

RELIABILITY ANALYSIS OF PISTON MANUFACTURING PLANT

*Thesis submitted in partial fulfillment of the requirement for
The award of the degree of
Masters of Science*

In

Mathematics and Computing

Submitted by

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Under

the guidance of

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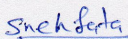
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CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled "**Reliability Analysis of Piston Manufacturing Plant**" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. A. K. Lal.

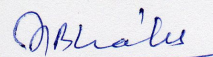
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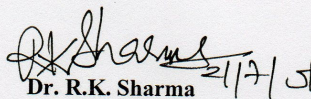

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.


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ABSTRACT

The Thesis entitled “RELIABILITY ANALYSIS OF PISTON MANUFACTURING PLANT” is an attempt to develop a mathematical model for analyzing the reliability of piston manufacturing plant. The piston is widely used in almost all the vehicles. This study has industrial significance to determine the quality characteristics for customers and is used for products, systems, processes and components of system.’

The Thesis has been divided into three chapters. The brief outline of the research work presented chapter wise in this thesis is as follows:

First chapter is introductory in nature. In this chapter, definition of reliability and basic concepts are discussed. A brief summary of literature available on the subject and summary of the work presented in the thesis also appears in this chapter.

In the second chapter, the reliability of piston manufacturing plant has been analyzed. The plant consists of fifteen sub-systems, namely Fixture Seat Machining, Rough Grooving and Turning, Rough Pin Hole Boring, Oil Hole Drilling, Finishing Grooving, Finish Profile Turning, Finish Pin Hole Boring, Finish Crown and Cavity, Valve Milling, Chamfering or RADIUSING, Circlip grooving, Deburring, Cleaning, Surface Treatment, Final Inspection. Rough Pin Hole Boring, Finish Pin Hole Boring and Finishing Grooving can also work in reduced state. The mathematical formulation of the problem is carried out using Markov-method. The Chapman-Kolomorgov differential equations thus developed and are solved by numerically to find out the reliability. The effect of constant failure and repair rates on the reliability has been studied in detail with the help of tables.

In third chapter, we have presented industrial significance of the present work. The limitations and future scope of the research work are also discussed in this chapter.

SYMBOLS USED IN THESIS

SYMBOLS	DESCRIPTION
α_i	Failure rates of the sub-systems
β_i	Repair rates of the sub-systems
$MTBF$	Mean time before failure
$R(t)$	Reliability at time t
X	Represents that the sub-system X is in good state
\bar{X}	Represents that the sub-system X is in reduced state
x	Represents that the sub-system X is in failed state
t	Total test time

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CHAPTER 1

INTRODUCTION

This chapter is introductory in nature. In this chapter, we firstly discuss the industrial significance for studying the reliability of process industry. Some important definitions are explained. In section 1.1 we present in brief the literature available on the subject. The summary of the present work is presented in 1.2. Finally, the definition and basic concepts of reliability are determined in section 1.3.

The study has been undertaken in an industry, which is situated in Punjab and manufactures pistons of almost all vehicles, compressors and engines available in India. The pistons are made with casting carried out in the foundry. Subsequently these castings are machined to finish pistons in the machine shop. The machining operation involves fifteen sub-systems. Firstly, fixture seat machining operation is performed for clamping the piston for other operations. After clamping the piston, rough grooves are made on the piston using the tool carbide tip. Two holes are made on the piston so that we can fit a pin in those holes. Further, the next machining operation is oil hole drilling, on this machine one hole is made on the piston which is used to pass the oil in piston. In the next operation, finishing is given to the rough grooves. In the next machining operation, ovality shape is given to the piston using carbide tool so that it can run smoothly in the cylinder. After that finishing is given to the rough pinhole portion using diamond tool. Next operation is finish crown and cavity, in these machine two operations are performed, first rough cavity is put and then finish cavity is put on the piston. After that valves are made on the piston using diamond tool and rounding off operation is performed on the corners of piston. In the next operation circlip grooves are made on the piston. In this operation, no coolant and no air pressure should be there. After that deburring, cleaning and surface treatment operations are performed. In the end, final inspection is done because some items may need operational tests before their packing.

1.1 Literature Review

Realizing the importance of the subject, reliability theory has been developed and a number of authors have been given a good contribution. In 1960, first textbook on reliability by Dummer and Griffen appeared in literature. Barlow and Hunter (1961) analyzed the reliability of a one-unit system. In India Professor R.S. Verma and his students Garg, Mohan and Singla (1962) at Delhi University initiated the study of reliability technology. Hosford (1960) considered a one unit system with two

states, namely, operating and failed, failure and repair being both of stationary type and found the probabilities of the system being in either state at an instant of time. Mohan and Garg (1962) considered a two unit system with general failure and exponential repair time distributions and solved the problem using the supplementary variable technique verifying the result of Hosford (1960) as special case. An extensive bibliography for these methods was given by Barlow, Hunter and Proschan (1967). Linton and Saw (1974) considered reliability analysis of the (k, n) system. Kapoor and Kapoor (1976) discussed the effect of standby redundancy on system reliability. Singh (1980) considered a similar system with imperfect switchover device. Arrarwal and Kumar (1980) considered a review of standby system. Kumar et al. (1991, 92) and Singh et al. (1991, 92) considered reliability of some complex industrial systems. Complexity also occurs in automobile systems. Such systems have been discussed by William (1959) and Dhilon et al. (1981). The earlier researchers in the field of reliability analyzed the system using Laplace transform. Most of these workers discussed the systems exhibiting Markovian properties. The systems having non-Markovian property can be converted to a system having markovian nature using some techniques, introducing some new variables called supplementary variables, Cox (1995) analyzed the non-Markovian systems using supplementary variables.

Fredrick and Hunziker (1990) have given details of butter industry. Lambert (2000) has considered the overview of small-scale milk collections and processing plant. Habchi (2002) discussed an improved method of reliability assessment for suspended tests and Gunes and Deveci (2002) have studied the reliability of service systems and its application in student office. When a system consists of more than one unit there is a possibility for complete failure of the system due to single cause. Such failures are termed as common cause failures and these are highly effective as far as reliability analysis of the system is considered. Gagloft (1975) defined the common cause failure in a system when multiple units or components at any instant fail due to single common cause. Flemming (1975) and Dhilon (1977) analyzed the problem of system reliability with identical units having statistically independent and dependent (common cause) failures. Cate et al. (1977) extended this idea to complex systems and developed a computer-based approach. Dhilon and Misra (1984) introduced the important factor of critical human error, which causes complete failure of the system. Dhilon (1980) provided extensive bibliography for human reliability. Chung (1990) extended the idea to a repairable system, which is subject to failures due to

common cause and critical human errors. Such systems are very common in our daily life. Pawan Gupta (2003) has discussed Mathematical analysis of reliability and availability of some process industries. Using numerical approach Pawan Gupta et al.(2005) investigated the behavioral study of the Cement manufacturing plant-A Pawaan et al. (2007) also analyse the Reliability and Availability of Serial Processes of Plastic - Pipe Manufacturing Plant-a case study using matrix calculus method.

In the present study a method is proposed to analyse reliability of Piston manufacturing plant situated in Patiala. A chapter wise summry of this work is presented in subsequent section.

1.2 Summary of the present work

In the present work, we have discussed the reliability of pistons manufacturing plant - Federal Mogul Company situated in Bahadurgarh near Patiala. The whole process consists of fifteen sub-systems, namely Fixture Seat Machining, Rough Grooving and Turning, Rough Pin Hole Boring, Oil Hole Drilling, Finish Grooving, Finish Profile Turning, Finish Pin Hole Boring, Finish Crown and Cavity, Valve Milling, Chamfering or Radiusing, Circlip grooving, Deburring, Cleaning, Surface Treatment and Final Inspection. Rough Pin Hole Boring, Finish Pin Hole Boring and Finishing Grooving can also work in reduced state. The mathematical model of this process industry has been formulated with the help of Markovian method and Chapman-Kolomorgov differential equations Assuming constant repair and failure rates of their sub-systems, the problem has been solved numerically to compute the reliability of the piston manufacturing plant. The effect of failure rates and repair rates on the reliability of the system has been studied in detail with the help of tables and graphs.

Before formulating the problem mathematically, we present in brief the basic concepts and relevant definition required in further analysis

1.3 Definition and basic concepts

Reliability is the probability of device performing its purpose adequately for the period intended under the given operating conditions. According to Srinath (2005), this definition brings into focus four important factors, namely,

- (a) The reliability of a device is expressed as a probability

- (b) The device is required to give sufficient performance
- (c) The duration of sufficient performance is specified
- (d) The environmental or operating conditions are prescribed.

These four factors are very important in the determination of reliability. We shall briefly discuss each one of these.

Reliability expressed as probability: Probability is the ratio of the number of times we can expect an event to occur to the total number of trials undertaken. Similarly a reliability factor of zero would mean that in almost all cases the equipment would fail to meet the required performance level. It is necessary to keep in mind that reliability factor of one would not mean that there is a guarantee that every unit of the device which is used will perform satisfactorily. It is possible that out of 100 trials, 1 or 2 units may fail, but the ratio of the number of failures to the number of success is expected to approach zero as the number of trials increases.

Sufficient performance: This is the second element in the definition of reliability. It describes in unambiguous terms, what is expected of a device or system. There may be instances, however, of a system giving satisfactorily performance even when one or two of its components may not be functioning. As an example, consider the spark-plugs of an eight-cylinder automobile. If one plug is not functioning properly, the automobile may still reach its destination in the prescribed time and thus meet the requirement of satisfactorily performance. On the other hand, its performance would be unsatisfactory there were a complete breakdown of the engine. Hence, a definite criterion must be established to clearly describe that what is considered to be satisfactorily performance.

Duration of sufficient performance: This is one of the most important elements in the definition since it represents a measure of the period for which the performance is satisfactory. Most units or systems fail when operated over long periods. Deterioration of materials and parts is natural and consequently the performance level of the unit will also go down with time. Therefore, it is necessary to specify a time limit within which the performance of a unit is to be assessed as being satisfactory or unsatisfactory.

Operating conditions: The environmental or operating conditions such as temperature, humidity, shock, vibration and so on should be specified in which we expect a device to function sufficiently. For example, an air conditioner which

performs satisfactorily in temperature zone may not have the same characteristics for hot and arid climate conditions.

Failure Rate: This is the ratio of the number of failures of the items undergoing test to the test time. Failure rate is expressed in terms of number of failures per unit time, for example, failures per hour or failures per day or failures per year. Failure rate is also known as the hazard rate. Sometimes, it is called the instantaneous failure rate.

Repair Rate: This is the ratio of the number of repairs of the items undergoing test to the test time. Repair rate is expressed in terms of number of repairs per unit time, for example, repairs per hour or repairs per day or repairs per year. Sometimes, it is called the instantaneous repair rate.

Mean Time before Failures: Mean time before failures, *MTBF* is the statistical average time before failure of the systems. When failure rate is fairly constant, then *MTBF* is defined as

$$MTBF = \int_0^{\infty} R(t)dt \quad (1)$$

Series System: In a series system the components are connected in such away that if any one of the components fails the entire system fails.

Let R_X, R_Y, R_Z denote the reliability of component X, Y and Z respectively. If the components are assumed to be independent, then the reliability of the system is given by

$$R_{system} = R_X \cdot R_Y \cdot R_Z \quad (2)$$

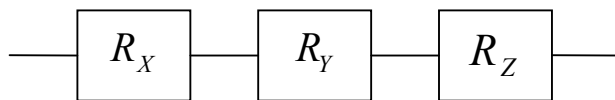


Fig. 1.1: Series system

Parallel System: A parallel system is a configuration such that, as long as not all of the system components fail, the entire system works. Conceptually, in a parallel configuration the total system reliability is higher than the reliability of any single system component. For three independent switches as shown in figure, we have eight possible outcomes.

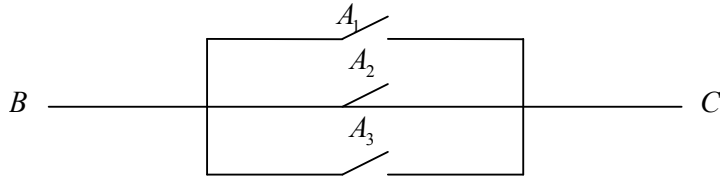


Fig. 1.2: Parallel system

Reliability (R_p) of this parallel system is given by the sum of the probabilities of realizing the seven events, i.e.

$$\begin{aligned}
 R_p &= R_1R_2R_3 + R_1R_2(1 - R_3) + R_1R_3(1 - R_2) + R_2R_3(1 - R_1) + R_1(1 - R_2)(1 - R_3) + \\
 &\quad R_2(1 - R_3)(1 - R_1) + R_3(1 - R_1)(1 - R_2) \\
 &= (1 - R_1)(1 - R_2)(1 - R_3) = 1 - \prod_{i=1}^3 (1 - R_i) \quad (3)
 \end{aligned}$$

Complex systems: These are the systems having more complex configuration than the series/parallel systems. Additional evaluation techniques are needed to determine the reliability of such systems.

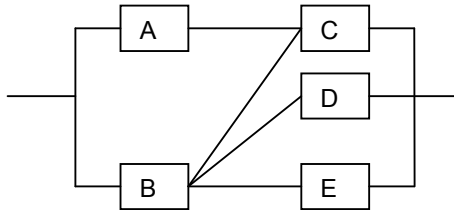


Fig.1.3: Complex system

CHAPTER 2

BEHAVIOUR ANALYSIS OF PISTON MANUFACTURING PLANT

In this chapter, the reliability of piston manufacturing plant has been analyzed with the actual data taken from the industry Federal Mogul situated in Patiala. In section 2.1, piston manufacturing plant has been discussed with the help of process flow chart. The assumptions taken into account are discussed in section 2.2. In section 2.3, we developed mathematical model determining reliability of the piston manufacturing plant. As this plant has a complex process structure, we have divided it into two parts and accordingly carried out mathematical formulation. In section 2.3.1, we discussed the mathematical formulation of the sub-system which gives the oval shapes of the desired piston and then the mathematical formulation of the sub-system which finally produces piston is discussed in section 2.3.2. The behavior study of the system has been presented in section 2.4. Finally, in section 2.5 certain conclusions based on the present study has been discussed with the help of graphs and tables.

The study has been undertaken in an industry, which is situated in Punjab and manufactures pistons of almost all vehicles, compressors and engines available in India. The pistons are made with casting carried out in the foundry. In the foundry shop, pistons are made with the help of semi-automatic die machines. The piston are produced by gravity die casting process i.e., no external pressure is applied during casting operation. The casted pistons are sent to cutting section for the cutting of risers and runners. Further, the casted pistons are checked for strength, hardness and foundry defects. A number of pistons are placed in piston blank and then the machining operations on the piston are started to make it complete product, it consists of following fifteen sub-systems. Firstly, fixture seat machining operation is performed for clamping the piston for other operations. After clamping the piston, rough grooves are made on the piston using the tool carbide tip. Two holes are made on the piston so that we can fit a pin in those holes. Further, the next machining operation is oil hole drilling, on this machine one hole is made on the piston which is used to pass the oil in piston. In the next operation, finishing is given to the rough grooves. In the next machining operation, ovality shape is given to the piston using carbide tool so that it can run smoothly in the cylinder. After that finishing is given to the rough pinhole portion using diamond tool. Next operation is finish crown and cavity, in these machine two operations are performed, first rough cavity is put and then finish cavity is put on the piston. After that valves are made on the piston using diamond tool and rounding off operation is performed on the corners of piston. In the next operation circlip grooves are made on the piston. In this operation, no coolant

and no air pressure should be there. After that deburring, cleaning and surface treatment operations are performed. In the end, final inspection is done because some items may need operational tests before their packing.

Rough Pin Hole Boring, Finish Pin Hole Boring and Finishing Grooving can also work in reduced state. Markov method is used for mathematical formulation. The Chapman-Kolomorgov differential equations thus developed and are solved by using Runge-Kutta fourth order method. The effect of failure rates and repair rates on the reliability has been studied in this chapter in detail using tables and graphs.

We are dividing the whole system into two sub-systems. The sub-system which gives the oval shapes of the piston is denoted by $R1$ and the sub-system which produces piston finally is denoted by $R2$. Further the sub-system $R1$ is divided into six sub-systems namely A, B, C, D, E and F and the sub-system $R2$ is divided into nine sub-systems namely G, H, I, J, K, L, M, N and O . The complete description of these symbols has been discussed below in section 2.1. The process flow chart of the piston manufacturing process is given below in figure 2.1.

2.1 System and Notations

2.1.1 System which gives oval shapes to the piston (R1)

1. Sub-system A (Fixture Seat Machining): On this machine, fixture seat operation is performed which is further used for the clamping of pistons while other operations are performed on the piston. The sub-system is subject to major failure.
2. Sub-system B (Rough Grooving and Turning): On this machine, rough grooves are made on piston, which are further used to tight the rings on it. Turning is also done on this machine i.e., to bring the dia of piston to proper size. The sub-system is subject to

major failure.

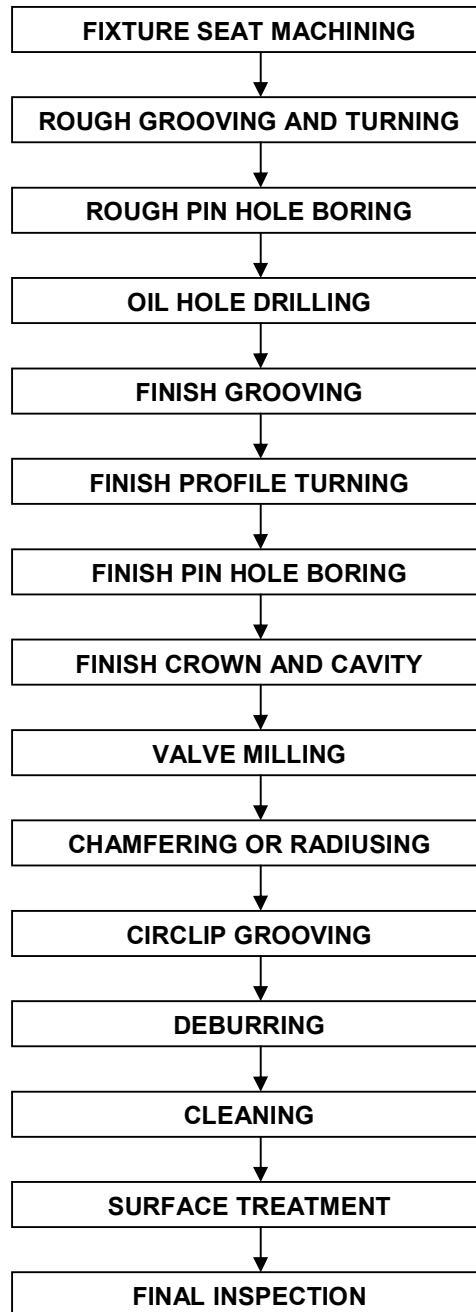


Fig 2.1: Flow chart of piston manufacturing

3. Sub-system *C* (Rough Pin Hole Boring): On this machine, pin hole boring is done on piston i.e., proper size is given to holes. That holes are used to fit the pin, by which rod is connected to crankshaft. The sub-system is subject to major and minor failure.
4. Sub-system *D* (Oil Hole Drilling): On this machine, one hole is done on piston,

through drilling, so that the oil can be supplied in piston. That oil is used to move piston in cylinder smoothly. The sub-system is subject to major failure.

5. Sub-system *E* (Finishing Grooving): On this machine, the finishing is given to rough grooves. The sub-system is subject to major and minor failure.

6. Sub-system *F* (Finish Profile Turning): This is the final overturning operation in which ovality is given to piston. As the piston operates at high temperature conditions, in the engine, it expands. To overcome expansion problems during working, oval shape is given to piston, so that it runs smoothly. The sub-system is subject to major failure.

2.1.2 System which produces piston (R2)

7. Sub-system *G* (Finish Pin Hole Boring): On this machine, the finish boring operation is done over the pinhole portion. On a single machine, work is done on two pistons. The sub-system is subject to major and minor failure.

8. Sub-system *H* (Finish Crown and Cavity): On this machine, finishing is given to the crown of piston i.e., the upper part of the piston. The sub-system is subject to major failure.

9. Sub-system *I* (Valve Milling): In this machine valve recess is made on the piston. Tool which is used for this purpose is diamond and at a single time work can be done on two pistons. The sub-system is subject to major failure.

10. Sub-system *J* (Chamfering or RADIUSING): On this machine, we round off the corners of the piston so that it can run smoothly in the cylinder. The sub-system is subject to major failure.

11. Sub-system *K* (Circlip grooving): On this machine, circlip grooves are made on the piston. The sub-system is subject to major failure.

12. Sub-system *L* (Deburring): On this machine, deburring or we can say brushing operation is performed on piston. As such, it can be considered that it never fails.

13. Sub-system *M* (Cleaning): On this machine, outside and inside cleaning of the piston is done. We perform the cleaning operation because no extra material should be on piston before surface treatment. The sub-system is subject to major failure.

14. Sub-system *N* (Surface Treatment): The next operation is surface treatment in which the Phosphate Coating Graphite Coating, Tin Coating, Anodizing & Moly-Coating of the piston is done. It is considered as never failed sub-system.

15. Sub-system *O* (Final Inspection): After the surface treatment, the final inspection of the piston is done. Some items may need operational tests before being packed. This type of Inspection is very essential to insure that no faulty product is dispatched to the customer, which otherwise will bring down to reputation, goodwill & hence the market share of manufacturer.

2.2 Assumptions

We are dividing the whole system into two sub-systems *R1* and *R2*. Further *R1* is divided into six sub-systems namely *A, B, C, D, E* and *F*. *R2* is divided into nine sub-systems namely *G, H, I, J, K* and *L, M, N, O*.

We are using the following notations for sub-system *R1*:

A, B, C, D, E and *F* denote good conditions of the sub-systems *R1*. \bar{C} and \bar{E} indicate that sub-systems *C* and *E* are working in reduced states.

α_i (i=1 to 8) represents respective constant failure rates of sub-systems *A, B, D, F, \bar{C} , \bar{E} , C, E*.

β_i (i=1 to 8) represents respective constant repair rates of sub-systems *A, B, D, F, \bar{C} , \bar{E} , C, E*.

$P_j(t)$ = Probability that system is in j^{th} state at time *t*, (j=1, 2, 3, ..., 24).

Small letters *a, b, c, d, e* and *f* indicate the failed states of the sub-systems *A, B, C, D, E* and *F* respectively.

We are using the following notations for sub-system *R2*:

G, H, I, J, K and *M* denotes good conditions of the sub-systems *R2*. \bar{G} indicate that sub-system *G* is working in reduced state.

α_i (i=1 to 7) represents respective constant failure rates of sub-systems *H, I, J, K, M, \bar{G} , G*.

β_i (i=1 to 7) represents respective constant repair rates of sub-systems *H, I, J, K, M, \bar{G} , G*.

$P_j(t)$ = Probability that system is in j^{th} state at time *t*, (j=1, 2, 3, ..., 13).

Small letters *g, h, i, j, k* and *m* indicate the failed states of the sub-systems *G, H, I, J, K* and *M* respectively.

Assumptions for the sub-systems R1 and R2 are as follows:

- (i) Failure and repair rates are independent with each other and their unit is per hour.
- (ii) There are no simultaneous failures among the sub-systems.
- (iii) Sub-systems C and E and G fail only through reduced states.
- (iv) Repair of C and E and G in reduced state does not make these sub-systems completely functional.
- (v) Switching to stand by components is perfect.
- (vi) The sub-systems *L*, *N* and *O* are considered as never failed sub-systems.

Following the above notations and assumptions the transition diagram of the sub-systems R1 and R2 are given in Figure 2.2 and 2.3.

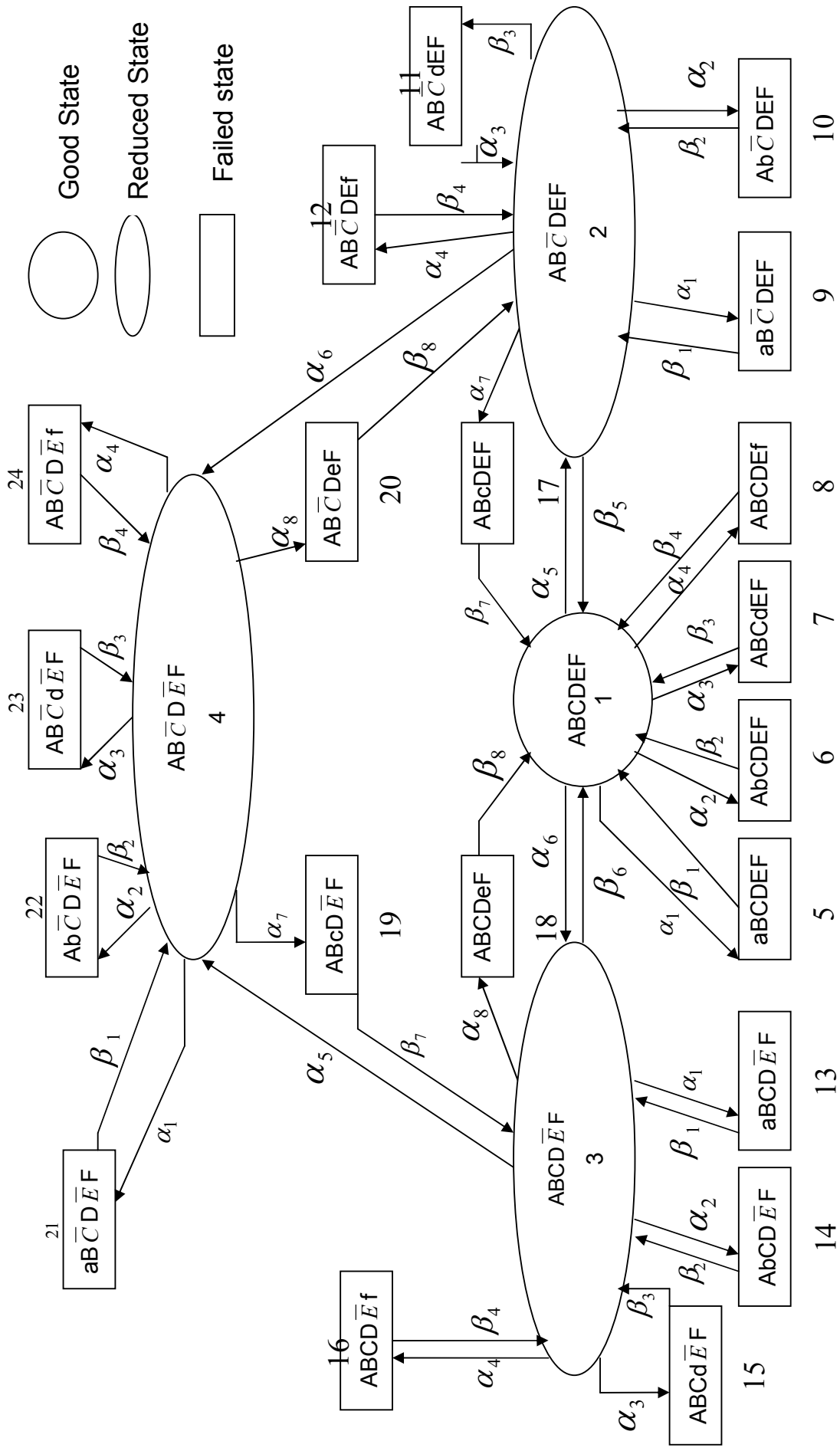


Fig 2.2: Transition Diagram for sub-system R1

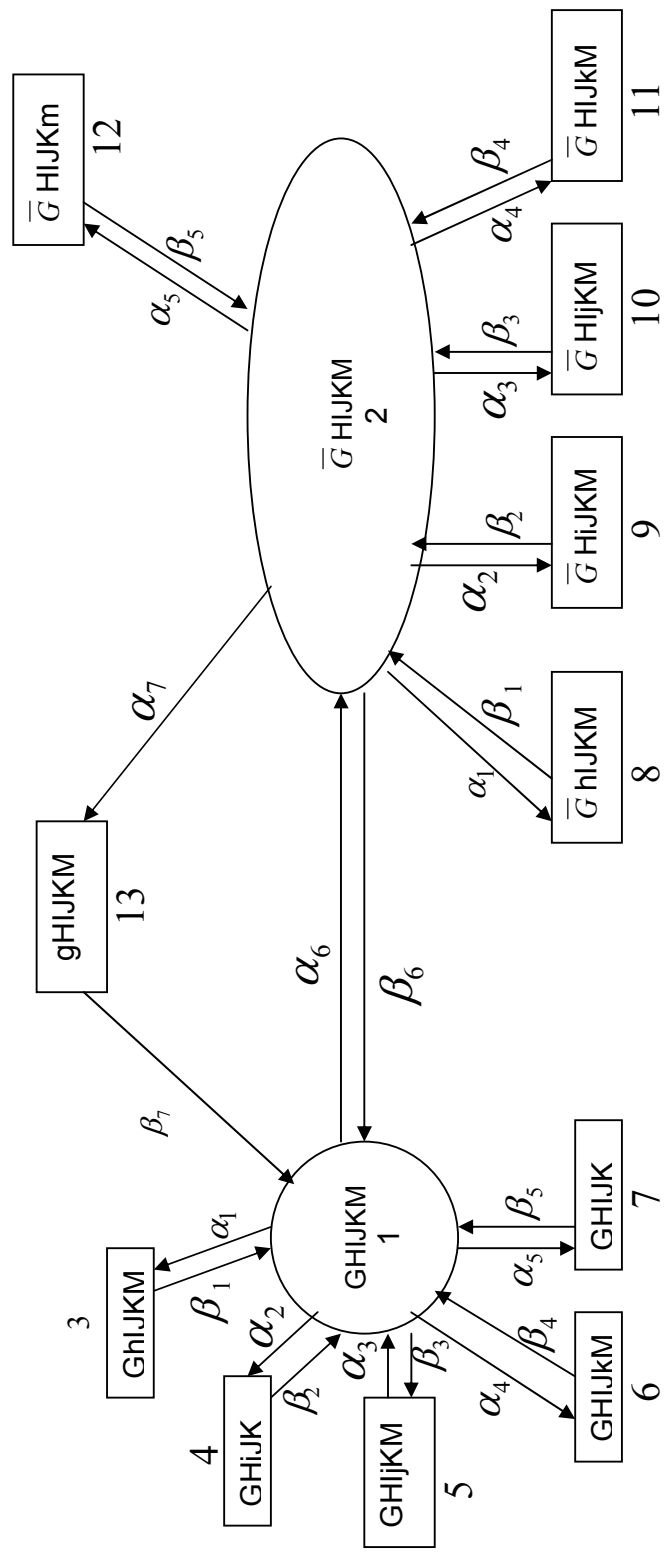
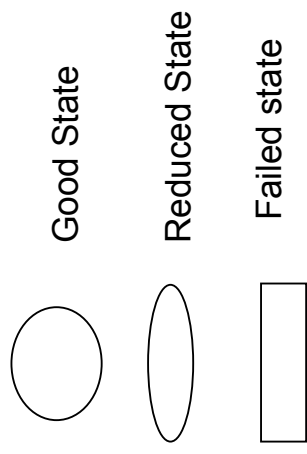


Fig: 2.3 Transition Diagram for sub-system R2

2.3 Mathematical model of piston manufacturing plant

2.3.1 Mathematical Formulation of the sub-system R1

In the transient state, probability considerations give the following first order differential equations associated with the transition diagram of the system.

$$P_1'(t) + \lambda_1 P_1(t) = \beta_1 P_5(t) + \beta_2 P_6(t) + \beta_3 P_7(t) + \beta_4 P_8(t) + \beta_5 P_2(t) + \beta_6 P_3(t) + \beta_7 P_{17}(t) + \beta_8 P_{18}(t) \quad (2.1)$$

$$P_2'(t) + \lambda_2 P_2(t) = \beta_1 P_9(t) + \beta_2 P_{10}(t) + \beta_3 P_{11}(t) + \beta_4 P_{12}(t) + \beta_8 P_{20}(t) + \alpha_5 P_1(t) \quad (2.2)$$

$$P_3'(t) + \lambda_3 P_3(t) = \beta_1 P_{13}(t) + \beta_2 P_{14}(t) + \beta_3 P_{15}(t) + \beta_4 P_{16}(t) + \beta_7 P_{19}(t) + \alpha_6 P_1(t) \quad (2.3)$$

$$P_4'(t) + \lambda_4 P_4(t) = \beta_1 P_{21}(t) + \beta_2 P_{22}(t) + \beta_3 P_{23}(t) + \beta_4 P_{24}(t) + \alpha_5 P_3(t) + \alpha_6 P_2(t) \quad (2.4)$$

$$P_5'(t) + \beta_1 P_5(t) = \alpha_1 P_1(t) \quad (2.5)$$

$$P_6'(t) + \beta_2 P_6(t) = \alpha_2 P_1(t) \quad (2.6)$$

$$P_7'(t) + \beta_3 P_7(t) = \alpha_3 P_1(t) \quad (2.7)$$

$$P_8'(t) + \beta_4 P_8(t) = \alpha_4 P_1(t) \quad (2.8)$$

$$P_9'(t) + \beta_1 P_9(t) = \alpha_1 P_2(t) \quad (2.9)$$

$$P_{10}'(t) + \beta_2 P_{10}(t) = \alpha_2 P_2(t) \quad (2.10)$$

$$P_{11}'(t) + \beta_3 P_{11}(t) = \alpha_3 P_2(t) \quad (2.11)$$

$$P_{12}'(t) + \beta_4 P_{12}(t) = \alpha_4 P_2(t) \quad (2.12)$$

$$P_{13}'(t) + \beta_1 P_{13}(t) = \alpha_1 P_3(t) \quad (2.13)$$

$$P_{14}'(t) + \beta_2 P_{14}(t) = \alpha_2 P_3(t) \quad (2.14)$$

$$P_{15}'(t) + \beta_3 P_{15}(t) = \alpha_3 P_3(t) \quad (2.15)$$

$$P_{16}'(t) + \beta_4 P_{16}(t) = \alpha_4 P_3(t) \quad (2.16)$$

$$P_{17}'(t) + \beta_7 P_{17}(t) = \alpha_7 P_2(t) \quad (2.17)$$

$$P_{18}'(t) + \beta_8 P_{18}(t) = \alpha_8 P_3(t) \quad (2.18)$$

$$P_{19}'(t) + \beta_7 P_{19}(t) = \alpha_7 P_4(t) \quad (2.19)$$

$$P_{20}'(t) + \beta_8 P_{20}(t) = \alpha_8 P_4(t) \quad (2.20)$$

$$P_{21}'(t) + \beta_1 P_{21}(t) = \alpha_1 P_4(t) \quad (2.21)$$

$$P_{22}'(t) + \beta_2 P_{22}(t) = \alpha_2 P_4(t) \quad (2.22)$$

$$P_{23}'(t) + \beta_3 P_{23}(t) = \alpha_3 P_4(t) \quad (2.23)$$

$$P'_{24}(t) + \beta_4 P_{24}(t) = \alpha_4 P_4(t) \quad (2.24)$$

Where,

$$\lambda_1 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 \quad (2.25)$$

$$\lambda_2 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_6 + \alpha_7 + \beta_5 \quad (2.26)$$

$$\lambda_3 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_8 + \beta_6 \quad (2.27)$$

$$\lambda_4 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_7 + \alpha_8 \quad (2.28)$$

with initial conditions $P_1(0)=1$ and 0, otherwise.

As the system of differential equations (2.1-2.28) is very complex, it is difficult to find its analytical solution, particularly using Laplace transformation method. Therefore, above equations have been solved numerically earlier used by Pawan et al (2005, 2007). The numerical integration is carried out with $t = 0.005$ assuming it as one day. Thus in order to get the reliability upto 360 hours for various combinations of the failure rates and repair rates, we have performed 72000 iterations in the Runge-Kutta Fourth Order Method. The reliability function $R(t)$ of the system is finally computed by

$$R(t) = \sum_{i=1}^4 P_i(t) \quad (2.29)$$

The effects of failure and repairs rates of the subsystem on the Reliability of the system have been computed and presented in tables 2.1(a)- 2.1(l).

2.3.2 Mathematical Formulation of the sub-system R2

Keeping in view the notations, assumptions and probability considerations of the systems R2 in transient state, the system of differential equations associated with the transition diagram of the system. (fig 2.2) is given by

$$P'_1(t) + \lambda_1 P_1(t) = \beta_1 P_3(t) + \beta_2 P_4(t) + \beta_3 P_5(t) + \beta_4 P_6(t) + \beta_5 P_7(t) + \beta_6 P_2(t) + \beta_7 P_{13}(t) \quad (3.1)$$

$$P'_2(t) + \lambda_2 P_2(t) = \beta_1 P_8(t) + \beta_2 P_9(t) + \beta_3 P_{10}(t) + \beta_4 P_{11}(t) + \beta_5 P_{12}(t) + \alpha_6 P_1(t) \quad (3.2)$$

$$P'_3(t) + \beta_1 P_3(t) = \alpha_1 P_1(t) \quad (3.3)$$

$$P'_4(t) + \beta_2 P_4(t) = \alpha_2 P_1(t) \quad (3.4)$$

$$P'_5(t) + \beta_3 P_5(t) = \alpha_3 P_1(t) \quad (3.5)$$

$$P'_6(t) + \beta_4 P_6(t) = \alpha_4 P_1(t) \quad (3.6)$$

$$P_7'(t) + \beta_5 P_7(t) = \alpha_5 P_1(t) \quad (3.7)$$

$$P_8'(t) + \beta_1 P_8(t) = \alpha_1 P_2(t) \quad (3.8)$$

$$P_9'(t) + \beta_2 P_9(t) = \alpha_2 P_2(t) \quad (3.9)$$

$$P_{10}'(t) + \beta_3 P_{10}(t) = \alpha_3 P_2(t) \quad (3.10)$$

$$P_{11}'(t) + \beta_4 P_{11}(t) = \alpha_4 P_2(t) \quad (3.11)$$

$$P_{12}'(t) + \beta_5 P_{12}(t) = \alpha_5 P_2(t) \quad (3.12)$$

$$P_{13}'(t) + \beta_7 P_{13}(t) = \alpha_7 P_2(t) \quad (3.13)$$

Where,

$$\lambda_1 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 \quad (3.14)$$

$$\lambda_2 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_7 + \beta_6 \quad (3.15)$$

with initial conditions $P_1(0)=1$ and 0, otherwise.

The system of differential equations (3.1 - 3.15) together with boundary conditions have been solved numerically in same manner as discussed in section 2.3. The reliability function $R(t)$ of the system (R2) is computed by

$$R(t) = \sum_{i=1}^2 P_i(t)$$

for various values of the failure rates and repair rates and their effects on reliability are presented in tables 2.2(a)-2.2(l).

2.4 Behaviour Study

2.4.1 Behaviour Study of sub-system R1

The system of above differential equations with initial conditions has been solved numerically using Runge-Kutta Fourth Order Method. The computations have been performed upto 360 hours for various values of failure and repair rates of the sub-systems, by taking the data for failure rates and repair rates of the sub-systems as per hour. The effect of various parameters on reliability of R1 is studied in this section. If the failure and repair rates are altered, the reliability is affected.

(i) Effect of failure rate of fixture seat machining on reliability of the system: The reliability of the system is computed by using the following combinations of failure and repair rates. Reliability is presented in Table 2.1(a) The parameter α_1 is

considered at the level of 0.0014, 0.0054, 0.0104, 0.0154, 0.0204 and other parameters are taken as: $\alpha_2= 0.0006$, $\alpha_3= 0.0009$, $\alpha_4= 0.0009$, $\alpha_5= 0.0208$, $\alpha_6= 0.0208$, $\alpha_7= 0.0009$, $\alpha_8= 0.0039$, $\beta_1= 1.08$, $\beta_2=0.043$, $\beta_3= 0.5$, $\beta_4= 0.286$, $\beta_5= 0.154$, $\beta_6= 0.25$, $\beta_7= 0.059$, $\beta_8= 0.087$. The table shows the effect of fixture seat machining (α_1) on the reliability of the system. The reliability of the system decreases by approximately 1.02% with the increase in failure rate of fixture seat machining from 0.0014 to 0.0204 and by approximately 0.4% with the increase in time from 30 to 360 hours.

Table 2.1(a): EFFECT OF FAILURE RATE OF FIXTURE SEAT MACHINING (α_1) ON RELIABILITY OF THE SYSTEM

$\alpha_1 \rightarrow$ Hours \downarrow	.0014	.0054	.0104	0.154	0.0204
30	0.977684	0.974165	0.969802	0.965478	0.961193
60	0.970475	0.967020	0.962736	0.958489	0.954279
90	0.966324	0.962905	0.958666	0.954463	0.950297
120	0.963442	0.960046	0.955836	0.951662	0.947523
150	0.961268	0.957889	0.953698	0.949543	0.945424
180	0.959577	0.956209	0.952032	0.947890	0.943785
210	0.958248	0.954887	0.950718	0.946587	0.942490
240	0.957198	0.953842	0.949679	0.945554	0.941464
270	0.956368	0.953015	0.948857	0.944736	0.940649
300	0.955712	0.952361	0.948206	0.944087	0.940003
330	0.955193	0.951843	0.947689	0.943572	0.939489
360	0.954782	0.951433	0.947279	0.943162	0.939081
MTBF	307.5288	306.4458	305.1029	303.7716	302.4518

(ii) Effect of failure rate of rough grooving and turning on reliability of the system: Effect of failure rate of sub-system, rough grooving and turning on the reliability of the system is studied by varying its values as $\alpha_2= 0.0006$, 0.0007, 0.0008, 0.0009, 0.0010 and other parameters are taken as: $\alpha_1= 0.0014$, $\alpha_3= 0.0009$, $\alpha_4= 0.0009$, $\alpha_5= 0.0208$, $\alpha_6= 0.0208$, $\alpha_7= 0.0009$, $\alpha_8= 0.0039$, $\beta_1= 1.08$, $\beta_2= 0.043$, $\beta_3= 0.5$, $\beta_4= 0.286$, $\beta_5= 0.154$, $\beta_6= 0.25$, $\beta_7= 0.059$, $\beta_8= 0.087$. Table

2.1(b) shows the effect of rough grooving and turning (α_2) on the reliability of the system. The reliability of the system decreases by approximately 0.2% with the increase in failure rate from 0.0006 to 0.0010 but it decreases by approximately 0.5% with the increase in time from 30 to 360 hours.

Table 2.1(b): EFFECT OF FAILURE RATE OF ROUGH GROOVING AND TURNING (α_2) ON RELIABILITY OF THE SYSTEM

$\alpha_2 \rightarrow$ Hours \downarrow	0.0006	0.0007	0.0008	0.0009	0.0010
30	0.977684	0.976059	0.974438	0.972821	0.971208
60	0.970475	0.968444	0.966420	0.964404	0.962395
90	0.966324	0.964202	0.962088	0.959983	0.957887
120	0.963442	0.961307	0.959181	0.957065	0.954957
150	0.961268	0.959138	0.957018	0.954906	0.952804
180	0.959577	0.957455	0.955341	0.953237	0.951142
210	0.958248	0.956131	0.954023	0.951925	0.949836
240	0.957198	0.955085	0.952982	0.950888	0.948803
270	0.956368	0.954258	0.952158	0.950066	0.947985
300	0.955712	0.953604	0.951505	0.949416	0.947336
330	0.955193	0.953086	0.950989	0.948901	0.946822
360	0.954782	0.952676	0.950579	0.948491	0.946413
MTBF	307.5288	306.8594	306.1928	305.5291	304.8682

(iii) **Effect of failure rates of rough pin hole boring (α_7) on reliability of the system:** Effect of failure rate of sub-system, rough pin hole boring on the reliability of the system is studied by varying its values as $\alpha_7 = 0.0009, 0.0010, 0.0011, 0.0012, 0.0013$ and other parameters are taken as: $\alpha_1 = 0.0014, \alpha_2 = 0.0006, \alpha_3 = 0.0009, \alpha_4 = 0.0009, \alpha_5 = 0.0208, \alpha_6 = 0.0208, \alpha_8 = 0.0039, \beta_1 = 1.08, \beta_2 = 0.043, \beta_3 = 0.5, \beta_4 = 0.286, \beta_5 = 0.154, \beta_6 = 0.25, \beta_7 = 0.059, \beta_8 = 0.087$. The table 2.1(c) shows the effect of rough pin hole boring (α_7) on the reliability of the system. It can be noticed that reliability of the system decreases by approximately 0.06% with the increase of failure rate from 0.0009 to 0.0013 but the reliability decreases by approximately 0.4% with the increase in time from 30 to 360 hours.

Table 2.1(c): EFFECT OF FAILURE RATE OF ROUGH PIN HOLE BORING (α_7) ON RELIABILITY OF THE SYSTEM

$\alpha_7 \rightarrow$ Hours \downarrow	0.0009	0.0010	0.0011	0.0012	0.0013
30	0.977684	0.977502	0.977321	0.977139	0.976958
60	0.970475	0.970169	0.969863	0.969558	0.969254
90	0.966324	0.965938	0.965553	0.965171	0.964790
120	0.963442	0.963002	0.962565	0.962132	0.961701
150	0.961268	0.960793	0.960321	0.959854	0.959391
180	0.959577	0.959079	0.958585	0.958097	0.957613
210	0.958248	0.957734	0.957227	0.956726	0.956230
240	0.957198	0.956676	0.956161	0.955653	0.955152
270	0.956368	0.955842	0.955323	0.954812	0.954309
300	0.955712	0.955184	0.954665	0.954153	0.953649
330	0.955193	0.954666	0.954147	0.953636	0.953134
360	0.954782	0.954256	0.953739	0.953231	0.952731
MTBF	307.5288	307.3814	307.2355	307.0914	306.9487

(iv) Effect of failure rates of oil hole drilling on reliability of the system: We have now calculated the reliability of the system by varying the failure rate of oil hole drilling. We have taken $\alpha_3 = 0.0009, 0.0010, 0.0011, 0.0012, 0.0013$ and other parameters are taken as: $\alpha_1 = 0.0014, \alpha_2 = 0.0006, \alpha_4 = 0.0009, \alpha_5 = 0.0208, \alpha_6 = 0.0208, \alpha_7 = 0.0009, \alpha_8 = 0.0039, \beta_1 = 1.08, \beta_2 = 0.043, \beta_3 = 0.5, \beta_4 = 0.286, \beta_5 = 0.154, \beta_6 = 0.25, \beta_7 = 0.059, \beta_8 = 0.087$. The table shows the effect of oil hole drilling (α_3) on the reliability of the system. The reliability of the system decreases by approximately 0.02% with the increase in failure rate of oil hole drilling from 0.0009 to 0.0013 and by approximately 0.4% with the increase in time from 30 to 360 hours.

Table 2.1(d): EFFECT OF FAILURE RATE OF OIL HOLE DRILLING (α_3) ON RELIABILITY OF THE SYSTEM

$\alpha_3 \rightarrow$ Hours \downarrow	0.0009	0.0010	0.0011	0.0012	0.0013
30	0.977684	0.977493	0.977302	0.977111	0.976921
60	0.970475	0.970288	0.970101	0.969914	0.969727
90	0.966324	0.966139	0.965953	0.965769	0.965583
120	0.963442	0.963257	0.963074	0.962889	0.962706
150	0.961268	0.961085	0.960902	0.960719	0.960536
180	0.959577	0.959395	0.959212	0.959029	0.958848
210	0.958248	0.958065	0.957883	0.957701	0.957519
240	0.957198	0.957016	0.956834	0.956652	0.956471
270	0.956368	0.956186	0.956005	0.955823	0.955642
300	0.955712	0.955530	0.955349	0.955167	0.954986
330	0.955193	0.955011	0.954829	0.954648	0.954467
360	0.954782	0.954601	0.954419	0.954238	0.954057
MTBF	307.5288	307.4700	307.4114	307.3527	307.2942

(v) Effect of failure rates of finish grooving (α_8) on reliability of the system:

Effect of failure rate of sub-system, namely, finish grooving on reliability of the system is studied by varying its values as $\alpha_8 = 0.0039, 0.0040, 0.0041, 0.0042, 0.0043$ and other parameters are taken as: $\alpha_1 = 0.0014, \alpha_2 = 0.0006, \alpha_3 = 0.0009, \alpha_4 = 0.0009, \alpha_5 = 0.0208, \alpha_6 = 0.0208, \alpha_7 = 0.0009, \beta_1 = 1.08, \beta_2 = 0.043, \beta_3 = 0.5, \beta_4 = 0.286, \beta_5 = 0.154, \beta_6 = 0.25, \beta_7 = 0.059, \beta_8 = 0.087$. The table shows the effect of oil hole drilling (α_8) on the reliability of the system. The reliability of the system decreases by approximately 0.01% with the above mentioned increase in failure rate of finish grooving and the reliability decreases by approximately 0.4% with the increase in time from 30 to 360 hours.

Table 2.1(e): EFFECT OF FAILURE RATE OF FINISH GROOVING (α_8) ON RELIABILITY OF THE SYSTEM

$\alpha_8 \rightarrow$ Hours \downarrow	0.0039	0.0040	0.0041	0.0042	0.0043
30	0.977684	0.977572	0.977460	0.977348	0.977237
60	0.970475	0.970292	0.970109	0.969927	0.969745
90	0.966324	0.966095	0.965867	0.965640	0.965415
120	0.963442	0.963184	0.962928	0.962674	0.962422
150	0.961268	0.960994	0.960722	0.960452	0.960185
180	0.959577	0.959294	0.959014	0.958738	0.958464
210	0.958248	0.957961	0.957679	0.957399	0.957124
240	0.957198	0.956912	0.956630	0.956352	0.956078
270	0.956368	0.956085	0.955806	0.955531	0.955260
300	0.955712	0.955432	0.955157	0.954886	0.954620
330	0.955193	0.954918	0.954647	0.954381	0.954119
360	0.954782	0.954512	0.954246	0.953984	0.953727
MTBF	307.5288	307.4464	307.3649	307.2844	307.2048

(vi) Effect of failure rate of finish profile turning (α_4) on reliability of the system: We consider the different parameters as: $\alpha_1 = 0.0014$, $\alpha_2 = 0.0006$, $\alpha_3 = 0.0009$, $\alpha_5 = 0.0208$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.0009$, $\alpha_8 = 0.0039$, $\beta_1 = 1.08$, $\beta_2 = 0.043$, $\beta_3 = 0.5$, $\beta_4 = 0.286$, $\beta_5 = 0.154$, $\beta_6 = 0.25$, $\beta_7 = 0.059$, $\beta_8 = 0.087$. Effect of failure rate of finish profile turning on reliability of the system is studied by varying its values as $\alpha_4 = 0.0009, 0.0010, 0.0011, 0.0012, 0.0013$ and the table shows the effect of oil hole drilling (α_4) on the reliability of the system. The reliability of the system decreases by approximately 0.03% with the increase in failure rate but the reliability decreases by approximately 0.39% with the increase in time from 30 to 360 hours.

Table 2.1(f): EFFECT OF FAILURE RATE OF FINISH PROFILE TURNING (α_4) ON RELIABILITY OF THE SYSTEM

$\alpha_4 \rightarrow$ Hours \downarrow	0.0009	0.0010	0.0011	0.0012	0.0013
30	0.977684	0.977350	0.977016	0.976682	0.976350
60	0.970475	0.970148	0.969820	0.969493	0.969166
90	0.966324	0.965999	0.965676	0.965353	0.965029
120	0.963442	0.963120	0.962798	0.962477	0.962156
150	0.961268	0.960948	0.960628	0.960308	0.959989
180	0.959577	0.959258	0.958939	0.958621	0.958302
210	0.958248	0.957929	0.957611	0.957293	0.956975
240	0.957198	0.956880	0.956562	0.956245	0.955928
270	0.956368	0.956050	0.955733	0.955416	0.955099
300	0.955712	0.955394	0.955077	0.954760	0.954444
330	0.955193	0.954875	0.954558	0.954242	0.953925
360	0.954782	0.954465	0.954148	0.953831	0.953515
MTBF	307.5288	307.4261	307.323	307.2212	307.1188

We now consider the effect of the repair rates of sub-systems of R1 on the reliability of the system.

(vii) Effect of repair rate of fixture seat machining (β_1) on reliability of the system:

We again consider the failure and repair rates of different components as: $\alpha_1 = 0.0014$, $\alpha_2 = 0.0006$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0009$, $\alpha_5 = 0.0208$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.0009$, $\alpha_8 = 0.0039$, $\beta_2 = 0.043$, $\beta_3 = 0.5$, $\beta_4 = 0.286$, $\beta_5 = 0.154$, $\beta_6 = 0.25$, $\beta_7 = 0.059$, $\beta_8 = 0.087$ with values of $\beta_1 = 1.08, 1.18, 1.28, 1.38, 1.48$. The table shows that the reliability of the system increases by approximately 0.001% with the increase of the repair rate from 1.08 to 1.12, but decreases by approximately 0.39% with the increases in time from 30 to 360 hours.

Table 2.1(g): EFFECT OF REPAIR RATE OF FIXTURE SEAT MACHINING (β_1) ON RELIABILITY OF THE SYSTEM

$\beta_1 \rightarrow$ Days \downarrow	1.08	1.18	1.28	1.38	1.48
30	0.977684	0.977788	0.977876	0.977952	0.978018
60	0.970475	0.970578	0.970665	0.970739	0.970803
90	0.966324	0.966425	0.966511	0.966585	0.966648
120	0.963442	0.963542	0.963627	0.963700	0.963764
150	0.961268	0.961368	0.961453	0.961526	0.961588
180	0.959577	0.959677	0.959762	0.959834	0.959897
210	0.958248	0.958347	0.958432	0.958504	0.958566
240	0.957198	0.957297	0.957381	0.957454	0.957516
270	0.956368	0.956467	0.956551	0.956623	0.956686
300	0.955712	0.955811	0.955895	0.955967	0.956029
330	0.955193	0.955292	0.955376	0.955448	0.955510
360	0.954782	0.954881	0.954965	0.955037	0.955100
MTBF	307.5288	307.5608	307.5880	307.6113	307.6315

(viii) Effect of repair rate of rough grooving and turning (β_2) on reliability of the system: The effect of repair rate of rough grooving and turning (β_2) on the reliability of the system is studied by varying its values as $\beta_2 = 0.043, 0.044, 0.045, 0.046, 0.047$ and other parameters are taken as: $\alpha_1 = 0.0014, \alpha_2 = 0.0006, \alpha_3 = 0.0009, \alpha_4 = 0.0009, \alpha_5 = 0.0208, \alpha_6 = 0.0208, \alpha_7 = 0.0009, \alpha_8 = 0.0039, \beta_1 = 1.08, \beta_3 = 0.5, \beta_4 = 0.286, \beta_5 = 0.154, \beta_6 = 0.25, \beta_7 = 0.059, \beta_8 = 0.087$. The table shows that the reliability increases by approximately 0.01% with the increase in the repair rate of rough grooving and turning (β_2) but decreases by approximately 0.37% with the increases in time.

Table 2.1(h): EFFECT OF REPAIR RATE OF ROUGH GROOVING AND TURNING (β_2) ON RELIABILITY OF THE SYSTEM

β_2 → Hours ↓	0.043	0.044	0.045	0.046	0.047
30	0.977684	0.977799	0.977912	0.978024	0.978134
60	0.970475	0.970698	0.970914	0.971123	0.971327
90	0.966324	0.966593	0.966853	0.967103	0.967344
120	0.963442	0.963727	0.964001	0.964263	0.964516
150	0.961268	0.961557	0.961835	0.962101	0.962356
180	0.959577	0.959867	0.960145	0.960411	0.960666
210	0.958248	0.958537	0.958814	0.959079	0.959333
240	0.957198	0.957486	0.957763	0.958028	0.958281
270	0.956368	0.956656	0.956932	0.957196	0.957449
300	0.955712	0.956000	0.956275	0.956539	0.956792
330	0.955193	0.955480	0.955756	0.956019	0.956272
360	0.954782	0.955070	0.955345	0.955608	0.955861
MTBF	307.5288	307.6162	307.70028	307.7808	307.8583

(ix) Effect of repair rate of rough pin hole boring (β_7) on reliability of the system: Effect of repair rate of rough pin hole boring (β_7) on the reliability of the system has now been considered. We have considered values as $\beta_7 = 0.059, 0.060, 0.061, 0.062, 0.063$ and other parameters are taken as: $\alpha_1 = 0.0014, \alpha_2 = 0.0006, \alpha_3 = 0.0009, \alpha_4 = 0.0009, \alpha_5 = 0.0208, \alpha_6 = 0.0208, \alpha_7 = 0.0009, \alpha_8 = 0.0039, \beta_1 = 1.08, \beta_2 = 0.043, \beta_3 = 0.5, \beta_4 = 0.286, \beta_5 = 0.154, \beta_6 = 0.25, \beta_8 = 0.087$. The table shows that the reliability of the system increases by approximately 0.001% with the increase of the repair rate from 0.059 to 0.063, but decreases by approximately 0.39% with the increases in time from 30 to 360 hours.

Table 2.1(i): EFFECT OF REPAIR RATE OF ROUGH PIN HOLE BORING (β_7) ON RELIABILITY OF THE SYSTEM

$\beta_7 \rightarrow$ Hours \downarrow	0.059	0.060	0.061	0.062	0.063
30	0.977684	0.977698	0.977712	0.977726	0.977740
60	0.970475	0.970512	0.970548	0.970584	0.970618
90	0.966324	0.966378	0.966431	0.966483	0.966533
120	0.963442	0.963509	0.963575	0.963639	0.963701
150	0.961268	0.961346	0.961421	0.961495	0.961566
180	0.959577	0.959663	0.959746	0.959827	0.959906
210	0.958248	0.958340	0.958429	0.958516	0.958600
240	0.957198	0.957295	0.957389	0.957481	0.957569
270	0.956368	0.956469	0.956567	0.956662	0.956754
300	0.955712	0.955816	0.955917	0.956015	0.956110
330	0.955193	0.955299	0.955403	0.955503	0.955600
360	0.954782	0.954890	0.954996	0.955098	0.955196
MTBF	307.5288	307.5543	307.5791	307.6032	307.6265

(x) Effect of failure rates of rough oil hole drilling (β_3) on reliability of the system: The reliability of the system has been computed by using the following combinations of failure and repair rates of different sub-systems and is given in Table. Here, we have taken $\beta_3 = 0.5, 0.6, 0.7, 0.8, 0.9$ and other parameters are taken as: $\alpha_1 = 0.0014$, $\alpha_2 = 0.0006$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0009$, $\alpha_5 = 0.0208$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.0009$, $\alpha_8 = 0.0039$, $\beta_1 = 1.08$, $\beta_2 = 0.043$, $\beta_4 = 0.286$, $\beta_5 = 0.154$, $\beta_6 = 0.25$, $\beta_7 = 0.059$, $\beta_8 = 0.087$. It can be noticed from this table that the reliability of the system increases by approximately 0.02% with the increase in repair rate from 0.5 to 0.9 and decreases by approximately 0.39% with the increases in time from 30 to 360 hours.

Table 2.1(j): EFFECT OF REPAIR RATE OF OIL HOLE DRILLING (β_3) ON RELIABILITY OF THE SYSTEM

$\beta_3 \rightarrow$ Hours \downarrow	0.5	0.6	0.7	0.8	0.9
30	0.977684	0.977970	0.978175	0.978329	0.978448
60	0.970475	0.970756	0.970957	0.971108	0.971225
90	0.966324	0.966602	0.966800	0.966950	0.967066
120	0.963442	0.963717	0.963915	0.964063	0.964178
150	0.961268	0.961542	0.961739	0.961886	0.962001
180	0.959577	0.959851	0.960046	0.960193	0.960308
210	0.958248	0.958520	0.958716	0.958862	0.958976
240	0.957198	0.957470	0.957665	0.957811	0.957925
270	0.956368	0.956640	0.956835	0.956981	0.957095
300	0.955712	0.955984	0.956178	0.956324	0.956438
330	0.955193	0.955464	0.955659	0.955805	0.955919
360	0.954782	0.955054	0.955248	0.955394	0.955508
MTBF	307.5288	307.6167	307.6796	307.7268	307.7636

(xi) Effect of repair rate of finish grooving (β_8) on reliability of the system: We now consider the effect of repair rate of finish grooving on reliability of the system. Following value combinations of failure and repair rates of various sub-systems have been considered as $\alpha_1 = 0.0014$, $\alpha_2 = 0.0006$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0009$, $\alpha_5 = 0.0208$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.0009$, $\alpha_8 = 0.0039$, $\beta_1 = 1.08$, $\beta_2 = 0.043$, $\beta_3 = 0.5$, $\beta_4 = 0.286$, $\beta_5 = 0.154$, $\beta_6 = 0.25$, $\beta_7 = 0.059$ and five levels of $\beta_8 = 0.087, 0.097, 0.107, 0.117, 0.127$. We have computed the reliability of the system using these parameters and it is given in Table. The results show that the reliability of the system increases by approximately 0.025% with the increase in the repair rate from 0.087 to 0.127 and decreases by approximately 0.33% with the increases in time from 30 to 360 hours.

Table 2.1(k): EFFECT OF REPAIR RATE OF FINISH GROOVING (β_8) ON RELIABILITY OF THE SYSTEM

$\beta_8 \rightarrow$ Hours \downarrow	0.087	0.097	0.107	0.117	0.127
30	0.977684	0.978003	0.978285	0.978535	0.978758
60	0.970475	0.971162	0.971742	0.972236	0.972662
90	0.966324	0.967291	0.968094	0.968771	0.969349
120	0.963442	0.964627	0.965605	0.966425	0.967122
150	0.961268	0.962625	0.963740	0.964673	0.965464
180	0.959577	0.961069	0.962293	0.963314	0.964179
210	0.958248	0.959845	0.961154	0.962246	0.963169
240	0.957198	0.958879	0.960255	0.961402	0.962372
270	0.956368	0.958115	0.959544	0.960735	0.961741
300	0.955712	0.957511	0.958982	0.960206	0.961241
330	0.955193	0.957033	0.958537	0.959788	0.960846
360	0.954782	0.956655	0.958184	0.959457	0.960532
MTBF	307.5288	307.9746	308.3411	308.6474	308.9073

(xii) Effect of repair rate of finish profile turning (β_4) on reliability of the system: The reliability of the system has been computed by using the following combinations of failure and repair rates of different sub-systems: $\alpha_1 = 0.0014$, $\alpha_2 = 0.0006$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0009$, $\alpha_5 = 0.0208$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.0009$, $\alpha_8 = 0.0039$, $\beta_1 = 1.08$, $\beta_2 = 0.043$, $\beta_3 = 0.5$, $\beta_5 = 0.154$, $\beta_6 = 0.25$, $\beta_7 = 0.059$, $\beta_8 = 0.087$ and five levels of β_4 have been computed by the increment of 0.01 as $\beta_4 = 0.286$, 0.296, 0.306, 0.316, 0.326. The results show that the reliability of the system increases by approximately 0.01% with the increase in the repair rate from 0.286 to 0.326 and decreases by approximately 0.39% with the increases in time from 30 to 360 hours.

Table 2.1(i): EFFECT OF REPAIR RATE OF FINISH PROFILE TURNING (β_4) ON RELIABILITY OF THE SYSTEM

$\beta_4 \rightarrow$ Hours \downarrow	0.286	0.296	0.306	0.316	0.326
30	0.977684	0.977785	0.977880	0.977969	0.978053
60	0.970475	0.970575	0.970668	0.970755	0.970837
90	0.966324	0.966422	0.966514	0.966601	0.966682
120	0.963442	0.963539	0.963631	0.963716	0.963797
150	0.961268	0.961365	0.961456	0.961541	0.961621
180	0.959577	0.959674	0.959765	0.959850	0.959929
210	0.958248	0.958344	0.958434	0.958519	0.958599
240	0.957198	0.957294	0.957384	0.957469	0.957549
270	0.956368	0.956464	0.956554	0.956639	0.956718
300	0.955712	0.955808	0.955898	0.955983	0.956062
330	0.955193	0.955289	0.955379	0.955463	0.955543
360	0.954782	0.954878	0.954968	0.955053	0.955132
MTBF	307.5288	307.5599	307.5890	307.6163	307.6420

2.4.2 Behaviour study of sub-system R2

The system of above differential equations with initial conditions has been solved numerically using Runge-Kutta Fourth Order Method. The computations have been performed upto 360 days for various values of failure and repair rates of the sub-systems, by taking the data for failure rates and repair rates of the sub-systems per hour. The effect of various parameters on reliability of R2 is studied in this section. If the failure and repair rates are altered, the reliability is affected.

(i) Effect of failure rate of finish crown and cavity (α_1) on reliability of the system: The reliability of the system is computed by using the following combinations of failure and repair rates. Reliability is presented in Table 2.2(a). The parameter α_1 is considered at the level of 0.0014, 0.0054, 0.0104, 0.0154, 0.0204 and other parameters are taken as: $\alpha_2 = 0.0003$, $\alpha_3 = 0.0001$, $\alpha_4 = 0.0003$, $\alpha_5 = 0.0001$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.004$, $\beta_1 = 0.33$, $\beta_2 = 0.5$, $\beta_3 = 0.67$, $\beta_4 = 0.035$, $\beta_5 = 3.03$, $\beta_6 = 0.222$, $\beta_7 = 0.125$. The table shows the effect of finish crown and cavity (α_1) on the reliability of the system. The reliability of the system decreases by approximately

0.01% with the increase of failure rate of finish crown and cavity from 0.0014 to 0.0204 and by approximately 0.03% with the increase in time from 30 to 360 hours.

Table 2.2(a): EFFECT OF FAILURE RATE OF FINISH CROWN AND CAVITY (α_1) ON RELIABILITY OF THE SYSTEM

$\alpha_1 \rightarrow$ Hours \downarrow	.0014	.0054	.0104	0.154	0.0204
30	0.991363	0.990468	0.989380	0.988324	0.987299
60	0.989342	0.988472	0.987413	0.986385	0.985385
90	0.988681	0.987820	0.986772	0.985755	0.984766
120	0.988452	0.987595	0.986551	0.985537	0.984552
150	0.988373	0.987517	0.986474	0.985461	0.984477
180	0.988345	0.987489	0.986447	0.985435	0.984452
210	0.988336	0.987480	0.986438	0.985426	0.984443
240	0.988333	0.987477	0.986435	0.985423	0.984440
270	0.988332	0.987476	0.986434	0.985422	0.984439
300	0.988332	0.987475	0.986434	0.985422	0.984439
330	0.988331	0.987475	0.986433	0.985421	0.984438
360	0.988331	0.987475	0.986433	0.985421	0.984438
MTBF	316.3501	316.0751	315.7403	315.4152	315.0993

(ii) Effect of failure rate of valve milling on reliability of the system: Effect of failure rate of sub-system, valve milling on the reliability of the system is studied by varying its values as $\alpha_2 = 0.0003, 0.0004, 0.0005, 0.0006, 0.0007$ and other parameters are taken as: $\alpha_1 = 0.0014, \alpha_2 = 0.0003, \alpha_3 = 0.0001, \alpha_4 = 0.0003, \alpha_5 = 0.0001, \alpha_6 = 0.0208, \alpha_7 = 0.004, \beta_1 = 0.33, \beta_2 = 0.5, \beta_3 = 0.67, \beta_4 = 0.035, \beta_5 = 3.03, \beta_6 = 0.222, \beta_7 = 0.125$. The table shows the effect of valve milling (α_2) on the reliability of the system. The reliability of the system decreases by approximately 0.002% with the increase of failure rate from 0.0003 to 0.0007 but the reliability decreases by approximately 0.03% with the increase in time from 30 to 360 hours.

Table 2.2(b): EFFECT OF FAILURE RATE OF VALVE MILLING (α_2) ON RELIABILITY OF THE SYSTEM

$\alpha_2 \rightarrow$ Hours \downarrow	0.0003	0.0004	0.0005	0.0006	0.0007
30	0.991363	0.991348	0.991333	0.991318	0.991303
60	0.989342	0.989327	0.989313	0.989298	0.989284
90	0.988681	0.988666	0.988652	0.988637	0.988623
120	0.988452	0.988438	0.988424	0.988409	0.988395
150	0.988373	0.988359	0.988344	0.988330	0.988316
180	0.988345	0.988331	0.988317	0.988303	0.988288
210	0.988336	0.988322	0.988307	0.988293	0.988279
240	0.988333	0.988319	0.988304	0.988290	0.988276
270	0.988332	0.988318	0.988303	0.988289	0.988275
300	0.988332	0.988317	0.988303	0.988288	0.988274
330	0.988331	0.988317	0.988302	0.988288	0.988274
360	0.988331	0.988317	0.988302	0.988288	0.988274
MTBF	316.3501	316.3455	316.3409	316.3363	316.3317

(iii) Effect of failure rate of Chamfering or RADIUSING (α_3) on reliability of the system: We have now calculated the reliability of the system by varying the failure rate of Chamfering or RADIUSING. We have taken $\alpha_3 = 0.0001, 0.0002, 0.0003, 0.0004, 0.0005$ and other parameters are taken as: $\alpha_1 = 0.0014, \alpha_2 = 0.0003, \alpha_4 = 0.0003, \alpha_5 = 0.0001, \alpha_6 = 0.0208, \alpha_7 = 0.004, \beta_1 = 0.33, \beta_2 = 0.5, \beta_3 = 0.67, \beta_4 = 0.035, \beta_5 = 3.03, \beta_6 = 0.222, \beta_7 = 0.125$. The table shows the effect of Chamfering or RADIUSING (α_3) on the reliability of the system. It can be noticed that reliability of the system decreases by approximately 0.02% with the increase of failure rate from 0.0001 to 0.0005 but the reliability decreases by approximately 0.03% with the increase in time from 30 to 360 hours.

Table 2.2(c): EFFECT OF FAILURE RATE OF CHAMFERING OR RADIUSING (α_3) ON RELIABILITY OF THE SYSTEM

$\alpha_3 \rightarrow$ Hours \downarrow	0.0001	0.0002	0.0003	0.0004	0.0005
30	0.991363	0.991217	0.991071	0.990925	0.990779
60	0.989342	0.989196	0.989051	0.988906	0.988760
90	0.988681	0.988535	0.988390	0.988245	0.988100
120	0.988452	0.988307	0.988162	0.988017	0.987872
150	0.988373	0.988228	0.988083	0.987938	0.987793
180	0.988345	0.988200	0.988055	0.987910	0.987765
210	0.988336	0.988191	0.988046	0.987901	0.987756
240	0.988333	0.988188	0.988043	0.987898	0.987753
270	0.988332	0.988186	0.988042	0.987897	0.987752
300	0.988332	0.988186	0.988041	0.987896	0.987751
330	0.988331	0.988186	0.988041	0.987896	0.987751
360	0.988331	0.988186	0.988041	0.987896	0.987751
MTBF	316.3501	316.3036	316.2572	316.2108	316.1643

(iv) Effect of failure rates of circlip grooving (α_4) on reliability of the system: We consider the different parameters as: $\alpha_1 = 0.0014$, $\alpha_2 = 0.0003$, $\alpha_3 = 0.0001$, $\alpha_5 = 0.0001$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.004$, $\beta_1 = 0.33$, $\beta_2 = 0.5$, $\beta_3 = 0.67$, $\beta_4 = 0.035$, $\beta_5 = 3.03$, $\beta_6 = 0.222$, $\beta_7 = 0.125$. Effect of failure rate of circlip grooving on reliability of the system is studied by varying its values as $\alpha_4 = 0.0003, 0.0004, 0.0005, 0.0006, 0.0007$ and the table shows the effect of oil circlip grooving (α_4) on the reliability of the system. The reliability of the system decreases by approximately 0.18% with the increase of failure rate of circlip grooving from 0.0003 to 0.0007 and by approximately 0.34% with the increase in time from 30 to 360 hours.

TABLE 2.2(d): EFFECT OF FAILURE RATE OF CIRCLIP GROOVING (α_4) ON RELIABILITY OF THE SYSTEM

$\alpha_4 \rightarrow$ Hours \downarrow	0.0003	0.0004	0.0005	0.0006	0.0007
30	0.991363	0.989539	0.987719	0.985904	0.984093
60	0.989342	0.986895	0.984459	0.982032	0.979614
90	0.988681	0.986021	0.983374	0.980740	0.978119
120	0.988452	0.985719	0.983001	0.980297	0.977607
150	0.988373	0.985615	0.982873	0.980145	0.977432
180	0.988345	0.985579	0.982828	0.980092	0.977372
210	0.988336	0.985567	0.982813	0.980074	0.977351
240	0.988333	0.985563	0.982808	0.980068	0.977344
270	0.988332	0.985561	0.982806	0.980066	0.977342
300	0.988332	0.985560	0.982805	0.980065	0.977341
330	0.988331	0.985560	0.982805	0.980065	0.977341
360	0.988331	0.985560	0.982805	0.980065	0.977340
MTBF	316.3501	315.4901	314.6347	313.7836	312.9371

(v) Effect of failure rate of cleaning machine (α_5) on reliability of the system: We consider the different parameters as: $\alpha_1 = 0.0014$, $\alpha_2 = 0.0003$, $\alpha_3 = 0.0001$, $\alpha_4 = 0.0003$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.004$, $\beta_1 = 0.33$, $\beta_2 = 0.5$, $\beta_3 = 0.67$, $\beta_4 = 0.035$, $\beta_5 = 3.03$, $\beta_6 = 0.222$, $\beta_7 = 0.125$. Effect of failure rate of cleaning machine on reliability of the system is studied by varying its values as $\alpha_5 = 0.0001, 0.0002, 0.0003, 0.0004, 0.0005$ and the table shows the effect of cleaning machine (α_5) on the reliability of the system. The reliability of the system decreases by approximately 0.003% with the above mentioned increase in failure rate of cleaning machine and the reliability decreases by approximately 0.15% with the increase in time from 30 to 360 hours.

Table 2.2(e): EFFECT OF FAILURE RATE OF CLEANING MACHINE (α_5) ON RELIABILITY OF THE SYSTEM

$\alpha_5 \rightarrow$ Hours \downarrow	0.0001	0.0002	0.0003	0.0004	0.0005
30	0.991363	0.991331	0.991298	0.991266	0.991234
60	0.989342	0.989310	0.989277	0.989245	0.989213
90	0.988681	0.988648	0.988616	0.988584	0.988552
120	0.988452	0.988420	0.988388	0.988356	0.988324
150	0.988373	0.988341	0.988309	0.988277	0.988245
180	0.988345	0.988313	0.988281	0.988249	0.988217
210	0.988336	0.988304	0.988272	0.988240	0.988208
240	0.988333	0.988301	0.988269	0.988236	0.988204
270	0.988332	0.988300	0.988268	0.988235	0.988203
300	0.988332	0.988300	0.988267	0.988235	0.988203
330	0.988331	0.988299	0.988267	0.988235	0.988203
360	0.988331	0.988299	0.988267	0.988235	0.988203
MTBF	316.3501	316.3399	316.3295	316.3192	316.3090

(vi) Effect of failure rate of finish pin hole boring(α_7) on reliability of the system: We consider the different parameters as: $\alpha_1 = 0.0014$, $\alpha_2 = 0.0003$, $\alpha_3 = 0.0001$, $\alpha_4 = 0.0003$, $\alpha_5 = 0.0001$, $\alpha_6 = 0.0208$, $\beta_1 = 0.33$, $\beta_2 = 0.5$, $\beta_3 = 0.67$, $\beta_4 = 0.035$, $\beta_5 = 3.03$, $\beta_6 = 0.222$, $\beta_7 = 0.125$. Effect of failure rate of finish grooving on reliability of the system is studied by varying its values as $\alpha_7 = 0.004, 0.005, 0.006, 0.007, 0.008$ and the table shows the effect of finish pin hole boring (α_7) on the reliability of the system. The reliability of the system decreases by approximately 0.06% with the increase of failure rate and the reliability decreases by approximately 0.03% also with the increase in time from 30 to 360 hours.

Table 2.2(f): EFFECT OF FAILURE RATE OF FINISH PIN HOLE BORING (α_7) ON RELIABILITY OF THE SYSTEM

$\alpha_7 \rightarrow$ Hours \downarrow	.004	.005	.006	.007	.008
30	0.991363	0.990746	0.990135	0.989529	0.988929
60	0.989342	0.988700	0.988064	0.987435	0.986811
90	0.988681	0.988039	0.987404	0.986775	0.986152
120	0.988452	0.987812	0.987177	0.986548	0.985925
150	0.988373	0.987732	0.987098	0.986469	0.985846
180	0.988345	0.987705	0.987071	0.986442	0.985819
210	0.988336	0.987696	0.987061	0.986433	0.985810
240	0.988333	0.987692	0.987058	0.986429	0.985806
270	0.988332	0.987691	0.987057	0.986428	0.985805
300	0.988332	0.987691	0.987056	0.986428	0.985805
330	0.988331	0.987691	0.987056	0.986428	0.985805
360	0.988331	0.987691	0.987056	0.986428	0.985805
MTBF	316.3501	316.1453	315.9424	315.7415	315.5423

We now consider the effect of the repair rates of sub-systems of R2 on the reliability of the system.

(vii) Effect of repair rate of finish crown and cavity (β_1) on reliability of the system: We again consider the failure and repair rates of different components as: $\alpha_1 = 0.0014$, $\alpha_2 = 0.0003$, $\alpha_3 = 0.0001$, $\alpha_4 = 0.0003$, $\alpha_5 = 0.0001$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.004$, $\beta_2 = 0.5$, $\beta_3 = 0.67$, $\beta_4 = 0.035$, $\beta_5 = 3.03$, $\beta_6 = 0.222$, $\beta_7 = 0.125$ with values of $\beta_1 = 0.33, 0.34, 0.35, 0.36, 0.37$. The table shows that the reliability of the system increases by approximately 0.0008% with the increase of the repair rate from 0.33 to 0.37, but decreases by approximately 0.03% with the increases in time from 30 to 360 hours.

Table 2.2(g): EFFECT OF REPAIR RATE OF FINISH CROWN AND CAVITY (β_1) ON RELIABILITY OF THE SYSTEM

β_1 → Hours ↓	0.33	0.34	0.35	0.36	0.37
30	0.991363	0.991372	0.991381	0.991390	0.991397
60	0.989342	0.989351	0.989359	0.989368	0.989375
90	0.988681	0.988690	0.988698	0.988706	0.988714
120	0.988452	0.988461	0.988470	0.988477	0.988485
150	0.988373	0.988382	0.988390	0.988398	0.988406
180	0.988345	0.988354	0.988363	0.988371	0.988378
210	0.988336	0.988345	0.988353	0.988361	0.988369
240	0.988333	0.988342	0.988350	0.988358	0.988365
270	0.988332	0.988340	0.988349	0.988357	0.988364
300	0.988332	0.988340	0.988348	0.988356	0.988364
330	0.988331	0.988340	0.988348	0.988356	0.988364
360	0.988331	0.988340	0.988348	0.988356	0.988364
MTBF	316.3501	316.3529	316.35565	316.3582	316.3606

(viii) Effect of repair rate of valve milling (β_2) on reliability of the system: The effect of repair rate of valve milling (β_2) on the reliability of the system is studied by varying its values as $\beta_2 = 0.5, 0.6, 0.7, 0.8, 0.9$ and other parameters are taken as: $\alpha_1 = 0.0014, \alpha_2 = 0.0003, \alpha_3 = 0.0001, \alpha_4 = 0.0003, \alpha_5 = 0.0001, \alpha_6 = 0.0208, \alpha_7 = 0.004$, $\beta_1 = 0.33, \beta_3 = 0.67, \beta_4 = 0.035, \beta_5 = 3.03, \beta_6 = 0.222, \beta_7 = 0.125$. The table shows that the reliability increases by approximately 0.0006% with the increase in the repair rate of valve milling (β_2) but decreases by approximately 0.03% with the increases in time.

Table 2.2(h): EFFECT OF REPAIR RATE OF VALVE MILLING (β_2) ON RELIABILITY OF THE SYSTEM

$\beta_2 \rightarrow$ Hours \downarrow	0.5	0.6	0.7	0.8	0.9
30	0.991363	0.991371	0.991376	0.991380	0.991383
60	0.989342	0.989349	0.989354	0.989358	0.989361
90	0.988681	0.988688	0.988693	0.988697	0.988700
120	0.988452	0.988459	0.988464	0.988468	0.988471
150	0.988373	0.988380	0.988385	0.988389	0.988392
180	0.988345	0.988353	0.988358	0.988362	0.988365
210	0.988336	0.988343	0.988348	0.988352	0.988355
240	0.988333	0.988340	0.988345	0.988349	0.988352
270	0.988332	0.988339	0.988344	0.988348	0.988351
300	0.988332	0.988338	0.988343	0.988347	0.988350
330	0.988331	0.988338	0.988343	0.988347	0.988350
360	0.988331	0.988338	0.988343	0.988347	0.988350
MTBF	316.3501	316.3524	316.3540	316.3552	316.3562

(ix) Effect of repair rate of Chamfering or RADIUSING (β_3) on reliability of the

system: The reliability of the system has been computed by using the following combinations of failure and repair rates of different sub-systems and is given in Table. Here, we have taken $\beta_3 = 0.67, 0.68, 0.69, 0.70, 0.71$ and other parameters are taken as: $\alpha_1 = 0.0014, \alpha_2 = 0.0003, \alpha_3 = 0.0001, \alpha_4 = 0.0003, \alpha_5 = 0.0001, \alpha_6 = 0.0208, \alpha_7 = 0.004, \beta_1 = 0.33, \beta_2 = 0.5, \beta_4 = 0.035, \beta_5 = 3.03, \beta_6 = 0.222, \beta_7 = 0.125$. The table shows that the reliability of the system increases by approximately 0.0002% with the increase of the repair rate from 0.67 to 0.71, but decreases by approximately 0.03% with the increases in time from 30 to 360 hours.

Table 2.2(i): EFFECT OF REPAIR RATE OF CHAMFERING OR RADUISING (β_3) ON RELIABILITY OF THE SYSTEM

$\beta_3 \rightarrow$ Hours \downarrow	0.67	0.68	0.69	0.70	0.71
30	0.991363	0.991365	0.991367	0.991369	0.991371
60	0.989342	0.989344	0.989346	0.989348	0.989350
90	0.988681	0.988683	0.988685	0.988687	0.988689
120	0.988452	0.988454	0.988456	0.988458	0.988460
150	0.988373	0.988375	0.988377	0.988379	0.988381
180	0.988345	0.988348	0.988350	0.988352	0.988354
210	0.988336	0.988338	0.988340	0.988342	0.988344
240	0.988333	0.988335	0.988337	0.988339	0.988341
270	0.988332	0.988334	0.988336	0.988338	0.988340
300	0.988332	0.988333	0.988335	0.988337	0.988339
330	0.988331	0.988333	0.988335	0.988337	0.988339
360	0.988331	0.988333	0.988335	0.988337	0.988339
MTBF	316.3501	316.3508	316.3514	316.3520	316.3527

(x) Effect of repair rate of circlip grooving (β_4) on reliability of the system: The reliability of the system has been computed by using the following combinations of failure and repair rates of different sub-systems: $\alpha_1 = 0.0014$, $\alpha_2 = 0.0003$, $\alpha_3 = 0.0001$, $\alpha_4 = 0.0003$, $\alpha_5 = 0.0001$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.004$, $\beta_1 = 0.33$, $\beta_2 = 0.5$, $\beta_3 = 0.67$, $\beta_5 = 3.03$, $\beta_6 = 0.222$, $\beta_7 = 0.125$ and five levels of β_4 have been computed by the increment of 0.01 as $\beta_4 = 0.035, 0.045, 0.055, 0.065, 0.075$. It can be noticed from this table that the reliability of the system increases by approximately 0.5% with the increase in repair rate from 0.035 to 0.075 and decreases by approximately 0.06% with the increases in time from 30 to 360 hours.

Table 2.2(j): EFFECT OF REPAIR RATE OF CIRCLIP GROOVING (β_4) ON RELIABILITY OF THE SYSTEM

$\beta_4 \rightarrow$ Hours \downarrow	0.035	0.045	0.055	0.065	0.075
30	0.991363	0.991988	0.992512	0.992955	0.993330
60	0.989342	0.990620	0.991567	0.992286	0.992842
90	0.988681	0.990297	0.991408	0.992206	0.992801
120	0.988452	0.990215	0.991378	0.992195	0.992797
150	0.988373	0.990194	0.991373	0.992193	0.992797
180	0.988345	0.990189	0.991372	0.992193	0.992797
210	0.988336	0.990187	0.991371	0.992193	0.992797
240	0.988333	0.990187	0.991371	0.992193	0.992797
270	0.988332	0.990187	0.991371	0.992193	0.992797
300	0.988332	0.990187	0.991371	0.992193	0.992797
330	0.988331	0.990187	0.991371	0.992193	0.992797
360	0.988331	0.990187	0.991371	0.992193	0.992797
MTBF	316.3501	316.8987	317.2590	317.5134	317.7022

(xi) Effect of repair rate of cleaning machine (β_5) on reliability of the system:

The effect of repair rate of cleaning machine (β_5) on the reliability of the system is studied by varying its values as $\beta_5 = 3.03, 3.13, 3.23, 3.33, 3.43$ and other parameters are taken as: $\alpha_1 = 0.0014, \alpha_2 = 0.0003, \alpha_3 = 0.0001, \alpha_4 = 0.0003, \alpha_5 = 0.0001, \alpha_6 = 0.0208, \alpha_7 = 0.004, \beta_1 = 0.33, \beta_2 = 0.5, \beta_3 = 0.67, \beta_4 = 0.035, \beta_6 = 0.222, \beta_7 = 0.125$. The results show that the reliability of the system increases by approximately 0.0001% with the increase in the repair rate from 3.03 to 3.43 and decreases by approximately 0.02% with the increases in time from 30 to 360 hours.

Table 2.2(k): EFFECT OF REPAIR RATE OF CLEANING MACHINE (β_5) ON RELIABILITY OF THE SYSTEM

$\beta_5 \rightarrow$ Hours \downarrow	3.03	3.13	3.23	3.33	3.43
30	0.991363	0.991364	0.991365	0.991366	0.991367
60	0.989342	0.989343	0.989344	0.989345	0.989346
90	0.988681	0.988682	0.988683	0.988683	0.988684
120	0.988452	0.988453	0.988454	0.988455	0.988456
150	0.988373	0.988374	0.988375	0.988376	0.988377
180	0.988345	0.988347	0.988347	0.988348	0.988349
210	0.988336	0.988337	0.988338	0.988339	0.988340
240	0.988333	0.988334	0.988335	0.988336	0.988336
270	0.988332	0.988333	0.988333	0.988334	0.988335
300	0.988332	0.988332	0.988333	0.988334	0.988335
330	0.988331	0.988332	0.988333	0.988334	0.988335
360	0.988331	0.988332	0.988333	0.988334	0.988335
MTBF	316.3501	316.3504	316.3507	316.3510	316.3513

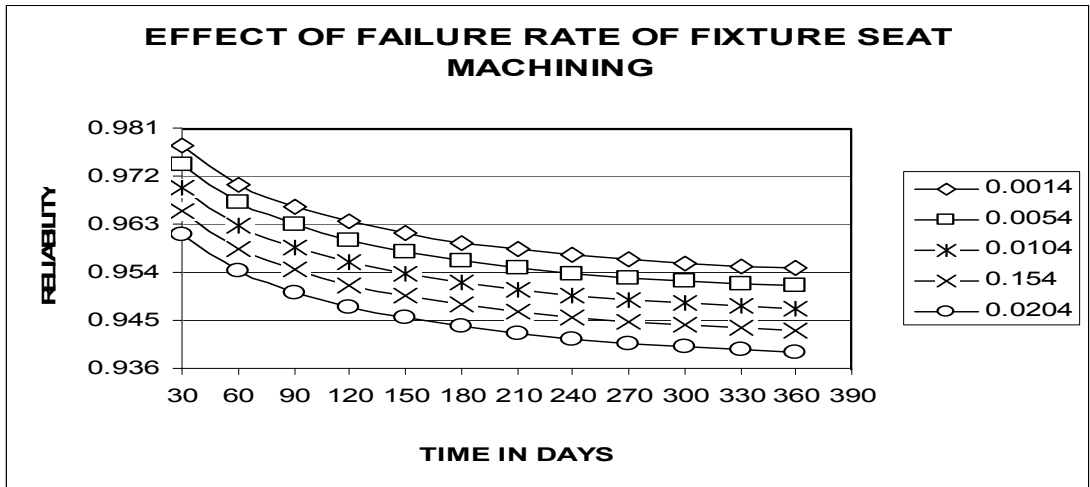
(xii) Effect of repair rate of finish pin hole boring (β_7) on reliability of the system: The reliability of the system has been computed by using the following combinations of failure and repair rates of different sub-systems: $\alpha_1 = 0.0014$, $\alpha_2 = 0.0003$, $\alpha_3 = 0.0001$, $\alpha_4 = 0.0003$, $\alpha_5 = 0.0001$, $\alpha_6 = 0.0208$, $\alpha_7 = 0.004$, $\beta_1 = 0.33$, $\beta_2 = 0.5$, $\beta_3 = 0.67$, $\beta_4 = 0.035$, $\beta_5 = 3.03$, $\beta_6 = 0.222$, $\beta_7 = 0.125$ and five levels of β_7 have been computed by the increment of 0.01 as $\beta_7 = 0.125, 0.135, 0.145, 0.155, 0.165$. The results of this computation are presented in table. The results show that the reliability of the system increases by approximately 0.02% with the increase in the repair rate from 0.125 to 0.165 and decreases by approximately 0.03% with the increases in time from 30 to 360 hours.

Table 2.2(l): EFFECT OF REPAIR RATE OF FINISH PIN HOLE BORING (β_7) ON RELIABILITY OF THE SYSTEM

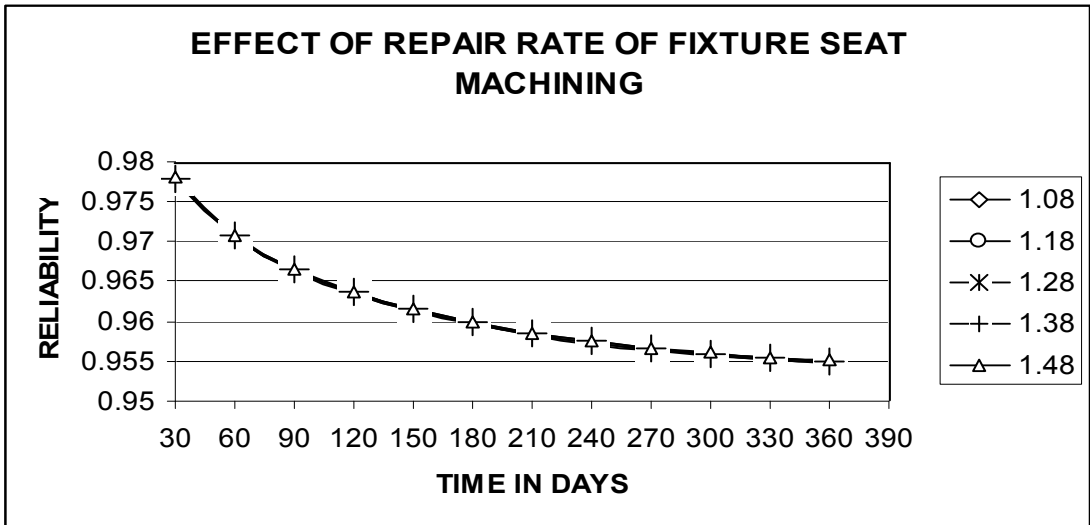
$\beta_7 \rightarrow$ Hours \downarrow	.125	.135	.145	.155	.165
30	0.991363	0.991527	0.991674	0.991805	0.991922
60	0.989342	0.989536	0.989703	0.989849	0.989978
90	0.988681	0.988875	0.989043	0.989189	0.989318
120	0.988452	0.988647	0.988814	0.988960	0.989089
150	0.988373	0.988567	0.988735	0.988881	0.989009
180	0.988345	0.988540	0.988707	0.988853	0.988982
210	0.988336	0.988530	0.988698	0.988844	0.988972
240	0.988333	0.988527	0.988695	0.988841	0.988969
270	0.988332	0.988526	0.988693	0.988839	0.988968
300	0.988332	0.988525	0.988693	0.988839	0.988967
330	0.988331	0.988525	0.988693	0.988839	0.988967
360	0.988331	0.988525	0.988693	0.988839	0.988967
MTBF	316.3501	316.4119	316.4653	316.5119	316.5529

2.5 Analysis of Results

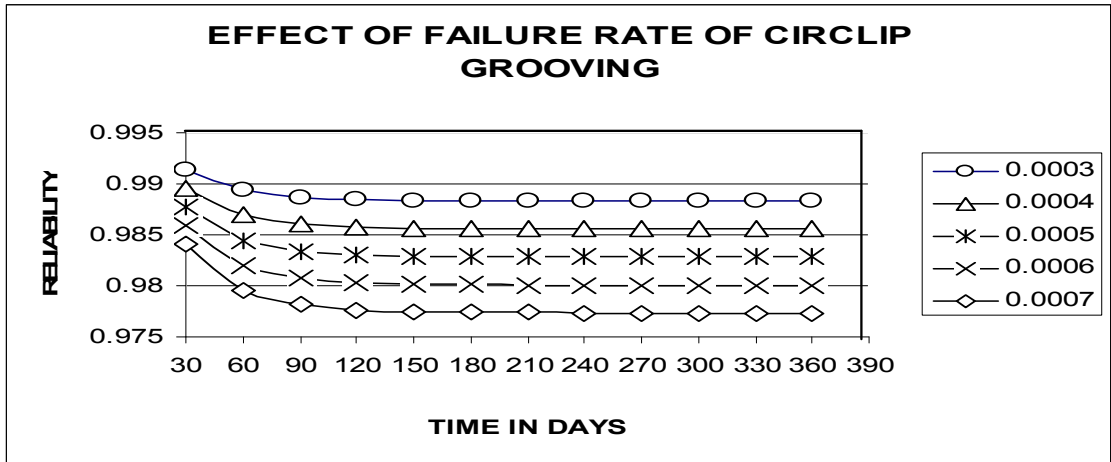
As mentioned above, the whole system is divided into two sub-systems R1 and R2. An independent study is performed for both sub-systems R1 and R2. The comparative study of the tables 2.1(a) to 2.1(l) shows that the sub-system A, that is, fixture seat machining affects the reliability of the sub-system R1 more than any of the other sub-systems. However, the comparative study based on the tables 2.2(a) to 2.2(l) indicates that the sub-system K, that is, Circlip grooving affects the reliability of the sub-system R2 more than any of the other sub-systems. The other sub-systems are almost equally effect the reliability, which is clear from the above tables. The effect of failure and repair rates of sub-system A of R1 on the reliability of sub-system R1 can be verified with the help of Graphs 2.1 and 2.2. Also the effect of failure and repair rates of sub-system K of R2 on the reliability of sub-system R2 can be verified with the help of Graphs 2.3 and 2.4. Therefore, it is recommended that management should pay more attention to sub-systems A of R1 and sub-system K of R2 to increase the reliability of the piston manufacturing plant.



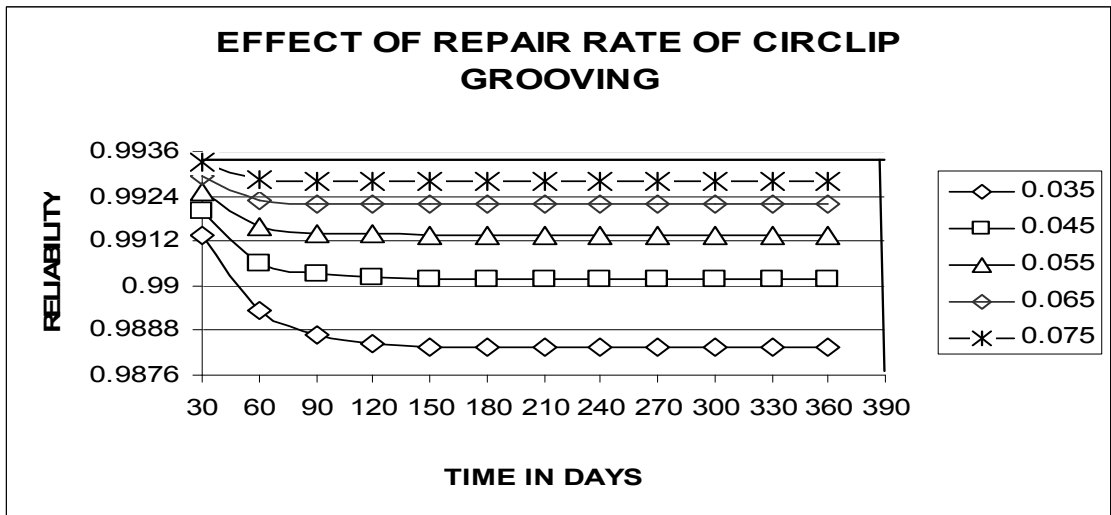
GRAPH 2.1



GRAPH 2.2



GRAPH 2.3



GRAPH 2.4

CHAPTER 3

CONCLUDING OBSERVATIONS

In this chapter, we have presented industrial significance of the present study as well as limitation of the work. In section 3.1 discusses the significance of studying reliability of piston manufacturing plant. The limitations and future research work of the present study are presented in section 3.2

3.1 Industrial Significance of This Study

As Reliability becomes very important factor of every product in industry, the techniques discussed in this thesis are designed for studying the problems of reliability. As reliability, MTBF are often calculated in industrial problems. These factors help in future planning so that industry can get more profit and also helps in the manufacture of more reliable products. Our analysis of the result is based on the actual data of failure and repair rates of the sub-systems. The conclusion drawn from the analysis of our results is consistent with the actual performance of the manufacturing plant. Our suggestions will help them a lot to increase over-all performance of their system.

3.2 Limitations and Future Scope of the Present Study

In our present study, we have divided the system into two sub-systems R1 and R2 because of the complexity of the system. An independent study is performed for both sub-systems R1 and R2. By the comparative study of tables and with the help of graphs, we have observed that the sub-system Fixture seat machining and the sub-system Circlip grooving affect the reliability of sub-systems R1 and R2, respectively more than any other sub-system. However, this result can be justified by studying the the reliability of the whole system instead of dividing it into two parts. Of course this is a difficult task to be performed. In our present study, we have presented the reliability of sub-systems R1 and R2 for 360 hours only. In future, one can study the reliability of the piston manufacturing plant for one year, two years etc.

In the present work, to solve the differential equations with given initial conditions, we have used Runge-Kutta fourth order method. The reliability of the system has been calculated for 360 hours. The calculations are performed by taking $t = 0.005$. One can use another numerical method for solving this problem.

We have not studied the long run availability of the system in the present study. In future, we can find the long run availability of the system that will be more

beneficial for future planning. While computing the reliability of manufacturing plant, we are not considering the cost required for the repair of sub-systems. The cost factor plays very important role to get the maximum reliability of a system. In future one can study the optimal reliability of the system by the consideration of cost factor and achieve the maximum reliability with minimum cost.

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