

**MUTUAL INFORMATION AND BIT ERROR RATE  
ANALYSIS OF MIMO SYSTEM UNDER CORRELATED  
FADING CHANNEL**

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**IN**

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Submitted by

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## DECLARATION

I, **Vikas Shivrain**, hereby certify that the work which is being presented in this thesis entitled "MUTUAL INFORMATION AND BIT ERROR RATE ANALYSIS OF MIMO SYSTEM UNDER CORRELATED FADING CHANNEL" by me in partial fulfillment of the requirements for the award of degree of Master of Engineering in Electronics and Communication Engineering from Thapar University (Deemed University), Patiala, is an authentic record of my own work carried out under the supervision of **Mr. Ankush Kansal**.

The matter presented in this thesis has not been submitted in any other University / Institute for the award of any other degree.

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It is certified that the above statement made by the student is correct to the best of my knowledge and belief.

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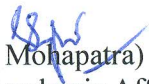
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## ABSTRACT

Multiple Input Multiple Output (MIMO) wireless links has recently emerged as one of the most significant technical breakthrough in modern communication. In this thesis, an overview of MIMO system has been discussed. After presenting brief idea of MIMO transmissions in the introduction, the system Capacity and Bit Error Rate of MIMO designs have been explained. MIMO system with multiple transmitter and receiver antennas have been recognized as the vital breakthrough for wireless system to achieve higher data rates and improved quality of service with limited bandwidth and power resources in influence of multipath fading. On the other hand, traditionally, multiple antennas have been used to increase diversity for combating channel fading. Hence, MIMO system emerges out as the solution for spatial multiplexing, capacity and bit error rate.

Wireless relaying networks have recently been given considerable attention due to their various advantages over traditional communication system. The relaying terminals forward the information from the source to the destination mainly using the well-known method Amplify-Forward since MIMO systems can provide better system capacity than SISO, SIMO and MISO systems. MIMO relays aim to provide improved system capacity, increases in range, and better diversity gain. In this thesis the ergodic capacity of an AF MIMO two-hop system under Rayleigh fading channel has been analyzed and compared with earlier techniques of MIMO system.

The main contribution of this work is to derive an exact expression for an AF MIMO two-hop system capacity and cumulant function of Mutual Information. The expression derived is unified, and it can be used for arbitrary numbers of antennas at the source, relay and destination. The simulation results have been presented to validate the analysis. Mutual information has shown improvement up to 30% in the capacity as compared to the MIMO system implemented without AF.

Further the capacity benefits of MIMO system under Rician fading channel have been presented. Here, in this thesis slow and frequency nonselective Rician fading channel is used for analyzing, the effect of Rician factor ( $K$ ) and correlation parameter ( $rr$ ) for the capacity of correlated MIMO channels. From the comparison it has been found that the outage capacity is higher than the ergodic capacity. From the study and simulation result presented in this thesis, it has been observed that by increasing the number of antennas of MIMO system and

with increase in signal to noise ratio the capacity of MIMO system degrades; when observed for different Rician factor coefficient.

The bit error rate analysis of MIMO system have been analyzed using BPSK modulation scheme over Rayleigh and Rician wireless fading channels. The BER performance characteristics investigation of 2x2 antenna configuration has done by using different type of linear and non-linear equalizer techniques namely ML, ZF and MMSE for MIMO system. The bit error rate characteristics for the two transmitting and two receiving antenna has been simulated. By comparing the Rayleigh and Rician fading simulation result it has been observed that there has been improvement in bit error rate up to 35% for ML detection, up to 24% in ZF detection and up to 18% in MMSE detection technique by keeping  $BER \sim 10^{-4}$  with respect to SNR under Rician fading channel.

Finally, the result achieved in this thesis have been compared with the BER produced on MIMO system by using IEEE802.11n protocol as benchmark, the significant improvement in BER for each technique have been achieved as compared to BER achieved from IEEE802.11n standard and reported in this thesis.

The simulation result have shown that ZF equalizer based receiver remove inter symbol interference much better as compared to the receivers implemented using ML and MMSE based equalizer. Whereas MMSE based equalizer found to be robust even in presence of channel nulls and noise because of its property directly minimizing the bit error rate.

# Table of Content

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<i>Declaration</i> .....	i
<i>Acknowledgements</i> .....	ii
<i>Abstract</i> .....	iii
<i>Table of Content Contents</i> .....	v
<i>List of Figures</i> .....	viii
<i>List of Table</i> .....	ix
<i>List of Abbreviation</i> .....	x
<b>1. Introduction</b> .....	1
1.1. Evolution of Wireless Communication.....	1
1.2. Types of Channel - SISO, SIMO, MISO, MIMO.....	2
1.3. Capacity of MIMO system.....	6
1.4. Capacity of MIMO Systems under the Rician fading Channels.....	7
1.4.1. MIMO Channel model and fading due to Rician fading.....	7
1.4.2. Determination of MIMO Capacity for channel unknown at transmitter.....	8
1.5. Multi-Hop Amplify and Forward MIMO System Mutual Information.....	9
1.6. MIMO detection technique.....	11
1.6.1 Zero Force (ZF) Detector.....	12
1.6.2. Maximum Likelihood (ML).....	13
1.6.3. Minimum-Mean Square Error (MMSE).....	14

1.7. Objective of thesis.....	15
1.8. Methodology of thesis.....	15
1.9. Organization of thesis.....	15
<b>2. Literature Survey.....</b>	<b>16</b>
2.1. MIMO Capacity.....	16
2.2. Amplify-Forward MIMO Mutual Information.....	20
2.3. Signal Detection in MIMO.....	25
<b>3. MIMO System.....</b>	<b>28</b>
3.1 MIMO Transmission.....	29
3.2 MIMO channel Correlation Matrix (H).....	30
3.3 Noise.....	31
3.4 Fading channel.....	32
3.5 MIMO channel Power Distribution and SNR.....	33
3.6 MIMO Channel Model Classification.....	34
3.7 Physical Model.....	36
3.8 Analytical Model.....	38
<b>4. MIMO System Mutual-Information Analysis.....</b>	<b>41</b>
4.1. Amplify-Forward MIMO Mutual Information.....	41
4.1.1. Introduction.....	42
4.1.2. MIMO Mutual Information Mathematical analysis.....	43
4.1.3. Simulation Result and Discussion.....	47

4.1.4. MIMO MI Analysis.....	48
4.2. Capacity of MIMO under Correlated Rician fading channel.....	48
4.2.1. MIMO Channel and fading due to Rician fading.....	49
4.2.2. Determination of MIMO Capacity for Channel unknown at Transmitter.....	50
4.2.3. Simulation Result and Discussion.....	52
4.2.4. MIMO Capacity Analysis.....	53
<b>5. Analysis of BER for MIMO System.....</b>	<b>55</b>
5.1. Introduction.....	55
5.2. System Model.....	55
5.2.1 Maximum Likelihood (ML) detection.....	57
5.2.2 Zero Force Equalizer (ZF).....	57
5.2.3 Minimum Mean Square Error-Estimator (MMSE).....	59
5.3. Simulation Result and Discussion.....	60
5.4. MIMO BER Analysis.....	63
<b>6. Conclusion and Future Work.....</b>	<b>64</b>
<b>References.....</b>	<b>66</b>
<b>Paper Communicated from this Thesis.....</b>	<b>73</b>

## LIST OF FIGURES

Figure No 1.1 Single Input Single Outputs (SISO).....	2
Figure No 1.2 Single-Input Multiple Output (SIMO).....	3
Figure No 1.3 Multiple Input Single Output (MISO).....	4
Figure No 1.4 Multiple Input Multiple Outputs (MIMO).....	5
Figure No 1.5 Block Diagram of System with Equalizer.....	11
Figure No 3.1 N-element MIMO Systems.....	28
Figure No 3.2 Data Transmissions in MIMO System.....	29
Figure No 3.3 Classifications of MIMO Channels and Propagation Model.....	35
Figure No 3.4 (a) propagation scenario (gray shading indicates buildings).....	37
(b) Corresponding visibility tree.....	37
Figure No 4.1 Block Diagram of MIMO Amplify-Forward Relay System.....	41
Figure No 4.2 Mutual information versus SNR for MIMO relay system with large number of antenna.....	47
Figure No 4.3 Ergodic Capacity versus SNR for MIMO system with Rician factor $K= 0$ and 10 and Correlation parameter $r_r = r_t = 0$ and 0.7.).....	52
Figure No 4.4 1 % Outage Capacity versus SNR for MIMO system with Rician factor $K= 0$ and 10 and Correlation parameter $r_r = r_t = 0$ and 0.7.).....	53
Figure No 5.1 BER versus SNR for MIMO system with Rician fading ( $K= 1$ and Correlation parameter $r_r = r_t = 0.7$ ) using 2x2 BPSK modulation Scheme.....	60
Figure No 5.2 BER versus SNR for MIMO system with Rayleigh fading using 2x2 BPSK modulations Scheme .....	61
Figure No 5.3 BER versus SNR for MIMO system with Rician fading ( $K= 5$ and Correlation parameter $r_r = r_t = 0.7$ ) using 2x2 BPSK modulation Scheme.....	62

## LIST OF TABLES

Table No 5.1	Comparison of detection technique Scheme for MIMO.....	62
Table No 5.2	Comparison of BER for MIMO under BPSK modulation scheme for Rayleigh and Rician fading using detection technique.....	63

## LIST OF ABBREVIATIONS

MIMO	Multiple Input Multiple Output
1G	First Generation
FDMA	Frequency Division Multiple Access
TDMA	Time Division Multiple Access
CDMA	Code Division Multiple Access
3G	Third Generation
QOS	Quality of Service
SVD	Singular Value Decomposition
SISO	Single Input Single Output
SIMO	Single Input Multiple Output
MISO	Mingle Input Multiple Output
SM	Spatial Multiplexing
PAN	Personal Area Network
LAN	Local Area Network
WAN	Wireless Area Network
MAN	Metropolitan Area Network
OFDM	Orthogonal Frequency Division Multiplex
<i>i. i. d</i>	Independent and Identically Distributed
LOS	Line of Sight
SNR	Signal to Noise Ratio
MI	Mutual Information
BER	Bit Error Rate
AF	Amplify and Forward
ZF	Zero forcing
ML	Maximum likelihood
MMSE	Minimum Mean Square Error-Estimator
IEEE	Institute of Electrical and Electronics Engineers
BPSK	Binary Phase Shift Key
ZMCSCG	Zero mean Circularly Symmetric Complex Gaussian Random Variables

CSI	Channel State Information
IRQ	Improved Rate Quantization
ISI	Inter Symbol Interference
DOD	Direction of Departure
DOD	Direction of Arrival
GSCM	Geometry based Stochastic Channel Models
RT	Ray Tracing
WLAN	Wireless Local Area Network
SRD	Source, Relay and Destination
AWGN	Adaptive White Gaussian Noise
MS-MMSE	Multi-Stage Minimum Mean Square Error Estimation

# CHAPTER 1

## INTRODUCTION

---

In this chapter an overview of Multiple Input Multiple Output (MIMO) communication system is presented, which commences with a description of the high capacity demands in future wireless communication networks and how high capacity can be achieved from MIMO system.

### **1.1 Evolution of Wireless Communication**

Wireless communication systems have made large advances since the first generation (1G) Systems. Frequency Division Multiple Access (FDMA) was the main techniques used at that level. Similarly, Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA) multiple access techniques are widely used in today's communication systems [1]. CDMA is also used in the third generation (3G) systems. Three multiple access schemes are used to allow users to simultaneously share radio spectrum, which is an finite and essential resource of all wireless communications technologies. Now a day, the wireless system designers are facing several challenges: like the limitation of the radio spectrum, the complexity of wireless propagation environment, the increasing demand for better quality of service (QoS) and higher transmission data rate. Traditional wireless communication systems have been made more spectrally efficient through the use of complex coding techniques and algorithms. By keeping in mind the constraint of limited bandwidth the MIMO systems is a hot topic of research over the past years, due to its ability to greatly increase spectral efficiencies.

As compared to traditional wireless systems, which are using one transmitting and one receiving antenna, MIMO systems use array of multiple antennas at both ends of the communication link, all operating at the same frequency at the same time [2],[3]. This introduces spatial diversity into the system, which can be used to tackle the problem of multipath fading. In wireless communications system, such as point to point radio links, radio waves do not simply propagate from the transmit antenna to the receive antenna. Rather they bounce and scatter off objects, this effect is known as multipath. This effect is regarded as an

impediment to the accurate transmission of data in traditional wireless links [4]. MIMO systems exploit multipath by using the rich scattering environment to increase the spectral efficiency of the wireless system.

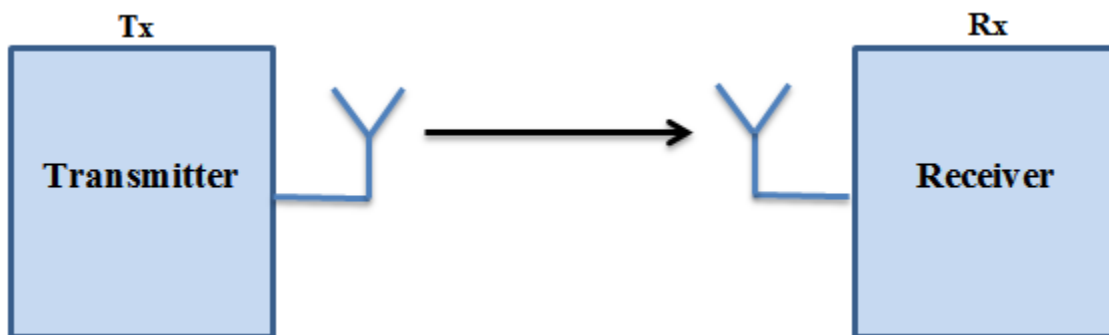
The modeling of radio waves on a large scale can be very complex. At high frequencies radio waves can be approximated as travelling along localized paths, this is similar to the geometrical treatment of light rays in optics. Complex radio environments can be modeled by using different deterministic and stochastic models. At the receiver and transmitter of MIMO system encoding and decoding of signal is done respectively by using SVD and correlation for calculating eigen value [5],[6]. In this thesis, an attempt is made to demonstrate the effect of correlation between the elements of the channel matrix for improvement in system performance criteria's such as channel capacity and bit error rate.

## 1.2 Types of Transmission Channel

In the wireless communication the Transmission channel classified according to number of input-output at the transmitter and receiver. They are classified in four categories as discussed below:

### 1.2.1 Single Input Single Output (SISO)

Radio transmissions traditionally use one antenna at the transmitter and one antenna at the receiver. This system is termed Single Input Single Output (SISO) shown in Figure-1.1



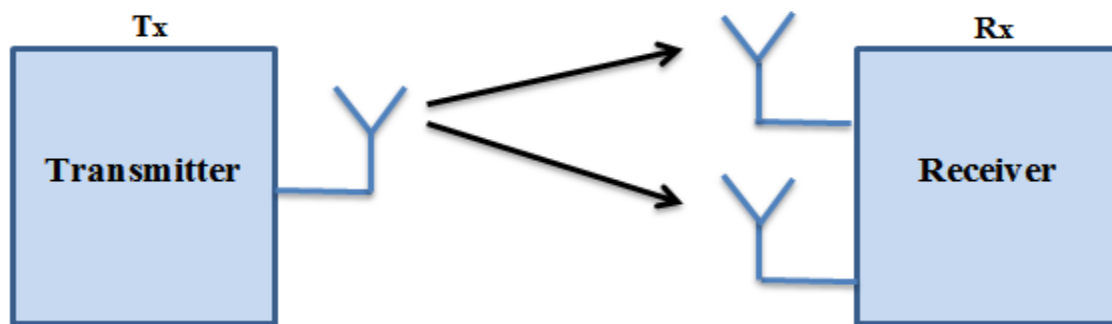
**Figure-1.1 Single Input Single Output (SISO)**

Both the transmitter and the receiver have one RF channel (that's coder and modulator). SISO is relatively simple and cheap to implement and it has been used age long since the birth of

radio technology. This technique is known as switched diversity or selection diversity. SISO employs no diversity technique. The advantage of a SISO system is its simplicity; it requires no processing in terms of the various forms of diversity that may be used. However the SISO channel is limited in its performance [7]. Interference and fading will impact the system more complex than MIMO system use diversity technique and in SISO channel bandwidth is limited by Shannon's law because the throughput being dependent upon the channel bandwidth and the signal to noise ratio. It is used in radio and TV broadcast and personal wireless technologies (e.g. Wi-Fi and Bluetooth).

### 1.2.2 Single Input Multiple Output (SIMO)

To improve performance, a multiple antenna technique has been developed. A system which uses a single antenna at the transmitter and multiple antennas at the receiver is named Single Input Multiple Output (SIMO) as shown in Figure-1.2. The receiver can either choose the best antenna to receive a stronger signal or combine signals from all antennas in such a way that maximizes SNR. SIMO Employs a receive diversity technique. This is also known as receiver diversity [8]. It is often used to enable a receiver system that receives signals from a number of independent sources to combat the effects of fading. It has been used for many years with short wave listening or receiving stations to combat the effects of ionosphere fading and interference. This technique is based on maximal ratio combining (MRC).



**Figure-1.2 Single Input Multiple Output (SIMO), 1x2**

SIMO has the advantage that it is relatively easy to implement although it does have some disadvantages. It requires complex processing at the receiver. The use of SIMO may be quite

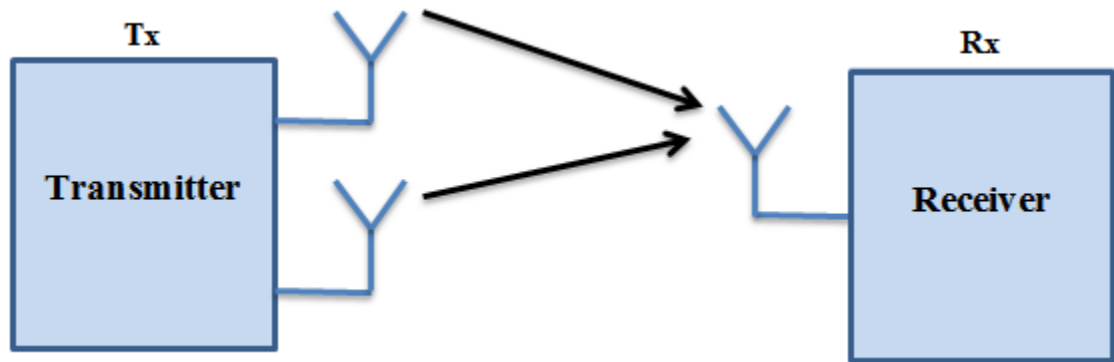
acceptable in many applications, but where the receiver is located in a mobile device such as a cell phone handset, the levels of processing may be limited by size, cost and battery drain.

There are two forms of SIMO that can be used [9]:

- **Switched diversity SIMO:** This form of SIMO looks for the strongest signal and switches to that antenna.
- **Maximum ratio combining SIMO:** This form of SIMO takes both signals and sums them to give the combination. In this way, the signals from both antennas contribute to the overall signal.

### 1.2.3 Multiple Input Single Output (MISO)

A system which uses multiple antennas at the transmitter and a single antenna at the receiver is named Multiple Input Single Output (MISO) as shown in Figure-1.3.



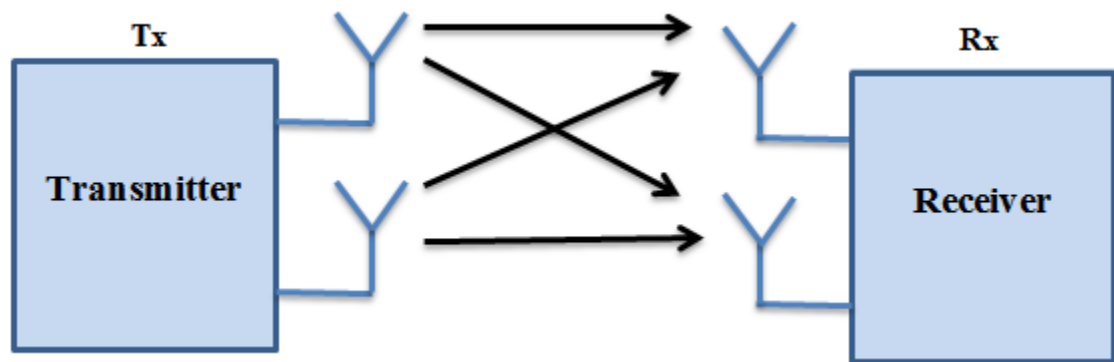
**Figure-1.3 Multiple Input Single Output (MISO), 2x1**

Alamouti STC (Space Time Coding) is employed at the transmitter with two antennas [10]. STC allows the transmitter to transmit signals (information) both in time and space, meaning the information is transmitted by two antennas at two different times consecutively. Multiple antennas (each with an RF channel) of either SIMO or MISO are usually placed at a base station (BS). This way, the cost of providing either a receiver diversity (in SIMO) or transmitter diversity (in MISO) can be shared by all subscriber stations (SS) served by the base station (BS) [8].

The advantage of MISO is that the multiple antennas and the redundancy coding processing are moved from the receiver to the transmitter. In instances such as cellphone UEs, this can be a significant advantage in terms of space for the antennas and reducing the level of processing required in the receiver for the redundancy coding. This has a positive impact on size, cost and battery life as the lower level of processing requires less battery consumption. MISO system does not take the advantage of the spatial multiplexing that use in MIMO system.

#### 1.2.4 Multiple Input Multiple Output (MIMO)

To multiply throughput of a radio link, multiple antennas (and multiple RF channel accordingly) are put at both the transmitter and the receiver. This system is referred to as MIMO as shown in Figure-1.4. A MIMO system with similar count of antennas at both the transmitter and the receiver in a point-to-point (PTP) link is able to multiply the system throughput linearly with every additional antenna. For example, a 2x2 MIMO will double the throughput.



**Figure-1.4 Multiple Input Multiple Outputs (MIMO), 2x2**

MIMO often employs Spatial Multiplexing (SM) to enable signal (coded and modulated data stream) to be transmitted across different spatial domains. Meanwhile, Mobile Wi-MAX supports multiple MIMO modes, that is using either SM or STC or both to maximize spectral efficiency (increase throughput) without shrinking the coverage area. The dynamic switching between these modes based on channel conditions is called Adaptive MIMO Switching (AMS) [11]. If combined with AAS (Adaptive Antenna System), MIMO can further boost Wi-MAX performance. MIMO is widely used in today wireless communications because all

wireless technologies (PAN, LAN, MAN, and WAN) try to add it to increase data rate multiple times to satisfy their bandwidth-hungry broadband users. Techniques that will most likely make 4G reality are MIMO, OFDM MIMO system will deliver multimedia services (VoIP, video, Internet) at high speed (100 Mbps or more) over end-to-end IP network infrastructure and it will enable seamless handoff between mobile wireless WAN and fixed wireless LAN [2].

### 1.3 Capacity of MIMO System

H.Foschini et al. [12] analyzed the information-theoretic capacity of a multiple-antenna point to point wireless system in a narrow-band Rayleigh fading environment. They assume independent and identically distributed (*i.i.d.*) fading at different antenna elements, and assume that the transmitter does not know the channel while the receiver is able to track the channel perfectly. With  $T$  transmitting and  $R$  receiving antennas, the system is described by the matrix equation

$$Y = \sqrt{\frac{E_S}{T}}HX + n \quad (1.1)$$

where  $E_S$  is the total energy available at the transmitter,  $Y$  is the  $R \times 1$  vector of signals received on the  $R$  antennas,  $X$  is the  $T \times 1$  vector of signals transmitted on the  $T$  transmit antennas,  $n$  is the  $R \times 1$  noise vector consisting of independent complex Gaussian distributed elements with zero mean and variance  $\sigma^2$ , and  $H$  is the  $R \times T$  channel matrix with components modeled as *i.i.d.* zero mean unit variance circularly symmetric complex Gaussian random variables (ZMCSCG). The capacity for this system is shown to be

$$C = E_H\{\log_2 \det(I_R + \frac{\rho}{T}HH^H)\} \quad (1.2)$$

With  $\rho = \frac{E_S}{\sigma^2}$

Where  $E_H\{\cdot\}$  denote the expectation over  $H$ , and the operator  $H$  indicates the hermitian of the matrix  $H$ . Foschini et al. [12] have been shown that the capacity  $C$  grows linearly with  $\min(T, R)$  for a given fixed transmitted power and bandwidth. In other words, without increasing the transmit power or bandwidth the capacity of the wireless channel can be increased by simply increasing the number of transmitter and receiver antennas. This is an enormous improvement compared to a logarithmic increase in more traditional systems

utilizing receiver diversity or no diversity. While the linear growth of capacity with the  $\min(T, R)$  is indicative of the tremendous potential of multiple antenna systems, the result is limited in scope by the assumptions it makes. The true benefits of multiple antennas depend on how well these assumptions hold up in practice and, if the assumptions are not realistic, then what capacity benefits of multiple-antenna systems are possible under more realistic assumptions. The performance estimation of a MIMO system under spatially correlated fading characterizes by two parameter the Rician factor and correlation parameter [15].

#### **1.4 Capacity of MIMO Systems under the Rician Fading Channels**

The use of antenna arrays at both sides of the wireless communication link can result in high channel capacity provided the propagation medium is highly scattered and uncorrelated fading is there. Consider a single user MIMO system that has  $T$  and  $R$  number of antennas at the transmitter and receiver, respectively. It has been shown that, for a given fixed average transmitter power and bandwidth, if the fading between pairs of transmit-receive antenna elements are independent and identically distributed (*i. i. d*) Rayleigh, the average channel capacity increases linearly with  $m = \min(T, R)$ [12]. This large capacity growth occurs even if the transmitter has no channel knowledge. Logarithmic increase has been achieved in MIMO system as compared to traditional systems utilizing receive diversity or no diversity. While the linear growth of capacity with in MIMO due to ‘ $m$ ’ is indicative of the tremendous potential of multiple antenna systems, the result is limited in scope by the assumptions it made. Most of the research effort has been focused on *i. i. d* Rayleigh channels. However, in real world propagation environment, the fading is not independent due, to insufficient spacing between antenna elements. It has been shown [13],[14] that correlated fading reduces the channel capacity. Secondly, there is a possibility that the line-of-sight (*LOS*) component may exist in addition to scattered component, then; the fading will follow the Rician distribution.

##### **1.4.1 MIMO Channel model and fading due to Rician fading**

Consider a single user MIMO system with  $T$  antennas at the transmitter and  $R$  antennas at the receiver. For simplicity we consider only frequency flat fading; i.e., the fading is not frequency selective. The transmitted signal in each symbol period is represented by a  $T \times 1$  column matrix  $S$ , where  $i^{th}$  component  $S_i$ , refers to the transmitted signal from antenna  $i$ . The

channel  $H$  is described by a  $R \times T$  complex matrix where the  $ij^{th}$  component of the matrix  $H$ , denoted by  $h_{ij}$ , is the channel response between the  $j^{th}$  transmit antenna and the  $i^{th}$  receive antenna. The additive white Gaussian noise vector  $n$  at the receiver is described by a  $R \times 1$  column matrix. Thus, the system is described by the matrix equation [16].

$$Y = \sqrt{\frac{E_S}{T}} HX + n \quad (1.3)$$

And the channel matrix,  $H$  can be denoted as shown below

$$H = \begin{pmatrix} h_{1,1} & \dots & h_{1,T} \\ \vdots & \ddots & \vdots \\ h_{R,1} & \dots & h_{R,T} \end{pmatrix} \quad (1.4)$$

In the Rician fading the elements of  $H$  are non-zero mean complex Gaussians. Hence we can express in matrix notation  $H$  [17]

$$H = aH^{sp} + bH^{sc} \quad (1.5)$$

Where the specular and scattered components of  $H$  are denoted by superscripts,  $a > 0$ ,  $b > 0$  and  $a^2 + b^2 = 1$ .  $H^{sp}$  is a matrix of unit entries denoted as  $H_1$ . If there is no correlation at the transmitter or at the receiver side then the entries of  $H^{sc}$  are independent and identically distributed (*i. i. d*) complex Gaussian random variables with zero mean and unit magnitude variance, usually denoted by  $H_\omega$ . If there is correlated fading then the  $H^{sc}$  matrix can be modeled as [18].

$$H^{sc} = R_r^{1/2} H_\omega R_t^{1/2} \quad (1.6)$$

Where  $R_t$  and  $R_r$  are the correlation matrix at the transmitter and at the receiver side, respectively. The correlation matrix  $R$  is defined by exponential correlation model [13].

#### 1.4.2 Determination of MIMO Capacity for Channel unknown at Transmitter

Channel knowledge acquiring at the transmitter is very difficult in practical systems. In general the channel is assumed perfectly known to the receiver where the channel state information at the transmitter is available or not. Furthermore, ergodic block fading channel model where the channel remains constant over a block of consecutive symbols, and changes in an independent fashion across blocks. The average SNR at each of the receive antennas is

given by  $E_S/\sigma^2$ , where  $E_S$  is the power of the transmitted signal and  $\sigma^2$  is the power spectral density of the noise. In fading channel two types capacity has been described: ergodic capacity and outage capacity [12],[16].

**Ergodic Capacity:** This is the time-averaged capacity of a stochastic channel. It is found by taking the mean of the capacity values obtained from a number of independent channel realizations.

**Outage Capacity:** The  $q\%$  outage capacity  $C_{out,q}$  is defined as the capacity that is guaranteed for  $(100 - q)\%$  of the channel realizations, i.e.

$$(P(C < C_{out,q}) = q\% \quad (1.7)$$

The MIMO channel capacity with  $\rho=E_S/\sigma^2$  [12],[14]

$$C = E_H\{\log_2 \det(\mathbf{I}_m + \frac{\rho}{T} \mathbf{W})\} \quad (1.8)$$

Where  $E_H \{.\}$  denote the expectation over  $H$ ,  $m = \min(T_x, R_x)$ ,  $\mathbf{I}_m$  is the  $m \times m$  identity matrix,  $\rho$  is the average signal-to-noise ratio (SNR) per receive antenna, and the  $m \times m$  matrix  $\mathbf{W}$  is given by

$$\mathbf{W} = \begin{cases} \mathbf{H}\mathbf{H}^H, & R_x \leq T_x \\ \mathbf{H}^H\mathbf{H}, & T_x < R_x \end{cases} \quad (1.9)$$

Where the operator  $\mathbf{H}^H$  indicates the hermitian of the matrix  $\mathbf{H}$ . Using singular value decomposition (SVD), equation 1.8 can be re written as

$$C = E_H\{\sum_i^k \log_2 (I_m + \frac{\rho}{T_x} \lambda_i)\} \quad (1.10)$$

Where  $(k \leq m)$  is the rank of  $\mathbf{H}$ , and  $\lambda_i$  ( $i = 1, 2, \dots, k$ ) denotes the positive eigenvalues of  $\mathbf{W}$ .

### 1.5 Multi-Hop Amplify and Forward MIMO System Mutual Information

In Multi-Hop Relay Networks a network have been considered where the source cannot reach the destination via a direct link. Wireless relaying networks have recently been given considerable attention due to their many advantages. Apart from increasing the range, relaying networks can also achieve better diversity by using cooperative transmission from the source and relays terminals [19]. The relaying terminals forward the information from the source to the destination mainly using the two well-known methods: Amplify Forward and Delay Forward. MIMO relays aim to provide improved system capacity, increases in range

and better diversity as traditional communication system. The source signal propagates through multiple layers of relays and the propagation paths are assumed to have equal lengths. Using large random matrix theory the ergodic capacity results of some particular relaying schemes have been established for large networks [20]. Recently, the study has been extended to multiple layers of relays and with multiple source-destination pairs. The scaling of capacity for relay channel is  $N/2 \log K$  with single source and destination. In this chapter AF MIMO relay channel without line of sight (LOS) path between source and destination is considered. Large networks have shown increase in ergodic capacity by using large random matrix theory. The main concern is to calculate the mean of MI of channel. In most case the Capacity calculation have been done on the basis of deterministic Eigen value decomposition using  $\log \det$ -formula. The analytical expression of mutual information MIMO relay channel inspired by MI of single hop MIMO relay channel under statistical channel state information.

### 1.5.1 MUTUAL INFORMATION FOR CHANNEL

In this thesis work the two-hop MIMO relay channel is considered having a source, a relay and a destination terminal equipped with  $M_s$ ,  $M_r$  and  $M_d$  numbers of antennas. The whole process of transmission is implemented in two phases. In first phase source terminal transmit a vector signal to relay this is single hop and in second phase relay terminal transmit the received vector signal after modifying the parameter to destination terminal that is second hop. First phase channel matrix is represented by  $H_0 \in \mathbb{C}^{M_r \times M_s}$  and the second phase channel matrix represented by the  $H_1 \in \mathbb{C}^{M_d \times M_r}$ . and the forward matrix of relay channel represented by  $G \in \mathbb{C}^{M_r \times M_r}$ . The received signal at the Relay terminal ( $Y_0$ ) and destination terminal ( $Y_1$ ) is given by

$$Y_0 = H_0 X + n_0 \quad (1.11)$$

$$Y_1 = H_1 G Y_0 + n_1 \quad (1.12)$$

Where  $n_0$  and  $n_1$  are the relay and destination noise vector. The  $X \in \mathbb{C}^{M_s}$  is a transmitted vector with zero mean and covariance  $E[xx^\dagger] = \rho/n$  and the noise at relay and destination assumed to be additive white Gaussian with unit variance. The resultant output can be written as

$$Y = H_1 G(H_0 X + n_0) + n_1 \quad (1.13)$$

Where  $H_0$  and  $H_1$  will be assumed to be zero mean circulate symmetric complex Gaussian random variable with covariance matrix as defined under statically CSI by Kronecker model [21] and  $G = \sqrt{\beta/n}$  is forward matrix where  $\beta$  overall power gain of relay channel that depend on the distance and path loss between source and relay terminal.

$$H_l = R_l^{1/2} W_l T_l^{1/2} \text{ for } l = 0, 1 \quad (1.14)$$

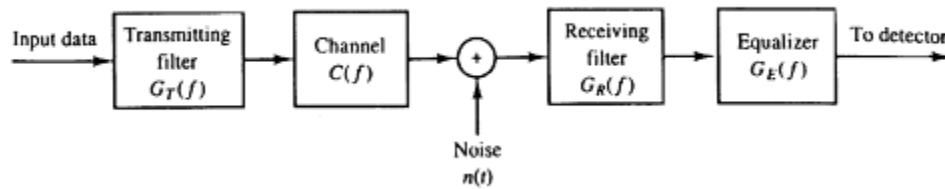
Where  $W_l$  denote the channel matrix with i.i.d complex Gaussian random entries. The  $R_l \in \mathbb{C}^{M \times M}$  and  $T_l \in \mathbb{C}^{M \times M}$  are the receiver and transmitter spatial correlation matrix. If  $H_l$  is perfectly known at the destination terminal then the mutual information between  $X$  and  $Y$  will be given by

$$I(X, Y) = \log \left( \det \left( I + \frac{\beta_i}{n_i} H_1 H_1^\dagger + \frac{\beta_s}{n_t} H_1 H_2 H_2^\dagger H_1^\dagger \right) \right) - \log \left( \det \left( I + \frac{\beta_i}{n_i} H_1 H_1^\dagger \right) \right) \quad (1.15)$$

For the simulation  $H_l$  and  $I$  are generated using random variable.

### 1.6 MIMO Detection Technique

There are numerous detection techniques available with combination of linear and non-linear detectors; the generalized block diagram of MIMO detection technique is shown in Figure 1.5. The most common detection techniques are ZF, MMSE and ML detection technique [22]. These detectors were compared using different antenna configurations and modulation techniques.



**Figure 1.5. Block Diagram of System with Equalizer**

### 1.6.1 Zero Forcing (ZF)

The ZF is a linear estimation technique, which inverse the frequency response of received signal, the inverse is taken for the restoration of signal after the channel. The estimation of strongest transmitted signal is obtained by nulling out the weaker transmit signal. The strongest signal has been subtracted from received signal and proceeds to decode strong signal from the remaining transmitted signal [23]. ZF equalizer ignores the additive noise and may significantly amplify noise for channel. The major advantages of ZF linear equalizer is that it simply eliminates ISI by forcing the overall pulse, which is the convolution of the channel and the equalizer to make a unit-impulse response. Although the noise power and covariance function does not need to be estimated in ZF even than its perform poor than that of ML [24],[25]. It is because ZF equalizer will further enhance the noise in channel where already deep fading is affecting the channel, which degrades its performance than ML. For a channel with frequency response  $C(f)$  the zero forcing equalizer  $E(f)$  is constructed by combining the equalizer and channel which gives a flat frequency and linear phase if and only if  $C(f) \times E(f) = 1$ . The channel response is  $H(s)$  then the input signal is multiplied by the reciprocal of  $H(s)$ . The basic Zero force equalizer of 2x2 MIMO channel can be modeled by taking received signal  $y_1$  during first slot at receiver antenna as:

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (1.16)$$

The received signal  $y_2$  at the second slot receiver antenna is:

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (1.17)$$

Where  $i = 1,2$  in  $x_i$  is the transmitted symbol and  $i = 1,2$  and  $j = 1,2$  in  $h_{i,j}$  is correlated matrix of fading channel, with  $j$  represented transmitted antenna and  $i$  represented receiver antenna,  $n_1$  and  $n_2$  is the noise of first and second receiver antenna. Equivalently, the ZF equalizer is given by the pseudo inverse [26] of  $H$ , i.e.

$$W_{ZF} = (H^H H)^{-1} H^H \quad (1.18)$$

Where  $W_{ZF}$  is equalization matrix and  $H$  is a channel matrix. ZF detector will produce favorable result only if  $W_{ZF} X H = 1$  conditionis satisfied, and diagonal element of pseudo

inverse matrix should not be zero. Assuming  $M_R \geq M_T$  and  $H$  has full rank, the result of ZF equalization before quantization is written as

$$y_{ZF} = (H^H H)^{-1} H^H y \quad (1.19)$$

### 1.6.2 Maximum Likelihood (ML)

The Maximum Likelihood (ML) detection is a scheme well known to be very robust and well suited for practical implementation whereas linear receiver suffers from numerical complexity [26]. ML detector is the optimal receiver in terms of bit error rate (detector performance) but it is a nonlinear detector with a high complexity. The ML detection rule is given by

$$R = \operatorname{argmin} |y - H \widehat{x}_i|^2 \quad (1.20)$$

Where element of  $R$  are optimized variable and  $|y - H \widehat{x}_i|^2$  is objective function. After minimized  $R$  are, we get

$$R = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \end{bmatrix} \right\|^2 \quad (1.21)$$

In case of BPSK, the possible value of  $\widehat{x}_1$  and  $\widehat{x}_2$  is +1 or -1, to find the minima from the all combination of  $\widehat{x}_1$  and  $\widehat{x}_2$ . The estimation of the transmitted symbol is done on the basis of the following values

$$R_{+1,+1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ +1 \end{bmatrix} \right\|^2 \quad (1.22)$$

$$R_{+1,-1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ -1 \end{bmatrix} \right\|^2 \quad (1.23)$$

$$R_{-1,+1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} -1 \\ +1 \end{bmatrix} \right\|^2 \quad (1.24)$$

$$R_{-1,-1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\|^2 \quad (1.25)$$

The estimate of the transmit symbol is chosen based on the minimum value from the above four values with  $R_{+1,+1} \Rightarrow [1 \ 1]$  ,  $R_{+1,-1} \Rightarrow [1 \ 0]$ ,  $R_{-1,+1} \Rightarrow [0 \ 1]$  and  $R_{-1,-1} \Rightarrow [0 \ 0]$ .

### 1.6.3 Minimum Mean Square Error-Estimator (MMSE)

Minimum mean square error equalizer minimizes the mean –square error between the output of the equalizer and the transmitted symbol, which is a stochastic gradient algorithm with low complexity. Unlike a ZF equalizer, an MMSE equalizer maximizes the signal–to–distortion ratio by penalizing both residual ISI and noise enhancement. Instead of removing ISI completely, an MMSE equalizer allows some residual ISI to minimize the overall distortion. Compared with a ZF equalizer, an MMSE equalizer is much more robust in presence of deepest channel nulls and noise. Most of the finite tap equalizers are designed to minimize the mean square error performance metric but MMSE directly minimizes the bit error rate. The channel model for MMSE is same as ZF [27],[28]. The MMSE equalization is

$$W_{MMSE} = \arg. \min_G E_{x,n} [\|x - x^\wedge\|^2] \quad (1.26)$$

By using orthogonality principle the  $W_{MMSE}$  is written as

$$W_{MMSE} = E_{x,n} [xy^H] (E_{x,n} [yy^H])^{-1} = H^H (HH^H + n_o I_n)^{-1} \quad (1.27)$$

$$W_{MMSE} = H^H \left\{ n_o \left( \frac{1}{n_o} HH^H + I_n \right) \right\}^{-1} I_n = H^H (HH^H + n_o I_n)^{-1} \quad (1.28)$$

Where  $W_{MMSE}$  is equalization matrix,  $H$  channel correlated matrix and  $n$  is channel noise.

$$y_{MMSE} = H^H (HH^H + n_o I_n)^{-1} y \quad (1.29)$$

From the equation 1.29 it has been shown that only factor  $n_o I_n$  is found to be additional than that of ZF in MMSE. This factor is the instrumental force in improving the performance of MMSE equalizer over ZF equalizer.

## 1.7 Objectives of Thesis

Primary objectives of this thesis are:

- ❖ To study and simulate the MIMO System Capacity and Bit Error Rate.
- ❖ To study and analyse the performance of Capacity gain of MIMO system in correlation Rician fading.
- ❖ To optimize the Mutual information of MIMO system improves by Amplify and Forward techniques.
- ❖ To study and analyse the impact of Bit Error Rate of MIMO system by linear and non-linear detection techniques and compare it with IEEE802.11n standard protocol.

## 1.8 Methodology of Thesis

In this thesis, the overview of MIMO system as well as the capacity and bit-error-rate has been thoroughly investigated. Further, the techniques have been described with mathematical modelling and closed form expressions of mutual cumulant of Mutual Information for Rayleigh Channels then the Capacity for Rician channels has been described. Capacity under Rayleigh fading channel has been experimentally computed using Amplify and Forward technique, cumulant function of Mutual Information of MIMO over Rayleigh channels have been presented by using integral identity and replica method. Then linear and non-linear detection based techniques are utilized for calculating and analysis of bit error rate. Finally the result obtained from above technique for BER has been compared with IEEE802.11n standard protocol as benchmark.

## 1.9 Organization of Thesis

The thesis is organized into six chapters. Chapter 1: introduces the MIMO system MI and BER introduction with fading channel. It provides a brief background into methods used throughout the thesis (MIMO models, ML, MMSE and ZF detection technique). Chapter 2: discusses the literature survey of MIMO channel with MI and BER. In Chapter 3. The overview of MIMO system model has been discussed. Chapter 4: discusses the analysis and simulation of MIMO Mutual information with cumulant function under Rayleigh fading and Capacity for correlated Rician fading channel. Chapter 5: discusses the analysis and simulation of bit-error-rate for MIMO (BPSK) using linear and non-linear detection technique. Chapter 6: provides a Conclusion and gives recommendations for future study.

# CHAPTER 2

## Literature Survey

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### Literature Review:

#### 2.1 MIMO Capacity

H. Foschini et al. [12] and Teletar [14] had done intense research in the area of MIMO systems.

H.Foschini et al. [12] analyzed the information-theoretic capacity of a multiple-antenna point to point wireless system in a narrow-band slowly Rayleigh-fading environment. They assume independent and identically distributed (i.i.d.) fading at different antenna elements with  $T$  transmit and  $R$  receiver antennas; the system is described by the matrix equation

$$Y = \sqrt{\frac{E_S}{T}} HX + n \quad (2.1)$$

Where  $H_{R \times T}$  channel matrix with components is modeled as *i.i.d* zero mean unit variance circularly symmetric complex Gaussian random variables (ZMCSCG). The capacity for this system is shown to be

$$C = E_H \{ \log_2 \det(\mathbf{I}_m + \frac{\rho}{T} H H^H) \} \quad (2.2)$$

The matrix  $H$  solved using singular value decomposition (SVD), can be given as

$$C = E_H \{ \sum_i^K \log_2 (\mathbf{I}_m + \frac{\rho}{T} \lambda_i) \} \quad (2.3)$$

Where  $k, (k \leq m)$  the rank of  $H$  and  $\lambda_i (i = 1, 2, \dots, k)$  denotes the positive eigenvalues of  $H \times H^H$ . Foschini and Gans shown that the capacity  $C$  grows linearly with  $\min(T, R)$  for a given fixed transmitter power and bandwidth. In other words, without increasing the transmitted power or bandwidth the capacity of the wireless channel can be increased by simply increasing the number of transmitter and receiver antennas. This is an enormous improvement compared to a logarithmic increase in more traditional systems utilizing receiver diversity or no diversity.

The channel state information is available only at the receiver and transmitter shown that for *i.i.d* slowly Rayleigh fading channels with  $T$  transmitter and  $R$  receiver antennas,

- Capacity  $C$  grows linearly with  $\min(T, R)$  for a given fixed transmitter power and bandwidth.
- For  $T = 1$ , Capacity increases logarithmically with the increase in the number of receive antennas  $R$ .
- For  $R = 1$ , Capacity does not increase at all with the increase in the number of transmitting antenna  $T$ .

Teletar et al. [14] proposed that when the channel parameters are known at the transmitter, i.e., if the channel state information (CSI) is available at the transmitter, the capacity given by (2.4) can be increased by assigning the transmitted power to various antennas according to the “water-filling” algorithm.

$$C = E_H\{\sum_{i=1}^k \log_2(\mu\lambda_i)\} \quad (2.4)$$

Where  $\mu$  is chosen to satisfy:

$$C = E_H\{\sum_{i=1}^k (\mu - \lambda_i)^+\} \quad (2.5)$$

and ‘+’ denotes only those terms has been considered which are positive.

The effects of fading correlations in multi-element antenna (MEA) communication systems have been investigated by Da-Shan et al. [18]. They characterize the fading correlation for narrow band Rayleigh fading. They modeled correlated fading using one-ring scattering model as

$$vec(H) = R^{\frac{1}{2}}vec(H_\omega) \quad (2.6)$$

Where  $vec(H)$  is the vector form of the  $H$  matrix and  $R = cov(vec(H))$  and  $H_\omega$  represents independent and identically distributed (*i.i.d*) Rayleigh fading channel. They have shown that the effective degree of freedom i.e., the number of independent paths reduces as the correlation increases thus leading to the reduction in the system capacity.

While Da-Shan et al. [18] contribution provides a useful insight; the results are limited to the case of one end (i.e., either transmitter or receiver) correlation only. Analysis accounts for both transmitter and receiver correlations using the eigenvalue decomposition technique, and modeled the MIMO correlated fading as

$$H_{corr} = R_r^{1/2} H_\omega R_t^{1/2} \quad (2.7)$$

Where  $R_t$  and  $R_r$  are the correlation matrix at the transmitter and at the receiver side, respectively given by[13]

The MIMO channel capacity in correlated channels using the uniform and exponential correlation matrix model has been investigated by Lokya et al. [29]. Using uniform correlation matrix model, the correlation matrix  $R$  is defined as.

$$r_{ij} = \begin{cases} r, & i \neq j \\ 1, & i = j \end{cases}, |r| \leq 1 \quad (2.8)$$

While using exponential correlation matrix model, the correlation matrix  $R$  is defined as

$$r_{ij} = \begin{cases} r^{j-1}, & i \leq j \\ r^*_{ji}, & i > j \end{cases}, |r| \leq 1 \quad (2.9)$$

Where  $r$  is the correlated fading parameter between any two adjacent antennas, and ‘\*’ denotes the complex conjugate. Using the Jensen’s inequality and approximations the capacity of  $n \times n$  MIMO Rayleigh fading channel in the presence of correlation has found out to be

$$C \approx n \cdot \log_2(1 + \frac{\rho}{n}(1 - |r|^2)) \quad (2.10)$$

Then two models were configured and shown that the exponential model predicts better MIMO performance. The other findings were as follows

- As the correlation increases, the capacity decreases. In other words, the increase in correlation is equivalent to the decrease in SNR.
- For the exponential correlation model, the MIMO capacity decrease significantly for  $r > 0.6$  which is in accordance with the measurement of MIMO channels.

Farrokh et al. [30] modeled the Rician fading channel as

$$H = aH^{sp} + bH^{sc} \quad (2.11)$$

Where the specular and scattered components of  $H$  are denoted by superscripts,  $a > 0$ ,  $b > 0$  and  $a^2 + b^2 = 1$ . The Rician factor,  $K$  is defined as  $a^2/b^2$ . Thus, the above  $H$  matrix can be written as

$$H = \sqrt{\frac{1}{1+K}} H_S + \sqrt{\frac{K}{1+K}} H_d \quad (2.12)$$

The upper bound on the average Rician channel capacity for  $n \times n$  MIMO system with CSI available at the receiver only was derived by Ayadi's et al. [31]. Ayadi represented that a limit of this capacity is given by the sum of the capacities corresponding to the LOS and Rayleigh components when they are considered separately. The upper bound is given as

$$C_{Rk} \leq C_{Los} + C_{Ray} \quad (2.13)$$

Where

$$C_{LOS} = \log_2 \left\{ \det \left[ \left( I_n + \left( \frac{\rho}{n} \right) a^2 H_{LOS}^H H_{LOS} \right) \right] \right\} \quad (2.14)$$

$$C_{Ray} = \log_2 \left\{ \det \left[ \left( I_n + \left( \frac{\rho}{n} \right) a^2 H_{Ray}^H H_{Ray} \right) \right] \right\} \quad (2.15)$$

Two cases have been considered, one when the antenna elements are uncorrelated and the other when they are perfectly correlated. It has been found that:

- Average upper bound on the Rician channel capacity is almost achieved for small values of SNR.
- For low values of Rician factor, the average upper bound seems to be less tight than the one obtained in the case of high values of Rician factor.

A brief overview of MIMO wireless technology covering channel models, capacity, coding, receiver design, performance limits has been presented by A. J. Paulraj et al. [32]. Space time code and receiver design with particular focus on iterative decoding and sphere decoding allowing low complexity implementation. The system design implications of fundamental performance tradeoffs (such as rate versus PER versus SNR) is required for better understanding. If the product of bandwidth (hertz) and spectral efficiency (measured in bits per second per hertz) is equal to 109 then the data rate up to 1-Gb/s with a single-transmit single-receive antenna wireless system was achieved by A. J. Paulraj et al. [32].

Different metrics should be applied if the underlying MIMO channel supports predominantly beam forming, spatial multiplexing or diversity. The number of envisaged antennas plays an important role. A good channel model is a model that renders correctly the relevant aspects of the MIMO system to be deployed. If no specific channel property is in focus, a good MIMO channel model reflects the spatial structure of the channel in general, that determines the benefits of MIMO. In an indoor environment, Huseyin et al. [33] assessed the analytical

MIMO models by three different metrics, viz. double directional angular power spectrum, average mutual information, and the Diversity Measure. The Weichselberger model performs best with respect to the analyzed metrics, even though it is inaccurate for joint angular power spectrum (APS) and Diversity Measure in case of large antenna numbers and the Kronecker model applicable for limited numbers of antennas.

The minimum energy per information bit is the same for both channels while their wideband slopes differs significantly. The interference degrades the capacity by increasing the required minimum energy per information bit and reducing the wideband slope. Exact expressions for the minimum energy per information bit  $E_B/N_{Omin}$  and for both channels wideband slope  $S_o$  were derived by Caijun Zhong et al. [34], which provide a much efficient way to evaluate the ergodic capacity of the system at low SNR as compared to the Monte-Carlo simulation method. For both MIMO Rician fading and Rayleigh-product channels interference degrades the capacity performance by increasing  $E_B/N_{Omin}$  and reducing  $S_o$ . The capacity of MIMO systems with  $N_r$  receiving antennas in the presence of a single interferer is no worse than that of MIMO systems with  $N_r - 1$  receiving antennas in interference-free environment, regardless of the interference power level for MIMO Rician channels the structure of mean matrix and Rician factor  $k$  will affect the ergodic capacity. In the MISO case high value of  $k$  does not necessarily lead to an improvement on the capacity. In fact the Rician factor  $k$  which affects the capacity is closely related to the interference level  $\rho_I$ . In the presence of a single interferer, the capacity of MIMO Rayleigh-product channels upper bounded by the capacity of a MIMO Rayleigh fading channel with the same number of transmit and receive antennas.

## 2.2 Amplify-Forward MIMO Mutual Information

The ergodic mutual information is a representative figure for ergodic channels but it is insufficient to properly describe the capacity versus outage tradeoff in non-ergodic systems. E. Riegler et al. [35] analyzed that the mutual information probability distribution approaches the Gaussian distribution asymptotically (as the number of transmit, interfering, and receive antennas grow large, with their ratios approaching finite constants).

MIMO can effectively increase channel capacity without aggrandize the transmitting power or signal bandwidth (higher spectral efficiency) by using uniform angular distribution and the correlation matrix. The general capacity formula suggested that the augment of correlation can contribute to the decrease of SNR, and the radius of circular receive antenna array or angle spread is the dominating factor for determining channel capacity. The capacity of MIMO system channel mainly depends on the correlation between its sub-channels. When the correlation parameter is zero, the capacity of channel in MIMO system reaches its maximum value. Z. Xinyu et al. [36] analyze the capacity of the channel in MIMO system through the uniform angular energy distribution and correlation matrix that give the higher value of correlation between the channels and the smaller value of SNR. The expansion of radius and angle of the receiving antenna array is the key point of capacity of channel in MIMO system.

B. Holter [17] analyzed that MIMO systems channel and found that MIMO offer a significant capacity gain over a traditional single-input single-output (SISO) channel. There is a linear increase in spectral efficiency compared to a logarithmic increase in more traditional systems which utilizing receiver diversity or no diversity. The high spectral efficiencies attained by a MIMO system are enabled in a rich scattering environment, the signals from each individual transmitter appear highly uncorrelated at each of the receive antennas. When the transmitted signals are conveyed through uncorrelated channels through the transmitter and receiver, the signals corresponding to each of the individual transmit antennas have attained different spatial signatures. The receiver can use these differences in spatial signature simultaneously and at the same frequency separate the signals that originated from different transmit antennas.

In MIMO systems, it is difficult to gain the ideal ergodic capacity for the time-varying fading channels except using adaptive modulation and power water-filling in space domains over every MIMO sub-channel. A novel improved Rate Quantization (IRQ) algorithm using water-filling theory for power allocation and adaptive modulation is proposed with low complexity to analyze the capacity by L. Ren et al. [37]. High transmit power utilization and greater spectral efficiency was obtained by using IRQ. Under average transmit power constraint, The novel IRQ algorithm for water-filling proposed by L. Ren et al. [37] transmit

power among multiple transmit antennas, and use low complexity rate quantization scheme for adaptive QAM modulation and mobile cellular communication system to avoid the m-channel interference to a certain extent.

G. Taricco et al. [38] provided an asymptotic method to derive a very good approximation to the ergodic capacity of a MIMO communication system that was affected by additive noise, interference, Rician fading and spatial correlation (Kronecker model). The ergodic capacity are highly suboptimal if the interference power level is relatively high compared to the signal power level. The main reasons for the difficulty in analyzing MIMO system is the large number of parameters involved, which can affect the system performance in an unpredictable way. Therefore, the proposed algorithms to calculate the mutual information and the capacity give a valuable tool for the MIMO system design and pave the way to parameter optimization in the complex distributed MIMO environment.

For multiple-antenna arrays communication systems, it is important to analyze the capacities of such systems in realistic situations, which may include spatially correlated channels and correlated noise, as well as correlated interferers with known channel at the receiver, here A. L. Moustakas et al. [39] provided analytic expressions for the statistics, i.e. the moments of the distribution, of the mutual information of multiple-antenna systems with arbitrary correlations, interferers and noise. The analytic approach provides the framework and a simple tool to accurately analyze the statistics of throughput for even small arrays in the presence of arbitrary channel and noise correlations as well as interferers with known channel at the receiver. It can be used in multiuser detection and dirty paper coding, where at each stage of detection the un-decoded users are treated as interferers.

S. Jin et al. [40] derive an expression for the exact ergodic capacity, closed-form expressions for the high SNR regime, and tight closed-form upper and lower bounds. The results are made possible by employing recent tools from finite-dimensional random matrix theory, which are used to derive new closed-form expressions for various statistical properties of the equivalent AF MIMO dual-hop relay channel, such as the distribution of an unordered eigenvalue and certain random determinant properties. The analytical results were made

possible by first employing random matrix theory techniques to derive new expressions for the probability density function of an unordered eigenvalue.

The Cumulant of the mutual information of the flat Rayleigh fading amplify-and-forward MIMO relay channel without direct link between source and destination are derived by A. Wittneben et al. [39] for large array limit. Replica trick use for analysis that covers both spatially independent and correlated fading in the first and the second hop, while beam forming at all terminals is restricted to deterministic weight matrix. In case of ordinary point-to-point MIMO channels, the observation result shown that all cumulant moments of order larger than two vanish as the antenna array sizes grow large and concluded that the respective mutual information is Gaussian distributed.

A new AF relaying scheme has been developed by F. Youhua et al. [41] under a total relay power constraint with the objective of maximizing the information rate of two-hop MIMO networks. The constrained optimization problem is in general not convex so that arithmetic means inequality leading to a nearly optimal relaying solution. Owing to the cooperative allocation of the total relay power, the proposed relaying scheme has a much better spectral efficiency than some of the existing methods when the relay power is moderate to high as confirmed by Monte-Carlo simulations.

The relay channel is a basic model for multiuser communications in wireless networks. Loosely speaking, parameter captures the cooperation between the source node and the relay node and leads to solving the maximization problem using convex programming. It is somewhat surprising that the upper bound can meet the lower bound under certain conditions. In particular case for identify sufficient conditions to achieve the ergodic capacity when all nodes have the same number of antennas, B. Wang et al. [42] analyzed that this can be achieved by using the source node and relay node as a function of virtual transmit antenna array when the relay node is located close to the source node.

S. Jin et al. [43] gives new closed-form upper and lower bounds that present the ergodic capacity of dual-hop AF MIMO relay channels under the assumption that CSI is available only at the destination terminal but not for the exact characterization of ergodic capacity on the relays or the source terminal. The analytical results were made possible by first

employing random matrix theory techniques to derive new expressions for the probability density function of an unordered eigenvalue, as well as random determinant results for certain product of finite-dimensional independent complex Gaussian matrix.

The multiuser multi-hop MIMO relay communication systems with correlated MIMO fading channels have been taken into the consideration for the practical scenario shown by Y. Rong et al. [44]. The channel fading is fast and thus the instantaneous channel state information (CSI) is only available at the destination node, but unknown at all users and all relay nodes. The structure of the optimal user pre-coding matrix and relay amplifying matrix is the main concern that maximizes the users-destination ergodic mutual information. Compared with existing works multiuser scenario MIMO relays with a finite dimension with noise vector at each relay node take into account.

N. Rajatheva et al. [45] investigated the performance of MIMO AF relay network over asymmetric Rayleigh and Rician channels. By considering general correlation matrix structure they derived the closed form expressions and analyzed the system in high SNR. The results shown that antenna correlation degrades the performance; however it does not affect the diversity gain. Moreover, results can be effectively used to quantify the performance loss due to antenna correlation of a relay network with asymmetric Rayleigh and Rician fading environment.

The performance of multi-antenna Amplify-and-Forward (AF) relays for a future cellular system where the destination and the source form a MIMO system analyzed by two scenarios, 1<sup>st</sup> the multi-relay scenario i.e. distributed relays in which each terminal equipped with multiple antenna. 2<sup>nd</sup> one-relay scenario i.e. a single relay in a sector where each relay is equipped with at least one antenna. The optimal filter coefficients applied at the relays are derived and recommended for both cases by M. Qingyu et al. [46]. The throughput of the single link showing the AF relay matched filter based performs best in the multi-relay scenario. On other hand the relay based singular value decomposition (SVD) is more suitable for the one-relay scenario.

The practical scenario for multiuser multi-hop multiple-input multiple-output (MIMO) relay systems with correlated MIMO fading channels considered where the channel fading is fast

and thus the instantaneous channel state information (CSI) is only available at the destination node, but unknown at all users and all relay nodes. Y. Rong et al. [47] derive the structure of the optimal user pre-coding matrix and relay amplifying matrix that maximizes the users-destination mutual information. The optimal structure of user pre-coding matrix and relay amplifying matrix has been proposed to optimize the mutual information of multiuser multi-hop MIMO relay system. The proposed power loading algorithm only has a small performance degradation compared with the optimal user and relay design using the instantaneous CSI, but greatly reduced computational complexity and signaling overhead.

### **2.3 Signal Detection in MIMO System**

R. D. Murch et al. [26] analyzed the performance of MLD over flat fading channels in a wireless MIMO antenna system. A tight union bound with an asymptotic form on the probability of SER for MIMO MLD systems with two-dimensional signal constellations has been introduced. Using this analytic bound, performance of the MIMO antenna system is demonstrated quantitatively with respect to channel estimation, constellation size, and antenna configuration. It is shown that a very high data rate can be achieved with little SNR penalty when the number of receive antennas becomes large then the diversity order of MLD is equal to number of receiver antennas.

In the presence of multipath fading the problem of blind estimation of multiple digital communication channels using an antenna array is studied. A fast sequential-estimation algorithm for separating multiuser signals based on the geometric observation develop by M. Torlak et al. [24]. When the signals are constrained to a finite alphabet then visualization of geometric problem can be exploited to sequentially extract the digital communication channel. The data processed with known pseudo-inverse processing (ZF). One of the well-known disadvantages of a zero-forcing equalizer is that it enhances the noise.

The comparative study of Detection algorithms for single user wireless communication using multiple antennas at both the transmitter and receiver in a Rayleigh fading environment has been discussed using the MMSE detector and its BLAST versions. The system includes  $N$  transmitting antenna and  $M$  receiving antennas ( $N \leq M$ ). The effects of coding and error propagation on algorithm performance are investigated for BPSK and QAM modulation scheme. Z. Catherine et al. [27] analyzed that if no coding is implemented, the performance

of the MMSE detector be improved by implementing its D-BLAST version and further improved by implementing its V-BLAST version. If repetition coding used, the BER versus SNR performance of the MMSE detector is better than of its BLAST versions.

The performance of the MIMO systems using zero-forcing (ZF) detectors is studied over a Rician fading channel. Wishart distribution is used to derive the bit error rate (BER) of the system. A closed-form expression of BER is derived by Xu Rongtao et al. [48]. They conclude that the performance of the MIMO system over a Rician fading channel degrades compared to that over a Rayleigh fading channel.

A new orthogonal coded Multi-Input-Multi-Output (MIMO) system using Walsh codes is proposed for Spatial Multiplexing system that offers equivalent bit error rate performance as compared to the conventional un-coded MIMO system, while increasing data transmission rate by two folds. A ML detector offers high computational complexity; the linear zero forcing (ZF) and minimum mean square error (MMSE) detectors have been presented for comparative study. I. T. Tasneem et al. [49] proposed that the system offers promising results as compared to the conventional MIMO system. Optimal ML, Linear ZF and MMSE detectors for the proposed WC-MIMO system were compared with the encoded MIMO detectors and shown that the data rate of the proposed WC-MIMO system increase while keeping the BER at par with the existing MIMO system.

X. Zhang et al. [28] present that employing multiple antennas at both the transmitter and receiver ends offers a promising channel capacity. Equalizers that can deliver better theoretical capacity performance usually incur higher implementation cost. The fact that ZF filters incur much simpler implementation than MMSE that's why ZF equalizer represents a very promising and feasible design tradeoff between performance and cost. It is worth noting that the tradeoff will become more important with the presence of ISI since the implementation cost will become more expensive.

ZF detection is a simple and effective technique for retrieving multiple transmitted data streams at the receiver in MIMO wireless antenna systems. However the detection requires accurate channel state information (CSI) which may not be available in practice. C. Wang et al. [25] demonstrated that the impact of imperfect CSI on the performance of MIMO ZF

receiver over uncorrelated Rayleigh flat fading channel is analytically assessed, also the tight approximation of the post processing SNR distribution is derived and system performance in terms of outage probability and the BER of MPSK modulation is obtained in closed-form.

The necessary accuracy for channel estimation is to achieve the desired BER performance in a MIMO channel under certain constraints like number of antennas, modulation and channel estimation scheme. In Rician channels a LOS increases the capacity, while the BER is nearly unaffected by the additional signal power. This is explained because the additional correlated signal power from the LOS improves the best sub-channel and leaves the poorest sub-channels which cause most of the bit errors nearly unchanged. E. Jorswieck et al. [50] have shown that the estimation of the MIMO channel capacity is not affected by channel estimation errors of the previously mentioned methods.

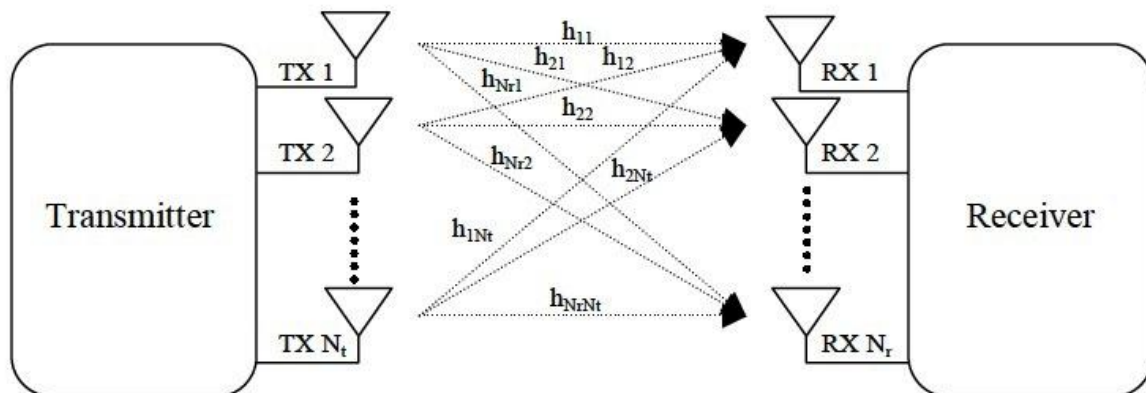
A detection algorithm for MIMO system with non-binary modulation scheme proposed by D. Yuehua et al. [51]. The method is based on rectangular searching that restricts the searching area to a certain rectangle when searching the candidate symbols for each symbol. It has considerably reduced computational complexity compared to ML (Maximum Likelihood) algorithm because only a small part of candidate constellation points are located in the searching rectangle. Simulation results have shown that the BER performance of the proposed method outperforms the ML (Maximum Likelihood) algorithm.

Combined multi-stage minimum mean-square error (MSMMSE-ML MUD) is applied in the underdetermined MIMO systems communicating over Gaussian fading channels. For MIMO system with  $M$  transmitting antennas, in multi stage detection first MMSE detection is operated for the first  $(M-T)$  users after that ML detection is executed for the last  $T$  users, where the parameter  $T$  is adopted to adjust the detection complexity of the last users. Simulation results show that the proposed method has a higher BER performance than the MS- MMSE operated in the system. K. Liu et al. [52] analyzed that it is better to use additional computation of modified scheme as compared to improvement achieved in performance. Therefore, the proposed scheme can be efficiently used for a practical underdetermined MIMO receiver implementation to get better performance of system.

## CHAPTER 3

### MIMO System

MIMO exploits multipath, traditionally a pitfall in wireless communications to enhance rather than degrade the signal. MIMO systems consist of multiple transmitters and multiple receivers; the statistical nature of wireless communications and its various forms of appropriate description confronts in the case of MIMO systems with an even more difficult problem. To be able to do a stringent analysis of MIMO systems and to make statements about performance gains, adequate description of the underlying channel and its properties in terms of fading, time variance, linearity and correlation are required. For MIMO systems to be most effective, a rich multipath scattering environment is needed to create independent propagation channels. It is the rich scattering in the propagation channel, which offers multiple parallel sub channels at the same frequency, therefore giving higher capacities over the same bandwidth. An adequate description of a MIMO channel is a research area in itself [16] and many publications have investigated the classification and description of MIMO transmission phenomena and their impact on MIMO performance parameters.



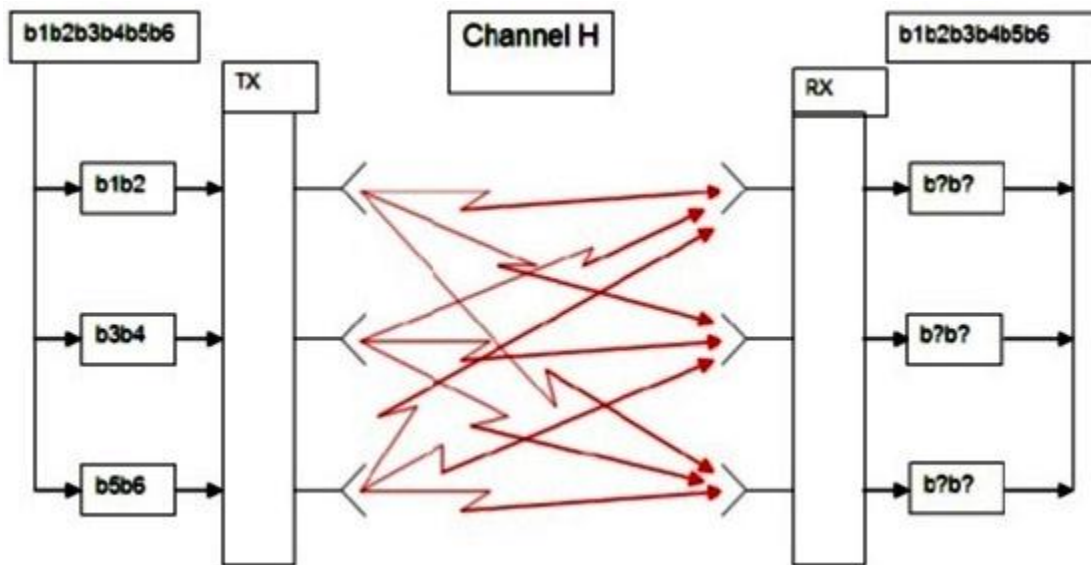
**Figure 3.1 N element MIMO systems**

The Figure 3.1 above shows MIMO transmission system consisting of N transmitting antennas and N receiving antennas. The channel 'H' is presumed to be a rich scattering environment. MIMO uses the multi antenna spatial diversity at both ends of the link, treating the multiplicity of the different scattering paths as separate parallel sub channels.

This chapter explains the MIMO transmission model, its analogies to a real communications environment, and the necessary assumptions to verify the choice of this representation. Furthermore, statistical properties have been investigated of this model and derive necessary properties for a basic information theoretic analysis of MIMO systems. In addition, fundamental issues of mutual information and bit error rate have been explained.

### 3.1 MIMO Transmission

A point-to-point link has established on a single user communication model where the transmitter is equipped with  $N_T$  antennas and the receiver employs  $N_R$  antennas, assuming that no Inter-Symbol-Interference (ISI) occurs. This implies that the bandwidth of the transmitted signal is very small and can be assumed flat frequency (narrowband assumption), so that each signal path can be represented by a complex-valued gain factor. For practical purposes, it is common to model the channel as flat frequency whenever the bandwidth of the system is smaller than the inverse of the delay spread of the channel; hence in case of wideband system operating where the delay spread is fairly small may sometimes be considered as frequency flat.



**Figure 3.2 Data transmission in MIMO systems**

The Figure 3.2 shown above demonstrates data transmission in MIMO system by using 6-bit data stream. This data stream is broken down (de-multiplexed) into  $N$  equal rate data streams,

where  $N$  is the number of transmitting antennas considered here. All bits are transmitted at the same time and at the same frequency, therefore they mix together in the channel. Each of the receive antennas picks up all of the transmitted signals superimposed upon one another. If the channel ' $H$ ' is a sufficiently rich scattering environment, each of the superimposed signals will have propagated over slightly different paths and differ the spatial signatures. The spatial signatures exist due to the spatial diversity at both ends of the link, and therefore create independent propagation channels. Each transmit receive antenna pair can be treated as parallel sub channels (i.e. a single-input single-output (SISO) channel). Since the data is being transmitted over parallel channels, one channel for each antenna pair, this increases the channel capacity in proportion to the number of transmitter-receiver pairs.

### 3.2 The MIMO Channel Correlation Matrix ( $H$ )

Since each of the receive antennas detects all of the transmitted signals, there are  $N_t \times N_r$  independent propagation paths, where  $N_t$  and  $N_r$  are transmitter and receiver antennas. This allows the channel to be represented as a  $N_t \times N_r$  matrix. Each of the elements of the channel matrix is an independent propagation path. Figure 3.1 and Figure 3.2 presents the path from transmitter antenna ' $i$ ' to receiver antennas. The transmitted and received signal can be represented as a vector.

Now let  $h_{i,j}$  be the complex-valued path gain from transmit antenna ' $j$ ' to receive antenna ' $i$ ' (the fading coefficient). If at a certain time instant the complex-valued signals  $(s_1, s_2, \dots, \dots, s_{n_T})$  are transmitted via the  $n_T$  antennas, respectively. Then the received signal at antenna ' $i$ ' can be expressed as

$$y_i = \sum_{j=1}^{n_T} h_{i,j} s_j + n_i \quad (3.1)$$

Where  $n_i$  is representing additive noise. This linear relation can easily be written in a matrix framework. Thus, let  $s$  be a vector of size  $n_T$  containing the transmitted values, and  $y$  be a vector of size  $n_R$  containing the received values, respectively. Certainly, we have  $s \in \mathbb{C}^{n_T}$  and  $y \in \mathbb{C}^{n_R}$ . The channel transfer matrix  $H$ [17] written as

$$H = \begin{pmatrix} h_{1,1} & \dots & h_{1,I} \\ \vdots & \ddots & \vdots \\ h_{R,1} & \dots & h_{R,I} \end{pmatrix}$$

Further, this can be written as

$$y = Hs + n \quad (3.2)$$

Where  $y$  is received vector,  $H$  is Channel Matrix,  $S$  is Transmitted signal vector and  $n$  is noise.

The relation shown above denoting transmission of symbol over one symbol interval, it is easily adapted to the case of several consecutive vectors  $\{s_1, s_2, \dots, s_l\}$  transmission (here,  $l$  denotes the total number of symbol intervals used for transmission) over the channel. Therefore, the transmitted vector, the received vector and the noise vectors of the matrix can be written as

$$Y = [y_1, y_2, y_3 \dots \dots \dots y_i] n = [n_1, n_2, n_3 \dots \dots \dots n_i] s = [s_1, s_2, s_3 \dots \dots \dots s_i] \quad (3.3)$$

The transmitted signals in the vector  $y$  are complex signals because of complex nature of  $H$  and  $S$  in equation 3.2. The complex form in each of the elements in the vectors represents the power of the signal and its phase delay. The complex form of the elements of the channel matrix ' $H$ ' represents the attenuation and phase delay associated with propagation path. The decoding of the received signal shown in the associated block transmission model will be given by:

$$\begin{pmatrix} y_{11} & \dots & y_{1,L} \\ \vdots & \ddots & \vdots \\ y_{n_R,1} & \dots & y_{n_R,L} \end{pmatrix} = \begin{pmatrix} h_{11} & \dots & h_{1,L} \\ \vdots & \ddots & \vdots \\ h_{n_R,1} & \dots & h_{n_R,L} \end{pmatrix} \begin{pmatrix} s_{11} & \dots & s_{1,L} \\ \vdots & \ddots & \vdots \\ s_{n_R,1} & \dots & s_{n_R,L} \end{pmatrix} + \begin{pmatrix} n_{11} & \dots & n_{1,L} \\ \vdots & \ddots & \vdots \\ n_{n_R,1} & \dots & n_{n_R,L} \end{pmatrix}$$

### 3.3 Noise

After stating the general linear input-output relation of the MIMO channel under general assumptions, the noise term of the transmission model (3.1) is elaborated here. In this thesis,

the noise vectors  $n_l$  will be assumed to be spatially white circular Gaussian random variables with zero-mean and variance  $\sigma^2$  per real and imaginary component. Thus,

$$n_l \sim \mathcal{N}_{\mathbb{C}}(0, 2\sigma_N^2 I) \quad (3.4)$$

Where  $N_{\mathbb{C}}$  stands for a complex-valued multivariate Gaussian probability density function. Because we will need an exact definition of the complex-valued multivariate Gaussian probability density function.

Let  $X \in \mathbb{C}^M$  then the probability density function (*pdf*)  $f_X(\xi)$  of  $X$  is given by

$$f_X(\xi) = \frac{1}{\det(\pi C_x)} \exp\left[-(\xi - \mu_x)^H C_x^{-1} (\xi - \mu_x)\right] \quad (3.5)$$

Where  $C_x \cong \mathbb{E}\{(\xi - \mu_x)(\xi - \mu_x)^H\}$  denote the covariance of  $X$ ,  $\mu_x = \mathbb{E}\{\xi\}$  denotes the mean vector of  $X$  and  $(.)^H$  stand for complex conjugate ( Hermitian transpose ). There are at least two strong reasons for making the Gaussian assumption of the noise. First, Gaussian distributions tend to yield mathematical expressions that are relatively easy to deal. Second, a Gaussian distribution of a disturbance term can often be motivated via the central limit theorem.

### 3.4. Fading Channel

The elements of the matrix  $H$  correspond to the complex valued channel gains between each transmitter and receiver antenna. For the purpose of assessing and predicting the performance of a communication system, it is necessary to postulate a statistical distribution of these elements [13]. This is also true to some degree for the design of well performing receivers, in the sense that knowledge of the statistical behavior of  $H$  could potentially be used to improve the performance of receivers. Throughout this thesis, the elements of the channel matrix  $H$  will be assumed as zero-mean complex-valued Gaussian random variables with unit variance. This assumption is made to model the fading effects induced by scattering in the absence of line-of-sight components. Consequently, the magnitudes of the channel gains  $h_{i,j}$  have a Rayleigh distribution, or equivalently  $|h_{i,j}|^2$  are exponentially distributed [22]. The presence of line-of-sight components can be modeled by letting  $h_{i,j}$  have a Gaussian distribution with a non-zero mean (this is also called Rician fading) [32].

The elements of  $H$  are statistically independent. Although this assumption again tends to yield mathematical expressions that are easy to deal and allows the identification of

fundamental performance limits, it is usually a rough approximation. In practice, the complex path gains  $h_{i,j}$  are correlated by an amount that depends on the propagation environment as well as the polarization of the antenna elements and the spacing between antennas. The channel correlation has a strong impact on the achievable system performance.

The fading itself will be modeled as block-fading, which means that the elements of  $H$  stay constant during the transmission of  $L$  data vectors  $s$  (or equivalently: during the whole transmission duration of  $s$ ) and change independently to another realization for the next block of  $L$  symbol periods. In practice, the duration  $L$  has to be shorter than the coherence time of the channel, although in reality the channel path gains will change gradually.

### 3.5 MIMO Channel Power Distribution and SNR

The MIMO transmission model already investigated, now the focus is toward the transmitted power. Furthermore, the signal-to-noise ratio (SNR) expression as a function at the receiver have to be defined in terms of the already introduced quantities.

In the theoretical literature of MIMO systems, it is common to specify the power constraint on the input power in terms of an average power over the  $n_T$  transmit antennas. This may be written as

$$\frac{1}{n_T} \sum_{j=1}^{n_T} |s_{i,l}|^2 = E_s \text{ for } l = 1, \dots, L, \quad (3.6)$$

Where  $E_s$  is power at each transmitting antenna. Here  $E_s$  denotes the mean symbol energy, i.e.  $E_s = \mathbb{E}\{|s^i|^2\}$  (here,  $i$  denotes the time index of the sent symbol), where the expectation is carried out over the symbol sequence for all value of ' $i$ ', which in case of a white symbol sequence reduces to an averaging over the symbol alphabet.

Although power assumption is a very common in MIMO system but in general case, there is a variety of similar assumption made which are having same basic information like theoretic MIMO transmission system [53]. The power constraints can be written as

1.  $\mathbb{E}\{|s^i|^2\} = E_s$ , for  $i = 1, \dots, n_T$ , and  $l = 1, \dots, L$ , where no averaging over the transmit antennas is performed.
2.  $\frac{1}{L} \sum_{l=1}^L |s_{i,l}|^2 = E_s$ , for  $i = 1, \dots, n_T$  here averaging is performed over time instead of space.

In most cases derived mathematical expressions and curve depends on the SNR at receiving and transmitting antennas. Adapted MIMO transmission model used to redefine the power assumptions. The expression of the average signal-to-noise ratio motivated by an arbitrary receive antenna, here transmitted power  $n_T E_s$  over a channel with an average path gain having unit magnitude and noise power  $2\sigma_N^2$  at each receiving antenna. The SNR at receiving antenna can be written as  $\rho = n_T E_s / (2\sigma_N^2)$ . This would have the negative aspect, that total transmitted power (and thus the receive SNR) depends on the number of transmitting antennas. The transmitted power normalized by the transmitting antennas  $n_T$ . Adapted MIMO transmission model receiving antenna signal is given by:

$$Y = \sqrt{\frac{\rho}{n_T}} HS + n \quad (3.7)$$

In this context, these following assumptions on the MIMO transmission model have been used:

1. Average magnitude of the channel path gains  $\mathbb{E} \{ \text{tr} SS^H \} = n_R n_T$ ,
2. Average transmit power  $\mathbb{E} \{ \text{tr} SS^H \} = n_T L$
3. Average noise variance  $\mathbb{E} \{ \text{tr} NN^H \} = n_R L$

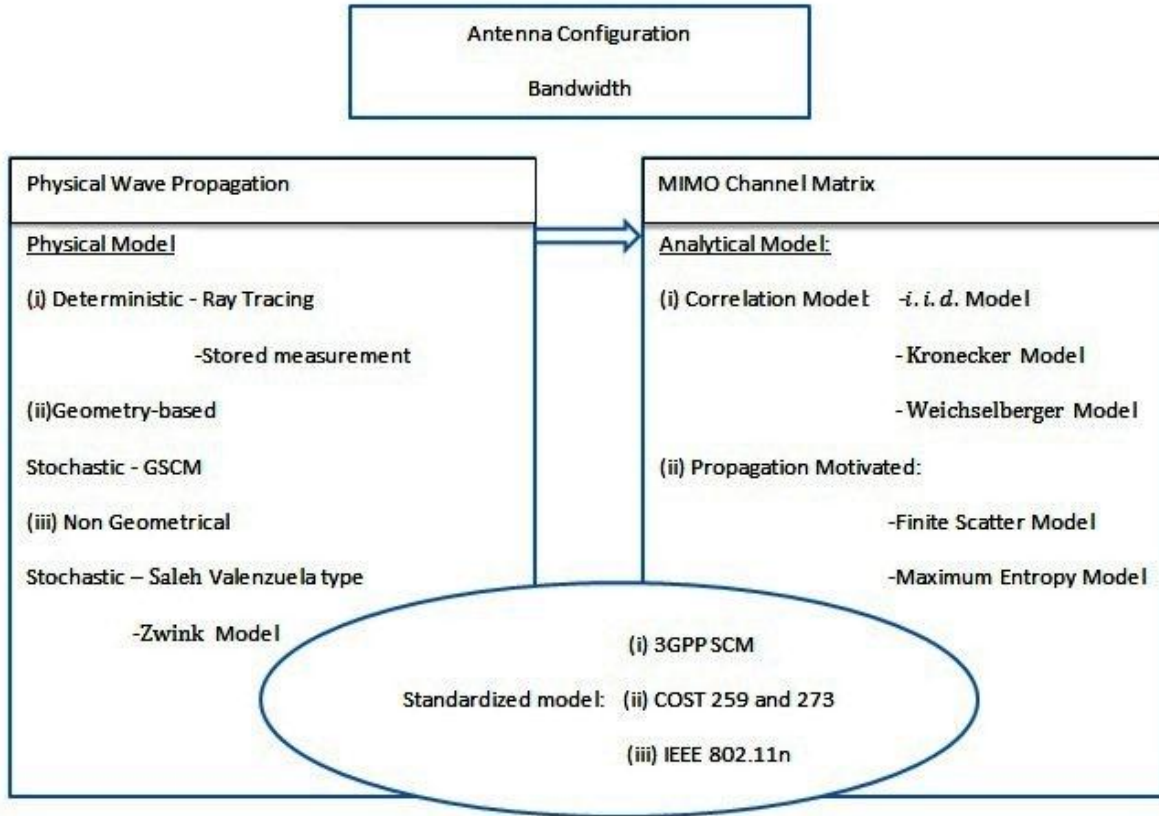
If these assumptions are fulfilled, then the factor  $\sqrt{\rho/n_T}$  ensures that  $\rho$  is the average SNR at a receive antenna and is independent of the number of transmit antennas.

### 3.6 MIMO Channel Model Classification

The data rate of MIMO systems grows linearly with the number of transmitting antenna. In general, however, the maximum transmission rate in a given bandwidth (i.e., the spectral efficiency) that can be exploited in MIMO systems depends on a number of parameters observed at the receiver, including the average received power of the desired signal, thermal and system related noise, as well as co-channel interference. Most of MIMO channel models presented are based on measurements. Generally, important requirements for such a model are [33]:

1. The representation of real-life MIMO channel statistics according to the targeted radio environment and system parameters like antenna spacing, polarization, antenna element directionalities.
2. The possibility to easily cover a wide range of best-case to worst-case scenarios.

- The possibility to convey the relevant parameters between various groups of researchers to reliably compare existing results.



**Figure 3.3 Classifications of MIMO Channels and Propagation Model [54]**

A potential way of distinguishing the individual model dependent on the type of channel that is being considered, narrowband (flat fading) versus wideband (frequency selective) models, time-varying versus time-invariant models [35]. Narrowband MIMO channels are completely characterized in terms of their spatial structure. In contrast, wideband (frequency-selectivity) channels require additional modeling of the multipath channel characteristics. Time varying channels require additional model for the channel evolution according to Doppler characteristics.

The fundamental distinction has been presented by physical models and analytical models. Physical channel models characterize an environment on the basis of electromagnetic wave propagation by describing the double-directional multipath propagation between the location of the transmitter array and the location of the receiver array [54]. They explicit model wave

propagation parameters like the complex amplitude, direction of departure (DOD), direction of arrival (DOA), and delay of a model predictive control (MPC). Physical models are independent of antenna configurations (antenna pattern, number of antennas, array geometry, polarization, mutual coupling) and system bandwidth. Physical MIMO channel models can further be split into deterministic models, geometry-based stochastic models, and non-geometric stochastic models. Deterministic models characterize the physical propagation parameters in a completely deterministic manner (examples are ray tracing and stored measurement data). With geometry-based stochastic channel models (GSCM), the impulse response is characterized by the laws of wave propagation applied to specific transmitter, receiver and scatter geometries, which are chosen in a stochastic (random) manner. In contrast, non-geometric stochastic models describe and determine physical parameters (DOD, DOA, delay, etc.)

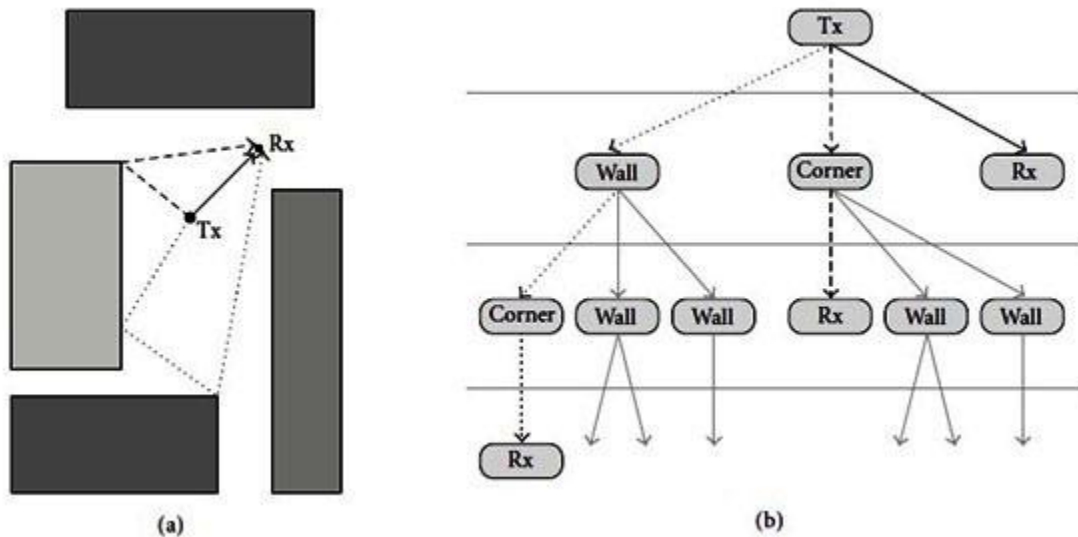
Analytical channel models characterize the impulse response (equivalently, the transfer function) of the channel between the individual transmitter and receiver antennas in a mathematical or analytical way without explicitly accounting wave propagation. Analytical models are very popular for synthesizing MIMO matrix in the context of system and algorithm development and verification. Analytical models can be further subdivided into propagation motivated models and correlation-based models. The first subclass model has been modeled by the channel matrix via propagation parameters. Examples are the finite scatter model [37] the maximum entropy model [38] and the virtual channel representation [39]. Correlation-based models characterize the MIMO channel matrix statistically in terms of the correlations between the matrix entries. Popular correlation-based analytical channel models are the Kronecker model and the Weichselberger model [54].

### **3.7 PHYSICAL MODELS**

#### **3.7.1 Deterministic Physical model**

Physical propagation models are termed as deterministic if they aim at reproducing the actual physical radio propagation process for a given environment. Buildings are usually represented as polygonal prisms with flat tops, that is, they are composed of flat polygons (walls) and piecewise rectilinear edges. Deterministic models are physically meaningful and

potentially accurate. However, they are only representative for the environment considered. Hence, in many cases, multiple runs using different environments are required.



**Figure 3.4** (a) propagation scenario (gray shading indicates buildings)  
 (b) Corresponding visibility tree [54]

### 3.7.2 Ray-tracing algorithm

With Ray-Tracing (RT) algorithms, initially the transmitter and receiver positions are specified and then all possible paths (rays) from the transmitter to the receiver are determined according to geometric considerations with the help of geometrical optics. Usually, a maximum number  $N_{max}$  of successive reflections or diffractions (often called prediction order) are prescribed. This geometric “ray tracing” core is by far the most critical and time consuming part of the RT procedure. In general, the strategy adopted to capture the individual propagation paths is called visibility tree as shown in Figure 3.4. The visibility tree consists of nodes, branches and a layered structure. Each node of the tree represents an object like a building wall, a wedge, the receiver antenna whereas each branch represents a line-of sight (LOS) connection between two nodes or objects. The root node corresponds to the transmitting antenna. For the  $i^{th}$  ray, a complex, vectorial electric field amplitude  $E_i$  is associated, which is computed by taking into account the transmitter emitted field, free space path loss, the reflections and diffractions. Reflections are accounted for by applying the Fresnel reflection coefficients, whereas for diffractions: the field vector is multiplied by

appropriate diffraction coefficients obtained from the uniform geometrical theory of diffraction .The distance-decay law (divergence factor) may vary along the way due to diffractions. The resulting field vector at the receiver position is composed of the fields for each of the  $N_r$  rays as  $E^{Rx} = \sum_{i=0}^{N_r} E_i^{Rx}$  with

$$E_i^{Rx} = I_i B_i E_j^{Tx} \text{ with } B_i = A_{i,N_i} A_{in_{i-1}} \dots \dots A_{i,1} \quad (3.8)$$

Here  $I_i$  is the overall diverge factor for  $i^{th}$  path,  $A_{i,j}$  is a Rank-one matrix that decompose the field into orthogonal component at the  $j^{th}$  node.

### 3.8 ANALYTICAL MODELS

#### 3.8.1 Correlation-based analytical models

Various narrowband analytical models are based on a various complex Gaussian distribution of the MIMO channel coefficients (i.e., Rayleigh or Rician fading) [55]. The channel matrix can be split into a zero-mean stochastic part  $H_S$  and a purely deterministic part  $H_d$  according to

$$H = \sqrt{\frac{1}{1+K}} H_S + \sqrt{\frac{K}{1+K}} H_d \quad (3.9)$$

Where  $K \geq 0$  denotes the Rician factor. The matrix  $H_d$  accounts for *LoS* components and other nonfading contributions. The number of *LoS* components are characterized by the Gaussian matrix  $H_S$ . For simplicity, we thus assume  $K = 0$ , that is  $H = H_S$ . In most general form, the zero-mean multivariate complex Gaussian distribution of  $h = \text{vec}\{H\}$  is given by

$$f(h) = \frac{1}{\pi^{nm} \det\{R_H\}} \exp(-h^H R_H^{-1} h) \quad (3.10)$$

The  $n \times n$  matrix is

$$R_H = E\{hh^H\} \quad (3.11)$$

is known as full correlation matrix and describes the spatial MIMO channel statistics. It contains the correlations of all channel matrix elements. Realizations of MIMO channels with distribution (3.8) can be obtained by

$$H = \text{unvec}\{h\} \text{ with } h = R_H^{1/2} g \quad (3.12)$$

### 3.8.2 The Identically Independent Distributed (*i. i. d*) Model

The simplest analytical MIMO model is the *i. i. d.* model (sometimes referred to as canonical model). Here  $R_H = \rho^2 I$ , that is, all elements of the MIMO channel matrix  $H$  are uncorrelated (and hence statistically independent) and have equal variance  $\rho^2$ . Physically, this corresponds to a spatially white MIMO channel which occurs only in rich scattering environments characterized by independent MPCs uniformly distributed in all directions. The *i. i. d.* model consists of a single parameter (the channel power  $\rho^2$ ) and is often used for theoretical considerations like the information theoretic analysis of MIMO systems [12].

### 3.8.3 The Kronecker model

Kronecker Model was used for capacity analysis before being proposed by L. Thomas et al. [53] in the framework of the European Union SATURN project [56]. It assumes that spatial transmitter and receiver correlation are separable, which is equivalent to restricting to correlation matrix that can be written as Kronecker product

$$R_H = R_{Tx} \otimes R_{Rx} \quad (3.13)$$

With the transmitter and receiver correlation matrix

$$R_{Tx} = E\{HH^H\}, R_{Rx} = E\{HH^H\} \quad (3.14)$$

The Kronecker model is not able to reproduce the coupling of a single *DoD* with a single *DoA*, which is an elementary feature of MIMO channels with single-bounce scattering.

### 3.8.4 The Weichselberger model

The Weichselberger model [53] aims at obviating the restriction of the Kronecker model to separable *DoA* – *DoD* spectra that neglects significant parts of the spatial structure of MIMO channels. Its definition is based on the eigenvalue decomposition of the  $T_x$  and  $R_x$  correlation matrix,

$$R_{Tx} = U_{Tx} \Lambda_{Tx} U_{Tx}^H, \quad R_{Rx} = U_{Rx} \Lambda_{Rx} U_{Rx}^H, \quad (3.15)$$

Here,  $U_{Tx}$  and  $U_{Rx}$  are unitary matrix whose columns are the eigenvectors of  $R_{Tx}$  and  $R_{Rx}$ , respectively,  $\Lambda_{Tx}$  and  $\Lambda_{Rx}$  are diagonal matrix with the corresponding eigenvalues. The model itself is given by

$$H = U_{Rx} (\Omega \odot G) U_{Tx}^H \quad (3.16)$$

Where  $G$  is again an  $n \times m$  *i. i. d.* MIMO matrix, denotes the Schur-Hadamard product (element wise multiplication), and  $\Omega$  is the element wise square root of a  $n \times m$  coupling

matrix  $\Omega$  whose (real-valued and nonnegative) elements determine the average power coupling between  $T_x$  and  $R_x$  Eigen-modes. This coupling matrix allows for joint modeling of  $T_x$  and Rx channel correlations. The Weichselberger model requires specification of  $T_x$  and  $R_x$  eigen-modes ( $U_{T_x}$  and  $U_{R_x}$ ) and of the coupling matrix  $\Omega$ .

### **3.8.5 IEEE 802.11n**

IEEE802.11n standard employs MIMO-OFDM transmission technology to ensure high throughput communication for up to 600 Mbps in a 40 MHz channel bandwidth. The wireless fading channel varies with time therefore link adaptation must be employed to sustain reliable communications and maximize throughput. The efficient and practical link adaptation techniques are needed to be developed for IEEE802.11n WLAN.

The TGn channel model of IEEE 802.11 was developed for indoor environments in the 2GHz and 5 GHz bands, with a focus on MIMO WLANs [57]. The 802.11 TGn model is a physical model that uses a non-geometric stochastic approach. For the Rayleigh fading and Rician fading channel, a Kronecker model is chosen for the bit error rate analysis. Here in this thesis IEEE802.11n standard has been used as benchmark. The BER analysis done in this thesis has been compared with the results reported in [58], for ieee802.11n as benchmark.

## CHAPTER 4

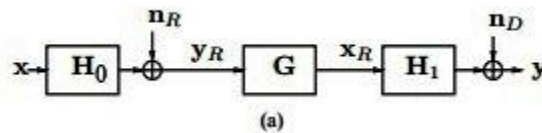
# MIMO System Mutual Information Analysis

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In this chapter the simulation and analyzing of MIMO system Capacity is done. The capacity of MIMO system are analyzed by two techniques:- in first the capacity (Mutual Information) is analyzed using Amplifying and Forward method under Rayleigh fading channel and in second the capacity of MIMO system under correlated Rician fading channel is analyzed.

### 4.1 Amplifying-Forward MIMO Mutual-Information

Wireless relaying networks have recently been given considerable attention due to their many advantages. Apart from increasing the range, relaying networks can also achieve better diversity by using cooperative transmission from the source and several relays [19]. The block diagram of MIMO AF relay system is shown in Figure. 4.1 below



**Figure.4.1 Block Diagram of MIMO Amplify-Forward Relay System**

The relaying terminals forward the information from the source to the destination mainly using the well-known method AF since MIMO systems can provide better system capacity than SISO, SIMO and MISO systems, relaying has recently been extended to MIMO scenarios [39]. MIMO relays aim to provide improved system capacity, increases in range, and better diversity as traditional communication system. The ergodic capacity of an AF MIMO two-hop system under Rayleigh fading channel is analyzed. Most of the capacity results on two-hop MIMO relays have been derived by employing asymptotic methods Furthermore, the random matrix results required for the MIMO relay capacity have been analyzed. The main contribution of this chapter is to derive an exact expression for the AF MIMO two-hop system capacity and cumulant function of Mutual Information. The expression derived in this chapter is unified, and it can be used for arbitrary numbers of

antennas at the source, relay and destination. The simulation results have been used to validate the analysis and improvement in capacity due to the relay terminal.

#### **4.1.1. Introduction**

Wireless network constitutes an important component of future information application. In wireless communication use of multiple antennas means higher complexity at transmitter and receiver. In cellular network direct transmission between base station and user close to cell boundary are demanding high transmission power [12],[14]. The Relaying technique with MIMO has been turnout potential technique to enhance diversity and transmission rate and cost expensive in terms of power transmission in wireless channel [19]. MIMO relay channel act as active scatter to suppress the problem of scattering that enhance the quality of service. A two-hop non-regenerative Amplifying-Forward (AF) MIMO relay channel consists of source, relay and destination terminal each having multiple antennas. Relay terminal employs an amplifying matrix. Large network have shown increase in ergodic capacity by using large random matrix theory [20],[59]. The upper bound and lower bound on Capacity of fading relay for correlated channel is demonstrated in [33]. The scaling of capacity for relay channel is  $N/2 \log K$  with single source and destination.

In this this chapter AF MIMO relay channel without line of sight (*LOS*) path between source and destination is considered. AF relay strategy is used to avoid interference. The main concern is to calculate the mean of MI of channel [39]. In most cases the Capacity calculation has been done on the basis of deterministic Eigen value decomposition using *det*-formula. For large antenna array without LOS, mean of mutual information of MIMO AF relay has been discussed in [60]. The analytical mutual information expression of MIMO relay channel inspired by MI of single hop MIMO relay channel under statistical channel state information. Multi-array channel analysis has been done by replica method based on spin glass theory is given in [61]. Replica method has been used in capacity analysis for large random matrix extraction. Evaluation of cumulate generating function of mutual information of MIMO relay channel under Rayleigh fading have been studied in [62],[63]. The basic purpose of this chapter is to improve MI for multiple users in MIMO system by using AF-MIMO. With the implementation of AF we have been able to improve MI up to 30% as compared to existing technique reported in literature survey.

### 4.1.2. MIMO Mutual Information Mathematical Analysis

In this thesis work the two-hop MIMO relay channel is considered having a source, a relay and a destination terminal equipped with  $M_s$ ,  $M_r$  and  $M_d$  numbers of antennas. The whole process of transmission is implemented in two phases. In first phase source terminal transmit a vector signal to relay, this is single hop and in second phase relay terminal transmit the received vector signal after modifying the parameter to destination terminal that is second hop. Here channel is modeled with no line of sight path and flat fading. First phase channel matrix is represented by  $H_0 \in \mathbb{C}^{M_r \times M_s}$  and the second phase channel matrix represented by the  $H_1 \in \mathbb{C}^{M_d \times M_r}$  .and the forward matrix of relay channel represented by  $G \in \mathbb{C}^{M_r \times M_r}$ .The received signal at the Relay terminal ( $Y_0$ ) and destination terminal( $Y_1$ ) is given by

$$Y_0 = H_0 X + n_0 \quad (4.1)$$

$$Y_1 = H_1 G Y_0 + n_1 \quad (4.2)$$

Where  $n_0$  and  $n_1$  are the relay and destination noise vector. The  $X \in \mathbb{C}^{M_s}$  is a transmitted vector with zero mean and covariance  $E[xx^H] = \rho/n$ , the noise at relay and destination assumed to be additive white Gaussian with unit variance. The resultant output can be written as

$$Y = H_1 G (H_0 X + n_0) + n_1 \quad (4.3)$$

Where  $H_0$  and  $H_1$  will be assumed to be zero mean circulate symmetric complex Gaussian random variable with covariance matrix as defined under statically CSI by Kronecker model [33] and  $G = \sqrt{\beta/n}$  is forward matrix where  $\beta$  overall power gain of relay channel that depend on the distance and path loss between source and relay terminal.

$$H_l = R_l^{1/2} W_l T_l^{1/2} \text{ for } l = 0, 1 \quad (4.4)$$

Where  $W_l$  denote the channel matrix with *i.i.d* complex Gaussian random entries.  $R_l \in \mathbb{C}^{M \times M}$  and  $T_l \in \mathbb{C}^{M \times M}$  are the receiver and transmitter spatial correlation matrix. If  $H_l$  is perfectly known at the destination terminal then the mutual information between  $X$  and  $Y$  will be given by

$$I(X, Y) = \log \left( \det \left( I + \frac{\beta_i}{n_i} H_1 H_1^\dagger + \frac{\beta_s}{n_t} H_1 H_2 H_2^\dagger H_1^\dagger \right) \right) - \log \left( \det \left( I + \frac{\beta_i}{n_i} H_1 H_1^\dagger \right) \right) \quad (4.5)$$

For the simulation  $H_l$  and  $I$  are generated using random variable. The mean of the MI  $I(X, Y)$  the mutual asymptotic mean is given by

$$\lim_{n \rightarrow \infty} E[I(X, Y)] = \ln(1 + \rho \mathfrak{R}_0) + \ln(1 + \beta \mathfrak{R}_1) + \ln(1 + \mathcal{B}_0 + \mathcal{B}_0 \mathcal{B}_1) - \ln(1 + \beta t) - \ln(1 + x) - (\mathcal{B}_0 \mathcal{B}_1 + \mathcal{B}_1 \mathfrak{R}_1 - xt) \quad (4.6)$$

Where the coefficient are calculated by nonlinear equation system that are real and positive given below

$$q_s = \beta_i/1 + \beta_i r_s, \quad r_s = q_t/1 + q_s + q_s q_t, \quad q_t = \beta_i/1 + \beta_i r_t, \quad r_t = 1 + q_s/1 + q_s + q_s q_t$$

$$x = \beta_i / (1 + \beta_i t), \quad t = 1/1 + x \quad (4.7)$$

#### (a) Calculation of mean MI

The mean of MI can be calculated using Replica analysis and integral identity [39],[60]. The mathematical framework introduced for deriving the cumulant moment of  $I$ . the generating function  $g(v)$  of  $I$  is given by

$$g(v) = E \left[ \log \left( \det \left( I + \frac{\beta_i}{n_i} H_0 H_0^\dagger + \frac{\beta_s}{n_t} H_0 H_1 H_1^\dagger H_0^\dagger \right) \right) \right]^{-v}$$

$$- E \left[ \log \left( \det \left( I + \frac{\beta_i}{n_i} H_0 H_0^\dagger \right) \right) \right]^{-v} \quad (4.8)$$

$$g(v) = E[e^{-vI}] = 1 - vE[(I)] - v^2/2 E[(I^2)] + \dots \dots \quad (4.9)$$

$$\log(v) = -vE[(I)] + \sum_{p=2}^{\infty} \frac{(-v)^p}{p!} c_p \quad (4.10)$$

Where  $c_p$  is the  $p^{th}$  cumulant of  $I$ .

#### (b) Integral Identity

In this chapter two types of general integral identities are dealt over matrix element. In the first type integrating over the real and imaginary part of a complex matrix  $X \in \mathbb{C}^{M_s \times M_r}$  will be done by using the integral identity

$$dX = \prod_{a=1}^{Ms} \prod_{a=1}^{Mr} \frac{dReX_{a,a} dImX_{a,a}}{2\pi} \quad (4.11)$$

For  $M \in \mathbb{C}^{n \times n}$ ,  $N \in \mathbb{C}^{n \times n}$  positive definite X, A and B can be written as

$$\begin{aligned} \text{Positive define X, A and B} &= \int \exp\left(-\frac{1}{2} \text{Tr}(NX^\dagger MX) + A^\dagger X - X^\dagger B\right) dX = \\ &\exp\left(-\frac{1}{2} \text{Tr}(N^{-1}A^\dagger B M^{-1})\right) / \det(N \otimes M) \end{aligned} \quad (4.12)$$

The second type of integration over pair of complex square matrix X and Y can be express as

$$d\mu(X, Y) = \prod_{a=1}^{Ms} \prod_{a=1}^{Mr} \frac{dReX_{a,a} dImX_{a,a}}{2\pi i} \quad (4.13)$$

The integration of element of X and Y along to real and imaginary axis is given by

$$\int \exp(\text{Tr}(XY - XA - BY)) d\mu(X, Y) = \exp(-\text{Tr}(AB)) \quad (4.14)$$

According to replica analysis

$$\begin{aligned} E[I(X, Y)] &= E\left[\log\left(\det\left(I + \frac{\beta_i}{n_i} H_0 H_0^\dagger + \frac{\beta_s}{n_t} H_0 H_1 H_1^\dagger H_0^\dagger\right)\right)\right] - E\left[\log\left(\det\left(I + \right.\right.\right. \\ &\left.\left.\left. \frac{\beta_i}{n_i} H_0 H_0^\dagger\right)\right)\right] \triangleq \varphi - \vartheta \end{aligned} \quad (4.15)$$

Where the first term  $\varphi$  represents mean Mutual information of overall signal plus noise covariance matrix at the destination and second term  $\vartheta$  is mean of overall noise covariance matrix at the destination.  $\vartheta$  is given by

$$\vartheta = \ln((1 + \beta_s t)(1 + x)) - xt \quad (4.16)$$

The first term  $\varphi$  evaluate using cumulant moment, given by

$$\varphi = -\frac{\partial}{\partial v} \log g(v) /_{v=0} \quad (4.17)$$

Using identity moment generating function  $g(v)$  express as

$$\begin{aligned} g(v) &= E\left[\int \exp\left[-\frac{1}{2} \text{Tr}\left[XX^\dagger + YY^\dagger + ZZ^\dagger + Y^\dagger H_0^\dagger H_1^\dagger X - X^\dagger H_1^\dagger H_0^\dagger Y + \frac{\beta_i}{n_i} Z^\dagger H_1^\dagger X - \right.\right.\right. \\ &\left.\left.\left. X^\dagger H_1 Z\right]\right] dX dY dZ \right] \end{aligned} \quad (4.18)$$

Repeat these identities to split the  $H_0$  and  $H_1$  with  $T_0, T_1, T^\dagger \in \mathbb{C}^{n \times v}$  and integrating over  $H_0$  and  $H_1$  will yield

$$g(v) = \mathbb{E} \left[ \int \exp \left[ -\frac{1}{2} \text{Tr} \left[ \begin{array}{c} XX^\dagger + YY^\dagger + ZZ^\dagger \\ + T_0 T_0^\dagger + T_1 T_1^\dagger \\ + Y^\dagger H_0^\dagger H_1^\dagger X \\ - X^\dagger H_1^\dagger H_0^\dagger Y \\ + \frac{1}{2} \left( -\frac{\beta_i}{n_i} YY^\dagger T_0^\dagger T_1 + \frac{\beta_s}{n_t} XX^\dagger T_1^\dagger Z \right) \\ - \frac{\beta_i}{n_i} X^\dagger X T_1^\dagger Z + \frac{\beta_i}{n_i} X^\dagger X Z Z^\dagger \end{array} \right] \right] dT_0 dT_1 dX dY dZ \right] \quad (4.19)$$

Where integration over  $H_0$   $A=Y^\dagger T_0$  and  $B = \frac{\beta_i}{n_i} Y^\dagger T_1$  and over  $H_1$   $A=X^\dagger(Z + T_1)$  and  $B = \frac{\beta_i}{n_i} (T_1 X^\dagger + Z X^\dagger)$  Equation a is used to split the quadratic terms with  $\mathfrak{R}_i, \mathcal{B}_i \in \mathbb{C}^{v \times v}$   $i = 0, 1, 2, \dots$

$$g(v) = \mathbb{E} \left[ \int \exp \left[ -\frac{1}{2} \text{Tr} \left[ XX^\dagger + YY^\dagger + ZZ^\dagger + T_0 T_0^\dagger + T_1 T_1^\dagger - \sum_{i=0}^4 \mathfrak{R}_i \mathcal{B}_i + \frac{\beta_i}{n_i} \mathfrak{R}_1 Y^\dagger Y - \mathcal{B}_0 T_0^\dagger T_1 + \frac{\beta_s}{n_t} \sum_{i=1}^4 \mathfrak{R}_i X X^\dagger + \mathcal{B}_1 T_1^\dagger T_0 - \mathcal{B}_2 Z^\dagger T_0 - \mathcal{B}_3 T_1^\dagger Z + \mathcal{B}_4 Z Z^\dagger \right] \right] dT_0 dT_1 dX dY dZ d\gamma \right] \quad (4.20)$$

Where  $d\gamma \triangleq \prod_{i=0}^4 d\mu(\mathfrak{R}_i, \mathcal{B}_i)$  integration again over all the parameter using some approximation [39]  $g(v)$  can be written as

$$g(v) = \int \exp(-S) d\gamma \quad (4.21)$$

$$S = -\text{Tr}(\sum_{i=0}^4 \mathfrak{R}_i \mathcal{B}_i) + \log \det \left( I + \frac{\beta_i}{n_i} \mathfrak{R}_1 \right) + \log \det \left( I + \frac{\beta_s}{n_t} \sum_{i=1}^4 \mathfrak{R}_i \right) + \log \det \left( I - (I + \mathcal{B}_0 \mathcal{B}_1)^{-1} \mathcal{B}_1 \mathcal{B}_2 \mathcal{B}_3 + \mathcal{B}_4 \right) (I + \mathcal{B}_0 \mathcal{B}_1) \quad (4.22)$$

Where  $I \in \mathbb{C}^v$  According to assumption of replica symmetry all complex matrices need to be proportional to  $I$ .  $\sum_{i=0}^4 \mathfrak{R}_i = \mathfrak{R}_i I$  and  $\sum_{i=0}^4 \mathcal{B}_i = \mathcal{B}_i I$ . The  $S_0$  is simplifies by differentiating with respect to  $\mathfrak{R}_i, \mathcal{B}_i$ . So that the entire coefficient are positive and real valued. Hence

$$S_0 = v \left\{ \begin{array}{l} \ln(1 + \rho \mathfrak{R}_0) + \ln(1 + \beta \mathfrak{R}_1) + \ln(1 + \mathcal{B}_0 + \mathcal{B}_0 \mathcal{B}_1) - \\ (\mathcal{B}_0 \mathcal{B}_1 + \mathcal{B}_1 \mathfrak{R}_1 - xt) \end{array} \right\} \quad (4.23)$$

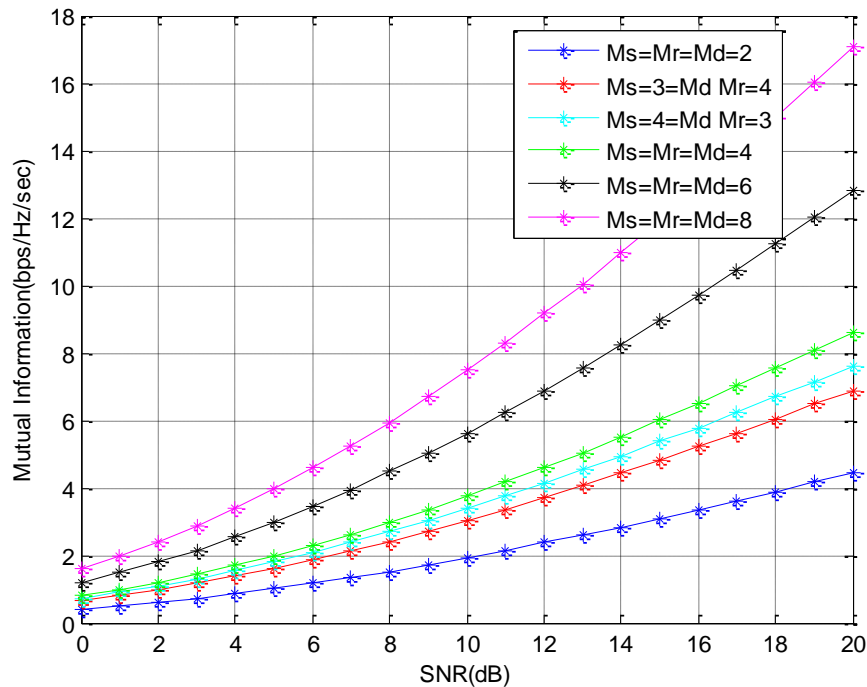
Using saddle point method [18],[19] identity

$$\varphi = \ln(1 + \rho\mathfrak{R}_0) + \ln(1 + \beta\mathfrak{R}_1) + \ln(1 + \mathcal{B}_0 + \mathcal{B}_0\mathcal{B}_1) - (\mathcal{B}_0\mathcal{B}_1 + \mathcal{B}_1\mathfrak{R}_1 - xt) \quad (4.24)$$

Hence the cumulant generating function of MI has been derived by both Gaussian integral identity and saddle point method and comes out to be same.

### 4.1.3. Simulation Result and Discussion

The ergodic MI results of MIMO relay system have been achieved by using Monte Carlo simulation. For asymptotic MI of MIMO relay has been simulated by adopting Rayleigh fading channel.



**Figure.4.2 Mutual information versus SNR for MIMO relay system with large number of antenna. (Symmetrical and non-symmetrical)**

In the simulation 1,00,000 Monte Carlo simulations are used for different array of antenna at source, relay and destination (SRD) terminal and according result have achieved are shown above in Figure.4.2. After comparing the result with [60], it was found that in case of two antenna asymptotic result are reasonable same and nearly equal to same as that of analytical mutual information but when compared to four and eight antenna at SRD terminal, simulation result has shown improvement in MI as high as up to 30% by increase number of

antenna up to eight for SRD in symmetric and non-symmetric way. When the number of antenna at the relay terminal are more as compare to the source and destination terminal this significantly increase the MI of channel. Lastly it has been observed with the increased no of antenna MI result will increases exponentially with increasing SNR.

#### 4.1.4. MIMO MI Analysis

The mutual information for spatially correlated MIMO amplifying forward relay channel has been derived. It has been observed that there is significant increase in MI for the value of  $(M_s, M_d, M_r)$  to be (3,3,4) as compared to (3,3,3). Whereas the response gets worse with (4,4,3) as compared to (4,4,4). The drawback of this technique has been founded that when the number of antenna at relay terminal are less than source and destination terminal then response of MI get worse. With the implementation of AF we have been improved MI by 30%, which will help in higher data rate in multi user MIMO transmission. Further it is observed that when  $(M_s, M_d, M_r)$  to be (8,8,8) then MIMO MI at SNR = 20dB is 17bps/Hz (MI), which is nearly 12bps/Hz higher than  $(M_s, M_d, M_r)$  to be (2,2,2). The replica method and integral identity plays central role in analyzing the mutual information cumulate generating function.

#### 4.2 Capacity of MIMO under Correlated Ricain fading channel

The use of antenna arrays at both sides of the wireless communication link can result in high channel capacity provided the propagation medium is highly scattered and uncorrelated. Consider a single user MIMO system that has  $T$  and  $R$  antennas at the transmitter and receiver, respectively. It has been shown [12] that, for a given fixed average transmitter power and bandwidth, if the fading between pairs of transmit-receive antenna elements are independent and identically distributed (*i. i. d*) Rayleigh, the average channel capacity increases linearly with  $m = \min(T, R)$ . This large capacity growth occurs even if the transmitter has no channel knowledge. Logarithmic increase has been achieved in MIMO system as compared to traditional systems utilizing receive diversity or no diversity as reported in literature survey. While the linear growth of capacity with in MIMO due to 'm' is indicative of the tremendous potential of multiple antenna systems, the result is limited in scope by the assumptions it made. Most of the research effort has been focused on *i. i. d* Rayleigh channels. However, in real world propagation environment, the fading is not

independent mainly due to insufficient spacing between antenna elements. It has been shown [13],[14] that correlated fading reduces the channel capacity. Secondly, there is a possibility that the line-of-sight (LOS) component may exist in addition to scattered component, then; the fading will follow the Rician distribution.

#### 4.2.1 MIMO Channel model and fading due to Rician fading

Channel capacity is defined as the maximum rate at which data can be transmitted at an arbitrarily small error probability. The capacity of MIMO channels has been well studied for the *i. i. d.* Rayleigh scenario. On the other hand, in practice, MIMO channels do not always satisfy the *i. i. d.* Rayleigh fading condition. In reality, there is often a line-of-sight (LOS) path between the transmitter and the receiver, and in such fading conditions, the channel is described by the Rician fading model. Mathematically, the random channel matrix in a MIMO Rician fading channel is a complex Gaussian matrix with a nonzero mean matrix, unlike in a *i. i. d.* Rayleigh-faded MIMO channel where the channel matrix is of zero mean. The Rayleigh fading model can be viewed as a special case of the Rician fading model by setting the mean to zero.

Consider a single user MIMO system with  $T$  antennas at the transmitter and  $R$  antennas at the receiver. For simplicity flat fading is considered. The transmitted signal in each symbol period is represented by a  $T \times 1$  column matrix ‘ $S$ ’, where  $i^{th}$  component  $S_i$ , refers to the transmitted signal from antenna  $i$ . The channel ‘ $H$ ’ is described by a  $R \times T$  complex matrix where the  $ij^{th}$  component of the matrix ‘ $H$ ’, denoted by  $h_{ij}$ , is the channel response between the  $j^{th}$  transmit antenna and the  $i^{th}$  receive antenna. The additive white Gaussian noise vector  $n$  at the receiver is described by a  $R \times 1$  column matrix. Thus, the system is described by the matrix equation [16].

$$Y = \sqrt{\frac{E_S}{T}} HX + n \quad (4.25)$$

And the channel matrix,  $H$  can be denoted as shown below

$$H = \begin{pmatrix} h_{1,1} & \dots & h_{1,T} \\ \vdots & \ddots & \vdots \\ h_{R,1} & \dots & h_{R,T} \end{pmatrix} \quad (4.26)$$

In Rician fading the elements of  $H$  are non-zero mean complex Gaussians. Hence  $H$  can express in matrix notation [30]

$$H = aH^{sp} + bH^{sc} \quad (4.27)$$

Where the specular and scattered components of  $H$  are denoted by superscripts,  $a > 0$ ,  $b > 0$  and  $a^2 + b^2 = 1$ .  $H^{sp}$  is a matrix of unit entries denoted by  $H_1$ . If there is no correlation at the transmitter or at the receiver side then the entries of  $H^{sc}$  are independent and identically distributed (i.i.d) complex Gaussian random variables with zero mean and unit magnitude variance, usually denoted by  $H_\omega$ . If there is correlated fading then the  $H^{sc}$  matrix can be modeled [17] as

$$H^{sc} = R_r^{1/2} H_\omega R_t^{1/2} \quad (4.28)$$

Where  $R_t$  and  $R_r$  are the correlation matrix at the transmitter and at the receiver side, respectively. The correlation matrix  $R$  is defined by exponential correlation model [13].

#### 4.2.2 Determination of MIMO Capacity for Channel unknown at Transmitter

Channel knowledge acquiring at the transmitter is very difficult in practical systems. In general the channel is assumed perfectly known to the receiver where the channel state information at the transmitter is available or not. Furthermore, ergodic block fading channel model where the channel remains constant over a block of consecutive symbols, and changes in an independent fashion across blocks have been considered. The average SNR at each of the receive antennas is given by  $E_S/\sigma^2$ , where  $E_S$  is the power of the transmitted signal and  $\sigma^2$  is the power spectral density of the noise. In fading channel two types capacity has been described: ergodic capacity and outage capacity [12],[22].

**Ergodic Capacity:** This is the time-averaged capacity of a stochastic channel. It is calculated by taking the mean of the capacity values obtained from a number of independent channel realizations.

**Outage Capacity:** The  $q\%$  outage capacity  $C_{out,q}$  is defined as the capacity that is guaranteed for  $(100 - q)\%$  of the channel realizations, i.e.

$$P(C < C_{out,q}) = q\% \quad (4.29)$$

##### (a) Channel Unknown at the Transmitter

Channel knowledge acquiring at the transmitter is in general very difficult in practical systems. When the transmitter has no channel state information, it is optimal to

evenly distribute the available power  $\rho$  among the transmit antennas. The MIMO channel capacity with  $\rho = E_S/\sigma^2$  [12],[14] is given by

$$C = E_H\{\log_2 \det(\mathbf{I}_m + \frac{\rho}{T} \mathbf{W})\} \quad (4.30)$$

Where  $E_H \{.\}$  denote the expectation over  $H$ ,  $m = \min(T_x, R_x)$ ,  $\mathbf{I}_m$  is the  $m \times m$  identity matrix,  $\rho$  is the average signal-to-noise ratio (SNR) per receive antenna, and the  $m \times m$  matrix  $\mathbf{W}$  is given by

$$\mathbf{W} = \begin{cases} \mathbf{H}\mathbf{H}^H, & R_x \leq T_x \\ \mathbf{H}^H\mathbf{H}, & T_x < R_x \end{cases} \quad (4.31)$$

Where the operator  $\mathbf{H}^H$  indicates the hermitian of the matrix  $\mathbf{H}$ . Using singular value decomposition (SVD) the equation (4.30) can be written as

$$C = E_H\{\sum_i^K \log_2 (\mathbf{I}_m + \frac{\rho}{T_x} \lambda_i)\} \quad (4.32)$$

Where  $k$ , ( $k \leq m$ ) is the rank of  $\mathbf{H}$ , and  $\lambda_i$  ( $i = 1, 2, \dots, k$ ) denotes the positive eigenvalues of  $\mathbf{W}$ .

### 4.2.3 Simulation Result and Discussion

The correlation matrix coefficient is random and depends on the type of fading channel, here for simulation Rician fading channel is taken into account. The correlation matrix at transmitter and receiver are generated by exponential correlation model and uniform complex correlation model but in this thesis only exponential correlation model have been used to generate correlation matrix. Four antennas are employed at the transmitter and receiver terminal, according to that result have shown in Figure.4.3 and Figure.4.4.

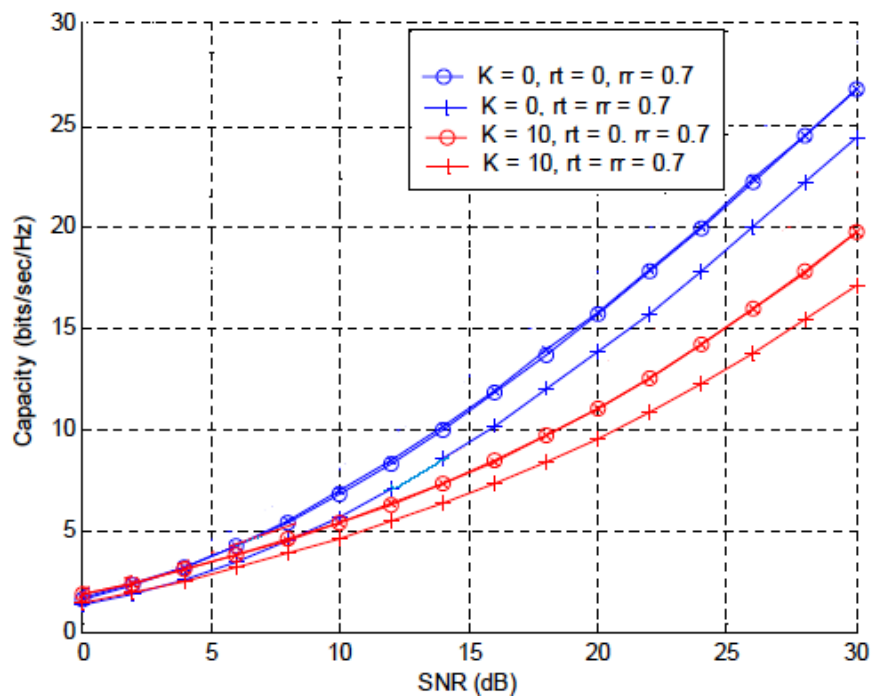
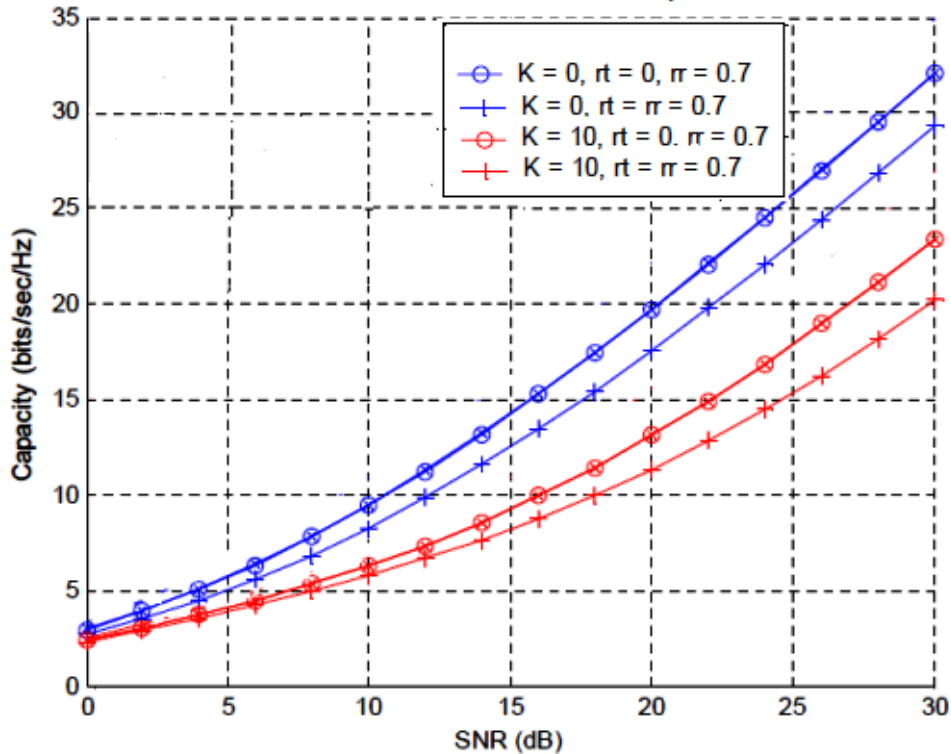


Figure.4.3 Ergodic Capacity versus SNR for MIMO system with Rician factor  $K=0$  and  $10$  and Correlation parameter  $rr = rt = 0, 0.7$ .)



**Figure.4.4: 1 % Outage Capacity versus SNR for MIMO system with Rician factor  $K=0$  and  $10$  and Correlation parameter  $rr = rt = 0, 0.7$ .)**

In case of four antennas, asymptotic result of outage capacity is much better than the ergodic capacity; i.e. at SNR = 20dB the outage capacity observed 20% higher than ergodic capacity where  $K=0$ ,  $rt = 0$  and  $rr = 0.7$ . When the Rician factor  $K = 10$ ,  $rt = 0$  and  $rr = 0.7$  than at SNR = 20dB the outage capacity is 10% higher than ergodic capacity. When SNR and number of antenna increases with respect to increase in the Rician factor than capacity of MIMO degraded.

#### 4.2.4 MIMO Capacity Analysis

The Capacity for spatially correlated MIMO channel has been simulated using Monte Carlo simulation. It has been observed that there is significant increase in Capacity for the small number of antennas but when SNR increases the capacity degraded with increases in the Rician factor. The correlation matrix at transmitter and receiver are generated by exponential correlation model. From the result it is observed that when correlation parameter at transmitter  $rt = 0$  and receiver  $rr = 0.7$ , then capacity of MIMO channel is higher compared to ( $rr = rt = 0.7$ ). The ergodic and outage capacity both are same but ergodic is higher compared

to outage when number of antenna increases with respect to SNR. The results are computed by setting the correlation parameter value to zero at the transmitter and 0.7 at the receiver, in this scenario, capacity of MIMO channel is observed to be 20% higher at SNR 20dB as compared to second scenario when ( $r_r = r_t = 0.7$ ). Experimental results demonstrate that reducing the correlation parameter and Rician factor to zero shows significant improvement in capacity, this will help in higher data rate in multi user MIMO transmission.

## CHAPTER 5

### Analysis of Bit Error Rate for MIMO System

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In this chapter MIMO system bit error rate have been analyzed using BPSK modulation technique under Rayleigh and Rician wireless fading channels. The BER of MIMO system is analyzed by two detection methods:-Linear and Non-Linear detection technique like ZF, ML and MMSE detector. The simulation and discussion of BER are made by comparing the simulated result to previous existing technique. Here the comparative study of both Rayleigh fading and Rician fading are discussed in this chapter. Finally the result achieved has been compared with IEEE802.11n standard protocol.

#### 5.1 Introduction

Symbol error can be calculated with difference between received symbols and original symbols. In the similar method, bit error can be calculated. The bit error rate is the number of bit errors divided by the total number of transmitted bits during an observed time interval. The bit error probability  $p_e$  is the expected value of the BER. The BER can be considered as an approximate estimate of the bit error probability. In a communication system, BER may be affected by transmission channel noise, interference, distortion, bit synchronization problems, attenuation and wireless multipath fading, etc. The BER may be improved by increasing the signal strength or by applying pre-coding schemes such as space-time block coding etc.

#### 5.2 System Model

The MIMO channel with  $M_T$  transmitter antennas and  $M_R$  receiver antennas are considered, where  $m^{th}$  data stream is directly transmit on  $m^{th}$  transmit antenna. The  $M_T \times 1$  transmit vector at a particular time instant is written as  $s$  and consists of BPSK bit with a constellation size two and average bit energy  $E_b$ , the  $M_T \times M_R$  channel matrix  $H$  and the  $M_R \times 1$  received vector .The received vector is given by

$$Y = Hx + n \quad (5.1)$$

Where  $H$  is an channel matrix for the fading channel, whose element are independent zero-mean complex Gaussian random variable with nonzero variance, and  $n$  are the sample of

nondependent complex additive Gaussian noise (AWGN). The fading channel considers in this chapter are Rayleigh fading and Rician fading [12]. The Rayleigh fading channel matrix  $H$  is composed by scattered component of multipath propagation. The Rician channel matrix  $H$  is composed of two components; the line-of-sight ( $LOS$ ) component (deterministic and constant), and a component resulting from multipath propagations (randomly varying) [13]. For the sake of simplicity, under assumption the receiver to be located at highly scattered propagation environment so that the fading at the receiver is spatially uncorrelated while the transmitter is located at a high altitude. The channel matrix  $H$  that, contains transmit antenna correlation but no receive correlation, is given by

$$H = aH^{sp} + bH^{sc} \quad (5.2)$$

Where the specular and scattered components of  $H$  are denoted by superscripts,  $a > 0$ ,  $b > 0$  and  $a^2 + b^2 = 1$ .  $H^{sp}$  is a matrix of unit entries denoted as  $H_1$ . The above equation can be written as

$$H = \sqrt{\frac{k}{k+1}} H_{Los} + \sqrt{\frac{1}{k+1}} H_w \quad (5.3)$$

Where  $H_{Los} \triangleq E[H]$  represent the channel mean and  $k$  define the Rician factor, which include the strength of LOS component relative to multipath component and  $H_w$  consists of *i. i. d.* circularly symmetric Gaussian random variable with zero mean and unit variance. If there is no correlation at the transmitter or at the receiver side then the entries of  $H^{sc}$  are independent and identically distributed (*i. i. d.*) complex Gaussian random variables with zero mean and unit magnitude variance, usually denoted by  $H_\omega$ . If there is correlated fading then the  $H^{sc}$  matrix can be modeled as [17].

$$H^{sc} = R_r^{1/2} H_\omega R_t^{1/2} \quad (5.4)$$

Where  $R_t$  and  $R_r$  are the correlation matrix at the transmitter and at the receiver side, respectively. The correlation matrix  $R$  is defined by exponential correlation method [13]. The receiver can be configured by variety of detection technique have been included linear, successive and no linear, here ML, ZF, MMSE detection technique are discussed.

### 5.2.1 Maximum Likelihood (ML) detection

The Maximum Likelihood (ML) detection is a scheme well known to be very robust and well suited for practical implementation whereas linear receiver suffers from numerical complexity. ML detector is the optimal receiver in terms of bit error rate (detector performance) but it is a nonlinear detector with a high complexity. The ML detection is given by [22]

$$R = \operatorname{argmin} |y - H\widehat{x}_l|^2 \quad (5.5)$$

Where element of  $R$  are optimized variable and  $|y - H\widehat{x}_l|^2$  is objective function. After minimized  $R$ , we get

$$R = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \end{bmatrix} \right|^2 \quad (5.6)$$

In case of BPSK, the possible value of  $\widehat{x}_1$  and  $\widehat{x}_2$  is used between +1 or -1, to find the minimum from the all combination of  $\widehat{x}_1$  and  $\widehat{x}_2$ . The estimation of the transmitted symbol is choose based upon the following value

$$R_{+1,+1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ +1 \end{bmatrix} \right|^2 \quad (5.7)$$

$$R_{+1,-1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ -1 \end{bmatrix} \right|^2 \quad (5.8)$$

$$R_{-1,+1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} -1 \\ +1 \end{bmatrix} \right|^2 \quad (5.9)$$

$$R_{-1,-1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right|^2 \quad (5.10)$$

### 5.2.2 Zero Force Equalizer (ZF)

The ZF is a linear estimation technique, which inverse the frequency response of received signal. The inverse is taken for the restoration of signal after the channel. The estimation of strongest transmitted signal is obtained by nulling out the weaker transmitted signal. The strongest signal is than subtracted from received signal and proceeded to decode strong

signal from the remaining transmitted signals [23]. ZF equalizer ignores the additive noise and may significantly amplify noise for channel. The major advantages of ZF linear equalizer is that it simply eliminates ISI by forcing the overall pulse, which is the convolution of the channel and the equalizer to make a unit-impulse response. Although the noise power and covariance function does not need to be estimated in ZF ever than its perform poor than that of ML [24],[25]. For a channel with frequency response  $C(f)$  the zero forcing equalizer  $E(f)$  is constructed by combining the equalizer and channel which gives a flat frequency and linear phase if and only if  $C(f) X E(f) = 1$ . The channel response is  $H(s)$  then the input signal is multiplied by the reciprocal of  $H(s)$ . The basic Zero force equalizer of 2x2 MIMO channel can be modeled by taking received signal  $y_1$  during first slot at receiver antenna is:

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (5.11)$$

The received signal  $y_2$  at the second receiver antenna

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (5.12)$$

Where  $i = 1,2$   $x_i$  is the transmitted symbol and  $i = 1,2$  and  $j = 1,2$   $h_{i,j}$  is correlated matrix of fading channel, with  $j$  represent transmitted antenna and ' $i$ ' is receiver antenna,  $n_1$  and  $n_2$  are the noise of first and second receiver antenna. Where,  $y_1$  and  $y_2$  are the received symbol on the first and second antenna respectively,  $h_{1,1}$  is the channel from 1<sup>st</sup> transmit antenna to 1<sup>st</sup> receive antenna,  $h_{1,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 1<sup>st</sup> receive antenna,  $h_{2,1}$  is the channel from 1<sup>st</sup> transmit antenna to 2<sup>nd</sup> receive antenna,  $h_{2,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 2<sup>nd</sup> receive antenna, Equivalently, ZF equalizer is given by the pseudo inverse [26] of  $H$ , i.e.

$$W_{ZF} = (H^H H)^{-1} H^H \quad (5.13)$$

Where  $W_{ZF}$  is equalization matrix and  $H$  is channel matrix. ZF detector will produce favorable result only if  $W_{ZF} X H = 1$  condition is satisfied and the diagonal element of pseudo inverse matrix should not be zero. Assuming  $M_R \geq M_T$  and  $H$  has full rank, the result of ZF equalization before quantization is written as

$$y_{ZF} = (H^H H)^{-1} H^H y \quad (5.14)$$

### 5.2.3 Minimum Mean Square Error-Estimator (MMSE)

Minimum mean square error equalizer minimizes the mean –square error between the output of the equalizer and the transmitted symbol, which is a stochastic gradient algorithm with low complexity. Unlike a ZF equalizer, an MMSE equalizer maximizes the signal–to–distortion ratio by penalizing both residual ISI and noise enhancement. Instead of removing ISI completely, an MMSE equalizer allows some residual ISI to minimize the overall distortion. Compared with a ZF equalizer, an MMSE equalizer is much more robust in presence of deepest channel nulls and noise. Most of the finite tap equalizer are designed to minimize the mean square error performance metric but MMSE directly minimizes the bit error rate. The channel model for MMSE is same as ZF [27],[28]. The MMSE equalization is

$$W_{MMSE} = \arg. \min_G E_{x,n} [\|x - x^\wedge\|^2] \quad (5.15)$$

By using orthogonality principle the  $W_{MMSE}$  is written as

$$W_{MMSE} = E_{x,n} [xy^H] (E_{x,n} [yy^H])^{-1} = H^H (HH^H + n_o I_n)^{-1} \quad (5.16)$$

A proof of equivalence between equation (5.16) and (5.17) proceed as follows

$$W_{MMSE} = H^H (HH^H + n_o I_n)^{-1} = \left\{ \frac{1}{n_o} I_n - \frac{1}{n_o} H^H \left( \frac{1}{n_o} HH^H + I_n \right)^{-1} \frac{1}{n_o} H \right\} H^H \quad (5.17)$$

$$W_{MMSE} = \frac{1}{n_o} H^H - \frac{1}{n_o} H^H \left( \frac{1}{n_o} HH^H + I_n \right)^{-1} HH^H \quad (5.18)$$

$$W_{MMSE} = \frac{1}{n_o} H^H \left( \frac{1}{n_o} HH^H + I_n \right)^{-1} \left( \frac{1}{n_o} HH^H + I_n \right) - \frac{1}{n_o} H^H \left( \frac{1}{n_o} HH^H + I_n \right)^{-1} \frac{1}{n_o} HH^H \quad (5.19)$$

$$W_{MMSE} = \frac{1}{n_o} H^H \left( \frac{1}{n_o} HH^H + I_n \right)^{-1} \left( \frac{1}{n_o} HH^H + I_n - \frac{1}{n_o} HH^H \right) \quad (5.20)$$

$$W_{MMSE} = H^H \left\{ n_o \left( \frac{1}{n_o} HH^H + I_n \right) \right\}^{-1} I_n = H^H (HH^H + n_o I_n)^{-1} \quad (5.21)$$

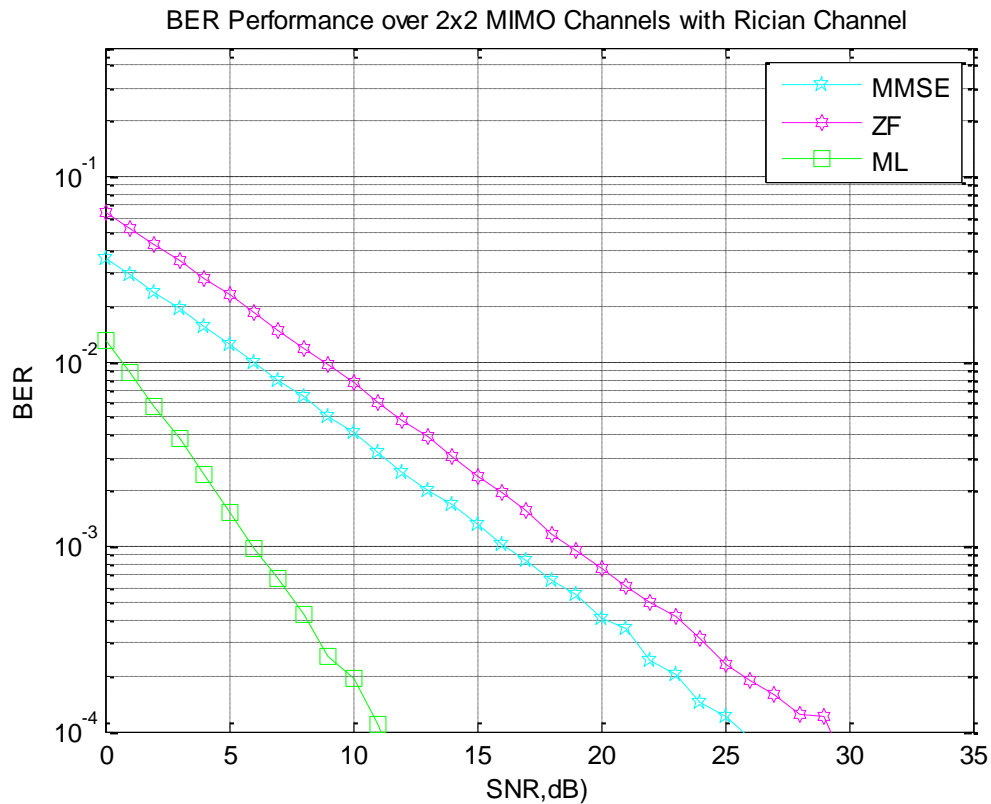
Where  $W_{MMSE}$  is equalization matrix,  $H$  is channel correlated matrix and  $n$  is channel noise.

$$y_{MMSE} = H^H (HH^H + n_o I_n)^{-1} y \quad (5.22)$$

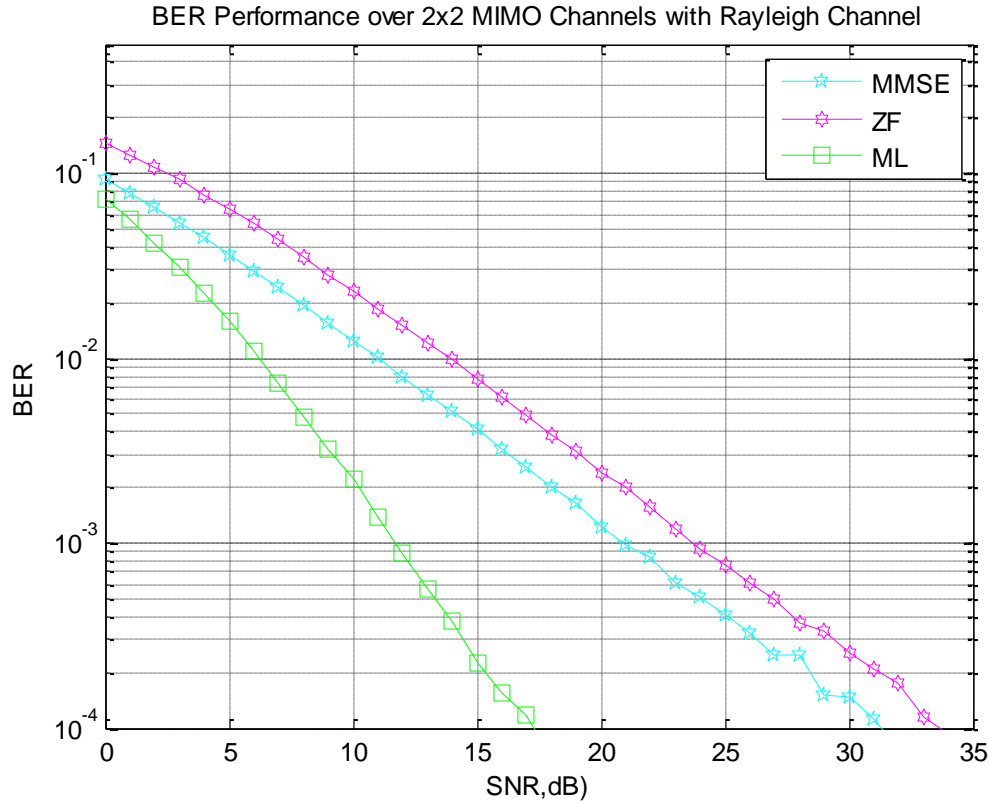
From the equation 5.23 it has been shown that only factor  $n_o I_n$  is found to be additional than that of ZF in MMSE. This factor is the instrumental force in improving the performance of MMSE equalizer over ZF equalizer.

### 5.3 Simulation Result and Discussion

The Bit Error Rate results of MIMO system for different detection technique (ZF, ML and MMSE) have been achieved by using Monte Carlo simulation. For analyzing the BER of MIMO channel Rayleigh fading channel and Rician fading channel has been considered. In the simulation 1,00,000 Monte Carlo simulations is used for different array of antennas at transmitter and receiver terminal and according result have achieved are shown in Figure.5.1 and Figure.5.2 and Figure 5.3 below:

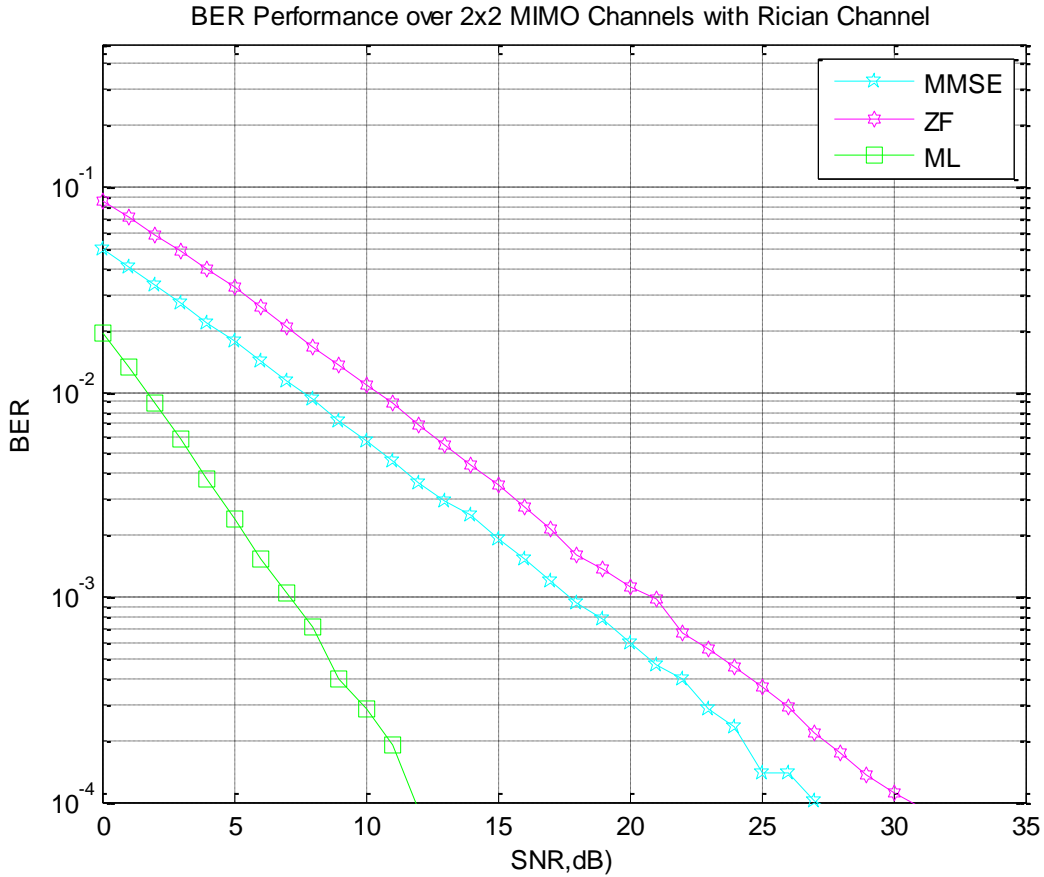


**Figure 5.1 BER versus SNR for MIMO system with Rician fading (K= 1 and Correlation parameter  $r_r = r_t = 0.7$ ) using 2x2 BPSK Modulation Scheme**



**Figure 5.2 BER versus SNR for MIMO system with Rayleigh fading using 2x2 BPSK Modulations Scheme**

After comparing the result with [49], it was found that in case of 2x2 antenna configuration using BPSK modulation asymptotic result are reasonable improved compared to all existing technique and are nearly same as that of analytical results. By comparing the Rayleigh and Rician fading, the result at  $BER \sim 10^{-4}$  with respect to SNR are improved in Rician fading up to 35% for ML detection, for ZF improvement is 24% and for MMSE detection technique result are excessively improved up to 18%. The simulation results shows that at  $BER \sim 10^{-4}$ , the SNR value of IEEE802.11n standard protocol is slightly lower than that of Rayleigh fading but still higher than that of Rician fading. In case of Rician fading, when the Rician fading parameter value increased form  $K = 1$  to  $K = 5$  then SNR also increases.



**Figure 5.3 BER versus SNR for MIMO system with Rician fading ( $K= 5$  and Correlation parameter  $r_r = r_t = 0.7$ ) using 2x2 BPSK Modulation Scheme**

The comparison table of result has been shown in Table 5.1 below.

**Table 5.1 Comparison of detection technique Scheme for MIMO**

Scheme	Performance	Complexity	Error enhancement
ZF	Worst	Very Low/ Linear	Extra High
MMSE	Poor	Low/Linear	High
ML	Optimum	Exponentially	Minimal

Lastly it has been observed that MMSE receiver detection technique performs better compared to other existing technique and promotes achieving better performances for digital transmission.

#### 5.4. MIMO Bit Error Rate Analysis

The BER of MIMO channel using ZF, ML and MMSE technique for 2x2 BPSK modulation scheme has been derived. Channel inversion is based on channel knowledge at the transmitter and receiver therefore the BER performance suffers more degradation if the CSI is imperfect than with detection technique. MMSE receiver for MIMO wireless system is more balanced linear equalizer, which not eliminate ISI completely but instead minimizes the total power of the noise and ISI components in the output. The MMSE method plays central role to get better result of BER for MIMO comparing to other technique but regarding computational complexity the ML perform better. For the case of Rician fading these techniques simulation result were compared with standard IEEE80.11n protocol [58] and there was significant improvement observed.

**Table 5.2 Comparison of BER for MIMO under BPSK modulation scheme for Rayleigh and Rician fading using detection technique**

ML	MMSE	ZF	BER scale
17	31	34	$10^{-4}$ Rayleigh 2x2
13	21	24	$10^{-3}$ Rayleigh 2x2
6	16	18	$10^{-3}$ Rician (K=1) 2x2
11	25	29	$10^{-4}$ Rician (K=1) 2x2
7	18	21	$10^{-3}$ Rician (K=5) 2x2
12	26	30	$10^{-4}$ Rician (K=5) 2x2
13	23	29	$10^{-3}$ IEEE 802.11n (K=5)
16	27	33	$10^{-4}$ IEEE 802.11n (K=5)

The simulation results shows that at  $BER \sim 10^{-4}$ , the SNR value of IEEE802.11n standard protocol is slightly lower than that of Rayleigh fading but still higher than that of Rician fading. In case of Rician fading when the Rician fading parameter value increased from  $K = 1$  to  $K = 5$  then SNR value increases, it is also shown in Table 5.2. The main focus on two transmit antennas for BER analysis, but the proposed scheme can be extended to the general system having any number of transmit antennas in a straight forward manner.

## CHAPTER 6

### Conclusion and Future Work

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#### Conclusion

The ergodic MI results of MIMO relay system have been achieved by using Monte Carlo simulation. For asymptotic MI of MIMO relay has been simulated for Rayleigh fading channel. In the simulation 1,00,000 Monte Carlo simulations are used for different array of antenna at source, relay and destination (SRD) terminal. After comparing the result with [60], it was found that in case of two antenna asymptotic result are reasonable same and nearly equal to same as that of analytical mutual information but when compared to four and eight antenna at SRD terminal, simulation result has been shown improvement in MI as high as up to 30% by increase number of antenna up to eight for SRD in symmetric and non-symmetric way. Further it has been observed that when the number of antenna at the relay terminal are more as compare to the source and destination terminal this significantly increase the MI of channel. It has been observed that there is significant increase in MI for the value of  $(M_s, M_d, M_r)$  to be (3,3,4) as compared to (3,3,3). Whereas the response gets worse with (4,4,3) as compared to (4,4,4).

In MIMO systems, the increase in value of Rician factor  $K$  and the growing number of antennas located at the transmitter and receiver end leads to the capacity degradation. From the simulation results, it is observed that the ergodic capacity is higher than the outage capacity. Also, the difference between the ergodic and the outage capacity degrades with the increasing value of Rician factor. The results are computed by setting the correlation parameter value to zero at the transmitter and 0.7 at the receiver, in this scenario, capacity of MIMO channel is observed to be 20% higher at SNR 20dB as compared to second scenario when  $(r_r = r_t = 0.7)$ . Experimental results demonstrate that reducing the correlation parameter and Rician factor to zero shows significant improvement in capacity.

The Bit Error Rate results of MIMO system for different detection technique (ZF, ML and MMSE) have been achieved by using Monte Carlo simulation. After comparing the result with [49], it was found that in case of 2x2 antenna configuration using BPSK modulation bit-

error-rate under Rician fading asymptotic result are reasonable improved compared to bit-error-rate under Rayleigh fading. By comparing the Rayleigh and Rician fading, the result at  $BER \sim 10^{-4}$  with respect to SNR are improved up to 7% for ML detection, for ZF improvement is 10% and for MMSE detection technique result are excessively improved up to 20%.

These detection techniques presented above have been further compared with the result produced from standard IEEE802.11n protocol [58] by using all parameter defined by IEEE 802.11n standard protocol, there was significant improvement observed. The MMSE method plays central role to get better result of BER for MIMO comparing to other technique but regarding complexity the ML perform better.

### **Future Work**

The work presented in this thesis can be extended to the case of multiple users. Multiple users add another dimension to multi-antenna systems. For the multi-user MIMO channel, the problem is somewhat more complex. This work can be extended to find the mutual information variance of Amplify-forward dual hop MIMO system with Rician fading (with and without separated correlation). Also the capacity of MIMO broadcast uncorrelated Rayleigh fading channels with CSI can be found. The combined multi-stage minimum mean-square error (MS-MMSE-ML) multi user detector will be used in the MIMO communicating systems over fading channels. Another extension to this work is to obtain analytical solutions to present capacity for deterministic models .

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