

# **TAMING INVERSE RESPONSE**

*A Thesis submitted in partial fulfillment of the  
requirements for the award of degree of*

**Master of Engineering**

**in**

**Electronic Instrumentation and Control**



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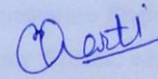
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## DECLARATION

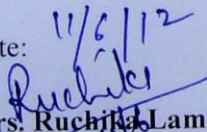
I hereby certify that the work which is being presented in the thesis entitled “ **Taming Inverse Response**” in partial fulfillment of award of degree of **Master of Engineering in Electronics Instrumentation and Control** submitted in electrical and instrumentation engineering department, Thapar University, Patiala is an authentic record of my own work carried under the supervision of **Mrs. Ruchika Lamba**, Assistant Professor, Department of Electrical and Instrumentation Engineering, Thapar University, Patiala, Punjab and **Dr. Mandeep Singh**, Assistant Professor, Department of Electrical and Instrumentation Engineering, Thapar University, Patiala, Punjab.

Date: 11/6/12

  
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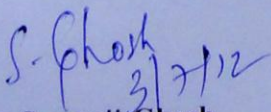
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## **ABSTRACT**

Until recent years, inverse response has always been the largest problem in a boiler plant. Only the largest boilers could justify sophisticated boiler controls. Its high fuel costs and limited fuel availability make it necessary to improve boiler efficiency. Drum level controls have become more important because boiler loads are being varied to meet needs, rather than operating at full capacity and wasting fuel and steam.

An inverse response has always been an evergreen challenge for the instrumentation engineers. The efficiency of an industrial plant like in boiler plants directly depends upon the way inverse response is handled. The inverse response is controlled by manipulating its parameters so that the errors and the offset are minimal. This paper presents a novel method to compensate the inverse response of the process comprising of two opposite first order systems with a delay element. The compensator for inverse response designed with both accurately and inaccurately estimated parameters. This is simulated in MATLAB SIMULINK and checked for its efficacy.

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## LIST OF SYMBOLS AND ABBREVIATIONS

<b>PID</b>	Proportional Integral Derivative
<b>T</b>	Torque
<b>T</b>	Time
<b>E(t)</b>	Error
<b>PSI</b>	Pound Square Inch
<b>PI</b>	Proportional Integral
<b>PV</b>	Process Variable
<b>K<sub>c</sub></b>	Controller Gain
<b>K<sub>p</sub></b>	Proportional Gain
<b>K<sub>i</sub></b>	Integral Gain
<b>K<sub>d</sub></b>	Derivative Gain
<b>T<sub>i</sub></b>	Integral Time
<b>T<sub>d</sub></b>	Derivative Time
<b>PB</b>	Proportional Band
<b>H/H<sub>inv</sub></b>	Minimum phase systems
<b>G<sub>a</sub>(j<math>\omega</math>)</b>	Minimum phase system
<b>G<sub>b</sub>(j<math>\omega</math>)</b>	Non-minimum phase system
<b><math>\Phi(\omega)</math></b>	Phase response
<b>LSE</b>	Least square error
<b>F</b>	Flow rate of water
<b><math>\mu</math></b>	Viscosity of water
<b><math>\Delta P</math></b>	Pressure difference between two pipes
<b>H</b>	Time delay
<b>T<sub>s</sub></b>	Settling Time
<b>M<sub>p</sub></b>	Peak Overshoot
<b>ISE</b>	Integral Square Error

<b>IAE</b>	Integral Absolute Error
<b>ITAE</b>	Integral of time multiplexed absolute error
<b>ISE</b>	Integral square error
<b>ITSE</b>	Integral of time multiplexed square error
<b><math>P_0(s)</math></b>	Process model
<b>SP</b>	Set point
<b>GUI</b>	Graphical user interface

# CHAPTER- 1

## INTRODUCTION

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### 1.1 Overview

An inverse response occurs due to the presence of right half plane zeroes or when the initial response is in the opposite direction with respect to the ultimate steady-state value. The common ‘boiler swell’ problem is an example of it.

A simple drum boiler is considered in which the level is controlled by manipulating the boiler feed water into the drum. A constant heat supply is provided to the boiler. Water vapors exit from the ‘riser’ pipes from the top of the boiler whereas liquid is drained downward from the bottom pipes. [1]

The vapor pressure is kept constant if the feed water is significantly colder than the temperature of steam in the drum. Now if on increasing the flow of feed water causes a temperature drop which decreases the volume of the entrained vapor bubbles. This leads to the following first-order behavior i.e.  $-\frac{k_2}{\tau s+1}$  and liquid level decreases initially. As the constant heat is supplied, the liquid level will start increasing in an integral form leading to a pure capacitive system. The result of two opposing effects is given by equation 1.1.

$$\frac{k_1}{s} - \frac{k_2}{\tau s+1} = \frac{(k_1\tau - k_2)s + k_1}{s(\tau s+1)} \quad - (1.1)$$

and for

$$K_1 * t < K_2 \quad - (1.2)$$

The system shows inverse response only if the above condition satisfies. The transfer

function has a positive zero at the point  $s = \frac{-k_1}{k_1 t - k_2} > 0$  - (1.3)

On decreasing the flow of feed water, the volume of liquid increases abruptly for a short time. Now if the volume of bubbles increases the cold water supply reduces leading to first order behavior. Since the steam supply remains constant and consequently the liquid level of the boiling water decreases in a pure capacitive response,  $K_2/s$ . The result of two opposing effects is given by equation 1.4.

$$\frac{k_2}{\tau s + 1} - \frac{k_1}{s} = \frac{(k_2 - k_1 \tau)s - k_1}{s(\tau s + 1)} \quad - (1.4)$$

and for  $k_1 \tau < k_2$  - (1.5)

If above condition is satisfied only then the system possess an inverse response. [1]

The transfer function has a positive zero at the point  $S = \frac{k_1}{k_2 - k_1 \tau} > 0$ . - (1.6)

## 1.2 Motivation

As per this problem of inverse response inverse response compensators are designed by many researchers. Dead time is also a problem for control engineers. So for this a dead time compensator is also designed. Here in this report a novel method is introduced which compensate the inverse response with dead time. The inverse-response process means the process showing a step response in the opposite direction initially to where it eventually ends up. Such a dynamic behavior is called inverse response which is used by processing units, such as drum boiler and distillation column. The essential characteristic of the process with inverse response is that the process transfer function has one or an odd number of zeros in the open right half plane. This is a class of non-minimum phase (NMP) processes. Generally, these processes are particularly difficult to control and thus require special attention.

## 1.3 Objective of dissertation

The problem of inverse response has been an evergreen challenge for control engineers. So if the parameters are estimated then a compensator may be properly designed. We estimate the parameters for an inverse response of a system made up of a purely integrative process and a first order processes that act in opposite directions. We have a graph through which we need to evaluate the process parameters i.e.  $K_1$ ,  $K_2$ ,  $\tau$ . Then we, design a inverse reponse compensator for the transfer function. We already got the parameters of the transfer function we are using this parametric estimation of a process with inverse response can be done progressively in three stages given as

1. Modelling the system as comprised of a capacitive system and a first order system without delay. The parameters to be estimated here are  $k_1$ ,  $k_2$ ,  $\tau$ .
2. Two first order systems obtaining the parameters  $k_1$ ,  $k_2$ ,  $\tau_1$ ,  $\tau_2$ .
3. Two first order systems with delay obtaining the parameters  $k_1$ ,  $k_2$ ,  $\tau_1$ ,  $\tau_2$ ,  $D$ .

The inverse response compensator with delay increases the efficiency of the overall output of the plant by decreasing the inverse response generating in it.

1. Study of dead time compensator
2. Study and implementation of inverse response compensator
3. Designing of inverse response dead time compensator

## **1.4 Organization of thesis**

### **Chapter 1**

This chapter describes the introduction part of the inverse response and its compensator with the highlighted view on objective work which is to be carried out in thesis.

### **Chapter 2**

In this chapter the process of inverse response, how it occurs and various controllers which are used in its compensation are discussed.

### **Chapter 3**

This chapter explains about the inverse response compensator. The work is done in MATLAB SIMULINK. The work carried out by various researchers is observed.

### **Chapter 4**

This chapter explains the literature cited in this field.

### **Chapter 5**

This chapter presents most of the results of the inverse response compensator with delay. The results followed the results of many researchers who designed the inverse response compensator without delay.

### **Chapter 6**

This section deals with analysis of all conclusions and discussions which has been done in the presented work so far.

## CHAPTER- 2

### INVERSE RESPONSE AND CONTROLLERS

---

#### 2.1 Inverse response

The problem of inverse response is a challenge for all control engineers. Inverse response occurs due to two main reasons:

- when the response is in opposite direction with respect to the ultimate steady state value
- presence of right half plane zeros for any other reason as well

The examples where this process is used are like in distillation columns, drum boiler, boost converter, etc. In this paper a whole model is simulated in MATLAB SIMULINK. The process used is the arrangement of two first order transfer functions with a delay element. Their output goes to the controller. The feedback is given to complete the loop. The controller used is a PI controller which is tuned according to the model. The compensation is basically done on the basis of four parameters i.e. integral square error(ISE), integral of time weighted square error(ITSE), integral of absolute error(IAE), integral of time weighted absolute error(ITAE).

#### 2.2 Conditions Yielding Inverse Response

Assume that there are two opposing first-order stable processes whose transfer functions is given by  $g_1(s)$  and  $g_2(s)$  given by equation

$$g_1(s) = \frac{k_1}{\tau_1 s + 1}, \quad g_2(s) = \frac{k_2}{\tau_2 s + 1} \quad - (2.1)$$

where  $k_1$ ,  $k_2$ ,  $\tau_1$  and  $\tau_2$  are positive constants. Figure 2.1 shows the inverse response occurring by two opposing effects. Figure 2.2 shows the block diagram of the inverse response.

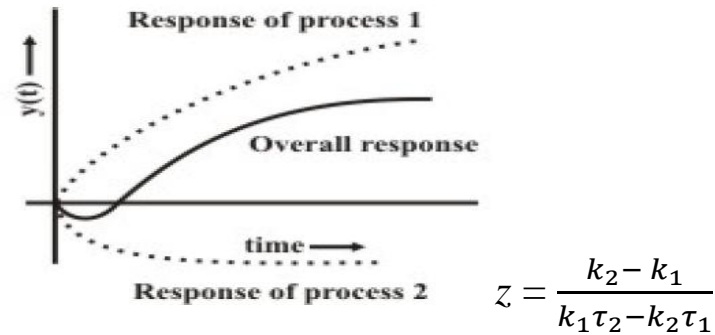


Figure 2.1 Inverse response of two first order systems

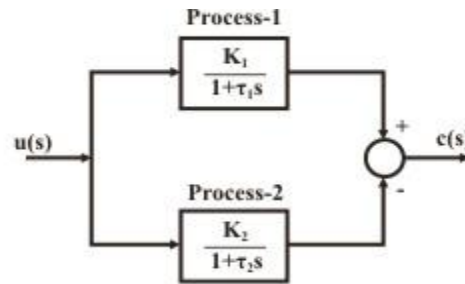


Figure 2.2 Block diagram of two opposing first order systems

The overall response equals to

$$y(s) = G(s) u(s) = (G1(s) - G2(s)) u(s)$$

$$= \left( \frac{k_1}{\tau_1 s + 1} - \frac{k_2}{\tau_2 s + 1} \right) u(s) \quad - (2.2)$$

or

$$y(s) = \frac{(k_1 \tau_2 - k_2 \tau_1) s + (k_1 - k_2)}{(\tau_1 s + 1) (\tau_2 s + 1)} \quad - (2.3)$$

where  $G(s)$  is the overall transfer function. We have inverse response when  $\frac{\tau_1}{\tau_2} > K1/K2 > 1$ , i.e., initially process 2, which reacts faster than process 1, dominates the response of the overall system, but ultimately process 1 reaches a higher steady-state value than process 2 and forces the response of the overall systems in the opposite direction . Here, the system transfer function has a zero in the open right half plane. This is a typical inverse-response process. [1] In process control, inverse response (IR) processes can be commonly encountered and classical examples are illustrated in textbooks (Tyner and May(1968)[2], Vidyasagar (1985)[3]).

Consider a simple drum boiler in which the level is controlled by manipulating the boiler feed water into the drum. A constant heat supply is provided to the boiler, water vapors exit from

the 'riser' pipes from the top of the boiler whereas liquid is drained downward from the bottom pipes. Keeping the vapor pressure constant if the feed water is significantly colder than the temperature of steam in the drum, then increasing the flow of feed water causes a temperature drop which decreases the volume of the entrained vapor bubbles, following first-order behavior i.e.  $-K_2/(\tau s+1)$  and liquid level decreases initially. With constant heat supply, the liquid level will start increasing in an integral form leading to a pure capacitive system. The result of two opposing effects is given by 2.4.

$$\frac{k_1}{s} - \frac{k_2}{\tau s+1} = \frac{k_1\tau - k_2}{s(\tau s+1)} \quad - (2.4)$$

and for

$$K_1\tau < K_2$$

The system shows inverse response only if the above condition satisfies.

$$\text{The transfer function has a positive zero at the point } s = -K_1 / (K_1\tau - K_2) \geq 0 \quad - (2.5)$$

If on decreasing the flow of feed water, the volume of liquid increases abruptly for a short time. The first order behavior reaches when the volume of bubbles increases. Since the steam supply remains constant and consequently the liquid level of the boiling water decreases in a pure capacitive response,  $K_2/s$ . [4]

The result of two opposing effects is given by 2.6.

$$\frac{k_2}{\tau s+1} - \frac{k_1}{s} = \frac{(k_2 - k_1\tau)s - k_1}{s(\tau s+1)} \quad - (2.6)$$

And for

$$K_1\tau < K_2 \quad - (2.7)$$

If above condition is satisfied only then the system possess an inverse response.

The transfer function has a positive zero at the point  $s = K_1 / (K_2 - K_1\tau) \geq 0$ . [1]

### 2.3 Boilers

Boiler is a closed vessel in which water or other fluid is heated. All of these types can generate hot water or steam by absorbing heat from another fluid. On heating, the vaporized fluid exits the boiler for use in various processes or heating applications. Wherever a source of steam is required a boiler or steam generator is employed.

The steam drum is an integral part of a boiler. This vessel's primary function is to provide a surface area and volume near the top of the boiler where separation of steam from water can occur. High level reduces the surface area, and can lead to water and dissolved solids entering the steam distribution system. So, steam drum water level controlled. The objective of the drum level control system is to maintain the water-steam interface at the specified level and provide a continuous mass balance by replacing every pound of steam and water removed with a pound of feed water. As steam pressure changes, there is transient change in level due to the effect of pressure on entrained steam bubbles. As pressure drops, a rise in level, called swell, occurs because the trapped bubbles enlarge. As pressure rises, a drop in level occurs. This is called shrink.

There are three types of drum level control systems; single-element, two-element and three-element. Their application depends upon the specific boiler size and load changes.

The hot water heater used in our home is not considered as a boiler unless it holds more than 120 gallons. Another limit on the size of a boiler is an internal diameter of 6 inches or less. Any application that heats air or any other gas for that matter that does not contain the heated fluid in an enclosed vessel is called a furnace. If the fluid is air or another gas and it is under pressure then it does meet the definition of a boiler. There are many boilers used in many industries. Asphalt heaters, flux heaters (a raw material that becomes asphalt), many forms of waste heat boilers and equipment like recovery boilers (used in the paper industry) which convert product can be encountered by burning it. The heated or vaporized fluid exits the boiler for use in various processes or heating applications. It is a device used to create steam by applying heat energy to water. Although the definitions are somewhat flexible, it can be said that older steam generators were commonly termed boilers and worked at low to medium pressure (1-300 psi), but at pressures above that figure it is more usual to speak of a steam generator. Figure 2.3 shows the industrial boiler used in the control industries. [5]



Fig 2.3 Industrial boiler

### 2.3.1 Applications

Boilers have many applications:

- They can be used in stationary applications to provide heat, hot water, or steam for domestic use, or in generators.
- They can be used in mobile applications to provide steam for locomotion in applications such as trains, ships, and boats.
- Some steam boats, particularly smaller types such as river launches, were designed around a vertical boiler.
- Using a boiler is a way to transfer stored energy from the fuel source to the water in the boiler, and then finally to the point of end use.

### 2.4 Level Control Systems

Single-element drum level control system is the simplest approach of level control of boiler. It measures level and regulates feed water flow to maintain the level. This system is only effective for smaller boilers supplying steady processes which have slow and moderate load changes. It is because shrink and swell causes an incorrect initial control reaction. As steam demand increases, it lowers the pressure, the drum level increases sending a false control signal to reduce feed water flow when actually the feed water flow should increase to maintain mass balance. When steam demand decreases, the drum level decreases sending a false control signal to increase the feed water flow. When actually the feed-water flow should

decrease to maintain mass balance, it decreases. More complex systems are required to handle significant shrink and swell effects. [5]

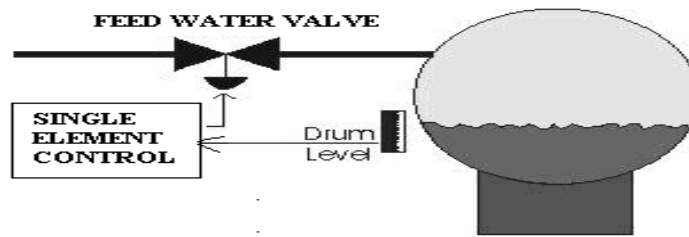


Figure 2.4 Single Element Drum Level Control

Figure 2.4 shows the control scheme of single element drum level control of boiler. Two-element drum level control system is suitable for processes with moderate load swings and can be used on any size boiler. Steam flow load changes are fed forward to the feed water control valve. A one pound change in steam flow results in a one pound change in feed water flow by matching the steam flow range and feed water flow range. The summer combines the steam flow signal with the feedback action of the drum level controller to compensate for unmeasured blow down losses and steam flow measurement errors. Two-element control is adequate for load changes of moderate speed and magnitude, and it can be applied to any size boiler. It has a drawback that it cannot adjust for pressure or load disturbances in the feed water system. If these disturbances are a concern, than three-element drum level control can correct the drawbacks.

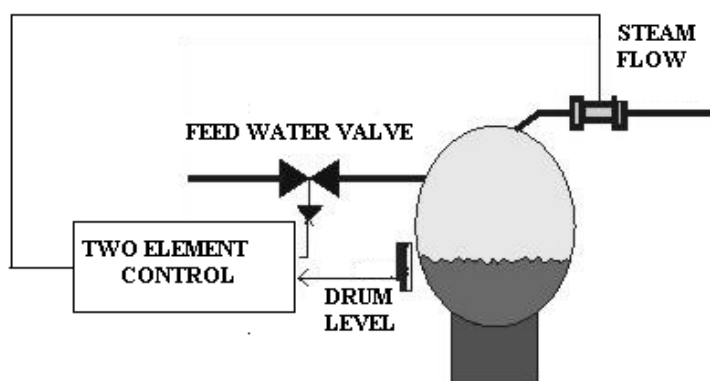


Figure 2.5 Two Element Drum Level Control

Figure 2.5 shows the control scheme of two element drum level control of industrial boiler. In a three-element drum level control system, there is an additional third variable, feed water flow rate, to manipulate the feed water control valve. This system basically cascades the

summer output of the two element system to the feed water flow controller as a remote set point signal. The additional feed water secondary loop assures an immediate correction for feed water disturbances. The drum level controller compensates for effects of smaller unmeasured flows such as blow down and mismatches between the two flows measurements. In the two-element system, nearly all the compensation for load changes is handled by the feed forward portion while the drum level feedback loop provides only trimming action. Regardless of boiler capacity, this system can handle large and rapid load changes and feed water disturbances. This method is required on multiple boilers having a common feed water supply. It is appropriate for plants with both batch and continuous processes where sudden and unpredictable steam demand changes are common.

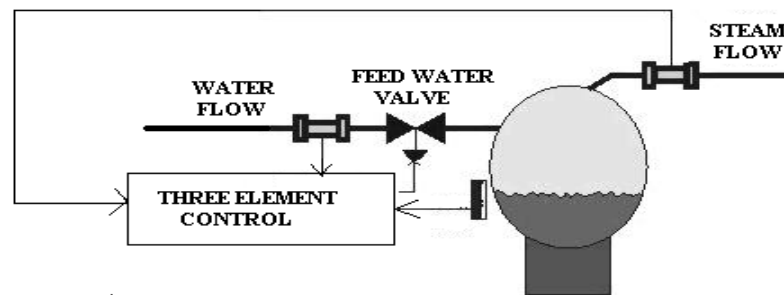


Figure 2.6 Three Element Drum Level Control

Figure 2.6 shows the control scheme of three element drum level control of boiler.

## 2.5 Types of controller

### 2.5.1. Feedback Controller

In this type of controller, error is calculated by comparing output with the input. The output is fed back to the input so that an appropriate control action should be taken as a function of output and input. The feedback is always negative to reduce the error. Negative feedback gives better stability in steady state, rejects any disturbance signal and also has a low sensitivity to the parameter variations. [6]

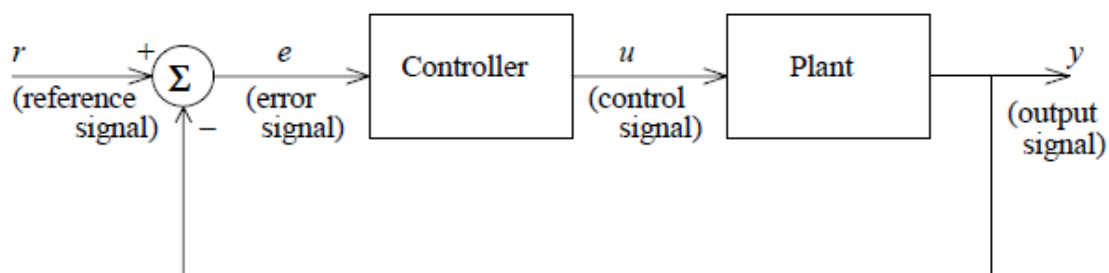


Figure 2.7 Feedback Control Loop

### 2.5.1.1 Advantages

- Corrects the controlled variable regardless of the source and type of disturbance.
- Requires minimal knowledge about the process.

### 2.5.1.2 Disadvantages

- Corrective action is taken only after a deviation in the controlled variable occurs.
- It does not provide predictive control action to compensate for the effects of known or measurable disturbances.
- May not be satisfactory for processes with large time constants and/or long time delays.
- Not feasible where the controlled variable cannot be measured on-line.

### 2.5.1.3 Application of Feedback Control

- Flow control
- Liquid level control
- Pressure and Temperature control

### 2.5.2 Feed forward Controller

The basic concept of the feed forward controller is to measure the disturbances and take corrective action before the disturbance can upset the process as shown in figure 2.8. [4]

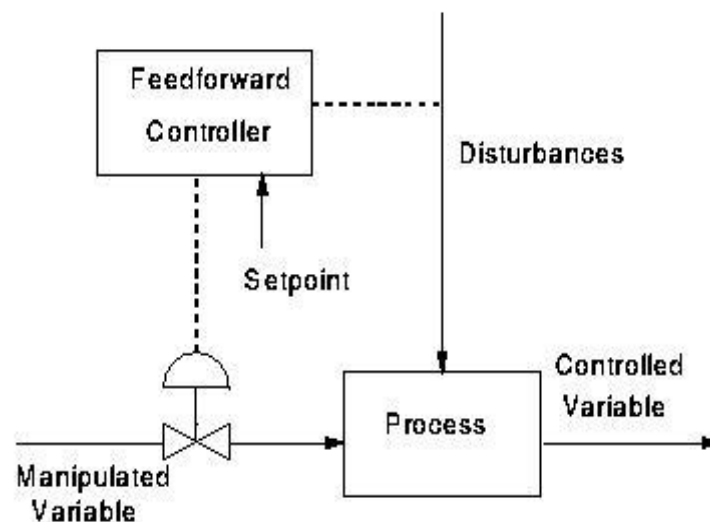


Figure 2.8 Schematic Diagram of Feed forward Control

### 2.5.2.1 Disadvantages

- The disturbance variable must be measured on line. In many applications it is not feasible.
- For feed forward controller at least an approximate process model should be available.
- Ideal feed forward controller which is theoretically possible may not be practically implemented.

### Feed forward Controller

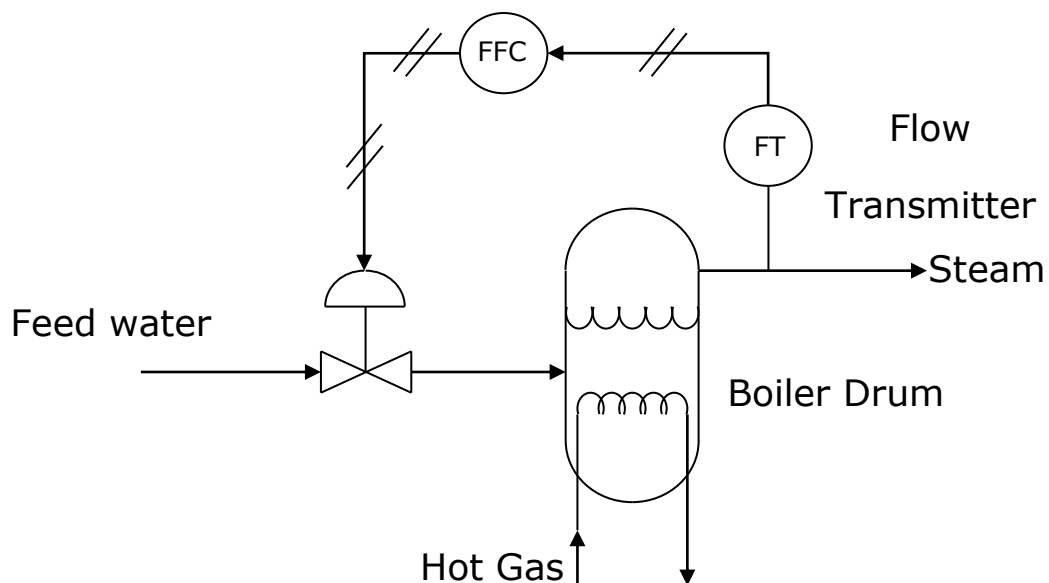


Figure 2.9 Example of Feed forward Control

Figure 2.9 explains the feed forward control. In this example of boiler drum the set point measure the disturbance and take corrective actions which do not upset the whole process.

### 2.5.3 Cascade Controller

In this type of control, there are two (or more) controllers of which one controller's output determines the set point of another controller. For example, a level controller driving the set point of a flow controller to keep the level at its set point. The flow controller, in turn, drives a control valve to match the flow with the set point the level controller is requesting. The controller driving the set point is called the primary, outer, or master controller. The controller that receives the set point is called the secondary, inner or slave controller. The block diagram is shown in figure 2.10.

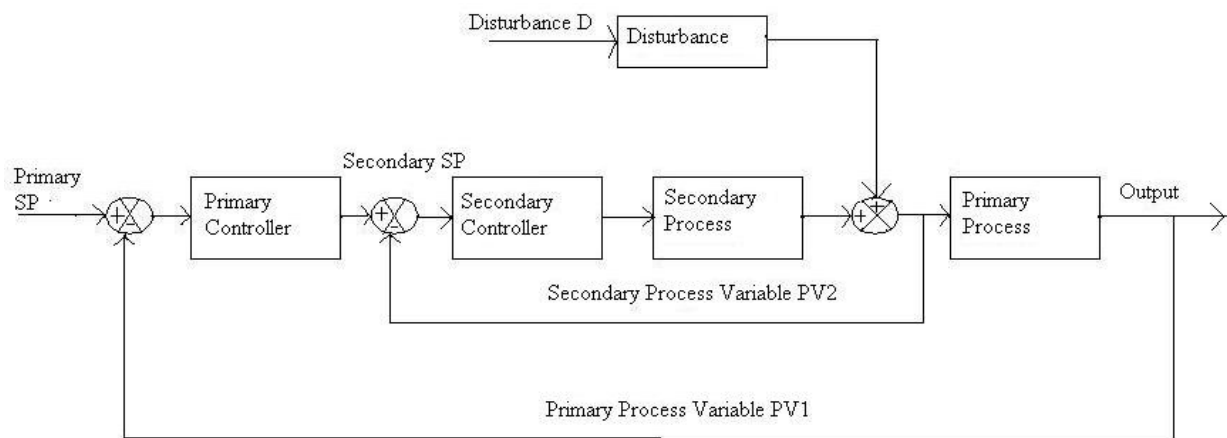


Figure 2.10 Schematic Diagram of Cascade Control

### 2.5.3.1 Advantages

- Better control of the primary variable
- Primary variable less affected by disturbances
- Faster recovery from disturbances

### 2.5.3.2 Applications

- The cascade controller is used in a shell and tube heat exchanger
- Temperature is controlled via steam flow control
- Temperature is controlled via pressure control [6]

## 2.6 Conventional Controllers

Controllers can be divided mainly into two groups:

- Conventional controllers
- Unconventional controllers

Conventional controllers are the ones known for years now, such as P, PI, PD, PID, Otto-Smith, all their different types and realizations, and other controller types. It is a characteristic of all conventional controllers that one has to know mathematical model of the process in order to design a controller. Unconventional controllers on the other side, utilize a new approach to the controller design in which knowledge of a mathematical model of a process generally is not required. Examples of unconventional controller are a fuzzy controller and neuro or neuro-fuzzy controllers.

- Many industrial processes are nonlinear and thus complicate to describe mathematically. However, it is known that a good many nonlinear processes can satisfactorily control PID controllers providing that controller parameters are tuned well.

It is advisable to use a larger number of simple PID controllers instead of a small number of complex controllers to control simpler processes in an industrial assembly in order to automate the certain more complex process. PID controller and its different types such as P, PI and PD controllers are today basic building blocks in control of various processes. In spite their simplicity; they can be used to solve even a very complex control problems, especially when combined with different functional blocks, filters (compensators or correction blocks), selectors etc. A continuous development of new control algorithms insure that the time of PID controller has not past and that this basic algorithm will have its part to play in process control in foreseeable future. It can be expected that it will be a backbone of many complex control systems. [8]

### 2.6.1 Basic controller types

Practical experience shows that this type of controller has a lot of sense since it is simple and based on 3 basic behavior types:

P - Proportional

I - Integrative

D - Derivative.

While proportional and integrative modes are also used as single control modes, a derivative mode is rarely used on its own in control systems.

Combinations such as PI and PD control are very often in practical systems. It can be also Shown that PID controller is a natural generalization of a simplest possible controller - On-off controller.

### 2.7 On-off controller

On-off controller algorithm is defined as:

$$\begin{aligned}
 U(t) &= U_{\max} ; e(t) > 0 \\
 &= U_{\min} ; e(t) < 0
 \end{aligned}$$

where:

$e(t)$  – control error (for unit feedback)

$u(t)$  – control signal (controller output)

Static characteristic of On-off controller is given in figure below

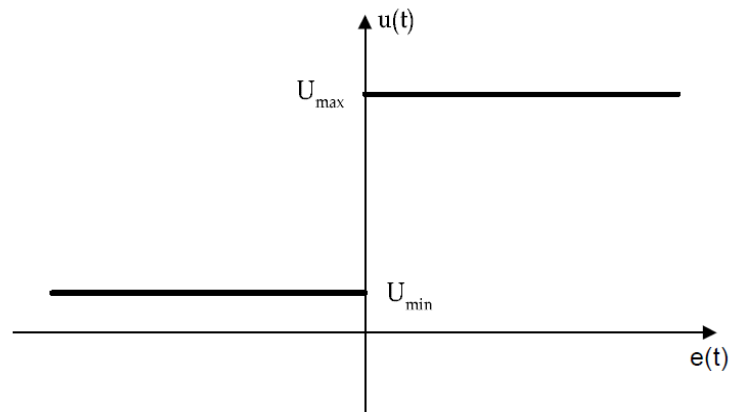


Figure 2.11: Static characteristic of On-off controller

Depending on the error (it can be positive or negative), control signal  $u(t)$  can have only two possible values, high  $U_{max}$  or low level  $U_{min}$ .

Assuming that process (controlled plant) has a positive static gain, high-level control signal will cause increase in controlled variable value. The main idea in this way of control, with only two control levels is to achieve the desired value of the controlled variable in shortest time possible. [8]

A shortfall in this approach of control is that control signal oscillates which may cause control variable to oscillate around desired value. Sometimes there is no solution for this problem. For example, if level of liquid in tank is controlled using valve with only two possible states (open or closed) the level will always oscillates around desired value.

On-off controller is very simple since there are only two possible control signal values, no matter what is the value of control error. Process is forced to oscillate since  $u(t)$  is never zero (it is either  $U_{max}$  or  $U_{min}$ ). The only way to avoid these forced oscillations is to reduce gain for small values of control error  $e(t)$ . That can be achieved by introducing a proportional mode that will be active for certain values of control errors.

## 2.8 P controller

P controller control algorithm is given with:

$$U(t) = \begin{cases} U_{\max} & ; e(t) > e_0 \\ U_0 + k e(t) & \text{for } -e_0 < e(t) < e_0 \\ U_{\min} & ; e(t) < -e_0 \end{cases}$$

where:

$u_0$  – amplitude of control signal when control error is equal 0

$K$  – P controller gain for P mode nominal area  $e(t) < e_0$

Many industrial controllers have defined a proportional band (PB) instead of gain:

$$PB = \frac{100}{K} [\%], \quad - (2.12)$$

It should be noted that for  $K=1$  a proportional band is equal  $PB = 100\%$ . Static characteristic of P controller is given in figure 2.12.

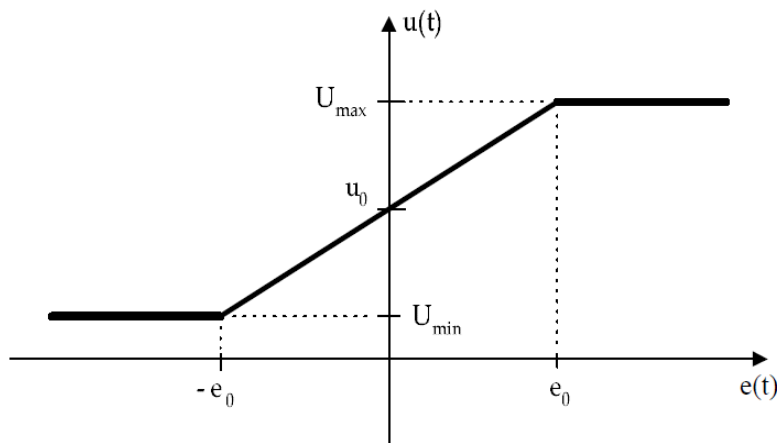


Figure 2.12: Static characteristic of P controller

P controller can eliminate forced oscillations caused by use of on-off controller.

However, it induces another problem. There exists now a steady state error.

A relationship between control signal and error inside area  $e(t) < e_0$  is given with:

$$u(t) = u_0 + Ke(t) \quad - (2.13)$$

Error is then:

$$e(t) = \frac{u(t) - u_0}{K} \quad - (2.14)$$

For a properly designed control system steady state error should be zero. With P controller that is possible if:

- a)  $K = \varphi$
- b)  $u(t) = u_0$

The first alternative ( $K = \varphi$ ) cannot be physically realized in any proportional band (PB) except for  $PB = 0$  [%] which leads back to on-off controller and forced oscillations. [12]

The second alternative ( $u(t) = u_0$ ) implies that it is possible to find  $u_0$  at every moment and that it is possible to satisfy condition  $u(t) = u_0$  for every given reference value  $r(t)$ . This can be achieved if integral mode is added to P controller.

P controller transfer function (unit step response) for  $K > 1$  is shown in Fig. 2.13(a-b).

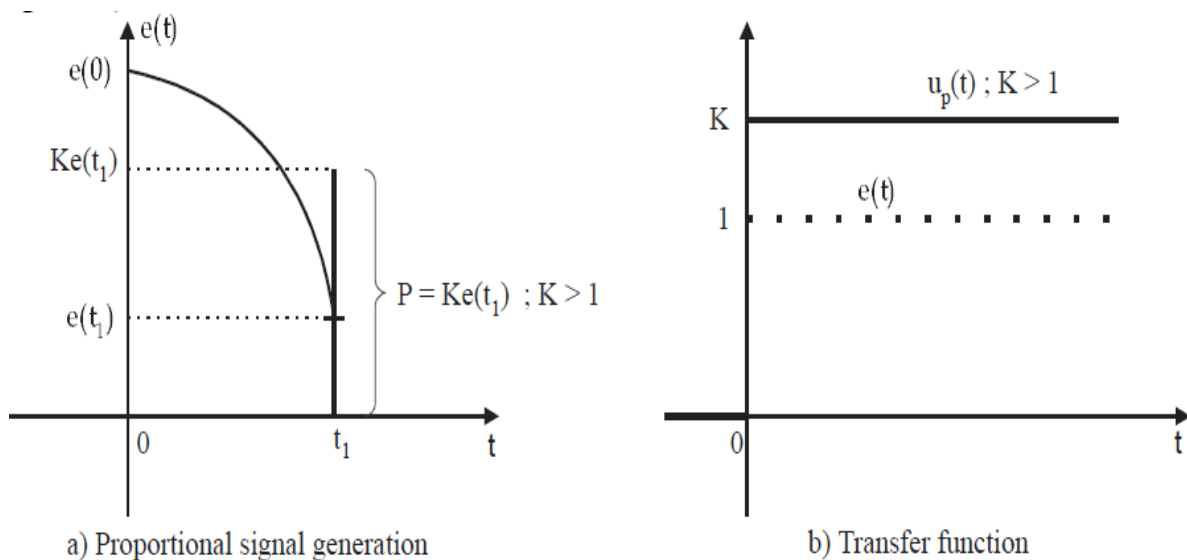


Fig. 2.13: Proportional signal generation and P controller transfer function

In general it can be said that P controller cannot stabilize higher order processes.

For the processes with one energy storage (1<sup>st</sup> order processes), a large increase in gain can be tolerated. Proportional controller can stabilize only 1<sup>st</sup> order unstable process. Changing

controller gain  $K$  can change closed loop dynamics. A large controller gain will result in control system with:

- a) Smaller steady state error, i.e. better reference following
- b) Faster dynamics, i.e. broader signal frequency band of the closed loop system and larger sensitivity with respect to measuring noise
- c) Smaller amplitude and phase margin

### 2.8.1 PI controller

PI controller forms control signal in the following way:

$$U(t) = K \left[ e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau \right] \quad - (2.15)$$

where:

$T_i$  – integral time constant of PI controller

This is graphically shown in Fig. 2.14 assuming  $K = 1$  and  $T_i = 1$ .

Constant  $K_i = K / T_i$  is called "reset mode". Integral control is also sometimes called reset control.

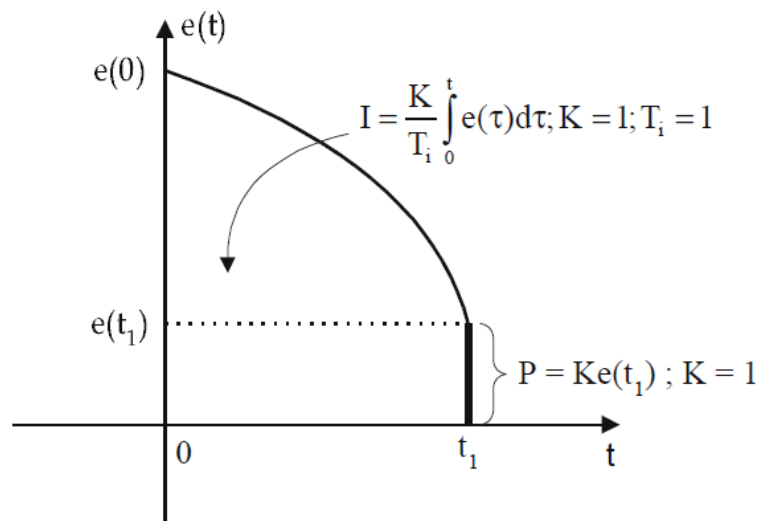


Fig. 2.14: PI controller signal generation

The name of this controller is derived from the term "manual reset" which marks a manual change of operating point or of "bias"  $u_0$  in order to eliminate error. PI controller performs this function automatically.

If control signal of P controller in proportional area is compared with PI controller

Output signal it can be seen that constant signal  $u_0$  is replaced with signal proportional

With the area under error curve: [8]

$$u_0 = \frac{K}{T_i} \int_0^t e(\tau) d\tau \quad - (2.16)$$

As  $u_0$  is replaced with an integral value, it allows PI controller to eliminate steady state error. Also, P controller cannot eliminate steady state error as it does not have any algorithm that would allow for the controller to increase control signal  $u(t)$  in order to increase controlled variable  $y(t)$  (assuming positive process gain) if in some moment  $t_1$  error  $e(t_1) = \text{const.} > 0$ . Proportional control law stays constant in this case and it will not try to change a controlled variable in such manner that control error is diminished.

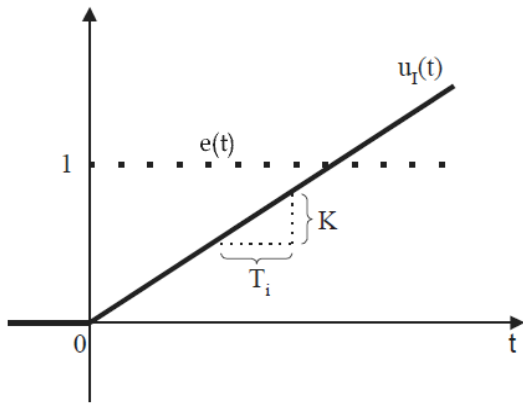
PI controller on the other hand will increase control signal when error  $e(t_1) = \text{const.} > 0$ . To the proportional part of the signal (P in Fig. 6.5) will be added integral part (I in Fig. 6.5) proportional to the area under curve  $e(t)$ , so, overall signal would be bigger.

$$U(t) = Ke(t) + \frac{K}{T_i} \int_0^t e(\tau) d\tau = U_P(t) + U_I(t) \quad - (2.17)$$

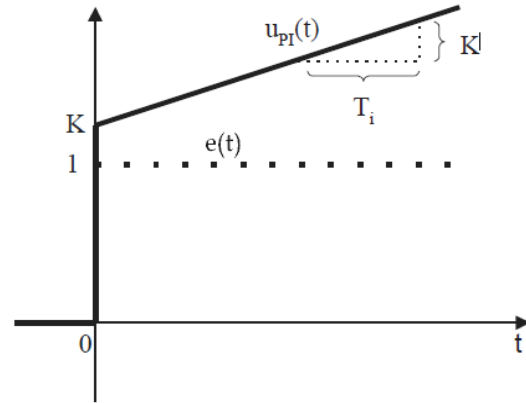
Assuming positive process gain, increase in control signal will result in increase in controlled variable and error will tend toward zero. When  $e(t) < 0$ , control signal will decrease, control variable will also decrease and error will tend toward zero. PI controller will not be active only when  $e(t) = 0$ . In all other situations PI controller will act to lead steady state control error to zero.

It can be concluded that PI controller will eliminate forced oscillations and steady state error resulting in operation of on-off controller and P controller respectively. However, introducing integral mode has a negative effect on speed of the response and overall stability of the system.

PI controllers are very often used in industry, especially when speed of the response is not an issue. Deceleration of response can be seen from transfer function of integrator shown in Fig. 2.15 a). [14]



a) Transfer function of integrator

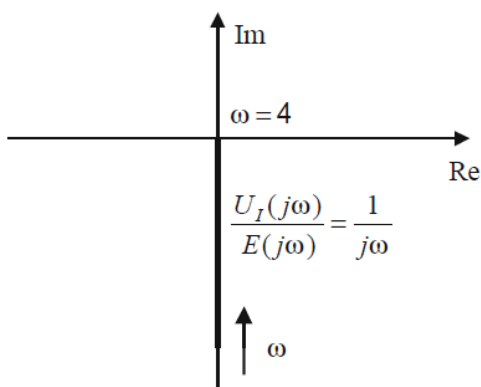


b) PI controller transfer function

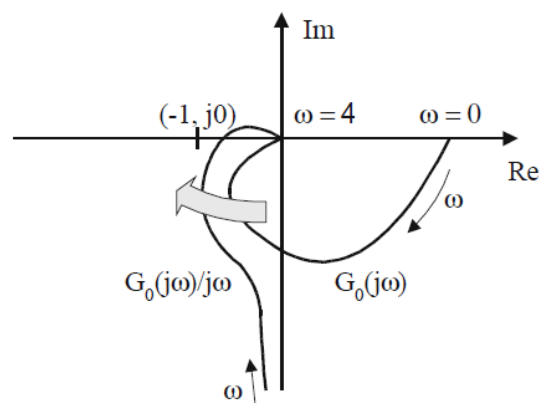
Figure 2.15 Transfer function of integrator and PI controller

As it can be seen from Fig. 2.15(a) a sudden change in input signal (step) will result in gradual change of the output signal (ramp). Transfer function of PI controller is given in Fig. 2.15( b). It can be seen that step change of the output is a result of proportional action, not integral.

Degradation of stability can be seen in frequency (Nyquist) characteristic where phase shift caused by integrator for all frequencies is  $-90^\circ$  (Fig. 2.16a)), thus the frequency characteristic moves closer to the critical point  $(-1, j0)$  (Fig. 2.16b)).



a) Frequency characteristic of integrator



b) Destabilizing effect of integrator

Figure. 2.16: Frequency characteristic and destabilizing effect of integrator

Frequency characteristic of PI controller is given in Fig. 4.8. We observe that phase lagging caused by PI controller is smaller than phase lag caused by pure integrator.

Phase lag is the biggest at low frequencies and decreases with the rise of frequency. Hence behaves in an inversely proportional manner.

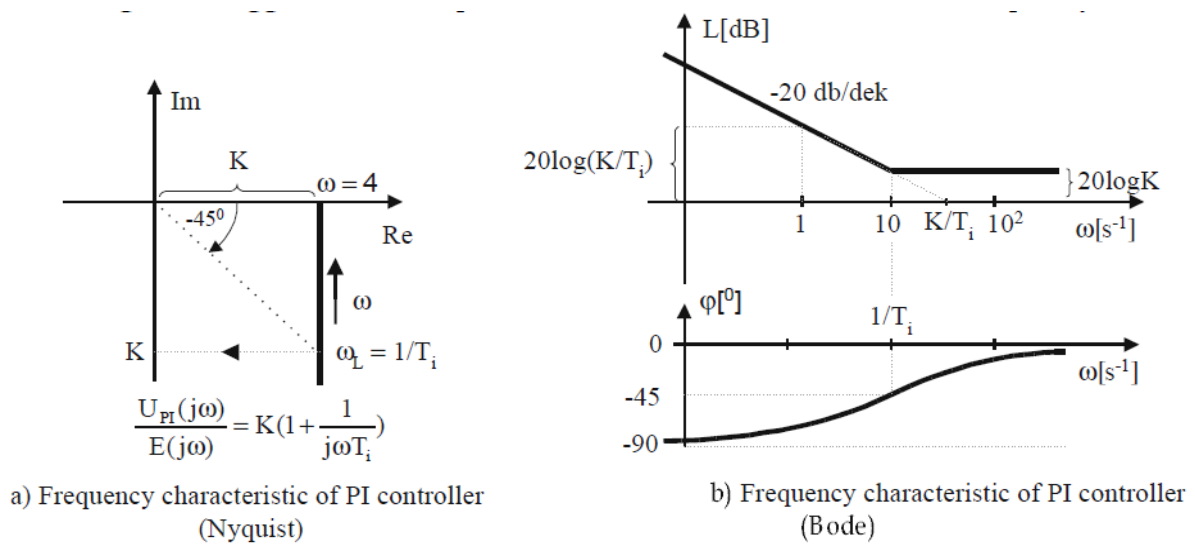


Figure 2.17: Frequency characteristic of PI controller

PI controller does not have means to predict what will happen with the error in near future. Hence we can say that the PI controller will not increase the speed of response. However, derivative mode has the ability to predict what will happen with the error in near future and thus can be used to decrease a reaction time of the controller.

Integral action can occur in the controller only on purpose, by design. Integral action can be noted on the other parts of the control system (actuators, plant etc.). These components may help in diminishing steady state error, but control system designer generally cannot tune this components.

## 2.9 PID Controller

Even though significant development have been made in advanced control theory in past five decades, the Proportional-Integral-Derivative (PID) controllers have been the most commonly used controllers. According to a survey conducted by Japan Electric Measuring Instrument Manufacturers Association in 1989, 90 % of the control loops in industries are of the PID type. The proportional action adjusts controller output according to the size of the error, the integral action eliminates the steady state offset and the future is anticipated via derivative action. These useful functions are sufficient for a large number of process applications and the transparency of the features leads to wide acceptance by the users. PID controller also deals with important practical issues such as actuator saturation and integrator

windup. PID controllers perform well for a wide class of processes and they give robust performance for a wide range of operating conditions and are easy to implement using analog or digital hardware.

A large industrial process may have hundreds of PID controllers. To achieve the desired response characteristics, proper tuning of the controllers is very crucial. They have to be tuned individually to match the process dynamics in order to provide good and robust control performance. If done manually, the tuning procedure is very tedious and time consuming activity. The resultant system performance mainly depends on the experience and the process knowledge of the engineers. It is recognized that in practice, many industrial control loops are poorly tuned. However with the advent of the auto-tuning of PID controller concept, this problem has been solved to a considerable extent. Automatic tuning techniques thus draw more and more attention of the researchers and practicing engineers. By automatic tuning, we mean a method which enables the controller to be tuned automatically on demand from an operator or an external signal. Basically, the user will either push a button or send a command to the controller. Industrial experience has clearly indicated that this is highly desirable and useful feature. The equation which describes the PID controller is given as 2.8.

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt} + K_i \int_0^t e(t) dt \quad - (2.8)$$

PID control is a name commonly given to three-term control. The mnemonic PID refers to the first letters of the names of the individual terms that make up the standard three-term controller. These are P for the proportional term, I for the integral term and D for the derivative term in the controller. Three-term or PID controllers are probably the most widely used industrial controller. Even complex industrial control systems may comprise a control network whose main control building block is a PID control module. The three-term PID controller has had a long history of use and has survived the changes of technology from the analogue era into the digital computer control system age quite satisfactorily. It was the first controller to be mass produced for the high-volume market that existed in the process industries. With the introduction of the Laplace transform to study the performance of feedback control systems supported its technological success in the engineering community. The theoretical basis for analyzing the performance of PID control is considerably aided by the simple representation of an Integrator by the Laplace transform,  $1/s$ , and a Differentiator using  $s$ . conceptually, the PID controller is quite sophisticated and three different

representations can be given. First, there is a symbolic representation, where each of the three terms can be selected to achieve different control actions. Secondly, there is a time domain operator form, and finally, there is a Laplace transform version of the PID controller. This gives the controller an s-domain operator interpretation and allows the link between the time domain and the frequency domain to enter the discussion of PID controller performance.

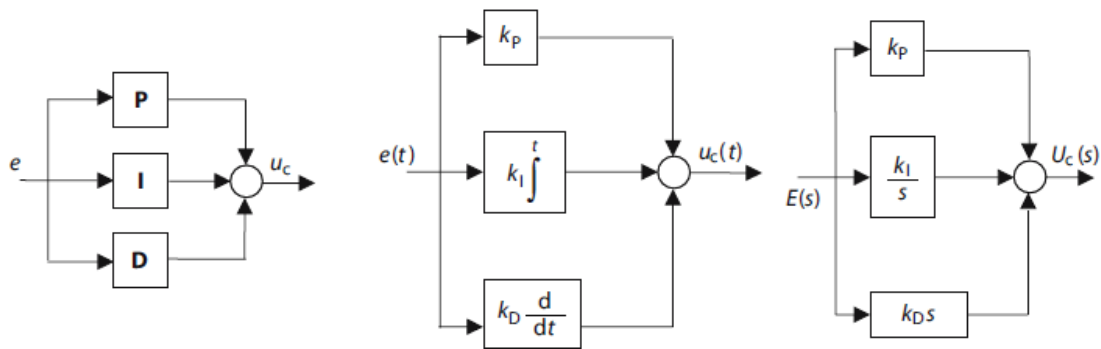


Figure 2.18 PID Controller Architecture in Time Domain and Laplace Domain

Here in this report PI controller is used as PID controller needs to get tuned. It is a long process which uses different methods for tuning of PID. Whereas PI tuning is less complex than PID tuning. So for the compensation of parameters we used PI controller. [7]

### 2.10 PI controller

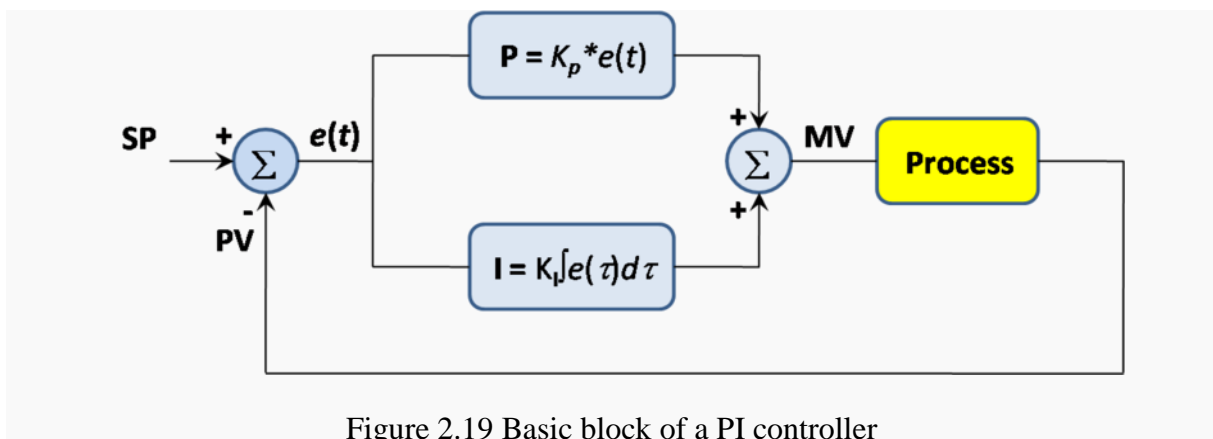


Figure 2.19 Basic block of a PI controller

A **PI Controller** (proportional-integral controller) is a special case of the PID controller in which the derivative (D) of the error is not used.

The controller output is given by

$$k_p \Delta + k_i \int \Delta dt \tag{2.9}$$

Where  $\Delta$  is the error or deviation of actual measured value (**PV**) from the set point (**SP**).

$$\Delta = SP - PV \quad - (2.10)$$

A PI controller can be modeled easily in software such as Simulink using a "flow chart" box involving Laplace operators:

$$C = \frac{G(1+\tau s)}{\tau s} \quad - (2.11)$$

Where

$G = k_p$  = proportional gain

$G/\tau = k_i$  = integral gain

Setting a value for  $G$  is often a tradeoff between decreasing overshoot and increasing settling time. The lack of derivative action may make the system steadier in the steady state in the case of noisy data. This is because derivative action is more sensitive to higher-frequency terms in the inputs.

Without derivative action, a PI-controlled system is less responsive to real (non-noise) and relatively fast alterations in state and so the system will be slower to reach setpoint and slower to respond to perturbations than a well-tuned PID system. [8]

## 2.11 RESPONSE OF SYSTEM

### Minimum phase response and non-minimum phase response-

In control theory and signal processing, a linear, time-invariant system is said to be **minimum-phase** if the system and its inverse are causal and stable.

When we impose the constraints of causality and stability the inverse system is unique; and the system  $\mathbb{H}$  and its inverse  $\mathbb{H}_{inv}$  are called **minimum-phase**. The causality and stability constraints in the discrete-time case are the following (for time-invariant systems where  $h$  is the system's impulse response). [6]

#### 2.11.1 Causality

Causality is defined as the output of a function at any time depends only on past and present values of input. A system that has some dependence on input values from the future (in addition to possible dependence on past or current input values) is termed a non-causal or acausal system, and a system that depends *solely* on future input values is an anti-causal system. The causality of systems also plays an important role in digital signal processing, where filters are often constructed so that they are causal. For a causal system, the impulse response of the system must be 0 for all  $t < 0$ . That is the necessary and sufficient condition for causality of a system, linear or non-linear.

Mathematically, a system mapping X to Y is causal if and only if, for any pair of input signals  $X_1(t)$  and  $X_2(t)$  such that

$$X_1(t) = X_2(t), \text{ for } t \leq t_0 \quad - (2.18)$$

And for corresponding outputs satisfy

$$Y_1(t) = Y_2(t), \text{ for } t \leq t_0 \quad - (2.19)$$

### 2.11.2 Stability

When the output of the system approaches infinity the system is unstable. Also, systems which become unstable often incur a certain amount of physical damage, which can become costly. Negativeness of any coefficient of a characteristic polynomial indicates that the system is either unstable or at most marginally stable. If any coefficient is zero/negative then we can say that the system is unstable.

$$\sum_{n=-\infty}^{\infty} |h(n)| = \|h\|_1 < \infty \quad - (2.20)$$

And

$$\sum_{n=-\infty}^{\infty} |h_{inv}(n)| = \|h_{inv}\|_1 < \infty \quad - (2.21)$$

### 2.11.3 Non minimum phase response-

Systems that are causal and stable whose inverses are causal and unstable are known as non-minimum-phase systems. A given non-minimum phase system will have a greater phase contribution than the minimum-phase system with the equivalent magnitude response.

### 2.11.4 Systems with minimum and non-minimum phase behavior

Stable systems without dead time, which are described by the transfer function

$$G(s) = \frac{N(s)}{D(s)}$$

and which do not have zeros in the right half plane, are called *minimum phase* systems. They are characterized by the fact that for a known amplitude response  $A(\omega) = |G(j\omega)|$  in the range of  $0 \leq \omega < \infty$  the corresponding phase response  $\varphi(\omega)$  can be calculated from  $A(\omega)$  and that the value of  $\varphi(\omega)$  determined has its minimum modulus for the

given  $A(\omega)$ . If a transfer function has poles and/or zeros in the right half  $s$  plane then this system shows non-minimum phase behavior. The modulus of the phase response is then always larger than that for a system with minimum phase behavior, which has the same amplitude response. In order to illustrate the non-minimum phase behavior, two systems will be considered, which have in fact the same amplitude response  $A(\omega)$  but differ considerably in the phase response. The transfer functions of the two systems are

$$G_a(s) = \frac{1 + sT}{1 + sT_1} \quad \text{and} \quad G_b(s) = \frac{1 - sT}{1 + sT_1} \quad - (2.22)$$

with  $0 < T < T_1$

The distributions of the poles and zeros of  $G_a(s)$  and  $G_b(s)$  in the  $s$  plane is shown in Figure 2.20. The

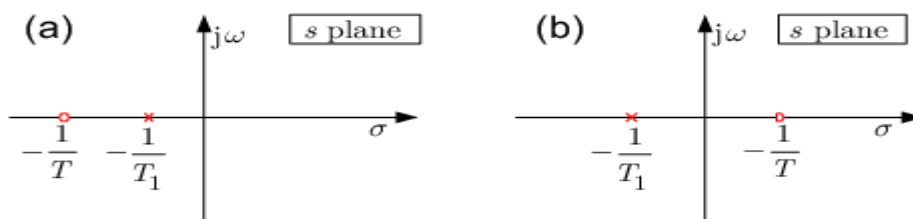


Figure 2.20. Distribution of poles and zeros in the  $s$  plane (a)  $G_a(s)$  and (b)  $G_b(s)$

Amplitude response of the corresponding frequency responses  $G_a(j\omega)$  (minimum phase system) and  $G_b(j\omega)$  (non-minimum phase system) is in both cases the same, as

$$A_a(\omega) = A_b(\omega) = \sqrt{\frac{1 + (\omega T)^2}{1 + (\omega T_1)^2}} \quad - (2.23)$$

For the phase responses one obtains

$$\varphi_a(\omega) = - \arctan \frac{\omega(T_1 - T)}{1 + \omega^2 T_1 T} \quad - (2.24)$$

And

$$\varphi_b(\omega) = - \arctan \frac{\omega(T_1 + T)}{1 - \omega^2 T_1 T} \quad - (2.25)$$

a different result, which is shown in Figure 2.20. For  $\varphi_b(\omega)$  the ambivalence of the  $\arctan$  function has to be observed. Here the minimum phase response of  $\varphi_a(\omega)$  can be clearly seen. [6]

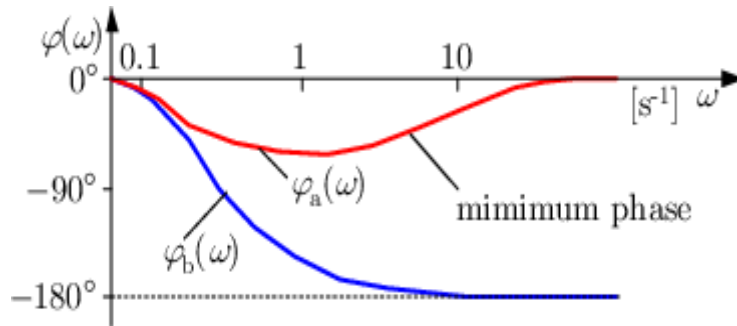


Figure 2.21 Phase response of two transfer functions with identical amplitude, but with minimum and non-minimum phase behavior:  $|\varphi_a| < |\varphi_b|$

The first uses a PID controller with Ziegler-Nichols tuning, and the second uses an inverse-response compensator. Unfortunately, both of them are empirical methods and cannot provide satisfactory performance. The well-known Smith predictor used for the control of processes with time delay is extended to the control of processes with inverse response. The quantitative step response is obtained. It is shown that the new structure can be further simplified to avoid unnecessary complications. This will result in a PID controller. The controller provides not only the optimal performance but also an adjustable performance and robustness. [9]

## 2.12 State space analysis

In control engineering, a **state space representation** is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors, and the differential and algebraic equations are written in matrix form (the last one can be done when the dynamical system is linear and time invariant). The state space representation (also known as the "time-domain approach") provides a convenient and compact way to model and analyze systems with multiple inputs and outputs. With  $p$  inputs and  $q$  outputs, we would otherwise have to write down  $q \times p$  Laplace transforms to encode all the information about a system. Unlike the frequency domain approach, the use of the state space representation is not limited to systems with linear components and zero initial conditions. "State space" refers to the space whose axes are the state variables. The state of the system can be represented as a vector within that space.

The internal state variables are the smallest possible subset of system variables that can represent the entire state of the system at any given time. The minimum number of state

variables required to represent a given system,  $n$ , is usually equal to the order of the system's defining differential equation. If the system is represented in transfer function form, the minimum number of state variables is equal to the order of the transfer function's denominator after it has been reduced to a proper fraction. It is important to understand that converting a state space realization to a transfer function form may lose some internal information about the system, and may provide a description of a system which is stable, when the state-space realization is unstable at certain points. In electric circuits, the number of state variables is often, though not always, the same as the number of energy storage elements in the circuit such as capacitors and inductors. The state variables defined must be linearly independent; no state variable can be written as a linear combination of the other state variables or the system will not be able to be solved. [6]

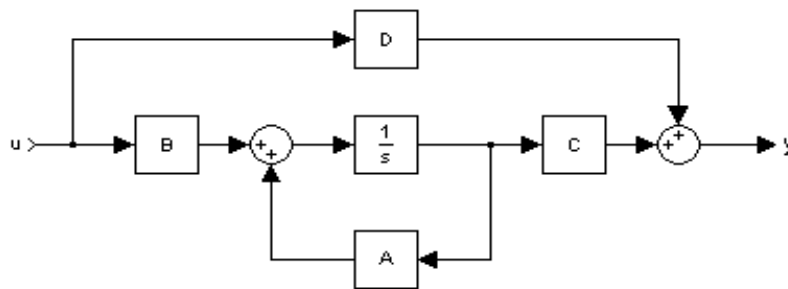


Figure 2.22 State space models

### 2.12.1 Controllability

Thus, state controllability condition implies that it is possible by admissible inputs to steer the states from any initial value to any final value within some finite time window. A continuous time-invariant linear state-space model is **controllable** if and only if

$$\text{rank}[ B \ AB \ A^2B \ \dots \ \dots \ A^{n-1} ] = n$$

Where rank is the number of linearly independent rows in a matrix.

### 2.12.2 Observability

Observability is a measure for how well internal states of a system can be inferred by knowledge of its external outputs. The observability and controllability of a system are mathematical duals (i.e., as controllability provides that an input is available that brings any initial state to any desired final state, observability provides that knowing an output trajectory provides enough information to predict the initial state of the system).

A continuous time-invariant linear state-space model is **observable** if and only if

$$\text{Rank} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = n$$

### 2.13 Closed loop stability concept

Consider the case where loop has been opened. Assume that the set point applied to ‘open loop’ system is a sine wave. Also assume the controller has been tuned so that the output lags the set point by 180 degree and has the same amplitude as the set point. Realize the  $-y(t)$  is exactly 180 degrees out of phase with  $y(t)$  which means that  $-y(t)$  is equal to  $r(t)$ . Now consider the case where the set point signal  $r(t)$  is suddenly stopped and simultaneously the loop is closed. This means that error will simply be  $-y(t)$ , which is identical with  $r(t)$ . Since it is identical to  $r(t)$ , then every signal on the control loop diagram remains the same. The output continues to oscillate with the same frequency and magnitude as before the loop was closed. We refer to this control loop as nominally stable. [6]

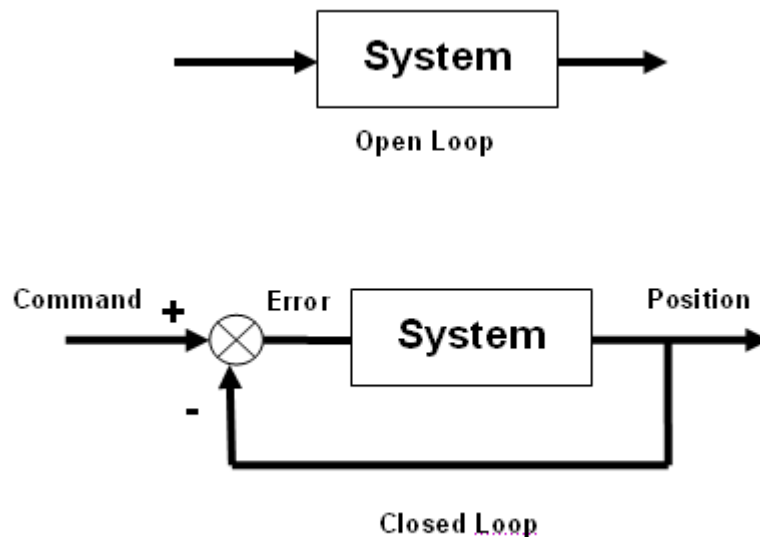


Fig 2.23: Open loop and closed loop systems

## CHAPTER- 3

### INVERSE RESPONSE COMPENSATOR

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#### 3.1 Predictive Control

In a chemical process control time delay comes into picture affecting the model and performance of the process. Processes having large time delays are normally difficult to control. As a result of a change in set point, performance of the closed loop control system is normally sluggish. Single change in set point or disturbance will give rise to large oscillations of the output before coming to a steady state value.

The scheme for taking a predictive action in presence of transportation delay in the system is better known as *Smith Predictor*. [10]

Let us consider that the transfer function of the process is given by:

$$G_p(s) = e^{-\tau s} G(s) \quad - (3.1)$$

where  $G(s)$  represents the system model without the delay. The basic scheme for Smith Predictor is shown in Fig 3.1. Here  $G(s)$  is the conventional PID controller designed for the process  $G(s)$ . Since the time delay is zero, the controller will see the effect of control action much earlier and the efficiency of response of the system will improve. The outer loop comes into play if the model of the process is not exact (normally that is expected). Fig. 3.1 can be redrawn as shown in Fig. 3.2 with the actual controller shown inside the dashed line. [11]

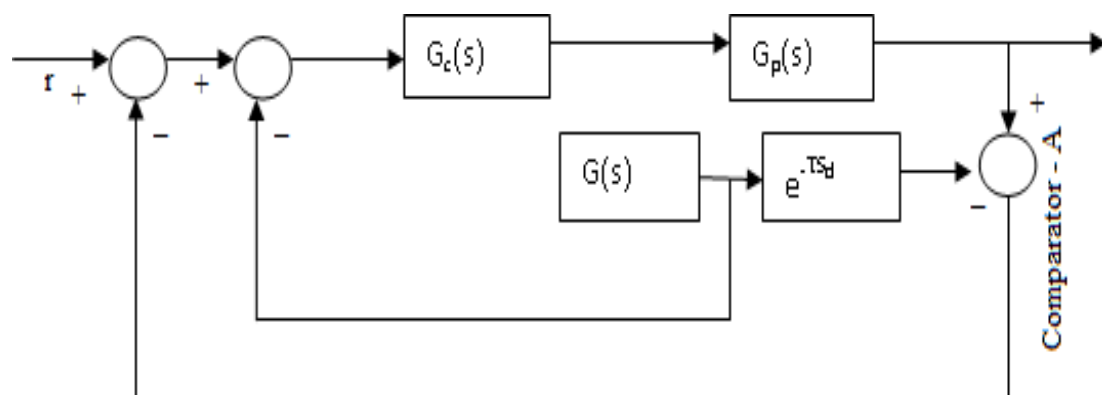


Figure 3.1 Smith predictor

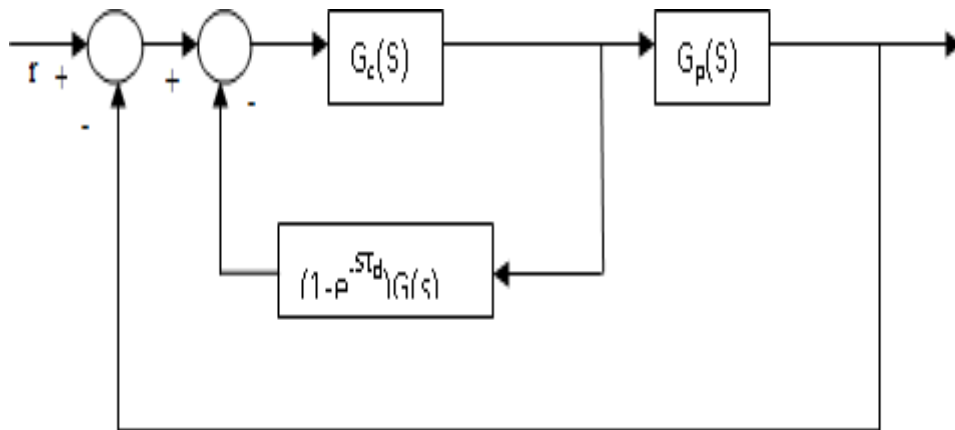


Fig 3.2 Equivalent scheme of controller

### 3.2 Systems with Inverse Response

We have seen different types of system response so far: first order system, second order system, etc. Another type of system can also be classified with its typical step response pattern: system with inverse response. It is essentially a system whose transfer function is having a zero on the right half plane. This type of system is also called, *nonminimum phase system*. [13]

#### 3.2.1 Example of a system with inverse response

Consider the dynamic characteristics of a boiler drum in a water tube boiler of a steam power plant. It is very important to control the water level of the drum at a desired level, which is done by controlling the feed water flow, with the varying demand of steam. The schematic arrangement can be shown as in Figure 2.3 [13]

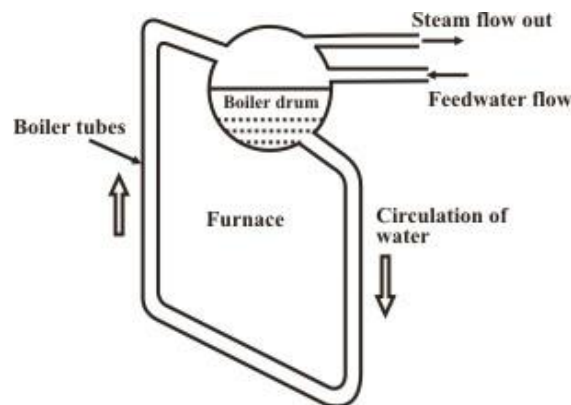


Figure 3.3 a boiler drum

### 3.3 Shrink/Swell Effect

Dynamic shrink/swell is a phenomenon that produces variations in the level of the liquid surface in the steam drum whenever boiler load occur. This behavior is strongly influenced by the actual arrangement of steam generating tubes in the boiler. The steam is always produced during operation. Steam rises in the tube and boiler water also rises and discharged into the steam drum. Tubes that are not producing significant steam flow have a net downward flow of boiler water from the steam drum to the mud drum. [14]

The tubes producing large quantities of steam are termed risers and those principally carrying water down to the mud drum from the steam drum are termed down comers. This mechanics of steam flow in the drum is the origin of shrink/swell problem. A pressure drop occurs in the steam drum as sudden steam load increases. Sudden increase of steam load is due to the firing rate which is unable to match the steam production rate at the new demand level. As the pressure in the drum drops, it has a dramatic effect on the natural convection within the boiler. Small fraction of saturated water immediately vaporizes producing large amount of boil up. This is due to the pressure drop in the steam. During the transient, most of the tubes temporarily become risers. Now the level in the steam drum rises in the combustion chamber. Now due to this rise in level an inverse response occurs. Now the boiler water should increase as the steam rate has gone up due to the mass of water in the boiler has fall. As the level controller senses a rise in the level of the steam drum it calls for a reduction in the flow of feed-water to the boiler. This sudden load increase is called dynamic swell. Dynamic swell effect is more hazardous than shrink effect as shrinking does not disrupt the natural convection circulation of the boiler. Consequently, the reduction in level produced by a sudden decrease in load is typically much smaller and of shorter duration than the effect produced by dynamic swell. [14]

### 3.4 Performance indices

The impulse response of the overall system is compared on the basis of four performance indices. These indexes differentiate the system in different manner. Generally, it so happens that during the control system design process one or more parameters are selected to give best performance. For this purpose a measure called performance index is build.

The system with adjusted parameter is called optimal control system.

The various criteria for performance indices in terms of error function  $e(t)$  are as follows:

1) Integral square error criterion :

$$ISE = \int_0^{\infty} e^2(t) dt$$

2) Integral of time multiplexed square error criterion :

$$ITSE = \int_0^{\infty} te^2(t) dt$$

3) Integral absolute error criterion :

$$IAE = \int_0^{\infty} |e(t)| dt$$

4) Integral of time multiplexed absolute error criterion :

$$IATE = \int_0^{\infty} t |e(t)| dt$$

The error signal in control system is given by

$$e(t) = r(t) - c(t) \text{ where } r(t) \text{ and } c(t) \text{ are input and output respectively.}$$

The error index approaches infinity if

$$\lim_{t \rightarrow \infty} e(t) \neq 0$$

However, for the case  $\lim_{t \rightarrow \infty} e(t) = 0$  does not approach zero then,

$$e(t) = c(\infty) - c(t)$$

As per above explanation the performance index will be determined as finite number for a stable system. [15]

### 3.5 Control of a System with Inverse Response

The control of a system with inverse response is difficult like in the case of control of a system with large time lag. For a closed loop time lag system with a simple feedback controller, the controller does not see any effect of control action till a time  $\tau_d$  has elapsed. On the other hand, for an inverse system, the controller will see an opposite effect to the expected one. So a special arrangement, similar to a Smith Predictor scheme is needed for control of inverse systems. Such an arrangement is shown in Figure 3.4.

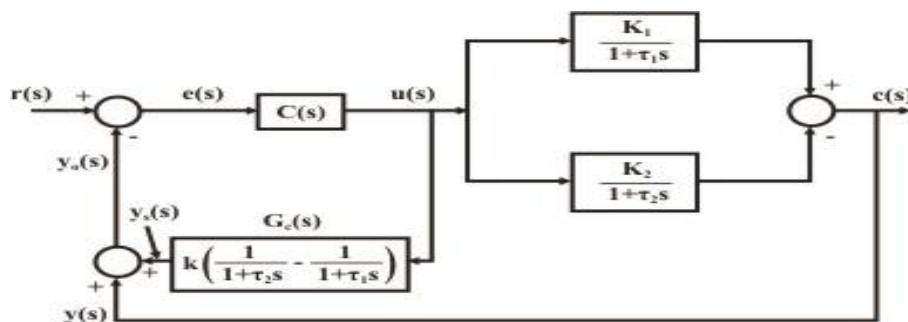


Figure 3.4 Scheme for controlling a system with inverse response

$$Y(S) = \frac{c(S)(k_1\tau_2 - k_2\tau_1)s + (k_1 - k_2)}{(1 + \tau_1 s)(1 + \tau_2 s)} e(S) \quad - (3.2)$$

To eliminate the effect of inverse response, one additional measurement signal must be added that excludes the information of inverse response. This can be achieved by the loop through the compensator  $G_c(s)$  that gives an additional output as shown in equation 3.3. [15]

$$y_s(s) = c(s) G_c(s) e(s) \quad - (3.3)$$

$$= c(S) = k\left(\frac{1}{1 + \tau_2 s} - \frac{1}{1 + \tau_1 s}\right)e(S) \quad - (3.4)$$

Combining both the equations (3.2 & 3.3)

$$y_0(s) = y(s) + y_s(s) \quad - (3.5)$$

$$c(S) \frac{(k_1\tau_2 - k_2\tau_1)s + k(\tau_1 - \tau_2)s + (k_1 - k_2)}{(1 + \tau_1 s)(1 + \tau_2 s)} e(s) \quad - (3.6)$$

Now the loop transfer function, i.e. transfer function between  $y_0$  and  $e$  will have a zero on the left half of s-plane, if the coefficient of  $s$  in the numerator of (3.2) is positive.

$$k \geq \frac{k_2\tau_1 - k_1\tau_2}{\tau_1 - \tau_2} \quad - (3.7)$$

It can be also easily seen that the compensator in Figure 2.6 has a similar configuration as in Fig. 2.1 for smith predictor. In other words, the compensator in Smith Predictor predicts the dead time behavior of the process, while in the present case; the compensator  $G_c(s)$  predicts the inverse behavior of the process. The basic controller is normally chosen of P-I type. [15]

### 3.6 Obtaining the transfer function parameters

For obtaining the transfer function parameters the mathematical and graphical method is used. This helps to estimate the transfer function parameters of the process from its unit step response. The procedure is based on graphical evaluation of the given process with suitable mathematical equations. We are required to estimate the parameters for an inverse response of a system made up of a purely integrative process and a first order processes that act in opposite directions. This method of parametric estimation of Inverse Response is primarily dependent on the unit step of the process. For this purpose, we have to simulate this response

using MATLAB SIMULINK. This is done by primarily following the equations along with the constraint  $K_1 \tau < K_2$ . The values of  $K_1$ ,  $K_2$  and  $\tau$  are chosen for six different cases and the transfer function is formed. From the response in the form of the simulated graph, the values of  $t_2$ ,  $t_1$  and  $Y_{\min}$  are obtained. [16]

This method is easy to use and gives reliable process parameters. The proposed model and estimating procedure is suitable for application in industrial environment because of its simplicity. The results show that it can be applied in the estimation of large number of processes giving better estimation in less time. The parameters so estimated are used for designing a suitable compensator so as to deal with the inverse response problem.

This method is used for a first order and a capacitive system. Now another method is used for two first order systems with inverse response. For this purpose inverse response is compensated by estimating parameters of comprising of two first order systems.

Classical method for this estimation involves simultaneous solution of three non-linear equations containing exponential terms. Most of the available tools fail to achieve this solution. This proposes a novel iterative method to overcome this limitation.

The system does not contain any delay. The function reaches its steady state value after some time. The derivative of the function remains zero at steady state. The following equation (3.8) expresses the output function in time domain which was earlier expressed in frequency domain. [4]

$$Y(t) = K_1 (1 - e^{-t/\tau_1}) - K_2 (1 - e^{-t/\tau_2}) \quad - (3.8)$$

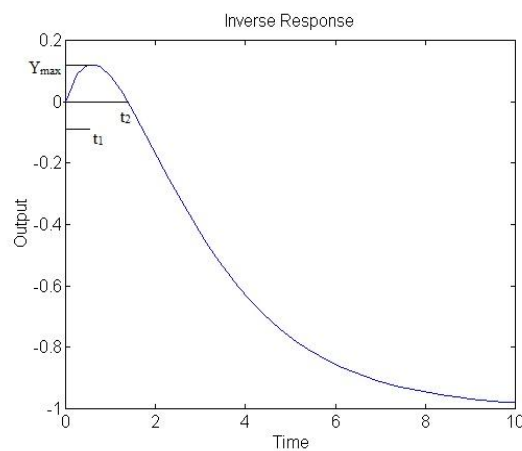


Figure 3.5 Graphical representation of output function showing required parameters

The derivative of the plot in figure 3.5 is given by the equation

$$\frac{dy}{dt} = \frac{K_1}{\tau_1} * e^{-\frac{t}{\tau_1}} - \frac{K_2}{\tau_2} * e^{-\frac{t}{\tau_2}} \quad - (3.9)$$

The value at the function at its minima is given as

$$Y_{\min} = K_1(1 - e^{-t_1/\tau_1}) - K_2(1 - e^{-t_1/\tau_2}) \quad - (3.10)$$

Derivative of the function holds zero value at its minima

$$\frac{dy}{dt} = \frac{K_1}{\tau_1} * e^{-\frac{t_1}{\tau_1}} - \frac{K_2}{\tau_2} * e^{-\frac{t_1}{\tau_2}} = 0 \quad - (3.11)$$

The value of the function is zero at time ( $t=t_2$ ). So this can be written as

$$Y(t_2) = K_1(1 - e^{-t_2/\tau_1}) - K_2(1 - e^{-t_2/\tau_2}) = 0 \quad - (3.12)$$

The final equation can be obtained at the steady state value where the function holds a constant value. This can be done by putting a limit on  $Y(t)$  to get a desired value at infinite time. It is stated as below

$$\lim_{t \rightarrow \infty} Y(t) = K_1 - K_2 \quad - (3.13)$$

This is because the exponential term vanishes as time approaches infinity. So we are just left with the difference of the proportionality gain constants of the two functions i.e.  $K_1 - K_2$ . [17]

So all four equations can be written as,

$$Y_{\max} = K_1(1 - e^{-t_1/\tau_1}) - K_2(1 - e^{-t_1/\tau_2}) \quad - (3.14a)$$

$$\frac{dy}{dt} = \frac{K_1}{\tau_1} * e^{-\frac{t_1}{\tau_1}} - \frac{K_2}{\tau_2} * e^{-\frac{t_1}{\tau_2}} = 0 \quad - (3.14b)$$

$$Y(t_2) = K_1(1 - e^{-t_2/\tau_1}) - K_2(1 - e^{-t_2/\tau_2}) = 0 \quad - (3.14c)$$

$$\lim_{t \rightarrow \infty} Y(t) = K_1 - K_2 \quad - (3.14d)$$

The above stated four equations (3.14a-3.14d) are sufficient to solve four variables ( $K_1$ ,  $K_2$ ,  $\tau_1$ ,  $\tau_2$ ). The remaining variables which are  $t_1$ ,  $t_2$ ,  $Y_{\max}$  or  $Y_{\min}$ ,  $\frac{dy}{dt}$  can be estimated graphically to get an approximate value of the unknown variables. The solution cannot be obtained by using the 'solve' command because the unmodified form of the three exponential equations does not return any solution using this command. This kind of problem cannot be tackled in MATLAB and so we use an alternative approach to it. [17]

Firstly we use substitution technique in which we replace the value of  $K_1$  by  $K_2$  in all equations using equation (3.13) as  $\lim_{t \rightarrow \infty} Y(t)$  holds a given unique value. Now we are left with three unknown parameters  $K_2$ ,  $\tau_1$  and  $\tau_2$  and three simultaneous equations. Since the MATLAB shows an error in solving three nonlinear algebraic equations, we use two non-algebraic equations whose result is always unique. For the third variable we use iterative method. The third variable is chosen to be  $\tau_1$  because it is in exponential form and any small variation in its value will cause a large change in the value of output function thereby limiting the number of considerations. The basic approach used is '**LEAST SQUARE ERROR (L.S.E.)**'. This L.S.E. method is done for a large number of values of  $\tau_1$ . The L.S.E. curve is now plotted and by number of experiments we observe a global minimum for different given iterations. The optimized value is actually the one occurring at the minima of the L.S.E. curve. User's concern is to identify the minima boundary limits and then using simple iterative method we can approximate the position of minima thereby evaluating the corresponding unknown variables. The error will reflect the accuracy of the solution. [17]

The procedure described above gives a highly reliable method which is easy to use and implement. The estimating procedure is suitable for application in industrial environment because of its simplicity. It has its application in dead time compensation and inverse response compensation. This method is helpful in designing a suitable compensator, by which the fine tuning of the PID (Proportional Integral and Derivative) controller. This is possible only if the process parameters are known to us.

### **3.7 Inverse response compensator**

To overcome the difficulty of inverse response which degrades the overall efficiency of the boiler plant, many methods of process control are included in the design of the controller for IR processes. Though a lot of methods, such as the state space method, can be used for controlling the inverse-response process, there are only two popular ways controlling such a

plant in the past in the context of process control (Stephanopoulos, 1984; Tyner and May, 1968; Wang and Zhu, 1991): the first uses a PID controller with Ziegler-Nichols tuning (Waller and Nygardas, 1975), and the second uses an inverse-response compensator (Iinoya and Altpeter, 1962). In this report, the well-known Smith Predictor used for the control of processes with time delay is used to compensate the values of inverse response. In our method the inverse response parameters are already obtained by the mathematical and graphical methods which are used in the transfer function and are directly taken in the inverse response compensator. The MAT LAB SIMULINK is used in this process for making models. [18]

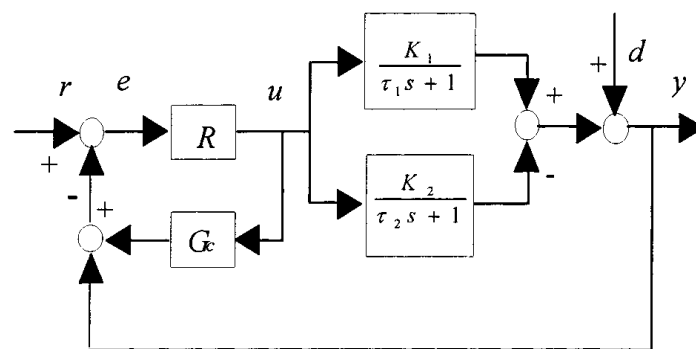


Figure 3.6 Compensation control structure

The above model shows the compensation control model which is used in MATLAB SIMULINK. The results are shown in the results and discussions section.

The specialized control scheme is known as time-delay compensation because; the minor loop was introduced to compensate for the presence of the time delay. It is referred to as a compensator. Smith predictor has no real effect to disturbance rejection improvement. It has some demerits like

- Stability
- Disturbance rejection
- Transient characteristic
- Robustness

### 3.7.1 Time delay

Time delay is the property of a physical system by which the response to an applied force (action) is delayed in its effect. Whenever material, information or energy is physically transmitted from one place to another, there is a delay associated with the transmission. The value of the delay is determined by the distance and the transmission speed. Some delays are short, some are very long. The presence of long delays makes system analysis and control design much more complex.

Time delays occur frequently in process control loops due to distance velocity lags, recycle loops, delay in measurements, etc. The principal difficulty with the time-delay systems is in the increased phase lag, which limits the possible amount of control action. The complication involved with the time delays further increases in multivariable systems due to the multivariable nature, where different time delays are present in different control loops and due to interactions. [19]

#### 3.7.1.1 Shower

A simple example of a time-delay system from everyday life is the shower. Most people have experienced the difficulty in adjusting the water temperature; it gets too cold or too warm. The actual temperature often overshoots the desired and, sometimes, it takes a while to get the right temperature. This is because it takes time for the increased (or decreased) hot/cold water to flow from the tap to the shower head (or the human body). This time is a delay, which depends on the water pressure and the length of the pipe. The change of the faucet position is almost immediate; however, the change of the water temperature has to wait until the delay has elapsed. If the faucet position is constantly adjusted according to the currently perceived temperature, then it is very likely that the temperature will fluctuate. Assume that the water is an incompressible fluid and stationary flow. According to the Poiseuille law, the flow rate of water is

$$F = \frac{\pi R^4}{8\mu l} \Delta P \quad - (3.15)$$

where  $\mu = 0.01$  is the kinematic viscosity of water,  $R$  is the radius of the pipe,  $l$  is the length of the pipe and  $\Delta p$  is the pressure difference between the two ends of the pipe. The time delay  $h$  can then be found as given by Eq. 3.16.

$$h = \frac{\pi R^2}{F} l \quad - (3.16)$$

using the equation 3.15

$$h = \frac{8\mu}{\Delta P} \left(\frac{l}{R}\right)^2 \quad - (3.17)$$

Here a smith predictor is designed in which delay is introduced in constant terms. Then this delay is reduced with the help of the compensator used in the smith predictor or the inverse response compensator. [20]

It is an experiment for improving the output of the smith predictor. It also removes the demerits of the predictor by compensating or optimizing the output.

#### 4.1 Traditional Control Methods

In the recent years, there were two popular ways to control the process with inverse response in process control. One is a PID controller with Ziegler-Nichols tuning, and the other is an inverse-response compensator.

Weidong Zhang et.al, [1] Extended the Smith predictor, known as the dead-time compensator to the control of a class of nonminimum phase processes which are called as inverse-response processes. An analytical design procedure is based on  $H^\infty$  theory. The resulting controller could provide not only the suboptimal frequency domain performance but also the quantitative time domain specifications for step response.

Tyner M et.al [2] explained the problem of inverse response that how the inverse response occurs and what effects it gives in boiler plant or distillation columns.

Vidyasagar. M [3] suggested the ways by which inverse response is compensated and explained the theory of non-minimum phase zeros.

Zan.W et.al [4] described how the inverse response can be controlled and suggested various methods for its compensation.

The well-known internal model controller (IMC) given by F. G. Shinskey[5] constitutes another control configuration suitable for the control of inverse response processes. The IMC structure is illustrated in Fig. 4.3, where  $P(s)$  denotes the actual plant,  $P_o(s)$  is the nominal model and  $Q(s)$  is the IMC controller. It is an open-loop structure in the nominal case ( $P_o(s) = P(s)$  and  $d = 0$ ) and thus allowing a direct design of the  $Q(s)$  controller by factoring the process model  $P_o(s)$  in a minimum phase,  $M^{-1}(s)$ , and in a non-minimum phase, all pass part  $N(s)$ , as

$$P_o(s) = N(s)M^{-1}(s) \quad - (4.7)$$

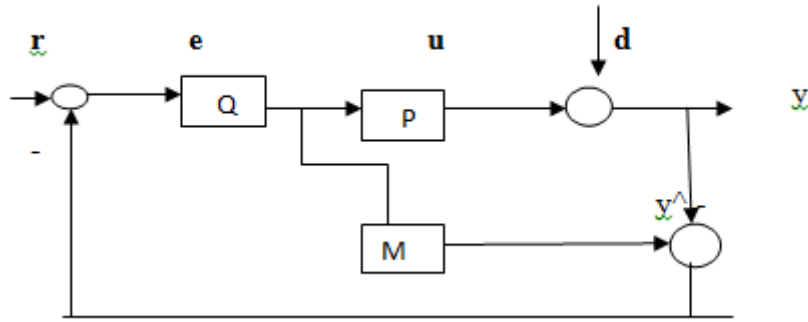


Figure 4.1 Internal model control structure

A simple factorization is

$$P_o(s) = \left( \frac{-as+1}{as+1} \right) \frac{as+1}{(\tau_1s+1)(\tau_2s+1)} \quad - (4.8)$$

the IMC controller is given by

$$Q(s) = M(s)F(s) \quad - (4.9)$$

where  $F(s)$  is a low-pass filter. For step input, it has the following structure:

$$F(s) = 1/(\lambda s + 1)^n \quad - (4.10)$$

If on increasing  $\lambda$ , the robustness margins increase at the expense of slower response.

For the plant IMC controller reads as

$$Q(s) = \frac{(\tau_1s+1)(\tau_2s+1)}{(as+1)(\lambda s+1)} \quad - (4.11)$$

The IMC structure can be equivalent to the feedback scheme. If we chose

$$C(s) = Q(s)/1 - P_o(s)Q(s) \quad - (4.12)$$

B. Wayne Bequette [6] suggested about the inverse response, controllers used for the system, controller tuning and the control system involves in an industrial plant.

Peiyong Chen et.al [7] described that inverse response processes are difficult to be controlled due to presence of right half plant (RHP) zeros. In this paper, firstly the RHP zeros are approximated to the form of dead time in terms of Pade approximation theory. Thus, the controlled processes can be regarded as general stable processes plus dead time without RHP zeros. Then based on the Internal Model Control (IMC) structure, an analytical PID controller design procedure is developed for the transformed inverse response processes.

Vukic Zoran et.al [8] explained and provided the basic PI controller lectures in which all the traditional methods and topologies are explained.

Lyuben W.L [9] described the tuning of Proportional controllers for processes with dead time and inverse response.

Luyben, W.L [10] explained the ways for simulating the models in process industry.

A. Seshagiri Rao et.al[11]

He suggested a modified smith predictor in which decoupling method is used. The classical multivariable Smith predictor is extended to multivariable non square systems with multiple time delays where the number of process inputs is higher than the process outputs. The input and output variables are paired using block relative gain technique. Decouplers are designed using simplified decoupling method. The decoupled processes are modelled as first-order plus time-delay model or second-order plus time delay model with positive/negative zero. The controllers are designed individually based on the identified models.

Seborg D. E et.al [12] explained all the process dynamics and various methods to control the disturbances occurring by inverse response processes.

Riggs J.B [13] describes about the inverse response processes and the various techniques for its compensation in chemical process control . He also explained about the controllers and their tuning methods.

Houtz D.Allen [14] explained about the shrink and swelling effect of the boilers. He also explained about the various boiler level controls.

G.Stephanopoulos [15] explained all the schemes used for compensating the inverse response and dead time introduced in the system.

Mandeep Singh and Dhruv Saksena [16] suggested the methods for evaluating the transfer function parameters using the mathematical and graphical equations and graphs. This is done for first order system and a pure capacitive system.

Mandeep Singh and Abhay Sharma [17] evaluated the various parameters for two first order transfer functions. These parameters help in inverse response compensation.

S.Alcantara et.al, [18] presented a control configuration for inverse response processes inspired in the Smith Predictor scheme. It is based on an inner-outer factorization and it also removes the non-minimum phase dynamics of the feedback loop. Then, the design of the feedback controller is done considering just the minimum phase part of the plant. To show the applicability of the proposed control configuration two common approaches are used in simulation.

Zhong et.al [19] explained about the control methods for the time delay and how the delay occurs in the system.

Aidan O Dwyer et.al [20] classified for time delay processes in which structures optimised controllers are used. Structurally optimised controllers are those in which the controller structure and parameters are adapted optimally to the structure and parameters of the process model.

Waller et.al [21] demonstrated numerically that the Ziegler-Nichols classical tuning of a PID controller could yield good control for systems with inverse response.

Iinoya et.al [22] utilized the concept of the Smith predictor to cope with the inverse response of a process and proposed a compensation scheme referred as an inverse compensator. Consider the unity feedback control system of Figure 4.2.

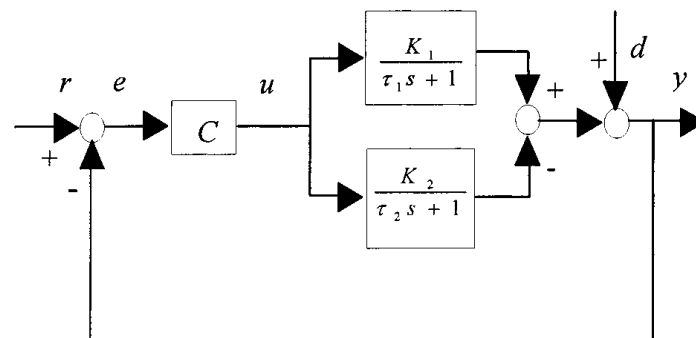


Figure 4.2 Unity feedback control system

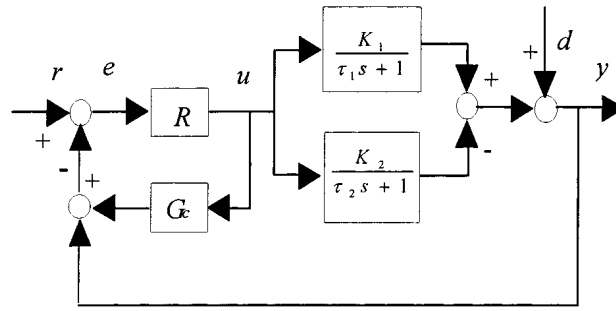


Figure 4.3 Compensation control structure

$$y(s) = C(s) \frac{(k_1\tau_2 - k_2\tau_1)s + (k_1 - k_2)}{(\tau_1s + 1)(\tau_2s + 1)} e(s) \quad - (4.1)$$

It has a zero in the open right half plane. To eliminate the inverse response, what the measurement signal reflects must exclude the information of inverse response. This is possible if in the open-loop response  $y(s)$  we add the quantity  $ys(s)$  given by

$$ys(s) = C(s) Gc(s) e(s) \quad - (4.2)$$

$$= C(s) k \left( \frac{1}{\tau_2s + 1} - \frac{1}{\tau_1s + 1} \right) e(s) \quad - (4.3)$$

Thus we can easily find that

$$y0(s) = y(s) + ys(s) \quad - (4.4)$$

$$= C(s) \frac{[(k_1\tau_2 - k_2\tau_1) + K(\tau_1 - \tau_2)]s + (k_1 - k_2)}{(\tau_1s + 1)(\tau_2s + 1)} e(s) \quad - (4.5)$$

And for

$$K \geq \frac{k_1\tau_2 - k_2\tau_1}{\tau_2 - \tau_1}$$

We find that the zero of the open-loop transfer function is in the open left half plane

$$z = \frac{(k_2 - k_1)}{(k_1\tau_2 - k_2\tau_1) + k(\tau_1 - \tau_2)} \quad - (4.6)$$

Adding the signal  $ys(s)$  to the main feedback signal  $y(s)$  means the creation of a partial loop  $Gc(s)$  with the gain  $K$  as shown in Figure 4.2. This local loop is referred as the inverse-response compensator. It predicts the inverse behavior of the process and provides a corrective signal to eliminate it.

## CHAPTER – 5

### RESULTS AND DISCUSSIONS

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The models are designed under MATLAB SIMULINK environment. The PI controller is tuned according to each model. There are two models for each parameter. First model is the main model and the second model is designed by using the dead time compensator. Then these two models are compared on the basis of these four parameters i.e. ISE, ITSE, IAE, ITAE. [21]

The comparison graphs are shown with the models.

These models are categorised by following three methods:

- 1) In first method a system, a controller, a delay with a feedback is designed using the traditional method. And then, it is again designed using dead time compensator. Finally it is compared.
- 2) In second method a system with inverse response, a controller, with a feedback is designed using the traditional method. And then, it is again designed using dead time compensator. Finally it is compared.
- 3) In third method a system with inverse response, a controller, a delay with a feedback is designed using the traditional method. And then, it is again designed using dead time compensator. Finally it is compared.

We can see the compensator results which are shown in this chapter later. [22]

#### 5.1 Results by first method:

##### Case 1: On the basis of IAE

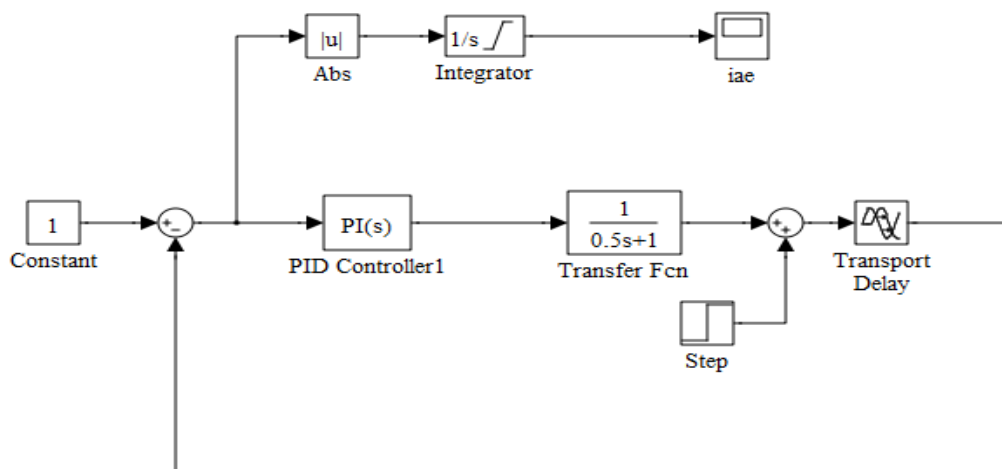


Figure 5.1 IAE with a transfer function and a delay

In all the models transfer function is same in first method. In this  $k=1$ ,  $\tau=0.5$ .

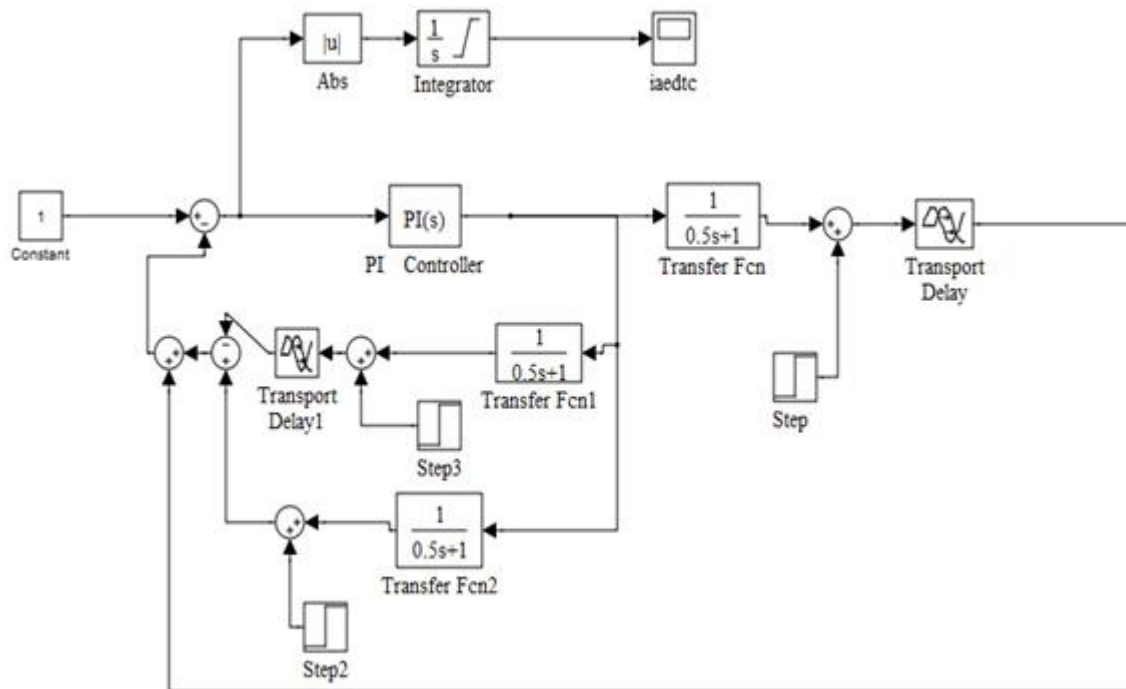


Figure 5.2 IAE with a dead time compensator with a transfer function and a delay

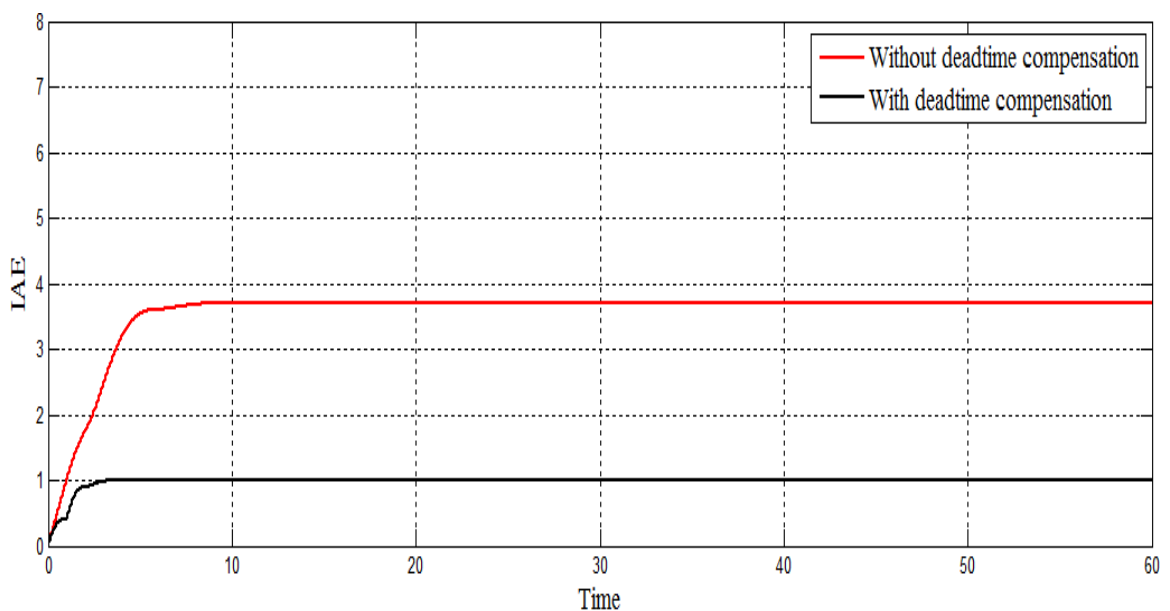


Figure 5.3 Comparison of IAE of two Systems

This figure is showing the result of two models designed by the second method. The black line is showing the compensated model. And the red line is showing the uncompensated model. In this a first order transfer function with delay is used as the system. The evaluation is done for the first parameter i.e. integral absolute error.

**Case 2: On the basis of ITAE**

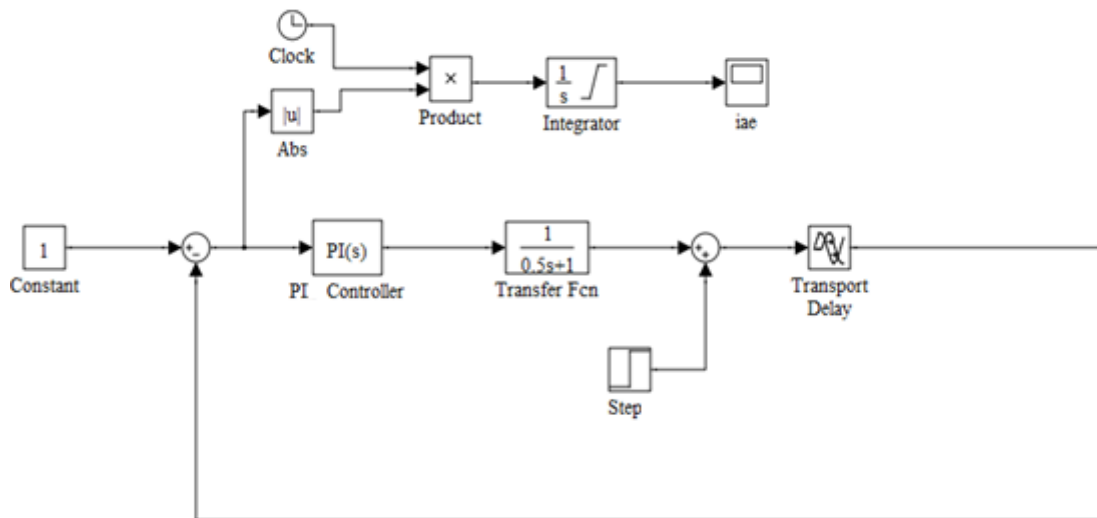


Figure 5.4 ITAE with a transfer function and a delay where  $k=1$  &  $\tau = 0.5$

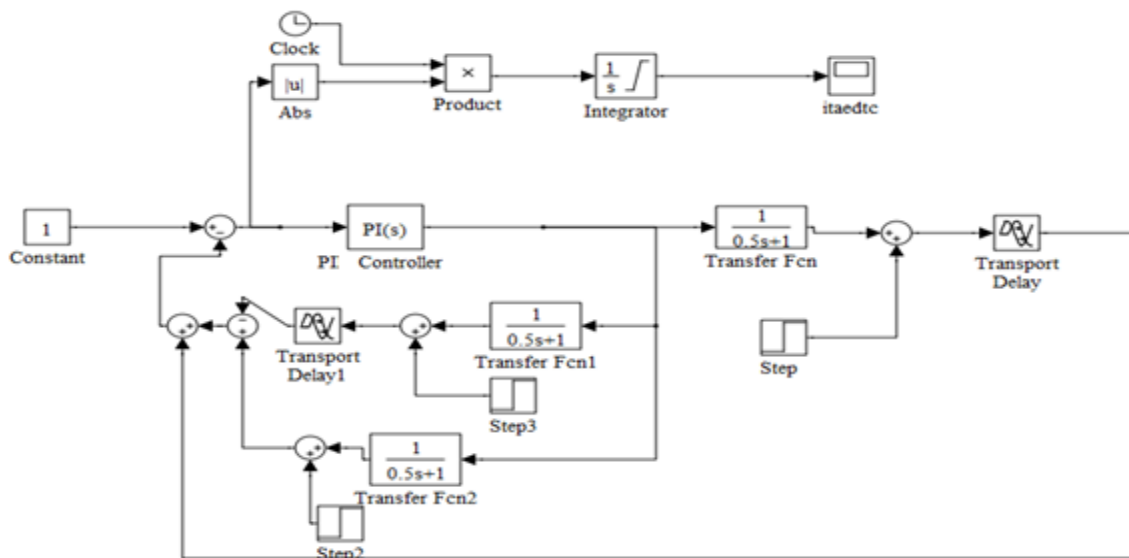


Figure 5.5 ITAE with dead time compensator, a transfer function and a delay

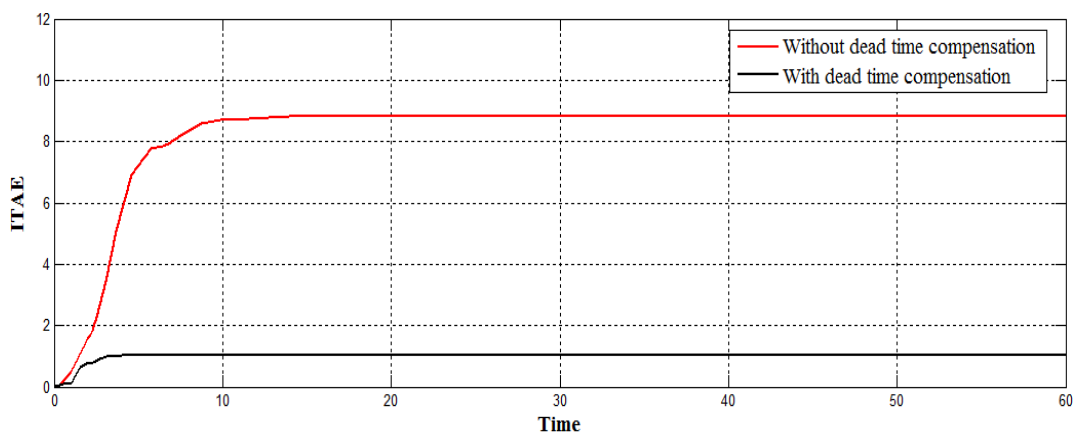


Figure 5.6 Comparison of ITAE of two Systems

This figure is showing the result of two models designed by the second method. The black line is showing the compensated model. And the red line is showing the uncompensated model. In this a first order transfer function with delay is used as the system.

The evaluation is done for second parameter i.e. integral of time multiplexed absolute error.

**Case 3: On the basis of ISE**

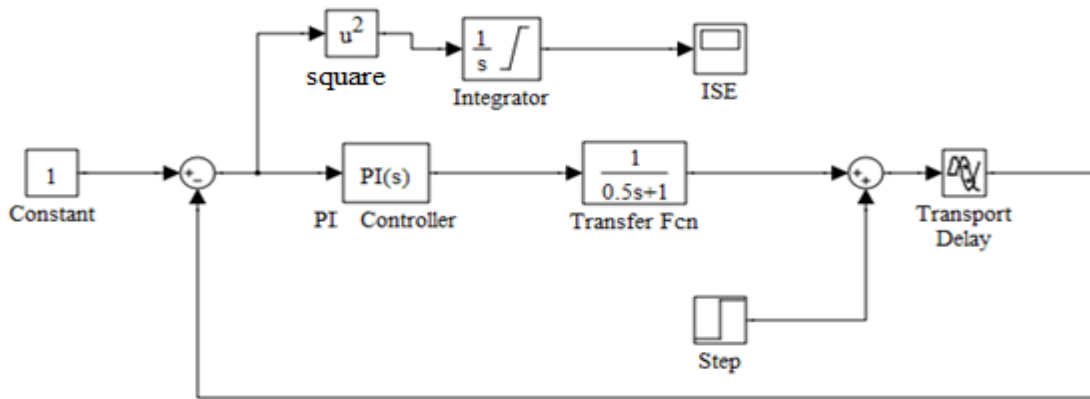


Figure 5.7 ISE with a transfer function and a delay where  $k=1, \tau=0.5$

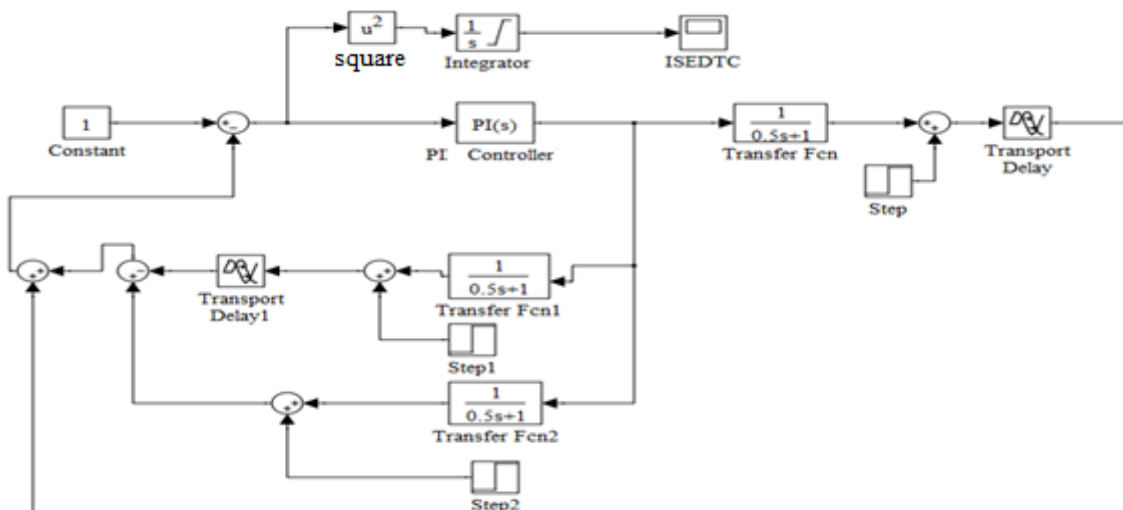


Figure 5.8 ISE with a dead time compensator, a transfer function and a delay

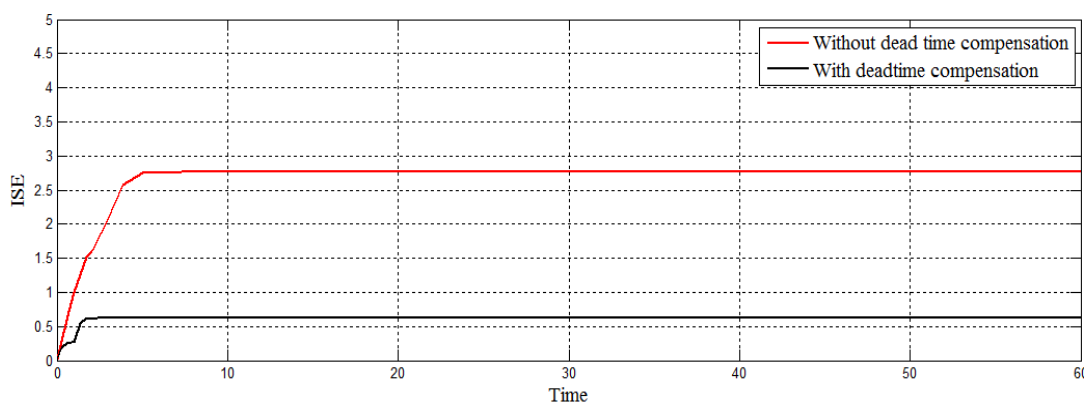


Figure 5.9 Comparison of ISE of two Systems

This figure is showing the result of two models designed by the second method. The black line is showing the compensated model. And the red line is showing the uncompensated model. In this a first order transfer function with delay is used as the system.

The evaluation is done for the third parameter i.e. integral square error.

**Case 4: On the basis if ITSE**

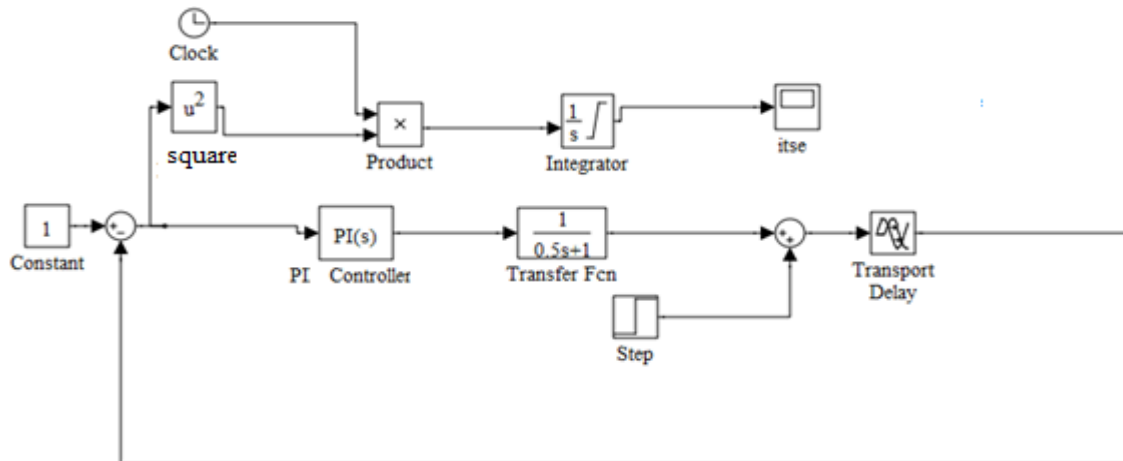


Figure 5.10 ITSE with a transfer function and a delay where  $k=1, \tau=0.5$

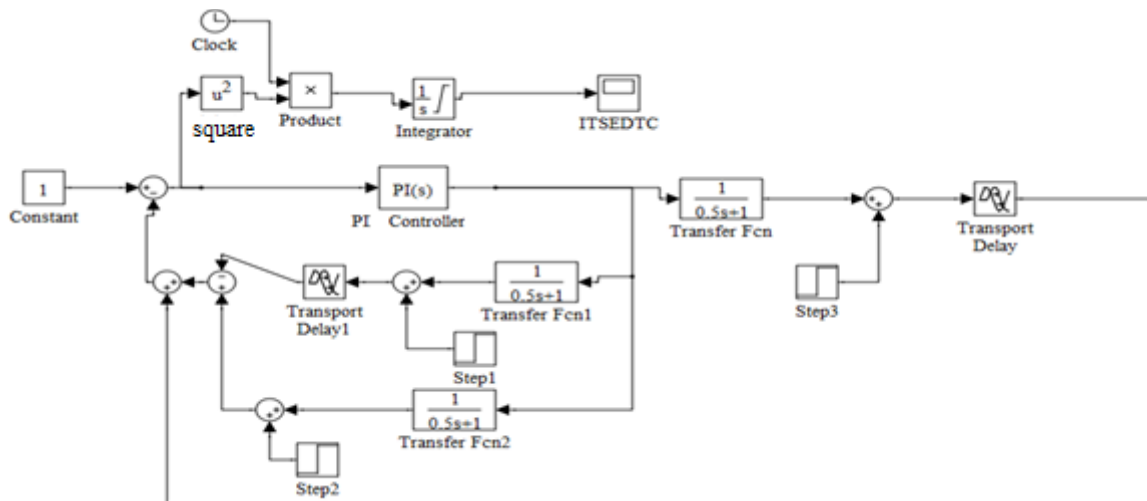


Figure 5.11 ITSE with a dead time compensator, a transfer function and a delay

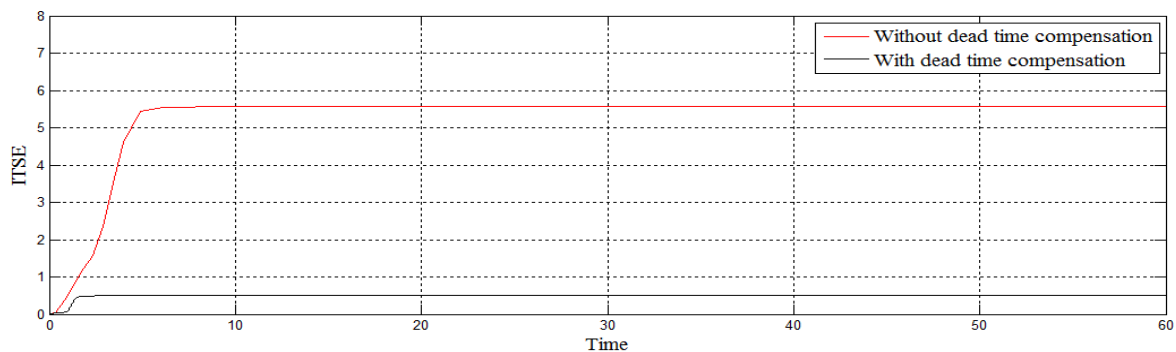


Figure 5.12 Comparison of ITSE of two Systems

This figure is showing the result of two models designed by the second method. The black line is showing the compensated model. And the red line is showing the uncompensated model. In this a first order transfer function with delay is used as the system.

The evaluation is done for the fourth parameter i.e. integral of time multiplexed square error.

### 5.2 Results by second method:

#### Case 1: On the basis of IAE

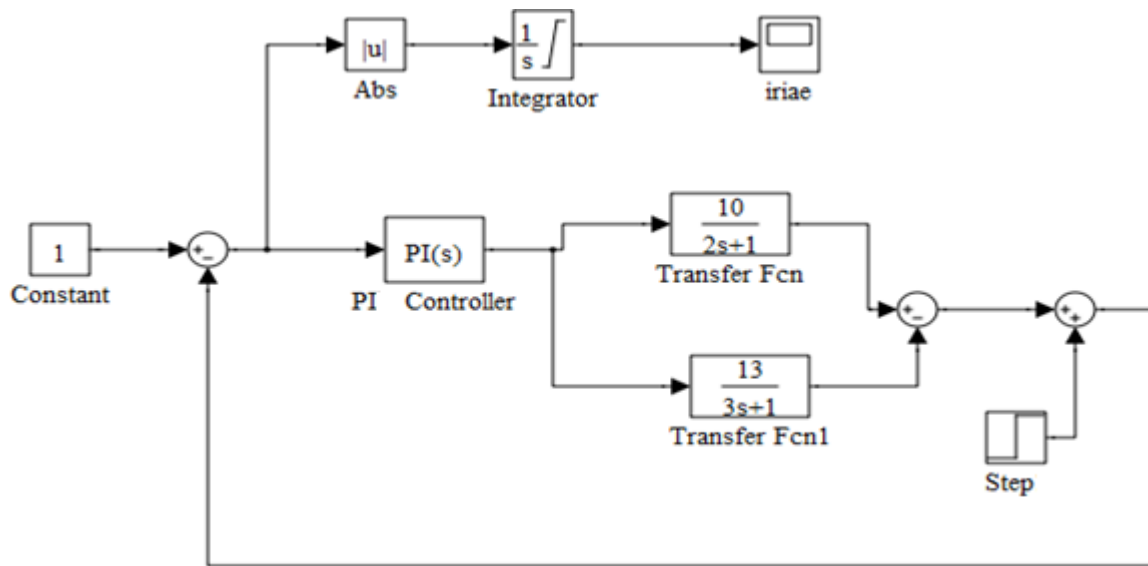


Figure 5.13 IAE with inverse response without delay

In this the two first order transfer functions are used as a system. Where  $k_1=10$ ,  $k_2=13$ ,  $\tau_1=2$ ,  $\tau_2=3$

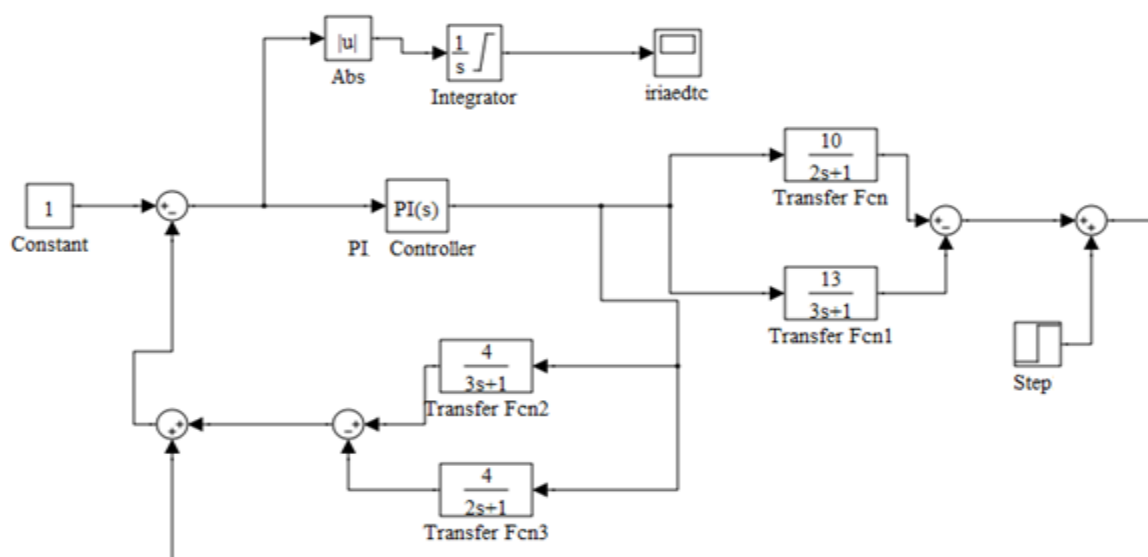


Figure 5.14 IAE with dead time compensator with inverse response, without delay

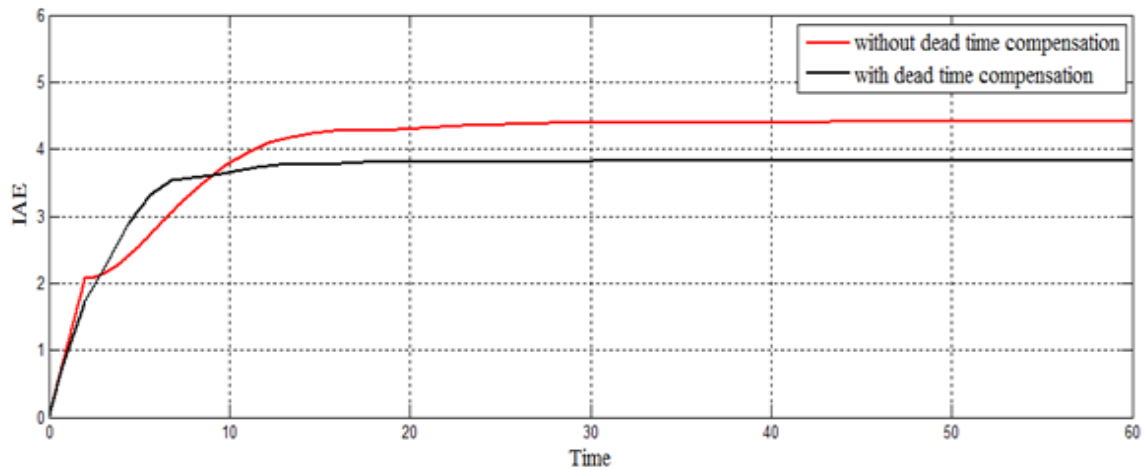


Figure 5.15 Comparison of IAE of two Systems

This figure is showing the result of two models designed by the second method. The black line is showing the compensated model. And the red line is showing the uncompensated model. In this a transfer function with inverse response without delay is used as the system. The evaluation is done for the first parameter i.e. integral absolute error.

**Case 2: On the basis of ITAE**

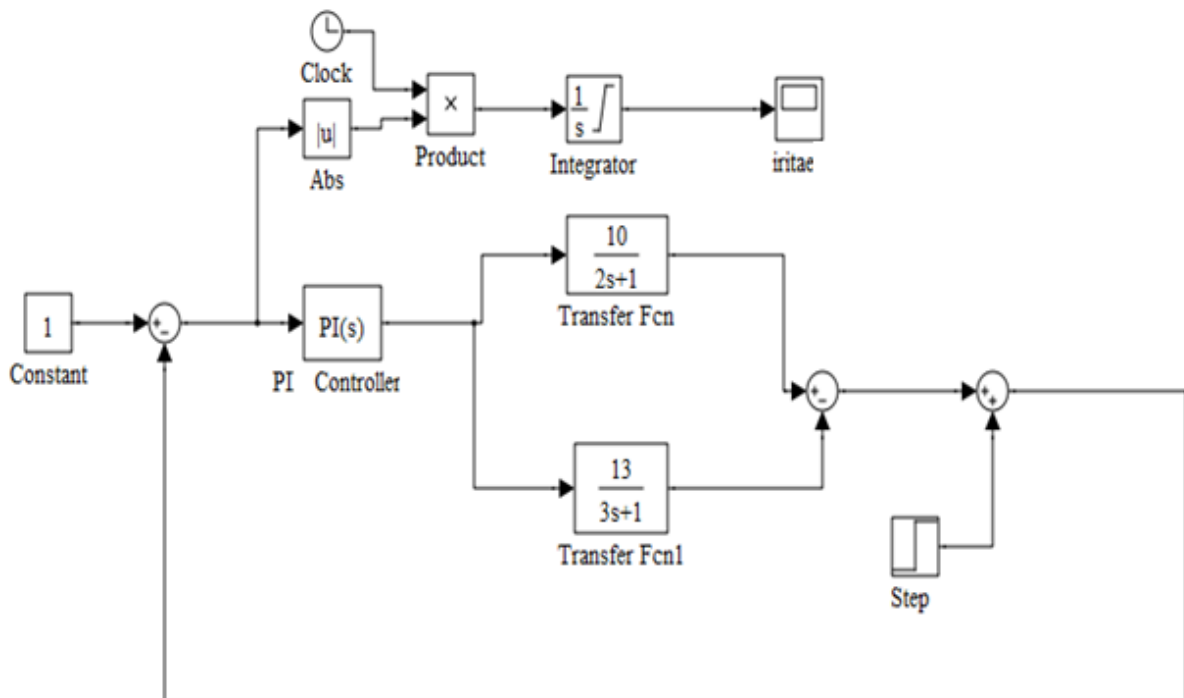


Figure 5.16 ITAE with inverse response without delay Where  $k_1=10$ ,  $k_2=13$ ,  $\tau_1=2$ ,  $\tau_2=3$

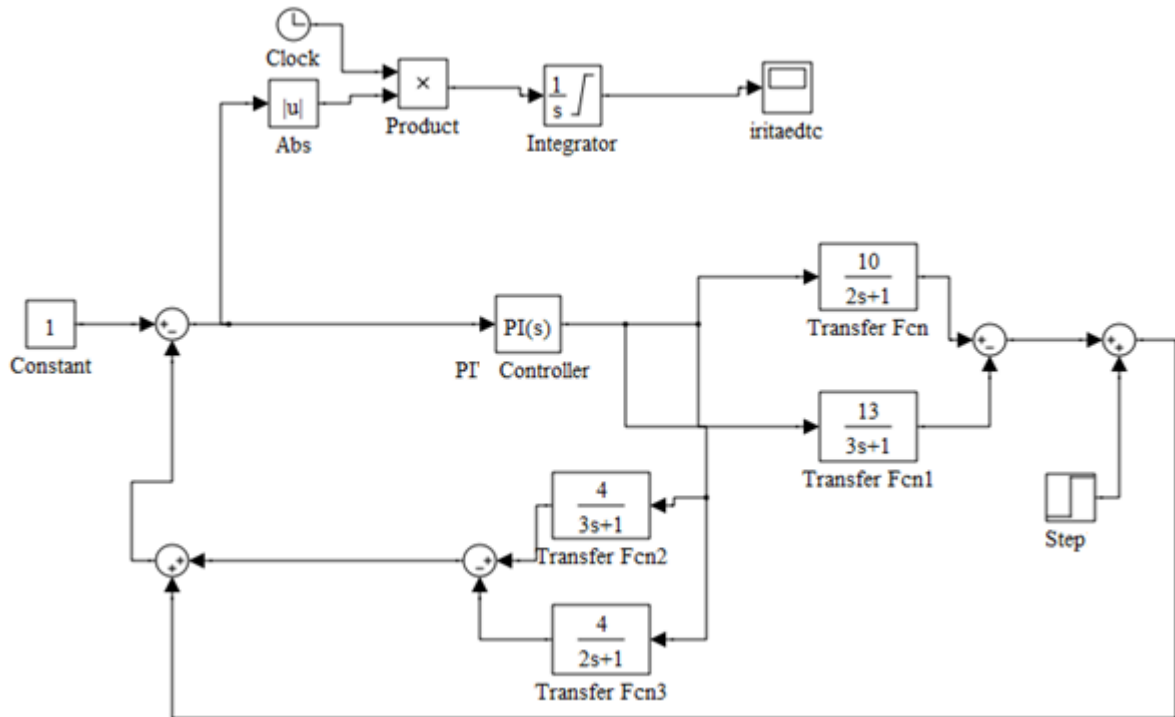


Figure 5.17 ITAE with dead time compensator with inverse response without delay

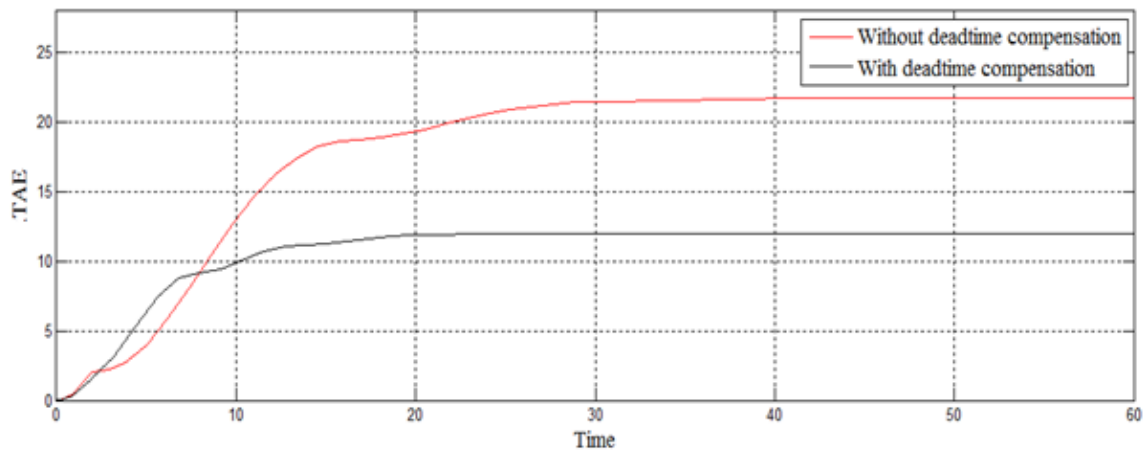


Figure 5.18 Comparison of ITAE of two Systems

This figure is showing the result of two models designed by the second method. The black line is showing the compensated model. And the red line is showing the uncompensated model. In this a transfer function with inverse response without delay is used as the system. The evaluation is done for the second parameter i.e. integral of time multiplexed absolute error.

**Case 3: On the basis of ISE**

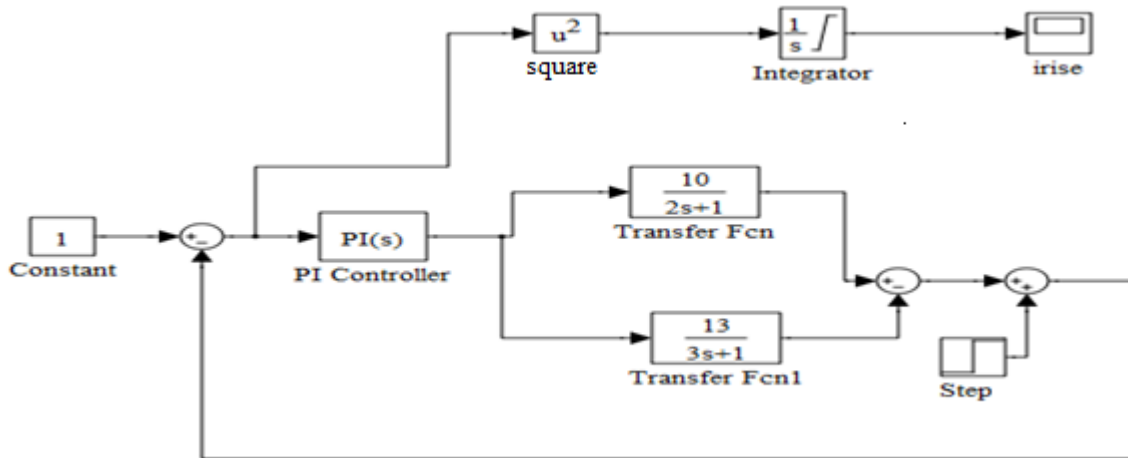


Figure 5.19 ISE with inverse response with delay Where  $k_1= 10, k_2= 13, \tau_1= 2, \tau_2= 3$

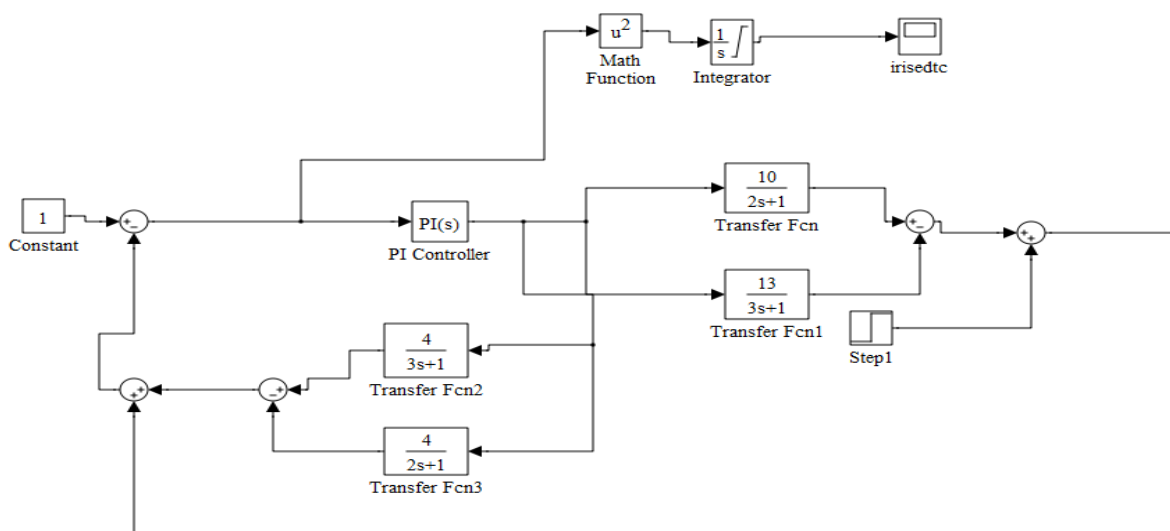


Figure 5.20 ISE with dead time compensator with inverse response without delay

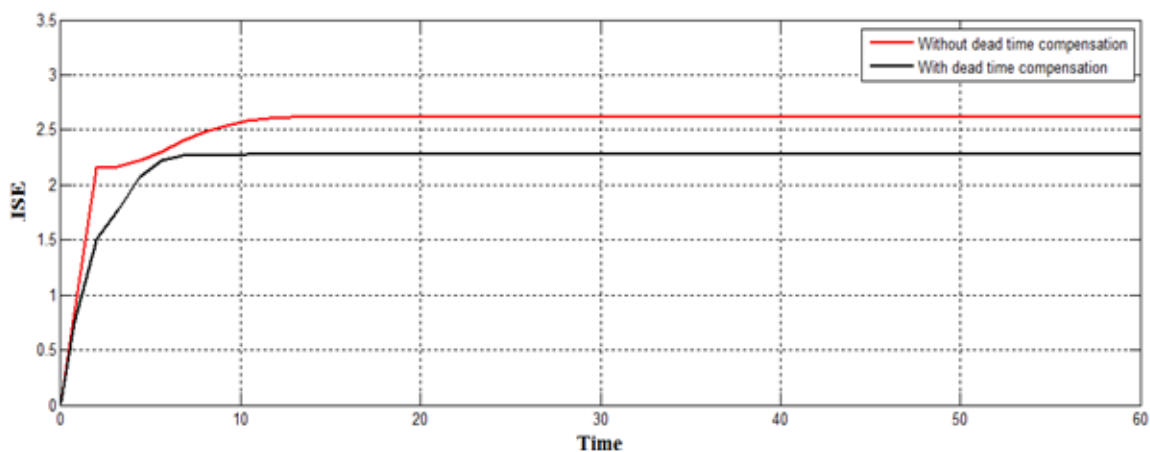


Figure 5.21 Comparison of ISE of two Systems

This figure is showing the result of two models designed by the second method. The black line is showing the compensated model. And the red line is showing the uncompensated model. In this a transfer function with inverse response without delay is used as the system. The evaluation is done for the third parameter i.e. integral square error.

**Case 4: On the basis of ITSE**

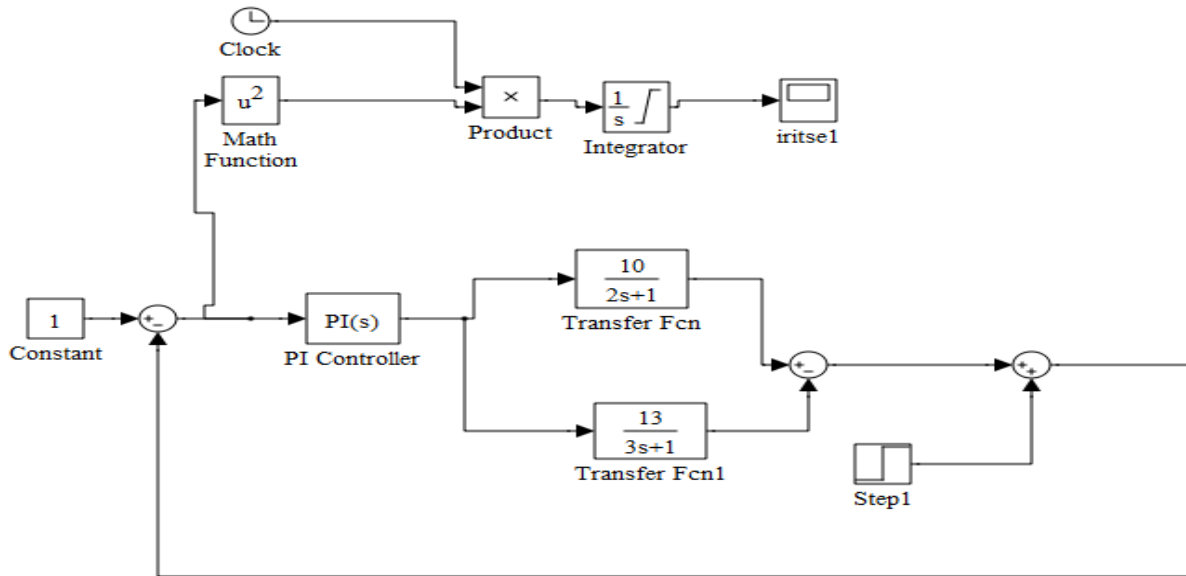


Figure 5.22 ITSE with inverse response without delay Where  $k_1= 10, k_2= 13, \tau_1= 2, \tau_2= 3$

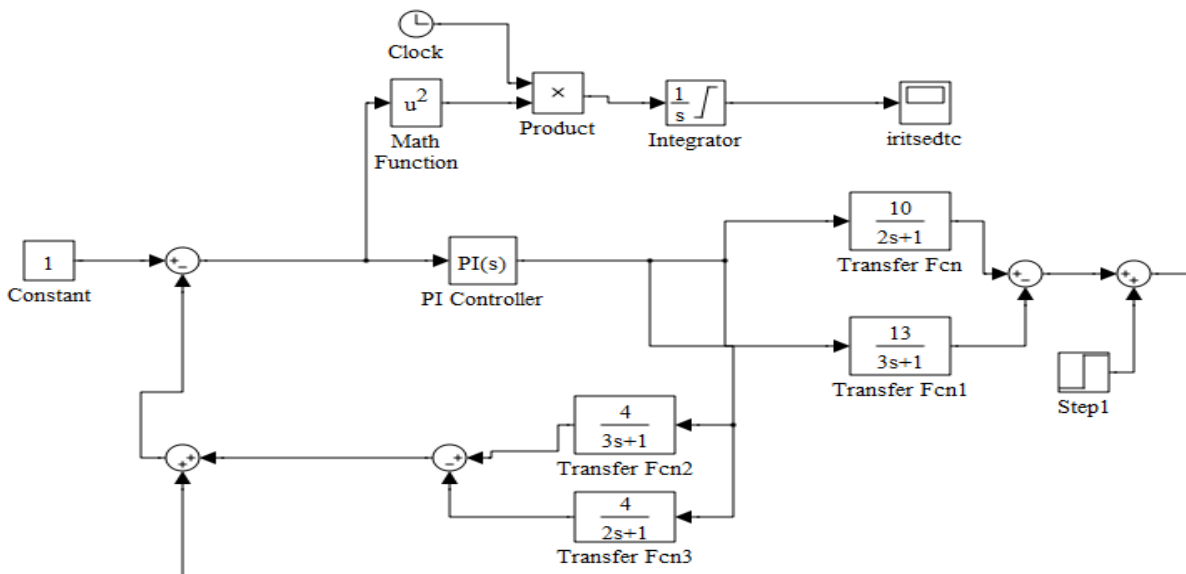


Figure 5.23 ITSE with dead time compensator with inverse response without delay

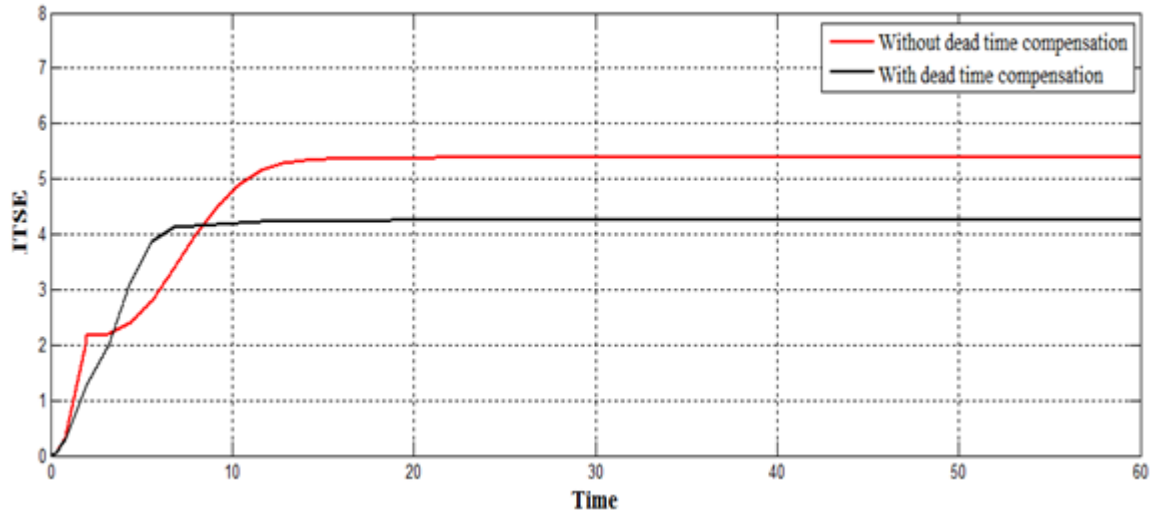


Figure 5.24 Comparison of ITSE of two Systems

This figure is showing the result of two models designed by the second method. The black line is showing the compensated model. And the red line is showing the uncompensated model. In this a transfer function with inverse response without delay is used as the system.

The evaluation is done for the fourth parameter i.e. integral of time multiplexed square error.

### 5.3 Result by third method:

#### Case 1: On the basis of IAE

As process as described in equation 1 with the actual parameters as  $k_1= 10$ ,  $k_2= 13$ ,  $\tau_1= 2$ ,  $\tau_2= 3$  and  $\tau_d= 1$  is simulated, along with the transportation delay of 1 sec is simulated on MATLAB SIMULINK as shown in figure 5.25. Necessary blocks are incorporated to compute and plot IAE of the uncompensated system in response to the step change loading of the system.

Suitable compensation is designed by estimating the parameters as proposed by Mandeep Singh and Dhruv Saksena and Mandeep Singh and Dhruv Saksena and Mandeep Singh and Abhay Sharma. We have deliberately introduced a slight error in the estimated parameters. To study the effect of increasing gain of compensator  $K$ . the value of  $\tau_1$  and  $\tau_2$  are therefore taken as 1.5 and 2.5 respectively instead of 2 and 3. Further the value of transport delay  $\tau_d$  is estimated as 1.5 instead of actual value 1. The compensated system block as simulated in MATLAB SIMULINK as shown in figure 5.26. The comparative graph of IAE for the uncompensated and the compensated system, as simulated for the lowest limit of  $K= 4$  as shown in figure 5.27.

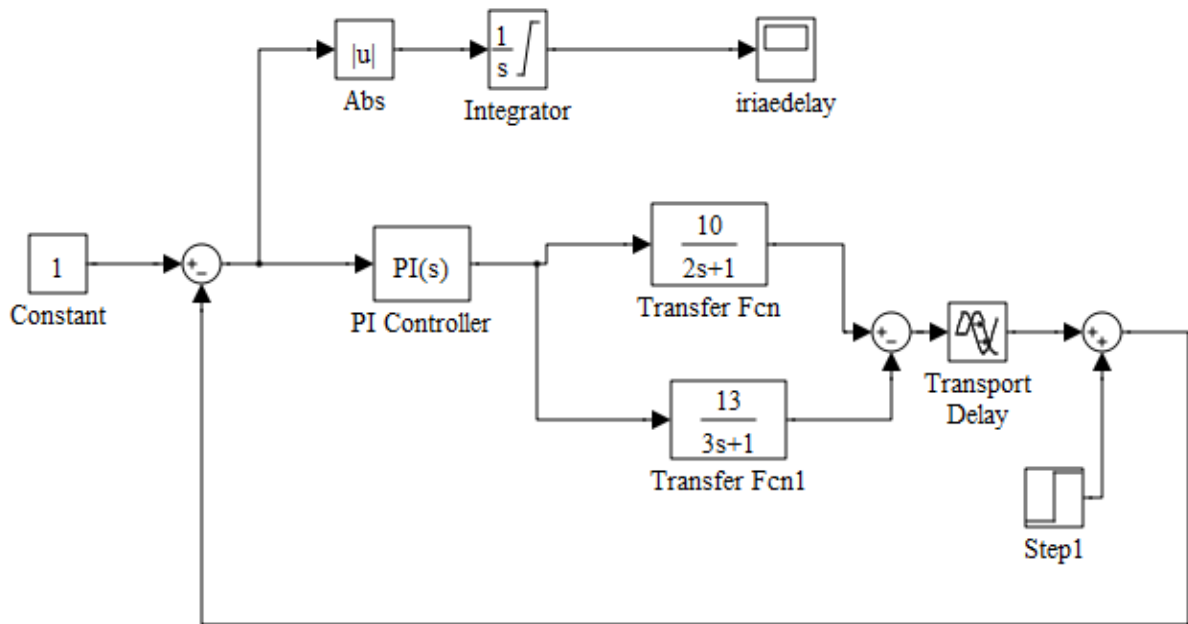


Figure 5.25 IAE with inverse response with delay Where  $k_1 = 10$ ,  $k_2 = 13$ ,  $\tau_1 = 2$ ,  $\tau_2 = 3$

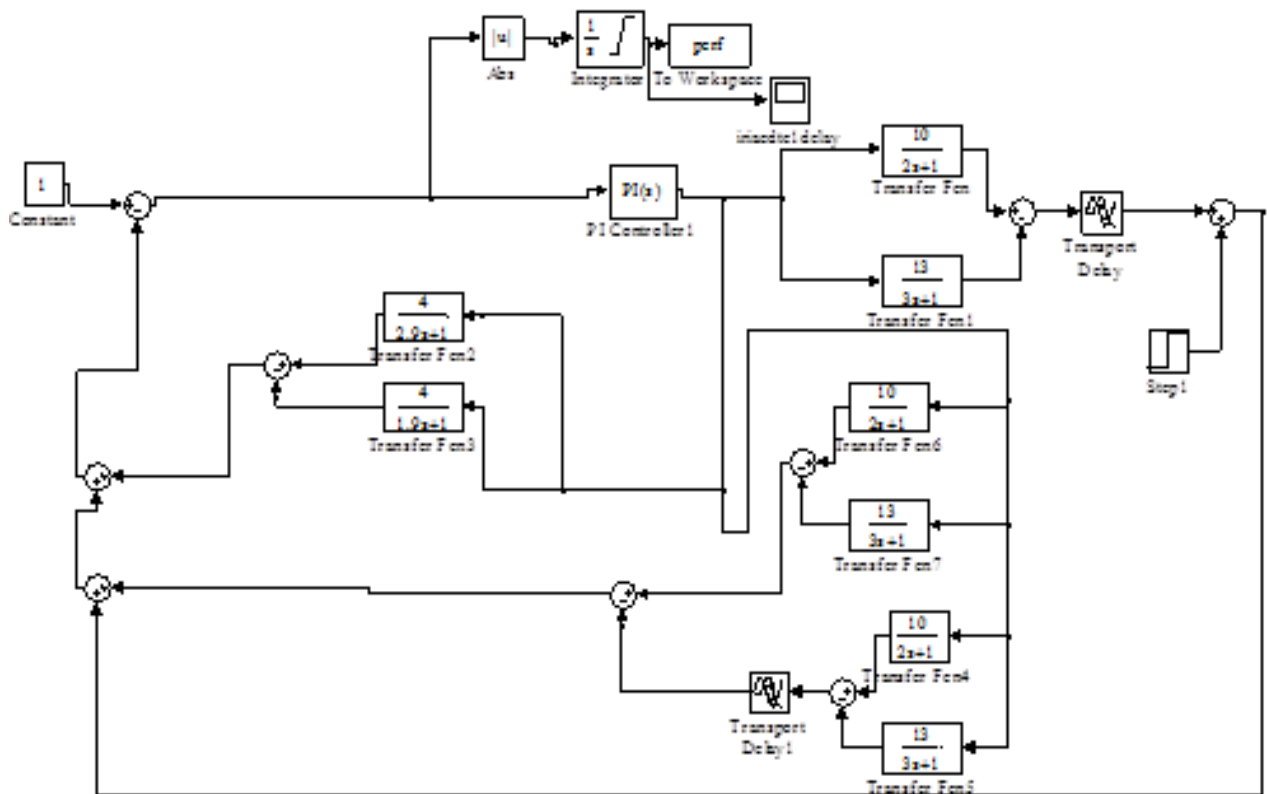


Figure 5.26 IAE with dead time compensator with inverse response with delay

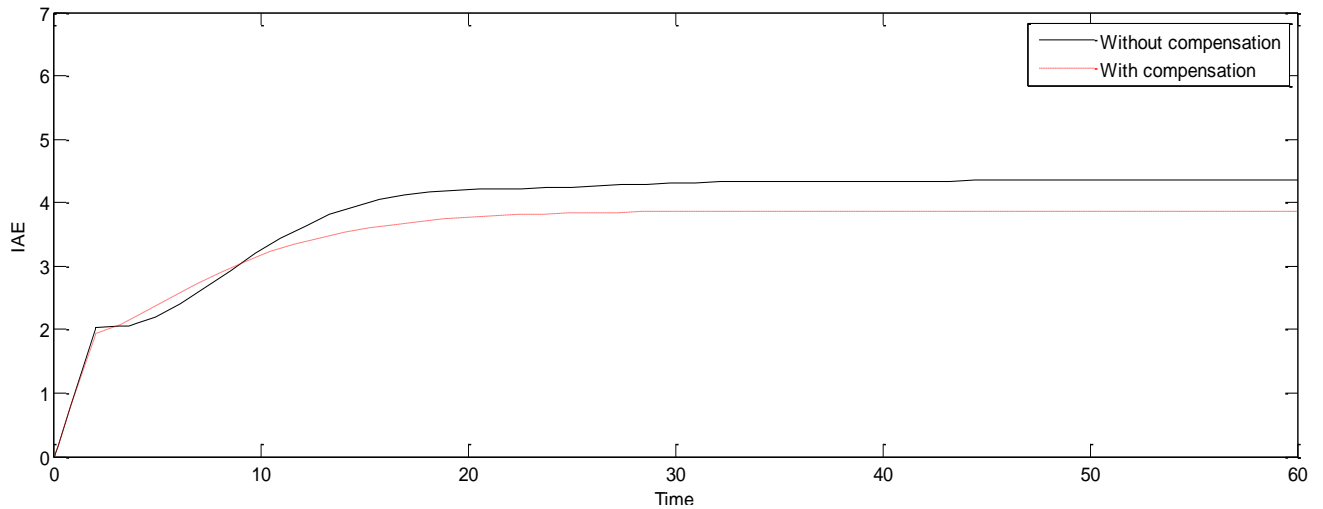


Figure 5.27 Comparison of IAE of two Systems

This figure is showing the result of two models designed by the third method. The red line is showing the compensated model. And the black line is showing the uncompensated model. In this a transfer function with inverse response with delay is used as the system.

The evaluation is done for the first parameter i.e. integral of absolute error.

**Case 2: On the basis of ITAE**

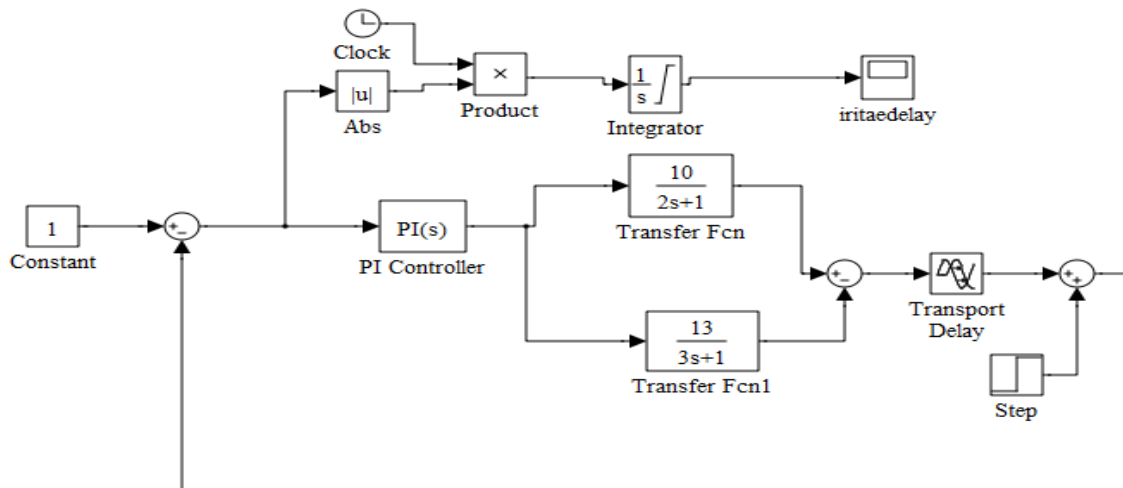


Figure 5.28 ITAE with inverse response with delay Where  $k_1=10, k_2=13, \tau_1=2, \tau_2=3$

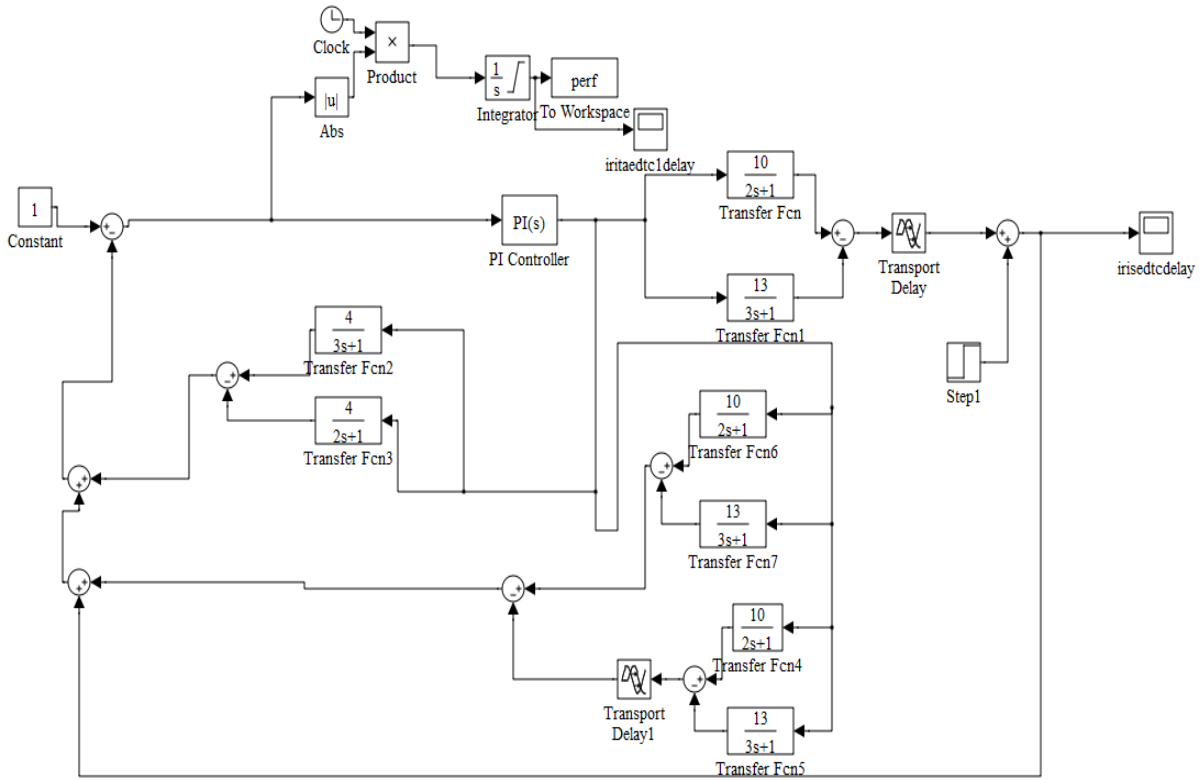


Figure 5.29 ITAE with dead time compensator with inverse response with delay

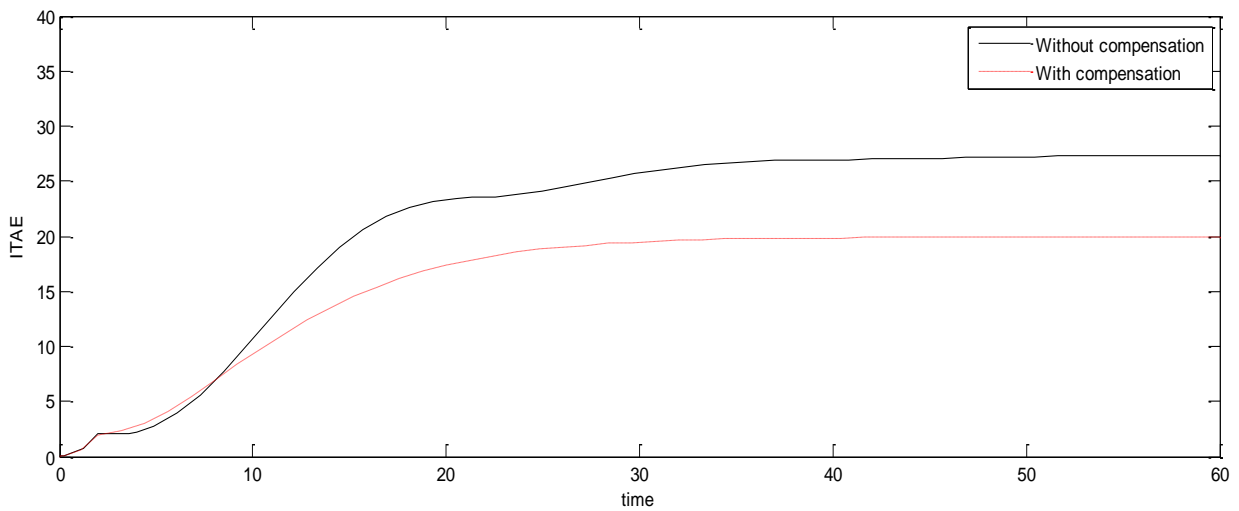


Figure 5.30 Comparison of ITAE of two Systems

This figure is showing the result of two models designed by the third method. The red line is showing the compensated model. And the black line is showing the uncompensated model. In this a transfer function with inverse response with delay is used as the system.

The evaluation is done for the second parameter i.e. integral of time multiplexed absolute error.

**Case 3: On the basis of ISE**

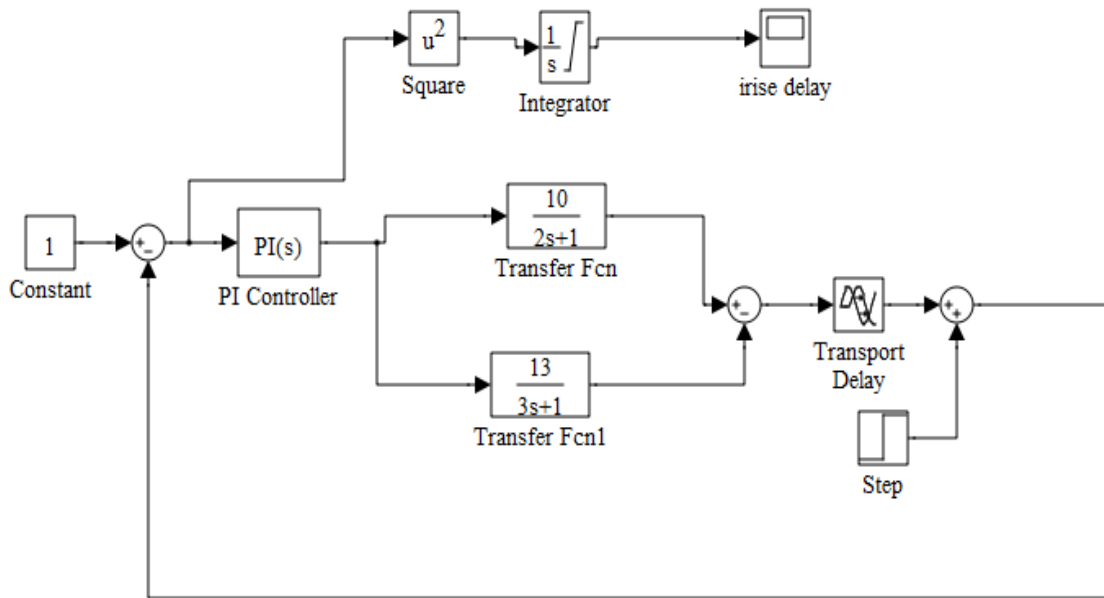


Figure 5.31 ISE with inverse response with delay Where  $k_1= 10, k_2= 13, \tau_1= 2, \tau_2= 3$

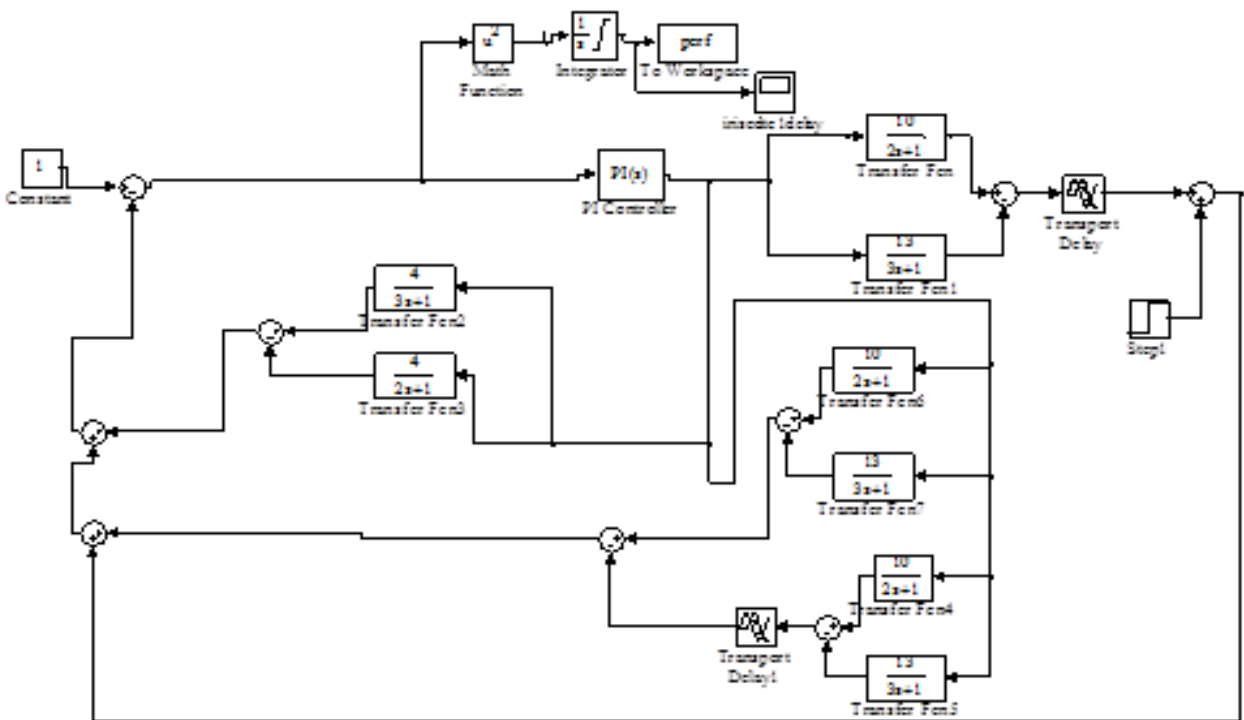


Figure 5.32 ISE with dead time compensator with inverse response with delay

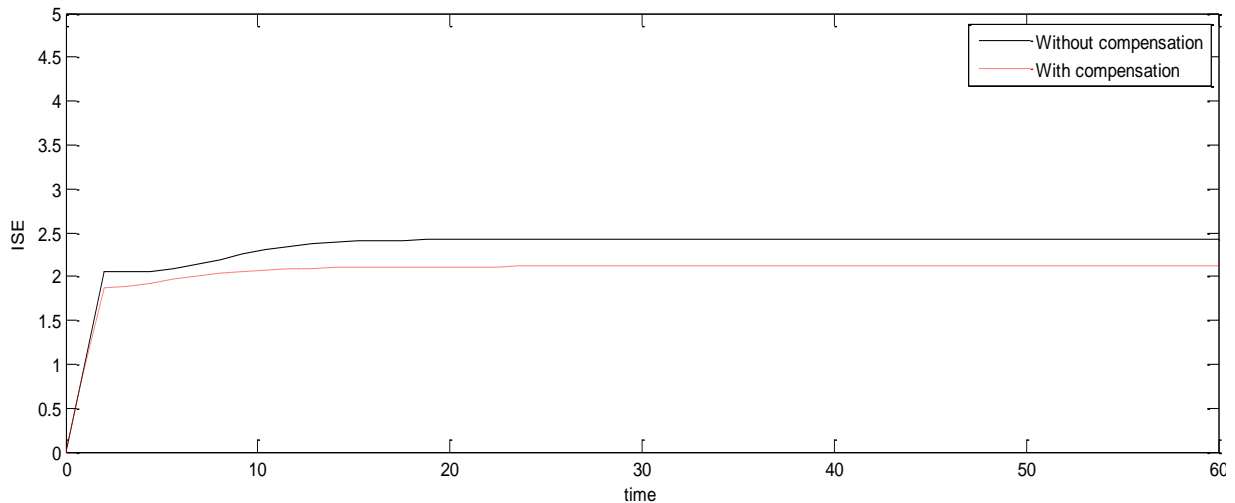


Figure 5.33 Comparison of ISE of two Systems

This figure is showing the result of two models designed by the third method. The red line is showing the compensated model. And the black line is showing the uncompensated model. In this a transfer function with inverse response with delay is used as the system.

The evaluation is done for the third parameter i.e. integral of square error.

#### Case 4: On the basis of ITSE

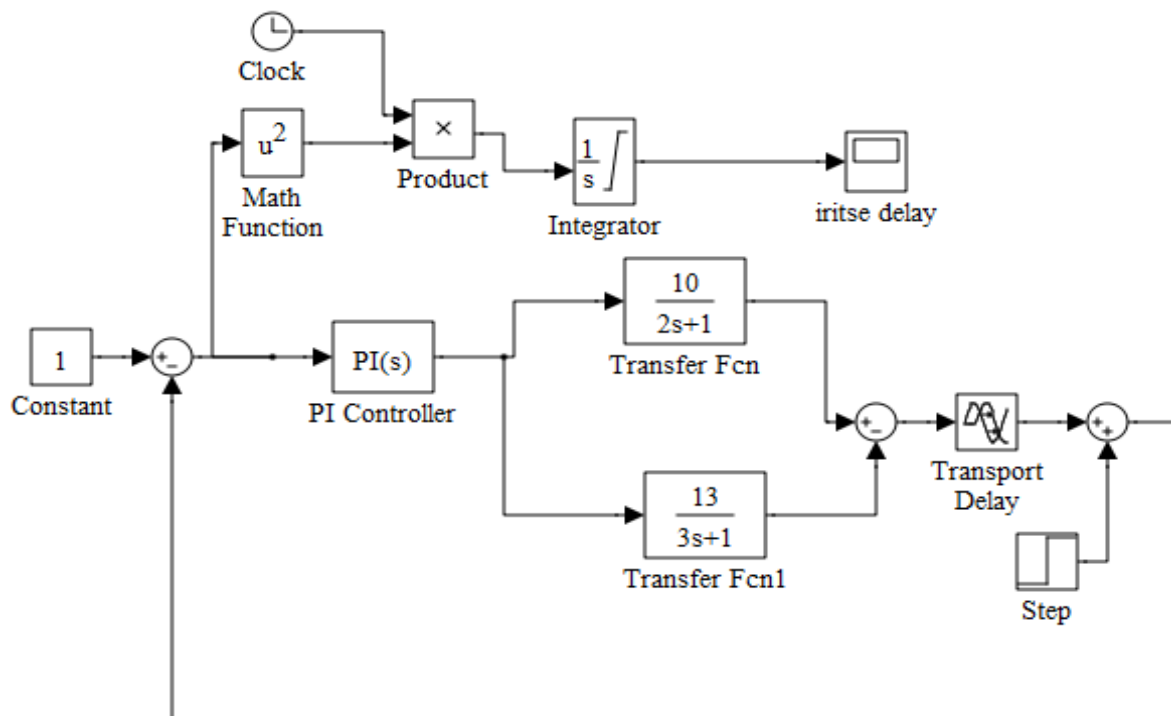


Figure 5.34 ITSE with inverse response with delay Where  $k_1=10$ ,  $k_2=13$ ,  $\tau_1=2$ ,  $\tau_2=3$

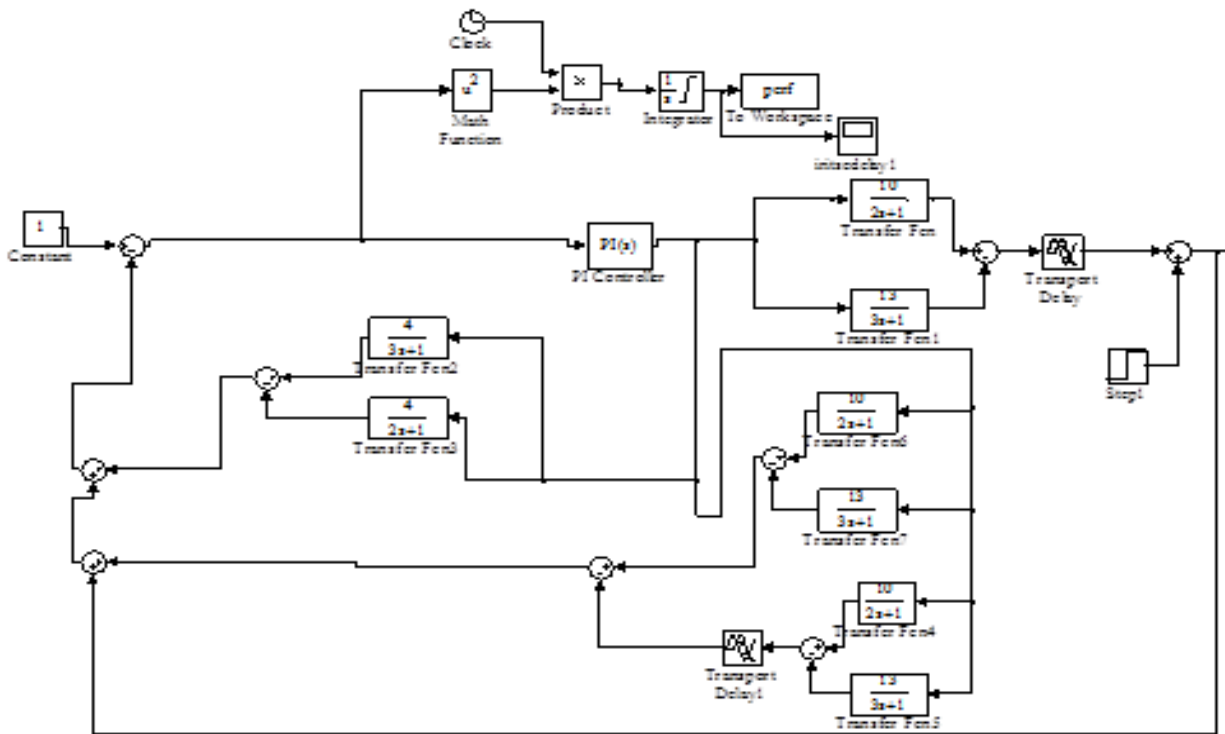


Figure 5.35 ITSE with dead time compensator with inverse response with delay

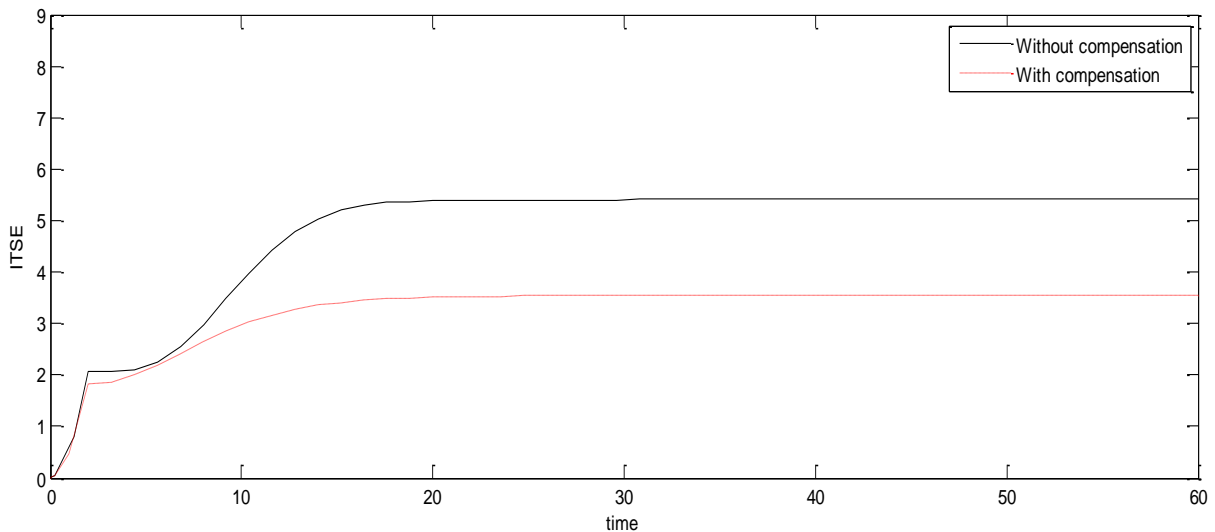


Figure 5.36 Comparison of ITSE of two systems

This figure is showing the result of two models designed by the third method. The red line is showing the compensated model. And the black line is showing the uncompensated model. In this a transfer function with inverse response with delay is used as the system. The evaluation is done for the fourth parameter i.e. integral of time multiplexed square error.

## CHAPTER – 6

### CONCLUSION

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Inverse response in any process creates a challenge for control engineers. When transportation delay is added to this process, the challenge becomes more forbidding. Many researchers have proposed compensators for these two aspects separately. In the scheme proposed in this thesis, the inverse response and the transport delay are taken together, compensated for these two, and the effect of increasing compensation gain ( $K$ ) is observed. It may be concluded that if the parameters are estimated accurately, then increasing the value of  $K$  decreases the error. On the other hand, if the parameters are estimated inaccurately, then increasing the value of  $K$  increases the error. Hence, it is always safer to keep the value of  $K$  as small as possible, though slightly higher or equal to the minimum value calculated.

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