Study of Continuous Casting Plant Using Regenerative Point Technique

Dissertation submitted in partial fulfilment of the requirement for
the award of the degree of
Master of Science
in
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Submitted by
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CERTIFICATE

I hereby certify that the work which is being presented in the dissertation entitled " **Study of Continuous Casting Plant Using Regenerative Point Technique"** in partial fulfilment of the requirements for the award of degree of Master of Science (Mathematics and Computing), School of Mathematics and Computer Applications (SMCA), Thapar University, Patiala is authentic record of my own work carried out under the supervision of **Dr. Jitender Kumar.**

The matter presented in this dissertation has not been submitted for the award of any other degree of this or any other university.

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This is to certify that the above statement made by the candidate is correct and true to the best of our knowledge.

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My special thanks to my friends who devoted their valuable time and helped me in the completion of this work. I thank all those who have contributed directly or indirectly to this work.

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July 13, 2012

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ABSTRACT

The whole range of our work reported in this dissertation is covered into four chapters which are described as:

Chapter I:

A review of literature pertaining to the theory of reliability and modelling is given in this chapter. This also includes some basic concepts such as probability distributions, stochastic process, profit function, etc.

Chapter II:

This chapter has been designed with a view to study reliability modelling and analysis of single unit Continuous Casting (CC) Plant. It suggests a modelling strategy of CC plant. The operative unit is subject to transit to any of the failed states depending on the type of failure/breakdown. The failed unit is attended by the repairman as soon as it fails. The system regenerates and works like a new one after each repair, replacement or reconditioning and reinstallation. This chapter outlines the modelling strategy embedded by the types of failures actually depicted into data and important reliability indices such as the mean time to system failure and steady state availability are obtained using semi-Markov processes and regenerative point techniques.

Chapter III.

In this chapter, we study reliability modelling and analysis of a two unit Continuous Casting plant. There are systems wherein the two-units are operating in parallel with full-reduced installed capacity and the concept of inspection is used to identify the type of failure. There is a single server who comes immediately at the complete failure of the unit for doing its repair. Various Reliability characteristic such as Reliability, Mean Time to System Failure(MTSF), Steady State Availability, Busy Period Analysis, Expected number of visits by server are obtained and finally the profit function is determine by using Semi-Markov process and Regenerative point technique.

Chapter IV.

In this chapter, we discuss the finding of chapter – II and chapter – III.

List of Contents

Notations and Symbols

$q_{ij}(t), Q_{ij}$: pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t].

p_{ij} : direct (one step) transition probability from state Si to Sj without passing into any other state.

$\Phi_i(t)$: cdf of the first passage time from regenerative state i to a failed state.

m_{ij} : contribution to mean sojourn time in state S_i when the system transits directly to state S_j ,(S_i ,S_j Є E) so that $\mu_i = \sum_i mij$ where $m_{ij} = \int q_{ij}(t)dt = \int dQ_{ij}(t)$ and μ_i is the mean sojourn time in state S_i Є E.

$A_i(t)$: probability that the system is up at instant t, given that system entered into regenerative state i to t=0.

$B_i(t)$: probability that the server is busy at an instant t given that system entered into the regenerative state i at t=0.

$M_i(t)$: probability that the system, initially up in the regenerative state i, is up at time t without passing through any other regenerative state.

$W_i(t)$: probability that the server is busy in the state S_i upto time t without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative state.

E_o : state of the system at time t=0.

E : set of regenerative states {Si}.

$\overline{E}$: set of non regenerative states {Si}.

L.S.T. : Laplace Stieltjes Transform.

L.T. : Laplace Transform.

pdf : probability density function.

cdf : cumulative distribution function.

(S) : symbol for Stieltjes convolution, e.g.

$$A(t)(S)B(t) = \int_0^t B(t-u)dA(u)$$

© : symbol for Laplace convolution, e.g.

$$f(t) © g(t) = \int_0^t g(t-u)df(t)$$

* : symbol for L.T.

CHAPTER-I

INTRODUCTION

Reliability is both a desirable as well as necessary factor in the present day technology for achieving healthy economic progress of a nation.Now a days, reliability is not only a subject of study for scientists and academicians but also a serious concern to the practicising engineers, manufacturer, economists and government leaders as well. Reliability considerations make more effective use of resources and it results in an increase in productivity and decrease in wastage of money, material and manpower. In the innovation of new and improved technological systems, reliability is acquiring special importance as one among the many important system measures such as performance, cost, etc. Unreliability usually results in prohibiting high cost of repair, maintenance as well as down time and as such reliability is an economic necessity, more so far developing countries.

Reliability has now a well established and a well formulated science of predicting, estimating and optimizing the probability of survivable of an equipment or system or mission. In the past not much importance was attached to reliability and it was recognized only in qualitative sense. It was only after certain studies, carried out during the past Second World War period, revealed many surprising results and the attention of scientists and engineers was drawn for further serious investigation towards it with technological advancement and increase in complexity and sophistication of system, reliability has acquired prime importance in the present age.

Today, is an era of industrial growth. Reliability consideration has occupied an increasing important place in all engineering disciplines. As the demand for the system that perform better and cost less have increase, so there is a economical requirement to minimize the probability of failures whether the failures simply increase cost and inconvenience or threaten the public safety. However, to think of a system without failure is quite impossible. The system might fail in its operation due to various reasons natural, human and mechanical. But even after these errors, a system can be made desirable and reliable to use by providing proper maintenance and repair facilities at certain level of damages. Thus, sincere efforts and these precautions can maximize the reliability of the system with minimum costs.

During last few decades, the reliability model of maintained systems operating under strict control of weather conditions have been discussed by various scholars and engineers due to their importance in varied of areas such as military, industry, health and the environment. In [1953], Epstein and Sobel began work in the field of life testing. After [1956], system maintainability problems were also considered besides reliability. Gaver [1963] was the first who generalized repair time distribution to analyses his model.

An excellent account of the early development of the mathematical theory of reliability has been given by Barlow and Proschan [1965].

Branson and Shah [1971] applied semi-Markov method when repair time distribution was general with exponential failure time. Srinivasan and Gopalan [1973] highlighted the regenerative point technique for analyzing a two-unit system with warm standby and single repair facility. Nakagawa [1976] analyzed the system with replacement of the unit at certain level of damage. Gopalan and Marathe [1978] evaluated the availability of one server two dissimilar unit system with slow switch.

Ramamurthy and Jaiswal [1982] analyzed a two dissimilar unit cold standby system with allowed down time. Murari and Goyal [1984] made a comparison of two-unit cold standby reliability models with three types of repair facility. Singh [1989] evaluated the profit of two-unit cold standby system with random appearance and disappearance time of the service facility. Gupta and Bansal [1991] studied profit analysis of a two-unit priority standby system subject to degradation. Tuteja and Malik [1992] discussed single-unit reliability models with different types of repair policies by using regenerative point technique. Gupta et al. [1993] discussed profit analysis of a two-unit priority standby system subject to degradation and random shocks. Yang and Dhillon [1995] analyzed a general standby system with constant human error and arbitrary system repair rates. Mokaddis et al. [1997] discussed a two-unit warm standby system subject to degradation.

Grewal [2003] has proposed reliability models of non-identical units with different repair policies of server. Kadyan et al. [2004] has made a stochastic analysis of non-identical units reliability models with priority and different modes of failure. Chander [2005] investigated reliability models with priority for operation and repair with arrival time of server. Chander and Singh [2005] have evaluated profit and reliability of an electric supply system. Chand et al. [2007] discussed availability analysis of cotton mill. Malik et al. [2008] discussed stochastic analysis of an operating system with two types of inspection subject to degradation. Chander and Singh [2009] developed reliability model for a 2-out-of-3 redundant system subject to degradation. Jitender Kumar [2010] developed Reliability modeling and cost-benefit analysis of redundant systems subject to degradation and inspection. Renbin Liu and Zaiming Liu.[2011] developed Reliability analysis of a one-unit system with finite vacations,Management Science Industrial Engineering (MSIE). Kumar, J. and Kadyan, M. S. [2012] developed Profit anaiysis of a system of non identical units with degradation and replacement.

RELIABILITY ENGINEERING

Reliability engineering is the discipline of ensuring that a system (or a device in general) will perform its intended function (s) when operated in a specified manner for a specified length of time. Reliability engineering is performed throughout the entire life cycle of a system, including development, test, production and operation.

Reliability may be defined in several ways:

- The idea that something is fit for purpose with respect to time;
- The capacity of a device or system to perform as designed;
- The resistance to failure of a device or system;
- The ability of a device or system to perform a required function under stated conditions for a specified period to time;
- The probability that a functional unit will perform its required function for a specified interval under stated conditions.

Reliability engineers rely heavily on statistics, probability theory, and reliability theory. Many engineering techniques are used in reliability engineering, such as reliability prediction, Weibull analysis, thermal management, reliability testing and accelerated life testing. Because of the large number or reliability techniques, their expense, and the varying degrees of reliability required for different situations, most projects develop a reliability program plan to specify the reliability tasks that will be performed for that specific system. The function of reliability engineering is to develop the reliability requirements for the product, establish an adequate reliability program, and perform appropriate analyses and tasks to ensure the product will meet its requirements. These tasks are managed by a reliability engineer, who has additional reliability specific education and training. Reliability engineering is closely associated with maintainability engineering and logistics engineering. Many problems from other fields, such as security engineering, can also be approached using reliability engineering techniques.

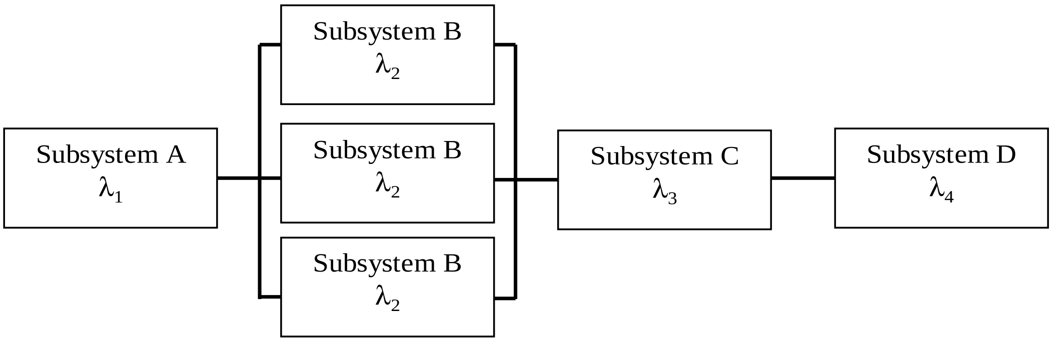

Fig. 1.1 A Reliability Block Diagram

Reliability theory is the foundation of reliability engineering. For engineering purposes, reliability is defined as:

> **the probability that a device(unit) will perform its intended function adequately for a given period of time under stated conditions or environment.**

Mathematically, if T is time till the failure of a unit occurs, this may be expressed as,

$$R(t) = \Pr(T>t) = \int_t^\infty f(x)dx$$

where

f(x) is the failure probability density function and *t* is the length of the period of time (which is assumed to start from time zero).

Reliability engineering is concerned with four key elements of this definition:

- First, reliability is a probability. This means that there is always some chance for failure. Reliability engineering is concerned with meeting the specified probability of success, at a specified statistical confidence level. Since it is a probability, its numerical value is always between one and zero, i.e.

  $$R(0)=1, \quad R(\infty)=0$$

  And R(t) is a non-increasing function between these limits.
- Second, reliability is predicated on “intended function:” Generally, this is taken to mean operation without failure. However, even if no individual part of the system fails, but the system as a whole does not do what was intended, then it is still charged against the system reliability. The system requirements specification is the criterion against which reliability is measured.
- Third, reliability applies to a specified period of time. In practical terms, this means that a system has a specified chance that it will operate without failure before time t. Reliability engineering ensures that components and materials will meet the requirements during the specified time. Units other than time may sometimes be used. The automotive industry might specify reliability in terms of miles, the military might specify reliability of a gun for a certain number of rounds fired. A piece of mechanical equipment may have a reliability rating value in terms of cycles of use.
- Fourth, reliability is restricted to operation under stated conditions. This constraint is necessary because it is impossible to design a system for unlimited conditions.

Reliability program plan

Many tasks, methods, and tools can be used to achieve reliability. Every system requires a different level of reliability. A commercial airliner must operate under a wide range of conditions, the consequences of failure are grave, but there is a correspondingly higher budget. A pencil sharpener may be more reliable than an airliner, but has a much different set of operational conditions, mild consequences of failure, and correspondingly lower budget.

A reliability program plan is used to document exactly what tasks, methods, tools, analyses and tests are required for a particular system. For complex systems, the reliability program plan is a separate document. For simple systems, it may be combined with the systems engineering management plan. The reliability program plan is essential for a successful reliability program and is developed early during system development. It specifies not only what the reliability engineer does, but also the tasks performed by others. The reliability program plan is approved by top program management.

RELIABILITY REQUIREMENTS

System reliability parameters

Requirements are specified using reliability parameters. The most common reliability parameter is the mean time between failures (MTBF), which can also be specified as the failure rate or the number of failures during a given period. These parameters are very useful for systems that are operated on a regular basis, such as most vehicles, machinery and electronic equipment. Reliability increases as the MTBF increases. The MTBF is usually specified in hours; but can also be used with any unit of duration such as miles or cycles.

In other cases, reliability is specified as the probability of mission success. For example, reliability of a scheduled aircraft flight can be specified as a dimensionless probability or a percentage.

A special case of mission success is the single-shot device or system. These are devices or systems that remain relatively dormant and only operate once. Examples include automobile airbags, thermal batteries and missiles. Single-shot reliability is specified as a probability of success, or is subsumed into a related parameter. Single-shot missile reliability may be incorporated into a requirement for the probability of hit.

In addition to system level requirements, reliability requirements may be specified for critical subsystems. In all cases, reliability parameters are specified with appropriate statistical confidence intervals.

It is a general praxis to model the early failure rate with an exponential distribution. This less complex model for the failure distribution has only one parameter: the constant failure rate.

DESIGN FOR RELIABILITY

Reliability must be “designed in” to the system. During system design, the top-level reliability requirements are allocated to subsystems by design engineers and reliability engineers working together.

Reliability design begins with the development of a model. Reliability models use block diagrams and fault trees to provide a graphical means of evaluating the relationships between different parts of the system. These models incorporate predictions based on parts-count failure rates taken from historical data. While the predictions are often not accurate in an absolute sense, they are valuable to assess relative differences in design alternatives.

One of the most important design techniques is redundancy. This means that if one part of the system fails, there is an alternate success path, such as a backup system. For example, an automobile brake light might use two light bulbs. If one bulb fails, the brake light still operates using the other bulb. Redundancy significantly increases system reliability, and is often the only viable means of doing so. However, redundancy is difficult and expensive, and is therefore limited to critical parts of the system. Another design technique, physics of failure, relies on understanding the physical processes of stress, strength and failure at a very detailed level. Then the material or component can be re-designed to reduce the probability of failure. Another common design technique is component derating: Selecting components whose tolerance significantly exceeds the expected stress, as using a heavier gauge wire that exceeds the normal specification for the expected electrical current.

Many tasks, techniques and analyses are specific to particular industries and applications. Commonly these include:

- Built-in test (BIT)
- Failure mode and effects analysis (FMEA)
- Reliability simulation modeling
- Thermal analysis
- Fault tree analysis
- Sneak circuit analysis
- Weibull analysis
- Electromagnetic analysis

- Statistical interference

Results are presented during the system design reviews and logistics reviews. Reliability is just one requirement among many system requirements. Engineering trade studies are used to determine the optimum balance between reliability and other requirements and constraints.

FAILURE RATE

Failure rate is the frequency with which a system or component fails, expressed for example in failures per hour. It is often denoted by the Greek letter λ (lambda) and is important in reliability theory. In practice, the reciprocal rate MTBF is more commonly expressed and used for high quality components or systems.

Failure rate is usually time dependent, and an intuitive corollary is that both rates change over time versus the expected life cycle of a system. For example, as an automobile grows older, the failure rate in its fifth year of service may be many times greater than its failure rate during its first year of service – one simply does not expect to replace an exhaust pipe, overhaul the brakes, or have major power plant-transmission problems in a new vehicle. So in the special case when the likelihood of failure remains constant with respect to time (for example, in some product like a brick or protected steel beam), failure rate is simply the inverse of the mean time between failure (MTBF), expressed for example in hours per failure. MTBF is an important specification parameter in all aspects of high importance engineering design – such as naval architecture, aerospace engineering, automotive design, etc. – in short, any task where failure in a key part or the whole of a system needs be minimized and severely curtailed, particularly where lives might be lost if such factors are not taken into account. These factors account for many safety and maintenance practices in engineering and industry practices and government regulations, such as how often certain inspections and overhauls are required on an aircraft. A similar ratio used in the transport industries, especially in railways and trucking is ‘Mean Distance Between Failure’, a variation which attempts to correlate actual loaded distances to similar reliability needs and practices. Failure rates and their projective manifestations are important factors in insurance, business, and regulation practices as well as fundamental to design of safe systems throughout a national or international economy.

Failure rate in the discrete sense

In words appearing in an experiment, the failure rate can be defined as **“The total number of failures within an item population, divided by the total time expended by that population, during a particular measurement interval under stated conditions”.**

Here failure rate λ (t) can be thought of as the probability that a failure occurs in a specified interval, given no failure before time t. It can be defined with the aid of the reliability function or survival function R(t), the probability of no failure before time t, as:

$$\lambda = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)} = \frac{R(t) - R(t + \Delta t)}{\Delta t.R(t)}$$

where t_1 (or t) and t_2 are respectively the beginning and ending of a specified interval of time spanning Δt. Note that this is a conditional probability, hence the R(t) in the denominator.

Failure rate in the continuous sense (Instantaneous Hazard rate)

By calculating the failure rate for smaller intervals of time Δt, the interval becomes infinitesimally small. This results in the hazard function, which is the instantaneous failure rate at any point in time:

$$r(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t.R(t)}$$

Continuous failure rate depends on a failure distribution, F(t), which is a cumulative distribution function that describes the probability of failure prior to time t,

$P(T \leq t) = F(t) = 1 - R(t), t \geq 0.$

where T is the failure time. The failure distribution function is the integral of the failure density function, f(x),

$$F(t) = \int_0^t f(x)dx.$$

Now, the hazard function can be defined as

$$r(t) = \lim_{\Delta t \to 0} \frac{R(t + \Delta t) - R(t)}{R(t).\Delta t} = \frac{-R'(t)}{R(t)}$$

$$r(t) = \frac{f(t)}{R(t)}$$

There are many failure distributions. A common failure distribution is the exponential failure distribution.

$$F(t) = \int_0^t \lambda e^{-\lambda x} dx = 1 - e^{-\lambda t} ,$$

which is based on the exponential density function. This leads to a constant hazard rate. For other distributions, such as the Weibull distribution, log-normal distribution or bathtub curve, the hazard function is not constant, which means that the failure rate varies with time.

Units of Failure Rate

Failure rates can be expressed using any measure of time, but hours is the most common unit in practice. Other units, such as miles, revolutions, etc., can also be used in place of “time” units.

Failure rates are often expressed in engineering notation as failures per million, or 10^6, especially for individual components, since their failure rates are often very low.

SYSTEM CONFIGURATIONS

By system, we mean an arbitrary device having several units/sub systems/components assuming that their reliabilities are known which help us to predict the reliability of whole system. It is now important that the system structure be known. Various system structures have been considered as follows:

Series Configuration

A system having n-units is said to have series configuration if the failure of an arbitrary unit, say i^{th} unit causes the entire system failure. The examples of the series configurations are :-

(i) The aircraft electronic system consists of a sensor sub system, a guidance subsystem, computer subsystem and the fire control subsystem. These systems can only operate successfully if all these operate simultaneously.

(ii) Deepawali or Christmas glow bulb, where if one bulb fails the whole lead fails. The block diagram of a series system is shown in fig. 1.2.

Let $R_i(t)$ be the reliability of i^{th} component, then the system reliability is given by

$$R(t) = \Pr[T > t] = \Pr[\min(T_1, T_2, T_3\ldots\ldots\ldots, T_n) > t]$$
$$= \prod_{i=1}^{n} P[T_i > t] = \prod_{i=1}^{n} R_i(t)$$

where

T_i is the life time of the i^{th} unit of the system. The system hazard rate, therefore is

$$r(t) = \sum_{i=1}^{n} r_i(t)$$

where

$r_i(t)$ is the instantaneous failure rate of i^{th} unit.

Parallel Configuration

In this configuration, all the units are connected in parallel i.e. the failure of the system occurs only when all the units of system fail. For example, four engined aircraft which is still able to fly with only two engines working. Block diagram representing a parallel configuration is shown in fig. 1.3.

Suppose $R_i(t)$ and T_i be the reliability of i^{th} components and the life time of the i^{th} unit in time t respectively, then the system reliability is given by

$$R(t) = \Pr[T > t]$$
$$= \Pr[\max.(T_1, T_2, T_3, \dots\dots\dots, T_n) > t]$$
$$= 1 - P[T_1 \leq t, T_2 \leq t, T_3 \leq t, \dots\dots\dots\dots\dots T_n \leq t]$$

If the units function independently, then

$$R(t) = 1 - [1-R_1(t)]\,[1-R_2(t)]\,[1-R_3(t)]\dots\dots\dots[1-R_n(t)]$$
$$= 1 - \prod_{i=1}^{n}[1-R_i(t)]$$

Standby Redundant Configuration

To assure high reliability of a system, redundancy is incorporated. In redundant system more units than the required are used so that when failures occur in a system, it does not stop functioning. In standby redundant system with n units, only one unit is on-line at a time. When it fails, it is replaced manually or automatically by a standby unit. This process continues until all (n-1) standby units have been exhausted. For example, consider a cinema hall in a city where power supply is irregular. In order to ensure uninterrupted supply of power apart from the regular source of supply, a generator is kept as standby. The generator is switched on as and when the main supply is resumed. A block diagram of such system is shown in fig. 1.4.

Gnedenko et al. (1969) classified the standby units as follows :-

a) If the off-line unit can fail and is loaded in exactly the same way as the operating unit. It is called the hot standby unit.

b) If the off-line unit can fail and can diminish the load, it is called warm standby unit. The probability of failure for a warm standby is less than the failure of operative unit.

c) If the off-line unit cannot fail and is completely unloaded, it is called cold standby.

Reliability R(t) of an n-unit standby system at any time instant t is given as

$$R(t) = \Pr\left[\sum_{i=1}^{n} T_i > t\right]$$

where

T_i is the life time of i^{th} unit and all the n units are independent.

k-out-of-n-Configuration

In many problems, the system operates if atleast k-out-of-n-units function e.g. a bridge supported by n cables, k of which are necessary to support the maximum load. If each of n-units are identical with the same reliability $R_0(t)$ (say), then the system reliability becomes

$$R(t) = \sum_{i=k}^{n} {}^{n}C_{i} R_{o}^{i}(t)[1 - R_{o}]^{n-1}$$

Series (k = n) and parallel (k = 1) system are subclasses of k-out-of-n structure. There are many other configurations as series-parallel, parallel series, mixed-parallel, etc. some of them are depicted in Fig. 1.5 to Fig. 1.7.

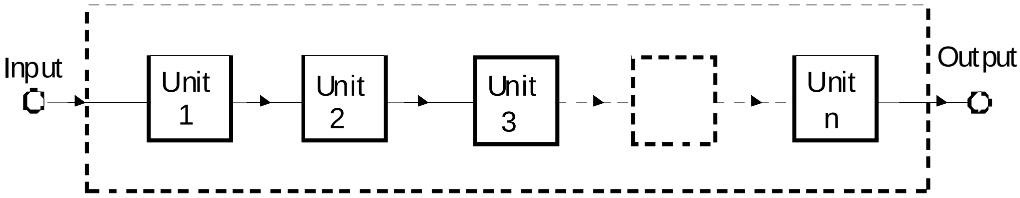

Fig. 1.2 Series Configuration

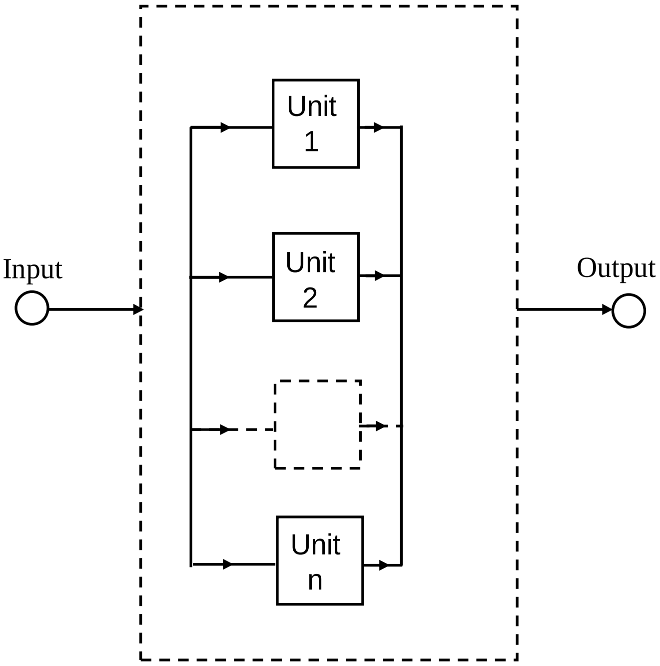

Fig. 1.3 Parallel Configuration

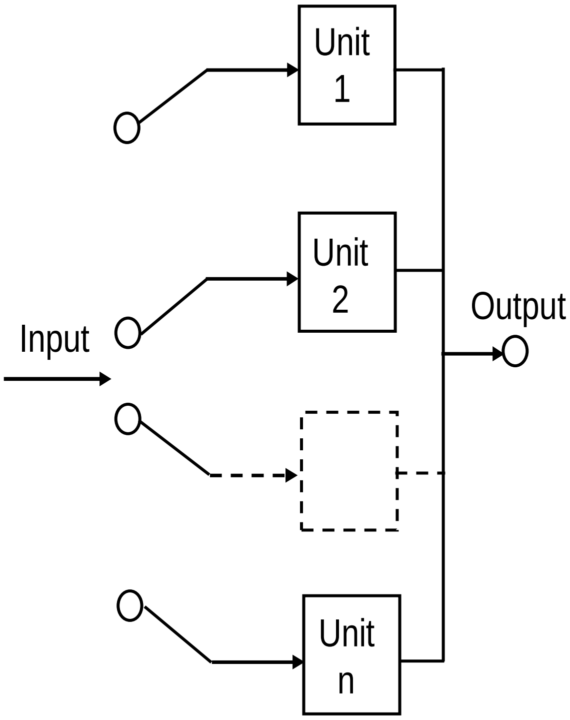

Fig. 1.4 Standby redundant Configuration

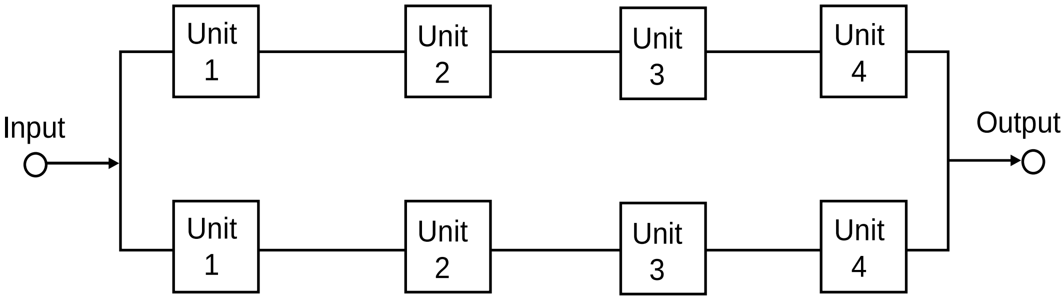

Fig. 1.5 Series Parallel Configuration

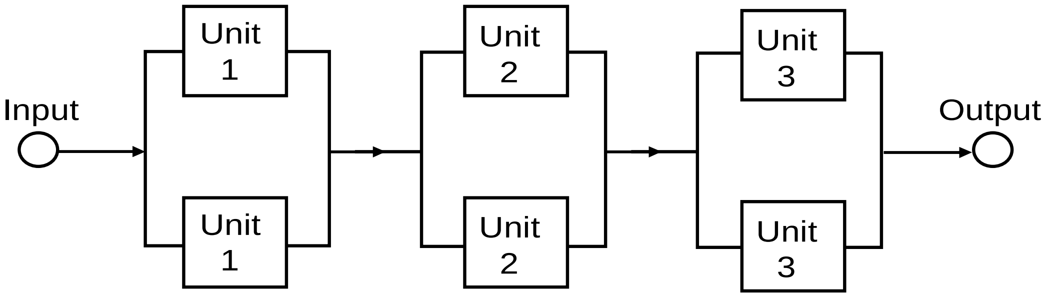

Fig. 1.6 Parallel Series Configuration

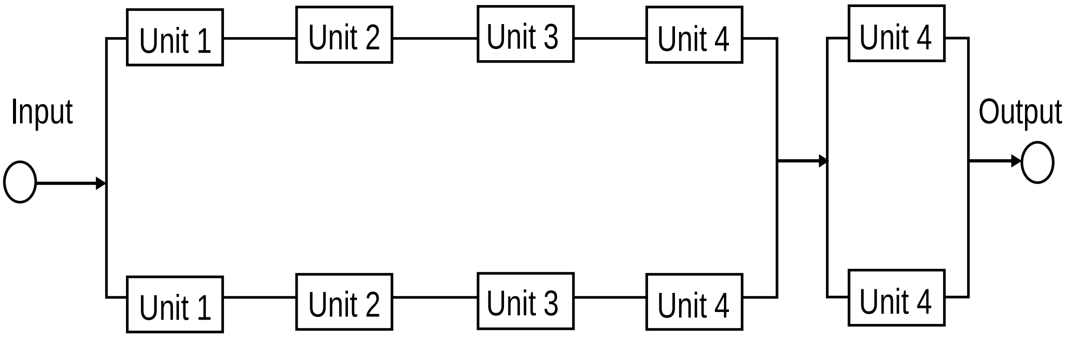

Fig. 1.7 Mixed Parallel Configuration

TRANSFORMS AND CONVOLUTIONS

Laplace Transforms

A transform is merely a mapping or function from one space to another. While it may be very difficult to solve certain equations directly for a particular function of interest, it is often easier to solve a corresponding equation in terms of a transform of the function and then invert the transform to obtain the function. One particular transform, that is, very useful for solving some types of differential equations as well as certain integral equations, is the Laplace transform (L.T.).

Let f(t) be a function of positive real variable t. Then the Laplace transform of f(t) is denoted by f*(s) and defined as

$$L[f(t)] = f^*(s) = \int_0^\infty e^{-st} f(t)\, dt.$$

For the range of value of s for which the integral exists. Here f(t) is called an inverse Laplace transform of f*(s) and we write $f(t) = L^{-1}\{f^*(s)\}$. The following are some important properties of Laplace transform:-

i) $L\left[\sum_{i=1}^{n} c_i f_i(t)\right] = \sum_{i=1}^{n} c_i f_i^*(s)$

ii) $L[t^n f(t)] = \frac{(-1)^n d^n f^*(s)}{ds^n}$

iii) $L\left[\int_o^t f(u)du\right] = \frac{f^*(s)}{s}$

iv) $\lim_{t-0} f(t) = \lim_{s\to\infty} sf^*(s)$ (initial value theorem)

v) $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sf^*(s)$ (final value theorem)

vi) $\lim_{s\to\infty} sf^*(s) = 0$

vii) $\lim_{s\to 0} f^*(s) = 1$ if f*(s) is L.T. of a p.d.f.

Laplace Stieltjes Transforms

Let X be a non-negative random variable with distribution function

$F(x) = \Pr[X \le x]$

then Laplace Stieltjes transform of F(x) is defined for s > 0 by

$$F^{**}(s) = \int_0^\infty e^{-SX}\, dF(X)$$

under certain regular conditions, we have

$$F^{**}(s) = s\int_0^{\infty} e^{-SX}\, F(X)dX = sF^*(s) \text{ and}$$

$$F^{**}(s) = \int_0^{\infty} e^{-SX}\, f(X)dX = f^*(s)$$

where

$$f(x) = \frac{dF(x)}{dx}$$

CONVOLUTION

Let f(t) and g(t) be two real valued non-negative continuous functions t, then the integral

$$\int_0^t f(t-u)g(u)du = \int_0^t g(t-u)f(u)du$$

$$= f(t)© g(t)$$

$$= L^{-1}\,[f^*(s)\, g^*(s)]$$

is called Laplace convolution of the functions f(t) and g(t). If F(t) and G(t) be two real valued distribution functions defined for $t \geq 0$, the resulting convolution is again a distribution and integral

$$\int_0^t F(t-u)dG(u) = \int_0^t G(t-u)dF(u) = F(t)®G(t)$$

is known as Stieltjes convolution of F(t) and G(t).

MEAN SOJOURN TIME IN A STATE

The expected time taken by the system in a particular state before transiting to any other state in known as mean sojourn time or mean survival time in that state. If T_i be the sojourn time in state i, then the mean sojourn time in state i is

$$\mu_i = \int_0^{\infty} \Pr(T_i > t)dt$$

FIRST PASSAGE TIME

Suppose that a system starts with the state j, then time taken to reach a given state k for the first time from state j is called first passage time. In general, first passage time is a measure of how long it takes to reach a given state from another state.

MEAN TIME TO SYSTEM FAILURE (MTSF)

It is defined as the expected time for which the system is in operation before it completely fails.

Let f(t) be the probability density function of life time of the system, then we have

$$\text{MTSF} = E(T) = \int_0^\infty t f(t)dt = \int_0^\infty R(t)dt$$

Also

$$\lim_{s\to 0} R^*(s) = \int_0^\infty R(t)dt$$

$$\Rightarrow \qquad \text{MTSF} = \lim_{s\to 0} R^*(s)$$

Let $\phi_0(t)$ be the cumulative distribution function of the first passage time from initial state to a failed state, then

$$R^*(s) = \frac{1-\phi_0^{**}(s)}{s}$$

from above equations, we have

$$\text{MTSF} = \lim_{s\to 0} R^*(s) = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s}$$

Where

$R^*(s)$ and $\phi_0^{**}(s)$ are respectively the Laplace transform and Laplace Stieltjes transform of R(t) and ϕ_0 (t).

AVAILABILITY

Availability is well established in the literature of stochastic modeling and optimal maintenance. Barlow and Proschan [1975] define availability of a repairable system as “the probability that the system is operating at a specified time t” and in reliability theory, the term availability has the following meanings:

The degree to which a system, subsystem or equipment is operable and in a committable state at the start of a mission, when the mission is called for at an unknown, i.e. a random time. Simply put, availability is the proportion of time a system is in a functioning condition. In general, we may categorize this measure as:-

a) Instantaneous Availability

This is the probability that the system will be able to operate within the tolerances at a given instant of time t(say). Let this probability be denoted by A(t).

Let X(t) = 1 if the system is operable at time t and X(t) = 0 when it is not operable. The availability A(t) of the system at time t is given by

$$A(t) = \Pr[X(t) = 1 \mid X(0) = 1]$$

b) Average (Interval) Availability

It is the expected fraction of a given interval of time that the system will be able to operate within tolerances.

Suppose the given interval of time is (0, t] then interval availability H(0, t] of this interval is given by :-

$$H(0, t] = \frac{1}{t}\int_0^t A(u)du = \frac{\mu_{up}(t)}{t}$$

When $\mu_{up}(t)$ = expected up time of the system during (0, t].

c) Steady-state (Limited Interval) Availability

The long run or steady-state availability is defined as the proportion of the time during which an equipment is available for use.

Mathematically, it is the limiting value of the point wise availability when t become finitely large i.e.

$$A = \lim_{t\to\infty} A(t)$$

RELIABILITY AND AVAILABILITY

The availability function A(t) is defined as the probability that the equipment is operating at time t. Although this definition appears to be very similar to the reliability function R(t), the two have different meaning. While reliability places emphasis on failure – free operation up to time t, availability is concerned with the status of the equipment at time t. The availability function does not say anything about the number of failures that occurred during time t. This means that two equipments A and B can have different number of failures in a given time interval and can still have the same availability.

MAINTAINABILITY

Maintainability is the probability that the system will be restored to operational effectiveness within a specified time when the maintenance action is taken in accordance with prescribed conditions. Maintenance is one of the effective ways of increasing the reliability of a system. Maintenance action can be classified in several categories: preventive maintenance, corrective maintenance and repair maintenance.

Preventive maintenance includes actions such as lubrication, replacement of a nut or screw of some part of the system, refueling, cleaning, etc. It is designed to minimize the limit that the system will spend in degraded states, it is a sort of repair that is done before a unit actually fails. Corrective maintenance deals with the system performance when it gives wrong result and it involves minor repairs that may creep up between inspections.

Repair maintenance is also concerned with increasing the system availability. In order to increase availability, failed unit upon failure is returned to operation by sending it to a repair facility if available, otherwise waits for repair. There may be two types of repair policies:-

a) Repeat repair policy

Due to certain reasons the repair of a failed unit has to be stopped. When the repair is begun again, it is started all over again.

b) Resume repair policy

The repair of failed component is terminated before completion due to one reason or the other. When it begins again, it is started from the stage where it was prior to the termination of repair.

BUSY PERIOD OF THE REPAIRMAN/SERVER WITH THE SYSTEM

Let B(t) be the probability that a repairman/server is busy with the system in the interval (0, t], then in the long run the total fraction of time for which a repairman is busy is given by :-

$$B = \lim_{t \to \infty} B(t)$$

EXPECTED NUMBER OF VISITS BY THE SERVER

Let N(t) be a random variable representing the number of times, the repairman has visited the system in the interval (0, t], then the expected number of visits by the repairman to the system in (0, t], is E[N(t)] and in the long run this number per unit time is given by

$$N = \lim_{t \to \infty} \frac{E[N(t)]}{t}$$

PROFIT ANALYSIS

Any manufacturing industry is basically a profit making organization and no organization can survive for long without minimum financial returns for its investment. There must be an optimal balance between the reliability aspect of a product and its cost. The major items contributing to the total cost are research and development, production, spares and maintenance. How the cost of these individual items varies with reliability shown in fig. 1.8. In order to increase the reliability of the products, we would require a correspondingly high investment in the research and development activities. The production cost also would increase with the requirement of greater reliability.

The revenue and cost function lead to the profit function of a firm, as the profit is excess of revenue over the cost of production. The profit function in time t is given by:-

$$P(t) = \text{Expected revenue in } (0, t] - \text{Expected total cost in } (0, t]$$

In general, the optimal policies can more easily be derived for an infinite time span or compared to a finite time span. The profit per unit time, in infinite time span is expressed as

$$\lim_{t \to \infty} \frac{P(t)}{t}$$

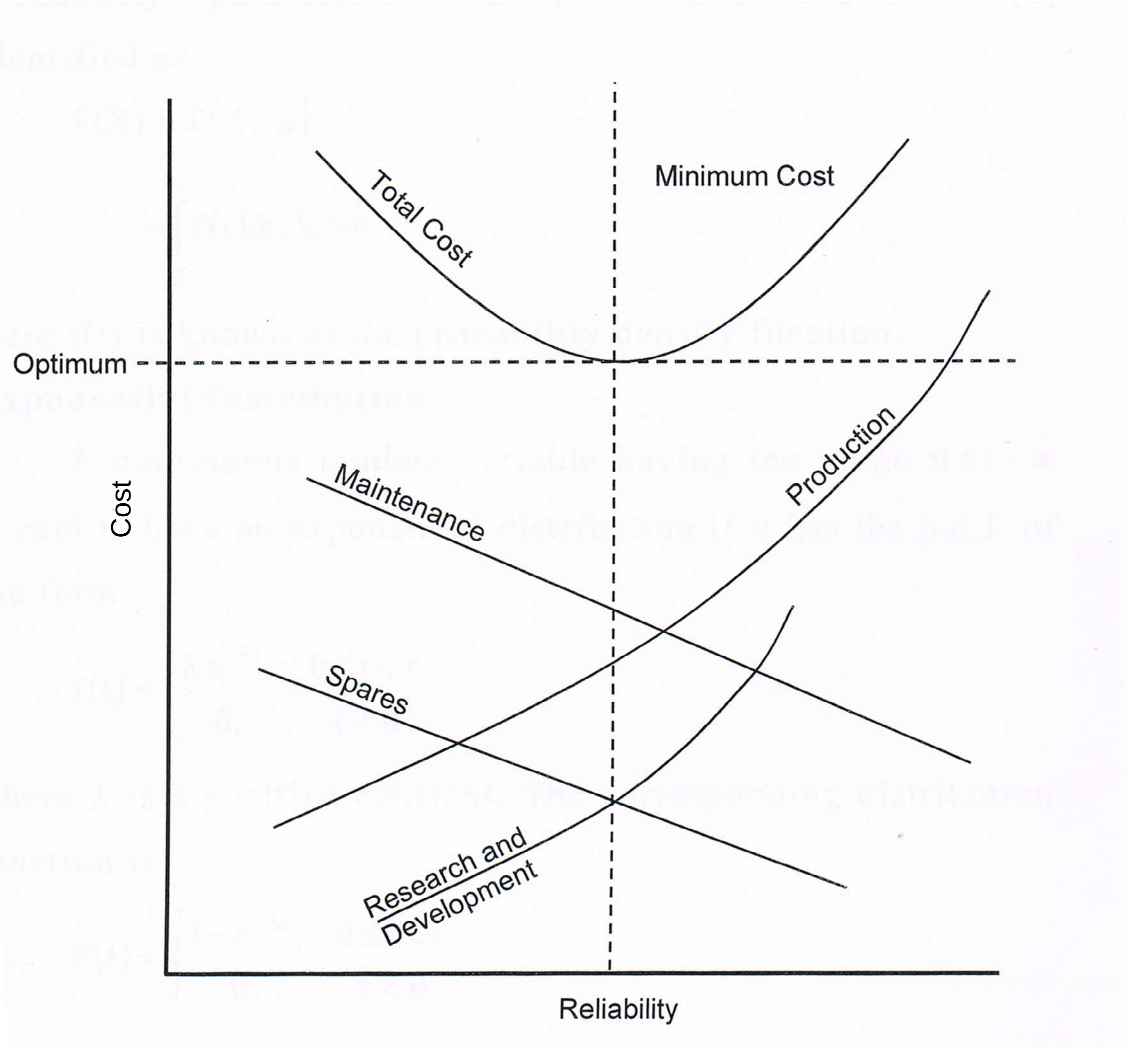

Fig. 1.8 Reliability vs. Cost

i.e. profit per unit time = total revenue per unit time – total cost per unit time. Considering the various costs, the profit equation is given as

$$P = K_1A_0 - K_2B_0 - K_3N_0$$

where

P = Profit per unit time incurred to the system

K_1 = Revenue per unit up time of the system

A_0 = Total fraction of time for which the system is up

K_2 = Cost per unit time for which server is busy

B_0 = Total fraction of time for which the server is busy

K_3 = Cost per visit by the server

N_0 = Expected number of visits per unit time for the server

EXPONENTIAL DISTRIBUTION

Characterization

Probability density function

The probability density function (pdf) of an exponential distribution has the form

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & , \ x \geq 0, \\ 0 & , \ x < 0. \end{cases}$$

Where $\lambda > 0$ is a parameter of the distribution, often called the rate parameter. The distribution is supported on the interval $[0,\infty)$. If a random variable X has this distribution, we write X ~ Exp. (λ).

STOCHASTIC PROCESS

A stochastic process is a family of random variables $\{X(t) \mid t \in T\}$, defined on a given probability space, indexed by the parameter t, where t varies over an index set T. Both the parametric set and state space can be independently either discrete or continuous.

In stochastic process $\{X(t), t\in T\}$, where X(t), t and T respectively, the state space, parameter (generally taken to be time) and the index set if T is a countable set as $T=\{0,1,2,3\ldots\ldots\ldots.\}$, then the stochastic process is said to be a discrete parameter process and if $T=\{t : -\infty < t < \infty\}$, the stochastic process is said to be a continuous parametric process. The state space is classified as discrete if it is finite or countable and continuous if it consists of an interval on the real line. In the present study, we have only dealt with discrete state space continuous time parameter stochastic processes.

MARKOV PROCESS

If $\{X(t), t\in T\}$ is a stochastic process such that given the value of X(s), the value of X(t), t>s do not depend on the values of X(u), u<s i.e. for t>s, $i\in s$.

$$\Pr\{X(t)=i|X(u), 0\leq u\leq s\} = \Pr\{X(t) = i|X(s)\}$$

Then the process $\{X(t), t \in T\}$ is a Markov process.

MARKOV CHAIN

A discrete parameter Markov process is known as a Markov Chain. The stochastic process $\{X_n, n = 0, 1, 2, \ldots\ldots\}$ is called a Markov chain, if, for j, k, j_1, $j_2 \ldots\ldots\ldots$, $j_{n-1} \in N$,

$\Pr [X_n = k/X_{n-1} = j, X_{n-2} = j_1, \ldots\ldots\ldots\ldots, X_0 = j_{n-1}]$

$= \Pr \{ X_n = k \mid x_{n-1} = j\}$

$= p_{jk}$ (say)

The conditional probability p_{jk} is called transition probability from the state j at(n-1)th trial to the state k at nth trial. If the transition probability p_{jk} is independent of n, the Markov Chain is said to be homogenous; and if it is dependent of n, the chain is said to be non-homogeneous.

SEMI-MARKOV PROCESS

In the above, assume that the process is time homogeneous, i.e.

$\Pr \{X_{n+1} = j, t_{n+1} - t_n \leq t \mid X_n = i\} = Q_{ij}(t)$, $i, j \in s$, is independent of n, then there exist limiting transition probabilities.

$p_{ij} = \lim\limits_{t\to\infty} Q_{ij}(t) = \Pr \{X_{n+1} = j \mid Xn = i\}$

then $\{X_n, n = 0, 1, 2, \ldots\ldots)$ constitute a Markov Chain with state space $E=\{0,1,2,\ldots\}$ and transition probability matrix (t.p.m.)

$P = [p_{ij}]$

The continuous parameter stochastic process Y(t) with state space E defined by

$Y(t) = X_n, t_n < t < t_{n+1}$

is called a semi-Markov process. The Markov Chain X_n is said to be an embedded Markov chain of the semi-Markov process.

In other words, we define the semi-Markov process is a process in which transition from one state to another is governed by the transition probabilities of a Markov process but the time spent in each state before a transition occurs, is a random variable depending upon the last transition made. Thus at transition instants the semi-Markov process behaves just like a Markov process. However, the times at which transitions occur are governed by a different probability mechanism.

REGENERATIVE PROCESS

Regenerative stochastic process was defined by Smith (1955) and has been crucial in the analysis of complex system. In this, we take time points at which the system history prior to the time points is irrelevant to the system conditions. These points are called regenerative points. Let X(t) be the state of the system of epoch. If t_1, t_2,… are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process $\{X(t), t = t_1, t_2\ldots\ldots\ldots\}$ is called regenerative process.

CHAPTER-II

Reliability Modelling and Analysis of Single Unit Continuous Casting Plant

Standby systems with various repair facilities are commonly used in industries. Thus, researchers have spent a great deal of effort in analyzing such systems. Goel and Murari [1990] studied a two unit system subject to checking, corrective maintenance and system replacement with repairable and non repairable type of failure, Gopalan and Muralidhar [1991] wrote about a repairable system subject to online preventive maintenance. Significant research has been further done in this area due to potential importance into industries, wherein researchers have established various concepts for system analysis, such as partial failure with inspections, stochastic analysis of a three unit system, system with regular repairman who is not always available, perfect repair at partial or complete failure, concept of accident and so forth. However in all this research work, the researchers have only analyzed and studied the system under various hypothetical failure and repair situations. The particular case analysis is based on the assumed numerical values of various failure and repair rates/costs. The missing part thus, is a significant balance between the modeling strategy adopted and its practical usefulness to some case examples in the manufacturing units; the research lacks a modeling strategy embedded by the types of breakdown /repair actually seen in the industries.

In view of practical applications, here we study a modeling strategy of a CC plant. The operative unit is subject to transit to any of the failed states depending on the type of failure/breakdown. The failed unit is attended by the repairman as soon as it fails. The system regenerates and works like a new one after each repair, replacement or reconditioning and reinstallation. This chapter outlines the modelling strategy embedded by the types of failures actually depicted into data and important reliability indices such as the mean time to system failure and steady state availability are obtained using semi-Markov processes and regenerative point techniques.

The collected data gives the following estimation:

Probability of repairable failure of type 1 category, p_1=0.191.

Probability of repairable failure of type 2 category, p_2=0.234.

Probability of replaceable failure of type 3 category, p_3=0.085.

Probability of replaceable failure of type 4 category, p_4=0.212.

Probability of recondition and reinstallation failure of type 5 category, p_5=0.276.

Estimated value of failure rate, $\lambda = 0.003$ per hour.

Estimated value of repair rate of type 1 category,$\alpha_1 = 0.092$ per hour.

Estimated value of repair rate of type 2 category, α_2 = 0.687 per hour

Estimated value of replacement rate of type 3 category, β_1 = 0.108 per hour.

Estimated value of replacement rate of type 4 category, β_2 = 0.046 per hour.

Estimated value of recondition and reinstallation rate of type 5 category, β = 0.136 per hour.

The system is analyzed using semi markov process and regenerative point technique, and the following reliability indices of the system effectiveness are obtained

- Mean time to system failure.
- Steady state availability.

MODEL ASSUMPTIONS

The unit is initially operative at state 0 and transits probabilistically depending on the type of failure to any of the five states 1 to 5 with probabilities p_1, p_2, p_3, p_4 and p_5 respectively (refer Fig.2.1).

1. All failure times are assumed to have exponential distribution with failure rate (λ) whereas the repair times have general distributions.
2. After each repair/replacement/reconditioning and reinstallation at states 1 to 5,the system works as good as new and returns back to state 0.
3. Breakdowns are self announcing.
4. The repairman comes as soon as the unit fails.

NOTATIONS

O : Operative unit.

λ : Constant failure rate of the unit.

p1 : Probability of failure (repairable failure of type 1 category).

p2 : Probability of failure (repairable failure of type 2 category).

p3 : Probability of failure (replaceable failure of type 3 category).

p4 : Probability of failure (replaceable failure of type 4 category).

p5 : Probability of failure (recondition and reinstallation of type 5 category).

$r_1(t)$: Unit is under repair of type 1 category.

r_2 (t) : Unit is under repair of type 2 category.

rep_1 (t) : Unit is under replacement of type 3 category.

rep_2 (t) : Unit is under replacement of type 4 category.

$r_i(t)$: Unit is under reconditioning and reinstallation of type 5 category

© : Convolution.

$p_{ij}, Q_{ij}(t)$: p.d.f., c.d.f. of first passage time from a regenerative state *i* to *j* or to a failed state j in (0, t].

$\square_i(t)$: c.d.f. of first passage time from a regenerative state *i* to a failed state *j*.

* : Laplace Transforms (LT), i.e., for any f(t) and g(t);

$f(t) * g(t) = \int_0^t f(t-u)g(u)du$

$g_1(t), G_1(t)$: p.d.f., c.d.f. of repair time of failed unit of type 1 category.

$g_2(t), G_2(t)$: p.d.f., c.d.f. of repair time of failed unit of type 2 category.

$h_1(t), H_1(t)$: p.d.f., c.d.f. of the replacement of type 3 category

$h_2(t), H_2(t)$: p.d.f., c.d.f. of the replacement of type 4 category.

$h(t), H(t)$: p.d.f., c.d.f. of reconditioning and reinstallation of type 5 category.

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A transition diagram showing the various states of transition of the system is shown in Fig 1.The epochs of entry into states 0,1,2,3,4 and 5 are regenerative points and hence these states are regenerative states .The states 1,2,3,4 and 5 are failed states. The transition probabilities are given by:

$dQ_{01} = p_1 \lambda e^{-\lambda t} dt$, $dQ_{02} = p_2 \lambda e^{-\lambda t} dt$,

$dQ_{03} = p_3 \lambda e^{-\lambda t} dt$, $dQ_{04} = p_4 \lambda e^{-\lambda t} dt$,

$dQ_{05} = p_5 \lambda e^{-\lambda t} dt$, $dQ_{10} = g_1(t)dt$,

$dQ_{20} = g_2(t)dt$, $dQ_{30} = h_1(t)dt$,

$dQ_{40} = h_2(t)dt$, $dQ_{50} = h(t)dt$ (1) – (10)

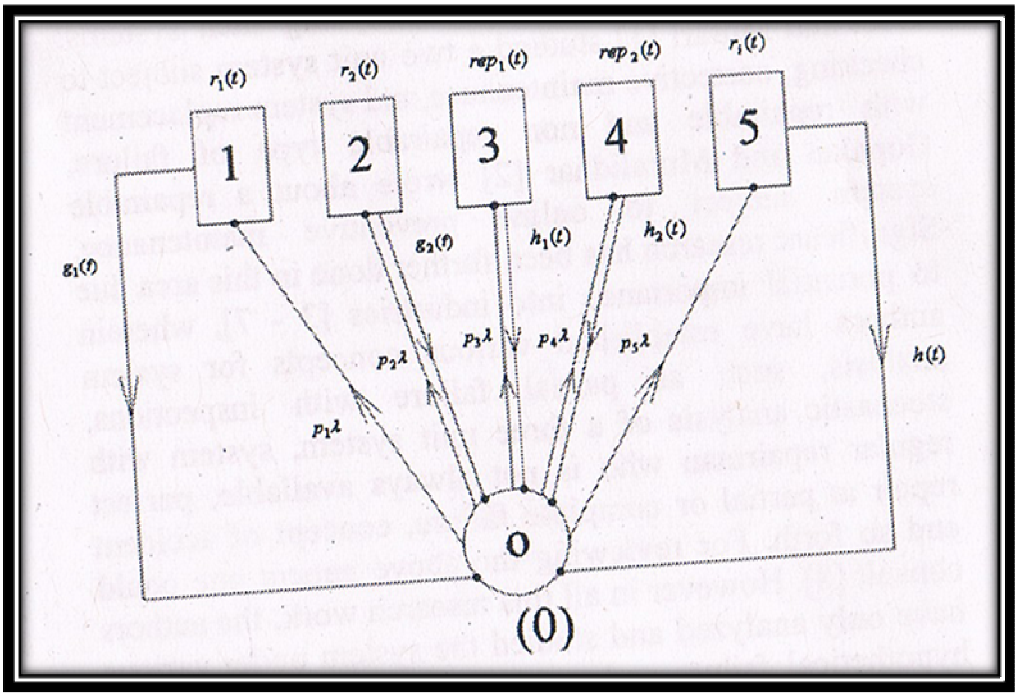

Fig. 2.1 Transition states of the system

The non-zero elements p_{ij} are given below:

$p_{01} = p_1$, $p_{02} = p_2$, $p_{03} = p_3$, $p_{04} = p_4$, $p_{05} = p_5$

$p_{10} = 1$, $p_{20} = 1$, $p_{30} = 1$, $p_{40} = 1$, $p_{50} = 1$ (11) - (20)

By these transition probabilities it is also verified that:
$p_{01} + p_{02} + p_{03} + p_{04} + p_{05} = 1$ (21)
$p_{10} = p_{20} = p_{30} = p_{40} = p_{50} = 1$ (22)

The mean sojourn time (μ_i) in the regenerative state

'*i*' is defined as the time of stay in that state before transition to any other state. If *T* denotes the sojourn time in the regenerative state *i,* then:

$\mu_i = E(T) = Pr[T > t]dt$

Thus: $\mu_0 = \int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda}$; $\mu_1 = \int_0^\infty \bar{G}_1(t)dt$;

$\mu_2 = \int_0^\infty \bar{G}_2(t)dt$; $\mu_3 = \int_0^\infty \bar{H}_1(t)$;
$\mu_4 = \int_0^\infty \bar{H}_2(t)dt$; $\mu_5 = \int_0^\infty \bar{H}(t)dt$; (23) – (28)

The unconditional mean time taken by the system to transit for any regenerative state *'j'* when it (time) is counted from the epoch of entrance into state *'i'* is mathematically stated as:

$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -q_{ij}{}^{*\prime}(0)$ (29)

Thus, $m_{10} = \mu_1$, $m0_1 + m_{02} + m_{03} + m_{04} + m_{05} = \mu_0$,
$m_{20} = \mu_2$, $m_{30} = \mu_3$, $m_{40} = \mu_4$ and $\mu_{50} = \mu_5$

MATHEMATICAL ANALYSIS

A. Reliability and Mean Time To System Failure

Let U_i be the random variable denoting the time to system failure, when the system starts from state i(i=0) then,the reliability of the system is given by:

$R_i(t) = P[U_i>t]$.

Taking the failed states 1, 2, 3, 4 and 5 as absorbing states and employing the arguments used for regenerative processes, we have the following recursive relation for R_0 (t) ,

$R_0(t) = q_{01}(t) + q_{02}(t) + q_{03}(t) + q_{04}(t) + q_{05}(t)$ (30)

By solving the equation (30) using the Laplace Transforms (L.T.) technique one can easily get the expression for R_0 (t) in terms of its L.T, i.e., $R_0{}^*(t)$, now using the usual formula for mean time to system failure (MTSF), we obtain:

$E(T_0) = MTSF = \lim_{s\to 0} Ro*(s) = \frac{N}{D}$ (31)

where $N = \mu_0$ and $D = 1$

B. *Availability Analysis*

Using the probabilistic arguments and defining A_i (t) as the probability that the system is in upstate at the instant *t*, given that the system entered into regenerative state *i* at *t = 0*, we have the following recursive relations:

$$A_0(t) = M_0(t)+q_{01}(t)©A_1(t)+q_{02}©A_2(t)+q_{03}©A_3(t)+q_{04}©A_4(t)+q_{05}©A_5(t) \tag{32}$$

$$A_1 (t) = q_{10} (t)©A_0 (t) \tag{33}$$

$$A_2 (t) = q_{20} (t)©A_0 (t) \tag{34}$$

$$A_3 (t) = q_{30} (t)©A_0 (t) \tag{35}$$

$$A_4 (t) = q_{40} (t)©A_0 (t) \tag{36}$$

$$A_5 (t) = q_{50} (t)©A_0 (t) \tag{37}$$

where $M_0 (t) = e^{-\lambda t}$.

Taking Laplace Transforms (L.T.) of the above equations and solving them for A_0 *(s) , we get:

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{38}$$

The steady state availability of the system is given by:

$$A_0 = \lim_{s \to 0} sAo*(s) = \frac{N1}{D1} \tag{39}$$

where

$N_1(s) = \mu_0$, $D_1 = \mu_0 + p_1\mu_1 + p_2\mu_2 + p_3\mu_3 + p_4\mu_4 + p_5\mu_5$

PARTICULAR CASE

The following particular case is considered:

$g_1(t) = \alpha_1 e^{-\alpha 1t}$, $g_2(t) = \alpha_2 e^{-\alpha 2t}$, $h_1(t) = \beta 1 e^{-\beta 1t}$, $h_2(t) = \beta_2 e^{-\beta 2t}$, $h(t) = \beta e^{-\beta t}$

$p_{01} = p_1$, $p_{02} = p_2$, $p_{03} = p_3$, $p_{04} = p_4$, $p_{05} = p_5$,

$p_{10} = 1$, $p_{20} = 1$, $p_{30} = 1$, $p_{40} = 1$, $p_{50} = 1$

$\mu_0 = \frac{1}{\lambda}$, $\mu_1 = \frac{1}{\alpha 1}$, $\mu_2 = \frac{1}{\alpha 2}$, $\mu_3 = \frac{1}{\beta 1}$, $\mu_4 = \frac{1}{\beta 2}$,

$\mu_5 = \frac{1}{\beta}$

Using the values estimated from the collected data as shown in Section I and the expressions (31) and (39); the mean time to system failure and availability are estimated as:

Mean Time to System Failure: 330.885 hours

Availability: 0.971154608

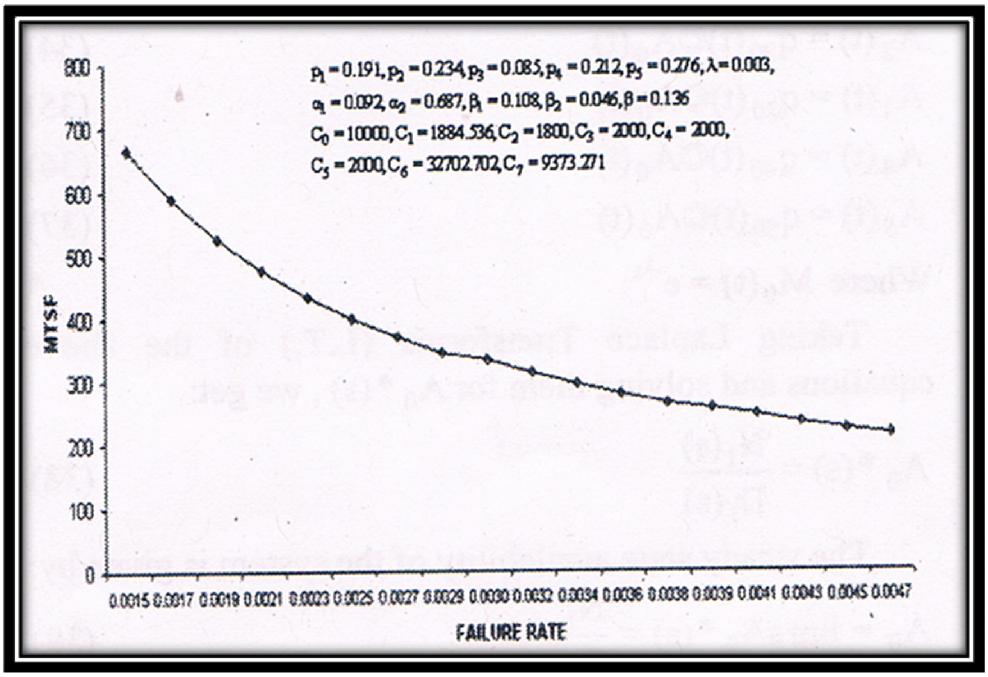
$p_1 = 0.191, p_2 = 0.234, p_3 = 0.085, p_4 = 0.212, p_5 = 0.276, \lambda = 0.003,$
$\alpha_1 = 0.092, \alpha_2 = 0.687, \beta_1 = 0.108, \beta_2 = 0.046, \beta = 0.136$
$C_0 = 10000, C_1 = 1884.536, C_2 = 1800, C_3 = 2000, C_4 = 2000,$
$C_5 = 2000, C_6 = 32702.702, C_7 = 9373.271$
MTSF
800
700
600
500
400
300
200
100
0
0.0015 0.0017 0.0019 0.0021 0.0023 0.0025 0.0027 0.0029 0.0030 0.0032 0.0034 0.0036 0.0038 0.0039 0.0041 0.0043 0.0045 0.0047
FAILURE RATE

Fig. 2.2

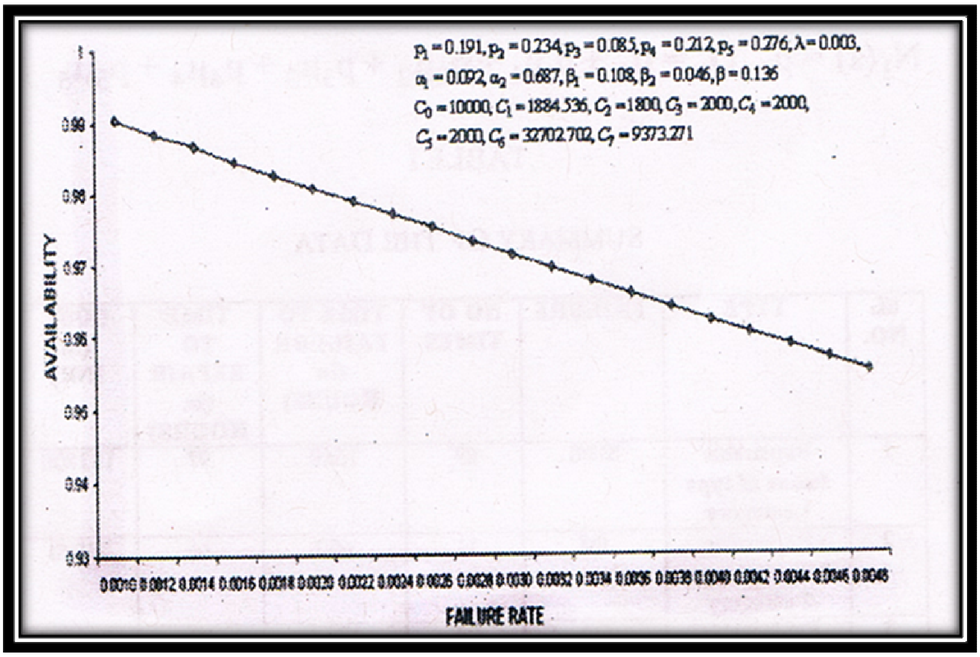
$p_1 = 0.191, p_2 = 0.234, p_3 = 0.085, p_4 = 0.212, p_5 = 0.276, \lambda = 0.003,$
$\alpha_1 = 0.092, \alpha_2 = 0.687, \beta_1 = 0.108, \beta_2 = 0.046, \beta = 0.136$
$C_0 = 10000, C_1 = 1884.536, C_2 = 1800, C_3 = 2000, C_4 = 2000,$
$C_5 = 2000, C_6 = 32702.702, C_7 = 9373.271$
AVAILABILITY
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.0010 0.0012 0.0014 0.0016 0.0018 0.0020 0.0022 0.0024 0.0026 0.0028 0.0030 0.0032 0.0034 0.0036 0.0038 0.0040 0.0042 0.0044 0.0046 0.0048
FAILURE RATE

Fig. 2.3

Chapter-III

Reliability Modelling and Analysis of a Two Unit Continuous Casting Plant

Various authors including Gopalan and Naidu (1982), Natesan (1984), Singh (1989) and Yang and Dhillon (1995), Malik et al. (2008) and Kumar et al. (2010) have discussed two-unit reliability models with different repair and inspection policies. However, not much works pertaining to the cost-benefit analysis of two-unit system have been reported so far in the subject. Hence reliability model of a two unit continuous casting plant with inspection is under taken for study. This paper has been proposed by A.G. Mathew et al. in 2010. Four years downtime maintenance data, on electrically operated overhead traveling (EOT) cranes (called as critical equipment/two-unit system) from a steel production plant, currently operational at Durgapur, India, have been collected. Three types of failures were noted in the plant, as depicted into the data viz., repairable, replaceable and reconditioning/reinstallation. The failure, repair and replacement rates along with the probabilities of various failures of the critical equipments of the CC plant have been estimated from the data. The failure situations considered in the present modeling are the same as those depicted in the data and the analysis is carried out by using the real estimated values of various rates and probabilities. Thus, the paper offers a new contribution to the reliability literature in terms of a real case analysis of a two-unit parallel operating system of a CC plant with full-reduced installed capacity of units.

The system regenerates and works as good as new after every maintenance task is performed. Hence, a robust model has been developed, which leads to the analysis for obtaining the important reliability indices by using semi-Markov processes and regenerative point techniques. The following measures of plant effectiveness in terms of reliability indices are obtained Mean time to system failure, Plant availability, Expected busy period of the repairman for inspection, Expected busy period of the repairman for repair, Expected busy period of the repairman for replacement, Expected busy period of the repairman for reconditioning/reinstallation, Expected number of visits by the repairman, Expected number of repairs, Expected number of replacements, Expected number of reconditioning/reinstallation and Profit incurred to the system.

Graphical study is also carried out to demonstrate the results.

The data are summarized as under:

Probability that the failed unit I needs repair p_1=0.377777777.

Probability that the failed unit I needs replacement p_2=0.4.

Probability that the failed unit I needs reconditioning/reinstallation p_3=0.222222222.

Probability that the failed unit II needs repair p_4=0.2.

Probability that the failed unit II needs replacement p_5=0.52.

Probability that the failed unit II needs reconditioning/reinstallation p_6=0.28.

Estimated value of failure rate for unit I λ_1=0.001301179 per hour.

Estimated value of failure rate for unit II λ_2=0.000720378 per hour.

Estimated value of repair rate of unit I α_1 =0.298245614 per hour.

Estimated value of replacement rate of unit I α_2=0.04347826 per hour.

Estimated value of reconditioning/reinstallation rate of unit I α_3=0.285714285 per hour.

Estimated value of repair rate of unit II β_1=0.045871559 per hour.

Estimated value of replacement rate of unit II β_2=0.110169491 per hour.

Estimated value of reconditioning/reinstallation rate of unit II β_3=0.077777777 per hour.

SYSTEM DESCRIPTION AND ASSUMPTIONS

- The CC plant has two units: unit I and unit II.
- Unit I and unit II constitute a parallel system.
- Repairman is called to carry out the inspection, repair, replacement and reconditioning.
- As soon as a unit fails, inspection is carried out to reveal the type of failure.
- Priority for operation is given to unit I.
- Unit II works at full installed capacity if unit I is failed otherwise it works at reduced installed capacity.
- The failure rate of unit II is lower than the unit I.
- During the inspection, the other unit does not fail.
- Failure times are exponentially distributed.
- After each repair, the system works as good as new.

Notation

Oc : Operative state of the CC plant

□ : Failed state of the CC plant

O : Cranes working under full installed capacity.

⌂ : Cranes working under reduced installed capacity.

α : Inspection rate

$\lambda 1/ \lambda 2$: Failure rate of either crane of unit I/II

p1/p4 : Probability of repairable failure of unit I/II

p2/p5 : Probability of replaceable failure of unit I/II

p3/p6 : Probability of reconditioning failure of unit I/II

FIi/FIIi : Failed unit I/II is under inspection

FIr1/FIIr1 : Failed unit I/II is under repair

FIr2/FIIr2 : Failed unit I/II is under replacement

FIr3/FIIr3 : Failed unit I/II is under reconditioning/reinstallation.

FIR1/FIIR1 : Failed unit I/II under repairable failure continues from previous state

FIR2 /FIIR2 : Failed unit I/II under replaceable failure continues from previous state

FIR3/FIIR3 : Failed unit I/II under reinstallation failure continues from previous state

FIwi /FIIwi : Failed unit I/II is waiting for inspection

Ao : Steady state availability of the system

Io : Busy period of the repairman for inspection

BRo/BRR0 : Busy period of the repairman for replaceable/reinstallation failure

Co : Revenue per unit uptime

C1 /C2 /C3 /C4 : Cost per unit uptime for which the repairman is busy for inspection repairable failure/ replaceable failure/reinstallation failure

C5 : Cost per visit of repairman

C6/C7/C8 : Cost per unit repair/replacement/ reinstallation

STATES OF THE SYSTEM:-

$S_o = (OI,⌂II)$, $S_1 = (F_{Ii},O_{II})$, $S_2 = (O_I,F_{IIi})$,

$S_3 = (F_{Ir1},O_{II})$, $S_4 = (F_{Ir2},O_{II})$, $S_5 = (F_{Ir3},O_{II})$,

$S_6 = (O_I,F_{IIr1})$, $S_7 = (O_I,F_{IIr2})$, $S_8 = (O_I,F_{IIr3})$,

$S_9 = (F_{IR1},F_{IIwi})$, $S_{10} = (F_{IR2},F_{IIwi})$, $S_{11} = (F_{IR3},F_{IIwi})$,

$S_{12} = (F_{Iwi},F_{IIR1})$, $S_{13} = (F_{Iwi},F_{IIR2})$, $S_{14} = (F_{Iwi},F_{IIR3})$

S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, and S_8 are regenerative states and S_9, S_{10}, S_{11}, S_{12}, S_{13} and S_{14} are failed states.

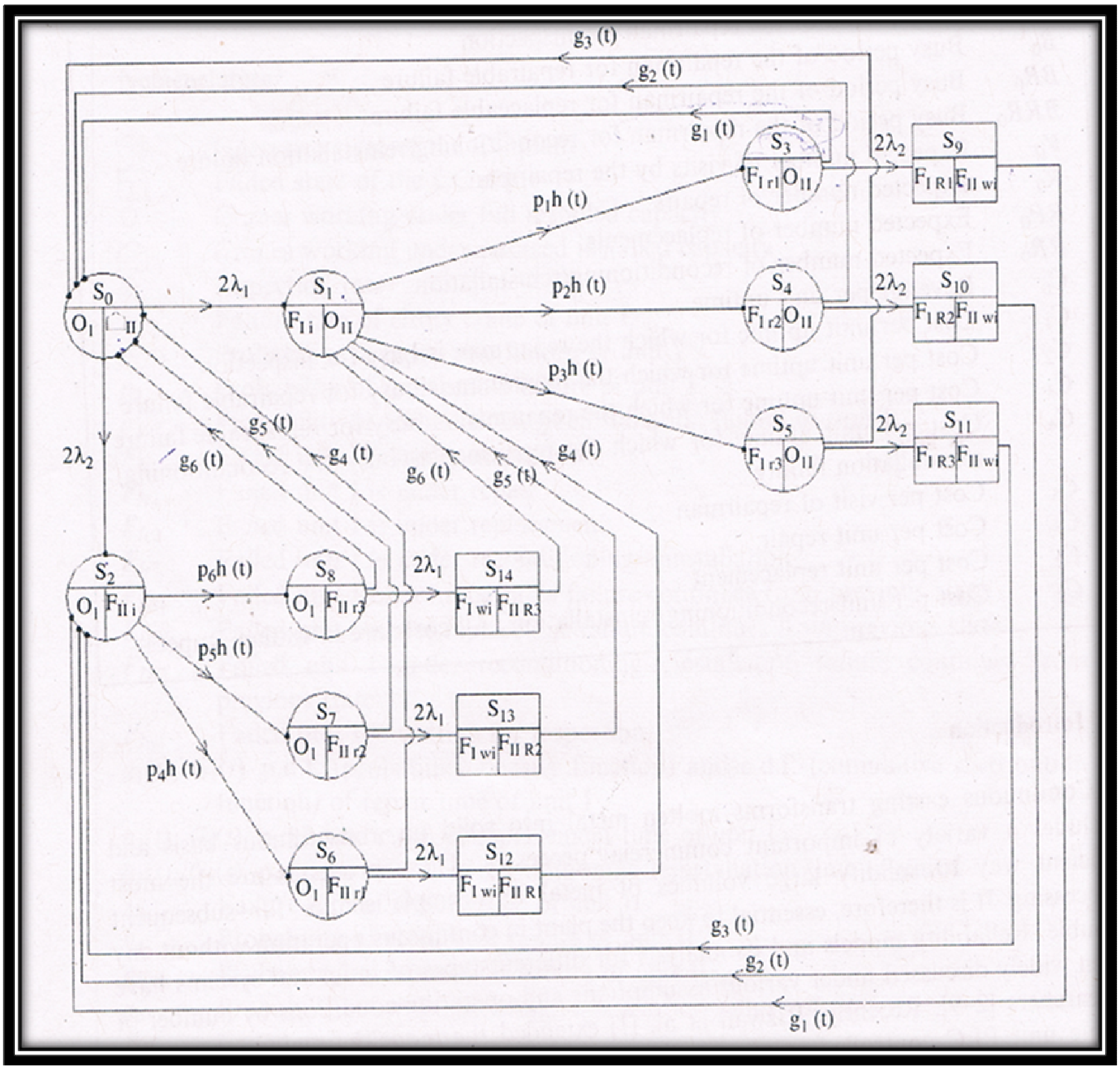

Fig. 3.1 Transition states of the system

Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state *i* to a failed state.

$$\Phi_0(t) = Q_{01}(t)(S)\Phi_1(t) + Q_{02}(t)(S)\Phi_2(t)$$

$$\Phi_1(t) = Q_{13}(t)(S)\Phi_3(t)+Q_{14}(t)(S)\Phi_4(t)+Q_{15}(t)(S)\Phi_5(t)$$

$$\Phi_2(t) = Q_{26}(t)(S)\Phi_6(t)+Q_{27}(t)(S)\Phi_7(t)+Q_{28}(t)(S)\Phi_8(t)$$

$$\Phi_3(t) = Q_{30}(t)(S)\Phi_0(t)+Q_{39}(t)$$

$$\Phi_4(t) = Q_{40}(t)(S)\Phi_0(t)+Q_{4,10}(t)$$

$$\Phi_5(t) = Q_{50}(t)(S)\Phi_0(t)+Q_{5,11}(t)$$

$$\Phi_6(t) = Q_{60}(t)(S)\ \Phi_0(t)+Q_{6,12}(t)$$

$$\Phi_7(t)=Q_{70}(t)(S)\ \Phi_0(t)+Q_{7,13}(t)$$

$$\Phi_8(t) = Q_{80}(t)(S)\ \Phi_0(t)+Q_{8,14}(t)$$

Transition Probabilities

$$p_{01} = \frac{2\lambda 1}{2(\lambda 1+\lambda 2)}\ , \qquad p_{02} = \frac{2\lambda 2}{2(\lambda 1+\lambda 2)}\ ,$$

$p_{13} = p_1$, $p_{14} = p_2$, $p15 = p_3$, $p26=p_4$, $p_{28}=p_6$,

$p_{30}=g_1*(2\lambda_2)$, $p_{39}=1-g_1*(2\lambda_2)$, $p_{32}^{(9)}=1-g_1*(2\lambda_2)$,

$p_{40}=g_2*(2\lambda_2)$, $p_{4,10}=1-g_2*(2\lambda_2)$, $p_{42}^{(10)}=1-g_2*(2\lambda_2)$,

$p_{50}=g_3*(2\lambda_2)$, $p_{5,11}=1-g_3*(2\lambda_2)$, $p_{52}^{(11)}=1-g_3*(2\lambda_2)$,

$p_{60}=g_4*(2\lambda_1)$, $p_{6,12}=1-g_4*(2\lambda_1)$, $p_{61}^{(12)}=1-g_4*(2\lambda_1)$,

$p_{70}=g_5*(2\lambda_1)$, $p_{7,13}=1-g_5*(2\lambda_1)$, $p_{71}^{(13)}=1-g_5*(2\lambda_1)$,

$p_{80}=g_6*(2\lambda_1)$, $p_{8,14}=1-g_6*(2\lambda_1)$, $p_{81}^{(14)}=1-g_6*(2\lambda_1)$

By these transition probabilities it can be verified that

$$\begin{aligned} & p_{01}+p_{02}=p_{13}+p_{14}+p_{15}=p_{26}+p_{27}+p_{28}=p_{30}+p_{39} \\ & =p_{30}+p_{32}^{(9)} = p_{40}+p_{4,10} = p_{40}+p_{42}^{(10)} \\ & =p_{50}+p_{5,11} = p_{50}+p_{52}^{(11)} = p_{60}+p_{6,12} \\ & =p_{60}+p_{61}^{(12)} = p70+p_{7,13} = p_{70}+p_{71}^{(13)} \\ & =p_{80}+p_{8,14} = p_{80}+p_{81}^{(14)} = 1 \end{aligned}$$

The unconditional mean time taken by the system to transit from any state S_i when time is counted from epoch at entrance into state S_j is stated as:

$$m_{ij} = \int t\, dQ_{ij}(t) = -q_{ij}{*}'(0)$$

$m_{01}+m_{02}=\mu_0;$ $m_{13}+m_{14}+m_{15}=\mu_1;$

$m_{26}+m_{27}+m_{28}=\mu_2;$ $m_{30}+m_{39}=\mu_3;$

$m_{40}+m_{4,10}=\mu_4;$ $m_{50}+m_{5,11}=\mu_5;$

$m_{60}+m_{6,12}=\mu_6;$ $m_{70}+m_{7,13}=\mu_7;$

$m_{80}+m_{8,14}=\mu_8;$ $m_{30}+m_{32}^{(9)}=k_3$(say)

$m_{40}+m_{42}^{(10)}=k_4$ (say) $m_{50}+m_{52}^{(11)}=k_5$(say)

$m_{60}+m_{61}^{(12)}=k_6$(say) $m_{70}+m_{71}^{(13)}=k_7$(say)

$m_{80}+m_{81}^{(14)}=k_8$(say) (*)

The reliability can be obtained by taking Laplace inverse transform of (*).

$$\text{MTSF}=\lim_{s\to 0}\frac{1-\phi 0(s)}{s}=\frac{N}{D}$$

where

$N=m01+m02+p01+p02+\mu 2$

$D=1-p01p13p30+p01p14p40+p01p15p50+p02p13p60+p02p14p70+p02p15p80$

Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant *t* given that the system entered regenerative state *i* at *t=0*.

$A_0(t) = M_0(t)+q_{01}(t)©A_1(t)+q_{02}(t)©A_2(t)$

$A_1(t) = M_1(t)+q_{13}(t)©A_3(t)+q_{14}(t)©A_4(t)+q1_5(t)©A_5(t)$

$A_2(t) = M_2(t)+q_{26}(t) ©A_6(t)+q_{27}(t) ©A_7(t)+q_{28}(t) ©A_8(t)$

$A_3(t) = M_3(t)+q_{30}(t) ©A_0(t)+q_{32}^{(9)}(t) ©A_2(t)$

$A_4(t) = M_4(t)+q_{40}(t) ©A_0(t)+q_{42}^{(10)}(t) ©A2(t)$

$A_5(t) = M_5(t)+q_{50}(t) ©A_0(t)+q_{52}^{(11)}(t) ©A_2(t)$

$A_6(t) = M_6(t)+q_{60}(t) ©A_0(t)+q_{61}^{(12)}(t) ©A_1(t)$

$A_7(t) = M_7(t)+q_{70}(t) ©A_0(t)+q_{71}^{(13)}(t) ©A_1(t)$

$A_8(t) = M_8(t)+q_{80}(t) ©A_0(t)+q_{81}^{(14)}(t) ©A_1(t)$ (**)

Taking LT of relations (**) and solving for $A_0^*(s)$.

The steady-state availability of the system can be given by

$$Ao=\lim_{s\to 0} sAo*(s)=\frac{N1}{D1}$$

where

$N_1=\mu_0[1-p_1p_{39}(p_1p_{6,12}+p_2p_{7,13}+p_3p_{8,14})-p_2p_{4,10}(p_1p_{6,12}+p_2p_{7,13}+p_3p_{8,14})-$
$p_3p_{5,11}(p_1p_{6,12}+p_2p_{7,13}+p_3p_{8,14})]$

$D_1=m_{01}(p_{13}p_{30}+p_{14}p_{40}+p_{15}p_{50}+p_{26}p_{60})+(u1+u2+u3+u4+u5+u6+u7+u8+u9+u9+u10+u11+u12+u13+u14+u15)(p_{26}p_{60}+p_{27}p_{70}+p_{28}p_{80})+(u16+u17+u18+u19+u20)(p_{13}p_{30}+p_{14}p_{40}+p_{15}p_{50})$

where

u1=p01μ1p13 u2 =p01μ2p13p39 u3=m01p13p39
u4=p01p13 $m_{32}^{(9)}$ u5=p01p13p39 u6=p01μ1p14p4,10
u7=m01p14p4,10 u8=p01μ2p14p4,10 u9=p01u3=m01p13p39
u10=p01p14p4,10 u11=m01p15p5,11 u12=p01μ1p15p5,11
u13=p01μ2p15p5,11 u14=p01p15 $m_{52}^{(11)}$ u15=p01p15p5,11
u16=m01p26p6,12 u17=p02μ1p26p6,12 u18=p02p26 $m_{61}^{(12)}$
u19=p02μ2p26p6,12 u20=p02p26p6,12

Busy Period Analysis of Repairman(Inspection Time Only)

Let $B_i(t)$ be the probability that the server is busy at an instant *t* given that the system entered regenerative state *i* at *t* = 0.

The following are the recursive relations for $B_i(t)$

$$I_i(t)=W_k(t)+\sum_j q_{i,j}^{(n)}(t)©I_j(t) \tag{1}$$

where *j* is a subsequent regenerative state to which state *i* transits through *n*≥1(natural number) transitions.

By Taking L.T. of eqn (1) solving for $I_0^*(s)$

$$I_0 = \lim_{s\to 0} sIo*(s)=\frac{N2}{D1}$$

where

$N_2=p01(\mu1+p1\mu2\ p_{32}^{(9)}+p2\mu2\ p_{42}^{(10)}+p3\mu2\ p_{52}^{(11)})+p02(\mu2+p4\mu1\ p_{61}^{(12)}+p5\mu1\ p_{71}^{(13)}+p6\mu1\ p_{81}^{(14)})$

and

D_1 is already specified.

Busy Period Analysis of Repairman (Repair Time Only)

Let $B_i(t)$ be the probability that the server is busy in repair at an instant *t* given that the system entered regenerative state *i* at *t* = 0. The following are the recursive relations for $B_i(t)$

$$B_i(t)=W_k(t)+\sum q_{i,j}^{(n)}(t)©B_j(t) \tag{2}$$

where *j* is a subsequent regenerative state to which state *i* transits through *n*≥1(natural number) transitions.

Taking L.T. of eqn (2) solving for $B_0^*(s)$

$B_0 = \lim_{s\to 0} sB_o*(s) = \frac{N_3}{D_1}$

where

$N_3 = p_{01}p_1\left(\frac{1}{\alpha 1} + p_4\, p_{32}^{(9)}\frac{1}{\beta 1}\right) + p_4\frac{1}{\beta 1}\left(p_{02} + p_{01}p_2\, p_{42}^{(10)} + p_{01}p_3\, p_{52}^{(11)}\right) + p_{02}p_1\frac{1}{\alpha 1}\left(p_4\, p_{61}^{(12)} + p_5 p_{71}^{(13)} + p_6\, p_{81}^{(14)}\right)$

and

D_1 is already specified.

Busy Period Analysis of Repairman (Replacement Time Only)

Let BRi(t) be the probability that the server is busy in replacement at an instant t given that the system entered regenerative state i at t = 0. The following are the recursive relations for Bi(t)

$$BR_i(t) = W_k(t) + \sum q_{i,j}^{(n)}(t) \copyright BR_j(t) \qquad (3)$$

where *j* is a subsequent regenerative state to which state *i* transits through *n*≥1(natural number) transitions.

Taking L.T. of eqn (3)

$BR_0 = \lim_{s\to o} sBR_o*(s) = \frac{N_4}{D_1}$

where

$N_4 = p_2\frac{1}{\alpha 2}\left(p_{01} + p_{02}p_4\, p_{61}^{(12)} + p_{02}p_5\, p_{71}^{(13)} + p_{02}\, p_6\, p_{81}^{(14)}\right)$

$\quad + p_5\frac{1}{\beta 2}\left(p_{02} + p_{01}p_1\, p_{32}^{(9)} + p_{01}p_2\, p_{42}^{(10)} + p_{01}p_3\, p_{52}^{(11)}\right)$

and

D_1 is already specified.

Busy Period Analysis of Repairman (Reinstallation Time Only)

Let BRRi(t) be the probability that the server is busy for reinstallation at an instant t given that the system entered regenerative state i at t = 0. The following are the recursive relations for BRRi(t)

$$BRR_i(t) = W_k(t) + \sum q_{i,j}^{(n)}(t) \copyright BRR_j(t) \qquad (4)$$

where *j* is a subsequent regenerative state to which state *i* transits through *n*≥1(natural number) transitions.

Taking L.T. of eqn (4)

$BRR_0 = \lim_{s\to 0} sBRR_o*(s) = \frac{N_5}{D_1}$

where $N_5 = p_3\frac{1}{\alpha 3}\left(p_{01} + p_{02}\, p_4\, p_{61}^{(12)} + p_{02}\, p_5\, p_{71}^{(13)} + p_{02}p_6\, p_{81}^{(14)}\right) p_6\frac{1}{\beta 3}\left(p_{02} + p_{01}p_1\, p_{32}^{(9)} + p_{01}p_2\, p_{42}^{(10)}\right.$

$\left. + p_{01}p_3\, p_{52}^{(11)}\right)$

and D_1 is already specified.

Expected number of visits by repairman

Let $V_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state *i* at *t=0*. We have the following recursive relations for $V_i(t)$:

$$V_i(t) = \sum_j Q_{i,j}(t) \quad (S) \quad [\delta_j + N_j(t)] \qquad (5)$$

where j is any regenerative state to which the given regenerative state *i* transits and δ_j =1, if *j* is the regenerative state where the server does job afresh, otherwise δ_j = 0.

Taking L.T. of eqn (5)

$$V_0 = \lim_{s\to 0} sVo^{**}(s) = \frac{N6}{D1}$$

where

$N_6=(p_{01}+p_{02})-(p_{01}+p_{02})p_4p_{6,12}(p_1p_{39}+p_2p_{4,10}+p_3p_{5,11})-(p_{01}+p_{02})p_5p_{7,13}(p_1p_{39}+p_2p_{4,10}+p_3p_{5,11})$
$-(p_{01}+p_{02})p_6p_{8,14}(p_1p_{39}+p_2p_{4,10}+p_3p_{5,11})$

and

D_1 is already specified.

Profit Analysis

Profit incurred to the system model in steady state is given by

$P=C_0A_0-C_1I_0-C_2B_0-C_3BR_0-C_4BRR_0-C_5V_0$

Particular Case

For the particular case, the rate of repairable failure and reinstallation failure and and inspection is assumed to be exponentially distributed i.e.

$g_1(t) = \alpha_1 e^{-\alpha_1 t}$; $g_2(t) = \alpha_2 e^{-\alpha_2 t}$; $g_3(t) = \alpha_3 e^{-\alpha_3 t}$

$g_4(t) = \beta_1 e^{-\beta_1 t}$; $g_5(t) = \beta_2 e^{-\beta_2 t}$; $g_6(t) = \beta_3 e^{-\beta_3 t}$

$h(t) = \alpha e^{-\alpha t}$

where

$\alpha_1 = 0.298245614$; $\alpha_2 = 0.04347826$; $\alpha_3 = 0.285714285$

$\beta_1 = 0.045871559$; $\beta_2 = 0.110169491$; $\beta_3 = 0.077777777$

$p_1=0.377777777$; $p_2=0.4$; $p_3=0.222222222$,

$p_4=0.2$; $p_5=0.52$; $p_6=0.28$

$\lambda_1=0.001301179$; $\lambda_2=0.000720378$

- MTSF : 7326.241655h
- Plant Availability : 0.772138258
- Expected busy period for inspection I_0 : 0.007921351
- Expected busy period for repairable failure B_0:0.007480351
- Expected busy period for replaceable failure BR_0:0.007184409
- Expected busy period for reinstallation failureBRR_0:0.00561197
- Expected number of visits by the repairman V_0 :0.003126125

Graphical Interpretation

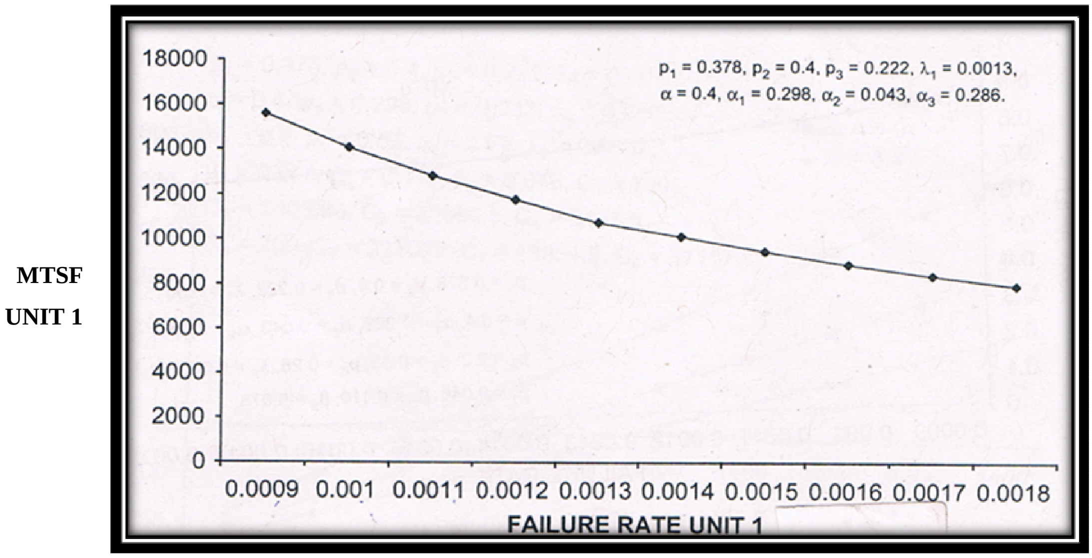

Fig. 3.2

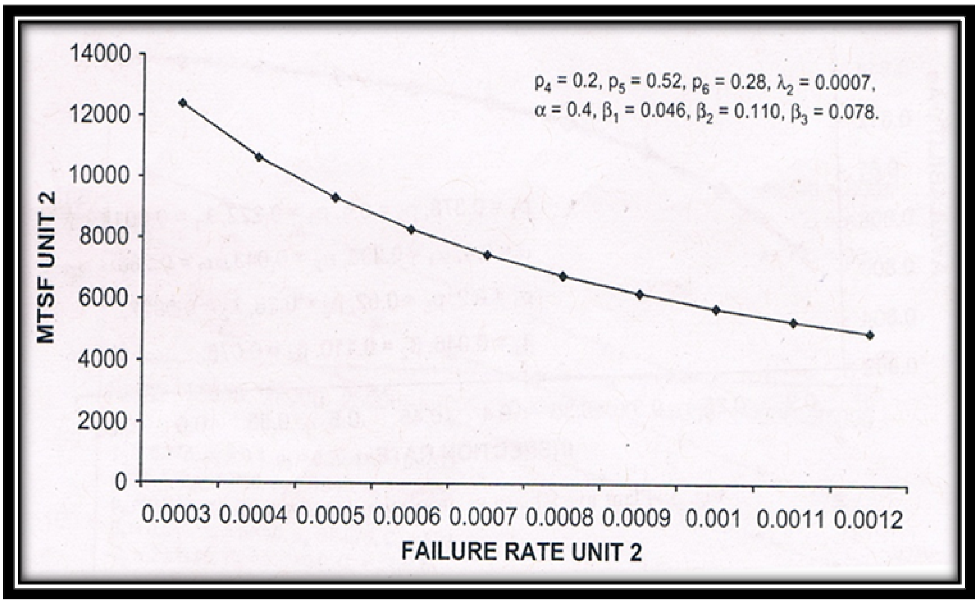

Fig. 3.3

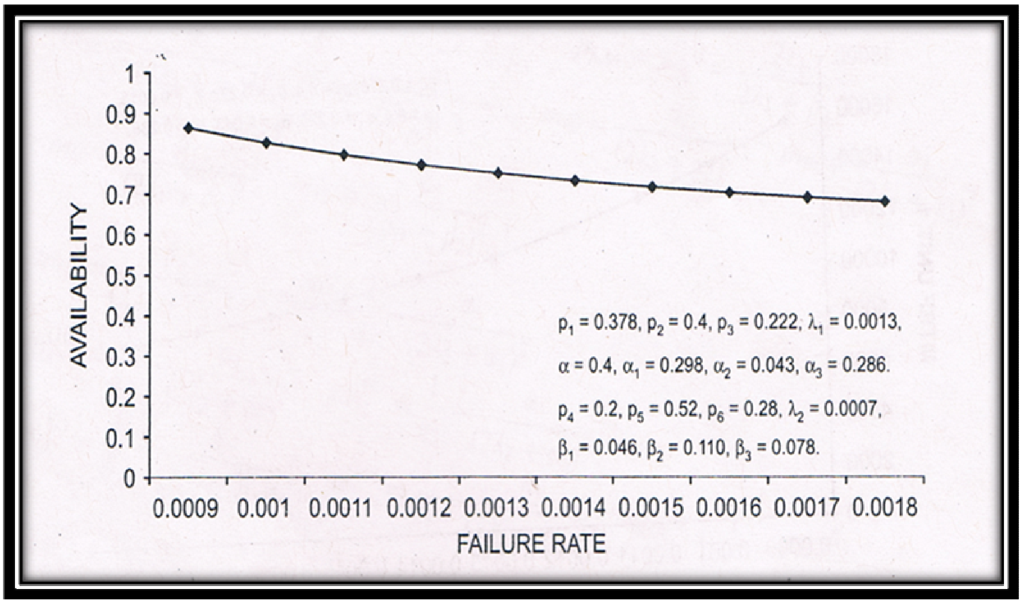
AVAILABILITY
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.0009
0.001
0.0011
0.0012
0.0013
0.0014
0.0015
0.0016
0.0017
0.0018
FAILURE RATE
$p_1 = 0.378$, $p_2 = 0.4$, $p_3 = 0.222$, $\lambda_1 = 0.0013$,
$\alpha = 0.4$, $\alpha_1 = 0.298$, $\alpha_2 = 0.043$, $\alpha_3 = 0.286$.
$p_4 = 0.2$, $p_5 = 0.52$, $p_6 = 0.28$, $\lambda_2 = 0.0007$,
$\beta_1 = 0.046$, $\beta_2 = 0.110$, $\beta_3 = 0.078$.

Fig. 3.4

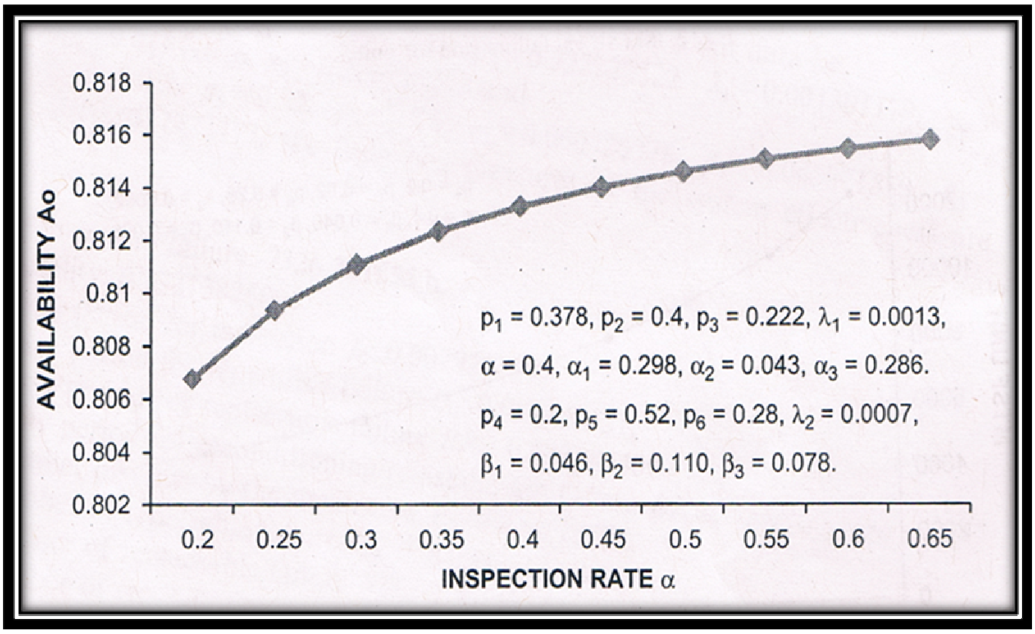
AVAILABILITY Ao
0.818
0.816
0.814
0.812
0.81
0.808
0.806
0.804
0.802
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
INSPECTION RATE α
$p_1 = 0.378$, $p_2 = 0.4$, $p_3 = 0.222$, $\lambda_1 = 0.0013$,
$\alpha = 0.4$, $\alpha_1 = 0.298$, $\alpha_2 = 0.043$, $\alpha_3 = 0.286$.
$p_4 = 0.2$, $p_5 = 0.52$, $p_6 = 0.28$, $\lambda_2 = 0.0007$,
$\beta_1 = 0.046$, $\beta_2 = 0.110$, $\beta_3 = 0.078$.

Fig. 3.5

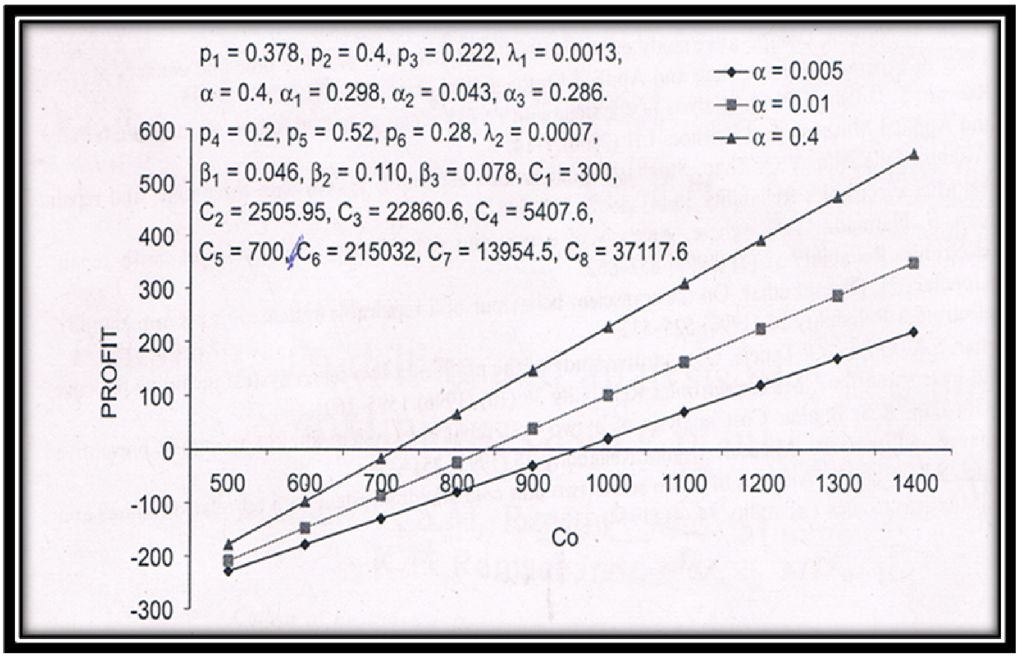

Fig. 3.6 Profit (P) versus revenue per unit uptime (C_0) for different values of inspection rate (α)

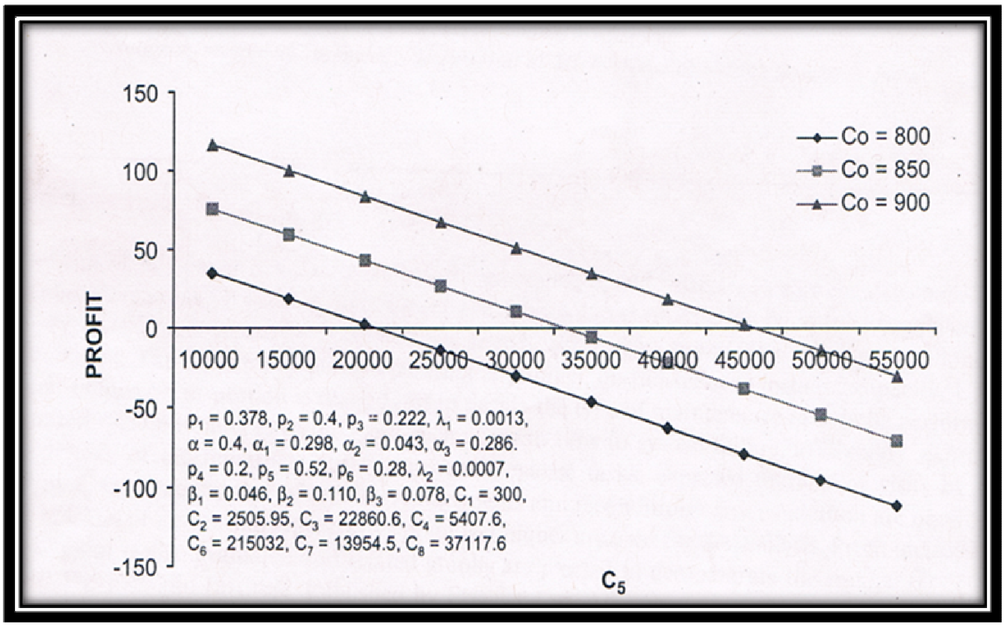

Fig. 3.7 Profit (P) versus revenue per unit uptime (C_5) for different values of revenue per unit uptime (C_0)

Chapter IV

Conclusion

From chapter II, Reliability modeling proves to be a powerful tool for analyzing plant effectiveness. Given a risk factor, the model predicts the breakdown and repair possibilities and offers a scientific basis on which the optimum reliability results are achieved.

The important reliability indices such as MTSF, availability are estimated numerically. A declining trend of MTSF and Availability with respect to the failure rate can be seen in Fig.2.2 and Fig.2.3.

Figures 3.2, 3.3 and 3.4 reveal the pattern of MTSF and plant availability with respect to the failure rate. A declining trend of MTSF and plant availability with respect to increase in failure rate is noted. Fig. 3.5 reveals the pattern of the plant availability (A_0) with respect to the inspection rate (α). It is noted, plant availability increases with the increase in inspection rate. Fig. 3.6 demonstrates the pattern of profit (P) with respect to revenue per unit uptime (C_0) for different values of inspection rate. The following interpretation could be achieved from this graph:

(i) The profit increases with increase in the values of revenue per unit uptime and has higher values for higher values of inspection rate.

(ii) For $\alpha = 0.005$, the profit is positive or zero or negative according as C_0> or = or <925.00 .

(iii)For $\alpha = 0.01$, the profit is positive or zero or negative according as C_0> or = or <790.00 .

(iv)For $\alpha = 0.4$, the profit is positive or zero or negative according as C_0> or = or <680.00 .

Fig. 3.7 demonstrates the pattern of profit (P) with respect to cost per visit of repairman (C_5) for different values of revenue per unit uptime (C_0). The following interpretation could be achieved from this graph:

(i) The profit decreases with increase in values of cost per visit of repairman (C_5).

(ii) For C0 = 800, the profit is positive or zero or negative accordingly as C5> or = or <17,500.00.

(iii) For C0 = 850, the profit is positive or zero or negative accordingly as C5> or = or <31,000.00.

(iv) For C0 = 900, the profit is positive or zero or negative accordingly as C5> or = or <42,500.00.

Given a risk factor the model predicts the failure and repair situations and offers a scientific basis on which the optimum reliability results are achieved.

Measures of system effectiveness of a CC plant in terms of reliability indices such as mean time to system failure, plant availability, expected busy periods of the repairman for various maintenance tasks, expected number of visits by the repairman, expected number of repairs, replacements and reconditioning/reinstallation are obtained numerically. Fig.3.2, 3.3, 3.4, 3.5 elucidates the result graphically.

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