

SOME METHODS FOR ANALYZING THE RELIABILITY OF SERIES-PARALLEL SYSTEMS USING GENERALIZED INTUITIONISTIC FUZZY NUMBER

A Thesis

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled "*Some methods for analyzing the reliability of series-parallel systems using generalized intuitionistic fuzzy number*" in partial fulfillment of the requirement for the award of degree of Master of Science, School of Mathematics (SoM), Thapar University, Patiala is an authentic record of my own carried out under the supervision of Dr. Harish Garg, Assistant Professor, SoM, Thapar University Patiala.

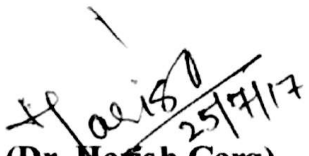
The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.


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Date: July 25, 2017


(Dr. Harish Garg)

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Abstract

With the growing complexity of an industrial system, it is difficult for the decision makers to get the accurate decision within a reasonable time. In order to maintain the performance of the systems, reliability is one of the importance factor. Reliability is measured as the ability of a system's to perform its intended function, without fail, for a specified period, under predetermined conditions. But uncertainties is an unavoidable phenomenon in most of the system. So it is difficult for the system analyst to get the correct decision without handling these uncertainties.

In the present thesis, some new approaches have been presented for analyzing the reliability of the series-parallel configuration systems in which the uncertainties which are present in the data are handled with the help of the generalized trapezoidal fuzzy numbers. In the approach, preferences of the decision makers, as well as, the highest level of confidence of domain experts are added into the analysis and hence the corresponding data are represented in the form of the generalized trapezoidal intuitionistic fuzzy numbers (GTrIFN). Based on these information, some methods for analyzing the behavior of the system have been analyzed at the different levels of the uncertainties.

The present thesis is organized into five chapters which are briefly summarized as follows:

A brief account of the related work of various authors in the evaluation of reliability analysis of some industrial system under fuzzy and intuitionistic fuzzy environment is presented in the first chapter. In **Chapter 2**, the basic and preliminaries related to the reliability theory and the fuzzy, as well as, the intuitionistic fuzzy set theory that are used in the subsequent chapters are given.

Chapter 3 presents a method for ranking the generalized trapezoidal fuzzy numbers

(GTrIFN). In it, the ranking of the two different GTrIFNs has been done by using a new defuzzification method based on the centroid point of the intuitionistic fuzzy numbers (IFNs). Some special cases of the proposed one have also been obtained. The method has been demonstrated through some examples for comparing the two different IFNs.

In **Chapter 4**, a method for analyzing the reliability of the series, series-parallel system has been presented under the generalized intuitionistic fuzzy number environment. In it, by using the α -cut of the membership and non-membership functions of the GTrIFN, reliability of some system configuration has been computed at different level of the uncertainties. A comparative studies with some of the existing approaches has also been done.

In **Chapter 5**, a method for analyzing the reliability of the industrial system has been presented by using GTrIFNs. For it, the preference of the decision makers have been taken in the form of the components of the systems and then various reliability parameters which affect the system behavior directly have been computed. The approach has been illustrated with a case study of the washing unit of a paper mill, a complex repairable industrial system.

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(Priyanka)

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Chapter 1

Introduction

With the growing complexities of the system day-by-day, reliability of the system is one of the most important factor on which system analyst pay more attentions. Traditionally, reliability of the system has been analyzed by means of some probability measures. However, due to the uncertainties arises in the data, the estimation of the reliability in terms of their probabilistic approach becomes very difficult to give the correct decision. Thus, probabilistic based approach are unable to enhance the reliability of the system up to desired level. To overcome these difficulties, methodologies based on fuzzy set (FS) theory [39] is one of the powerful tools to handle the impreciseness and vagueness in the data. Since their existence, various researchers have used the FS theory into the reliability analysis. For instance, Singer [34] estimated the reliability of the system by using L-R (left-right) type fuzzy numbers. Later on, Cheng and Mon [6] proposed confidence interval method for analyzing the system reliability under the fuzzy environment. Chen [3] analyzed the reliability of the system using fuzzy arithmetic operations instead of ordinary arithmetic operations while Chen [4] analyzed the system behavior by using α -cut arithmetic operations of the fuzzy number. In continuation with these, Mon and Cheng [32] analyzed the reliability of the system by using different types of membership functions rather than triangular function. Utkin and Gurov [36] measuring the system performance in terms of steady-state availability and unavailability by considering the failure and repair process of the system. Huang et al. [27] presented a method based on the artificial neural network to estimate the parameters of the reliability and hence analyzed the performance of the system.

After the successful applications of fuzzy set theory, Atanassov [1] developed the concept of intuitionistic fuzzy sets (IFSs), which are characterized by a membership function and a non-membership function, thus generalizing fuzzy sets which only assign a membership degree to each element. The concept of IFS can be viewed as an alternative approach in which the degrees of membership and non-membership functions have been used for expressing the uncertainties in the data such that their sum is less than one. From the reliability field, Knezevic and Odoom [29] presented a theory, called as fuzzy lambda-tau (FLT), by using Petri net (PN) and FS theory for analyzing the behavior of the system in terms of several parameters. Further, this theory has been used by the researchers and apply them to investigate the behavior of the different industrial systems [25, 33]. Recently, due to its greater flexibility in handling vagueness or uncertainty, IFSs theory has been widely used in many areas, such as medical diagnosis [7], pattern recognitions [31], clustering analysis [38], and decision making [14, 15, 17, 19, 37]. As far as reliability field is concerned, a lot of work has been investigated by the authors for enhancing the reliability of the system. For more details, about such types of work, we may refer to Chang et al. [2], Chen [5], Garg [15, 18], Garg and Rani [20], Garg et al. [21], Garg, Rani, Sharma and Vishwakarma [23, 24], Taheri and Zarei [35] and their corresponding references.

Since all the above studies have expressed the obtained data in terms of fuzzy/intuitionistic fuzzy numbers which usually follows an exponential distribution along with the same type of the numbers. However, in real-life situations, these types of situation is rarely occurring because failure is an unavoidable phenomenon. Therefore, there is a need to investigate these issues and to include some other types of membership function into the analysis so that the plant personnel may use the technique for the variable rate also. Further, there is a need to add the highest level of the confidence of domain expert into the analysis. By keeping all these features in mind, in the present thesis, an attempt has been made to present some methods for analyzing the reliability of the series-parallel systems using generalized intuitionistic fuzzy numbers. In the approach, preferences of the decision makers, as well as, the highest level of confidence of domain experts are added into the analysis and hence the corresponding data are represented in the form of the generalized trapezoidal intuitionistic fuzzy numbers (GTrIFN).

The present thesis is organized into five chapters including the present one that contains mainly the literature review. The rest of the chapters are described below:

In **Chapter 2**, the basic and preliminaries related to the reliability theory and the fuzzy, as well as, the intuitionistic fuzzy set theory that are used in the subsequent chapters are given.

Chapter 3 presents a method for ranking the generalized trapezoidal fuzzy numbers (GTrIFN). In it, the ranking of the two different GTrIFNs has been done by using a new defuzzification method based on the centroid point of the intuitionistic fuzzy numbers (IFNs). Some special cases of the proposed one have also been obtained. The method has been demonstrated through some examples for comparing the two different IFNs.

In **Chapter 4**, a method for analyzing the reliability of the series, series-parallel system has been presented under the generalized intuitionistic fuzzy number environment. In it, by using the α -cut of the membership and non-membership functions of the GTrIFN, reliability of some system configuration has been computed at different level of the uncertainties. A comparative studies with some of the existing approaches has also been done.

In **Chapter 5**, a method for analyzing the reliability of the industrial system has been presented by using GTrIFNs. For it, the preference of the decision makers have been taken in the form of the components of the systems and then various reliability parameters which affect the system behavior directly have been computed. The approach has been illustrated with a case study of the washing unit of a paper mill, a complex repairable industrial system.

Chapter 2

Preliminaries

This chapter presents some of the fundamental definitions and mathematical theory for fuzzy set theory. The focus is on defining the fuzzy set, α -cuts, convex and normal fuzzy set, fuzzy numbers and sigmoidal fuzzy numbers.

2.1 Reliability

Reliability is defined as the probability that a system or component will perform its required function for a given period of time $(0, t]$, when used under stated operating conditions [8]. Therefore, reliability a function of time t , denoted by $R(t)$, is the probability that no failure at system level will occur in the time interval $(0, t]$. Define a continuous random variable T to be the time to failure of the system/component; $T \geq 0$ then the equation of reliability becomes as [8]:

$$R(t) = \Pr(T \geq t) = 1 - \int_0^t f(u) du \quad (2.1.1)$$

where $R(t) \geq 0$, $R(0) = 1$ and $\lim_{t \rightarrow \infty} R(t) = 0$ and $f(t)$ failure probability function.

The continuous function $F(t)$ defined by

$$F(t) = 1 - R(t) = \Pr(T \leq t) \quad (2.1.2)$$

is termed as the cumulative distribution function of the failure distribution function.

An important function, mean time to failure (MTTF) which gives the expected value of the probability density function $f(t)$ of the failure distribution is expressed as

$$MTTF = \int_0^\infty t f(t) dt = \int_0^\infty R(t) dt \quad (2.1.3)$$

Another function known as failure rate or hazard rate function, denoted by $\lambda(t)$ has major significance in the field of reliability engineering. It measures an instantaneous rate of failure of system/component and is given as [8]

$$\lambda(t) = \frac{-dR(t)}{dt} \cdot \frac{1}{R(t)} = \frac{f(t)}{R(t)} \quad (2.1.4)$$

Based on these hazard rate function, the reliability function can be derived as

$$R(t) = \exp \left[- \int_0^t \lambda(u) du \right] \quad (2.1.5)$$

2.2 Availability

Availability is defined as the probability of a product or system working satisfactorily at any given point of time when used under the given conditions of use. Mathematically, for any system whose failure and repair rates are denoted by λ_s and μ_s respectively then at any given time t , the system will be operational if the following conditions are met [8]:

$$A_s(t) = \frac{\mu_s}{\lambda_s + \mu_s} + \frac{\lambda_s}{\lambda_s + \mu_s} e^{-(\lambda_s + \mu_s)t} \quad (2.2.1)$$

The steady state availability, A of a system is the limit of the instantaneous availability function as time approaches to infinity and is represented as

$$A = \lim_{t \rightarrow \infty} A_s(t) = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \quad (2.2.2)$$

where MTTR and MTBF are the mean time to repair and the mean time failures of the system/component respectively.

2.3 Crisp reliability of a system

In this section, the crisp reliability evaluation methods for series system and parallel system are given.

2.3.1 Crisp reliability of a series system

The components in a system are said to be in series configuration from the reliability point of view if they all are needed to work for a system success or only one is needed to fail for system failure [8].

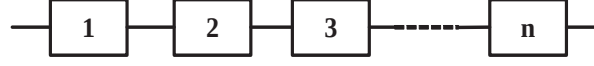


Figure 2.1: Configuration of series system

Consider a series system consisting of ‘ n ’ mutually independent components as shown in Figure 2.1. Then the crisp reliability R_s of the series system is evaluated as follows:

$$R_s = \prod_{i=1}^n R_i \quad (2.3.1)$$

where R_i represents the reliability of i^{th} component.

2.3.2 Crisp reliability of a parallel system

The components, in a system, are said to be in parallel configuration from the reliability point of view if only one is needed to work for system success or all are needed to fail for system failure[8].

Consider a parallel system consisting of ‘ n ’ mutually independent components as shown in Figure 2.2. Then the crisp reliability R_p of the parallel system is evaluated as follows

$$R_p = 1 - \prod_{i=1}^n (1 - R_i) \quad (2.3.2)$$

where R_i represents the reliability of i^{th} component.

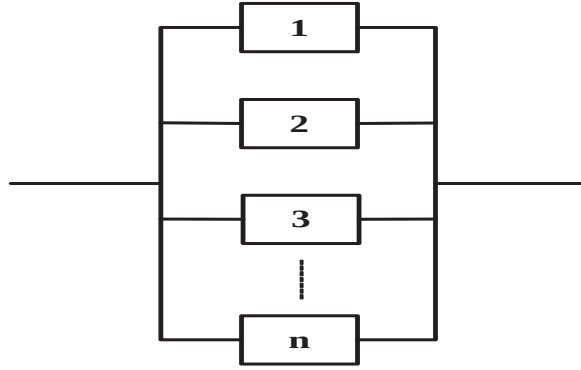


Figure 2.2: Configuration of parallel system

2.4 Fuzzy set theory

2.4.1 Classical Sets (Crisp sets)

A classical set A is a collection of distinct objects $X = \{x_1, x_2, \dots, x_n\}$ whose membership function is defined as follows

$$\chi_A(x) = \begin{cases} 1 & \text{; if } x \in A \\ 0 & \text{; if } x \notin A \end{cases} \quad (2.4.1)$$

2.4.2 Fuzzy set theory

Zadeh [39] extended and generalized the concept of crisp set by allowing the partial membership i.e between 0 and 1 and named as fuzzy sets (FSs). A fuzzy set A defined on X is a set of ordered pairs of the element and its membership function which is represented as follows[39] :

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}} \rangle \mid x \in X\} \quad (2.4.2)$$

where $\mu_{\tilde{A}}(x)$ is the degree of membership of x in set $\tilde{A}$ such that $\mu_{\tilde{A}}(x) \in [0, 1]$.

2.4.3 Convex fuzzy set

A fuzzy set $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x) \rangle \mid x \in X\}$ is said to be convex fuzzy set [28] if the following inequality holds for $x_1, x_2 \in X$

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)] \quad (2.4.3)$$

If above inequality does not hold then that is said to be non-convex fuzzy set.

2.4.4 Normal fuzzy set

A fuzzy set is said to be normal fuzzy set [28] if there exist at least one element in the universal set U such that their corresponding membership function is unity.

2.4.5 Fuzzy number

A convex, normal membership function on the real line R is called a fuzzy number [28] i.e if its membership function is piecewise continuous and there exist at least one $x_0 \in U$

such that $\mu_{\tilde{A}}(x_0) = 1$. The corresponding membership function defined on $[a, b] \neq \emptyset$ is given as

$$\mu_{\tilde{A}}(x) = \begin{cases} f(x) & ; x \in (-\infty, a) \\ 1 & ; x = [a, b] \\ g(x) & ; x \in (b, \infty) \\ 0 & ; \text{otherwise} \end{cases} \quad (2.4.4)$$

where f and g monotonic, continuous from the right and left, nondecreasing and non-increasing functions such that $f(x) = 0$ for $x \in (-\infty, a)$ and $g(x) = 0$ for $x \in (b, \infty)$. A fuzzy number $A = (a, b, c, d)$ is called a trapezoidal fuzzy number if its membership function μ_A is given by

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & ; a \leq x < b \\ 1 & ; b \leq x < c \\ \frac{d-x}{d-c} & ; c \leq x < d \\ 0 & ; \text{otherwise} \end{cases} \quad (2.4.5)$$

The α - cut of the number $A = (a, b, c, d)$ is the closed interval

$$A_\alpha = [A_\alpha^L, A_\alpha^R] = [a + \alpha(b - a), d + \alpha(c - d)], \quad \alpha \in (0, 1]$$

2.5 Intuitionistic fuzzy set theory

An intuitionistic fuzzy set (IFS) is a finite universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ is given by [1]

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\} \quad (2.5.1)$$

where $\mu_A(x)$ and $\nu_A(x)$ respectively the grades of membership and non-membership of an element x with the conditions that $0 \leq \mu_A(x), \nu_A(x) \leq 1$ and $\mu_A(x) + \nu_A(x) \leq 1$. IFS reduces to fuzzy set when $\mu_A(x) + \nu_A(x) = 1 \forall x \in X$.

2.5.1 Convex intuitionistic fuzzy set

An IFS set $\tilde{A}$ in universe X is said to be convex if and only if the membership function of $\mu_A(x)$ of $\tilde{A}$ is convex and non-membership function of $\nu_A(x)$ of $\tilde{A}$ is concave.

2.5.2 Normal IFS

An IFS $\tilde{A}$ is said to be normal if there exist at least two points $x_1, x_2 \in X$ such that $\mu_{\tilde{A}}(x_1) = 1$ and $\nu_{\tilde{A}}(x_2) = 0$.

2.5.3 Intuitionistic fuzzy number

An intuitionistic fuzzy subset $A = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \rangle \mid x \in R\}$ of real line R is called an IFN if

- (i) A is convex and normal.
- (ii) $\mu_{\tilde{A}}$ (or $\nu_{\tilde{A}}$) is upper (or lower) semi-continuous.
- (iii) $\text{Supp } \tilde{A} = (x \in X \mid \mu_A(x) \leq 1)$ is bounded.

2.5.4 Generalized Trapezoidal intuitionistic fuzzy number

Let $\tilde{A} = \langle (a, b, c, d); [\omega_r, \omega_t], [u_r, u_t] \rangle$ be a set where $a, b, c, d \in R$ then the set $\tilde{A}$ is said to be Generalized Trapezoidal intuitionistic fuzzy number (GTrIFN) if its membership function is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x < a \\ \omega_t \left(\frac{x-a}{b-a} \right) & ; a \leq x < b \\ \frac{(\omega_r - \omega_t)(x-b)}{c-b} + \omega_t & ; b \leq x < c \\ \omega_r \left(\frac{x-d}{c-d} \right) & ; c \leq x < d \\ 0 & ; x > d \end{cases} \quad (2.5.2)$$

and non membership function is defined as follows

$$\nu_{\tilde{A}}(x) = \begin{cases} 1 & ; x < a \\ \frac{(b-x)+u_r(x-a)}{b-a} & ; a \leq x < b \\ u_r + \frac{(u_t - u_r)(x-b)}{c-b} & ; b \leq x < c \\ \frac{u_t(x-d)+(c-x)}{c-d} & ; c \leq x < d \\ 1 & ; x > d \end{cases} \quad (2.5.3)$$

2.5.5 α -cuts

The α -cut sets of GTrIFNs $\tilde{A} = ((a, b, c, d); [\omega_r, \omega_t], [u_r, u_t])$ is a crisp subset of R , which is defined as $\tilde{A}_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$, where $0 \leq \alpha \leq \omega_r$ and denoted by $A_\alpha = [L_{\tilde{A}}(\alpha), M_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)]$.

From (2.5.2), we can compute the α cut of the GTrIFN A as

$$\begin{aligned} A_\alpha &= [L_{\tilde{A}}(\alpha), M_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)] \\ &= \left[a + \frac{\alpha}{\omega_t}(b - a), b + \frac{c - b}{(\omega_r - \omega_t)}(\alpha - \omega_t), d + \frac{\alpha}{\omega_r}(c - d) \right] \end{aligned}$$

2.5.6 β -cuts

The β -cut set of a GTrIFN $\tilde{A} = ((a, b, c, d); [\omega_r, \omega_t], [u_r, u_t])$ is a crisp subset of R which is defined as $\tilde{A}_\beta = \{x \mid \nu_A(x) \leq \beta\}$ where $u_r \leq \beta \leq 1$. and denoted by $A_\beta = [L_{\tilde{A}}(\beta), M_{\tilde{A}}(\beta), R_{\tilde{A}}(\beta)]$.

From (2.5.3), we can compute the β cut of the GTrIFN A as

$$\begin{aligned} A_\beta &= [L_{\tilde{A}}(\beta), M_{\tilde{A}}(\beta), R_{\tilde{A}}(\beta)] \\ &= \left[\frac{\beta(b - a) - (b - au_r)}{(u_r - 1)}, \frac{(c - b)(\beta - u_r) + b(u_t - u_r)}{(u_t - u_r)}, \frac{\beta(c - d) + du_t - c}{(u_t - 1)} \right] \end{aligned}$$

Chapter 3

Ranking of the Generalized Intuitionistic Fuzzy number by Centroid Points

In this chapter, we have computed the defuzzified value of the generalized IFS with different heights. Further, based on these defuzzified values, some special cases have been deduced.

3.1 Ranking Method

Let $A = ([a, b, c, d]; [\omega_t, \omega_r], [u_r, u_t])$ be a GTrIFN whose membership and non-membership functions are defined as follows:

$$\mu_A(x) = \begin{cases} 0 & ; x < a \\ \omega_t \left(\frac{x-a}{b-a} \right) & ; a \leq x < b \\ \frac{(\omega_r - \omega_t)(x-b)}{c-b} + \omega_t & ; b \leq x < c \\ \omega_r \left(\frac{x-d}{c-d} \right) & ; c \leq x < d \\ 0 & ; x > d \end{cases} \quad (3.1.1)$$

and

$$\nu_A(x) = \begin{cases} 1 & ; x < a \\ \frac{(b-x)+u_r(x-a)}{b-a} & ; a \leq x < b \\ u_r + \frac{(u_t - u_r)(x-b)}{c-b} & ; b \leq x < c \\ \frac{u_t(x-d)+(c-x)}{c-d} & ; c \leq x < d \\ 1 & ; x > d \end{cases} \quad (3.1.2)$$

where $0 \leq \omega_t \leq \omega_r \leq 1$; $0 \leq u_r \leq u_t \leq 1$; $\omega_r + u_t \leq 1$; $a, b, c, d \in \mathbb{R}$. In particular, if $b = c$, then GTrIFN transform to TIFN. On the other hand, if $\omega_r = \omega_t = 1$ and $u_r = u_t = 0$ then GTrIFN becomes TrIFN.

The representation of this GTrIFN A is shown in Fig. 3.1(a). Here, in the following, we have presented a new method for ranking the GTrIFN by determining the centroid point of intuitionistic fuzzy numbers. In order to achieve this, the area under membership and non-membership functions have been considered together. First of all, the considered Fig. 3.1(a) is divided into the five rectangles namely, ARUP, REBU, EFCB, FSVC, and SDQV where co-ordinates of corner points of rectangles have been given below:

$$A : (a, 0), B : (b, \omega_t), C : (c, \omega_r), D : (d, 0)$$

$$E : (b, 0), F : (c, 0), P : (a, 1), Q : (d, 1)$$

$$R : \left((a\omega_t - au_r + b)/(\omega_t - u_r + 1), 0 \right)$$

$$S : \left((d\omega_r - du_t + c)/(\omega_r - u_t + 1), 0 \right)$$

$$U : \left((a\omega_t - au_r + b)/(\omega_t - u_r + 1), (\omega_t/\omega_t - u_r + 1) \right)$$

$$V : \left((d\omega_r - du_t + c)/(\omega_r - u_t + 1), (\omega_r/\omega_t - u_t + 1) \right)$$

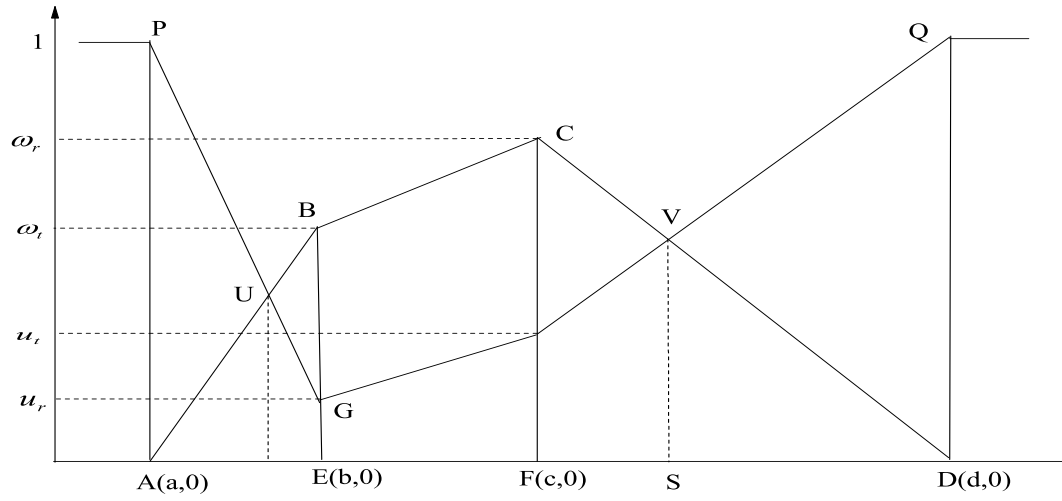
Now, the centroid point of the GTrIFN is found by using the centroid formulae $X = \int \frac{xf(x) dx}{f(x)}$ and $Y = \int \frac{yg(y) dy}{g(y)}$ where the specific region bounded by continuous function $f(x)$ and $g(y)$ respectively. The centroid point (X_A, Y_A) of GTrIFNs A has been given below:

$$X_A = \frac{X_1}{X_2} \tag{3.1.3}$$

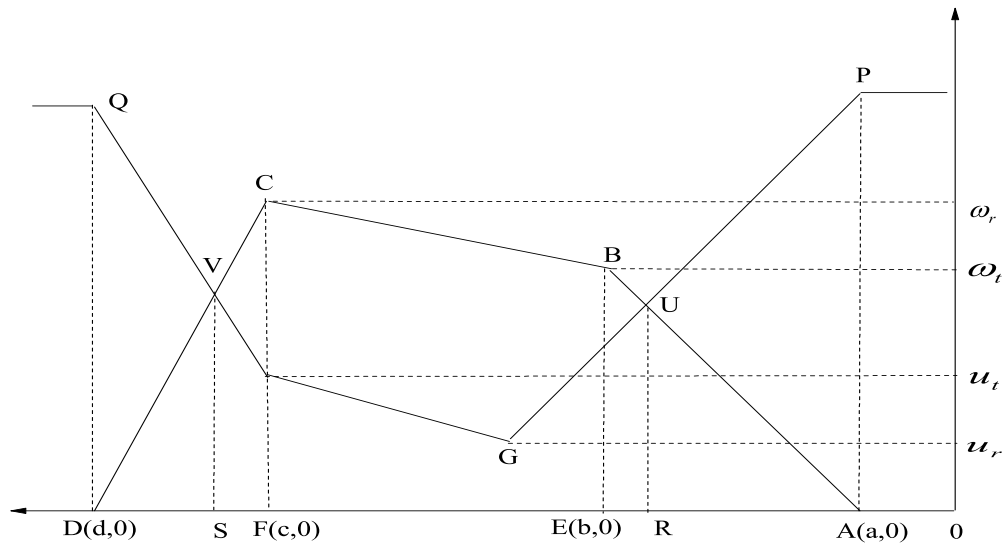
$$\text{and } Y_A = \frac{Y_1}{Y_2} \tag{3.1.4}$$

where

$$\begin{aligned} X_1 &= \text{area(ARUP)} + \text{area(REBU)} + \text{area(EFCB)} + \text{area(FSVC)} + \text{area(SDQV)} \\ &= \int_a^{\frac{a\omega_t - au_r + b}{\omega_t - u_r + 1}} xg_L dx + \int_{\frac{a\omega_t - au_r + b}{\omega_t - u_r + 1}}^b xf_L dx + \int_b^c xf_M dx + \int_c^{\frac{d\omega_r - du_t + c}{\omega_r - u_t + 1}} xf_R dx + \int_{\frac{d\omega_r - du_t + c}{\omega_r - u_t + 1}}^d xg_R dx \end{aligned}$$



(a)



(b)

Figure 3.1: (a) Generalized Trapezoidal intuitionistic fuzzy number, (b) its Inverse function

and

$$\begin{aligned}
X_2 &= \text{area(ARUP)} + \text{area(REBU)} + \text{area(EFCB)} + \text{area(FSVC)} + \text{area(SDQV)} \\
&= \int_a^{\frac{a\omega_t - au_r + b}{\omega_t - u_r + 1}} g_L dx + \int_{\frac{a\omega_t - au_r + b}{\omega_t - u_r + 1}}^b f_L dx + \int_b^c f_M dx + \int_c^{\frac{d\omega_r - du_t + c}{\omega_r - u_t + 1}} f_R dx + \int_{\frac{d\omega_r - du_t + c}{\omega_r - u_t + 1}}^d g_R dx \\
Y_1 &= \text{area(ABCD)} + \text{area(DQV)} + \text{area(UAP)} \\
&= \left[\int_0^{\omega_r} y h_R dy - \int_{\omega_t}^{\omega_r} y h_M dy - \int_0^{\omega_t} y h_L dy \right] + \left[\int_0^1 y d dy - \int_0^{\frac{\omega_r}{\omega_r - u_t + 1}} y h_R dy - \int_{\frac{\omega_r}{\omega_r - u_t + 1}}^1 y k_R dy \right] \\
&\quad + \left[\int_0^{\frac{\omega_t}{\omega_t - u_r + 1}} y h_L dy + \int_{\frac{\omega_t}{\omega_t - u_r + 1}}^1 y k_L dy - \int_0^1 a y dy \right]
\end{aligned}$$

and

$$\begin{aligned}
Y_2 &= \text{area(ABCD)} + \text{area(DVQ)} + \text{area(UAP)} \\
&= \left[\int_0^{\omega_r} h_R dy - \int_{\omega_t}^{\omega_r} h_M dy - \int_0^{\omega_t} h_L dy \right] + \left[\int_0^1 d dy - \int_0^{\frac{\omega_r}{\omega_r - u_t + 1}} h_R dy - \int_{\frac{\omega_r}{\omega_r - u_t + 1}}^1 k_R dy \right] \\
&\quad + \left[\int_0^{\frac{\omega_t}{\omega_t - u_r + 1}} h_L dy + \int_{\frac{\omega_t}{\omega_t - u_r + 1}}^1 k_L dy - \int_0^1 a dy \right]
\end{aligned}$$

where $f_L : [a, b] \rightarrow [0, \omega_t]$, $f_M : [b, c] \rightarrow [\omega_t, \omega_r]$, $f_R : [c, d] \rightarrow [0, \omega_r]$ are the left, middle and right membership part of the GTrIFNs A . $g_L : [a, b] \rightarrow [1, u_r]$, $g_M : [b, c] \rightarrow [u_r, u_t]$, $g_R : [c, d] \rightarrow [u_t, 1]$ are the left and the right part of the non-membership function which have been shown in the Fig. 3.1(a) and are defined as $f_L = \frac{\omega_t}{(b-a)}(x-a)$, $f_M = \frac{(\omega_r - \omega_t)}{(c-b)}(x-b) + \omega_t$, $f_R = \frac{\omega_r}{(c-d)}(x-d)$, $g_L = \frac{(b-x) + u_r(x-a)}{(b-a)}$, $g_M = u_r + \frac{(u_t - u_r)}{(c-b)}(x-b)$, and $g_R = \frac{u_t(x-d) + (c-x)}{(c-d)}$. On the other hand, $h_L : [0, \omega_t] \rightarrow [a, b]$, $h_M : [\omega_t, \omega_r] \rightarrow [b, c]$, $h_R : [0, \omega_r] \rightarrow [c, d]$ are the inverse functions of functions are f_L, f_M , and f_R respectively. Similarly, $k_L : [0, u_r] \rightarrow [a, b]$, $k_M : [u_r, u_t] \rightarrow [b, c]$, $k_R : [u_t, 1] \rightarrow [c, d]$ are the inverse functions of functions are g_L, g_M , and g_R respectively which have been shown in Fig. 3.1(b) and are defined as $h_L = a + \frac{y}{\omega_t}(b-a)$, $h_M = b + \frac{(c-b)(y-\omega_t)}{(\omega_r - \omega_t)}$, $h_R = d + \frac{y}{\omega_r}(c-d)$, $k_L = \frac{y(b-a) + (au_r - b)}{(u_r - 1)}$, $k_M = b + \frac{(c-b)(y-u_r)}{(u_t - u_r)}$ and $k_R = \frac{y(c-d) + (u_t d - c)}{(u_t - 1)}$.

From Fig. 3.1(a), we compute that

$$\begin{aligned}
X_1 &= \text{area}(\text{ARUP}) + \text{area}(\text{REBU}) + \text{area}(\text{EFCB}) + \text{area}(\text{FSVC}) + \text{area}(\text{SDQV}) \\
&= \int_a^{\frac{a\omega_t - au_r + b}{\omega_t - u_r + 1}} x g_L dx + \int_{\frac{a\omega_t - au_r + b}{\omega_t - u_r + 1}}^b x f_L dx + \int_b^c x f_M dx + \int_c^{\frac{d\omega_r - du_t + c}{\omega_r - u_t + 1}} x f_R dx + \int_{\frac{d\omega_r - du_t + c}{\omega_r - u_t + 1}}^d x g_R dx \\
&= \int_a^{\frac{a\omega_t - au_r + b}{\omega_t - u_r + 1}} x \left(\frac{(b-x) + u_r(x-a)}{b-a} \right) dx + \int_{\frac{a\omega_t - au_r + b}{\omega_t - u_r + 1}}^b x \left(\omega_t \frac{(x-a)}{(b-a)} \right) dx \\
&\quad + \int_b^c x \left(\frac{(\omega_r - \omega_t)(x-b)}{(c-b)} + \omega_t \right) dx + \int_c^{\frac{d\omega_r - du_t + c}{\omega_r - u_t + 1}} x \left(\omega_r \frac{(x-d)}{(c-d)} \right) dx \\
&\quad + \int_{\frac{d\omega_r - du_t + c}{\omega_r - u_t + 1}}^d x \left(\frac{(x-c) + u_t(d-x)}{(d-c)} \right) dx \\
&= \frac{1}{6(b-a)} \left[\frac{(a\omega_t - au_r + b)^2}{(\omega_t - u_r + 1)^3} \left((3b - 2a - au_r)(\omega_t - u_r) + b + (2b - 3a)u_r \right) \right. \\
&\quad \left. + u_r a^3 - a^2(3b - 2a) \right] + \frac{\omega_t}{6(b-a)} \left[b^2(2b - 3a) - \left(\frac{(a\omega_t - au_r + b)^2}{(\omega_t - u_r + 1)^3} (-a(\omega_t - u_r) \right. \right. \\
&\quad \left. \left. + (2b - 3a)) \right) \right] + \frac{1}{6(c-b)} (\omega_r - \omega_t)(2c^3 - 3bc^2 + b^3) + \omega_t \frac{(c^2 - b^2)}{2} \\
&\quad + \frac{1}{6(c-d)} \omega_r \left[\frac{(d\omega_r - du_t + 1)^2}{(\omega_r - u_t + 1)^3} \left(-d(\omega_r - u_t) + (2c - 3d) \right) - c^2(2c - 3d) \right] \\
&\quad + \frac{1}{6(d-c)} \left[d^2(2d - 3c) + u_t d^3 - \frac{(d\omega_r - du_t + c)^2}{(\omega_r - u_t + 1)^3} \left((\omega_r - u_t)(2d - 3c + du_t) \right. \right. \\
&\quad \left. \left. + (3d - 2c)u_t - c \right) \right]
\end{aligned}$$

Similarly, we can compute

$$\begin{aligned}
X_2 &= \text{area(ARUP)} + \text{area(REBU)} + \text{area(EFCB)} + \text{area(FSVC)} + \text{area(SDQV)} \\
&= \int_a^{\frac{a\omega_t - au_r + b}{\omega_t - u_r + 1}} \frac{(x - b) + u_r(a - x)}{(a - b)} dx + \int_{\frac{a\omega_t - au_r + b}{\omega_t - u_r + 1}}^b \frac{\omega_t}{(b - a)}(x - a) dx \\
&\quad + \int_b^c \left(\omega_t + \frac{(\omega_r - \omega_t)(x - b)}{(c - b)} \right) dx + \int_c^{\frac{d\omega_r - du_t + c}{\omega_r - u_t + 1}} \omega_r \left(\frac{x - d}{c - d} \right) dx \\
&\quad + \int_{\frac{d\omega_r - du_t + c}{\omega_r - u_t + 1}}^d \frac{(x - c) + u_t(d - x)}{(d - c)} dx \\
&= \frac{(a - b)}{2(\omega_t - u_r + 1)^2} [(\omega_t - u_r)^2 - u_r - (\omega_t - u_r + 1)^2] \\
&\quad + \frac{\omega_t(b - a)}{2(\omega_t - u_r + 1)^2} \left((\omega_t - u_r + 1)^2 - 1 \right) + \frac{(\omega_r - \omega_t)(c - b)}{2} \\
&\quad + \omega_t(c - b) + \frac{\omega_r(c - d)}{2(\omega_r - u_t + 1)^2} \left(1 - (\omega_r - u_t + 1)^2 \right) \\
&\quad + \frac{(d - c)}{2(\omega_r - u_t + 1)^2} \left((\omega_r - u_t + 1)^2 - (\omega_r - u_t)^2 - u_t \right) \\
&= \frac{(a - b)}{2(\omega_t - u_r + 1)^2} \left((\omega_t - u_r)^2 - u_r - (\omega_t - u_r + 1)^2 - \omega_t \left((\omega_t - u_r + 1)^2 - 1 \right) \right) \\
&\quad + \frac{(\omega_r - \omega_t)}{2} (c - b) + \omega_t(c - b) + \frac{(c - d)}{2(\omega_r - u_t + 1)^2} \left(\omega_r(1 - (\omega_r - u_t + 1)^2) \right. \\
&\quad \left. - (\omega_r - u_t + 1)^2 + (\omega_r - u_t)^2 + u_t \right)
\end{aligned}$$

Hence, by Eq. (3.1.3), we get X_A .

To find the y-coordinate of the centroid, we compute Y_1 and Y_2 . For it consider the inverse function of the GTrIFN A as shown in Fig. 3.1(b) and hence, based on this figure, we can compute the centroid of figure as $Y_A = \frac{Y_1}{Y_2}$ where

$$\begin{aligned}
Y_1 &= \text{area(ABCD)} + \text{area(DQV)} + \text{area(UAP)} \\
&= \int_0^{\omega_r} y \left(d + \frac{(c - d)y}{\omega_r} \right) dy - \int_{\omega_t}^{\omega_r} y \left(b + \frac{(c - b)(y - \omega_t)}{(\omega_r - \omega_t)} \right) dy - \int_0^{\omega_t} y \left(a + \frac{(b - a)y}{\omega_t} \right) dy
\end{aligned}$$

$$\begin{aligned}
& + \int_0^1 y d \, dy - \int_0^{\frac{\omega_r}{\omega_r - u_t + 1}} y \left(d + \frac{(c-d)y}{\omega_r} \right) dy - \int_{\frac{\omega_r}{\omega_r - u_t + 1}}^1 \left(\frac{y(c-d) + (u_t d - c)}{u_t - 1} \right) dy \\
& + \int_0^{\frac{\omega_t}{\omega_t - u_r + 1}} y \left(a + \frac{y(b-a)}{\omega_t} \right) dy + \int_{\frac{\omega_t}{\omega_t - u_r + 1}}^1 y \left(\frac{(b-a) + (au_r - b)}{(u_r - 1)} \right) dy - \int_0^1 a y \, dy \\
& = \omega_r^2 \left(\frac{d}{2} + \frac{c-d}{3} \right) - \left(\frac{b}{2} (\omega_r^2 - \omega_t^2) + \frac{c-b}{6(\omega_r - \omega_t)} (2\omega_r^3 - 3\omega_t \omega_r^2 + \omega_t^3) \right) - \omega_t^2 \left(\frac{a}{2} + \frac{b-a}{3} \right) \\
& + \frac{d}{2} \left(1 - \frac{\omega_r^2}{(\omega_r - u_t + 1)^2} \right) - \frac{c-d}{3} \left(\frac{\omega_r^2}{(\omega_r - u_t + 1)^3} \right) - \left(\frac{1}{u_t - 1} \right) \left[\frac{c-d}{3} \left(1 - \frac{\omega_r^3}{(\omega_r - u_t + 1)^3} \right) + \frac{(du_t - c)}{2} \left(1 - \frac{\omega_r^2}{(\omega_r - u_t + 1)^2} \right) \right] + \frac{a}{2} \left(\frac{\omega_t^2}{(\omega_t - u_r + 1)^2} - 1 \right) \\
& + \frac{(b-a)}{3} \frac{\omega_t^2}{(\omega_t - u_r + 1)^3} + \left(\frac{1}{u_r - 1} \right) \left[\frac{b-a}{3} \left(1 - \frac{\omega_t^3}{(\omega_t - u_r + 1)^3} \right) + \frac{(au_r - b)}{2} \left(1 - \frac{\omega_t^2}{(\omega_t - u_r + 1)^2} \right) \right]
\end{aligned}$$

and

$$\begin{aligned}
Y_2 & = \text{area(ABCD)} + \text{area(DVQ)} + \text{area(UAP)} \\
& = \left\{ \int_0^{\omega_r} \left(d + \frac{y(c-d)}{\omega_r} \right) dy - \int_{\omega_t}^{\omega_r} \left(b + \frac{(c-b)(y - \omega_t)}{(\omega_r - \omega_t)} \right) dy - \int_0^{\omega_t} \left(a + \frac{y(b-a)}{\omega_t} \right) dy \right\} \\
& + \left\{ \int_0^1 d \, dy - \int_0^{\frac{\omega_r}{\omega_r - u_t + 1}} \left(d + \frac{y(c-d)}{\omega_r} \right) dy - \int_{\frac{\omega_r}{\omega_r - u_t + 1}}^1 \frac{y(c-d) + (u_t d - c)}{(u_t - 1)} dy \right\} \\
& + \left\{ \int_0^{\frac{\omega_t}{\omega_t - u_r + 1}} \left(a + \frac{y}{\omega_t} (b-a) \right) dy + \int_{\frac{\omega_t}{\omega_t - u_r + 1}}^1 \frac{y(b-a) + (au_r - b)}{(u_r - 1)} dy - \int_0^1 a \, dy \right\} \\
& = \omega_r \left(d + \frac{(c-d)}{2} \right) - \left[b(\omega_r - \omega_t) + \frac{(c-b)}{2(\omega_r - \omega_t)} (\omega_r^2 - 2\omega_r \omega_t + \omega_t^2) \right] - \omega_t \left[a + \frac{(b-a)}{2} \right] \\
& + d \left(1 - \frac{\omega_r}{\omega_r - u_t + 1} \right) - \frac{(c-d)}{2} \left(\frac{\omega_r}{(\omega_r - u_t + 1)^2} \right) - \left(\frac{1}{u_t - 1} \right) \left[\frac{(c-d)}{2} \left(1 - \frac{\omega_r^2}{(\omega_r - u_t + 1)^2} \right) + (du_t - c) \left(1 - \frac{\omega_r}{\omega_r - u_t + 1} \right) \right] + a \left(\frac{\omega_t}{\omega_t - u_r + 1} - 1 \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{(b-a)}{2} \frac{\omega_t}{(\omega_t - u_r + 1)^2} + \left(\frac{1}{u_r - 1} \right) \left[\frac{(b-a)}{2} \left(1 - \frac{\omega_t^2}{(\omega_t - u_r + 1)^2} \right) \right. \\
& \left. + (au_r - b) \left(1 - \frac{\omega_t}{\omega_t - u_r + 1} \right) \right]
\end{aligned}$$

and hence by using Eq. (3.1.4), we can obtain Y_A .

Hence, after obtaining the centroid of the GTrIFN (X_A, Y_A) , we can rank the IFN. Since X_A represents the location of the IFN A on the real line and Y_A represents the average height of the IFN. Now, ranking of the two different IFNs A and B has been done with the following rule.

- (i) If $X_A > X_B$, then $A > B$;
- (ii) If $X_A = X_B$, then
 - (a) If $Y_A > Y_B$, then $A > B$;
 - (b) If $Y_A = Y_B$, then $A = B$.

3.2 Special cases

From the above computed study, it has been observed that some of the special cases can be obtained by assigning a particular value to the ω_r, ω_t, u_r and u_t which has been explained as follows:

(Case 1:) when $\omega_t = \omega_r = \omega$ and $u_r = u_t = u$, then the values of X_1, X_2, Y_1 and Y_2 becomes

$$\begin{aligned}
X_1 = & \frac{1}{6(b-a)} \left[\frac{(aw - au + b)^2}{(w - u + 1)^3} \left((3b - 2a - au + aw)(w - u) + b \right. \right. \\
& \left. \left. + (2b - 3a)(u - w) \right) + a^3u - a^2(3b - 2a) + b^2w(2b - 3a) \right] + w \frac{(c^2 - b^2)}{2} \\
& + \frac{1}{6(c-d)} \left[\frac{(dw - du + c)^2}{(w - u + 1)^3} \left((-dw + 2d - 3c + du)(w - u) + (2c - 3d)w \right. \right. \\
& \left. \left. + (3d - 2c)u - c \right) - wc^2(2c - 3d) - d^2(2d - 3c) - ud^3 \right]
\end{aligned}$$

$$\begin{aligned}
X_2 &= \frac{(a-b)}{2(w-u+1)^2} \left[(w-u)^2 - u - (w-u+1)^2 - w((w-u+1)^2 - 1) \right] + \\
&\quad w(c-b) + \frac{(c-d)}{2(w-u+1)^2} \left[w(1 - (w-u+1)^2) - (w-u+1)^2 \right. \\
&\quad \left. + (w-u)^2 + u \right] \\
Y_1 &= w^2 \left(\frac{d}{2} + \frac{c-d}{3} \right) - w^2 \left(\frac{a}{2} + \frac{b-a}{3} \right) + \frac{d}{2} \left(1 - \frac{w^2}{(w-u+1)^2} \right) \\
&\quad - \frac{(c-d)}{3} \left(\frac{w^2}{(w-u+1)^3} \right) - \left(\frac{1}{u-1} \right) \left[\frac{(c-d)}{3} \left(1 - \frac{w^3}{(w-u+1)^3} \right) \right. \\
&\quad \left. + \frac{(ud-c)}{2} \left(1 - \frac{w^2}{(w-u+1)^2} \right) \right] + \frac{a}{2} \left(\frac{w^2}{(w-u+1)^2} - 1 \right) + \\
&\quad \frac{(b-a)}{3} \frac{w^2}{(w-u+1)^3} + \left(\frac{1}{u-1} \right) \left(\frac{(b-a)}{3} \left(1 - \frac{w^3}{(w-u+1)^3} \right) \right. \\
&\quad \left. + \frac{(au-b)}{2} \left(1 - \frac{w^2}{(w-u+1)^2} \right) \right) \\
Y_2 &= w \left(d + \frac{(c-d)}{2} \right) - w \left(a + \frac{(b-a)}{2} \right) + d \left(1 - \frac{w}{w-u+1} \right) \\
&\quad - \frac{(c-d)}{2} \left(\frac{w}{(w-u+1)^2} \right) - \left(\frac{1}{u-1} \right) \left[\frac{(c-d)}{2} \left(1 - \frac{w^2}{(w-u+1)^2} \right) \right. \\
&\quad \left. + (ud-c) \left(1 - \frac{w}{w-u+1} \right) \right] + a \left(\frac{w}{w-u+1} - 1 \right) + \frac{(b-a)}{2} \frac{w}{(w-u+1)^2} \\
&\quad + \left(\frac{1}{u-1} \right) \left[\frac{(b-a)}{2} \left(1 - \frac{w^2}{(w-u+1)^2} \right) + (au-b) \left(1 - \frac{w}{w-u+1} \right) \right]
\end{aligned}$$

(Case 2:) When $\omega_t = \omega_r = 1$ and $u_r = u_t = 0$ then, we get

$$\begin{aligned}
X_A &= \frac{3a^2 + b^2 - c^2 - 3d^2}{2(3a + b - c - 3d)} \\
Y_B &= \frac{7(d-a) + 5(c-b)}{18(d-a) + 6(c-b)}
\end{aligned}$$

3.3 Illustrative Examples

The above mentioned approach for ranking the different GTrIFNs are illustrated with some numerical examples as follows.

Example 3.3.1. If $A = \langle (0.57, 0.73, 0.81, 0.83); [0.71, 0.73], [0.20, 0.21] \rangle$ and $B = \langle (0.58, 0.74, 0.79, 0.81); [0.72, 0.75], [0.19, 0.20] \rangle$ be two GTrIFNs then, we get $X_A = 0.6396$, $Y_A = 0.0052$, $X_B = 0.6173$ and $Y_B = 0.0064$. Since $X_A > X_B$ and thus by using comparison law, we get the GTrIFN A is larger than GTrIFN B .

Example 3.3.2. If $A = \langle (0.56, 0.74, 0.80, 0.90); [0.50, 0.60], [0.10, 0.40] \rangle$ and $B = \langle (0.50, 0.70, 0.85, 0.95); [0.40, 0.50], [0.20, 0.30] \rangle$ be two GTrIFNs then $X_A = 0.7966$, $Y_A = 0.0387$, $X_B = 0.7540$, $Y_B = 0.0632$. Since $X_A > X_B$ and thus we get the GTrIFN A is larger than GTrIFN B .

Example 3.3.3. If $A = \langle (0.45, 0.65, 0.70, 0.75); [0.3, 1], [0.0, 0.6] \rangle$ and $B = \langle (0.55, 0.60, 0.70, 0.75); [0.4, 1], [0.0, 0.6] \rangle$ and be two GTrIFNs then, $X_A = 0.5414$, $Y_A = 0.1224$, $X_B = 0.5803$ and $Y_B = 0.0862$. Since $X_B > X_A$ and thus we get the GTrIFN B is larger than GTrIFN A .

Example 3.3.4. If $A = \langle (6, 1, 1.5, 2); [0.3, 0.7], [0.2, 0.5] \rangle$ and $B = \langle (-6, 1, 1.5, 2); [0.5, 0.6], [0.3, 0.4] \rangle$ be two GTrIFNs then, $X_A = -2.6830$, $Y_A = 0.3249$, $X_B = -2.5293$ and $Y_B = 0.3009$. Thus, from these values, we conclude that GTrIFN B is greater than GTrIFN A .

3.4 Conclusion

In this chapter, some more generalized ranking method to rank the different GTrIFN has been presented. For it, different heights have been taken corresponding to the crisp values of the input and hence based on the centroid point of the IFN, we compute the defuzzified values of GTrIFN. Some special cases of the proposed approach has also been discussed here. Finally, some numerical examples have been provided to compare the two different IFNs. This ranking approach may be applicable to the different fields such as cluster analysis, fuzzy optimization, decision-making problems.

Chapter 4

Reliability analysis of series-parallel systems based on generalized intuitionistic fuzzy number

This chapter addresses the fuzzy system reliability analysis of series, series-parallel, and sump-pump systems, using different types of GTrIFNs. For it, failure rate of the system components' are taken as a constant and represented in the form of the generalized intuitionistic fuzzy numbers.

4.1 Overview

In the literature, failure rate of the system components have taken as a constant and same types of fuzzy/intuitionistic fuzzy numbers during the analysis of the reliability of the system. However, in real-life situation, this kind of situation is rarely occurs. For handling it, it is necessary to consider the situations where the input related to the system behavior is represented in the form of the different types of the intuitionistic fuzzy failure rates. Furthermore, for analyzing the behavior of the series-parallel systems, it is generally assumed that the highest level of confidence of domain experts is assumed to be 1. But, in real-life situations, it lies between $[0,1]$ or the subset of $[0,1]$. Keeping this in mind, Garg [9] presented an approach for analyzing the system behavior with some confidence level of the expert's knowledge lies between $[0,1]$. In this chapter, an approach has been presented

in which the expert's knowledge has been taken in the form of the subset of $[0,1]$ instead of real numbers, in the form of the degrees of the membership and non-membership. Based on these preferences, the expression of the series, series-parallel system has been computed under the intuitionistic fuzzy environment where the input parameters are taken in the form of the generalized intuitionistic fuzzy numbers.

4.2 Fuzzy reliability function

In terms of failure rate function, system reliability function $R(t)$ is

$$R(t) = \exp \left[- \int_0^t \lambda(t') dt' \right], \quad t > 0 \quad (4.2.1)$$

where $\lambda(t)$ is failure rate function.

Assume that the failure rate function be constant i.e., $\lambda(t) = \lambda$, then reliability function is

$$R(t) = e^{-\lambda t}$$

Let the system failure rate be represented in the form of GTrIFN defined on X as

$$\tilde{\lambda} = \{x, (\lambda_1, \lambda_2, \lambda_3, \lambda_4; [\omega_r, \omega_t], [u_r, u_t]) \mid x \in X\} \quad (4.2.2)$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in R$ then the set $\tilde{\lambda}$ is said to be generalized trapezoidal intuitionistic fuzzy number if its membership function is defined as follows

$$\mu_{\tilde{\lambda}}(\lambda) = \begin{cases} 0 & ; \lambda < \lambda_1 \\ \omega_t \left(\frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} \right) & ; \lambda_1 \leq \lambda < \lambda_2 \\ \frac{(\omega_r - \omega_t)(\lambda - \lambda_2)}{\lambda_3 - \lambda_2} + \omega_t & ; \lambda_2 \leq \lambda < \lambda_3 \\ \omega_r \left(\frac{\lambda - \lambda_4}{\lambda_3 - \lambda_4} \right) & ; \lambda_3 \leq \lambda < \lambda_4 \\ 0 & ; \lambda > \lambda_4 \end{cases} \quad (4.2.3)$$

and

$$\nu_{\tilde{\lambda}}(\lambda) = \begin{cases} 1 & ; \lambda < \lambda_1 \\ \frac{(\lambda_2 - \lambda) + u_r(\lambda - \lambda_1)}{\lambda_2 - \lambda_1} & ; \lambda_1 \leq \lambda < \lambda_2 \\ u_r + \frac{(u_t - u_r)(\lambda - \lambda_2)}{\lambda_3 - \lambda_2} & ; \lambda_2 \leq \lambda < \lambda_3 \\ \frac{u_t(\lambda - \lambda_4) + (u_r - u_t)(\lambda_3 - \lambda)}{\lambda_3 - \lambda_4} & ; \lambda_3 \leq \lambda < \lambda_4 \\ 1 & ; \lambda > \lambda_4 \end{cases} \quad (4.2.4)$$

α -cut sets of a GTrIFN $\tilde{\lambda}$ corresponding to the membership function is a crisp subset of R , which is defined as $\tilde{\lambda}_\alpha = \{x \mid \mu_\lambda(\lambda) \geq \alpha\}$, where $0 \leq \alpha \leq \omega_r$. We denote the α -cut of this set as $\tilde{\lambda}_\alpha = [L_{\tilde{\lambda}}(\alpha), M_{\tilde{\lambda}}(\alpha), R_{\tilde{\lambda}}(\alpha)]$ and can be calculated as follows:

$$\begin{aligned}\tilde{\lambda}_\alpha &= [L_{\tilde{\lambda}}(\alpha), M_{\tilde{\lambda}}(\alpha), R_{\tilde{\lambda}}(\alpha)] \\ &= \left[\lambda_1 + \frac{\alpha}{w_t}(\lambda_2 - \lambda_1), \lambda_2 + \frac{\lambda_3 - \lambda_2}{(w_r - w_t)}(\alpha - w_t), \lambda_4 + \frac{\alpha}{w_r}(\lambda_3 - \lambda_4) \right]\end{aligned}$$

where $L_{\tilde{\lambda}}(\alpha)$ is increasing function of α and $R_{\tilde{\lambda}}(\alpha)$ is decreasing function of α .

On the other hand, β -cut set of a GTrIFN $\tilde{\lambda}_1$ corresponding to non-membership function is a crisp subset of R which is defined as $\tilde{\lambda}_\beta = \{x \mid \nu_\lambda(\lambda) \leq \beta\}$ where $u_r \leq \beta \leq 1$. We denote the β -cut of this set as $\tilde{\lambda}_\beta = [L_{\tilde{\lambda}}(\beta), M_{\tilde{\lambda}}(\beta), R_{\tilde{\lambda}}(\beta)]$ and can be calculated as follows:

$$\begin{aligned}\tilde{\lambda}_\beta &= [L_{\tilde{\lambda}}(\beta), M_{\tilde{\lambda}}(\beta), R_{\tilde{\lambda}}(\beta)] \\ &= \left[\frac{\beta(\lambda_2 - \lambda_1) - (\lambda_2 - \lambda_1)u_r}{(u_r - 1)}, \frac{(\lambda_3 - \lambda_2)(\beta - u_r) + \lambda_2(u_t - u_r)}{(u_t - u_r)}, \frac{\beta(\lambda_3 - \lambda_4) + \lambda_4 u_t - \lambda_3}{(u_t - 1)} \right]\end{aligned}$$

where $L_{\tilde{\lambda}}(\beta)$ is decreasing function of β and $R_{\tilde{\lambda}}(\beta)$ is increasing function of β .

Thus, the basic arithmetic operations of GTrIFNs depends on the arithmetic of the interval of confidence. The major arithmetic operations on two GTrIFNs $\tilde{A}_1 = \langle (a_1, b_1, c_1, d_1); [\omega_t^{(1)}, \omega_t^{(1)}], [u_r^{(1)}, u_t^{(1)}] \rangle$ and $\tilde{A}_2 = \langle (a_2, b_2, c_2, d_2); [\omega_t^{(2)}, \omega_t^{(2)}], [u_r^{(2)}, u_t^{(2)}] \rangle$ described by the α - and β -cuts are shown in Table 4.1.

Table 4.1: Operations on GTrIFNs

Operation	Crisp	Intuitionistic fuzzy
Addition	$A_1 + A_2$	$\tilde{A}_1 + \tilde{A}_2 = [L_{A_1}(\alpha) + L_{A_2}(\alpha), M_{A_1}(\alpha) + M_{A_2}(\alpha), R_{A_1}(\alpha) + R_{A_2}(\alpha)]$
Subtraction	$A_1 - A_2$	$\tilde{A}_1 - \tilde{A}_2 = [L_{A_1}(\alpha) - R_{A_2}(\alpha), M_{A_1}(\alpha) - M_{A_2}(\alpha), R_{A_1}(\alpha) - L_{A_2}(\alpha)]$
Multiplication	$A_1 \cdot A_2$	$\tilde{A}_1 \cdot \tilde{A}_2 = [L_{A_1}(\alpha) \cdot L_{A_2}(\alpha), M_{A_1}(\alpha) \cdot M_{A_2}(\alpha), R_{A_1}(\alpha) \cdot R_{A_2}(\alpha)]$
Division	$A_1 \div A_2$	$\tilde{A}_1 \div \tilde{A}_2 = [L_{A_1}(\alpha) \div R_{A_2}(\alpha), M_{A_1}(\alpha) \div M_{A_2}(\alpha), R_{A_1}(\alpha) \div L_{A_2}(\alpha)]$

4.3 Illustrative Example

The above defined operation for GTrIFNs have been applied to analyze reliability of the series, series-parallel and sump-pump systems, where component parameters are represented

by different types of generalized fuzzy numbers.

4.3.1 Series System

Consider a series configuration system which consists of ‘ n ’ components as shown in Fig. 4.1. Each component has failure rate, denoted by $\lambda_i (i = 1, 2, \dots, n)$ and are represented in the form of GTrIFNs. Assume that $R_i (i = 1, 2, \dots, n)$ be their corresponding reliability of the system at time t , then system reliability is defined as

$$R_s(t) = \prod_{i=1}^n R_i(t)$$

where $R_i(t) = \exp(-\lambda_i \cdot t)$ is the reliability of the i^{th} component.



Figure 4.1: Structure of the series system

Example : Consider three independent components of the system which are arranged in the series configuration such that each of them must have to work for proper functioning the system [16, 30]. Assume that the failure rate of these components are in the form of different types of GTrIFNs λ_1 , λ_2 and λ_3 whose values are given in Table 4.2. By

Table 4.2: Input data for series system

Generalized Intuitionistic fuzzy failure rate of i^{th} component of series system	Types of the intuitionistic fuzzy numbers
$\lambda_1 = \langle (0.02, 0.03, 0.04 ; 0.01, 0.03, 0.05); [0.6, 0.7], [0.1, 0.2] \rangle$	Triangular
$\lambda_2 = \langle (0.03, 0.04, 0.05, 0.06 ; 0.02, 0.04, 0.05, 0.07); [0.6, 0.7], [0.1, 0.2] \rangle$	Trapezoidal
$\lambda_3 = \langle (0.02, 0.05, 0.08, 0.12 ; 0.01, 0.04, 0.09, 0.14); [0.6, 0.7], [0.1, 0.2] \rangle$	Trapezoidal

considering the different mission times $t = 5, 10$ and 15 hrs, the α - and β - cuts of the failure rates have been computed and hence system reliability $R_s(t)$ is computed. The results corresponding to the different level of uncertainties ranging from 0 to 1 with an increment of 0.1 are summarized in Table 4.3 along with the existing technique results [30] which shows that the proposed approach follows the same trend as that of previous methods and hence it is conservative in nature. The complete variation of the reliability at the different mission times is summarized in Fig. 4.2.

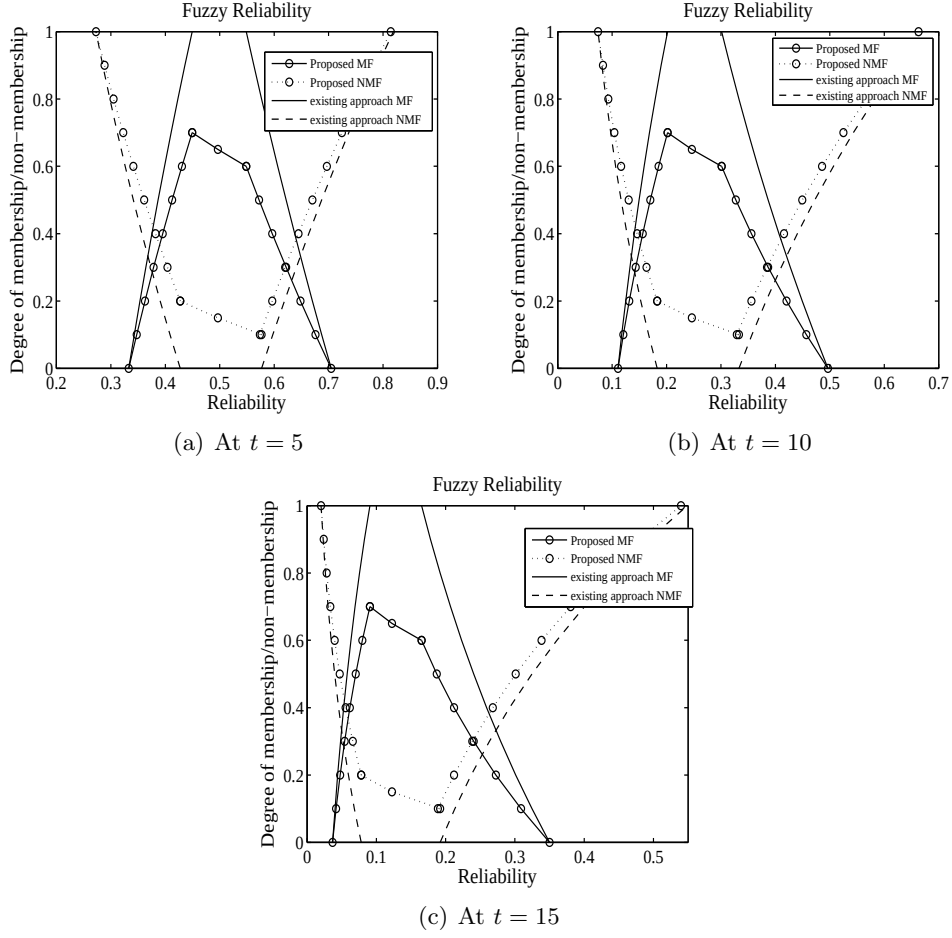


Figure 4.2: Fuzzy reliability plot of the series system

4.3.2 Series-parallel system

Consider a set of four components A_1, A_2, A_3 and A_4 which are connected in series-parallel configuration as shown in Fig. 4.3. Let R_1, R_2, R_3, R_4 respectively be the reliability of these components which are measured in the form of GTrIFNs given in Table 4.4, then the reliability of the system $\tilde{R}_{sp}$ is

$$\begin{aligned}
 R_{sp} &= (R_1 \cup R_2) \cap (R_3 \cup R_4) \\
 &= \{1 - (1 - R_1)(1 - R_2)\} \{1 - (1 - R_3)(1 - R_4)\} \\
 &= (R_1 + R_2 - R_1 R_2)(R_3 + R_4 - R_3 R_4)
 \end{aligned}$$

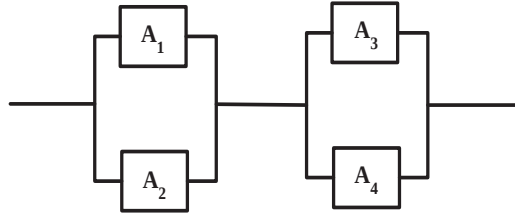


Figure 4.3: Series-parallel system

Table 4.4: Input data for series-parallel system

Generalized Intuitionistic fuzzy reliability of i^{th} component of the system	Type of intuitionistic fuzzy numbers
$R_1 = \langle (0.03, 0.04, 0.05 ; 0.02, 0.04, 0.07); [0.6, 0.7], [0.1, 0.2] \rangle$	Triangular
$R_2 = \langle (0.02, 0.03, 0.04, 0.06 ; 0.01, 0.03, 0.04, 0.07); [0.6, 0.7], [0.1, 0.2] \rangle$	Trapezoidal
$R_3 = \langle (0.03, 0.06, 0.08, 0.11 ; 0.02, 0.04, 0.09, 0.14); [0.6, 0.7], [0.1, 0.2] \rangle$	Trapezoidal
$R_4 = \langle (0.02, 0.04, 0.06 ; 0.01, 0.04, 0.08); [0.6, 0.7], [0.1, 0.2] \rangle$	Triangular

By using the different level of α - and β - cut ranging from 0 to 1 of the considered data, the corresponding system reliability R_{sp} in the form of membership and non-membership values are summarized in Table 4.5 along with the existing technique result [30]. The variation of these values are plotted graphically in Fig. 4.4. Again it has been followed that they have reduced range of prediction as compared to other.

Table 4.5: Series-parallel system reliability values

α, β	Membership function values					Non-Membership functions				
	Kumar and Yadav [30] approach		Proposed approach			Kumar and Yadav [30] approach		Proposed approach		
	L	R	L	M	R	L	R	L	M	R
0	0.00244	0.01748	0.00244		0.01748	0.00539	0.00991			
0.1	0.00279	0.01654	0.00303		0.01614	0.00478	0.01135	0.00539	0.00539	
0.2	0.00315	0.01562	0.00367		0.01485	0.00420	0.01287	0.00463	0.00991	0.00991
0.3	0.00353	0.01472	0.00435		0.01361	0.00366	0.01449	0.00400		0.01172
0.4	0.00394	0.01385	0.00509		0.01242	0.00315	0.01619	0.00342		0.01367
0.5	0.00435	0.01301	0.00588		0.01128	0.00268	0.01798	0.00288		0.01576
0.6	0.00479	0.01219	0.00671	0.00671	0.01019	0.00225	0.01986	0.00238		0.01798
0.7	0.00525	0.01139		0.00916	0.00916	0.00185	0.02182	0.00193		0.02034
0.8	0.00572	0.01062				0.00149	0.02387	0.00152		0.02284
0.9	0.00621	0.00988				0.00117	0.02600	0.00117		0.02546
1	0.00671	0.00916				0.00089	0.02821	0.00089		0.02821

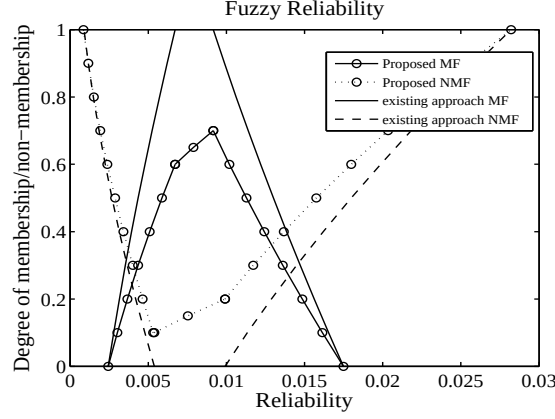


Figure 4.4: Fuzzy reliability plot of the series-parallel system

4.3.3 Sump pump system

This example states that “a building basement equipped with a sump pump system may be flooded during heavy rainstorms if the rate of inflow water exceeds the normal pump capacity. However, a smaller rainstorm may cause a flooding in the basement if the sump pump system is not operating properly. For this, a system [16] has been considered which consists of a primary pump operated by the utility power and a back-up battery-operated pump that will be automatically activated on power failure shown in Fig. 4.5(a). The failure of the pump system will occur if both primary pump and the back-up pump fail to operate during periods of ground water inflow from a storm. However, the primary pump will fail if it breaks down or if there is a power outage while the back-up pump will fail if it malfunctions or if the battery is drained. On the other hand, if both the periods of power outage and inflow exceed the battery capacity then the battery will be drained. Thus the various components/events $E_i, i = 1, 2, \dots, 12$ associated with the system are defined as below”.

E_1 : “failure of occupant to take remedial action”.

E_2 : “period of power outage exceeds battery capacity”.

E_3 : “period of inflow exceeds the battery capacity”.

E_4 : “battery drained”.

E_5 : “back-up pump malfunctions”.

E_6 : “back-up pump fails”.

- E_7 : “power outage”.
 E_8 : “primary pump malfunctions”.
 E_9 : “primary pump fails”.
 E_{10} : “inflow to basement sump”.
 E_{11} : “failure of pump system”.
 E_{12} : “rate of inflow exceeds pump capacity”.

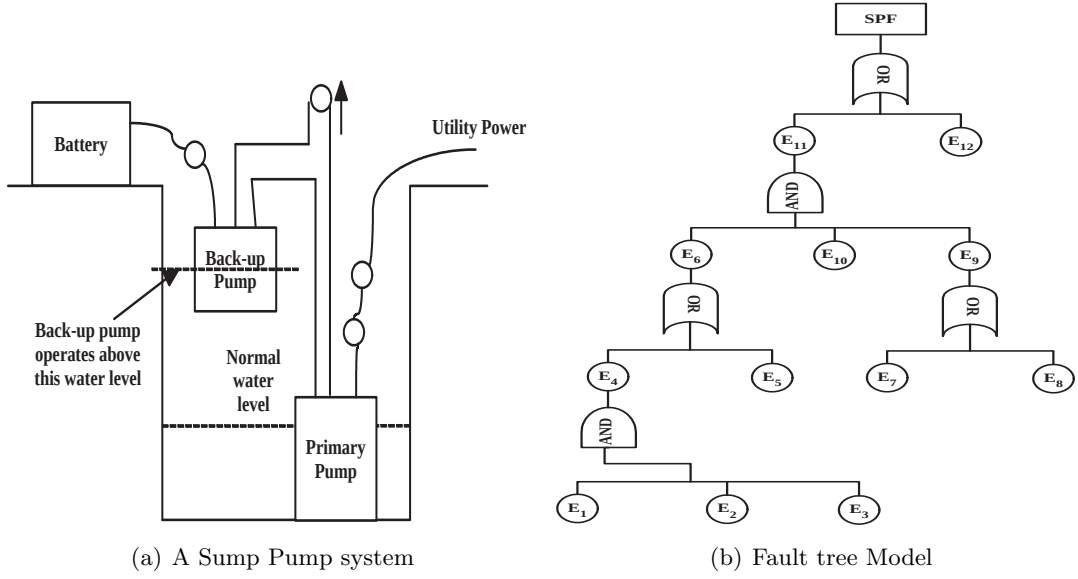


Figure 4.5: Systematic diagram and Fault tree model of a sump pump system

The systematic diagram, as well as, the fault tree model of the considered system have been given in Fig. 4.5(a) and 4.5(b) respectively. The data related to main components of the system are represented in the form of GTrIFNs given in Table 4.6 [16], where R_i represents the reliability of i^{th} events in the form of intuitionistic fuzzy numbers.

Table 4.6: Input data for the main component of the pump

Reliability of the main components of the pump in IFNs	
$R_1 = (0.02, 0.05, 0.08, 0.13; 0.01, 0.03, 0.09, 0.14)$	$R_2 = (0.20, 0.33, 0.36, 0.41; 0.09, 0.31, 0.37, 0.42)$
$R_3 = (0.04, 0.07, 0.10, 0.14; 0.03, 0.05, 0.11, 0.15)$	$R_5 = (0.05, 0.08, 0.11, 0.16; 0.04, 0.06, 0.12, 0.17)$
$R_7 = (0.10, 0.13, 0.16, 0.21; 0.09, 0.11, 0.17, 0.22)$	$R_8 = (0.06, 0.09, 0.12, 0.17; 0.05, 0.07, 0.13, 0.18)$
$R_{10} = (0.10, 0.12, 0.15, 0.17; 0.08, 0.11, 0.16, 0.19)$	$R_{12} = (0.05, 0.08, 0.11, 0.14; 0.03, 0.07, 0.12, 0.16)$
$\omega_t = 0.6, \omega_r = 0.7, u_t = 0.2$ and $u_r = 0.1$	

Thus, based on Fig. 4.5(b), the reliability of the main events is written as

$$R_4 = R_1 \cap R_2 \cap R_3 \quad ; \quad R_6 = R_4 \cup R_5 \quad ; \quad R_9 = R_7 \cup R_8 \quad ; \quad R_{11} = R_6 \cap R_9 \cap R_{10}$$

while the whole system reliability is computed as

$$R_s = R_{11} \cup R_{12}$$

Based on the given information, the ranges of the system reliability values at different cut levels are summarized in Table 4.7 while their variations with degree of membership are plotted in Fig. 4.6 along with existing approaches result. Based on these results, a system analyst may take appropriate action for better managerial decision-making and estimate success interval more flexible than previous methods.

Table 4.7: Reliability of basement flooding system at different values of cuts α, β

α, β	Membership function values					Non-Membership functions				
	Kumar and Yadav [30] approach		Proposed approach			Kumar and Yadav [30] approach		Proposed approach		
	L	R	L	M	R	L	R	L	M	R
0	0.05073	0.14837	0.05073		0.14837	0.07107	0.12482			
0.1	0.05385	0.14494	0.05593		0.14347	0.06700	0.12938	0.07107	0.07107	
0.2	0.05697	0.14150	0.06112		0.13856	0.06294	0.13394	0.06655	0.09800	0.12482
0.3	0.06008	0.13807	0.06631		0.13364	0.05888	0.13849	0.06204	0.12482	0.13052
0.4	0.06319	0.13463	0.07150		0.12872	0.05481	0.14304	0.05752		0.13622
0.5	0.06631	0.13118	0.07668		0.12379	0.05075	0.14758	0.05301		0.14190
0.6	0.06942	0.12774	0.08186	0.08186	0.11886	0.04669	0.15211	0.04849		0.14758
0.7	0.07253	0.12429		0.11392	0.11392	0.04262	0.15664	0.04398		0.15325
0.8	0.07564	0.12083				0.03855	0.16117	0.03946		0.15890
0.9	0.07875	0.11738				0.03449	0.16569	0.03494		0.16456
1	0.08186	0.11392				0.03042	0.17020	0.03042		0.17020

4.4 Conclusion

In this chapter, a systematic procedure for computing the reliability of the series, series-parallel and mixed configuration structure have been presented by using α cuts. Traditionally, it is assumed that the failure rates of components of a system follow the same type of fuzzy set or IFS. In this chapter, we have addressed a approach for analyzing the fuzzy reliability of a system with components following different types of generalized intuitionistic fuzzy failure rate. The variations of the reliability at different levels of the

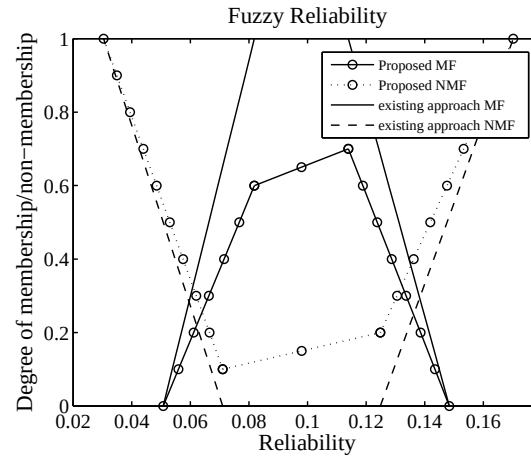


Figure 4.6: Fuzzy reliability plot of the sump-pump system

uncertainties are computed and plotted/summarized in the respective tables. Also, approach has been compared with some of the existing approaches. The major advantage of using GTrIFNs over fuzzy numbers is that GTrIFNs separate the positive and negative evidence for membership of an element in the set.

Chapter 5

Fuzzy reliability analysis of an industrial system using generalized trapezoidal intuitionistic fuzzy number

The objective of this chapter is to present a new approach for analyzing the reliability of an industrial systems by using generalized trapezoidal intuitionistic fuzzy numbers and then illustrated with a case study of washing unit of a paper mill, a complex repairable industrial system.

5.1 Introduction

Today in real-world decision, reliability is one of the most significant component for increasing the performance of the system. The system designer is always wants to increase the production of the system by maintaining the component of the system for a longer period. But the uncertainties play a dominant role during the analysis and hence without handling the uncertainties, it is difficult for the system analyst to maintain the system performance[11, 24]. However, it is difficult for a decision maker to give his assessments in crisp values due to ambiguity and incomplete information. Instead, it has become popular that these assessments are presented by fuzzy set or extensions of the fuzzy set. Fuzzy set (FS)[39], proposed by Zadeh, is a powerful tool to deal with vagueness and has received much attention. After it, intuitionistic fuzzy set (IFS) [1] is one of the successful extension

of fuzzy set to handle the uncertainties in the data. In the field of the reliability theory, researchers [6, 9, 10, 12, 13, 16, 22, 26] have proposed different approaches for analyzing the reliability of the system under IFS environment by taking triangular linear membership functions. In contrast to these, in this chapter, a concept of generalized trapezoidal intuitionistic fuzzy number based methodology for analyzing the behavior of the industrial systems has been presented. Finally, the approach has been illustrated with a case study.

5.2 Methodology

In this section, we have presented an approach for illustrating the behavior of an industrial system under intuitionistic fuzzy environment. For it, the following assumptions have been taken as follows:

- (i) component parameters are independent.
- (ii) maintenance facility is separate for each component.
- (iii) after repairs, component is assumed to be as new.
- (iv) standby components are of same nature as of active units.

Based on these assumptions, the various step involved in the approach are summarized as follows:

- (Step 1:) In this step, the information related to various major component of the system has been extracted in the form of the failure rate λ_i 's and repair time τ_i 's either from the sheets or from their personal experiences.
- (Step 2:) In order to handle the uncertainties in the obtained data, the crisp information is converted into the trapezoidal form of the intuitionistic fuzzy numbers with some known spread say 15%, 25% and 50% as suggested by the decision makers towards the data and by taking the level of significance in the form of interval $[\omega_t, \omega_r] = [0.5, 0.7]$ for membership functions and $[u_r, u_t] = [0.1, 0.3]$ for non-membership functions.

(Step 3:) The α – and β – cuts obtained from the above step have been used for finding the overall system fuzzy numbers by using basic expression of the system as given in Table 5.1. Based on these expressions, various reliability indices, listed

Table 5.1: Basic Expressions of Lambda Tau Methodology

Gate	λ_{AND}	τ_{AND}	λ_{OR}	τ_{OR}
Expression	$\prod_{j=1}^n \lambda_j \left[\sum_{i=1}^n \prod_{\substack{j=1 \\ i \neq j}}^n \tau_j \right]$	$\frac{\prod_{i=1}^n \tau_i}{\sum_{j=1}^n \left[\prod_{\substack{i=1 \\ i \neq j}}^n \tau_i \right]}$	$\sum_{i=1}^n \lambda_i$	$\frac{\sum_{i=1}^n \lambda_i \tau_i}{\sum_{i=1}^n \lambda_i}$

in Table 5.2, are quantitatively in terms of evaluating the membership functions at different levels of α – and β – cuts.

Table 5.2: Some Reliability Parameters

Parameters	Expressions
Failure rate	$MTTF_s = \frac{1}{\lambda_s}$
Repair time	$MTTR_s = \frac{1}{\mu_s} = \tau_s$
Mean Time Between Failures (MTBF)	$MTBF_s = MTTF_s + MTTR_s$
Reliability	$R_s = e^{-\lambda_s t}$
Availability	$A_s = \frac{\mu_s}{\lambda_s + \mu_s} + \frac{\lambda_s}{\lambda_s + \mu_s} e^{-(\lambda_s + \mu_s)t}$
Expected numbers of failures (ENOF)	$W_s(0, t) = \frac{\lambda_s \mu_s t}{\lambda_s + \mu_s} + \frac{\lambda_s^2}{(\lambda_s + \mu_s)^2} [1 - e^{-(\lambda_s + \mu_s)t}]$

(Step 4:) The obtained information from the above step is fuzzified in nature. Now, in order to implement it in the real life, it is necessary to defuzzify it into the crisp number. For it, a defuzzification equation as given in (3.1.3) of Chapter 4 has been used for each reliability index.

5.3 Case study

In this section, a brief description of the system namely washing unit of the repairable paper mill has been presented. The washing unit is one of the most important functioning unit of the system whose brief description is summarized as below.

5.3.1 System description

The considered washing system consists of four main components which are defined as below.

- **Filter (A):** It is a single unit and used to drain black liqueur from the cooked pulp.
- **Cleaners (B):** It consists of three units connected in parallel configuration whose main action is to clean the pulp by centrifugal action. Failure of anyone will reduce the efficiency of the system as well as quality of paper.
- **Screeners (C):** It consists of two units arranged in series and used to remove oversized, uncooked and odd shaped fibers from pulp through straining action.
- **Deckers (D):** It consists of two units arranged in parallel configuration. The main function of it is to reduce the blackness of pulp. Complete failure of decker occurs when both the components will fail.

The systematic diagram of the system are shown in Fig. 5.1.

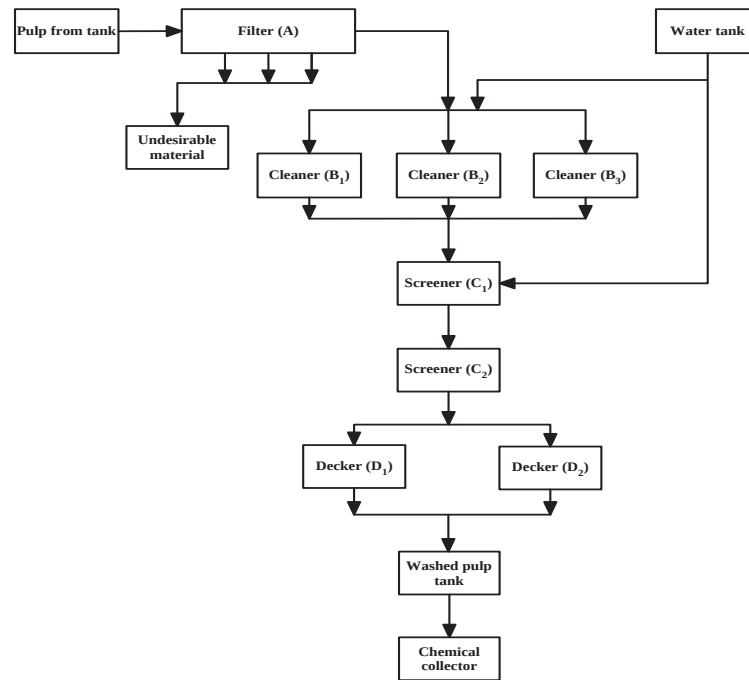


Figure 5.1: Systematic diagram of Washing system

5.3.2 Behavior analysis

The procedure steps used for conducting the analysis by using proposed methodology are given as below.

(Step 1:) The data related to main components' of the system are extracted in the form of failure rates (λ_i 's) and repair times (τ_i 's) and are summarized in Table 5.3.

Table 5.3: Input data for the main components of the Washing system

Components→	Filter	Cleaners	Screeners	Deckers
	($i = 1$)	($i = 2, 3, 4$)	($i = 5, 6$)	($i = 7, 8$)
Failure rate λ_i (hrs ⁻¹)	$[1, 2] \times 10^{-3}$	$[3, 4] \times 10^{-3}$	$[5, 6] \times 10^{-3}$	$[5, 6] \times 10^{-3}$
Repair time τ_i (hrs)	$[3, 4]$	$[2, 3]$	$[3, 4]$	$[3, 4]$

(Step 2:) To handle the uncertainties in the data, they are fuzzified into TrIFNs with some known spread or support, $\pm 15\%$ (also at $\pm 25\%$, $\pm 50\%$) on both sides of data, as suggested by system analyst/ decision makers.

(Step 3:) Based on these data and the systematic representation of the system, the system expression for systems' failure rate (λ_s) and repair time (τ_s) are obtained using Table 5.1, as below

$$\begin{aligned}\lambda_s &= \lambda_1 + \lambda_2 \lambda_3 \lambda_4 (\tau_2 \tau_3 + \tau_3 \tau_4 + \tau_4 \tau_2) + \lambda_5 + \lambda_6 + \lambda_7 \lambda_8 (\tau_7 + \tau_8) \\ \tau_s &= \frac{\lambda_1 \tau_1 + \lambda_2 \lambda_3 \lambda_4 \tau_2 \tau_3 \tau_4 + \lambda_5 \tau_5 + \lambda_6 \tau_6 + \lambda_7 \lambda_8 \tau_7 \tau_8}{\lambda_s}\end{aligned}$$

Based on it, the various reliability parameters have been computed at different confidence levels ranging from 0 to 0.7 for membership while 1 to 0.1 to non-membership, at the mission time $t = 10$ (hrs) with left and right spread. These computed results are depicted graphically in Fig. 5.2 for $\pm 15\%$ spreads along with the existing techniques results while their corresponding membership and non-membership values of each α - and β - cuts are tabulated in Tables 5.4 and 5.5.

(Step 4:) The defuzzified value by using Eq. (3.1.3) at different level of the spread $\pm 15\%$, $\pm 25\%$ and $\pm 50\%$ are computed and are summarized in Table 5.6. It shows that when uncertainty level in the form of spread increases from $\pm 15\%$ to $\pm 25\%$ and further $\pm 50\%$, the variation in defuzzified values for almost all the reliability indices are not so much.

Table 5.5: Non-Membership values for various reliability indices values

β	Failure rate ($\times 10^{-3}$)			Repair time			MTBF		
	Garg approach			Garg approach			Garg approach		
	L	R	M	L	R	M	L	R	M
0	11.150	14.290		2.979	3.959		92.663	73.935	
0.1	10.978	14.513	11.150	2.935	4.017	2.979	94.021	72.916	92.663
0.2	10.807	14.737	10.959	2.891	4.075	2.930	95.422	71.928	82.131
0.3	10.635	14.961	10.769	2.847	4.133	2.881	96.868	70.969	73.935
0.4	10.464	15.186	10.578	2.803	4.191	2.832	98.361	70.039	72.489
0.5	10.293	15.411	10.388	2.759	4.249	2.783	99.903	69.135	71.105
0.6	10.123	15.637	10.199	2.714	4.307	2.734	101.497	68.258	69.778
0.7	9.952	15.862	10.009	2.670	4.365	2.685	103.145	67.405	68.506
0.8	9.782	16.089	9.820	2.626	4.422	2.636	104.850	66.576	67.285
0.9	9.612	16.316	9.631	2.581	4.480	2.586	106.615	65.770	66.112
1	9.442	16.543	9.442	2.537	4.538	2.537	108.444	64.985	64.985
Reliability									
α	Garg approach			Garg approach			Garg approach		
	L	R	M	L	R	M	L	R	M
	L	R	M	L	R	M	L	R	M
0	0.8945	0.8668		0.9688	0.9502		0.1089	0.1379	
0.1	0.8960	0.8649	0.8945	0.9697	0.9489	0.9688	0.1073	0.1400	0.1089
0.2	0.8976	0.8630	0.8962	0.9706	0.9475	0.9698	0.1057	0.1420	0.1235
0.3	0.8991	0.8610	0.8979	0.9714	0.9462	0.9707	0.1041	0.1440	0.1379
0.4	0.9006	0.8591	0.8996	0.9722	0.9449	0.9717	0.1025	0.1461	0.1408
0.5	0.9022	0.8572	0.9013	0.9730	0.9435	0.9726	0.1008	0.1481	0.1437
0.6	0.9037	0.8552	0.9030	0.9739	0.9422	0.9735	0.0992	0.1502	0.1467
0.7	0.9053	0.8533	0.9048	0.9747	0.9408	0.9744	0.0976	0.1522	0.1496
0.8	0.9068	0.8514	0.9065	0.9755	0.9395	0.9753	0.0960	0.1542	0.1525
0.9	0.9084	0.8495	0.9082	0.9762	0.9381	0.9762	0.0944	0.1563	0.1554
1	0.9099	0.8475	0.9099	0.9770	0.9367	0.9770	0.0928	0.1583	0.1583
Availability									
α	Garg approach			Garg approach			Garg approach		
	L	R	M	L	R	M	L	R	M
	L	R	M	L	R	M	L	R	M
0	0.8945	0.8668		0.9688	0.9502		0.1089	0.1379	
0.1	0.8960	0.8649	0.8945	0.9697	0.9489	0.9688	0.1073	0.1400	0.1089
0.2	0.8976	0.8630	0.8962	0.9706	0.9475	0.9698	0.1057	0.1420	0.1235
0.3	0.8991	0.8610	0.8979	0.9714	0.9462	0.9707	0.1041	0.1440	0.1379
0.4	0.9006	0.8591	0.8996	0.9722	0.9449	0.9717	0.1025	0.1461	0.1408
0.5	0.9022	0.8572	0.9013	0.9730	0.9435	0.9726	0.1008	0.1481	0.1437
0.6	0.9037	0.8552	0.9030	0.9739	0.9422	0.9735	0.0992	0.1502	0.1467
0.7	0.9053	0.8533	0.9048	0.9747	0.9408	0.9744	0.0976	0.1522	0.1496
0.8	0.9068	0.8514	0.9065	0.9755	0.9395	0.9753	0.0960	0.1542	0.1525
0.9	0.9084	0.8495	0.9082	0.9762	0.9381	0.9762	0.0944	0.1563	0.1554
1	0.9099	0.8475	0.9099	0.9770	0.9367	0.9770	0.0928	0.1583	0.1583
ENOF									
α	Garg approach			Garg approach			Garg approach		
	L	R	M	L	R	M	L	R	M
	L	R	M	L	R	M	L	R	M
0	0.8945	0.8668		0.9688	0.9502		0.1089	0.1379	
0.1	0.8960	0.8649	0.8945	0.9697	0.9489	0.9688	0.1073	0.1400	0.1089
0.2	0.8976	0.8630	0.8962	0.9706	0.9475	0.9698	0.1057	0.1420	0.1235
0.3	0.8991	0.8610	0.8979	0.9714	0.9462	0.9707	0.1041	0.1440	0.1379
0.4	0.9006	0.8591	0.8996	0.9722	0.9449	0.9717	0.1025	0.1461	0.1408
0.5	0.9022	0.8572	0.9013	0.9730	0.9435	0.9726	0.1008	0.1481	0.1437
0.6	0.9037	0.8552	0.9030	0.9739	0.9422	0.9735	0.0992	0.1502	0.1467
0.7	0.9053	0.8533	0.9048	0.9747	0.9408	0.9744	0.0976	0.1522	0.1496
0.8	0.9068	0.8514	0.9065	0.9755	0.9395	0.9753	0.0960	0.1542	0.1525
0.9	0.9084	0.8495	0.9082	0.9762	0.9381	0.9762	0.0944	0.1563	0.1554
1	0.9099	0.8475	0.9099	0.9770	0.9367	0.9770	0.0928	0.1583	0.1583

L: left non-membership ; M: middle non-membership; R: right non- membership

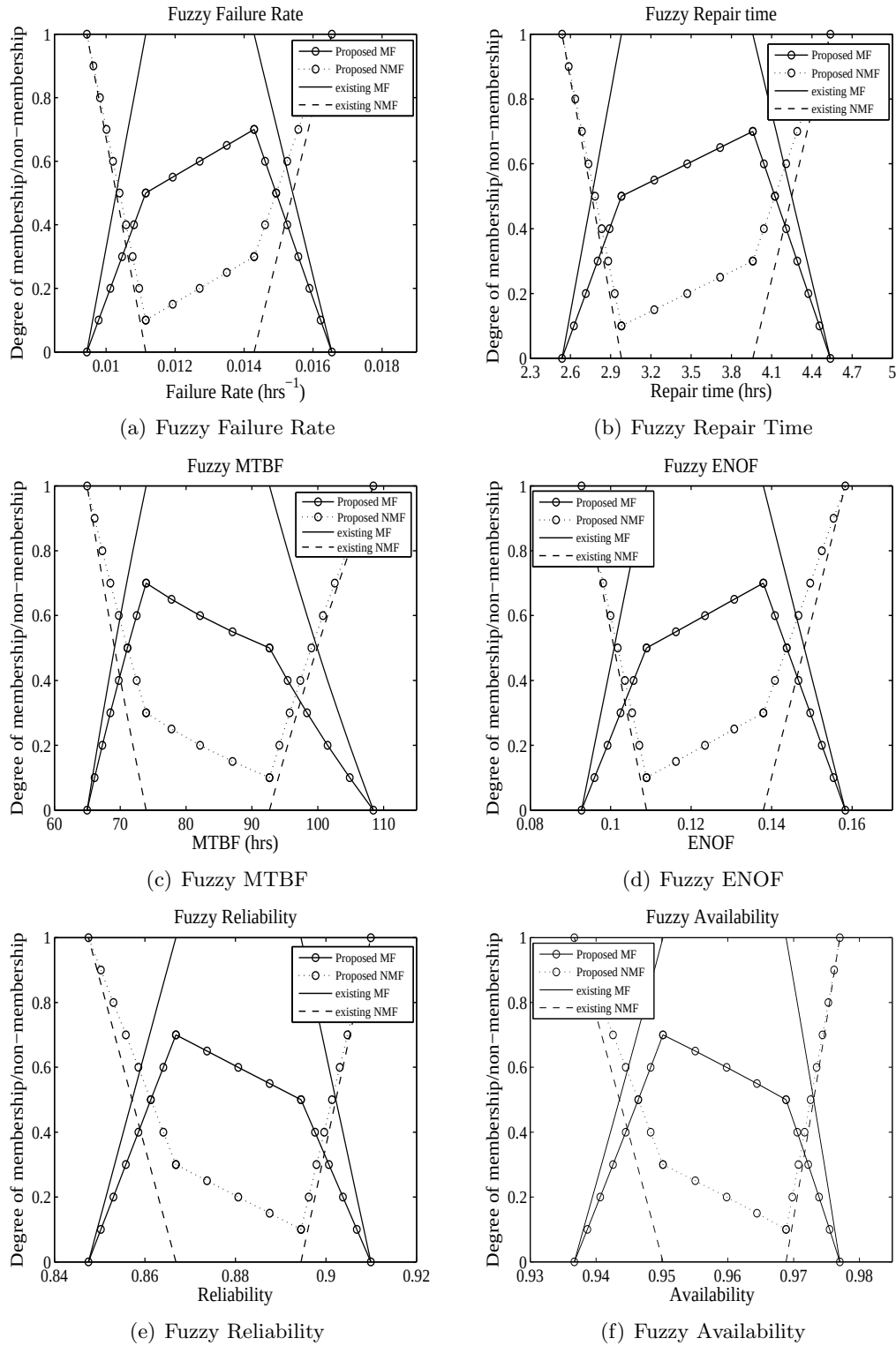


Figure 5.2: Fuzzy Reliability Indices Plots for Washing system

Table 5.6: Crisp and defuzzified values of the system

Reliability parameters	Crisp values	Defuzzified values at spread		
		$\pm 15\%$	$\pm 25\%$	$\pm 50\%$
Failure rate	0.0137	0.0131	0.0127	0.0118
Repair time	3.7335	3.5861	3.4874	3.2394
MTBF	88.2308	90.5087	93.6291	110.7123
Reliability	0.8807	0.8797	0.8791	0.8776
ENOF	0.1326	0.1272	0.1236	0.1146
Availability	0.9595	0.9582	0.9570	0.9533

5.4 Conclusion

In this chapter, we have analyzed the reliability of the industrial system by using the generalized trapezoidal intuitionistic fuzzy numbers. In it, firstly, the information related to the various components of the system are extracted in the form of the failure and repair rates and then corresponding to basic expression of the series-parallel system, we have analyzed the behavior of the system. Different reliability indices are computed at different levels of spread and then their results are plotted in the Figure and compared with the existing approaches. Their corresponding variations in their membership and non-membership values are summarized in their respective tables. Finally, the defuzzified values corresponding to various indices are presented for implementing these parameters into the real-life decision.

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