DQM FREE VIBRATION ANALYSIS OF MULTI SPAN COMPOSITE LAMINATED PLATES

A Thesis report submitted

in partial fulfillment of the requirements for

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IN

(CAD/CAM & ROBOTICS)

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CERTIFICATE

This is to certify that the thesis titled, **"DQM FREE VIBRATION ANALYSIS OF MULTI SPAN COMPOSITE LAMINATED PLATES"**, being submitted by **Mr. RAJU YADAV,** in partial fulfillment of the requirement for the award of degree of **MASTER OF ENGINEERING (CAD/CAM & ROBOTICS)** at **Mechanical Engineering Department**, **Thapar University, Patiala,** is a bonafide work carried out by him under our guidance and supervision and that no part of this thesis has been submitted for the award of any other degree.

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NOMENCLATURE

u, v, w = Displacement in x, y, z direction

ψ_x, ψ_y = Rotations about x and y directions

Q_{ij} = Material constants

$\overline{Q}_{ij}$ = Transformed material constants

N_x, N_y = Number of grid points in x and y direction

M_x, M_y = Moment resultant in x and y direction

Q_x , Q_y = Shear force in x and y direction respectively

k_{ij} = Shear correction factor = 5/6

N = Number of layers

$C_{ij}^{(n)}$ = Weighting coefficients for n_{th} order

ν, E , G = poisson's ratio, young's modulus, shear modulus

D = Plate flexural rigidity = $Eh^3/12(1-\nu^2)$

r = Radial coordinate

p_z = Load in thickness direction

A_{ij} = Extensional stiffness

D_{ij} = Bending stiffness

B_{ij} = Bending-extensional coupling stiffness

t = Thickness of plate

NOS= Number of layers

ω = Frequency parameter

K_w = Elastic Lateral Edge Parameter

S_ϕ = Elastic Edge Parameter

LIST OF FIGURES

ABSTRACT

A global numerical technique, the differential quadrature (DQ) method, is examined here for its suitability to solve the boundary-value problem of symmetric and asymmetric cross-ply laminates using the first-order shear deformation plate theory.

The frequency parameters of symmetric and asymmetric cross-ply laminates, subject to elastic restraints constraints, are investigated. In this study, the method is used to transform the sets of governing differential equations and boundary conditions of the laminated plates into sets of linear algebraic equations. Boundary conditions along the edges are implemented through the discrete grid points by constraining the displacements, bending moments and rotations.

In this study the eigen values and mode shapes are investigated for different variables.

Chapter 1
INTRODUCTION

The continuing quest for improved performance, specified in terms of weight reduction, high strength and low cost has led to develop a new class of materials called composite materials.

A composite material consists of combining two or more constituents called matrix and reinforcement. The constituents are combined at microscopic level and are not soluble to each other. Matrix phase materials are generally continuous while the reinforcing phase may be in the form of fibers, particles or flakes.

The composite materials possess characteristic properties, such as high stiffness, high strength, low weight, high temperature performance, good corrosion resistance, high hardness and conductivity that are not possible in any of its constituents alone. Analysis of these properties reveals that they depend on the following:

a) Properties of the individual constituents.
b) Relative amounts of the constituents.
c) Size and shape of the constituents (i.e. Morphology).
d) Degree of bonding between constituents.
e) Orientation of the various constituents.

The composite materials possess high specific modulus and specific strength as compared to conventional materials. Specific Modulus is defined as the ratio of Young Modulus and density whereas Specific Strength is defined as the ratio of strength and density of materials. As an example the strength of a graphite/epoxy unidirectional composite material is same as that of steel but specific strength of this composite is three times that of steel thereby saving in cost of material and energy [1].

1.1 PARAMETERS FOR SELECTION OF COMPOSITES

For selecting a composite material for a particular application, the following parameters are to be considered:

(i) Strength
(ii) Toughness

(iii) Formadibility

(iv) Weldeability

(v) Corrosion

(vi) Wear Resistance

(vii) Affordability.

1.2 FACTORS AFFECTING MECHANICAL PERFORMANCE

The mechanical performance of composite materials depends on number of factors such as:

I) Fiber factor

II) Matrix factor

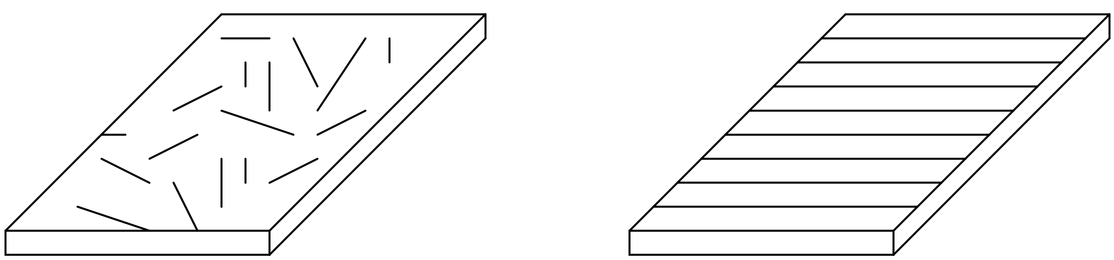
III) Other factors (Fiber Matrix interface, etc.)

I) Fiber factor:

It involves the following four parameters.

a) Length
b) Orientation
c) Shape
d) Material

a) Length: - Fiber can be either short or long in the length as shown in Fig.1.2



(a) **(b)**

FIG.1.2 (a) SHORT FIBER COMPOSITE, (b) LONG FIBER COMPOSITE

Long continuous fibers are easy to orient and process while short fibers cannot be fully controlled or oriented properly. Further, long fibers provide several benefits over short fibers including high impact resistance, low shrinkage, improved surface finish and dimensional stability. On the other hand, short fibers provide low cost, easy to work with and have fast cycle time fabrication procedures. Short fibers possess few flaws, therefore leading to higher strength.

b) Orientation: - The distribution of fiber in a matrix can be random or aligned in a specific direction to achieve very high stiffness and strength in that direction. If fibers are oriented in more than one direction in matrix, the composite will exhibit possess high stiffness and strength along these directions.

c) Shape: - The most common shape of fibers is circular because of ease of handling and manufacturing. Hexagonal and square-shaped fibers are possible but their advantages of strength and high packing factors do not out weigh the difficulty in handling and processing.

d) Material: - The material of fiber directly influences the mechanical performance of a composite. Fibers are generally expected to have high elastic modulus and strength. This expectation along with the low cost are the key factors that graphite, aramids and glass fibers dominate the fiber market for composites.

II) Matrix Factors

Fibers are used as reinforcement to matrix. The matrix functions include binding of the fibers together, protecting fibers from environment, shielding from damage during handling and load transfer from matrix to fibers. In general, the matrix possesses inferior properties compared fibers.

III) OTHER FACTORS

Apart from fiber and matrix there are several other factors, which can affect the mechanical performance of composite materials. The fiber-matrix interface is an important factor, which determines how well the matrix transfers the loads to the fibers. The fiber-matrix interfacial bonding is of following three types:

a) Chemical Bonding
b) Mechanical Bonding

c) Reaction Bonding

a) Chemical Bonding

Chemical bonding is formed between the fiber surface and the matrix. Some fibers bond naturally to the matrix while others do not. Coupling agents are often added to form a chemical bond.

b) Mechanical Bonding

Natural roughness and etching of the fiber surface causing interlocking may form a mechanical bond between the fiber and the matrix. If the coefficient of thermal expansion of the matrix is higher than that of fiber, and the manufacturing temperatures are higher than the operating temperatures, the matrix will shrink more than the fiber, causing the compression of matrix around the fiber.

c) Reaction Bonding

Reaction bonding occurs when atom or molecules of fiber and matrix diffuse into each other at interface. This inter diffusion often creates a distinct interfacial layer, which has different properties than that of fiber or matrix. Though, this thin layer helps to form a bond but it also forms micro cracks in the fiber. These micro-cracks reduce the strength of fiber, which ultimately leads to poor strength of composite materials.

1.3 TYPES OF COMPOSITE MATERIALS

omposite materials are commonly formed in three different types:

a) Fiber Composites
b) Particulate Composites
c) Laminated Composites

a) Fiber Composites

It consists of matrix reinforced by short (discontinuous) or long (continuous) fibers as shown in Figs. 1.2. Fibers are generally anisotropic. Typical examples of matrices are resins such as epoxy, metals such as aluminum and ceramics such as calcium-alimino-silicate.

b) Particulate Composites

It consists of particles reinforced in matrices such as alloy and ceramics as shown in Fig.1.3(a). They are usually isotropic since the particles are randomly distributed. Particulate composite has advantages such as improved strength, increased operating temperature and oxidation resistance etc. Examples include use of aluminum particles in rubber matrix, silicon particles in aluminum matrix etc.

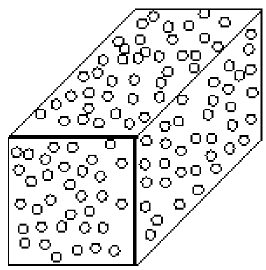


FIGURE.1.3(a) PARTICULATE COMPOSITE

c) Laminated Composites

It consists of flake shaped reinforcement such as glass, mica, silica, silver etc in matrices as shown in Fig.1.3(b). Flake composites provide advantages such as high out of plane flexural modulus, higher strength and low cost. However, flakes can not be oriented easily therefore a limited number of materials are available for use.

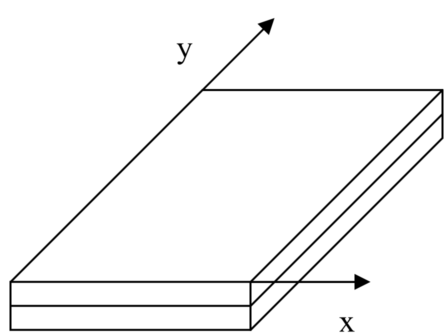


FIG.1.3(b) LAMINATED COMPOSITE

1.4 LIMITATIONS OF COMPOSITES

Composite materials are having the following limitations.

a) High cost of fabrication of composite materials. For example, a part made-up of graphite/epoxy composite may cost up to 10 to 15 times of materials cost.
b) Mechanical characterization of composite structure is more complex than that of a metal structure. Composite materials are not isotropic (i.e. do not possess same properties in all the direction). For example, a single layer of graphite/epoxy composite requires nine stiffness and strength constants for conducting mechanical analysis while in the case of steel only four stiffness and strength constant are required.
c) Repair of composite is not a simple process as compared to metals.
d) Composites do not have good combination of strength and fracture toughness as compared to metals.
e) Composites do not necessarily give higher performance in the all the property used for material selection.

Chapter 2

THEORIES OF LAMINATED COMPOSITE PLATES

2.1 THE CLASSICAL LAMINATED PLATE THEORY [3]

Assumptions

The classical laminated plate theory is an extension of the classical plate. Theory to composite laminates .In the classical laminated plate theory e (CLPT) it is assumed that the Kirchoff's hypothesis holds:

1) Straight line perpendicular to the midsurface (i.e., transverse normal) before deformation the main straight after deformation
2) The transverse normal do not experience elongation (i.e., they are inextensible).
3) The transverse narmals rotates such that they remain perpendicular to the mid surface after deformation.

The first two assumptions implies that the transverse displacement is independent of the transverse. (or thickness) coordinate and transverse normal strain ε_{zz} is zero. The third assumption results in zero transverse shear strain, $\varepsilon_{xz} = 0$, $\varepsilon_{yz} = 0$.

It is the simplest equivalent single-layer laminated theory. It is based on the Kirchoff's plate theory, which is assumed that straight lines normal to the x-y plane before deformation remain straight and normal to the mid-surface after deformation. Thus it is neglects the effect of both transverse shear and transverse normal. The displacement components (u, v, w) are given by [3]:

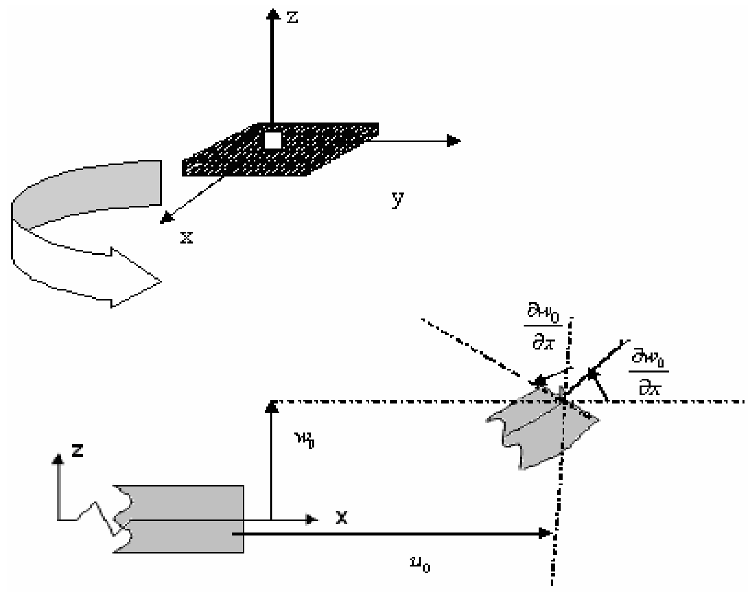


FIGURE 2.1. UNDEFORMED AND DEFORMED GEOMETRIES OF AN EDGE OF A PLATE UNDER THE KIRCHHOFF HYPOTHESIS

$$u(x, y, z, t) = u_0(x, y, t) - z\frac{\partial w0}{\partial x}$$

$$v(x, y, z, t) = v_0 (x, y, t) - z\frac{\partial w0}{\partial y}$$

$$w(x, y, z, t) = w_0 (x, y, t)$$

Where t denotes time (u_0, w_0, v_0) are the displacement components along the (x, y, z) coordinates directions, respectively, of a point on the mid plane (i.e. z=0)

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 - z\frac{\partial^2 w_0}{\partial x^2}$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right) - 2z\frac{\partial^2 w_0}{\partial x \partial y}$$

$$\varepsilon_{yy} = \frac{\partial u_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - z\frac{\partial^2 u_0}{\partial y^2}$$

$$\varepsilon_{xz} = 0, \varepsilon_{yz} = 0, \varepsilon_{zz} = 0$$

2.2 THE FIRST ORDER SHEAR DEFORMATION THEORY

It is also one of the equivalent single layer laminat theories. This theory assume that transverse normal do not remain perpendicular to the mid surface after deformation. This amounts to include the effect of transverse shear .the displacement field theory is given by

$u(x, y, z, t) = u_0(x, y, t) + z\phi_x (x, y, t)$

$v(x, y, z, t) = v_0(x, y, t) + z\phi_y (x, y, t)$

$w(x, y, z, t) = w_0(x, y, t)$

Where u, v, w are the displacement component along x, y, z axies, (u_0, v_0, w_0) are the displacement component along the x, y, z coordinate direction respectively, of a point on the mid plane (i.e. z = 0), $\phi x = \frac{\partial u}{\partial z}$ and $\phi y = \frac{\partial v}{\partial z}$ are the rotations of a transverse normal about y and x axes respectively.

Linear strain-displacement relation yield strain as:

$$\varepsilon_x = \frac{du^0}{dx} + z\frac{d\psi_x}{dx}$$

$$\varepsilon_y = \frac{dv^0}{dy} + z\frac{d\psi_y}{dy},$$

$$\varepsilon_z = 0$$

$$\gamma_{xy} = \frac{du^0}{dy} + \frac{dv^0}{dx} z\left(\frac{d\psi_x}{dy} + \frac{d\psi_y}{dx}\right)$$

$$\gamma_{yz} = \psi_y + \frac{dw^0}{dy}$$

$$\gamma_{zx} = \psi_x + \frac{dw^0}{dx}$$

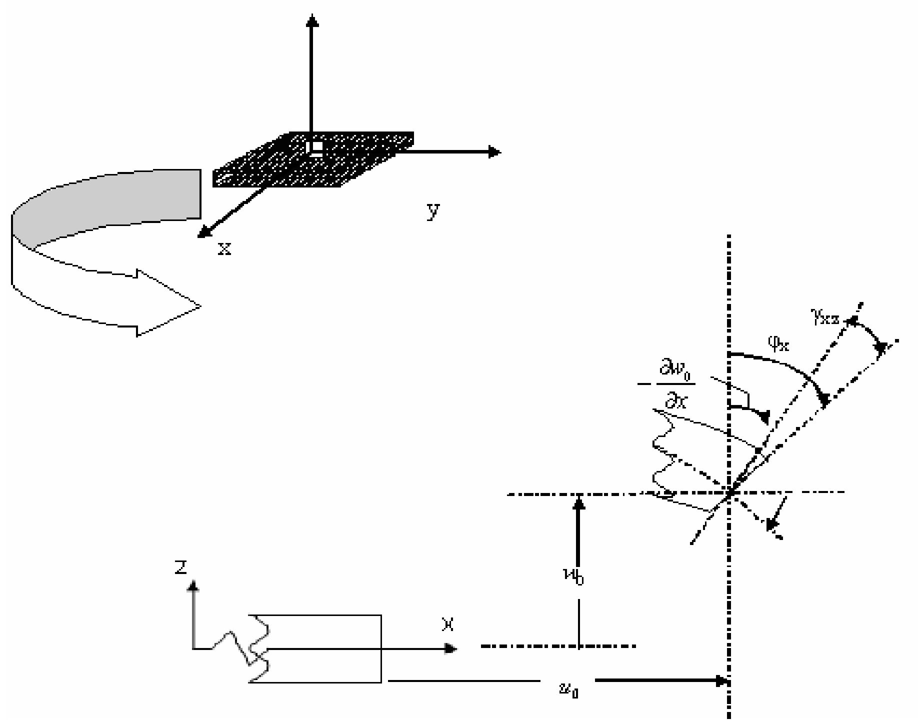


FIGURE 2.2. UNDEFORMED AND DEFORMED GEOMETRIES OF AN EDGE OF A PLATE UNDER THE ASSUMPTIONS OF FIRST ORDER SHEAR DEFORMATION THEORY

Where:

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ =Linear strain

And $\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ =shear strains

2.2.1 Laminate constitutive equations

Here we derive the constitutive equations that relate the force and moment resultants to the strains of a laminate. To this we assume that each layer is orthotropic with respect to its material symmetry lines and obeys Hook's law, holds for the k^{th} lamina in the problem coordinates. For the moment we consider the case in which the temperature and piezoelectric effects are not included. Although the strains are continuous through the thickness,(i.e. ,each lamina). Hence, the integration of stresses through the laminate thickness requires laminawise integration. The force resultants are given by

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} z dz$$

$$= \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_{xx}^{0} + z\varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{0} + z\varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{0} + z\gamma_{xy}^{(1)} \end{Bmatrix} z dz$$

or

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{0} \\ \varepsilon_{yy}^{0} \\ \gamma_{xy}^{0} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{1} \\ \varepsilon_{yy}^{1} \\ \gamma_{xy}^{1} \end{Bmatrix}$$

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} dz$$

$$= \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_{xx}^{0} + z\varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{0} + z\varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{0} + z\gamma_{xy}^{(1)} \end{Bmatrix} dz$$

or

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{0} \\ \varepsilon_{yy}^{0} \\ \gamma_{xy}^{0} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{1} \\ \varepsilon_{yy}^{1} \\ \gamma_{xy}^{1} \end{Bmatrix}$$

Where A_{ij} are called extentional stiffnesses, D_{ij} the bending stiffnesses, and B_{ij} the bending-extensional coupling stiffnesses, which are defined in terms of the lamina stiffnesses $\overline{Q_{ij}^{(k)}}$ as material coefficients are given by

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{\frac{-h}{2}}^{\frac{h}{2}} \overline{Q_{ij}}(1, z, z^2)dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)}(1, z, z^2)dz$$

or

$$A_{ij} = \sum_{k=1}^{N} \overline{Q}_{ij}^{(k)}(z_{k+1} - z_k), \qquad B_{ij} = \frac{1}{2}\sum_{k=1}^{N} \overline{Q}_{ij}^{(k)}(z_{k+1}^2 - z_k^2),$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} \overline{Q}_{ij}^{(k)}(z_{k+1}^3 - z_k^3)$$

$$\overline{Q_{11}} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta$$
$$\overline{Q_{12}} = (Q_{11} + Q_{22} + 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta)$$
$$\overline{Q_{22}} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta$$
$$\overline{Q_{16}} = (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta\cos\theta$$
$$\overline{Q_{26}} = (Q_{11} - Q_{12} - 2Q_{66})\sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\cos^3\theta$$
$$\overline{Q_{66}} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta)$$
$$\overline{Q_{44}} = (Q_{44}\cos^2\theta + Q_{55}\sin^2\theta$$
$$\overline{Q_{45}} = (Q_{55} - Q_{44})\sin\theta\cos\theta$$
$$\overline{Q_{55}} = (Q_{55}\cos^2\theta + Q_{44}\sin^2\theta$$

2.3 THIRD ORDER THEORY

The displacement field given by this theory is given by

$$u(x,y,z,t) = u_0(x, y, t) + z\phi_x(x, y, t) - c_1 z^3(\phi_x + c_0\frac{\partial w_0}{\partial x})$$

$$v(x, y, z, t) = v_0(x, y, t) + z\phi_y(x, y, t) - c_1 z^3(\phi_y + c_0\frac{\partial w_0}{\partial y}$$

$$w(x, y, z, t) = w_0(x, y, t)$$

Where u, v, w are the displacement component along x, y, z axes, (u_0, v_0, w_0) are the displacement component along the (x, y, z) coordinate directions, respectively of a point on the midplane (i.e. z = 0), $\phi x = \frac{\partial u}{\partial z}$ and $\phi y = \frac{\partial v}{\partial z}$ are the rotations of a transverse normal about y and x axes respectively.c_1, c_0 are parameters called racers, which are introduced to include the displacement field of CLPT and FSDT.

Nonlinear strain displacement relations

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}(\frac{\partial w_0}{\partial x})^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}(\frac{\partial w_0}{\partial x})^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \phi x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{\partial \phi x}{\partial y} + \frac{\partial \phi y}{\partial x} \end{Bmatrix} + z^3 * -c \begin{Bmatrix} (\frac{\partial \phi x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}) \\ (\frac{\partial \phi y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}) \\ \frac{\partial \phi x}{\partial y} + \frac{\partial \phi y}{\partial x} + 2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varphi_y + \frac{\partial w_0}{\partial y} \\ \varphi_x + \frac{\partial w_0}{\partial x} \end{Bmatrix} + z^2 * -c \begin{Bmatrix} \varphi_y + \frac{\partial w_0}{\partial y} \\ \varphi_x + \frac{\partial w_0}{\partial x} \end{Bmatrix}$$

2.4 DIFFERENT LAMINATION SCHEMES

Lamination schems for a composite laminate are generally classified into four categories angle-ply, cross ply,symmetric and Antisymmetric laminates[reddy].

(i) Angle ply laminate

General angle-ply laminat have ply orientation between the rang 0°≤θ≤90°, with at least one layer having an orientation other than 0°or 90°. An example of angle-ply laminate is provided by (15/-30/0/90/45/-45).

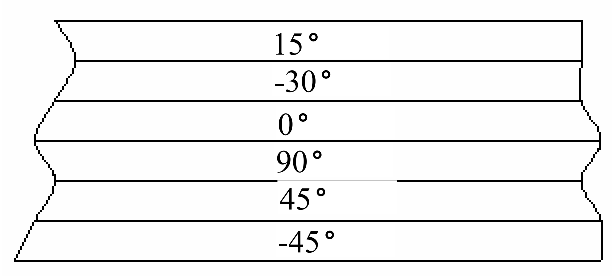


FIGURE 2.4.(a). A GENERAL ANGLE PLY LAMINATE

(ii) Cross ply laminate

Cross ply laminates are those laminates which have ply orientation of 0°or 90°. A example of cross ply laminate is (0/90/90/0/0/90).

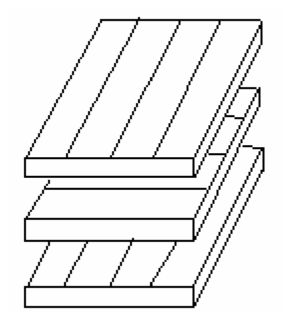


FIGURE 2.4.(b). A CROSS PLY LAMINATED PLATE WITH 0° AND 90° ORIENTED LAYERS

(iii) Symmetric Laminate

Symmetric laminate are those laminates in which stacking sequence, material and geometry are symmetric about the mid plane of the laminate.

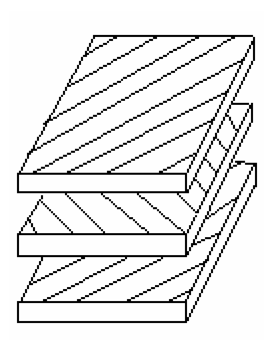


FIGURE 2.4.(c). SYMMETRIC LAMINATE

Chapter 3

DIFFERENTIAL QUADRATURE METHOD

The following example describe the DQM method by taking example of vibration analysis of beam, for simplicity Bernoulli-Euler beam is considered.

3.1 GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

For a beam, three problems are often encountered, i.e., the bending, vibration and column buckling analysis. For a Bernoulli-Euler beam of varying cross-section with length L, the non-dimentional governing equations for vibration is

$$s(X)\frac{d^4W}{dX^4}+2\frac{ds(X)}{dX}\frac{d^3W}{dX^3}+\frac{d^2s(X)}{dX^2}\frac{d^2W}{dX^2}-\Omega^2W=0 \tag{3.1}$$

Where $s(X)=EI/EI_0$, $\Omega^2=\rho AL^4\omega^2/EI_0$, $X=x/L$, EI is the beam flexural rigidity, ρA is the mass per unit length, ω is the dimentional frequency. For a beam of varying cross section, EI is function of the coordinate x. the governing equation for a beam is a 4th order ordinary differential equation. For well posed problem, ir requires four boundary conditions. These can be obtained by specifying two boundary conditions at the end X=0, and another two boundary conditions at the end X=1.

3.2 NUMERICAL DISCRETIZATION

For the numerical computation, the continuous solution is approximated by the functional values at discreat points. Now, we assume that the computational domain $0 \le X \ge 1$ is divided into (N-1) intervals with coordinates of the grid points given as X_1, X_2, …, X_n. Although a uniform mesh can be used for the analysis, it is recommended that a non-uniform mesh be used. Here, we adopt the mesh point distribution used by Shu[17].

$$X_i=\frac{1}{2}\left(1-\cos\left(\frac{i\pi}{n}\right)\right)\times\Delta x \qquad \text{for } i=0,1,2,\ldots\ldots\ldots\ldots\ldots\ldots\ldots N; \tag{3.2}$$

With the coordinates of mesh points given by equation (3.2), the weighting coefficient can be easily computed. This weigthing coefficients can then be used to discretize equation (3.1). Using DQ method, Equation (3.1) can be discretized as

$$s(X_i)\sum_{k=1}^{N} c_{ik}^{(4)}W_k + 2s^{(1)}(X_i)\sum_{k=1}^{N} c_{ik}^{(3)}.W_k + s^{(2)}(X_i)\sum_{k=1}^{N} c_{ik}^{(2)}W_k = \Omega^2 W_i \quad (3.3)$$

where W_i ,i=1,2,….N, is the functional value at the grid point X_i, $s^{(2)}(X_i)$ and $s^{(1)}(X_i)$ are respectively the second and first order derivatives of $s(X)$ at X_i, $c_{ik}^{(n)}$, n=2,3,4 are the DQ weighting coefficients of the nth order derivative. With proper implementation of the boundary conditions, equation (3.3) can be put in matrix form as

$$[A_v]\{W\} = \Omega^2\{W\} \quad (3.4)$$

where $[A_v]$ is a matrix, $\{W\}$ is a vector of unknowns.

Equation (3.4) is eigenvalue systems. The frequency of a free vibration problem can be obtained from the eigenvalues of equation (3.4).

3.3 IMPLEMENTATION OF BOUNDARY CONDITIONS

We will use the vibration problem to illustrate the implementation of boundary conditions.

a) The δ-technique

The δ-technique was proposed by Jang,Bert and Striz(1989) to eliminate the difficulties in implementing two conditions at a single boundary point. It applies the Dirichlet condition (W=0) at the boundary point itself, and derivative condition at its adjacent point which is at a distance δ from the boundary point.

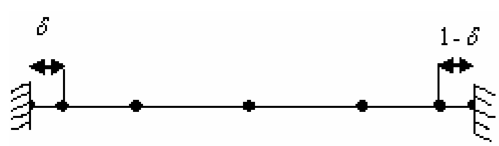


FIGURE 3.1 ILLUSTRATION OF δ -POINTS FOR A BEAM PROBLEM

As shown in figure 3.1, the δ-points are actually the grid points $X_2 = \delta$ and $X_{N-1} = 1-\delta$. As an example, when the simply supported condition is specified at the two ends, the derivate condition $\partial^2 W / \partial X^2 = 0$ is discretized at mesh point X_2 and X_{N-1}, which gives

$$\sum_{k=1}^{N} c_{2,k}^{(2)}.W_k = 0 \quad (3.5)$$

$$\sum_{k=1}^{N} c_{N-1,k}^{(2)}.W_k = 0 \tag{3.6}$$

Clearly, the derivative condition is not accurately approximated by the δ-technique. There are two major draw backs to the δ-technique. One drawback arises from the implementation of the derivative condition at the δ-point. Since it is an approximation to the true boundary condition that should be implemented at the boundary, one can expect that the numerical results may depend on the choice of the δ-value. To obtain an accurate numerical solution, the values of δ should be chosen to be very small (possibly not greater than 0.0001). however, this small value of δ would will result in the second drawback of this technique. When one mesh spacing (δ) is much smaller than the others, the DQ weighting coefficient matrices may become highly ill-conditioned, which then causes the solution to oscillate. As a result, the numerical solution is less accurate.

Now to calculate the weighting coefficients we are having the following formula.

$$C_{ik}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_k)M^{(1)}(x_k)} \quad \text{i, k=1,2........N,} \quad \text{but } k \neq i,$$

where,

$$M^{(1)}(x_i) = \prod_{k=1, k\neq i}^{N} (x_i - x_k)$$

and

$$C_{ik}^{(n)} = n\left(C_{ik}^{(n-1)} C_{ik}^{(1)} - \frac{C_{ik}^{(n-1)}}{x_i - x_k} \right) \quad \text{for} \quad \text{i, k=1,2........N,} \quad \text{but } k \neq i,$$

$$C_{ik}^{(n)} = -\sum_{i=1, k\neq i}^{N} C_{ik}^{(n)}$$

b) Modification of Weighting Coefficient Matrices

This technique was presented by Wang and Bert(1993) to overcome the drawbacks of the δ-technique. For this approach, only one boundary condition is numerically implemented. The other boundary condition(derivative condition) is built into the DQ weighting coefficient matrices. To demonstrate this approach, we consider the following boundary condition(C-C).

$W = 0$, $dW/dX = 0$, at $X = 0$ (3.7a)

$W = 0$, $dW/dX = 0$, at $X = 1$ (3.7b)

Let $[A^{(n)}]$ and $\left[\tilde{A}^{(n)}\right]$ be respecvely the original and modified weighting coefficient matrices of the nth order derivative. Since

$$\frac{d^n W}{dX^n} = \frac{d}{dX}\left(\frac{d^{n-1}W}{dX^{n-1}}\right) \tag{3.8}$$

we than have

$$[A^{(n)}]=[A^{(1)}]\,[A^{(n-1)}]=[A^{(2)}]\,[A^{(n-2)}]=\ldots\ldots\ldots=[A^{(n-1)}]\,[A^{(1)}] \tag{3.9}$$

Now, consider the DQ analog of the derivative dW/dX at all mesh points

$$\begin{Bmatrix} (dW/dX)_1 \\ (dW/dX)_2 \\ \cdot \\ \cdot \\ \cdot \\ (dW/dX)_{N-1} \\ (dW/dX)_N \end{Bmatrix} = \begin{bmatrix} c_{1,1}^{(1)} & c_{1,2}^{(1)} & \cdot & \cdot & \cdot & c_{1,N-1}^{(1)} & c_{1,N}^{(1)} \\ c_{2,1}^{(1)} & c_{2,2}^{(1)} & \cdot & \cdot & \cdot & c_{2,N-1}^{(1)} & c_{2,N}^{(1)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{N-1,1}^{(1)} & c_{N-1,2}^{(1)} & \cdot & \cdot & \cdot & c_{N-1,N-1}^{(1)} & c_{N-1,N}^{(1)} \\ c_{N,1}^{(1)} & c_{N,2}^{(1)} & \cdot & \cdot & \cdot & c_{N,N-1}^{(1)} & c_{N,N}^{(1)} \end{bmatrix} \begin{Bmatrix} W_1 \\ W_2 \\ \cdot \\ \cdot \\ \cdot \\ W_{N-1} \\ W_N \end{Bmatrix} \tag{3.10}$$

To satisfy the derivative condition of equation (3.7a), equation (3.10) should be modified by zeroing the first row of the matrix

$$\begin{Bmatrix} (dW/dX)_1 \\ (dW/dX)_2 \\ \cdot \\ \cdot \\ \cdot \\ (dW/dX)_{N-1} \\ (dW/dX)_N \end{Bmatrix} = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ c_{2,1}^{(1)} & c_{2,2}^{(1)} & \cdot & \cdot & \cdot & c_{2,N-1}^{(1)} & c_{2,N}^{(1)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{N-1,1}^{(1)} & c_{N-1,2}^{(1)} & \cdot & \cdot & \cdot & c_{N-1,N-1}^{(1)} & c_{N-1,N}^{(1)} \\ 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \end{bmatrix} \begin{Bmatrix} W_1 \\ W_2 \\ \cdot \\ \cdot \\ \cdot \\ W_{N-1} \\ W_N \end{Bmatrix} \tag{3.11}$$

Obviously, the matrix in equation (3.10) is $[A^{(1)}]$, and matrix in equation (3.11) is $[\overline{A}^{(1)}]$. To satisfy the boundary condition (3.7a), the second order weighting coefficient matrix $[\overline{A}^{(2)}]$ can be derived from equation (3.9).

$$[\overline{A}^{(2)}] = [A^{(1)}][\overline{A}^{(1)}] \tag{3.12}$$

It is noticed that the derivatives conditions given in equation (3.9) have been completely built into the matrix $[\overline{A}^{(2)}]$. With $[\overline{A}^{(2)}]$, the modified weigthing coefficient matrices $[\overline{A}^{(3)}]$ and $[\overline{A}^{(4)}]$ can be computed by.

$$[\overline{A}^{(3)}]=[A^{(1)}].[\overline{A}^{(2)}] \tag{3.13}$$

$$[\overline{A}^{(4)}]=[A^{(2)}].[\overline{A}^{(2)}] \tag{3.14}$$

In the above process, the weighting coefficient matrices are modified through the matrix form. This process is very simple since it just zeros some elements of the matrix. However, for the non- homogeneous derivative condition, this process cannot be simply implemented.

It should be indicated that similar to the finite element method and since only one condition is implemented at the boundary point, the discrete governing equation has to be applied to the interior points, $2 \le i \ge N-1$. So, the dimension of the equation system using this approach is $(N-2)\times(N-2)$.

c) Differential Quadrature Analysis of Plate

In similar way, we can solve the two dimensional problem. The polynomials are approximate by the following way.

$$f(x,y)=\sum_{i=1}^{N_x}\sum_{j=1}^{N_y} f(x_i,y_j)r_i(x)s_j \tag{3.15}$$

where $r_i(x)$ and $s_j,(y)$ are the Lagrange interpolation polynomials along the x and y directions, respectively, and given in the following forms :

$$r_i(x)=\frac{(x-x_1)(x-x_2)...............(x-x_{N_x})}{M^{(1)}(x_i)} \tag{3.16}$$

$$s_j(y)=\frac{(y-y_1)(y-y_2)...............(y-y_{N_y})}{P^{(1)}(y_i)} \tag{3.17}$$

3.4 LITERATURE REVIEW

K. M. LIEW, *et. al.* [5] analyzed the axisymmetric free vibrations of moderately thick circular plates which are described by the linear shear-deformation Mindlin theory by the differential quadrature (DQ) method. The first fifteen natural frequencies of vibration are calculated for uniform circular plates with free, simply-supported and clamped edges. Through these computations, the capability and simplicity of the differential quadrature method for moderately thick plate eigen value analysis is demonstrated, and convergence and accuracy are thoughtfully examined. They also considered the case of a rigid point support at the plate centre in the present paper, for which special attention is paid to the capability and convergence of the current method.

The equations governing the axisymmetric free vibration of a uniform circular plate of isotopic material can be derived using Mindlin's theory

$$D\left(\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}-\frac{\psi}{r^2}\right)-kGh\left(\frac{\partial w}{\partial r}+\psi\right)-\frac{\rho h^3}{12}\frac{\partial^2\psi}{\partial t^2}=0,$$

$$kGh\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial\psi}{\partial r}+\frac{\psi}{r}\right)-\rho h\frac{\partial^2 w}{\partial t^2}=0,$$

where w is the transverse deflection, ψ is the rotation of the normal about the r-axis. $D = Eh^3/[12(1-v^2)]$, E, G and v are the plate flexural rigidity, Young's modulus, shear modulus and Poisson's ratio respectively, ρ and k are the density of the plate material and the shear correction factor respectively.

S. Moradi, F. Taheri [6] modeled the delamination buckling response of a composite panel containing through the width using a solution that is based on the differential quadrature method (DQM). The composite is modeled as a general one-dimensional beam plate having a through the width delamination that can be at any arbitrary location through its thickness. Hence, dividing the domain into four regions. The DQM is applied to each region and with the imposition of appropriate boundary conditions, the problem is transformed into a standard eigen value problem. Numerical results are presented, illustrating the stability and validity of the method. The results also demonstrate the efficiency of the method in treating this class of engineering problem.

H. ZENG, C. W. BERT [7] presented a differential quadrature analysis of free vibration of plates with eccentric stiffeners. Structures consisting of thin plates stiffened by a system of ribs or diaphragms form a class of structural elements of practical importance in various engineering applications. The plate and the stiffeners are treated separately. Simultaneous governing differential equations are derived from the plate dynamic equilibrium, the stiffener dynamic equilibrium, and equilibrium and compatibility conditions along the interface of a plate segment and a stiffener. The plate and the stiffeners have displacements in three dimensions. Shear forces and in plane forces in the plate are considered to satisfy the compatibility at the interface of a plate segment and a stiffener. Meanwhile in plane inertia effects in the plate and in the stiffener are ignored. The application of the differential quadrature method is demonstrated by three examples: a simply supported plate with central eccentric stiffener, a clamped square plate with central eccentric stiffener, and a double-ribbed plate with all edges clamped. The natural frequencies are compared with the experimental results.

Y. Xiang, G. W. Wei [8] developed the first-known exact solutions for the vibration of multi-span rectangular Mindlin plates with two opposite edges simply supported. The Levy type solution method and the state-space technique are employed to develop an analytical approach to deal with the vibration of rectangular Mindlin plates of multiple spans. Exact vibration frequencies are obtained for two-span square Mindlin plates with varying span ratios and two, three and four-equal-span rectangular Mindlin plates. The influence of the span ratios, the number of spans and plate boundary conditions on the vibration behavior of square and rectangular Mindlin plates is examined.

The natural frequency of a plate is expressed in terms of a non-dimensional frequency parameter

$$\lambda = (wL^2 / \pi^2)\sqrt{\rho h / D}$$

Where D is the flexural rigidity, L is the plate width, ρ is mass density, w is vibration frequency and h is plate thickness.

A rectangular plate is considered to have two parallel edges simply supported while the other two edges may have any combination of free, simply supported or clamped conditions. The internal line supports that divided the plate into multiple spans are arranged to be perpendicular to the two simply supported parallel edges. An analytical model based on the Levy solution method and the state- space technique is developed for the vibration analysis of multi- span rectangular Mindlin plates.. The influence of the internal line support on the frequency parameters of a square Mindlin plate is examined. It is observed that the optimal location of the internal line support in strengthening the plate against vibration varies from plate to plate and from mode to mode. However, for fundamental frequency, the best location of the internal line support is at the plate center for symmetric Levy square plates. For equal-span rectangular plates, the frequency parameters decrease with increasing number of spans. The frequency parameters also decrease as the plate thickness ratio increases due to the influence of transverse shear deformation and rotary inertia.

G. Karami, ***et. al.*** [9] applied the differential quadrature method for static, free vibration and stability analysis of skewed and trapezoidal composite thin plates. To mechanize the modelling procedure, a general transformation scheme is employed to transfer the variation of the variables in the computational to the physical domain and vice versa. Examples are shown to show the accuracy and convergence of the solutions under different geometrical parameters and boundary conditions. The accuracy is demonstrated by comparing the results with those of other numerical methods.

The general governing equation of a thin elastic plate in a general coordinates x y is written as:

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}-N_x\frac{\partial^2 w}{\partial x^2}$$
$$-N_y\frac{\partial^2 w}{\partial y^2}-2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}+kw+\rho h\frac{\partial^2 w}{\partial t^2}=p(x,y,t)$$

Where w, N_x, N_y, N_{xy} and p(x,y,t) are the transverse displacement , in plane normal and shear forces in x and y directions , and the intensity of transverse distributed load respectively, also D_{ij}, ρ, h, k are respectively, the flexural rigidity coefficients, density, thickness and elastic stiffness of support.

Y. Xiang, ***et. al.*** [10] presented the Levy method to investigate the vibration behavior of multi-span rectangular plates. Multi-span plates are often encountered in various engineering fields. For instance, the aeroplane surfaces, slabs in house construction, bridge decks and glass window panels can all be modeled as plates with multiple spans. The problem of free vibration of multi-span rectangular plates has attracted the attention of many researchers. Theoretically, multi-span plates are often treated as plates with internal line supports.

The Levy method is applicable and analytical for rectangular plates with at least two parallel simply supported edges. The continuity at an interface between two spans is maintained by imposing both the essential and natural boundary conditions along the interface. The impact of the internal line supports on the vibration behavior of the plates is investigated by varying both the number of internal lines and the line positions. Results for the vibration of two and three span rectangular plates are presented, in which the 3rst-known exact solutions for plates involving free edges are included.

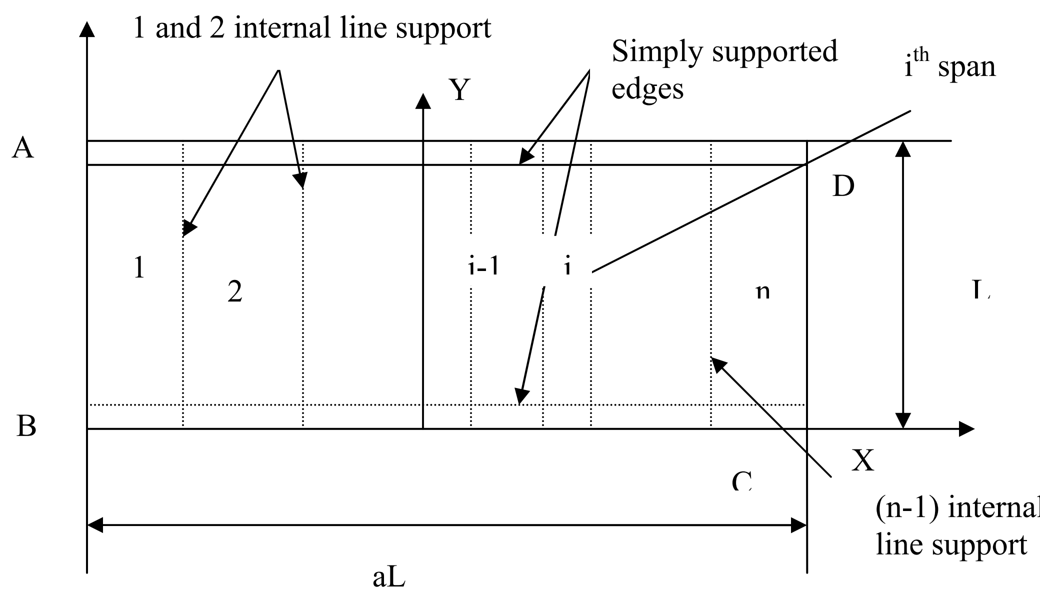


FIGURE 3.2. GEOMETRY AND COORDINATE SYSTEM FOR LEVY PLATE WITH (N-1) INTERNAL LINE SUPPORT

An isotropic rectangular plate of n spans is shown in Fig. The plate is of uniform thickness h, width L, length aL, Young's modulus E, Poisson's ratio v and shear modulus G. The origin of the coordinate system is set at the centre of the bottom edge BC of the plate as shown in Fig. . The plate is of $(n - 1)$ internal line supports that provide the constraint of zero transverse displacement along the line supports The problem at hand is to determine the vibration frequencies of the multi-span rectangular plate.

The governing differential equation for free vibration of the i^th span is given by

$$\frac{\partial^4 w_i}{\partial x^4}+2\frac{\partial^4 w_i}{\partial x^2 \partial y^2}+\frac{\partial^4 w_i}{\partial y^4}=\frac{\rho h \omega^2}{D}w_i$$

G. Karami, P. Malekzadeh [11] employed the differential quadrature (DQ) methodology for the static and stability analysis of irregular quadrilateral straight-sided thin plates. A four-noded super element is used to map the irregular physical domain into a square computational domain. Second order transformation schemes with relative ease and low computational effort are employed to transform the fourth order governing equations of thin plates between the domains. Within the domain, the displacements are the only degrees of freedom whereas, along the boundaries, the displacements as well as the second order derivatives of the displacements with respect to the associated normal coordinate variables in the computational domain are the two sets of degrees of freedom. The implementation procedures for different boundary conditions including free-edge boundaries have been formulated by them. The results were compared with those of other DQMs as well as other numerical techniques.

L.J.Gray, *et. al.* [12] developed the algorithms for the direct evaluation of singular Galerkin boundary integrals for three-dimensional anisotropic elasticity. The integral of the traction kernel is defined as a boundary limit, and (partial) analytic evaluation is employed to compute the limit. The spherical angle components of the Green's function and its derivatives are not known in closed form, and thus the analytic integration requires a splitting of the kernel, into `singular' and `non-singular' terms. For the coincident singular integral, a single analytic evaluation suffices to isolate the potentially divergent term, and to show that this term self-cancels. The implementation for a linear element is considered in detail, and the extension to higher order curved interpolation is also discussed. Three linear shape functions are given below:

$$\psi_1(\eta,\varepsilon) = \frac{\sqrt{3(1-\eta)-\varepsilon}}{2\sqrt{3}}$$

$$\psi_2(\eta,\varepsilon) = \frac{\sqrt{3(1+\eta)-\varepsilon}}{2\sqrt{3}}$$

$$\psi_3(\eta,\varepsilon) = \frac{\varepsilon}{\sqrt{3}}$$

The focus herein is on the evaluation of singular integrals that arise in a Galerkin approximation of the displacement boundary integral equation, with most attention on the integrals containing the first derivative of the Green's function. This is a necessary first step in addressing the more difficult hyper singular traction equation and its second order derivatives. The lack of a simple form for the Green's function is clearly an impediment for singular integration algorithms.

S.H. Ju, H.T. Lin [13] investigated the resonant characteristics of three-dimensional bridges when high-speed trains pass them. Multi-span bridges with high piers and simply supported beams were used in the dynamic finite element analysis. The dominated train frequencies proposed in this study can be clearly seen from the finite element result. To avoid resonance, the dominated train frequencies and the bridge natural frequencies should be as different as possible, especially for the first dominated train frequency and the first bridge natural frequency in each direction. If the two first frequencies are similar, the bridge resonance can be serious.

A suitable axial stiffness between two simple beams can reduce vibrations at a near-resonance condition. The axial stiffness of the continuous railway and the friction of the bearing plate should be enough to obtain this axial stiffness. However, extra-axial links or dampers can be installed to reduce vibrations if the stiffness from the above items is not enough. This small modification of the bridge is probably the cheapest way to reduce vibrations for simple beam bridges.

To evaluate the dominated frequencies of the train loads, the wheel loading of the moving train passing a point is assumed to be a periodic function P(t) as follows:

$$P(t) = P_{wheel}[\delta(t-t_1) + \delta(t-t_2) + \delta(t-t_3) + \delta(t-t_4)]$$

Where P_{wheel} is the load of pair of wheel and $\delta(t-t_i)$ is the unit impulse function for i=1 to 4 and $t_i = s_i/v$, v is the train velocity and s_i is distance traveled by i^{th} wheel.

R. Lassoured, M. Guenfoud [14] presented the computational procedure to calculate natural frequency of multi span plates. The frequencies obtained are proportional to the number of mode considered. The general vibratory movement of the structure considered is the sum of all the modal movements.

Many researchers have studied the vibrations of plates and their displacement because of its importance in engineering applications. Indeed, rectangular plates are commonly used as structural components in many branches of modern technology namely mechanical, aerospace, electronic, optical, marine and structural engineering. So, there is a particular need for access to highly accurate eigen values for plates and beams.

The bilinear and linear structures considered simulate a bridge. The dynamic behavior of this one is analyzed by using the theory of the orthotropic plate simply supported on two sides and free on the two others. The plate can be excited by a convoy of constant or harmonic loads. The determination of the dynamic response of the structures considered requires knowledge of the free frequencies and the shape modes of vibrations. The formulation is based on the determination of the solution of the differential equations of vibrations. The boundary conditions corresponding to the shape modes permit to lead to a homogeneous system. Determination of the non commonplace solutions of this system led to a nonlinear problem in Eigen frequencies. The governing equations of motion of this orthotropic plate can be written as:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + C\frac{\partial w}{\partial t} + \rho h \frac{\partial^2 w}{\partial t^2} = F(x, y, t)$$

Where D_x ,D_y and D_{xy} are the flexural rigidity, ρ is mass density, h is plate thickness, w is the displacement and F is the external load.

They also develop a computer code for the determination of the eigen values. It is based on a method of bisection with interpolation whose precision reaches 10 -12.

Moreover, to determine the corresponding modes, the calculation algorithm that we develop uses the method of Gauss with a partial optimization of the "pivots" combined with an inverse power procedure. The Eigen frequencies of a plate simply supported along two opposite sides while considering the two other free sides are thus analyzed.

F.M. Li, ***et. al.*** [15] investigated the problem of wave localization in disordered periodic multi- span rib-stiffened plates based on the theory of elastic dynamics. Plates are commonly used in many engineering applications such as airplanes, buildings, bridges and ships. In order to enhance the ability of resisting axial instability and transverse rigidity of plates, ribs are often added in them. If the ribs are added to them in period form, the plates became periodic rib-stiffened ones. In complete periodic rib-stiffened plates, waves can propagate throughout all the structures. But disorder can lead to the appearance of localization of elastic waves in mistuned periodic structures. Localization leads to a spatial decay of wave amplitude, and the associated exponential decay constant is known as localization factor. So localization factor characterizes the average exponential rates of decay of wave amplitudes in disordered periodic structures.

The transfer matrix method is employed to obtain the transfer matrix of the system, and the method for calculating the Lyapunov exponents in continuous dynamical systems presented by Wolf is used to determine the localization factors in discrete dynamical systems. The equation of motion of flexural waves in plate uniformly compressed in x-direction can be written as the following form:

$$D\Delta^2\Delta^2 w + N\frac{\partial^2 w}{\partial x^4} + \rho h\frac{\partial^2 w}{\partial t^2} = q$$

where w is the transverse displacement, D= $Eh^3/12(1-v^2)$ is the bending stiffness of plate, ρ is mass density, h is thickness of plate, N the magnitude of compressive force per unit length of the edge in x-direction of plate, Δ Laplacian operator, t the time and q the transverse load.

The main findings of this work are as follows:

1. Tuned periodic multi-span rib-stiffened plates have the properties of frequency passband and stopband. Localization phenomenon can occur in mistuned periodic multi-span plates and the larger the degree of disorder, the larger the degree of localization.

2. With the increase of the dimensionless torsional and flexural rigidities of the rib, the passband and the stopband will become narrower and wider, respectively. So, the degree of localization will be increased.

3. For different dimensionless wave number, the variation of the localization factor versus the dimensionless axial compressive force is very complicated, and it is necessary to study the problem of wave localization according to the actual structural dynamical status.

4. Applying these properties of periodic structures, disordered periodic structures can be designed according to different purposes to localize the amplitudes of elastic waves and vibration, to reduce the vibration of important substructures and to realize the structural vibration control.

C.F. Lu, *et. al.* [16] presented the exact analysis for free vibration of long-span continuous rectangular plates based on the classical Kirchhoff plate theory, using the state space approach associated with joint coupling matrices. Levy-type solution is adopted to model the field variation in the direction perpendicular to the pair of simply supported edges. The series of internal rigid line supports are parallel to the remaining pair of edges, which can be of an arbitrary combination of simply supported, clamped and free edges. Transfer relationship is derived in the span direction by the state space approach. The joint coupling matrices are employed to avoid numerical instability that exists in the conventional state space approach for high- frequency calculation or long-span geometry. Numerical calculation is carried out to validate effectiveness and efficiency of the present method. Influence of location of internal line supports on natural frequencies of multi-span plates with large aspect ratios is investigated and discussed.

They investigated the free vibration of long span continuous rectangular Kirchhoff plates using the state space approach (transfer matrix method) in conjunction with joint coupling matrices. The series of internal rigid line supports, which are the constraints of zero transverse displacement, are perpendicular to the pair of simply supported edges.

The remaining two edges can be of an arbitrary combination of simply supported, clamped and free edges. First, the state equation is derived with state vector composed of deflection, slop, moment and equivalent shear force, which facilitate precise expression of boundary conditions and internal line support conditions. Second, the joint coupling matrices concept is adopted to establish the frequency equation of plate. This is particularly important for prevention of numerical instability associated with the conventional state space approach (transfer matrix method) when the frequency is very high or the plate span is very large.

In free vibration, the governing equations for an individual span of Kirchhoff plate are as follows:

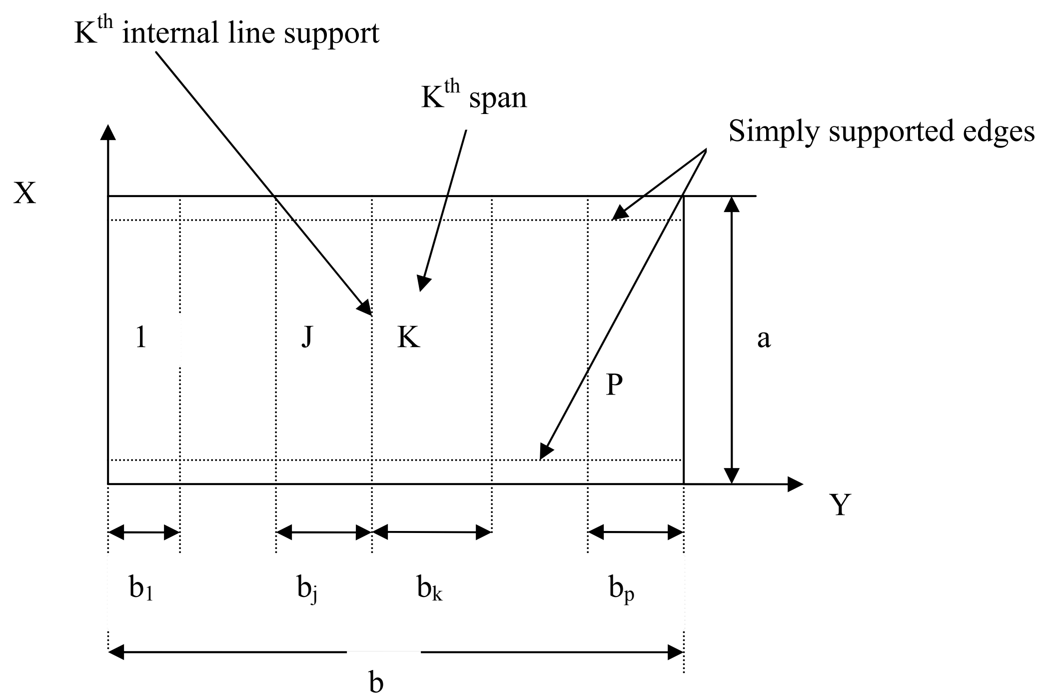


FIGURE 3.3. GEOMETRY OF RECTANGULAR PLATE WITH INTERNAL LINE SUPPORT

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = \rho h \frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0$$
$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = 0$$

where ρ and w are respectively, the material density and transverse displacement (defection), Q_x and Q_y are the transverse shear forces, M_x and M_y the bending moments, and M_{xy} the twisting moment. These resultant quantities, defined as the forces and moments per unit length of the section on which they act, can be expressed in terms of the transverse displacement as:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right)$$
$$M_y = -D\left(v\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$
$$M_{xy} = -(1- v)\, D\frac{\partial^2 w}{\partial x \partial y}$$
$$Q_x = -D\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right)$$
$$Q_y = -D\left(\frac{\partial^3 w}{\partial x\ \partial y} + \frac{\partial^3 w}{\partial y^3}\right)$$

Where $D = Eh^3/12(1-v^2)$ is the flexural rigidity.

S. Hatami, *et. al.* [17] did the study of free vibration of axially moving symmetrically laminated plates subjected to in-plan forces by classical plate theory. This category includes symmetric cross ply and angle-ply laminates and anisotropic plates. Firstly, an exact method is developed to analyze vibration of multi-span traveling cross-ply laminates, and then a semi-analytical finite strip method is extended for moving symmetric laminated plates in general, with arbitrary boundary conditions. By the finite strip method intermediate elastic or rigid supports can also be added to the model of the moving plate. The supports may be in the form of point, line or local distributed supports. Axially moving materials are of technological importance and are present in various industrial applications. For example the cable, belt, and chain in power transmission lines, the conveyor belts, the paper and plastic sheets in process, the steel strip in a thin steel sheet production line, the band saw blade etc. In many of these instances, the moving material is not isotropic, but is a single-layer orthotropic material or consists of several orthotropic layers. In axially traveling systems, the transverse vibration of the moving material often becomes a serious problem in achieving good quality. The axial speed of a structure may significantly affect the dynamic characteristics of the system even at low velocity, giving rise to variations of natural frequencies and complex modes. In a certain critical speed, first natural frequency vanishes and the axially moving structure may experience severe vibrations and structural instability. Thus, accurately prediction of dynamic characteristics of such structures is requisite for the analysis and optimal design of a broad class of technological devices.

Assume a symmetrically laminated composite plate moving with constant velocity v in the x-direction. Consider a domain with area A in coordinates (x, y) fixed in the space. By using Hamilton's principle for the piece of the plate fill this domain in time t, the equation of motion for transverse displacement in the fixed coordinate w(x, y, t) can be derived.

However, Hamilton's principle in its traditional form is formulated for a closed system. In this consideration the particles, which fill the domain of interest, are changing all the time and the energy flux across the boundaries must be taken into account in the formulation. Assuming that the boundaries are fixed both at the inlet and the outlet, the fluxes in and out are equal and the net energy flux through the boundaries is zero. Therefore Hamilton's principle takes the familiar form:

$$\delta\int_{t_i}^{t_f}(U-T)dt=0$$

Where U is the total potential energy of the particles fill the domain and T is the kinetic energy of structure mass. U includes the strain energy due to bending U_b and the effect of in-plane forces on the transverse deflection U_g.

$$U = U_b + U_g$$

Tushar V. Ugale, ***et. al.*** [18] in recent years, single or multi cell reinforced concrete box girder bridges have been widely used as economic and aesthetic solution for today's highway systems. The main advantage of this type of bridge lies in the high rigidity in torsion. This is due to the closed box section and convenience in varying the depth along the span. High torsion stiffness gives them better stability and load distribution characteristics and also makes this form particularly suitable for grade separations. This hollow section also can be used to accommodate services and has added advantage of less weight.

Here methods available for analysis of Multi cell box girder and the parametric studies for alternatives of multi cell with three, four, and five cells have been incorporated. Study also includes effect on multi cell with and without inclined web. For analysis IRC-6 vehicle loading has been considered. The software modeling and FEM analysis carried out and obtained the coefficient of moment and shear distribution for each cell. The coefficients of live load distribution obtained for span: depth ratio varies from10:24. This paper is suggested to the most effective alternative for upcoming concrete box girder bridges.

Bridge construction today has achieved a world – wide level of importance. The search for bridge structures with such good torsional as well as flexural strength that the supporting piers of elevated highways could be unobtrusive in an urban environment has led to wide adoption of spine (box) beam bridges. These will refer as Cellular Bridges Single or multi cell reinforced and pre-stressed concrete box-girder bridges have been widely used due to economic and aesthetic solutions for over crossing, separation structures and viaducts and are found today's modern highway systems.

For analysis and design of such multi cell structures. The basic two methods available for analysis of box girder 1 Elastic analysis 2. Experimental studies on the elastic response of box girder bridges.

Elastic analysis of box girder bridges- In the design of bridges, analysis is usually simplified by means of assumptions that establish the relationship between the behaviors of single elements in the integrated structure. The combined response of these single elements is assumed to represent the response of the whole structure. The accuracy of such solutions depends on the validity of the assumptions made.

Finite difference method- In this method analysis, the deck is notionally divided into grids of arbitrary mesh size and the deflection values at the grid points are treated as unknown quantities. The usual governing differential equation of an orthotropic plate is considered in the finite difference method. The differential equation and accompanying boundary conditions are expressed in terms of these unknown deflections. The resulting sets of linear simultaneous equations are then solved for these unknown deflections. Finally moments and shear forces are determined from the known deflection pattern. The curved deflection profile of the deck is approximated by a series of straight lines and for getting accurate results can be expected only if fine girds are used. However, the efforts needed to analyses a cellular deck by this method is cumbersome and is generally not recommended.

Finite-strip method- The finite-strip method may be regarded as a special form of the displacement formulation of the finite-element method. In this method, structure is assumed that to be discretised into a number of strips. Each strip has constant thickness, as per condition the thickness can vary from strip to strip. The strip is traded as a beam. The stiffness matrix of a strip with pre-set end conditions is formulated. In principle, it employs the minimum total potential energy theorem to develop the relationship between unknown nodal displacement parameters and the applied load. In this method, the box girders and plates are discretised into annular finite strips running from one end support to the other and connected transversely along their edges by longitudinal nodal lines. The displacement functions of the finite strips are assumed as a combination of harmonics varying longitudinally and polynomials varying in the transverse direction. Compared to the finite-element method, the finite-strip method yields considerable savings in both computer time and effort, because only a small number of unknowns are generally required in the analysis. However, the drawback of the finite-strip method is that the method is limited to simply support prismatic structures with simple line.

Chapter 4
PROBLEM FORMULATION

4.1 GOVERNING EQUATIONS

The governing equations of symmetric angle-ply laminated plates based on the first order shear deformation theory are given as [19].

$$D_{11}\frac{\partial^2\phi^x}{\partial x^2}+D_{12}\frac{\partial^2\phi^y}{\partial x\partial y}+D_{16}\left(2\frac{\partial^2\phi^x}{\partial x\partial y}+\frac{\partial^2\phi^y}{\partial x^2}\right)+D_{26}\frac{\partial^2\phi^y}{\partial y^2}+D_{66}\left(\frac{\partial^2\phi^x}{\partial y^2}+\frac{\partial^2\phi^y}{\partial x\partial y}\right)-k_{55}A_{55}\left(\phi^x+\frac{\partial w}{\partial x}\right)$$

$$k_{45}A_{45}\left(\phi^y+\frac{\partial w}{\partial y}\right)=I_2\frac{\partial^2\phi^x}{\partial t^2}, \tag{4.1}$$

$$D_{16}\frac{\partial^2\phi^x}{\partial x^2}+D_{66}\left(2\frac{\partial^2\phi^x}{\partial x\partial y}+\frac{\partial^2\phi^y}{\partial x^2}\right)+D_{12}\frac{\partial^2\phi^x}{\partial x\partial y}+D_{22}\frac{\partial^2\phi^y}{\partial y^2}+D_{26}\left(\frac{\partial^2\phi^x}{\partial y^2}+2\frac{\partial^2\phi^y}{\partial x\partial y}\right)-k_{45}A_{45}\left(\phi^x+\frac{\partial w}{\partial x}\right)$$
$$k_{44}A_{44}\left(\phi^y+\frac{\partial w}{\partial y}\right)=I_2\frac{\partial\phi^y}{\partial t^2}, \tag{4.2}$$

$$k_{45}A_{45}\left(\frac{\partial\phi^x}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+k_{45}A_{45}\left(\frac{\partial\phi^y}{\partial x}+\frac{\partial\phi^x}{\partial y}+2\frac{\partial^2 w}{\partial x\partial y}\right)+k_{44}A_{44}\left(\frac{\partial^2\phi^y}{\partial y}+\frac{\partial^2 w}{\partial y^2}\right)=I_0\frac{\partial^2 w}{\partial t^2} \tag{4.3}$$

where A_{ij} and D_{ij} are stretching and bending stiffness; k_{ij} (i, j = 4,5) are the shear correction factors; w is transverse displacement, ϕ^x and ϕ^y are bending rotations about y-axis and x-axis; t is time, and I_i (i = 0,2) are the mass inertias of the plate defined as[3].

$$I_i=\int_{-\frac{t}{2}}^{\frac{t}{2}}\rho z^i\,dz, \qquad i=0,2.$$

Here ρ and t denote the mass per unit volume of each lamina and the total thickness of the plate, respectively.

Considering a plate with all edges elastically restrained against translation and rotation, as shown in Fig. 4.1, the boundary conditions along each edges are as follows:

Boundary conditions along edges x = 0 and a

$$Q_x + n_{ix} k_w w = 0, \qquad M_x + n_{ix} k_\phi \phi^x = 0, \qquad M_{xy} = 0, \tag{4.4}$$

where i = 1 for edge x = 0 and i = 3 for edge x = a (see Fig. 4.1). n_{ix}, k_w and k_ϕ are the x-component of unit normal along edges i, the coefficients of transverse and torsional support at edges i (i = 1,3), respectively.

Boundary conditions along edges y = 0 and b

$$Q_y + n_{jy} k_w w = 0, \qquad M_y + n_{jy} k_\phi \phi^y = 0, \qquad M_{xy} = 0, \tag{4.5}$$

Where j = 2 for y = 0 and j = 4 for edge y = b (see Fig. 4.1). n_{jy} (j = 2, 4) are the y component of unit normal along edges j.

Corner boundary conditions

$$Q_x + n_{ix} k_w w = 0, \qquad Q_y + n_{jy} k_\phi w = 0, \qquad M_{xy} = 0, \tag{4.6}$$

where the values of i and j are similar to those given in Eqs. (4.4) and (4.5).

In the above equations, M_x and M_y are bending moments per unit length about y- and x-axis, M_{xy} is the twisting moment, Q_x and Q_y are the shear force per unit length along x and y edges, respectively. The bending moments and shear forces can be expressed in terms of the displacement gradients using the constitutive relations as [3]

$$\left.\begin{aligned}
&\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \phi^x}{\partial x} \\ \dfrac{\partial \phi^y}{\partial y} \\ \dfrac{\partial \phi^x}{\partial y} + \dfrac{\partial \phi^y}{\partial x} \end{Bmatrix} \\
&\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} k_{55} A_{55} & k_{45} A_{45} \\ k_{45} A_{45} & k_{44} A_{44} \end{bmatrix} \begin{Bmatrix} \phi^x + \dfrac{\partial w}{\partial x} \\ \phi^y + \dfrac{\partial w}{\partial w} \end{Bmatrix}
\end{aligned}\right\} \tag{4.7}$$

where A_{ij} and D_{ij} are the extensional and bending stiffness of the plate. Using (4.4) and (4.5) and without any additional formulations, a wide spectrum of boundary conditions can be developed by allowing the coefficients of elastic supports to take the their natural limiting values of zeros and infinity.

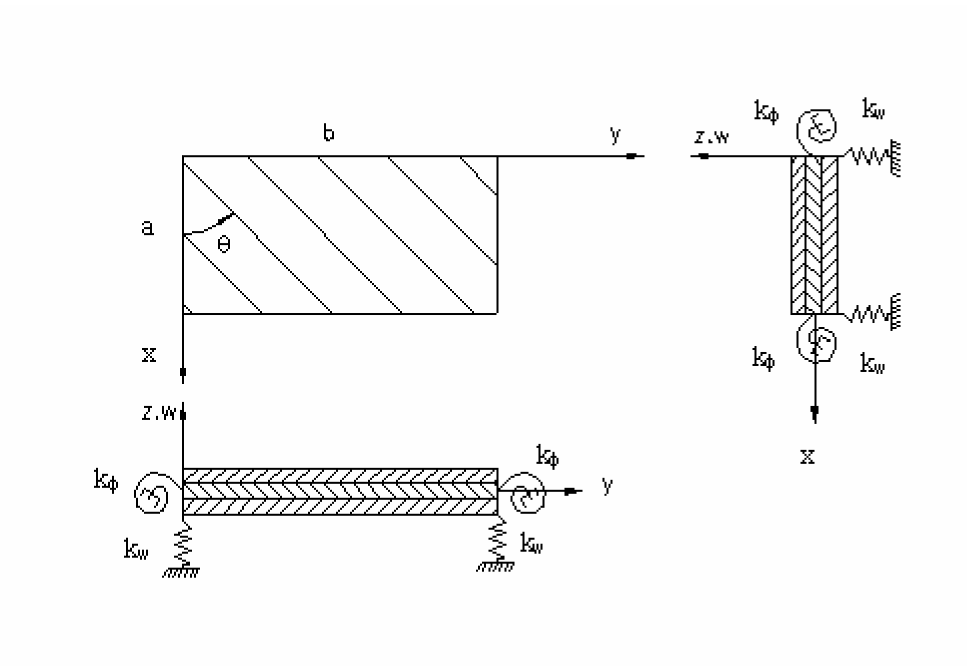


FIGURE 4.1. THE GEOMETRY AND BOUNDARY CONDITIONS OF A LAMINATED PLATE.

A zero transverse displacement and rotation conditions along the boundaries can be obtained by approaching k_w and k_ϕ to infinity in (4.4)–(4.6). In this work, values of $K_w = k_w(b^3/D_{22})$ and $K_w = k_w\,(a^3/D_{22})$ are used to simulate zero transverse displacement along edges x = 0, a and along edges y = 0, b, respectively. Also, zero rotation along edges x = 0, a and along edges y = 0,b are simulated by using $S_\phi = k_\phi\,(b/D_{22})$ and $S_\phi = k_\phi\,(a/D_{22})$ along x = 0,a and y = 0,b edges, respectively. Here S_w and S_ϕ are nondimentional form of k_w and k_ϕ respectively. Different types of boundary conditions considered in the present work are as follows:

E_{RV}: Along edges x =0 and x = a:

$$M_x + n_{ix} k_\phi \phi^x = 0, \qquad Q_x + n_{ix} k_w w = 0, \qquad \phi^y = 0,$$

Along edges y =0 and y = b:

$$M_y + n_{jy} k_\phi \phi^y = 0, \qquad Q_y + n_{jy} k_w w = 0, \qquad \phi^x = 0, \tag{4.8}$$

Since the free vibration of plates is a harmonic motion, one can assume the following periodic form for the field variables (w, ϕ^x , ϕ^y)

$$w(x,y,t) = W(x,y)e^{i\omega t} \text{ , } \phi^x(x,y,t) = \psi^x(x,y)e^{i\omega t} \text{ , } \phi^y(x,y,t) = \psi^y(x,y)e^{i\omega t} \text{ ,} \tag{4.9}$$

4.2 DIFFERENTIAL QUADRATURE ANALOGS

To formulate the eigenvalue equations, the governing differential equations with their associated boundary conditions are transformed into algebric equations by using DQ-rules. The derivatives of the field variables may be transformed into computational domain efficiently by modifying the weighting coefficients as

$$\left.\begin{aligned} A_{ij}^x &= \frac{\overline{A_{ij}^x}}{a}, \qquad B_{ij}^x = \frac{\overline{B_{ij}^x}}{a^2}, \\ A_{ij}^y &= \frac{\overline{A_{ij}^y}}{b}, \qquad B_{ij}^y = \frac{\overline{B_{ij}^y}}{b^2}, \end{aligned}\right\} \tag{4.10}$$

where, $\overline{A_{ij}^x}$, $\overline{A_{ij}^y}$ and $\overline{B_{ij}^x}$, $\overline{B_{ij}^y}$ are the weighting coefficients of first and second order derivatives in x- and y-directions, respectively. Details on calculations of weighting coefficients can be found in the work of Shu and Richards [20]. In this study, a cosine rule is used for generation of grid points in x- and y-directions [21]. Using (4.9) and (4.10), the DQ analogs of the governing equations and boundary conditions take the following forms, respectively:

Governing equations

$$D_{11}\left(\sum_{m=1}^{N_x} B_{im}^x \psi_{mj}^x\right) + D_{12}\left(\sum_{m=1}^{N_x}\sum_{n=1}^{N_y} A_{im}^x A_{jn}^y \psi_{mj}^y\right) + D_{16}\left[2\left(\sum_{m=1}^{N_x}\sum_{n=1}^{N_y} A_{im}^x A_{jn}^y \psi_{mn}^y\right) + \sum_{m=1}^{N_x} B_{im}^x \psi_{mj}^y\right]$$

$$+ D_{26}\sum_{n=1}^{N_y} B_{jn}^y \psi_{in}^y + D_{66}\left(\sum_{n=1}^{N_y} B_{jn}^x \psi_{in}^x + \sum_{m=1}^{N_x}\sum_{n=1}^{N_y} A_{im}^x A_{jn}^y \psi_{mn}^y\right) - k_{55}A_{55}\left(\psi_{ij}^x + \sum_{m=1}^{N_x} A_{im}^x W_{mj}\right)$$

$$-k_{45}A_{45}\left(\psi_{ij}^{y}+\sum_{n=1}^{N_y}A_{jn}^{y}W_{in}\right)+I_2\omega^2\psi_{ij}^{x}=0, \tag{4.11}$$

$$D_{16}\left(\sum_{m=1}^{N_x}B_{im}^{x}\psi_{mj}^{x}\right)+D_{66}\left(\sum_{m=1}^{N_x}\sum_{n=1}^{N_y}A_{im}^{x}A_{jn}^{y}\psi_{mn}^{x}+\sum_{m=1}^{N_x}B_{im}^{x}\psi_{mj}^{y}\right)+D_{12}\left(\sum_{m=1}^{N_x}\sum_{n=1}^{N_y}A_{im}^{x}A_{jn}^{y}\psi_{mn}^{x}\right)$$

$$D_{22}\left(\sum_{n=1}^{N_y}B_{im}^{x}\psi_{in}^{y}\right)+D_{26}\left(\sum_{n=1}^{N_y}B_{jn}^{y}\psi_{in}^{x}+2\sum_{m=1}^{N_x}\sum_{n=1}^{N_y}A_{im}^{x}A_{jn}^{y}\psi_{mn}^{y}+\right)-k_{45}A_{45}\left(\psi_{ij}^{x}+\sum_{m=1}^{N_x}A_{im}^{x}W_{mj}\right)$$

$$-k_{44}A_{44}\left(\psi_{ij}^{y}+\sum_{n=1}^{N_y}A_{jn}^{y}W_{in}\right)+I_2\omega^2\psi_{ij}^{x}=0, \tag{4.12}$$

$$k_{55}A_{55}\left(\sum_{m=1}^{N_x}A_{im}^{x}\psi_{mj}^{x}+\sum_{m=1}^{N_x}B_{im}^{x}W_{mj}\right)+k_{45}A_{45}\left(\sum_{m=1}^{N_x}A_{im}^{x}\psi_{mj}^{y}+\sum_{n=1}^{N_y}A_{jn}^{y}\psi_{in}^{x}+2\sum_{m=1}^{N_x}\sum_{n=1}^{N_y}A_{im}^{x}A_{jn}^{y}W_{mn}\right)$$

$$k_{44}A_{44}\left(\sum_{n=1}^{N_y}A_{jn}^{y}\psi_{in}^{y}+\sum_{n=1}^{N_y}B_{jn}^{y}\psi_{in}^{y}\right)+I_0\omega^2W_{ij}=0, \tag{4.13}$$

4.3 DESCRITIZATION OF BOUNDARY CONDITION:

The DQ analogous of boundary conditions are obtained as follows:

Bending moment along edges x=0 or a:

$$D_{11}(\sum_{m=1}^{N_x}A_{im}^{x}\psi_{mj}^{x})+D_{12}(\sum_{m=1}^{N_y}A_{jm}^{y}\psi_{im}^{y})+D_{16}(\sum_{m=1}^{N_y}A_{jm}^{y}\psi_{im}^{x}+\sum_{m=1}^{N_x}A_{im}^{x}\psi_{mj}^{y})+n_{Ix}k_{\phi}\psi_{ij}^{x}=0,$$

For i=1, i =1 or i=N_x, i =2; j=2 …N_y-1 (4.14)

Bending moment along edges y=0 or b:

$$D_{21}(\sum_{m=1}^{N_x}A_{im}^{x}\psi_{mj}^{x})+D_{22}(\sum_{m=1}^{N_y}A_{jm}^{y}\psi_{im}^{y})+D_{26}(\sum_{m=1}^{N_y}A_{jm}^{y}\psi_{im}^{x}+\sum_{m=1}^{N_x}A_{im}^{x}\psi_{mj}^{y})+n_{Jx}k_{\phi}\psi_{ij}^{y}=0$$

For j=1, J=3 or j=N_y, j =4; i=2 …N_x-1 (4.15)

Shear force along edges x=0 or a:

$$k_{55}A_{55}(\psi_{ij}^{x}+\sum_{m=1}^{N_x}A_{im}^{x}\psi_{mj}^{x})+k_{45}A_{45}(\psi_{ij}^{y}+\sum_{n=1}^{N_y}A_{jn}^{y}\psi_{in}^{y})+n_{Ix}k_{Iw}W_{ij}=0,$$

For i=1, i =1 or i=N_x, i =2; j=2 …N_y-1 (4.16)

Shear force along edges y=0 or b:

$$k_{45}A_{45}(\phi_{ij}^{x}+\sum_{m=1}^{N_x}A_{im}^{x}w_{mj})+k_{44}A_{44}(\phi_{ij}^{y}+\sum_{n=1}^{N_y}A_{jn}^{y}w_{in})+n_{jx}k_{Jw}W_{ij}=0,$$

For j=1, J=3 or j=N_y, j =4; i=2 …N_x-1 (4.17)

Boundary conditions for multispan problem $i=const.,\ j=1,2,........N_y$

$$w_{ij}=0,\ \phi_{ij}{}^{x}=0,\ \phi_{ij}{}^{y}=0,$$

At corner grid points, where the numbers of equations are greater than the number of unknowns, two of the boundary conditions can be removed. There is no sensitivity on which of the two that can be removed. In the analyses, the shear forces and twisting moments are applied at corner grid points. To establish the eigenvalue system of equations, the degrees of freedom are separated into the boundary and the domain degrees of freedom as

$$\{U\}_b=\begin{Bmatrix}W\\ \psi^{x}\\ \psi^{y}\end{Bmatrix}_b,\qquad \{U\}_d=\begin{Bmatrix}W\\ \psi^{x}\\ \psi^{y}\end{Bmatrix}_d \tag{4.18}$$

Where subscripts ‘d’ and ‘b’ stand for the domain and boundary degrees of freedom.

Rearranging the discretized governing equations, the assembled form of them in a matrix form becomes,

$$[S]\{U\}_d - \omega^2 [M]\{U\}_d = 0, \tag{4.19}$$

Where S= stiffness matrix.

By solving the above equations, natural frequencies as well as mode shapes can be obtained.

Chapter 5
RESULT AND DISCUSSION

To illustrate the convergence and accuracy of the DQM here we are taking two cases for two dimensional laminated plates.

1. With two equally spaced span.
2. With three equally spaced span.

The layout of two span plates is shown in fig. 5.1. The location of internal line support is a/2.

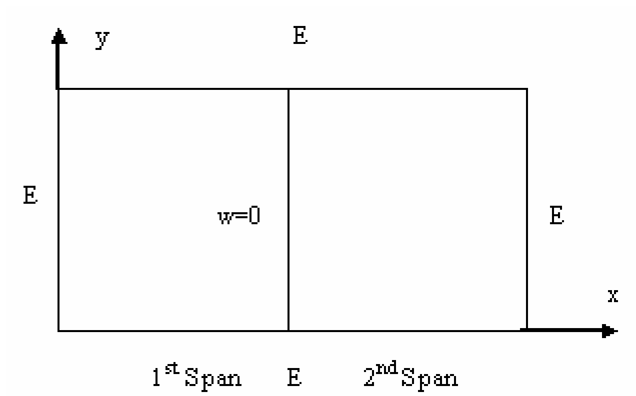


FIGURE 5.1. LAMINATED PLATE N=9 WITH ELASTICALLY RESTRAINT EDGE HAVING ONE SUPPORT AT MID OF THE SPAN.

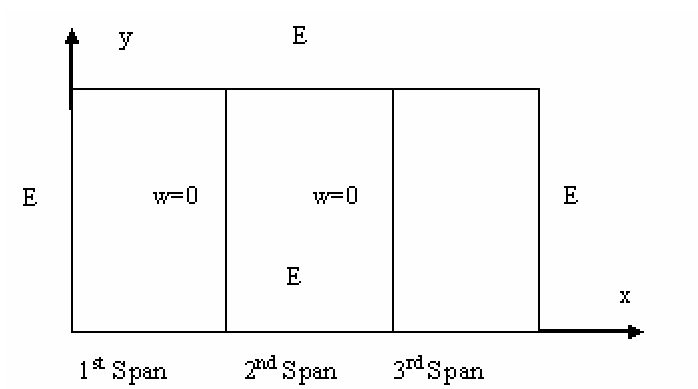


FIGURE 5.2. LAMINATED PLATE N=9 WITH ELASTICALLY RESTRAINT EDGE HAVING TWO EQUALLY SPACED SUPPORT AT MID OF THE SPAN.

Moderately thick angle-ply laminates with elastically restrained edges is taken as example. The material properties of each layer are

1. $\dfrac{E_{11}}{E_{22}} = 40, \qquad \dfrac{G_{12}}{E_{22}} = \dfrac{G_{13}}{E_{22}} = 0.6, \dfrac{G_{23}}{E_{22}} = 0.5, \quad \upsilon_{12} = 0.25.$
2. a/b=1, t/b=0.1, k=5/6, N=9.
3. $K_w = k_w b^3 / D_{22}$, $S_\phi = k_\phi b / D_{22}$

Where t is the total thickness of the laminated plates. While a and b are the length of plates in x- direction and y-direction respectively.

The convergence behaviors of the first three mode functions are tested in table 5.1 for laminates with elastically restrained edges. As shown in tables the convergence achieved is very good.

The frequency parameters are calculated here for grid points 7,9,11,13,15,17 and 19 in x-direction. At the same time the grid points in y-direction kept constant. In the table NOS denotes the number of span while N_x and N_y denotes the number of grid point in x and y direction respectively. The $\overline{\omega_1}$, $\overline{\omega_2}$, and $\overline{\omega_3}$ are the first three frequency parameters. From the table 5.1 it can be seen that convergence from above occurs as the grid points are increased and highly accurate values are obtained for N_x=13

NOS	N_x	N_y	$\overline{\omega_1}$	$\overline{\omega_2}$	$\overline{\omega_3}$
2	7	9	1.02458	1.14639	1.67842
	9	9	1.02339	1.12413	1.67774
	11	9	1.02336	1.11257	1.67773
	13	9	1.02337	1.1069	1.67773
	15	9	1.02337	1.10245	1.67773
	17	9	1.02337	1.09968	1.67773
	19	9	1.02337	1.09727	1.67773
3	10	9	1.05521	1.69475	1.69758
	13	9	1.04853	1.69329	1.69614
	16	9	1.04374	1.69022	1.69582
	19	9	1.04123	1.68861	1.69518
	22	9	1.03918	1.6873	1.69488
	25	9	1.03785	1.68645	1.69445
	28	9	1.03671	1.68572	1.69419

TABLE 5.1. THE FIRST THREE NATURAL FREQUENCY PARAMETERS $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})$ OF (EEEE) MODERATELY THICK ANISOTROPIC PLATES (0/90/0/90/0/90/0/90/0), (t/b=0.1, k=5/6, a/b=1).

A numerical example of cross-ply plates with elastic restrained boundary conditions is examined here. To define the boundary conditions along the edges the alphabet symbolism is used, so that EEEE indicates that all four edges of plate are elastically restraints. To show the computational efficiency of this DQM, a thin square plate composed with nine orthotropic layers (0/90/0/90/0/90/0/90/0) is considered. The material properties of each layer are

1. $\frac{E_{11}}{E_{22}} = 40, \quad \frac{G_{12}}{E_{22}} = \frac{G_{13}}{E_{22}} = 0.6, \frac{G_{23}}{E_{22}} = 0.5, \quad \upsilon_{12} = 0.25.$
2. k=5/6, N=9.
3. $K_w = k_w b^3 / D_{22}$, $S_\phi = k_\phi b / D_{22}$
4. a/b=1, t/b=0.1

For different three different values of t/b with three different values of S_ϕ, the Non-dimensional frequency parameter $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})$ is plotted against elastic restraint parameter K_w in the following figures (5.1-5.4).

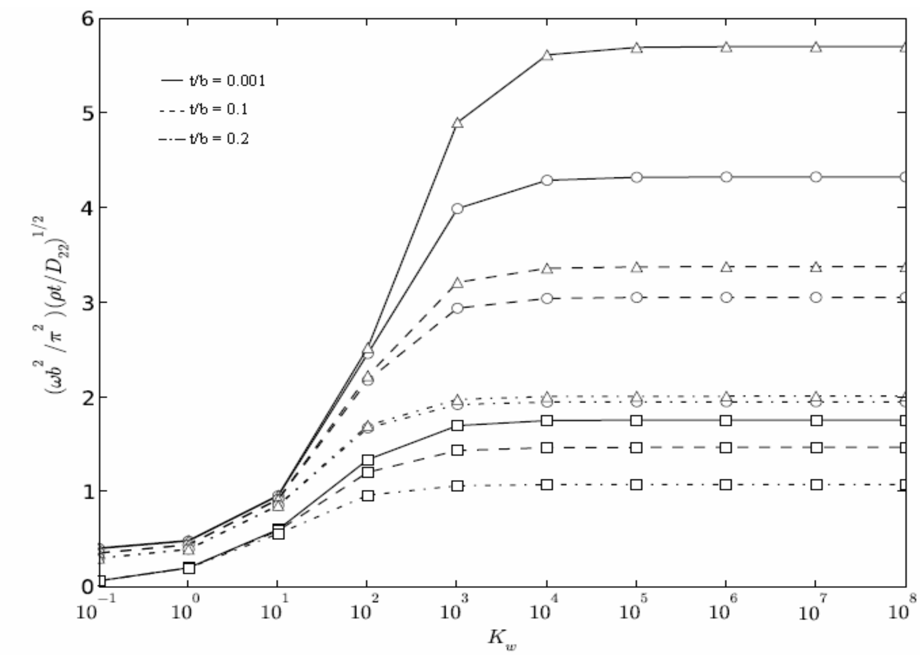


FIGURE 5.3. FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})$, VERSUS ELASTIC LATERAL EDGE PARAMETER K_w FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (a/b=1, S_ϕ=0,□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

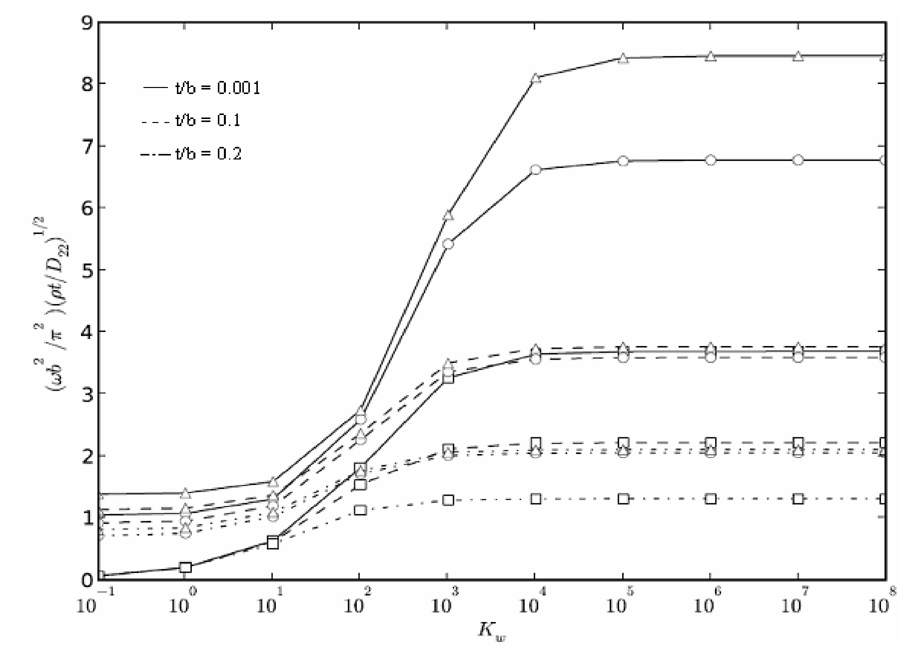


FIGURE 5.4. FREQUENCY PARAMETER, $(\omega b^2 / \pi^2)(\sqrt{\rho t / D_{22}})$, VERSUS ELASTIC LATERAL EDGE PARAMETER K_w FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (a/b=1, S_ϕ=100,□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

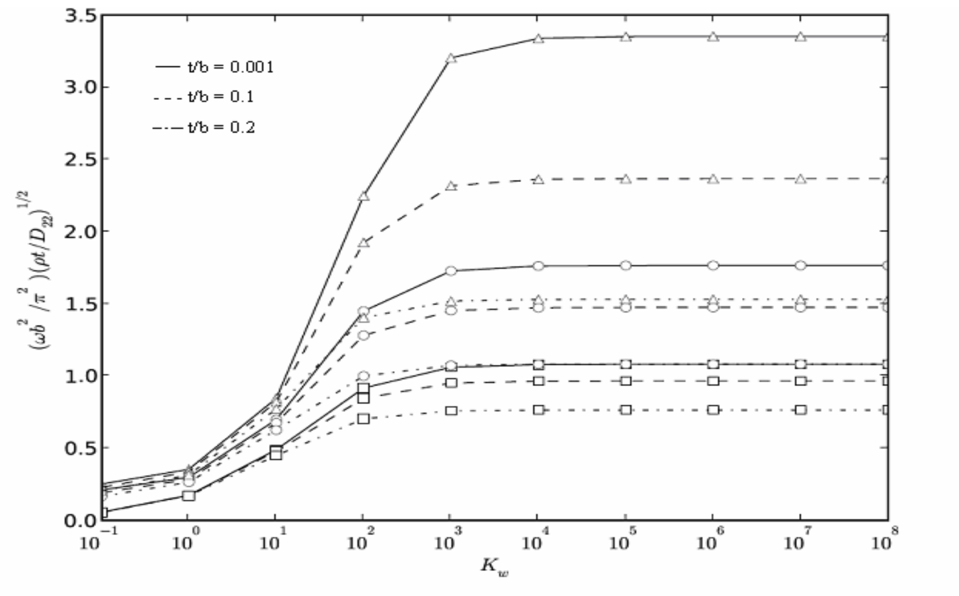


FIGURE 5.5. FREQUENCY PARAMETER, $(\omega b^2 / \pi^2)(\sqrt{\rho t / D_{22}})$, VERSUS ELASTIC LATERAL EDGE PARAMETER K_w FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (a/b=2, S_ϕ=0,□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

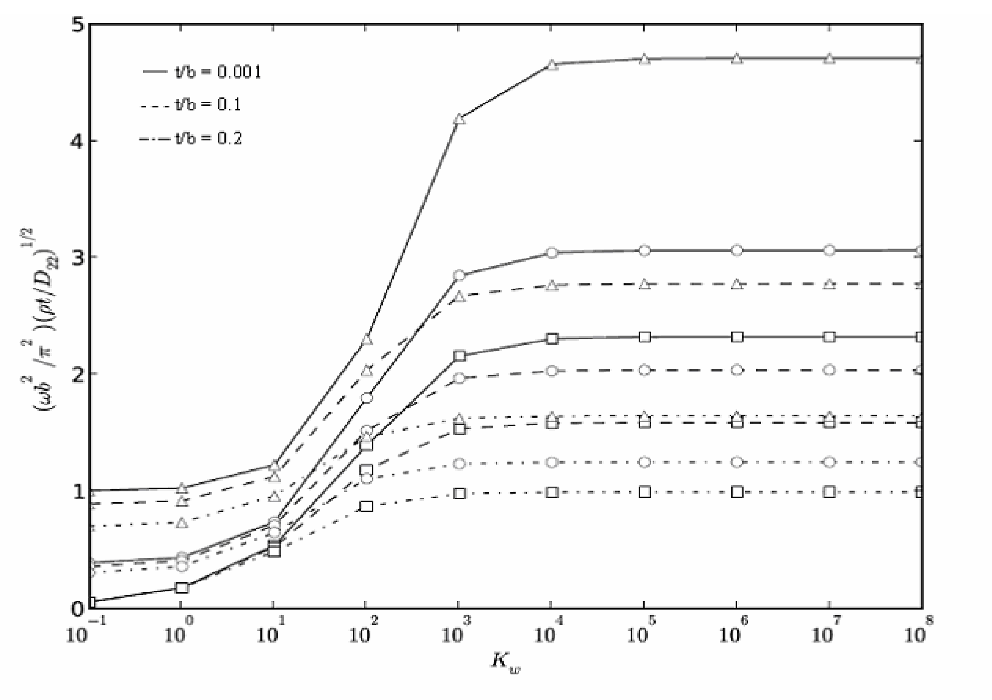


FIGURE 5.6. FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})$, VERSUS ELASTIC LATERAL EDGE PARAMETER K_w FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (a/b=2, S_ϕ=100,□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

From the observation of the figure (5.3-5.6), we can conclude that for the certain rang of K_w the non-dimensional frequency rise suddenly, after this range non-dimensional frequency parameter is near about constant.

The analysis of the hybrid laminated is done for elastically restrained edge boundary condition(EEEE). The fig 5.7-5.9 presents variations of first three frequency parameter λ against number of span.

The material properties are taken same as taken for convergence study.

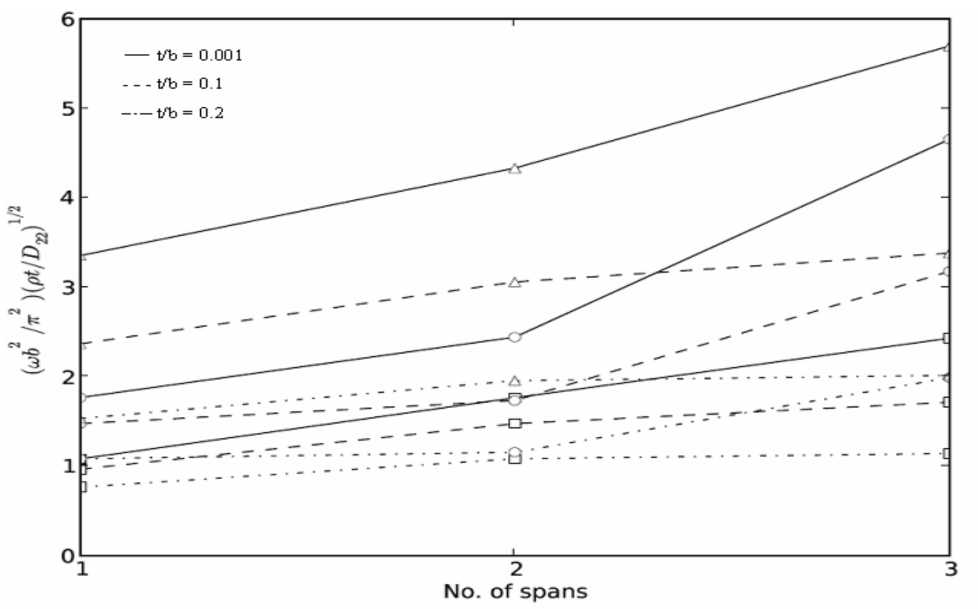


FIGURE 5.7. FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})$, VERSUS NO. OF SPAN, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (a/b=1, S_ϕ=0, □ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

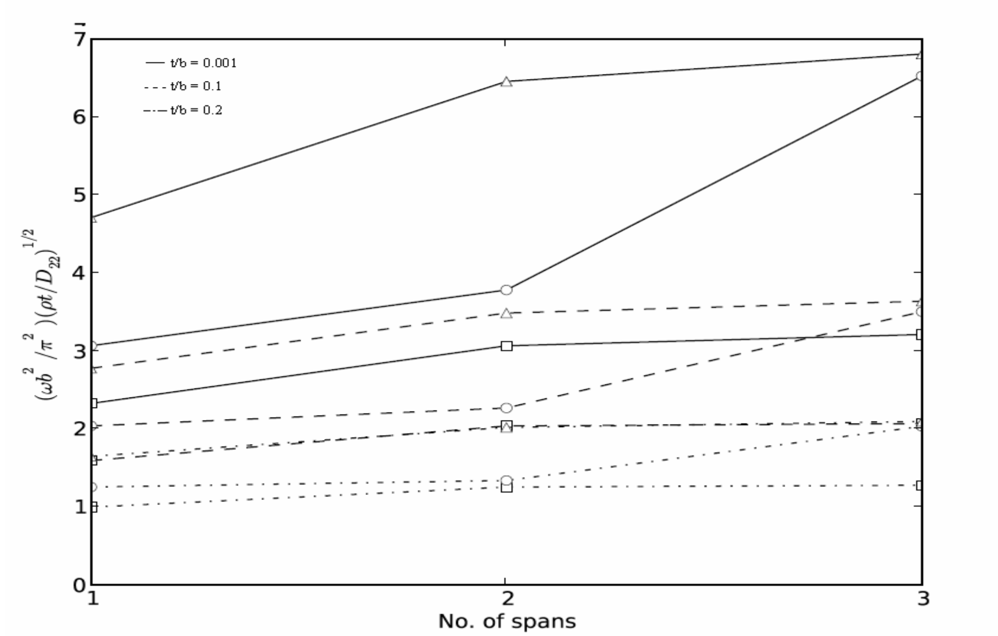


FIGURE 5.8 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})$, VERSUS NO. OF SPAN, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (a/b=1, S_ϕ=100,□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

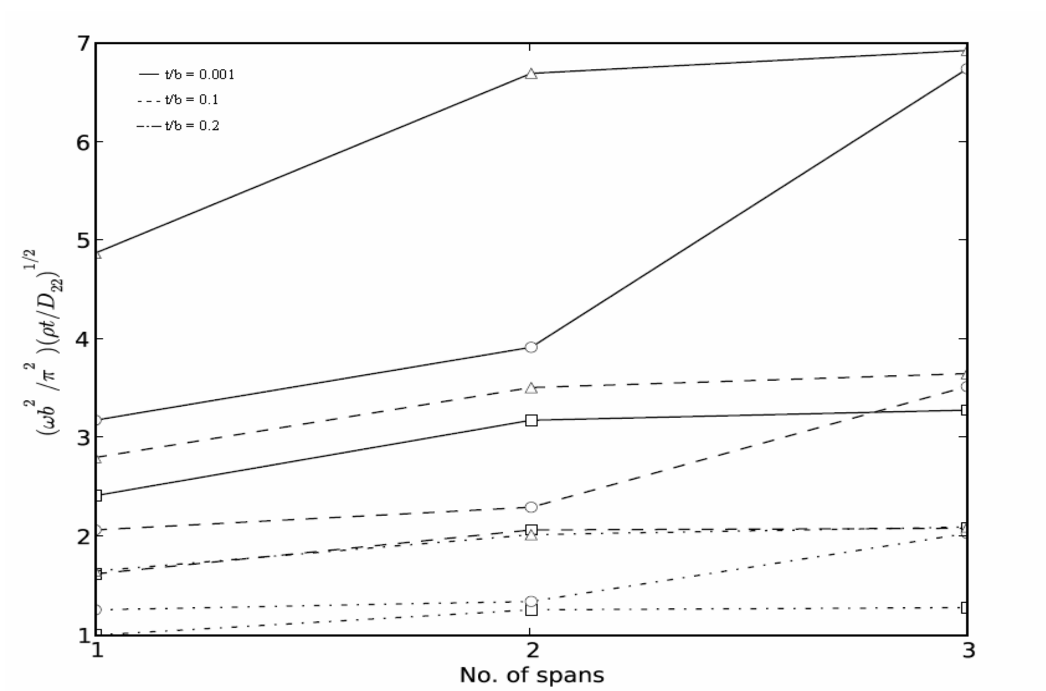


FIGURE 5.9 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})$, VERSUS NO. OF SPAN, FOR SYMMETRIC CROSS-PLY LAMINATED PLATE (a/b=1, $S_\phi=10^8$, □ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

From the observation of the figure (5.7-5.9), we can conclude that

1. As the value of ratio of t/b is increases, the value of frequency parameter is decreases.
2. As the number of span increases the value of frequency parameter is increases.

The same research has been done for asymmetric laminated plate having N=4.

1. $\frac{E_{11}}{E_{22}} = 40, \qquad \frac{G_{12}}{E_{22}} = \frac{G_{13}}{E_{22}} = 0.6, \frac{G_{23}}{E_{22}} = 0.5, \quad \upsilon_{12} = 0.25.$
2. a/b=2, k=5/6, N=4.

The analysis of the hybrid laminated is done for elastically restrained edge boundary condition(EEEE). The fig 5.8-5.10 presents variations of first three frequency parameter λ against number of span.

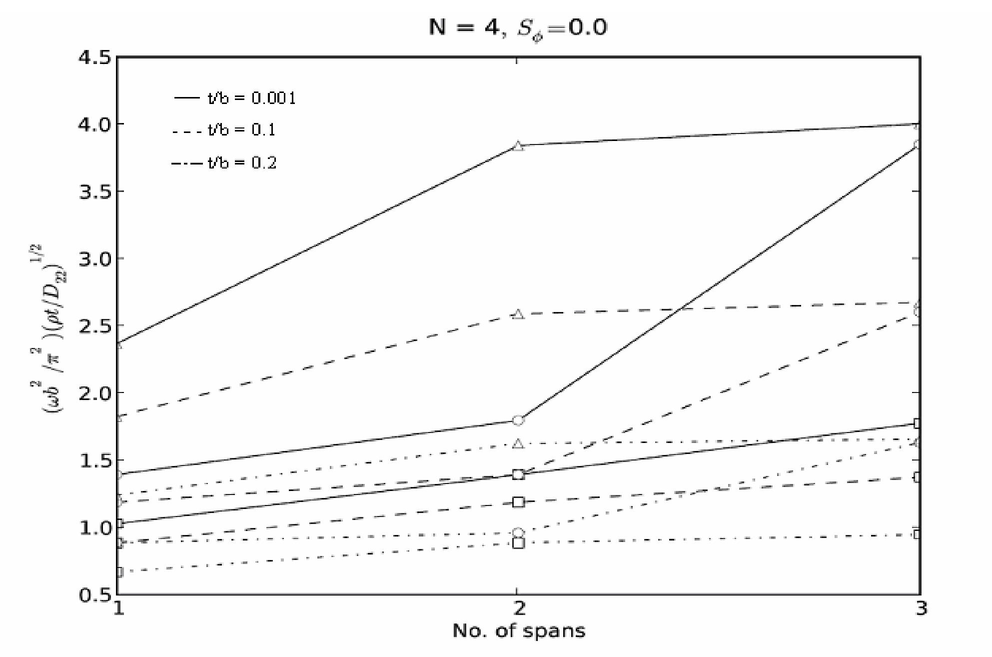


FIGURE 5.10 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})$, VERSUS NO. OF SPAN, FOR ASYMMETRIC CROSS-PLY LAMINATED PLATE (a/b=1, $S_\phi=0$, □ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

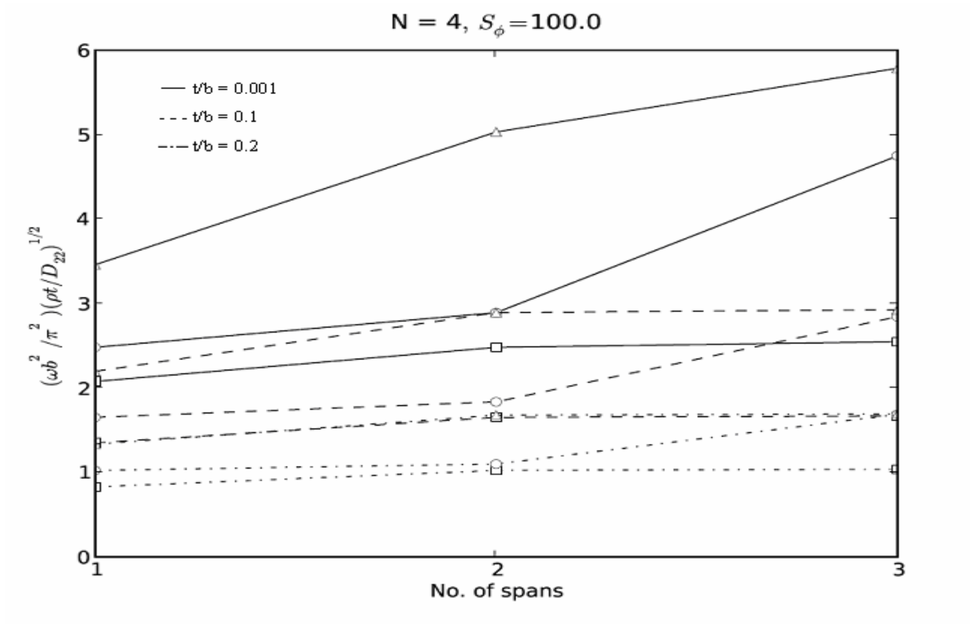


FIGURE 5.11 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})$, VERSUS NO. OF SPAN, FOR ASYMMETRIC CROSS-PLY LAMINATED PLATE (a/b=1, S_ϕ=100,□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

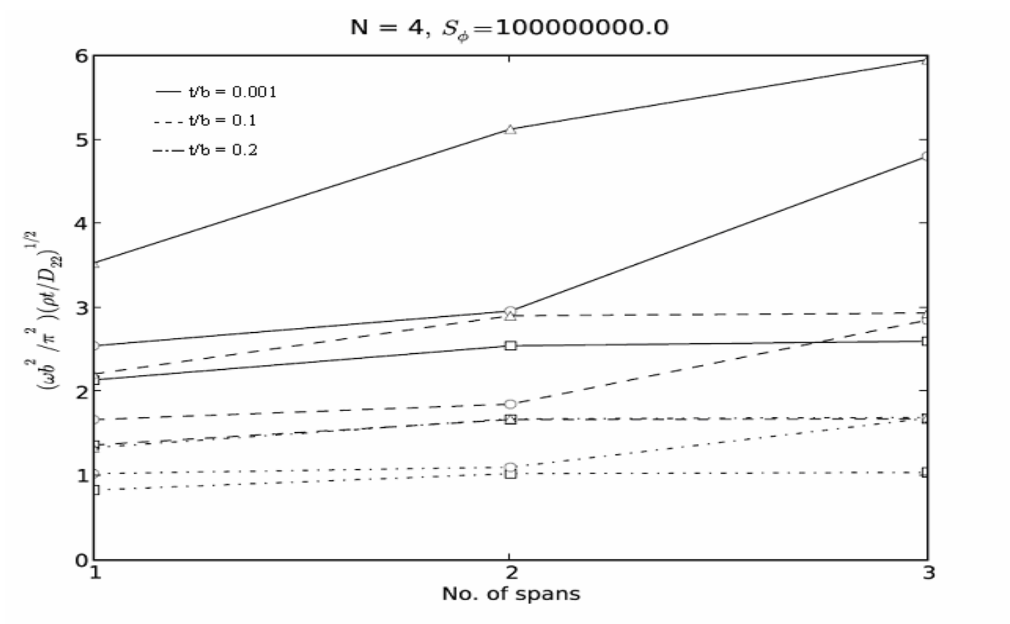


FIGURE 5.12 FREQUENCY PARAMETER, $(\omega b^2/\pi^2)(\sqrt{\rho t/D_{22}})$, VERSUS NO. OF SPAN, FOR ASYMMETRIC CROSS-PLY LAMINATED PLATE (a/b=1, $S_\phi=10^8$,□ FIRST MODE; O SECOND MODE; AND Δ THIRD MODE)

From the observation of the figure (5.10-5.12), we can conclude that

1. As the value of ratio of t/b is increases, the value of frequency parameter is decreases.
2. As the number of span increases the value of frequency parameter is increases.
3. There is rapid increment in the value of second frequency parameter as the number of span increases.

The normalized mode shapes in the x-direction for first and the third modes of an asymmetric square plate shown in figure 5.13.

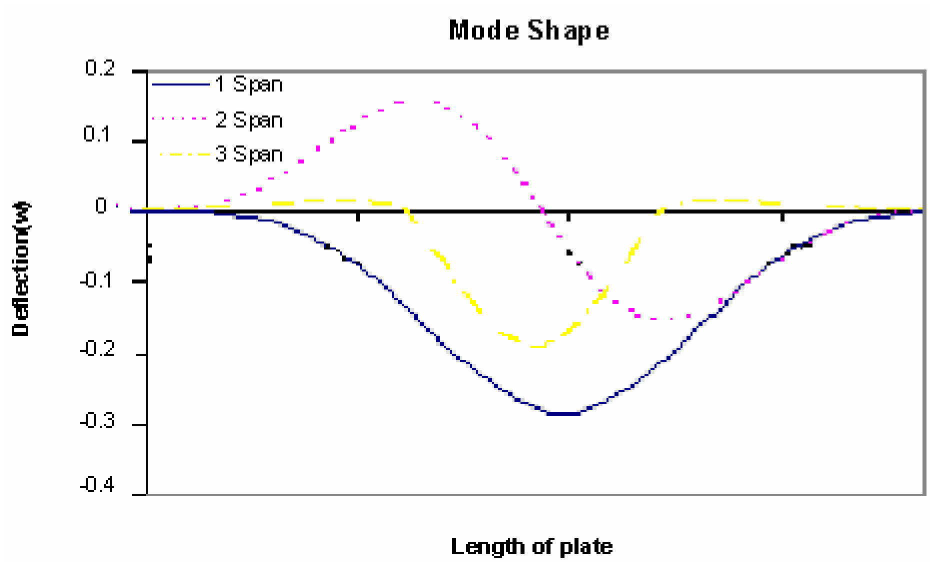


FIGURE 5.13 DEFLECTION, VERSES LENGTH OF PLATE FOR DIFFERENT SPAN OF PLATE.

From the figure 5.13 we can say that for single span the deflection is higher near about the half distance of the plate and in negative direction when only mass load is working on it. It should also be notice that deflection is at the x=0 and x=a. is near about zero.

For two span problem, the deflection is positive for first span and negative for second span. At the x=0, x=a, and midline support, the deflection is near about zero.

For three span problem, the deflection for first and third span is very low as compare to deflection at second span. Here at the edge and at internal support the deflection is zero same as for two span problem.

CHAPTER 6

CONCLUSION AND FUTURE SCOPE

6.1 CONCLUSION

The DQ method is applied to angle-ply laminated plate having elastic edge restrained elastically in against translation and rotation, and with internal support. Here we have shown the trend of frequency parameter against K_w and number of span for different values of aspect ratio t/b and S_ϕ. After observing the curve we can say that

1. The values of frequencies of the plates is increased when the nuber of span is increased.
2. As the ratio of t/b is increased, the values frequencies are decreased.

The result and convergence study state that DQM can yield very accurate result for multi span elastically restrained laminate plate.

6.2 FUTURE SCOPE

1. Further Analysis can be done with different homogeneous boundary conditions like clamp (All four boundaries are clamed(C-C-C-C)) or simply supported (All four boundaries are simply supported (S-S-S-S)).
2. Same analysis can be done with non homogeneous boundary conditions i.e. combination of two or more.(C-S-C-S, C-C-F-F, S-F-S-F).
3. Forced vibration analysis can be done as a extension of this study.
4. Stress analysis can also be done in similar manner.

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