Analytic and Computational Studies of Structures and Oscillations of Rotating Stars and Stars in Binary Systems

(Ph. D. Thesis)

Submitted by

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A Thesis

submitted in fulfillment of the
requirement for the award of the degree of

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CANDIDATE'S DECLARATION

I hereby certify that the work being presented in the thesis entitled **"Analytic and Computational Studies of Structures and Oscillations of Rotating Stars and Stars in Binary Systems"** in fulfillment of the requirements for the award of the degree of Doctor of Philosophy, submitted in the School of Mathematics and Computer Applications of Thapar University, Patiala, is an authentic record of my own work carried out during a period from January, 2005 to May 2009 under the supervision of Dr. A. K. Lal.

The matter embodied in this thesis has not been submitted by me for the award of any other degree:

Date: **(Ankush Pathania)**

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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Abstract

In the present thesis an attempt has been made to extend the work of Mohan et al (108, 109) and Lal (70). This work relates to the analytic studies of the problems of computing the equilibrium structure and the periods of small adiabatic oscillations of rotating stars and stars in binary systems. While computing the effects of rotational and tidal distortions on the equilibrium structures and the eigenfrequencies of radial and nonradial modes of oscillations of rotating stars and stars in binary systems, Mohan and Saxena (99, 100) and Mohan et al (108, 109, 104, 107) did not explicitly account for the effect of Coriolis force and they analyze these problems in inertial frame of reference only. In the present study we have tried to investigate the effects of inclusion of Coriolis force on the equilibrium structures as well as periods of small adiabatic barotropic modes of oscillations of rotating stars and stars in binary systems. In the case of some of the observed rotating stars and stars in binary systems, rotation is not only expected to be uniform but also differential in which angular velocity of rotation varies from point to point. In this thesis we have also investigated the effect of Coriolis force on the equilibrium structures and periods of small adiabatic barotropic oscillations of differentially rotating stars.

Thesis consists of seven chapters. Chapter I is introductory in nature. In this chapter we briefly discuss the astrophysical significance of the problem of determining the equilibrium structures and periods of small adiabatic oscillations of rotationally and tidally distorted stellar models of the stars. Mohan, Saxena and Agarwal (108, 109) approach of determining the effects of rotation and/or tidal distortions on the equilibrium structure and the eigenfrequencies of radial and nonradial modes of oscillations of the theoretical models of the stars is also discussed. A brief survey of the literature available on the subject and summary of the work presented in the succeeding chapters of the thesis are also given in this chapter.

In chapter II we first develop an expression for the Roche equipotential of a rotating star in a binary system in a rotating frame of reference to explicitly include the effect of Coriolis force besides the centrifugal and gravitational forces. Next, we use it to determine the effects of Coriolis force on the shapes of Roche equipotential surfaces and position of Roche limit for different types of binary stars. Numerical computations have been performed to determine the shapes of Roche equipotentials and the values of Roche limits for different values of mass ratio of the companion stars and the values of angular velocity of rotation of

the primary star. The results thus obtained have been compared with the earlier corresponding results in which the effect of Coriolis force had not been considered. For comparison the shapes of various Roche equipotential surfaces of rotating stars and stars in binary systems have also been drawn in certain cases both in the presence as well as absence of Coriolis force.

Analytic expression for Roche equipotentials developed in chapter II which incorporates the effects of Coriolis force also, has been used in chapter III in conjunction with Mohan, Saxena and Aggarwal (108) approach to obtain the system of differential equations governing the equilibrium structures of rotationally and tidally distorted stellar models. The method has been applied to obtain the equilibrium structures and certain other observable parameters of rotationally and tidally distorted polytropic models of indices 1.5 and 3.0 for different choices of the values of rotational and tidal distortion parameters. The results thus obtained are compared with the results earlier computed by Mohan and Saxena (99) in which the effect of Coriolis force had not been considered. Shapes of the outer most surfaces of certain polytropic models of stars have also been drawn to highlight the effect of Coriolis force on the shapes of stars.

In chapter IV we develop eigenvalued boundary value problem which determines the periods of small adiabatic barotropic modes of oscillations of rotating stars and stars in binary systems incorporating the effects of Coriolis force besides the gravitational and centrifugal forces. The method has been then used to determine the effects of Coriolis force on the eigenfrequencies of various pseudo – radial and nonradial modes of oscillations of polytropic models of stars. The eigenvalued boundary value problem, determining the radial and nonradial modes of oscillations have then been solved numerically for various polytropic models of indices 1.5 and 3.0. The results thus obtained are compared with the results earlier obtained by Mohan and Saxena (100) and certain other authors.

In chapter V we have used the methodology developed in chapters II and chapter III to determine the effect of Coriolis force on the equilibrium structures of differentially rotating stars in binary systems using the law of differential rotation earlier used by Clement (15). Expressions for the volumes, surface areas and other physical parameters of polytropic models, rotating differentially according to the above law have also been obtained. Numerical computations have also been performed to compute the equilibrium structures of certain differentially rotating polytropic models of stars of indices 1.5 and 3.0. The numerical results thus obtained are compared with the results earlier obtained by Mohan et al (104) in which the effects of Coriolis force were not considered.

In chapter VI we study the effect of Coriolis force on the eigenfrequencies of small adiabatic barotropic modes of oscillations of differentially rotating stars in binary systems assuming a law of differential rotation used in chapter V. The eigenvalued boundary value problems, determining the eigenfrequencies of small adiabatic pseudo radial and nonradial modes of oscillations of differentially rotating stellar models have been formulated taking into account the effects of Coriolis force. The eigenvalued problems have then been solved numerically to compute the eigenfrequencies of pseudo radial and nonradial modes of oscillations of certain differentially rotating polytropic models of the stars whose equilibrium structures were earlier obtained in chapter V. The numerical results so obtained have been compared with the results earlier reported in Mohan et al (107) which do not explicitly account for the effects of Coriolis force.

Conclusions based on the present study are finally drawn in the concluding chapter VII. The limitations and scope for future work are also discussed in this chapter.

A research paper entitled **"Effect of Coriolis force on the shapes of rotating stars and stars in binary systems"** based on the contents of the work presented in chapter II has been published in the International Journal of *Astrophysics and Space Science*, Springer, 319, 45 – 53, 2009. A research paper entitled **"Effect of Coriolis force on the equilibrium structures of rotating stars and stars in binary systems"** based on the contents of the work presented in chapter III has been published in the International Journal of *Astrophysics and Space Science*, Springer, 315, 157 – 165, 2008.

CONTENTS

CHAPTER – I

INTRODUCTION

This chapter is introductory in nature. In section 1.1 we first explain in brief the astrophysical significance of the theoretical study of the problem of determining the effects of rotation and/or tidal distortions on the equilibrium structures and the periods of small adiabatic oscillations of gaseous spheres. A brief survey of the literature available on the subject is presented in Section 1.2. Analytic studies of the problems of equilibrium structures and periods of small adiabatic oscillations of rotating stars and stars in binary systems have been discussed in section 1.3. In subsection 1.3.1 we present the fundamental system of differential equations which governs the equilibrium structure of a gaseous sphere in hydrostatic and thermal equilibrium. In subsection 1.3.2 we show how Kippenhahn and Thomas (58) used an averaging technique to determine the equilibrium structures of rotationally and tidally distorted gaseous spheres. The concepts of Roche equipotentials and Roche limits are then introduced in subsection 1.3.3. Certain results obtained by Kopal (63) and Mohan et al (108, 109) for the Roche equipotentials are also presented in this subsection. In section 1.4 we next present how Mohan et al (108, 109) used Kippenhahn and Thomas (58) approach in conjunction with certain results on Roche equipotentials to obtain the system of differential equations governing the equilibrium structures and small adiabatic modes of oscillations of rotationally and tidally distorted gaseous spheres. In subsection 1.4.1, Mohan, Saxena and Agarwal's approach for determining the equilibrium structures of rotationally and tidally distorted gaseous spheres have been discussed. Mohan, Saxena and Agarwal's approach for determining the eigenfrequencies of pseudo-radial modes of oscillations of rotationally and tidally distorted gaseous spheres is then explained in section 1.4.2. In section 1.4.3 we next present their approach for setting up an eigenvalued boundary value problem which determines the effects of rotation and tidal distortions on the eigenfrequencies of nonradial modes of oscillations of gaseous spheres. A brief summary of the work presented in the succeeding chapters of this thesis is finally presented in section 1.5.

1.1 ASTROPHYSICAL SIGNIFICANCE OF THE PROBLEM OF DETERMINING THE EFFECTS OF ROTATIONAL AND TIDAL DISTORTIONS ON THE EQUILIBRIUM STRUCTURES AND THE PERIODS OF OSCILLATIONS OF GASEOUS SPHERES

The theoretical model of a star is essentially a self gravitating gaseous sphere in hydrostatic and thermal equilibrium. Theoretical studies of the problems of the equilibrium structure of gaseous spheres have often been carried out to understand the nature of the internal structures, responsible for various observed phenomena of the stars. Whereas some of the stars are observed as single stars others are observed in groups of two or more stars. Observations also show that some of the stars are rotating about their axes of rotation. This rotation may be a solid body rotation or a differential rotation. Many of the stars in binary and multiple systems are also known to be rotating about their axes as well as revolving around each other. Thus if we assume the equilibrium model of a single nonrotating star as a gaseous sphere, the equilibrium model of a rotating star will be rotationally distorted gaseous sphere. Similarly, the equilibrium model of a star appearing in a binary or a multiple system will be a tidally distorted gaseous sphere if it is not rotating and a rotationally and tidally distorted gaseous sphere if the star is rotating as well.

The brightness of certain observed stars varies with time. These stars are called variable stars. In some of these variable stars, the variations in luminosity are periodic. It is reasonable to assume in the case of such regular variable stars that these are pulsating gaseous spheres in which the variation in luminosity is being caused by the periodic contraction and expansion of the gaseous mass. The regular variable stars gained importance in astrophysics when it was discovered that there exists a definite relation between the periods of pulsation and the luminosities of such stars. This relationship has been utilized to determine the distance of stars. Such an important use of the regular variable stars motivated theoretical astrophysicists to investigate the problems of small oscillations of the equilibrium models of the variable stars so as to have a clear idea of the mechanism which could possibly be sustaining pulsations in these stars. Such investigations are also expected to help us in better understanding the nature of the internal structure of the stars. In most of these theoretical studies, the variable star is represented by a gaseous sphere undergoing radial and nonradial oscillations.

Observations, however, show that some of the variable stars are also rotating stars. The theoretical models of such rotating stars can be regarded as rotationally distorted gaseous spheres performing small oscillations about their equilibrium configuration. Similarly some of the variable stars have also been observed in binary and multiple stellar systems. The theoretical model of such stars can be regarded as rotationally and tidally distorted gaseous spheres performing small oscillations about their equilibrium configuration.

Analytic studies of the problems of rotating stars and stars in binary systems have engaged the attention of astrophysicist since long with a view to analyze and understand the observational behavior of such stars. A large number of stars observed in the sky are rotating stars or binary stars. In a binary system of stars the two stars normally rotate about their own axis as well as revolve about their common center of mass. In a majority of binary stars, one star called primary, is generally more massive compared to its companion star. Contact binaries also been observed in which outermost surfaces of the two stars just touch each other.

Keeping this in view an attempt has been made in the present thesis to investigate certain aspects of the problems of equilibrium structures and small oscillations of rotationally and tidally distorted gaseous spheres which still need investigations.

1.2 BRIEF SURVEY OF THE LITERATURE.

Most of the theoretical studies about the equilibrium structures and oscillations of the stars have been carried out in literature by assuming the star to be an undistorted spherical gaseous sphere. Extensive literature is now available on this subject (see for instance Chandrasekhar (11), Rosseland (127), Schwarzschild (135), Eddington (32), Mentzel et al. (93), Cox and Giuli (22), Kippenhahn and Weigert (59), Clement (16), Kopal (64, 66), Tassoul (151), Cox (21), Bohm-Vitense (4), Abhyankar (1), Horedt (50) and Unno et al (157).

In the case of some stars, it is observed that brightness varies with time. These are called variable stars. Whereas variation in brightness of some of the stars is regular and periodic in others it is not so. An important class of regular variable stars is Cepheid variables. The regular variable stars, gained importance in astrophysics in the year 1912, when Miss Leavitt discovered that there exists a definite relation between the periods of pulsation and the luminosities of such stars and the relationship could be utilized to determine the distance of these stars. In most of the theoretical studies of such stars, the variable star is represented by a gaseous sphere, both in hydrostatic and thermal equilibrium, undergoing small periodic oscillations. These oscillations can be radial as well as nonradial.

In case of a regular variable star which is a nonrotating and exists in isolation it may be reasonable to represent it as a gaseous sphere performing radial and/or nonradial oscillations. Some of the variable stars are also observed to be rotating stars. In such cases the rotation of the star will effect the equilibrium structure as well as the modes of its oscillations. However, if the pulsating star is a member of the binary or a multiple system, then not only its equilibrium structure but also its modes of oscillations will also get affected by the rotational and tidal forces. Mathematical models of such stars will obviously have to be rotationally and tidally distorted gaseous spheres performing pseudo-radial or nonradial or some entirely different types of oscillations. The mathematical study of the problem of equilibrium structure and periods of small adiabatic modes of oscillations of such stars becomes quite complex.

The analytic study of the problem of oscillations of gaseous spheres was initiated by Ritter. He was perhaps the first to suggest in the year 1879 that the periodic variations in the luminosity of a variable star may be due to radial oscillations. Extensive studies have since been made to analyze the problems of small adiabatic radial modes of oscillations of gaseous spheres. Investigators such as Prasad (120), Hurley et al (52), Prasad and Mohan (121) studied the problems of oscillations of gaseous spheres with no rotation. Later on Gurm (43), Clement (14), Simon (139), Kochar and Trehan (60), Lee and Saio (77), Saio and Jeffery (131), Das et al (25), Lee and Baraffe (76), Soofi et al (147), Dintrans and Rieutord (28) and Townsend (156) studied the effect of rotation on stellar pulsations.

In the case of regular variables, the high symmetry of their observed properties favors the hypothesis of purely radial oscillations. However, purely radial oscillations may not be able to explain many other phenomena observed in the case of certain variables stars. Ledoux and Walraven (75) pointed out that the dynamical instability leading to explosions in the stars might be easier to reach for some modes of nonradial oscillations. Chandrasekhar and Lebovitz (13) were of the view that it might be possible to explain variability of Beta Canis major type stars on the basis of resonance between the radial and nonradial modes of oscillations. Dalsgaard and Gough (24) suggested that certain observed phenomena in the outer layer of sun could be explained on the basis of certain nonradial modes of oscillations of the sun. Smith (145) studied zero-age main sequence B star and found that this star is pulsating nonradially.

Theoretical studies of the problem of nonradial oscillations commenced with Kelvin's investigation of the oscillatory modes of an incompressible gas sphere. But the proper formulation of the problem was given by Perkeris (117) who derived the fourth order linear differential equation governing the adiabatic nonradial modes of oscillations of a compressible self- gravitating gaseous sphere. Since then, the theoretical studies of the problem of nonradial oscillations of spherical models have been carried out by many investigators. Several authors such as Cowling (19), Kopal (62), Ledoux (74), Owen (115), Smeyers and Denis (143), Singh (140), Hansen et al (45), Saio (130), Clement (17), McDermott et al. (91), Bhatia (3), Lee and Strohmayer (78) and Savonije (132) have made significant contributions to the studies of the problems of nonradial pulsations of stars.

Cox and Cahn (20) calculated representative radial and nonradial pulsation modes of five Wolf- Rayet star models. Chandrasekhar and Ferrari (12) have proposed a complete theory of the nonradial oscillations of a static spherical symmetric distribution of matter described in terms of energy density and isotropic pressure on the premise that the oscillations are excited by incident gravitational waves. Rosenwald and Rabaey (126) have given an application of the continuous orthonormalization and adjoint methods to the computation of star eigenfrequencies and eigenfrequency sensitivities. This method integrates an eight-order nonlinear system of ordinary differential equations which define the linear adiabatic nonradial oscillatory modes of the sun. Telting and Schrijvers (154) used a model of a nonradially, adiabatically pulsating rotating star to generate time series of absorption line profiles. Clement (18) also discussed normal modes of oscillations for rotating stars using a new numerical method for computing nonradial eigenfunctions. This technique for calculating the normal modes of spherical stellar models is generalized to two dimensions. Lignieres et al (79) developed a new non perturbative method to compute accurate oscillations modes in rapidly rotating stars. They computed the axisymmetric p-modes in uniformly rotating polytropic models of stars.

The theoretical investigation related to the problems of determining the equilibrium structures and stability of rotating and self gravitating objects, possibly begun with the work of Newton. He was the first to realize the importance of the law of gravitation for explaining the figures of celestial bodies. Later on Maclaurin, Clairaut, Laplace, Legendre, Jacobi, Poincare etc. contributed ideas, necessary for the development of the general theory of rotating bodies. Maclaurin, Jacobi, Kelvin and Jeans investigated in detail the problem of structure and stability of rotating liquid masses assuming uniform rotation.

In the year 1923, Edward Arthur Milne developed a technique for constructing the first detailed model for a slowly rotating star in pure radiative equilibrium. Later on in the year 1933, the technique of Milne was generalized and applied to slightly distorted polytrope by Chandrasekhar. The effect of uniform rotation on slow rotating Cowling star obeying simple Kramer's opacity has been studied by Sweet and Roy (150). Much of the work on the effect of rotation on stellar interiors is summarized in the review article of Strittmatter (149). Authors such as Kruszewski (68), Limber (80), Roberts (124), James (54), Hurley and Roberts (51), Roxburgh and Durney (128), Martin (90), Sackmann and Anand (129), Linnell (81, 82), Endal and Sofia (35), Smith (144), Lubow (86), Kopal (65), Singh and Singh (141), Durney (29), Geroyannis and Valvi (38), Deupree (26), Einsel and Spurzem (34), Maeder and Zahn (88), Reyniers and Smeyers (123) and Sood and Singh (146) have also investigated the problems of equilibrium structures of rotating stars. Meynet and Meader (94) studied the effects of rotation on the equilibrium structure and evolution of massive stars. Mender et al (92) investigated the theoretical models of low mass pre main sequence rotating stars. Zeng (162) has developed more powerful evolutionary models for rotating stars. Reese et al (122) have studied the acoustic oscillations of rapidly rotating stars taking into account the effects of Coriolis force and centrifugal forces. Lovekin and Deupree (84) have studied the radial and nonradial modes of oscillations of rapidly rotating stars using the 2D stellar models and 2D finite differences integration of the linearized pulsation equation.

Whereas many of the observed rotating stars may be having solid body rotation some of the stars are observed to be rotating differentially. In such type of stars different parts of the star are rotating about the axis of rotation with different angular velocities. Problems of differentially rotating stellar models have also been studied in literature. Stoeckly (148) obtained the numerical solution of the hydrostatic equilibrium equation for non uniformly rotating stellar models having no meridional currents and axial rotation. With pressure density relation of the type $P \propto \rho^{3/2}$, Peraiah (116) showed that synchronism between orbital and rotational angular velocities of binary stars may not hold in many cases in the presence of differential rotation. However it is trivially true as a differentially rotating star has a number of angular velocities depending on where one is in the star and orbital velocity cannot equal all of them. Ireland (53) presented results for gravity darkening and limb darkening in a rapidly rotating Roche model of a star subject to non uniform rotation and demonstrated that the effects of small uniform rotation are likely to be of greater significance than the actual values of rotational velocities themselves. Schmitz (134) studied the equilibrium structures and stability of differentially rotating self gravitating gaseous spheres. Komatsu et al (61) applied the numerical method developed for Newtonian gravity models to general relativistic differentially rotating bodies including ring-like structures. He also obtained equilibrium structures for polytropes of indices 0.5 and 1.5. Goode et al (42) also tried to analyze the nature of differential rotation in the interior of the sun for the study of its 5 – min oscillations. Mohan et al (107) have proposed a method to compute the eigenfrequencies of small adiabatic barotropic modes of oscillations of differentially rotating stars polytropic models of stars. Karino et al (57) computed the r – mode oscillations of differentially and rapidly rotating Newtonian polytropic stars. Macgregor et al (87) have studied the structure and properties of differentially rotating, main-sequence stars in the $1-2\,M_{\odot}$ range. Authors such as Goldreich and Schubert (41), Harris and Clement (47), Hansen et al (46), Geroyannis et al (39), Bruning (5), Geroyannis and Antonakopoulus (37), Glatzmaier and Gilman (40), Durney (30), Pinsonneault et al (118), Galli (36), Mohan et al (104, 105, 106), Shibata et al (138), Karino (55), Kuker and Rudiger (69) and Miesch (95) have also analyzed the problems of differential rotation in stars.

Equilibrium structures of stars which appear in binary and multiple systems are likely to be affected by both the rotational as well as the tidal effects of the companion stars. Attempts have been made in literature to determine the effects of rotation and tidal distortions on the equilibrium structure and modes of oscillations of the stars in binary and multiple systems. In a series of papers Chandrasekhar (8, 9, 10) developed a first order analysis which he applied to the study of the rotational problem, the tidal problem and the binary star problem. The method, however, was found unsuitable when the separation between the components is only a few times the undisturbed radius of the primary. Monaghan (110) modified it to get more accurate results near the surface.

The method of Monaghan and Roxburgh (111) to study the structure of the primary component of a synchronous close binary was further extended by Naylor and Anand (112). Kippenhahn and Thomas (58) suggested a practical way of analyzing the effects of rotation and tidal distortions on the equilibrium structures of stars by approximating the actual equipotentials surfaces of the star by Roche equipotentials.

Chan and Chau (6) developed a method which allows an efficient and accurate investigation of the structure and evolution of a rotationally and tidally distorted star in close binary systems. Tassoul and Tassoul (152) considered the meridional circulation in rotating stars and mean steady motions in rotationally and tidally distorted stars. Later Tassoul and Tassoul (153) extended the earlier work to study the reflection effects in close binaries when there is meridional circulation in rotating stars. Nelson et al (113) have discussed the evolution of rotationally and tidally distorted low-mass close binary systems. Marten and Smeyers (89) investigated the problem of linear adiabatic oscillations of a uniformly and synchronously rotating component of a binary system. Lopezorti et al (83) analyzed the equilibrium configurations of close binary systems by expanding the auto gravitational, centrifugal and tidal potentials in Clairaut coordinates. Chandra and Bhatnagar (7) studied the systematic regularity of escape with the formation of a binary with low perturbation velocities for equal masses in the evolution of stellar system in a plane. Lal et al (72) have discussed the equilibrium structures of rotationally and tidally distorted primary component of binary stars taking into account the effect of mass variation inside the star.

Mohan and Singh (103) have used the Kippenhahn and Thomas (58) averaging technique in conjunction with certain results of Kopal (63) on Roche equipotential to study the effect of rotation and tidal distortions on the small adiabatic oscillations of stars in binary system. Hachisu (44) formulated a new-three dimensional method for obtaining structure of a rapidly rotating star and multiple stellar system including binaries. Rocca (125) studied effect of slow uniform rotation on the tidal effects in close binary system. Todaran (155) has used the time dependent potential function to study the equipotentials surfaces in close binary systems. Deupree and Karkas (27) studied the structure and evolution of close binary stars using the two-dimensional stellar structure algorithm. They have calculated a series of solar composition stellar evolution sequences of binary. Sepenisky et al (137) investigated the existence and properties of equipotential surfaces and Lagrangian points in non-synchronous, eccentric binary star and planetary systems under the assumption of quasi-static equilibrium.

Orlov (114) have generalized the Roche model as is applied in the case of double star. In this model the point nuclei of the Roche model has been substituted by polytropic gas nuclei of finite dimensions. Plavec (119) presented tables of Roche model for the use of investigators in close binary systems. Kopal and Ali (67) studied the integrability of the

Roche coordinates. Avni and Schiller (2) studied the Roche potential systems where the stellar rotation axis is not aligned with the orbital revolution axis. Eggleton (33) computed the effective radii of Roche lobes and compared the results with the earlier results available in literature. Mochnacki (97) accurately integrated Roche model for close binary system in synchronous rotation to give volume, radii, surface area, mean gravities and mean inverse gravities in normalized form. Seidov (136) derived the exact analytical formula for the potential and mass ratio as a function of Lagrangian point's position in the classical Roche model of the close binary stars. Csataryova and Skopal (23) derive approximate analytical formulas for the basic parameters of the Roche lobe, its radius and the position of the L_1 point for asynchronously rotating component in a binary system. Lal et al (73) studied the validity of series expression being used for determining the position of a point on a Roche equipotential in case of rotating stars and stars in binary systems.

The simple hypothesis of the pulsating model of a regular variable star is made all the more complicated by the fact that some of the variable stars observed to be rotating stars or stars in binary or multiple systems. The eigenfrequencies of small oscillations of such stars are expected to be influenced by rotation and tidal effects of companion stars.

Most of the authors have studied pulsations of stars having solid body rotation. However, there are several variable stars which are suspected to be rotating differentially. Clement (15) has shown that by assuming a particular form of differential rotation the discrepancy that existed between observations and earlier calculations based on the assumption of uniform rotation could be removed. Vorontsov (160) developed a perturbation theory to calculate the influence of slow differential rotation on the adiabatic nonradial modes of stellar oscillations. He analyzed the effects of Coriolis force and ellipticity simultaneously using the perturbation technique for Hamiltonian operator which is developed up to second order in eigenfrequencies and to first order in eigenfunctions. Woodard (161) considered the effect on eigenfrequencies and eigenfunctions of slow, axisymmetric differential rotation which is also mirror symmetric across the solar equatorial plane. Dziembowski and Goode (31) developed a complete formalism, valid through second order in differential rotation and applied it to calculate the frequencies of stellar oscillations. Urpin et al (158) studied the problem of differential rotation, circulation and turbulence in radiative zones of stars. Karino and Eriguchi (56) considered the linear stability analysis of some differentially rotating polytropes. Lovekin et al (85) investigated the effects of uniform rotation and a specific model for differential rotation on the pulsation frequencies of $10 M_{\odot}$ stellar models.

Kopal (63) introduced a system of coordinates, which he called Roche coordinates, to study the problems of rotating stars and stars in binary system. Mohan and Singh (101, 102) considered the use of Roche coordinates in solving the problems of small adiabatic oscillations of rotationally and tidally distorted stellar models. Mohan and Saxena (99, 100) used the Kippenhahn and Thomas (58) averaging technique in conjunction with Kopal's results on Roche equipotentials to determine the combined effects of rotation and tidal distortions on the equilibrium structures and oscillations of the polytropic models of the stars. This approach is presented in detail by Saxena (133). Later this approach was also used by Mohan and Agarwal (98) to study the effects of rotation and tidal distortions on the structure and periods of small adiabatic oscillations of composite models of stars. The technique was subsequently formalized by Mohan et al (108, 109) and used to study the problems of equilibrium structures and oscillations of rotationally and tidally distorted main sequence stars. Lal (70) studied in detail the equilibrium structures and periods of oscillations of differentially rotating stellar models. Later on Singh and Sharma (142) also studied the oscillations of differentially rotating stars in binary system. Lal et al (71) applied this technique to study the equilibrium structures of differentially rotating and tidally distorted white dwarf models of stars.

Whereas the properties of equilibrium structures and periods of small adiabatic radial and nonradial modes of oscillations of undistorted gaseous sphere have been investigated in detail in literature, the effect of rotation and tidal distortions on the equilibrium structures and the modes of oscillations of gaseous spheres has still not been fully understood. For instance comments have generally been made that the approaches based on Roche equipotential for computing the equilibrium structures and periods of oscillations of stars in binary systems carry out their studies in fixed frame of reference and do not account for Coriolis force which is expected to arise in such cases when rotating frames of reference are used. Earlier studies of Mohan and Saxena (100), Mohan et al (107) based on this approach show effect of rotation is to decrease the eigenvalues of g – modes of nonradial oscillations whereas results of some other authors (Clement (17)) show contrary results. In the present work we have addressed ourselves to the analytic and computational studies of problems related to this field.

1.3 ANALYTIC STUDIES OF THE PROBLEMS OF EQUILIBRIUM STRUCTURES AND PERIODS OF SMALL ADIABATIC OSCILLATIONS OF ROTATING STARS AND STARS IN BINARY SYSTEMS.

The general problem of determining the effects of rotation and/or tidal distortions on the equilibrium structure and the eigenfrequencies of small adiabatic oscillations of realistic models of stars is quite complex. No general approach is as yet available which can determine exactly the combined effects of rotation and tidal distortions on the equilibrium structure and eigenfrequencies of the modes of oscillations of gaseous spheres. Most of the attempts in this direction have generally tried to investigate some particular aspects of this problem in certain approximate ways. In its present state, the problem of determining the effects of rotation and tidal distortions on the structure, stability and the eigenfrequencies of small adiabatic oscillations of gaseous spheres thus far not satisfactorily solved. Keeping in view its importance in astrophysics, there is still need for further investigations in this direction.

1.3.1 BASIC EQUATIONS DETERMINING THE EQUILIBRIUM STRUCTURE OF A GASEOUS SPHERE

The system of basic equations which governs the equilibrium structure of a gaseous sphere in hydrostatic and thermal equilibrium, especially as they pertain to the problems of the equilibrium structure of stellar models, are by now well established in literature. Let P and ρ denote the pressure and the density respectively at a point distant r from the center of the sphere and $M(r)$ the mass contained in a sphere of radius r. Also let T be the temperature at a point distant r from the center of the sphere, $L(r)$ the net amount of energy crossing a spherical surface of radius r per second and ε the rate of energy generation from thermo – nuclear processes per gram per second, then the equation of conservation of mass states is

$$\frac{dM(r)}{dr}=4\pi r^2\rho \tag{1.1}$$

The equation of hydrostatic equilibrium gives

$$\frac{dp}{dr} = -G\frac{M(r)}{r^2}\rho \tag{1.2}$$

where G is the gravitational constant.

The luminosity equation is

$$\frac{d\,L(r)}{dr} = 4\pi r^2 \rho\,\varepsilon \tag{1.3}$$

and the energy transport equation states

$$\frac{dT}{dr} = -\frac{3\kappa}{4ac}\frac{\rho}{T^3}\frac{L(r)}{4\pi r^2} \tag{1.4}$$

in case of radiative equilibrium and for convective equilibrium,

$$\frac{dT}{dr} = (1-\frac{1}{\gamma})\frac{T}{P}\frac{dP}{dr} \tag{1.5}$$

where κ is the mass absorption coefficient of the gas, c is the velocity of light, a is the Stefan – Boltzmann constant and γ is an adiabatic exponent which is equal to the ratio of specific heats at constant pressure and volume (for a perfect non degenerate particle gas in the absence of radiation ($\gamma = 1.67$).

In addition to the above four differential equations we also require three more explicit relations which characterize more specifically the behavior of the interior gas. They are the equation of state, the equation for the mass absorption for energy generation by thermonuclear processes. These equations may be formally represented by

$$P = P(\rho, T, \text{chemical composition}) \tag{1.6a}$$

$$\kappa = \kappa(\rho, T, \text{chemical composition}) \tag{1.6b}$$

and

$$\varepsilon = \varepsilon(\rho, T, \text{chemical composition}) \tag{1.6c}$$

Equations (1.1 – 1.5) are to be satisfied in every layer of the gaseous sphere. In addition certain boundary conditions are also to be satisfied. It is obvious from the definitions of $M(r)$ and $L(r)$, that at the center we have

$$r = 0 \;:\; M(r) = 0,\; L(r) = 0 \tag{1.7a}$$

$$r = R:\; M(r) = M \text{ and } L(r) = L \tag{1.7b}$$

where R is the radius of the gaseous sphere, M the total mass and L the total energy radiated by the gaseous sphere. The pressure and the temperature at the outermost surface of a gaseous sphere, taken as the representative model of star, are often approximated by what are generally known as zero boundary conditions. Under these conditions we take

$$r = R : P = 0 \text{ and } T = 0 \tag{1.7c}$$

Conditions (1.7c) are reasonably accurate for most of the theoretical stellar models. If moreover, more accurate computations are to be done then we may replace (1.7c) by

$$r = R : P = P_s \text{ and } T = T_s \tag{1.7d}$$

where P_s is determined from the pressure of the upper atmospheric layers and T_s is determined from the effective temperature of the star.

In problems where the thermal properties of the model are either not important or are not to be investigated, the equilibrium structure of the gaseous sphere may be determined by solving equations (1.1 – 1.2) using some suitable equation of state together with boundary conditions

At the center $\quad r = 0 : M(r) = 0 \qquad$ (1.8a)

At the surface $\; r = R : \; M(r) = M, \; P = 0$ or P_s, $\; \rho = 0$ or $\rho_s \qquad$ (1.8b)

A number of the theoretical as well as numerical studies regarding the equilibrium structures of gaseous spheres, particularly those which have particular reference to the problems of the equilibrium structures of the stars are available in literature (Chandrasekhar (11), Schwarzschild (135), Eddington (32), Menzel et al. (93), Cox and Giuli (22), Kippenhahn and Weigert (59)).

1.3.2 AVERAGING TECHNIQUE OF KIPPENHAHN AND THOMAS

In order to study the effects of rotation and tidal distortions on the equilibrium structure of gaseous spheres, Kippenhahn and Thomas (58) developed the concept of topologically equivalent spherical surfaces corresponding to actual equipotential surfaces of a rotationally and tidally distorted model. They define on these equivalent spherical surfaces, quantities such as $\overline{f}, \overline{g}$ etc. which denote certain averages of the quantities f, g, respectively on the actual equipotential surfaces. If ψ denotes the total potential (gravitation, rotation and tidal forces) of a rotationally and tidally distorted model at an arbitrary point $P(x, y, z)$ then $\psi(x, y, z)$ = constant is an equipotential surface. Let V_ψ be the volume enclosed by the equipotential surface. ψ = constant and S_ψ the surface area of this equipotential surface. For any function $f(x,y,z)$ they define $\bar{f}$ as its mean value over the equipotential surfaces ψ = constant by the relation

$$\bar{f} = \frac{1}{S_\psi} \int_{\psi = const.} f \, d\sigma \tag{1.9}$$

where $d\sigma$ denotes the surface element of the equipotential surface ψ = constant. Clearly $\bar{f}$ is a function of equipotential surface ψ only and can be obtained as equation (1.9) for each equipotential surface ψ = constant. Kippenhahn and Thomas also define a variable r_ψ in analogy with the radius of sphere by the relation

$$V_\psi = \frac{4}{3}\pi r_\psi^3 \tag{1.10}$$

Also by definition

$$S_\psi = \int_{\psi = constant} d\sigma \tag{1.11}$$

Obviously, in general, S_ψ is not equal to $4\pi r_\psi^2$. Kippenhahn and Thomas (58) define a function $g(x,y,z)$ by the relation

$$g = \frac{d\psi}{dn} \tag{1.12}$$

This g corresponds to the force of gravity of a sphere. The distance dn between two neighboring surfaces ψ = constant and $\psi + d\psi$ = constant is, in general, not constant (i.e. not same at all points of the surface). They used (1.12) to compute the mean values $\bar{g}$ and $\overline{g^1}$ with the help of relations

$$\left.\begin{aligned} \bar{g} &= \frac{1}{S_\psi} \int_{\psi = const} \frac{d\psi}{dn} d\sigma \\ \overline{g^{-1}} &= \frac{1}{S_\psi} \int_{\psi = const} \left(\frac{d\psi}{dn}\right)^{-1} d\sigma \end{aligned}\right\} \tag{1.13}$$

Both $\bar{g}$ and $\overline{g^{-1}}$ are functions of ψ alone and represent the value of g and g^{-1} respectively over the topologically equivalent spherical surface. The volume dV_ψ between the surface ψ = constant and $\psi + d\psi$ = constant is given by

$$dV_\psi = \int_{\psi=const} dn\, d\sigma = \int_{\psi=const} (\frac{d\psi}{dn})^{-1} dn \; = S_\psi \overline{g^{-1}}\, d\psi \tag{1.14}$$

Kippenhahn and Thomas also defined nondimensional parameters u, v and w as

$$u = \frac{S_\psi}{4\pi r_\psi^2}, \; v = \frac{\bar{g}\, r_\psi^2}{GM_\psi}, \; w = \frac{\overline{g^{-1}}\, GM_\psi}{r_\psi^2} \tag{1.15}$$

where M_ψ is the mass enclosed by equipotential surface ψ =constant.

We may thus regard the equipotential surface ψ = constant to be topologically equivalent to a sphere of radius r_ψ for which various functions are defined by the above relations. (It may be noticed that if ψ is the gravitational potential of a sphere then the surface ψ = constant is spherical surface with $r_\psi = r$ for which u =1 and $g = GM_\psi / r_\psi^{\,2}$ is constant on these spheres and therefore v and w are constants and equal to 1).

Equations (1.9) to (1.15) are purely mathematical definitions, which have been applied by Kippenhahn and Thomas (58) to gravitational fields of gaseous spheres distorted by rotational and tidal forces. In hydrostatic equilibrium the equipotential surfaces are also surface of equipressure and equidensity. Therefore, on an equipotential surface the pressure P_ψ and the density ρ_ψ are also constant. Using these concepts, Kippenhahn and Thomas (58) obtained the equations governing the equilibrium structure of a rotationally and tidally distorted stellar model in the following manner

From equation (1.10) the mass dM_ψ between the equipotential surface ψ = constant and $\psi + d\psi$ = constant is given by

$$dM_\psi = dV_\psi\, \rho_\psi = 4\pi\, r_\psi^2 \rho_\psi\, dr_\psi \tag{1.16}$$

Thus, we get

$$\frac{dM_\psi}{dr_\psi} = 4\pi\, r_\psi^2 \rho_\psi \tag{1.17}$$

From equation (1.14) and (1.16) we have

$$d\psi = \frac{d\psi}{dV_\psi} dV_\psi = (\frac{dV_\psi}{d\psi})^{-1} \frac{dM_\psi}{\rho_\psi} = \frac{dM_\psi}{S_\psi\, \overline{g^{-1}}\, \rho_\psi} \tag{1.18}$$

Using relations (1.15), we get

$$d\psi = \frac{G\,M_\psi\, dM_\psi}{4\pi\, r_\psi^4 \rho_\psi\, u\, w} \tag{1.19}$$

The conditions for hydrostatic equilibrium, $dP_\psi / d\psi = -\rho_\psi$, can now be written with equation (1.15) in the form

$$\frac{dP_\psi}{dM_\psi} = -\frac{G\,M_\psi}{4\pi\, r_\psi^4} f_p \tag{1.20}$$

where

$$f_P = \frac{1}{u\,w} = \frac{4\pi\, r_\psi^4}{G\,M_\psi} \frac{1}{S_\psi\, \overline{g^{-1}}}$$

The factor f_P is a function of ψ only. If ψ is known the equipotential surface can be determined, and then consequently values of $S_\psi, r_\psi, \overline{g}$ and $\overline{g^{-1}}$ for each equipotential surface can be obtained simply from the geometry of the equipotentials. The mass M_ψ which depends on the density distribution ρ_ψ can be determined by integrating the equation (1.17). Similarly the other structure equations derived by Kippenhahn and Thomas (58), which includes the effects of rotation and tidal distortions on the equilibrium structure of gaseous spheres are as follows.

For chemically homogenous spheres, the nuclear energy generation rate ε depends only upon density ρ_ψ and the temperature T_ψ and are, therefore, constant on equipotential surface. Thus, if L_ψ is the energy which passes per second through the equipotential surface $\psi =$ constant, then

$$\frac{dL_\psi}{dM_\psi} = \varepsilon \tag{1.21}$$

Using equation (1.17), it can be written as

$$\frac{dL_\psi}{dr_\psi} = 4\pi\, r_\psi^2 \rho_\psi\, \varepsilon \tag{1.22}$$

If the energy is transported by radiation, then the energy transport equation is

$$F_\psi = -\frac{4\,a\,cT_\psi^3}{3\kappa} \frac{d\psi}{dn} \frac{dT_\psi}{dM_\psi} \frac{4\pi\, r_\psi^4\, u\, w}{G\,M_\psi} \tag{1.23}$$

where F_ψ is the radiative flux on the equipotential surface $\psi =$ constant. By integrating F_ψ over the equipotential surface ψ = constant, we get

$$L_\psi = \int_{\psi = constant} F_\psi \, d\sigma$$

$$= -\frac{4\,a\,c T_\psi^{\;3}}{3\kappa}\frac{d\,T_\psi^{\;3}}{d\,M_\psi} u\,w \frac{4\,\pi\,r_\psi^{\;4}}{G\,M_\psi} \int_{\psi = constant} (\frac{d\psi}{dn}) \, d\sigma \tag{1.24}$$

$$= -\frac{64\,\pi^2 a\,c T_\psi^{\;3} r_\psi^{\;4}}{3\kappa} u^2\,v\,w \frac{d\,T_\psi}{d\,M_\psi}$$

so that

$$\frac{d\,T_\psi}{d\,M_\psi} = -\frac{3\,\kappa\,L_\psi}{64\,\pi^2 a\,c T_\psi^{\;3} r_\psi^{\;4}} f_T \tag{1.25}$$

Using equation (1.16), this equation can be expressed as

$$\frac{dT_\psi}{dr_\psi} = -\frac{3\,\kappa\,\rho_\psi\,L_\psi}{16\,\pi\,a\,c\,T_\psi^3\,r_\psi^2} f_T \tag{1.26}$$

where

$$f_T = \frac{1}{u^2\,v\,w}$$

Equations (1.17), (1.20), (1.21) and (1.25) which are the four basic equations governing the equilibrium structure of a gaseous sphere distorted by rotational and tidal forces may now be collected together and written as.

$$\frac{dM_\psi}{dr_\psi} = 4\pi\,r_\psi^2\,\rho_\psi \tag{1.27a}$$

$$\frac{dP_\psi}{dM_\psi} = -\frac{G\,M_\psi}{4\,\pi\,r_\psi^4} f_p \tag{1.27b}$$

$$\frac{dL_\psi}{dM_\psi} = \varepsilon \tag{1.27c}$$

and

$$\frac{d\,T_\psi}{d\,M_\psi} = -\frac{3\,\kappa\,L_\psi}{64\,\pi^2 a\,c T_\psi^{\;3} r_\psi^4} f_T \tag{1.27d}$$

where

$$f_P = \frac{1}{u\,w} \qquad \text{and} \qquad f_T = \frac{1}{u^2\,v\,w}.$$

These reduce to the normal equations used for determining the equilibrium structures of spherical models of stars when distortion parameters u, v, w are set one each. The boundary conditions which the above equations has to satisfy are

$$M_\psi = 0, \;\; L_\psi = 0 \tag{1.28a}$$

at the center $r_\psi = 0$

$$M_\psi = M_0, \; L_\psi = L_{\psi S}$$
$$P_\psi = 0, \; T_\psi = 0 \;\; or \;\; P_\psi = P_{\psi s}, \; T_\psi = T_{\psi s}$$

at the free surface $r_\psi = R_\psi$ (1.28b)

where M_0 is the total mass of the model and $L_{\psi s}$, $P_{\psi s}$, $T_{\psi s}$ are the values of L_ψ, P_ψ, T_ψ respectively, on the outermost equipotential surface.

1.3.3 ROCHE EQUIPOTENTIAL

Roche equipotentials have often been used to analyze the problems of rotationally and tidally distorted models of stars. In order to introduce the concept of Roche equipotential, we assume two components of a close binary system known as primary and secondary star. The primary star is supposed to be much more massive than the secondary which is assumed as a point mass causing tidal effects on the more massive primary component. Both the components of binary system are assumed to be rotating about their axes as well as revolving about their common center of mass. Following Kopal (63), Mohan and Singh (101, 102), and Mohan et al (108, 109) certain results on Roche equipotential which are of practical interest to the present study, are summarized below:

Suppose that M_0 and M_1 are the masses of the two components of a close binary system separated by distance *D*. Further suppose that the primary component of this system of mass M_0 is much larger than its companion star of mass $M_1 (M_0 \geq M_1)$ which can be regarded as a point mass. Next suppose that the position of the two components is referred to a rectangular system of Cartesian coordinates with origin at the center of gravity of mass

M_0, the $x-$ axis along the line joining the mass centers of two components and $z-$axis perpendicular to the plane of the orbit of the two components (Fig. 1.1). Then the total potential ψ due to the gravitational and disturbing force acting at an arbitrary point $P(x, y, z)$, which is not inside any of these two gaseous spheres is given by:

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_2} + \frac{\Omega^2}{2}[(x - \frac{M_1 D}{M_0 + M_1})^2 + y^2] \tag{1.29}$$

where $r^2 = x^2 + y^2 + z^2$ and $r_2^2 = (D-x)^2 + y^2 + z^2$ represent the squares of the distances of P from the center of gravity of the two components, Ω denotes the angular velocity of rotation of the system about an axis perpendicular to the *xy*- plane and passing through the center of gravity of the system and G the constant of gravitation.

The first, second and third term on the right hand side of equation (1.29) respectively represent the potential which arises due to the mass M_0 of the primary component, the disturbing potential of its companion of mass M_1 and the potential arising from the centrifugal force respectively. Equation (1.29) strictly holds at points which are outside the components of binary system.

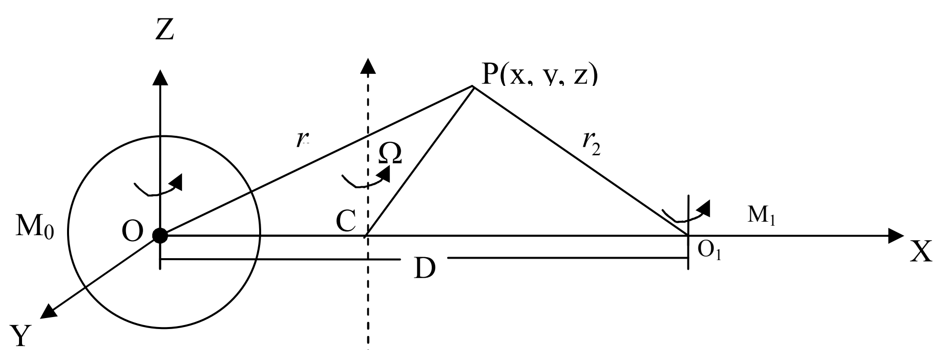


Fig 1.1: Axis of Reference for a binary system

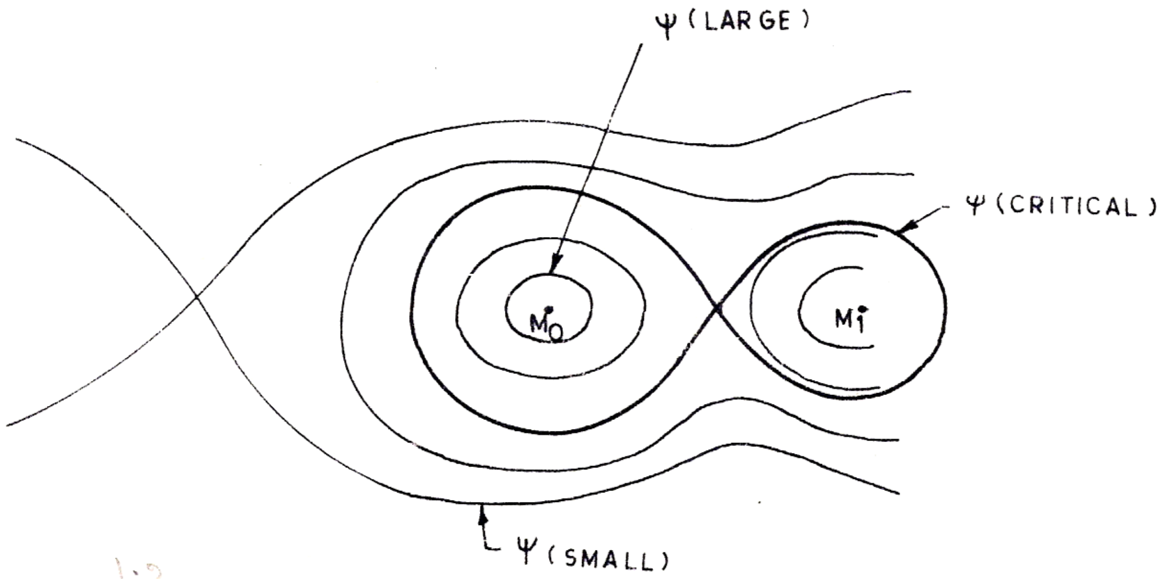


Fig 1.2 Roche equipotential surfaces (two dimensional)

In case we assume Roche model for the primary (In Roche model it is assumed that the total mass of a star is concentrated at its center and this point mass is surrounded by an evanescent envelope in which density varies inversely as the square of the distance from its center) and a point mass for the secondary component, equation (1.29) holds everywhere. Also if we assume that the angular velocity Ω is identical with Keplerian angular velocity, that is,

$$\Omega^2 = \Omega_k^2 = G\frac{M_0 + M_1}{D^3} \qquad (1.30)$$

then we get a relation of the type

$$n = \frac{q+1}{2} \qquad (1.31)$$

where $q = M_1/M_0$ is a nondimensional parameter representing the ratio of mass of the secondary over primary and $2n$ represents the square of the normalized angular velocity Ω. Equation (1.29) can be expressed in nondimensional form as

$$\psi^* = \frac{1}{r^*} + q[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^*] + nr^{*2}(1-\nu^2) \qquad (1.32)$$

where

$$\psi^* = \frac{D\psi}{GM_0} - \frac{M_1^2}{2M_0(M_0 + M_1)} \tag{1.33}$$

is the nondimensional form of total potential ψ and $r^* = r/D$ is nondimensional form of r. Also $\lambda = \sin\theta\cos\phi$, $\mu = \sin\theta\sin\phi$ and $\nu = \cos\theta$ (r, θ, ϕ being the polar spherical coordinate of the point *P*. The equation (1.29) reduces to the potential of a purely rotating spherical model if $q = 0$. For $n = 0$, it reduces to the potential of a non-rotating spherical model distorted by the tidal effects of the companion alone.

The surfaces generated by setting ψ = constant on the left hand side of equation (1.29) are referred to as Roche equipotentials. Roche equipotentials in nondimensional form may be represented by ψ^*= constant where ψ^* is same as defined in equation (1.32). The form of Roche-equipotential depends entirely upon the values of ψ . If ψ is large the corresponding equipotentials consist of two separate ovals, closed around each of the two mass points (Fig. 1.2). For specified values of M_0, M_1, Ω and D the right hand side of equation (1.29) can be large only if r and r_2 becomes small. Therefore, large values of ψ correspond to equipotentials which differ little from spheres surrounding each of the two mass centers. With decreasing values of ψ, these spherical equipotential surfaces become oval shaped and get elongated in the direction of the center of gravity of the system until for a certain critical value of ψ, which is characteristic of each mass ratio, both oval shaped surfaces unite at a single point on the x-axis to form a dumbbell like configuration. These limiting values of ψ are called Roche limits. For certain mass ratios Kopal (63) computed the numerical values of Roche limits in the case of synchronous binary stars for values of q ranging from zero to one.

Defining a non-dimensional variable r_0 by the relation

$$r_0 = \frac{1}{\psi^* - q} \tag{1.34}$$

Kopal (63) has also shown that on the surface of Roche equipotentials, (r, θ, ϕ) are connected through the relation

$$r^* = r_0 [1 + C_3 r_0^3 + C_4 r_0^4 + C_5 r_0^5 + C_6 r_0^6 + C_7 r_0^7 + C_8 r_0^8 + C_9 r_0^9 + ...] \tag{1.35}$$

where

$$C_3 = q\,P_2 + n(1-\nu^2),\ C_4 = q\,P_3,\ C_5 = q\,P_4$$
$$C_6 = q\,p_5 + 3C_3^{\,2},\ C_7 = q\,P_6 + 7\,q\,C_3^{\,2}\,P_3$$
$$C_8 = q\,P_7 + 8q\,C_3\,P_4 + 4q^2 P_3^2$$
$$C_9 = q\,P_8 + 9\,q\,C_3\,P_5 + 9\,q^2 P_3\,P_4$$

Here, $P_j = P_j(\lambda)$ are the Legendre polynomials and terms up to second order of smallness in n and q have been retained in equation (1.35). This relation helps to obtain the shape of a Roche equipotentials ψ^* = constant.

The volume enclosed by the equipotential surface ψ^* = constant is given by

$$V_\psi = \frac{2}{3}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^3}{\mu}\,d\lambda\,d\nu \tag{1.36}$$

Kopal has shown that the explicit expression of V_ψ in terms of r_0 defined by equation (1.34), can be represented as

$$V_\psi = \frac{4}{3}\pi D^3 r_0^3\,[1 + 2n\,r_0^3 + (\frac{12}{5}q^2 + \frac{8}{5}n\,q + \frac{32}{5}\,n^2)\,r_0^6 + \frac{15}{7}q^2\,r_0^{\,8} + 2q^2\,r_0^{\,10} + ...] \tag{1.37}$$

where terms up to second order of smallness in n and q are retained.

Following the approach of Kopal (63), Mohan and Singh (103) obtained the explicit expressions for the surface area S_ψ and the values of averages or parameters r_ψ, $\overline{g}$ and $\overline{g^{-1}}$ on the Roche equipotential ψ^* = constant. These are

$$\begin{aligned} S_\psi &= 2\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^2}{\mu}\,d\lambda\,d\nu \\ &= 4\,\pi\,D^2\,r_0^2\,[1 + \frac{4n}{3}r_0^{\,3} + (\frac{7}{5}q^2 + \frac{14}{15}n\,q + \frac{56}{15}\,n^2)\,r_0^6 + \frac{9}{7}q^2\,r_0^{\,8} + \frac{11}{9}q^2\,r_0^{\,10} + ...] \end{aligned} \tag{1.38}$$

$$\begin{aligned} r_\psi &= (\frac{3V_\psi}{4\pi})^{1/3} \\ &= D\,r_0\,[1 + \frac{2\,n}{3}r_0^3 + (\frac{4}{5}q^2 + \frac{8}{15}n\,q + \frac{76}{45}n^2)\ r_0^6 + \frac{5}{7}q^2\,r_0^{\,8} + \frac{2}{3}q^2\,r_0^{\,10} + ...] \end{aligned} \tag{1.39}$$

$$\overline{g} = \frac{2}{S_\psi}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} (\frac{d\psi}{dn})\frac{r^2}{\mu}d\lambda\, d\nu$$

$$= \frac{GM_\psi}{D^2 r_0^2}[1-\frac{8n}{3}r_0^3 - (3q^2 + 2nq + \frac{40}{9}n^2)r_0^6 - \frac{51}{14}q^2 r_0^8 - \frac{13}{3}q^2 r_0^{10} + ...] \qquad (1.40)$$

and

$$\overline{g^{-1}} = \frac{2}{S_\psi}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} (\frac{d\psi}{dn})^{-1}\frac{r^2}{\mu}d\lambda\, d\nu$$

$$= \frac{D^2 r_0^2}{GM_\psi}[1+\frac{8n}{3}r_0^3 + (\frac{31}{5}q^2 + \frac{62}{15}nq + \frac{584}{45}n^2)r_0^6 + \frac{101}{14}q^2 r_0^8 + \frac{25}{3}q^2 r_0^{10} + ...] \qquad (1.41)$$

Inverting the relation (1.39) they also obtain

$$r_0 = r_\psi^* [1-\frac{2n}{3}r_\psi^{*3} - [\frac{4}{5}q^2 + \frac{8}{15}nq - \frac{4}{45}n^2]r_\psi^{*6} - \frac{5}{7}q^2 r_\psi^{*8} - \frac{2}{3}q^2 r_\psi^{*10} - ...] \qquad (1.42)$$

where $r_\psi^* = r_\psi / D$, r_ψ^* being the nondimensional form r_ψ. In all the above expressions terms up to second order of smallness in n and q have been retained.

1.4 MOHAN, SAXENA AND AGARWAL'S APPROACH FOR COMPUTING THE EFFECTS OF ROTATIONAL AND TIDAL DISTORTIONS ON THE EQUILIBRIUM STRUCTURES AND PERIODS OF OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED GASEOUS SPHERES

Mohan, Saxena and Agarwal (108, 109) used the concept of Roche equipotentials proposed by Kopal in conjunction with Kippenhahn and Thomas's averaging approach to explicitly obtain equations governing the equilibrium structures and periods of radial and nonradial oscillations of rotationally and/or tidally distorted stars and applied these to analyze the problems of rotating stars and stars in binary systems. In this section we briefly review their approach.

1.4.1 MOHAN, SAXENA AND AGARWAL'S APPROACH FOR DETERMINING THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED GASEOUS SPHERES

In order to determine the inner structure of a rotationally and tidally distorted gaseous sphere, the system of equations (1.27) has to be integrated numerically subject to the boundary conditions (1.28) specified therein. Therefore, the evaluation of the actual equipotential surface of a rotationally and tidally distorted gaseous sphere is complicated. Kippenhahn and Thomas (58) proposed that for evaluation of the distortion parameters u, v, w, f_P, f_T etc., the actual equipotential surface may be replaced by Roche equipotential surface (It may be noted that this approximation is reasonably valid for most of the models of the actual stars. In fact as far back as 1933, Chandrasekhar had shown that for stars whose central density bears to the mean density a ratio of 100 or more, the Roche model of a rotating configuration will represent the actual equipotential surfaces of the star within an error of less than one percent).

Once the equipotential surfaces of a rotationally and tidally distorted star are approximated by the Roche equipotentials, the results obtained by Kopal (63, 64) and Mohan and Singh (101, 102) may be used to evaluate explicitly the values of the distortion parameters u, v, w, f_P and f_T appearing in stellar structure equations (1.20) and (1.26). Using equations (1.15), (1.20), (1.26) and (1.37 – 1.42) the explicit expressions of the distortions parameters u, v, w, f_P and f_T on the equipotential surface as obtained by Mohan et al (108) are

$$u = 1 - (\frac{1}{5}q^2 + \frac{2}{15}nq + \frac{4}{45}n^2) r_\psi^{*6} - \frac{1}{7}q^2 r_\psi^{*8} - \frac{1}{9}q^2 r_\psi^{*10} + \ldots \quad (1.43a)$$

$$v = 1 - \frac{4n}{3} r_\psi^{*3} - (\frac{7}{5}q^2 + \frac{14}{15}nq + \frac{68}{45}n^2) r_\psi^{*6} - \frac{31}{14}q^2 r_\psi^{*8} - 3q^2 r_\psi^{*10} - \ldots \quad (1.43b)$$

$$w = 1 + \frac{4n}{3} r_\psi^{*3} + (\frac{23}{5}q^2 + \frac{16}{15}nq + \frac{212}{45}n^2) r_\psi^{*6} + \frac{81}{14}q^2 r_\psi^{*8} + 7q^2 r_\psi^{*10} + \ldots \quad (1.43c)$$

$$f_P = 1 - \frac{4n}{3} r_\psi^{*3} - (\frac{22}{5}q^2 + \frac{44}{15}nq + \frac{128}{45}n^2) r_\psi^{*6} - \frac{79}{14}q^2 r_\psi^{*8} - \frac{62}{9}q^2 r_\psi^{*10} - \ldots \quad (1.43d)$$

and

$$f_T = 1 - (\frac{14}{5}q^2 + \frac{28}{15}nq + \frac{56}{45}n^2) r_\psi^{*6} - \frac{46}{14}q^2 r_\psi^{*8} - \frac{34}{9}q^2 r_\psi^{*10} - \ldots \quad (1.43e)$$

where $r_\psi^\bullet = r_\psi / D$ is the nondimensional form of r_ψ and terms up to second order of smallness in n and q are retained.

The values of M_ψ, P_ψ, L_ψ etc. on the various equipotential surfaces of a rotationally and tidally distorted gaseous spheres may now be obtained by solving the system of differential equations (1.27) with boundary conditions (1.28) and using the values of distortion parameters f_P and f_T as given in (1.43).

It may be noted that while approximating the equipotential surfaces of a rotationally and tidally distorted model by Roche equipotentials, the structure of the star is not approximated by the structure of a Roche model. In the case of no distortion (*n=q=0),* equation (1.43) gives $u = v = w = f_P = f_T = 1$ and the system of differential equations (1.27) reduce to the equations governing the equilibrium structure of the original undistorted star and not of the Roche model.

Usual methods for stellar structure equations such as Henyey et al (48) method can be now used to integrate the system of differential equation (1.27) governing the equilibrium structure of a rotationally and tidally distorted gaseous sphere. At every step, the values of the distortion parameters u, v, w, f_P and f_T have to be computed using (1.43).

In case the thermal properties are not considered important and only hydrostatic equilibrium of a rotationally and tidally distorted gaseous spheres is to be investigated then we need only to integrate equation (1.17) and (1.20) subject to the boundary conditions

At the center $r_\psi = 0$

$$M_\psi = 0 \tag{1.44a}$$

and at the free surface $r_\psi = R_\psi$

$$\begin{aligned} &M_\psi = M_0, P_\psi = 0 \\ &\rho_\psi = 0 \ \ or \ \ P_\psi = P_{\psi s}, \rho_\psi = \rho_{\psi s} \end{aligned} \tag{1.44b}$$

In case the star is being distorted by rotational forces alone (or tidal forces alone) we may set $q = 0$ ($n = 0$) in (1.43) and still use the above approach to determine the equilibrium structure of corresponding purely rotationally distorted or purely tidally distorted model. For obtaining the structure of the primary component of a synchronous binary system we may set $n = (q+1)/2$.

Mohan and Saxena (99) found it more convenient to work with r_0 in place of M_ψ or r_ψ as independent variable by using (1.34) which is connected with variable r_ψ through relation (1.42). Saxena (133) expressed the system of differential equations governing the equilibrium structure of a rotationally and tidally distorted stellar model as

$$\frac{dM_{\psi}}{dr_0} = 4\pi D^3 \rho_{\psi} r_0^2 f_1, \tag{1.45a}$$

$$\frac{dP_{\psi}}{dr_0} = -\frac{GM_{\psi}}{Dr_0^2} \rho_{\psi} f_2, \tag{1.45b}$$

$$\frac{dL_{\psi}}{dr_0} = 4\pi \varepsilon D^3 \rho_{\psi} r_0^2 f_1, \tag{1.45c}$$

and

$$\frac{dT_{\psi}}{dr_0} = -\frac{3\kappa L_{\psi}}{16\pi D ac T_{\psi}^3} \frac{\rho_{\psi}}{r_0^2} f_3. \tag{1.45d}$$

Here f_1, f_2 and f_3 are functions of n, q and r_0 incorporating the effects of rotation and tidal distortions on the equilibrium structure equations of a stellar model. The explicit expressions for these distortion parameters as given by Saxena (133) are

$$f_1 = 1 + 4n r_0^3 + (\frac{36}{5}q^2 + \frac{24}{5}nq + \frac{96}{5}n^2) r_0^6 + \frac{55}{7}q^2 r_0^8 + \frac{26}{3}q^2 r_0^{10} + ... \tag{1.46a}$$

$$f_2 = 1 - (\frac{2}{5}q^2 + \frac{4}{15}nq + \frac{16}{15}n^2) r_0^6 - \frac{9}{14}q^2 r_0^8 - \frac{8}{9}q^2 r_0^{10} + ... \tag{1.46b}$$

and

$$f_3 = 1 + \frac{4n r_0^3}{3} + (\frac{6}{5}q^2 + \frac{4}{5}nq + \frac{224}{45}n^2) r_0^6 + \frac{24}{14}q^2 r_0^8 + \frac{20}{9}q^2 r_0^{10} + ... \tag{1.46c}$$

In these above expressions terms up to second order of smallness in n, q and up to r_0^{10} in r_0 are retained. The boundary conditions governing the system of differential equations (1.45) are:

At the center $r_0 = 0$,

$$M_{\psi} = 0, \; L_{\psi} = 0 \tag{1.47a}$$

and at the free surface $r_0 = r_{0s}$

$$\begin{aligned} &M_{\psi} = M_0, \; L_{\psi} = L_{\psi s} \\ &P_{\psi} = 0, \; \rho_{\psi} = 0, \; T_{\psi} = 0 \;\; or \;\; P_{\psi} = P_{\psi s}, \; \rho_{\psi} = \rho_{\psi s}, T_{\psi} = T_{\psi s} \end{aligned} \tag{1.47b}$$

where r_{0s} being the value of r_0 at the free surfaces.

In fact

$$r_{0s} = \frac{1}{\psi_s^* - q} \tag{1.48}$$

where $\psi_s^\bullet$ is the nondimensional form of the total potential ψ on the outermost equipotential surface of a rotationally and tidally distorted stellar model. In the case of no distortion $f_p = f_T = 1$ and the above equations reduce to the usual equations governing the equilibrium structure of an undistorted gaseous sphere.

1.4.2 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EIGENFREQUENCIES OF SMALL ADIABATIC PSEUDO – RADIAL MODES OF OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED GASEOUS SPHERES

Mohan et al (109) also formulated eigenvalue problems which determine the eigenfrequencies of small adiabatic pseudo-radial and nonradial modes of oscillations of rotationally and tidally distorted stellar models. The approach was later used by Lal (70) to determine the eigenfrequencies of small adiabatic pseudo-radial and nonradial modes of oscillations of certain differentially rotating and tidally distorted stars.

Assuming that during oscillations the fluid elements on an equipotential surface oscillate in unison, the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of the actual rotationally and tidally distorted model can be obtained from its topologically equivalent spherical model developed on the basis of the averaging technique of Kippenhahn and Thomas (58). Following this approach, the equation determining the eigenfrequencies of pseudo-radial modes of oscillations of rotationally and tidally distorted stellar model has been expressed by Mohan and Saxena (100) as:

$$\frac{d^2\kappa}{dr_{0\psi}^2} + \frac{4-\mu}{r_{0\psi}}\frac{d\kappa}{dr_{0\psi}^2} + [\frac{\rho_{0\psi}}{\gamma P_{0\psi}}\sigma^2 - (3-\frac{4}{\gamma})\frac{\mu}{r_{0\psi}^2}]\kappa = 0 \tag{1.49}$$

where

$$\mu = -\frac{r_{0\psi}}{P_{0\psi}}\frac{dP_{0\psi}}{dr_{0\psi}}$$

Here $r_{0\psi}$, $\rho_{0\psi}$ and $P_{0\psi}$ are the values of r_ψ, ρ_ψ and P_ψ on the equipotential ψ^* = constant in its equilibrium position, σ the eigenfrequency of oscillation and κ some average of the relative amplitudes of pulsation of the fluid elements on the equipotential surface ψ^* =

constant. Using r_ψ, ρ_ψ and P_ψ in place of $r_{0\psi}$, $\rho_{0\psi}$ and $P_{0\psi}$ to denote the equilibrium values on the equipotential surfaces and taking $r_0 = 1/(\psi - q)$ in place of r_ψ as the independent variable, equation (1.49) governing the small adiabatic pseudo-radial modes of oscillations of a rotationally and tidally distorted gaseous sphere has been expressed as:

$$A(n,q,r_0)\frac{d^2\kappa}{dr_0^2} + [\frac{4-\mu}{r_0}B(n,q,r_0) - C(n,q,r_0)]\,\frac{d\kappa}{dr_0}$$
$$+[\frac{D^2\sigma^2\rho_\psi}{\gamma P_\psi} - \left(3-\frac{4}{\gamma}\right)\frac{\mu}{r_0^2}E(n,q,r_0)]\;\kappa = 0 \qquad (1.50)$$

where

$$A(n,q,r_0) = 1 - \frac{16}{3}nr_0^{\,3} - (\frac{56}{5}q^2 + \frac{112}{15}nq + \frac{104}{45}n^2)\,r_0^6 - \frac{90}{7}q^2r_0^{\,8} - \frac{44}{3}q^2r_0^{\,10} + \ldots$$

$$B(n,q,r_0) = 1 - \frac{10n}{3}r_0^{\,3} - (\frac{32}{5}q^2 + \frac{64}{15}n\,q + \frac{188}{45}n^2)\,r_0^6 - \frac{50}{7}q^2r_0^{\,8} - 8\,q^2r_0^{\,10} + \ldots$$

$$C(n,q,r_0) = \frac{1}{r_0}\,[8n\,r_0^{\,3} + (\frac{168}{5}q^2 + \frac{112}{5}n\,q + \frac{104}{15}n^2)\,r_0^6 + \frac{360}{7}q^2r_0^{\,8} + \frac{220}{3}q^2r_0^{\,10} + \ldots]$$

and

$$E(n,\,q,r_0) = 1 - \frac{4n}{3}r_0^{\,3} - (\frac{8}{5}q^2 + \frac{16}{15}nq + \frac{92}{45}n^2)\,r_0^6 - \frac{10}{7}q^2r_0^{\,8} - \frac{4}{3}q^2r_0^{\,10} + \ldots$$

Also $\quad \mu = -\frac{r_\psi}{P_\psi}\frac{dP_\psi}{dr_0}\frac{dr_0}{dr_\psi} = -F\,(n,\,q,\,r_0)\frac{r_0}{P_\psi}\frac{dP_\psi}{dr_0}$

where

$$F(n,q,r_0) = 1 - 2nr_0^{\,3} - (\frac{24}{5}q^2 + \frac{16}{5}nq + \frac{72}{15}n^2)r_0^6 - \frac{40}{7}q^2r_0^{\,8} - \frac{20}{3}q^2r_0^{\,10} + \ldots$$

In the absence of any distortion, $\mathrm{n}=0,\ \mathrm{q}=0, D=R,\ P_\psi = P,\ \rho_\psi = \rho,\ r_0 = x$, the above equation reduces to

$$\frac{d^2\kappa}{dx^2} + \frac{4-\mu}{x}\,\frac{d\kappa}{dx} + [\frac{R^2\sigma^2\rho}{\gamma P} - (3-\frac{4}{\gamma})\frac{\mu}{x^2}]\kappa = 0 \qquad (1.51)$$

with

$$\mu = -\frac{x}{P}\frac{dP}{dx}$$

which is the usual equation determining the eigenfrequencies of small adiabatic radial modes of oscillations of a gaseous sphere (cf. Rosseland (127) p.30 with $(\gamma = 1.67)$).

Equation (1.50) forms an eigenvalued boundary value problem in the eigenfrequency of oscillation σ. This eigenvalue problem is of Sturm-Liouville type having singularities both at the centre and the surface of the model. It has to be solved subject to the boundary conditions which require κ to be finite at the centre as well as at the free surface.

In reality equation (1.50) determines the periods of small adiabatic radial modes of oscillations of the topologically equivalent spherical model. However, since equipotential surfaces of the actual rotationally and tidally distorted model are also the surfaces of equipressure and equidensity, the values of pressure and density on the equipotential surfaces of the rotationally and tidally distorted stars are same as on the corresponding equipotential surfaces of the equivalent spherical model. Hence the eigenfrequencies of the radial modes of oscillations determined by solving the eigenvalue problem for the topologically equivalent spherical model are indeed the eigenfrequencies of the radial modes of oscillation of the undistorted model which has got distorted by the rotational and tidal effects. However, the values of the eigenfunction κ obtained on solving equation (1.50) are not the actual values of amplitudes of pulsation κ for the distorted model but rather some average of the true values of eigen function κ at various points on the corresponding equipotential surface of the rotationally and tidally distorted model.

We may thus use equation (1.50) to determine the effects of rotation and tidal distortions on the periods of small adiabatic radial modes of oscillations of a stellar model. The effects of rotation and tidal distortions have been incorporated through introduction of terms $A(n, q, x)$, $B(n, q, x)$, $C(n, q, x)$, $E(n, q, x)$ and $F(n, q, x)$ and dependence of ρ_ψ and P_ψ on ψ. The present method, in fact, incorporates the effects of distortional forces both while computing the equilibrium structure (in computing the values of P_ψ, ρ_ψ etc.) as well as in the coefficients *A, B* and *C* of the equation (1.50).

The eigenvalue problem (1.50) together with the boundary conditions which require κ to be finite both at the centre as well as the free surface of the star has been solved numerically in the usual manner as is done in the case of undistorted models. For the numerical work authors found it convenient to set

$$\kappa = \frac{\varsigma}{r_0} \quad \text{and} \quad r_0 = x\, r_{OS} \tag{1.52}$$

(r_{OS} being the value of r_0 on the outermost surface) in equation (1.50) and treat x as independent variable and ς as the dependent variable. With these substitution x becomes zero at the centre and one at the free surface. The boundary condition κ = finite at the centre gets replaced by ς = 0 at the centre. The boundary condition κ = finite at the free surface now becomes ς finite at $x=1$. Using relation (1.52), equation (1.50) gets transformed in terms of the variables ς and x and as

$$A^*(n,q,x)\frac{d^2\varsigma}{dx^2} + B^*(n,q,x)\frac{d\varsigma}{dx} + C^*(n,q,x)\varsigma = 0 \tag{1.53}$$

where

$$A^*(n,\, q,\, x) = A(n,\, q,\, x\, r_{OS})$$

$$B^*(n,\, q,\, x) = \frac{4-\mu}{x} B(n,\, q,\, x\, r_{OS}) - r_{OS} C(n,\, q,\, x\, r_{OS}) - \frac{2}{x} A(n,\, q,\, x\, r_{OS})$$

and

$$C^*(n,\, q,\, x) = \frac{r_{OS}^2 D^2 \rho_\psi}{\gamma P_\psi}\sigma^2 - (3 - \frac{4}{\gamma})\frac{\mu}{x^2} E(n,\, q,\, x\, r_{OS}) - \frac{1}{x} B^*(n,\, q,\, x\, r_{OS})$$

The boundary conditions now are:

$$\varsigma = 0 \text{ at the centre } \mathrm{x} = 0 \tag{1.54a}$$

and

$$\varsigma = \text{finite at the surface } \mathrm{x} = 1 \tag{1.54b}$$

For computing an eigenvalue σ, equation (1.53) has to be solved numerically subject to the boundary conditions specified in (1.54). Since the centre and the free surface of the star are the singularities of this differential equitation so it has been found advisable to write the series solutions of equation (1.53) near the singularities to start numerical integrations.

1.4.3 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EIGENFREQUENCIES OF SMALL ADIABATIC NONRADIAL MODES OF OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED GASEOUS SPHERES

Mohan et al (109) have also formulated eigenvalued boundary value problem to determine the eigenfrequencies of nonradial modes of oscillations of rotationally and tidally distorted gaseous spheres. As in the earlier case, values of the physical parameters ρ_ψ and P_ψ on the equipotential surfaces of the distorted model are assumed to be same as those on the corresponding equipotential surfaces of the topologically equivalent spherical model. This topologically equivalent spherical model has then been used to determine the eigenfrequencies of nonradial modes of oscillations of the rotationally and tidally distorted gaseous spheres. Saxena (133) has expressed the eigenvalue problem determining the eigenfrequencies of nonradial modes of oscillations of rotationally and tidally distorted gaseous spheres in an explicit form convenient for computational work as

$$\left.\begin{aligned} &\frac{d\zeta}{dx} + B_1\zeta + (B_2 + \frac{1}{\sigma^2}B_3)\eta + \frac{1}{\sigma^2}B_3\phi = 0 \\ &\frac{d\eta}{dx} + \left(E_1\sigma^2 + E_2\right)\zeta + E_3\eta + E_4\phi + \frac{d\phi}{dx} = 0 \\ &\frac{d^2\phi}{dx^2} + F_1\frac{d\phi}{dx} + F_2\zeta + F_3\eta + F_4\phi = 0 \end{aligned}\right\} \tag{1.55}$$

where

$$B_1 = \frac{l+1}{x} + \frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx}$$

$$\begin{aligned} B_2 &= \frac{2\pi G\rho_c}{Dx}\frac{\rho_\psi}{\gamma P_\psi}r_\psi^2\frac{dr_\psi}{dx} \\ &= \frac{2\ \pi\ G\rho_c}{\gamma\, P_\psi}D^2\ \rho_\psi\ r_{0s}^3\ x[1+4n(xr_{0s})^3 + (\frac{36}{5}q^2 + \frac{864}{45}n^2 + \frac{72}{15}nq)(xr_{0s})^6 \\ &\quad + \frac{55}{7}q^2(xr_{0s})^8 + \frac{26}{3}q^2(xr_{0s})^{10} + ...] \end{aligned}$$

$$B_3 = -\frac{l(l+1)}{Dx}\frac{dr_\psi}{dx}2\pi G\rho_c$$

$$= -\frac{l(l+1)}{x}2\pi G\rho_c r_{0s}[1+\frac{8n}{3}(xr_{0s})^3 + (\frac{28}{5}q^2+\frac{532}{45}n^2+\frac{56}{15}nq)(xr_{0s})^6$$

$$+\frac{45}{7}q^2(xr_{0s})^8+\frac{22}{3}q^2(xr_{0s})^{10}+...]$$

$$E_1 = -\frac{1}{2\pi G\rho_c}\frac{Dx}{r_\psi^2}\frac{dr_\psi}{dx}$$

$$= -\frac{1}{2\pi G\rho_c r_{0s}x}[1+\frac{4n}{3}(xr_{0s})^3+(4q^2+\frac{56}{9}n^2+\frac{8}{3}nq)(xr_{0s})^6$$

$$+5q^2(xr_{0s})^8+6q^2(xr_{0s})^{10}+...]$$

$$E_2 = \frac{1}{2\pi G\rho_c}\frac{A_\psi}{\rho_\psi}\frac{dP_\psi}{dx}\frac{Dx}{r_\psi^2}$$

$$= \frac{1}{2\pi G\rho_c D^2}\frac{1}{\rho_\psi}(\frac{1}{\rho_\psi}\frac{d\rho_\psi}{dx}-\frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx})\frac{dP_\psi}{dx}\frac{1}{xr_{0s}^3}[1-4n(xr_{0s})^3-(\frac{36}{5}q^2+\frac{144}{45}n^2$$

$$+\frac{72}{15}nq)(xr_{0s})^6-\frac{55}{7}q^2(xr_{0s})^8-\frac{26}{3}q^2(xr_{0s})^{10}-...]$$

$$E_3 = \frac{l}{x}+A_\psi\frac{dr_\psi}{dx} = \frac{l}{x}+(\frac{1}{\rho_\psi}\frac{d\rho_\psi}{dx}-\frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx}),\quad E_4=\frac{l}{x}$$

$$F_1 = \frac{2l}{x}-\frac{d^2r_\psi/dx^2}{dr_\psi/dx}+\frac{2}{r_\psi}\frac{dr_\psi}{dx}$$

$$= \frac{1}{x}[2(l+1)-4n(xr_{0s})^3-(24q^2+32n^2+16nq)(xr_{0s})^6$$

$$-40q^2(xr_{0s})^8-60q^2(xr_{0s})^{10}-...]$$

$$F_2 = 2\frac{\rho_\psi}{\rho_c}\frac{A_\psi Dx}{r_\psi^2}(\frac{dr_\psi}{dx})^2$$

$$= 2\frac{\rho_\psi}{\rho_c}(\frac{1}{\rho_\psi}\frac{dP_\psi}{dx}-\frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx})\frac{1}{xr_{0s}}[1+\frac{4n}{3}(xr_{0s})^3+(4q^2+\frac{56}{9}n^2+\frac{8}{3}nq)(xr_{0s})^6$$

$$+5q^2(xr_{0s})^8+6q^2(xr_{0s})^{10}+...]$$

$$F_3 = -\frac{4\pi G\rho_\psi^2}{\gamma P_\psi}\left(\frac{d\,r_\psi}{d\,x}\right)^2$$

$$= -\frac{4\pi G r_{0s}^2\, D^2 \rho_\psi^2}{\gamma P_\psi}[1 + \frac{16n}{3}(xr_{0s})^3 + (\frac{56}{5}q^2 + \frac{1384}{45}n^2 + \frac{112}{15}nq)(xr_{0s})^6$$

$$+\frac{90}{7}q^2\ (xr_{0s})^8 + \frac{44}{3}q^2(xr_{0s})^{10} + ...]$$

$$F_4 = \frac{l(l+1)}{x^2} - \frac{l}{x}\left(\frac{d^2 r_\psi}{dx^2}\right)/\left(\frac{dr_\psi}{dx}\right) + \frac{2l}{x}\left(\frac{1}{r_\psi}\frac{dr_\psi}{dx}\right) - \frac{l(l+1)}{r_\psi^2}\left(\frac{dr_\psi}{dx}\right)^2$$

$$= -\frac{l}{x^2}\{[8n(xr_{0s})^3 + (\frac{168}{5}q^2 + \frac{2412}{45}n^2 + \frac{336}{15}nq)(xr_{0s})^6 + \frac{360}{7}q^2\,(xr_{0s})^8$$

$$+\frac{220}{3}q^2(xr_{0s})^{10}....] + l[4n(xr_{0s})^3\ + (\frac{48}{5}q^2 + \frac{972}{45}n^2\ + \frac{96}{15}nq)(xr_{0s})^6$$

$$+\frac{80}{7}q^2\ (xr_{0s})^8 + \frac{40}{3}q^2(xr_{0s})^{10} + ...]\}$$

Again σ is the eigenfrequency of oscillations and $x = r_0/r_{0s}$ is the nondimensional form of a distance of a fluid element from the center of the star.

Also $\zeta = \frac{r_\psi^2 \delta r_\psi}{D^3 x^{l+1}}$, $\eta = \frac{\mathrm{P}'_\psi}{2\,\pi\,\mathrm{G}\,\rho_\mathrm{c}\,D^2 x^l \rho_\psi}$ and $\phi = \frac{\psi'_g}{2\pi\,\mathrm{G}\rho_\mathrm{c}\,D^2 x^l}$ (1.56)

δr_ψ being an average of the amplitudes of Lagrangian variations in the radial direction and P'_ψ, ψ'_g the amplitudes of Lagrangian variation in pressure and gravitational potential on the equipotential surface ψ^* = constant. In the above expressions terms up to second order of smallness in n and q and up to order r_0^{10} in r_0 have been retained.

The eigenvalue problem (1.55) determining the eigenfrequencies of nonradial modes of oscillations of rotationally and tidally distorted gaseous spheres is to be solved subject to the boundary conditions at the centre and the free surface. Boundary conditions at the centre require $\delta r_\psi = 0$, $P'_\psi/\rho_\psi = 0$ and $\psi'_g = 0$ for $r_\psi = 0$. These requirements lead to the analytic conditions

$$\eta + \phi = \frac{\sigma^2}{2\pi G\rho_c\, l\, r_{OS}}\zeta,\ \frac{d\phi}{dx} = 0 \tag{1.57}$$

at the centre $x = 0$.

If the pressure P_ψ on the free surface is taken to be zero, then δP_ψ, the Lagrangian variation in pressure, should be zero at the outer surface. This leads to the condition

$$2\pi G\rho_c\, r_\psi^2\, \rho_\psi \frac{dr_\psi}{dx}\eta + D\frac{dP_\psi}{dx}\zeta = 0$$

or

$$2\pi G\,\rho_c\rho_\psi D^2 r_{0s}^3[1+4n\,r_{0s}^3 + (\frac{36}{5}q^2 + \frac{864}{45}n^2 + \frac{72}{15}nq)r_{0s}^6 + \frac{55q^2}{7}r_{0s}^8 + \frac{26q^2}{3}r_{0s}^{10} + ...]\eta + \frac{dP_\psi}{dx}\zeta = 0. \qquad (1.58a)$$

The condition requiring gravitational potential to be continuous across the free surface gives

$$\frac{d\phi}{dx} + [l + \frac{(l+1)}{r_\psi}\frac{dr_\psi}{dx}]\phi + \frac{2D\rho_\psi}{\rho_c r_\psi^2}\frac{dr_\psi}{dx}\zeta = 0$$

or

$$\frac{d\phi}{dx} + \phi\{l + (l+1)\,[1 + 2n\,r_{0s}^3 + (\frac{24q^2}{5} + \frac{396n^2}{45} + \frac{48nq}{15})r_{0s}^6 + \frac{40q^2}{7}r_{0s}^8 + \frac{20q^2}{3}r_{0s}^{10} + ...]\} + \frac{2\rho_\psi}{\rho_c\, r_{0s}}[1 + \frac{4}{3}n\,r_{0s}^3 + (4q^2 + \frac{8}{3}n\,q + \frac{56}{9}n^2)r_{0s}^6 + 5q^2 r_{0s}^8 + 6q^2 r_{0s}^{10} + ...]\zeta = 0 \qquad (1.58b)$$

at the surface $x = 1$

Thus, in terms of the nondimensional eigenfunctions ζ, η and ϕ the problem of determining the eigenfrequencies of nonradial modes of oscillation of rotationally and tidally distorted gaseous spheres reduces to solving the system of differential equation (1.55) subject to the boundary conditions (1.57) at the centre and the boundary conditions (1.58) at the free surface.

1.5 THE PRESENT WORK

In case of stars in binary systems besides gravitational and centrifugal forces, Coriolis force also comes into picture. While computing the effects of rotational and tidal distortions on the equilibrium structures and the eigenfrequencies of radial and nonradial modes of oscillations of rotating stars and stars in binary systems, Mohan and Saxena (99, 100) and Mohan et al (108, 109, 104, 107) approach does not explicitly accounts for the effects of

Coriolis force. Comments have been made in this regard particularly when it was observed that using this approach there is decrease in the values of eigenfrequencies of g – modes of nonradial oscillations whereas results of other workers are contrary to it. Moreover in this approach the problem is analyzed in a stationary frame of reference whereas in case of analysis of rotation in a binary system, rotating frame of reference is used. Keeping this in view we have analyzed in the present study the effects of inclusion of Coriolis force, besides the gravitational and centrifugal forces, on the equilibrium structures as well as periods of small adiabatic barotropic oscillations of rotating stars and stars in binary system using a rotating frame of reference. Keeping in view the fact that in the case of some observed rotating stars and stars in binary systems, rotation is considered to be not uniform but differential (in which velocity of rotation varies from point to point), we have also investigated the effect of Coriolis force on the equilibrium structures and periods of small adiabatic barotropic oscillations of differentially rotating binary stars.

Chapterwise summary of the work presented in the subsequent chapters of this thesis is as follows:

In chapter II we first develop an expression for the Roche equipotential of a rotating star in a binary system in a rotating frame of reference to explicitly include the effect of Coriolis force besides the effect of centrifugal and gravitational forces. Next we use it to determine the effects of Coriolis force on the shapes of Roche equipotential surfaces and position of Roche limit of binary stars. Numerical computations have been performed to determine the shapes of Roche equipotentials and the values of Roche limits for different values of mass ratio of the companion stars as well as for different values of the angular velocity of rotation of the primary star. The results thus obtained have been compared with the earlier corresponding results in which the effects of Coriolis force had not been considered. Shapes of various Roche equipotentials surfaces of rotating stars and stars in binary systems have also been drawn in certain cases both in the presence as well as absence of Coriolis force.

Analytic expression for Roche equipotentials developed in chapter II which incorporates the effects of Coriolis force also, has been used in chapter III in conjunction with Mohan, Saxena and Aggarwal (108) approach to obtain the system of differential equations governing the equilibrium structures of rotationally and tidally distorted stellar models. The use of the method in computing the equilibrium structures of rotationally and tidally distorted stars has been next illustrated by applying it to obtain the equilibrium structures and certain other observable parameters of rotationally and tidally distorted polytropic models of stars of indices 1.5 and 3.0 for different choices of the values of rotational and tidal distortion parameters. The results thus obtained are compared with the results earlier computed by Mohan and Saxena (99) in which the effects of Coriolis force had not been considered. Shapes of the outer most surfaces of certain polytropic models of stars have also been drawn to highlight the effect of Coriolis force on the shapes of stars.

In chapter IV we develop eigenvalued boundary value problem which determines the periods of small adiabatic barotropic modes of oscillations of rotating stars and stars in binary systems incorporating besides the gravitational and centrifugal forces, the effects of Coriolis force as well. The method has been then used to determine the effects of Coriolis force on the eigenfrequencies of various pseudo radial and nonradial modes of oscillations of polytropic models of stars. The eigenvalued problem determining the radial and nonradial modes of oscillations have been solved numerically for various polytropic models of indices 1.5 and 3.0. The results thus obtained are compared with the results earlier obtained by Mohan and Saxena (100) and certain other authors.

In chapter V we use the methodology developed in chapters II and chapter III to determine the effect of Coriolis force on the equilibrium structures of differentially rotating stars in binary systems using a law of differential rotation proposed by Clement (15). Expressions for the volumes, surface areas and other physical parameters of polytropic models rotating differentially according to this law have also been obtained. Numerical computations have also been performed to compute the equilibrium structures of certain differentially rotating polytropic models of stars of indices 1.5 and 3.0. The numerical results thus obtained are compared with the results earlier obtained by Mohan et al (104) in which the effects of Coriolis force were not considered.

In chapter VI we study the effect of Coriolis force on the eigenfrequencies of small adiabatic barotropic modes of oscillations of differentially rotating stars in binary systems assuming a law of differential rotation used in chapter V. The eigenvalued boundary value problem, determining the eigenfrequencies of small adiabatic pseudo radial and nonradial modes of oscillations of differentially rotating stellar models have been formulated taking into account the effects of Coriolis force. These eigenvalued problems have then been solved numerically to compute the eigenfrequencies of pseudo radial and nonradial modes of oscillations of certain differentially rotating primary components of binary stars assuming these to have polytropic structures whose equilibrium structures were earlier obtained in chapter V. The numerical results thus obtained have been compared with the results earlier reported in Mohan et al (107) which do not explicitly account for the effects of Coriolis force.

Conclusions based on the present study are finally drawn in the concluding chapter VII. The limitations and scope for future work are also discussed in this chapter.

CHAPTER – II

EFFECT OF CORIOLIS FORCE ON THE SHAPES OF ROTATING STARS AND STARS IN BINARY SYSTEMS

It is thus evident that theoretical investigations of determining the effects of rotation and tidal forces on the equilibrium structures and the periods of small radial and nonradial modes of oscillations of gaseous spheres can help in better understanding the observed phenomena of the rotating stars and stars in binary and multiple systems. Such studies are also expected to help in better understanding the problems of stellar stability in general and the problems of stellar variability of rotating stars and stars in binary or multiple systems in particular.

Kopal (63) introduced the concept of Roche equipotentials to analyze the problems of rotating stars and stars in binary systems. Since then several authors such as Kopal (64, 65, 66), Mohan and Singh (101, 102), Eggleton (33), Mohan and Saxena (99), Mohan et al (108, 104, 106) and Lal et al (72) have used this concept to analyze the problems of rotationally and/or tidally distorted stars. In this approach Roche approximation for the inner structure of a star is used to obtain an expression for the potential of a rotating star and star in a binary system. This approach, accounts for the effects of gravitational and centrifugal forces. However, it does not explicitly take into account the effect of Coriolis force.

In the present chapter we have first developed an expression for Roche equipotential of the primary component of a binary star in a rotating frame of reference which explicitly accounts for the effects of Coriolis force besides gravitational and centrifugal forces. This expression is then used to determine the shapes of various Roche equipotentials and position of Roche limits (the point on the line joining centers of two stars where the surfaces of the two stars just touch each other) in case of binary stars. The fact that close binaries in which one, or both, components have attained their Roche limit are observed in the sky underlines the importance of the study of the geometry of Roche limits in binary systems of different mass ratios.

The expression for the Roche equipotential surface which incorporates the effect of Coriolis force in addition to the centrifugal and gravitational force has been obtained in section 2.1. The modified expression for Roche equipotential is then used to compute the shapes of Roche equipotentials in section 2.2. In this section, mathematical expressions which can be used to determine the shapes of Roche equipotentials, position of Roche limit and various other important parameters have been derived. In section 2.3, the expression for Roche equipotentials determined in section 2.2 has been then used to numerically determine the shapes of Roche equipotentials, position of Roche limit and certain other important parameters. Numerical results so obtained are also compared with the results in which the effect of Coriolis force is not taken into account. Certain conclusions based on the present study are finally drawn in section 2.4.

2.1 ROCHE EQUIPOTENTIALS OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS

A rotating star is a star rotating about an axis passing through its center. In case of a binary system of stars we have two stars which are rotating about their axes passing through their centers as well as revolving about their common center of mass. In the case of most of the observed binary stars, one star (usually called the primary component) is more massive than the other star (called the secondary component).

In a binary system following Kopal (63), let M_0 and M_1 be the masses of the two components separated by distance *D*. The primary component of mass M_0 of this binary system is supposed to be much massive than its companion star of mass M_1 $(M_0 >> M_1)$ which for all practical purposes is regarded as a point mass. The primary is supposed to have its normal configuration. However, its inner structure is approximated by Roche model for the purpose of computation of the potential of the system. (In case of Roche model it is assumed that the total mass of a star is concentrated at its center and this point mass is surrounded by an evanescent envelope in which density varies inversely as the square of the distance from its center). This approximation is reasonably valid for majority of the realistic stars of main sequence and post main sequence stages (c.f. Chandrasekhar (11)).

Now suppose that the position of the two components of such a binary system is referred to a rectangular system of cartesian coordinates with origin at the center of gravity of the primary star of mass M_0, $x-$ axis along the line joining the mass centers of the two stars, and $z-$axis perpendicular to the plane of the orbit of the two components (Fig 2.1).

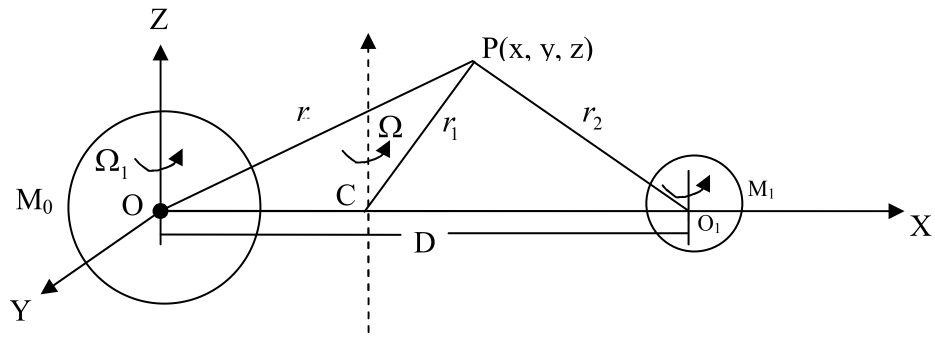


Fig. 2.1: Axis of Reference for a binary system

In the figure $r=\sqrt{x^2+y^2+z^2}$, $r_2=\sqrt{(D-x)^2+y^2+z^2}$ and $r_1=\sqrt{(x-d_1)^2+y^2+z^2}$, represent the distances of a point P(x, y, z) from the centers of gravity of the primary star with center at O, secondary star with center at O_1 and the center of gravity C ($(d_1, 0, 0)$ where $d_1 = M_1 D/(M_0+M_1)$) of the system respectively. Let Ω denote the angular velocity of revolution of the system about a line parallel to z - axis which passes through the center of gravity C of the system and is perpendicular to the xy - plane. Also let Ω_1 be the angular velocity of rotation of the primary about z – axis.

In a binary system, the primary star (which is of interest to us in our present study) is rotating about axis OZ with angular velocity Ω_1 as well as revolving about an axis parallel to z – axis passing through the common center of mass C with angular velocity Ω. The point P when it is inside the primary will experience the effects of Coriolis force besides the gravitational and centrifugal forces. Using the concepts of classical dynamics, for the system described above (Fig 2.1), the total potential at a point P inside the primary component, which experiences the effect of Coriolis force besides the gravitational and centrifugal forces, is given by

$$\psi=\frac{GM_0}{r}+\frac{GM_1}{r_2}+\frac{1}{2}(\vec{\Omega}\times\vec{r}_1)\cdot(\vec{\Omega}\times\vec{r}_1)+\vec{V}\cdot(\vec{\Omega}\times\vec{r}_1) \tag{2.1}$$

Here $\vec{V}$ is the velocity of particle of unit mass at point P(x, y, z) with respect to a rotating frame of reference, rotating with the angular velocity of system. The first two terms in equation (2.1) correspond to the gravitational potential which arises due to the primary and secondary component of binary system and third term is due to centrifugal force. These three terms are same as earlier obtained by Kopal (63) in his studies to the problems of Roche Model and its applications to close binary system. The fourth term $\vec{V}\cdot(\vec{\Omega}\times\vec{r}_1)$ represents the contribution of the Coriolis force to the potential at point P, where $\vec{V}$ is the tangential component of velocity of this particle in the rotating frame of reference. Points inside the rotating star will be subjected to Coriolis force even when such a point is not having any external velocity because of the differences in the velocity of the rotation of the primary and angular velocity of revolution of the nonsynchronous binary system. (This difference of course vanishes in case of synchronous binary stars where velocity of rotation is same as that of revolution). In the earlier studies carried out on Roche equipotentials by Kopal (63) and Mohan and Saxena (99), the contribution of this last term $\vec{V}\cdot(\vec{\Omega}\times\vec{r})$, which arises on account of Coriolis force, has been neglected. In the present study we have tried to incorporate its effect on the subsequent studies.

If we write $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ to represent the position vector of the point P(x, y, z) inside the star rotating about its axis with angular velocity $\vec{\Omega}_1$, the term $\vec{V}\cdot(\vec{\Omega}\times\vec{r}_1)$ can be simplified as follows.

In the rotating frame of reference, the point P(x, y, z) rotates in a circle of radius PL with angular velocity $\Omega_P = \Omega_1 - \Omega$ (Fig 2.2).

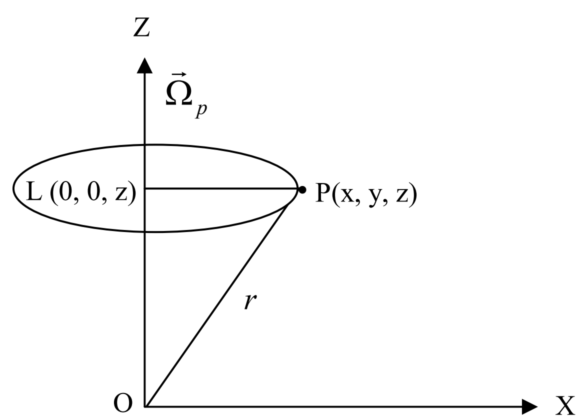


Fig. 2.2: Rotation of point P about z- axis

We thus have

$$\vec{V} = \vec{\Omega}_P \times \overrightarrow{LP} \tag{2.2}$$

where

$$\vec{\Omega}_P = (\Omega_1 - \Omega)\,\hat{k} \tag{2.3}$$

and

$$\overrightarrow{LP} = x\,\hat{i} + y\,\hat{j} \tag{2.4}$$

Using (2.3) and (2.4) in (2.2), we get

$$\vec{V} = -(\Omega_1 - \Omega)(y\,\hat{i} - x\,\hat{j})\,. \tag{2.5}$$

Also

$$\vec{\Omega} \times \vec{r}_1 = -\Omega(y\hat{i} - (x - d_1)\hat{j}) \tag{2.6}$$

so the term $\vec{V} \cdot (\vec{\Omega} \times \vec{r}_1)$ in (2.1) becomes

$$\vec{V} \cdot (\vec{\Omega} \times \vec{r}_1) = (\Omega\,\Omega_1 - \Omega^2)\,(x^2 + y^2 - x d_1) \tag{2.7}$$

which is the contribution to potential at point P(x, y, z) inside the star due to Coriolis force (For a point P outside the primary star this will be zero unless P has some external velocity). Using equation (2.7) in equation (2.1), we get the modified expression for potential at a point P inside the star in cartesian form as

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_2} + \frac{\Omega^2}{2}[(x - d_1)^2 + y^2] + (\Omega\Omega_1 - \Omega^2)(x^2 + y^2 - x d_1) \tag{2.8}$$

Multiplying both sides of (2.8) by D/GM_0 and following Kopal (63), equation (2.8) may be expressed in nondimensional form as

$$\begin{aligned}\psi^* = {} & \frac{1}{r^*} + \frac{q}{\sqrt{1 - 2\lambda r^* + r^{*2}}} + \frac{\Omega^{*2}}{2}[r^{*2}\,(1 - \nu^2) \; + \; d_1^{*2} - 2\lambda r^* d_1^*] \\ & + \; (\Omega_1^*\,\Omega^* - \Omega^{*2})\,[r^{*2}\,(1 - \nu^2) - \lambda r^* d_1^*]\end{aligned} \tag{2.9}$$

where

$$\psi^* = \frac{D\psi}{GM_0}\,,\ \Omega^{*2} = \frac{D^3\Omega^2}{GM_0}\,,\ \Omega_1^*\,\Omega^* = \frac{D^3(\Omega_1\Omega)}{GM_0} \text{ and } d_1^* = \frac{M_1}{M_0 + M_1} \tag{2.10}$$

In the above expressions $r^* = r/D$ is nondimensional form of r, and $\lambda = \sin\theta\cos\varphi$, $\mu = \sin\theta\sin\varphi$, $\nu = \cos\theta$ (r, θ, φ being the polar spherical coordinates of the point *P*). Moreover, $q = M_1/M_0$ is a nondimensional parameter representing the ratio of the mass of the secondary component over the primary component (we assume $q << 1$).

Equation (2.9) is the most general expression of the potential at point P which incorporates the effect of Coriolis force in addition to centrifugal and gravitational forces. If we assume that the angular velocity Ω is identical with Keplerian angular velocity Ω_K where $\Omega_k^2 = G(M_0 + M_1)/D^3$ then in terms of the nondimensional variables used by us, we get a relation $\Omega^{*2} = 2n = (q+1)$ .Using this relation, equation (2.9) can be written as

$$\begin{aligned} \psi^* = \frac{1}{r^*} + \frac{q}{\sqrt{1-2\lambda r^* + r^{*2}}} + nr^{*2}(1-\nu^2) + \frac{q^2}{2(1+q)} - \lambda r^* q \\ + 2(\sqrt{n_1 n} - n)\,[r^{*2}(1-\nu^2) - \frac{\lambda r^* q}{1+q}] \end{aligned} \qquad (2.10a)$$

On rearranging the terms in the above equation we get

$$\begin{aligned} \psi^{**} = \frac{1}{r^*} + q[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^* \{1 + \frac{1}{n}(\sqrt{n_1 n} - n)\}] \\ + nr^{*2}(1-\nu^2)[1 + \frac{2}{n}(\sqrt{n_1 n} - n)] \end{aligned} \qquad (2.10b)$$

which can be written in more simplified form as

$$\psi^{**} = \frac{1}{r^*} + q[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \alpha\lambda r^*] + \beta n r^{*2}(1-\nu^2) \qquad (2.11)$$

where

$$\alpha = \sqrt{n_1/n}\,,\ \beta = 2\sqrt{n_1/n} - 1\,,\ \Omega_1^{*2} = 2n_1 \text{ and } \psi^{**} = \frac{D\psi}{GM_0} - \frac{q^2}{2(1+q)} \qquad (2.12)$$

For binary system rotating synchronously, the angular velocity due to rotation and revolution are same, that is, $\vec{\Omega}_1 = \vec{\Omega}$. Hence there will be no explicit term for Coriolis force. In such cases equation (2.9) reduces to

$$\psi^{**} = \frac{1}{r^*} + q[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^*] + n r^{*2}(1-\nu^2) \qquad (2.13)$$

This expression (2.13) is same as earlier obtained by Kopal (63).

Also there is no Coriolis force in pure tidal case and therefore expression for ψ^{**} in this case is identical to its earlier expression obtained by Kopal (63) and can be obtained directly from (2.13) by putting $n=0$.

In case of pure rotation also, Coriolis force is not generated as there is no revolution of the center of the star and hence no rotating frame of reference. In such a case the expression for Roche equipotential for a purely rotating star which is not subject to tidal effects of the companion star becomes

$$\psi^{*} = \frac{1}{r^{*}} + n r^{*2} (1 - \nu^{2}) \tag{2.14}$$

which can be obtained from (2.13) by setting $q = 0$ or from (2.11) by setting $q = 0, \beta = 1$. This expression is same as earlier discussed by Kopal (63) for pure rotating stars.

Thus on explicit inclusion of Coriolis force, expression for Roche equipotential gets modified from the earlier one obtained by Kopal (63) only in the case of nonsynchronous binaries. In case of synchronous binaries, purely rotating and purely tidally distorted stars there is no change in it.

2.2 EFFECT OF INCLUSION OF CORIOLIS FORCE ON THE SHAPES OF BINARY STARS

Surfaces generated by setting ψ^{**}=constant on the left hand side of equation (2.11) (or (2.13)) are referred to as Roche equipotentials. Roche equipotential surfaces corresponding to the outermost surface of the star depicts its shape. In case of a binary system, for large values of $\psi^{**} = \xi\,(say)$, equation (2.11) (or equation (2.13)) represents Roche equipotential surfaces which consist of two separate ovals (which differ little from spheres) around each of the two mass centers of the binary system. But as the value of ξ decreases, shapes of the Roche equipotential surfaces get elongated towards the center of gravity of the system. This process continues until for some critical value of ξ (say ξ_1) these oval shaped surfaces get united at single point on the axis joining the centers of two masses to form a dumb bell like configuration. Such a critical value of ξ $(= \xi_1)$ which is characteristic of each mass ratio, is called the Roche limit. For $\xi < \xi_1$, the two initial oval shaped surfaces coalesce into a single dumb bell surface surrounding both the stars.

Using equation (2.13), Kopal (66) computed the shapes of Roche equipotentials and Roche limits of binary systems of stars for different mass ratios. In the present section we have computed the shapes of Roche equipotential surfaces of binary systems for different values of mass ratios and angular velocity of rotation, using expression (2.11) for Roche equipotentials which incorporates the effect of Coriolis force in addition to the gravitational and the centrifugal forces.

In order to determine the position of Roche limit in a binary system, the approach earlier adopted by Kopal (66) has been followed. In this method, the first task is to determine the values of $\psi^{**} = \xi$ for which the two loops of the equipotential (Fig 2.3) develop a common point of contact at point P_1. However, its determination presupposes a knowledge of the position of P_1 on the x- axis. The location of this point is characterized by the vanishing of the total potential force at that point. In other words, at this point

$$\xi_x = \xi_y = 0 \tag{2.15}$$

The vanishing of ξ_x gives x- coordinate of P_1 (say x_1) which is a root of the equation

$$(q+1)\beta x^5 - \{q(\alpha+2\beta)+2\beta\}x^4 + \{q(2\alpha+\beta)+\beta\}x^3 + (q(1-\alpha)-1)x^2 + 2x - 1 = 0 \tag{2.16}$$

This equation has five roots. The root of this equation which lies between 0 and 1 is the desired position of Roche limit (Fig 2.3). Once a sufficiently accurate value of x_1 has been obtained, the actual value of ξ corresponding to this critical equipotential is given by

$$\xi_1 = \xi(x_1, 0, 0) \tag{2.17}$$

which on simplification gives

$$\xi_1 = \frac{1}{x_1} + q[\frac{1}{1-x_1} - \alpha x_1] + \beta n x_1^2 \tag{2.18}$$

The points $P_{4,5}$ in the xy- plane (Fig 2.3) are characterized by the vanishing of the derivative dy/dx for this value of x_1 (which is the position of the Roche limit). Their coordinates $x_{4,5}$ and $y_{4,5}$ can, therefore, be obtained by solving the simultaneous system of algebraic equations

$$\xi(x, y, 0) = \xi_1 \tag{2.19}$$

$$\xi_x(x, y, 0) = 0 \tag{2.20}$$

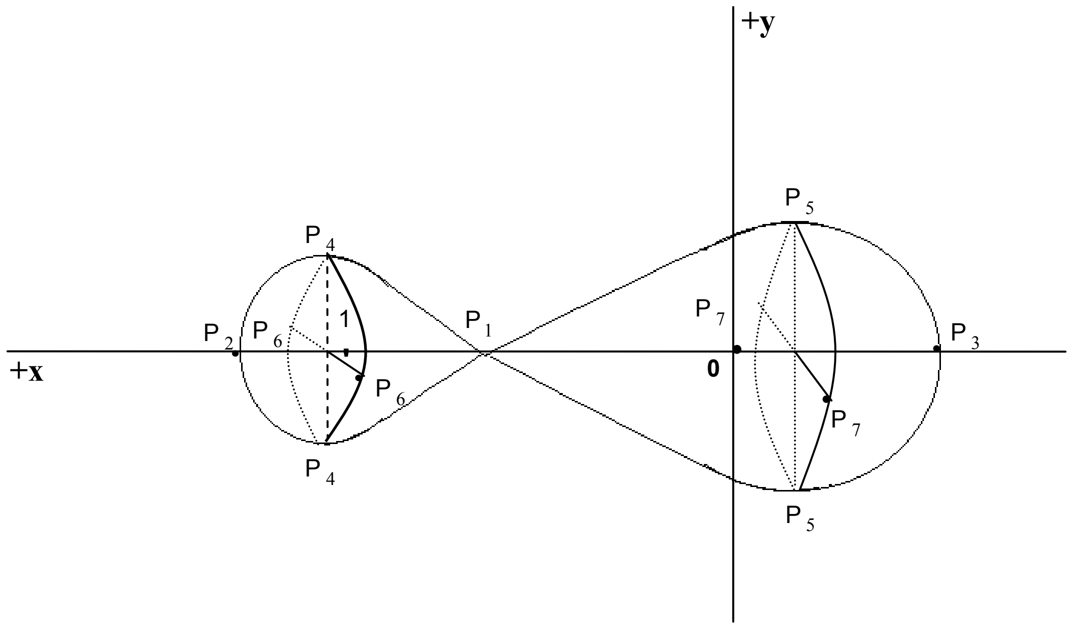


Fig. 2.3: Schematic view of a contact binary at the Roche limit (In order to exhibit the essential features of the geometry of this model, the diagram has not been drawn to scale for any particular mass ratio; and certain features of it (such as the distance of the P_5P_7 - plane from the origin) have been exaggerated)

On simplification, $\xi(x, y, 0) = \xi_1$ becomes

$$\xi_1 \;=\; \frac{1}{\sqrt{x^2+y^2}} + q[\frac{1}{\sqrt{(1-x)^2+y^2}} - \alpha x] + \beta n(x^2+y^2) \tag{2.21}$$

while $\xi_x(x, y, 0) = 0$ gives

$$-x + (x^2+y^2)^{3/2}\,[q\left(\{(1-x)/[(1-x)^2+y^2]^{3/2}\} - \alpha\right) + 2\beta n x] \;=\; 0 \tag{2.22}$$

Once the values of $x_{4,5}$ have been found, the z -coordinates of points $P_{6,7}$ in the xz - plane (Fig 2.3) can be obtained as the roots of the equation

$$\xi(x_{4,5}, 0, z) = \xi_1 \tag{2.23}$$

which in the present case gives

$$\xi_1 \;=\; \frac{1}{\sqrt{x^2+z^2}} + q[\frac{1}{\sqrt{(1-x)^2+z^2}} - \alpha x] + \beta n\, x^2 \tag{2.24}$$

For synchronous binaries $\alpha = 1$ and $\beta = 1$. For these values of α and β, the above equations reduce to the form as given by Kopal (66) for synchronous binaries since in the case of synchronous binaries angular velocity of rotation is same as the angular velocity of revolution and as a result last term on the right hand side of equation (2.9) (or (2.8) which corresponds to Coriolis force becomes zero.

2.3 NUMERICAL RESULTS

Using expressions (2.15 – 2.24) obtained in above section, we have computed the values of $\xi_1, x_1, x_{4,5}, y_{4,5}$ and $z_{6,7}$ for Roche limits corresponding to different values of mass ratios and angular velocities of rotation. The value of these parameters, when the effect of Coriolis force is not considered, has also been computed. Results are presented in Table 2.1. We have also tried to analyze the effect of variation in n_1 (angular velocity of rotation of primary) keeping n (angular velocity of revolution) and q (mass ratio) fixed in certain cases. The results of this study are presented in Table 2.2. Roche equipotential surfaces of nonsynchronous binary stars incorporating the effect of Coriolis force have also been drawn (Fig 2.4).

2.4 ANALYSIS OF THE RESULTS AND CONCLUSIONS

Our results on the shapes of Roche equipotentials for nonsynchronous binary stars as depicted in Fig 2.4 show that inclusion of Coriolis force does not produce any appreciable difference in the shapes of nonsynchronous binary stars. The figures in the last four cases which correspond to synchronous binaries and purely rotating stars show that as expected there is no effect of Coriolis force on their shapes.

Our results in the case of Roche limits of binary stars given in Table 2.1, however, show that the value of Roche limit (ξ_1) is greater than its corresponding value in the cases where the effect of Coriolis force is not considered. As a result the shapes of the limiting Roche equipotentials when the effects of Coriolis force is considered are more extended than the corresponding Roche equipotentials when the effects of Coriolis force are not considered.

As a result of inclusion of the Coriolis force, the values of x_1 (the position of the Roche limit) decrease. Comparison of the corresponding values of $x_{4,5}, y_{4,5}$ and $z_{6,7}$ in

Table 2.1 shows that whereas the values of y_4, x_5, y_5 and z_6 increases in magnitude with the inclusion of Coriolis force, the values of z_7 decreases in magnitude. The values of x_4 are not effected to any appreciable extend.

It is also observed from the Table 2.1 that with the inclusion of Coriolis force the percentage difference in the values of the Roche limit (ξ_1) and the position of the Roche limit (x_1) is less than 4% for the Roche limit and up to 1% for the position of Roche limit.

Table 2.2 shows the effect of varying the value of parameter n_1 of rotation for fixed values of parameter n of revolution and parameter q of tidal effects. The results show that when the value of parameters n_1 and n are same then the two corresponding values (when the effect of Coriolis force are considered and when they are not) are equal. However, as the difference between the values of two parameters increases, the percentage difference between the corresponding values of Roche limit increases (less than 5%), position of Roche limit increases (less than 1.5%) and in the values of other parameters also increases.

Our present study has shown the effects of Coriolis force on the shapes of Roche equipotential surfaces and position of Roche limit. It shows that the inclusion of Coriolis force does not produce any effect in case of purely rotating stars and synchronous binaries. In the case of nonsynchronous binaries there is not any appreciable change. However, appreciable effect is noticed when the difference between the parameters of rotation and revolution is large.

Table 2.1 Values of ξ_1 , x_1 , $x_{4,5}$, $y_{4,5}$ and $z_{6,7}$ in case of certain nonsynchronous binary systems

Model No.	n	q	n_1	x_1	ξ_1	x_4	$\pm y_4$	$-x_5$	$\pm y_5$	$\pm z_6$	$\pm z_7$
1	0.10	0.10	0.15	0.7546 (0.7570)	1.7228 (1.7141)	0.9732 (0.9726)	0.1435 (0.1408)	0.0174 (0.0105)	0.6330 (0.6291)	0.1428 (0.1404)	0.6109 (0.6140)
2	0.10	0.05	0.15	0.8081 (0.8115)	1.5432 (1.5228)	0.9773 (0.9769)	0.1098 (0.1068)	0.0129 (0.0083)	0.6987 (0.6978)	0.1093 (0.1066)	0.6660 (0.6751)
3	0.10	0.01	0.15	0.8994 (0.9027)	1.3175 (1.2830)	0.9862 (0.9863)	0.0552 (0.0529)	0.0052 (0.0035)	0.8258 (0.8288)	0.0551 (0.0528)	0.7636 (0.7842)
4	0.10	0.002	0.15	0.9510 (0.9532)	1.2211 (1.1808)	0.9927 (0.9929)	0.0262 (0.0249)	0.0016 (0.0011)	0.9093 (0.9124)	0.0261 (0.0249)	0.8200 (0.8480)
5	0.05	0.05	0.10	0.8133 (0.8157)	1.5003 (1.4897)	0.9768 (0.9766)	0.1054 (0.1033)	0.0167 (0.0082)	0.7075 (0.7022)	0.1052 (0.1032)	0.6855 (0.6904)
6	0.05	0.02	0.10	0.8703 (0.8732)	1.3479 (1.3236)	0.9826 (0.9826)	0.0715 (0.0694)	0.0098 (0.0052)	0.7838 (0.7830)	0.0713 (0.0693)	0.7509 (0.7647)
7	0.05	0.01	0.15	0.9035 (0.9062)	1.2723 (1.2421)	0.9864 (0.9866)	0.0524 (0.0506)	0.0062 (0.0034)	0.8325 (0.8335)	0.0523 (0.0506)	0.7909 (0.8101)
8	0.05	0.002	0.10	0.9536 (0.9553)	1.1722 (1.1353)	0.9930 (0.9932)	0.0246 (0.0236)	0.0018 (0.0010)	0.9138 (0.9159)	0.0246 (0.0236)	0.8542 (0.8820)
9	0.05	0.001	0.10	0.9666 (0.9679)	1.1485 (1.1102)	0.9949 (0.9950)	0.0176 (0.0169)	0.0010 (0.0006)	0.9367 (0.9386)	0.0176 (0.0169)	0.8712 (0.9014)
10	0.02	0.02	0.05	0.8742 (0.8755)	1.3083 (1.3007)	0.9827 (0.9827)	0.0687 (0.0678)	0.0114 (0.0051)	0.7902 (0.7858)	0.0687 (0.0678)	0.7738 (0.7783)
11	0.02	0.01	0.05	0.9069 (0.9082)	1.2313 (1.2174)	0.9866 (0.9867)	0.0502 (0.0494)	0.0070 (0.0033)	0.8381 (0.8363)	0.0502 (0.0494)	0.8173 (0.8266)
12	0.01	0.01	0.10	0.9068 (0.9088)	1.2252 (1.2092)	0.9866 (0.9868)	0.0503 (0.0490)	0.0178 (0.0032)	0.8482 (0.8372)	0.0503 (0.0490)	0.8215 (0.8323)

Note: Values in the parenthesis are obtained when the effect of Coriolis is not considered.

Table 2.2 Values of ξ_1, x_1, $x_{4,5}$, $y_{4,5}$ and $z_{6,7}$ in case of certain nonsynchronous binary systems

Model No.	n_1	x_1	ξ_1	x_4	$\pm y_4$	$-x_5$	$\pm y_5$	$\pm z_6$	z_7
	$n=0.05,\ q=0.10$								
1	0.01	0.7625 (0.7610)	1.6954 (1.6853)	0.9715 (0.9719)	0.1351 (0.1369)	0.0040 (0.0105)	0.6202 (0.6327)	0.1351 (0.1367)	0.6210 (0.6249)
2	0.03	0.7616 (0.7610)	1.6894 (1.6853)	0.9717 (0.9719)	0.1362 (0.1369)	0.0043 (0.0105)	0.6274 (0.6327)	0.1361 (0.1367)	0.6232 (0.6249)
3	0.05	0.7610 (0.7610)	1.6853 (1.6853)	0.9719 (0.9719)	0.1369 (0.1369)	0.0105 (0.0105)	0.6327 (0.6327)	0.1367 (0.1367)	0.6249 (0.6249)
4	0.07	0.7605 (0.7610)	1.6820 (1.6853)	0.9720 (0.9719)	0.1375 (0.1369)	0.0159 (0.0105)	0.6372 (0.6327)	0.1373 (0.1367)	0.6262 (0.6249)
5	0.10	0.7599 (0.7610)	1.6778 (1.6853)	0.9722 (0.9719)	0.1383 (0.1369)	0.0234 (0.0105)	0.6431 (0.6327)	0.1380 (0.1367)	0.6280 (0.6249)
	$n=0.10,\ q=0.10$								
6	0.01	0.7643 (0.7570)	1.6870 (1.7141)	0.9712 (0.9725)	0.1334 (0.1408)	0.0071 (0.0105)	0.6188 (0.6290)	0.1335 (0.1404)	0.6240 (0.6140)
7	0.05	0.7602 (0.7570)	1.7026 (1.7141)	0.9718 (0.9725)	0.1376 (0.1408)	0.0024 (0.0105)	0.6244 (0.6290)	0.1373 (0.1404)	0.6182 (0.6140)
8	0.10	0.7570 (0.7570)	1.7141 (1.7141)	0.9725 (0.9725)	0.1408 (0.1408)	0.0105 (0.0105)	06290 (0.6290)	0.1404 (0.1404)	0.6140 (0.6140)
9	0.15	0.7546 (0.7570)	1.7228 (1.7141)	0.9732 (0.9725)	0.1435 (0.1408)	0.0174 (0.0105)	0.6330 (0.6290)	0.1428 (0.1404)	0.6109 (0.6140)
10	0.20	0.7525 (0.7570)	1.7301 (1.7141)	0.9738 (0.9725)	0.1458 (0.1408)	0.0237 (0.0105)	0.6367 (0.6290)	0.1450 (0.1404)	0.6083 (0.6140)
	$n=0.10,\ q=0.05$								
11	0.01	0.8210 (0.8115)	1.4596 (1.5228)	0.9764 (0.9769)	0.0990 (0.1068)	0.0039 (0.0083)	0.6962 (0.6978)	0.0990 (0.1065)	0.7049 (0.6751)
12	0.05	0.8156 (0.8115)	1.4960 (1.5228)	0.9766 (0.9769)	0.1033 (0.1068)	0.0027 (0.0083)	0.6970 (0.6978)	0.1032 (0.1065)	0.6874 (0.6751)
13	0.10	0.8115 (0.8115)	1.5228 (1.5228)	0.9769 (0.9769)	0.1068 (0.1068)	0.0083 (0.0083)	0.6978 (0.6978)	0.1065 (0.1065)	0.6751 (0.6751)
14	0.15	0.8081 (0.8115)	1.5432 (1.5228)	0.9773 (0.9769)	0.1098 (0.1068)	0.0129 (0.0083)	0.6987 (0.6978)	0.1093 (0.1065)	0.6660 (0.6751)
15	0.20	0.8053 (0.8115)	1.5602 (1.5228)	0.9777 (0.9769)	0.1125 (0.1068)	0.0171 (0.0083)	0.6996 (0.6978)	0.1119 (0.1065)	0.6587 (0.6751)

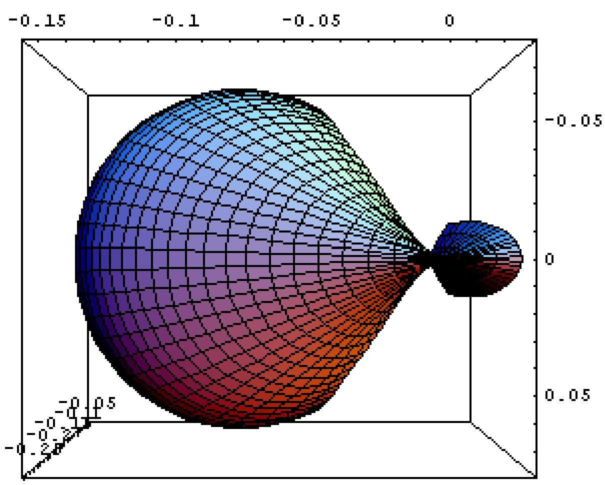
Note: Values in the parenthesis are obtained when the effect of Coriolis is not considered.

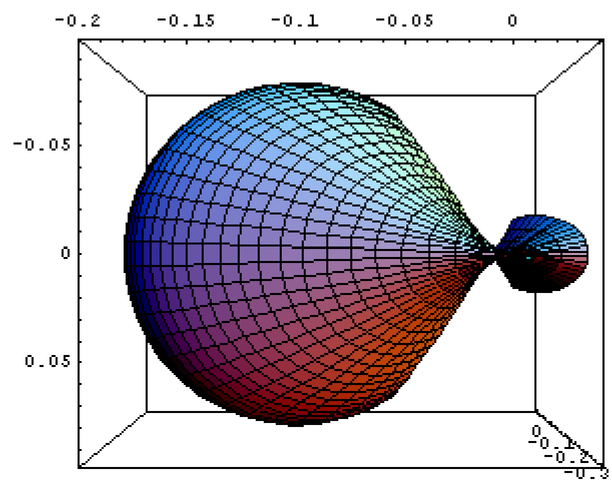
Fig 2.4 Effect of Coriolis force on the shapes of binary and rotating stars

Shapes of various equipotential surfaces when Coriolis force is not included

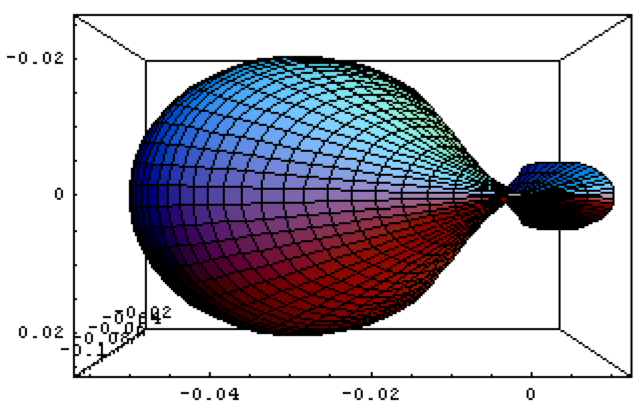
Shapes of various equipotential surfaces when Coriolis force is included

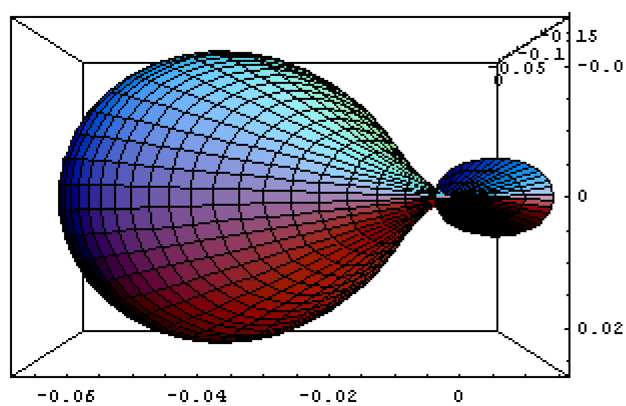
(1) $n = 0.1$, $q = 0.01$, $n_1 = 0.15$ (Nonsynchronous binary)



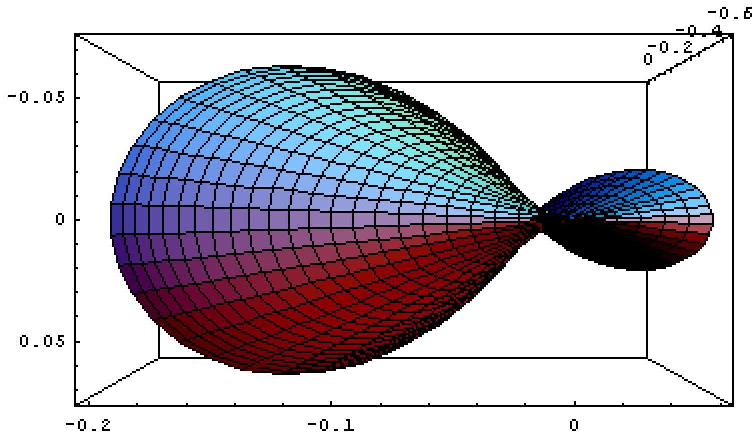


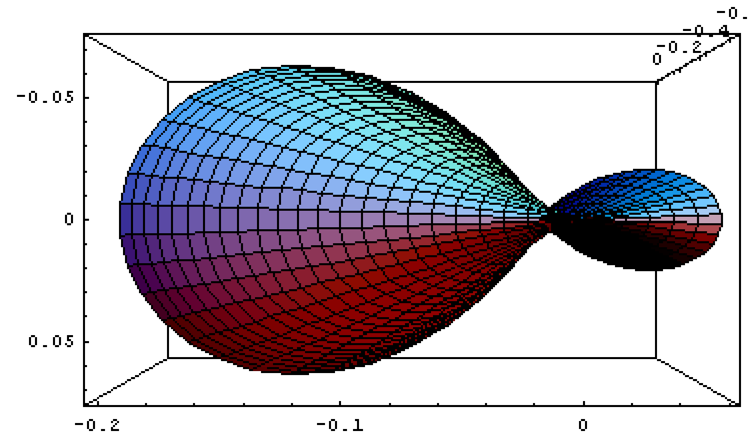
(2) $n = 0.05$, $q = 0.001$, $n_1 = 0.1$ (Nonsynchronous binary)



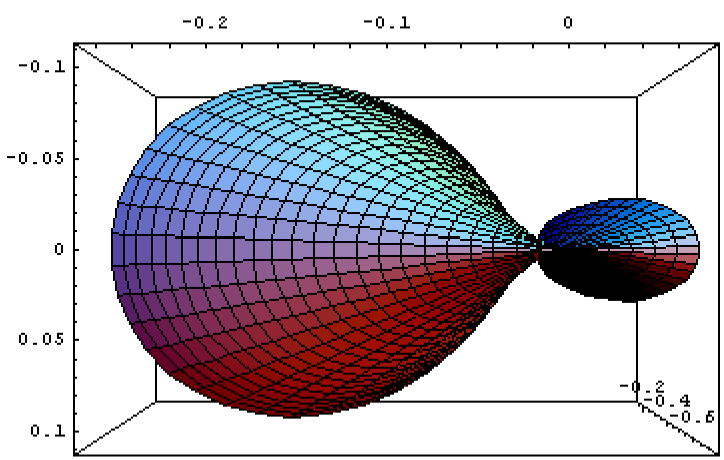


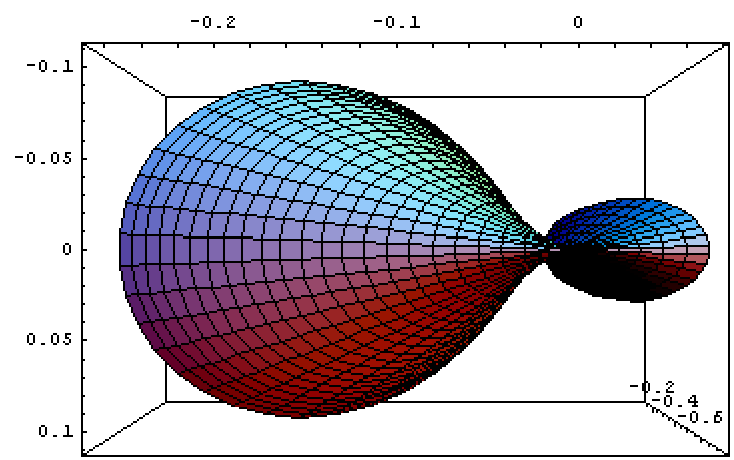
(3) $q = 0.01$, $n = n_1 = 0.505$ (Synchronous binary)



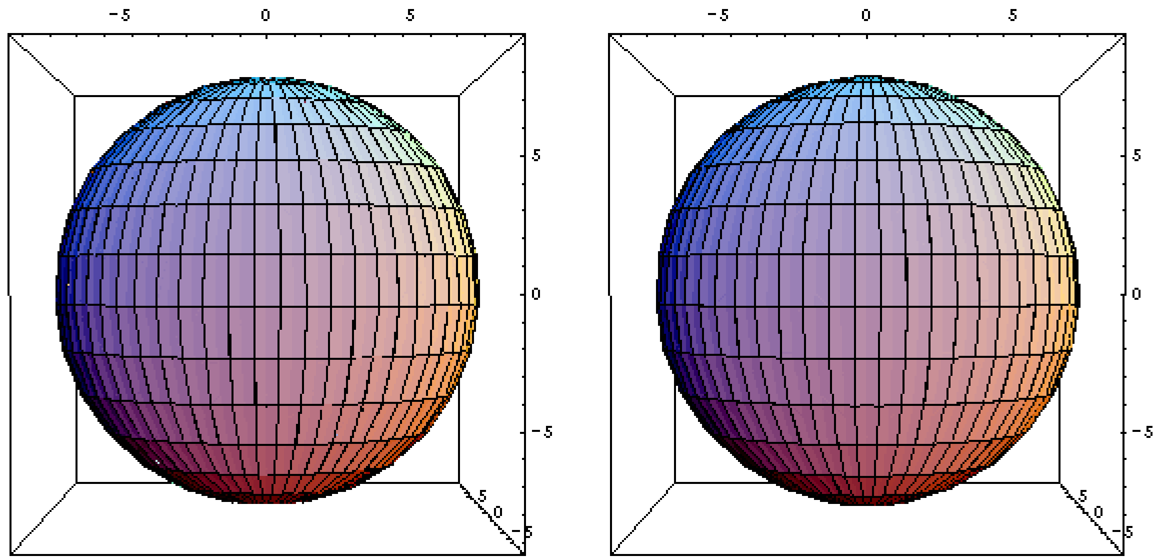


(4) $q = 0.05$, $n = n_1 = 0.525$ (Synchronous binary)

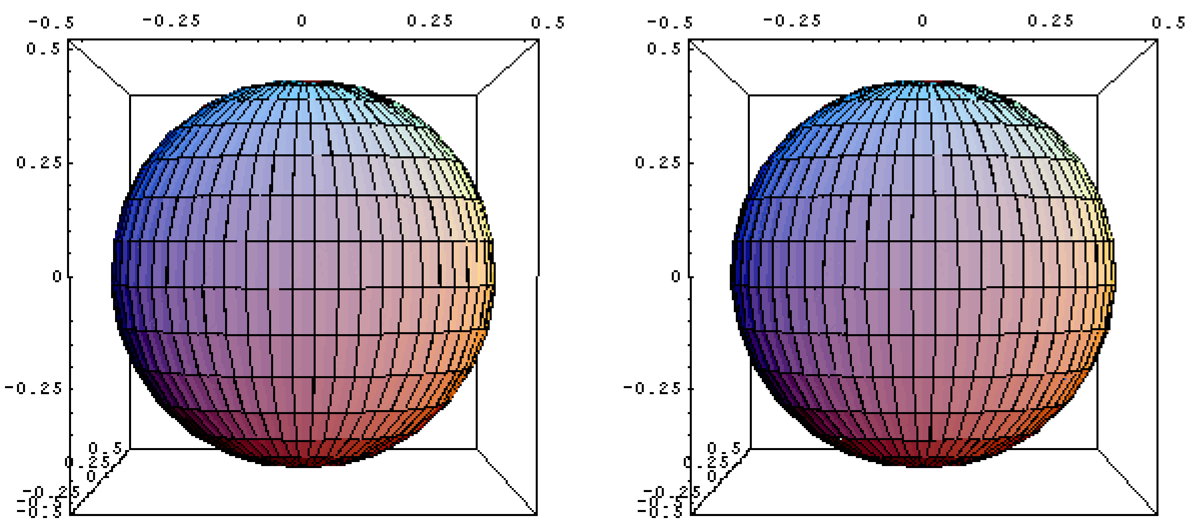




(5) $n = 0.1,\ q = 0.0,\ \psi^{\bullet} = 10$ (Pure Rotation)



(6) $n = 0.01,\ q = 0.0,\ \psi^{\bullet} = 1.5$ (Pure Rotation)



CHAPTER – III

EFFECT OF CORIOLIS FORCE ON THE EQUILIBRIUM STRUCTURES OF POLYTROPIC MODELS OF ROTATING STARS AND STARS IN BINARY SYSTEMS

Mohan and Saxena (99, 100), Mohan et al (108, 109, 104, 106) and Lal et al (71, 72) used Kopal's concept of Roche equipotentials in conjunction with the averaging technique of Kippenhahn and Thomas (58) to incorporate the rotational and tidal effects in the equations of stellar structure and stellar oscillations of rotating stars and stars in binary systems. The method has been extensively used by them to compute the equilibrium structures and periods of small oscillations of a variety of rotationally and tidally distorted stellar models. However their studies are based on expression for Roche equipotential obtained in a nonrotating frame of reference which does not explicitly account for the effect of the Coriolis force.

In the present chapter analytic expression for Roche equipotential developed in chapter II which accounts for Coriolis force besides the gravitational and centrifugal forces, has been incorporated in conjunction with Mohan, Saxena and Agarwal (108) approach to obtain the system of differential equations governing the equilibrium structures of rotationally and tidally distorted stellar models in general and polytropic models of stars in particular. The expressions for modified Roche equipotential surfaces of rotationally and tidally distorted stellar models which account for the effects of Coriolis force besides the effects of gravitational and centrifugal forces are presented in section I. These are then used in conjunction with Mohan, Saxena and Agarwal (108) approach to obtain the equations governing the equilibrium structures of rotationally and tidally distorted stellar models in section 3.2. The concept of polytropic models of stars is next presented in section 3.3. The methodology developed in section 3.2 is then used in section 3.4 to compute the inner structures of rotationally and tidally distorted polytropic models of rotating stars and stars in binary systems. In section 3.5 mathematical expressions for the volumes, surface areas, shapes and some other physical parameters of rotationally and tidally distorted polytropic models of stars are obtained. Numerical computations have next been performed in section 3.6 to obtain the inner structures, shapes and values of other physical parameters of certain rotationally and tidally distorted polytropic models of stars of polytropic indices 1.5 and 3.0. The numerical results thus obtained have next been compared with the results obtained earlier by Mohan and Saxena (99) who do not account for the effect of Coriolis force. For comparison, shapes of the outer surfaces of various rotationally and tidally distorted polytropic models of stars have also been drawn in both the cases when effects of Coriolis force are explicitly included (not included). Conclusions based on the present analysis are summarized in section 3.7.

3.1 EXPRESSION FOR ROCHE EQUIPOTENTIAL OF A ROTATIONALLY AND TIDALLY DISTORTED STAR

We derived in section 2.1 (eqn. (2.11)) of Chapter II, an expression for the Roche equipotential which explicitly incorporates the effects of Coriolis force besides the centrifugal and gravitational forces. This expression in nondimensional form is

$$\psi^{**} = \frac{1}{r^*} + q\left[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \alpha\lambda r^*\right] + \beta n r^{*2}(1-\nu^2) \tag{3.1}$$

where

$$\alpha = \sqrt{n_1/n}\,,\ \beta = 2\alpha - 1,\ \Omega^{*2} = 2n\,,\ \Omega_1^{*2} = 2n_1 \text{ and } \psi^{**} = \frac{D\psi}{GM_0} - \frac{q^2}{2(1+q)}$$

As pointed out in section 2.1 for binary systems rotating synchronously, since the angular velocity due to rotation (Ω_1) and revolution (Ω) is same, hence $\alpha = \beta = 1$ and in such cases (3.1) reduces to

$$\psi^{**} = \frac{1}{r^*} + q\left[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^*\right] + n r^{*2}(1-\nu^2) \tag{3.2}$$

Expression (3.2) is same as earlier obtained by Kopal (63).

In pure tidal case there is no rotation or revolution. Hence there is no Coriolis force. Therefore, expression for ψ^{**} in this case can be derived directly from equation (3.2) by putting $n=0$ which is identical to its earlier expression obtained by Kopal (63). Similarly, in the case of a purely rotating star, there will be no Coriolis force as there is no revolution of the center of the star and hence no rotating frame of reference. In such a case the expression for Roche equipotential for a purely rotating star which is not subject to tidal effects of the companion star becomes

$$\psi^{*} = \frac{1}{r^*} + n r^{*2}(1-\nu^2) \tag{3.3}$$

which can be obtained from (3.2) by setting $q=0$ or from (3.1) by setting $q=0, \beta=1$. This expression is same as earlier discussed by Kopal (63) for pure rotating stars.

Adopting an approach similar to the one adopted by Kopal (63) and Mohan et al (108), we may define a nondimensional variable r_0 by the relation

$$r_0 = \frac{1}{\psi^{**} - q} \tag{3.4}$$

Following Kopal (63), variables (r, θ, ϕ) on the surfaces of the modified Roche equipotentials (3.1) can be shown to be connected through the relation

$$\begin{aligned} r = r_0 [1 &+ \lambda q t r_0^2 + a_0 r_0^3 + (q P_3 + 2\lambda^2 q^2 t^2) r_0^4 + (q P_4 + 5 a_0 \lambda q t) r_0^5 + (q P_5 + 3 a_0^2 + 6\lambda q^2 t P_3) r_0^6 \\ &+ (q P_6 + 7 a_0 q P_3 + 7\lambda q^2 t P_4) r_0^7 + (q P_7 + 8 a_0 q P_4 + 8\lambda q^2 t P_5 + 4 q^2 P_3^2) r_0^8 \\ &+ (q P_8 + 9 a_0 q P_5 + 9\lambda q^2 t P_6 + 9 q^2 P_3 P_4) r_0^9 + (q P_9 + 10 a_0 q P_6 + 10\lambda q^2 t P_7 \\ &+ 5 q^2 \{P_4^2 + 2 P_3 P_5\}) r_0^{10} + \cdots] \end{aligned} \tag{3.5}$$

where $a_0 = q P_2 + \beta n (1 - \nu^2)$, $t = 1 - \alpha$ and $P_j = P_j(\lambda)$ denote Legendre polynomials. As earlier terms up to second order of smallness in n, n_1 and q and terms up to r_0^{10} in r_0 are retained in (3.5). Relation (3.5) incorporates the effect of Coriolis force and can be used to obtain the shapes of various Roche equipotential surfaces ψ^{**} = constant.

Again following Kopal (63) and Mohan et al (108), expressions for volume V_ψ, surface area S_ψ and value of radial distance r_ψ of a point on the equipotential surface ψ^{**} = constant inside the primary can be shown to be

$$\begin{aligned} V_\psi = \frac{4\pi}{3} D^3 r_0^3 [1 &+ 2(\beta n) r_0^3 + 3 q^2 t^2 r_0^4 + (\frac{12}{5} q^2 + \frac{32}{5} (\beta n)^2 + \frac{8}{5} (\beta n) q) r_0^6 \\ &+ \frac{15}{7} q^2 r_0^8 + 2 q^2 r_0^{10} + \ldots] \end{aligned} \tag{3.6}$$

$$\begin{aligned} S_\psi = 4\pi D^2 r_0^2 [1 &+ \frac{4}{3} (\beta n) r_0^3 + \frac{5}{3} q^2 t^2 r_0^4 + (\frac{7}{5} q^2 + \frac{56}{15} (\beta n)^2 + \frac{14}{15} (\beta n) q) r_0^6 \\ &+ \frac{9}{7} q^2 r_0^8 + \frac{11}{9} q^2 r_0^{10} + \ldots] \end{aligned} \tag{3.7}$$

and

$$r_\psi = D r_0 [1 + \frac{2}{3} (\beta n) r_0^3 + q^2 t^2 r_0^4 + (\frac{4}{5} q^2 + \frac{76}{45} (\beta n)^2 + \frac{8}{15} (\beta n) q) r_0^6 + \frac{5}{7} q^2 r_0^8 + \frac{2}{3} q^2 r_0^{10} + \ldots] \tag{3.8}$$

Inverting (3.8) we have

$$r_0 = r_\psi^* [1 - \frac{2}{3}(\beta n) r_\psi^{*3} - q^2 t^2 r_\psi^{*4} - (\frac{4}{5}q^2 - \frac{4}{45}(\beta n)^2 + \frac{8}{15}(\beta n) q) r_\psi^{*6} - \frac{5}{7} q^2 r_\psi^{*8} - \frac{2}{3} q^2 r_\psi^{*10} + ...] \tag{3.9}$$

where $r_\psi^* = r_\psi / D$, r_ψ^* being the nondimensional form of r_ψ. Similarly using equations (1.31) and (1.32) of chapter I, the explicit expressions for the values of $\bar{g}$ and $\overline{g^{-1}}$, in the presence of Coriolis force, at points inside the primary are given as

$$\bar{g} = \frac{GM_\psi}{D^2 r_0^2}[1 - \frac{8}{3}(\beta n) r_0^3 - 2q^2 t^2 r_0^4 - (2q^2 + \frac{28}{9}(\beta n)^2 + \frac{4}{3}(\beta n) q) r_0^6 - \frac{15}{7} q^2 r_0^8 - \frac{7}{3} q^2 r_0^{10} + ...] \tag{3.10}$$

$$\overline{g^{-1}} = \frac{D^2 r_0^2}{GM_\psi}[1 + \frac{8}{3}(\beta n) r_0^3 + 5q^2 t^2 r_0^4 + (\frac{26}{5}q^2 + \frac{524}{45}(\beta n)^2 + \frac{52}{15}(\beta n) q) r_0^6 + \frac{40}{7} q^2 r_0^8 + \frac{19}{3} q^2 r_0^{10} + ...] \tag{3.11}$$

where $t = 1 - \alpha$.

As in earlier studies in obtaining the above expressions, terms up to second order of smallness in n, n_1, q and terms up to r_0^{10} in r_0 have been retained. In absence of the effect of Coriolis force ($\alpha = \beta = 1$) these reduce to expressions earlier obtained by Mohan et al (108).

3.2 EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED STARS INCORPORATING THE EFFECT OF CORIOLIS FORCE

Following the approach presented in section 1.4 of chapter I and using the modified expressions for Roche equipotential and other parameters obtained in section 3.1 which incorporate the effects of Coriolis force besides gravitational and centrifugal forces, the values of distortions parameters u, v, w, f_p and f_t now become

$$u = [1 - \frac{1}{3} q^2 t^2 r_\psi^{*4} - (\frac{1}{5} q^2 + \frac{4}{45}(\beta n)^2 + \frac{2}{15}(\beta n) q) r_\psi^{*6} - \frac{1}{7} q^2 r_\psi^{*8} - \frac{1}{9} q^2 r_\psi^{*10} - ...] \tag{3.12}$$

$$v = [1 - \frac{4}{3}(\beta n) r_\psi^{*3} - (\frac{2}{5} q^2 + \frac{8}{45}(\beta n)^2 + \frac{4}{15}(\beta n) q) r_\psi^{*6} - \frac{5}{7} q^2 r_\psi^{*8} - q^2 r_\psi^{*10} - ...] \tag{3.13}$$

$$w = [1 + \frac{4}{3}(\beta n) r_\psi^{*3} + 3q^2 t^2 r_\psi^{*4} + (\frac{18}{5} q^2 + \frac{152}{45}(\beta n)^2 + \frac{12}{5}(\beta n) q\ ) r_\psi^{*6}$$
$$+ \frac{30}{7} q^2 r_\psi^{*8} + 5\, q^2 r_\psi^{*10} + ...] \tag{3.14}$$

$$f_P = [1 - \frac{4}{3}(\beta n) r_\psi^{*3} - \frac{8}{3} q^2 t^2 r_\psi^{*4} - (\frac{17}{5} q^2 + \frac{68}{45}(\beta n)^2 + \frac{34}{15}(\beta n) q) r_\psi^{*6} - \frac{29}{7} q^2 r_\psi^{*8} - \frac{44}{9} q^2 r_\psi^{*10} - ...] \tag{3.15}$$

$$f_T = [1 - \frac{7}{3} q^2 t^2 r_\psi^{*4} - (\frac{14}{5} q^2 + \frac{56}{45}(\beta n)^2 + \frac{28}{15}(\beta n) q)\ r_\psi^{*6} - \frac{23}{7} q^2 r_\psi^{*8} - \frac{34}{9} q^2 r_\psi^{*10} - ...] \tag{3.16}$$

where $r_\psi^* = r_\psi / D$ is the nondimensional form of r_ψ and terms up to second order of smallness in n, n_1, q and up to r_ψ^{10} in r_ψ are retained. These reduce to expressions earlier obtained in Mohan et al (108) by setting $\alpha = \beta = 1$. The values of P_ψ, ρ_ψ, L_ψ etc. on the various equipotential surfaces of a rotationally and tidally distorted gaseous sphere may now be obtained by solving the system of differential equations (1.27) subject to the boundary conditions (1.28) and using the values of the distortion factors f_P and f_T as given in equations (3.13- 3.16).

It may be noted that approximating the equipotential surfaces of a rotationally and tidally distorted model by Roche equipotentials, the structure of the star is not approximated by the structure of a Roche model. This is evident from the fact that in the case of no distortion ($n = n_1 = q = 0$), equations (3.13 -3.16) yields $u = v = w = f_P = f_T = 1$ and the system of differential equations (1.27) reduce to the equations governing the equilibrium structure of original undistorted star and not of the undistorted Roche model. This is evident as pointed out earlier in literature (Saxena (133))

Usual numerical methods, which are used for solving the stellar structure equations, can be applied here to integrate the system of given differential equations which govern the equilibrium structures of rotationally and tidally distorted gaseous spheres. However, at each step the values of the distortion parameters u, v, w, f_P and f_T has to be computed using equations (3.13 – 3.16).

In case the thermal properties are not considered important and only hydrostatic equilibrium of a rotationally and tidally distorted gaseous sphere is to be investigated then we need only to integrate equations (1.27a) and (1.27b) subject to the boundary conditions

At the center $r_\psi = 0$:

$$M_\psi = 0 \tag{3.17a}$$

and at the free surface $r_\psi = R_\psi$:

$$M_\psi = M_0,\ P_\psi = 0,\ \rho_\psi = 0 \ \ or \ \ P_\psi = P_{\psi s},\ \rho_\psi = \rho_{\psi s} \tag{3.17b}$$

The present analysis is valid for the rotationally and tidally distorted models in which the distorting forces causing rotational and tidal distortions are not too large. Keeping this in view we have retained terms up to second order of smallness in n, n_1 and q in expressions (3.13 – 3.16).

For computational work, sometimes it is found more convenient to work with r_0 in place of r_ψ or M_ψ as the independent variable. Expression for r_0 is given by (3.4) and is connected with variable r_ψ through the relation (3.9). By using these relations in equations (1.27), the system of differential equations governing the equilibrium structure of a rotationally and tidally distorted model which incorporates the effects of Coriolis force besides the gravitational and centrifugal forces can be expressed as

$$\frac{dM_\psi}{dr_0} = 4\pi D^3 \rho_\psi r_0^2 f_1 \tag{3.18a}$$

$$\frac{dP_\psi}{dr_0} = -\frac{GM_\psi}{Dr_0^2}\rho_\psi f_2 \tag{3.18b}$$

$$\frac{dL_\psi}{dr_0} = 4\pi\,\varepsilon\, D^3 \rho_\psi r_0^2 f_1 \tag{3.18c}$$

$$\frac{dT_\psi}{dr_0} = -\frac{3\kappa L_\psi\,\rho_\psi}{16\pi DacT_\psi^3 r_0^2} f_3 \tag{3.18d}$$

with

$$r_0 = \frac{1}{\psi^{**} - q}$$

Here f_1, f_2, f_3 are certain functions of distortion parameters n, n_1, q and r_0 and incorporate the effects of Coriolis force in addition to centrifugal and gravitational forces on the equilibrium structure equations of rotationally and tidally distorted stellar model (In case of no distortion $f_1 = f_2 = f_3 = 1$). Explicit expressions for distortion parameters $f_1 = \frac{dr_\psi}{dr_0}\frac{r_\psi^2}{D^3}$, $f_2 = \frac{f_p}{r_\psi^2}\frac{dr_\psi}{dr_0} D$, $f_3 = \frac{f_t}{r_\psi^2}\frac{dr_\psi}{dr_0} D$ and r_0 when terms up to second order of smallness in n, n_1, q and up to r_0^{10} in r_0 are retained, are given as

$$f_1 = [1 + 4(\beta n) r_0^3 + 7q^2t^2r_0^4 + (\frac{36}{5}q^2 + \frac{96}{5}(\beta n)^2 + \frac{24}{5}(\beta n) q) r_0^6 + \frac{55}{7}q^2 r_0^8 + \frac{26}{3}q^2 r_0^{10} + ...] \tag{3.19a}$$

$$f_2 = [1 + \frac{1}{3}q^2 t^2 r_0^4 + (\frac{3}{5}q^2 + \frac{4}{15}(\beta n)^2 + \frac{2}{5}(\beta n) q) r_0^6 + \frac{6}{7}q^2 r_0^8 + \frac{10}{9}q^2 r_0^{10} + ...] \tag{3.19b}$$

$$f_3 = [1 + \frac{4}{3}(\beta n) r_0^3 + \frac{2}{3}q^2t^2r_0^4 + (\frac{6}{5}q^2 + \frac{224}{45}(\beta n)^2 + \frac{4}{5}(\beta n) q) r_0^6 + \frac{12}{7}q^2 r_0^8 + \frac{20}{9}q^2 r_0^{10} + ...] \tag{3.19c}$$

and

$$r_0 = r_\psi^* D\,[1 - \frac{2}{3}(\beta n) r_\psi^{*3} - q^2t^2 r_\psi^{*4} - (\frac{4}{5}q^2 - \frac{4}{45}(\beta n)^2 + \frac{8}{15}(\beta n) q) r_\psi^{*6} - \frac{5}{7}q^2 r_\psi^{*8} - \frac{2}{3}q^2 r_\psi^{*10} - ...] \tag{3.19d}$$

where r_ψ^* is the nondimensional value of the radius of topologically equivalent spherical surface. Effects of Coriolis force appear in these expressions through α and β. The boundary conditions given in equation (1.28) now becomes

At the center $r_0 = 0$:

$$M_\psi = 0, \; L_\psi = 0 \tag{3.20a}$$

and at the free surface $r_0 = r_{0s}$:

$$\begin{aligned} &M_\psi = M_0, \; L_\psi = L_{\psi s} \\ &P_\psi = 0 \; or \; P_{\psi s}, \; T_\psi = 0 \; or \; T_{\psi s} \end{aligned} \tag{3.20b}$$

Here M_0 is the total mass of the model and $L_{\psi s}, P_{\psi s}, T_{\psi s}$ are the values of L_ψ, P_ψ, T_ψ respectively, on the outermost equipotential surface, ψ^{**} = constant.

At the free surface, $r_0 = r_{0s}$

where $r_{0s} = \dfrac{1}{\psi_s^{**} - q}$ (3.21)

ψ_s^{**} being the nondimensional value of the total potential ψ^{**} on the outermost equipotential surface of the rotationally and tidally distorted stellar model.

3.3 POLYTROPIC MODELS OF STARS

Polytropic models have frequently been used in literature to depict the inner structures of realistic stars. A polytropic model is a model in which the quantity of heat supplied (say dq) is directly proportional to the instantaneous change of temperature (say dt), so that dq/dt is constant. In a polytropic model the pressure P and the density ρ at an arbitrary point in the model are given by the relation

$$P = P_c\, \theta^{N+1} \text{ and } \rho = \rho_c\, \theta^N \tag{3.22}$$

where P_c and ρ_c are, respectively the pressure and density at the center of the model and θ ($0 \le \theta \le 1$) is a parameter depending upon the distance of the chosen point from the center. The index N used in relation (3.22) is called the polytropic index of the model. For cases of practical interest to the problems of stellar structure N lies between zero and five. Index N is a measure of the central concentration of the model (larger the value of N greater is central condensation). Whereas a polytropic model of index zero has a homogenous structure in which the density is uniform throughout the model, a polytropic model of index five is a very highly centrally condensed model whose radius extends to infinity.

The equilibrium structure of a polytropic model of index N is determined by the solution of the nonlinear differential equation

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\theta}{d\xi}\right) = -\,\theta^N \tag{3.23}$$

subject to the boundary conditions

$$\theta = 1,\ \frac{d\theta}{d\xi} = 0 \text{ at the center } \xi = 0 \tag{3.24a}$$

and

$$\theta = 0 \text{ at the surface } \xi = \xi_s \tag{3.24b}$$

Equation (3.23) is known as the Lane – Emden equation. Analytical solutions of this equation are only possible for N = 0, 1 and 5. Chandrasekhar (11) and several other investigators have obtained numerical solutions of the Lane – Emden equation (3.23) which satisfy conditions (3.24) for various values of the polytropic index N lying between zero and five.

Once the solution of Lane – Emden equation (3.23) has been obtained which satisfies condition (3.24), the values of the various physical parameters of the star having inner structure as a polytrope of index N can be determined.

3.4 EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

In this section we consider the feasibility of using the approach developed in section 3.2 of this chapter to determine the inner structures and equilibrium configurations of certain rotationally and tidally distorted polytropic models of stars which incorporate the effects of Coriolis force besides the centrifugal and gravitational forces.

Suppose a polytropic model is subject to rotation and tidal distortion then its structure becomes a rotationally and tidally distorted polytropic model. Following the approach of section 3.2 we may approximate the equipotential surfaces of such a distorted model by modified Roche equipotential surfaces which incorporate the effects of Coriolis force besides centrifugal and gravitational forces.

Let P_ψ denote the pressure and ρ_ψ the density on the equipotential surface ψ^{**} = constant of the distorted model. Then the value of the density and the pressure on the equivalent surface of the corresponding topologically equivalent spherical model will also be ρ_ψ and P_ψ, respectively. We assume that the distorted model behaves like a polytropic model so that in its case also ρ_ψ and P_ψ are connected through the polytropic relations of the type

$$P_\psi = P_{c\psi}\,\theta_\psi^{N+1} \text{ and } \rho_\psi = \rho_{c\psi}\,\theta_\psi^{N} \tag{3.25}$$

where N is the polytropic index and $P_{c\psi}$, $\rho_{c\psi}$ the values of P_ψ and ρ_ψ respectively at the center. Here θ_ψ represents some average of the value of the polytropic parameter θ at various points on the equipotential surface ψ^{**} = constant. In the case of polytropic models, the equations (3.18a) and (3.18b) which govern the hydrostatic equilibrium structure of rotationally and tidally distorted gaseous sphere can be combined together with equation (3.25) to yield

$$\frac{1}{r_0^2}\frac{d}{dr_0}\left(\frac{r_0^2}{f_2}\frac{d\theta_\psi}{dr_0}\right) = -\frac{R^2}{\ell^2} f_1\,\theta_\psi^N \tag{3.26}$$

where

$$\ell^2 = \frac{(N+1)P_{c\psi}}{4\pi G\rho_{c\psi}^2},$$

and f_1, f_2 are the distortion parameters as given in equation (3.19).

As regards the boundary conditions, P_ψ and ρ_ψ must be maximum at the center and zero at the free surface. These require θ_ψ to be maximum at the center and zero at the free surface. This leads to the conditions: $\theta_\psi = 1$ and $\frac{d\theta_\psi}{dr_o} = 0$ at the center and $\theta_\psi = 0$ at the free surface. Thus the boundary conditions which equation (3.26) has to satisfy are

$$\theta_\psi = 1,\ \frac{d\theta_\psi}{dr_0} = 0,\ \text{at the center } r_0 = 0 \tag{3.27a}$$

and

$$\theta_\psi = 0,\ \text{at the surface } r_0 = r_{0s}. \tag{3.27b}$$

where r_{0s} is the value of r_0 at surface.

In the absence of any distortions ($f_1 = f_2 = 1$), equation (3.26) reduces to the usual Lane – Emden equation governing the equilibrium structure of an undistorted polytropic model of index N in nondimensional form.

The quantity ℓ as defined in equation (3.26) is of the dimension of length. If we set $r_\psi = \ell\,\xi$, then ξ is a nondimensional variable defined for the topologically equivalent spherical model. It corresponds to the usual Emden variable ξ of the Lane – Emden equation for an undistorted spherical polytropic model.

If we set $R = \ell\,\xi_u$ (where ξ_u is the value of ξ at the outermost surface of the undistorted polytropic model) in equation (3.26), the differential equation governing the equilibrium structure of a rotationally and tidally distorted polytropic model may be written in nondimensional form as

$$\frac{d}{dr_0}[A(r_0,n,q)\frac{d\theta_\psi}{dr_0}] = -\frac{\xi_u^2}{K^2}\theta_\psi^N r_0{}^2 B(r_0,n,q) \qquad (3.28)$$

where

$$A(r_0,n,n_1,q) = \frac{r_0^2}{f_2} = r_0{}^2[1-\frac{1}{3}q^2t^2r_0^4 - (\frac{3}{5}q^2+\frac{4}{15}(\beta n)^2+\frac{2}{5}(\beta n)q\,)r_0^6 - \frac{6}{7}q^2r_0^8 - \frac{10}{9}q^2r_0^{10}+...]$$

$$B(r_0,n,n_1,q) = f_1 = [1+4(\beta n)r_0{}^3+7q^2t^2r_0{}^4+(\frac{36}{5}q^2+\frac{96}{5}(\beta n)^2+\frac{24}{5}(\beta n)q+)r_0{}^6$$
$$+\frac{55}{7}q^2\ r_0{}^8+\frac{26}{3}q^2r_0^{10}+...]$$

with

$$r_0 = \frac{1}{\psi^{**}-q},\ \ t=1-\alpha\ ,\ \alpha=\sqrt{n_1/n}\ \text{ and }\ \beta=2\alpha-1.$$

In above expressions terms up to second order of smallness in n, n_1, q and up to r_0^{10} in r_0 are retained. The dimensionless constant K in equation (3.28) is the ratio of the undistorted radius R_ψ of the primary to the separation D between the centers of the primary and secondary star. In fact

$$\frac{D}{\ell} = \frac{D\,\xi_u}{\ell\,\xi_u} = \frac{D}{R_\psi}\xi_u = \frac{1}{K}\xi_u \qquad (3.29)$$

where ξ_u is the value of ξ at the outermost surface of the undistorted polytropic model.

In the absence of Coriolis force (on setting $\alpha=\beta=1$) the expressions $A(r_0,n,n_1,q)$ and $B(r_0,n,n_1,q)$ reduce to their corresponding expressions earlier reported in literature (Mohan and Saxena (99)) when the effects of Coriolis force were not explicitly considered.

Equation (3.28) subject to the boundary conditions (3.27) determines the equilibrium structure of a rotationally and tidally distorted polytropic model which accounts for the effect of Coriolis force in addition to centrifugal and gravitational forces on its equipotential surfaces.

In order to determine the numerical solution of the second – order nonlinear differential equation (3.28) subject to the boundary conditions (3.27), we integrate equation

(3.28) for certain choice of the values of N, ξ_u, K, n, n_1 and q with boundary conditions (3.27). The integration may be continued till θ_ψ first becomes zero. The value of r_0 (i.e. r_{0s}) when θ_ψ first becomes zero determines the outermost free surface of the topologically equivalent spherical model. Once the solutions of equation (3.28) are obtained, we know the values of θ_ψ for various values of the nondimensional independent variable r_0 varying from zero to r_{os}. The pressure P_ψ and the density ρ_ψ on various equipotentials of the distorted model may now be obtained through the relations (3.25) in the same manner as is done for undistorted polytropic model. Also, the radius r_ψ of the topologically equivalent spherical surface corresponding to the equipotential surface ψ^{**} = constant can be determined from equation (3.8) and (3.29) and is given as

$$\begin{aligned} r_\psi = \left(\frac{\ell\,\xi_u}{K}\right) r_0\,[1 + \frac{2}{3}(\beta n)\,r_0^3 + q^2 t^2 r_0^4 + \left(\frac{4}{5}q^2 + \frac{76}{45}(\beta n)^2 + \frac{8}{15}(\beta n)\,q\right) r_0^6 \\ + \frac{5}{7}q^2\,r_0^8 + \frac{2}{3}q^2\,r_0^{10} + \ldots] \end{aligned} \tag{3.30}$$

3.5 COMPUTATION OF VOLUME, SURFACE AREA AND OTHER PHYSICAL PARAMETERS OF DISTORTED POLYTROPIC MODELS OF STARS

In this section we present explicit expressions for computing the volume, surface area and the shape of a rotationally and tidally distorted polytropic model. Following the approach given in section 3.2 the total volume enclosed by a rotationally and tidally distorted polytropic model which accounts for Coriolis force as well is given by

$$\begin{aligned} V_\psi = \frac{4\pi}{3}(\frac{\ell\xi_u}{K})^3 r_{0s}^3[1 + 2(\beta n)r_{0s}^3 + 3q^2t^2r_{0s}^4 + (\frac{12}{5}q^2 + \frac{32}{5}(\beta n)^2 + \frac{8}{5}(\beta n)\,q+)r_{0s}^6 \\ + \frac{15}{7}q^2\,r_{0s}^8 + 2q^2r_{0s}^{10} + \ldots] \end{aligned} \tag{3.31}$$

and the surface area of this rotationally and tidally distorted polytropic model is given by

$$\begin{aligned} S_\psi = 4\pi(\frac{\ell\xi_u}{K})^2\,{r_{os}}^2[1 + \frac{4}{3}(\beta n)r_{0s}^3 + \frac{5}{3}q^2t^2\,r_{0s}^4 + (\frac{7}{5}q^2 + \frac{56}{15}(\beta n)^2 + \frac{14}{15}(\beta n)q)r_{0s}^6 \\ + \frac{9}{7}q^2 r_{0s}^8 + \frac{11}{9}q^2 r_{0s}^{10} + \ldots] \end{aligned} \tag{3.32}$$

The shape of the outermost equipotential surface of such a rotationally and tidally distorted polytropic model may be obtained using

$$r = \left(\frac{\ell\,\xi_u}{k}\right) r_{0s}\,[1 + \lambda q t r_{0s}^2 + a_0 r_{0s}^3 + (qP_3 + 2\lambda^2 q^2 t^2) r_{0s}^4 + (qP_4 + 5a_0\lambda q t) r_{0s}^5 + (qP_5 + 3a_0^{\,2}$$
$$+ 6\lambda q^2 t P_3) r_{0s}^6 + (qP_6 + 7a_0 q P_3 + 7\lambda q^2 t P_4) r_{0s}^7 + (qP_7 + 8a_0 q P_4 + 8\lambda q^2 t P_5$$
$$+ 4q^2 P_3^{\,2}) r_{0s}^8 + (qP_8 + 9a_0 q P_5 + 9\lambda q^2 t P_6 + 9q^2 P_3 P_4) r_{0s}^9 + (qP_9 + 10a_0 q P_6$$
$$+ 10\lambda q^2 t P_7 + 5q^2\{P_4^{\,2} + 2P_3 P_5\}) r_{0s}^{10} + \ldots] \tag{3.33}$$

where $a_0 = qP_2 + \beta n(1-\nu^2)$, $t = 1-\alpha$ and $P_j = P_j(\lambda)$ denote Legendre polynomial.

In the absence of Coriolis force ($\alpha = \beta = 1$), the expressions for V_ψ, S_ψ and r reported in equations (3.31 – 3.33), respectively reduce to their corresponding expressions obtained by Kopal (63) and Mohan and Saxena (99) who did not explicitly account the effects of Coriolis force.

The relations (3.31-3.33) determine the volume, surface area and the shape of a rotationally and tidally distorted polytropic model when terms up to second order of smallness in n, n_1, q and terms up to r_{0s}^{10} in r_{0s} are retained. In case we need the volume, the surface area or the shape of some inner equipotential surface of the distorted model then we need only replace r_{0s} by the appropriate value of r_0 for that surface in the above relations (3.31 - 3.33).

3.6 NUMERICAL COMPUTATIONS

To obtain the inner structure, the shape, the volume and the surface area of a rotationally and tidally distorted polytropic model, equation (3.28) has to be integrated numerically subject to the boundary conditions (3.27) for the specified values of the parameters N, ξ_u, n, n_1, q and K which denote respectively the polytropic index, the radius of the undistorted polytropic model, the nondimensional measure of angular velocity of rotation, the ratio of the mass of the companion to the mass of the primary and the ratio of the undistorted radius of the primary to the distance between the centers of the primary and secondary. For a polytropic model distorted by rotational forces alone $K = 1$. In the case of a polytropic model which is the primary component of a binary system the value of K has to be chosen such that the outermost surface of the primary component lies well within the Roche lobe otherwise the two components of the binary will coalesce (cf. Kopal (63), page 11).

For obtaining the numerical solutions, equation (3.28) has been integrated numerically using fourth-order Runge-Kutta method for the specified values of the input parameters. A series solution similar to the one available for undistorted polytropic models (see Chandrasekhar (11) page 95) was developed to start integrations at points near the centre. This series solution is given by

$$\begin{aligned}\theta_\psi = 1 - \frac{\xi_u^2}{6} r_0^2 + \frac{N\xi_u^4}{120} r_0^4 - \frac{2(\beta n)\xi_u^2}{15} r_0^5 - \left[\frac{N(8n-5)\xi_u^6}{15120} + \frac{q^2 t^2}{6}\left(1 - \frac{\xi_u^2}{9}\right)\xi_u^2\right] r_0^6 \\ + \left[\frac{(\beta n) N \xi_u^4}{70} - \frac{q^2 t^2\, \xi_u^2}{504}\right] r_0^7 + \ldots\end{aligned} \tag{3.34}$$

Taking starting values from this series solution at $r_0 = 0.005$, numerical integration of equation (3.28) was then carried forward using Runge-Kutta method of order four. Using a step length of 0.005, numerical integration was continued till θ_ψ first became zero. Relations (3.31 – 3.33) were then used to determine the volume, surface area and shape of the distorted polytropic model.

Numerical results obtained for different values of the input parameters are tabulated in Tables 3.1 to 3.3. The value of the parameter K has been taken as one for the rotationally distorted model and 0.5 for rotationally and tidally distorted models. (The chosen value of K provides the outer-most surface of the model well within Roche lobe for each considered case). In Table 3.1 we present the values of θ_ψ, P_ψ and ρ_ψ for various types of distorted polytropic models of indices 1.5 and 3.0. The values of the volumes and surface areas obtained for certain distorted models are next presented in Table 3.2. In Table 3.3 we have analyzed the effect of different choices for values of n, n_1 and q on the volumes and surface areas of rotationally and tidally distorted polytropic models of stars. For comparison we have also given in these tables the values when the effects of Coriolis force had not been considered (Mohan and Saxena (99)).

Equation (3.34) has been used to draw the shapes of outermost Roche equipotential surfaces in certain cases. These are depicted in Fig 3.1.

3.7 ANALYSIS OF THE RESULTS

Our results in Table 3.1 show that in the case of nonsynchronous binaries the values of θ_ψ, P_ψ and ρ_ψ increase with the incorporation of the effect of Coriolis force. In the case of purely rotating stars and synchronous binaries there is no effect of Coriolis force and results match with the results obtained earlier by Mohan and Saxena (99) who did not take into account the effect of Coriolis force..

As regards the volumes and surface areas of polytropic models, our results in Table 3.2 show that with the inclusion of Coriolis force, the values of volumes and surface areas in the case of nonsynchronous binaries do not change to any appreciable extent as compared to the corresponding values earlier obtained by Mohan and Saxena (99). In case of pure rotation and synchronous binaries the present results are more or less identical with the corresponding values of Mohan and Saxena (99) (Theoretically these should have been exactly the same. The marginal difference is due to different computational techniques used)

Results in Table 3.3 show that, in the case of nonsynchronous binaries when the values of parameter n of revolution and parameter q of tidal effects are kept fixed then with an increase in the value of parameter n_1, the angular velocity of primary, there is an increase in the values of volumes and surface areas of polytropic models of stars. However, the volumes and surface areas are lesser than the values in the case of undistorted model when the value of n_1 is less than $n/4$. However when parameters n_1 and q are kept fixed, then for values of $n \le 0.05$ and $n > 0.1$, there is decrease in the values of volumes and surface areas whereas for $n > 0.05$ and $n \le 0.1$ there is increase in these values.. When the value of n becomes greater than $4n_1$ these values become even less than their corresponding values for the undistorted models. Again when the rotational parameters n and n_1 are kept fixed and value of q is increased the volumes and surface areas of polytropic models increase but not to any appreciable extent.

In Fig 3.1 we have drawn the shapes of the outermost Roche equipotentials in the case of polytropic models for both cases when the effects of Coriolis force are accounted for as well when these are not considered. Figures show that whereas in case of pure rotation and synchronous binaries, there is, as expected, no change in the shapes, even in the case of nonsynchronous binaries inclusion of Coriolis force does not cause any appreciable change in the shapes.

Thus, our present study has shown that whereas earlier results of Kopal (63) and Mohan and Saxena (99) already account for the effects of Coriolis force in the case of purely rotating stars and synchronous binaries, even in the case of nonsynchronous binaries explicit inclusion of Coriolis force in the equation of structure does not produce any appreciable effect on the equilibrium structure and shapes of polytropic models of such stars.

Table 3.1 Effect of inclusion of Coriolis force on the equilibrium structures of rotating stars and stars in binary systems having polytropic structure

x	Uniformly Rotating			Non synchronous binary			Synchronous binary		
	$n=0.1,\ q=0.0$ $n_1=0.1$			$n=0.1,\ q=0.05$ $n_1=0.2$			$n=0.55,\ q=0.1$ $n_1=0.55$		
	θ_ψ	P_ψ	ρ_ψ	θ_ψ	P_ψ	ρ_ψ	θ_ψ	P_ψ	ρ_ψ
Polytropic index N= 1.5									
0.0	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)
0.1	0.9780 (0.9780)	0.9458 (0.9458)	0.9671 (0.9671)	0.9783 (0.9781)	0.9517 (0.9462)	0.9707 (0.9674)	0.9789 (0.9789)	0.9530 (0.9530)	0.9716 (0.9716)
0.2	0.9144 (0.9144)	0.7996 (0.7996)	0.8744 (0.8744)	0.9156 (0.9151)	0.8111 (0.8011)	0.8819 (0.8754)	0.9180 (0.9180)	0.8160 (0.8160)	0.8851 (0.8851)
0.4	0.6936 (0.6936)	0.4006 (0.4006)	0.5776 (0.5776)	0.6983 (0.6967)	0.4170 (0.4052)	0.5916 (0.5816)	0.7053 (0.7053)	0.4272 (0.4272)	0.6003 (0.6003)
0.5	0.5573 (0.5573)	0.2319 (0.2319)	0.4161 (0.4161)	0.5649 (0.5630)	0.2471 (0.2378)	0.4322 (0.4224)	0.5734 (0.5734)	0.2564 (0.2564)	0.4419 (0.4419)
0.6	0.4182 (0.4182)	0.1131 (0.1131)	0.2704 (0.2704)	0.4292 (0.4272)	0.1254 (0.1193)	0.2878 (0.2793)	0.4381 (0.4381)	0.1320 (0.1320)	0.2967 (0.2967)
0.8	0.1646 (0.1646)	0.0110 (0.0110)	0.0668 (0.0668)	0.1838 (0.1826)	0.0156 (0.0142)	0.0823 (0.0780)	0.1895 (0.1895)	0.0168 (0.0168)	0.0862 (0.0862)
0.9	0.0608 (0.0608)	0.0009 (0.0009)	0.0150 (0.0150)	0.0835 (0.0829)	0.0023 (0.0020)	0.0262 (0.0239)	0.0863 (0.0863)	0.0025 (0.0025)	0.0275 (0.0275)
1.0	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
Polytropic index N = 3.0									
0.0	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)
0.1	0.9260 (0.9260)	0.7352 (0.7352)	0.7940 (0.7940)	0.9265 (0.9263)	0.7586 (0.7362)	0.8129 (0.7948)	0.9276 (0.9276)	0.7619 (0.7619)	0.8155 (0.8155)
0.2	0.7531 (0.7531)	0.3217 (0.3217)	0.4271 (0.4271)	0.7547 (0.7540)	0.3411 (0.3233)	0.4464 (0.4287)	0.7577 (0.7577)	0.3465 (0.3465)	0.4516 (0.4516)
0.4	0.4047 (0.4047)	0.0268 (0.0268)	0.0663 (0.0663)	0.4078 (0.4069)	0.0296 (0.0274)	0.0714 (0.0674)	0.4118 (0.4118)	0.0308 (0.0308)	0.0735 (0.0735)
0.5	0.2819 (0.2819)	0.0063 (0.0063)	0.0224 (0.0224)	0.2856 (0.2849)	0.0072 (0.0066)	0.0246 (0.0231)	0.2890 (0.2890)	0.0075 (0.0075)	0.0255 (0.0255)
0.6	0.1902 (0.1902)	0.0013 (0.0013)	0.0069 (0.0069)	0.1944 (0.1938)	0.0015 (0.0014)	0.0078 (0.0073)	0.1969 (0.1969)	0.0016 (0.0016)	0.0081 (0.0081)
0.8	0.0688 (0.0688)	0.0000 (0.0000)	0.0003 (0.0003)	0.0736 (0.0734)	0.0000 (0.0000)	0.0004 (0.0004)	0.0747 (0.0747)	0.0000 (0.0000)	0.0005 (0.0005)
0.9	0.0277 (0.0277)	0.0000 (0.0000)	0.0000 (0.0000)	0.0327 (0.0326)	0.0000 (0.0000)	0.0000 (0.0000)	0.0332 (0.0332)	0.0000 (0.0000)	0.0000 (0.0000)
1.0	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)

1. Here $x = r_0 / r_{0s}$ and the values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered.
2. Here pressure P_ψ and density ρ_ψ are in the units of P_c and ρ_c respectively.

Table 3.2 Volumes and Surface areas of rotating stars and stars in binary systems having polytropic structures

Type of model	n	n_1	q	Volume $V_\psi \times 10^{-2}$			Surface Area $S_\psi \times 10^{-2}$		
				Present values	Mohan and Saxena (99)	% difference	Present values	Mohan and Saxena (99)	% difference
Polytropic index N= 1.5									
Undistorted	0.0	0.0	0.0	2.0432	2.0432	0.0	1.6776	1.6776	0.0
Uniformly rotating	0.02	0.02	0.0	2.0887	2.0903	0.08	1.7024	1.7035	0.06
	0.05	0.05	0.0	2.1626	2.1672	0.2	1.7425	1.7452	0.2
Nonsynchronous Binary	0.05	0.01	0.2	2.0472	2.0621	0.7	1.6791	1.6879	0.5
	0.05	0.05	0.2	2.0600	2.0621	0.1	1.6865	1.6879	0.08
	0.05	0.1	0.2	2.0734	2.0621	0.5	1.6936	1.6879	0.3
Synchronous binary	0.525	0.525	0.05	2.2035	2.2032	0.01	1.7646	1.7646	0.0
	0.550	0.550	0.1	2.2127	2.2214	0.4	1.7695	1.7744	0.3
Polytrope index $N = 3.0$									
				Volume $V_\psi \times 10^{-3}$			Surface Area $S_\psi \times 10^{-2}$		
Undistorted	0.0	0.0	0.0	1.3747	1.3747	0.0	5.9774	5.9774	0.0
Uniformly rotating	0.02	0.02	0.0	1.4174	1.4185	0.08	6.1021	6.1061	0.07
	0.05	0.05	0.0	1.4898	1.4935	0.2	6.3087	6.3199	0.2
Nonsynchronous Binary	0.05	0.01	0.2	1.3774	1.3910	1.0	5.9842	6.0260	0.7
	0.05	0.05	0.2	1.3896	1.3910	0.1	6.0210	6.0260	0.08
	0.05	0.1	0.2	1.4022	1.3910	0.8	6.0566	6.0260	0.5
Synchronous binary	0.525	0.525	0.05	1.5313	1.5302	0.07	6.4261	6.4240	0.03
	0.550	0.550	0.1	1.5406	1.5486	0.5	6.4522	6.4754	0.4

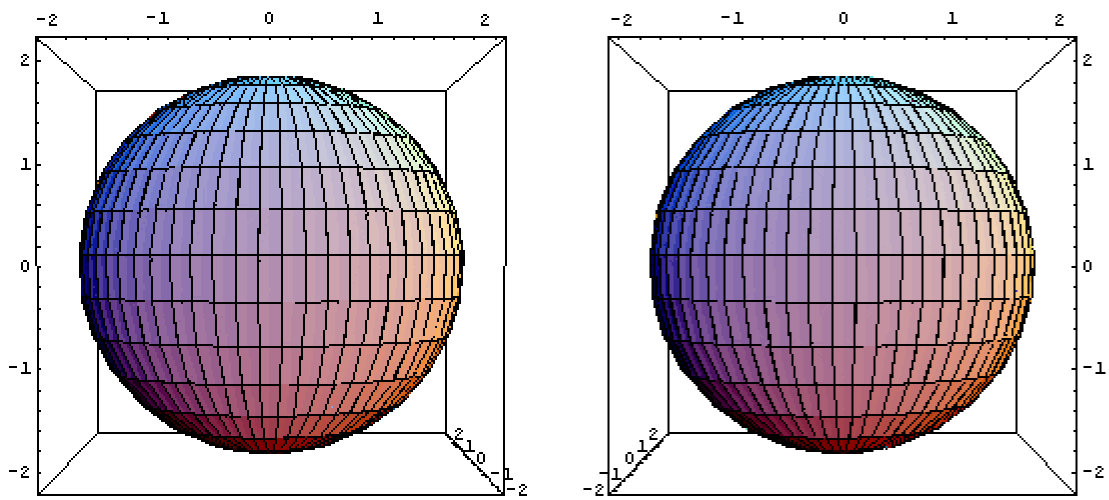
Table 3.3 Effect of the choice of values of n, n_1 and q on the volumes and surface areas of rotationally and tidally distorted polytropic models of stars

Model No.		Volume ($V_\psi \times 10^{-2}$)	Surface Area ($S_\psi \times 10^{-2}$)	Volume ($V_\psi \times 10^{-3}$)	Surface Area ($S_\psi \times 10^{-2}$)
		Polytropic index 1.5		Polytropic index 3.0	
Undistorted Model		2.0432	1.6776	1.3742	5.9774
	n_1	$n=0.10$, $q=0.20$			
1	0.00	2.0284	1.6678	1.3592	5.9279
2	0.03	2.0504	1.6810	1.3804	5.9936
3	0.05	2.0583	1.6855	1.3880	6.0159
4	0.07	2.0651	1.6893	1.3945	6.0350
5	0.10	2.0744	1.6944	1.4033	6.0605
6	0.12	2.0801	1.6974	1.4087	6.0760
7	0.15	2.0882	1.7017	1.4164	6.0977
8	0.20	2.1005	1.7083	1.4282	6.1311
	n	$n_1=0.10$, $q=0.20$			
1	0.01	2.1002	1.7025	1.4262	6.1049
2	0.03	2.0743	1.6934	1.4029	6.0562
3	0.05	2.0734	1.6936	1.4022	6.0566
4	0.07	2.0740	1.6941	1.4029	6.0590
5	0.10	2.0745	1.6944	1.4033	6.0605
6	0.20	2.0702	1.6919	1.3992	6.0483
7	0.40	2.0482	1.6797	1.3783	5.9872
8	0.50	2.0343	1.6721	1.3652	5.9490
	q	$n=0.10$, $n_1=0.15$			
1	0.00	2.0842	1.7000	1.4132	6.0899
2	0.01	2.0843	1.7000	1.4132	6.0900
3	0.05	2.0847	1.7002	1.4135	6.0907
4	0.10	2.0855	1.7005	1.4142	6.0923
5	0.15	2.0866	1.7011	1.4151	6.0947
6	0.20	2.0882	1.7017	1.4164	6.0977
7	0.30	2.0923	1.7036	1.4198	6.1061
8	0.40	2.0978	1.7061	1.4243	6.1173

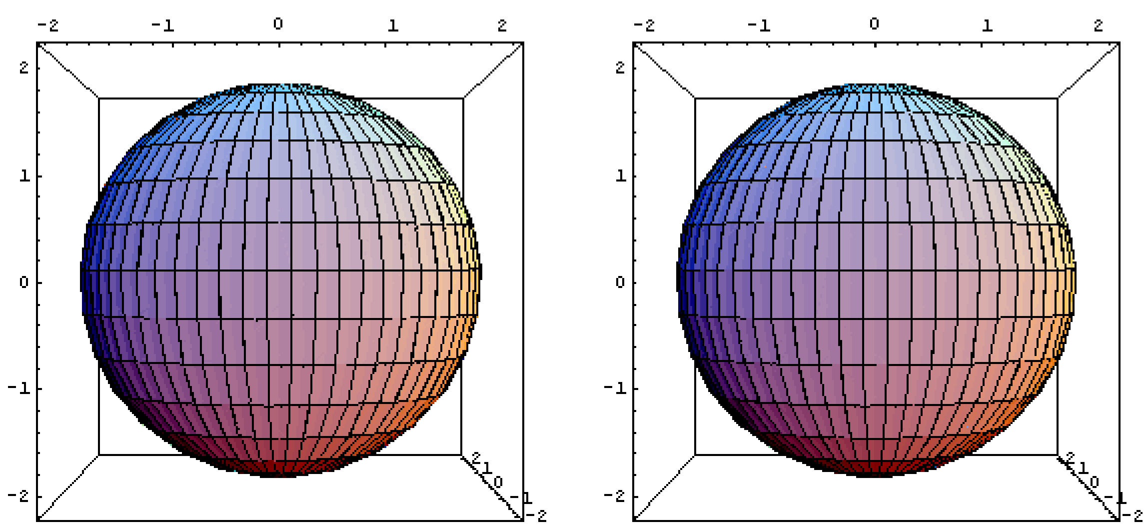
Fig 3.1 Effect of inclusion of Coriolis force on the shapes of rotating stars and stars in binary systems (N=3.0)

Shape when Coriolis force is not included	Shape when Coriolis force is included

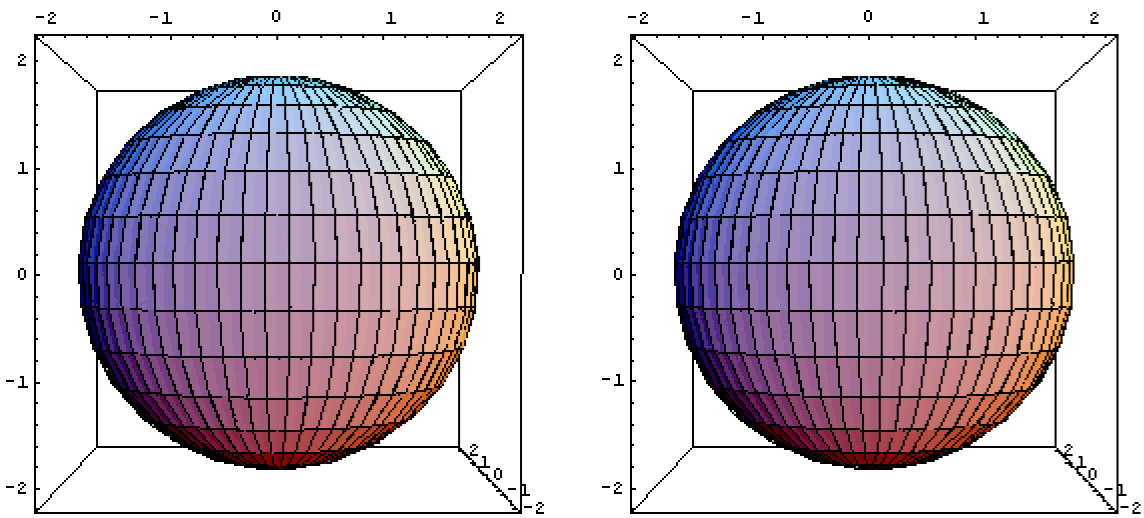
1. $n = 0.05$, $n_1 = 0.1$, $q = 0.2$ (Nonsynchronous binaries)



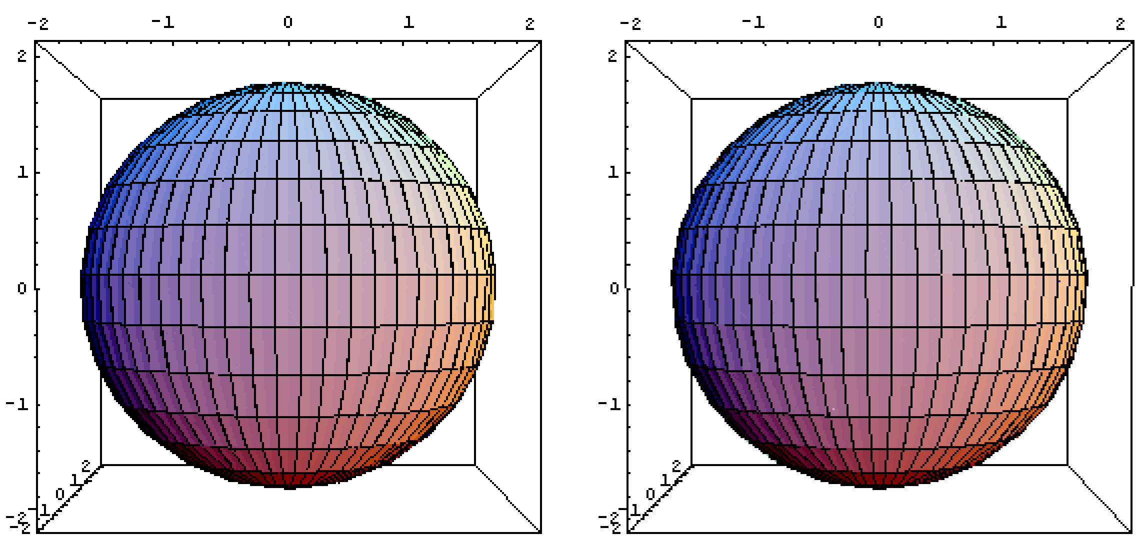
2. $n = 0.05$, $n_1 = 0.05$, $q = 0.2$ (Nonsynchronous binaries)



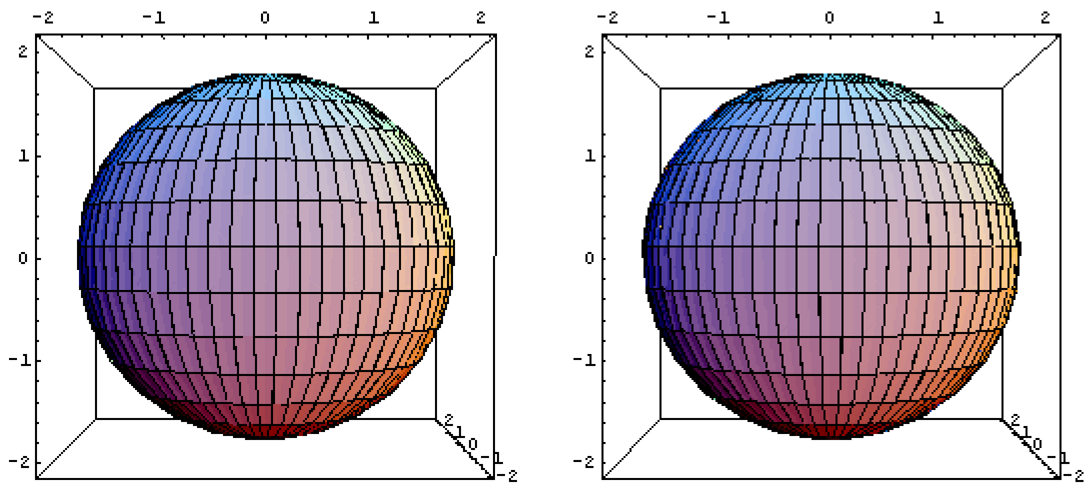
3. $n = 0.05$, $n_1 = 0.01$, $q = 0.2$ (Nonsynchronous binaries)



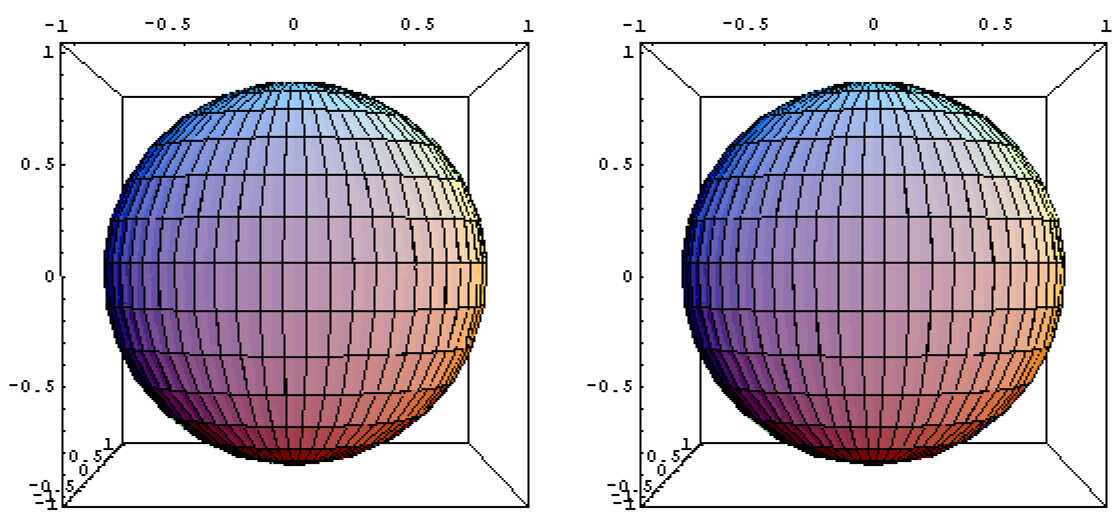
4. $n = 0.525$, $n_1 = 0.525$, $q = 0.05$ (Synchronous binaries)



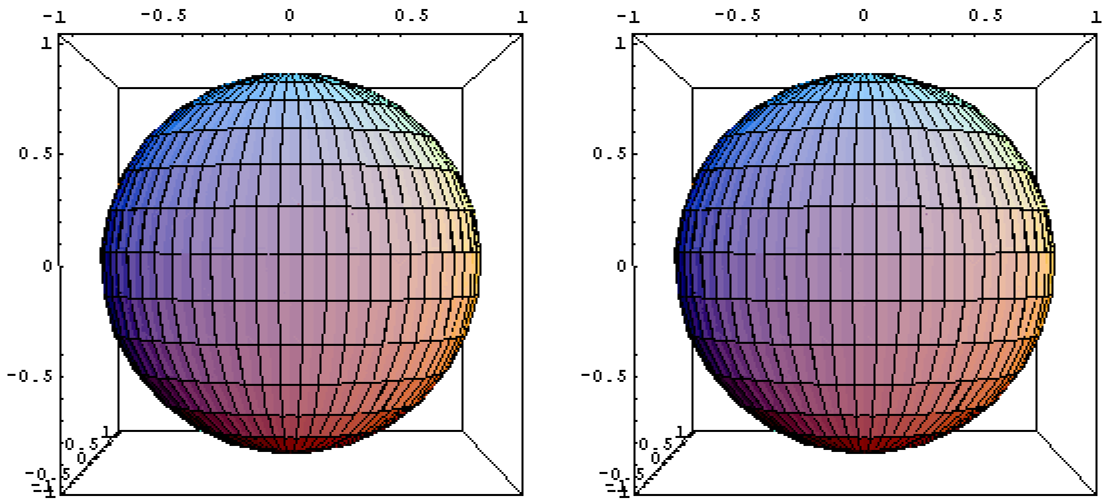
5. $n=0.55$, $n_1=0.55$, $q=0.1$ (Synchronous binaries)



6. $n=0.05$, $q=0.0$ (Pure Rotation)



7. $n = 0.02$, $q = 0.0$ (Pure Rotation)



CHAPTER – IV

EFFECT OF CORIOLIS FORCE ON THE EIGENFREQUENCIES OF SMALL ADIABATIC BAROTROPIC MODES OF OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

In this chapter we use the averaging approach developed in section 3.1, 3.2 and 3.4 of chapter III to compute the effects of Coriolis force on the eigenfrequencies of small adiabatic pseudo – radial and nonradial modes of oscillations of rotationally and tidally distorted stars.

Using the approach developed in sections 3.2 and 3.4 of chapter III, an eigenvalued boundary value problem determining the effect of Coriolis force on the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted stellar models is formulated in section 4.1. An eigenvalued boundary value problem which determines the effect of Coriolis force on the eigenfrequencies of nonradial modes of oscillations of the stellar model has next been formulated in section 4.2. In section 4.3 and 4.4, the methodologies of section 4.1 and 4.2 have been used respectively to formulate the eigenvalue problems of determining the eigenfrequencies of pseudo – radial and nonradial modes of oscillations of rotationally and tidally distorted polytropic models of stars in the presence of Coriolis force. The eigenvalue problems developed in section 4.3 and 4.4 for rotationally and tidally distorted polytropic models have been then solved numerically in section 4.5 to obtain the eigenfrequencies of small adiabatic pseudo-radial and nonradial modes of oscillations of certain rotationally and tidally distorted polytropic models of stars whose inner structures were earlier obtained in chapter III. Numerical results have been then analyzed in section 4.6 and certain conclusions drawn.

4.1 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EIGENFREQUENCIES OF SMALL ADIABATIC PSEUDO – RADIAL MODES OF OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS

In order to determine the effects of rotation and tidal distortions on the eigen-frequencies of stars in binary systems, Mohan and Singh (103) formulated an eigenvalued boundary value problem which determines the periods of small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted Roche model. Mohan et al (109) used this approach to formulate eigenvalue problem which determine the eigenfrequencies of small adiabatic pseudo-radial and nonradial modes of oscillation of rotationally and tidally distorted gaseous spheres in general. The approach adopted by them was later used by Lal (70) to set up the eigenvalue problems which determine the eigenfrequencies of small adiabatic pseudo-radial and nonradial modes of oscillations of certain differentially rotating and tidally distorted stars.

In this approach assuming that during oscillations the fluid elements on an equipotential surface oscillate in unison, the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of a rotationally and tidally distorted star have been obtained from its topologically equivalent spherical model developed on the basis of the averaging technique of Kippenhahn and Thomas (58). Using the approach of Mohan et al (109), the equation determining the eigenfrequencies of pseudo-radial modes of oscillations of rotationally and tidally distorted stellar models may be expressed as

$$\frac{d^2\kappa}{dr_{0\psi}^2}+\frac{4-\mu}{r_{0\psi}}\frac{d\kappa}{dr_{0\psi}}+\left[\frac{\rho_{0\psi}}{\gamma P_{0\psi}}\sigma^2-\left(3-\frac{4}{\gamma}\right)\frac{\mu}{r_{0\psi}^2}\right]\kappa=0 \tag{4.1}$$

where $\mu=-\dfrac{r_{0\psi}}{P_{0\psi}}\dfrac{dP_{0\psi}}{dr_{0\psi}}$

Here $r_{0\psi}$, $\rho_{0\psi}$ and $P_{0\psi}$ are the values of r_ψ, ρ_ψ and P_ψ on the equipotential ψ^{**} = constant in its equilibrium position, σ the eigenfrequency of oscillation and κ some average of the relative amplitudes of pulsation of the fluid elements on the equipotential surface ψ^{**} = constant. Using r_ψ, ρ_ψ and P_ψ in place of $r_{0\psi}$, $\rho_{0\psi}$ and $P_{0\psi}$ to denote the equilibrium values on the equipotential surfaces and taking $r_0=1/(\psi-q)$ in place of $r_{0\psi}$ as the independent variable, the equation (4.1) governing the small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted stellar model incorporating the effects of Coriolis force as well, may be expressed as

$$\begin{aligned}A(n,n_1,q,r_0)\frac{d^2\kappa}{dr_0^2}&+\left[\frac{4-\mu}{r_0}B(n,n_1,q,r_0)-C(n,n_1,q,r_0)\right]\frac{d\kappa}{dr_0}\\&+\left[\frac{R^2\sigma^2\rho_\psi}{\gamma P_\psi}-\left(3-\frac{4}{\gamma}\right)\frac{\mu}{r_0^2}E(n,n_1,q,r_0)\right]\kappa=0\end{aligned} \tag{4.2}$$

where

$$\begin{aligned}A(n,n_1,q,r_0)=1&-\frac{16}{3}(\beta n)r_0^3-10\,q^2t^2r_0^4-(\frac{56}{5}q^2+\frac{104}{45}(\beta n)^2+\frac{112}{15}(\beta n)q)r_0^6\\&-\frac{90}{7}q^2r_0^8-\frac{44}{3}q^2r_0^{10}-\ldots\end{aligned}$$

$$B(n,n_1,q,r_0)=1-\frac{10}{3}(\beta n)r_0^3-6q^2t^2r_0^4-(\frac{32}{5}q^2+\frac{188}{45}(\beta n)^2+\frac{64}{15}(\beta n)q)r_0^6$$
$$-\frac{50}{7}q^2r_0^8-8q^2r_0^{10}-...$$

$$C(n,n_1,q,r_0)=\frac{1}{r_0}[8(\beta n)r_0^3+20q^2t^2r_0^4+(\frac{168}{5}q^2+\frac{104}{15}(\beta n)^2+\frac{112}{5}(\beta n)q)r_0^6$$
$$+\frac{360}{7}q^2r_0^8+\frac{220}{3}q^2r_0^{10}+...]$$

$$E(n,n_1,q,r_0)=1-\frac{4}{3}(\beta n)r_0^3-2q^2t^2r_0^4-(\frac{8}{5}q^2+\frac{92}{45}(\beta n)^2+\frac{16}{15}(\beta n)q)r_0^6-\frac{10}{7}q^2r_0^8-\frac{4}{3}q^2r_0^{10}-...$$

and

$$\mu=-\frac{r_\psi}{P_\psi}\frac{dP_\psi}{dr_0}\frac{dr_0}{dr_\psi}=-F(n,q,r_0)\frac{r_0}{P_\psi}\frac{dP_\psi}{d\,r_0}$$

where

$$F(n,n_1,q,r_0)=1-2(\beta n)r_0^3-4q^2t^2r_0^4-(\frac{24}{5}q^2+\frac{72}{15}(\beta n)^2+\frac{16}{5}(\beta n)q)r_0^6-\frac{40}{7}q^2r_0^8$$
$$-\frac{20}{3}q^2r_0^{10}-...$$

and

$$t=1-\alpha,\ \ \alpha=\sqrt{n_1/n}\ \text{ and }\ \beta=2\alpha-1$$

In the absence of Coriolis force (on setting $\alpha=1$ and $\beta=1$) the above equation reduces to expression as earlier obtained by Mohan et al (109) in which the effects of Coriolis force were not taken into account.

Equation (4.2) forms an eigenvalue problem in the eigenfrequency of oscillation σ. As usual, this eigenvalue problem is of Sturm-Liouville type having singularities both at the centre and the surface of the model. It has to be solved subject to the boundary conditions which require κ to be finite at the centre as well as at the free surface.

In reality equation (4.2) determines the periods of small adiabatic pseudo-radial modes of oscillations of the topologically equivalent spherical model incorporating the effects of Coriolis force. However, since equipotential surfaces of the actual rotationally and tidally distorted model are also the surfaces of equipressure and equidensity, the values of pressure and density on the equipotential surfaces of the rotationally and tidally distorted star are same as on the corresponding equipotential surfaces of the equivalent spherical model. Hence the eigenfrequencies of the radial modes of oscillations determined by solving the eigenvalue problem for the topologically equivalent spherical model are indeed the eigenfrequencies of the radial modes of oscillation of the undistorted model which have got influenced by the rotational and tidal effects. However, the values of the eigenfunction κ obtained on solving (4.2) for the equivalent spherical model are not the actual values of amplitudes of pulsation κ for the distorted model but rather some averages of the true values of eigenfunctions κ on the rotationally and tidally distorted model.

We may thus use equation (4.2) to determine the effects of Coriolis force on the periods of small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted stellar model. The effect of Coriolis force appear through the expressions $A(n,n_1,q,r_0)$, $B(n,n_1,q,r_0)$, $C(n,n_1,q,r_0)$, $E(n,n_1,q,r_0)$, $F(n,n_1,q,r_0)$ and its influence is incorporated through the values evaluating ρ_ψ and P_ψ for the equilibrium model**.** The present method in fact incorporates the effect of Coriolis force both while computing the equilibrium structure (in computing the values of P_ψ, ρ_ψ etc.) as well as in the coefficients *A, B* and *C* of the equation (4.2) which determines the periods of small adiabatic pseudo-radial modes of oscillations.

4.1.1 COMPUTATION OF EIGENVALUES OF RADIAL OSCILLATIONS

The eigenvalue problem (4.2) together with the boundary conditions which require κ to be finite both at the centre as well as the free surface of the star may be solved numerically in the usual manner as is done in the case of undistorted models. For convenience in numerical work it is sometimes found convenient to set

$$\kappa = \frac{\varsigma}{r_0} \quad \text{and} \quad r_0 = x\, r_{OS} \tag{4.3}$$

(r_{OS} being the value of r_0 on the outermost surface) in equation (4.2) and treat x as the independent variable and κ as the dependent variable. With these substitutions, x is now zero at the centre and one at the free surface. The boundary condition κ = finite at the centre is now replaced by $\varsigma = 0$ at the centre whereas the boundary condition κ = finite at the free surface becomes ς = finite at $x=1$. Using (4.3), equation (4.2) gets transformed in terms of the variables ς and x as

$$A^*(n,n_1,q,x)\frac{d^2\varsigma}{dx^2}+B^*(n,n_1,q,x)\frac{d\varsigma}{dx}+C^*(n,n_1,q,x)\varsigma=0 \tag{4.4}$$

where

$$A^*(n,n_1,q,x)=A(n,n_1,q,xr_{0s})$$

$$B^*(n,n_1,q,x)=\frac{4-\mu}{x}B(n,n_1,q,xr_{0S})-r_{0S}C(n,n_1,q,xr_{0S})-\frac{2}{x}A(n,n_1,q,xr_{0S})$$

$$C^*(n,n_1,q,x)=\frac{r_{0S}^2R^2\rho_\psi}{\gamma P_\psi}\sigma^2-(3-\frac{4}{\gamma})\frac{\mu}{x^2}E(n,n_1,q,xr_{0S})-\frac{1}{x}B^*(n,n_1,q,xr_{0S})$$

The boundary conditions now are

$\varsigma=0$ at the centre $\mathrm{x}=0$

and (4.5)

$\varsigma=$ finite at the free surface $\mathrm{x}=1$

For computing an eigenvalue σ, equation (4.4) has to be solved numerically subject to the specified boundary conditions (4.5). Since the centre and the free surface of the star are the singularities of this differential equation, it is advisable to write series solutions of (4.4) near the singularities to start numerical integrations. If we assume ς to be normalized to have value one at the free surface, we can assume a series solutions of the type

$$\varsigma=\sum_{J=0}^{\infty}a_Jx^J \tag{4.6}$$

near the centre x = 0 and

$$\varsigma=1+\sum_{J=0}^{\infty}b_J(1-x)^J \tag{4.7}$$

near the surface x =1, to start the integration of (4.4) near these two singularities.

For obtaining an eigenfrequency of pseudo-radial mode of oscillation, the equation (4.4) has to be integrated numerically for trial values of σ till a value of σ is obtained for which both the boundary conditions are satisfied. One way to achieve this objective could be to integrate equation (4.4) numerically from the surface towards the centre using say fourth-order Runge-Kutta method. Starting values near the surface may be obtained from series solution (4.7). Similarly we can integrate equation (4.4) numerically outwards from the centre starting from a point near the centre. The starting values near the centre may be obtained from the series solution (4.6). Trials with different values of σ may be continued till a value of σ is found for which the value $\varsigma/(d\varsigma/dx)$ from the inward and outward integrations match to desired accuracy at some suitably selected point inside the model.

For determining the eigenfrequencies it is recommended that ρ_ψ, P_ψ be first converted into suitable nondimensional forms keeping in view the physical nature of the model under investigation.

4.2 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EIGENFREQUENCIES OF SMALL ADIABATIC NONRADIAL MODES OF OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS

Mohan et al (109) also formulated an eigenvalued boundary value problem to determine the eigenfrequencies of the nonradial modes of oscillations of rotationally and tidally distorted gaseous spheres. As in the radial case since the values of the physical parameters ρ_ψ and P_ψ on the equipotential surfaces of the distorted model are same as those on the corresponding equipotential surfaces of a topologically equivalent spherical model, we may use this topological equivalent spherical model to compute the eigenfrequencies of nonradial modes of oscillations of rotationally and tidally distorted gaseous spheres as well. Following Mohan et al (109), the eigenvalue problem determining the eigenfrequencies of nonradial modes of oscillations of rotationally and tidally distorted gaseous spheres which incorporates the effects of Coriolis force as well may be expressed in an explicit form convenient for computational work as

$$\left.\begin{aligned} &\frac{d\zeta}{dx} + B_1\zeta + \left(B_2 + \frac{1}{\sigma^2}B_3\right)\eta + \frac{1}{\sigma^2}B_3\,\phi = 0 \\ &\frac{d\eta}{dx} + \left(E_1\sigma^2 + E_2\right)\zeta + E_3\eta + E_4\,\phi + \frac{d\phi}{dx} = 0 \\ &\frac{\mathrm{d}^2\phi}{dx^2} + F_1\frac{d\phi}{dx} + F_2\zeta + F_3\eta + F_4\,\phi = 0 \end{aligned}\right\} \tag{4.8}$$

where

$$B_1 = \frac{l+1}{x} + \frac{1}{\gamma P_\psi}\frac{dP\psi}{dx}$$

$$B_2 = \frac{2\pi G\rho_c}{Dx}\frac{\rho_\psi}{\gamma P_\psi} r_\psi^2 \frac{dr_\psi}{dx}$$

$$= \frac{2\pi G\rho_c}{\gamma P_\psi} D^2\ \rho_\psi\ r_{0s}^3\ x[1+4(\beta n)(xr_{0s})^3 + 7q^2t^2(xr_{0s})^4 + (\frac{36}{5}q^2 + \frac{864}{45}(\beta n)^2$$

$$+\frac{24}{5}(\beta n)q)(xr_{0s})^6 + \frac{55}{7}q^2(xr_{0s})^8 + \frac{26}{3}q^2(xr_{0s})^{10} + ...]$$

$$B_3 = -\frac{l(l+1)}{D\,x}\frac{dr_\psi}{dx} 2\pi G\rho_c$$

$$= -\frac{l(l+1)}{x} 2\pi G\,\rho_c r_{0s}\,[1+\frac{8}{3}(\beta n)(xr_{0s})^3 + 5q^2t^2\,(xr_{0s})^4 + (\frac{28}{5}q^2 + \frac{532}{45}(\beta n)^2$$

$$+\frac{56(\beta n)q}{15})(xr_{0s})^6 + \frac{45}{7}q^2(xr_{0s})^8 + \frac{22}{3}q^2(xr_{0s})^{10} + ...]$$

$$E_1 = -\frac{1}{2\pi G\,\rho_c}\frac{D\,x}{r_\psi^2}\frac{dr_\psi}{dx}$$

$$= -\frac{1}{2\pi G\,\rho_c\, r_{0s}\, x}[1+\frac{4}{3}(\beta n)(xr_{0s})^3 + 3q^2t^2\,(xr_{0s})^4 + (4q^2 + \frac{56}{9}(\beta n)^2$$

$$+\frac{8}{3}(\beta n)q)(xr_{0s})^6\ + 5q^2(xr_{0s})^8 + 6q^2(xr_{0s})^{10} + ...]$$

$$E_2 = \frac{1}{2\pi G\rho_c}\frac{A_\psi}{\rho_\psi}\frac{dP_\psi}{dx}\frac{D\,x}{r_\psi^2}$$

$$= \frac{1}{2\pi G\,\rho_c\,D^2}\frac{1}{\rho_\psi}(\frac{1}{\rho_\psi}\frac{d\rho_\psi}{dx} - \frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx})\frac{dP_\psi}{dx}\frac{1}{x\,r_{0s}^3}[1-4(\beta n)(xr_{0s})^3 - 7q^2t^2\,(xr_{0s})^4$$

$$-(\frac{36}{5}q^2 + \frac{144}{45}(\beta n)^2\ + \frac{72}{15}(\beta n)q)(xr_{0s})^6\ - \frac{55}{7}q^2\,(xr_{0s})^8 - \frac{26}{3}q^2(xr_{0s})^{10} - ...]$$

$$E_3 = \frac{l}{x} + A_\psi\frac{dr_\psi}{dx}\ = \frac{l}{x} + (\frac{1}{\rho_\psi}\frac{d\rho_\psi}{dx} - \frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx}),$$

$$E_4 = \frac{l}{x}$$

$$F_1 = \frac{2l}{x} - \frac{d^2r_\psi/dx^2}{dr_\psi/dx} + \frac{2}{r_\psi}\frac{dr_\psi}{dx}$$

$$= \frac{1}{x}[\,2(l+1) - 4(\beta n)(xr_{0s})^3 - 7q^2t^2\,(xr_{0s})^4 - (\,24q^2 + 32(\beta n)^2$$

$$+16(\beta n)q\,)(xr_{0s})^6\ - 40q^2(xr_{0s})^8 - 60q^2(xr_{0s})^{10} - ...]$$

$$F_2 = 2\frac{\rho_\psi}{\rho_c}\frac{A_\psi\, D\, x}{r_\psi^2}\left(\frac{dr_\psi}{dx}\right)^2$$

$$= 2\frac{\rho_\psi}{\rho_c}\left(\frac{1}{\rho_\psi}\frac{dP_\psi}{dx} - \frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx}\right)\frac{1}{x\, r_{0s}}[1 + \frac{4}{3}(\beta n)(xr_{0s})^3 + 3q^2t^2(xr_{0s})^4 + (4q^2$$

$$+\frac{56}{9}(\beta n)^2 + \frac{8}{3}(\beta n)q)(xr_{0s})^6 + 5q^2(xr_{0s})^8 + 6q^2(xr_{0s})^{10} + ...]$$

$$F_3 = -\frac{4\pi G\rho_\psi^2}{\gamma P_\psi}\left(\frac{d\, r_\psi}{d\, x}\right)^2$$

$$= -\frac{4\pi G\, r_{0s}^2\, D^2\rho_\psi^2}{\gamma P_\psi}[1 + \frac{16}{3}(\beta n)(xr_{0s})^3 + 10q^2t^2(xr_{0s})^4 + (\frac{56}{5}q^2$$

$$+\frac{1384}{45}(\beta n)^2 + \frac{112}{15}(\beta n)q)(xr_{0s})^6 + \frac{90}{7}q^2(xr_{0s})^8 + \frac{44}{3}q^2(xr_{0s})^{10} + ...]$$

$$F_4 = \frac{l(l+1)}{x^2} - \frac{l}{x}\left(\frac{d^2r_\psi}{dx^2}\right)/\left(\frac{dr_\psi}{dx}\right) + \frac{2l}{x}\left(\frac{1}{r_\psi}\frac{dr_\psi}{dx}\right) - \frac{l(l+1)}{r_\psi^2}\left(\frac{dr_\psi}{dx}\right)^2$$

$$= -\frac{l}{x^2}\{[8(\beta n)(xr_{0s})^3 + 15q^2t^2(xr_{0s})^4 + (\frac{168}{5}q^2 + \frac{2412}{45}(\beta n)^2$$

$$+\frac{336}{15}(\beta n)q)(xr_{0s})^6 + \frac{360}{7}q^2(xr_{0s})^8 + \frac{220}{3}q^2(xr_{0s})^{10} + ...] + l[4(\beta n)(xr_{0s})^3$$

$$+8q^2t^2(xr_{0s})^4 + (\frac{48}{5}q^2 + \frac{972}{45}(\beta n)^2 + \frac{96}{15}(\beta n)q)(xr_{0s})^6 + \frac{80}{7}q^2(xr_{0s})^8$$

$$+\frac{40}{3}q^2(xr_{0s})^{10} + ...]\}$$

Here σ is the eigenfrequency of oscillations, $x = r_0/r_{0s}$, $t = 1-\alpha$ and

$$\zeta = \frac{r_\psi^2\delta r_\psi}{D^3x^{l+1}},\ \eta = \frac{\mathrm{P}'_\psi}{2\,\pi\,\mathrm{G}\,\rho_\mathrm{c}\,D^2x^l\rho_\psi}\ \text{and}\ \phi = \frac{\psi'_g}{2\pi\,\mathrm{G}\rho_\mathrm{c}\,D^2x^l}$$

δr_ψ being an average of the amplitudes of Lagrangian variations in the radial direction and P'_ψ, ψ'_g the amplitudes of Lagrangian variation in pressure and gravitational potential on the equipotential surface ψ^{**} = constant.

In the above expressions terms up to second order of smallness in n, n_1, q and up to order r_0^{10} in r_0 have been retained. In the absence of Coriolis force ($\alpha = 1$ and $\beta = 1$) the above expressions reduce to the corresponding ones obtained by Mohan et al (109) who did not take into account the effect of Coriolis force.

The eigenvalue problem (4.8) determining the eigenfrequencies of nonradial modes of oscillations of rotationally and tidally distorted gaseous sphere has to be solved subject to the boundary conditions at the centre and the free surface. Boundary conditions at the centre require $\delta r_\psi = 0$, $P'_\psi / \rho_\psi = 0$ and $\psi'_g = 0$ for $r_\psi = 0$. These requirements lead to the analytic conditions

$$\eta + \phi = \frac{\sigma^2}{2\pi G \rho_c \, l \, r_{OS}} \zeta , \qquad \frac{d\phi}{dx} = 0 \tag{4.9}$$

at the centre $x = 0$.

If the pressure P_ψ on the free surface $(r_\psi = R_\psi)$ is taken to be zero, then δP_ψ the Lagrangian variation in pressure should also be zero at the outer surface. This leads to the condition

$$2\pi G \rho_c \, r_\psi^2 \, \rho_\psi \frac{d\, r_\psi}{d\, x} \eta + D \frac{d\, P_\psi}{d\, x} \zeta = 0$$

or

$$\begin{aligned} &2\pi G \rho_c \rho_\psi D^2 r_{0s}^3 [1 + 4(\beta n) r_{0s}^3 + 7 q^2 t^2 r_{0s}^4 + (\frac{36}{5} q^2 + \frac{864}{45} (\beta n)^2 \\ &+ \frac{72}{15} (\beta n) q) r_{0s}^6 + \frac{55}{7} q^2 r_{0s}^8 + \frac{26}{3} q^2 r_{0s}^{10} + ...] \eta + \frac{dP_\psi}{dx} \zeta = 0. \end{aligned} \tag{4.10a}$$

The condition requiring gravitational potential to be continuous across the free surface gives

$$\frac{d\phi}{dx} + \left[l + \frac{(l+1)}{r_\psi} \frac{dr_\psi}{dx} \right] \phi + \frac{2 D \rho_\psi}{\rho_c r_\psi^2} \frac{dr_\psi}{dx} \zeta = 0$$

or

$$\begin{aligned} &\frac{d\theta_\psi}{dx} + \phi \{ l + (l+1) \, [1 + 2(\beta n) r_{0s}^3 + 4 q^2 t^2 \, r_{0s}^4 + (\frac{24}{5} q^2 + \frac{396}{45} (\beta n)^2 + \frac{48}{15} (\beta n) q) \, r_{0s}^6 \\ &+ \frac{40}{7} q^2 r_{0s}^8 + \frac{20}{3} q^2 r_{0s}^{10} + ...] \} + \frac{2\rho_\psi}{\rho_c \, r_{0s}} [1 + \frac{4}{3} (\beta n) r_{0s}^3 + (4 q^2 + \frac{56}{9} (\beta n)^2 \\ &+ \frac{8}{3} (\beta n) q) r_{0s}^6 + 5 q^2 r_{0s}^8 + 6 q^2 r_{0s}^{10} + ...] \zeta = 0 \end{aligned} \tag{4.10b}$$

at the surface $x = 1$.

4.2.1 COMPUTATION OF EIGENVALUES OF NONRADIAL OSCILLATIONS

In order to determine the effects of Coriolis force on the eigenfrequencies of nonradial modes of oscillation of rotationally and tidally distorted gaseous spheres, system of differential equations (4.8) has to solved subject to the boundary conditions (4.9) at the centre and the boundary conditions (4.10) at the free surface.

For determining the eigenfrequencies, it is recommended that in all the above expressions ρ_ψ, P_ψ be first converted into suitable nondimensional forms keeping in view the physical nature of the model under investigation.

The numerical method which has been used to solve the eigenvalued boundary value problem of this section is discussed in an appendix to this chapter.

4.3 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EIGENFREQUENCIES OF SMALL ADIABATIC PSEUDO – RADIAL MODES OF OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

The eigenvalued boundary value problem which determines the effect of Coriolis force on the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted gaseous sphere has been formulated in section 4.1. In order to use this formulation to determine the effect of Coriolis force on the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted polytropic model, we have to use the values of ρ_ψ and P_ψ for the appropriate rotationally and tidally distorted polytropic model in this eigenvalue problem.

On substituting in equation (4.2) the values of P_ψ and ρ_ψ as defined by relations (3.23) of chapter III, we get the eigenvalued boundary value problem determining the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted polytropic model which accounts for the effects of Coriolis force besides the centrifugal and gravitational forces in nondimensional form as

$$H_1 \frac{d^2\kappa}{dr_0^{\,2}} + H_2 \frac{d\kappa}{dr_0} + (H_3\omega^2 - H_4)\kappa = 0 \tag{4.11}$$

where

$$H_1 = 1 - \frac{16}{3}(\beta n)r_0^3 - 10q^2t^2r_0^4 - (\frac{56}{5}q^2 + \frac{104}{45}(\beta n)^2 + \frac{112}{15}(\beta n)q)r_0^6 - \frac{90}{7}q^2r_0^8 - \frac{44}{3}q^2r_0^{10} - ...$$

$$H_2 = [(4 - \frac{64}{3}(\beta n)r_0^3 - 44q^2t^2r_0^4 - (\frac{296}{5}q^2 + \frac{1064}{45}(\beta n)^2 + \frac{592}{15}(\beta n)q)r_0^6$$
$$- \frac{560}{7}q^2r_0^8 - \frac{316}{3}q^2r_0^{10} - ...) + (N+1)(\frac{1}{\theta_\psi}\frac{d\theta_\psi}{dx})r_0 H_1]/r_0$$

$$H_3 = \frac{(N+1)\xi_u^{\,2}K}{3\gamma r_{os}^{\;3}}\left(\frac{\bar{\rho}}{\rho_c}\right)\frac{1}{\theta_\psi}$$

$$H_4 = -(3 - \frac{4}{\gamma})(N+1)(\frac{1}{\theta_\psi}\frac{d\theta_\psi}{dr_0})\frac{1}{r_0}[1 - \frac{10}{3}(\beta n)r_0^3 - 6q^2t^2r_0^4 - (\frac{32}{5}q^2 + \frac{188}{45}(\beta n)^2 + \frac{64}{15}(\beta n)q)r_0^6$$
$$- \frac{50}{7}q^2r_0^8 - 8q^2r_0^{10} - ...]$$

and $\omega^2 = \dfrac{D^3 r_{os}^3 \sigma^2}{GM_0}$

ω being the nondimensional form of the eigenfrequency σ. In the above expressions, values of the parameters ξ_u, $\bar{\rho}$, ρ_c and K are to be taken for the original undistorted polytropic model.

Equation (4.11) is the general equation in nondimensional form which determines the effects of Coriolis force on the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted polytropic model when terms up to second order of smallness in n, n_1, q and up to r_0^{10} in r_0 are retained. For numerical evaluation of the eigenfrequencies, the second order differential equation (4.11) is to be solved numerically subject to the boundary conditions which require κ to be finite at points corresponding to the centre $(r_0 = 0)$ and the free surface $(r_0 = r_{OS})$ of the model.

In the absence of Coriolis force (on setting $\alpha = 1$ and $\beta = 1$) equation (8.11) reduces to the equation as obtained earlier by Mohan and Saxena (100) who did not account for Coriolis force.

4.4 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EIGENFREQUENCIES OF SMALL ADIABATIC NONRADIAL MODES OF OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

System of equation (4.8) with the boundary conditions (4.9 - 4.10) constitutes the eigenvalued boundary value problem which determines the effects of Coriolis force on the eigenfrequencies of nonradial modes of oscillations of rotationally and tidally distorted gaseous spheres. In order to use this eigenvalue problem to determine the effects of Coriolis force on the eigenfrequencies of nonradial modes of oscillations of polytropic models the values of P_ψ and ρ_ψ appearing in these equations are to be taken from relation (3.23) of Chapter III. Using these, the system of differential equations (4.8) governing the nonradial modes of oscillations of rotationally and tidally distorted polytropic models incorporating the effects of Coriolis force may be expressed as

$$\left.\begin{aligned} &\frac{d\zeta}{dx} + B_1\zeta + (B_2 + \frac{B_3}{\omega^2})\eta + \frac{B_3}{\omega^2}\phi = 0 \\ &\frac{d\eta}{dx} + (E_1\omega^2 + E_2)\zeta + E_3\eta + E_4\phi + \frac{d\phi}{dx} = 0 \\ &\frac{d^2\phi}{dx^2} + F_1\frac{d\phi}{dx} + F_2\zeta + F_3\eta + F_4\phi = 0 \end{aligned}\right\} \tag{4.12}$$

where

$$B_1 = \frac{l+1}{x} + \frac{N+1}{\gamma}(\frac{1}{\theta_\psi}\frac{d\theta_\psi}{dx})$$

$$B_2 = \frac{(N+1)\xi_u^{\,2}\, xr_{0s}^3}{2\gamma K^2\theta_\psi}[1+4(\beta n)(xr_{0s})^3 + 7q^2t^2(xr_{0s})^4 + (\frac{36}{5}q^2 + \frac{864}{45}(\beta n)^2$$
$$+\frac{24}{5}(\beta n)q)(xr_{0s})^6 + \frac{55}{7}q^2(xr_{0s})^8 + \frac{26}{3}q^2(xr_{0s})^{10} + ...]$$

$$B_3 = \frac{-3l(l+1)r_{0s}^4}{2K^3x}(\frac{\rho_c}{\overline{\rho}})[1+\frac{8}{3}(\beta n)(xr_{0s})^3 + 5q^2t^2(xr_{0s})^4 + (\frac{28}{5}q^2 + \frac{532}{45}(\beta n)^2$$
$$+\frac{56}{15}(\beta n)q)(xr_{0s})^6 + \frac{45}{7}q^2(xr_{0s})^8 + \frac{22}{3}q^2(xr_{0s})^{10} + ...]$$

$$E_1 = \frac{-2k^3}{3xr_{0s}^4}(\frac{\overline{\rho}}{\rho_c})[1+\frac{4}{3}(\beta n)(xr_{0s})^3 + 3q^2t^2(xr_{0s})^4 + (4q^2 + \frac{56}{9}(\beta n)^2$$
$$+\frac{8}{3}(\beta n)q)(xr_{0s})^6 + 5q^2(xr_{0s})^8 + 6q^2(xr_{0s})^{10} + ...]$$

$$E_2 = \frac{2k^2}{\xi_u^{\ 2}}(N - \frac{N+1}{\gamma})\frac{1}{\theta_\psi}(\frac{d\theta_\psi}{dx})^2 \frac{1}{xr_{0s}^3}[1 - 4(\beta n)(xr_{0s})^3 - 7q^2t^2(xr_{0s})^4 - (\frac{36}{5}q^2$$

$$+ \frac{144}{45}(\beta n)^2 + \frac{72}{15}(\beta n)q)(xr_{0s})^6 - \frac{55}{7}q^2(xr_{0s})^8 - \frac{26}{3}q^2(xr_{0s})^{10} - ...]$$

$$E_3 = \frac{l}{x} + (N - \frac{N+1}{\gamma})\frac{1}{\theta_\psi}(\frac{d\theta_\psi}{dx}), \qquad E_4 = \frac{l}{x}$$

$$F_1 = \frac{1}{x}[2(l+1) - 4(\beta n)(xr_{0s})^3 - 7q^2t^2(xr_{0s})^4 - (24q^2 + 32(\beta n)^2$$

$$+ 16(\beta n)q)(xr_{0s})^6 - 40q^2(xr_{0s})^8 - 60q^2(xr_{0s})^{10} - ...]$$

$$F_2 = \frac{2}{xr_{0s}}(N - \frac{N+1}{\gamma})\theta_\psi^{\ N-1}\frac{d\theta_\psi}{dx}[1 + \frac{4}{3}(\beta n)(xr_{0s})^3 + 3q^2t^2(xr_{0s})^4 + (4q^2$$

$$+ \frac{56}{9}(\beta n)^2 + \frac{8}{3}(\beta n)q)(xr_{0s})^6 + 5q^2(xr_{0s})^8 + 6q^2(xr_{0s})^{10} + ...]$$

$$F_3 = -\frac{(N+1)}{\gamma}\frac{\xi_u^{\ 2}}{k^2}\theta_\psi^{\ N-1}r_{0s}^2[1 + \frac{16}{3}(\beta n)(xr_{0s})^3 + 10q^2t^2(xr_{0s})^4 + (\frac{56}{5}q^2$$

$$+ \frac{1384}{45}(\beta n)^2 + \frac{112}{15}(\beta n)q)(xr_{0s})^6 + \frac{90}{7}q^2(xr_{0s})^8 + \frac{44}{3}q^2(xr_{0s})^{10} + ...]$$

$$F_4 = -\frac{l}{x^2}\{[8(\beta n)(xr_{0s})^3 + 15q^2t^2(xr_{0s})^4 + (\frac{168}{5}q^2 + \frac{2412}{45}(\beta n)^2 + \frac{336}{15}(\beta n)q)(xr_{0s})^6$$

$$+ \frac{360}{7}q^2(xr_{0s})^8 + \frac{220}{3}q^2(xr_{0s})^{10} + ...] + l[4(\beta n)(xr_{0s})^3 + 8q^2t^2(xr_{0s})^4$$

$$+ (\frac{48}{5}q^2 + \frac{972}{45}(\beta n)^2 + \frac{96}{15}(\beta n)q)(xr_{0s})^6 + \frac{80}{7}q^2(xr_{0s})^8 + \frac{40}{3}q^2(xr_{0s})^{10} + ...]\}$$

and $\omega^2 = \dfrac{Dr_{0s}^{\ 3}\sigma^2}{GM_0}$,

ω being the nondimensional form of the eigenfrequency σ. As mentioned in the radial case, values of the parameters ξ_u, ρ_c, $\bar{\rho}$ and K are to be taken from the original undistorted polytropic model.

The boundary conditions (4.9) at the centre ($x=0$) for the case of distorted polytropic model become

$$\eta + \phi = \frac{2K^3\omega^2}{3l\,r_{0s}^4}\left(\frac{\bar{\rho}}{\rho_c}\right)\zeta, \qquad \frac{d\phi}{dx} = 0 \tag{4.13}$$

On substituting the values of P_ψ and ρ_ψ from equation (3.23) in the boundary conditions (4.10) at the free surface ($x=1$), the boundary conditions at the free surface in the case of polytropic models become

$$\eta {r_{0s}}^3[1+4(\beta n)(xr_{0s})^3+7q^2t^2(xr_{0s})^4+(\frac{36}{5}q^2+\frac{864}{45}(\beta n)^2+\frac{72}{15}(\beta n)q)(xr_{0s})^6 + \frac{55}{7}q^2(xr_{0s})^8+\frac{26}{3}q^2(xr_{0s})^{10}+...]+2\frac{K^2}{\xi_u^2}\frac{d\theta_\psi}{dx}\zeta=0 \tag{4.14a}$$

and

$$\frac{d\theta_\psi}{dx}+\phi\{l+(l+1)\,[1+2(\beta n)(xr_{0s})^3+4q^2t^2\,(xr_{0s})^4+(\frac{24}{5}q^2+\frac{396}{45}(\beta n)^2 + \frac{48}{15}(\beta n)q)\,(xr_{0s})^6+\frac{40}{7}q^2(xr_{0s})^8+\frac{20}{3}q^2(xr_{0s})^{10}+...]\}=0 \tag{4.14b}$$

at the surface $x=1$.

The system of differential equations (4.12) together with the boundary conditions (4.13 – 4.14) constitutes the eigenvalued boundary value problem determining the effects of Coriolis force on the eigenfrequencies of nonradial modes of oscillations of polytropic models of stars.

In the absence of Coriolis force ($\alpha=\beta=1$) equation (4.13 – 4.14) reduce to their corresponding expressions obtained by Mohan and Saxena (100) who did not account for the effects of Coriolis force.

4.5 NUMERICAL COMPUTATION OF THE EIGENFREQUENCIES OF PSEUDO – RADIAL AND NONRADIAL MODES OF OSCILLATIONS OF DISTORTED POLYTROPIC MODELS OF STARS INCORPORATING THE EFFECTS OF CORIOLIS FORCE

The eigenvalue problem of section 4.3 is of Sturm – Liouville type. For determining the eigenfrequencies of small adiabatic pseudo- radial oscillations of rotationally and tidally distorted polytropic models, equation (4.11) is to be integrated numerically subject to the boundary conditions which require κ to be finite at points corresponding to the center and the free surface of the model. The numerical integration can be performed using the approach suggested in section 4.1. The values of θ_ψ and $d\theta_\psi/(dx)$ needed at various points are to be taken from the numerical solution of the equation (3.30) of chapter III. Computations are started with some trial value of ω^2. For this chosen value of ω^2 series solution is first developed at a point close to the center. This solution is then used to carry integration of the pulsation equation (4.11) outwards using fourth order Runge-Kutta method. Using the same numerical value of ω^2, series solution is also developed at points near the surface which is then used to carry integration of the equation (4.11) inwards. The value of $\kappa/(d\kappa/dx)$ obtained from the outward and inward integrations of (4.11), is then matched at some preselected point in the interior of the model. Process is iteratively continued with different choices of the value of ω^2, till a value of ω^2 is found for which the two solutions agree to specified accuracy. To start integrations from points near the centre and the surface, series solutions were developed at x = 0.01 and x = 0.99. Outward and inward integrations were performed using a step length x = 0.01. Trials with different values of ω^2 were continued till the absolute difference in the value $\kappa/(d\kappa/dx)$ at the preselected point in the interior of the model from the outward and inward integrations was found to be less than 0.0005.

Computations have been performed to compute the effect of Coriolis force on the eigenvalues of the fundamental and the first mode of pseudo-radial oscillations of rotationally and tidally distorted polytropic models of indices 1.5 and 3.0 for which equilibrium structures were earlier obtained in Chapter III for the same values of distortion parameters n, n_1 and q. The results are presented in Table 4.1. For comparison we also present in this table the corresponding results when effects of Coriolis force are not considered.

The eigenfrequencies of the nonradial modes of oscillations - incorporating the effects of Coriolis force - of some of these rotationally and tidally distorted polytropic models have also been computed using Chebyshev polynomial expansion technique earlier used by Mohan et al (109). The essential details of the method are given in Saxena (133) and are presented for ready reference as an appendix to this chapter. The boundary condition (4.14a) was used as the discriminate condition and $\zeta = 1$ at the centre was used as the normalization condition. The values of θ_ψ and $(d\theta_\psi/dx)$ needed at various points in the interior of the model were obtained from the solutions of the structure equation (3.30) of these models earlier obtained in Chapter III. For polytropic indices 1.5 and 3.0 we ordinarily used 10 and 15 collocation points, respectively. However, for determining the eigenfrequencies of certain higher modes of nonradial oscillations, the number of collocation points was increased to achieve the desired accuracy of 0.0001 for the polytropic models of indices 1.5 and 3.0 in getting the discriminant condition to be satisfied. The number of collocation points used in determining a specific mode of nonradial oscillation of a distorted polytropic model was same as used in determining this mode for the corresponding undistorted model.

The numerical results for the nonradial modes of oscillations of rotationally and tidally distorted polytropic models are presented in Table 4.2 for polytropic indices 1.5 and 3.0, respectively. The number of nodes appearing in the eigenfunctions ζ and η are also shown in parenthesis in these tables. The corresponding results when the effects of Coriolis force are not considered are also presented in these tables for comparison.

4.6 ANALYSIS OF THE RESULTS

The results in Table 4.1 and Table 4.2 show that with the inclusion of Coriolis force there is not any appreciable change in the eigenfrequencies of rotationally and tidally distorted primary components of nonsynchronous binary systems as compared to the corresponding eigenfrequencies of Mohan and Saxena (100) who did not consider the effect of Coriolis force. The two are nearly same for $n_1 = n$. For synchronously rotating primary component of binary system or pure rotation there is no Coriolis force and hence values obtained by us are same as to the values obtained earlier by Mohan and Saxena (100).

It is also observed from Table 4.2, that in the case of nonradial modes of oscillations, pressure p - modes shows more percentage change than gravitational g - modes.

On comparing our results with the corresponding results of undistorted model in Table 4.1 and 4.2, we find that there is decrease in the eigenfrequencies of radial modes of oscillations and f, g, p modes of nonradial oscillations with the inclusion of Coriolis force. This difference between the two corresponding values of eigenfrequencies increases with increase in the value of n_1.

In Table 4.3 and Table 4.4 we have analyzed the effect of inclusion of Coriolis force on the values of eigenfrequencies of primary component of nonsynchronous binaries with change in the values of parameters n , n_1 and q. Our results show that when the rotational parameter n and tidal parameter q are kept fixed, then with an increase in the value of parameter n_1, there is a decrease in the values of the eigenfrequencies of polytropic models of stars. These values become even less than the corresponding values of undistorted model when $n_1 \geq n/4$. However, when the parameters n_1 and q are kept fixed, then for small values of $n \leq 0.1$, with an increase in the value of parameter n, there is a decrease in the values of eigenfrequencies but for large values of n there is an increase in the eigenvalues. For $n > 4n_1$, the present values become greater than the values in case of undistorted model. Again when both the rotational parameters n and n_1 are kept fixed, then with an increase in the value of q, there is a decrease in the values of eigenfrequencies but not to an appreciable extent.

Thus, our present study has shown that whereas as expected, there is no effect of Coriolis force on the eigenfrequencies of radial and nonradial modes of oscillations in case of synchronous binaries and purely rotating stars, the effect in the case of even nonsynchronous binaries is also not very appreciable.

Table 4.1 Eigenfrequencies ω_0^2 for the fundamental mode and ω_1^2 for the first mode of pseudo – radial oscillations of rotationally and tidally distorted polytropic models of stars

n	n_1	q	k	$N=1.5$		$N=3.0$	
				ω_0^2	ω_1^2	ω_0^2	ω_1^2
Undistorted model							
0.00	0.00	0.00	1.0	2.7059 (2.7059)	12.5338 (12.5338)	9.2550 (9.2550)	16.9844 (16.9844)
Uniformly rotating							
0.01	0.01	0.00	1.0	2.6623 (2.6623)	12.2408 (12.2408)	9.1140 (9.1140)	16.6197 (16.6197)
0.05	0.05	0.00	1.0	2.4865 (2.4865)	11.0429 (11.0429)	8.5192 (8.5192)	15.1000 (15.1000)
Nonsynchronous binaries							
0.10	0.05	0.10	0.5	2.6820 (2.6502) [1.2]	12.3704 (12.1553) [1.8]	9.1783 (9.0751) [1.1]	16.7830 (16.5141) [1.6]
0.10	0.10	0.10	0.5	2.6502 (2.6502) [0.0]	12.1553 (12.1553) [0.0]	9.0751 (9.0751) [0.0]	16.5141 (16.5141) [0.0]
0.10	0.15	0.10	0.5	2.6252 (2.6502) [0.9]	11.9859 (12.1553) [1.4]	8.9928 (9.0751) [0.9]	16.3055 (16.5141) [1.3]
Synchronous binaries							
0.525	0.525	0.05	0.5	2.4157 (2.4157)	10.5457 (10.5457)	8.2649 (8.2649)	14.4539 (14.4539)
0.550	0.550	0.10	0.5	2.4000 (2.4000)	10.4283 (10.4283)	8.2069 (8.2069)	14.2988 (14.2988)

1. Entries in the parenthesis are the eigenvalues when the effect of Coriolis force is not considered.
2. Entries in the square brackets represent the percentage difference in the values of eigenfrequencies with the inclusion of the effect of Coriolis force.

Table 4.2 Eigenfrequencies ω^2 for the nonradial modes of oscillations of rotationally and tidally distorted polytropic models of stars

Mode	Undistorted	Uniformly rotating		Nonsynchronous Binaries			Synchronous binaries	
	$n=0.0$ $n_1=0.0$ $q=0.0$	$n=0.01$ $n_1=0.01$ $q=0.0$	$n=0.05$ $n_1=0.05$ $q=0.0$	$n=0.10$ $n_1=0.05$ $q=0.10$	$n=0.10$ $n_1=0.10$ $q=0.10$	$n=0.10$ $n_1=0.15$ $q=0.10$	$n=0.525$ $n_1=0.525$ $q=0.05$	$n=0.55$ $n_1=0.55$ $q=0.1$
Polytrope of index 1.5								
p_3	41.296 (41.296)	40.254 (40.254)	36.206 (36.206)	40.700 (39.949) [1.9]	39.949 (39.949) [0.0]	39.357 (39.949) [1.5]	34.700 (34.700)	34.400 (34.400)
p_2	23.516 (23.516)	22.914 (22.914)	20.536 (20.536)	23.178 (22.738) [1.9]	22.738 (22.738) [0.0]	22.393 (22.738) [1.5]	19.630 (19.630)	19.432 (19.432)
p_1	10.291 (10.291)	10.025 (10.025)	8.976 (8.976)	10.142 (9.947) [2.0]	9.947 (9.947) [0.0]	9.795 (9.947) [1.5]	8.577 (8.577)	8.490 (8.490)
f	2.423 (2.423)	2.364 (2.364)	2.132 (2.132)	2.390 (2.347) [1.8]	2.347 (2.347) [0.0]	2.313 (2.347) [1.4]	2..045 (2.045)	2.026 (2.026)
Polytrope of index 3.0								
p_3	41.475 (41.475)	40.365 (40.365)	35.853 (35.853)	40.854 (40.038) [2.0]	40.038 (40.038) [0.0]	39.397 (40.038) [1.6]	34.064 (34.064)	33.666 (33.666)
p_2	26.728 (26.728)	26.032 (26.032)	23.198 (23.198)	26.340 (25.828) [2.0]	25.828 (25.828) [0.0]	25.426 (25.828) [1.6]	22.063 (22.063)	21.808 (21.808)
p_1	15.283 (15.283)	14.917 (14.917)	13.430 (13.430)	15.080 (14.811) [1.8]	14.811 (14.811) [0.0]	14.600 (14.811) [1.4]	12.835 (12.835)	12.702 (12.702)
f	8.246 (8.246)	8.128 (8.128)	7.641 (7.641)	8.182 (8.095) [1.1]	8.095 (8.095) [0.0]	8.027 (8.095) [0.8]	7.442 (7.442)	7.397 (7.397)
g_1	4.927 (4.927)	4.892 (4.892)	4.751 (4.751)	4.908 (4.883) [0.5]	4.883 (4.883) [0.0]	4.863 (4.883) [0.4]	4.692 (4.692)	4.679 (4.679)
g_2	2.833 (2.833)	2.816 (2.816)	2.748 (2.748)	2.823 (2.811) [0.4]	2.811 (2.811) [0.0]	2.800 (2.811) [0.4]	2.719 (2.719)	2.713 (2.713)
g_3	1.836 (1.836)	1.825 (1.825)	1.784 (1.784)	1.830 (1.822) [0.4]	1.822 (1.822) [0.0]	1.816 (1.822) [0.3]	1.766 (1.766)	1.762 (1.762)

1. Entries in the parenthesis are the values of eigenfrequencies when the effect of Coriolis force is not taken into account.
2. Values in the square brackets represent the percentage difference in the values of eigenfrequencies with the inclusion of the effect of Coriolis force.

Table 4.3 Effect of variation in the values of n, n_1 and q on the eigenfrequencies of the pseudo – radial modes of oscillations of rotationally and tidally distorted polytropic model of primary component of binary system (N = 3.0)

Model No.		ω_0^2	ω_1^2		ω_0^2	ω_1^2		ω_0^2	ω_1^2
Undistorted		9.2550	16.9844		9.2550	16.9844		9.2550	16.9844
	n= 0.1, q =0.1			n_1 = 0.1, q =0.1			n= 0.1, n_1 =0.15		
	n_1			n			q		
1	0.00	9.4148	17.3971	0.01	9.1020	16.5679	0.00	8.9979	16.3231
2	0.03	9.2333	16.9289	0.05	9.0885	16.5480	0.01	8.9977	16.3223
3	0.05	9.1783	16.7830	0.07	9.0794	16.5251	0.05	8.9961	16.3170
4	0.07	9.1382	16.6651	0.10	9.0751	16.5140	0.10	8.9928	16.3055
5	0.10	9.0751	16.5140	0.15	9.0838	16.5364	0.15	8.9879	16.2886
6	0.12	9.0404	16.4244	0.25	9.1335	16.6664	0.20	8.9813	16.2663
7	0.15	8.9928	16.3055	0.35	9.2073	16.8580	0.30	8.9635	16.2053
8	0.20	8.9217	16.1214	0.45	9.2944	17.0843	0.40	8.9391	16.1218

Table 4.4 Effect of variation in the values of n, n_1 and q on the eigenfrequencies of the nonradial modes of oscillations of rotationally and tidally distorted polytropic model of primary component of binary system (N = 3.0)

Model No.		g_3	g_2	g_1	f	p_1	p_2	p_3
Undistorted		1.8359	2.8334	4.9265	8.2461	15.2827	26.7282	41.4747
n= 0.1, q =0.1								
	n_1							
1	0.00	1.8475	2.8506	4.9654	8.3798	15.6918	27.5022	42.7065
2	0.03	1.8339	2.8291	4.9212	8.2276	15.2223	26.6111	41.2871
3	0.07	1.8261	2.8176	4.8971	8.1435	14.9619	26.1153	40.4962
4	0.12	1.8200	2.8000	4.8742	8.0661	14.7222	25.6585	39.7676
5	0.15	1.8162	2.8000	4.8631	8.0266	14.6003	25.4259	39.3965
6	0.20	1.8114	2.8000	4.8460	7.9681	14.4193	25.0806	38.8461
n_1 = 0.1, q =0.1								
	n							
1	0.01	1.8238	2.8143	4.8898	8.1174	14.8675	25.9296	40.1959
2	0.05	1.8227	2.8126	4.8860	8.1060	14.8453	25.8930	40.1409
3	0.15	1.8224	2.8121	4.8848	8.1021	14.8336	25.8709	40.1060
4	0.25	1.8262	2.8175	4.8972	8.1441	14.9633	26.1179	40.5000
5	0.35	1.8315	2.8260	4.9148	8.2058	15.1546	26.4823	41.0813
6	0.45	1.8383	2.8362	4.9359	8.2788	15.3812	26.9133	41.7685
n= 0.1, n_1 =0.15								
	q							
1	0.00	1.8173	2.8026	4.8643	8.0311	14.6177	25.4614	39.4544
2	0.05	1.8172	2.8024	4.8639	8.0295	14.6116	25.4490	39.4337
3	0.10	1.8169	2.8021	4.8631	8.0266	14.6003	25.4259	39.3965
4	0.20	1.8162	2.8000	4.8605	8.0169	14.5618	25.3476	39.2676
5	0.30	1.8151	2.7990	4.8564	8.0000	14.5022	25.2265	39.0697
6	0.40	1.8135	2.7961	4.8509	7.9814	14.4219	25.0632	38.8025

APPENDIX

A Computational method for determining the eigenfrequencies of nonradial oscillations using Chebyshev polynomial expansions

The eigenvalued boundary value problem which determines the effects of rotation and tidal distortions and which incorporates the effects of Coriolis force besides the effects of gravitational and centrifugal forces on the eigenfrequencies of nonradial modes of oscillations of gaseous spheres is governed by the set of differential equations (4.8) together with the boundary conditions (4.9 – 4.10). This eigenvalued boundary value problem does not yield analytical solutions even for simple density distribution laws. In this appendix we present a method based on the use of Chebyshev polynomial expansions which can be used to solve such problems.

Hurley et al (52) were perhaps the first to use Chebyshev polynomial expansion to obtain the eigenfrequencies of various nonradial modes of oscillations of polytropic models. Later on, the same technique with slight modifications was used by Singh (140), Saxena (133) and Lal (70) to compute the eigenfrequencies of nonradial oscillations of various stellar models. Following this approach the Chebyshev polynomial expansion may be explained as follows:

Chebyshev polynomials are orthogonal polynomials related to the trigonometric functions. A simple way to define a Chebyshev polynomial $T_n(x)$ of order n is given by the relation $T_n(x) = \cos(n\theta)$, where $\theta = \cos^{-1} x$.

In Chebyshev polynomial expansion technique, the unknown functions are represented as linear combinations of Chebyshev polynomials containing a number of unspecified expansion parameters. The substitution of these expansions of unknown functions in the set of differential equations converts the problem of solving the set of linear differential equation into the problem of solving a set of linear simultaneous algebraic equations.

In order to solve the system of differential equations (4.8) with the boundary conditions (4.9 – 4.10), the system of equations together with the boundary conditions are transformed using the substitution $x = (z+1)/2$ $(-1 \le z \le 1)$ so that the range of integration is renormalized from (0, 1) to (-1, 1). The above substitution transforms the set of equations (4.8) to the form

$$\left.\begin{array}{l} \dfrac{d\zeta}{dz}+\dfrac{1}{2}B_1[\zeta+(B_2+\dfrac{1}{\sigma^2}B_3)\eta+\dfrac{1}{\sigma^2}B_3\,\phi]=0, \\ \dfrac{d\eta}{dz}+\dfrac{1}{2}[(E_1\sigma^2+E_2)\zeta+E_3\eta+E_4\,\phi]+\dfrac{d\phi}{dz}=0, \\ \text{and} \\ \dfrac{\mathrm{d}^2\phi}{dz^2}+\dfrac{1}{2}F_1\dfrac{d\phi}{dx}+\dfrac{1}{4}[F_2\zeta+F_3\eta+F_4\,\phi]=0 \end{array}\right\} \tag{4.15}$$

where B_1, B_2, B_3, E_1, E_2 etc are the same functions as defined in (4.8) with x replaced by $(z+1)/2$. Now we assume Chebyshev polynomial expansions for $\dfrac{d\zeta}{dz}, \dfrac{d\eta}{dz}$ and $\dfrac{d^2\phi}{dz^2}$ truncated after 'm' terms as

$$\frac{d\zeta}{dz}=\sum_{j=2}^{m+1}a_j\,T_{j-2},\quad \frac{d\eta}{dz}=\sum_{j=2}^{m+1}b_j\,T_{j-2}\,,\quad \frac{d^2\phi}{dz^2}=\sum_{j=3}^{m+2}d_j\,T_{j-3} \tag{4.16}$$

where a_j, b_j $(j=2,3,...,m+1)$ and d_j $(j=3,4,...,m+2)$ are unknown expansion parameters and $T_j(z)=T_j(\cos\theta)=\cos(j\theta)$.

Integrating the above expression analytically, we get

$$\left.\begin{array}{l} \zeta=a_1+a_2T_1+a_3\dfrac{T_2}{4}+\displaystyle\sum_{j=4}^{m+1}\dfrac{a_j}{2}(\dfrac{T_{j-1}}{j-1}-\dfrac{T_{j-3}}{j-3}) \\ \eta=b_1+b_2T_1+b_3\dfrac{T_2}{4}+\displaystyle\sum_{j=4}^{m+1}\dfrac{b_j}{2}(\dfrac{T_{j-1}}{j-1}-\dfrac{T_{j-3}}{j-3}) \\ \dfrac{d\phi}{dz}=d_2+d_3T_1+d_4\dfrac{T_2}{4}+\displaystyle\sum_{j=5}^{m+2}\dfrac{d_j}{2}(\dfrac{T_{j-2}}{j-2}-\dfrac{T_{j-4}}{j-4}) \\ \phi=d_1+d_2T_1+d_3\dfrac{T_2}{4}+\dfrac{d_4}{24}(T_3-3T_1)+\dfrac{d_5}{48}(T_4-8T_2) \\ \quad +\displaystyle\sum_{j=6}^{m+2}\dfrac{d_j}{4}(\dfrac{T_{j-1}}{(j-1)(j-2)}-\dfrac{2T_{j-3}}{(j-2)(j-4)}+\dfrac{T_{j-5}}{(j-4)(j-5)}) \end{array}\right\} \tag{4.17}$$

where a_1, b_1, d_1 and d_2 are constants of integration.

Substituting above expressions in the system of differential equations (4.15), we get in their place three algebraic equations which contain $a_1, a_2, ..., a_{m+1}$, $b_1, b_2, ..., b_{m+1}$ and

$d_1, d_2, ..., d_{m+1}$, that is, in all $3m+4$ unknown expansion parameters. To evaluate these unknown parameters, the above equations may now be forced to be satisfied at m collocation points in the range of integration. Zeros of Chebyshev polynomials of order m are often preferred as the collocation points. On satisfying the above equations at m collocation points in the range of integration we get $3m$ linear algebraic equations in $3m+4$ unknown expansion parameters. Now, if we add to these $3m$ equations, the four linear equations which can be obtained from the four boundary conditions (4.9 – 4.10) by converting them to Chebyshev polynomial expansion forms, we get a set of $3m+4$ linear homogeneous algebraic equations in as many unknowns.

In order to determine the eigenfrequencies an approach similar to one followed by Singh (140) may now be used. In this approach we withhold one of the boundary conditions and replace it by a normalization condition which assigns a non – zero value to one of the unknown eigenfunction ζ or η at the center or the surface of the model. The withhold boundary condition is now used as a discriminant condition for determining the value of the nondimensional eigenfrequency σ. Trials with different values of σ are made till a value of σ is found which satisfies the withheld condition to the desired accuracy. (In our subsequent investigations in this thesis, we have used $\zeta=1$ at the center as normalization condition and the boundary condition (4.10a) at the surface as the discriminant condition. The set of linear algebraic simultaneous equations have been solved using Gaussian elimination method with pivotal condensation).

CHAPTER – V

EFFECT OF CORIOLIS FORCE ON THE EQUILIBRIUM STRUCTURES OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED STELLAR MODELS

Most of the stars in binary systems are known to be rotating about their axes as well as revolving around their common center of mass. Some of the stars in binary systems are expected to be rotating differentially. This differential rotation is likely to influence the inner structure and the equilibrium configurations of such stars. It is also expected that the equilibrium structures of differentially rotating stars in binary systems will be influenced by the combined effects of differential rotation as well as the tidal forces of the companion star.

In the present chapter we investigate the problem of determining the effects of Coriolis force on the equilibrium structures of differentially rotating stellar models which are the primary components of a binary system. In their studies, Mohan et al (104, 106) did not account for the effects of Coriolis force. It may be of interest to note that even though Coriolis force does not arise in case of a rotating star having solid body rotations (the angular velocity at each point of the rotating body is same): it does arise in a star having differential rotation (angular velocity at each point of the rotating body is different). Moreover even in case of synchronous binaries whereas in case of solid body rotation the effect of Coriolis force does not appear explicitly in expression of its Roche equipotential, it does appear in case of differential rotation. It may be also of our interest to determine how the inclusion of Coriolis force affects inner structure of stars having differential rotation. In the present chapter we have essentially updated the work of earlier authors (Mohan et al (104, 106)) by including the effects of Coriolis force while computing the equilibrium structures of differentially rotating and tidally distorted stellar models in general and on polytropic models in particular.

In section 5.1 we present in brief the differential rotation laws commonly used in literature. The law of differential rotation adopted by us for present study is presented in section 5.2. In section 5.3 we obtain expression for Roche equipotential of a differentially rotating and tidally distorted stellar model which is primary component in a binary system. This expression takes into account the effect of Coriolis force in addition to the effects of gravitational and centrifugal forces. In section 5.4 we use the approach presented in section 1.4 of chapter I to obtain the system of differential equations which govern the equilibrium structure of a differentially rotating and tidally distorted stellar model that incorporate the effects of Coriolis force besides the centrifugal and gravitational forces. The methodology developed in this section has been next used in section 5.5 to compute the equilibrium structures of differentially rotating and tidally distorted polytropic models, considered as primary components of a binary system. Numerical computations have been next performed in section 5.6 to obtain the inner structure, the shapes and other relevant physical parameters of certain differentially rotating and tidally distorted polytropic models of stars of polytropic indices 1.5 and 3.0. The numerical results thus obtained have been compared with the corresponding results in which the effects of Coriolis force are not considered. Certain conclusions based on this study are finally drawn in section 5.7.

5.1 LAWS OF DIFFERENTIAL ROTATION

By differential rotation we mean rotation of a gaseous sphere in which all the fluid elements of the sphere do not rotate with the same angular velocity. Different authors have used different laws of differential rotation to account for some of the observed features of differentially rotating stars. Theoretically the general form of a law of differential rotation for a star rotating with angular velocity Ω about an axis passing through its centre should be of the type $\Omega = \Omega(s, z)$. In other words the angular velocity of rotation Ω should be a function of both the distance s of the element from the axis of rotation and its latitude z. In fact some of the authors such as Von Zeipel (159), Hoiland (49), Mohan et al (105) etc have used such types of general laws in their studies. However, according to Tassoul (151) it is perhaps not possible to build a chemically homogenous stellar model in radiative equilibrium with a general law of rotation of the type $\Omega = \Omega(s, z)$. According to him since in the zones of efficient convection the transport of energy is not by radiation so in such a case arguments of Von Zeipel do not apply and therefore, in practice for such a differentially rotating star to be in equilibrium, laws of differential rotation of the form $\Omega = \Omega(s)$ should only be used.

For a differentially rotating stellar model to be stable against local perturbations, the assumed law of differential rotation should satisfy stability criteria against local perturbations. Stoeckly (148) obtained one such criterion. According to this criterion a model rotating differentially according to the law $\Omega = \Omega(s)$ is stable if

$$\frac{d}{d\,s}[s^2\,\Omega(s)] > 0 \tag{5.1}$$

for all s from the centre to the surface.

In the category of laws of differential rotation of the type $\Omega(s)$, Faye as early as 1865 assumed a law of differential rotation

$$\Omega = b_1 + b_2\, s^2 \tag{5.2}$$

to account for differential rotation of the Sun's surface (where Ω is the angular velocity of rotation of a fluid element at a distance s from the axis of rotation and b_1, b_2 are suitably chosen constants). In 1965 Stoeckly constructed axisymmetric models of differentially rotating polytropes of index 1.5 assuming a law of a differential rotation of the type

$$\Omega(s) = \Omega_c\, e^{-\left(a\, s^2/\xi_e^2\right)} \tag{5.3}$$

where s is measured from the axis of rotation, Ω_c denotes the angular velocity of rotation of an element on the axis of rotation and ξ_e the equatorial radius of the polytropic model. Also a is a suitably chosen constant. Ireland (53) calculated the results for gravity – darkening and limb – darkening in a rapidly rotating Roche model of a star subject to the non-uniform rotation assuming a general law of differential rotation $\Omega = \Omega(s)$, where Ω is the angular velocity of the star and s the distance of a fluid element from axis of rotation. Clement (15) proposed a law of differential rotation in which the angular velocity $\Omega(s)$ of a fluid element is given by

$$\Omega(s) = \left(\sum_{i=1}^{3} a_i\, e^{-c_i\, s^2} \right)^{1/2} \tag{5.4}$$

where s is a suitably modified nondimensional cylindrical coordinate and a_i, c_i constants. Komatsu et al. (61) computed equilibrium structures of differentially rotating relativistic polytropes of indices 0.5 and 1.5 using a rotation law determined by specifying the angular momentum $J(\Omega)$. Although, theoretical choice of $J(\Omega)$ is arbitrary, stability criteria impose some constraints on its selection and thus

$$J(\Omega) = A^2\left(\Omega_c - \Omega\right) \tag{5.5}$$

where A is a positive constant and Ω_c the angular velocity at the centre of the coordinate system (Ω_c depends implicitly on the value of A which is called rotation parameter). For the Newtonian case, this leads to a rotation law of the type

$$\Omega = \frac{\Omega_c A^2}{A^2 + s^2} \tag{5.6}$$

where $s = r\sin\theta$. When $A \to \infty$, Ω approaches rigid rotation whereas when $A \to 0$, it becomes a J – constant rotation (that is, the specifying angular momentum is constant in space). Woodard (161) considered a law of differential rotation of the type

$$\Omega(x) = B_0 + B_1 x^2 + B_2 x^4, \tag{5.7}$$

where Ω is an even function of latitude x.

While studying the effects of rotation and tidal distortions on the equilibrium structures and small adiabatic oscillations of stellar models Mohan et al (104, 106) used differential rotation law of the type

$$\Omega^2 = b_0^* + b_1^* s^2 + b_2^* s^4 \tag{5.8}$$

where s is a nondimensional measure of the distance of a fluid element from the axis of rotation passing through its center and b_0^*, b_1^*, b_2^* are suitably chosen arbitrary constants in units of Ω^2. This law may be regarded as a Taylor series expansion of a general law of differential rotation of the form $\Omega^2 = f(s^2)$ in which terms up to second order of smallness in Taylor series expansion of Ω^2 are retained. The law includes Faye's law (5.2) as a special case and ensures symmetry of angular velocity of rotation about the axis of rotation.

The law of differential rotation (5.4) used by Clement (15) assumes that velocity of rotation to be a function of s^2, the square of the distance of the fluid element from the axis of rotation.(velocity is assumed to be function of s^2 and not s so as to ensure symmetry about axis of rotation for stability). On writing right hand side of (5.4) in a series form and retaining terms up to s^4 it reduces to (5.8) with constants b_0^*, b_1^* and b_2^* as

$$\left.\begin{aligned} b_0^* &= a_1 + a_2 + a_3 \\ b_1^* &= -(a_1 c_1 + a_2 c_2 + a_3 c_3) \\ b_2^* &= \frac{1}{2}(a_1 {c_1}^2 + a_2 {c_2}^2 + a_3 {c_3}^2) \end{aligned}\right\} \tag{5.9}$$

This type of law of differential rotation may be considered to be a more general form of Faye's law of the type

$$\Omega = b_0 + b_1 s^2 \tag{5.10}$$

with

$$b_0^2 = b_0^*,\ 2b_0\, b_1 = b_1^* \text{ and } b_1^2 = b_2^*$$

Thus essentially Faye's simple law of differential rotation of the type (5.10) may be regarded as a special case of Clement's law (5.4).

5.2 THE LAW OF DIFFERENTIAL ROTATION USED IN PRESENT STUDY

Keeping above in view we have preferred to use a law of differential rotation of the type (5.10) in our present study. It can be used to not only generate a variety of differential rotations commonly observed in stars, but is also in a form in which it can be conveniently incorporated in the mathematical formulations which we use to compute the effects of Coriolis force on the structures and periods of oscillations of rotating stars and stars in binary systems. The law of differential rotation of the type (5.10) assures that there is symmetry about the axis of rotation (angular velocity varies with s^2 but is same on the surface of a cylinder of radius s whose axis is the axis of rotation). According to this law the angular velocity along the axis of rotation is constant and equal to b_0.

Various models of differentially rotating stars can be generated using law of differential rotation (5.10) by giving different values to b_0 and b_1. For studying the effect of Coriolis force on the equilibrium structures of differentially rotating stars and stars in binary systems we have chosen four models. These models are shown in Table 5.1. The local stability of these models has also been checked using Stoeckly's (148) criteria.

5.3 ROCHE EQUIPOTENTIALS OF A DIFFERENTIALY ROTATING AND TIDALLY DISTORTED STELLAR MODEL INCORPORATING THE EFFECTS OF CORIOLIS FORCE

A differentially rotating star is a star rotating about an axis passing through its center such that angular velocity of rotation at different points in the star is different. In our present study we assume the primary component (which is of prime interest to us) of a binary system to be rotating differentially according to law (5.10).

Following the approach developed in section 2.1 of chapter II, the expression for potential at a point P inside the differentially rotating primary component of a binary system may be expressed in cartesian form as

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_2} + \frac{\Omega^2}{2}[(x-d_1)^2 + y^2] + (\Omega\Omega_1 - \Omega^2)(x^2 + y^2 - xd_1) \tag{5.11}$$

where Ω_1 is the angular velocity of rotation of the primary component rotating differentially according to the law of differential rotation (5.10). Following Kopal (63) above expression can be written in nondimensional form as

$$\psi^* = \frac{1}{r^*} + \frac{q}{r_2^*} + \frac{\Omega^{*2}}{2}[r^{*2}(1-\nu^2) + (\frac{q}{1+q})^2 - \frac{2\lambda r^* q}{1+q}] + (\Omega_1^*\Omega^* - \Omega^{*2})[r^{*2}(1-\nu^2) - \frac{\lambda r^* q}{1+q}] \tag{5.12}$$

where

$$\psi^* = \frac{D\psi}{GM_0},\ r^* = \frac{r}{D},\ r_2^* = \frac{r_2}{D},\ q = \frac{M_1}{M_0},\ \Omega^{*2} = \frac{D^3\Omega^2}{GM_0} \text{ and } \Omega_1^*\Omega^* = \frac{D^3(\Omega_1\Omega)}{GM_0}$$

For differential rotation we assume a law of differential rotation of type (5.10). Substituting the value of Ω_1 from (5.10) in (5.12), we get after some simplifications

$$\psi^{**} = \frac{1}{r^*} + q[\frac{1}{(1-2\lambda r^* + r^{*2})^{1/2}} - \frac{(b_0 + b_1 s^2)}{(2n)^{1/2}}\lambda r^*] + n r^{*2}(1-\nu^2)[(b_0 + b_1 s^2)(2/n)^{1/2} - 1] \tag{5.13}$$

where $\psi^{**} = \psi^* - \dfrac{q^2}{2(1+q)}$ and $2n = \Omega^{*2}$.

In (5.13), $s = [r^{*2}(1-\nu^2)]^{1/2} = r^*\sin\theta$ is the nondimensional measure of distance of a particle at point P from the axis of rotation of the primary component of the binary system in units of R_e (where R_e is the equatorial radius of the model under investigation). Also, as earlier $\lambda = \sin\theta\cos\varphi$, $\mu = \sin\theta\sin\varphi$, $\nu = \cos\theta$ (r, θ, φ being the polar spherical coordinates of the point *P*, θ being measured from the axis of rotation). Thus, (5.13) is the general expression for potential at point P of a differentially rotating primary component of a binary system. This expression incorporates the effect of Coriolis force in addition to the centrifugal and gravitational forces.

On substituting $b_0 = \Omega_1^* = (2n_1)^{1/2}$ and $b_1 = 0$ in (5.13), we get the expression of Roche equipotential similar to the expression obtained for solid body rotation (equation (2.11) of chapter II).

In case of differential rotation of the primary, the Coriolis force will be generated even in case of synchronous binaries as well as pure rotation due to the difference in the angular velocities at the center of the primary and at other points inside the primary.

In case of a synchronous binary, if we assume the angular velocity of revolution Ω to be same as the angular velocity of rotation at the center of the primary, (i.e., $b_0 = \Omega = (2n)^{1/2}$), (5.13) after some simplifications yields

$$\psi^{**} = \frac{1}{r^*} + q[\frac{1}{(1-2\lambda r^* + r^{*2})^{1/2}} - (1 + \frac{b_1 r^{*2}(1-\nu^2)}{(2n)^{1/2}})\lambda r^*] + n r^{*2}(1-\nu^2)[1 + \frac{2b_1 r^{*2}(1-\nu^2)}{(2n)^{1/2}}] \tag{5.14}$$

On substituting $b_1 = 0$ in above expression we get the expression of Roche equipotential for solid body rotation (equation (2.13) of chapter II) in which Coriolis force does not appear explicitly.

In case of pure rotation of a star there is no secondary component and hence no angular velocity of revolution. Putting $q = 0$ and $\Omega = b_0$ (angular velocity at the center of the star) in (5.13) we get the expression for Roche equipotential at point P inside a differentially rotating star as

$$\psi^* = \frac{1}{r^*} + \frac{1}{2} r^{*2}(1-\nu^2)[b_0^2 + 2b_0 b_1 r^{*2}(1-\nu^2)] \tag{5.15}$$

This incorporates the effects of Coriolis force. On substituting $b_1 = 0$ and $b_0^2 = 2n$ in (5.15), we get expression of Roche equipotential for a purely rotating star having solid body rotation (equation (2.14) of chapter II) in which terms due to the effect of Coriolis force disappear.

In the absence of Coriolis force there will be no fourth term on the right hand side of (5.11) and angular velocity Ω will now be governed by differential rotation law (5.10). In this case expression (5.11) can be solved to yield total potential at point P (in a manner discussed earlier by Kopal (63)) as

$$\psi^{**} = \frac{1}{r^*} + q[\frac{1}{(1-2\lambda r^* + r^{*2})^{1/2}} - \frac{(b_0 + b_1 s^2)^2}{1+q} \lambda r^*] + \frac{1}{2}(b_0 + b_1 s^2)^2 r^{*2}(1-\nu^2) \tag{5.16}$$

On substituting $b_0 = (2n)^{1/2}$, $b_1 = 0$ and $n = (q+1)/2$ in (5.16), we get an expression similar to the one for solid body rotations (equation (2.13) of chapter II) which does not account for the effects of Coriolis force.

In the absence of Coriolis force, the total potential at point P for purely rotating stars can be obtained by simplifying first and third terms of expressions (5.11) to yield

$$\psi^* = \frac{1}{r^*} + \frac{1}{2}(b_0 + b_1 s^2)^2 r^{*2}(1-\nu^2) \tag{5.17}$$

which can also be obtained directly from (5.16) by putting $q=0$. On substituting $b_0 = (2n)^{1/2}$, $b_1 = 0$ in (5.17), it reduces to the expression ((2.14) of chapter II) obtained earlier for purely rotating stars having solid body rotations when the effects of Coriolis force are not considered.

Thus inclusion of Coriolis force in the Roche equipotential of a differentially rotating primary component of a binary system, expressions for Roche equipotential get modified in all the three cases of nonsynchronous binaries, synchronous binaries as well as purely rotating stars, whereas in the case of solid body rotation the expression for Roche equipotential gets modified only in case of nonsynchronous binaries. In case of synchronous binaries and purely rotating stars there is no change in the expressions.

We define a nondimensional variable r_0 by the relation

$$r_0 = \frac{1}{\psi^{**} - q} \tag{5.18}$$

Following the approach adopted in section 1.3 of chapter I, the variables (r, θ, ϕ) on the surfaces of the modified Roche equipotentials (5.13) can be shown to be connected through the relation

$$\begin{aligned} r = r_0 [1 &+ \lambda q h r_0^2 + a_0 r_0^3 + (qP_3 + 2\lambda^2 q^2 h^2) r_0^4 + (q P_4 + 5a_0 \lambda q h) r_0^5 + (qP_5 + 3a_0^{\,2} \\ &+ 6\lambda q^2 h P_3) r_0^6 + (qP_6 + 7a_0 q P_3 + 7\lambda q^2 h P_4) r_0^7 + (qP_7 + 8a_0 q P_4 + 8\lambda q^2 h P_5 \\ &+ 4q^2 P_3^2) r_0^8 + (q P_8 + 9a_0 q P_5 + 9\lambda q^2 h P_6 + 9q^2 P_3 P_4) r_0^9 + (q P_9 + 10 a_0 q P_6 \\ &+ 10\lambda q^2 h P_7 + 5q^2\{P_4^2 + 2P_3 P_5\}) r_0^{10} + \cdots] \end{aligned} \tag{5.19}$$

where $a_0 = q P_2 + [(b_0 + b_1 s^2)(2n)^{1/2} - n](1-\nu^2)$, $h = 1 - [(b_0 + b_1 s^2)/(2n)^{1/2}]$ and $P_j = P_j(\lambda)$ denote Legendre polynomials. Relation (5.19) incorporates the effect of Coriolis force and can be used to obtain the shapes of various Roche equipotential surfaces ψ^{**} = constant.

Again following the approach of section 1.3 of chapter I , explicit expressions for volume V_ψ, surface area S_ψ and value of radial distance r_ψ of a point on the equipotential surface ψ^{**} = constant inside the differentially rotating and tidally distorted primary component can be shown to be

$$V_\psi = \frac{4\pi}{3} D^3 r_0^3 [1 + 2c\, r_0^3 + 3q^2 h^2 r_0^4 + (\frac{12}{5} q^2 + \frac{32}{5} c^2 + \frac{8}{5} cq) r_0^6 + \frac{15}{7} q^2 r_0^8 + 2q^2 r_0^{10} + ...] \quad (5.20)$$

$$S_\psi = 4\pi D^2 r_0^2 [1 + \frac{4}{3} c\, r_0^3 + \frac{5}{3} q^2 h^2 r_0^4 + (\frac{7}{5} q^2 + \frac{56}{15} c^2 + \frac{14}{15} cq) r_0^6 + \frac{9}{7} q^2 r_0^8 + \frac{11}{9} q^2 r_0^{10} + ...] \quad (5.21)$$

$$r_\psi = D\, r_0 [1 + \frac{2}{3} c r_0^3 + q^2 h^2 r_0^4 + (\frac{4}{5} q^2 + \frac{76}{45} c^2 + \frac{8}{15} cq) r_0^6 + \frac{5}{7} q^2 r_0^8 + \frac{2}{3} q^2 r_0^{10} + ...] \quad (5.22)$$

Here

$$h = 1 - [(b_0 + b_1 s^2)/(2n)^{1/2}] \text{ and } c = [(b_0 + b_1 s^2)(2n)^{1/2} - n]. \quad (5.23)$$

Inverting (5.22) we have

$$r_0 = r_\psi^* [1 - \frac{2}{3} c\, r_\psi^{*3} - q^2 h^2 r_\psi^{*4} - (\frac{4}{5} q^2 - \frac{4}{45} c^2 + \frac{8}{15} cq) r_\psi^{*6} - \frac{5}{7} q^2 r_\psi^{*8} - \frac{2}{3} q^2 r_\psi^{*10} + ...] \quad (5.24)$$

where $r_\psi^* = r_\psi / D$, r_ψ^* being the nondimensional form of r_ψ. Similarly using equations (1.31) and (1.32) of chapter I, explicit expressions for the values of $\bar{g}$ and $\overline{g^{-1}}$, in case of differential rotation which take into account the effects of Coriolis force as well, at points inside the primary are given as

$$\bar{g} = \frac{GM_\psi}{D^2 r_0^2} [1 - \frac{8}{3} c r_0^3 - 2q^2 h^2 r_0^4 - (2q^2 + \frac{28}{9} c^2 + \frac{4}{3} cq) r_0^6 - \frac{15}{7} q^2 r_0^8 - \frac{7}{3} q^2 r_0^{10} + ...] \quad (5.25)$$

$$\overline{g^{-1}} = \frac{D^2 r_0^2}{GM_\psi} [1 + \frac{8}{3} c r_0^3 + 5q^2 h^2 r_0^4 + (\frac{26}{5} q^2 + \frac{524}{45} c^2 + \frac{52}{15} cq) r_0^6 + \frac{40}{7} q^2 r_0^8 + \frac{19}{3} q^2 r_0^{10} + ...]$$

(5.26)

The angular velocity of rotation in case of a differentially rotating star is different at different points. The polar angular velocity Ω_p and the equatorial angular velocity Ω_e of such a star with the law of differential rotation of the type (5.10) is given as

$$\Omega_p = b_0 \text{ and } \Omega_e = b_0 + b_1 R_e^2 \quad (5.27)$$

Here R_e (in units of $(GM_0)/R$) is the equatorial radius. The values of polar radius and equatorial radius can also be computed using

$$R_p = r_{0s} D \quad (5.28)$$

$$R_e = r_{0s} D[1 + qhr_{0s}^2 + a_0 r_{0s}^3 + (q + 2q^2 h^2) r_{0s}^4 + (q + 5a_0 qh) r_{0s}^5 + (q + 3a_0{}^2 + 6q^2 h) r_{0s}^6$$
$$+ (q + 7 a_0 q + 7q^2 h) r_{0s}^7 + (q + 8 a_0 q + 8q^2 h + 4q^2) r_{0s}^8 \quad (5.29)$$
$$+ (q + 9 a_0 q + 9q^2 h + 9 q^2) r_{0s}^9 + (q + 10 a_0 q + 10q^2 h + 15q^2 r_{0s}^{10} + \cdots]$$

where

$$h = 1 - [(b_0 + b_1 s^2) / (2n)^{1/2}] \text{ and } c = [(b_0 + b_1 s^2)(2n)^{1/2} - n].$$

In case effects of Coriolis force are not considered, the corresponding expressions for the various physical parameters of the star can be obtained by replacing h and c in the above expressions by $h = 1 - [(b_0 + b_1 s^2)^2 / (1 + q)]$ and $c = (b_0 + b_1 s^2)^2 / 2$ respectively.

5.4 EQUILIBRIUM STRUCTURES OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED STELLAR MODELS INCORPORATING THE EFFECTS OF CORIOLIS FORCE

Using the expressions of Roche equipotential and other parameters obtained in section 5.3 and following the approach as presented in section 1.4 of chapter I, the values of various distortions parameters u, v, w, f_P and f_T on an equipotential surface of a differentially rotating and tidally distorted stellar model, which incorporates the effects of Coriolis force besides the gravitational and centrifugal forces, now become

$$u = [1 - \frac{1}{3} q^2 h^2 r_\psi^{*4} - (\frac{1}{5} q^2 + \frac{4}{45} c^2 + \frac{2}{15} c\, q) r_\psi^{*6} - \frac{1}{7} q^2 r_\psi^{*8} - \frac{1}{9} q^2 r_\psi^{*10} - ...] \quad (5.30)$$

$$v = [1 - \frac{4}{3} c r_\psi^{*3} - (\frac{2}{5} q^2 + \frac{8}{45} c^2 + \frac{4}{15} cq) r_\psi^{*6} - \frac{5}{7} q^2 r_\psi^{*8} - q^2 r_\psi^{*10} - ...] \quad (5.31)$$

$$w = [1 + \frac{4}{3} c\, r_\psi^{*3} + 3q^2 h^2 r_\psi^{*4} + (\frac{18}{5} q^2 + \frac{152}{45} c^2 + \frac{12}{5} c\, q) r_\psi^{*6} + \frac{30}{7} q^2 r_\psi^{*8} + 5 q^2 r_\psi^{*10} + ...] \quad (5.32)$$

$$f_P = [1 - \frac{4}{3} c\, r_\psi^{*3} - \frac{8}{3} q^2 h^2 r_\psi^{*4} - (\frac{17}{5} q^2 + \frac{68}{45} c^2 + \frac{34}{15} cq) r_\psi^{*6} - \frac{29}{7} q^2 r_\psi^{*8} - \frac{44}{9} q^2 r_\psi^{*10} - ...] \quad (5.33)$$

$$f_T = [1 - \frac{7}{3} q^2 h^2 r_\psi^{*4} - (\frac{14}{5} q^2 + \frac{56}{45} c^2 + \frac{28}{15} cq)\, r_\psi^{*6} - \frac{23}{7} q^2 r_\psi^{*8} - \frac{34}{9} q^2 r_\psi^{*10} - ...] \quad (5.34)$$

where $h = 1 - [(b_0 + b_1 s^2) / (2n)^{1/2}]$, $c = [(b_0 + b_1 s^2)(2n)^{1/2} - n]$ and $r_\psi^* = r_\psi / D$ is the nondimensional form of r_ψ.

For computational work, we find it convenient to work with r_0 in place of r_ψ or M_ψ as the independent variable. Expression for r_0 is given by (5.18) and is connected with variable r_ψ through the relation (5.24).

Using the above relations in equations (1.27) of chapter I, the system of differential equations governing the equilibrium structure of a differentially rotating and tidally distorted model which incorporates the effects of Coriolis force besides the gravitational and the centrifugal forces can be expressed as

$$\frac{dM_\psi}{dr_0} = 4\pi D^3 \rho_\psi r_0^{\ 2} f_1 \tag{5.35a}$$

$$\frac{dP_\psi}{dr_0} = -\frac{GM_\psi}{Dr_0^{\ 2}} \rho_\psi f_2 \tag{5.35b}$$

$$\frac{dL_\psi}{dr_0} = 4\pi\,\varepsilon\, D^3 \rho_\psi r_0^{\ 2} f_1 \tag{5.35c}$$

$$\frac{dT_\psi}{dr_0} = -\frac{3\kappa L_\psi\,\rho_\psi}{16\pi Dac T_\psi^{\ 3} r_0^{\ 2}} f_3 \tag{5.35d}$$

with

$$r_0 = \frac{1}{\psi^{**} - q}$$

Here f_1, f_2 and f_3 are certain functions of distortion parameters b_0, b_1, n, q and r_0 that incorporate the effects of Coriolis force in addition to centrifugal and gravitational forces on the equilibrium structure equations of differentially rotating and tidally distorted stellar model. Explicit expressions for these distortion parameters when terms up to second order of smallness in b_0, b_1, n, q and up to r_0^{10} in r_0 are retained, are

$$f_1 = [1 + 4cr_0^{\ 3} + 7q^2h^2r_0^{\ 4} + (\frac{36}{5}q^2 + \frac{96}{5}c^2 + \frac{24}{5}c\,q)r_0^{\ 6} + \frac{55}{7}q^2r_0^{\ 8} + \frac{26}{3}q^2r_0^{\ 10} + ...] \tag{5.36a}$$

$$f_2 = [1 + \frac{1}{3}q^2\,h^2r_0^{\ 4} + (\frac{3}{5}q^2 + \frac{4}{15}c^2 + \frac{2}{5}cq)r_0^{\ 6} + \frac{6}{7}q^2r_0^{\ 8} + \frac{10}{9}q^2r_0^{\ 10} + ...] \tag{5.36b}$$

$$f_3 = [1 + \frac{4}{3}c\,r_0^{\ 3} + \frac{2}{3}q^2h^2r_0^{\ 4} + (\frac{6}{5}q^2 + \frac{224}{45}c^2 + \frac{4}{5}cq)r_0^{\ 6} + \frac{12}{7}q^2r_0^{\ 8} + \frac{20}{9}q^2r_0^{\ 10} + ...] \tag{5.36c}$$

and

$$r_0 = r_\psi^* \, D \, [1 - \frac{2}{3} c \, r_\psi^{*3} - q^2 h^2 \, r_\psi^{*4} - (\frac{4}{5} q^2 - \frac{4}{45} c^2 + \frac{8}{15} cq) \, r_\psi^{*6} - \frac{5}{7} q^2 r_\psi^{*8} - \frac{2}{3} q^2 r_\psi^{*10} - ...] \qquad (5.36d)$$

Here r_ψ^* is the nondimensional value of the radius of topologically equivalent spherical surface. Effects of Coriolis force appear in these expressions through c and h. The boundary conditions given in equation (1.28) of chapter I now become

$$M_\psi = 0, \; L_\psi = 0 \qquad (5.37a)$$

at the center $r_0 = 0$

and

$$\begin{aligned} &M_\psi = M_0, \;\; L_\psi = L_{\psi s} \\ &P_\psi = 0 \; or \; P_{\psi s}, \;\; T_\psi = 0 \; or \; T_{\psi s} \end{aligned} \qquad (5.37b)$$

at the free surface $r_0 = r_{0s}$.

In the above expression, M_0 is the total mass of the star and $L_{\psi s}$, $P_{\psi s}$, $T_{\psi s}$ are the values of L_ψ, P_ψ, T_ψ respectively, on the outermost equipotential surface, ψ^{**} = constant.

At the free surface of the star, $r_0 = r_{0s}$, where

$$r_{0s} = \frac{1}{\psi_s^{**} - q} \qquad (5.38)$$

ψ_s^{**} being the nondimensional value of the total potential ψ^{**} on the outermost equipotential surface of the rotationally and tidally distorted stellar model

In the absence of Coriolis force, expression for the various parameters can be obtained by writing in above expressions h and c as $h = 1 - [(b_0 + b_1 s^2)^2 / (1+q)]$ and $c = (b_0 + b_1 s^2)^2 / 2$ respectively.

5.5 EQUILIBRIUM STRUCTURES OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

Following the approach as given in section 3.4 of chapter III, the equations (5.35a – 5.35b) can be combined to obtain the equilibrium structure of a differentially rotating and tidally distorted stellar model as

$$\frac{1}{r_0^{\,2}} \frac{d}{dr_0} (\frac{r_0^2}{\rho_\psi \, f_2} \frac{dP_\psi}{dr_0}) = -4 \pi G R^2 \rho_\psi f_1 \qquad (5.39)$$

On substituting the values of P_ψ and ρ_ψ from equation (3.25) of chapter III, the differential equation governing the equilibrium structure of a differentially rotating and tidally distorted polytropic model may be written in nondimensional form as

$$\frac{d}{dr_0}[A(b_0,b_1,n,q,r_0)\frac{d\theta_\psi}{dr_0}] = -\frac{\xi_u^2}{K^2}\theta_\psi^N r_0^{\,2} B\,(b_0,b_1,n,q,r_0) \tag{5.40}$$

where

$$A(r_0,\,n\,,q) = \frac{r_0^2}{f_2} = r_0^{\,2}[1-\frac{1}{3}q^2h^2r_0^4 - (\frac{3}{5}q^2+\frac{4}{15}c^2+\frac{2}{5}cq\,)r_0^6 - \frac{6}{7}q^2r_0^8 - \frac{10}{9}q^2r_0^{10} + ...]$$

$$B\,(r_0,\,n,\,q) = f_1 = [1+4c\,r_0^{\,3}+7q^2h^2r_0^{\,4}+(\frac{36}{5}q^2+\frac{96}{5}c^2+\frac{24}{5}cq+)\,r_0^{\,6}$$

$$+\frac{55}{7}q^2\;r_0^{\,8}+\frac{26}{3}q^2r_0^{10}+...]$$

with

$r_0 = 1/(\psi^{**}-q)$, $h = 1-[(b_0+b_1s^2)/(2n)^{1/2}]$ and $c = [(b_0+b_1s^2)(2n)^{1/2}-n]$.

In above expressions terms up to second order of smallness in b_0, b_1, n, q and up to r_0^{10} in r_0 are retained. As earlier, the dimensionless constant K is the ratio of the undistorted radius R_ψ of the primary to the separation D between the centers of the primary and secondary star. In fact

$$\frac{D}{\ell} = \frac{D\,\xi_u}{\ell\,\xi_u} = \frac{D}{R_\psi}\xi_u = \frac{1}{K}\xi_u \tag{5.41}$$

where ξ_u is the value of ξ at the outermost surface of the undistorted polytropic model. The quantity ℓ is of the dimension of length. The boundary conditions which equation (5.40) has to satisfy are

$$\theta_\psi = 1,\ \frac{d\theta_\psi}{dr_0} = 0,\ \text{at the center } r_0 = 0 \tag{5.42a}$$

and

$$\theta_\psi = 0,\ \text{at the surface } r_0 = r_{0s}. \tag{5.42b}$$

where r_{0s} is the value of r_0 at surface.

In the absence of Coriolis force, above expression can be obtained by replacing h and c by $h = 1-[(b_0 + b_1 s^2)^2 / (1+q)]$ and $c = (b_0 + b_1 s^2)^2 / 2$ respectively.

Equation (5.40) subject to the boundary conditions (5.42) determines the equilibrium structure of a differentially rotating and tidally distorted polytropic model which accounts for the effect of Coriolis force in addition to centrifugal and gravitational forces on its equipotential surfaces.

5.6 NUMERICAL COMPUTATIONS

To obtain the inner structures, the shapes, the volumes and the surface areas of differentially rotating and tidally distorted polytropic models, equation (5.40) has to be integrated numerically subject to the boundary conditions (5.42) for the specified values of the parameters N, ξ_u, n, q and K which denote respectively the polytropic index, the radius of the undistorted polytropic model, the nondimensional measure of angular velocity of rotation, the ratio of the mass of the secondary component to the mass of the primary and the ratio of the undistorted radius of the primary to the distance between the centers of the primary and secondary. For a polytropic model distorted by rotational forces alone $K = 1$. In the case of a polytropic model which is the primary component of a binary system the value of K has to be chosen such that the outermost surface of the primary component lies well within the Roche lobe otherwise the two components of the binary will coalesce (cf. Kopal (63), page 11).

For obtaining the numerical solutions, equation (5.40) has been integrated numerically as earlier using fourth-order Runge-Kutta method for the specified values of the input parameters. A series solution similar to the one available for undistorted polytropic models (see Chandrasekhar (11) page 95) was developed to start integrations at points near the centre. This series solution in the present case is given by

$$\theta_\psi = 1 - \frac{\xi_u^2}{6} r_0^2 + \frac{N\xi_u^4}{120} r_0^4 - \frac{2c\,\xi_u^2}{15} r_0^5 - [\frac{N(8n-5)\,\xi_u^6}{15120} + [\frac{q^2 h^2}{6}(1 - \frac{\xi_u^2}{9})\,\xi_u^2]\; r_0^6 + [\frac{cN\xi_u^4}{70} - \frac{q^2 h^2\, \xi_u^2}{504}] r_0^7 + ... \qquad (5.43)$$

where $h = 1-[(b_0 + b_1 s^2)/(2n)^{1/2}]$ and $c = [(b_0 + b_1 s^2)(2n)^{1/2} - n]$

Taking the starting values from this series solution at $r_0 = 0.005$, the numerical integration of equation (5.40) was then carried outward using Runge-Kutta method of order four with a step length of 0.005. Numerical integration was continued till θ_ψ first became zero.

Once we obtain r_{0s}, the value of r_0 where θ_ψ first became zero, relation (5.19) can be used to determine its outer shape by replacing r_0 by r_{0s} and writing $D = \ell\,\xi_u / K$. Similarly we can determine the volumes and surface areas of polytropic models of star by replacing r_0 by r_{0s} and writing $D = \ell\,\xi_u / K$ in relations (5.20) and (5.21) respectively. The value of the parameter K has been taken as one for the rotationally distorted model and 0.5 for rotationally and tidally distorted models. (The chosen value of K provides the outer-most surface of the model well within Roche lobe for each considered case).

Numerical results obtained for different values of the input parameters are presented in Tables 5.2 to 5.4. In Table 5.2 we present the obtained values of θ_ψ, P_ψ and ρ_ψ for various types of distorted polytropic models of indices 1.5 and 3.0. The values of volumes and surface areas obtained for some differentially rotating polytropic models are next presented in Tables 5.3.1 – 5.3.6. For comparison we have also present in these tables the corresponding values when the effects of Coriolis force are not considered.

In Table 5.4 we have compared our results in which the effect of Coriolis force has been considered with the corresponding results obtained by using the law (5.8) of differential rotation and methodology earlier used by Mohan et al (104) in which the effect of Coriolis force has not been considered. The values of various parameters used by Mohan et al (104) in the present case becomes, Model 1 $b_0^* = 0.01$, $b_1^* = 0.002$ and $b_2^* = 0.0001$, Model 2 $b_0^* = 0.01$, $b_1^* = 0.02$ and $b_2^* = 0.01$, Model 3 $b_0^* = 0.01$, $b_1^* = 0.04$ and $b_2^* = 0.04$ and Model 4 $b_0^* = 0.01$, $b_1^* = -0.01$ and $b_2^* = 0.0025$.

5.7 ANALYSIS OF THE RESULTS

Our results in Table 5.2 show that the values of θ_ψ, P_ψ and ρ_ψ obtained by us when the effects of Coriolis force are included, do not change to any appreciable extent in comparison with the corresponding values in which the effects of Coriolis force are not taken into account.

For nonsynchronous and synchronous binaries (Table 5.3.1 – 5.3.4) there is no appreciable change in the volumes, surface areas and equatorial angular velocity (Ω_e) of various differentially rotating polytropic models in the presence of Coriolis force. However, in the case of nonsynchronous binaries some change (less than 2%) can be noticed in the values of equatorial angular velocity (Ω_e) for models (2 and 3) which have large angular velocities of rotation.

In case of purely differentially rotating stars (Table 5.3.5 – 5.3.6) with the inclusion of Coriolis force the values of different parameters do not change to any appreciable extent for models in which the angular velocity of rotation is small. However for model (3) which has large angular velocity of rotation there is some noticeable change in the values of volumes, surface areas and equatorial angular velocity (Ω_e) near surface with percentage difference in each case being less than 3%.

From Table 5.4, we find that the in case of purely rotating stars, present results (Coriolis force included) are similar to the corresponding results obtained by using the law of differential rotation (5.8) and approach earlier used by Mohan et al (104) in which the effect of Coriolis force has not been considered.

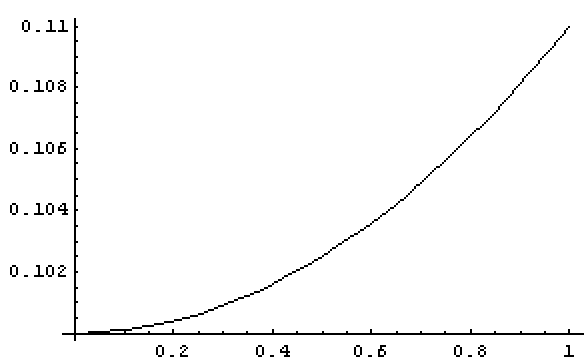
Thus our present study has shown that with the inclusion of Coriolis force there is not any appreciable change in the equilibrium structures of slow rotating polytropic models of differentially rotating stars and stars in synchronous and nonsynchronous binary systems. However, some change is noticed in these values for rotating stars in case of models which have large angular velocities of rotation at points near the surface.

Table 5.1 Various models of differentially rotating stars

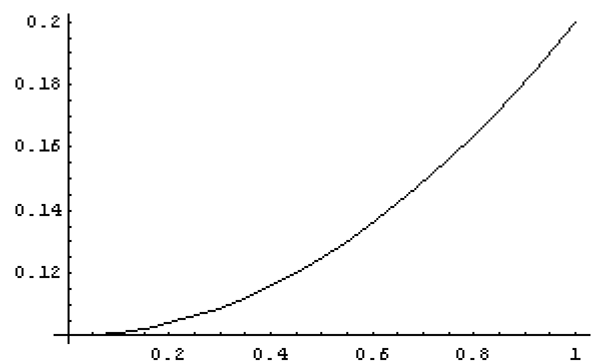
Model No.	Values of various parameters in the law of differential rotation		Behavior of square of the angular velocity Ω^2 from the axis of rotation ($s = 0$) to equator ($s = 1$) in the equatorial plane/(from pole to equator on the surface), for differentially rotating model in which s is in the units of equatorial radii R_e	Stability of the model according to Stoeckly (148) criterion
	b_0	b_1		
1	0.1	0.01	Ω^2 increases gradually from 0.1 at center to 0.11 at surface	Stable
2	0.1	0.1	Ω^2 increases gradually from 0.1 at center to 0.2 at surface	Stable
3	0.1	0.2	Ω^2 increases gradually from 0.1 at center to 0.3 at surface	Stable
4	0.1	-0.05	Ω^2 decreases gradually from 0.1 at center to 0.05 at surface	Stable

Graphs of Ω^2 versus s in above differentially rotating stellar models

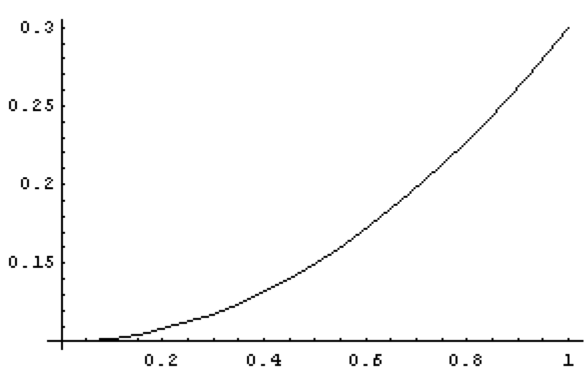
Model 1



Model 2



Model 3



Model 4

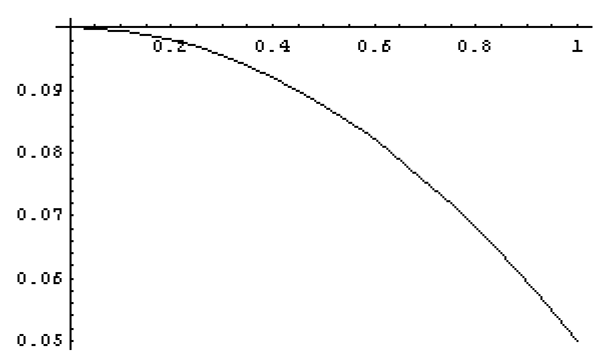


Table 5.2 Values of $\theta_\psi, P_\psi, \rho_\psi$ (N = 3.0)

θ	$x = \frac{r_0}{r_{0s}}$	Purely Rotating stars			Nonsynchronous Binary			Synchronous Binary		
		θ_ψ	P_ψ	ρ_ψ	θ_ψ	P_ψ	ρ_ψ	θ_ψ	P_ψ	ρ_ψ
		$b_0 = 0.1,\ b_1 = 0.1$ $n = 0.005,\ q = 0.0$			$b_0 = 0.1,\ b_1 = 0.1$ $n = 0.005,\ q = 0.05$			$b_0 = 1.0247,\ b_1 = 0.1$ $n = 0.525,\ q = 0.05$		
0	0.0	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)
	0.2	0.7535 (0.7535)	0.3391 (0.3392)	0.4444 (0.4444)	0.7533 (0.7533)	0.3387 (0.3388)	0.4439 (0.4440)	0.7575 (0.7575)	0.3461 (0.3461)	0.4512 (0.4512)
	0.4	0.4063 (0.4063)	0.0292 (0.0292)	0.0706 (0.0706)	0.4060 (0.4060)	0.0291 (0.0291)	0.0704 (0.0705)	0.4115 (0.4115)	0.0307 (0.0307)	0.0734 (0.0734)
	0.6	0.1934 (0.1934)	0.0015 (0.0015)	0.0077 (0.0077)	0.1932 (0.1933)	0.0015 (0.0015)	0.0077 (0.0077)	0.1967 (0.1967)	0.0016 (0.0016)	0.0081 (0.0081)
	0.8	0.0732 (0.0732)	0.0000 (0.0000)	0.0004 (0.0004)	0.0731 (0.0732)	0.0000 (0.0000)	0.0004 (0.0004)	0.0746 (0.0746)	0.0000 (0.0000)	0.0005 (0.0005)
	1.0	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
30	0.0	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)
	0.2	0.7537 (0.7537)	0.3394 (0.3394)	0.4447 (0.4447)	0.7532 (0.7533)	0.3387 (0.3387)	0.4439 (0.4440)	0.7577 (0.7577)	0.3464 (0.3464)	0.4516 (0.4516)
	0.4	0.4065 (0.4065)	0.0293 (0.0293)	0.0707 (0.0708)	0.4060 (0.4060)	0.0291 (0.0291)	0.0705 (0.0705)	0.4118 (0.4118)	0.0308 (0.0308)	0.0735 (0.0735)
	0.6	0.1935 (0.1936)	0.0015 (0.0015)	0.0077 (0.0077)	0.1932 (0.1932)	0.0015 (0.0015)	0.0077 (0.0077)	0.1969 (0.1969)	0.0016 (0.0016)	0.0081 (0.0081)
	0.8	0.0732 (0.0733)	0.0000 (0.0000)	0.0004 (0.0004)	0.0731 (0.0732)	0.0000 (0.0000)	0.0004 (0.0004)	0.0746 (0.0746)	0.0000 (0.0000)	0.0005 (0.0005)
	1.0	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
60	0.0	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)
	0.2	0.7540 (0.7542)	0.3400 (0.3403)	0.4452 (0.4455)	0.7533 (0.7534)	0.3388 (0.3388)	0.4440 (0.4441)	0.7582 (0.7582)	0.3472 (0.3472)	0.4523 (0.4523)
	0.4	0.4069 (0.4071)	0.0294 (0.0294)	0.0710 (0.0711)	0.4060 (0.4061)	0.0291 (0.0291)	0.0705 (0.0705)	0.4124 (0.4124)	0.0310 (0.0310)	0.0738 (0.0738)
	0.6	0.1938 (0.1939)	0.0015 (0.0015)	0.0077 (0.0077)	0.1933 (0.1933)	0.0015 (0.0015)	0.0077 (0.0077)	0.1972 (0.1972)	0.0016 (0.0016)	0.0081 (0.0082)
	0.8	0.0734 (0.0734)	0.0000 (0.0000)	0.0004 (0.0004)	0.0732 (0.0732)	0.0000 (0.0000)	0.0004 (0.0004)	0.0748 (0.0748)	0.0000 (0.0000)	0.0005 (0.0005)
	1.0	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
90	0.0	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)
	0.2	0.7542 (0.7545)	0.3402 (0.3408)	0.4454 (0.4460)	0.7534 (0.7534)	0.3388 (0.3389)	0.4441 (0.4442)	0.7584 (0.7584)	0.3476 (0.3476)	0.4527 (0.4527)
	0.4	0.4071 (0.4076)	0.0294 (0.0296)	0.0711 (0.0713)	0.4061 (0.4061)	0.0291 (0.0291)	0.0705 (0.0705)	0.4127 (0.4127)	0.0310 (0.0311)	0.0740 (0.0740)
	0.6	0.1939 (0.1942)	0.0015 (0.0015)	0.0077 (0.0078)	0.1933 (0.1933)	0.0015 (0.0015)	0.0077 (0.0077)	0.1974 (0.1974)	0.0016 (0.0016)	0.0082 (0.0082)
	0.8	0.0734 (0.0735)	0.0000 (0.0000)	0.0004 (0.0004)	0.0732 (0.0732)	0.0000 (0.0000)	0.0004 (0.0004)	0.0749 (0.0749)	0.0000 (0.0000)	0.0005 (0.0005)
	1.0	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)

1. First column of the table shows the values of angle θ in degrees.
2. P_ψ and ρ_ψ are in the units of P_c and ρ_c respectively
3. The values shown in the parenthesis are the corresponding values when the effect of Coriolis force is not considered.

Table 5.3.1 Values of certain structure parameters and related quantities for differentially rotating and tidally distorted polytropic models (N = 1.5)

Model	θ	r_{0s}	$V_{\psi} \times 10^{-2}$	$S_{\psi} \times 10^{-2}$	Ω_p	Ω_e
Nonsynchronous Binaries						
$n = 0.005,\ q = 0.1$						
1	0	0.499879 (0.499754)	2.0452 (2.0475)	1.6787 (1.6795)	0.1000 (0.1000)	0.1026 (0.1028)
	30	0.499874 (0.499749)	2.0453 (2.0475)	1.6787 (1.6796)	0.1000 (0.1000)	0.1026 (0.1028)
	60	0.499863 (0.499739)	2.0455 (2.0477)	1.6788 (1.6796)	0.1000 (0.1000)	0.1026 (0.1028)
	90	0.499858 (0.499734)	2.0455 (2.0477)	1.6788 (1.6797)	0.1000 (0.1000)	0.1026 (0.1028)
2	0	0.499879 (0.499754)	2.0452 (2.0475)	1.6787 (1.6795)	0.1000 (0.1000)	0.1264 (0.1279) [1.2]
	30	0.499824 (0.499700)	2.0461 (2.0482)	1.6791 (1.6799)	0.1000 (0.1000)	0.1260 (0.1279 [1.5]
	60	0.499677 (0.499557)	2.0486 (2.0502)	1.6803 (1.6811)	0.1000 (0.1000)	0.1254 (0.1280) [1.2]
	90	0.499585 (0.499467)	2.0502 (2.0515)	1.6811 (1.6818)	0.1000 (0.1000)	0.1250 (0.1280) [2.0]
3	0	0.499879 (0.499754)	2.0452 (2.0475)	1.6787 (1.6795)	0.1000 (0.1000)	0.1528 (0.1558) [1.9]
	30	0.499756 (0.499634)	2.0472 (2.0491)	1.6796 (1.6805)	0.1000 (0.1000)	0.1514 (0.1559) [2.9]
	60	0.499364 (0.499252)	2.0544 (2.0545)	1.6829 (1.6834)	0.1000 (0.1000)	0.1488 (0.1560) [4.6]
	90	0.499094 (0.498988)	2.0596 (2.0583)	1.6852 (1.6855)	0.1000 (0.1000)	0.1476 (0.1561) [5.4]
4	0	0.499879 (0.499754)	2.0452 (2.0475)	1.6787 (1.6795)	0.1000 (0.1000)	0.0868 (0.0860)
	30	0.499902 (0.499776)	2.0449 (2.0471)	1.6785 (1.6794)	0.1000 (0.1000)	0.0868 (0.0860)
	60	0.499938 (0.499812)	2.0444 (2.0466)	1.6782 (1.6791)	0.1000 (0.1000)	0.0868 (0.0860)
	90	0.499952 (0.499825)	2.0442 (2.0465)	1.6781 (1.6790)	0.1000 (0.1000)	0.0868 (0.0860)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 5.3.2 Values of certain structure parameters and related quantities for differentially rotating and tidally distorted polytropic models (N = 3.0)

Model	θ	r_{0s}	$V_{\psi} \times 10^{-3}$	$S_{\psi} \times 10^{-2}$	Ω_p	Ω_e
Nonsynchronous Binaries						
$n=0.005,\ q=0.1$						
1	0	0.499932 (0.499862)	1.3760 (1.3779)	5.9825 (5.9869)	0.1000 (0.1000)	0.1026 (0.1028)
	30	0.499929 (0.499859)	1.3761 (1.3780)	5.9827 (5.9871)	0.1000 (0.1000)	0.1026 (0.1028)
	60	0.499924 (0.499854)	1.3762 (1.3781)	5.9831 (5.9875)	0.1000 (0.1000)	0.1026 (0.1028)
	90	0.499921 (0.499851)	1.3763 (1.3782)	5.9833 (5.9877)	0.1000 (0.1000)	0.1026 (0.1028)
2	0	0.499932 0.499862)	1.3760 (1.3779)	5.9825 (5.9869)	0.1000 (0.1000)	0.1264 (0.1279) [1.2]
	30	0.499902 (0.499832)	1.3768 (1.3787)	5.9846 (5.9890)	0.1000 (0.1000)	0.1260 (0.1279) [1.5]
	60	0.499822 (0.499754)	1.3791 (1.3806)	5.9906 (5.9945)	0.1000 (0.1000)	0.1254 (0.1280) [2.0]
	90	0.499772 (0.499705)	1.3806 (1.3818)	5.9945 (5.9980)	0.1000 (0.1000)	0.1251 (0.1280) [2.3]
3	0	0.499932 (0.499862)	1.3760 (1.3779)	5.9825 (5.9869)	0.1000 (0.1000)	0.1528 (0.1559) [2.0]
	30	0.499865 (0.499796)	1.3778 (1.3795)	5.9874 (5.9915)	0.1000 (0.1000)	0.1514 (0.1559) [2.9]
	60	0.499652 (0.499588)	1.3843 (1.3846)	6.0039 (6.0063)	0.1000 (0.1000)	0.1489 (0.1561) [4.6]
	90	0.499505 (0.499445)	1.3891 (1.3882)	6.0154 (6.0167)	0.1000 (0.1000)	0.1477 (0.1562) [5.4]
4	0	0.499932 (0.499862)	1.3760 (1.3779)	5.9825 (5.9869)	0.1000 (0.1000)	0.0868 (0.0860)
	30	0.499945 (0.499874)	1.3757 (1.3777)	5.9816 (5.9860)	0.1000 (0.1000)	0.0868 (0.0860)
	60	0.499964 (0.499893)	1.3752 (1.3772)	5.9802 (5.9847)	0.1000 (0.1000)	0.0868 (0.0860)
	90	0.499972 (0.499900)	1.3750 (1.3770)	5.9797 (5.9841)	0.1000 (0.1000)	0.0868 (0.0860)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 5.3.3 Values of certain structure parameters and related quantities for differentially rotating and tidally distorted polytropic models (N = 1.5)

Model	θ	r_{0s}	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$	Ω_p	Ω_e
Synchronous Binaries						
$n=0.525,\ q=0.05$						
$b_0=1.0247$ $b_1=0.01$	0	0.489659 (0.489659)	2.2035 (2.2035)	1.7646 (1.7646)	1.0247 (1.0247)	1.0275 (1.0275)
	30	0.489155 (0.489149)	2.2044 (2.2044)	1.7651 (1.7651)	1.0247 (1.0247)	1.0275 (1.0275)
	60	0.488148 (0.488092)	2.2060 (2.2060)	1.7660 (1.7660)	1.0247 (1.0247)	1.0276 (1.0276)
	90	0.487645 (0.487546)	2.2069 (2.2069)	1.7664 (1.7664)	1.0247 (1.0247)	1.0276 (1.0276)
$b_0=1.0247$ $b_1=0.1$	0	0.489659 (0.489659)	2.2035 (2.2035)	1.7646 (1.7646)	1.0247 (1.0247)	1.0532 (1.0532)
	30	0.489609 (0.489609)	2.2119 (2.2120)	1.7691 (1.7692)	1.0247 (1.0247)	1.0533 (1.0533)
	60	0.489508 (0.489507)	2.2285 (2.2295)	1.7782 (1.7787)	1.0247 (1.0247)	1.0536 (1.0535)
	90	0.489457 (0.489456)	2.2370 (2.2386)	1.7828 (1.7837)	1.0247 (1.0247)	1.0537 (1.0536)
$b_0=1.0247$ $b_1=0.2$	0	0.489659 (0.489659)	2.2035 (2.2035)	1.7646 (1.7646)	1.0247 (1.0247)	1.0816 (1.0816)
	30	0.488651 (0.488626)	2.2202 (2.2206)	1.7737 (1.7739)	1.0247 (1.0247)	1.0822 (1.0821)
	60	0.486640 (0.486418)	2.2537 (2.2575)	1.7919 (1.7940)	1.0247 (1.0247)	1.0832 (1.0830)
	90	0.485638 (0.485244)	2.2705 (2.2772)	1.8011 (1.8047)	1.0247 (1.0247)	1.0837 (1.0836)
$b_0=1.0247$ $b_1=-0.05$	0	0.489659 (0.489659)	2.2035 (2.2035)	1.7646 (1.7646)	1.0247 (1.0247)	1.0105 (1.0105)
	30	0.489911 (0.489910)	2.1994 (2.1994)	1.7624 (1.7624)	1.0247 (1.0247)	1.0105 (1.0105)
	60	0.490416 (0.490402)	2.1911 (2.1913)	1.7579 (1.7580)	1.0247 (1.0247)	1.0106 (1.0105)
	90	0.490668 (0.490644)	2.1870 (2.1874)	1.7556 (1.7558)	1.0247 (1.0247)	1.0106 (1.0106)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 5.3.4 Values of certain structure parameters and related quantities for differentially rotating and tidally distorted polytropic models (N = 3.0)

Model	θ	r_{0s}	$V_{\psi} \times 10^{-3}$	$S_{\psi} \times 10^{-2}$	Ω_p	Ω_e
Synchronous Binaries						
$n=0.525,\ q=0.05$						
$b_0=1.0247$ $b_1=0.01$	0	0.494305 (0.494305)	1.5313 (1.5313)	6.4261 (6.4261)	1.0247 (1.0247)	1.0276 (1.0276)
	30	0.494276 (0.494276	1.5321 (1.5321)	6.4285 (6.4285)	1.0247 (1.0247)	1.0276 (1.0276)
	60	0.494220 (0.494220)	1.5338 (1.5338)	6.4333 (6.4334)	1.0247 (1.0247)	1.0276 (1.0276)
	90	0.494192 (0.494191)	1.5347 (1.5347)	6.4358 (6.4358)	1.0247 (1.0247)	1.0276 (1.0276)
$b_0=1.0247$ $b_1=0.1$	0	0.494305 (0.494305)	1.5313 (1.5313)	6.4261 (6.4261)	1.0247 (1.0247)	1.0539 (1.0539)
	30	0.494022 (0.494018)	1.5398 (1.5399)	6.4504 (6.4507)	1.0247 (1.0247)	1.0541 (1.0540)
	60	0.493455 (0.493423)	1.5572 (1.5582)	6.4997 (6.5024)	1.0247 (1.0247)	1.0544 (1.0544)
	90	0.493170 (0.493114)	1.5660 (1.5678)	6.5246 (6.5295)	1.0247 (1.0247)	1.0546 (1.0545)
$b_0=1.0247$ $b_1=0.2$	0	0.494305 (0.494305)	1.5313 (1.5313)	6.4261 (6.4261)	1.0247 (1.0247)	1.0831 (1.0831)
	30	0.493738 (0.493724)	1.5485 (1.5489)	6.4749 (6.4762)	1.0247 (1.0247)	1.0838 (1.0837)
	60	0.492601 (0.492475)	1.5838 (1.5878)	6.5750 (6.5862)	1.0247 (1.0247)	1.0852 (1.0851)
	90	0.492030 (0.491805)	1.6018 (1.6090)	6.6260 (6.6462)	1.0247 (1.0247)	1.0860 (1.0859)
$b_0=1.0247$ $b_1=-0.05$	0	0.494305 (0.494305)	1.5313 (1.5313)	6.4261 (6.4261)	1.0247 (1.0247)	1.0101 (1.0101)
	30	0.494446 (0.494445)	1.5270 (1.5270)	6.4140 (6.4140)	1.0247 (1.0247)	1.0102 (1.0101)
	60	0.494728 (0.494720)	1.5185 (1.5187)	6.3900 (6.3906)	1.0247 (1.0247)	1.0102 (1.0102)
	90	0.494869 (0.494855)	1.5143 (1.5147)	6.3780 (6.3791)	1.0247 (1.0247)	1.0103 (1.0103)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 5.3.5 Values of certain structure parameters and related quantities for differentially rotating polytropic models (N = 1.5)

Model	θ	r_{0s}	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$	Ω_p	Ω_e
Purely Rotating stars						
$n = 0.005,\ q = 0.0$						
1	0	0.998442 (0.998442)	2.0542 (2.0542)	1.6836 (1.6836)	0.1000 (0.1000)	0.1101 (0.1101)
	30	0.998364 (0.998363)	2.0548 (2.0548)	1.6839 (1.6839)	0.1000 (0.1000)	0.1101 (0.1101)
	60	0.998207 (0.998199)	2.0559 (2.0559)	1.6845 (1.6846)	0.1000 (0.1000)	0.1101 (0.1101)
	90	0.998130 (0.998114)	2.0564 (2.0565)	1.6848 (1.6849)	0.1000 (0.1000)	0.1101 (0.1101)
2	0	0.998442 (0.998442)	2.0542 (2.0542)	1.6836 (1.6836)	0.1000 (0.1000)	0.2007 (0.2007)
	30	0.997661 (0.997563)	2.0598 (2.0605)	1.6867 (1.6871)	0.1000 (0.1000)	0.2011 (0.2011)
	60	0.996096 (0.995215)	2.0712 (2.0777)	1.6929 (1.6964)	0.1000 (0.1000)	0.2018 (0.2022)
	90	0.995313 (0.993744)	2.0770 (2.0887)	1.6960 (1.7024)	0.1000 (0.1000)	0.2021 (0.2029)
3	0	0.998442 (0.998442)	2.0542 (2.0542)	1.6836 (1.6836)	0.1000 (0.1000)	0.3014 (0.3014)
	30	0.996879 (0.996488)	2.0655 (2.0683)	1.6898 (1.6913)	0.1000 (0.1000)	0.3028 (0.3032)
	60	0.993744 (0.990207)	2.0887 (2.1158) [1.3]	1.7024 (1.7171)	0.1000 (0.1000)	0.3058 (0.3093) [1.1]
	90	0.992173 (0.985877)	2.1006 (2.1500) [2.3]	1.7089 (1.7356) [1.5]	0.1000 (0.1000)	0.3073 (0.3138) [2.7]
4	0	0.998442 (0.998442)	2.0542 (2.0542)	1.6836 (1.6836)	0.1000 (0.1000)	0.0497 (0.0497)
	30	0.998832 (0.998807)	2.0514 (2.0516)	1.6821 (1.6822)	0.1000 (0.1000)	0.0497 (0.0497)
	60	0.999612 (0.999392)	2.0459 (2.0475)	1.6791 (1.6799)	0.1000 (0.1000)	0.0499 (0.0499)
	90	1.000000 (0.999612)	2.0432 (2.0459)	1.6776 (1.6791)	0.1000 (0.1000)	0.0500 (0.0499)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered
3. Values in the square bracket represent the percent difference of two corresponding values. Only those values are given in which the percentage difference is greater than 1%.

Table 5.3.6 Values of certain structure parameters and related quantities for differentially rotating polytropic models (N = 3.0)

Model	θ	r_{0s}	$V_{\psi} \times 10^{-3}$	$S_{\psi} \times 10^{-2}$	Ω_p	Ω_e
Purely Rotating stars						
$n = 0.005,\ q = 0.0$						
1	0	0.999153 (0.999153)	1.3846 (1.3846)	6.0075 (6.0075)	0.1000 (0.1000)	0.1101 (0.1101)
	30	0.999110 (0.999109)	1.3851 (1.3851)	6.0090 (6.0090)	0.1000 (0.1000)	0.1101 (0.1101)
	60	0.999025 (0.999020)	1.3862 (1.3862)	6.0121 (6.0122)	0.1000 (0.1000)	0.1101 (0.1101)
	90	0.998983 (0.998975)	1.3867 (1.3868)	6.0136 (6.0139)	0.1000 (0.1000)	0.1101 (.01101)
2	0	0.999153 (0.999153)	1.3846 (1.3846)	6.0075 (6.0075)	0.1000 (0.1000)	0.2008 (0.2008)
	30	0.998728 (0.998675)	1.3899 (1.3905)	6.0228 (6.0248)	0.1000 (0.1000)	0.2013 (0.2013)
	60	0.997876 (0.997395)	1.4007 (1.4069)	6.0540 (6.0719)	0.1000 (0.1000)	0.2022 (0.2027)
	90	0.997448 (0.996592)	1.4062 (1.4174)	6.0699 (6.1021)	0.1000 (0.1000)	0.2026 (0.2035)
3	0	0.999153 (0.999153)	1.3846 (1.3846)	6.0075 (6.0075)	0.1000 (0.1000)	0.3017 (0.3017)
	30	0.998302 (0.998089)	1.3953 (1.3980)	6.0383 (6.0462)	0.1000 (0.1000)	0.3034 (0.3039)
	60	0.996592 (0.994654)	1.4174 (1.4436) [1.8]	6.1021 (6.1770) [1.2]	0.1000 (0.1000)	0.3071 (0.3114) [1.4]
	90	0.995732 (0.992268)	1.4289 (1.4472) [1.3]	6.1350 (6.2728) [2.2]	0.1000 (0.1000)	0.3089 (0.3170) [2.6]
4	0	0.999153 (0.999153)	1.3846 (1.3846)	6.0075 (6.0075)	0.1000 (0.1000)	0.0496 (0.0496)
	30	0.999365 (0.999352)	1.3819 (1.3821)	6.0000 (6.0004)	0.1000 (0.1000)	0.0497 (0.0497)
	60	0.999789 (0.999670)	1.3767 (1.3782)	5.9848 (5.9891)	0.1000 (0.1000)	0.0499 (0.0498)
	90	1.000000 (0.999789)	1.3742 (1.3767)	5.9774 (5.9848)	0.1000 (0.1000)	0.0500 (0.0499)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered
3. Values in the square bracket represent the percent difference of two corresponding values. Only those values are given in which the percentage difference is greater than 1%.

Table 5.4 Comparison of certain structure parameters and related quantities for differentially rotating polytropic models

Model	r_{0s}	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$	Ω_p	Ω_e
Purely Rotating stars					
$n = 0.005,\ q = 0.0$					
$N = 1.5$					
1	0.998130 (0.998374)	2.0564 (2.0556)	1.6848 (1.6844)	0.1000 (0.1000)	0.1101 (0.1101)
2	0.995313 (0.997657)	2.0770 (2.0717)	1.6960 (1.6932)	0.1000 (0.1000)	0.2021 (0.2020)
3	0.992173 (0.996630)	2.1006 (2.0986)	1.7089 (1.7077)	0.1000 (0.1000)	0.3073 (0.3080)
4	1.000000 (0.998746)	2.0432 (2.0486)	1.6776 (1.6806)	0.1000 (0.1000)	0.0500 (0.0498)
$N = 3.0$					
	r_{0s}	$V_\psi \times 10^{-3}$	$S_\psi \times 10^{-2}$	Ω_p	Ω_e
1	0.998983 (0.999133)	1.3867 (1.3857)	6.0136 (6.0107)	0.1000 (0.1000)	0.1101 (0.1101)
2	0.997448 (0.998931)	1.4062 (1.3987)	6.0699 (6.0487)	0.1000 (0.1000)	0.2026 (0.2023)
3	0.995732 (0.998660)	1.4289 (1.4206)	6.1350 (6.1112)	0.1000 (0.1000)	0.3089 (0.3090)
4	1.000000 (0.999247)	1.3742 (1.3799)	5.9774 (5.9941)	0.1000 (0.1000)	0.0500 (0.0498)

1. Values of various parameters are taken at surface (θ = 90)
2. The values in the parenthesis are the corresponding values obtained by using the law of differential rotation earlier adopted by Mohan et al (104) in which the effect of Coriolis force is not considered.

CHAPTER – VI

EFFECT OF CORIOLIS FORCE ON THE EIGENFREQUENCIES OF SMALL ADIABATIC BAROTROPIC MODES OF OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

In the previous chapter we considered the problem of computing the effects of Coriolis force on equilibrium structures of differentially rotating stars In the present Chapter we consider the problem of determining the effects of Coriolis force on the eigenfrequencies of small adiabatic pseudo radial and nonradial modes of oscillations of differentially rotating and tidally distorted stellar models.

Using the averaging approach developed in section 1.4 of Chapter I, section 4.1 and 4.2 of chapter IV and using a law of differential rotation of the type (5.10), an eigenvalued boundary value problem determining the eigenfrequencies of small adiabatic pseudo – radial mode of oscillations of differentially rotating and tidally distorted stellar models have been formulated in Section 6.1. This eigenvalue problem incorporates the effects of Coriolis force as well besides the gravitational and centrifugal forces. An eigenvalued boundary value problem which determines the effects of Coriolis force on the eigenfrequencies of nonradial modes of oscillations of the stellar models has next been formulated in Section 6.2. In section 6.3 and section 6.4, analysis of sections 6.1 and 6.2 respectively has been used to formulate the eigenvalue problems which determine the pseudo – radial and nonradial modes of oscillations of differentially rotating and tidally distorted polytropic models. The numerical computations for determining the eigenvalues of radial and nonradial modes of oscillations of differentially rotating and tidally distorted polytropic models have been done for polytropic models of indices 1.5 and 3.0. Analysis of the numerical results has finally been carried out in Section 6.5 and conclusions drawn.

6.1 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EFFECTS OF CORIOLIS FORCE ON THE EIGENFREQUENCIES OF SMALL ADIABATIC PSEUDO – RADIAL MODES OF OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED STELLAR MODELS

Using the approach presented in section 1.4 of chapter I and section 4.1 of chapter IV, the equation determining the eigenfrequencies of small adiabatic pseudo – radial modes of oscillations of differentially rotating and tidally distorted stellar models obeying the differential rotation law (5.10) and which incorporates the effects of Coriolis force besides the effects of centrifugal and gravitational forces may be expressed as

$$A(b_0,b_1,n,q)\frac{d^2\kappa}{dr_0^2}+\left[\frac{4-\mu}{r_0}B(b_0,b_1,n,q)-C(b_0,b_1,n,q)\right]\frac{d\kappa}{dr_0}$$
$$+\left[\frac{R^2\sigma^2\rho_\psi}{\gamma P_\psi}-\left(3-\frac{4}{\gamma}\right)\frac{\mu}{r_0^2}E(b_0,b_1,n,q\right]\kappa=0 \tag{6.1}$$

where

$$A(b_0,b_1,n,q,r_0)=1-\frac{16}{3}cr_0^3-10q^2h^2r_0^4-(\frac{56}{5}q^2+\frac{104}{45}c^2+\frac{112}{15}cq)r_0^6-\frac{90}{7}q^2r_0^8-\frac{44}{3}q^2r_0^{10}-...$$

$$B(b_0,b_1,n,q,r_0)=1-\frac{10}{3}cr_0^3-6q^2h^2r_0^4-(\frac{32}{5}q^2+\frac{188}{45}c^2+\frac{64}{15}cq)r_0^6-\frac{50}{7}q^2r_0^8-8q^2r_0^{10}-...$$

$$C(b_0,b_1,n,q,r_0)=\frac{1}{r_0}[8cr_0^3+20q^2h^2r_0^4+(\frac{168}{5}q^2+\frac{104}{15}c^2+\frac{112}{5}cq)r_0^6+\frac{360}{7}q^2r_0^8$$
$$+\frac{220}{3}q^2r_0^{10}+...]$$

$$E(b_0,b_1,n,q,r_0)=1-\frac{4}{3}cr_0^3-2q^2h^2r_0^4-(\frac{8}{5}q^2+\frac{92}{45}c^2+\frac{16}{15}cq)r_0^6-\frac{10}{7}q^2r_0^8-\frac{4}{3}q^2r_0^{10}-...$$

and

$$\mu=-\frac{r_\psi}{P_\psi}\frac{dP_\psi}{dr_0}\frac{dr_0}{dr_\psi}=-F(n,q,r_0)\frac{r_0}{P_\psi}\frac{dP_\psi}{d\,r_0}$$

where

$$F(b_0,b_1,n,q,r_0)=1-2cr_0^3-4q^2h^2r_0^4-(\frac{24}{5}q^2+\frac{72}{15}c^2+\frac{16}{5}cq)r_0^6-\frac{40}{7}q^2r_0^8-\frac{20}{3}q^2r_0^{10}-...$$

and

$$h=1-[(b_0+b_1s^2)/(2n)^{1/2}] \text{ and } c=[(b_0+b_1s^2)(2n)^{1/2}-n].$$

In the absence of Coriolis force, the equation determining the eigenfrequencies of small adiabatic pseudo – radial modes of oscillations of differentially rotating and tidally distorted stellar models obeying the differential rotation law (5.10) can be obtained by substituting $h=1-[(b_0+b_1s^2)^2/(1+q)]$ and $c=(b_0+b_1s^2)^2/2$ in (6.1).

In the above expressions terms up to second order of smallness in b_0,b_1,n,q and up to order r_0^{10} in r_0 have been retained. Equation (6.1) is an eigenvalue problem in the square of eigenfrequency of oscillation σ. As usual, this eigenvalue problem is of Sturm-Liouville type having singularities both at the centre and the surface of the model. It has to be solved subject to the boundary conditions which require κ to be finite at the centre as well as at the free surface. As discussed in chapter IV, for convenience in numerical work we find it more convenient to substitute

$$\kappa = \frac{\varsigma}{r_0} \quad \text{and} \quad r_0 = x\, r_{OS} \tag{6.2}$$

(r_{0s} being the value of r_0 on the outermost surface) in equation (6.2) and treat x as the independent variable and κ as the dependent variable. With this substitution, equation (6.1) can now be written in terms of the variables ς and x as

$$A^*(n,q,x)\frac{d^2\varsigma}{dx^2} + B^*(n,q,x)\frac{d\varsigma}{dx} + C^*(n,q,x)\varsigma = 0 \tag{6.3}$$

where

$$A^*(b_0,b_1,n,q,\ x) = A(b_0,b_1,n,q,xr_{0s})$$

$$B^*(b_0,b_1,n,q,\ x) = \frac{4-\mu}{x}B(b_0,b_1,n,q,\ xr_{0S}) - r_{0S}C(b_0,b_1,n,q,xr_{0S}) - \frac{2}{x}A(b_0,b_1,n,q,xr_{0S})$$

$$C^*(b_0,b_1,n,q,x) = \frac{r_{0S}^2 R^2 \rho_\psi}{\gamma P_\psi}\sigma^2 - (3-\frac{4}{\gamma})\frac{\mu}{x^2}E(b_0,b_1,n,q,xr_{0S}) - \frac{1}{x}B^*(b_0,b_1,n,q,xr_{0S})$$

The boundary conditions now are

$$\varsigma = 0 \text{ at the centre } x = 0 \quad \text{and} \quad \varsigma = \text{finite at the surface } x = 1 \tag{6.4}$$

The equation (6.3) subject to the boundary conditions (6.4) constitute an eigenvalue problem which incorporates the effects of Coriolis force on the eigenfrequencies of pseudo – radial modes of oscillations of differentially rotating and tidally distorted stellar models.

6.2 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EFFECTS OF CORIOLIS FORCE ON THE EIGENFREQUENCIES OF SMALL ADIABATIC NONRADIAL MODES OF OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED STELLAR MODELS

Similarly following the approach of section 1.4 of Chapter I and section 4.2 of chapter IV, the eigenvalue problem determining the eigenfrequencies of nonradial modes of oscillations of differentially rotating and tidally distorted stellar models obeying a law of differential rotation of type (5.10) and which incorporates the effects of Coriolis force besides the effects of centrifugal and gravitational forces may be expressed in an explicit form convenient for computational work as

$$
\left.\begin{aligned}
&\frac{d\zeta}{dx}+B_1\zeta+(B_2+\frac{1}{\sigma^2}B_3)\eta+\frac{1}{\sigma^2}B_3\,\phi=0\\
&\frac{d\eta}{dx}+(E_1\sigma^2+E_2)\zeta+E_3\eta+E_4\,\phi+\frac{d\phi}{dx}=0\\
&\frac{\mathrm{d}^2\phi}{dx^2}+F_1\frac{d\phi}{dx}+F_2\zeta+F_3\eta+F_4\,\phi=0
\end{aligned}\right\}\qquad(6.5)
$$

where

$$B_1=\frac{l+1}{x}+\frac{1}{\gamma P_\psi}\frac{dP\psi}{dx}$$

$$
\begin{aligned}
B_2&=\frac{2\pi G\rho_c}{Dx}\frac{\rho_\psi}{\gamma P_\psi}r_\psi^2\frac{dr_\psi}{dx}\\
&=\frac{2\pi G\rho_c}{\gamma P_\psi}D^2\rho_\psi\,r_{0s}^3\,x[1+4c(xr_{0s})^3+7q^2h^2(xr_{0s})^4+(\frac{36}{5}q^2+\frac{864}{45}c^2\\
&\quad+\frac{24}{5}cq)(xr_{0s})^6+\frac{55}{7}q^2(xr_{0s})^8+\frac{26}{3}q^2(xr_{0s})^{10}+...]
\end{aligned}
$$

$$
\begin{aligned}
B_3&=-\frac{l(l+1)}{D\,x}\frac{dr_\psi}{dx}2\pi G\rho_c\\
&=-\frac{l(l+1)}{x}2\pi G\rho_c r_{0s}[1+\frac{8}{3}c(xr_{0s})^3+5q^2h^2(xr_{0s})^4+(\frac{28}{5}q^2+\frac{532}{45}c^2\\
&\quad+\frac{56}{15}cq)(xr_{0s})^6+\frac{45}{7}q^2(xr_{0s})^8+\frac{22}{3}q^2(xr_{0s})^{10}+...]
\end{aligned}
$$

$$
\begin{aligned}
E_1&=-\frac{1}{2\pi G\rho_c}\frac{D\,x}{r_\psi^2}\frac{dr_\psi}{dx}\\
&=-\frac{1}{2\pi G\rho_c\,r_{0s}\,x}[1+\frac{4}{3}c(xr_{0s})^3+3q^2h^2(xr_{0s})^4+(4q^2+\frac{56}{9}c^2\\
&\quad+\frac{8}{3}cq)(xr_{0s})^6+5q^2(xr_{0s})^8+6q^2(xr_{0s})^{10}+...]
\end{aligned}
$$

$$
\begin{aligned}
E_2&=\frac{1}{2\pi G\rho_c}\frac{A_\psi}{\rho_\psi}\frac{dP_\psi}{dx}\frac{D\,x}{r_\psi^2}\\
&=\frac{1}{2\pi G\rho_c D^2}\frac{1}{\rho_\psi}(\frac{1}{\rho_\psi}\frac{d\rho_\psi}{dx}-\frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx})\frac{dP_\psi}{dx}\frac{1}{x\,r_{0s}^3}[1-4c(xr_{0s})^3-7q^2h^2(xr_{0s})^4\\
&\quad-(\frac{36}{5}q^2+\frac{144}{45}c^2+\frac{72}{15}cq)(xr_{0s})^6-\frac{55}{7}q^2(xr_{0s})^8-\frac{26}{3}q^2(xr_{0s})^{10}+...]
\end{aligned}
$$

$$E_3=\frac{l}{x}+A_\psi\frac{dr_\psi}{dx}=\frac{l}{x}+(\frac{1}{\rho_\psi}\frac{d\rho_\psi}{dx}-\frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx})$$

$$E_4 = \frac{l}{x}$$

$$F_1 = \frac{2l}{x} - \frac{d^2 r_\psi / dx^2}{dr_\psi / dx} + \frac{2}{r_\psi}\frac{dr_\psi}{dx}$$

$$= \frac{1}{x}[\,2(l+1) - 4c\,(xr_{0s})^3 - 7q^2h^2\,(xr_{0s})^4 - (24\,q^2 + 32\,c^2$$

$$+16\,c\,q\,)(xr_{0s})^6 \; - 40\,q^2\,(xr_{0s})^8 - 60\,q^2\,(xr_{0s})^{10} - ...]$$

$$F_2 = 2\frac{\rho_\psi}{\rho_c}\frac{A_\psi\,D\,x}{r_\psi^2}\left(\frac{dr_\psi}{dx}\right)^2$$

$$= 2\frac{\rho_\psi}{\rho_c}(\frac{1}{\rho_\psi}\frac{dP_\psi}{dx} - \frac{1}{\gamma\,P_\psi}\frac{dP_\psi}{dx})\frac{1}{x\,r_{0s}}[1 + \frac{4}{3}c\,(\,xr_{0s})^3 + 3q^2h^2\,(xr_{0s})^4 + (4q^2$$

$$+\frac{56}{9}c^2 + \frac{8}{3}cq)\,(xr_{0s})^6 \; + 5q^2\,(xr_{0s})^8 + 6q^2\,(xr_{0s})^{10} - ...]$$

$$F_3 = -\frac{4\pi\,G\rho_\psi^2}{\gamma\,P_\psi}(\frac{d\,r_\psi}{d\,x})^2$$

$$= -\frac{4\pi\,G\,r_{0s}^2\,D^2\rho_\psi^2}{\gamma\,P_\psi}[1 + \frac{16}{3}c\,(\,xr_{0s})^3 + 10q^2h^2\,(xr_{0s})^4 + (\frac{56}{5}q^2$$

$$+\frac{1384}{45}c^2 + \frac{112}{15}cq)\,(xr_{0s})^6 + \frac{90}{7}q^2\,(xr_{0s})^8 + \frac{44}{3}q^2\,(xr_{0s})^{10} + ...]$$

$$F_4 = \frac{l\,(l+1)}{x^2} - \frac{l}{x}(\frac{d^2 r_\psi}{dx^2})/(\frac{dr_\psi}{dx}) + \frac{2l}{x}(\frac{1}{r_\psi}\frac{dr_\psi}{dx}) - \frac{l\,(l+1)}{r_\psi^2}(\frac{dr_\psi}{dx})^2$$

$$= -\frac{l}{x^2}\{[\,8\,c\,(\,xr_{0s})^3 + 15q^2h^2\,(xr_{0s})^4 + (\frac{168}{5}q^2 + \frac{2412}{45}c^2$$

$$+\frac{336}{15}cq\,)\,(xr_{0s})^6 + \frac{360}{7}q^2\,(xr_{0s})^8 + \frac{220}{3}q^2\,(xr_{0s})^{10} + ...] + l\,[\,4\,c\,(\,xr_{0s})^3$$

$$+8q^2h^2\,(xr_{0s})^4 + (\frac{48}{5}q^2 + \frac{972}{45}c^2 \; + \frac{96}{15}cq\,)\,(xr_{0s})^6 \; + \frac{80}{7}q^2\,(xr_{0s})^8$$

$$+\frac{40}{3}q^2\,(xr_{0s})^{10} + ...]\}$$

and

$$h = 1 - [(b_0 + b_1 s^2)/(2n)^{1/2}\,] \text{ and } c = [(b_0 + b_1 s^2)\,(2n)^{1/2} - n\,].$$

Also as earlier σ is the eigenfrequency of oscillations, $x = r_0/r_{0s}$ and

$$\zeta = \frac{r_\psi^2\,\delta r_\psi}{D^3 x^{l+1}}\,,\; \eta = \frac{\mathrm{P}'_\psi}{2\,\pi\,\mathrm{G}\,\rho_\mathrm{c}\,D^2 x^l\,\rho_\psi} \text{ and } \phi = \frac{\psi'_g}{2\,\pi\,\mathrm{G}\rho_\mathrm{c}\,D^2 x^l}$$

δr_ψ being an average of the amplitudes of Lagrangian variations in the radial direction and P'_ψ, ψ'_g the amplitudes of Lagrangian variation in pressure and gravitational potential on the equipotential surface ψ^{**} = constant. In the above expressions all other symbols have their earlier meanings and terms up to second order of smallness in b_0, b_1, n, q and up to order r_{0s}^{10} in r_{0s} have been retained.

The eigenvalue problem (6.5) determining the eigenfrequencies of nonradial modes of oscillations of differentially rotating and tidally distorted stellar model has to be solved subject to the boundary conditions:

$$\eta + \phi = \frac{\sigma^2}{2\pi G \rho_c\, l\, r_{OS}} \zeta , \qquad \frac{d\phi}{dx} = 0 \tag{6.6}$$

at the centre $x = 0$ and

$$2\pi G \rho_c \rho_\psi D^2 r_{0s}^3 [1 + 4c\, r_{0s}^3 + 7q^2 h^2 r_{0s}^4 + (\frac{36}{5} q^2 + \frac{864}{45} c^2 + \frac{72}{15} cq) r_{0s}^6 + \frac{55}{7} q^2 r_{0s}^8 + \frac{26}{3} q^2 r_{0s}^{10} + ...]\eta + \frac{dP_\psi}{dx}\zeta = 0. \tag{6.7a}$$

and

$$\frac{d\theta_\psi}{dx} + \phi\{l + (l+1)\, [1 + 2c\, r_{0s}^3 + 4q^2 h^2\, r_{0s}^4 + (\frac{24}{5} q^2 + \frac{396}{45} c^2 + \frac{48}{15} cq) r_{0s}^6 + \frac{40}{7} q^2 r_{0s}^8 + \frac{20}{3} q^2 r_{0s}^{10} + ...]\} + \frac{2\rho_\psi}{\rho_c\, r_{0s}} [1 + \frac{4}{3} c\, r_{0s}^3 + (4q^2 + \frac{56}{9} c^2 + \frac{8}{3} cq) r_{0s}^6 + 5q^2 r_{0s}^8 + 6q^2 r_{0s}^{10} + ...]\zeta = 0 \tag{6.7b}$$

at the free surface $x = 1$.

In case effects of Coriolis force are not considered, the equation determining the eigenfrequencies of nonradial modes of oscillations of differentially rotating and tidally distorted stellar models can be obtained by replacing h and c in the above expressions by $h = 1 - [(b_0 + b_1 s^2)^2 / (1+q)]$ and $c = (b_0 + b_1 s^2)^2/2$ respectively

The system of differential equations (6.5) subject to the boundary conditions (6.6) at the centre and (6.7) at the free surface constitute an eigenvalue problem which incorporates the effects of Coriolis force on the eigenfrequencies of nonradial modes of oscillation of differentially rotating and tidally distorted stellar models in general.

6.3 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EFFECTS OF CORIOLIS FORCE ON THE EIGENFREQUENCIES OF SMALL ADIABATIC PSEUDO – RADIAL MODES OF OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

On substituting the values of P_ψ and ρ_ψ as defined by relation (3.22) of chapter III in equation (6.1), the eigenvalued boundary value problem determining the small adiabatic pseudo – radial modes of oscillations of differentially rotating and tidally distorted polytropic models can now be expressed as

$$H_1 \frac{d^2\kappa}{dr_0^{\,2}} + H_2 \frac{d\kappa}{dr_0} + (H_3\omega^2 - H_4)\kappa = 0 \tag{6.8}$$

where

$$H_1 = 1 - \frac{16}{3} c r_0^{\,3} - 10q^2h^2r_0^{\,4} - (\frac{56}{5}q^2 + \frac{104}{45}c^2 + \frac{112}{15}cq)r_0^{\,6} - \frac{90}{7}q^2r_0^{\,8} - \frac{44}{3}q^2r_0^{\,10} - ...$$

$$H_2 = [(4 - \frac{64}{3} c r_0^{\,3} - 44q^2h^2r_0^{\,4} - (\frac{296}{5}q^2 + \frac{1064}{45}c^2 + \frac{592}{15}cq)r_0^{\,6} - \frac{560}{7}q^2r_0^{\,8}$$
$$- \frac{316}{3}q^2r_0^{\,10} - ...) + (N+1)(\frac{1}{\theta_\psi}\frac{d\theta_\psi}{dx})r_0\, H_1]/r_0$$

$$H_3 = \frac{(N+1)\xi_u^{\,2}}{3\gamma r_{os}^{\,3}} K \left(\frac{\bar{\rho}}{\rho_c}\right)\frac{1}{\theta_\psi}$$

$$H_4 = -(3 - \frac{4}{\gamma})(N+1)(\frac{1}{\theta_\psi}\frac{d\theta_\psi}{dr_0})\frac{1}{r_0} \times$$
$$[1 - \frac{10}{3} c r_0^{\,3} - 6q^2h^2r_0^{\,4} - (\frac{32}{5}q^2 + \frac{188}{45}c^2 + \frac{64}{15}cq)r_0^{\,6} - \frac{50}{7}q^2r_0^{\,8} - 8q^2r_0^{\,10} - ...]$$

Also $\omega^2 = \dfrac{D^3 r_{os}^3 \sigma^2}{GM_0}$

and

$$h = 1 - [(b_0 + b_1 s^2)/(2n)^{1/2}] \text{ and } c = [(b_0 + b_1 s^2)(2n)^{1/2} - n].$$

Here ω is the nondimensional form of the eigenfrequency.

Equation (6.8) is the general equation in nondimensional form which determines the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of differentially rotating and tidally distorted polytropic model when terms up to second order of smallness in b_0, b_1, n, q and up to r_0^{10} in r_0 are retained. This equation incorporates the effects of Coriolis force besides the effects of gravitational and centrifugal forces.

The eigenvalued boundary value problem (6.8) which determines the eigenfrequencies of small adiabatic pseudo – radial modes of oscillations of differentially rotating and tidally distorted polytropic models is of Sturm – Liouville type. As explained earlier equation (6.8) has to be integrated numerically subject to the boundary conditions which require κ to be finite at points corresponding to the center and the free surface of the model. The values of θ_ψ and $d\theta_\psi/(dx)$ needed at various points are to be taken from the numerical solution of the equation (5.40) of chapter V.

Following the procedure as explained in section 4.5 of chapter IV computations have been performed to compute the effect of Coriolis force on the eigenvalues of the fundamental and the first mode of pseudo – radial oscillations of differentially rotating and tidally distorted polytropic models of indices 1.5 and 3.0 for those values of distortion parameters b_0, b_1, n, q for which equilibrium structures were earlier obtained in Chapter V. The results are presented in Table 6.1 – 6.3. For comparison we also present in this table the corresponding results when effects of Coriolis force are not considered.

6.4 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EFFECTS OF CORIOLIS FORCE ON THE EIGENFREQUENCIES OF SMALL ADIABATIC NONRADIAL MODES OF OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

On substituting the values of P_ψ and ρ_ψ from relations (3.22) of Chapter III in (6.5), the system of differential equations governing the nonradial modes of oscillations of a differentially rotating and tidally distorted polytropic model, may be expressed as

$$\left.\begin{aligned}
&\frac{d\zeta}{dx} + B_1\zeta + (B_2 + \frac{B_3}{\omega^2})\eta + \frac{B_3}{\omega^2}\phi = 0 \\
&\frac{d\eta}{dx} + (E_1\omega^2 + E_2)\zeta + E_3\eta + E_4\phi + \frac{d\phi}{dx} = 0 \\
&\frac{\mathrm{d}^2\phi}{dx^2} + F_1\frac{d\phi}{dx} + F_2\zeta + F_3\eta + F_4\phi = 0
\end{aligned}\right\} \qquad (6.9)$$

where

$$B_1 = \frac{l+1}{x} + \frac{N+1}{\gamma}\left(\frac{1}{\theta_\psi}\frac{d\theta_\psi}{dx}\right)$$

$$B_2 = \frac{(N+1)\xi_u^{\,2}\, xr_{0s}^3}{2\gamma K^2 \theta_\psi}[1+4c(xr_{0s})^3+7q^2h^2(xr_{0s})^4+(\frac{36}{5}q^2+\frac{864}{45}c^2$$

$$+\frac{24}{5}cq)(xr_{0s})^6+\frac{55}{7}q^2(xr_{0s})^8+\frac{26}{3}q^2(xr_{0s})^{10}+...]$$

$$B_3 = \frac{-3l(l+1)r_{0s}^4}{2K^3x}(\frac{\rho_c}{\overline{\rho}})[1+\frac{8}{3}c(xr_{0s})^3+5q^2h^2(xr_{0s})^4+(\frac{28}{5}q^2+\frac{532}{45}c^2$$

$$+\frac{56}{15}cq)(xr_{0s})^6+\frac{45}{7}q^2(xr_{0s})^8+\frac{22}{3}q^2(xr_{0s})^{10}+...]$$

$$E_1 = \frac{-2K^3}{3xr_{0s}^4}(\frac{\overline{\rho}}{\rho_c})[1+\frac{4}{3}c(xr_{0s})^3+3q^2h^2(xr_{0s})^4+(4q^2+\frac{56}{9}c^2$$

$$+\frac{8}{3}cq)(xr_{0s})^6+5q^2(xr_{0s})^8+6q^2(xr_{0s})^{10}+...]$$

$$E_2 = \frac{2K^2}{\xi_u^{\,2}}(N-\frac{N+1}{\gamma})\frac{1}{\theta_\psi}(\frac{d\theta_\psi}{dx})^2\frac{1}{xr_{0s}^3}[1-4c(xr_{0s})^3-7q^2h^2(xr_{0s})^4-(\frac{36}{5}q^2$$

$$+\frac{144}{45}c^2+\frac{72}{15}cq)(xr_{0s})^6-\frac{55}{7}q^2(xr_{0s})^8-\frac{26}{3}q^2(xr_{0s})^{10}+...]$$

$$E_3 = \frac{l}{x}+(N-\frac{N+1}{\gamma})\frac{1}{\theta_\psi}(\frac{d\theta_\psi}{dx}),$$

$$E_4 = \frac{l}{x}$$

$$F_1 = \frac{1}{x}[2(l+1)-4c(xr_{0s})^3-7q^2h^2(xr_{0s})^4-(24q^2+32c^2$$

$$+16cq)(xr_{0s})^6-40q^2(xr_{0s})^8-60q^2(xr_{0s})^{10}+...]$$

$$F_2 = \frac{2}{xr_{0s}}(N-\frac{N+1}{\gamma})\theta_\psi^{\,N-1}\frac{d\theta_\psi}{dx}[1+\frac{4}{3}c(xr_{0s})^3+3q^2h^2(xr_{0s})^4+(4q^2$$

$$+\frac{56}{9}c^2+\frac{8}{3}cq)(xr_{0s})^6+5q^2(xr_{0s})^8+6q^2(xr_{0s})^{10}+...]$$

$$F_3 = -\frac{(N+1)}{\gamma}\frac{\xi_u^{\,2}}{K^2}\theta_\psi^{\,N-1}r_{0s}^2[1+\frac{16}{3}c(xr_{0s})^3+10q^2h^2(xr_{0s})^4+(\frac{56}{5}q^2$$

$$+\frac{1384}{45}c^2+\frac{112}{15}cq)(xr_{0s})^6+\frac{90}{7}q^2(xr_{0s})^8+\frac{44}{3}q^2(xr_{0s})^{10}+...]$$

$$F_4 = -\frac{l}{x^2}\{[8c(xr_{0s})^3 + 15q^2h^2(xr_{0s})^4 + (\frac{168}{5}q^2 + \frac{2412}{45}c^2 + \frac{336}{15}cq)(xr_{0s})^6$$
$$+\frac{360}{7}q^2(xr_{0s})^8 + \frac{220}{3}q^2(xr_{0s})^{10} + ...] + l[4c(xr_{0s})^3 + 8q^2h^2(xr_{0s})^4$$
$$+(\frac{48}{5}q^2 + \frac{972}{45}c^2 + \frac{96}{15}cq)(xr_{0s})^6 + \frac{80}{7}q^2(xr_{0s})^8 + \frac{40}{3}q^2(xr_{0s})^{10} + ...]\}$$

and $\omega^2 = \dfrac{Dr_{0s}{}^3\sigma^2}{GM_0}$ (ω being the nondimensional form of the eigenfrequency σ)

Also as earlier

$$h = 1 - [(b_0 + b_1 s^2)/(2n)^{1/2}] \text{ and } c = [(b_0 + b_1 s^2)(2n)^{1/2} - n].$$

The boundary conditions (6.6) at the centre ($x = 0$) for the case of the distorted polytropic model become

$$\eta + \phi = \frac{2\omega^2}{3 l r_{0s}^4}\left(\frac{\bar{\rho}}{\rho_c}\right)\zeta, \qquad \frac{d\phi}{dx} = 0 \tag{6.10}$$

On substituting the values of P_ψ and ρ_ψ from equation (3.22) in the boundary conditions (6.7) at the free surface ($x = 1$), the boundary conditions at the free surface in the case of polytropic models become

$$\eta r_{0s}^3[1 + 4c(xr_{0s})^3 + 7q^2h^2(xr_{0s})^4 + (\frac{36}{5}q^2 + \frac{864}{45}c^2 + \frac{72}{15}cq)(xr_{0s})^6$$
$$+\frac{55}{7}q^2(xr_{0s})^8 + \frac{26}{3}q^2(xr_{0s})^{10} + ...] + 2\frac{K^2}{\xi_u^2}\frac{d\theta_\psi}{dx}\zeta = 0 \tag{6.11a}$$

and

$$\frac{d\theta_\psi}{dx} + \phi\{l + (l+1)[1 + 2c(xr_{0s})^3 + 4q^2h^2(xr_{0s})^4 + (\frac{24}{5}q^2 + \frac{396}{45}c^2$$
$$+\frac{48}{15}cq)(xr_{0s})^6 + \frac{40}{7}q^2(xr_{0s})^8 + \frac{20}{3}q^2(xr_{0s})^{10} + ...]\} = 0 \tag{6.11b}$$

at the surface $x = 1$.

The system of differential equations (6.9) together with the boundary conditions (6.10 – 6.11) constitutes the eigenvalued boundary value problem which may be used to compute the effects of Coriolis force on the eigenfrequencies of nonradial modes of oscillations of polytropic models of stars.

In order to compute the eigenfrequencies of nonradial modes of oscillation of differentially rotating and tidally distorted stellar models which incorporates the effects of

Coriolis force, system of differential equations (6.9) has to solved subject to the boundary conditions (6.10) at the centre and the boundary conditions (6.11) at the free surface. The values of θ_ψ and $d\theta_\psi/(dx)$ needed at various points in the interior of the model were obtained from the solutions of the structure equation (5.40) of these models earlier obtained in Chapter V.

Following the procedure as explained in section 4.5 of chapter IV computations have been performed to compute the effect of Coriolis force on the eigenfrequencies of the fundamental, gravitational and pressure modes of nonradial oscillations of differentially rotating and tidally distorted polytropic models of indices 1.5 and 3.0 for those values of distortion parameters b_0, b_1, n, q for which equilibrium structures were earlier obtained in Chapter V. The results are presented in Table 6.1 – 6.9. The corresponding results when the effects of Coriolis force are not considered are also presented in these tables for comparison. In Table 6.10 – 6.11 we have compared our results when the effect of Coriolis force has been considered with the corresponding results obtained by using the law (5.8) of differential rotation and methodology earlier used by Mohan et al (107) in which the effect of Coriolis force has not been considered.

6.5 ANALYSIS OF THE RESULTS

Results in Tables 6.1 – 6.2, 6.4 – 6.5 and 6.7 – 6.8 show that with the inclusion of Coriolis force, the eigenfrequencies of radial and nonradial modes of oscillations of differentially rotating and tidally distorted polytropes in synchronous and nonsynchronous binaries do not show any appreciable change as compared to the corresponding values when the effect of Coriolis force is not considered. It is true for all the differentially rotating and tidally distorted polytropic models of polytropic indices 1.5 and 3.0.

However in case of purely rotating stars (Table 6.3, 6.6 and 6.9) some change (up to 6.2%) can be noticed in the values of eigenfrequencies for the model (3) which has high angular velocity of rotation near the surface.

It has also been observed from the Table 6.9, that the percentage change in the values of p – modes is more than the values of g - modes for nonradial modes of oscillations of differentially rotating polytropic models of indices 3.0.

From Table 6.10 – 6.11, we find that in case of purely rotating stars, there is not any appreciable difference in the present results when the effect of Coriolis force has been taken into account as compared to the corresponding results obtained by using the approach of Mohan et al. (107) in which the effect of Coriolis force has not been considered.

Thus our present study has shown that, in general, with the inclusion of Coriolis force there is not any appreciable change in the values of eigenfrequencies of radial and nonradial modes of oscillations of differentially rotating and tidally distorted polytropic models in purely rotating stars as well as synchronous and nonsynchronous binaries. However some effect is observed in the case of purely rotating stars which has high angular velocity of rotation near the surface.

Table 6.1 Eigenfrequencies ω_0^2 for the fundamental mode and ω_1^2 for the first mode of pseudo – radial oscillations of differentially rotating and tidally distorted polytropic models of stars

Model	θ	ω_0^2	ω_1^2	ω_0^2	ω_1^2
		$N=1.5$		$N=3.0$	
Nonsynchronous Binaries					
$n=0.005,\ q=0.1$					
1	0	2.7023 (2.6986)	12.5076 (12.4796)	9.2438 (9.2325)	16.9544 (16.9202)
	30	2.7022 (2.6985)	12.5066 (12.4787)	9.2433 (9.2320)	16.9544 (16.9190)
	60	2.7019 (2.6982)	12.5047 (12.4768)	9.2429 (9.2311)	16.9513 (16.9167)
	90	2.7018 (2.6980)	12.5036 (12.4758)	9.2424 (9.2307)	16.9488 (16.9155)
2	0	2.7023 (2.6986)	12.5076 (12.4796)	9.2438 (9.2325)	16.9544 (16.9202)
	30	2.7007 (2.6971)	12.4966 (12.4695)	9.2391 (9.2276)	16.9389 (16.9075)
	60	2.6961 (2.6931)	12.4638 (12.4425)	9.2243 (9.2147)	16.8987 (16.8736)
	90	2.6931 (2.6906)	12.4421 (12.4256)	9.2146 (9.2066)	16.8721 (16.8530)
3	0	2.7023 (2.6986)	12.5076 (12.4796)	9.2438 (9.2325)	16.9544 (16.9202)
	30	2.6987 (2.6953)	12.4820 (12.4571)	9.2325 (9.2216)	16.9209 (16.8920)
	60	2.6856 (2.6846)	12.3879 (12.3851)	9.1906 (9.1871)	16.8059 (16.8026)
	90	2.6762 (2.6772)	12.3191 (12.3355)	9.1605 (9.1632)	16.7222 (16.7392)
4	0	2.7023 (2.6986)	12.5076 (12.4796)	9.2438 (9.2325)	16.9544 (16.9202)
	30	2.7030 (2.6992)	12.5118 (12.4838)	9.2458 (9.2345)	16.9577 (16.9255)
	60	2.7039 (2.7002)	12.5175 (12.4906)	9.2487 (9.2377)	16.9648 (16.9340)
	90	2.7041 (2.7006)	12.5190 (12.4931)	9.2496 (9.2389)	16.9668 (16.9371)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 6.2 Eigenfrequencies ω_0^2 for the fundamental mode and ω_1^2 for the first mode of pseudo – radial oscillations of differentially rotating and tidally distorted polytropic models of stars

Model	θ	ω_0^2	ω_1^2	ω_0^2	ω_1^2
		$N=1.5$		$N=3.0$	
Synchronous Binaries					
$n=0.525,\ q=0.05$					
$b_0=1.0247$ $b_1=0.01$	0	2.4157 (2.4157)	10.5457 (10.5457)	8.2649 (8.2649)	14.4564 (14.4564)
	30	2.4143 (2.4143)	10.5356 (10.5356)	8.2596 (8.2596)	14.4387 (14.4387)
	60	2.4114 (2.4114)	10.5154 (10.5153)	8.2490 (8.2490)	14.4124 (14.4123)
	90	2.4100 (2.4099)	10.5053 (10.5051)	8.2438 (8.2437)	14.3993 (14.3990)
$b_0=1.0247$ $b_1=0.1$	0	2.4157 (2.4157)	10.5457 (10.5457)	8.2649 (8.2649)	14.4564 (14.4564)
	30	2.4013 (2.4012)	10.4446 (10.4433)	8.2125 (8.2119)	14.3201 (14.3185)
	60	2.3725 (2.3709)	10.2406 (10.2292)	8.1045 (8.0985)	14.0509 (14.0355)
	90	2.3581 (2.3552)	10.1377 (10.1163)	8.0496 (8.0387)	13.9129 (13.8855)
$b_0=1.0247$ $b_1=0.2$	0	2.4157 (2.4157)	10.5457 (10.5457)	8.2649 (8.2649)	14.4564 (14.4564)
	30	2.3869 (2.3862)	10.3429 (10.3378)	8.1588 (8.1561)	14.1862 (14.1795)
	60	2.3290 (2.3225)	9.9301 (9.8835)	7.9384 (7.9131)	13.6307 (13.5664)
	90	2.2998 (2.2882)	9.7197 (9.6356)	7.8242 (7.7777)	13.3416 (13.2224)
$b_0=1.0247$ $b_1=-0.05$	0	2.4157 (2.4157)	10.5457 (10.5457)	8.2649 (8.2649)	14.4564 (14.4564)
	30	2.4229 (2.4229)	10.5960 (10.5957)	8.2912 (8.2910)	14.5182 (14.5179)
	60	2.4372 (2.4368)	10.6964 (10.6936)	8.3434 (8.3420)	14.6515 (14.6476)
	90	2.4444 (2.4437)	10.7464 (10.7415)	8.3693 (8.3668)	14.7153 (14.7088)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 6.3 Eigenfrequencies ω_0^2 for the fundamental mode and ω_1^2 for the first mode of pseudo – radial oscillations of polytropic models of differentially rotating stars

Model	θ	ω_0^2	ω_1^2	ω_0^2	ω_1^2
		$N=1.5$		$N=3.0$	
Purely Rotating stars					
$n=0.005,\ q=0.0$					
1	0	2.6841 (2.6841)	12.3876 (12.3876)	9.1853 (9.1853)	16.8029 (16.8029)
	30	2.6830 (2.6830)	12.3802 (12.3802)	9.1818 (9.1817)	16.7930 (16.7975)
	60	2.6809 (2.6807)	12.3656 (12.3647)	9.1747 (9.1743)	16.7753 (16.7745)
	90	2.6798 (2.6796)	12.3582 (12.3568)	9.1712 (9.1705)	16.7687 (16.7674)
2	0	2.6841 (2.6841)	12.3876 (12.3876)	9.1853 (9.1853)	16.8029 (16.8029)
	30	2.6732 (2.6719)	12.3143 (12.3051)	9.1500 (9.1456)	16.7151 (16.7003)
	60	2.6514 (2.6391)	12.1672 (12.0843)	9.0787 (9.0384)	16.5276 (16.4252)
	90	2.6405 (2.6186)	12.0935 (11.9456) [1.2]	9.0429 (8.9706)	16.4366 (16.2512) [1.1]
3	0	2.6841 (2.6841)	12.3876 (12.3876)	9.1853 (9.1853)	16.8029 (16.8029)
	30	2.6623 (2.6569)	12.2407 (12.2040)	9.1144 (9.0966)	16.6194 (16.5762)
	60	2.6186 (2.5692) [1.9]	11.9456 (11.6104) [2.9]	8.9706 (8.8056) [1.9]	16.2512 (15.8263) [2.7]
	90	2.5967 (2.5086) [3.5]	11.7970 (11.1954) [5.4]	8.8975 (8.5970) [3.5]	16.0623 (15.2954) [5.0]
4	0	2.6841 (2.6841)	12.3876 (12.3876)	9.1853 (9.1853)	16.8029 (16.8029)
	30	2.6896 (2.6892)	12.4242 (12.4219)	9.2029 (9.2018)	16.8496 (16.8465)
	60	2.7005 (2.6974)	12.4973 (12.4767)	9.2361 (9.2282)	16.9404 (16.9146)
	90	2.7059 (2.7005)	12.5339 (12.4973)	9.2550 (9.2380)	16.9885 (16.9404)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered.
3. Values in the square bracket represent the percent difference of two corresponding values. Only those values are given in which the percentage difference is greater than 1%.

Table 6.4 Eigenfrequencies ω^2 for the nonradial modes of oscillations of differentially rotating and tidally distorted polytropic models of stars

Model	θ	f	p_1	p_2	p_3
$N=1.5$					
Nonsynchronous Binaries					
$n=0.005,\ q=0.1$					
1	0	2.4172 (2.4114)	10.2663 (10.2402)	23.4596 (23.4000)	41.2000 (41.1000)
	30	2.4170 (2.4112)	10.2654 (10.2394)	23.4576 (23.3981)	41.2000 (41.1000)
	60	2.4166 (2.4108)	10.2636 (10.2377)	23.4535 (23.3944)	41.1866 (41.0838)
	90	2.4164 (2.4106)	10.2626 (10.2368)	23.4514 (23.3924)	41.1833 (41.0806)
2	0	2.4172 (2.4114)	10.2663 (10.2402)	23.4596 (23.4000)	41.2000 (41.1000)
	30	2.4150 (2.4094)	10.2562 (10.2311)	23.4368 (23.3796)	41.1579 (41.0580)
	60	2.4083 (2.4040)	10.2262 (10.2068)	23.3687 (23.3248)	41.0398 (40.9631)
	90	2.4039 (2.4006)	10.2064 (10.1916)	23.3238 (23.2902)	40.9609 (40.9000)
3	0	2.4172 (2.4114)	10.2663 (10.2402)	23.4596 (23.4000)	41.2000 (41.1000)
	30	2.4120 (2.4069)	10.2429 (10.2200)	23.4066 (23.3541)	41.1000 (41.0148)
	60	2.3929 (2.3926)	10.1573 (10.1551)	23.2116 (23.2076)	40.7677 (40.7608)
	90	2.3792 (2.3827)	10.0959 (10.1104)	23.0717 (23.1064)	40.5253 (40.5855)
4	0	2.4172 (2.4114)	10.2663 (10.2402)	23.4596 (23.4000)	41.2000 (41.1000)
	30	2.4181 (2.4122)	10.2700 (10.2440)	23.4680 (23.4087)	41.2000 (41.1000)
	60	2.4192 (2.4135)	10.2751 (10.2501)	23.4798 (23.4225)	41.2320 (41.1328)
	90	2.4194 (2.4141)	10.2763 (10.2523)	23.4825 (23.4278)	41.2371 (41.1413)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 6.5 Eigenfrequencies ω^2 for the nonradial modes of oscillations of differentially rotating and tidally distorted polytropic models of stars

Model	θ	f	p_1	p_2	p_3
$N=1.5$					
Synchronous Binaries					
$n=0.525,\ q=0.05$					
$b_0=1.0247$ $b_1=0.01$	0	2.0446 (2.0448)	8.5771 (8.5771)	19.6301 (19.6301)	34.7000 (34.7000)
	30	2.0429 (2.0430)	8.5692 (8.5692)	19.6120 (19.6119)	34.6843 (34.6839)
	60	2.0395 (2.0396)	8.5535 (8.5534)	19.5464 (19.5762)	34.6261 (34.6257)
	90	2.0377 (2.0378)	8.5456 (8.5455)	19.5587 (19.5583)	34.6000 (34.6000)
$b_0=1.0247$ $b_1=0.1$	0	2.0446 (2.0448)	8.5771 (8.5771)	19.6301 (19.6301)	34.7000 (34.7000)
	30	2.0274 (2.0273)	8.4987 (8.4977)	19.4520 (19.4499)	34.4240 (34.4205)
	60	1.9933 (1.9916)	8.3439 (8.3354)	19.1000 (19.0809)	33.8578 (33.8266)
	90	1.9765 (1.9733)	8.2676 (8.2526)	18.9269 (18.8927)	33.5808 (33.5271)
$b_0=1.0247$ $b_1=0.2$	0	2.0446 (2.0448)	8.5771 (8.5771)	19.6301 (19.6301)	34.7000 (34.7000)
	30	2.0103 (2.0096)	8.4209 (8.4171)	19.2753 (19.2668)	34.1389 (34.1249)
	60	1.9434 (1.9363)	8.1170 (8.0841)	18.5847 (18.5099)	33.0400 (32.9227)
	90	1.9109 (1.8984)	7.9694 (7.9121)	18.2497 (18.1194)	32.5168 (32.3158)
$b_0=1.0247$ $b_1=-0.05$	0	2.0446 (2.0448)	8.5771 (8.5771)	19.6301 (19.6301)	34.7000 (34.7000)
	30	2.0533 (2.0534)	8.6165 (8.6163)	19.7195 (19.7190)	34.8597 (34.8593)
	60	2.0708 (2.0704)	8.6959 (8.6937)	19.9000 (19.8946)	35.1549 (35.1464)
	90	2.0796 (2.0789)	8.7358 (8.7320)	19.9904 (19.9816)	35.3000 (35.3000)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 6.6 Eigenfrequencies ω^2 for the nonradial modes of oscillations of differentially rotating polytropic models of stars

Model	θ	f	p_1	p_2	p_3
$N = 1.5$					
Purely Rotating stars					
$n = 0.005,\ q = 0.0$					
1	0	2.3931 (2.3931)	10.1582 (10.1582)	23.2157 (23.2157)	40.7749 (40.7749)
	30	2.3916 (2.3916)	10.1516 (10.1515)	23.2006 (23.2004)	40.7485 (40.7485)
	60	2.3887 (2.3885)	10.1382 (10.1375)	23.1705 (23.1687	40.6963 (40.6932)
	90	2.3872 (2.3869)	10.1316 (10.1303)	23.1554 (23.1524)	40.6705 (40.6653)
2	0	2.3931 (2.3931)	10.1582 (10.1582)	23.2157 (23.2157)	40.7749 (40.7749)
	30	2.3784 (2.3765)	10.0917 (10.0833)	23.0651 (23.0462)	40.5143 (40.4814)
	60	2.3488 (2.3324)	9.9585 (9.8836)	22.7635 (22.5940)	39.9933 (39.7000)
	90	2.3342 (2.3049) [1.3]	9.8919 (9.7589) [1.4]	22.6127 (22.3114) [1.4]	39.7336 (39.2152) [1.3]
3	0	2.3931 (2.3931)	10.1582 (10.1582)	23.2157 (23.2157)	40.7749 (40.7749)
	30	2.3637 (2.3563)	10.0251 (9.9917)	22.9143 (22.8388)	40.2536 (40.1234)
	60	2.3049 (2.2394) [2.9]	9.7589 (9.4615) [3.1]	22.3115 (21.6374) [3.1]	39.2152 (38.0633) [3.0]
	90	2.2757 (2.1606) [5.3]	9.6264 (9.1039) [5.7]	22.0111 (20.8261) [5.7]	38.7000 (36.6919) [5.5]
4	0	2.3931 (2.3931)	10.1582 (10.1582)	23.2157 (23.2157)	40.7749 (40.7749)
	30	2.4004 (2.4000)	10.1915 (10.1894)	23.2909 (23.2862)	40.9051 (40.8966)
	60	2.4151 (2.4110)	10.2579 (10.2393)	23.4412 (23.3990)	41.1656 (41.0923)
	90	2.4225 (2.4151)	10.2911 (10.2579)	23.5163 (23.4412)	41.2954 (41.1656)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered
3. Values in the square bracket represent the percent difference of two corresponding values. Only those values are given in which the percentage difference is greater than 1%.

Table 6.7 Eigenfrequencies ω^2 for the nonradial modes of oscillations of differentially rotating and tidally distorted polytropic models of stars

Model	θ	g_3	g_2	g_1	f	p_1	p_2	p_3
$N=3.0$								
Nonsynchronous Binaries								
$n=0.005,\ q=0.1$								
1	0	1.8345 (1.8336)	2.8303 (2.8289)	4.9236 (4.9208)	8.2364 (8.2264)	15.2504 (15.2158)	26.6649 (26.5975)	41.3727 (41.2641)
	30	1.8345 (1.8335)	2.8302 (2.8288)	4.9236 (4.9207)	8.2360 (8.2260)	15.2492 (15.2147)	26.6625 (26.5954)	41.3695 (41.2610)
	60	1.8344 (1.8334)	2.8301 (2.8288)	4.9233 (4.9205)	8.2352 (8.2252)	15.2467 (15.2124)	26.6579 (26.5910)	41.3612 (41.2537)
	90	1.8343 (1.8333)	2.8300 (2.8287)	4.9231 (4.9204)	8.2349 (8.2249)	15.2454 (15.2112)	26.6553 (26.5887)	41.3575 (41.2501)
2	0	1.8345 (1.8336)	2.8303 (2.8289)	4.9236 (4.9208)	8.2364 (8.2264)	15.2504 (15.2158)	26.6649 (26.5975)	41.3727 (41.2641)
	30	1.8341 (1.8330)	2.8296 (2.8284)	4.9224 (4.9197)	8.2321 (8.2223)	15.2367 (15.2033)	26.6387 (26.5728)	41.3307 (41.2265)
	60	1.8334 (1.8325)	2.8280 (2.8269)	4.9189 (4.9166)	8.2195 (8.2115)	15.1962 (15.1698)	26.5609 (26.5101)	41.2061 (41.1243)
	90	1.8326 (1.8318)	2.8268 (2.8259)	4.9166 (4.9140)	8.2114 (8.2047)	15.1694 (15.1489)	26.5091 (26.4703)	41.1230 (41.0614)
3	0	1.8345 (1.8336)	2.8303 (2.8289)	4.9236 (4.9208)	8.2364 (8.2264)	15.2504 (15.2158)	26.6649 (26.5975)	41.3727 (41.2641)
	30	1.8334 (1.8332)	2.8289 (2.8277)	4.9208 (4.9182)	8.2264 (8.2174)	15.2186 (15.1879)	26.6040 (26.5444)	41.2750 (41.1798)
	60	1.8303 (1.8300)	2.8241 (2.8237)	4.9104 (4.9095)	8.1913 (8.1885)	15.1029 (15.0986)	26.3808 (26.3746)	40.9168 (40.9090)
	90	1.8282 (1.8284)	2.8208 (2.8210)	4.9037 (4.9040)	8.1660 (8.1686)	15.0193 (15.0369)	26.2194 (26.2572)	40.6584 (40.7213)
4	0	1.8345 (1.8336)	2.8303 (2.8289)	4.9236 (4.9208)	8.2364 (8.2264)	15.2504 (15.2158)	26.6649 (26.5975)	41.3727 (41.2641)
	30	1.8348 (1.8337)	2.8305 (2.8291)	4.9241 (4.9213)	8.2380 (8.2281)	15.2556 (15.2210)	26.6748 (26.6076)	41.3887 (41.2799)
	60	1.8351 (1.8341)	2.8308 (2.8296)	4.9249 (4.9222)	8.2405 (8.2308)	15.2626 (15.2294)	26.6881 (26.6235)	41.4095 (41.3056)
	90	1.8351 (1.8341)	2.8309 (2.8297)	4.9251 (4.9224)	8.2413 (8.2318)	15.2645 (15.2325)	26.6913 (26.6293)	41.4147 (41.3148)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 6.8 Eigenfrequencies ω^2 for the nonradial modes of oscillations of differentially rotating and tidally distorted polytropic models of stars

Model	θ	g_3	g_2	g_1	f	p_1	p_2	p_3
$N=3.0$								
Synchronous Binaries								
$n=0.525,\ q=0.05$								
$b_0=1.0247$ $b_1=0.01$	0	1.7659 (1.7659)	2.7191 (2.7191)	4.6921 (4.6921)	7.4417 (7.4416)	12.8352 (12.8352)	22.0632 (22.0632)	34.0640 (34.0640)
	30	1.7655 (1.7655)	2.7186 (2.7185)	4.6908 (4.6908)	7.4377 (7.4377)	12.8232 (12.8232)	22.0403 (22.0404)	34.0284 (34.0279)
	60	1.7647 (1.7647)	2.7174 (2.7174)	4.6884 (4.6883)	7.4297 (7.4294)	12.7993 (12.7991)	21.9949 (21.9947)	33.9573 (33.9561)
	90	1.7643 (1.7643)	2.7169 (2.7169)	4.6872 (4.6872)	7.4254 (7.4254)	12.7874 (12.7871)	21.9723 (21.9718)	33.9217 (33.9208)
$b_0=1.0247$ $b_1=0.1$	0	1.7659 (1.7659)	2.7191 (2.7191)	4.6921 (4.6921)	7.4417 (7.4416)	12.8352 (12.8352)	22.0632 (22.0632)	34.0640 (34.0640)
	30	1.7621 (1.7621)	2.7137 (2.7137)	4.6798 (4.6796)	7.4011 (7.4004)	12.7157 (12.7143)	21.8358 (21.3830)	33.7087 (33.7044)
	60	1.7555 (1.7551)	2.7000 (2.7000)	4.6556 (4.6543)	7.3191 (7.3146)	12.4773 (12.4641)	21.3821 (21.3571)	33.0813 (32.9646)
	90	1.7517 (1.7514)	2.7000 (2.7000)	4.6434 (4.6410)	7.2779 (7.2699)	12.3583 (12.3348)	21.1561 (21.1115)	32.7432 (32.5849)
$b_0=1.0247$ $b_1=0.2$	0	1.7659 (1.7659)	2.7191 (2.7191)	4.6921 (4.6921)	7.4417 (7.4416)	12.8352 (12.8352)	22.0632 (22.0632)	34.0640 (34.0640)
	30	1.7587 (1.7585)	2.7000 (2.7000)	4.6675 (4.6668)	7.3602 (7.3581)	12.5963 (12.5905)	21.6087 (21.5978)	33.3544 (33.3374)
	60	1.7455 (1.7432)	2.6855 (2.6829)	4.6186 (4.6132)	7.1951 (7.1768)	12.1210 (12.0684)	20.7054 (20.6058)	31.9624 (31.8106)
	90	1.7396 (1.7367)	2.6736 (2.6690)	4.5938 (4.5838)	7.1117 (7.0786)	11.8846 (11.7917)	20.2569 (20.0809)	31.2831 (31.0194)
$b_0=1.0247$ $b_1=-0.05$	0	1.7659 (1.7659)	2.7191 (2.7191)	4.6921 (4.6921)	7.4417 (7.4416)	12.8352 (12.8352)	22.0632 (22.0632)	34.0640 (34.0640)
	30	1.7678 (1.7678)	2.7217 (2.7216)	4.6976 (4.6976)	7.4619 (7.4617)	12.8949 (12.8946)	22.1769 (22.1763)	34.2423 (34.2412)
	60	1.7717 (1.7717)	2.7270 (2.7269)	4.7094 (4.7092)	7.5025 (7.5011)	13.0146 (13.0113)	22.4047 (22.3987)	34.6003 (34.5907)
	90	1.7738 (1.7735)	2.7297 (2.7294)	4.7158 (4.7147)	7.5228 (7.5207)	13.0745 (13.0687)	22.5187 (22.5075)	34.7796 (34.7622)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 6.9 Eigenfrequencies ω^2 for the nonradial modes of oscillations of purely rotating polytropic models of stars

Model	θ	g_3	g_2	g_1	f	p_1	p_2	p_3
$N=3.0$								
Purely rotating stars								
$n=0.005,\ q=0.0$								
1	0	1.8307 (1.8307)	2.8234 (2.8234)	4.9094 (4.9094)	8.1872 (8.1872)	15.1004 (15.1004)	26.3812 (26.3812)	40.9216 (40.9217)
	30	1.8305 (1.8305)	2.8231 (2.8230)	4.9086 (4.9087)	8.1843 (8.1843)	15.0913 (15.0912)	26.3639 (26.3635)	40.8940 (40.8938)
	60	1.8299 (1.8299)	2.8222 (2.8222)	4.9068 (4.9068)	8.1784 (8.1781)	15.0730 (15.0720)	26.3289 (26.3270)	40.8380 (40.8354)
	90	1.8298 (1.8296)	2.8218 (2.8217)	4.9063 (4.9060)	8.1754 (8.1748)	15.0639 (15.0620)	26.3115 (26.3081)	40.8104 (40.8048)
2	0	1.8307 (1.8307)	2.8234 (2.8234)	4.9094 (4.9094)	8.1872 (8.1872)	15.1004 (15.1004)	26.3812 (26.3812)	40.9216 (40.9217)
	30	1.8281 (1.8279)	2.8194 (2.8191)	4.9009 (4.8999)	8.1577 (8.1540)	15.0089 (14.9975)	26.2069 (26.1852)	40.6436 (40.6092)
	60	1.8230 (1.8200)	2.8115 (2.8071)	4.8838 (4.8741)	8.0983 (8.0649)	14.8251 (14.7215)	25.8567 (25.6591)	40.0853 (39.7702)
	90	1.8203 (1.8152)	2.8075 (2.8000)	4.8750 (4.8576)	8.0685 (8.0085)	14.7329 (14.5480) [1.3]	25.6811 (25.3285) [1.4]	39.8053 (39.2428) [1.4]
3	0	1.8307 (1.8307)	2.8234 (2.8234)	4.9094 (4.9094)	8.1872 (8.1872)	15.1004 (15.1004)	26.3812 (26.3812)	40.9216 (40.9217)
	30	1.8254 (1.8243)	2.8154 (2.8134)	4.8924 (4.8881)	8.1281 (8.1130)	14.9171 (14.8712)	26.0321 (25.9443)	40.3650 (40.2251)
	60	1.8153 (1.8035)	2.8000 (2.7789)	4.8576 (4.8180)	8.0085 (7.8724) [1.7]	14.5480 (14.1300) [3.0]	25.3285 (24.5316) [3.2]	39.2428 (37.9721) [3.3]
	90	1.8100 (1.7892) [1.2]	2.7904 (2.7544) [1.3]	4.8401 (4.7689) [1.5]	7.9481 (7.7033) [3.2]	14.3626 (13.6170) [5.5]	24.9749 (23.5535) [6.0]	38.6788 (36.4174) [6.2]
4	0	1.8307 (1.8307)	2.8234 (2.8234)	4.9094 (4.9094)	8.1872 (8.1872)	15.1004 (15.1004)	26.3812 (26.3812)	40.9216 (40.9217)
	30	1.8320 (1.8319)	2.8255 (2.8253)	4.9138 (4.9136)	8.2020 (8.2011)	15.1461 (15.1433)	26.4682 (26.4628)	41.0603 (41.0517)
	60	1.8345 (1.8337)	2.8295 (2.8283)	4.9223 (4.9199)	8.2314 (8.2232)	15.2373 (15.2117)	26.6417 (26.5928)	41.3372 (41.2593)
	90	1.8359 (1.8345)	2.8315 (2.8295)	4.9266 (4.9223)	8.2461 (8.2314)	15.2827 (15.2373)	26.7282 (26.6417)	41.4746 (41.3372)

1. Second column of the table shows the values of angle θ in degrees.
2. The values in the parenthesis are the corresponding values when the effect of Coriolis force is not considered

Table 6.10 Comparison of the eigenfrequencies of pseudo – radial modes of oscillations of polytropic models of differentially rotating stars

Model	ω_0^2	ω_1^2	ω_0^2	ω_1^2
	$N=1.5$		$N=3.0$	
Purely Rotating stars				
$n=0.005,\ q=0.0$				
1	2.6798 (2.6827)	12.3582 (12.3737)	9.1712 (9.1809)	16.7687 (16.7866)
2	2.6405 (2.6664)	12.0935 (12.2110)	9.0429 (9.1322)	16.4366 (16.6052)
3	2.5967 (2.6415) [1.7]	11.7970 (11.9457) [1.2]	8.8975 (9.0570) [1.8]	16.0623 (16.3058) [1.5]
4	2.7059 (2.6904)	12.5339 (12.4443)	9.2550 (9.2043)	16.9885 (16.8673)

3. Values of various parameters are taken at surface (θ = 90)
4. The values in the parenthesis are the corresponding values obtained by using the law of differential rotation earlier adopted by Mohan et al (107) in which the effect of Coriolis force is not considered
5. Values in the square bracket represent the percent difference of two corresponding values. Only those values are given in which the percentage difference is greater than 1%.

Table 6.11 Comparison of the eigenfrequencies of nonradial modes of oscillations of polytropic models of differentially rotating stars

Model	g_3	g_2	g_1	f	p_1	p_2	p_3
Purely Rotating stars							
$n = 0.005,\ q = 0.0$							
$N = 1.5$							
1	-	-	-	2.3872 (2.3903)	10.1316 (10.1445)	23.1554 (23.1841)	40.6705 (40.7176)
2	-	-	-	2.3342 (2.3576)	9.8919 (9.9788)	22.6127 (22.7911)	39.7336 (39.7792)
3	-	-	-	2.2757 (2.3042) [1.2]	9.6264 (9.7034)	22.0111 (22.1041)	38.7000 (37.8147) [2.3]
4	-	-	-	2.4225 (2.4038)	10.2911 (10.2133)	23.5163 (23.3398)	41.2954 (40.9250)
$N = 3.0$							
1	1.8298 (1.8306)	2.8218 (2.8231)	4.9063 (4.9089)	8.1754 (8.1834)	15.0639 (15.0836)	26.3115 (26.3468)	40.8104 (40.8649)
2	1.8203 (1.8291)	2.8075 (2.8205)	4.8750 (4.9017)	8.0685 (8.1399)	14.7329 (14.8846)	25.6811 (25.9284)	39.8053 (40.1359)
3	1.8100 (1.8269)	2.7904 (2.8164)	4.8401 (4.8910)	7.9481 (8.0715) [1.5]	14.3626 (14.5569) [1.3]	24.9749 (25.2182)	38.6788 (38.8967)
4	1.8359 (1.8313)	2.8315 (2.8245)	4.9266 (4.9124)	8.2461 (8.2036)	15.2827 (15.1696)	26.7282 (26.5182)	41.4746 (41.1323)

1. Values of various parameters are taken at surface (θ = 90)
2. The values in the parenthesis are the corresponding values obtained by using the law of differential rotation earlier adopted by Mohan et al (107) in which the effect of Coriolis force is not considered
3. Values in the square bracket represent the percent difference of two corresponding values. Only those values are given in which the percentage difference is greater than 1%.

CHAPTER – VII

CONCLUDING OBSERVATIONS

In this chapter we have critically reviewed in brief the work done in the earlier chapters of this thesis and have drawn certain conclusions. In the present thesis we have primarily investigated the effects of inclusion of Coriolis force on the equilibrium structures as well as periods of small adiabatic barotropic modes of oscillations of rotating stars and stars in binary systems using the approach adopted by Kopal (63) and Mohan et al (108, 109). Our main conclusions are as follows.

7.1 EFFECT OF CORIOLIS FORCE ON THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS

In chapter II we developed an expression for Roche equipotential of a rotating star in a binary system which explicitly incorporates the effects of Coriolis force besides the effects of centrifugal and gravitational forces. This expression has been then used to determine the shapes of Roche equipotential surfaces and position of Roche limit for different types of binary stars. Expression for Roche equipotential developed in chapter II which incorporates the effects of Coriolis force has been next used in chapter III in conjunction with Mohan, Saxena and Aggarwal (108) approach, to obtain the equilibrium structures and certain other observable parameters of rotationally and tidally distorted polytropic models of stars. The methodology developed in chapters II and chapter III has been next used in chapter V to determine the effect of Coriolis force on the equilibrium structures of differentially rotating stars in binary systems using the law of differential rotation of the type (5.10).

Our results in chapter II show that as expected explicit inclusion of Coriolis force do not produce any effect on the shapes of Roche equipotential surfaces and position of Roche limit in case of purely rotating stars, synchronous binaries and nonsynchronous binaries. However, it does produce some effect in case of nonsynchronous binaries particularly when the difference between the angular velocity of rotation and revolution is large. Result in chapter III show that there is no appreciable effect on the equilibrium structures and shapes of purely rotating stars, stars in synchronous binaries and even nonsynchronous binaries with the inclusion of Coriolis force. Results in chapter V shows that Coriolis force does not cause any appreciable change on the equilibrium structures of majority of differentially rotating stars and stars in synchronous and nonsynchronous binaries. However, some change is noticed in stars which have large angular velocity of rotation near the points at the surface.

7.2 EFFECT OF CORIOLIS FORCE ON THE PERIODS OF SMALL OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS

In chapter IV we developed eigenvalued boundary value problem which determines the periods of small adiabatic barotropic radial and nonradial modes of oscillations of rotating stars and stars in binary systems incorporating the effects of Coriolis force besides the centrifugal and gravitational forces. The method has been then used to determine the effects of Coriolis force on the eigenfrequencies of various pseudo radial and nonradial modes of oscillations of polytropic models of stars. In chapter VI we study the effect of Coriolis force on the eigenfrequencies of small adiabatic barotropic modes of oscillations of differentially rotating stars in binary systems assuming a law of differential rotation of the type (5.10).

Our results in chapter IV show that inclusion of Coriolis force does not cause any appreciable change in the values of eigenfrequencies of radial and nonradial modes of oscillations of purely rotating stars and stars in synchronous binaries. Even in the case for nonsynchronous binaries, no appreciable effect has been observed. Results of chapter VI also show that with the inclusion of Coriolis force no significant change is observed in the values of the eigenfrequencies of radial and nonradial modes of oscillations of differentially rotating stars and stars in synchronous and nonsynchronous binaries. However some appreciable effect is noticed in the case of purely rotating stars which have large angular velocity of rotation near the surface.

7.3 OVERALL CONCLUSIONS

We had undertaken this study with the object of determining the effect of Coriolis force in the study of equilibrium structure and periods of small oscillations of rotating stars and stars in binary systems as comments had been received on the earlier work carried in this direction by authors Mohan and Saxena (100) and Mohan et al (107) that they are not accounting for the effects of Coriolis force in their studies of problems of rotating stars and stars in binary system. According to the comments besides centrifugal and gravitational forces, Coriolis force is also generated in the case of rotation. As a result, the conclusions arrived on the basis of their study may not be totally valid (Using the Mohan, Saxena and Agarwal's approach there is decrease in the values of eigenfrequencies of g – modes of nonradial oscillations whereas results of other workers (for instance Clement (17)) are contrary to it.). However, our present study has shown that conclusions arrived earlier do not get altered when Coriolis force is taken into account.

7.3.1 SYNCHRONOUS BINARIES

Although earlier workers had not mentioned explicitly the inclusion of the effect of Coriolis force in their studies, our studies has shown that its inclusion does not in any way modify the analytic expressions derived for computing the equilibrium structures and periods of small oscillations of synchronous binaries having solid body rotations. Hence the earlier obtained results already account for Coriolis force.

However, in case of synchronous binaries in which primary component have differential rotation, there is some effect. However this effect is also only marginal and does not alter the overall decisions arrived when the effect of Coriolis force was not considered.

7.3.2 ROTATING STARS

Coriolis force does not arise in case of a single rotating star having solid body rotation. Therefore in the case of rotating stars having solid body rotations earlier results and conclusions are still valid.

However, Coriolis force does arise in the case of differentially rotating stars. Our study has shown that this effect is also marginal for slow rotating stars and does not in any way alter the earlier conclusions arrived when the effect of Coriolis force was neglected. However, noticeable change is observed in case of stars which have large angular velocity of rotation near the surface.

7.3.3 NONSYNCHRONOUS BINARIES

Effect of Coriolis force appears in case of nonsynchronous binaries even when rotation of the primary is solid body. Our study has shown that this effect is not large enough to alter the conclusions arrived earlier by authors who did not account for the effect of Coriolis force. However, appreciable change is noticed in the results when the difference between angular velocity of rotation and revolution is large.

Coriolis force is also generated when the rotation of primary is differential. However, even in these cases there is not any appreciable change in the results to justify any alterations in the conclusions arrived earlier. Moreover nonsynchronous binaries are not so common.

7.4 SCOPE FOR FUTURE WORK

In the present analysis, while investigating the problems of rotating stars in binary systems it has been assumed that the angular velocity of rotation as well as the mass of the companion star causing tidal distortions is not large so that terms beyond second order of smallness in rotational parameter (n, n_1 for solid body rotations and b_0, b_1, n for differential rotations) and tidal parameter (q) can be neglected. However, it will be worth while to extend the analysis further to include higher order terms so that method can be used to study the more realistic problems of stars in which the angular velocity of rotation and the mass of the companion star causing tidal distortions cannot be considered small.

It has also been assumed that the axis of rotation is perpendicular to the line joining the mass centers of the two stars. It may be of interest to analyze the problems in which the axis of rotation is not perpendicular but inclined at some angle to the line joining the mass centre (as is expected to be the case in some of the observed binary systems).

It would be worth while to develop techniques which account for exact potential in place of approximating it by Roche equipotential. It should be possible in view of efficient computational softwares now available.

From the astrophysical view point, it will be worth while to incorporate the present methodology into certain available computer codes for stellar structure and stellar pulsations and apply it to determine the equilibrium models and trace the evolutionary tracks of certain realistic models of rotating stars and stars in binary system. It will also be of interest to apply the present methodology and results to analyze the problems of helio and stellar seismology.

BIBLIOGRAPHY

1. Abhyankar, K. D., Astrophysics Stars and Galaxies*,* Tata McGraw Hill Publishing Company Ltd., 1992.
2. Avni, Y. and Schiller, N., Generalized Roche potential for misaligned binary systems: Properties of the critical lobe, Astrophysical Journal, 257, 703 - 714, 1982.
3. Bhatia, P. K., Nonradial oscillations of a star, Astrophysics and Space Science, 166(2), 219 – 228, 1990.
4. Bohm-Vitense, E., Introduction to Stellar Astrophysics Volume 1 – Basic stellar observations and data, Cambridge University Press Publication, 1989.
5. Brunning, D. H., Fourier analysis of differential rotation in Late-type stars, Astrophysical Journal, 248, 274 – 278, 1981.
6. Chan, K. L. and Chau, W. Y., Structure and evolution of very close binary systems. I. structure equations including rotational and tidal distortions and calculations for 2, 1 and 0.6 $M_{\odot}$ Zero-age main sequence stars, Astrophysical Journal, 233, 950 – 960, 1979.
7. Chandra, N and Bhatnagar, K. B., Escape with the formation of a binary in two-dimensional three-body problem – I, Astronomy and Astrophysics, 346, 652 – 662, 1999.
8. Chandrasekhar, S., The equilibrium of distorted polytropes (I) The rotational problem, Monthly Notices of Royal Astronomical Society, 93, 390 - 406, 1933.
9. Chandrasekhar, S., The equilibrium of distorted polytropes (II) The tidal problem, Monthly Notices of Royal Astronomical Society, 93, 449 - 461, 1933.
10. Chandrasekhar, S., The equilibrium of distorted polytropes (III) The double star problem, Monthly Notices of Royal Astronomical Society, 93, 462 - 471, 1933.
11. Chandrasekhar, S., An Introduction to the study of Stellar Structure, The University of Chicago Press, Chicago, 1939.

12. Chandrasekhar, S., and Ferrari, V., On the non-radial oscillations of stars, Royal Soc. (London), Proceedings, Series A – Mathematical and Physical Science, 432, 247 – 279, 1991.

13. Chandrasekhar, S. and Lebovitz, N. R., On the oscillations and the stability of rotating gaseous masses III. The distorted polytropes, Astrophysical Journal, 136, 1082 – 1104, 1962.

14. Clement, M. J., The radial and nonradial oscillations of slowly rotating gaseous masses, Astrophysical Journal, 141, 210 – 231, 1965.

15. Clement, M. J., Non-radial Oscillations and the Beta Canis Majoris Stars II. The effect of differential rotation, Astrophysical Journal, 150, 589 – 606, 1967.

16. Clement, M. J., In A. Slettebak (ed.), Stellar Rotation, D. Reidel Publ. Co., Dordrecht, Holland, 1970.

17. Clement, M. J., Normal modes of oscillations for rotating stars. II Variational solutions, Astrophysical Journal, 276, 724 – 736, 1984.

18. Clement, M. J., Normal modes of oscillations for rotating stars. V. A new numerical method for computing nonradial eigenfunctions, Astrophysical Journal Supplement Series, 116, 57 – 74, 1998.

19. Cowling, T. G., The nonradial oscillations of polytropic stars, Monthly Notices of Royal Astronomical Society, 101, 367 – 375, 1941.

20. Cox, A. N. and Cahn, J. H., The pulsational stability of Wolf-Rayet stars, Astrophysical Journal, 326, 804 – 812, 1988.

21. Cox, J. P., Theory of Stellar Pulsations, Princeton Univ. Press, Princeton, 1980.

22. Cox, J. P. and Giuli, R. T., Principle of stellar structure, Vol. I and II, Gordon and Breach, New York, 1968.

23. Csataryova, M. and Skopal, A., Notices to investigation of symbiotic binaries III. Approximation of the Roche lobe parameters for asynchronously rotating star in a binary system, Contrib. Astron. Obs. Skalnate Pleso (CoSka), 35, 17 – 22, 2005.

24. Dalsgaard, C. J. and Gough, D. L., Towards a heliological inverse problem, Nature, 259, 89 – 92, 1976.

25. Das, M. K., Singh, H. P. and Tondon, J. N., On multimode pulsations of degenerate stellar models, Astrophysical Journal, 346, 289 – 298, 1989.

26. Deupree, R. G., Stellar evolution with arbitrary rotation laws: I Mathematical Technique and Test Cases, Astrophysical Journal, 357, 175 – 187, 1990.

27. Deupree, R. G. and Karakas, A. I., The structure of close binaries in two dimensions, Astrophysical Journal, 633, 418 – 423, 2005.

28. Dintrans, B. and Rieutord, M., Oscillations of a rotating star, a non perturbative theory, Astronomy and Astrophysics, 354, 86 – 98, 2000.

29. Durney, B. R., On theory of rotating convection zones, Astrophysical Journal, 297, 787 – 798, 1985.

30. Durney, B. R., Observational constraints on theories of the solar differential rotation, Astrophysical Journal, 378, 378 – 397, 1991.

31. Dziembowski, W. A. and Goode, P. R., Effects of differential rotation on stellar oscillations: A second order theory, Astrophysical Journal, 394, 670 – 687, 1992.

32. Eddington, A. S., The internal constitution of the star, Dover Publication, Newyork, 1959.

33. Eggleton, P. P., Approximations to the radii of Roche lobes, Astrophysical Journal, 268, 368 – 369, 1983.

34. Einsel, C. and Spurzem, R., Dynamical evolution of rotating stellar systems – I. Pre collapse, equal mass system, Monthly Notices of Royal Astronomical Society, 302, 81 – 95, 1999.

35. Endal, A. S. and Sofia, S., The evolution of rotating stars. I. Method and exploratory Calculations for 7 $M_{\odot}$ star, Astrophysical Journal, 210, 184 – 198, 1976.

36. Galli, D., Constant - Mass sequence of differentially - rotating polytropes, Astrophysics and Space Science, 188, 241 – 255, 1992.

37. Geroyannis, V. S. and Antonakopoulus, G. A., Structure distortion of polytropic stars by differential rotation: Third order perturbation theory, Astrophysics and Space Science, 74, 97 – 110, 1981.

38. Geroyannis, V. S. and Valvi, F. N., Third-order perturbation theory: structure parameters of uniformly rotating polytropic stars and improvement and accuracy of the results, Astrophysical Journal Supplement Series, 62, 363 – 371, 1986.

39. Geroyannis, V. S., Tokis, J. N. and Valvi, F. N., A second order perturbation theory for differentially rotating gaseous polytropes, Astrophysics and Space Science, 64, 359 – 389, 1979

40. Glatzmaier, G. A. and Gilman, P. A., Compressible convection in a rotating spherical shell. V. Induced differential rotation and meridional circulations, Astrophysical Journal, 256, 316 – 330, 1982.

41. Goldreich, P. and Schubert, G., Differential rotation in stars, Astrophysical Journal, 150, 571 – 588, 1967.

42. Goode, R. P., Dziembowski, W. A., Korzennik, S. G. and Rhodes, E. J. Jr., What we know about the sun's internal rotation from solar oscillations, Astrophysical Journal, 367, 649 – 657, 1991.

43. Gurm, H. S., On the pulsation of fluid masses, PhD Thesis, Roorkee Univ., Roorkee, India, 1961.

44. Hachisu, I., A versatile method for obtaining structures of rapidly rotating stars, II. Three-dimensional self consistent field method, Astrophysical Journal Supplement Series, 62, 461 - 499, 1986.

45. Hansen, C. J., Cox, J. P. and Carroll, B. W., The quasi – adiabatic analysis of nonradial modes of stellar oscillation in the presence of slow rotation, Astrophysical Journal, 226, 210 – 221, 1978.

46. Hansen, C. J., Cox, J. P. and Horn, H. M. V., The effects of differential rotation on the splitting of nonradial modes of stellar oscillation, Astrophysical Journal, 217, 151 – 159, 1977.

47. Harris, W. E. and Clement, M. J., Differential rotation in stars on the upper main sequence II, Astrophysical Journal, 167, 321 - 325, 1971.

48. Henyey, L. G., Forbes, J. E. and Gould, N. L., A new method of automatic computation of stellar evolution, Astrophysical Journal, 139, 306 – 317, 1964

49. Hoiland, E., Avhandliger Norske Videnskaps – Akademi i Oslo, I., math. naturv. Klasse, 11, 1-24, 1941.

50. Horedt, G. P., Polytropes, Application in astrophysics and related fields, Kluwer academic publishers, Dordrecht, 2004.

51. Hurley, M. and Roberts, P. H., On highly rotating polytropes. IV, Astrophysical Journal Supplement Series, 11, 95 – 119, 1965.

52. Hurley, M., Roberts, P. H. and Wright, K., The oscillations of gas spheres, Astrophysical Journal, 143, 535 – 551, 1966.

53. Ireland, J. G., On the effect of non-uniform rotation on stellar luminosity, Zeitschrift fur Astrophysik, 65, 123 – 132, 1967.

54. James, R. A., The structure and stability of rotating gas masses, Astrophysical Journal, 140, 552 – 582, 1964.

55. Karino, S., Importance of perturbed gravitational potentials in differentially rotating Newtonian stars, Monthly Notices of Royal Astronomical Society, 343, 175 – 180, 2003.

56. Karino, S. and Eriguchi, Y., Linear stability analysis of differentially rotating polytropes: New Results for m=2 f-mode dynamical instability, Astrophysical Journal, 592, 1119 – 1123, 2003.

57. Karino, S, Yoshida, S. and Eriguchi, Y., R – mode oscillations of differentially and rapidly rotating Newtonian polytropic stars, Physical Review D, 64, 024003, 2001.

58. Kippenhahn, R. and Thomas, H. C., A simple method for the solution of the stellar structure equation including rotation and tidal forces, Stellar Rotation, Proceedings of IAU Colloq 4, Ohio State Univ, Columbus, ed. Slettebak, A., D., Gordon and Breach Science Publishers, 1970.

59. Kippenhahn, R. and Weigert, A., Stellar Structure and Evolution, Springer – Verlag publ., 1990.

60. Kochar, R. K. and Trehan, S. K., Adiabatic pulsations and convective instability of uniformly rotating gaseous masses, Astrophysics and Space Science, 26, 271 – 287, 1974.

61. Komatsu, H., Eriguchi, Y. and Hachisu, I., Rapidly rotating general relativistic stars-II. Differentially rotating polytropes, Monthly Notices of Royal Astronomical Society, 239, 153 - 171, 1989.

62. Kopal, Z., Nonradial oscillations of the standard model, Astrophysical Journal, 109, 509 – 527, 1949.

63. Kopal, Z., Roche Model and Its Application to Close Binary Systems, Advances in Astronomy and Astrophysics, 9, 1 – 65, 1972.

64. Kopal, Z., Dynamics of close Binary Systems, D. Reidel Publ. Co., Dordrecht, Holland, 1978.

65. Kopal, Z., Effects of rotation on Internal Structure of the Stars, Astrophysics and Space Science, 93, 149 – 175, 1983.

66. Kopal, Z., The Roche problem and its significance for double star astronomy, Kluwer Academic Publishers, Dordrecht, Netherland, 1989.

67. Kopal, Z. and Ali, A. K. M. Sekender., On the integrability of the Roche coordinates, Astrophysics and Space Science, 11, 423 – 429, 1971.

68. Kruszewski, A., Exchange of matter in close binary systems I. Equilibrium configurations in the case of deviations from synchronism, Acta Astronomica, 13, 106 – 117, 1963.

69. Kuker, M. and Rudiger, G., Differential rotation of main sequence F – stars, Astronomy and Astrophysics, 433, 1023 – 1030, 2005.

70. Lal, A. K., Structure oscillations and stability of differentially rotating and tidally distorted stellar models, PhD Thesis, Roorkee University, Roorkee, India, 1993.

71. Lal, A. K., Mohan, C. and Singh, V. P., Equilibrium structures of differentially rotating and tidally distorted white dwarf models of stars, Astrophysics and Space Science, 301, 51 – 60, 2006.

72. Lal, A. K., Saini, S., Mohan, C. and Singh, V. P., Equilibrium structures of rotationally and tidally distorted polytropic models of stars, Astrophysics and Space Science, 306, 165 – 169, 2006.

73. Lal, A. K., Saini, S., Mohan, C. and Singh, V. P., On the validity of series expansion being used for the position of a point on a Roche equipotential, Astrophysics and Space Science, 310, 317 – 320, 2007.

74. Ledoux, P., The nonradial oscillations of gaseous stars and the problems of βeta Canis Majoris, Astrophysical Journal, 114, 373 – 384, 1951.

75. Ledoux, P. and Walraven, T., Variable Stars, Handbuch der Physik, 51, 353 – 604, 1958.

76. Lee, U. and Baraffe, I., Pulsational stability of rotating main sequence stars, the second order effects of rotation on the nonadiabatic oscillations, Astronomy and Astrophysics, 301, 419 – 432, 1995.

77. Lee, U. and Saio, H., Low frequency oscillations of uniformly rotating stars, Monthly Notices of Royal Astronomical Society, 224, 513 – 526, 1987.

78. Lee, U. and Strohmayer, T. E., Nonradial oscillations of rotating neutron stars: the effects of the Coriolis force, Astronomy and Astrophysics, 311, 155 – 171, 1996.

79. Lignieres F., Rieutord, M. and Reese, D., Acoustic oscillations of rapidly rotating polytropic stars. II. Effects of the centrifugal distortions, Astronomy and Astrophysics, 455, 607 – 620, 2006.

80. Limber, D. N., Surface forms and mass loss for the components of close binaries, General case of nonsynchronous rotation, Astrophysical Journal, 138, 1112 – 1133, 1963.

81. Linnell, A. P., Rotational distortion of polytropes by analytic approximation, Astrophysics and Space Science, 37, 73 – 86, 1975.

82. Linnell, A. P., The volume and surface area of a uniformly rotating polytrope, Astrophysics and Space Science, 80, 501 – 511, 1981.

83. Lopezorti, J. A., Garcia, A. L. and Machi, R. L., Figures of equilibrium in close binary systems, Celestial Mechanics and Dynamical Astronomy, 53, 311 – 322, 1992.

84. Lovekin, C. C. and Deupree, R. G., Radial and nonradial oscillations modes in rapidly rotating stars, Astrophysical Journal, 679, 1499 – 1508, 2008.

85. Lovekin, C. C., Deupree, R. G. and Clement, M. J., Effects of uniform and differential rotation on stellar pulsations, Astrophysical Journal, 693, 677 – 690, 2009.

86. Lubow, S. H., Equilibrium states of nonsynchronous stars in detached binaries, Astrophysical Journal, 229, 1008 – 1022, 1979.

87. Macgregor, K. B., Jackson, S., Skumanich, A. and Metcalfe, T. S., On the structure and properties of differentially rotating main sequence stars in the 1 - 2 $M_{\odot}$ range, Astrophysical Journal, 663, 560 – 572, 2007.

88. Maeder, A. and Zahn, J. P., Stellar evolution with rotation III. Meridional circulation with μ - gradients and non stationarity, Astronomy and Astrophysics, 334, 1000 – 1006, 1998.

89. Marten, L. and Smeyers, P., Rotational and tidal perturbations of linear isentropic oscillations of a gaseous star, Astronomy and Astrophysics, 115, 211 – 226, 1986.

90. Martin, P. G., A study of the structure of rapidly rotating close binary systems, Astrophysics and Space Science, 7, 119 – 138, 1970.

91. McDermott, P. N., Horn, H. M. V. and Hansen, C. J., Nonradial oscillations of neutron stars, Astrophysical Journal, 325, 725 – 748, 1988.

92. Mender, L. T. S., Antona, F. D. and Mazzitelli, I., Theoretical model of low mass premain sequence rotating stars I. The effects on lithium depletion, Astronomy and Astrophysics, 341, 174-180, 1999

93. Mentzel, D. H., Bhatnagar, P. L. and Sen, H. K., Stellar Interiors, Chapman and Hall Ltd., London, 1963.

94. Meynet, G. and Maeder, A., Stellar evolution with rotation: I. The Computational Method and the inhibiting effect of the μ-gradient, Astronomy and Astrophysics, 321, 465 – 476, 1997.

95. Miesch, M. S., Differential rotation and meridional circulation in global models of solar convection, Astronomische Nachrichten, 328, 998 – 1001, 2007.

96. Milne, E. A., The equilibrium of a rotating star, Monthly Notices of Royal Astronomical Society, 83, 118 – 147, 1923.
97. Mochnacki, S. W., Accurate integrations of the Roche model, Astrophysical Journal Supplement Series, 55, 551 – 561, 1984.
98. Mohan, C. and Agarwal, S. R., Effects of rotation and tidal distortions on the periods of adiabatic oscillations of composite models of stars, Astrophysics and Space Science, 129, 73 – 94, 1987.
99. Mohan, C. and Saxena, R. M., Effects of rotation and tidal distortions on the structure of polytropic stars, Astrophysics and Space Science, 95, 369 – 381, 1983.
100. Mohan, C. and Saxena, R. M., Effects of rotation and tidal distortions on the periods of small adiabatic oscillations of the polytropic models of the stars, Astrophysics and Space Science, 113, 155 – 169, 1985.
101. Mohan, C. and Singh, V. P., Use of Roche coordinates in the problems of small oscillations of rotationally distorted stellar models, Astrophysics and Space Science, 54, 293 – 304, 1978.
102. Mohan, C. and Singh, V. P., Use of Roche Coordinates in the problems of small oscillations of tidally distorted stellar models, Astrophysics and Space Science, 56, 109 – 117, 1978.
103. Mohan, C. and Singh, V. P., Effect of rotational and tidal distortions on the periods of small adiabatic oscillations of stellar models, Astrophysics and Space Science, 85, 83 – 98, 1982.
104. Mohan, C., Lal, A. K. and Singh, V. P., Equilibrium structure of differentially rotating polytropic models of the stars, Astrophysics and Space Science, 193, 69 – 85, 1992.
105. Mohan, C., Lal, A. K., and Singh, V. P., Equilibrium structure of stars obeying a generalized differential rotation law, Astrophysics and Space Science, 215, 111 – 130, 1994.
106. Mohan, C., Lal, A. K. and Singh, V. P., Equilibrium structures of differentially rotating primary components of binary stars, Astrophysics and Space Science, 254, 97 – 109, 1997.

107. Mohan, C., Lal, A. K. and Singh, V. P., Effects of differential rotation on the eigenfrequencies of small adiabatic barotropic modes of oscillations of the stars, Indian J pure appl. Math, 29 (2), 199 – 220, 1998.

108. Mohan, C., Saxena, R. M. and Agarwal, S. R., Equilibrium structures of rotationally and tidally distorted stellar models, Astrophysics and Space Science, 163, 23 – 39, 1990.

109. Mohan, C., Saxena, R. M. and Agarwal, S. R., Eigenfrequencies of small adiabatic barotropic models of oscillations of rotationally and tidally - distorted stars, Astrophysics and Space Science, 178, .89 – 105, 1991.

110. Monaghan, F. F., The accuracy of first order perturbation theory for uniformly rotating polytropes, Zeitschrift fur Astrophysik, 67, 222 - 226, 1967.

111. Monaghan, F. F. and Roxburgh, I. W., The structure of rapidly rotating polytropes, Monthly Notices of Royal Astronomical Society, 131, 13 – 22, 1965.

112. Naylor, M. D. T. and Anand, S. P. S., Structure of close binaries II Theory of uniformly rotating binaries, Astrophysics and Space Science, 16, 137 – 150, 1972.

113. Nelson, L. A., Chau, W. Y. and Rosenblum, A., Evolution of rotationally and tidally distorted Low-mass close binary systems: Implications for the minimum orbital period of cataclysmic variables, Astrophysical Journal, 299, 658 – 667, 1985.

114. Orlov, A. A., A generalization of the Roche model, Soviet Astronomy, 4, 845 – 849, 1961.

115. Owen, J. W., The nonradial oscillations of centrally condensed stars, Monthly Notices of Royal Astronomical Society, 117, 384 – 392, 1957.

116. Peraiah, A., Rotation of the components in close binary systems, Zeitschrift fur Astrophysik, 62, 48 - 53, 1965.

117. Perkeris, C. L., Nonradial oscillations of stars, Astrophysical Journal, 88, 189 - 199, 1938.

118. Pinsonneault, M. H., Deliyannis, C. P. and Demarque, P., Evolutionary models of Halo stars with rotation I. Evidence for differential rotation with depth in stars, Astrophysical Journal, 367, 239 – 252, 1991.

119. Plavec, M., Tables for the Roche model of close binaries, Bulletin of the Astronomical Institute of Czechoslovakia, 15, 165 – 170, 1964.

120. Prasad, C., Radial oscillations of a stellar model, Monthly Notices of Royal Astronomical Society, 109, 103 – 107, 1949.

121. Prasad, C. and Mohan, C., Pulsational properties of massive star in the helium burning phase-I, Monthly Notices of Royal Astronomical Society, 141, 389 – 395, 1968.

122. Reese, D., Lignieres, F. and Rieutord, M., Acoustic oscillations of rapidly rotating polytropic stars II. Effects of the Coriolis force and centrifugal accelerations, Astronomy and Astrophysics, 455, 621 – 637, 2006.

123. Reyniers, K. and Smeyers P., Tidal perturbations of linear isentropic oscillations in components of circular-orbit close binaries II. Validity of the perturbation method applied to equilibrium tides, Astronomy and Astrophysics, 409, 677 – 688, 2003.

124. Roberts, P. H., On highly rotating polytropes, I., Astrophysical Journal, 137, 1129 – 1141, 1963.

125. Rocca, A., Tidal effects in rotating close binaries, Astronomy and Astrophysics, 213, 114 – 126, 1989.

126. Rosenwald, R. D. and Rabaey, G. F., Application of the continuous Orthonormalization and adjoint methods to the computation of solar eigenfrequencies and eigenfrequency sensitivities, Astrophysical Journal Supplement Series, 77, 97 – 117, 1991.

127. Rosseland, S., The Pulsation Theory of Variable Stars, Oxford, Clarendon Press, 1949.

128. Roxburgh, I. W. and Durney, B. R., Structure, oscillations and stability of rotating white dwarfs, Zeitschrift fur Astrophysik, 64, 504 – 511, 1966.

129. Sackmann, I. J. and Anand, S. P. S., Structure and evolution of rapidly rotating B-type stars, Astrophysical Journal, 162, 105 – 124, 1970.

130. Saio, H., Rotational and tidal perturbation of nonradial oscillations in a polytropic star, Astrophysical Journal, 244, 299 – 315, 1981.
131. Saio, H. and Jeffery, C. S., Radial pulsation in luminous hot helium stars, Astrophysical Journal, 328, 714 – 725, 1988.
132. Savonije, G. J., Nonradial oscillations of the rapidly rotating Be star HD 163868, Astronomy and Astrophysics, 469, 1057 – 1062, 2007.
133. Saxena, R. M., The effects of rotation and tidal distortions on the structure and periods of small oscillations of gaseous spheres, PhD Thesis, Roorkee University, Roorkee, India, 1984.
134. Schmitz, F., Equilibrium structures of differentially rotating self-gravitating gases, Astronomy and Astrophysics, 200, 127 – 134, 1988.
135. Schwarzschild, M., Structure and Evolution of the Stars, Princeton University Press, Princeton, 1958.
136. Seidov, Z. F., The Roche Problem: Some analytics, Astrophysical Journal, 603, 283 – 284, 2004.
137. Sepinsky, J. F., Willems, B. and Kalogera, V., Equipotential surfaces and Lagrangian points in nonsynchronous, Eccentric binary and planetary systems, Astrophysical Journal, 660, 1624 – 1635, 2007.
138. Shibata, M., Karino, S and Eriguchi, Y., Dynamical instability of differentially rotating stars, Monthly Notices of Royal Astronomical Society, 334, 127 – 131, 2002.
139. Simon, R., Rotational perturbation of a radial oscillation in a gaseous star, Astronomy and Astrophysics, 2, 390 – 397, 1969.
140. Singh, K., Nonradial oscillations of gaseous spheres, PhD Thesis, University. of Roorkee, Roorkee, India, 1976.
141. Singh, M. and Singh, G., The structure of the tidally and rotationally distorted polytropes, Astrophysics and Space Science, 96, 313 – 319, 1984.
142. Singh, V. P. and Sharma, M. K., Effect of differential rotation on the periods of small adiabatic oscillations of stellar models, Indian J Pure & Appl. Math., 26 (1), 69 – 79, 1995.

143. Smeyers, P. and Denis, J., Second order rotational perturbation of nonradial oscillations of a star, Astronomy and Astrophysics, 14, 311 – 318, 1971.

144. Smith, B. L., Strained coordinate methods in rotating stars I. Polytropes, Astrophysics and Space Science, 43, 411 – 424, 1976.

145. Smith, M. A., Nonradial pulsation in the zero-age main sequence upsilon Orionis (0.95V), Astrophysical Journal, 248, 214 – 221, 1981.

146. Sood, N. K. and Singh, K, Equilibrium of self gravitating polytropes, Astrophysics and Space Science, 283, 87 – 96, 2003.

147. Soofi, F., Goupil, M. J. and Dziembowski, W. A., The effects of moderate rotation on stellar pulsation I. Third Order perturbation formalism, Astronomy and Astrophysics, 334, 911 – 924, 1998.

148. Stoeckly, R., Polytropic models with fast, non-uniform rotation, Astrophysical Journal, 142, 208 – 228, 1965.

149. Strittmatter, P.A., Stellar rotation, Annual Reviews of Astronomy and Astrophysics, 7, 665 – 684, 1969.

150. Sweet, P. A. and Roy, A. E., The structure of rotating stars I., Monthly Notices of Royal Astronomical Society, 113, 701 – 715, 1953.

151. Tassoul, J. L., Theory of Rotating Stars, Princeton University Press, Princeton, 1978.

152. Tassoul, J. L. and Tassoul, M., Meridional Circulation in rotating stars II, Mean Steady motion in rotationally and tidally distorted stars, Astrophysical Journal, 261, 265 – 272, 1982.

153. Tassoul, J. L and Tassoul, M., Meridional circulation in rotating stars III, The reflection effect in close binaries, Astrophysical Journal, 261, 273 – 276, 1982.

154. Telting J. H. and Schrijvers, C., Line-Profile Variations of nonradial adiabatic pulsations of rotating stars III. On the alleged misidentification of tesseral modes, Astronomy and Astrophysics, 317, 742 – 748, 1997.

155. Todoran, I., Equipotential surfaces in close binary systems - Remarks on the time-dependent potential functions, Astrophysics and Space Science, 187, 119 – 126, 1992.

156. Townsend, R. H. D., Influence of the Coriolis force on the instability of slowly pulsating B stars, Monthly Notices of Royal Astronomical Society, 360, 465 – 476, 2005.

157. Unno, W., Osaki, Y., Ando, H. and Shibahashi, H., Nonradial Oscillations of Stars, University of Tokyo Press, 1989.

158. Urpin, V. A., Shalybkov, D. A. and Spruit, H. C., Differential rotation, Circulation and turbulence in radiative zones of stars, Astronomy and Astrophysics, 306, 455 – 463, 1996.

159. Von Zeipel, H., The Radiative equilibrium of a rotating system of gaseous masses, Monthly Notices of Royal Astronomical Society, 84, 665 – 683, 1924.

160. Vorontsov, S. V., Adiabatic oscillations of a differentially rotating star, second order perturbation theory, Solar Physics, 82, 379 – 382, 1983.

161. Woodard, M. F., Distortion of high-degree solar p-mode eigenfunctions by Latitudinal differential rotation, Astrophysical Journal, 347, 1176 – 1182, 1989.

162. Zeng, Y. R., A more powerful evolution model for rotating stars, Astronomy and Astrophysics, 394, 965 – 969, 2002.