/3}} - C_A^{1/3} \right] \qquad (3.12)$$

With X = N or Z, M_i-1 < X < Mi and Mi as the magic numbers 2, 8, 14 (or 20), 28, 50, 82, 126 and 184 for both neutrons and protons. The constants C = 5.8 MeV and c = 0. 26. In this work, we refer to the use of magic numbers 14 or 20 as MS14 or MS20 parameterizations.

3.2.3 Temperature dependent shell effects with a realistic effective Hamiltonian

The study of thermal excitations in finite nuclei has received much attention since there is great interest in nuclear properties far from the `ground` state [22]. A significant amount of work has been devoted both to schematic and realistic

descriptions at finite temperature within Hartree-Fork [23-25], field treatments [26], and collective models [27-29].

Recently, realistic effective Hamiltonians have been adapted to the finite temperature Hartree- Fock (FTHF) method [30]. Hereinefter , we will address the role of finite temperature shell corrections in understanding the smooth and fluctuating contributions to the free energy surfaces [31-33]. We also address the consequences of self-consistency effects on the level density parameter.

Some of these temperature dependent effects have been investigated previously [32-33] in a non-self-consistent scheme, namely, by assuming the constancy of single particle energies over an extended temperature range. Although the collapse of shell corrections has been shown to occur, and in the level density parameter has also been observed in the calculations [32-33], the question about the persistence of these results in a self-consistent treatment has not been answered yet. Within the present approach, the answer to this question is associated with the thermal sensitivity of surface related quantities.

This problem is investigated by performing calculations of shell corrections at finite temperature within self-consistent finite temperature Hartree-Fock descriptions of ^{16} O and ^{40} Ca.

Here we shall briefly review some aspects of the realistic effective interaction. The effective Hamiltonian H_{eff} consists of three two-body operators each evaluated in a basis of harmonic oscillator states. First, there is the relative kinetic energy operator acting between all nucleon pairs. Second, we include a G matrix based on the Reid soft-core interaction and the lowest order folded diagram for our no-core model space. Finally, the coulomb interaction between protons is included; All the results reported here, unless specifically noted otherwise, are obtained with this H_{eff} evaluated for the lowest six major oscillator shells. The same model space was used extensively [30] to evaluate some thermal properties of ^{16} O and ^{40}Ca .for the purposes of examining the role of larger model spaces, we add a phenomenological single-particle Hamiltonian acting on shells above the first six oscillator shells. This Hamiltonian consists of the single-particle kinetic energy operator, a Woods-Saxon potential of strength-

60 MeV, diffuseness 0.6 fm, and radius 1.1 A $^{1/3}$ fm. In addition, to achieve a smooth matching with the HF spectrum at T=0, an additional overall positive shift of 20 MeV was added to the diagonal terms of the phenomenological Hamiltonian.

Following the notation of ref[30] , we treat H_{eff} in the FTHF approximation, which is to minimize the free energy.

$$F = H_{\text{eff}} - TS \ , \tag{3.13}$$

where T is the temperature and S in the single-particle entropy

$$S = -\sum_{\nu} [f_{\nu} \ln f_{\nu} + (1 - f_{\nu}) \ln (1 - f_{\nu})] \ , \tag{3.14}$$

f_{ν} being the thermal occupation probabilities for fermions in orbitals labeled by v, namely:

$$f_{\nu} = \left[1 + \exp \left[\frac{e_{\nu} - \mu}{T} \right] \right]^{-1} . \tag{3.15}$$

The chemical potential μ are determined separately for neutrons and protons and e_{ν} are the self-consistent single-particle energies, which are the solutions of FTHF equations,

$$h \mid \nu \rangle = e_{\nu} \mid \nu \rangle \ , \tag{3.16}$$

with

$$\langle \alpha \mid h \mid \beta \rangle = \sum_{\gamma} \langle \alpha\gamma \mid H_{\text{eff}} \mid \beta\gamma \rangle f_{\gamma} \ . \tag{3.17}$$

in this context, we can write the FTHF energy as:-

$$\begin{aligned} E_{\text{HF}} &= \sum_{\alpha} e_{\alpha} f_{\alpha} - \tfrac{1}{2} \sum_{\alpha < \beta} \langle \alpha\beta \mid H_{\text{eff}} \mid \alpha\beta \rangle f_{\alpha} f_{\beta} \\ &= \tfrac{1}{2} \sum_{\alpha} e_{\alpha} f_{\alpha} \ . \end{aligned} \tag{3.18}$$

with this single particle basis, we can calculate finite temperature shell corrections to the energy . The formalism has been presented in ref [33] and here we shell follow the same procedure. We can therefore introduce a smooth energy

$$\tilde{E} = \sum_{\nu} \int_{-\infty}^{\infty} de \, \bar{g}_{\nu}(e) e f(e) \ , \tag{3.19}$$

and a smooth entropy

$$\tilde{S} = -\sum_{\nu} \int_{-\infty}^{\infty} de\, \bar{g}_{\nu}(e)\{ f(e)\ln f(e) + [1-f(e)]\ln[1-f(e)]\} \,, \tag{3.20}$$

where

$$\bar{g}_{\nu}(e) = \frac{1}{\gamma(\pi)^{1/2}} \exp\left[-\left[\frac{e-e_{\nu}}{\gamma} \right]^2 \right] \times \sum_{m=0}^{M} a_{2m} H_{2m}\left[\frac{e-e_{\nu}}{\gamma} \right] , \tag{3.21}$$

is the smoothing function and f(e) are the fermion occupation numbers defined as continuous functions of the energy e. consequently, we can write for the smooth free energy the following expressions:

$$\tilde{F} = \tilde{E} - T\tilde{S} \,, \tag{3.22}$$

and we can relate this quantity with the FTHF free energy, namely [33]

$$F_{\mathrm{HF}} = \tilde{F} + \delta\tilde{F}_{\mathrm{shell}} \,. \tag{3.23}$$

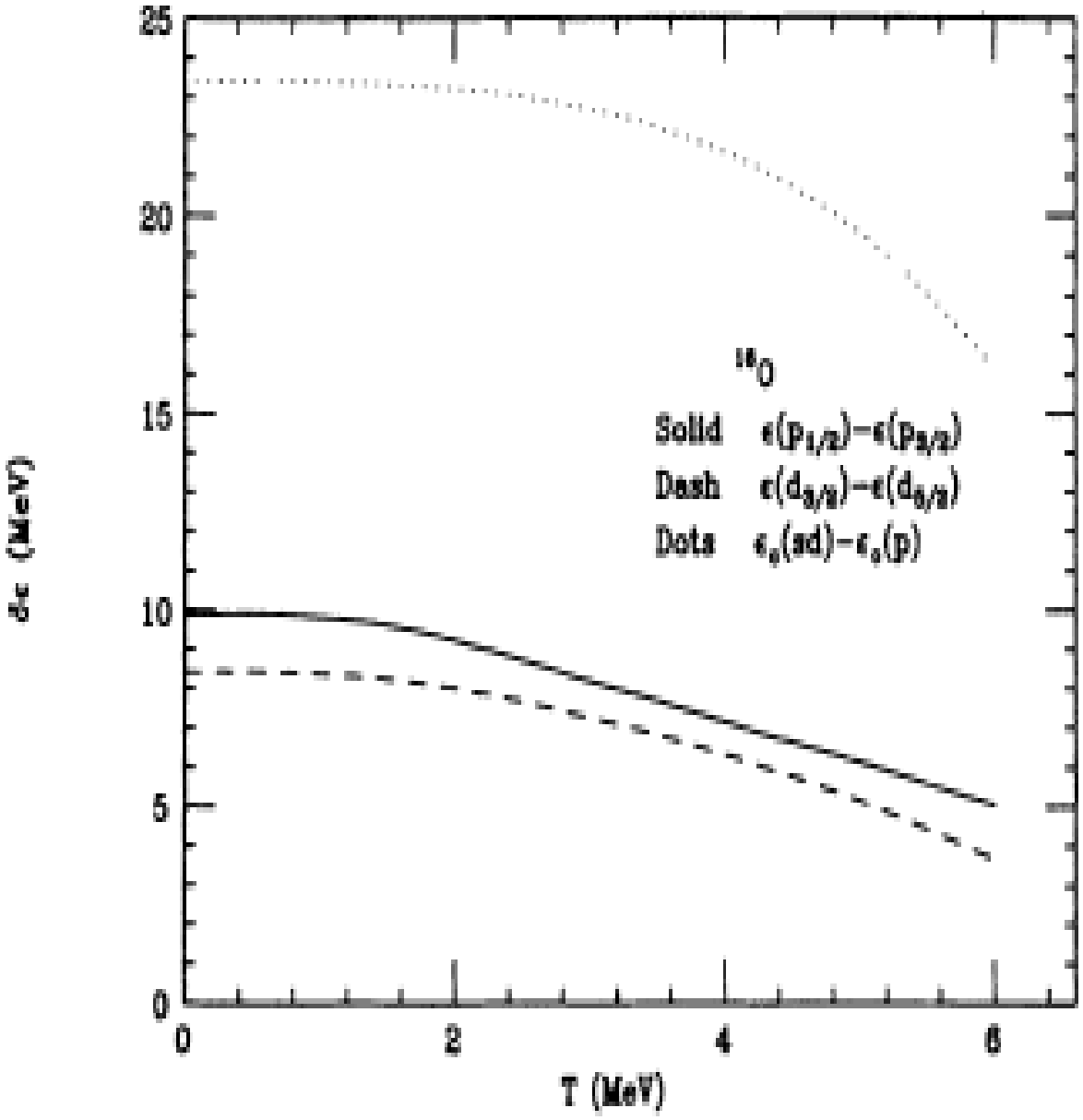
equation (23) is the finite temperature definition of the shell correction term $\delta\tilde{F}_{\mathrm{shell}}$, in full analogy with the zero temperature case, where for the energy we have

$$E_{\mathrm{HF}} = \tilde{E} + \delta\tilde{E}_{\mathrm{shell}} \,. \tag{3.24}$$

3.2.4 Results and discussion

The results for single particle energies, matrix elements of the effective interaction, and related global thermodynamical quantities have been presented in ref [30]. As well as new calculations in larger model spaces in order to obtain the single-particle bases which are needed for the present estimates of shell effects in ^{16}O and ^{40}Ca as a function of temperature. The most striking feature associated with temperature dependent effects, is the washing out of the spin-orbit splitting as shown in fig 3.5. Note that for both nuclei the spin-orbit splitting decreases by an amount between 30% and 50% as T increases from 0 to 6 MeV. The collapse of spin-orbit splittings occur for levels both above and below the

Fermi energy. These results are in marked contrast with those obtained from the Skyrme Hamiltonian [33-37], where virtually no T dependence of the single-particle energies was reported in this temperature range.



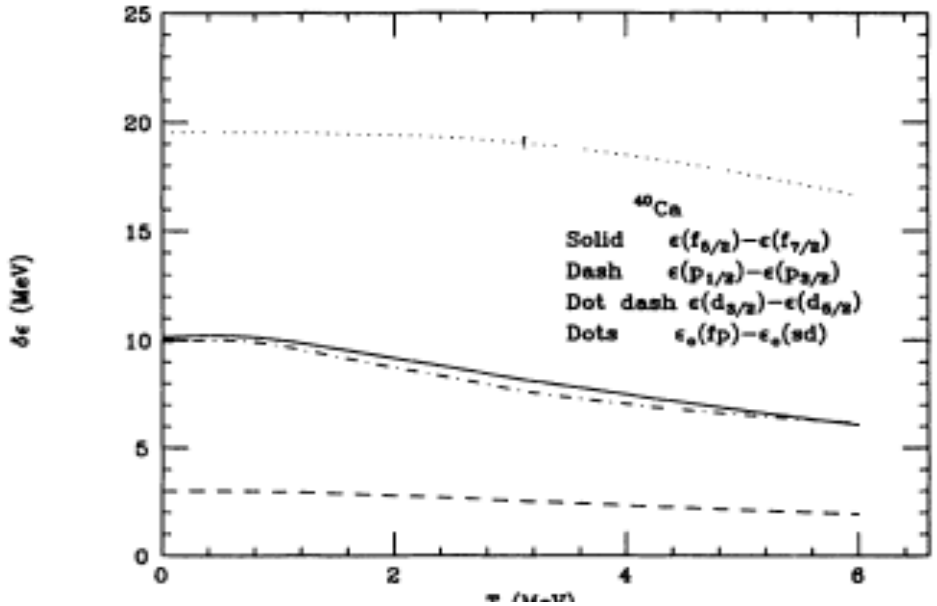


Figure 3.5:- self consistent spin-orbit splittings and gaps between shell centroids $\acute{\varepsilon}_c$ for 16 O and 40 Ca as a function of temperature T. all results are for the neutron single-particle energies.

Fitting of seeger's constants:-

Although Seeger's formula was quite successful in explaining the available literature of its time (1961) but our knowledge regarding the understanding of nuclear properties has grown, consequently the constants used in this model need major revival in order to meet the present day requirements.

Actually the aim is not to fit the constants of LDM, but to include the temperature dependence explicitely on the recent binding energies. Therefore, our group[44] have fitted only the bulk α(0) and neutron proton asymmetry a_a constants of Seeger's formula at T=0. This refitting of model parameters is extremely useful in view of availability of enormous amount of binding energy data in [3],[4]. This work has been partly done upto Z=56 in [5],[6],for experimental data provided at that time [7]. But later refitted the bulk, α(0) and neutron proton asymmetry, a_a constants in reference to improved experimental data [3]. In the recent work not only previous fittings are improved but also extended upto Z=118. the domain of this work has been further extended for neutron deficient and neutron excess

nuclides (for which experimental binding energies are not available). Such nuclear systems have their own importance due to significant change in various nuclear properties corresponding to these combinations. For these neutron deficient/neutron excess nuclei calculations were made in reference to the theoretical binding energies made available by Moller-Nix[4]. These fitted constants have been successfully used in the recent calculations to study the decay of heavy compound nucleus ^{246} Bk* [8] formed in heavy ion reactions at different incident centre of mass energies for two entrance channels ^{11} B+ ^{235} U and ^{14} N+ ^{232} Th, ^{202}pb* formed in 48Ca+ 154Sm reaction [44].

Our group have refitted the seeger's constant in reference to the experimental binding energies [15] and the theoretical binding energies [16] (only for neutron excess and neutron deficient nuclide for which experiment data is not available). The constants required to be fitted are the bulk constant α(0), working as an overall scaling factor, and the proton-neutron asymmetry constant a_a, controlling the curvature of the experimental parabola.

The role of these refitted constants is depicted in Figure 3.6 [15], [16] for Z=97 nuclides. The Figure 3.6 shows the excellent agreement between the present fits (cross and down open triangle) corresponding to experimental data of [15] (solid circles) and theoretical data [16] (open circle), respectively. The fits are obtained between 0-1.5 MeV of the available binding energies (experimental and theoretical). In Fig. 3.6 [15], [16] the calculated binding energies using Seeger's constants are also shown (hollow square), stressing the requirement and extent of fitting can clearly be anticipated.

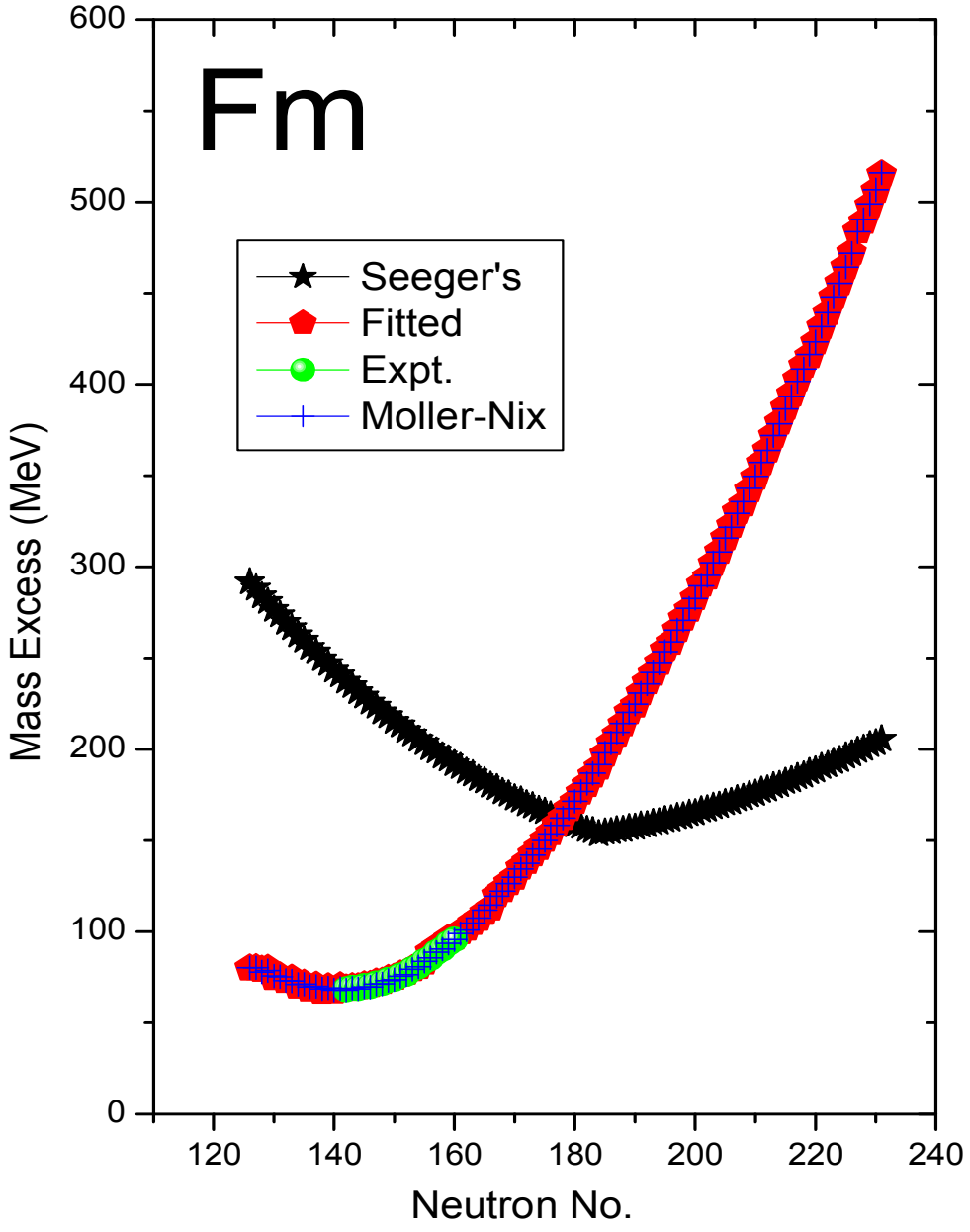


Figure 3.6 The mass excess ΔM (=M_A –A = NM_n +ZM_p+ B(Z,N) -A) in MeV as a function of neutron number N for Z=100, calculated by using the experimental data [15] and theoretical data [16], with newly fitted constants and calculated with the 1961 Seeger's constants[2]

References

[1] N. J. Davidson, S. S. Hsiao, J. Markram, H. G. Miller and Y. Tzeng, Nucl. Phys. A 570, 61c (1994).

[2] P. A. Seeger. Nucl. Phys., 25, 1 (1961).

[3] G. Audi, A.H. Wapstra and C. Thiboult, Nucl. Phys. A 729, 337 (2003).72.

[4] P. Moller, J. R. Nix, W. D. Myers and W. J. Swiatecki, At. Nucl. Data Tables 59, 185 (1995).

[5] R. K. Gupta, R. Kumar, N. K. Dhiman, M. Balasubramaniam, W. Scheid and C. Beck, Phys. Rev. C 68, 014610 (2003).

[6] M. Balasubramaniam, R. Kumar, R. K. Gupta, C. Beck and W. Scheid, J. Phys.G: Nucl. Part. Phys. 29, 2703 (2003).

[7] G. Audi and A.H. Wapstra Nucl. Phys. A 595, 4 (1995).

[8] B. B. Singh, M. K. Sharma and R. K. Gupta, Phys. Rev. C 77, 054613 (2008).

[9] A.Gilbert and A. G. W. Cameron. Can. J. Phys, 43, 1446 (1965).

[10]R. K. Gupta, M Balasubramaniam, R. Kumar, D. Singh and C. Beck Nucl. Phys.A738, 479c (2004).

[11] K. Gupta, M. Balasubramaniam, R. Kumar, D. Singh, C. Beck and W. Greiner, Phys. Rev. C 71, 014601 (2005).

[12] K. Gupta, M. Balasubramaniam, R. Kumar, D. Singh, S. K. Arun and W. Geiner, J. Phys. G: Nucl. Part. Phys. 32, 345

[13] B. Singh, M. K. Sharma, R. K. Gupta and Walter Greiner Int. J. Mod. Phys. E 15, 699 (2006).

[14] R. K. Gupta,M. Balasubramaniam, C. Mazzocchi, M. La Commara and C. Beck, Phys. Rev. C 65, 024601 (2002).

[15] G. Audi, A.H. Wapstra and C. Thiboult, Nucl. Phys. A 729, 337 (2003).72

[16] P. Moller, J. R. Nix, W. D. Myers and W. J. Swiatecki, At. Nucl. Data Tables 59, 185 (1995).

[17] S. DeBenedetti, Nuclear Interactions (New York: Wiley) (1964)

[18] E. E. Salpeter. Private communication

[19] W. Myers and W.J. Swiatecki, Nucl. Phys. 81, 1 (1966).

[20] S. DeBenedetti, Nuclear Interactions (New York: Wiley) (1964).

[21] J.M. Lattimer and F. D. Swesty. Nucl.Phys. , A535, 331 (1991)
[22] M. Lefort, Nucl.Phys. A387,3C(1982).
[23] M . Brack and P. Quentin, Nucl.Phys.A361, 35 (1981).
[24] A. L. Goodman, Nucl. Phys. A352, 30 (1981); A352, 45 (1981); A369, 365 (1981); A370, 90 (1981);A402, 189 (1983).
[25] A.L. Goodman, Phys. Rev. C29, 1887(1984)
[26] A. K. Kerman, S. Levit, and T. Troudet, Ann. Phys. 148, 436 (1983)
[27] W. Besold, P-G. Reinhard, and C. Toepffer, Nucl.Phys.A431,1 (1985).
[28] N. Vinh Mau and D. Vautherin, Nucl. Phys. A445, 245 (1985).
[29] P. Bonche, S. Levit, and D. Vautherin, Nucl. PHYS. A427, 278(1984).
[30] G. Bozzolo and J.P. Vary, Phys, Rev. C31, 1909 (1985).
[31] Y. Alhassid and J.Zingman, Phys. Rev. C30,684 (1984).
[32] O. Civitarese and A. L. De Paoli, Nucl. Phys. A440, 480 (1985).
[33] O. Civitarese et al., Z. Phys. A 305, 341 (1982); 309,177 (1982);311, 317 (1983).
[34] P. Armbruster, in: R.A. Broglia, G.F. Bertsch (Eds.), Proc. Int. School on Heavy Ion Physics, Erice, Italy, 1986,
Plenum, New York, 1988, p. 153.
[35] Z. Patyk, A. Sobiczewski, P. Armbruster, K.-H. [36] A.H. Wapstra, G. Audi, Nuclear Phys. A 432 (1985) 1.
[37] R. Smola´nczuk, A. Sobiczewski, in: Low Energy Nuclear Dynamics, St. Petersburg, Russia, 1995, World Scientific, Singapore, 1995, p. 313.
[38] Z. Patyk, J. Skalski, A. Sobiczewski,
[39] A. Sobiczewski, F.A. Gareev, B.N. Kalinkin, Phys. Lett. 22 (1966) 500
[40] H. Meldner, Ark. Fys. 36 (1967) 593.
[41] U. Mosel, W. Greiner, Z. Phys. 222 (1969) 261.
[42] Z. Patyk, A. Sobiczewski, Nuclear Phys. A 533 (1991) 132.
[43] Z. Patyk, A. Sobiczewski, Phys. Lett. B 256 (1991) 307.
[44] S. Kanwar, M K Sharma, B B Singh, R K Gupta, W Griever, journ. Of Mod. Phys. E. (accepted) 2009.

Chapter-4

Summery

The model parameters, bulk constant α(0) and the proton-neutron asymmetry constant a_a of Seeger's formula at T=0 are refitted in view of availability of a larger data set for binding energies. These fitted constants can be used in order to understand the structure and dynamics related aspects of excited nuclear systems.

Actually the aim of refitting these constants is simply to include the T-dependence on experimental binding energies, and not to obtain the new parameter set of V_{LDM}. In view of this the readjustment in bulk constant α(0) and the proton-neutron asymmetry constant a_a enables us to obtain experimental binding energies B_{expt} within <1.5 MeV accuracy. Actually this refitting of seeger's constants (1961) plays extremely useful role to establish temperature dependent binding energies (macroscopic part of nuclear potential) in reference to Davidson etel [1994] work. The availability of these refitted constants for large number of nuclear isotopes extends a possibility to investigate the heavy ion reactions at extreme conditions of temperature angular momentum, deformation and orientation etc.

The table of refitted constants is uploaded in Thapar University library database. These refitted constants are successfully applied to test the problems releted to nuclear structure and dynamics for a variety of nuclear systems ranging from low medium to heavy mass systems in recent times by our group and collaborators.

It is strongly anticipated that this bunch of refitted constants is extremely useful in reference to present and future planned activities related to nuclear dynamics in general and nuclear structure in particular. This thesis is an overview of developments related to the temperature dependent potentials and its possible relevance. I am extremely thankful to all the authors whose work is utilized and reffered in order to achieve this task.