A STUDY ON ASSIGNMENT PROBLEM AND ITS EXTENSIONS

IN

FUZZY ENVIRONMENT

Thesis submitted in partial fulfillment of the requirement for

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In

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Submitted by

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DEDICATED

TO

GOD, MY PARENTS AND MEHAR

CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled **"A study on assignment problems and its extensions in fuzzy environment"** in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications (SMCA), Thapar University, Patiala is an authentic record of my own work carried out under the supervision of **Dr. Amit Kumar**.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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Chapter 1

Literature review

The assignment problem is a special type of linear programming problem in which our objective is to assign n number of jobs to n number of persons at a minimum cost (time). The mathematical formulation of the problem suggests that this is a 0-1 programming problem and is highly degenerate. All the algorithms developed to find optimal solution of transportation problem are applicable to assignment problem. However, due to its highly degeneracy nature, a specially designed algorithm, widely known as Hungarian method proposed by Kuhn [19], is used for its solution.

The generalized assignment problem, unlike assignment problem, does not have one-to-one correspondence between the jobs and workers.The jobs are assigned to workers in such a way that each job is assigned to exactly one worker, but a worker may be assigned more than one job, subject to availability of time at his/her disposal. In existing literature, several researchers have developed different methodologies for solving generalized assignment problems. Among these, one may refer to the works of Ross and Soland [31], Cattrysse and Wassenhove [7], Amini and Racer [2], Lorena and Narciso [25], Chu and Beasley [10], Diaz and Fernandez [11] and Haddadi and Ouzia [16]. Airline crew-scheduling, nurse-scheduling, project assignment etc. problems are solved as special cases of generalized assignment problem by several researchers like Aickelin and Dowsland [1] and Harper et al.[17].

Travelling salesman problem is a well-known NP-hard problem in combinatorial optimization. In the ordinary form of travelling salesman problem, a map of cities is given to the salesman and he has to visit all the cities only once and return to the starting point to complete the tour in such a way that the length of the tour is the shortest among all possible tours for this map. The data consists of weights assigned to the edges of a finite complete graph and the objective is to find a cycle passing through all the vertices of the graph while having the minimum total weight. There are different approaches for solving travelling salesman problem. Almost every new approach for solving engineering and optimization problems has been tried on travelling salesman problem. Many methods have been developed for solving travelling salesman problem. These methods consist of heuristic methods and population based optimization algorithms etc. Heuristic methods like cutting planes and branch and bound can optimally solve only small problems whereas the heuristic methods such as 2-opt, 3-opt, Markov chain, simulated annealing and tabu search are good for large problems. Population based optimization algorithms are a kind of nature based optimization algorithms. The natural systems and creatures which are working and developing in nature are one of the interesting and valuable sources of inspiration for designing and inventing new systems and algorithms in different fields of science and technology. Particle Swarm Optimization, Neural Networks, Evolutionary Computation, Ant Systems etc. are a few of the problem solving techniques inspired from observing nature.

Among all the aforesaid works, to the best of my knowledge, deterministic real numbers are used in concerned real life problems. However, in real life situations, the parameters in real life problems should be imprecise number instead of fixed real number as time/cost for doing a job by a worker might vary due to different reasons. Zadeh [36] introduced the concept of fuzzy sets to deal with imprecision and vagueness in real life situations. Since then, significant advances have been made on the development of numerous methodologies and their applications to various decision problems.

Fuzzy assignment problems have received great attention in recent years. For instance, Chen [9] proposed a fuzzy assignment model that did not consider the differences of individuals, and also proved some theorems. Wang [33] solved a similar model by graph theory. Dubois and Fortemps [12] proposed a flexible assignment problem, which combines with fuzzy theory, multiple criteria decision-making and constraint-directed methodology. They also demonstrated and solved an example of fuzzy assignment problem. Sakawa et al. [32] dealt with actual problems on production and work force assignment of a housing material manufacturer and formulated two-level linear and linear fractional programming problems according to profit and profitability maximization respectively. By applying interactive fuzzy programming for two-level linear and linear fractional programming problems, they derived satisfactory solutions to the problems and compared the results. Lin and Wen [22] proposed an efficient algorithm based on the labeling method for solving fuzzy assignment problems. The algorithm begins with primal feasibility and proceeds to obtain dual feasibility while maintaining complementary slackness until the primal optimal solution is found.

Feng and Yang [14] investigated a two objective-cardinality assignment problem. A chance-constrained goal programming model is constructed for the problem and tabu search algorithm based on fuzzy simulation is used to solve the problem. Majumdar and Bhunia [26] proposed an elitist genetic algorithm to solve generalized assignment problem with imprecise cost/time. Ye and Xu [35] proposed an effective method on priority-based genetic algorithm to solve fuzzy vehicle routing assignment when there is no genetic algorithm which can give clearly procedure of solving it. Liu and Gao [24] proposed an equilibrium optimization problem and extended the assignment problem to the equilibrium multi-job assignment problem, equilibrium multi-job quadratic assignment problem and used genetic algorithm to solve the proposed models. Bai et al. [4] proposed a method for solving fuzzy generalized assignment problem.

Chapter 2

Methods for solving fuzzy assignment problems and fuzzy travelling salesman problems

Mukherjee and Basu [28] proposed a method for solving fuzzy assignment problems. Kumar and Gupta [20] chosen some fuzzy assignment problems and fuzzy travelling salesman problems, which cannot be solved by using the fore-mentioned method and proposed a new method for solving such type of fuzzy assignment problems and fuzzy travelling salesman problems. In this chapter, the existing method [20] is presented.

2.1 Preliminaries

In this section, some basic definitions and Yager's ranking approach for the ranking of fuzzy numbers are presented.

2.1.1 Basic definitions

In this section, some basic definitions are presented.

Definition 2.1 [13] The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within specified range i.e. $\mu_{\tilde{A}} : X \rightarrow [0, 1]$. The assigned value indicate the membership grade of the element in the set A.

The function $\mu_{\tilde{A}}$ is called membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}) : x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2 [13] A fuzzy set $\tilde{A}$, defined on universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{A}$ is convex i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \text{minimum}(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))\ \forall\ x_1, x_2 \in R, \forall\ \lambda \in [0,1]$

(ii) $\tilde{A}$ is normal i.e., $\exists\ x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$

(iii) $\mu_{\tilde{A}}$ is piecewise continous.

Definition 2.3 [13] A fuzzy number $\widetilde{A} = (m, n, \alpha, \beta)_{L-R}$ is said to be an $L-R$ flat fuzzy number if

$$\mu_{\tilde{A}}(x) = \begin{cases} L(\frac{m-x}{\alpha}), & x \leq m,\ \alpha > 0 \\ R(\frac{x-n}{\beta}), & x \geq n,\ \beta > 0 \\ 1, & \text{otherwise} \end{cases}$$

If $m = n$ then $\widetilde{A} = (m, n, \alpha, \beta)_{L-R}$ will be converted into $\widetilde{A} = (m, \alpha, \beta)_{L-R}$ and is said to be an $L-R$ fuzzy number.

L and R are called reference functions, which are continuous, non-increasing functions that define the left and right shapes of $\mu_{\tilde{A}}(x)$ respectively and $L(0) = R(0) = 1$. Two special cases are triangular and trapezoidal fuzzy number, for which $L(x) = R(x) = \text{maximum } \{0, 1-|x|\}$, are linear functions. Three commonly used nonlinear reference functions with parameters q, denoted as RF_q, are summarized as follows:

power: $RF_q(x) = \text{maximum } (0, 1-|x|^q), \quad q \geq 0,$

exponential power: $RF_q(x) = e^{-|x|^q}, \quad q \geq 0,$

rational: $RF_q(x) = \frac{1}{(1+|x|^q)}, \quad q \geq 0.$

Definition 2.4 [13] Let $\widetilde{A} = (m, n, \alpha, \beta)_{LR}$ be an $L-R$ flat fuzzy number and λ be a real number in the interval $[0,1]$, then the crisp set $A_\lambda = \{x \in X : \mu_{\tilde{A}}(x) \geq \lambda\} =$

$[m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$ is said to be λ-cut of $\widetilde{A}$.

2.1.2 Yager's ranking approach

A number of approaches have been proposed for the ranking of fuzzy numbers. In this chapter, Yager's method [34] is used for the ranking of fuzzy numbers. Yager's method [34] involves relatively simple computation and is easily understandable. This method involves a procedure for ordering fuzzy sets in which a ranking approach $\Re(\tilde{A})$ is calculated for the fuzzy number $\widetilde{A} = (m, n, \alpha, \beta)_{LR}$ from its λ-cut $A_\lambda = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$ according to the following formula:

$$\Re(\tilde{A}) = \frac{1}{2}(\int_0^1 (m - \alpha L^{-1}(\lambda))\ d\lambda + \int_0^1 (n + \beta R^{-1}(\lambda))\ d\lambda)$$

Since $\Re(\tilde{A})$ is calculated from the extreme values of λ-cut of $\tilde{A}$ i.e., $m - \alpha L^{-1}(\lambda)$ and $n + \beta R^{-1}(\lambda)$ rather than its membership function, so it is not necessary to know the explicit form of the membership functions of the fuzzy numbers to be ranked. Unlike most of the ranking methods that require the knowledge of the membership functions of all fuzzy numbers to be ranked, the Yager's ranking approach can be applied even if the explicit form of membership function of the fuzzy number is unknown.

Let $\widetilde{A}$ and $\widetilde{B}$ be two fuzzy numbers. Then,

(i) $\widetilde{A} \succ \widetilde{B}$ if $\Re(\widetilde{A}) > \Re(\widetilde{B})$.

(ii) $\widetilde{A} \approx \widetilde{B}$ if $\Re(\widetilde{A}) = \Re(\widetilde{B})$.

(iii) $\widetilde{A} \succeq \widetilde{B}$ if $\Re(\widetilde{A}) \geq \Re(\widetilde{B})$.

2.1.2.1 Linearity property of Yager's ranking index

Let $\widetilde{A} = (m_1, n_1, \alpha_1, \beta_1)_{L_1 - R_1}$ and $\widetilde{B} = (m_2, n_2, \alpha_2, \beta_2)_{L_2 - R_2}$ be two $L - R$ flat fuzzy numbers and k_1, k_2 be two non negative real numbers.

Using Definition 2.4, the λ-cut A_λ and B_λ corresponding to $\widetilde{A}$ and $\widetilde{B}$ are

$A_\lambda = [m_1 - \alpha_1 L_1^{-1}(\lambda), n_1 + \beta_1 R_1^{-1}(\lambda)]$ and $B_\lambda = [m_2 - \alpha_2 L_2^{-1}(\lambda), n_2 + \beta_2 R_2^{-1}(\lambda)]$

Using the property, $(\delta_1 A_1 + \delta_2 A_2)_\lambda = \delta_1 (A_1)_\lambda + \delta_2 (A_2)_\lambda$, $\forall\ \delta_1, \delta_2 \in R$ (R is a set of real numbers), the λ-cut$(k_1 A + k_2 B)_\lambda$ corresponding to$(k_1 \widetilde{A} \oplus k_2 \widetilde{B})$ is

$(k_1 A + k_2 B)_\lambda = [k_1 m_1 + k_2 m_2 - k_1 \alpha_1 L_1^{-1}(\lambda) - k_2 \alpha_2 L_2^{-1}(\lambda), k_1 n_1 + k_2 n_2 + k_1 \beta_1 R_1^{-1}(\lambda) + k_2 \beta_2 R_2^{-1}(\lambda)]$

Using Section 2.1.2, the Yager's ranking index $\Re(k_1 \widetilde{A} \ \oplus \ k_2 \widetilde{B})$ corresponding to fuzzy number $(k_1 \widetilde{A} \ \oplus \ k_2 \widetilde{B})$ is

$$\begin{aligned} \Re(k_1 \widetilde{A} \ \oplus \ k_2 \widetilde{B}) &= \tfrac{1}{2}\ k_1\ [\textstyle\int_0^1 (m_1 - \alpha_1 L_1^{-1}(\lambda))\ d\lambda + \int_0^1 (n_1 + \beta_1 R_1^{-1}(\lambda))\ d\lambda] \\ &\quad + \tfrac{1}{2}\ k_2\ [\textstyle\int_0^1 (m_2 - \alpha_2 L_2^{-1}(\lambda))\ d\lambda + \int_0^1 (n_2 + \beta_2 R_2^{-1}(\lambda))\ d\lambda] \\ &= k_1 \Re(\widetilde{A}) \ + \ k_2 \Re(\widetilde{B}) \end{aligned}$$

Similarly, it can be proved that $\Re(k_1 \widetilde{A} \ \oplus \ k_2 \widetilde{B}) = k_1 \Re(\widetilde{A}) \ + \ k_2 \Re(\widetilde{B})\ \forall\ k_1,\ k_2 \in R$.

2.2 Linear programming formulations of fuzzy assignment problems and fuzzy travelling salesman problems

In this section, linear programming formulations of fuzzy assignment problems and fuzzy travelling salesman problems are presented [28].

2.2.1 Linear programming formulation of fuzzy assignment problems

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do one job at a time and each job can be assigned to one person only. Let $\tilde{c}_{ij}$ be the fuzzy cost (payment) if j^{th} job is assigned to i^{th} person. The problem is to find an assignment x_{ij} (whether i^{th} job is assigned to j^{th} person or not) so that the total cost for performing all the jobs is minimum.

The chosen fuzzy assignment problem may be formulated into the following fuzzy linear programming problem:

Minimize $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$

subject to

$$\begin{aligned} &\sum_{i=1}^{n} x_{ij} = 1, && j = 1,2,...,n \\ &\sum_{j=1}^{n} x_{ij} = 1, && i = 1,2,...,n \qquad (P_{2.1}) \\ &x_{ij} = 0 \text{ or } 1, \ \forall\ i,j \end{aligned}$$

where

$\tilde{c}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$: Fuzzy payment to i^{th} person for doing j^{th} job.

where $L(x) = R(x) = \text{maximum}\{0, 1 - |x|\}$.

$\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$: Total fuzzy cost for performing all the jobs.

2.2.2 Linear programming formulation of fuzzy travelling salesman problems

Suppose a salesman has to visit n cities. He starts from a particular city, visit each city once and then returns to the starting point. The fuzzy travelling costs from i^{th} city to j^{th} city is given by $\tilde{c}_{ij}$. The objective is to select the sequence (tour) in which the cities are visited in such a way that the total travelling cost is minimum.

The chosen fuzzy travelling salesman problem may be formulated into the following fuzzy linear programming problem

Minimize $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}\ x_{ij}$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \;\; j = 1, 2, ..., n \text{ and } j \neq i \qquad (1)$$

$$\sum_{j=1}^{n} x_{ij} = 1, \;\; i = 1, 2, ..., n \text{ and } i \neq j \qquad (2)$$

$$x_{ij} + x_{ji} \leq 1, \;\; 1 \leq i \neq j \leq n \qquad (3)$$

$$x_{ij}+x_{jk}+x_{ki} \leq 2, \; 1 \leq i \neq j \neq k \leq n \qquad (4) \qquad\qquad (P_{2.2})$$

$$\vdots$$

$$x_{ip_1} + x_{p_1p_2} + x_{p_2p_3} + ... + x_{p_{(n-2)}i} \leq (n-2),$$

$$1 \leq i \neq p_1 \neq ... \neq p_{(n-2)} \leq n \qquad (5)$$

where $\tilde{c}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{L-R}$: Fuzzy travelling cost from i^{th} city to j^{th} city, and $L(x) = R(x) = \text{maximum}\{0, 1 - |x|\}$

$\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}\; x_{ij}$: The total fuzzy travelling cost of completing the tour

$x_{ij} = 1$ if the salesman visits city j immediately after visiting city i and $x_{ij} = 0$ otherwise. Constraints (1) and (2) ensure that each city is visited only once. Constraints (3) is known as subtour elimination constraint and eliminates all $2-$city subtours. Constraints (4) eliminates all $3-$city subtours. Constraints (5) eliminates all $(n-1)-$city subtours. For a feasible solution of travelling salesman problem, the solution should not contain subtours. So, for a $5-$city travelling salesman problem, we should not have subtours of length $2, 3$ and 4. For a $6-$city travelling salesman problem, we should not have subtours of length $2, 3, 4$ and 5. Similarly, for a $n-$city travelling salesman problem, we should not have subtours of length 2 to $n-1$.

2.2.3 Optimal solution of fuzzy assignment problems

The optimal solution of the fuzzy assignment problem ($P_{2.1}$) is the set of non-negative integers $\{x_{ij}\}$ which satisfies the following characteristics:

(i) $\sum_{i=1}^{n} x_{ij} = 1,\ j = 1, 2, ..., n$ and $\sum_{j=1}^{n} x_{ij} = 1,\ i = 1, 2, ..., n$

(ii) If there exist any set of non-negative integers $\{x'_{ij}\}$, such that $\sum_{i=1}^{n} x'_{ij} = 1$, $j = 1, 2, ..., n$ and $\sum_{j=1}^{n} x'_{ij} = 1$, $i = 1, 2, ..., n$ then $\Re(\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}) < \Re(\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij} x'_{ij})$

2.2.4 Optimal solution of fuzzy travelling salesman problems

The optimal solution of the fuzzy travelling salesman problem ($P_{2.2}$) is the set of non-negative integers $\{x_{ij}\}$ which satisfies the following characteristics:

(i) $\sum_{i=1}^{n} x_{ij} = 1,\ j = 1, 2, ..., n, j \neq i$ and $\sum_{j=1}^{n} x_{ij} = 1,\ i = 1, 2, ..., n, i \neq j$ and also satisfies subtour elimination constraints.

(ii) If there exist any set of non-negative integers $\{x'_{ij}\}$, such that $\sum_{i=1}^{n} x'_{ij} = 1$, $j = 1, 2, ..., n, j \neq i$ and $\sum_{j=1}^{n} x'_{ij} = 1$, $i = 1, 2, ..., n, i \neq j$ and also satisfies subtour elimination constraints, then $\Re(\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}\ x_{ij}) < \Re(\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}\ x'_{ij})$

2.3 Limitations of existing method

In this section, the limitations of existing method [28] are discussed.

(i) The existing method [28] can be applied only to solve the following type of fuzzy assignment problems.

Example 2.1

The fuzzy assignment problem, solved by Mukherjee and Basu [28], may be formulated into the following fuzzy linear programming problem:

Minimize $((5,6,2,1)_{L-R})x_{11} \oplus ((8,11,3,1)_{L-R})x_{12} \oplus ((10,11,1,4)_{L-R})x_{13} \oplus ((8,10,3,1)_{L-R})x_{14} \oplus ((8,10,1,1)_{L-R})x_{21} \oplus ((5,6,2,1)_{L-R})x_{22} \oplus ((8,10,2,2)_{L-R})x_{23} \oplus ((8,9,3,1)_{L-R})x_{24} \oplus ((4,5,2,1)_{L-R})x_{31} \oplus ((7,10,2,1)_{L-R})x_{32} \oplus ((11,13,3,2)_{L-R})x_{33} \oplus ((6,7,2,3)_{L-R})x_{34} \oplus ((8,10,2,2)_{L-R})x_{41} \oplus ((5,6,3,1)_{L-R})x_{42} \oplus ((7,10,2,1)_{L-R})x_{43} \oplus ((4,5,2,2)_{L-R})x_{44}$

subject to

$$x_{11}+x_{12}+x_{13}+x_{14}=1 \qquad x_{11}+x_{21}+x_{31}+x_{41}=1$$

$$x_{21}+x_{22}+x_{23}+x_{24}=1 \qquad x_{12}+x_{22}+x_{32}+x_{42}=1$$

$$x_{31}+x_{32}+x_{33}+x_{34}=1 \qquad x_{13}+x_{23}+x_{33}+x_{43}=1$$

$$x_{41}+x_{42}+x_{43}+x_{44}=1 \qquad x_{14}+x_{24}+x_{34}+x_{44}=1$$

$$x_{ij}=0 \text{ or } 1, \ \forall\ i=1,2,3,4 \quad \text{and} \quad j=1,2,3,4$$

where $L(x)=R(x)=$maximum$\{0,1-|x|\}$

The existing method [28] cannot be used for solving the following type of fuzzy assignment problems.

Minimize $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$

subject to

$$\sum_{i=1}^{n} x_{ij}=1, \qquad j=1,2,...,n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1, 2, ..., n \qquad (P_{2.3})$$

$$x_{ij} = 0 \text{ or } 1, \ \forall \ i, j$$

where

$\tilde{c}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{L_{ij}-R_{ij}}$: Fuzzy payment to i^{th} person for doing j^{th} job.

$\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}x_{ij}$: Total fuzzy cost for performing all the jobs.

Example 2.2

The fuzzy assignment problem for which the fuzzy costs $\tilde{c}_{ij}$, left shape function $L_{ij}(x)$, right shape function $R_{ij}(x)$, shown in Table 2.1, may be formulated into the following fuzzy linear programming problem:

$$\text{Minimize}((1, 1.5, 1, 1)_{L_{11}-R_{11}})\ x_{11} \oplus ((6, 9, 2, 3)_{L_{12}-R_{12}})\ x_{12} \oplus ((2, 3, 1, 2)_{L_{13}-R_{13}})\ x_{13} \oplus$$
$$((6, 7, 1, 2)_{L_{21}-R_{21}})\ x_{21} \oplus ((5, 5, 1, 1)_{L_{22}-R_{22}})\ x_{22} \oplus ((9, 9, 1, 1)_{L_{23}-R_{23}})\ x_{23} \oplus$$
$$((8, 9, 2, 4)_{L_{31}-R_{31}})\ x_{31} \oplus ((4, 4, 2, 2)_{L_{32}-R_{32}})\ x_{32} \oplus ((3, 4, 2, 3)_{L_{33}-R_{33}})\ x_{33}$$

subject to

$$x_{11} + x_{12} + x_{13} = 1 \qquad x_{11} + x_{21} + x_{31} = 1$$
$$x_{21} + x_{22} + x_{23} = 1 \qquad x_{12} + x_{22} + x_{32} = 1$$
$$x_{31} + x_{32} + x_{33} = 1 \qquad x_{13} + x_{23} + x_{33} = 1$$
$$x_{ij} = 0 \ \text{ or } \ 1, \ \forall \ \ i = 1, 2, 3 \ \text{ and } \ j = 1, 2, 3$$

Table 2.1: The fuzzy costs $\tilde{c}_{ij}$, left shape function $L_{ij}(x)$, right shape function $R_{ij}(x)$

$\tilde{c}_{ij}$	$L_{ij}(x)$	$R_{ij}(x)$
$\tilde{c}_{11} = (1, 1.5, 1, 1)_{L_{11}-R_{11}}$	$L_{11}(x) =$max$\{0, 1 - x^2\}$	$R_{11}(x) =$max$\{0, 1 - x\}$
$\tilde{c}_{12} = (6, 9, 2, 3)_{L_{12}-R_{12}}$	$L_{12}(x) =$max$\{0, 1 - x^2\}$	$R_{12}(x) = e^{-x^2}$
$\tilde{c}_{13} = (2, 3, 1, 2)_{L_{13}-R_{13}}$	$L_{13}(x) =$max$\{0, 1 - x^4\}$	$R_{13}(x) = e^{-x}$
$\tilde{c}_{21} = (6, 7, 1, 2)_{L_{21}-R_{21}}$	$L_{21}(x) = e^{-x^2}$	$R_{21}(x) =$max$\{0, 1 - x^2\}$
$\tilde{c}_{22} = (5, 5, 1, 1)_{L_{22}-R_{22}}$	$L_{22}(x) =$max$\{0, 1 - x\}$	$R_{22}(x) =$max$\{0, 1 - x^4\}$
$\tilde{c}_{23} = (9, 9, 1, 1)_{L_{23}-R_{23}}$	$L_{23}(x) =$max$\{0, 1 - x^4\}$	$R_{23}(x) = e^{-x}$
$\tilde{c}_{31} = (8, 9, 2, 4)_{L_{31}-R_{31}}$	$L_{31}(x) =$max$\{0, 1 - x^4\}$	$R_{31}(x) =$max$\{0, 1 - x^2\}$
$\tilde{c}_{32} = (4, 4, 2, 2)_{L_{32}-R_{32}}$	$L_{32}(x) =$max$\{0, 1 - x^2\}$	$R_{32}(x) =$max$\{0, 1 - x^4\}$
$\tilde{c}_{33} = (3, 4, 2, 3)_{L_{33}-R_{33}}$	$L_{33}(x) =$max$\{0, 1 - x\}$	$R_{33}(x) =$max$\{0, 1 - x^4\}$

(ii) The existing method [28] can be used for solving following type of fuzzy travelling salesman problems.

Example 2.3.

The fuzzy travelling salesman problem, solved by existing method [28], may be formulated into the following fuzzy linear programming problem:

Minimize $((8, 11, 3, 1)_{L-R})x_{12} \oplus ((10, 11, 1, 4)_{L-R})x_{13} \oplus ((8, 10, 3, 1)_{L-R})x_{14} \oplus$

$((8, 10, 1, 1)_{L-R})x_{21} \oplus ((8, 10, 2, 2)_{L-R})x_{23} \oplus ((8, 9, 3, 1)_{L-R})x_{24} \oplus$

$((4, 5, 2, 1)_{L-R})x_{31} \oplus ((7, 10, 2, 1)_{L-R})x_{32} \oplus ((6, 7, 2, 3)_{L-R})x_{34} \oplus$

$((8, 10, 2, 2)_{L-R})x_{41} \oplus ((5, 6, 3, 1)_{L-R})x_{42} \oplus ((7, 10, 2, 1)_{L-R})x_{43}$

subject to

$$x_{12}+x_{13}+x_{14}=1 \qquad x_{21}+x_{31}+x_{41}=1$$

$$x_{21}+x_{23}+x_{24}=1 \qquad x_{12}+x_{32}+x_{42}=1$$

$$x_{31}+x_{32}+x_{34}=1 \qquad x_{13}+x_{23}+x_{43}=1$$

$$x_{41}+x_{42}+x_{43}=1 \qquad x_{14}+x_{24}+x_{34}=1$$

$$x_{12}+x_{21}\leq 1 \qquad x_{13}+x_{31}\leq 1 \qquad x_{14}+x_{41}\leq 1$$

$$x_{23}+x_{32}\leq 1 \qquad x_{24}+x_{42}\leq 1 \qquad x_{34}+x_{43}\leq 1$$

$$x_{ij}=0 \quad \text{or} \quad 1,\ \forall\ i=1,2,3,4 \quad \text{and} \quad j=1,2,3,4$$

where $L(x)=R(x)=$maximum$\{0,1-|x|\}$

The existing method [28] cannot be used for solving the following type of fuzzy travelling salesman problems.

Minimize $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}\ x_{ij}$

subject to

$$\sum_{i=1}^{n} x_{ij}=1,\ \ j=1,2,...,n \text{ and } j\neq i$$
$$\sum_{j=1}^{n} x_{ij}=1,\ \ i=1,2,...,n \text{ and } i\neq j$$
$$x_{ij}+x_{ji}\leq 1,\ \ 1\leq i\neq j\leq n$$
$$x_{ij}+x_{jk}+x_{ki}\leq 2,\ 1\leq i\neq j\neq k\leq n \qquad (P_{2.4})$$
$$\vdots$$
$$x_{ip_1}+x_{p_1p_2}+x_{p_2p_3}+...+x_{p_{(n-2)}i}\leq (n-2),\ \ 1\leq i\neq p_1\neq ...\neq p_{(n-2)}\leq n$$

where

$\tilde{c}_{ij}=(m_{ij},n_{ij},\alpha_{ij},\beta_{ij})_{L_{ij}-R_{ij}}$: Fuzzy travelling cost from i^{th} city to j^{th} city, $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}\ x_{ij}$: The total fuzzy travelling cost of completing the tour

$x_{ij}=1$ if the salesman visits city j immediately after visiting city i and $x_{ij}=0$

otherwise.

Example 2.4.

The fuzzy travelling salesman problem for which the fuzzy costs $\tilde{c}_{ij}$, left shape function $L_{ij}(x)$, right shape function $R_{ij}(x)$, shown in Table 2.2, may be formulated into the following fuzzy linear programming problem :

$$\begin{aligned}\text{Minimize }\; &(((1,1.5,1,1)_{L_{12}-R_{12}})x_{12}\oplus((6,9,2,3)_{L_{13}-R_{13}})x_{13}\oplus((2,3,1,2)_{L_{14}-R_{14}})x_{14}\oplus\\ &((6,7,1,2)_{L_{21}-R_{21}})x_{21}\oplus((9,11,3,4)_{L_{23}-R_{23}})x_{23}\oplus((5,5,1,1)_{L_{24}-R_{24}})x_{24}\oplus\\ &((9,9,1,1)_{L_{31}-R_{31}})x_{31}\oplus((8,9,2,4)_{L_{32}-R_{32}})x_{32}\oplus((4,4,2,2)_{L_{34}-R_{34}})x_{34}\oplus\\ &((10,11,3,4)_{L_{41}-R_{41}})x_{41}\oplus((3,4,2,3)_{L_{42}-R_{42}})x_{42}\oplus((7,8,1,3)_{L_{43}-R_{43}})x_{43})\end{aligned}$$

subject to

$$\begin{aligned} x_{12}+x_{13}+x_{14}&=1 & x_{21}+x_{31}+x_{41}&=1\\ x_{21}+x_{23}+x_{24}&=1 & x_{12}+x_{32}+x_{42}&=1\\ x_{31}+x_{32}+x_{34}&=1 & x_{13}+x_{23}+x_{43}&=1\\ x_{41}+x_{42}+x_{43}&=1 & x_{14}+x_{24}+x_{34}&=1\end{aligned}$$

$$x_{12}+x_{21}\le 1 \qquad x_{13}+x_{31}\le 1 \qquad x_{14}+x_{41}\le 1$$

$$x_{23}+x_{32}\le 1 \qquad x_{24}+x_{42}\le 1 \qquad x_{34}+x_{43}\le 1$$

$$x_{ij}=0 \;\text{ or }\; 1,\ \forall\;\; i=1,2,3,4 \;\text{ and }\; j=1,2,3,4$$

Table 2.2: The fuzzy costs $\tilde{c}_{ij}$, left shape function $L_{ij}(x)$, right shape function $R_{ij}(x)$

$\tilde{c}_{ij}$	$L_{ij}(x)$	$R_{ij}(x)$
$\tilde{c}_{12} = (1, 1.5, 1, 1)_{L_{12}-R_{12}}$	$L_{12}(x) = \max\{0, 1-x^2\}$	$R_{12}(x) = \max\{0, 1-x\}$
$\tilde{c}_{13} = (6, 9, 2, 3)_{L_{13}-R_{13}}$	$L_{13}(x) = \max\{0, 1-x^2\}$	$R_{13}(x) = e^{-x^2}$
$\tilde{c}_{14} = (2, 3, 1, 2)_{L_{14}-R_{14}}$	$L_{14}(x) = \max\{0, 1-x^4\}$	$R_{14}(x) = e^{-x}$
$\tilde{c}_{21} = (6, 7, 1, 2)_{L_{21}-R_{21}}$	$L_{21}(x) = e^{-x^2}$	$R_{21}(x) = \max\{0, 1-x^2\}$
$\tilde{c}_{23} = (9, 11, 3, 4)_{L_{23}-R_{23}}$	$L_{23}(x) = \max\{0, 1-x^2\}$	$R_{23}(x) = \max\{0, 1-x\}$
$\tilde{c}_{24} = (5, 5, 1, 1)_{L_{24}-R_{24}}$	$L_{24}(x) = \max\{0, 1-x\}$	$R_{24}(x) = \max\{0, 1-x^4\}$
$\tilde{c}_{31} = (9, 9, 1, 1)_{L_{31}-R_{31}}$	$L_{31}(x) = \max\{0, 1-x^4\}$	$R_{31}(x) = e^{-x}$
$\tilde{c}_{32} = (8, 9, 2, 4)_{L_{32}-R_{32}}$	$L_{32}(x) = \max\{0, 1-x^4\}$	$R_{32}(x) = \max\{0, 1-x^2\}$
$\tilde{c}_{34} = (4, 4, 2, 2)_{L_{34}-R_{34}}$	$L_{34}(x) = \max\{0, 1-x^2\}$	$R_{34}(x) = \max\{0, 1-x^4\}$
$\tilde{c}_{41} = (10, 11, 3, 4)_{L_{41}-R_{41}}$	$L_{41}(x) = \max\{0, 1-x^2\}$	$R_{41}(x) = \max\{0, 1-x\}$
$\tilde{c}_{42} = (3, 4, 2, 3)_{L_{42}-R_{42}}$	$L_{42}(x) = \max\{0, 1-x\}$	$R_{42}(x) = \max\{0, 1-x^4\}$
$\tilde{c}_{43} = (7, 8, 1, 3)_{L_{43}-R_{43}}$	$L_{43}(x) = \max\{0, 1-x\}$	$R_{43}(x) = e^{-x^2}$

2.4 Existing method to find the optimal solution of fuzzy assignment problems and fuzzy travelling salesman problems

In this section, the existing method [20] to find the optimal solution of fuzzy assignment problem and fuzzy travelling salesman problem is presented.

The steps of existing method [20] are as follows:

Step 1 Check that the chosen problem is fuzzy assignment problem or fuzzy travelling salesman problem.

Case (i) If the chosen problem is fuzzy assignment problem, then formulate it as $(P_{2.3})$.

Case (ii) If the chosen problem is fuzzy travelling salesman problem, then formulate it as $(P_{2.4})$.

Step 2 Convert fuzzy linear programming problem ($P_{2.3}$ or $P_{2.4}$), obtained in Step 1, into the following crisp linear programming problem:

Minimize $\sum_{i=1}^{n}\sum_{j=1}^{n} \Re(\tilde{c}_{ij})\ x_{ij}$

subject to respective constraints.

Step 3 Solve crisp linear programming problem, obtained in Step 2, to find the optimal solution $\{x_{ij}\}$ and Yager's ranking index $(\sum_{i=1}^{n}\sum_{j=1}^{n} \Re(\tilde{c}_{ij})\ x_{ij})$ corresponding to minimum total fuzzy cost.

2.5 Optimal solution of fuzzy assignment problem

In this section, fuzzy assignment problem chosen in Example 2.2, which cannot be solved by using the existing method [28], is solved by using the existing method [20].

The optimal solution of fuzzy assignment problem, chosen in Example 2.2, may be obtained by using the following steps of the existing method [28]:

Step 1 The fuzzy linear programming problem of the fuzzy assignment problem, chosen in Example 2.2 is:

$$\text{Minimize } ((1,1.5,1,1)_{L_{11}-R_{11}})\ x_{11} \oplus ((6,9,2,3)_{L_{12}-R_{12}})\ x_{12} \oplus ((2,3,1,2)_{L_{13}-R_{13}})\ x_{13} \oplus$$
$$((6,7,1,2)_{L_{21}-R_{21}})\ x_{21} \oplus ((5,5,1,1)_{L_{22}-R_{22}})\ x_{22} \oplus ((9,9,1,1)_{L_{23}-R_{23}})\ x_{23} \oplus$$
$$((8,9,2,4)_{L_{31}-R_{31}})\ x_{31} \oplus ((4,4,2,2)_{L_{32}-R_{32}})\ x_{32} \oplus ((3,4,2,3)_{L_{33}-R_{33}})\ x_{33}$$

subject to

$$x_{11} + x_{12} + x_{13} = 1 \qquad x_{11} + x_{21} + x_{31} = 1$$
$$x_{21} + x_{22} + x_{23} = 1 \qquad x_{12} + x_{22} + x_{32} = 1$$
$$x_{31} + x_{32} + x_{33} = 1 \qquad x_{13} + x_{23} + x_{33} = 1$$
$$x_{ij} = 0 \;\text{ or }\; 1,\ \forall\, i = 1,2,3 \quad \text{and} \quad j = 1,2,3$$

Step 2 Using Step 2 of the existing method [20], the formulated fuzzy linear programming is converted into the following crisp linear programming problem:

Minimize $(\Re(1,1.5,1,1)_{L_{11}-R_{11}})\ x_{11} + (\Re(6,9,2,3)_{L_{12}-R_{12}})\ x_{12} + (\Re(2,3,1,2)_{L_{13}-R_{13}})$
$x_{13} + (\Re(6,7,1,2)_{L_{21}-R_{21}})\ x_{21} + (\Re(5,5,1,1)_{L_{22}-R_{22}})\ x_{22} + (\Re(9,9,1,1)_{L_{23}-R_{23}})$
$x_{23} + (\Re(8,9,2,4)_{L_{31}-R_{31}})\ x_{31} + (\Re(4,4,2,2)_{L_{32}-R_{32}})\ x_{32} + (\Re(3,4,2,3)_{L_{33}-R_{33}})\ x_{33}$

subject to

$$x_{11} + x_{12} + x_{13} = 1 \qquad x_{11} + x_{21} + x_{31} = 1$$
$$x_{21} + x_{22} + x_{23} = 1 \qquad x_{12} + x_{22} + x_{32} = 1$$
$$x_{31} + x_{32} + x_{33} = 1 \qquad x_{13} + x_{23} + x_{33} = 1$$
$$x_{ij} = 0 \;\text{ or }\; 1,\ \forall\, i = 1,2,3 \quad \text{and} \quad j = 1,2,3$$

Step 3 Using Definition 2.4 and Section 2.1.2, the values of $\Re(c_{ij}),\ \forall\, i,j$ are $\Re(\tilde{c}_{11}) = 1.1667$, $\Re(\tilde{c}_{12}) = 8.1623$, $\Re(\tilde{c}_{13}) = 3.1$, $\Re(\tilde{c}_{21}) = 6.7337$, $\Re(\tilde{c}_{22}) = 5.15$, $\Re(\tilde{c}_{23}) = 9.1$, $\Re(\tilde{c}_{31}) = 9.03333$, $\Re(\tilde{c}_{32}) = 4.13333$, $\Re(\tilde{c}_{33}) = 4.2$

Using the values of $\Re(\tilde{c}_{ij})$, the crisp linear programming problem obtained in Step 2 may be written as:

Minimize $(1.1667\ x_{11} + 8.1623\ x_{12} + 3.1\ x_{13} + 6.7337\ x_{21} + 5.15\ x_{22} + 9.1\ x_{23} +$
$9.03333\ x_{31} + 4.13333\ x_{32} + 4.2\ x_{33})$

subject to

$$x_{11} + x_{12} + x_{13} = 1 \qquad x_{11} + x_{21} + x_{31} = 1$$

$$x_{21} + x_{22} + x_{23} = 1 \qquad x_{12} + x_{22} + x_{32} = 1$$

$$x_{31} + x_{32} + x_{33} = 1 \qquad x_{13} + x_{23} + x_{33} = 1$$

$$x_{ij} = 0 \quad \text{or} \quad 1, \ \forall \ \ i = 1, 2, 3 \quad \text{and} \quad j = 1, 2, 3$$

Step 4 Solving the crisp linear programming problem, obtained in Step 3, the optimal solution is $x_{11} = 1$, $x_{22} = 1$, $x_{33} = 1$ and Yager's ranking index corresponding to minimum total fuzzy cost is 10.5167

2.6 Optimal solution of fuzzy travelling salesman problem

In this section, fuzzy travelling salesman problem chosen in Example 2.4, which cannot be solved by using the existing method [28], is solved by using the the existing method [20].

The optimal solution of fuzzy travelling salesman problem, chosen in Example 2.4, may be obtained by using the following steps of the existing method [20].

Step 1 The fuzzy linear programming problem of the fuzzy travelling salesman problem, chosen in Example 2.4, is:

$$\begin{aligned} \text{Minimize} \ \ & (((1, 1.5, 1, 1)_{L_{12}-R_{12}})x_{12} \oplus ((6, 9, 2, 3)_{L_{13}-R_{13}})x_{13} \oplus ((2, 3, 1, 2)_{L_{14}-R_{14}})x_{14} \oplus \\ & ((6, 7, 1, 2)_{L_{21}-R_{21}})x_{21} \oplus ((9, 11, 3, 4)_{L_{23}-R_{23}})x_{23} \oplus ((5, 5, 1, 1)_{L_{24}-R_{24}})x_{24} \oplus \\ & ((9, 9, 1, 1)_{L_{31}-R_{31}})x_{31} \oplus ((8, 9, 2, 4)_{L_{32}-R_{32}})x_{32} \oplus ((4, 4, 2, 2)_{L_{34}-R_{34}})x_{34} \oplus \\ & ((10, 11, 3, 4)_{L_{41}-R_{41}})x_{41} \oplus ((3, 4, 2, 3)_{L_{42}-R_{42}})x_{42} \oplus ((7, 8, 1, 3)_{L_{43}-R_{43}})x_{43}) \end{aligned}$$

subject to

$$x_{12} + x_{13} + x_{14} = 1 \qquad x_{21} + x_{31} + x_{41} = 1$$

$$x_{21} + x_{23} + x_{24} = 1 \qquad x_{12} + x_{32} + x_{42} = 1$$

$$x_{31}+x_{32}+x_{34}=1 \qquad x_{13}+x_{23}+x_{43}=1$$

$$x_{41}+x_{42}+x_{43}=1 \qquad x_{14}+x_{24}+x_{34}=1$$

$$x_{12}+x_{21}\leq 1 \qquad x_{13}+x_{31}\leq 1 \qquad x_{14}+x_{41}\leq 1$$

$$x_{23}+x_{32}\leq 1 \qquad x_{24}+x_{42}\leq 1 \qquad x_{34}+x_{43}\leq 1$$

$$x_{ij}=0 \text{ or } 1,\ \forall\ i=1,2,3,4 \text{ and } j=1,2,3,4$$

Step 2 Using Step 2 of existing method [20], the formulated fuzzy linear programming problem is converted into the following crisp linear programming problem:

Minimize $((\Re(1,1.5,1,1)_{L_{12}-R_{12}})x_{12}+(\Re(6,9,2,3)_{L_{13}-R_{13}})x_{13}+(\Re(2,3,1,2)_{L_{14}-R_{14}})$
$x_{14}+(\Re(6,7,1,2)_{L_{21}-R_{21}})x_{21}+(\Re(9,11,3,4)_{L_{23}-R_{23}})x_{23}+(\Re(5,5,1,1)_{L_{24}-R_{24}})$
$x_{24}+(\Re(9,9,1,1)_{L_{31}-R_{31}})x_{31}+(\Re(8,9,2,4)_{L_{32}-R_{32}})x_{32}+(\Re(4,4,2,2)_{L_{34}-R_{34}})$
$x_{34}+(\Re(10,11,3,4)_{L_{41}-R_{41}})x_{41}+(\Re(3,4,2,3)_{L_{42}-R_{42}})x_{42}+(\Re(7,8,1,3)_{L_{43}-R_{43}})x_{43})$

subject to

$$x_{12}+x_{13}+x_{14}=1 \qquad x_{21}+x_{31}+x_{41}=1$$

$$x_{21}+x_{23}+x_{24}=1 \qquad x_{12}+x_{32}+x_{42}=1$$

$$x_{31}+x_{32}+x_{34}=1 \qquad x_{13}+x_{23}+x_{43}=1$$

$$x_{41}+x_{42}+x_{43}=1 \qquad x_{14}+x_{24}+x_{34}=1$$

$$x_{12}+x_{21}\leq 1 \qquad x_{13}+x_{31}\leq 1 \qquad x_{14}+x_{41}\leq 1$$

$$x_{23}+x_{32}\leq 1 \qquad x_{24}+x_{42}\leq 1 \qquad x_{34}+x_{43}\leq 1$$

$$x_{ij}=0 \text{ or } 1,\ \forall\ i=1,2,3,4 \text{ and } j=1,2,3,4$$

Step 3 Using Definition 2.4 and Section 2.1.2, the values of $\Re(c_{ij}),\ \forall\ i,j$ are:

$\Re(\tilde{c}_{12})=1.1667$, $\Re(\tilde{c}_{13})=8.1623$, $\Re(\tilde{c}_{14})=3.1$, $\Re(\tilde{c}_{21})=6.7337$, $\Re(\tilde{c}_{23})=10$, $\Re(\tilde{c}_{24})=5.15$, $\Re(\tilde{c}_{31})=9.1$, $\Re(\tilde{c}_{32})=9.03333$, $\Re(\tilde{c}_{34})=4.13333$ $\Re(\tilde{c}_{41})=10.5$, $\Re(\tilde{c}_{42})=4.2$, $\Re(\tilde{c}_{43})=8.549$

Using the values of $\Re(\tilde{c}_{ij})$, the crisp linear programming problem obtained in Step 2 may be written as:

Minimize $((1.1667)x_{12} + (8.1623)x_{13} + (3.1)x_{14} + (6.7337)x_{21} + (10)x_{23} + (5.15)x_{24} + (9.1)x_{31} + (9.03333)x_{32} + (4.13333)x_{33} + (10.5)x_{41} + (4.2)x_{42} + (8.549)x_{43})$

subject to

$$x_{12} + x_{13} + x_{14} = 1 \qquad x_{21} + x_{31} + x_{41} = 1$$

$$x_{21} + x_{23} + x_{24} = 1 \qquad x_{12} + x_{32} + x_{42} = 1$$

$$x_{31} + x_{32} + x_{34} = 1 \qquad x_{13} + x_{23} + x_{43} = 1$$

$$x_{41} + x_{42} + x_{43} = 1 \qquad x_{14} + x_{24} + x_{34} = 1$$

$$x_{12} + x_{21} \leq 1 \qquad x_{13} + x_{31} \leq 1 \qquad x_{14} + x_{41} \leq 1$$

$$x_{23} + x_{32} \leq 1 \qquad x_{24} + x_{42} \leq 1 \qquad x_{34} + x_{43} \leq 1$$

$$x_{ij} = 0 \quad \text{or} \quad 1, \ \forall \ i = 1,2,3,4 \quad \text{and} \quad j = 1,2,3,4$$

Step 4 The optimal solution of the crisp linear programming problem, obtained in Step 3, is $x_{13} = 1$, $x_{34} = 1$, $x_{42} = 1$, $x_{21} = 1$ and Yager's ranking index corresponding to minimum total travelling cost is 23.2293.

2.7 Results and discussion

To compare the existing [28] and the existing method [20], the results of fuzzy assigment problems and fuzzy travelling salesman problems chosen in Example 2.1, Example 2.2, Example 2.3 and Example 2.4 obtained by using the existing method [20] are shown in Table 2.3:

Table 2.3: Comparison of results obtained by using existing method [28] and the existing method [20]

Examples	Existing method [28]	Existing method [20]
Example 2.1	$J_1 \to P_3, J_2 \to P_2$ $J_3 \to P_1, J_4 \to P_4$ and Minimum total fuzzy cost= (23,27,7,8)	$J_1 \to P_3, J_2 \to P_2$ $J_3 \to P_1, J_4 \to P_4$ and Minimum total fuzzy cost= (23,27,7,8)
Example 2.2	Not applicable	$J_1 \to P_1$ $J_2 \to P_2$ $J_3 \to P_3$ and Yager's ranking index corresponding to minimum total fuzzy cost=10.5167
Example 2.3	$1 \to 4 \to 2 \to 3 \to 1$ and Minimum total fuzzy cost= (25,31,10,5)	$1 \to 4 \to 2 \to 3 \to 1$ and Minimum total fuzzy cost= (25,31,10,5)
Example 2.4	Not applicable	$1 \to 3 \to 4 \to 2 \to 1$ and Yager's ranking index corresponding to minimum total fuzzy cost=23.2293

It is obvious from the results shown in Table 2.3 that irrespective of whether we use existing method [28] or existing method [20], same results are obtained for Example 2.1 and Example 2.3. While, Example 2.2 and Example 2.4 can be solved only by using the existing method [20]. On the basis of above results, it can be suggested that it is better to use the existing method [20] instead of existing method [28] to solve fuzzy assignment problems and fuzzy travelling salesman problems.

2.8 Conclusions

On the basis of the presented study, it can be concluded that, it is better to use existing method [20] as compared to the existing method [28] for solving fuzzy assignment and fuzzy travelling salesman problems.

Chapter 3

A new method for solving fuzzy generalized assignment problems

Bai et al. [4] proposed a method for solving fuzzy generalized assignment problem. Kumar and gupta [21] pointed out the limitation of existing method and proposed method. To overcome the limitation, In this chapter, the existing method [21] for solving the generalized fuzzy assignment problem is presented.

3.1 Optimal solution of fuzzy generalized assignment problems

In this section, the existing method [21] to find the optimal solution of fuzzy generalized assignment problems, occurring in real life situations in which such parameters are represented by different type of $L-R$ fuzzy numbers is presented. The steps of existing method [21] are as follows:

Step 1 Formulate the given fuzzy generalized assignment problem as $P_{3.1}$.

Minimize $\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}\tilde{t}_{ij}x_{ij}$

subject to

$$\begin{aligned} &\sum_{j=1}^{m} x_{ij} = 1, && i = 1, 2, \ldots, n \\ &\sum_{i=1}^{n} \tilde{t}_{ij}\; x_{ij} \preceq \tilde{a}_j, && j = 1, 2, \ldots, m \qquad (P_{3.1}) \\ &x_{ij} = 0 \text{ or } 1,\ \forall\ i, j. \end{aligned}$$

where,

$\tilde{t}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{L_{ij}-R_{ij}}$ is fuzzy time required to perform i^{th} job by j^{th} person.

$\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}\tilde{t}_{ij}x_{ij}$ = total fuzzy time for performing all the jobs.

Step 2 Convert fuzzy linear programming problem $P_{3.1}$, obtained in Step 1, into the following crisp linear programming problem:

Minimize $\sum_{i=1}^{n}\sum_{j=1}^{m} \Re(\tilde{t}_{ij})\ x_{ij}$

subject to

$$\sum_{j=1}^{m} x_{ij} = 1, \qquad i = 1, 2, ..., n$$
$$\sum_{i=1}^{n} \Re(\tilde{t}_{ij})\ x_{ij} \leq \Re(\tilde{a}_j), \qquad j = 1, 2, ..., m$$
$$x_{ij} = 0 \text{ or } 1 \quad \forall\ i, j.$$

Step 3 Solve crisp linear programming problem, obtained in Step 2, to find the optimal solution $\{x_{ij}\}$ and Yager's ranking index $(\sum_{i=1}^{n}\sum_{j=1}^{m} \Re(\tilde{t}_{ij})\ x_{ij})$ corresponding to minimum total fuzzy time.

3.2 Optimal solution of fuzzy generalized assignment problem

In this section, fuzzy generalized assignment problem, chosen in Example 3.1, which cannot be solved by using the existing method [4], is solved using the existing method [21].

Example 3.1

Suppose there are five jobs J_1 to J_5 to be performed and two persons P_1 and P_2 are available for doing these jobs. Assume that each job can be assigned to one person only and each person can do more than one job according to availability of time $\tilde{a}_j$ with person P_j. From past records, the time t_{ij} that person P_j takes to do J_i job are represented by different type of $L-R$ fuzzy numbers and are shown in following Table 3.1. The problem is to find an assignment x_{ij} (which job should be assigned to which person) so that the total time for performing all the jobs is minimum.

Table 3.1: The fuzzy times $\tilde{t}_{ij}$, left shape function $L_{ij}(x)$, right shape function $R_{ij}(x)$

$\tilde{t}_{ij}$	$L_{ij}(x)$	$R_{ij}(x)$
$\tilde{t}_{11} = (1, 1.5, 1, 1)_{L_{11}-R_{11}}$	$L_{11}(x) = \max\{0, 1 - x^2\}$	$R_{11}(x) = \max\{0, 1 - x\}$
$\tilde{t}_{12} = (6, 9, 2, 3)_{L_{12}-R_{12}}$	$L_{12}(x) = \max\{0, 1 - x^2\}$	$R_{12}(x) = e^{-x^2}$
$\tilde{t}_{21} = (2, 3, 1, 2)_{L_{21}-R_{21}}$	$L_{21}(x) = \max\{0, 1 - x^4\}$	$R_{21}(x) = e^{-x}$
$\tilde{t}_{22} = (6, 7, 1, 2)_{L_{22}-R_{22}}$	$L_{22}(x) = e^{-x^2}$	$R_{22}(x) = \max\{0, 1 - x^2\}$
$\tilde{t}_{31} = (9, 11, 3, 4)_{L_{31}-R_{31}}$	$L_{31}(x) = \max\{0, 1 - x^2\}$	$R_{31}(x) = \max\{0, 1 - x\}$
$\tilde{t}_{32} = (5, 5, 1, 1)_{L_{32}-R_{32}}$	$L_{32}(x) = \max\{0, 1 - x\}$	$R_{32}(x) = \max\{0, 1 - x^4\}$
$\tilde{t}_{41} = (9, 9, 1, 1)_{L_{41}-R_{41}}$	$L_{41}(x) = \max\{0, 1 - x^4\}$	$R_{41}(x) = e^{-x}$
$\tilde{t}_{42} = (8, 9, 2, 4)_{L_{42}-R_{42}}$	$L_{42}(x) = \max\{0, 1 - x^4\}$	$R_{42}(x) = \max\{0, 1 - x^2\}$
$\tilde{t}_{51} = (4, 4, 2, 2)_{L_{51}-R_{51}}$	$L_{51}(x) = \max\{0, 1 - x^2\}$	$R_{51}(x) = \max\{0, 1 - x^4\}$
$\tilde{t}_{52} = (10, 11, 3, 4)_{L_{52}-R_{52}}$	$L_{52}(x) = \max\{0, 1 - x^2\}$	$R_{52}(x) = \max\{0, 1 - x\}$
$\tilde{a}_1 = (14, 16, 3, 4)_{L_1-R_1}$	$L_1(x) = \max\{0, 1 - x^2\}$	$R_1(x) = \max\{0, 1 - x\}$
$\tilde{a}_2 = (11, 13, 3, 4)_{L_2-R_2}$	$L_2(x) = \max\{0, 1 - x^2\}$	$R_2(x) = \max\{0, 1 - x\}$

Step 1 The fuzzy linear programming problem of fuzzy generalized assignment problem, chosen in Example 3.1, is:

$$\text{Minimize } (((1, 1.5, 1, 1)_{L_{11}-R_{11}})\, x_{11} \oplus ((6, 9, 2, 3)_{L_{12}-R_{12}})\, x_{12} \oplus ((2, 3, 1, 2)_{L_{21}-R_{21}})\, x_{21} \oplus ((6, 7, 1, 2)_{L_{22}-R_{22}})\, x_{22} \oplus ((9, 11, 3, 4)_{L_{31}-R_{31}})\, x_{31} \oplus ((5, 5, 1, 1)_{L_{32}-R_{32}})\, x_{32} \oplus ((9, 9, 1, 1)_{L_{41}-R_{41}})\, x_{41} \oplus ((8, 9, 2, 4)_{L_{42}-R_{42}})\, x_{42} \oplus ((4, 4, 2, 2)_{L_{51}-R_{51}})\, x_{51} \oplus ((10, 11, 3, 4)_{L_{52}-R_{52}})\, x_{52})$$

subject to

$$x_{11} + x_{12} = 1 \qquad x_{21} + x_{22} = 1 \qquad x_{31} + x_{32} = 1$$

$$x_{41} + x_{42} = 1 \qquad x_{51} + x_{52} = 1$$

$((1, 1.5, 1, 1)_{L_{11}-R_{11}})\ x_{11} \oplus ((2, 3, 1, 2)_{L_{21}-R_{21}})\ x_{21} \oplus ((9, 11, 3, 4)_{L_{31}-R_{31}})\ x_{31} \oplus$
$((9, 9, 1, 1)_{L_{41}-R_{41}})\ x_{41} \oplus ((4, 4, 2, 2)_{L_{51}-R_{51}})\ x_{51} \preceq (14, 16, 3, 4)_{L_1-R_1}$

$((6, 9, 2, 3)_{L_{12}-R_{12}})\ x_{12} \oplus ((6, 7, 1, 2)_{L_{22}-R_{22}})\ x_{22} \oplus ((5, 5, 1, 1)_{L_{32}-R_{32}})\ x_{32} \oplus$
$((8, 9, 2, 4)_{L_{42}-R_{42}})\ x_{42} \oplus ((10, 11, 3, 4)_{L_{52}-R_{52}})\ x_{52} \preceq (11, 13, 3, 4)_{L_2-R_2}$

$x_{ij} = 0$ or $1,\ \forall\ i = 1, 2, \ldots, 5$ and $j = 1, 2$

Step 2 Using Step 2 of existing method [21], the formulated fuzzy linear programming problem is converted into the following crisp linear programming problem:

Minimize $(\Re((1, 1.5, 1, 1)_{L_{11}-R_{11}})\ x_{11} + \Re((6, 9, 2, 3)_{L_{12}-R_{12}})\ x_{12} + \Re((2, 3, 1, 2)_{L_{21}-R_{21}})$
$x_{21} + \Re((6, 7, 1, 2)_{L_{22}-R_{22}})\ x_{22} + \Re((9, 11, 3, 4)_{L_{31}-R_{31}})\ x_{31} + \Re((5, 5, 1, 1)_{L_{32}-R_{32}})\ x_{32} +$
$\Re((9, 9, 1, 1)_{L_{41}-R_{41}})\ x_{41} + \Re((8, 9, 2, 4)_{L_{42}-R_{42}})\ x_{42} + \Re((4, 4, 2, 2)_{L_{51}-R_{51}})\ x_{51} +$
$\Re((10, 11, 3, 4)_{L_{52}-R_{52}})\ x_{52})$

subject to

$$x_{11} + x_{12} = 1 \qquad x_{21} + x_{22} = 1 \qquad x_{31} + x_{32} = 1$$

$$x_{41} + x_{42} = 1 \qquad x_{51} + x_{52} = 1$$

$(\Re((1, 1.5, 1, 1)_{L_{11}-R_{11}})\ x_{11} + \Re((2, 3, 1, 2)_{L_{21}-R_{21}})\ x_{21} + \Re((9, 11, 3, 4)_{L_{31}-R_{31}})\ x_{31} +$
$\Re((9, 9, 1, 1)_{L_{41}-R_{41}})\ x_{41} + \Re((4, 4, 2, 2)_{L_{51}-R_{51}})\ x_{51}) \leq \Re(14, 16, 3, 4)_{L_1-R_1}$

$(\Re((6, 9, 2, 3)_{L_{12}-R_{12}})\ x_{12} + \Re((6, 7, 1, 2)_{L_{22}-R_{22}})\ x_{22} + \Re((5, 5, 1, 1)_{L_{32}-R_{32}})\ x_{32} +$

$\Re((8,9,2,4)_{L_{42}-R_{42}})\ x_{42} + \Re((10,11,3,4)_{L_{52}-R_{52}})\ x_{52}) \leq \Re(11,13,3,4)_{L_2-R_2}$

$x_{ij} = 0$ or 1, $\forall\ i = 1,2,\ldots,5$ and $j = 1,2$

Step 3 Using and Section 2.1.2, the values of $\Re(\tilde{t}_{ij})$ and $\Re(\tilde{a}_j)\ \forall\ i,j$ are:

$\Re(\tilde{t}_{11}) = 1.1667$, $\Re(\tilde{t}_{12}) = 8.1623$, $\Re(\tilde{t}_{21}) = 3.1$, $\Re(\tilde{t}_{22}) = 6.7337$, $\Re(\tilde{t}_{31}) = 10$, $\Re(\tilde{t}_{32}) = 5.15$, $\Re(\tilde{t}_{41}) = 9.1$, $\Re(\tilde{t}_{42}) = 9.0333$, $\Re(\tilde{t}_{51}) = 4.1333$, $\Re(\tilde{t}_{52}) = 10.5$, $\Re(\tilde{a}_1) = 15$ and $\Re(\tilde{a}_2) = 12$.

Using the values of $\Re(\tilde{t}_{ij})$ and $\Re(\tilde{a}_j)$ the crisp linear programming problem, obtained in Step 2, may be written as:

Minimize $(1.1667\ x_{11} + 8.1623\ x_{12} + 3.1\ x_{21} + 6.7337\ x_{22} + 10\ x_{31} + 5.15\ x_{32} + 9.1\ x_{41} + 9.0333\ x_{42} + 4.1333\ x_{51} + 10.5\ x_{52})$

subject to

$$x_{11} + x_{12} = 1 \qquad x_{21} + x_{22} = 1 \qquad x_{31} + x_{32} = 1$$

$$x_{41} + x_{42} = 1 \qquad x_{51} + x_{52} = 1$$

$1.1667\ x_{11} + 3.1\ x_{21} + 10\ x_{31} + 9.1\ x_{41} + 4.1333\ x_{51} \leq 15$

$8.1623\ x_{12} + 6.7337\ x_{22} + 5.15\ x_{32} + 9.0333\ x_{42} + 10.5\ x_{52} \leq 12$

$x_{ij} = 0$ or 1, $\forall\ i = 1,2,\ldots,5$ and $j = 1,2$.

Step 4 Solving the crisp linear programming problem, obtained in Step 3, the optimal solution is $x_{11} = 1$, $x_{22} = 1$, $x_{32} = 1$, $x_{41} = 1$, $x_{51} = 1$ and Yager's ranking index corresponding to minimum total fuzzy time is 26.2837.

3.3 Results and discussion

To compare the existing method [4] and the existing method [21], the results of fuzzy generalized assigment problem, chosen in Example 3.1, obtained by using the existing method [4] and the existing method [21] are shown in Table 3.2:

Table 3.2: Comparison of results obtained by using existing method [4] and existing method [21]

Example	Existing method [4]	Existing method [21]
Example	It cannot be used here as fuzzy time variables are represented by continuous fuzzy numbers. It can only be applied when fuzzy time variables are represented by discrete fuzzy numbers.	$J_1 \to P_1, J_4 \to P_1, J_5 \to P_1$ $J_2 \to P_2, J_3 \to P_2$ and Yager's ranking index corresponding to minimum total fuzzy time=26.2837

It is obvious from the results shown in Table 3.2 that the chosen problem may not be solved by using the existing method while it can be solved by using the proposed method. On the basis of above results, it can be suggested that it is better to use the fuzzy time variables as continuous fuzzy numbers, because they represent real life problems in real manner and proposed method to solve fuzzy generalized assignment problems with fuzzy time variables as continuous fuzzy variables.

3.4 Conclusions

On the basis of the presented study it can be concluded that it is better to use the existing method [21] as compared to the existing method [4] for generalized fizzy assignment problems.

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