Multi-Objective Decision-Making Framework for Transportation Investments using Surrogate Worth Trade-off Method

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AND

MY SUPERVISOR

CERTIFICATE

I hereby certify that the work which is being presented in this thesis entitled "Trade-off Analysis of Multi-Objective Framework for Transportation Investments" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Mahesh Kumar Sharma.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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ABSTRACT

This thesis presents a multi-objective decision framework to support decision-making process in transportation investment analysis. One important capability of the multi-objective decision framework is that it allows many intangible objectives that are difficult to express on an absolute numerical scale to be considered without the need to convert the units into monetary scale.

The multi-objective decision framework allows a decision maker to select a reduced number of alternatives from a larger number of all available alternatives while ensuring that the selected alternatives are the best possible options. The framework, which is based on the Surrogate Worth Trade-off analysis, could be applied to both discrete and continuous decision-problem scenarios. In a discrete problem, a predefined set of alternatives is available, whereas continuous problems are not characterized by a pre-defined set of alternatives. This framework is applied with the data generated for a Capital Beltway Corridor investment study by Chowdhury and Tan [2].

This thesis consists of two chapters. Chapter one is introductory in nature, Non-dominated solutions, ideal solutions, surrogate worth trade-off method applied to both discrete and continuous problem, a fast algorithm for finding a non-dominated set in multi-objective optimization, and a brief survey of literature have been also presented in this chapter. In chapter two, the problem of multi-objective decision making framework for transportation investment considered by Chowdhury and Tan [2] is reviewed by reducing the problem into three objectives and applied with the data generated by Beltway Corridor project. Non-dominated solutions are obtained with the approach used by Mishra and Harit [15]. Tradeoff evaluation also has been done in this chapter.

CONTENTS

CHAPTER-I

INTRODUCTION

The single-objective model was the first to be developed and thus it was received considerably more exposure, been put to more use, and is generally considered to be a relatively high level of refinement. Thus, the implication is simple, well-tested tool is available and we may be inclined to fit the problem to this model despite the assumptions required. But in real life there are many problems with more than one objective for which the multi-objective models are required.

Dantzig's initial concept was centered about the development of the linear programming model but with a single objective. This so set the tone for the development of traditional linear programming that many (if not most) linear programming texts completely ignore even the possibility of more than one objective. Unlike the traditional single objective optimization problem wherein it is settle on optimizing single objective function, there is no single universally accepted approach for solving the multi-objective optimization problems due to usually conflicting nature of objective functions leading to the situation where the optimization of one of these may adversely affect the optimization of others. So in the case of multi-objective optimization problems, there is no need to access the decision maker's utility function that may vary from decision maker to decision maker.

The process of decision-making is the selection of an act or courses of action from among alternative acts or courses of actions such that it will produce optimal results under some criteria of optimization. In decision analysis of complex systems, such terms as "multiple criteria", "multiple objectives", or "multiple attributes" are used to describe decision situations. Often, these terms are used interchangeably. Certainly, there are no universal definitions of these terms. Multiple criteria decision- making (MCDM) has seemed to emerge as the accepted nomenclature for all models and techniques dealing with multiple objectives decision-making (MODM) or multiple attribute decision making (MADM). These are the two brand categories of MCDM problems. MODM methods are often used with reference to problems with large set of alternatives, while MADM methods are meant to select the best from a small explicit list of alternatives. MODM, therefore, is a problem of design, and mathematical techniques of optimization are needed in solving it. On the other hand, MADM is a problem of choice and classical mathematical programming tools need not be used.

A necessary condition of MCDM is the presence of more than one criterion. The sufficient condition is that the criteria must be conflicting in nature. Therefore the following definition can be stated: "A problem can be considered as that of MCDM if and only if there appears at least two conflicting criteria and there are at least two alternatives solutions." Criteria are said to be in conflict if the full satisfaction of one will result in impairing or precluding the full satisfaction of the other(s). Conflict may arise due to intrapersonal and interpersonal reasons. A consumer purchasing a car is often confronted with conflicting criteria caused intrapersonally. On the other hand, a family looking for a house to reside is a typical example where conflicts of criteria may be due to interpersonal reasons. In view of the conflicting nature of the criteria involved in MCDM, choosing the "best" alternative is indeed a difficult task for the decision maker. Consequently there is a need for methods to systematically resolve the conflicts among criteria (or objectives) in order to reach acceptable compromises and come up with satisfying (or often as "satisficing") solutions.

Most decision-making scenarios in transportation problem involve multiple objectives that often conflict. Multi-objective optimization is not purely a maximizing or minimizing problem. It is a mixture of several conflicting maximum or minimum problems, which boils down to that of "satisficing" these conflicting objectives. Multi-objective optimization is the process of simultaneously optimizing two or more conflicting objective subject to certain constraints.

Formulation of Multi-objective optimization

Multi-objective optimization can be defined as:

"a vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions. Hence the term "optimizes" means finding such a solution which would give the values of all the objective functions acceptable to the decision maker."

This model can be formulated as:

Optimize $f(X) = (f_1(X), f_2(X), ..., f_k(X))$

Subject to

$$g_j(X) \le = \ge b_j \ , \ j = 1, 2, ..., m$$

$$X \ge 0$$

$$X = (x_1, x_2, ..., x_n)^T$$

where, $f(X)$ is the objective function to optimize. $f_1(X), f_2(X), ..., f_k(X)$ are k number of distinct objective functions subject to m constraints. X is a vector consists of decision variables $x_1, x_2, ..., x_n$.

1.1 OPTIMAL AND EFFICIENT SOLUTIONS:

Optimal solution

An optimal solution is one which attains the maximum value of all the objectives simultaneously. The solution y^* is optimal to some problem iff $y^* \in S$ and $f_k(y^*) \geq f_k(y)$ for all k and for all $x \in S$, where S is the feasible region. In general, there is no optimal solution to a multi-objective problem. Therefore, optimality replaced by the concept of "satisficing" or the best compromise solution, which depends on the decision makers preferences with respect to the objectives. Therefore, one can obtain only efficient solutions in multi-objective problem.

Efficient or Non-Dominated solutions

A set of solutions is said to be efficient if there exits no solution that is superior to it with respect to at least one objective function but is not inferior to it with respect to any of the objective functions.

If x_1 and x_2 are two solutions, then these can have any of two possibilities-one dominates the other or non-dominates the other. In a minimization problem, without the loss of generality, a solution x_1 dominates x_2 iff the following two conditions are satisfied:

$\forall i \in \{1, 2, ..., N_{obj}\} : f_i(x_1) \leq f_i(x_2)$

$\exists j \in \{1, 2, ..., N_{obj}\} : f_j(x_1) < f_j(x_2)$

where, $f(x_1)$ *and* $f(x_2)$ are the objective functions

If any of the above conditions is violated, the solution x_1 does not dominate the solution x_2. If x_1 dominates the solution x_2, x_1 is called the non-dominated solution within the set $\{x_1, x_2\}$. The solutions that are non-dominated within the entire search space are denoted as Pareto-optimal and constitute the Pareto-optimal set or Pareto-optimal front. From the entire set of efficient (non-dominated) solutions the decision maker can select the solution one believed most attractive.

1.2 IDEAL SOLUTIONS

Due to the conflicting nature of the objectives in multi-objective optimization, it is not at all possible to achieve the individual optimum of each objective by a single solution. Each objective function has its own "ideal" solution which is different for all other objectives. Hence, using one of the "ideals" as the solution for the multi-objective problem would only mean that one objective achieves its individual optimum while all the rest only achieve partial satisfaction with respect to their individual optima. The matrix, or generally known as "payoff" matrix, shown in Fig.1.1 helps to clarify this phenomenon. The diagonal of the matrix constitutes individual optimal values of the k objectives functions. The x^{h*}*'s* are the individual optimal solutions and each of these are used to determine the values of other individual objective functions, thus the payoff matrix is developed.

	x^{1*}	x^{2*}	………	x^{h*}	………	x^{k*}
Z_1	$f_1(x^{1*})$	$f_1(x^{2*})$	………	$f_1(x^{h*})$	………	$f_1(x^{k*})$
Z_2	$f_2(x^{1*})$	$f_2(x^{2*})$	………	$f_2(x^{h*})$	………	$f_2(x^{k*})$
⋮	⋮	⋮	⋱	⋮	⋱	⋮
Z_l	$f_l(x^{1*})$	$f_l(x^{2*})$	………	$f_l(x^{h*})$	………	$f_l(x^{k*})$
⋮	⋮	⋮	⋱	⋮	⋱	⋮
Z_k	$f_k(x^{1*})$	$f_k(x^{2*})$	………	$f_k(x^{h*})$	………	$f_k(x^{k*})$

Fig.1.1

1.3 DISCRETE AND CONTINUOUS PROBLEM

A typical decision problem in transportation investment could be continuous, discrete, or a combination of both. In a discrete problem, a predefined set of alternatives is available. Many transportation investment projects have this characteristic, as project alternative are selected from a number of possible alternatives (such as the selection of a project site from several possible sites). Continuous problems are not characterized by a pre-defined set of alternatives. Instead, for a continuous problem (such as miles of a particular road that need to be reconstructed), a mathematical model including decision variables, constraints, and multiple objective functions must be formulated to generate alternatives. The decision alternatives are not pre-defined in a continuous problem while a finite number exist in discrete problems.

1.4 SURROGATE WORTH TRADE-OFF METHOD

Haimes, et al., [9] introduced the SWT method and it has been applied in water resources systems. The SWT method is used to generate surrogate worth functions. The method is composed of several consecutive phases in two major steps. The first step generates the non-dominated solutions using the constraint (ε) method. The constraint method suggested by Haimes (1971) is applied by optimizing one objective while all of the other objectives are constrained to some value (ε). The solution to the problem largely depends on the chosen ε vector. It must be chosen so that it lies within the minimum or maximum values of the individual objective function.

The second step in the SWT method, known as the interactive process, includes direct interaction between the analyst and the decision maker. It can guide the decision maker to develop trade-off among objectives from the set of feasible solutions generated earlier to find the preferred solution. In many situations, transportation projects consider both economic factors and financial return on the investment and other factors such as quality of life and preservation of the environment. However, even though the generation of non-dominated solutions meets those criteria, decision maker need to make trade-off among such criteria.

1.5 A FAST ALGORITHM FOR FINDING THE NON DOMINATED SET IN MULTI-OBJECTIVE OPTIMIZATION

Mishra and Harit [15] has proposed an approach which is very different from existing algorithms. In existing algorithms, there is ability to classify solutions only after finding all dominated solutions. So to find non dominated solutions, we need to search entire set repeatedly.

In this algorithm, only non-dominated solutions are to be stored. A few comparisons need to classify a point in to either dominated set or non-dominated set. First sort population according to the descending order of importance to the first objective value. In this way the solutions which are good in first objective will come first, in the list those having bad value will come in last.

Initialize a set S_1, for keeping non-dominated solutions only. Then start with the first solution and add this solution to non-dominated set S_1. Since first point is best in terms of first objective so no point can dominate this point in first objective, so it will be non-dominated. Now compare every other solution of the list with this set S_1 and update this set when find another non-dominated solution and skip on those solutions which are dominated by any element of the set. For example if solutions in the list are unique in first objective function value, then for second point it need only one comparison to decide whether this is dominated or non-dominated.

The reason can be explained as follows, this solution can be dominated by only first solution (which is best in first objective). It cannot be dominated by other solutions because its value for first objective function is greater than all solutions except first. Similarly for the third solution it need at most two comparisons from fist and second point. And for the last point of list it needs to compare this solution to all non-dominated solutions. If solutions in list are not unique in first objective function value, then make certain modification in proposed algorithm. Like checking every solution to its immediate successors, if any immediate solution dominates this solution then remove this point from the non- dominated set S_1. Finally display the non-dominated solutions.

1.6 BRIEF SURVEY OF THE LITERATURE

The multi-objective analysis is gaining prominence due to its ability to handle multiple conflicting objectives in an efficient manner. Study on multi-objective analysis is reported by Cohon [4], Hwang [11], Goicoechea et al. [8], Deb [6], in both traditional and non-traditional optimization environment. In traditional multi-objective optimization methods, namely, Weightage Method and Constraint Method that generate non-inferior/non-dominated alternatives whereas numerous methods are developed in non-traditional environment provided by Raju and Kumar [16]. Recent development in the field of optimization (single objective and multi-objective) play a major role to appraise decision situations systematically and effectively in a structured and holistic manner given by Deb, K.[6].

Most of the decision-making scenarios in the transportation field are complex and include multiple and often conflicting objectives. These objectives are sometimes difficult to measure in monetary units alone, so traditional economic methods such as cost-benefit analysis may not be sufficient. Sakarada [17] suggested the purpose of cost-benefit analysis, if a policy maker has to decide between several mutually exclusive projects, cost-benefit analysis is a useful economic tool for comparing projects and deciding which one is optimal. However, cost-benefit analysis requires a common unit of measurement, and most typically used common unit is money. Monetary valuation of benefits for cost-benefit analysis may be difficult to do and unreliable in its result for many decision scenarios. Multi-objective analysis enables evaluation of the alternatives without the need to convert the objectives into monetary units.

Tabucanon [18] suggested multi-objective analysis provides a set of best solutions from a larger set of available options, and provides an objective framework to eliminate a large number of possible options from any further consideration. At the same time, the framework provides an acceptable confidence bound where most desirable solutions are included in the set of best solutions.

Tantawy and Sallam [20] considered the Multiple objective linear programming problems with the same objective space and they suggested that under the linear mapping efficient extreme points in decision space of multiple objectives linear programming may not map to non-dominated extreme points in objective space, condition that two or even more multiple objective linear programming problems to have the same objective space is given, the important of this study is that the Decision-Maker may depends on extreme points of the set of the objective space than that of the decision space since in most practical problems the number of objective is small compared to the number of the decision variables and so they have fewer extreme points.

Buchanan and Daellenbach [1] proposed a comparative evaluation of interactive solution methods for multiple objective decision models. Their work describes a laboratory experiment which compares the performance from the user's point of view of four different solution methods for multiple objective decision models with continuous variables. The results highlight a need for solution methods to accommodate the decision making characteristics of the user.

Debeljak et al. [5] outlines the MCDM process and then presents two of the more well-known methods: the analytic hierarchy process and the surrogate worth trade-off method. They proposed an integrated approach which attempts to capture the advantages of both methods in order to develop one robust solution strategy.

A typical decision problem in transportation investment could be continuous, discrete, or a combination of both. In a discrete problem, a predefined set of alternatives is available. Many transportation investment projects have this characteristic, as project alternatives are selected from a number of possible alternatives (such as the selection of a project site from several possible sites). Continuous problems are not characterized by a pre-defined set of alternatives. Instead, for a continuous problem (such as miles of a particular road that need to be reconstructed), a mathematical model including decision variables, constraints, and multiple objective functions must be formulated to generate alternatives. The decision alternatives are not pre-defined in a continuous problem while a finite number exist in discrete problems.

Chowdhury and Tan [2] developed a framework based on multi-objective optimization that can be used to generate and analyze the most desirable transportation investment options based on their objectives and constraints. The framework which is based on the Surrogate Worth Trade-off analysis could be applied to both discrete and continuous decision-problem scenarios. In discrete problem, a pre-defined set of alternatives is available, whereas continuous problem are not characterized by pre-defined set of alternatives.

Several studies applied different multi-objective methods to develop tools for making decisions in transportation-related issues, mostly for discrete problems. Many of these applications have been limited to a utility-based approach, rather than optimization-based approach, where various weights are assigned to different decision criteria. This type of decision-making approach was used in a study to identify the critical highway safety needs of special population groups (Dissanayake et al. [7]).

The optimization-based approach is more objective than the utility-based approach for multi-objective decision analysis. The utility-based approach is greatly reliant upon decision makers' input and criteria weighting, when final output and project selection could be influenced by personnel changes among the decision makers. The utility-based approach includes the decision maker's input as a part of developing the output. Input is sought through a set of questionnaires on the relative importance of selected measures of effectiveness (MOEs).

Haimes et al. [9] applied a powerful optimization-based approach called the Surrogate Worth Trade-off (SWT) method for decision making in water resource systems (Haimes et al., 1975). The SWT method is used to generate surrogate worth functions. The method is composed of several consecutive phases in two major steps. The first step generates the non-dominated solutions using the constraint (ε) method. In a non-dominated solution, any improvement of one objective can be achieved only at the degradation of the other. The constraint method is applied by optimizing any one of the objectives from n number of objectives while all of the other objectives (n-1) are constrained to some value (ε). The solution to the problem largely depends on the chosen ε_k vector, which is chosen between the minimum and maximum values of the k^{th} objective function.

The second step in the SWT method, known as the interactive process, includes direct interaction between the analyst and the decision maker. It can guide the decision maker to develop trade-offs among objectives from the set of feasible solutions generated earlier to find the preferred solution. In many situations, transportation projects consider both economic factors and financial return on the investment and other factors such as quality of life and preservation of the environment. However, even though the generation of non-dominated solutions meets those criteria, decision makers need to make trade-offs among such criteria.

Haimes et al. [9], has developed a framework for evaluating the safety of alternative automobile designs in terms of the likelihood of crash occurrence and severity of likely injury and they used a multi-objective decision analysis approach called "Partitioned Multi-objective Risk Method" to develop the framework for evaluating the vehicle-based crash avoidance and worthiness technologies based on the expected and worst-case outcomes. This methodology permitted evaluation based on unconditional expected events as well as worst-case outcomes. The proposed framework included the SWT method to assess the preference of the decision maker/design engineer for competing design alternatives by interviewing him or her and communicating the possible outcomes and corresponding trade-offs.

A study by Chowdhury et al. [3], presented an Interactive Multi-Objective Resource Allocation (IMRA) tool to help decision makers minimize the frequency and severity of vehicular crashes by selecting countermeasures and allocating resources optimally among various competing highways. This methodology also illustrated the trade-off between various decision options and how to set priorities for a variety of potential crash countermeasures. The main objective of this research was to develop a tool that would aid in optimal resource allocation to improve highway safety. The IMRA tool supported interaction between the analyst and the decision maker that would help the decision maker select the best among various options.

Korhonen et al. [12] proposed a formal man-machine interactive approach to multiple criteria optimization with multiple decision makers. The approach is based on some earlier research findings in multiple criteria decision making. A discrete decision space is assumed. The same framework may readily be used for multiple criteria mathematical programming problems. The results of the experiments indicate that their approach is a potentially useful decision aid for group decision-making and bargaining problems.

1.7 PRESENT WORK

This thesis presents a multi-objective decision framework to support decision-making process in transportation investment analysis. One important capability of the multi-objective decision framework is that it allows many intangible objectives that are difficult to express on an absolute numerical scale to be considered without the need to convert the units into monetary scale.

The multi-objective decision framework allows a decision maker to select a reduced number of alternatives from a larger number of all available alternatives while ensuring that the selected alternatives are the best possible options. The framework, which is based on the Surrogate worth Trade-off analysis, could be applied to both discrete and continuous decision-problem scenarios. In a discrete problem, a predefined set of alternatives is available, whereas continuous problems are not characterized by a pre-defined set of alternatives. This framework is applied with the data generated for a Capital Beltway Corridor investment study by Chowdhury and Tan [2].

This thesis consists of two chapters. Chapter one is introductory in nature, Non-dominated solutions, ideal solutions, surrogate worth trade-off method applied to both discrete and continuous problem, a fast algorithm for finding a non-dominated set in multi-objective optimization, and a brief survey of literature have been also presented in this chapter. In chapter two, the problem of multi-objective decision making framework for transportation investment considered by Chowdhury and Tan [2] is reviewed by reducing the problem into three objectives and applied with the data generated by Beltway Corridor project. Non-dominated solutions are obtained with the approach used by Mishra and Harit [15]. Tradeoff evaluation also has been done in this chapter.

CHAPTER-II

TRADE-OFF ANALYSIS OF MULTI-OBJECTIVE FRAMEWORK FOR TRANSPORTATION INVESTMENTS

INTRODUCTION

In this chapter the work of Chowdhury and Tan [2], has been reviewed in detail by reducing the problem in three objectives. A fast algorithm has been used to find the set of non-dominated solutions. In decision making scenarios, a transportation agency always want to make the best use of available resources for completing projects. But many problems arise to agency like limited resources, selection of an alternative from competing projects etc. Decision Maker has to select the best alternative for completing the project such that the available limited resources can be used in an optimal manner.

Most decision- making scenarios in transportation problem involve multiple objectives that often conflict. These objectives are sometimes difficult to measure in monetary units. A traditional economical method cost benefit analysis is also not sufficient because it requires a common unit of measurement and mostly used common unit is money. Therefore, this approach requires that all costs and benefits have to be transformed into a single monetary dimension. Since, all objectives such as customer satisfaction and environmental quality cannot be transformed into monetary units. So this approach leads to many difficulties in practical. The decision making process led to the development of multi-objective decision making concepts in the late 1970s and early 1980s. In this development, all objectives are approached on an equal basis, regardless of whether they can be estimated in monetary or non-monetary terms.

Multi-objective decision making analysis can help the decision making process for transportation projects. Two major players are involved in transportation investment: decision maker(s) and analyst(s). The decision maker makes important judgments regarding the relative significance of the objectives. The analyst generates alternatives and trade-offs among objectives using mathematical tools.

There are two approaches to implementing multi-objective decision analysis. The first emphasizes preference assessment and the second is based on generating solutions or alternatives.

The preference assessment approach requires an explicit statement of preferences by decision maker prior to the solution process. The preferences are expressed in many different forms, including weighting or use of a multi-attribute utility function. The decision maker can then determine how to allocate resources among a set of projects.

The solution generating approach emphasizes generation of a range of solutions and trade-offs to be presented to the decision maker for consideration. The main purpose of this approach is to produce the entire set of non-dominated or effective solutions. The analyst then helps decision maker choose among possible solutions. The solution chosen is called the most preferred solution or the best compromise solution. Surrogate Worth Trade-off Method is such a technique of multi-objective optimization, which is used with this approach.

The multi-objective decision framework allows a decision maker to select a reduced number of alternatives from a larger number of all available alternatives while ensuring that the selected alternatives are the best possible options.

The framework, which is based on the Surrogate Worth Trade-off analysis, could be applied to both discrete and continuous decision-problem scenarios. This framework allows a decision maker to select a reduced number of alternatives from a larger number of all available alternatives while ensuring that the selected alternatives are the best possible options. This framework is applied with the data generated for a Capital Beltway Corridor investment study. The detailed case study follows a discussion of the framework.

2.1 Framework of the Multi-objective Methodology

The multi-objective framework includes a total of six steps. These steps are discussed below.

Step 1: Identify Objectives

The first activity is to identify the objectives to be measured. An objective is a statement about the desired state of the system under consideration. Objectives should be specific and cover the main goal or need, such as minimizing delay time, minimizing cost, and maximizing safety. Although some projects have a single objective, most transportation projects have multiple objectives. The analyst should consider all of the objectives in this process.

Step 2: Select Measures of Effectiveness

In this step, the objectives to be measured are defined. The effectiveness of each project alternative is measured according to the performance of these alternatives on all of the objectives specified in Step1.

Step 3: Formulate a Mathematical Model

The mathematical model is expressed as a mathematical function that represents the problem. The model is used to generate the value of decision variables and maximize or minimize the objective function subject to the specified constraints. If there are n-related decisions to be made, they are represented as decision variables ($x_1,x_2,...,x_n$) whose respective values are to be determined.

The appropriate measures of effectiveness, such as cost and travel time, are then expressed as a mathematical function of these decision variables. This function is called the objective function. For example, in the scenario given above the objective is to minimize cost and the decision variable (x_i) is number of miles of road to build in area *i*, where i = 1, 2, and *n*. The identification of the decision variable leads to essential answers to the questions the decision-maker is seeking. The constant value (C_i) in this case may be the cost per mile of road built in area *i*, where i = 1, 2,…,*n*. So, the objective function would be total cost, $C = C_1x_1 + C_2x_2 +\ldots+ C_nx_n$. Any restrictions on the values that can be assigned to these decision variables are also expressed mathematically, typically by means of inequality or equality. This mathematical restriction is called a constraint function. A constraint function can restrict or reduce the number of alternatives.

The decision variable (x_i), objective function (Z_j), and constraint function are used to represent the decision-making problems by transforming them into a mathematical model. The decision variable is used to differentiate the mathematical model between a continuous and discrete problem. In a continuous problem, the decision variables will be continuous, such as the case where decision variable x_{ij} represents miles of pavement type *i* in area *j* that will be built.

In a discrete problem, the decision maker simply decides which projects are to be chosen. The mathematical model for this type of problem uses the following decision variables:

$$x_i = \begin{cases} 1 & \text{if a project } i \text{ is selected, } i=1,2,...,m \\ 0 & \text{if project } i \text{ is not selected} \end{cases}$$

Each x_i is a binary variable, which has value of 0 or 1. Binary variables are important in mathematical models because they represent a "yes" or "no" decision. In this case, a yes/no decision means project *i* is or is not selected. An example is provided below where minimizing cost is the objective, subject to a set of constraints such as travel time and emissions that are less than some amount or number, ε_k.

The mathematical model can be expressed as follows:

Minimize Cost $$Z = \sum_{i=1}^{m} x_i c_i$$

Subject to,

$$\sum_{i=1}^{m} x_i * T_i \le \varepsilon_T$$

$$\sum_{i=1}^{m} x_i * E_i \leq \varepsilon_E$$

Where, c_i = cost of project *i*

T_i = travel time of project *i*

E_i = emissions from project *i*

where, $x_i = 0$ *or* 1. ε_T *and* ε_E are constants, which are given or acceptable limit of travel time and emissions, respectively.

Step 4: Generate Solutions Using Surrogate Worth Trade-off Method (SWT)

The SWT method is a multi-objective method used to generate a set of solutions and provide a technique that incorporates the decision maker's preferences in choosing the optimal solution. The following tasks are performed in this step:

Construct a Payoff Table

A payoff table (Table 1) consists of all objective values, when each objective is optimized subject to constraints. The first row in the table shows that the result $Z_1(X^1)$ represents the objective values for the first optimization run, X^1, optimizing objective Z_1. This process (optimization run) is repeated for a number of times equal to the number of objectives, Z_1, Z_2,.., Z_p. For example, when the total numbers of objectives are three (Z_1, Z^2, *and* Z^3), the optimization should be run three times to construct a payoff table. X^P refers to the number of optimization runs for each *Z*. The maximum (max) and minimum (min) refer to the optimization runs with the highest and lowest values of *Z*.

The purpose of developing a payoff table is to help formulate the constraint model in the next task by determining the lower and upper bounds for the constraint ε value, such as lower and upper bounds of cost or travel time constraints.

Table 1: Payoff Table

	$Z_1(X^k)$	$Z_2(X^k)$	&	$Z_P(X^k)$
X^1	$Z_1(X^1)$	$Z_2(X^1)$	&	$Z_p(X^1)$
X^2	$Z_1(X^2)$	$Z_2(X^2)$	&	$Z_p(X^2)$
.	.	.	&	.
.	.	.	&	.
.	.	.	&	.
X^p	$Z_1(X^p)$	$Z_2(X^p)$	&	$Z_p(X^p)$
Max	Maximum value of $Z_1(X^k)$	Maximum value of $Z_2(X^k)$		Maximum value of $Z_P(X^k)$
Min	Minimum value of $Z_1(X^k)$	Minimum value of $Z_2(X^k)$		Minimum value of $Z_P(X^k)$

Transform a Multi-Objective Problem into a Single Objective Problem

This task involves considering one objective as primary and transforming other objectives as constraints. The general form of a multi-objective problem with p objectives and m constraints is shown below transformed into a constraint model (Cohon, [4]).

Maximize or minimize $Z_1(X_1, X_2, ..., X_n), Z_2(X_1, X_2, ..., X_n), ..., Z_p(X_1, X_2, ..., X_n)$

Subject to: $c_1\ (X_1, X_2, ..., X_n) \leq 0,$

$$c_2\ (X_1, X_2, ..., X_n) \leq 0,$$

$$..., c_m\ (X_1, X_2, ..., X_n) \leq 0$$

$$X_j \geq 0,\ j = 1, 2, ..., n$$

The primary objective is (Z_h) where the h^{th} objective is chosen arbitrarily for following optimization model:

$$
\begin{aligned}
&\textit{Maximize} \quad Z_h(X_1, X_2, ..., X_n) \\
&\textit{Subject to}: \quad c_1\ (X_1, X_2, ..., X_n)\ \le\ 0, \\
&\qquad c_2\ (X_1, X_2, ..., X_n)\ \le\ 0, \\
&\qquad ..., c_m\ (X_1, X_2, ..., X_n)\ \le\ 0 \\
&\qquad Z_h(X_1, X_2, ..., X_n) = \varepsilon_k \\
&\qquad k = 1, 2, ..., h - 1, h + 1, ..., p \\
&\qquad X_j \ge 0,\ j\ =\ 1, 2, ..., n
\end{aligned}
$$

Choose the Different Values of ε_k from the Range of Minimum and Maximum Values for Each Objective (identified in Step 1)

The minimum and maximum values (for each column representing $Z_1(X^k)$, $Z_2(X^k)$,..., $Z_p(X^k)$ in the payoff table) are derived from the payoff table. Feasible solutions to the constraint model will exist when ε_k is chosen between the minimum and maximum limit. The selection of constraint values, ε_k, between the minimum and maximum limit, ensure that feasible solutions to the constraint problem could be generated. Each solution of the constraint model, with a selected combination of ε_k values between the minimum and maximum limit, will produce a non-dominated solution when all the objective constraints are binding.

Solve the Constraint Model for Every Combination of Values for the ε_k

The mathematical models (constraint model) with every combination of constraint values, ε_k, are solved in this task to generate a set of non-dominated solutions. The model may be solved mathematically or by using commercially available optimization software packages.

Step 5: Trade-off Evaluation

The analyst can leave the selection of final decision from the set of non-dominated solutions to the decision maker. So, from Step 4 one can go directly to step 6. Alternatively, the analyst can pursue Step 5 to guide the decision maker to a solution from the set of non-dominated solutions.

The following tasks are included in this step:

Generate Trade-off Value (λ)

The decision maker's preference from a set of generated solutions is constructed through trade-off evaluation between objectives. The trade-off value, λ_{ij}, is computed as the change in value between a pair of alternatives in objective *i* given changes in value between a pair of alternatives in objective *j*. Z_i^1 means value of objective *i* for alternative 1 and Z_i^2 means value of objective *i* for alternative 2. The following equation illustrates the calculation:

$$\lambda_{ij} = (Z_i^1 - Z_i^2) / (Z_j^1 - Z_j^2) = \Delta Z_i / \Delta Z_j$$

Interact with Decision Maker to Get Worth Value (*W*)

By interacting with the decision maker, the analyst can identify the regression function between the worth, W_{ij} and trade-off value, λ_{ij}. W_{ij} ranges from –10 to +10, where +10 indicates that the decision maker prefers changing the λ_{ij} unit for objective *i*, Z_i, over one unit change of objective *j*, Z_j. However, -10 indicates the opposite extreme - that the decision maker does not prefer the change of λ_{ij} unit of Z_i to one-unit change of Z_j. A zero indicates indifference or signifies equal preference between objective Z_i and Z_j.

The optimum solution would result in average worth values closest to zero, signifying the point at which the decision maker cannot trade between objectives. The worth function can be determined by asking the decision maker a set of questions to determine how much of λ_{ij} he or she has the greatest desire (W_{ij} = +10) to change Z_i by per one unit change of Z_j. The questions would continue for W_{ij} values of +5, -5, and –10 to get the worth regression function.

Step 6: Choose the Preferred Solution

The preferred solution is defined as the solution with an average worth value close to zero (computed in Step 5). This indicates that the decision maker is indifferent to the various objectives.

2.2 An Example Application of the Multi-Objective Framework

The multi-objective decision-making framework discussed above is applied to the Capital Beltway Project (Chowdhury and Tan [2]) in the metropolitan Washington, DC, area to demonstrate the practical application of the methodology to actual transportation projects.

The Capital Beltway corridor is located in the metropolitan Washington, DC, area. It is the only circumferential route in the area, connecting many radial routes. A study conducted by the Maryland State Highway Administration found that the projected high increase in travel demand within the Beltway corridor in the year 2025 requires that both High Occupancy Vehicle (HOV) lanes and rail transit will be needed to handle the projected traffic. This Maryland study recommended that both HOV lane and rail transit alternatives be considered. HOV lanes and rail transit would perform different functions. It would serve different markets within the region and corridor. HOV lanes are added to concurrent lanes by adding one lane in each direction, providing commuters who are willing to carpool or take a bus with one lane on the Beltway that operates without too much congestion. It is concluded that even when rail transit is available, a large percentage of total trips in the corridor would be made by automobile. It is further concluded that the HOV lanes would help to improve travel conditions for HOV users. The Maryland Department of Transportation conducted separate impact studies for the HOV lane and the rail transit alternatives for the study area. The HOV lane corridor is divided into five segments. Rail transit was divided into six different alignments (P_1, P_2, P_3, P_4, P_5, P_6), in which P_1, P_2, and P_3 are aligned for heavy rail and P_4, P_5, and P_6 for light rail transit.

Decision Problem

In this study case, the decision-making problem is solved using the Surrogate Worth Trade-off Method. The best solution has been achieved by combining both HOV lane and rail transit options and deciding how many miles of HOV lane are needed for each segment. Additionally, an alignment is to be selected with the corresponding light or heavy rail transit.

Problem Solving

Step 1:

The identification of objectives is the first step in the decision making process. The objectives for the Capital Beltway Corridor project are as follows:

- Support regional mobility and address travel demand,
- Minimize incremental costs while maximizing transportation capacity.

Step 2:

Measure of Effectiveness uses three different criteria to evaluate each alternative.

- Total costs
- Annual ridership
- Daily new ridership

Each HOV segment and transit alternative is evaluated and measured for their effectiveness. "Annual ridership" represents current ridership without any improvements and "daily new ridership" represents the increase in ridership because of improvements. The measures of effectiveness are shown in Tables 2 and 3. These values are obtained for this case study from a Maryland Department of Transportation Capital Beltway Corridor Transportation study. Ridership values in Tables 2 and 3 are approximations as they are based on the assumption that they are not affected by the HOV length or type of transit selected.

Each of the criteria is measured in terms of cost, annual ridership, and daily new ridership. This case study is basically a combination of a continuous (HOV) and a discrete (rail transit) problem. The goal is to generate how many HOV miles need to be built and decide which rail transit alignment should be chosen.

The rail transit alternatives are based on alignment and rail transit type (light and heavy rail):

- P_1 = heavy rail transit alternative with alignment 1
- P_2 = heavy rail transit alternative with alignment 2
- P_3 = heavy rail transit alternative with alignment 3
- P_4 = light rail transit alternative with alignment 4
- P_5 = light rail transit alternative with alignment 5
- P_6 = light rail transit alternative with alignment 6

Table 2: Measures of Effectiveness for HOV Lanes

Segment	Cost per mile (in millions of $)	Annual Ridership per mile (in thousands)	Daily New Ridership per mile
1	$17	399	1,347
2	$17	25	83
3	$41	297	1,002
4	$43	96	324
5	$24	86	292

Table 3: Measures of Effectiveness for Rail Transit

Total Cost (In Millions of $)

Segment	Alignment					
	P_1	P_2	P_3	P_4	P_5	P_6
1	$2,136	$1,659	$2,136	$786	$857	$786
2	$3,766	$3,019	$3,066	$1,254	$1,257	$1,430
3	$2,599	$2,599	$2,599	$766	$630	$630
4	$1,418	$1,418	$1,418	$470	$423	$423

Annual Ridership

Segment	Alignment					
	P_1	P_2	P_3	P_4	P_5	P_6
1	5,935,403	4,382,770	5,228,753	3,507,785	5,376,469	3,492,959
2	33,810,886	40,724,948	28,604,913	53,625,924	30,363,296	39,989,544
3	11,133,474	10,531,544	10,789,939	10,251,554	15,392,336	12,122,800
4	4,522,762	4,338,398	4,317,583	6,005,887	9,396,844	8,627,636

Daily New Ridership

Segment	Alignment					
	P_1	P_2	P_3	P_4	P_5	P_6
1	10,028	7,401	8,830	5,925	9,085	5,900
2	30,847	32,054	26,090	42,332	27,696	31,822
3	10,156	9,606	9,840	9,360	14,036	11,063
4	7,636	7,325	7,291	10,148	15,870	14,578

Step 3: Develop a Mathematical Formulation. There are two types of decision variables in this model. One, which is continuous, is for the HOV alternative, representing how many miles of road need to be built for each segment. The HOV alternative is divided into five different segments:

X_i = miles of road in segment i, $i = 1,2,\ldots,5$

The second, which is discrete, is for rail transit representing a "yes" or "no" decision (1 = yes and 0 = no) based on six different alignments (P_j with $j = 1,2,...,6$) and type of rail transit; heavy rail (P_1, P_2, P_3) or light rail (P_4, P_5, P_6).

$$P_j = \begin{cases} 1, \text{ if a project } j \text{ is selected,} \\ j = 1,2,...,6 \\ 0, \text{ if a project is not selected.} \end{cases}$$

Three objectives are considered based on the MOEs selected earlier,

1. Minimize Total Cost,

$$Z_1 = \sum_{i=1}^{5} X_i \ C_i + \sum_{j=1}^{6} P_j \ C_j$$

2. Maximize Annual Ridership,

$$Z_2 = \sum_{i=1}^{5} X_i \ A_i + \sum_{j=1}^{6} P_j \ A_j$$

3. Maximize Daily New Ridership,

$$Z_3 = \sum_{i=1}^{5} X_i \ D_i + \sum_{j=1}^{6} P_j \ D_j$$

- C_i = total cost per mile for HOV in segment i
- C_j = total cost for rail transit alternative j
- A_i = annual ridership per mile for HOV in segment i
- A_j = annual ridership for rail transit alternative j
- D_i = daily new ridership per mile for HOV in segment i
- D_j = daily new ridership for rail transit alternative j

Constraints:

- $X_1 \leq 4.0$ miles
- $X_2 \leq 2.6$ miles
- $X_3 \leq 8.6$ miles
- $X_4 \leq 8.3$ miles
- $X_5 \leq 16.6$ miles
- $P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 1$
- $X_i \geq 0$

There is a restriction for P_j (j = 1,2,...,6) where the sum of P_j should equal 1, representing mutually exclusive alternatives (i.e., only one rail transit alternative needs to be chosen). In addition, the total HOV mileage should be less than or equal to the total segment length.

Step 4: Generate a non-dominated solution using the SWT, including the following tasks:

Construct a Payoff Table: The first task is to construct a payoff table (Table 4) by optimizing each of the three objectives separately (cost, annual ridership, daily new ridership) to obtain maximum or minimum values.

Table 4: Payoff Table of Capital Beltway Corridor

		Z_1	Z_2	Z_3
Min Cost	X^1	$3,167,000,000	60,528,945	66,687
Max Annual Ridership	X^2	$4,496,100,000	79,830,750	89,522.40
Max Daily New Ridership	X^3	$4,496,100,000	79,830,750	89,522.40

Formulate a Constraint Model: The second task is to transform a multi-objective problem into a single objective problem using the constraint method. In this case, the maximized daily new ridership objective (Z_3) is chosen as a primary objective and all other objectives (Z_1 and Z_2) are transformed as constraints, as shown below:

Daily New Ridership,

$$Z_3 = \sum_{i=1}^{5} X_i D_i + \sum_{j=1}^{6} P_j D_j$$

Subject to Constraints:

$$Z_1 = \sum_{i=1}^{5} X_i C_i + \sum_{j=1}^{6} P_j C_j \leq L_1$$

$$Z_2 = \sum_{i=1}^{5} X_i A_i + \sum_{j=1}^{6} P_j A_j \leq L_2$$

$X_1 \leq 4.0$ *miles*

$X_2 \leq 2.6$ *miles*

$X_3 \leq 8.6$ *miles*

$X_4 \leq 8.3$ *miles*

$X_5 \leq 16.6$ *miles*

$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 1$

$X_i \geq 0$

Choose Constraint Values: The *L* values (Table 5) are chosen arbitrarily from different values of the range of minimum and maximum for objectives 1 and 2 from the payoff table (Table 4). By choosing *L* between the minimum and maximum values, the feasible solutions for the above constraint problem can be generated. Each solution of the constraint model, with a selected combination of *L* values between the minimum and maximum limit, will produce a non-dominated solution when all the objective constraints are binding. The algorithm given by Mishra and Harit [15], is used for the selection of non-dominated solutions in multi-objective optimization.

Table 5: Constraint Values of Capital Beltway Corridor

Constraint	Selected Constraint Values		
L_1	\$3,167,000,000	\$3,831,500,000	\$4,496,100,000
L_2	60,528,945	70,179,800	79,830,750

Derive Solutions from the Model: The final task is to solve the constraint problem by maximizing the daily new ridership subject to all constraints for every combination of values for L_1 and L_2. The optimization process shows that $P_j = 1$ when project *J* is selected. If the result shows $P_j = 0$, project *J* is not selected. Seven project alternatives (Y_1, Y_2, Y_3, Y_4, Y_5, Y_6 and Y_7) exist as combinations of HOV and rail transit projects by solving the constraint model presented in this step. Using commercially available optimization software packages, such as TORA software, may solve the model. The combinations are shown in Table 6.

Table 6: Generated Alternatives and Associated Decision Variables from the Solved Constraint Model

Project Selection	**HOV**					**Rail Transit**
	X_1	X_2	X_3	X_4	X_5	
Y_1	0.00	0.00	0.00	0.00	0.00	P_5
Y_2	4.00	0.00	8.60	0.00	10.16	P_5
Y_3	4.00	0.00	8.60	0.00	5.91	P_6
Y_4	4.00	0.00	8.60	0.00	5.62	P_4
Y_5	4.00	2.60	8.60	8.30	16.60	P_5
Y_6	4.00	2.60	8.60	8.30	16.60	P_6
Y_7	4.00	2.60	8.60	8.30	16.60	P_4

In Table 6, Y_1 represents the result of the optimization where HOV decision variable values of X_1, X_2, X_3, X_4, and X_5 equaled 0, 0, 0, 0, and 0 miles, respectively and the rail transit decision variable values of P_1, P_2, P_3, P_4, P_5, and P_6 equaled 0, 0, 0, 0, 1, and 0, respectively.

Based on the decision variable values generated in Table 6, the objective values (Z_1, Z_2 and Z_3) for each alternative are shown in Table 7.

Table 7: Objective Value of Generated Non-Dominated Solution of Capital Beltway Corridor

	$Z_{1(min\ cost)}$	$Z_{2(max\ annual\ ridership)}$	$Z_{3(max\ daily\ new\ ridership)}$
Y_1	3,167,000,000	60,528,945	66,687
Y_2	3,831,440,000	65,552,905	83,659.65
Y_3	3,831,440,000	68,891,399	79,094.65
Y_4	3,831,480,000	78,024,670	83,411.48
Y_5	4,387,100,000	66,968,545	88,444.4
Y_6	4,489,100,000	70,672,539	85,120.4
Y_7	4,496,100,00	79,830,750	89,522.4

Step 5:

Trade-off analysis as part of Surrogate Worth Trade-off Method involves the following tasks:

Generate Trade-off Value

As part of the SWT method, a trade-off evaluation between objectives is needed to choose the preferred solution among the seven generated alternatives (Y_1, Y_2,…,Y_7). These values are based on the objective values from generated alternatives shown in Table 7. The analyst presents the trade-off values between different alternatives to help the decision maker select an alternative. The trade-off value is obtained from the Daily new ridership divided by the difference of the other objective value (Table 8). The value can be negative or positive. A positive value means that by increasing the primary objective, in this case daily new ridership, the other objective will increase. A negative value means that by increasing the primary objective, the other objective will decrease.

Table 8: Trade-off Value of Capital Beltway Corridor

Project	$\lambda_{3,1}$**/10^5**	$\lambda_{3,2}$**/10^3**
Y_2/Y_1	2.5	3.3
Y_3/Y_1	1.8	1.4
Y_4/Y_1	2.5	9.5
Y_5/Y_1	1.7	3.3
Y_6/Y_1	1.4	1.8
Y_7/Y_1	1.7	1.2
Y_2/Y_3	~	-1.3
Y_2/Y_4	620.42	-0.01
Y_2/Y_5	0.86	3.3
Y_2/Y_6	0.22	0.28
Y_2/Y_7	0.88	0.41
Y_3/Y_4	10792.07	0.47
Y_3/Y_5	-1.6	-4.8
Y_3/Y_6	0.91	3.3
Y_3/Y_7	1.5	0.95
Y_4/Y_5	-0.9	-0.45
Y_4/Y_6	-0.25	-0.23
Y_4/Y_7	0.91	3.3
Y_5/Y_6	3.2	-0.89
Y_5/Y_7	0.98	0.08
Y_6/Y_7	62.8	0.48

In the Table 8, the trade-off value for project Y_2/Y_1, if the daily new ridership (Z_3) increases by 3.3, the annual ridership (Z_2) will also increase to 1,000 riders. For project Y_2/Y_3, if the daily new ridership (Z_3) increases by 1.3, the annual ridership (Z_2) will decrease to 1,000 riders. The ~ Symbol means that a trade-off between two objectives is not influenced by another particular objective.

Interact with Decision Maker to Get Worth Values (W_{ij})

A regression analysis is done to develop the relationship between objective trade-off value and worth values based on decision maker opinion, which is facilitated by question-and-answer process between the analysis and decision maker. The Values for the $W_{3,1}$ and $W_{3,2}$ which are shown in Table 13 can be find by putting the values of $\lambda_{3,1}$ and $\lambda_{3,2}$ (Table 8) in regression lines.

Table 9

$W_{3,1}$	$\lambda_{3,1}$
10	-1
5	-0.5
-5	1
-10	0.5

Table 10

$W_{3,2}$	$\lambda_{3,2}$
10	2
5	1.6
-5	-0.6
-10	-0.01

Regression line can be formulated using the following equation:

$$Y = a + b_1 X + b_2 X^2$$

Evaluate the following equations using principle of least square on above regression line:

$$\sum y_i = na + b_1 \sum x_i + b_2 \sum x_i^2$$

$$\sum x_i y_i = a \sum x_i + b_1 \sum x_i^2 + b_2 \sum x_i^3$$

$$\sum x_i^2 y_i = a \sum x_i^2 + b_1 \sum x_i^3 + b_2 \sum x_i^4$$

For given set of points (x_i , y_i) *:* $i = 1,2,\ldots,n$, above equations can be solved for a, b_1 and b_2 , and with these values of a, b_1 and b_2 in regression line, it is the parabola of best fit.

By taking worth values shown in table 9 and table 10 as *Y* variable and trade-off values shown in table 9 and 10 as *X* variable, evaluate the regression line with these values of *X* and *Y* variables.

Table 11

X	***Y***	X^2	X^3	X^4	$X^2 Y$	$X Y$
-1	10	1	-1	1	10	-10
-0.5	5	0.25	-0.125	0.0625	1.25	-2.5
1	-5	1	1	1	-5	-5
0.5	-10	0.25	0.125	0.0625	-2.5	-5
$\sum x_i = 0$	$\sum y_i = 0$	$\sum x_i^2 =$ 2.5	$\sum x_i^3 = 0$	$\sum x_i^4 =$ 2.125	$\sum x_i^2 y_i =$ 3.75	$\sum x_i y_i =$ - 22.5

$$4a + b_1 (0) + b_2 (2.5) = 0$$

$$a(0) + b_1 (2.5) + b_2 (0) = -22.5$$

$$a(2.5) + b_1 (0) + b_2 (2.125) = 3.75$$

On solving these equations the values for a, b_1 and b_2 are:

$$a = -4.166 ,\ b_1 = -9 ,\ b_2 = 6.666$$

Finally, the regression line for table 9 is:

$$\boldsymbol{W_{3,1} = 6.7\ (\lambda_{3,1})^2 - 9(\lambda_{3,1}) - 4.1667}$$

Table 12

X	***Y***	***X²***	***X³***	***X⁴***	***X² Y***	***X Y***
2	10	4	8	16	40	20
1.6	5	2.56	4.096	6.5536	12.8	8
-0.6	-5	0.36	-0.216	0.1296	-1.8	3
-0.01	-10	0.0001	- .000001	0.00000001	-0.001	0.1
$\sum x_i =$ 2.99	$\sum y_i = 0$	$\sum x_i^2 =$ 6.9201	$\sum x_i^3 =$ 11.8799	$\sum x_i^4 =$ 22.68	$\sum x_i^2 y_i =$ 50.999	$\sum x_i y_i =$ 31.1

$$4a + b_1(2.99) + b_2(6.9201) = 0$$

$$a(2.99) + b_1(6.9201) + b_2(11.879) = 31.1$$

$$a(6.9201) + b_1(11.879) + b_2(22.683) = 50.999$$

On solving these equations the values for a, b_1 and b_2 are:

$$a = -8.810,\ b_1 = -1.72,\ b_2 = 5.830$$

Finally, the regression line for table 10 is:

$$W_{3,2} = 5.84(\lambda_{3,2})^2 - 1.72(\lambda_{3,2}) - 8.8$$

Step 6:

The final stage of this decision scenario is to select the preferred solution based on alternatives with an average value of decision maker weight, *W*, closest to zero. Table 13 shows that alternatives Y_1 and Y_3 have a trade-off value closest to zero, 0.78. As a result, the decision maker can select either Y_1 or Y_3 based on preference. Since, the best solution is needed as the combination of both HOV lane and rail transit options and deciding how many miles of HOV lane are needed for each segment. Therefore, Y_3 is selected as best alternative.

Table 13: Worth Value for Capital Beltway Corridor

Project	$W_{3,1}$	$W_{3,2}$	Average
Y_2/Y_1	15.2083	49.1216	32.16495
Y_3/Y_1	1.3413	0.2384	0.78985
Y_4/Y_1	15.2083	501.92	258.5642
Y_5/Y_1	-0.1037	49.1216	24.50895
Y_6/Y_1	-3.6347	7.0256	1.69545
Y_7/Y_1	-0.1037	-2.4544	-1.27905
Y_2/Y_3	~	3.3056	3.3056
Y_2/Y_4	2573424	-8.78222	1286708
Y_2/Y_5	-6.95138	49.1216	21.08511
Y_2/Y_6	-5.82242	-8.82374	-7.32308
Y_2/Y_7	-6.89822	-8.5235	-7.71086
Y_3/Y_4	7.8E+08	-8.31834	3.9E+08

Y_3/Y_5	27.3853	134.0096	80.69745
Y_3/Y_6	-6.80843	49.1216	21.15659
Y_3/Y_7	-2.5917	-5.1634	-3.87755
Y_4/Y_5	9.3603	-6.8434	1.25845
Y_4/Y_6	-1.49795	-8.09546	-4.79671
Y_4/Y_7	-6.80843	49.1216	21.15659
Y_5/Y_6	35.6413	-2.64334	16.49898
Y_5/Y_7	-6.55202	-8.90022	-7.72612
Y_6/Y_7	25854.36	-8.28006	12923.04

2.3 Conclusions

Multi-objective decision making framework for transportation investment considered by Chowdhury and Tan [2] is reviewed by reducing the problem into three objectives and applied with the data generated by them for Beltway Corridor project. Most decision-making in transportation agencies involves multiple objectives that often conflict and cannot be measured in monetary units. This makes the use of traditional investment analysis tools, such as benefit-cost analysis, difficult. This study presented a multi-objective framework that could be applied under different decision scenarios in transportation investment processes. Instead of transforming all different project alternatives or objectives into monetary values, these alternatives or objectives can be approached on an equal basis in their own measures of effectiveness, either in monetary or non-monetary terms. The proposed framework permits objective decision analysis for any transportation investment.

The proposed framework addresses both discrete and continuous decision problems and a combination of the two. This makes the proposed framework applicable to a wide range of decisions that are required in transportation investment scenarios.

The application of the proposed framework in the Capital Beltway Corridor investment study demonstrated the suitability of the methodology. The framework presented in this study could be considered by public agencies as an alternative or complement to traditional economic analysis and integrated with agency funding processes and management systems. The proposed decision-analysis framework, which is general in nature, could also be applied to other transportation areas, such as aviation, rail, and water.

The approaches could lead to new practices for decision analysis and decision-making in the transportation industry. They allow interaction between the analyst and the decision maker in the selection of final projects, enhancing objective project selection.

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