LOVE WAVE DISPERSION RELATION IN REINFORCED AND INHOMOGENEOUS ANISOTROPIC MEDIUM

Dissertation submitted in partial fulfillment of the requirements for the award of the degree of

Master of Science

in

Mathematics and Computing

Submitted by

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July 2019

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CERTIFICATE

This is to certify that the thesis entitled "**Love wave dispersion relation in reinforced and inhomogeneous anisotropic medium**", being presented in partial fulfillment of the requirements for the award of the degree of Masters of Science in Mathematics and Computing and submitted to **School of Mathematics**(SOM), Thapar Institute of Engineering and Technology, Patiala, is an authentic record of my own work carried out under the supervision of **Dr. Pramod Kumar Vaishnav.**

The matter presented in this thesis has not been submitted for the award of any other degree from this or any other institute.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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Acknowledgement

This thesis marks the end of the beautiful journey to achieve my Masters degree. Throughout this journey I have been supported and guided by several people. I would like to take this opportunity to express my gratitude to all those people.

My first and sincere appreciation goes to **Dr. Pramod Kr. Vaishnav**, my supervisor, for all I have learnt from him and for his continuous help and support in all stages of this thesis. His insights and clarity of thoughts have been presented at every moment of this work. I would like to thank him for encouraging and helping me to shape my interests and ideas. It is with immense gratitude that I acknowledge **Dr. Satish Kumar**, Head, SOM, Thapar Institute of Engineering and Technology, Patiala. His advices and discussions were in-valuable to me and his attitude towards research always inspired me. I really appreciate him for always being so supportive.

I express my gratitude to all the faculty members and staff of the School of Mathematics, Thapar Institute of Engineering and Technology, for their support. Above all I would like to thank my parents for their love, blessings, support, encouragement, sacrifice, and unwavering belief in me. Without them, I would not be the person I am today. I would also like to convey my sincere thanks to my brother **Money** for his moral support that helped me to work harder each day and finally. I thank and pay my regards to the Almighty for his love and blessings.

Date: 02/08/2019

Place: Patiala

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Abstract

The dispersion relation of Love wave propagation in reinforced and inhomogeneous anisotropic medium has been derived in the present work. On Love wave's phase velocity, the effect of reinforced parameters and initial stress has been explained graphically. In particular cases, the standard dispersion relation of Love wave has been obtained in both the problems. This thesis entitled "Love wave relation in reinforced and inhomogeneous anisotropic medium" is carried out for the study of propagation of Love wave in detail. The thesis contains three major chapters in addition to chapter Bibliography in end.

Chapter 1 deals with the introduction for the relevant problems. It gives the explanation of studies in the field of theoretical seismology.

In **Chapter 2**, we considered a reinforced layer of finite thickness lying over an inhomogeneous semi-infinite medium. For the displacement in the layer and half-space, we used separation of variable method(for solving the second order partial differential equation i.e. governing equation of motion). The presence of reinforced parameters and heterogeneity in the obtained dispersion relation make a significance contribution in the study of Love wave propagation. For the validation of problem, we obtained standard Love wave dispersion relation in the particular cases. The significant effect of reinforced parameter and heterogeneity has been noticed in graphical section. This study plays an important role in the field of material science and mining engineering for searching minerals inside the earth.

In **Chapter 3**, Love wave propagation in an inhomogeneous anisotropic medium has been discussed. In this earth model, we take an inhomogeneous anisotropic superficial layer lying over an initially stressed orthotropic semi-infinite medium. Love wave propagation direction is taken along $x-$direction. The displacement along $y-$direction has been obtained by solving the second order P.D.E. with the help of separation of variable method. The effect of initial stress and heterogeneity parameter is obtained in graphical section. We also obtained the standard Love wave dispersion relation in particular cases. This study is useful for geologists and seismologists.

Contents

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Nomenclature

F	Body force per unit volume
H	Thickness of the layer
N,L	Directional rigidities of the layer
$\delta(x)$	Dirac-delta function
$\triangle$	Cubical dilation
ρ	Density of the medium
u,v,w	Displacement components
σ_{ij}	Stress components
e_{ij}	Strain components
x_i	Cartesian co-ordinates
t	Time parameter
A,B,C,D	Arbitrary constants

Chapter 1

Introduction

The study of earth and earthquake waves/seismic waves is called seismology. The motion we feel on the surface of the earth during an earthquake comes from the energy released deep within the earth. This energy is transmitted to the surface of the earth by earthquake waves or seismic waves. Earthquakes[1, 2] occur when rocks deep underground suddenly break under pressure or slip along a fault. The point of release is known as the focus of the earthquake. All natural earthquakes take place in the lithosphere. Lithosphere refers to the outermost, rigid portion of the earth. It is deep up to 200 kilometers from the earth's surface. Seismograph, an instrument, records the waves which reach the surface of the earth. Seismologists are those who study the interior structure of the earth and earthquakes[3].

While the earthquake and seismic waves are dangerous, they are critical in the development of our understanding of the layers of the earth. P and S-waves are the primary tools that scientists use to understand what the properties are, of each of the layer of the earth, as these waves travel differently through solids and liquid. The earthquake waves can be used to understand the layers of the earth and their mechanical properties. So, these waves are crucial in understanding the interior of the earth as this is a place that no human has ever been to. So, it is through these earthquake waves that we know what we know about the center of the earth.

1.1 Wave

A wave is a molecular disturbance in a medium. It transfers energy through matter or space from one point to another point in the medium, with limited or no mass transport. The particles do not move in the direction of waves. They do not go and carry energy with them, they simply vibrate at their own positions i.e. a particle only oscillates about its mean position. So, its only the energy which is transported. Most common examples of waves are sound waves and water waves. Sound waves are used to transport sound energy from one point to another point. Sound can travel through air, solids and liquids. Air, solids and liquids are examples of medium through which sound travels. Similarly, radio waves transport electromagnetic wave energy from the transmitter to the receiver. This wave travels through air and even vacuum.

1.2 Seismic Waves

Seismic waves are produced in the interior part of the earth. They carry energy away from the focus of the earthquake towards the surface where we experience that as ground shaking or earthquake. These waves are outcome of earthquake, large slides, volcanic eruptions and large man made explosions. The devices used to record these waves are seismometer, hydrophone (in water) or accelerometer. These waves are crucial in understanding the interior of the earth.

Fundamentally, seismic waves have two types- body waves and surface waves. Body waves, as the name suggest travel through the interior of the earth. On the other hand, surface waves move along the earth's surface. These waves play an important role in understanding the interior of the earth. When an earthquake happens, the seismologists can use the time taken by P and S-waves to reach other locations around the earth to estimate the properties of the earth between those two locations(the earthquake's originated point and the point where the earthquake waves have been detected). Also, because S-waves cannot travel though liquids, seismologists can determine which material has formed the layers of the earth and how big they are.

1. **Body Waves:**

   Body waves' vibrations travel through earth's interior. These are generated due to the energy released at the focus called epicenter and move in all the directions travelling through the body of the earth. As these waves travel internally within the earth's surface and move the interior of the earth so these are referred to as body waves. The effect of body waves is similar to the effect of refraction of light. These waves move inside the earth along paths controlled by the properties of the material in terms of density and modulus (stiffness). Body waves can be further classified into two categories: P waves and S waves.

   - **P Waves:** Primary waves or P-waves are compressional body waves that force the ground to move backward and forward as they get compressed and expanded. Primary waves can travel through any medium and every layer of the earth. These waves behave like sound waves in air. These waves are longitudinal in nature, that is, the displacement of the material is in the same direction as the wave is travelling. P-waves are pressure waves. Their speed is faster than other waves and reaches the seismograph station firstly, hence they are given the name "Primary". P-waves are 1.7 times faster than S-waves. P-waves can pass through solids, liquids and any type of medium. In air, they take the form of sound waves, so they travel at the speed of sound. Their velocity in air, water and granite is 330 m/s, 1450 m/s and 5000 m/s respectively.

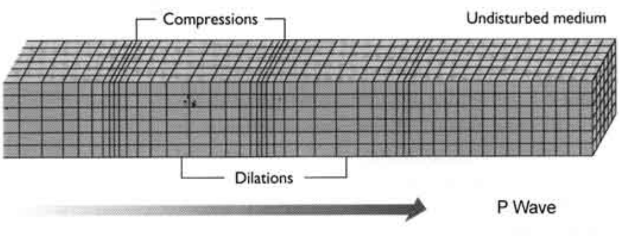

Figure 1.1: Primary wave

   - **S Waves:** Secondary waves or S-waves are the secondary waves to leave an earthquake and they travel much slower than P-waves. These are transverse waves, so the displacement of medium is upright to the wave motion. These waves move the surface back and forth, vertical to the wave direction. Just as P-waves are classified as a compressional waves, S-waves are classified as shear waves. After an earthquake event, S-waves are the secondary waves to arrive at seismograph stations following the primary waves. These can only pass through solids and are unable to travel through fluids because fluids (liquids and gases) do not support any shear stresses. Speed of S-wave is typically 60% of that of P-wave in the same medium or material.

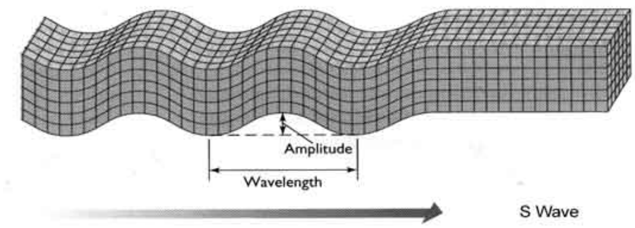

Figure 1.2: Secondary wave

2. **Surface Waves:**
Seismic surface waves move along the surface of the earth. When body waves move towards the surface, they come in contact with surface rocks and generate a new set of waves called Surface waves. The surface waves report last on the seismograph. Surface waves are more destructive and spread across the earth's surface. They cause the displacement of rocks and hence, the collapse of structure occur. As compared to P and S-waves, surface waves travel more gradually. These waves can have high amplitude and long wavelength in large earthquakes. Rayleigh wave, Love wave and Torsional wave are the types of surface waves.

   - **Rayleigh Waves:** Rayleigh waves can be viewed as the ground rolls as these waves are the kind of ground moving up and down. These are surface waves that move like ripples with movements similar to those of waves on the surface of water. These are called transverse waves as the movement of the waves is perpendicular to the direction of wave propagation. Lord Rayleigh, in 1885, predicted the existence of Rayleigh waves. These are slower than the waves of the body, about 90% of the velocity of the S-waves. These are generated by the interaction of Primary and SV waves with free surface. They can propagate in both homogeneous half-space and layered one.

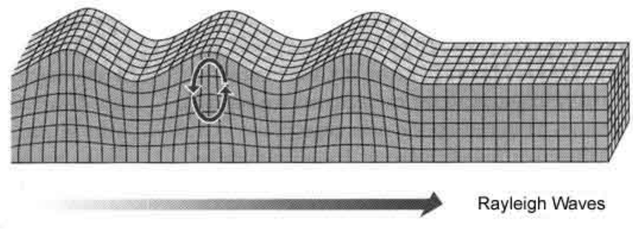

Figure 1.3: Rayleigh wave

- **Love Waves:** Love waves are SH waves(horizontally polarized shear-waves) that travel in the presence of superficial layer of finite thickness overlying semi-infinite medium. These are surface seismic waves that cause horizontal shifting of the earth during an earthquake. These are named by a British mathematician, A.E.H. Love[4]. In 1911, he gave a mathematical model of these surface waves known as Love waves. These waves travel with a lower velocity than P and S-waves but faster than Rayleigh waves. These have the largest amplitude and near about 90% of the S-waves' velocity.

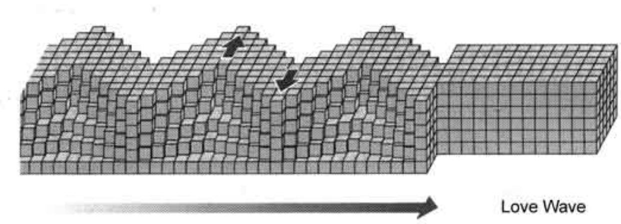

Figure 1.4: Love wave

- **Torsional Waves:** Twisting or torsional waves are the third type of surface waves. In torsion waves, the medium vibrations are rotational movements around the direction of wave propagation. It means that the elements of the medium carrying the wave are oscillating around an axis which is parallel to the propagation direction. For example, if we take a flexible helical string and twist it gently at one end, the twist will be transferred to the other end as a wave pulse.

1.3 Standard Love Wave Dispersion Relation

Consider the wave is propagating through an isotropic layer of finite thickness H overlying homogeneous isotropic semi-infinite medium. The wave propagation direction is taken along $x-$axis, displacement along $y-$axis and $z-$axis is taken vertically downwards perpendicular to the wave propagation direction as shown in the following figure.

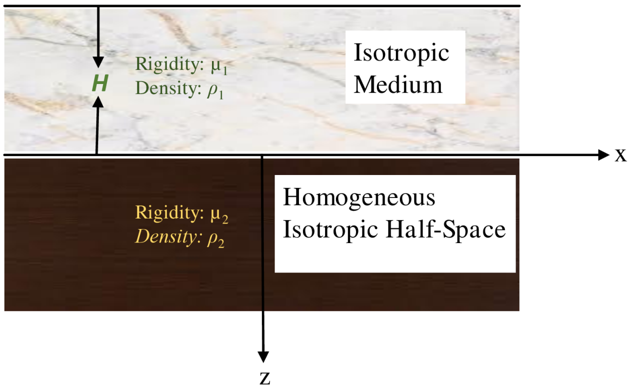

Figure 1.5: Geometry for Seismic Love Wave

The governing equations of motion for Love wave propagation in the absence of body forces are[5]

$$\frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2} \tag{1.3.1}$$

$$\frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \tag{1.3.2}$$

$$\frac{\partial \sigma_{31}}{\partial x} + \frac{\partial \sigma_{32}}{\partial y} + \frac{\partial \sigma_{33}}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2} \tag{1.3.3}$$

where σ_{ij} = incremental stress components, u, v and w = displacement components in semi-infinite medium along x, y and $z-$ directions respectively. ρ = density of the material.

According to Hooke's law, the stress-strain relations in an isotropic semi-infinite medium are written below

$$\sigma_{ij} = \lambda_1 \triangle \delta_{ij} + 2\mu_1 e_{ij} \tag{1.3.4}$$

where λ_1, μ_1 = Lame's constants and e_{ij} = strain components.

Love wave is travelling along the $x-$direction therefore, $w_1 = u_1 = 0$ and $v_1 = v_1(x,z,t)$. The displacement components in the isotropic medium are u_1, v_1 and w_1 along x, y and $z-$directions respectively.

The cubical dilation ($\triangle$) is the change in volume/unit volume. For small strain $\triangle = e_{11} + e_{22} + e_{33}$, the stress component will be obtained as

$$\sigma_{11} = 0, \sigma_{12} = \mu_1 \frac{\partial v_1}{\partial x}, \sigma_{22} = 0, \sigma_{13} = 0, \sigma_{23} = \mu_1 \frac{\partial v_1}{\partial z}, \sigma_{33} = 0 \tag{1.3.5}$$

Using stress-strain relations and standard Love wave conditions in the governing equation of motion

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} = 1/c_1^2 \frac{\partial^2 v_1}{\partial t^2} \tag{1.3.6}$$

where $c_1 = \sqrt{\mu_1/\rho_1}$ is the shear wave velocity.

We can consider the harmonic solution of (1.3.6) along the $x-$direction as $v_1(x,z,t) = V_1(z)e^{ik(x-ct)}$, where c = phase velocity of the Love wave and k = wave number. Thus,(1.3.6) takes the form

$$\frac{d^2 V_1}{dz^2} + k^2 m_1^2 V_1 = 0 \tag{1.3.7}$$

where $m_1 = \sqrt{c^2/c_1^2 - 1}$

The solution of (1.3.7) is given as

$$v_1(x,z,t) = (A \cos km_1 z + B \sin km_1 z)e^{ik(x-ct)} \tag{1.3.8}$$

1.4 Solution of Homogeneous Isotropic Half-Space

Let rigidity and density of the medium are μ_2 and ρ_2 respectively. Then by using the above procedure we get the differential equation as

$$\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial z^2} = 1/c_2^2 \frac{\partial^2 v_2}{\partial t^2} \tag{1.4.1}$$

where $c_2 = \sqrt{\mu_2/\rho_2}$ is the shear wave velocity.

One can consider the harmonic solution of (1.4.1) along the $x-$direction as $v_2(x,z,t) = V_2(z)e^{ik(x-ct)}$, Love wave's phase velocity has been taken as c and wave number as k. Thus,(1.4.1) takes the form

$$\frac{d^2 v_2}{dz^2} + k^2 m_2^2 v_2 = 0 \tag{1.4.2}$$

Solution of (1.4.2) is given by

$$V_2 = Ce^{-km_2 z} + De^{km_2 z} \tag{1.4.3}$$

In the homogeneous half-space the displacement is given by

$$v_2 = Ce^{-km_2 z}e^{ik(x-ct)} \tag{1.4.4}$$

1.5 Boundary Conditions

The suitable boundary conditions are:

1. The surface of the geometry is free of stress at $z = -H$ i.e.,

$$\mu_1 \frac{\partial v_1}{\partial z} = 0 \tag{1.5.1}$$

2. At the interface, the stress and displacement components are continuous, so $z = 0$ i.e.

$$v_1 = v_2 \tag{1.5.2}$$

$$\mu_1 \frac{\partial v_1}{\partial z} = r_2 \frac{\partial v_2}{\partial z} \tag{1.5.3}$$

Now, applying boundary conditions, we get the following phase velocity equations with arbitrary constants

$$A \sin km_1 H + B \cos km_1 H = 0 \tag{1.5.4}$$

$$A = C \tag{1.5.5}$$

and

$$\mu_1 m_1 B = -\mu_2 m_2 C \tag{1.5.6}$$

Eliminating arbitrary constants A, B and C from (1.5.4), (1.5.5), (1.5.6)

$$\begin{vmatrix} \sin km_1H & \cos km_1H & 0 \\ 1 & 0 & -1 \\ 0 & \mu_1 m_1 & \mu_2 m_2 \end{vmatrix} = 0$$

$$\tan\left\{kH\sqrt{\frac{c^2}{c_1^2}-1}\right\} = \frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{1.5.7}$$

This is the standard dispersion relation for Love wave.

1.6 Literature Survey

Various discussions on waves and its propagation have been made by researchers. It has been brought to light that the elastic properties of the medium in which the wave is propagating, effect the propagation of elastic surface waves(Achenbach)[6]. Belfield et al.[7] established the theory on propagation of wave in fibre-reinforced medium. Abd-Alla and Ahmed [8] pointed out Love wave propagation in an initially stressed non-homogeneous orthotropic layer. He used Fourier transform method to obtain dispersion relation. The effect of heterogeneity and initial stress was shown on Love wave's phase velocity. In the beginning of 21^{st} century, many seismologists started developing their theories on propagation of waves in fibre-reinforced medium as it was the era when these began to be fabricated. Chattopadhyay and Choudhury[9] discussed about the magnetoelastic shear waves; their reflection, propagation and transmission. He considered two self-reinforced medium, one medium overlying over medium. Biot[10] gave a theory on elasticity and consolidation for a porous anisotropic solid. He explained the particular cases for transverse and complete isotropy. Chattopadhyay and Kar[11] established Love-type wave propagation in an initially stressed visco-elastic layer and half-space, the former overlying the latter, considering irregularities at the interface. Gupta and Chattopadhyay[12] pointed out propagation of Love wave in an inhomogeneous anisotropic layer with rigid boundary overlying initially stressed inhomogeneous quartz semi-infinite medium and discussed the effect of these two parameters on wave propagation. Kumar and Kumar[13] analyzed surface wave propagation in an orthotropic thermoelastic material with voids and isotropic semi-infinite medium. Abd-Alla et al. [14, 15] clarified the propagation of surface waves in different structure of the earth. Vaishnav and Kundu[16] described Love wave propagation in porous medium overlying an orthotropic half-space. The rectangular irregularity were considered at the interface of layer and semi-infinite medium. Kundu et al. [17] pointed out the effect of point source on Love wave propagation. They considered heterogeneous superficial layer overlying inhomogeneous half space. Saha et al. [18] established the theory on SH wave propagation in a layer sandwiched by two orthotropic half-spaces. Vaishnav and Kundu[19] discussed Love wave's behavior in reinforced medium between two half-space, one lying above and other lying below. The former being an orthotropic semi-infinite medium and the latter being an isotropic half-space.

Chapter 2

Love Wave Dispersion Relation in Reinforced Medium lying over Inhomogeneous Semi-Infinite Medium

2.1 Objective

This chapter deals with the Love wave propagation in a fiber-reinforced medium lying over an inhomogeneous anisotropic semi-infinite medium. The displacement in the semi-infinite medium and reinforced medium has been calculated by solving the governing equation of motion. The separation of variables method is used to solve the second order partial differential equation. The Love wave dispersion relation in reinforced medium is derived in explicit form. In particular cases, we obtained the standard dispersion relation of Love wave which validates this problem. The presence of reinforced parameters and heterogeneity parameter in the obtained dispersion relation approved the empirical impact on the phase velocity of Love wave. Results of this analytical study are significant in the field of theoretical seismology.

2.2 Mathematical Formulation of the Problem

For the Love wave propagation, a fiber-reinforced medium is considered over inhomogeneous semi-infinite medium in the presence of initial stress and reinforced parameters. The thickness of fiber-reinforced medium is taken as H. σ_{ij} are the stress components in reinforced medium and ρ_1 is the density of the medium. The rigidity of the fiber-reinforced medium is taken as μ_1. The cartesian coordinate system is considered such that, the Love wave propagation is along $x-$direction, displacement is along $y-$direction and $z-$axis is vertically taken downwards i.e. upright to Love wave propagation as shown in figure below. The inhomogeneity of half-space is taken as $N^* = N_0^*(1+\alpha z)$, $\rho^* = \rho^*(1+\alpha z)$ and $L^* = L_0^*(1+\alpha z)$, where α is a constant having dimensions that are inverse of length.

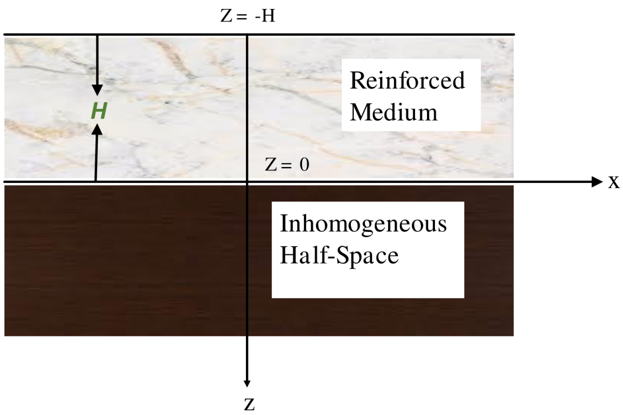

Figure 2.1: Geometry for Love Wave

2.3 Solution of Fiber-Reinforced Medium

The relation given by Belfield et al.[7] for transversely isotropic linear elastic material is

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki})$$
$$+\beta a_k a_m e_{km} a_i a_j \tag{2.3.1}$$

for all $1 \leq i,\ j,\ k,\ m \leq 3$ and e_{ij}, σ_{ij} are the strain and stress components respectively, δ_{ij} is the Kronecker delta(named after Leopold Kronecker) whose value is 1 if variables are equal and 0 otherwise. $\overrightarrow{a} = (a_1, a_2, a_3)$ = preferred direction of reinforcement and $a_1^2 + a_2^2 + a_3^2 = 1$. The elastic constants are taken as λ, α and β. Also, μ_L and μ_T are the longitudinal and transverse shear moduli respectively. The displacement is along the $y-$direction. So, $a_2 = 0$ i.e. the direction of reinforcement may be taken as $(a_1, 0, a_3)$.

The Love wave displacement components are:

$$u_1^* = 0 = w_1^*, v_1^* = v_1^*(x,z,t),\ \frac{\partial}{\partial y} \equiv 0 \tag{2.3.2}$$

All other equation of motion vanishes as $u_1^* = 0 = w_1^*$ except

$$\frac{\partial \sigma_{12}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} = \rho_1 \frac{\partial^2 v_1^*}{\partial t^2} \tag{2.3.3}$$

where ρ_1 is the density of the reinforced medium. Using stress-strain relation and Love wave condition in Eq. (2.3.3)

$$p\frac{\partial^2 v_1^*}{\partial x^2} + 2q\frac{\partial^2 v_1^*}{\partial x \partial z} + r\frac{\partial^2 v_1^*}{\partial z^2} = \rho_1 \frac{\partial^2 v_1^*}{\partial t^2} \tag{2.3.4}$$

where $q = a_1 a_3(\mu_L - \mu_T)$, $p = \mu_T + (\mu_L - \mu_T)a_1^2$, $r = \mu_T + (\mu_L - \mu_T)a_3^2$

Let us take the solution of Eq. (2.3.4) as

$$v_1^*(x,z,t) = V_1^*(z)e^{ik(x-ct)} \tag{2.3.5}$$

$V_1^*(z)$ can be obtained from the below mentioned differential equation

$$r\frac{d^2 V_1^*}{dz^2} + 2ikq\frac{dV_1^*}{dz} + k^2(\rho_1 c^2 - p)V_1^*(z) = 0 \tag{2.3.6}$$

Thus, the solution for $V_1^*(z)$ is given by

$$V_1^*(z) = B_1 e^{ikX_1 z} + B_2 e^{ikX_2 z} \tag{2.3.7}$$

where B_1, B_2 are arbitrary constants

Hence, in the fiber-reinforced layer, the displacement is given by

$$v_1^*(z) = \left(B_1 e^{ikX_1 z} + B_2 e^{ikX_2 z}\right) e^{ik(x-ct)} \tag{2.3.8}$$

where $X_1 = \frac{1}{r}\{-q+M\}$, $X_2 = -\frac{1}{r}\{q+M\}$ and $M = \sqrt{q^2 + r(\rho_1 c^2 - p)}$.

2.4 Solution of Inhomogeneous Semi-Infinite Medium

Let u be the displacement component in $x-$direction and v, w be in y, $z-$direction respectively. As Love wave is propagating along $x-$ direction and displacement is along $y-$direction, $w = 0 = u$ and $v = v_2^*(x,z,t)$. The equation of motion in the absence of body forces given by Biot[20] is

$$N^* \frac{\partial^2 v_2^*}{\partial x^2} + \frac{\partial}{\partial z}\left(L^* \frac{\partial v_2^*}{\partial z}\right) = \rho^* \frac{\partial^2 v_2^*}{\partial t^2} \tag{2.4.1}$$

Love wave is propagating along $x-$direction, we may take

$$v_2^* = V_2^*(z) e^{ik(x-ct)} \tag{2.4.2}$$

Substituting Eq. (2.4.2) into Eq. (2.4.1), we obtain

$$\frac{d^2 V_2^*}{dz^2} + \frac{1}{L^*}\frac{dL^*}{dz}\frac{dV_2^*}{dz} + \frac{k^2}{L^*}(c^2\rho^* - N^*)V_2^* = 0 \tag{2.4.3}$$

Now, put $V_2^* = \frac{V_1^*}{L^*}$ in Eq. (2.4.3)

$$\frac{d^2 V_1^*}{dz^2} - \frac{1}{2L^*}\frac{d^2 L^*}{dz^2} V_1^* + \frac{1}{4(L^*)^2}\left(\frac{dL^*}{dz}\right)^2 V_1^* + \frac{k^2}{L^*}(c^2\rho^* - N^*)V_1^* = 0 \tag{2.4.4}$$

We assume variations in the density and rigidities as

$$N^* = N_0^*(1+\alpha z)^2, \rho^* = \rho^*(1+\alpha z)^2, L^* = L_0^*(1+\alpha z)^2 \tag{2.4.5}$$

here α is any constant. Putting Eq. (2.4.5) into Eq. (2.4.4), we obtain

$$\frac{d^2 V_1^*}{dz^2} + m_1^2 V_1^* = 0 \tag{2.4.6}$$

where $m_1^2 = \frac{k^2}{L_0^*}(c^2\rho^* - N_0^*)$

The solution of Eq. (2.4.6) may be considered as

$$V_1^* = Ce^{im_1 z} + Be^{-im_1 z} \tag{2.4.7}$$

Hence, in an inhomogeneous anisotropic semi-infinite medium, the displacement is given by

$$v_2^* = V_2^*(z)e^{ik(x-ct)} = \frac{Be^{-im_1 z}}{\sqrt{L_0^*}(1+\alpha z)}e^{ik(x-ct)} \tag{2.4.8}$$

2.5 Boundary Conditions

The appropriate boundary conditions according to the mathematical formulation of our problem are

1. The layer's upper surface is free of stress; i.e.

$$r\frac{\partial v_1^*}{\partial z} + q\frac{\partial v_1^*}{\partial x} = 0 \tag{2.5.1}$$

at $z = -H$

2. When displacement in reinforced medium is same as that of displacement in half-space $v_1^* = v_2^*$; i.e.

$$(B_1 e^{ikX_1 z} + B_2 e^{ikX_2 z})e^{ik(x-ct)} = \frac{Be^{-im_1 z}}{\sqrt{L_0^*}(1+\alpha z)}e^{ik(x-ct)} \tag{2.5.2}$$

at $z = 0$ and

$$r\frac{\partial v_1^*}{\partial z} + q\frac{\partial v_1^*}{\partial x} = L_0^*\frac{\partial v_1^*}{\partial z} \tag{2.5.3}$$

at $z = 0$

2.6 Dispersion Relation

Using boundary conditions, we get the following phase velocity equations

$$B_1(rX_1 e^{-ikX_1 H} + qe^{-ikX_1 H}) + B_2(rX_2 e^{-ikX_2 H} + qe^{-ikX_2 H}) = 0 \tag{2.6.1}$$

$$B_1 + B_2 = \frac{B}{\sqrt{L_0^*}} \tag{2.6.2}$$

$$(rikX_1 + qik)B_1 + (rikX_2 + qik)B_2 = B(-im_1\sqrt{L_0^*} - \alpha\sqrt{L_0^*}) \tag{2.6.3}$$

Now, eliminating the arbitrary constants B, B_1 and B_2 from Eq. (2.6.1), Eq. (2.6.2) and Eq. (2.6.3) as:

$$\begin{vmatrix} rX_1e^{-ikX_1H} + qe^{-ikX_1H} & rX_2e^{-ikX_2H} + qe^{-ikX_2H} & 0 \\ 1 & 1 & \frac{-1}{\sqrt{L_0^*}} \\ rikX_1 + qik & rikX_2 + qik & im_1\sqrt{L_0^*} + \alpha\sqrt{L_0^*} \end{vmatrix} = 0 \tag{2.6.4}$$

Above determinant gives us the solution as

$$\sqrt{L_0^*}(im_1 + \alpha)\left\{e^{-ik(\frac{M}{r})H} + e^{ik(\frac{M}{r})H}\right\} + \frac{ikM}{\sqrt{L_0^*}}\left\{e^{ik(\frac{M}{r})H} - e^{-ik(\frac{M}{r})H}\right\} = 0 \tag{2.6.5}$$

Closed dispersion relation can be determined as

$$\tan\left[\frac{kH\sqrt{\left(a_1a_3(\frac{\mu_L}{\mu_T}-1)\right)^2 + \left(1+(\frac{\mu_L}{\mu_T}-1)a_3^2\right)\left(\frac{c^2}{c_1^2}-1+(1-\frac{\mu_L}{\mu_T})a_1^2\right)}}{1+(\frac{\mu_L}{\mu_T}-1)a_3^2}\right]$$
$$= \frac{L_0^*\left(\sqrt{\frac{N_0^*}{L_0^*}-\frac{c^2}{c_2^2}} - \frac{\alpha}{k}\right)}{\mu_T\sqrt{\left(a_1a_3(\frac{\mu_L}{\mu_T}-1)\right)^2 + \left(1+(\frac{\mu_L}{\mu_T}-1)a_3^2\right)\left(\frac{c^2}{c_1^2}-1+(1-\frac{\mu_L}{\mu_T})a_1^2\right)}} \tag{2.6.6}$$

It is a generalized Love wave dispersion relation on the alleged Earth model.

2.7 Validation of the Problem

Case-I: Setting $\mu_T \to \mu_L \to \mu_1$ hence $p \to \mu_1, r \to \mu_1$, we have acquired the dispersion relation as:

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right] = \frac{L_0^*}{\mu_1}\frac{\sqrt{\frac{N_0^*}{L_0^*}-\frac{c^2}{c_2^2}} - \frac{\alpha}{k}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{2.7.1}$$

Case-II: If $N_0^* \to L_0^* \to \mu_2$, then the previous relation takes the form:

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right] = \frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}-\frac{\alpha}{k}}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{2.7.2}$$

Case-III: For standard Love wave dispersion, taking into account inhomogeneous semi-infinite medium with a plain surface, the heterogeneity parameter $\frac{\alpha}{k} = 0$. So, above reduced to a standard wave propagation relationship such as:

$$\tan\left\{kH\sqrt{\frac{c^2}{c_1^2}-1}\right\} = \frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{2.7.3}$$

This is the standard dispersion relation of Love wave in an isotropic homogeneous layer and half-space.

2.8 Numerical Computation and Discussions

Following are the discussions and observations of how on varying the value of different factors, the wave velocity gradually increases or decreases i.e. the impact of different parameters on phase velocity. An interesting change in the behavior of phase velocity in the presence of these parameters has been noticed. Taking non-dimensional wave number kH along $x-$axis and non-dimensional phase velocity c^2/c_1^2 along $y-$axis, curves have been plotted. We have taken the numerical data from Gubbins[21].

Fig. (2.2) illustrates the effect of α/k on Love wave's phase velocity. Curve 1, curve 2, curve 3 have been plotted for α/k = 0.2, α/k = 0.4, α/k = 0.6 respectively. It can be clearly seen from the figure that as the value of α/k increases, the Love wave's phase velocity decreases; keeping the other parameters constant. Dimensionless phase velocity is maximum when α/k is minimum.

Fig. (2.3) demonstrates the effect of distinct values of reinforced parameters on Love wave's phase velocity. It is clear from the figure that for the increase in value of a_1^2 and respective decrease in value of a_3^2 the phase velocity of Love wave increases keeping the other parameters constant. The variations in phase velocity of Love wave verses wave number for distinct values of reinforced parameters i.e. (a_1^2, a_3^2) = $(0.25, 0.75)$, $(0.35, 0.65)$, $(0.45, 0.55)$, $(0.55, 0.45)$, $(0.65, 0.35)$ for curves 1, 2, 3, 4, 5 respectively. Dimensionless phase velocity is maximum for ($a_1^2 = 0.65$, $a_3^2 = 0.35$).

Fig. (2.4) is demonstrating the effect of reinforced parameters on Love wave's phase velocity. As it can be clearly seen from the figure that for the increase in value of a_3^2 and respective decrease in value of a_1^2 the phase velocity verses non-dimensional wave number decreases keeping the other parameters constant and dimensionless phase velocity is maximum for (a_1^2 = 0.75, a_3^2 = 0.25). The variations in phase velocity of Love wave verses wave number for distinct values of reinforced parameters i.e. (a_1^2, a_3^2) = $(0.75, 0.25)$, $(0.65, 0.35)$, $(0.55, 0.45)$, $(0.45, 0.55)$, $(0.35, 0.65)$ for curves 1, 2, 3, 4, 5 respectively.

Fig. (2.5) Non-dimensional phase velocity c^2/c_1^2 as function of non-dimensional wave number kH evaluated L_0^*/μ_T = $1.0, 1.2, 1.3, 1.4$. As it is clear from the figure that for the increase in value of L_0^*/μ_T, Love wave's phase velocity also increases slightly, keeping the reinforced parameters and other parameters constant. Dimensionless phase velocity and dimensionless wave number is maximum for (L_0^*/μ_T = 1.4).

Fig. (2.6) demonstrates the effect of reinforced parameters on Love wave's phase velocity. Curves have been plotted for distinct values of reinforced parameters i.e. (a_1^2, a_3^2) = $(1,0), (0,1)$ for curve 1 and curve 2 respectively. Curve 1 has been drawn in the absence of reinforced parameter a_3^2 and curve 2 has been drawn in the absence of a_1^2. Drastic change has been observed. Dimensionless phase velocity of Love wave is maximum in the absence of a_3^2 i.e. ($a_1^2 = 1$, $a_3^2 = 0$).

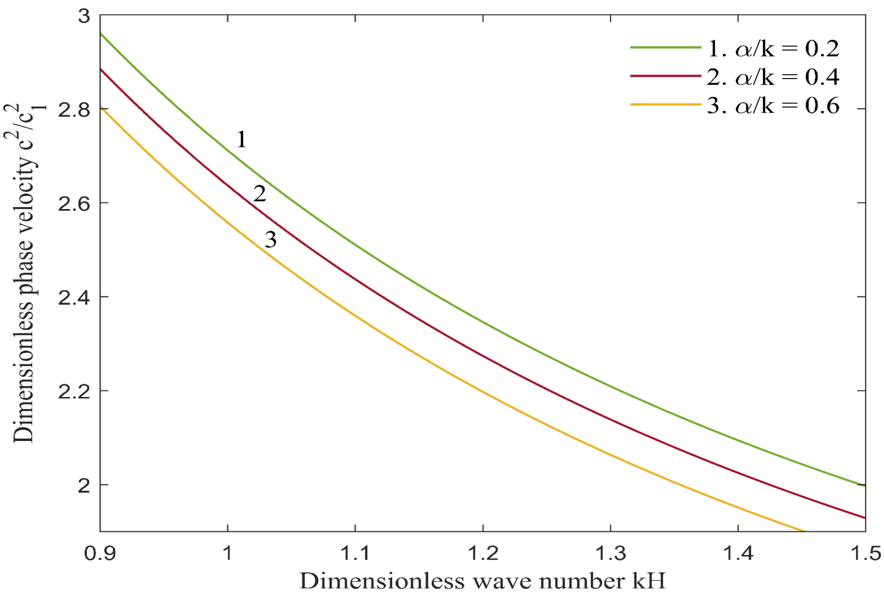

Figure 2.2: Non-dimensional wave number kH verses non-dimensional phase velocity c^2/c_1^2 with the effect of $\alpha/k = (0.2, 0.4, 0.6)$.

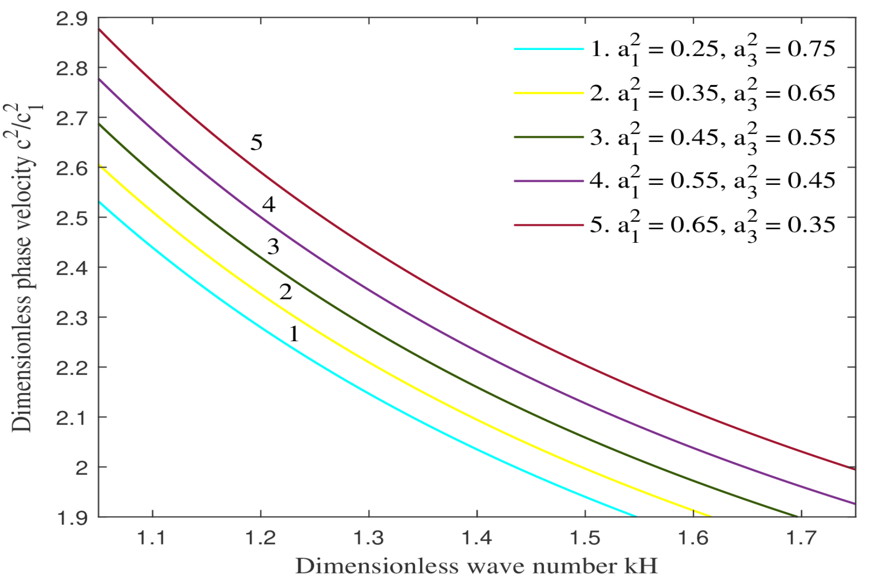

Figure 2.3: Non-dimensional wave number kH verses non-dimensional phase velocity c^2/c_1^2 with the effect of reinforced parameters (a_1^2, a_3^2) = $(0.25, 0.75)$, $(0.35, 0.65)$, $(0.45, 0.55)$, $(0.55, 0.45)$, $(0.65, 0.35)$.

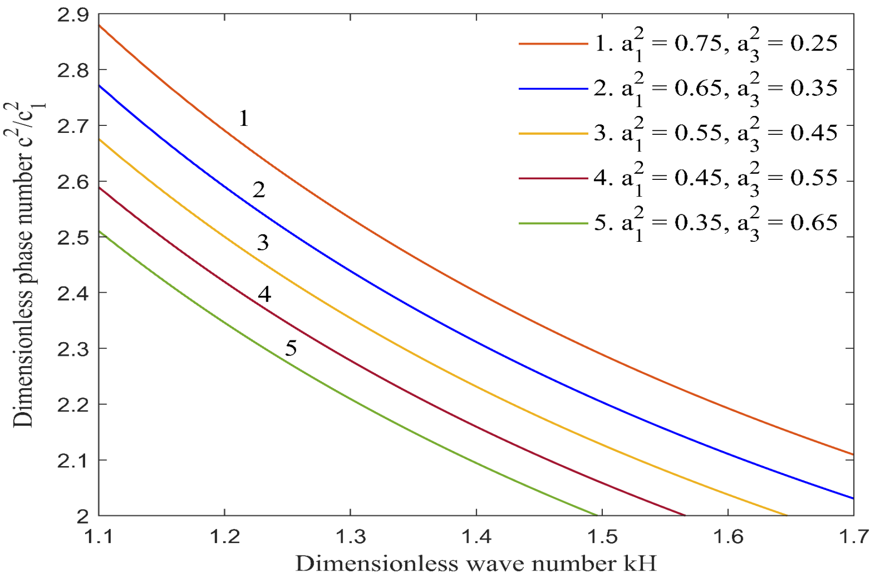

Figure 2.4: Non-dimensional wave number kH verses non-dimensional phase velocity c^2/c_1^2 with the effect of reinforced parameters (a_1^2, a_3^2) = $(0.75, 0.25)$, $(0.65, 0.35)$, $(0.55, 0.45)$, $(0.45, 0.55)$, $(0.35, 0.65)$.

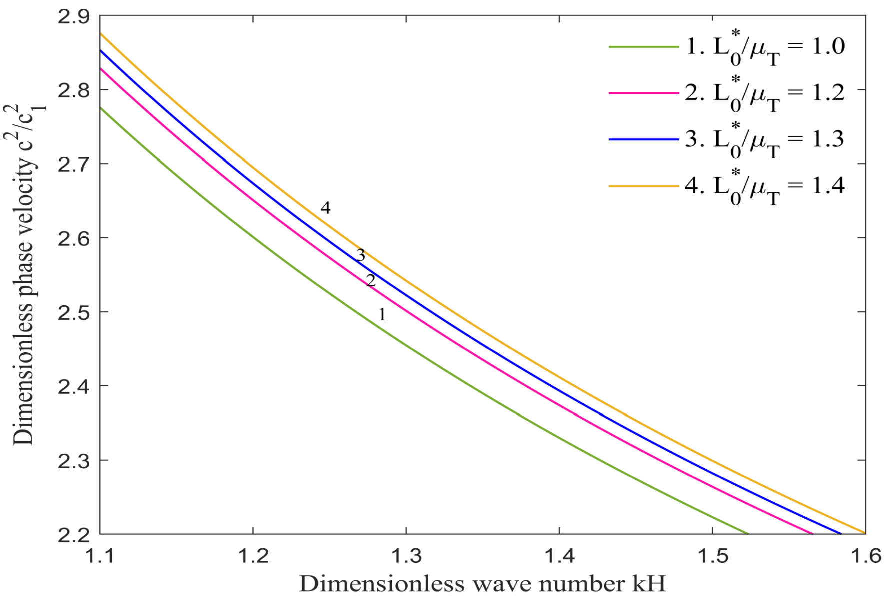

Figure 2.5: Non-dimensional wave number kH verses non-dimensional phase velocity c^2/c_1^2 with the effect of $L_0^*/\mu_T = 1.0, 1.2, 1.3, 1.4$.

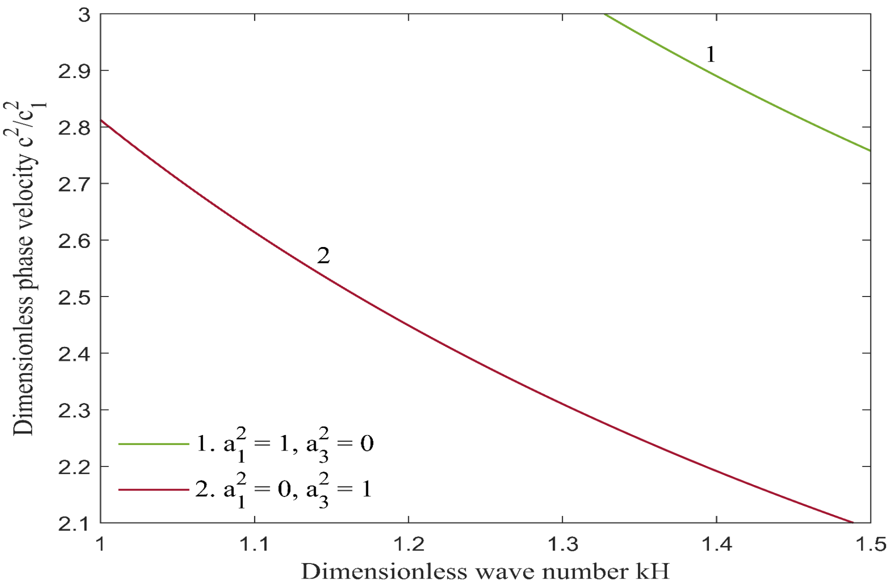

Figure 2.6: Non-dimensional wave number kH verses non-dimensional phase velocity c^2/c_1^2 with the effect of reinforced parameters $(a_1^2, a_3^2) = (1,0), (0,1)$.

2.9 Conclusion

The Love wave dispersion relation in reinforced medium in the presence of heterogeneity has been derived analytically. The standard Love wave dispersion relation has been derived in particular cases for the validation of the problem. The effect of reinforced parameters and other parameters on the dispersion curve of Love wave propagation has been noticed graphically. The effect of above mentioned parameters has been explained as follows:

1. For the increase in value of heterogeneity parameter α/k, the phase velocity decreases.

2. As the value of reinforced parameters a_1^2 increases and a_3^2 decreases, the phase velocity increases.

3. Love wave's phase velocity decreases as the value of reinforced parameters a_1^2 decreases and a_3^2 increases in an inhomogeneous half space.

4. The phase velocity increases marginally for the increase in value of L_0^*/μ_T.

5. Love wave's phase velocity decreases in random presence of reinforced parameters i.e. $a_1^2 = 1.0$ and $a_3^2 = 0.0$ and vice versa.

Chapter 3

Love Wave Dispersion Relation in Inhomogeneous Anisotropic Layer lying over Initially Stressed Orthotropic Semi-Infinite Medium

3.1 Objective

In this chapter, we use an inhomogeneous anisotropic layer of finite thickness (H) lying over initially stressed orthotropic semi-infinite medium for the propagation of Love wave. The prior concern is to show the effect of initial stress and heterogeneity on Love-type wave propagation. By solving the governing equation of motion, the displacement in inhomogeneous anisotropic medium and initially stressed orthotropic semi-infinite medium has been calculated. The classic Love wave dispersion relation is obtained in particular cases which validates our problem construction. The significant effects of distinct parameters on the wave propagation is explained in detail in the graphical section. Thus, this study may play an important role in the development of our understanding of the layers of the earth and the damages occurred during an earthquake.

3.2 Mathematical Formulation

For Love wave propagation, an inhomogeneous anisotropic medium of finite thickness H is considered over an orthotropic semi-infinite medium in the presence of initial stress. ρ is the density and N and L are the rigidities at any point in the superficial layer. $N = N_0(1+\alpha z)^2$, $\rho = \rho_0(1+\alpha z)^2$, $L = L_0(1+\alpha z)^2$, are the inhomogeneity of the superficial layer. Here α is any constant. ρ_2 is the density of semi-infinite medium. We considered the cartesian coordinate system such that the Love wave propagation is along $x-$direction, displacement of Love wave is along $y-$direction and $z-$axis is taken vertically downwards i.e. upright to the direction of wave propagation, as shown in figure below.

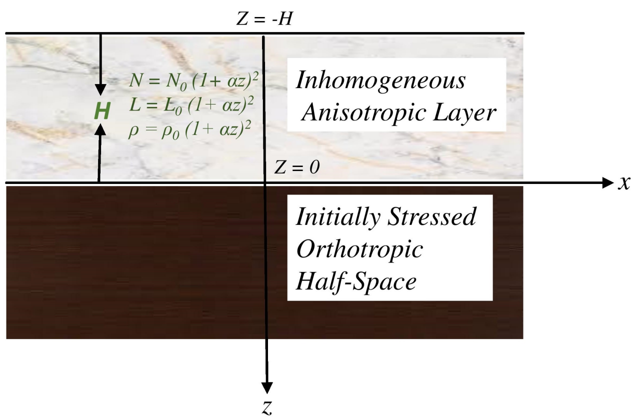

Figure 3.1: Geometry for Love Wave

3.3 Solution of Inhomogeneous Anisotropic Layer

Let u be the displacement component in $x-$direction and v, w be in y, $z-$direction respectively. The wave propagation direction is taken along $x-$ direction and displacement is along $y-$direction, $w = 0 = u$ and $v = v_1(x,z,t)$. The equation of motion in the absence of body forces given by Biot[20] is

$$N\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial}{\partial z}\left(L\frac{\partial v_1}{\partial z}\right) = \rho\frac{\partial^2 v_1}{\partial t^2} \tag{3.3.1}$$

Love wave is propagating along $x-$direction, we may take

$$v_1 = v(z)e^{ik(x-ct)} \tag{3.3.2}$$

Substituting Eq. (3.3.2) into Eq. (3.3.1), we obtain

$$\frac{d^2v}{dz^2} + \frac{1}{L}\frac{dL}{dz}\frac{dv}{dz} + \frac{k^2}{L}(c^2\rho - N)v = 0 \tag{3.3.3}$$

Putting $v = \frac{V}{L}$ in Eq. (3.3.3)

$$\frac{d^2V}{dz^2} - \frac{1}{2L}\frac{d^2L}{dz^2}V + \frac{1}{4L^2}\left(\frac{dL}{dz}\right)^2 V + \frac{k^2}{L}(c^2\rho - N)V = 0 \tag{3.3.4}$$

We assume variations in the density and rigidities as

$$N = N_0(1+\alpha z)^2, \rho = \rho_0(1+\alpha z)^2, L = L_0(1+\alpha z)^2 \tag{3.3.5}$$

here α is any constant. Putting Eq. (3.3.5) into Eq. (3.3.4), we obtain

$$\frac{d^2V}{dz^2} + s_1^2 V = 0 \tag{3.3.6}$$

where

$$s_1^2 = \frac{k^2}{L_0}(c^2\rho_0 - N_0)$$

The solution of Eq. (3.3.6) may be considered as

$$V = Be^{is_1 z} + Ce^{-is_1 z} \tag{3.3.7}$$

Hence, in an inhomogeneous anisotropic layer, the displacement is obtained as

$$v_1 = v(z)e^{ik(x-ct)} = \frac{Be^{is_1 z} + Ce^{-is_1 z}}{\sqrt{L}_0(1+\alpha z)}e^{ik(x-ct)} \tag{3.3.8}$$

3.4 Solution of Orthotropic Semi-Infinite Medium

For orthotropic medium, the equations of motion in the absence of body forces given by Gupta[22] are

$$\left.\begin{aligned}
&\frac{\partial\sigma_{11}}{\partial x} + \frac{\partial\sigma_{12}}{\partial y} + \frac{\partial\sigma_{13}}{\partial z} - P_2\left(\frac{\partial w_z}{\partial y} - \frac{\partial w_y}{\partial z}\right) = \rho_2\frac{\partial^2 u_1}{\partial t^2}\\
&\frac{\partial\sigma_{21}}{\partial x} + \frac{\partial\sigma_{22}}{\partial y} + \frac{\partial\sigma_{23}}{\partial z} - P_2\frac{\partial w_z}{\partial x} = \rho_2\frac{\partial^2 v_2}{\partial t^2}\\
&\frac{\partial\sigma_{31}}{\partial x} + \frac{\partial\sigma_{32}}{\partial y} + \frac{\partial\sigma_{33}}{\partial z} - P_2\frac{\partial w_y}{\partial x} = \rho_2\frac{\partial^2 w_3}{\partial t^2}
\end{aligned}\right\} \tag{3.4.1}$$

where u_1 is the displacement component in the $x-$diection and v_2 and w_3 be in y and $z-$ direction respectively in the orthotropic medium. w_y, w_x, w_z are the rotational components along $y, x, z-$direction respectively. Here, σ_{ij} and ρ_2 are stress and the density of the material in the orthotropic half space.

In an semi-infinite medium, the stress-strain relations are

$$\left.\begin{aligned}
\sigma_{11} &= B_{11}e_{11} + B_{12}e_{22} + B_{13}e_{33} \\
\sigma_{12} &= 2G_3e_{12} \\
\sigma_{22} &= B_{21}e_{11} + B_{22}e_{22} + B_{23}e_{33} \\
\sigma_{23} &= 2G_1e_{23} \\
\sigma_{33} &= B_{31}e_{11} + B_{32}e_{22} + B_{33}e_{33} \\
\sigma_{31} &= 2G_2e_{31}
\end{aligned}\right\} \tag{3.4.2}$$

where the incremental normal elastic coefficient are B_{ij} and G_i are shear moduli. e_{ij} be strain components.

All other equation of motion vanishes for semi-infinite medium as $u_1 = 0 = w_3$ except

$$\frac{\partial}{\partial x}\left(G_3\frac{\partial v_2}{\partial x}\right) + \frac{\partial}{\partial z}\left(G_1\frac{\partial v_2}{\partial z}\right) - P_2\frac{\partial}{\partial x}\left(\frac{1}{2}\frac{\partial v_2}{\partial x}\right) = \rho_2\frac{\partial^2 v_2}{\partial t^2} \tag{3.4.3}$$

$$\left(G_3 - \frac{P_2}{2}\right)\frac{\partial^2 v_2}{\partial x^2} + G_1\frac{\partial^2 v_2}{\partial z^2} = \rho_2\frac{\partial^2 v_2}{\partial t^2} \tag{3.4.4}$$

and the stress components $\sigma_{12} = 2G_3e_{12}$, $\sigma_{23} = 2G_1e_{23}$, all components other than these will be zero.

As propagation of Love wave is along $x-$ direction, let us take the solution as

$$v_2 = V(z)e^{ik(x-ct)} \tag{3.4.5}$$

where wave number is denoted by k and phase velocity by c, then Eq. (3.4.3) takes the form

$$\frac{d^2V}{dz^2} - s_2^2 V = 0 \tag{3.4.6}$$

where $s_2^2 = (k^2/G_1)\left[(G_3 - P_2/2) - c^2\rho_2\right]$

Hence, in the initially stressed orthotropic half-space, the displacement is obtained as

$$v_2 = Ae^{-s_2 z}e^{ik(x-ct)} \tag{3.4.7}$$

3.5 Boundary Conditions

The appropriate boundary conditions according to the mathematical formulation of our problem are

1. The upper surface of inhomogeneous anisotropic layer is free of stress; i.e.

$$L\frac{\partial v_1}{\partial z} = 0 \tag{3.5.1}$$

at $z = -H$

2. When displacement in inhomogeneous anisotropic layer is same as that of displacement in orthotropic semi-infinite medium, $v_1 = v_2$, i.e.

$$\frac{Be^{is_1 z} + Ce^{-is_1 z}}{\sqrt{L_0}(1+\alpha z)} e^{ik(x-ct)} = Ae^{-s_2 z} e^{ik(x-ct)} \tag{3.5.2}$$

at $z = 0$ and

$$L\frac{\partial v_1}{\partial z} = Q_1 \frac{\partial v_2}{\partial z} \tag{3.5.3}$$

at $z = 0$

3.6 Dispersion Relation

Using boundary conditions, we get the following phase velocity equations with constants as

$$Be^{-is_1 H}(is_1(1-\alpha H) - \alpha) - Ce^{is_1 H}(is_1(1-\alpha H) + \alpha) = 0 \tag{3.6.1}$$

$$B + C = A\sqrt{L_0} \tag{3.6.2}$$

$$B\left[\frac{L}{\sqrt{L_0}}(is_1 - \alpha)\right] - C\left[\frac{L}{\sqrt{L_0}}(is_1 + \alpha)\right] = -AQ_1 s_2 \tag{3.6.3}$$

Now eliminating the arbitrary constants A, B and C from Eq. (3.6.1), Eq. (3.6.2) and Eq. (3.6.3) as:

$$\begin{vmatrix} e^{-is_1H}[is_1(1-\alpha H)-\alpha] & -e^{is_1H}[is_1(1-\alpha H)+\alpha] & 0 \\ 1 & 1 & -\sqrt{L_0} \\ \sqrt{L_0}(is_1-\alpha) & -\sqrt{L_0}(is_1+\alpha) & Q_1s_2 \end{vmatrix} = 0$$

i.e.

$$\begin{aligned} & e^{-is_1H}(-\alpha+is_1(1-\alpha H))((Q_1s_2-\alpha L_0)-is_1L_0) \\ & +e^{is_1H}(\alpha+is_1(1-\alpha H))((Q_1s_2-\alpha L_0)+is_1L_0)=0 \end{aligned} \tag{3.6.4}$$

which reduces to

$$P_1\left\{e^{is_1H}-e^{-is_1H}\right\}+iP_2\left\{e^{is_1H}+e^{-is_1H}\right\}=0 \tag{3.6.5}$$

where, $P_1 = \alpha(Q_1s_2-\alpha L_0)-s_1^2(1-\alpha H)L_0$ and

$P_2 = \alpha s_1L_0+s_1(1-\alpha H)(Q_1s_2-\alpha L_0)$

Dispersion relation can be determined as

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-\frac{N_0}{L_0}}\right]$$

$$=\frac{\frac{\alpha}{k}\sqrt{\frac{c^2}{c_1^2}-\frac{N_0}{L_0}}+(1-\frac{\alpha}{k}(kH))\sqrt{\frac{c^2}{c_1^2}-\frac{N_0}{L_0}}\left(\frac{Q_1}{L_0}\sqrt{\frac{Q_3}{Q_1}-\frac{P_2}{2Q_1}-\frac{c^2}{c_2^2}}-\frac{\alpha}{k}\right)}{(\frac{c^2}{c_1^2}-\frac{N_0}{L_0})(1-\frac{\alpha}{k}(kH))-\frac{\alpha}{k}\left(\frac{Q_1}{L_0}\sqrt{\frac{Q_3}{Q_1}-\frac{P_2}{2Q_1}-\frac{c^2}{c_2^2}}-\frac{\alpha}{k}\right)} \tag{3.6.6}$$

It is a generalized Love wave dispersion relation.

3.7 Validation of the Problem

Case-I: Setting $L_0 \to N_0 \to \mu_1$, $Q_1 \to Q_3 \to \mu_2$, we have acquired the dispersion relation as:

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right]=\frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{P_2}{2\mu_2}-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{3.7.1}$$

Case-II: Now taking $P_2/2\mu_2 = 0$, then the previous relation takes the following form:

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right] = \frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{3.7.2}$$

This is the standard dispersion relation of Love wave in an isotropic homogeneous layer and half-space.

3.8 Numerical Computation and Results

To study the effect of heterogeneity α/k and initial stress $P_2/2\mu_2$ on Love wave's phase velocity, numerical computations have been done for distinct values of these parameters. All the figures below show the variation of non-dimensional wave number kH against non-dimensional phase velocity c^2/c_1^2. We have taken the numerical data from Gubbins[21].

Fig. (3.2) depicts the effect of α/k on Love wave's phase velocity. Curves have been plotted for distinct values of α/k i.e. for curve 1 the value of $\alpha/k = 0.10$, similarly for curve 2, 3, 4 the value of $\alpha/k = 0.15, 0.20, 0.25$ respectively. It is clear from the figure that as the value of α/k increases the Love wave's phase velocity also increases taking the other parameters constant. Dimensionless phase velocity is maximum when $\alpha/k = 0.25$ i.e. maximum.

Fig. (3.3) demonstrates the effect of initial stress on Love wave's phase velocity. Curves 1, 2, 3 have been plotted for distinct values of $P_2/2\mu_2$. Figure shows that for the increase in value of $P_2/2\mu_2$, Love wave's phase velocity moderately increases keeping the other parameters constant. The variations in phase velocity of Love wave verses wave number for distinct values of initial stress i.e. $P_2/2\mu_2 = 0.1, 0.3, 0.5$ for curves 1, 2, 3 respectively. Dimensionless phase velocity is maximum for $P_2/2\mu_2 = 0.5$.

Fig. (3.4) shows the effect of initial stress $P_2/2\mu_2$ in the absence of heterogeneous parameter on Love wave's phase velocity. Keeping $\alpha/k = 0$ and values of $P_2/2\mu_2$ as 0.1, 0.3, 0.5, Love wave's phase velocity increases as it was increasing in the presence of α/k shown in fig. (3.3). Phase velocity is maximum for $(\alpha/k, P_2/2\mu_2)$ = $(0, 0.5)$. Three curves have been plotted taking values $(\alpha/k, P_2/2\mu_2)$ = $(0, 0.1)$, $(0, 0.3)$, $(0, 0.5)$ for curves 1, 2, 3 respectively.

Fig. (3.5) Non-dimensional phase velocity c^2/c_1^2 as function of non-dimensional wave number kH evaluated $(\alpha/k, P_2/2\mu_2)$ = $(0.10, 0)$, $(0.15, 0)$, $(0.20, 0)$, $(0.25, 0)$ i.e. the effect of α/k in the absence of initial stress. Phase velocity is maximum for $(\alpha/k, P_2/2\mu_2)$ = $(0.25, 0)$.

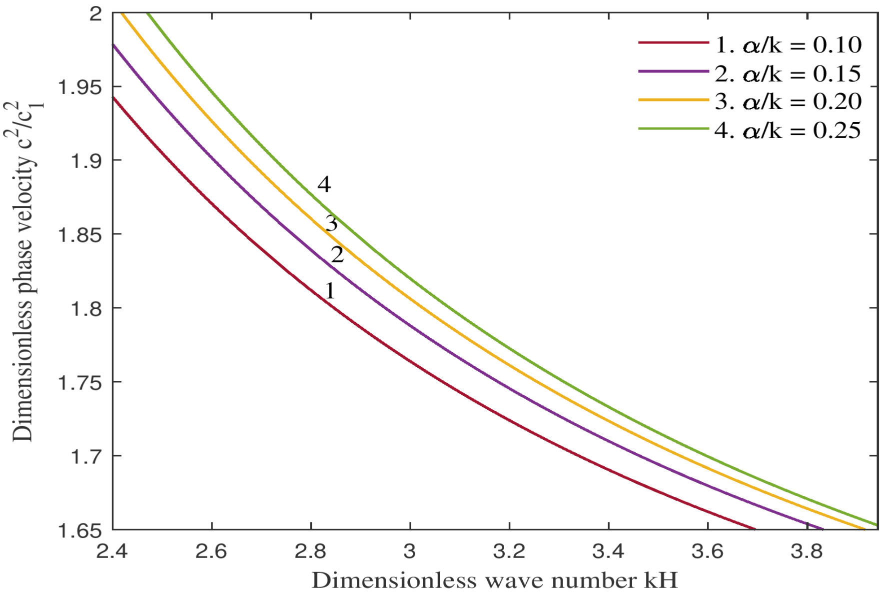

Figure 3.2: Dimensionless wave number kH verses dimensionless phase velocity c^2/c_1^2 with the effect of heterogeneity parameter $\alpha/k = (0.10, 0.15, 0.20, 0.25)$.

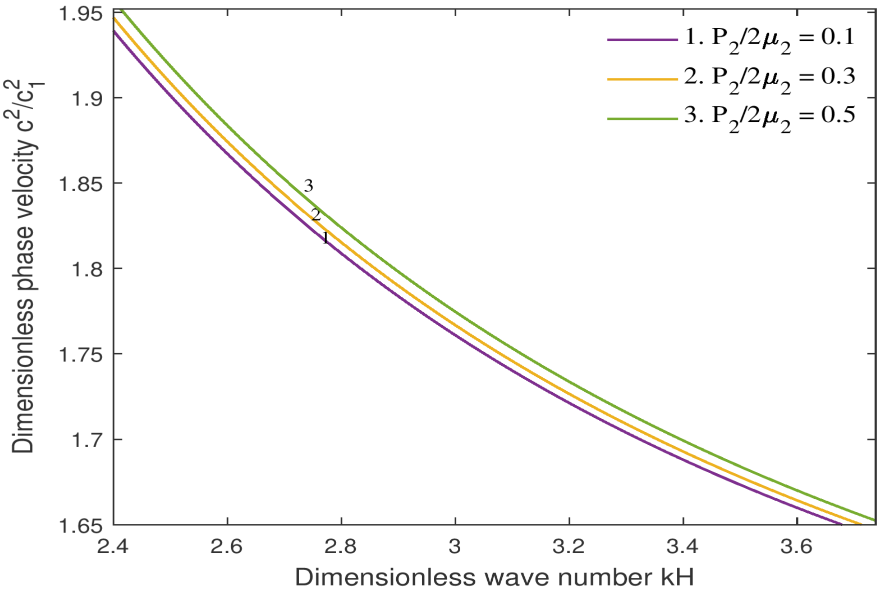

Figure 3.3: Dimensionless wave number kH verses dimensionless phase velocity c^2/c_1^2 with the effect of initial stress $P_2/2\mu_2 = (0.1, 0.3, 0.5)$.

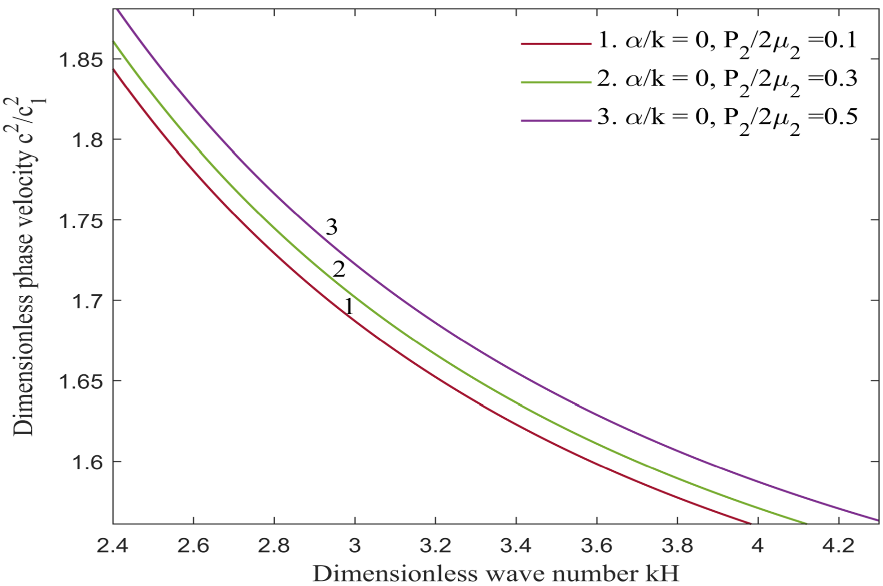

Figure 3.4: Dimensionless wave number kH verses dimensionless phase velocity c^2/c_1^2 for different values of $P_2/2\mu_2 = (0.1, 0.3, 0.5)$ when $\alpha/k = 0$.

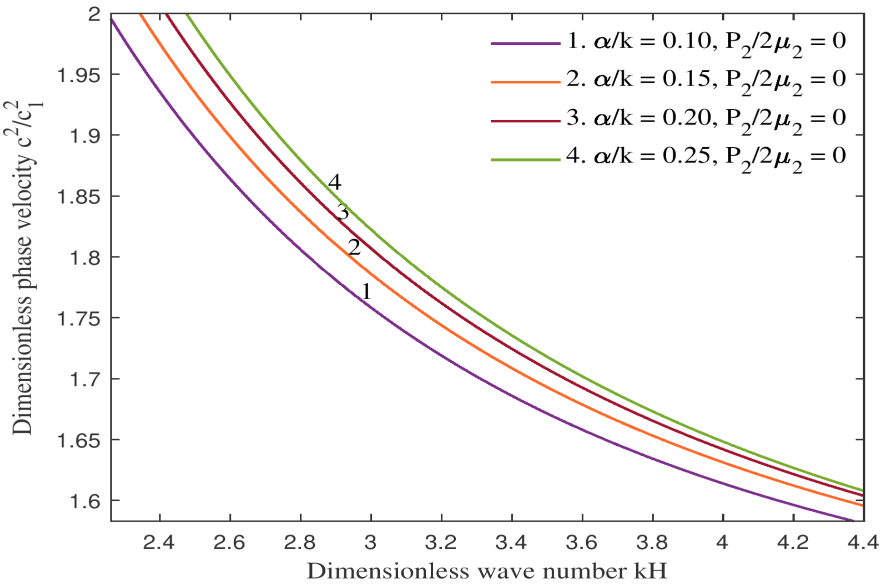

Figure 3.5: Dimensionless wave number kH verses dimensionless phase velocity c^2/c_1^2 for different values of $\alpha/k = (0.10, 0.15, 0.20, 0.25)$ when $P_2/2\mu_2 = 0$.

3.9 Conclusion

The dispersion relation of Love wave in an inhomogeneous anisotropic layer over initially stressed orthotropic semi-infinite medium has been obtained. The classic Love wave dispersion relation has been obtained in particular cases which validates our problem construction. The significant effect of distinct parameters on the wave propagation has been explained in detail in the graphical section. The effect of above mentioned parameters has been explained as follows:

1. As the value of heterogeneity parameter α/k increases, Love wave's phase velocity also increases.

2. As the value of initial stress $P_2/2\mu_2$ increases, the phase velocity increases moderately.

3. The phase velocity of Love wave increases as the value of initial stress $P_2/2\mu_2$ increases in the absence of heterogeneity parameter α/k.

4. The phase velocity increases for the increase in value of heterogeneity parameter α/k in the absence of initial stress $P_2/2\mu_2$.

The obtained results are very crucial in various branches of geology, civil engineering etc. Seismologists can use this information to predict the location of earthquakes and their mechanism.

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