

A

THESIS REPORT

On

**Mass Balancing of 3-Cylinder and 4-Cylinder Inline Engines
with Balancer Shafts using Bond Graph Approach**

*Submitted in partial fulfillment of the requirement
for the award of degree of*

Master of Engineering

in

CAD/CAM Engineering

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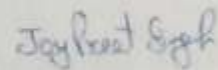
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DECLARATION

I hereby declare that work done in this thesis report entitled, "Mass Balancing of 3-Cylinder and 4-Cylinder Inline Engines with Balancer Shafts using Bond Graph Approach" submitted towards partial fulfillment of requirement for award of Master of Engineering degree in CAD/CAM Engineering in Mechanical Engineering Department of Thapar University, Patiala, is an authentic record of work carried out by me under the supervision and guidance of Dr. Tarun Kumar Bera, Associate Professor of Mechanical Engineering Department, Thapar University, Patiala along with Mr. Jasvir Singh Bisht, Senior Manager, Product Design and Development, VE Commercial Vehicles Ltd. Pithampur, Madhya Pradesh.

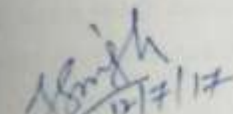
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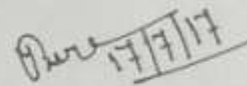
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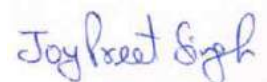
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Abstract

The internal combustion engines are widely used as a source of power generation in automobiles. These engines are subjected to noise, vibrations and harshness (NVH) due to unbalanced inertia forces and moments. To minimize the unbalance, various engine configurations are analyzed for the unbalanced forces and moments. Any unbalance within an engine can result in component fatigue, excessive vibration, and residual noise especially at higher engine speeds. Engines of trucks and buses develop high inertia forces which make it more essential that engine is well balanced. A well-balanced engine is more durable and operates smoothly with increases ride comfort for driver and passengers. In this thesis report, mass balancing of 3-cylinder and 4-cylinder inline engines is studied and counterweights of crankshaft and balancing masses of balancer shafts are designed in order to neutralize these inertia forces and moments developed in the engine and to make the engine well balanced as a whole. Bond graph approach is utilized for modelling of engine balancing. First, 3-cylinder inline engine is modelled, further balancer shaft model has been developed and attached to main engine model. Results from simulation shows that with the introduction of balancer shaft, the engine gets balanced i.e. rocking couple of engine disappears. Going further, two balancer shafts have been modelled and are integrated with the 4-cylinder inline engine model. The simulation results are validated with theoretical results.

Key words: 3-cylinder and 4-cylinder engine; Noise, vibration and harshness; Mass balancing; Slider crank mechanism; Balance factor; Bob weight; Lanchester balancing technique; Balancer shaft; Bond graph

List of acronyms

BDC	Bottom Dead Center: Position of the crankshaft when the piston is at the bottom position in the cylinder.
CAD	Computer Aided Design
COG	Center of Gravity
MOI	Moment of Inertia
NVH	Noise, Vibrations and Harshness
RPM	Revolutions Per Minute
TDC	Top Dead Center: Position of the crankshaft when the piston is at the top position in the cylinder.

Nomenclature

a	Cylinder spacing
b	Axial distance between two balance masses COG of balancer shaft
B_u	Unbalance for balance weights of balancer shaft
C_{di}	Coefficient of discharge of inlet valve
C_{do}	Coefficient of discharge of outlet valve
C_v	Specific heat at constant volume
CW_u	Unbalance for counterweight of crank shaft
d	Width of piston
F_P	Primary inertia force
F_S	Secondary inertia force
J	Moment of inertia
J_f	Moment of inertia of flywheel
K	Stiffness
l	Connecting rod length
l_{cg}	Distance between point E_c and center of gravity of cylinder
l_p	Distance between point E_p and center of gravity of piston
l_{pg}	Distance between point E_p and center of gravity of piston rod
m_b	Mass of engine block
M_b	Moment generated by balancer shaft
m_{bf}	Front balancing mass of balancer shaft
m_{br}	Rear balancing mass of balancer shaft
m_c	Mass of cylinder
m_{cr}	Mass of connecting rod assembly
m_{crb}	Connecting rod big end rotating mass, located at crank pin (m_A)
m_{crs}	Connecting rod small end reciprocating mass, located at piston pin (m_B)
m_{CW}	Mass of counter weight of crankshaft

M_P	Primary reciprocating unbalanced moment
m_{pa}	Mass of piston assembly (piston, piston pin, piston rings, 2 circlips)
m_{re}	Reciprocating mass: mass of piston assembly + connecting rod small end mass
m_{ro}	Rotational mass: mass of rotating crank + connecting rod big end mass
m_{roc}	Rotational mass of crank pin journal, cut section of crank cheek and crank web
M_S	Secondary reciprocating unbalanced moment
N	RPM
P_a	Cross-sectional area of piston
P_{atm}	Atmospheric pressure
r	Crank radius
R	Gas constant
r_{bf}	Radial distance of front balancing mass COG
r_{br}	Radial distance of rear balancing mass COG
r_{CW}	Radial distance of counter weight COG
r_{ro}	Radial distance of rotating mass COG
T_{atm}	Atmospheric temperature
V_{ia}	Area of intake valve
V_{oa}	Area of exhaust valve
x	Displacement in x-direction
y	Displacement in y-direction
\dot{x}	Velocity in x-direction
\dot{y}	Velocity in y-direction
θ	Crank angle
$\dot{\theta}$	Angular velocity about z-axis
μ	Transformer modulus
ω	Angular velocity of crankshaft ($2\pi N/60$)
λ	Connecting rod ratio $\left(\frac{r}{l}\right)$
λ_{ba}	Overall heat transfer coefficient between body and environment

λ_{gb}

Overall heat transfer coefficient between gas and body

γ

Ratio of specific heats

Subscripts

a	Area
atm	Atmosphere
b	Block
B	Balancer shaft
ba	Body and environment
bf	Front balancing mass of balancer shaft
br	Rear balancing mass of balancer shaft
c	Cylinder
cg	Center of gravity of cylinder
cr	Connecting rod
crb	Connecting rod big end
crs	Connecting rod small end
CW	Counterweight of crankshaft
di	Discharge of intake valve
do	Discharge of exhaust valve
f	Front
F	Force
fw	Flywheel
H	Horizontal
i	Intake valve
ia	Intake valve area
max	Maximum
o	Outlet/exhaust valve
oa	Outlet/exhaust valve area
p	Piston
P	Primary
pa	Piston assembly
pg	Center of gravity of piston

r	Rear
re	Reciprocating part
ro	Rotating part
roc	Rotational portion of crank
S	Secondary
U	Unbalance
v	Volume
V	Vertical
0	Initial
1,2	Contact point number

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Chapter 1

Introduction

The objective of this thesis report is to study that how engine mass balancing can be achieved in inline 3, 4 and 6-cylinder internal combustion engines. The main focus has kept on 3-cylinder and 4-cylinder inline engines as the 6-cylinder inline engine is self balanced. Balancing is the act of redistributing the masses of rotating and reciprocating components in an engine, such that the conditions for static as well as dynamic balancing are achieved. The act of balancing is a means of evening out unwanted vibrations and forces in the engine caused by an unbalanced mass. Engine needs to be balanced so that vibrations during operating speed do not result in resonance.

In 3-dimensional system, an object is said to be in static as well as in dynamic equilibrium if summation of all the forces and moments in all the 3 mutually perpendicular directions (x , y and z) are zero. i.e. $\sum F_x = \sum F_y = \sum F_z = 0$ and $\sum M_x = \sum M_y = \sum M_z = 0$ [1].

Balancing of a constant rotating force can easily be done by an equal and opposite constant rotating force [2] such that $F_2 = -F_1$ in Fig. 1.1

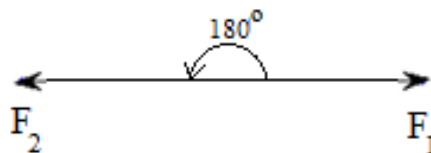


Fig. 1.1 Balancing of a constant rotating force

1.1 Background and motivation

The study of inline internal combustion engine mass balancing has been very useful in overcoming engine noise, vibration and harshness (NVH). The unbalanced reciprocating and rotating forces and moments gets developed inside the engine as it runs. Their magnitude increases as the engine speed increases. These unbalanced forces and moments are prime cause of engine NVH. Any unbalance within an engine can result in component fatigue, excessive vibration, and residual noise. In order to reduce them, engine mass balancing is done. Balancer shafts are used to counteract unbalanced reciprocating forces and moments. Counterweights of crankshaft are responsible to counteract the rotational unbalanced

moments. A well balanced engine is more durable and operates smoothly with increases ride comfort for driver and passengers. In this thesis, mass balancing of 3-cylinder and 4-cylinder inline engines has been studied and counterweights of crankshaft and balancing masses of balancer shafts have designed in order to neutralize the inertia forces and moments developed inside the engine, so as to make the engine well balanced as a whole. Further, bond graph approach has been well utilized for inline engine mass balancing. Symbols Shakti 2.0, a modelling and simulation software had provided to be highly beneficial in this thesis. Its results were validated with classical methodology.

1.2 Types of balancing

There are different types of balancing as shown in Fig. 1.2. The main concentration is on dynamic mass balancing i.e. summation of all the inertia forces and the unbalanced moments is zero. The moving parts in an engine are either rotating (crankshaft, balancer shaft, big end of connecting rod etc.) or reciprocating parts (piston assembly, small end of connecting rod), these should be so placed that they counterbalance each other and minimise the vibrations. Mechanical balancing comprises of static balancing and dynamic balancing [3].

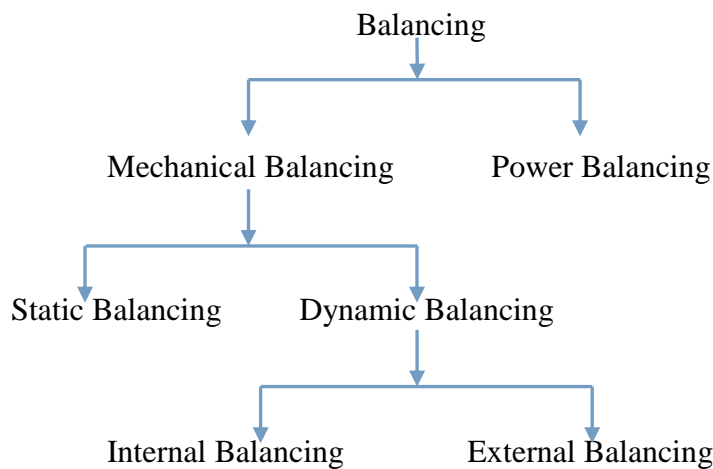


Fig. 1.2 Types of balancing

1.2.1 Static balancing

In static balancing, the combined centre of mass of the entire system lies on the rotational axis of the system. The weights of all parts on crankshaft are to be distributed equally around the central axis. If crankshaft tends to roll when placed over knife edges, it shows that it is out of balance, to bring it in balance, metal is to be drilled from counterweights until the crankshaft does not tend to rotate when placed in any orientation.

Also, the sum of all the forces on the rotating system must be zero ($\Sigma F=0$) i.e. every mass is coupled with a mass that results in the same force on the opposite side of the rotational axis. Static balancing is also known as single-plane balancing i.e. the masses which are producing the inertia forces are in or nearly in, the same plane.

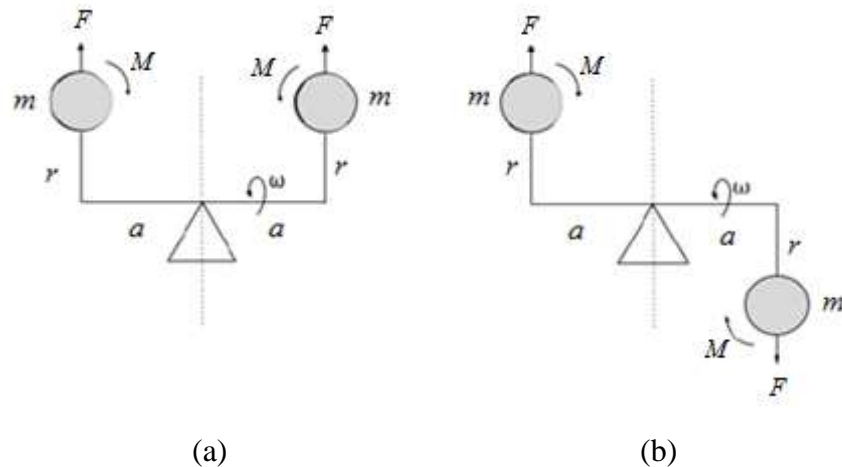


Fig. 1.3 Two examples of static balancing

In Fig. 1.3(a), the crankshaft is statically balanced in moments but not in forces, where as in Fig. 1.3(b), forces are balanced but not moments. These unbalanced moments create a rocking couple as shown in Fig. 1.4 [4].

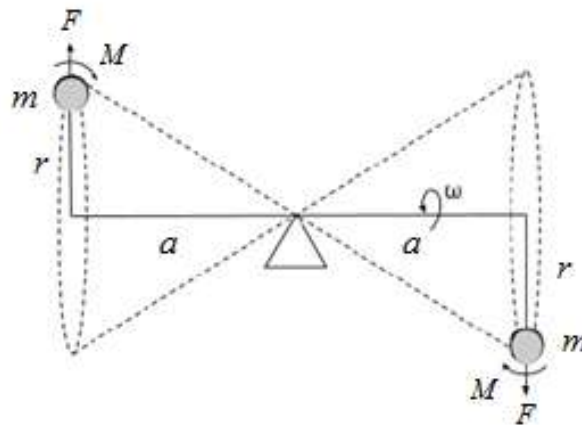


Fig. 1.4 Statically force-balanced crankshaft set in motion creates a rocking couple due to unbalanced moments

This rocking couple is balanced by adding another couple of the same magnitude but on opposite side of the rotational axis as shown in Fig. 1.5 (Dynamic balancing)

1.2.2 Dynamic balancing

Engine and its parts must not vibrate while it is running. In order to achieve this, weights has been placed directly opposite to the weights which are to be balanced. The centrifugal forces

forms couple when several masses rotate in different planes. A system of rotating masses is said to be in dynamic balancing when there does not exist any resultant centrifugal forces or resultant couple [5]. It is achieved by installing counterweights such that the rocking couple disappears. From the balancing point of view the manufacturers spin each crankshaft and bob weights on a dynamic balancing machine and remove mass from the counterweights.

Dynamic balancing: $\Sigma F=0$ (static balancing) and $\Sigma M=0$. Dynamic balanced object must be statically balanced but reverse is not true. A crankshaft is dynamic balanced when both the sum of forces and the sum of moments about the COG are zero. A dynamic balanced crankshaft is shown in Fig. 1.5

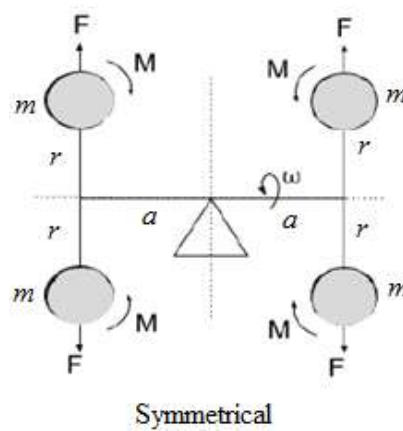


Fig. 1.5 Dynamic balanced crankshaft

Dynamic balancing is of two types: Internal balancing and external balancing. In an internally balanced engine, the counterweights alone can be made to balance the crankshaft where as in external balanced engine; the counterweights are not heavy enough to fully balance the engine components, so the flywheel and damper must be mounted on the crankshaft prior to balancing.

To convert an externally balanced engine into an internal balance, some metal have been removed from the counterweights and cylindrical slug of a much heavier metal has substituted as shown in Fig. 1.6. The substance is “Mallory metal”, an alloy of tungsten. “Densalloy” is another material. They are added to counterweights such that axis of cylindrical slug is parallel to crankshaft rotational axis, to prevent their tendency to come out under the effect of centrifugal forces acting on them if they are installed perpendicular to crankshaft rotational axis.



Fig. 1.6 Heavier material is added into the counterweights [6]

1.3 Forces in the engine

The crankshaft is subjected to forces from the rotating and reciprocating masses of the engine. The orientation of the reciprocating forces i.e. inertia forces changes with the translation of the piston inside the cylinder from TDC to BDC and vice versa. The rotating masses cause forces i.e. centrifugal forces that rotate with the crankshaft as shown in Fig. 1.7

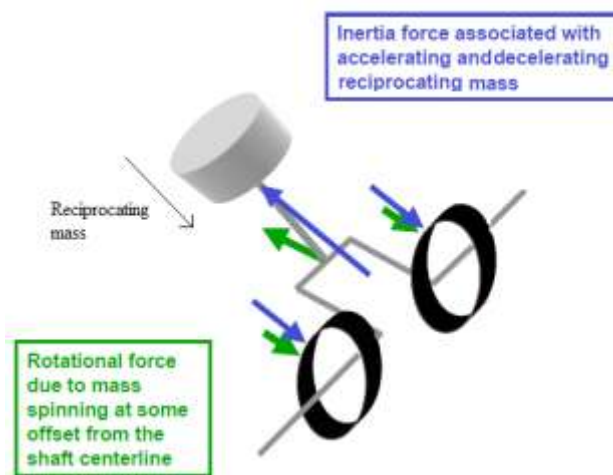


Fig. 1.7 Reciprocating and rotational forces in single cylinder engine [7]

1.3.1 Primary and secondary reciprocating forces

As the piston moves, the connecting rod transfers the reciprocating motion into rotational motion of the crankshaft. Primary reciprocating forces varies periodically once per crankshaft rotation where as secondary reciprocating forces varies periodically twice per crankshaft rotation, both are shown in Fig. 1.8

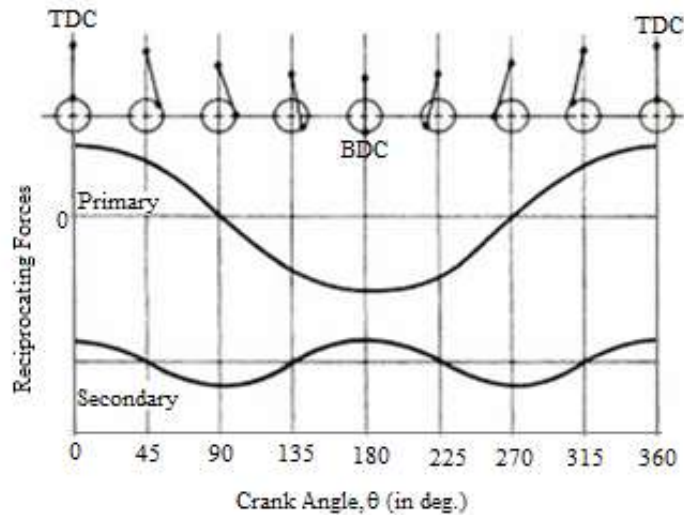


Fig. 1.8 Reciprocating forces in single cylinder engine [7]

1.3.2 How secondary forces are generated?

In Fig. 1.9, the stroke is of 1 unit length and the connecting rod is equal to 2 units are assumed, resulting in a total length of 3 units (when piston is at TDC). Intuitively, one would think that when the crankshaft has rotated to 90° after TDC (ATDC), the piston would have to travel half the stroke, but the simple geometric calculations prove otherwise. When the crankshaft has rotated to 90° after TDC (ATDC), $h = 2.4635$ and not 2.5 (using Pythagoras theorem).

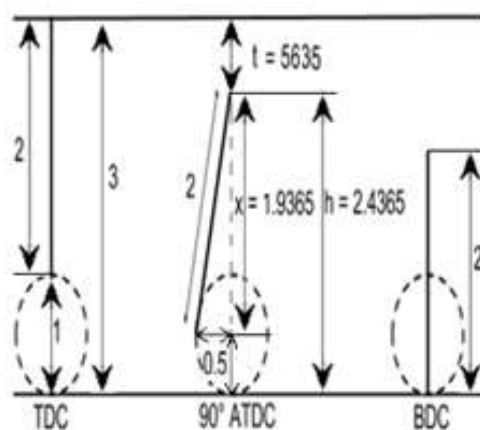


Fig. 1.9 The connecting rod travels greater distance from TDC to 90° ATDC than from 90° ATDC to BDC in the same amount of time [8]

The piston has travelled more than half its stroke when the crankshaft has rotated from TDC to 90° ATDC, shown graphically in Fig. 1.10 [8]. The piston translation, t , from TDC to 90° ATDC: $t = 3 \text{ units} - h = 3 \text{ units} - 2.4365 \text{ units} = 0.5635 \text{ units}$, i.e. Piston has travelled

more than half of its stroke when the crankshaft has rotated from TDC to 90° ATDC, as $\text{Stroke}/2 = 0.5 \text{ units} < 0.5635 \text{ units}$. This is also shown in Fig. 1.10

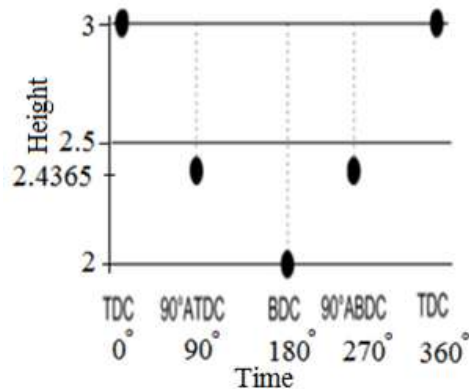


Fig. 1.10 The piston travels greater distance from TDC to 90° ATDC than from 90° ATDC to BDC in the same amount of time. This is repeated on the way up from 90° ABDC to TDC

Hence, the piston travels longer distance in the 1st and 4th quadrant of the rotating circle, the crankshaft is accelerated, i.e. piston is moving faster at the top than at the bottom of rotational circle as shown in Fig. 1.11. The changes of acceleration twice in one crankshaft rotation give rise to secondary inertia forces i.e. forces with frequency twice the crankshaft rotation.

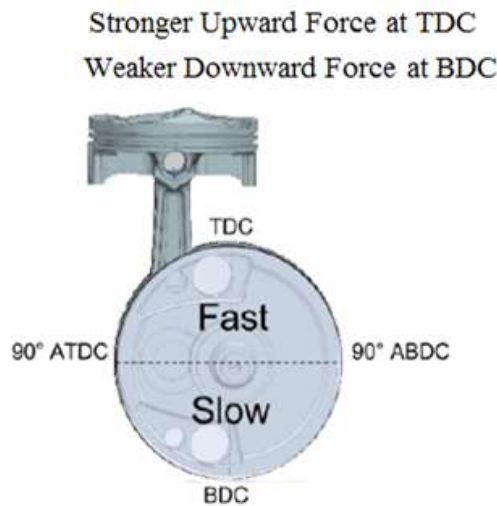


Fig. 1.11 The piston accelerated from TDC to 90° ATDC to TDC, and slows down from 90° ATDC to 90° ABDC, resulting in secondary forces.

The piston velocity is not sinusoidal, but is skewed, with higher velocities near TDC than BDC [7] as shown in Fig. 1.12

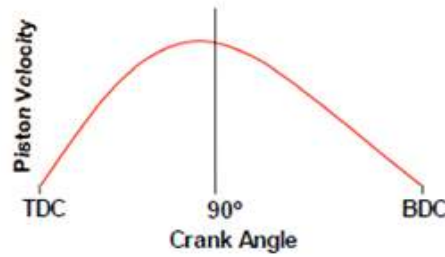


Fig. 1.12 Piston velocity vs crank angle

1.4 Dynamic balancing machines

Such machines are used to measure the unbalanced force couple in crankshafts. Mass is removed from the counterweights to balance the forces created by the imbalance. Both the magnitude and angular location are displayed by the machine from where the mass is required to be drilled out so to made the crankshaft bob weights assembly dynamically balanced. The mass of the piston assembly and the connecting rod assembly is added to the crankshaft as bob weights, and the crankshaft is rotated. Bob weight: 100% connecting rod rotational mass + 50% total reciprocating [9]. When a crankshaft is balanced, the actual connecting rods and pistons cannot be used in the dynamic balancing machine, so they must be simulated. This simulated weight is called the “Bob weight”. They are mounted on crankpins.

1.4.1 Calculation of bob weight [10]

Connecting rods have a big end (considered to be part of the rotating weight) and a small end (considered to be part of the reciprocating weight) after the connecting rod has been reconditioned. These two ends are weighted separately. Various steps for bob weight calculations have been shown in Fig. 1.13.



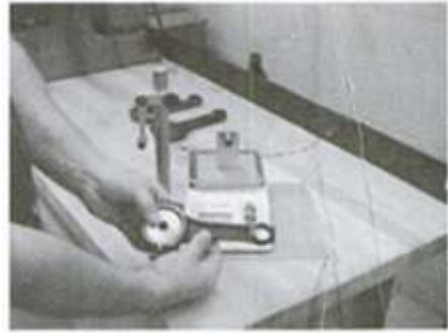
(a)



(b)



(c)



(d)



(e)



(f)



(g)



(h)



(i)



(j)



(k)

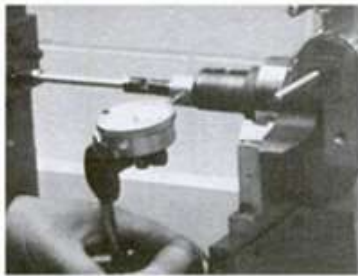


(l)

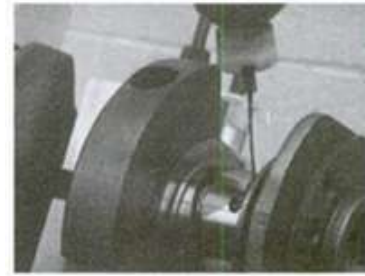
Fig. 1.13 Bob weight calculation

1.4.2 Steps to achieve final balancing [10]

Crankshaft and bob weights assembly is spinned on dynamic balancing machine at known RPM. The software used on this machine shows that how much material is to be removed from the counterweights and from where i.e. angular location so as to achieve final balancing. The material removal process does not happen in one go but it is an iterative process i.e. after removal of some material from the counterweight, the crankshaft is again rotated on the machine and the software again tells how much material is needed to be removed and so on as shown in Fig. 1.14.



(a)



(b)



(c)



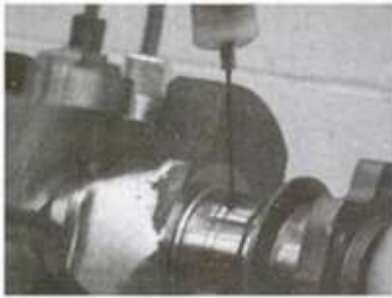
(d)



(e)



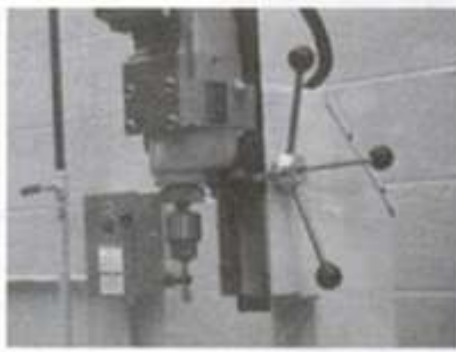
(f)



(g)



(h)



(i)



(j)

Fig. 1.14 Steps to achieve final balancing

1.5 Slider crank mechanism

The single cylinder reciprocating engine is subjected to rotational forces from the crankshaft and reciprocating forces from the piston assembly. The complex motion of the connecting rod is one of the challenges when working with the system. The big end of the connecting rod, which is attached to crank pin has a purely rotating motion and the small end of the connecting rod, which is attached to piston pin has purely reciprocating motion. The rest of the connecting rod will move in a motion that is neither purely rotational nor purely reciprocal. Unless properly balanced, these forces may negatively affect the engine performance, as well as causing vibrations that may be harmful to the vehicle and the rider.

1.5.1 Balancing of the slider crank mechanism

Connecting rod exhibits both rotation as well as reciprocating motion as shown in Fig. 1.15, It is tedious to analysis it. Therefore, its dynamic equivalence is created. A particular object is said to be “dynamically equivalent” to another object if same set of forces and moments produces the same set of motions.

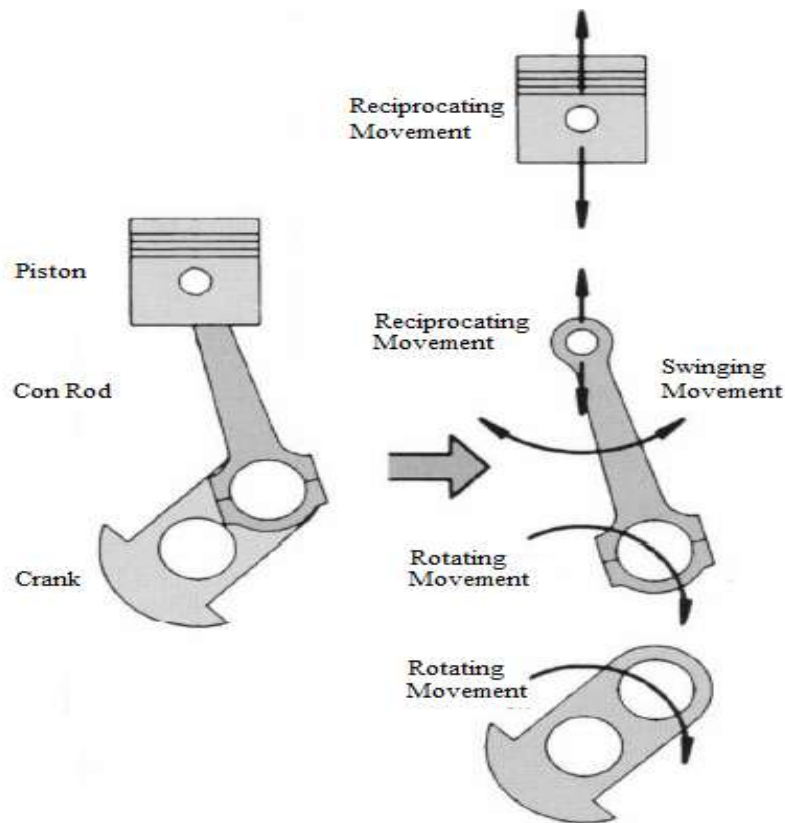


Fig. 1.15 Movements of crank, connecting rod and piston [11]

The actual connecting rod has been replaced by a hypothetical connecting rod. The hypothetical connecting rod is divided into two concentrated masses, one placed at piston end (purely reciprocating mass: m_B) and other at crank pin end (purely rotating mass: m_A), making the connecting rod itself mass-less. The connecting rod simplification is illustrated in Fig. 1.16.

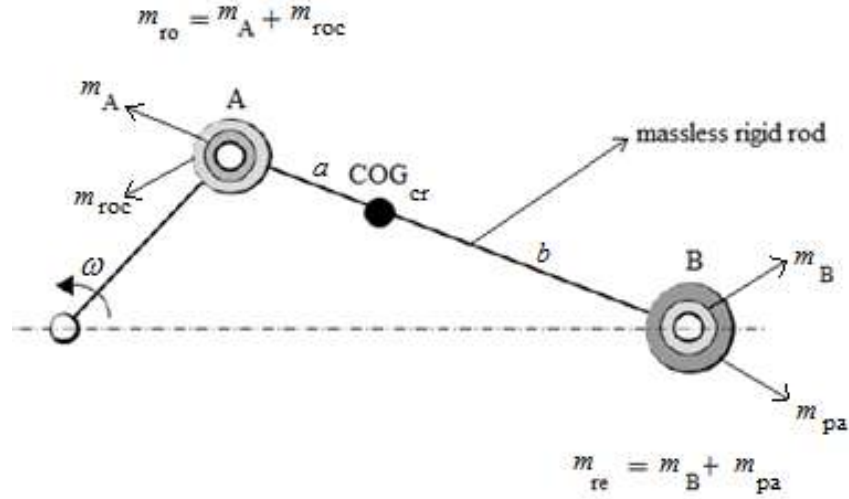


Fig. 1.16 Dynamic equivalent system of connecting rod assembly

The simplified connecting rod must fulfill 3 conditions (for dynamic equivalence) as follows:

1. The combined mass of connecting rod big-end (m_A) and connecting rod small-end (m_B) must be equal to the mass of the entire connecting rod assembly (m_{cr}). Connecting rod assembly consists of connecting rod, small end bushing, big end lower and upper shell, two bolts:

$$m_A + m_B = m_{cr} \quad (1.1)$$

2. The location of COG in the two systems remains same so that the moments produced by COG also remain same. The centre of gravity of the connecting rod assembly is unaffected:

$$m_A \times a = m_B \times b \quad (1.2)$$

where a and b are the distances from m_A to COG_{cr} and m_B to COG_{cr} respectively.

3. Same set of moment about COG_{cr} should produce same angular acceleration. This is possible when MOI of equivalent system about COG will be same as MOI of original connecting rod about its COG. The moment of inertia is unaffected:

$$m_A \times a^2 + m_B \times b^2 = I_{cr} \quad (1.3)$$

Total mass of original connecting rod assembly remains same and location of COG_{cr} also remains same. As there are only two unknown parameters (m_A and m_B) in these three equations, its only possible to satisfy two out of the three conditions. According to Prof. Amitabha Ghosh [12], it should be sufficient to satisfy only Eq 1.1 and Eq 1.2. Therefore, the

rotating mass consists of the crank pin journal; cut section of crank cheek, crank web and big-end of the connecting rod:

$$m_{ro} = m_{roc} + m_A \quad (1.4)$$

and, the reciprocating mass consists of the piston assembly (piston, piston pin, piston rings, two circlips) and small-end of the connecting rod:

$$m_{re} = m_{pa} + m_B \quad (1.5)$$

These masses produce unbalanced forces when the engine is operating. To counteract them, counterweight is added 180° from the crankpin. Purely rotating force (produced by m_{ro}) can be taken care of placing counterweights placed at opposite ends of crankpin journal, balances not only the original crank pin but also big end mass of connecting rod.

1.5.2 Forces developed in slider crank mechanism

Reciprocating Inertia Forces are unbalanced forces due to inertia of reciprocating parts. These forces are transmitted from the piston along the connecting rod, through the crankshaft and onto the main journal where it causes severe vibrations. Hence, these forces are needed to be balanced.

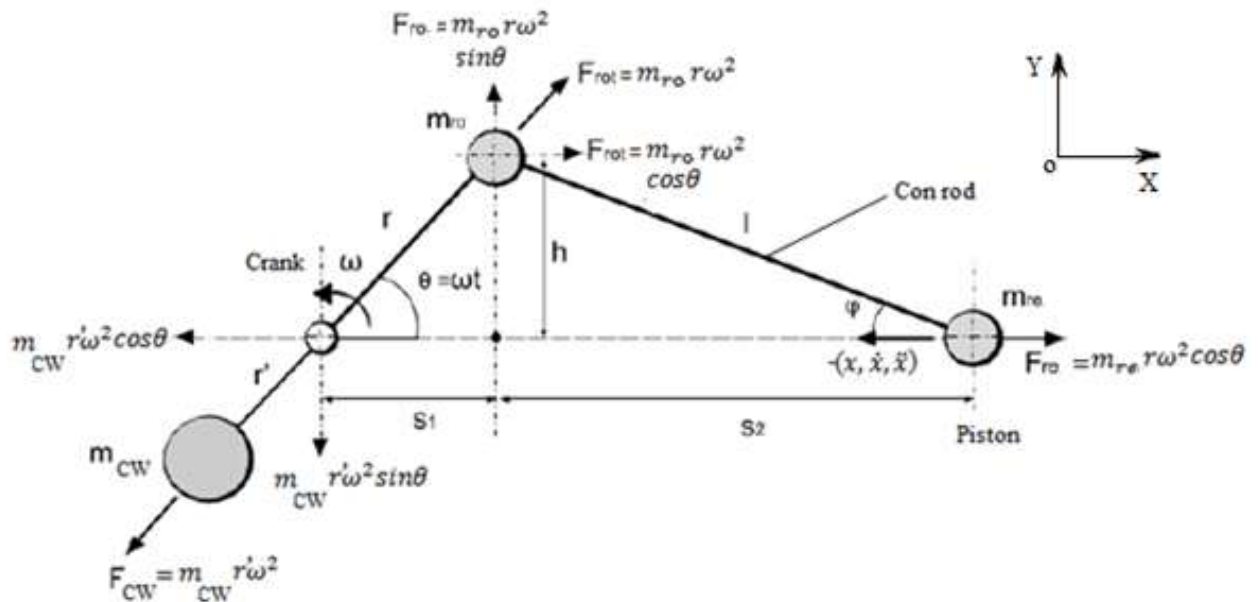


Fig. 1.17 Forces produced in slider crank mechanism

The reciprocating force in slider crank mechanism or, single cylinder engine is then expressed as [13]:

$$F_{re} \cong \underbrace{m_{re} \omega^2 r \cos(\omega t)}_{\text{Primary}} + \underbrace{\frac{\lambda m_{re}}{4} (2\omega)^2 r \cos(2\omega t)}_{\text{Secondary}} \quad (1.6)$$

Both these components are along X direction only (along cylinder line). \cong sign is used because higher order terms has been neglected. It is the horizontal component of Inertia Force (F_x).

The vertical component of primary inertia force (F_y) is given by:

$$F_y = m_{re} \omega^2 r \sin(\omega t) \quad (1.7)$$

F_x and F_y can be balanced by installing counterweights on the cranshaft.

Balancing of secondary reciprocating forces requires use of two secondary balancer shafts. Primary reciprocating force's frequency is equal to the rotational frequency of crankshaft i.e. they goes through one full cycle for each revolution of crankshaft. Secondary reciprocating force's frequency is equal to twice the rotational frequency of crankshaft i.e. they goes through two full cycles for each revolution of crankshaft.

Both primary and secondary inertia forces along with their resultant force are shown in Fig. 1.18. [14]

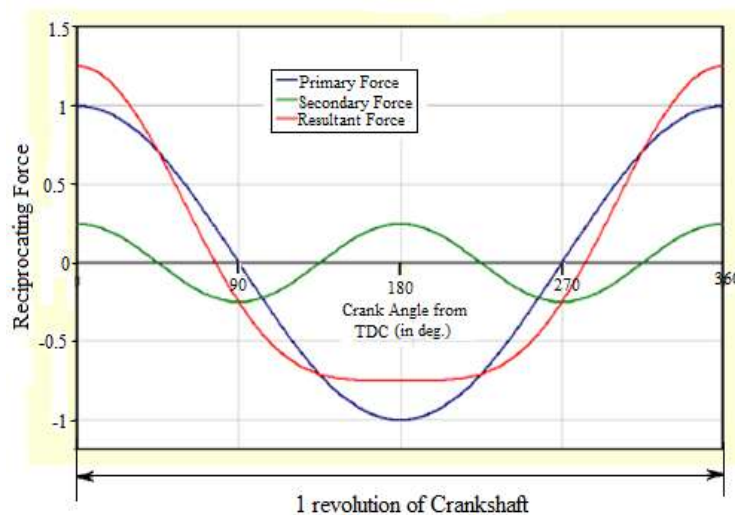


Fig. 1.18 Plot of primary, secondary and resultant of reciprocating forces w.r.t crank angle

Single cylinder engine produces unbalanced forces in a matter that part of it can be balanced by counterweights. The reciprocating masses produces an unbalanced reciprocating forces along the cylinder centre line. Now, to neutralize primary and secondary components

of unbalanced inertia forces, there is a requirement of an active system which produces some balancing forces matching exactly the magnitude and been opposite in direction.

Hence, multi-cylinder engines are designed in such a manner that unbalanced forces of individual cylinder neutralize each other, then the whole engine thus becomes free from unbalanced forces. Each unit i.e. piston assembly and connecting rod assembly are identical to one another in their masses and dimensions i.e. slider crank mechanism of each and every component of the engine is identical. Centrelines of each and every slider crank mechanism are parallel, gap between each centre line is fixed (say, a). Each crank radius and connecting rod length are r and l respectively.

When the engine is running such that at one instant only one cylinder gets fired, therefore, each firing of individual cylinders are uniformly distributed over time, for 4-stroke engine (crankshaft rotates twice for completion of 1 cycle).

To distribute the firing uniformly, the angle of rotation of crankshaft between consecutive firings will be [15]:

$$\text{Firing interval, } \theta = \frac{4\pi}{\text{no. of cylinders}} \quad (1.8)$$

Replace 2 with 4 in Eq. 1.8 for 2-stroke engines.

Hence, crank has to be so designed that the firing takes place at uniform interval. At TDC and BDC, velocity of piston is zero, but with maximum acceleration and deceleration. Due to this, crankshaft is subjected to shocks which are called primary inertia forces.

1.6 Balance Factor (f)

It is the fraction of reciprocating mass to be added into counterweight. The reciprocating force acting along the cylinder axis (in-line) can be balanced by adding fraction (f) of m_{re} to the counterweight. The primary reciprocating force will then be completely balanced when the piston is at TDC and BDC. However, adding m_{re} to the counterweight will also create a force in transverse direction to cylinder axis, with its peak at mid-stroke ($\sin 90^\circ = 1$). Instead of eliminating the unbalance force, it is shifted 90° , illustrated in Fig. 1.19

$$F_{ret} = m_{re} r' \omega^2 \sin \theta \quad (1.9)$$

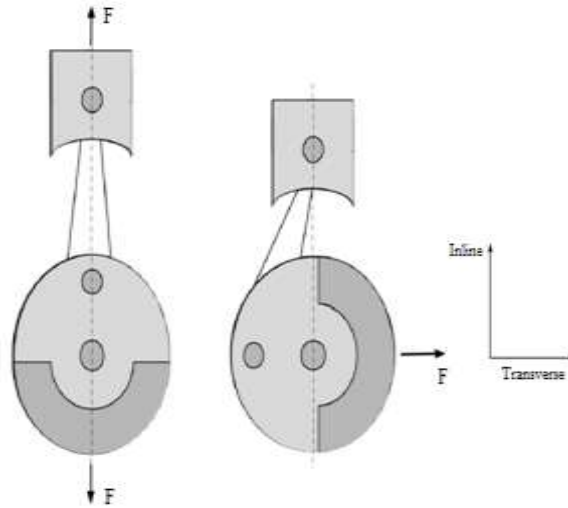


Fig. 1.19 Adding m_{re} to counterweight balances the reciprocating forces at TDC and BDC, but introduces a transverse force during the rest of stroke (max. at $\theta = 90^\circ$ and 270°)

The reciprocating force can be reduced along the cylinder axis by adding percentage of the reciprocating mass to the counterweight. This percentage is called the balance factor (f). A low “ f ” will give higher in-line forces while a high “ f ” will give higher transverse forces.

Hence, mass of counterweight [16]:

$$m_{CW} = m_{ro} + f \times m_{re} \quad (1.10)$$

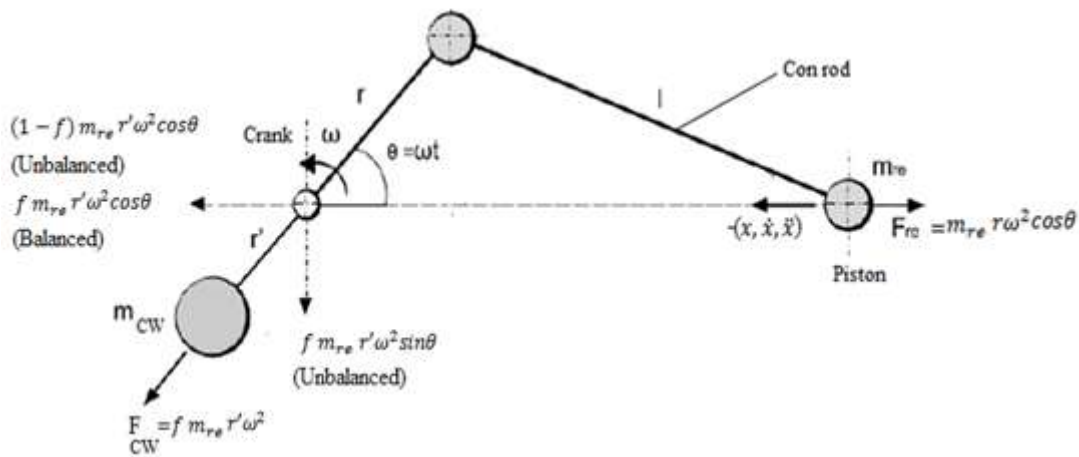


Fig. 1.20 F.B.D of unbalanced forces in slider crank mechanism

Referring Fig. 1.20, If “ f ” is the fraction of the reciprocating mass, primary force balanced by the CW:

$$F_{pb} = f m_{re} r' \omega^2 \cos \theta \quad (1.11)$$

Primary force unbalanced by the CW:

$$F_{\text{pub}} = (1 - f)m_{\text{re}}r'\omega^2 \cos \theta \quad (1.12)$$

Vertical component of centrifugal force which remains unbalanced:

$$F_{\text{vub}} = fm_{\text{re}}r'\omega^2 \sin \theta \quad (1.13)$$

Resultant unbalanced force at any instant:

$$F_{\text{res}} = \sqrt{[(1 - f)m_{\text{re}}r'\omega^2 \cos \theta]^2 + [fm_{\text{re}}r'\omega^2 \sin \theta]^2} \quad (1.14)$$

For minima resultant unbalanced force, differentiate F_{res} w.r.t θ and equate it to zero.

$$\frac{dF_{\text{res}}}{d\theta} = 0 \quad (1.15)$$

On solving Eq. 1.15, “ f ” comes out to be 0.5 or 50%. Thus, the resultant unbalanced force is minimum when balance factor (f)= 0.5. Hence, mass of counterweight:

$$m_{\text{CW}} = m_{\text{ro}} + 0.5 \times m_{\text{re}} \quad (1.16)$$

Therefore, a balance factor of 50% is often used as it results in the lowest resultant forces.

1.7 Bond graph approach

Bond graph modelling technique is a technique which utilizes the law of conservation of energy to develop a generalized model which is capable of representing systems from different energy-domains. It was developed in 1959, by Prof. H.M. Paynter. The technique works on a model built up by arranging two types of junctions i.e. 1-junction and 0-junction which are further branched and connected with seven types of elements as explained below.

The basic seven elements that are used for representing a system in bond graph technique are C, I, R, SE, SF, TF and GY. A line segment known as a bond connects all these elements. These bonds represent the flow of power. Flow of power means effort and flow in opposite directions. In one direction effort flows and in the other direction flow flows. Classification of a bond graph model can be done as follows:

➤ Single port passive elements

Compliance element (C), Inertial element (I), and resistive element (R) are single port passive elements. These may be active or passive. Active elements are usually sources of power to the system while passive elements only interact with the system power. Single port elements are as explained follows:

Compliance element:

The compliance element represented by 'C' is connected to a junction element 1 or 0. It always has an integral causality. It is an energy storage element of the system. It may be graphically as represented in Fig. 1.21.

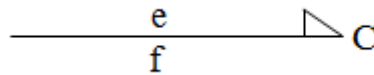


Fig. 1.21 Representation of compliance element

Inertial element:

Inertial element represented by 'I' is also an energy storage element. Relationship between effort and flow is given by this I element in such a way that flow can be obtained by integrating effort. It is graphically as represented in Fig. 1.22

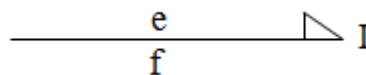


Fig. 1.22 Representation of inertance element

Resistive element:

Resistive element represented by 'R', unlike I and C elements which are energy storing elements is an energy dissipater element. R-element may be integrally or differentially causalled depending on the junction it is connected to. It may be graphically represented as shown in Fig. 1.23.

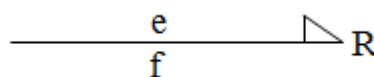


Fig. 1.23 Representation of resistance element

Source of effort:

Source of effort represented by 'SE' gives power to the system in the form of effort acting external to the system. Graphically, it may be represented as shown in Fig. 1.24.

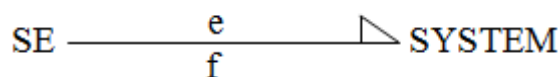


Fig. 1.24 Representation of source of effort element

Source of flow:

Source of flow represented as 'SF' gives external flow to system. Graphically, it may be represented as shown in Fig. 1.25.

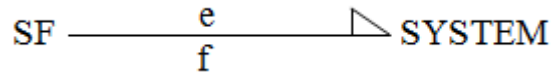


Fig. 1.25 Representation of source of flow element

➤ Two port elements

Two port elements are connected to the system at two points. The two port elements are converters which can convert flow into effort or vice versa or simply multiplies one value to form other depending on the junctions they connect and on the desired values. The 2-port elements in bond graph theory are the 'transformer' and the 'gyrator'. These are represented by 'TF' and gyrator is 'GY', respectively.

Transformer:

The transformer neither creates nor destroys energy. It simply redistributes the flow and effort information in between the bond junctions. It magnifies effort from one side to the. The flow multiplication takes place in the direction of arrow and effort multiplication in the opposite. It is as shown in Fig. 1.26.

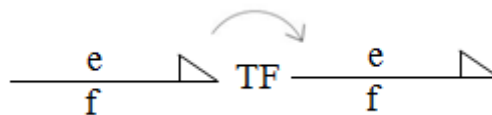


Fig. 1.26 Representation of transformer element

Gyrator:

The Gyrator also neither creates nor destroys energy. It simply redistributes the flow and effort information in between the bond junctions. The gyrator element can convert flow into effort and effort into flow. It is usually assigned to a junction representing change in domain. A gyrator element is as shown in Fig. 1.27.

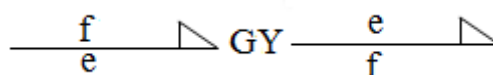


Fig. 1.27 Representation of gyrator element

. The advantages of bond graph are as follows:

- Need of only 7 elements: SE (Source of Effort), SF (Source of Flow) [Both Active]; I(Inertial), C(Compliance), R(Resistive) [All Passive]; TF (Transformer), GY (Gyrator) [Two Port Elements].
- Multi-disciplinary systems (e.g. mechanical, electrical, hydraulic etc.) can be modelled and results can be simulated.
- C++ language basis and mathematical modelling.
- State equations (Ordinary Differential Equations) are generated by the software itself.
- No “black box” model: Expressions and relations are inserted in the software as per the need.
- Identification of unknown parameters by positioning of sensors (flow and/or effort) to know the kinematic (velocity, mass flow rate etc.) and/or dynamic quantities (force, pressure etc.)
- Faults can be localisable and detectable by positioning detectors.
- Applications: Automobile Engineering, Robotics, Aerospace, AGVs, Biomedical Engineering, Mechatronics, Electrical Engineering, Manufacturing, Vehicle Dynamics etc.

1.8 Organization of the thesis

This thesis work is divided into 5 chapters which can be summarized as follows:

Chapter 1 explains engine mass balancing and its different types, as well as the forces that arise in a reciprocating engine. The dynamic balancing machine, which is still widely used and recommended, is described. The slider-crank mechanism which is often used as a simplification of the single cylinder reciprocating engine is discussed. Balance factor (f) and bond graph technique for modelling systems is also discussed.

Chapter 2 presents literature review done in order to complete this thesis mostly on inline engine mass balancing and bond graph approach of internal combustion engines.

Chapter 3 deals with the three-cylinder inline engine mass balancing. Expressions for various forces and moments, both reciprocating (primary and secondary) and resultant of rotating moments are derived; along with vector diagrams are discussed. What is the need of balancer shaft is discussed. The optimum profile of counter weights and balancer weights are obtained in CAD. Going further, free body diagrams of rotating masses (balancing masses, counter weights and rotating parts of crankshaft) and reciprocating masses are drawn and

successive tables are used comprising of various forces and moments acting on the engine. CAD (Creo 3.0) is utilized to create 3-D models and to obtain their mass properties (mass, COG location, mass moment of inertia etc.). 3-cylinder inline engine mass balancing using bond graph approach is also done using modelling and simulation software i.e. Symbols Shakti 2.0. Also, 6-cylinder inline engine study has been carried out.

Chapter 4 is dedicated to the mass balancing of 4-cylinder inline engine. Lanchester balancing technique is used for mass balancing of such an engine configuration. For modelling of 4-cylinder inline engine, firstly combustion model of engine, then piston-cylinder model and finally two balancer shafts were modelled in Symbols Shakti 2.0 software and then they are arranged in 4-cylinder inline configuration. Simulations were performed with the given parameters and balancing results from software were validated with classical formulae.

Chapter 5 presents conclusions obtained from the research work conducted during thesis work. In addition to this, this chapter also deals with future scopes of the thesis.

Chapter 2

Literature Review

This chapter discusses the literature review done for the completion of the thesis and the objectives of the thesis. Literature review was done in two areas: (i) Mass balancing of internal combustion engines and (ii) Bond graph technique.

2.1 Literature survey on mass balancing

The internal combustion engines are widely used as a source of power generation in automobiles. These engines are subjected to Noise, Vibrations and Harshness (NVH) due to unbalanced inertia forces and moments. To minimize the unbalance, mass balancing of the engines has to be done [1]. There are different types of engine balancing like static and dynamic balancing [3]. In I.C. engines masses are divided as rotating and reciprocating masses. Rotating masses consist of big end of connecting rod assembly and rotational portions of the crankshaft (crank pin, crank web and crank cheek) whereas reciprocating masses consist of small end of the connecting rod assembly and the piston assembly [1]. Slider crank mechanism is the basis for engine mass balancing. It is a single cylinder engine. The reciprocating inertia force is due to the motion of reciprocating components within the engine. It is an unbalanced force and has two components i.e. primary and secondary [13]. The secondary forces in an engine are developed due to the acceleration and deceleration of the piston assembly [8, 16]. Balance factor is the fraction of reciprocating mass (usually 50%) to be added into counterweight [16].

Lanchester balancing technique is used for complete dynamic balancing of the engine i.e. primary as well as secondary balancing. It is an elaborate system and done in special cases. It is the basis for four-cylinder inline engine mass balancing [17]. In four-cylinder inline engines, there are only secondary reciprocating forces and no primary inertia forces. These secondary reciprocating forces are balanced by using two balancer shafts [18], both rotate at twice the speed of crankshaft. Also, they rotate in opposite direction to one another. Also, due to the symmetric configuration of such an engine there are no moments developed whether it may be primary or secondary [19]. The six-cylinder engine is also symmetric about central plane hence neither it develops unbalanced forces nor unbalanced moments [19]. Crankshaft manufacturers use dynamic balancing machines to balance the crankshafts along with bob

weights assembly [10]. Bob weight consists of 100% connecting rod rotational mass and 50% total reciprocating mass [9]. When a crankshaft is balanced, the actual connecting rods and pistons cannot be used on the dynamic balancing machine, so they must be simulated. This simulated weight is called the bob weight. They are mounted on crankpins.

In order to balance the primary unbalanced moments in 3-cylinder inline engines, a mechanical rotating component known as balancer shaft is used [10]. It is placed parallel to the crankshaft and rotates with same angular velocity as that of crankshaft but in opposite direction. 3-cylinder inline engine has crankpins oriented with 120 degrees with respect to each other. In such configuration, there are no inertia forces but primary and secondary unbalanced moments due to reciprocating masses exist. Also, unbalanced moments due to rotating masses are present [20]. In order to balance these moments, counterweights are designed in such a way that they balance 100% of rotating unbalanced moments along with 50% of primary reciprocating moments. The rest of primary reciprocating moments are balanced by the balancer shaft [21]. The purpose of balancer shaft in 3-cylinder engine is to counteract the rocking/pitching couple. This rocking couple is generated due to the reciprocating forces [22].

2.2 Literature survey on bond graph modelling technique

Bond graph technique has become a popular approach since its initiation in early 1960's for modelling a vast variety of systems. It has found applications in automobile engineering, robotics, aerospace, AGVs, biomedical engineering, mechatronics, electrical engineering, manufacturing, vehicle dynamics etc. The most highlighting feature of this modelling technique is that it is comparatively very easy for modelling multi-energy subsystems of a very complicated system. The most fascinating feature is that systems need not to be of the same origin. While one subsystem is a mechanical one, other may be hydraulic, electrical or pneumatic one. All these can be merged in a single bond graph connected by power bonds. With these power bonds, flow and effort variables from one system can very easily be converted to the other by manipulations using transformers and gyrators.

The most important literature [23] is modelling of the prismatic joint i.e. piston-cylinder arrangement. In it modelling of the engine is also done. Based on this literature, a 4-cylinder inline engine is first modelled and then two balancer shafts are attached with it for the purpose of engine mass balancing. Also, a 3-cylinder inline engine is modelled with a single

balancer shaft. Symbols Shakti 2.0 software is used for modelling and its simulation results are validated with the classical formulae.

2.3 Observations from literature

It is observed from the literature survey that a lot of work has been carried out in the engineering field on internal combustion engine mass balancing. If engine is not properly balanced, it leads to excessive noise, vibrations and harshness. Inertia forces and unbalanced moments results due to reciprocating and rotating masses. Their magnitude increases as the engine RPM increases. Study of slider crank mechanism gives the expression for inertia force. This inertia force is needed to be balanced. Counterweights are attached on the engine crankshaft to balance the inertia forces and moments due to rotating engine components. From literature it has been observed that there exist secondary inertia forces and moments also although their magnitude is less as compared to primary. Secondary balancing is done for high speed engines like aero engines. Lanchester balancing technique is the basis for designing balancer shafts. Balancer shafts are used to balance reciprocating forces and moments. It is a shaft with masses at its two ends, phased 180° to one another. The required profile designing of balancing weights of balancer shaft and the counterweights of crankshaft is a challenge in manufacturability and engine packaging point of view. Crankshaft manufacture uses dynamic balancing machine for engine mass balancing. Bob weight is installed on the crankshaft and this assembly is rotated on this machine to check balancing. Bob weight simulates the piston assembly and connecting rod assembly. In symmetric crankshafts like 6-cylinder inline engine, inertia forces and moments cancel among themselves. Also, in 4-cylinder inline engine due to the symmetric configuration the primary inertia forces and the moments (both primary and secondary) balances among themselves only secondary inertia forces exists, which are balanced by using two counter rotating balancer shafts, rotating with twice the speed of crankshaft. In this thesis, bond graph modelling technique has been applied for the engine mass balancing.

2.4 Objectives of the present work

The objectives of the work conducted in this thesis are:

- To study the need of engine balancing.
- To study various unbalanced forces and moments acting of different configurations of inline I.C. engines, starting from single cylinder engine (slider crank mechanism), 3-cylinder inline engine, 4-cylinder inline engines.

- To calculate the unbalance value ($m \times r$) required for counterweights of crankshaft and balancing weights on balancer shaft.
- To design the profiles of counterweights and balancing weights in CAD.
- To model 3-cylinder inline engine.
- To model 4-cylinder inline engine.
- To model balancer shafts. These are used for engine mass balancing.

Chapter 3

Mass Balancing of Three-Cylinder Inline Engine

This chapter deals with the detailed explanation of mass balancing of 3-cylinder inline engine and bond graph based modelling of 3-cylinder inline engine along with balancer shaft. The modelling and simulation are done in Symbols Shakti 2.0 software. The plots clearly shows that with the introduction of balancer shaft, the 3-cylinder inline engine gets balanced i.e. rocking/ pitching couple disappears.

First step in 3-cylinder inline engine for mass balancing is to study and derive the expressions for various reciprocating forces (primary and secondary) as well as unbalanced moments caused by these reciprocating forces. In addition, resultant of rotating unbalanced moments expression is required. The three crank throws in three-cylinder inline engine are arranged at 120° apart.

3.1 Total primary reciprocating forces

The primary reciprocating forces are developed in the engine due to the reciprocating movements of piston and reciprocating part of connecting rod i.e. small end of connecting rod. These are inertia forces. There are two methods to find the total primary reciprocating forces in three-cylinder inline engine. These are analytical method and vector diagram approach. Both of these methods are explained in following sub sections:

3.1.1 Analytical method

The expression for total primary reciprocating forces can be derived analytically. Line diagram for 3-cylinder engine crankshaft is shown in Fig. 3.1. The three crankpins are having an angle of 120° with respect to each other. The reference is centre line of cylinder 1, cylinder 2 lags 120° behind and cylinder 3 240° behind the reference cylinder as shown in Fig. 3.1 (b). So, the total primary reciprocating force for all the three cylinders [21] is given by

$$\begin{aligned} F_p &= m_{re} r \omega^2 (\cos \theta + \cos(\theta - 120^\circ) + \cos(\theta - 240^\circ)) \\ &= m_{re} r \omega^2 (\cos \theta + (\cos \theta \cos 120^\circ + \sin \theta \sin 120^\circ) + (\cos \theta \cos 240^\circ + \sin \theta \sin 240^\circ)) \\ &= -m_{re} r \omega^2 (\cos \theta + (-0.5 \cos \theta + 0.866 \sin \theta) + (-0.5 \cos \theta - 0.866 \sin \theta)) \end{aligned}$$

$$= 0 \quad (3.1)$$

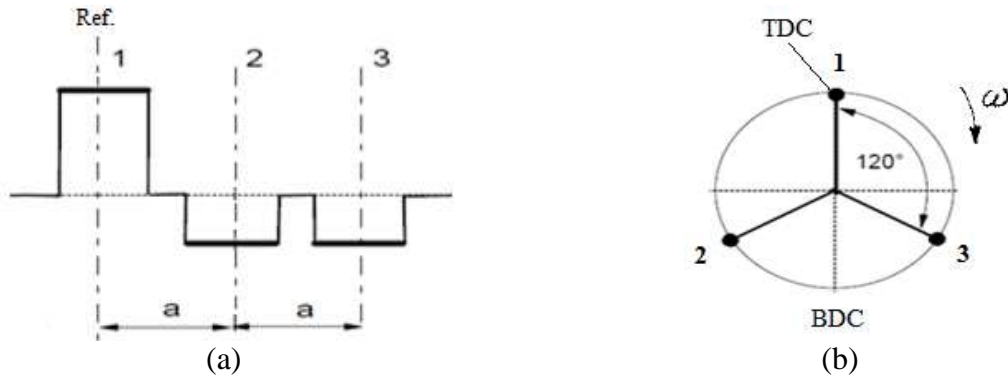


Fig. 3.1 (a) Line diagram for 3-cylinder engine crankshaft and (b) Crankpin orientation

3.1.2 Vector diagram

The total primary reciprocating forces can also be found out with the help of vector diagram as shown in Fig. 3.2. The vertical components i.e. cosine components are the primary reciprocating forces. The horizontal components (sin components) cancel out among themselves. Let us consider crank angle is zero degree i.e. piston of cylinder 1 is at TDC. It can easily be seen in Fig. 3.2 (a) that the cosine components acting on piston 2 and 3 adds up together and gives $(m_p r \omega^2)$ which is same as the reciprocating force acting on piston 1 in opposite direction. Hence, total primary reciprocating force acting on three-cylinder inline engine is zero. The direction of these forces is shown in Fig. 3.2 (b) i.e. if the primary reciprocating force on piston 1 acts in upwards direction, the direction of primary reciprocating forces on pistons 2 and 3 is in downward direction such that the net force is zero always.

3.2 Total secondary reciprocating forces

The secondary reciprocating forces are developed in the engine due to the acceleration and deceleration of piston and reciprocating part of connecting rod i.e. small end of connecting rod. These forces occur twice the frequency of crankshaft rotation. These are inertia forces. There are two methods to find the total secondary reciprocating forces in three-cylinder inline engine. These are analytical method and vector diagram procedure. Both of these methods are explained in following sub sections:

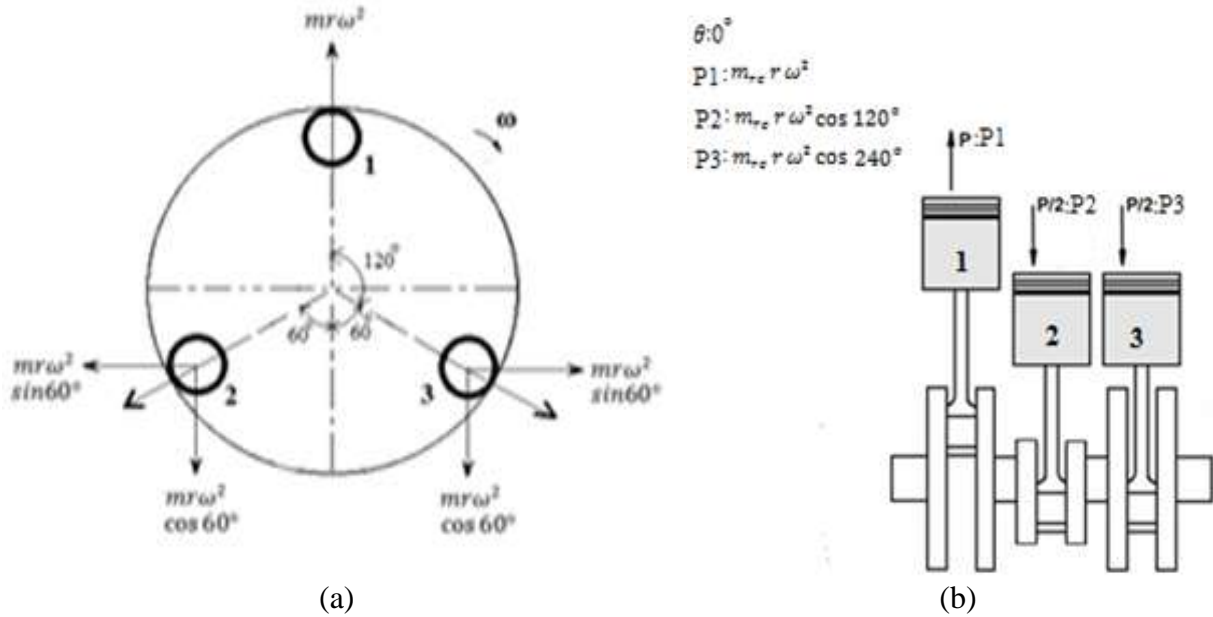


Fig. 3.2 Vector diagram for total primary reciprocating forces

3.2.1 Analytical method

The expression for total secondary reciprocating forces can be derived analytically. The total secondary reciprocating force for all the three cylinders varies periodically twice per crankshaft rotation ($2\omega t$) or (2θ) [20] is given by

$$\begin{aligned}
 F_s &= \frac{m_{re} r \omega^2}{l} (\cos 2\theta + \cos(2\theta - 240^\circ) + \cos(\theta - 480^\circ)) \\
 &= \frac{m_{re} r \omega^2}{l} (\cos 2\theta + (\cos 2\theta \cos 240^\circ + \sin 2\theta \sin 240^\circ) + (\cos 2\theta \cos 480^\circ + \sin 2\theta \sin 480^\circ)) \\
 &= \frac{-m_{re} r \omega^2}{l} (\cos 2\theta + (-0.5 \cos 2\theta + 0.866 \sin 2\theta) + (-0.5 \cos 2\theta - 0.866 \sin 2\theta)) \\
 &= 0
 \end{aligned} \tag{3.2}$$

3.2.2 Vector diagram

The total secondary reciprocating forces can also be found out with the help of vector diagram as shown in Fig. 3.3. The vertical components i.e. cosine components are the secondary reciprocating forces. The horizontal components (sin components) cancel out among themselves. Let us consider crank angle is zero degree i.e. piston of cylinder 1 is at TDC. It can easily be seen in Fig. 3.3 (a) that the cosine components acting on piston 2 and 3 adds up together and gives $(\lambda m_{re} r \omega^2)$ which is same as the reciprocating force acting on piston 1 in opposite direction, where, $\lambda = \frac{r}{l}$. Hence, total secondary reciprocating force acting

on three-cylinder inline engine is zero. The direction of these forces is shown in Fig. 3.3 (b) i.e. if the secondary reciprocating force on piston 1 acts in upwards direction, the direction of secondary reciprocating forces on pistons 2 and 3 is in downward direction such that the net force is always zero.

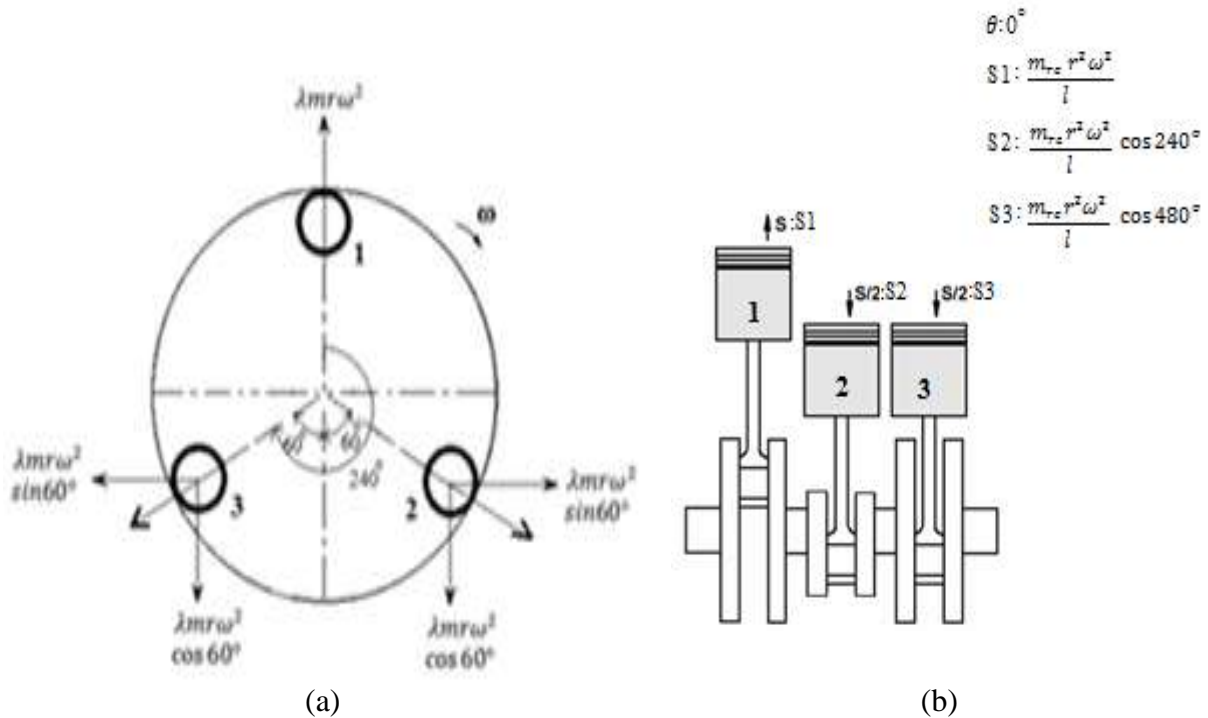


Fig. 3.3 Vector diagram for total secondary reciprocating forces

3.3 Primary reciprocating unbalanced moments

The moment is the product of reciprocating force and position vector (perpendicular distance from the reference plane to the line of action of the reciprocating force). There are two methods to find the total primary reciprocating moments in three-cylinder inline engines. These are analytical method and vector diagram approach. Both of these methods are explained in following sub sections:

3.3.1 Analytical method

The expression for primary reciprocating unbalanced moment can be derived analytically. Line diagram for 3-cylinder engine crankshaft is shown in Fig. 3.4. The three crankpins are having an angle of 120° with respect to each other. The reference is centre line of cylinder 1, cylinder 2 lags 120° behind and cylinder 3 240° behind the reference cylinder as shown in Fig. 3.4 (b).

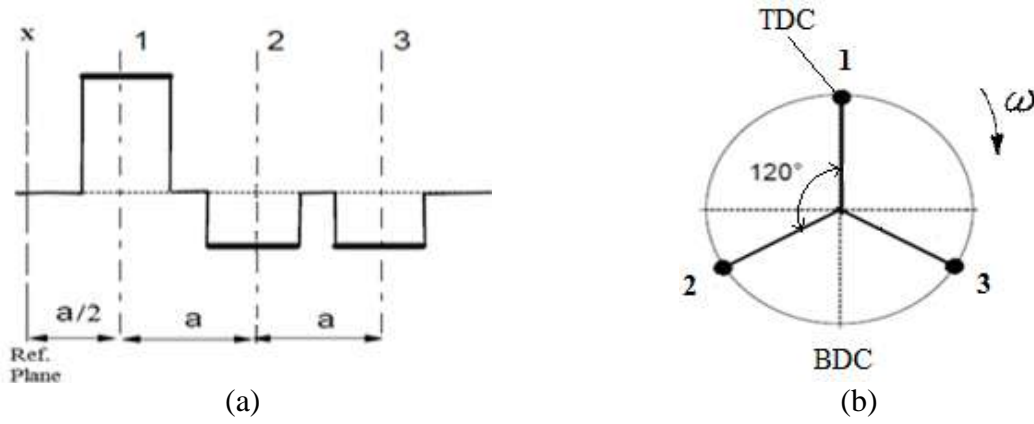


Fig. 3.4 (a) Line diagram for 3-cylinder engine crankshaft and (b) Crankpin orientation

Although reciprocating inertia forces are in same plane, they do not have same line of action. Therefore, it gives rise to unbalanced moments in plane of cylinders. Let us consider the primary reciprocating moments about X-axis as shown in Fig. 3.4 (a) [21]. Cylinder 2 and 3 lag 120° and 240° , respectively from cylinder 1 as shown in Fig. 3.4. The primary reciprocating moments about X-axis is given by

$$\begin{aligned}
 M_p &= -m_{re} r \omega^2 \left(\frac{a}{2} \cos \theta + \frac{3a}{2} \cos(\theta - 120^\circ) + \frac{5a}{2} \cos(\theta - 240^\circ) \right) \\
 &= -m_{re} r a \omega^2 [(0.5 \cos \theta + 1.5(\cos \theta \cos 120^\circ + \sin \theta \sin 120^\circ) \\
 &\quad + 2.5(\cos \theta \cos 240^\circ + \sin \theta \sin 240^\circ)] \\
 &= -m_{re} r a \omega^2 [(0.5 \cos \theta + 1.5(-0.5 \cos \theta + 0.866 \sin \theta) + 2.5(-0.5 \cos \theta - 0.866 \sin \theta)] \\
 &= -m_{re} r a \omega^2 (-1.5 \cos \theta - 0.866 \sin \theta) \\
 &= m_{re} r a \omega^2 (1.5 \cos \theta + 0.866 \sin \theta)
 \end{aligned} \tag{3.3}$$

Now, for maxima of ' θ ', differentiating the Eq. 3.3 and equate it to zero.

$$\begin{aligned}
 \frac{d(M_p)}{d\theta} &= 0 \\
 \Rightarrow m_{re} r a \omega^2 (-1.5 \sin \theta + 0.866 \cos \theta) &= 0 \\
 \Rightarrow \theta &= \tan^{-1} \left(\frac{0.866}{1.5} \right)
 \end{aligned}$$

Hence, ' θ ' comes out to be 30° or 210° from TDC for cylinder 1.

So, the maximum value of primary reciprocating unbalanced moment is given by

$$\begin{aligned}
 M_{p_{\max}} &= -m_{re} r a \omega^2 (-1.5 \cos 30^\circ - 0.866 \sin 30^\circ) \\
 &= \sqrt{3} m_{re} r a \omega^2
 \end{aligned} \tag{3.4}$$

$$\begin{aligned}
\frac{d^2(M_p)}{d\theta^2} &= \frac{d(m_{re} r a \omega^2 (-1.5 \sin \theta + 0.866 \cos \theta))}{dx} \\
&= -m_{re} r a \omega^2 (-1.5 \cos \theta - 0.866 \sin \theta) \\
&= m_{re} r a \omega^2 (-1.5 \cos 30^\circ - 0.866 \sin 30^\circ) \\
&= -\sqrt{3} m_{re} r a \omega^2
\end{aligned} \tag{3.5}$$

which is -ve, hence the condition for maxima.

3.3.2 Vector diagram

The primary reciprocating unbalanced moments can also be found out with the help of vector diagram as shown in Fig. 3.5. Let us consider crank angle is zero degree i.e. piston of cylinder 1 is at TDC, where 'a' is cylinder spacing. The centre line of cylinder 1 is taken as reference and the moments are calculated about this reference. The moment due to primary reciprocating force of cylinder 1 is zero (reference). It can be seen in Fig. 3.5 (b) that the resultant of unbalanced moments due to primary reciprocating forces developed by number pistons 2 and 3 is Za acting at an angle 210° from TDC for cylinder 1 [13].

$$\begin{aligned}
M_p &= \sqrt{(Za)^2 + (Z2a)^2 + 2ZaZ2a \cos 120^\circ} \\
&= \sqrt{Z^2 a^2 + 4Z^2 a^2 - 2Z^2 a^2} \\
&= \sqrt{3} Za
\end{aligned} \tag{3.6}$$

where, $Z = m_{re} r \omega^2$. Hence, primary reciprocating unbalanced moment $= -\sqrt{3} m_{re} r a \omega^2$

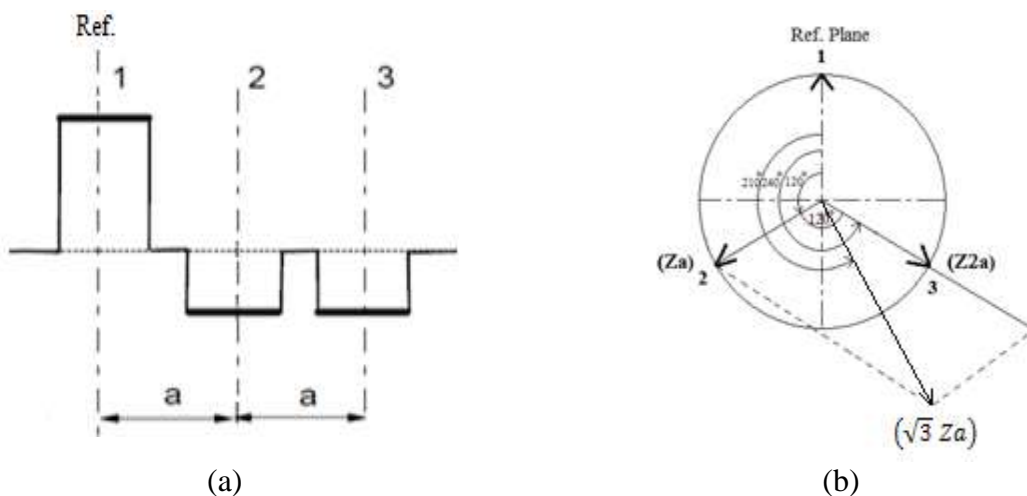


Fig. 3.5 (a) Line diagram for 3-cylinder engine crankshaft and (b) Vector diagram for primary reciprocating unbalanced moment's magnitude and direction

3.4 Secondary reciprocating unbalanced moments

The secondary reciprocating unbalanced moments are due to the secondary reciprocating forces. These forces varies periodically twice per crankshaft rotation. There are two methods to find the total primary reciprocating moments in three-cylinder inline engines. These are analytical method and vector diagram approach. Both of these methods are explained in following sub sections:

3.4.1 Analytical method

The expression for secondary reciprocating unbalanced moment can be derived analytically. Line diagram for 3-cylinder engine crankshaft is shown in Fig. 3.6. The three crankpins are having an angle of 120° with respect to each other. The reference is centre line of cylinder 1. Let us consider the secondary reciprocating moments about X-axis [21]. Cylinder 2 and 3 lags 240° and 480° , respectively from cylinder 1 as shown in Fig. 3.6 (b).

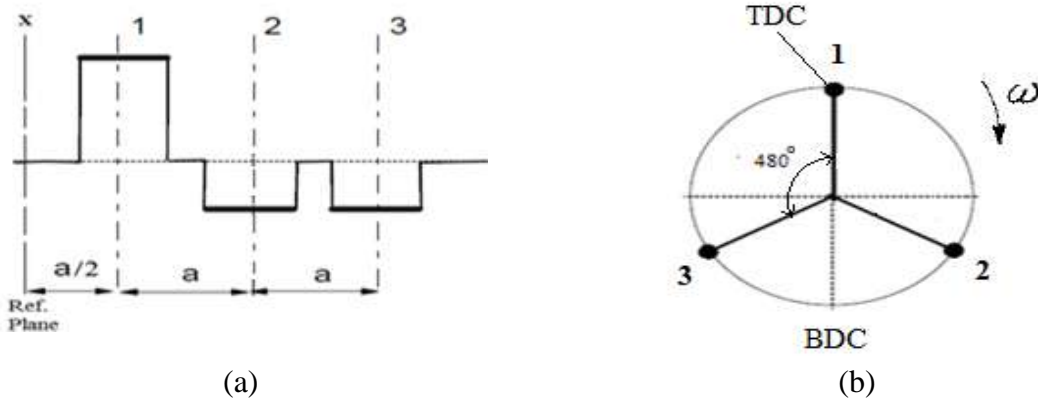


Fig. 3.6 (a) Line diagram for 3-cylinder engine crankshaft and (b) Crankpin orientation

Secondary reciprocating moments about X-axis is given by

$$\begin{aligned}
 M_s &= \frac{-m_{re} r^2 \omega^2}{l} \left(\frac{a}{2} \cos 2\theta + \frac{3a}{2} \cos(2\theta - 240^\circ) + \frac{5a}{2} \cos(2\theta - 480^\circ) \right) \\
 &= \frac{-m_{re} r^2 a \omega^2}{l} [(0.5 \cos 2\theta + 1.5(\cos 2\theta \cos 240^\circ + \sin 2\theta \sin 240^\circ) \\
 &\quad + 2.5(\cos 2\theta \cos 120^\circ + \sin 2\theta \sin 120^\circ)] \\
 &= \frac{-m_{re} r^2 a \omega^2}{l} [(0.5 \cos 2\theta + 1.5(-0.5 \cos 2\theta + 0.866 \sin 2\theta) \\
 &\quad + 2.5(-0.5 \cos 2\theta + 0.866 \sin 2\theta)] \\
 &= \frac{-m_{re} r^2 a \omega^2}{l} (-1.5 \cos 2\theta - 0.866 \sin 2\theta)
 \end{aligned}$$

$$= \frac{m_{re} r^2 a \omega^2}{l} (1.5 \cos 2\theta + 0.866 \sin 2\theta) \quad (3.7)$$

Now, for maxima 'θ', differentiate the expression 3.7 and equate it to Zero.

$$\begin{aligned} \frac{d(M_s)}{d\theta} &= \frac{m_{re} r^2 a \omega^2}{l} (-3 \sin 2\theta - 1.732 \cos 2\theta) = 0 \\ \Rightarrow 2\theta &= \tan^{-1} \left(\frac{-1.732}{3} \right) \end{aligned} \quad (3.8)$$

Hence, 'θ' comes out to be -15° or 165° from TDC for cylinder 1.

So, the maximum value of secondary reciprocating unbalanced moment is given by

$$\begin{aligned} M_{S_{\max}} &= \frac{-m_{re} r^2 a \omega^2}{l} (-1.5 \cos 2\theta + 0.866 \sin 2\theta) \\ &= \frac{\sqrt{3} m_{re} r^2 a \omega^2}{l} \\ \frac{d^2(M_s)}{d\theta^2} &= \frac{d \left(\frac{m_{re} r^2 a \omega^2 (-3 \sin 2\theta - 1.732 \cos 2\theta)}{l} \right)}{d\theta} \\ &= \frac{m_{re} r^2 a \omega^2}{l} (-6 \cos 2\theta + 3.464 \sin 2\theta) \\ &= \frac{m_{re} r^2 a \omega^2}{l} (-6 \cos 30^\circ - 3.464 \sin 30^\circ) \\ &= \frac{-6.928 m_{re} r^2 a \omega^2}{l} \end{aligned} \quad (3.10)$$

which is -ve, hence, this is the condition for maxima.

3.4.2 Vector diagram

The secondary reciprocating unbalanced moments can also be found out with the help of vector diagram as shown in Fig. 3.7. Let us consider crank angle is zero degree i.e. piston of cylinder 1 is at TDC, where 'a' is the cylinder spacing. The centre line of cylinder 1 is taken as reference. The moments are calculated about this reference. The moment due to secondary reciprocating force by piston 1 is zero (reference). It can be seen in Fig. 3.7 (b) that the resultant of unbalanced moments due to secondary reciprocating forces developed by pistons

2 and 3 is $\sqrt{3} \lambda Z a$ acting at an angle 165° from TDC for cylinder 1 as shown in Fig. 3.7 [13] where, $\lambda = \frac{r}{l}$.

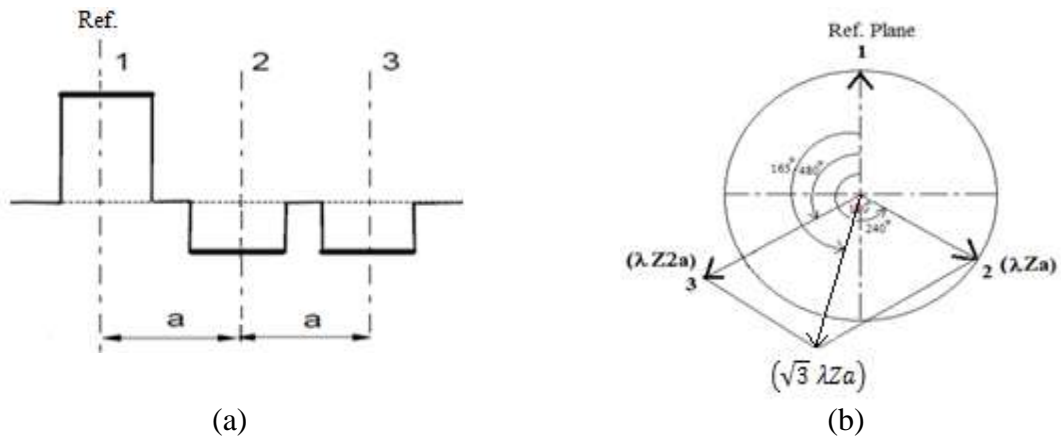


Fig. 3.7 (a) Line diagram for 3-cylinder engine crankshaft and (b) Vector diagram for secondary reciprocating unbalanced moments

Secondary reciprocating unbalanced moment is given by

$$\begin{aligned}
 M_s &= \sqrt{(\lambda Z a)^2 + (\lambda Z 2a)^2 + 2\lambda Z a \lambda Z 2a \cos 120^\circ} \\
 &= \sqrt{\lambda^2 Z^2 a^2 + 4\lambda^2 Z^2 a^2 - 2\lambda^2 Z^2 a^2} \\
 &= \sqrt{3} \lambda Z a
 \end{aligned} \tag{3.11}$$

where, $Z = m_{re} r \omega^2$ and $\lambda = \frac{r}{l}$. Hence, secondary reciprocating unbalanced moments

$$M_s = \frac{\sqrt{3} m_{re} r^2 a \omega^2}{l}$$

3.5 Resultant of rotating unbalanced moments

The rotating unbalanced moments are developed due to the centrifugal forces acting on the rotational parts of crankshaft. The rotational parts consist of connecting rod big end and rotational portion of crank (crank pin journal and the cut section of crank cheek and crank web). The line and vector diagrams for rotating unbalance are shown in Fig. 3.8.

The rotating unbalanced moments can be calculated about reference plane which for convenience has been taken at a distance of $a/2$ from the centre line of cylinder 1, where 'a' is cylinder spacing [21].

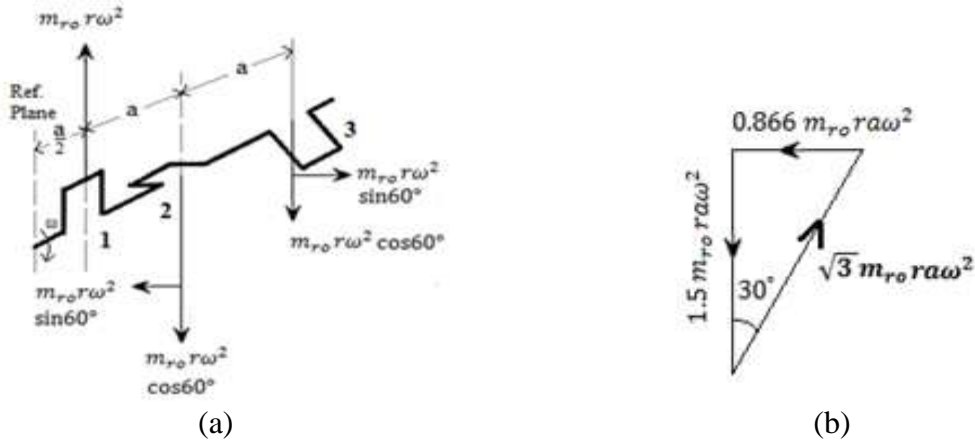


Fig. 3.8 (a) Line diagram for rotating out of balance for 3-cylinder engine and its (b) Vector diagram

Total moments from vertical projection (considering clockwise moments as +ve) is given as

$$\begin{aligned}
 M_V &= m_{r_0} r \omega^2 \left(-\frac{a}{2} + \frac{3a}{2} \cos 60^\circ + \frac{5a}{2} \cos 60^\circ \right) \\
 &= m_{r_0} r \omega^2 \left(-\frac{a}{2} + \frac{3a}{4} + \frac{5a}{4} \right) \\
 &= +1.5 m_{r_0} r a \omega^2
 \end{aligned} \tag{3.12}$$

From horizontal projection, the moment is given by

$$\begin{aligned}
 M_H &= m_{r_0} r \omega^2 \left(0 \cdot \frac{a}{2} - \frac{3a}{2} \sin 60^\circ + \frac{5a}{2} \sin 60^\circ \right) \\
 &= m_{r_0} r a \omega^2 \left(-\frac{3}{2} + \frac{5}{2} \right) \sin 60^\circ \\
 &= +0.866 m_{r_0} r a \omega^2
 \end{aligned} \tag{3.13}$$

The resultant of rotating unbalanced moment is expressed as

$$\begin{aligned}
 M_{r_0} &= (1.5^2 + 0.866^2)^{1/2} \\
 &= \sqrt{3} m_{r_0} r a \omega^2
 \end{aligned} \tag{3.14}$$

whose angle from crank 1 is given by

$$\tan^{-1} \left(\frac{0.866}{1.5} \right) = 30^\circ$$

All these forces and moments are listed in Table 3.2. In order to balance the engine, these reciprocating and rotating out of balance needs to be eliminated i.e. reciprocating and rotating masses satisfy the static and dynamic equilibrium conditions. In order to do that counterweights of crankshaft and balancing masses of balancer shaft are designed such that counterweights balances 100% rotating moments along with 50% primary inertia moments. The remaining 50% primary inertia moments is balanced by primary balancer shaft.

Table 3.1 Values for various parameters for 3-cylinder inline engine

Parameters	Description	Values
a	Cylinder spacing (mm)	100
l	Connecting rod length (mm)	150
m_{cr}	Mass of connecting rod assembly (kg)	1.5
m_{crb}	Connecting rod big end rotating mass, located at crank pin (m_A) (kg)	1
m_{crs}	Connecting rod small end reciprocating mass, located at piston pin (m_B) (kg)	0.5
m_{cw}	Mass of counter weight of crankshaft (kg)	1.5
m_{pa}	Mass of piston assembly (piston, piston pin, piston rings, 2 circlips) (kg)	1.5
m_{re}	Reciprocating mass: mass of piston assembly + connecting rod small end mass (kg)	2
m_{ro}	Rotational mass: mass of rotating crank + connecting rod big end mass (kg)	3.2
m_{roc}	Rotational mass of crank pin journal, cut section of crank cheek and crank web (kg)	2.2
N	Over speed (rpm)	5000
r	Crank radius (mm)	45
r_{ro}	Radial distance of rotating mass COG (kg)	51
r_{cw}	Radial distance of counter weight COG (kg)	61
ω	Angular velocity of crankshaft ($2\pi N/60$) (rad/s)	523.59

Table 3.2 Amplitude of reciprocating unbalanced forces and moments in 3-cylinder engine

Forces and moments	Complete expressions	Final expression
Primary unbalanced forces		Zero
Secondary unbalanced forces		Zero
Primary reciprocating unbalanced moments	$m_{re} r a \omega^2 (1.5 \cos \theta + 0.866 \sin \theta)$ $\theta = 30^\circ \text{ or } 210^\circ \text{ from TDC for no. 1 cyl.}$	$\sqrt{3} m_{re} r a \omega^2$
Secondary reciprocating unbalanced moments	$= \frac{-m_{re} r^2 a \omega^2}{l} (-1.5 \cos 2\theta - 0.866 \sin 2\theta)$ $\theta = -15^\circ \text{ or } 165^\circ \text{ from TDC for no. 1 cyl.}$	$\frac{\sqrt{3} m_{re} r^2 a \omega^2}{l}$
Resultant of rotating unbalanced moments	Angle from crank 1 = 30°	$\sqrt{3} m_{ro} r a \omega^2$

3.6 Three-cylinder inline engine balancing

In three-cylinder engine, there are no primary and secondary reciprocating forces but there are primary and secondary reciprocating moments which are to be balanced. Also, there are rotating unbalanced moments. Half of the primary reciprocating unbalanced moments are balanced by balancer shaft and rest by counterweights on crankshaft. Also, counterweights balance the rotating unbalanced moments.

3.6.1 Need of balancer shaft

It is a mechanical rotating shaft, invented by British engineer Frederick Lanchester in 1904. It is driven by crankshaft and arranged parallel to the crankshaft to dampen the reciprocating moments of the engine. It rotates with same speed as that of the crankshaft but in opposite direction. It consists of two balancing weights attached at its two ends (one at each end), phased 180° to one another. These two weights on the ends move in the direction opposite to that of end pistons. When the piston goes up, the balancing weight goes down and vice versa. This reduces the end to end rocking/pitching action on 3-cylinder inline engines

[9]. When piston on cylinder 1 is at TDC, front balancing weight is 180° opposite to it. These two balancing weights set up a couple whose magnitude is half of the total reciprocating moment in opposite direction, i.e. it neutralizes 50% of reciprocating moments, (rest 50% is being balanced by counterweights on crankshaft). Balancer shaft is statically balanced (i.e. $\Sigma \text{ Forces} = 0$) but is not dynamically balanced (i.e. $\Sigma \text{ Moments} \neq 0$). Hence, it creates a rocking couple which is in opposite sense to rocking couple generated by engine because of reciprocating moments. Hence, as a whole, engine is balanced in reciprocating moments with the help of balancer shaft. If rotation of crankshaft is considered clockwise when seen from rear (flywheel end), pitching couple is generated in 3-cylinder inline engine as explained below [22]:

1. The upward inertia force due to piston 1 equals to the combined downward inertia force due to pistons 2 and 3 (both being 60° from BDC). Due to these opposing offset forces along the crankshaft, a vertical pitching couple in clockwise direction with front end lifted upward and rear end pressed downward.

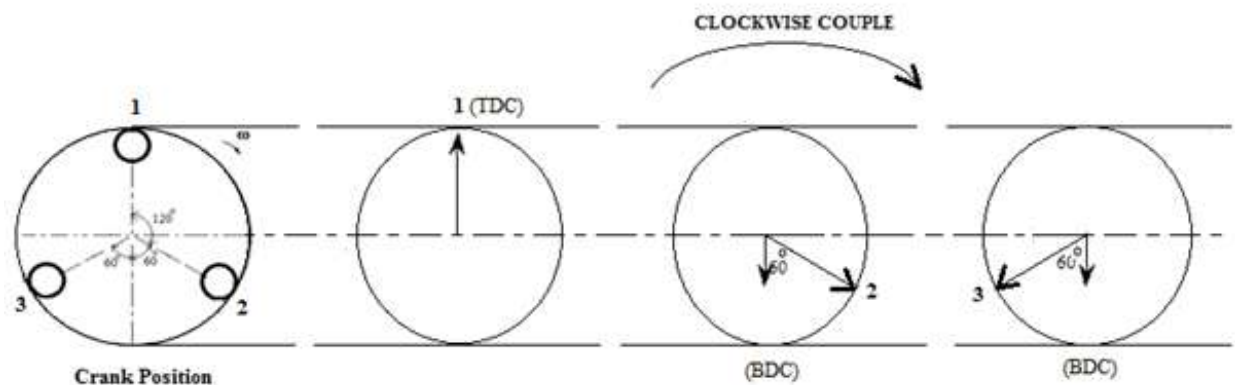


Fig. 3.9 (a) Pitching couple in clockwise direction

2. The piston 2 is at BDC and pistons 1 and 3 are at 60° from TDC. Upward inertia force components are equal to downward inertia force produced. There will be no pitching couple.

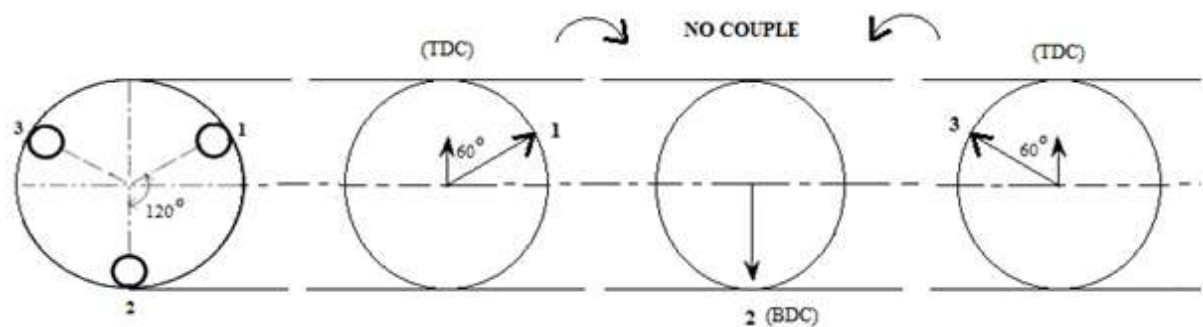


Fig. 3.9 (b) No Pitching couple

- Rotating further, piston 3 reaches TDC, pistons 1 and 2 move to 60° either side of BDC. The pistons 1 and 2 produces downward inertia force components equal to that produced by piston 3 in upward direction. But, as all pistons are in different planes, a counterclockwise pitching couple is produced. It tends to lift the engine downwards at the front and upward at the rear.

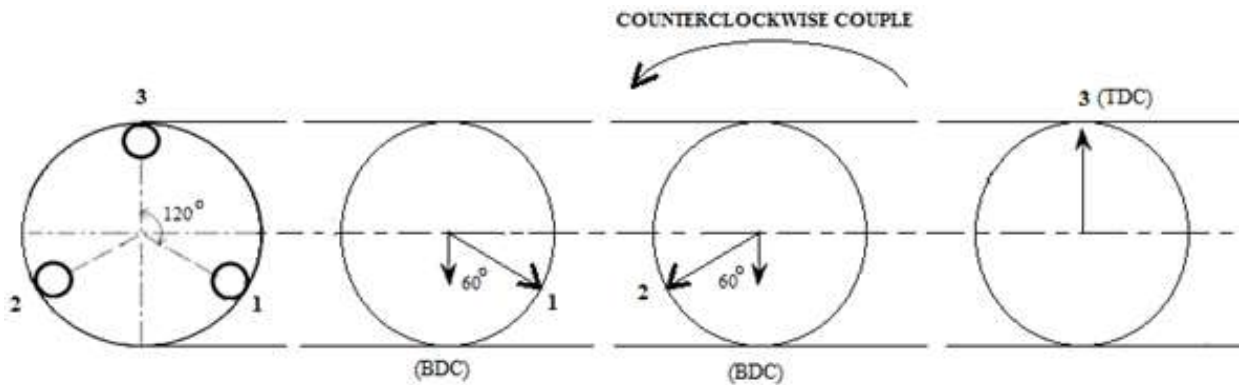


Fig. 3.9 (c) Pitching couple in counter-clockwise direction

- Piston 1 is at BDC, piston 2 approaching the TDC and piston 3 is moving downward. Pitching couple is formed as in (1) but in anticlockwise direction.

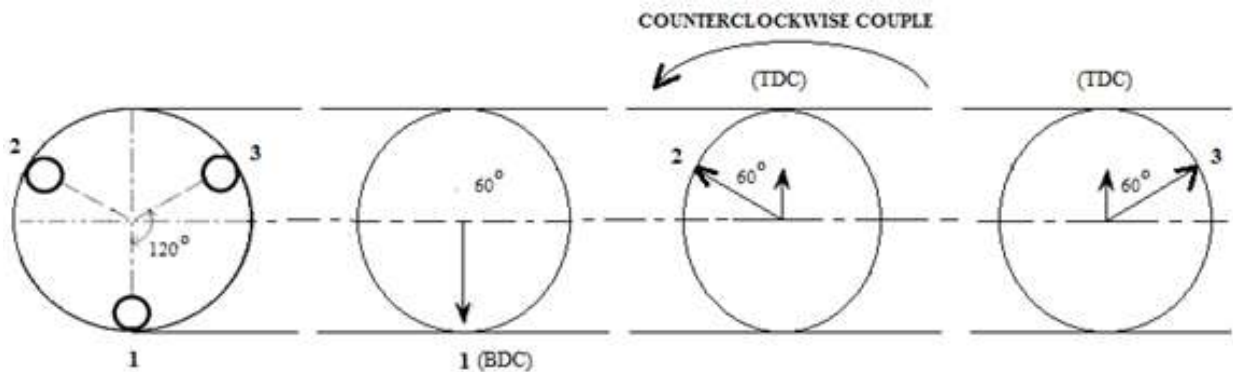


Fig. 3.9 (d) Pitching couple in counter-clockwise direction

- Considering further 60° movement of crankshaft, the piston 2 reaches TDC. The piston 3 is moving downwards and piston 1 is moving upwards. The inertia forces upwards and downwards balances each other and there is no pitching couple.

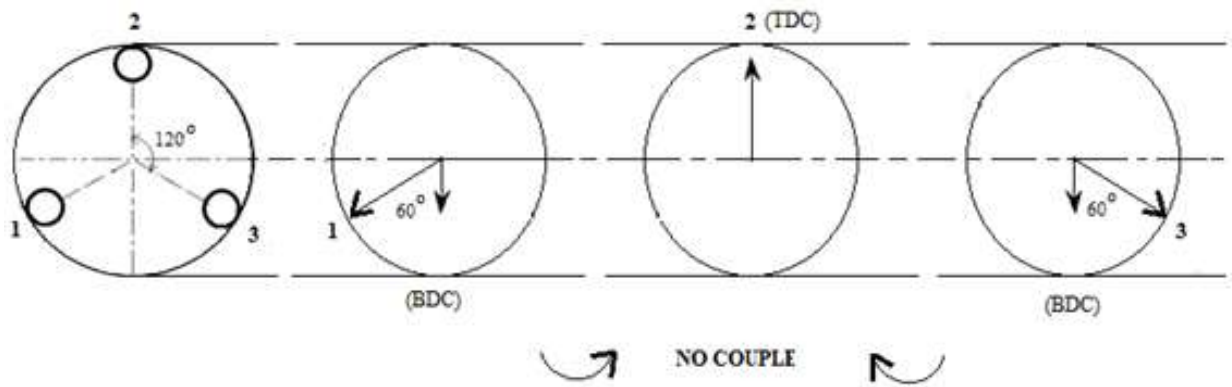


Fig. 3.9 (e) No Pitching couple

6. The next 60° movement places the piston 3 at BDC and piston 1 moves upwards and piston 2 downwards. This gives rise to clockwise pitching couple due to inertia force acting in opposite direction.

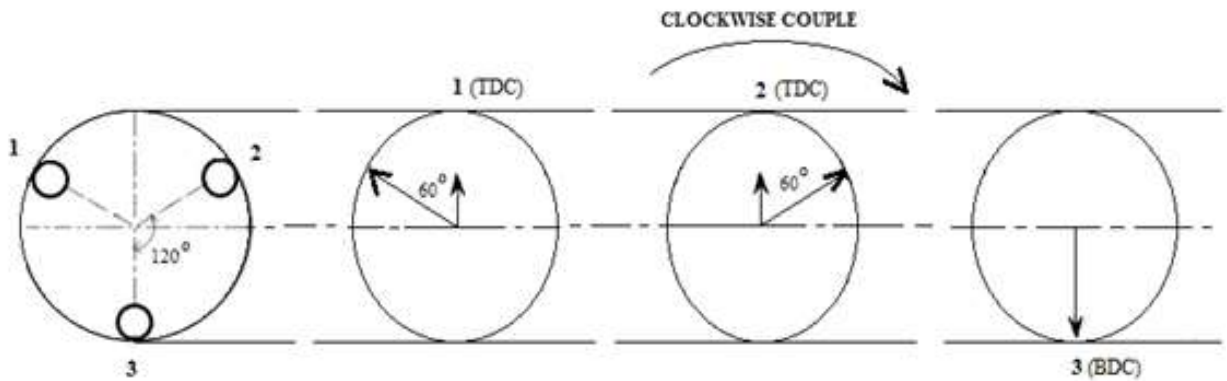


Fig. 3.9 (f) Pitching couple in clockwise direction

The methods 4, 5, 6 are similar to 1, 2, 3 except that directions of pitching couples are opposite and their effects on engine are also opposite in nature.

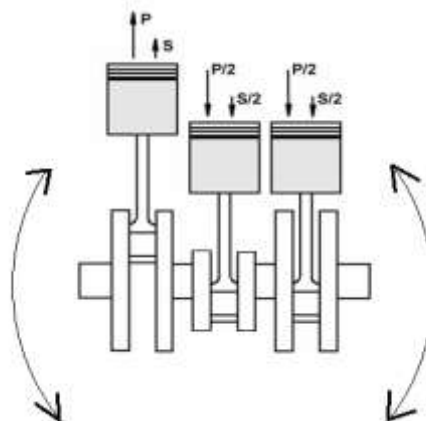


Fig. 3.10 Pitching couples in 3-cylinder inline engine

3.6.2 Balancer weights on balancer shaft

The value of unbalance required for balance weights of balancer shaft is given by 50% of primary reciprocating unbalanced moments divided by axial distance between two balance masses COG of balancer shaft 'b' [21], (using values from Table 3.1 and Table 3.3)

$$B_U = \frac{\sqrt{3}m_r r a}{2b} = 19.48 \text{kgmm} = m_B r_B \quad (3.15)$$

The desired value is 19.48 kg mm. In CAD, few modifications were done in the balancer weights to get the result.



Fig. 3.11 (a) Front balancer mass and (b) Rear balancer mass of balancer shaft, obtained from Creo 3.0 software

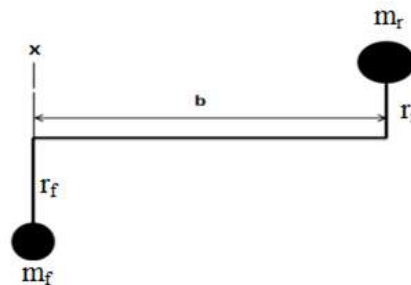


Fig. 3.12 Primary balancer shaft

Moment generated by primary balancer shaft (from Fig. 3.12) is given by

$$M_B = -m_r r_r b \omega^2 = -2135 \text{Nm} \quad (3.16)$$

(Values are taken from Table 3.3)

Hence, 50% of primary reciprocating unbalanced moment (2136.76 Nm) is balanced by primary balancer shaft. Therefore, primary balancer shaft generates a counter rocking couple to prevent engine rocking. It is statically balanced ($\sum \text{Forces} = 0$) but dynamically, it is unbalanced ($\sum \text{Moments} \neq 0$).

Table 3.3 Balancer shaft values for three-cylinder inline engine, obtained from Creo 3.0 software

Parameters	Description	Values
b	Axial distance between two balance masses COG of balancer shaft (mm)	400
m_{bf}	Front balancing mass of balancer shaft (kg)	0.85
m_{br}	Rear balancing mass of balancer shaft (kg)	1
r_{bf}	Radial distance of front balancer mass COG (mm)	22.92
r_{br}	Radial distance of rear balancer mass COG (mm)	19.46

3.6.3 Counter weights on crankshaft

Crankshaft counterweights are designed to balance the effects of reciprocating and rotating masses in engine. The reciprocating mass consists of piston assembly (piston, piston pin, piston rings, 2 circlips) and the small-end of connecting rod assembly, and rotating mass consists of the crank pin journal, cut section of crank cheek, crank web and big-end of the connecting rod assembly. The required unbalance for counterweights [21] (values taken from Table 3.1) is given by

$$CW_U = m_{CW} r_{CW} = \left((1 \times m_{ro} r_{ro}) + (0.5 \times m_{re} r) \right) \frac{\cos 30^\circ}{2} = 88.76 \text{ kg mm} \quad (3.17)$$



Fig. 3.13 (a) Rotational mass of crank for one throw and (b) Connecting rod big end, obtained from Creo 3.0 software



Fig. 3.14 Counterweight profile with 90 kg mm unbalance value, obtained from Creo 3.0 software

The counterweights on crankshaft has 50% balance factor and the balancer shaft creates further 50% to balance the reciprocating moments.

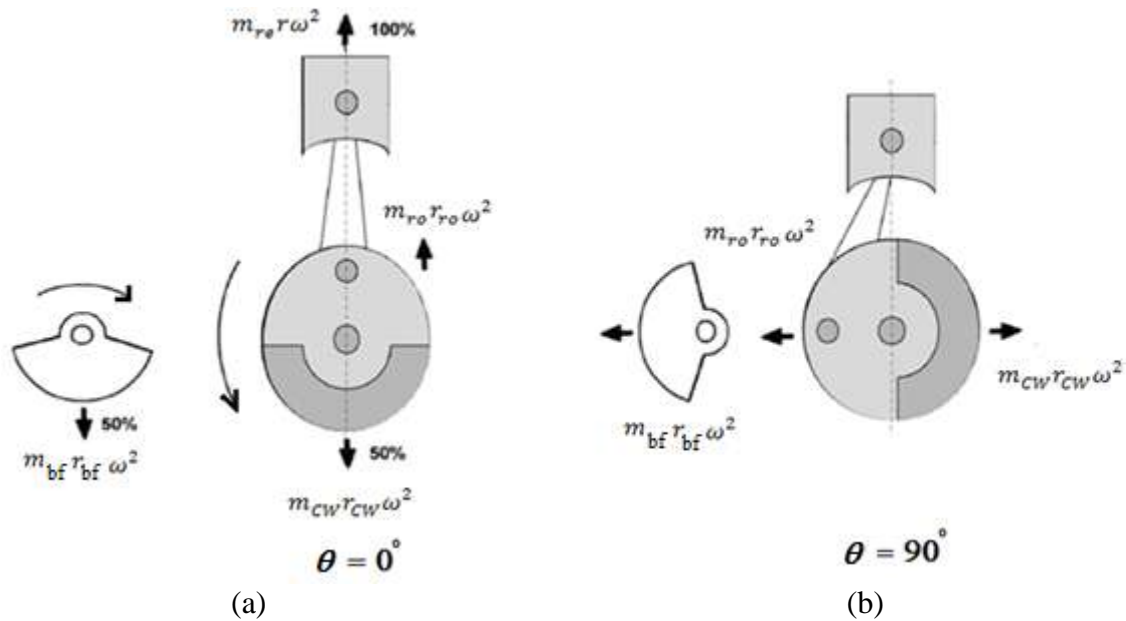


Fig. 3.15 Forces generated by reciprocating and rotating components in 3-cylinder engine with primary balancer shaft, (a) and (b) when $\theta = 0^\circ$ and $\theta = 90^\circ$, respectively

When $\theta = 0^\circ$, ($\cos 0^\circ = 1$ (max.)), the inline components forces produced by reciprocating masses, rotating masses are being balanced by 2 counterweights and front balancer weight.

$$0.5m_{re} r \omega^2 + m_{ro} r_{ro} \omega^2 = 2m_{CW} r_{CW} \omega^2 + m_{bf} r_{bf} \omega^2 \quad (3.18)$$

or, $205 \approx 202.5$ (values from Table 3.1 and 3.3)

On substituting the values in Eq. 3.18, L.H.S comes out to be approximately equals to R.H.S. Hence, the engine is balanced. Now, for $\theta = 90^\circ$, ($\sin 90^\circ = 1$ (max.)), the transverse components forces produced by rotating masses are being balanced by 2 counterweights and front balancer weight.

$$m_{ro} r_{ro} \omega^2 + m_{bf} r_{bf} \omega^2 = 2m_{CW} r_{CW} \omega^2 \quad (3.19)$$

or, $182.7 \approx 183$ (values from Table 3.1 and 3.3)

On substituting the values in Eq. 3.19, L.H.S comes out to be approximately equals to R.H.S. Hence, the engine is balanced.

3.7 Free body diagrams (analytical approach)

In this approach, FBD of various masses are drawn i.e. balancing weights, counter weights, rotating masses and reciprocating masses of the three-cylinder inline engine are drawn and from them various forces and moments are calculated. In following sub sections, these are studied one by one.

3.7.1 Balancing weights

The balance weights on balancer shafts are rotating when the engine is running and hence, centrifugal forces act on them. There are two balancing masses, one at front end and the other is at rear end of the balancer shaft. The resolution of centrifugal forces is shown in Fig. 3.16. The various forces and moments due to the balancing weights are studied in Table 3.4

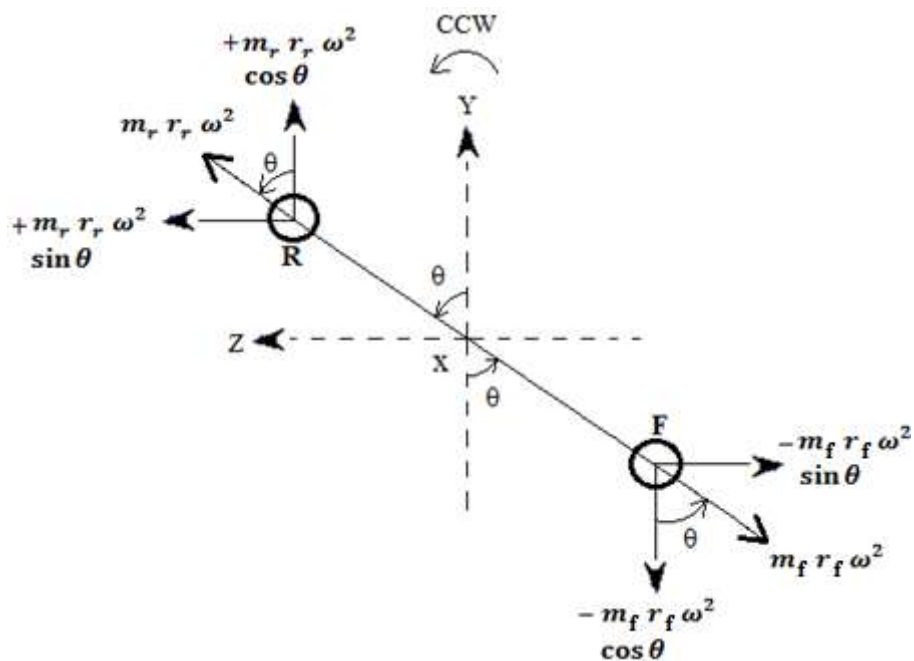


Fig. 3.16 FBD of balancing weights

Moment is the cross product of position vector and the force i.e.

$$\vec{M} = \vec{r} \times \vec{F} \quad (3.20)$$

Also, if \hat{i} , \hat{j} , and \hat{k} are unit vectors then:

$$\hat{i} \times \hat{j} = \hat{k} \quad (3.21)$$

$$\hat{i} \times \hat{k} = -\hat{j} \quad (3.22)$$

Table 3.4 Forces and moments due to balancing weights

	$F_y (\hat{j})$	$F_z (\hat{k})$	r (\hat{i})	M_y	M_z
FRONT	$-m_f r_f \omega^2 \cos \theta$	$-m_f r_f \omega^2 \sin \theta$	0	0	0
REAR	$m_r r_r \omega^2 \cos \theta$	$m_r r_r \omega^2 \sin \theta$	$-b$	$bm_r r_r \omega^2 \sin \theta$	$-bm_r r_r \omega^2 \cos \theta$
	$\sum_{i=1}^2 F_y$	$\sum_{i=1}^2 F_z$		$\sum M_y = bm_r r_r \omega^2 \sin \theta$	$\sum M_z = bm_r r_r \omega^2 \cos \theta$
	Resultant F = $\frac{\sqrt{(\sum F_y)^2 + (\sum F_z)^2}}{N} = 0$ (statically balanced)			Resultant M = $\sqrt{(\sum M_y)^2 + (\sum M_z)^2} = 2133.96 \text{ Nm}$	

3.7.2 Counter weights

The counter weights on crank shafts are rotating when the engine is running and hence, centrifugal forces act on them. There are four counter weights in the crank shaft of three-cylinder inline engine. The resolution of centrifugal forces is shown in Fig. 3.17. The various forces and moments due to the counter weights are studied in Table 3.5

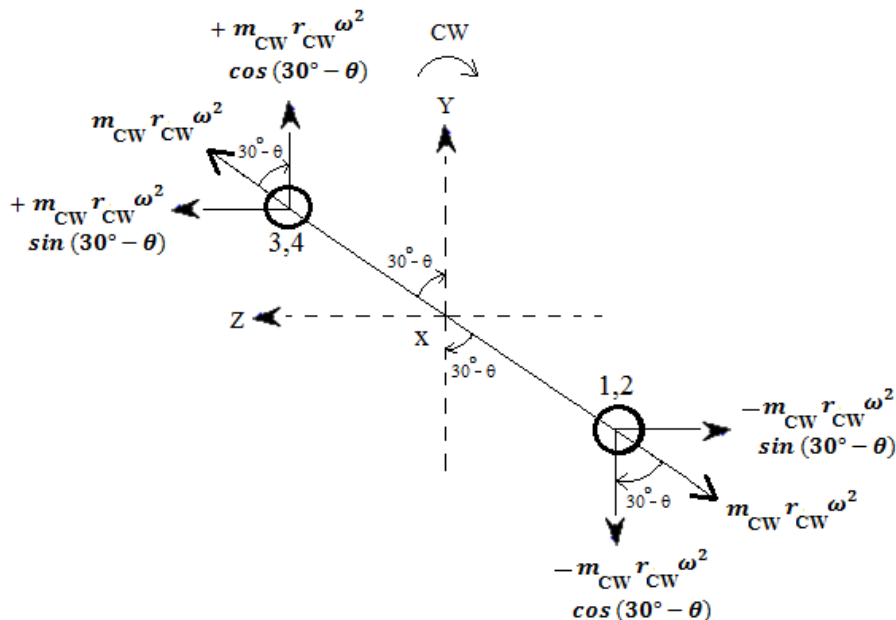


Fig. 3.17 FBD of counter weights

Table 3.5 Forces and moments in counter weights

	$F_y (\hat{j})$	$F_z (\hat{k})$	$r (\hat{i})$	M_y	M_z
1.	$-m_{CW}r_{CW}\omega^2 \cos(30^\circ - \theta)$	$-m_{CW}r_{CW}\omega^2 \sin(30^\circ - \theta)$	$-y_1$	$-y_1m_{CW}r_{CW}\omega^2 \sin(30^\circ - \theta)$	$-y_1m_{CW}r_{CW}\omega^2 \cos(30^\circ - \theta)$
2.	$-m_{CW}r_{CW}\omega^2 \cos(30^\circ - \theta)$	$-m_{CW}r_{CW}\omega^2 \sin(30^\circ - \theta)$	$-y_2$	$-y_2m_{CW}r_{CW}\omega^2 \sin(30^\circ - \theta)$	$-y_2m_{CW}r_{CW}\omega^2 \cos(30^\circ - \theta)$
3.	$m_{CW}r_{CW}\omega^2 \cos(30^\circ - \theta)$	$m_{CW}r_{CW}\omega^2 \sin(30^\circ - \theta)$	$-y_3$	$y_3m_{CW}r_{CW}\omega^2 \sin(30^\circ - \theta)$	$y_3m_{CW}r_{CW}\omega^2 \cos(30^\circ - \theta)$
4.	$m_{CW}r_{CW}\omega^2 \cos(30^\circ - \theta)$	$m_{CW}r_{CW}\omega^2 \sin(30^\circ - \theta)$	$-y_4$	$y_4m_{CW}r_{CW}\omega^2 \sin(30^\circ - \theta)$	$y_4m_{CW}r_{CW}\omega^2 \cos(30^\circ - \theta)$
	$\sum_{i=1}^4 F_y = 0$	$\sum_{i=1}^4 F_z = 0$		$\sum_{i=1}^4 M_y$	$\sum_{i=1}^4 M_z$
	Resultant F = $\sqrt{(\sum_{i=1}^4 F_y)^2 + (\sum_{i=1}^4 F_z)^2} = 0$			Resultant M = $\sqrt{\left(\sum_{i=1}^4 M_y\right)^2 + \left(\sum_{i=1}^4 M_z\right)^2}$ $= 10345.79 \text{ Nm}$	

3.7.3 Rotating masses

The centrifugal forces act on the rotating masses of the crank shaft. The rotating masses consist of the crank pin journal, cut section of crank cheek, crank web and big end of the connecting rod assembly. The resolution of centrifugal forces is shown in Fig. 3.18. The various forces and moments due to the counter weights are studied in Table 3.6

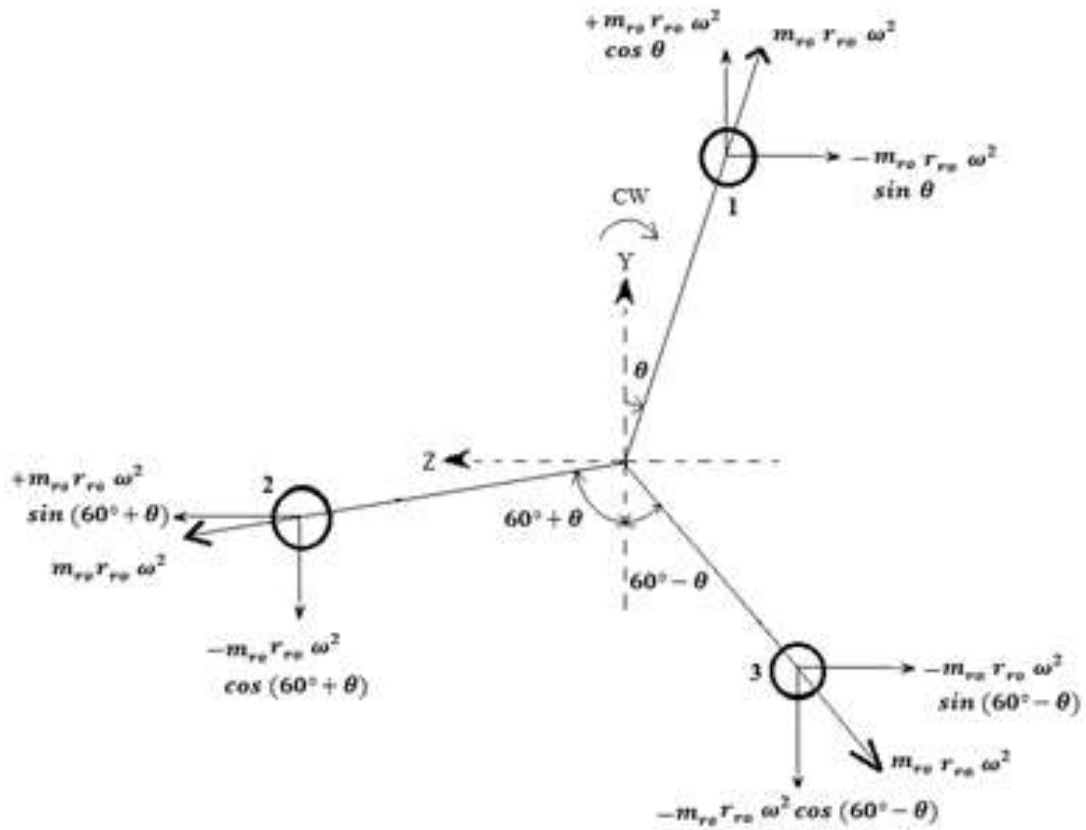


Fig. 3.18 FBD of rotating masses

Table 3.6 Forces and moments in rotating masses

	$F_y (\hat{j})$	$F_z (\hat{k})$	$r (\hat{i})$	M_y	M_z
1.	$m_{r0} r_{r0} \omega^2 \cos \theta$	$-m_{r0} r_{r0} \omega^2 \sin \theta$	$-x_1$	$-x_1 m_{r0} r_{r0} \omega^2 \sin \theta$	$-x_1 m_{r0} r_{r0} \omega^2 \cos \theta$
2.	$-m_{r0} r_{r0} \omega^2 \cos(60^\circ + \theta)$	$m_{r0} r_{r0} \omega^2 \sin(60^\circ + \theta)$	$-x_2$	$x_2 m_{r0} r_{r0} \omega^2 \sin(60^\circ + \theta)$	$x_2 m_{r0} r_{r0} \omega^2 \cos(60^\circ + \theta)$
3.	$-m_{r0} r_{r0} \omega^2 \cos(60^\circ - \theta)$	$-m_{r0} r_{r0} \omega^2 \sin(60^\circ - \theta)$	$-x_3$	$-x_3 m_{r0} r_{r0} \omega^2 \sin(60^\circ - \theta)$	$x_3 m_{r0} r_{r0} \omega^2 \cos(60^\circ - \theta)$
	$\sum_{i=1}^3 F_y = 0$	$\sum_{i=1}^3 F_z = 0$		$\sum_{i=1}^3 M_y$	$\sum_{i=1}^3 M_z$
	Resultant F = $\sqrt{(\sum_{i=1}^3 F_y)^2 + (\sum_{i=1}^3 F_z)^2} = 0$			Resultant M = $\sqrt{\left(\sum_{i=1}^3 M_y\right)^2 + \left(\sum_{i=1}^3 M_z\right)^2}$ = 8214.28 Nm	

3.7.4 Reciprocating masses

The reciprocating unbalanced forces act on the engine due to the movements of the reciprocating masses. The reciprocating masses consist of the piston assembly (piston, piston pin, piston rings, 2 circlips) and small end of the connecting rod assembly. The various forces and moments due to the reciprocating masses are studied in Table 3.7.

Table 3.7 Forces and moments in reciprocating masses

	$F_y (\hat{j})$	$r (\hat{i})$	M_z
1.	$m_{re} r \omega^2 \cos \theta$	0	0
2.	$m_{re} r \omega^2 \cos(\theta - 120^\circ)$	$-z_2$	$-z_2 m_{re} r \omega^2 \cos(\theta - 120^\circ)$
3.	$m_{re} r \omega^2 \cos(\theta - 240^\circ)$	$-z_3$	$-z_3 m_{re} r \omega^2 \cos(\theta - 240^\circ)$
	Resultant $F = \sum_{i=1}^3 F_y = 0$		Resultant $M = \sum_{i=1}^3 M_z = 4273.32 \text{ Nm}$

Table 3.8 Results for 3-cylinder inline engine moments

Unbalanced moments	Values (Nm)	Calculations (Nm)	Results (Nm)
Balancer shaft (rotating)	2133.96		
Primary inertia (reciprocating)	4273.32	$0.5 \times 4273.32 = 2136.66$	$2133.96 \approx 2136.66$
Crankshaft rotational (crank + connecting rod big end)	8214.28		
Counter weights (rotating)	10345.79	$(1 \times 8214.28) + (0.5 \times 4273.32) = 10351$	$10345.79 \approx 10351$

Hence, counterweights balance 100% moments due to rotating masses and further 50% moments due to reciprocating masses. The remaining 50% moments due to reciprocating masses are being balanced by balancer shaft.

3.8 Bond graph modelling of three-cylinder inline engine

Modelling of three-cylinder engine consists of piston-cylinder model and combustion inside the cylinder. The three piston-cylinder models are arranged inline with each other. From balancing point of view, a balancer shaft is modelled and attached with the main engine model.

3.8.1 Modelling of piston-cylinder arrangement

The schematic view of piston-cylinder arrangement is shown in Fig. 3.19, based on it, modelling is done in Symbol Shakti 2.0 software. The velocities of the end point of the cylinder i.e. $E_C(x_1, y_1)$ and the end point of the piston i.e. $E_P(x_2, y_2)$ are expressed in terms of the linear velocities of the cylinder and connecting rod centre of gravity, respectively, in the horizontal plane (x-y) and the rotational velocities of the piston-cylinder about the z-axis are given as follows [23]:

$$\dot{x}_1 = \dot{x}_{cg} + l_{cg} \dot{\theta}_{cg} \sin \theta_{cg} \quad (3.23)$$

$$\dot{y}_1 = \dot{y}_{cg} - l_{cg} \dot{\theta}_{cg} \cos \theta_{cg} \quad (3.24)$$

$$\dot{x}_2 = \dot{x}_{pg} - l_{pg} \dot{\theta}_{pg} \sin \theta_{pg} \quad (3.25)$$

$$\dot{y}_2 = \dot{y}_{pg} + l_{pg} \dot{\theta}_{pg} \cos \theta_{pg} \quad (3.26)$$

Differentiating contemporary length (l) between the two points E_C and E_P with respect to time and is given by

$$\dot{l} = \left(\frac{x_1 - x_2}{l} \right) (\dot{x}_1 - \dot{x}_2) + \left(\frac{y_1 - y_2}{l} \right) (\dot{y}_1 - \dot{y}_2) \quad (3.27)$$

Normal velocities at point 1 (as shown in Fig. 3.19) on cylinder and on piston are, respectively

$$V_{C1} = -\sin \theta_{cg} \dot{x}_{cg} + \cos \theta_{cg} \dot{y}_{cg} + (l - l_p - l_{cg} - d/2) \dot{\theta}_{cg} \quad (3.28)$$

$$V_{P1} = -\sin \theta_{pg} \dot{x}_{pg} + \cos \theta_{pg} \dot{y}_{pg} - (l_p - l_{pg} + d/2) \dot{\theta}_{pg} \quad (3.29)$$

Normal velocities at point 2 (as shown in Fig. 3.19) on cylinder and on piston are, respectively

$$V_{C2} = -\sin \theta_{cg} \dot{x}_{cg} + \cos \theta_{cg} \dot{y}_{cg} + (l - l_p - l_{cg} + d/2) \dot{\theta}_{cg} \quad (3.30)$$

$$V_{P2} = -\sin \theta_{pg} \dot{x}_{pg} + \cos \theta_{pg} \dot{y}_{pg} - (l_p - l_{pg} - d/2) \dot{\theta}_{pg} \quad (3.31)$$

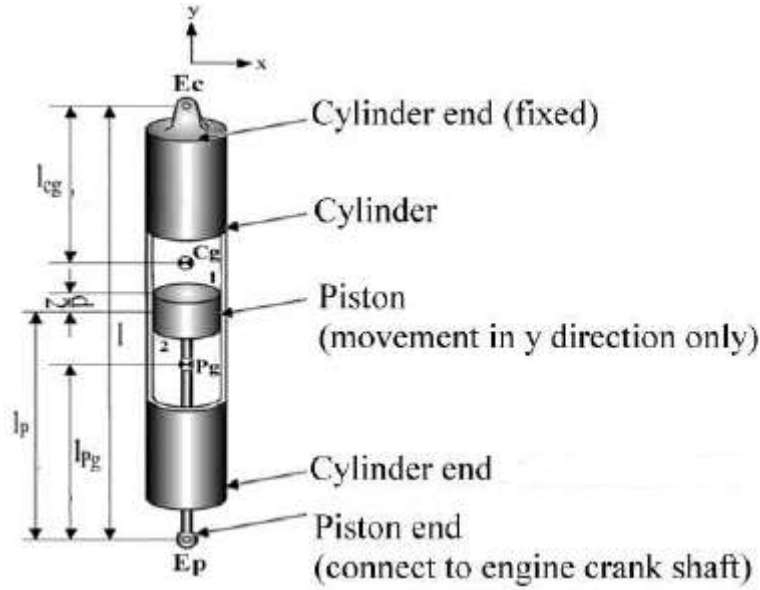


Fig. 3.19 Schematic diagram of piston cylinder arrangement [23]

Eqs. (3.23) to (3.31) are used to model the bond graph of piston-cylinder arrangement as shown in Fig. 3.20. The different multipliers in Eqs. (3.23) to (3.31) are used as transformer moduli of MTF elements [23] and they are given by

$$\mu_1 = l_{cg} \sin \theta_{cg}, \mu_2 = -l_{cg} \cos \theta_{cg}, \mu_3 = \frac{-1}{\sin \theta_{cg}}, \mu_4 = \frac{1}{\cos \theta_{cg}}, \mu_5 = \frac{1}{l - l_p - l_{cg} - \frac{d}{2}}$$

$$\mu_6 = \frac{1}{l - l_p - l_{cg} + \frac{d}{2}}, \mu_7 = \frac{x_1 - x_2}{l}, \mu_8 = \frac{y_1 - y_2}{l}, \mu_9 = -l_{pg} \sin \theta_{pg}, \mu_{10} = l_{pg} \cos \theta_{pg}$$

$$\mu_{11} = -\sin \theta_{pg}, \mu_{12} = \cos \theta_{pg}, \mu_{13} = -\left(l_p - l_{pg} + \frac{d}{2}\right), \mu_{14} = -\left(l_p - l_{pg} - \frac{d}{2}\right)$$

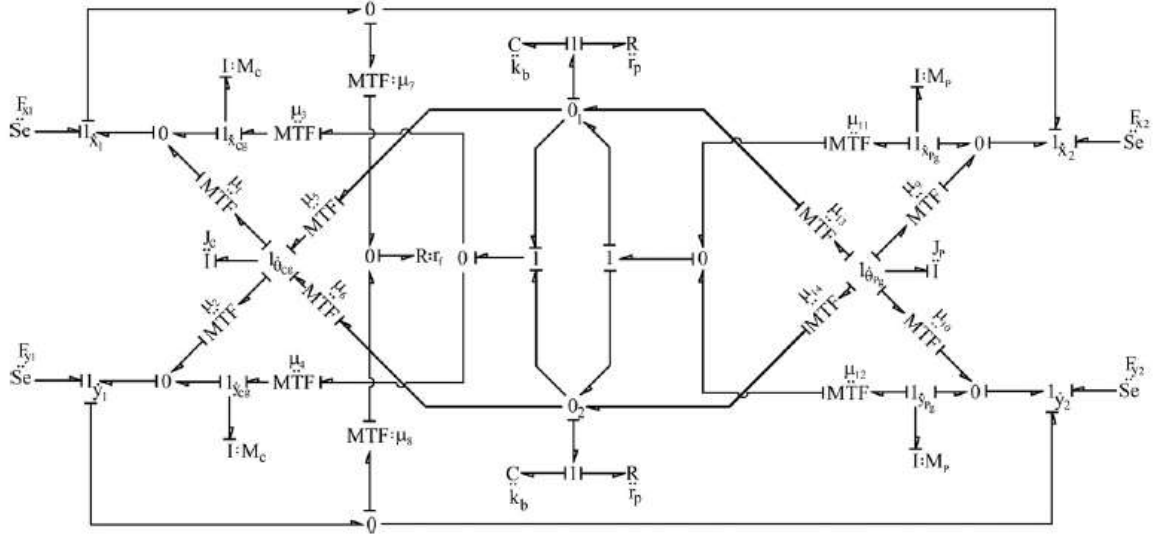


Fig. 3.20 Bond graph model of piston-cylinder arrangement [23]

3.8.2 Modelling to engine (combustion)

The flow variables are rate of mass transfer (\dot{m}) heat transfer (\dot{E}) and volumetric change (\dot{V}). The effort variables are temperature (T) and pressure (P) of fuel-air mixture. They depend on the mass (m), total internal energy (E) and volume (V) and of fuel-air mixture. There is a three port C-field. The three ports indicate three energy transfer mechanisms, i.e., mechanical work, heat transfer and mass transfer. This C-field receives three flow-information ($\dot{m}, \dot{V}, \dot{E}$) and computes the effort variables [23]:

$$T(t) = \frac{\int_0^t \dot{E}(\tau) d\tau + E_0}{\left(\int_0^t \dot{m}(\tau) d\tau + m_0 \right) C_v}, P(t) = \frac{R \left(\int_0^t \dot{E}(\tau) d\tau + E_0 \right)}{\left(\int_0^t \dot{V}(\tau) d\tau + V_0 \right) C_v} \quad (3.32)$$

The state variables (m, V and E) are time integrals of flow variables. Initial values for the state variables are given as m_0, V_0 and E_0 , respectively.

The output from the engine is used to rotate the crankshaft. The angular velocity of the crankshaft ($\dot{\theta}_1$) in terms of piston velocity is given as

$$\dot{\theta}_1 = \mu_T v_p \quad (3.33)$$

where transformer modulus, $\mu_T = \frac{1}{r_c} \left[\frac{(r_c / l_c) \sin 2\theta_1}{2[1 - (r_c / l_c)^2 \sin^2 \theta_1]^{1/2}} + \sin \theta_1 \right]^{-1}$

The forces acting on piston assembly are forces due to atmospheric pressure (on the free side), combustion pressure, connecting rod reaction force and viscous damping forces. The total force acting on the piston assembly is

$$F_{pa} = P_{atm}A_p - PA_p - R_p\dot{x}_p - \mu_T\tau_c \quad (3.34)$$

where τ_c is the load torque

The mass flow rate of fuel-air mixture through intake valve is given by

$$\dot{m}_i = \frac{A_{vi}C_{di}P_0}{\sqrt{T}} \sqrt{\frac{2\gamma}{R'(\gamma-1)} \left[P_r^{2/\gamma} - P_r^{(\gamma-1)/\gamma} \right]} \quad (3.35)$$

where $P_r = P/P_0$

The mass flow rate of fuel-air mixture through exhaust valve is given by

$$\dot{m}_o = \frac{A_{vo}C_{do}P}{\sqrt{T}} \sqrt{\frac{2\gamma}{R'(\gamma-1)} \left[P_r^{2/\gamma} - P_r^{(\gamma-1)/\gamma} \right]} \quad (3.36)$$

The energy balance equation is

$$\dot{E} = \dot{Q}_c + \dot{Q}_i - \dot{Q}_o - P\dot{V} - \lambda_{gb}(T - T_b) \quad (3.37)$$

where \dot{Q}_c is the heat released due to combustion, $\dot{Q}_i = \dot{m}_i C_v T_0$ and $\dot{Q}_o = \dot{m}_o C_v T$ are the rate of energy convection during intake and exhaust stroke, respectively, $P\dot{V}$ is the mechanical work output and $\lambda_{gb}(T - T_b)$ is the heat transferred to the bodies. The body also exchanges heat with the environment. The heat balance equation for the body is given by

$$T_b = T_0 + \frac{1}{m_b C_{pb}} \int_0^t (\lambda_{gb}(T(\tau) - T_b(\tau)) - \lambda_{ba}(T_b(\tau) - T_0)) d\tau \quad (3.38)$$

where T_0 is the initial temperature, $m_b C_{pb}$ is the total heat capacity of the body and the first and second terms within the integral are the heat transfer between the gas and the body and between the body and the environment, respectively.

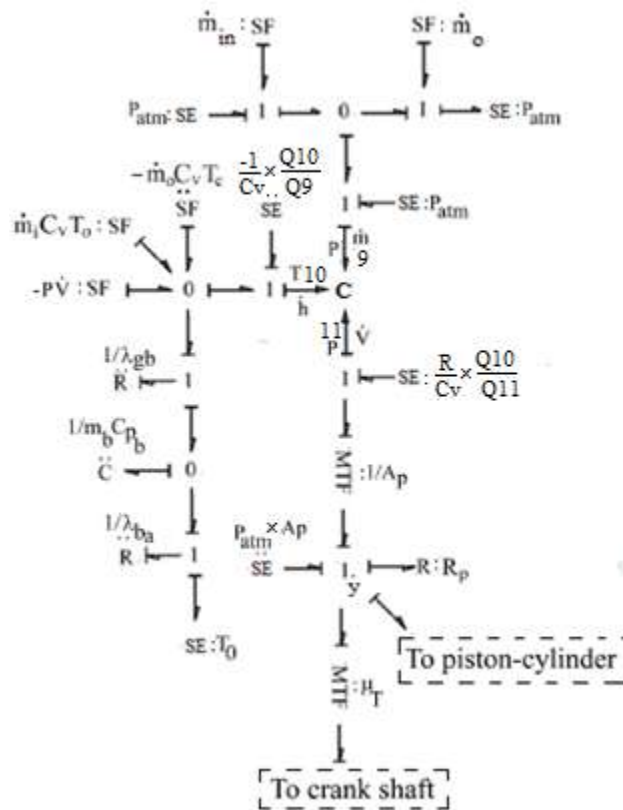


Fig. 3.21 Bond graph of engine (combustion)

The bond graph model of combustion and heat transfer is shown in Fig. 3.21. The heat transfer is modelled by elements $R: 1/\lambda_{gb}$ (thermal resistance between body and gas), $C: 1/m_b c_{pb}$ (heat capacity of the body) and $R: 1/\lambda_{ba}$ (thermal resistance between body and environment). The total heat input is fed to the thermal port of the C-field. The mechanical work output port of the C-field is connected to the crankshaft model. The engine output is the torque applied onto the crankshaft.

3.8.3 Modelling of balancer shaft

Bond graph of 3-cylinder inline engine with balancer shaft is shown in Fig. 3.22. Balancer shaft is modelled and attached to the main engine model in such a way that its angular velocity is same that of crankshaft but it rotates in opposite direction to crankshaft. It is used to prevent rocking of the engine.

software. The conclusions are drawn that the engine rotates back and forth without balancer shaft and with the introduction of balancer shaft, this rocking couple disappears. Proper value of unbalance of the two balancing masses and the length of balancer shaft are inserted in the software such that it introduces proper counter couple. Results are shown in Fig. 3.26 and 3.27.

Table 3.9 Parameters used in Symbols Shakti 2.0 software

Parameters	Values	Units	Parameters	Values	Units
C_{di}	0.2	-	m_b	80	kg
C_{do}	1.0	-	m_c	5	kg
C_{pm}	460	$\text{Jkg}^{-1}\text{K}^{-1}$	m_{pa}	2	kg
C_v	720	$\text{Jkg}^{-1}\text{K}^{-1}$	P_a	$7e^{-3}$	m^2
d	0.1	m	P_{atm}	101325	N/m^2
J_c	$3e^{-3}$	kgm^2	r	0.07	m
J_{fw}	3	kgm^2	R	287.05	$\text{Jkg}^{-1}\text{K}^{-1}$
J_p	$2e^{-3}$	kgm^2	T_{atm}	300	K
k_{bend}	$1e^5$	N/m	V_{ia}	$1.4e^{-3}$	m^2
k_{pad}	$1e^5$	Ns/m	V_{oa}	$1e^{-3}$	m^2
l	0.2	m	γ	1.41	-
l_{cg}	0.1	m	λ_{ba}	25	Wm^{-2}/K
l_p	0.2	m	λ_{gb}	1	Wm^{-2}/K
l_{pg}	0.15	m			

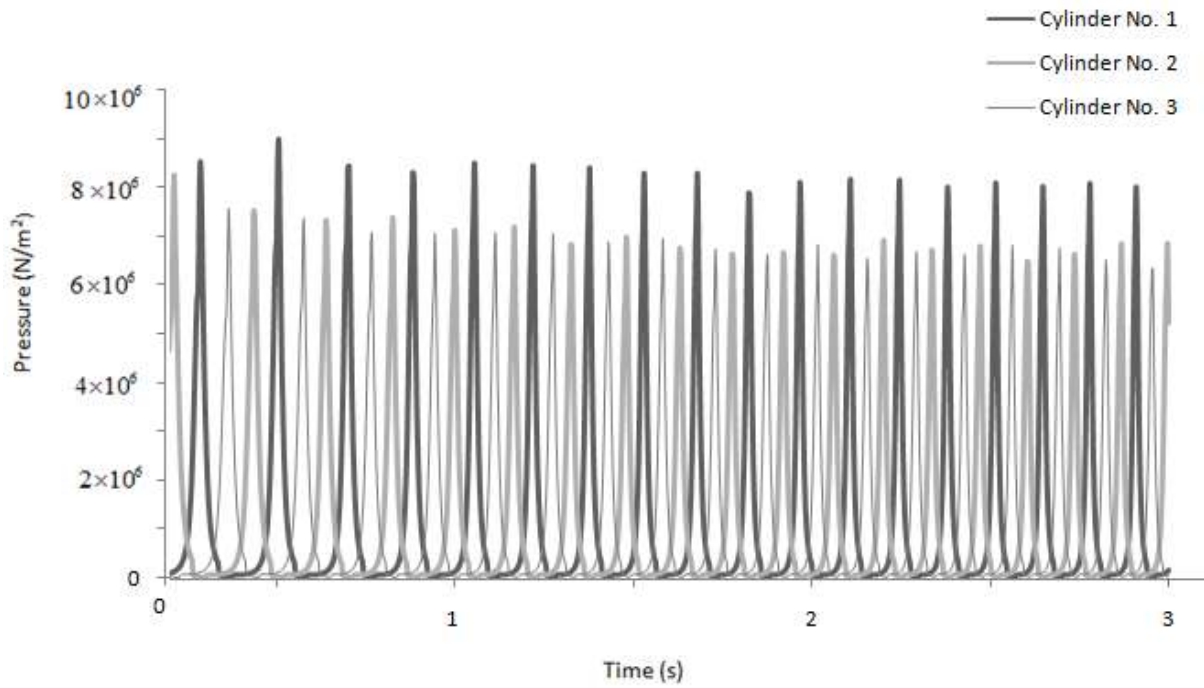


Fig. 3.23 Pressure developed inside the cylinders

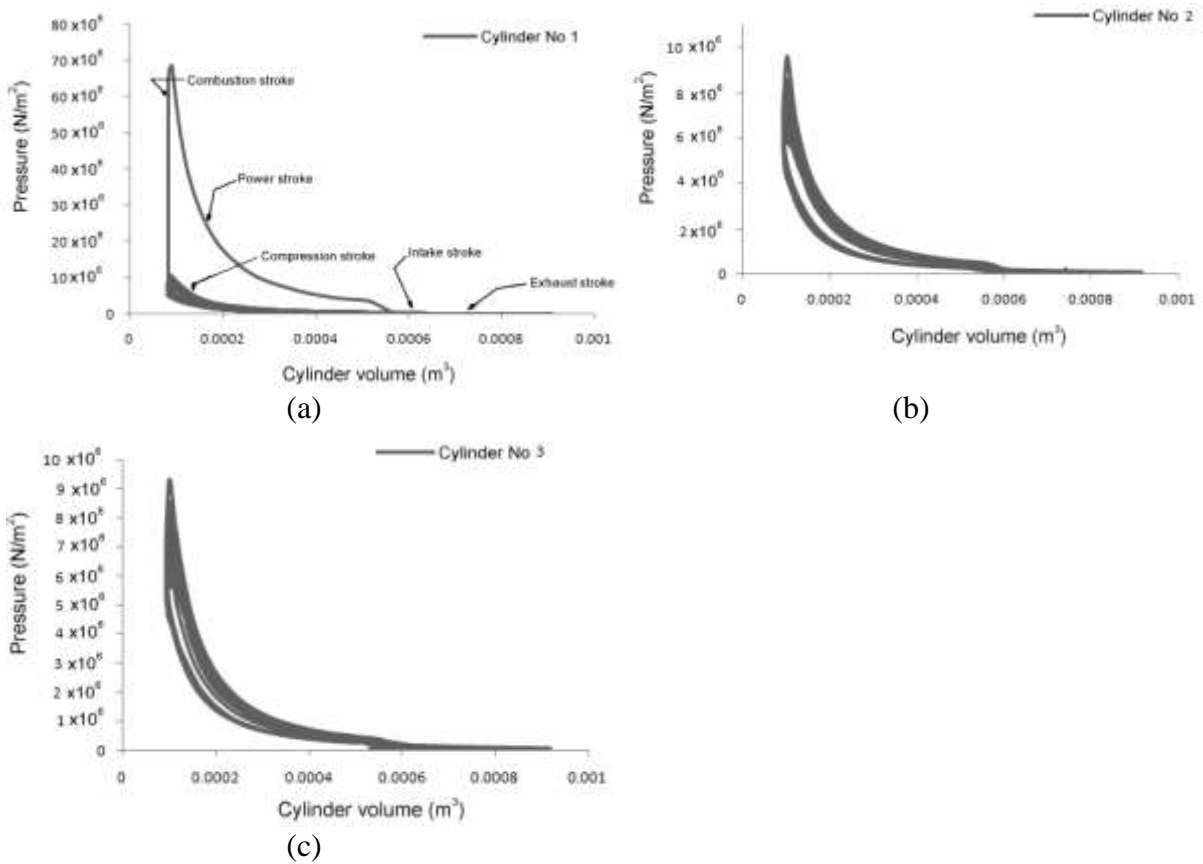
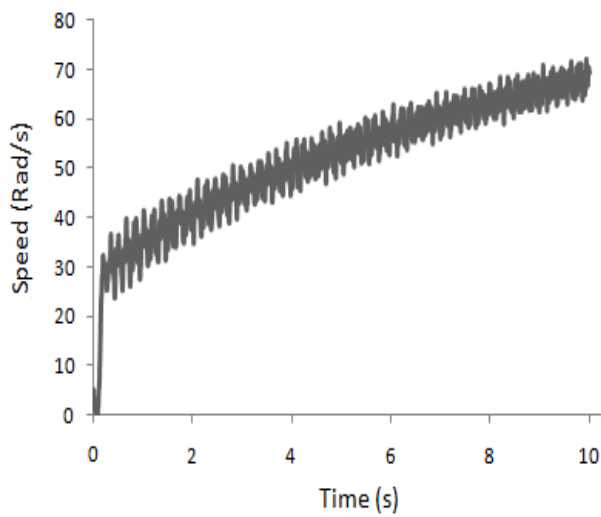
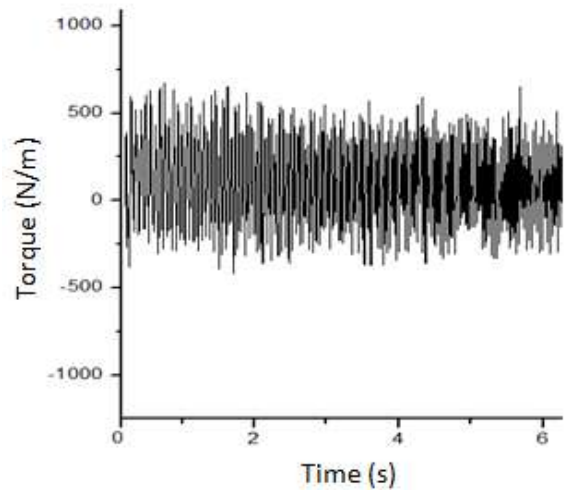


Fig. 3.24 Pressure-volume diagram of (a) Cylinder 1, (b) Cylinder 2 and (c) Cylinder 3

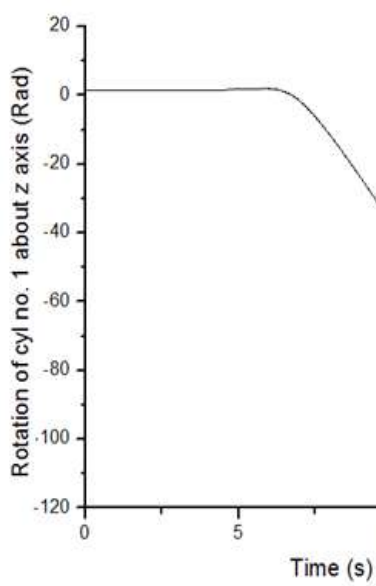


(a)

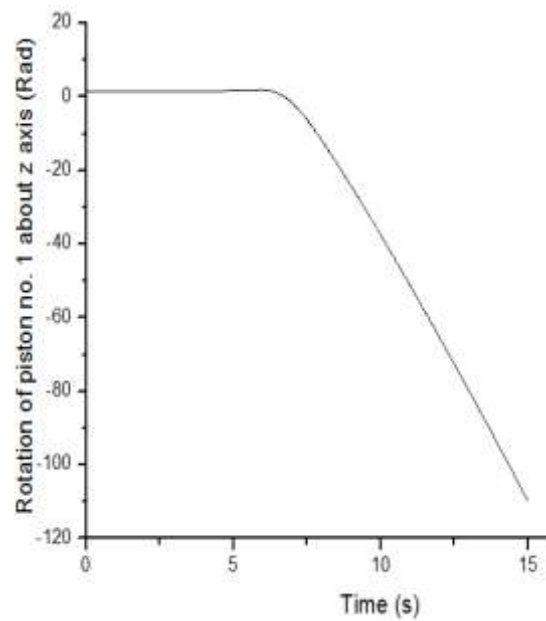


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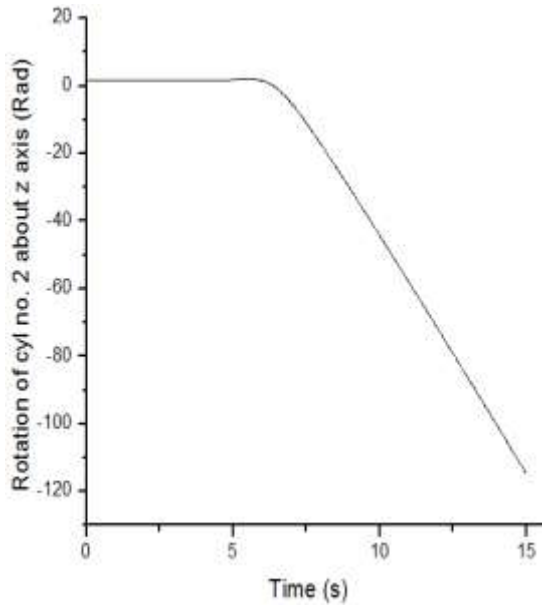
Fig. 3.25 (a) Speed of crankshaft with respect to time and (b) Engine torque with respect to time



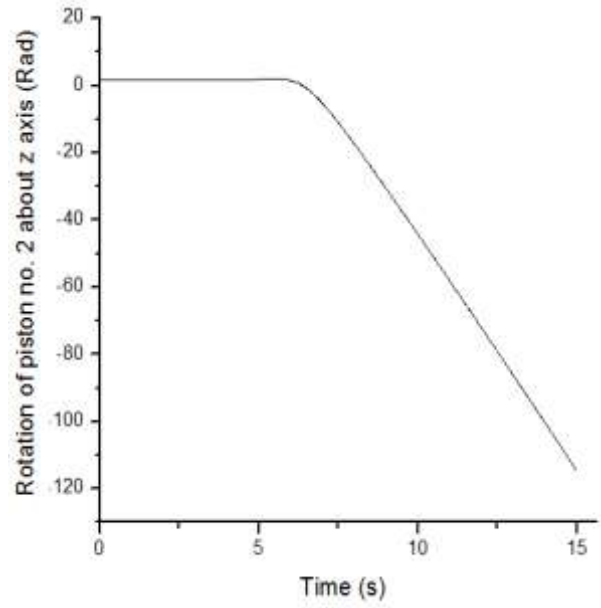
(a)



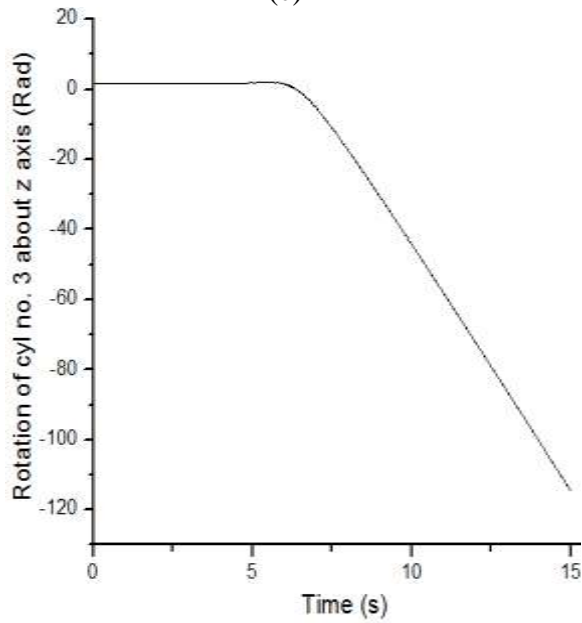
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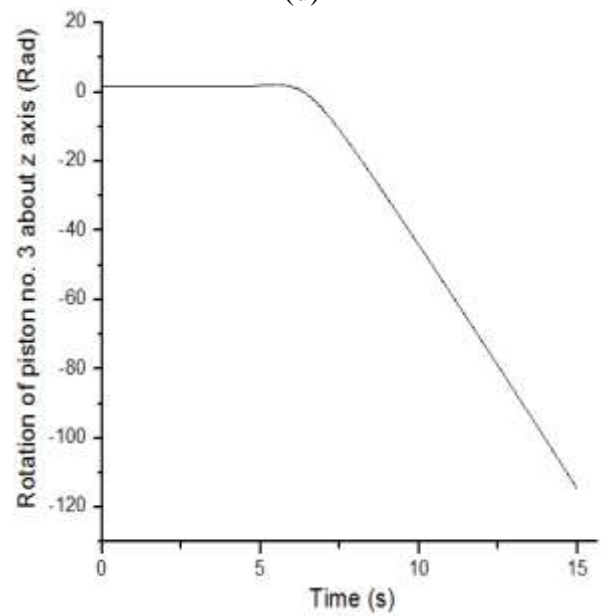
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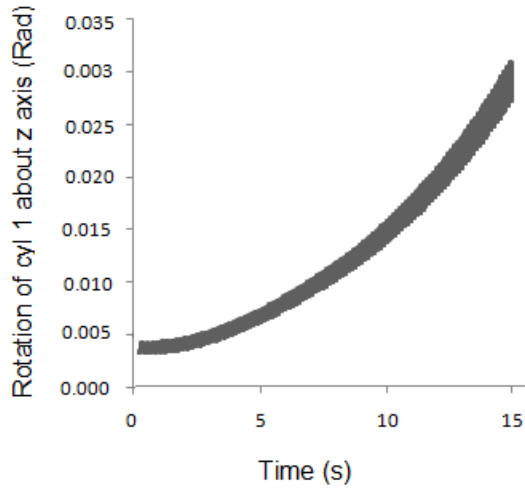


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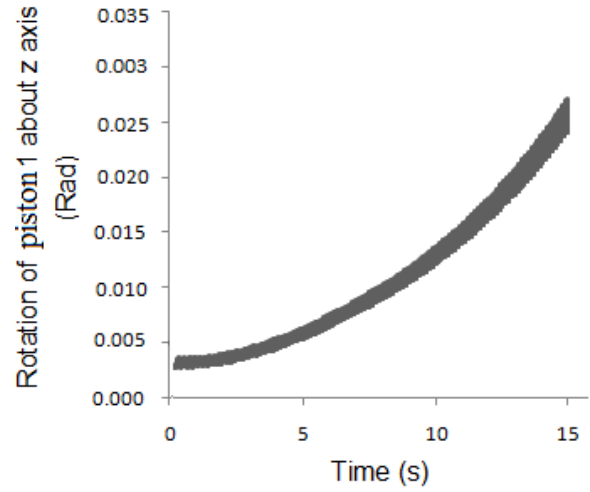


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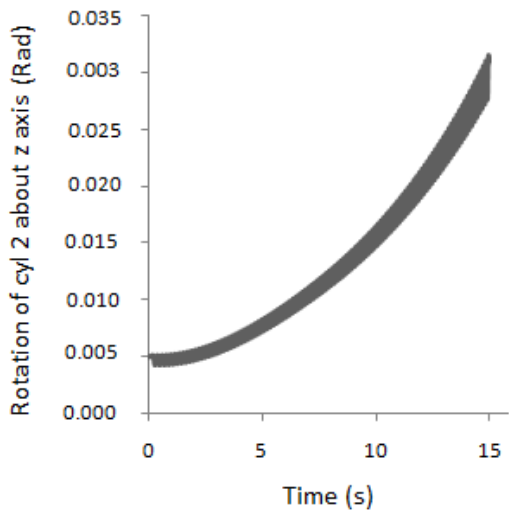
Fig. 3.26 Plots showing that without balancer shaft engine rocks (a,b) Rotation of cylinder-1 and piston-1 about z axis, (c,d) Rotation of cylinder-2 and piston-2 about z axis and (e,f) Rotation of cylinder-3 and piston-3 about z axis



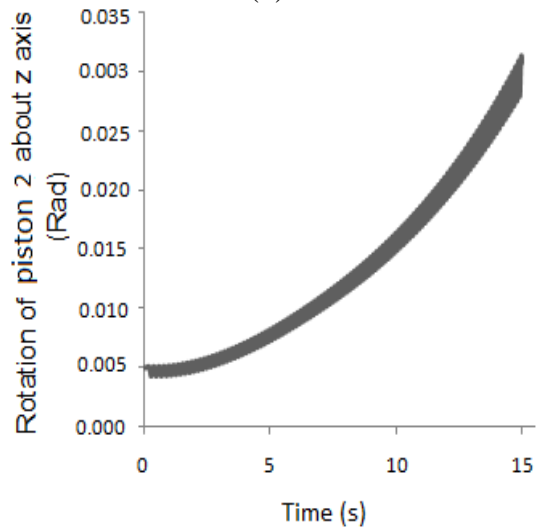
(a)



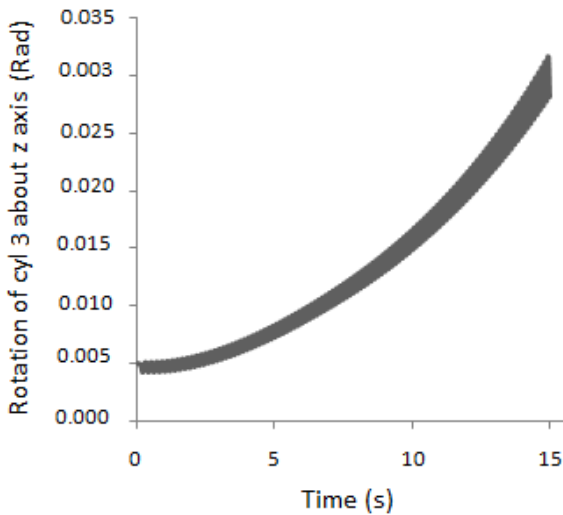
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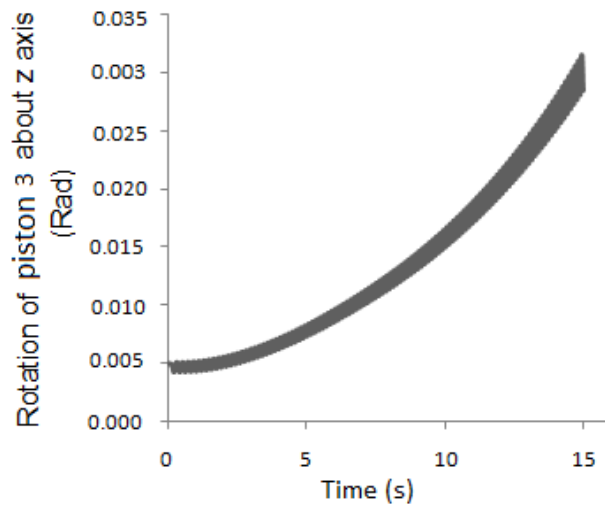
(c)



(d)



(e)



(f)

Fig. 3.27 Plots showing that with balancer shaft engine rocking disappears (a,b) Rotation of cylinder-1 and piston-1 about z axis, (c,d) Rotation of cylinder-2 and piston-2 about z axis and (e,f) Rotation of cylinder-3 and piston-3 about z axis

3.10 Six-cylinder inline engine

6-cylinder inline engine's crankshaft can be seen as two 3 cylinder crankshafts placed end to end, but with one half reversed. It is symmetric about centrally placed vertical plane. Such configuration results in "completely balanced" the crankshaft in reciprocating forces and moments as well as in rotating forces and moments.

Resultant primary inertia forces of reciprocating masses and their moments are zero i.e.
 $\Sigma F_I = \Sigma M_I = 0$

Resultant secondary inertia forces of reciprocating masses and their moments are zero i.e.
 $\Sigma F_{II} = \Sigma M_{II} = 0$

Hence, the 6-Cylinder inline engine is "completely balanced" as shown in Fig. 3.28

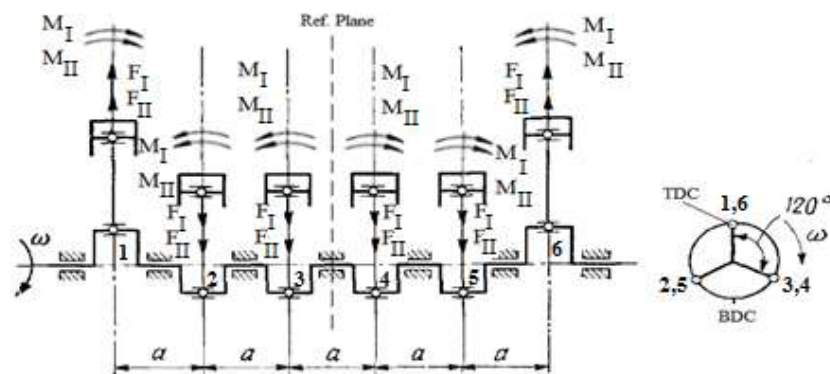


Fig. 3.28 Inertia forces and moments acting in 6-cylinder inline engine [19]

3.11 Need of counterweights in symmetric crankshafts

- To prevent deformation of crankshaft at main bearings as counterweights pulls the crankshaft in opposite direction to connecting rod-piston movements.
- The primary couples tries to bend the crankshafts with deflection increasing with rpm as shown in Fig. 3.29. Without counterweights, this deflection causes fatigue and main bearing failures.
- The counterweights produce couple in the crankshaft that counters the primary couple and pull the crankshaft straighten to sustain higher rpms. Hence, no bending loads on crankshaft. Also, bearing shells life increases.

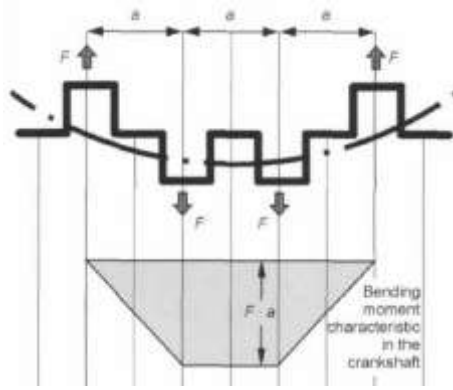


Fig. 3.29 Inertia forces tries to bend the crankshaft in 4-cylinder inline engine [11]

Chapter 4

Mass Balancing of Four-Cylinder Inline Engine

This chapter deals with four-cylinder inline engine and its mass balancing with the help of bond graph approach. Firstly, the configuration of 4-cylinder inline engine is studied i.e. what types of inertia forces and moments are acting on such an engine while it is running, then it is studied that how to balance these inertia forces i.e. with the help of two secondary balancer shafts. Lanchester balancing technique is used for modelling of secondary balancer shafts. Further, bond graph of 4-cylinder inline engine is modelled along with two secondary balancer shafts in Symbols Shakti 2.0 software. Simulations from this software are obtained and studied.

4.1 Introduction

The crankshaft of four-cylinder inline engine is symmetrical about central plane. Hence, such configuration is internally balanced by their design.

Pistons 1 and 4 moves in direction opposite to 2 and 3. Therefore, these two pairs of pistons balance each other and neutralizes primary inertia forces as shown in Fig. 4.1. But, secondary inertia exists in 4-cylinder inline engine, especially at high speeds as shown in Fig. 4.2. In order to dampen them two secondary balancer shafts are used.

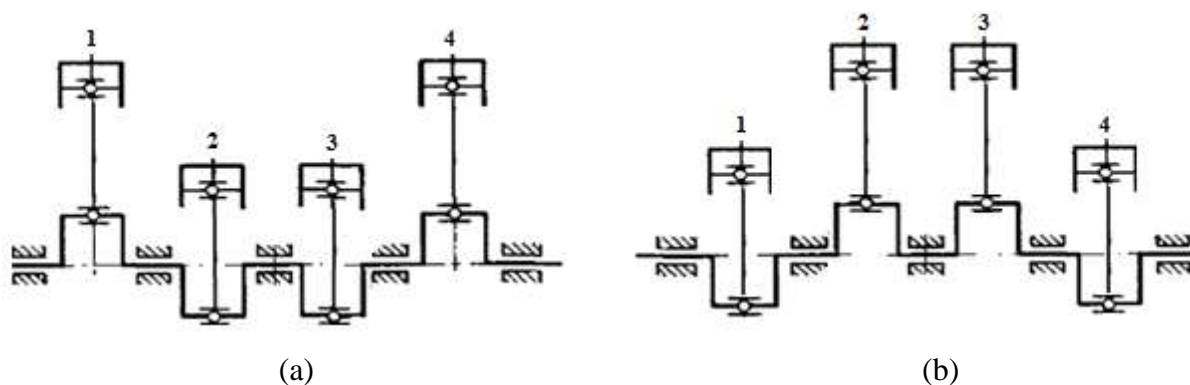


Fig. 4.1 Two pairs of pistons (1,4 and 2,3) moves in opposite directions and neutralizes primary inertia forces

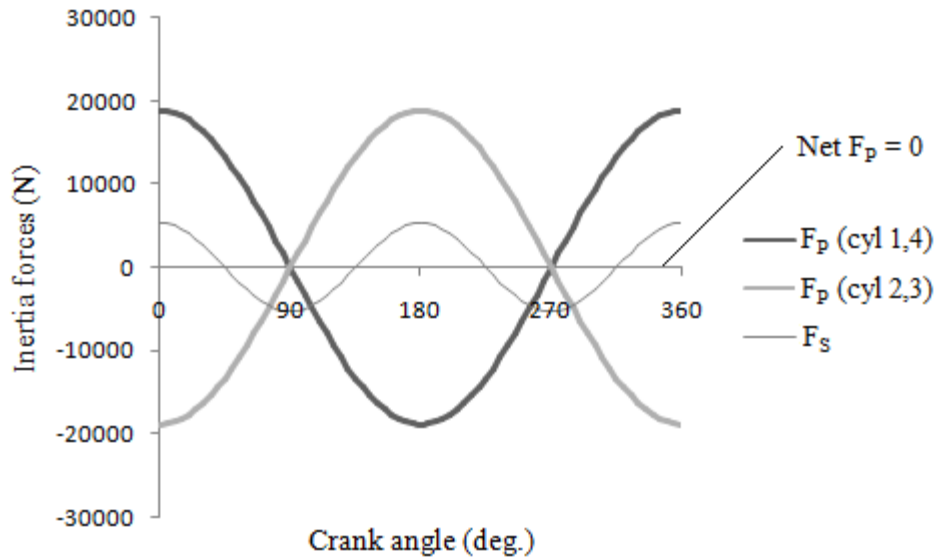


Fig.4.2 Net primary inertia forces is 0, but secondary inertia forces exists, obtained from Microsoft Excel 2007 F_p

Firing in an engine must be uniform with fixed intervals in such a way so as to distribute the forces generated due to combustion uniformly on the engine. In four-cylinder inline engine firing order is 1-3-4-2 [24], as shown in Table 4.1. Engine balancing is also affected due to this firing order.

Table 4.1: Occurrence of power strokes in 4-cylinder inline engines

Cylinder No.	1	3	4	2
1st Stroke	P	C	I	E
2nd Stroke	E	P	C	I
3rd Stroke	I	E	P	C
4th Stroke	C	I	E	P
	180° 180° 180° 180° 			
	 1 rev. (360°) 1 rev. (360°)			
	 720° of Crankshaft Rotation (1 complete cycle)			

4.1.1 Inertia forces and moments in 4-cylinder inline engine (analytical approach)

Let us consider a plane passing through the middle bearing about which the arrangement is symmetrical as the reference plane. Cylinder spacing is expressed as a . The angular positions for the cranks are θ for the first, $180^\circ + \theta$ for the second, $180^\circ + \theta$ for the third and θ for the fourth as shown in Fig. 4.3 (a). Secondary cranks are having angular positions as 2θ for the first, $360^\circ + 2\theta$ for the second, $360^\circ + 2\theta$ for the third and 2θ for the fourth as shown in Fig. 4.3 (b).

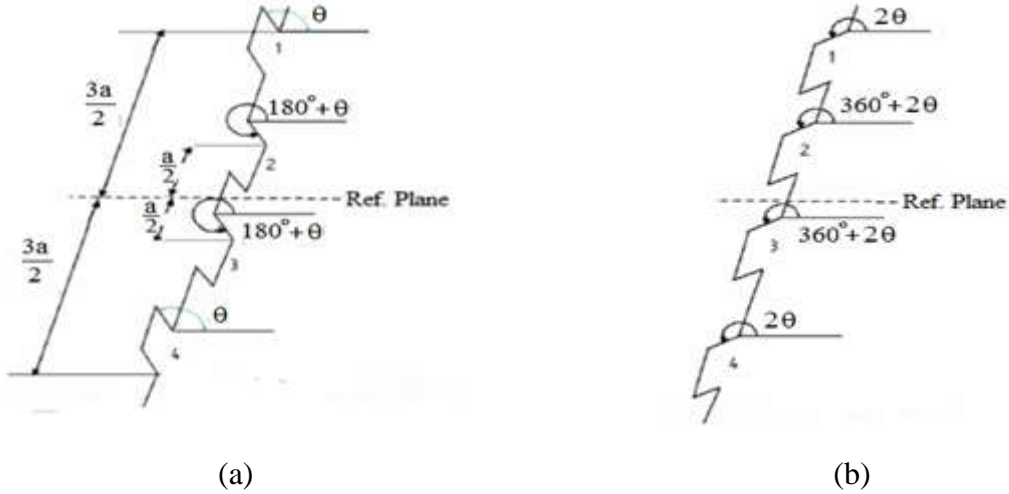


Fig. 4.3 (a) Primary and (b) Secondary cranks

Primary inertia force is given by

$$\begin{aligned}
 F_p &= m_{re} r \omega^2 [\cos \theta + \cos(180^\circ + \theta) + \cos(180^\circ + \theta) + \cos \theta] \\
 &= m_{re} r \omega^2 [2 \cos \theta + 2 \cos 180^\circ \cos \theta - 2 \sin 180^\circ \sin \theta] = 0
 \end{aligned} \tag{4.1}$$

Let us consider clockwise moment positive, then primary moment is given by

$$M_p = m_{re} r \omega^2 \left[\left(\frac{3a}{2} \right) \cos \theta + \left(-\frac{a}{2} \right) \cos(180^\circ + \theta) + \left(\frac{a}{2} \right) \cos(180^\circ + \theta) + \left(-\frac{3a}{2} \right) \cos \theta \right] = 0 \tag{4.2}$$

Secondary inertia force is given by

$$\begin{aligned}
 F_s &= \frac{m_{re} r^2 \omega^2}{l} [\cos 2\theta + \cos(360^\circ + 2\theta) + \cos(360^\circ + 2\theta) + \cos 2\theta] \\
 &= \frac{m_{re} r^2 \omega^2}{l} [2 \cos 2\theta + 2 \cos 360^\circ \cos 2\theta - 2 \sin 360^\circ \sin 2\theta] \\
 &= \frac{m_{re} r^2 \omega^2}{l} [2 \cos 2\theta + 2 \cos 2\theta] = \frac{4m_{re} r^2 \omega^2}{l} \cos 2\theta
 \end{aligned} \tag{4.3}$$

Maximum secondary inertia force is given by

$$F_{S_{max}} = \frac{4m_{re} r^2 \omega^2}{l} \cos 2\theta, \text{ at } 2\theta = 0^\circ, 180^\circ, 360^\circ, 540^\circ \text{ or } \theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ$$

Secondary moment is given by

$$M_s = \frac{m_{re} r^2 \omega^2}{l} \left[\left(\frac{3a}{2} \right) \cos 2\theta + \left(\frac{a}{2} \right) \cos(360^\circ + 2\theta) + \left(-\frac{a}{2} \right) \cos(360^\circ + 2\theta) + \left(-\frac{3a}{2} \right) \cos 2\theta \right] = 0 \quad (4.4)$$

Thus, the engine is not balanced in secondary inertia forces as shown in Fig. 4.4

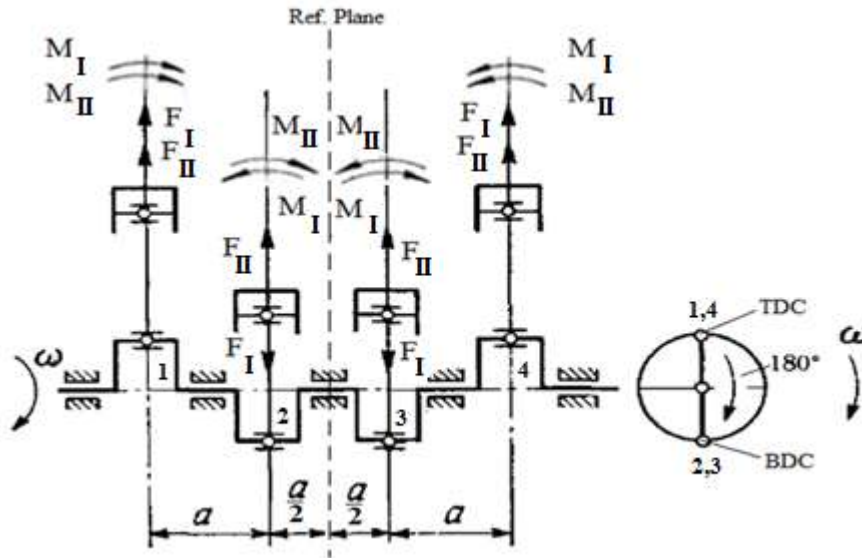


Fig. 4.4 Inertia forces and moments acting in 4-cylinder inline engine [19]

Table 4.2 Amplitude of reciprocating inertia forces and couples in 4-cylinder engines

Forces and couples	Expressions
Primary reciprocating inertia force	Zero
Secondary reciprocating inertia force	$\frac{4m_{re} r^2 \omega^2}{l} \cos 2\theta$
Primary couple	Zero
Secondary couple	Zero

Table 4.3 Values of various parametric for 4-cylinder inline engine

Parameters	Description	Values
l	Connecting rod length (m)	0.2
m_{cr}	Mass of connecting rod assembly (kg)	2
m_{crb}	Connecting rod big end rotating mass, located at crank pin (m_A) (kg)	1.5
m_{crs}	Connecting rod small end reciprocating mass, located at piston pin	0.5

	(m_B) (kg)	
m_{pa}	Mass of piston assembly (piston, piston pin, piston rings, 2 circlips) (kg)	1.5
m_{re}	Reciprocating mass: mass of piston assembly + connecting rod small end mass (kg)	2
N	Over speed (rpm)	4225
ω	Angular velocity of crankshaft ($2\pi N/60$) (rad/s)	442.44
r	Crank radius (m)	0.06
λ	Connecting rod ratio $\left(\frac{r}{l}\right)$	0.3

4.1.2 Secondary balancer shaft design for 4-cylinder inline engine

To balance secondary inertia forces in 4-cylinder inline engines, two secondary balancer shafts are used, both rotating with twice the crankshaft speed [25] as shown in Fig. 4.5. The two secondary balancer shafts rotate in the opposite direction with respect to one another [10]. It is necessary to use two secondary balancer shafts instead of one because rotating two shafts in opposite direction will cancel their lateral forces i.e. side to side vibrations in horizontal direction as shown in Fig. 4.5. If we use only one shaft, the lateral component of the force generated by secondary shafts remains unbalanced. The location of secondary balancer shaft with respect to crankshaft is shown in Fig. 4.6.



Fig. 4.5 Two balancer shafts cancel out lateral forces

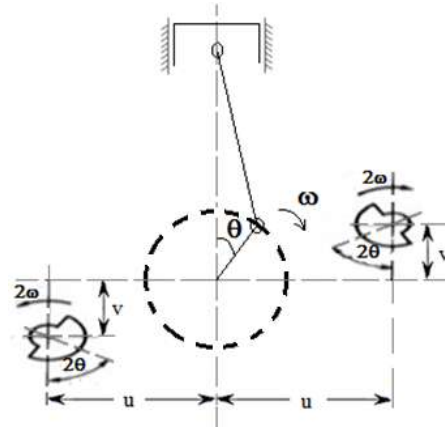


Fig. 4.6 Location of secondary balancer with respect to crankshaft [26]

The unbalance value ($m_B r_B$) is calculated as [21]

$$\frac{4m_{re} r^2 \omega^2}{l} = 2m_B r_B (2\omega)^2 \quad (4.5)$$

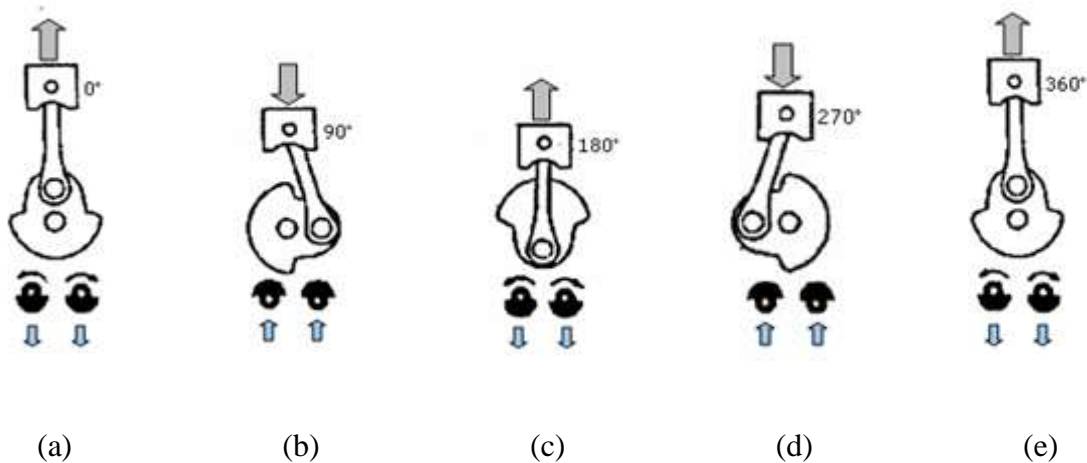


Fig. 4.7 Two counter rotating secondary balancer shafts

Twin secondary balancer shafts on 4-cylinder inline engine are used to reduce secondary inertia forces [18]. The two counter rotating balancer shafts having offset masses are rotated at twice the speed of crankshaft [27]. The crankshaft drives these two shafts. Both the balancer shafts are timed in such a way with respect to the crankshaft that when the piston is at TDC both masses exert force in downward direction. The shafts exert an opposing force only when it is necessary. For 4-cylinder inline engine this balancing force is maximum at 0° , 90° , 180° , 270° and 360° rotation of the crankshaft. In Fig. 4.7 (a), (c) and (e), this balancing force is acting in downward direction and in Fig. 4.7 (b) and (d), upwards. In

between these crank angles, the two balancing masses oppose each other causing a neutral effect. The engine attains a balance state in these neutral positions. Fig. 4.8 shows secondary balancer shafts position with respect to crankshaft.

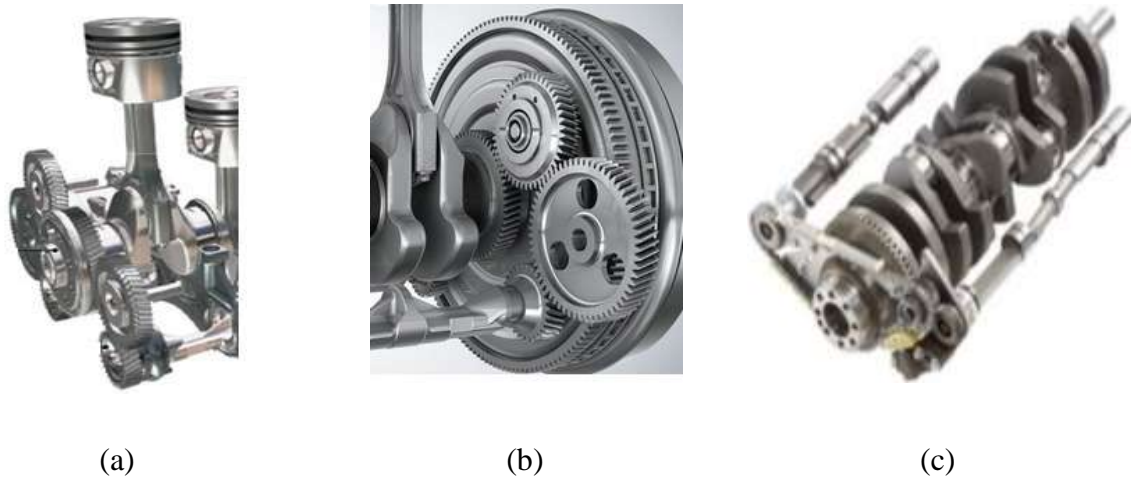


Fig. 4.8 Secondary balancer shafts position w.r.t crankshaft [14, 28]

4.2 Lanchester balancing technique

This technique is used for complete dynamic balancing of the engine [17]. In Fig. 4.9, gear 2 is identical to crank gear (gear 1) and gear 3 and 2 are attached to the same shaft. Hence, speed of gears 1, 2 and 3 is same ω , but gears 2 and 3 rotate in opposite direction to gear 1. Also, gear 4 is identical to gear 3 and so rotates with ω , and with same direction as that of gear 1. Forces generated by of gears 3 and 4 (horizontal components combined) balances primary reciprocating inertia force produced by piston assembly and connecting rod small end. The vertical components neutralizes among themselves.

$$\frac{(m' r')}{2} + \omega^2 \cos(\omega t) + \frac{(m' r')}{2} + \omega^2 \cos(\omega t) = m' r' \omega^2 \cos(\omega t) \quad (4.6)$$

$m' r'$ is so calculated such that, $m' r' = m_{re} r$

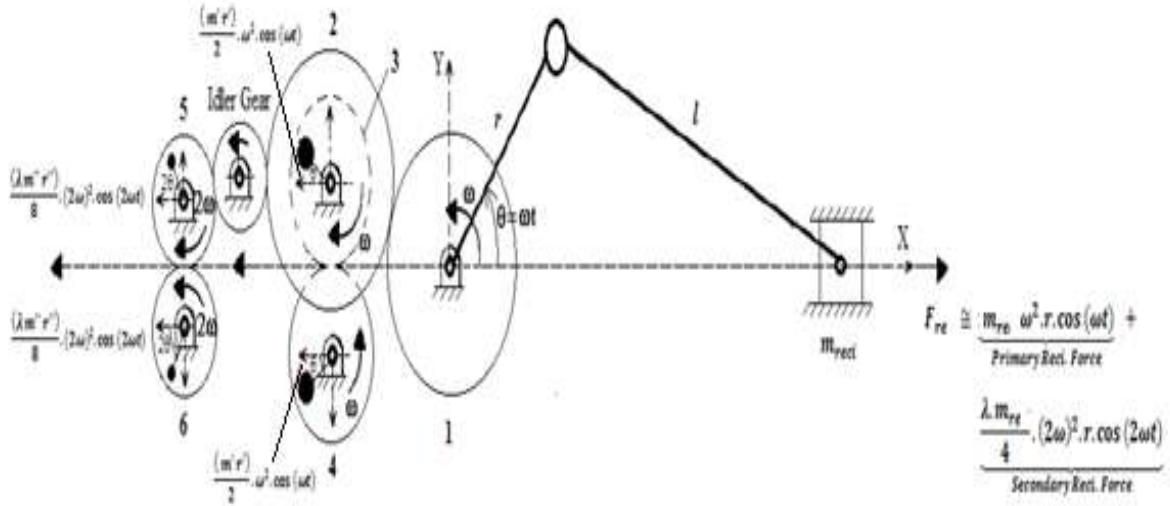


Fig. 4.9 Lanchester balancing technique

Further, to balance secondary reciprocating inertia force of piston assembly and connecting rod small end, we use two more identical gears (5 and 6), such that their radii is half of gear 1 as shown in Fig. 4.9. Hence, they rotate with twice the speed of gear 1. An idler gear is used between gear 2 and 5 to make the rotational direction of gear 5 opposite to gear 1. Gear 6 rotates in same direction as that of gear 1. Forces generated by of gears 5 and 6 (horizontal components combined) balances secondary reciprocating inertia force produced by piston assembly and connecting rod small end. The vertical components neutralizes among themselves.

$$\frac{(\lambda m'' r'')}{8} + (2\omega)^2 \cos(\omega t) + \frac{(\lambda m'' r'')}{8} + (2\omega)^2 \cos(\omega t) = \frac{(\lambda m'' r'')}{4} (2\omega)^2 \cos(2\omega t) \quad (4.7)$$

$m'' r''$ is so calculated such that, $m'' r'' = m_{re} r$

4.3 Bond graph modelling of 4-cylinder inline engine with two secondary balancer shafts

First of all, 4-cylinder inline engine is modelled [23] in Symbols Shakti 2.0 software, shown in Fig. 4.11. Also, two balancer shafts are modelled in such a way that their angular velocity is twice that of crankshaft. Based on Lanchester balancing technique (Section 4.2, Fig. 4.9), the horizontal components of the force generated by the two balancer shafts adds up and cancels the affects of the secondary inertia forces developed due to the reciprocating masses of the engine. The vertical components neutralized among themselves. Fig. 4.10 shows the arrangement of the two secondary balancer shafts with respect to crankshaft and the vector

diagrams. In Fig. 4.10 (a), 2, 3 and 5 are idler gears. Radii of gear 4 and 6 is half of the gear 1 (crank gear). The secondary balancer shafts are attached to the gear 4 and 6. Hence, they rotate at twice the rotational velocity of the crankshaft.

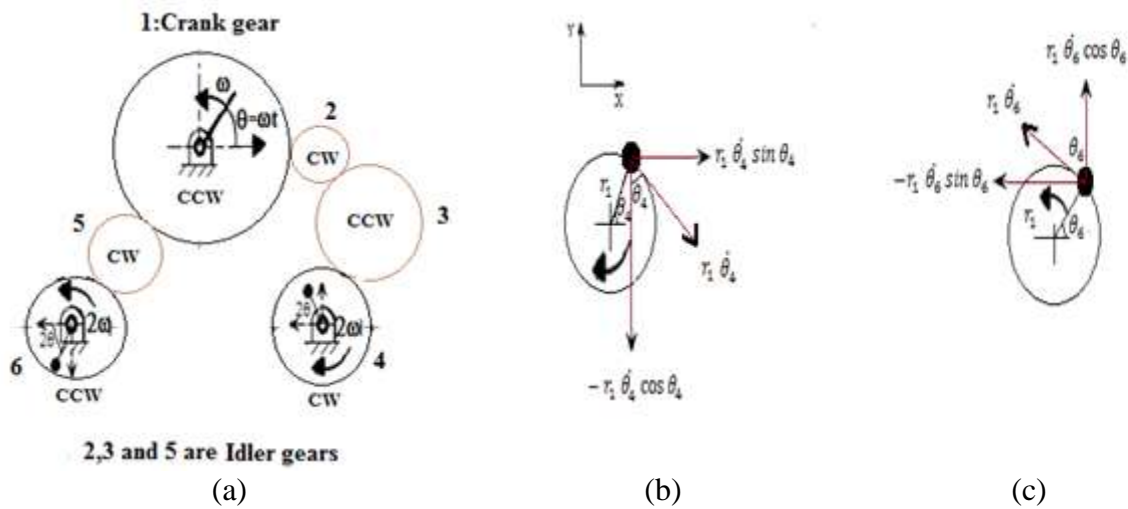


Fig. 4.10 (a) Arrangement of the two secondary balancer shafts with respect to crankshaft and (b), (c) Vector diagrams

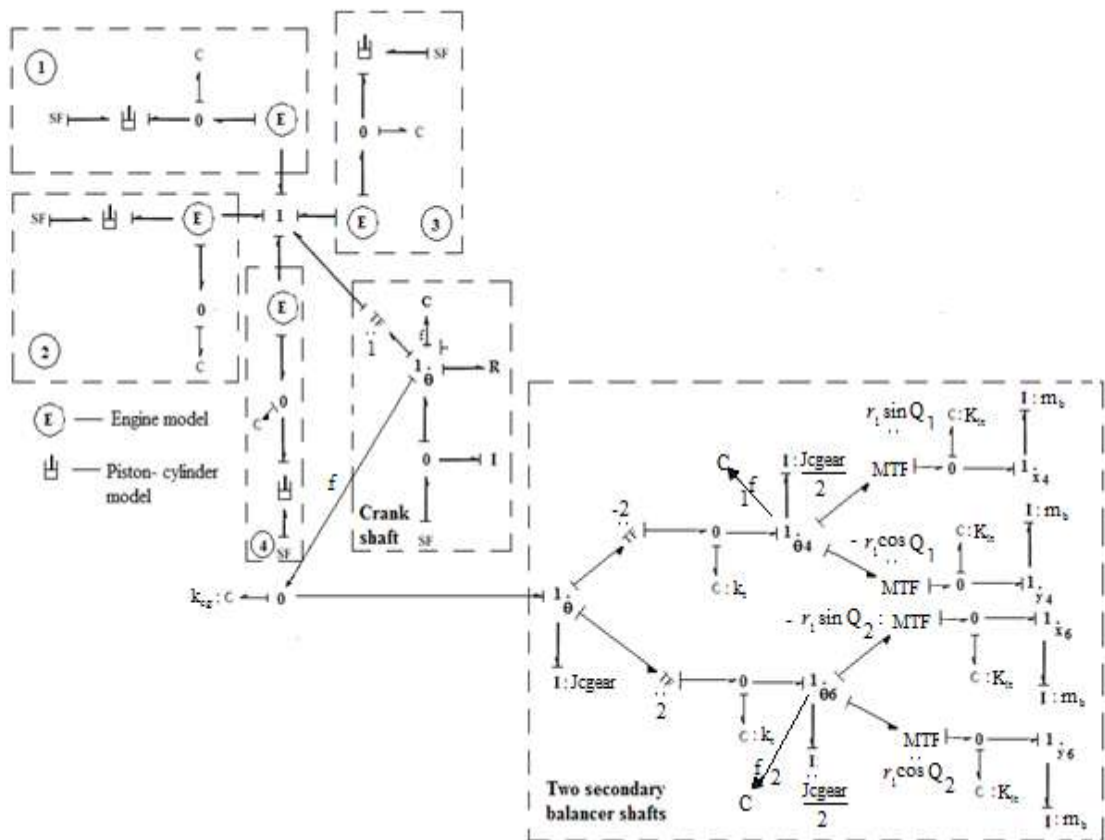


Fig. 4.11 Bond graph of 4-cylinder inline engine with two secondary balancer shafts

4.4 Parameter values and simulation results

The 4-cylinder engine parameters are inserted in the software as given in Table 3.9 of chapter 3. The Fig. 4.12 shows the pressure developed inside the cylinder as per the firing order 1-3-4-2. The pressure-volume diagram for all the four cylinders is shown in Fig. 4.13.

Crankshaft angular velocity ($\dot{\theta}$) is plotted with respect to time as shown in Fig. 4.14 (a) and the torque developed by the engine with time is shown in Fig. 4.14 (b).

The secondary inertia forces are plotted for each of the 4 actuators (pistons) with respect to time as shown in Fig. 4.15. The secondary inertia forces generated by the engine are due to reciprocating masses. These unbalanced forces are needed to be balanced. This is done by installing two balancer shafts. These balancer shafts produces equal forces but in opposite direction so as to balance the engine.

The forces produced by the two balancer shafts (their vertical and horizontal components) are plotted with respect to time as shown in the Fig. 4.16. The two vertical components cancel among themselves. But, horizontal components add up and balance the net secondary inertia forces developed by the engine.

The unbalance value ($m_B r_B$) of the one balancer mass is calculated as [21]

$$\frac{4m_r r^2 \omega^2}{l} = 2m_B r_B (2\omega)^2 \quad (4.8)$$

where, m_B = balancing mass attached to balancer shaft and r_B =radial distance from COG of balancing mass to axis of rotation of the balancer shaft.

For, 4-cylinder engine, it comes out to be 0.018 kg m. Table 4.4 shows the results comparisons between standard expressions and software outputs.

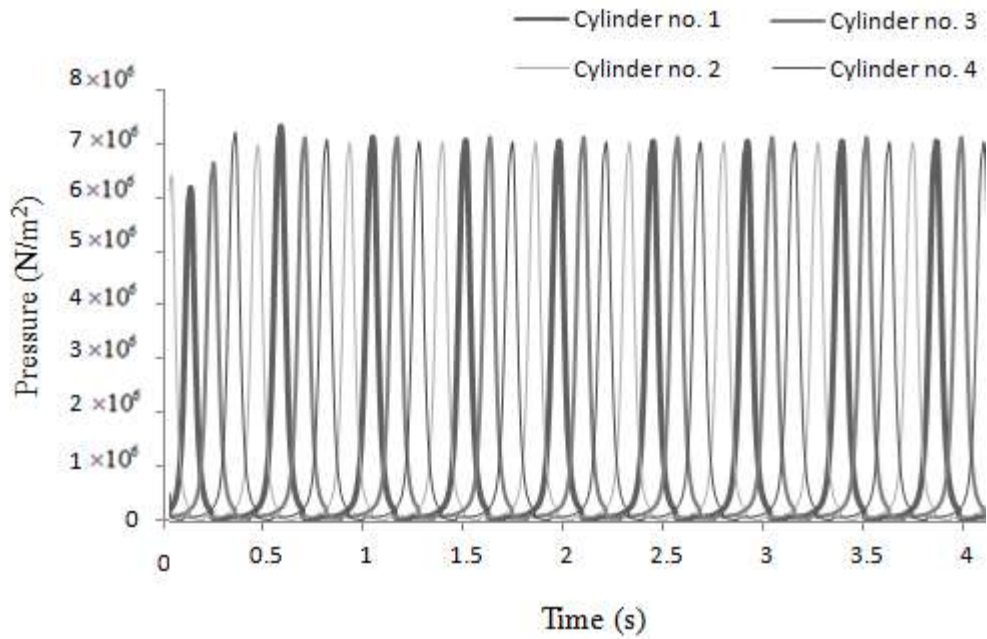
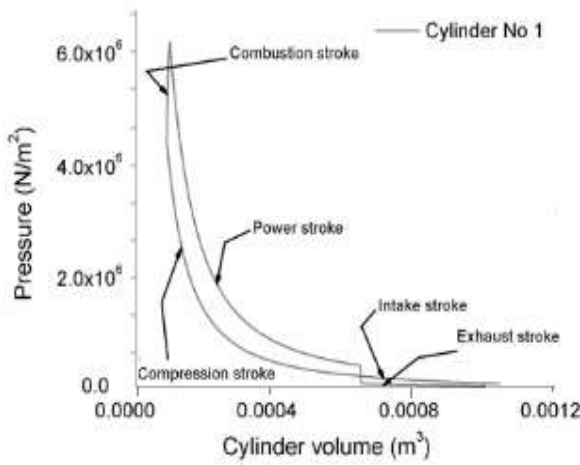
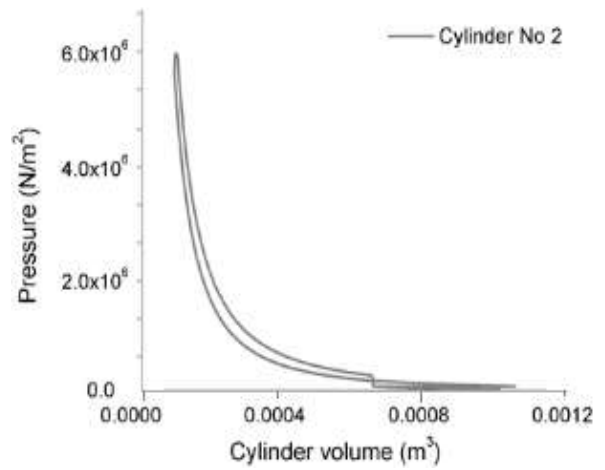


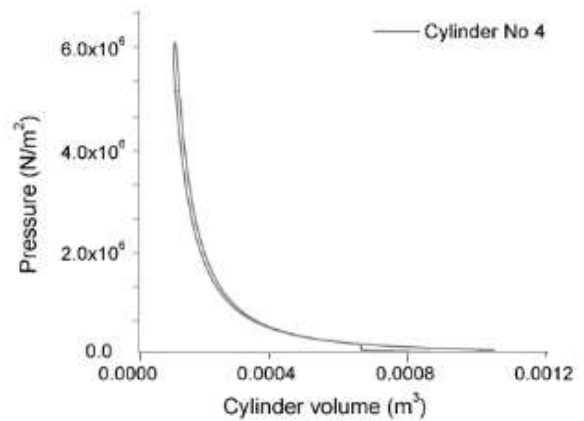
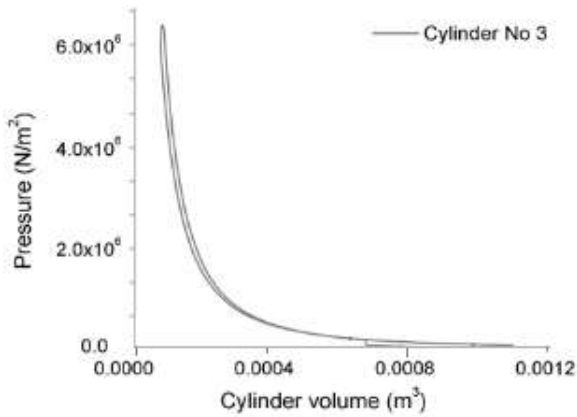
Fig. 4.12 Pressure developed inside the cylinders



(a)



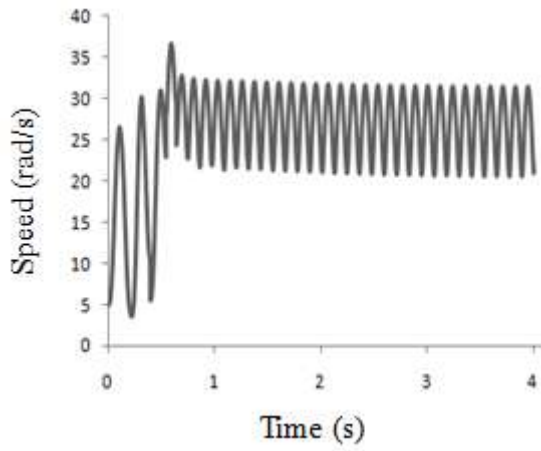
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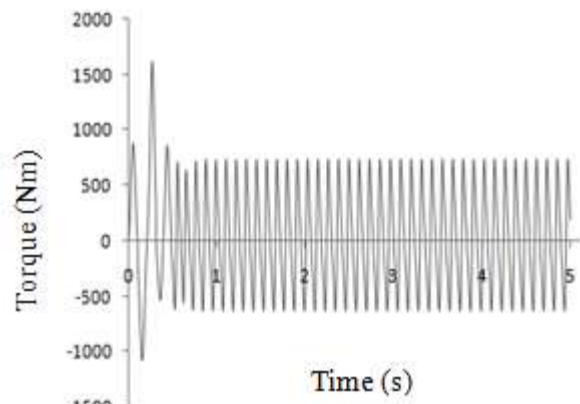
(c)

(d)

Fig. 4.13 Pressure-volume diagram of (a) Cylinder 1, (b) Cylinder 2, (c) Cylinder 3 and (d) Cylinder 4

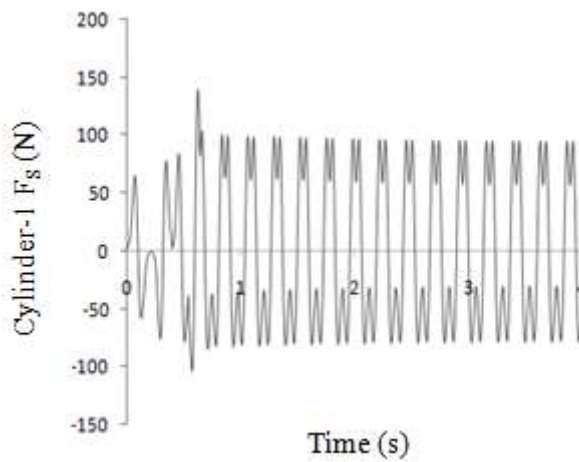


(a)

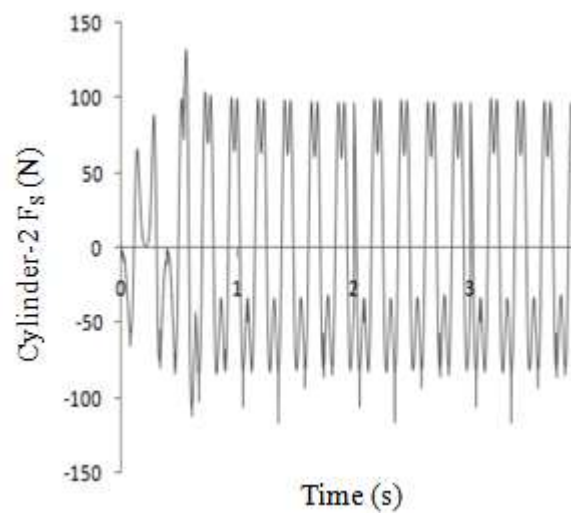


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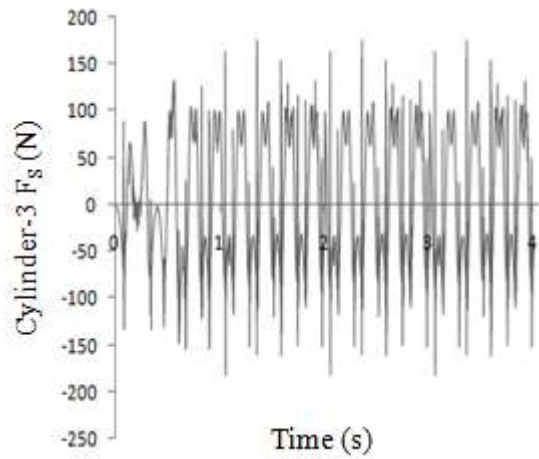
Fig. 4.14 (a) Speed of crankshaft with respect to time and (b) Engine torque with respect to time



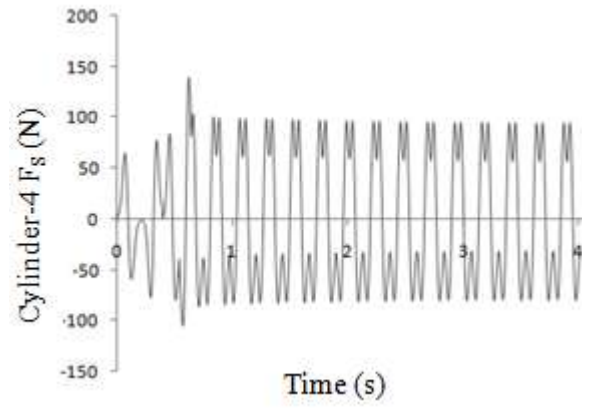
(a)



(b)

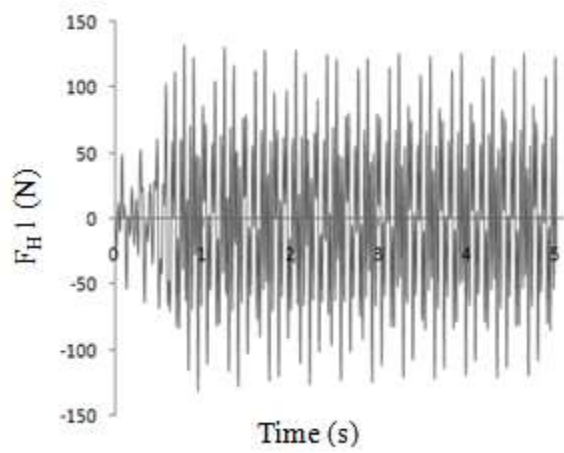


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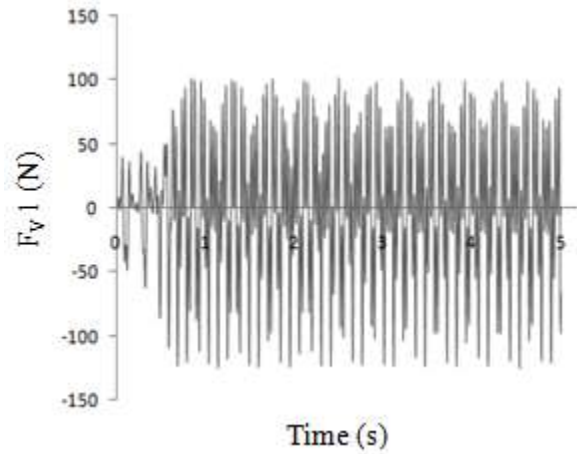


(d)

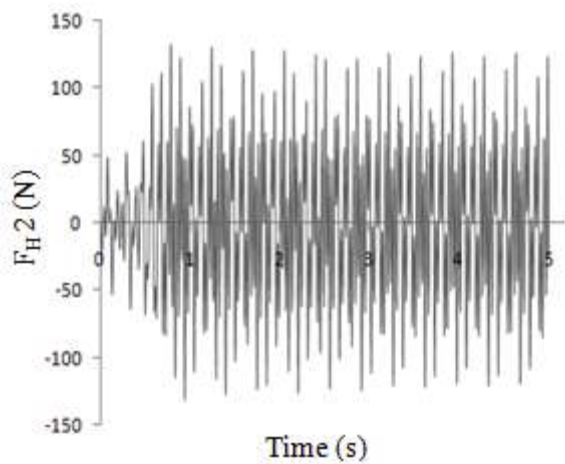
Fig. 4.15 Secondary inertia force of (a) Cylinder 1, (b) Cylinder 2, (c) Cylinder 3 and (d) Cylinder 4



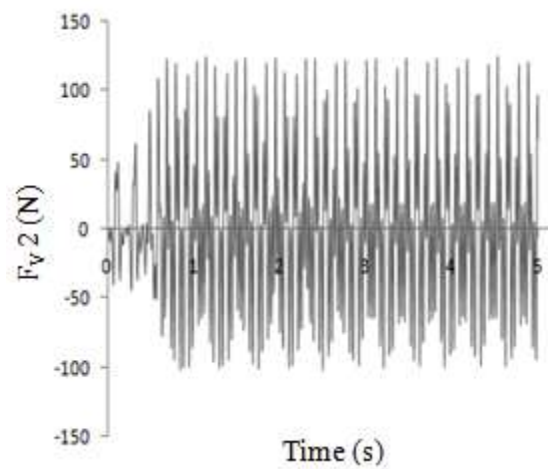
(a)



(b)



(c)



(d)

Fig. 4.16 Plots of (a) Horizontal component of force produced by balance shaft 1 with time, (b) Vertical component of force produced by balance shaft 1 with time, (c) Horizontal component of force produced by balance shaft 2 with time and (d) Vertical component of force produced by balance shaft 2 with time

Table 4.4 Results for 4-cylinder inline engine

Quantities	Results from expression	Software results
Net secondary inertia forces (N)	61.82 (for $\omega=20.72$ rad/s)	62.5
Unbalance required (kg m)	0.018	0.02
Forces generated by 2 balancer shafts (N)	-62	-63

Chapter 5

Conclusions

5.1 Conclusions

The objective of this thesis is to show the applicability of bond graph technique in modelling the inline-engines for mass balancing. Bond graph models for 3-cylinder inline-engine with single balancer shafts and 4-cylinder inline-engine with two balancer shafts were studied and analyzed. The following conclusions can be drawn from the work done in this thesis:

- The simulations of the models developed in this work present good quality results. The results of engine mass balancing are very much similar to the other theoretical ones.
- Mass balancing of various inline engine configurations (3 and 4) was studied.
- Various unbalanced forces and moments of 3 and 4-cylinder inline engines were calculated.
- Optimum design of balancing weights of primary balancer shaft for three-cylinder inline engine was done.
- Optimum design of counter weights of crank shaft for three-cylinder inline-engine was done.
- Further, optimum design of balancing weights of secondary balancer shafts for four-cylinder inline engine was done.
- Modelling of 3-cylinder inline engine with balancer shaft is done in Symbols Shakti 2.0 software and balancing results are being simulated. Results show that with the introduction of balancer shaft, rocking of the engine disappears i.e. engine becomes balanced. Rotation of each cylinder-piston arrangement about z-axis is reduced considerably with the introduction of balancer shaft.
- Plots of the pressure developed inside the cylinders with respect to time for 3-cylinder inline engine clearly shows that the firing order is 1-3-2 i.e. firstly cylinder-1 fires, followed with cylinder-3 and thereafter cylinder-2 and the cycle repeats.

- Plots of crankshaft speed with respect to time and the engine torque with respect to time for 3-cylinder inline engine were studied. The results were close to theoretical results.
- Modelling of 4-cylinder inline engine with two balancer shafts is done in Symbols Shakti 2.0 software.
- Plots of the pressure developed inside the cylinders with respect to time for 4-cylinder inline engine clearly shows that the firing order is 1-3-4-2 i.e. firstly cylinder-1 fires, followed with cylinder-3, then cylinder-4 and thereafter cylinder-2 and the cycle repeats.
- Plots of crankshaft speed with respect to time and the engine torque with respect to time for 4-cylinder inline engine were studied. The results were close to theoretical results.
- Plots of secondary inertia forces developed in 4-cylinder inline engine due to unbalanced reciprocating masses with respect to time were studied.
- Design of balancing weights of secondary balancer shaft has been validated on this software with the classical formulae. This is done by studying the plots of horizontal and vertical components of the force produced by the two balancer shafts with respect to time.
- It can be seen that the horizontal components of the force produced by the two balancer shafts combined is equal to the net secondary inertia force developed by the engine but in opposite direction. Hence, these balancer shafts balance the engine. Also, the vertical components of the force produced by the two balancer shafts cancel among themselves.
- Symbols Shakti 2.0 modelling and simulation tool proved to be highly beneficial in evaluating this work done in this thesis.
- Further, unbalance value ($\text{mass} \times \text{radial distance from rotational axis}$) were calculated for the crankshaft's counterweights and balancer shaft's balancing weights. Based on these unbalanced values, modifications in the design is done in Creo 3.0 software.
- An overbalance is maintained in the crankshaft's counterweights which enable final balancing to be done by the manufacturer by drilling holes into the counterweights.

5.2 Future Scope

Based on the work presented in this thesis, the following work is suggested for future:

- Without balancer shaft, 3-cylinder inline engine balancing is to be studied with the help of Symbols Shakti 2.0 software, along with engine mount optimisation (as done by Ford 1.0 L Eco-boost balancing strategy).
- In such type of 3-cylinder engine, additional weight of balancer shaft gets reduced. There will be one less component to be driven by crankshaft. Hence, it will lead to improvement in engine acoustics and higher fuel efficiency.

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