NOVEL HYBRID ALGORITHM FOR HYDROTHERMAL GENERATION SCHEDULING

A Thesis

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DOCTOR OF PHILOSOPHY

Submitted by

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Dedicated to my

Great Grandparents

S. Hazara Singh Ji and Bibi Kesar Kaur Ji

S. Bhagat Singh Ji and Bhabi Pritam Kaur Ji

Grandparents

S. Diwan Singh Ji and Bibi Bhagwant Kaur Ji

S. Baldev Singh Ji and Biji Partap Kaur Ji

ਵਾਹਿਗੁਰੂ ਜੀ ਕਾ ਖਾਲਸਾ॥ ਵਾਹਿਗੁਰੂ ਜੀ ਕੀ ਫਤਿਹ॥

With Special Thanks

To

Respected Parents

Prof. Harbinder Singh

(Retd. Principal GNDPC, Ludhiana)

and

Mrs. Jasbir Kaur

CERTIFICATE

Certified that the thesis entitled "***Novel Hybrid Algorithm For Hydrothermal Generation Scheduling***", being submitted by Manbir Kaur (Regd. No. 950904015) in fulfillment of the requirements of Doctor of Philosophy to Electrical and Instrumentation Engineering Department, Thapar Institute of Engineering & Technology, Patiala (Punjab) is a bona-fide record of the candidate's own work carried out by her under our supervision and guidance. The matter contained in this thesis has been submitted neither in part nor in full to any other university or institute for award of any degree.

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SYNOPSIS

Electric power systems have experienced continuous growth in all three major sectors of the power system, namely, generation, transmission, and distribution. Electricity generation cannot be stored economically, but it involves a dynamic system that needs a balance of its supply and demand for 24 hours. Conventionally, nearly 70-75% of total installed capacity, the power generation systems include both fixed head hydro and fuel-fired thermal units world-wide. Owing to the low operational cost and minimum start-up time, the hydro units facilitate peak load shaving by replacing high fuel cost thermal units. The designed infrastructure of hydroelectric power plants considers the units to be located on the same stream or/and on different streams. On the other hand, the selection of the thermal units as a base, middle or peak one is based on their efficiency, fuel cost and the dynamic response. The efficient scheduling of available energy resources for satisfying load demand has become an important task in the modern power system. The generation scheduling problem aims at determining the optimal operation strategy for the next scheduling interval subject to a variety of constraints. For a short-term hydrothermal generation, planning period is an hour or a day or half or a week as the load demand on the power system exhibits cyclic variations. Short-term hydrothermal scheduling is an important planning task in power system operation with two aspects; a day-ahead scheduling for short-term profitable market contracts and hourly scheduling for power balance requirements, hence it is the subject of intensive investigation.

Major hydro units have high capital cost, whereas run-off river plants and pumped storage plants are not to be relied upon over short term planning of generation allocation to meet the load demand. The well-timed allocation of hydro energy resources requires probabilistic analysis and long-term considerations, as the excess water utilization in the present period, may lead to optimum availability in the future, increasing in this way the future operating costs.

Hydrothermal generation scheduling is more complex than thermal unit commitment owing to various physical and technical operational constraints. Other than system constraints and specifications of hydrothermal units, the stochastic nature of water availability, limited energy storage capability of water reservoirs, and uncertainty in load demand over the scheduling period make its solution a more challenging task.

Also, the increased concern of clean environment requirements, the low priced fuel of thermal units cannot meet the guidelines of environment act. The harmful emission production limit from the thermal unit must be satisfied simultaneously. This non-linear problem aims at determining the optimal hourly water discharge from hydro reservoirs and power generation from thermal units to meet load demand over the operation period while utilizing the limited amount of water available to its fullest extent, such that the total system production cost and pollutants emission of committed thermal units are minimized while satisfying thermal, hydraulic and other network constraints. Short term hydrothermal generation scheduling is a constrained multi-objective optimization problem with conflicting objectives. The conventional search methods are applied to solve the hydrothermal coordination problem made use of straightforward single point search procedures of exploration through pattern moves, to make the optimization problem simpler. But simplifying assumptions to a practical problem may extend to a suboptimal solution. The input-output characteristics of practical models of thermal-units and hydro-units are non-smooth, non-convex, and discontinuous. Direct search and /or gradient methods are although efficient ones, yet find a tedious task to manage non-linearity of the hydrothermal coordination problem. On the other hand, population-based heuristic (P-heuristics) search methods have paved a way to address various issues of optimization problems like complexity, multimodality, dimensionality, constraints handling, global solution search, convergence speed, etc.. However, the fitness measure of P-heuristics is influenced by the potential of the initial population, selection of control parameters, mutation and crossover techniques to balance exploration and exploitation and method of selection of a population for next generation. The interactive optimization techniques generate a set of non-inferior solutions and search the best possible solution for conflicting objectives of the problem with a trade-off between the computational effort and the quality of the solution.

In recent years, a growing interest and awareness in the successful, inexpensive, and efficient application of stochastic algorithms are witnessed. According to no free lunch theorem, no algorithm is universally the best optimizer. There is a scope of improvement in optimization tools to improve convergence behaviour, the robustness of the global solution, dependence on control parameters, and strategies to prevent entrapping in sub-optimal solutions and optimal management of computational space and time for large scale real-world optimization problems.

The scope of this thesis work is focused on the proposal of a novel multi-objective metaheuristic optimization technique integrated with promising strategies for better desirable results to constrained multi-objective real-time problems. In the first phase of the investigation, the exploitation capability of a global search technique is enhanced to identify the potential and quality solution search space. In the second phase, the global search technique is integrated with local search techniques for a better exploration and problems like premature convergence and stagnation to local minima are alleviated through adaptive learning. In the third phase, a strategy is adopted to archive elicit solutions.

The *differential evolution (DE)* is undertaken as a global search technique. The DE technique is based on a population-based evolutionary algorithm with two control parameters, evolved through three evolutionary operators' mutation, crossover, and selection. Basic DE suffers from occasional stagnation and premature convergence, where the solution converges to suboptimal solutions, but it generates a new candidate solution during evolution. To overcome the drawbacks of basic DE to some extent, three modifications in the DE strategy are proposed. Regeneration of promising initial population is proposed by exploring the uniformly distributed population members and their opposite ones simultaneously.

The chaotic search strategy is one of the promising memetic approaches to strengthen the exploitation ability of DE for a quality solution and to accelerate its convergence speed. Among various chaotic sequence variations, namely Logistic map, Tent map, Gauss map, and Sine map, Circle map etc., a chaotic sequence of control parameters of DE, mutation constant, and crossover rate, is generated by Logistic mapping in this study.

Differential evolution is a self-referential algorithm. In the first attempt of modification to DE strategy, crisscross mutation and crisscross crossover operations are proposed in both horizontal and vertical directions alternately of the population for a better exploration and to escape from local optima. This strategy suggests a dual mechanism of interaction across the three-dimensional problem space and supports the generation of qualified potential solutions through competition. This supplementary attribute to chaotic DE eliminates the comparatively weak solutions from a parent population to increase the convergence rate and global search capability with accuracy.

Further, a competitive computational intelligence strategy, opposition-based learning is incorporated to avoid stagnation of solution in local minima. The jumping rate is selected based on pre-specified non-dominated solutions in the current generation. The proposed concept contributes to enhancing the efficient convergence of the search algorithm. The proposed optimization techniques are applied to search optimum solution of the fixed head, multi-chain short-term hydrothermal generation scheduling problems. Multi-objective generation scheduling problem undertakes two conflicting objectives that are the operating cost and emission of gaseous pollutants Compromised solution is selected by finding the aggregated effect of participating objectives using their membership functions. In this work, the constraints are satisfied by perturbing the decision variables in a systematic procedure. The proposed approach is tested on four fixed head hydrothermal power system. The proposed optimization techniques are also validated on generalized benchmark test functions. In the research work, an interactive fuzzy method is used to solve the multi-objective optimization problem with two conflicting objectives. The decision-maker decides the solution interactively by considering the unified effect of participating objectives by exploiting their membership functions. The highest cardinal priority ranking provides maximum satisfaction level of the participating objectives.

The penalty function method lacks flexibility and undergoes multiple runs to fine-tune the penalty factor leading to a high computational cost. In the second phase of solving the multi-objective constrained optimization problem, the equality constraints are handled using a two-step approach in hydrothermal system problems. In the first step, a variable elimination approach with heuristic repair strategy is applied to handle equality constraints and subsequently in second step fuzzy model is recommended to handle unpredicted residues in violations of operational constraints; those result in due to the stochastic nature of variables and algorithm-dependent parameters. Residues are fuzzy quantified and are considered as an objective to be optimized. The best-compromised solution of the three interdependent objectives of the *hydrothermal generation scheduling* (HTGS) problem is achieved by an interactive fuzzy decision model. The results obtained are satisfactory and demonstrate the appropriateness of each algorithm not only for handling non-convexities, discontinuity, and non-differentiable functions but are also well suitable for constrained optimization problems of the power sector.

Stochastic optimization population-based algorithms generate several random solutions and improve during the process of optimization. This bounded search space may contain discontinuities, more than one local minima, global optimum located on the boundaries of search space, deceptive valleys, etc.. It is important to equip the optimization algorithms with suitable mathematical functions to update the position of random solutions in the promising regions for handling these difficulties to find the global optimum.

In the second attempt, the Sine-Cosine algorithm (SCA) that uses trigonometric functions to redefine the search space and allows to guarantee the exploitation of space between two solutions is incorporated. The range of distance and direction of movement in the cyclic pattern of the sine-cosine function is updated adaptively to emphasize the exploitation as the iteration counter grows.

Chaotic differential evolution algorithm is hybridized with the sine-cosine algorithm. The performance of the hybrid algorithm is tested through simulations on generalized benchmark test functions and hydrothermal systems.

In the third attempt, the self-adaptive differential evolution algorithm, a global search technique is integrated with a Simplex search method, a local search technique. Chaotic differential evolution algorithm is responsible for diversification whereas Simplex search is dedicated to the exploitation of search space. The accuracy of the elite solution is improved by invoking the simplex technique for neighborhood search. The proposed hybrid algorithm identifies the solution close to the Pareto front and supports the selection of suitable compromising solution among the available options of conflicting objectives hydrothermal system problem.

In the fourth attempt, the artificial neural network (ANN) training algorithm is hybridized with differential evolution. ANN-based methodologies possess some interesting qualities like learning from experience, ability to handle uncertainties and ambiguity in data set, modelling the nonlinear and complex relationship between input-output data, and store information in the form of the weights of connections between the network layers. The major challenge with ANN is in dealing with adaptive learning and to maintain the balance between recently acquired knowledge with information already embodied in the network. The selection of geometrical configuration, learning parameter values, and learning strategy in multi-layered neural networks greatly influence the convergence rate and accuracy of an optimal solution. In this study, the feedforward neural network (FNN) model is proposed for the mapping of variables. The training time and global search capability of FNN are influenced by the initial population set and the learning parameters. The proposition of a Salp chain mechanized behavior of interaction is enabled for a search around the individual, individual best, and the global best position to generate a promising population that accelerates the convergence speed. The chaotic differential evolution is responsible for diversification and is adopted to optimize the learning strategy of FNN. The external elite approach is used to archive the non-dominant solution.

In the thesis, the validity and effectiveness of the proposed novel metaheuristic techniques developed to solve multi-objective constrained optimization problems have been extensively verified and numerically tested on two categories of optimization problems. In the first category, a set of twenty diverse benchmark test functions unimodal and multimodal groups is selected to investigate the exploitation and exploration capability respectively, of proposed hybrid techniques. The quantitative performance analysis of each problem is estimated for its fitness value in terms of the best, average, the worst, the standard deviation, whereas qualitative performance is observed from the search history, trajectory of a population member, average fitness of the population, and convergence behavior in the relevant parametric search space. The capability of the proposed algorithm is evaluated against well-established state-of-the-art meta-heuristic algorithms, reported in the literature for a fair comparison.

In the second category of numerical optimization of real-world application, hydrothermal generation scheduling problems of varied dimensions are selected. Fuzzy model of handling uncertain violations in the equality and inequality constraints due to the stochastic nature of decision variables and variations in load demand and water availability in reservoirs is the contribution of this thesis work.

For a fair comparison of the efficacy of the proposed algorithms, the quantitative results obtained after the application of the proposed variants of DE in terms of the minimum cost, minimum emission are compared with the state-of-the-art metaheuristic techniques. Cardinality ranking operator is preferred to rank the best-compromised solutions obtained from the compared the most utilized meta-heuristic stochastic optimization algorithms applied to respective test systems. The robustness and the accuracy of the global solution obtained from the suggested algorithms are measured through statistical significance tests.

Through the investigations in this study, it is concluded that no single strategy can improve all the performance metrics of differential evolution when applied to a variety of optimization problems. However, the major contributions to P-heuristics when applied to solve non-linear multi-objective optimization problem are summed up as:

Three strategies to improve the initial population are applied. Firstly, by exploring the opposite population that improves the diversification of population. Secondly, to randomize the population with the sine-cosine strategy that cyclically repositions the population member around the other and supports exploitation of space between two solutions. This strategy takes care of the problem of isolation of global optimum. Thirdly, the use of the Salp chain behavior, the individual member of the population is guided towards the global best position. This strategy has drastically increased the convergence rate.

Two strategies at mutation and crossover levels of DE are explored. Chaotic tuned mutation constant and crossover rate controls diversification and intensification within feasible space. Crisscross mutation and crisscross crossover operations are employed in orthogonal directions alternately in three-dimensional search space. This strategy prevents the stagnation of local minima. To archive elite solutions, opposition based learning is incorporated to generate potential Pareto set of non-dominant solutions. In the second phase of the study, the investigation of differential evolution hybrid with Simplex search confirms the robustness of the algorithm. In the third phase, feedforward neural network a nonlinear optimizer is trained using differential evolution. This meta-optimization strategy will justify the computational expenses in terms of space and time when applied to large scale optimization problems, in case the issue of parameter values is resolved.

LIST OF PUBLICATIONS

Publications from Research work (Chapter wise)

1. **Chapter 3**

   Manbir Kaur, J.S. Dhillon and D.P. Kothari, Crisscross Differential Evolution Algorithm for Constrained Hydrothermal Scheduling, ***Applied Soft Computing****:93, 106393, 2020.* (**SCI Impact Factor: 4.873)**

2. **Chapter 5**

   Manbir Kaur, J.S. Dhillon and D.P. Kothari, Short-term Hydro-thermal Generation Scheduling using Differential Evolution Trained Neural Network, Aegaeum Journal, **8(7): 2020 (UGC Care list Approved Journal Scopus indexed)**

Publications under Review

1. **Chapter 4**

   Manbir Kaur, J.S. Dhillon and D.P. Kothari, Multi-objective Fixed Head Hydrothermal Scheduling using Simplex Integrated Self-adaptive Differential Evolution Optimization, **Soft Computing.** (Submitted) (SCI Impact Factor 1.632)

2. **Chapter 5**

   Manbir Kaur, J.S. Dhillon and D.P. Kothari, Meta-Optimization of Hydrothermal System with Self-referential Differential Evolution Trained Neural Network, **Neurocomputing** (Revision Received) (SCI Impact Factor 4.438)

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LIST OF ABBREVIATIONS

Notation	Description
ABC	Artificial Bee Colony
ACO	Ant Colony Optimization algorithm
AFS	Artificial Fish swarm
AI	Artificial Intelligence
AIS	Artificial Immune system
ALO	Ant lion optimization
ANN	Artificial Neural Network
ANOVA	Analysis of Variance
AR	Acceleration Rate
BA	Bat algorithm
BBBC.	Big-bang big crunch algorithm
BFOA	Bacterial Foraging Optimization algorithm
BH	Black Hole theory
BPNN	Backpropagation Neural Network
BSO	BrainStorm Optimization
BTA	Backtracking search algorithm
CA	Cultural algorithm
CDE	Chaotic Differential Evolution
CCDE	Crisscross Differential Evolution
CCDESC	Crisscross Differential Evolution hybrid with Sine-Cosine
CMODE	Chaotic Multi-objective Differential Evolution
CRO	Chemical Reaction Based optimization
CS	Cuckoo Search Algorithm
CSA	Clonal Selection Algorithm
CSO	Civilized Swarm Optimization
DA	Dragonfly algorithm
DE	Differential Evolution
DEO	Dolphin echolocation Optimization
DG	Distributed Generation
DM	Decision-Maker

LIST OF ABBREVIATIONS (Continued)

Notation	Description
EA	Evolutionary Algorithm
EM	Electromagnetism algorithm
EP	Evolutionary programming
FA	Firefly Algorithm
FACTS	Flexible Alternating Current Transmission System
FLC	Fuzzy Logic controller
FNN	Feedforward Neural Network
FOA	Fruit Fly Optimization algorithm
FPO	Flower pollination algorithm
GA	Genetic Algorithms
GSA	Gravitational search algorithm
GWO	Grey wolf optimizer
HAS	Harmony search algorithm
HCMSPSO	Hierarchical Cluster-based Multi-Species Particle-Swarm Optimization
HGSA	Henry gas solubility based algorithm
HNN	Hopfield Network
HTGS	Hydrothermal Generation Scheduling
IMO	Ion motion optimizer
IMCA	Imperialist Competitive algorithm
IPSO	Improved Particle Swarm Optimization
KHO	Krill Herd optimizer
KKT	Karush-Kuhn-Tucker
k-s	Kolmogorov-Simonov Test
LP	Linear Programming
LS	Local Search
LSA	Lightning Search algorithm
LSHADE	Linear population size and success history-based DE parameter
LUBE	Lower upper bound estimation
MEO	multi-objective evolutionary optimization

LIST OF ABBREVIATIONS (Continued)

Notation	Description
MOCA	Multi-objective cultural Algorithm
MS	Monkey search algorithm
MSE	Mean Square Error
NSGA	Non-dominated Sorting Genetic Algorithm
NGS	Neighborhood guided selection
NFE	Number of Function Evaluations
NFL	No Free Lunch
NLP	Non-linear Programming
NP	Non-Polynomial
ODE	Opposition Differential Evolution
PMOGA	Parallelized Multi-objective Genetic Algorithm
PPO	Predator-Prey Optimization
PSO	Particle Swarm Optimization
PV	Photovoltaic
QP	Quadratic Programming
QPSO	Quantum behaved Particle Swarm optimizer
QRSO	Quasi-reflected symbiotic organisms optimization
RPS	Robotic Parallel System
RNFS	reinforcement neural fuzzy surrogate
SA	Simulated Annealing
SaDE	Self-adaptive Differential Evolution
SaDESM	Self-adaptive differential Evolution hybrid with Simplex search algorithm
SCA	Sine-Cosine Algorithm
SPEA	Strength Pareto Evolutionary Algorithm
SDENN	Self-adaptive differential Evolution hybrid with Neural network
SFLA	Shuffled Frog leap algorithm
SMA	Slime mould algorithm
SPO	Spiral Optimization algorithm
SQP	Sequential Quadratic Programming

LIST OF ABBREVIATIONS (Continued)

Notation	Description
SR	Success Rate
TLBO	Teacher Learner based Optimization
TS	Tabu Search
TSK	Takagi-Sugeno-Kang fuzzy
WCA	Water Cycle Algorithm
WHA	Whale optimization

NOMENCLATURE

Notation	Description
a_i, b_i, c_i, d_i *and* e_i	operating cost coefficients
B_{00}, B_{i0}, B_{ij}	Transmission Loss coefficients
$C_{1j}, C_{2j}, C_{3j}, C_{4j}, C_{5j}, C_{6j}$	Hydraulic coupling coefficients.
CR	crossover rate
D	Problem dimensions
F	Mutation Constant
F_C	Constraint objective function
F_{opp}	Opposition factor.
$f_i(\cdot)$	Scalar-valued design objective function
$g_k(\cdot)$	Scalar-valued inequality constraint function
$h_j(\cdot)$	Scalar-valued equality constraint function
i	Index to a decision variable
j	Index to a decision variable
k	Index to a decision variable
n	Number of control/design variables
M	Number of objectives
p	Probability value
T	scheduling period.
U	Utility function
X	Vector of decision variables
I_{kj}	Water inflow rate in j^{th} reservoir in k^{th} time-interval
F_n^{min} and F_n^{max}	upper and lower bounds of the n^{th} objective function obtained
N_H	number of hydro unis
N_G	total number of generating units
.N_T	number of thermal units
N_P	Population size
N_{Pw}	Neural network weight population size
N_{GM}	Number of neurons in hidden layer
P_{Dk}	load demand in k^{th} time-interval

NOMENCLATURE (Continued)

Notation	Description
P_{ki}	power generation of i^{th} thermal unit
P_{ks}	active power generation for slack generator.
P_{Lk}	transmission losses during k^{th} sub-interval,
P_i^{min} and P_i^{max}	lower and upper limits of active power generation of i^{th} hydro or thermal unit
q_{hj}	Water discharge from j^{th} hydro unit in h^{th} time-interval
q_{kj}	water discharge rate from reservoir of j^{th} unit in k^{th} sub-interval
q_j^{min} and q_j^{max}	minimum and maximum water discharge rate limits of j^{th} hydro unit respectively.
r_1 and r_2, r_3 and r_4	randomly selected numbers
R_H	number of reservoirs on upper side
R_{jki}	uniform random number
S_T	violations in the total reservoir volumes utilized by hydro units at the end of scheduling period,
S_V	violation in the volume of water in for each reservoir in all subintervals
S_{P_D}	violation in load demand balance
S_{PH}	average of membership functions of the violations of hydro generation limits over the planning period of all the hydro units
S_{PT}	average of membership functions of the violations of thermal generation limits over the scheduling period of all the thermal units
S_{kj}	Water Spillage from j^{th} unit in k^{th} sub-interval
T_{DEW}	Number of iteration of learning loop for weight learning
T_{MAX}	Maximum number of iterations of the main loop
$v_j(\cdot)$	Violation in j^{th} constraint
V_{kj}	storage volume, of j^{th} unit in k^{th} sub-interval
$V_{(k-1)j}$	storage volume, of j^{th} unit in $(k-1)^{th}$ sub-interval

NOMENCLATURE (Continued)

Notation	Description
V_{1j}	Initial volume in j^{th} reservoir
V_{T+1j}	final storage volume of j^{th} reservoir at the end of the scheduling period
V_j^{min} and V_j^{max}	Lower and upper limits of storage capacity of j^{th}reservoir
x^b	Basepoint
x^e	Expansion point
x^r	Reflection point
x^w	Worst point
$x_1, x_2, \dots, x_n$	Set of design/control variables
x^*	Non-inferior solution
x_i^{min}, x_i^{max}	Minimum and maximum limits to i^{th} component of design variable x
X_{jki}^t	base vector
X_{Bki}^t	Current-best individual in t^{th} generation
Z^*	Pareto optimal set
Z_F	Pareto optimal Front
α	Level of significance
$\alpha_1, \alpha_2, \alpha_3$	Salp chain coefficients
β_{ih}	Slope of sigmoidal activation function between input to hidden layer
β_{ho}	Slope of sigmoidal activation function between hidden to output layer
w_{ij}	Weight of connection between j^{th} node in l layer to i^{th} *node in* $(l-1)$ layer
$\alpha_i, \beta_i, \gamma_i, \eta_i$ and δ_i	pollutant's emission coefficients of i^{th} thermal unit.
ϵ	Convergence criteria
μ_D	unified objective function or cardinal priority ranking.
$\boldsymbol{\mu_{F1}}$	Membership function of objective function F1
μV_{ki}	violation of reservoir volume in each subinterval of the scheduling period.
ϵ_{PD}	tolerance of error in power demand mismatch during k^{th} sub-interval

NOMENCLATURE (Continued)

Notation	Description
$\mu(\mathrm{P_{kj}})$	violation of limit of active power from j^{th} hydro unit in k^{th} sub-interval
$\mu(\Delta P_{Dk})$	violation in the load demand mismatch in each subinterval
$\mu(\mathrm{P_{ki}})$	violation in the active power output of i^{th} thermal unit.
$\mu(F_n)$	Membership function of objective function
$\mathcal{X}_{jki}^{t} H_{jki}^{t}, V_{jki}^{t}, U_{jki}^{t}\ or\ W_{jki}^{t}$	trial vector representation
ϵ_{Vol_i}	tolerance of error in satisfying the total available water constraint of i^{th} hydro unit reservoir for the scheduled time
$\epsilon_{Vol_i}^{U}$	upper range to decide the satisfaction level of the total available water constraint of i^{th} hydro unit reservoir.
$\mathbb{S}$	Feasible search space
τ_m	water transport time delay from m^{th} upper reservoir to lower reservoir 'j'

CHAPTER - 1

HYDROTHERMAL SCHEDULING OPTIMIZATION

1.1 INTRODUCTION

In real-world power system operation, the integrated operation of hydro and thermal units for optimum utilization of energy resources in the most economical and environmentally safe manner is envisaged on account of reduction in energy reserves and environmental degradation due to excessive use of conventional fossil-fuels. However, the active power output from generating units essentially depend upon the input energy (Saiprasad *et al.,* 2018). The active power generation from thermal units is primarily responsible for the electricity cost on account of fuel choices from the economy, energy security and environmental considerations (Chandler *et al.,* 1953), whereas the source of hydropower is natural water resource. Owing to the insignificant marginal cost of hydro plants, the optimum generation scheduling of the hydrothermal system is essentially focussed to the operational cost of the committed thermal units. In the operation of thermal units, throttling losses in steam turbine valves during their opening and closing to control the steam inlet and vibrations in shaft due to load variations lead to variation in incremental heat rate characteristics. This variation is considered as valve-point loading and amounts to increased operational cost. Therefore, the input-output characteristics of the thermal unit are non-linear and discontinuous. Further, *Environment Protection Amendment act* recommends the lessening of the fatal and harmful emissions such as oxide by-products of carbon, sulphur and nitrogen, *etc.* from fossil fuel-fired thermal plants to the permissible limits reported in the proceedings (Conf. Proc., 2018). Therefore the generation scheduling of hydrothermal (HTGS) systems is viewed as a bi-objective optimisation problem of conflicting nature.

Also, fixed head major hydropower units are constrained by the amount of water available in the reservoirs, prohibited bounds of reservoir volumes, limits on spillage of water and water to be released or to be drawn down during planning period, (Farhat and El-Hawary, 2009) etc.. In general, the relation between input water energy and output power is linear for fixed head plants. The large capacity hydro plants are designed as cascaded units for effective utilisation of water energy. The water transport delay in the downstream hydraulically coupled units, the prohibited regions of operation and the amount of available water in downstream reservoirs change the slope of incremental cost curve of hydro units for increased loads. Supplementary constraints associated with hydro generation units such as flood checks and control due to incessant rains, seasonal irrigation and other water supply requirements, fishing and other recreational activities, etc. may be considered as constraints depending upon contracts or legal obligations. However, the load demand and water inflows are taken as pre-specified attributes in hydrothermal generation scheduling (Wood *et al.,* 2012).

In the light of these observations, the scheduling of hydrothermal system over the specified time-horizon is revealed as a non-linear, discontinuous NP-hard and complex numerical optimization problem with a mix of linear/non-linear constraints. The objective of solving the hydrothermal generation scheduling problem is to find the generation dispatch from committed hydro and thermal units to meet energy demand over the scheduling period, with the minimum operational cost of thermal units and with minimum emission of pollutant gases to the environment in a given period while preserving all physical and technical constraints. However, it is somewhat difficult to solve this complicated optimisation problem under the considerations of dynamic hydraulic and operational constraints of hydrothermal system.

1.2 OPTIMIZATION ALGORITHMS

Optimization is an important research area in the economic operation of an interconnected power systems. The primary aim in solving the optimization problem is determining the objective function that states the relations between the physical system parameters and the underlying system constraints. In numerical optimization of power system operation, problems are large scale, dynamic and constrained ones. The objective functions may be non-linear, non-separable and discontinuous and are usually designed in a way that defines the optimum solution of the objective function as the global minima. The process of searching for global optima in the problem-domain search space of the optimisation problem is called global optimisation. The desirable features of optimisation algorithms include the capability to search a problem's global minima with the minimum number of function evaluations, less computational cost and complexity, the ability to set algorithm-dependent control parameters, robustness and flexibility in generalization with a variety of problem models (Deb, 2012).

Mathematically, an optimisation problem can be expressed as:

Minimize $f_i(x)$; $(i = 1,2, \dots, M)$ and $x \in \Re^D$ (1.1)

Subjected to $h_j(x) = 0$; $(j = 1,2, \dots J)$ (1.2)

$g_k(x) \leq 0$; $(k = 1,2, \dots K)$ (1.3)

where, $f_i(x)$, $h_j(x)$ and $g_k(x)$ are scalar-valued design vector functions,

$X = [x_1, x_2, \dots, x_D]^T$ represents a set of design/control/decision variables. These variables may be continuous, real discrete, or a combination of both.

D denotes the dimensions of an optimisation problem.

The functions, $f_i(x)$ where $(i = 1,2, \dots, M)$ are termed as objective functions and the case $M = 1$ represents a single objective function optimisation problem.

The area covered by boundaries of decision variables is called search space $\Re^D$ and the area formed by objective function is called feasible solution space $\mathbb{S}$, such that $\mathbb{S} \subseteq \Re^D$; $h_j(x)$ and $g_k(x)$ are equality and inequality constraints.

The searchable design space designed by may have maximum and minimum limits of decision variables called side or soft constraints defined as:

$X^{min} = \left[x_1^{min}, x_2^{min}, \dots, x_D^{min}\right]^T$ and

$X^{max} = [x_1^{max}, x_2^{max}, \dots, x_D^{max}]^T$

An optimisation problem with one or more equality and/or the inequality constraints is solved as a constrained optimisation problem and the optimum solution of the problem must satisfy these constraints. An unconstrained optimisation problem does not have any hard constraints but is committed to side constraints.

Optimisation problems are classified based on:

- Number of control variables: Univariate ($D = 1$), Multi-variate ($D > 1$)
- Type of control variables: Continuous, Discrete, Mixed Integer
- Type of objective functions: Linear (LP), Quadratic (QP), Non-linear function of control variables (NLP)
- Type of problem: Constrained, Unconstrained

Some important terms about optimisation problem are summarised as:

Feasible and Infeasible Solution

A point $\hat{x}$ in the feasible space is considered as a feasible solution if all the equality and inequality constraints and the decision variable bounds are satisfied at that point. All other points, those do not satisfy the equality and inequality constraints are termed as infeasible solutions.

Optimal Solution

An optimal solution x^* is a point in $\mathbb{S}$ that satisfies: $f(x^*) \leq f(x), \forall x \in \mathbb{S}$ An optimal solution may not exist or may not be unique.

Local and Global minima

For an optimisation problem defined in Eq. (1.1), a point $\bar{x}$ is said to be

- Local minimum; if $\bar{x} \in \mathbb{S}$ and $\exists \epsilon > 0, such\ that\ f(\bar{x}) \leq f(x), \forall x \in \mathbb{S}, \forall x \in B(\bar{x}, \epsilon) \cap \mathbb{S}$
- Strict Local Minimum: if $\bar{x} \in \mathbb{S}$ and $\exists \epsilon > 0, such\ that\ f(\bar{x}) \leq f(x), \forall x \in \mathbb{S}, \forall x \in B(\bar{x}, \epsilon) \cap \mathbb{S},\ and\ x \neq \bar{x}$
- Global Minimum: if $\bar{x} \in \mathbb{S}$ and $\exists \epsilon > 0, and\ if\ \ f(\bar{x}) \leq f(x), \forall x \in \mathbb{S}$.
- Strict Global Minimum: if $\bar{x} \in \mathbb{S}\ and\ if\ \ f(\bar{x}) \leq f(x), \forall x \in \mathbb{S}, \forall x \in \mathbb{S},\ and\ x \neq \bar{x}$

where according to 2-norm definition $B(\bar{x}, \epsilon) = \{x |\ \|x - \bar{x}\| \leq \epsilon\}$

Conditions of Optimality

For convex functions, the necessary and sufficient condition for the unconstrained optimisation problem, the first-order derivative is zero such that $\nabla f\overline{(x) = 0}$ for local and global optimality.

For non-convex functions, the necessary condition for local optimality is that second-order derivative is positive such that $\nabla^2 f(x) \geq 0$.

NP-hard Combinatorial Problem

An optimisation problem is said to be an NP-hard problem when an optimisation technique allocated to solve the NP-hard problem takes non-deterministic polynomial time.

Optimisation techniques are classified as presented in Figure 1.1 and are selected based on computational requirements, convergence properties, and the nature of the objective function of the problem to be solved. Based on the type of obtained solution, analytical methods are used to derive optimality condition equations to find out an optimal solution. This method is preferred when the number of decision variables is less (one or two), otherwise it will be very complex to solve optimality conditions. Broadly there are four types of computational models to solve optimisation problems namely direct search models, dynamic search model, stochastic search model and intelligent computational techniques.

Direct search methods, a family of derivative-free algorithms are characterized by the absence of the construction of a model of the objective function. Hooke and Jeeves, 1961 coined the phrase "*direct search*" to designate the sequential analysis of trial solutions involving comparison of each trial solution with the "best" obtained up to that time together with a strategy for determining, the next possible trial solution.

Gradient search methods are first-order iterative procedures and respond fast in a continuous space. These methods face difficulty with multiple local minima, when there is a complex relationship between parameters and the underlying objective function of the problems, as it will make calculations of derivatives difficult. Univariate methods search the direction with one variable at a time hence is limited to a problem with one or two variables. Pattern search methods are multi-direction search methods (Lewisa, *et al.,* 2000) and can steer the search direction towards a global solution with the proper selection of parameters.

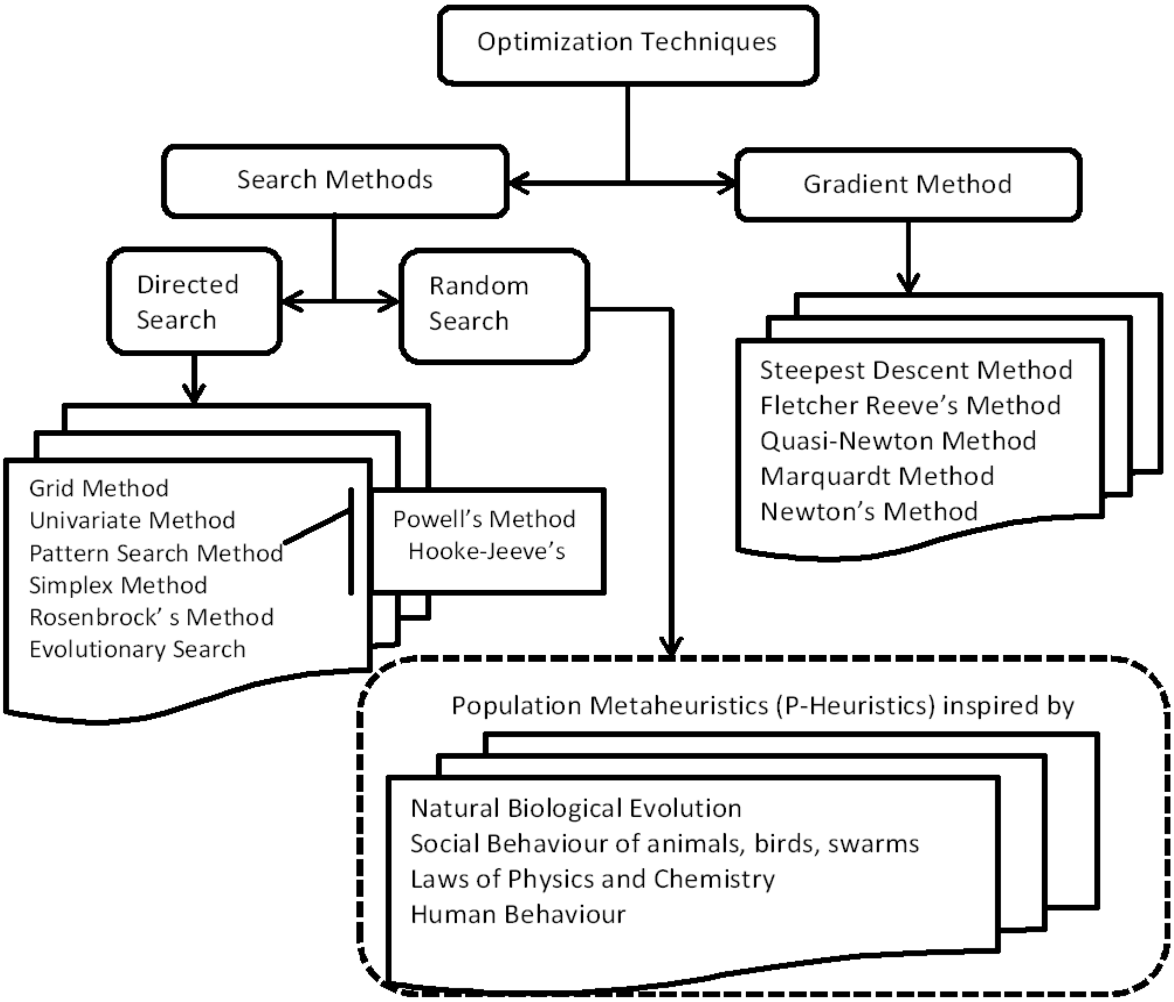


Fig. 1.1 Classification of optimisation techniques

Rosenbrock's method proceeds in several stages and takes the number of exploratory searches along with a set of directions that are fixed for the given stage, but which are updated from stage to stage to make use of information acquired about the objective. It does not prefer repeating the search process with the same set of orthogonal vectors, as is done for

the method of local variations. Rosenbrock rotates the set of directions to capture information about the objective ascertained during the early stages of search moves.
When the number of decision variables and constraints is more and the objective function is non-linear such that it is difficult to find the derivative of the function, then gradient-free search methods are preferred.

Since decades, there are too many optimisation techniques proposed to solve hydrothermal generation scheduling problems and are summarised in the following sections.

1.2.1 Conventional Optimisation Techniques

Several classical mathematical optimisation techniques are applied to solve the hydrothermal generation scheduling problem. Conventional individual-based direct search techniques such as variational calculus method (Grake *et al.*, 1962), maximum principal (Papageorgiou, 1985), conjugate gradient search method (Carvalho, 1987), progressive optimality method (Nanda *et al.*,1986), functional analysis for the variable head system (Solima and Christensen, 1986), dynamic programming for unit commitment (Yang and Chen, 1989), Lagrange multiplier method (Rashid and Nor, 1993), network flow method (Oliveira and Soares, 1995), Lagrange relaxation method (Guan and Luh, 1995), modified Lagrange relaxation with decomposition technique (Ruzic *et al.*, 1996) to handle non-linearity and non-convexity of HTGS, non-linear programming method (Xia *et al.*, 1998), progressive successive approximation method (Ferrero, 1998) for long term scheduling etc., have been applied to solve hydrothermal dispatch problem. Maximum principle and variation calculus are the direct function methods those face difficulty to handle the non-linearity of the problem. Progressive optimality method and network flow method suffer from getting entrapped in local minima easily. On the other hand, Lagrange multiplier and gradient methods assume mathematical models of hydro and thermal systems as piecewise linear or polynomial approximations of monotonic increasing nature, suffers slow convergence and selection of multipliers is a challenging task as well.

Relaxation method relaxes hard constraints and considers the problem as a dual problem and carries out separate scheduling of hydro and thermal units. The dual solution in the separable condition is generally infeasible. Dynamic programming technique faces dimensionality difficulties and large memory requirements and time. The gradient optimisation method is fast provided the objective function is differentiable but lacks a global solution. Further, these methods require complete information about the objective function, dependence on each control variable, and the nature of the function restrict their application to multi-dimensional optimization problems. Some assumptions are also practised to realize the function as a continuous one at the cost of solution accuracy.

To overcome the aforesaid difficulties of conventional direct and dynamic search techniques, researchers suggested the modified versions and applied to solve the generation scheduling problem with a reasonably acceptable solution. Kold *et al.,* (2003) applied new perspective approaches in classical search methods. Among local search techniques, Powell's method (El-Hawary, 1982), Simplex method (Irving, 1983) and other pattern search methods (Sherif and Boice, 1994), are the non-gradient type local search methods and are capable of solving non-linear unconstrained optimisation problems successfully. Powell's method is based on the concept of conjugate directions, while the Simplex algorithm makes use of a pivot simplex and a set of simple rules that reflects the worst vertex through the centroid of the simplex. Pattern search method carries out an exploratory search in the improved direction. Also, dual-quadratic programming (Finardi, 2006) method that finds a search direction based on a quadratic approximation of the objective function and linear approximations of the constraint functions. Bender's decomposition (Santos *et al.,*2009), gradient descent methods, Lagrange relaxation (Rodrigues *et al.,* 2012), mixed-integer linear/nonlinear programming (Jian *et al.,* 2019) can handle both discrete and continuous variables along with nonlinear objective functions.

To conclude, these point by point search methods generates one solution in one run and needs some rough and impractical assumptions to simplify the problem or to break a bigger problem into smaller ones at the cost of the accuracy of the global solution. Although simple in implementation when appropriately selected for the specific problem, but are time-consuming as there is no support of exploration of search space, and needs more computational effort since its convergence behaviour depends upon the initial starting point and the nature of the objective function. No doubt, conventional optimisation methods are applied to solve the HTGS problems and generate a locally optimal solution efficiently but lack in global solution accuracy on account of single solution methodology besides premature phenomena and time complexity in solving large scale problems. Their modified versions offer better exploration with pattern search stages, yet converge to a suboptimal solution in multimodal problems.

1.2.2 Stochastic Heuristic Techniques

There are many challenges in finding an optimum solution to real-world problems with deterministic optimization. The fundamental reasons may be erroneous measured data or uncertainty in data representing forecasted information about an inflow of water in hydro plant reservoirs, load demand or fuel price etc. for the future period. Besides, the presence of noise in the objective function evaluations, the difficulties in distinguishing a globally optimal solution from locally optimal solutions, the dimensionality problem, the difficulties associated with non-trivial constraints, and the lack of stationarity in the solution as a result of the dynamic conditions of the problem, pose challenges to the conventional optimization models. On the other hand, robust optimization techniques can generate an optimum solution in the feasible space, when only parameters limits are known (Datta and Deb, 2010).

Stochastic optimization methods take advantage of the fact that probability distributions of data are known and can be estimated as well. This aspect is especially useful in treating some of the challenges of conventional techniques (Spall, 2012). There is a great challenge to search optimum solution to a multi-dimensional, non-linear, non-convex, constrained hydrothermal generation scheduling problem using deterministic optimization models. Linearization of function and constraints lead to a sub-optimal solution and ultimately loss in revenue. To solve the hydrothermal generation scheduling problem, a robust, flexible and versatile strategy seeking a globally optimal solution is required.

Metaheuristic algorithms form an important part of contemporary global optimization algorithms, computational intelligence and soft computing. Resolution by metaheuristics can avoid significant financial loss.

In the recent decade, there is a flood of a variety of stochastic population-based search numerical optimization techniques. The researchers have invested in immense efforts and have developed many methods mimicking natural evolution of living beings, imitating the social behaviour of human beings, animals, birds, and swarms, and others based on laws of nature, physics and chemistry etc.. These meta-heuristics are derivative-free methods and generate multiple non-dominated solutions for Pareto-front in a single run. These metaheuristics are well adapted to solve optimisation problems irrespective of their core mathematical model (Qu *et al.*, 2018). The era of evolutionary algorithms (EAs) started in 1989 as an effective stochastic tool extended to explore the problem's search space (Venkatesh *et al.*, 2003). Among the variety of stochastic global optimisers, *Genetic algorithms* (GA), *Differential Evolution* (DE) and *Particle swarm optimisation* (PSO) achieve remarkable popularity. The classical form of these optimisers has been applied in a variety of applications such as GA (Gil, 2003), PSO (Yu *et al.*, 2007), DE (Huang *et al.*, 2006) and many more reported in the literature in the previous decade. These algorithms are prone to the tuning of plausible control parameters and suffer the convergence stagnation in multimodal large scale problem and computational time. Some of the variants are fuzzy clustering-based PSO to solve non-convex economic dispatch problem (Agarwal *et al*, 2008), real coded GA with wavelet mutation for stochastic optimisation (Dhillon *et al.,* 2011), small population-based PSO (Zhang *et al.*, 2012), self-organising Hierarchal PSO (Mandal, 2012), Parallel population and bi-segmented crossover in GA (Feng, 2017), PSO-hybrid with PPO (Narang *et al.,* 2014), fully informed PSO to solve non-cascaded HTGS problem (Fakhar *et al.,* 2015), hybridised with GA (Ali, 2017) with improved global search capability and many more and many more. Further inspired from natural evolution, the search capability of classical DE has been reviewed by (Wu *et al.,* 2018), and is attributed its performance to cooperation and competition among individual vectors, parameter selection, differential mutation, crossover and greedy decision, but suffers from weak local search.

There is a lot of research on new meta-heuristic optimizers with specific global and local search strategies and providing choices to researchers. Each one of reported population-based strategy flutters on one of its achievement in terms of improvement in either of the traits of optimisation problem solution; convergence speed, flexibility, solution accuracy, avoidance of premature convergence, exploration of local search, exploitation of global search, tuning of control parameters, simple operating mechanism. Some of the P-heuristics employed for all hydrothermal generation scheduling are summarised in Table 1.1.

Table 1.1 Application of stochastic algorithms to the hydrothermal scheduling problem

Stochastic Algorithm	**Background**			**Hydrothermal scheduling applications**	
	Researcher, Year	**Inspired by**	**Features**	**Author, Year**	**Salient features of the approach**
Swarm based Optimization Algorithms: Inspired from social behaviour of animals, birds and swarms behaviour					
Ant Colony Optimisation algorithm (ACO)	Marco Dorigo, (1992)	Swarm of ants (distance and amount of pheromone on paths)	Evaluate the quality of paths and choose between them with a random probability	Huang, (2001)	Implementation of the state transition rule, local pheromone-updating and global pheromone-updating rule on collection of cooperative agents to avoid stagnation
Particle swarm optimization (PSO)	J. Kennedy and R. Eberhart, (1995)	Swarm behaviour of particles	Social behaviour and movement dynamics of birds	Amjady and Soleymanpour, (2010)	Adaptive controlled PSO parameters using tree topology
Bacterial Foraging Optimisation algorithm (BFOA)	Kevin Passino, (2002)	Nature-inspired of foraging strategy of a swarm of escherichia-coli bacteria	Mimicking the chemotactic movement of virtual bacteria	Farhat and El-Hawray M.E., (2011)	An efficient strategy to manage large dimension conflicting objectives

Stochastic Algorithm	Background			Hydrothermal scheduling applications	
	Researcher, Year	Inspired by	Features	Author, Year	Salient features of the approach
Artificial Fish Swarm (AFS) algorithm	Li Xiaolei, (2002)	A collective movement of the fish and their instinctive social behaviours	Mixed crossover strategy, lack of balance between global and local search	Fang *et al.* (2014)	High convergence speed, flexibility, fault-tolerant and high accuracy but suffers from high time complexity
Artificial Bee Colony (ABC)	Devis Karaboga, (2005)	Inspired by intelligent honey bee colonies	Adaptive mutation /selection and local search	Lia *et al.*, (2013)	Adaptive chaotic ABC to escape from local minima and premature convergence
Firefly Algorithm (FA)	Xin-She Yang, (2008)	Flashing pattern of tropical fireflies	Fireflies mutual attract based on light attenuation over distance, adaptive subdivision	Dekhici *et al.*, (2012)	Economic dispatch of thermal units with emission, dimensionality problem but handles multimodality
Monkey Search algorithm (MS)	Ruiqing and Wansheng, (2008)	Bio-inspired from a swarm of monkeys	Climb process, watch-jump process, and somersault process	Ehteram *et al.*, (2018)	Multi reservoir systems
Cuckoo Search (CS)	Xin-she Yang and Suash Deb, (2009)	Brooding parasitism of the cuckoo species.	Neighbourhood double nest search procedure	Nguyen *et al.*, (2018)	Cost as objective, solution dependency on probability parameter and localized search in abandoned nest
Bat Algorithm (BA)	Xin-She Yang (2010)	The sound system of bats, emission and loudness.	Chaotic sequence-based	Yang *et al.*, (2016)	Economic dispatch with multiple fuels, prohibited zones and with ramp rate limits, an abrupt shift to exploitation phase leads to premature convergence
Krill Herd Optimizer (KHO)	A.H. Gandomi, and A.H. Alavi, (2012)	The behaviour of Herd of ocean Krills	Neighbourhood search procedure, foraging activity, and (random diffusion.	Roy *et al.*, (2015)	Cost as objective, discarding of the population on violation of inequality constraint, suffers from poor exploitation due to dependency on random walk steps
Grey Wolf Optimizer (GWO)	Seyedali Mirjalili, Syed M. Mirjalili, and A. Lewis (2014)	Leadership hierarchy and hunting mechanism of grey *wolves*	Hybridized real and integer decision variables	Chowshun *et al.*, (2019),	Best Pareto front, suffers from the fixed positioning of wolves in contrast with social hierarchy
Ant Lion optimizer (ALO)	Seyedali Mirjalili, (2015)	Mimics hunting mechanism of antlions	Random walk of ants, building traps, entrapment of ants in traps, catching preys, and	Dubey *et al.*, (2016)	Exploration with a random walk and roulette wheel operation with elitism for wind integrated model suffers from unbalanced exploration

Stochastic Algorithm	Background			Hydrothermal scheduling applications	
	Researcher, Year	**Inspired by**	**Features**	**Author, Year**	**Salient features of the approach**
			rebuilding taps.		due to more number of computational steps
Whale optimization (WHA)	S. Mirjalili and A. Lewis, (2016)	Hunting strategy of whales, bubble-net attacking behaviour for exploitation	Polynomial mutation to improve population diversity	Yin *et al.*,(2019)	Solar and wind power considering multi-reservoir cascaded hydro plants.
Quasi-reflected symbiotic organisms optimization (QRSO)	T.T. Dieu and J. Lee (2017)	An emotional relationship between two different specie	Three phases of operation mutualism, commensalism and parasitism	Das *et al.*, (2018)	Economic cost as objective, slack generator procedure for constraints handling, suffers from the tuning of the degree of adaptation of favour between mandatory and voluntary symbiotic interactions
Fruit Fly Optimization algorithm (FOA)	W.T. Pan, (2011)	Inspired from food foraging behaviour of fruit fly swarms	Inspiring osphresis and vision behaviour of the fruit flies,	Geruna *et al.*, (2017)	Economic dispatch of all thermal units, suffers from low convergence precision, easily trapped in a local optimum at the later evolution stage.
Biological/ Neurobiological based Optimization Algorithms					
Evolutionary Programming (EP)	Lawrence J. Fogel, (1960)	Natural Law of evolution	Clonal real coded quantum-inspired,	Wong *et al.*, (2012)	Applied heuristic constraint handling, Cauchy mutation,
Genetic algorithm (GA)	John H. Holland, (1960)	Biological evolution	Real coded with random transfer vectors-based mutation	Haghrah *et al.*, (2015)	Minimum thermal power generation and maximizing profit scene, Selection of parameters and population size
Hopfield Neural Network (HNN)	John J. Hopfield, (1982)	Bio-Neurons	Energy function based on augmented Lagrangian relaxation	Dieu and Ongakasul, (2009)	Dynamic selection of Lagrange multipliers, more computation burden
Cultural algorithm (CA)	R.G. Reynolds, (1994)	Societal evolution	Imitates the Lineal evolution of socio-cultural transition	Liu *et al.*, (2011)	Penalty less best Pareto front, management of brief space, population space on account of solution accuracy
Shuffled Frog leap algorithm (SFLA)	M. Eusuff, (2006)	Natural memetic	Cooperative search metaphor inspired by natural memetics. contains elements of local search and global information exchange	Yang *et al.*, (2018)	Binary and real coded variables skip global solution due to hopping. performs an independent local search on each memetic.

Stochastic Algorithm	**Background**			**Hydrothermal scheduling applications**	
	Researcher, Year	**Inspired by**	**Features**	**Author, Year**	**Salient features of the approach**
Brain-Storm Optimization (BSO)	Y. Shi, (2011)	Human brainstorming process	Core ideas of intelligent solution postponing judgment, bold hypotheses, cross-referencing and winning by the quantity	Jordehi, (2015)	Optimal location of FACTS devices
Dolphin echolocation Optimisation (DEO)	A. Kaveh, and Neda Farhoudi, (2013)	Intelligent behaviour of dolphins	Based on biological sonar used by dolphins for navigation, hunting	Rosselan M. Z and Sulaiman, (2018)	For optimal sizing of photovoltaic grid
Clonal Selection Algorithm (CSA)	L. N. de Castro and F. J. Von Zuben, (2016)	Bio-inspired	Inverse mutation Premature convergence and poor solution accuracy in multimodal problems	Roy, (2016)	
Based on Physics/Chemistry/Mathematics based Laws					
Simulated Annealing (SA)	S. Kirkpatrick, C.D. Gelatt, Jr., and M.P. Vecchi, (1983)	Heating and controlled cooling of a material to reduce progressive atomic movement till lowest energy state is achieved	generates a new potential solution (or neighbour of the current state) to the problem considered by altering the current state, according to a predefined criterion.	Wong and Wong (1994)	Constraints are handled using the relaxation method.
Tabu Search	Fred W, Glover, (1987)	Inspired by artificial intelligence	Advanced local search method using depth and breadth search process	Bai and Shahidepour, (1996)	Large scale problem handling with increased computation accuracy of dynamic programming
Electromagnetism algorithm (EM)	S.I. Birbil and Ş.C., Fang, (2003)	Mimics the attraction-repulsion mechanism between charged particles in a magnetic field.	Random local search. based on a pattern search method, and a population shrinking strategy to improve efficiency.	Guo *et al.*, (2011)	Dynamics water spillage strategy, non-preference DM
Big-bang big crunch algorithm (BBBC).	Erol and Eksin, (2006)	The evolution of the universe, namely the big bang and	Energy dissipation and randomly distributed particles Problem to trap in local minima	Sedighizadeh *et al.*, (2014)	DG allocation and emission control, the mutation for better exploration,

Stochastic Algorithm	**Background**			**Hydrothermal scheduling applications**	
	Researcher, Year	**Inspired by**	**Features**	**Author, Year**	**Salient features of the approach**
		big crunch theory			
Gravitational search algorithm (GSA)	E. Rashedi, H. Nezamabadi-pour, and S. Saryazdi, (2009)	Law of gravity and law of motion	Improved exploration and exploitation, performance is measured by weight.	Nadakudi ti *et al.*, (2016)	Non-dominated sorting, with a disruptive operator, fuzzy decision making and use of a slack variable to handle water dynamics and load balance. better performance in binary encoded problems
Chemical Reaction Based optimisation (CRO)	A.Y.S. Lam and Li VOK, (2010)	Chemistry law of molecular collisions and interaction	Real coded and Oppositional real coded for decision variables	Bhattacha rjee *et al.*, (2014)	Single objective, valve point loading effect, slack hydro generator selection, poor global search capability on account of tuning of more operators
Water Cycle Algorithm (WCA)	H. Eskandar, A Sadollah, A. Bahreininejad and M. Hamdi, (2012)	Evaporation rate	mimics the flow of rivers and streams toward the sea, Lacks in randomisation of the population that affects diversification	Haroon and Mallik, (2017)	Weight pattern method, prohibited limits for inequality constraints,
Ion motion optimizer (IMO)	B. Javidy, A. Hatamlou, and S.Mirjalili, (2015)	mimics the attraction and repulsion of anions and cations	Liquid phase and crystal phase of ions for exploration and exploitation, tendency to struck in local minima due to less movement of ions in the crystal phase	Das *et al.*, (2017)	Single objective optimisation, slack generator method of constraint handling, selection of best solution from two phases of ions fitness,
Spiral Optimization algorithm (SPO)	K. Tamura and K. Yasuda, (2016)	The spiral phenomenon of nature	Use no gradients but spiral trajectories, Lacks in diversity, computation effort to shape the spiral trajectory	Benasla *et al.*, (2014)	Penalty price method for combined cost and emission optimization
Black Hole theory (BH)	Katie Bouman, (2016)	The movement law of the "*black hole*" celestial body in the universe	Computational complexity	Liu and Luo, (2014)	Trapped in local minima, difficulty in handling problems with high and low dimensions
Sine-Cosine Algorithm (SCA)	Seyedali Mirjalili, (2016)	Based on a mathematical model using trigonometric	Adaptively balances exploration and exploitation to search the optimal	Das *et al.*, (2018)	Offers computational efficiency and a better quality solution.

Stochastic Algorithm	Background			Hydrothermal scheduling applications	
	Researcher, Year	**Inspired by**	**Features**	**Author, Year**	**Salient features of the approach**
		functions	solution		
Henry Gas Solubility based algorithm (HGSA)	F.A. Hashim E.H. Houssein, M. Mabrouk, W.Al.Atabany and S.Mirjalili, (2019)	based on Henry's law of chemical phenomenon.	Imitates the huddling behaviour of gas to balance exploitation and exploration in the search space	Hashim *et al.*, (2018)	Single objective, slack generator selection, best solution out of the two phases of ions fitness
Human-driven Optimization Algorithms					
Harmony search algorithm (HAS)	Zong Woo Geem, (2000)	Inspired from music and controlled by random search, harmony memory considering rule, and pitch adjusting rule	Adaptive parameters balanced combination of exploration and exploitation and ease of application.	Herris *et al.*, (2018)	The Penalty function for constraints and is solved as a single objective problem. Suffers from an improper balance of global and local search due to poor coordination of pitch adjustment, random selection and memorisation rules of HAS.
Imperialist Competitive algorithm (IMCA)	E. Atashpaz-Gargari and C. Lucas, (2007)	Imperial competition	Colonies with highest cost move toward their imperialist.	Ahgdam and Hagh, (2019)	Security constrained unit commitment
Seeker optimization algorithm (SOA)	C. Dai, (2007)	Concept of simulating human search behaviour	The search direction is based on the empirical gradient by evaluating the response to the position changes. The step length is based on uncertainty reasoning on historical and social experience.	Dia *et al.*, (2009)	Reactive power dispatch with voltage stability criteria.
Teaching-learning based optimization (TLBO)	R.V. Rao, V.J. Savsani, and D.P. Vakharia, (2011)	Emotional behaviour of human	Teaching-learning Emotional mode Characteristics	Roy, (2016)	Adjustment on the violation, the survival of the fittest, lacks trade-off between local and global search
Artificial Immune system (AIS)	J. Zhang, (2011)	Bio-inspired human characteristics	Free from control parameters	Basu, (2011)	Use of the slack bus system suffers from time complexity of social learning
Backtracking search algorithm (BSA)	P. Civicioglu, (2013)	Human behaviour of memorizing	Use sorting procedure, Poor local exploration capability that	Delshad and Rahim, (2016)	Single objective problem with penalty function

Stochastic Algorithm	Background			Hydrothermal scheduling applications	
	Researcher, Year	**Inspired by**	**Features**	**Author, Year**	**Salient features of the approach**
			affects convergence speed		
Nature-inspired Optimization Algorithms					
Flower pollination algorithm (FPA)	Xin-She Yang, (2012)	Pollination of flowers	Effective control of global and local search that supports exploration and exploitation	Dubey *et al.*, (2015)	Smooth cost function and valve point loading models with and without prohibited discharge zones
Lightning Search algorithm (LSA)	H. Shareef, A.A. Ibrahim, and A.H. Mutlag, (2015)	Natural mechanism of lightning and step leader propagation	High convergence rate, reliable but low convergence, accuracy and easy fall to local optimum	Sirjani and Shareef, (2016)	Used as an optimizer in the fuzzy logic controller for PV inverter
Slime mould algorithm	S. Li, H. Chen, M. Wang, Ali A. Heidari, and S. Mirjalili, (2020)	Oscillation mode of slime mould in nature	Use adaptive weights to simulate the process of producing positive and negative feedback of the propagation wave of slime mould. Based on bio-oscillator to form the optimal path for connecting food with excellent exploratory ability and exploitation propensity.	LI *et al.*, (2020)	To solve benchmark test functions of nature unimodal, multimodal, composite etc.

Each one form of global search technique has its own merits and demerits. Ultimately, the global optimum search capability and convergence rate are two conflicting metrics to review any global optimiser. To improve both the metrics simultaneously, researchers intend to improve the current algorithm or recommend a new one.

1.2.3 Differential Evolution

Differential evolution (DE), introduced by Storn and Price in 1995, is a simple and efficient stochastic evolutionary algorithm with the generate-and-test feature for global optimization. The significant difference between DE and other evolutionary algorithms is that it exploits the solutions differences in mutation operator from the current population. Among DE's advantages are its simple structure, ease of use and robustness, it is the most popular among research community to find the global optimum of non-linear, non-convex, multimodal and non-differentiable functions defined in the continuous parameter space with a better quality solution. The search capability of classical DE has been reviewed by Das *et al.* (2016). DE has the advantage of not being biased towards any prior defined distribution for sampling mutational step sizes and its selection operator follows a hill-climbing process. According to the literature, the DE algorithm performs very well in the initial stages however as the search proceeds, the convergence rate of DE slows down gradually. The performance of DE is attributed to cooperation and competition between individual vectors, parameter selection, differential mutation, crossover and greedy decision, but suffers from weak local search. This shortcoming of DE leads to poor convergence speed and premature convergence. Since its inception, various DE variants have been proposed based on novel mutation and crossover strategies, techniques for maintenance of population diversity, an adaption of control parameters, hybridization with other stochastic techniques and intelligent combination of multiple search strategies (Wu *et al.*, 2018).

Table 1.2 Differential evolution (DE) and its variants

DE and its variants	**Variety of Variations with DE**	**Author(s), Year**	**Application**
Population Settings	Opposite population initialization	Rahnamayan *et al.*, (2008)	Applied opposition based learning strategy for population initialisation to handle global solution accuracy and validated through benchmark functions
	Orthogonal population initialization	Xu *et al.*, (2012)	Hydrothermal scheduling
	Population size-reduction	Tanabe *et al.*, (2014)	Benchmark functions
	Dual-population	Gao *et al.*, (2015)	Uses two sub-populations separately for cost function and constraint function and evolved by DE. Information-sharing strategy to exchange search information between the different subpopulations like team cooperation is used. The strategy is validated on benchmark test functions.
	Cooperative DE with multiple populations	Wang *et al.*, (2016)	(M+1) populations for M objectives implemented on multi-objective benchmark test functions
	Small population	Zhang *et al.*, (2017)	Two parallel processes of DE are implemented; one through the small population itself and the other through the communication mechanism among different running processes to enhance diversity and are implemented to solve optimal power flow problem.
	Multi-role population	Gui *et al.*, (2019)	The entire population is divided into multiple small-sized groups, and each group is assigned with different roles in each generation according to their fitness. Based on the assigned role, an individual selects its trial vector generation strategies and select control parameters from a pool to produce offspring for next trial solution.

DE and its variants	Variety of Variations with DE	Author(s), Year	Application
New Mutation Strategy	Trigonometric mutation	Fan and Lampinen, (2003)	Sine-cosine trigonometric mutation strategy is applied to improve the trade-off between convergence rate and robustness of DE. It is applied to solve two test functions and training of the neural network.
	Chaotically tuned mutation and crossover	Yuan *et al.*, (2008)	Chaos theory is applied to obtain self-adaptive parameter settings in DE and is applied to solve the HTGS problem
	Neighbourhood search based mutation operator	Das *et al.*, (2009)	A family of improved variants of the DE/target-to-best/1/bin scheme used the concept of the neighbourhood of each population member. The idea of a small neighbourhood is defined over the index-graph of parameter vectors. It is applied to solve benchmark functions and two real-life problems *viz.*, parameter estimation for frequency-modulated sound waves and spread spectrum radar poly-phase code design.
	Self-adaptive Chaotic parameters	Lu *et al.*, (2010)	Three chaotic sequences are generated to generate three Pareto-fronts to solve hydrothermal scheduling problem.
	Time-varying mutation	Sharma *et al.*, (2010)	Dynamic adaptation of mutation constant and crossover rate is generated to solve the economic dispatch problem.
	Neighbourhood guided selection search	Cai *et al.*, (2011)	Neighbourhood guided selection (NGS) employing operator construction, grouping, ranking and selection to extract the promising search directions to guide DE mutation.
	Novel mutation and crossover strategy	Islam *et al.*, (2012)	DE/current-to-gr-best/1; uses the best of a group of randomly selected solutions from the current generation to perturb the parent vector, and for recombination, a biased parent selection scheme has been incorporated by letting each mutant undergo binomial crossover with one of the top-ranked individuals from the current population. It is applied to 23 benchmark test functions.
	The multi-objective sorted mutation operator	Wang *et al.*, (2014)	Individuals in the current population are sorted according to their fitness and diversity. Parents in the mutation operators are proportionally selected according to their rankings based on fitness and diversity, This strategy is validated through 43 benchmark and 12 real-world problems.
	Ranked mutation operators	Gong *et al.*, (2015)	The population is partitioned in three sets of solution regions. Infeasible solution population is ranked based on constraint violation, semi-infeasible region population is ranked based on transformed fitness function and feasible region population is ranked based on objective function to be optimised. Adaptive mutation ranked DE is applied to solve CEC2006 and CEC2010 benchmark functions.
	Selective pressure strategy	Vladimir *et al.*,	Rank-based and tournament selection mutation

DE and its variants	Variety of Variations with DE	Author(s), Year	Application
		(2018)	strategy for solving CEC2017 functions
	Ordered mutation	Bidgoli *et al.*, (2019)	This approach orders candidate individuals using popular ranking measures of multi-objective optimization problems to utilize the ordered solutions in mutation operator. The best one of three randomly selected solutions is considered as the parent, and two others are applied as second and third candidate solutions in DE mutation and is applied to multi-objective benchmark functions.
Control Parameter Adaptation	L-SHADE: Linear population size and success history-based DE parameter adaptation	Piotrowski, (2018)	Population-wide inertia based on averaged direction and size of successful moves is added to mutation strategy. This strategy is the winner of IEEE-CEC competitions.
	Adaptive Mutation operator	Mohamed and Suganthan, (2018)	Triangular mutation operator based on the convex combination vector of the triplet defined by the three randomly chosen vectors and the difference vectors between the best, better and the worst individuals among the three randomly selected vectors. It supports the search for a better balance between the global exploration ability and the local exploitation tendency as well as enhancing the convergence rate of CEC 2013 test functions.
	Intersect mutation operator	Yinzhi *et al.*, (2013)	Intersect mutation operation divides all individuals into the better part and the worse part according to their fitness. Mutation and crossover operations are applied to generate new individuals. It is applied to benchmark functions.
DE Strategy Variant	An ensemble of DE variants	Wu. *et al.*, (2018)	Variety of population initialization, mutation strategies and selection procedures are reviewed and tested on IEEE CEC data
DE hybridization for the balance of exploration and exploitation	Mixed-integer programming	Su and Lee (2003)	DE is hybridized with mixed-integer programming to solve distribution network reconfiguration problem with minimum loss as objective.
	Hill-climbing Algorithm	Noman and Iba (2008)	A crossover-based adaptive local search (LS) operation to solve benchmark functions is proposed in which LS scheme adaptively adjust the length of the search, using a hill-climbing heuristic.
	Biogeography Algorithm	Gong *et al.*, (2010)	Exploration of DE and exploitation of biogeography (migration operator for information exchange) is used to solve 23 benchmark functions.
	Sequential Quadratic Programming	Sivsubramani and Swarup (2011)	DE is used as a global optimizer and sequential quadratic programming (SQP) method as a local optimizer to fine-tune the solution to a hydrothermal scheduling problem.
	Artificial Immune System	Yildiz, (2013)	Hybrid with positive properties of the Taguchi's method to the differential evolution algorithm for minimizing the production cost associated with multi-pass turning problems.
	K-means clustering	Tvrdik and	Population clustering for classification of objects

DE and its variants	Variety of Variations with DE	Author(s), Year	Application
		Krivy (2015)	
	Harmony search	Zhang *et al.*, (2016)	Harmony search as local search method hybrid with DE to obtain optimal day-ahead scheduling model for a microgrid system with photovoltaic cells, wind turbine units, diesel generators and battery storage systems.
	Powell's Search	Singh *et al.*, (2017)	DE is hybridized with Powell search for economic load dispatch of multi-fuel system.
	Particle Swarm Optimisation	Bingyan *et al.*, (2018)	Forward kinematics of a 3-RPS parallel manipulator
Strategy Adaptation for global optimization	External archive strategy	Zhang and Sanderson (2009)	New mutation strategy DE/current-to-best/rand with optional external archive and adaptively updating control parameters. To solve 20 unconstrained benchmark functions.
	Elite guided composite strategy	Cui *et al.*, (2018)	Adaptive elite strategy to guide potential trial vector generation during mutation is used for solving CEC2014 benchmark functions.

Among all meta-heuristic techniques, exploration and exploitation are the common phrases in their search process irrespective of the nature of the algorithm. Exploration phase generates a new candidate solution over the entire search space and exploitation phase will generate improved solution than the previously generated solutions and hence leads to convergence. However, there is a need for heuristic optimization technique based on the integration of global and local search technique to achieve better speed, efficiency and reliable convergence.

1.2.4 Hybrid Optimization Techniques

The most challenging task in the development of meta-heuristic techniques is to find a proper balance between diversification and intensification propensity due to the stochastic nature of the optimization process (Hashim *et al.,* 2019). Since two decades, the use of stochastic algorithms to solve large and complex optimization problems especially for NP-hard or NP-complete problems has been emphatically recommended and their successful outcome is significantly influenced by population cooperation and competition of individuals. Although metaheuristic optimization algorithms are popular owing simplicity and flexibility and exhibit the exclusive advantages like the quality of a solution, reliable performance and robustness, yet during diversification and global solution search process, there are unexpected changes in the problem-domain population. This may lead to some limitations like premature convergence, falling to a sub-optimal solution, poor solution accuracy, low ability to solve multimodal problems. EA shows slow convergence in multimodal problems near optimum, SA needs an appropriate setting of control parameters, a challenging task for real-world problems. Tabu search is a local search method but has little tendency to trap in local minima due to the associative memory concept but the selection of appropriate depth and breadth in the search process slows down the convergence. Among evolutionary algorithms, GA deals with mutation and crossover operators to produce new off-springs but is affected by the initial population and its size, in addition to the selection of crossover probabilities. The diversification of search space is a major problem with the classical evolutionary DE approach and the issue of stagnation comes while dealing with large scale problems. On the other hand, AIS is an efficient algorithm, less number of control parameters, but poor selection of ageing operator to eliminate old antibodies sometimes leads to premature convergence in large-scale problems. PSO has emerged out as a widely accepted near-global optimizer, but it is computationally expensive (Yildiz, 2013), (Derrac *et al.*, 2014), (Das *et al.*, 2016).

There can be three possible options to overcome such difficulties in general, one; design a new algorithm second; apply the variations in current algorithms to overcome the shortcomings and to enhance their best features to improve the performance, Third; exploit the best features of stochastic heuristics and combine them to design a superior algorithm that will use merits of multiple algorithms to solve complex optimization problems of different nature and type with better performance metrics.

Despite the symbolic amount of new algorithms in recent years, logically the question arises in the minds of researchers in this field, about the need for any other new optimization algorithm. However, the current algorithms are designed in a definite set of optimization problems and may not present acceptable solutions to problems different in nature and type.
Referring to No Free Lunch (NFL) theorem, researchers are allowed to propose new techniques to solve a wide range of problems, Wolpert and Macready, (1997).

Acceding to the second option, several researchers have applied variants respective to current algorithms and solved optimization problems with better solution accuracy and speed and have been reported about the improved population initialization strategies and other approaches to improve the performance of basic operators under a variety of stochastic algorithms. Rahnamayan *et al.,*(2007) proposed opposition based learning strategy to initialize the population for an evolutionary algorithm to accelerate convergence. Kazimipour *et al.,* (2014) explored initialization of population in EAs from three different perspectives. Zhang *et al.,* (2017) proposed a small-population based parallel differential evolution approach to overcome the low diversity problem of DE. In the proposed approach, a large population is divided into several small size subpopulations evolving independently but in parallel to search for the optimal solution independently. (Chuang *et al.*, 2011) introduced chaotic maps into catfish particle swarm optimization to enhance the search capability via the chaos approach. Haghrah *et al.*, (2015) proposed random transfer vectors-based mutation to produce better quality off-springs in real coded genetic algorithm for short-term hydrothermal scheduling. Ratnaweera *et al.*, (2004) worked on self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients that control the velocity cognitively and suggested time-varying step size mutation. Krashy *et al.*, (2019) proposed the variation in traditional WCA by increasing the C-value to reduce the search space that effects on the balance between explorative and exploitative phases, gradually during the iterative process to find the global minimum. Orthogonal designs (Cai *et al.*, 2017), dual population, (Gao *et al.*, 2015), chaotic procedures for control parameters settings (Singh *et al.*, 2017), elite guided search (Cui *et al.*, 2018), dynamic neighbourhood learning (Rasoulzadeh, 2015), fully informed PSO (Fakhar *et al.*, 2015), disruption factor-based GSA (Gouthamkumar *et al.*, 2015), fuzzy-assisted stochastic algorithms (Fraile and Zufiria, 2007), (Chen and Hsin, 2015) and many more presented the effectiveness of the modified stochastic algorithms.

Thirdly, developing hybrid algorithm has been envisaged as a successful strategy to overcome the shortcomings of individual ones. This methodology to combine different methods synergistically with the incorporation of domain knowledge greatly enhances the problem-solving capabilities of the plagiarized algorithm. In the literature, several hybrid algorithms have been reported. Zhang *et al.*, (2006) applied a hybrid of DE with GA for improved crossover strategy to the multi-objective optimization problem. A stochastic fuzzy neural network trained by GA for optimal reservoir operations has been proposed by Chaves and Kojiri, (2007), Graciela *et al.*, (2008) used GA and PSO hybridization for power system parameter identification. Liao, (2010) has proposed two-hybrid algorithms of DE, as a local search operator, i.e., random walk with direction exploitation, and harmony search, to cooperate with the differential evolution algorithm to produce the desirable synergetic effect and tested on engineering design problems. Basu, (2010) proposed DE hybrid with real coded GA to solve economic emission dispatch problem of HTGS. Karaboga, (2011) has applied the clustering approach to ABC. Lu *et al.*, (2011) have hybridized cultural algorithm with DE for HTGS problem. Yildiz, (2013) has proposed the Taguchi method based on statistical and sensitivity analysis for the optimal set of control parameters of DE and is successfully applied to robotic operations. Jiang and Zhang, (2013) applied DE-TLBO for constrained HTGS problem with incommensurable objectives. Panda *et al.*, (2013) preferred BFOA–PSO algorithm for automatic generation control of linear and nonlinear interconnected power systems. Jiang *et al.,* (2014) used PSO with GSA for enviro-economic load dispatch (EELD) of the hydrothermal system. Fang *et al.,* (2014) hybridized GA with artificial fish optimization algorithm for optimal scheduling of the hydrothermal system. Siddique *et al.,* 2014 combined PSO and GSA for finding the optimal values of weights and biases for HNN. Narang *et al.,* (2014) have proposed predator-prey with Powell search method for HTGS problem. Muhsen *et al.,* (2015) have proposed adaptive DE combined with electromagnetism algorithm for extraction of PV cell parameters. Nguyen *et al.,* (2015) proposed a hybrid algorithm based on PSO and CRO for multi-object problems. Narang, (2017), has applied CSO hybrid with a predator-prey search for profit based hydrothermal scheduling. Wu *et al.,* (2017) combined black hole algorithm with PSO for profit analysis in optimal HTGS. Zhou *et al.,* (2017) have proposed a hybridized lighting search algorithm with the simplex method for global optimization of benchmark functions of variety. Padhy and Panda, (2017) have proposed a hybrid stochastic fractal search and pattern search technique based cascade PI-PD controller for AGC in an electric vehicle. Chen and Zhang, (2019) have presented Lagrange-relaxation based alternative iterative (AI) algorithm to solve combined heat and power scheduling problem. Carlos *et al.,* (2019) have proposed adaptive simulated annealing hybrid with genetic operators for economic load dispatch and many more. The authors have restricted their study of hybrid techniques to hydrothermal systems only.

1.2.5 Multi-objective Optimization

Multi-objective optimizations state that there is more than one objective to be minimized and/or maximized. There is a set of solutions that define the best trade-off between competing objectives. Mathematically, the multi-objective optimization problem can be formulated as:

Minimize/Maximize $F(x) = [f_1(x), f_2(x), \ldots f_M(x)]^T$

$$\text{Subjected to } h_j(x) = 0 \quad (j = 1,2, \ldots J) \tag{1.2}$$

$$g_k(x) \le 0 \quad (k = 1,2, \ldots K) \tag{1.3}$$

where $X = [x_1, x_2, \ldots, x_n]^T$ is a vector of design/control/decision variables. The searchable design space is designed by may have maximum and minimum limits of decision variables called side constraints and are defined as:

$X^{min} = \left[x_1^{min}, x_2^{min}, \ldots, x_n^{min}\right]^T$ and $X^{max} = [x_1^{max}, x_2^{max}, \ldots, x_n^{max}]^T$

To solve the multi-objective optimization problem, the conceptual definitions include;

Feasible Solution set

If x such that $[\forall x \in X]$, satisfies all constraints defined by equations (1.2) and (1.3), then x is a feasible solution and set of all feasible solutions is the feasible set denoted by X_j , where $X_j \subseteq X$

Domination

Given the two feasible solutions, x_1 and x_2 , the solution x_1 is said to dominate other solution x_2 , if both the conditions are true:

Condition 1: $f_i(x_1) \leq f_i(x_2), \; \forall i \in [1,2,\dots,M]$ (solution x_1 is not worse than x_2 in all objectives)

Condition 2: $f_{\bar{\imath}}(x_1) < f_{\bar{\imath}}(x_2), \exists i \in [1,2,\dots,M]$ (solution x_1 is strictly better than x_2 in at least one objective)

Pareto-Optimal Set

The non-dominated set of the feasible decision space is called the Pareto-optimal set.

Mathematically, for a feasible solution $x \subseteq X$, if there does not exist another feasible solution $x' \subseteq X$ satisfying $x' \prec x$, then x is non-dominated for X . This feasible solution is defined as a Pareto-optimal solution as x^*. The set of all such Pareto-optimal solutions is defined as the Pareto-optimal set Z^*such that $Z^* = \{x^*, \exists x \in X : x \prec x^*\}$

Pareto-optimal Front:

The boundary defined by the set of all points mapped from the Pareto optimal set is called the Pareto-optimal front.

Mathematically,

$Z_F = \{F(x^*) = |f_1(x^*),\; f_2(x^*), \dots, f_M(x^*)\}; x^* \in Z^*$

Multi-objective Generation Scheduling

There are several classical methods, applied to solve multi-objective hydrothermal generation optimization problem. Owing to the variety of sources such as coal, gas, oil, river water, pumped storage etc., the choice of the source for electricity generation is not only driven by economics, technical constraints and geographical locations of the plant but also essentially by carbon emission targets. Primarily, the operating cost of running thermal units is quite high in comparison with the marginally negligible operating cost of major hydro plants. On the other hand, thermal units are not the optimum choice to cater to peak load demands due to more starting time and slow dynamic response to high load variations. From the perspective of operational planning of electric power system, it is essential to optimize the energy resources, taking into account the energy storage facilities such as water reservoirs. The integrated operation of thermal and hydro units is essential for flexible, reliable and stable operation of the interconnected network. Inadvertently, the objective function of the hydrothermal generation scheduling is the minimization of cost and pollutant emission simultaneously with due consideration to the satisfaction of physical and operational constraints. There is a sheer need for the optimum operating strategy in a multi-objective framework that can ensure the best-compromised solution between two conflicting objectives of cost and emission. Principally methodologies for solving multi-objective problems differ on two points: one, the procedure used to generate non-inferior solutions, second; information shared with decision-maker. The decision-maker may use a defined utility function for optimization problem or make use of an effective tool to achieve the best alternatives.

Augusto, *et al.,* 2012 reported in their work about Edgeworth (1881) as the first academic to define an optimization problem involving multiple criteria. The problem, elaborated in the context of two consumers called **P** and π can be defined as: "*it is required to find a point (x, y) such that, in whatever direction we take an infinitely step, P and π do not increase together, but, while one increases, the other decreases*".

Since the 1970s, the role of decision-maker in solving multi-objective optimization problem has been the subject of intensive investigations. The theoretical concepts like *Karush –Kuhn-Tucker* (KKT) conditions, Pareto solution etc. are derived from mathematical programming. The role of decision-maker is classified in four ways according to Hwang and Masud, (1979) and Miettinen, (1998).

No preference method: This method is used when no preference information about the solution is available. There is no role of decision-maker; a simple neutral compromise solution is identified.

Priori method: In this method, the decision-maker articulates his preferences without knowing the limitations and outcomes of the problem in the solution search

Posteriori method: As the name suggests, the decision-maker selects the most preferred solution from the Pareto optimal set generated. But in case of more than one criteria to search the most preferred solution, it is computationally expensive to generate the Pareto set and time consuming for decision-maker to analyse the large set of alternatives.

Interactive method: In this method, the solution pattern is developed and is repeated until the decision-maker is satisfied with the most preferred weighting method, ε-constraint method etc. solution. This method is computationally less expensive as the solution patterns are generated according to the decision maker's preference.

Different methods are followed to convert multiobjective hydrothermal generation scheduling optimization problem into a scalar objective optimisation problem and decision maker's preferences. The methods in use are weight-pattern search methods (Bath *et al.*, 2006), (Basu, 2010), (Narang *et al.*, 2014), interactive decision-making using reference direction method (Deb *et al.*, 2007), goal attainment method for individual objective of HTGS (Basu, 2010) using ε-constraint method, quadratic approximation with value trade-off approach (Lu and Sun, 2011), lexicographic optimization with the fuzzy approach of DM to solve the enviro-economic issue (Norouzi *et al.*, 2014), normal boundary intersection technique for the compromised solution of bi-objective hydrothermal scheduling problem (Ahmadi *et al.*, 2015) through multi-Pareto-fronts, hybridized direct search methods (Custodio and Madeira, 2018), decomposition-based multi-objective HTGS solution using the multiple-preference model (Santos and Diniz, 2009) and decomposition of MO problem into scalar optimisation problems and are simultaneulosly optimised using EA with neighbourhodd relationship approach (Zhang and Li, 2007), logarithmic size mixed integer programming for hydrothermal commitment (Jian *et al.*, 2019), interactive fuzzy-based decision objectives (Basu, 2004) and many more. Invariably penalty function method has been used to handle water dynamics and load balance constraints explicitly in direct search methods as reviewed by Coello, (2000). These efficient classical methods perform single solution computation, but are not capable enough to provide the true global optimum solution to large scale multi-objective problem on account of physical limitations and simplifying assumptions, such as weight pattern assignment to individual objective, goal preference, relaxation in soft constraints to save computation time, static/dynamic penalty parameter tuning and hence suffers from slow convergence. Some researchers have tried new methods to alleviate these problems. Binary performance measure by (Zilzler and Kunzli, 2004) and the integration of user preference to resolve trade-off among conflicting objectives by (Branke and Deb, 2005) have been proposed in EMO decision making. Singh *et al.*, (2006) have proposed a solution methodology for the determination of best objective weights to solve multi-objective thermal power dispatch problem. The weights are calculated by conventional statistical measures, which characterize the correlation coefficients matrix evolution. Petcharaks and Ongsakul, (2007) have proposed a hybrid enhanced Lagrange relaxation method and quadratic programming to solve HTGS problem. Zitzler and Kunzli (2004) presented indicator-based evolutionary algorithm (IBEA) to introduce a total order between solutions by generalizing the dominance relation and preference information is taken into account through a reference point and has been applied to solve bi-objective flow-shop scheduling problem. Yalcinoz and Koksoy, (2007) have presented progressive articulation of preference information based optimization algorithm to solve the multi-objective environmental economic load dispatch problem. Although these quantitative trade-off ways are easy to implement yet are time-consuming and lack engineering viability (Coello, 2006).

Optimal hydrothermal scheduling is an NP-hard constrained optimisation problem with conflicting objectives; and population-based stochastic algorithms generate a set of multiple feasible solutions to a multi-objective problem in a single run and are expected to achieve a balance between exploration and exploitation for improved global solution and convergence rate as reported in the literature (Deb, 2010). Multi-objective evolutionary algorithms proposed to generate Pareto solutions in a single run such as non-sorting genetic algorithm (NSGA-II) that uses ranking selection and niching techniques to find non-dominated solutions and maintain the diversity of population and strength-Pareto evolutionary algorithms (SPEA2) (Qu *et al.*, 2018), non-dominated sorting gravitational search algorithm (Tian *et al.*, 2014) that uses crowding distance to estimate the density of solutions near each solution. Feng *et al.*, (2017) proposed a parallelized multi-objective genetic algorithm (PMOGA) to generate Pareto optimal solutions with different trade-offs by assigning different processor in a different region of search space to explore Pareto solutions. These Pareto-dominance sorting based approaches sometimes become ineffective in finding quality solution due to non-uniformity of Pareto-front or convergence to premature non-inferior solution in multi-modal problems. However, to assess the quality of Pareto-front, quantitative quality measures such as C-measure, D-measure and U-measure etc. are used. Beume *et al.*, (2007) prefered hypervolume metric to assess the quality of Pareto front. This quality indicator ranks the distribution of points toward the Pareto front and discards the worst-ranked. Singh *et al.*, (2017) applied U-measure to evaluate Pareto front of multi-objective economic load dispatch problem. Singh *et al.*, (2018) have investigated four different configurations of hybrid renewable energy systems in a multi-objective environment subjected to reliability and economic constraints for isolated and grid mode connections of applications.

Decision making is quite an important aspect of multi-objective evolutionary algorithms. The decision-maker selects the best-compromised non-inferior solution from a set of non-dominated solutions. Among decision making, Wang *et al.*, (2016) proposed a cooperative multi-objective DE to generate Pareto by using a nested approach to exploit decomposition of population, co-evolution and parallelism to reduce the complexity of multi-objective decision making. Goal programming based consistency model to decision making in multiplicative preference relations by Zhang and Pedrycz, (2018), the generic fuzzy approach (Bahari *et al.*, 2018), Singh et al., (2019) proposed metaheuristic firefly algorithm for robust tuning of excitation controller while minimising maximum overshoot and settling time simultaneously to enhance the stability of the system, a non-interactive surrogate worth approach in multi-objective decision making (Singh *et al.*, 2017) to overcome the preference issue, co-evolutionary hybrid decomposition technique for bi-objective optimisation (Chaabani *et al.*, 2019) *etc*., are employed to select the best-compromised from a set of non-inferior solutions, subjected to satisfaction of constraints in HTGS problems.

1.3 ARTIFICIAL INTELLIGENCE APPROACH

For the reliable and efficient power supply and to avoid an impact on the environment, the close monitoring of the power system equipment, generation and consumption are required. It needs artificial intelligent (AI) based computational methods which make the system highly reliable, accurate and automated systems such as energy management systems, intelligent sub-station ornamented by high-speed protection, monitoring, control and communication protocols. The field of artificial intelligence includes artificial neural networks, fuzzy logic approach, expert systems and evolutionary computing with different areas of applications in power system industry. Since the early 1980s, much of the effort in power systems analysis has turned away from the methodology of formal mathematical modelling which came from the fields of and evolutionary computing. Various techniques of artificial neural network and fuzzy logic approach with an application in hydrothermal generation scheduling have been reviewed in the following section

1.3.1 Artificial Neural Network

Artificial neural networks (ANN) are knowledge-based intelligent paradigms representing biological neuron system. Invariably the ANN learning algorithms provide local search solutions when dealing with numerical optimization. Miranda, *et al.,* 1995 have proposed ANN training model for the load dispatch based on the cost to the less rigorous techniques of artificial intelligence (AI). Today, the main AI techniques found in power systems applications are not only utilising the logic and knowledge representations of expert systems, fuzzy systems and the derivatives of the transient energy margin value to ensure transient stability of power system under contingencies but has strengthened to the extent of adaptive ANN structure and parameter learning to extract the advantages of ANN in synergism with other evolutionary algorithms. Cells and Rylander, (2002) proposed the structure learning of ANN with PSO. Lahiri and Chakravorty, (2005) solved 3D electrode spacer optimisation problem by coupling a trained neural network with the simulated annealing algorithm, Flores *et al.,* (2005) presented an electricity market decision-making model to predict electricity production from wind system, in which ANN model has been proposed for average wind speed prediction and GA for active power generation from doubly fed induction generator-wind systems. Mirjalili *et al.,* (2012) trained ANN with PSO and GSA, Quan *et al.,* (2014) have proposed a novel lower upper bound estimation (LUBE), strategy extended to develop predictive interval PIs using NN models to quantify the potential uncertainties in load, wind and solar forecasts. Particle swarm optimization (PSO) integrated with the mutation operator is used to solve the multi-objective problem. Agarwal and Bawane, (2015) have proposed multi-objective PSO based adaption of neural network topology for pixel classification in satellite imagery. Kaalam *et al.,* (2017) have exhibited the optimum design procedure to tune controller parameters for grid-connected distributed generation system based on cuckoo search algorithm (CSA) and ANN has been used to model PV unit to handle non-linearity. Zhang *et al.,* (2018) presented wind speed prediction using improved particle swarm optimization (IPSO) algorithm to globally optimize the weights and thresholds of BP neural network, and overcome the problem of local minimum value.

Further, the literature survey regarding applications of ANN in the power systems and about other memetic ANN algorithm has been included in chapter 5.

1.3.2 Fuzzy Logic Approach

Bellman and Zadeh, 1970 wrote: "*Much of the decision-making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely*". In 1965, L.A. Zadeh laid the foundations of fuzzy set theory as a method to deal with the imprecision of practical systems.

Uncertainties due to approximations in modelling, imprecision in data measurement, unpredicted variations in parameter values, inaccuracies due to subjective assessment etc., creep in a variety of ways within deterministic modelling of a large and complex power system. Much of this imprecision or uncertainty is neither measurable nor random. But it can be due to vague and ambiguous information collected for data about the problem and modelling of uncertainty. Stochastic models for random load variations, noise in measurements for state estimation, fluctuations in model parameters, randomised values of decision variables in stochastic optimisation problems etc. give rise to random information and data to be handled for deterministic decisions in power systems (Klir and Folger, 1988).

To deal with the inaccuracies and uncertainties embedded in data, a fuzzy set comprising membership functions representing the degree of belonging to the set is defined. In general, some useful capabilities and features provided by modelling in fuzzy set approaches are summarised as:

- Representation methods for natural language statements
- Models of uncertainty where statistics are unavailable or imprecise
- Information models of subjective statements
- Measures of the quality of subjective statements
- Integration between logical and numerical methods
- Models for soft constraints
- Models for resolving multiple conflicting objectives

Given these supportive attributes of the fuzzy logic approach, the power system applications can be broadly classified into three groups; rule-based fuzzy logic system, fuzzy controllers and fuzzy decision making.

Mathematical models have been developed for manipulating numerical values with linguistic uncertainty. Miranda and Saraiva, (1992) presented fuzzy logic approach for power system optimal flow operation with uncertainties in load and generations modelled as fuzzy numbers and system behaviour under known injections is dealt with by a DC fuzzy power flow model. Luciano and Savastano, (1997) proposed the fuzzy-based unsupervised function approximation model for identification of non-linear systems. Kozma and Kasabov, (1998), proposed a novel approach with the combined use of the fuzzy rule-based system with structural learning results in a powerful and efficient tool of automatic extraction of meaningful rules. Chaotic adaptation has been included for both the membership functions and rules of the fuzzy system and tested the modelling and prediction of the Mackey-Glass data set successfully. Ertugrul and Cheok, (2000) have developed sensor-less indirect rotor angle position estimator for switched reluctance motor that can provide accurate and continual position data over a wide range of speeds and can also function accurately at different operating conditions like chopping, single pulse mode, steady-state and transient operation. You *et al.,* (2003) presented an adaptive neuro-fuzzy controller for effective damping out of oscillations in a multi-area system. Lee et al. (2007), presented a fuzzy hybrid method for temperature prediction based on fuzzy rule-based systems and utilized genetic algorithms for fuzzification of variables. Altin and Sefa, (2012), proposed dSpace based hybrid neuro-fuzzy controller for interactive grid operation. Fuzzy systems utilize linguistic information and develop the model to represent system behaviour in diagnostic monitoring applications. Adam *et al.,* (2014) presented the fuzzy model for diagnostics of power circuit breaker and simulated the model in LABVIEW platform to increase the operational reliability and hence the safety of the power system. Hannan *et al.,* (2019) comprehensively reviewed the FLC-based inverter control system to minimize photovoltaic (PV) output fluctuations, which cause inverter issues related to output harmonics, power factor, switching schemes, losses, and system implementation for recommendations in future for PV applications and generation. The precision in the fuzzy membership functions depends on the valid range of inputs and the general response characteristics of the system. Juang and Bui, (2019) proposed a new reinforcement neural fuzzy surrogate (RNFS) assisted multi-objective evolutionary optimization (RNFS-MEO) algorithm to boost the learning efficiency of data-driven fuzzy controllers (FCs). The RNFS-MEO is applied to evolve a population of FCs in a multi-objective robot wall-following control problem to reduce the number of time-consuming control trials and the implementation time of learning. Within power systems, fuzzy logic controllers have been proposed primarily for stabilization control.

There is a wide span of problems that involve decision-making and optimization in power system planning and operation like; transmission planning, security analysis, optimal power flow, state estimation, generation scheduling and unit commitment, and many more. The problem arises from attempting to fit practical problems into deterministic models of the system to be optimized. Real-world problems include either nonlinear objective function or non-linear constraints or both. This results in a reduction in information either in the form of simplified constraints or objectives. The simplifications of the system model and subjectivity of the objectives may often be represented as uncertainties in the fuzzy model.

Chang, (1994), proposed a fuzzy model to enhance fuzzy decision making and presented two multi-objective fuzzy-logic control algorithms for controlling power system static/dynamic security are presented and analysed. The first algorithm is based on the successive inferences of fuzzy implication rules for each objective, and the second algorithm is an application of the method of fuzzy linear programming. Surmann, (1996) has presented a genetic algorithm approach to optimize a fuzzy rule-based system for charging such high-power Ni-Cd batteries. For the optimization of the fuzzy system, the entropy energy function is considered as the single objective function of a fuzzy system is formulated. Yuan and Luh, (1997) proposed a fuzzy optimization-based algorithm and the Lagrangian relaxation technique to solve fuzzy mixed integer programming problem for daily scheduling of thermal units and uncertain purchase transactions. Basu, (2004) has presented an interactive fuzzy satisfying method for solving fixed head, cascaded reservoir multi-objective generation scheduling problem. Dhillon *et al., (*2006) have applied the fuzzy decision-making theory to obtain the compromised solution of stochastic multi-objective load dispatch problem. Brar *et al.* (2008) have proposed the utility of GA to search for the active and reactive power scheduling for a multi-objective load dispatch problem and applied fuzzy decision-making methodology to decide the fitness function corresponding to each string of active and reactive power generations. Juang *et al.,* (2009) proposed a hierarchical cluster-based multispecies particle-swarm optimization (HCMSPSO) algorithm to learn Takagi-Sugeno-Kang (TSK) type fuzzy rules with high accuracy for fuzzy-system optimization. Moeini *et al.,* (2011) presented a fuzzy rule-based model for release from the reservoir based on ideal or target storage levels. Reservoir storage, inflow, and time-period as premises and the water release as the consequence have been considered. For an aggregation of rules, the interpolation method is adopted. To overcome the sparsity of fuzzy rules, Fazzolari *et al.,* (2013) reviewed multi-objective evolutionary fuzzy systems, describing the main contributions on this field and providing a two-level taxonomy of the existing proposals, to outline a well-established framework for future promising directions. Chen and Hsin, (2014) proposed weighted fuzzy interpolative reasoning method using PSO-based weights-learning algorithm that outperforms the existing methods to deal with the computer activity prediction problem, the multivariate regression problems, and the time series prediction problems. Masteri *et al.,* (2018), proposed a multi-objective algorithm that uses fuzzy optimization technique to handle investment cost and maximizing the reliability performance as contradicting objectives with battery energy storage systems (BESS) as an asset solution included for distribution system planning. Singh *et al.,* (2017) suggested surrogate worth, non-interactive fuzzy model, for solving conflicting objectives of multi-fuel economic load dispatch problem. Maulik and Das, (2018), presented a combination of particle swarm optimisation and fuzzy max-min technique as optimal power dispatch strategy for simultaneous reduction of cost and emission from generation activities in an AC-DC hybrid microgrid under load and generation uncertainties. Wang *et al.,* (2018) have proposed shape-ratio and value-at-risk ratio in fuzzy environments to evaluate investment risks from different perspectives in a multi-objective model to evaluate their joint impact on portfolio selection. The proposed model is solved by a fuzzy simulation-based multi-objective particle swarm optimization algorithm. Zhou *et al.,* (2019) have proposed an optimisation model of power reference of the wind farm in power restoration based on the fuzzy chance constraints model. The model maximises the power support of a wind farm based on the safety of the restored system.

Zhao *et al.,* (2019) presented trade-offs at three stages. First; the trade-off between computing efficiency and model accuracy in the spatiotemporal discrete analysis of nonlinear modelling of a pumped storage system. Secondly, a trade-off between algorithm convergence and searching diversity is enabled in neighbourhood-search chaotic mutation multi-objective gravitational search algorithm, integrating multiple constraints, performed to solve the Pareto front. Thirdly, the fuzzy analytic hierarchy process is innovatively introduced to select the most compatible solution, which comprehensively considers quantitative and qualitative factors. Mohamad *et al.,* (2019) proposed a method to optimize the penetration of solar/wind energy, followed by the optimization of the energy storage capacity in the second part. The fuzzy decision-making method is utilized to select a preferred solution from the Pareto front based on the assignment of the membership function values to reflect the operator's preferences.

In multi-objective hydrothermal generation optimisation problem, objectives are conflicting in nature. No solution can satisfy all the objectives in all respects. Application of the fuzzy methodology to choose the best-compromised solution undoubtedly provides maximum satisfaction level among the participating objectives.

1.4 CONSTRAINT HANDLING IN STOCHASTIC ALGORITHMS

Stochastic evolutionary algorithms have successfully handle real-world optimization problems, but there is no provision for constraint handling in their original formulation. These nature-inspired algorithms perform an unconstrained search. The mutation and recombination (move) operators perform on feasible individuals without any apprehension about constraints but cannot necessarily generate feasible off-springs. Old literature survey suggests that by using repair techniques or by using special move operators or decoders, it is possible to generate feasible candidate solutions in simpler constrained optimization problems. However, for complex (combinatorial) constrained optimization, additional mechanisms need to be incorporated to handle constraints (Datta and Deb, 2015). There are four different types of constraint handling mechanisms in the meta-heuristic algorithm such as penalty, repair, separatist and hybrid approaches (Da Silva *et al.,* 2011).

The penalty function method is the oldest method proposed by Courant, (1943) and penalty functions are classified as multiplicative and additive. The penalty function transforms constrained optimization problem into an unconstrained with predefined or adaptive weights which indicate a preference between the constraints and the objectives.

In multiplicative case, penalty function amplifies the fitness function by positive penalty factor $p(v(x), T)$

$$F(x) = p(v(x), T) \times f(x) \tag{1.4}$$

If $p\big(v(x)\big) = 1$, there is a feasible solution. If $p\big(v(x)\big) > 1$, the solution is not feasible. the penalty function is added to fitness function as

$$F(x) = f(x) + \frac{1}{k} p\big(v(x)\big)\;;\ \text{Interior penalty function} \tag{1.5}$$

$$F(x) = f(x) + kp\big(v(x)\big)\;;\ \text{Exterior penalty function} \tag{1.6}$$

Such that $p\big(v(x)\big) = 0$, for a feasible solution, otherwise $p\big(v(x)\big) > 0$.

The violation in constraint is computed as:

$$v_j(x) = \begin{cases} \max\{0, |h_j(x)| - \varepsilon\} & ;\ \text{for equality constraint} \\ \max\{0, -g_{j(x)}\} & ;\ \text{otherwise} \end{cases} \tag{1.7}$$

The most popular method to compute the penalty function introduced by Leunberger and Ye, (2008) is defined as:

$$p(x) = \sum_{j=1}^{m} \left(v_j(x)\right)^2 \tag{1.8}$$

Adaptive penalty methods require multiple runs to fine-tune the penalty factor leading to a high computational cost. The inclusion of the penalty function also distorts the objective function. The distortion in the objective function is small for small values of the penalty parameter. For a large value of penalty parameter, the optimum of the objective function may have artificial local optimal solutions. Barbosa and Lemonge, (2003) proposed an adaptive penalty method with genetic algorithms where each penalty value has been adopted proportional to the violation of the corresponding constraint.

In the *repair approach*, an infeasible solution is converted to a feasible one by using a repair operator (Bai *et al.*, 1996), (Bu *et al.*, 2014), (Sanchos, 2009).

In the *separatist approach,* objectives and constraints are handled separately *viz;* multi-objective-based constraint handling (Arturo *et al.*, 2004), (Karthikeyan *et al.*, 2009), (Abido *et al.*, 2010), (Bath *et al.*, 2006), co-evolutionary-based Mohamed and Sabry, (2012), (Gao *et al.*, 2015), constrained-domination principle (CDP) (Carlos *et al.*, 2002), (Sifuentes, Vargas, 2007), stochastic ranking (SR) (Ali *et al.*, 2012), α-constrained method to nonlinear simplexes by Takjahama and Sakai, (2005), and epsilon constraint methods (Basu, 2010) have been reported in the literature.

In the *hybrid approach*, representative methods include Lagrangian multipliers (Chen and Zhang, 2019), multi-objective DE based dynamic hybrid constraint handling by Lin *et al.,* (2019), fuzzy logic (Jiminez *et al.,* 2005), (Dhillon *et al.,* 2011), (Narang, 2017).

Several strategies are reported in survey reports on constraint handling published by renowned researchers like Kramer, (2010), and Coello, (2019).

1.5 STATISTICAL SIGNIFICANCE TEST METHODS

In a multimodal real-world optimisation problem, the stability and quality of the solution are quite significant metrics for the assessment of the algorithm. The use of statistical tests to improve the evaluation process of performance of the new algorithm has become a widespread technique in computational intelligence (Johnson *et al.,* 2005).

Benchmarking in evolutionary computation is a task that must be performed when comparing a new algorithm to existing ones. There are three important aspects of the thought process and needs to be considered equally; problems to be chosen, experimental setups and methods to evaluate the performance of the compared algorithms. In continuation with the literature review on computational techniques for solving constrained multi-objective hydrothermal scheduling problem, different statistical approaches are used for comparison, from the experimental simulations and illustrations that are obtained by applying the techniques. One of the common methods to use statistical tests as a comparison technique is based on the obtained solutions values of the fitness function for hypothesis testing, where the distribution of the obtained solutions in the search space is neglected (Derrac, *et al.,* 2014). However, the information about the solutions' search space distribution provides the user an understanding of the spread of the solution space as diversity justifies the quality of solution whereas clustering indicates more stability. Besides, this information also supports the strengths and weaknesses of the compared algorithms. *Eftimov and Korosec*, (2019) have compared scenarios for the comparison of the obtained solutions according to their values and their distribution in the search space as:

- As the compared algorithms are not statistically significant about the obtained solutions values and their distribution, the compared algorithms have the same exploration and exploitation power. There is no statistical significance between the performances to values, but there is a statistical significance to the distribution of the obtained solutions in the search space. The algorithm with the sparser distribution of obtained solutions has better exploration power.

- In case, there is a statistical significance between the compared algorithms about the obtained solutions values, but no statistical significance as to the distribution then the compared algorithms have the same exploration power but have different exploitation powers and one algorithm may be able to find statistically better solutions than the other and indicates that the trailing algorithm must improve its exploitation power.
- As the compared algorithms have statistically significant performance in their obtained solutions values and their distributions as well, there is an opportunity for poorer performing algorithms to improve exploration power, as its exploitation power cannot be assessed.

A significance test consists of three essential ingredients: a test statistic, a distribution of test statistics, and a significance level based on the value of test statistic.

There are two types of significance tests: Parametric tests and Non-parametric tests

Parametric Significance Tests consider the data that follows a specific distribution typically the normal distribution or Gaussian distribution and require a small sample size. These tests have been commonly used in the analysis of experiments in computational intelligence. Common parametric tests include t-test (paired or unpaired), ANOVA (one-way; two-way, three-way), linear regression and Pearson rank correlation.

Non-Parametric Tests are performed, when the distribution is not normal or the distribution is skewed or the distribution is not known, or the sample size is too small (<30) to assume a normal distribution. These methods make fewer assumptions, they are more flexible, more robust, and applicable to non-quantitative data. Kolmogorov-Simonov (k-s) test is performed to check the normality of the data. The results of a high value of D-score and low value of p-value less than the level of significance obtained from the k-s test gives clear evidence that the data under test is not normally distributed.

In general, hypothesis tests rely on the assumption that the population follows a normal distribution with parameters means and standard deviation. The null hypothesis for this test is that there is no difference between the median values for the two groups of observations.

Hypothesis Testing is applied to draw inferences about one or more populations from given samples. For these two hypotheses, the null hypothesis and the alternative hypothesis are defined. The null hypothesis is a statement of no effect or no difference, whereas the alternative hypothesis represents the presence of a difference between algorithms. (Derrac *et al.,* 2011)

When a statistical procedure is applied to test the hypothesis, a level of significance (α) determines at which hypothesis may be rejected. Unlike the *p*-value, the α-level is not derived from any observational data and does not depend on the underlying hypothesis; the value of is instead set by the researcher before examining the data. Conventionally, it is commonly set to 0.05, 0.01, 0.005, or 0.001.

p-value: This probability of the experimental criterion score given the distribution created by null hypothesis is also known as the p-value. When the significance level is low, the researcher can feel comfortable in rejecting the null hypothesis. The *p*-value is used in the context of null hypothesis testing to quantify the idea of statistical significance of evidence. The null hypothesis is rejected p-value is less than or equal to a small, fixed but arbitrarily pre-defined threshold value α, which is referred to as the level of significance (α). The one-sample t-test is used to test whether the mean of a population is greater than, less than, or not equal to a specific value. Because the t distribution is used to calculate critical values for the test, this test is often called the one-sample t-test. The t-test assumes that the population standard deviation is unknown and is estimated.

Wilcoxon signed-rank test is a nonparametric analogue of the t-test that aims to detect significant differences between two sample means *i.e.* the behaviour of two algorithms. Outliers have less effect on Wilcoxon test than on t-test.

Friedman Test is a nonparametric test analogue of the repeated measures ANOVA. It is a multiple comparisons test that aims to detect significant differences between the behaviour of two or more algorithms. The null hypothesis for Friedman's test states the equality of medians between the populations (Garcia *et al.,* 2010).

Hence it is concluded that the algorithms are not compared only according to the obtained solutions values, but also according to the distribution of the obtained solutions in the search space for its stability, reliability and robustness.

1.6 SCOPE OF WORK

Practically, decisions for hourly optimal generation output from thermal units and water release from hydro units over the scheduling period are constrained by power balance requirements, and water management necessities such as water head, the amount of water available in the reservoirs, water to be drawn down, and generation limits etc. as applicable to the scheduling problem. In the case of hydraulically cascaded units, it needs to be coordinated in time as well, due to the dependency of water storage in reservoirs, which may take a long period for utilization. Besides valve point loading of multi-valve steam turbines of thermal units, diversified fuel options and other system inaccuracies and uncertainties, etc. may neither fit the assumptions of a single artificial intelligence computation technique or stochastic algorithms nor be effectively solved. Several global optimizers have been developed but their improvement in terms of robustness and efficiency is still a challenge. Secondly, in multi-objective optimization of the *hydrothermal generation scheduling* (HTGS) problem, the decision to seek the most preferred solution from the Pareto set of non-inferior solutions to the conflicting objectives of the problem is of paramount importance.

- Mathematical programming methods, conventional search methods are quite efficient in finding a Pareto set of solutions but does not yield an entire set of global solutions (Carvalho and Soares, 1987). These techniques have limitations of linearization, continuity, and differentiability of an objective function, sensitivity to initial guess, constraint handling, large dimensionality, and hence prone to local optimality, etc. Therefore they suffer from the drawback of slow convergence at times and result in unrealistic conclusions due to the stochastic nature of parameters like load demand, fuel prices, unit availability, and reservoir inflows.
- Stochastic heuristic algorithms offer a flexible and well-balanced mechanism to local exploration and exploitation to accomplish global tasks exceedingly well in an efficient manner but offer a slow convergence rate because of sensitivity towards tuning of control parameters (Spall, 2012).
- An artificial neural network has a parallel and distributed computational power of handling non-linear data but the selection of network architecture and weight parameters is a challenging task. Owing to its features of inductive learning and recognizing efficiently, ANN has renewed (gained) the insight of researchers and is established that ANNs are promising alternatives to classical techniques. Convergence to a locally optimal solution is a fundamental limitation of any local search based training approach. Evolutionary algorithms belong to a global search group that can evolve topology and weight coefficients individually or simultaneously (Carlos *et al.*, 2010).

- Evolutionary and swarm algorithms demand computational cost due to the time-consuming trial and error parameter and operator setting process in the complex search space (Narang *et al.,* 2012).
- Self-adaptive control mechanism used in stochastic strategy can enhance the performance of the algorithm (Wu *et al.,* 2018)
- The application of hybrid systems based on computational intelligence in power system problems is a novel development, which represents a class of memetic algorithms that hybridize multi-objective meta-heuristic techniques with local search techniques and offer the globosity and robustness of the evolution approach (Lara *et al.,* 2010). Meta-optimization offers low-level learning, human-like thinking and computational power of neural networks to P-metaheuristics and brings the diversification and intensification along with high-level interactions (learning) capability of stochastic algorithms to neural networks, to generate a quality solution in less computation time.

However, there exists a challenging task to resolve several related crucial issues such as the selection of local search technique, multi-objective stochastic algorithm, the optimum structure of hybridization, individual parameters to be tuned, learning algorithm of parameter tuning, convergence criteria of individual parameter tuning. It is intended to investigate the solution methodology of multi-objective optimization of short-term hydrothermal system by exploring interactive and non-preference solution procedures. The fuzzy logic approach is proposed to be incorporated to handle constraints and for decision making among conflicting objectives. It is further proposed to explore an optimal strategy for hydrothermal generation scheduling that will comprehend multilayer artificial neural network trained by differential evolution technique to obtain a near-optimal solution. Further, it is envisaged to hybridize global and local search techniques that will improve the solution quality of the multi-objective hydrothermal generation scheduling problem. There is great scope to consider the violation in the equality constraints due to the stochastic search of decision variables. The proposed strategy of optimization procedures will be a definite future trend in power system operation and control.

1.6.1 Objectives of Research

The proposed research aims to solve the hydrothermal generation scheduling problem using a memetic algorithm approach for solution methodology in the multi-objective framework with due consideration to the operational and physical constraints of a realistic power system. The objectives of the proposed research work are outlined below:

- To solve multi-objective hydrothermal generation scheduling problems using *artificial neural networks* (ANN) employing memetic algorithm including *differential evolution* (DE), *opposite differential evolution* (ODE), and *local search technique.*
- To achieve the best solution from the set of Pareto optimal solution of a formulated multi-objective optimization problem

1.7 ORGANIZATION OF THESIS

The thesis intends to solve the hydrothermal generation scheduling problem using a memetic algorithm approach in the multi-objective framework with due consideration to the operational and physical constraints of a realistic power system. The thesis is organized into seven chapters. The description of the chapters is outlined below:

Chapter 1 presents the prominent milestones in the development of optimization techniques. The optimization techniques for single and multiple objectives covering direct search methods, gradient methods, stochastic heuristics methods, hybrid methods, artificial neural optimization are reviewed addressing their salient features and limitations. The significant contributions of various researchers in the area of hydrothermal generation scheduling are reviewed in brief.

In **Chapter 2** differential evolution as a global optimiser is introduced to solve a single-objective hydrothermal generation scheduling problem. To improve the performance of classical DE, twelve variations in *mutation strategies* are explored and their effectiveness is validated on twenty benchmark test functions and scalar objective hydrothermal generation scheduling problem.

Chapter 3 elaborates on the development of a *novel chaotic-crisscross DE algorithm*, in the light of the global solution accuracy and convergence rate of stochastic algorithms that are significantly affected by parameter-tuning, exploration, and exploitation strategies. Broadly, two variations in the DE algorithm are explored. Firstly, DE is integrated with dual crisscross mechanism orthogonally with chaotic DE to balance diversification and intensification in three-dimensional problem search space. The horizontal crisscross supports the exploration at the boundaries of hypercube space and the vertical crisscross facilitates the perturbation of stagnant population, to escape out from local minima. This double search strategy of information exchange in the competitive mode is envisaged with significant advantages in convergence rate and global solution accuracy. Secondly, the exploitation capability of CDE is combined with the exploration capability of the *Sine-cosine algorithm* (SCA). This population-based algorithm establishes random candidate solutions and entails them to oscillate around the best solution derived by a trigonometric function mathematical model for better exploitation of space defined by two functions.

To investigate the robustness of the proposed algorithm these are applied to realize an optimal generation schedule of a multi-chain short-term hydrothermal system over 24 hrs. time-horizon in a multi-objective framework, considering conflicting economic-environmental aspects of thermal units. The equality constraints of active power balance and the amount of available water, are independently handled using the variable elimination method. However, the statistical uncertainties in the violations of constraints are quantified within their prescribed bound using a fuzzy approach. An interactive unified fuzzy satisfying function is aimed to solve the conflict of three objectives. The numerical results show improvement in unified satisfying objective function and convergence performance metrics over the existing methods. The competence of the proposed algorithm is confirmed through illustrations on benchmark functions and is substantiated through statistical significance tests.

Chapter 4 integrates *Chaotic differential evolution and crisscross differential evolution integrated with Simplex search method* respectively to solve the HTGS problem. The simplex method is a fast, direct, and derivative-free search method with strong local optimization ability. The probability of having located global optimum increases in the local search methods by multiple starts or by random restarts from different points repeatedly. Secondly, the three operations; reflection, expansion, and compression of the Simplex method initialize the new simplex from the current best vertex and improves the position of the worst solutions. The iterative implementation of these operations guides the search towards a global solution with improved convergence speed and accuracy has motivated the authors to hybridize DE with the simplex method (SaDESM) and is demonstrated through illustrations on benchmark functions of variety as well.

Chapter 5 presents the neural network meta-optimization approach based on differential evolution (DE) to solve short-term economic-emission hourly dispatch of the hydrothermal system for a time-horizon of 24 hours. The *feedforward neural network* (FNN) model is proposed for the mapping of decision variables. The selection of geometrical configuration, network parameters and learning strategy in the multi-layered network significantly impact its convergence rate and global solution accuracy. Besides the dimensions of the problem, the training time, and global search ability of FNN are influenced by the initial population and learning strategy. Inspired by the Salp swarm chain, a novel reproduction proposition is enabled to generate a diversified promising population to accelerate convergence by avoiding local minima. Self–referential chaotic DE algorithm is responsible for the robustness of the search process and is recommended to optimise FNN parameters. The external elite approach is used to archive a non-dominant solution. Operating limit violation by a slack hydro and slack thermal unit is taken care of using the exterior penalty method and squared error of violations in constraints is added as an objective to be optimised. The multi-objective HTGS problem is solved for cost and emission objectives while making attempts to simultaneously minimize fuel cost and pollutant emission. The results obtained from simulations of three test problems demonstrate the effectiveness of the proposed approach in case of fuel cost and emission compared to other reported results.

In **Chapter 6**, the summary and conclusions of the research work undertaken are presented**.** It observed that the *global-best mutation strategy and opposition-based-learning mutation strategy* have emerged out as the promising mutation strategies. Further, Crisscross dual mechanism outperforms classical DE and chaotic DE in terms of convergence rate and global solution search capability. CCDE based hybrid approaches as CCDESC and SaDESM provides either compatible or better solutions within feasible search space. The number of function evaluations taken to test benchmark functions reveals that the CCDE based algorithm gives fast convergence and handle modality, complexity, and dimensionality very well. The performance of CCDE based algorithms in solving hydrothermal scheduling problem is robust and efficient. Fuzzy model to handle equality constraints reduces the mismatch in violations to the order of 10^{-8}. In the SDENN algorithm, it is observed that there is a remarkable improvement in the objective function when considered independently. From the results of statistical significance tests, it is concluded that all algorithms qualify non-parametric tests. The descriptive and the cardinality ranking obtained from each algorithm conclude that the standard deviation is quite low with CCDE based algorithms and SaDESM and CCDE algorithms present the best-compromised solution among all. The future scope of the research is also presented.

CHAPTER - 2

DIFFERENTIAL EVOLUTION AS AN OPTIMISER

2.1 INTRODUCTION

The real-world problems frequently encountered in the field of economics, health, management, sports, science and engineering etc., are multi-dimensional, complex, dynamic and constrained/unconstrained ones. Since decades, engineers, mathematician's and researchers committed in the field of optimization have invested immense resources and worked upon plenty of prominent algorithms. Keeping a focus on engineering problems, there exist several non-commensurable objectives subjected to soft and hard constraints. The urge of the optimisation procedure is to search for the best satisfying and feasible solution to the objectives of the underlying problem. On the contrary, there is no possibility of the improvement in the non-inferior set of solutions called efficient solutions, without sacrificing the best solution to one or more objectives and /or accepting the satisfaction of hard constraints to the reasonable extent of tolerance. Several deterministic optimization techniques such as Lamda-gamma iterative method, gradient search, Lagrange multiplier method, dynamic programming, mixed integer programming, Powell's method, Simplex method, decomposition method etc., are reported in the literature with their proficiencies and deficiencies in solving NP-hard combinatorial problems.

Since two decades, metaheuristic algorithms get more attention and are dominating the market of the solution to optimization problems. These algorithms inspired by nature, social and ecological behaviour of living beings, are flexible, robust and generate multiple solutions in a single run. Each one of the reported population-based metaheuristic strategies flutters on one of its achievements commonly in terms of improvement in either of the traits of the solution to a combinatorial optimization problem; avoidance of premature convergence, convergence speed, diversification of space, exploration and exploitation of global search, flexibility, solution accuracy, simplicity of the operating mechanism, tuning of control parameters or averting loss of information. However, evolutionary-based algorithms have received more attention by engineers regarding their potential as global optimization techniques as these algorithms utilize the learning process of an individual member of the population (Coello, 2006). Some of the most famous of these algorithms like EA, GA, PSO, and DE and their improved versions and many more, have been applied to obtain the solution to power system optimisation problems successfully to a certain extent.

Based on two operators, the EA has been introduced in 1965 as an approach to handle artificial intelligence and been subsequently applied as a global optimization tool. The mutation is the key operator that generates new off-springs of the current parent population and selection operator decides the parent for next-generation based on survival of fittest. However, EA is prone to slow convergence near-global optimum solution in the later stages of the progress in generations. Several modifications are applied such as improved initialization of population (Kazimipour *et al.,* 2014) and (Carrano, *et al.,* 2011), Cauchy mutation procedure (Wong *et al.,* 2012) and (Gong et l., 2015), integration with other stochastic algorithms such as clonal selection (Venkatesh, *et al.,* 2003), neural network (Abbass, 2004), (Carlos, 2010) and fuzzy approach (Fazzolari *et al.,* 2013) and has been successfully applied to the diverse applications such as thermal load dispatch (Padhy *et al.,* 2003), hydrothermal scheduling Basu, 2004), power system stabilizer, feature classification (Carlos et al., 2010), robot learning control (Juang *et al.*, 2020) as reported in the literature.

Genetic algorithms (GA) introduced in (1975) based on natural evolution, a computational costly stochastic algorithm lacks in diversity, computational efficiency and hence converges to local optima (Deb, 2010). Over the years, several modifications have been applied to GA such as dominance-based tournament selection (Carlos, 2002), real coded GA to overcome encoding problem (Dhillon *et al.*, 2011) for multi-objective economic load dispatch, transfer vector-based mutation (Haghrah, 2015), economic environmental scheduling with NSGA (BAsu, 2011), parallel population evolution (Feng *et al.,* 2017),), hybrid with artificial fish algorithm (Fang, 2014), hybrid with neuro-fuzzy controller for stabiliser control design (Fraile, *et al.,* 2014), population partitioning GA-PSO hybrid for large scale optimization (Ali and Tawhid, 2017), propriety based GA based hybrid with local search and many more are reported in the literature.

Particle Swarm Optimization (PSO) algorithm introduced in 1995 based on the simulation of social behaviour of swarms and has been successfully emerged as a robust stochastic algorithm. Unlike EAs, PSO is free from the encoding process and can be easily programmed. In PSO, each particle moves to the next generation by searching the direction based on its history of current and best-fit moves in reference with more particles in search space. However, its performance is largely dependent on the proper setting of the velocity update parameter and other inertia constants. Several procedures have been adopted to improve the performance of PSO such as the use of self-organising hierarchical topologies (Mandal and Chakarborty, 2012), informed PSO (Fakhar *et al.*, 2015), predator-prey PSO (Narang *et al.*, 2012), hybrid with Powell search (Narang *et al.*, 2014), DA-PSO (Khankutti *et al.*, 2019), civilised PSO (Narang, 2017), hybrid with neural network (Zhang, *et al.*, 2018), etc. and has been successfully applied in diverse application like AGC control (Panda *et al.*, 2013), wind speed predictor (Zhang *et al.*, 2013), hydrothermal scheduling (Narang, 2017), pixel classification in satellite imaginary (Agrawal *et al.*, 2015), transmission loss estimation (Hemparuva *et al.*, 2017) and many more as reported in the literature.

Differential evolution (DE) is a stochastic direct search evolutionary technique, widely used to solve global optimisation problems. Proposed by Storn and Price in 1995, as a heuristic method, it offers fast convergence behaviour and exhibits ability to find true global solution Like other evolutionary algorithms, DE also faces some challenges to find a solution when applied to multimodal large scale optimization problems. The main characteristic of DE different from other evolutionary algorithms is the mutant operation that generates new mutant individual by adding the weighted difference between two individuals to a third one. The crossover operator is applied to combine the mutant individual with the parent individual to generate offspring individual. In the selection process, the survival of the better individuals from parent and offspring population are selected to the next generation. These three operations are repeated to search for the global optimum. Because of its simplicity and effectiveness, DE has been successfully applied to solve optimisation problems in diverse fields such as network reconfiguration (Su *et al.*, 2003), hydrothermal scheduling (Qin *et al.*, 2010), structural design (Liao, 2010), energy resource scheduling (Sharma *et al.*, 2011), system stability control (Vakula *et al.*, 2012), PV cell parameter extraction (Muhsen *et al.*, 2015), multi-pass milling operation control (Yildiz, 2013), optimal clustering (Tvrdik and Krivy, 2015), multi-fuel economic dispatch (Singh *et al.*, 2017), energy storage system management (Muhsen *et al.*, 2015). Irrespective of simplicity in operation and only two control parameters to shape the search direction, DE succumbs to premature convergence in large scale problems. Das *et al.*, (2016) have discussed in detail about the concepts, its variants and applications to multi-objective optimization problems. The success of DE in solving specific problems is largely dependent on the trial vector selection strategies and setting of control parameters to manage a balance between expansion and exploitation of search space.

Based on the above facts and arguments, it is empirically significant to investigate the performance of DE by incorporating chaotic sequences for control parameters settings and mutation strategies for the best trial vector selection. For validating the suggested concepts of investigation, benchmark test functions and single objective hydrothermal scheduling problem are taken for simulations.

2.2 DIFFERENTIAL EVOLUTION

The operational strategy of DE involves extracting the distance and direction information from the current population to guide the population to search the global best generation. Storn and Price, (1995) have proposed DE with a single mutation strategy based on the differential term. Initially, the difference term is very large and as the solution approaches the global minimum, the difference term approaches to a small value. Differential evolution method can search in very large spaces of candidate solutions with few assumptions about the optimisation problem under consideration. DE strategy offers a three-fold advantage in comparison with population-based optimisation techniques; one it is easy to code, second; it requires a lesser number of control parameters to tune. Third; it maintains the multiplicity of population. Since the first publication about the DE algorithm, various significant DE variants have been proposed to enhance its performance. The performance of the DE algorithm is significantly affected by the population size, mutation strategy, mutation factor, and crossover rate.

Broadly there are three directions in which DE has experienced noticeable progress since its inception and summarised in Table 1.2. One; design of novel mutation and crossover strategies (Das, et al., 2009), (Qin *et al.,* 2010), (Gng and Cai, 2011), (Da Silva et al. 2011) etc. for better exploration of search space. Two; an adaptation of control parameters based on the previous experience of generating promising solutions for the maintenance of population diversity (Pan et al., 2011), (Zhoa et al., 2016,), (Mohamed and Suganthan, 2016), (Singh *et al.,* 2017), (Zhang *et al.,* 2013). Three; hybridisation with other evolutionary algorithms (Zhang et al., 2006), and local search techniques such as simplex search, Powell search etc., in which improvements are reported in the literature with varying degree of success.
The procedure of classical differential evolution is described as:

2.2.1 Initialization of Population

Classical DE algorithm is based on n-dimensional vectors $X_k^t = (x_{k1}^t, \ x_{k2}^t, \ x_{k3}^t, \dots \ x_{kn}^t)$ in a population for t^{th} generation of an algorithm, NP is the population size. The initial population is randomly generated which is uniformly distributed over the problem space bounded by limits of decision variables. The population of size

$(N_P \times N_G)$, is generated in the feasible space of the problem using the Equation (2.1). The i^{th} individual vector of the population in t^{th} generation is denoted by x_{ki}^t, such that

$$X_{ki}^t = X_i^{min} + R_{ki}\left(X_i^{max} - X_i^{min}\right); \qquad (i = 1,2,\ldots,N_G; \quad k = 1,2,\ldots,N_P) \qquad (2.1)$$

where R_{jki} is a uniform random number that varies between 0 and 1 and X_i^{min} and X_i^{max} are the minimum and maximum limit of the decision variable X_i

2.2.2 Mutation

In mutation operation, a new mutant vector is produced for the individual current population by mutating a target vector with weighted differential generalised as

$DE((a1/a2)/a3)$.

where $a1$ refers to the method of generating a parent vector (rand/best). $a2$ indicates the number of different vectors in mutation operation (normally set to 1 or 2). $a3$ indicates crossover scheme to create a trial vector population (binomial/exponential)

The basic mutation strategy is expressed as $DE/rand/1$

$$V_{ki}^t = X_{r1i}^t + F(X_{r2i}^t - X_{r3i}^t) \qquad ; r_1 \neq r_2 \neq r_3 \neq k \in (1, N_P) \qquad (2.2)$$

V_{ki}^t is mutant vector in t^{th} generation generated from one difference term of two population members randomly selected from feasible search space. F is scaling factor called mutation constant and controls the amplification of differential variation. Its value is primarily selected between [0, 1].

2.2.3 Crossover

To increase the diversity of individual mutant vector, a trial vector is generated with binomial crossover operation and is expressed as:

$$U_{ki}^t = \begin{cases} V_{ki}^t & ; \ if\ (r_i < CR^t) or\ (j \neq R) \\ X_{ki}^t & ; \qquad otherwise \end{cases} \qquad ; i \in [1, N_G], k \in [1, N_P] \qquad (2.3)$$

where r_i is a random number from dimension N_G of variable and R is random population. A careful selection of population index will help the selection of offspring as a new parent in the population, otherwise, the population will remain unaltered and the solution may not converge to a global optimum. $CR \in [0,1]$, is the crossover rate.

2.2.4 Selection

Selection operation constructs the population for next-generation deterministically. Non-dominated sorting is performed on the current generation of population size 2*NP. Trial vector will be selected for next-generation based on survival of the fittest principle. This mechanism will keep the average population size the same in the next generation. Mathematically

$$X_{ki}^{t+1} = \begin{cases} U_{ki}^{t} & ; if\ f_j^t(U_{ki}^t) > F_j^t(X_{ki}^t) \\ X_{ki}^{t} & ;\ otherwise \end{cases} \quad ;\ (i \in [1, N_G], k \in [1, N_P] \qquad (2.4)$$

The selection operation ensures that the individual vectors in the next generation will be of better quality or at least of the same quality to that of the current generation.

2.3 DIFFERENTIAL EVOLUTION VARIANTS

There are two important aspects of DE algorithms: self-adaptive parameter control and hybridization of DE with other strategies. Among the four basic operations of classical DE, mutation operation plays an important role in generating a potential trial vector to be a part of next-generation for exploration. The history of the decision variable over the generations and the interactions among individual members in all search directions of the problem landscape can improve the success rate of generating quality solutions remarkably. In this chapter, the variations at concept level in simple mutation strategies are presented for solving large scale, multimodal and constrained optimization problem.

2.3.1 Mutation Strategies

The classical DE mutation strategies represented by Equations (2.5a) to (2.5d) have different characteristics and are selected based on the problem domain.

a) DE/rand/1

$$V_k = X_{r1} + F(X_{r2i} - X_{r3}) \qquad (2.5a)$$

b) DE/best/1

$$V_k = X_{best} + F(X_{r1} - X_{r2}) \qquad (2.5b)$$

c) DE/best/2

$$V_k = X_{best} + F(X_{r1} - X_{r2}) + F(X_{r3} - X_{r4}) \qquad (2.5c)$$

d) DE/current-to-best/2

$$V_k = X_i + F(X_{best} - X_{r1}) + F(X_{r2} - X_{r3}) \qquad (2.5d)$$

where X_{best}represents the best individual in the current generation of population and $r_1 \neq r_2 \neq r_3 \neq r_4 \neq k\ \ and\ r_2, r_3, r_4 \in \{1, N_P\}$

These strategies utilize the random or current-best or global-best solution to guide the mutation and can converge very fast on simple unimodal problems. However, such a greedy strategy may suffer from premature convergence when it deals with complex multimodal problems. The most appropriate configuration of DE to solve different optimization problems efficiently can be significantly different. Mutation strategies have been revised to generate a trial vector for next-generation from current-best individual to global-best individual candidate solution to support convergence rate. An appropriate combination of multiple strategies into one DE variant attracts increasing attention recently (Islam *et al.,* 2012). Multi-population based approach to realize an ensemble of multiple strategies simultaneously results in new DE variants named multi-population ensemble DE. Each constituent mutation strategy has one indicator of the subpopulation. In this manner, multiple locally best solutions are selected from different groups of solutions, each of which is mutated under the directions of its respective locally best solution. The concept of the local and global neighbourhood of each population member has been proposed to generate current best population member. The entire population is likely to be engrossed by multiple good solutions chosen from subpopulations and the problem of premature convergence can be better alleviated. Qin *et al.* (2010) proposed the self-adaptive DE (SaDE) algorithm in which both mutant strategies and their associated control parameter values were adaptively selected by learning from their previous experiences and appropriate parameter setting is carried out matching with search evolution process. Wang *et al.* (2011) and Xiang *et al.,* (2015) worked on the composite mutant strategies and control parameters algorithm to generate promising solutions.

In this investigation, various state-of-art mutation strategies defined in the three-dimensional search space of real-world problem listed in Table 2.1 have been explored. Some of these strategies are based on the current-best, global best and history of a variable in the previous generation and others related to the exploration of the population in the opposite direction as well.

X_{kji}^{t} represents the candidate solution of i^{th} decision variable in j^{th} time domain interval from k^{th} population in t^{th} generation

$$R_1 \neq R_2 \neq R_3 \neq R_4 \neq k \quad and\; R_1, R_2, R_3, R \in \{1, N_P\}$$

$Pbest_i$ represents the global best solution selected from the combination of the population of the archived set of non-inferior solutions collected over the previous generations and the current best population.

X_{kji}^{t-1} represents the individual candidate solution of i^{th} decision variable in j^{th} time domain interval from k^{th} population in $(t-1)^{th}$ generation to incorporate the effect of previous experience of the decision variable generating potential solutions.

The objective of exploring different DE mutation strategies is to search for the best appropriate strategy to solve optimization problems efficiently while achieving the global optimum.

Table 2.1: List of mutation strategies

Strategy Number	Mutation Strategy Type	Updating variable X_{kji}^{t}
C1	DE/best/1	$X_{kji}^{t} = Pbest_i + F \times \left(X_{R1ji}^{t} - X_{R2ji}^{t}\right)$
C2	DE/best/2	$X_{kji}^{t} = Pbest_i + F \times \left(X_{R1ji}^{t} + X_{R2ji}^{t} - X_{R3ji}^{t} - X_{R4ji}^{t}\right)$
C3	DE/rand/1	$X_{kji}^{t} = X_{R1ji}^{t} + F \times \left(X_{R2ji}^{t} - X_{R3ji}^{t}\right)$
C4	DE/rand-current-best/2	$X_{kji}^{t} = X_{kji}^{t-1} + F \times \left(Pbest_i - X_{kji}^{t-1}\right)$ $X_{kji}^{t} = X_{kji}^{t} + F \times \left(X_{R1ji}^{t} - X_{R2ji}^{t}\right)$
C5	DE/rand-current-best/3	$X_{kji}^{t} = X_{kji}^{t-1} + F \times \left(Pbest_i - X_{kji}^{t-1}\right)$ $X_{kji}^{t} = X_{kji}^{t} + F \times \left(X_{R1ji}^{t} - X_{R2ji}^{t}\right)$ $X_{kji}^{t} = X_{kji}^{t} + F \times \left(X_{R3ji}^{t} - X_{R4ji}^{t}\right)$
C6	DE/rand/2	$X_{kji}^{t} = X_{kji}^{t-1} + F \times \left(X_{R1ji}^{t} - X_{R2ji}^{t}\right)$ $X_{kji}^{t} = X_{kji}^{t} + F \times \left(X_{R3ji}^{t} - X_{R4ji}^{t}\right)$
C7	DE/current-best/1	$X_{kji}^{t} = Pbest_i + F \times \left(Pbest_i - X_{kji}^{t-1}\right)$
C8	DE/rand-current-best/2	$X_{kji}^{t} = Pbest_i + F \times \left(Pbest_i - X_{kji}^{t}\right)$ $X_{kji}^{t} = X_{kji}^{t} - F \times \left(X_{R1ji}^{t} + X_{R2ji}^{t}\right)$
C9	DE/current-best/2	$X_{kji}^{t} = Pbest_i + F \times \left(Pbest_i - X_{kji}^{t-1}\right)$ $X_{kji}^{t} = X_{kji}^{t} + F \times \left(X_{R1ji}^{t} - X_{R2ji}^{t}\right)$
C10	DE/current-best/2	$X_{kji}^{t} = Pbest_i + F \times \left(Pbest_i + X_{kji}^{t-1}\right) - F \times (X_{R1ji}^{t} + X_{R2ji}^{t}\big)$
C11	DE/best/1	$X_{kji}^{t} = Pbest_i + F \times (Pbest_i - Pbest_{i-1})$
C12	ODE/rand/1	$X_{kji}^{t} = \left(X_i^{max} + X_i^{min}\right) - F \times X_{kji}^{t-1}$

2.3.2 Chaotic Mapping of Control Parameters

The classical mutation operator in DE explores the search space in a fixed model. The population diversity decreases as the evolution proceeds and it almost converges to local optima. In case, local optima is not a global solution then differential evolution will suffer from premature convergence. Being differential evolution a strong global optimiser player among evolutionary algorithms, it is worth to invest in the computational cost to find the self-adaptive alternatives that are easy to implement and relatively stable. There are five control parameters in the DE algorithm namely; population size (N_P), mutation constant (F), crossover probability (CR) and number of function evaluations (N_{FE}) and the number of maximum generations (T_{MAX}). The terminal criteria of the stochastic algorithm fix the maximum number of function evaluations or the maximum number of generations. The self-adaptive mechanism to tune the other three control parameters at the population level is a tedious task for efficient computation of the problem. In general, the setting of population size-specific to a problem is empirically taken as 5 to 10 times the dimensions of the considered problem. However, deterministic factors like several function evaluations, population generation member may be used at the population level to control parameters but are not sufficiently effective as fitness value is not used. Otherwise at the population level, feedback information like fitness value, population diversity is considered for adaptive tuning of the population as population increment strategy and population cut strategy proposed by Zhu *et al*., (2013). Islam *et al.,* (2012) proposed the adaptive strategy to control parameters to maintain the strong explorative ability as well as to eradicate the maximum probability of premature convergence and overcome the high risk of stagnation. In response to challenges faced by DE, several approaches have been presented in the literature that is applied successfully to solve a variety of real-life optimisation problems.

In this work, a chaotic sequence strategy adaptively generates parameters *viz;* mutation constant and crossover rate. A chaotic search strategy is one of the promising memetic approaches to strengthen the exploitation ability of DE for a quality global solution and accelerate its convergence speed. It is a nonlinear dynamical system known for its properties of ergodicity, randomness and sensitivity. The ergodicity of the chaotic system can support more diversity to enhance exploration and exploitation capability of evolutionary algorithms. Due to this nature, chaos is a deterministic, non-period, non-converging one-dimensional random process dynamical system that can randomly generate a long-time sequence but highly sensitive dependence on the initial condition. A chaotic system can traverse through every state of the system and every state is generated only once if given a long enough period (Sharma *et al.,* 2011). The utilization of chaotic mappings to control parameters enlarge the search space when it converges to optima and the chaotic factor supports the search trajectory to escape out from the local optimal region when it is trapped in local optima, more easily than the traditional ones. Among various chaotic sequence variations, namely Logistic map, Tent map, the Gauss map, Sine map, Chebyshev and circle map etc., the logistic map is a polynomial of degree 2 that shows chaotic behaviour of rather simple nonlinear difference equation stated as:

$$y^t = \mu y^{t-1}(1 - y^{t-1}) \quad (2.6)$$

where μ is the logistic parameter with values in the interval [1,4] for chaotic mapping. y^{t-1} is distributed in the range [0,1].

The value of μ determines whether chaotic sequence stabilizes at a constant size and oscillates between a limited sequence of sizes, or behaves chaotically in an unpredictable pattern. Equation (2.6) is deterministic. y^0 is initialized to an appropriate value so that maximal Lyapunov exponent achieves value close to or more than 1.0. The chaotic behaviour of the logistic map for μ=4 and $y^t \notin \{0, 0.25, 0.5, 0.75, 1\}$ is depicted in Figure 2.1. Maximal Lyapunov exponent (MLE), $\lambda(\cdot) > 0$, stated in Equation (2.7) determines the notion of predictability of chaotic system and verifies the quality of a chaotic map. Larger the value of this MLE, better is the quality of chaos map.

$$\lambda(y^0) = \lim_{n \to \propto} \frac{1}{n} \sum_{i=1}^{\propto} ln\left|f'\left(y^i\right)\right| \tag{2.7}$$

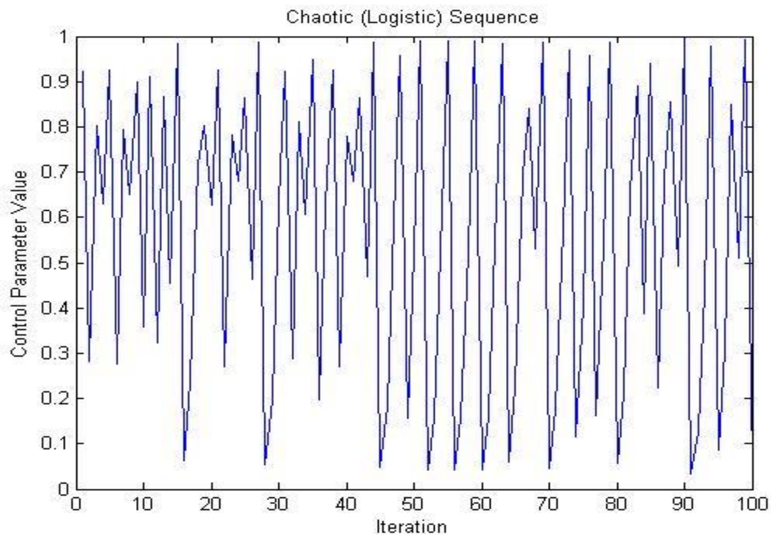


Fig. 2.1: Chaotic behaviour with Logistic mapping

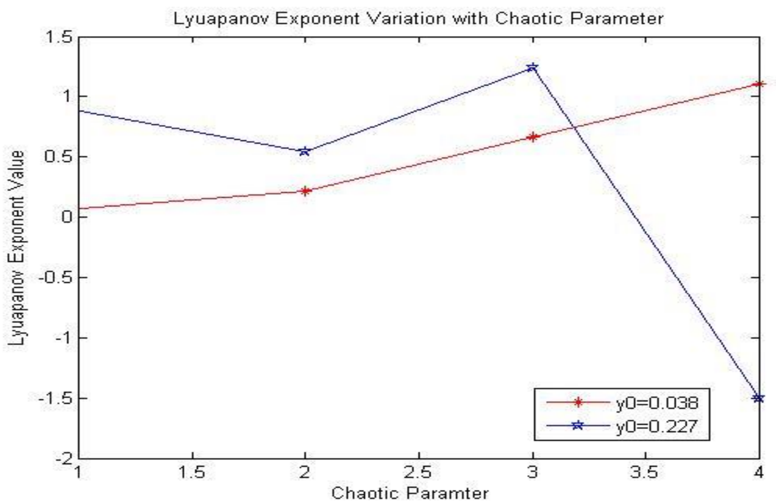


Fig. 2.2 Lyapunov exponent spectrum

Lyapunov exponent spectrum for the logistic map with two different initial values of y^0 is illustrated in Figure 2.2.

From the spectrum, it is evident that maximal Lyapunov exponent of the Logistic map is good and is positive and is close to 1.0 at $\mu \to 4$. Values of MLE from the spectrum for two different initial values are presented in Table 2.2

Table 2.2: Maximal Lyapunov Exponent (MLE) values for different initial values

Chaotic Parameter	**Initial Values y^0**	**Maximal Lyapunov Exponent (MLE)**	**Comment**
$\mu \to 4$	0.038	1.106	Good
$\mu \to 4$	0.227	-1.509	Not good
$\mu \to 3$	0.227	1.243	

The mutation constant, F and crossover rate, CR is generated by Logistic mapping using the following relations.

$$F^t = F^L + y^t(F^U - F^L) \tag{2.8}$$

$$CR^t = CR^L + y^t(CR^U - CR^L) \tag{2.9}$$

where $F^L and\ F^U$ are lower and upper limits of mutation constant respectively. Also $CR^L\ and\ CR^U$ are lower and upper limits of crossover rate respectively

Mutation constant controls the amplification of differential variation and selection of F^t greatly affects the positioning of the moderated solution in the neighbourhood of optimum solution or near the periphery of the hypercube search space, allowing a gradual exploration of the search space. It is important to understand the sensitiveness of the initial condition of a chaotic system that can lead to different search paths and escape from local optima for enhancing its search performance.

There are two different strategies to apply the chaotic maps in the differential evolution algorithm.

- Replace the random numbers needed by the algorithm by chaotic sequence generated by a chaotic map. Coello, (2008) has suggested chaotic sequences based on Zaslavskii map for mutation operator in quantum-behaved particle swarm optimization (QPSO). Zhang and Zhang, (2013) suggested three different chaotic sequences namely logistic, circle and tent with differential evolution to solve hydrothermal scheduling problems.

This is a powerful strategy to diversify the population and improve the performance in preventing premature convergence to local minima.

- Apply chaos optimization as an operator, when there is no improvement in the objective function over the iterations. Under this strategy, the given candidate solution population is arranged in descending order and the chaotic system can be applied to reinitialize the nearly 50% of the weak population. It will replace the worst part of the population with the promising one keeping the best portion as such and supports to escape from local optimum. whereas the best portion is kept unchanged. Wang and Zhang [19] employed chaos analogously to crossover and population perturbation.

In this study, both strategies are successfully applied in the following chapters.

2.3.3 Opposition Based Learning Strategy

The concept of opposition computational is inspired by the concept of opposition in real-world applications defined as:

Assume $X_i^t = \left(x_{i1}^t,\ \ x_{i2}^t,\ \ x_{i3}^t, \dots\ x_{iN_G}^t\right)$ be a point in N_G-dimensional space, and $x_i \in [a_i, b_i]$; $i = 1,2, \dots N_P$. The opposite of X_i^t is defined as $\left(\overline{x_{i1}}, \dots, \overline{x_{iN_G}}\right)$ such that

$$\overline{x_i} = a_i + b_i - x_i \tag{2.10}$$

Graphically, the point x and its opposite in one, two and three-dimensional space are as indicated in Figure 2.3 (Mahadevi *et al.*, 2018). There are two categories of oppositional concept.

Type 1: Opposition is defined according to the relationship between the points in search space but no consideration of objective function.

Type –II: Opposition is defined according to objective function space. For every point X_i^t in N_G-dimensional space, the opposite of X_i^t defined as $(\overline{x_{i1}}, \dots, \overline{x_{in}})$ such that

$$\overline{x_i} = \{x | \bar{y} = y_{min} + y_{max} - y\} \tag{2.11}$$

The opposition based concept was applied by Rahnamayan *et al.*, (2006) with the DE algorithm to improve is performance. This technique can be applied in two stages:

Initialization Stage: The uniformly distributed initial population and the opposite population are simultaneously evaluated for its fitness value. From the population of size (2*N_P,), the top 50%, as the best population is accepted for the next operation.

Algorithm: *Initialization*

Step 1: Input (N_P, NFE, N_G)

Step 2: Generate initial population uniformly distributed over feasible search space.

Step 3: Calculate opposite population using Equation (2.10).

Step 4: Calculate the fitness' value and pick up the top 50% of the population as the initial population.

Step 5: Increment $iteration\ count\ t = t + 1$

While($t < Tmax$)

Step 6: Perform Mutation.

Step 7: Perform Crossover

Step 8: Perform Selection

ENDDO

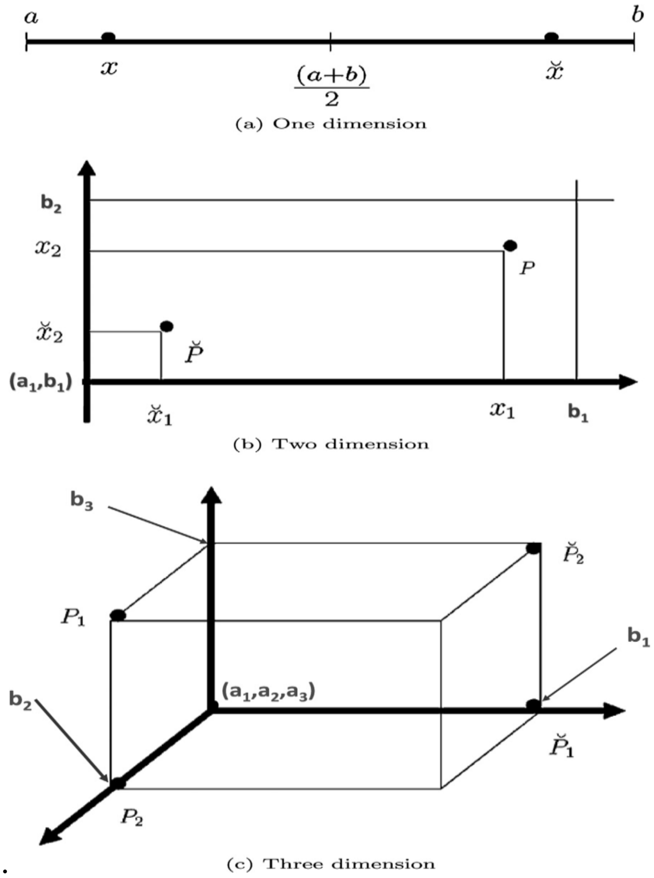


Fig. 2.3: Type –I opposite to point x in one, two and three-dimensional space

Evolutionary Stage: The opposite point of each variable within its limits in the current population is defined in Equation (2.10). In case, the solution does not improve for a certain number of iterations, then the opposition learning is applied using the equation defined as:

$$X_{ki}^{t+1} = \begin{cases} x_i^{min} + x_i^{max} - x_{kji}^{t+1} & ;\ F_{opp} \leq 0.5 \\ x_i^{min} + R_{ki}\left(x_i^{max} - x_i^{min}\right) & ;\ F_{opp} > 0.5 \end{cases} \quad ;(i = 1,2 \dots N_G;\ \ k = 1,2 \dots N_P) \quad (2.12)$$

where F_{opp} is the opposition factor.

Algorithm: *Evolutionary*

Step 1: Input a population A of candidate solutions obtained after selection.

Step 2: If $F_{opp} \leq 0.5$

Step 2: Calculate opposite population B of the current population of step 1

Step3: Pick (N_p*population from A∪B as the current population*

ENDIF

The global search capability of DE strategy is further expedited using opposition-based learning in an evolutionary stage. For keeping the elite solutions, the retention mechanism is integrated with the selection operation of DE. For several non-dominated solutions generated from populations by mutation, crossover and selection, the opposite population is generated and is evaluated. The fittest individuals are selected from the unification of the current-best population and opposite population. This proposal speeds up the convergence towards the global solution

2.4 CRISSCROSS OPERATIONS

In metaheuristic algorithms, the performance metrics convergence speed and global solution accuracy are conflicting. Some algorithms manage population diversity at the cost of convergence speed, others manage premature convergence at the cost of computational efficiency. Some algorithms need to tune control parameters to manage a balance between exploration and exploitation to achieve global solution accuracy. It is indeed a challenging task to develop a stochastic random search algorithm that can manage all prominent performance metrics. Since a decade several researchers are working on the problem of differential evolution leading to premature convergence. In this investigation crisscross mechanism inspired by the Confucian doctrine of the gold mean, a dual search mechanism is initiated to search the potential trial vectors in DE. The crisscross search strategy operates in both the horizontal and vertical directions and the moderate solutions for the next generation are selected through completion among moderate solutions obtained from an alternate sequence of mutation and crossover operations in orthogonal directions within a generation (Meng *et al.*, 2015). The horizontal crisscross enhances diversity, whereas vertical crisscross prevents the trapping in local optima. The procedure for the crisscross mechanism is summarised as:

Algorithm: *Crisscross*

Step 1: Initialise the population

While $(t < Tmax)$

DO

Step 2: Perform a horizontal mutation operation to obtain a set of trial vectors.

Step 3: Perform vertical mutation to obtain a set of trial vectors

Step 4: Perform a horizontal crossover operation to obtain a set of trial vectors.

Step 5: Perform a vertical crossover to obtain a set of trial vectors.

Step 6: Select the best solution from the four solutions

ENDDO

2.4.1 Crisscross Mutation

The horizontal crisscross mutation is performed on all the dimensions between two different individual members of a decision variable for the given population.

$$H_{ki}^{t} = x_{kr_1}^{t} + F^{t} \times \left(x_{kr_2}^{t} - x_{kr_3}^{t}\right) \qquad ; r_1, r_2, r_3 \neq i \in \{1, N_G\}, \forall k \in \{1, N_P\} \tag{2.13}$$

The vertical crisscross mutation is performed on all the dimensions between two different individual members of the population for a given decision variable

$$U_{ki}^{t} = x_{r_4 i}^{t} + F^{t} \times \left(x_{r_5 i}^{t} - x_{r_6 i}^{t}\right) \qquad (r_4, r_5, r_6 \neq k \in \{1, N_P\}, \forall i \in \{1, N_G\}) \tag{2.14}$$

where F^t is chaotically generated mutation constant.

2.4.2 Crisscross Crossover

The horizontal crossover exchange information among the decision variables and sub-intervals of a population. The horizontal binomial crossover is defined below to generate variable U_{jki}^{t}.

$$V_{ki}^{t} = \begin{cases} U_{ki}^{t} & ; if\ (r_i < CR^t) or\ (k = R1) \\ \mathcal{X}_{ki}^{t} & ;\ otherwise \end{cases} \qquad (i = 1,2, \ldots, N_G; k = 1,2, \ldots, N_P) \tag{2.15}$$

The random integer numbers are generated such that $r_i \in [1, N_G]$, and $R1 \in [1, N_P]$.

The vertical crossover exchange information among the population and subintervals. The vertical binomial crossover is defined below to generate variable, W_{jki}^{t}.

$$W_{ki}^{t} = \begin{cases} V_{ki}^{t} & ; if\ (r_i < CR^t)\ or\ (i = R1)\ or\ (k = R2) \\ \mathcal{X}_{ki}^{t} & ;\ otherwise \end{cases}$$

$$(i = 1,2, \ldots, N_G;\ k = 1,2, \ldots, N_P) \tag{2.16}$$

where CR^t is chaotically generated crossover rate and where the random integer numbers are generated such that $r_i \in [1, N_P], R1 \in [1, N_G]$.

The recombination operation is performed among all four trial vector solutions to select the best one as

$$\mathcal{X}_{ki}^{t+1} = \begin{cases} y_{ki}^{t} & ; if\ f_j^t(y_{ki}^{t})\ > F_j^t(\mathcal{X}_{ki}^{t}) \\ \mathcal{X}_{ki}^{t} & ;\ otherwise \end{cases} \qquad (\ k = 1,2 \ldots N_P) \tag{2.17}$$

where $f_j^t(y_{ki}^t) = \max\{F_j^t(H_{ki}^t), F_j^t(V_{ki}^t), F_j^t(U_{ki}^t), F_j^t(W_{ki}^t)\}$

The simplicity of this competition allows the population to move rapidly in the search space with better fitness and speeds up the convergence rate to the global optimal solution.

2.5 SINGLE OBJECTIVE OPTIMIZATION

The problem of hydrothermal generation scheduling (HTGS) aims at obtaining the minimum operating cost of thermal units over the scheduling period by optimal utilization of water resources while satisfying various operational constraints. Mathematically the objective function and constraints are formulated as:

Quadratic Model of Thermal Unit

$$\text{Minimize operating cost}, F_1 = \sum_{j=1}^{T}\left(\sum_{i=1}^{N_T}\left(a_i P_{ji}^2 + b_i P_{ji} + c_i\right)\right) \tag{2.18}$$

Quadratic Model of Hydro Unit

In the multi-chain model, the power generation of a hydro unit at any time is a function of water discharge rate and reservoir volume (Kothari and Dhillon, 2010) stated as:

$$P_{ji} = C_{1i}V_{ji}^2 + C_{2i}q_{ji}^2 + C_{3i}V_{ji} + C_{4i}q_{ji} + C_{5i}V_{ji}q_{ji} + C_{6i} \quad (i = 1,2,\dots N_H, j = 1,2,\dots T) \tag{2.19}$$

Subjected to constraints

(i) To ensure power balance, an equality constraint during each subinterval is imposed.

$$\sum_{i=1}^{N_G} P_{ji} = P_{Dj} + P_{Lj} \qquad (j = 1,2,\dots,T,\;\; N_G = N_T + N_H) \tag{2.20}$$

The transmission losses are represented by Kron's loss formula (Kothari and Dhillon, 2010) through B-coefficients.

$$P_{Lj} = B_{00} + \sum_{m=1}^{N_G} B_{m0}P_{jm} + \sum_{m=1}^{N_G}\sum_{n=1}^{N_G} P_{jm}B_{mn}P_{jn} \qquad (j = 1,2,\dots,T) \tag{2.21}$$

(ii) The volume dynamics in the hydro reservoir (Kothari and Dhillon, 2010)

$$V_{ji} = V_{(j-1)i} + I_{ji} - q_{ji} - S_{ji} + \sum_{m=1}^{R_H}\left(q_{(j-\tau_{mi})m} + S_{(j-\tau_{mi})m}\right)$$

$$(i = 1,2 \dots N_H, j = 1,2 \dots T) \tag{2.22}$$

(iii) The hydraulic (storage) constraint for a hydro unit reservoir at the end of the scheduling period (Kothari and Dhillon, 2010)

$$V_{(T+1)i} = V_{1i} + \sum_{j=1}^{T} I_{ji} - \sum_{j=1}^{T} q_{ji} - \sum_{j=1}^{T} S_{ji} + \sum_{j=1}^{T}\sum_{m=1}^{R_H}\left(q_{(j-\tau_{mi})m} + S_{(j-\tau_{mi})m}\right)$$

$$(i = 1,2 \ldots N_H) \qquad (2.23)$$

Inequality Constraints

(i) Active power generation limits on each hydro and thermal unit

$$P_i^{min} \le P_{ji} \le P_i^{max} \qquad (i = 1,2,\ldots,N_G;\ j = 1,2,\ldots,T) \qquad (2.24)$$

(ii) Water discharge rate limits on hydro units

$$q_i^{min} \le q_{ji} \le q_i^{max} \qquad (i = 1,2,\ldots,N_H;\ j = 1,2,\ldots,T) \qquad (2.25)$$

(iii) Reservoir volume limits on hydro units

$$V_i^{min} \le V_{ji} \le V_i^{max} \qquad (i = 1,2,\ldots N_H; j = 1,2,\ldots,T) \qquad (2.26)$$

where $a_i, b_i, c_i, d_i\ and\ e_i$ are operating cost coefficients, of i^{th} thermal unit. N_T is the number of thermal units, N_H is the number of hydro units and N_G represents the total number of committed hydrothermal generating units. T is the scheduling period. $P_{ki}\ and\ P_{kj}$ are active power generation of i^{th} thermal and j^{th} hydro units, respectively, P_{Dk} is the load demand and P_{Lk} represents transmission losses during k^{th} sub-interval. B_{00}, B_{m0}, B_{mn} are termed as B-coefficients. $C_{1i}, C_{2i}, C_{3i}, C_{4i}, C_{5i}, C_{6i}$ are hydro unit coefficients of i^{th} hydro unit. $V_{kj}, I_{kj}, q_{kj}\ and\ S_{kj}$ are storage volume, water inflow rate, water discharge rate and spillage from the reservoir of j^{th} unit in k^{th} subinterval respectively, R_H is the number of reservoirs on the upper side, τ_m represents water transport time delay from m^{th} upper reservoir to lower reservoir 'j'. V_{1j} is an initial volume and $V_{(T+1)j}$ is the final storage volume of j^{th} reservoir at the end of the scheduling period. $V_j^{min}\ and\ V_j^{max}$ are lower and upper limits of the storage capacity of the reservoir and $q_j^{min}\ and\ q_j^{max}$ are minimum and maximum water discharge rate limits of the j^{th} hydro unit, respectively. $P_i^{min}\ and\ P_i^{max}$ are lower and upper limits of active power generation of i^{th} hydro or thermal unit, respectively

2.5.1 Solution Methodology

To perform hydrothermal generation scheduling, active power generation for thermal units ($P_{ji};\ j = 1,2,\ldots,T;\ i = 1,2,\ldots,\mathrm{N}_T$) and the water discharge rate for hydro units ($Q_{ji};\ j = 1,2,\ldots,T;\ i = 1,2,\ldots,\mathrm{N}_H$) during each sub-interval are searched using the proposed Chaotic DE algorithm. For j^{th} population member of decision variables defined as:

$$X_k = \begin{bmatrix} P_{k11} & P_{k12} & P_{k1N_T} & Q_{k11} & Q_{k12} & Q_{k1N_H} \\ P_{k21} & P_{k22} & P_{k2N_T} & Q_{k21} & Q_{k22} & Q_{k2N_H} \\ P_{k31} & P_{k32} & P_{k3N_T} & Q_{k31} & Q_{k32} & Q_{k3N_H} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ P_{kT1} & P_{kT2} & P_{kTN_T} & Q_{kT1} & Q_{kT2} & Q_{kTN_H} \end{bmatrix}_{T\times(N_T+N_H)} \quad (\mathrm{k} = 1,2,\dots,N_P) \qquad (2.27)$$

The minimum and maximum limits of decision variables are represented as:

$$X_i^{min} = \left[P_1^{min}, P_2^{min}, \dots, P_{N_T}^{min}, Q_1^{min}, Q_2^{min}, \dots, Q_{N_H}^{min}\right]^T$$

$$X_i^{max} = \left[P_1^{max}, P_2^{max}, \dots, P_{N_T}^{max}, Q_1^{max}, Q_2^{max}, \dots, Q_{N_H}^{max}\right]^T$$

The augmented objective function is rewritten as:

$$F_T = \begin{cases} F_1\,; & x_i^{min} \le x_{kji} \le x_i^{max} \\ F_1 + F_c; & otherwise \end{cases} \qquad (2.28)$$

where F_1 is the combined objective function of cost and emission and F_c is the function that corresponds to the violation of constraints and is defined as:

$$F_c = P_F \times \langle viol \rangle \qquad (2.29)$$

where P_F is the exterior penalty factor having a large value of the order of 10^3 to 10^5.

and total constraint violation value $\langle viol \rangle$ is described as:

$$\langle viol \rangle = \sum_{j=1}^{T}\sum_{i=1}^{N_H} min\left(0, \left(V_{kji} - V_i^{min}\right)^2, \left(V_i^{max} - V_{kji}\right)^2\right) + \sum_{i=1}^{N_H}\left(V_{(T+1)i} - V_{1i}\right)^2 + \sum_{j=1}^{T}\left(P_{Dj} + P_{Lj} - \sum_{i=1}^{N_G} P_i\right)^2 + \sum_{j=1}^{T}\sum_{i=1}^{N_H} min\left(0, \left(P_{kji} - P_i^{min}\right)^2, \left(P_i^{max} - P_{kji}\right)^2\right) \qquad (2.30)$$

The computed dependent variable x_{ki} in the k^{th} sub-interval) is taken equal to its nearest boundary if it violates its respective boundary limits expressed as:

$$x_{kji} = \begin{cases} x_i^{min} & ;\ \text{if } x_{kji} < x_i^{min} \\ x_i^{max} & ;\ \text{if } \mathrm{x_{kji}} > x_i^{max} \\ x_{kji} & ;\ \text{if } x_i^{min} \le \mathrm{x_{kji}} \le x_i^{max} \end{cases} \quad (j \in [1, T]) \qquad (2.31)$$

Equality Constraints

If both the violation in the equality constraints are within pre-set values then, continue with better population for fitness function, otherwise, violation in the equality constraints is handled through the exterior penalty method defined in Equation (2.30) and (2.31). At the end of the scheduling period, violation in total reservoir volume of i^{th} hydro unit, $|\Delta V_i| = \left|V_{(T+1)i} - V_{1i}\right|$

Violation in the power balance in j^{th}, sub-interval will be: $\left|\Delta P_{Dj}\right| = \left|\sum_{i=1}^{N_G} P_{ji} - P_{Dj} - P_{Lj}\right|$

Inequality Constraints Handling

For inequality constraints expressed by Equations (2.24), (2.25) and (2.26), any design control variable x_{kji}^{t} as a candidate solution is kept in the feasible problem-space region. Any violation is the constraint on hydropower generation, reservoir volume and water discharge rate is computed in Equations (2.30)

2.6 SIMULATION OF RESULTS AND DISCUSSION

In this section, the feasibility and the effectiveness of the proposed mutation strategies, for chaotic differential evolution (CDE) algorithm have been demonstrated by the generalized benchmark test functions (Appendix-B) and the real-world problem of hydrothermal generation scheduling problem.

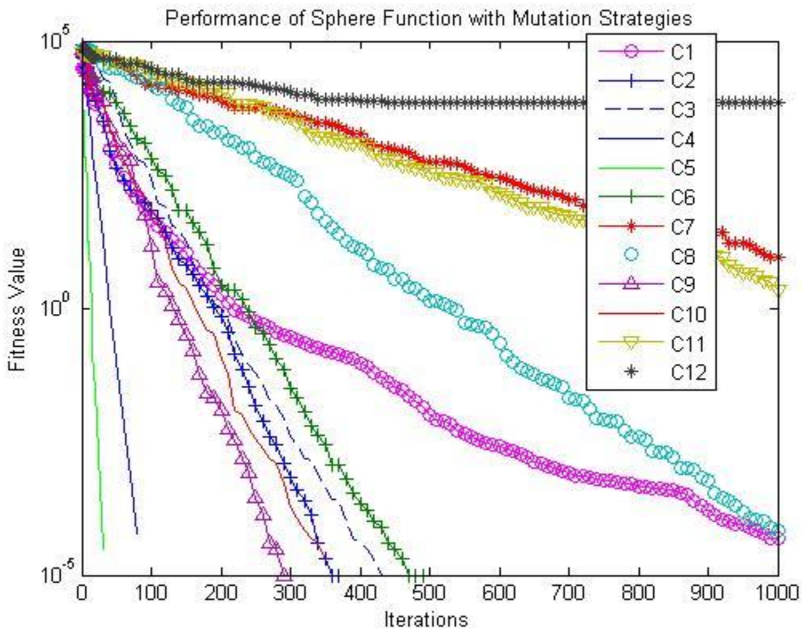
Experimental Set up

For simulation experiments on each benchmark test functions, the number of independent trial runs is set to 30. For every run, the population size, N_P is taken as 50 and the maximum number of iterations is set to 1000. The number of function evaluations (N_{FE}), are considered to assess the efficiency of the algorithm. However, for calculating the success rate and acceleration rate, the fitness value-to-reach (VTR) is taken as 10^{-10}. The trial is said to be successful when the algorithm results in fitness value not worse than VTR. The initial values of mutation constant are randomly chosen in the range [0,0.5] and crossover rate is chosen on the range [0, 0.995] and in subsequent iterations, both the constants are chaotically updated. The termination criterion of the algorithm is set to achieve the fitness value of the respective test function or the maximum number of iterations, otherwise.

2.6.1 Generalized Benchmark Test Functions

The characteristics of undertaken benchmark test functions are unimodal and multimodal continuous and discontinuous, differentiable and non-differentiable, scalable and non-scalable, separable and non-separable. In this study, the performance investigation focuses on the quantitative and qualitative comparison of twelve mutation strategies for thirteen benchmarks test functions $(F_1, F_2, \ldots, F_{13})$ of a variety and the description of each of the thirteen benchmark functions is given in appendix-B. The logistic map chaotic sequence is applied to adjust the mutation factor F^t and crossover rate CR^t to investigate the behaviour.

The quantitative results obtained from simulations for the maximum number of evaluations N_{FE} = 60030 are presented in Table 2.3. The qualitative results show the convergence behaviour of all twelve strategies applied to a test function. The convergence curves are obtained in the *semilog* scale that gives a better comparison of performance. Figure 2.4 depicts the behaviour of mutation strategies for selective benchmark functions in the category unimodal (U), multimodal (M), shifted (SH), differentiable (D), non-differentiable (ND), continuous (C), non-continuous (NC), separable (S) and non-separable (NS).



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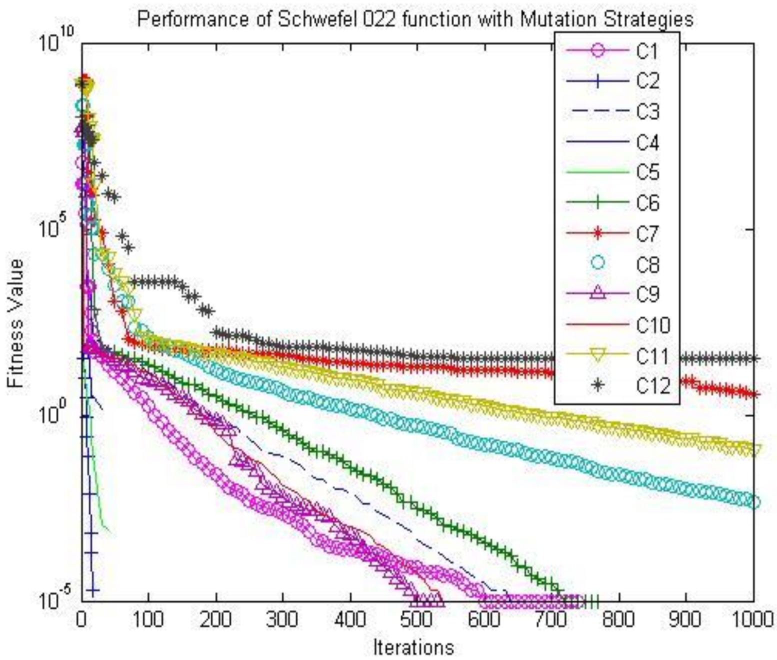
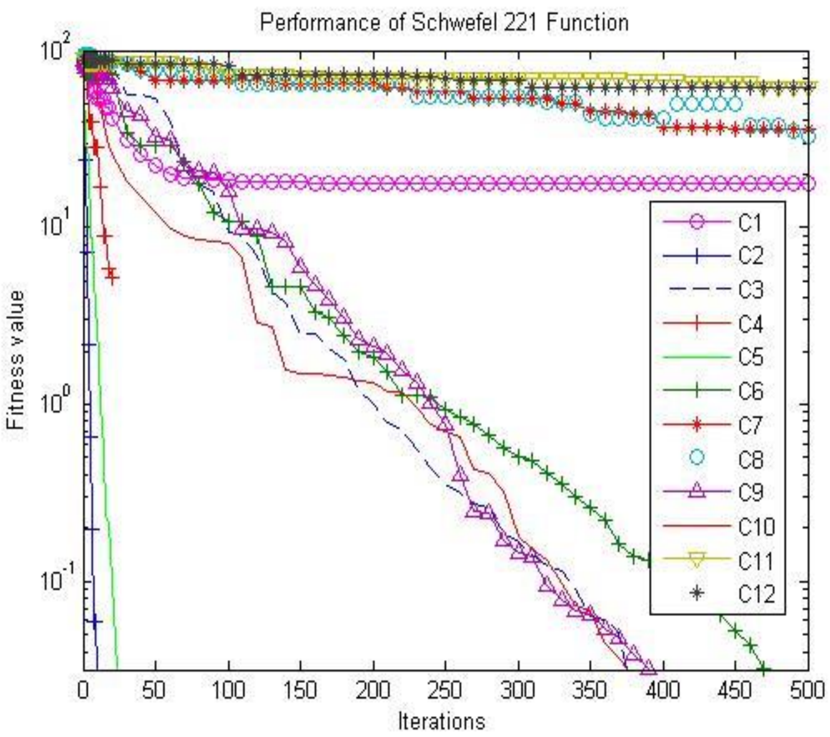
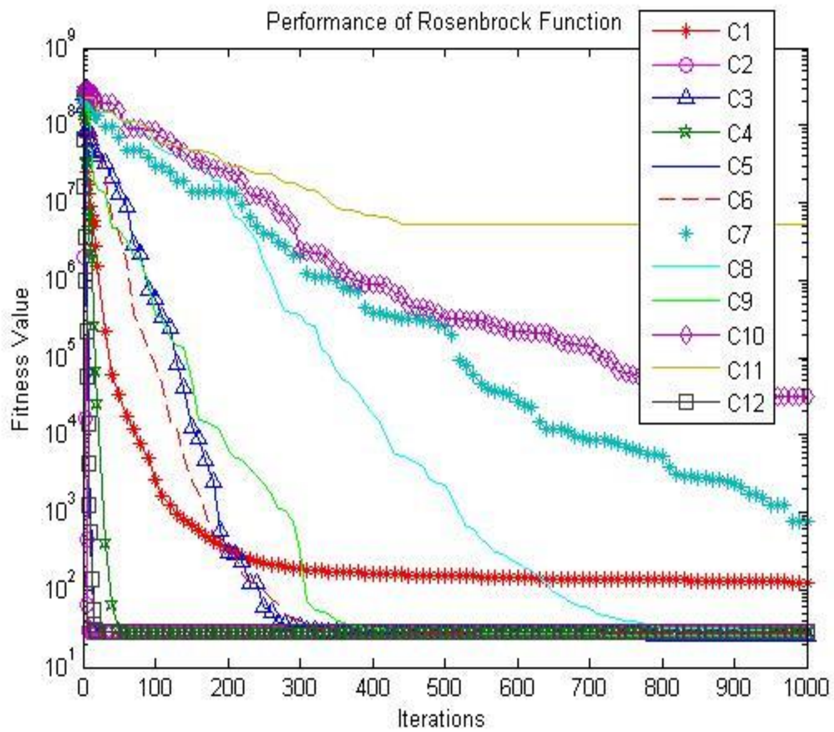
F_1 : Sp**h**ere Function (C, D, S, U)



F_2 :Schwefel 022 Function (C, D, NS, U)

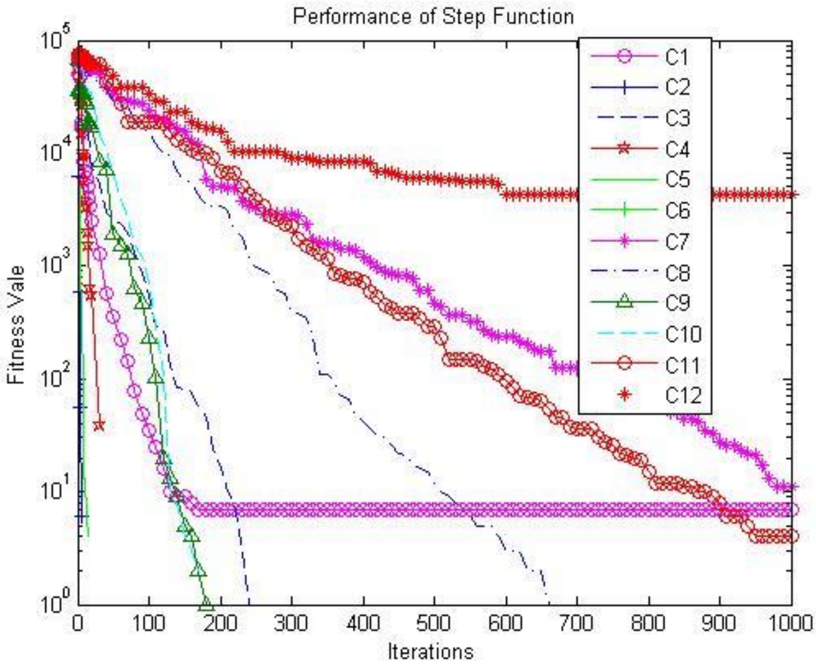


F_4 :Schwefel 221 Function (C, ND, S, U)

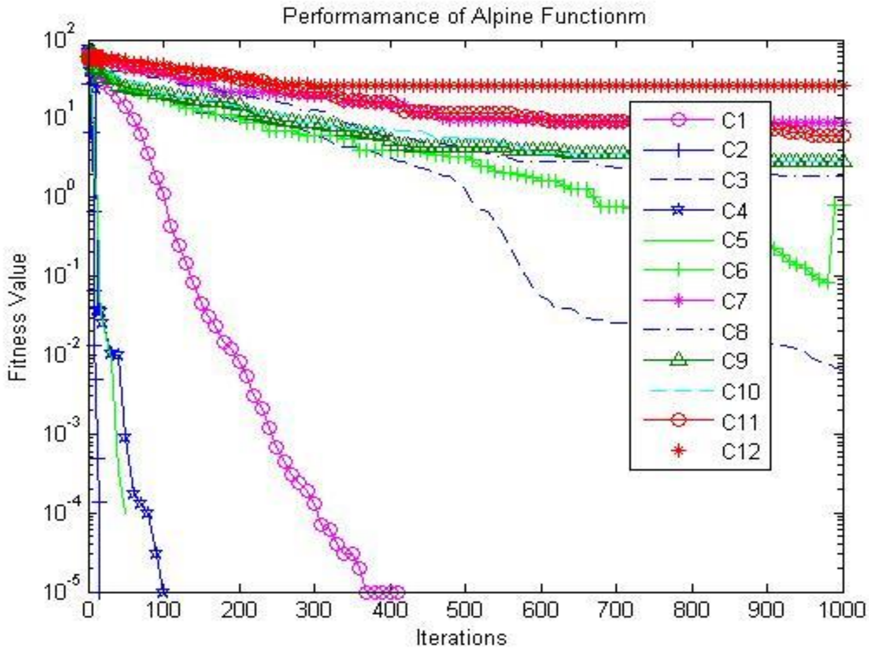


F_5 :Rosenbrock Function (C, D, NS, U)

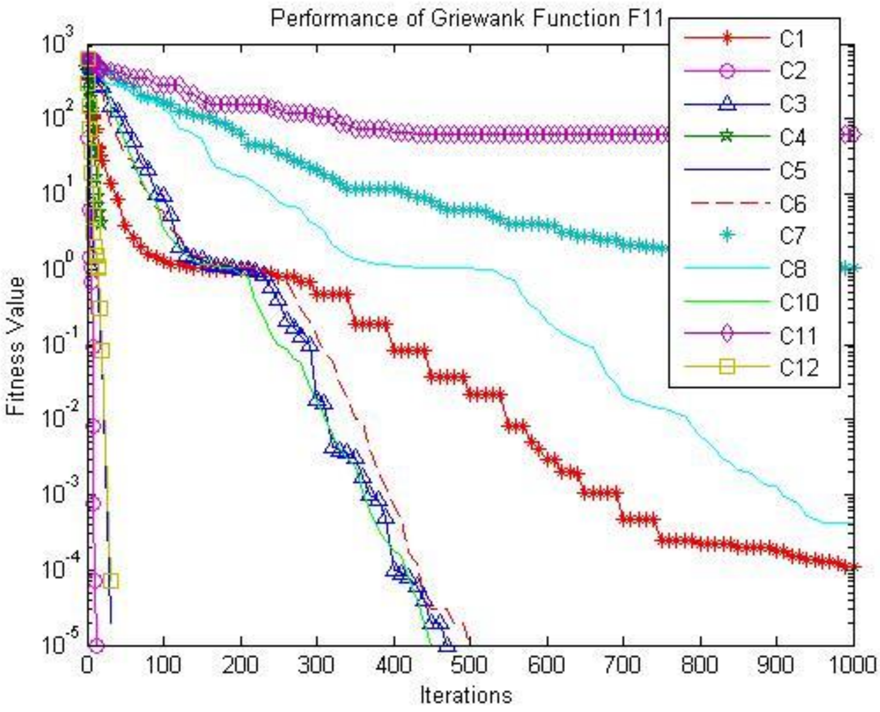
(a)



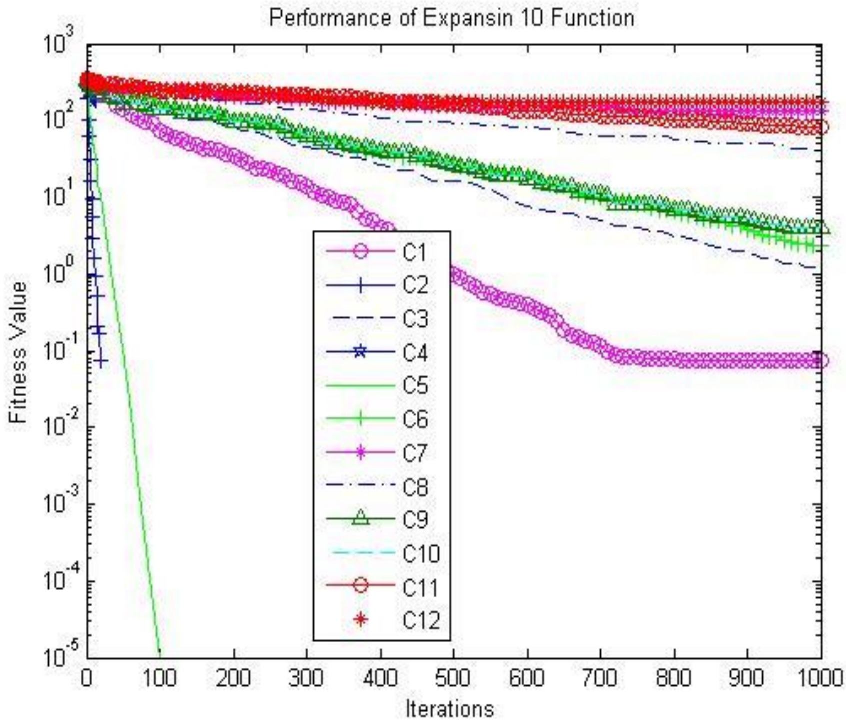
F_6:Step Function (NC, ND, S, U, SH)



F_{10} : Alpine Function (C, ND, S, M)



F_{11} Griewank Function (C, D, NS, M, R)



F_{13}: Expansion 10 Function (C, D, S, M)

Fig. 2.4 Convergence behaviour of Test functions in response to mutation strategies

The performance criteria include the best value, standard deviation, success rate, convergence graph and CPU time to evaluate the mutation strategy. From the results summarized Table

2.3 it is quite evident that mutation strategy C2, C4 and C5 and C12 are excellent among all with 100% success rate and satisfy the performance criteria set for the evaluation of strategies. The strategy C2 gives a tough competition to strategy C4 in terms of the best value and convergence trajectory.

Table 2.4: Outcome of chaotic mutation strategies

No.	Nomenclature	Strategy Description	Priority
C2	CDE/best/2	$X_{kji}^{t} = Pbest_i + F^t \times \left(X_{R1ji}^{t} + X_{R2ji}^{t} - X_{R3ji}^{t} - X_{R4ji}^{t}\right)$	1
C4	CDE/rand-current-best/2	$X_{kji}^{t} = X_{kji}^{t} + F^t \times \left(Pbest_i - X_{kji}^{t-1}\right)$ $X_{kji}^{t} = X_{kji}^{t} + F^t \times \left(X_{R1ji}^{t} - X_{R2ji}^{t}\right)$	1
C5	CDE/rand-current-best/3	$X_{kji}^{t} = X_{kji}^{t-1} + F^t \times \left(Pbest_i - X_{kji}^{t-1}\right)$ $X_{kji}^{t} = X_{kji}^{t} + F^t \times \left(X_{R1ji}^{t} - X_{R2ji}^{t}\right)$ $X_{kji}^{t} = X_{kji}^{t} + F^t \times \left(X_{R3ji}^{t} - X_{R2ji}^{t}\right)$	1
C12	ODE/rnd/1	$X_{kji}^{t} = \left(X_{i}^{max} + X_{i}^{min}\right) - F^t \times X_{kji}^{t-1}$	2

The quantitative results observe the best, the worst, mean value, standard deviation and CPU time taken. From the results, it is evident that strategy C4 has emerged out as one mutation strategy that is capable to generate quality solutions to the majority of the benchmark test functions. The convergence is very fast with C4, although C12 achieves VTR, convergence is slow as evident from graphs of selected benchmark functions. The number of function evaluations in C4 is less than that assigned as it achieves VTR much earlier in most of the cases. On the other hand, with strategy, C5 convergence speed is very high but takes more NFE to achieve VTR in comparison with C4.

2.6.2 Hydrothermal System

This system consists of three thermal units and four multi-chain hydro units. The quadratic model is used to represent the characteristics of thermal units. This system is simulated for without transmission losses. The system details and load data are taken from the work referred to as the work (Narang *et.al*, 2014). The system is simulated for twelve mutation strategies and the quantitative results are presented in Table 2.5.

In this problem, the active power generation from thermal units and water release from hydro units are decision variables. The hydrothermal system is simulated for a different combination of mutation strategies applied to a set of decision variables of the hydro and thermal unit; $P = [P_1, P_2, \dots, P_i]$, $\forall i \in N_T$ and $Q = \left[Q_1, Q_2, \dots, Q_j\right]$ $\forall j \in N_H$.

Table 2.3: Performance of twenty benchmark functions mutation strategies

Function	Attribute	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
F1	Best	5.25E-5	0.00	2.188E-18	**0.00**	0.00	5.567E-16	8.98	6.687E-5	3.57E-24	2.24	6779.00	0.00
	Average	2.10	9.65E-2	1.01E-16	**0.00**	0.00	7.666E-15	14.94	1.219E-2	7.12E-20	58.07	19105.00	0.00
	Maximum	18.96	0.99	6.934E-16	**0.00**	0.00	2.993E-14	33.46	5.446E-2	1.4E1-18	1251.00	40515.00	0.00
	STD	4.40	0.26	1.621E-16	**0.00**	0.00	8.582E-15	5.04	1.337E-2	2.53E-19	222.18	8414.00	0.00
	Time (Sec)	1.51	1.48	1.45	**1.30**	1.83	1.49	1.49	1.49	1.50	1.49	1.49	1.86
F2	Best	8.27E-7	**0.00**	2.37E-9	0.00	0.00	4.380-8	3.95	2.48E-3	5.52E-12	0.11	32.10	4.2E-44
	Average	8.75E-2	**0.00**	2.58E-8	0.00	0.00	1.41E-7	6.10	9.72E-3	9.79E-10	0.74	7.92E+9	4.2E-45
	Maximum	1.88	**0.00**	1.92E-7	0.00	0.00	3.8E-7	8.50	2.68E-2	1.56E-8	14.47	2.31E+11	4.2E-46
	STD	0.34	**0.00**	3.41E-8	0.00	0.00	7.82E-8	1.00	5.48E-3	2.95E-9	2.55	4.414E+10	0.00
	Time (Sec)	1.26 Sec	**1.13**	1.25	1.19	1.33	1.28	1.28	1.29	1.18	1.27	1.24	2.80
F4	Best	17.68	0.00	2.08E-8	**0.00**	0.00	10.0E-6	22.88	10.00	1.93E-8	57.00	61.60	31.50
	Average	25.85	0.00	3.7E-7	**0.00**	0.00	1.64E-4	28.67	43.40	1.53E-5	70.00	74.45	44.87
	Maximum	35.92	0.00	3.89E-6	**0.00**	0.00	9.1E-4	35.90	71.25	2.02E-4	86.00	86.03	49.43
	STD	4.32	0.00	7E-7	**0.00**	0.00	1.58E-4	3.28	17.42	4E-5	7.18	7.17	3.62
	Time (Sec)	1.24	1.23	1.26	**1.20**	1.34	1.77	1.24	1.25	1.24	1.28	1.21	1.26
F5	Best	125.00	28.82	26.29	28.60	28.81	26.95	751	28.65	26.73	3.06E+4	5.22E+6	28.75
	Average	586.00	28.89	27.75	28.80	28.86	27.84	1222	1.29E+4	28.15	2.28E+6	6.18E+7	28.80
	Maximum	3390.00	28.95	28.35	29.00	28.92	28.30	1821	1.68E+5	28,65	1.71E+7	1.72E+8	28.85
	STD	766.00	2.96E-2	0.55	0.08	0.03	0.34	254.00	3.65E+4	0.44	4E+6	4.8E+7	0.014
	Time (Sec)	1.93	1.95	1.96	1.98	1.94	1.95	1.95	2.34	1.85	1.83	1.83	1.83
F6	Best	7.00	0.00	0.00	0.00	**0.00**	0.00	11.00	0.00	0.00	4.00	4.3E+3	0.00
	Average	58.36	0.00	0.00	0.00	**0.00**	0.00	21.00	0.00	0.00	19.80	1.87E+4	0.00
	Maximum	304.00	0.00	0.00	0.00	**0.00**	0.00	42.00	2.00	0.00	92.00	4.05E+4	0.00
	STD	75.52	0.00	0.00	0.00	**0.00**	0.00	7.05	0.70	0.00	17.00	8.92E+3	0.00
	Time (Sec)	1.40	1.35	1.54	1.29	**1.27**	1.37	1.48	1.49	1.42	1.54	1.30	1.28
F7	Best	0.019	1.8E-4	3.06E-3	6.1E-4	2.51E10-4	0.004	0.450	0.100	1.14E-3	1.270	7.260	1.93E-4
	Average	0.049	3.14E-4	1.13E-2	1.47E-3	5.04E10-4	0.016	0.624	0.250	5.29E-3	6.110	33.150	3.87E-4
	Maximum	0.083	6.18E-4	3.48E-2	2.92E-3	1.17E10-3	0.036	0.782	0.110	1.6E-2	24.630	78.620	7.09E-4
	STD	0.016	1.5E-4	6.77E-3	6E10-4	2.31E10-4	0.007	0.107	0.210	3.83E-3	7.080	21.530	1.72E-4
	Time (Sec)	1.35	1.39	1.37	1.42	1.41	1.390	1.380	1.520	1.390	1.400	1.410	1.390

Table 2.3: Performance of twenty benchmark functions mutation strategies (continued)

Function	Attribute	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
F8	Best	38.24	0.00	40.00	0.00	0.00	47.650	95.500	91.600	58.370	87.320	251.000	0.000
	Average	45.33	0.00	53.38	0.00	0.00	54.760	112.520	109.100	79.260	117.900	338.430	19.100
	Maximum	55.40	0.00	66.51	0.00	0.00	67.200	130.100	143.400	106.700	200.580	397.750	44.780
	STD	4.54	0.00	6.88	0.00	0.00	4.58	6.31	13.24	10.62	28.36	35.64	17.10
	Time (Sec)	1.53	1.49	1.75	1.83	1.52	1.51	1.52	1.55	1.50	1.50	1.47	1.50
F9	Best	1172.56	4608.00	3400.00	4813.00	**3773.00**	3504.00	3745.00	3057.00	3953.00	4881.50	6508.00	8319.00
	Average	2545.70	5377.00	4120.00	5413.00	**5010.00**	4053.00	4204.00	4580.00	4519.00	6284.00	7723.50	9202.00
	Maximum	3456.70	6286.00	4670.00	5925.00	**6135.00**	4485.00	4514.00	7882.00	5898.00	7404.00	9035.00	9880.00
	STD	514.35	423.10	326.80	243.25	**526.67**	243.67	189.67	943.67	359.11	507.40	702.86	3495.00
	Time (Sec)	1.22	1.35	1.30	1.41	**1.27**	1.25	1.25	1.33	1.26	1.41	1.26	1.26
F10	Best	1.15E-9	0.00	6.47E-3	**0.00**	0.00	0.08	8.95	1.88	2.85	6.12	25.73	0.00
	Average	2.63E-3	0.00	0.019	**0.00**	0.00	0.72	11.96	6.59	5.55	11.81	45.57	0.00
	Maximum	0.01	0.00	0.139	**0.00**	0.00	1.77	14.05	16.05	11.04	23.63	59.80	0.00
	STD	4.44E-3	0.00	0.023	**0.00**	0.00	0.49	1.46	3.39	2.00	3.72	8.75	0.00
	Time (Sec)	1.25	2.51	1.15	**1.13**	2.76	1.19	1.21	1.21	1.22	1.20	1.21	2.13
F11	Best	1.12E-4	**0.00**	0.00	0.00	**0.00**	0.00	1.07	4.2E10-4	0.00	0.92	62.00	0.00
	Average	0.16	**0.00**	5E-4	0.00	**0.00**	4.3E-7	1.14	0.10	1.17E-3	1.48	173.00	0.00
	Maximum	0.61	**0.00**	7.5E-3	0.00	**0.00**	5.3E-6	1.24	0.44	0.01	12.25	365.00	0.00
	STD	0.16	**0.00**	1.86E-3	0.00	**0.00**	1.19E-6	0.30	0.11	3.55E-3	2.00	75.70	0.00
	Time (Sec)	1.68	**1.66**	1.66	1.72	**1.66**	1.65	1.68	1.67	1.67	1.63	1.72	1.71
F12	Best	0.94	**0**	3E-10	0.00	0.00	5.8E-9	5.58	4.25E-3	6.58E-13	4.74	13.93	3E-15
	Average	2.60	**0.00**	4.4E-9	0.00	0.00	3.4E-8	17.42	10.66	1.8E-7	19.45	19.58	3E-15
	Maximum	4.67	**0.00**	2E-8	0.00	0.00	9.2E-8	20.00	20.00	5.4E-6	20.00	20.30	3E-15
	STD	0.78	**0.00**	4.5E-9	0.00	0.00	1.85E-8	4.17	10.00	9.7E-7	2.73	1.27	0.00
	Time (Sec)	1.51	**1.56**	1.59	1.61	1.58	1.62	1.53	1.63	1.59	1.62	1.57	1.59
F13	Best	0.07	0.00	1.15	0.00	0.00	2.28	131.88	41.54	4.04	79.56	170.66	1.33E-21
	Average	0.96	0.00	1.95	0.00	0.00	3.41	155.80	57.95	28.38	96.56	248.42	1.33E-22
	Maximum	6.53	0.00	3.23	0.00	0.00	4.65	173.00	80.83	61.73	128.41	308.97	1.33E-23
	STD	1.30	0.00	0.52	0.00	0.00	0.61	9.78	10.29	16.80	9.45	31.74	0.00
	Time (Sec)	2.28	1.63	2.40	1.72	1.91	2.44	2.44	2.54	2.42	2.46	2.41	3.29

Table 2.5 Performance of Hydrothermal system with three thermal and four hydro units

	STATUS	QC1	QC2	QC3	QC4	QC5	QC6	QC7	QC8	QC9	QC10	QC11	QC12
PC1	Best	48161.21	47289.89	47780.4	48299.17	47199.43	47257.42	47408.39	47926.53	48662.12	48426.75	48663.81	48370.58
	Mean	49062.34	48118.08	49106.34	49072.91	49059.75	48372.99	48887.52	49045.91	49461.55	49213.79	48996.65	48844.78
	Worst	50444.74	50389.83	50036.7	50444.24	50389.83	50389.83	50444.24	50389.83	50444.24	50212.22	50444.24	50389.83
PC2	Best	48584.13	**46996**	47437.05	48525.32	47155.4	47879.11	48541.72	48417.64	48425.07	48049.45	48822.45	48753.34
	Mean	49216.55	**47840.04**	48901.29	49151.73	49086.5	48679.59	49220.86	49168.48	49168.48	49282.24	49057.62	49126.53
	Worst	50589.93	**50389.83**	50298.91	50444.24	50389.83	50444.24	50225.04	50352.27	50444.24	50212.22	50444.24	50225.04
PC3	Best	48652.5	47433.27	48370.58	48348.04	48236.8	47712.66	48702.84	48039.41	48442.98	48324.94	48600.5	47906.68
	Mean	49246.14	48724.86	48874.02	48807.84	49041.77	48461.08	49325.28	49068.17	49280.24	49267.58	49124.14	49347.86
	Worst	50589.93	50389.83	50389.83	50218.18	50324.41	50444.24	50444.24	50352.27	50444.24	50212.22	50389.83	50129.43
PC4	Best	47760.82	47355.93	48269.89	**46219.24**	**46259.17**	47341.84	48775.25	48343.12	48276.14	48627.07	48516.18	**47174.54**
	Mean	49105.67	48205.22	48901.29	**48989.32**	**48142.82**	49034.96	49410.99	49051.84	49462.5	49395.51	49096.08	**49242.41**
	Worst	50225.04	50389.83	50444.24	**50218.18**	**50389.83**	50444.24	50444.24	50389.83	50444.24	50212.22	50444.24	**50389.83**
PC5	Best	48216.5	47295.98	48480.41	48511.21	**46915.88**	47710.95	48517.46	48712.8	48308.2	48399.6	48393.34	48685.53
	Mean	48849.57	48229.94	49362.26	49193.69	**49218.11**	48447.15	49124.92	48995.03	49136.89	49256.72	49037.03	49031.83
	Worst	50225.04	50389.83	50444.24	50389.83	**50389.83**	50444.24	50444.24	50298.91	50444.24	50212.22	50444.24	50444.24
PC6	Best	48195.77	47491.82	48436.43	48200.79	48696.74	47756.61	48292.57	48613.45	48135.37	48465.93	48162.76	47994.39
	Mean	48865.78	48128.36	49150.74	49397.11	49265.3	49469.82	49070.65	49141.72	49284.83	49291.39	49001.64	49291.39
	Worst	50212.22	50389.83	50444.24	50389.83	50389.83	50444.24	50444.24	50589.83	50444.24	50212.22	50389.83	50302.12

Table 2.5 Performance of Hydrothermal system with three thermal and four hydro units (continued)

	STATUS	QC1	QC2	QC3	QC4	QC5	QC6	QC7	QC8	QC9	QC10	QC11	QC12
PC7	Best	48147.58	47022.99	48168.14	48350.49	48098.05	47585.71	48551.09	48085.85	48066.8	48390.73	48650.08	48441.95
	Mean	48986.5	47707.16	49188.86	49115.88	49249.7	47923.98	49087.17	49046	49229.18	49192.01	49113.59	49250.74
	Worst	50212.22	50389.83	50389.83	50444.24	50389.83	50444.24	50389.83	50389.83	50444.24	50212.22	50444.24	50444.24
PC8	Best	48133.6	47245.78	48262.15	48192.33	48189.63	47511.33	48507.52	48452.95	48180.32	48457.63	48419.37	48247.95
	Mean	48851.54	48226.83	49194.61	49393.24	49178.84	48580.38	48987.43	49162.65	48962.73	49079.21	49039.41	49133.07
	Worst	50225.04	50389.83	50389.83	50389.83	50324.41	50444.24	50444.24	50389.83	50444.24	50212.22	50444.24	50444.24
PC9	Best	48080.94	47243.56	48042.66	48536.12	47963.94	47751.78	48542.46	48376.21	48583.7	48760.57	48555.24	48239.05
	Mean	48940.45	48038.9	48998.08	49069.5	49068.36	48330.28	49078.9	49999.39	49227.04	49470.29	49042.06	49329.24
	Worst	50444.24	50389.83	50444.24	50389.83	50389.83	50444.24	50444.24	50389.83	50444.24	50212.22	50389.83	50444.24
PC10	Best	48281.96	47129.41	48463.23	48124.55	48078.68	47408.39	48602.27	48531.89	48247.95	48704.84	48085.26	48491.18
	Mean	49501.05	47871.42	49095.59	49277.42	48791.1	48887.52	49411.42	49080.76	49423.24	49312.17	49242.59	49144.55
	Worst	50444.24	50389.83	50444.24	50444.24	50389.83	50444.24	50434.27	50389.83	50444.24	50389.83	50389.83	50389.83
PC11	Best	48412.09	47137.63	48332.78	47505.12	48204.1	48155.76	48291.68	48698.22	48247.95	48423.71	48033.16	47391.91
	Mean	49140.56	48053.45	48776.05	48630.11	49328.07	49005.71	49161.82	49075.22	49133.07	49655.2	49410.89	49382.16
	Worst	50389.83	50389.83	50389.83	50036.7	50389.83	50389.83	50389.83	50444.24	50444.24	50444.24	50389.83	50316.4
PC12	Best	47161.24	**46548.66**	47264.8	**46908.68**	**47206.56**	47240.35	47330.25	47865.97	47786.73	47477.63	47836.79	**46139.25**
	Mean	48827.34	**47383.82**	48976.17	**48579.3**	**48526.84**	47951.9	49137.03	48452.37	48677.04	48962.79	48704.31	**49380.08**
	Worst	50389.83	**50316.4**	50444.24	**50444.24**	**50389.83**	50444.24	50389.83	50444.24	50444.24	50316.4	50298.91	**50444.24**

In the simulation of single-objective constrained hydrothermal system, strategy C4, C5 and C12 compete and gives the best value of operating cost. Although the best value is not the global best value, however, based on the best value achieved for operating cost of thermal units, C12 gives a better result.

2.7 CONCLUSIONS

Differential evolution is a versatile population-based global optimization algorithm. There are several DE variants reported in the literature, however, it is very difficult to choose the best one to a specific problem. In this investigation number of DE, variants are proposed and experimental demonstrations have been conducted on twenty benchmark functions of variety and one real-word problem of constrained hydrothermal scheduling over time-frame of 24 hours. Among 12 mutation strategies, C4 and C12 have emerged out as the best. The formulation of these two strategies is opposite to each other. Experimental results presented in Table 2.3 show that C4 takes less number of function evaluations to achieve VTR so it is computationally efficient, whereas C5 is computationally costly.

In the hydrothermal system, C12 gives a better quality solution in comparison with C4 and C5. For real power generation vector P of the thermal unit and water discharge vector Q of the hydro unit, the combination of PC12 with QC2, P C12 with QC12 has emerged out with satisfying results. It is concluded that C2, C4, C5 and C12 are significant mutation strategies. These four strategies achieve the specified VTR successfully and CPU time is also low consistently for C2, C4 and C5 showing fast convergence. However, strategy C12 does not show consistent performance in convergence although it achieves VTR in a specified number of function evaluations. In the next chapters, mutation strategy C2 and C4 are employed for mutation operation to save computational cost and strategy C12 is applied for perturbation of population in the opposite region as well during the selection of a non-inferior population for next generation. The perturbation of population during selection will be included to not to leave any chance to drop better solution in the generation.

CHAPTER - 3

MULTI-OBJECTIVE SHORT-TERM HYDROTHERMAL SCHEDULING USING CRISSCROSS DIFFERENTIAL EVOLUTION

3.1 INTRODUCTION

The national electricity policy aims to provide cost-effective and quality electricity to consumers and to increase per capita consumption. An electric power system is a dynamic system that needs to balance its supply and demand round the clock (24 hours). Practically, decisions for hourly optimal generation output from thermal units and water release from hydro units over the scheduling period are constrained by power balance requirements, and water management necessities *i.e*. water head, the amount of water available in the reservoirs, water to be drawn down, and generation limits *etc*. In the case of hydraulically cascaded units, it needs to be coordinated in time as well, due to the dependency of water storage in reservoirs, this may take a long period for utilization. Hence, the problem of hydrothermal generation scheduling (HTGS) is visualized as an NP-hard multi-objective optimization problem that aims at determining the optimal active power generation schedule to meet the load demand over the operation period with the best-compromised solution to the total operating cost and pollutant emission level (Kothari and Dhillon, 2010).

Researchers have used direct and gradient optimization techniques to solve the HTGS problem (Farhat and El-Hawary, 2009). These conventional techniques and their modified versions provide a locally optimal solution but are quite straight forward and efficient procedures. However, on account of simplifying assumptions, these methods have limited ability to manage combinatorial NP-hard real-world HTGS problems. Since three decades, population-based meta-heuristic random search methods have experienced notable progress in the scenario of engineering optimization. These optimization algorithms are summarised in Table 1.1. They exhibit multi-start, derivative-free computation and can provide a Pareto set of solutions to address the challenges of global solution search to HTGS problem. The success of the random search methods is largely dependent on the promising initial population and a good balance between exploration and exploitation of the problem domain space (Mirjalili, 2016). Several researchers have developed new stochastic search engines (algorithms) to handle the optimization problems of a specific application with success (Yang, 2008). The advent of new global and local search concepts specific to applications inspires researchers to explore a variety of choices in the application of their interest Researchers have taken note of the high level of perspective advantages of existing metaheuristic methods as well and ensemble algorithms by reallocation of operators, mixing of search components and related concepts, hybridisation etc. to deal with the problems of flexibility in search, selection of control parameters, premature convergence, convergence speed and global solution accuracy of complex optimization problems but at the cost of computational complexity.

Differential evolution (DE) a stochastic global search evolutionary optimization method, and is prone to trap in suboptimal solution and affects the convergence speed, like other population-based stochastic algorithms for large dimensional and multimodal problems, where the exploration and exploitation capability of the classical DE cannot be well balanced. In the previous chapter, several mutation strategies have been explored to solve unconstrained optimisation problems of a variety of fitness landscape. However, the issues of multiple objectives and constrained optimization problems, uncertainty in the problem data, unpredicted violation in constraints, dimensions of the problem etc., are grey areas in the multi-faceted DE to be investigated upon. Inspired from the simplicity of differential evolution (DE), and its ability to maintain the multiplicity of the population to support the neighbourhood search, the researchers have the motivation to explore the modifications in DE in terms of concepts and hybridization with other components from metaheuristics. In this chapter, the researchers have tried to alleviate the problem of entrapping in local minima and to improve the global solution accuracy when applied to solve multimodal constrained non-linear HTGS problems of high dimensionality.

In the literature, Xiang *et al.,* (2015) have proposed mixing of mutation strategies to expand search space for a better balance between exploration and exploitation during the search. Gong *et al.,* (2015) have suggested a surrogate model select the best off-spring solution for the next trial solution operation. Wu. *et al.,* (2018) have presented to a variety of operational improvements in DE to solve nonlinear optimisation problems. Meng *et al.,* (2015*)* have proposed crisscross crossover operator in GA to improve population diversity. Peng *et al.,* (2015) have solved the distributed generation allocation problem with crisscross crossover operation. The explorative mutation operator tends to locate the minima of the objective function while the exploitative property makes the objective function rapidly converging to an optimum as explained by Singh *et al.,* (2017).

On the other hand, the application of non-linear dynamics for mixing of operators has drawn the attention of researchers. The empirical studies have suggested that the chaotic procedure of sequence generation for parameters have a high level of mixing capability. In the literature, the successful application of chaotic maps to generate the control parameters are reported such as Kozma *et al.,* (1998) for selection of fuzzy rules in neuro-fuzzy controller application, Lu *et al.*, (2010) to tune DE parameters, Lia, *et al.*, (2013) for artificial bee colony algorithm parameters, Tian *et al.,* (2014) for a chaotic mutation in gravitational search algorithm, Yang *et al*., (2016) with bat algorithm, Mustaffa *et al.,* (2018) to tune parameters of local search method and many more. It is expected that the application of chaotic maps to adaptively tune the DE control parameters can offer flexibility in search with better diversification and can improve the performance of DE search in deceptive valley regions of search space to avoid premature convergence

Despite the important thought given by individual researchers to improve the performance of basic DE algorithm, there exists no specific DE version substantially to achieve the best solution for all types of optimization problems because the exploration and the exploitation often mutually contradict in reality (Das *et al.,* 2016).

In the light of these observations, a novel chaotic-crisscross DE algorithm is proposed. This algorithm suggests a dual crisscross mechanism that supports the information exchange across the dimensions among generations. The crisscross mechanism is applied orthogonally (horizontally and vertically) at both mutation and crossover level of DE to search for new potential solutions alternatively. The horizontal crisscross supports the exploration at the boundaries of hypercube space and the vertical crisscross facilitates the stagnant population, to jump out from local minima. In this competitive operation, only those moderate solutions survive for the next generation who outperform the comparatively weak solutions in the competition among qualified solutions. This double search strategy of information exchange in the competitive mode is envisaged with significant advantages in convergence rate and global solution accuracy.

Further, to ensure beneficial diversity in the population, the abilities of the two algorithms are blended into one. In this respect, sine-cosine a global search optimization algorithm is memetically blended with CCDE to support better location of the global optimum solution with improved convergence rate. The sine-cosine algorithm is a population-based metaheuristic algorithm that generates cyclic patterns for movement around the best solution position in the feasible search space (Nenavath *et al.,* 2017). The movements in the patterns are controlled by the selection of parameter that controls the balance of exploration and exploitation in the circular space around best solution located at the centre of space.

. Further, opposition-based learning is incorporated in the selection phase to guide the localized search in the neighbourhood of the population of a quality solution to get close to a better optimal solution. The proposed novel chaotic crisscross concept can contribute towards the efficient convergence of the global search DE algorithm with improved accuracy when applied to solve the HTGS problem.

To select the best-compromised solution from a set of non-inferior solutions of multi-objective hydrothermal problem subjected to satisfaction of constraints, the focus is extended to solve DE based multi-objective problem. The significant work includes the use of Pareto fronts (Abbass *et al.,* 2001), penalty function method (Coello, 2008), weighting method (Adeyemo and Otieno, 2009), goal-attainment method (Basu, 2010), ε-constraints approach (Yuan, 2008) *etc.* have been reported. The parametric methods are easy to implement and provide multiple set of Pareto solutions. The selection of a single dominating solution among all is subjective and quite time-consuming. Multi-objective DE generates a single Pareto front using non-dominated sorting and other elite solution sorting procedures (Deb, 2011). HTGS is a conflicting objective optimization problem. This leads to a set of trade-off approach to finding the best-compromised solution set of non-inferior solutions. An interactive fuzzy approach is a well-versed approach to handle uncertainties in decision variables and other parameters related to each of the objective functions. The fuzzy parameters in the description of the objective functions and the constraints reflect the DM's ambiguous understanding of the nature of the parameters in the nonconvex problem.

To solve multi-objective constrained hydrothermal scheduling problem over a time–horizon of 24 hours, constraint handling is another challenge as the violations in the equality and inequality constraints are uncertain. Wang *et al.,* (2018) suggested constraint-domination approach to handle inequality constraints that eliminated the use of penalty factors. Hajinassary *et.al* (2014) have preferred composite constraint handling approach for bi-level hydrothermal coordination problem. Li *et al.,* (2016) motivated the use of interval type-2 Sugeno model to handle parameter uncertainties in non-linear communication network control.

In this chapter, a novel strategy of constraint handling is discussed where the statistical uncertainties called residues those arise due to infringements of equality constraints while adjusting the violated dependent variables within their boundaries are repaired using fuzzy approach. These residues are fuzzy quantified within their prescribed bounds and are embedded as objectives to be optimized.

This chapter intends to study chaotic-crisscross based differential evolution (CCDE) approach to solve constrained multi-objective short-term hydrothermal generation scheduling problem. Two real-world case studies of the constrained hydrothermal generation scheduling optimization problems viz., seven-unit system and twenty-two-unit system are solved as multi-objective constrained optimization problems to determine the active power output of thermal and hydro units and water discharge from hydro units in each subinterval of the 24 hour scheduling period. The highlights of the work in this chapter are as follows:

- Chaotic-Crisscross differential evolution (CCDE) algorithm is proposed to solve multi-objective short-term constrained hydrothermal scheduling problem over a time horizon of 24 hours.
- Crisscross strategy is proposed orthogonally to reinforce chaotically tuned DE mutation and crossover operations for better information exchange in the hypercube search space. It helps to avoid stagnation in local minima and improves global search capabilities and convergence rate.
- Opposition based learning is incorporated to explore the promising neighbourhood and preserves good solutions. This strategy helps to avoid premature convergence and improves the accuracy of the global solution.
- CCDE algorithm is blended with a sine-cosine algorithm to accelerate exploration and exploitation.
- The variable elimination method is used to handle equality constraints and fuzzy set approach is proposed to mitigate the dependency of unpredicted deviations in the satisfaction of equality and inequality constraints.
- A unified fuzzy satisfying function is proposed to resolve the conflict between the objectives.
- The performance of the proposed algorithm is illustrated on generalized test functions and two practical hydrothermal system problems and robustness of the algorithm is validated through statistical significance tests.

3.2 MULTI-OBJECTIVE HYDROTHERMAL GENERATION SCHEDULING

Short term hydrothermal generation scheduling problem is formulated as a multi-objective optimization problem with operating cost and emission as two conflicting objectives of thermal units, owing to no incremental cost of hydro units. The aim of short term HTGS is to obtain the generation schedule of hydro and thermal units with maximum use of the amount of water available to draw down in each subinterval of the planning period. The active power output of the thermal unit and water discharge of the hydro unit are considered as decision variables. The power output of the hydro unit in the multi-chain hydro system is taken as the dependent variable. For simplification, the unit generation output is assumed to be adjusted smoothly and instantaneously. The objective function is developed from the mathematical models of hydro and thermal units. Hydraulic constraints include water flow rate, series and parallel cascading of hydro units, water transport delay, and storage capacity of reservoirs. Electrical constraints include time-varying load demand, the loading capacity of hydro and thermal units, and energy balance of the system. Mathematically, the multi-objective hydrothermal generation scheduling problem is stated as an optimization problem:

3.2.1 Cascaded Reservoir Hydro model

A chain of multi-reservoir cascaded fixed head hydro unit system with the water transport delay between reservoirs is taken for study. The active power generation of a hydro unit is a function of water discharge rate, reservoir volume and net hydraulic head. In the multi-chain model, the power generation of a hydro unit at any time is a function of water discharge rate and reservoir volume (Kothari and Dhillon, 2010) stated as:

$$P_{kj} = C_{1j}V_{kj}^2 + C_{2j}q_{kj}^2 + C_{3j}V_{kj} + C_{4j}q_{kj} + C_{5j}V_{kj}q_{kj} + C_{6j} \quad (j = 1,2,\dots N_H, k = 1,2,\dots T) \qquad (3.1)$$

where $C_{1j}, C_{2j}, C_{3j}, C_{4j}, C_{5j}, C_{6j}$ are hydro unit coefficients of j^{th} hydro unit. V_{kj}, *and* q_{kj} are storage volume, and water discharge rate from the reservoir of j^{th} hydro unit in k^{th} sub-interval respectively and P_{kj} is active power generation from j^{th} hydro units in k^{th} sub-interval, N_H is number of hydro units.

3.2.2 Thermal Unit Model

Thermal unit is modelled with a non-smooth fuel cost curve considering valve-point loading effect and emission of pollutant (Kothari and Dhillon, 2010). Ripples in cost function are due to throttling losses, which occurs when the steam valves are opened to admit the steam in the turbine. This is called valve-point loading and is taken care by adding a sine function term to quadratic term of the cost function. According to *Environment Protection amendment act*, the lethal pollutant emissions from fossil-fuel based thermal plants shall not exceed the permissible limits. The amount of emission of the thermal unit is modelled as a function of active power output and is expressed as a sum of its quadratic and exponential function terms stated in Equation 3.3.

Objective Functions

$$\text{Minimize operating cost}, F_1 = \sum_{k=1}^{T}\left(\sum_{i=1}^{N_T}\left(a_i P_{ki}^2 + b_i P_{ki} + c_i + \left|d_i sin\{e_i\left(P_i^{min} - P_{ki}\right)\}\right|\right)\right)$$

(3.2)

$$\text{Minimize Emission of Pollutants}, F_2 = \sum_{k=1}^{T}\left(\sum_{i=1}^{N_T}\left(\alpha_i P_{ki}^2 + \beta_i P_{ki} + \gamma_i + \eta_i exp(\delta_i P_{ki})\right)\right)$$

(3.3)

where $a_i, b_i, c_i, d_i\ and\ e_i$ are operating cost coefficients, $\alpha_i, \beta_i, \gamma_i, \eta_i\ and\ \delta_i$ are gaseous pollutant's emission coefficients of i^{th} thermal unit. N_T is the number of thermal units, and. T is the scheduling period. P_{ki} are active power generation of i^{th} thermal units.

Equality Constraints

(i) To ensure power balance, an equality constraint during each subinterval is imposed.

$$\sum_{i=1}^{N_T} P_{ki} + \sum_{j=1}^{N_H} P_{kj} = P_{Dk} + P_{Lk} \quad (k = 1,2,\ldots,T) \tag{3.4}$$

The transmission losses are represented by Kron's loss formula (Kothari and Dhillon, 2010) through B-coefficients.

$$P_{Lk} = B_{00} + \sum_{i=1}^{N_G} B_{i0} P_{ki} + \sum_{i=1}^{N_G}\sum_{j=1}^{N_G} P_{ki} B_{ij} P_{kj} \tag{3.5}$$

P_{Dk} is the load demand and P_{Lk} represents transmission losses during k^{th} sub-interval, N_G represents the total number of committed hydrothermal generating units and B_{00}, B_{i0}, B_{ij} are termed as B-coefficients

(ii) The volume dynamics in the hydro reservoir (Kothari and Dhillon, 2010)

$$V_{kj} = V_{(k-1)j} + I_{kj} - q_{kj} - S_{kj} + \sum_{m=1}^{R_H}\left(q_{(k-\tau_{mj})m} + S_{(k-\tau_{mj})m}\right)$$

$(j = 1,2 \ldots N_H, k = 1,2 \ldots T)$ (3.6a)

(iii) The hydraulic (storage) constraint for a hydro unit reservoir at the end of the scheduling period (Kothari and Dhillon, 2010)

$$V_{(T+1)j} = V_{1j} + \sum_{k=1}^{T} I_{kj} - \sum_{k=1}^{T} q_{kj} - \sum_{k=1}^{T} S_{kj} + \sum_{k=1}^{T}\sum_{m=1}^{R_H}\left(q_{(k-\tau_{mj})m} + S_{(k-\tau_{mj})m}\right)$$

$(j = 1,2 \ldots N_H)$ $(3.6b)$

Where, R_H is the number of reservoirs on the upper side, τ_m represents water transport time delay from m^{th} upper reservoir to lower reservoir 'j'. V_{1j} is an initial volume and $V_{(T+1)j}$ is the final storage volume of the j^{th} reservoir at the end of the scheduling period, V_{kj}, I_{kj}, q_{kj} *and* S_{kj} are storage volume, water inflow rate, water discharge rate and spillage from the reservoir of j^{th} unit in k^{th} sub-interval respectively,

Inequality Constraints

(i) Active power generation limits on each hydro and thermal unit

$$P_i^{min} \leq P_{ki} \leq P_i^{max} \qquad (i = 1,2, \dots, N_G;\ k = 1,2, \dots, T) \qquad (3.7a)$$

(ii) Water discharge rate limits on hydro units

$$q_j^{min} \leq q_{kj} \leq q_j^{max} \qquad (j = 1,2, \dots, N_H;\ k = 1,2, \dots, T) \qquad (3.7b)$$

(iii) Reservoir volume limits on hydro units

$$V_j^{min} \leq V_{kj} \leq V_j^{max} \qquad (j = 1,2, \dots N_H; k = 1,2, \dots, T) \qquad (3.7c)$$

P_i^{min} *and* P_i^{max} are lower and upper limits of active power generation of i^{th} hydro or thermal unit, q_j^{min} *and* q_j^{max} are minimum and maximum water discharge rate limits of j^{th} hydro unit respectively.

V_j^{min} *and* V_j^{max} are lower and upper limits of the storage capacity of j^{th} reservoir and, respectively.

3.3 CONSTRAINT HANDLING MECHANISM

In constrained multi-objective hydrothermal generation problem, there exist equality and inequality constraints. In this research work, the explicit variable elimination method is applied to handle two equality constraints namely; active power balance and the amount of available water in the reservoir, independently. This method not only reduces the number of decision variables but also several inequality constraints. One or more variables selected among decision variables for elimination are called dependent variables and the remaining are independent variables.

3.3.1 Equality Constraints Handling

The generation must meet the power demand and utilization of the available water to its maximum extent is another requirement.

Power Balance Constraint

To solve power balance, equality constraint, a dependent variable is the active power generation, randomly selected among committed hydro and thermal unit for a sub-interval. Rewriting power balance Equation (3.4) for slack generator 's' and is stated as:

$$X_k P_{sk}^2 + Y_k P_{sk} + Z_k = 0 \qquad (k = 1,2,\dots,T) \qquad (3.8)$$

where

$$Y_k = \left\{ \sum_{\substack{i=1 \\ i \neq s}}^{N_G} (B_{is} + B_{si}) P_{ik} + B_{s0} - 1 \quad ; P_{Lk} \neq 0 \right.$$

$$Z_k = \left\{ B_{00} + P_{Dk} + \sum_{\substack{i=1 \\ i \neq s}}^{N_G} \sum_{\substack{j=1 \\ j \neq s}}^{N_G} P_{ik} B_{ij} P_{jk} + \sum_{\substack{i=1 \\ i \neq s}}^{N_G} B_{i0} P_{ik} - \sum_{\substack{i=1 \\ i \neq s}}^{N_G} P_{ik} \; ; P_{Lk} \neq 0 \right.$$

Solution of the quadratic Equation (3.8), P_{sk} in each subinterval $(k = 1,2,\dots,T)$ is obtained as:

$$P_{ks} = \left\{ \frac{-Y_k \pm \left(Y_k^2 - 4X_k Z_k\right)^{1/2}}{2X_k} \right. \qquad (P_{Lk} \neq 0) \qquad (3.9)$$

where $\left(Y_k^2 - 4X_k Z_k\right) \geq 0$

On solving, only positive and real roots are considered as a solution for the active power generation for slack generator. In case slack generator unit violates its generation limits during an interval, then violated limit is reassigned as:

$$P_{ks} = \begin{cases} P_s^{min} & ; \text{ if } P_{ks} < P_s^{min} \\ P_s^{max} & ; \text{ if } P_{ks} > P_s^{max} \\ P_{ks} & ; \text{ if } P_s^{min} \leq P_{ks} \leq P_s^{max} \end{cases} \qquad (k = 1,2,\dots,T) \qquad (3.10)$$

This procedure of satisfying inequality constraint of the active power of each thermal unit may perturb the equality constraint to an extent. The uncertainty in the violation of active power generation limits of thermal units is modelled using fuzzy set theory described in section 3.4.

Available Water Equality Constraint

To solve water balance, equality constraint, water discharge from i^{th} hydro unit is taken as the dependent variable for randomly selected h^{th} sub-interval of the scheduling period. Using available water balance constraint for j^{th} hydro unit in h^{th} sub-interval is rewritten as:

$$q_{hj} = V_{1j} - V_{Fj} + \sum_{k=1}^{T} I_{kj} - \sum_{\substack{k=1 \\ k \neq h}}^{T} q_{kj} - \sum_{k=1}^{T} S_{kj} + \sum_{k=1}^{T} \sum_{m=1}^{R_H} \left(q_{(k-\tau_{mj})m} + S_{(k-\tau_{mj})m} \right)$$

$$(j = 1,2,\dots,N_H) \qquad (3.11)$$

The violation of the limits of water discharge of hydro units is adjusted using the following equation.

$$q_{hi} = \begin{cases} q_i^{min} & ;\ \text{if } q_{hi} < q_i^{min} \\ q_i^{max} & ;\ \text{if } q_{hi} > q_i^{max} \\ q_{hi} & ;\ \text{if } q_i^{min} \le q_{hi} \le q_i^{max} \end{cases} \qquad (h = 1,2,\dots,T) \qquad (3.12)$$

Like the active power generation, the uncertainty in the violation of water discharge from the hydro units and violations in water volume storage of reservoirs henceforth is modelled with a fuzzy approach within acceptable limits as described in section 3.4.

3.3.2 Inequality Constraints Handling

During each subinterval, limits of water volume in each reservoir, generation limits of generating units other than slack generating unit and reservoir storage capacity at the end of the scheduling period during a search for optimum solution are checked. In case, the updated thermal and/or hydropower violates the limits, then these variables are set to the corresponding limit. To guarantee power balance equality constraint, a thermal unit is chosen as a slack (dependent) generator during each sub-interval. The power generated from the slack thermal unit is determined using power balance constraint given by Equation (3.9). To guarantee available water equality constraint, hydro units are chosen at a slack (dependent) sub-interval. On the other hand, the water discharge from the slack hydro generator is calculated from available water constraint given by Equation (3.11). The power output from hydro units is determined using Equation (3.5). On violation of their respective generation limits, the dependent slack unit may be fixed to the corresponding limit using Equation (3.10).

3.4 FUZZY APPROACH FOR HANDLING VIOLATION IN CONSTRAINTS

The adjustment of the variables within their respective limits on its violation may lead to a violation in the equality constraints. This statistical uncertainty in violation of equality constraints arise due to adjustment of dependent variables within limits is called the residue. These unpredicted violations occur due to the inherently stochastic nature of decision variables and the other randomness present in algorithm-dependent parameters. These uncertain residues are recommended to be resolved using a fuzzy approach. These residues are quantified by their membership functions, defined within acceptable limits of prescribed tolerance applicable to the variables of the hydrothermal problem. The zero value of the membership function gives full violation of the constraint whereby one value of the membership function gives no violation of the constraint. The mathematical formulation of the fuzzy approach for violation in constraint handling is described as:

3.4.1 Reservoir Volume

A membership function $\mu(\mathrm{V_{Ti}})$, is defined that gives the violation of available water volume constraint. The membership function $\mu(\mathrm{V_{Ti}})$, for violation in the total reservoir volume of the hydro unit at the end of the scheduling period, is defined as a monotonic decreasing function as stated below.

$$\mu(\mathrm{V_{Ti}}) = \begin{cases} 0 & ; |\Delta \mathrm{V_i}| \geq \epsilon_{Vol_i}^{U} \\ \dfrac{\left(\epsilon_{V_i}^{U} - |\Delta \mathrm{V_i}|\right)}{\left(\epsilon_{V_i}^{U} - \epsilon_{Vol_i}\right)} & ; \epsilon_{Vol_i} < |\Delta \mathrm{V_i}| < \epsilon_{Vol_i}^{U} \\ 1 & ; |\Delta \mathrm{V_i}| \leq \epsilon_{Vol_i} \end{cases} \qquad (i = 1,2, \dots, N_H) \qquad (3.13)$$

where $|\Delta \mathrm{V_i}| = \left|V_{(T+1)i} - V_{1i}\right|$ and ϵ_{Vol_i} is tolerance of error in satisfying the total available water constraint of i^{th} hydro unit reservoir for the scheduled time. $\epsilon_{Vol_i}^{U}$ is the upper range to decide the satisfaction level of the total available water constraint of i^{th} hydro unit reservoir.

The overall membership function, S_T of violations in the total reservoir volumes utilized by hydro units at the end of scheduling period, is defined by average membership functions of the residues (unutilized water volume of reservoirs) of hydro units and is stated below:

$$S_T = \frac{1}{\mathrm{N}_H}\left(\sum_{\mathrm{i}=1}^{\mathrm{N}_H} \mu(\mathrm{V_{Ti}})\right) \qquad (3.14)$$

The value of S_T will vary from 0 to 1. '1' value means the volume constraints of all hydro units are satisfied.

3.4.2 Availability of Water

The membership function $\mu(\mathrm{V_{ki}})$, is defined for violation of reservoir volume in each subinterval of the scheduling period. The membership function $\mu(\mathrm{V_{ki}})$, for violation of available water constraint of the hydro unit at the end of the interval, is defined by trapezoidal function and is expressed by Equation (3.15). $\epsilon_{V_{ji}}$ is tolerance of error range to decide the satisfaction level of the lower and upper limits of the volume during j^{th} sub-interval for i^{th} hydro unit assumed in the range of 3-5%.

$$\mu(V_{ji}) = \begin{cases} 0 & ; \left(V_i^{max} + \epsilon_{V_{ji}}\right) \leq V_{ji} \leq \left(V_i^{min} - \epsilon_{V_{ji}}\right) \\ \dfrac{\left(V_{ji} - V_i^{min} + \epsilon_{V_{ki}}\right)}{\epsilon_{V_{ji}}} & ; \left(V_i^{min} - \epsilon_{V_{ji}}\right) < V_{ji} < V_i^{min} \\ \dfrac{\left(V_i^{max} - V_{ji} + \epsilon_{V_{ki}}\right)}{\epsilon_{V_{ji}}} & ; V_i^{max} < V_{ji} < \left(V_i^{max} + \epsilon_{V_{ji}}\right) \\ 1 & ; V_i^{min} \leq V_{ji} \leq V_i^{max} \end{cases}$$

$$(i = 1,2, \dots, \mathrm{N}_H; j = 1,2, \dots, T) \qquad (3.15)$$

The overall membership function, S_V of violation in the volume of water in for each reservoir in all subintervals is stated by the average of membership functions of the violations of reservoir limits over the planning period of all the hydro units and is given in Equation (3.16):

$$S_V = \frac{1}{\mathrm{N}_H}\left(\sum_{i=1}^{\mathrm{N}_H}\left(\frac{1}{T}\sum_{j=1}^{T}\mu(V_{ji})\right)\right) \qquad (3.16)$$

The value of S_V defined within 0 and 1 as '1' value means the volume constraints of all hydro units are satisfied.

3.4.3 Power Mismatch

The membership function $\mu(\Delta P_{Dj})$, is defined for violation in the load demand mismatch in each subinterval. The membership function $\mu(\Delta P_{Dj})$ is stated by monotonic decreasing function and is given as:

$$\mu(\Delta P_{Dj}) = \begin{cases} 0 & ; \left|\Delta P_{Dj}\right| \geq \epsilon_{PD}^U \\ \dfrac{\left(\epsilon_{PD}^U - \left|\Delta P_{Dj}\right|\right)}{\left(\epsilon_{PD}^U - \epsilon_{PD}\right)} & ; \epsilon_{PD} < \left|\Delta P_{Dj}\right| < \epsilon_{PD}^U \quad (j = 1,2, \dots, T) \\ 1 & ; \left|\Delta P_{Dj}\right| \leq \epsilon_{PD} \end{cases} \qquad (3.17)$$

where $\left|\Delta P_{Dj}\right| = \left|\sum_{i=1}^{\mathrm{N}_G} P_{ji} - P_{Dj} - P_{Lj}\right|$ and ϵ_{PD} is tolerance of error in power demand mismatch during j^{th} sub-interval and is chosen in the range of 5% of the load requirement. ϵ_{PD}^U is the upper range to decide the satisfaction degree of attainment of the power demand met during j^{th} subinterval. The overall membership function, S_{P_D} that gives violation in load demand balance equation, is expressed by the average membership functions of the violation of power demands over the planning period as:

$$S_{P_D} = \frac{1}{T}\sum_{j=1}^{T}\mu(\Delta P_{Dj}) \qquad (3.18)$$

The value of S_{P_D}will vary from 0 to 1. '1' value means the all power demand constraints in each sub-interval are satisfied.

3.4.4 Active Power of Hydro Unit

The membership function $\mu(\mathrm{P_{ji}})$, is defined for violation of the limit of active power from i^{th} hydro unit in j^{th} sub-interval. The membership function $\mu(\mathrm{P_{ji}})$ is defined by monotonic decreasing function and is stated as in Equation (3.19).

$$\mu(P_{ji}) = \begin{cases} 0 & ; P_{ji} \geq \left(P_j^{max} + \epsilon_{PH}\right) \\ \dfrac{\left(P_i^{max} - P_{ji} + \epsilon_{PH}\right)}{\epsilon_{PH}} & ; P_j^{max} < P_{ji} < \left(P_j^{max} + \epsilon_{PH}\right) \\ 1 & ; P_j^{min} \leq P_{ji} \leq P_j^{max} \end{cases}$$

$$(j = 1,2, \dots, T; i = (i + \mathrm{N}_T), \dots, \mathrm{N}_G \qquad (3.19)$$

where ϵ_{PH} is tolerance of error range to decide the satisfaction level of the lower and upper limits of power generation of the i^{th} hydro unit during j^{th} sub-interval. ϵ_{PH} is taken as 5% of P_j^{max}.

The average membership function, S_{PH} accounts for violation in the active power output from a hydro unit. S_{PH} is defined by the average of membership functions of the violations of hydro generation limits over the planning period of all the hydro units and is given in Equation (3.20):

$$S_{PH} = \frac{1}{T}\left(\sum_{j=1}^{T}\left(\frac{1}{\mathrm{N}_H}\sum_{i=1+\mathrm{N}_T}^{\mathrm{N}_G} \mu(P_{ji})\right)\right) \qquad (3.20)$$

The value of S_{PH} will be within 0 and 1 value. '1' value means the limits of the hydro generation of all hydro units are satisfied.

3.4.5 Active Power of Thermal Unit

The membership function $\mu(\mathrm{P_{ki}})$, is defined for the violation in the active power output of i^{th} thermal unit. The membership function $\mu(\mathrm{P_{ji}})$ is defined by trapezoidal function as stated below.

$$\mu(P_{ji}) = \begin{cases} 0 & ; (P_i^{max} + \epsilon_{PT}) \leq P_{ji} \leq (P_i^{max} - \epsilon_{PT}) \\ \dfrac{\left(P_{ji} - P_i^{min} + \epsilon_{PT}\right)}{\epsilon_{PT}} & ; \left(P_i^{min} - \epsilon_{PT}\right) < P_{ji} < P_i^{min} \\ \dfrac{\left(P_i^{max} - P_{ji} + \epsilon_{PT}\right)}{\epsilon_{PT}} & ; P_i^{max} < P_{ji} < (P_i^{max} + \epsilon_{PT}) \\ 1 & ; P_i^{min} \leq P_{ji} \leq P_i^{max} \end{cases}$$

$$(j = 1,2 \dots T;\ i = 1,2, \dots, \mathrm{N}_T) \qquad (3.21)$$

where ϵ_{PT} is tolerance (of the order of 5%) of error range to decide the satisfaction level of the lower and upper limits of the i^{th} thermal power generation during j^{th} sub-interval.

The zero value of the membership function gives the full violation of the permissible limits whereby one value of the membership function gives no violation of active power limit of i^{th} thermal unit in the j^{th} sub-interval.

The overall membership function, S_{PT} of the violation in the active power output of thermal units over a scheduling period. S_{PT} is defined by the average of membership functions of the violations of thermal generation limits over the scheduling period of all the thermal units and is stated as:

$$S_{PT} = \frac{1}{T}\left(\sum_{j=1}^{T}\left(\frac{1}{\mathrm{N}_T}\sum_{i=1}^{\mathrm{N}_T}\mu(P_{ji})\right)\right) \tag{3.22}$$

The value of S_{PT} will vary from 0 to 1. '1' value means the active thermal generation constraints of all the thermal units are satisfied.

The augmented fuzzy penalty function, F_{C}, representing the violations in the equality and inequality constraints is considered as an objective function that is minimized along with other objective functions. The objective to be minimized is stated below:

$$F_{\mathrm{C}} = P_f\left((1-\mathrm{S}_T)^2 + (1-S_V)^2 + \left(1-S_{P_D}\right)^2 + (1-S_{PH})^2 + (1-S_{PT})^2\right) \tag{3.23}$$

where P_f is the exterior penalty factor with high value, of the order of 10^3 to 10^5.

The constraint function, F_{C}, as stated in Equation (3.24) is taken as an objective to be minimized and is converted to maximize its value within the range 0 and 1, expressed as:

$$\mu(F_C) = \frac{1}{1+\left(F_C/P_f\right)} \tag{3.24}$$

3.5 FORMULATION OF UNIFIED OBJECTIVE FUNCTION

The degree of satisfaction of objectives (the best-compromised solution) is the maximum value of all membership value of three objectives. This is achieved through a max-min composition model of decision making under uncertainty environment for hydrothermal system and is defined as:

$$\mu_D = max\left(min\left(\mu(F_1), \mu(F_2), (1-\mu(F_C)\right)\right) \tag{3.25}$$

where F_1 is the fuel cost objective function, F_2, is the objective function representing gaseous pollutant emission from thermal units and F_C, is the objective function representing constraints and μ_D, is the unified objective function or cardinal priority ranking. The value of μ_D, can be used for decision making.

3.6 INTERACTIVE FUZZY MODEL OF DECISION MAKING

The objective functions of a thermal unit are of conflicting nature. The decision-maker assigns fuzzy trade-off values to the available non-inferior solutions obtained. The solution with the maximum degree of satisfaction among the conflicting objectives generates the final solution. To handle the imprecision in the judgment of decision-maker, the best-compromised solution among Pareto solutions of the conflicting objectives of the problem is obtained by transforming them to fuzzy representation in this thesis. Linear membership function values are assigned to each solution that reflects the linear distance between the points of a solution on a Pareto curve to the point of its extreme value. Mathematically, linear fuzzification of the objective function is stated in Equation (3.26):

$$\mu(F_n) = \begin{cases} 0 & F_n \leq F_n^{min} \\ \dfrac{F_n - F_n^{min}}{F_n^{max} - F_n^{min}} & F_n^{min} \leq F_n \leq \ F_n^{max} \\ 1 & F_n \geq F_n^{max} \end{cases} \qquad (n = 1,2, \dots M) \qquad (3.26)$$

where F_n^{min} and F_n^{max} are upper and lower bounds of the n^{th} objective function obtained in the trial runs. The overall objective function of the hydrothermal generation problem is the unification of all objectives and the objective function corresponding to constraints to find the effect of all objectives simultaneously.

3.7 SOLUTION METHODOLOGY

To perform hydrothermal generation scheduling, active power generation for thermal units (P_{ki}; $k = 1,2, \dots, T$; $i = 1,2, \dots \mathrm{N}_T$) and the water discharge rate for hydro units (Q_{ji}; $j = 1,2, \dots, T$; $i = (1 + \mathrm{N}_T), 2, \dots \mathrm{N}_G$) during each sub-interval are the decision variables and are searched using proposed crisscross differential evolution technique.

For hydrothermal system, k^{th} member of the population to be searched is represented in matrix form. Decision variables and their limits of the system are expressed in Equations (3.27), (3.28) and (3.29) respectively.

$$X_k = \begin{bmatrix} P_{k11} & P_{k12} & P_{k1N_T} & Q_{k11+\mathrm{N}_T} & Q_{k12} & Q_{k1N_G} \\ P_{k21} & P_{k22} & P_{k2N_T} & Q_{k21+\mathrm{N}_T} & Q_{k22} & Q_{k2N_G} \\ P_{k31} & P_{k32} & P_{k3N_T} & Q_{k31+\mathrm{N}_T} & Q_{k32} & Q_{k3N_G} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ P_{kT1} & P_{kT2} & P_{kTN_T} & Q_{kT1+\mathrm{N}_T} & Q_{kT2} & Q_{kTN_G} \end{bmatrix} \qquad (\ k = 1,2, \dots, \mathrm{N}_P) \qquad (3.27)$$

Above variable X_k stated in Equation (3.27) can be rewritten in indexed form as:

$$X_{kj}^{t} = \left[P_{kj1}^{t}, P_{kj2}^{t}, \dots, P_{kjN_T}^{t}, Q_{kj1+\mathrm{N}_T}^{t}, Q_{kj2+\mathrm{N}_T}^{t}, \dots, Q_{kjN_G}^{t}\ \right]^{T}$$

The minimum and maximum limits of decision variables are represented as:

$$X_{i}^{min} = \left[P_{1}^{min}, P_{2}^{min}, \dots, P_{N_T}^{min}, Q_{1+\mathrm{N}_T}^{min}, Q_{2+\mathrm{N}_T}^{min}, \dots, Q_{N_G}^{min}\ \right]^{T} \tag{3.28}$$

$$X_{i}^{max} = \left[P_{1}^{max}, P_{2}^{max}, \dots, P_{N_T}^{max}, Q_{1+\mathrm{N}_T}^{max}, Q_{2+\mathrm{N}_T}^{max}, \dots, Q_{N_G}^{max}\ \right]^{T} \tag{3.29}$$

3.7.1 Initialization of Population

The initial population is randomly generated which is uniformly distributed over the problem space bounded by limits of decision variables. The population of size $(\mathrm{N}_P \times T \times N_G)$, is generated in the feasible space of the hydrothermal problem using the following equation.

$$x_{kji}^{0} = x_{i}^{min} + R_{kji}\left(x_{i}^{max} - x_{i}^{min}\right) \quad (i = 1,2, \dots, N_G; j = 1,2, \dots, T; k = 1,2, \dots, \mathrm{N}_P) \tag{3.30}$$

where R_{kji} is a uniform random number that varies between 0 and 1.
The fitness function for k^{th} member of the population is stated as:

$$F_j\left(x_{kji}\right) = \mu_D\left(x_{kji}\right) \qquad (k = 1,2, \dots, \mathrm{N}_P) \tag{3.31}$$

3.7.2 Crisscross Mutation

Crisscross mutation and crisscross crossover operations are offered in both horizontal and vertical directions alternately for information exchange within-population map. Horizontal crisscross recombines the trial crisscross mutant vector and parent vector of randomly selected t^{th} generation to produce offspring trial vector, U_i. Within the generation, after the horizontal search, the moderated solutions compete with their corresponding parent population. The winners are saved in the updated population and serve as the dominated parent population of vertical crisscross and vice versa. Vertical crisscross takes place in different dimensions and facilitates stagnate solutions to jump out of local minima and simultaneously does not destroy probable global optimum dimension. This competitive search mechanism supports DE with gain in convergence rate and improvement in global search capability.

The population during mutation and crossover is generated using crisscross with a random selection of variables (characteristic of the member that is a dimension) in the horizontal direction and a random selection of a member of the population in the vertical direction. The random selection of subinterval is considered for variation with characteristics of member (*i.e.* variable) and member of the population.

Crisscross Horizontal Mutation: The horizontal mutation is performed over the different decision variables of an individual population to generate mutant vector according to the following equation:

$$H_{kji}^{t} = \mathcal{X}_{Bji}^{t} + F^{t}\left(\mathcal{X}_{Bji}^{t} + \mathcal{X}_{kji}^{t} - \mathcal{X}_{kr_1r_3}^{t} - \mathcal{X}_{kr_2r_4}^{t}\right)$$

$$(i = 1,2,\dots,N_G \ and \ r_3 \neq r_4 \neq i;\ j = 1,2,\dots,T;\ r_1 \neq r_2 \neq j; k = 1,2,\dots,\mathrm{N}_P) \qquad (3.32)$$

where X_{Bki}^{t} is the best individual in the current generation and X_{jki}^{t} is base vector, $r_1 \ and \ r_2$ are randomly selected population member in j^{th} sub-interval randomly selected from the set of the time intervals and $r_3 \ and \ r_4$ are randomly selected from a set of decision variables for i^{th} decision variable. Mutation constant, F^t is uniformly distributed between 0 and 1 and is chaotically updated as:

$$F^t = F^L + y^t(F^U - F^L) \qquad (3.33)$$

The Logistic map is a polynomial of degree 2 that shows chaotic behaviour of simple nonlinear difference equation as stated in Equation (3.34):

$$y^t = \mu y^{t-1}(1 - y^{t-1}) \qquad (3.34)$$

where μ is the logistic parameter with values in the interval [1,4] for chaotic mapping. y^0 is set to 0.2027 and $y^t \notin \{0, 0.25, 0.5, 0.75, 1\}$, $F^L \ and \ F^U$ are set to 0.45 and $0.975,$ respectively in this study.

Crisscross Vertical Mutation: The vertical mutation is performed over the different population in different time intervals for a given decision variable according to the Equation (3.35):

$$V_{kji}^{t} = \mathcal{X}_{Bji}^{t} + F^{t}\left(\mathcal{X}_{Bji}^{t} + \mathcal{X}_{kji}^{t} - \mathcal{X}_{r_1r_3i}^{t} - \mathcal{X}_{r_2r_4i}^{t}\right)$$

$$(i = 1,2,\dots,N_G; j = 1,2,\dots,T;\ r_3, r_4 \neq j; k = 1,2,\dots,\mathrm{N}_P;\ r_1, r_2 \neq k) \qquad (3.35)$$

where $r_1 \ and \ r_2$ are randomly selected population from an NP set of population and $r_3 \ and \ r_4$ are randomly selected sub-intervals in a time horizon of j sub-intervals of the scheduling period for i^{th} the dimension of the decision variable.

Mutation constant F^t, controls the amplification of differential variation and its selection greatly affects the positioning of the moderated solution in the neighbourhood of optimum solution or near the periphery of the hypercube search space, allowing a gradual exploration of the search space (Das *et al.*, 2016).

3.7.3 Crisscross Crossover

Crisscross crossover operator provides the opportunity of competition of survival between the offspring and its parent population. The vertical crossover probability is less than the horizontal crossover probability to find as many possible better solutions during the horizontal crossover. It will reduce the number of solutions getting trapped into local minima. The simplicity of the competition operation between horizontal and vertical crossover that makes the population move the search region rapidly with better solutions and accelerates the convergence rate to the global optima. The horizontal crossover exchange information among the decision variables and sub-intervals of a population. The horizontal binomial crossover to generate variable is defined as:

$$U_{kji}^{t} = \begin{cases} H_{kji}^{t} \, ; if \ (r_i < CR^t) or \ (j = R1) \ or \ (k = R2) \\ \mathcal{X}_{kji}^{t} \quad ; \ otherwise \end{cases}$$

$$(i = 1,2, \ldots, N_G; j = 1,2, \ldots, T; k = 1,2, \ldots, \mathrm{N}_P) \quad (3.36)$$

The random integer numbers are generated such that $r_i \in [1, N_G], R1 \in [1, T]$ and $R2 \in [1, \mathrm{N}_P]$. The control parameter CR^t, is the crossover rate and is generated by logistic chaotic mapping defined as:

$$CR^t = CR^L + y^t(CR^U - CR^L) \quad (3.37)$$

where $CR^U \ and \ CR^L$ are set to 0.01 and 0.5 respectively in this study.

The vertical crossover exchange information among the population and subintervals. The vertical binomial crossover is defined below to generate variable, W_{kji}^{t}.

$$W_{kji}^{t} = \begin{cases} V_{kji}^{t} \, ; if \ (r_i < CR^t) or \ (i = R1) \ or \ (j = R2) \\ \mathcal{X}_{kji}^{t} \quad ; \ otherwise \end{cases}$$

$$(i = 1,2, \ldots, N_G; j = 1,2, \ldots, T; \ k = 1,2, \ldots, \mathrm{N}_P) \quad (3.38)$$

where the random integer numbers are generated such that $r_i \in [1, NP], R1 \in [1, N_G]$ and $R2 \in [1, T]$.

3.7.4 Selection

For the selection of better off-springs as a member of the population of the next generation, the cost of each trial vector is compared with that of the parent target vector. To decide whether the vector $H_{kji}^{t}, V_{kji}^{t}, U_{kji}^{t} \ or \ W_{kji}^{t}$ shall be a member of the population comprising the next generation, these are compared with the corresponding vector, $\mathcal{X}_{kji}^{t}$. The trial vector is selected when the fitness value of the trial vector is better than the current or target vector, then

$$\mathcal{X}_{kji}^{t+1} = \begin{cases} y_{kji}^{t} \quad ; if \ f_k^t(y_{kji}^{t}) > F_j^t(\mathcal{X}_{kji}^{t}) \\ \mathcal{X}_{kji}^{t} \quad ; \ otherwise \end{cases} \qquad (\ k = 1,2 \ldots \mathrm{N}_P) \quad (3.39)$$

where $f_k^t\left(y_{kji}^t\right) = \max\left\{F_k^t\left(H_{kji}^t\right), F_k^t\left(V_{kji}^t\right), F_k^t\left(U_{kji}^t\right), F_k^t\left(W_{kji}^t\right)\right\}$ and its corresponding best population is y_{kji}^t $(i = 1,2,\dots,N_G; j = 1,2,\dots,T; k = 1,2,\dots,\mathrm{N}_P)$.

The fitness value, $F_k^{t+1}\left(\mathcal{X}_{kji}^{t+1}\right)$ is evaluated for population $\mathcal{X}_{kji}^{t+1}$; $(i \in N_G; j \in T; k \in \mathrm{N}_P)$

3.7.5 Opposition Learning-based Selection

The global search capability of proposed CCDE strategy is further expedited using opposition-based learning. For keeping the elite solutions, the retention mechanism is integrated with the selection operation of DE.

For several non-dominated solutions generated from populations by crisscross mutation, crisscross crossover and selection, the opposite population is generated and is evaluated. The fittest individuals are selected from the unification of the current-best population and opposite population. This proposal speeds up the convergence towards the global solution. The opposite point of each decision variable within its limits in the current population is defined in Equation (3.40). In case, the solution does not improve for a certain number of iterations, then the opposition learning is applied using the following equation.

$$X_{kji}^{t+1} = \begin{cases} x_i^{min} + x_i^{max} - x_{kji}^{t+1} & ; F_{opp} \leq 0.5 \\ x_i^{min} + R_{kji}\left(x_i^{max} - x_i^{min}\right) & ; F_{opp} > 0.5 \end{cases} \qquad (i \in N_G; j \in T; k \in \mathrm{N}_P) \tag{3.40}$$

where F_{opp} is the opposition factor.

The largest fitness value of the individual is termed as the best solution among the population at $(t+1)^{th}$ iteration:

$$F_B^{t+1}(X_{Bji}^{t+1}) = \max\left(F_k^{t+1}\left(X_{kji}^{t+1}\right)\right) (k = 1,2,\dots \mathrm{N}_P)$$

The global best is obtained with the unification of new best candidate and the current–best solution as:

$$X_{Gji}^{t+1} = \begin{cases} X_{Bji}^{t+1} & ; if\ F_B^t(X_{Bji}^{t+1}) > F_G^t\left(X_{Gji}^t\right) \\ X_{Gji}^t \quad ; ibest = ibest + 1 & ; Otherwise \end{cases} \tag{3.41}$$

***Termination Criterion**:* The iterative process terminates when the maximum number of evaluations of function is achieved or the maximum number of iterations are performed.

Computational Complexiety: For the comparison with conventional DE, the number of function evaluations (NFE) is $(\mathrm{N}_P + 5 \times \mathrm{N}_P \times ITMAX)$, for crisscross DE whereas conventional DE follows NFE as $(\mathrm{N}_P + 2 \times \mathrm{N}_P \times ITMAX)$.

3.8 HYPOTHESIS OF CRISSCROSS DIFFERENTIAL EVOLUTION ALGORITHM

The crisscross DE algorithm is elaborated as:

1. **Input Data:** Parameter Set Up: Input population size N_P, number of decision variables N_G, upper and lower limits of each decision variable X, Mutation constant F, crossover rate CR, convergence criteria parameter N_{FE} and the maximum number of iterations $ITMAX$.
2. **Initialization**: Initialize the value of i^{th} decision variable X in k^{th} population using Equation (3.30).
3. Set iteration count, $t = 0$
4 Evaluate the fitness function value $F_k^t(x_{kji}^t)$ using Equation (3.31) for every individual according to the objective function.
5. Select the global best solution, $F_G^t(x_{Gji}^t) = \max\left(F_k^{t+1}\left(X_{kji}^{t+1}\right)\right);\ j \in N_P$ among the population members.
6 Set the updating counter for the best solution, $\text{ibest} = 0$

   WHILE $(\boldsymbol{t} < \boldsymbol{ITMAX})$ ***DO***
7 Increment the iteration counter, $t = t + 1$

   FOR $\boldsymbol{k = 1, N_P}$ ***(population size)***
6. Horizontal Mutation: Perform horizontal mutation operation to generate a trial vector using the *current-best* mutation strategy to generate a trial vector, H_{kji}^t, using Equations (3.32). Evaluate fitness function value $F_j^t(H_{kji}^t)$ using Equation (3.31).
7 Vertical Mutation: Perform vertical mutation operation to generate a trial vector using the *current-best* mutation strategy to generate a trial vector, V_{kji}^t, using Equations (3.35). Evaluate fitness function value $F_j^t(V_{kji}^t)$ using Equation (3.31).
6 Horizontal Crossover: Apply horizontal binomial crossover or recombination operation to recombine the trial mutant vector and parent vector to produce offspring trial vector, U_{jki}^t using Equations (3.36). Evaluate fitness function value $F_j^t(U_{kji}^t)$ using Equation (3.31).
7 Vertical Crossover: Apply vertical binomial crossover operation to recombine the trial mutant vector and parent vector to produce offspring trial vector, W_{kji}^t, using Equations (3.38). Evaluate fitness function value $F_j^t(W_{kji}^t)$ using Equation (3.31).

8 Selection: Compute the fittest solution among the moderated solutions: $f_j^t\left(y_{kji}^t\right) = \max\left\{F_k^t\left(H_{kji}^t\right), F_k^t\left(V_{kji}^t\right), F_k^t\left(U_{kji}^t\right), F_k^t\left(W_{kji}^t\right)\right\}$ and save corresponding trial vector individual y_{jki}^t $(i = 1,2, \dots, N_G; j = 1,2, \dots, T; k = 1,2, \dots, \mathrm{N}_P)$ to $f_k^t\left(y_{kji}^t\right)$

9. Compute the individual for the next generation $t = t + 1$ using Equation (3.39)

END FOR

10 Select the best solution, $F_B^{t+1}\left(X_{Bji}^{t+1}\right) = \max\left(F_k^{t+1}\left(X_{kji}^{t+1}\right)\right) (j = 1,2, \dots, \mathrm{N}_P)$

13 Find the global best point.

$$\mathcal{X}_{Gji}^t = \begin{cases} \mathcal{X}_{Bji}^{t+1} & ;\ if\ F_B^t(X_{Bji}^{t+1}) > F_G^t\left(\mathcal{X}_{Gji}^t\right) \\ \mathcal{X}_{Gji}^t\ ;\ ibest = ibest + 1 & ;Otherwise \end{cases}$$

14: IF $(\boldsymbol{MOD}(\boldsymbol{ibest}, \mathbf{5}) = \mathbf{0})$, THEN

Apply opposition based learning with known F_{opp} using Equation (3.40).

ENDIF

ENDDO

3.9 MODIFIED CRISSCROSS DIFFERENTIAL EVOLUTION WITH SIN-COSINE TECHNIQUE

In recent research reported in the literature, the use of mathematical operators to strengthen the balance between exploration and exploitation of stochastic algorithms is attracting the attention of researchers such as chaos map for mutation and crossover operator (Yuan, *et al.*, 2008), Cauchy mutation (Qin *et al.*, 2010), disruption factor (Gouthakumar *et al.*, 2015), crisscross in GA crossover (Meng, 2015), orthogonal crossover in DE (Wang *et al.*, 2012), intensification operator for interaction in PSO (Singh *et al.*, 2017), and sine-cosine function by Mirjalili, (2016).

On the other front, the blending of two algorithms into one to exploit the advantages of each algorithm and use of one algorithm to overcome the deficiency of another one is an excellent way to address the challenging task of handling shortcomings of evolutionary algorithms. There are several examples in the literature of hybridization such as predator-prey with PSO (Narang *et al.*, 2012), sine cosine with PSO (Nenavath *et al.,* 2017), selective pressure strategy in DE mutation (Vladimir *et al.,* 2018),

In this chapter adaptive DE is integrated with a sine-cosine algorithm (SCA) to improve the position update in search space for better and precise exploration and exploitation. The sine-cosine is a population-based metaheuristic algorithm introduced by Mirjalili, (2016). This method is derived from mathematical trigonometric functions sine and cosine. The application of a sine-cosine strategy produces multiple random candidate solutions and moves in a cyclic pattern around a closed circular space. The best solution is assumed to lie at the centre of search space and other candidate solutions follow a cyclic pattern of movement to reach the best solution. SCA supports the abrupt and fast exploration initially and then slows down as it reaches near destination point to avoid local optima. This integration guides the search solution towards an optimal point with reasonably good accuracy in comparison with adaptive differential evolution algorithm.

The switching between set of equations as expressed in equation (3.41) i.e. selection of sine function or cosine function is controlled by key parameter $r_4 \in [0,1]$ as:

$$X_{kji}^{t+1} = \begin{cases} X_{kji}^{t} + r_1 \times sin(r_2) \times \left| r_3 X_{Bkj}^{t} - X_{kji}^{t} \right| & ;\ r_4 < 0.5 \\ X_{kji}^{t} + r_1 \times cos(r_2) \times \left| r_3 X_{Bji}^{t} - X_{kji}^{t} \right| & ;r_4 \geq 0.5 \end{cases} \quad (i \in N_G; j \in T; k \in \mathrm{N}_P) \tag{3.41}$$

The sine-cosine metaheuristic technique is elaborated in section A.2 (Appendix-A). The pseudo-code of sine-cosine algorithm is as given :

Algorithm: *Sin-Cosine*

1. *Intilization of population using Equation (3.30)*

2. *Perform CCDE mutation operation using Equation (3.32) and Equation (3.35)*

3. *Perform CCDE crossover operation using Equation (3.36)and Equation (3.38).*

4. *Select the population best solution.*

5. *Set Sin-Cosine iteration count SCA,* $g = 0$

WHILE $(g < ITSCA)$

DO

6. *Set increment counter,* $g = g + 1$

7. *Perturb the population for the key parameter* $r_4 \in [0,1]$*, using equation (3.41)*

8. Select the best solution, $F_B^{t+1}(X_{Bji}^{t+1}) = \max\left(F_k^{t+1}\left(X_{kji}^{t+1}\right)\right)$; $k = 1,2,\ldots,\mathrm{N}_P$

9. *Check and assign the best solution* $\mathcal{X}_{Gki}^{t} = \begin{cases} \mathcal{X}_{Bji}^{t+1}\ ;\ if\ F_B^{t}(X_{Bji}^{t+1}) > F_G^{t}\left(\mathcal{X}_{Gji}^{t}\right) \\ \mathcal{X}_{Gji}^{t}\ ; Otherwise \end{cases}$

10. *Calculate the error:* $MSE = \left\|(F_j^{t}\left(X_{kji}^{t}\right) - F_B^{t}\left(X_{Bji}^{t}\right)\right\|$

11. *Check if* $MSE \leq 10^{-3}$ *then it concludes that similar to the previous best pattern is achieved then exit SIN COS procedure.*

ENDDO

3.10 SIMULATION OF TEST PROBLEMS AND RESULTS

In this section, the feasibility and effectiveness of the proposed CCDE and CCDESC algorithms are described in section 3.8 and 3.9 have been illustrated using numerical experiments. In this study, the generalized benchmark test functions and standard hydrothermal test systems are considered to investigate the performance of CDE and CCDE and CCDESC.

3.10.1 Benchmark Test Functions

In this thesis, the performance evaluation is focussed on the comparison of convergence behaviour, assessment of exploitation and exploration on varieties of benchmark functions grouped in categories namely: unimodal, multimodal, continuous and discontinuous, differentiable and non-differentiable, separable and non-separable and compared with state-of-the-art heuristic algorithms (Nenavath *et al.,* 2017). Two performance metrics are defined to assess the convergence speed and frequency of convergence that reflects the search history as:

Acceleration Rate (AR) that assesses the convergence speed of the algorithm is defined as.

Acceleration Rate $(AR) = \frac{NFE_{Alg1}}{NFE_{Alg2}}$;if $AR > 1$, then algorithm 2 (Alg2) is faster than algorithm1.

Success Rate (SR) that estimates the number of times; value-to-reach (VTR), is achieved before reaching the maximum number of function evaluations by the algorithm and is defined as:

Success rate $(SR) = \frac{Number\ of\ times\ \ VTR\ is\ achieved}{Number\ of\ Trials}$

For simulation experiments on each benchmark test functions, the number of trial runs $(IRUN)$, is set to 30 to ensure the reliability of the results. For every run, the population size N_P is taken as 50 and the maximum number of iterations is set to 1000. The performance analysis of each test function is assessed for its fitness value in terms of the best, average, the worst, and the standard deviation (SD). The termination criteria of the algorithm are set to achieve the fitness value of the respective test function or the maximum number of iterations, otherwise. The number of function evaluations (N_{FE}), are considered to assess the efficiency of the algorithm. However, for calculating the success rate and acceleration rate, the fitness value-to-reach (VTR) is taken as 10^{-10}. In case, $AR > 1$, it is marked as 'Y'. The performance outcome from twenty generalized benchmark test functions with CDE, CCDE and CCDESC algorithms is presented in Table 1. The acceleration rate indicates that the number of function evaluations for CCDE is much less than taken in CDE to attain the same VTR. For 30 trial-runs of each of the proposed strategy, CCDE shows higher success-rate than CDE and computation time is also less in general except for non-differential functions.

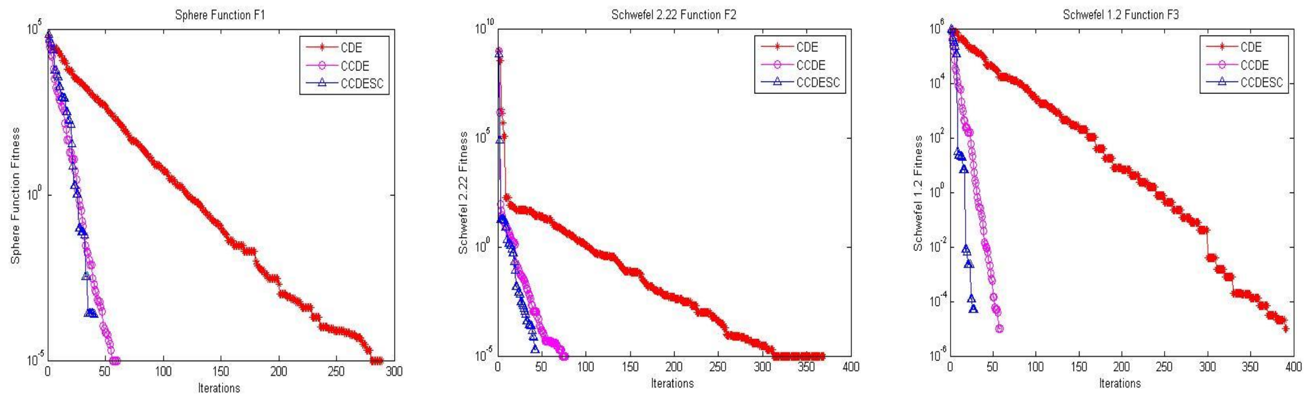
Table 3.1: Performance analysis of generalized benchmark test functions

S. No.	Test Function	Type of Function*	Algorithm	NFE	CPU Time (Sec)	Fitness value				% SR	AR
						Best	Average	Worst	SD		
1	Sphere	C, D, S, U	CDE	100050	2.16	3.14×10^{-31}	7.8×10^{-21}	1.93×10^{-20}	9.1×10^{-21}	100	Y
			CCDE	60050	1.76	0.0	0.0	0.0	0.0	100	
			CCDESC	75000	3.16	0.0	0.0	0.0	0.0	100	
2	Schwefel 2.22	C, D, NS, U	CDE	100050	1.71	1.51×10^{-17}	2.24×10^{-11}	5.04×10^{-11}	1.98×10^{-11}	100	Y
			CCDE	10050	0.333	0.0	0.0	0.0	0.00	100	
			CCDESC	75000	9.85	0.0	0.0	0.0	0.0	100	
3	Schwefel 1.2	C, ND, NS, U	CDE	100050	8.99	9.24×10^{-27}	1.43×10^{-19}	5.91×10^{-19}	2.28×10^{-19}	100	Y
			CCDE	70050	13.13	0.0	0.0	0.0	0.0	100	
			CCDESC	75000	6.86	0.0	0.0	0.0	0.0	100	
4	Schwefel 2.21	C, ND, S, U	CDE	100050	17.21	3.42×10^{-3}	4.46×10^{-1}	1.91×10^{-2}	7.37×10^{-1}	0.0	Y
			CCDE	60050	1.57	0.0	0.141×10^{-44}	0.2802×10^{-44}	0.0	100	
			CCDESC	75000	3.35	1.03×10^{-4}	5.21	11.82	3.95	0.0	
5	Rosenbrock	C, D, NS, U,	CDE	100050	4.88	11.59	15.78	21.78	4.05	0.0	Y
			CCDE	60050	2.04	0.0	14.21	18.46	2.224	42.5	
			CCDESC	75000	3.96	16.28	19.71	24.35	2.87	0.0	
6	Step	NC, ND, S, U, SH	CDE	100050	2.06	0.0	0.4	1.0	0.489	60	Y
			CCDE	10050	0.27	0.0	0.0	0.0	0.0	100	
			CCDESC	75000	4.82	0.0	0.0	0.0	0.0	100	
7	Quartic	C, D, S, U	CDE	100050	2.17	7.05×10^{-3}	1.12×10^{-2}	1.74×10^{-2}	3.974×10^{-3}	0.0	Y
			CCDE	100050	2.35	1.87×10^{-3}	6.67×10^{-3}	7.87×10^{-3}	2.41×10^{-3}	0.0	
			CCDESC	75000	10.73	1.73×10^{-3}	4.55×10^{-3}	9.58×10^{-3}	2.44×10^{-3}	0.0	
8	Rastrigin	C, D, NS, M	CDE	100050	2.40	2.98	10.34	24.87	7.59	0.0	Y
			CCDE	40050	1.48	0.0	0.0	0.0	0.0	100	
			CCDESC	75000	1.60	0.0	0.0	0.0	0.0	100	
9	Schwefel 2.26	C, D, S, M, SH	CDE	100050	3.71	2.37×10^{4}	1.04×10^{5}	1.67×10^{5}	4.67×10^{4}	0.0	N
			CCDE	60050	1.14	-1.25×10^{-2}	-1.25×10^{-2}	-1.25×10^{-2}	0.0	100	
			CCDESC	75000	1.28	217.21	963.33	1796.38	568.64	0	
10	Alpine	C, ND, S, M	CDE	100050	1.76	1.79×10^{-8}	7.18×10^{-7}	2.46×10^{-6}	7.79×10^{-7}	0	Y
			CCDE	80050	3.30	3.11×10^{-42}	9.89×10^{-10}	2.47×10^{-9}	1.22×10^{-11}	100	
			CCDESC	75000	7.45	0.0	0.0	0.0	0.0	100	
11	Griewank	C, D, NS, M, R	CDE	100050	2.51	2.22×10^{-16}	1.48×10^{-3}	7.39×10^{-3}	2.95×10^{-3}	100	Y
			CCDE	30050	0.82	0.0	0.0	0.0	0.0	100	
			CCDESC	75000	4.67	0.0	0.0	0.0	0.0	0.0	
12	Ackley	C, D, NS, M, R	CDE	100050	2.64	2.41×10^{-11}	11.9	19.96	9.77	0	Y
			CCDE	60050	1.55	-5.87×10^{-16}	2.25×10^{-15}	2.96×10^{-15}.	1.43×10^{-15}	100	
			CCDESC	75000	3.45	-5.87×10^{-16}	-5.87×10^{-16}	-5.87×10^{-16}	0.0	100	
13	Penalised 1	C, D, NS, M	CDE	100050	2.96	8.06×10^{-15}	1.07×10^{-14}	2.14×10^{-14}	5.36×10^{-15}	12	Y
			CCDE	60050	1.26	8.0×10^{-15}	8.0×10^{-15}	8.0×10^{-15}	0.0	100	
			CCDESC	75000	1.98	8.00×10^{-15}	2.00×10^{-4}	0.2073	6.02×10^{-02}	100	
14	Penalised 2	C,D,NS,M	CDE	100050	3.85	7.64×10^{-15}	1.97×10^{-2}	9.89×10^{-2}	3.96×10^{-2}	22	Y
			CCDE	60050	1.54	7.64×10^{-15}	7.64×10^{-15}	7.64×10^{-15}	0.0	100	
			CCDESC	75000	7.45	7.64×10^{-15}	0.295	2.84	0.849	50	
15	Neumaier 3	C, D, NS, M, SH	CDE	100050	2.16	4.95×10^{3}	5.37×10^{3}	6.632×10^{3}	5.55×10^{2}	0	Y
			CCDE	20050	1.12	4.95×10^{3}	4.95×10^{3}	4.95×10^{3}	4.95×10^{-2}	0	
			CCDESC	75000	1.61	4.95×10^{3}	5.192×10^{3}	6.22×10^{3}	3.92×10^{2}	0	

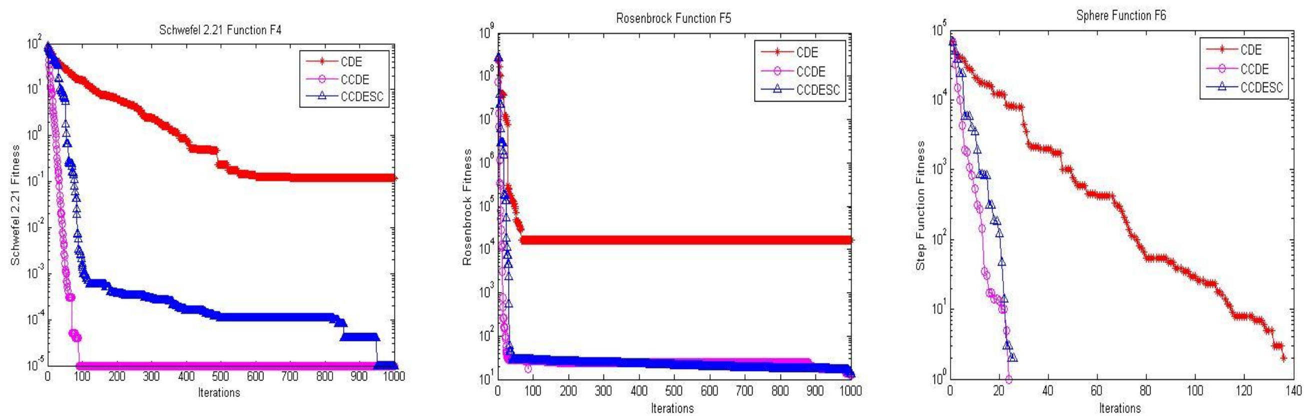
16	Salomon	C, D, NS, M	CDE	100050	2.25	2.99×10^{-1}	4.79×10^{-1}	5.99×10^{-1}	1.16×10^{-1}	0	N
			CCDE	60050	1.67	9.98×10^{-2}	9.98×10^{-2}	9.98×10^{-2}	6.64×10^{-2}	0	
			CCDESC	75000	1.53	0.00	0.109	0.299	0.083	25	
17	Levy	C, D, NS, M	CDE	100050	2.75	6.87×10^{-14}	6.87×10^{-14}	6.87×10^{-14}	0.0	100	Y
			CCDE	100050	3.34	6.87×10^{-14}	3.13×10^{-6}	1.56×10^{-5}	6.24×10^{-6}	72	
			CCDESC	75000	1.90	6.87×10^{-14}	2.19×10^{-02}	0.1096	4.4×10^{-02}	80	
18	Zakharov	C, D, NS, M	CDE	100050	2.15	2.97×10^{-11}	2.18×10^{2}	5.54×10^{2}	2.67×10^{1}	60	Y
			CCDE	100050	2.98	0.0	1.02×10^{-42}	5.10×10^{-42}	0.00000	100	
			CCDESC	75000	1.45	5.31×10^{-06}	4.062	21.16	8.084	20	
19	Expansion F10	C, D, S, M	CDE	100050	4.22	7.92×10^{-2}	2.33×10^{-1}	5.76×10^{-1}	4.89×10^{-1}	0	Y
			CCDE	20050	1.27	0.0	0.0	0.0	0.0	100	
			CCDESC	75000	2.17	0.0	0.0	0.0	0.0	100	
20	Weierstrass	C, D, S, M, SH, R	CDE	100050	15.16	-6.34×10^{-4}	-1.11×10^{-4}	1.43×10^{-3}	7.85×10^{-4}	0	Y
			CCDE	60050	5.59	-7.42×10^{-4}	-7.42×10^{-4}	-7.42×10^{-4}	0.0	0	
			CCDESC	75000	9.40	-7.35×10^{-4}	-6.86×10^{-4}	-5.822×10^{-4}	4.22×10^{-5}	0	

C: Continuous, NC: Not continuous, S: Separable, NS: Non-separable, D: Differentiable, ND: Not differentiable, SH: Shifted, R: Rotated, U: Unimodal, M: Multimodal

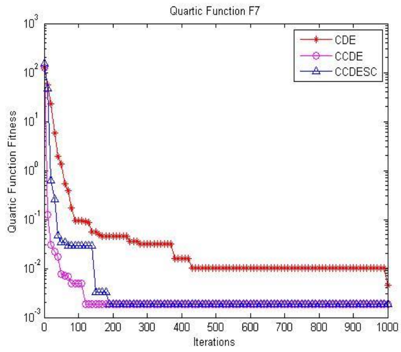
For further comparison of three algorithms, the convergence plots for the best trial run of all 20 benchmark functions are presented in Figure 3.1 (a) to (t) respectively. For further comparison of three algorithms, the convergence plots for the best trial run of all 20 benchmark functions are presented in Figure 3.1 (a) to (t) respectively.



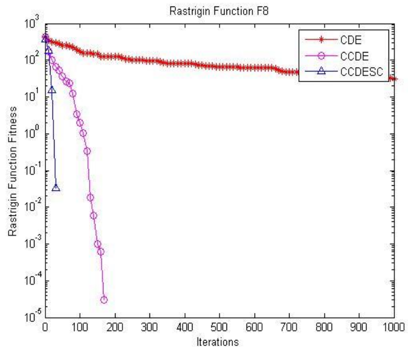
(a) Sphere Function (b) Schwefel 2.22 (c)Schwefel 1.2



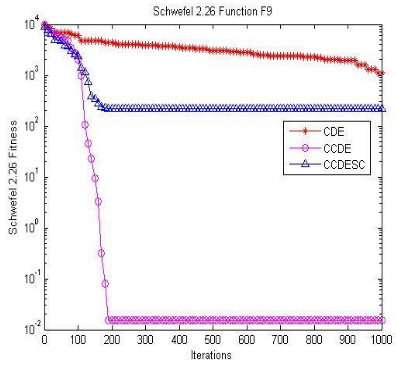
(d) Schwefel 2.21 I Rosenbrock (f) Step



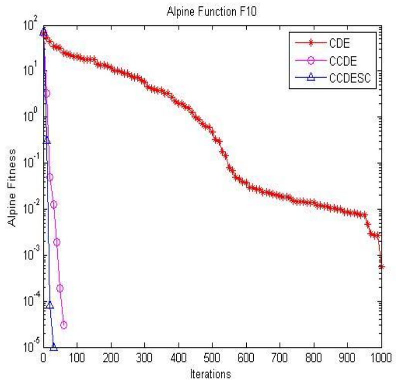
(g) Quartic



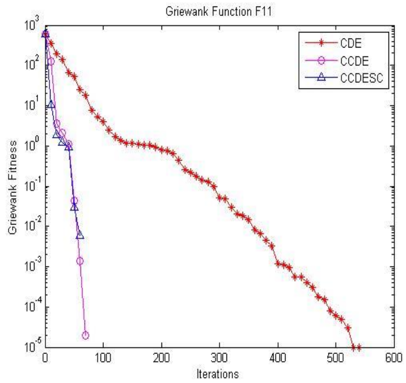
(h) Rastrigin



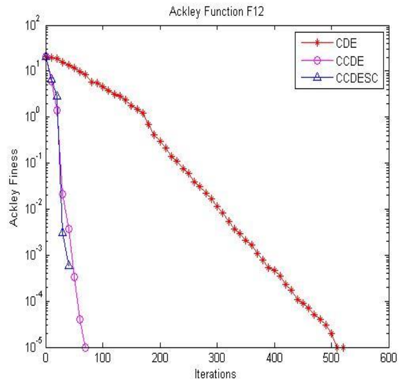
(i) Schwefel 2.26



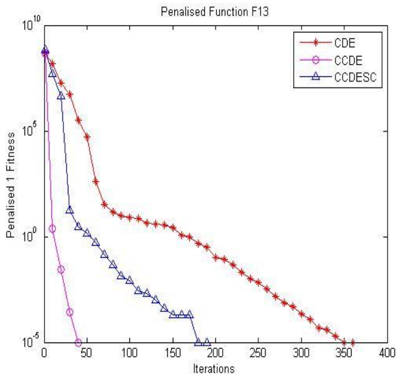
(j) Alpine



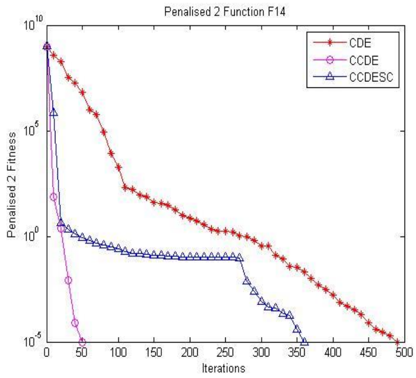
(k) Griewank



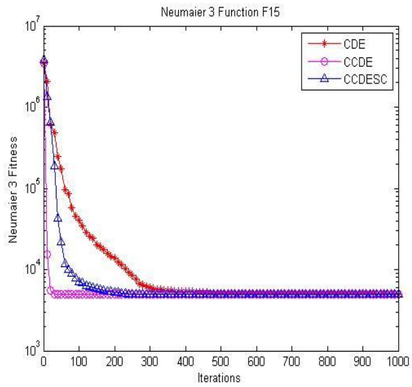
(l) Ackley



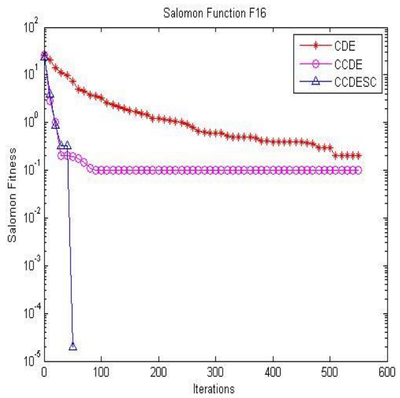
(m) Penalised 1



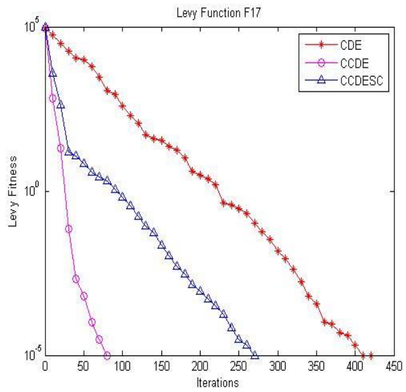
(n) Penalised 2



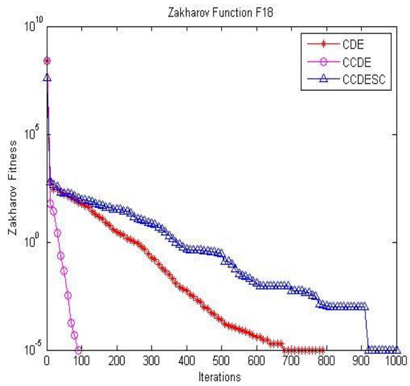
(o) Neumaier 3



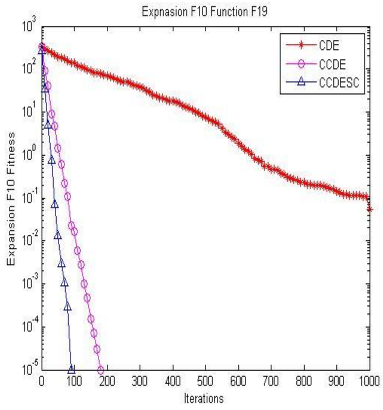
(p) Salomon



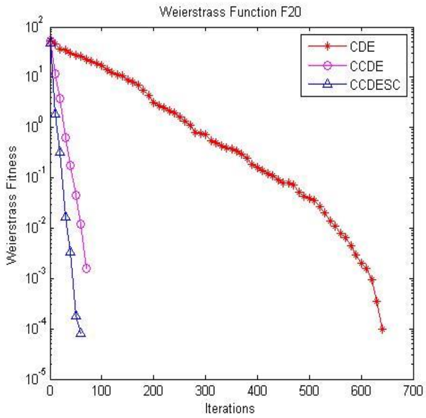
(q) Levy Function



I Zakharov



(s) Expansion F10



(t) Weierstrass

Fig. 3.1 (a) to (t): Convergence characteristics of generalized benchmark functions

Table 3.2: Comparison of benchmark functions

S. No.	Test Function	CCDE		PSO		GA		SCA		FPA		CDEPS-1
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean
1	Sphere	0.0	0.0	0.0003	0.0011	0.8078	0.4393	0.0	0.00	0.2111	0.0717	7.67E-19
2	Schwefel 2.22	0.0	0.0	0.0693	0.2164	0.5406	0.2363	0.0	0.0001	0.919	0.7804	2.41E-16
3	Schwefel 1.2	0.0	0.0	0.0157	0.0158	0.5323	0.2423	0.0371	0.1372	0.2016	0.1225	--
4	Schwefel 2.21	0.0	0.0	0.0936	0.4282	0.8837	0.7528	0.0965	0.5823	0.816	0.5618	--
5	Rosenbrock	0.0	1.132	0.0	0.0	0.6677	0.4334	0.0005	0.0017	0.0813	0.0426	0.0
6	Step	0.0	0.0	0.0004	0.0033	0.7618	0.7443	0.0002	0.0001	0.2168	0.1742	0.0
7	Quartic	0.00187	0.0024	0.0398	0.0634	0.5080	0.1125	0.00	0.0014	0.3587	0.2104	--
8	Rastrigin	0.0	0.0	0.3582	0.8795	1.0000	0.6881	0.0	0.7303	0.8714	0.8665	206.574
9	Schwefel 2.26	0.0125	0.0	1.0000	0.0036	1.0000	0.0055	1.0000	0.0036	1.000	0.0029	--
10	Griewank	0.0	0.0	0.0521	0.0448	0.7679	0.2776	0.00	0.00	0.2678	0.0706	2.47E-16
11	Ackley	0.0	0.0	0.1045	0.0541	0.8323	0.0686	0.3804	1.000	1.000	0.0162	--
12	Penalised 1	0.0	0.0	0.0	0.0	0.4573	0.4222	0.0	0.00	0.0008	0.0015	--
13	Penalised 2	0.0	0.0	0.0	0.0	0.6554	0.8209	0.0	0.00	0.0187	0.0375	--

For plots other than presented in Figure 3.1. (d), (g), (h), (i), (j) and (o), the response is restricted to the number of iterations for no improvement in results thereafter. In general, CCDE and CCDESC show better convergence speed than chaotic DE irrespective of the type of benchmark function. The encouraging trend in this graphical comparison shows that crisscross differential evolution (CCDE) presents a stable and better solution in terms of less number of function evaluations, better best-so-far fitness value and fast convergence whereas it is a very significant conclusion that for non-separable and multimodal functions like Griewank, Rosenbrock, Ackley, Expansion-10, CCDESC outperforms CCDE and gives encouraging convergence trends both in convergence rate and speed.

Further, to validate the performance of CCDE and CCDESC algorithms, it is compared with the published results obtained by other reputed heuristic algorithms such as Particle swarm optimization (PSO), Genetic algorithm (GA), Sine-Cosine algorithm (SCA) and Flower pollination algorithm (FPA), reported in the literature (Nenavath, *et al.*, 2017) and with hybrid Chaotic DE-Powell Search method (CDEPS-I) (Singh *et al.*, 2018) The statistical results (mean and standard deviation) of these algorithms are compared for specified benchmark functions and are presented in Table 3.2.

It is evident to conclude that CCDE algorithm outperforms other heuristic algorithms in the majority of test functions and gives good competition to SCA in multimodal functions. The performance of CCDE for unimodal functions concludes that CCDE has better exploitation capability and convergence accuracy.

3.10.2 Hydrothermal Systems

The feasibility of the proposed strategies CDE, CCDE and CCDESC has been tested on two hydrothermal system problems. Hydrothermal System-I is comprised of seven units (three-thermal units and four-hydro units) for 24 hours load demand (Appendix C.1) and hydrothermal system – II is comprised of twenty-two units (six-thermal and sixteen-hydro units) with 24 hours load demand (Appendix C.2). The transmission loss is not taken into account in both test systems. The details of the hydrothermal system data are mentioned in Appendix-C. For thermal units, hourly active power generation is considered as decision variables and for hydro units, hourly water discharge is taken as decision variables.

Hydrothermal System-I

The short-term HTGS problem is evaluated in three sub-problems, (i) total operating cost of thermal units (ii) total emission from thermal units (iii) multi-objective for an enviro-economic generation. The results of cost and emission for thermal units in all three sub-problems are presented in Table 3.3. The population size is taken as 50 and the number of trial runs as 60. The comparison of convergence characteristics of three algorithms DE, CDE, CCDE and CCDESC for a multi-objective problem in terms of cost as the objective is presented in Figure 3.2. Figure 3.3 depicts the comparison of the Pareto front obtained from CCDE and CCDESC algorithms. Pareto

Table 3.3: Comparison of cost and emission for hydrothermal system –I

Algorithm (Reference)	Cost as an objective		Emission as objective		Multi-objective scheduling		
	Cost ($)	**Emission (lbs).)**	**Cost ($)**	**Emission (lbs.)**	**Cost ($)**	**Emission (lbs.)**	μ_D
QPSO (Chiang, 2007)	42359	31298	45271	17767	44259	18229	0.492
PSO (Mandal, 2008)	44730	--	--	--	--	--	--
DE (Mandal et.al, 2009)	43500	21092	51449	18257	44914	19615	0.392
EVA (Basu, 2011)	45063	48797	59228	16554	--	--	--
IQPSO (Sun et.al, 2011)	43278	17984	47871	17019	44344	17408	0.479
ε-ODEMO (E. Xu *et.al*, 2012)	--	--	--	--	44672	16653	0.429
NSGA (Basu, 2014)	42474	28132	48263	16928	43280	17899	0.644
PSO (Narang *et. al* , 2012)	43076	25384	48570	16199	45906	18620	0.238
PPO (Narang *et. al*, 2012)	42042	27961	48913	15728	44111	17473	0.515
MOCA+PSO (Zhang, 2012)	42009	16842	44962	16242	43873	16222	0.552
CDE (Zhang *et.al*, 2013)	41872	17726	45049	16221	42198	17711	0.811
PPO+PS (Narang *et. al.*, 2014)	41530	28757	48920	15716	42836	17254	0.712
DE (Author)	43065	27944	46461	18809	45327	19327	0.327
CDE (Proposed)	42845	25892	46172	16300	43316	18340	0.638
CCDE (Proposed)	**40975**	**19778**	**47447**	**14810**	**41708**	**16896**	**0.886**
CDECS (Proposed)	43153.1	21385	47127	16896.2	42949	17954	0.636
CCDESC (Proposed)	**41028**	**21180**	**47537**	**14800**	**42363**	**16148**	**0.7885**

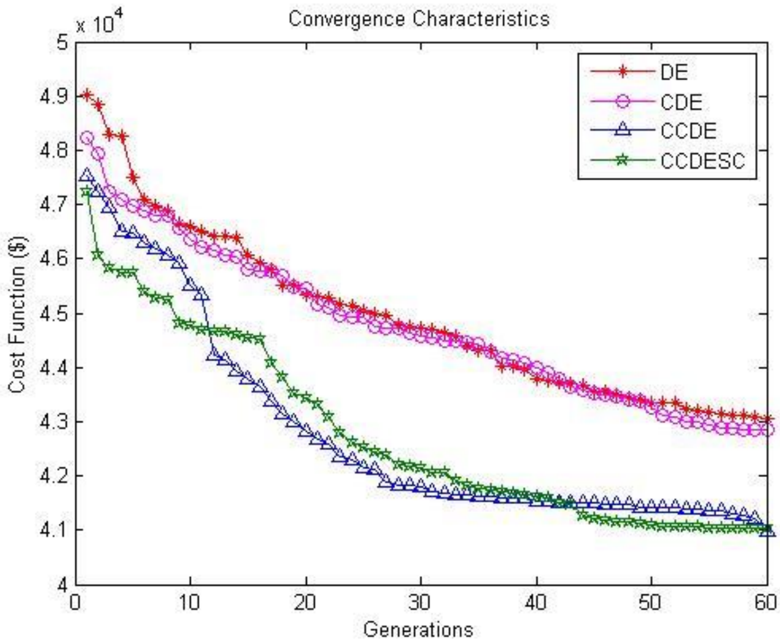


Fig. 3.2 Comparison cost characteristics among DE, CDE, CCDE and CCDESC for hydrothermal system-I

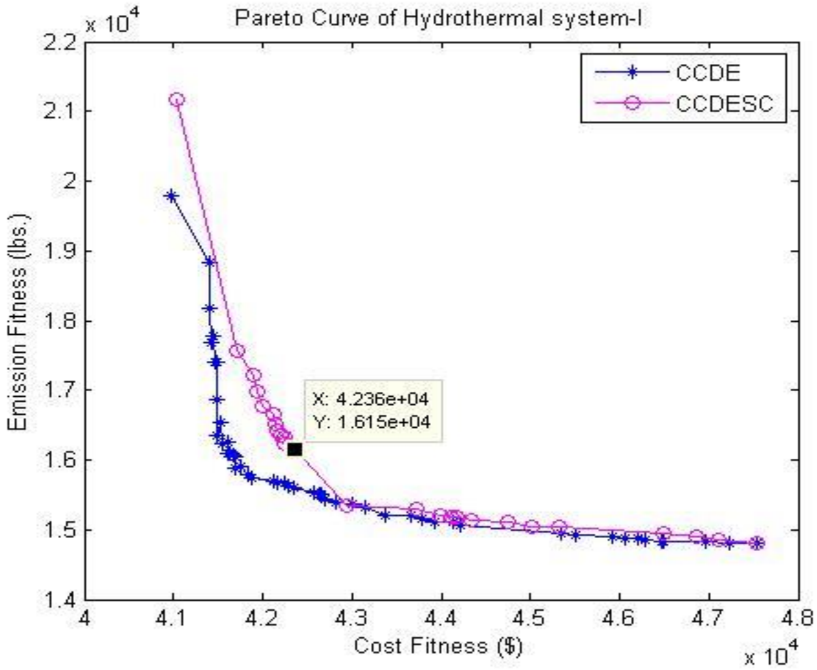


Fig. 3.3 Pareto curve of Hydrothermal system –I with CCDE and CCDESC

front of CCDE algorithm shows more non-convexity than with CCDESC. The corresponding an hourly water release from each of the four reservoirs of hydro units over 24-hour scheduling period for hydrothermal system-I is presented in Figure 3.4. From Table 3.3, the comparison shows that CCDE can obtain a compromised solution of good satisfaction level, as its cardinality function value, $\boldsymbol{\mu_D}$ is better than CDE and also in comparison with established algorithms are undertaken for comparison for the hydrothermal system -I.The active power generations from thermal power and hydropower generations of a hydrothermal system –I in all the above three cases over a scheduling period of 24-time intervals are presented In Table 3.4.

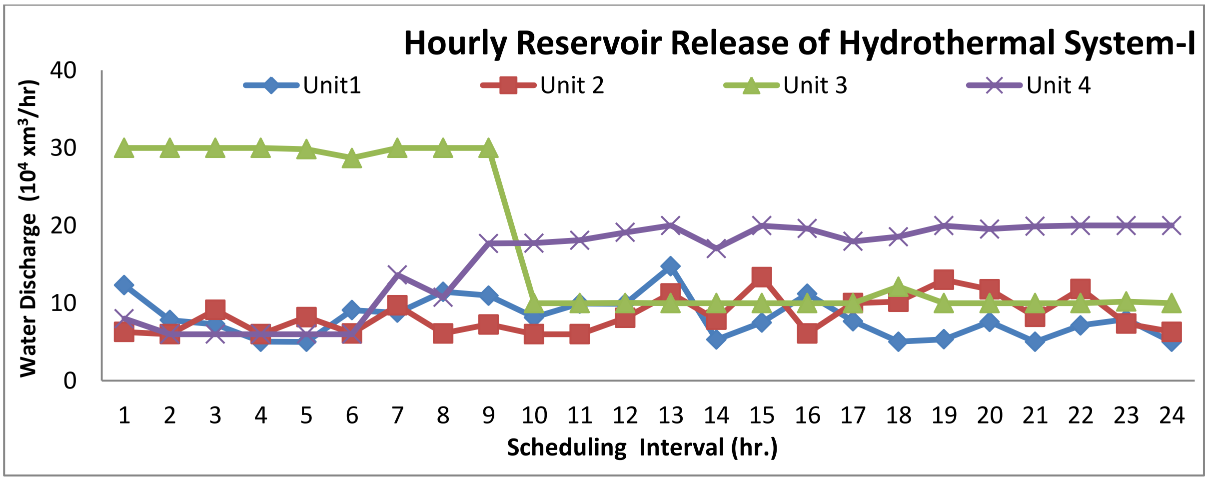


Fig. 3.4: Water discharge ($10^4 \times m^3/hr$) for multi-objective hydrothermal system-I

Table 3.5: Power generation from thermal and hydro units of hydrothermal system -I

Time sub-interval (k) Hrs.	Load Deman P_{Dk} (MW)	Cost as an Objective								Emission as Objective							Multi-objective						
		Thermal Power Generation P_{iT} (MW)			Hydro Power Generation P_{iH} (MW)				Error	Thermal Power Generation P_{iT} (MW)			Hydro Power Generation P_{iH} (MW)				Thermal Power Generation P_{iT} (MW)			Hydro Power Generation P_{iH} (MW)			
		P_{1T}	P_{2T}	P_{3T}	P_{1H}	P_{2H}	P_{3H}	P_{4H}	PERR	P_{1T}	P_{2T}	P_{3T}	P_{1H}	P_{2H}	P_{3H}	P_{4H}	P_{1T}	P_{2T}	P_{3T}	P_{1H}	P_{2H}	P_{3H}	P_{4H}
1	750	172.23	294.33	50	52.5	49	0	131.93	0.01	173	200.4	130.1	65.6	49	0	131.9	102.8	205.4	138.8	94.4	51	0	157.6
2	780	102.56	300	141.29	57.18	50.18	0	129.06	0.01	172	228.7	131.9	68.1	50.2	0	129.1	175	214.6	139	73.2	51	0	127.2
3	700	175.0	213.29	50	80.61	55.12	0	125.80	0.00	170.9	167.5	107.1	77.5	51.3	0	125.7	174.8	210.7	50	69.94	70.66	0	123.9
4	650	102.69	205.38	50	65.76	66.65	0	159.51	0.01	153.8	161.8	96	63.8	52.9	0	121.6	172.3	204.4	50	52.55	51	0	119.75
5	670	102.58	297.35	50	54	53.08	0	112.95	0.04	154.3	162	111.4	71.9	54.5	0	115.9	175	211.2	50	52.82	67.13	0	113.85
6	800	102.48	300	139.82	59.15	61.61	0	136.94	0	172.3	214.6	134.6	84.8	55.7	0	137.98	174.8	213.5	139	82.8	53.23	0	136.67
7	950	102.46	300	235.47	95.38	62.85	0	153.82	0.02	174.9	250.1	149.21	92.64	67.62	0	215.44	102.8	209.7	236	79.82	75.73	0	245.65
8	1010	102.75	296.65	229.73	94.7	94.09	0	192.08	0.0	174.9	247.4	170.8	87.6	73.2	0	256.1	102.8	300	235	91.8	51.54	0	228.86
9	1090	101.65	300	230.64	78.16	51.85	0	327.69	0.01	175	270.1	179.5	85.37	71.1	0	308.9	175	220	233	88.94	60.12	0	312.25
10	1080	102.56	300	230.43	54.24	49.88	0.8	342.09	0.0	175	265	156.9	85.45	72.3	2.5	322.6	174.7	212	237.7	75	52	6.45	322.15
11	1100	174.46	209.79	230.27	102.3	51.57	6.63	324.93	0.0	174.6	266.7	161.7	82.1	76.7	7.4	330.9	175	209	230.3	85.55	53.65	13.6	332.7
12	1150	102.44	207.41	324.48	86.56	53.20	14.55	361.36	0	174.5	261.9	178	88.3	74.9	12.6	359.71	174.4	209.8	244.5	86	68.9	17.55	348.77
13	1110	102.47	204.33	233.75	94.97	95.13	16.08	363.27	0	173.6	225.3	173	83.9	70.25	18.07	365.87	174.6	227.6	139.3	99	85	21.4	362.9
14	1030	102.52	300	50	101.2	80.92	22.94	372.42	0.00	174.3	196.5	118.9	80.2	65	23.95	371.13	102.3	300	139.7	54.4	65.7	26	341.8
15	1010	102.42	207.06	140.86	102	63.57	26.87	367.24	0.02	165.2	186.6	122.6	75.2	66.2	29.59	364.67	102.5	209	139.7	73	91	30	364.55
16	1060	102.68	295.42	138.79	53.89	77.75	31.26	360.2	0.01	170.2	219.9	122.9	80.34	71.16	34.35	361	173.3	210	138	93.68	52.3	36.48	355.5
17	1050	102.76	297.34	139.63	70.88	47.58	39.26	352.55	0	168.1	216.2	119.5	75.32	78	38	354.92	174.6	209.8	138.5	74.75	76.8	39.7	335.89
18	1120	102.15	295.22	229.81	54.7	48.76	44.91	344.46	0.01	173.4	255.9	140.6	82.13	78.96	40.74	348.26	174.4	298.5	138.8	54.13	75.71	43	335.33
19	1070	101.86	300	139.65	90.76	49.83	46.47	341.42	0.01	175	224.1	126.1	81	79	43.55	341.28	102.7	300	139	57.3	83.6	46.6	339.78
20	1050	102.4	200.07	229.98	54.71	80.12	49.05	333.67	0	170.7	203.8	143.9	77.1	74.56	46.49	333.34	173.6	209.7	139	75.4	75.4	47.95	328.5
21	910	102.43	205.23	50	93.89	84.02	49.43	325	0	157.5	147.6	95.4	66.89	69.69	48.38	324.6	168.7	208.5	50	54.3	56.5	49.5	322.45
22	860	102.18	209.21	50	57.82	73.42	51.58	315.83	0.00	155.6	137.2	92.3	54.76	53.7	51.19	315.27	102.5	196.8	50	72.15	73.4	51.3	313.8
23	850	102.60	119.45	140.22	84.79	44.75	52.28	305.9	0.01	152.9	132.9	88.4	54.35	62.57	53.6	305.33	102.5	209.5	50	77.78	49.95	54.25	305.97
24	800	175	125.46	50	54.71	43.27	56.22	295.34	0	137.9	127.9	74.6	54.7	54.78	55.32	294.79	175	126	50	54.7	43.55	55.3	295.5
		Cost 40975 $								**Emission: 14810.4 lbs.**							**Cost: 41708 $, Emission 16896 lbs.**						

Table 3.6: Optimal hydro water discharge ($Q_{iH} \times 10^4 m^3/hr$) for hydrothermal system –I

Time Interval (k)	**Water Discharge of Hydro Units (Cost as Objective)**				**Water Discharge of Hydro Units (Emission as objective)**				**Water Discharge of Hydro Units (Multi-Objective)**			
	Q_{1H}	Q_{2H}	Q_{3H}	Q_{4H}	Q_{1H}	Q_{2H}	Q_{3H}	Q_{4H}	Q_{1H}	Q_{2H}	Q_{3H}	Q_{4H}
1	5	6	30	6	6.61	6	30	6	12.31	6.28	30	7.99
2	5.43	6	29.89	6	6.84	6	30	6	7.81	6.0	30.0	6.0
3	8.51	6.58	29.94	6	8.13	6	30	6	7.26	9.13	30.0	6.0
4	6.4	8.14	29. 91	9.09	6.22	6	30	6	5.02	6.0	30.0	6.0
5	5	6	29.94	6	7.28	6	30	6.1	5.0	8.19	29.83	6.0
6	5.57	7.08	30	6.14	9.43	6.1	30	6.02	9.09	6.09	28.69	6.0
7	11.35	7.28	30	6	11.42	7.72	30	10.6	8.76	9.71	30	13.62
8	11.46	15	29.98	7.58	10.44	8.82	30	12.86	11.46	6.1	30.0	10.75
9	8.25	6.42	28.41	18.72	10.1	8.62	30	17	10.95	7.25	30	17.69
10	5.1	6	10.1	19.49	10.1	8.89	10.44	17.48	8.16	6	10	17.72
11	13.52	6	10	16.65	9.29	9.72	11.16	17.46	9.93	6	10	18.1
12	9.42	6	10.59	19.92	10.43	9.44	10.56	19.98	9.9	8.1	10.02	19.1
13	11.17	15	10	19.45	9.51	8.74	10.5	20	14.73	11.25	10	20
14	13.18	11.58	10.1	20	8.73	7.91	10.8	19.98	5.28	7.84	10	17.03
15	13.79	8.28	10.09	20	7.76	8	10.26	19.82	7.5	13.36	10	19.98
16	5	11.08	10	19.85	8.46	8.75	10	20	11.18	6.1	10	19.6
17	6.95	6.1	10.1	19.62	7.64	10.18	10	20	7.67	10	10	17.96
18	5.0	6.1	10	19.42	8.64	10.82	10	20	5.03	10.17	12.11	18.54
19	9.94	6.25	10	19.95	8.48	11.62	10	20	5.33	13	10	19.98
20	5	11.96	10.1	19.98	7.92	11.29	10.66	20	7.58	11.77	10	19.54
21	10.67	14.42	10	19.99	6.55	10.67	10	20	5	8.23	10	19.89
22	5.37	11.98	10.16	20	5.09	7.73	10.02	20	7.12	11.83	10	20
23	8.97	6.63	10.16	20	5	9.18	10	20	7.9	7.35	10.2	20
24	5	6.25	11.1	19.98	5	7.9	10.08	20	5	6.3	10	20

Table 3.5 presents the water discharge from four-hydro units of test System-I in all three sub-problems. The tabulated results depict that all equality and inequality constraints are satisfied, and average power balance error is within an acceptable range. The operating cost

and pollutant emission values are better in CCDE than CDE algorithm when considered scheduling problem for individual objectives and combined objectives.

Hydrothermal System-II

The effectiveness of the proposed algorithms has been further emphasized through a demonstration on Hydrothermal system-II. This active power generation and hourly water discharge over the scheduling period are obtained for three sub-problems, viz. (i) Scheduling with total operating cost as objective (ii) Scheduling with emission as objective (iii) multi-objective for enviro- economic generation.

Table 3.7: Comparison of cost and emission for hydrothermal system -II

Algorithm	**Cost as an objective**		**Emission as objective**		**Multi-objective scheduling**		
	Cost ($)	**Emission (tons)**	**Cost ($)**	**Emission (tons)**	**Cost ($)**	**Emission (tons)**	μ_D
PSO (Narang *et.al* (2012)	1,365,733	26,235	1,390,559	24,871	1,364,068	25,340	0.524
PPO (Narang *et al*. 2012)	1,349,846	26,967	1,391,413	23,711	1,360,188	25,064	0.572
PSO+Powell (Narang *et al*. 2014)	1,337,500	27,225	1,406,200	23,184	1,359,100	24,289	0.586
CDE (Proposed)	1,347,912	28,287	1,400,148	26,615	1,357,148	27,094	0.612
CCDE (Proposed)	**1,325,825**	**27,271**	**1,388,496**	**23,961**	**1,356,946**	**25,364**	**0.623**
CCDESC (Proposed)	**1.325,322**	**27,238**	**1,390,064**	**24,042**	**1,351,912**	**25,711**	**0.589**

The simulation results of cost and emission obtained by CDE and CCDE techniques in the three cases are tabulated in Table 3.6 and are compared with the established techniques. The population size is taken as 100 and the number of trial runs as 30.

The comparison of convergence cost characteristics, while solving a hydrothermal system –II as a single objective problem with CDE and CCDE are shown in Figure 3.5. Solution with CDE gets stagnated as compared to CCDE. Moreover, CCDE offers a better solution at a faster rate than CDE. The improvement in the cardinality priority ranking μ_D represents the best quality of solution as compared to the solution obtained by standard PSO and PPO [12]. Figure 3.5 depicts the variation in membership function of cost and emission as function. Thermal power generation corresponding to the best-compromised solution with a cardinality priority value $\mu_D = 0.623$ is obtained by CCDE for test system–II and are presented in Table 3.7.

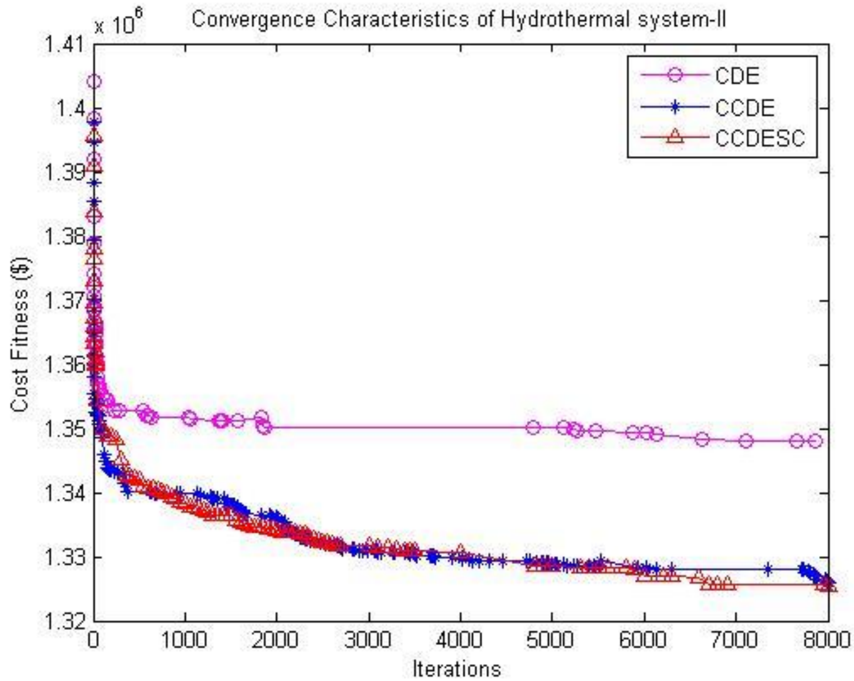


Fig. 3.5: Comparison of cost function with CDE, CCDE and CCDESC for hydrothermal system-II

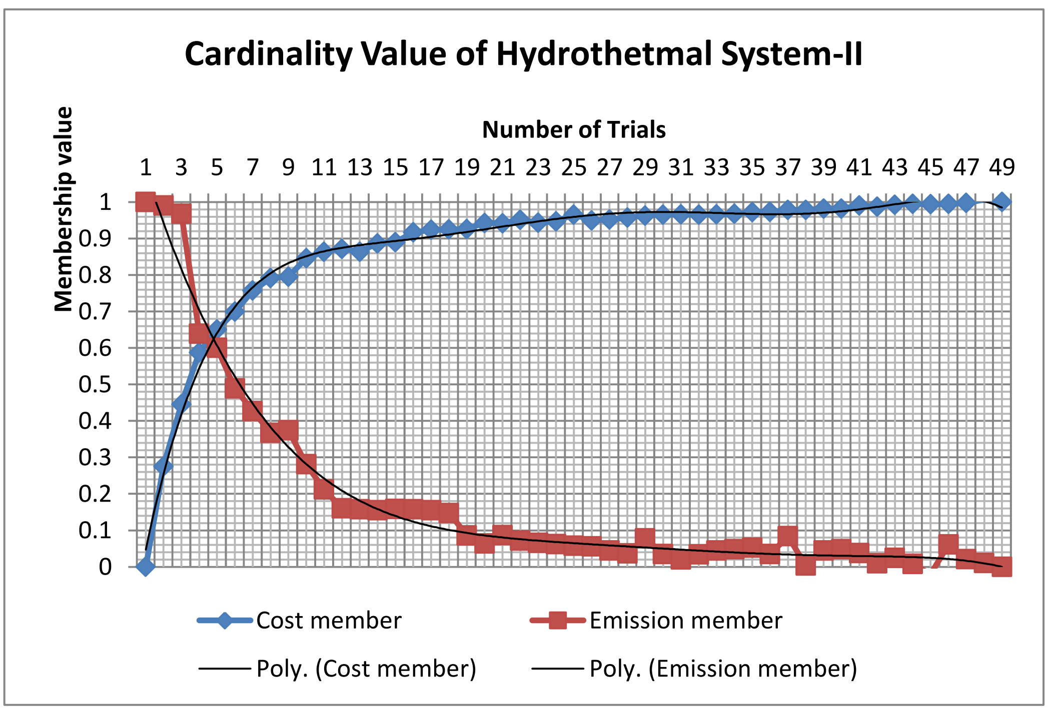


Fig. 3.6 Cardinality ranking of hydrothermal system –II

Table 3.7: Thermal power generation of six thermal units for Case 3 of hydrothermal system-II

Hourly Time sub-interval (k)	**Load Demand P_{Dk} (MW)**	**Thermal Power Generation P_{iT} (MW)**						**Power Mismatch ΔP_k (MW)**
		P_{1T}	P_{2T}	P_{3T}	P_{4T}	P_{5T}	P_{6T}	**-0.00012**
1	1374	99.65	148.5	193.8	200	106.3	317.37	0.0
2	1278	79.5	86.7	181.4	197.5	286.22	325	0.0
3	1372	82.1	130.7	196.1	190.4	312.2	325	0.0
4	1058	73.1	10	114.1	178.95	296.25	245	0.00012
5	1358	63.3	123.4	204.8	212.7	306.37	309.1	0.0
6	1282	75.65	105.4	147.75	217.12	287.6	312.5	0.0
7	1130	38.75	77.65	193.4	145.68	275.3	259.92	0.0
8	1200	34.3	62.6	150.47	185.62	315	314.58	-0.00012
9	1248	30.95	127.04	181.4	179.65	313.13	277.47	-0.00012
10	1364	82.41	139.68	210	206.46	315	263.75	0.0
11	1146	67.17	71.52	161.5	150.84	313.16	238.55	-0.00012
12	1262	74.62	71.45	209.12	200.8	308.73	238.73	0.0000
13	1246	82.3	54.21	203	183.27	299.85	283.1	-0.00012
14	1140	76.42	37.4	163.2	177.22	293.63	294.62	-0.00012
15	1226	28.13	61.65	177.46	204.92	302.41	310.1	0.0
16	1314	52.15	125.9	189.33	187.17	289.68	318.14	0.0
17	1426	105.78	150	207.64	213.42	285.64	308.95	-0.00012
18	1028	80.52	13.89	72.71	208.3	281.32	236.2	0.0000
19	1148	33.97	89.7	171.7	166.1	305	233.12	-0.00012
20	1256	78.77	96.13	171.25	184.97	273.35	299.84	0.0
21	1348	118.52	134.65	196.44	202.52	261	281.54	0.0
22	1424	115.1	150	210	215.4	303.42	293.5	0.0
23	1070	90.22	10	131.67	150.33	315	228.23	0.0
24	1118	59.3	15.11	169.3	162.78	315	259.4	0.0

Figure 3.7 and Figure 3.8 represent the hourly water discharge and reservoir volume for each hydro unit of test problem-II while solving for the best-compromised solution in terms of

optimizing operating cost and emission pollutants' simultaneously. All inequality constraints are certainly satisfied and equality constraints are achieved up to satisfaction level.

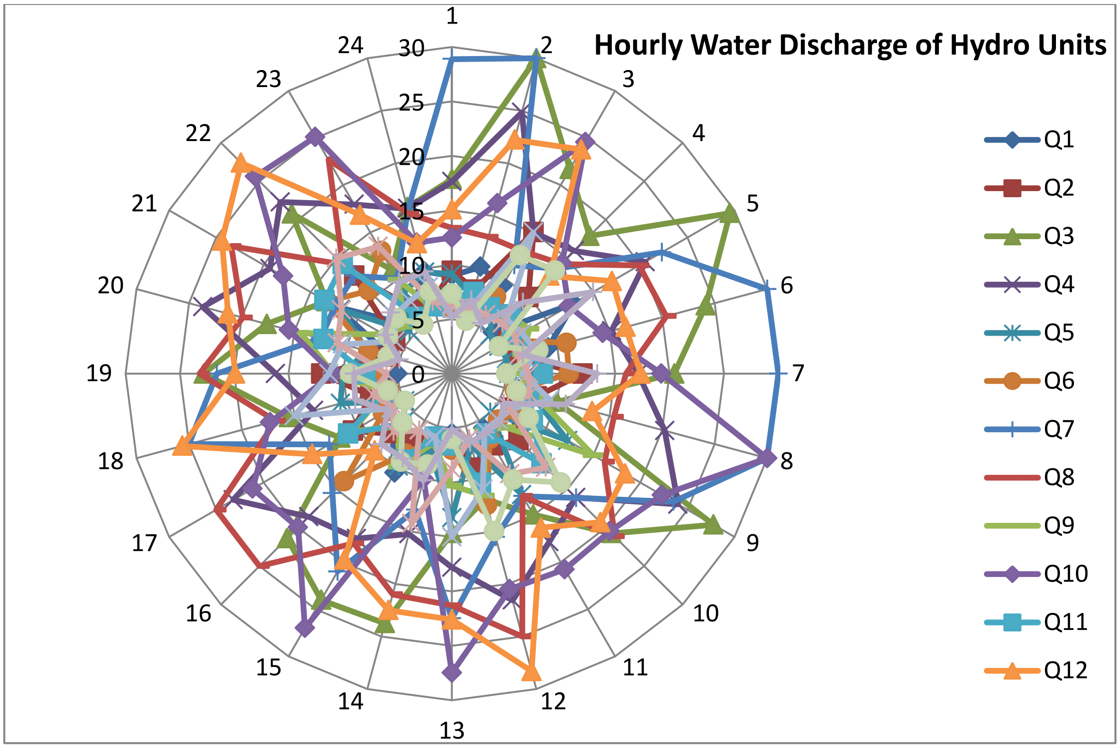


Fig.3.7: Hourly water discharge ($10^4 \times m^3/hr$) for Case 3 of hydrothermal system –II

The proposed algorithms are tested on small and medium dimensional hydrothermal systems. There are 168 (7×24) number of decision variables in test System-I whereas, in test system-II, the number of decision variables is 528, higher than test System-I. The results of the hydrothermal system-I are compared with other heuristic algorithms like PSO, PPO, Integrated PPO, NSGA, QPSO, DE, EVA, MOCA. The convergence characteristics in two cases depicted in Figure 3.2 and Figure 3.4 show that both CCDE and CCDESC are faster than CDE to achieve the specific fitness value and do not stagnate into local sub-optimal solution. However, CCDE gives a better-compromised solution to two conflicting objectives in both test problems. The simulation results obtained for both the test systems of hydrothermal generation scheduling demonstrate the effectiveness and supports their ability to solve large scale constrained multi-objective optimization problems.

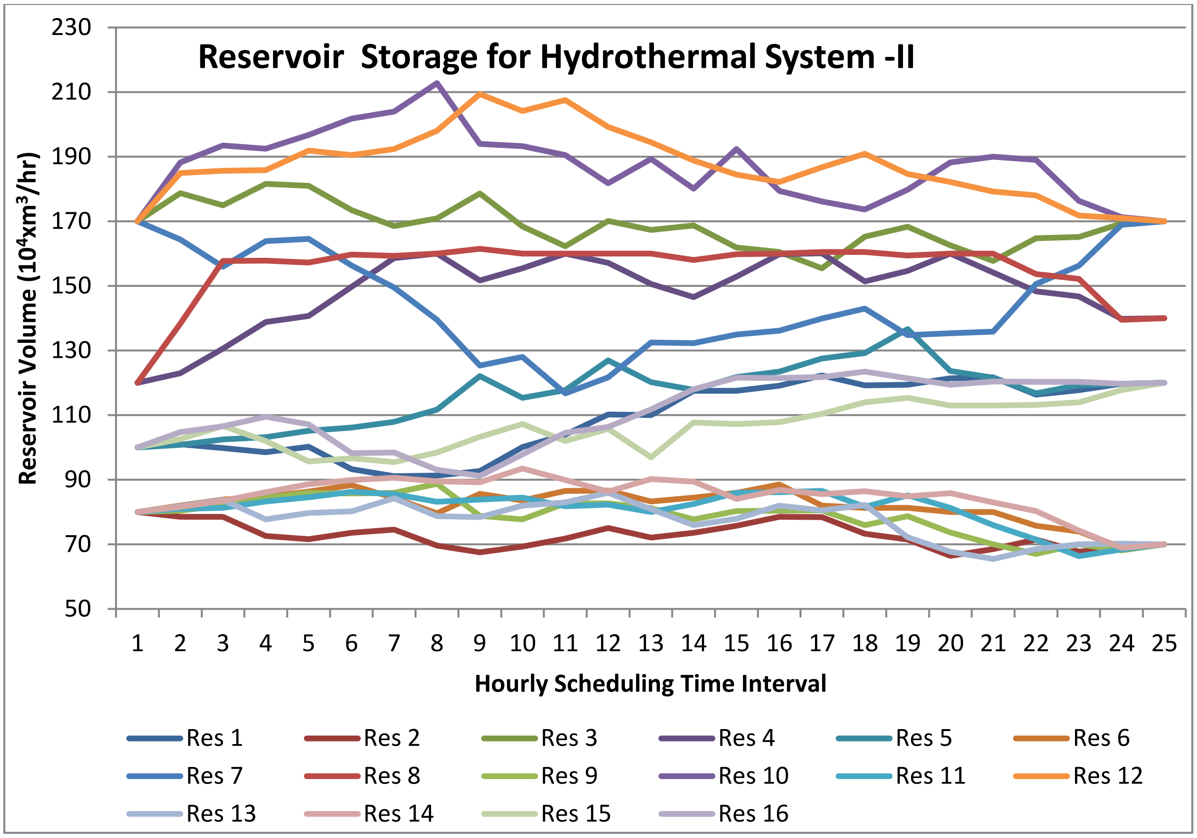


Fig. 3.8: Hourly Reservoir storage ($10^4 \times m^3/hr$) for Case 3 of hydrothermal System –II

3.10 Statistical Measurement and Quality Solution

Non-parametric statistical analysis has been carried out to assess the robustness and quality of results, obtained from the proposed CCDE algorithm. Pairwise comparison tests are considered to compare the performance of two algorithms when applied to a common set of problems.

For benchmark test functions, the p-values of the results of CCDE and compared algorithms presented in Table 2, are obtained using a paired wise comparison of means and standard deviations.

The null hypothesis H_0 and an alternative hypothesis H_A are defined as:

H_0: $\mu_1 = \mu_2$ and H_A: $\mu_1 < \mu_2$

The proposed hypothesis is tested at a 5% level of significance. The results of significance tests on benchmark functions are presented in Table 3.8 and shows that CCDE gives competing results to PSO and SCA. The null hypothesis is rejected and the performance of

CCDE gives superior results for unimodal and multimodal functions. These results strongly conclude that CCDE has better exploitation and convergence capability.

To decide the significance of the results from the proposed algorithms for the practical multi-objective constrained hydrothermal problem, Wilcoxon's signed-rank test, Mann-Whitney’s are applied for pairwise comparisons on hydrothermal system –I. The proposed hypothesis is tested at a 5% level of significance. The null hypothesis H_0 and an alternative hypothesis H_A are defined as:

H_0: Two samples follow the same distribution ($\mu_1 = \mu_2$)

H_A: Two samples follow different distributions ($\mu_1 \neq \mu_2$)

Table 3.8: p-values of paired wise comparisons for benchmark test functions

Function	**PSO**	**GA**	**FPA**	**SCA**
Sphere	0.0567	0.0001	<0.0001	1
Schwefel 2.22	0.0258	<0.0001	<0.0001	1
Schwefel 1.2	<0.0001	<0.0001	<0.0001	0.0058
Schwefel 2.21	0.1254	<0.0001	<0.0001	0.002441
Rosenbrock	1	0.0002	0.613	0.9975
Step	0.3935	<0.0001	<0.0001	<0.0001
Quartic	0.0001	<0.0001	<0.0001	<0.0001
Rastrigin	0.0049	<0.0001	<0.0001	1
Schwefel 2.26	<0.0001	<0.0001	<0.0001	<0.0001
Griewank	<0.0001	<0.0001	<0.0001	1
Ackley	<0.0001	<0.0001	<0.0001	0.0084
Penalised 1	0.9441	<0.0001	0.0003	0.9441
Penalised 2	0.9442	<0.0001	0.0006	0.9442

Table 3.9: Descriptive statistics for hydrothermal system-I with DE, CDE,CCDE and CCDESC

Algorithms	**Sample size**	**Minimum ($)**	**Mean ($)**	**Maximum ($)**	**Median ($)**	**Standard Deviation ($)**	**p-value**	**Inter Quartile Range**
DE	60	43063	44864.78	49028	44659	1564.7	0.1994	2500
CDE	60	42845	44764.37	48234	44558	1429.1	0.0674	2276
CCDE	60	40975	42253.68	45747	41607.5	1300.52	0.00007	958
CCDESC	60	41028	42759.98	47229	42112	1713.57	0.0394	658.26

Table 3.10: Mann-Whitney's test applied to hydrothermal system-I

Statistical Parameters	DE versus CDE	DE versus CCDE	CCDE vs CCDESC
p-value	0.013	< 0.0001	0.0746
Z-Score	0.366	9.940	2.828
U	1862	3232.5	1746

The descriptive statistics of trial solutions are obtained and are compared with classical DE and are presented in Table 3.9 and Table 3.10.

It is evident from Table 3.9, that CCDE method can obtain better minimum, average and maximum cost and standard deviation than CDE and basic DE with fast convergence. Pairwise comparisons are performed using Mann-Whitney's Test. The computed p-value is lower than the considered value of significance α=5%, in each case. This concludes that the null hypothesis H_0 is rejected and accepts the alternative hypothesis H_A, means two sample algorithms are of different distributions and have different means.

The stability of metaheuristic algorithms is established by using the interquartile range from box plot analysis. The experimental outcomes of sixty independent trials of the hydrothermal system –I are plotted in Figure 3.9 for all three algorithms.

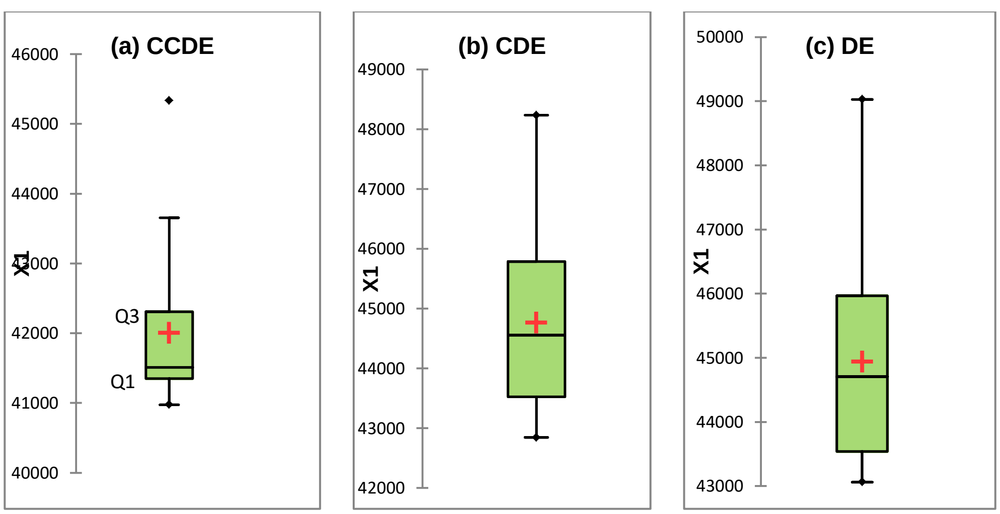


Fig.3.9: Robustness analysis of hydrothermal system –I (Quartile Distribution)

It is observed from the box-plot statistics that the values of the first and third quartile $Q_1 = 41349$ and $Q_3 = 42308$ respectively, are lower in CCDE than CDE and DE. This indicates that discrepancy in the solution generated by the CCDE algorithm is smaller in comparison with CDE and DE algorithms. However, in comparison with CCDESC, the interquartile range is lowest in CCDESC that confirms the robustness of the algorithm. From the descriptive statistics, it is observed that the standard deviation in the solution obtained by the CCDE algorithm is smaller than obtained by CCDECS, CDE and DE. Among all algorithms, the mean values of CDE and DE are very close, whereas CCDE has the smallest mean value. Hence it is concluded that CCDE algorithm attributes to a high-quality solution.

3.11 CONCLUSIONS

The hydrothermal scheduling problem is a multi-objective and non-linear constrained optimization problem with conflicting objectives *viz.* operating cost and gaseous pollutant emission. With cascaded hydro reservoirs, water transport delay among units and constrained regulation of water discharge from reservoirs are considered simultaneously over the operation period. The equality constraints of active power balance and the amount of available water are independently handled using the variable elimination method. The unpredicted deviations arise due to infringements of equality constraints while adjusting the violated dependent variables within their limits. These residues are quantified by defining the membership function within their prescribed limits and are incorporated as objectives to be optimized. The three conflicting objectives of the hydrothermal system are optimised through an interactive fuzzy satisfying function. To solve the problem, chaotic-crisscross differential evolution (CCDE) algorithm has been successfully employed to obtain hourly generation schedule of the hydrothermal system over 24 hours' time-horizon. The main modifications of CCDE algorithm based on conventional DE are; first, the regulation of mutation constant and crossover rate using a chaotic logistic mapping that supports the better diversification of search space to enhance exploration, second; the crisscross strategy that has been applied orthogonally in DE operations to exchange information in hypercube search space of hydrothermal problem to improve convergence rate and global searchability. The horizontal crisscross interaction improves population diversity of DE and the vertical crisscross interaction supports stagnated solutions to jump out from sub-optimal location. For further improvement in the exploitation of the global solution and to increase search efficiency, Opposition-based learning has been integrated to collect elite solutions during the selection phase. This novel competition among updated mutation, crossover and selection operations in CCDE strategy enhances global solution accuracy, convergence rate and protects the DE algorithm from premature convergence. The proposed CCDE algorithm is tested on two hydrothermal systems with varied dimensions to observe the global search efficiency of the algorithm. The results compared with conventional DE, Chaotic DE and other heuristic methods reported in the literature have indicated that proposed algorithm is better than the compared methods in terms of minimum cost and high unified cardinality value that justifies better global solution capability of CCDE. The results also show that the convergence rate of proposed CCDE in both test system problems is better than DE and CDE. The proposed CCDE algorithm has powerful optimisation capability in the benchmark functions with the more complex landscape as well.

In the second phase of solution methodology, sine cosine algorithm has been embedded to adaptive differential evolution sine-cosine algorithm (CCDESC). The modified strategy supports the exploration of search space around the best solution position and has exhibited its capability in maintaining the best solution repeatedly. The successful application of CCDE and CCDESC to twenty benchmark test functions of varied nature and hydrothermal generation scheduling problems is well demonstrated and indicates that the CCDE has a great potential in solving large scale complex constrained optimization problems at the cost of time. Though the number of function evaluations per iteration of CCDE is more than the convention DE and CDE methods despite this it achieves a solution in a lesser total number of evaluations ensuring its good search efficiency. On the other hand, the comparison of CCDE with CCDESC through numerical results and convergence trajectories confirm the stability and effectiveness of two strategies in the majority of simulation studies. The non-parametric tests measurements justify the robustness of the suggested algorithms.

CHAPTER - 4

SELF-ADAPTIVE DIFFERENTIAL EVOLUTION AND SIMPLEX HYBRID APPROACH FOR MULTI-OBJECTIVE HYDROTHERMAL GENERATION SCHEDULING

4.1 INTRODUCTION

The search capability of classical differential evolution (DE) has been reviewed by Das *et.al.*, (2016) and is attributed its performance to cooperation and competition between individual vectors, parameter selection, differential mutation, crossover and greedy decision, but suffers from weak local search. This shortcoming of DE leads to poor convergence speed and premature convergence. Several variations from various aspects of DE have been investigated by researchers to enhance its performance such as the adaptive ranking of mutation operator (Gong *et al.*, 2015), economic load dispatch optimization with DE incorporating time-varying mutation in DE by Pandit *et al.*, (2015), neighbourhood guided search in DE (Cai *et al.*, 2017), multi-search DE with parallel subpopulation evaluation (Xiangto *et al.*, 2017), multi-role aspects of DE by Gui, *et al.*, (2019). Wu *et al.*, (2018) have reviewed successful variations in DE. Ghosh *et al.*, (2020) proposed two-phase DE with success history-based parameter to fine-tune search in promising landscapes identified in the first phase of search and many more reviewed in the literature summarized in Chapter 1 under hybrid optimization techniques. It is observed from the review of literature that DE and its improved versions are extended to solve NP-hard optimisation problems and reasonably dominate among a variety of stochastic evolutionary algorithms as a better global optimiser on account of its simplicity and specific memory ability to track the current search. With this extensive literature review, there are still many queries that are unanswered like the selection of mutation constant and crossover rate, population size, elite selection mechanism for a global solution. Hence, improving the global search solution of a combinatorial and numerical optimisation without impairing the convergence speed, is continually a challenge for optimization algorithms.

In computational intelligence, there is an extensive account of hybrid algorithms to solve hard optimisation problems. Several methods for global and local search have been integrated into DE framework to enhance the capability of search in potential regions. Su and Lee, (2003) proposed a mixed-integer programming hybrid with DE to solve network configuration problem of the electrical distribution system. Zhang et. al (2006) have reported hybridisation of DE with GA for crossover operation and tested the adaptive GA for variety of multi-objective optimisation problems. Noman and Iba, (2008) have crossover based adaptive local search method to enhance the capability of DE. In this hybridisation, the length of the local search method is adjusted using a hill-climbing heuristic. Yuan, *et al.*, (2008) have used three different chaotic operations hybridised with DE for optimising the cost of hydrothermal systems. Sivsubramani and Swarup, (2011) combined sequential quadratic programming with DE to solve non-convex HTGS problem. Wang *et al.*, (2011) proposed Nelder and Mead algorithm for parameter identification of chaotic systems in DE. Basu (2011) used DE for multi-objective optimisation of the hydrothermal system and applied non-sorting GA for generating Pareto solutions. Chakarborty *et al.*, (2012) have proposed a modified harmony search hybridised with modified DE mutation operator to solve combined economic emission dynamic dispatch problem. Vakula and Sudha, (2012) proposed DE for tuning of optimal parameters of fuzzy logic controller designed for power system stabilizer and reported saving in evaluation time. Yildiz, (2013) opted Taguchi method for parameter calibration in multi-pass turning operation in machining and milling production cost has been optimised using DE. Chandrasekaran *et al.*, (2014) have proposed a hybrid cuckoo search method to solve emission reliable economic multi-objective dispatch problem. Tvrdik and Krivy, (2015), have combined DE with k-means clustering technique for classification of objects. Singh *et al.*, (2017) have proposed chaotic DE hybridised with Powell search method to solve multi-fuel thermal unit dispatch problem. Mustafa *et al.*, (2018) have proposed chaotically tuned parameters of the local search method for optimal allocation of distributed generators. Chaabani *et al.*, (2019) have proposed decomposition method for the partitioning of population and all subpopulations are evolved simultaneously for bi-objective optimisation. The interaction among the populations supports the optimisation of the convergence speed of the algorithm. Researchers have explored different local search methods combined with differential evolution to solve different types of optimisation problems successfully to an acceptable extent.

In this chapter, a hybrid self-adaptive differential evolution simplex search algorithm is investigated to provide an optimal solution to constrained multi-objective short-term hydrothermal generation scheduling (HTGS) problem. The simplex method is a fast, direct and derivative-free search method with strong local optimisation ability. The probability of having located global optimum increases in the local search methods by multiple starts or by random restarts from different points repeatedly. Secondly, the three operations; reflection, expansion and compression of simplex method initialise the new simplex from the current best vertex and improves the position of the worst solutions. The accuracy of the elite solution is improved by invoking simplex technique for neighbourhood search. The iterative implementation of these operations guides the search towards a global solution with improved convergence speed and accuracy has motivated the authors to hybridise DE with the simplex method. Variable elimination approach with heuristic repair strategy is applied to handle equality constraints. Further trapezoidal fuzzy model is recommended to handle unpredicted residues in violations of operational constraints; those result in due to the stochastic nature of variables. Residues are fuzzy quantified and are considered as an objective to be optimised. The best-compromised solution of three interdependent objectives of HTGS problem is achieved by an interactive fuzzy decision model. The effectiveness of the proposed algorithm is established through illustrations on benchmark functions as well and is substantiated through statistical significance tests. The numerical results reveal improvement in the unified satisfying objective function and competitive convergence performance metrics over the existing methods.

4.2 MULTI-OBJECTIVE HYDRO-THERMAL GENERATION SCHEDULING PROBLEM

The short-term multi-objective hydrothermal generation scheduling problem aims to obtain the hourly optimal generation schedule of a constrained multi-chain hydrothermal system over the time horizon of 24 hours in a multi-objective framework. The generation cost and harmful emissions from thermal units are to be minimised simultaneously while satisfying the physical and technical constraints of the hydrothermal system. Physical constraints include multi-reservoir water flow rate, water transport delay due to series and parallel cascading of hydro units, storage capacity of reservoirs, water discharge rate and major technical constraints include loading capacity of hydro and thermal units and energy balance of the system and forecasted time-varying load demand. Thermal unit active power output and hydro unit's water discharge are considered as decision variables and the power output of the hydro unit is taken as the dependent variable in the feasible search space of the problem.

Mathematically, the multi-objective hydrothermal generation scheduling problem is stated as an optimization problem and the objective functions and constraint are formulated using mathematical models of hydro and thermal units (Dhillon and Kothari, 2010) as stated below:

Objective Functions

Minimize operating cost:

$$F_1 = \sum_{k=1}^{T}\left(\sum_{i=1}^{N_T}\left(a_i P_{ki}^2 + b_i P_{ki} + c_i + \left|d_i sin\left\{e_i\left(P_i^{min} - P_{ki}\right)\right\}\right|\right)\right) \tag{4.1}$$

Minimize Emission of Pollutant:

$$F_2 = \sum_{k=1}^{T}\left(\sum_{i=1}^{N_T}\left(\alpha_i P_{ki}^2 + \beta_i P_{ki} + \gamma_i + \eta_i exp(\delta_i P_{ki})\right)\right) \tag{4.2}$$

The active power generation of each hydro unit is a function of water discharge rate, reservoir volume and net hydraulic head and is computed using the following relation

$$P_{kj+N_T} = C_{1j}V_{kj}^2 + C_{2j}q_{kj}^2 + C_{3j}V_{kj} + C_{4j}q_{kj} + C_{5j}V_{kj}q_{kj} + C_{6j}$$

$$(j \in [1, N_G], k \in [1, T]) \qquad (4.3)$$

Subjected to equality and inequality constraints, defined in chapter 3.

Equality Constraints

(i) To ensure power balance, equality constraint during each subinterval is imposed (Equation 3.4).

(ii) The volume dynamics in the hydro reservoir (Equation 3.6a)

(iii) The hydraulic (storage) constraint for the hydro unit reservoir at the end of the planning period (Equation 3.6b)

Inequality Constraints

(i) Active power generation limits on each hydro and thermal unit (Equation 3.7a)

(ii) Water discharge rate limits on hydro units (Equation 3.7b)

(iii) Reservoir volume limits on hydro units (Equation 3.7c)

4.2.1 Non-Inferior Solution Set Reformation

The Pareto front set obtained from the feasible solutions is modified by incorporating non-dominated solutions generated in each generation. The imprecise nature of DM's judgement, the best- compromised solution among Pareto solutions of the conflicting objectives of the problem are transformed to linear membership function fuzzy representation by considering extremum of objective functions as stated in Equation (4.4): $\mu(F_n(P)) =$

$$\begin{cases} 0 & F_n(P) \leq F_n^{min} \\ \frac{F_n(P)-F_n^{min}}{F_n^{max}-F_n^{min}} & F_n^{min} \leq F_n(P) \leq F_n^{max} \\ 1 & F_n(P) \geq F_n^{max} \end{cases} \quad (n = 1,2, \dots, M) \tag{4.4}$$

where F_n^{min} and F_n^{max} are maximum and minimum values of the n^{th} objective function obtained in the trial runs. The objective function is accepted if its membership value is 1 and is rejected otherwise.

The behaviour of membership function $\mu(F_n(P))$ is presented in Figure 4.1.

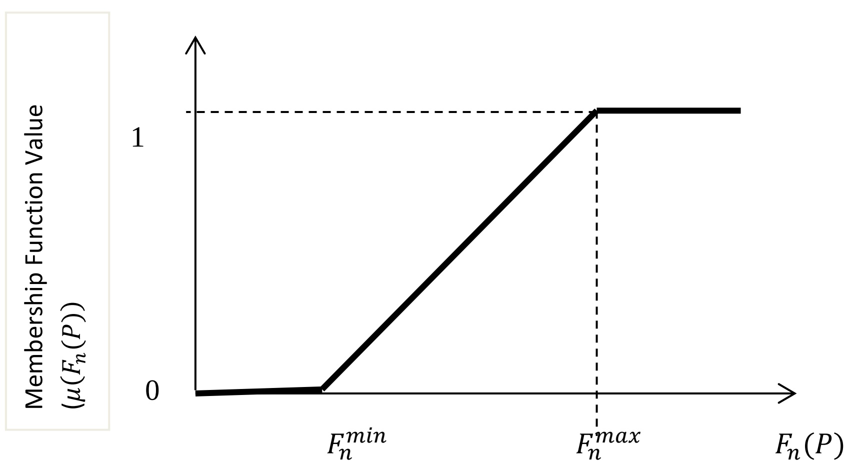


Fig.4.1: Objective function fuzzy behaviour

The overall objective function of the hydrothermal generation problem is the unification of thermal unit objectives and constraint function. The degree of satisfaction of objectives is the maximum value of all membership value of three objectives. This is achieved through a max-min composition model of decision making under uncertainty environment for hydrothermal system and is defined as:

$$\mu_D = max\left(min\left(\mu(F_1), \mu(F_2), (1-\mu(F_C)\right)\right) \tag{4.5}$$

where F_1 is the cost objective function, F_2, is the objective function representing gaseous pollutant emission from thermal units and F_C, is the objective function representing constraints. μ_D, represents the unified objective function called cardinal priority ranking and its value is used for decision making.

The equality constraints are non-linear in the constrained multi-objective HTGS problem. In this research work, the variable elimination method is used for satisfying equality constraints. This method reduces the number of decision variables and the number of constraints as well.)

4.2.2 Fuzzy Modelling of Handling Constraint Violation

During equality constraint handling variable elimination procedure, the modification in water discharge of hydro unit can affect hydropower and reservoir volume coupled with water continuity dynamics imbalance and also system load balance violation together with modification in thermal power. The limits of reservoir volume, generation of units other than the slack generating unit in each subinterval and final reservoir storage at the end of the scheduling period are systematically checked individually and adjusted within prescribed limits. The computed slack dependent variable x_{ks} in the sub-interval is taken equal to its nearest boundary if it violates its respective boundary limits expressed as:

$$x_{js} = \begin{cases} x_s^{min} & ; \text{ if } x_{js} < x_s^{min} \\ x_s^{max} & ; \text{ if } \mathrm{x_{js}} > x_s^{max} \\ x_{js} & ; \text{ if } x_s^{min} \le \mathrm{x_{js}} \le x_s^{max} \end{cases} \qquad (j \in [1, T]) \tag{4.6}$$

This procedure of satisfying limits violation in slack decision variables using Eq. (4.6) cannot avoid violation of the coupled constraints due to stochastic selection of variables and it may leave a residue. However, these independent adjustments generate an infeasible solution as the search for the optimal solution of a hydrothermal system with cascaded multi-reservoir hydro unit proceeds. This statistical uncertainty in violation of equality constraints and readjustment of decision variables to satisfy inequality constraints in each sub-interval is a time-consuming effort.

In this study, the trapezoidal fuzzy approach is recommended to resolve the unpredictable residue of violations. These residues are quantified with their membership functions, defined within acceptable limits of tolerance in lower and upper bounds as applicable to the variables of the hydrothermal problem. In each fuzzy membership function, the zero value of the membership function gives full violation of the constraint whereby one value of the membership function gives no violation of constraint is taken for study. Membership functions of inequality constraints are represented by trapezoidal function as shown in Fig.4.2 and 4.3 respectively. Mathematically, membership function, $\mu(x_{ki})$ of variable x_{ki} is represented as below:

$$\mu(x_{ji}) = \begin{cases} 0 & ; (x_i^{max} + \varepsilon) \le x_{ji} \le (x_i^{min} - \varepsilon) \\ \dfrac{(x_{ji} - x_i^{min} + \varepsilon)}{\epsilon_{V_{ki}}} & ; (x_i^{min} - \varepsilon) < x_{ji} < x_i^{min} \\ \dfrac{(x_i^{max} - x_{ji} + \varepsilon)}{\varepsilon} & ; x_i^{max} < x_{ji} < (x_i^{max} + \varepsilon) \\ 1 & ; x_i^{min} \le x_{ji} \le x_i^{max} \end{cases}$$

$$(i \in [1, N_G]; j \in [1, T]) \qquad (4.7)$$

where ε is the tolerance to decide the satisfaction level of the lower and upper limits of the variable during j^{th} sub-interval for i^{th} unit assumed in the range of 3-5%.

Active power generation limits on each hydro and thermal units, and reservoir volume limits on hydro units, are treated as inequality constraints.

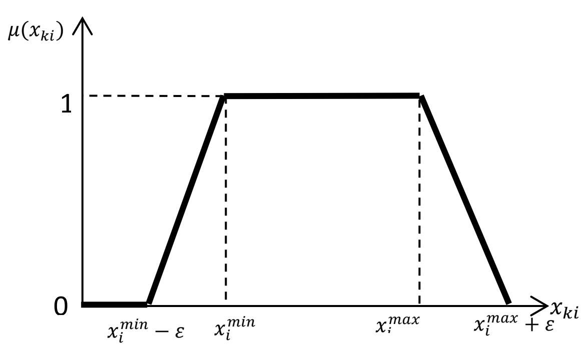


Fig. 4.2 Fuzzification of inequality constraint

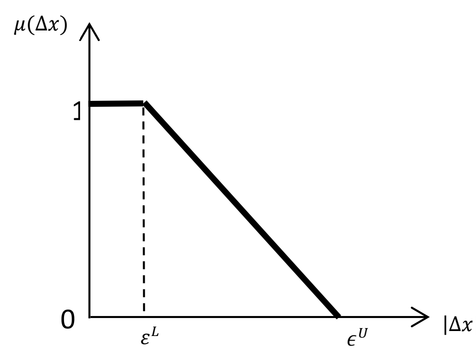


Fig. 4.3uzzification of equality constraint

Membership functions of equality constraints are represented by linearly monotonic decreasing function as shown in Figure 4.3 Mathematically, membership function, $\mu(\Delta x)$, of deviation in the equality constraint, $|\Delta x|$ is represented as:

$$\mu(\Delta x_i) = \begin{cases} 0 & ; |\Delta \mathrm{x_i}| \geq \varepsilon^U \\ \dfrac{(\varepsilon - |\Delta \mathrm{x_i}|)}{(\varepsilon^U - \varepsilon)} & ; \varepsilon < |\Delta \mathrm{x_i}| < \varepsilon^U \\ 1 & ; |\Delta \mathrm{x_i}| \leq \varepsilon \end{cases} \qquad (i \in [1, N_G]) \tag{4.8}$$

where $|\Delta \mathrm{x_i}|$ is a deviation in meeting equality equation and ε is tolerance of error in satisfying the equality constraint of i^{th} variable for the scheduled time. ε^U is the upper range to decide the satisfaction level of the equality constraint of i^{th} variable in the T scheduling time-period.

Total reservoir volume of the hydro unit, $|\Delta \mathrm{V_i}| = |V_{(T+1)i} - V_{1i}|$ at the end of the scheduling period and violation in power balance mismatch, $|\Delta P_{Dj}| = \left|\sum_{i=1}^{\mathrm{N}_G} P_{ji} - P_{Dj} - P_{Lj}\right|$ are the equality constraints. All the membership functions are unified to act as the third objective to be minimized and are stated below:

$$F_{\mathrm{C}} = P_f \left((1 - \mathrm{S}_T)^2 + (1 - S_V)^2 + \left(1 - S_{P_D}\right)^2 + (1 - S_{PH})^2 + (1 - S_{PT})^2 \right) \tag{4.9}$$

where P_f is the exterior penalty factor with a high value of the order of 10^3 to 10^5.

The overall membership function, S_T of violations in the total reservoir volumes utilized by hydro units at the end of scheduling period, is defined by average membership functions of the residues (unutilized water volume of reservoirs) of hydro units and is stated below:

$$\mathrm{S}_T = \frac{1}{\mathrm{N}_H} \left(\sum_{\mathrm{i=1}}^{\mathrm{N}_H} \mu(\mathrm{V_{Ti}}) \right) \tag{4.10}$$

where, $\mu(\mathrm{V_{Ti}})$ is the membership function for violation in the total reservoir volume of the hydro unit at the end of the scheduling period is defined by a monotonic decreasing function.

The average of membership functions of the violations of reservoir limits over the scheduled period of all the hydro units acts as the overall membership function, S_V of violation in the volume of water in for each reservoir in all sub-interval and is stated below:

$$S_V = \frac{1}{\mathrm{N}_H}\left(\sum_{i=1}^{\mathrm{N}_H}\left(\frac{1}{T}\sum_{j=1}^{T}\mu\left(V_{ji}\right)\right)\right) \tag{4.11}$$

where $\mu(\mathrm{V_{ki}})$ is the membership function for violation of available water constraint of the hydro unit at the end of the interval, is defined by trapezoidal function

The overall membership function, S_{P_D} that gives violation in load demand balance equation and is expressed by the average membership functions of the violation of power demands over the scheduling period as:

$$S_{P_D} = \frac{1}{T}\sum_{k=1}^{T}\mu(\Delta P_{Dk}) \tag{4.12}$$

where $\mu(\Delta P_{Dk})$ the membership function for violation in power balance mismatch is stated by a monotonic decreasing function

The average membership function, S_{PH} accounts for violation in the active power output from hydro unit, and is defined by the average of membership functions of the violations of hydro generation limits over the scheduling period of all the hydro units

$$S_{PH} = \frac{1}{T}\left(\sum_{j=1}^{T}\left(\frac{1}{\mathrm{N}_H}\sum_{i=1}^{\mathrm{N}_H}\mu(P_{ji})\right)\right) \tag{4.13}$$

where $\mu(\mathrm{P_{ji}})$ is the membership function for violation of limit of active power from i^{th} hydro unit in j^{th} sub-interval is defined by monotonic decreasing function.

The overall membership function, S_{PT} of the violation in the active power output of thermal units is defined as the average of membership functions of the violations of thermal generation limits over the scheduling period:

$$S_{PT} = \frac{1}{T}\left(\sum_{j=1}^{T}\left(\frac{1}{\mathrm{N}_T}\sum_{i=1}^{\mathrm{N}_T}\mu(P_{ji})\right)\right) \tag{4.14}$$

where $\mu(\mathrm{P_{ji}})$ is the membership function for the violation in the active power output of an i^{th} thermal unit, defined by trapezoidal function

The value of $S_T, S_V\ S_{P_D}, S_{PH}$ and S_{PT} vary from 0 to 1. '1' value means the constraints are satisfied. The constraint function, F_C , as stated in Equation (4.9) is taken as an objective to be minimized and is converted to maximize its value within the range 0 and 1, expressed as:

$$\mu(F_C) = \frac{1}{1 + (F_C/P_f)} \tag{4.15}$$

4.3 SOLUTION METHODOLOGY

To improve the balance between diversification and exploitation of stochastic DE approach, a hybrid SaDESM algorithm is adopted for the investigation. In the first phase, Chaotic DE (CDE) hybridised with Simplex method is considered and is applied to solve benchmark test functions and hydrothermal systems. The algorithm does not show competitive behaviour on convergence trends, convergence speeds and solution obtained on the given set of examples. The shortcomings of these algorithms are alleviated by adding three elite strategies as:

First: Implementation of crisscross mutation and crossover, that offers information exchange in three-dimensional hyperspace,

Second: Use of opposition based learning to archive the current best solution and

Third: Hybridised with simplex local search method to expedite the global search

The performance of the proposed hybrid DE-simplex algorithm is envisaged for a better global solution.

Initialization of Population: To perform hydrothermal generation scheduling, active power generation for thermal units ($P_{ji};\ j = 1,2,\dots,T;\ i = 1,2,\dots,\mathrm{N}_T$) and the water discharge rate for hydro units ($Q_{ji};\ j = 1,2,\dots,T;\ i = 1,2,\dots,\mathrm{N}_H$) during each sub-interval are searched using the proposed SaDESM algorithm. For hydrothermal system, j^{th} member of the population of problem decision variables is represented in matrix form as:

The initial population is randomly generated which is uniformly distributed over the problem space bounded by limits of decision variables. The population of size $(N_P \times T \times N_G)$, is generated in the feasible space of the hydrothermal problem using the following equation.

$$x_{kji}^0 = x_i^{min} + R_{kji}(x_i^{max} - x_i^{min}) \qquad (i \in [1, N_G]; j \in [1, T]; jk \in [1, N_P\ ]) \tag{4.17}$$

where R_{jki} is a uniform random number that varies between 0 and 1.

The fitness function for j^{th} member of the population is stated as:

$$F_j(x_{kji}) = \mu_D(x_{kji}) \qquad (k = 1,2,\dots,N_P) \qquad (4.18)$$

Chaotic Strategy: Logistic mapping, is applied to generate a chaotic sequence of mutation constant and crossover rate within a specified boundary as stated in Equation (3.33) and (3.37) respectively.

Crisscross Strategy: The double search strategy supports the generation of qualified solutions and hence eliminates the comparatively weak solutions from a parent population.

Horizontal Crisscross Mutation: The horizontal crisscross mutation is performed over the different decision variables of an individual population to generate mutant vector according to the following equation:

$$H_{kji}^t = \mathcal{X}_{Bji}^t + F^t(\mathcal{X}_{Bji}^t + \mathcal{X}_{kji}^t - \mathcal{X}_{kr_1r_3}^t - \mathcal{X}_{kr_2r_4}^t)$$

$$(i \in [1, N_G] \text{ and } r_3 \neq r_4 \neq i;\ j \in [1, T];\ r_1 \neq r_2 \neq j; k \in [1, N_P\ ]) \qquad (4.19)$$

where X_{Bji}^t is the best individual in the current generation and X_{kji}^t is base vector r_1 *and* r_2 are randomly selected population from j set of time intervals and r_3 *and* r_4 are randomly selected from a set of decision variables for k^{th} population.

Vertical Crisscross Mutation: The vertical mutation is performed over the different population in different time intervals for a given decision variable according to the Equation (4.21):

$$V_{kji}^t = \mathcal{X}_{Bji}^t + F^t(\mathcal{X}_{Bji}^t + \mathcal{X}_{kji}^t - \mathcal{X}_{r_1r_3i}^t - \mathcal{X}_{r_2r_4i}^t)$$

$$(\forall i \in N_G; \forall j \in T;\ r_3, r_4 \neq j;\ \forall k \in N_P\ ;\ r_1, r_2 \neq k\ ) \qquad (4.20)$$

where r_1 *and* r_2 are randomly selected population from an N_P set of population and r_3 *and* r_4 are randomly selected sub-interval from the set of j intervals of the scheduling period for i^{th} variable.

Horizontal Crisscross Crossover: The horizontal crossover exchange information among the decision variables and subintervals of a population. The horizontal binomial crossover is defined below to generate variable, U_{kji}^t.

$$U_{kji}^t = \begin{cases} H_{kji}^t\ ; if\ (r_i < CR^t) or\ (j = R1)\ or\ (k = R2) \\ \mathcal{X}_{kji}^t \quad ;\ otherwise \end{cases} \quad (i \in [1, N_G]; j \in [1, T];\ \ k \in [1, N_P]) \quad (4.21)$$

The random integer numbers are generated such that $r_i \in [1, N_G], R1 \in [1, T]$ and $R2 \in [1, N_P]$.

Vertical Crisscross Crossover: The vertical crossover exchange information among the population and subintervals. The vertical binomial crossover is defined below to generate variable, W_{kji}^t.

$$W_{kji}^t = \begin{cases} V_{kji}^t & ; if\ (r_i < CR^t) or\ (i = R1)\ or\ (j = R2) \\ \mathcal{X}_{kji}^t & ;\ otherwise \end{cases}$$

$$(i \in [1, N_G]; k \in [1, T];\ j \in [1, N_P]) \qquad (4.22)$$

where the random integer numbers are generated such that $r_i \in [1, N_P], R1 \in [1, N_G]$ and $R2 \in\ [1, T]$

Selection: To decide whether the vector $H_{kji}^t, V_{kji}^t, U_{kji}^t\ or\ W_{kji}^t$ should be a member of the population comprising the next generation, greedy selection strategy search for the best vector$\mathcal{X}_{jki}^t$, is applied to speed up the convergence.

$$\mathcal{X}_{kji}^{t+1} = \begin{cases} y_{kji}^t & ; if\ f_j^t\left(y_{kji}^t\right) > F_k^t\left(\mathcal{X}_{kji}^t\right) \\ \mathcal{X}_{kji}^t & ;\ otherwise \end{cases} \qquad ; (k = 1,2 \dots N_P) \qquad (4.23)$$

where $f_k^t\left(\gamma_{kji}^t\right) = \max\{F_k^t\left(H_{kji}^t\right), F_k^t\left(V_{kji}^t\right), F_k^t\left(U_{kji}^t\right), F_k^t\left(W_{kji}^t\right)\}$ and its corresponding best population is $\gamma_{kji}^t \quad ; \quad \forall i \in N_G; \forall j \in T; \forall k \in N_P$

The fitness value, $F_k^{t+1}\left(\mathcal{X}_{kji}^{t+1}\right)$ is evaluated corresponding to population $\mathcal{X}_{kji}^{t+1}$; $\forall i \in N_G; \forall j \in T; \forall k \in N_P$

Opposition Learning-based Selection: For non-dominated solutions generated from populations by crisscross mutation, crisscross crossover and selection, the elite individuals are selected from the unification of the current-best population and opposite population using Equation (3.40) and (3.41) and are retained.

4.3.1 Simplex Search Method

Nelder and Mead introduced a simplex search algorithm as a derivative-free direct search method that makes it quite suitable to solve multi-dimensional, non-smooth and discontinuous functions based constrained problems. In an $N-$dimensional search space, a simplex is a $(N + 1)$ polytope of (N+1) vertices, such that each vertex represents a decision variable x_i; $i = 1,2, \dots, (N + 1)$. The point is designated a *base point* from which a pattern move is initiated and the direct search procedure is conceived as fundamentally proceeding from one base point to a new point in search space which has a desirable value of the objective function and replaces least desirable point in a feasible set of solutions.

This method is comparable to the evolutionary technique in terms of selection of $(N+1)$ individuals and replacement strategy (Wang *et.al*, 2011)

The selection operation is viewed with three special variations called transformations; namely reflection, contraction and expansion. It explores the space in the best forward direction with expansion and contraction operations and replaces the worst individual solution with a better solution until the best solution is obtained or termination criteria is met.

$$x_{ji}^{N}=\begin{cases}(1+\alpha)x_{kji}^{C}-\alpha X_{wji}^{t} & ;f\left(x_{ji}^{r}\right)<F_{b}^{t}\left(X_{bji}^{t}\right)\\(1-\beta)x_{kji}^{C}+\beta X_{wji}^{t} & ;f\left(x_{ji}^{r}\right)\geq F_{w}^{t}\left(X_{wji}^{t}\right)\\(1+\beta)x_{kji}^{C}+\beta X_{wji}^{t} & ;\ f\left(x_{ji}^{r}\right)<F_{h}^{t}\left(X_{hji}^{t}\right)\\x_{ji}^{r} & ;otherwise\end{cases}\quad (i\in[1,N_G];j\in[1,T];\ \ k\in[1,N_P])\qquad(4.24)$$

where Minimum, maximum and second to maximum function value are evaluated as $F_b^t\left(X_{bji}^t\right)=min\left\{F_k^t\left(\mathcal{X}_{kji}^t\right);\ k=1,2,\ldots,N_P\right\}$, $F_w^t(X_{wji}^t)=max\left\{F_k^t\left(\mathcal{X}_{kji}^t\right);\ j=1,2,\ldots,N_P\right\}$ and $F_h^t(X_{hji}^t)=max\left\{F_k^t\left(\mathcal{X}_{kji}^t\right);\ k=1,2,\ldots,N_P,k\neq w\right\}$, respectively. α and β are expansion and contraction factors. The reflected point and centroid are computed as:

$$x_{ji}^{r}=(1+\alpha)x_{ki}^{C}-\alpha X_{wki}^{t}\qquad (i\in[1,N_G];j\in[1,T])\qquad(4.25)$$

$$x_{ji}^{C}=\frac{1}{(NP-1)}\sum_{\substack{k=1\\k\neq w}}^{N_P}x_{kji}^{t}\qquad(4.26)$$

This pattern search improves the exploitation capability of search space at the cost of convergence speed on account of an increased number of function evaluations in the simplex call.

Termination Criterion: The iterative process terminates when the maximum number of evaluations of function is achieved or the maximum number iterations are performed. The number of function evaluations (NFE) is $(N_P+5\times N_P\times ITMAX)$, for SaDESM whereas conventional DE follows NFE=$(N_P+2\times N_P\times ITMAX)$.

4.4 HYBRID SELF-ADAPTIVE DIFFERENTIAL EVOLUTION-SIMPLEX ALGORITHM (SaDESM)

The Hybrid Crisscross Differential Evolution-Simplex Algorithm is elaborated as:

1 ***Input Data****: Parameter Set Up: Input population sizeNP, number of decision variables N_G, upper and lower limits of each decision variable X, Mutation constant F, crossover* *rate CR, convergence criteria NFE, Maximum number of iterations $ITMAX$, Maximum number of simplex iterations $ITSIM$.*

2 ***Initialization**: Initialize the value of i^{th} decision variable X in j^{th} population using Equation (4.17).*

3 *Set iteration count, $t = 0$*

4 ***Evaluate** the fitness function value $F_j^t(x_{kji}^t)$ using Equation (4.18) for every individual according to the objective function.*

5. *Select the global best solution, $F_G^t(x_{Gji}^t) = max\{F_k^t(\mathcal{X}_{kji}^t);\ k = 1,2,\ldots,N_P\}$ among the population members.*

6 *Set the updating counter for the best solution, $ibest = 0$*

WHILE** $(t < ITMAX)$ **DO

7 *Increment the iteration counter, $t = t + 1$*

***FOR** $k = 1, N_P$ (population size)*

6. ***Horizontal Mutation:** Perform horizontal mutation operation to generate a trial vector using the current-best mutation strategy to generate a trial vector, H_{kji}^t, using Equations (4.19).*

7 ***Vertical Mutation:** Perform vertical mutation operation to generate a trial vector using the current-best mutation strategy to generate a trial vector, V_{kji}^t, using Equations (4.20).*

6 ***Horizontal Crossover**: Apply horizontal binomial crossover or recombination operation to recombine the trial mutant vector and parent vector to produce offspring trial vector, U_{kji}^t using Equations (4.21).*

7 ***Vertical Crossover**: Apply vertical binomial crossover operation to recombine the trial mutant vector and parent vector to produce offspring trial vector, W_{kji}^t, using Equations (4.22).*

8 ***Selection:** Compute the fittest solution among the moderated solutions:*

$f_j^t(y_{kji}^t) = max\{F_k^t(H_{kji}^t), F_k^t(V_{kji}^t), F_k^t(U_{kji}^t), F_k^t(W_{kji}^t)\}$ *and save corresponding trial vector individual* y_{jki}^t $(i = 1,2,\ldots,N_G; k = 1,2,\ldots,T; k = 1,2,\ldots,N_P)$ *to* $f_k^t(y_{kji}^t)$

9. *Compute the individual for the next generation $t = t + 1$ using Equation (4.23)*

END FOR

10 *Select the best solution,* $F_B^{t+1}(X_{Bji}^{t+1}) = max\{F_k^{t+1}(\mathcal{X}_{kji}^{t+1});\ k = 1,2,\ldots,N_P\}$.

13 *Find the global best point.*

$$\mathcal{X}_{Gji}^{t} = \begin{cases} \mathcal{X}_{Bji}^{t+1} & ;\ if\ F_B^t(X_{Bji}^{t+1}) > F_G^t(\mathcal{X}_{Gji}^{t}) \\ \mathcal{X}_{Gji}^{t};\ ibest = ibest + 1 & ;Otherwise \end{cases}$$

14: ***IF*** $(\boldsymbol{MOD}(\boldsymbol{ibest}, \mathbf{5}) = \mathbf{0})$, ***THEN***

Apply opposition based learning with known F_{opp} *using Equation (3.40).*

ENDIF

15. *Arrange the obtained solution in ascending order and compute three points* x^b, x^w, x^h *as the best, the worst and the second-worst solution in the population as:*

16. *Set counter for simplex,* $g = 0$

WHILE $(g < ITSIM)$ ***DO***

17. *Increment counter for simplex,* $g = g + 1$

18. *Compute the base point as a centroid* x_{ji}^C *of all points except the worst point* x_{hji}^t*, using Eq.(4.26)*

Evaluate the objective function corresponding to the base point x_{ki}^C *of the decision variable.*

19. *Check and assign as:* $x_{jki}^C = \begin{cases} \mathcal{X}_{Bji}^{t+1} & ;\ if\ F_B^t(X_{Bji}^{t+1}) > F_C^t(\mathcal{X}_{ji}^C) \\ \mathcal{X}_{kji}^0; & ;Otherwise \end{cases}$

20. *Calculate the error:* $MSE = \|(F_k^t(X_{kji}^t) - F_B^t(X_{Bji}^t)\|$

21. *Check if* $MSE \leq 10^{-3}$ *then it concludes that similar to the previous best pattern is achieved then exit simplex procedure.*

22. *Calculate the reflection point* x_{kji}^r *of worst value about base point using Eq.(4.25)*

23. *Find the new point,* x_{ji}^N *using Eq.(4.24).*

24. *Check whether* x_{ji}^N *is better than the worst point* X_{wji}^t *, if* $\left(f(x_{ji}^N) < f(X_{wji}^t)\right)$ *then replace worst point with expansion point as* $X_{wji}^t = x_{ji}^N$*, else replace* $X_{wji}^t = x_{ji}^N$ *and go to step 17.*

END DO

END DO

Computational Complexity

The computational complexity of SaDESM algorithm depends upon the process of initialisation, fitness evaluation and mechanism of updating the variables. For NP population size of decision variables, initialisation complexity is $\mathcal{O}(N_P)$, space complexity is $\mathcal{O}(2 \times N_P \times D)$ and computational complexity of updating mechanism that includes the search of the best location and updating the decision variable vector for all generations is $\mathcal{O}(ITMAX \times N_P) + \mathcal{O}(ITMAX \times N_P \times D)$. However, the time computational complexity to search the best fitness value is $\mathcal{O}(ITMAX \times (N_P \times D + N_P \times FF))$, where $ITMAX\ is$ maximum number of iterations, D denotes the problem dimensions and FF is the fitness value of an objective function.

4.5.1.1 SIMULATION OF TEST PROBLEMS AND RESULTS

The numerical efficacy of the proposed SaDESM algorithm is evaluated by solving optimisation problems of two categories: benchmark test functions and real-world engineering problems. The quantitative performance analysis of each problem is estimated for its fitness value in terms of the best, average, the worst, the standard deviation (SD), whereas qualitative performance is observed from the convergence trends and the behaviour during exploration and exploitation phases of search in the relevant parametric space. The capability of the proposed algorithm is evaluated against well-established state-of-the-art meta-heuristic algorithms, reported in the literature for a fair comparison.

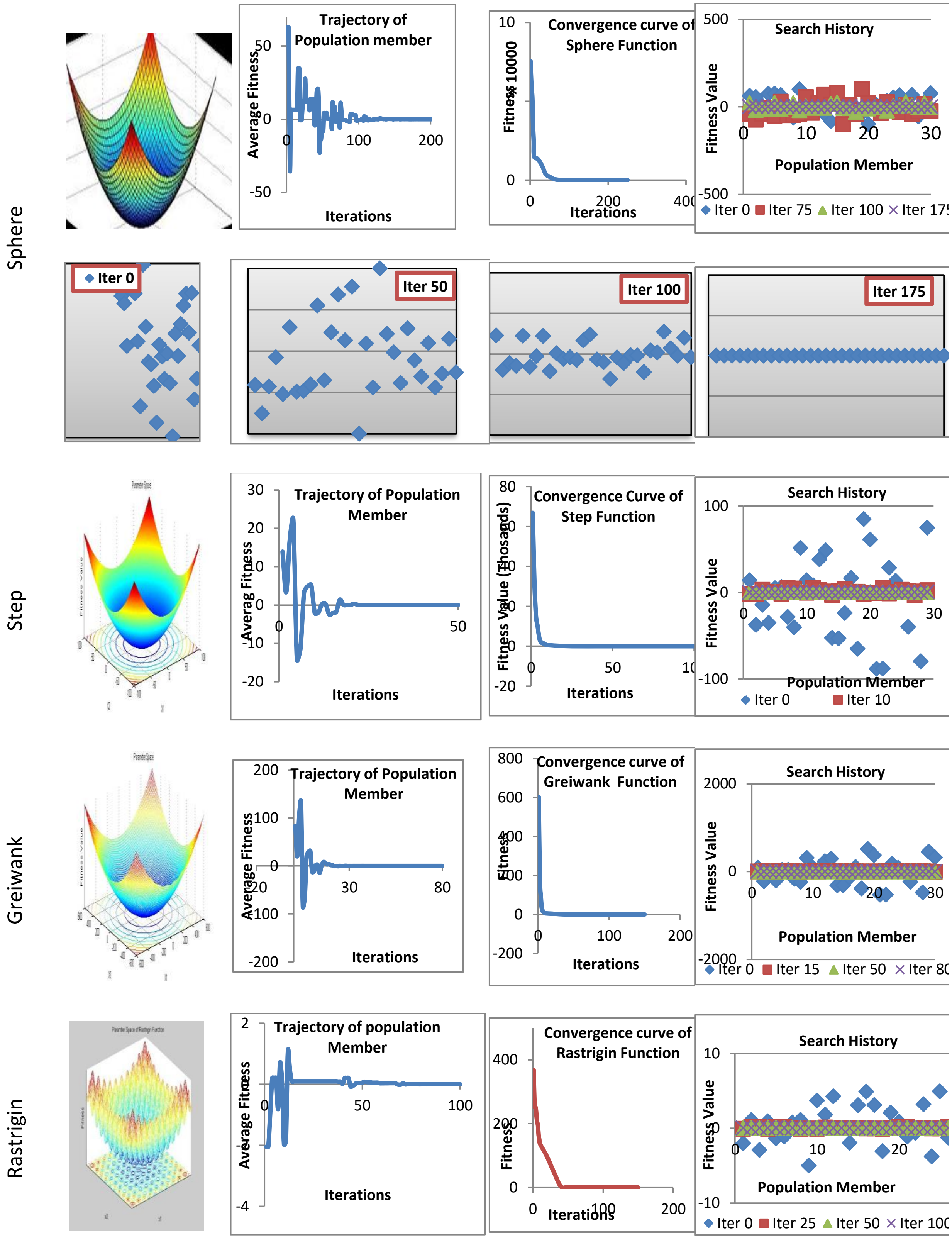
4.5.1 Benchmark Test Functions

In this chapter, the performance investigation of SaDESM is focussed on the convergence behaviour of a set of twenty diverse benchmark functions. Unimodal functions with unique global optima and no local minima are used to assess exploitation capability, on the other hand, multimodal functions have many local minima, and are used to assess diversification capability, with a potential to avoid the trap in local minima. However, the variants in the multimodal category such as fixed global optima but with many local minima like Sphere, Griewank functions, a flat surface landscape of Penalised functions, separable/non-separable functionality in Schwefel series, scalability of Weierstrass and Zakharov functions etc., pose a challenging environment to assess the ability of the proposed algorithm.

In the first phase, the qualitative results of randomly selected five benchmark functions are monitored over the thirty independent runs to assess the exploration and exploitation capability of algorithm and are demonstrated in Figure 4.4. For every run of all functions, the population size is taken as 50 and the maximum number of iterations is set to 1000. The results include performance metrics: search history, the trajectory of a population member, average fitness of population, and convergence behaviour. In each generation, the fitness is evaluated at the current location of each member and collection of these data points makes a database, called history.

Over the generations, the population member initially diversifies and explores the favourable locations of feasible solution space and then trends to the neighbourhood of the best locations. For sphere function optimisation, the details of search history after every 50-75 iterations are shown in Figure 4.4 as a case study. Trajectory map of population member shows the turbulence in the movement during the initial phase of search and then its improvement towards the best location in the current generation. The average fitness monitors the variation of fitness of the population over the complete run. The convergence behaviour supports the visualisation of the search procedure towards the target position. From the search history, the similar patterns of search by SaDESM algorithm for the test functions with diverse properties are observed. In all cases, the algorithm supports exploration during the initial phase and exploitation in the later phase of the search process. On the other hand, the trajectory map of population member supports the conclusion that as the search progresses, the amplitude of the fluctuations in the fitness decreases and eventually settles to a stable value. This means that SaDESM supports exploitation in the promising regions of space. The convergence curve in all different cases shows that the acceleration of the average fitness falls sharply within the first few iterations. Overall SaDESM reveals the dynamical shift from exploration to exploitation and accelerated convergence.

Further in this study, the quantitative results for twenty benchmark functions are obtained and are presented in Table 4.1. For robustness measure, mean fitness and the standard deviation are estimated.


Sphere
Trajectory of Population member
Average Fitness
Iterations
Convergence curve of Sphere Function
Fitness 10000
Iterations
Search History
Fitness Value
Population Member
Iter 0
Iter 75
Iter 100
Iter 50
Iter 175
Step
Trajectory of Population Member
Averag Fitness
Iterations
Convergence Curve of Step Function
Iterations
Search History
Fitness Value
Population Member
Iter 0
Iter 10
Greiwank
Trajectory of Population Member
Average Fitness
Iterations
Convergence curve of Greiwank Function
Iterations
Search History
Fitness Value
Population Member
Iter 0
Iter 15
Iter 50
Rastrigin
Trajectory of population Member
Average Fitness
Iterations
Convergence curve of Rastrigin Function
Search History
Fitness Value
Population Member
Iter 0
Iter 25
Iter 50

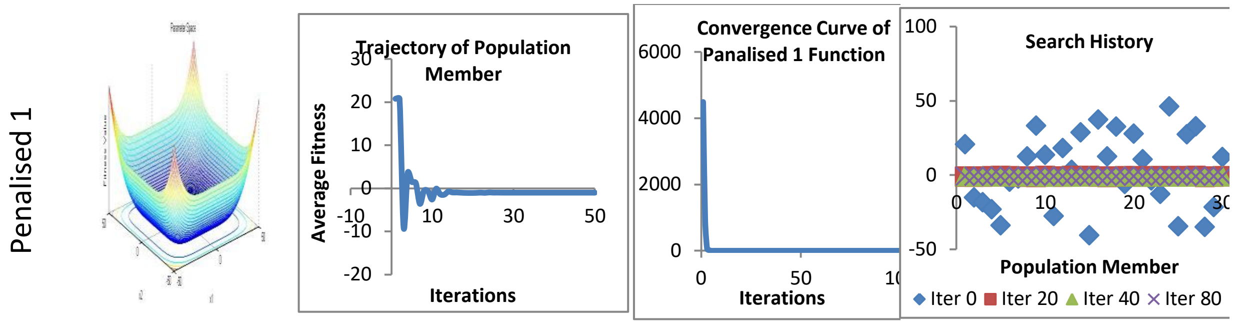


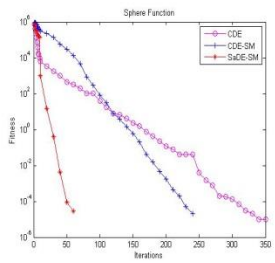
Fig. 4.4: Simulation results of benchmark functions

The number of function valuations (N_{FE}), is considered for estimating the efficiency of the algorithm. However, for calculating the success rate and acceleration rate, the fitness value-to-reach is taken as 10^{-15}. The acceleration rate indicates that the numbers of function evaluations for SaDESM are more than taken in CDE and CDESM to achieve the same value for thirty trial-runs in all functions. Also, SaDESM shows higher success-rate than CDE and CDESM showing its capability of maintaining a good balance between exploration and exploitation. Also, SaDESM maintains its performance consistently better even when realising complex multimodal functions. However, the total CPU time taken by SaDESM algorithm for the resulting number of evaluations is a remarkable achievement mark of efficiency in both unimodal and multimodal cases. The qualitative stability of the proposed algorithm is investigated using convergence characteristics as illustrated in Figure 4.5 for all test functions. The fitness value is taken on a semi-log scale for more clarity to conclude comparison with CDE and CDESM algorithms about the convergence trends towards the quality solution. It is evident from graphical results, that SaDESM exhibits superior exploitation capability for unimodal functions and indicates better exploration ability. SaDESM algorithm extends support to avoid the trap in local minima in multimodal functions as well.

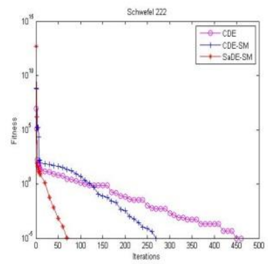
For a fair comparison of the efficacy of the proposed algorithm, the mean values and standard deviation of test functions are compared with the most utilised meta-heuristic stochastic optimisation algorithms is applied to evaluate benchmark function reported in the literature such as *Particle swarm optimiser (PSO), Genetic algorithms (GA), Flower pollination algorithm (FPA)* (Nevanth *et al., 2017), Cuckoo search algorithm (CSA),*

Table 4.1: Performance analysis of generalized benchmark test functions

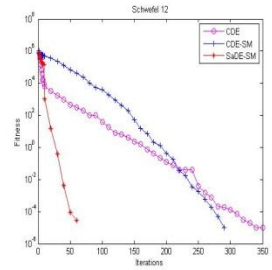
S. No.	Test Function	Type of Function*	Algorithm	NFE	Time (Sec)	Objective Function value				Statistics	% SR	AR
						Best	Average	Worst	SD	p-value		
1	Sphere	C, D, S, U	CDE	100050	2.16	3.14×10^{-31}	7.8×10^{-21}	1.93×10^{-20}	9.1×10^{-21}	<0.00001	100	Y
			CDESM	151440	4.24	3.147×10^{-40}	3.42×10^{-18}	3.46×10^{-17}	1.03×10^{-17}	0.01171	100	
			SaDESM	200190	3.79	0.0	0.0	0.0	0.0	NA	100	
2	Schwefel 2.22	C, D, NS, U	CDE	100050	1.71	1.51×10^{-17}	2.24×10^{-11}	5.04×10^{-11}	1.98×10^{-11}	<0.00001	82	Y
			CDESM	160840	4.15	1.559×10^{-22}	5.606×10^{-14}	$2..678\times 10^{-13}$	1.0×10^{-13}	0.000121	100	
			SaDESM	200136	3.01	0.0	0.0	0.0	0.0	NA	100	
3	Schwefel 1.2	C, ND, NS, U	CDE	100050	8.99	9.24×10^{-27}	1.43×10^{-19}	5.91×10^{-19}	2.28×10^{-19}	0.000026	100	Y
			CDESM	157713	15.17	8.57×10^{-41}	1.011×10^{-11}	5.06×10^{-11}	2.02×10^{-11}	0.000445	76	
			SaDESM	200189	22.27	0.0	0.0	0.0	0.0	NA	100	
4	Schwefel 2.21	C, ND, S, U	CDE	100050	17.21	3.42×10^{-3}	4.46×10^{-1}	1.91×10^{-2}	7.37×10^{-1}	0.000043	0.0	Y
			CDESM	166111	4.71	$2..718\times 10^{-8}$	8.3×10^{-4}	4.15×10^{-3}	1.66×10^{-3}	0.000312	0.0	
			SaDESM	200100	3.17	0.0	0.0	0.0	0.0	NA	100	
5	Rosenbrock	C, D, NS, U,	CDE	100050	4.88	11.59	15.78	21.78	4.05	0.00199	0.0	Y
			CDESM	150050	5.72	3.0	13.46	23.69	7.2344	0.000059	0.0	
			SaDESM	200050	6.36	0.0	20.38	27.2	10.26	<0.00001	12	
6	Step	NC, ND, S, U, SH	CDE	100050	2.06	0.0	0.4	1.0	0.489	<0.00001	60	Y
			CDESM	155778	4.09	0.0	0.0	0.0	0.0	NA	100	
			SaDESM	200243	3.20	0.0	0.0	0.0	0.0	NA	100	
7	Quartic	C, D, S, U	CDE	100050	2.17	7.05×10^{-3}	1.12×10^{-2}	1.74×10^{-2}	3.97×10^{-3}	<0.00001	0.0	Y
			CDESM	284960	8.99	1.18×10^{-3}	2.156×10^{-3}	3.081×10^{-3}	7.945×10^{-4}	<0.00001	0.0	
			SaDESM	335410	6.42	2.00×10^{-4}	4.15×10^{-4}	6.64×10^{-4}	1.78×10^{-4}	<0.00001	0.0	
8	Rastrigin	C, D, NS, M	CDE	100050	2.40	2.98	10.34	24.87	7.59	<0.00001	0.0	Y
			CDESM	179965	6.57	0.0	1.59	2.985	1.014	<0.00001	34	
			SaDESM	200309	3.56	0.0	0.0	0.0	0.0	NA	100	
9	Schwefel 2.26	C, D, S, M, SH	CDE	100050	3.71	2.37×10^{4}	1.04×10^{5}	1.67×10^{5}	4.67×10^{4}	<0.00001	0.0	Y
			CDESM	156862	4.23	1.07×10^{4}	1.42×10^{5}	1.66×10^{5}	2.31×10^{5}	0.000017	0.0	
			SaDESM	279336	5.60	-0.0125	-0.0125	-0.0125	0.0	NA	0.0	
10	Alpine	C, ND, S, M	CDE	100050	1.76	1.79×10^{-8}	7.18×10^{-7}	2.46×10^{-6}	7.79×10^{-7}	<0.00001	0.0	Y
			CDESM	168068	4.92	1.154×10^{-9}	2.023×10^{-7}	4.103×10^{-8}	1.792×10^{-7}	<0.00001	0.0	
			SaDESM	200139	5.22	0.0	1.23×10^{-15}	2.47×10^{-13}	1.4×10^{-14}	NA	64	
11	Griewank	C, D, NS, M, R	CDE	100050	2.51	2.22×10^{-16}	1.48×10^{-3}	7.39×10^{-3}	2.95×10^{-3}	0.000435	38	Y
			CDESM	150157	4.90	0.0	4.433×10^{-3}	0.0148	5.9×10^{-4}	<0.00001	100	
			SaDESM	200226	3.76	0.0	0.0	0.0	0.0	NA	100	
12	Ackley	C, D, NS, M, R	CDE	100050	2.64	2.41×10^{-11}	11.9	19.96	9.77	<0.00001	0	Y
			CDESM	155210	5.31	6.52×10^{-15}	3.418×10^{-5}	1.69×10^{-5}	6.75×10^{-6}	<0.00001	100	
			SaDESM	200211	3.57	5.88×10^{-15}	-5.88×10^{-15}	-5.88×10^{-15}	0.0	NA	100	
13	Penalised 1	C, D, NS, M	CDE	100050	2.96	8.06×10^{-15}	1.07×10^{-14}	2.14×10^{-14}	5.36×10^{-15}	0.000416	12	Y
			CDESM	157169	5.45	8×10^{-15}	4.9558×10^{-13}	2.39×10^{-12}	9.49×10^{-13}	0.000335	100	
			SaDESM	255817	5.43	8×10^{-15}	8×10^{-15}	8×10^{-15}	0.0	NA	100	
14	Penalised 2	C,D,NS,M	CDE	100050	3.85	7.64×10^{-15}	1.97×10^{-2}	9.89×10^{-2}	3.96×10^{-2}	0.000476	27	Y
			CDESM	154971	4.94	4.95×10^{3}	04.95×10^{3}	4.9598×10^{3}	1.464×10^{3}	<0.00001	100	
			SaDESM	276912	7.84	7.64×10^{-15}	7.64×10^{-15}	7.64×10^{-15}	0.0	NA	100	
15	Neumaier 3	C, D, NS, M, SH	CDE	100050	2.16	4.95×10^{3}	5.37×10^{3}	6.632×10^{3}	5.55×10^{2}	<0.00001	0	Y
			CDESM	158494	6.97	0.764	7.64×10^{-15}	7.64×10^{-15}	0.0	<0.00001	0	
			SaDESM	264670	5.56	4944	4944	4944	49	NA	0	
16	Salomon	C, D, NS, M	CDE	100050	2.25	2.99×10^{-1}	4.79×10^{-1}	5.99×10^{-1}	1.16×10^{-1}	<0.00001	0	Y
			CDESM	169329	8.40	0.1998	0.3198	0.5	0.116	<0.00001	0	
			SaDESM	269500	6.21	0.09	0.09	0.09	0.0	NA	0	
17	Levy	C, D, NS, M	CDE	100050	2.75	6.87×10^{-14}	6.87×10^{-14}	6.87×10^{-14}	0.0	NA	0.0	Y
			CDESM	155510	5.39	6.86×10^{-14}	5.28×10^{-14}	2.63×10^{-10}	1.05×10^{-10}	0.4994	0.0	
			SaDESM	284296	6.15	6.86×10^{-14}	6.86×10^{-14}	6.86×10^{-14}	0.0	NA	0.0	
18	Zakharov	C, D, NS, M	CDE	100050	2.15	2.97×10^{-11}	2.18×10^{2}	5.54×10^{2}	2.67×10^{1}	<0.00001	0.0	Y
			CDESM	150650	4.19	1.44×10^{-11}	6.528×10^{-2}	0.3263	0.131	0.000454	0.0	
			SaDESM	200140	3.69	0.0	0.0	0.0	0.0	0.5	100	
19	Expansion F10	C, D, S, M	CDE	100050	4.22	7.92×10^{-2}	2.33×10^{-1}	5.76×10^{-1}	4.89×10^{-1}	0.000739	0	Y
			CDESM	173120	9.03	4.38×10^{-8}	0.0489	0.1441	0.6146	0.28814	0.0	
			SaDESM	200131	4.90	0.0	0.0	0.0	0.0	0.5	100	
20	Weierstrass	C, D, S, M, SH, R	CDE	100050	15.16	-6.34×10^{-4}	-1.11×10^{-4}	1.43×10^{-3}	7.85×10^{-4}	<0.00001	0	Y
			CDESM	195520	32.22	-7.15×10^{-4}	-6.95×10^{-4}	6.738×10^{-4}	1.537×10^{-5}	<0.00001	0	
			SaDESM	219317	21.14	-7.42×10^{-4}	-7.42×10^{-4}	-7.42×10^{-4}	0.0	NA	0	



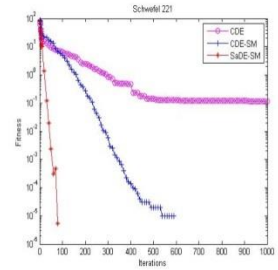
(a) Sphere



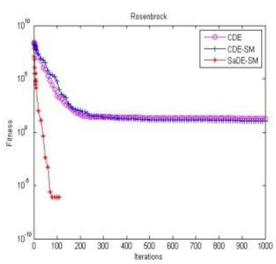
(b) Schwefel 2.22



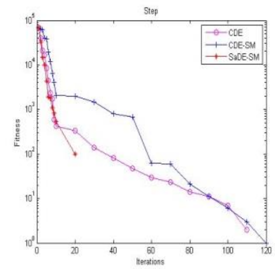
(c) Schwefel 1.2



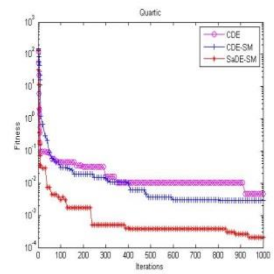
(d) Schwefel 2.21



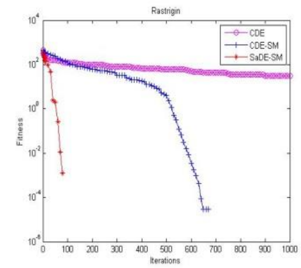
(e) Rosenbrock



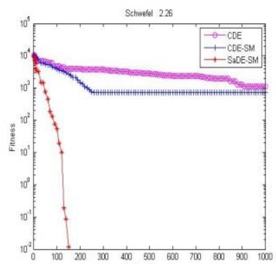
(f) Step



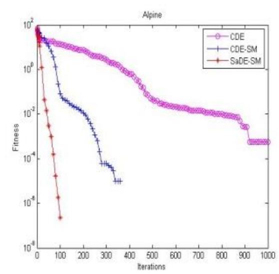
(g) Quartic



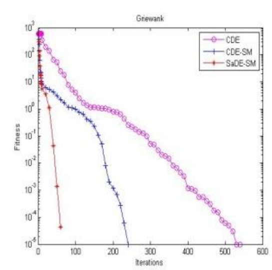
(h) Rastrigin



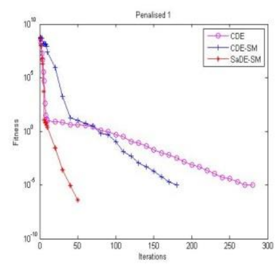
(i) Schwefel 2.26



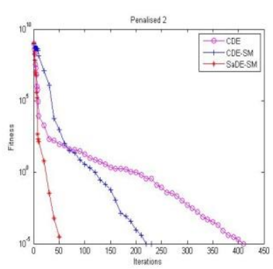
(j) Alpine



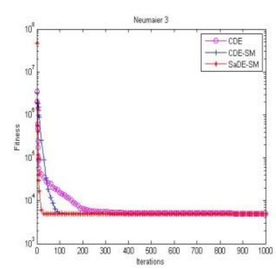
(k) Griewank



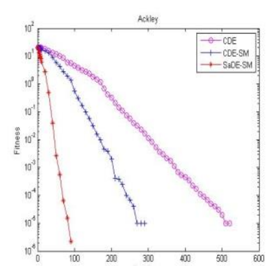
(l) Ackley



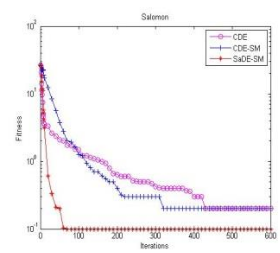
(m) Penalised 1



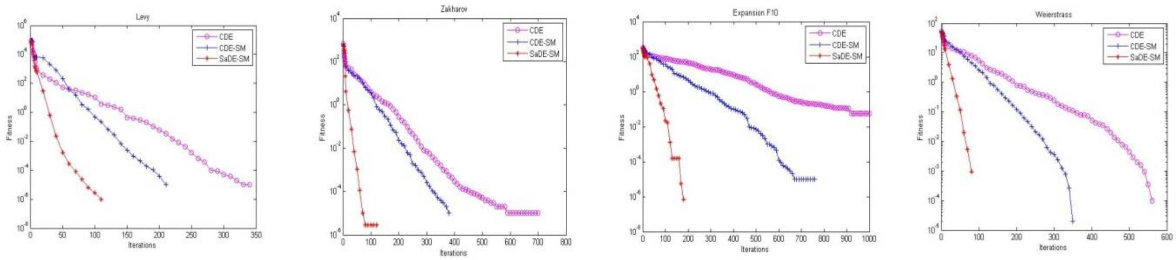
(n) Penalised 2



(o) Neumaier 3



(p) Salomon



(q) Levy Function (r) Zakharov (s) Expansion F10 (t) Weierstrass

Fig. 4.5: Convergence characteristics comparison of CDE, CDESM and SaDESM for benchmark test functions

Gravitational search algorithm (GSA), Grey wolf optimiser (GWO), Simulated annealing (SA), Elephant herding optimiser (EHO), hybrid PSO (SCA-PSO) and Henry gas solubility optimisation (HGSO) (Hashim *et al.*, 2019) and are summarised in Table 4.2.

Further to investigate the significant differences of SaDESM with results attained with other algorithms, non-parametric Wilcoxon's statistical test suggested by Derrac *et al.*. (2011) with a 5% degree of level of significance.

The null hypothesis H_0 and an alternative hypothesis H_A are defined as:

H_0: $\mu_1 = \mu_2$ and

H_A: $\mu_1 < \mu_2$

The paired wise comparison of means and standard deviations from the results of SaDESM algorithm and other compared algorithms is carried out to obtain p-values and are presented in Table 4.2.

It is found that SaDESM demonstrates better performance in finding the best solutions compared to other PSO, GA and FPA. It is worth to note that SaDESM gives a competitive solution to SCA-PSO and HGSO algorithms for solving unimodal and complex multimodal benchmark test functions. From the results of significance tests, it is concluded that in the majority of cases p-value is less than 0.05, so the null hypothesis stating the zero difference of means is rejected and performance of SaDESM gives superior results for unimodal and multimodal functions in competition with SCA-PSA and HGSO algorithms. These results strongly conclude that SaDESM has the capability of better exploitation and convergence to obtain a quality solution.

Table 4.2: Comparison of benchmark functions

Test Function	Metric	CS	EHO	GA	GSA	GWO	FPA	HGSO	PSO	SA	SCA-PSO	SaDESM
Sphere	Mean	2.4E-05	2.52E-06	0.8078	1.9E-17	3.75E-73	0.2111	0.0	6.37E-38	3.11E-07	0.0	**0.0**
	SD	8.96E-06	4.77E-07	0.4393	5.52E-18	5.47E-73	0.0717	0.0	3.49E-37	3.91E-07	0.0	**0.0**
	p-val.	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	NA	<0.0001	<0.0001	NA	**--**
Schwefel 2.22	Mean	1.00E+10	1.49E-02	0.5406	55.77	9.84E-40	0.9190	0.0	4.77E-02	4.27E-05	0.0	**0.0**
	SD	0.00	1.0E-03	0.2363	37.25	9.73E-40	0.7804	0.0	1.44E-01	4.41E-05	0.0	**0.0**
	p-val.	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	NA	0.0212	<0.0001	NA	
Schwefel 1.2	Mean	0.9201	1.49E-02	0.5323	2.28E-08	4.14E-40	0.2016	0.0	0.0157	4.3E-05	0.0	**0.0**
	SD	0.1684	8.59E-04	0.2423	3.66E-09	5.63E-40	0.1225	0.0	0.0158	6.13E-05	0.0	**0.0**
	p-val.	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	NA	<0.0001	<0.0001	1.0	
Schwefel 2.21	Mean	3.1308	1.20E-03	0.8837	3.30E-09	2.12E-17	0.8160	0.00	9.33E-02	3.19E-05	0.0	**0.0**
	SD	1.9807	9.64E-05	0.7528	6.98E-10	5.01E-17	0.5618	0.0	6.52E-02	3.81E-05	0.0	**0.0**
	p-val.	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	NA	<0.0001	<0.0001	NA	
Rosenbrock	Mean	1.5E-03	--	0.6677	--	26.244	0.0813	0.0	91.57	--	5.9438	**10.26**
	SD	2.7E-03	--	0.4334	--	0.70447	0.0426	0.0	74.50	--	0.3587	**20.38**
	p-val.	0.000839		0.001237		<0.0001	0.000633	0.0006	<0.0001		0.13752	
Step	Mean	0.00	0.00	0.7618	0.00	0.00	0.2168	0.0	0.267	12.9	0.0	**0.0**
	SD	0.00	0.00	0.7443	0.00	0.00	0.1742	0.0	0.868	1.65	0.0	**0.0**
	p-val.	NA	NA	0.1375	NA	NA	<0.0001	NA	<0.0001	<0.0001	NA	
Quartic	Mean	3.28E-02	3.60E-05	0.5080	1.98E-02	5.83E-04	0.3587	3.96E-05	1.98E-02	1.62E-03	1.5E-04	**4.15E-04**
	SD	1.09E-02	2.83E-05	0.1125	8.80E-03	3.32E-04	0.2104	2.47E-05	4.83E-03	4.56E-04	6.4E-04	**1.78E-04**
	p-val.	<0.0001	<0.0001	<0.0001	<0.0001	0.002782	<0.0001	<0.0001	<0.0001	<0.0001	0.006913	
Rastrigin	Mean	83.0	5.02E-04	1.0000	15.2	0.938	0.8714	0.0	48.28	284	0.0	**0.0**
	SD	11.4	9.83E-04	0.6881	3.50	4.00	0.8665	0.0	16.1	38.9	0.0	**0.0**
	p-val.	<0.0001	0.00072	<0.0001	<0.0001	0.100374	<0.0001	NA	<0.0001	<0.0001	NA	
Schwefel 2.26	Mean	--	--	1.0000	--	--	1.0000	--	0.1	--	1.25E-02	**1.25E-02**
	SD	--	--	0.0055	--	--	0.0029	-	0.316	--	0.0	**0.0**
	p-val.			<0.00001			<0.00001		0.5319		NA	
Alpine	Mean	1.65	9.88E-05	-	1.91E-01	1.27E-14	-	0.00	9.07E-04	12.40	-	**1.23E-15**
	SD	1.42E-01	6.66E-06	-	4.23E-02	3.50E-15	-	0.00	1.60E-03	1.91	-	**1.40E-14**
	p-val.	<0.0001	<0.0001	--	<0.0001	<0.00001	-	<0.00001	0.000123	<0.00001	--	
Griewank	Mean	7.55E-02	1.88E-05	0.7679	3.88	7.97E-04	0.2678	0.0	0.0521	1.33	0.0	**0.0**
	SD	2.73E-02	3.46E-06	0.2776	1.17	3.00E-03	0.0706	0.0	2.21E-02	3.00E-02	0.0	**0.0**
	p-val.	<0.0001	<0.00001	<0.00001	<0.00001	0.063285	<0.00001	NA	<0.00001	<0.00001	NA	
Ackley	Mean	6.66	4.75E-04	0.8323	2.32E-09	1.46E-04	1.0000	0.0	4.03E-02	7.14	8.88E-16	**5.88E-15**
	SD	1.11	5.03E-05	0.0686	4.41E-10	4.77E-04	0.0162	0.0	0.192	0.443	0.0	**0.0**
	p-val.	<0.00001	<0.00001	<0.00001	<0.00001	0.03287	<0.00001	NA	0.14099	<0.00001	NA	
Penalised 1	Mean	5.98E-06	--	0.4573	--	2.09E-02	0.0008	--	2.44	--	2.24E-02	**8.0E-15**
	SD	1.11E-05	--	0.4222	--	1.15E-02	0.0015	--	1.12	--	7.24E-03	**0.0**
	p-val.	0.000247	--	<0.00001	--	<0.00001	0.000278	--	<0.00001	--	<0.00001	
Penalised 2	Mean	2.62E-05	--	0.6554	--	3.50E-01	0.0187	--	3.18E-01	--	5.71E-02	**7.64E-15**
	SD	4.63E-05	--	0.8209	--	1.21E-01	0.0375	--	3.46E-02	--	4.25E-02	**0.0**
	p-val.	0.000122	--	<0.0001	--	<0.00001	0.000643	--	<0.00001	--	<0.00001	
Zakharov	Mean	50.1	3.06E-04	--	54.2	4.14E-27	--	0.0	7.79E-04	3.37E-01	--	**0.0**
	SD	11.6	1.39E-04		12.4	7.75E-27	--	0.0	1.48E-03	3.41E-02	--	**0.0**
	p-val.	<0.00001	<0.00001		<0.00001	0.000272		NA	0.000334	<0.00001		

4.5.2 Hydrothermal System-I

In this section, the feasibility and effectiveness of the proposed meta-heuristic algorithm are verified through its implementation to two HTGS problems. In this study, thermal power and water discharge are considered as decision variables and both HTGS problems are solved for three different cases; Case 1.

Total fuel cost as an objective to be minimised, Case 2: Total emission from thermal units as an objective to be minimised, Case 3: As a multi-objective problem for the best-compromised solution between cost and emission. In all three cases, transmission losses are neglected, however, the relevant equality and inequality constraints of HTGS are considered. The unified satisfying objective function is obtained for three interdependent objectives of HTGS using an interactive fuzzy decision-making model expressed by Equation (4.24). Three variations of the DE algorithm; CDE, CDESM and SaDESM are applied to solve two HTGS test systems.

Experimental Set up: With stochastic global optimisers, the setting of parameters plays a significant role in the quality of a solution. It is a challenging task, yet important. For all three cases of DE algorithms, the number of trials runs is set to 30, the population size (NP) is taken as 50 in test problem-I and 100 in test problem-II. Mutation constant (F) and crossover rate (CR) are chaotically tuned. Chaotic logistic map constant is set to 3.967 with a variation of ±0.1%. Maximum number of simplex procedure loop to achieve the stable results is set to 100. All three cases of the algorithm have been implemented under FORTRAN 99 on a computer with a Windows 10, 64-bit processor.

Hydrothermal system-I of HTGS consists of three thermal and four hydro units. The scheduling period is of 24 hours with one-hour sub-interval. The specifications of thermal and hydro system data, the hydraulic relationships, and load demand details have been referred from the work by Narang *et., al,* (2012). To investigate the effect of initial random solution, the technique has been run for thirty independent trials to obtain the best, the worst and average solution in respective cases under study. The quantitative results obtained after the application of the proposed variants of DE on Test system-I in terms of the minimum cost, minimum emission are compared with the state-of-the-art metaheuristic techniques like *Evolutionary algorithm (EVA), Quadratic approx DE (QADEVT), Non-sporting Genetic algorithm (NSGA), Cultural algorithm (MOCA), Civilised swarm optimiser (IPCSO), Chaotic DE (CDE), ε-dominated orthogonal DE (ε-ODEMO), PSO and Self organised hierarchal PSO viz. QPSO, SOHPSO, CS-modes of DE, versions of PSO with predator-prey* *viz.QPSO, PSO, PPO, MOCA with PSO, hybrid PPO* applied to solve hydrothermal system-I and are summarised with references in Table 4.3.

Table 4.3: Comparison of cost and emission for hydrothermal system -I

Algorithms	**Author, Year**	**Cost as an objective**		**Emission as objective**		**Multi-objective scheduling**		**Cardinality Ranking μ_D**
		Cost ($)	**Emission. (lbs.)**	**Cost ($)**	**Emission (lbs.)**	**Cost ($)**	**Emission (lbs.)**	
EVA	Basu, (2010)	45063	48797	59228	16554	47906	26234	0.37459
QADEVT	Lu and Sun, (2011)	41762	30710	45971	16654	42939	17918	0.62866
NSGA	Deb, *et.al*, (2007)	42474	28132	48263	16928	43280	17899	0.63096
HMOCA	Lu, *et. al*, (2011)	41805	16841	46744	15914	43593	16204	0.60393
IPCSO	Narang, 2017	40881	--	--	--	--	--	NA
DE	Mandal and Chakraborty, (2009)	43500	21092	51449	18257	44914	19615	0.40129
CDE	Yuan et.al, (2008)	41872	17726	45049	16221	42198	17711	0.65364
ε-ODEMO	Xu et. al, (2012)	--	--	--	--	44672	16653	0.43841
QPSO	Mandal and Chakraborty, (2012)	42543	31205	46288	17735	44122	18102	0.52278
SOHPSO		41983	24842	44432	16803	43045	17003	0.68799
MODE	Zhang et. al, (2013)	42198	17711	45157	16241	43569	19871	0.39305
LM-MODE		41872	17726	45049	16221	43277	16684	0.65240
TM-MODE		42051	17861	45054	16091	43377	16517	0.63706
CM-MODE		42309	17697	45084	16248	43279	16603	0.65209
IQPSO	Narang et. al, (2012)	43278	17984	47871	17019	44344	17408	0.48873
PSO		43076	25384	48570	16199	45906	18620	0.24912
PPO		42042	27961	48913	15728	44111	17473	0.52447
MOCA+PSO	Narang, *et. al*, (2014)	42009	16842	44962	16242	43873	16222	0.56098
PPO+PS		41530	28757	48920	15716	42836	17254	0.70872
DE (Author)		43065	27944	46461	18809	45327	19327	0.33794
CDE (Author)		42845	25892	46172	16300	43316	18340	0.57775
CDESM(Proposed)		**41145**	**19396**	**47426**	**15793**	**42237**	**17950**	**0.62408**
SaDESM (Proposed)		**41011**	**23129**	**47530**	**14840**	**42025**	**16831**	**0.75980**

The comparison of obtained results presented in Table 4.3, shows that SaDESM can obtain a compromised solution of good satisfaction level. The cardinality ranking function value, $\boldsymbol{\mu_D}$ in case of SaDESM is better than CDE and CDESM, and also in comparison with established algorithms undertaken for comparison for hydrothermal system-I. Quantitatively, the operating cost and pollutant emission values are better in SaDESM than CDESM and CDE algorithms, when considered scheduling problem for individual objectives and combined objectives.

This suggested SaDESM algorithm supports the better quality solution as it is evident from the saving in the thermal fuel cost and emission cost as well.

The hourly generation schedule from thermal and hydro units over 24 hour time period, obtained for the best- compromised solution of hydrothermal system-I is presented in Table 4.4.

Table 4.4: Optimal output from hydro and thermal units in Case 3 of Test System –I with SaDESM

Time interval (hr.)	**Load Demand**	**Power Mismatch**	**Thermal Power Generation (MW)**			**Hydro Power Generation (MW)**				**Water Discharge ($Q_{iH} \times 10^4 m^3/hr$)**			
	P_D (MW)	**$\|\Delta P_D\|$ (MW)**	P_{1T}	P_{2T}	P_{3T}	P_{1H}	P_{2H}	P_{3H}	P_{4H}	Q_{1H}	Q_{2H}	Q_{3H}	Q_{4H}
1	750	0.0000	170.85	295.55	50	52.6	49.15	0	131.85	5.1	6	30	6
2	780	0.0000	174.95	300	54.25	71.55	50.15	0	129.1	7.26	6	30	6.1
3	700	0.0000	174.95	214.15	50	83.85	51.3	0	127.75	9.15	6	30	6
4	650	0.0000	160.45	209.45	50	53.65	54.75	0	121.7	5	6.26	30	6
5	670	0.0000	102.6	210.15	50	100.35	90.15	0	116.75	13.56	12.92	30	6.07
6	800	0.0000	102.55	208.7	139.75	75.9	73	0	200.1	8.13	9.47	30	11.16
7	950	0.0000	169.6	295.75	229.55	52.25	50.35	0	152.5	5	6	30	6
8	1010	0.00006	174.75	210	232	84.65	50.35	2.5	255.75	9.66	6	28.1	12.88
9	1090	0.0000	174.75	228.65	231.4	93.45	66.45	16.4	278.95	11.81	8.41	10	13.89
10	1080	0.0000	174.7	209.75	228	78.8	71	4.8	312.95	8.65	9.24	26.31	16.22
11	1100	0.0000	174.15	300	139.9	72.9	50.55	11	351.5	7.58	6	12.5	19.66
12	1150	0.0000	174.6	215.1	233.3	90.8	59.5	15.2	360.5	10.6	7.06	10	20
13	1110	0.0000	125	209.5	232.05	80.95	76.35	20.6	365.55	8.7	9.92	10.25	20
14	1030	0.0000	174.7	209.8	139.75	63.75	58.8	25.65	357.55	6.16	7.05	10	19.74
15	1010	0.0000	103.25	209.5	140	83.6	78.1	30.25	365.3	8.82	10.28	10.72	20
16	1060	0.000012	170.85	295.55	50	52.6	49.15	0	131.85	10.31	6	10	19.88
17	1050	0.0000	175	209.7	139.8	86.5	53.5	36.55	349	9.32	6	10	19.28
18	1120	0.0000	132.15	295.5	140.2	87.55	77.1	39.45	348.05	9.44	9.86	10	20
19	1070	0.0000	172.9	225.15	139.65	62	87.25	43.7	339.35	5.84	13.07	10	19.84
20	1050	0.0000	175	209.8	138.65	73.85	75.75	45.75	331.2	7.31	19.95	10.16	19.74
21	910	0.0000	102.75	208.75	50	100.75	75.7	47.55	324.5	12.75	11.4	10.10	20
22	860	0.0000	173.85	172.95	50	53.8	44.75	49.5	315.15	5	6	10	20
23	850	0.0000	174.9	127.4	50	54.25	85.55	52.7	305.2	5	14.92	10.2	20
24	800	0.0000	174.8	124.5	50	54.85	49.85	55.05	290.95	5	7.15	10	19.36
						Total Reservoir Volume Error				**0.0**	**0.0**	**1.1E-05**	**-1.6E-05**

It is observed that equality constraints of the hydrothermal system –I; load balance and hydro continuity, and inequality constraints are consistently satisfied in each hour. The load balance mismatch and total error in reservoir volume at the end of the scheduling period are significantly low, that may account for the error in the recording of the values after decimal positions and hence ignored.

For further comparison, the qualitative results obtained in case 3 with proposed techniques for hydrothermal system-I are demonstrated in Figure 4.6 and 4.7 of water discharge, reservoir storage trajectories of hydrothermal system-I. The results depicted in Figure 4.7 and 4.8 represent convergence curves, Pareto curve of hydrothermal system-I.

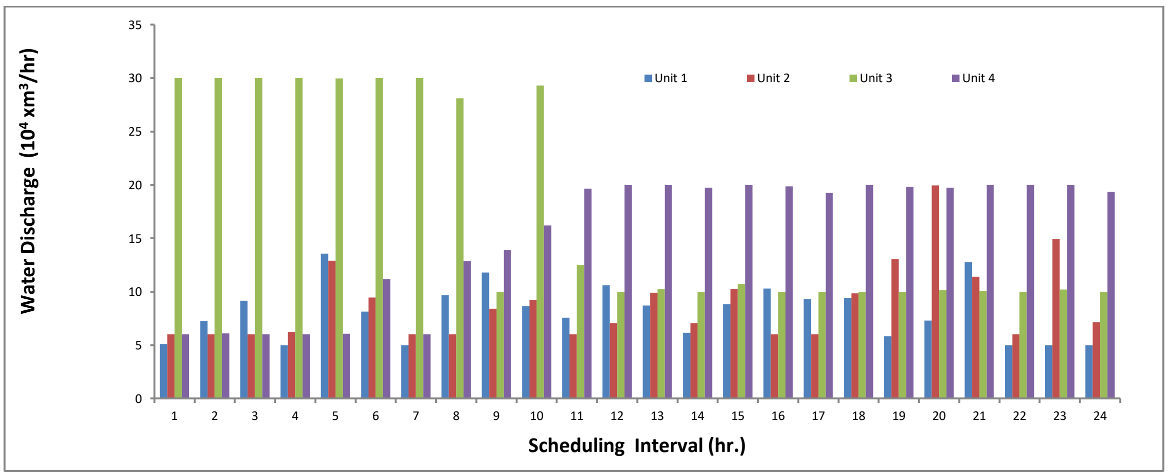


Fig. 4.6: Water discharge ($10^4 \times m^3/hr$) for case 3 of hydrothermal system-I

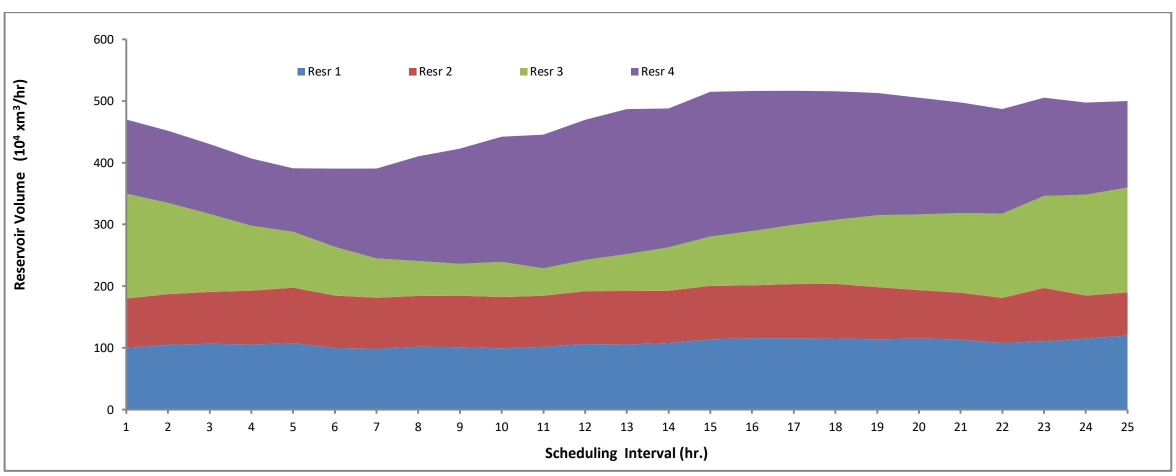


Fig. 4.7: Hourly reservoir storage reservoir for case 3: hydrothermal system-I

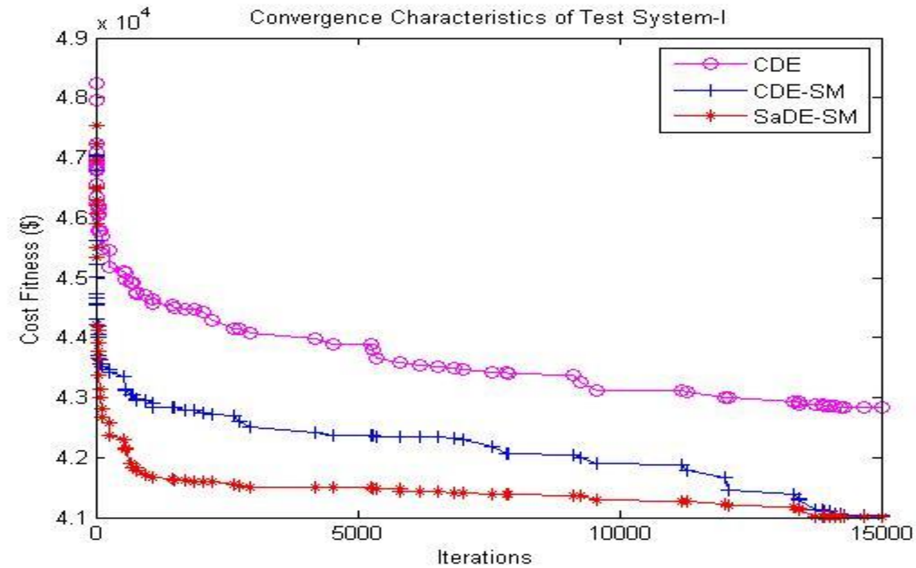


Fig. 4.8: Trajectory cost curve comparison of case 3: hydrothermal system-I

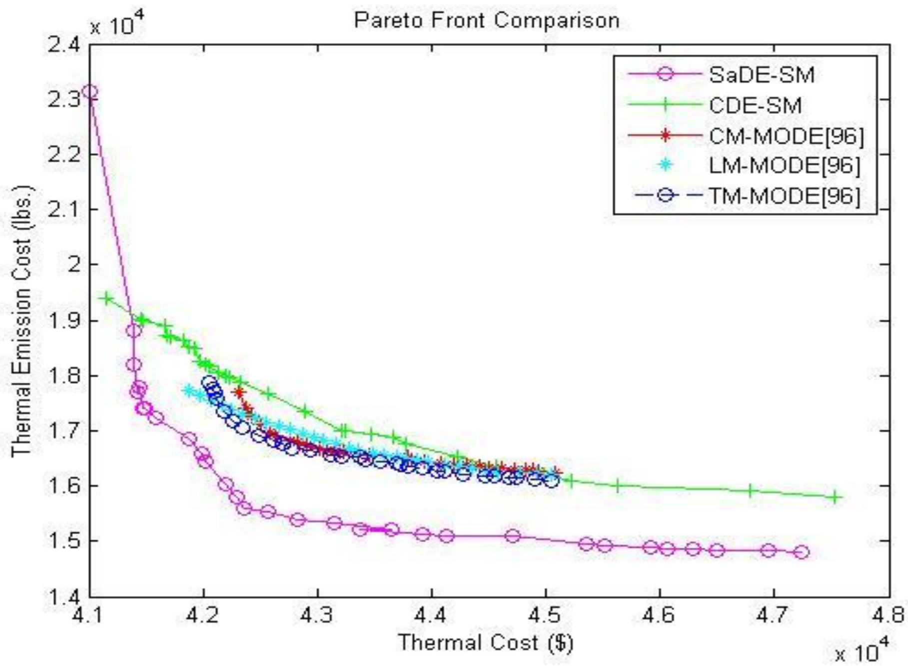


Fig. 4.9: Pareto front comparison case 3: hydrothermal system-I

This convergence trends in the comparison reveal that all three algorithms exhibit excellent acceleration towards promising locations during the initial phase; however, SaDESM algorithm is more efficient than CDE and CDESM in obtaining the better quality solution. The reservoir storage capacity of four reservoirs is under control in all sub-intervals of the entire scheduling period. Reservoir 3 and 4 have relatively high storage volume in comparison with reservoir 1 and 2 due to their downstream positions in cascaded units.

Figure 4.8 depicts the convergence trajectory of SaDESM algorithm and is compared with CDE and CDESM algorithms for the solution of hydrothermal system-I. To investigate the potential of better exploration of SaDESM algorithm, Pareto front comparison of the algorithms understudy with three other algorithms CM-MODE, LM-MODE, and TM-MODE of referred work by Zhang *et. al, (*2013), applied to solve Case 3 (multi-objective) of the hydrothermal system –I, are presented in Figure 4.9. It is clear from the comparison that convergence property and diversity distribution of Pareto front attained from proposed SaDESM is much better than presented by other compared algorithms.

4.5.3 Hydrothermal System-II

Hydrothermal system-II of HTGS consists of six thermal and sixteen hydro units. The scheduling period is of 24 hours with one-hour sub-interval. The specifications of thermal and hydro system data, the hydraulic relationships, and load demand details have been referred from the work by Narang *et. al*, (2014). The best quantitative results are obtained over thirty independent trials on hydrothermal system-II for Case 1, Case2 and Case 3, respectively, and are presented in Table 4.5. The comparison with the state-of-the-art metaheuristic techniques like PSO, PPO, PSO hybridised with Powell Search is reported in Table 4.5.

Table 4.5: Comparison of cost and emission for hydrothermal system -II

Algorithm	**Author, Year**	**Cost as an objective**		**Emission as objective**		**Multi-objective scheduling**		**Cardinality Ranking μ_D**
		Cost ($)	**Emission (tons)**	**Cost ($)**	**Emission (tons)**	**Cost ($)**	**Emission (tons)**	
CSO	Narang (2017)	1360241	--	--	--	--	--	Cost:0.4338
PCSO		1348902	--	--	--	--	--	Cost=0.6217
IPCSO		1337390	--	--	--	--	--	Cost=0.8126
PSO	Narang *et al.*, (2014)	13,65,733	26,235	13,90,559	24,871	13,64,068	25,340	0.369829
PPO		13,49,846	26,967	13,91,413	23,711	13,60,188	25,064	0.434058
PSO and Powell		13,37,500	27,225	14,06,200	23,184	13,59,100	24,289	0.452068
CDE (Authors)		13,47,912	28,287	14,00,148	26,615	13,62,611	24,648	0.393948
CDESM (Authors)		**13,29,621**	**27,104**	**13,80,708**	**24,318**	**13,57,061**	**25,365**	**0.485822**
SaDESM (Proposed)		**13,26,000**	**27,209**	**13,86,409**	**23,552**	**13,47,595**	**25,012**	**0.600766**

It is concluded that there is saving in the fuel cost when the algorithms under study are applied to hydrothermal system-II for economic cost. It is evident from the results that SaDESM performs better in the operating cost and pollutant emission values than CDESM and CDE algorithms and other compared algorithms when considered scheduling problem for individual objectives and combined objectives. This suggested approaches support the better quality solution as it is evident from the saving in the thermal fuel cost and emission cost in hydrothermal system-II as well. The convergence curves are obtained for Case 1 of hydrothermal system-II and are presented in Figure 4.10.

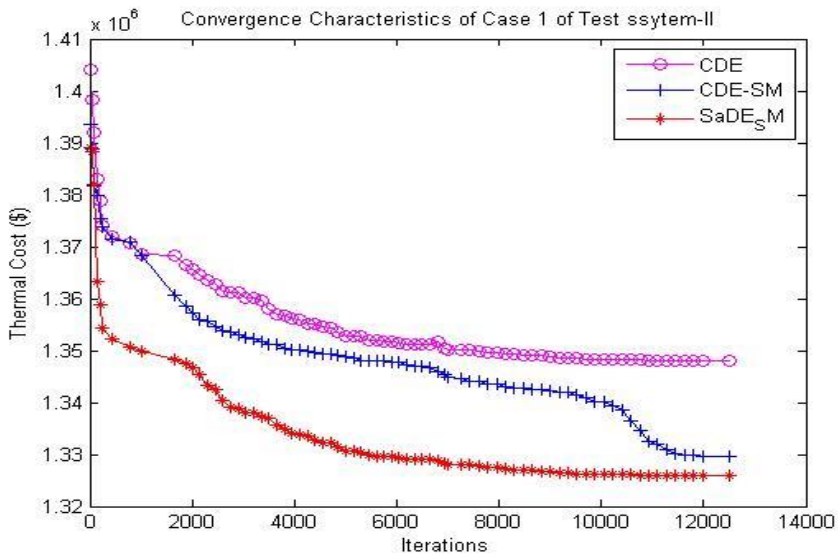


Fig. 4.10: Trajectory cost curve comparison of case 1: hydrothermal system-II

The trajectory trends demonstrate acceleration towards the promising location during the initial phase in all three techniques, and this pattern gradually decreases in the succeeding iterations.

However, SaDESM technique exhibits the remarkable shift from exploration to exploitation and provides better quality solution among CDE, CDESM and SaDESM, for the given number of iterations. The property of inclination towards exploitation and escaping from local minima establish the efficacy of SaDESM algorithm to solve non-linear constrained optimisation problems.The thermal power generation from six thermal units and water discharge from sixteen hydro units for hydrothermal system-II in each subinterval over the 24 hour scheduling period in Case 1: Minimum Cost is presented in Tables 4.6, and 4.7 respectively.

Table 4.6: Optimal output thermal units in case 1 of hydrothermal system-II with SaDESM

Hourly sub-interval (k)	**Load Demand P_{Dk} (MW)**	**Thermal Power Generation P_{iT} (MW)**						**Power Mismatch ΔP_k (MW)**
		P_{1T}	P_{2T}	P_{3T}	P_{4T}	P_{5T}	P_{6T}	
1	1374	94.329	88.919	210.000	200	313.756	324.804	0.0
2	1278	47.114	44.0177	206.205	224.888	314.984	324.954	0.0
3	1372	91.523	78.707	207.592	224.921	315.000	325	0.0
4	1058	28.316	10	157.7923	151.088	314.314	271.368	0.00008
5	1358	65.508	84.738	205.495	224.532	314.881	324.88	0.0
6	1282	46.417	31.988	201.933	225	315	324.8	0.0
7	1130	342.74	10.012	170.678	175.198	275.3	322.442	0.0
8	1200	39.443	10.000	189.552	184.267	315	324.58	-0.00009
9	1248	29.1178	10.086	205.695	218.096	313.13	325	-0.00012
10	1364	62.726	69.928	210	206.46	315	322.60	0.0
11	1146	39.910	10	136.51	19.75	313.16	323.14	-0.00004
12	1262	34.502	39.273	162.045	225	314	325	0.0000
13	1246	46.263	10.837	209.040	191.132	304.658	324.19	-0.00001
14	1140	27.384	10.024	129.674	201.383	315	305.15	0.0
15	1226	30.81580	10.00446	168.00860	219.19380	311.76970	325	0.0
16	1314	45.56738	23.67	209.33	224.95	315	323.75	0.0
17	1426	87.00952	80.15	209.64	224.94	315	324.9	0.00001
18	1028	80.52	13.89	72.71	208.3	281.32	236.2	0.0
19	1148	42.2	10.00	143.27	174.31	297.8	322.45	0.00001
20	1256	46.72	15.53	186.4	203	314	323.5	0.0
21	1348	64.1	40.4	209.7	224.84	313.27	325	0.0
22	1424	85.83	89.4	209.85	224.15	315	324.8	0.0
23	1070	47.56	10	116.92	168.5	314.95	252.29	0.0
24	1118	36.25	10	133.25	197.9	291.88	297.92	0.0

4.6 STATISTICAL TESTS AND RESULTS

The non-parametric Wilcoxon statistical test and Mann Whitney's test with 5% degree of significance are performed to assess the significant differences between the results obtained of different techniques in both the test systems of hydrothermal generation scheduling problem. The results are obtained from significance tests for the sample size of 30 in each case of hydrothermal system-I and hydrothermal system-II and are presented in Table 4.8.

Table 4.7: Optimal hydro water discharge ($Q_{iH} \times 10^4 m^3/hr$) for hydrothermal system –II

Sub Interval	**Hourly Water Discharge in Case 1: Minimum Cost**															
	Q_{1h}	Q_{2h}	Q_{3h}	Q_{4h}	Q_{5h}	Q_{6h}	Q_{7h}	Q_{8h}	Q_{9h}	Q_{10h}	Q_{11h}	Q_{12h}	Q_{13h}	Q_{14h}	Q_{15h}	Q_{16h}
1	9.5	6.44	30	13.3	9.75	6.2	29.35	13.15	13.75	6	6.04	10.55	4.21	6.15	7.17	10.88
2	8.9	6.78	29.65	13.1	5.47	6.15	29.85	13.36	6.35	12.95	6.6	17.53	6.05	6.14	5.4	6.55
3	8.2	6.18	18.34	13.1	8.6	6.85	28.9	13	7.22	16.1	6.3	18.1	6.12	6.48	9.33	6.1
4	6.9	6	29.33	13	7.8	8.75	22.75	13.5	6.1	21.72	7.1	14.1	6	6.5	5.94	5.1
5	8.3	7.57	17.45	14.85	12.15	6.25	29.85	13.8	7.7	11.2	6.12	15.12	6	6.3	5.84	6.53
6	7.5	6.11	21	13.33	11.45	6.65	19.95	13.1	7.62	16.4	6.45	15.75	6	6.5	5.45	5.94
7	8.3	7.8	30	14.6	7.66	6.35	29.2	15.83	6.11	17.52	6.1	22.23	6.1	7.4	5.72	7.25
8	7.5	8.75	29.35	13.94	9.7	6.2	28.55	15.13	6	1`4.1	6	17.45	6.15	6.1	8.1	6.75
9	9.1	7.27	29.98	18	8.4	11.15	28.85	16.55	7.25	21.19	6	18.63	7.35	6.15	7.25	5.7
10	10	7.11	19.5	19.19	7.25	9.5	15.4	20.35	7.75	18.75	6.22	17.7	6.75	6.65	7.15	8
11	9.8	6.1	14.4	16.7	6.8	6	13.25	13.97	6.35	18.65	6.35	17.35	8.35	6.15	12.9	6.75
12	11.5	6.95	27.65	21.5	8.85	9.34	10.5	21.35	7.75	20.61	7.25	20.75	10.3	6.16	9.05	5.1
13	6.6	10.7	14.35	20.4	5.9	6.1	16.45	17.1	6	18	9.06	19.87	6.2	6.375	7.85	12.85
14	7.6	8.3	16.3	16.1	6.31	7.85	18.1	17	6.5	22.21	8.1	17.6	6.35	7	7.25	7.17
15	7.5	6.45	17.25	18.1	9.7	11.6	14.75	20.5	8.77	20	8.2	19	7.42	6.4	5.25	6.5
16	10.2	8.7	14.8	22.2	8.1	11.11	12.45	21.65	6.36	21.5	6.85	19.6	7.15	9.75	9.36	9.48
17	7.8	13.5	13	25	10.87	9	14.55	24.72	8.95	19.5	14.55	17.25	10.3	7.25	11.19	11.3
18	7.9	8.82	14.85	22.98	5.62	7.13	12.25	20.54	8.7	25.82	8.15	21.45	12.8	8.32	9.85	10.9
19	5	8.1	11.25	19.5	5.84	6.2	16.32	23.07	11.95	25.82	9.43	29.58	10.2	14.5	9.4	10.9
20	5.9	9.4	11.4	24.2	7.25	9.91	14.25	25	9.97	21.7	9.31	20.25	11.5	12.55	6.9	6.55
21	7.0	11.7	10.17	24.95	5.88	13.31	14.25	23.92	14.65	20.36	13.65	20.55	13.4	14.75	7.32	7.89
22	7.7	11.3	10	24.65	7.85	11.1	13	24.75	15	18.7	14.55	17.75	15	15	11.1	11.61
23	7.7	13.5	11.75	23.1	8.95	11.58	10	24.1	14.14	26.55	15	29.7	12.6	14	10.58	10.14
24	8.5	8.6	17.6	20.81	8.85	7.75	16.65	20.75	8.75	15.2	8.82	18.9	8.52	9.45	9.77	10.15

P-value: Wilcoxon's sign test is performed for paired wise comparison among algorithms CDE, CDESM and SaDESM to compute the smallest level of significance that may result into rejection of a null hypothesis H_0. The results computed for samples of both hydrothermal systems I and II, presented in Table 4.9 concludes that the p-value is much smaller than 0.05, hence null hypothesis is taken as rejected.

Table 4.8: Significance statistical test results for Case 1 of hydrothermal system-I and II

Test System	Algorithm	Descriptive Statistics					Inter-quartile Range	Paired wise Algorithms writ SaDESM	Wilcoxon's test		Mann Whitney's Test
		Min	Max	Mean	Median	SD	IQR		p-value	z-score	U-Statistics
I	CDE	42,845	43,709	43,270	43,195	234.67	363.75	CDE	<0.0001	-4.7821	900
	CDESM	41,145	43,525	41,707	41,568	489.79	356.25	CDESM	<0.0001	4.8788	780
	SaDE-SM	41,011	42,792	42,299	42,778	347.91	400		<0.0001	--	32
II	CDE	13,47,912	13,58,500	13,51,458	13,52,009	2792.5	4729	CDE	<0.00001	-6.6456	923
	CDESM	13,29,621	13,34,516	13,31,381	13,31,030	1350.2	2592.5	CDESM	<0.001	-6.4795	890
	SaDE-SM	13,26,000	13,30,144	13,27,874	13,27,704	1101	1643		-0.3364	--	10

Z-score: The critical range of z-score within 95% confidence interval is (-1.96, 1.96). For a given sample from Test System-I, the comparison of CDE-SaDESM and CDE –SM with SaDESM depicts z-score greater than 1.96 in magnitude for hydrothermal systems I and II.

Table 4.9: Significance statistical Friedman's test results for case 1 of hydrothermal systems - I and II

Test System	Paired wise with SaDESM	Friedman's statistics			Hypothesis rejected
		χ_r^2,	F ratio	p-value	
I	CDE	30	153.11	<0.00001	H_0
	CDESM	16.17	27.30	<0.00001	H_0
	CDE, CDESM	49.4	126.01	<0.00001	
II	CDE	30	1562.8	<0.00001	H_0
	CDESM	30	121.51	<0.00001	H_0
	CDE, CDESM	60	1167.85	<0.00001	

Z-statistics: This statistic signifies that the null hypothesis is rejected; there is a significant difference in means of samples of the compared algorithms.

From the measurements of Wilcoxon's test, although it has been recommended to reject the null hypothesis. However p-value is much smaller than the level of significance, so to ascertain the significance of hypothesis rejection, Mann-Whitney's test and Friedman's test are performed.

U-Statistics: Value of U-statistic from Whitney's test, CDE group reports high U-statistics than CDESM group in both hydrothermal systems I and II. Hence CDE group depicts a high difference in means than CDESM group and hence against an alternative hypothesis H_1.

χ_r^2, F *Statistics*: Friedman's test is the most well- known procedure for testing the differences between more than two related samples measures across the same data sets. In this study, the hydrothermal systems I and II are solved using three algorithms, CDE, CDESM and SaDESM.

For the computed Friedman's statistics χ_r^2 , F-ratio and the p-value, it recommends the rejection of the null hypothesis.

From the statistical analysis of proposed algorithm, it is concluded that low value of standard deviation in hybrid algorithm indicates the consistence of performance of algorithm over number of trials to achieve the best solution. Further low value of F-ratio suggests that there is improvement in the accuracy of the global solution.

4.7 CONCLUSIONS

A *new differential evolution-simplex search approach (SaDESM),* a hybrid approach has been presented to find the hourly optimal generation schedule for the short term hydrothermal scheduling (HTGS) problem over a time horizon of 24 hours. Three-layered robust methodologies that include chaotically controlled parameters of DE, orthogonally crisscross mutation and crossover operations and opposition based learning selection procedure has been adopted to handle the diversity of the small population, to accelerate the convergence and to obtain the quality solution to a constrained multi-criteria STHS problem. Simplex search method has been hybridised with self-adaptive DE algorithm to repair the candidate solutions that optimise the set of non-dominated solutions and hence improved the global solution. Here, instead of forming a simplex of dimension plus one, the whole population is considered, that improves the exploitation and global solution with less SD is achieved. A special repair strategy has been employed to handle equality constraints of hydrothermal system power balance and volume of each reservoir and total storage of hydro units. However, the uncertainties in violations that arise during the attainment of operational constraints are coarsely repaired first followed by fuzzy modelling of residues due to violations in constraints. The best-compromised solution of multi-objective HTGS problem has been achieved by an interactive fuzzy decision-making model. Owing to the adoption of a robust methodology of handling DE operations along with two-fold constraint handling approach, *SaDESM* approach can explore the optimum solution with reduced computational effort. The proposed approach has been tested on twenty benchmark functions and two hydrothermal test systems. The results have been compared with those obtained by other well-known meta-heuristic algorithms reported in the literature. It is quite evident from the comparisons that the proposed SaDESM approach provides competitive performance in terms of quality of optimal solution as well as computational time and effort.

CHAPTER-5

META-OPTIMIZATION OF HYDROTHERMAL SYSTEM WITH SELF-ADAPTIVE DIFFERENTIAL EVOLUTION TRAINED NEURAL NETWORK

5.1 INTRODUCTION

Mathematical optimisation and artificial intelligence techniques are established tools for numerical optimization, yet face challenges in dealing with large scale, multi-modal, non-continuous and non-differential real-world problems with great flexibility and efficiency. In a modern power system operation, it is a challenging task to allocate the optimal utilisation of water and fuel energy resources, intending to optimise the overall operating cost of a mixed hydro-thermal system. The purpose of short-term hydrothermal generation scheduling is dual optimization problem to minimise the thermal fuel cost, emission of gaseous pollutants from thermal units and effective use of reservoir water by optimising power generation in each sub-interval of 24 hour planning period. Hence finding an optimum schedule of HTGS is a problem of research interest from several decades.

It is learnt that the design methodology of global optimisers incorporate multi-starts, random variations, covetous selection features, and exhibit capability to generate a better global solution with limited prior knowledge and favours the interaction between candidate solutions to handle issues like local optima, the bias of search space, isolation of minima to overcome premature convergence and their probability of entrapping in sub-optimal solution reduces (Tanabe *et.al,* 2014). Broadly, conventional optimisation techniques and meta-heuristic stochastic search models employed to solve short term hydrothermal scheduling problem are discussed in chapter 2. In chapter 3, the application of crisscross differential evolution model to solve the HTGS problem is discussed.

The proposed heuristic search technique explores the appropriate balance between exploration and exploitation to enhance convergence behaviour. The proficiency of crisscross differential evolution (CCDE) is validated by its implementation on two sets of hydrothermal systems of varying dimensions. In chapter 4, a hybridised model with simplex local search is discussed and is applied successfully to short-term hydrothermal systems. The purpose of a hybrid stochastic algorithm is to guide the search towards the promising region to enhance local search ability of *differential evolution* (DE) and this combination results into more effective optimisation algorithm for global optimisation. Straightforwardly, these stochastic or hybrid algorithms lack in co-operating with uncertain and imprecise information and making decisions based on experience and prior accumulated knowledge like a human.

In the category of intelligent computational techniques, *artificial neural networks* (ANN), is inspired by the biological human nerve system. The domain knowledge is distributed among neurons and information processing is carried out in a parallel distributed manner. ANN is employed to solve broad range problems under supervised or unsupervised learning mode such as pattern recognition, clustering and classification, function approximation, control, bioinformatics, signal and speech processing applications etc. (Abbass, 2004). ANN applications to power systems are categorized under three main areas: regression, classification and combinatorial optimization. Some major applications of ANNs and their hybrids are summarized in Table 5.1.

Owing to its features of inductive learning and recognizing efficiently, ANN has renewed (gained) the insight of researchers and is established that ANNs are promising alternatives to classical techniques.

In general, there are two basic ANN configurations: feedforward network and feedback network. Feedforward network learning employs a forward pass of information and employ steepest descent approach to update the weights. In feedback structure, like recurrent network employs steepest descent approach in supervised mode to minimize the error function and the state transition energy function in unsupervised mode to update the weights. In batch learning, the fitness measure is aggregated over the epochs of training patterns, although it is simple, it increases computation time and may stagnate to local minima, therefore global search capability cannot be assessed (Yao and Liu, 1997). In another example, Liang and Hsu, (1996) proposed a two-stage Hopfield network for hydro system scheduling. The local optimal solution is achieved with no guarantee of global solution and heuristic search strategy is applied to satisfy the violation in system constraints. The magnitude and direction of constraint violations are used to update the states of the Hopfield network until the energy function is reduced to a preset value

Table 5.1: Neural Networks and their applications in power systems

Application	**Neural Network Topology and variants**	**Author(s)**	**Year**
Dynamic Stability Assessment	Perceptron Model	Sobajic and Pao	1989
Load forecasting	Functional Neural network	*Peng, et al.*	1992
Economic load Dispatch	Hopfield Network	*Park, et al.*	1993
	Back Propagation Neural Network (BPNN)	Singh *et al.* Ching and Chen	1995 2000
Load Flow Analysis	Two-phase Neural network	Nguyen	1995
Fault detection and classification	Kohonen Map NN	Chowdhary and Wang	1996
Power System Stabilizer	Recurrent Neural Network (RNN)	He and Mallik	1997
Security assessment	BPNN	Victor *et al.*	1998
Hydrothermal scheduling	Two-phase Neural Network	Naresh and Sharma	1999
Prediction load-ability margins	Parallel self-organising hierarchical NN (PSHNN)	Srivastava and Singh	2000
Combined economic-emission	BPNN	Kulkarni *et al.*	2000
Optimal power Flow	Modified Hopfield Network	Hartati and El-Hawray	2001
Voltage stability monitoring	BPNN	Sahari *et al.*	2003
Hydrothermal scheduling	Hopfield Neural Network	Basu	2003
Consumer load profile allocation	Probabilistic neural network	Gerbec *et al.*	2005
Solar Tracking	Steepest descent-based ANN	Ocran *et al.*	2005
Transient Stability Analysis	ARTMAP recurrent neural Network	Ferreira *et al.*	2006
Contingency selection and ranking	Cascaded Neural network	Singh and Srivastava	2006
PMU based Rotor angle estimation	Multilayer perceptron model	Del *et al.*	2007
Power System Security Evaluation	Cascaded Neural network	Dangi *et al.*	2010
State Estimation	BPNN	Manitas *et al.*	2012
AC-DC Power flow Controller	Wavelet Network	Song *et al.*	2013
Hybrid algorithms			
Real-time Swing Prediction	Fuzzy Hyper-rectangular composite NN	Liu *et al.*	1999
Hydrothermal scheduling	GA and BPNN	Xingping *et al.*	2000
MPPT tracking	GA and Radial basis function network	Zhang *et al.*	2002
Unit Commitment	Evolutionary Neural Network	Chen and Chen	2006
Power System Stabiliser	Neuro-fuzzy and GA	Fraile and Zufiria	2007
Short term Load Forecasting	Bacterial Foraging and Wavelet NN	Ulagammai *et al.*	2007
Transmission loss estimation with generation scheduling	PSO and BPNN	Hemparuva *et al.*	2017
Wind energy low current controller	Antlion-Recurrent Neural Network	Sekhar and Kumar	2019
PV Battery Charger controller	ANFIS and GA	Bendary and Ismail	2019

Similarly, Naresh and Sharma, (1999), faced the issues of parameter settings to achieve a global solution in the two-phase neural network model with an analytical calculation of Lagrangian energy function for hydrothermal scheduling. The problem of local minima can be efficiently avoided for small networks by using repeated training and randomly initialized weight values. But, the computational complexity increases enormously in time and memory space for training of large networks even with efficient algorithms like Quasi-Newton, Levenberg-Marquardt ass suggested by Daniel *et al.,* (2018), etc. to solve large scale optimization problems. Besides, there is no uniform fitness performance measure to optimize the weights and bias values that can be computed independently of a given data set and ANN sizes as reported by (Lin *et al.,* 2009) and they preferred to use cooperative behaviour of swarms from PSO and features of the cultural algorithm to guide the search within limited search space. The combination of two algorithms trained functional neural network for prediction applications.

ANN optimizer is classified into two groups: local search optimizer and global search optimizer. Backpropagation neural network (BPNN) belongs to the local search group that modify the weight coefficients only, for the topology specified through expert knowledge. Topology can modify through the iterative process of growth and pruning procedures. Convergence to a locally optimal solution is a fundamental limitation of any local search based training approach. Evolutionary algorithms belong to global search group that can evolve topology and weight coefficients individually or simultaneously (Carlos *et al.,* 2010).

To exploit the potential advantage of Pareto-dominance based learning approach of P-heuristics, the ensemble of neural network with stochastic global optimizers is envisaged with four advantages; better exploration, Pareto-dominance set, the accuracy of a global solution, simultaneous optimization of conflicting objectives. This ensemble of ANN and P-metaheuristics is termed as Meta-optimization. Meta-optimisation offers low-level learning, human-like thinking and computational power of neural networks to P-metaheuristics and in turn, P-metaheuristics bring the diversification and intensification along with high-level interactions (learning) capability of stochastic algorithms to neural networks, to generate a quality solution in less computation time. Meta-heuristics can formulate FNN components such as structure, weights, number of neurons in the hidden layer as a multi-objective optimisation problem.

Zhang *et al.*, (2002), proposed GA based radial basis function network for self-growth of hidden layer and exploited the property of no prior knowledge requirement about PV cell characteristics for MPPT tracking. Ulagammai *et al.*, (2006), proposed bacteria foraging optimization for tuning of parameters of translation and dilation in the wavelet nodes and the weighting factors in a neural network for successful short term load forecasting in the custom based application. Dangi *et al.*, (2010), proposed a cascaded neural network controller to assess power system security. A few applications of stochastic algorithms in an artificial neural network for structure learning and/or parameter learning is gaining attention from researchers in the field of power systems summarised in Table 5.1.

In this investigation, differential evolution (DE) is selected as a gradient-free global search engine to train an artificial neural network, based on its versatility and simplicity among stochastic algorithms, Apart from providing multiple solutions in a single run, DE maintains population size over generations and above all the self-referential reproduction and replacement scheme does not allow the population to diverge or to lose the best solution.

In this study, the constrained self-adaptive differential evolution with neural network (SDENN) problem is formulated as a scalar objective function. Operating limit violation by the slack hydro and slack thermal generating unit is taken care using exterior penalty method and squared error of violation in constraints is added as an objective to be optimised. *Feedforward neural network* (FNN) is recommended for the approximation of decision variables to optimise an objective function. The chaotic differential evolution strategy is adopted for generating and exploring effective search between best-so-far weights and randomly selected individuals before locating weights for the global solution in the feasible problem-domain.

5.2 MULTI-OBJECTIVE HYDRO-THERMAL GENERATION SCHEDULING PROBLEM FORMULATION

The SDENN problem aims to obtain the optimal hourly generation schedule of a constrained multi-chain hydrothermal system over the time horizon of 24 hours with optimal use of water resources and optimising the generation cost and harmful emissions from thermal units while satisfying the physical and technical constraints of the hydrothermal system.

To perform hydrothermal generation scheduling, active power generation for thermal units (P_{ji}; $j = 1,2,\dots,T$; $i = 1,2,\dots,\mathrm{N}_T$) and the water discharge rate for hydro units (Q_{ji}; $j = 1,2,\dots,T$; $i = 1,2,\dots,\mathrm{N}_H$) during each sub-interval are searched using the proposed SDENN algorithm.

Mathematically, the HTGS problem is stated as an optimization problem and the objective functions and constraint are formulated using mathematical models of hydro and thermal units (Dhillon and Kothari, 2010) as stated below:

5.2.1 Mathematical Model

Objective Functions

(i) Quadratic Model

Minimize operating cost:

$$F_1 = \sum_{k=1}^{T}\left(\sum_{i=1}^{N_T}(a_i P_{ki}^2 + b_i P_{ki} + c_i)\right) \tag{5.1}$$

(ii) Valve Point Loading Model

Minimize operating cost,

$$F_1 = \sum_{k=1}^{T}\left(\sum_{i=1}^{N_T}\left(a_i P_{ki}^2 + b_i P_{ki} + c_i + \left|d_i sin\left\{e_i\left(P_i^{min} - P_{ki}\right)\right\}\right|\right)\right) \tag{5.2}$$

(iii) Environmental Impact

Minimize Emission of Pollutant:

$$F_2 = \sum_{k=1}^{T}\left(\sum_{i=1}^{N_T}\left(\alpha_i P_{ki}^2 + \beta_i P_{ki} + \gamma_i + \eta_i exp(\delta_i P_{ki})\right)\right) \tag{5.3}$$

The active power generation of each hydro unit is a function of water discharge rate, reservoir volume and net hydraulic head and is computed using the following relation

$$P_{kj+N_T} = C_{1j}V_{kj}^2 + C_{2j}q_{kj}^2 + C_{3j}V_{kj} + C_{4j}q_{kj} + C_{5j}V_{kj}q_{kj} + C_{6j}$$

$$(j \in [1, N_G], k \in [1, T]) \tag{5.4}$$

The objective functions are subjected to equality and inequality constraints, defined in chapter 3 as:

Equality Constraints

(i) To ensure power balance, equality constraint during each subinterval is imposed (Equation 3.4).

(ii) The volume dynamics in the hydro reservoir (Equation 3.6a)

(iii) The hydraulic (storage) constraint for the hydro unit reservoir at the end of the planning period (Equation 3.6b)

Inequality Constraints

(i) Active power generation limits on each hydro and thermal unit (Equation 3.7a)

(ii) Water discharge rate limits on hydro units (Equation 3.7b)

(iii) Reservoir volume limits on hydro units (Equation 3.7c)

5.2.2 Fuzzy Decision Making

For a multi-objective optimisation problem, the solution method involves two important steps. First to generate a non-dominated set of solutions, Second; the trade-off operator for the best-compromised solution. The decision-maker (DM) assigns fuzzy tradeoff to select the best-compromised solution of the available non-inferior solutions obtained for conflicting objectives of thermal units. The solution with a maximum degree of satisfaction among the three interdependent objectives generates the final solution. To handle the imprecision in the judgment of DM, the best- compromised solution among Pareto solutions of the conflicting objectives of the problem is quantified using a fuzzy approach as follows:

$$\mu(\bar{F_n}) = \begin{cases} 1 & F_n \leq F_n^{min} \\ \dfrac{F_n^{max} - F_n}{F_n^{max} - F_n^{min}} & F_n^{min} \leq F_n \leq F_n^{max} \\ 0 & F_n \geq F_n^{max} \end{cases} \qquad (n = 1, .., 3) \qquad (5.5)$$

where F_n^{min} and F_n^{max} are extremum values of the n^{th} objective function obtained in the trial runs.

The degree of satisfaction of objectives μ_D will be its maximum value and is achieved through non-preference function representing the satisfaction level of one objective for others. The Pareto solutions of HTGS problem are found using an objective function with equal weightage to all the three objectives desired to be minimized simultaneously using Equations (5.8a) and (5.8b)

Mathematically it is defined as:

$$U_t = \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\mu\big(F_i(X)\big)}{\mu\Big(F_j(X)\Big)}\right) \tag{5.6}$$

The desired satisfying function among partially comparable objectives is defined as:

$$\mu_D = \begin{cases} \mu_C U_t & ; U_t \le 1 \\ \mu_C U_t^{-1} & ; U_t > 1 \end{cases} \tag{5.7}$$

Such that μ_C is cardinality priority ranking defined as: $\mu_C = min(\mu(F_1), \mu(F_2), \mu(F_c))$

The overall objective function of the hydrothermal generation problem reformulated as fuzzy sets is the unification of thermal unit objectives and constraint function stated as:

The augmented objective function is rewritten as:

$$F_T = \begin{cases} \mu_D\,;\ x_i^{min} \le x_{jki} \le x_i^{max} \\ \mu_D + F_c; \quad otherwise \end{cases} \tag{5.8a}$$

where FF is the combined objective function of cost and emission and F_c is the function that corresponds to a violation of constraints and is defined as:

$$F_c = P_F \sum_{j=1}^{m} \emptyset_j \tag{5.8b}$$

where P_F is the exterior penalty factor having large value, $\emptyset_j$ are equality constraints of the optimisation problem

Such that

$$\emptyset_1 = \sum_{j=1}^{T}\left(\Delta P_{err,j}\right)^2 \text{ and } \Delta P_{err,j} = P_{Dj} + P_{Lj} - \sum_{i=1}^{N_G} P_i \tag{5.9a}$$

$$\emptyset_2 = \sum_{i=1}^{N_H}(\Delta VOLER_i)^2 \text{ and } \Delta VOLER_i = V_{(T+1)i} - V_{1i} \tag{5.9b}$$

$$\emptyset_3 = \sum_{j=1}^{T}\sum_{i=1}^{N_H}\left(VO_{ji}\right)^2 \tag{5.9c}$$

$$\emptyset_4 = \sum_{j=1}^{T} \sum_{i=N_T+1}^{N_G} \left(P_{ji}\right)^2 \tag{5.9d}$$

$$\emptyset_5 = \sum_{j=1}^{T} \left(Q_{ji}\right)^2 \tag{5.9e}$$

$$\emptyset_7 = \sum_{i=1}^{N_T} \left(P_{ji}\right)^2 \tag{5.9f}$$

The constraint function represents the violation of equality and inequality constraints and is dependent on the value of decision variables.

5.3 CONSTRAINTS HANDLING TECHNIQUE

The equality constraints of HTG problem are non-linear. In the first phase of the procedure, the variable elimination method is used for satisfying equality constraints.

5.3.1 Equality Constraints Handling

(i) *Power balance*

To solve power balance, equality constraint, a slack thermal unit is selected at random and the active power generation of slack unit P_{sk} in k^{th} sub-interval taken as a dependent variable is computed by using a quadratic equation (5.10).

$$B_{00} + P_{Dk} + \sum_{\substack{i=1 \\ i\neq s}}^{N_G} \sum_{\substack{j=1 \\ j\neq s}}^{N_G} P_{ik}\, B_{ij}\, P_{jk} + \sum_{\substack{i=1 \\ i\neq s}}^{N_G} B_{i0}\, P_{ik} - \sum_{\substack{i=1 \\ i\neq s}}^{N_G} P_{ik} = 0 \quad (P_{Lk} \neq 0, k \in [1,T]) \tag{5.10}$$

(ii) *Water balance equality constraint*

For water balance, equality constraint, water discharge from i^{th} hydro unit is taken as the dependent variable for randomly selected h^{th} sub-interval of the scheduling period.

Using available water balance constraint for i^{th} hydro unit in h^{th} subinterval, taken as a dependent (slack) and is rewritten as:

$$q_{hi} = V_{1i} - V_{Fi} + \sum_{j=1}^{T} I_{ki} - \sum_{\substack{j=1 \\ j\neq h}}^{T} q_{ji} - \sum_{j=1}^{T} S_{ji} + \sum_{j=1}^{T} \sum_{m=1}^{R_H} \left(q_{(j-\tau_{mi})m} + S_{(j-\tau_{mi})m}\right)$$

$$(i \in [1, N_H]) \tag{5.11}$$

The computed slack dependent variable x_{ks} in the k^{th} sub-interval by Eq. (5.10) and (5.11) is taken equal to its nearest boundary if it violates its respective boundary limits expressed as in Equation (4.6):

At the end of the scheduling period, violation in total reservoir volume of i^{th} hydro unit, $|\Delta V_i| = \left|V_{(T+1)i} - V_{1i}\right|$

Violation in the power balance in j^{th}, sub-interval will be: $\left|\Delta P_{Dj}\right| = \left|\sum_{i=1}^{N_G} P_{ji} - P_{Dj} - P_{Lj}\right|$

If both the violation values are within pre-set values then, continue with better population for fitness function, otherwise, violation in the equality constraints is handed through the exterior penalty method defined in Equation (5.8a) and (5.8b)

5.3.2 Inequality Constraints Handling

For inequality constraints, any design control variable x_{ji}^{t} as a candidate solution is kept in the feasible problem-space region. Any violation is the constraint on hydropower generation, reservoir volume and water discharge rate is computed using Equations (5.12a), (5.12b) and (5.12c) respectively.

$$\langle P_{kji} \rangle = \begin{cases} \left(P_{kji} - P_i^{min}\right)^2 & ;\ P_{kji} < P_i^{min} \\ \left(P_i^{max} - P_{kji}\right)^2 & ;\ P_{kji} > P_i^{max} \\ 0 & ;\ P_i^{min} \le P_{kji} \le P_i^{max} \end{cases}$$

$$\left(\mathrm{i} \in [\mathrm{N_T} + 1, \mathrm{N_G}];\ \ \mathrm{j} \in [1, \mathrm{T}]; \mathrm{k} \in [1, \mathrm{N_p}]\right) \qquad (5.12a)$$

$$\langle V_{kji} \rangle = \begin{cases} \left(V_{kji} - V_i^{min}\right)^2 & ;\ V_{kji} < V_i^{min} \\ \left(V_i^{max} - V_{kji}\right)^2 & ;\ V_{kji} > V_i^{max} \\ 0 & ;\ V_i^{min} \le V_{kji} \le V_i^{max} \end{cases}$$

$$\left(i \in [1, N_H];\ \ j \in [1, T]; k \in [1, N_p]\right) \qquad (5.12b)$$

$$\langle q_{kji} \rangle = \begin{cases} \left(q_{kji} - q_i^{min}\right)^2 & ;\ q_{kji} < q_i^{min} \\ \left(q_i^{max} - q_{kji}\right)^2 & ;\ q_{kji} > q_i^{max} \\ 0 & ;\ q_i^{min} \le q_{kji} \le q_i^{max} \end{cases}$$

$$\left(i \in [1, N_H];\ \ j \in [1, T]; k \in [1, N_p]\right) \qquad (5.12c)$$

5.4 FEEDFORWARD NEURAL NETWORK

Inspired by human brain neuron system, McCulloh-Pitts in 1943 introduced the perceptron computational model and initiated the research on *artificial neural network* (ANN). ANNs are capable of generalising the data through learning by examples and can solve complex problems in the area of recognition, classification and optimisation. The structure of an ANN consists of several neurons called processing units and arranged in layers such that the neurons in a layer have connections from the neurons at its previous layer. These connections are assigned weights. Fundamentally, an ANN optimization (learning/training) is met by searching an appropriate network structure called function, and the weights called parameters of the functions. So finding a suitable network structure includes the determination of appropriate neurons (nodes) in terms of activation functions, the number of neurons and the arrangement of neurons. A proper design of ANN significantly affects its learning. A single layer network consists of an input and output layer. Such networks cannot solve nonlinear separable problems. Therefore, a multi-layer ANN structure is explored by researchers that consist of one or more hidden layers other than input and can accurately approximate continuous functions and derivatives (Haykins, 2009) using gradient optimisation.

The other ANN models, like radial basis function and support vector machine, are a special class of three-layer FNNs. These structures operate in supervised mode and are employed for classification and regression applications. On the other side, adaptive resonance theory, Kohonen's self-organizing map and learning-vector-quantization are two-layer FNNs. These structures operate in unsupervised mode and handle application like pattern recognition, data compression *etc.* using Winner-take-all learning rule. Feedback network model like a recurrent neural network, Boltzman’s machine, Hopfield network are effectively used to predict sequences of information or associations among data using gradient optimisation or energy function for state transition. Conventionally learning algorithms are applied to optimise weights only for predefined network structure.

These learning paradigms of feedforward or feedback network are time-consuming procedures and may fall in a local minima solution. However, these algorithms do not guarantee a global solution. Having these limitations of traditional ANNs optimisation, it is essential to devise the techniques that can be used to train the system parameters to find the global solution while optimising the network structure as well.

The feedforward network is designed with one hidden layer as shown in Figure 5.1 and each node (neuron) receives activated aggregated weighted input from the output of the nodes of the previous layer described as:

$$Y_j^l = f_j\left(\sum_{i=1}^{n}\left(w_{ji}x_i^{l-1}\right) + b_j^l\right) \qquad ; j \in [1, h] \tag{5.13}$$

where Y_j^l is input to j^{th} node in l^{th} from layer $(l-1)^{th}$, layer and b_j^l is external bias input to to j^{th} node in l^{th} layer, x_i^{l-1} is input to i^{th} node in $(l-1)^{th}$layer, w_{ji} is the weight of connection from i^{th} node in $(l-1)^{th}$layer to j^{th} node in l^{th}layer, n is the number of nodes in $(l-1)^{th}$layer. $f_j(\cdot)$ is a threshold function.

External Bias: The bias neuron supports the shifting of the activation function left, right, up, or down. In ANN, a weight is associated with each input provided to an artificial neuron (processing element). Weight increases the steepness of activation function to decide the speed of the activation function to trigger whereas bias is used to delay the triggering of the activation function. The **bias** value also reflects how well the model fits the training set. A high bias means the neural network is not able to generate correct predictions even for the training patterns it trained on.

Activation Function: Activation functions provide a transformation to map the input signals into output signals that are needed for the neural network to function. It determines the output of a neural learning model, its accuracy and the computational efficiency of training the network. Besides, activation functions have a major impact on the neural network's ability to converge and the convergence speed. In general, neural networks use non-linear activation functions, which can help the network to learn complex data, compute and learn almost any input function representing and provide accurate predictions (Haykins, 2009). There are three types of activation functions as:

Binary Step Function: A binary step function is a threshold-based activation function as shown in Figure 5.2(a). If the aggregated input value at the neuron is above or below a certain threshold, the neuron is activated and outputs the same signal to the next layer. This activation function does not allow multi-valued output and hence cannot be used to classify the input in several ways.

Linear activation function: It considers the aggregated weighted inputs for each neuron, and generates an output from the neuron in proportion to the input. Unlike step function, a linear function allows multiple outputs and is better than a step function. A linear function cannot be used with backpropagation algorithm as the first-order derivative of linear function will be constant and second-order derivative will result in zero. Secondly, with linear activation function, the output will be linear and multilayer neural network will behave like a single layer network. Linear activation function representation is shown in Figure 5.2(b).

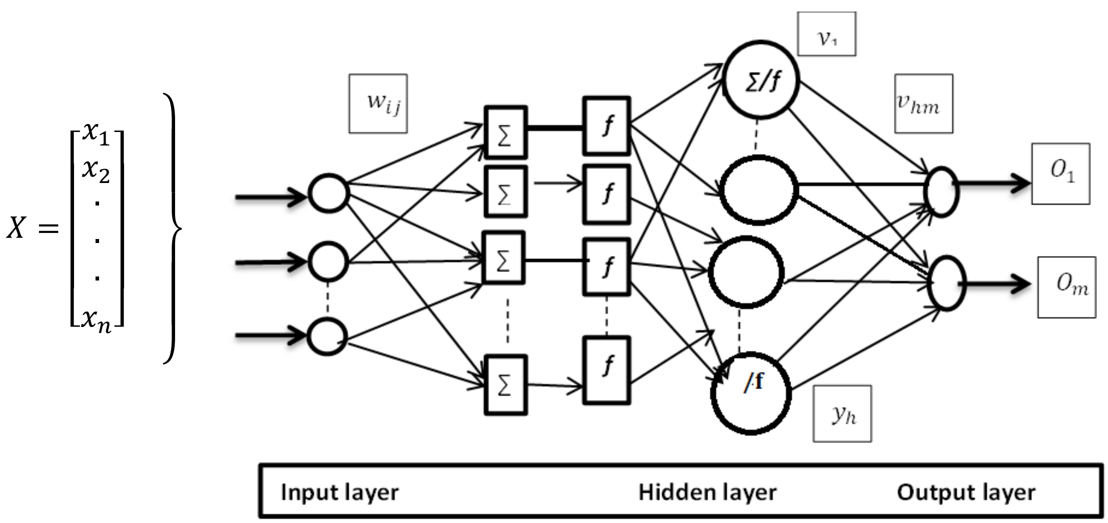


Fig. 5.1: Feedforward neural network representation

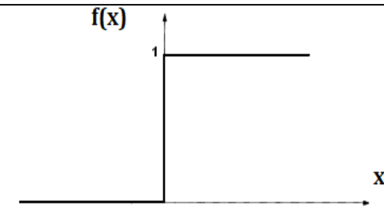
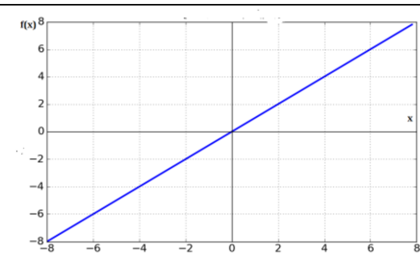
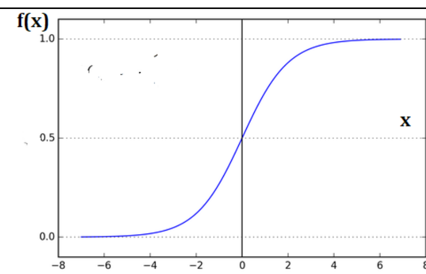
Non-linear activation function: This allows nonlinear mapping between input and output for better learning and modelling of complex data such as images, speech signals, nonlinear and of high dimensions. Nonlinear activation function allows backpropagation and stacking of multiple layers with more than one hidden layer for handling complex data with a high level of accuracy. Various types of nonlinear activation functions include sigmoid function, tangent-hyperbolic function, Softmax, rectified linear unit activation function etc.. In this investigation sigmoid activation function is shown in Figure 5.2(c), is chosen, although it is computationally expensive, yet can easily handle derivatives and results into clear predictions of output.

Activated output at m^{th} node in the output layer

$$O_j^l = f_j\left(\sum_{i=1}^{h}\left(v_{ji}y_i^{l-1}\right) + b_j^l\right) \qquad ; j \in [1, m] \tag{5.15}$$

In matrix notation: Input vector $X = [x_1, x_2, \dots, x_n]^T$ and weight vector associated with the connection between i^{th} node in the input layer and j^{th} node in the hidden layer is $W_j = \left[w_{j1}, w_{j2}, \dots, w_{jn}\right]^T$. Weight vector associated with connection from j^{th}node in the hidden layer to m^{th}node in the output layer will be $V_m = [v_{m1}, v_{m2}, \dots, v_{mh}]^T$

Table 5.2 Activation functions: (a) Binary Step (b) Linear (c) Non-linear

Activation Function	**Representation**	**Graphic representation**
Binary Step (Threshold)	$f(x) = \begin{cases} 0; x < 0 \\ 1; x \geq 0 \end{cases}$ $\forall x \in [-\propto, \propto]$	
Linear	$f(x) = m * x$ $m > 0, \forall x \in [-\propto, \propto]$	
Non-Linear Function (Sigmoid)	$f(x) = 1/(1 + exp^{(-\beta x)})$ $\beta > 0, \forall x \in [-1,1]$	

The non-linear sigmoidal activation function is defined as:

$$f_j\left(z_j\right) = \frac{1}{1 + e^{-\beta z_j}} \tag{5.14}$$

where β is the slope of sigmoidal function and

$$z_j = \begin{cases} \sum_{i=1}^{n} \left(w_{ji} x_i^{l-1}\right) + b_j^l & ; hidden\ layer \\ \sum_{i=1}^{h} \left(v_{ji} y_i^{l-1}\right) + b_j^l & ; output\ layer \end{cases} \tag{5.15}$$

5.5 HYPOTHESIS ABOUT HYBRID SELF-ADAPTIVE DIFFERENTIAL EVOLUTION NEURAL NETWORK (SDENN) ALGORITHM

This section describes the basic concepts concerning differential evolution and the cooperative algorithm evolve the population space inconsistent with the learning property of feedforward neural network and guide the search to an optimal solution.

5.5.1 Initialization of Population of Decision Variables

In the absence of prior information about the solution, meta-heuristics start with random initial solutions and the process of search to find the new individuals with improved performance is terminated, when predefined criteria are satisfied. The distance of the initial guess from optimal solution affects computation time. In the uniformly distributed population, although the population is evenly distributed over the feasible space yet, the probability theory suggests that there are chances of 50% of this initial guess, be far from the solution than the opposite guess. In this study, a novel procedure is adopted to explore for the best fit population among uniformly distributed initial one and its opposite one simultaneously, to improve the diversity of population and search capability of the initial population.

Hypothetically the proposition of activating the quasi-random sequence follows two mechanisms. One, it enables each member of the population to search around its individual best position and global best position such that the individual member is guided by quality solutions to achieve the global best position. Second; the individual best is combined with randomly generated population member and opposite ones and learning strategy is implemented. In both cases, the distance between individual best and global best solution exponentially decreases and can support accelerated convergence.

A population that represents a solution vector of decision variables as defined in Equation (5.11) of size $(N_P \times \mathrm{T} \times \mathrm{N_G})$, uniformly distributed in the feasible space of the hydrothermal problem is generated using Equation (5.16) and its opposite one of size $(N_P \times \mathrm{T} \times \mathrm{N_G})$ using Equation [5.17] respectively.

$$x_{kji}^{0} = x_i^{min} + R_{kji}\left(x_i^{max} - x_i^{min}\right) \qquad (i \in [1, N_G]; j \in [1, T]; k \in [1, N_P]) \qquad (5.16)$$

where R_{jki} is a uniform random number that varies between 0 and 1.

$$x_{k+N_P ji}^{0} = x_i^{min} + x_i^{max} - x_{kji}^{0} \qquad (i \in [1, N_G]; j \in [1, T]; k \in [1, N_P]) \qquad (5.17)$$

Modified Population Reproduction Strategy

Inspired by Salp swarm reproduction behaviour, the population members are connected in a chain that starts as a coil around the solitary member. It grows over time, giving rise to the beginning of the colony of members. Everyone within the chain reproduces sexually. An attempt has been made never to lose the best solution, even if the whole population deteriorates. It has a single parameter to control. Its value goes on decreasing for every successive iteration. Therefore, it explores the search space in the initial stage after that it performs exploitation (Kansal and Dhillon, 2020).

Mathematically, the position of individual x_{jki}^{t} member of the population concerning global best member x_{Gki} is updated as:

$$x_{kji}^{t} = \begin{cases} x_{Gji} + A_1 x_{kji}^{t} & ; k \le NL \ \ and \ R1 < 0.5 \\ x_{Gji} - A_1 x_{kji}^{t} & ; k \le NL \ \ and \ R1 \ge 0.5 \\ 0.5\left(x_{kji}^{t} - x_{(k-1)ji}^{t}\right) & ; k > NL \end{cases}$$

$$; \forall i \in [1, N_G]; \ j \in [1, T], k \in [1, N_P] \qquad (5.18)$$

where $x_{kji}^{t} = x_i^{min} + r_{kji}\left(x_i^{max} - x_i^{min}\right); \, r_{kji} \in [0,1]$

$$For \ NL = 0.45 \times N_P \ and \ \ A_1^t = \alpha_3 \exp\left(-\left(\frac{\alpha_2 t}{T_{DEW}}\right)^2\right) ; \alpha_2, \alpha_3 \ > 1$$

Selection of α_2, α_3 greater than 1.0 control the exponential decrease in step size between individual to global best and will save time in selecting the best population as shown in Figure 5.2.

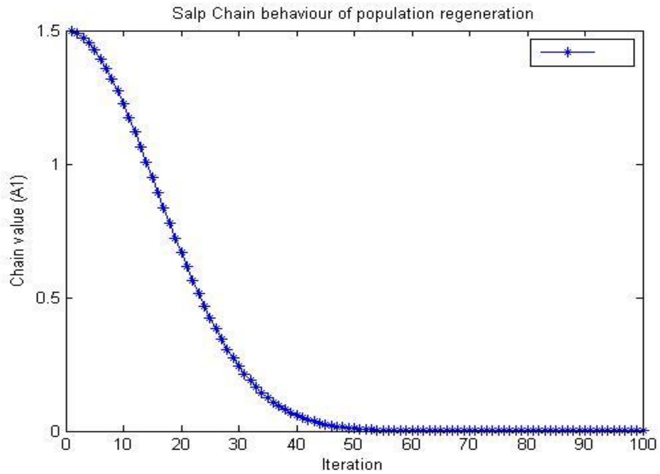


Fig. 5.2: Salp chain behaviour of population regeneration

$R1$ is randomly generated number, NL represents the percentage of a population selected for reproduction and T_{DEW} is the number of iteration, the population perturbation loop is repeated. x_{Gji} represents the global best candidate solution of i^{th} decision variable in t^{th} sub-interval.

5.5.2 Differential Evolution

Differential evolution is a competitive P-heuristic optimiser that combines simple arithmetic operators with the three classical measures of mutation, crossover, and selection on the vector of a randomly generated population of individuals to search for the final individual solution in a generation of the problem domain. In this study, the key idea of DE is focussed on generating a trial weight vector for the learning of artificial feedforward neural networks. Classical DE strategy is described as:

Initialization of Population of weights: The initial population is randomly generated which is uniformly distributed over the problem space bounded by limits of decision variables. The population of size $(N_{PW} \times T \times N_{GM} \times N_G)$, is generated in the feasible space [-1.1] representing the weights of connections between the input layer and the hidden layer of the feed-forward neural network using the following equation.

$$w_{kjhi}^0 = (2R_{kjhi} - 1) \quad ; (i \in [1, N_G]; h \in [1, N_{GM}]; j \in [1, T]; k \in [1, N_{PW}]) \qquad (5.19a)$$

where R_i is a uniform random number that varies between 0 and 1.

Similarly, for the weights between the hidden and output layer of feedforward network are:

$$v_{kjih}^{0} = (2R_{kjih} - 1) \qquad ; (h \in [1, N_{GM}]; i \in [1, N_G]; j \in [1, T]; k \in [1, N_{PW}]) \qquad (5.20b)$$

Chaotic mutation: The mutation is performed over the different decision variables of an individual weight population to generate mutant vector according to the following equation:

$$U_{kjhi}^{t} = w_{Bjhi}^{t} + F^{t}\left(w_{Bjhi}^{t} + w_{kjhi}^{t} - w_{r_1 r_{3hi}}^{t} - w_{r_2 r_{4hi}}^{t}\right)$$

$$(\forall i \in [1, N_G]; h \in [1, N_{GM}];\ j \in [1, T]; r_3 \neq r_4 \neq j; k \in [1, N_{PW}]; r_1 \neq r_2 \neq k) \quad (5.21)$$

where W_{Bjhi}^{t} is the best individual in the current generation and W_{kjhi}^{t} is base vector r_1 *and* r_2 are randomly selected population from j set of time intervals and r_3 *and* r_4 are randomly selected from a set of weight variable for k^{th} population.

Mutant vector for weight variable defined between hidden to output layer will be

$$L_{kjih}^{t} = V_{Bjih}^{t} + F^{t}\left(V_{Bjih}^{t} + V_{kjih}^{t} - V_{r_1 r_3 ih}^{t} - V_{r_2 r_4 ih}^{t}\right)$$

$$(\forall h \in N_{GM}, i \in N_G; j \in T;\ r_3, r_4 \neq j;\ k \in [1, N_{PW}];\ r_1, r_2 \neq k\ ) \qquad (5.21b)$$

where r_1 *and* r_2 are randomly selected population from an N_{PW} set of population and r_3 *and* r_4 are randomly selected sub-interval from a set of j sub-intervals of the scheduling period for k^{th} population. Mutation constant, F^{t} is chaotically updated.

Chaotic Crossover: The crossover exchange information among the weight variables and subintervals of a population. The binomial crossover performed for weights between the input layer to the hidden layer to generate variable, N_{jkhi}^{t} and weights of hidden to output layer to generate variable, M_{jkih}^{t}. are defined in Equations (5.27a) and (5.27b) respectively.

$$N_{kjhi}^{t} = \begin{cases} U_{kjhi}^{t} & ; \quad if\ (r_i < CR^{t}) or (h = R1) or (j = R2)\ or\ (k = R3) \\ W_{kjhi}^{t} & ; \quad otherwise \end{cases}$$

$$(i \in [1, N_G]; h \in [1, N_{GM}];\ j \in [1, T];\ \ k \in [1, N_{PW}]) \qquad (5.22a)$$

The random integer numbers are generated such that $r_i \in [1, N_G], R1 \in [1, N_{GM}],\ R2 \in [1, T]$and $R3 \in [1, N_{PW}]$.

$$M_{kjih}^{t} = \begin{cases} L_{kjih}^{t} & ; \quad if\ (r_h < CR^{t}) or (i = R1) or\ (j = R2)\ or\ (k = R3) \\ V_{kjih}^{t} & ; \quad otherwise \end{cases}$$

$$(h \in [1, N_{GM}];\ i \in [1, N_G]; j \in [1, T];\ \ k \in [1, N_{PW}]) \qquad (5.22b)$$

where the random integer numbers are generated such that $r_h \in [1, N_{GM}], R1 \in [1, N_G],\ R2 \in [1, T]$and $R3 \in [1, N_{PW}]$.

The control parameter CR^{t}, is the crossover rate and is generated by logistic chaotic mapping.

Selection: To decide whether the vector $L^t_{jkih}, or\ M^t_{jkih}$ should be a member of the population comprising the next generation, greedy selection strategy search for the best vector $\mathcal{X}^t_{jki}$, is applied to speed up the convergence.

$$\mathcal{X}^{t+1}_{kji} = \begin{cases} \gamma^t_{kji} & ; if\ f^t_k\left(\gamma^t_{kji}\right) > F^t_k\left(\mathcal{X}^t_{kji}\right) \\ \mathcal{X}^t_{kji} & ;\ otherwise \end{cases} \qquad (\ k = 1,2 \dots N_P) \tag{5.23}$$

where $f^t_k\left(\gamma^t_{kji}\right) = \max\left\{F^t_k\left(X^t_{kji}\left(U^t_{kjhi}, L^t_{kjih}\right)\right),\ F^t_k\left(X^t_{kji}\left(N^t_{kjhi}, M^t_{kjih}\ \right)\right)\right\}$

and its corresponding best population is γ^t_{jki} ($\forall i \in N_G; \forall j \in T; \forall k \in N_P$)

The fitness value, $F^{t+1}_k\left(\mathcal{X}^{t+1}_{kji}\right)$ is evaluated corresponding to candidate population as $\mathcal{X}^{t+1}_{kji} = \gamma^t_{kji}$; $(\forall i \in [1, N_G]; j \in [1, T]; k \in [1, N_P])$

5.5.3 Opposition Learning-based Selection

The global explorative capability of the proposed SDENN strategy is expedited using opposition-based learning. The population is improved by taking into account the opposite points of the randomly generated population defined by Equation (5.24). For non-dominated solutions generated from populations by crisscross mutation, crisscross crossover, and selection, the elite individuals are selected from the unification of the current-best population and opposite population using Equation (5.25). This proposal speeds up the convergence towards the global solution.

$$W^{t+1}_{kjhi} = \begin{cases} w^{min}_{hi} +\ w^{max}_{hi} - W^{t+1}_{kjhi} & ;\ F_{opp} \geq 0.5 \\ w^{min}_{hi} + R_{kjhi}\left(w^{max}_{hi} - w^{min}_{hi}\right); & F_{opp} < 0.5 \end{cases}$$

$$(\forall i \in N_G; h \in N_{GM}; j \in T; k \in N_{PW}) \tag{5.24}$$

$$V^{t+1}_{kjih} = \begin{cases} v^{min}_{ih} +\ v^{max}_{ih} - V^{t+1}_{kjih} & ;\ F_{opp} \geq 0.5 \\ v^{min}_{ih} + R_{kjih}\left(v^{max}_{ih} - v^{min}_{ih}\right); & F_{opp} < 0.5 \end{cases}$$

$$(\forall i \in N_G; h \in N_{GM}; j \in T; k \in N_{PW}\ \ ) \tag{5.25}$$

where F_{opp} is the opposition factor.

The largest fitness value of the individual is termed as the best solution among the population at $(t + 1)^{th}$ iteration

$$F^{t+1}_B(X^{t+1}_{Bki}\left(W^{t+1}_{Bjhi}, V^{t+1}_{Bjih}\right)) = max\left\{F^{t+1}_k\left(X^{t+1}_{kji}\right);\ k = 1,2, \dots, NP\right\}$$

The global best is obtained with the unification of new best candidate and the current–best solution as:

$$X_{Gki}^{t+1} = \begin{cases} X_{Bki}^{t+1} & ; \quad if\ F_B^t\left(X_{Bki}^{t+1}\left(\left(W_{Bjhi}^{t+1}, V_{Bjih}^{t+1}\right)\right)\right) > F_G^t\left(X_{Gji}^{t}\left(\left(W_{Gjhi}^{t}, V_{Gjih}^{t}\right)\right)\right) \\ X_{Gki}^{t} & ; \quad ibest = ibest + 1\ ; Otherwise \end{cases}$$

Termination criterion: The iterative process terminates when the maximum number of evaluations of function is achieved or the maximum number iterations are performed. The number of function evaluations (NFE) is $(2 \times N_P + MXIN \times N_P + N_{PW} + (T_{MAX} \times 3 \times N_{PW} \times T_{DEW}))$, for SDENN whereas conventional DE follows NFE=$(N_P + 2 \times N_P \times T_{MAX})$ Conventional EBP follows $= (N_P \times T_{DEW} \times T_{MAX})$.

5.6 HYBRID SELF-ADAPTIVE DIFFERENTIAL EVOLUTION-NEURAL NETWORK (SDENN) ALGORITHM

The Hybrid Self-adaptive neural network with differential evolution is elaborated as:

1. ***Input Data****: Parameter Set Up: Input population size N_P, number of decision variables N_G, upper and lower limits of each decision variable X, Mutation constant F, crossover rate CR, convergence criteria NFE, Maximum number of iterations T_{MAX},*
2. ***ANN Set up:*** *Number of input layer neurons N_G, Number of hidden layers =1, number of neurons in the hidden layer $N_{GM} = 2 \times N_G$ Weight Population N_{PW}, number of DE learning iterations T_{DEW}. the slope of threshold function* β
3. ***Initialization of problem-domain population:*** *Initialize the value of i^{th} decision variable X in j^{th} candidate in the population* of size $(N_P \times T \times N_G)$, uniformly distributed in the feasible space of the hydrothermal problem is generated *using Equation (5.16) and its opposite one of size $(N_P \times T \times N_G)$ using Equation [5.17] respectively and update the population of decision variables as defined in Equation(5.18).*

FOR $\boldsymbol{k = 1}, N_P$

4 *For the vector of decision variables $\mathcal{X}_{kji}^0$ and $F_k^0\left(\mathcal{X}_{kji}^0\right)$ using Equations (5.8).*

ENDFOR

5 *Select the global best solution, $F_G(x_{Gji}) = max\left\{F_k^t\left(\mathcal{X}_{kji}^1\right);\ k = 1,2,\ldots,2N_P\right\}$ among the population members and retain best N_P.*

t=0

While *$(t \leq T_{max})$* ***DO***

6.1 t=t+1

6.2 Find A_1^t

6.3 Update $\mathcal{X}_{kji}^t$ *and* $F_k^t(\mathcal{X}_{kji}^t)$ *using Equations (5.18) and (5.8), respectively*

6.4. Select the global best solution, $F_B(x_{Bji}) = max\{F_k^t(\mathcal{X}_{kji}^t);\ k = 1,2,\ldots,2N_P\}$ *among the population members.*

Set $x_{Gji} = \begin{cases} x_{Bji} & ; \quad IF \ \ F_B(x_{Bji}) < F_G(x_{Gji}) \\ x_{Gji} & ; \quad \quad Otherwise \end{cases}$

ENDDO

7. *Set iteration count,* $t = 0$
8. *Set the updating counter for the best solution,* $ibest = 0$
9. *Training of ANN: Initialize the value of* l^{th} *weight variable* W *in* j^{th} *candidate in the population* of size $(N_{PW} \times T \times N_G \times N_{GM})$, uniformly distributed in the feasible space of the ANN connections between the input layer and the hidden layer. And the population of weight variable V *in* j^{th} *candidate in the population* of size $(N_{PW} \times T \times N_{GM} \times N_G)$, uniformly distributed in the feasible space for ANN connections between the hidden layer and output layer

WHILE $(t < T_{MAX})$ ***DO***

10. *Increment the iteration counter,* $t = t + 1$

FOR $k = 1, N_{PW}$ *(ANN population size)*

FOR $j = 1, T$ *(scheduling period)*

11. *Present* $\mathcal{X}_{kji}^t$; $(i = 1,2,\ldots,N_G)$*;* $(N_G$ *is the number of nodes in the input layer) as the input of the feedforward network*
12. *Compute output of hidden layer using Equation (5.13).*
13. *Compute output of output layer using Equation (5.14) as:* $\mathcal{X}_{kji}^{nt}$

ENDFOR

14. *Compute the fitness as* $F_k^{nt}\left(\mathcal{X}_{kji}^{nt}\right)$; $(k = 1,2,\ldots,N_P)$

ENFOR

15. *Select the best solution,* $F_B^t(X_{Bki}^t) = max\{F_k^t(\mathcal{X}_{kji}^{nt});\ k = 1,2,\ldots,N_P\}$.
16. *Find the global best point.*

$$\mathcal{X}_{Gji}^{t} = \begin{cases} \mathcal{X}_{Bji}^{t} & ;\ if\ F_B^t(X_{Bji}^t) > F_G^t(\mathcal{X}_{Gji}^t) \\ \mathcal{X}_{Gji}^{t};\ ibest = ibest + 1 & ;Otherwise \end{cases}$$

17. ***IF*** $(\boldsymbol{MOD}(\boldsymbol{ibest}, \mathbf{5}) = \mathbf{0})$, ***THEN***

Perturb the weights using DE operators

FOR k =1, $\boldsymbol{N_{PW}}$

Mutation: *Perform horizontal mutation operation to generate a trial vector using the current-best mutation strategy to generate a trial vector,* U_{kjhi}^t, L_{kjih}^t *using Equations (5.21a) and (5.21b) respectively*

Crossover*: Apply horizontal binomial crossover or recombination operation to recombine the trial mutant vector and parent vector to produce offspring trial vector,* N_{kjhi}^t, M_{kjih}^t *using Equations (5.22a) and (5.22b) respectively.*

Selection: *Compute the fittest solution among the moderated solutions:*

$f_k^t(y_{kji}^t) = max\left\{F_k^t\left(X_{kji}^t(U_{kjhi}^t, L_{kjih}^t)\right),\ F_k^t\left(X_{kji}^t(N_{kjhi}^t, M_{kjih}^t)\right)\right\}$ *and*

save corresponding trial vector individual y_{kji}^t $(i = 1,2, \dots, N_G; j = 1,2, \dots, T; k = 1,2, \dots, N_P)$ *to* $f_k^t(y_{kji}^t)$

Apply opposition based learning with known F_{opp} *using Equation (5.24) and (5.25)*

18. *Compute the individual for the next generation* $t = t + 1$ *using Equation (5.26)*

ENDFOR

ENDIF

19 *Select the best solution,* $F_B^{t+1}(X_{Bji}^{t+1}) = max\{F_k^{t+1}(\mathcal{X}_{kji}^{t+1});\ k = 1,2, \dots, N_P\}$.

20 *Find the global best point.*

$$\mathcal{X}_{Gji}^{t} = \begin{cases} \mathcal{X}_{Bji}^{t+1} & ;\ if\ F_B^t(X_{Bji}^{t+1}) > F_G^t(\mathcal{X}_{Gji}^t) \\ \mathcal{X}_{Gji}^{t};\ ibest = ibest + 1 & ;Otherwise \end{cases}$$

END DO

Computational Complexity: The algorithm complexity is defined by three fundamental components; population size problem dimensions, and the number of iterations. The computational complexity of the SDENN algorithm depends upon the process of initialization, fitness evaluation, and mechanism of updating the variables.

Total SDENN computational complexity can be defined as:

$$\mathcal{O}(SaDE - NN) = \mathcal{O}(N_P) + \mathcal{O}(N_{PW}) + \mathcal{O}(N_P \times T_{MAX}) + \mathcal{O}(N_{PW} \times T_{DEW}) +$$

$$\mathcal{O}(N_P \times T_{MAX} \times D) + \mathcal{O}(N_{PW} \times T_{DEW} \times N_G \times N_{GM})$$

The time complexity of SDENN can be defined as:

$\mathcal{O}(N_{PW} \times T_{DEW} \times T_{MAX})$. $\mathcal{O}(T_{MAX} \times (N_P \times D + N_P \times T_{DEW} \times T_{MAX} \times FF))$

where T_{MAX} is the maximum number of iterations for problem-optimization, $D = N_G \times T$ denotes the problem dimensions T_{DEW} is the maximum number of weight update iterations through DE and FF is the fitness value of an objective function.

5.7 SIMULATION OF TEST PROBLEMS AND RESULTS

The feasibility and effectiveness of the proposed SDENN, meta-optimization algorithm are verified through simulations on three hydrothermal systems and the details of the obtained results are presented in the next section.

The scheduling period of 24 hours with a subinterval of one hour is considered. The details of the thermal units, hydro units, and load data for these test systems have been taken from the work as referred below:

Hydrothermal System-

This system consists of three thermal units and four multi-chain hydro units The emission component and valve point loading are taken into consideration. This system is simulated for both with and without transmission losses. The system details and load data are taken from the work referred to as the work (Narang *et al.*, 2014)

Hydrothermal System –III

This system includes one thermal unit and four cascaded hydro units. The fuel cost curve is a quadratic one. The system details and load data is referred to from the work (Naresh and Sharma, 1999)

Hydrothermal System-IV

This system includes three thermal units and four multi-chain hydro units. The emission component and valve point loading are taken into consideration. This system is simulated for both with and without transmission losses. The system details and load data are taken from the work referred to in the work (Basu, 2010)

The hourly schedule of thermal power and water discharge is computed over the planning period of 24 hours when the problems are solved for the different cases;

Case 1: As a single objective problem with total operating cost as an objective to be minimized. This is implemented for all three hydrothermal system test problems.

Case2: As a single objective problem with total gaseous emission as an objective to be minimized. The relevant equality and inequality constraints of HTGS are considered reflected in Eq. (3.6a) (3.6b) , and Equations ((3.7a) to (3.7c)

Case 3: As multi-objective problem to obtain the best-compromised solution for the two conflicting objectives of operating cost and gaseous emission in test problem I and IV. The best-compromised solution is obtained using a fuzzy model reflected in Equations (5.5) and (5.6).

The quantitative performance analysis of each problem is estimated from its best fitness value and allocation of generation among hydro and thermal systems for the best utilization of energy resources in the relevant parametric space. The qualitative performance is observed from the convergence trends and the variation of the fitness value over the number of trial runs of the algorithm to assess the stability of the solution and the convergence rate to achieve the same. The capability of the proposed algorithm is evaluated against well-established state-of-the-art meta-heuristic algorithms, reported in the literature for a fair comparison.

5.7.1 Experimental Set up

The optimized structure of the feedforward neural network is selected through 50 simulations of training and 10 simulations of testing using neural network toolbox with different topologies from the feed-forward network (FNN) and recurrent neural network (RCN). The mean squared error and training time obtained for possible topologies of FNN and RCN are presented in Table 5.3. The selected topology #2 includes one hidden layer with nearly two times the input layer neurons and is programmed for simulation of three problems of hydrothermal systems in this study.

Parameter Initialization: The initial value of the parameters used in the SDENN approach in solving hydrothermal problems as mentioned in Table 5.4. However, the parameters are adjusted through trial and error within a variation of ±0.1%. in this simulation study. For all the cases under study, the number of trials runs is set to 20, Mutation constant (F), and crossover rate (CR) are chaotically tuned. With these parameter settings, the proposed

SDENN meta-optimization method was implemented using FORTRAN 99 code on a computer with a Windows 10 Pro, 64-bit processor to solve enviro-economic dispatch problem of three test hydrothermal systems 20-30 times from diverse initial populations and the best result is selected towards the final solution of respective problems.

Table 5.3 Results for selection of ANN structure

Topology No.	Structure	Number of Hidden Layers	Number of Neurons in Hidden Layer	Mean Square Error (MSE)	Training Time (sec)
1	FNN	1	7	3.7864	26
2	FNN	1	15	0.6527	20
3	FNN	1	21	0.9457	34
4	FNN	2	7	0.9982	28
5	FNN	2	15	0.0843	38
6	FNN	2	21	13.6459	123
7	RCN	1	7	2.1652	21
8	RCN	1	15	3.678	114
9	RCN	1	21	3.2458	324
10	RCN	2	7	1.0983	42
11	RCN	2	15	14.4532	623
12	RCN	2	21	6.8735	1560

Table 5.4: Initial values of parameters

α_1	α_2	α_3	β_{ih}	β_{mh}	μ	FM	CR	N_P	N_{PW}	T_{MAX}	T_{DEW}
0.45	4.5	1.5	0.3-0.4	0.3-0.4	3.96	0.02	0.4	50	100-150	100	100-2000

5.7.2 Hydrothermal Test System-III

The attained optimal results for the hourly schedule of thermal and hydro generations with SDENN at the lowest fuel cost for hydrothermal system-III are presented in Table 5.5. The power balance in each sub-interval is achieved. The other relevant equality and inequality constraints are satisfied. The water reservoir volume trajectories over the scheduling period are shown in Figure 5.3. The allocation of hourly generation to the hydro and thermal units is presented in Figure 5.4. The spread of generation allocation over 24 hours depicts that in each hour major load is shared by hydro units. This saves fuel costs. The best cost function value as an optimization solution to hydrothermal system-III is selected and is compared with the results obtained when solved by three different search techniques.

Table 5.5: Optimal output from hydro and thermal units in Case 1 of hydrothermal system -III with SDENN

Time interval (hr.)	**Load Demand (MW)**	**Thermal Power Generation (MW)**	**Hydro Power Generation (MW)**				**Water Discharge ($Q_{iH} \times 10^3 m^3/hr$)**			
	P_D	P_{1T}	P_{1H}	P_{2H}	P_{3H}	P_{4H}	Q_{1H}	Q_{2H}	Q_{3H}	Q_{4H}
1	190	54.45	23.5	16.55	45.5	50	5.15	6.11	18.98	13.02
2	170	40.45	23.75	17.55	42.2	46.05	5.02	6.26	18.92	13.02
3	170	44.00	25.5	18	40.5	42.00	5.06	6.16	20.92	13.03
4	190	63.00	26.35	18.57	38.58	43.50	5.03	6.22	21.2	13.08
5	190	62.30	27.32	19.65	35.33	45.40	5.08	6.30	16.77	13.02
6	210	80.65	28.7	18.9	34	47.75	5.15	6.08	17.75	13.05
7	230	87.12	40.45	20.38	31.65	50.40	7.85	6.40	16.33	13.03
8	250	94.25	49.55	23	32.25	50.95	10.32	7.11	20.47	13.06
9	270	109.75	55.65	21.7	30.55	52.35	12.34	6.61	18.96	13.07
10	310	137.40	58	31.5	28.6	54.50	13.44	9.98	15.83	13.58
11	350	161.80	60.35	36.25	31.4	60.2	14.8	12.43	18.74	15.46
12	310	132.20	53.6	32.05	33.15	59.00	12.65	11.07	16.67	14.53
13	350	156.00	57.9	35.25	34.12	66.74	14.72	13.14	17.41	18.08
14	350	153.20	56.85	34.45	37.3	68.2	14.93	13.24	20.26	19.06
15	310	122.00	54.44	28.9	38.9	65.75	14.55	11.28	18.44	17.89
16	290	109.10	47.13	27.75	40.92	65.1	12.08	11.19	19.82	17.68
17	270	89.30	41.5	27.2	43.15	68.64	10.14	11.89	19.26	19.05
18	250	71.20	43.5	22.82	44.93	67.55	10.94	10.48	22.13	18.74
19	230	75.50	34.48	14.38	37.28	68.35	8.07	7.22	14.26	19.11
20	210	53.50	30.45	14.52	42.82	68.7	6.91	7.03	17.11	19.45
21	210	34.60	44.1	17.64	44.76	68.9	10.78	7.85	18.52	19.71
22	210	63.30	23.75	11.48	39.57	71.9	5.00	6.00	14.77	23.82
23	190	42.11	24.9	12.08	40.97	69.94	5.00	6.00	15.55	25.00
24	190	53.74	26.9	12.70	29.00	67.65	5.00	6.00	10.00	25.00
Reservoir Volume Error at the end of Scheduling Period:							**0.762939 E-08**	**0.22888 E-06**	**0.915527 E-09**	**0.839233 E-05**

Table 5.6: Comparison of thermal generation cost for hydrothermal system-III

Algorithms	**[Ref]**	**Thermal Plant Cost as an objective ($)**
Augmented Lagrange	Naresh and Sharma, (1999)	1,54,739.0
Two-phase Neural Network	Naresh and Sharma, (1999)	1,54,808.5
CDE	Yuan *et al.*, (2008)	1,54,338.1
SDENN (Proposed)		**1.54,089.6**

The economic cost solution of hydrothermal system-III is solved by direct search method using augmented Lagrange method, intelligent computational model using *two-phase neural network* (Naresh, 1999) and with P-heuristic model using *Chaotic DE* (Yuan, 2008) and the results are presented in Table 5.6 for comparison which shows the saving in fuel cost.

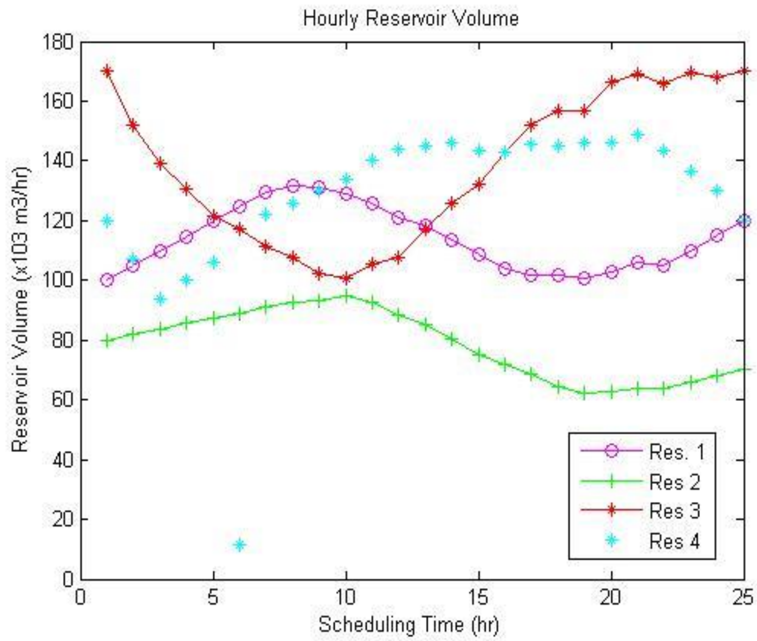


Fig.5. 3 Hourly reservoir volume of hydrothermal system-III

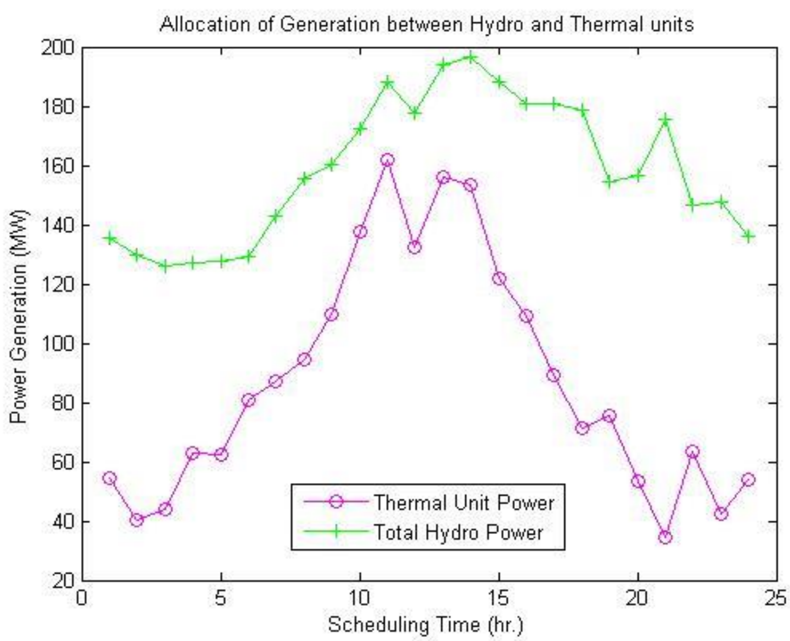


Fig.5.4 Allocation of generation of hydrothermal system-III

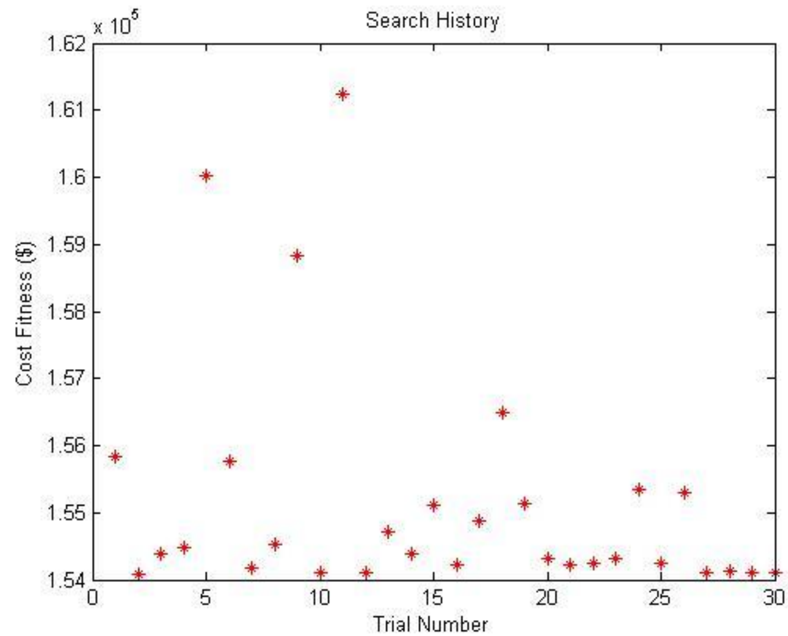


Fig.5.5 Fitness function Search over 30 trial runs of Hydrothermal system-III

It is evident from Figure 5.5 that there is a small variation in the near best value of cost function over the trial runs. This supports the stability and effectiveness of the proposed SDENN approach to generate a better quality solution.

5.7.2 Hydrothermal Test System –IV

In this case, thermal units are considered with valve point loadings and their emission curve is modelled using the exponential term. For the implementation of SDENN method on the test system -II, the population size is set as N_P as 50 and N_{PW} as 150. The maximum number of iterations is set with T_{MAX} as 500 and T_{DEW} as 500 respectively. The losses are computed using Kron's loss coefficient method. The optimal results for the hourly schedule of thermal and hydro generations with SDENN for test system are obtained and are presented in Table 5.7.

The relevant equality and inequality constraints are satisfied. Figures 5.6 and 5.7 show the convergence curve of hydrothermal system-IV with cost as objective and emission as objective respectively and depicts that convergence rate is high in the first 10 iterations and stabilizes afterwards. Figure 5.8 depicts the distribution of fitness value when the algorithm is 20 times with different initial populations considering improved initial population strategy and otherwise. The mean value and standard deviation are computed in both cases. The maximum value of cost fitness, its minimum value, mean and standard deviation are presented in Table 5.8. It is concluded that with improved initial population strategy, the standard deviation is reduced and hence ensures a better quality solution. Also, the smaller mean value implies that the algorithm searches more accurately. For fair justification, the obtained results are compared with other well-established techniques including D*E*, *Real coded GA (RCGA, PSO, PPO, and PPO with Powell search*, (references are given in Table 5.9 and constraints are handled with different techniques for all three cases. It is observed from Table 5.9 that the proposed SDENN approach yields better results for thermal operating cost and gaseous emission. The results show the net profit in cost and reduced emission level when hydrothermal system-IV is solved with and without transmission losses.

Table 5.7: Optimal output from hydro and thermal units in Case 2 of hydrothermal system-IV with SDENN

Time interval (hr.)	**Load Demand**	**Minimum Emission**							
		Power Generation (MW)				**Water Discharge ($Q_{iH} \times 10^4 m^3/hr$)**			
	P_D **(MW)**	P_{1T}	P_{2T}	P_{3T}	P_{HTot}*	Q_{1H}	Q_{2H}	Q_{3H}	Q_{4H}
1	750	140	110.71	216	283.04	8.72	7.69	29.62	6.39
2	780	175	112.42	221.3	271.26	9.76	6.47	28.9	6.72
3	700	123.96	102.32	195.82	277.91	8.79	8.15	28.99	7.35
4	650	114.39	91.9	174.65	269.02	7.55	7.41	29.33	6.74
5	670	124.26	69.43	212.19	264.12	8.96	7.52	29.21	7.01
6	800	175	114.42	232.54	278.03	9.45	6.34	27.72	9.43
7	950	175	138.33	312.2	324.46	6.41	7.92	28.28	13.25
8	1010	175	132.62	297.38	405	11.92	6.98	14.03	12.88
9	1090	175	136.38	341.75	436.87	10	0.32	15.22	16.66
10	1080	175	133.27	294.3	477.4	9.11	9.22	15.38	19.86
11	1100	166.18	127.93	294.1	511.78	7.74	9.82	27.95	19.98
12	1150	174.59	140.14	322.67	512.62	10.56	9.33	12.31	20
13	1110	175	128.05	286.05	520.08	5.63	12.5	9.99	20
14	1030	142.28	116.45	249.28	522	9.16	7.65	10	20
15	1010	141.33	121.38	225.34	521.94	8.81	7.25	10	20
16	1060	175	117.8	243.31	523.87	9.76	6.52	10	20
17	1050	145.48	117.5	233.33	553.67	8.32	11.52	10	20
18	1120	158.1	130.7	284.45	546.03	8.56	10.6	10	20
19	1070	152.49	132.23	252.01	533.26	9.35	8.66	10	20
20	1050	175	119.62	235.31	520.06	7.45	10.03	10	20
21	910	127.26	110.54	188	483.99	5.11	8.3	10	20
22	860	119.25	95.05	163.55	481.24	5	9.02	10	20
23	850	114.6	85.84	182.36	467.19	5	7.75	10	20
24	800	105.48	95.99	151.42	446.61	5	6	10	20

$*$: P_{HTot} represents the total power generation from four hydro units.

Table 5.8: Effect of initial population improvement strategy in hydrothermal system-IV

Strategy Status	Maximum ($)	Mean ($)	Minimum ($)	Std. Deviation
Without Population strategy	70,230	66,601.6	64,113	2,164.34
With Population strategy	67,484	65,095.4	63,709	1,170.86

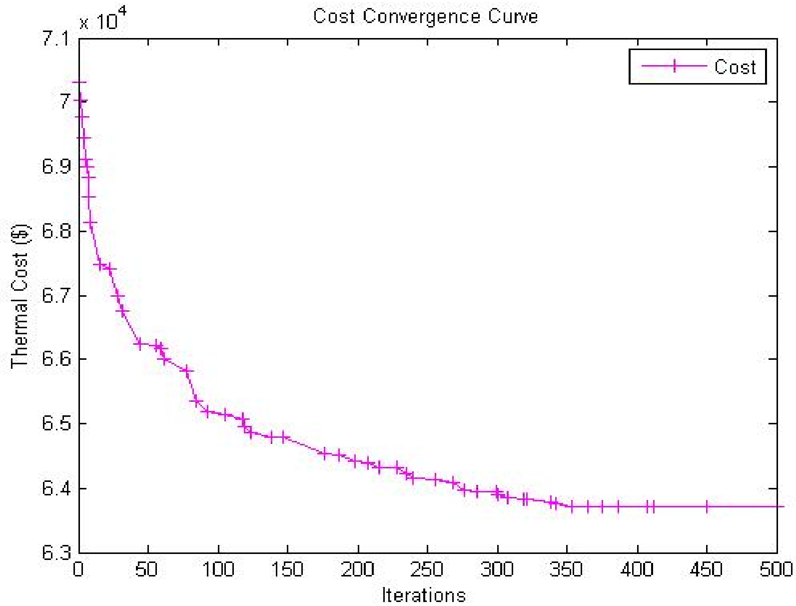


Fig. 5.6 Cost convergence curve hydrothermal system-IV

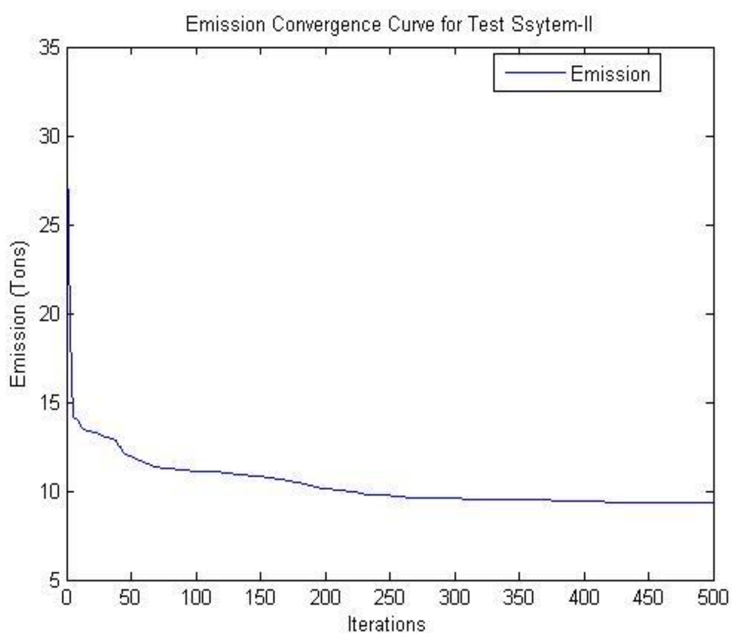


Fig. 5.7 Emission convergence curve of hydrothermal system-IV

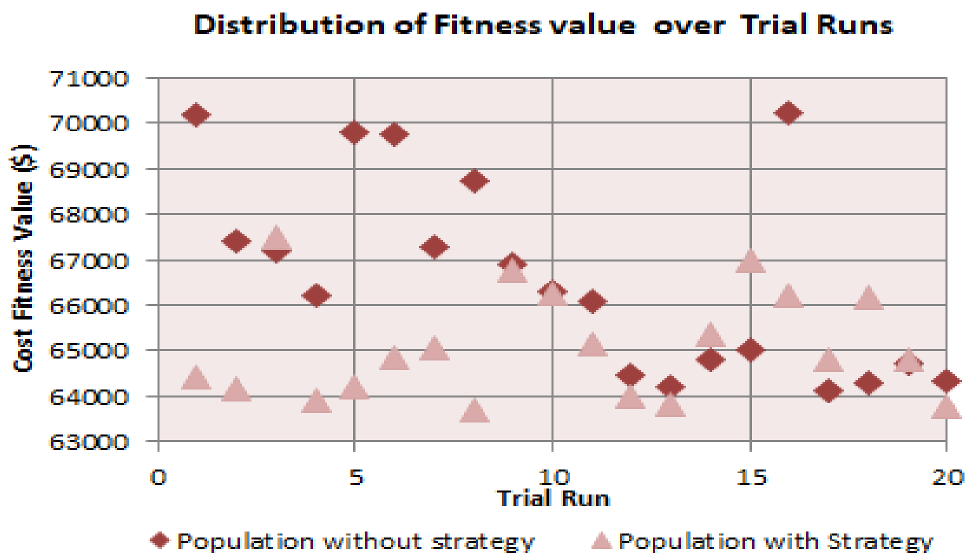


Fig. 5.8 Fitness distribution in hydrothermal system-IV

Table 5.9: Comparison of cost and emission for hydrothermal system-IV

Algorithms	Author, Year	Cost as an objective		Emission as objective		Multi-objective	
		Cost ($)	Emission (tons)	Cost ($)	Emission (tons)	Cost ($)	Emission (tons)
DE	Basu, (2010)	110810	51.3742	161370	11.4904	126820	17.7019
RCGA	Basu, (2010)	112940	49.8731	160040	11.6256	127200	18.9605
PSO	Narang *et.al*, (2014)	96099	15.444	163028	11.0854	114128	13.4016
PPO		85760	31.4622	139898	10.9158	108451	11.801
Improved PPO		72151	137.2621	145219	10.1704	81826	21.757
PPO+ Powell		69351	145.1997	139677	10.0397	81279	18.4126
MO	Lu *et.al*, (2011)	--	--	--	--	84404	12.6820
SDENN*		63709	129.55	1,19,283	9.313171	79893	86.14
SDENN**		63,794	141.74	1,37,938	10.1978	--	--

*: Without Transmission, Losses **: With Transmission Losses

The Pareto solutions of hydrothermal system-IV are obtained with the weightage of 0.5 to each of the three objectives considered to be minimised simultaneously

5.7.4 Hydrothermal Test System –I

The simulated results of the hydrothermal system -I are presented in Table 5.10 and 5.11. The Pareto optimal solutions are obtained for the objective functions using Equations (5.8a), (5.8b). for three different cases of different weighting factors. To confirm the robustness of the obtained solution, each case simulation is carried out 20 times with a different set of population.

Case 1: The optimum solution of minimum cost is obtained by discarding emission minimization such that weighting factors are taken as: $w_1 = 1.0\ and\ w_2 = 0.0$ for cost and emission respectively. The best solution for cost as objective found in 20 runs is presented in Table 5.10.

Case 2: The optimum solution of minimum emission is obtained by discarding cost minimization such that weighting factors are taken as $w_1 = 0.0\ and\ w_2 = 1.0$ for cost and emission respectively. The best solution of emission as objective found in 20 runs is presented in Table 5.10.

Case 3: The Pareto optimal solution of all three objective functions is obtained by considering the same weighting factor to all $(w_1 = 1.0, w_2 = 1.0\ and\ w_c = 1.0$. The simulation results for three different cases of weighting factor are presented in Table 5.10.

It is observed from Table 5.10 that the total cost can be reduced to 41396 ($) when emission objective is rejected however the emission, in that case, is 18187 (lbs.). On the other hand, when cost objective is rejected, the total emission generation is reduced to 14794 (lbs.) at the risk of an increase in the cost of generation to 47146 ($). In other words, in a multi-objective solution of the given problem, for the minimization of emission of the order of 3393 (lbs.) for the increase in cost to the order of 5750($). This state demonstrates the conflicting nature of two objective functions. The preferred Pareto solutions obtained for hydrothermal system-I with the equal weightage to both the objectives is illustrated in Figure 5.9

The most preferred solution among Pareto solutions is selected using the fuzzy satisfying method. Table 5.11 represents the results obtained for the objective functions wherein the membership value implies the degree of optimality. For the same weighting factor, the highest membership value is obtained using compromised cardinality ranking as discussed in section 5.2.2. For the degree of satisfaction obtained as 0.6948, the best-compromised solution gives the cost of generation as 42380 ($) and corresponding emission as 15810 (lbs.).

Compared with the results obtained for case 1 and case 2, presented in Table 5.11, it is observed that the best-compromised cost is more than the cost obtained in case 1 and less than that obtained in case 2. Similarly, the pollutant emission generation with that of the best compromised solution is less than obtained in case 1 but more than that obtained in case 2. The decision taken by the decision-maker to choose the trade-offs between conflicting objectives as equal weighting factor is well justified.

Table 5.10: Optimum solution of hydrothermal system-I with equal weighting factors

Weight			**Cost ($)**	**Emission (lbs.)**	**Violation in Constraint**	μ_{F1}	μ_{F2}	μ_{Fc}	μ_C	μ_D
w_1	w_2	w_c								
Case1: Minimum Cost										
1	0	1	41396	18187	3.79E-08					
Case 2: Minimum Emission										
0	1	1	47146	14794	1.49E-08					
Case 3: Pareto Solution s with Uniform weighting factor										
1	1	1	41486	17398	4.45E-08	0.9843	0.2325	0.9979	0.2325	0.1269
1	1	1	41694	16228	4.51E-08	0.9482	0.5774	0.9992	0.5774	0.5279
1	1	1	41712	16129	3.99E-09	0.9451	0.6065	1.01E-16	1.01E-16	7E-32
1	1	1	41713	16120	7.45E-09	0.9449	0.6092	0.5078	0.5078	0.4597
1	1	1	41729	16098	2.61E-08	0.9421	0.6157	0.9284	0.6157	0.5813
1	1	1	41881	15982	2.24E-08	0.9157	0.6499	0.9007	0.6499	0.6262
1	1	1	41917	15923	1.52E-08	0.9094	0.6673	0.8079	0.6673	0.6513
1	1	1	41957	15962	1.91E-08	0.9024	0.6558	0.8669	0.6558	0.6364
1	1	1	41964	15940	1.51E-08	0.9013	0.6622	0.806	0.6622	0.6464
1	1	1	42003	16153	1.17E-08	0.8944	0.5995	0.7216	0.5995	0.5761
1	1	1	42030	15923	2.78E-08	0.8898	0.6673	0.9387	0.6673	0.6453
1	1	1	42083	15905	2.98E-08	0.8805	0.6726	0.9493	0.6726	0.651
1	1	1	42272	16117	3.75E-08	0.8477	0.6101	0.9795	0.6101	0.5757
1	1	1	42336	16026	2.24E-08	0.8365	0.6369	0.9007	0.6369	0.6162
1	**1**	**1**	**42380**	**15810**	**1.52E-08**	**0.8289**	**0.7006**	**0.8079**	**0.7006**	**0.6948**
1	1	1	42402	16308	1.15E-08	0.8251	0.5538	0.7151	0.5538	0.5319
1	1	1	42667	16148	2.64E-08	0.7789	0.6009	0.9304	0.6009	0.5729
1	1	1	43211	16972	3.75E-08	0.6844	0.3581	0.9795	0.3581	0.2803
1	1	1	42030	16257	3.12E-08	0.8898	0.5688	0.9503	0.5688	0.5269

Further, the cost fitness and emission fitness values obtained by SDENN are compared with the published results of other P-heuristics like *Chaotic DE (CDE), Evolutionary algorithm (EVA), Quadratic approx. DE (QADEVT), Hybrid Cultural algorithm (HMOCA), PSO, and Self-organised hierarchal PSO viz. QPSO, SOHPSO, CS-modes of DE, versions of PSO with predator-prey viz. QPSO, PSO, PPO, MOCA with PSO, hybrid PPO, Civilised swarm optimizer (IPCSO*), The problem is solved by considering transmission losses as well and the results are placed in Table 5.10.

It is concluded that there is saving in the fuel cost when the algorithms The investigation of hydrothermal system-I with SDENN reveals that the attained minimum cost is not the best value whereas when the emission is taken as objective, it gives the best results in comparison with compared results. This may be due to a random selection of parameter values understudy when applied to hydrothermal system-I for economic cost.

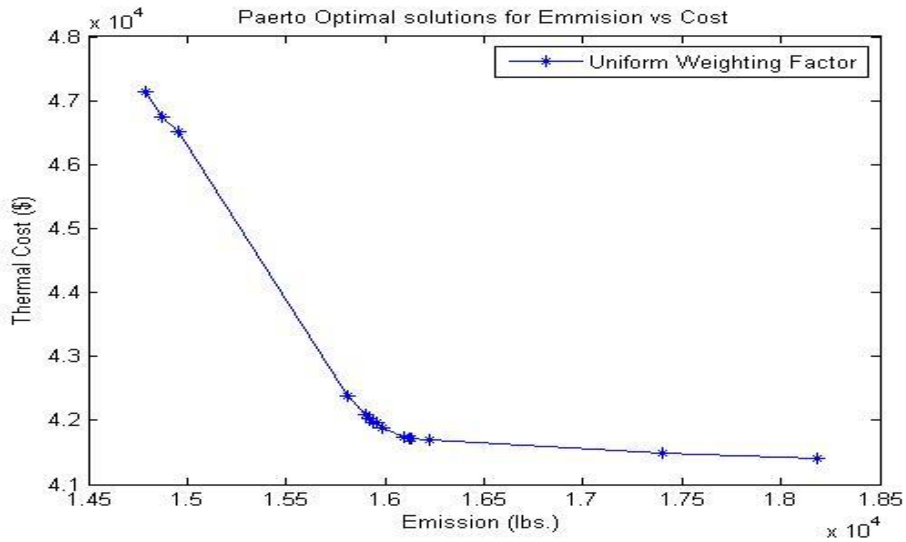


Fig. 5.9: Pareto optimal solutions for emission vs cost in hydrothermal system-I

Table 5.11: Comparison of cost and emission for hydrothermal system-I

Algorithms	Author(s), Year	Cost as an objective		Emission as objective		Multi-objective scheduling		Cardinality Ranking $\mu_C = Average(\mu_{F1}, \mu_{F2})$
		Cost ($)	Emission (lbs.)	Cost ($)	Emission (lbs.)	Cost ($)	Emission (lbs.)	
QPSO	Chiang, (2007)	42359	31298	45271	17767	--	--	--
CDE	Yuan *et al.*, (2008)	41872	17726	45049	16221	42198	17711	0.5004
QADEVT	Lu and Sun, (2011)	41762	30710	45971	16654	42939	17918	0.4055
HMOCA	Lu *et al.*, (2011)	41805	16841	46744	15914	43593	16204	0.6012
QPSO	Mandal, Chakraborty, (2012)	42543	31205	46288	17735	44122	18102	0.2755
SOHPSO		41983	24842	44432	16803	43045	17003	0.5311
LM-MODE	Zhang *et al.*, (2013)	41872	17726	45049	16221	43277	16684	0.5578
TM-MODE		42051	17861	45054	16091	43377	16517	0.5738
CM-MODE		42309	17697	45084	16248	43279	16603	0.5697
PSO	Narang *et al.*, (2012)	43076	25384	48570	16199	45906	18620	0.1717
PPO		42042	27961	48913	15728	44111	17473	0.3691
MOCA+PSO	Zhang *et al.*, (2012)	42009	16842	44962	16242	43873	16222	0.5742

PPO+PS	Narang *et al.*, (2014)	41530	28757	48920	15716	42836	17254	0.5123
IPCSO**	Narang, (2017)	41636	--	--	--	--	--	--
SDENN* (Proposed)		**41396**	**18187**	**47146**	**14794**	42380	15810	**0.6098**
SDENN** (Proposed)		42885	21924	47333	15248	43283	18652	--

*: Without Transmission, Losses   **: With Transmission Losses

It is evident from the results that SDENN performs better in the operating cost and pollutant emission values than other compared algorithms when considered scheduling problem for individual objectives and combined objectives.

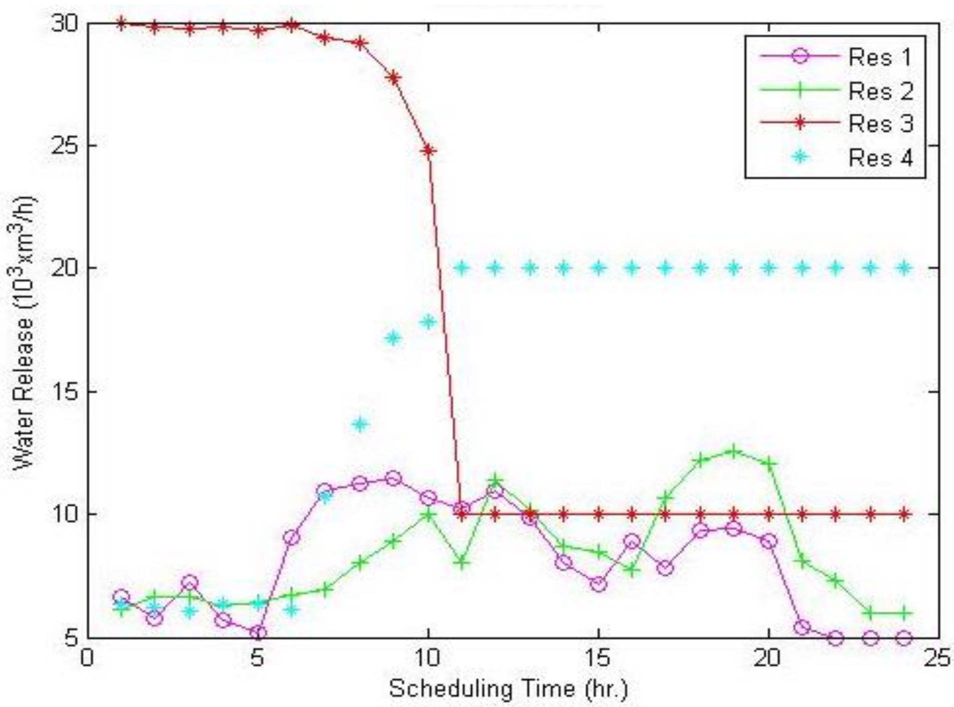


Fig. 5.10 Water release from reservoirs (x10^4m^3/h) of hydrothermal system-I

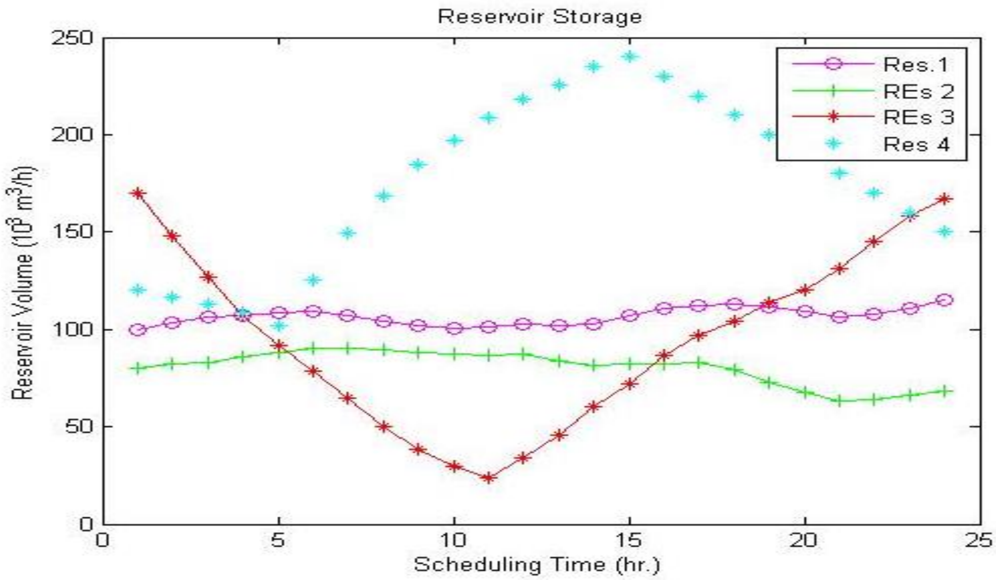


Fig. 5.11 Reservoir volume (x$10^4$$m^3$/h) of hydrothermal system-I

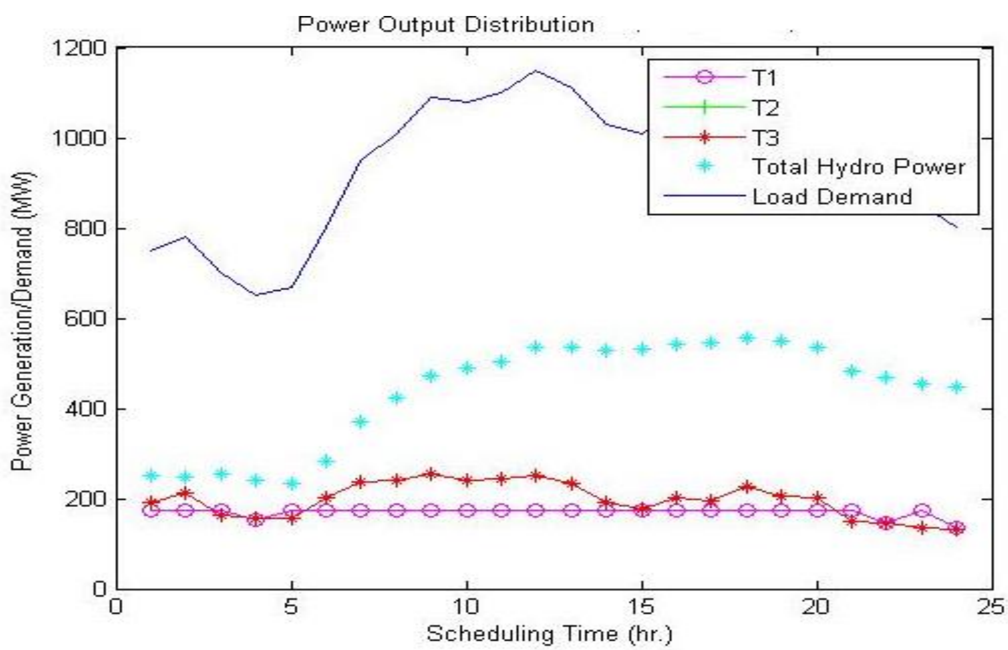


Fig 5.12 Power generation distribution of hydrothermal system-I

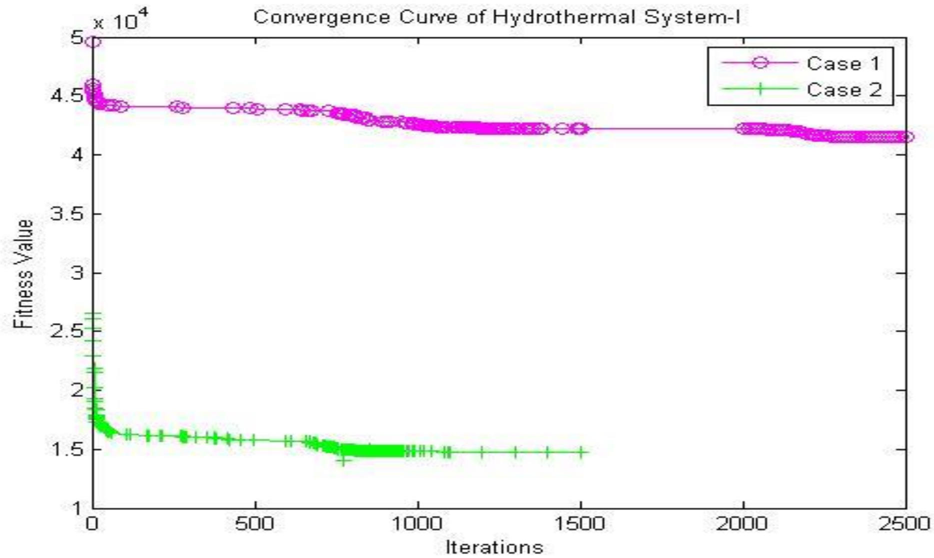


Fig. 5.13 Learning curves for best performance of hydrothermal system-I

These suggested approaches support the better quality solution as it is evident from the improvement in the cardinality ranking μ_C as well. Figure 5.10 and 5.11 exhibit hourly water release and reservoir storage over the 24 hour scheduling time. The distribution of power among the seven generators is shown in Figure 5.12. Figure 5.13 exhibits learning curves for the best performance of the hydrothermal system –I; in case cost is minimized (ii) emission is minimized. It shows the fitness value converges to a low value quickly and stabilizes in successive iterations. The active power generation and water discharge from thermal and hydro units for the best-compromised solution of the hydrothermal system –I without and with losses are presented in Table 5.12 and 5.13 respectively. It reveals that all equality and inequality constraints are satisfied.

Table 5.12: Optimal output from hydro and thermal units in Case 3 (without losses) of hydrothermal system-I

Time interval (hr.)	Load Demand P_D (MW)	Best compromised solution (without Losses)							
		Power Generation (MW)				Water Discharge ($Q_{iH} \times 10^4 m^3/hr$)			
		P_{1T}	P_{2T}	P_{3T}	P_{HTot}*	Q_{1H}	Q_{2H}	Q_{3H}	Q_{4H}
1	750	175	169.87	139.75	265.37	6.386	8.454	29.65	6.372
2	780	174.8	209.8	134.26	261.14	6.409	8.055	28.698	6.4009
3	700	173.8	128.09	139.76	258.35	8.027	6.145	28.442	6.409
4	650	112.83	125.04	139.15	272.96	7.823	8.948	29.439	6.571
5	670	175	124.12	132.55	238.33	6.426	6.555	28.99	6.353

6	800	174.68	209.25	138.28	277.78	8.133	6.648	27.077	6.651
7	950	175	296.75	143.61	334.64	9.571	11.151	27.323	7.155
8	1010	175	300	229.52	305.46	9.425	6.207	29.313	6.307
9	1090	175	291.26	139.75	483.98	11.239	7.304	28.805	19.78
10	1080	175	276.73	139.75	488.52	8.8737	8.816	27.676	18.989
11	1100	175	276.02	138.98	509.95	10.843	7.856	10.01	19.019
12	1150	175	300	135.76	539.24	10.742	11.979	9.898	20.00
13	1110	171.98	279.83	138.65	516.54	8.466	7.973	9.698	20.00
14	1030	172.5	208.52	132.19	516.79	6.985	7.536	10.912	20.00
15	1010	175	155.85	139.75	539.40	8.396	7.775	12.84	20.00
16	1060	175	213.98	141.09	529.93	7.426	7.138	10.025	20.00
17	1050	175	201.06	134.95	538.99	9.365	7.028	11.004	20.00
18	1120	175	267.19	138.97	538.84	8.603	8.295	9.897	20.00
19	1070	175	212.11	139.76	543.13	7.577	12.463	10.00	20.00
20	1050	175	209.7	139.2	526.10	8.424	9.297	10.00	20.00
21	910	111.45	128.92	139.42	530.14	8.305	12.512	10.008	20.00
22	860	107.22	124.92	139.76	488.10	7.691	6.694	9.998	20.00
23	850	102.67	124.69	138.02	484.61	5.0005	11.188	10.001	20.00
24	800	102.67	124.9	126.47	445.96	5.00	6.00	10.00	20.00
		Error in Reservoir Volume :				**0.0E+00**	**0.0E+00**	**0.11444E-06**	**0.11923E-07**
Cost: 42380 \$, Emission: 15810 lbs. , Total Penalty: 1.157E-09									

Table 5.13: Optimal output from hydro and thermal units in Case 3 (with losses) of hydrothermal system -I

Time interval (hr.)	**Load Demand**	**Best compromised solution (with Losses)**							
		Power Generation (MW)				**Water Discharge ($Q_{iH} \times 10^4 m^3/hr$)**			
	P_D **(MW)**	P_{1T}	P_{2T}	P_{3T}	P_{HTot}*	Q_{1H}	Q_{2H}	Q_{3H}	Q_{4H}
1	750	175	211.154	51.134	312.732	10.317	6.708	29.824	9.154
2	780	175	296.393	50.3	258.307	8.233	6.442	29.547	6.215
3	700	175	210.11	51.438	263.462	7.398	7.683	29.89	6.592
4	650	175	128.3	52.322	294.384	10.275	8.565	29.879	7.681
5	670	175	216.072	51.909	227.027	6.206	6.172	29.95	6.16
6	800	175	219.5	51.66	353.853	13.145	6.316	12.15	9.088
7	950	175	300	148.12	326.9	13.702	9.282	29.97	6.814
8	1010	175	300	229.63	305.393	5.891	9.084	27.316	6.977
9	1090	175	300	320.83	294.197	5.8	6.246	29.828	6.572
10	1080	175	300	139.03	465.995	13.439	6.347	26.439	16.338
11	1100	175	300	142.16	482.864	6.157	6.642	10.12	19.28

12	1150	175	300	139.477	535.551	12.817	7.691	10	20
13	1110	175	300	133.814	501.21	5.847	6.362	10	20
14	1030	175	219.95	53.2	581.877	12.814	13.131	10	20
15	1010	107	292.753	55.936	554.345	13.846	6.142	10	20
16	1060	175	289.24	53.751	542.035	9.115	7.016	10	20
17	1050	175	300	64.671	510.368	5	6.789	10	20
18	1120	175	300	139.48	505.549	5	6.125	10	20
19	1070	175	300	56.326	538.707	5	14.132	10	20
20	1050	175	295.8	51.6	527.634	5	14.427	10	20
21	910	175	205.19	51.5	478.365	5	6.19	10	20
22	860	175	135.9	51.561	497.549	5	10.797	10	20
23	850	175	141.563	54.426	479.025	5	8,.717	10	20
24	800	20	230.824	56.708	492.478	5	15	10	20
			Reservoir Volume Error			**.76294E-05**	**.22889E-04**	**.15259E-05**	**.15279E-04**
			Cost: 43283 $, Emission: 18652 lbs. , Total Penalty: 2.655E-08						

$*$: P_{HTot} represents the total power generation from four hydro units.

5.8 SENSITIVITY ANALYSIS

The selection of control parameters of DE and NN is a challenging task. In this study, the simulations are carried out by considering impartial dependence as an initial value to a parameter. Mutation constant and crossover rate in DE are chaotically generated and the weak candidate solutions are not moved to a non-dominated set. DE adapts the best solution by the rejection of the same. In FNN, the slope β of the logistic sigmoid activation function plays an important role in the trap to a sub-optimal solution. The activation function varies between scaled linear function (β→0) and Heaviside step function (β→5). The candidate solution shows no improvement as the weighted sum at the neuron input reaches the maximum value.

Table 5.14: Sensitivity of parameter variations on Fitness value

β_{ih}	β_{hm}	Fitness Value	Percentage Variation	β_{ih}	β_{hm}	Fitness Value	Percentage Variation
0.1	0.002	--	--	0.51	0.152	64500	1.24
0.2	0.011	74638	32.85	0.53	0.163	64134	0.66
0.25	0.022	70647	10.89	0.56	0.19	64539	1.3
0.30	0.033	64204	0.77	0.61	0.21	64312	0.95
0.33	0.053	64953	1.95	0.7	0.3	64756	1.64
0.35	0.063	65392	2.64	0.8	0.4	66479	4.35
0.37	0.073	64953	1.95	1.0	0.5	64153	0.67

0.39	0.083	63995	0.44	1.15	0.6	64231	0.82
0.41	0.1	64387	1.06	1.1	0.7	65273	2.45
0.43	0.11	64202	0.77	1.1	0.8	64156	0.70
0.45	0.123	64666	1.5	1.5	0.8	65250	2.42
0.47	0.132	63942	0.365	1.5	0.9	65934	3.49
0.49	0.14	64387	1.06	1.5	1.0	66056	3.68
0.50	0.15	64244	0.84	1.5	1.1	66078	3.70

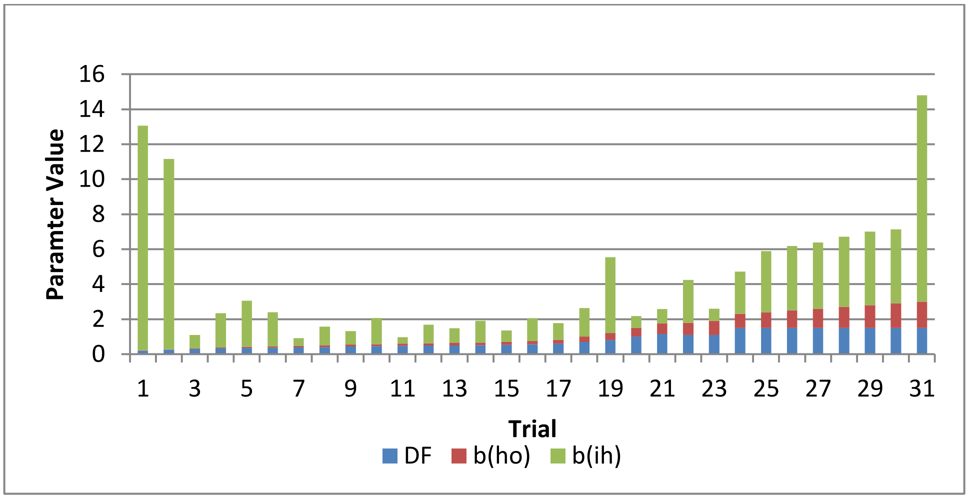


Fig. 5.14 Activation function parameter sensitivity analysis

DF: Percentage variation in fitness value, b(ho) parameter between hidden to the output layer, b(ih) parameter between the input to the hidden layer

In this study, a quantitative sensitivity analysis on the fitness of hydrothermal system-I is carried out for different values of the logistic sigmoid parameter of FNN and the results are presented in Table 5.14 of possible values. The activation function slope between the input to the hidden layer β_{ih} is randomly varied between 0.1 to 1.5 and slope of activation function between hidden to the output layer β_{hm} is varied between 0.022 to 1.5 to observe the effect of the fitness function value. Figure 5.14 depicts the percentage variation of fitness with the parameter values, but it does not give any definite trend although percentage variation is increasing on either side of the minimum value but not by the ratio of variation of slope values between the input to hidden and hidden to the output layer. Therefore this is the grey area of research to be explored upon further.

5.9 CONCLUSIONS

In this chapter, meta-optimization algorithm to solve the hydrothermal generation scheduling optimization problem is investigated. In the suggested algorithm, it is aimed to apply neural networks to perform computations with an efficiency comparable to that of the brain. Though metaheuristic techniques work in parallel through interactions between solutions but without interaction between variables. Interconnected parallel computation leads to relative explorability beneficial to achieve a global solution. DE-algorithm is applied for derivative-free computation of neural network parameters. The coupling constraints of hydro units and the load balance constraints violate with the initial set of population, yet this infeasible state trajectory is handled with penalty price. The variation in the optimal solution converges to small values as the search progresses due to regeneration of quality initial population. In this chapter, inspired by Salp swarm chain behaviour the concurrent interactions among population members, individual best and global best member in enabled to explore uniformly distributed population and its opposite one simultaneously. The chaotic DE responsible for robustness of the search process is recommended to optimise FNN parameters and it reduces training time. The accuracy and consistency of the SDENN algorithm have been confirmed by the good agreement in its performance when applied to three hydrothermal system test problems and also with compared results obtained from different search techniques. Further studies should employ the SDENN algorithm to address large-scale optimization problems and multi-objective capability. Also, different modifications will be made to SDENN algorithms, such as NN structure optimization, and techniques to resolve issues of tuning of parameters of DE and NN. The main challenge faced during training is to adjust the minimum and maximum limits of the weights and to decide the parameters of the activation function.

CHAPTER 6

CONCLUSIONS AND FUTURE SCOPE

6.1 INTRODUCTION

This chapter presents the salient and significant contributions of the thesis as well as a brief statement concerning some recommendations for further research. The significant original contributions are listed and deliberated.

6.2 ANALYSIS OF RESULTS FOR HYDROTHERMAL SYSTEM-I

The results obtained for day-long hourly scheduling of constrained hydrothermal system-I when solved with the differential evolution and its suggested variants; chaotic differential evolution (CDE), crisscross differential evolution (CCDE), CDE with trigonometric initialisation and mutation with sine-cosine function (CDESC) and crisscross based DE embedded with sine-cosine algorithm (CCDESC), CDE and CCDE synergistically hybridised with Simplex search method as CDSM and SaDESM and meta-optimisation algorithm SDENN are summarised in this chapter as a case study.

Quantitative results obtained of all investigated algorithms include the best, the mean and the standard deviation (SD) when the problem is solved for the objective as minimum cost and minimum emission simultaneously. The global solution search capability of the algorithms is assessed from the comparison of the cardinality ranking value. The highest value of cardinality ranking reveals the best-compromised solution obtained for the problem. The quantitative results are summarised in Table 6.1. The standard deviation is low in case of crisscross based DE and other CCDE based hybrid algorithms. This justifies that crisscross technique improves the quality of the solution. SDENN algorithm also gives good competition to other variants of DE based on crisscross operations.

Statistical significance non-parametric tests are performed to confirm the robustness of algorithms. Wilcoxon signed-rank test, Kolmogorov-Smirnov test of normality statistics are summarised in Table 6.1. The paired-wise significance tests Friedman’s test and Mann

Whitney's test, are performed and the descriptives of the statistics are presented in Table 6.2. It is observed that the p-value is less than 0.05 except in CDE and CDESC but D-statistics is reported high. All algorithms pass the normality test successfully, whereas the sample of CDE and CDESC exhibit nearly normal distribution. Paired-wise results indicate the low value of rank-sum for CCDE and its variants, this shows that CCDE and its hybrid algorithms offer high global solution accuracy and present better exploration and exploitation.

Table 6.1: Comparison of DE algorithms for hydrothermal system –I.

Algorithm	**Minimum Cost ($)**			**Minimum Emission (lbs.)**			**CPU time for 1000 iterations**	**Cardinality Ranking**	**p-Value**	**K-S test (D Statistics)**	**Interquartile range (IQR)**
	Best	Mean	SD	Best	Mean	SD					
DE	43065	43952.23	736.11	17271.41	17411.84	105.88	29 sec.	0.3368	0.02127	0.2684	1256.425
CDE	42845	43498.13	425.292	16300	16411	75.98	31 sec	0.6433	0.0627	0.2341	546.45
CCDE	40975.9	41631.84	224.715	14814.6	14846.33	22.565	58 sec	0.8402	0.0133	0.2678	301.70
**CDESC*	43153.1	43433.45	177.278	16896.2	17277	116.08	12.04 min	0.6992	0.1773	0.1953	292.83
**CCDESC*	41028.1	41505.6	196.676	14800	14832.57	16.53	2.10 min	0.7885	0.00178	0.33347	243.5
**CDESM*	41145.4	41775.55	462.85	15793	16311.77	309.77	3.38 min	0.76	0.00034	0.3703	496.7
**SaDESM*	41011.8	41494.74	336.32	14799	14848.65	23.308	52 sec.	0.8399	0.0491	0.1469	377.70
**SDENN*	41396.8	42801.35	566.68	14794.6	14961.54	160.177	68 min	0.7859	0.04872	0.24256	899.89

* all hybrid algorithms take 100 iterations of the inner loop

Table 6.2: Paired wise comparison of statistical significance tests

Pairwise Algorithms	**Friedman Test**		**Mann Whitney Test**	
	Rank Sum	χ_r^2	**U**	**z-score**
DE, CDE	DE: 48, **CDE: 42**	1.2	317	1.95894.
DE,CDE,CCDE	DE: 78, CDE: 72, **CCDE:30**	45.6	-	6.6456
CDE,CCDE	CDE: 59, CCDE:31	26.133	56	5.8177
CCDE, CDESC	**CCDE:30**, CDESC: 60	30	30	6.45093
CCDE, CDSC, CCDESC	CCDE: 52, CDESC: 90, **CCDESC: 30**	48.2667	--	--
CCDE, CCDESC	**CCDE: 42**, CDESM: 48	1.2	286	2.41725.
CCDE CDESM, SaDESM	CCDE: 64, CDESM:71, **SaDESM: 30**	12.0667	--	--
CCDE, CDESM	**CCDE: 38**, CDESM:52	6.5333	357	-1.36756.
CCDE, SaDESM	CCDESC:44, SaDESM: 46	0.1333	272	2.62424.
CCDE, SDENN	CCDE:60, **SDENN: 30**	30	262	2.7728
CCDE, CCDESC, SaDESM, SDENN	**CCDE:74, SaDESM:73, SDENN: 36**	**65.8**	--	--

For qualitative comparison of the investigated algorithms, the convergence trajectory, convergence rate and Pareto curve are compared and are presented in Figure 6.1 and 6.2

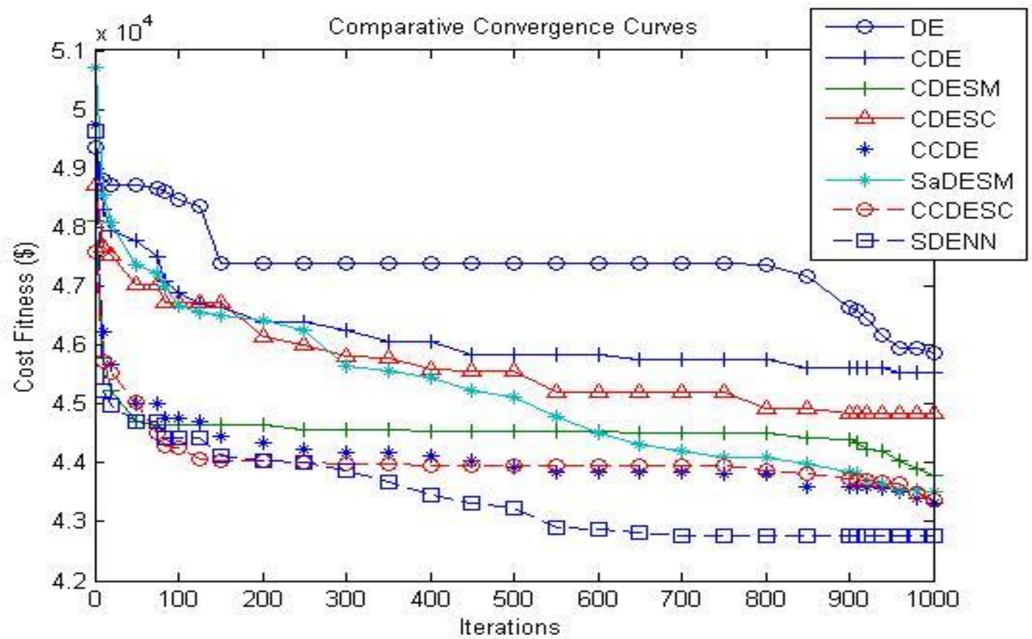


Fig. 6.1: Comparison of convergence curves of DE variants

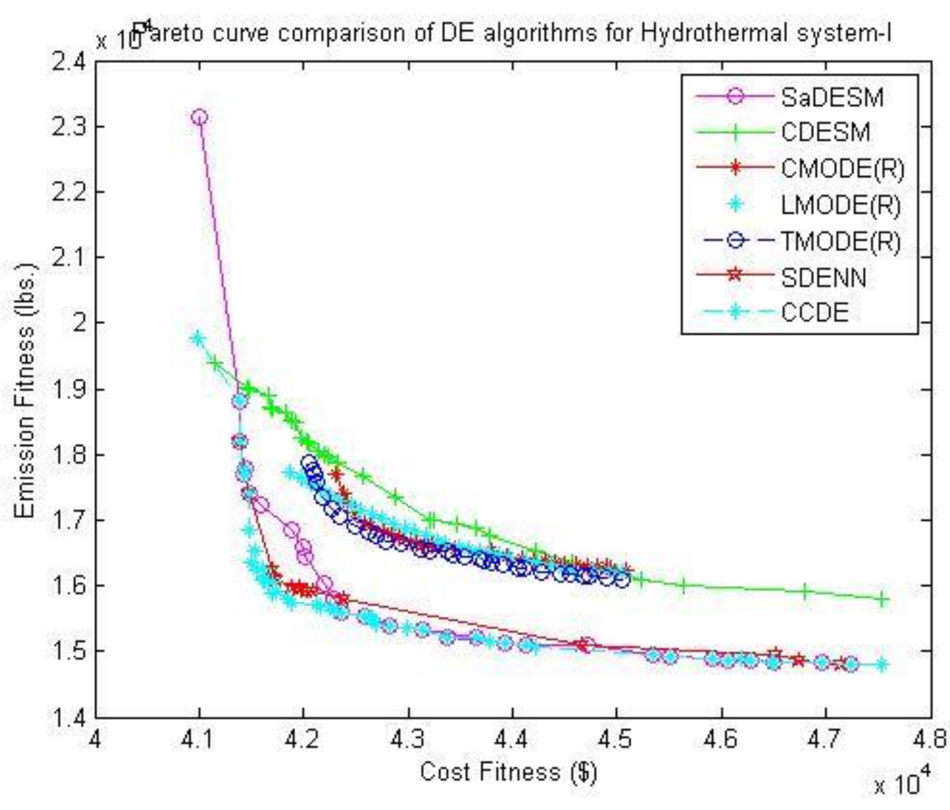


Fig. 6.2: Pareto front comparison of DE variants

. The convergence rate is more in the initial stages of convergence for CCDE, CCDESC and SDENN algorithms in comparison with SaDESM. The simplex search hybrid algorithm follows gradual convergence and concludes with a good quality solution. The Pareto front obtained from CCDE algorithm is the steepest among all, however, SaDESM and SDENN algorithms share remarkable performance.

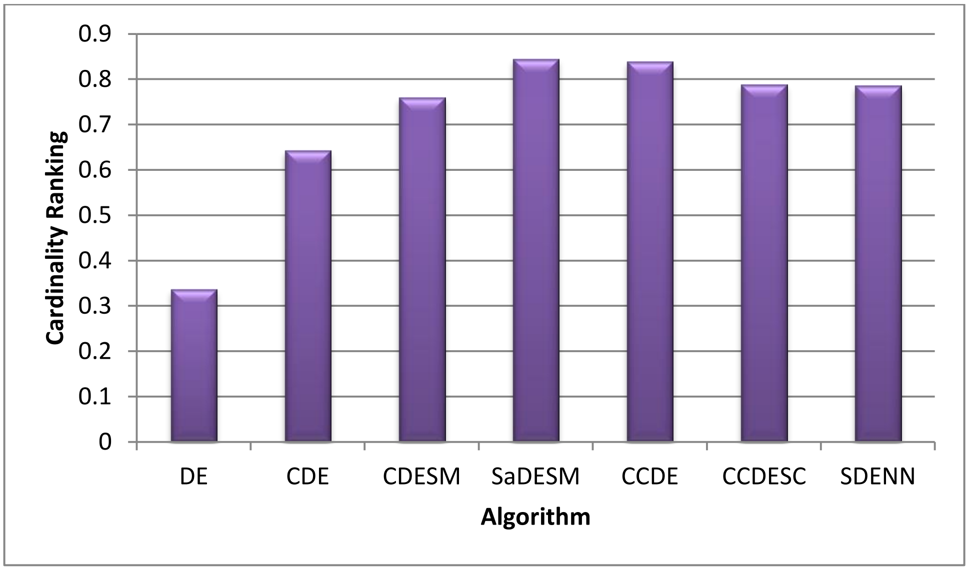


Fig. 6.3: Cardinality ranking comparison of DE variants

Cardinality ranking chart presented in Figure 6.3 shows that both CCDE and SaDESM are in a competitive race for a quality solution to hydrothermal system-I, It is concluded that the optimum solution of the hydrothermal system-I problem solved with crisscross differential evolution hybrid with simplex method achieves highest cardinality ranking. This qualifies for the quality of the best near-global solution, whereas ANN gives the minimum number of function evaluations, although the time taken is quite large. It is concluded from the comparison that the performance of SDENN is a competitive algorithm to solve real-world problems.

To summarise the case study, the hybrid of CCDE with simplex search method improves exploration and exploitation balance of the algorithm and better convergence rate when applied to solve hydrothermal system-I.

6.3 SIGNIFICANT CONTRIBUTIONS

In this study, derivative-free and global optimiser, differential evolution stochastic evolutionary algorithm has been selected to solve single objective and multi-objective global optimisation problems in the unconstrained and constrained form. Exploiting the exclusive advantages of global search and local search techniques, these have been integrated with differential evolution to alleviate its shortcomings.

Benchmark test functions of variety that includes unimodal, multimodal, low and high dimension, separable, non-separable, rotated and shifted functions have been considered as unconstrained optimisation problems.

Four highly constrained hydrothermal systems are investigated using the proposed hybrid population-based meta-heuristic methods. The hydrothermal systems include, single objective, bi-objective of conflicting nature, non-linear characteristics of thermal units with and without valve point loading, quadratic emission function and non-polynomial emission objective function with exponential terms, hydraulic coupling, and scheduling period of 24 hours, in addition to wide varying dimensions of the hydrothermal system are considered to investigate the proposed novel algorithms.

Through the investigations in this study, it is concluded that no single strategy can improve all the performance metrics of differential evolution when applied to a variety of optimization problems. However, the major contributions to P-heuristics when applied to solve non-linear multi-objective optimization problem are summed up in two phases as:

1. Three strategies to improve the initial population and logical movements of search in multi-direction search space have been invoked with differential evolution as an optimizer and have emerged out with success as:
    - Exploring the opposite population that improves the diversification of population,
    - Randomization of the population with the sine-cosine strategy that cyclically repositions the population member around the other and supports exploitation of space between two solutions. This strategy has taken care of the problem of isolation of global optimum at the final stage of the algorithm.
    - Using the Salp chain behaviour, the individual member of the population is guided towards the global best position through interactions among individual member, current best and global best members. ***This strategy has drastically increased the convergence speed in the initial iterations of the solution search***.
2. For the best extraction from logical movements of candidate solutions in multidirectional search, twelve mutation strategies have been investigated. The

mutation strategy **DE/rand-current-best/2** and **DE/rand-current-best/3 and ODE/rand/1** have emerged out as the better strategies and showed better performance in terms of convergence trajectory, convergence speed, and global optimum search capability however in application to the constrained hydrothermal system –I, strategy **DE/best/1** has shown competitive results**.**

3. For better mixing capability and balance of exploration and exploitation, two mechanisms have been investigated at mutation and crossover levels of DE.
   - The chaotic sequence of mutation constant and crossover rate dynamically controls diversification and intensification within feasible space and have supported the expansion of search space for better explorative search with a tendency to reduce the chances of skipping optimum solution.
   - Crisscross mutation and crisscross crossover operations are employed in orthogonal directions alternately in three-dimensional search space. This strategy has guided an effective diversification to achieve an efficient exploration of the search domain with disjointed feasible sub-regions and strength to pull out weak solutions from deceptive valleys of search space. ***Crisscross strategy has exhibited excellent results in the improvement of global solution search capability***.
4. In the final stage of the algorithm, opposition based learning is incorporated to generate potential Pareto set of non-dominant solutions and to archive elite solutions.

After the investigations of first phase modifications in differential evolution, a robust self-adaptive crisscross differential evolution algorithm (CCDE) has emerged out and the performance has been validated through illustrations on benchmark test functions and constrained hydrothermal systems.

5. In the second phase of the study, two different hybridisations of adaptive differential evolution have been investigated.
   - A novel optimization algorithm called hybrid sine-cosine with CCDE has been proposed for solving benchmark test functions and two constrained hydrothermal generation scheduling problems for minimum operating cost and minimum gaseous emission simultaneously. The feasibility and effectiveness of the proposed techniques are demonstrated on two electric power hydro-thermal systems for three

different types of scheduling; minimum cost, minimum emission and multi-objective. The first hydrothermal system has 3-thermal units and the 4-hydro and second hydrothermal system have 6-thermal units and 16-hydro units.

- The equality constraints are handled using the variable elimination method. The fuzzy logic approach is implemented to handle the uncertain violations in constraints that arise due to randomness in the decision variables, unpredicted variations in hydro and thermal resources and random values of other parameters of stochastic algorithms. The violations in inequality constraints are quantified using trapezoidal membership functions and violations in the equality constraints are quantified using linear membership functions. The quantified constraint function is added to the overall objective function to be optimized using the stochastic evolutionary technique. To decide the 'best' generation schedule for multi-objective scheduling problem, fuzzy cardinal priority ranking is compared to various reported results. ***The results reveal that the proposed constraint handling strategy generates better quality solutions by reducing cost and emission level and a high degree of satisfaction of equality constraints.***

  The proposed hybridization CCDESC has accelerated the convergence to a solution with good accuracy and exhibit stability as well.

- In the second attempt of hybridization, CCDE is hybridized with simplex local pattern search method. The pattern search method extricates metaheuristics from wandering around the optimal solution. The feasibility and effectiveness of the proposed techniques are demonstrated on two electric power hydro-thermal systems for three cases of scheduling; minimum cost, minimum emission and multi-objective. The effect of hybridization is investigated through simulations on benchmark test functions as well. This hybridization SaDESM exhibits satisfactory results.

After the investigations of hybridised algorithms, it has been observed that simplex local search method (SaDESM) accelerated the convergence during the initial phase of evolution search and support in the final stage of metaheuristics to obtain good accuracy quickly. However, in comparison, the other memetic hybridisation with a sine-cosine algorithm (CCDESC) shows gradual convergence but satisfactory to a good extent.

- In the third phase of the study, chaotic differential evolution (CDE) is hybridized with an artificial neural network, in which CDE is applied to learn the weights of feedforward neural network. Salp chain population regeneration strategy is incorporated with SDENN algorithm. The feasibility and effectiveness of this combination are validated through illustration on three hydrothermal systems. The third hydrothermal system with one thermal and four hydro units is scheduled for 24 hours for minimum cost only. The fourth hydrothermal system with 3-thermal and 4- hydro units are hourly scheduled for 24 hours for minimum cost and minimum emission with and without transmission losses. The multi-objective solution is obtained with consideration of 50% weightage to each conflicting objectives. The first hydrothermal system with three thermal and four hydro units is hourly scheduled for 24 hours for minimum cost, minimum emissions, multi-objective scheduling with consideration of equal weightage to each of the objective. The results obtained give are comparable with other hybrid algorithms of DE and other compared results. This meta-optimization strategy will justify the computational expenses in terms of space and time when applied to large-scale optimization problems, in case the issue of parameter values is resolved.

The proposed methods are promising in practice and compete with the other compared methods in terms of computational costs and the success of obtaining global solutions.

6.4 SUGGESTIONS FOR FUTURE RESEARCH

The presented results have demonstrated the efficacy of the proposed P-heuristic optimization algorithm and their hybrids to solve unconstrained benchmark test functions and constrained multi-objective hydro-thermal generation scheduling; still, there is a lot of scope to improve the performance of P-heuristic optimization techniques and these heuristic techniques can be applied to solve different optimization problems. Some of the possible further research directions are summarised as:

- The stochastic model of a multi-chain hydro model can be implemented.
- Generation maintenance scheduling can be added in a coordinated hydrothermal scheduling problem.
- Stochastic constraint handling can be implemented.

- P-heuristics and their hybrids can be applied to solve the combined operation of conventional hydrothermal systems with renewable energy sources.
- The further scope of research may be to investigate optimal hydro-thermal scheduling with renewable resources in a market-driven competitive environment of energy contracts.
- Hyper-heuristics can be explored to solve hydrothermal generation scheduling problem.
- Constrained benchmark test functions can be included in the study.
- Deep statistical comparisons can be included for comparison of P-heuristic techniques.

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APPENDIX – A

ALGORITHMS AND STATISTICAL METHODS

A.1 DIFFERENTIAL EVOLUTION OPTIMIZATION TECHNIQUE

In a population-based meta-heuristic search method, it is indeed a challenging task to maintain the balance between the global search capability and the convergence speed of the algorithm. For solving non-linear, non-convex, multi-modal real-world optimization problems, the mechanism to select the control parameters, a variety of combinations of evolution strategies, dual search approach etc. are proposed in the literature with varying degree of success to achieve improved solution accuracy, convergence rate and computation time.

Similar to evolutionary algorithms, the trial vectors with variation from one generation to the next generation are achieved with two main operators namely mutation and crossover in differential evolution (DE), however it differs from these evolutionary algorithms to determine search distances and directions in the sense that:

- the mutation is applied first to generate a trial vector, followed by a crossover operator to produce one offspring
- mutation step sizes are not sampled from a prior known probability distribution function.

The central limit theorem states that the probability distribution governing a random variable approaches the normal distribution as the number of samples of that random variable tends to infinity.

The population of individuals is generated by uniform random initialization representing the entire search space bounded by the limits of decision variables, such that an individual vector $\mathcal{X}_i^t$, is genetated by using equation (A.1) as:

$$x_{ij}^t \sim U(x_i^{min}, x_i^{max}), \quad \ldots \text{(A.1)}$$

where x_i^{min} and x_i^{max} are the minimum and maximum values of the decisional variable defined in the problem domain.

The position of individuals provides information about the fitness of the landscape. The classical DE has been developed for searching through continuous-valued landscapes as depicted in Figure A.1.

The diversity of the current population is assessed from the distances between individuals and the order of magnitude of the mutation step sizes those are taken for the population to contract to one point. The deviation of this distribution is determined by the magnitude of the difference of individual vectors. It is emphasized to use more than one differential to determine the mutation step size.

The total number of differential perturbations will be defined as :

$$\binom{N_p}{2n} 2n! = \mathcal{O}\left(N_p{}^{2n}\right), \qquad \text{…. (A.2)}$$

where n is the number of difference vectors and N_p is the population size.

Equation (A.2) expresses the total number of directions that can be explored per generation. To increase the exploration power of DE, the number of directions can be increased by increasing the population size and/or the number of differentials used.

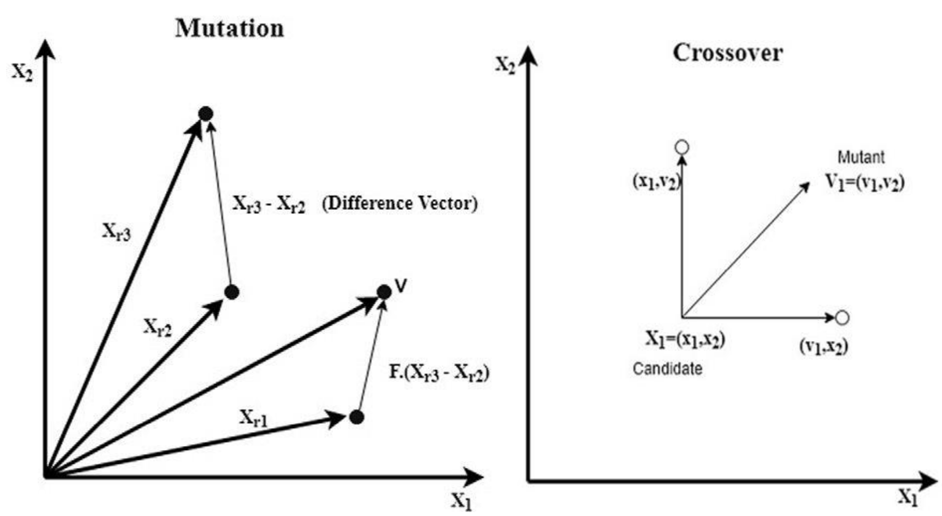

Fig. A.1: Graphical representation of differential evolution mutation and crossover

Mutation: The DE mutation operator produces a trial vector for each vector of the current population by mutating a target vector from the population with a weighted differential vector, such that

$$U_i^t = x_{i1}^t + F(x_{i2}^t - x_{i3}^t); \quad i \neq i_1 \neq i_2 \neq i_3 \quad \text{and} \quad i_1, i_2, i_3 \in [1, N_p] \qquad \text{…. (A.3)}$$

Where U_i^t is the new trial vector, x_{i1}^t is the target vector from a population, x_{i2}^t, x_{i3}^t are two randomly selected vectors from the population, $F \in (0, \propto)$ is the scale factor that controls the magnitude of the difference of vectors.

Crossover: This trial vector generated in Equation (A.3) will be used by the crossover operator to produce offspring. DE crossover operator implements discrete recombination of the trial vector U_i^t, and the parent vector, x_i^t, to produce offspring, V_i^t defined as:

$$V_{ij}^t = \begin{cases} U_i^t & ; U(0,1) < CR \;\; or \; j \neq j_{rd} \; and \;\; j_{rd} \in [1, D] \\ x_{ij}^t & ; \;\; otherwise \end{cases} \qquad \text{...(A.4)}$$

x_{ij}^t is the j^{th} an element of the vector x_i^t, j_{rd} is the set of crossover points (elements that will undergo perturbation and its value is determined by either of two crossover methods.

Binomial crossover: The crossover points are randomly selected from the set of possible crossover points, $j_{rd} \in$ (1, 2,...,D), where D is the dimension of the problem. CR defines the probability that the considered crossover point will be included. The larger the value of CR, the more crossover points will be selected compared to a smaller value. This means that more elements of the mutant trial vector will be used to produce the offspring and less of the parent vector.

Exponential crossover: From a randomly selected index, the exponential crossover operator selects a sequence of adjacent crossover points, representing a circular list of potential crossover points.

Selection

The population for the next generation is selected with the deterministic selection method. According to this, the offspring replaces the parent if the fitness of the offspring is better than its parent; otherwise, the parent survives to the next generation. This ensures that the average fitness of the population does not deteriorate and is defined as:

$$Z_{ij}^{t+1} = \begin{cases} V_{ij}^t & ; if \; f(V_{ij}^t) \leq \; f(x_{ij}^t) \\ x_{ij}^t & ; \qquad otherwise \end{cases} \qquad \text{... (A.5)}$$

The procedural steps of classical DE algorithm are presented as:

Read Input data and constants of an algorithm.

1. do-while ($t < ITmax$)
2. Generate initial population of variable $X(i, j); i \in (1, Np) and \; j \in (1, D)$ using equation (A.1)
3. do-while ($i < Np$)
    - 3.1 Generate random numbers $r_1, r_2, r_3, r_4 \in Np \;\; such \; that \; r_1, \neq, r_3, \neq r_4 \neq i$
    - 3.2 Perform mutation operation and generate trial vector U_i^t using equation (A.3)

3.3 Compute fitness value.
For $k \in (1, D)$
3.4 Generate a random number $j_{rn} \in (1, D)$
3.5 Perform crossover and generate trial vector V_{ij}^{t} using equation (A.4)
3.6 Compute fitness value.
end-for
3.7 Perform a selection of population individual for next-generation using equation (A.5)
enddo
enddo
4. Stop

The performance of classical DE is influenced by population size, intilisation of two control parameters scale factor F and crossover probability CR and their upgradation.

Population size: The size of the population has a direct influence on the exploration ability of DE algorithms. More the individuals in the population, the more differential vectors are produced. This will increase the number of directions to be explored. However, it should be kept in mind that the computational complexity per generation increases with the size of the population. For n number of difference vectors, $2n$ number of individuals are needed. The lower bound on the individuals in the population (minimum size of the population) will be $N_p > (2n + 1)$, such that after n combinations, the additional individual represents the target vector.

Scaling factor: The scaling factor, $F \in (0, \propto)$ controls the amplification of the differential variations. The smaller the value of F, the smaller the mutation step sizes, and the algorithm will take more time to converge. On the other hand, larger values for F facilitate exploration but may cause the algorithm to overshoot good optima. The value of F should be small enough to allow differentials to explore deceptive valleys, and large enough to maintain diversity. As the population size increases, the scaling factor should decrease. With more individuals in the population, it will reduce the magnitude of the difference vectors, and the individuals will be closer to one another. Therefore, smaller step sizes can be used to explore local areas. The larger population size will reduce the need for large mutation step sizes.

Recombination Probability: The probability of recombination, CR has a direct influence on the diversity of DE. This parameter controls the number of elements of the parent vector, that will change. The higher the probability of recombination, the more variation is introduced in the new population, thereby increasing diversity and increasing exploration. Increasing CR results in faster convergence, while decreasing CR increases search robustness.

A.2 SINE-COSINE ALGORITHM

Stochastic optimization algorithms offer high flexibility and alleviate the solution to optimization problems randomly. These algorithms benefit from local minima avoidance largely as compared to conventional optimization algorithms. Simple mathematical functions are also used to design optimization algorithms such as criss-cross technique, sine-cosine algorithm, radial basis function algorithm, newtonian theory-based algorithms.

The sine-cosine algorithm is a population-based optimization algorithm proposed by Seyedali Mirjalili, (2015). This search method differs from other stochastic optimization techniques in a way, that it has two phases of the optimization process. In the exploration phase, it combines the random solutions in the set of solutions abruptly with a high rate of randomness to search the potential regions of feasible space. In the exploitation process, there are gradual changes in random solutions. The sine–cosine algorithm uses trigonometric sine and cosine functions to set random solutions to oscillate around the best solution. The cyclic pattern of sine cosine functions allows the solution to be repositioned around another solution and this property strongly emphasize the better exploitation of space between two solutions (Nenvath *et al.*, 2017), For exploring a search space, it is important to search solutions outside the space between their corresponding destinations. This is achieved by changing the range of sine and cosine functions. The current position of potential solutions is updated by using equations (A.6) and (A.7) expressed as:

$$X_i^{t+1} = X_i^t + r_1 \times sin(r_2) \times |r_3 P_i^t - X_i^t| \quad \ldots. \quad \text{(A.6)}$$

and

$$X_i^{t+1} = X_i^t + r_1 \times cos(r_2) \times |r_3 P_i^t - X_i^t| \quad \ldots. \quad \text{(A.7)}$$

where X_i^t is the position of the current solution of the i^{th} decision variable in the t^{th} iteration. $r_1, r_2, and\; r_3$ are the random numbers and P_i^t is the position of target position of the i^{th} decision variable in the t^{th} iteration. Mathematical operator |-| specifies the absolute value.

The switching between equations (A.6) and (A.7) that selects the sine function or cosine function, is controlled by key parameter $r_4 \in [0,1]$ as:

$$X_i^{t+1} = \begin{cases} X_i^t + r_1 \times sin(r_2) \times |r_3 P_i^t - X_i^t| & ; r_4 < 0.5 \\ X_i^t + r_1 \times cos(r_2) \times |r_3 P_i^t - X_i^t| & ; r_4 \geq 0.5 \end{cases} \quad (A.8)$$

Sine-Cosine mathematical model involves four parameters r_1, r_2, r_3, r_4, although the values are randomly generated.

The parameter r_1, decides about the next position's movement direction, which could either be in space amidst the solution and the destination or outside this space.

r_2, decides about the distance of the movement from or towards the destination.

r_3, decides the weight of influence of the destination in describing the distance of movement. For the value of $r_3 < 1$, it is stochastically less emphasized and $r_3 > 1$, it is stochastically more emphasized.

r_4 is the key parameter that equally switches between sine or cosine function.

Figure A.2 describes the effect of sine and cosine function. A conceptual model of the effects of sine and cosine function is illustrated within the range in [-2.2]. For defining the random location inside or outside is achieved by defining parameter r_2 in the range [0, 2π].

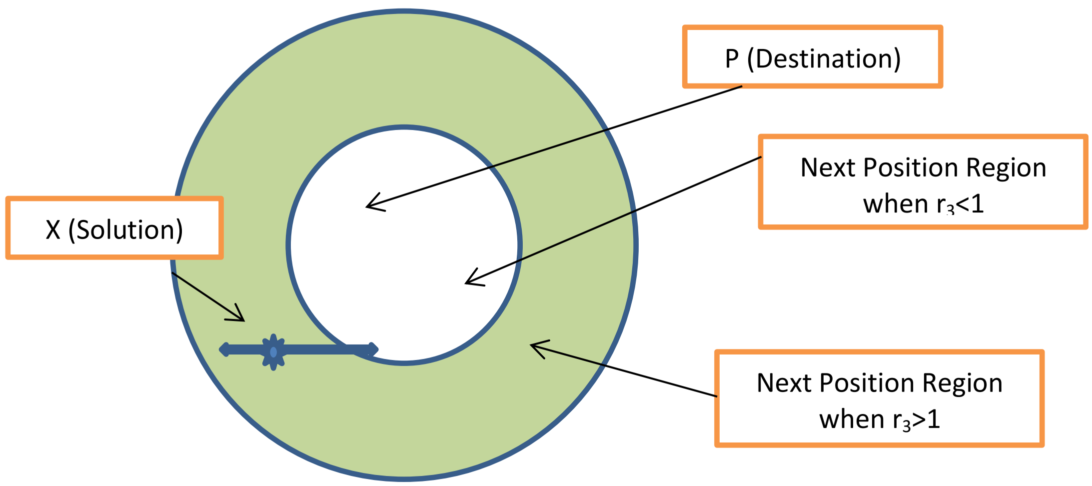

Fig. A.2: Effect of Sine and Cosine Function on new position

To locate the potential region of feasible search space, it is necessary to strike a balance between exploration and exploitation process to target the global optimum position. To achieve this, the range of sine and cosine function can be changed adaptively as defined by equation (A.9)

$$r_1 = \alpha - t\frac{\alpha}{ITMAX} \qquad (A.9)$$

where t is the current iteration, ITMAX is the maximum number of iterations and α is a constant. The procedural steps of sine cosine algorithm are presented in algorithm A.2 as:

ALGORITHM-A.2: Sine-Cosine Algorithm

1. Initialise asset of random solutions (X)
2. Start the movement counter *t*=1
3. *DO*

   3.1 Compute the fitness value of all parent vectors and compute the minimum objective function among the parent population.

3.2 Update the current best solution obtained as (P) using equations (A.6) and (A.7)
3.3 Generate parameters r_1, r_2, r_3, r_4 randomly, in the corresponding appropriate range
3.4 Update the position of solutions using Equation (A.8)
WHILE ($t \leq$ Maximum number of iterations)
4. Return the best solution obtained so far as global optimum.
5. STOP

The Sine-Cosine algorithm determines global optimum as it is intrinsically benefiting high exploration and avoid local optima as compared to an individual-based solution. For values in the range [-1.1], promising regions of the search space are exploited. When parameter r_1 is adaptively updated, the transition from exploitation to the exploration is smooth. Destination point stores, the best approximation of global optimum. The solutions update their positions around the best solution obtained so far, there is more tendency of updating of position towards the best region in the search space.

A.3 SIMPLEX SEARCH METHOD

Simplex search method introduced by George Dantzig in 1947, is a linear programming method to solve linearly constrained linear problem. Nelder and Mead, (1965), have introduced a simplex search algorithm as a direct search method for multidimensional unconstrained function minimization. This algorithm is simple, easy to use and require less memory storage. It does not require derivatives of function that makes it quite suitable to solve problems with non-smooth and discontinuous functions (Kold *et al.*, 2003). In an $N-$ dimensional search space, a simplex is $(N+1)$ polytope of $(N+1)$ vertices, such that each vertex represents the decision variable $x_i\,; i = 1,2, \ldots, (N+1)$. The point from which a pattern move is initiated called a *base point*, and the direct search procedure is conceived as fundamentally proceeding from one base point to a new point in search space which has a desirable value for the objective function and replaces least desirable point in a feasible set of solutions. This method is comparable to the evolutionary method in terms of selection of $(N+1)$ individuals and replacement strategy (Takjahama and Sakai, 2005).

The selection operation is viewed with three special variations called transformations; namely reflection, contraction and expansion as depicted in Figure A.3 (Nelder, 1965). The simplex technique looks for the perspective individuals along the line starting from the centroid towards the best point. It explores the space in the best forward direction with expansion and contraction operations and replaces the worst individual solution with a better solution until the best solution is obtained or termination criteria is met.

1. Initialize a set S of a simplex by generating n+1 points as the population of individual decision variables $x_{ki}; i = 1,2 \ldots N\,, k = 1,2 \ldots NP$
2. Find three points x^w, x^b, x^h in the population for which the function has the best, the worst and second-worst values defined as follows:

$$x^b = \overbrace{min\ f(x_k)}^{k}$$

$$x^w = \overbrace{max\ f(x_k)}^{k}\ ; \qquad \ldots \text{(A.10)}$$

$$x^h = \overbrace{max\ f(x_k)}^{k \neq w}$$

3. Find base point as a centroid x^0 of all points except the worst point x_{kh}, using Equation (A.11)

$x_i^0 = \frac{1}{n}\sum_{\substack{k=1 \\ k\neq h}}^{n} x_{ki} \quad \ldots$ ….(A.11)

4. Calculate the reflection point x^r of worst value about base point using Equation (A.12)

   $x^r = (1+\alpha)x^0 - \alpha x^w \quad ; \alpha > 0$ ….(A.12)

5. Check whether x^r is better than the best point x^b , if $\left(f(x^r) < f(x^b)\right)$ then go to step 6 else go to step 7.
6. Calculate the expansion point x^e in the direction from base point to reflected point using Equation (A.13)

   $x^e = (1-\beta)x^0 + \beta x^r \quad \beta > 1$ … (A.13)

7. Check whether x^e is better than the best point x^b, if $\left(f(x^e) < f(x^b)\right)$ then replace worst point with expansion point as $x^w = x^e$, else replace $x^w = x^r$ and go to step 2.
8. Repeat till the termination criteria are satisfied.

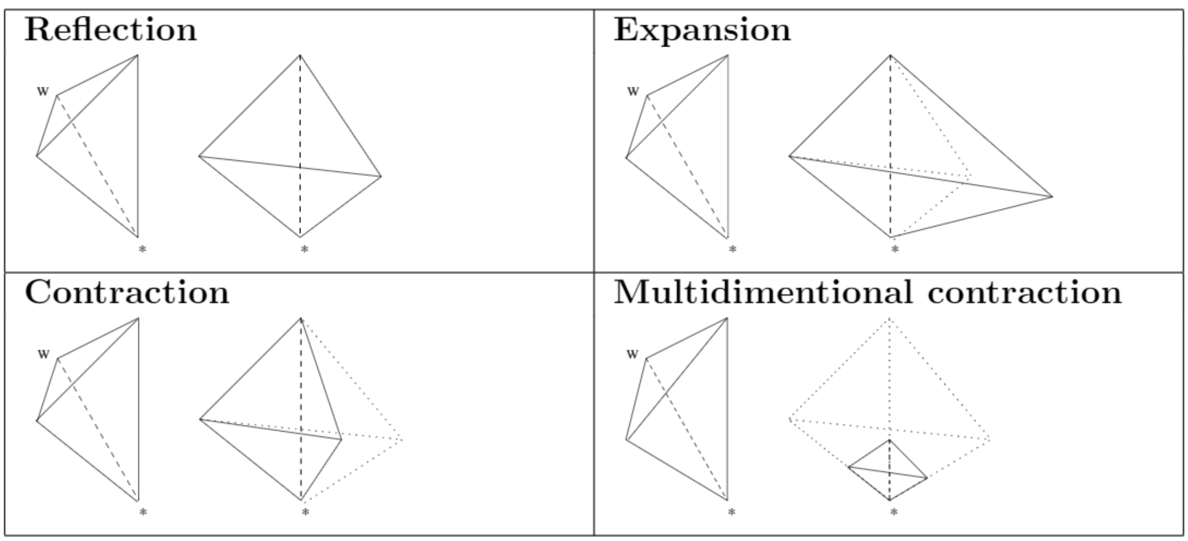

Fig. A.3: Graphical representation of Simplex

ALGORITHM-A.3: Simplex search Method

1. Initialize simplex surface of a population of N+1 points.
2. Set population counter *NP=1*
3. *Compute three points* x^b, x^w, x^h *as the best, the worst and the second-worst solution in the population using Eq. (A.10)*

DO

3.1 Compute base point using Eq. (A.11)

3.2 Compute the reflected point x^r using Eq. (A.12)

If the reflected point is better than base point i.e. $f(x^r) < f\left(x^b\right)$

Compute expanded point x^e

If x^e is better than the best point x^b then $x^w = x^e$
Else $x^w = x^r$

4. Set coordinate search direction index *j=0*
5. Save decision variable for pattern search direction as Z_i .

DO

6. Increment, coordinate search direction index, *j=j+1*
7. Randomly generate the step length λ_i by implying Eq. (A.11)
8. *IF* (*j =N+1*)
   8.1 Pattern search direction is obtained using Eq. (A.13)
   8.2 *m=m+1*
   ELSE
   8.3 Decision variable modified as per Eq. (A.11)
   8.4 If the objective function converges, update variable by implying Eq. (A.13)

ENDIF
WHILE (j< N+1)
WHILE (m < N)

9. Increment movement counter, $IT_P = IT_P+1$

WHILE ($IT_p \leq IT_{pmax}$)
STOP

Direct Search methods are used to provide solutions to some problems, which cannot be solved using classical (gradient) method.

- Search methods work even if the objective function is discontinuous and non-differentiable at some points.
- Search methods provide faster solutions to problems as compared to classical methods.
- Search methods are easily implemented on computers being repetitions of identical arithmetic operations with simple logic.
- Search methods are capable of providing approximate solutions at every step during the process of problem-solving.
- Search methods are used to find the global minimum when the objective function possesses several relative minima.

A.4 NONPARAMETRIC INFERENCE TESTS

Statistical procedures are developed to carry out statistical analyses on the evaluation process to improve its performance. It is important to carry out experimental analysis to confirm whether the newly proposed method offers a significant improvement over the existing one or not. Statistical procedures are categorized in two classes depending upon the type of substantial data employed for an application (Garcia *et al.*, 2010) namely; parametric and non-parametric.

Parametric statistical tests are commonly used in the analysis of experiments in computational intelligence with some assumptions, *viz* independence, normality and homoscedasticity and then the test will have a lower value of Type I error and higher power. In this statistical test, it is assumed that data comes from a type of probability distribution and inferences are made about the parameters of the distribution.

Further, the analysis of results is based on one of the two alternatives: single problem analysis and multiple-problem analysis. In single problem analysis, it corresponds to the study of the performance of several algorithms over a unique problem case whereas multiple problem analysis pertains to the study of several algorithms over more than one problem simultaneously. The parametric test, in the latter case, reach erroneous conclusions.

Non-parametric tests are inference procedures those do require the population distribution to be normal or some specified in terms of parameters (*Corder and Foreman, 2014).*

Statistical inferences may be based on hypotheses testing on one or more applications. There are two hypotheses defines. One; null hypothesis that means no difference in algorithms and other; alternative hypothesis means there is a significant difference in the algorithms. When applying a statistical procedure to reject a hypothesis, a level of significance α is defined to determine at which level the hypothesis may be rejected. Instead, of setting a certain level of significance to reject the hypothesis, it is more significant to calculate the lowest level of significance that results in rejection of the null hypothesis. This is called p-value that is defined as the probability of obtaining a result at least as extreme as the one that was observed assuming that the null hypothesis is true. Smaller the p-value, the stronger is the evidence against the null hypothesis.

The non-parametric test normally deals with nominal data and also can be applied to continuous data by making some adjustments according to test requirements. It can perform two types of analysis; pairwise comparison and multiple comparisons. Pairwise statistical procedures perform individual comparisons between two algorithms obtaining in each application a p-value independent from the other one. These are the simplest one. Signed test and Wilcoxon signed test are the examples of simple, robust non-parameter test for pairwise statistical comparisons.

A.4.1 Wilcoxon Signed-Rank Test

The Wilcoxon's signed-rank test was introduced by F. Wilcoxin in 1945. It is a statistical hypothesis test to assess the difference in the population mean ranks in the comparison of two related samples. This test is analogous to the paired t-test for dependent samples when the population is assumed to be normally distributed.

A Wilcoxon signed-rank test is a non-parametric test that can be used to determine whether two dependent samples were selected from populations having the same distribution. The assumptions to perform this test are:

- Data is a paired sample from the same population
- Each pair is chosen randomly and independently
- Data is ordinal.

Wilcoxon' test is defined as:

Assume a sample size of N number of pairs such that there are $2N$ data points such that $X_{1i}\ and\ X_{2i}\ (i = 1,2, \dots N)$ be the measurements for paired samples.

Assume the hypothesis as :

H_0: Difference between the pairs follows a symmetric distribution around zero.

H_1: Difference between the pairs does not follow a symmetric distribution around zero.

The procedure of Wilcoxon Test Measurement:

1. Calculate the diffrence $|X_{1i} - X_{2i}|$ and $sgn(X_{1i} - X_{2i})$, for $(i = 1,2, \dots N)$
2. Eliminate the pairs with a difference $|X_{1i} - X_{2i}|$ equal to zero and the size of paired samples will be N_R.
3. DO
   3.1 Arrange the absolute difference of the sample pairs in ascending order.
   3.2 Assign the rank R_i to the pairs with the smallest different as rank 1. In case there is a tie, rank is assigned equal to the average of ranks corresponding to their span.
   WHILE $(i = 1,2, \dots N_R)$
4. Calculate the test statistic as the sum of signed ranks. $W = \sum_{i=1}^{N_R} sgn(X_{1i} - X_2 i) \times R_i$

5. If $|W| < W_{critical,N_R}$ then reject the null hypothesis H_0
6. In case $N_R > 20$, compute z-sore using equation D.2

   $z = \frac{W}{\sigma_w}$., $where\ \ \sigma_w = \sqrt{\frac{N_R(N_R+1)(2N_R+1)}{6}}$

   In case If $|z| > z_{critical,N_R}$ then reject the null hypothesis H_0
6. For the one-sided tail test, calculate the p-value.

A.4.2 Mann-Whitney' s Test

This is a non-parametric test of the null hypothesis. It does not require the data sample to be normally distributed like t-test but it is as efficient as the t-test. This test is performed on two independent samples from populations to determine whether they have the same distribution (Derrac *et al.*, 2011). The assumptions to perform this test are:

- Two data samples are independent of each other.
- Data is ordinal.

Mann-Whitney's test is defined as:

Assume a sample size of N number of pairs such that there are $2N$ data points such that $X_{1i}\ and\ X_{2i}\ (i = 1,2, \ldots N)$ be the measurements for independent samples.

Assume the hypothesis as :

H_0: Both Samples have the same distribution

H_1: Both samples have a different distribution.

The procedure of Mann-Whitney's Test Measurement:

This test involves the calculation of the statistics called U. The maximum value of the statistics U is the product of the size of two samples.

1. Combine the observations of both groups and arrange them in ascending order.
2. Assign a rank starting from the smallest value of observation.
3. In case of a tie, assign them same rank adjusted ranks.
4. Add the ranks of the observations of two groups separately as $R_1 and\ R_2$.bsuch that sum of all ranks $R_1 + R_2 = \frac{N(N+1)}{2}$. Where $N = N_1 + N_2$
5. Compute the statistic U for each group as

   $U_1 = R_1 - \frac{N_1(N_1+1)}{2}$; N_1 is the number of observations in group 1 and

   $U_2 = R_2 - \frac{N_2(N_2+1)}{2}$; N_2 is the number of observations in group 2.
6. Compute statistics $U = U_1 + U_2 = N_1 \times N_2$
7. Find $U_S = \max(U_1, U_2)$
8. Find the critical value of statistic U from the table of critical values for different population sizes at the level of significance α(may be 0.01,0or 0.05) for a given value of $N_1\ and\ N_2$, as U_c.
9. Reject null hypothesis H_0; if $P(U_S < U_c) \geq \alpha$
10. Conclude.

A.4.3 Friedman. Test

The Friedman is a non-parametric test developed by Milton Friedman. This hypothesis test is to test the difference between three or more paired groups when the dependent variable is at least ordinal. This test is preferred where the same parameter has been measured under different conditions on the same subject. The Friedman test is to test the k number of paired samples (k>2) of n size, from the same population or the random samples from populations having similar properties.

The procedure to conduct the Friedman test is as follows:

1. Rank each row together and independently of the other rows. When there are ties, consider the average ranks of the observations.
2. Sum the ranks for each treatments column and then sum the squared columns
3. Compute the test statistic using

$$Q = \frac{12}{nk(k+1)} \sum_{i=1}^{k} r_i^2 - 3n(k+1)$$

where r_i is the sum of the ranks for i^{th} sample.
n is the number of independent sample size.
k is the number of groups

7. Determine the critical value using a Chi-square distribution table with (k-1) degree of freedom.
8. Formulate the decision and conclude.

APPENDIX-B

BENCH MARK TEST FUNCTIONS

The benchmark functions used to test the proposed algorithm are given in Table B.1.

Table B.1: Benchmark functions

S. No	Name of Function	Test Function Representation	Dim. 'D'	Domain 'S'	$f_{min}{}'$
1	Sphere	$f_{01} = \sum_{i=1}^{D} x_i^2$	30,50	[-100, 100]	0
2	Schwefel 2.22	$f_{02} = \sum_{i=1}^{D} \lvert x_i \rvert + \prod_{i=1}^{D} \lvert x_i \rvert$	30,50	[-10, 10]	0
3	Schwefel 1.2	$f_{03} = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} x_j \right)^2$	30,50	[-100, 100]	0
4	Schwefel 2.21	$f_{04} = max_i\{\lvert x_i \rvert, 1 \le i \le D\}$	30, 50	[-100, 100]	0
5	Rosenbrock	$f_{05} = \sum_{i=1}^{D-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30, 50	[-30, 30]	0
6	Step	$f_{06} = \sum_{i=1}^{D-1} (\lvert x_i + 0.5 \rvert)^2$	30, 50	[-100, 100]	0
7	Quartic	$f_{07} = \sum_{i=1}^{D} ix_i^4 + random(0,1)$	30, 50	[-1.28, 1,28]	0
8	Rastrigin	$f_{08} = \sum_{i=1}^{D} (x_i^2 - 10cos(2\pi x_i) + 10)$	30, 50	[-5.12, 5.12]	0
9	Schwefel 2.26	$f_{09} = \sum_{i=1}^{D} \left(-x_i sin\left(\sqrt{\lvert x_i \rvert}\right)\right) + 418.9829 * D$	30, 50	[-500, 500]	0
10	Alpine	$f_{10} = \sum_{i=1}^{D} \lvert x_i sinx_i + 0.1x_i \rvert$	30, 50	[-10, 10]	0

Table B.1: Benchmark functions (continued)

S. No	Name of Function	Test Function Representation	Dim. 'D'	Domain 'S'	$'f_{min}'$
11	Griewank	$f_{11} = \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30, 50	[-600, 600]	0
12	Ackley	$f_{12} = -20exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - exp\left(\frac{1}{D}\sum_{i=1}^{D} cos(2\pi x_i)\right) + 20 + \exp(1)$	30, 50	[-32, 32]	0
13	Penalised 1	$f_{13} = \frac{\pi}{D}\left\{10sin^2(\pi y_i) + \sum_{i=1}^{D-1}(y_i - 1)^2[1 + 10sin^2(\pi y_{i+1})] + (y_D - 1)^2\right\} + \sum_{i=1}^{D} u(x_i, 10,100,4)$ Where $y_i = \left(1 + \frac{x_i+1}{4}\right)$ and $u(x_i, \alpha, k, m) = \begin{cases} k(x_i - \alpha)^m & ; x_i > \alpha \\ 0 & ; -\alpha < x_i < \alpha \\ k(-x_i - \alpha)^m & ; x_i < -\alpha \end{cases}$	30, 50	[-50, 50] Where $\alpha = 10;$ $k = 100;$ $m = 4.$	0

Table B.1: Benchmark functions (continued)

S. No	Name of Function	Test Function Representation	Dim. 'D'	Domain 'S'	$'f_{min}'$
14	Penalised 2	$f_{14} = \frac{1}{10}\left\{sin^2(3\pi y x_1) + \sum_{i=1}^{D-1}(x_i-1)^2[1+sin^2(3\pi x_{i+1})] + (x_D-1)^2[1+sin^2(2\pi x_D)]\right\} + \sum_{i=1}^{D} u(x_i,5,100,4)$	30, 50	[-50, 50]	0
15	Neumaier 3	$f_{15} = \sum_{i=1}^{D}(x_i-1)^2 + \sum_{i=2}^{D} x_i x_{i-1} + \frac{D(D+4)(D-1)}{6}$	30,50	[- D^2, D^2]	0
16	Salomon	$f_{16} = 1 - cos(2\pi\lVert x_i\rVert) + 0.1\lVert x_i\rVert$ $where\ \lVert x_i\rVert = \sum_{i=1}^{D} x_i$	30,50	[-100, 100]	0
17	Levy	$f_{17} = sin^2(3\pi x_1) + \sum_{i=1}^{D-1}(x_i-1)^2[1+sin^2(3\pi x_{i+1})] + (x_D-1)^2[1+sin^2(3\pi x_D)]$	30,50	[-100.100]	0
18	Zakharov	$f_{18} = \sum_{i=1}^{D} x_i^2 + \left(\sum_{i=1}^{D} 0.5 i x_i\right)^2 + \left(\sum_{i=1}^{D} 0.5 i x_i\right)^4$	30.50	[-5,10]	0

Table B.1: Benchmark functions (continued)

S. No	Name of Function	Test Function Representation	Dim. 'D'	Domain 'S'	$'f_{min}'$
19	Expansion F10	$f_{19} = \sum_{i=2}^{D}\{[(x_{i-1}^2 + x_i^2)]^{0.25} \times sin(50((x_{i-1}^2 + x_i^2)^{0.1})^2 + 1)\} + \{[(x_D^2 + x_1^2)]^{0.25} \times (\sin(50 \times (x_D^2 + x_1^2)^{0.1})^2 + 1\}$	30,50	[-100,100]	0
20	Weierstrass	$f_{20} = \sum_{i=1}^{D}\sum_{n=0}^{n_{max}}\left[a^n\left(cos\left(2\pi b^n(x_i + a)\right)\right)\right] - D\sum_{n=0}^{n_{max}}[a^n cos(\pi b^n)]$	30,50	[-0.5,0.5] Where $a = 0.5,$ $b = 3,$ $n^{max} = 30$	0

APPENDIX-C

HYDROTHERMAL SYSTEMS

C.1 DATA FOR HYDROTHERMAL SYSTEM-I

Hydrothermal system-I consists of 3-thermal and 4-hydro units with hydraulic coupling, having 24-hour scheduling period divided into 24 sub-intervals of 1 hour each. Thermal unit cost coefficients and gaseous pollutant emission coefficients of 3-thermal units undertaken are given in Table C.1 and C.2 respectively. A cascaded hydraulic system network is shown in Figure C.1. Hydropower generation coefficients are given in Table C.3. A set of water transport characteristics of hydro units and the time delays, physical and technical operating limits of hydro units are given in Table C.4, C.5 respectively. Reservoirs inflow is given in Table C.6. Hourly load demand over 24 hours scheduling period taken for the given study is tabulated in Table C.7.

Table C.1: Cost coefficients and operating limits of thermal units of hydrothermal system-I

Unit (i)	a_i $(\$/MW^2h)$	b_i $(\$/MWh)$	c_i $(\$/h)$	d_i $(\$/h)$	e_i (MW^{-1})	P_i^{min} (MW)	P_i^{max} (MW)
1	0.0012	2.45	100	160	0.038	20	175
2	0.0010	2.32	120	180	0.037	40	300
3	0.0015	2.10	150	200	0.035	50	500

Table C.2: Emission coefficients of hydrothermal system-I

Unit (i)	α_i (lb/MW^2h)	β_i (lb/MWh)	γ_i (lb/h)	η_i (lb/h)	δ_i (MW^{-1})
1	0.0105	-1.355	60	0.4968	0.01925
2	0.0080	-0.600	45	0.4860	0.01694
3	0.0120	-0.555	30	0.5035	0.01478

Table C.3 Hydropower generation coefficients of hydrothermal system -I

Unit (i)	C_{1i}	C_{2i}	C_{3i}	C_{4i}	C_{5i}	C_{6i}
1	-0.0042	-0.42	0.030	0.90	10.0	-50
2	-0.0040	-0.30	0.015	1.14	9.5	-70
3	-0.0016	-0.30	0.014	0.55	5.5	-40
4	-0.0030	-0.31	0.027	1.44	14.0	-90

Table C.4 Hydraulic Coupling of hydrothermal system –I

Plant	1	2	3	4
$\boldsymbol{U_i}$	0	0	2	1
$\boldsymbol{t_{ir}}$	2	3	4	0

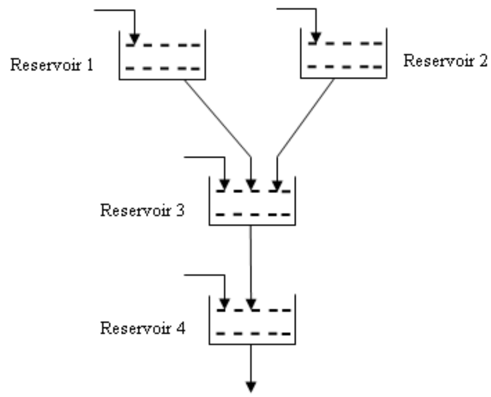

Figure C.1: Hydraulic system network: Hydrothermal system-i

Table C.5: Physical and technical operating limits of hydro units: Hydrothermal system -I

Unit $(\boldsymbol{i})$	$\boldsymbol{V_i^{min}}$	$\boldsymbol{V_i^{max}}$	$\boldsymbol{V_{iI}}$	$\boldsymbol{V_{iI}}$	$\boldsymbol{q_i^{min}}$	$\boldsymbol{q_i^{max}}$	$\boldsymbol{P_i^{min}}$ $(\boldsymbol{MW})$	$\boldsymbol{P_i^{max}}$ $(\boldsymbol{MW})$
	$(\times 10^4 m^3)$				$(\times 10^4 m^3/h)$			
1	80	150	100	120	5	15	0	500
2	60	120	80	70	6	15	0	500
3	100	240	170	170	10	30	0	500
4	70	160	120	140	6	20	0	500

Table C.6: Reservoir inflows $(\times 10^4 m^3/h)$: Hydrothermal system -I

Interval	**Reservoir**				**Interval**	**Reservoir**				**Interval**	**Reservoir**			
$\boldsymbol{k}$	1	2	3	4	$\boldsymbol{k}$	1	2	3	4	$\boldsymbol{k}$	1	2	3	4
1	10	8	8.1	2.8	9	10	8	1	0	17	9	7	2	0
2	9	8	8.2	2.4	10	11	9	1	0	18	8	6	2	0
3	8	9	4	1.6	11	12	9	1	0	19	7	7	1	0
4	7	9	2	0	12	10	8	2	0	20	6	8	1	0
5	6	8	3	0	13	11	8	4	0	21	7	9	2	0
6	7	7	4	0	14	12	9	3	0	22	8	9	2	0
7	8	6	3	0	15	11	9	3	0	23	9	8	1	0
8	9	7	2	0	16	10	8	2	0	24	10	8	0	0

Table C.7: Load Demand (MW): Hydrothermal System -I

$\boldsymbol{k}$	$\boldsymbol{P_{Dk}}$	$\boldsymbol{k}$	$\boldsymbol{P_{Dk}}$	$\boldsymbol{k}$	$\boldsymbol{P_{Dk}}$
1	750	9	1090	17	1050
2	780	10	1080	18	1120

3	700	11	1100	19	1070
4	650	12	1150	20	1050
5	670	13	1110	21	910
6	800	14	1030	22	860
7	950	15	1010	23	850
8	1010	16	1060	24	800

C.2. DATA FOR HYDROTHERMAL SYSTEM-II

Hydrothermal system-II consists of 6-thermal and 16-hydro units with hydraulic coupling, having 24-hour scheduling period divided into 24 sub-intervals of one hour each. Cost coefficients and gaseous pollutant emission coefficients of 6-thermal units undertaken are given in Table C.8 and C.9 respectively. A cascaded hydraulic system network is shown in Fig C.2. Hydropower generation coefficients are given in Table C.10. A set of water transport characteristics of hydro units and the time delays, physical and technical operating limits of hydro units are given in Table C.11, C.12 respectively. Reservoirs inflow is given in table C.13. Hourly load demand over 24 hours scheduling period taken for the given study is tabulated in Table C.14.

Table C.8: Cost coefficients and operating limits of thermal units: Hydrothermal System-II

Unit (i)	a_i $(\$/MW^2h)$	b_i $(\$/MWh)$	c_i $(\$/h)$	d_i $(\$/h)$	e_i (MW^{-1})	P_i^{min} (MW)	P_i^{max} (MW)
1	0.15247	38.5390	756.7988	-	-	10	125
2	0.10587	46.1591	451.3251	-	-	10	150
3	0.03546	38.3055	1243.5311	-	-	35	210
4	0.02803	40.3965	1049.9977	-	-	35	225
5	0.01799	38.2704	1356.6592	-	-	125	315
6	0.02111	36.3278	1658.5696	-	-	130	325

Table C.9: Emission coefficients: Hydrothermal System-II

Unit (i)	α_i (ton/MW^2h)	β_i (ton/MWh)	γ_i (ton/h)	η_i (ton/h)	δ_i (MW^{-1})
1	0.00419	0.32767	13.8593	-	-
2	0.00419	0.32767	13.8593	-	-
3	0.00683	-0.54551	40.2669	-	-
4	0.00683	-0.54551	40.2669	-	-
5	0.00461	-0.51116	42.8955	-	-
6	0.00461	-0.51116	42.8955	-	-

Table C.10: Load demand (MW): Hydrothermal System -II

k	P_{Dk}	k	P_{Dk}	k	P_{Dk}
1	1374	9	1248	17	1426
2	1278	10	1364	18	1028
3	1372	11	1146	19	1148
4	1058	12	1262	20	1256
5	1358	13	1246	21	1348

k	P_{Dk}	k	P_{Dk}	k	P_{Dk}
6	1282	14	1140	22	1424
7	1130	15	1226	23	1070
8	1200	16	1314	24	1118

Table C.11: Hydraulic Coupling: Hydrothermal system –II

Plant	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
U_i	0	0	2	1	0	0	2	1
t_{ir}	0	0	0	0	0	0	0	0
Plant	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
U_i	0	1	0	1	0	0	0	0
t_{ir}	0	0	0	0	0	0	0	0

Table C.12: Hydropower generation coefficients of hydrothermal system -II

Unit (i)	C_{1i}	C_{2i}	C_{3i}	C_{4i}	C_{5i}	C_{6i}
1	-0.00042	-0.042	0.0030	0.090	1.0	-5
2	-0.00040	-0.030	0.0015	0.114	0.95	-7
3	-0.00016	-0.030	0.0014	0.055	0.55	-4
4	-0.00030	-0.031	0.0027	0.144	1.4	-9
5	-0.00042	-0.042	0.0030	0.090	1.0	-5
6	-0.00040	-0.030	0.0015	0.114	0.95	-7
7	-0.00016	-0.030	0.0014	0.055	0.55	-4
8	-0.00030	-0.031	0.0027	0.144	1.4	-9
9	-0.00040	-0.030	0.0015	0.114	0.95	-7
10	-0.00016	-0.030	0.0014	0.055	0.55	-4
11	-0.00040	-0.030	0.0015	0.114	0.95	-7
12	-0.00016	-0.030	0.0014	0.055	0.55	-4
13	-0.00040	-0.030	0.0015	0.114	0.95	-7
14	-0.00040	-0.030	0.0015	0.114	0.95	-7
15	-0.00042	-0.042	0.0030	0.090	1.0	-5
16	-0.00042	-0.042	0.0030	0.090	1.0	-5

Table C.13: Physical and technical operating limits of hydro units: Hydrothermal System -II

Unit (i)	V_i^{min}	V_i^{max}	V_{iI}	V_{iI}	q_i^{min}	q_i^{max}	P_i^{min} (MW)	P_i^{max} (MW)
	$(\times 10^4 m^3)$				$(\times 10^4 m^3/h)$			
1	80	150	100	120	5	15	0	100
2	60	120	80	70	6	15	0	100
3	100	240	170	170	10	30	0	100
4	70	160	120	140	13	25	0	100
5	80	150	100	120	5	15	0	100
6	60	120	80	70	6	15	0	100
7	100	240	170	170	10	30	0	100

8	70	160	120	140	13	25	0	100
9	60	120	80	70	6	15	0	100
10	100	240	170	170	10	30	0	100
11	60	120	80	70	6	15	0	100
12	100	240	170	170	10	30	0	100
13	60	120	80	70	6	15	0	100
14	60	120	80	70	6	15	0	100
15	80	150	100	120	5	15	0	100
16	80	150	100	120	5	15	0	100

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Table C.14: Reservoir inflows ($\times 10^4 m^3/h$) : Hydrothermal System -II

k	**Reservoir** $(\boldsymbol{i})$															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	10.0	8.0	8.1	2.8	10.0	8.0	8.1	2.8	8.0	23.1	8.0	23.1	8.0	8.0	10.0	10.0
2	9.0	8.0	8.2	2.4	9.0	8.0	8.2	2.4	8.0	15.2	8.0	15.2	8.0	8.0	9.0	9.0
3	8.0	9.0	4.0	1.6	8.0	9.0	4.0	1.6	9.0	17.0	9.0	17.0	9.0	9.0	8.0	8.0
4	7.0	9.0	2.0	0.0	7.0	9.0	2.0	0.0	9.0	11.0	9.0	11.0	9.0	9.0	7.0	7.0
5	6.0	8.0	3.0	0.0	6.0	8.0	3.0	0.0	8.0	9.3	8.0	9.3	8.0	8.0	6.0	6.0
6	7.0	7.0	4.0	0.0	7.0	7.0	4.0	0.0	7.0	9.7	7.0	9.7	7.0	7.0	7.0	7.0
7	8.0	6.0	3.0	0.0	8.0	6.0	3.0	0.0	6.0	12.6	6.0	12.6	6.0	6.0	8.0	8.0
8	9.0	7.0	2.0	0.0	9.0	7.0	2.0	0.0	7.0	12.4	7.0	12.4	7.0	7.0	9.0	9.0
9	10.0	8.0	1.0	0.0	10.0	8.0	1.0	0.0	8.0	6.6	8.0	6.6	8.0	8.0	10.0	10.0
10	11.0	9.0	1.0	0.0	11.0	9.0	1.0	0.0	9.0	8.7	9.0	8.7	9.0	9.0	11.0	11.0
11	12.0	9.0	1.0	0.0	12.0	9.0	1.0	0.0	9.0	8.0	9.0	8.0	9.0	9.0	12.0	12.0
12	10.0	8.0	2.0	0.0	10.0	8.0	2.0	0.0	8.0	12.3	8.0	12.3	8.0	8.0	10.0	10.0
13	11.0	8.0	4.0	0.0	11.0	8.0	4.0	0.0	8.0	11.0	8.0	11.0	8.0	8.0	11.0	11.0
14	12.0	9.0	3.0	0.0	12.0	9.0	3.0	0.0	9.0	12.7	9.0	12.7	9.0	9.0	12.0	12.0
15	11.0	9.0	3.0	0.0	11.0	9.0	3.0	0.0	9.0	8.0	9.0	8.0	9.0	9.0	11.0	11.0
16	10.0	8.0	2.0	0.0	10.0	8.0	2.0	0.0	8.0	9.0	8.0	9.0	8.0	8.0	10.0	10.0
17	9.0	7.0	2.0	0.0	9.0	7.0	2.0	0.0	7.0	9.0	7.0	9.0	7.0	7.0	9.0	9.0
18	8.0	6.0	2.0	0.0	8.0	6.0	2.0	0.0	6.0	13.0	6.0	13.0	6.0	6.0	8.0	8.0
19	7.0	7.0	1.0	0.0	7.0	7.0	1.0	0.0	7.0	9.4	7.0	9.4	7.0	7.0	7.0	7.0
20	6.0	8.0	1.0	0.0	6.0	8.0	1.0	0.0	8.0	6.0	8.0	6.0	8.0	8.0	6.0	6.0
21	7.0	9.0	2.0	0.0	7.0	9.0	2.0	0.0	9.0	9.7	9.0	9.7	9.0	9.0	7.0	7.0
22	8.0	9.0	2.0	0.0	8.0	9.0	2.0	0.0	9.0	7.0	9.0	7.0	9.0	9.0	8.0	8.0
23	9.0	8.0	1.0	0.0	9.0	8.0	1.0	0.0	8.0	10.0	8.0	10.0	8.0	8.0	9.0	9.0
24	10.0	8.0	0.0	0.0	10.0	8.0	0.0	0.0	8.0	5.0	8.0	5.0	8.0	8.0	10.0	10.0

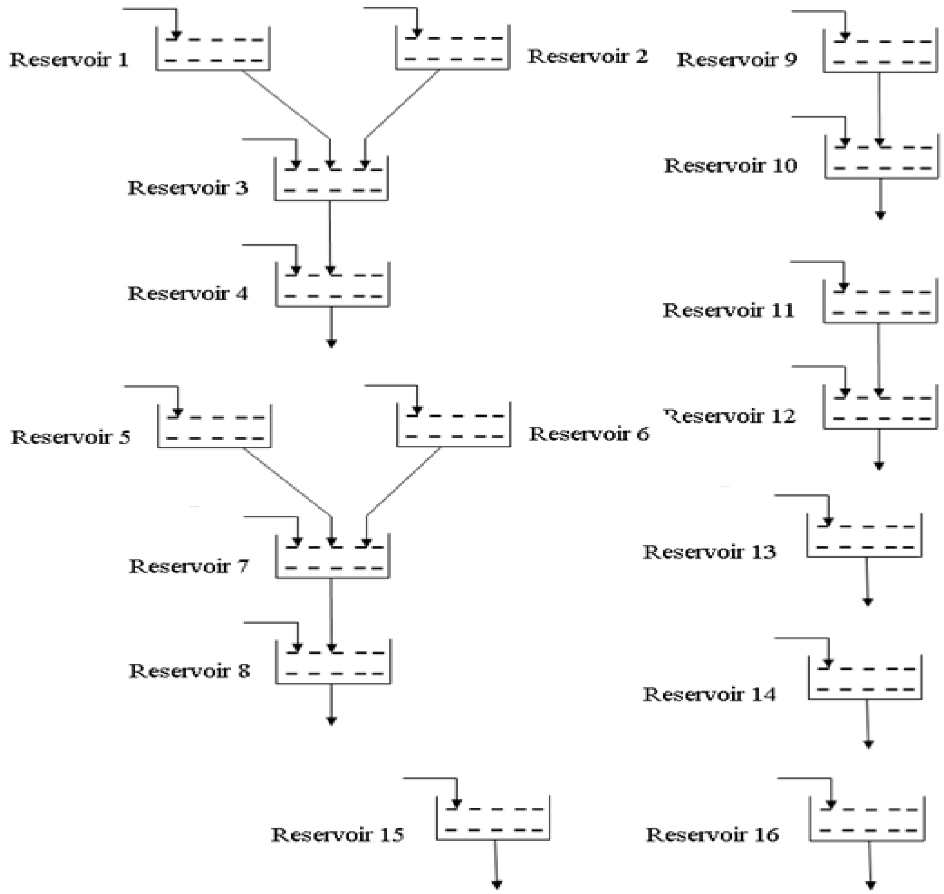

Figure C.2: Hydraulic System Network: Hydrothermal System-II

C.3. DATA FOR HYDROTHERMAL SYSTEM-III

Hydrothermal system-III consist of 1-unit equivalent to a thermal plant and 4-hydro units with hydraulic coupling, having 24-hour scheduling period divided into 24 sub-intervals of one hour each. Cost coefficients of 2-thermal units undertaken are given in Table C.15. A cascaded hydraulic system network is shown in Fig C.3. Hydropower generation coefficients are given in Table C.16. A set of water transport characteristics of hydro units and the time C.18 respectively. Reservoirs inflow is given in Table C.19 and hourly load in Table C. 20.

Table C.15: Cost coefficients and operating limits of thermal units: Hydrothermal System-III

Unit (i)	a_i ($\$/MW^2h$)	b_i ($\$/MWh$)	c_i ($\$/h$)	d_i ($\$/h$)	e_i (MW^{-1})	P_i^{min} (MW)	P_i^{max} (MW)
1	0.5	10	1000	-	-	10	160

Table C.16: Hydropower generation coefficients: Hydrothermal system -III

Unit (i)	C_{1i}	C_{2i}	C_{3i}	C_{4i}	C_{5i}	C_{6i}
1	-0.001	-0.1	0.01	0.40	4.0	-30
2	-0.001	-0.1	0.01	0.38	3.5	-30
3	-0.001	-0.1	0.01	0.30	3.0	-30
4	-0.001	-0.1	0.01	0.38	3.8	-30

Table C.17: Hydraulic Coupling: Hydrothermal system –III

Plant	1	2	3	4
U_i	0	0	2	1
t_{ir}	1	2	2	0

Table C.18: Physical and Technical Operating limits of Hydro units: Hydrothermal System -III

Unit (i)	V_i^{min}	V_i^{max}	V_{iI}	V_{iI}	q_i^{min}	q_i^{max}	P_i^{min} (MW)	P_i^{max} (MW)
	($\times 10^3 m^3$)				($\times 10^3 m^3/h$)			
1	80	150	100	120	5	15	0	200
2	60	120	80	70	6	15	0	200
3	100	240	170	170	10	30	0	200
4	70	160	120	120	6	25	0	200

Table C.19: Reservoir inflows ($\times 10^4 m^3/h$) : Hydrothermal System -III

Interval k	Reservoir 1	2	3	4	Interval k	Reservoir 1	2	3	4	Interval k	Reservoir 1	2	3	4
1	10	8	1	0	9	10	8	1	0	17	10	8	1	0
2	10	8	1	0	10	10	8	1	0	18	10	8	1	0
3	10	8	1	0	11	10	8	1	0	19	10	8	1	0
4	10	8	1	0	12	10	8	1	0	20	10	8	1	0
5	10	8	1	0	13	10	8	1	0	21	10	8	1	0
6	10	8	1	0	14	10	8	1	0	22	10	8	1	0
7	10	8	1	0	15	10	8	1	0	23	10	8	1	0

8	10	8	1	0	16	10	8	1	0	24	10	8	1	0

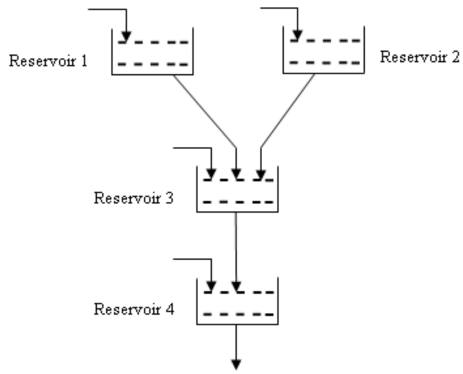

Figure C.3: Hydraulic system network: Hydrothermal system-III

Table C.20: Load demand (MW): Hydrothermal System -III

$\boldsymbol{k}$	$\boldsymbol{P_{Dk}}$	$\boldsymbol{k}$	$\boldsymbol{P_{Dk}}$	$\boldsymbol{k}$	$\boldsymbol{P_{Dk}}$
1	190	9	270	17	270
2	170	10	310	18	250
3	170	11	350	19	230
4	190	12	310	20	210
5	190	13	350	21	210
6	210	14	350	22	210
7	230	15	310	23	190
8	250	16	290	24	190

C.4 DATA FOR HYDROTHERMAL SYSTEM-IV

Hydrothermal system-IV consists of 3-thermal and 4-hydro units with hydraulic coupling, having 24-hour scheduling period divided into 24 sub-intervals of 1 hour each. Thermal unit cost coefficients and gaseous pollutant emission coefficients of 3-thermal units undertaken are given in Table C.21 and C.22 respectively. A cascaded hydraulic system network is shown in Fig C.1. Hydropower generation coefficients are given in Table C.3. A set of water transport characteristics of hydro units, and reservoirs inflow are given in Table C.4, C.5 respectively. Physical and technical operating limits of hydro units are given in Table C.6. Hourly load demand over 24 hours scheduling period taken for the given study is tabulated in Table C.7. Loss B-coefficients data is given in Table C.23.

Table C.21: Cost coefficients and operating limits of thermal units: Hydrothermal System-IV

Unit (i)	a_i ($/MW^2h$)	b_i ($/MWh$)	c_i ($/h$)	d_i ($/h$)	e_i (MW^{-1})	P_i^{min} (MW)	P_i^{max} (MW)
1	0.0037	2.00	10	18	0.0370	20	175
2	0.0175	1.75	10	16	0.0380	40	300
3	0.0625	1.00	20	14	0.0400	50	500

Table C.22: Emission coefficients: Hydrothermal System-IV

Unit (i)	$\alpha_i \times 10^{-6}$ (ton/MW^2h)	$\beta_i \times 10^{-4}$ (ton/MWh)	$\gamma_i \times 10^{-2}$ (ton/h)	$\eta_i \times 10^{-4}$ (ton/h)	$\delta_i \times 10^{-2}$ (MW^{-1})
1	6.490	-5.554	4.091	2.0	2.857
2	5.638	-6.047	2.543	5.0	3.333
3	4.586	-5.094	4.258	0.01	0.8

Table C.23: Loss coefficients: Hydrothermal System-IV

B_{00}	[0.0]
B_{0i}	[0.0 0.0 0.0 0.0 0.0 0.0 0.0]
B_{ij}	$\begin{bmatrix} 0.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.1 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1 \end{bmatrix} \times 10^{-6}$