Effect of the radiation pressure on the dimensions of the primary component of the binary system of the stars

A thesis submitted in partial fulfilment of the requirements for the award of degree of

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Submitted by:

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I dedicate this work to my parents,

'Sh. Mukesh Sachdeva and Smt. Meenu Sachdeva'

and my younger brother

'Ashutosh Sachdeva'

and my friends who encouraged and supported me.

CERTIFICATE

I hereby declare that the work which has been presented in this thesis entitled **"Effect of the radiation pressure on the dimensions of the primary component of the binary system of the stars"** is an authentic record of my own work carried out for the partial fulfilment of the requirement for the award of the degree of Masters of Science in Mathematics and Computing at Thapar Institute of Engineering and Technology (TIET), Patiala (Punjab), under the guidance of **Dr. Ankush Pathania,** Associate Professor, School of Mathematics (SOM) and refers other researcher's work which are duly listed in the reference section. The intellectual content of this thesis is the product of my own work and contains no material which to a substantial extent has been accepted for the award of any other degree at this or any other educational institution, except where due acknowledgment is made in the thesis.

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Date: 12/07/19

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It is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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ABSTRACT

In this dissertation, we have studied the dimensions of the primary component of the binary system of the stars under the effect of radiation pressure using the methodology of Kopal (1959) and modified Roche potential as given by Schuerman (1972).

Kopal (1959) introduced the concept of Roche potential to analyse various problems of the rotating stars and stars in the binary systems. Using the concept of Roche potential, author also studied the dimensions and other parameters of the binary system of the stars. However, the Roche potential that has been used in these studies consist of only gravitational and centrifugal forces, whereas, the forces like radiation pressure has not been considered.

It has been observed that during certain stages of evolution of the binary stars, the effect of radiation pressure becomes significant and affects the shapes of the stars, binary configuration and the mass flow. While studying the effect of radiation pressure on the shapes of the stars in close binary systems, Schuerman (1972) obtained an expression for the modified Roche potential that takes into account the effect of radiation pressure from the primary component of the binary system.

In the present work, we have used the modified expression of Roche potential as given by Schuerman (1972) and utilized the methodology of Kopal (1959) to determine the effect of radiation pressure on the dimensions of the primary component of the binary system of the stars.

Thesis consists of 2 chapters. Chapter I is preliminary in nature. In this chapter, literature survey, Roche potential and methodology of Kopal (1959) (to obtain the dimensions of the primary component and certain other parameters of the binary system of the stars) has been discussed.

In chapter II, the modified Roche potential as given by Schuerman (1972) that incorporates the effect of radiation pressure has been used along with the methodology of Kopal (1959) to obtain

the modified expression for the dimensions of the primary component and certain other parameters of the binary system of the stars. The numerical results has been then obtained for various models of the stars in the binary system and finally certain conclusions has been drawn.

Various parameters used in the manuscript

S. No.	Parameter	Definition
1	n	Rotation parameter that represents the distortions due to rotation forces
2	q	Tidal parameter that represents the distortions due to tidal forces
3	Ω	Angular velocity of rotation of the star
4	ω	Angular velocity of revolution of the binary system
5	δ	Radiation parameter that represents the effect due to the radiation pressure
6	x_1	Position of the first Lagrangian point (L_1) of the primary component
7	ξ_1	Roche limit (value of the Roche potential at the first Lagrangian point (L_1))
8	x_2	Back radius of the primary component of the binary system
9	y_3	Side radius of the primary component of the binary system
10	x_3	x-coordinate corresponding to the side radius of the primary component
11	z_4	Pole radius of the primary component of the binary system
12	x_5	Position of the second Lagrangian point (L_2)
13	ξ_5	Value of the Roche potential at the second Lagrangian point (L_2)
14	x_6	Position of the third Lagrangian point (L_3)
15	ξ_6	Value of the Roche potential at the third Lagrangian point (L_3)

CONTENTS

CHAPTER – 1

Introduction

1.1 Overview

Structures and oscillations of the stars consist of one of the major problems of astrophysics. The analytic studies of the problems of the rotating stars and rotating stars in the binary systems have engaged the attention of astrophysicists since long with a view to analyse and understand the observational behaviour of such stars. Research over the past decades shows that more than 80% of all stars in the space are a part of multiple star systems and the most common multiple star systems are the binary star systems. Observations shows that more than 50% of all stellar systems are the binary systems.

In a binary system of stars, the two stars normally rotate about their own axis of rotation as well as revolve about their common centre of mass. The brighter star is called primary and dimmer star is called secondary component of the binary system. Furthermore, in the majority of binary stars, one star called primary is generally more bigger as compared to its companion (secondary) star. On the basis of their orbit the binary stars are classified as wide binaries and close binaries. In wide binaries the two stars have wide orbits that keep them spread apart from one another. These stars develop separately with very little impact from their companions. Close binaries are close enough to transfer matter by tidal forces. They are those in which orbits of the system are small and the components are strongly bounded to each other by gravitational force.

This chapter is introductory in nature. The relevant literature related to the current work has been reviewed in section 1.2. In section 1.3, classical Roche potential equation has been discussed. Using the methodology of Kopal (1959) expression for the dimensions of the primary component and certain other parameters of the binary system has been obtained in section 1.4.

1.2 Literature review

Kopal (1959) introduced the concept of Roche potential to analyse the problems of rotating stars and stars in the binary system. Furthermore, the Roche potential has been frequently used in literature to study the structures, oscillations, mass flow and the evolutionary process of the stars in the binary system. In the work of Kopal (1959) classical Roche model has been used. Roche model is a model of star in which the whole mass of a star is assumed to be concentrated at the centre and this point mass is surrounded by an evanescent envelope in which density varies inversely as square of distance from its centre.

While deriving the expression for the Roche potential – using the concept of Roche model – various assumptions that has been made in the work of Kopal (1959) are: uniform rotation (angular velocity of rotation is same at each point of a star), synchronous rotation (angular velocity of rotation of two components is equal to the angular velocity of revolution of the system), circular orbits (components of the binary system are revolving in circular orbits), aligned axis (axis of the rotation of the stars is at right angle to the line joining the centres of two components) and no radiation pressure (effect of radiation pressure due to two components of binary system are not considered).

However, appropriate modifications has been incorporated in the Roche potential by certain authors for studying the various problems of the stars. Limber (1963) and Kruszewski (1963) has included the effects of non-synchronous rotation, Schuerman (1972) considered the effects of radiation pressure, Wilson (1979) have taken into account the effects for eccentric orbits, Avni and Schiller (1982) considered misaligned binaries and Mohan et al. (1992) has incorporated the effects of differential rotation in the expression of Roche potential while investigating various problems of the rotating stars and stars in the binary system.

In literature – using the concept of Roche potential – the dimensions of the components of close binary system has been studied in detail by Kuiper and Johnson (1956), Kopal (1959), Plavec and Kratochvil (1964) and Mochnacki (1984). Sepinsky et al. (2007) has studied the equipotential surfaces and lagrangian points in nonsynchronous eccentric binary stars. Lal et al. (2009) investigated the effect of coriolis force on the shapes of the components of the binary system of the stars. Pathania and Medupe (2014) has discussed the effect of nonsynchronous rotation on the dimensions of the primary component of the nonsynchronous binary system of the stars.

Schuerman (1972) was first to study the effects of the radiation pressure on the Roche potential of the close binary system. In his work he has shown that the radiation pressure can affect the shape of the critical Roche potential surfaces. Vanbeveren (1978) and Howarth (1997) has studied the influence of the radiation pressure on the Roche equipotential surfaces of the binary system taking into account the effect of radiation pressure from both components of the binary system. While studying the radiation pressure effects in early type close binaries, Drechsel et al. (1994) has shown that the force due to radiation pressure cannot be neglected for close luminous binary system of the stars. Dermine et al. (2009) has investigated the effect of the radiation pressure on the Roche lobes of the binary system.

Effects of the radiation pressure are important as they affect the shapes of the stars and the binary configuration. These effects cannot be neglected for close luminous binary system of the stars. So taking these points into consideration – in the present study – an attempt has been made to analyse the effect of radiation pressure from the primary component of the binary system on its dimensions using the modified Roche potential as given by Schuerman (1972) and the methodology of Kopal (1959). The main objective of this work is to check how the inclusion of the radiation pressure in the expression of Roche potential affects the dimensions of the primary component of the binary system of the stars.

1.3 Roche potential

Roche potential has been widely used in literature to study the structures, oscillations and evolution of the rotating stars and stars in the binary systems. In order to derive expression for the Roche potential consider a binary system in which the two stars are rotating about their own axis and also revolving about an axis that passes through the centre of the mass (point C) of the binary system (see figure 1).

Let m_1 and m_2 be the masses of the two components (primary and secondary, respectively) in a close binary system parted by distance R. The position of two components in a binary system is taken in cartesian coordinate system. Origin of the coordinate system is taken at the centre of the mass (point O) of the primary component. The line passing through the centre of the mass of two stars is taken as x-axis and perpendicular to the plane is taken as z-axis. Let angular velocity of the rotation of these two components about an axis perpendicular to xy-plane is Ω and angular velocity of revolution of systems is ω.

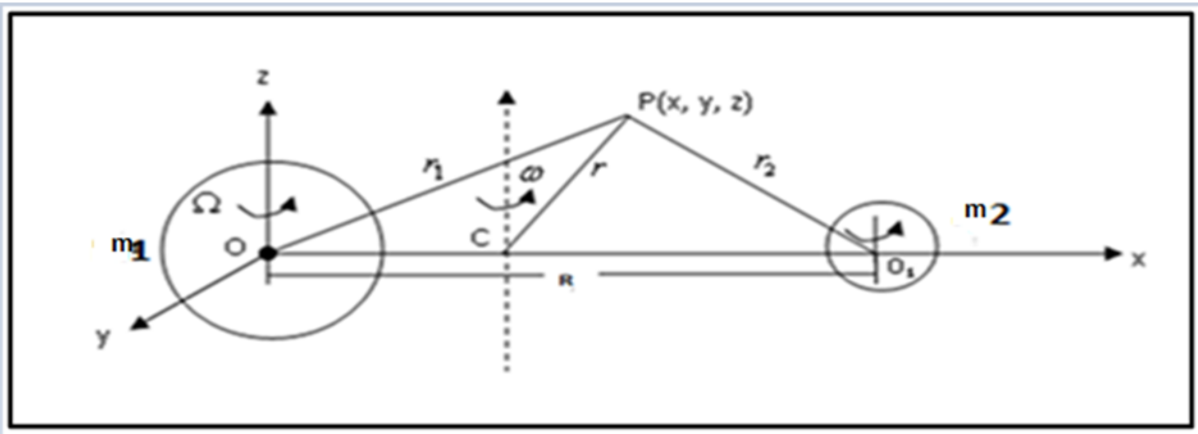

Fig. 1 – Binary system of stars

From figure (1), C (d_1, 0, 0) is the centre of the mass of the system where $d_1 = \frac{m_2 R}{m_1+m_2}$. Also

$r_1^2 = x^2 + y^2 + z^2$, $r_2^2 = (R-x)^2 + y^2 + z^2$, $r^2 = (x-d_1)^2 + y^2 + z^2$ represents the distance of any arbitrary point $P(x, y, z)$ from the centre of the mass of the primary star, secondary star and the binary system, respectively.

Now for such a binary system as described in fig 1, the total potential at a point P(x,y, z) due to all forces is given by

$$\psi = \frac{Gm_1}{r_1} + \frac{Gm_2}{r_2} + \frac{\omega^2}{2}\left\{\left(x - \frac{m_2 R}{m_1+m_2}\right)^2 + y^2\right\} \tag{1.1}$$

Here $\frac{Gm_1}{r_1}$ is the potential arising from the primary component, $\frac{Gm_2}{r_2}$ is the potential arising from the secondary component and third term in equation (1.1) represents the potential due to the centrifugal force.

While deriving (1.1), primary star is assumed to be more massive and luminous than the secondary star. Also, it has been assumed that the angular velocity of rotation (Ω) and revolution (ω) are same (synchronous binary system). Now, if the angular velocity of rotation is assumed to be equal to keplerian angular velocity:

$$\omega_k^2 = \frac{G(m_1+m_2)}{R^3} \tag{1.2}$$

then we get a relation of the type

$$\omega^{*^2} = 2n = 1 + q \tag{1.3}$$

where $q = \frac{m_2}{m_1}$ is a tidal parameter that represents the distortions due to the tidal forces, n is a rotational parameter that represents the distortions due to the rotational forces and $\omega^{*^2} = \frac{\omega^2 R^3}{Gm_1}$ is the normalized angular velocity.

Multiply both side of (1.1) by $\frac{R}{Gm_1}$ and using (1.3), the equation (1.1) in the non-dimensional form can be written as:

$$\Psi^* = \frac{1}{r_1{}^*} + q\left(\frac{1}{r_2{}^*} - x^*\right) + \left(\frac{1+q}{2}\right)(x^{*2}+y^{*2}) \tag{1.4}$$

where $\psi^* = \frac{\psi R}{Gm_1} - \frac{q^2}{2(1+q)}$, $r_1^* = \frac{r_1}{R}$, $r_2^* = \frac{r_2}{R}$, $x^* = \frac{x}{R}$, $y^* = \frac{y}{R}$.

Equation (1.4) represents the total potential (also called Roche potential) at point P in the non-dimensional form for the binary system of the stars. This is the classical Roche potential that has been derived under the assumptions of uniform and synchronous rotation, circular orbits, aligned axis and no radiation pressure.

Now if we put $\Psi^*(= \xi\ say) = constant$ in (1.4) then it represents the equipotential surfaces also called Roche equipotential surfaces. Roche equipotential surfaces will be in the form of two separate spherical ovals around each centre of mass for large values of ξ. With the diminishing values of ξ, these spherical ovals become elongated along the line joining the centres of the gravity of the binary system. This process continues until for some critical value of ξ these oval surfaces get united at a point on x-axis to form a dumbbell like structure. This critical value of ξ is called Roche limit. Below this value of ξ the two components of the binary system merge into one another.

1.4 Methodology of Kopal (1959) to determine the dimensions of the primary component of the binary system of the stars

In this section, we will determine the mathematical expressions for the dimensions of the primary component and some other parameters of the binary system of stars using the methodology of Kopal (1959). Here we will determine the mathematical expressions for computing the point radius (position of the first lagrangian point (L_1), that is, the distance of point P_1 from the centre of the mass (O) of the primary component, see figure (2)), back radius (distance between the point O and point P_2 where the lobe of the primary is intersected by the X-axis in the negative direction), side radius (perpendicular distance of the point P_3 (lying in XY-plane)from X-axis)and the pole radius (perpendicular distance of point P_4 (above XY-plane) of the primary component of the binary system. We will also determine the position of the second and the third lagrangian points and potential at L_1, L_2 (second lagrangian point) and L_3 (third lagrangian point) points.

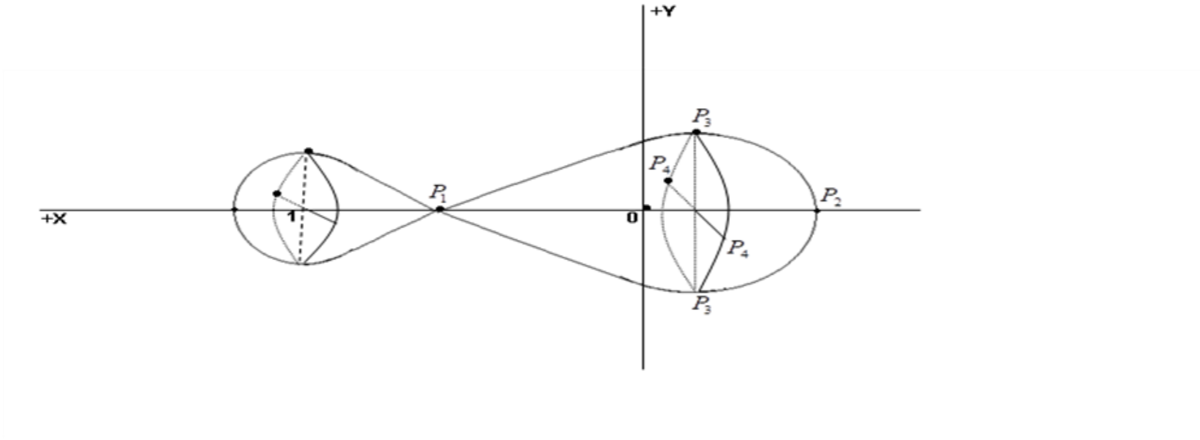

Fig. 2 – Schematic view of a contact binary at the Roche limit

(in order to exhibit the essential features of the geometry of this model, the diagram has not been drawn to scale for any particular mass ratio; and certain features of it (such as the distance of the P_3 P_4- plane from the origin) have been exaggerated)

For our convenience we consider equation (1.4) as:

$$\xi = \frac{1}{r_1} + q\left(\frac{1}{r_2} - x\right) + \left(\frac{1+q}{2}\right)(x^2 + y^2) \qquad \textbf{(1.5)}$$

Using the values of r_1 and r_2, this equation can be written as:

$$\xi = \frac{1}{\sqrt{x^2+y^2+z^2}} + q\left(\frac{1}{\sqrt{(R-x)^2+y^2+z^2}} - x\right) + \left(\frac{1+q}{2}\right)(x^2 + y^2) \qquad \textbf{(1.6)}$$

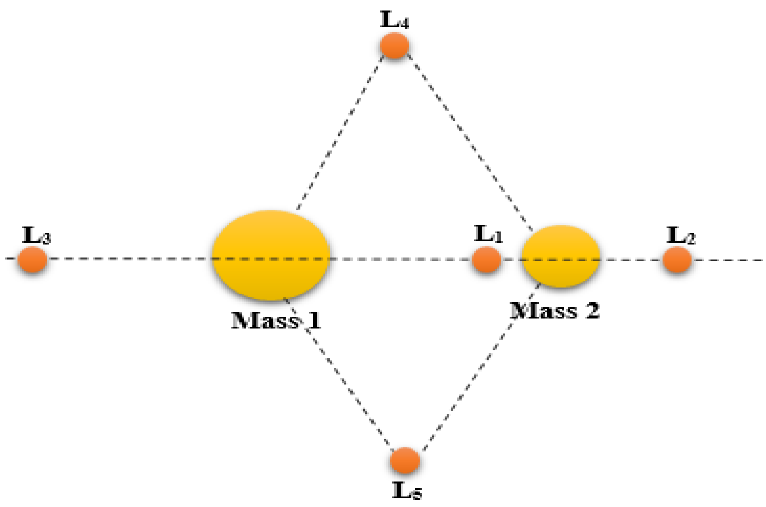

Fig. 3 – Lagrangian points

Point Radius: The position of the first lagrangian point L_1 (also called the point radius of the primary component) represented by $P_1(x_1,0,0)$ in figure (2) is characterized by vanishing of the total potential force. That is, at this point

$$\xi_x = \xi_y = 0 \qquad \textbf{(1.7)}$$

The vanishing of ξ_x gives

$$\xi_x = (q+1)x^5 - (3q+2)\, x^4 + (3q+1)x^3 - x^2 + 2x - 1 = 0 \qquad \textbf{(1.8)}$$

The root of this equation that lies between 0 and 1 gives the desired coordinate x_1(point radius) of the first lagrangian point (L_1). The value of Roche potential ξ_1(also called the Roche limit), corresponding to the first lagrangian point is given by

$$\xi_1 = \xi_1(x_1, 0, 0) \qquad \textbf{(1.9)}$$

which on simplification gives

$$\xi_1 = \frac{1}{x_1} + q\left(\frac{1}{1-x_1} - x_1\right) + \left(\frac{q+1}{2}\right)(x_1^2) \qquad \textbf{(1.10)}$$

Back Radius: From figure (2), we can see that the lobe of primary component is again intersected by x-axis at point $P_2(-x_2,0,0)$. The value of x_2 (called back radius of the primary component) can be obtained by solving the equation

$$\xi(-x, 0,0) = \xi_1 \qquad \textbf{(1.11)}$$

On simplification equation (1.11) gives

$$\left(\frac{q+1}{2}\right) x^4 + \left(\frac{q+1}{2} + q\right) x^3 + (q-\xi_1)x^2 + (q+1-\xi_1)x + 1 = 0 \qquad \textbf{(1.12)}$$

Side Radius: Now consider the side radius of the primary in direction perpendicular to the X-axis, that is, y-coordinate of the point $P_3(x_3, \pm y_3, 0)$. The coordinates of the point P_3 in the XY-plane are obtained by solving the system of equations

$$\xi(x, y, 0) = \xi_1 \quad \textbf{(1.13)}$$

$$\xi_x(x, y, 0) = 0 \quad \textbf{(1.14)}$$

On simplification, $\xi(x, y, 0) = \xi_1$ gives

$$\xi_1 = \frac{1}{\sqrt{x^2+y^2}} + q\left(\frac{1}{\sqrt{(1-x)^2+y^2}} - x\right) + \left(\frac{1+q}{2}\right)(x^2 + y^2) \quad \textbf{(1.15)}$$

and $\xi_x(x, y, 0) = 0$ gives

$$- x + (x^2 + y^2)^{3/2} \left(q\left(\frac{1-x}{((1-x)^2+y^2)^{3/2}} - 1\right) + (q + 1)x\right) = 0 \quad \textbf{(1.16)}$$

Pole Radius: Once the value of x_3 have been found, the value along Z-axis (also called the pole radius) in the XZ-plane, that is, the z-coordinate of point P_4 can be obtained by solving the equation

$$\xi(x_3, 0, z_4) = \xi_1 \quad \textbf{(1.17)}$$

which on simplification gives

$$\xi_1 = \frac{1}{\sqrt{x^2+z^2}} + q\left(\frac{1}{\sqrt{(1-x)^2+z^2}} - x\right) + \left(\frac{1+q}{2}\right)(x^2) \quad \textbf{(1.18)}$$

Second lagrangian point (L_2):

At second lagrangian point say, ($P_5(x_5,0,0)$) equation of the Roche potential can be written as:

$$\xi = \frac{1}{x} + q\left(\frac{1}{x-1} - x\right) + \left(\frac{1+q}{2}\right)(x^2) \quad \textbf{(1.19)}$$

The position of the second lagrangian point is characterized by vanishing of the total potential force, that is, at this point

$$\xi_x = \xi_y = 0 \quad \textbf{(1.20)}$$

The vanishing of ξ_x gives

$$\xi_x = (q+1)x^5 - (3q+2)x^4 + (3q+1)x^3 - (2q + 1)x^2 + 2x - 1 = 0 \quad \textbf{(1.21)}$$

The root of this equation that is greater than 1 gives the desired coordinate x_5 of the second lagrangian point. The value of Roche potential ξ_5 corresponding to second lagrangian point is given by

$$\xi_5 = \xi(x_5, 0, 0) \quad \textbf{(1.22)}$$

which on simplification gives

$$\xi_5 = \frac{1}{x_5} + q\left(\frac{1}{x_5 - 1} - x_5\right) + \left(\frac{q+1}{2}\right)(x_5^2) \quad \textbf{(1.23)}$$

Third lagrangian point (L_3):

At third lagrangian point say, ($P_6(-x_6,0,0)$) equation of the Roche potential can be written as:

$$\xi = \frac{1}{x} + q\left(\frac{1}{1+x} + x\right) + \left(\frac{1+q}{2}\right)(x^2) \quad \textbf{(1.24)}$$

which on simplification gives

$$\xi_x = (q+1)x^5 + (3q+2)x^4 + (3q+1)x^3 - x^2 - 2x - 1 = 0 \quad \textbf{(1.25)}$$

The value of Roche potential ξ_6 corresponding to third lagrangian point is given by

$$\xi_6 = \xi(-x_6, 0, 0) \quad \textbf{(1.26)}$$

which on simplification gives

$$\xi_6 = \frac{1}{x_6} + q\left(\frac{1}{x_6 + 1} + x_6\right) + \left(\frac{q+1}{2}\right)(x_6^2) \quad \textbf{(1.27)}$$

CHAPTER - 2

Effect of the radiation pressure on the dimensions of the primary component of the binary system of the stars

2.1 Overview

Studies has shown that during certain stages of evolution of the binary system of the stars, radiation pressure becomes important and cannot be neglected. There is a significant interaction between radiation and matter for early type binary systems where radiation pressure increases rapidly with the temperature. When the strength of radiation pressure cannot be neglected as compared to force of the gravity of the star then it leads to deviation in the geometry of Roche equipotential surfaces and shifting of the position of lagrangian points (L_1 , L_2 and L_3). This affects the overall configuration of the system and have immediate consequence on the mass transfer (through L_1) or mass loss (through L_2 or L_3) and hence on the evolution of the binary system.

In the present chapter we have considered the effect of radiation pressure on the dimensions of the primary component of the binary system of the stars. For this purpose, we have used the modified Roche potential that incorporates the effect of radiation pressure as given by Schuerman (1972) along with the methodology of Kopal (1959).

In section 2.2, we have presented an expression for the modified Roche potential – that includes the effect of radiation pressure – as given in the work of Schuerman (1972). This expression is then used in section 2.3 to obtain mathematical equations for determining the dimensions of the primary component and some other parameters of the binary system. In section 2.4, numerical results are obtained and finally conclusions of the present work are discussed in section 2.5.

2.2 Modified Roche potential including the effect of radiation pressure

In addition to gravitational and centrifugal forces the particles in the atmosphere of luminous or radiating star will experience force due to radiation. Schuerman (1972) was first to address this issue in case of the binary stars and derived an effective potential, taking into account the radiation pressure effects. According to Schuerman (1972) if we consider a binary system configuration as described in figure (1) (already discussed in detail in section 1.3 of chapter 1) then for the primary component of the binary system, the force per unit mass due to radiation pressure (or radiation force field of mass m_1) is given by

$$F_{rad} = \frac{\delta G m_1}{r_1^2} \quad \textbf{(2.1)}$$

where δ (= $\frac{F_{rad}}{F_{grav}}$) is the dimensionless ratio of the force due to radiation pressure and gravitational force per unit mass. Now this parameter δ can be used to characterize the strength and efficiency of its radiation pressure. On solving further Schuerman (1972) has shown that the radiation parameter δ can be written as

$$\delta = \frac{1}{4\pi c G m_1} \int_0^\infty \kappa_\nu L_\nu \, d\nu \quad \textbf{(2.2)}$$

where κ_ν is the absorption coefficient per unit mass at frequency ν, L is the luminosity of the primary component in the frequency range ν to ν +dν, c is the speed of light, m_1 is the mass of the primary component and G is the universal gravitational constant. Also, the value of radiation parameter δ lies in the range, 0≤δ<1, for the cases of practical interest.

So, using equation (2.1), the modified expression for the Roche potential – that incorporates the effects of radiation pressure – as given by Schuerman (1972) is

$$\xi = \frac{(1-\delta)}{r_1} + q\left(\frac{1}{r_2} - x\right) + \left(\frac{1+q}{2}\right)(x^2 + y^2) \quad \textbf{(2.3)}$$

where all other symbols has same meaning as described earlier. Further equation (2.3) can be written as

$$\xi = \frac{(1-\delta)}{\sqrt{x^2+y^2+z^2}} + q\left(\frac{1}{\sqrt{(R-x)^2+y^2+z^2}} - x\right) + \left(\frac{1+q}{2}\right)(x^2 + y^2) \quad \textbf{(2.4)}$$

2.3 Dimensions of the primary component of the binary system of the stars under the effect of the radiation pressure

In this section we will determine the dimensions of the primary component and some other parameters of the binary system using the modified expression of the Roche potential (equation 2.3 and 2.4, that takes into account the effect of radiation pressure) and methodology of Kopal (1959) as discussed earlier in chapter 1. Here we will determine the mathematical expressions for computing the point radius, back radius, side radius and the pole radius of the primary component of the binary system of the stars under the effect of radiation pressure. We will also determine the position of the second and the third lagrangian points and corresponding potential at first, second and third lagrangian points.

Point Radius: The position of the first lagrangian point L_1 (also called the point radius of the primary component) represented by $P_1(x_1,0,0)$ in figure (2) is characterized by vanishing of the total potential force. That is, at this point

$$\xi_x = \xi_y = 0 \quad \textbf{(2.5)}$$

The vanishing of ξ_x gives

$$\xi_x = (q+1)x^5 - (3q+2)\, x^4 + (3q+1)x^3 - (1-\delta)x^2 + 2x(1-\delta) - 1 + \delta = 0 \quad \textbf{(2.6)}$$

The root of this equation that lies between 0 and 1 gives the desired coordinate x_1(point radius) of the first lagrangian point (L_1). The value of Roche potential ξ_1(also called the Roche limit), corresponding to the first lagrangian point is given by

$$\xi_1 = \xi_1(x_1, 0, 0) \quad \textbf{(2.7)}$$

which on simplification gives

$$\xi_1 = \frac{(1-\delta)}{x_1} + q\left(\frac{1}{1-x_1} - x_1\right) + \left(\frac{q+1}{2}\right)(x_1^2) \quad \textbf{(2.8)}$$

Back Radius: From figure (2), we can see that the lobe of primary component is again intersected by X-axis at point $P_2(-x_2,0,0)$. The value of x_2 (called back radius of the primary component) can be obtained by solving the equation

$\xi(-x,0,0) = \xi_1$ **(2.9)**

On simplification equation (2.9) gives

$$\left(\frac{q+1}{2}\right)x^4+\left(\frac{q+1}{2}+q\right)x^3+(q-\xi_1)x^2+(q+1-\delta-\xi_1)x+1-\delta=0 \quad \textbf{(2.10)}$$

Side Radius: Now consider the side radius of the primary in direction perpendicular to the X-axis, that is, y-coordinate of the point $P_3(x_3,\pm y_3,0)$. The coordinates of the point P_3 in the XY-plane are obtained by solving the system of equations

$\xi(x,y,0) = \xi_1$ **(2.11)**

$\xi_x(x,y,0) = 0$ **(2.12)**

On simplification, $\xi(x,y,0) = \xi_1$ gives

$$\xi_1=\frac{(1-\delta)}{\sqrt{x^2+y^2}} + q\left(\frac{1}{\sqrt{(1-x)^2+y^2}} - x\right)+ \left(\frac{1+q}{2}\right)(x^2+y^2) \quad \textbf{(2.13)}$$

and $\xi_x(x,y,0) = 0$ gives

$$-x(1-\delta)+(x^2+y^2)^{3/2}\left(q\left(\frac{1-x}{((1-x)^2+y^2)^{3/2}}-1\right)+(q+1)x\right)=0 \quad \textbf{(2.14)}$$

Pole Radius: Once the value of x_3 have been found, the value along Z-axis (also called the pole radius) in the XZ-plane, that is, the z-coordinate of point P_4 can be obtained by solving the equation

$\xi(x_3, 0, z_4)=\xi_1$ **(2.15)**

which on simplification gives

$$\xi_1=\frac{(1-\delta)}{\sqrt{x^2+z^2}} +q\left(\frac{1}{\sqrt{(1-x)^2+z^2}} - x\right) + \left(\frac{1+q}{2}\right)(x^2) \quad \textbf{(2.16)}$$

Second lagrangian point (L_2):

At second lagrangian point say, ($P_5(x_5,0,0)$) , equation of the Roche potential can be written as:

$$\xi = \frac{(1-\delta)}{x} + q\left(\frac{1}{x-1} - x\right)+ \left(\frac{1+q}{2}\right)(x^2) \quad \textbf{(2.17)}$$

The position of the second lagrangian point is characterized by vanishing of the total potential force, that is, at this point

$$\xi_x = \quad \xi_y = 0 \qquad \textbf{(2.18)}$$

The vanishing of ξ_x gives

$$\xi_x = (q+1)x^5\text{-}(3q+2)x^4 + (3q+1)x^3\text{-}(2q+1+\delta)\, x^2 + 2x(1-\delta) - 1 + \delta = 0 \qquad \textbf{(2.19)}$$

The root of this equation that is greater than 1 gives the desired coordinate x_5 of the second lagrangian point. The value of Roche potential ξ_5 corresponding to second lagrangian point is given by

$$\xi_5 = \xi(x_5, 0, 0) \qquad \textbf{(2.20)}$$

which on simplification gives

$$\xi_5 = \frac{(1-\delta)}{x_5} + q\left(\frac{1}{x_5-1} - x_5\right) + \left(\frac{q+1}{2}\right)(x_5^2) \qquad \textbf{(2.21)}$$

Third lagrangian point (L_3):

At third lagrangian point say, ($P_6(-x_6,0,0)$) , equation of the Roche potential can be written as:

$$\xi = \frac{(1-\delta)}{x} + q\left(\frac{1}{1+x} + x\right) + \left(\frac{1+q}{2}\right)(x^2) \qquad \textbf{(2.22)}$$

which on simplification gives

$$\xi_x = (q+1)x^5 + (3q+2)x^4 + (3q+1)x^3 + (\delta-1)\, x^2 + 2x(\delta-1) - 1 + \delta = 0 \qquad \textbf{(2.23)}$$

The value of Roche potential ξ_6 corresponding to third Lagrangian point is given by

$$\xi_6 = \xi(-x_6, 0, 0) \qquad \textbf{(2.24)}$$

which on simplification gives

$$\xi_6 = \frac{(1-\delta)}{x_6} + q\left(\frac{1}{x_6+1} + x_6\right) + \left(\frac{q+1}{2}\right)(x_6^2) \qquad \textbf{(2.25)}$$

2.4 Numerical computations

The equations (2.6 – 2.16) have been used to determine the dimensions of the primary component of the binary system under the effect of radiation pressure. The calculations have been performed for various values of the tidal parameter q and radiation parameter δ. The corresponding results are shown in the tables (1-3) (here the value of q varies whereas the value of δ is fixed) and tables (7-11) (here the value of δ varies whereas the value of q is fixed).

We have also used the expressions (2.17 – 2.25) to find the coordinates and potential at second and third lagrangian point under the effect of radiation pressure. We have computed the values of coordinates and potential at L_2 and L_3 point for the various values of the parameters q and δ. The results so obtained are presented in tables (4-6) and (12-16).

We have used the software Mathematica 7.0 for computing various values of the dimensions of the primary component and other parameters of the binary system of the stars. For the ready reference of the reader the mathematica code has been given in annexure I. Some of the numerical results obtained in the tables (1-6) have been depicted graphically in figures (4-12). These figures show the variation in the values of various parameters with change in the value of q when the value of δ is kept fixed. Also, some of the results obtained in the tables (7-16) have been depicted graphically in figures (13-21). These figures show the variation in the values of various parameters with change in the value of δ when the value of q have been kept fixed. Percentage change in the value of the dimensions of the primary component has also be depicted graphically in figures (22-29).

We have considered 10 models (q= 0.001, 0.005, 0.01, 0.05, 0.1, 0.2, 0.4, 0.5, 0.6, 0.8) of the binary system and the value of the radiation parameter δ has been taken as: 0, 0.1, 0.3, 0.5, 0.7 and 0.9 in the present study. Also, the distance R between centres of two stars has been taken as unity in the numerical computations.

2.5 Conclusions

From tables (1-3), we can conclude that – when the effect of radiation pressure is not considered (that is, δ=0) – then with increase in the value of q, the values of x_1 (point radius), x_2 (back radius), y_3 (side radius) and z_4 (pole radius) of the primary component of the binary system decreases; whereas, the value of ξ_1(Roche limit) increases. These results are in accordance with the results – earlier obtained by Kopal (1959). Further, from table 4 (when δ=0) the value of x_5 (position of L_2 point), ξ_5 (potential at L_2) and ξ_6 (potential at L_3) increases; whereas, the value of x_6 (position of L_3 point) decreases with increase in the value of q.

From tables (1-6), we can also observe that – when the effect of radiation pressure is considered – then the values of x_1, x_2, y_3, z_4, ξ_1,x_5, ξ_5, x_6 and ξ_6 shows similar trend (as shown when the effect of the radiation pressure is not considered) with increase in the value of q. However, it has been noticed that with increasing value of q , there is corresponding less percentage change in the dimensions of the primary component when the higher values of δ are considered (see figures 22-25).

Results in the tables (7-16) show that for the particular value of q, with increase in the value of δ, there is decrease in the values of x_1, x_2, y_3, z_4, x_6, ξ_1 and ξ_6, whereas, the values of x_5 and ξ_5 increases. However, the value of ξ_5(potential at L_2) does not follow the trend upto some value of δ for models q= 0.1, 0.2, 0.4 and 0.5 and it shows opposite trend of decrease for most values of δ when q= 0.6 and 0.8. It can also be observed from these tables that there is corresponding less percentage change in the values of various dimensions of the primary component when higher values of q are taken (see figures 26-29).

So, we can conclude that as the effect of the radiation pressure increases, there is decrease in the value of the point radius, side radius, back radius and pole radius of the primary component of the binary system. Hence, from the present study we have found that with increase in the effect of radiation pressure the overall dimensions of the primary component tend to decrease.

The present work is based on the assumptions of the Roche model (circular orbits, uniform rotation, synchronous binaries and aligned axis). So, it will be worthwhile to see the effects of other conditions (eccentric orbits, differential rotation, non-synchronous binaries and misaligned axis) on the dimensions, structures and oscillations of the primary component of the binary system.

In the present analysis we have used the modified expression of Roche potential as given by Schuerman (1972) and methodology of Kopal (1959) to study the effect of radiation pressure on the dimensions of the primary component of the binary system. For our future study – using the methodology of Mohan and Saxena (1983, 1985) - we intend to investigate the effect of radiation pressure on the structures and oscillations of the rotating stars and stars in the binary system.

Table 1 Dimensions of the primary component of the binary system

δ=0							
Model No.	q	x_1	ξ_1	$-x_2$	$-x_3$	$\pm y_3$	$\pm z_4$
1	0.001	0.932309	1.521480	0.889945	0.001366	0.888157	0.657615
2	0.005	0.886350	1.562560	0.822574	0.003272	0.818067	0.641700
3	0.01	0.858525	1.599100	0.784304	0.004574	0.777787	0.628675
4	0.05	0.768745	1.788850	0.671883	0.008593	0.658041	0.572911
5	0.1	0.717513	1.959100	0.614139	0.010337	0.596094	0.534511
6	0.2	0.658556	2.232730	0.552382	0.011630	0.529893	0.487144
7	0.4	0.592948	2.678100	0.488548	0.012132	0.461892	0.432775
8	0.5	0.570752	2.875840	0.467937	0.012090	0.440095	0.414334
9	0.6	0.552343	3.063440	0.451176	0.011982	0.422443	0.399089
10	0.8	0.522950	3.416970	0.424985	0.011679	0.395011	0.374910
δ=0.1							
1	0.001	0.918190	1.413450	0.874077	0.001604	0.871948	0.637119
2	0.005	0.872830	1.448900	0.807696	0.003585	0.802686	0.622981
3	0.01	0.845083	1.481740	0.769806	0.004905	0.762711	0.610905
4	0.05	0.755261	1.657650	0.658367	0.008862	0.643823	0.557623
5	0.1	0.704013	1.818440	0.601164	0.010515	0.582423	0.520322
6	0.2	0.645139	2.079340	0.540066	0.011680	0.516947	0.474062
7	0.4	0.579805	2.507580	0.477032	0.012038	0.449877	0.420838
8	0.5	0.557752	2.698650	0.456710	0.011953	0.428424	0.402778
9	0.6	0.539483	2.880290	0.440196	0.011811	0.411066	0.387854
10	0.8	0.510354	3.223450	0.414413	0.011460	0.384123	0.364193

1. x_1, x_2, y_3 and z_4 represent the values of point radius, back radius, side radius and pole radius respectively.
2. x_3 is the value of x-coordinate of the side radius of the primary component.
3. ξ_1 represent the value of the Roche limit.

Table 2 Dimensions of the primary component of the binary system

δ=0.3							
Model No.	q	x_1	ξ_1	$-x_2$	$-x_3$	$\pm y_3$	$\pm z_4$
1	0.001	0.868954	1.190250	0.826324	0.002137	0.823365	0.588536
2	0.005	0.832526	1.214790	0.767329	0.004274	0.761101	0.578284
3	0.01	0.807205	1.240030	0.731884	0.005604	0.723438	0.568477
4	0.05	0.720700	1.386950	0.625132	0.009370	0.608983	0.521361
5	0.1	0.670361	1.527700	0.569836	0.010794	0.549542	0.486862
6	0.2	0.612425	1.761580	0.510754	0.011660	0.486272	0.443418
7	0.4	0.548318	2.153340	0.449934	0.011703	0.421748	0.393078
8	0.5	0.526761	2.330150	0.430375	0.011525	0.401189	0.375974
9	0.6	0.508940	2.499110	0.414501	0.011313	0.384587	0.361840
10	0.8	0.480604	2.820150	0.389759	0.010868	0.358876	0.339447
δ=0.5							
1	0.001	0.786523	0.949226	0.752195	0.002544	0.748392	0.527232
2	0.005	0.765002	0.965122	0.706063	0.004882	0.698417	0.520449
3	0.01	0.745997	0.983192	0.675883	0.006242	0.665764	0.513174
4	0.05	0.670167	1.099960	0.579257	0.009706	0.561222	0.474014
5	0.1	0.622927	1.218990	0.527615	0.010836	0.505550	0.443426
6	0.2	0.567700	1.423220	0.472028	0.011343	0.446065	0.403971
7	0.4	0.506339	1.774680	0.414708	0.011039	0.385501	0.357688
8	0.5	0.485736	1.935720	0.396293	0.010767	0.366262	0.341908
9	0.6	0.468729	2.090610	0.381363	0.010487	0.350753	0.328865
10	0.8	0.441751	2.387140	0.358124	0.009954	0.326789	0.308203

1. x_1, x_2, y_3 and z_4 represent the values of point radius, back radius, side radius and pole radius respectively.
2. x_3 is the value of x-coordinate of the side radius of the primary component.
3. ξ_1 represent the value of the Roche limit.

Table 3 Dimensions of the primary component of the binary system

δ=0.7							
Model No.	q	x_1	ξ_1	$-x_2$	$-x_3$	$\pm y_3$	$\pm z_4$
1	0.001	0.666558	0.674778	0.642611	0.002622	0.638109	0.445188
2	0.005	0.656177	0.684816	0.610653	0.005030	0.601643	0.441004
3	0.01	0.645084	0.696928	0.588202	0.006378	0.576396	0.436173
4	0.05	0.589056	0.783676	0.509534	0.009457	0.489451	0.406827
5	0.1	0.548973	0.879048	0.464753	0.010236	0.440822	0.381847
6	0.2	0.500000	1.050000	0.415499	0.010347	0.388110	0.348389
7	0.4	0.444485	1.355490	0.364181	0.009704	0.334206	0.308282
8	0.5	0.425763	1.498410	0.347657	0.009352	0.317101	0.294509
9	0.6	0.410310	1.637140	0.334258	0.009021	0.303328	0.283106
10	0.8	0.385824	1.905430	0.313417	0.008434	0.282089	0.265032
δ=0.9							
1	0.001	0.463183	0.324674	0.476938	0.001934	0.446225	0.308906
2	0.005	0.459406	0.330679	0.434453	0.003764	0.424865	0.306829
3	0.01	0.454944	0.338126	0.422193	0.004797	0.409589	0.304338
4	0.05	0.426476	0.395824	0.374270	0.007025	0.353131	0.287516
5	0.1	0.401406	0.464662	0.343824	0.007417	0.319162	0.271602
6	0.2	0.367427	0.595848	0.308418	0.007216	0.281021	0.248896
7	0.4	0.326412	0.844213	0.270176	0.006448	0.241240	0.220474
8	0.5	0.312266	0.964265	0.257691	0.006110	0.228548	0.202291
9	0.6	0.300530	1.082470	0.247532	0.005812	0.218325	0.189153
10	0.8	0.281871	1.314790	0.231692	0.005315	0.202570	0.153985

1. x_1, x_2, y_3 and z_4 represent the values of point radius, back radius, side radius and pole radius respectively.
2. x_3 is the value of x-coordinate of the side radius of the primary component.
3. ξ_1 represent the value of the Roche limit.

Table 4 Coordinates and potential at the second and third lagrangian point

Model No.	q	x_5	ξ_5	$-x_6$	ξ_6
δ=0					
1	0.001	1.070890	1.520810	0.999417	1.502000
2	0.005	1.122980	1.559220	0.997098	1.509990
3	0.01	1.156220	1.592440	0.994224	1.519950
4	0.05	1.273190	1.755820	0.972216	1.598770
5	0.1	1.346990	1.893800	0.946927	1.695200
6	0.2	1.438080	2.105140	0.902498	1.882360
7	0.4	1.545380	2.434110	0.831800	2.237620
8	0.5	1.582380	2.577260	0.803028	2.407750
9	0.6	1.613040	2.712370	0.777524	2.573830
10	0.8	1.661470	2.966560	0.734139	2.895840
δ=0.1					
1	0.001	1.445100	1.668800	0.964921	1.400190
2	0.005	1.448070	1.679130	0.962659	1.407940
3	0.01	1.451710	1.691850	0.959857	1.417610
4	0.05	1.478690	1.787090	0.938407	1.494110
5	0.1	1.507910	1.893540	0.913779	1.587800
6	0.2	1.555760	2.079440	0.870565	1.769570
7	0.4	1.625270	2.392420	0.801926	2.115210
8	0.5	1.651770	2.532390	0.774034	2.280950
9	0.6	1.674530	2.665490	0.749330	2.442860
10	0.8	1.711750	2.917450	0.707346	2.757110

1. x_5 and x_6 represent the position of the second and third lagranigan point.
2. ξ_5 *and* ξ_6 represent the value of the Roche potential at the second and third lagrangian point.

Table 5 Coordinates and potential at the second and third lagrangian point

Model No.	q	x_5	ξ_5	$-x_6$	ξ_6
δ=0.3					
1	0.001	1.655140	1.793900	0.887369	1.184370
2	0.005	1.656330	1.800530	0.885239	1.191610
3	0.01	1.657810	1.808780	0.882601	1.200640
4	0.05	1.669200	1.873390	0.862428	1.272120
5	0.1	1.682380	1.951110	0.839314	1.359760
6	0.2	1.705720	2.098330	0.798872	1.530110
7	0.4	1.743250	2.369670	0.734929	1.855090
8	0.5	1.758590	2.497340	0.709040	2.011390
9	0.6	1.772170	2.621200	0.686154	2.164360
10	0.8	1.795180	2.860250	0.647345	2.462000
δ=0.5					
1	0.001	1.784160	1.872950	0.793207	0.946607
2	0.005	1.784790	1.878300	0.791242	0.953263
3	0.01	1.785580	1.884980	0.788810	0.961566
4	0.05	1.791640	1.937890	0.770237	1.027370
5	0.1	1.798760	2.002830	0.749015	1.108180
6	0.2	1.811640	2.129300	0.712034	1.265640
7	0.4	1.833040	2.371740	0.653925	1.567370
8	0.5	1.842020	2.489020	0.630516	1.713070
9	0.6	1.850090	2.604280	0.609872	1.856020
10	0.8	1.864000	2.830010	0.574971	2.135060

1. x_5 and x_6 represent the position of the second and third lagranigan point.
2. ξ_5 *and* ξ_6 represent the value of the Roche potential at the second and third lagrangian point.

Table 6 Coordinates and potential at the second and third lagrangian point

δ=0.7					
Model No.	q	x_5	ξ_5	$-x_6$	ξ_6
1	0.001	1.882530	1.932350	0.668997	0.673703
2	0.005	1.882840	1.937000	0.667261	0.679666
3	0.01	1.883230	1.942800	0.665113	0.687107
4	0.05	1.886220	1.989020	0.648751	0.746151
5	0.1	1.889760	2.046320	0.630140	0.818835
6	0.2	1.896220	2.159510	0.597912	0.960991
7	0.4	1.907120	2.381390	0.547755	1.235260
8	0.5	1.911760	2.490550	0.527702	1.368490
9	0.6	1.915960	2.598780	0.510085	1.499670
10	0.8	1.923280	2.812940	0.480430	1.756900
δ=0.9					
1	0.001	1.963800	1.980180	0.463827	0.324420
2	0.005	1.963890	1.984360	0.462506	0.329435
3	0.01	1.964000	1.989580	0.460875	0.335697
4	0.05	1.964860	2.031330	0.448513	0.385514
5	0.1	1.965880	2.083390	0.434593	0.447145
6	0.2	1.967760	2.187170	0.410822	0.568605
7	0.4	1.970950	2.393580	0.374579	0.806013
8	0.5	1.972330	2.496320	0.360313	0.922625
9	0.6	1.973580	2.598810	0.347873	1.038140
10	0.8	1.975760	2.803150	0.327110	1.266510

1. x_5 and x_6 represent the position of the second and third lagranigan point.
2. ξ_5 *and* ξ_6 represent the value of the Roche potential at the second and third lagrangian point.

Table 7 Dimensions of the primary component of the binary system

S. No.	δ	x_1	ξ_1	$-x_2$	$-x_3$	$\pm y_3$	$\pm z_4$
q=0.001							
1	0	0.932309	1.52148	0.889945	0.001366	0.888157	0.657615
2	0.1	0.918190	1.41345	0.874077	0.001605	0.871948	0.637119
3	0.3	0.868954	1.190250	0.826324	0.002137	0.823365	0.588536
4	0.5	0.786523	0.949226	0.752195	0.002544	0.748392	0.527232
5	0.7	0.666558	0.674778	0.642611	0.002622	0.638109	0.445188
6	0.9	0.463183	0.324674	0.476938	0.001934	0.446225	0.308906
q=0.005							
1	0	0.886350	1.562560	0.822574	0.003272	0.818067	0.641700
2	0.1	0.872830	1.448900	0.807696	0.003585	0.802686	0.622981
3	0.3	0.832526	1.214790	0.767329	0.004274	0.761101	0.578284
4	0.5	0.765002	0.965122	0.706063	0.004882	0.698417	0.520449
5	0.7	0.656177	0.684816	0.610653	0.005030	0.601643	0.441004
6	0.9	0.459406	0.330679	0.434453	0.003764	0.424865	0.306829

1. x_1, x_2, y_3 and z_4 represent the values of point radius, back radius, side radius and pole radius respectively.
2. x_3 is the value of x-coordinate of the side radius of the primary component.
3. ξ_1 represent the value of the Roche limit.

Table 8 Dimensions of the primary component of the binary system

q=0.01							
S. No.	δ	x_1	ξ_1	$-x_2$	$-x_3$	$\pm y_3$	$\pm z_4$
1	0	0.858525	1.59910	0.784304	0.004574	0.777787	0.628675
2	0.1	0.845083	1.48174	0.769806	0.004905	0.762711	0.610905
3	0.3	0.807205	1.240030	0.731884	0.005604	0.723438	0.568477
4	0.5	0.745997	0.983192	0.675883	0.006242	0.665764	0.513174
5	0.7	0.645084	0.696928	0.588202	0.006378	0.576396	0.436173
6	0.9	0.454944	0.338126	0.422193	0.004797	0.409589	0.304338
q=0.05							
1	0	0.768745	1.788850	0.671883	0.008593	0.658041	0.572911
2	0.1	0.755261	1.657650	0.658367	0.008862	0.643823	0.557623
3	0.3	0.720700	1.386950	0.625132	0.009370	0.608983	0.521361
4	0.5	0.670167	1.099960	0.579257	0.009706	0.561222	0.474014
5	0.7	0.589056	0.783676	0.509534	0.009457	0.489451	0.406827
6	0.9	0.426476	0.395824	0.374270	0.007025	0.353131	0.287516

1. x_1, x_2, y_3 and z_4 represent the values of point radius, back radius, side radius and pole radius respectively.
2. x_3 is the value of x-coordinate of the side radius of the primary component.
3. ξ_1 represent the value of the Roche limit.

Table 9 Dimensions of the primary component of the binary system

q=0.1							
S. No.	δ	x_1	ξ_1	$-x_2$	$-x_3$	$\pm y_3$	$\pm z_4$
1	0	0.717513	1.95910	0.614139	0.010337	0.596094	0.534511
2	0.1	0.704013	1.81844	0.601164	0.010515	0.582423	0.520322
3	0.3	0.670361	1.52770	0.569836	0.010794	0.549542	0.486862
4	0.5	0.622927	1.21899	0.527615	0.010836	0.505550	0.443426
5	0.7	0.548973	0.879048	0.464753	0.010236	0.440822	0.381847
6	0.9	0.401406	0.464662	0.343824	0.007417	0.319162	0.271602
q=0.2							
1	0	0.658556	2.232730	0.552382	0.011630	0.529893	0.487144
2	0.1	0.645139	2.079340	0.540066	0.011680	0.516947	0.474062
3	0.3	0.612425	1.761580	0.510754	0.011660	0.486272	0.443418
4	0.5	0.567700	1.423220	0.472028	0.011343	0.446065	0.403971
5	0.7	0.500000	1.050000	0.415499	0.010347	0.388110	0.348389
6	0.9	0.367427	0.595848	0.308418	0.007216	0.281021	0.248896

1. x_1, x_2, y_3 and z_4 represent the values of point radius, back radius, side radius and pole radius respectively.
2. x_3 is the value of x-coordinate of the side radius of the primary component.
3. ξ_1 represent the value of the Roche limit.

Table 10 Dimensions of the primary component of the binary system

q=0.4							
S. No.	δ	x_1	ξ_1	$-x_2$	$-x_3$	$\pm y_3$	$\pm z_4$
1	0	0.592948	2.678100	0.488548	0.012132	0.461892	0.432775
2	0.1	0.579805	2.507580	0.477032	0.012038	0.449877	0.420838
3	0.3	0.548318	2.153340	0.449934	0.011703	0.421748	0.393078
4	0.5	0.506339	1.774680	0.414708	0.011039	0.385501	0.357688
5	0.7	0.444485	1.355490	0.364181	0.009704	0.334206	0.308282
6	0.9	0.326412	0.844213	0.270176	0.006448	0.241240	0.220474
q=0.5							
1	0	0.570752	2.875840	0.467937	0.012090	0.440095	0.414334
2	0.1	0.557752	2.698650	0.456710	0.011953	0.428424	0.402778
3	0.3	0.526761	2.330150	0.430375	0.011525	0.401189	0.375974
4	0.5	0.485736	1.935720	0.396293	0.010767	0.366262	0.341908
5	0.7	0.425763	1.498410	0.347657	0.009352	0.317101	0.294509
6	0.9	0.312266	0.964265	0.257691	0.006110	0.228548	0.210546

1. x_1, x_2, y_3 and z_4 represent the values of point radius, back radius, side radius and pole radius respectively.
2. x_3 is the value of x-coordinate of the side radius of the primary component.
3. ξ_1 represent the value of the Roche limit.

Table 11 Dimensions of the primary component of the binary system

S. No.	δ	x_1	ξ_1	$-x_2$	$-x_3$	$\pm y_3$	$\pm z_4$
q=0.6							
1	0	0.552343	3.063440	0.451176	0.011982	0.422443	0.399089
2	0.1	0.539483	2.880290	0.440196	0.011811	0.411066	0.387854
3	0.3	0.508940	2.499110	0.414501	0.011313	0.384587	0.361840
4	0.5	0.468729	2.090610	0.381363	0.010487	0.350753	0.328865
5	0.7	0.410310	1.637140	0.334258	0.009021	0.303328	0.283106
6	0.9	0.300530	1.082470	0.247532	0.005812	0.218325	0.202291
q=0.8							
1	0	0.522950	3.416970	0.424985	0.011679	0.395011	0.374910
2	0.1	0.510354	3.223450	0.414413	0.011460	0.384123	0.364193
3	0.3	0.480604	2.820150	0.389759	0.010868	0.358876	0.339447
4	0.5	0.441751	2.387140	0.358124	0.009954	0.326789	0.308203
5	0.7	0.385824	1.905430	0.313417	0.008434	0.282089	0.265032
6	0.9	0.281871	1.314790	0.231692	0.005315	0.202570	0.189153

1. x_1, x_2, y_3 and z_4 represent the values of point radius, back radius, side radius and pole radius respectively.
2. x_3 is the value of x-coordinate of the side radius of the primary component.
3. ξ_1 represent the value of the Roche limit.

Table 12 Coordinates and potential at the second and third lagrangian point

q=0.001					
S. No.	δ	x_5	ξ_5	$-x_6$	ξ_6
1	0	1.070890	1.520810	0.999417	1.50200
2	0.1	1.445100	1.668800	0.964921	1.40019
3	0.3	1.655140	1.793900	0.887369	1.18437
4	0.5	1.784160	1.872950	0.793207	0.946607
5	0.7	1.882530	1.932350	0.668997	0.673703
6	0.9	1.963800	1.980180	0.463827	0.324420
q=0.005					
1	0	1.122980	1.559220	0.997098	1.509990
2	0.1	1.448070	1.679130	0.962659	1.407940
3	0.3	1.656330	1.800530	0.885239	1.191610
4	0.5	1.784790	1.878300	0.791242	0.953263
5	0.7	1.882840	1.937000	0.667261	0.679666
6	0.9	1.963890	1.984360	0.462506	0.329435

1. x_5 and x_6 represent the position of the second and third lagranigan point.
2. ξ_5 *and* ξ_6 represent the value of the Roche potential at the second and third lagrangian point.

Table 13 Coordinates and potential at the second and third lagrangian point

S. No.	δ	x_5	ξ_5	$-x_6$	ξ_6
q=0.01					
1	0	1.156220	1.592440	0.994224	1.519950
2	0.1	1.451710	1.691850	0.959857	1.417610
3	0.3	1.657810	1.808780	0.882601	1.200640
4	0.5	1.785580	1.884980	0.788810	0.961566
5	0.7	1.883230	1.942800	0.665113	0.687107
6	0.9	1.964000	1.989580	0.460875	0.335697
q=0.05					
1	0	1.273190	1.755820	0.972216	1.598770
2	0.1	1.478690	1.787090	0.938407	1.494110
3	0.3	1.669200	1.873390	0.862428	1.272120
4	0.5	1.791640	1.937890	0.770237	1.027370
5	0.7	1.886220	1.989020	0.648751	0.746151
6	0.9	1.964860	2.031330	0.448513	0.385514

1. x_5 and x_6 represent the position of the second and third lagranigan point.
2. ξ_5 *and* ξ_6 represent the value of the Roche potential at the second and third lagrangian point.

Table 14 Coordinates and potential at the second and third lagrangian point

q=0.1					
S. No.	δ	x_5	ξ_5	$-x_6$	ξ_6
1	0	1.346990	1.893800	0.946927	1.69520
2	0.1	1.507910	1.893540	0.913779	1.58780
3	0.3	1.682380	1.951110	0.839314	1.35976
4	0.5	1.798760	2.002830	0.749015	1.10818
5	0.7	1.889760	2.046320	0.630140	0.818835
6	0.9	1.965880	2.083390	0.434593	0.447145
q=0.2					
1	0	1.438080	2.105140	0.902498	1.882360
2	0.1	1.555760	2.079440	0.870565	1.769570
3	0.3	1.705720	2.098330	0.798872	1.530110
4	0.5	1.811640	2.129300	0.712034	1.265640
5	0.7	1.896220	2.159510	0.597912	0.960991
6	0.9	1.967760	2.187170	0.410822	0.568605

1. x_5 and x_6 represent the position of the second and third lagranigan point.
2. ξ_5 *and* ξ_6 represent the value of the Roche potential at the second and third lagrangian point.

Table 15 Coordinates and potential at the second and third lagrangian point

q=0.4					
S. No.	δ	x_5	ξ_5	$-x_6$	ξ_6
1	0	1.545380	2.434110	0.831800	2.237620
2	0.1	1.625270	2.392420	0.801926	2.115210
3	0.3	1.743250	2.369670	0.734929	1.855090
4	0.5	1.833040	2.371740	0.653925	1.567370
5	0.7	1.907120	2.381390	0.547755	1.235260
6	0.9	1.970950	2.393580	0.374579	0.806013
q=0.5					
1	0	1.582380	2.577260	0.803028	2.407750
2	0.1	1.651770	2.532390	0.774034	2.280950
3	0.3	1.758590	2.497340	0.709040	2.011390
4	0.5	1.842020	2.489020	0.630516	1.713070
5	0.7	1.911760	2.490550	0.527702	1.368490
6	0.9	1.972330	2.496320	0.360313	0.922625

1. x_5 and x_6 represent the position of the second and third lagranigan point.
2. ξ_5 *and* ξ_6 represent the value of the Roche potential at the second and third lagrangian point.

Table 16 Coordinates and potential at second and third lagrangian point

q=0.6					
S. No.	δ	x_5	ξ_5	$-x_6$	ξ_6
1	0	1.613040	2.712370	0.777524	2.573830
2	0.1	1.674530	2.665490	0.749330	2.442860
3	0.3	1.772170	2.621200	0.686154	2.164360
4	0.5	1.850090	2.604280	0.609872	1.856020
5	0.7	1.915960	2.598780	0.510085	1.499670
6	0.9	1.973580	2.598810	0.347873	1.038140
q=0.8					
1	0	1.661470	2.966560	0.734139	2.895840
2	0.1	1.711750	2.917450	0.707346	2.757110
3	0.3	1.795180	2.860250	0.647345	2.462000
4	0.5	1.864000	2.830010	0.574971	2.135060
5	0.7	1.923280	2.812940	0.480430	1.756900
6	0.9	1.975760	2.803150	0.327110	1.266510

1. x_5 and x_6 represent the position of the second and third lagranigan point.
2. ξ_5 *and* ξ_6 represent the value of the Roche potential at the second and third lagrangian point.

Variation in the values of various parameters with change in the value of tidal parameter q

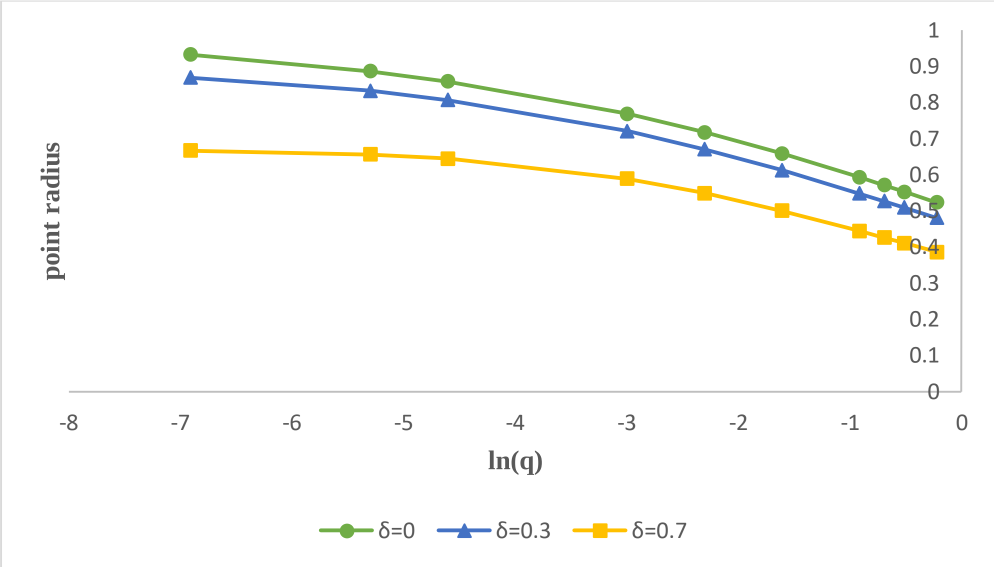

Fig. 4. Point radius (first lagrangian point) of the primary component of the binary system

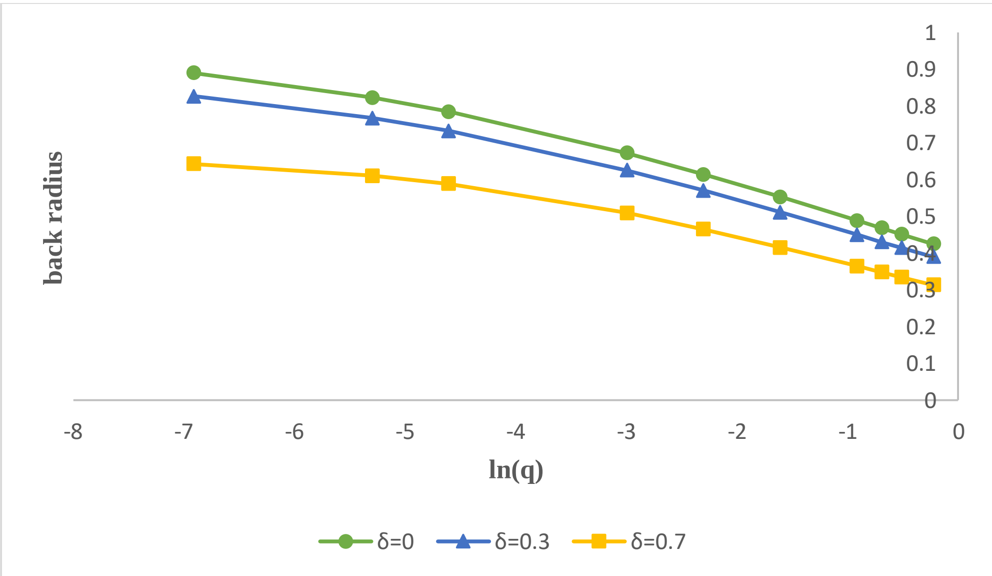

Fig. 5. Back radius of the primary component of the binary system

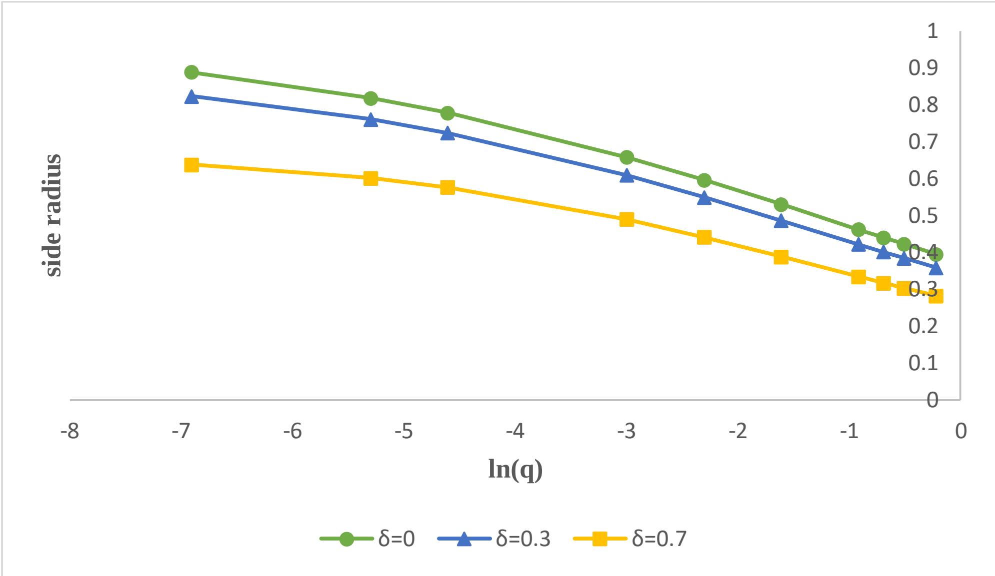

Fig. 6. Side radius of the primary component of the binary system

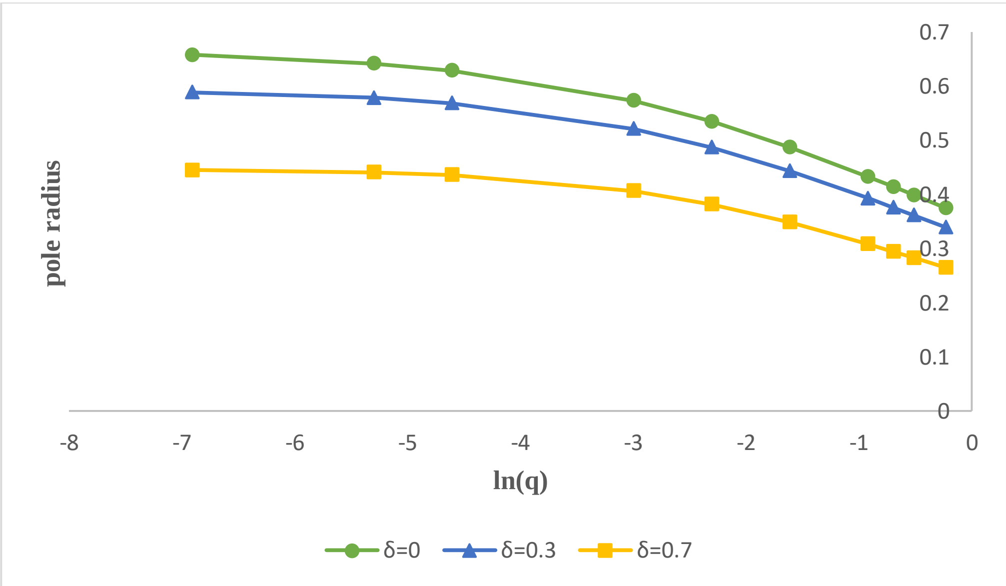

Fig. 7. Pole Radius of the primary component of the binary system

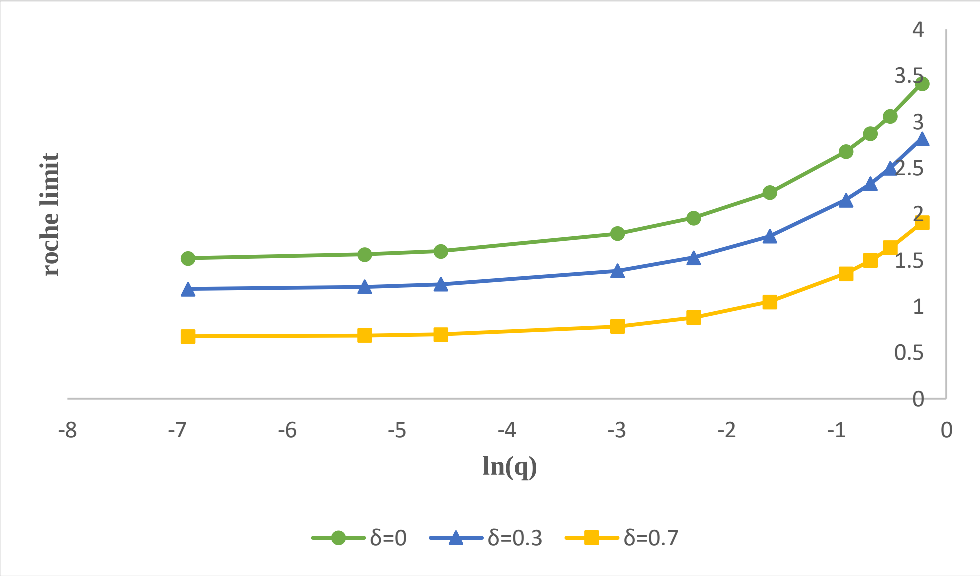

Fig. 8. Roche limit of the primary component of the binary system

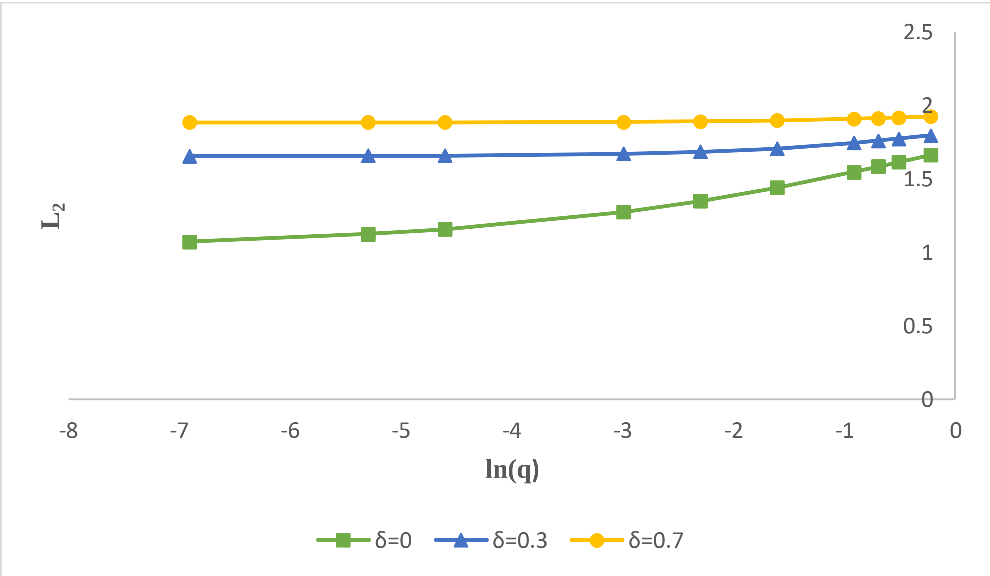

Fig. 9. Position of the L_2 point (x-coordinate)

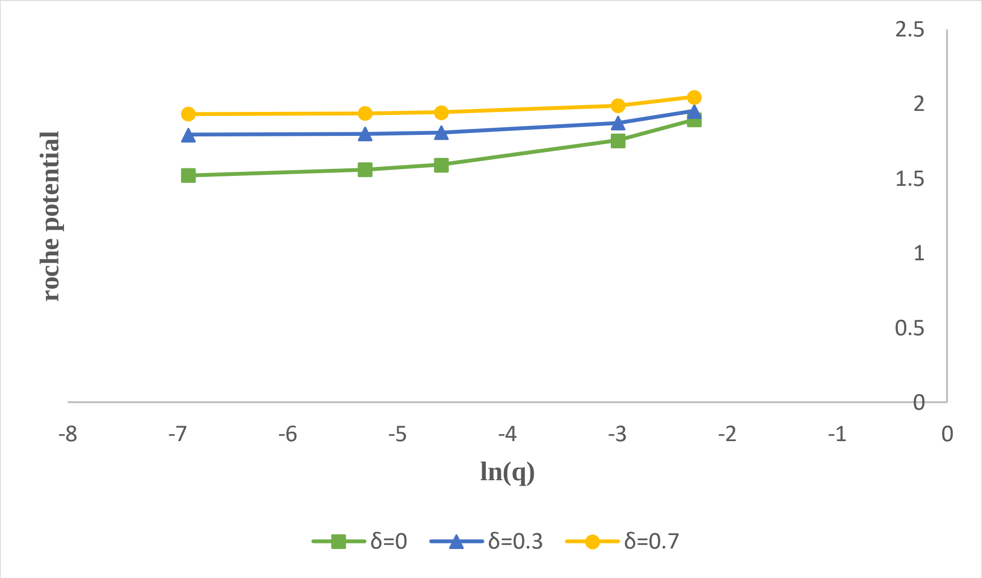

Fig. 10. Roche potential at the L_2 point

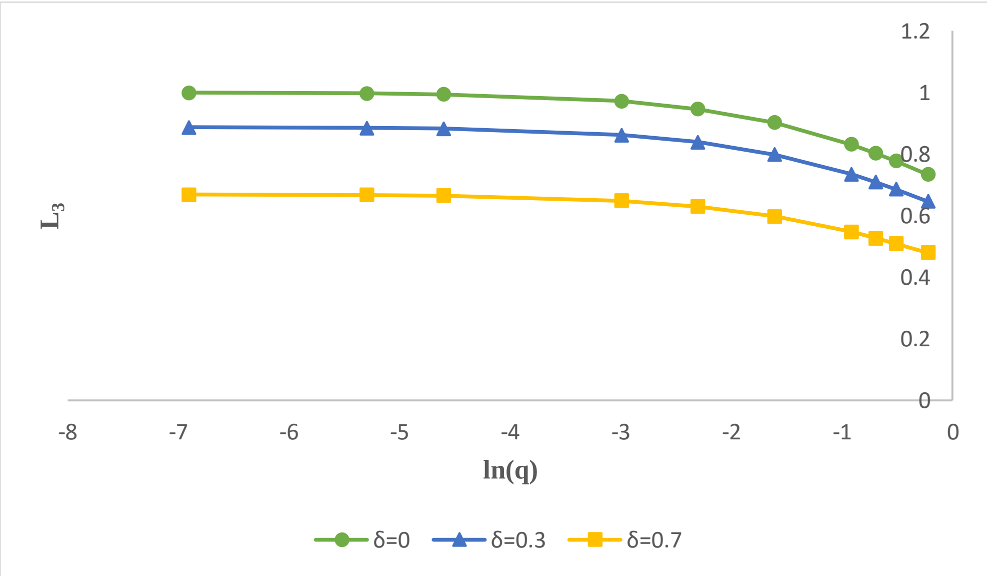

Fig. 11. Position of the L_3 point (x-coordinate)

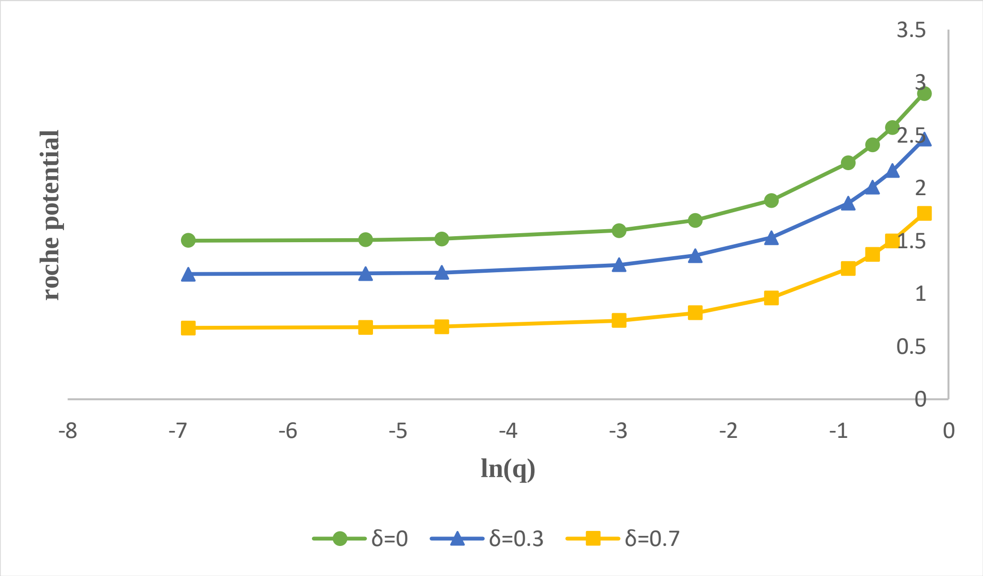

Fig. 12. Roche potential at the L_3 point

Variation in the values of various parameters with change in the value of radiation parameter δ

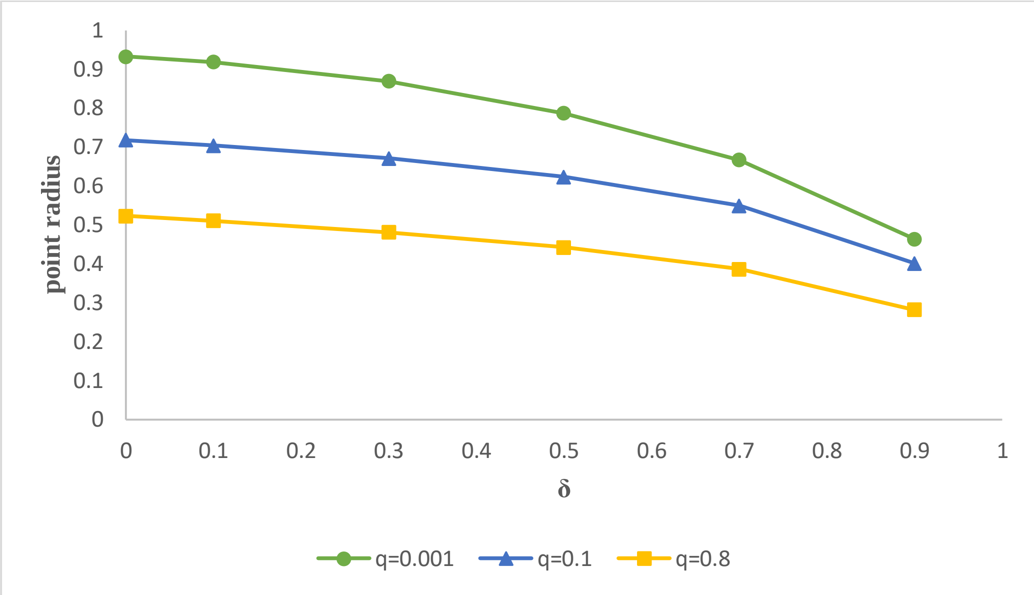

Fig. 13. Point radius of the primary component of the binary system

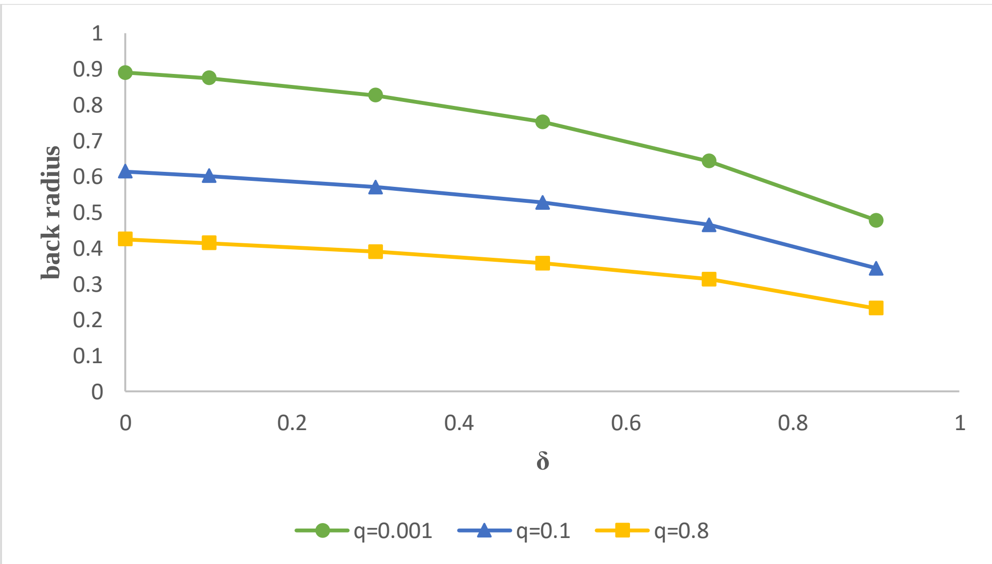

Fig. 14. Back radius of the primary component of the binary system

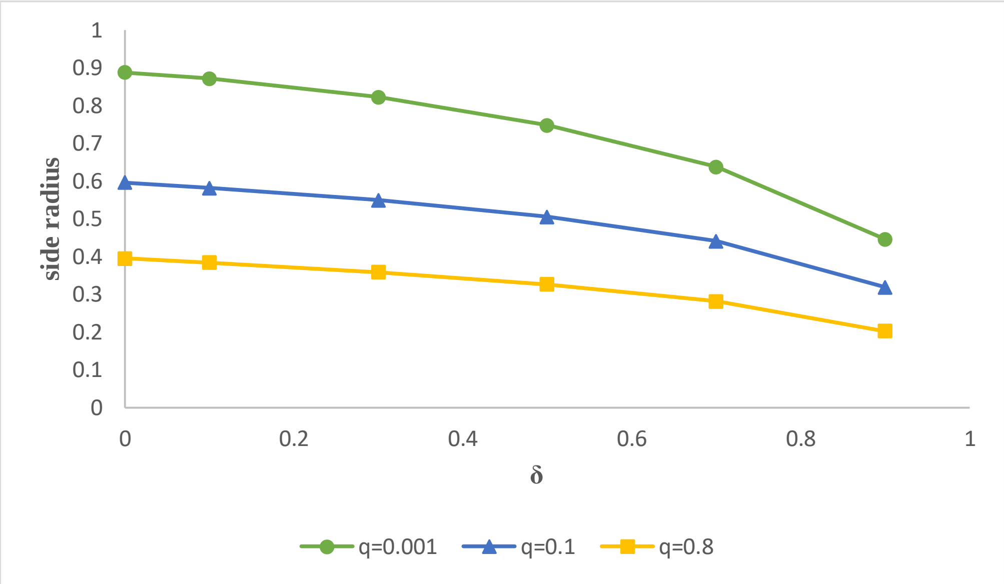

Fig. 15. Side radius of the primary component of the binary system

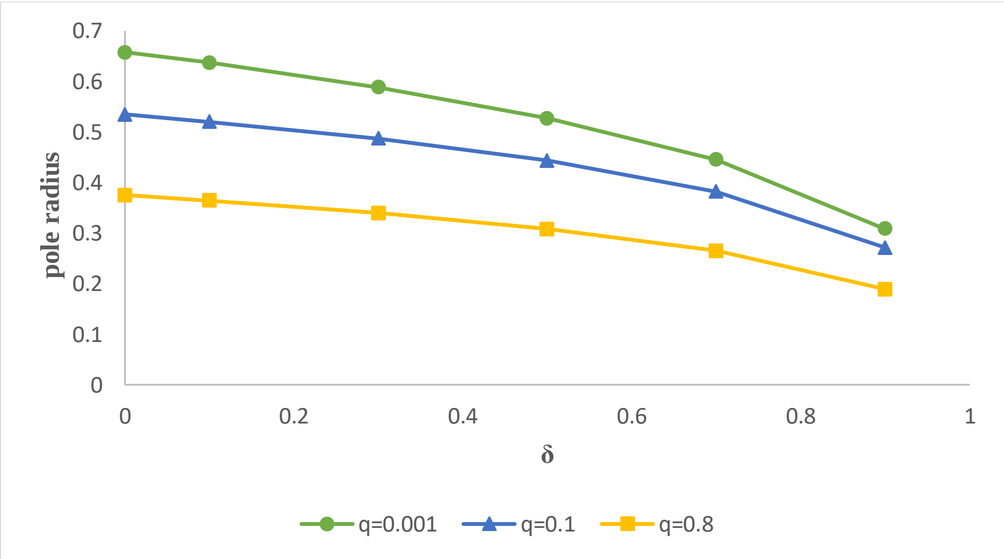

Fig. 16. Pole radius of the primary component of the binary system

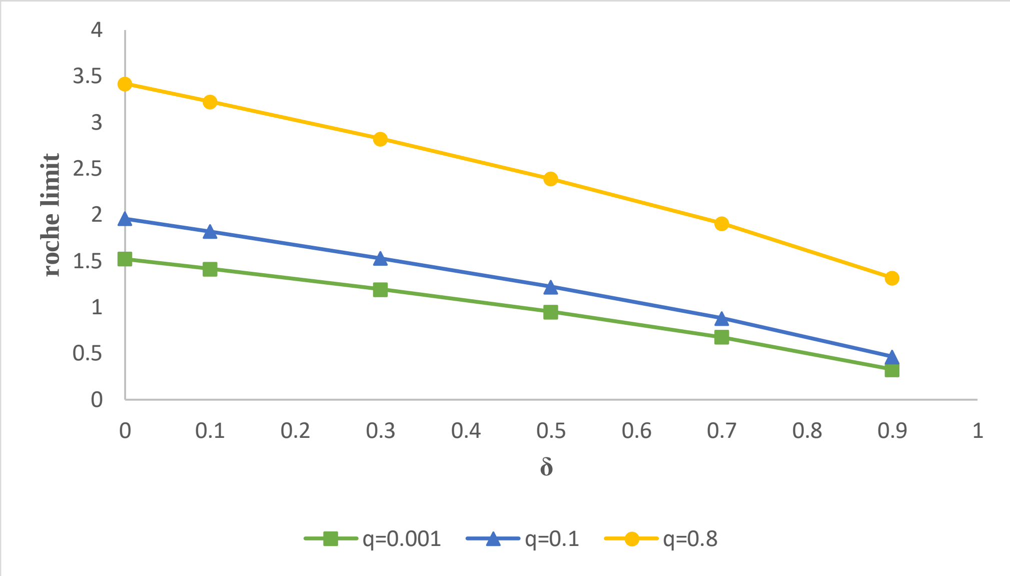

Fig. 17. Roche limit of the primary component of the binary system

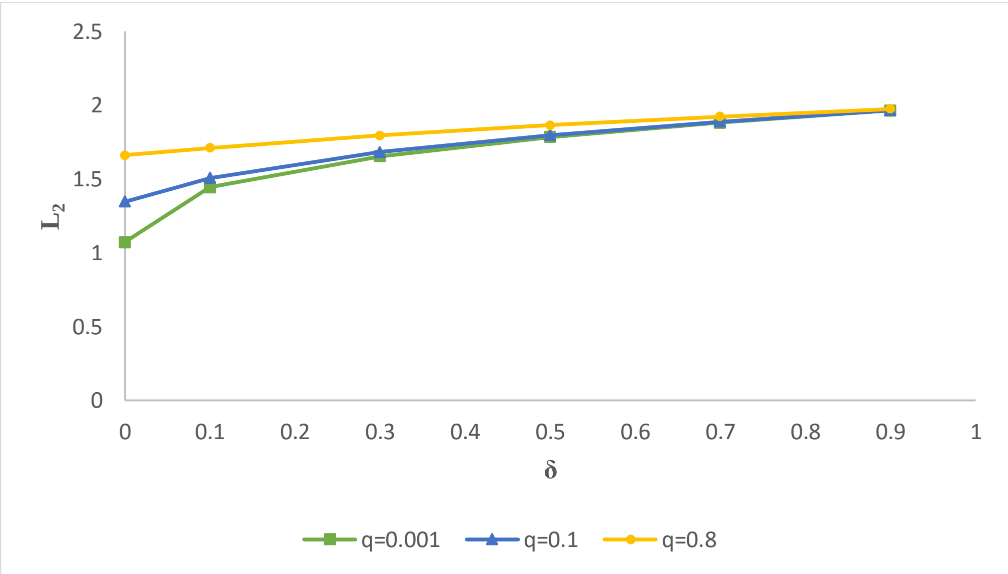

Fig. 18. Position of the L_2 point (x-coordinate)

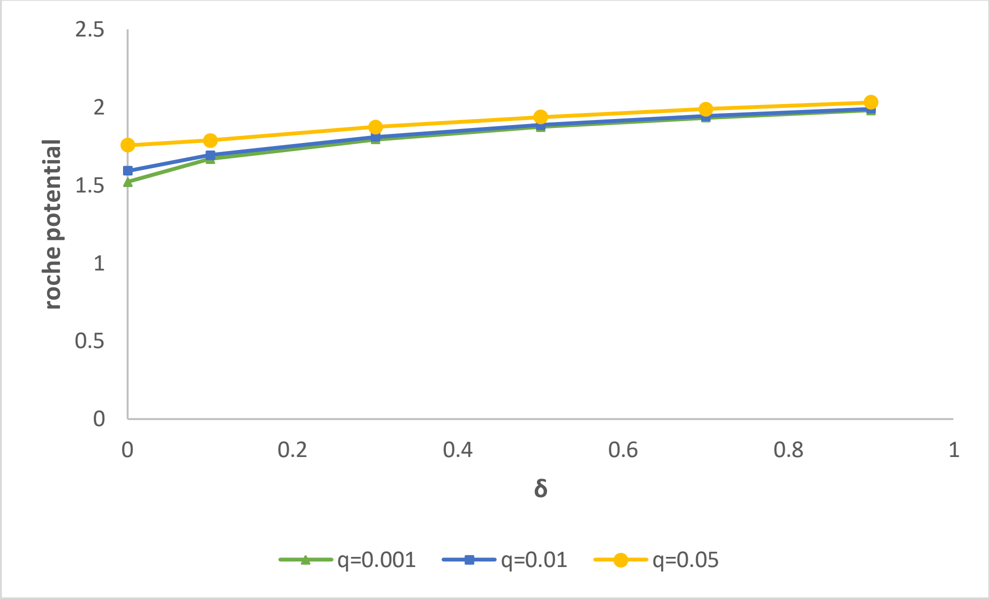

Fig. 19. Roche potential at L_2 point

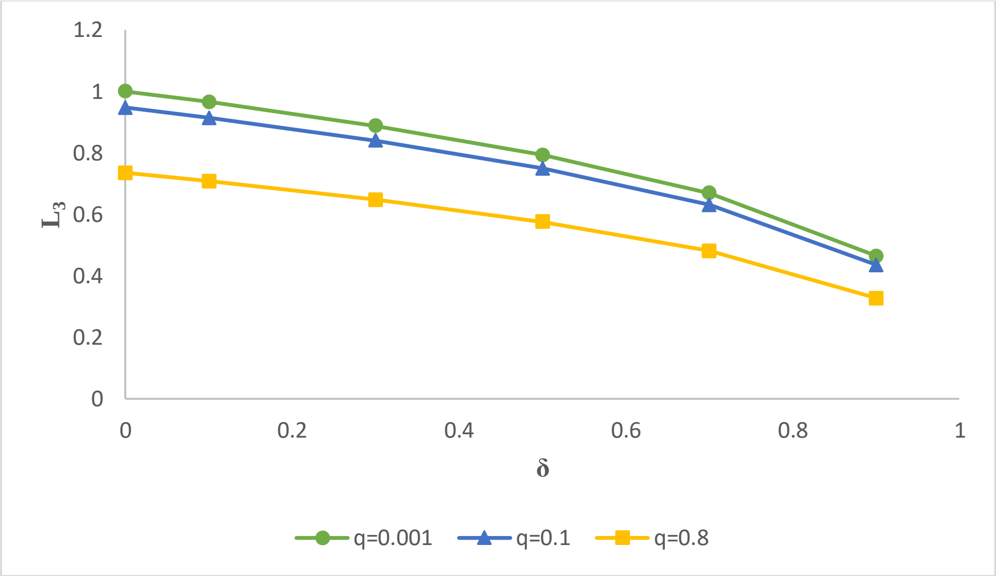

Fig. 20. Position of the L_3 point (x-coordinate)

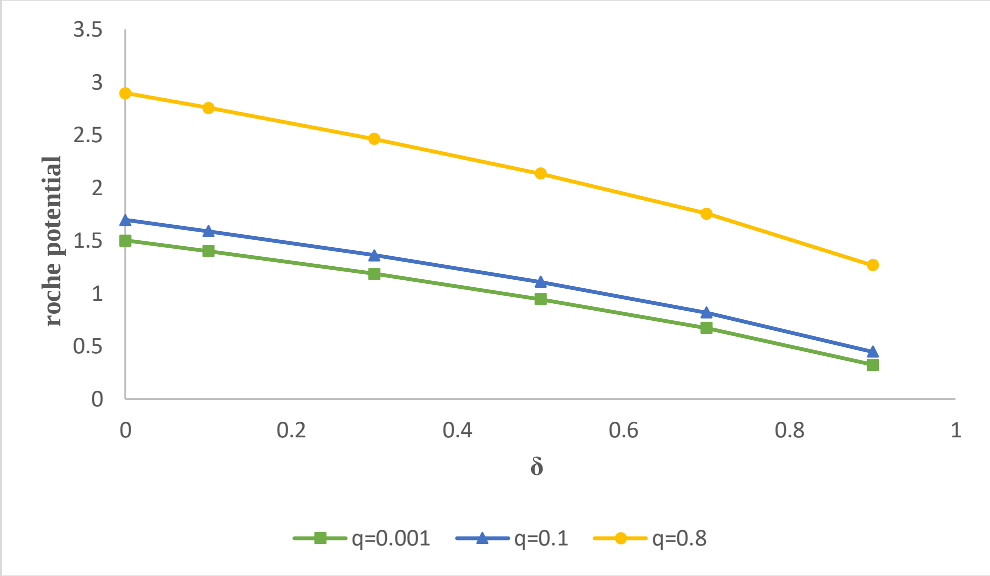

Fig. 21. Roche potential at the L_3 point

Percentage change in the values of various parameters with change in the value of tidal parameter q

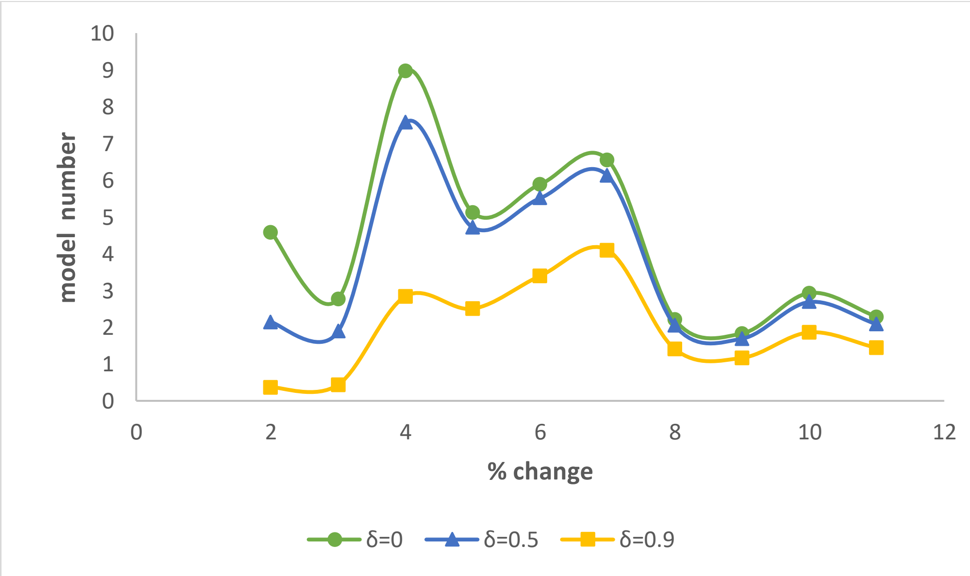

Fig. 22. Percentage change in the values of the point radius

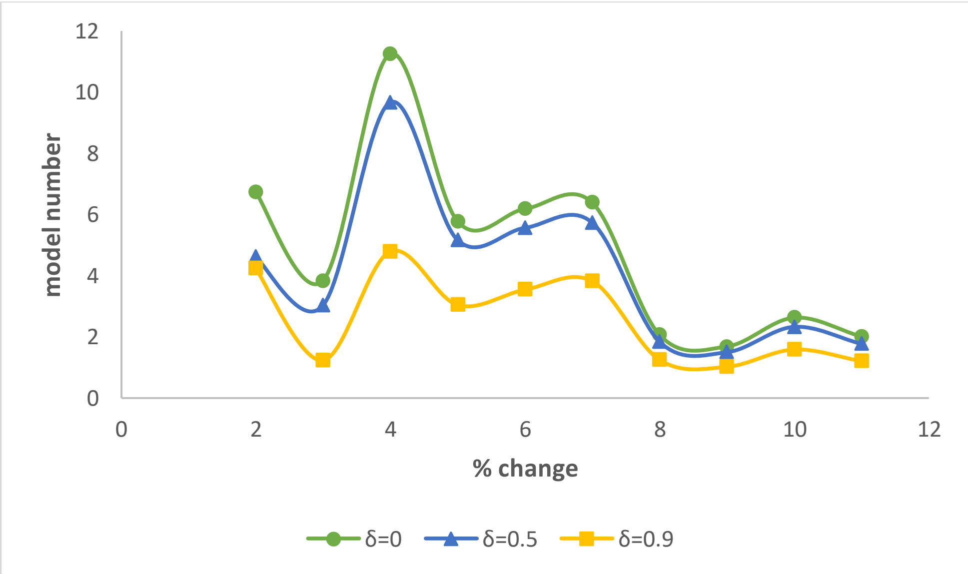

Fig. 23. Percentage change in the values of the back radius

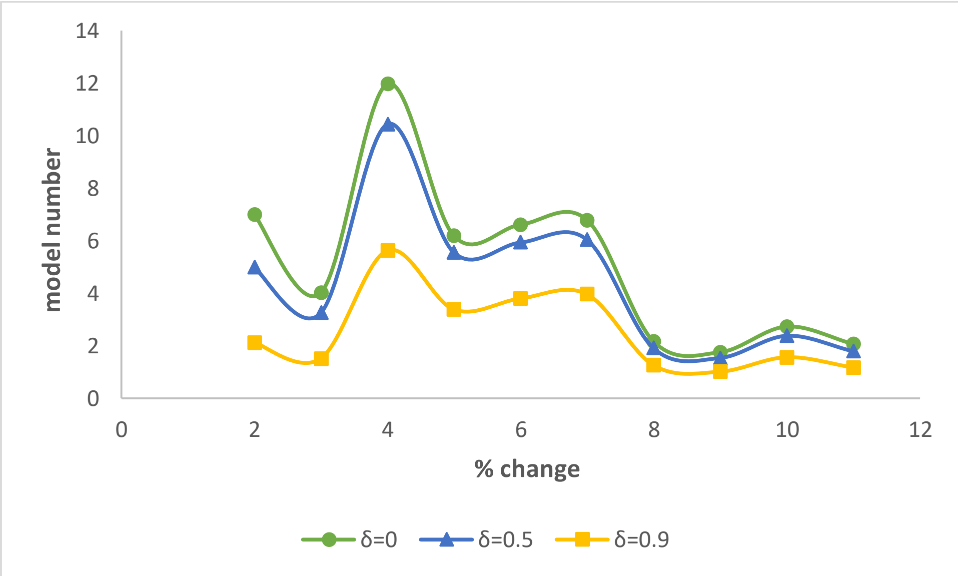

Fig. 24. Percentage change in the values of the side radius

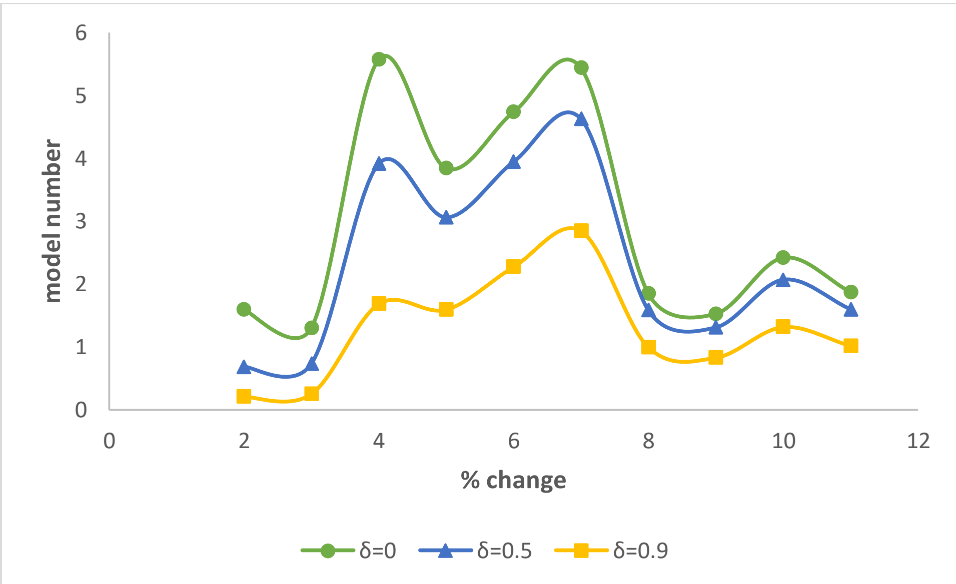

Fig. 25. Percentage change in the values of the pole radius

Percentage change in the values of various parameters with change in the value of radiation parameter δ

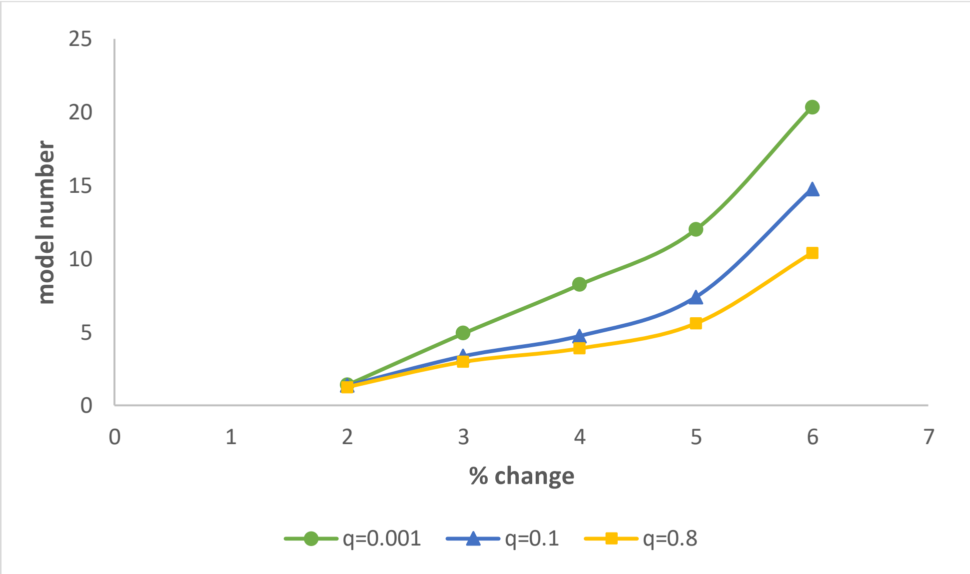

Fig. 26. Percentage change in the values of the point radius

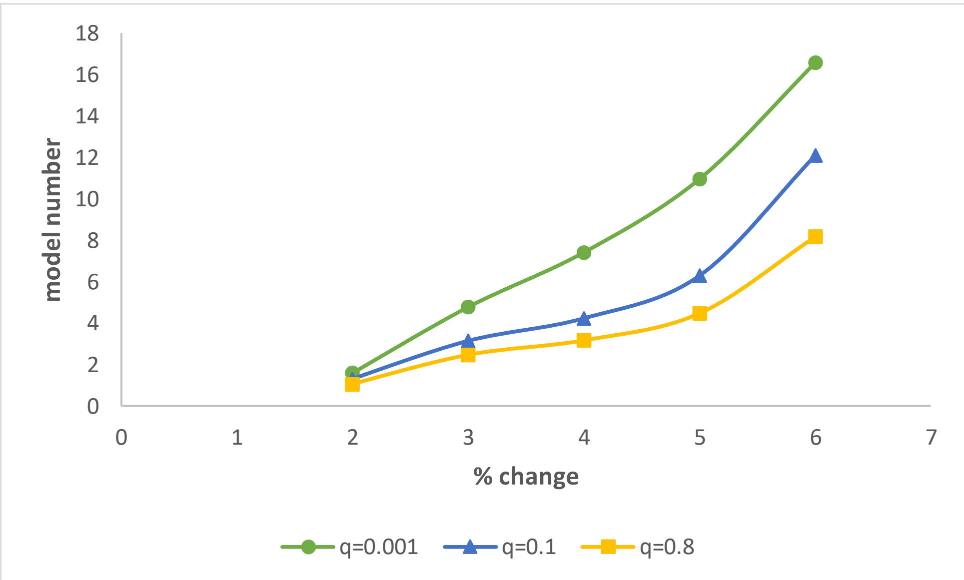

Fig. 27. Percentage change in the values of the back radius

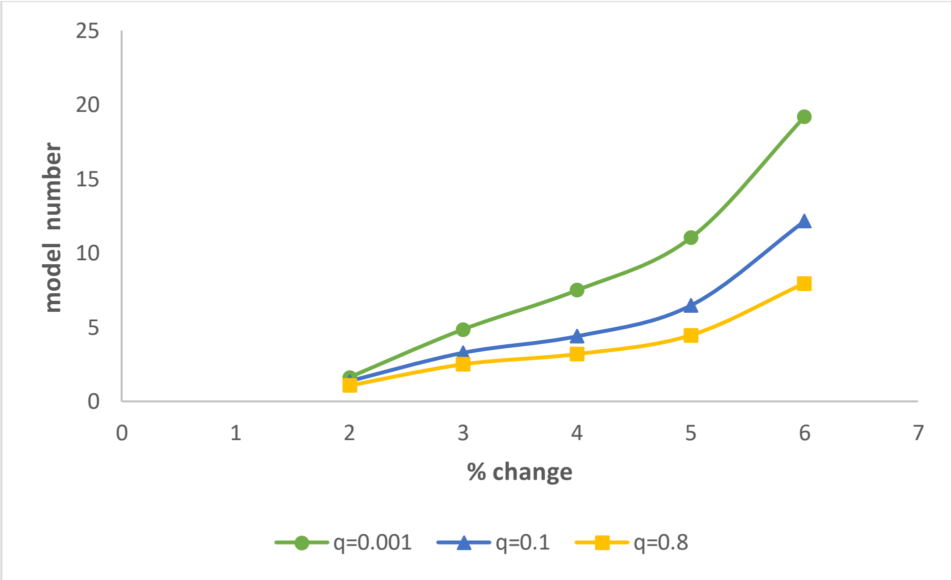

Fig. 28. Percentage change in the values of the side radius

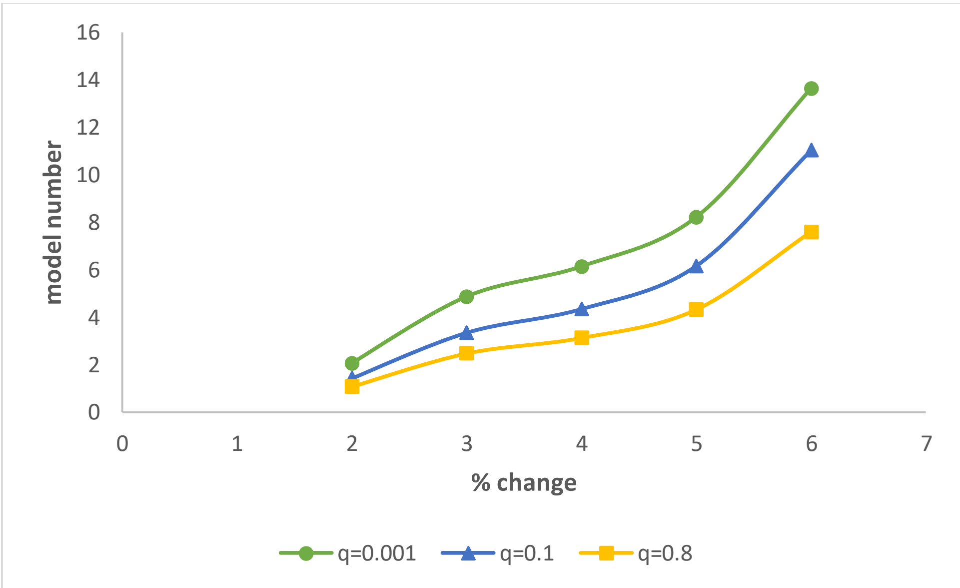

Fig. 29. Percentage change in the values of the pole radius

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Annexure – I

Mathematica code for determining dimensions and various other parameters of the primary component of the binary system;

```
Clear[q, x, x1, y, n, ξ1, ξ5, ξ6, x6, δ];
 Coordinates of First lagrangian point (L1);
q = 0.001; δ = 0;
FL = (q + 1) * x^5 - (3 * q + 2) * x^4 + (3 * q + 1 ) * x^3 - x^2 * (1 - δ) + 2 * x * (1 - δ) - 1 + δ
```

$-1 + 2x - x^2 + 1.003 x^3 - 2.003 x^4 + 1.001 x^5$

```
NSolve[FL == 0, x]
```

{{x → -0.499556 - 0.865641 i}, {x → -0.499556 + 0.865641 i},
 {x → 0.932309}, {x → 1.0339 - 0.0614281 i}, {x → 1.0339 + 0.0614281 i}}

```
FindRoot[FL, {x, 0}]
```

{x → 0.932309}

```
x1 = x /. FindRoot[FL, {x, 0}]
```

0.932309

VALUE OF THE POTENTIAL AT L1;

$$\xi 1 = \frac{(1-\delta)}{x1} + q * \left(\frac{1}{1 - x1} - x1\right) + \frac{(q+1)}{2} * (x1)^2$$

1.52148

Back coordinates of Primary component;

$$BR1 = \frac{(q+1)}{2} * x^4 + \left(\frac{(q+1)}{2} + q\right) * x^3 + (q - \xi 1) * x^2 + (q + 1 - \delta - \xi 1) * x + 1 - \delta;$$

```
NSolve[BR1 == 0, x]
```

{{x → -2.00997}, {x → -0.999506}, {x → 0.889945}, {x → 1.11753}}

```
FindRoot[BR1 == 0, {x, 0.5, 1}]
```

{x → 0.889945}

VALUE OF THE SIDE COORDINATES X3 Y3 OF THE PRIMARY COMPONENT OF BINARY SYSTEM;

$$SR1 = \frac{(1-\delta)}{\sqrt{x^2+y^2}} + q\left(\frac{1}{\sqrt{(1-x)^2+y^2}} - x\right) + \frac{(q+1)}{2} * (x^2+y^2) - \xi 1;$$

$$SR2 = \frac{-x\,(1-\delta)}{(x^2+y^2)^{\frac{3}{2}}} + q\left(\frac{1-x}{((1-x)^2+y^2)^{\frac{3}{2}}} - 1\right) + \frac{(q+1)}{2} * 2 * x;$$

NSolve[{SR1 == 0, SR2 == 0}, {x, y}, WorkingPrecision → 2 MachinePrecision]

{{x → -20.829918907552962027383880585 79, y → 20.805917844205468616269931124 31 ⅈ},
{x → -20.82991890755296202738388058579, y → -20.80591784420546861626993112431 ⅈ},
{x → 0.002429370813310439707186570583471, y → -1.120406042032515639085832751467},
{x → 0.002429370813310439707186570583471, y → 1.120406042032515639085832751467},
{x → -0.001366052234603223928511355623810, y → 0.888156690740491817248994926659 7},
{x → -0.001366052234603223928511355623810, y → -0.8881566907404918172489949266597},
{x → 1.074547166491731676317748074213, y → 0.02261482822525523749676026310860 ⅈ},
{x → 1.074547166491731676317748074213, y → -0.02261482822525523749676026310860 ⅈ},
{x → 1.000406103710096883147568179808, y → -0.04550040343240037683430364546116},
{x → 1.000406103710096883147568179808, y → 0.04550040343240037683430364546116},
{x → 0.9323089895399696287185771097095, y → 1.152657271578775020361617909043 × 10^{-8} ⅈ},
{x → 0.9323089895399696287185771097095, y → -1.152657271578775020361617909043 × 10^{-8} ⅈ}}

VALUES OF THE Z COORDINATES Z4 OF THE PRIMARY COMPONENT;

Clear[x1, x2, pp, ξ1, n, q];
q = 0.001; δ = 0;
ξ1 = 1.52148; xp = -0.001366;

$$pp = \frac{(1-\delta)}{\sqrt{xp^2+z^2}} + q\left(\frac{1}{\sqrt{(1-xp)^2+z^2}} - xp\right) + \frac{(q+1)}{2} * xp^2 - \xi 1;$$

Solve[pp == 0, z]

{{z → -0.657615}, {z → 0.657615}}

Coordinates of Second lagrangian point L2;

$$SL = (q+1) * x^5 - (3*q+2) * x^4 + (3*q+1) * x^3 - (2*q+1+\delta) * x^2 + 2*(1-\delta) * x - 1 + \delta$$

Solve[SL == 0, x]

$$-1 + 2\,x - 1.002\,x^2 + 1.003\,x^3 - 2.003\,x^4 + 1.001\,x^5$$

{{x → -0.499445 - 0.865833 ⅈ}, {x → -0.499445 + 0.865833 ⅈ},
{x → 0.964498 - 0.0586521 ⅈ}, {x → 0.964498 + 0.0586521 ⅈ}, {x → 1.07089}}

FindRoot[SL, {x, 2}]

{x → 1.07089}

x5 = x /. FindRoot[SL == 0, {x, 1, 2}]

1.07089

VALUE OF THE POTENTIAL AT L2;

$$\xi 5 = \frac{(1-\delta)}{x5} + q\left(\frac{1}{x5-1} - x5\right) + \frac{(q+1)}{2} * x5^2$$

1.52081

COORDINATES OF THIRD LAGRANGIAN POINT L3;

$$TL = (q+1) * x^5 + (3*q+2) * x^4 + (3*q+1) * x^3 + (\delta-1) * x^2 + 2*(\delta-1) * x - 1 + \delta$$

$$-1 - 2x - x^2 + 1.003\, x^3 + 2.003\, x^4 + 1.001\, x^5$$

Solve[TL == 0, x]

{{x → −0.999875 − 0.0223673 i}, {x → −0.999875 + 0.0223673 i},
{x → −0.500333 − 0.865448 i}, {x → −0.500333 + 0.865448 i}, {x → 0.999417}}

x6 = x /. FindRoot[TL == 0, {x, 1, 2}]

0.999417

VALUE OF THE POTENTIAL AT L3;

$$\xi 6 = \frac{(1-\delta)}{x6} + q\left(\frac{1}{x6+1} + x6\right) + \frac{(q+1)}{2} * x6^2$$

1.502