Seminar

on

" PRACTICAL ANALYSIS OF CONCRETE BUILDING FRAMES FOR VERTICAL LOADS "

By

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CERTIFICATE

Certified that the special problem (Seminar/ Design project) on " Practical Analysis of concrete Building frames for vertical loads " which is being submitted by Mr.Yougesh Kumar Gupta, in partial fulfilment for the award of Post-Graduate Diploma in Civil (Structures) Engg. of the Punjabi University, Patiala, is a record of student's own work carried out by him under my supervision and guidance.

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(Yougesh Kumar Gupta)

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S Y N O P S I S

The purpose of this study is to explain briefly a method which has been found useful in analyzing building frames which are statically indeterminate. The determination of Maximum moments and shears in beams which are quite satisfactory from design point of view has been discussed extensively. The determination of design moments for columns and point of inflection in beams have also been discussed.

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1.0 INTRODUCTION :-

The building frame analysis has assumed a position so prominent that it is now one of the most important subjects with which a structural engineer is confronted. The moment analysis of such frames by rigors methods with hand computations becomes too laborious and time consuming due to high degree of indetermency. So some practicle methods of analysis of such frames were thought of from time to time by engineers. The practical method described here is known as " two cycle method of moment distribution". This method is based on moment distribution method presented by Prof. Hardy Cross in 1929. In this method the original moment distribution has been limited to what is called " two cycles " and this limitation, slight as it may seem, opens up opportunities for introducing a labour and time saving arrangement of recording. But this special type of recording should not be misinterpreted and construed to signify that moment distribution method has been changed or improved . It has merely been condensed. The procedure for condensation was presented by Portland Cement Association in 1941. This method is identically the same as described in Appendix 2 of the *Joint Committee Report 1940, where formulas are derived for the solution of moment distribution, but these are rather complicated. The results are same as the Joint Committee procedure goes, but the arrangement of calculations is changed and provision is made for determination of mid span moments and column moments.

* "Joint Committee on Standard specification for concrete & reinforced concrete" of America.

2.0 PRECISION OF ANALYSIS

The two cycle method can be safly adopted for determining design moments. Because the so called "exact" methods are laborious and the use of such methods is not justified in concrete structure for several reasons given below :-

(i) E_c. The modulus of elasticity of concrete vary considerably in the same structure with different condition of age and moisture.

(ii) I_c. As the concrete in slabs is generally monolithic with beams and it is uncertain how much of the slab is acting as flange. A much wider portion of slab acts than that limited amount available for tee in stress computation as specified in v-arious codes.

(iii) A_s. There is a growing practice to determine M.O.I from the gross section of concrete ignoring the steel reinforcement which is not known before hand.

(iv) Variable I. Most of the concrete beams are monolithic with slabs so they act as Tee-beams for +ve moment and Rectangular beams for -ve moments. As the character of B.M changes in beam of a frame so the M.O.I is also variable.

(v) Loads. In computations loads are customarily placed in those positions that will produce absolute maximum values, regardless of the fact that such loading conditions may never be realized in the life of the structure.

(vi) Discrepancies. Minor discrepancies will occure in construction . Column centre will vary a small fraction of a cm. and cross sections of members may vary quite a bit from the theoratical sizes. Such variations do not amount to anything with statically determinate structures, but the effect in continuous frames may be to change considerably the assumed ratios of member stiffness.

On the other hand it should be noted that tests of models and fullsized structures have demonstrated that reinforced concrete frames are elastic in their behavior to a surprising degree, and that designs made by the principles of continuity have proved capable of severe overloading and satisfactory in all respects. The above argument is merely to demonstrate that no hair-splitting is practicable and that a reasonably accurate approximation is all that is necessary or desired.

3.0 LOADING PATTERNS FOR MAXIMUM MOMENTS.

The moments, shears and axial loads are brought about in part by the weight of the structure (dead load) and in part by the live load. While the farmer is constant, live loads can be placed in various ways, some of which will result in larger effects than others.

In figures 1(a) only span CD is loaded by live load. The distortion of various frame members are seen to be

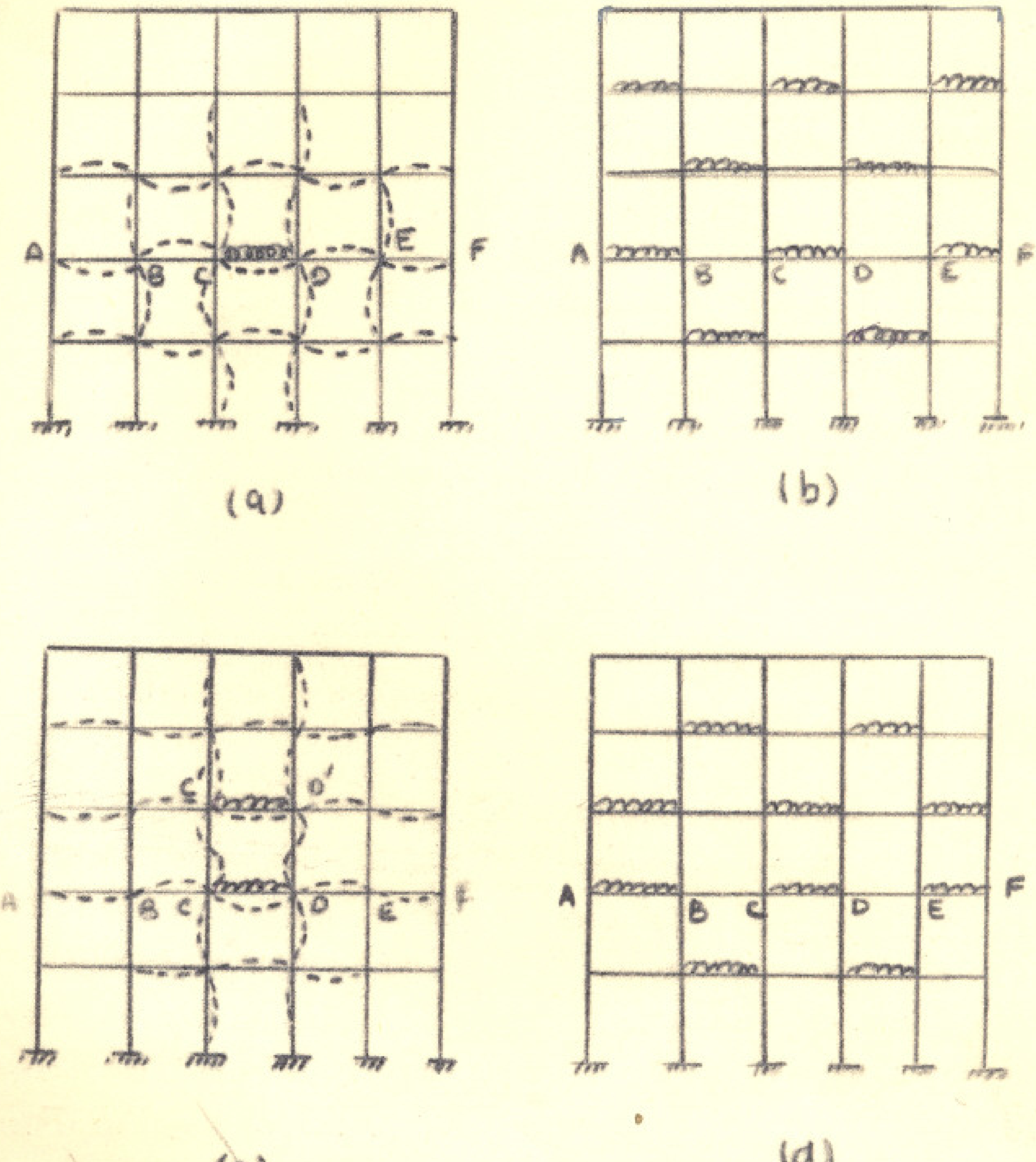


FIG (1)

largest in and immediately adjacent to the loaded span,and decrease rapidly with increasing distance from the load. Since B.M is proportional to curvatures, the moments in more remote members are correspondingly smaller than those in , or close to, loaded span. However, the loading of figure 1 (a) does not produce maximum moments in CD. In fact the"checker board pattern " of live load of figure 1 (b) produces largest possible positive moments not only in CD but in all loaded spans. Hence two such checker board patterns are required to obtain the maximum positive moments in all spans. In addition to maximum span moments the loading pattern in figure I (b) gives minimum span moments in unloaded spans.

Maximum negative moments at the supports of the girders are obtained , on the other hand, if the loads are placed on two spans adjacent to the particular support and in a corresponding pattern on the more remote girders.

While the loading of figure 1 (c) results in large moments at the ends of the column cc¹ and DD '. These moments are further augmented if additional loads are placed as shown in figure 1 (d).

4.0 MOMENT OF INERTIA OF BEAMS AND COLUMNS

Regarding the stiffness the IS: 456- 1978 stipulates:

21.3 Stiffness

21.3.1 Relative stiffness- The relative stiffness of the members may be based on the M.O.I of the section determined on the basis of any one of the following definations:

a) Gross Section- The cross-section of the member ignoring reinforcement ;

b) Transformed section- The concrete cross-section plus the area of reinforcement transformed on the basis of modular ratio ; or

c) Cracked section- The area of concrete in compression plus the area of reinforcement transformed on the basis of modular ratio;

The assumption made shall be consistent for all the members of the structure throughout any analysis.

According to ACI : 318- 1971:

8.5.3.1 Any reasonable assumptions may be adopted for computing the relative flexural and torsional stiffnessess of columns, walls, floors and roof systems. The assumption made shall be consistent throughout the analysis.

The M.O.I bassed on Gross section is very simple to calculate and is practicable as it eliminates the design of reinforcement and the determination of centroid of the section.

Fortunately, in the moment analysis of a continuous structure, the only use to which M.O.I is put is in the computation of the stiffness of the members and of distribution factors at the joints of the structure. But the distribution factors at a joint can be obtained by using the relative I's as well as by using exact I's. And the relative I's of the members can be obtained in a simpler manner than the exact I's.

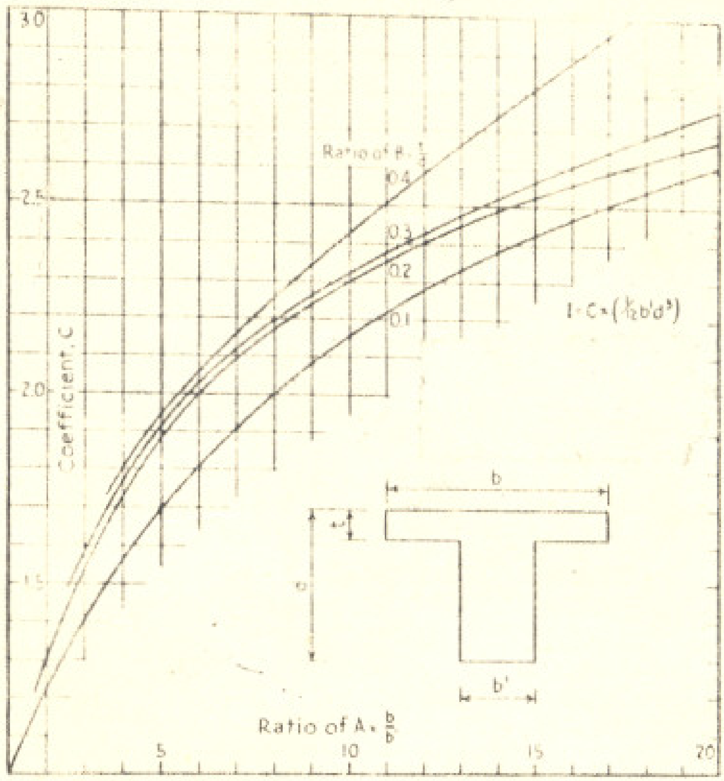
For the purpose of moment analysis the I for a rectangular section is $\frac{bh^3}{12}$, considering only gross concrete section.

The I's obtained in this manner are always larger than the exact I's and the ratios of the relative I's to the exact I's are not constant. However, small variation in M.O.I and stiffness have little effect on the moments, and therefore the use of the relative I's in moment analysis is justified.

For T- shaped sections, typical of reinforced concrete beams, an allowance must be made for the effect of the flange. If the reinforcement is neglected, the I of the section shown in figure 2 is given by the formula

$$I = C \cdot \frac{b' h^3}{12}$$

in which $c = 1 + (b/b'-1)(t/h)^3 + \frac{3(1-t/h)^2}{1 + 1/\frac{t}{h}(b/b'-1)}$



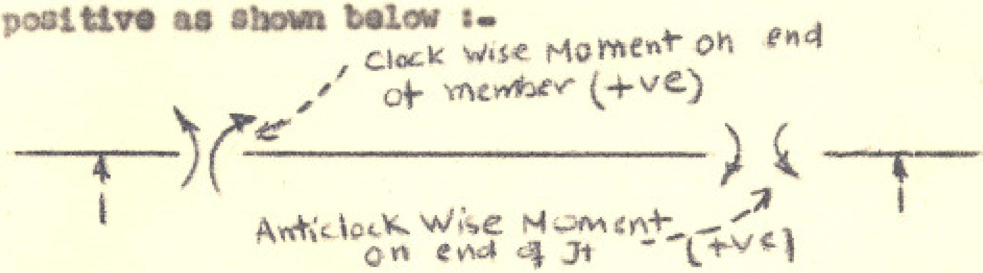
Coefficients for moment of inertia of T-beams.

FIG. 2

Figure 2 shows the value of C, for various values of t/h, plotted as ordinates against abscissas of b/b'. It can be seen that, for the common values of t/h between 0.2 and 0.4 and of b/b' between 5 and 6 values of C deviate but little from 2. Therefore, in the preliminary analysis, a convenient way to compute the I of a T- section is to double the I of the rectangular section b'h.

5.0 SIGN CONVENTION

It is assumed that clockwise end moment on the ends of the members or anticlockwise on the ends of the joints are positive as shown below :-



The distributed moments have sign opposite to that of unbalanced moments.

The carry over moments have the same signs as the distributed moments or the opposite signs to that of unbalanced moments.

6.0 CALCULATION OF MAXIMUM MOMENTS IN BEAMS

6.1 Basic assumptions:-

For calculating the maximum moments in the beams following simplifying assumptions as specified in various codes are made.

IS : 456 - 1978 :

21.4.2 Substitute Frame - For determining the moments and shears at any floor or roof level due to gravity loads, the beams at that level togather with columns above and below with their far ends fixed may be considered to constitute the frame.

ACI : 318 - 1971 :

8.5.1 The live load may be considered to be applied only to the floor or roof under consideration, and the far ends of the columns may be assumed as fixed.

6.2 Procedure:-

The procedure for computing maximum moments can be explained as below :-

i) Isolate the floor to be designed from the entire frame.

ii) Calculate the relative stiffness of each member, i.e, I/L value.

iii) Calculate the distribution factors at the joints. i.e $F_{ij} = \frac{K_{ij}}{\Sigma K_{ij}}$

iv) Calculate the fixed end moments for the Dead Load and total Load on the member separately.

v) After calculating the quantities described above analysis is done for

a) Maximum Negative Moments at the support.

b) Maximum Positive moments at the center.

c) Minimum moments at the center as described below :-

6.2.1 CALCULATION OF MAXIMUM SUPPORT MOMENTS

The calculation of maximum moments at support can be done easily by the procedure explained in Table 1.

T A B L E - 1

Calculation of Maximum Support [illegible]

1. Dist. factor	r_{AB}		r_{BA}	r_{BC}		r_{CB}	r_{CD}		r_{DC}
2. F.E.M. D.L.	-		$+M_{DL_1}$	$-M_{DL_2}$		$+M_{DL_2}$	$-M_{DL_3}$		-
3. F.E.M. T.L.	$-M_{TL_1}$		$+M_{TL_1}$	$-M_{TL_2}$		$+M_{TL_2}$	$-M_{TL_3}$		$+M_{TL_3}$
4. Dist. & C.O	Q_{A_1}	$-(M_{TL_1}-M_{DL_1})\, r_{BA}/2$ $-(-M_{TL_1}-0)\, r_{AB}/2$	Q_{B_1}	Q_{B_2}	$-(M_{TL_2}-M_{DL_3})r_{CB}/2$ $-(-M_{TL_2}+M_{DL_1})r_{BC}/2$	Q_{C_2}	Q_{C_3}	$-(+M_{TL_3}+0)\, r_{DC}/2$ $-(-M_{TL_3}+M_{DL_2})r_{CD}/2$	Q_{D_3}
5. Addition (3 + 4)	$X_{AB} = -M_{TL_1} + Q_{A_1}$		$X_{BA} = +M_{TL_1} + Q_{B_1}$	X_{BC}		X_{CB}	X_{CD}		X_{DC}
6. Dist.	$-r_{AB} \cdot X_{AB}$		$-r_{BA} \cdot (X_{BA} + X_{BC})$	$-r_{BC} \cdot (X_{BC} + X_{BA})$		$-r_{CB} \cdot (X_{CB} + X_{CD})$	$-r_{CD} \cdot (X_{CD} + X_{CB})$		$-r_{DC} \cdot X_{DC}$
7. Max. moments (5 + 6)	$-M_{AB}$	+	$+M_{BA}$	$-M_{BC}$		$+M_{CB}$	$-M_{CD}$		$+M_{DC}$
8. Max. col. moments									

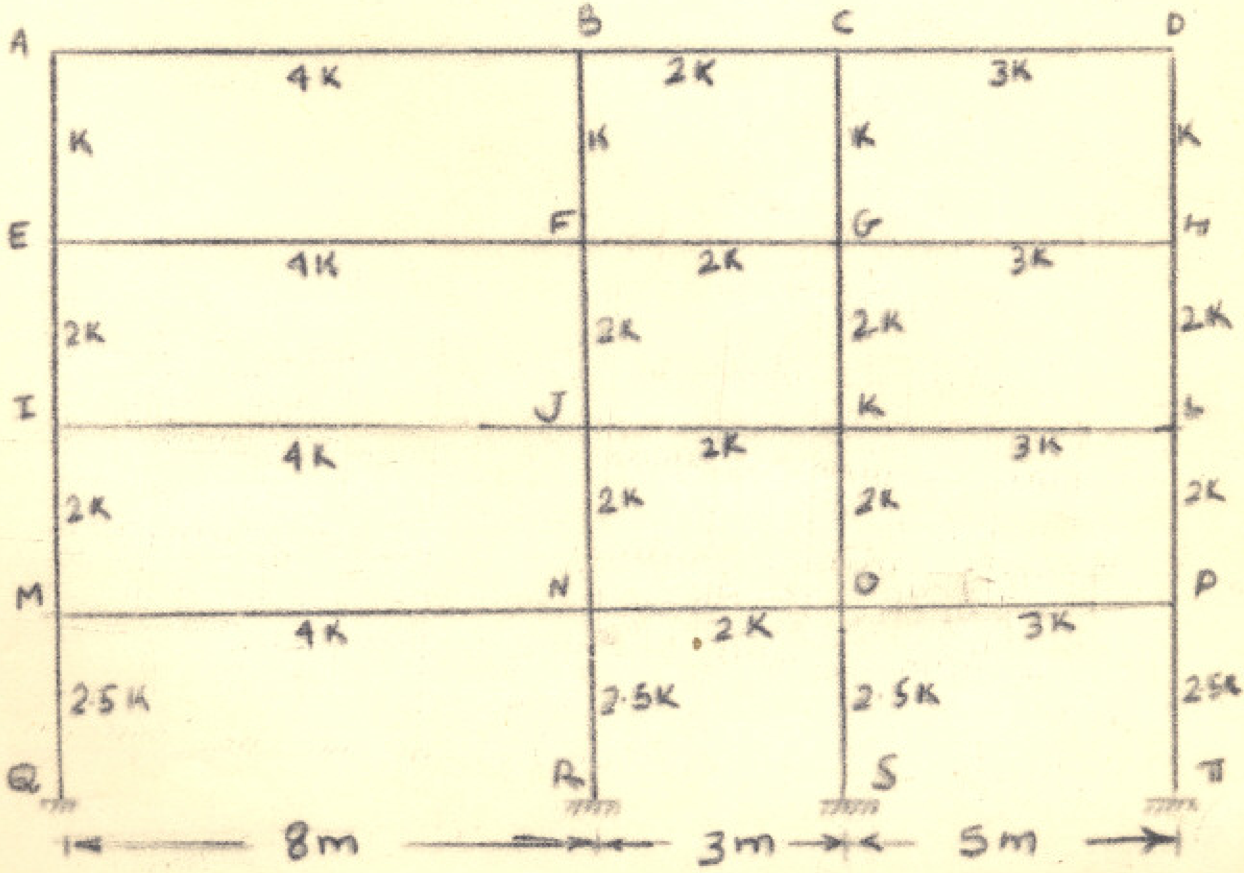
NOTATIONS: M_{DL_n} = F.E.M due to D.L in n[th] span.
M_{TL_n} = " " T.L " "

r_{BA} = Distribution factor at joint B for member BA.
Q_{A_n} = C.O at joint A where subscript n shows the span.

The operations illustrated in the Table I actually covers two complete cycles of distribution, which in the ordinary type of recording means that moments are distributed twice. Yet only one distribution is in evidence. This is because the usual two distributions have been combined in one operation. Moments are included with F E M' s before the distribution is made. The operation in Table 1 can be easily explained with the help of an example.

ILLUSTATIVE EXAMPLE:

Analyse the frame shown in figure given below .The frames are at 4 m interval. Dead load is 350 kg/ m^2 and live load is 500 kg/ m^2 .



Solution:-

Second floor analysis of the frame is done here. Self wt. of the beams IJ, JK and KL are assumed 500 kg/m , 300 kg/m, 400 kg/m respectively.

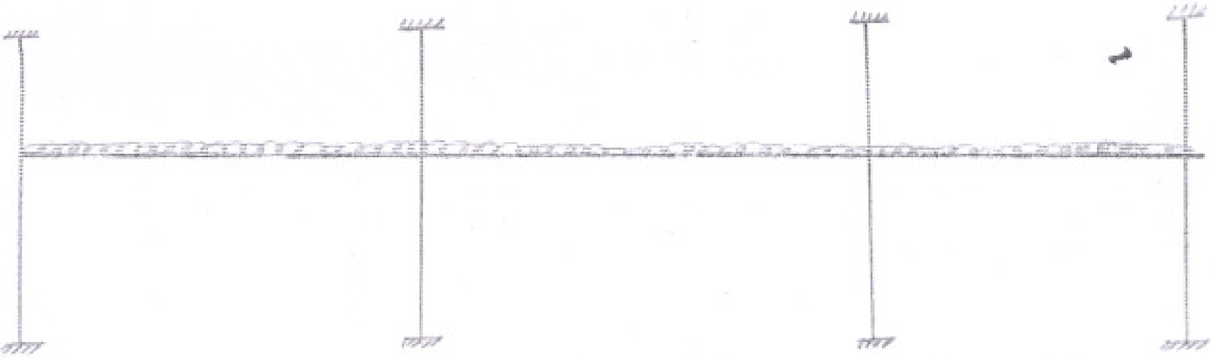
D.L from slab/ m of girder = 350 x 4 = 1400 kg/m

L.L " " " = 500 x 4 = 2000 kg/m

F.E.M for D.L and T.L for girder IJ, JK and KL are shown in table below:

Span	D.L.F.E.M	T.L.F.E.M.
IJ	10133 kg-m	20800 kg-m
JK	1275 "	2775 "
KL	3750 "	7917 "

The second floor is isolated from the whole frame. To determine the maximum end moment at I place Total load on IJ and Dead load on JK as shown in figure 3. As long as J is considered fixed, end moment at J is + 20800 to left and - 1275 to right. The unbalanced moment is U = (20800 - 1275) = + 19525. When J is released, the moment distributed to the left of J is - U x D.F = - 19525 x 0.4 and the moment carried over to I while I remains fixed is - U x D.F / 2'= - 19525 x 0.4 /2 = 3905. This value is written as shown in figure 3 but neither U or - U x D.F is recorded. Joint J is then relocked in its new position. Now turn to I which so far has been considered locked. Original F.E.M. is-20800



1. Dist. factor	½		0.4				1/3		3/7
2. D.L.F.E.M		TL		-1275	DL DL	+1275		TL	
3. T.L.F.E.M	-20800		+20800				-7917		+7917
		-0.4x½						-1/3x½	
4. Dist.& C.O	-3905								+1107
5. Addition	-24705								+9024
6. Dist.	+12353								-3867
7. Final Max.	-12352								+ 5157

D.F	½		0.4	0.2		2/9		
D.L.F.E.M		TL			TL		-3750	DL
T.L.F.E.M	-20800		+20800	-2775		+2775		
Dist.& C.O.		-½ x ½	+ 5200	+108	-2/9 x ½			
Addition			+26000	-2667				
Dist.			-9334	-4667				
Final.max.			+16666	-7334				

FIGURE 3

but release and rotation of J transfers an additional moment to I, and at this stage the modified total F.E.M is (- 20800 - 3905) = - 24705. Since there is no F.E.M to the left of I, unbalanced moment U at I equals - 24705. Releasing I and permitting it to rotate induces a distributed moment at I equal to - U x D.F = -(-24705 x $\frac{1}{2}$) = + 12353. Thus Final maximum moment at I is given by (- 24705 + 12353) = - 12352 kg-m . Then to determine maximum -ve moment at J total load is placed on I J and J K with D.L on others. Now begin with releasing I and K . The figure 3 clearly shows how to compute the two moments + 5200 and + 108. While J is still considered fixed the modified total F.E.M at J are + 26000 and - 2667. Then release joint J. Multiply the unbalanced moment of (26000 - 2667) = 23333 by distribution factors i.e 23333 x 0.4 =+9334 and 23333 x 0.2=+ 4667. These values with negative sign give the distributed moments. Then addition of these to above gives final maximum moments. The computations presented in figure 3 in expanded form can be condensed into a single table shown in figure 4. A comparison of the two methods of recording shows that all the numbers in different tables of expanded examples fits into the blank spaces of the other tables, and the overlapping occurs only when the same number is used in two different tables. In a condensed version such a number is written once and is used successively in several operations.

	I			J			K		L
Dist. factor	1/2		0.4	0.2		2/9	1/3		3/7
D.L.F.E.M	-10133		+10133	-1275		+1275	-3750		+3750
T.L.F.E.M	-20800		+20800	-2775		+2775	-7917		+7917
Dist. & C.O.	- 3905		+ 5200	+ 108		- 736	-1697		+1107
Addition	-24705		+26000	-2667		+2039	-9614		+9024
Distribution	+12353		-9334	-4667		+1638	+2575		-3807
Final Max. moments	-12352		+16666	-7334		+3677	-7089		+5157
	+6176								-2578

FIGURE - 4.

6.2.2 MAXIMUM MOMENTS AT THE INTER MEDIATE POINTS IN THE SPAN :-

The procedure for computing maximum moments at inter-mediate points can be developed as follows:

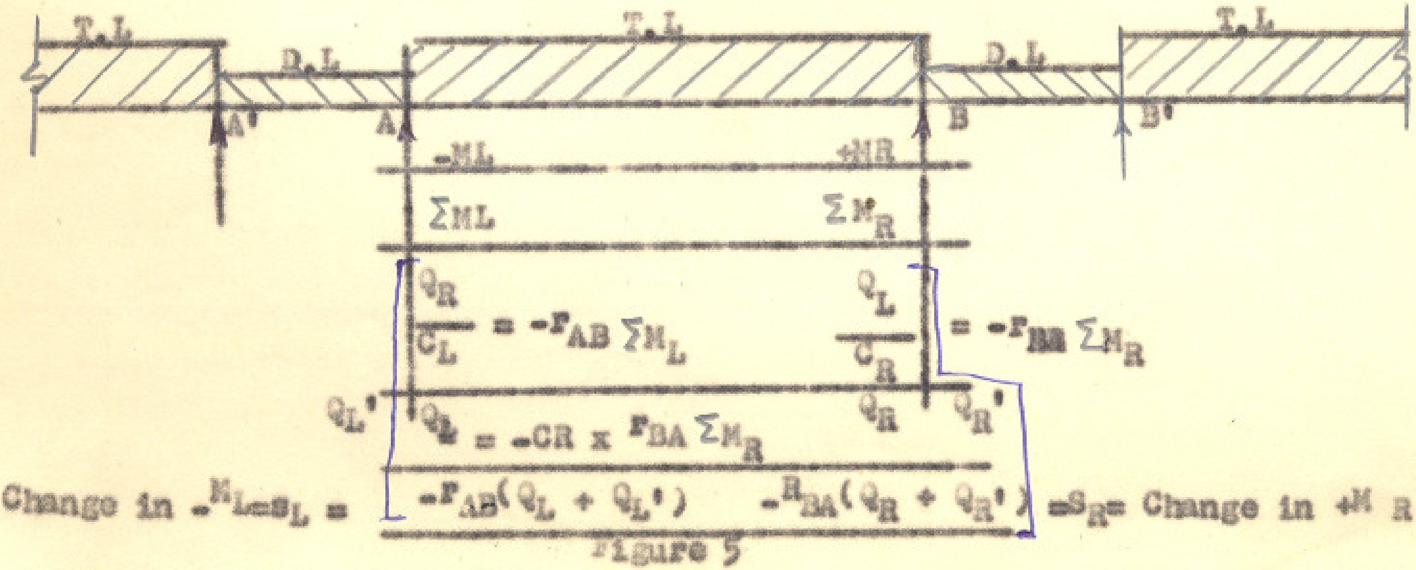


Figure 5

Notation :

M = F.E.M

ΣM = Unbalanced moment

r = Distribution factor

c = Carry over factor

Q = C. r. ΣM

S = Change in F.E.M.

L; R = Subscripts indicating the left and right ends of span.

Figure 5 shows the five spans and the standard moment distribution operations carried out to determine the changed in the initial fixed end moments in the central span. The above notation is used to indicate the various quantities at the ends of the central span.

The sum, s , of the last three terms at either end of the span is equal to the change in the F.E.M at that end.

A little reflection will show that a positive change in S_L , its sign based on moment distribution convention, produces a numerical decrease in $-M_L$; and that the effect of this decrease, represented diagrametically by lowering of the left

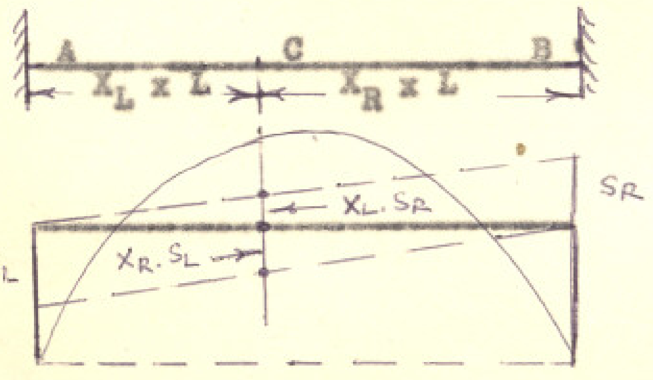


FIG 6

end of the axis of fixed beam moment diagram, drawn on the basis of the structural convention, is to produce positive changes in all the ordinates of the moment diagram. In a similar manner, a positive change S_R produces a numerical increase in $+M_R$, which, reflected in the rising of the right end of the axis, produces negative changes in the ordinates of the moment diagram.

Let AB (figure 6) be the fixed central span in which S_L and S_R occure , and let it be required to find the corresponding change Δ in the fixed beam moment at any point C. If X_L and X_R are the ratios of AC and CB to AB , it follows from the preceding paragraph that

$$\Delta = X_R \cdot S_L - X_L \cdot S_R$$

Substituting from figure 5

$$\Delta = X_R \left[\frac{Q_R}{C_L} + Q_L - r_{AB} (Q_L + Q_L') \right] - X_L \left[\frac{Q_L}{C_R} + Q_R - r_{BA} (Q_R + Q_R') \right]$$

Factoring

$$\Delta = - Q_L \left[\frac{X_L}{C_R} - X_R (1 - r_{AB}) \right] + Q_R \left[\frac{X_R}{C_L} - X_L (1 - r_{BA}) \right] - Q_L' \cdot X_R \cdot r_{AB} + Q_R' \cdot X_L \cdot r_{BA}$$

In equations above the Q and Q' are the moments obtained after the first distribution and carry-over ; Comparison of moments denoted by Q in figure 5 and with those recorded in line 3, Table 1 shows they are identical, both with regard to the arrangement of loads and to the procedure of computations. The moments denoted by Q ' do not appear in table 1 due to loading arrangement which is exactly opposite to that which results in Q values.

In beams of constant x - section $C_L = C_R = 1/2$. Therefore for such beams

$$\Delta = - Q_L (2 x_L - X_R (1 - r_{AB}) + Q_R \; 2 x_R - X_L (1 - r_{BA})$$

$$- Q_L' X_R \; r_{AB} + Q_R' \; X_L \; r_{BA}$$

.... (1)

At the centre of span

$X_L = X_R = 1/2$

$$\Delta = - \frac{Q_L (1 + r_{AB})}{2} + Q_R \frac{(1 + r_{BA})}{2} - Q_L' \frac{r_{AB}}{2}$$

$$+ Q_R' \frac{r_{BA}}{2}$$

.... (2)

The far ends of the adjacent spans contributes little towards the moment distribution at near end so the value of Q_L' or Q_R ' being very small can be ignored. Then

Δ = Change in moment at centre under fixed condition

$$= - \frac{(1 + r_{AB})}{2} . \; Q_L + Q_R \frac{(1 + r_{BA})}{2}$$

Generally in most of the building frames maximum positive moments occures at the mid span, the procedure for which is shown in Table 2 and explained with the help of previous example. For span IJ multiply - 3905 (i.e Q_L) at I

T A B L E - 2

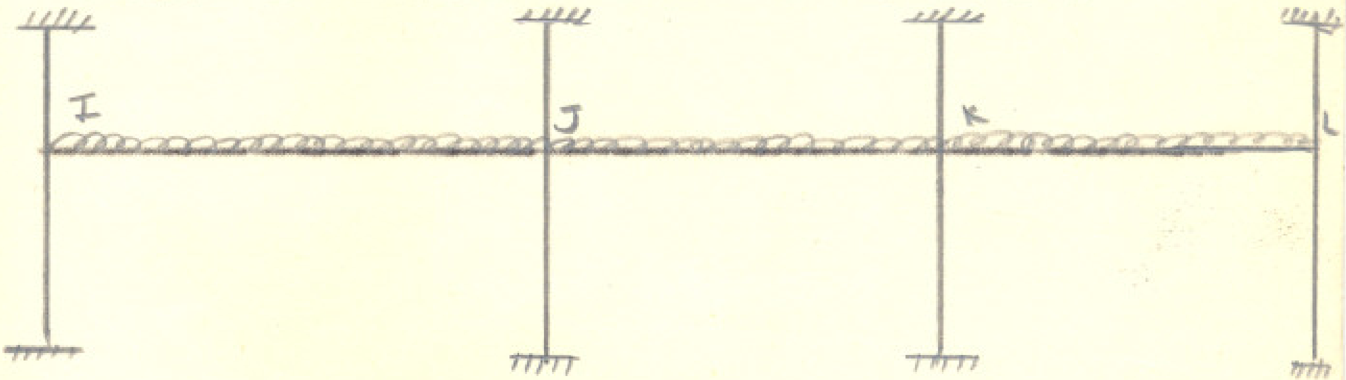
Computation of Maximum Positive Moment at Mid span.

1. Dist. factor	r_{AB}		r_{BA}	r_{BC}		r_{CB}	r_{CD}		r_{DC}
2. F.E.M.D.L.	-		$+M_{DL_1}$	$-M_{DL_2}$		$+M_{DL_2}$	$-M_{DL_3}$		-
3. F.E.M.T.L	$-M_{TL_1}$	i) $+M_{FC_1}$	$+M_{TL_1}$	$-M_{TL_2}$	i) $+M_{FC_2}$	$+M_{TL_2}$	$-M_{TL_3}$	i) M_{FC_3}	$+M_{TL_3}$
4. Dist.& C.O.	Q_{A_1}	ii) $-\frac{1}{2}(1+r_{AB})Q_{A_1}$ iii) $+\frac{1}{2}(1+r_{BA})Q_{B_1}$	Q_{B_1}	Q_{B_2}	ii) $-\frac{1}{2}(1+r_{BC})Q_{B_2}$ iii) $+\frac{1}{2}(1+r_{CB})Q_{C_2}$	Q_{C_2}	Q_{C_3}	ii) $-\frac{1}{2}(1+r_{CD})Q_{C_3}$ iii) $+\frac{1}{2}(1+r_{DC})Q_{D_3}$	$Q_{D\ 3}$
5. Addition (3+4)	X_{AB}		X_{BA}	X_{BC}		X_{CB}	X_{CD}		X_{DC}
6. Dist.	$-r_{AB}.X_{AB}$		$-r_{BA}.\left(X_{BA} + X_{BC}\right)$	$-r_{BC}.(X_{BA} + X_{BC})$		$-r_{CB}.(X_{CD} + X_{CB})$	$-r_{CD}.\left(X_{CD} + X_{CB}\right)$		$-r_{DC}.X_{DC}$
7. Max.moments (5 + 6)	$-M_{AB}$	M_{C_1}=i)+ii)+iii)	$+M_{BA}$	$-M_{BC}$	M_{C_2}= i)+ii)+iii)	$+M_{CB}$	$-M_{CD}$	M_{C_3}=i)+ii)+iii)	$+M_{DC}$
8. Max.Col. moments									

NOTATION : -

M_{FC_n} = fixed beam mid span moment in n^{th} span.

by $- \frac{1}{2} (1 + r_{IJ}) = - \frac{1}{2} (1 + \frac{1}{2})$ and record the result + 2929. Multiply + 5200 at J (Right end) by $+ \frac{1}{2} (1 + .4)$ and record the result = + 3640. All other corrections are determined in the same manner. Then the sum + 10400 + 2929 + 3640 = 16969 is the maximum moment at mid span. It is to be noted that the factor for multiplication of Q_L is $-\frac{1}{2}(1+ r_{AB})$ i.e with -ve sign and that at right is $+ \frac{1}{2}(1 + r_{BA})$ i.e +ve sign. This assumption is same as that for F.E.M.'S.



D.F	½		0.4	0.2		2/9	1/3		3/7
D.L.F.E.M	-10133		+10133	-1275		+1275	-3750		+3750
T.L.F.E.M	-20800	+10400	+20800	-2775	+1388	+2775	-7917	+3959	+7917
Dist.& C.O.	- 3905	+ 2929 + 3640	+ 5200	+ 108	- 65 - 450	- 736	-1697	+1131 + 791	+1107
Add.	-24705		+26000	-2667		+2039	-9614		+9024
Dist.	+12353		- 9334	-4667		+1638	+2525		-3867
Max. moments	-12352	+16969	+16666	-7334	+873	+3677	-7089	+5881	+5157

FIGURE-2.

If however it is required to find maximum moment at an intermediate point which is generally a quarter point , the mid point or the one third point, then the formulas for the coefficients of Q at these points in span of constant cross-section are given below :

Point	X_L	X_R	Intermediate Point Factors for QL	Intermediate Point Factors for QR
Left ¼ point	¼	¾	$-\frac{3r_L - 1}{4}$	$\frac{3r_R + 5}{4}$
Right ¼ point	¾	¼	$-\frac{r_L + 5}{4}$	$\frac{3r_R - 1}{4}$
Left 1/3 point	1/3	2/3	$-(2/3)r_L$ -	$\frac{r_R}{3} + 1$
Right 1/3 point	2/3	1/3	$-(\frac{r_L}{3} + 1)$	$\frac{2}{3} r_R$
Mid point	½	½	$-\frac{1+r_L}{2}$	$+\frac{1+r_R}{2}$

6.2.3. MINIMUM MOMENTS IN BEAMS

The term"minimum moments" denotes a moment produced at a point in structure when live load are placed only in those positions which, in the computation of maximum moments at the same point are kept free of live load. The word " minimum" is used because, ordinarily the minimum moment is the smallest moment that may occure at the point; it is usually of same sign as the maximum moment.

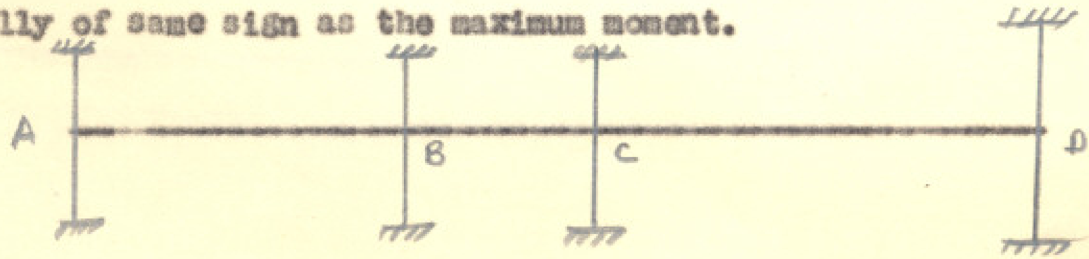


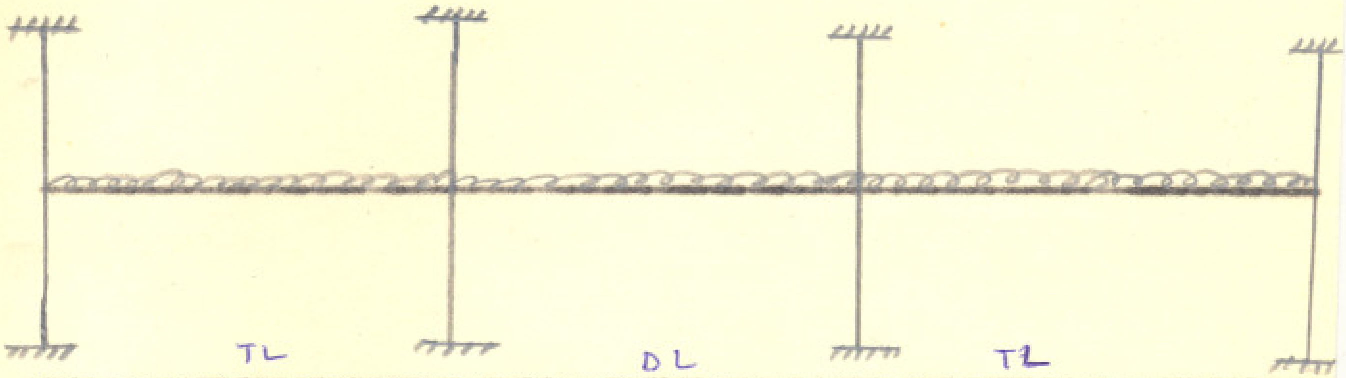
FIG. 8

It is conceivable, however, that if a short span such as BC in figure 8 is flanked by two relatively longer spans, and if the live load is placed only on the side spans, the moment at the mid point of BC may not only become negative, but also be numerically larger than the maximum positive moment which can develop at that point when live load is placed on BC.

The minimum span moments in all the beams can be computed in a manner similar to as described in the Table 2 with the only difference that 2nd and 3rd rows i.e. F.E.M due to D.L and F.E.M due to live load are to be interchanged.

In most of the cases the D.L present on all spans prevents the live load from creating negative moments in the Central segments of spans. Therefore the usual effect of placing live load on the side spans of the beam is to reduce positive center moment in middle span. The calculation of this is not important from design point of view. Occassionally, however, if the live load produces a negative moment at the mid point, then the procedure explained in table 3 may be used for that span only which will also be used for calculation of maximum moments in interior columns as described in the proceeding article.

Figure 9 shows the calculation of minimum moments for the middle span of the given example.



D.F	½		0.4	0.2		2/9	1/3		3/7
F.E.M. TL or DL	-20800		+20800	-1275	+638	+1275	-7917		+7917
Dist.& C.O.			+ 5200	+ 738	-443 -1193	-1953	-1697		
Addition			+26000	-537		- 678	-9614		
Dist.			-10185	-5093		+2287	+3031		
Minimum beam moments			+15815	-5630	-998	+1609	-6583		
Col. moments	—		-5093			+2487			—

FIGURE 9

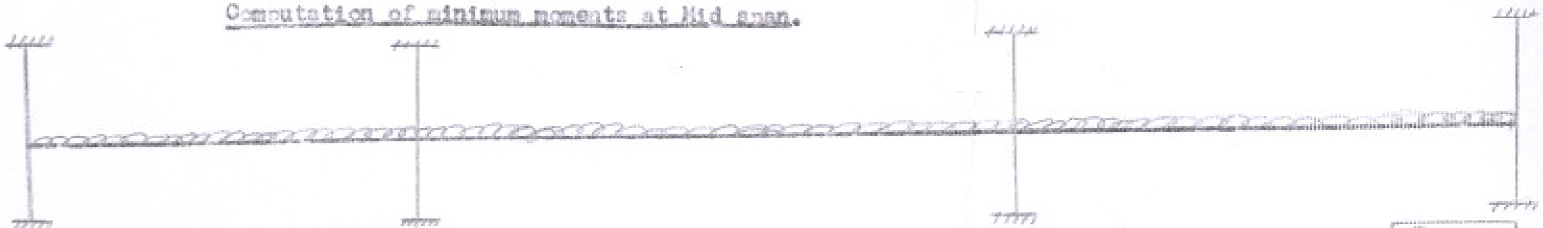
7.0 SHEAR IN BEAMS:

Shear at the end of a beam that is part of a frame is determined as the sum of the shear in the beam considered simply supported and a correction due to difference between end moments produced by the frame action. The correction is usually small compared with simple beam shear, especially in interior spans.

In end spans the correction may be obtained from the moment calculation in Table 2.

TABLE 3

Computation of minimum moments at Mid span.



1. Dist. factor	r_{AB}	r_{BA}	r_{BC}		r_{CB}	r_{CD}	r_{DC}
2. - -	-						
3. F.E.M. T.L. and D.L	$-M_{TL_4}$	$+M_{TL_1}$	$-M_{DL_2}$	i) $+M_{FC}$	$+M_{DL_2}$	$-M_{TL_3}$	$+M_{TL_3}$
4. Dist. & C.O.		Q_{B_1}	Q_{B_2}	ii) $-\frac{1}{2}(1+r_{BC})\,Q_{B_2}$ iii) $+\frac{1}{2}(1+r_{CB})\,Q_{C_2}$	Q_{C_2}	Q_{C_3}	
5. Addition (3 + 4)		X_{BA}	X_{BC}		X_{CB}	X_{CD}	
6. Dist.		$-r_{BA}\cdot\left(X_{BA}+X_{BC}\right)$	$-r_{BC}\cdot\left(X_{BA}+X_{BC}\right)$		$-r_{CB}\cdot\left(X_{CB}+X_{CD}\right)$	$-r_{CD}\cdot\left(X_{CD}+X_{CB}\right)$	
7. Min. beam moments. (5 + 6)		$+M_{BA}$	$-M_{BC}$		$+M_{CB}$	$-M_{CD}$	
8. Col. moments							

For illustration in end span IJ, the end moments are 12352 and 16666. The difference between them is (16666 - 12352) = 4314 kg-m.

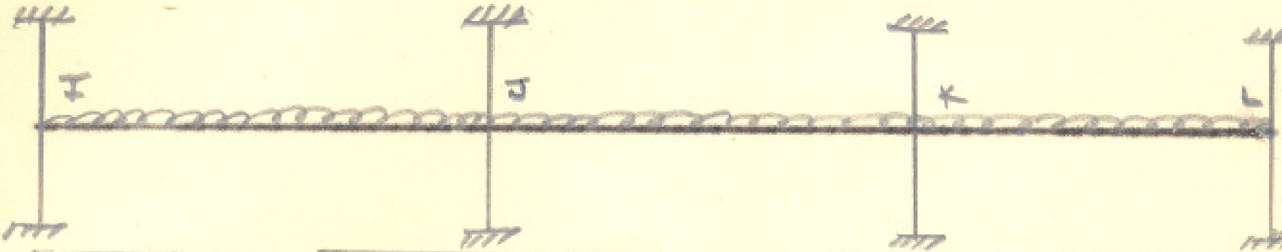
The shear correction is 4314 divided by span length.

i.e = 4314/ 8 = 539.25.

The end shear in the beam considered simply supported is 850x8 /2 = 3400 kg. The total shear at J is 3400 + 539.25 = 3939.25 kg and that at I = 3400- 539.25 = 2860.75 kg.

For interior beams the loading condition for maximum moments are not quite as favourable for determination of maximum shears. For illustration, consider the problem to determine maximum shear at J. The shear in the simply supported beam is 850 x 3/2 = 1275 kg. 7334 kg-m is the maximum moment at J, but 3677 kg-m at K is not the moment due to loading which will result in maximum shear at J. The moment at K is too large . So computing the shear correction as $\frac{7334 - 3677}{3} = 1219$ kg is not on safe side. But this correction is usually so small compared with the value it modifies that it is generally sufficient to use some rough approximation such as twice its value. In this case shear would be (850 x 3/2) + 2 x 1219 = 3713 kg.

If it is necessary under special circumstances, when the correction is not small as compared to shear due to simply supported beam as in the present case, to determine the shear correction accuratly . The end moment to be used instead of 3677 may be computed simply in quickly as in figure below :-



		TL	0.4	0.2	TL	2/9	1/3	DL	3/7
F.E.M TL or DL	-20800		+20800	-2775		+2775 -1803	-3750 - 804		+3750
						+ 972 + 796	-4554		
						+1768			

FIGURE 10

The shear correction now is

equal to $= \frac{7334 - 1768}{3}$

= 1855 kg.

Maximum shear = 1275 + 1855

= 3130 kg.

which is less than the approximate value.

8.0 BENDING IN COLUMNS

On the subject of how to determine bending moments in columns, the ACI code 1971 says in section 8.5.4.1 "In computing moments in columns, the far ends may be considered fixed. Columns shall be designed to resist the axial forces from loads on all floors plus the maximum bending due to loads on a single adjacent span of the floor under consideration.

" Resistance to bending moments at any floor level shall be provided by distributing the moment between the columns immediately above and below the given floor in proportion to their relative stiffnesses conditions of restraint."

First of all, it is without any doubt simplest to use the moments obtained from regular beam analysis such as illustrated in figure 4. If granted that loadings other than those in figure 4 produces greater moments in the exterior columns, but it is a question whether the extra effort spent in obtaining maximum column is justifiable.

It is generally conceded that moments cannot be determined in columns nearly the same degree of accuracy as in beams. A beam moment is obtained as the sum of F.E.M and an additional term or a correction derived by analysis. But a column moment simply equals the corrections obtained by analysis and is therefore, as a rule, for more senstive to changes in assumptions and much more susceptible to faulty analysis.

Another argument is that columns appear to have a marked ability to throw off moments they are unable to support. Consider for example a column supporting an axial load and assume that one end of it being subjected to gradually increasing rotation. At a certain stage of rotation, the column section may be overstressed, and it may crack or yeild in some way. When this happens there is a sudden drop in moment required to produce to rotation. One might say that columns in buildings tend to select the amount of moments they are capable of supporting.

The importance of accurate moment determination for girders as compared to columns can be discussed this way also. In girders the required cross-sections are dictated exclusively by moments and shears and for this reason a relatively accurate determination of these quantities is called for. But columns, on the otherhand, must resist axial loads from overlying parts of the structure, in addition to bending moments induced in them by rigdly connected girders. Some inaccuracy in determining these moments affects, therefore, only one of the two factors (loads and moments) which determine the required cross-section and, for this reason, has small overall effects than similar inaccuracies in determining girder moments.

From the above arguments following conclusions may be drawn :

The elastic theory is not at present closely enough in accordance with facts to justify a too elaborate procedure for determination of moments in columns. Granting that larger moments may be obtained from theoratical analysis, it is felt to be satisfactory to compute column moments under the same assumption as that used for beam moments which is that far ends of columns are fixed above and below the floor at which moments are to be determined.

8.1 DETERMINATION OF COLUMN MOMENTS

The consideration in the last section appear to justify the recommendations that a column moment to be determined on basis of the assumption underlying the calculation made for beam in figure 4. For illustration, the moment at the exterior end of the beam is - 12352. This moment must equal the sum of the moments in the columns at I and should be distributed to them in proportion to their stiffness ratio or distribution factor. The moments in the interior columns are not recorded in figure 4 because all end moments there are based upon live load on both sides of each individual joint, whereas most codes specify that columns moments be computed for unbalanced loading i.e Live load on one side only.

Figure 9 which was used to determine minimum moments at mid span, serves the additional purpose of obtaining moments in interior columns produced by unbalanced floor loading i.e live load is placed on the alternate long spans.

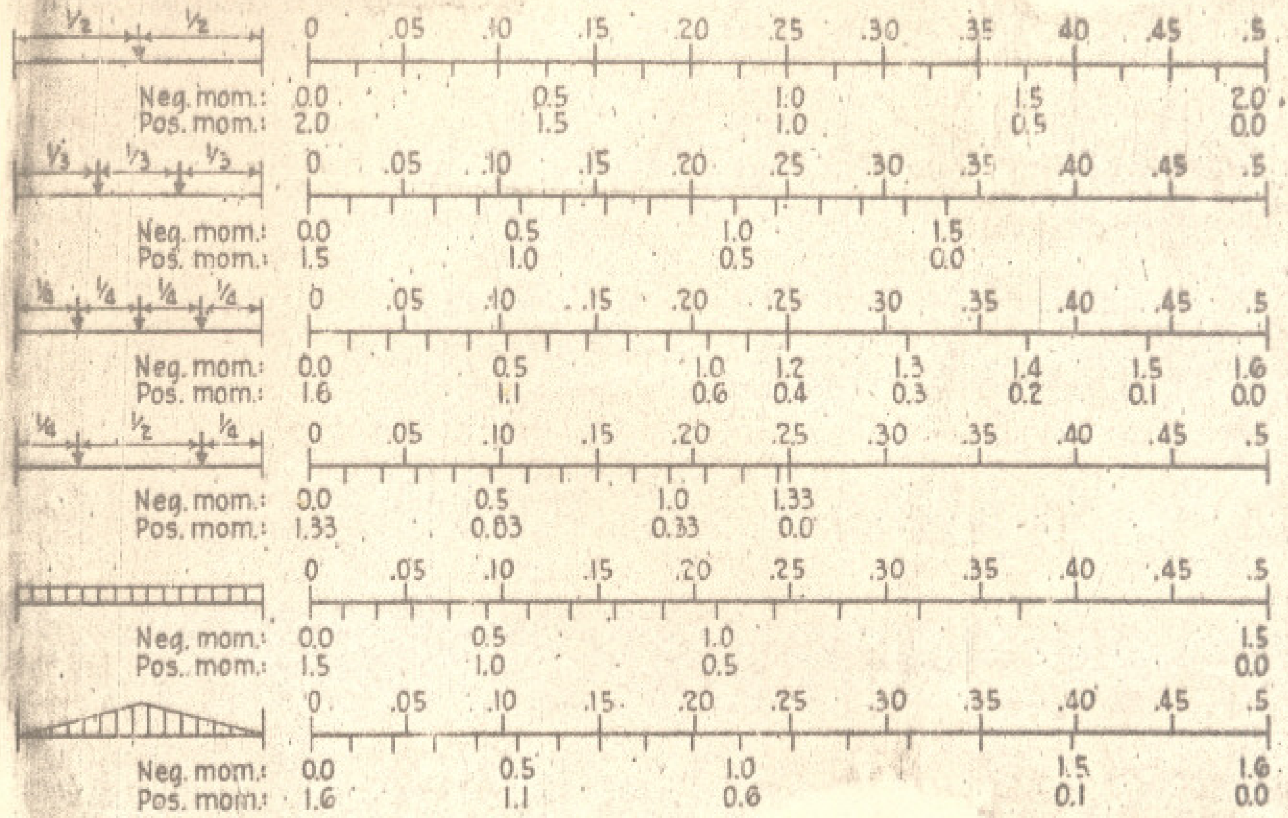
If irregularities in span and loading are large enough to call for an analysis for beams more extensive than in figure 9 the general form of moment distribution may be used in which case it should be employed for both columns and beams.

9.0 POINT OF INFLECTION:

It is necessary to specify where to bend up bars and how far negative reinforcement shall extend into adjacent spans. The rule generally adopted is that such reinforcing bars shall be extended. atleast 12 diameters beyond the point of inflection or beyond the point at which they are no longer needed to resist stress. Table 4 can be used for calculation of point of inflexion in continuous beams. Taking again the case of beam in the above example. Let the problem be to determine the point of inflection for negative moments near J.

The final maximum moment M_{JI} is + 16666 and the original F.E.M is + 20800. Compute the ratio 16666/20800 = .80 locate this type of loading in Table 4 and proceed

Points of Inflection.



in the line marked " Negative moment " to the right until the ratio of 0.8 is reached. Just above that point on the adjacent scale, the value of 0.16 appears which signifies that the point of inflextion is a distance of 0.16 L from the support, L being span length.

If the span is particularly short compared with the adjacent spans, then under such circumstances it is possible that a greater distance to the point of inflextion may be obtained with minimum loading as on BC.

The construction of scales in Table 4 can be illustreated as below :

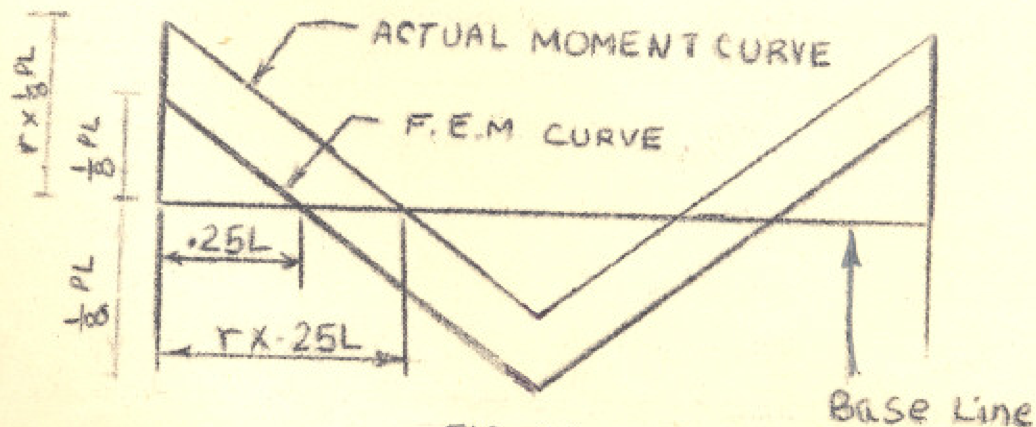


FIG 11

Figure 11 illustrates the method of construction for a concentrated load at mid span. The heavy black line is the moment curve in a beam with fixed ends, and the point of inflextion for this curbe is at the quarter point. If M^F is the fixed end moment and r. M^F is the final moment in the beam then the distance to the point of inflection must be 0.25 r x L. This determines the relationship between the scales in Table 4.

Distances to the point of inflection for positive moments are determined in a similar way, and data are given in Table 4, for several type of loading. In all instances actual moments whether at ends or at mid span are to be divided by fixed end moments. It should be noted that the data in table 4 are correct for the case only in which moment curves are symmetrical. But it is usually satisfactory to use Table 4 also for cases of dissymmentry. The table is applicable to members of constant or variable M.O.I. Table 4 is also useful for determining where a certain percentage of total reinforcement is no longer needed.

10.0 COMPARISION OF RESULTS.

The accuracy of the two cycle procedure is illustrated in figure 12 by comparing the results obtained by two cycle moment distribution and by doing the ordinary moment distribution of whole frame for various conditions of loading and ignoring the sway in the frame due to unsymetrical loading which in building frames is usually very small.

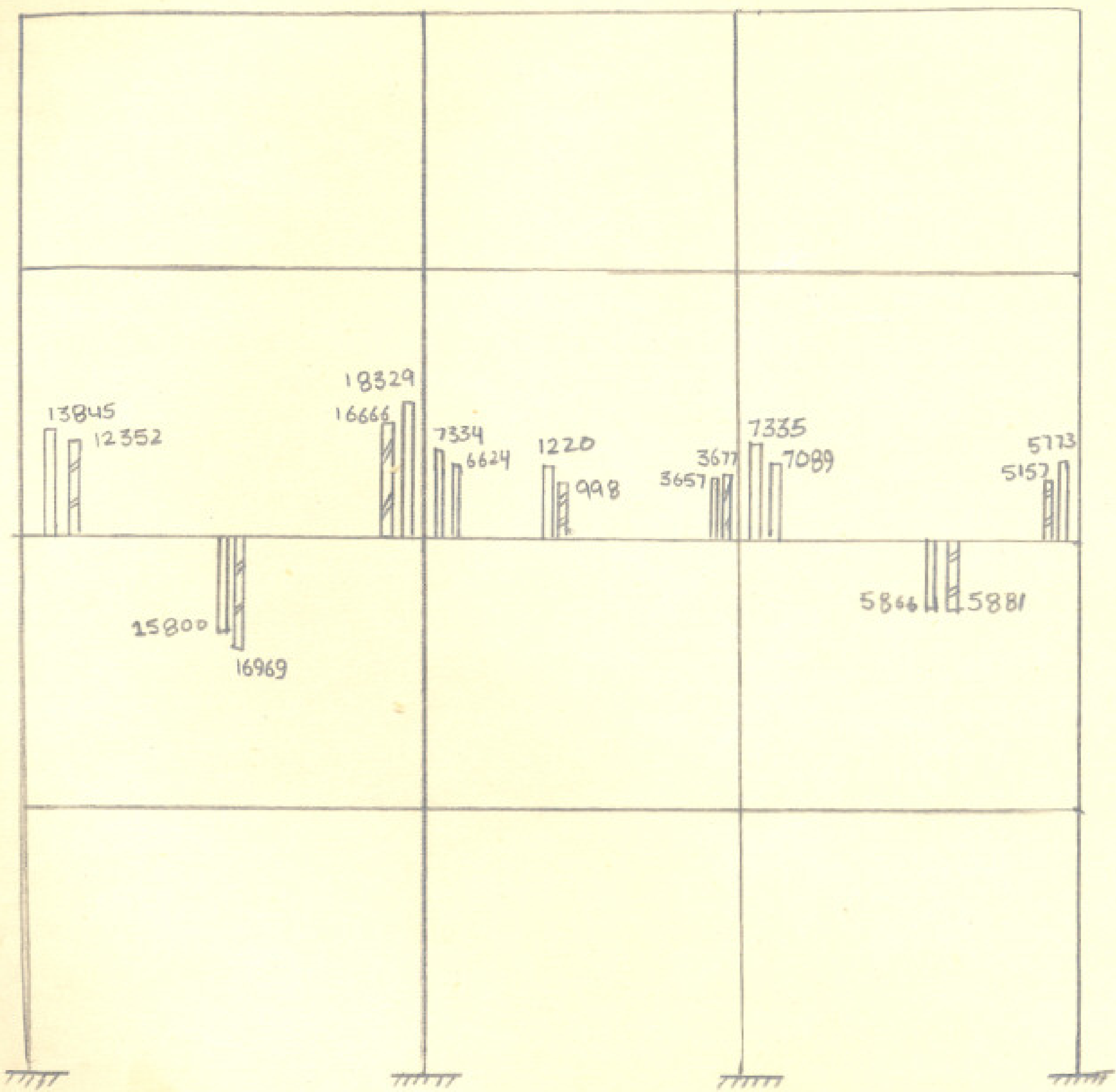


FIGURE 12

BY TWO CYCLE MOMENT DISTRIBUTION

BY DOING MOMENT DISTRIBUTION OF WHOLE FRAME (FOUR CYCLES)

The results in figure 12 shows that the results obtained by two cycle procedure are fairly accurate and can be used for design due to the reasons already explained under the heading " Precesion of Analysis". In the example it is seen that the percentage variation is not more than ten percent.

11.0 ADVANTAGES OF TWO CYCLE METHOD

The two cycle method though not as accurate as the more exact methods has got the following advantages :

1. This method limits the analytical work to just that which is required for reasonable accuracy.
2. The computation of moments by this method saves considerable time, effort and paper.
3. This method is perfectly general. It can be applied to any type of loading whether it is uniform or concentrated, symmetrical or unsymmetrical. It works with equal facility for any combination of stiffness for various beams and columns.
4. It can be used for haunced beams and flared columns.

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