NEUTRINO MASS MATRICES WITH TWO TEXTURE ZEROS

Thesis submitted in partial fulfillment of the requirement for
The award of the degree of
Masters of Science
In
PHYSICS

Under
the guidance of
Dr. Sanjeev Kumar

Submitted by
Neha Bansal
Roll no. - 30704010

School of physics and Material Science
Thapar University
Patiala – 147004 (PUNJAB)
INDIA

CERTIFICATE

This is to certify that Ms. Neha Bansal, Roll No. 30704010, has worked on this thesis report as a partial fulfillment for award of the degree of **MASTERS OF SCIENCE** in physics. I certify that the matter embodied in this report is of candidate's own record and not submitted to any other university in any part or full form for the award of such a degree.

8.7.09

(Dr. Sanjeev Kumar)
Supervisor
SPMS, Thapar University
Patiala.

Countersigned by:

10/7/09

Dr. O.P. Pandey
(Prof. & Head)
Affairs
School of Physics and Materials Science,
Thapar University,
Patiala.

15/7/09

Dr. R.K. Sharma
Dean of academic

Thapar University,
Patiala.

Dedicated to my parents

ACKNOWLEDGEMENT

Knowledge in itself is a continuous process. I would have never succeeded in completing my task without cooperation, encouragement and help provided to me by various personalities.

I would like to give my thanks to **Dr. O. P. Pandey**, Professor and Head, School of Physics and Material Science, for his full motivation and appreciation to my work.

With deep sense of gratitude, I express my sincere thanks to my worthy superviser, **Dr. Sanjeev Kumar**, for his valuable guidance in caring out work under his effective supervision, encouragement and cooperation.

All my friends at the School of Physics and Material Sciences are acknowledged for providing me a friendly atmosphere and encouraging me throughout this work. Their assistance and partnership were of great pleasure. Their views were very insightful and helpful.

I am deeply thankful to my Family, their moral support and patience has bared fruit through completion of this Thesis which will result in award of the prestigious degree of M.Sc.

Above all I render my gratitude to the Almighty who bestowed self – confidence, ability, strength and path on me in accomplishing this work.

Neha Bansal

Roll No. 30704010

CONTENTS

CHAPTER -1
INTRODUCTION

NEUTRINOS

Neutrinos are elementary particles that travel almost with the speed of light, lack an electric charge, are able to pass through ordinary matter almost undisturbed and are thus are extremely difficult to detect. Neutrinos are created as a result of certain types of radioactive decay (beta decay) or nuclear reactions such as those that take place in the Sun, in nuclear reactors, or when cosmic rays hit atmosphere. The neutrino has half-integer spin ($\frac{1}{2}\hbar$) and is therefore are fermions. Neutrinos interact primarily through the weak force. The discovery of neutrino flavor oscillations implies that neutrinos have mass. The existence of a neutrino mass strongly suggests the existence of a tiny neutrino magnetic moment of the order of 10^{-19} Bohr magneton allowing the possibility that neutrinos may interact electromagnetically as well. There are three known types (*flavors*) of neutrinos: electron neutrino ν_e, muon neutrino ν_μ and tau neutrino ν_τ, named after their partner leptons. The question of neutrino velocity is closely related to their mass. According to relativity, if neutrinos are massless, they must travel at the speed of light. However, if they carry a mass, they cannot reach the speed of light. Phenomenon of neutrino oscillation requires neutrinos to have nonzero masses. There are three types, or "flavors", of neutrinos: electron neutrinos, muon neutrinos and tau neutrinos; each type also has an antimatter partner, called an antineutrino.

The neutrino was first postulated in 1930 by Wolfgang Pauli to preserve the laws of conservation of energy, conservation of momentum, and conservation of angular momentum in beta decay – the decay of a neutron into a proton, an electron and an antineutrino. At the time physicist were puzzled because nuclear beta decay appeared to violate energy conservation. In beta decay, a neutron in an unstable nucleus transforms into a proton and emits an electron, where the radiated electron was found to have a continuous spectrum. This was a great surprise because in other types of radioactivity involved gamma rays and alpha particles with discrete energies. Pauli postulated that some of the energy must have been taken away by a new particle emitted in decay process, the neutrino, which carries energy have spin ½ but is massless, electrically neutral and weakly interacting.

This energy carried by the particle could be in any ratio with other particles being formed in the reaction, due to which its difficult to predict their actual mass. The neutrino emitted is corresponding to its lepton i.e. if electron is being emitted during the reaction then electron neutrino will be emitted along with it. And like electron is a particle along with it antineutrino of electron neutrino will be emitted. Pauli theorized that an undetected particle was carrying away the observed difference between the energy, momentum, and angular momentum of the initial and final particles.

NEUTRINO OSCILLATION

Neutrino oscillation is a quantum mechanical phenomenon predicted by Bruno Pontecorvo where by a neutrino created with a specific lepton flavor (electron, muon or tau) can later be measured to have a different flavor. The probability of measuring a particular flavor for a neutrino varies periodically as it propagates. Neutrino oscillation is of theoretical and experimental interest since observation of the phenomenon implies that the neutrino has a non-zero mass.

SURVIVAL PROBABILITY

There is now convincing evidence that atmospheric, solar, and reactor neutrinos change from one flavour to another. Neutrino flavour change implies that neutrinos have masses and that they mix. Neutrinos are produced and detected as flavour eigenstates and they propagate through space as superposition of mass eigenstates. $v_{e,}$, $v_{\mu,}$ v_{τ} are flavour eigenstates which can be expressed as combination of mass eigenstates v_1, v_2, v_3 now for mathematical treatment we will consider the case for two neutrino flavours say v_e and v_μ and they will be the linear combination of two mass eigenstates v_1 and v_2. The transformation between flavor and mass eigenstates can be written using unitary transformation of matrices involving a mixing angle θ:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}.$$

In other words,

$$\nu_\mu = \nu_1 \cos\theta + \nu_2 \sin\theta,$$

$$\nu_e = -\nu_1 \sin\theta + \nu_2 \cos\theta.$$

The propagation of mass eigenstates is given by:

$$\nu_1(t) = \nu_1 e^{-itE_1},$$

$$\nu_2(t) = \nu_2 e^{-itE_2}.$$

Here, we are using natural system of units so

c=1, h=1.

The states v_1 and v_2 will have the fixed momentum p, so that if the masses are m_1 and m_2 , the above equations become:

$$\nu_1(t) = \nu_1(0) e^{-itw_1}$$

$$\nu_2(t) = \nu_2(0) e^{-itw_2}$$

Now let us consider two states at time t=0 and time t=t as initial and final states $|i>$ and $|f>$ respectively. Then, these can be written as:

$$|i> = |\nu_e(0)>$$

$$|f> = |\nu_e(t)>$$

Now,

amp $(\nu_e \rightarrow \nu_e) = <i|f> = <\nu_e(0)|\nu_e(t)>$

amp $(\nu_e \rightarrow \nu_\mu) = <i|f> = <\nu_e(0)|\nu_\mu(t)>$

amp $(\nu_e \rightarrow \nu_e) = cos^2\theta e^{-iw_1 t} + sin^2\theta e^{-w_2 t}$

Probability of transition:

$(\nu_e \rightarrow \nu_e) = |amp|^2 = Re^2 + Im^2$

Re = $cos^2\theta \cos w_1 t + sin^2\theta cosw_2 t$

Im = $cos^2\theta \sin w_1 t + sin^2\theta sinw_2 t$

P$(\nu_e \rightarrow \nu_e)$= $cos^4\theta + sin^4\theta + 2sin^2\theta cos^2\theta(cosw_1 t\ cosw_2 t + sinw_1 t\ sinw_2 t)$

P$(\nu_e \rightarrow \nu_e)$ = $cos^4\theta + sin^4\theta + 2sin^2\theta cos^2\theta \cos(w_2 - w_1)\, t$

Now as we know

$$cos^2\theta + sin^2\theta = 1$$

$$cos^4\theta + sin^4\theta + 2sin^2\theta cos^2\theta = 1$$

So,

P = 1 - $2sin^2\theta cos^2\theta + 2sin^2\theta cos^2\theta \cos(\Delta wt)$

P = 1 - $4sin^2\theta cos^2\theta\ sin^2\ (\Delta wt/2)$

P = 1 - $sin^2 2\theta\ sin^2\ (\Delta wt/2)$

Here, we get the probability relation for transition from $\nu_e \rightarrow \nu_e$. So,

P$(\nu_e \rightarrow \nu_e)$ = 1 - $sin^2 2\theta\ sin^2\ (\Delta wt/2)$

Also, we can write:

t = L/c

where L is mixing length and L is in km and the beam energy E is in GeV.
So, we will get:

P$(\nu_e \rightarrow \nu_e)$ = 1 - $sin^2 2\theta \, sin^2 \, (\Delta wL/2c)$

This is the expression for survival probability in two flavor neutrino oscillation case.

NUTRINO MIXING MATRIX

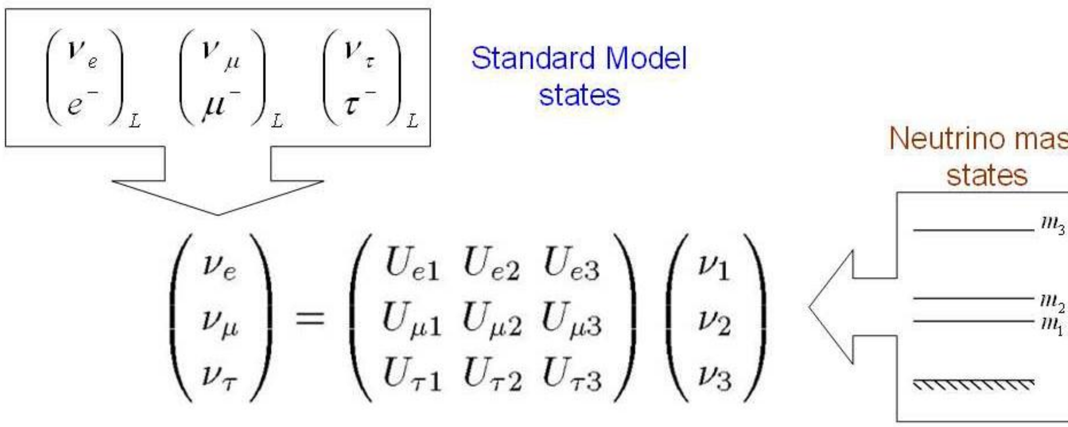


Figure 1: The mass states are not necessarily ordered as m1 < m2 < m3 as discussed later. The convention is chosen such that v_1 contains mostly v_e ,while v_3 contains mainly $v_{\mu,}$ and v_τ with very little $v_{e.}$ the state v_2 contain roughly equal amounts of $v_{e,}$, $v_{\mu,}$ v_τ . The matrix U is unitary, which implies that the probability that each of the states v_1, v_2, v_3 contains each of $v_{e,}$, $v_{\mu,}$ v_τ must sum to unity

The minimal neutrino sector required to account for the atmospheric and solar neutrino oscillation data thus consists of three light physical neutrinos with left-handed flavour eigenstates, υ_e, υ_μ, and υ_ζ , defined to be those states that share the same doublet as the charged lepton mass eigenstates e,μ,ζ (see Fig.1). Within the framework of three–neutrino oscillations, the neutrino flavor eigenstates υ_e, υ_μ, and υ_ζ are related to the neutrino mass eigenstates υ_1, υ_2, and υ_3 with mass m_1, m_2, and m_3, respectively, by a 3 × 3 unitary matrix called the lepton mixing matrix U [1]

$$\begin{pmatrix} \upsilon e \\ \upsilon \mu \\ \upsilon \tau \end{pmatrix} = = \begin{pmatrix} Ue1 & Ue2 & Ue3 \\ U\mu 1 & U\mu 2 & U\mu 3 \\ U\zeta 1 & U\zeta 2 & U\zeta 3 \end{pmatrix} \begin{pmatrix} \upsilon 1 \\ \upsilon 2 \\ \upsilon 3 \end{pmatrix}$$

If the light neutrinos are Majorana, U can be parameterized in terms of three mixing angles θ_{ij} and three complex phases. A unitary matrix has six phases but three of them are removed by the phase symmetry of the charged lepton Dirac masses. Since the neutrino masses are Majorana there is no additional phase symmetry associated with them, unlike the case of quark mixing where a further two phases may be removed. If we begin by assuming that the phases are zero, then the lepton mixing matrix may be parametrised by a product of three Euler rotations, as depicted in Fig.2 given below,

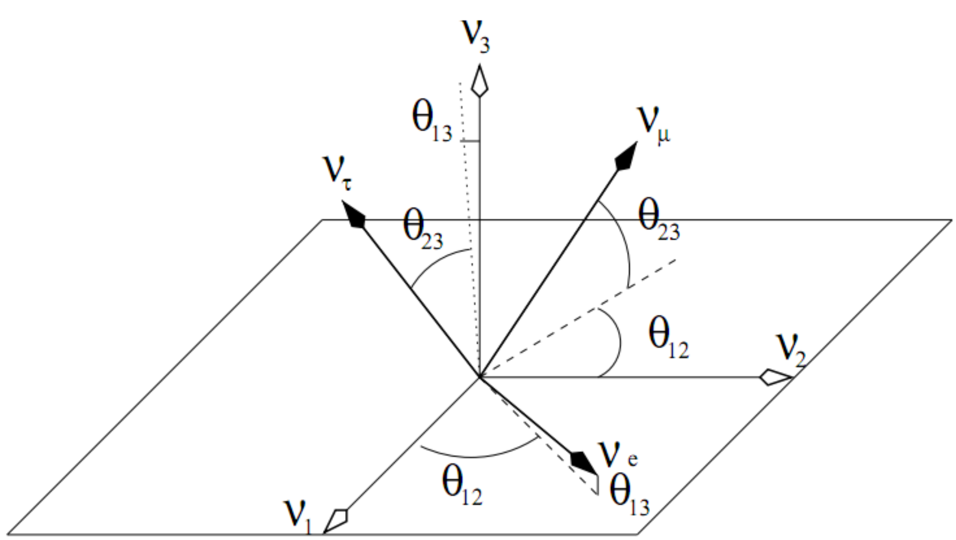


Figure 2: The relation between the neutrino weak eigenstates υ_e, υ_μ ,υ_ζ and and the neutrino mass eigenstates υ_1, υ_2, and υ_3 in terms of the three mixing angles θ_{12}, θ_{13}, θ_{23}. Ignoring phases, these are just the Euler angles respresenting the rotation of one orthogonal basis into another.[2]

and given by a product of three matrices:

$$U = R_{23}R_{13}R_{12}$$

where

$$R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c23 & s23 \\ 0 & -s23 & c23 \end{pmatrix}$$

$$R_{13} = \begin{pmatrix} c13 & 0 & s13 \\ 0 & 1 & 0 \\ -s13 & 0 & c13 \end{pmatrix}$$

$$R_{12} = \begin{pmatrix} c12 & s12 & 0 \\ -s12 & c12 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Note that the allowed range of the angles is $0 \leq \theta_{ij} \leq \pi/2$. The phases $\alpha_{1,2}$ are called Majorana phases since they are only present if the neutrino mass is Majorana . The phase δ is called the Dirac phase since it is always present even if neutrinos have Dirac mass. We have already seen that the first matrix in Fig.2 above is associated with Atmospheric neutrino oscillations.

The "LEPTON MIXING MASS MATRIX",U,relates the basic standard model neutrino states $v_{e,}$, $v_{\mu,}$ v_{τ} ,associated with the electron,muon, and tau to the neutrino mass states v_1, v_2, v_3 with mass m_1,m_2 and m_3 as shown in Fig. 1.

EXPERIMENTAL EVIDENCES

SOLAR NEUTRINO EXPERIMENTS

The first clues that neutrinos have mass came from an experiment deep underground, carried out by an American scientist Raymond Davis, detecting solar neutrinos. But it revealed only one third of number predicted by theories of how sun works. The result puzzled both solar and neutrino physicists. However some Russian scientists suggested that the solar the solar neutrinos might be changing into something else. Only electron neutrinos are emitted by the sun and they could be converting into muon and tau neutrinos which were not being detected by the experiments. This effect, called “neutrino oscillations”, as the types of neutrino interconvert over time from one kind to another and vice-versa, was first proposed some time earlier by Pontecorvo.

Super-Kamiokande is also sensitive to the electron neutrinos arriving from the Sun, the “solar neutrinos”, and has independently confirmed the reported deficit of such solar neutrinos long reported by other experiments. For example, Davis’s Homestake Chlorine experiment which began data taking in 1970 consists of 615 tons of tetrachloroethylene, and uses radiochemical techniques to determine the Ar^{37} production rate [3]. More recently, the SAGE and Gallex experiments, containing large amounts of Ga71 which is converted to Ge71 by low energy electron neutrinos arising from the dominant pp reaction in the Sun [3], have confirmed neutrino oscillation phenomena. The combined data from these and other experiments implies an energy dependent suppression of solar neutrinos which can be interpreted as due to flavour oscillations. Taken together with the atmospheric data, this requires that a second neutrino flavour has a non-zero mass.

Sundbury Neutrino Observatory (SNO) is a water Cerenkov detector like Super-Kamiokande, but instead of using normal water it uses heavy water, D_2O. The deuterons, D, in the heavy water are the most weakly bound of all nuclei, which gives SNO the chance to observe three different reactions induced by solar neutrinos. The first of these processes is the charged-current (CC) reaction $\upsilon_e + D \rightarrow p + p + e^-$, which is detected by observing Cerenkov photons from the energetic recoil electron, e^-. SNO also measures the neutral-current (NC) reaction $\upsilon_\beta + D \rightarrow p + n + \upsilon_\beta$. This is observed via the emitted neutrons, n, and is independent of the flavour of the incoming neutrino, υ_β. It therefore provides a way to normalize the total flux of neutrinos being emitted by the Sun. Finally SNO measures the elastic scattering (ES) reaction also measured in Super-Kamiokande, $\upsilon_\beta + e^- \rightarrow \upsilon_\beta + e^-$, which has some sensitivity to all neutrino flavours. SNO measurements of CC reaction on deuterium is sensitive exclusively to υ_e's, while the ES off electrons also has a small sensitivity to υ_μ's and υ_ζ 's. The CC ratio is significantly smaller than the ES ratio. This immediately disfavours oscillations of υ_e's to sterile neutrinos 3 which would lead to a diminished flux of electron neutrinos, but equal CC and ES ratios. On the other hand the different ratios are consistent with oscillations of υ_e's to active neutrinos υ_μ's and υ_ζ 's since this would lead to a larger ES rate since this has a neutral current component. The SNO analysis is nicely consistent with both the hypothesis that electron neutrinos from the Sun oscillate into other active flavors, and with the Standard Solar Model prediction.

ATMOSPHERIC NEUTRINO EXPERIMENTS

The neutrino oscillation hypothesis gained support from the Japanese experiment super-Kamiokande which in 1998 showed that there was deficit of muon neutrinos reaching earth which are produced when cosmic rays strike the upper atmosphere, the so called atmospheric neutrinos. The Super-Kamiokande experiment consists of thousands of tones of pure water in a tank deep underground, and was originally built to search for proton decay. However, its designers realized that the experiment might also be able to detect highly energetic neutrinos from the Sun that interact with electrons via scattering reactions. These electrons can travel faster than the local speed of light in the water, causing them to emit the optical equivalent of a sonic boom - a glow of blue light called Cerenkov radiation that can be detected by ultra-sensitive photomultiplier tubes around the tank. Super-Kamiokande also measured the number of electron and muon neutrinos that arrive at the Earth's surface as a result of cosmic ray interactions in the upper atmosphere, which are referred to as "atmospheric neutrinos". While the number and angular distribution of electron neutrinos is as expected, Super-Kamiokande showed that the number of muon neutrinos is significantly smaller than expected and that the flux of muon neutrinos exhibits a strong dependence on the zenith angle. These observations gave compelling evidence that muon neutrinos undergo flavour oscillations and this in turn implies that at least one neutrino flavour has a non-zero mass. The standard interpretation, well supported by current data, is that muon neutrinos are oscillating into tau neutrinos. Current atmospheric neutrino oscillation data are well described by simple two-state mixing.

$$\begin{pmatrix} \upsilon_\mu \\ \upsilon_\zeta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \upsilon_1 \\ \upsilon_2 \end{pmatrix}$$

and the two-state probability oscillation formula

$$P(\upsilon_\mu \rightarrow \upsilon_\zeta) = \sin^2 2\theta_{23} \sin^2(1.27\Delta m^2_{32} L/E)$$

where

$$\Delta m^2_{ij} \equiv m^2_i - m^2_j$$

and m_i are the physical neutrino mass eigenvalues associated with the mass eigenstates υ_i. Δm^2_{32} is in units of eV^2, the baseline L is in km and the beam energy E is in GeV. The atmospheric data results support maximal mixing, with best-fit two-neutrino oscillation parameters of

$$\sin^2 2\theta_{23} = 1,\ \Delta m^2_{32} = 2.6 \times 10^{-3} eV^2$$

.

The 90% C.L. range for Δm^2_{32} at $\sin^2 2\theta_{23} = 1$ is between 2.0 and 3.2 $\times 10^{-3} eV^2$. The approximately maximal mixing angle $\theta_{23} = 45^{\circ}$ means that we identify the heavy atmospheric neutrino eigenstate of mass m_3 as being approximately

$$\upsilon_3 \approx \upsilon_{\mu} + \upsilon_{\zeta} / \sqrt{2}$$

and in addition there is a lighter orthogonal combination of mass m_2, where

$$m^2_3 - m^2_2 = 2.6 \times 10^{-3}\ eV^2.$$

If $m_3 \gg m_2$ then this implies $m_3 \approx 0.05$ eV.

Since most neutrinos pass unhindered through the earth, this experiment was able to detect muon neutrinos coming from above and below, and found that while the correct number of muon neutrinos came from above, only about a half of the expected number came from below. The results were interpreted as half the muon neutrinos from below oscillating into tau neutrinos over an oscillating length L of the diameter of earth, with muon neutrinos from above having a negligible oscillating length, and so not having time to oscillate, yielding expected number of muon neutrinos from above. The Sudbury Neutrino Observatory (SNO) data revealed that physicist's theories of sun were correct after all, and and electron solar neutrinos were produced at standard rate but were oscillating into muon and tau neutrinos, with only about a third of original electron neutrino flux arriving at the earth.

Neutrino oscillations consistent with solar neutrino observations have been seen using man made neutrinos from the nuclear reactors at KamLAND in Japan, and neutrino oscillation consistent with atmospheric neutrino observations have been seen using neutrino beams fired over hundred of kilometres as in K2K experiment in Japan, a Fermilab-MINOS in US or the CERN-OPERA experiment in Europe.

Following these results, several research groups [4,5] showed that the electron neutrino has a mixing matrix element of $|U_{e2}| \approx 1/\sqrt{3}$ which is the quantum amplitude for υ_e to contain an admixture of the mass eigenstate υ_2 corresponding to a massive neutrino of mass $m_2 \approx 0.007$ electronvolts (eV) or greater (by comparison the electron has a mass of about half a mega electron volt (MeV)). The muon and tau neutrinos were observed to contain approximately equal amplitudes of a heavier neutrino υ_3 of mass $m_3 \approx 0.05$ eV or greater, $|U_{\mu 3}| \approx |U_{\zeta 3}| \approx 1/\sqrt{2}$, where a normalized amplitude of $1/\sqrt{2}$ corresponds to a 1/2 fraction of υ_3 in each of υ_μ and υ_ζ, leading to a maximal mixing and oscillation of $\upsilon_\mu \leftrightarrow \upsilon_\zeta$. However, according to the results from the CHOOZ nuclear reactor experiment [6], the electron neutrino must only mix very weakly (if at all) with this state, $|U_{e3}| < 0.2$. Neutrino oscillations are only sensitive to mass differences, and the lightest neutrino mass m_1 is not measured, so these mass values are only lower bounds. However, there are cosmological reasons to believe that none of the neutrino masses can exceed about 0.3 eV. Clearly, then, neutrino masses are much smaller than the other charged fermion masses, and this represents something of a puzzle. However, there is a more urgent question that must be faced since, unlike the case for quarks and charged leptons, the Standard Model actually predicts that neutrinos have no mass at all!

REACTOR NEUTRINO EXPERIMENTS

Reactor experiments detect the anti-electron neutrinos which are produced copiously in the cores of nuclear reactors, and interpret any deficit in the expected number of such particles in terms of neutrino oscillations. The solar neutrino background is low because the Sun produces electron neutrinos, with negligible numbers of anti-electron neutrinos. The CHOOZ reactor experiment in France failed to see any signal of anti-neutrino oscillations over the Super-Kamiokande mass range. CHOOZ data from $\bar{\upsilon}_e \rightarrow \bar{\upsilon}_e$ disappearance not being observed provides a significant constraint on θ_{13} over the Super-Kamiokande (SK) prefered range of Δm^2_{32} [6]:

$$\sin 2\,\theta_{13} < 0.04\ .$$

The CHOOZ experiment therefore limits $\sin\theta_{13} \leq 0.2$ or $\theta_{13} \leq 12^\circ$ over the favoured atmospheric range at 90% C.L. The experiment is currently being upgraded to Double CHOOZ, to increase the sensitivity on the angle θ_{13}. The phase δ also appears in the third matrix, and physically represents CP violation. Since, the angle θ_{13} has not yet been measured, it might seem somewhat premature to discuss the phases associated with this angle. Nevertheless, there is, in fact, a huge experimental effort under way to both measure the angle θ_{13} and the CP phase δ. However it should be emphasized that the CP-violation in the lepton sector is one of the most challenging frontiers in the future studies of neutrino mixing. Nevertheless the experimental searches for CP-violation in neutrino oscillations can help answer the fundamental question about the status of CP-symmetry in the lepton sector at low energy. The observation of leptonic CP-violation at low energies will have far reaching consequences, and can shed light, in particular, on the possible origin of the baryon asymmetry of Universe.

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric | Reactor | Solar | Majorana

Figure 3: The lepton mixing matrix with phases factorizes into a matrix product of four matrices, associated with the physics of Atmospheric neutrino oscillations, Reactor neutrino oscillations, Solar neutrino oscillations and a Majorana phase matrix.

The physics of the third matrix in Fig.3 is associated with Solar neutrino oscillations, as discussed above, and recently confirmed by the Japanese reactor experiment KamLAND, that measures υe's produced by several surrounding nuclear reactors [7]. KamLAND has already seen a signal of neutrino oscillations over the Solar neutrino LMA MSW mass range, and has recently confirmed the LMA MSW region "in the laboratory" [8]. KamLAND and SNO results when combined with other solar neutrino data especially that of Super- Kamiokande uniquely specify the large mixing angle (LMA) MSW [10] solar solution with three active light neutrino states, a large solar angle

$\sin_2 \theta_{12} \approx 0.30$, $\Delta m^2{}_{21} \approx 7.9 \times 10^{-5} eV^2$.

according to the most recent global fits [9]. KamLAND has thus not only confirmed solar neutrino oscillations, but has also uniquely specified the large mixing angle (LMA) solar solution, heralding a new era of precision neutrino physics. The physics of the fourth matrix in Fig.4 is associated with Majorana neutrino masses. These phases could in principle be measured in neutrinoless double beta decay. It is clear that neutrino oscillations, which only depend on $\Delta m^2_{ij} \equiv m^2_i - m^2_j$, give no information about the absolute value of the neutrino mass squared eigenvalues m^2_i, and there are basically two patterns of neutrino mass squared orderings consistent with the atmospheric and solar data as shown in Fig.4. Three family oscillation probabilities depend upon the time–of–flight (and hence the abaseline L), the Δm^2_{ij} , and U (and hence θ_{12}, θ_{23}, θ_{13}, and δ).

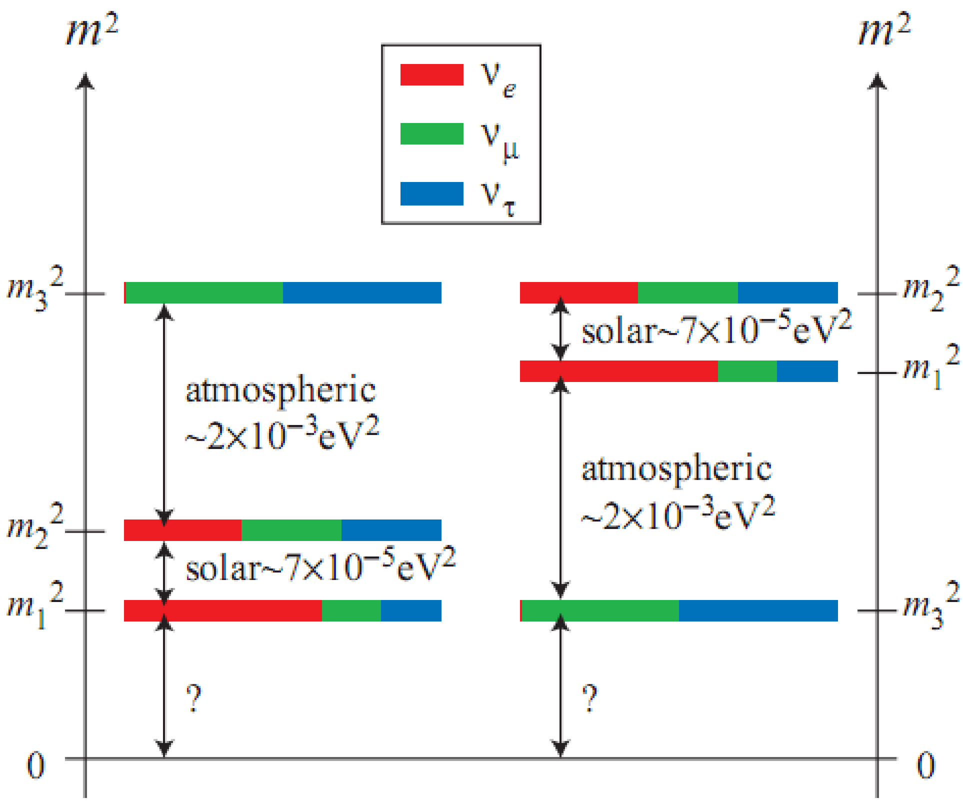


Figure 4: Alternative neutrino mass patterns that are consistent with neutrino oscillation expla-nations of the atmospheric and solar data. The pattern on the left (right) is called the normal (inverted) pattern. The coloured bands represent the probability of finding a particular weak eigen- state νe, νμ, and ντ in a particular mass eigenstate. The absolute scale of neutrino masses is not fixed by oscillation data and the lightest neutrino mass may vary from 0.0 − 0.3 eV [10]

The latest results from various experiments are given below:

parameter	best fit	2σ	3σ	4σ
Δm^2_{21} [10^{-5} eV^2]	7.9	7.3–8.5	7.1–8.9	6.8–9.3
Δm^2_{31} [10^{-3} eV^2]	2.6	2.2–3.0	2.0–3.2	1.8–3.5
$\sin^2\theta_{12}$	0.30	0.26–0.36	0.24–0.40	0.22–0.44
$\sin^2\theta_{23}$	0.50	0.38–0.63	0.34–0.68	0.31–0.71
$\sin^2\theta_{13}$	0.000	≤ 0.025	≤ 0.040	≤ 0.058

Table 1: Best-fit values, 2σ, 3σ, and 4σ intervals (1 dof) for the three–flavour neutrino oscillation parameters from global data including solar, atmospheric, reactor (KamLAND and CHOOZ) and accelerator (K2K and MINOS) experiments, taken from [9].

FUTURE EXPERIMENTAL PROSPECTS

Further experimental progress from SNO and KamLAND will consist of pinning down LMA MSW parameters to high accuracy. Neutrino physics has, now entered the precision era. Future neutrino oscillation experiments, will give accurate information about the mass squared splittings $\Delta m^2_{ij} \equiv m^2_i - m^2_j$, mixing angles, and CP violating phase. In the near future much better solar neutrino measurements will be available as KamLAND, SNO and Borexino furnish us with new and better data. The K2K long baseline (LBL) experiment from KEK to Super-Kamiokande has recently reported results in its phases I and II, which cover the atmospheric region and support the Super-Kamiokande results. In the longer term LBL experiments such as MINOS and eventually the CERN to Gran Sasso experiments will given more accurate determinations of the atmospheric parameters, eventually to 10%. J-PARC will be an “off-axis superbeam” over a LBL of 295 km to Super-Kamiokande due to start in 2008. Its first goal is to measure θ_{13} or set a limit on it of about 0.05 (as compared to the CHOOZ limit on

θ_{13} of about 0.2). Interestingly, MINOS over a LBL of 735 km is more sensitive than J-PARC to matter effects, so there should be some interesting complementarity between these two experiments, which could for example allow the sign of Δm^2_{32} to be determined. The ultimate goal of oscillation experiments however is to measure the CP violating phase δ. An upgraded J-PARC with a 4W proton driver and a 1 megaton Hyper-Kamiokande detector, or some sort of Neutrino Factory based on muon storage rings would seem to be required for this purpose [9]. Oscillation experiments are not capable of telling us anything about the absolute scale of neutrino masses. The Tritium beta decay experiment KATRIN will tell us about the absolute scale of neutrino mass down to about 0.35 eV. The neutrinoless double beta decay experiment GENIUS will probe the Majorana nature of the electron neutrino down to about 0.01 eV . Recent results from the 2dF galaxy redshift survey and WMAP, when combined with oscillation data, give the strong limit on the absolute mass of each neutrino species of about 0.23 eV [11, 12]. Turning to astrophysics, a galactic supernova could give valuable information about neutrino masses [13]. In future detection of energetic neutrinos from gamma ray bursts (GRBs), by neutrino telescopes such as ANTARES or ICECUBE could also provide important astrophysical information, and may provide another means of probing neutrino mass, and even quantum gravity [14].

REFERENCES

[1] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theo. Phys. 28 (1962) 247; B. W. Lee, S. Pakvasa, R. E. Shrock and H. Sugawara, Phys. Rev. Lett. 38 (1977) 937 [Erratum-ibid. 38 (1977) 1230].

[2] S.F.King, arXiv:0712.1750v1 [physics.pop-ph] 11 Dec 2007

[3] J. N. Bahcall, arXiv:physics/0406040.

[4] P. C. de Holanda and A. Y. Smirnov, arXiv:hep-ph/0309299; G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo and A. M. Rotunno, arXiv:hep-ph/0310012; A. Bandyopadhyay, S. Choubey, S. Goswami and S. T. Petcov, arXiv:hep-ph/0309236; M. Maltoni, T. Schwetz, M. A. Tortola and J. W. Valle, arXiv:hep-ph/0309130.

[5] M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. 6 (2004) 122 [arXiv:hep-ph/0405172].

[6] H. V. Klapdor-Kleingrothaus et al. [GENIUS Collaboration], arXiv:hep-ph/9910205

[7] J. Maricic and J. G. Learned, Contemp. Phys. 46 (2005) 1.

[8] K. Eguchi et al. [KamLAND Collaboration], Phys. Rev. Lett. 90 (2003) 021802 [arXiv:hep-ex/0212021].

[9] B. Autin, D. A. Harris, S. F. King, K. S. McFarland and O. Yasuda, arXiv:hep-ph/0302218.

[10] S.F. King., arXiv:hep-ph/0310204v2 27 Nov 2003

[11] O. Elgaroy et al., galaxy redshift survey," arXiv:astro-ph/0204152.

[12] A. Pierce and H. Murayama, arXiv:hep-ph/0302131.

[13] A. S. Dighe, M. T. Keil and G. G. Raffelt, arXiv:hep-ph/0303210.

[14] S. Choubey and S. F. King, Phys. Rev. D 67 (2003) 073005 [arXiv:hep-ph/0207260].

[15] L. Wolfenstein, Phys. Rev. D17 (1978) 2369; S. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. 42 (1985) 913.

CHAPTER-2
TWO TEXTURE ZEROS

The neutrino mass matrix contains all information about neutrino masses and mixings. However, the present neutrino data is not sufficient to determine the neutrino mass matrix. Therefore, we need some additional theoretical input to constraint the neutrino mass matrix. One of the moat attractive ideas to constraint the neutrino mass matrix is to introduce certain number of zeros in the mass matrix. These zeros needn't be algebraical zeros but they can be phenomenological zeros by which we mean that the elements are not exactly zeros but many order of magnitudes smaller than the other non zero elements.

We can make different combinations of zeroes by setting many elements of the neutrino mass matrix equal to zero. However, it was found that the presence of more than 2 zeroes in matrix is unable to reproduce the current neutrino masses and mixing data. So, we'll be discussing only two texture zeroes.

There has been a long standing interest in the texture zeros of the 3 × 3 quark mass matrices as a possible source of the observed hierarchies in their masses and mixing angles. Frampton, Glashow and Marfatia [6] have systematically compared the predictions of all the symmetric 3 × 3 neutrino mass matrices with two or more independent texture zeros with the neutrino mass and mixing parameters as derived from the oscillation data. A symmetric 3×3 mass matrix has in general 6 independent elements. They find no neutrino mass matrix with three or more texture zeros, which is compatible with the neutrino oscillation data. Moreover they find that 7 out of the 15 independent neutrino mass matrices with two texture zeros ($^6C_2 = 15$) are compatible with the oscillation data.

As usual the neutrino mass matrices shall be written in their flavour basis, which corresponds to the mass basis of the charged leptons. For simplicity we shall consider real mass matrices. A 3×3 real symmetric mass matrix with two texture zeros has four independent parameters. Three of them can be determined in terms of the three mass eigen values using the invariance of trace and determinant, i.e.

$$\text{Tr. M}= m_1 + m_2 + m_3,\ \text{Det. M}= m_1m_2m_3,$$
$$\text{Det.M}\times\text{TrM}^{-1} = m_1m_2 + m_2m_3 + m_3m_1. \quad (1)$$

We shall assume the maximal mixing angle for the atmospheric neutrino oscillation [2] to provide this input, i.e.

$$t_1=\tan\theta_{23}=1. \quad (2)$$

This represents by far the most robust result of the neutrino oscillation experiments so far. The most favored value of this mixing parameter has remained 1 over the years, while the error bar has shrunk steadily. Therefore we treat this as a manifestation of an underlying symmetry of the neutrino mass matrix. It is conceivable of course that the physical value of this parameter may not be exactly 1. Nonetheless it is fair to assume that this point is smoothly connected to the symmetry limit. Therefore we expect the phenomenological favored mass matrix to be compatible with the maximal mixing angle of eq. (2). 4 out of the 7 experimentally allowed mass matrices with two texture zeros are incompatible with this requirement.

EXPERIMENTAL CONSTRAINTS

We shall impose the following experimental constraints on the neutrino masses and mixing angles.

i) The atmospheric neutrino data implies

$\Delta a=|m^2_3-m^2_{1,2}|=(1.7-4)10^{-3}eV^2,$ (3)

$\tan^2\theta_{23}=t^2_1=0.9-1,$ (4)

ii) The solar neutrino data admits LMS (LOW) solution at 90 (99.7%) CL with [3]

$\Delta s=|m^2_2-m^2_1|\simeq 6\times 10^{-5}(1\times 10^{-7})eV^2,$ (5)

$t^2_3=\tan^2\theta_{12}\simeq 0.4(0.6).$ (6)

iii) The CHOOZ and Paoloverde atomic reactor experiments give the 90% CL limit [4]

$s_2=\sin\theta_{13}<0.16$ (7)

Thus we shall require the ratio of the solar and atmospheric neutrino mass differences to satisfy the inequality

$\Delta_s/\Delta_a=|m_2{}^2-m^2{}_1|/|m^2{}_3-m^2{}_{1,2}|< 2\times 10^{-2}.$ (8)

Neutrino mass matrices with two texture zeros

We will assume following constraints:

$\sin^2\theta_{13} = s^2_2 = 0$

$\cos^2\theta_{13} = c^2_2 = 1$
$\tan^2\theta_{23} = t^2_1 = 1$
$\sin\theta_{23} = s_1 = 1/\sqrt{2}$

The mass matrix can be expressed in terms of the above masses and mixing angles as

$$M = U \begin{pmatrix} m1 & 0 & 0 \\ 0 & m2 & 0 \\ 0 & 0 & m3 \end{pmatrix} U^T \qquad (9)$$

where

$$U = \begin{pmatrix} c2c3 & c2s3 & s2 \\ -c1s3 - s1s2c3 & c1c3 - s1s2s3 & s1c2 \\ s1s3 - c1s2c3 & -s1c3 - c1s2s3 & c1c2 \end{pmatrix} \qquad (10)$$

Substituting values from U to M, we'll get the value of respective matrix element of M: For example:

$M_{11} = e_1^2 m1 + e_2^2 m2 + e_3^2 m3$

where $e_1 = c_2c_3$, $e_2 = c_2s_3$, $e_3 = s_2$. Therefore, substituting these values, using trigonometric relations and the constraints mentioned above, we'll get,

$M_{11} = m_1c^2_3 + m_2s^2_3$

Similarly we can solve for other matrix elements of M, and together they are given by the following relations:

$M_{11} = m_1c^2_3 + m_2s^2_3$,

$M_{12} = -(m_1 - m_2)s_3c_3/\sqrt{2} + m_3s_2/\sqrt{2} - (m_1c^2_3 + m_2s^2_3)s_2/\sqrt{2}$,

$M_{13} = (m_1 - m_2)s_3c_3/\sqrt{2} + m_3s_2/\sqrt{2} - (m_1c^2_3 + m_2s^2_3)s_2/\sqrt{2}$,

$M_{22} = (m_1s^2_3 + m_2c^2_3)/2 + m_3/2 + (m_1 - m_2)s_2s_3c_3$,

$M_{33} = (m_1s^2_3 + m_2c^2_3)/2 + m_3/2 - (m_1 - m_2)s_2s_3c_3$,

$M_{23} = -(m_1s^2_3 + m_2c^2_3)/2 + m_3/2.$ (11)

The number n of the independent zeroes takes into account the symmetry of mass matrix: it counts instance that Mij = 0 where i>=j. we find that currently available data disfavour all cases with n>=3 but that there are seven empirically tolerable texture with n=2. From these expressions one can study the implications of setting different pairs of matrix elements to zero, as we see below.

(A) Hierarchical Solutions

1) $M_{11} = 0$, $M_{12} = 0$:

$M_{11}=m_1c^2{}_3+m_2s^2{}_3=0$

$=> m_1c^2{}_3= -m_2s^2{}_3$

$=> -m_1/m_2 = s^2{}_3/c^2{}_3 = t^2{}_3$ (a)

$M_{12} = -(m_1-m_2)s_3c_3/\sqrt{2}+m_3s_2/\sqrt{2} - (m_1c^2{}_3+m_2s^2{}_3)s_2/\sqrt{2}$

Now, $M_{12} = 0$.
Therefore,

$M_{12}=-(m_1-m_2)\ s_3c_3+m_3s_2=0$

⇨ $m_3s_2=(m_1-m_2)s_3c_3$

⇨ $s_2=[(m_1+m_2)/m_3]s_3c_3$

From equation (a)

$s^2{}_3/c^2{}_3= -m_1/m_2$

Adding (1) on both sides:

$s^2{}_3/c^2{}_3+1= -m_1/m_2+1$

$\Longrightarrow$ $s^2{}_3+c^2{}_3/c^2{}_3= m_2-m_1/m_2$

$\Longrightarrow$ $1/c^2{}_3= m_2-m_1/m_2$

$\Longrightarrow$ $c^2{}_3= m_2/m_2-m_1$ (b)

Similarly,

$s^2{}_3= m_1/m_1-m_2$ (c)

Multiplying (b) and (c), we get

$s_3c_3=\sqrt{}-m_1m_2/(m_1-m_2)^2$

$=\sqrt{-m_1m_2}/\ (m_1-m_2)$

Now, substituting value of s_3c_3 in s_2

$s_2= (m_1-m_2)\ s_3c_3/m_3$

$\Longrightarrow$ $s_2= \sqrt{-m_1m_2}\ /m_3$

Therefore the two solutions are:

$$t_{23}=-m_1/m_2,$$
$$s_2=\sqrt{-m_1 m_2}/m_3. \quad (12)$$

Combining these with eqs. (6) and (7) we get

$t^2{}_3 = (0.4)^2 = -m_1/m_2$

⇨ $-0.16\ m_2 = m_1$

Therefore, $m_2 > m_1$

Now,

$s_2 = (0.16) = \frac{\sqrt{-m1m2}}{m3}$

⇨ $0.0256\ m^2{}_3 = 0.16m^2{}_2$

⇨ $m^2{}_3 = 6.25\ m^2{}_2$

⇨ $m_3 = 2.5\ m_2$

therefore, $|m_1| < |m_2| \ll |m_3|$, i.e. hierarchical masses.

From eqs. (3) and (5) we get $|m_3| \simeq .05$, $|m_2| \simeq .009$, $|m_1| \simeq .004$ eV. In the former case, we get $s_2 \simeq 0.12$, i.e. close to the CHOOZ limit (7); i.e. s_2 can lie close to its present experimental limit. This prediction will become more precise when θ_3 is better measured. The subdominant angle θ_2 is likely to be measurable and neutrinos may display observable CP violation.

Note that for this solution, the υ_e Majorana mass,

$$M_{11}=\Sigma U^2{}_{ei}m_i, \quad (13)$$

is zero. So, the amplitude for neutrinoless double beta decay vanishes to lowest order in neutrino masses. If this case is realised in nature, the neutrinoless beta decay process simply cannot be detected. This is because the probability of a transition is propotional to matrix element square corresponding to the transition.

Thus, it predicts no 0υββ signal even at the level of .001 eV, which corresponds to the highest level of sensitivity expected at the proposed GENIUS experiment [5].

Finally, substituting (12) in (11) gives the explicit form of the mass matrix to first order in s_2. For example,

$M_{13} = (m_1 – m_2)\ s_3c_3/\sqrt{2}\ + m_3s_2/\sqrt{2}\ - (m_1c^2_3 + m_2s^2_3)\ s_2/\sqrt{2}$

We know that

$t^2_3 = s^2_3/c^2_3 = -m_1/m_2$

and

$m_2s^2_3 = -m_1c^2_3$

Substituting these results in the above equation, we have

$M_{13} = (m_1 - m_2)\ s_3c_3/\sqrt{2} + m_3s_2/\sqrt{2} – (m_1c^2_3 – m_1c^2_3)\ s_2/\sqrt{2}$

$= (m_1 - m_2)\ s_3c_3/\sqrt{2} + m_3s_2/\sqrt{2}$

Now we know that

$s_2 = \sqrt{-m1m2}\ /3$

Substituting s_2 in above equation we'll get:

$M_{13} = [(m_1 - m_2)s_3c_3 + \sqrt{-m1m2}\ ]/\ \sqrt{2}\ ]$

Now we know that,

$\frac{s_3^2}{c_3^2} + 1 = 1 - \frac{m_1}{m_2}$, $\quad (as\ \frac{s_3^2}{c_3^2} = -\ \frac{m1}{m2}\ )$

⇨ $c^2_3 = \frac{m2}{m2-m1}$

$= \frac{-m2}{m1-m2}$

Similarly,

$s^2_3 = \frac{m1}{m1-m2}$

Multiplying both sides we get,

$s^2_3\ c^2_3 = \frac{-m1m2}{(m1-m2)^2}$

$s_3c_3 = \frac{\sqrt{-m1m2}}{m1-m2}$

Substituting this in the equation

$M_{13} = [(m_1 - m_2)s_3c_3 + \sqrt{-m1m2}] / \sqrt{2}$,

we get

$M_{13} = \sqrt{-2m1m2}$

Similarly by using results of $M_{11} = 0$, $M_{12} = 0$, we can find other matrix elements and the results are:

$$M=\begin{pmatrix} 0 & 0 & \sqrt{-2m1m2} \\ 0 & \frac{m1+m2+m3}{2} & \frac{m3-m1-m2}{2} \\ \sqrt{-2\text{m}1\text{m}2} & \frac{m3-m1-m2}{2} & \frac{m1+m2+m3}{2} \end{pmatrix} \quad (14)$$

This solution is called A_1.

2) $M_{11} = 0$, $M_{13} = 0$:

$$M_{13}= (m_1\text{-}m_2)\ s_3c_3/\sqrt{2} + m_3s_2/\sqrt{2} - (m_1c^2{}_3+m_2s^2{}_3)s_2/\sqrt{2} = 0$$
$$M_{11}= m_1c^2{}_3 + m_2s^2{}_3 = 0$$

Therefore,

$$M_{13}= (m_1\text{-}m_2)\ s_3c_3/\sqrt{2} + m_3s_2\ /\sqrt{2}$$
$$\Longrightarrow\ s_2 = -\ (m_1\text{-}m_2)/m_3 * s_3c_3$$
$$= -\ \sqrt{-m_1m_2}/m_3$$

The phenomenological predictions for this case are identical to the previous one except for the change of sign of s_2. The mass matrix is simply obtained from (14) by interchanging the M_{12} and M_{13} elements. This solution is called A_2.

The neutrino mass matrices of type A_1 and A_2 together are called class A.

(B) Degenerate Solutions

1) $M_{22} = 0$, $M_{13} = 0$: Substituting these in eq. (11) imply for $t_1=1$

$$m^2_1=m^2_3+4s_2m^2_3/t_3,$$
$$m^2_2=m^2_3-4s_2m^2 3t_3, \qquad (15)$$

i.e. m^2_1, m^2_3, m^2_2 are nearly degenerate and occur in that order.

substracting the above two equations we'll get

$$m^2_2 - m^2_1 = m^2_3 - 4s_2m^2_3t_3 - m^2_3 + 4s_2m^2_3/t_3$$
$$= 4s_2m^2_3\,(t_3 + 1/t_3)$$

And,

$$m^2_3 - m^2_2 = 4s_2m^2_3t_3$$

Now, by substituting value of t_3, we'll find value of $(t_3 + 1/t_3) > 1$.

Therefore,

$$|m^2_2 - m^2_1| > |m^2_3 - m^2_2|$$

which implies, $\frac{\Delta m^2_{12}}{\Delta m^2_{23}} > 1$, which is in contradiction with solar and atmospheric results, since according to these experiments, this value is less than one.

Therefore here $\theta_{23} \neq 45^0$

In other words, this class is not supporting bimaximal mixing case.

As shown in [6], this mass matrix is compatible with the experimental constraint of Eq. (8) away from the maximal mixing points (2), i.e. for t_1 not equal to 1. In this case one gets

$$m^2_1= m^2_3[t^4_1+ 2s_2t^2_1(t_1 + 1/t_1)/t_3],$$
$$m^2_2=m^2_3[t^4_1-2s_2t^2_1(t_1+1/t_1)t_3]. \qquad (16)$$

Compatibility with the experimental constraint of eq. (8) can be achieved for

$$s_2 \sim 0.5\times10^{-2}(1-t^4_1)/(t_3+1/t_3)<4\times10^{-4}, \quad (17)$$

corresponding to the $t^2_1 \geq 0.9$ range of eq. (4). Thus the s_2 angle in this case is too small to be observed at the future long base line experiments. On the other hand this texture is also promising in regard to search for neutrinoless double beta decay. The υ_e Majorana mass is given by:

$$M_{11} \simeq t^2_1\sqrt{(\Delta a/(1-t_1^4))} \geq 0.10\text{eV}. \quad (18)$$

This is not too far below the present experimental upper limit of 0.2 eV [7], and will surely be measurable at GENIUS [5]. Note that this represents the common mass scale of degenerate neutrinos and it is cosmologically significant. Nonetheless, we consider the incompatibility of this solution with the maximal mixing region ($t_1 = 1$), favoured by the atmospheric neutrino data, to be a serious drawback of this model. The expressions for the matrix elements are rather long for t_1 is not equal to 1. Therefore we are not displaying the explicit form of the mass matrix. The above solution is called B_1.

2) $M_{33} = 0$, $M_{12} = 0$: Substituting these in eq. (11) imply

$$m^2_1 = m^2_3 - 4s_2m^2_3/t_3,$$
$$m^2_2 = m^2_3 + 4s_2m^2_3t_3, \quad (19)$$

i.e. nearly degenerate m^2_1, m^2_3, m^2_2 with $|m^2_2 - m^2_1| > |m^2_3 - m^2_2|$ as in the previous case. In fact the magnitudes of s_2 and t_3 are the same as above. The results for t_1 not equal to 1 are similar to the previous case with t_1 replaced by $-1/t_1$. Consequently, the predicted s_2 and υ_e Majorana mass are very close to those of Eqs. (17) and (18). This solution is called B_2.

3) $M_{22} = 0$, $M_{12} = 0$: The results are practically the same as in 2). The solution is called B_3.

4) $M_{33} = 0$, $M_{13} = 0$: The results are practically the same as in 1). The solution is called B_4.

The four types of mass matrices given above belong to class B.

(C) Inverted Hierarchy

$M_{22} = 0$, $M_{33} = 0$: Substituting these in eq. (11) gives

$M_{22} = 0 = (m_1 s_{23} + m_2 c_{23}) + m_3/2 + (m_1\text{-}m_2) s_2 s_3 c_3$
$M_{33} = 0 = (m_1 S_{23} + m_2 c_{23}) + m_3/2 - (m_1\text{-}m_2) s_2 s_3 c_3$

Subtracting the above 2 equations:

$$2(m_1\text{-}m_2)\ s_2 s_3 c_3 = 0$$

Now,

as, $m_1 \neq 0$, $m_2 \neq 2$

s_3 and c_3 are large

$s_2 = 0$ (d)

Now,

$m_1 s^2{}_1 + m_2 c^2{}_3 + m_3 = 0$

$\Rightarrow \quad m_1 s^2{}_3 + m_2 (1\text{-}s^2{}_3) + m_3 = 0$

$\Rightarrow \quad m_1 s^2{}_3 + m_2\text{-}m_2 s^2{}_3) + m_3 = 0$

$\Rightarrow \quad s^2{}_3 (m_1\text{-}m_2) + m_2 + m_3 = 0$

$\Rightarrow \quad s^2{}_3 = -(m_2 + m_3)/(m_1\text{-}m_2)$

Similarly,

$c^2{}_3 = (m_3 + m_1)/(m_1\text{-}m_2)$

Therefore, we get

$t^2{}_3 = s^2{}_3 / c^2{}_3 = -(m_2 + m_3)/(m_3 + m_1)$

Hence, the two solutions obtained in this case are:

$s_2=0$,

$t^2_3=-(m_2+m_3)/(m_1+m_3)$. (20)

The first equality implies of no observable CP violation in the neutrino sector. The consistency of the second with eqs. (6) and (8) implies an inverted hierarchy of masses, i.e.

$-m_2 \simeq m_1 \simeq (2-4)m_3$. (21)

Substituting this in eq. (3), we have $-m_2 \simeq m_1 \simeq .05$ eV and $m_3 \simeq .02$ eV. The predicted υ_e Majorana mass is

$M_{11} \simeq [(1-t^2_3)/2t_3]\sqrt{\Delta a} \simeq (.025-.013)$eV. (22)

This is an order of magnitude below the present upper limit of 0.2 eV [8], but will be measurable at GENIUS [5]. Substituting (20) in (11) gives the explicit form of the mass matrix. Let us calculate M_{11} :

$$M_{11} = m_1 c^2_3 + m_2 s^2_3$$

We know that

$$t^2_3 = s^2_3 / c^2_3 = -\left(\frac{m2+m3}{m1+m3}\right)$$

Adding 1 in above equation

$$\frac{s_3^2 + c_3^2}{c_3^2} = \frac{m1 + m3 - m2 - m3}{m1 + m3}$$

$$\Rightarrow \frac{s_3^2+c_3^2}{c_3^2} = \frac{m1-m2}{m1+m3}$$

$$\Rightarrow c^2_3 = \frac{m1+m3}{m1-m2}$$

Similarly,

$$s^2_3 = \frac{m2+m3}{m2-m1} = -\frac{m2+m3}{m1-m2}$$

Substituting c^2_3 and s^2_3 values in M_{11}, we get

$M_{11} = m_1+m_2+m_3$

Similarly, we can solve for other matrix elements and results can be written in matrix form. The mass matrix M will be given by:

$$\begin{pmatrix} m1+m2+m3 & -\sqrt{-(m1+m3)(m2+m3)/2} & \sqrt{-(m1+m3)(m2+m3)/2} \\ -\sqrt{-(m1+m3)(m2+m3)/2} & 0 & m3 \\ \sqrt{-(m1+m3)(m2+m3)/2} & m3 & 0 \end{pmatrix}$$

Fig. 5 below shows the two Hierarchies:

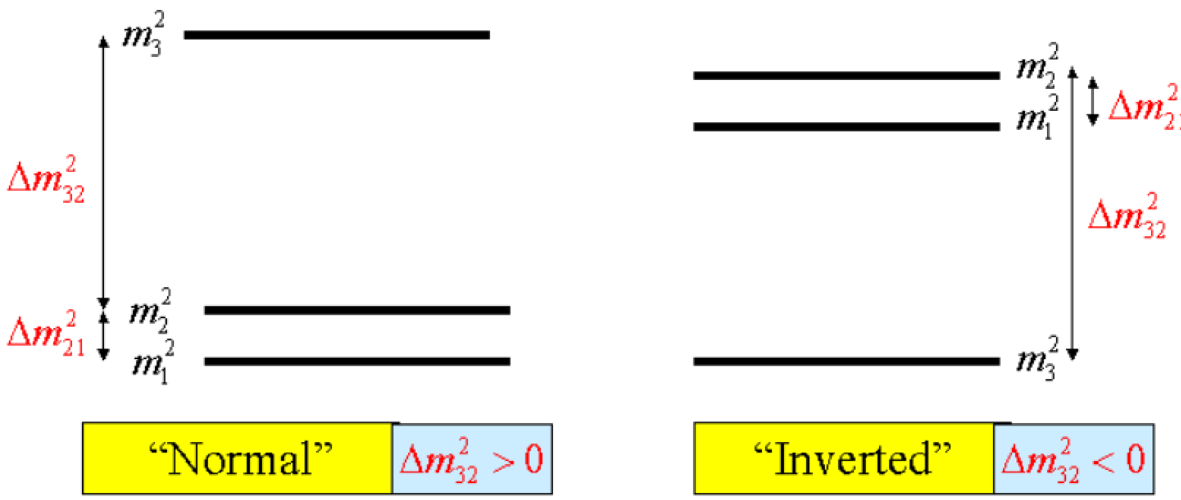


Figure 5 Alternative neutrino mass patterns that are consistent with neutrino oscillation expla-nations of the atmospheric and solar data. The absolute scale of neutrino masses is not fixed by oscillation data and the lightest neutrino mass may vary from 0.0-0.23 eV.

The neutrino mass matrix, in this case, is said to belong to class C.

(D) Disallowed Cases

Finally, let us briefly indicate why the remaining 8 mass matrices are experimentally disallowed.

1) $M_{12} = 0$, $M_{13} = 0$: $M_{12} - M_{13} = 0$ implies

$$\Rightarrow \quad -2(m_1-m_2)s_3c_3 = 0$$
$$\Rightarrow \quad (m_1-m_2)\sin 2\Theta_{12} = 0$$

Therefore in this condition

$$(m_1-m_2)\sin 2\theta_{12}=0, \qquad (24)$$

i.e. no solar neutrino oscillation.

2) M_{12} or $M_{13} = 0$, $M_{23} = 0$: $\sqrt{2}M_{12(13)} - 2s_2M_{23} = 0$ implies

$$|\tan 2\theta_{12}|=2|t_3|/1-t_{23}=2|s_2|<0.32, \qquad (25)$$

which is in gross disagreement with the solar neutrino result (6).

3) $M_{11} = 0$, M_{22} or M_{33} or $M_{23} = 0$: In each case one gets (neglecting $O(s_2)$ terms)

$$|m^2_2-m^2_1|=(1-t^4_3)m^2_2,$$
$$|m^2_3-m^2_2|=(2t^2_3 - t^4_3)m^2_2, \qquad (26)$$

in conflict with Eqs. (6) and (8).

4) M_{22} or $M_{33} = 0$, $M_{23} = 0$: $_{M22(33)}$ $+M_{23} = 0$ implies

$$m_3=-(m_1-m_2)s_2s_3c_3,$$
$$\text{i.e. } m_1 \simeq -m_2 \gg m_3 \qquad (27)$$

using Eqs. (7) and (8). Substituting this in $M_{23} = 0$ implies $t_{23} \simeq 1$, in conflict with eq. (6). It follows that there is no tolerable three zero texture. For example, a neutrino mass matrix with vanishing diagonal entries cannot yield a large enough value of R_υ unless solar neutrino oscillations are very nearly maximal [9]. A situation that appears to be strongly disfavoured by experiment.

Comparison with Quark Mass Matrices

As stated earlier, equation (1) was used in obtaining the structure of the up- and down quark mass matrices, U and D, with two and three texture zeroes, in terms of the mass eigenvalues [10]. In determining their matrix elements it was further assumed that the mixing angles were very small so as to be compatible with mass hierarchy [10], [11]. Among the available choices, the structures that fit the experiment the best, with two texture zeroes, were [10]

$$U=\begin{pmatrix} 0 & 0 & \sqrt{-m1m2} \\ 0 & m2 & \sqrt{m1m3} \\ \sqrt{-m1m2} & \sqrt{m1m3} & m3 \end{pmatrix} \qquad (28)$$

with $m_1 = m_u$, $m_2 = m_c$, $m_3 = m_t$
and

$$D=\begin{pmatrix} 0 & \sqrt{-m1m2} & 0 \\ \sqrt{-m1m2} & m2 & \sqrt{m1m3} \\ 0 & \sqrt{m1m3} & m3 \end{pmatrix} \qquad (29)$$

with $m_1 = m_d$, $m_2 = m_s$, $m_3 = m_b$.
These matrices together give the correct CKM angles [10].

Neutrino mass matrix (M) for hierarchical mass pattern is given by Eq. (14). One may conjecture that in that the texture of the charged-lepton mass matrix (L) should be the same as the D-matrix. Indeed, since the hierarchy of masses in L is similar to D, the entire structure of the two matrices may be similar. In other words, the best choice for the neutrino and charged-lepton mass matrices would be (14) and

$$L=\begin{pmatrix} 0 & \sqrt{-m e m \mu} & 0 \\ \sqrt{-m e m \mu} & m\mu & \sqrt{m e m \tau} \\ 0 & \sqrt{m e m \tau} & m\tau \end{pmatrix} \qquad (30)$$

Conclusions

The seven allowed two-zero neutrino textures fall into three classes: A (with two members), B (with four members), and C. The textures within each class are difficult or impossible to distinguish experimentally, but each of the three classes has radically different implications. For class A, the subdominant angle θ_2 is expected to be relatively large, but no-neutrino ββ decay is forbidden. For class B, the latter process should be measureable by the next generation of double beta decay experiments, whilst s_2 may or may not be large enough to be detected. However, if s_2 is comparable to its experimental upper limit, CP violation must be nearly maximal and should be readily detectable by proposed experiments. For class C, no-neutrino ββ decay is likely to be observable and θ_2 ought to be large enough to measure (unless $\theta_1 \equiv \pi/4$) and to permit a search for CP violation. It is surprising that such a great variety of textures of the neutrino mass matrix can fit what is presently known about neutrino masses and oscillations. Future data should reveal which, if any, of these textures should serve as a guide to the model builder.

We are aware that specific textures of the neutrino mass such as we discuss cannot be preserved to all orders in the unspecified interactions which generate neutrino masses. However, straightforward elaborations of the standard model can generate zeroes of M in tree approximation in such a manner that these entries remain tiny when radiatively corrected.

REFERENCES

[1] Bipin R. Desai, D.P. Roy and Alexander R. Vaucher, hep-ph/0209035v2 16sep2002

[2] Super-Kamiokande Collaboration: S. Fukuda et al., Phys. Rev. Lett. 85 (2000) 3999 and references therein.

[3] A. Bandyopadhay, S. Choubey, S. Goswami and D.P. Roy, Phys. Lett. B540 (2002) 14; V. Barger, D. Marfatia, K. Whisnant and B.P. Wood, Phys. Lett. B537 (2002) 179; J.N. Bahcall, M.C. Gonzalez-Garcia and C. Pena-Garay, hep-ph/0204314; M. Maltoni, T. Schwetz, M.A. Tortola, J.W.F. Valle, hep-ph/0207227 and hep-ph/0207157.

[4] M. Apollonio et al., Phys. Lett. B466 (1999) 415; F. Boehm et al., Phys. Rev. D64 (2001) 112001.

[5] H.V. Klapdor-Kleingrothaus et. al., hep-ph/9910205.

[6] P.H. Frampton, S.L. Glashow and D. Marfatia, Phys. Lett. B536 (2002) 79.

[7] L. Baudis et al., Phys. Rev. Lett. 83 (1999) 41.

[8] A. Kageyama, S. Kaneko, N. Shimoyama and M. Tanimoto, Phys. Lett. B538 (2002) 96.

[9] S.K. Kang and C.S. Kim, Phys. Rev. D63 (2001) 113010 .

[10] B.R. Desai, and A.R. Vaucher, Phys. Rev. D63 (2001)13001-1

[11] T.K. Kuo, S.W. Mansour, and G.H. Wu, Phys. Rev. D60 (1999)093004; Phys. Lett. B467 (1999) 116; S.H. Chin, T.K. Kuo, and G.H. Wu, Phys. Rev. D62 (2000) 053014

CHAPTER – 3
Numerical solution and results

Mass matrices provide important tools for the investigation of the underlying symmetries and the resulting dynamics. The first step in this direction is the reconstruction of the neutrino mass matrix in the flavour basis. However, the reconstruction results in a large variety of possible structures of mass matrices depending strongly on the mass scale, mass hierarchy and the Majorana phases. However, the relatively weak dependence on some oscillation parameters (θ_{23} and δ) results in the degeneracy of possible neutrino mass matrices. The mass matrix for Majorana neutrinos contains nine physical parameters including the three mass eigenvalues, three mixing angles and the three CP-violating phases. The two squared-mass differences (Δm^2_{12} and Δm^2_{13}) and the two mixing angles (θ_{12} and θ_{23}) have been measured in solar, atmospheric and reactor experiments. The third mixing angle θ_{13} and the Dirac-type CP-violating phase δ are expected to be measured in the forthcoming neutrino oscillation experiments. The possible measurement of the effective Majorana mass in neutrinoless double β decay searches will provide an additional constraint on the remaining three neutrino parameters viz. the neutrino mass scale and two Majorana-type CP-violating phases. While the neutrino mass scale will be independently determined by the direct beta decay searches and cosmological observations, the two Majorana phases will not be uniquely determined from the measurement of effective Majorana mass even if the absolute neutrino mass scale is known. Under the circumstances, it is natural to employ other theoretical inputs for there construction of the neutrino mass matrix. The possible form of these additional theoretical inputs are limited by the existing neutrino data. Several proposals have been made in the literature to restrict the form of the neutrino mass matrix and to reduce the number of free parameters which include presence of texture zeros [1, 2, 3, 4, 5], requirement of zero determinant [6], the zero trace condition

[7] to name just a few. However, the current neutrino oscillation data are consistent only with a limited number of texture schemes [1, 2, 3, 4, 5]. In particular, the current neutrino oscillation data disallow all neutrino mass matrices with three or more texture zeros in the flavor basis. Out of the fifteen possible neutrino mass matrices with two texture zeros, only seven are compatible with the current neutrino oscillation data. The seven allowed two texture zero mass matrices have been classified into three categories. The two class A matrices of the types A_1 and A_2 give normal hierarchy (NH) of neutrino masses. The class B matrices of types B_1, B_2, B_3 and B_4 yield a quasidegenerate (QD) spectrum of neutrino masses. The single class C matrix corresponds to inverted hierarchy (IH) of neutrino masses. In the absence of a significant breakthrough in the theoretical understanding of the fermion flavors, the phenomenological approaches are bound to play a crucial role in interpreting new experimental data on quark and lepton mixing. These approaches are expected to provide useful hints towards unraveling the dynamics of fermion mass generation, CP violation and identification of possible underlying symmetries of the lepton flavors from which realistic models of lepton mass generation and flavor mixing could be, hopefully, constructed. Even though the grand unification on its own does not shed any light on the flavor problem, the Grand Unified Theories (GUTs) provide the optimal framework in which possible solutions to the flavor problem could be embedded. This is because the GUTs predict definite group theoretical relations between the fermion mass matrices. For this purpose, it is useful to find out possible leading order forms of the neutrino mass matrix in a basis in which the charged lepton mass matrix is diagonal. Such forms of neutrino mass matrix provide useful hints for model building which will eventually shed important light on the dynamics of lepton mass generation and flavor mixing. For example, a phenomenological favored texture of quark mass matrix has been presented earlier [8]. In the spirit of quark-lepton similarity, the same texture has been prescribed for the charged lepton and Dirac neutrino mass matrices. The same texture for the right handed neutrino mass matrix in the see-saw mechanism might follow from universal flavor symmetry hidden in a more fundamental theory

of mass generation. Thus, the texture zeros in different positions of the neutrino mass matrix, in particular and fermion mass matrices, in general could be consequence of an underlying symmetry [9, 10]. Such universal textures of fermion mass matrices can, theoretically, be obtained in the context of GUTs based on SO(10) [11]. Moreover, neutrino mass matrices with texture zeros have important implications for leptogenesis [12]. In the present work, we examine phenomenological implications of all possible neutrino mass matrices with two texture zeros. Neutrino mass matrices within a class were thought to have identical phenomenological consequences [1, 2, 3] leading to degeneracy of mass matrices within a class. Thus, there is a two-fold degeneracy in class A while the neutrino mass matrices of class B exhibit an eight-fold degeneracy since normal/inverted hierarchies are practically indistinguishable because of quasi-degenerate spectrum of neutrino masses in this class. We study this degeneracy in detail and discuss possible ways to lift this degeneracy. It is found that the deviation of atmospheric mixing from maximality and the quadrant of the Dirac-type CP-violating phase δ can be used to distinguish the mass matrices within a class. We, also, note that the determination of hierarchy will have important implications for class B neutrino mass matrices. The prospects for the measurement of θ_{13} for class A neutrino mass matrices are quite optimistic since a definite lower bound on θ_{13} is obtained for this class. For neutrino mass matrices of class B, CP-violation will be near maximal if θ_{13} is large. Definite lower bounds on the effective Majorana mass M_{ee} are obtained for neutrino mass matrices of class B and C. Class D of neutrino mass matrices is disallowed in our analysis.

NEUTRINO MASS MATRIX

The neutrino mass matrix, M, can be parameterized in terms of three neutrino mass eigenvalues (m_1, m_2, m_3), three neutrino mixing angles (θ_{12}, θ_{23}, θ_{13}) and one Dirac-type CP violating phase, δ. If neutrinos are Majorana particles then there are two additional CP violating phases α, β in the neutrino mixing matrix. The complex symmetric mass matrix M can be diagonalized by a complex unitary matrix V :

$M = VM^{diag}{}_{\upsilon} V^{T}$

where M^{diag}_{υ} = Diag{m1,m2,m3}. The neutrino mixing matrix V [13] can be written as

$$V \equiv UP = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix}$$

where s_{ij} = sin θ_{ij} and c_{ij} = cos θ_{ij} . The matrix V is called the neutrino mixing matrix or PMNS matrix. The matrix U is the lepton analogue of the CKM quark mixing matrix and P contains the two Majorana phases. The elements of the neutrino mass matrix can be calculated from Eq. (1). Some of the elements of M, which can be equated to zero in the various allowed texture zero schemes, are given by

$M_{ee} = c^2{}_{13}c^2{}_{12}m_1 + c^2{}_{13}s^2{}_{12}m_2e^{2i\alpha} + s^2{}_{13}m_3e^{2i\beta}$,

$M_{e\mu} = c_{13}\{s_{13}s_{23}e^{i\delta} (e^{2i\beta}m_3 - s^2{}_{12}e^{2i\alpha}m_2) - c_{12}c_{23}s_{12}(m_1 - e^{2i\alpha}m_2) - c^2{}_{12}s_{13}s_{23}e^{i\delta}m_1\}$,

$M_{e\zeta} = c_{13}\{s_{13}c_{23}e^{i\delta}(e^{2i\beta}m_3 - s^2{}_{12}e^{2i\alpha}m_2) + c_{12}s_{23}s_{12}(m_1 - e^{2i\alpha}m_2) - c^2{}_{12}s_{13}c_{23}e^{i\delta}m_1\}$,

$M_{\mu\mu} = m_1(c_{23}s_{12} + e^{i\delta}c_{12}s_{13}s_{23})^2 + e^{2i\alpha}m_2(c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23})^2 + e^{2i(\beta+\delta)}m_3c^2{}_{13}s^2{}_{23}$

and

$M_{\zeta\zeta} = m_1(s_{23}s_{12} - e^{i\delta}c_{12}s_{13}c_{23})^2 + e^{2i\alpha}m_2(c_{12}s_{23} + e^{i\delta}s_{12}s_{13}c_{23})2 + e^{2i(\beta+\delta)}m_3c^2{}_{13}c2_{23}$.

It will be helpful to note from above equations that the transformation

T : $\theta_{23} \rightarrow \pi/2 - \theta_{23}$, $\delta \rightarrow \delta + \pi$

transforms $M_{e\mu}$ to $-M_{e\zeta}$ and $M_{\mu\mu}$ to $M_{\zeta\zeta}$. Therefore, if $M_{e\mu}$ vanishes for θ_{23} and δ, then $M_{e\zeta}$ vanishes for $\pi/2 - \theta_{23}$ and δ + π. Similarly, if $M_{\mu\mu}$ vanishes for θ_{23} and δ, then $M_{\zeta\zeta}$ vanishes for $\pi/2 - \theta_{23}$ and δ + π.

Type	Constraining Eqs.
A_1	$M_{ee}=0,\ M_{e\mu}=0$
A_2	$M_{ee}=0,\ M_{e\tau}=0$
B_1	$M_{e\tau}=0,\ M_{\mu\mu}=0$
B_2	$M_{e\mu}=0,\ M_{\tau\tau}=0$
B_3	$M_{e\mu}=0,\ M_{\mu\mu}=0$
B_4	$M_{e\tau}=0,\ M_{\tau\tau}=0$
C	$M_{\mu\mu}=0,\ M_{\tau\tau}=0$
D_1	$M_{\mu\mu}=0,\ M_{\mu\tau}=0$
D_2	$M_{\tau\tau}=0,\ M_{\mu\tau}=0$

Table 2: Allowed two texture zero mass matrices

The current best fit values of the oscillation parameters with 1, 2 and 3 σ [14] errors given below

$$\begin{aligned}
\Delta m^2_{12} &= 7.9\ ^{+0.3,\ 0.6,\ 1.0}_{-0.3,\ 0.6,\ 0.8}\ \times 10^{-5}\ eV^2\ , \\
\Delta m^2_{23} &= \pm 2.6\ ^{+0.2,\ 0.4,\ 0.6}_{-0.2,\ 0.4,\ 0.6}\ \times 10^{-3}\ eV^2 \\
s^2_{12} &= 0.30\ ^{+0.02,\ 0.06,\ 0.10}_{-0.02,\ 0.04,\ 0.06}\ , \\
s^2_{23} &= 0.50\ ^{+0.06,\ 0.13,\ 0.18}_{-0.05,\ 0.12,\ 0.16}\ , \\
s^2_{13} &< \ 0.012,\ 0.025,\ 0.040\ .
\end{aligned}$$

Here, '+' ('−') sign with the value of Δm^2_{23} is for normal (inverted) hierarchy. The analysis [14] incorporates not only the latest long baseline data for Δm^2_{13} from the MINOS collaboration [15] but also the updated KamLAND and SNO data [16]. Jarlskog rephasing invariant quantity [18] is given by

$J = s_{12}s_{23}s_{13}c_{12}c_{23}c^2_{13} \sin\delta$

which measure the leptonic CP violation.

CLASS A

C.L.	$m_1(10^{-3}eV)$	α(deg)	β(deg)	θ_{13}(deg)	J
1	2.6 - 4.5	83 - 97	-80 - 80	5.2 - 6.3	-0.025 - 0.025
2	2.1 - 8.9	77 - 103	-90 - 90	4.2 - 9.1	-0.037 - 0.037
3	1.7 - 13.4	73 - 107	-90 - 90	3.3 - 11.5	-0.046 - 0.046

Table 3: The predictions for neutrino mass matrices of class A

The results for m_1, α , β, θ_{13} and J for class A mass matrices have been summarized in Table 3 at various confidence levels. These quantities are the same for mass matrices of types A_1 and A_2. The range for the Majorana-type CP-violating phase β at 1 σ C.L. is found to be -80^0 - 80^0. However, if the neutrino oscillation parameters are allowed to vary beyond their present 1.2 σ C.L. ranges, the full range for β (-90^0 - 90^0) is allowed. The accuracies of the oscillation parameters required to distinguish between the theoretical predictions of neutrino mass matrices of types A_1 and A_2 depend on the upper bound on θ_{13}. With the current precision of the oscillation parameters, the neutrino mass matrices of types A_1 and A_2 can be distinguished at 1σ, 2σ and 3σ C.L. for $\theta_{13} < 5.7^0$, 5.2^0 and 4.8^0 respectively.

CLASS B

C.L.	α	M_{ee}	J
1 σ	-2.0^0-2.0^0	$\geq 0.064eV$	-0.025-0.025
2 σ	-7.7^0-7.7^0	$\geq 0.037eV$	-0.036-0.036
3 σ	-22^0-22^0	$\geq 0.023eV$	-0.045-0.045

Table 4: The predictions for neutrino mass matrices of class B.

B_1 and B_3 (or B_2 and B_4) will have, almost, identical predictions for θ_{23} if it is near maximality. However, B_1 and B_4 (or B_2 and B_3) can be distinguished from each other by their predictions for the octant of θ_{23} and B_1 and B_3 (or B_2 and B_4) can be distinguished from each other by their predictions for the quadrant of δ. Thus, the four-fold degeneracy in class B is, now, lifted. If $\theta_{23} < 45^0$, B_1 and B_3 give normal hierarchy while B_2 and B_4 give inverted hierarchy. Similarly, if $\theta_{23} > 45^0$, then B_1 and B_3 give inverted hierarchy while B_2 and B_4 give normal hierarchy.

Fig. 6 depicts the correlations which lift the degeneracy between the neutrino mass matrices of types B_1 (B_4) and B_2 (B_3). Since, the range of Dirac-type CP-violating phase, δ, is different for B_1 (B_4) and B_2 (B_3), the eightfold degeneracy in B type mass matrices has, now, been reduced to four-fold. The left panel depicts the correlation plots for both B_1 and B_4 and the right panel depicts the correlation plots for B_2 and B_3. It can be seen from Fig. 6(a,b) that the range of Dirac-type CP-violating phase, δ, is $90^0 \leq \_ \leq 270^0$ ($-90^0 \leq \_ \leq 90^0$) for B_1 and B_4 (B_2 and B_3) type mass matrices, as expected. Also, there is a strong correlation between Majorana-type CP violating phase, β, and Dirac-type CP-violating phase, δ reinforcing the zeroth order result ($\beta+\delta = (n + ½)\pi$. It can be seen from Fig. 4(c,d) that θ_{13} remains unconstrained in all cases because full range of θ_{13} is allowed at $\delta = 90^0$ and 270^0. However, as we deviate from these values of δ, θ_{13} decreases rapidly to very small values. So, if the CP violation is found to be non maximal, θ_{13} will be constrained to very small values. However, if CP violation is found to be nearly maximal, θ_{13} can be large. Thus, the suppression factor $s_{13} \cos\delta$ in the first order correction term in the Taylor expansion for (m1/m2) is small as expected from the zeroth order approximation for this class.

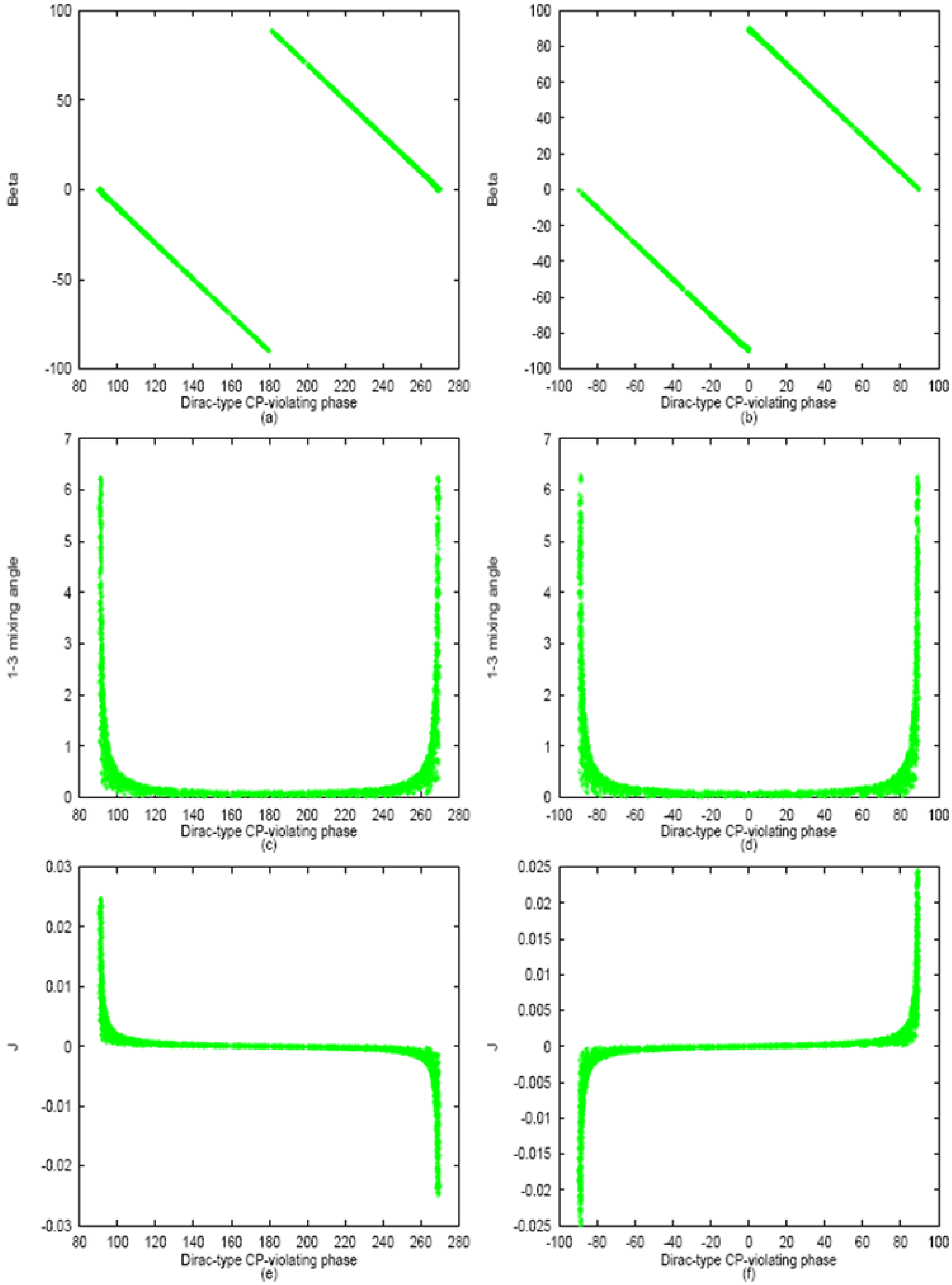
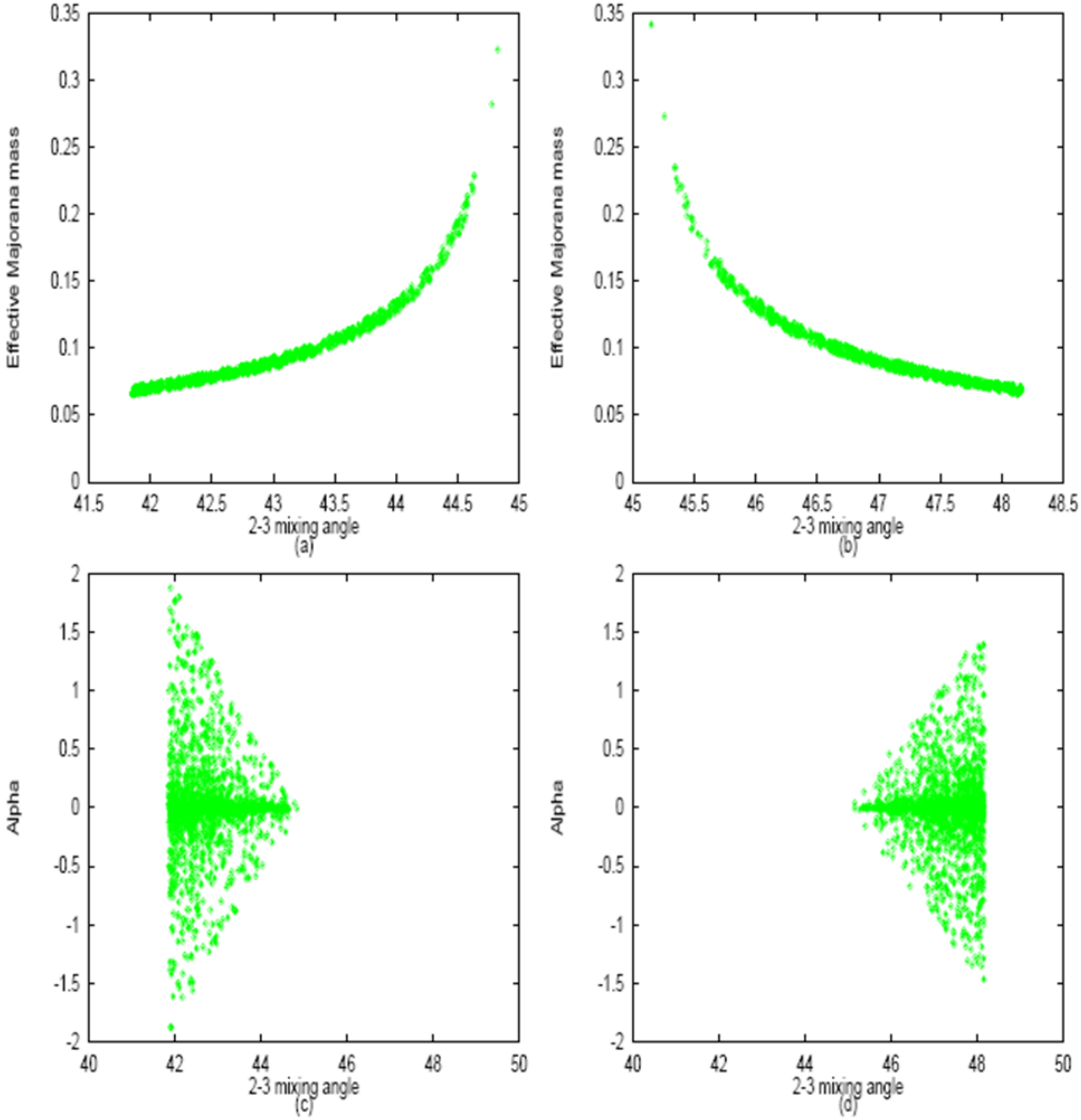


Fig. 6. Class B.

Fig. 7 depicts the correlations of M_{ee} and α with θ_{23}. The correlation plots with θ_{23} can be used to further reduce the remaining four-fold degeneracy in B type mass matrices to a two-fold degeneracy which will be lifted by the determination of the neutrino mass hierarchy. The left panel (right panel) depicts the correlation plots for B_1(NH) and B_2(IH) (B_1(IH) and B_2(NH)). The correlation plots of M_{ee} and α with θ_{23} for B_3(NH) and B_4(IH) (or B_3(IH) and B_4(NH)) will be, almost, identical to the plots given in Fig. 7 because B_1 and B_3 (or B_2 and B_4) have, almost, identical predictions for θ_{23} and, therefore, are not given here. It can be seen from Fig. 6(a,b) that the effective Majorana mass M_{ee} is strongly correlated with θ_{23}. Moreover, maximal value of θ_{23} is not allowed by the current neutrino oscillation data since M_{ee} diverges at $\theta_{23} = \pi/4$. Larger the deviation of θ_{23} from maximality, smaller is the value of Mee. Also, a large value of α implies large deviations of θ_{23} from maximality. The extent of the deviation of 2-3 mixing from maximality is governed by the magnitude of the upper bound on M_{ee} while the direction of the deviation from maximality is governed by the quadrant of δ and the neutrino mass hierarchy.



Effective Majorana mass
2-3 mixing angle
(a)
Effective Majorana mass
2-3 mixing angle
(b)
Alpha
2-3 mixing angle
(c)
Alpha
2-3 mixing angle
(d)

Fig.7. Class C.

REFERNCES

[1] Paul H. Frampton, Sheldon L. Glashow and Danny Marfatia, Phys. Lett. B 536, 79 (2002); Bipin R. Desai, D. P. Roy and Alexander R. Vaucher, Mod. Phys. Lett A 18, 1355 (2003).

[2] Zhi-zhong Xing, Phys. Lett. B 530 159 (2002).

[3] Wanlei Guo and Zhi-zhong Xing, Phys. Rev. D 67, 053002 (2003).

[4] Alexander Merle and Werner Rodejohann, Phys. Rev. D 73, 073012 (2006); S. Dev and Sanjeev Kumar, hep-ph/0607048.

[5] S. Dev, Sanjeev Kumar, Surender Verma and Shivani Gupta, hep-ph/0611313.

[6] G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim and T. Yanagida, Phys. Lett. B 562 265 (2003); Bhag C. Chauhan, Joao Pulido and Marco Picariello, Phys. Rev. D 73, 053003 (2006).

[7] Xiao-Gang He and A. Zee, hep-ph/0302201 v2.

[8] Suraj. N. Gupta and Subhash Rajpoot, “Quark Mass Matrices and the Top Quark Mass”, Wayne State University Preprint, September 1990; S. N. Gupta and J. M. Johnson, Phys. Rev. D 44, 2110 (1991); S. Rajpoot, Mod. Phys. Lett. A 7, 309 (1992); H. Fritzsch and Z.Z. Xing, Phys. Lett. B 353, 114 (1995).

[9] P. H. Frampton, M. C. Oh and T. Yoshikawa, Phys. Rev. D 66, 033007 (2002).

[10] Walter Grimus and Luis Lavoura, J. Phys. G 31, 693-702 (2005).

[11] H. C. Goh, R. N. Mohapatra and Siew-Phang Ng, Phys. Rev. D 68, 115008 (2003).

[12] S. Kaneko, M. Katsumata and M. Tanimoto, JHEP 0307, 025 (2003).

[13] G. L. Fogli et al,hep-ph/0506083 v1.

[14] M. Maltony, T. Schwetz, M. A. Tortola and J. W. F. Valle, hep-ph/0405172 v5.

[15] J. Nelson, Talk at Neutrino 2006, 1319 June 2006, Santa Fe, New Mexico, http://neutrinosantafe06.com.

[16] B. Aharmim et al. [SNO collaboration], Phys. Rev. C 72, 055502 (2005), nucl-ex/0502021.

[17] T. Araki et al. [KamLAND Collaboration], hep-ex/0406035; Talk by G. Gratta at Neutrino 2004, 1419 June 2004, Paris, http://neutrino2004.in2p3.fr.

[18] C. Jarlskog Phys. Rev. Lett. 55, 1039 (1985).

[19] S. Dev, Sanjeev Kumar, Surender Verma and Shivani Gupta, arXiv:hep-h/0612102v3 6 Aug 2007

Chapter 4
Summary

Neutrinos are elementary particles that often travel close to the speed of light, lack an electric charge, are able to pass through ordinary matter almost undisturbed and are thus extremely difficult to detect. Neutrinos are created as a result of certain types of radioactive decay (beta decay) or nuclear reactions such as those that take place in the Sun, in nuclear reactors, or when cosmic rays hit atoms.

Neutrino oscillation is a quantum mechanical phenomenon predicted by Bruno Pontecorvo in which a neutrino created with a specific lepton flavor (electron, muon or tau) can later be measured to have a different flavor. The probability of measuring a particular flavor for a neutrino varies periodically as it propagates. Neutrino oscillation is of theoretical and experimental interest since observation of the phenomenon implies that the neutrino has a non-zero mass.

Solar neutrino, atmospheric neutrino, reactor neutrino experiments confirm the phenomenon of oscillations and the existence of neutrino mass. The present experimental data is already sufficient to constraint or test theoretical models of neutrino mass matrix. For example, the present data does not support any neutrino mass matrix with three or more texture zeros. Further, there can be only 15 independent neutrino mass matrices with two texture zeros ($^6C_2 = 15$) and only 7 out of the 15 are compatible with the oscillation data. The physically allowed 7 mass matrices can be further divided into three classes:

(A) Hierarchical Solutions : $|m_1| < |m_2| \ll |m_3|$. The mass matrices are A1 and A2.

(B) Degenerate Solutions : The mass matrices B1, B2, B3 and B4 show degeneracy i.e. masses are are approximately equal and they do not support the bimaximal case.

(C) Inverted Hierarchy : The mass matrix C has the inverted hierarchy of neutrino masses.

In **chapter 2**, it was shown that predictions of neutrino mass matrices within a class are experimentally indistinguishable. For example the mass matrices A1 and A2 will have identical predictions. Similarly, the mass matrices of type B1 B2, B3 and B4 are same physically. This leads to a degeneracy in the neutrino mass matrices with two texture zeros. Later, we find in **Chapter 3** that the predictions are same within a class only for the measured neutrino parameters. As we will have measurements of presently unknown neutrino parameters in the future experiments, we will be able to lift the degeneracy which is present in the neutrino mass matrices with two texture zeros.