PROBABILISTIC LOAD FLOW ANALYSIS THROUGH LINEARIZED EQUATIONS

A dissertation submitted in partial fulfillment of the requirements for the award of the degree of

Master of Engineering

in

Power Systems

by

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Declaration

I hereby certify that the work presented in the dissertation entitled, **"PROBABILISTIC LOAD FLOW ANALYSIS THROUGH LINEARIZED EQUATIONS"**, in the partial fulfillment of the requirement for the award of the degree of **Master of Engineering** in **Power Systems**, submitted to the Electrical and Instrumentation Engineering Department of Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Sanjay K. Jain, Associate Professor, EIED. It also refers to the other researchers'work which are duly listed in the reference section. The matter contained in this dissertation has not been submitted, neither in part nor in full to any other university or institute except as reported in text and references.

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Abstract

For real time operation, control and security assessment of power system, traditional load flow cannot be used because it operates with fixed load parameters. The consequences of variation in injected power at the buses due to variation in load and generation cannot be accounted.

In this work, a probabilistic load flow method has been proposed which takes into account the load variation at the buses. The variations in load, resulting bus voltages and angles and injected power are represented by normal distribution and discrete distribution. Linearized load flow equation was used in a modified manner by generating random values from the PDF of injected active and reactive power at the buses to obtain the random values of bus voltage and its angle. The method was applied on IEEE 14-Bus, IEEE 30-Bus and IEEE 57-Bus systems. For IEEE 14-Bus system, two cases have been studied. In the first case, the loads are considered as continuous with normal distribution and in second case, the load has been considered discrete at 9th bus while continuous at other buses. The probabilistic load flow was analysed considering varying active and /or reactive load components and the performance has been analysed through distribution function PDF and CDF for bus voltage and angles having minimum and maximum deviation.

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List of Abbreviations

ADC	Active Distribution System
CDF	Cumulative Density Function
DDC	Dependent Discrete Convolution
LHS	Latin Hypercube Sampling
MCS	Monte Carlo Simulation
MLR	Multiple Linear Regression
NR	Newton Raphson
PDF	Probability Density Function
PLF	Probabilistic Load Flow

Chapter 1

Introduction

1.1 *Overview*

For planning and assessment of operation of power system on a daily basis, deterministic load flow is used. In this, the voltage and its angle at the buses and power flows are calculated without considering any change in load demand, power generation and network configuration. Nowadays, renewable energy sources have lead to increment in the power generation uncertainty. Deterministic load flow doesn't take into account all these uncertainties.

Probabilistic or stochastic load flow is a suitable method for addressing the problems of uncertainties while planning and assessment of operation of power system. In this method, the probability of bus bar voltage violating the operational constraints, the probability of line flows exceeding the thermal rating of the lines and probability of reactive power injections being in between two limits can be obtained. These are important for assessment of system performance. In probabilistic load flow, probability density functions for all the state variables and output network variables are evaluated for obtaining all the possible ranges of load flow results[1].

Traditionally, in system planning the worst operating conditions are identified first and then corrective measures are taken to ensure safety. For this, conventional load flow is applied which for each solution assumes nodal voltage, power generation and configuration of the network to be constant which is rarely the case in practical conditions. Though this type of load flow gives accurate results with these assumptions, if any of the conditions change, it fails to give accurate results. The generating units, transformers and the overhead lines are subjected to forced outages. The nodal loads can't be predicted accurately. Thus many operating conditions can be studied giving equal weightage to each condition. Based on past experience, events are required to be identified and probability of occurrence of these events are calculated. Statistical parameters are used to represent past performance and future performance can be predicted using probabilistic techniques. In this method, load flow equations are linearized about an expected value. Then basic input variables are transformed into output probability density functions using convolution process. Deterministic load flow uses nonlinear equations whereas in probabilistic load flow either linear or quadratic approximations of these nonlinear equations at an expected value of input variables are used [2].

1.2 *Literature Review*

The literature review gives a detailed survey on the various approach adopted by different authors for applying probabilistic load flow technique for various applications.

Hu and Wang [3] proposed a probabilistic load flow method in which uncertainties in power injection at the nodes and random branch outages were considered. Simulation of branch outages were carried out by injecting fictitious power at the nodes of the branch. For each kind of disturbance, variation in the angle was calculated and was compared with a defined threshold angle. Based on the comparison, variation in the desired random variables were calculated using linear equations and conventional load flow. Cumulants of these variables were calculated and converted into moments. Finally, PDF and CDF of the variables were calculated. The method was found to be effective and results were comparable with Monte Carlo simulation results. The calculation speed was much faster than MCS method .

Dong et al. [4] worked on a.c. probabilistic load flow analysis. Load fluctuation, unit outages and random branch outages were considered. The method was applied to analyze static state security of power system. Cumulants and Gram-Charlier Series expansion method were used for calculation of probability distribution of line flows and nodal voltages. The results obtained were compared by Monte Carlo Simulation results. The results were found to be accurate and calculation speed was found to be high .

Lu et al. [5] proposed a probabilistic load flow method in which contingency in the transmission network was considered. Each network configuration was considered as a separate case and probability density function of power flow for each case can be computed by applying conditional probability for that configuration. Finally, total probability theorem was applied considering all the mutually exclusive events. i.e. all the network configurations which have resulted due to contingencies .

Hatziargyriou et al. [6] proposed a probabilistic model to take into account variation in the velocity when operating wind turbines. Probabilistic modelling of loads, active power and reactive power absorbed by wind turbines was done. The effect of wind energy penetration in distribution network was assessed.

Su [1] proposed a probabilistic load flow method which was based on a point estimate method. Statistical moments were calculated for load flow solution distribution. Using the moments, probability density function was determined. The results were comparable with Monte Carlo Simulation results.

Villanueva et al. [7] used probabilistic load flow method in which uncertainties in power injection were studied. Probability density function of power injected by wind turbine into the network was calculated. Then DC power flow method was used.

Aien et al. [8] worked on probabilistic optimal load flow method for correlated hybrid Wind-Photovoltaic power systems. Modified two point estimate method (2 PEM) was developed since the original 2 PEM method can’t handle correlated uncertain variables. The method was found to be effective.

Siano and Mokryani [9] studied effect of integration of wind turbines with the distribution systems. Monte Carlo simulation method and market-based optimal power flow were used in the analysis. Variation in the wind power and load demand were modelled by MCS method and market-based OPF was used to maximize social welfare .

Fan et al. [10] proposed a PLF algorithm in which uncertainty in photovoltaic generation and load demand was considered. Cumulant method was used. Uncertainty in economic dispatch was analysed. Optimization of expected value of total cost was achieved in which probability of overload was used as a constraint .

Sexauer and Mohagheghi [11] worked on a PLF method to study the effect of uncertainty in renewable energy generation on voltage quality in distribution system. Load, wind speed and solar irradiation were modelled statistically.

Gupta [12] worked on a Probabilistic load flow method in which uncertainties in power injection were considered. Deterministic load flow was run at the expected value of the injected power and expected value of voltage, active and reactive power and power loss was calculated. Finally, probability density functions for all these desired variables were calculated. The PDF of reactive power at the generator buses was important for the assessment of the capability of maintaining the desired voltage levels.

Hatziargyriou et al. [13] applied PLF techniques to check the limit violation of voltages when wind power penetrates into the system. The main objective of the work was to find the positions at which reactive compensation was required so that the constraints are not violated. The method of linearized load flow equations was used for PLF formulation.

Usaola [14] proposed a PLF method based on cumulants and Cornish-Fisher expansion. Wind power injecton was taken into consideration. Since the PDF of injected power was non-gaussian in this case,the method was found to be more effective than Gram-Charlier series expansion method .

Ruiz-Rodriguez et al. [15] performed PLF by analytical technique in which combination of cumulant and Cornish-Fisher expansion was used. Probabilistic modelling of Photovoltaic distributed generation and load was performed. Better results were obtained as compared to MCS method. Convergence was achieved in small number of iterations .

Williams and Crawford [16] used Maximum entropy method and Gram-Charlier expansion for obtaining PDFs for voltage and its angle and PDF for the power flow.This work included application of moments, cumulants. Linearized load flow equations were used for genertaing random variables for desired parameters. When the results were compared with that of MCS results,the maximum entropy method was found to be more accurate and effective .

Bracale et al. [17] performed PLF analysis of three phase unbalanced distribution system by point estimate schemes. IEEE-34 bus system was used for study. A modified 2m+1 Hong's point estimate scheme was used in which moments of random variables were calculated and finally PDF was obtained. In this work, presence of wind was considered. The system configuration changes with variation in wind speed. Thus, due to different wind speeds, different configurations are obtained and discrete distribution can be used to represent these different system configurations.The results obtained from point estimate scheme were comparable with MCS results .

Kong et al. [18] used Stratified Latin Hypercube sampling PLF method to study the variation in electric vehicle charging. The traffic pattern changes with time so does the penetration level of electric vehicle. Kernel density function was used to represent the distribution of charging electrical vehicle load.

Valverde et al. [19] worked on non-Gaussian correlated variables. Generally these are either aggregate loads or generations like wind or solar. Gaussian mixture distribution was used to model non-Gaussian variables. Gaussian component combination method was used to find PDFs of line power flows .

Ruiz-Rodriguez et al. [20] used probabilistic radial load flow for studying the effect on distribution system which is integrated with Biomass energy systems. The load and heat value or calorific value of biomass were considered as random variables. Both these quantities were considered normally distributed. Cornish-Fisher expansion was used to find quantiles of random variable. With this method, it was possible to check if the voltages at the buses didn't violate their specified limits. A discrete PSO technique was used to find the best positions in the system where these Biomass energy sources could be connected .

Wu et al. [21] used Polynomial Chaos method for PLF analysis. The advantage of this method is that non linearity of load flow equations is preserved. The PLF equations are transformed into deterministic equations. Combination of generalized Polynomial Chaos and Galerkin method were used for the analysis.

Nimpitiwan and Chaiyabut [22] applied MCS method to check the voltage at the buses through their CDFs. The performance of different voltage control and reactive power control schemes were discussed. Variation in load, OLTC transformers,etc.were considered as a source of uncertainty .

Cui and Franchetti [23] proposed a Quasi-Monte technique for PLF analysis of radial distribution network. This method is suitable for high dimension problems and takes lesser time than MCS method. When in distribution network, there is large variance in load or generation, this method has been found to be effective.

Wang et al. [24] used Gaussian Mixture model to find joint PDF of active power. The injected power was considered as non-Gaussian. Linearized load flow equations and sufficent sample of random variable which represents injected power at the buses were used.

Ahmed et al. [25] used MCS method to study the variation in voltage and power flows in a radial distribution network. Forward/Backward Sweet method was used for performing deterministic load flow. In this type of deterministic load flow, Jacobian matrix is not required to be constructed. Thus, this method saves time and has good convergence than Newton Raphson method.

Melhorn et al. [26] proposed a PLF method for analysis of variable charging of plug-in electric vehicles and household load demand. Correlation of these two quantities was taken into account. Reactive power support was provided for reducing the voltage violations.

Pan et al. [27] used NR method and MCS method for analysis of islanded microgrid considering Droop controlled DGs.Both Photovoltaic system and wind turbine were included. In an islanded microgrid, frequency is a variable. According to load, the power output is changed by droop control. The buses connected with wind power source were considered as PQ buses. Loads were considered to be normally distributed. In the MCS method,the deterministic load flow was run 10,000 times.

Wan et al. [28] used PLF analysis in a market based power system. Cumulant and Gram charlier series expansion method were used in the analysis. Market factors such as bidding and dispatch process in a deregulated environment add to the uncertainties. In a competitive market, scheduling of generators is not done as in case of regulated environment. Generation is dependent on different dispatch strategies of different companies in a competitive environment. A considerable amount of effect of market factors on PLF results were found from study.

Ciapessoni et al. [29] proposed analytical method for multiterminal HVDC network integration of offshore wind power sources with AC onshore power system. Linearized load flow equations and cumulant method were used in the study.

Yuan et al. [30] applied cumulants and Gram-Charlier series expansion method to study a power system integrated with wind power. This method gave better results than the MCS method. Load was considered as deterministic while velocity of wind was considered as probabilistic. Weibull distribution function was used to model variable wind speed.

Xu and Yan [31] used LHS method with MLR for PLF analysis of correlated variables. The results were better than achieved by applying genetic algorithm and NORTA technique. The time consumption also reduced.

Wang et al. [32] worked on integration of ADS with REs in which the correlated uncertainties were studied using Copula function. DDC technique was used for the PLF analysis. The uncertainties are introduced in the ADS due to REs. PDFs for forecasting error in case of load and wind power was calculated.

Prusty and Jena [33] proposed a hybrid PLF method, in which probabilistic modelling of load, photovoltaic power and uncertain ambient temperature was achieved. Sensitivity matrix was used in the PLF analysis which was obtained from the Jacobian matrix by applying NR method. The results were compared with those obtained from analytical methods and MCS method and was found to be comparable.

1.3 *Need and Scope of Work*

The importance of probabilistic load flow lies in identifying the possible variations in the system due to various factors so that the performance of the system is not compromised. All the loads connected at the consumer work under specified conditions to give their best performance. Any change in these conditions can affect their performance. Thus to take preventive measures, it is very important to study the possible causes of variation and how these can affect the system parameters.

Many PLF techniques have been proposed to study these variations. With these techniques not only variation of load but variation in generation can be also be studied. The emergence of integrated hybrid energy systems which include different type of energy resources like the renewables can be easily studied by PLF analysis as deterministic load flow can't perform under variable conditions.

The PLF techniques use pure statistical and probability techniques for analysis. Many statistical techniques have been applied by the researchers to find best and accurate results and many more techniques will be used in future to achieve better results.

1.4 *Objective of the Work*

The main objective of this work is to study the effect of variation in load on the bus voltage and its angle. To apply probabilistic load flow using linearized load flow equations to study the maximum and minimum variation of bus voltage and angle through standard deviation and PDF and CDF curves which is important for the system planning.

1.5 *Organization of Dissertation*

The work can be summarized in following five chapters.

- **Chapter 1** discusses overview of the problem, literature review, objective of work and organization of thesis.
- **Chapter 2** discusses NR method, probability density and distribution functions, cumulative density function, PLF methods, applications of probabilistic load flow.
- **Chapter 3** describes methodology adopted to implement probabilistic power flow.
- **chapter 4** summarizes the results and discussions.
- **chapter 5** summarizes conclusion and future scope of the work.

Chapter 2

Probabilistic Power Flow Analysis

2.1 *Deterministic Load Flow*

For operation and control of power system in steady state conditions, deterministic load flow analysis is used. In this analysis, parameters such as voltage, current, injected active and reactive power, line power flows are obtained. The system losses can be calculated and can be minimized by taking suitable measures.

Conventional load flow uses non linear equations. It can be implemented using different methods such as Gauss-Seidel method, Newton Raphson method, Decoupled and Fast Decoupled load flow methods. In all these methods, the specified values of voltage,angle, injected active and reactive power are constant. Thus, variation in injected power at the buses is not considered.

The results obtained from deterministic load flow are as follows:

1. Voltage magnitude at the buses.
2. Phase angle of voltage.
3. Injected Real or active power.
4. Injected Reactive power.
5. Power flow through the lines.

Types of Buses

The buses in a power system can be classified into following categories.

1. Slack bus or Swing bus.
2. PV bus or Generator bus.
3. PQ bus or Load bus.

- Slack bus is the reference bus. In this bus, magnitude of voltage and phase angle are specified. This bus takes into account the losses in a power system.
- PV bus is called voltage controlled bus for which real power(P) and voltage(V) are specified. Reactive power(Q) and phase angle are to be determined.
- PQ bus is called load bus at which active power(P) and reactive power(Q) are specified. Voltage and angle are to be determined.

2.2 *Newton Raphson Method*

Newton Raphson method is the most accurate method. Even though it is time consuming but convergence is guaranteed.

The power flow equations can be repesented as following.

The injected active power is computed through following equation.

$$P_i = \sum_{k=1}^{nb} V_i V_k [G_{ik} cos(\delta_i - \delta_k) + B_{ik} sin(\delta_i - \delta_k)] \tag{2.1}$$

for $i = 2, 3, ...nb$, where nb is the number of buses.

The injected reactive power of PQ buses is computed through following equation.

$$Q_i = \sum_{k=1}^{nb} V_i V_k [G_{ik} sin(\delta_i - \delta_k) - B_{ik} cos(\delta_i - \delta_k)] \tag{2.2}$$

for $i = nv + 1, nv + 2, ...nb.$, where nv represents number of PV buses.

The following equation is used to update voltage and angle.

$$\begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{2.3}$$

where Jacobian matrix is J.

$$J = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \tag{2.4}$$

The Jacobian matrix elements can be computed by following equations.

when $i = k$

$$H_{ii} = -Q_i - B_{ii}V_i^2 \tag{2.5}$$

$$N_{ii} = P_i + G_{ii}V_i^2 \tag{2.6}$$

$$J_{ii} = P_i - G_{ii}V_i^2 \tag{2.7}$$

$$L_{ii} = Q_i - B_{ii}V_i^2 \tag{2.8}$$

When $i \neq k$

$$H_{ik} = V_iV_k[G_{ik}sin(\delta_i - \delta_k) - B_{ik}cos(\delta_i - \delta_k)] \tag{2.9}$$

$$N_{ik} = V_iV_k[G_{ik}cos(\delta_i - \delta_k) + B_{ik}sin(\delta_i - \delta_k)] \tag{2.10}$$

$$J_{ik} = V_iV_k[-G_{ik}cos(\delta_i - \delta_k) - B_{ik}sin(\delta_i - \delta_k)] \tag{2.11}$$

$$L_{ik} = V_iV_k[G_{ik}sin(\delta_i - \delta_k) - B_{ik}cos(\delta_i - \delta_k)] \tag{2.12}$$

The algorithm for NR method is as follows.

Step 1 Read the data for specified bus data including voltage, angle, active,reactive power at the buses and line data including branch admittances, tap setting of tap changing transformers, tolerance in convergence.

Step 2 Form the Y bus with the help of bus data and line data.

Step 3 Set iteration count as zero.

Step 4 Compute injected active and reactive power at the buses using the load flow equations.

Step 5 Compute the mismatch in active power by the following expression.

$$\Delta P = Pspecified - Pcalculated. \tag{2.13}$$

Step 6 Compute the mismatch in reactive power at the buses by the following expression.

$$\Delta Q = Qspecified - Qcalculated. \tag{2.14}$$

Step 7 If the absolute value of the active and reactive power mismatch is less than a tolerance value then directly calculate the slack bus active and reactive power and end the load flow. Otherwise, proceed to following steps.

Step 8 Compute the Jacobian matrix elements and hence the Jacobian matrix.

Step 9 With the help of Jacobian matrix calculate the update in voltage and angle.

Step 10 Update the angle of all the buses except the slack bus by following relation. $\delta_i = \delta_i + \Delta\delta_i$,for $i = 2, 3, ...nb$, where nb is the number of buses.

Step 11 Update the voltage of the PQ buses by following relation. $v_i = v_i + \Delta v_i$,for $i = nv + 1, nv + 2, ...nb.$, where nv is the number of PV buses.

Step 12 For the PQ buses,check the limits of voltage.

1. If V_i is less than or equal to V_{min} then set V_i as V_i=V_{min}
2. If V_i is greater than or equal to V_{max} then set V_i as V_i=V_{max}

Step 13 For PV buses, check the limits of Reactive power.

1. If Q_i is less than or equal to Q_{min} then set Q_i as Q_i=Q_{min}
2. If Q_i is greater than or equal to Q_{max} then set Q_i as Q_i=Q_{max}

Step 14 If the reactive power limits are violated,convert PV bus temporarily into PQ bus and update the voltage.

Step 15 Again start a new iteration.

Step 16 Load flow program will run until the mismatch of active and reactive power is less than the convergence value.

Step 17 Once the convergence is achieved,the slack bus active and reactive power are calculated.

2.3 *Probability and Statistical Background*

Random Variable

Whenever a random experiment is carried out, for each outcome a real number can be associated. Hence, from this experiment we get a set of variables called random variables. Depending on the number of values, random variable can be classified into discrete and continuous random variables. When the number of values are finite and countable, then it is called discrete random variable otherwise it is called continuous random variable.

A random variable can also be defined as a real valued function in which domain is taken as the sample space of the random experiment.

Mathematical expectation or mean

If x_1,x_2,......,x_n are random variables, then the mean or expectation can be found by simply taking arithmetic average of these random variables.

$$Mean = \frac{\sum_{i=1}^{n} x_i}{n} \tag{2.15}$$

Variance

If x_1,x_2,......,x_n are random variables, then variance can be found by the following expression.

$$Variance = \frac{\sum_{i=1}^{n} (x_i - x_{\text{exp}})^2}{n} \tag{2.16}$$

Standard deviation

If x_1,x_2,......,x_n are random variables, then Standard deviation can be found by the following expression.

$$\sigma = \sqrt{Variance} \tag{2.17}$$

where σ is the standard deviation of the random variable with respect to the mean or expected value.

Probability Distribution Function

The probability distribution function is defined for discrete variables.It can be defined by f(x) which can be expressed as:

$$P(X = x) = f(x) \tag{2.18}$$

where x is a random variable.

Each value of random variable has a probability of occurence associated with it. The sum of the respective probability of occurences of all the variables is one.

Mean of Discrete distribution

If a discrete random variable is given which can take values such as x_1,x_2,......,x_n. Then their respective probabilities can be expressed as p_1,p_2,p_3,.....,p_n. The mean of the discrete distribution is the sum of $p_i x_i$ where i varies from 1,2,3,....,n.

$$Mean = \sum_{i=1}^{n} p_i x_i \tag{2.19}$$

Mean is also called as expected value of the random variable and can be represented by E(X). This mean is same as obtained from probability distribution.

Examples of discrete distribution

The examples of discrete distribution are:

1. Binomial distribution
2. Poisson distribution
3. Uniform discrete distribution
4. Hypergeometric distribution
5. Geometric distribution
6. Multinomial distribution
7. Negative binomial distribution

- *Binomial Distribution*- In this distribution, random experiment is carried out under identical conditions repeatedly. The experiment has two outcomes.These are success and failure. This type of distribution can be applied when number of trials are finite and independent of each other. Success and failure should have constant probability in all trials. If p is the probability of success, q is the probability of failure and n is number of trials then the mean and variance can be found as:

$$mean = np \tag{2.20}$$

$$variance = npq \tag{2.21}$$

$$Standarddeviation = \sqrt{variance} \tag{2.22}$$

$$Standarddeviation = \sqrt{npq} \tag{2.23}$$

- *Poisson Distribution*- This distribution can be used if number of trials is infinite. In each trial, the probability of success tends to zero. It finds application when the events occur randomly in time and space and probability that the event will occur is very small. The distribution function f(x) can be represented as follows.

  $$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad x = 0, 1, 2... \tag{2.24}$$

  where λ is a constant and is a positive quantity.

Probability Density Function

A random variable is defined as continuous if it can take any value between specified limits. The probability density function is defined for continuous variables. With help of PDF, probability of finding a random variable in a particular range can be obtained.

The probability of finding a variable quantity in a particular range can be found by applying definite integral of the PDF over the specified limits. This would give the area bounded by the probability density curve between the limits. Hence, the magnitude of this area would be the required probability. This can be described by following expression.

$$\int_a^b f(x)dx = P \tag{2.25}$$

where P is the probability of finding the random variable within limits a and b.

PDF is a function of continuous variable. It has following properties.

1. It is always non negative,i.e
$$f(x) \geq 0 \tag{2.26}$$

2. The area under the PDF curve when definite integral od PDF is applied over infinite limits for all real values is one that is
$$\int_{-\infty}^{+\infty} f(x)dx = 1 \tag{2.27}$$

Mathematical expectation of a continuous variable

The mathematical expectation of a continuous random variable can be found by the integral of the product of random variable and PDF over infinite limits that is $\int_{-\infty}^{+\infty} xf(x)dx$ where x is the random variable and f(x) is the PDF of the continuous distribution function.

Examples of continuous distribution

Some examples of continuous distribution are as follows.

1. Normal distribution
2. Beta distribution
3. Gamma distribution
4. Kernel distribution
5. Weibull distribution
6. Rayleigh distribution
7. Uniform continuous distribution
8. Exponential distribution
9. Chi-Square distribution
10. Lognormal distribution

- *Normal Distribution*- De Moivre discovered normal distribution. It finds application when variables are continuous.It can be considered as limiting case of Binomial distribution.

The expression for PDF is given below.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{2.28}$$

where μ is the mean or expected value, σ is the standard deviation and f(x) represents the PDF.

It has following properties.

1. Number of trials is infinite.
2. Probability of success and failure are not very small.
3. The PDF curve is bell shaped and is symmetric about the mean of the distribution.
4. The maximum value of the PDF curve occurs at the mean value.
5. The probability density function and its curve depend on mean and standard deviation of the distribution.

- *Beta Distribution*- It is a type of continuous distribution which is defined for a finite range.

  It's PDF can be defined as following.

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad for\ \ x \in [0,1] \tag{2.29}$$

$$f(x) = 0, \quad for\ \ x \notin [0,1] \tag{2.30}$$

Here x is a the random variable of Beta distribution. α and β are the shape parameters of the distribution. Both α and β are positive.

The mean(μ) and variance(σ^2) of the distribution can be expressed as follows.

$$\mu = \frac{\alpha}{\alpha+\beta} \tag{2.31}$$

$$\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \tag{2.32}$$

Beta Distribution is used to represent PDF of solar radiation in probabilistic modelling of Photovoltaic integrated systems.

- *Weibull distribution*- It can be defined by following expression.

  $$f(x) = \frac{\beta}{\delta}(\frac{x}{\delta})^{\beta-1} e^{-(\frac{x}{\delta})^{\beta}}, \quad for \ x > 0 \tag{2.33}$$

  where δ is scale parameter and β is shape parameter. Both of these parameters are positive quantities. f(x) is PDF and x is random variable of Weibull distribution. When the value of shape parameter β is 2 then Weibull distribution becomes Rayleigh distribution.

  Weibull distribution finds application in probabilistic modelling of wind velocity in wind power integrated systems.

- *Gamma distribution*- The density function f(x) of Gamma distribution can be defined as follows.

  $$f(x) = \frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}, \quad for \ x > 0 \tag{2.34}$$

  $$f(x) = 0, \quad for \ x \leq 0 \tag{2.35}$$

  where $\Gamma(\alpha)$ represents Gamma function. The mean μ and variance σ^2 are defined as follows.

  $$\mu = \alpha\beta \tag{2.36}$$

  $$\sigma^2 = \alpha\beta^2 \tag{2.37}$$

Cumulative Density Function

CDF signifies the probability of a random variable being less than a specified value. The magnitude of CDF can vary between 0 and 1 since it is a probability. The expression below shows how CDF can be calculated.

$$\int_{-\infty}^{X} f(x)dx = CDF \tag{2.38}$$

The above equation gives value of CDF which is probability that random variable x is less than X.

2.4 *Introduction to Probabilistic Load Flow*

In system planning, it is highly important to assess possible variations in the system and their effect. Earlier, deterministic load flow used to be applied for operation and planning but since variation occurs in load and nowadays REs are being integrated in the grid because of which power injected at the buses is variable, conventional load flow is not an effective method as due to this variation, variation occurs in voltage, angle, reactive and active power flows, etc.

To address this issue, Borkowska first invented Probabilistic load flow. This method generally involves use of PDF of variable inputs to find the PDF of the desired output variables. The density function helps to determine the probable load, voltage , branch flows. It helps in load forecasting, in choosing the capacity of lines while designing of new transmission lines [34].

The uncertainties in a network can be due to load variation, generator outages, branch outages and integration of REs. The generator outages and load variation can be dealt with by considering variation in injected power at the buses. When a branch outage takes place, the power network configuration changes. For using linear equations simulation of each line outage is carried out. Fictitious powers are injected at both the ends of the line. The system state can be considered equivalent to that after the line outage when at each end of the line, the injected powers are equivalent to powers leaving from that end .The simulation of random branch outage can be done by two random power injections with normal distribution at each branch end. Hence, the linear equations can be used. Similarly, simulation of multiple branch can be done [3].

When uncertainties in load, generation and line outages are considered together, the bus voltages may violate lower limit and due to branch outages, the thermal rating of the lines may get violated. Hence, we can identify the buses and lines which are weak.

2.4.1 *Types of Distributions Used in PLF*

The probability distribution functions which can be used for application in PLF depend on type of variation in the random values of the quantities. The loads and generation may vary differently.

The following are the types of load variation [12].

1. *Normally distributed and continuous*- Generally in practical cases, when time interval considered is short, we find continuous loads, which can be represented by normal distribution PDF.

2. *Discrete* - The discrete data for loads can be represented by discrete PDF. The data can be obtained from substation data for either months or years. In this case, the PDF is obtained from the load duration curve. To find PDF of state variables,Gram -Charlier expansion series is used. To find CDF, Cornish-Fisher expansion series is used.

3. *Normal and Discrete*- When mixed loads are considered ,then both normal and discrete distribution are considered. In case of unavailability of the load data, normal distribution is assumed.

The possible distribution functions for representing variation in generation can be discrete, Weibull distribution in case of wind power, Beta distribution in case of photovoltaic power modelling. Whereas normal distribution can be used to simulate branch outages.

2.4.2 *PLF Methods*

Probabilistic load flow methods can be classified into three categories. These are Monte Carlo Simulation, analytical method and approximate methods [35].

- *MCS method*- It uses a set of random numbers to evaluate a deterministic model. In this method, a set of values are first taken from the probability density function of random variables and then simulation is carried out. Next, the same procedure is carried out with different set of values from the PDF. Hence, repeated simulations are carried out in the same manner to get the output. Many trials are required for the simulation procedure to converge. More is the number of simulations, more is the accuracy. Thus, it is a time consuming technique.

  In PLF analysis, to implement MCS method, for each simulation, the deterministic load flow is run with a set of random numbers generated for injected active and reactive power. These random numbers can be obtained from their PDF. Thus in the output we get random variables for the desired quantities such as voltage and angle,etc. Thus, standard deviation of these random variables from the expected values can be obtained. Thus, PDF and CDF can be obtained. Generally in PLF analysis by MCS method, 1000 simulations are run for accuracy.

- *Analytical method*- In this method, linearized equations are applied. It is more effective and less time consuming than Monte Carlo simulation technique. In these methods, generally the expected values of desired output variables are determined by deterministic load flow. Then using linearized load flow equations, the random variables and PDF of the desired variables are obtained.For this, deterministic load flow uses expected values of the injected power and in probabilistic load flow, the distribution function of the injected power is used to generate random variables.

- *Approximate methods*- These are unscented transformation method, Cornish-Fisher expansion technique, Point estimate methods. In these methods, deterministic routines are used. These methods are not capable of providing higher moments. So, it is difficult to obtain PDF of the output random variables. These methods are not suitable for complex systems such as systems in which random variables are correlated.

2.4.2.1 *Examples of PLF Techniques*

Some examples of PLF techniques are listed below.

- MCS method
- Linearized power flow equations
- Point estimate method
- Cumulant method and Gram-Charlier series expansion method
- Cornish-Fisher expansion method
- Stratified Latin Hypercube Sampling method
- Polynomial Chaos method

2.4.2.2 *Discussion on some PLF Methods*

- *Cumulant & Gram-Charlier series expansion method*- In this method, the variations in the desired output variables are calculated which can be obtained through linearized load flow equations or any other suitable method. Then the cumulants are calculated from the moments. Thus, from cumulants the coefficients of Gram-Charlier expansion is calculated and the PDF and CDF of the desired random variables can be obtained.

  If α_s represents moments of order s and β_s represents central moments of order s, then these can be calculated by following relations.

$$\alpha_s = \int_{-\infty}^{+\infty} x^s f(x) dx \tag{2.39}$$

$$\beta_s = \int_{-\infty}^{+\infty} (x - \alpha_1)^s f(x) dx \tag{2.40}$$

  where f(x) represents PDF and x represents random variable.

The moment generating function $\varphi(t)$can be calculate as

$$\varphi(t) = \int\limits_{-\infty}^{+\infty} e^{tx} f(x)dx \tag{2.41}$$

Hence, to obtain cumulants logarithm of $\varphi(t)$ is expanded using Maclaurin's series

$$\log \varphi(t) = \sum_{s=1}^{k} \frac{\gamma_s}{s!} t^v + 0(t^k) \tag{2.42}$$

Here γ_s represents cumulants of order s.

The Gram-Charlier expansion is described by following equations.

$$F(\xi) = c_0\Phi(\xi) + c_1\Phi^{'}(\xi) + c_2\Phi^{''}(\xi)... \tag{2.43}$$

$$f(\xi) = c_0\varphi(\xi) + c_1\varphi^{'}(\xi) + c_2\varphi^{''}(\xi) + ... \tag{2.44}$$

where $F(\xi)$ and $f(\xi)$ are the PDF and CDF respectively of the normalized output variable which are expanded by the above equations consisting of standard normal distribution and its derivatives. $\Phi(\xi)$ and $\varphi(\xi)$ are the PDF and CDF of standard normal distribution.

The coefficients in the expansion are calculated from central moments [28]&[30].

- *Cornish-Fisher expansion method*- Through this method, approximate quantile of CDF in terms of quantile of the Normal distribution and cumulants of CDF is obtained. It works better for non-Gaussian PDF than Gram-Charlier series expansion even though they are related to each other [14].
- *Point Estimate methods*- With the help of these methods, statistical moments can be calculated for a random variable which can be a function of either one or many random variables. The value of this random variable can be calculated on the sampled points of these random variables. This method can be categorized as 2m, 2m+1, km+1, 2^m Point estimate methods [32].

2.4.3 *Applications of Probabilistic Load Flow*

Probabilistic or stochastic load flow has diverse applications:

- In network security and expansion.

- It can be used to calculate reactive power requirement in generator buses so as to reduce voltage sags.
- It helps to find increment in the active power demand with variation in the loads.
- In case of distributed generation where renewable energy sources are used, variation in the power generated can be studied.
- In case of line contingencies or branch outages, PLF method helps to find the impact on power flow and voltage at the buses.
- For studying the impact of variation in the wind speed in wind farms on the optimal power flow.
- Assessment of voltage fluctuation for preventing overvoltage risk.
- Assessment of fluctuation in power flow to limit overloading probability of transmission lines considering output variation of the generators.
- Probabilistic harmonic power flow helps to analyse the influence of uncertain harmonic current injected into the grid which are generated by DGs integrated with the grid and can affect the power quality and power system operation.
- Impact of DGs on voltage quality (voltage limits, voltage flicker and voltage imbalance) of feeders.
- To study the uncertain characteristics of plug-in EV (Electric vehicle) charging loads integrated with the distributed networks.
- Analysis of transmission line temperature violation due to power fluctuations.

2.4.4 *Typical Applications of Probabilistic load flow*

- *Application of PLF when wind power is integrated with the system*- Wind speed is dependent on wind speed variation. The wind speed variation can be represented by Weibull distribution function and Rayleigh distribution function.The power generated by the wind turbine can be represented by a weibull distribution function whose parameters are function of the parameters of the wind speed PDF. Thus, the variation in injected power at the buses where wind turbine has been connected can be studied by applying suitable PLF method [7].
- *When photovoltaic source is integrated with the system*- In this case, the variation in light intensity can be represented by Beta distribution. Similarly the output photovoltaic power injected at the buses can be represented by the Beta distribution. Thus effect of variation in radiation intensity on the injected power can be studied [36].The peak of

the injected power is mainly in the day time.The radiation intensity varies all throughout the day and also varies with environmental conditions. Due to this variation, the active power balance is effected. To prevent this, automatic generation control and generation dispatch strategies can be used [10].

- *Variation in load*- The variation in load demand leads to the variation in the injected power at the buses. The factors responsible for the variation are changing customer demands, environmental conditions,type of electrical appliances, temperature. Time of the day also influences the load demand. The load variation can be studied from the load duration curve. The suitable distribution functions for load modelling are normal and discrete. Accordingly, the loads can be represented by their respective PDFs and CDFs [10].
- *Temperature variation due to power fluctuation*- When there is variation in power injected into the system, the temperature varies and may violate thermal limits. Probabilistic model of line temperature due to fluctuation in power can be obtained by the MCS method [37].
- *When Hybrid electric vehicles are connected to the Grid*- A plug-in hybrid electric vehicle consists of a battery which charges itself from an external source. The charging demand,time duration of charging ,different voltage and current levels lead to uncertainty in the system. Thus power demand of these hybrid vehicles can ne represented by PDFs and corresponding CDFs [38].
- *In estimating voltage unbalance in a distribution network*- If magnitude of voltages are different or 120 degree phase shift is not maintained among the three phases, then this condition can be described as voltage unbalance. The demerits of this unbalance are reduced efficiency and lifetime of the equipments, overheating of devices and losses. Nowadays different types of loads like single phase and dual phase load and storage make the unbalance more unpredictable. Distributed generation or integrating single phase REs can either reduce or increase the unbalance. Hybrid Electric vehicles are single phase loads which lead to unequal distribution of load per phase. The MCS method can be used for PLF analysis. The N-R method can be used to obtain expected values of phase voltages. Random variation in power factor can be used to model the load unbalance probabilistically. The power factor variation can be represented by normal distribution. This variation can be achieved by either varying active power or reactive power.Hence random values of power factors can be generated using their respective distribution function. For different power factor combinations, load flow is run. Large number of simulations by MCS method ensures accurate results[39] .
- *In railway traction systems*- The PLF techniques can be used for analysis of variation in power demand in electric railway traction systems. In these systems, the load demand

varies with time, position and operating mode of the locomotive. Techniques like MCS method can be used for studying the variation in load.The variation in train position can be represented by their PDFs. From the PLF analysis, PDF can be obtained for power demand, voltage, losses and line flows. Hence we can obtain probable values of these quantities from their PDFs and corresponding CDFs [40].

- *In system which is integrated with Biomass energy systems*- In this case the load and heat value or calorific value of biomass can be considered as random variables with normal distribution. Cornish-Fisher expansion can be used to find quantiles of random variable. With this method, it is possible to check if the voltages at the buses violate their specified limits. [20].

Chapter 3

Development of Probabilistic Load Flow Solution Method

3.1 *Load Flow Equation*

The basic load flow equation can be described as:

$$Y = f(X) \tag{3.1}$$

Here Y is input random vector of injected real and reactive power injections. X is state random vector of voltage and angle.

where random vector Y can be represented as,

$$Y = \begin{bmatrix} P \\ Q \end{bmatrix} \tag{3.2}$$

and random vector X is represented as,

$$X = \begin{bmatrix} V \\ \delta \end{bmatrix} \tag{3.3}$$

The load flow equation can also be represented by following equations.

$$P_i = \sum_{k=1}^{nb} V_i V_k [G_{ik} cos(\delta_i - \delta_k) + B_{ik} sin(\delta_i - \delta_k)] \tag{3.4}$$

for $i = 2, 3, ...nb$, where nb is the number of buses.

$$Q_i = \sum_{k=1}^{nb} V_i V_k [G_{ik} sin(\delta_i - \delta_k) - B_{ik} cos(\delta_i - \delta_k)] \tag{3.5}$$

for $i = nv + 1, nv + 2, ...nb.$, where i represents the PQ bus and nv is the number of PV buses.

3.2 *Linearization of Load Flow Equation*

When load flow equation is linearized, the vector X is linearized about the vector X_0 which is the vector of expected bus voltage and angle. Random vector Y is linearized with respect to Y_0 which is vector of expected injected active and reactive power. Matrix S which is inverse of Jacobian matrix which is calculated with the expected values of voltage and its angle obtained from deterministic load flow [3] and [41]. The linearization can be explained through following deduction.

The solution of deterministic load flow can be expressed by following equation.

$$Y_0 = f(X_0) \tag{3.6}$$

where vector Y_0 can be represented as,

$Y_0 = \begin{bmatrix} P_0 \\ Q_0 \end{bmatrix}$

and vector X_0 is represented as,

$X_0 = \begin{bmatrix} V_0 \\ \delta_0 \end{bmatrix}$

The linearization of load flow equation represented by equation [3.1] is described below.

Linearization of load flow equations is implemented with Taylor series. Taylor series can be represented by following equation.

$$f(x+\Delta x) = f(x) + f^{'}(x)(\Delta x) + \frac{f^{''}(x)}{2!}(\Delta x)^2 + \frac{f^{'''}(x)}{3!}(\Delta x)^3 + ... + \frac{f^{n}(x)}{n!}(\Delta x)^n \tag{3.7}$$

where $f^{'}(x) = \frac{df}{dx}$

Let $x = x_0 + \Delta x$, then from taylor series,neglecting higher order terms, the following expression can be obtained.

$$f(x_0 + \Delta x) = f(x_0) + f^{'}(x)(\Delta x) \tag{3.8}$$

or

$$f(x) = f(x_0) + (\frac{df}{dx}|_{x=x_0})(\Delta x) \tag{3.9}$$

Now, let there are be two functions $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$

Thus using Taylor series, following equations can be written.

$$f_1(x_1, x_2) = f_1({x_1}^0, {x_2}^0) + [\frac{\partial f_1}{\partial x_1}|_{x={x_1}^0}(\Delta x_1) + \frac{\partial f_1}{\partial x_2}|_{x={x_2}^0}(\Delta x_2)] \tag{3.10}$$

$$f_2(x_1, x_2) = f_2({x_1}^0, {x_2}^0) + [\frac{\partial f_2}{\partial x_1}|_{x={x_1}^0}(\Delta x_1) + \frac{\partial f_2}{\partial x_2}|_{x={x_2}^0}(\Delta x_2)] \tag{3.11}$$

$$\begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix} = \begin{bmatrix} f_1({x_1}^0, {x_2}^0) \\ f_2({x_1}^0, {x_2}^0) \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial x_1}|_{x={x_1}^0} & \frac{\partial f_1}{\partial x_2}|_{x={x_2}^0} \\ \frac{\partial f_2}{\partial x_1}|_{x={x_1}^0} & \frac{\partial f_2}{\partial x_2}|_{x={x_2}^0} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} \tag{3.12}$$

Let $x_1 = V$ and $x_2 = \delta$
then,
$f_1(x_1, x_2) = P(V, \delta)$
$f_2(x_1, x_2) = Q(V, \delta)$

$$\begin{bmatrix} P(V, \delta) \\ Q(V, \delta) \end{bmatrix} = \begin{bmatrix} P(V^0, \delta^0) \\ Q(V^0, \delta^0) \end{bmatrix} + \begin{bmatrix} \frac{\partial P}{\partial V}|_{V=V^0} & \frac{\partial P}{\partial \delta}|_{\delta=\delta^0} \\ \frac{\partial Q}{\partial V}|_{V=V^0} & \frac{\partial Q}{\partial \delta}|_{\delta=\delta^0} \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \delta \end{bmatrix} \tag{3.13}$$

$$\Delta P = P(V, \delta) - P(V^0, \delta^0) \tag{3.14}$$

$$\Delta Q = Q(V, \delta) - Q(V^0, \delta^0) \tag{3.15}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \delta \end{bmatrix} \tag{3.16}$$

$$\Delta Y = Y - Y_0 = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{3.17}$$

$$\Delta X = X - X_0 = \begin{bmatrix} \Delta V \\ \Delta \delta \end{bmatrix} \tag{3.18}$$

$$\begin{bmatrix} Y - Y_0 \end{bmatrix} = \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} X - X_0 \end{bmatrix} \tag{3.19}$$

$$\begin{bmatrix} \Delta Y \end{bmatrix} = \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} \Delta X \end{bmatrix} \tag{3.20}$$

$$\left[\Delta X\right] = \left[J\right]^{-1}\left[\Delta Y\right] \tag{3.21}$$

$\left[J\right]^{-1}$ can be repesented by $\left[S\right]$.

The matrix S is inverse of Jacobian matrix calculated at the expected values of voltage and angle and is expressed as $[(\frac{df}{dx})_{x=x_0}]^{-1}$.

$$\left[X - X_0\right] = \left[S\right]\left[\Delta Y\right] \tag{3.22}$$

$$\left[X\right] = \left[X_0\right] + \left[S\right]\left[\Delta Y\right] \tag{3.23}$$

3.3 *Algorithm for Probabilistic Load Flow*

For applying PLF, the specified values of injected power at the buses are considered as their expected values. The loads are assumed to have either continuous or discrete distribution.

At first, deterministic load flow is run to obtain the expected values of voltage and its angle at the buses. Following which, PLF analysis is applied. For injected active and reactive power using their expected values and standard deviation, normal distribution PDF is obtained. The standard deviation of injected power is same as assumed for the loads. Thus, PDF helps to generate random values which are used to form random vector of injected power. Thereafter, the inverse of Jacobian matrix at the expected values of voltage and its angle is calculated. The vector of expected values of voltage and angle can be formed from the results of deterministic load flow.

Hence, by applying linearized load flow equations, random vector of voltage and its angle is obtained. This process can be repeated with different set of random values of injected active and reactive power. Thus, as per requirement, samples of random values for voltage and its angle can be obtained. These random values can be used for calculation of variance and standard deviation with respect to the expected values for voltage and and its angle at the buses. Finally assuming normal distribution, PDF is obtained for voltage and angle using expected values and standard deviation. PDF and CDF curves are plotted for the buses at which loads are continuous. For buses with discrete loads, discrete values for voltage and its angle are obtained from PLF analysis and are represented by bar graphs.

The algorithm has been described below.

Step 1 Read the system data including bus data and line data.

Step 2 Consider the specified values of injected active and reactive power as their expected values.

Step 3 At the expected values of injected active and reactive power, run deterministic load flow by applying NR method.

Step4 From the deterministic load flow, expected values of voltage and its angle are obtained. Using these values, form the vector X_0 .

Step5 Form the vector Y_0 by the expected values of injected active power and reactive power.

Step6 Form the S matrix by taking inverse of the jacobian matrix calculated at the expected values of voltage and angle obtained from the deterministic load flow.

Step7 To obtain random vector Y, form the PDF of injected active and reactive power using the expected values and standard deviation which can be taken as ten percent of the loads. For this, Normal distribution is assumed as loads are normally distributed.

Step8 Generate random variables for injected active and reactive power using Normal distribution PDF to form vector Y.

Step9 Hence, using linearized load flow equation, obtain random vector X.

Step10 The random values of voltage and angle are obtained from vector X.

Step11 Repeat from Step 8 to Step 10 as many times as required to generate requisite number of random values.

Step12 Now for both voltage and its angle, variance and standard deviation of the random variables are calculated with respect to their respective expected values.

Step13 For buses with continuous loads, using the expected values and standard deviation and assuming normal distribution, plot the PDF and CDF curves.

Step14 For buses with discrete loads, plot the bar graphs for bus voltage and its angle.

The flowchart describing methodology has been shown (Fig 3.1).

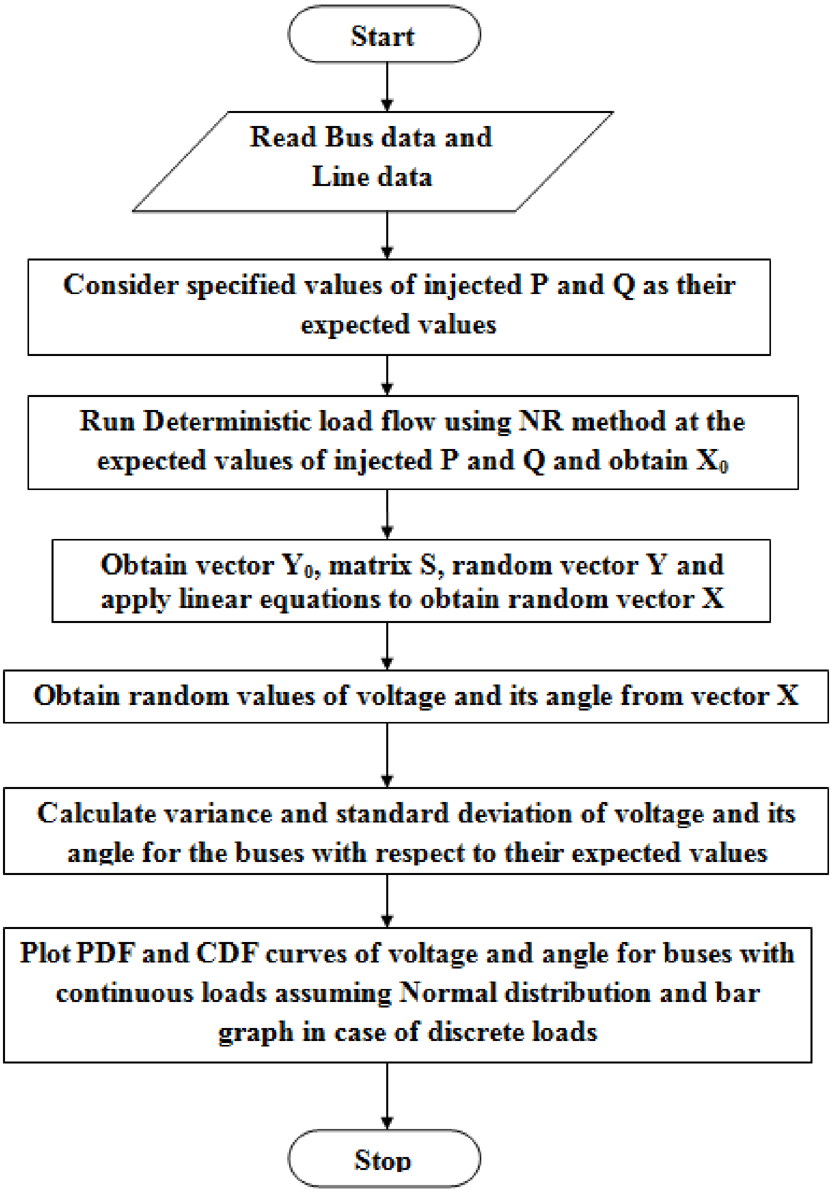


Figure 3.1: Flowchart of methodology

Chapter 4

Results and Discussions

The proposed probabilistic method was applied to 3 test systems. At first the expected values of voltage and angle were calculated by NR method (Deterministic load flow).Standard deviation of the desired variables was obtained with respect to expected values using Probabilistic load flow.The PLF method applied was based on analytical method in which linearized load flow equations were used. Hence, the PDF and CDF curves were plotted. The method was applied on following 3 test systems.

- 14-Bus System
- 30-Bus System
- 57-Bus System

The method was applied in two ways.At first the, for all three test systems, the loads were considered to be continuous and normally distributed. In second case,for 14 bus system,the load at the buses either had discrete or continuous distribution. The deterministic load flow analysis and PLF analysis were carried out under MATLAB environment.

4.1 *When loads are continuous*

4.1.1 *IEEE 14-Bus system*

Matpower case-14 test system data was used for PLF analysis. In this system, there were four generator buses and nine load buses and one slack bus. At the bus no.9, reactive load was connected. There are 20 line branches. In three of the lines, tap changing transformers are connected (refer to Table A.1.1 and Table A.1.2).

The standard IEEE 14-Bus system has been shown in the figure below (Fig 4.1).

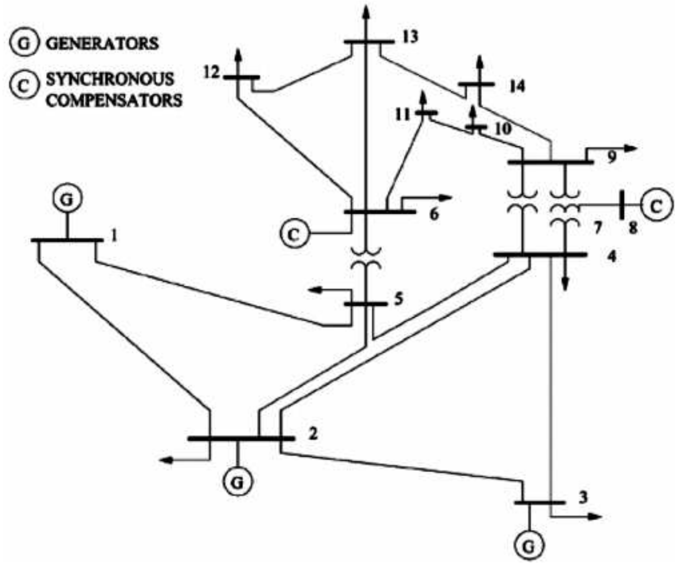


Figure 4.1: IEEE 14-Bus System

At first deterministic load flow was run by using NR method to obtain expected values of voltage and angle. The specified values of injected active and reactive power at the buses was considered as their expected values.

Table 4.1: Deterministic load flow results for 14-bus system with continuous loads

Bus no.	Voltage(p.u.)	angle(radians)	Injected P(MW)	Injected Q(MVAr)
1	1.0600	0	232.4014	-16.4453
2	1.0450	-0.0870	18.3000	31.1949
3	1.0100	-0.2221	-94.2000	6.2831
4	1.0173	-0.1799	-47.8000	3.9000
5	1.0193	-0.1531	-7.6000	-1.6000
6	1.0700	-0.2485	-11.2000	6.1279
7	1.0604	-0.2330	0.0000	0.0000
8	1.0900	-0.2330	0	18.3226
9	1.0537	-0.2605	-29.5000	2.4000
10	1.0491	-0.2634	-9.0000	-5.8000
11	1.0560	-0.2582	-3.5000	-1.8000
12	1.0550	-0.2634	-6.1000	-1.6000
13	1.0501	-0.2648	-13.5000	-5.8000
14	1.0341	-0.2799	-14.9000	-5.0000

After obtaining expected values of voltage and angle,the PLF analysis using linear equations was applied. Loads were considered continuous having normal distribution. About 10 percent standard deviation was considered in both active and reactive power of loads.

At first, the normal PDF of injected active power and reactive power was obtained by using their expected values and standard deviation. From the PDF, random values were generated for injected active and reactive power so that their random vector is formed. Now, expected vector of injected active and reactive power is formed from their specified values. Then, the inverse of Jacobian matrix was calculated at the expected values of voltage and angle. Thus using linearized equations of power flow, random vector of voltage and angle was obtained.Hence, standard deviation of voltage and its angle was calculated. PLF was run for 400 random values of bus voltage and angle. Following were the results obtained.

Table 4.2 and Table 4.3 show the results obtained from PLF analysis.

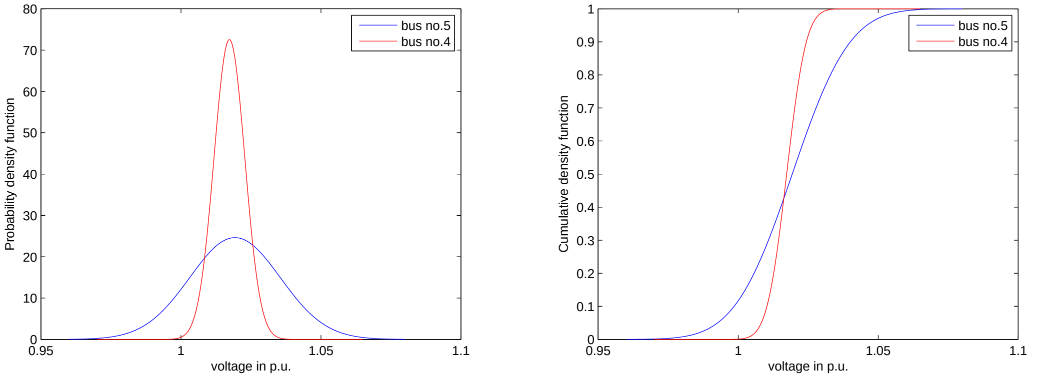
Table 4.2: PLF Results for voltage for 14-bus system with continuous loads

Bus no.	Expected value(p.u.)	Standard deviation
4	1.0173	0.0055
5	1.0193	0.0162
7	1.0604	0.0092
9	1.0537	0.0075
10	1.0491	0.0099
11	1.0560	0.0105
12	1.0550	0.0105
13	1.0501	0.0114
14	1.0341	0.0112

Table 4.3: PLF Results for angle for 14-bus system with continuous loads

Bus no.	Expected value(radians)	Standard deviation
2	-0.0870	0.0105
3	-0.2221	0.0104
4	-0.1799	0.0106
5	-0.1531	0.0117
6	-0.2485	0.0010
7	-0.2330	0.0009
8	-0.2330	0.0007
9	-0.2605	0.0010
10	-0.2634	0.0011
11	-0.2582	0.0007
12	-0.2634	0.0006
13	-0.2648	0.0009
14	-0.2799	0.0018

From the results of PLF analysis, it is observed that for bus no.5, the standard deviation in voltage was maximum and it was minimum for bus no.4. The standard deviation in bus voltage angle was maximum for bus no.5 and was found minimum for bus no.12.Based on the results obtained from the PLF analysis, the PDF and CDF curves are shown for the buses with maximum and minimum standard deviation in bus voltage and bus voltage angle.



(a) PDF curves of voltage for bus no. 5 and 4

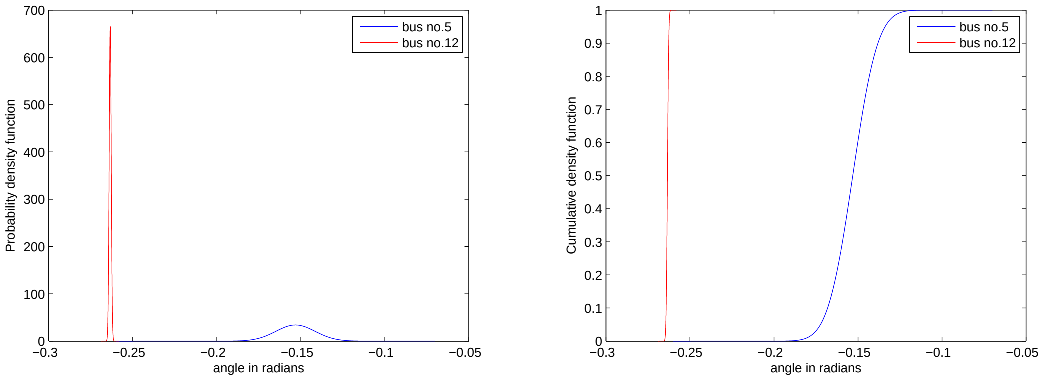
(b) CDF curves of voltage for bus no. 5 and 4

Figure 4.2: PDF and CDF curves of voltage for buses with maximum and minimum deviation for 14-bus system with continuous loads

From the curves (Fig 4.2), it can be observed that for bus no.5, the PDF has highest value approximately 24 at the expected value of its voltage which is 1.0193 p.u.. The PDF and CDF has a non zero value within the range 0.96 p.u. to 1.08 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 1.0193 p.u.,the probability of occurence of these values decreases.

For bus no.4, the PDF has highest value approximately 72 at the expected value of its voltage which is 1.0173 p.u.. The PDF and CDF has a non zero value within the range 0.99 p.u. to 1.04 p.u..Hence, the bus voltage will take up random values within this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 1.0173 p.u.,the probability of occurence of these values decreases.

Thus,the PDF value for bus no.4 is higher than that for bus no. 5 and range of random values is short in bus no.4 compared to bus no.5. This shows standard deviation in voltage for bus no.4 is lesser and more random values are likely to be around its expected value as compared to bus no.5.



(a) PDF curves of angle for bus no. 5 and 12 (b) CDF curves of angle for bus no. 5 and 12

Figure 4.3: PDF and CDF curves of angle for buses with maximum and minimum deviation for 14-bus system with continuous loads

From the curves(Fig 4.3), it can be observed that for bus no.5, the PDF has highest value approximately 34 at the expected value of its angle which is -0.1531 radians. The PDF and CDF has a non zero value within the range -0.18 radians to -0.13 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.1531 radians,the probability of occurence of these values decreases.

For bus no.12, the PDF has highest value around 680 at the expected value of its angle which is -0.2634 radians. The PDF and CDF has a non zero value within the range -0.266 radians to -0.261 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.2634 radians,the probability of occurence of these values decreases.

Thus, PDF of bus no.12 is very high as compared to bus no.5. Also, the range of random values of angles is short in case of bus no.12. Hence, it shows, the standard deviation in angle is less in case of bus no.12 and more number of random values are likely to be around its expected value as compared to bus no.5.

4.1.2 *IEEE 30-Bus system*

The standard IEEE 30-Bus system has been shown in the figure below (Fig 4.4).

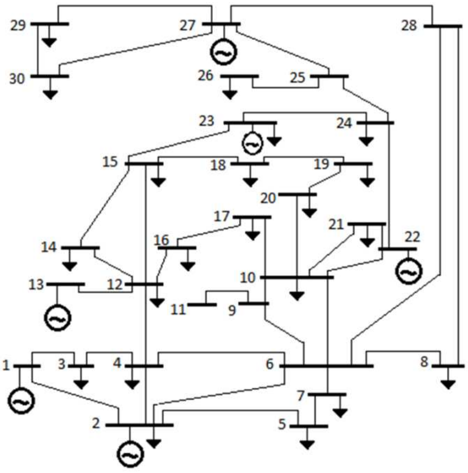


Figure 4.4: IEEE 30-Bus System

For the PLF analysis, the data was taken from Matpower case-30. In this system, there are 1 slack bus,5 generator buses, 24 load buses. There are 41 line branches.Reactive loads were connected at bus no.5 and 24. (refer to Table A.2.1 and Table A.2.2).

At first deterministic load flow was run by using NR method to obtain expected values of voltage and angle.After obtaining expected values of voltage and angle from deterministic load flow, the PLF analysis was carried out to obtain standard deviation of these quantities with respect to their expected values. In this case also, about 10 percent standard deviation in active and reactive power of loads at the buses was considered. Distribution considered for the loads was Normal. The PDF of injected power was used to generate random values for injected active and reactive power.Now, expected vector of injected active and reactive power is formed from their specified values. The inverse of Jacobian matrix was calculated at the expected values of voltage and angle. Thus using linearized equations of power flow, random vector of voltage and angle was obtained.Hence, standard deviation of voltage and angle was calculated. PLF was run for 400 random values of bus voltage and angle.

Table 4.4: Deterministic load flow results for 30-bus system

Bus no.	Voltage(p.u.)	angle(radians)	Injected P(MW)	Injected Q(MVAr)
1	1.0000	0	25.9738	-0.9989
2	1.0000	-0.0073	39.2700	19.2938
3	0.9831	-0.0266	-2.4000	-1.2000
4	0.9801	-0.0313	-7.6000	-1.6000
5	0.9824	-0.0325	0	0.1900
6	0.9732	-0.0396	0	0
7	0.9674	-0.0463	-22.8000	-10.9000
8	0.9606	-0.0476	-30.0000	-30.0000
9	0.9805	-0.0523	0	0
10	0.9844	-0.0589	-5.8000	-2.0000
11	0.9805	-0.0523	0	0
12	0.9855	-0.0268	-11.2000	-7.5000
13	1.0000	0.0258	37.0000	11.3527
14	0.9767	-0.0403	-6.2000	-1.6000
15	0.9802	-0.0403	-8.2000	-2.5000
16	0.9774	-0.0462	-3.5000	-1.8000
17	0.9769	-0.0592	-9.0000	-5.8000
18	0.9684	-0.0607	-3.2000	-0.9000
19	0.9653	-0.0691	-9.5000	-3.4000
20	0.9692	-0.0676	-2.2000	-0.7000
21	0.9934	-0.0609	-17.5000	-11.2000
22	1.0000	-0.0592	21.5900	39.5689
23	1.0000	-0.0277	16.0000	6.3505
24	0.9886	-0.0459	-8.7000	-6.6600
25	0.9902	-0.0295	0	0
26	0.9722	-0.0373	-3.5000	-2.3000
27	1.0000	-0.0145	26.9100	10.5400
28	0.9747	-0.0395	0	0
29	0.9796	-0.0371	-2.4000	-0.9000
30	0.9679	-0.0531	-10.6000	-1.9000

Table 4.5 and Table 4.6 show the results obtained from PLF analysis.

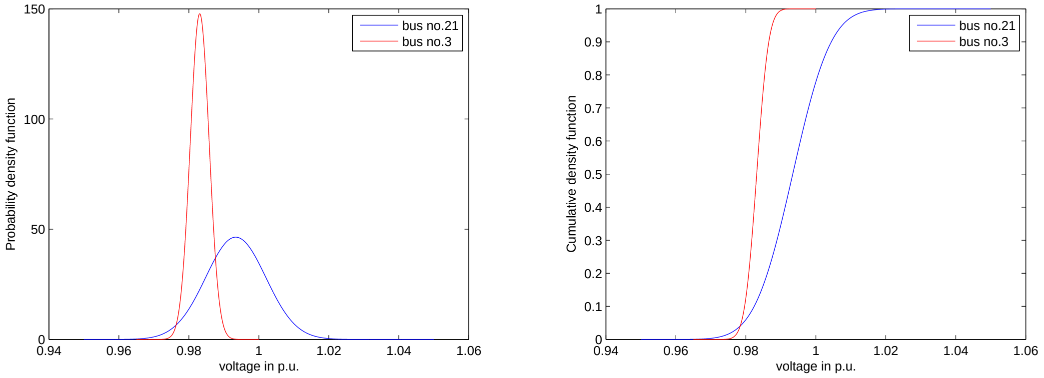
Table 4.5: PLF Results for voltage for 30-bus system

Bus no.	Expected value(p.u.)	Standard deviation
3	0.9831	0.0027
4	0.9801	0.0038
5	0.9824	0.0046
6	0.9732	0.0045
7	0.9674	0.0054
8	0.9606	0.0057
9	0.9805	0.0063
10	0.9844	0.0069
11	0.9805	0.0079
12	0.9855	0.0069
14	0.9767	0.0073
15	0.9802	0.0073
16	0.9774	0.0078
17	0.9769	0.0079
18	0.9684	0.0076
19	0.9653	0.0080
20	0.9692	0.0084
21	0.9934	0.0086
24	0.9886	0.0084
25	0.9902	0.0085
26	0.9722	0.0084
28	0.9747	0.0082
29	0.9796	0.0084
30	0.9679	0.0083

Table 4.6: PLF Results for angle for 30-bus system

Bus no.	Expected value(radians)	Standard deviation
2	-0.0073	0.0086
3	-0.0266	0.0083
4	-0.0313	0.0058
5	-0.0325	0.0099
6	-0.0396	0.0117
7	-0.0463	0.0009
8	-0.0476	0.0011
9	-0.0523	0.0011
10	-0.0589	0.0016
11	-0.0523	0.0019
12	-0.0268	0.0027
13	0.0258	0.0008
14	-0.0403	0.0005
15	-0.0403	0.0008
16	-0.0462	0.0006
17	-0.0592	0.0009
18	-0.0607	0.0006
19	-0.0691	0.0007
20	-0.0676	0.0009
21	-0.0609	0.0010
22	-0.0592	0.0013
23	-0.0277	0.0011
24	-0.0459	0.0003
25	-0.0295	0.0008
26	-0.0373	0.0006
27	-0.0145	0.0019
28	-0.0395	0.0017
29	-0.0371	0.0015
30	-0.0531	0.0028

From the results of PLF analysis, it is observed that standard deviation in voltage is maximum for bus no.21 and is minimum for bus no.3. The standard deviation in angle was maximum for bus no. 6 and was minimum for bus no.24. The PDF and CDF curves for the buses with maximum and minimum standard deviation in voltage and its angle are shown.



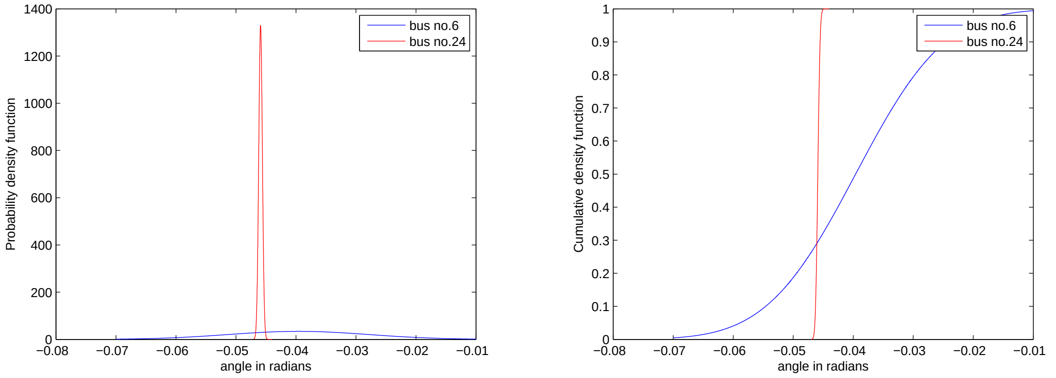
(a) PDF curves of voltage for bus no. 21 and 3 (b) CDF curves of voltage for bus no. 21 and 3

Figure 4.5: PDF and CDF curves of voltage for buses with maximum and minimum deviation for 30-bus system

From the curves (Fig 4.5), it can be observed that for bus no.21, the PDF has highest value approximately 47 at the expected value of its voltage which is 0.9934 p.u.. The PDF and CDF has a non zero value within the range 0.96 p.u. to 1.02 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 0.9934 p.u.,the probability of occurence of these values decreases.

For bus no.3, the PDF has highest value approximately 149 at the expected value of its voltage which is 0.9831 p.u.. The PDF and CDF has a non zero value within the range 0.974 p.u. to 0.993 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 0.9831 p.u.,the probability of occurence of these values decreases.

Thus,the PDF value for bus no.3 is higher than that for bus no.21 and range of random values is short in bus no.3 compared to bus no.21. This shows standard deviation in voltage for bus no.3 is lesser and more random values are likely to be around its expected value as compared to bus no.21.



(a) PDF curves of angle for bus no. 6 and 24

(b) CDF curves of angle for bus no. 6 and 24

Figure 4.6: PDF and CDF curves of angle for buses with maximum and minimum deviation for 30-bus system

From the curves (Fig 4.6), it can be observed that for bus no.6, the PDF has highest value approximately 34 at the expected value of its angle which is -0.0396 radians. The PDF and CDF has a non zero value within the range -0.07 radians to -0.01 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.0396 radians,the probability of occurence of these values decreases.

For bus no.24, the PDF has highest value approximately 1350 at the expected value of its angle which is -0.0459 radians. The PDF and CDF has a non zero value within the range -0.047 radians to -0.0449 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.0459 radians,the probability of occurence of these values decreases.

Thus,the PDF value for bus no.24 is higher than that for bus no.6 and range of random values is short in bus no.24 compared to bus no.6. This shows standard deviation in angle for bus no.24 is lesser and more random values are likely to be around its expected value as compared to bus no.6.

4.1.3 *IEEE 57- Bus system*

The standard IEEE 57-Bus system has been shown (Fig 4.7).

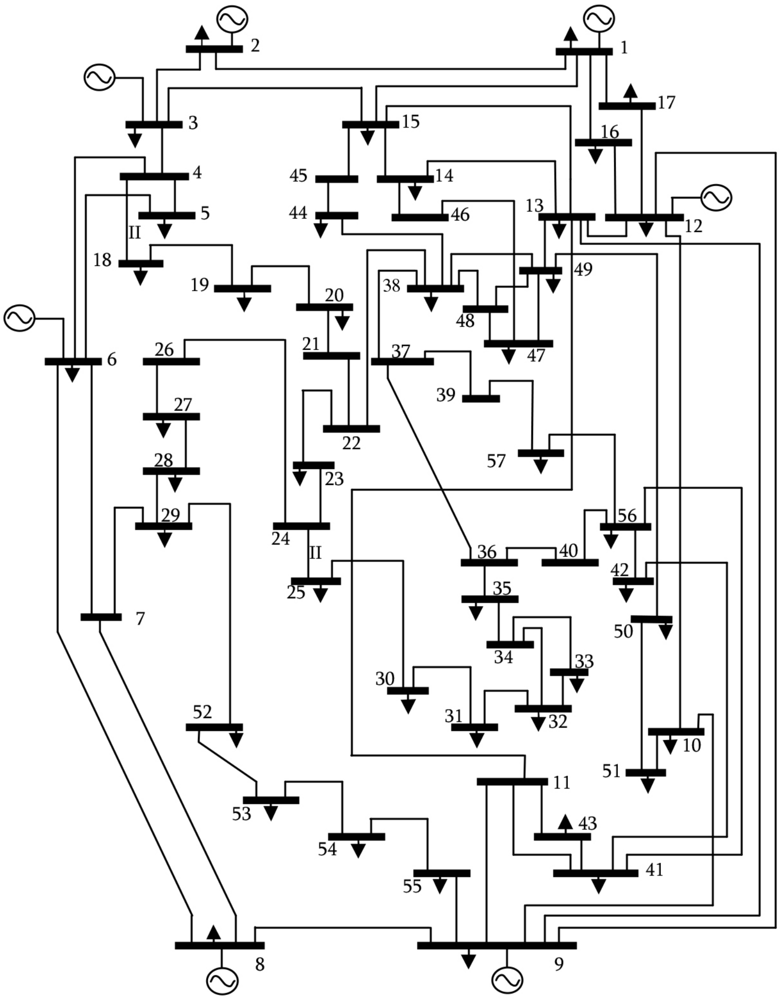


Figure 4.7: IEEE 57-Bus System

Matpower case-57 was used for analysis. There are 6 generator buses, 1 slack bus and 50 load buses in the system. There are 80 line branches in the system. Reactive loads were connected at bus no.18,25 and 53. Tap changing transformers were connected in 15 line branches (refer to Table A.3.1 and Table A.3.2). .

At first deterministic load flow was run by using NR method to obtain expected values of voltage and angle.The expected values obtained were used in the PLF analysis to obtain the distribution of voltage and angle. Both active and reactive loads were considered to be normally distributed with standard deviation of about 10 percent. The PDF of injected power was used to generate random values for injected active and reactive power. Thus using linearized equations of power flow, random vector of voltage and angle was obtained. PLF was run for 400 random values of bus voltage and angle. The following were the results obtained.

Table 4.7: Deterministic load flow results for 57-bus system

Bus no.	Voltage(p.u.)	angle(radians)	Injected P(MW)	Injected Q(MVAr)
1	1.0400	0	423.6455	111.8218
2	1.0100	-0.0207	-3.0000	-88.7553
3	0.9850	-0.1045	-1.0000	-21.9521
4	0.9808	-0.1281	0	0
5	0.9765	-0.1492	-13.0000	-4.0000
6	0.9800	-0.1514	-75.0000	-1.2374
7	0.9843	-0.1327	0	0
8	1.0050	-0.0782	300.0000	39.9391
9	0.9800	-0.1673	-121.0000	-23.9248
10	0.9863	-0.1998	-5.0000	-2.0000
11	0.9740	-0.1779	0	0
12	1.0150	-0.1827	-67.0000	104.5693
13	0.9789	-0.1711	-18.0000	-2.3000
14	0.9702	-0.1632	-10.5000	-5.3000
15	0.9881	-0.1255	-22.0000	-5.0000
16	1.0134	-0.1546	-43.0000	-3.0000
17	1.0175	-0.0942	-42.0000	-8.0000
18	1.0006	-0.2047	-27.2000	0.2000
19	0.9702	-0.2309	-3.3000	-0.6000
20	0.9639	-0.2347	-2.3000	-1.0000
21	1.0087	-0.2257	0	0
22	1.0099	-0.2247	0	0
23	1.0085	-0.2259	-6.3000	-2.1000
24	0.9998	-0.2322	0	0
25	0.9840	-0.3173	-6.3000	2.7000
26	0.9593	-0.2268	0	0
27	0.9819	-0.2010	-9.3000	-0.5000
28	0.9970	-0.1830	-4.6000	-2.3000
29	1.0105	-0.1706	-17.0000	-2.6000
30	0.9641	-0.3268	-3.6000	-1.8000
31	0.9372	-0.3382	-5.8000	-2.9000
32	0.9507	-0.3229	-1.6000	-0.8000
33	0.9484	-0.3236	-3.8000	-1.9000
34	0.9595	-0.2470	0	0
35	0.9665	-0.2427	-6.0000	-3.0000
36	0.9761	-0.2380	0	0
37	0.9851	-0.2347	0	0
38	1.0130	-0.2223	-14.0000	-7
39	0.9830	-0.2355	0	0
40	0.9730	-0.2384	0	0
41	0.9963	-0.2457	-6.3000	-3.0000
42	0.9666	-0.2711	-7.1000	-4.4000
43	1.0096	-0.1982	-2.0000	-1.0000
44	1.0169	-0.2069	-12.0000	-1.8000
45	1.0361	-0.1618	0	0
46	1.0599	-0.1940	0	0.0001
47	1.0334	-0.2184	-29.7000	-11.6000
48	1.0275	-0.2201	0	0
49	1.0363	-0.2258	-18.0000	-8.5000
50	1.0234	-0.2341	-21.0000	-10.5000
51	1.0523	-0.2187	-18.0000	-5.3000
52	0.9811	-0.2010	-4.9000	-2.2000
53	0.9719	-0.2143	-20.0000	-3.7000
54	0.9969	-0.2046	-4.1000	-1.4000
55	1.0310	-0.1885	-6.8000	-3.4000
56	0.9685	-0.2804	-7.6000	-2.2000
57	0.9650	-0.2895	-6.7000	-2.0000

Table 4.8 and Table 4.9 show the results obtained from PLF analysis.

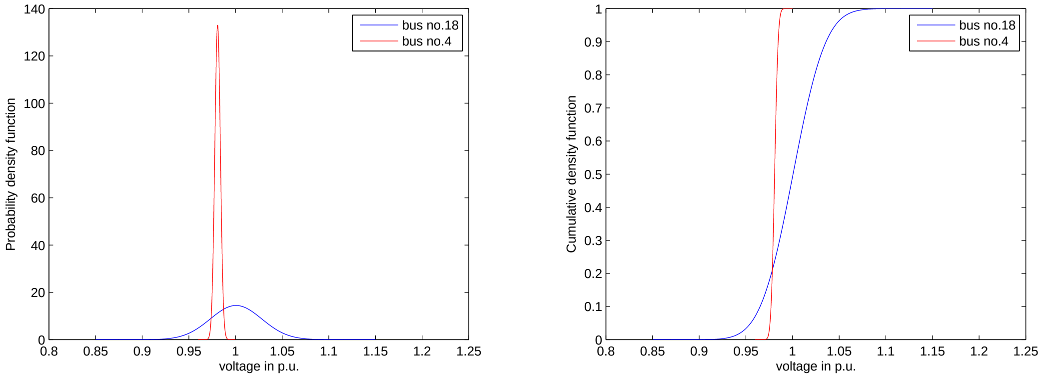
Table 4.8: PLF Results for voltage for 57-bus system

Bus no.	Expected value(p.u.)	Standard deviation
4	0.9808	0.0030
5	0.9765	0.0124
7	0.9843	0.0154
10	0.9863	0.0201
11	0.9740	0.0224
13	0.9789	0.0247
14	0.9702	0.0268
15	0.9881	0.0254
16	1.0134	0.0254
17	1.0175	0.0227
18	1.0006	0.0276
19	0.9702	0.0209
20	0.9639	0.0178
21	1.0087	0.0134
22	1.0099	0.0200
23	1.0085	0.0106
24	0.9998	0.0175
25	0.9840	0.0179
26	0.9593	0.0181
27	0.9819	0.0189
28	0.9970	0.0191
29	1.0105	0.0192
30	0.9641	0.0209
31	0.9372	0.0214
32	0.9507	0.0212
33	0.9484	0.0233
34	0.9595	0.0240
35	0.9665	0.0245
36	0.9761	0.0215
37	0.9851	0.0216
38	1.0130	0.0208
39	0.9830	0.0208
40	0.9730	0.0195
41	0.9963	0.0194
42	0.9666	0.0194
43	1.0096	0.0193
44	1.0169	0.0189
45	1.0361	0.0194
46	1.0599	0.0195
47	1.0334	0.0223
48	1.0275	0.0220
49	1.0363	0.0226
50	1.0234	0.0178
51	1.0523	0.0152
52	0.9811	0.0184
53	0.9719	0.0190
54	0.9969	0.0191
55	1.0310	0.0204
56	0.9685	0.0224
57	0.9650	0.0249

Table 4.9: PLF Results for angle for 57-bus system

Bus no.	Expected value(radians)	Standard deviation
2	-0.0207	0.0250
3	-0.1045	0.0255
4	-0.1281	0.0253
5	-0.1492	0.0254
6	-0.1514	0.0216
7	-0.1327	0.0214
8	-0.0782	0.0003
9	-0.1673	0.0004
10	-0.1998	0.0004
11	-0.1779	0.0007
12	-0.1827	0.0005
13	-0.1711	0.0006
14	-0.1632	0.0009
15	-0.1255	0.0006
16	-0.1546	0.0012
17	-0.0942	0.0015
18	-0.2047	0.0010
19	-0.2309	0.0015
20	-0.2347	0.0018
21	-0.2257	0.0017
22	-0.2247	0.0017
23	-0.2259	0.0017
24	-0.2322	0.0022
25	-0.3173	0.0041
26	-0.2268	0.0021
27	-0.2010	0.0015
28	-0.1830	0.0010
29	-0.1706	0.0008
30	-0.3268	0.0047
31	-0.3382	0.0059
32	-0.3229	0.0046
33	-0.3236	0.0047
34	-0.2470	0.0029
35	-0.2427	0.0027
36	-0.2380	0.0024
37	-0.2347	0.0022
38	-0.2223	0.0017
39	-0.2355	0.0022
40	-0.2384	0.0024
41	-0.2457	0.0018
42	-0.2711	0.0031
43	-0.1982	0.0008
44	-0.2069	0.0014
45	-0.1618	0.0007
46	-0.1940	0.0012
47	-0.2184	0.0019
48	-0.2201	0.0017
49	-0.2258	0.0016
50	-0.2341	0.0023
51	-0.2187	0.0012
52	-0.2010	0.0028
53	-0.2143	0.0038
54	-0.2046	0.0021
55	-0.1885	0.0005
56	-0.2804	0.0029
57	-0.2895	0.0032

From the results of PLF analysis, it is observed that standard deviation in voltage was maximum for bus no.18 and was minimum in bus no.4. The standard deviation in angle was maximum for bus no.3 and was minimum for bus no.8. The PDF and CDF curves are shown for the buses with maximum and minimum standard deviation in angle and voltage.



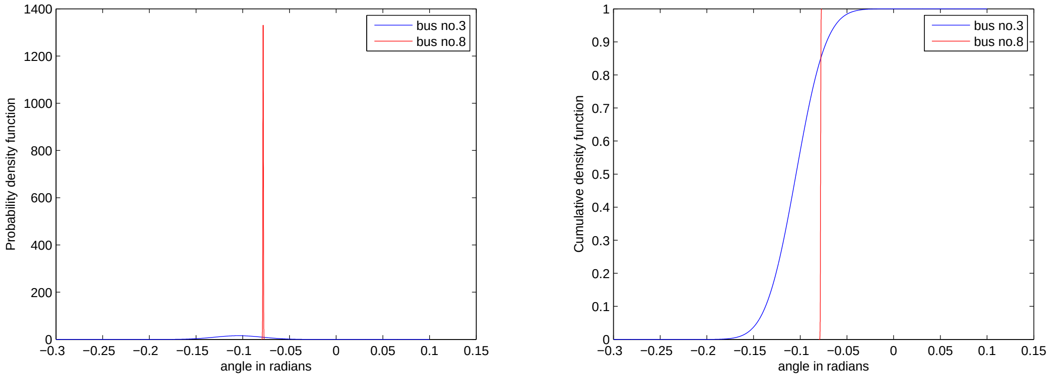
(a) PDF curves of voltage for bus no. 18 and 4 (b) CDF curves of voltage for bus no. 18 and 4

Figure 4.8: PDF and CDF curves of voltage for buses with maximum and minimum deviation for 57-bus system

From the curves (Fig 4.8), it can be observed that for bus no.18, the PDF has highest value 14 at the expected value of its voltage which is 1.0006 p.u.. The PDF and CDF has a non zero value within the range 0.9 p.u. to 1.1 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 1.0006 p.u.,the probability of occurence of these values decreases.

For bus no.4, the PDF has highest value 130 at the expected value of its voltage which is 0.9808 p.u.. The PDF and CDF has a non zero value within the range 0.97 p.u. to 0.993 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 0.9808 p.u.,the probability of occurence of these values decreases.

Thus,the PDF value for bus no.4 is higher than that for bus no.18 and range of random values is short in bus no.4 compared to bus no.18. This shows standard deviation in voltage for bus no.4 is lesser and more random values are likely to be around its expected value as compared to bus no.18.



(a) PDF curves of angle for bus no. 3 and 8

(b) CDF curves of angle for bus no. 3 and 8

Figure 4.9: PDF and CDF curves of angle for buses with maximum and minimum deviation for 57-bus system

From the curves (Fig 4.9), it can be observed that for bus no.3, the PDF has highest value approximately 15.9 at the expected value of its angle which is -0.1045 radians. The PDF and CDF has a non zero value within the range -0.15 radians to -0.05 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.1045 radians, the probability of occurence of these values decreases.

For bus no.8, the PDF has highest value approximately 1350 at the expected value of its angle which is -0.0782 radians. The PDF and CDF has a non zero value within the range -0.0793 radians to -0.0773 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.0782 radians,the probability of occurence of these values decreases.

Thus,the PDF value for bus no.8 is higher than that for bus no.3 and range of random values is short in bus no.8 compared to bus no.3. This shows standard deviation in angle for bus no.8 is lesser and more random values are likely to be around its expected value as compared to bus no.3.

4.2 *When loads are continuous and discrete*

4.2.1 *IEEE 14-Bus system*

IEEE 14 bus system data was taken.In this system, the load at the 9th bus was considered with discrete distribution while load at all other buses have been considered as normally distributed. (refer to Table A.4.1, Table A.4.2 and A.4.3).

4.2.1.1 Variation in both active and reactive power of loads

In this case, variation in both active and reactive loads was considered.The data has been referred in the appendix. At first deterministic load flow was run following NR method to obtain expected values of voltage and its angle at the buses. Following were the results obtained.

Table 4.10: Deterministic load flow results for 14-bus system with continuous and discrete loads with varying active and reactive power

Bus no.	Voltage(p.u.)	angle(radians)	Injected P(MW)	Injected Q(MVAr)
1	1.0600	0	232.4014	-16.4453
2	1.0450	-0.0870	18.3000	31.1949
3	1.0100	-0.2221	-94.2000	6.2831
4	1.0173	-0.1799	-47.8000	3.9000
5	1.0193	-0.1531	-7.6000	-1.6000
6	1.0700	-0.2485	-11.2000	6.1279
7	1.0604	-0.2330	0.0000	0.0000
8	1.0900	-0.2330	0	18.3226
9	1.0537	-0.2605	-29.5000	2.4000
10	1.0491	-0.2634	-9.0000	-5.8000
11	1.0560	-0.2582	-3.5000	-1.8000
12	1.0550	-0.2634	-6.1000	-1.6000
13	1.0501	-0.2648	-13.5000	-5.8000
14	1.0341	-0.2799	-14.9000	-5.0000

Hence, PLF analysis was applied through linearized load flow equations to study the variation in voltage and angle due to variation in load. Table 4.11 and Table 4.12 show the results obtained from PLF analysis.

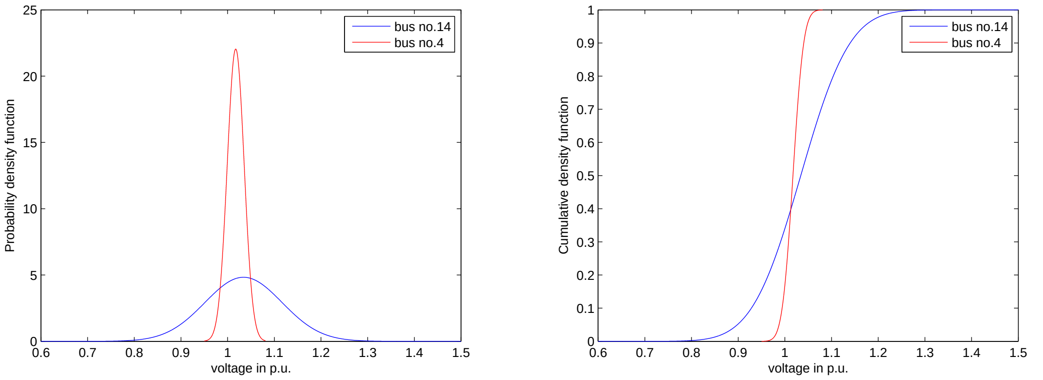
Table 4.11: PLF Results for voltage for 14-bus system with continuous and discrete loads with varying active and reactive power

Bus no.	Expected value(p.u.)	standard deviation
4	1.0173	0.0181
5	1.0193	0.0353
7	1.0604	0.0346
9	1.0537	0.0334
10	1.0491	0.0786
11	1.0560	0.0620
12	1.0550	0.0620
13	1.0501	0.0764
14	1.0341	0.0826

Table 4.12: PLF Results for angle for 14-bus system with continuous and discrete loads with varying active and reactive power

Bus no.	Expected value(radians)	standard deviation
2	-0.0870	0.0937
3	-0.2221	0.0857
4	-0.1799	0.0781
5	-0.1531	0.0884
6	-0.2485	0.0073
7	-0.2330	0.0067
8	-0.2330	0.0126
9	-0.2605	0.0245
10	-0.2634	0.0267
11	-0.2582	0.0217
12	-0.2634	0.0273
13	-0.2648	0.0164
14	-0.2799	0.0383

From the results of PLF analysis, it is observed that standard deviation in voltage is maximum for bus no.14 and is minimum for bus no.4. The standard deviation in angle was maximum for bus no. 2 and was minimum for bus no.7. The PDF and CDF curves for the buses with maximum and minimum standard deviation in voltage and its angle are shown.



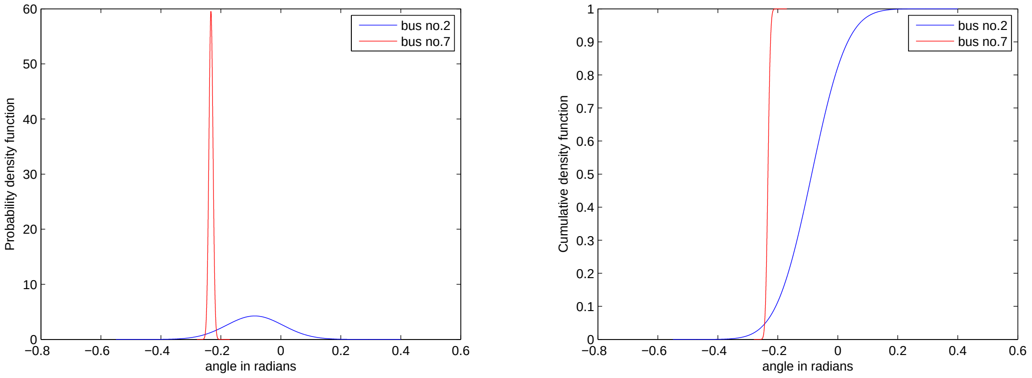
(a) PDF curves of voltage for bus no. 14 and 4

(b) CDF curves of voltage for bus no. 14 and 4

Figure 4.10: PDF and CDF curves of voltage for buses with maximum and minimum deviation for 14-bus system with continuous and discrete loads with varying active & reactive power

From the curves (Fig 4.10), it can be observed that for bus no.14, the PDF has highest value 4.9 at the expected value of its voltage which is 1.0341 p.u.. The PDF and CDF have non zero value within the range 0.8 p.u. to 1.3 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 1.0341 p.u.,the probability of occurence of these values decreases.

For bus no.4, the PDF has highest value 23 at the expected value of its voltage which is 1.0173 p.u.. The PDF and CDF has a non zero value within the range 0.96 p.u. to 1.08 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 1.0173 p.u.,the probability of occurence of these values decreases. Thus,the PDF value for bus no.4 is higher than that for bus no.14 and range of random values is short in bus no.4 compared to bus no.14. This shows standard deviation in voltage for bus no.4 is lesser and more random values are likely to be around its expected value as compared to bus no.14.



(a) PDF curves of angle for bus no. 2 and 7 (b) CDF curves of angle for bus no. 2 and 7

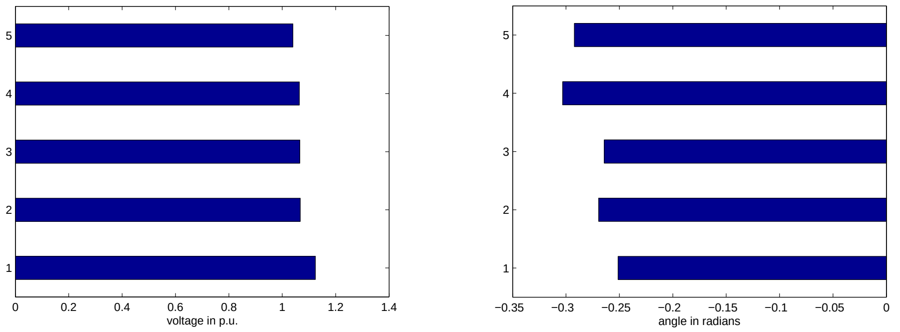
Figure 4.11: PDF and CDF curves of angle for buses with maximum and minimum deviation for 14-bus system with continuous and discrete loads with varying active & reactive power

From the curves (Fig 4.11), it can be observed that for bus no.2, the PDF has highest value approximately 4.3 at the expected value of its angle which is -0.0870 radians. The PDF and CDF has a non zero value within the range -0.3 radians to 0.2 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.0870 radians,the probability of occurence of these values decreases.

For bus no.7, the PDF has highest value approximately 59 at the expected value of its angle which is -0.2330 radians. The PDF and CDF has a non zero value within the range -0.26 radians to -0.21 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.2330 radians,the probability of occurence of these values decreases.

Thus,the PDF value for bus no.7 is higher than that for bus no.2 and range of random values is short in bus no.7 compared to bus no.2. This shows standard deviation in angle for bus no.7 is lesser and more random values are likely to be around its expected value as compared to bus no.2.

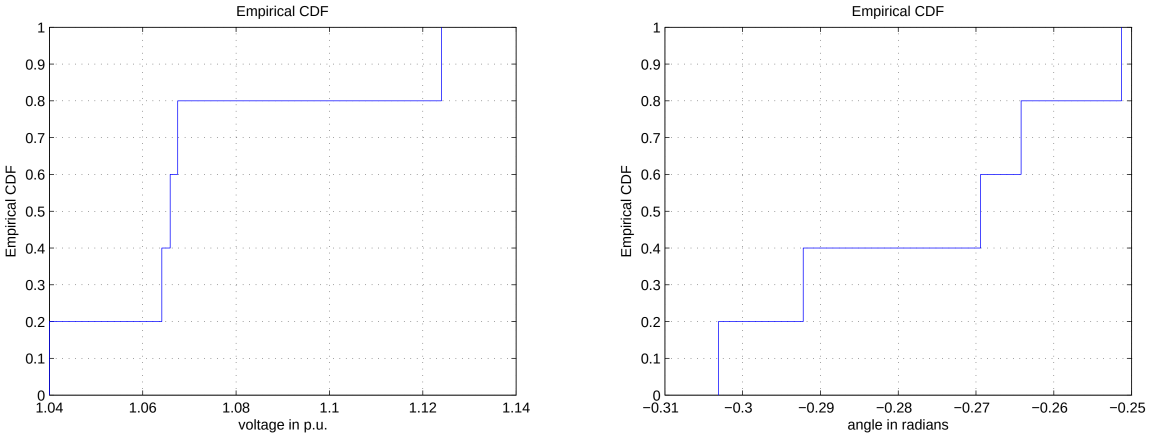
The load at 9th bus had discrete distribution. Hence, the bar graph plots are shown for bus voltage and its angle (Fig 4.12).



(a) Bar graph plot for voltage at bus no.9 (b) Bar graph plot for angle at bus no.9

Figure 4.12: Bargraph plot of bus voltage and its angle for bus no.9 for 14-bus system with continuous and discrete loads with varying active & reactive power

The empirical CDF plot for voltage and angle for 9th bus are shown (Fig 4.13).



(a) Empirical CDF plot for voltage at bus no.9 (b) Empirical CDF plot for angle at bus no.9

Figure 4.13: Empirical CDF plot of bus voltage and its angle for bus no.9 for 14-bus system with continuous and discrete loads with varying active & reactive power

The bar graphs show one bar for each discrete random value of voltage and angle respectively. The empirical CDF plots show the cumulative distribution function for the discrete voltage and angle.

The results obtained in the above analysis were compared to the results in [42] which are shown below. This PLF analysis was carried out using three different types of formulations in which linearized load flow equations were used. Thus standard deviations were obtained from the 3 formulations are represented by $\sigma 1$, $\sigma 2$, $\sigma 3$.

Table 4.13: PLF Results for voltage

Bus no.	$\sigma 1$	$\sigma 2$	$\sigma 3$
4	0.0009	0.0020	0.0009
5	0.0005	0.0016	0.0005
7	0.0025	0.0029	0.0027
9	0.0050	0.0052	0.0052
10	0.0042	0.0044	0.0044
11	0.0022	0.0023	0.0023
12	0.0005	0.0007	0.0005
13	0.0010	0.0012	0.0010
14	0.0033	0.0037	0.0034

Table 4.14: PLF Results for angle

Bus no.	$\sigma 1$	$\sigma 2$	$\sigma 3$
2	0.44	0.44	0.41
3	0.99	1.00	0.92
4	0.71	0.69	0.66
5	0.59	0.58	0.56
6	0.84	0.85	0.84
7	0.99	0.98	0.96
8	0.99	0.98	0.96
9	1.17	1.15	1.16
10	1.11	1.10	1.10
11	0.98	0.97	0.97
12	0.87	0.88	0.88
13	0.91	0.91	0.90
14	1.08	1.06	1.06

It is observed that in these results (refer to Table 4.13 and Table 4.14) in case of voltage, maximum standard deviation was in 9th bus and was minimum in 12th bus. In case of angle, the maximum, standard deviation was observed in 9th bus and minimum standard deviation was observed in bus no.2. For all the buses, standard deviation of voltage is less and it is more in case of angle when compared to the results which had been obtained in the present work(refer to Table 4.11 and Table 4.12).

4.2.1.2 Variation in only active power of the continuous loads and in both active and reactive power of discrete load

In this case, variation in only active power of the loads was considered for buses with continuous loads. The data has been referred in the appendix. In this data, standard deviation of injected reactive power was considered to be zero. At first deterministic load flow was run following NR method to obtain expected values of voltage and its angle at the buses.Hence, PLF analysis was applied through linearized load flow equations to study the variation in voltage and angle due to variation in active power requirement of load. Following were the results obtained.

Table 4.15: Deterministic load flow results for 14-bus system with continuous and discrete loads with varying active power of continuous loads

Bus no.	Voltage(p.u.)	angle(radians)	Injected P(MW)	Injected Q(MVAr)
1	1.0600	0	232.4014	-16.4453
2	1.0450	-0.0870	18.3000	31.1949
3	1.0100	-0.2221	-94.2000	6.2831
4	1.0173	-0.1799	-47.8000	3.9000
5	1.0193	-0.1531	-7.6000	-1.6000
6	1.0700	-0.2485	-11.2000	6.1279
7	1.0604	-0.2330	0.0000	0.0000
8	1.0900	-0.2330	0	18.3226
9	1.0537	-0.2605	-29.5000	2.4000
10	1.0491	-0.2634	-9.0000	-5.8000
11	1.0560	-0.2582	-3.5000	-1.8000
12	1.0550	-0.2634	-6.1000	-1.6000
13	1.0501	-0.2648	-13.5000	-5.8000
14	1.0341	-0.2799	-14.9000	-5.0000

Table 4.16: PLF Results for voltage for 14-bus system with continuous and discrete loads with varying active power of continuous loads

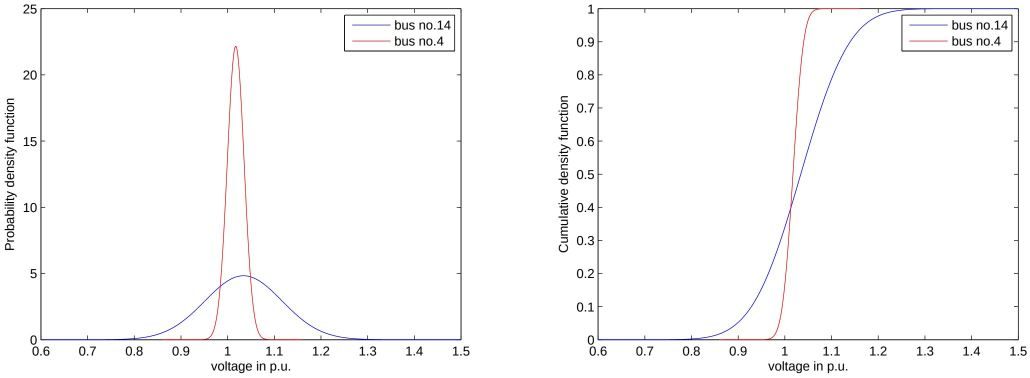
Bus no.	Expected value(p.u.)	Standard deviation
4	1.0173	0.0180
5	1.0193	0.0351
7	1.0604	0.0347
9	1.0537	0.0333
10	1.0491	0.0775
11	1.0560	0.0623
12	1.0550	0.0623
13	1.0501	0.0766
14	1.0341	0.0826

Table 4.17: PLF Results for angle for 14-bus system with continuous and discrete loads with varying active power of continuous loads

Bus no.	Expected value(radians)	Standard deviation
2	-0.0870	0.0927
3	-0.2221	0.0923
4	-0.1799	0.0822
5	-0.1531	0.0942
6	-0.2485	0.0058
7	-0.2330	0.0061
8	-0.2330	0.0091
9	-0.2605	0.0179
10	-0.2634	0.0157
11	-0.2582	0.0114
12	-0.2634	0.0112
13	-0.2648	0.0052
14	-0.2799	0.0152

From the results of PLF analysis, it is observed that standard deviation in voltage is maximum for bus no.14 and is minimum for bus no.4. The standard deviation in angle was maximum for bus no. 5 and was minimum for bus no.13. The PDF and CDF curves for the buses with maximum and minimum standard deviation in voltage and its angle are shown.

PDF and CDF curves of voltage for bus no.14 are shown.



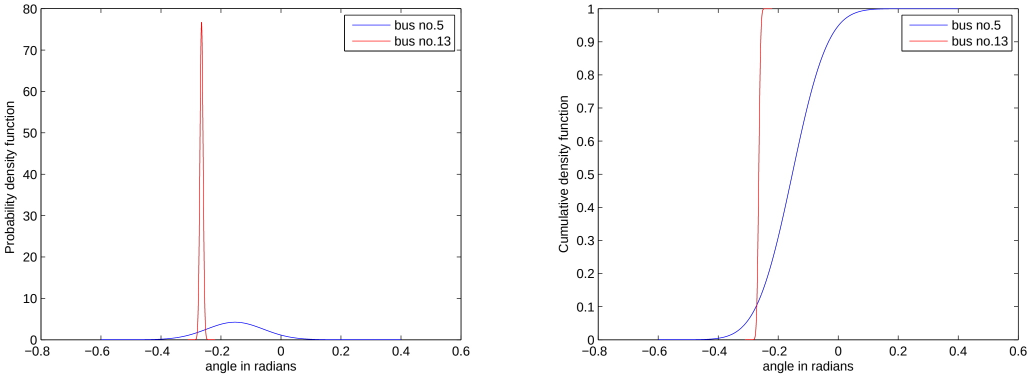
(a) PDF curves of voltage for bus no. 14 and 4

(b) CDF curves of voltage for bus no. 14 and 4

Figure 4.14: PDF and CDF curves of voltage for buses with maximum and minimum deviation for 14-bus system with continuous and discrete loads with varying active of continuous loads

From the curves (Fig 4.14), it can be observed that for bus no.14, the PDF has highest value 4.8 at the expected value of its voltage which is 1.0341 p.u.. The PDF and CDF have non zero value within the range 0.8 p.u. to 1.3 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 1.0341 p.u.,the probability of occurence of these values decreases.

For bus no.4, the PDF has highest value 23 at the expected value of its voltage which is 1.0173 p.u.. The PDF and CDF have non zero value within the range 0.95 p.u. to 1.08 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 1.0173 p.u.,the probability of occurence of these values decreases. Thus,the PDF value for bus no.4 is higher than that for bus no.14 and range of random values is short in bus no.4 compared to bus no.14. This shows standard deviation in voltage for bus no.4 is lesser and more random values are likely to be around its expected value as compared to bus no.14.



(a) PDF curves of angle for bus no. 5 and 13

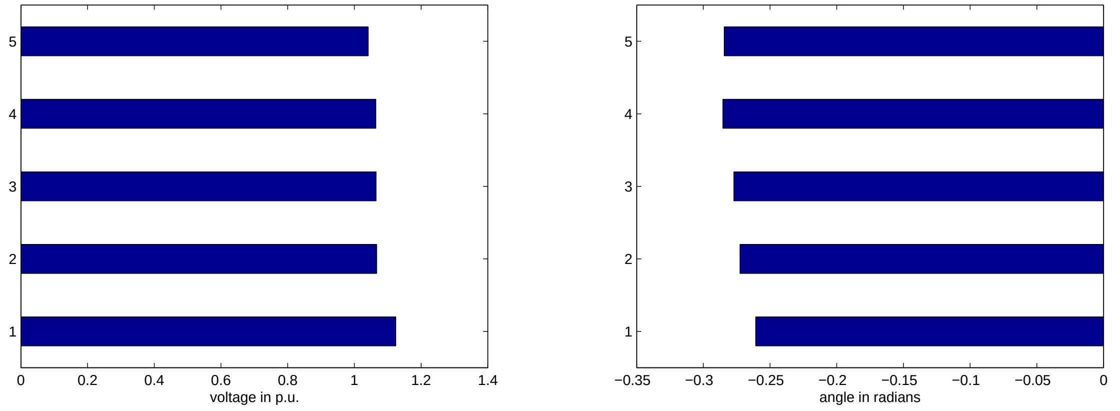
(b) CDF curves of angle for bus no. 5 and 13

Figure 4.15: PDF and CDF curves of angle for buses with maximum and minimum deviation for 14-bus system with continuous and discrete loads with varying active power of continuous loads

From the curves (Fig 4.15), it can be observed that for bus no.5, the PDF has highest value approximately 4.3 at the expected value of its angle which is -0.1531 radians. The PDF and CDF has a non zero value within the range -0.4 radians to 0.1 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.1531 radians,the probability of occurence of these values decreases.

For bus no.13, the PDF has highest value approximately 78 at the expected value of its angle which is -0.2648 radians. The PDF and CDF has a non zero value within the range -0.285 radians to -0.245 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.2648 radians,the probability of occurence of these values decreases. Thus,the PDF value for bus no.13 is higher than that for bus no.5 and range of random values is short in bus no.13 compared to bus no.5. This shows standard deviation in angle for bus no.13 is lesser and more random values are likely to be around its expected value as compared to bus no.5.

The load at 9th bus had discrete distribution. Hence, the bar graph plots are shown for bus voltage and its angle (Fig 4.16).

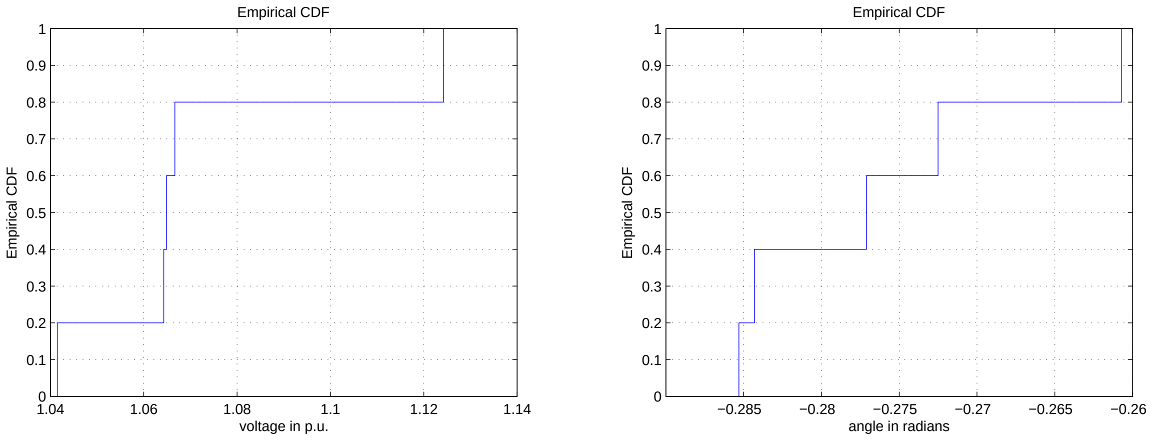


(a) Bar graph plot for voltage at bus no.9

(b) Bar graph plot for angle at bus no.9

Figure 4.16: Bargraph plot of bus voltage and its angle for bus no.9 for 14-bus system with continuous and discrete loads with varying active power of continuous loads

The empirical CDF plot for voltage and angle for 9th bus are shown (Fig 4.17).



(a) Empirical CDF plot for voltage at bus no.9

(b) Empirical CDF plot for angle at bus no.9

Figure 4.17: Empirical CDF plot of bus voltage and its angle for bus no.9 for 14-bus system with continuous and discrete loads with varying active power of continuous loads

The bar graphs show one bar for each discrete random value of voltage and angle respectively. The empirical CDF plots show the cumulative distribution function for the discrete voltage and angle.

4.2.1.3 Variation in only reactive power of continuous loads and in both active and reactive power of discrete load

In this case, variation in only reactive power of the loads was considered for buses with continuous loads. The data has been referred in the appendix. In this data, standard deviation of injected reactive power was considered to be zero. At first deterministic load flow was run following NR method to obtain expected values of voltage and its angle at the buses. Following were the results obtained.

Table 4.18: Deterministic load flow results for 14-bus system with continuous and discrete loads with varying reactive power of continuous loads

Bus no.	Voltage(p.u.)	angle(radians)	Injected P(MW)	Injected Q(MVAr)
1	1.0600	0	232.4014	-16.4453
2	1.0450	-0.0870	18.3000	31.1949
3	1.0100	-0.2221	-94.2000	6.2831
4	1.0173	-0.1799	-47.8000	3.9000
5	1.0193	-0.1531	-7.6000	-1.6000
6	1.0700	-0.2485	-11.2000	6.1279
7	1.0604	-0.2330	0.0000	0.0000
8	1.0900	-0.2330	0	18.3226
9	1.0537	-0.2605	-29.5000	2.4000
10	1.0491	-0.2634	-9.0000	-5.8000
11	1.0560	-0.2582	-3.5000	-1.8000
12	1.0550	-0.2634	-6.1000	-1.6000
13	1.0501	-0.2648	-13.5000	-5.8000
14	1.0341	-0.2799	-14.9000	-5.0000

Hence, PLF analysis was applied through linearized load flow equations to study the variation in voltage and angle due to variation in reactive power requirement of load.

Table 4.19: PLF Results for voltage for 14-bus system with continuous and discrete loads with varying reactive power of continuous loads

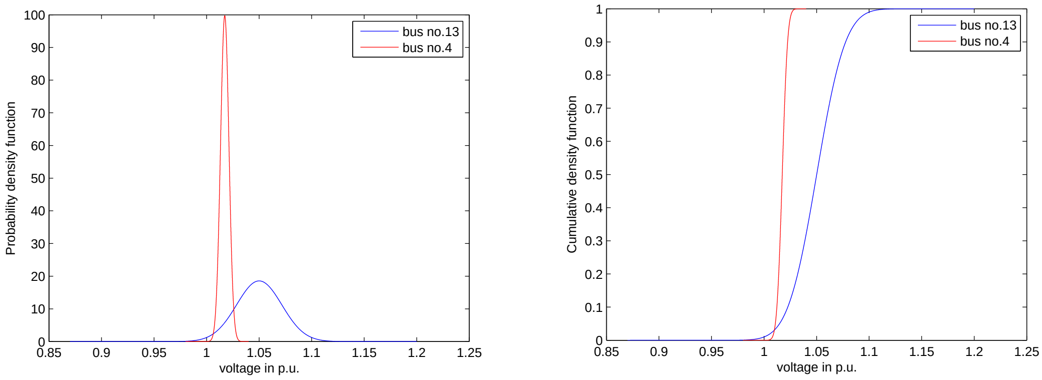
Bus no.	Expected value(p.u.)	Standard deviation
4	1.0173	0.0040
5	1.0193	0.0068
7	1.0604	0.0087
9	1.0537	0.0075
10	1.0491	0.0131
11	1.0560	0.0172
12	1.0550	0.0172
13	1.0501	0.0215
14	1.0341	0.0204

Table 4.20: PLF Results for angle for 14-bus system with continuous and discrete loads with varying reactive power of continuous loads

Bus no.	Expected value(radians)	Standard deviation
2	-0.0870	0.0176
3	-0.2221	0.0196
4	-0.1799	0.0155
5	-0.1531	0.0224
6	-0.2485	0.0061
7	-0.2330	0.0037
8	-0.2330	0.0090
9	-0.2605	0.0170
10	-0.2634	0.0150
11	-0.2582	0.0153
12	-0.2634	0.0136
13	-0.2648	0.0063
14	-0.2799	0.0162

From the results of PLF analysis, it is observed that standard deviation in voltage is maximum for bus no.13 and is minimum for bus no.4. The standard deviation in angle was maximum for bus no. 5 and was minimum for bus no.7. The PDF and CDF curves for the buses with maximum and minimum standard deviation in voltage and its angle are shown.

PDF and CDF curves of voltage for bus no.13 are shown.

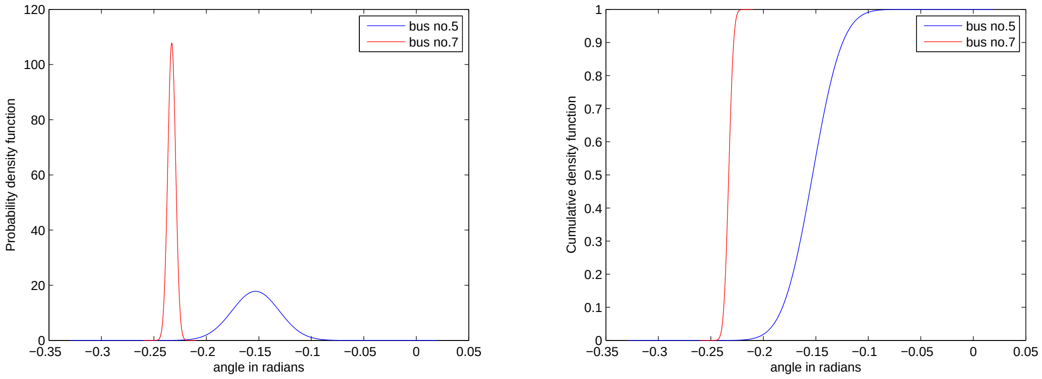


(a) PDF curves of voltage for bus no.13 and 4 (b) CDF curves of voltage for bus no. 13 and 4

Figure 4.18: PDF and CDF curves of voltage for buses with maximum and minimum deviation for 14-bus system with continuous and discrete loads with varying reactive power of continuous loads

From the curves (Fig 4.18), it can be observed that for bus no.13, the PDF has highest value 19 at the expected value of its voltage which is 1.0501 p.u.. The PDF and CDF have non zero value within the range 0.97 p.u. to 1.13 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 1.0501 p.u.,the probability of occurence of these values decreases.

For bus no.4, the PDF has highest value 100 at the expected value of its voltage which is 1.0173 p.u.. The PDF and CDF have non zero value within the range 1 p.u. to 1.035 p.u..Hence, the bus voltage will take up random values in this range. The voltage magnitude is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is 1.0173 p.u.,the probability of occurence of these values decreases. Thus,the PDF value for bus no.4 is higher than that for bus no.13 and range of random values is short in bus no.4 compared to bus no.13. This shows standard deviation in voltage for bus no.4 is lesser and more random values are likely to be around its expected value as compared to bus no.13.



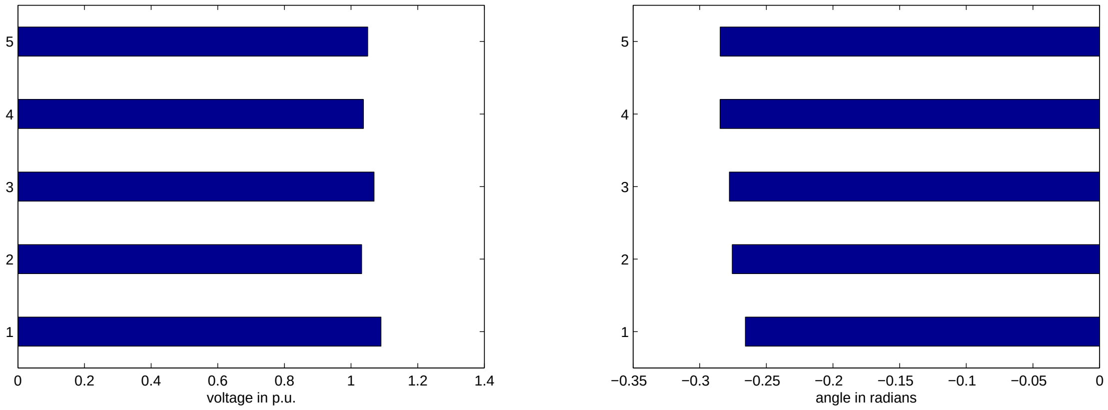
(a) PDF curves of angle for bus no. 5 and 7 (b) CDF curves of angle for bus no. 5 and 7

Figure 4.19: PDF and CDF curves of angle for buses with maximum and minimum deviation for 14-bus system with continuous and discrete loads with varying reactive power of continuous loads

From the curves (Fig 4.19), it can be observed that for bus no.5, the PDF has highest value approximately 17.9 at the expected value of its angle which is -0.1531 radians. The PDF and CDF has a non zero value within the range -0.23 radians to -0.07 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.1531 radians,the probability of occurence of these values decreases.

For bus no.7, the PDF has highest value approximately 110 at the expected value of its angle which is -0.2330 radians. The PDF and CDF has a non zero value within the range -0.24 radians to -0.07 radians. Hence, the angle will take up random values within this range. The angle is most likely to take up values around its expected value. As the random values either increase or decrease from expected value that is -0.2330 radians,the probability of occurence of these values decreases. Thus,the PDF value for bus no.7 is higher than that for bus no.5 and range of random values is short in bus no.7 compared to bus no.5. This shows standard deviation in angle for bus no.7 is lesser and more random values are likely to be around its expected value as compared to bus no.5.

The load at 9th bus had discrete distribution. Hence, Hence, the bar graph plots are shown for bus voltage and its angle (Fig 4.20).

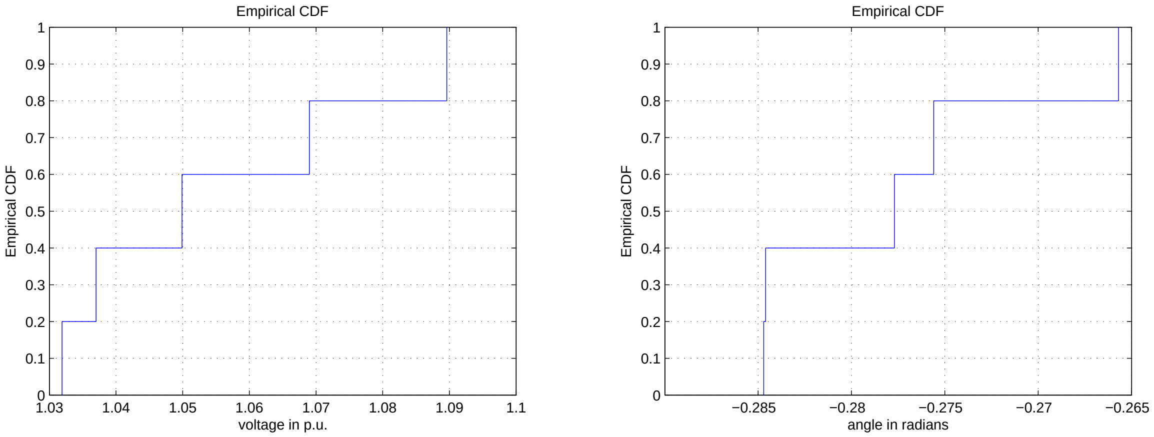


(a) Bar graph plot for voltage at bus no.9

(b) Bar graph plot for angle at bus no.9

Figure 4.20: Bargraph plot of bus voltage and its angle for bus no.9 for 14-bus system with continuous and discrete loads with varying reactive power of continuous loads

The empirical CDF plot for voltage and angle for 9th bus are shown (Fig 4.21).



(a) Empirical CDF plot for voltage at bus no.9

(b) Empirical CDF plot for angle at bus no.9

Figure 4.21: Empirical CDF plot of bus voltage and its angle for bus no.9 for 14-bus system with continuous and discrete loads with varying reactive power of continuous loads

The bar graphs show one bar for each discrete random value of voltage and angle respectively. The empirical CDF plots show the cumulative distribution function for the discrete voltage and angle.

Chapter 5

Conclusions and Future scope

5.1 *Conclusions*

It can be concluded that if there is variation in the injected active and reactive power at the buses,then it results into the variation in the bus voltage and its angle. The Probability density function obtained from the standard deviation with respect to the expected value shows this variation in the bus voltage and its angle. The buses for which the PDF value as observed from the PDF curves at the expected values of the voltage and its angle was very less compared to other buses, indicated that variation was maximum at that bus. The PDF and CDF curves can be used to check the probability of bus voltage and its angle in any specified range. Hence, most probable range of bus voltage and its angle can be found out. Thus, measures can be taken to check this variation so that equipments connected at the load end are least affected. Reactive power compensation can be provided at the buses with high voltage variation for voltage correction.

From the results the following can be concluded.

1. In 14-bus system when loads had continuous distribution, maximum variation in voltage was found in 5th bus. The variation in bus voltage angle was also maximum in case of 5th bus.
2. In 30-bus system loads had continuous distribution, variation in voltage was maximum for bus no.21 and variation in angle was maximum for bus no.6.
3. In 57-bus system loads had continuous distribution, the variation of voltage was maximum for bus no.18 and variation of bus voltage angle was maximum for bus no.3.
4. In 14-bus system, when load at one bus had discrete distribution and at other buses had continuous distribution,following are the conclusions for three cases.
    (a) When variation in both active and reactive power of load was considered, variation in voltage was maximum for bus no.14 and variation in bus voltage angle was maximum in case of bus no.2.
    (b) When variation was considered in only active power for continuous loads and both active and reactive power for discrete loads, variation in voltage was maximum for

bus no.14 having same magnitude of standard deviation as in first case and variation in bus voltage angle was maximum for bus no.5 with magnitude of standard deviation slightly higher than in first case where it was maximum for bus no.2

(c) When variation was considered in only reactive power for continuous loads and both active and reactive power for discrete load, variation in voltage was maximum for bus no.13 with magnitude of standard deviation lesser than in above two cases where it was maximum for 14th bus. Variation in bus voltage angle was maximum for bus no.5 but the magnitude of standard deviation was lesser than the first two cases.

5.2 *Scope for Future Work*

In future the following work is possible.

1. The results obtained from linearized model can be compared with Monte Carlo Simulation method.
2. The probabilistic load flow analysis can be extended to power system having FACTS devices.
3. Some of the buses can be connected to renewable sources of energy like wind and solar. Thus, variation in bus voltage and its angle can be studied with renewable energy system.

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Appendix

The bus types which have been used in the data for different bus systems are represented as following.

1. Type 1 is the slack bus.
2. Type 2 is the PV bus or generator bus.
3. Type 3 is the PQ bus or load bus.

A.1 *IEEE 14-Bus system data for continuous loads*

Table A.1.1: Bus data for IEEE 14-bus system

Bus no.	Type	Voltage(p.u.)	Angle(radians)	Injec.P(MW)	Injec.Q(MVAR)
1	1	1.06	0	0	0
2	2	1.045	0	18.3	-12.70
3	2	1.010	0	-94.20	-19
4	3	1	0	-47.80	3.90
5	3	1	0	-7.60	-1.60
6	2	1.070	0	-11.20	-7.50
7	3	1	0	0	0
8	2	1.090	0	0	0
9	3	1	0	-29.5	2.4
10	3	1	0	-9.00	-5.80
11	3	1	0	-3.50	-1.80
12	3	1	0	-6.10	-1.60
13	3	1	0	-13.50	-5.80
14	3	1	0	-14.90	-5

Table A.1.2: Line data for IEEE 14-bus system

From bus	To bus	R(p.u.)	X(p.u.)	B(p.u.)	tap setting
1	2	0.01938	0.05917	0.0528	1
1	5	0.05403	0.22304	0.0492	1
2	3	0.04699	0.19797	0.0438	1
2	4	0.05811	0.17632	0.034	1
2	5	0.05695	0.17388	0.0346	1
3	4	0.06701	0.17103	0.0128	1
4	5	0.01335	0.04211	0	1
4	7	0	0.20912	0	0.978
4	9	0	0.55618	0	0.969
5	6	0	0.25202	0	0.932
6	11	0.09498	0.19890	0	1
6	12	0.12291	0.25581	0	1
6	13	0.06615	0.13027	0	1
7	8	0	0.17615	0	1
7	9	0	0.11001	0	1
9	10	0.03181	0.08450	0	1
9	14	0.12711	0.27038	0	1
10	11	0.08205	0.19207	0	1
12	13	0.22092	0.19988	0	1
13	14	0.17093	0.34802	0	1

A.2 *IEEE 30-Bus system data for continuous loads*

Table A.2.1: Bus data for IEEE 30-bus system

Bus no.	Type	Voltage (p.u.)	Angle (radians)	P gen. (MW)	Pload (MW)	Qgen (MVAR)	Qload (MVAR)	G	B
1	1	1	0	23.54	0	0	0	0	0
2	2	1	0	60.97	21.7	0	12.7	0	0
3	3	1	0	0	2.4	0	1.2	0	0
4	3	1	0	0	7.6	0	1.6	0	0
5	3	1	0	0	0	0	0	0	0.19
6	3	1	0	0	0	0	0	0	0
7	3	1	0	0	22.8	0	10.9	0	0
8	3	1	0	0	30	0	30	0	0
9	3	1	0	0	0	0	0	0	0
10	3	1	0	0	5.8	0	2	0	0
11	3	1	0	0	0	0	0	0	0
12	3	1	0	0	11.2	0	7.5	0	0
13	2	1	0	37	0	0	0	0	0
14	3	1	0	0	6.2	0	1.6	0	0
15	3	1	0	0	8.2	0	2.5	0	0
16	3	1	0	0	3.5	0	1.8	0	0
17	3	1	0	0	9	0	5.8	0	0
18	3	1	0	0	3.2	0	0.9	0	0
19	3	1	0	0	9.5	0	3.4	0	0
20	3	1	0	0	2.2	0	0.7	0	0
21	3	1	0	0	17.5	0	11.2	0	0
22	2	1	0	21.59	0	0	0	0	0
23	2	1	0	19.2	3.2	0	1.6	0	0
24	3	1	0	0	8.7	0	6.7	0	0.04
25	3	1	0	0	0	0	0	0	0
26	3	1	0	0	3.5	0	2.3	0	0
27	2	1	0	26.91	0	0	0	0	0
28	3	1	0	0	0	0	0	0	0
29	3	1	0	0	2.4	0	0.9	0	0
30	3	1	0	0	10.6	0	1.9	0	0

Table A.2.2: Line data for IEEE 30-bus system for continuous loads

From bus	To bus	R(p.u.)	X(p.u.)	B(p.u.)	tap setting
1	2	0.02	0.06	0.03	1
1	3	0.05	0.19	0.02	1
2	4	0.06	0.17	0.02	1
3	4	0.01	0.04	0	1
2	5	0.05	0.2	0.02	1
2	6	0.06	0.18	0.02	1
4	6	0.01	0.04	0	1
5	7	0.05	0.12	0.01	1
6	7	0.03	0.08	0.01	1
6	8	0.01	0.04	0	1
6	9	0	0.21	0	1
6	10	0	0.56	0	1
9	11	0	0.21	0	1
9	10	0	0.11	0	1
4	12	0	0.26	0	1
12	13	0	0.14	0	1
12	14	0.12	0.26	0	1
12	15	0.07	0.13	0	1
12	16	0.09	0.2	0	1
14	15	0.22	0.2	0	1
16	17	0.08	0.19	0	1
15	18	0.11	0.22	0	1
18	19	0.06	0.13	0	1
19	20	0.03	0.07	0	1
10	20	0.09	0.21	0	1
10	17	0.03	0.08	0	1
10	21	0.03	0.07	0	1
10	22	0.07	0.15	0	1
21	22	0.01	0.02	0	1
15	23	0.1	0.2	0	1
22	24	0.12	0.18	0	1
23	24	0.13	0.27	0	1
24	25	0.19	0.33	0	1
25	26	0.25	0.38	0	1
25	27	0.11	0.21	0	1
28	27	0	0.4	0	1
27	29	0.22	0.42	0	1
27	30	0.32	0.6	0	1
29	30	0.24	0.45	0	1
8	28	0.06	0.2	0.02	1
6	28	0.02	0.06	0.01	1

A.3 *IEEE 57-Bus system data for continuous loads*

Table A.3.1: Bus data for IEEE 57-bus system for continuous loads

Bus no.	Type	Voltage (p.u.)	Angle (radians)	P gen. (MW)	Pload (MW)	Qgen (MVAR)	Qload (MVAR)	G	B
1	1	1.04	0	0	55	0	17	0	0
2	2	1.01	0	0	3	0	88	0	0
3	2	0.985	0	40	41	0	21	0	0
4	3	1	0	0	0	0	0	0	0
5	3	1	0	0	13	0	4	0	0
6	2	0.98	0	0	75	0	2	0	0
7	3	1	0	0	0	0	0	0	0
8	2	1.005	0	450	150	0	22	0	0
9	2	0.98	0	0	121	0	26	0	0
10	3	1	0	0	5	0	2	0	0
11	3	1	0	0	0	0	0	0	0
12	2	1.015	0	310	377	0	24	0	0
13	3	1	0	0	18	0	2.3	0	0
14	3	1	0	0	10.5	0	5.3	0	0
15	3	1	0	0	22	0	5	0	0
16	3	1	0	0	43	0	3	0	0
17	3	1	0	0	42	0	8	0	0
18	3	1	0	0	27.2	0	9.8	0	10
19	3	1	0	0	3.3	0	0.6	0	0
20	3	1	0	0	2.3	0	1	0	0
21	3	1	0	0	0	0	0	0	0
22	3	1	0	0	0	0	0	0	0
23	3	1	0	0	6.3	0	2.1	0	0
24	3	1	0	0	0	0	0	0	0
25	3	1	0	0	6.3	0	3.2	0	5.9
26	3	1	0	0	0	0	0	0	0
27	3	1	0	0	9.3	0	0.5	0	0
28	3	1	0	0	4.6	0	2.3	0	0
29	3	1	0	0	17	0	2.6	0	0
30	3	1	0	0	3.6	0	1.8	0	0
31	3	1	0	0	5.8	0	2.9	0	0
32	3	1	0	0	1.6	0	0.8	0	0
33	3	1	0	0	3.8	0	1.9	0	0
34	3	1	0	0	0	0	0	0	0

Bus no.	Type	Voltage (p.u.)	Angle (radians)	P gen. (MW)	Pload (MW)	Qgen (MVAR)	Qload (MVAR)	G	B
35	3	1	0	0	6	0	3	0	0
36	3	1	0	0	0	0	0	0	0
37	3	1	0	0	0	0	0	0	0
38	3	1	0	0	14	0	7	0	0
39	3	1	0	0	0	0	0	0	0
40	3	1	0	0	0	0	0	0	0
41	3	1	0	0	6.3	0	3	0	0
42	3	1	0	0	7.1	0	4.4	0	0
43	3	1	0	0	2	0	1	0	0
44	3	1	0	0	12	0	1.8	0	0
45	3	1	0	0	0	0	0	0	0
46	3	1	0	0	0	0	0	0	0
47	3	1	0	0	29.7	0	11.6	0	0
48	3	1	0	0	0	0	0	0	0
49	3	1	0	0	18	0	8.5	0	0
50	3	1	0	0	21	0	10.5	0	0
51	3	1	0	0	18	0	5.3	0	0
52	3	1	0	0	4.9	0	2.2	0	0
53	3	1	0	0	20	0	10	0	6.3
54	3	1	0	0	4.1	0	1.4	0	0
55	3	1	0	0	6.8	0	3.4	0	0
56	3	1	0	0	7.6	0	2.2	0	0
57	3	1	0	0	6.7	0	2	0	0

Table A.3.2: Line data for IEEE 57-bus system for continuous loads

From bus	To bus	R(p.u.)	X(p.u.)	B(p.u.)	tap setting
1	2	0.0083	0.028	0.129	1
2	3	0.0298	0.085	0.0818	1
3	4	0.0112	0.0366	0.038	1
4	5	0.0625	0.132	0.0258	1
4	6	0.043	0.148	0.0348	1
6	7	0.02	0.102	0.0276	1
6	8	0.0339	0.173	0.047	1
8	9	0.0099	0.0505	0.0548	1
9	10	0.0369	0.1679	0.044	1
9	11	0.0258	0.0848	0.0218	1
9	12	0.0648	0.295	0.0772	1
9	13	0.0481	0.158	0.0406	1
13	14	0.0132	0.0434	0.011	1
13	15	0.0269	0.0869	0.023	1
1	15	0.0178	0.091	0.0988	1
1	16	0.0454	0.206	0.0546	1
1	17	0.0238	0.108	0.0286	1
3	15	0.0162	0.053	0.0544	1
4	18	0	0.555	0	0.97
4	18	0	0.43	0	0.978
5	6	0.0302	0.0641	0.0124	1
7	8	0.0139	0.0712	0.0194	1
10	12	0.0277	0.1262	0.0328	1
11	13	0.0223	0.0732	0.0188	1
12	13	0.0178	0.058	0.0604	1
12	16	0.018	0.0813	0.0216	1
12	17	0.0397	0.179	0.0476	1
4	15	0.0171	0.0547	0.0148	1
18	19	0.461	0.685	0	1
19	20	0.283	0.434	0	1
21	20	0	0.7767	0	1.043
21	22	0.0736	0.117	0	1
22	23	0.0099	0.0152	0	1
23	24	0.166	0.256	0.0084	1
24	25	0	1.182	0	1
24	25	0	1.23	0	1
24	26	0	0.0473	0	1.043
26	27	0.165	0.254	0	1
27	28	0.0618	0.0954	0	1
28	29	0.0418	0.0587	0	1
7	29	0	0.0648	0	0.967

From bus	To bus	R(p.u.)	X(p.u.)	B(p.u.)	tap setting
25	30	0.135	0.202	0	1
30	31	0.326	0.497	0	1
31	32	0.507	0.755	0	1
32	33	0.0392	0.036	0	1
34	32	0	0.953	0	0.975
34	35	0.052	0.078	0.0032	1
35	36	0.043	0.0537	0.0016	1
36	37	0.029	0.0366	0	1
37	38	0.0651	0.1009	0.002	1
37	39	0.0239	0.0379	0	1
36	40	0.03	0.0466	0	1
22	38	0.0192	0.0295	0	1
11	41	0	0.749	0	0.955
41	42	0.207	0.352	0	1
41	43	0	0.412	0	1
38	44	0.0289	0.0585	0.002	1
15	45	0	0.1042	0	0.955
14	46	0	0.0735	0	0.9
46	47	0.023	0.068	0.0032	1
47	48	0.0182	0.0233	0	1
48	49	0.0834	0.129	0.0048	1
49	50	0.0801	0.128	0	1
50	51	0.1386	0.22	0	1
10	51	0	0.0712	0	0.93
13	49	0	0.191	0	0.895
29	52	0.1442	0.187	0	1
52	53	0.0762	0.0984	0	1
53	54	0.1878	0.232	0	1
54	55	0.1732	0.2265	0	1
11	43	0	0.153	0	0.958
44	45	0.0624	0.1242	0.004	1
40	56	0	1.195	0	0.958
56	41	0.553	0.549	0	1
56	42	0.2125	0.354	0	1
39	57	0	1.355	0	0.98
57	56	0.174	0.26	0	1
38	49	0.115	0.177	0.003	1
38	48	0.0312	0.0482	0	1
9	55	0	0.1205	0	0.94

A.4 *IEEE 14-Bus system data for continuous & discrete loads*

The following data for IEEE-14 bus was taken from [42].

Table A.4.1: Bus data for IEEE 14-bus system

Bus no.	Type	Voltage(p.u.)	Angle(radians)	Injec.P(MW)	P(σ)	Injec.Q(MVAR)	Q(σ)
1	1	1.06	0	0	0	0	0
2	2	1.045	0	18.3	9	-12.70	9.2
3	2	1.010	0	-94.20	10	-19	10.5
4	3	1	0	-47.80	11	3.90	9.7
5	3	1	0	-7.60	5	-1.60	5
6	2	1.070	0	-11.20	6	-7.50	6.3
7	3	1	0	0	0	0	0
8	2	1.090	0	0	0	0	0
9	3	1	0	-29.5	7.74	2.4	19.5
10	3	1	0	-9.00	10	-5.80	10
11	3	1	0	-3.50	9.50	-1.80	9.5
12	3	1	0	-6.10	7.60	-1.60	8.6
13	3	1	0	-13.50	10.50	-5.80	9.5
14	3	1	0	-14.90	8.60	-5	8.6

Bus data for 9th bus which is connected to discrete load has been shown in the table below which contains discrete data for injected active and reactive power. It is a load bus(PQ bus),hence the voltage was assumed to be 1 p.u. for applying load flow and bus angle was assumed to be 0 radian.

Table A.4.2: Bus data of 9th bus for IEEE 14-bus system

Injec.P(MW)	Injec.Q(MVAR)
-13.4	-7.5
-19.6	-11.0
-30.2	-17.0
-34.8	-19.6
-37.3	-21.0

Table A.4.3: Line data for IEEE 14-bus system

From bus	To bus	R(p.u.)	X(p.u.)	B(p.u.)	tap setting
1	2	0.01938	0.05917	0.0528	1
1	5	0.05403	0.22304	0.0492	1
2	3	0.04699	0.19797	0.0438	1
2	4	0.05811	0.17632	0.034	1
2	5	0.05695	0.17388	0.0346	1
3	4	0.06701	0.17103	0.0128	1
4	5	0.01335	0.04211	0	1
4	7	0	0.20912	0	0.978
4	9	0	0.55618	0	0.969
5	6	0	0.25202	0	0.932
6	11	0.09498	0.19890	0	1
6	12	0.12291	0.25581	0	1
6	13	0.06615	0.13027	0	1
7	8	0	0.17615	0	1
7	9	0	0.11001	0	1
9	10	0.03181	0.08450	0	1
9	14	0.12711	0.27038	0	1
10	11	0.08205	0.19207	0	1
12	13	0.22092	0.19988	0	1
13	14	0.17093	0.34802	0	1

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