BER Performance of Optimum Combining with BPSK Modulation in Multipath Fading Environments

Thesis report submitted towards the partial fulfillment of Requirements for the award of the degree of

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DECLARATION

I hereby declare that the thesis report entitled "**BER Performance of Optimum Combining with BPSK Modulation in Multipath Fading Environments**" is an authentic record of my own work carried out as requirements for the award of degree of M.E. (Electronics and Communication) at Thapar Institute of Engineering & Technology (Deemed University), Patiala, under the guidance of Mr. Rajesh khanna (Guide), Assistant Professor and Mr. Jagpal Singh Ubhi (C0-Guide), Assistant Professor during January to June, 2006.

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It is certified that the above statement made by the student is correct to the best of my knowledge and belief.

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Abstract

Wireless communications is, by any measure, the fastest growing segment of the communications industry. As such, it has captured the attention of the media and the imagination of the public. But, there are many technical challenges that must be overcome in order to make this vision a reality. A signal transmitted on a wireless channel is subject to Fading, Shadowing, Interference, Delay spread, Doppler spread, and Propagation path loss. There is always a greater demand for capacity, reliability and also need to integrate voice, data and other type of traffic over radio channels. There are many ways to solve all these problems, such as Space-time coding, Spread spectrum techniques, Antenna arrays.

Diversity techniques are used to mitigate all these above effects. Optimum combining is one of them which is used in space diversity reception is studied in digital cellular mobile radio communication systems with Rayleigh fading, Nakagami fading, & with Ricean fading with multiple cochannel interferers binary phase-shift keying (BPSK) modulation is considered in Rayleigh-fading, Nakagami fading, & with Ricean fading environment when the number of interferences L is no less than the number of antenna elements M ($L \geq M$). Exact mathematical framework is also developed to analyze the BER performance of an optimum combining diversity in correlated Nakagami-m channels, Rayleigh fading & with Ricean fading. Simulations are done to find out the BER performance of optimum combining diversity in correlated Nakagami-m channels, Rayleigh fading & with Ricean fading. Simulations are also done to compare BER performance for Multipath fading Environments. It is observed in thesis that for SNR 5dB the combiner in Nakagami fading channel requires 0.1953 BER less as compared to Rayleigh faded channel and 0.0717 BER as compared to Ricean faded channel. Also this thesis includes BER performance for Nakagami Fading with multiple cochannel interferes which is not done previously in any paper.

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List of Abbreviations

Pdf	Probability Density Function
Cdf	Cumulative Density Function
BER	Bit Error Rate
SNR	Signal to Noise Ratio
SC	Selection Combining
EGC	Equal Gain Combining
MRC	Maximal Ratio Combining
OC	Optimum Combining
LOS	Line of Sight
CCI	Cochannel Interference
BPSK	Binary Phase-Shift Keying
DOF	Degree of Freedom

Chapter-1

Introduction

1.1 Overview

The wireless era began around 1897 when Guglielmo Marconi first established a radio link to provide continuous contact with ships sailing the English Channel. Since then, mobile systems have developed and spread considerably. The use of wireless systems increased rapidly in the 1950's and 60's, when the number of users largely exceeded the small number of channels available. This trend showed a clear need for larger capacity as well as better roaming flexibility. Bell Labs addressed these issues in 1970's by proposing a new conceptual idea called the *cellular concept* – the concept of breaking a coverage zone into small cells, each of which reuses portions of the spectrum to increase the user capacity [1]. From then on, *wireless cellular communications* has been one of the fastest growing fields of technology in the world.

The first-generation (1G) cellular and cordless telephone networks, which were based on analog technology with FM modulation, have been successfully deployed throughout the world since the early and mid-1980s. But the remarkable growth of the market was such that a second-generation (2G) of wireless systems was quickly needed. Due to the well known advantages of digital transmission, especially in the use of digital modulation, speech coding, and spectrally efficient multiple access schemes such as frequency-division multiple-access (FDMA), time-division multiple-access (TDMA), and code-division multiple-access (CDMA), the 2G digital cellular radio showed a significant improvement in both system capacity and quality of service over the 1G systems. As the economy and technologies keep growing, the demand for a third-generation (3G) of wireless communication systems becomes increasingly urgent. These systems are evolving from the mature 2G networks, with the aim of providing universal access, global roaming, and multimedia high-speed high-quality wireless communications.

Wireless communications is, by any measure, the fastest growing segment of the communications industry. As such, it has captured the attention of the media and the imagination of the public. In addition, wireless local area networks currently supplement or replace wired networks in many homes, businesses, and campuses. Many new applications, including wireless sensor networks, automated highways and factories, smart homes and appliances, and remote telemedicine, are emerging from research ideas to concrete systems. The explosive growth of wireless systems coupled with the proliferation of laptop and palmtop computers indicate a bright future for wireless networks, both as stand-alone systems and as part of the larger networking infrastructure. However, many technical challenges remain in designing robust wireless networks that deliver the performance necessary to support emerging applications.

There are many technical challenges that must be overcome in order to make this vision a reality. These challenges transcend all levels of the overall system design including hard- ware, communication link, network and application design. One of the toughest challenges faced by wireless engineers and system designers is the bottleneck presented by the wireless link layer. Achieving high data rates on the wireless channel is a hard problem for several reasons. The wireless channel is a harsh time-varying propagation environment. A signal transmitted on a wireless channel is subject to:

- Fading
- Shadowing
- Interference
- Delay spread
- Doppler spread
- Propagation path loss

While it is possible to increase data rates by increasing the transmission bandwidth or using higher transmit power, both spectrum and transmit power are very constrained in a wireless system. The rapid proliferation of internet and the strong growth of mobile telephony promise a key role for “broadband” and “wireless” in future telecommunications. So there is always a greater demand for **capacity**, **reliability** and also need to **integrate voice**, **data** and other type of traffic over radio channels. The demands for higher bit rates combined with the ever-increasing number of users, however, introduces the need for clever and efficient usage of limited resources of the wireless channel. The bandwidth, or spectrum, is prohibitively expensive. Increasing transmit power adds interference to other systems and also reduces the battery life-time of mobile transmitters. Conventional antennas had to face several problems in wireless communications; a few of them includes

- Poor BER due to the range, which causes path loss.
- Poor BER due to uniformity of coverage, which causes fading.
- Poor BER due to Frequency reuse which causes co-channel interference.
- Need more capacity (reuse would affect the BER), which causes co-channel interference.

There are many ways to solve all these problems, such as

1. Space time coding
2. Spread spectrum techniques
3. Antenna arrays

Space-time coding is an effective transmit diversity technique to combat fading in wireless communications. Space-time codes are a highly bandwidth-efficient approach to signaling within wireless communication that takes advantage of the spatial dimension by transmitting a number of data streams using multiple co-located antennas. There are various approaches to the coding structures, including space-time trellis coded modulation, space-time turbo codes and also layered architectures. The central issue in all these various coding structures is the exploitation of multipath effects in order to achieve very high spectral efficiencies. Hence, space-time coding enables an increase in capacity by an order of magnitude [2].

Spread Spectrum communications is also used to increase capacity and bandwidth, which is distinguished by three key elements:

- The signal occupies a bandwidth much greater than that, which is necessary to send the information.
- The bandwidth is spread by means of a code, which is independent of the data
- The receiver synchronizes to the code to recover the data.

It was originally developed by the military as a method of communications that is less sensitive to intentional interference or jamming by third parties, but has become very popular in the realm of personal communications recently.

Antenna arrays has been suggested in recent years for mobile communications systems to overcome the problem of limited channel bandwidth, thereby satisfying an ever growing demand for a large number of mobiles on communications channels. It has been shown by many studies that when an array is appropriately used in a mobile communications system, it helps in improving the system performance by increasing channel capacity and spectrum efficiency, extending range coverage, tailoring beam shape, steering multiple beams to track many mobiles, and compensating aperture distortion electronically. It also reduces multipath fading, cochannel interferences, system complexity and cost, BER, and outage probability. It has been argued that adaptive antennas and the algorithms to control them are vital to a high-capacity communications system development [3].

Antenna arrays can improve reliability and capacity in two ways.

1. Adaptive beam forming
2. Diversity combining

Adaptive beam forming:

An array of antennas with the capability to form independent beams may be used at the base station .The array is used to find the location of each mobile, and then beams are formed to cover different mobiles or groups of mobiles. As the mobiles move, the different beams cover different clusters of mobiles, offering the benefit of transmitting the energy toward the mobiles. The arrangement is particularly useful in situations where the mobiles move in clusters or along confined paths, such as highways. It is envisaged in [70] that each mobile may be covered by a separate beam. Each beam then would follow the mobile, reducing the handoff problem.

Diversity combining

In most scattering environments, antenna diversity is a practical, effective and, hence, a widely applied technique for reducing the effect of multipath-fading. The number of antennas at the Transmitter or the Receiver will decide the type of the system that will finally be implemented. Space-time processing will either be "Receive Diversity" or "Transmit Diversity". In Transmit Diversity no of antennas are used at transmitter. In Receive Diversity the Channel can be estimated and there can be multiple antennas at the receiver.

The basic aim in receive diversity is to improve the resultant signal to noise ratio (SNR) at the receiver by intelligent combining linearly, SNR's received over the diversity channels. It is a powerful communication technique that provides wireless link improvement at relatively low cost. Unlike equalization, diversity requires no training overhead since a training sequence is not required at the transmitter. Furthermore, there are wide ranges of diversity implementation, many of which are very practical and provide significant link improvement with little added cost. The most common diversity combining schemes are Maximal ratios combining (MRC), equal gain combining (EGC), Selection combining (SC), Switched diversity (SWC) and general selection combining (which is basically hybrid SC/MRC) [1]. Depending on the propagation mechanisms, diversity may be categorized as spaced diversity, frequency diversity, angle diversity, polarization diversity, time diversity and multipath diversity. More recently, coded modulation has been regarded as a way of introducing time diversity

In particular, with optimum combining (OC), the received signals are weighted and combined to maximize the output signal-to-interference-plus-noise ratio (SINR). In the presence of interference, this technique provides substantial improvement in performance over maximal ratio combining where the received signals are combined to maximize the desired signal-to-noise ratio (SNR) only.

The major problem with using the receive diversity approach is the cost, size, and power of the remote units. The use of multiple antennas and radio frequency (RF) chains (or selection and switching circuits) makes the remote units larger and more expensive. As a result, diversity techniques have almost exclusively been applied to base stations to improve their reception quality.

Antenna Diversity at the handset in indoor environment results in improved radio link. It was incorporated to use adaptive antennas at handsets for improved performance and the effectiveness of a smart antenna system can be clearly examined by applying diversity and adaptive combining schemes at the handsets. Applying antenna diversity at the handsets can reduce capacity problems and co-channel interference.

A brief introduction to these diversity combining techniques is given in the proceeding chapters in this thesis. In this dissertation a diversity combining scheme is studied which combats the effects of multipath fading and interference and is referred to as Optimum Combining. The computer simulations is used to approximate channel characteristics so that a general idea can be obtained about the system performance (BER, SINR) at the input to the receiver.

1.2 Objectives of the thesis

- To calculate and simulate BER with Multiple cochannel interferers for Rayleigh fading channel with optimum combining with number of interferences is greater than number of antenna elements at receiver with BPSK modulation scheme.
- To calculate and simulate BER with Multiple cochannel interferers for Nakagami fading channel with optimum combining with number of antenna elements at receiver with BPSK modulation scheme.
- To calculate and simulate BER with Multiple cochannel interferers for Ricean fading channel with optimum combining with number of antenna elements at receiver with BPSK modulation scheme.
- To compare BER Performance with Multiple cochannel interference for Multiple Fading Environments with optimum combining with number of antenna elements at receiver with BPSK modulation scheme.

1.2 Thesis Outline

The Chapter 2 describes various types of diversity and various diversity combining techniques.

The Chapter 3 describes the performance of optimum diversity combining; calculate formulas for pdf, cdf and for BER.

In chapter 4, the performance of optimum diversity combining is described over Rayleigh and Ricean fading channels with multiple cochannel interference environments. Simulations are carried out with MATLAB to observe the performance of optimum combiner in solving the problem of co-channel interference and Multipath Fading.

In chapter 5, the performance of optimum diversity combining is described over Nakagami fading channels multiple cochannel interference environments. Simulations are carried out with MATLAB to observe the performance of optimum combiner in solving the problem of co-channel interference and Multipath Fading. Rayleigh and Ricean fading Performance has also been derived and simulated from Nakagami Fading model.

Finally in chapter 6 the conclusion part of the thesis is outlined which is a result of the iterative work is carried out on various Fading channel models.

Chapter-2

Spatial Diversity

2.1 Introduction

Next generation wireless systems are being designed to provide ubiquitous broadband link access to information infrastructure. Spatial Diversity techniques play a vital role in supporting such high-speed connections over radio channels by mitigating the detrimental effects of multi-user interference and multipath fading. It was incorporated verify improved performance in both the statistical and spatial channel models. Spatial Diversity is known to reduce channel fading and increase the reliability of the transmitted signal. Spatial Diversity is of two types, “Transmitter Diversity” and “Receiver Diversity”. The number of antennas at the Transmitter or the Receiver will decide the type of the system that will finally be implemented. In Transmit Diversity numbers of antennas are used at transmitter to obtain uncorrelated fading signals at receiver [4]. Total transmitted power is split among the antennas. In Receive Diversity the Channel can be estimated and there can be multiple antennas at the receiver to obtain independent fading.

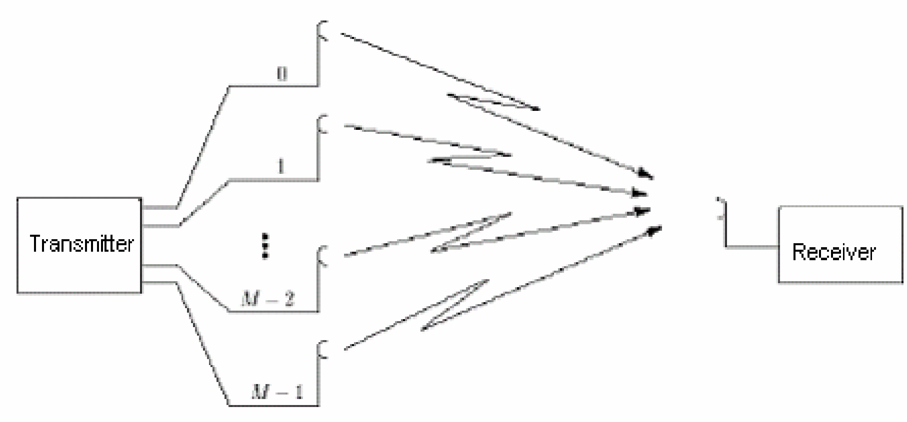

Figure- 2.1: Transmitter Diversity

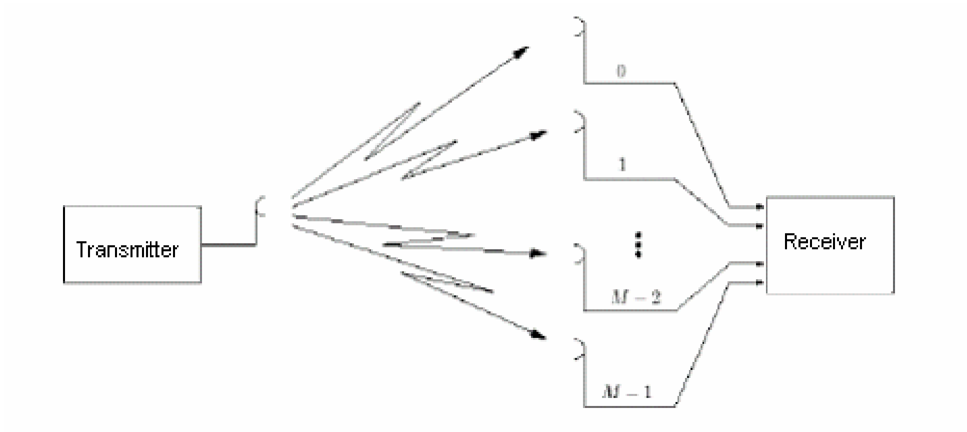

Figure- 2.2: Receiver diversity

Transmitter and receiver diversity are shown in Fig 2.1and Fig (2.2) respectively. Work is done with receive Diversity. It is assumed that the signals received at the receiver are uncorrelated. Several diversity schemes were available to choose from including space, polarization, angle, frequency, and time. The independent channels in a fading channel environment are often referred to as diversity branches..

2.2 Diversity Techniques

Diversity is a powerful communication receiver technique that provides wireless link improvement at relatively low cost. Unlike equalization, diversity requires no training overhead since the transmitter does not require a training sequence. Furthermore, there are wide ranges of diversity implementations, many of which are very practical and provide significant link improvement with little added cost.

The diversity concept can be explained simply. If one radio path undergoes a deep fade, another independent path may have a strong signal. By having more than one path to select from, both the instantaneous and average SNRs at the receiver may be improved.

2.3 Types of Diversity

In this section, we examine the type of diversity that can be used to provide the inputs to the diversity combiner. Most diversity systems are implemented in the receiver instead of the transmitter since no extra transmitter power is needed to implement the receiver diversity system. There are two general types of diversity schemes [1].

2.3.1 Macroscopic diversity scheme

The Macroscopic diversity scheme is used for combining two or more long-term lognormal signals, which are obtained via independently fading paths received from two or more different antennas at different base-station sites. The local mean strength varies because of variations of terrain between the mobile transmitter and the base station receiver. If only one antenna site is used, the traveling mobile unit may not be able to transmit a signal to the base station at certain geographical locations because of terrain variations such as hills or mountains. Therefore, two separated antenna sites can be used to receive two signals and to combine them to reduce long-term fading.

2.3.2 Microscopic diversity scheme

The Microscopic diversity scheme is used for combining two or more short-term Rayleigh signals, which are obtained via independently fading paths received from two or more different antennas but only at one receiving co site. Once the diversity branches are created, any of the combining methods can be used.

2.3.2.1 Time Diversity

Time diversity reception techniques are primarily applicable to the transmission of digital data over a fading channel. In time diversity, the same data are sent over the channel at time intervals of the order of the reciprocal of the baseband fade rate $f_b = 2f_m$. In mobile radio, the reciprocal fade rate can be expressed:

$$\tau \geq \frac{1}{2f_m} = \frac{1}{2\left(v/\lambda\right)} \tag{2.1}$$

The time separation increases as the fade rate decreases. Multiple diversity channels can be provided by successively transmitting the signal sample in each time slot. For M-branch diversity, the sampling rate must be (M x 8 kHz), since the transmission delay spread is usually less than 20μs, which is much less than the inverse of the sampling rate:

$$f_s < \frac{1}{\Delta} \tag{2.2}$$

Hence, the sampling rate f_s is not limited by the time-delay spread. However, the minimum time separation between samples shown in fig 2.3 for diversity application may cause a serious problem, since f_m is a Doppler frequency, expressed:

$$f_m = \frac{V}{\lambda} \tag{2.3}$$

When the vehicle is stationary, v=0, and thus f_m=0. This means that the time separation μ_s is infinite. Therefore, the advantages of time diversity are lost when the vehicle is not moving. This is in sharp contrast to other diversity schemes, in which the branch separation is not a function of vehicle speed and thus the two diversity signals are independent over any value of v.

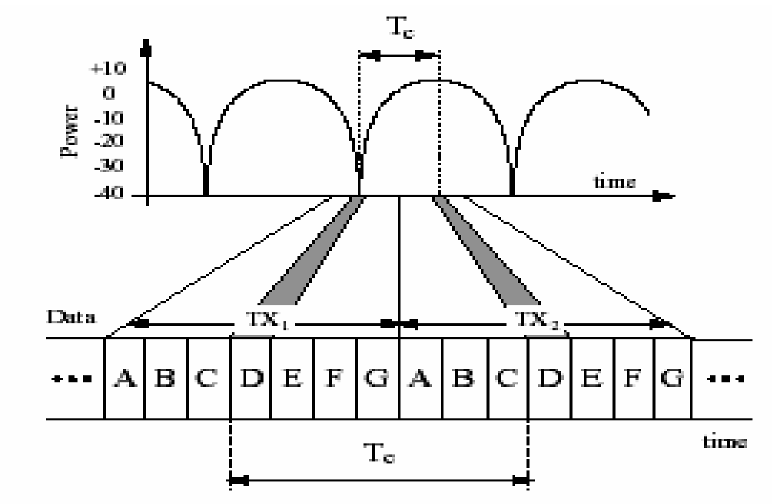

Figure- 2.3: Time Diversity

2.3.2.2 Frequency Diversity

Frequency diversity is implemented by transmitting information on more than one carrier frequency. The rationale behind this technique is that frequencies separated by more than the coherence bandwidth of the channel will be uncorrelated and will thus not experience the same fades. Theoretically, if the channels are uncorrelated, the probability of simultaneous fading will be product of the individual fading probabilities.

Frequency diversity is often employed in microwave line-of-sight links, which carry several channels in a frequency division multiplex mode (FDM). In practice, 1: N protection switching is provided by a radio licensee, wherein one frequency is nominally idle but is available on a stand-by basis to provide frequency diversity switching for any one of the N other carriers (frequencies) being used on the same link, each carrying independent traffic.

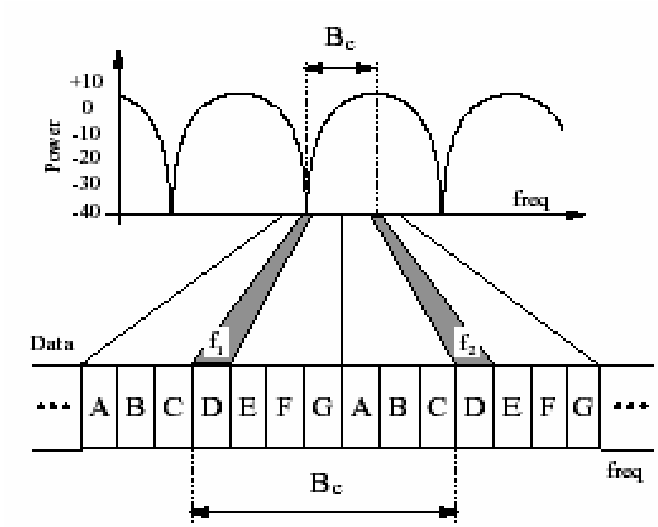

Figure- 2.4: Frequency Diversity

2.3.2.3 Polarization Diversity

Signals transmitted in either horizontal or vertical electric fields are uncorrelated at both the mobile and Base station receivers. The horizontal and vertical polarization components, E_x and E_y, transmitted by two polarized antennas at the base station and received by two polarized antennas at the mobile unit, can provide two uncorrelated fading signals. The decorrelation for the signals in each polarization is caused by multiple reflections in the channel between the mobile and base station. After sufficient random reflections, the polarization state of the signal will be independent of the transmitted polarization. In practice, however, there is some dependence of the received of the received polarization on the transmitted polarization.

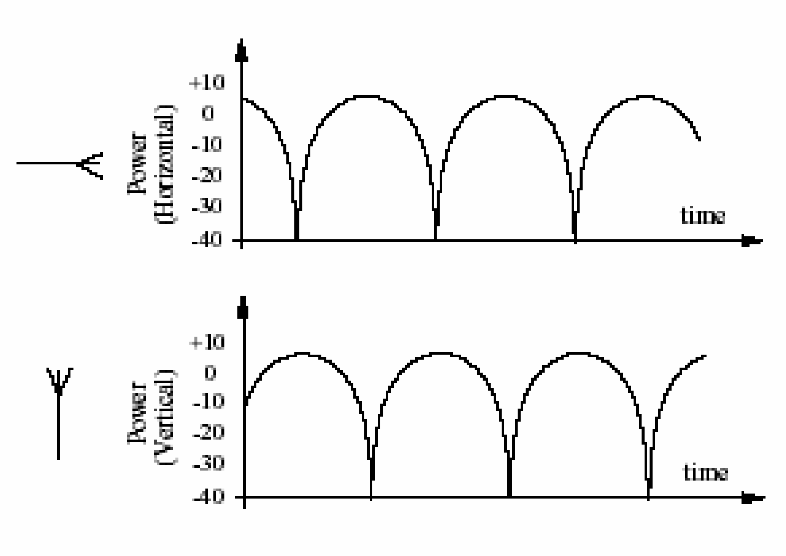

Figure- 2.5: Polarization Diversity

2.3.2.4 Angle Diversity

When the operating frequency is greater than 10 GHz, the scattering of a signal from transmitter to receiver generates received signals from different directions that are uncorrelated with each other. Thus, two or more directional antennas can be pointed in different directions at the receiving site and provide signals for a combiner.

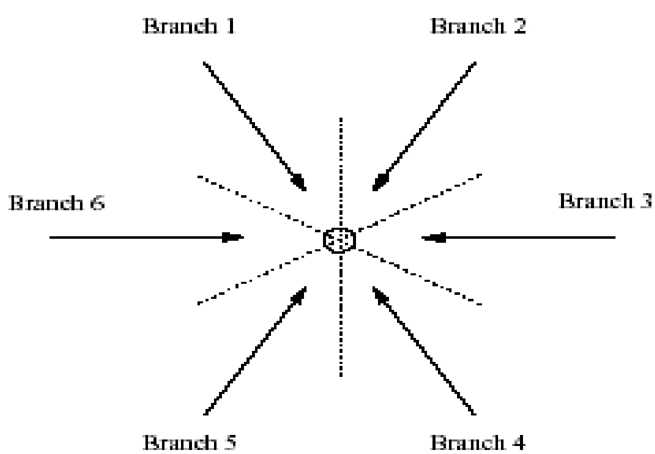

Figure- 2.6: Angle Diversity

2.3.2.5 Space Diversity

Space Diversity, also known as antenna diversity, is one of the most popular forms of diversity used in wireless systems. Conventional wireless systems consist of an elevated base station antenna and a mobile antenna close to the ground. The existence of a direct path between the transmitter and the receiver is not guaranteed and the possibility of a number of scatterers in the vicinity of the mobile suggests a Rayleigh fading signal. Typically a separation of a few wavelengths is enough to obtain uncorrelated signals.

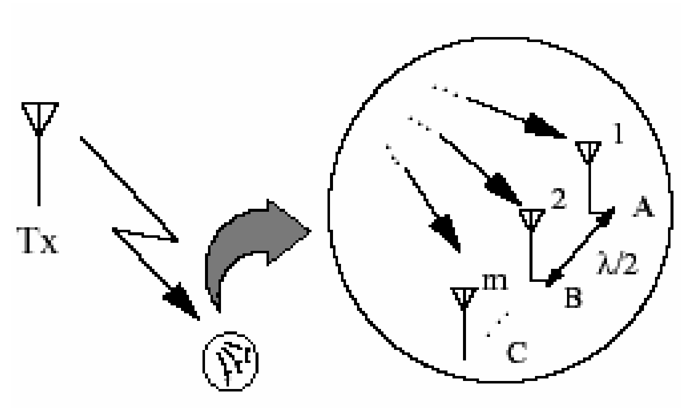

Figure- 2.7: Space Diversity

2.4 Basic Diversity Combining Methods

Diversity combining [5] is used to overcome the problem of fading in radio channels and utilizes the fact that the signals arriving at different locations fade at different rates. A system employing a diversity combiner uses signals induced on various antennas placed a few wavelengths apart at different locations and combines these signals in one of many ways. In general, increasing the number of antennas results in more reduction in channel fading. However, in practice, 10–20 antennas seem to provide satisfactory results [6]. Information on diversity receiver design may be found in.

Diversity combining is useful for combating fading and is efficient in the absence of a common interference on all the antennas. In the presence of cochannel interferences, however, the array processing methods are able to provide better performance by canceling interferences.

The collection of independently fading signal branches can then be combined in a variety of ways to improve the received SNR. Since the chance of having two deep fades from two uncorrelated signals at any instant is rare, combining them can reduce the effect of the fades. The three most prevalent space diversity-combining techniques are SC, EGC, MRC and OC. MRC co-phases the signal branches, weights them according to their respective SNR's, and then takes their sum. MRC is the most complex combining technique, but also yields the highest SNR. The analysis of all of these diversity techniques is presented here.

2.4.1 Selection Combining

Selection combining is the simplest of all the diversity schemes. It is based on the probability that the received signals are greater than a threshold. An ideal selection combiner chooses the signal with the highest instantaneous SNR of all the branches, so the output SNR is equal to that of the best incoming signal and makes it available to the receiver at all times. Multiple branches will improve the probability of having a larger SNR at the receiver [1]. A block diagram of SC is given in figure 2.8

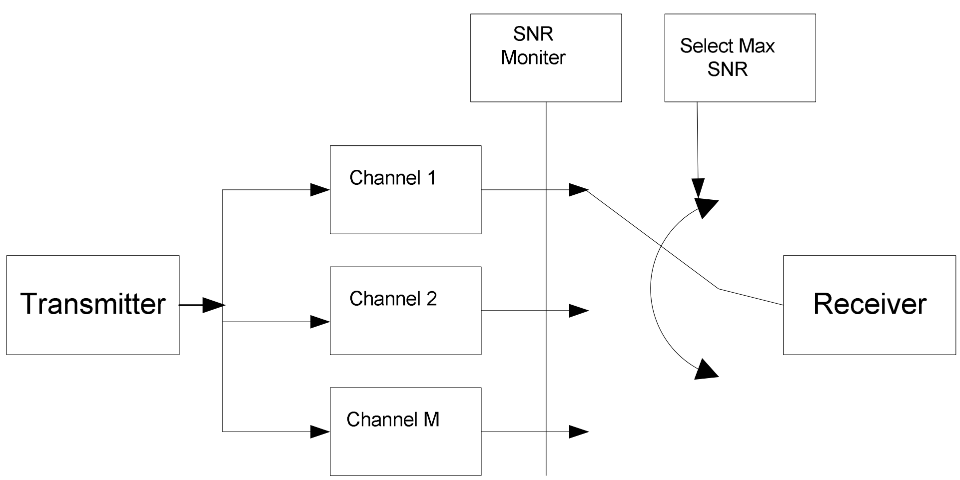

Figure- 2.8: Selection Combining

We assume that the signal received by each diversity branch is statistically independent of the signals in other branches and is Rayleigh distributed with equal mean signal power P_0. The probability density function of the signal envelope, on branch *i*, is given by

$$p(r_i) = \frac{r_i}{P_0} e^{-r_i^2 / 2P_0} \tag{2.4}$$

where $2P_0$ = mean-square signal power per branch = $< r_i >$ and

r_i^2 = Instantaneous power in the i-th branch.

Let $\xi_i = r_i^2 / 2P_0$ and $\xi_0 = (2P_0) / (2N_i)$, where N_i is the noise power in the i-th branch.

$$\therefore \frac{\xi_i}{\xi_0} = r_i^2 / 2P_0 \tag{2.5}$$

The probability density function for ξ_i is given by

$$p(\xi_i) = \frac{1}{\xi_0} e^{\left(-\xi_i / \xi_0\right)} \tag{2.6}$$

We assume that the signal in each branch has a constant mean; thus, the probability that the SNR on any one branch is less than or equal to any given value ξ_g is given y

$$P[\xi_i \le \xi_g] = \int_0^{\xi_g} p(\xi_i) d\xi_i = 1 - e^{\left(-\xi_g / \xi_0\right)} \tag{2.7}$$

Therefore, the probability that the SNRs in all branches are simultaneously less than or equal to ξ_g is given by

$$P_M(\xi_g) = P[\xi_1, \xi_2, \ldots, \xi_M \le \xi_g] = \left[1 - e^{\left(-\xi_g / \xi_0\right)}\right]^M \tag{2.8}$$

2.4.2 Maximal Ratio Combining

MRC presents the receiver with a signal-to- noise ratio that is the direct sum of all individual SNRs in the branches as shown in figure 2.9. The main drawback of using MRC is that the signal level and noise power at each branch needs to be correctly estimated for all instances in time [7].

The M signals are weighted proportional to their signal voltage-to-noise power ratios and then summed.

$$r_M = \sum_{i=1}^{M} a_i r_i(t) \tag{2.9}$$

Since noise in each branch is weighted according to noise power,

$$\overline{n_i^2(t)} = \sum_{j=1}^{M} \sum_{i=1}^{M} a_i a_j \overline{n_i(t) n_j(t)} \tag{2.10}$$

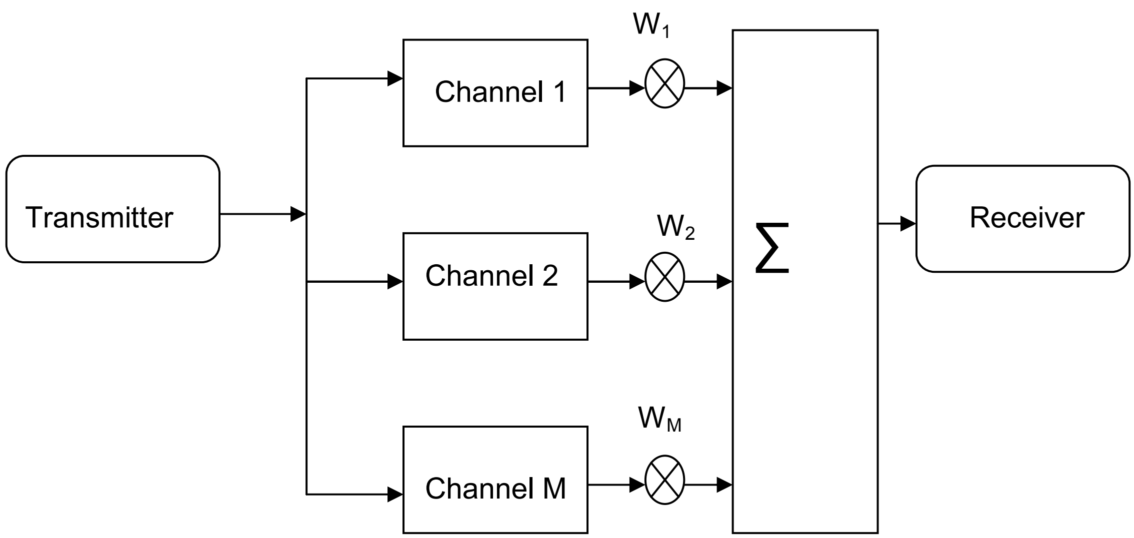

Figure- 2.9: Maximal Ratio Combining

The average noise power,

$$N_T = \sum_{i=1}^{M} a_i^2 \overline{n_i^2(t)} = 2\sum_{i=1}^{M} |a_i|^2 N_i \tag{2.11}$$

Where $\overline{n_i^2(t)} = 2N_i$

The probability that $\xi_M \le \xi_g$ is given by:

$$P(\xi_M \le \xi_g) = 1 - e^{\frac{-\xi_g}{\xi_0}} \sum_{K=1}^{M} \frac{\left(\frac{-\xi_g}{\xi_0}\right)^{K-1}}{(K-1)!} \tag{2.12}$$

$$P(\xi_M > \xi_g) = e^{\frac{-\xi_g}{\xi_0}} \sum_{K=1}^{M} \frac{\left(\frac{\xi_g}{\xi_0}\right)^{K-1}}{(K-1)!} \tag{2.13}$$

2.4.3 Equal Gain Combining

EGC diversity receiver is of practical interest because of its reduced complexity relative to optimum maximal ratio combining scheme while achieving near-optimal performance [1]. It is the sum of all the signals received in order to increase the available SNR at the receiver. The gain of all of the branches is set to a particular value that does not change which is in contrast to MRC. The block diagram of EGC is shown in figure 2.10

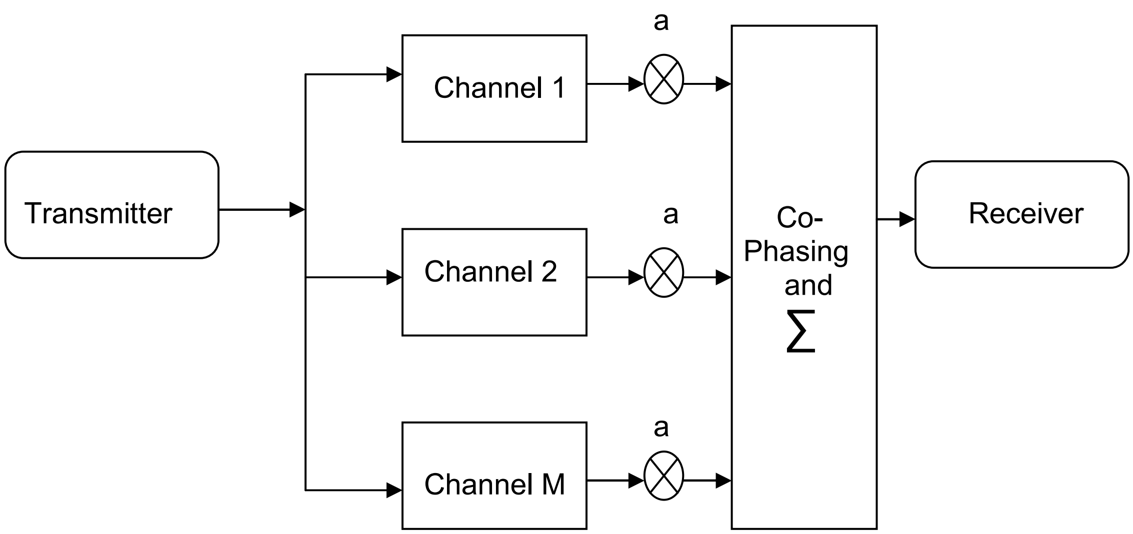

Figure- 2.10: Equal Gain Combining

EGC is similar to MRC, but there is no attempt to weight the signal before addition; thus $a_i = 1$. The envelope of the output signal with all $a_i = 1$ is given by

$$r = \sum_{i=1}^{M} r_i \tag{2.14}$$

and the mean output is given as:

$$\overline{\xi_M} = \frac{1}{2}\frac{\overline{\left[\sum_{i=1}^{M} r_i\right]^2}}{\sum_{i=1}^{M}\overline{N_i}} \tag{2.15}$$

For M=2, the probability P can be written in closed form as:

$$P\left(\xi_M \le \xi_g\right) = 1 - e^{-\left(\frac{2\xi_g}{\xi_0}\right)} - \sqrt{\pi\left(\frac{\xi_g}{\xi_0}\right)}e^{-\frac{\xi_g}{\xi_0}}.erf\sqrt{\frac{\xi_g}{\xi_0}} \tag{2.16}$$

For M > 2, the probability can be obtained by numerical integration techniques.

Hence, the basic idea of diversity reception is that, if two or more independent samples of a signal are taken, then these samples will fade in an uncorrelated manner. This means that the probability of all the samples being simultaneously below a given level is much less than the probability of any individual sample being below that level. The probability of M samples all being simultaneously below a certain level is p^M, where p is the probability that a single sample is below the level. Thus, it can be seen that a signal composed of a suitable combination of various samples will have much less severe fading properties than any individual sample alone.

These three diversity-combining techniques are effective to combat the effects of noise and multipath fading. But as the mobile communication system is more interference affected rather than fading or noise. So some way should be found to minimize the effect of co channel interference. In this dissertation a diversity combining method called as **Optimum combining** is described which can minimize the effects of fading as well as interference. Optimum combining is described in chapter 3. In this dissertation performance of optimum combining is evaluated over flat Rayleigh, Nakagami and Rician fading channels.

Summary of the chapter

This chapter describes the various diversity and diversity combining techniques. Diversity combining discussed in this chapter is useful in mitigating the effects of multipath fading. In proceeding chapter a combining technique called as Optimum combining is described which is useful to mitigate the effects of co-channel interference and multipath fading both. The concept of co-channel interference is of utmost importance because the mobile communication systems are interference limited rather than noise limited.

Chapter-3

Optimum Combining

3.1. Introduction

Antennas in general may be classified as omnidirectional, directional, phased array, adaptive, and optimal. An omnidirectional antenna has equal gain in all directions and is also known as an isotropic antenna. Directional antennas, on the other hand, have more gain in certain directions and less in others. The direction, in which the gains of these antennas are maximum, referred to as the boresight direction of the antenna. The gain of directional antennas in the boresight is more than that of the omnidirectional antennas and is measured with respect to the gain of the omnidirectional antennas. For example, a gain of 10 dBi (sometimes indicated by dBic or simply dB) means the power radiated by this antenna is 10 dB more than that radiated by an isotropic one. It should be noted that the same antenna may be used as a transmitting antenna or as a receiving antenna. The gain of an antenna remains the same in both cases. The gain of a receiving antenna indicates the amount of power it delivers to the receiver compared to an omnidirectional antenna.

A phased array antenna uses an array of simple antennas, such as omnidirectional antennas, and combines the signal induced on these antennas to form the array output. Each antenna forming the array is known as an element of the array. The direction where the maximum gain would appear is controlled by adjusting the phase between different antennas. The phases of signals induced on various elements are adjusted such that the signals due to a source in the direction where maximum gain is required are added in phase. It results, that the gain of the array (or equivalently, the gain of the combined antenna) is equal to the sum of the gains of all individual antennas.

The term adaptive antenna is used for the phased array when the gain and the phase of the signals induced on various elements are changed before combining to adjust the gain of the array in a dynamic fashion, as required by the system. In a way, the array adapts to the situation, and the adaption process is normally under the control of the system. A block diagram of a typical adaptive antenna array system is shown in Fig. 3.1.

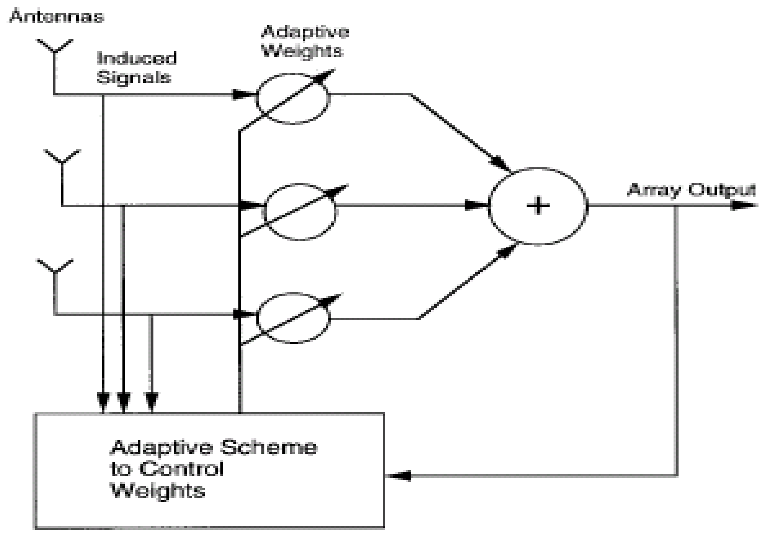

Figure- 3.1: Block diagram of a narrow-band adaptive antenna system.

An optimal antenna is one in which the gain and phase of each antenna element is adjusted to achieve the optimal performance of the array in some sense. For example, to obtain maximum output SNR by canceling unwanted interferences and receiving the desired signal without distortion may be one way of adjusting gains and phases of each element. This arrangement where the gain and phase of each antenna element is adjusted to obtain maximum output SNR (sometimes also referred to as signal-to-interference-and noise ratio, SINR) is also referred to as optimal combining in the mobile communications literature [8], [9], [10].

For high-speed data transmission over mobile radio channels, the optimum way of processing the diversity signals aims at minimizing the dispersion effect caused by the fading's frequency selectivity, instead of maximizing the received level. The optimum space diversity schemes under our consideration are selection diversity and combining diversity. As seen in Fig. 3.2, the optimum selection combining diversity structure consists of an optimum receiver on each diversity path. Each optimum receiver is a cascade of a matched filter and an infinite-length tapped-delay transversal filter. The basic algorithm for selection diversity is based on the principle of selecting the best signal with the least minimum-mean-square error among all the branches. The optimum combining diversity reception revealed by the theory is composed of a set of matched filters whose outputs are first summed and sampled. These samples are then passed through an infinite-length tapped-delay transversal filter.

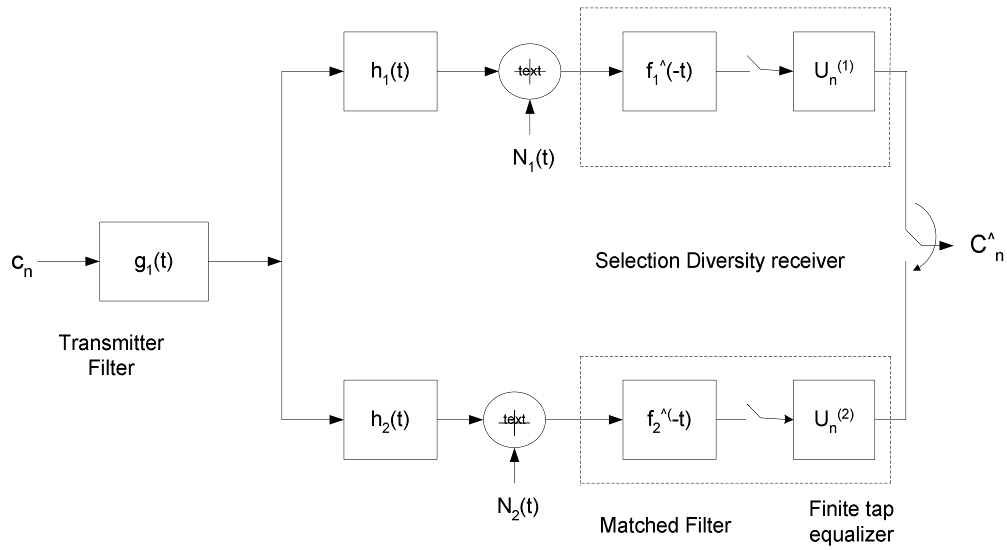

Figure-3.2: Optimum selection diversity and finite-length equalization

Winters [8] presents a good discussion of optimum combining at the base station and also provides simulation results. For the returns offered by optimum combining in the mobile communications case, the reader is referred to "*Winters' article*" .The best improvements (over other combining methods) occur when the interference levels are high relative to the wanted signal, and when the number of diversity antenna elements is large.

The application is in high-density cellular systems where the co-channel interference is high. A theoretical worst case occurs in a corner of a hexagonal cell where the average interference power is less than 5 dB below average signal power [9]. Local shadow effects such as caused by large buildings can make this figure worse still, and Winters [8] suggests that a signal-to-interference-plus-noise ratio (SINR) of + 5 dB to -5 dB is typical.

A disadvantage of optimum combining at the mobile rather than at the base station (using adaptive retransmission) is that the cost and complexity of implementation at every mobile is much greater over-all than that of implementing only at each base station. An advantage of many-branch systems at the mobile, however, is that the antennas can be realized in a compact manner. At the base station, even a six-element array becomes very expansive.

In the following, the mobile diversity antenna is treated as an array antenna. The terminology is also array antenna-oriented. For example, the multipath environment of mobile communications is referred to as a multiple source, or distributed source scenario since the distribution of sources effectively approaches a continuum in urban environments. The scenario is referred to as *stationary*, which is here taken to mean that its statistics are unchanging with time and the position of the mobile. In some sections, the scenario is referred to as *static*, indicating that the source positions are constant relative to the mobile. A real-world mobile scenario is rarely stationary or static, but the latter simplification does not seem too drastic for the short periods over which each optimum array solution is sought.

The distributed source scenario around the mobile in an urban environment must be considered unchanging for each adaptation cycle of the combining algorithm. With this in mind, this section considers constant (in location) sources only. In particular, signal correlation measurement intervals or a sequence of them-to establish the covariance matrix with sufficient accuracy during convergence-are taken over a static scenario. In practice, this will not be the case (unless the mobile and its surroundings are still): the adaptation algorithm will be chasing a changing solution, and its performance will be correspondingly degraded. If the adaptation is fast compared with the rate of change of the sought solution, the degradation will not be too much.

The array antenna has N branches with M incident signals from M sources in the field of view of the array. Here, a source is defined as a single point source; any incident signal which is derived (e.g., diffracted, reflected, or delayed) from the point source, is considered as a separate (point) source. In the mobile communications case, N i s limited to less than, say, six. M is effectively unlimited since the sources are considered distributed. Many of the M sources bear the same signal because of the multipath. If there are P different signals (one wanted signal and (P -1) interferers), then each of the P signals can be considered randomly allocated to many of the M sources.

With the time factor e^{jwt} understood, the complex envelope of the signal conveyed by the nth branch from the mth source is

$$x_{nm}(t) = a_{nm} m_m(t) e^{j\phi_{nm}} \tag{3.1}$$

Where $m_m(t)$ is the signal modulation of the mth source and ϕ_{nm} is the carrier phase of the mth source, in the nth branch. The total signal in the nth branch is thus

$$x_n(t) = \sum_{m=1}^{M} x_{nm}(t) \tag{3.2}$$

a_{nm} is real and represents the signal amplitude (or rather its mean over the modulation) in the branch from a particular source. The static scenario is characterized by the a and ϕ being independent of time. Some physical insight is offered for the mobile communication case if a and ϕ are thought of as functions of distance moved by the mobile. After the antenna has (hopefully) adapted to a set of a and ϕ, the mobile has moved, and adaptation to a new set of a and ϕ is required [9].

3.1.1 Array Branch Signals

Define the total (sum over all sources) branch signals in the usual way

$$X^T(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_N(t)] \tag{3.3}$$

and a vector of the RF (or IF) phase terms weighted by the amplitude of the source signal in the branch

$$T_m^{\ T} = [a_{1m} e^{j\phi_{1m}} a_{2m} e^{j\phi_{2m}} \cdots a_{nm} e^{j\phi_{Nm}}] \tag{3.4}$$

T_m are the vector contributions of the *m*th source to the array. When only the phases are included (a_{nm} = 1, all n) $T_m = S_m$ is the *source vector* to the mth source, a term from conventional array technology.

Now let all the sources be included in the total weighted source vector

$$Q = [T_1 \quad T_2 \cdots \quad T_M] \tag{3.5}$$

Note that a column of Q corresponds to a given source signal in all branches and a row corresponds to all the source signals in a given branch. Finally, define a vector containing the source modulations

$$A^T = [m_1(t) m_2(t) \cdots m_M(t)] \tag{3.6}$$

so that

$$X(t) = QA(t) \tag{3.7}$$

is the vector formulation of (3.1).

In the mobile communications case, let there be P different (uncorrelated) signals, with each signal conveyed by subsets of the M sources. The subsets contain $M_1, M_2, ... M_P$ sources. Now,

$$\sum_{i=1}^{P} M_i = M \tag{3.8}$$

and (3.2) can be written as

$$x_n(t) = \sum_{i=1}^{P} \sum_{m=1}^{M_i} a_{nm} m_{m_i}(t) e^{j\phi_{nm}} \tag{3.9}$$

$$\stackrel{\Delta}{=} \sum_{i=1}^{P} a_{nM_i} m_{M_i}(t) e^{j\phi_{nM_i}} \tag{3.10}$$

Where the summation is over $\boldsymbol{i}$ for all M_i. In (3.9), the inner summation is over each source bearing the *i*th signal, and the outer summation is over the difference source subsets. i.e., over the different signals. The subscript M_i serves as a reminder that the (subscripted) signal quantity is derived from several sources rather than an individual source in the scenario. Equation (3.10) shows that each branch can be considered to support P signal-bearing vectors, each of which correspond to the vector sum of the signals received from the appropriate sources. This is an important maxim as it allows branch signals to be combined such that certain signals are maximized and/or others minimized. The latter effect is the extension of maximum ratio combining to optimum combining. Equation (3.10) is used in the following in a vector formulation of the covariance matrix for the mobile communications case.

3.1.2 The Covariance Matrix

The output signal is usually written

$$y(t) = \mathbf{w}^T X(t) \tag{3.11}$$

Where *w* is the column vector of weights. The output power is the Hermitian form

$$\overline{|y(t)|^2} = \mathbf{W}^H R \mathbf{W} \tag{3.12}$$

In which the covariance matrix is defined as

$$R = \overline{X^*(t) X^T(t)} \tag{3.13}$$

$$= Q^* \overline{A^* A^T} Q^T \tag{3.14}$$

The over bar means time averaging in the presence of the static scenario.

It is evident that *R* must contain all cross products of both the weighted space vectors and modulations [9]. Some physical insight can be obtained by examining where these cross products occur in ***R*.** Denote a source modulation correlation

$$\rho_{mq} = \overline{m^*{}_m(t) m_q(t)} \tag{3.15}$$

in which the normalization is understood. Obviously, $\rho_{mq} = \rho^*_q$ and $\rho_{mm} = 1$. Now the inner term of the covariance matrix of (3.14) is

$$A^* A^t = \begin{bmatrix} 1 & \rho_{12} \cdots & \rho_{1M} \\ \rho^*_{12} & 1 & \vdots \\ \vdots & & \vdots \\ \rho^*_{1M} \ \ldots & & \end{bmatrix} \tag{3.16}$$

In the case where all the sources bear different (uncorrelated) signals, A^*AT becomes the identity matrix. In the mobile communications case, the off-diagonal elements will be randomly placed (other than complying with the Hermitian construction) ones or zeros. Each source adds one to the rank of A^*A which makes it of mathematical interest only except in simple scenarios, i.e., those with a manageable number of sources. In the mobile communications case, not even the rank of A^*A ' is known.

Inclusion of the weighted source vectors is facilitated by denoting

$$(Q)^*_{nm}(Q)_{qP} = a_{nm}a_{qP}e^{j(\phi_{qP}-\phi_{nm})} \tag{3.17}$$

$$= \pi_{nmqP}\ \phi_{qPnm} \tag{3.18}$$

$(Q)_{nm}$ is the nmth element of ***Q***. ϕ_{qPnm} is the phase difference between the pth source in the qth branch and the mth source in the nth branch. π_{nmqP} has the dimensions for power and denote $\pi_{nmnm} = \pi_{nm}$. From (3.l0), (3.14), and (3.18),

$$(R)_{nq} = \sum_{i=1}^{P} \pi_{nM_i qM_i} \phi_{qM_i nM_i} \tag{3.19}$$

and

$$(R)_{nn} = \sum_{i=1}^{P} \pi_{M_i} \tag{3.20}$$

The off-diagonal elements are seen to be a summation of vectors, each vector corresponding to one of the P signals from one of the subsets M, of the M incident sources.

In practice, the covariance matrix will also contain terms due to noise. The noise is uncorrelated between branches (any unwanted signal that is correlated between branches is defined as interference) so that only the principle diagonal in the covariance matrix becomes augmented. This is convenient because it ensures that R is always positive definite and the existence of $(R)^{-1}$ is assured. Conversely, if the noise level is very low relative to the source signal levels. Numerical problems may sometimes arise in seeking $(R)^{-1}$ from measured values of *R***.** Techniques for ill-conditioned *R* includes taking the generalized inverse instead of the conventional inverse.

3.1.3 The Optimum Antenna

In public service mobile communications, the average (over several fades) power of interferers would rarely be above that of the wanted signal. In each antenna branch or a single port antenna, however, the instantaneous level of the interferer could be a few tens of decibels above the wanted signal owing to the Rayleigh like fading. The optimum antenna (here the antenna is regarded as the elements, weights, and summing network) is defined to maximize the SINR. If covariance matrices *R* and *M* are defined to embody the wanted signals only and the interference-plus-noise, respectively, then the adaptive algorithm seeks an optimum weight vector W_{opt} which is well-known to satisfy

$$R_S W_{opt} = k' M W_{opt} \tag{3.21}$$

Where k' is a constant which is, in fact, the maximum SINR.

Denote a weighted space vector for the wanted source T_0**;** then

$$R_S = T^*{}_0 T^T{}_0 \tag{3.22}$$

In the mobile communications case, the wanted signal and interferers are distributed across many source vectors and the small size of the array will normally prevent the isolation of only one of these, the T_0 above. However, R_S can always be expressed as a dyadic when the sources are considered branch sources rather than spatial sources. Each branch source is the summation of the contributions from the actual sources which bear the same signal. Equation (3.19) depicts the situation. The weighted source vector of (3.4) becomes reformulated as branch signals,

$$T_{M_i} = \left\lfloor T_{1M_i} T_{2M_i} \cdots T_{NM_i} \right\rfloor \tag{3.23}$$

in which

$$T_{nM_i} = \sum_{m=1}^{M_i} T_m \quad (3.24)$$

and then the form

$$R_S = T_{M_0}^{*} \; T_{M_0}^{T} \quad (3.25)$$

is a valid representation even in the case of a spatially distributed set of (e.g., wanted) signal sources.

Substituting (3.25) into (3.21) gives

$$k' M W_{opt} = T_{M_0}^{*} T_{M_0}^{T} W_{opt} \quad (3.26)$$

$$= g_0 T_{M_0}^{*} \quad (3.27)$$

Where *go* is a constant representing the amplitude gain of the array to the wanted signal or to the wanted source in the case of an isolated source. In the latter case, the right side of (3.26) is $g_0 \pi_0 S^{*}{}_0 . S_0$ is called the steering vector, and π_0 is the total power (in all branches) of the desired source, quantities familiar from the conventional array case.

The optimality criteria can be expressed as the Wiener solution

$$M W_{opt} = k T_{M_0}^{*} \quad (3.28)$$

Where *k* is a scaling constant (= *go/k')* which has no effect on the SINR**.** If *M* is invertible (which can be arranged, if necessary, by adding noise), then the scaled weights of the optimum antenna are given by

$$W_{opt} = M^{-1} T_{M_0}^{*} \quad (3.29)$$

Equations (3.27) and (3.28) are the generalized version (applicable to the mobile communications case) of the results for conventional arrays in which T_{M_0} is replaced by $\pi_0 S_0$. The difference is that the conventional array employs purely phase weighting. Here, both phase and amplitude weighting are necessary.

In practice, $T^*_{M_0}$ must be known *a priori***,** or else it is measured, as is the covariance matrix. In mobile communications, *a priori* knowledge of $T^*_{M_0}$ is not possible and instead, knowledge of a characteristic of the wanted signal is used to estimate $T^*_{M_0}$ **.** To measure *M***,** the wanted signal must first be removed from each branch. Measurement of *R***,** which includes the wanted signal, is clearly more practical but its unentangled inclusion in the optimum solution of (3.28) requires special conditions. Specifically, if

$$R = M + R_S \qquad (3.30)$$

Then (3.21), (3.26), and (3.29) can be combined to yield the optimum scaled weights solution

$$W_{opt} = R^{-1} T^*_{M_m} \qquad (3.31)$$

This is known to be equivalent to (3.28), but features the more readily available total covariance matrix. The validity of (3.29) is the key resource: the desired signal and the interferers must be uncorrelated (ρ_{mp} *= 0, m # p)* or else orthogonal in the sense

$$T^T_{M_m} \; T^*_{M_P} = 0, \qquad m \neq p \qquad (3.32)$$

In the conventional array case, the analogy of (31) is the spatial orthogonality relationship

$$S^T_{M_m} \; S^*_{M_p} = 0, \qquad m \neq p \qquad (3.33)$$

Which is sought in null steering by retro beams (eigenbeams for subtraction from the quiescent array pattern)? In the mobile communications case, eigenbeams for preprocessing can be formulated mathematically (only at the expense of juggling signal powers, since T_{M_i} are not orthonormal), but the patterns are not in real space

3.2 Optimum Combiner

Space diversity provides an attractive means for improving the performance of mobile radio systems. With space diversity, the signals from the receiving antennas can be combined to combat multipath fading of the desired signal and reduce the relative power of interfering signals.

Fig.3.3 shows a block diagram of an *M* element space diversity combiner. The signal received by the *i*th element $y_i(t)$ is split with a quadrature hybrid into an in-phase signal $x_{I_i}(t)$ and **a** quadrature signal $x_{Q_i}(t)$ These signals are then multiplied by a controllable weight $w_{I_i}(t)$ or $w_{Q_i}(t)$**.** The weighted signals are then summed to form the array output $s_0(t)$**.**

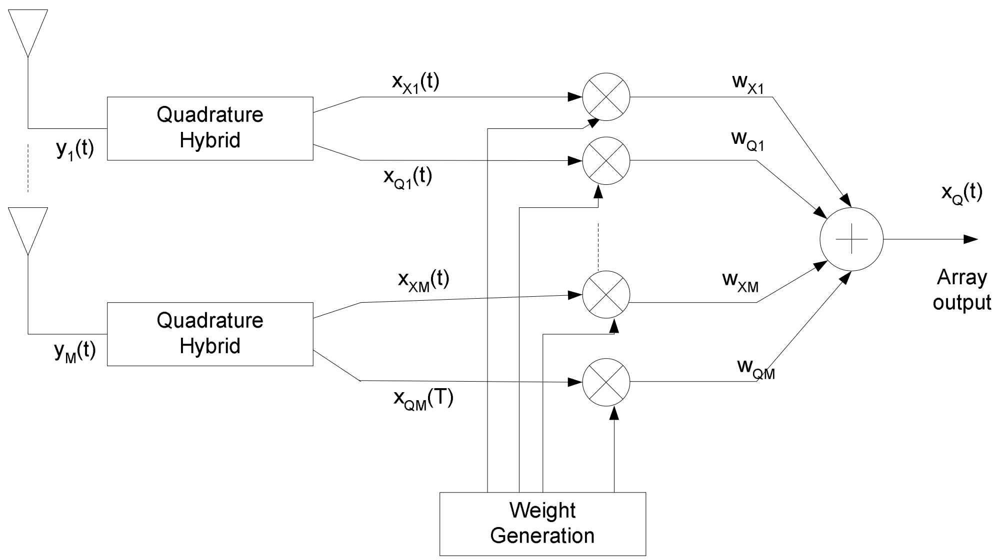

Figure-3.3: Block diagram of an M element space diversity combiner

The space diversity combiner can be described mathematically using complex notation [11]. Let the weight vector w be given by

$$w = \begin{bmatrix} w_{I_1} \\ \vdots \\ w_{I_M} \end{bmatrix} - j \begin{bmatrix} w_{Q_1} \\ \vdots \\ w_{Q_M} \end{bmatrix} \tag{3.34}$$

and the received signal vector x be given by

$$x = \begin{bmatrix} x_{I_1} \\ \vdots \\ x_{I_M} \end{bmatrix} + j \begin{bmatrix} x_{Q_1} \\ \vdots \\ x_{Q_M} \end{bmatrix} \tag{3.35}$$

The received signal consists of the desired signal, thermal noise, and interference and therefore, can be expressed as

$$x = x_d + x_n + \sum_{j=1}^{L} x_j \qquad (3.36)$$

Where x_d ,x_n, and x_j are the received desired signal, noise, and *j*th interfering signal vectors, respectively, and ***L*** is the number of interferers. Furthermore, let $s_d(t)$ and $s_j(t)$ be the desired and *j*th interfering signals as they are transmitted, respectively, with

$$\mathrm{E}[s^2{}_d(t)] = 1 \qquad (3.37)$$

and

$$\mathrm{E}[s^2{}_d(t)] = 1 \qquad for\ 1 \le j \le L \qquad (3.38)$$

Then x can be expressed as

$$x = u_d s_d(t) + x_n + \sum_{j=1}^{L} u_j s_j(t) \qquad (3.39)$$

Where u_d and u_j are the desired and jth interfering signal propagation vectors, respectively.

The general form of this propagation vector is given by

$$u_x = [\exp(j2\pi f_0 t_1(\theta_1)), \cdots, \exp(j2\pi f_0 t_x(\theta_x))]^T \qquad (3.40)$$

Where

f_0 = Source frequency,

$t_x(\theta_x)$ is time taken by a plane wave arriving from the *i*th source in direction (θ_i) and is given by

$$t_x(\theta_x) = \frac{d}{c}(M-1)\cos\theta_x \qquad (3.41)$$

Where c is the speed of propagation of the plane wave front, and d is the inter element spacing of an array.

The received interference-plus-noise correlation matrix is given by

$$R_{nn} = \mathrm{E}\left[\left(x_n + \sum_{j=1}^{L} x_j\right)^{*}\left(x_n + \sum_{j=1}^{L} x_j\right)^{T}\right] \qquad (3.42)$$

Where the superscripts * and ***T*** denote conjugate and transpose, Assuming the noise and interfering signals are uncorrelated, we can show that

$$R_{nn} = \sigma^2 \mathrm{I} + \sum_{j=1}^{L} \mathrm{E}\ [u_j^{*} u_j^{T}] \qquad (3.43)$$

Where σ^2 is the noise power and I is the identity matrix. In (3.43) the expected value is taken over a period much less than the reciprocal of the fading rate (e.g., several bit intervals). Note that we have assumed that the fading rate is much less than the bit rate. Finally, the equation for the weights that maximize the output SINR is (from [12])

$$w = \alpha R_{nn}^{-1} u_d^{*} \qquad (3.44)$$

Where α is a constant, and the superscript - 1 denotes the inverse of the matrix.

For the case of only one interferer the Noise plus Interference Correlation Matrix is given by

$$R_{nn} = \sigma^2 \mathrm{I} + \mathrm{E}\ [u_1^{*} u_1^{T}] \qquad (3.45)$$

And

$$\gamma_R = u_d^{T} R_{nn}^{-1} u_d^{*} \qquad (3.46)$$

Where $$\gamma_R = \frac{\textit{local mean desired signal power at the array output}}{\textit{mean noise plus interference power at the array output}} \qquad (3.47)$$

The probability density function of γ_R can be calculated to be given by [13]

$$p(\gamma_R) = \frac{e^{-\gamma_R/\Gamma_d}\left(\frac{\gamma_R}{\Gamma_d}\right)^{M-1}(1+M\Gamma_1)}{\Gamma_d (M-2)!}\int_0^1 e^{-((\gamma_R/\Gamma_d)M\Gamma_1)t}(1-t)^{M-2}\,dt \qquad (3.48)$$

Thus, the cumulative distribution function is given by [9]

$$P(\gamma_R)=\frac{\int_0^{\gamma_R/\Gamma_d} e^{-x}x^{M-1}(1+M\Gamma_1)}{(M-2)!}.\int_0^1 e^{-xM\Gamma_1 t}(1-t)^{M-2}\,dtdx \qquad (3.49)$$

BER for optimum combining with one interferer can be calculated to be given by

$$BER=\frac{(-1)^{M-1}(1+M\Gamma_1)}{2(M\Gamma_1)^{M-1}}\left\{\begin{array}{l} -\frac{M\Gamma_1}{1+M\Gamma_1}+\sqrt{\frac{\Gamma_d}{1+\Gamma_d}}-\frac{1}{1+M\Gamma_1}\sqrt{\frac{\Gamma_d}{1+M\Gamma_1+\Gamma_d}} \\ -\sum_{k=1}^{M-2}(-M\Gamma_1)^k\left[1-\sqrt{\frac{\Gamma_d}{1+\Gamma_d}}\left(1+\sum_{i=1}^{k}\frac{(2i-1)!!}{i!(2+2\Gamma_d)^i}\right)\right]\end{array}\right\} \qquad (3.50)$$

Where

$$(2i-1)!!=1\cdot 3\cdot 5\cdot\cdots\cdot(2i-1) \qquad (3.51)$$

$$\Gamma_d=\frac{mean\ \ received\ \ desired\ \ signal\ \ power\ \ per\ antenna}{mean\ received\ \ noise\ \ power\ \ per\ antenna} \qquad (3.52)$$

$$\Gamma_j=\frac{mean\ \ received\ \ jth\ \ \text{int}\,erferer\ signal\ \ power\ \ per\ antenna}{mean\ \ received\ noise\ power\ \ per\ antenna} \qquad (3.53)$$

Summary of the chapter

This chapter describes the performance of optimum combining. We calculated probability density function (pdf), cumulative distribution function (cdf), and Bit error Rate (BER). In the next chapter we will use this optimum combining with Rayleigh and Ricean Fading to improve BER.

4.1 Introduction

The mobile channel places fundamental limitations on the performance of wireless communication systems. The radio link between the transmitter and the receiver varies from simple line-of-sight to one that is severely obstructed by buildings, mountains, and hence suffers from severe multipath fading

Multi-path is a condition where the transmitted radio signal is reflected by physical features/structures, creating multiple signal paths between the base station and the user terminal. These multipath signals can interfere with the desired signal and make it harder for receiver to detect the original signal that was transmitted

When the waves of multi-path signals are out of phase, reduction in signal strength can occur. One such type of reduction is called a fade; the phenomenon is known as "Rayleigh fading" or "fast fading." A fade is a constantly changing, three-dimensional phenomenon. Fade zones tend to be small, multiple areas of space within a multi-path environment that cause periodic attenuation of a received signal for users passing through them.

4.2 Fading channel models

Multipath fading is due to the constructive and destructive combination of randomly delayed, reflected, scattered and diffracted signal components. This type of fading is relatively fast and is therefore responsible for the short-term signal variations. Depending on the nature of the radio propagation environment, there are different models describing the statistical behavior of the multipath-fading envelope. The Rayleigh, Ricean and Nakagami are the most commonly used statistical models to represent small-scale fading phenomenon.

4.2.1 Rayleigh Fading Channel

The Rayleigh distribution is the most widely used distribution to describe the received envelope value. The Rayleigh flat fading channel model assumes that all the components that make up the resultant received signal are reflected or scattered and there is no direct path from the transmitter to the receiver, which is shown in fig. 4.1. The Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. In the Rayleigh flat fading channel model, it is assumed that the channel induces amplitude, which varies in time according to the Rayleigh distribution.

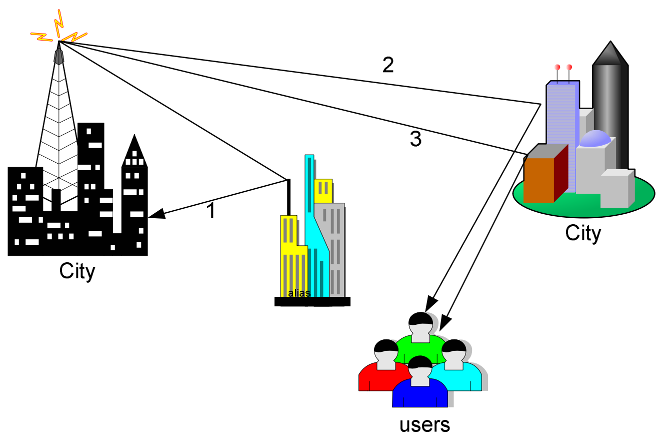

Figure-4.1: Rayleigh fading

When the channel impulse response is modeled as a zero-mean complex-valued Gaussian process, the envelope at any instant is Rayleigh-distributed. The Rayleigh distribution of a received complex envelope of a signal $z(t)=|x(t)|$ at any time t is given as

$$p_z(x)=\frac{x}{\sigma^2}e^{\left(\frac{-x^2}{2\sigma^2}\right)} \qquad (x \geq 0) \tag{4.1}$$

Where $$E\{x^2\}=2\sigma^2 \quad \text{and } x \geq 0 \tag{4.2}$$

Where σ is the root man square value of the received voltage signal before envelope detection, and σ^2 is the time-average power of the received signal before envelope detection [1]. It is well known that the envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution. This fading distribution could be described as follows:

- This represents the worst fading case because we do not consider having Line-of-Sight (LOS).
- It is caused by Doppler-shifted echoes with a Gaussian distribution,
- The power is exponentially distributed.
- The phase is uniformly distributed and independent from the amplitude.
- This is the most used signal model in wireless communications.

4.2.2 Ricean Fading Channel

In micro-cellular environments, there usually exists a dominant line of sight (LOS) path in addition to numerous diffused multipath components between the transmitter and receiver [1] In such a case, the other faded signal components are superimposed on the dominant component and the resultant signal amplitude follows Rician distribution with the ratio between the LOS and diffused components denoted by the Rice factor *K*. Rice fading is caused by Doppler-shifted echoes with a Gaussian distribution, but in addition there is always a direct path from the Tx antenna to the Rx antenna. Ricean fading is shown in fig. 4.2

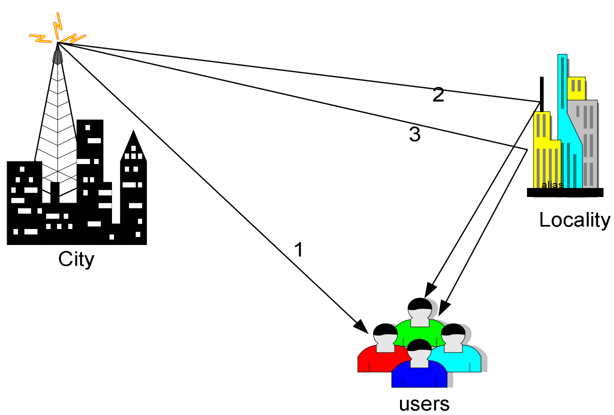

Figure-4.2: Ricean fading

The direct path 1 between the antennas considerably boosts the received field strength level. This path can in mobile reception only be influenced by the Doppler Effect. In addition to the direct path, many echoes are received. At the output of an envelope detector, this has the effect of adding a dc component to the random multipath. The effect of a dominant signal arriving with many weaker multipath signals gives rise to the Ricean distribution. As the dominant signal becomes weaker, the composite signal resembles a noise signal, which has an envelope that is Rayleigh. Thus, the Ricean distribution degenerates to a Rayleigh distribution when the dominant component fades away.

With fixed scatterers or signal reflectors in the medium, in addition to randomly moving scatterers, the channel impulse response will have a non-zero mean value and its envelope will have a Rice distribution. This channel is said to be a Ricean Fading Channel. For a multipath fading channel containing a specular or LOS component, the complex envelope of the received signal can be given by the Ricean distribution

$$p_z(x) = \begin{cases} \frac{x}{\sigma^2} e^{-\frac{(x^2+A^2)}{2\sigma^2}} I_0\left(\frac{Ax}{\sigma^2}\right) & \text{for } (A \geq 0, x \geq 0) \\ 0 & \text{for } (x < 0) \end{cases} \tag{4.3}$$

Where A denotes the peak amplitude of the dominant or LOS signal and I_0 (.) is the zeroth order modified Bessel function of the first kind. The Ricean distribution is often described in terms of a parameter K called Rician factor, which is defined as the ratio between the deterministic signal power and the variance of the multipath or can also be defined as $K = v^2/2\sigma^2$ is the relation between the power of the LOS component and the power of the Rayleigh component.

4.3 Generation of Rayleigh and Ricean Fading Parameters

The method of filtered noise is used to generate channel coefficients with the specified distribution and spectral power density. For each tap a set of complex zero-mean Gaussian distributed numbers is generated with a variance of 0.5 for the real and imaginary part, so that the total average power of this distribution is 1. This yields a normalized Rayleigh distribution (equivalent to Rice with K=0) for the magnitude of the complex coefficients. If a Ricean distribution (K>0 implied) is needed, a constant path component $|m|$ has to be added to the Rayleigh set of coefficients. The ratio of powers between this constant part and the Rayleigh (variable) part is specified by the K-factor. For this general case, we show how to distribute the power correctly by first stating the total power P is [13], [14], which is obtained at the receiver of Optimum combiner as shown in fig 3.3

$$P = |m|^2 + \sigma^2 \tag{4.4}$$

Where m is the complex constant and σ^2 the variance of the complex Gaussian set. Second, the ratio of powers is

$$K = \frac{|m|^2}{\sigma^2} \tag{4.5}$$

From these equations, we can find the power of the complex Gaussian and the power of the constant part as

$$\sigma^2 = P\frac{1}{K+1}; \qquad \text{and} \qquad |\mathrm{m}|^2 = P\frac{K}{K+1} \tag{4.6}$$

Now, with the help of the specified powers, Ricean channel coefficients can be calculated.

4.4 Optimum combining in Rayleigh Fading

Antenna arrays, with optimum combining, have been shown to combat both multipath fading of the desired signal and CCI, subsequently increasing the performance of mobile radio communication systems. Optimum combining was studied for both faded [9] and unfaded [15] communications. The concept of optimum combining is not new, and has been thoroughly investigated by the radar community, where it is usually referred to as adaptive array processing [15]. The use of adaptive arrays in radar has never managed to break out of niche applications, but the emerging applications to wireless might finally make the use of adaptive arrays more widespread. Current research shows that significant performance improvements can be obtained even with a modest number of antenna elements, thus increasing the chances of adaptive arrays to become a cost effective alternative to other ways of increasing capacity, such as reducing the cell size.

It is concerned with the flat (nondispersive), slowly fading, Rayleigh channel model, in which a desired signal competes with CCI with equal-power interferers. It is recognized that the assumption of equal-power interferers will result in overly pessimistic assessment of the array performance when compared to other more realistic models such as the exponential decay model. To further simplify the analysis, and since it is concerned with systems which are clearly interference limited, and also the effect of thermal noise is ignored. More recent contributions to the field include the analysis of the average bit-error rate (BER) for optimum combining with a single CCI [8], [9].

When the number of interferers exceeds the number of antenna elements, the array is unable to cancel every interfering signal. Still, this case is of practical interest, since a moderate increase in SINR at the output of the antenna array can result in a significant increase in system capacity. Analysis of the adaptive array is done when L≥M.

4.4.1 System Model

Consider the reverse link (mobile to base) of a digital cellular mobile radio communication system. The base station consists of an M element antenna array. The system employs binary phase-shift keying (BPSK) modulation and signals are assumed to be narrow band, hence they can be represented by samples of their complex envelopes. The channel is characterized by flat Rayleigh fading. It is further assumed that the fading is slow and that the receiver has perfect knowledge of the instantaneous channel realization and a coherent receiver can be implemented as shown in fig (3.3). The system is assumed interference limited. It is also assumed that the Degree of Freedom (DOF) is insufficient to suppress all interferers; hence, thermal noise is neglected.

After complex carrier demodulation, the received signal vector at the elements' outputs is given by

$$\boldsymbol{r}(\mathrm{t}) = \sqrt{\boldsymbol{P_s}}\,\boldsymbol{cs(t)} + \sum_{k=1}^{L}\sqrt{\boldsymbol{P_k}}\,\boldsymbol{c_k s_k}(t) \qquad (4.7)$$

Where s (t) and s_k (t) =± g (t) are the desired and the k^{th} interfering signals, respectively, c is fading coefficient and g (t) is the transmitted waveform with $\int_0^T |g(t)|^2\, dt = T$, where T is the symbol interval. The binary symbols of the desired user and CCI's are assumed time synchronized, equally probable and mutually independent. The vectors c_K, k=1… L, are independently identically distributed (i.i.d.), complex Gaussian, with $E[c_K]=0$ and covariance matrix $\sum = E[c_k c_k^H]$. The elements of c_k have σ^2 variance. The same parameters hold for c. Here we have, the number of antenna elements M and the number of cochannel interferers L are related L≥M. The powers of the desired signal and the kth interfering signal are P_s and P_k, respectively.

The weight vector that maximizes SIR at the output of the array is [15]

$$\boldsymbol{w} = \alpha \boldsymbol{R}^{-1} c \tag{4.8}$$

Where α is an arbitrary constant. The constant α does not affect the SIR at the array output. The interference covariance matrix, conditioned on channel vectors c_k , is

$$\boldsymbol{R} = \sum_{k=1}^{L} P_k \, \boldsymbol{c}_k \, \boldsymbol{c}_k^H \tag{4.9}$$

Where the superscript $\{\cdot\}$ stands for transpose complex conjugate. Note that R varies with the fading rate and that we have assumed that the fading rate is much less than the bit rate.

By definition, $\Sigma = \mathrm{E}[c_k c_k^H]$ is positive semidefinite and Hermitian. By assumption, it will be positive-definite; hence, its inverse exists. When independent fading at each antenna element is assumed and σ^2=1, then $\Sigma = I_M$, where I_M is an identity matrix of dimension M. When Σ>0, i.e., positive-definite, and L≥M, the matrix R is positive-definite with probability one. This exists in (4.8). The density of such random matrices was studied in [16]. Therein it is shown that the joint distribution of the elements R is given by

$$f_R(\mathrm{R}) = \begin{cases} \dfrac{|\boldsymbol{R}|^{L-M} |\boldsymbol{\Sigma}|^{-L} |\Lambda|^{-M} {}_0F_0^{(L)}(\Lambda^{-1}, -\boldsymbol{\Sigma}^{-1}\boldsymbol{R})}{\widetilde{\Gamma}_M(L)} & \boldsymbol{R}>0, \boldsymbol{\Sigma}>0 \\ 0, & \text{otherwise} \end{cases} \tag{4.10}$$

Where $|\cdot|$ denotes the determinant of a matrix. The complex multivariate gamma function $\widetilde{\Gamma}_M(L)$ is defined

$$\widetilde{\Gamma}_M(L) = \pi^{M(M-1)/2} \prod_{i=i}^{M} \Gamma(L-i+1) \tag{4.11}$$

Where $\Gamma(\cdot)$ is the standard gamma function. The quantity ${}_0F_0^{(L)}(\Lambda^{-1},\Sigma^{-1}R)$ is a hypergeometric function with matrix arguments [16]. Also ${}_pF_q$ can be defined as

$${}_pF_q\left(a_{1,} \cdots, a_p; b_{1,} \cdots, b_q; x\right) = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n}{(b_1)_n \cdots (b_q)_n} \frac{x^n}{n!} \tag{4.12}$$

The matrix $\Lambda = \text{diag}\ (P_{1,\dots,}P_{L},)$. For the special case of $P_K = P$, i.e., all interferers have equal power P, the density function in (4.10) reduces to the complex Wishart distribution.

$$f_R(R) = \begin{cases} \dfrac{|R|^{L-M} |\Sigma|^{-M} |\Lambda|^{-M} e^{-tr\left(\frac{1}{P}\Sigma^{-1} R\right)}}{\widetilde{\Gamma}_M(L) P^{LM}} & R>0,\Sigma>0 \\ 0, & otherwise \end{cases} \tag{4.13}$$

Where tr(.) denotes the trace of a matrix. In deriving (4.13) from (4.10), we have used the following relations:

$$\begin{aligned} {}_0F_0^{(L)}(\Lambda^{-1}, \Sigma^{-1}R) &= {}_0F_0^{(L)}(P^{-1}I, -\Sigma^{-1}R) \\ &= {}_0F_0^{(L)}(I, -P^{-1}\Sigma^{-1}R) \\ &= {}_0F_0(-P^{-1}\Sigma^{-1}R) \\ &= e^{-tr}(P^{-1}\Sigma^{-1}R) \end{aligned} \tag{4.14}$$

The distribution in (4.13) (apart from a constant factor) is the complex Wishart distribution with parameter Σ and L degrees of freedom. It is denoted as $CW_M(\Sigma, L)$. The Wishart distribution is the multivariate generalization of the chi-square distribution.

4.4.2 Average Probability of Bit Error

Computation of the bit error requires determining the distribution of the interference at the array output. Due to dissimilar fading effects, interferers are not identically distributed; hence the central limit theorem cannot be strictly invoked to claim the Gaussian property for the sum of binary random variables. Nevertheless, when the number of interferers is large, the Gaussian property is often assumed in such analyses. Thus, the conditional BER, i.e., the BER computed for a given value of μ, is simply

$$P_{\frac{e}{\mu}} = Q(\sqrt{2\mu}) \tag{4.14}$$

Where $Q(.)$ is the area under the tail of the Gaussian probability density function and is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{\frac{-t^2}{2}} dt \tag{4.15}$$

In integral tables used to obtain the results below, the related function erfc (.) (the complementary error function) is usually found. The relation between the two functions is $Q(\sqrt{2x}) = \frac{1}{2} erfc(\sqrt{x})$. The average BER, i.e., the one averaged over all the values of $P_e = \frac{1}{2}\int_0^\infty erfc(\sqrt{\mu}) f_\mu(\mu) d\mu$

$$= \frac{1}{2} \frac{\Gamma(L+1)}{\Gamma(M)\Gamma(L+1-M)} \left(\frac{P_s}{P}\right)^{L+1-M} \int_0^\infty erfc(\sqrt{\mu}) \frac{\mu^{M-1}}{\left(\frac{P_s}{P} + \mu\right)^{L+1}} d\mu \tag{4.16}$$

4.4.3 Simulation of BER of Optimum Combining in Rayleigh Fading

We evaluate the system performance for the worst case scenario only, i.e., the mobile transmitting the desired signal is at the point in the cell farthest from the base station, and the interfering mobiles in the surrounding cells are as close as possible to the base station of the desired mobile. Unless specified otherwise, the number of interferers assumed in the analysis is L=6. As explained in the introduction, we make the assumption of equal-power interferers. Due to this assumption, the results are pessimistic with respect to the case of unequal-power when the largest interferer is of the same power as the ones assumed here. The variance σ^2 of the channel coefficients was assumed one. Simulated results are shown in given figures from (4.3)-(4.16) for different values of M and with L=6, L=18.

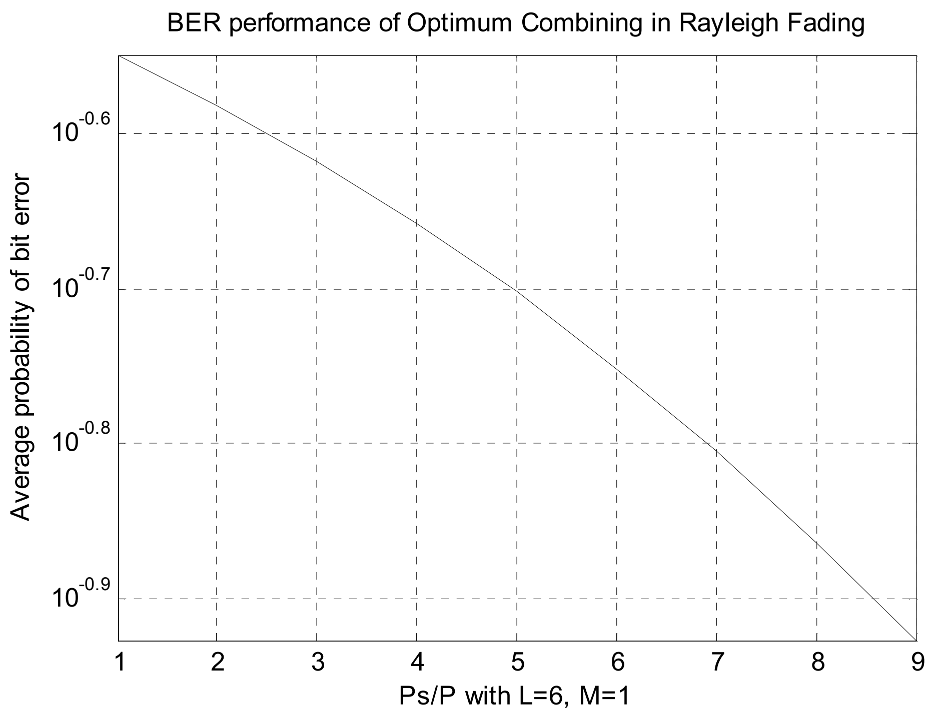

Figure-4.3: BER Performance in Rayleigh Fading with M=1, L=6

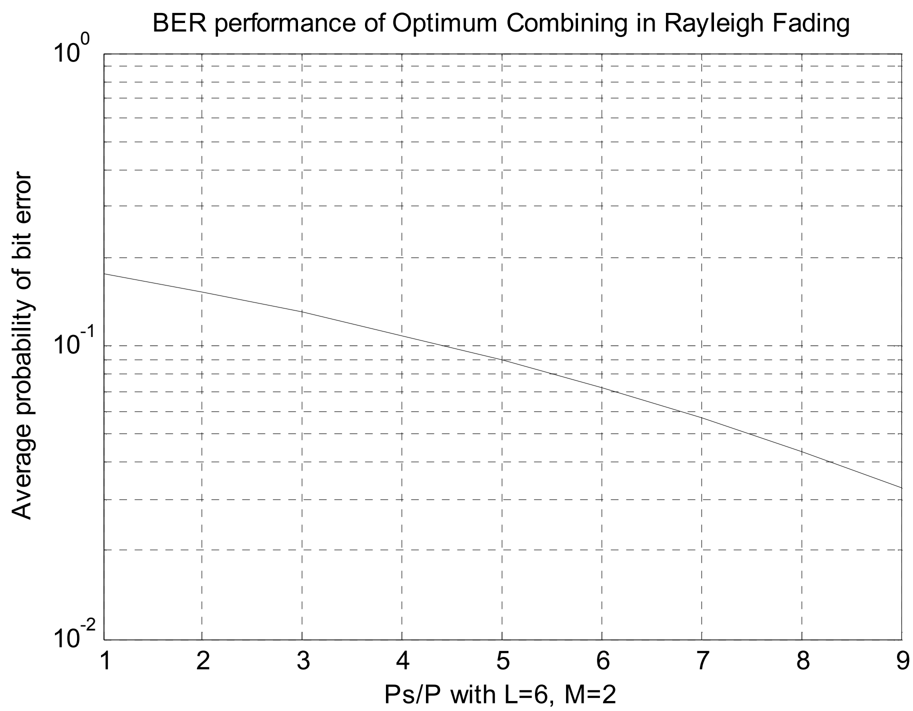

Figure-4.4: BER Performance in Rayleigh Fading with M=2, L=6

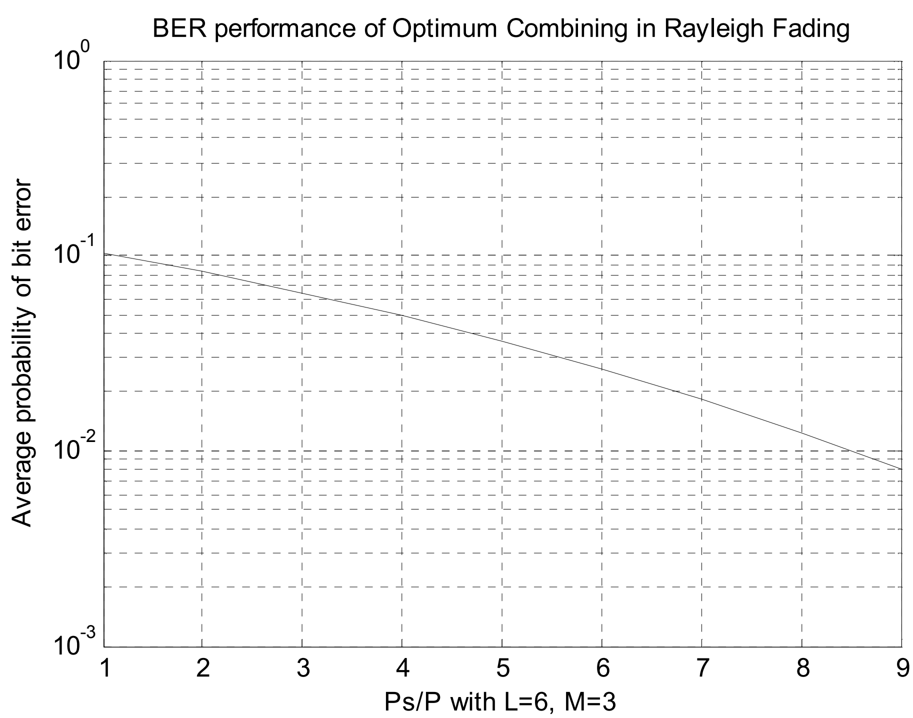

Figure-4.5: BER Performance in Rayleigh Fading with M=3, L=6

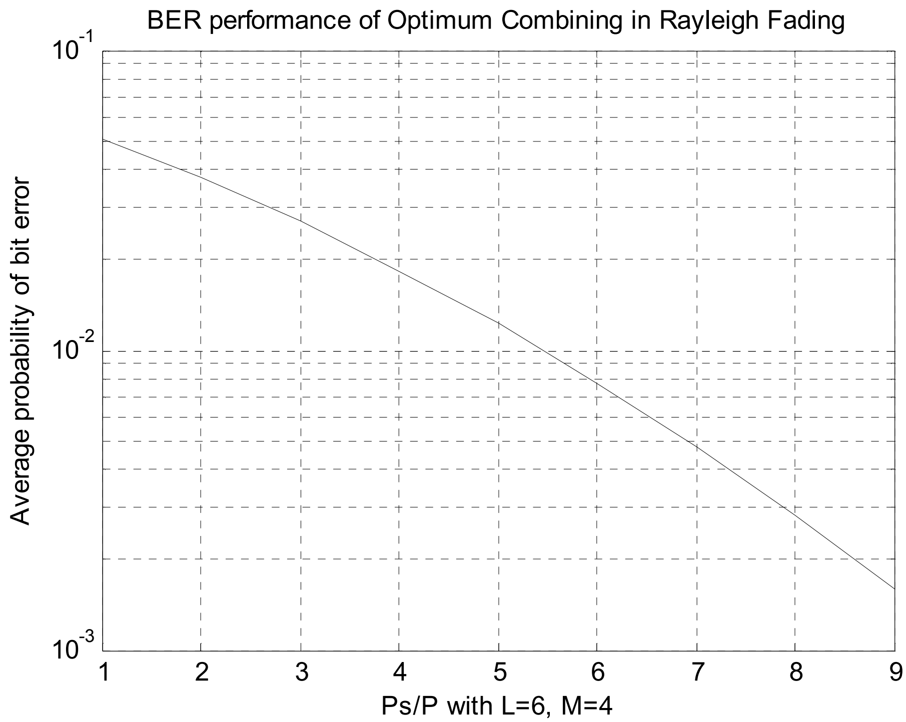

Figure-4.6: BER Performance in Rayleigh Fading with M=4, L=6

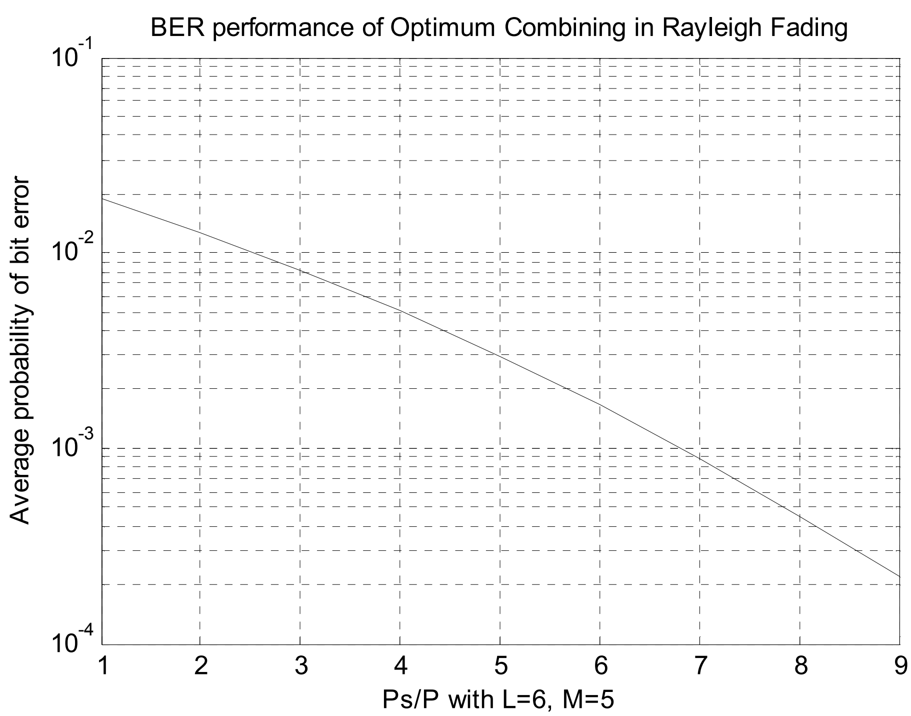

Figure-4.7: BER Performance in Rayleigh Fading with M=5, L=6

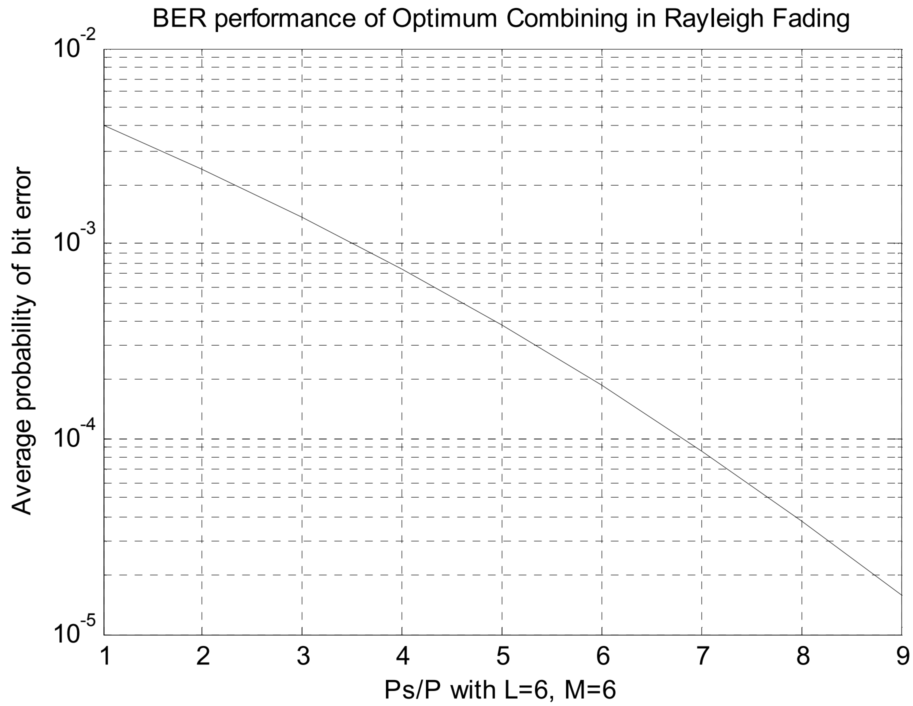

Figure-4.8: BER Performance in Rayleigh Fading with M=6, L=6

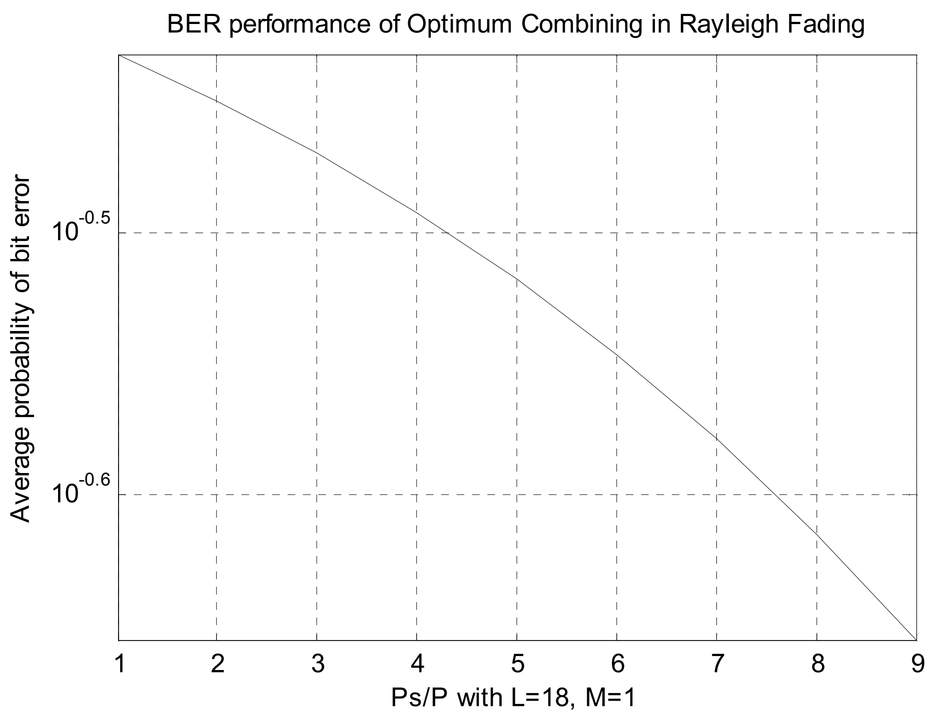

Figure-4.9: BER Performance in Rayleigh Fading with M=1, L=18

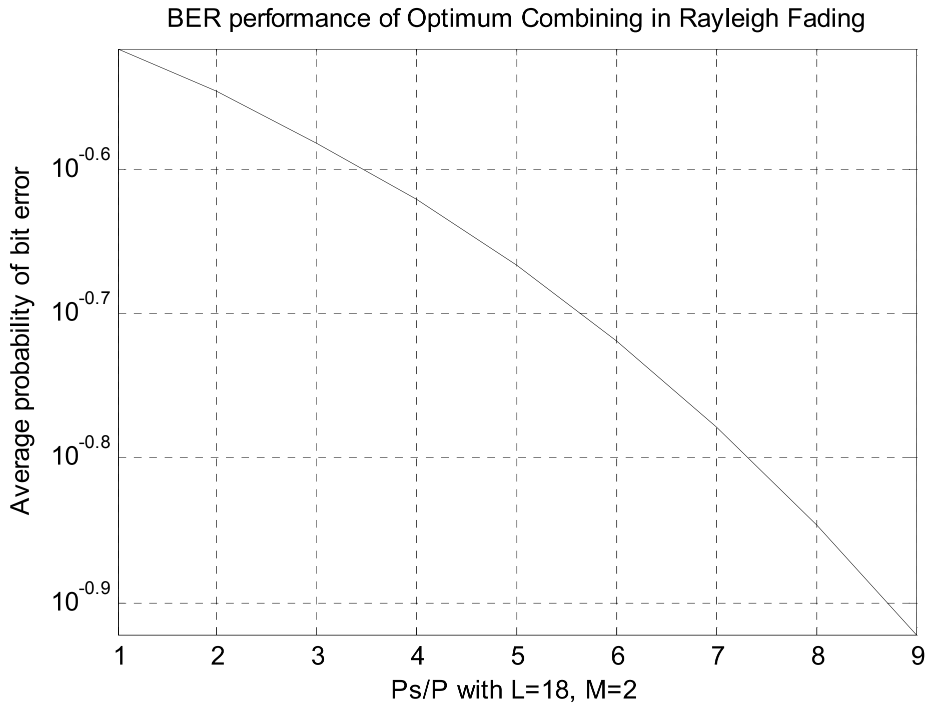

Figure-4.10: BER Performance in Rayleigh Fading with M=2, L=18

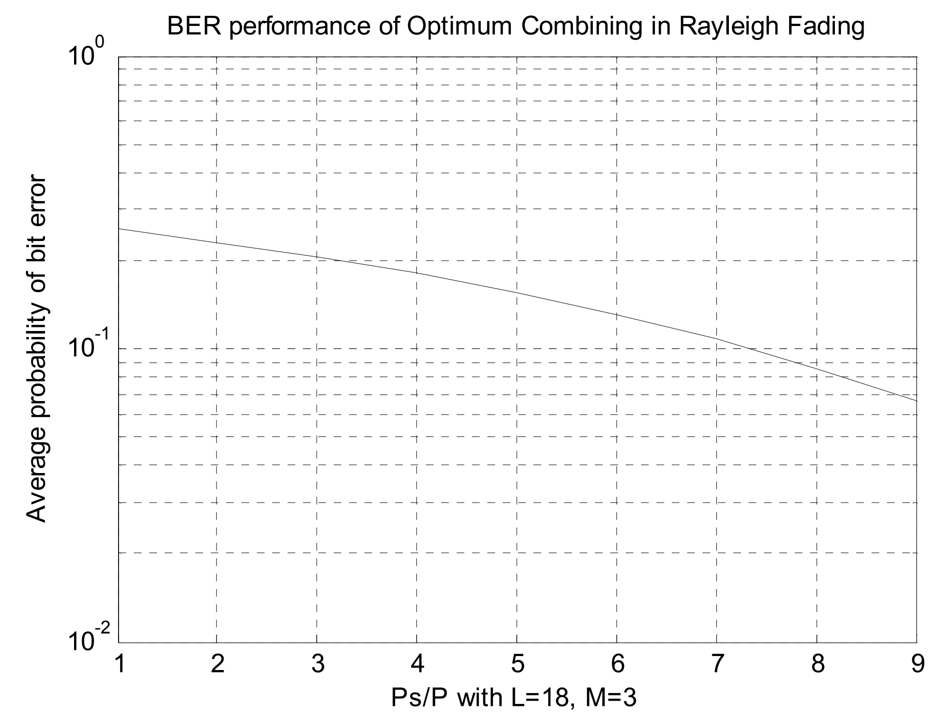

Figure-4.11: BER Performance in Rayleigh Fading with M=3, L=18

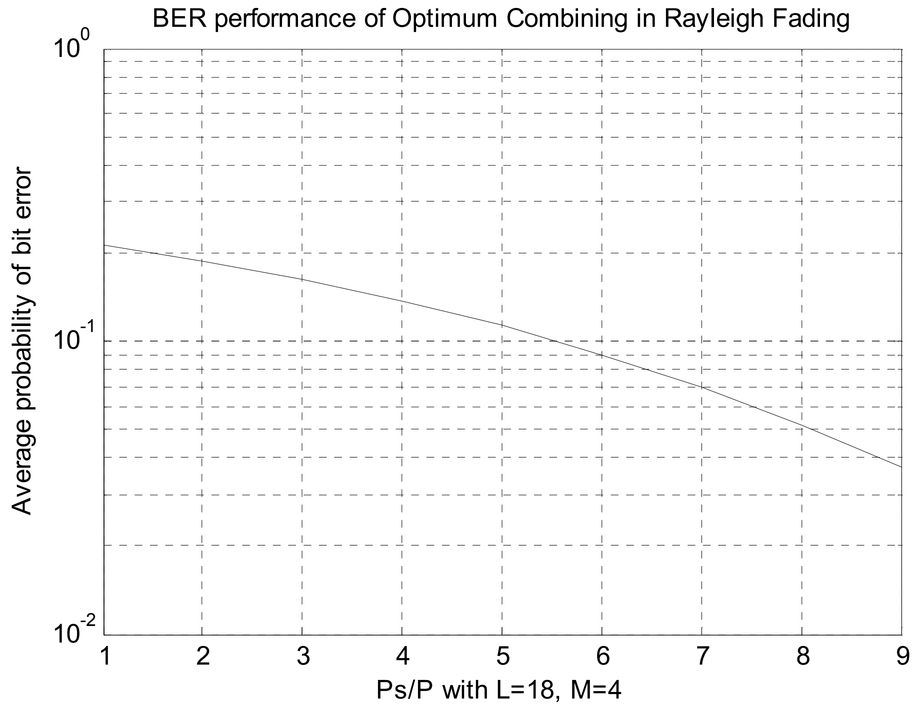

Figure-4.12: BER Performance in Rayleigh Fading with M=4, L=18

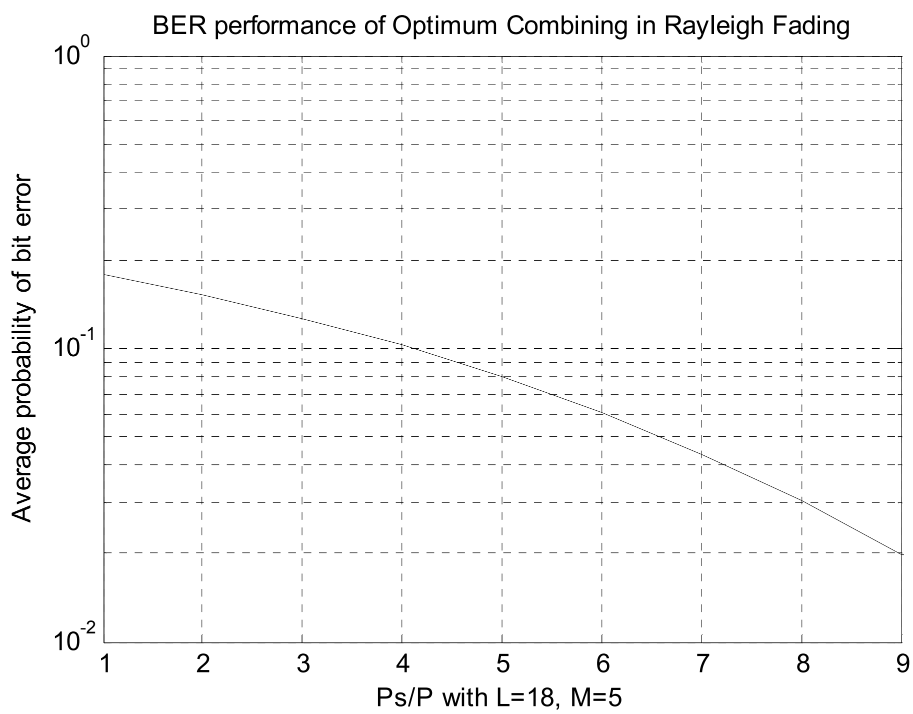

Figure-4.13: BER Performance in Rayleigh Fading with M=5, L=18

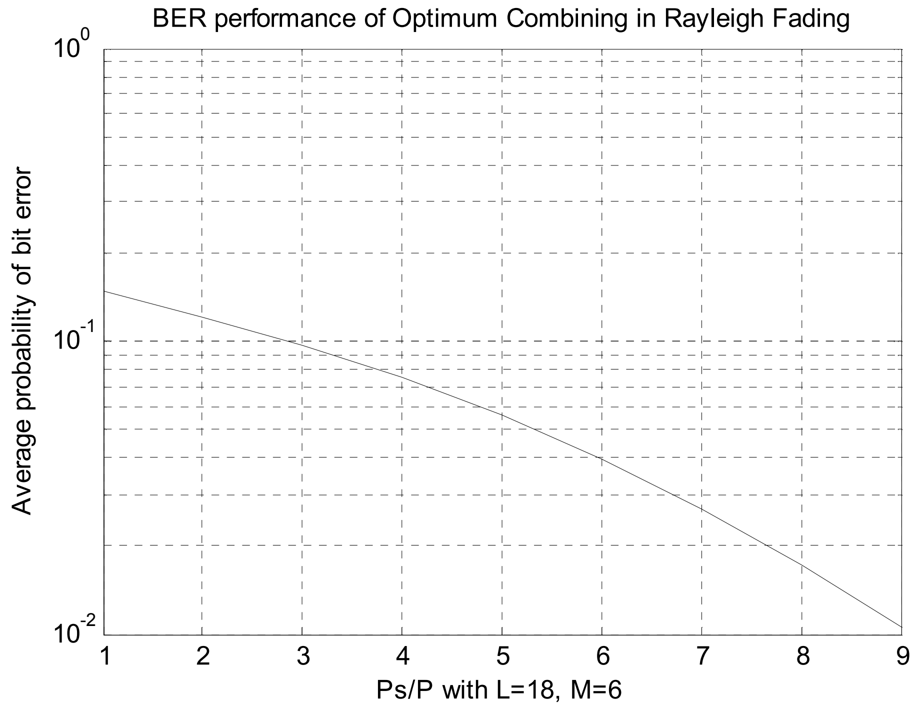

Figure-4.14: BER Performance in Rayleigh Fading with M=6, L=18

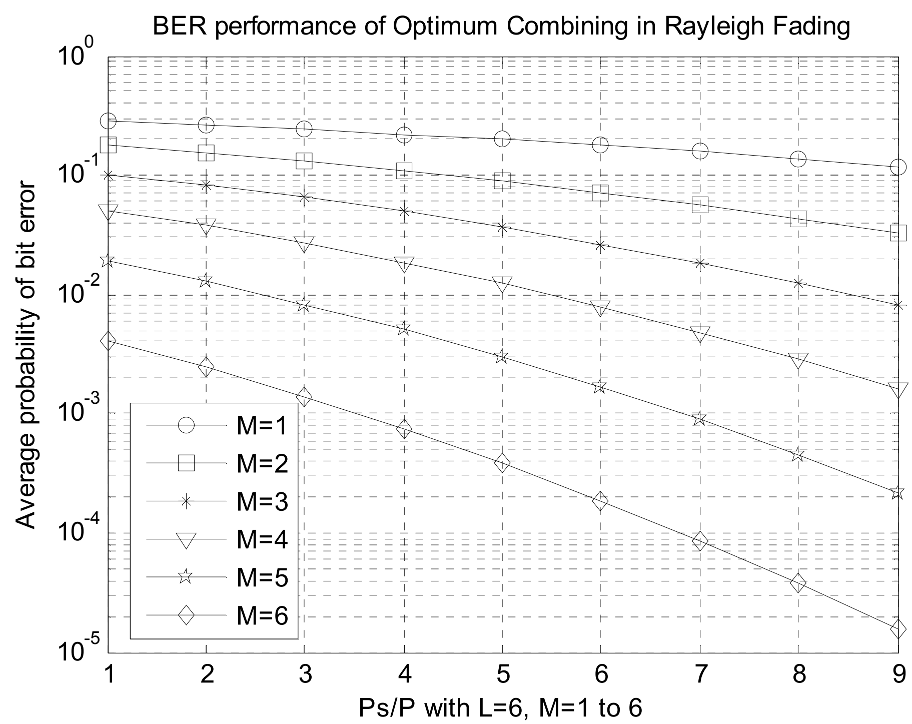

Figure-4.15: BER Performance in Rayleigh Fading with M=1 to 6, L=6

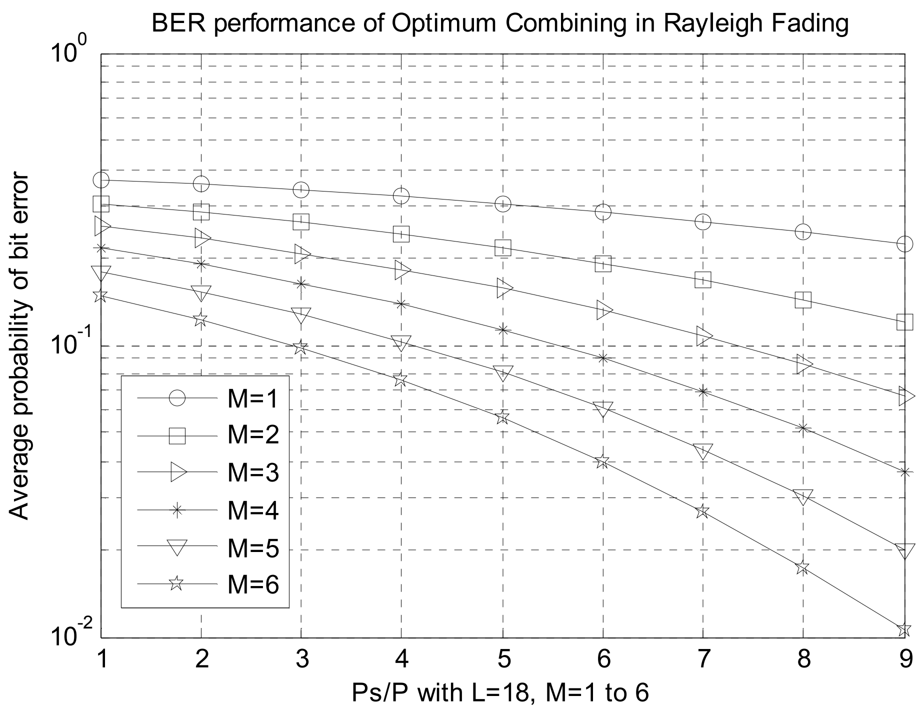

Figure-4.16: BER Performance in Rayleigh Fading with M=1 to 6, L=18

- Figure (4.3)-(4.8) shows for Rayleigh Fading, for a given SINR, with 6 cochannel interferes, also with increasing M reduces the required BER, which in turn, leads to an increase in the system performance.
- Figure (4.9)-(4.14) shows for Rayleigh Fading, for a given SINR, with 18 cochannel interferes, also with increasing M reduces the required BER, which in turn, leads to an increase in the system performance.
- Figure (4.15) shows the combined plot for Rayleigh Fading, for effect on SINR with changing the values of antenna elements, with 6 cochannel interferes, it also shows that with increasing M reduces the required BER, which in turn, leads to an increase in the system performance.
- Figure (4.16) shows the combined plot Rayleigh Fading, for effect on SINR with changing the values of antenna elements, with 18 cochannel interferes, it also shows that with increasing M reduces the required BER, which in turn, leads to an increase in the system performance.

Table 4.1 BER for Rayleigh Fading at L=6, SNR=5db

Serial no	No. of antenna elements M	BER at L=6,SNR=5db
1	M=1	0.1986
2	M=2	0.0893
3	M=3	0.0365
4	M=4	0.0122
5	M=5	0.0030
6	M=6	0.0004

Table 4.2 BER for Rayleigh Fading at L=18, SNR=5db

Serial no	No. of antenna Elements M	BER at L=18,SNR=5db
1	M=1	0.3038
2	M=2	0.2151
3	M=3	0.1559
4	M=4	0.1127
5	M=5	0.0803
6	M=6	0.0560

As from Table 4.1 and 4.2 it is concluded that BER get reduced with the increase in number of antenna elements at SNR 5db, with 6 and 18 cochannels interferes respectively, so the system performance will get improved.

Now tables 4.3 and 4.4 shows the comparison of SNR for a given BER is evaluated at L=6, and L=18 respectively

Table 4.3 SNR for Rayleigh Fading at L=6, BER$\leq 10^{-1}$

Serial no	No. of antenna elements M	SNR at L=6, BER$\leq 10^{-1}$
1	M=2	1dB
2	M=3	2dB
3	M=4	4dB

As from table 4.3, it is concluded that for L=6,BER$\leq 10^{-1}$, optimum combining with 4 antennas requires 1dB less power as compared to optimum combining with 3 antennas and 2dB less power as compared to optimum combining with 2 antennas.

Table 4.4 SNR for Rayleigh Fading at L=18, BER$\leq 10^{-1}$

Serial no	No. of antenna elements M	SNR at L=18, BER$\leq 10^{-1}$
1	M=3	7dB
2	M=5	4dB
3	M=6	3dB

From table 4.4, it is concluded that for L=18,BER$\leq 10^{-1}$, optimum combining with 6 antennas requires 1dB less power as compared to optimum combining with 5 antennas and 4dB less power as compared to optimum combining with 3 antennas.

4.3 Simulation of BER of Optimum Combining in Ricean Fading

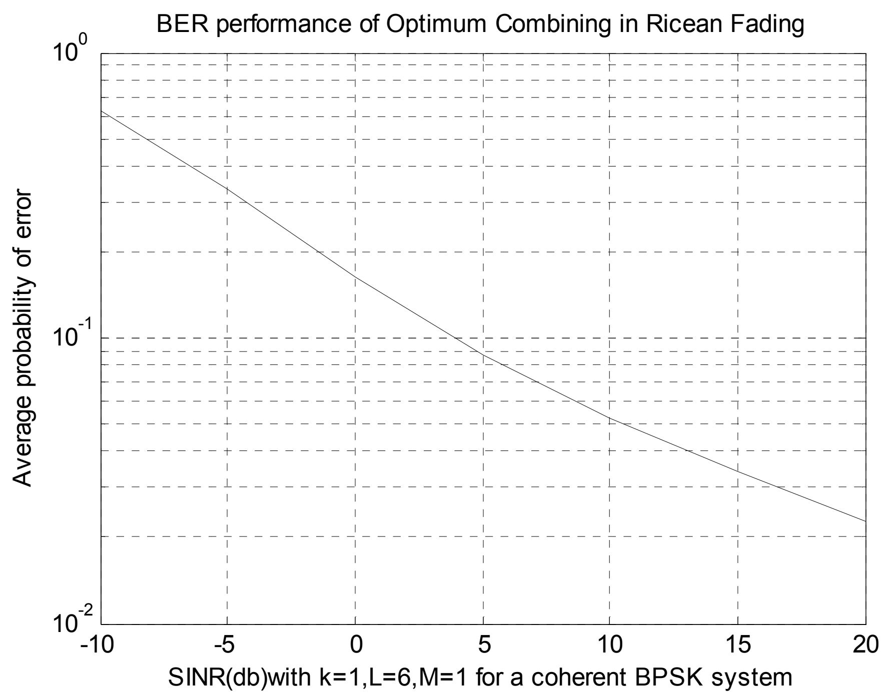

Figure -4.17: BER Performance in Ricean Fading with M=1, k=1

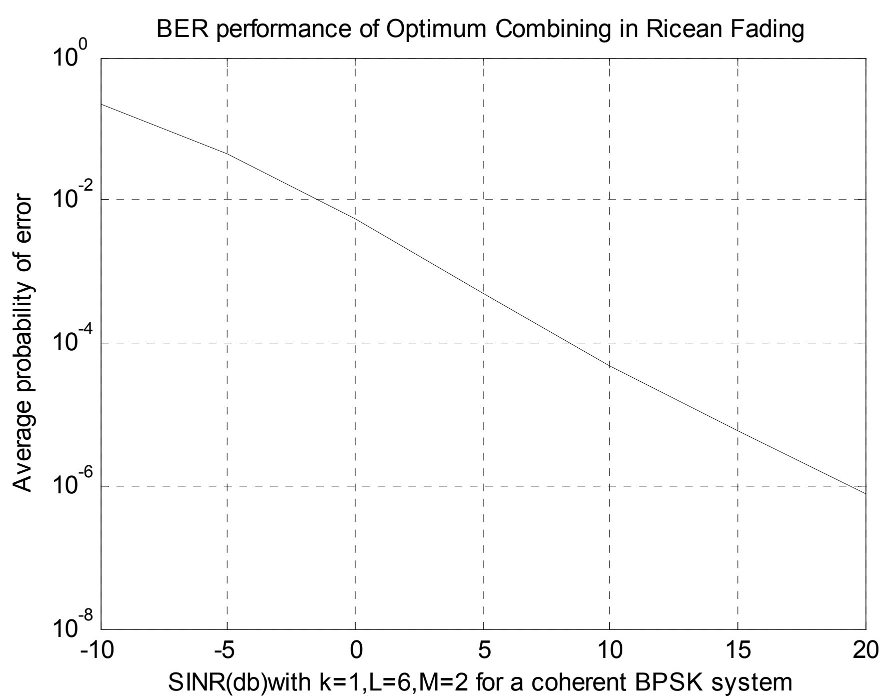

Figure -4.18: BER Performance in Ricean Fading with M=2, k=1

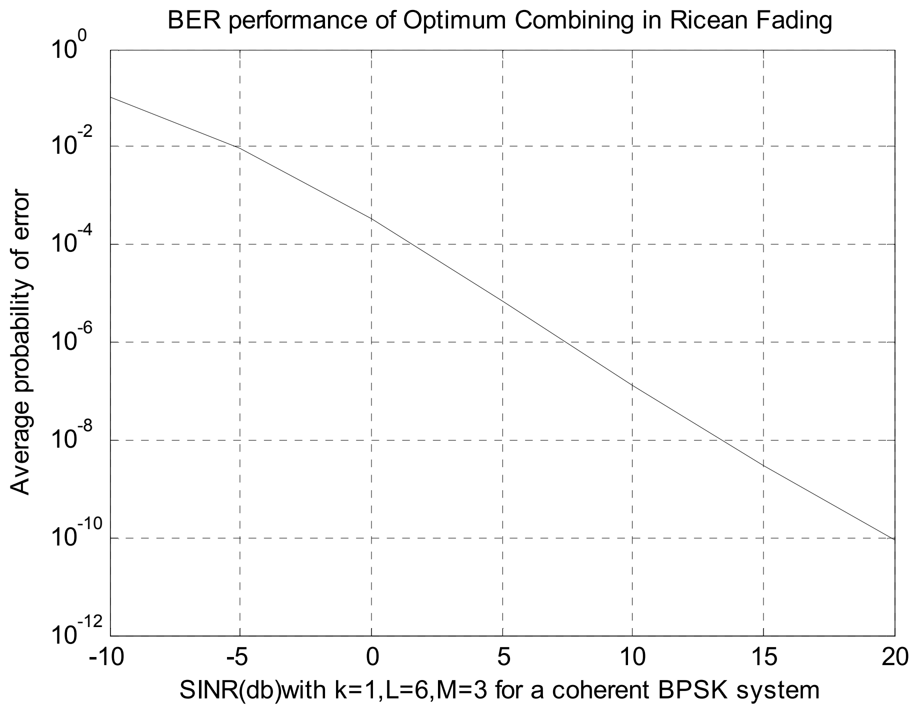

Figure -4.19: BER Performance in Ricean Fading with M=3, k=1

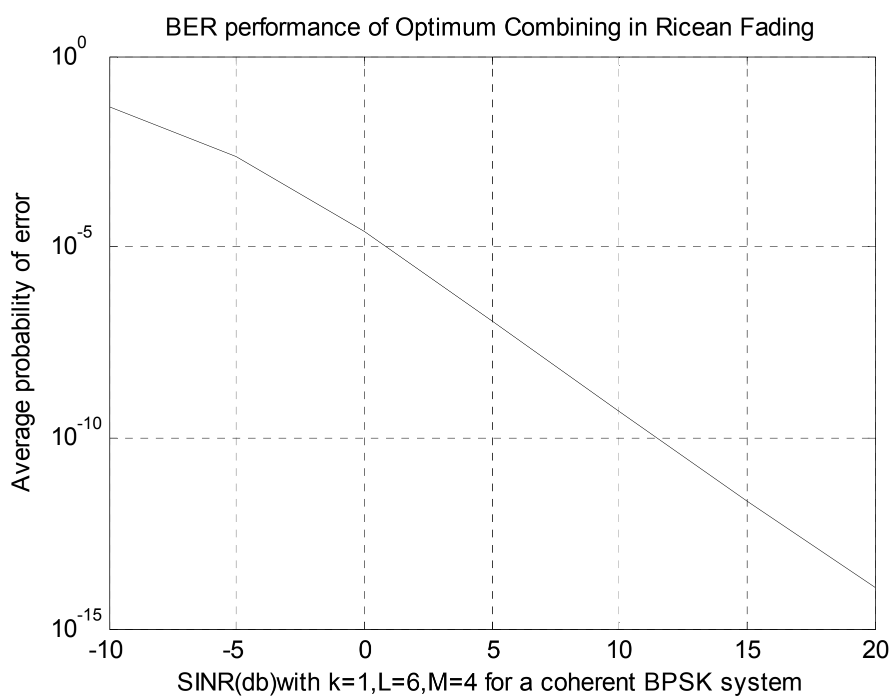

Figure -4.20: BER Performance in Ricean Fading with M=4, k=1

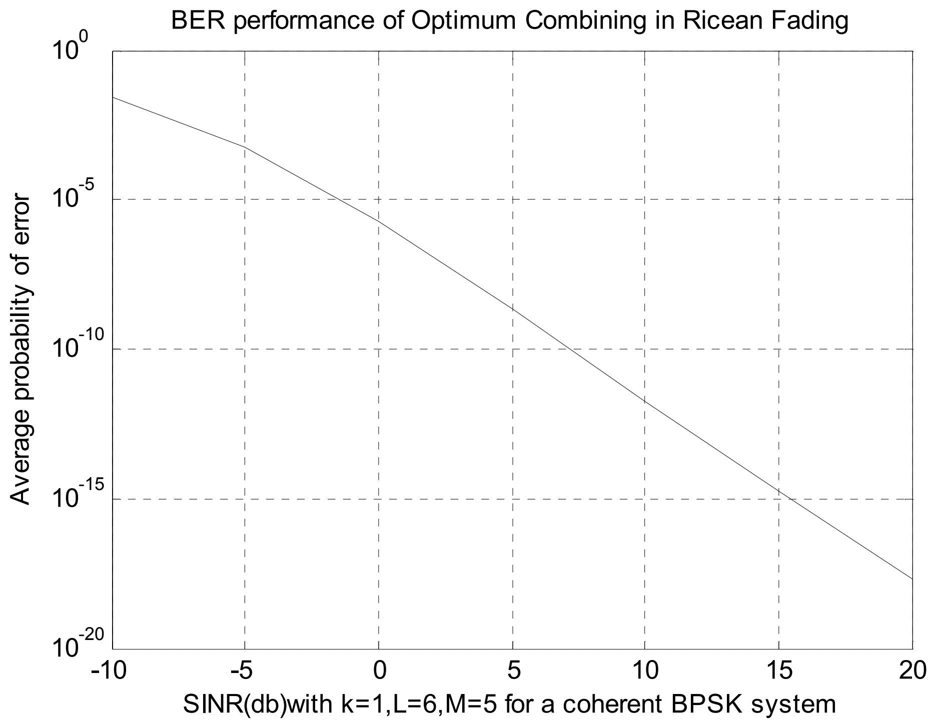

Figure -4.21: BER Performance in Ricean Fading with M=5, k=1

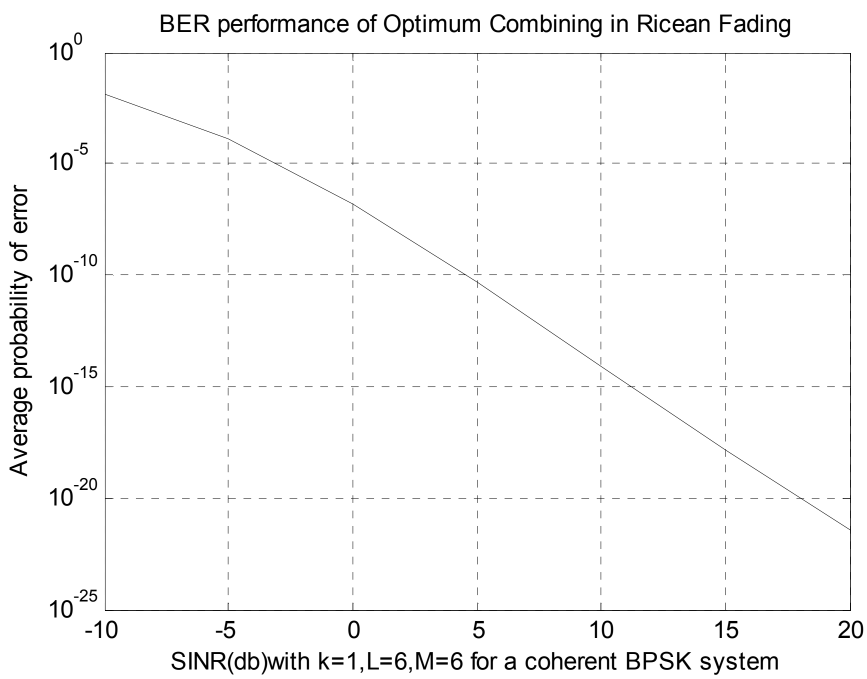

Figure -4.22: BER Performance in Ricean Fading with M=6, k=1

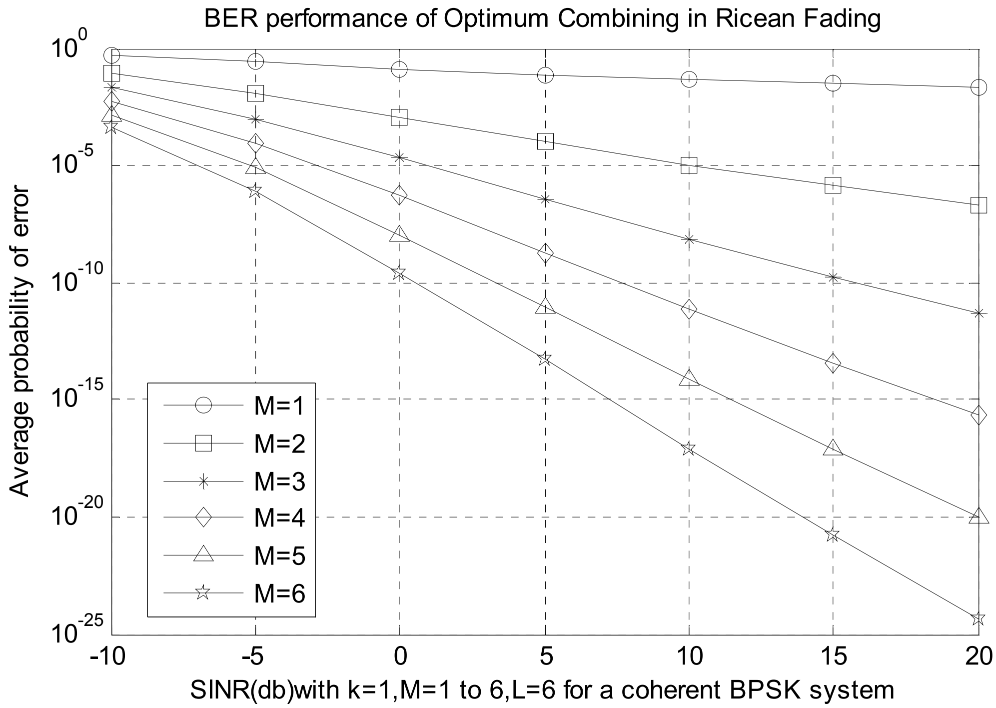

Figure-4.23: BER Performance in Ricean Fading with L=6, M=1 to 6, k=1

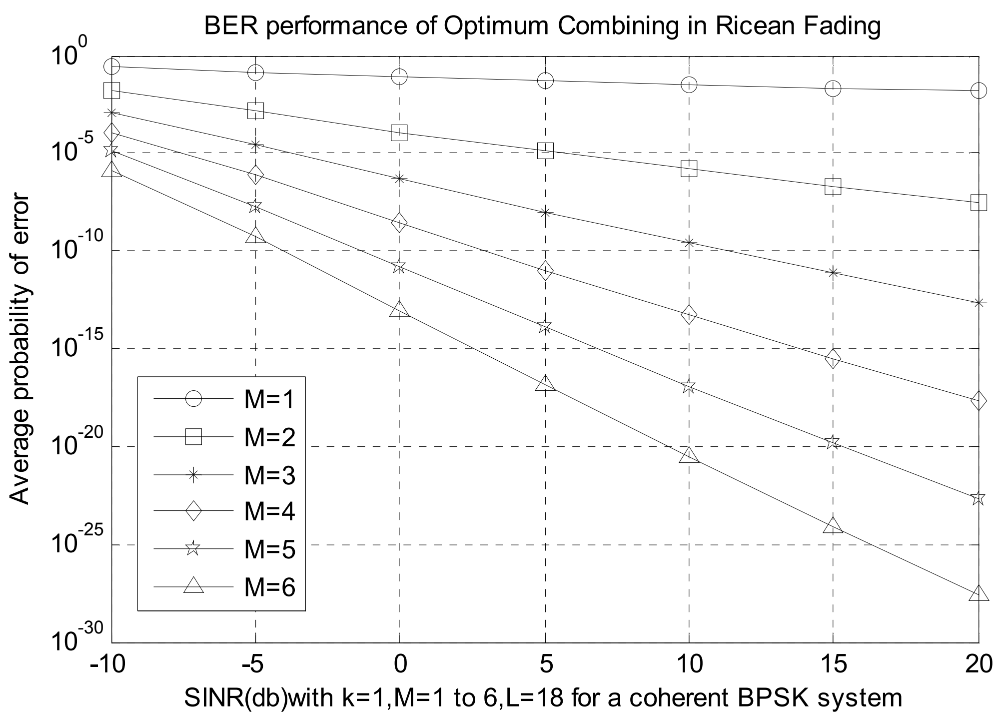

Figure-4.24: BER Performance in Ricean Fading with L=18, M=1 to 6, k=1

- Figure (4.18)-(4.22) shows plot for Ricean Fading, for a given SINR, for 6 cochannel interferes , also with increasing M reduces the required BER, which in turn, leads to an increase in the system performance.
- Figure (4.23)-(4.24) shows combined plot of Ricean Fading, for a given SINR, for multiple cochannel interferes with L=6 and L=18 respectively , also with increasing M reduces the required BER, which in turn, leads to an increase in the system performance.

Table 4.5 BER for Ricean Fading at L=6, SNR=5db

Serial no	No. of antenna elements M	BER at L=6,SNR=5db
1	M=1	0.0750
2	M=2	0.0001
3	M=3	0.0000
4	M=4	0.0000
5	M=5	0.0000
6	M=6	0.0000

As from Table 4.5 it is concluded that BER get reduced with the increase in number of antenna elements at SNR 5db, with 6 cochannels interferes, so the system performance will get improved.

Table 4.6 BER for Ricean Fading at L=18, SNR=5db

Serial no	No. of antenna elements M	BER at L=18,SNR=5db
1	M=1	0.0479
2	M=2	0.0000
3	M=3	0.0000
4	M=4	0.0000
5	M=5	0.0000
6	M=6	0.0000

As from Table 4.6 it is also concluded that BER get reduced with the increase in number of antenna elements at SNR 5db, with 18 cochannel interferes, so the system performance will get improved.

Table 4.7 SNR for Ricean Fading at L=6, BER≤ 10^{-1}

Serial no	No. of antenna elements M	SNR at L=6, BER≤10^{-1}
1	M=1	15dB
2	M=2	-10dB

As from table 4.7, it is concluded that for L=6, BER≤10^{-1}, optimum combining for Ricean Fading with 2 antennas requires 25dB less power as compared to optimum combining with 1 antenna.

Table 4.8 SNR for Ricean Fading at L=18, BER≤ 10^{-1}

Serial no	No. of antenna elements M	SNR at L=18, BER≤10^{-1}
1	M=1	20dB
2	M=2	-10dB

As from table 4.8, it is concluded that for L=18, BER≤10^{-1} , optimum combining for Ricean Fading with 2 antennas requires 30dB less power as compared to optimum combining with 1 antenna.

Summary of the chapter

This chapter describes the major factors that affect the performance of mobile communication systems. The major cause of these defects is multipath propagation. To reduce these defects Rayleigh and Ricean Fading environments are used with Optimum Combining .It is concluded from simulated results that BER get reduced as with increase in number of antenna elements. Also for given BER, power consumption will get reduced with the increase in number of antenna elements. Also both fading environments can be derived from Nakagami Fading Environment which is described in next chapter.

Chapter-5

Optimum combining in Nakagami Fading

5.1 Nakagami Fading Channel

A received signal envelope model which allows for different degrees of fading severity for the desired as well as for interfering signal envelopes, and which is widely used to model many mobile radio environments, is the Nakagami-m distribution.

The Nakagami-m distribution is a versatile statistical model, because it can model fading amplitudes that experience either less or severe fading than that of Rayleigh variates. It sometimes fits experimental data much better than a Rayleigh or Rician distribution [17], [18]. The Nakagami-m distribution of envelope of the received signal is given by [17].

Considering a receiver with M diversity branches, let σ is the root mean square value of received voltage signal before envelope detection, The Nakagami distribution describes the received envelope $z(t)=x(t)$ by a central chi-square distribution with m degrees of freedom, i.e. [19],

$$p_z(x)=\frac{2}{\Gamma(m_k)}\cdot\left(\frac{m_k}{2\sigma^2}\right)^{m_k}\cdot x^{2m_k-1}.e^{-\frac{m_k x^2}{2\sigma^2}}, \quad k=1,2,....M \tag{5.1}$$

Where $\Gamma(.)$ is the Gamma function, where $2\sigma^2$ = E $\{r^2\}$ $=\Omega_s$, and $m_k \geq 1\ 2$ is the fading figure (degrees of freedom related to the number of added Gaussian random variables). The derivation of Nakagami Fading is given in Appendix-A.

5.2 Average Probability of BER

Using the model documented in section 4.4.1 and4.4.1, the average BER of the output of Optimum Combining for Nakagami Fading is given by

$$\tilde{P}_e(\gamma_1) = \frac{1}{2\pi}\left(\frac{m_s}{m_s+\alpha\tilde{\gamma}_s}\right)^{m_s(M-1)} \sqrt{\frac{\alpha\tilde{\gamma}_s}{m_s+\alpha\tilde{\gamma}_s}} \times \left\{ \frac{\Gamma\left(Mm_s - m_s + \frac{1}{2}\right)}{\Gamma\left(Mm_s - m_s + 1\right)} \left(\frac{1+\gamma_1}{\gamma_s / m_s}\right)^{m_s - 1} \right.$$

$$\times {}_2F_1\left(1, m_s(M-1)+\frac{1}{2}; m_s(M-1)+1; \frac{m_s}{m_s+\tilde{\gamma}_s}\right)$$

$$- \sum_{i=0}^{m_s-1} \frac{\Gamma(i+1)\Gamma\left(Mm_s - i - \frac{1}{2}\right)}{\Gamma\left(Mm_s - i\right)} \times \left(\frac{1+\gamma_1}{1+\frac{\alpha\tilde{\gamma}_s}{m_s}}\right)^{m_s-1-i}$$

$$\left. \times {}_2F_1\left(m_s - i, Mm_s - i - \frac{1}{2}; Mm_s - i; -\frac{m_s\gamma_1}{m_s+\alpha\tilde{\gamma}_s}\right) \right\} \tag{5.2}$$

Where γ is the SINR, $\tilde{\gamma}_s = \Omega_s / N_0$ is average desired signal to noise ratio per diversity branch. $\Omega = E\left(r^2\right) = 2\sigma^2$, $\tilde{\gamma}_1 = \Omega_1 / N_0$ is average interference to noise ratio per diversity branch for the interfering signal, $\alpha = 1$ for coherent BPSK. m_s=m is the Nakagami distribution factor, which can have values from m=2 to m=5, means Nakagami distribution is defined for the value of m up to 5. Here results are obtained for m=2, but it can be obtained for m= 3, 4, and 5 also.

5.3 Simulation of BER of Optimum Combining in Nakagami Fading

We plot the average BER versus the average received SINR, defined as $\tilde{\gamma}_b = \tilde{\gamma}_s / (1 + L\tilde{\gamma}_1)$, for several values of *M*. As expected, the system performance improves as *M* increases. The results will show that system performance 'improves as the interference power increases. This is consistent with the results of [9] since in the presence of a strong interfering signal, optimum combining can more easily cancel out the effect of the interferer than in the presence of weak interference. The average BER is plotted versus the average received SINR for both coherent BPSK systems with multiple cochannel interference and for several values of the diversity order.

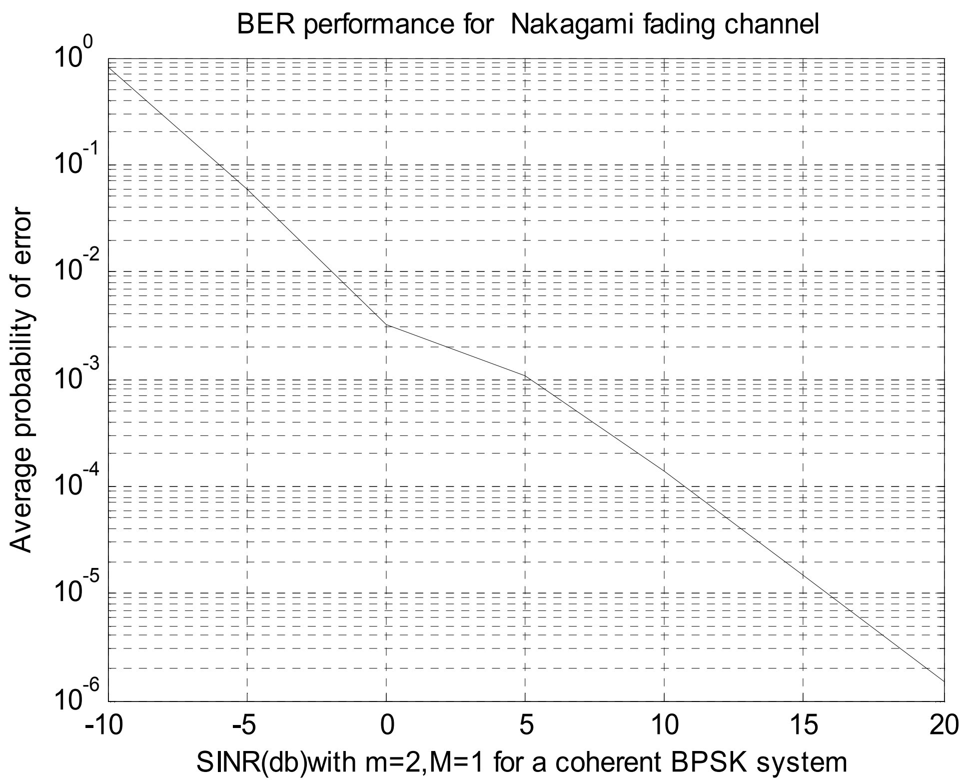

Figure-5.1: BER Performance in Nakagami Fading with M=1, m=2, L=6

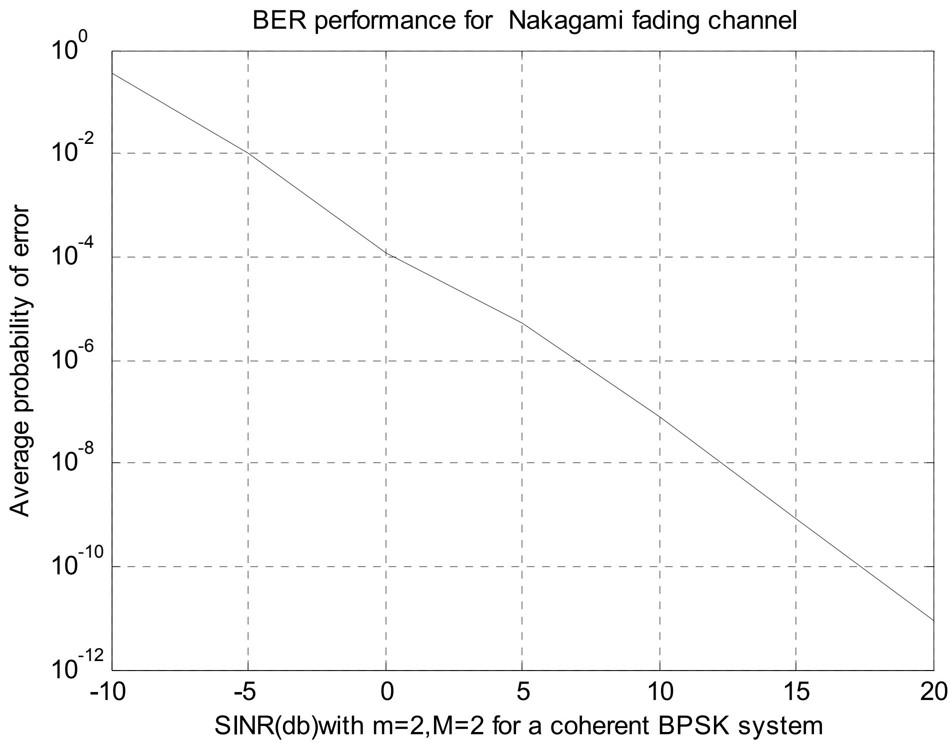

Figure-5.2: BER Performance in Nakagami Fading with M=2, m=2, L=6

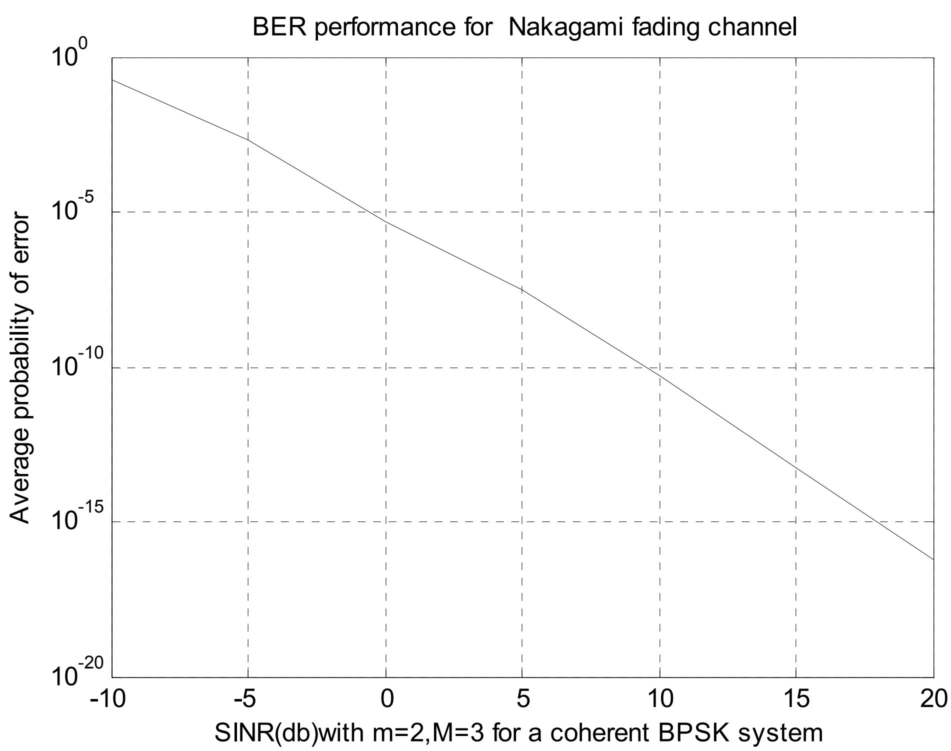

Figure-5.3: BER Performance in Nakagami Fading with M=3, m=2, L=6

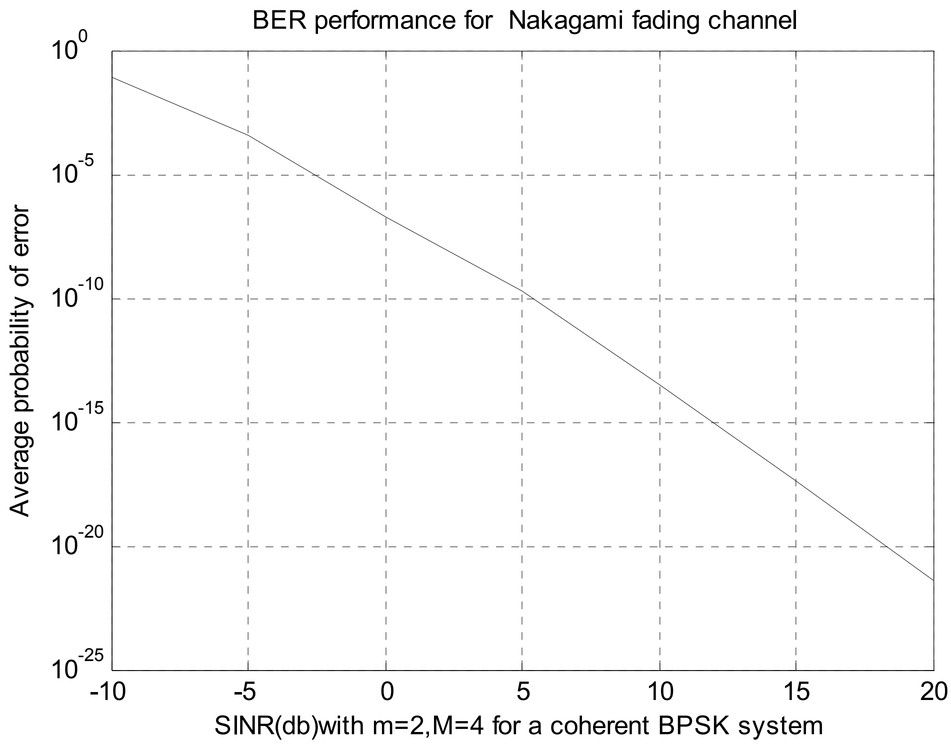

Figure-5.4: BER Performance in Nakagami Fading with M=4, m=2, L=6

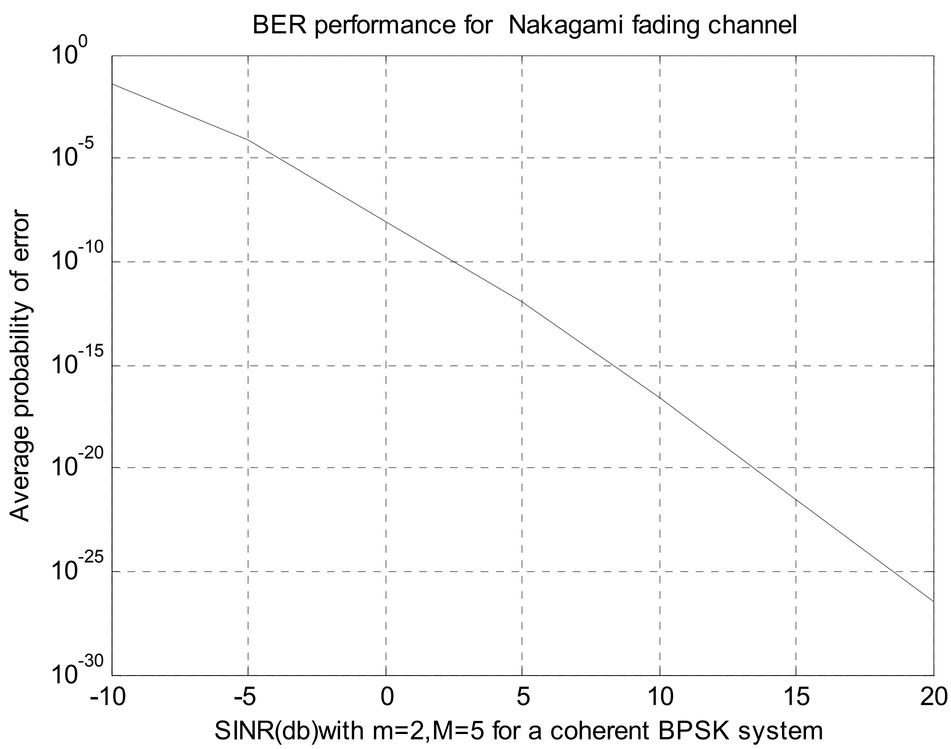

Figure-5.5: BER Performance in Nakagami Fading with M=5, m=2, L=6

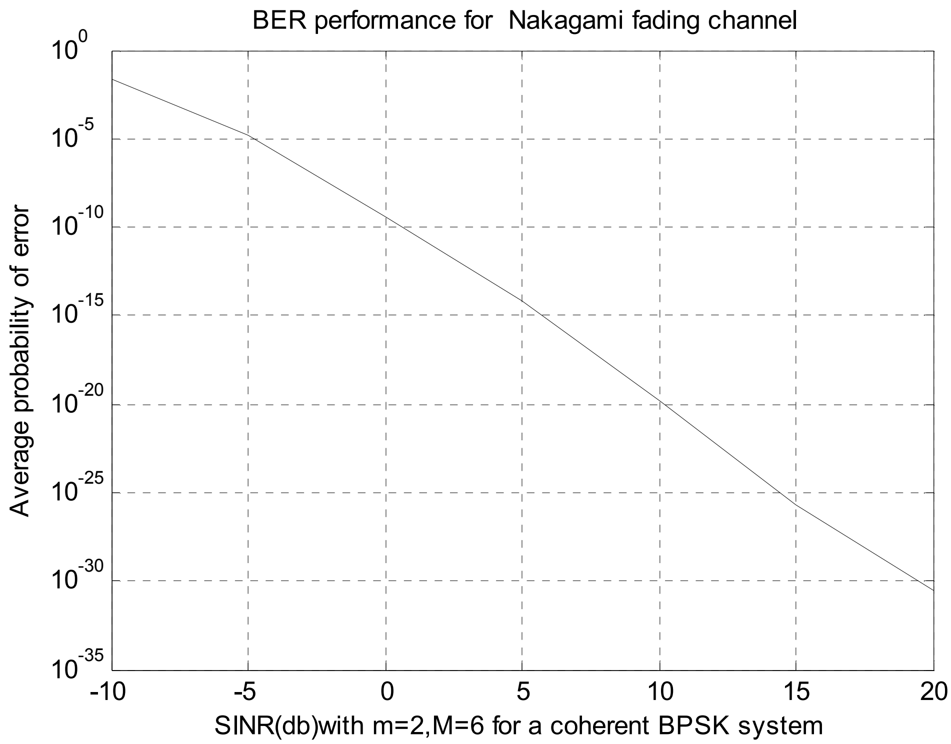

Figure-5.6: BER Performance in Nakagami Fading with M=6, m=2, L=6

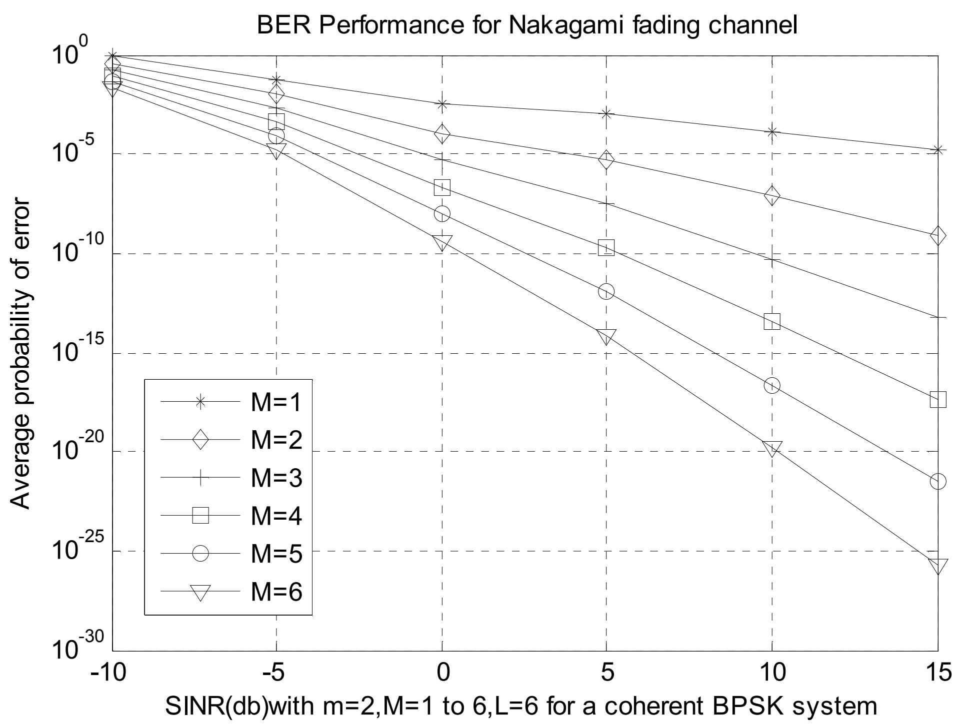

Figure-5.7: BER Performance in Nakagami Fading with L=6, M=1 to 6, m=2

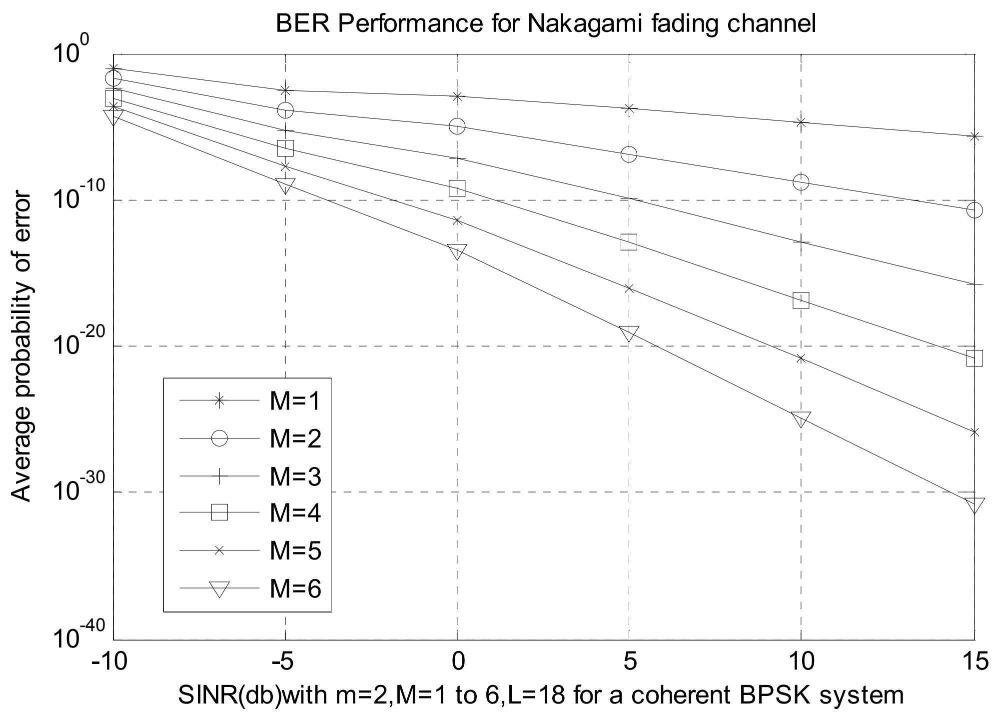

Figure-5.8: BER Performance in Nakagami Fading with L=18, M=1 to 6, m=2

- Fig. 5.1 to 5.6 shows for Nakagami Fading that with 6 cochannel interferers, BER decreases with increase in SINR, which results that system performance improves as the interference power decreases, with m=2, with changing number of antenna elements.
- Fig. 5.7 shows the combined plot of Nakagami Fading for m=2 with M=1 to 6, which shows that with 6 cochannel interferers, increase in number of antenna elements, performance of system will get improved.
- Fig. 5.8 shows the combined plot of Nakagami Fading for m=2 , with M=1 to 6, which shows that with 18 cochannel interferers, increase in number of antenna elements, performance of system will get improved.

Table 5.1 BER for Nakagami Fading at L=6, SNR=5db

Serial no	No. of antenna elements M	BER at L=6,SNR=5db
1	M=1	-0.0033
2	M=2	-0.0001
3	M=3	-0.0000
4	M=4	-0.0000
5	M=5	-0.0000
6	M=6	-0.0000

Table 5.2 BER for Nakagami Fading at L=18, SNR=5db

Serial no	No. of antenna elements M	BER at L=18,SNR=5db
1	M=1	-0.0002
2	M=2	-0.0000
3	M=3	-0.0000
4	M=4	-0.0000
5	M=5	-0.0000
6	M=6	-0.0000

As from Table 5.1 and 5.2 it is concluded that BER get reduced with the increase in number of antenna elements at SNR 5db, with 6 and 18 cochannel interferes respectively, so the system performance will get improved.

Now tables 5.3 and 5.4 shows the comparison of SNR for a given BER is evaluated at L=6, and L=18 respectively

Table 5.3 SNR for Nakagami Fading at L=6, BER≤ 10^{-1}

Serial no	No. of antenna elements M	SNR at L=6, BER≤10^{-1}
1	M=1	5dB
2	M=2	-5dB

As from table 5.3, it is concluded that for L=6, BER≤10^{-1} , optimum combining for Nakagami Fading with 2 antennas requires 10dB less power as compared to optimum combining with 1 antenna.

Table 5.4 SNR for Nakagami Fading at L=18, BER≤ 10^{-1}

Serial no	No. of antenna elements M	SNR at L=18, BER≤10^{-1}
1	M=1	10dB
2	M=2	-5dB

As from table 5.4, it is concluded that for L=6, BER≤10^{-1} , optimum combining for Nakagami Fading with 2 antennas requires 15dB less power as compared to optimum combining with 1 antenna.

5.3 Derivation of Rayleigh and Rician fading from Nakagami Fading Distribution

The Nakagami distribution is often used to model multipath fading as it can model fading conditions that are either more or less severe than Rayleigh fading. When m=1, Nakagami distribution becomes the Rayleigh distribution, when m=1/2 it becomes a one-sided Gaussian distribution and when $m \to \infty$ the distribution becomes an impulse (no fading).

Let us define m = $E[R] / \sqrt{E[R^2]}$ and g = $V[R^2]/(E[R^2])^2$, with $V[.]$ denoting the variance.

It is straightforward to show that

$$\mu = \frac{\sqrt{\pi}}{2}(K+1)^{-1/2}\exp(-K/2)\left[(K+1)I_0(K/2) + KI_1(K/2)\right] \tag{5.3}$$

$$\gamma = \frac{2K+1}{(K+1)^2} \tag{5.4}$$

Note that both μ and γ depend only on *K*, and the effect of Ω is canceled out by the proper definitions of the ratio of the moments. This enables us to estimate *K* and Ω separately. Based on the sample estimate of μ, an estimate of *K* can be obtained by solving the nonlinear equation in (5.3), numerically. However, *K* can be expressed in terms of γ explicitly as:

$$K = \frac{\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}} \tag{5.5}$$

γ turns out to be a useful quantity as it also provides a reliable and simple moment-based estimator for the *m* parameter of Nakagami fading distribution, which is $m = 1/\gamma$. Even Rice distribution can be closely approximated using Nakagami parameter *m* via the relationship $m = (K+1)^2/(2K+1)$. We can also simulate BER for Rayleigh and rice fading environment derived from Nakagami fading and then compare the results with Nakagami Fading.

5.4 Simulated Results for Rayleigh fading derived from Nakagami fading:

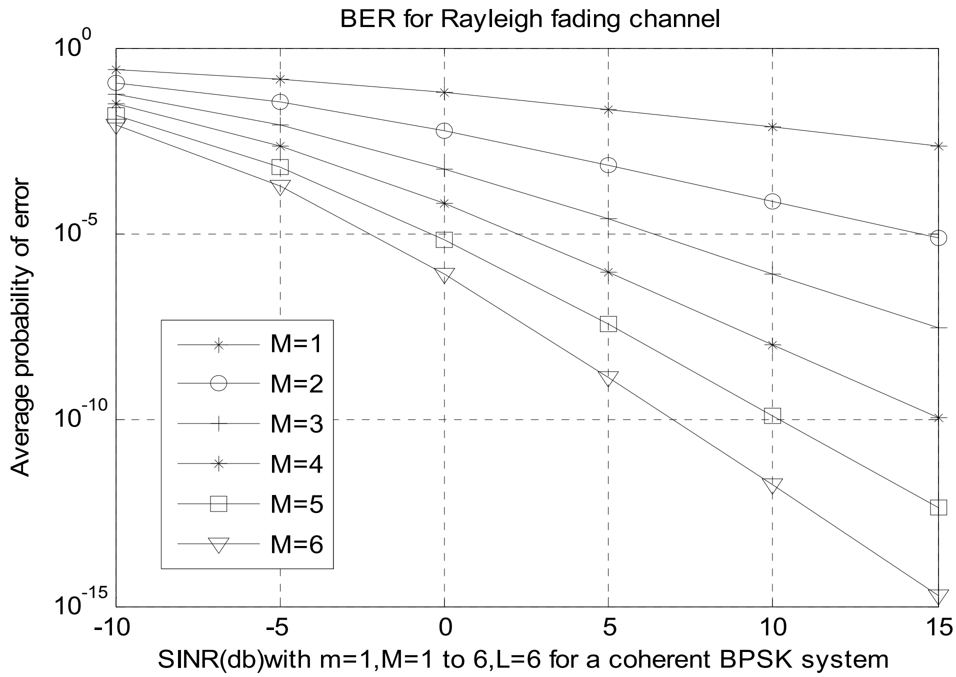

Figure-5.9: BER Performance in Rayleigh Fading with M=1 to 6, m=1, L=6

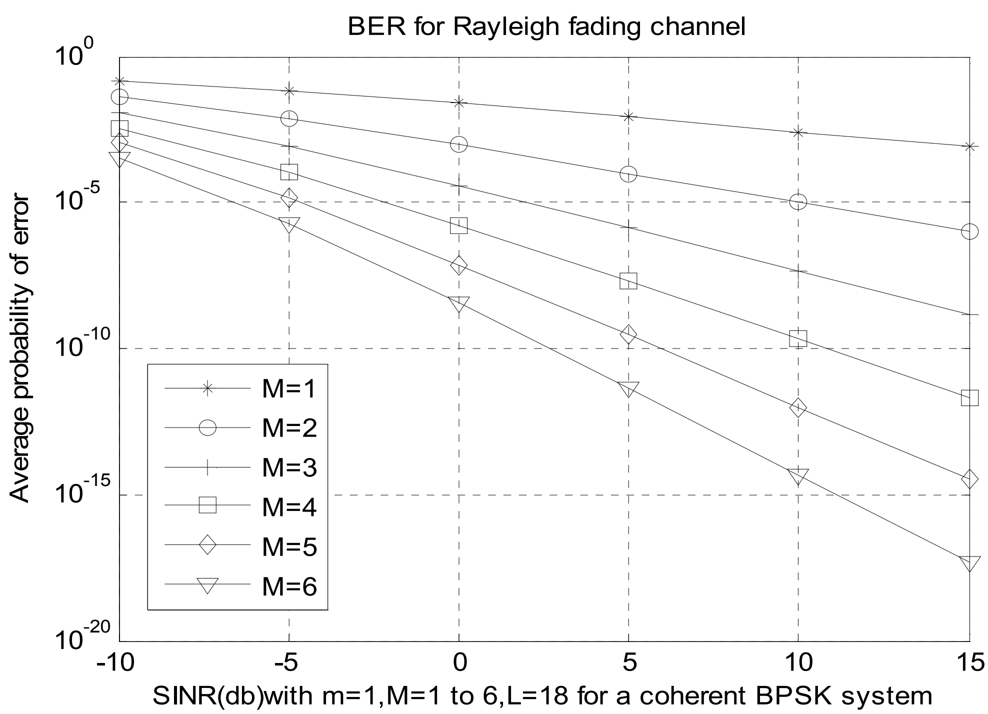

Figure-5.10: BER Performance in Rayleigh Fading with M=1 to 6, m=1, L=18

5.5 Simulated Results for Ricean fading derived from Nakagami fading:

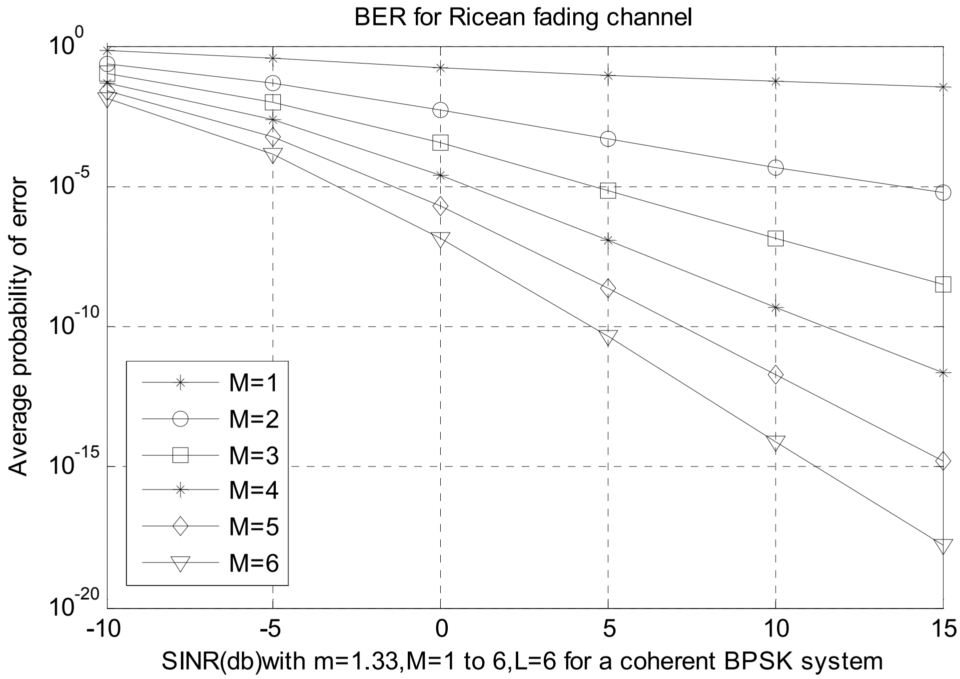

Figure-5.11: BER Performance in Ricean Fading with M=1 to 6, m=1.33, L=6

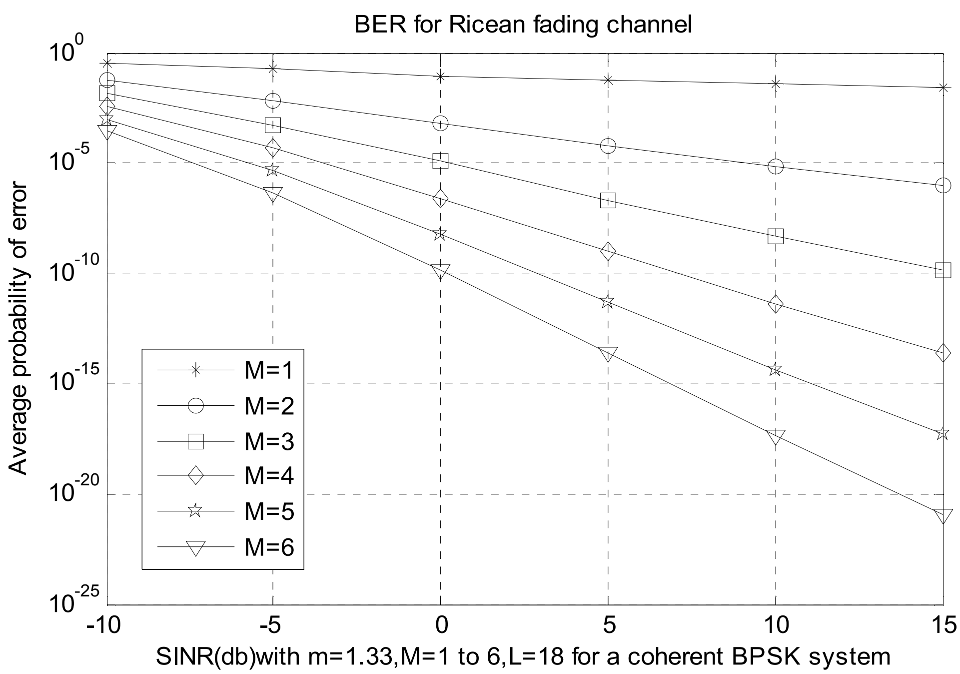

Figure-5.12: BER Performance in Ricean Fading with M=1 to 6, m=1.33, L=18

- Fig. (5.9)-(5.10) shows the combined plot for Rayleigh Fading for m=1 with M=1 to 6, which shows that with 6 and 18 cochannel interferers respectively, with increase in number of antenna elements, BER performance for Rayleigh Fading derived from Nakagami Fading of the system will get improved.
- Fig. (5.11)-(5.12) shows the combined plot for Ricean Fading for m=1 with M=1 to 6, which shows that with 6 and 18 cochannel interferers respectively, with increase in number of antenna elements, BER performance for Ricean Fading derived from Nakagami Fading of the system will get improved.

Comparison of BER performance is also done for multipath fading with 6 and 18 cochannel interferes, which is shown in fig 5.13 and fig 5.14 respectively.

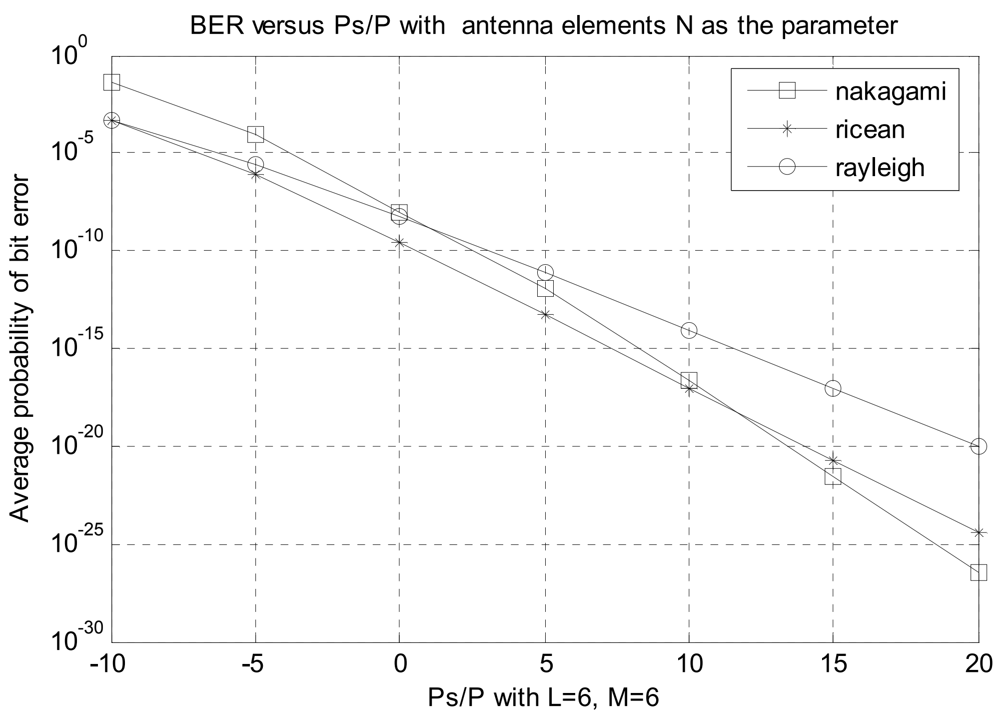

Figure-5.13: BER for Multiple Fading Environments with M=6, L=6

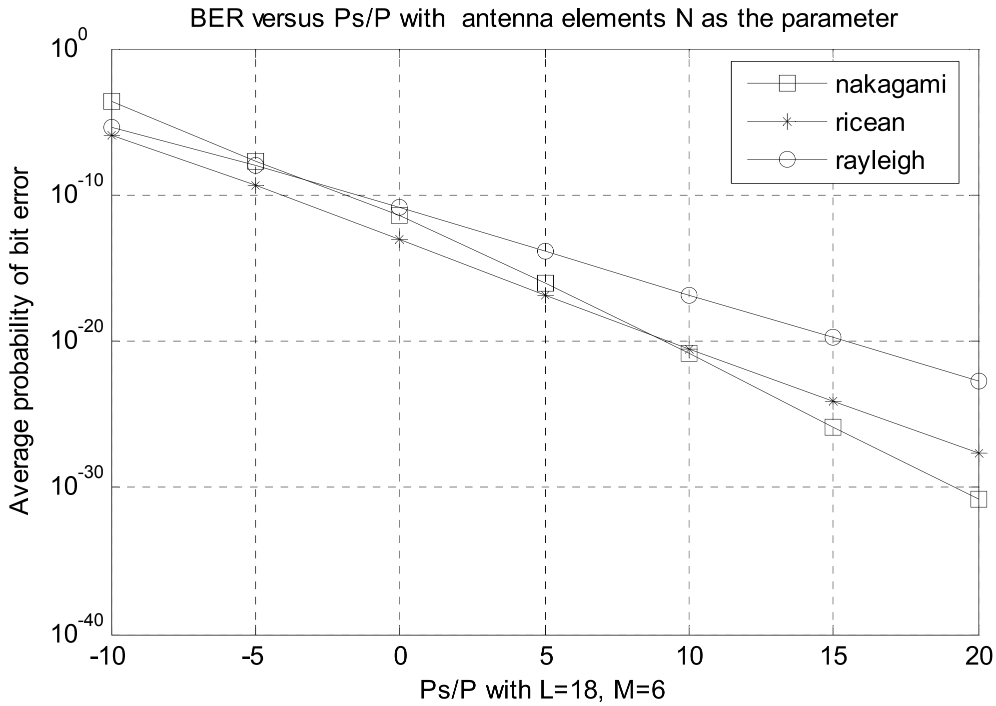

Figure-5.14: BER for Multiple Fading Environments with M=6, L=18

- Figure (5.13) and (5.14) shows for Multipath Fading Environments for a given SINR, for L=6 and L=18 cochannel interferes respectively. Nakagami Fading Distribution is best among other Distributions because it is giving less BER as compared to other Distributions.

Summary of chapter

This chapter describes Nakagami Fading Distribution, its BER expression, also Simulation results. In later sections we derived both Rayleigh and Ricean Fading Distribution from Nakagami Fading Distribution and compare their BER performance. The optimum combiner refers to the antenna arrays that maximizes signal to interference ratio at the output of the array. We derived, for BPSK signals in space diversity with optimum combining. In next chapter, we will discuss its conclusions and Future Scope.

Chapter-6

Conclusions and Future Scope

In this dissertation, a method is proposed called as Optimum combining which can mitigate the effects of co-channel interference and multipath fading. The project involves simulation of optimum diversity technique. Optimum diversity technique was simulated in MATLAB. The previous studied diversity combining techniques Selection Combining (SC), Equal Gain Combining (EGC), and Maximum Ratio Combining (MRC) were effective in the environment of no interference. But optimum combining is one technique which is effective against multipath fading and co-channel interference.

Optimum combining is used in digital cellular mobile radio communication systems with multiple cochannel interferers in the Rayleigh, Ricean, and Nakagami fading environments. The optimum combiner refers to the antenna arrays that maximizes signal-to-interference ratio at the output of the array. We derived, for BPSK signals in space diversity with optimum combining. Closed form expressions for the BER were presented for the case of multiple interferers. These closed form expressions provide a convenient tool for system analysis and a substitute for time consuming Monte Carlo simulations.

The results were used to plot Bit Error Rae for various fading channels. This chapter is included to compare the simulated results of Multipath Fading Environments for BER Performance. This Chapter also focuses on future scope on thesis work.

6.1 Conclusions

Rayleigh and Ricean fading environments are discussed in chapter 4th, Nakagami is discussed in 5th chapter. Simulated results has been shown in sections (4.4.3), (4.3) and (5.3) for Rayleigh, Ricean and Nakagami fading environments respectively. Simulated results show that BER get reduced by increasing number of antenna elements. The comparison of BER Performance for multipath fading is shown in Table 6.1.

Table 6.1 A BER Comparison for Multiple Fading Environments for L=6

Serial No	No of antennas elements	BER for Nakagami Distribution	BER for Ricean Distribution	BER for Rayleigh Distribution
1	M=1	-0.0033	0.0750	0.1986
2	M=2	-0.0001	0.0001	0.0893
3	M=3	-0.0000	0.0000	0.0365
4	M=4	-0.0000	0.0000	0.0122
5	M=5	-0.0000	0.0000	0.0030
6	M=6	-0.0000	0.0000	0.0004

Table 6.1 extracts a conclusion that BER is less for Nakagami Fading Distribution as compared to other Distributions, which is also shown in fig (5.13). In other words, it refers that optimum combining with Nakagami Fading Environment is best among other Distributions.

6.2 Future Scope

In future these diversity techniques can further be applied to the more realistic frequency-selective fading channel models. Moreover, the work can also be extended to other combining technique Adaptive Combining. Lots of work is been going on to enhance the mobile communication systems performance with the use of Smart Antenna technology. And also there are other methods with which BER of the system can be reduced like Equalization, channel coding. This work can also be extended with the use of these technologies.

Appendix-A

We assume that the propagation channel is such that the received signal envelope has a Nakagami-m distribution while the received signal phase is uniformly distributed. It can be shown that under some conditions, such a channel model belongs to the family of spherically invariant distributions [20]. Specifically, let the complex envelope of received signal be in the form

$$\tilde{r}(t) = R(t)e^{j\varphi(t)} \tag{A.1}$$

Where *R (t)* is the received signal envelope, the phase $\varphi(t)$ is uniformly distributed in (0, 2π) and R and φ are independent. We assume that the quadrature components of $\tilde{r}(t)$ *have* a circularly symmetric joint probability density function (pdf) of the form

$$f_{z_c, z_s}(z_c, z_s) = \frac{1}{2\pi} h_2(z_c^2 + z_s^2) \tag{A.2}$$

Where the function $h_2(.)$ belongs the class of 'admissible' functions for a spherically invariant random process (SIRP) [20]. The special case of h_2 (p) = exp (-p/2) leads to Gaussian distributed quadratures and a Rayleigh distributed envelope. It follows from (A.2) that the joint pdf of *R* and φ is given by

$$f_{R\varphi}(r,\varphi) = \frac{r}{2\pi} h_2(r^2) \tag{A.3}$$

Consequently, the pdf of the envelope is then given in terms of the function h_2 (.) as

$$f_R(r) = rh_2(r^2) \tag{A.4}$$

It is shown in [8] that an appropriate choice of the function is h_2 *(.)* is

$$h_2(w) = \frac{2}{\Gamma(v)} b^{2v} w^{v-1} \exp(-b^2 w) \tag{A.5}$$

Substituting (A.5) in (A.4) yields the Chi-distributed envelope

$$f_R(r) = \frac{2b}{\Gamma(v)} (br)^{2v-1} \exp(-b^2 r^2) \tag{A.6}$$

Where ***b*** denotes the scale parameter and *U* is a shape parameter of the distribution. We recognize that for the special case when $v = m$ and $\boldsymbol{b^2} = \frac{m}{\Omega}$, (A.6) reduces to the Nakagami-m pdf for the envelope of the received, signal given by

$$f_R(r) = 2\left(\frac{m}{\Omega}\right)^m \frac{r^{2m-1}}{\Gamma(m)} \exp(-\tfrac{m}{\Omega} r^2); r \geq 0 \tag{A.7}$$

Where the parameter $m \geq 1/2$ determines the severity of fading and $\Omega = E(R^2) = 2\sigma^2$. This distribution has been shown to fit experimental data from a variety of wireless environments. It has the Rayleigh envelope distribution (when m = 1) as a special case and -can approximate the Rice distribution in a practical range of Ω values [17].

References

1) T. S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice-Hall PTR, NJ, 1996.
2) J.Paulraj and C. B. Papadias, "Space-time processing for wireless communications," *IEEE Signal Processing Mag.*, pp. 49-82, Nov. 1997.
3) Saunders, S.; *Antennas and Propagation for Wireless Communication Systems*, Wiley, 2000.
4) S. M. Alamouti, "A simple transmit diversity technique for wireless communications," IEEE Journal on Select Areas in Communications, vol. 16, pp. 1451–1458, August 1998.
5) D. G. Brennan, "Linear diversity combining techniques," *Proc. IRE,* vol. 47, pp. 1075–1102, 1959.
6) R. G. Vaughan and N. L. Scott, "Closely spaced monopoles for mobile communications," *Radio Sci.,* vol. 28, pp. 1259–1266, 1993.
7) J. H. Winters, J. Salz, and R. D Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. Commun.,* vol. 42, pp. 1740–1751, 1994.
8) J. H. Winters, "Optimum combining for indoor radio systems with multiple users," *IEEE Trans. Commun.,* vol. COM-35, pp. 1222–1230, 1987.
9) J. H. Winters, "Optimum combining in digital mobile radio with co-channel interference," *IEEE J. Select. Areas Commun.,* vol. SAC-2, pp. 528–539, 1984.
10) R. G. Vaughan, "On optimum combining at the mobile," *IEEE Trans. Veh. Technol.,* vol. 37, pp. 181–188, 1988.
11) B.Widrow,J.McCooI; and M.Ball, "The complex LMS algorithm," Proc.IEEE,vol.63,p.719,apr.1975.
12) C. A. Baird,Jr. andC.L. Zahm. "Performance criteria for narrowband array processing," ***1971 Conf. Decision Contr.,*** Miami Beach, FL, Dec. 15-17, 1971, p. 564.
13) G. L. Stuber, *Principles of Mobile Communication*. Boston, MA: Kluwer, 1996.

14) K. K. Talukdar and W. D. Lawing, "Estimation of the parameters of the Rice distribution," *J. Acoust. Soc. Am.*, vol. 89, pp. 1193-1197, 1991.

15) R. A. Monzingo and T. W. Miller, *Introduction to Adaptive Arrays.*New York: Wiley, 1980.

16) C. G. Khatri, "On certain distribution problems based on positive definite quadratic functions in normal vectors," *Ann. Math. Stat.*, vol. 37, pp. 468–479, 1966.

17) N. Nakagami, "The m-distribution, a general formula for intensity distribution of rapid fading" , in *Statistical Methods in Radio Wave Propagation,* N. *G.* Hoffman, Ed., Pergamon, Oxford, England, 1960.

18) U. Charash, "Reception through Nakagami fading multipath channel with random delays", *IEEE Trans. Commun.,* ~01.27p, p.657-670, April 1979.

19) Gradshteyn and I. Ryzhik, *Tables* of *Integrals, Serzes* ***and*** *Products,* Academic Press. New York, 1980.

20) A 1 **.** Rangaswang, D**.** Weiner, and .A**.** Ozturk, "Non-Gaussian random vector identification using spherically invariant random processes". *IEEE Trans.Aerosp. Elect. Syst.,* vol. 29: pp.111-123. January 1993.

List of Publications

1. Rabina bansal, Amanpreet Kaur, Rajesh Khanna," CMOS IMAGE SENSORS-A REVIEW", published in National Conference on "Trends in Electronics, Computers and Communication", organized by Department of Electronics and Communication Engineering at Thanthai Periyar Government Institute of Technology Vallore-2, Tamil Nadu held on 24th April 2006.
2. Rabina bansal, Rajesh Khanna, "BER Performance of Optimum Combining with BPSK Modulation scheme in different fading environments", accepted in National Conference on "Wireless Networks and Embedded Systems" organized by Electronics and Communications Department at Chitkara Institute of engineering and Technology, Rajpura to be held on 28th July 2006.