A MODIFIED APPROACH FOR BI-CRITERION CALL CENTER PROBLEM

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Under

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DEDICATED

TO

GOD, MY PARENTS

AND

SUPERVISOR

CERTIFICATE

*I hereby certify that the work which is being presented in the thesis entitled "**A Modified Approach for Bi-criterion Call Center Problem**" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Mahesh Kumar Sharma.*

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.

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ABSTRACT

The present work deals with the problem of selecting sites for setup of call centers from among a given number of potential sites for assigning a given number of zones of call origin to them. The two objectives without being prioritized with several constraints are considered. The objectives are to minimize the total call charges and total call centers setup costs with the several constraints that each zone is assigned to a unique site at which a call center is setup and call centers at different potential sites have different capacities. There is no restriction in the number of zones to be assigned to a call center as long as its capacity is not exceeded.

The present thesis consists of three chapters. Chapter 1 is introducing in nature in which the concept of multi-objective optimization has been discussed. Also the literature related to the work presented in this chapter. In chapter 2, a bi-criterion call center problem through an evolved branch and bound algorithm (Prakash *et. al.* (2011)) has been reviewed. In chapter 3, a simple procedure to find efficient solutions has been developed by modifying the existing approach.

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CHAPTER 1

"INTRODUCTION"

1 INTRODUCTION

The single-objective model was the first to be developed and thus it was received considerably more exposure, been put to more use, and is generally considered to be a relatively high level of refinement. Thus, the implication is simple, well-tested tool is available and we may be inclined to fit the problem to this model despite the assumptions required. But in real life there are many problems with more than one objective for which the multi-objective models are required.

Dantzig's initial concept was centered about the development of a linear programming model but with a single objective. This so set the tone for the development of traditional linear programming that many (if not most) linear programming texts completely ignore even the possibility of more than one objective. Unlike the traditional single objective optimization problem wherein it is settle on optimizing single objective function, there is no single universally accepted approach for solving the multi-objective optimization problems due to usually conflicting nature of objective functions leading to the situation where the optimization of one of these may adversely affect the optimization of others. So in the case of multi-objective optimization problems, there is no need to access the decision maker's utility function that may vary from decision maker to decision maker.

The problem of selecting sites for setup of call centers from among a given number of potential sites for assigning a given number of zones of call origin to them subject to several constraints with more than one objective is a real life problem. As the name suggests, call centers are offices assigned to telephonic contact with customers. An official definition says: Call centers are "tools for organizing communication with customers with the help of telecommunication". It constitutes of a set off resources- typically personnel, computers, and telecommunication equipment which enable the delivery of services via the telephone. They are typically assisted by information about organization products, services and customer information from database and intranets.

1.1 MULTI OBJECTIVE OPTIMIZATION

Multi-objective optimization is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints also known as **multi-criteria** or **multi-attribute optimization**.

Multi-objective optimization can be defined as:

"a vector of decision variables which satisfies constraints and optimizes a vector function whose element represent the objective functions. Hence the term "optimizes" means finding the solution which would give the values of all the objective functions acceptable to the decision maker."

This model can be formulated as:

$$\begin{aligned} &\text{optimize } f(X) = (f_1(X), f_2(X), ..., f_k(X)) \\ &\text{subject to } g_j(X) \leq=\geq b_j,\ j = 1, 2, ..., m \\ &\qquad X \geq 0 \\ &\qquad X = (x_1, x_2, ..., x_n)^T \end{aligned}$$

Where, $f(X)$ is the objective function to optimize. $(f_1(X), f_2(X), ..., f_k(X))$ are k number of distinct objective functions subject to m constraints. X is a vector consists of decision making variables $x_1, x_2, ..., x_n$.

Multi-objective optimization problems can be found in various fields: product and process design, finance, aircraft design, the oil and gas industry, automobile design, or wherever optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives. Maximizing profit and minimizing the cost of a product; maximizing performance and minimizing fuel consumption of a vehicle; and minimizing weight while maximizing the strength of a particular component are examples of multi-objective optimization problems.

1.2 CONCEPT OF OPTIMAL AND EFFICIENT SOLUTIONS

1.2.1 Optimal Solution

An optimal solution in the classical sense is one which attains the maximum value of all the objectives simultaneously. The solution x^* is optimal to the problem defined if and only if $x^* \in S$ and $f_l(x^*) \geq f_l(x)$ for all l and for all $x \in S$, where S is the feasible region.

In general, there is no optimal solution to a multi-objective problem. Therefore, optimality replaced by the concept of “satisficing” or the best compromise solution, which depends on the decision makers preferences with respect to the object. Optimality is not illusion only when the objectives are non-conflicting. Therefore, one must be satisfied with obtaining efficient solutions in multi-objective problem.

1.2.2 EFFICIENT OR NON-DOMINATED SOLUTIONS

A tentative solution is called non-dominated, Pareto optimal, or Pareto efficient if it cannot be eliminated from consideration by replacing it with another solution which improves an objective without worsening another one. Finding such non-dominated solutions, and quantifying the trade-offs in satisfying the different objectives, is the goal when setting up and solving a multi-objective optimization problem and mathematically can be stated as

A set of solutions is said to be efficient if there exist no solution that is superior to it with respect to at least one objective function but is not inferior to it with respect to any of the objective functions.

If x_1 and x_2 are two solutions, then these can have any of two possibilities-one dominates the other or non-dominates the other. In a minimization problem, without the loss of generality, a solution x_1 dominates x_2 if and only if the following conditions are satisfied:

$$\forall\, i \in \{1, 2, ..., N_{obj}\} : f_i(x_1) \le f_i(x_2)$$
$$\exists\, j \in \{1, 2, ..., N_{obj}\} : f_j(x_1) < f_j(x_2)$$

Where, $f(x_1)$ and $f(x_2)$ are the objective functions.

If any of the above conditions is violated, the solution x_1 does not dominate the solution x_2. If x_1 dominates the solution x_2, x_1 is called the non-dominated solution with in the set $\{x_1, x_2\}$. The solutions that are non-dominated within the entire search space are denoted as Pareto-optimal and constitute the Pareto-optimal set or Pareto-optimal front. From the entire set off efficient (non-dominated) solutions the decision maker can select the solution one believed most attractive.

EXAMPLE OF EFFICIENT SOLUTION

MODES OF TRAVEL

	1	2	3	4	5	6	7	8
Travel cost	300	300	1400	1200	3000	3500	5000	5500
Travel time	30	35	18	17	3	4	2	2

1,4,5,7 are efficient solutions

1.3 BRANCH AND BOUND ALGORITHM

Branch and bound algorithms are a variety of adaptive partition strategies have been proposed to solve global optimization models. These are based upon the partition, sampling, and subsequent lower and upper bounding procedures, these operations are applied iteratively to the collection of active ("candidate") subsets within the feasible set. Their exhaustive search feature is guaranteed in similar spirit to the analogous integer linear programming methodology. Branch and bound subsumes many specific approaches, and allows for a variety of implementations. Branch and bound methods typically rely on some a priori structural knowledge about the problem.

The branch and bound algorithm is evolved to find the set of efficient solution of the problems. The general branch and bound methodology is applicable to broad classes of global optimization problems, e.g., in integer programming, combinatorial optimization, concave minimization, bi-criterion bulk transportation problem. Branch & bound algorithm in our life is very important to find set of efficient solutions.

1.4 LITERATURE REVIEW

The present work deals with multi-objective optimization problems. Multi-objective problems differ from single-objective problems in the sense that the former problems have more than one objective whereas the latter problems have only one objective. There are many approaches for solving multi-objective optimization problems, whereas there is appropriate approach seeking to optimize the multi-objective functions. The various approaches for solving multi-objective optimization problems are lexicographic / prioritized / and Pareto optimal / efficient / non-dominated solution approach. A discussion about them can be found in the works of Zeenlay (1974), Prakash (1981), Igznio (1982), Sharma and Prakash (1986), Steuer (1986), Prakash, Aggarwal and Shah (1988), Prakash and Pradeep (1991), Prakash *et. al.* (1999), Taha(2008), Prakash and Gupta (2006).

Krarup and Pruzan (1983) presented a simple plant location problem. Efroymson and Ray (1966) proposed a formulation of the un-capacitated plant location problem and the use of branch and bound algorithm to solve it. Khumawala (1972) utilized the special structure of UFLP to improve the branch and bound algorithm of Efroymson and Ray (1966). Bilde and Krarup (1977) and Erlenkotter (1978) developed a dual-based branch and bound procedure for the problem and this procedure has been widely accepted as an efficient known procedure. So, one of the main results has been the development of linear programming-based branch and bound algorithms. The standard reference in this area is the algorithm of Erlenkotter (1978), a branch and bound algorithm based on dual descent which is an efficient way to solve the un-capacitated warehouse location problem.

The capacitated warehouse problem consists of the well known transportation problem with the additional feature of affixed charge associated with each warehouse which is put to use. The problem is usually solved as a special type of mixed integer programme, so that relaxation and lower bounding are a vital part of any algorithm. A deeper insight into the relaxation process may eventually lead to more efficient algorithms for the problem. Baker (1982) has shown that the LP relaxation of the capacitated warehouse location problem can incorporate constraints of a much more general nature than those previously described.

Kelly *et. al.* (1982) presented an algorithm for finding a minimal cost warehouse system design wherein individual warehouse have limited capacities and exhibit economies of scale because earlier the mixed integer-linear models that have been used in most analysis of warehouse location problems fail to capture the potential operating efficiencies associated with large scale facilities. The iterative procedure defines and solves a series of conventional transportation problems in order to converge on the optimal system design.

Prakash and Om (1996) has developed an algorithm to obtain the set of non-dominated solutions of the two-objective problem of determining non-dominated programmes for augmentation of capacities of depots and shipment of buses from them to the starting points of routes along with determining the capacity in reserve and the number of buses to be parked at the respective depots after the augmentation of their capacities is considered with two objectives. One objective is to minimize the present value of the total cost of dead-travelling to be performed by buses between the depots and the starting points of their routes over a planning horizon plus the capital expenditure to be incurred in augmenting capacities of the depots. The other objective is to minimize the maximum distance among the distances traversed by individual buses from depots to the starting points of their respective routes. The two objectives are not accorded priorities.

A detailed discussion about efficient solution can be found in works of Ignizio (1982) and Steuer (1986). Some algorithms are described for solving plant location problems with non-linear warehousing costs. The heuristic procedures are flexible with respect to the type of warehousing cost structure permitted, and may be used to solve fixed *p*-median location problems as well as problems in which the numbers and locations of warehouses in solution are jointly determined as a trade-off between transportation and fixed and operating plant costs. Computational experience is reported by Whitaker (1985) on some well known problem sets in which the economies of scale in production are continuously concave. Comparisons with other solution methods indicate that the proposed procedures perform as well as, or better than any currently known on these standard test problems.

Several exact algorithms based on branch and bound have been proposed. The major differences among these algorithms are of the methods of solving the strategies to improve the lower bound. A discussion about branch and bound algorithm can be found in the works of Rao (1996), Rardin

(2002), Kasana and Kumar (2003), Pant (2004), Bronson and Naadimuthu (2004), Natarajan, Balasubramani and Tamilarasi (2006), Sharma (2007) from the literature.

Brigandi, Dargon and Sheenan (1990) have been considered many facility location problems relating to selection of sites for call centers of telemarketing customers of AT&T.

Ignizio and Cavalier (1994) have considered the problem of selecting upto a fixed number of sites from among a given number of potential sites for locating warehouses at them and clustering customers to the selected sites in such a way that each customer is assigned to a unique selected site. The single objective of this problem was to minimize the sum of distances from each customer to his/her assigned site. Praveena *et al.* (1999) have extended this problem which selects upto a fixed number of sites from among a given number of potential sites for warehouse for clustering ration shops to them subject to several constraints with two objectives. The two objectives are to minimize the total cost and duration of meeting the requirements of all the ration shops from their assigned warehouses at the selected sites. These two objectives are not accorded priorities. One constraint is that each ration shop should be clustered to a unique site, which is selected for locating a warehouse at it but there is no restriction on the number of ration shops to be clustered to a warehouse at the selected site. Other constraint is that the set up cost of the warehouses should not exceed a certain budgetary amount. This problem has been solved by finding the set of efficient solutions of it using heuristic method consisting of a combination of add and drop rules. Prakash *et al.* (2009) has developed a heuristic iterative algorithm incorporating Tabu search to find the set of efficient solutions of this problem.

Prakash and Aggarwal (1992) considered the problem of establishing a route between two specified nodes through a network with two objectives-one primary and another secondary is considered. The primary objective is to minimize the total cost of travel and the secondary objective is to minimize the duration of travel.

Prakash *et. at*. (1999) has developed an algorithm to obtain the set of non-dominated solutions of the two-objective problem. The problem of determining non-dominated schedules to take buses from depots to the starting points of their routes, spare capacity and the number of buses to be parked overnight at each depot is considered with two objectives without assigning priorities to them. The two objectives are to minimize the cumulative distance covered by all the buses and the maximum distance among the distances covered by individual buses from the depots to the starting points of their respective routes.

Prakash *et. al.* (2011) proposed that a cost-time trade-off bulk transportation problem with the objectives to minimize the total cost and duration of bulk transportation without according priorities to them is considered. The entire requirement of each destination is to be met from one source only, however a source can supply to any number of destinations subject to the availability of the commodity at it. Two new algorithms are provided to obtain the set of Pareto optimal solutions of this problem. This work extends and generalizes the work related to single-objective and prioritized two-objective bulk transportation problems while providing flexibility in decision making.

Prakash *et. al.* (2011) proposed that the problem of selecting sites for setup of call centers from among a given number of potential sites for assigning a given number of zones of call origin to them subject to several constraints with two objectives without being prioritized is considered. The constraints are that each zone is assigned to a unique site at which a call center is setup. Call centers at different potential sites have different capacities. Capacity of a call center at a site means the number of calls which can be handled at it. There is no restriction on the number of zones to be assigned to a call center as long as its capacity is not exceeded. The number of sites to be selected for setup of call centers is left open to choice depending on the need. The two objectives are to minimize the total call charges and total call charges and total call center setup costs. A branch and bound algorithm is evolved to find the set of efficient solutions for this problem.

1.5 PRESENT WORK

Call center is an office where a company's inbound calls are received or outbound calls are made. Call centers are increasingly popular in modern society, in which many companies have centralized customer service and support functions. Call center employ many staff members in customer service, sales and support functions. Call centers outsource the work of organizations through promoting their products, answering queries of the people at large, taking orders, recording complaints of customers for passing these to their respective organizations for action, and host of other types of work. The scope of the work of the call centers in the future is also likely to include providing knowledge about the processing of the products and many other types of work in addition to the existing work now being done Prakash *et. al*. (2011).

The present work is based on bi-criteria call center problem. Thesis contains three chapters. Chapter one is introducing in nature. In the second chapter a bi-criteria call center problem through an evolved branch and bound algorithm (Prakash *et. al.* (2011)) has been reviewed. In chapter three, a simple procedure to find efficient solutions has been developed by modifying the existing approach.

CHAPTER – 2

"A BI-CRITERION CALL CENTER PROBLEM THROUGH AN EVOLVED BRANCH AND BOUND ALGORITHM"

2.1 INTRODUCTION

Call center industry employs hundreds of thousands of people all over the world and is growing very rapidly. A call center is an office where people work using telephones and computers. Call centers outsource the work of organizations through promoting their products, answering queries of the people at large, taking orders, recording complaints of customers for passing these to their respective organizations for action and host of other types of work. The scope of the work of the call centers in the future is also likely to include providing knowledge about the processing of the products and many other types of work now being done.

The call centers handled telephone reservations and orders arising from many geographic zones. Since the telephone rates are dependent on the zone of call origin and the location of receiving center, site selection plays an important role in minimizing the total of call charges and call center setup costs. This is clarified and illustrated through an example by Rardin (2002). He has formulated and solved the problem of selection of sites for opening call centers from among a given number of potential sites for assigning zones of call origin to them subject to some constraints with a single objective. The constraints are that the calls originated from each zone are fully handled at one or more than one call center. Capacity of each call center lies within a specified range. Capacity of a call center is the number of calls which can be handled at it. The single objective function is to minimize the sum of total call charges and total call center setup costs.

Prakash *et. al.* (2011) modified and extended the work to a new situation leading a new problem. The new problem selects sites for setup of call centers and assigns zones of call origin to them subject to modified constraints with two objectives. The constraints are that each zone is assigned to a unique site at which a call center is setup wherein it can be assigned to one or more than one site. Capacity of each call center has an upper bound. There is no restriction on the number of zones to be assigned to a call center. The new problem has two objectives seeking to minimize the recurring total call charges cost and nonrecurring total call center setup cost. The two objectives are not prioritized.

A branch and bound algorithm is evolved to find out the set of efficient solutions of this problem. The evolved branch and bound algorithm is altogether new in its approach. Herein the sequences of bi-criterion prioritized problem which yield efficient solution of the formulated problem are not reduced to a single-objective problem but are treated as such. This results in reducing the computational work considerably and making the implementation of the algorithm easy. In the past, a branch and bound algorithm was developed to solve a bi-criterion bulk transportation problem with prioritized objective by Prakash and Ram (1995) and thereafter to solve this very problem without the objectives being prioritized by Prakash, Kumar, Prasad and Gupta (2008). In both the problems, the bi-criterion prioritized problems are reduced to single-objective problems through the introduction of preemptive priority factors before applying the branch and bound algorithm. A bi-criterion bulk transportation problem is also solved through an alternative branch and bound algorithm which focuses on the first priority objective keeping the second priority objective in abeyance and finds all the alternative optimal solutions of the single objective problem. Among all the alternative optimal solutions, that one is picked up for an efficient solution of the bi-criterion problem for which the second priority objective is minimum. There exists no other branch and bound algorithm other than the above mentioned ones to solve a bi-criterion problem. The process of reducing the bi-criterion prioritized problems into single-objective ones through the introduction of the preemptive priority factors or finding all alternative optimal solutions keeping the second priority objective in abeyance and picking up the one among them, involves a lot of computational work and time which the new algorithm dispenses with.

2.2 FORMULATION OF THE PROBLEM (Prakash *et. al.* (2011)

Suppose that there are m zones of call origin and n potential sites for setup of call centers. Selection of sites for setup of call centers from among a given number of potential sites for assigning a given number of zones subject to several constraints with two objectives is required. Each zone is assigned to a unique site at which a call center is setup. Call centers at different sites have different capacities. Capacity of a call center at a site means the number of calls which can be handled at it. There is no restriction on the number of zones to be assigned to a call center so long as its capacity is not exceeded. The number of sites to be selected for setup of call centers is left open to choice depending on the need. Let a_i be the number of bulk calls originated from zone Z_i $(i=1,2,...,m)$, b_j be the capacity of the call center at the potential site P_j $(j=1,2,...,n)$, c_{ij} be the cost of handling bulk calls originated from zone Z_i at the site P_j, s_j be the setup cost of the call center at the site P_j. Let x_{ij} be the variable assuming the value 0 or 1 according as the zone Z_i is not assigned or assigned to the potential site P_j and y_j be the variable assuming the value 0 or 1 according as the potential site P_j is not selected or selected for setup of a call center at it. Let $C(\bar{x})$ and $S(\bar{y})$ denote the recurring total call charges cost spread over an interval of time period of handling bulk calls from the zones at the call centers and the nonrecurring one time total call centers setup cost at the selected sites respectively. The two objective functions which are sought to be minimized are

$$C(\bar{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \qquad (2.1)$$

$$S(\bar{y}) = \sum_{j=1}^{n} s_j y_j \qquad (2.2)$$

without being prioritized subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad (i=1,...,m) \qquad (2.3)$$

$$x_{ij} - y_j \le 0 \qquad (i=1,...,m;\ j=1,...,n) \qquad (2.4)$$

$$\sum_{i=1}^{m} a_i x_{ij} \le b_j \qquad (j=1,...,n) \qquad (2.5)$$

$$x_{ij} = 0 \text{ or } 1;\quad y_j = 0 \text{ or } 1 \qquad (i=1,...,m;\ j=1,...,n) \qquad (2.6)$$

The constraints (2.3) ensure that each zone is assigned to a unique site while the constraints (2.4) ensure that the site to which a zone is assigned is the site selected for setup of a call center. The constraints (2.5) ensure that the sum of bulk calls from the zones assigned to a selected call center site does not exceed its capacity. It is required to find the set of efficient solutions of the problem given by (2.1) - (2.6). A solution $(\bar{X},\bar{Y})$ is efficient if no solution $(\bar{x},\bar{y})$ exists satisfying the conditions:

$(i)\ C(\bar{X}) \ge C(\bar{x})$ *and* $(ii)\ S(\bar{Y}) \ge S(\bar{y})$

with strict inequality holding in at least one of the conditions out of (i) and (ii). Here $\bar{x} = (x_{ij} : i = 1,...,m; j = 1,...,n)$ and $\bar{y} = (y_j : j = 1,...,n)$. For the purpose of listing the Efficient Solutions, an efficient solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ shall be called the 1st efficient solution of the formulated problem if it is the optimal solution with the minimization of $C(\bar{x})$ and $S(\bar{y})$ as the 1st and 2nd priority objectives respectively. An efficient solution shall be called the 2nd efficient solution if no efficient solution $(\bar{X}, \bar{Y})$ of the problem exists satisfying the conditions

$$(i)\ C(\bar{X}^{(1)}) < C(\bar{X}) < C(\bar{X}^{(2)}) \quad and \quad (ii)\ S(\bar{Y}^{(1)}) > S(\bar{Y}) > S(\bar{Y}^{(2)})$$

The 3rd and the subsequent efficient solutions are defined in the same way as is done for the 2nd efficient solution.

2.3 SOLUTION PROCEDURE

Prakash *et. al.* (2011) proposed a procedure for obtaining the set of efficient solution of the problem (2.1) – (2.6) which require a sequence of prioritized bi-criterion problem to be solved. The bi-criterion call center problem formulated above in section 2.2 is a binary integer problem, because a variables x_{ij}'s and y_j's are binary integers. A branch and bound algorithm is evolved for solving the sequence of prioritized bi-criterion problem has been explained in detail and given below.

The total number of the bi-criterion problems to be solved for obtaining the set of efficient solutions is only one more than the total number of the efficient solutions of the problem. Procedures for obtaining the 1st, 2nd and subsequent efficient solutions are explained below.

2.3.1 PROCEDURE FOR OBTAINING 1st EFFICIENT SOLUTION

The 1st efficient solution of the formulated bi criterion call center problem is the solution of the problem with the first and second priorities to the minimization of $C(\bar{x})$ provided by (2.1) and $S(\bar{y})$ provided by (2.2) respectively subject to constraints (2.3) – (2.6). This prioritized problem is designated as the 1st prioritized problem of the formulated problem. Its optimal solution would yield the 1st efficient solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ of the formulated problem.

For solving the 1st prioritized bi criterion problem, its tableau representation is required as shown in table 2.1. In this table, $c_{ij}\ (i=1,2,...,m;\ j=1,...,n)$ depicts the recurring handling call charges cost of the bulk calls originated from zone Z_i at potential site P_j. Since the number of zones is m and the bulk calls originated from each zone are handled at a unique call center setup at a selected site, the set consisting the number m of x_{ij}'s at level 1 each with a different value of subscript i and the y_j's at level 1 with subscripts j's in the x_{ij}'s at level 1, provide an optimal solution of the 1st prioritized problem if the constraints (2.3) - (2.6) are satisfied with the 1st and 2nd priorities to the minimization of $C(\overline{x})$ and $S(\overline{y})$ respectively.

Now a branch and bound algorithm is evolved for solving the 1st prioritized bi-criterion problem. Different branch and bound algorithms are evolved by choosing in different ways

(i) The lower bound of the objective function at a node,

(ii) The node at each stage for branching the set of solutions, and

(iii) The variables with respect to which branching at the chosen node to be done.

The evolved branch and bound algorithm for solving the 1st prioritized bi-criterion problem is a modification and extension of the branch and bound algorithm used for solving the single-objective binary integer problems. Unlike one lower bound for the objective functions of the single-objective problem. Two lower bounds – one for the recurring total call charges cost $C(\overline{x})$ and another for the non-recurring total call centers setup cost $S(\overline{y})$ at each node in the tree are provided. The various steps involved in solving the 1st prioritized bi-criterion problem are as follows

Table 2.1

	Potential call center sites						
Zones of call origin	P_1	P_2	…	P_j	…	P_n	Bulk calls originated from zones
Z_1	c_{11}	c_{12}	…	c_{1j}	…	c_{1n}	a_1
Z_2	c_{21}	c_{22}	…	c_{2j}	…	c_{2n}	a_2
…	…	…	…	…	…	…	…
Z_i	c_{i1}	c_{i2}	…	c_{ij}	…	c_{in}	a_i
…	…	…	…	…	…	…	…
Z_m	c_{m1}	c_{m2}	…	c_{mj}	…	c_{mn}	a_m
Capacity of call center at sites P_j	b_1	b_2	…	b_j	…	b_n	
Setup costs s_j of call centers at sites P_j	s_1	s_2	…	s_j	…	s_n	

(1st prioritized bi-criterion call center problem)

(a) Lower bounds of the objective functions at the node Lower bounds LBC of the objective function $C(\bar{x})$ and LBS of the objective function $S(\bar{y})$ at the topmost node 0 are the lower bounds of the objective functions of the set of all solutions of the problem and the procedures for their computations is as follows. First, the columns associated with the potential call center sites containing b_j's having values smaller than those of all the a_i's are deleted from Table 1, because the call centers at the sites cannot handle the bulk calls originated from any of the zones. Also every cell (i, j) corresponding to which $a_i > b_j$ is deleted from Table 1, because then the call center at the site P_j cannot handle the bulk calls originated from the zone Z_i. After this, the smallest entry in each corresponding to a zone is subtracted from all the entries of that row yielding a reduced cost table wherein each row corresponding to a zone has a zero cost in a cell. Then the two lower bounds at the topmost node 0 are obtained. The lower LBC of the objective function $C(\bar{x})$ at the topmost node 0 is the sum of the subtracted smallest entries of each row in the reduced cost table. And the lower bound LBS of the objective function $S(\bar{y})$ at it is set equal to 0. Computation of the lower bounds LBC and LBS at the node 0 in this way results into simplifying their computation at the succeeding nodes. Lower bounds at any node other than the topmost node 0 are obtained as follows. First we update the table of the node at which the lower bounds of the objective functions are to be obtained. By updating the Table of the node, we mean that each row corresponding to a zone whose bulk calls have been assigned to a call center is deleted from the Table. Then the bulk calls originated from the zones which have been assigned to a potential call centre site are subtracted from the capacity of the site. This means that if a zone Z_i is assigned to a call center at site P_j , then the capacity of the potential call center site P_j is reduced to $(b_j - a_i)$ in the updated table. Having deleted the rows corresponding to the zones and updated capacities of the potential call center sites, we delete each of the columns corresponding to a potential call center site whose updated capacity is less than the bulk calls originated from each of the zones. Also we delete each of the cells (i, j)'s corresponding to which the updated capacity is less then a_i . The table thus obtained is the updated table of the node. After this, the smallest entry in each row corresponding to a zone is subtracted from all the entries of that row in the reduced cost table of the updated table of the node. The lower bound LBC of the objective function $C(\bar{x})$ at the node other than the topmost node 0 is the lower bound of the immediately preceding node plus the sum of the subtracted smallest entries of each row of the updated table of the node. And the lower bound LBS of the objective function $S(\bar{y})$ at the node is the lower bound LBS of the immediately preceding node plus the setup cost of the call center at the newly selected if any.

(b) The node at each stage for branching the set of solutions The node chosen at each stage for branching the set of solutions is the node among the terminal unfathomed nodes having the smallest lower bound LBC of the objective function $C(\bar{x})$. In the case of tie, we branch the node which has the smallest lower bound LBS of the objective function $S(\bar{y})$ among the terminal unfathomed nodes having the same smallest lower bound LBC of the objective function $C(\bar{x})$.

(c) The variable with respect to which branching at the chosen node to be done The variable x_{ij} with respect to which branching at the chosen node is to be done is determined as follows. In the Reduced cost table of the chosen node, the least cost of exclusion is entered in the left bottom corner of each of the cells having zero costs. The least cost of exclusion for each cell having a zero cost in a row is the next lower cost in a cell of the row. However, if more than one cell in a row having zero cost exists, then the least cost of exclusion for each of these cells is zero. Further if there is only one cell having a zero cost in a row and all the other cells in that row have been deleted, then the least cost of exclusion for that cell is ∞. For if the bulk calls from the zones corresponding to the row are not handled at the call center setup at the site corresponding to the column containing the cell, then the bulk calls from the zone corresponding to the row containing the cell can never be handled. The least cost of exclusion at a cell (i, j) indicates the least cost to be incurred if the bulk calls a_i from the zone Z_i are not handled at the call center setup at the site P_j. Branching is done at the chosen node with respect to the variable x_{ij} corresponding to the cell (i, j) having the greatest least cost of exclusion among all the cells having zero costs.

(d) Termination of the procedure we shall terminate the procedure in either of the two situations. The first situation wherein we will terminate the procedure is that we have arrived at a terminal unfathomed node containing the number m of the variables x_{ij} 's at level 1 each with a different value of subscript i along the chain from the topmost node 0 to it with lexicographically minimum lower bounds LBC of $C(\bar{x})$ and LBS of $S(\bar{y})$ among all the terminal unfathomed nodes in the tree. The unfathomed terminal node provides the optimal solution $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ of the 1st prioritized bi criterion call center problem. Among several nodes each having lower bounds LBC and LBS with the first and second prioritized respectively, that node has lexicographically minimum lower bounds, wherein the value of LBC is either least of the value of LBS is least in the case of LBC having tied values. $\bar{X}^{(1)}$ consists of the number m of the x_{ij} 's at level 1 each with a different value of i and the remaining x_{ij} 's at level 0 while $\bar{Y}^{(1)}$ consists of the y_j 's level 1 corresponding to different values of subscripts j 's in the x_{ij} 's level 1 in $\bar{X}^{(1)}$ and the remaining y_j 's at level 0. Since the optimal solution of the 1st prioritized bi criterion problem is the 1st efficient solution of the formulated problem, $(\bar{X}^{(1)}, \bar{Y}^{(1)})$ is the 1st efficient solution of the formulated problem. The 2nd solution wherein we terminate the procedure is that we find that all the terminal nodes in the tree are fathomed without having arrived at a terminal unfathomed node containing the number m of x_{ij} 's at level 1 each with a different value of i along the chain from the topmost node 0 to it, indicating that no solution of the 1st prioritized bi criterion problem exists.

2.3.2 PROCEDURE FOR OBTAINING 2nd AND SUBSEQUENT EFFICIENT SOLUTION

After having obtained the 1st efficient solution $(\bar{X}^{(1)},\bar{Y}^{(1)})$ of the bi criterion call center problem, its 2nd efficient solution is obtained. To do this, the branch and bound algorithm explained above in Subsection 2.3.1 for obtaining the 1st efficient solution $(\bar{X}^{(1)},\bar{Y}^{(1)})$ is applied to the bi criterion problem with the first and second priorities to the minimization of the recurring total call charges and nonrecurring total call centers setup costs respectively with the stipulation that a terminal node at each stage is now fathomed whenever the lower bound at it for the nonrecurring total call centers setup cost is greater than or equal to $S(\bar{y}^{(1)})$ .The unfathomed node thus obtained containing the number m of the variables x_{ij} 's each with a different value of i having the lexicographically minimum lower bounds LBC of $C(\bar{x})$ and LBS of $S(\bar{y})$ among all the terminal unfathomed nodes along the chain from the topmost node 0 to it, yields the 2nd efficient solution $(\bar{X}^{(1)},\bar{Y}^{(1)})$. The 3rd and subsequent efficient solutions are obtained in the same way as in done for obtaining the 2nd efficient solution $(\bar{X}^{(1)},\bar{Y}^{(1)})$. The process of obtaining efficient solutions of the formulated problem is terminated after encountering a tree whose all terminal nodes at a stage are fathomed before arriving at a terminal unfathomed node containing the number m of x_{ij} 's at level 1 each with a different value of i along the chain from the topmost node 0 to it, indicating that it is not possible to find a new efficient solution with lesser nonrecurring total call centers setup cost.

2.4 NUMERICAL PROBLEM

The procedure is illustrate by applying it to obtain the set of efficient solutions of a numerical problem obtained by taking m =4, n =5 and assigning numerical values to all other parameters in the bi criterion call center problem formulated in section 2.2 . The data in the numerical problem are fictitious. Nevertheless the problem has some bearing to real – life because the problems similar to the one considered here are encountered in life. The tableau representation of the numerical problem is shown in Table 2.2. The numerical problem seeks to determine x_{ij} 's and y_j 's which minimize the two objective functions.

$$C(\bar{x}) = \begin{Bmatrix} 8000\,x_{11} + 6000\,x_{12} + 9200\,x_{13} + 9600\,x_{14} + 4000\,x_{15} \\ 3600\,x_{21} + 15000\,x_{22} + 12000\,x_{23} + 15600\,x_{24} + 12000 x_{25} \\ 7200\,x_{31} + 6900\,x_{32} + 3600\,x_{33} + 3300\,x_{34} + 1500\,x_{35} \\ 11000\,x_{41} + 11000\,x_{42} + 5000\,x_{43} + 9000\,x_{44} + 11500 x_{45} \end{Bmatrix} \tag{2.7}$$

$$S(\bar{y}) = 600000\,y_1 + 1000000\,y_2 + 900000\,y_3 + 900000\,y_4 + 1200000\,y_5 \tag{2.8}$$

Table 2.2

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	8,000	6,000	9,200	9,600	4,000	4,000
z_2	3,600	15,000	12,000	15,600	12,000	9,000
z_3	7,200	6,900	3,600	3,300	1,500	3,000
z_4	11,000	11,000	5,000	9,000	11,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

(Call charges costs of handling bulk calls from zones at potential call center sites, bulk calls originated from zones, capacities and setup costs of call center at sites)

Now the procedure is applied for obtaining the set of efficient solutions of the numerical problem. Since none of the potential call center have the capacity less than the bulk calls originated from each of the zone, thus no column is deleted from the table whereas the cells (2,2), (2, 3), (2, 4), (2, 5) and (4, 4) are blocked because $a_2 > b_2$, $a_2 > b_3$, $a_2 > b_4$, $a_2 > b_5$ and $a_4 > b_4$ shown in table 2.3.

Table 2.3

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	8,000	6,000	9,200	9,600	4,000	4,000
z_2	3,600	M	M	M	M	9,000
z_3	7,200	6,900	3,600	3,300	1,500	3,000
z_4	11,000	11,000	5,000	M	11,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Find the row reduced matrix and given in table 2.4.

Table 2.4

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_2	0	M	M	M	M	9,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	6,000	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 4000 + 3600 + 1500 + 5000 = 14100

Node 0

LBC=14100, LBS=0

Figure 2.1

It can be easily verified that the penalties using VAM for the zones Z_1, Z_2, Z_3, Z_4 are: (2000, M, 1800, 6000) corresponding to cells (1, 5), (2, 1), (3, 5), (4, 3) respectively.

Since the largest penalty corresponds to zone Z_2, so make an allocation in the cell (2, 1) and apply the branch and bound algorithm.

In the case of $x_{21} = 0$, then the cell (2, 1) will be blocked by M and make the reduce matrix given in table 2.5.

Table 2.5

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_2	M	M	M	M	M	9,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	6,000	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

This node is fathomed because no calls assigning in x_{21} .

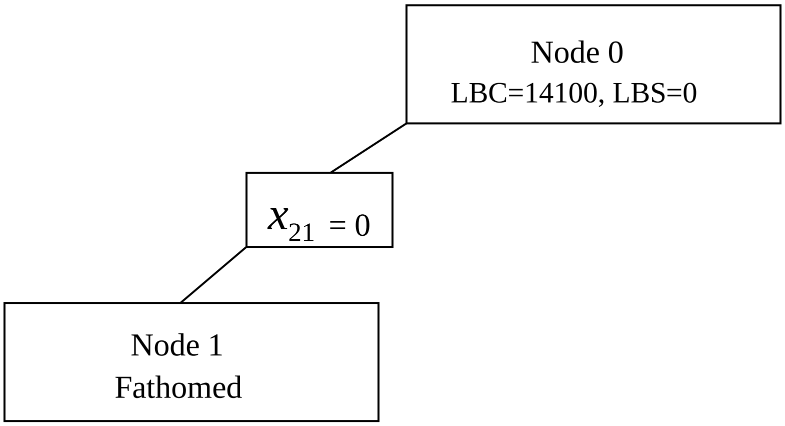

Figure 2.2

In case of $x_{21} = 1$, block the corresponding row and column and make the reduced matrix given in table 2.6.

Table 2.6

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	–	2,000	5,200	5,600	0	4,000
z_2	–	–	–	–	–	9,000
z_3	–	5,400	2,100	1,800	0	3,000
z_4	–	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 14100 + 0 = 14100 and LBS = 600,000

At level one, LBC=14100 and site y_1 is selected and LBS = 600,000 (figure 2.3).

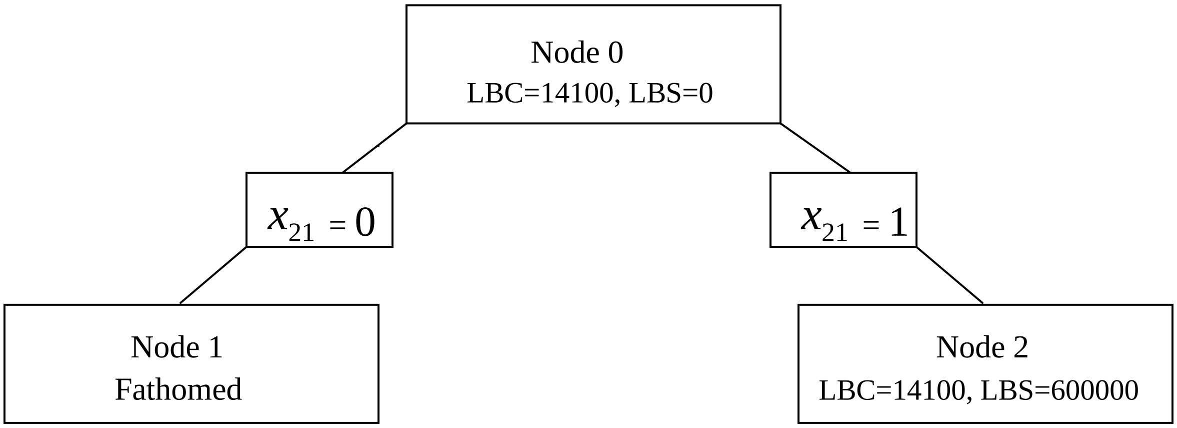

Figure 2.3

Procedure is repeated until the calls of all zones have been completed. Next solution for the lower bound is shown in Table 2.7.

Table 2.7

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	6,000	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Since none of the potential call center has the capacity less than the bulk calls originated from each of the zone, thus no column is deleted from the table whereas the cell (4, 1) is blocked by M because $a_4 > b_4$ shown in Table 2.8.

Table 2.8

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	M	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Recalculate the penalties using VAM in the table 2.8 and proceed in the same manner. It can be easily verified that the penalties using VAM for the zones Z_1, Z_3, Z_4 are: (2000, 1800, 6000) corresponding to cells (1, 5), (3, 5), (4, 3) respectively. Since the largest penalty corresponds to zone Z_4, so make an allocation in the cell (4, 3) and apply the branch and bound algorithm.

In the case of $x_{43} = 0$, then the cell (4, 3) will be blocked by M and make the reduce matrix given in table 2.9 and 2.9.1

Table 2.9

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	M	6,000	M	M	500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Table 2.9

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	M	0	M	M	500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 14100 + 6000 = 20100 and LBS = 600,000

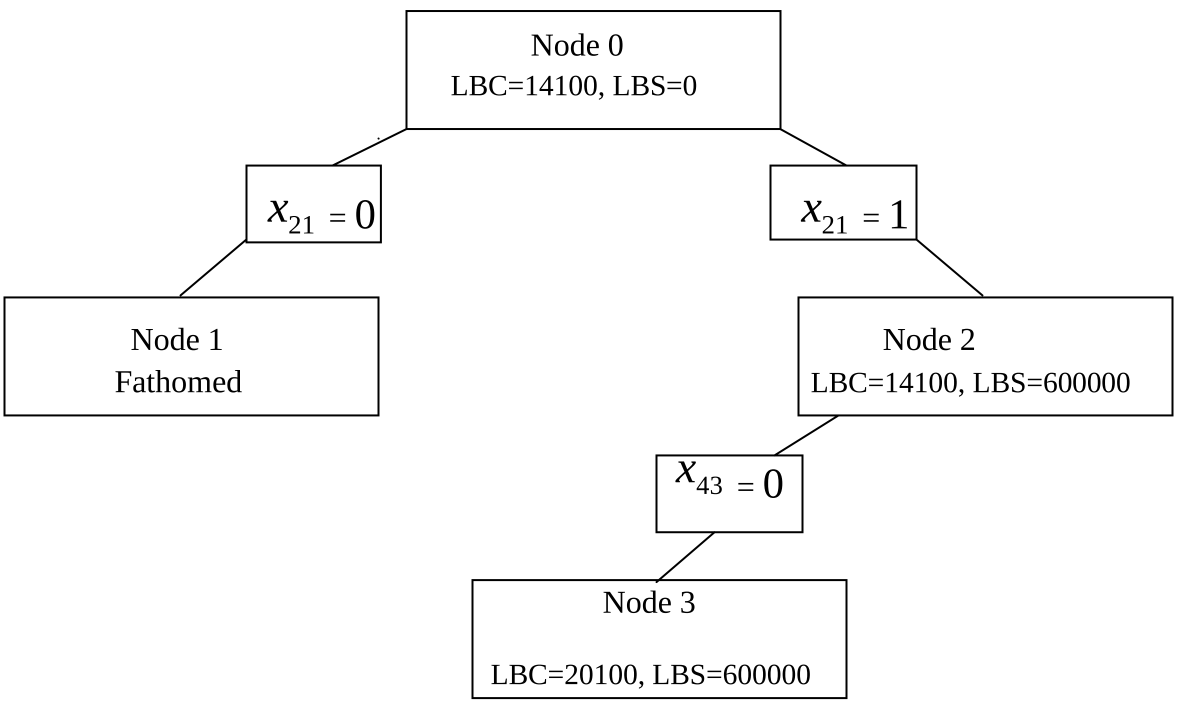

Figure 2.4

In case of x_{43} =1, block the corresponding row and column and make the reduced matrix given in table 2.10.

Table 2.10

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	–	5,600	0	4,000
z_3	5,700	5,400	–	1,800	0	3,000
z_4	–	–	–	–	–	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 14100 + 0 = 14100 and LBS = 600,000 + 900,000 = 1500000

At the next level, LBC=14100 and site y_3 is selected and LBS = 1500000 (figure 2.5).

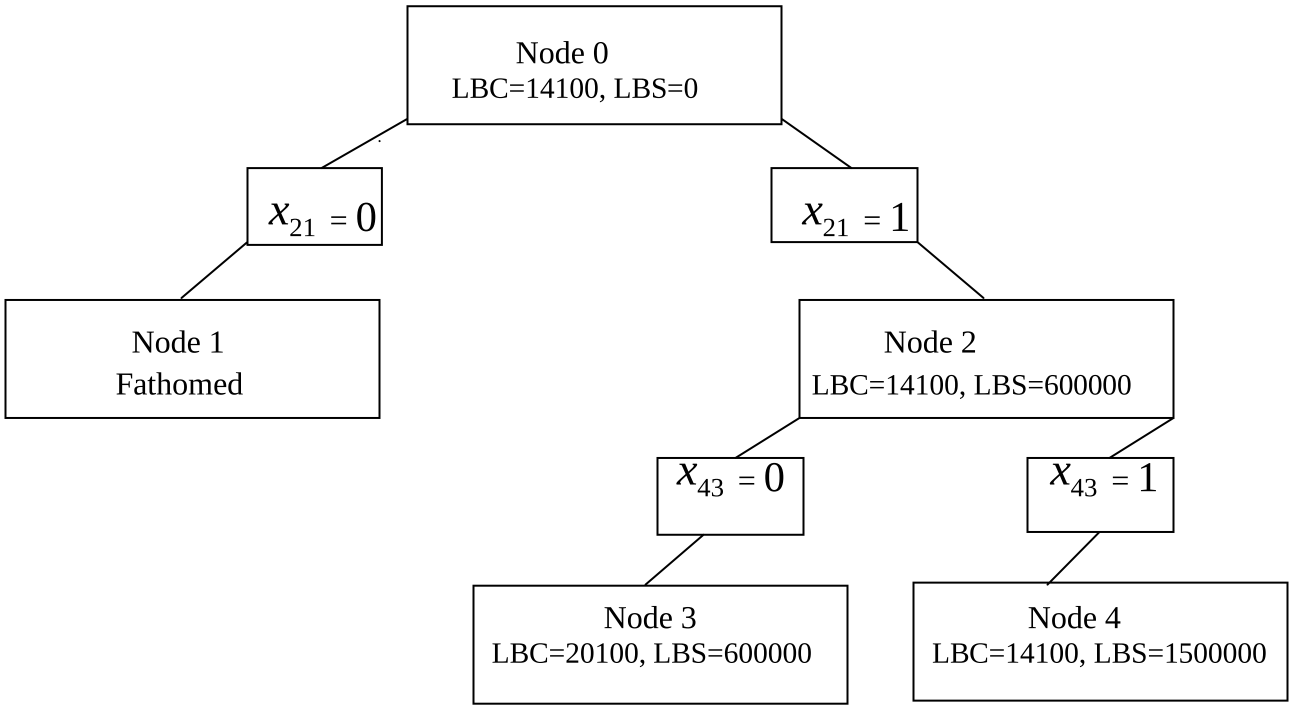

Figure 2.5

Table 2.11

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Apply the same procedure, the cell (1, 3) is blocked by M because $a_1 > b_3$ and the penalties for the zones Z_1 *and* Z_3 are: (2000, 1800) corresponding to cells (1, 5) and (3, 5). Highest penalty is on cell (1, 5).

In case of $x_{15} = 0$, then the cell (1, 5) will be blocked by M and make the reduced matrix given in table 2.12.

Table 2.12

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	M	3,600	M	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 14100 + 2000 = 16100 and LBS = 1500000

In case of $x_{15} = 1$, block the corresponding row and column and make the reduced matrix given in table 2.13.

Table 2.13

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	3,000	
Setup costs S_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC = 14100 +0=14100 and LBS = 1500000 + 1200000= 2700000

At the next level, LBC=14100 and site y_5 is selected and LBS = 2700000 (figure 2.6).

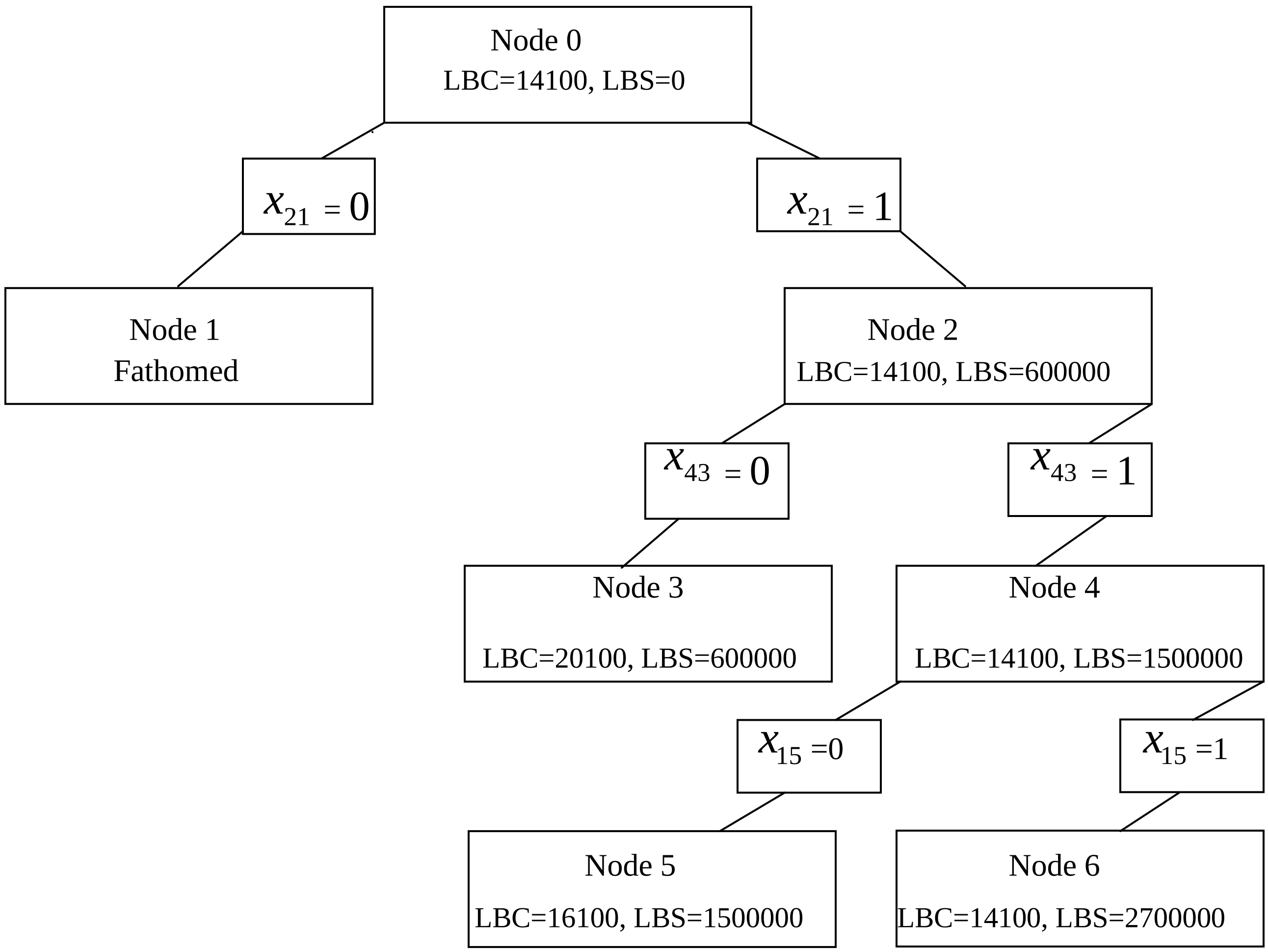

Figure 2.6

Similarly next allocation is obtained in the cell (3, 5).

In case of $x_{35} = 0$, then the cell (3, 5) will be blocked by M and make the reduced matrix given in table 2.14.

Table 2.14

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	3,900	3,600	300	0	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	3,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 14100 + 1800 = 15900 and LBS = 2700000

In case of $x_{35}=1$, block the corresponding row and column and make the reduced matrix given in table 2.15.

Table 2.15

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	3,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 14100 and LBS = 2700000

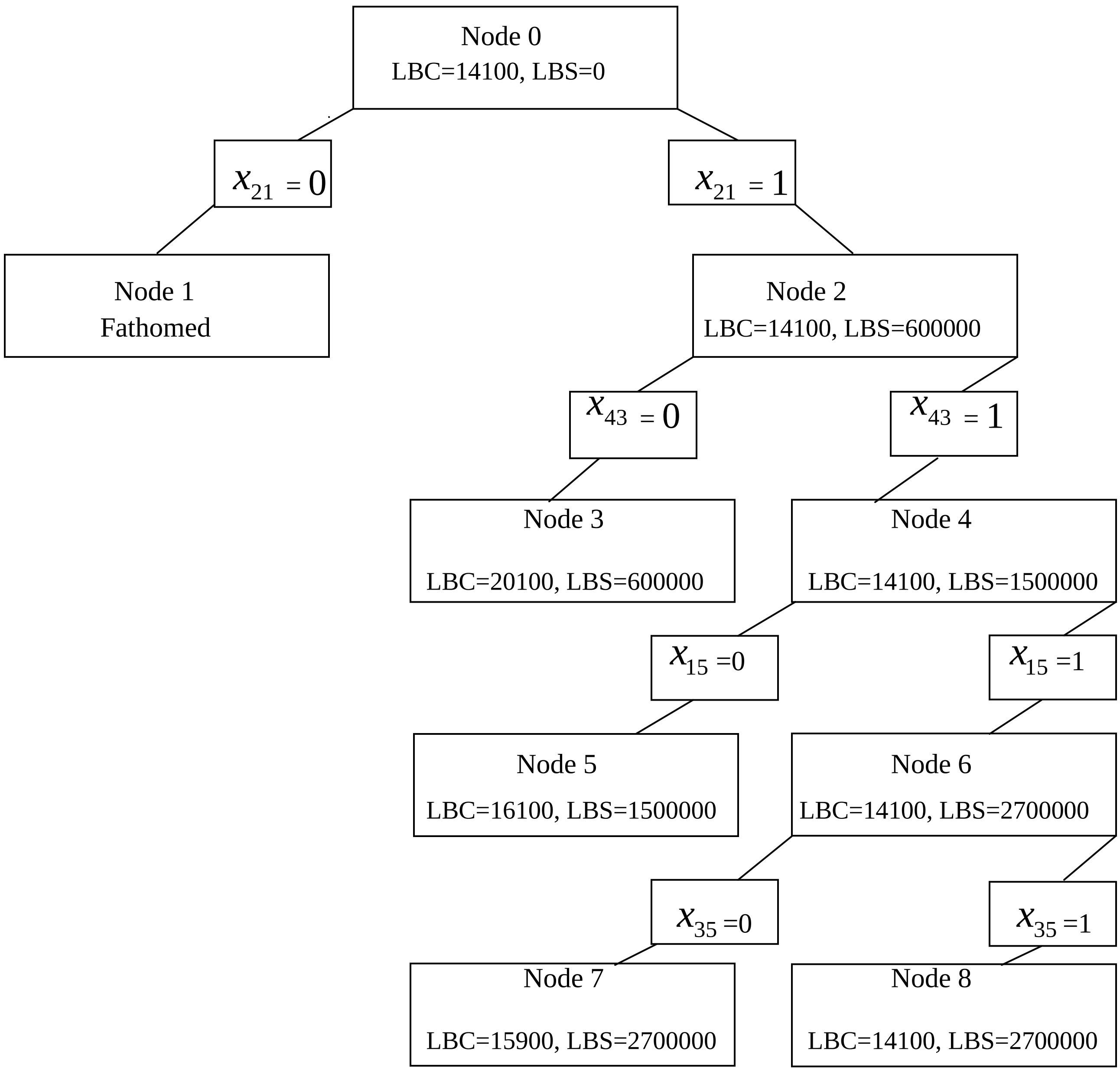

Figure 2.7 **(Tree drawn for obtaining 1st efficient solution of the numerical bi criterion call center problem)**

The optimal solution of the 1st prioritized bi criterion problem yields the efficient solution of the numerical problem.

The variables x_{21}, x_{43}, x_{15}, x_{35} at level 1 along the chain from the topmost node 0 to node 8 and the variables y_{1}, y_{3}, y_{5} at least one provide the optimal solution $(\overline{X}^{(1)}, \overline{Y}^{(1)})$ of the 1st prioritized bi-criterion with $C(\overline{X}^{(1)})$ = 3600 + 5000 + 4000 + 1500 =14100 and $S(\overline{Y}^{(1)})$ = 600000 + 900000 + 1200000 = 2700000. This optimal solution yields the 1st efficient solution of the numerical problem.

The remaining efficient solutions of the numerical problem are obtained by following the procedure outlined in subsection 2.3.2. For obtaining the 2nd efficient solution of the numerical problem, obtain the 2nd prioritized bi-criterion problem from the 1st one by stipulating the condition $S(\overline{y}) < S(\overline{Y}^{(1)})$ = 2700000. Apply the branch and bound algorithm for obtaining the optimal solution of this 2nd prioritized problem in the same way as done for obtaining the optimal solution of 1st one but fathoming a terminal node having LBS ≥ $S(\overline{Y}^{(1)})$ = 2700000 and draw a tree as shown in figure 2.8.

Now the variable $x_{15} = 1$ is fathomed because LBS ≥ $S(\overline{Y}^{(1)})$ = 2700000.

Apply the branch and bound algorithm to the node corresponding to $x_{15} = 0$ and is shown in table 2.16.

Table 2.16

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	M	3,600	M	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs S_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Now calculate the penalties of each row. In this case the penalties are: (2000, 1800) corresponding to cells (1, 2) and (3, 5). Highest penalty is on cell (1, 2).

In case of $x_{12} = 0$, then the cell (1, 2) will be blocked by M and make the reduced matrix given in table 2.17.

Table 2.17

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	0	M	M	1,600	M	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC=16100 + 2000=18100 and LBS= 1500000

Again applying the branch and bound algorithm to the node corresponding to $x_{12} = 0$ and in this case the penalties are: (1600, 1800) corresponding to cells (1, 1) and (3, 5).

Highest penalty is on cell (3, 5).

In case of $x_{35} = 0$, block the cell (3, 5) by M and make the reduced matrix given in table 2.19.

Table 2.19

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	0	M	M	1,600	M	4,000
z_3	3,900	3,600	300	0	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 18100 + 1800=19900 and LBS = 1500000

In case of $x_{35} = 0$ this node is fathomed because LBS $\geq$ $S(\overline{Y}^{(1)})$ = 2700000.

Now discuss the case when $x_{21} = 1$.

In case of $x_{12} = 1$ make the allocation in (1, 2) and update the table.

Table 2.20

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	1,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC=16100+0=16100 and LBS=1500000+1000000=2500000

On applying the same procedure the cell (3, 5) is selected.

In case of $x_{35} = 0$ the second column is deleted because the potential call center site b_2 have the value smaller than a_3.

Now block the cell (3, 5) by M and make the reduced matrix given in table 2.21.

Table 2.21

	Potential call center sites				
Zones of call origin	P_1	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	3,900	300	0	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	900,000	900,000	1,200,000	

LBC=16100+1800=17900 and LBS=2500000

Now the cell (3, 4) is selected. In case of $x_{34} = 1$, this node is fathomed because LBS ≥ $S(\overline{Y}^{(1)}) =$ 2700000.

In case of $x_{34} = 0$, block the cell (3, 4) by M and make the reduced matrix shown in table 2.22.

Table 2.22

	Potential call center sites				
Zones of call origin	P_1	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	3,600	0	M	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	900,000	900,000	1,200,000	

LBC=17900+300=18200 and LBS=2500000

Now the cell (3, 3) is selected. In case of $x_{33} = 0$ block the cell (3, 3) by M and make reduced matrix shown in table 2.23.

Table 2.23

	Potential call center sites				
Zones of call origin	P_1	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	0	M	M	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	900,000	900,000	1,200,000	

LBC=18200+3600=21800 and LBS=2500000

In case of $x_{33} = 1$, make the allocation in cell (3, 3) and update the table shown in table 2.24.

Table 2.24

	Potential call center sites				
Zones of call origin	P_1	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	3,600	0	M	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	900,000	900,000	1,200,000	

LBC=18200 and LBS=250000

The unfathomed terminal node 16 thus obtained containing the variables x_{12}, x_{21}, x_{33}, x_{43} at level 1 along the chain from the topmost node 0 to node 16 and the variables y_1, y_2, y_3 at level 1 provide the optimal solution $(\overline{X}^{(2)}, \overline{Y}^{(2)})$ of the 2nd prioritized bi criterion with $C(\overline{X}^{(2)})$ = 3600 + 5000 + 6000 + 3600 =18200 and $S(\overline{Y}^{(2)})$ = 600000 + 1000000 + 900000 = 2500000. This optimal solution yields the 2nd efficient solution of the numerical problem.

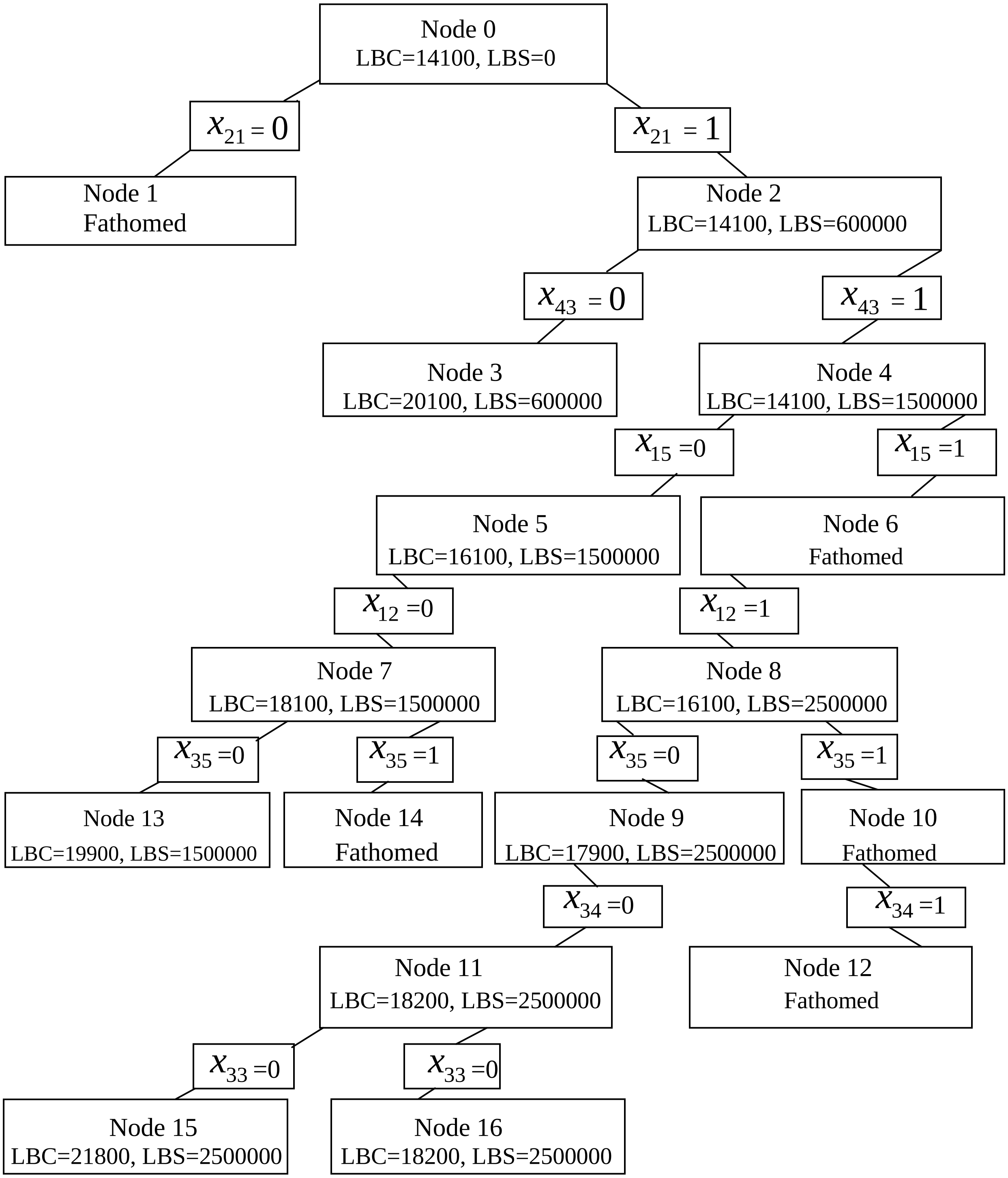

Figure 2.8: (Tree drawn for obtaining 2nd efficient solution of numerical bi criterion call center problem)

For obtaining the 3rd efficient solution of the numerical problem, obtain the 3rd prioritized bi criterion problem from the 2nd one by stipulating the condition $S(\bar{y}) < S(\overline{Y}^{(2)})$ = 2500000. Apply the branch and bound algorithm for obtaining the optimal solution of this 3rd prioritized problem in the same way as done for obtaining the optimal solution of 2nd one but fathoming a terminal node having LBS ≥ $S(\overline{Y}^{(2)}) = 2500000$ and draw a tree as shown in figure 2.9.

Apply the branch and bound algorithm to node corresponding to $x_{43} = 0$ and is shown in table 2.25.

Table 2.25

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	M	0	M	M	500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

The penalties are: (2000, 1800, 500) corresponding to cells (1, 5), (3, 5), (4, 2) respectively.

Highest penalty is on cell (1, 5). The cell (1, 5) is selected.

In case of $x_{15} = 0$ block the cell (1, 5) by M and make the reduced matrix shown in table 2.26.

Table 2.26

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	3,200	3,600	M	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	M	0	M	M	500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC=20100+2000=22100 and LBS=600000

In case of $x_{15} = 1$ make the allocation in (1, 5) and update the table shown in table 2.27.

Table 2.27

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	M	0	M	M	M	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	3,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC=20100+0=20100 and LBS=600000+1200000=1800000

Again applying the branch and bound algorithm. The cell (3, 5) is not selected because LBS ≥ $S(\overline{Y}^{(2)})$ = 2500000. Also the cell (4, 2) is not selected. Therefore these nodes are fathomed.

The nodes $x_{15} = 1$, $x_{12} = 1$, $x_{35} = 1$ are fathomed because LBS ≥ $S(\overline{Y}^{(2)})$ = 2500000.

Apply branch and bound algorithm to the node corresponding to $x_{34} = 0$ and is shown in table 2.28.

Table 2.28

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	0	M	M	1,600	M	4,000
z_3	3,900	3,600	300	0	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Penalties are: (1600, 300) corresponding to cells (1, 1) and (3, 4).

Highest penalty is on cell (1, 1).

In case of $x_{11} = 0$, block the cell by M and make the reduced matrix given in table 2.29.

Table 2.29

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	M	M	M	0	M	4,000
z_3	3,900	3,600	300	0	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC=19900+1600=21500 and LBS=1500000

In case of $x_{11} = 1$, make the allocation in the cell (1, 1) and update the table.

Table 2.30

	Potential call center sites				
Zones of call origin	P_1	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	3,600	300	0	M	3,000
Capacities b_j 's of call centers at sites P_j 's	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	900,000	900,000	1,200,000	

LBC=19900 and LBS=1500000

Now the cell (3, 4) is selected.

In case of $x_{34} = 0,$ block the cell by M and make reduced matrix shown in table 2.31.

Table 2.31

	Potential call center sites				
Zones of call origin	P_1	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	3,300	0	M	M	3,000
Capacities b_j 's of call centers at sites p_j 's	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites p_j 's	600,000	900,000	900,000	1,200,000	

LBC=19900+300=20200 and LBS=1500000

In case of $x_{34} = 1,$ make the allocation in cell (3, 4), we get

LBC= 19900+0= 19900 and LBS= 1500000+900000 = 2400000

The unfathomed terminal node 14 thus obtained containing the variables x_{11}, x_{21}, x_{34}, x_{43} at level 1 along the chain from the topmost node 0 to node 16 and the variables $y_{1\,,}\,y_{3\,,}\,y_4$ at level 1 provide the optimal solution $(\overline{X}^{(3)}, \overline{Y}^{(3)})$ of the 3rd prioritized bi criterion with $C(\overline{X}^{(3)})$ = 3600 + 5000 + 8000 + 3300 =19900 and $S(\overline{Y}^{(3)})$ = 600000 + 900000 + 900000 = 2400000. This optimal solution yields the 3rd efficient solution of the numerical problem.

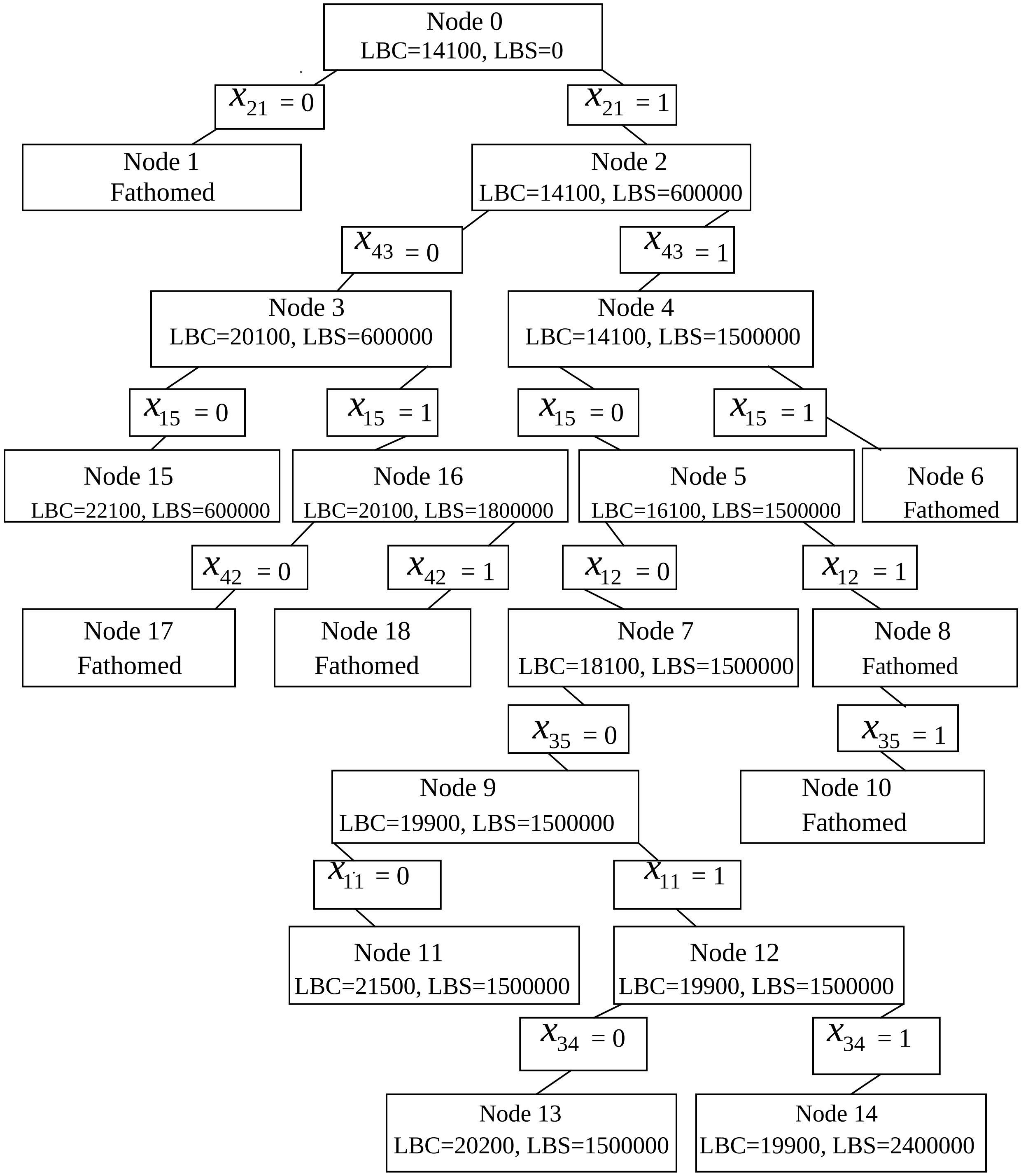

Figure 3.16: (Tree drawn for obtaining 3rd efficient solution of numerical bi criterion call center problem)

For obtaining the 4th efficient solution of the numerical problem, obtain the 3rd prioritized bi criterion problem from the 2nd one by stipulating the condition $S(\bar{y}) < S(\bar{Y}^{(3)})$ = 2400000. Apply the branch and bound algorithm for obtaining the optimal solution of this 3rd prioritized problem in the same way as done for obtaining the optimal solution of 2nd one but fathoming a terminal node having LBS $\geq S(\bar{Y}^{(3)}) = 2400000$ and draw a tree as shown in figure 2.10.

Apply the branch and bound algorithm to the node corresponding to $x_{34} = 0$, shown in table 2.30.

Table 2.32

	Potential call center sites				
Zones of call origin	P_1	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	3,300	0	M	M	3,000
Capacities b_j 's of call centers at sites P_j 's	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	900,000	900,000	1,200,000	

The cell (3, 3) is selected.

In case of $x_{33} = 0$, block the cell (3, 3) by M and make the reduced matrix shown in table 2.33.

Table 2.33

	Potential call center sites				
Zones of call origin	P_1	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	0	M	M	M	3,000
Capacities b_j 's of call centers at sites P_j 's	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	900,000	900,000	1,200,000	

LBC=20200+3300=23500 and LBS=1500000

In case of x_{33} =1, make the allocation in the cell (3, 3), we get LBC=20200 and LBS=1500000.

The unfathomed terminal node 20 thus obtained containing the variables $x_{11}, x_{21}, x_{33}, x_{43}$ at level 1 along the chain from the topmost node 0 to node 16 and the variables y_{1}, y_3 at level 1 provide the optimal solution $(\overline{X}^{(4)}, \overline{Y}^{(4)})$ of the 4th prioritized bi criterion with $C(\overline{X}^{(4)})$ = 3600 + 5000 + 8000 + 3600 =20200 and $S(\overline{Y}^{(4)})$ = 600000 + 900000 = 1500000. This optimal solution yields the 4th efficient solution of the numerical problem.

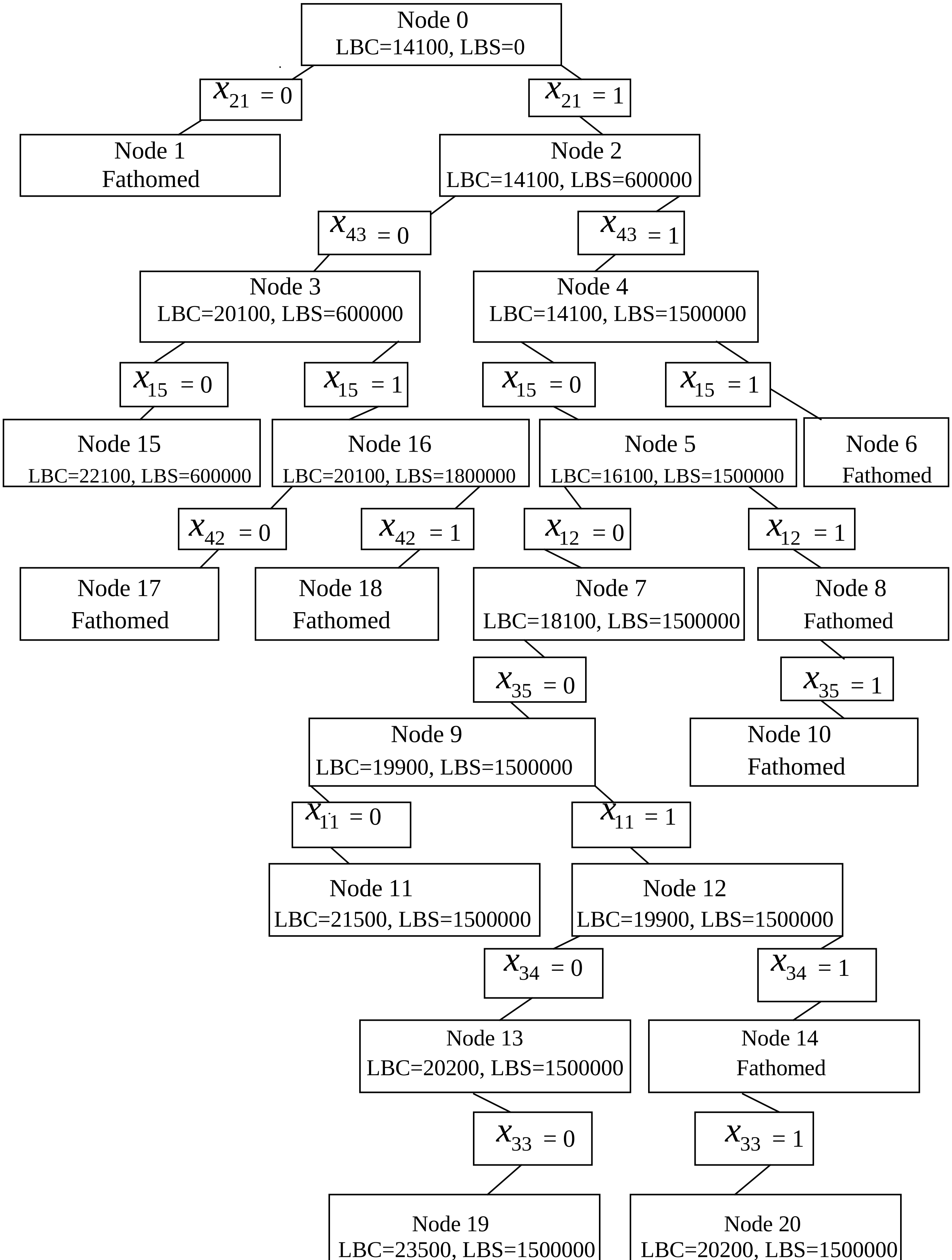

Figure 2.10: (Tree drawn for obtaining 4th efficient solution of numerical bi criterion call center problem)

For obtaining the next efficient solution, the above tree shows that it is not possible to find a new efficient with a lesser total call centers setup cost $S(\overline{Y}^{(4)})$ =1500000 and that the process of obtaining the further efficient solution should be terminated.

The numerical problem is found to have only four efficient solutions. The set of the numerical problem together with the variables x_{ij} 's y_i 's at level one, recurring total call charges cost $C(\overline{x})$'s and nonrecurring total call centers setup cost $S(\overline{y})$'s are shown in table 2.34.

Table 2.34

Efficient solution	Variables at level 1	Recurring total call charges cost	Non recurring total call centers setup cost
$(\overline{X}^{(1)}, \overline{Y}^{(1)})$	$x_{21}, x_{43}, x_{15}, x_{35}$; y_1, y_3, y_5	$C(\overline{X}^{(1)})$ =14100	$S(\overline{Y}^{(1)})$ = 2700000
$(\overline{X}^{(2)}, \overline{Y}^{(2)})$	$x_{12}, x_{21}, x_{33}, x_{43}$; y_1, y_2, y_3	$C(\overline{X}^{(2)})$ =18200	$S(\overline{Y}^{(2)})$ = 2500000
$(\overline{X}^{(3)}, \overline{Y}^{(3)})$	$x_{11}, x_{21}, x_{34}, x_{43}$; y_1, y_3, y_4	$C(\overline{X}^{(3)})$ = 19900	$S(\overline{Y}^{(3)})$ =2400000
$(\overline{X}^{(4)}, \overline{Y}^{(4)})$	$x_{11}, x_{21}, x_{33}, x_{43}$; y_1, y_3	$C(\overline{X}^{(4)})$ = 20200	$S(\overline{Y}^{(4)})$ =1500000

(Set of efficient solutions of the numerical problem)

2.5 CONCLUSION

Prakash *et. al.* (2011) concluded that the present work could be useful to a decision maker who is not able to assign priorities to his/her objectives in the context of selection of sites for setup of call centers from among a given number of potential sites for assigning zones to them with the objective to minimize the recurring call charges and nonrecurring total call center setup cost without priorities to them. In such a situation, help can be provided to the decision maker by presenting him/her with a set of efficient solutions. He/she can pickup that solution out of this set which suits him/her most regard to his/her objectives and liking. The only input data needed to use the model considered in this work are capacities and setup costs of the call centers, bulk call charges cost of handling bulk calls from zones at different potential call center sites, the bulk calls originated from different zones.

The model discussed in this chapter is versatile. The numerical problem of a small size is given in this chapter to illustrate the solution procedure but the model is capable of tackling large size problems after computerization. It can be used in areas other than the ones not involving call centers. This can be achieved if the terms "call center" is stretched to include any facility. With this generalization, the model can be used to solve problems involving selection of sites from among potential facility sites and assigning zones to them with the objectives and constraints similar to those associated with the model.

CHAPTER - 3

"A MODIFIED APPROACH FOR BI-CRITERION CALL CENTER PROBLEM"

In this chapter a bi-criterion call center problem through an evolved branch and bound algorithm, (2.4) has been revisited and modified to find better efficient solutions. Algorithm is as follows.

3.1 ALGORITHM

Step 1: Obtain the row reduced matrix (the matrix which contains at least one zero in each zone). Delete those columns wherein the capacity of potential call center sites $b_j 's$ is less than the availability $a_i 's$ for all i (The call centers at the sites cannot handle the bulk calls originated from any of the zones).

Step 2: In the row reduced matrix, block those cells for which capacity b_j is less than the availability a_i. Assigning M to those cells.

Step 3: The lower bound LBC of the objective function $C(\bar{x})$ at the topmost node 0 is the sum of the subtracted smallest entries of each row in the reduced cost table and the lower bound LBS of the objective function $S(\bar{y})$ at it is set equal to 0.

Step 4: Calculate the cost penalty of each zones (corresponding to those cells which contain zero cost) by taking the difference of minimum costs of corresponding to row and column.

Step 5: Select the cell for which the penalty is largest and allocate a_i to this cell. In case of tie at both these aspects, it is an arbitrary choice of the decision maker and branching is done at the chosen node with respect to the variable x_{ij} corresponding to the cell (i, j).

Step 6: The lower bound LBC of the objective function $C(\bar{x})$ at the node other than the top most node 0 is the lower bound of the immediately preceding node plus the sum of the subtracted smallest entries of each row of the updated table of the node and the lower bound LBS of the objective function $S(\bar{y})$ at the node is the lower bound LBS of the immediately preceding node plus the setup cost of the call center at the newly selected site if any.

Step 7: Drop the zone for further consideration. Block the cells for which capacity of potential call center sites b_j is less than the bulk call a_i $(a_i > b_j)$ and calculate the new penalties for the remaining zones as done in step 4. Repeat the same procedure till the calls of all zones are completed.

3.2 PROCEDURE FOR OBTAINING 2ND AND SUBSEQUENT EFFICIENT SOLUTION

After having obtained the 1st efficient solution $(\overline{X}^{(1)}, \overline{Y}^{(1)})$ of the bi criterion call center problem, 2nd efficient solution $(\overline{X}^{(2)}, \overline{Y}^{(2)})$ is obtained by deleting all the cells (i, j) corresponding to the LBS ≥ $S(\overline{Y}^{(1)})$. The resultant problem is designated the 2nd efficient solution $(\overline{X}^{(2)}, \overline{Y}^{(2)})$ Further the 3rd efficient solution is obtained by deleting those cells (i, j) in the 2nd bi criterion call center problem, that correspond to the LBS ≥ $S(\overline{Y}^{(2)})$ .Subsequent efficient solution are obtained by proceeding exactly in the same way as for $(\overline{X}^{(2)}, \overline{Y}^{(2)})$ and $(\overline{X}^{(3)}, \overline{Y}^{(3)})$.

3.3 NUMERICAL EXAMPLE

Consider a numerical problem, apply the algorithm for obtaining the set of efficient solutions of the numerical problem obtained by taking *m*=4, *n*=5 and assigning numerical values to all the other parameters in the bi-criterion call center problem (2.4).

Table 3.1

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	8,000	6,000	9,200	9,600	4,000	4,000
z_2	3,600	15,000	12,000	15,600	12,000	9,000
z_3	7,200	6,900	3,600	3,300	1,500	3,000
z_4	11,000	11,000	5,000	9,000	11,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Now the algorithm is applied to the matrix. Since none of the potential call center have the capacity less than the bulk calls originated from each of the zone, thus no column is deleted from the table whereas the cells (2, 2), (2, 3), (2, 4), (2, 5) and (4, 4) are blocked by a large number M because $a_2 > b_2$, $a_2 > b_3$, $a_2 > b_4$, $a_2 > b_5$ and $a_4 > b_4$ as shown in table 3.2.

Table 3.2

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	8,000	6,000	9,200	9,600	4,000	4,000
z_2	3,600	M	M	M	M	9,000
z_3	7,200	6,900	3,600	3,300	1,500	3,000
z_4	11,000	11,000	5,000	M	11,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Find the row reduced matrix and given in table 3.3.

Table 3.3

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_2	0	M	M	M	M	9,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	6,000	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 4000 + 3600 + 1500 + 5000 = 14100

Node 0
LBC=14100, LBS=0

Figure 3.1

Now calculate the penalties of each row (corresponding to those cells which contain zero cost) by taking difference of minimum costs of corresponding to rows and columns and in this case the penalties are: (2000, M, 1800, 3900) corresponding to cells (1, 5), (2, 1), (3, 5), (4, 3) respectively.

Choose the largest penalty allocate in the corresponding cell. The cell (2, 1) is selected because the penalty on this cell is largest and makes the allocation in cell (2, 1).

In the case of $x_{21} = 0$, then the cell (2, 1) will be blocked and make the reduce matrix given in table 3.4.

Table 3.4

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_2	M	M	M	M	M	9,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	6,000	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

This node is fathomed because no calls assigning in x_{21}.

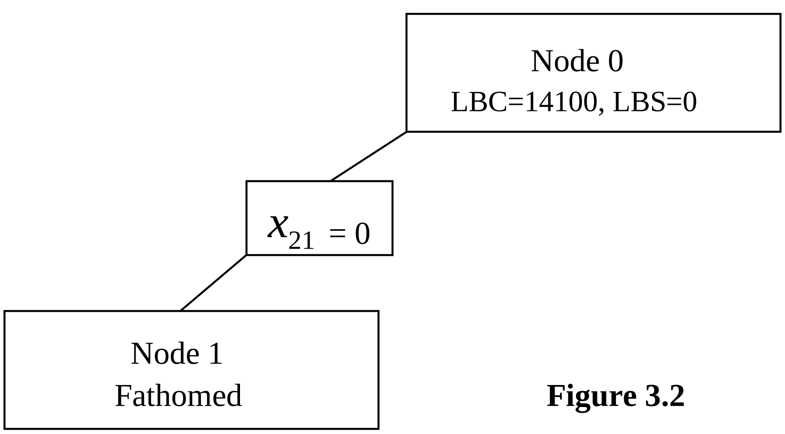

Figure 3.2

In case of $x_{21} = 1$, then apply the algorithm for the cell (2,1) and block the corresponding row and column and make the reduced matrix given in table 3.5.

Table 3.5

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	–	2,000	5,200	5,600	0	4,000
z_2	–	–	–	–	–	9,000
z_3	–	5,400	2,100	1,800	0	3,000
z_4	–	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Using step 5 to step 7, LBC and LBS are obtained.

$$\text{LBC} = 14100 + 0 = 14100 \text{ and LBS} = 600{,}000$$

At level one, LBC=14100 and site y_1 is selected and LBS = 600,000 and shown in figure 3.3.

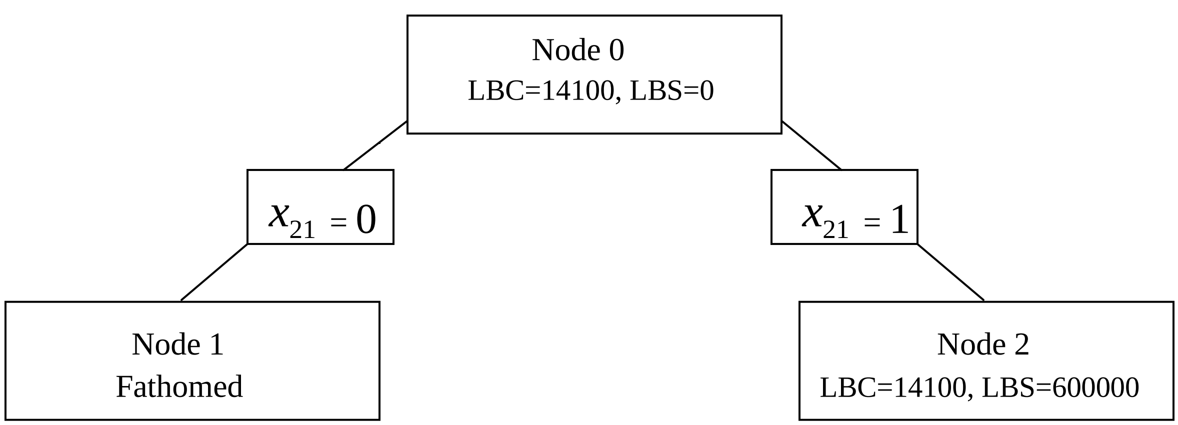

Figure 3.3

Procedure is repeated until the calls of all zones have been completed. Next solution for the lower bound is shown in table 3.6.

Table 3.6

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	6,000	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Since none of the potential call center have the capacity less than the bulk calls originated from each of the zone , thus no column is deleted from the table whereas the cell (4, 1) is blocked by M because $a_4 > b_4$ (table 3.7).

Table 3.7

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	M	6,000	0	M	6,500	5,000
Capacities $b_j's$ of call centers at sites $P_j 's$	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Now calculate the penalties of each row and in this case the penalties are: (2000, 1800, 3900) corresponding to cells (1, 5), (3, 5), (4, 3) respectively.

The cell (4, 3) is selected because the penalty on this cell is largest and makes allocation in the cell (4, 3).

In case of $x_{43} = 0$ then the cell (4, 3) will be blocked and make the reduced matrix (table 3.8).

Table 3.8

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	M	6,000	M	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Table 3.9 (row reduced matrix)

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
z_4	M	0	M	M	500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Using step 5 to step 7, LBC and LBS are obtained.

$$LBC = 14100 + 6000 = 20100 \text{ and } LBS = 600{,}000$$

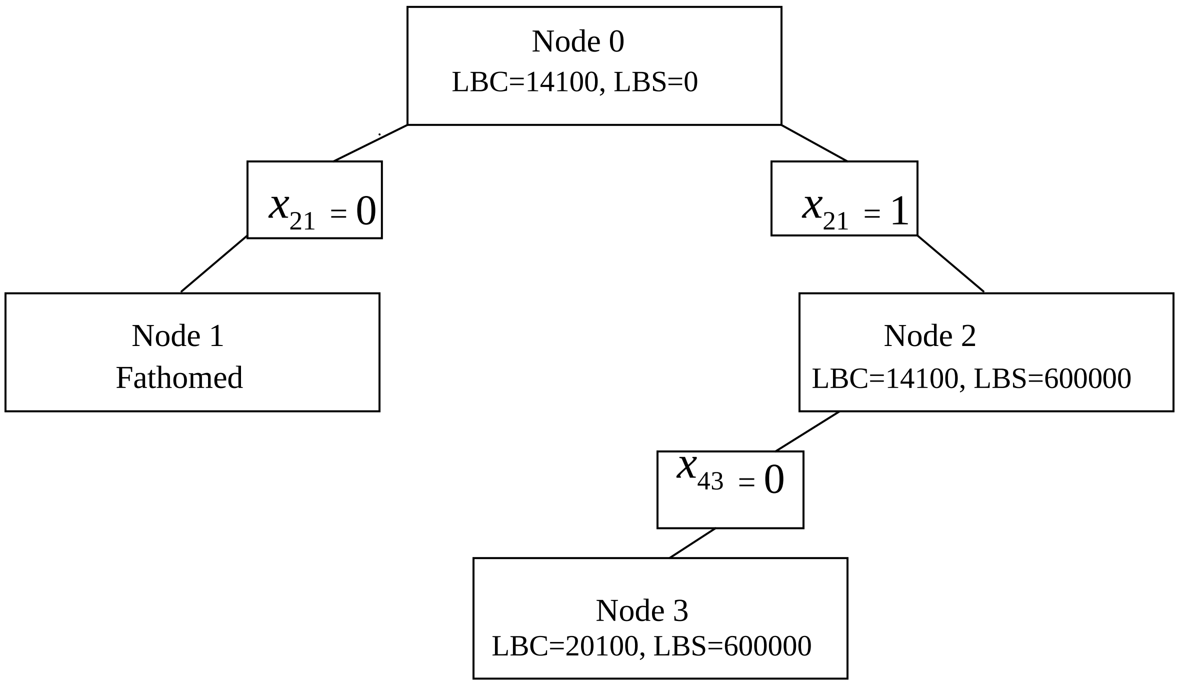

Figure 3.4

In case of $x_{43} = 1$ then apply the algorithm for the cell (4, 3) and block the corresponding row and column and make the reduced matrix given in table 3.10.

Table 3.10

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	–	5,600	0	4,000
z_3	5,700	5,400	–	1,800	0	3,000
z_4	–	–	–	–	–	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Using step 5 to step 7, LBC and LBS are obtained.

$$\text{LBC} = 14100 + 0 = 14100 \text{ and LBS} = 600{,}000 + 900{,}000 = 1500000$$

At the next level, LBC=14100 and site y_3 is selected and LBS = 1500000 (figure 3.5).

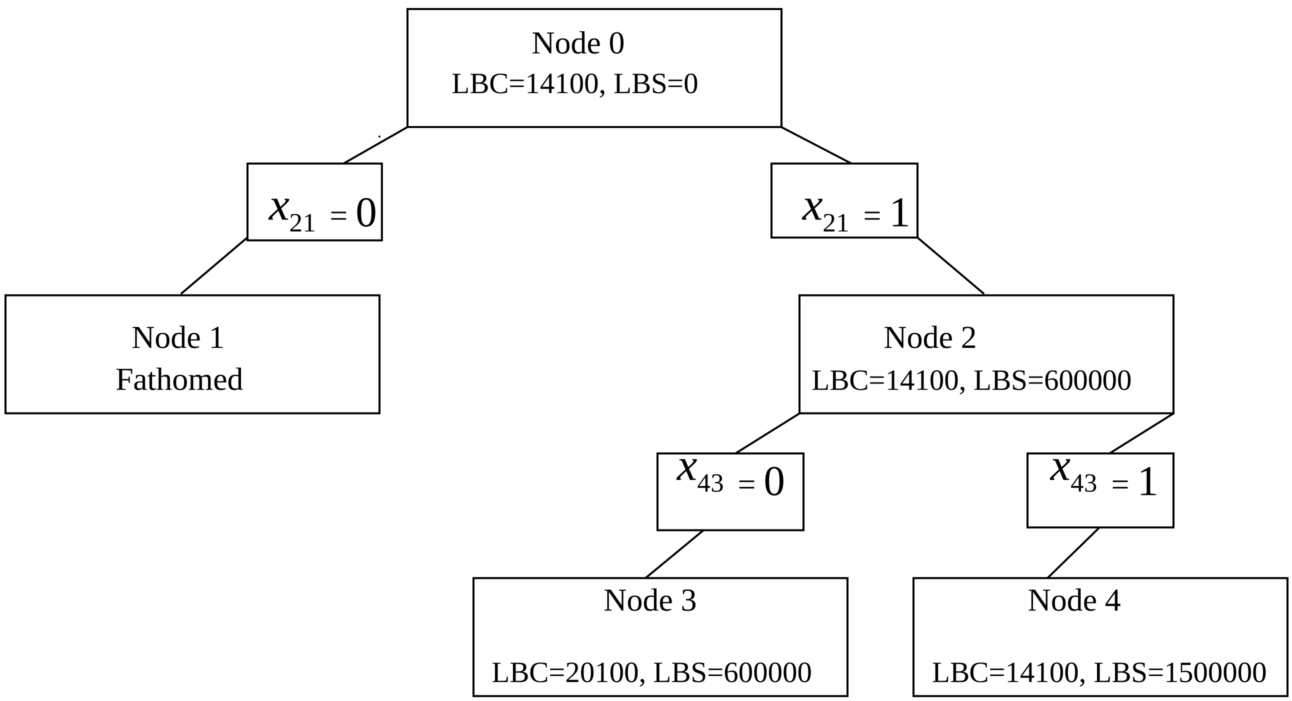

Figure 3.5

Table 3.11

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	4,000	2,000	5,200	5,600	0	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Since the largest penalty is corresponding to cell (1, 5).

In case of $x_{15} = 0$, then the cell (1, 5) will be blocked and make the reduced matrix (table 3.12).

Table 3.12

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	M	3,600	M	4,000
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 14100 + 2000 = 16100 and LBS = 1500000

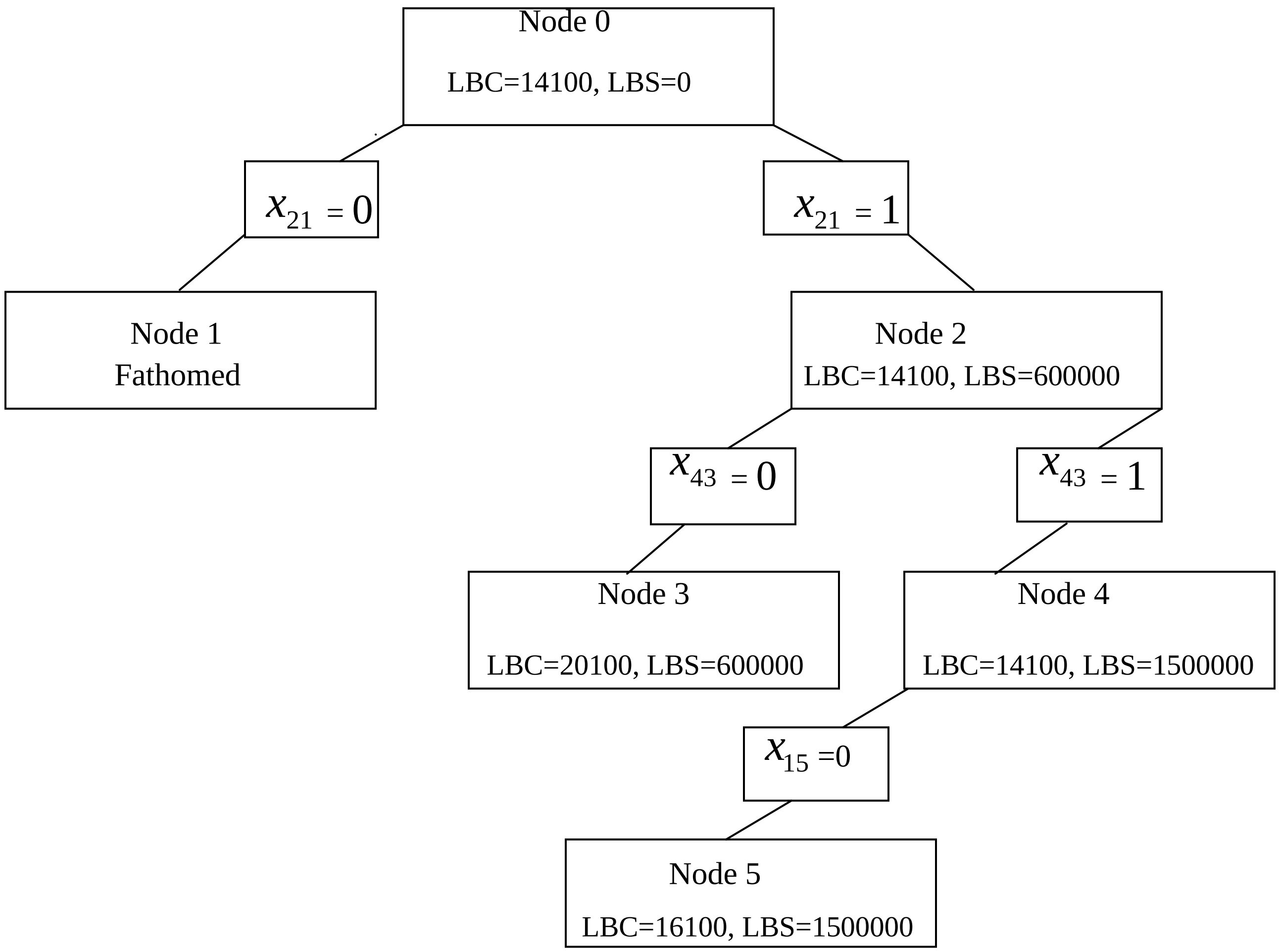

Figure 3.6

In case of $x_{15} = 1$, apply the algorithm for the cell (1, 5) and block the corresponding row and column and make the reduced matrix (table 3.13).

Table 3.13

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	3,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Using step 5 to step 7, LBC and LBS are obtained.

LBC= 14100 and LBS = 1500000 + 1200000 = 2700000

At the next level, LBC=14100 and site y_5 is selected and LBS = 1500000 (figure 3.7).

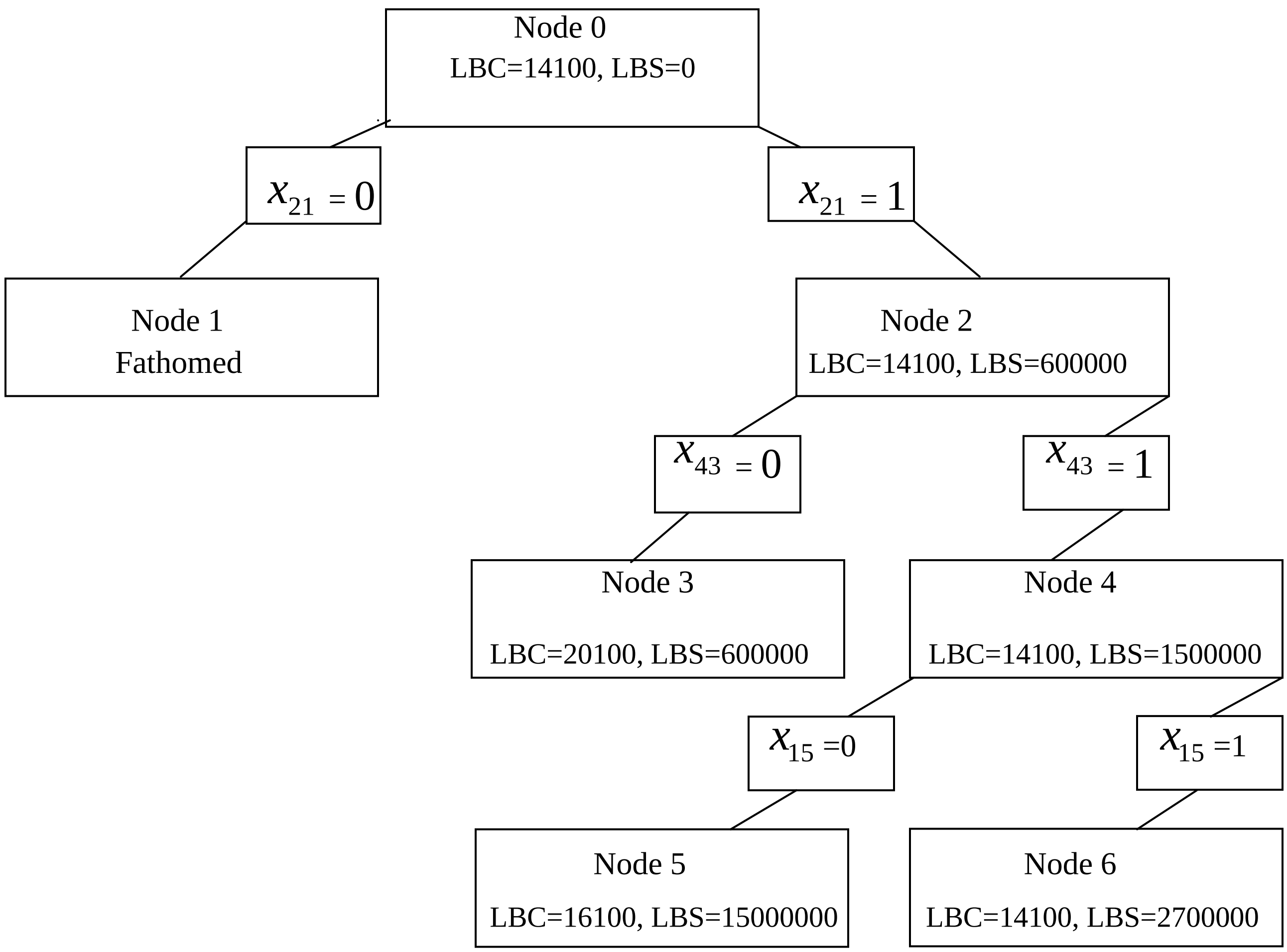

Figure 3.7

Similarly, the next allocation is corresponding to the cell (3, 5).

In case of $x_{35} = 0$, then the cell (1, 5) will be blocked and make the reduced matrix (table 3.14).

Table 3.14

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	3,900	3,600	300	0	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	3,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 14100 + 1800 = 15900 and LBS = 2700000

In case of $x_{35} = 1$, block the corresponding row and column and make the reduced matrix (table 3.15.

Table 3.15

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_3	5,700	5,400	2,100	1,800	0	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	3,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Using step 5 to step 7, LBC and LBS are obtained.

LBC= 14100 and LBS = 2700000

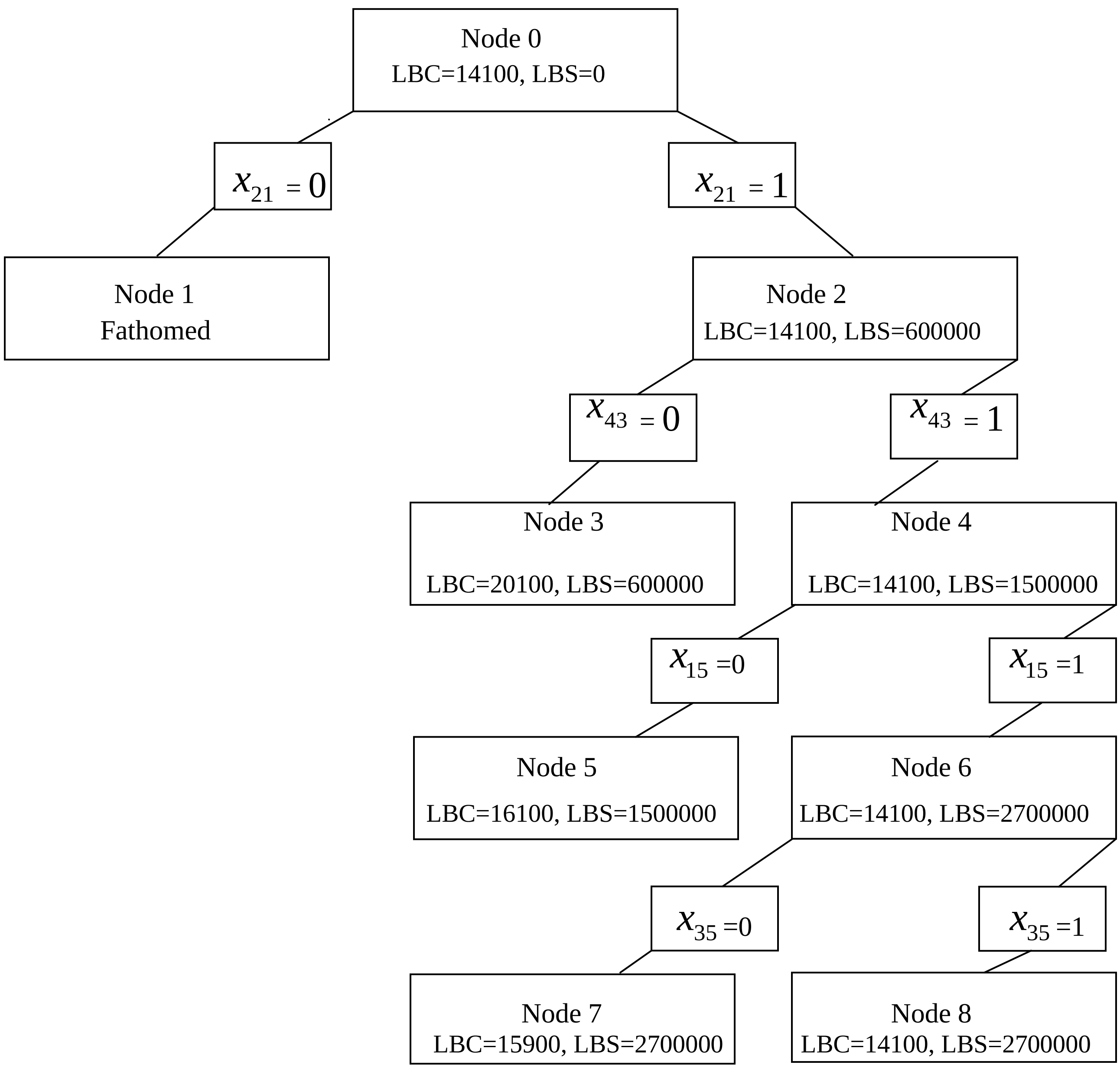

Figure 3.8: **(Tree drawn for obtaining 1st efficient solution of the numerical bi criterion call center problem)**

The optimal solution of the 1st prioritized bi criterion problem yields the efficient solution of the numerical problem.

The variables x_{21}, x_{43}, x_{15}, x_{35} at level 1 along the chain from the topmost node 0 to node 8 and the variables y_1, y_3, y_5 at least one provide the optimal solution $(\overline{X}^{(1)}, \overline{Y}^{(1)})$ of the 1st prioritized bi-criterion with $C(\overline{X}^{(1)})$ = 3600 + 5000 + 4000 + 1500 =14100 and $S(\overline{Y}^{(1)})$ = 600000 + 900000 + 1200000 = 2700000. This optimal solution yields the 1st efficient solution of the numerical problem. The remaining efficient solutions of the numerical problem are obtained by following the above procedure outlined in Subsection 3.2.

For obtaining the 2nd efficient solution of the numerical problem, obtain the 2nd prioritized bi-criterion problem from the 1st one by deleting all the variable x_{ij} corresponding to LBS ≥ $S(\overline{Y}^{(1)})$ = 2700000 and given in table 3.16.

Table 3.16

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	8,000	6,000	9,200	9,600	M	4,000
z_2	3,600	M	M	M	M	9,000
z_3	7,200	6,900	3,600	3,300	M	3,000
z_4	11,000	11,000	5,000	M	11,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Again apply the algorithm. Row Reduced matrix is given in table 3.17.

Table 3.17

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	3,200	3,600	M	4,000
z_2	0	M	M	M	M	9,000
z_3	3,900	3,600	300	0	M	3,000
z_4	6,000	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 6000 + 3600 + 3300 + 5000 = 17900

Node 0
LBC=17900, LBS=0

Figure 3.9

Now calculate the penalties of each row. In this case the penalties are: (1600, M, 3300, 5700) corresponding to cells (1, 2), (2, 1), (3, 4), (4, 3) respectively. The cell (2, 1) is selected and makes the allocation in cell (2, 1).

In the case of $x_{21} = 0$, then the cell (2, 1) will be blocked and make the reduce matrix given in table 3.18.

Table 3.18

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	3,200	3,600	M	4,000
z_2	M	M	M	M	M	9,000
z_3	3,900	3,600	300	0	M	3,000
z_4	6,000	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

This node is fathomed because no calls assigning in x_{21} .

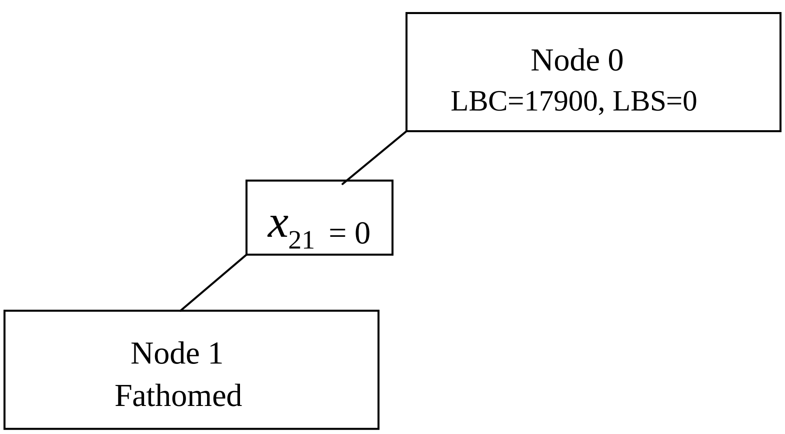

Figure 3.10

In case of x_{21} =1, then apply the algorithm for the cell (2, 1) and block the corresponding row and column and make the reduced matrix given in table 3.19.

Table 3.19

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	–	0	3,200	3,600	M	4,000
z_2	–	–	–	–	–	9,000
z_3	–	3,600	300	0	M	3,000
z_4	–	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	

LBC= 17900 + 0 = 17900 and LBS = 600,000

At level one, LBC=14100 and site y_1 is selected and LBS = 600,000 (figure 3.11).

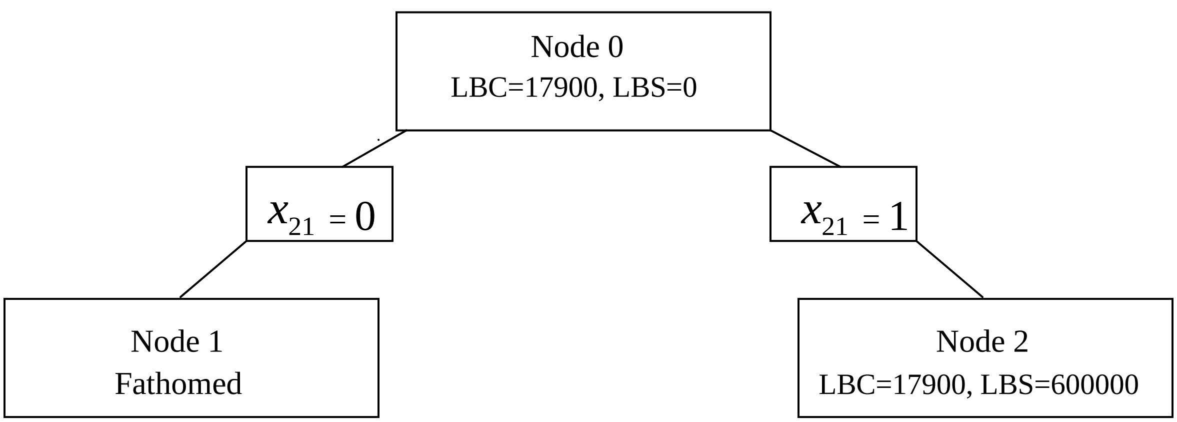

Figure 3.11

Next solution for the lower bound is shown in Table 3.20.

Table 3.20

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	3,200	3,600	M	4,000
z_3	3,900	3,600	300	0	M	3,000
z_4	6,000	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs S_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

The cell (4, 1) is blocked by M because $a_4 > b_4$ shown in Table 3.21.

Table 3.21

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	3,200	3,600	M	4,000
z_3	3,900	3,600	300	0	M	3,000
z_4	M	6,000	0	M	6,500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs S_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

Now calculate the penalties of each row and in this case the penalties are: (1600, 3300, 5700) corresponding to cells (1, 2), (3, 4), (4, 3) respectively. The cell (4, 3) is selected because the penalty on this cell is largest and makes allocation in the cell (4, 3).

In case of $x_{43} = 0$ then the cell (4, 3) will be blocked and make the reduced matrix (table 3.22).

Table 3.22

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	3,200	3,600	M	4,000
z_3	3,900	3,600	300	0	M	3,000
z_4	M	0	M	M	500	5,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	8,000	4,000	7,000	
Setup costs S_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 17900 + 6000 = 23900 and LBS = 600,000

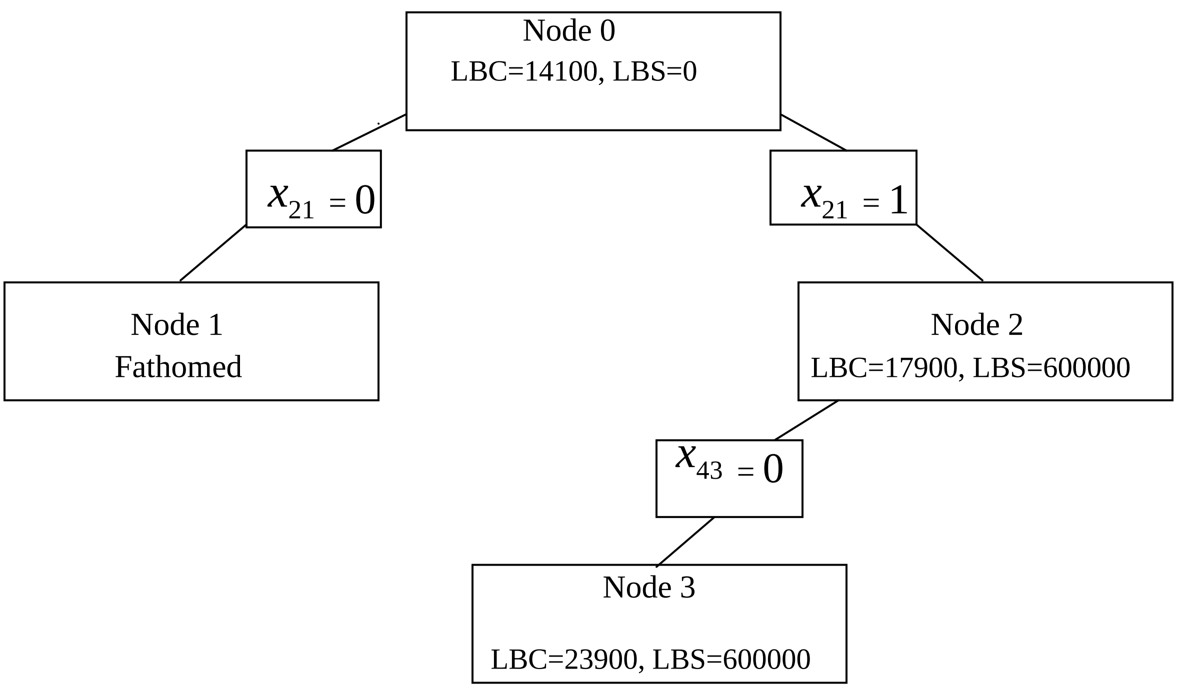

Figure 3.12

In case of $x_{43} = 1$ make the allocation in cell (4, 3) and update the table.

Table 3.23

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	3,200	3,600	M	4,000
z_3	3,900	3,600	300	0	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 17900 + 0 = 17900 and LBS = 600,000 + 900,000 = 1500000

At the next level, LBC=17900 and site y_3 is selected and LBS = 1500000 (figure 3.13).

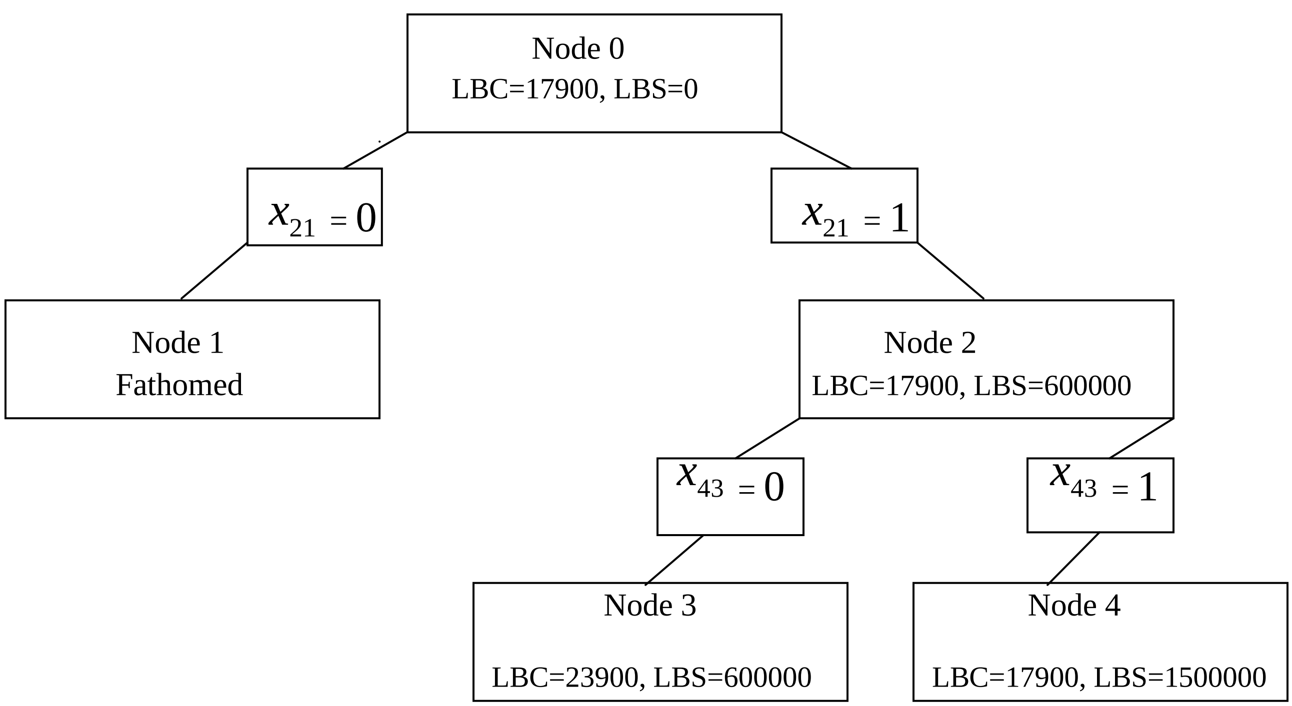

Figure 3.13

Table 3.24

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	M	3,600	M	4,000
z_3	3,900	3,600	300	0	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

The penalties are:(1600, 3300) corresponding to cell (1, 2) and (3, 4). Highest penalty is on (3, 4)

In case of $x_{34} = 0$, then the cell (3, 4) will be blocked and make the reduced matrix given in table 3.25.

Table 3.25

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	M	3,600	M	4,000
z_3	3,600	3,300	0	M	M	3,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 17900 +300 = 18200 and LBS = 1500000

In case of $x_{34} = 1$, make the allocation in cell (3, 4) and update the table.

Table 3.26

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	M	3,600	M	4,000
Capacities b_j's of call centers at sites P_j's	4,000	5,000	3,000	1,000	3,000	
Setup costs s_j of call centers at sites P_j's	600,000	1,000,000	900,000	900,000	1,200,000	

LBC= 17900 and LBS = 600000 + 900000 + 900000 = 2400000

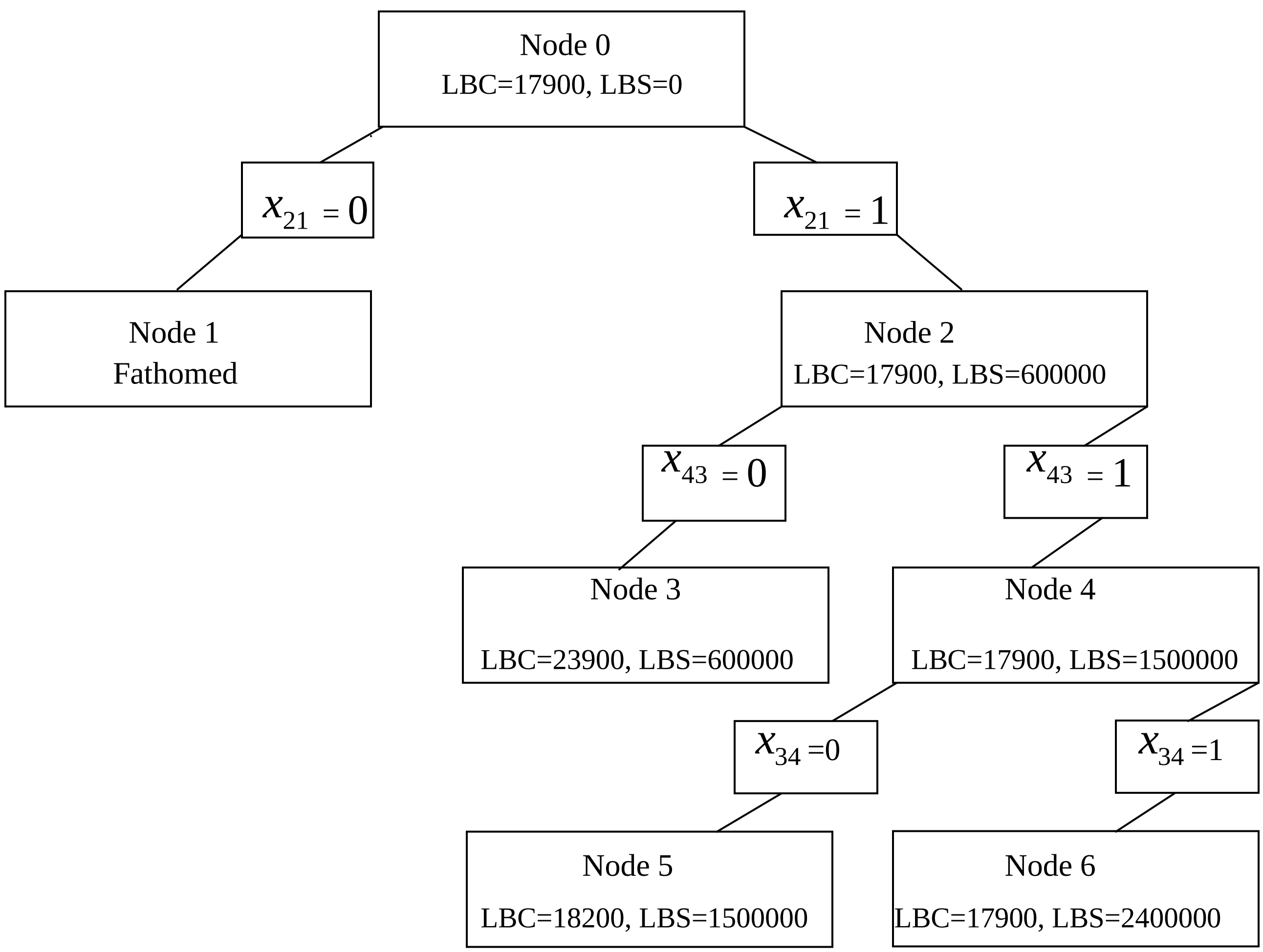

Figure 3.14

Table 3.27

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	2,000	0	M	3,600	M	4,000
Capacities b_j 's of call centers at sites P_j 's	4,000	5,000	3,000	1,000	3,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

In this case, we cannot find another solution for the lower bound because LBS $\geq$ 2700000.

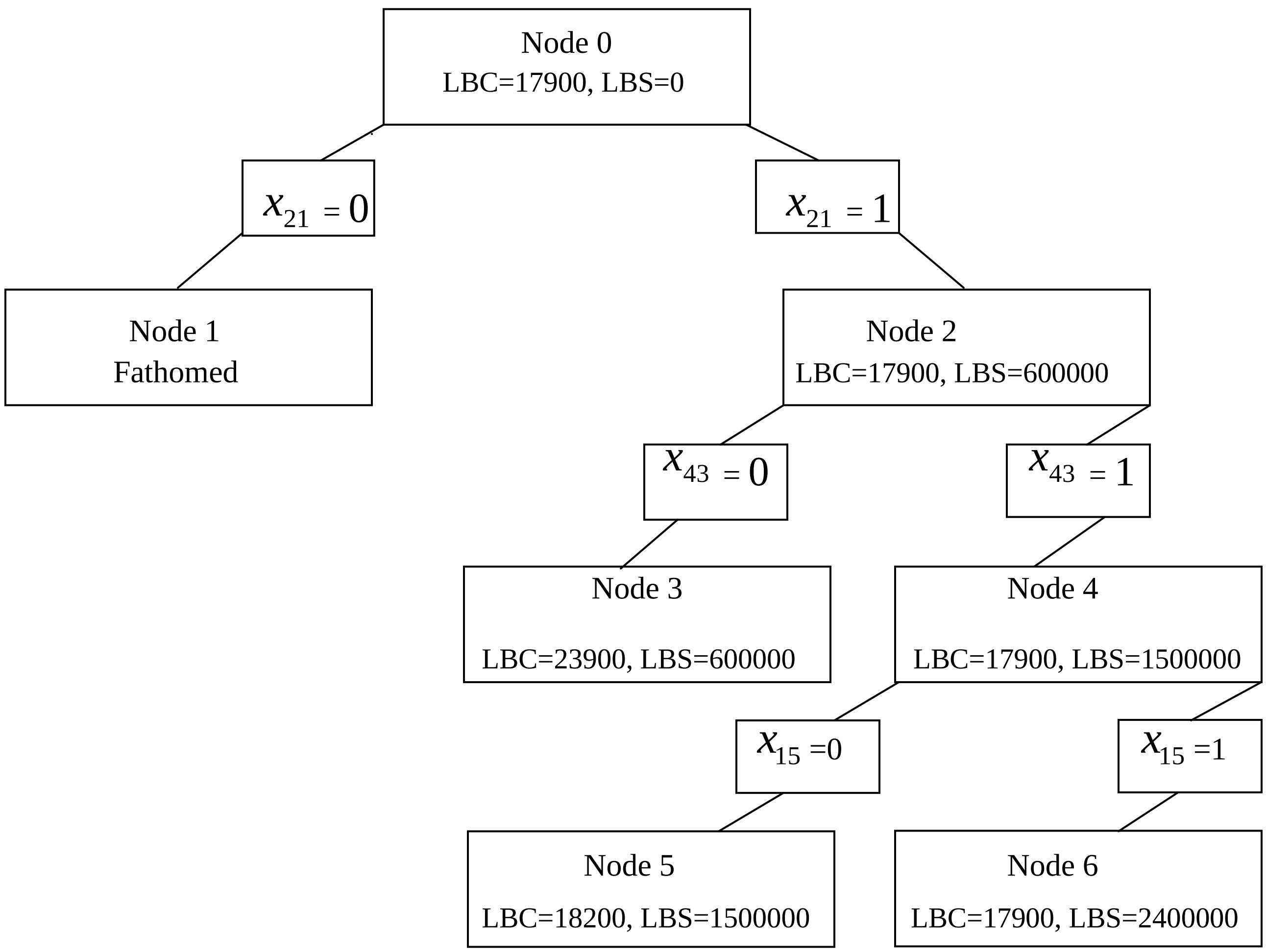

Figure 3.15: (Tree drawn for obtaining 2nd efficient solution of numerical bi criterion call center problem)

The unfathomed terminal node 6 thus obtained containing the variables x_{12}, x_{21}, x_{34}, x_{43} at level 1 and the variables y_{1}, y_{3}, y_{4} at level one provide the optimal solution $(\overline{X}^{(2)}, \overline{Y}^{(2)})$ of the 2nd prioritized bi criterion with $C(\overline{X}^{(2)})$ = 6000 + 3600 + 3300 + 5000 =17900 and $S(\overline{Y}^{(2)})$ = 600000 + 900000 + 900000 = 2400000. This optimal solution yields the 2nd efficient solution of the numerical problem.

Similarly we can obtain 3rd prioritized bi criterion problem $(\overline{X}^{(3)}, \overline{Y}^{(3)})$ the variable x_{ij} 's at level 1 are x_{12}, x_{21}, x_{33}, x_{43} at level 1 and the variables y_{1}, y_{3} at level one provide the optimal solution $(\overline{X}^{(2)}, \overline{Y}^{(2)})$ of the 3rd prioritized bi criterion with $C(\overline{X}^{(3)})$ = 6000 + 3600 + 3600 + 5000 =18200 and $S(\overline{Y}^{(3)})$ = 600000 + 900000 = 1500000. This optimal solution yields the 3rd efficient solution of the numerical problem.

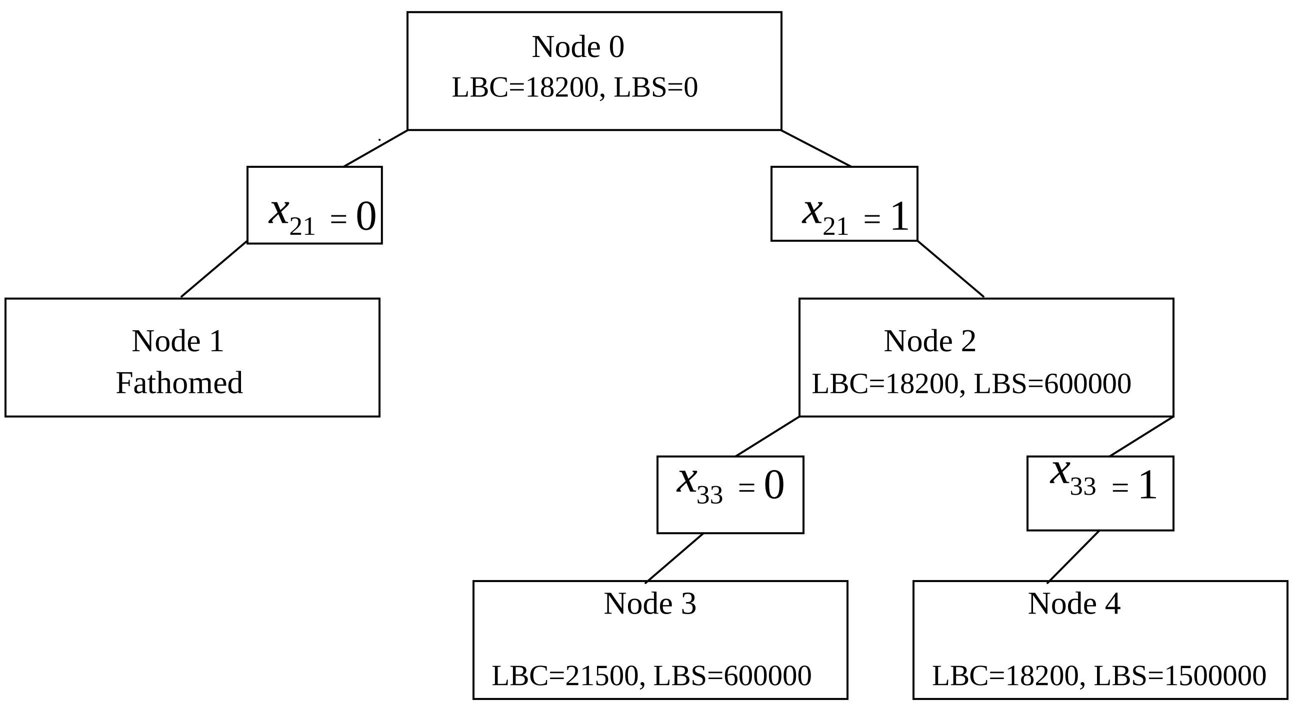

Figure 3.16: (Tree drawn for obtaining 3rd efficient solution of numerical bi criterion call center problem)

For obtaining the further optimal solution, we can obtain the 4th prioritized bi criterion problem from 3rd one by deleting all the variable x_{ij} corresponding to LBS $\geq S(\overline{Y}^{(3)})$ = 1500000 and given in table 3.28.

Table 3.28

	Potential call center sites					
Zones of call origin	P_1	P_2	P_3	P_4	P_5	Bulk calls a_i from zones z_i
z_1	8,000	6,000	9,200	9,600	M	4,000
z_2	3,600	M	M	M	M	9,000
z_3	7,200	6,900	M	M	M	3,000
z_4	11,000	11,000	5,000	M	11,500	5,000
Capacities b_j 's of call centers at sites P_j 's	13,000	5,000	8,000	4,000	7,000	
Setup costs s_j of call centers at sites P_j 's	600,000	1,000,000	900,000	900,000	1,200,000	

On applying the same procedure above, it is not possible to find a new efficient solution with a lesser total call centers setup cost $S(\overline{Y}^{(3)}) = 1500000$ and the process of obtaining further efficient solutions should be terminated.

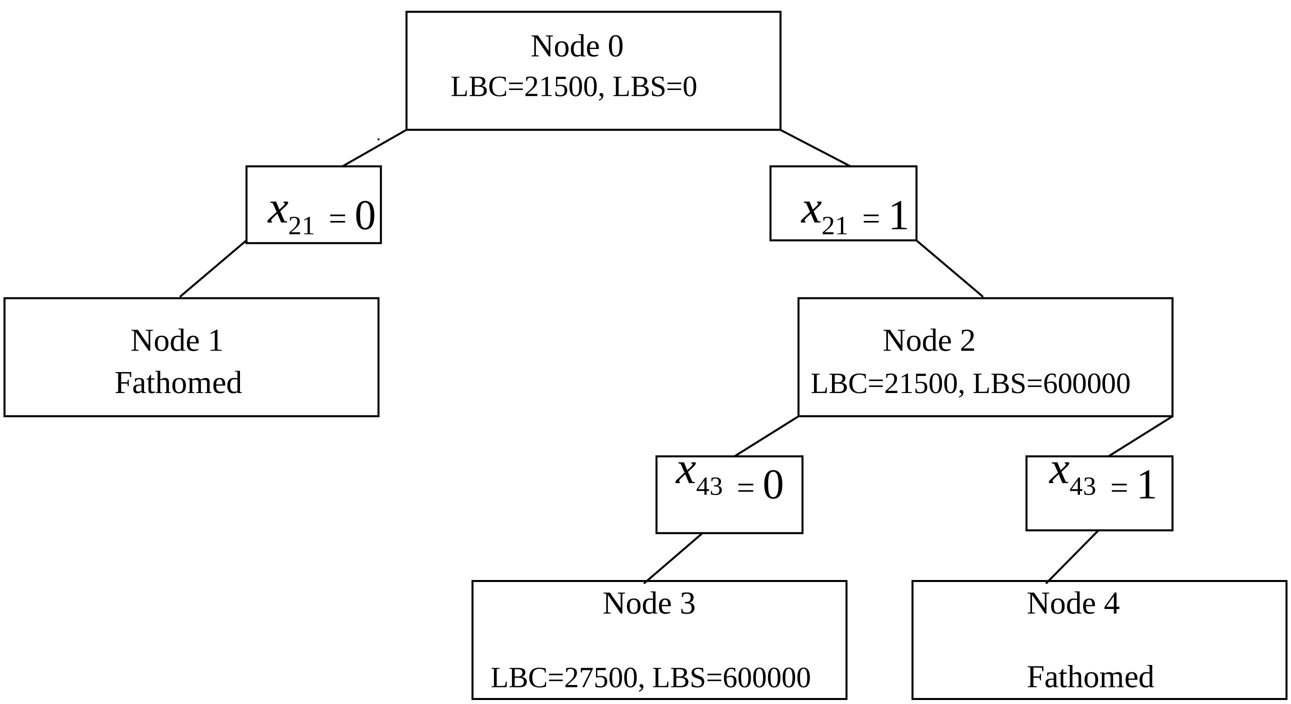

Figure 3.17: (Tree drawn showing no efficient solution of numerical bi criterion call center problem)

The numerical problem is found to have only three efficient solutions. The set of the numerical problem together with the variables x_{ij} 's y_i 's at level one, recurring total call charges cost $C(\bar{x})$'s and nonrecurring total call centers setup cost $S(\bar{y})$'s are shown in table 3.29.

Table 3.29 Set of efficient solutions of the numerical problem

Efficient solution	Variables at level 1	Recurring total call charges cost	Non recurring total call centers setup cost
$(\bar{X}^{(1)}, \bar{Y}^{(1)})$	$x_{21}, x_{43}, x_{15}, x_{35}; y_1, y_3, y_5$	$C(\bar{X}^{(1)})$ =14100	$S(\bar{Y}^{(1)})$ =2700000
$(\bar{X}^{(2)}, \bar{Y}^{(2)})$	$x_{12}, x_{21}, x_{34}, x_{43}; y_1, y_3, y_4$	$C(\bar{X}^{(2)})$ =17900	$S(\bar{Y}^{(2)})$ =2400000
$(\bar{X}^{(3)}, \bar{Y}^{(3)})$	$x_{12}, x_{21}, x_{33}, x_{43}; y_1, y_3$	$C(\bar{X}^{(3)})$ =18200	$S(\bar{Y}^{(3)})$ =1500000

Table 3.30 Comparison of results with the existing algorithm

Efficient solution	Variables at level 1	Recurring total call charges cost	Non-recurring total call centers setup cost
$(\bar{X}^{(1)}, \bar{Y}^{(1)})$	$x_{21}, x_{43}, x_{15}, x_{35}; y_1, y_3, y_5$ $x_{21}^*, x_{43}^*, x_{15}^*, x_{35}^*; y_1^*, y_3^*, y_5^*$	$C(\bar{X}^{(1)})$ =14100 $C(\bar{X}^{(1)}) = 14100^*$	$S(\bar{Y}^{(1)})$ = 2700000 $S(\bar{Y}^{(1)}) = 2700000^*$
$(\bar{X}^{(2)}, \bar{Y}^{(2)})$	$x_{12}, x_{21}, x_{33}, x_{43}; y_1, y_2, y_3$ $x_{12}^*, x_{21}^*, x_{34}^*, x_{43}^*; y_1^*, y_3^*, y_4^*$	$C(\bar{X}^{(2)})$ =18200 $C(\bar{X}^{(2)}) = 17900^*$	$S(\bar{Y}^{(2)})$ =2500000 $S(\bar{Y}^{(2)}) = 2400000^*$
$(\bar{X}^{(3)}, \bar{Y}^{(3)})$	$x_{11}, x_{21}, x_{34}, x_{43}; y_1, y_3, y_4$ $x_{12}^*, x_{21}^*, x_{33}^*, x_{43}^*; y_1^*, y_3^*$	$C(\bar{X}^{(3)})$ = 19900 $C(\bar{X}^{(3)}) = 18200^*$	$S(\bar{Y}^{(3)})$ =2400000 $S(\bar{Y}^{(3)}) = 1500000^*$
$(\bar{X}^{(4)}, \bar{Y}^{(4)})$	$x_{11}, x_{21}, x_{33}, x_{43}; y_1, y_3$	$C(\bar{X}^{(4)})$ = 20200	$S(\bar{Y}^{(4)})$ =1500000

$*$ = results given by modified approach.

3.4 CONCLUSION

A real life problem of selecting sites for setup of call centers from among a given number of potential sites for assigning a given number of zones of call origin to them subject to several constraints with two objectives without being prioritized is considered. A branch and bound algorithm given by Prakash *et. al.* (2011) has been reviewed and an attempt has been made to modify this approach to find better solutions. It has been shown that the existing approach yield four efficient solutions however the modified approach yield only three efficient solutions but the modify approach find two efficient solutions better than the existing approach and a comparison has been shown in table 3.30.

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