EFFECT OF ALIGNED AND NON SYNCHRONOUS ROTATION IN THE EQUILIBRIUM STRUCTURE OF BINARY SYSTEM

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Submitted by
Jaspinder kaur
Roll no: 301003011

Under the guidance of

Dr. A. K. Lal
Associate Professor

School of Mathematics and Computer Applications
Thapar University, Patiala- 147004 (Punjab), INDIA

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CERTIFICATE

This is to certify that the thesis " **EFFECT OF ALIGNED AND NON SYNCHRONOUS ROTATION ON THE EQUILIBRIUM STRUCTURE OF BINARY SYSTEM** " submitted by Ms. Jaspinder Kaur of M.Sc. (Mathematics and Computing), Thapar University, Patiala, was carried out by me under the supervision of Dr. A. K. Lal. She has not submitted this material for credit towards any other degree at Thapar University Patiala or any other University.

Jaspinder Kaur
(JASPINDER KAUR)

This is to certify that the above statement made by candidate is correct and true to the best of my knowledge.

Dr. A. K. Lal
Supervisor
School of Mathematics and Computer Applications
Thapar University, Patiala

Countersigned by:

Dr. S. S. Bhatia
Professor and Head
School of Mathematics and Computer Application
Thapar University, Patiala

Dr. S. K. Mahapatra
Dean of Academic Affairs
Thapar University, Patiala

Dedicated to my

Parents and God

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ABSTRACT

The problem to study the equilibrium structure of stars distorted by rotational and tidal effect has great importance in astrophysics. Such a problem will help better to understand inner structure of binary system. Therefore, there is need to study the equilibrium structure of gaseous sphere.

Analytic study of determining the equilibrium structure of rotationally and tidally distorted stellar models is quite complex. Therefore, investigators attempted to solve such problems in some approximate way. In one such attempt Mohan, Saxena and Aggarwal used Kippenhahn and Thomas averaging technique together with the results of Kopal on Roche equipotential, to determine the effects of rotation and tidal distortion on the equilibrium structure of binary stars. However, the problems of determining the equilibrium structure of aligned and non synchronously rotating stars have not been satisfactory tackled so far.

In the present thesis an attempt has been made to study the effect of aligned and non synchronous rotation on the equilibrium structure of rotation and tidal distortion on the binary system.

The thesis consists of three chapters and chapter wise summary of the work is presented below. Chapter one is introductory in nature where we discussed the astrophysical significance of studying the equilibrium structure of rotationally and tidally distorted stellar models. Chapter two deals with the concept of modified Roche equipotential of distorted stars which accounts for the aligned, non synchronous rotation of the binary system. The explicit expression for distortional parameters u, v, w, f_P and f_T are obtained using Kippenhahn and Thomas averaging technique along with the results of Roche equipotential. In this chapter we have also discussed how to find the equilibrium structure of rotationally and tidally distorted stellar models in presence of aligned and non synchronous rotation.

The methodology developed in Chapter-II is next used in chapter III to determine the equilibrium structure of non synchronous rotating and tidally distorted polytropic models of stars. Computation is carried out to determine the inner structure as well as various physical parameters of the rotationally and tidally distorted polytropic models with various values of degree of non synchronous parameter f. Conclusion based on the study has finally been discussed.

CONTENTS

CHAPTER – I

INTRODUCTION

This chapter is basically introductory in nature. In section 1.1 we study in brief the significance of the problem for determining the effects of rotational and tidal distortions on the equilibrium structures of gaseous spheres. A brief survey of literature available on this subject is next presented in section 1.2. Section 1.3 deals the with important Kippenhahn and Thomas averaging technique to derive system of differential equations determining the equilibrium structure of rotationally and tidally distorted binary system of stars. The summary of the work presented in succeeding chapter is finally discussed in section 1.4.

1.1 SIGNIFICANCE OF DETERMINING THE EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED GASEOUS SPHERES

The theoretical model of a star is essentially a self gravitating gaseous sphere in hydrostatic and thermal equilibrium. Theoretical study of the problems regarding the equilibrium structure of gaseous spheres has been carried out to understand the nature of the internal structures, responsible for various phenomena of the stars. Whereas some of the stars are observed as single stars or in groups of two or more stars. Observations show that some of the stars are rotating about their axes of rotation. Moreover, they are also revolving around each other. Thus, if we assume the equilibrium model of a single non-rotating star as a gaseous sphere, the equilibrium model of a rotating star will be rotationally distorted gaseous sphere. Similarly, if it is not rotating then the equilibrium model of a star in a binary system will be tidally distorted gaseous sphere and if it is rotating as well then the model will be rotationally and tidally distorted gaseous sphere.

Analytic studies of the problems of rotating stars and stars in binary system have drawn the attention of astrophysicist with a view to analyze the observational behavior of such stars. In a binary system of stars, the two stars normally rotate about their axis as well as revolve about their common centre of mass. If the rotational and revolutionary period are same, then the stars are said to be in synchronous rotation otherwise non synchronous rotation. Out of these two stars, first star is known as primary star which is supposed to be much more massive as compare to secondary star known as companion star

Keeping this in view an attempt has been made in the present thesis to investigate certain aspects of the problems of equilibrium structures of rotationally and tidally distorted gaseous spheres, subject to the aligned and non synchronous rotation.

1.2 BRIEF SURVEY OF THE LITERATURE

Most of the theoretical studies about the equilibrium structures of the stars have been carried out in literature by assuming the star to be an undistorted spherical gaseous sphere. Extensive literature is now available on this subject (see for instance Chandrasekhar (9), Eddington (19), Mentzel et al. (49), Cox and Giuli (14), Kippenhahn and Weigert (30), Clement (11), Kopal (32-34), Tassoul (70), Cox (13), Bohm-Vitense et al. (3).

The theoretical investigation related to the problems of determining the equilibrium structures and stability of rotating and self gravitating objects, possibly begun with the work of Newton. He was the first to realize the importance of the law of gravitation for explaining the figures of celestial bodies. Later on Maclaurin, Clairaut, Laplace, Legendre, Jacobi, Poincare etc. contributed ideas, necessary for the development of the general library theory of rotating bodies. Maclaurin, Jacobi, Kelvin and Jeans investigated in detail the problem of structure and stability of rotating liquid masses assuming uniform rotation.

In the year 1923, Edward Arthur Milne developed a technique for constructing the first detailed model for slowly rotating star in pure radiative equilibrium. Later on in the year 1933, the technique of Milne was generalized and applied to slightly distorted polytropes by Chandrasekhar. The effect of uniform rotation on slow rotating Cowling star obeying simple Kramer’s opacity has been studied by Sweet and Roy (69). Much of the work on the effect of rotation on stellar interiors is summarized in the review article of Authors such as Kruszewski (36), Limber (42), Roberts (60), James (28), Martin(46), Linnell(43,44), Endal and Sofia(22), Lubow(47), Kopal(33), Durney(18), Deupree(16), Einsel and Spurzem(21), Maeder and Zahn(50), and Sood and Singh(67) have also investigated the problems of equilibrium structures of rotating stars. Meynet and Meader(50) studied the effects of rotation on the equilibrium structure and evolution of massive stars. Mender et al.(48) investigated the theoretical models of low mass pre-main sequence rotating stars.

Equilibrium structures of stars which appear in binary systems are likely to be affected by both the rotational as well as the tidal effects of the companion stars. Attempts have been made in literature to determine the effects of rotation and tidal distortions on the equilibrium structure of the stars in the binary system. In a series of papers Chandrsaekhar(6,7,8) developed a first order analysis which he applied to the study of rotational problem, the tidal problem and the binary star problem. The method, however, was found unsuitable when the separation between the componentsm is only a few times the undisturbed radius of the primary.

The method to study the structure of the primary component of a synchronous close binary was further extended by Naylor and Anand(54). Kippenhahn and Thomas suggested a practical way of analyzing the effects of rotation and tidal distortions on the equilibrium structure of star by approximating the actual equipotentials surfaces of the star by Roche equipotentials.

After the discovewry of known-synchronous rotation by schles tinger in binary 8 lib in 1909, it was found in several binary stars by various author. According to Wilson(1985) and hamme(1992), the degree to which the rotation is non synchronous $f = \frac{\Omega_*}{\Omega}$ is rather high for these stars.

Chan and Chau(4) developed a method which allows an efficient and accurate investigation of the structure and evolution of a rotationally and tidally distorted star in close binary systems. Tassoul and Tassoul(72) considered the meridional circulation in rotating stars and mean study motions in rotationally and tidally distorted stars. Later, Tassoul and Tassoul(72) extended the earlier work to study the reflection effects in close binaries when there is meridional circulation in rotating stars. Lopezorti et al(45) analyzed the equilibrium configurations of close binary systems by expanding the auto gravitational, centrifugal and tidal potentials in Clairaut coordinates. Lal et al(39) have discussed the equilibrium structures of rotationally and tidally distorted primary component of binary stars taking into account the effect of mass variation inside the star.

Mohan and Singh(52) have used the Kippenhahn and Thomas(29) averaging technique in conjunction with certain results of Kopal(31) on Roche equipotential to study the effect of rotation and tidal distortions. Sepenisky (65) et al investigated the existence and properties of equipotential surfaces and Langrangian points in non-synchronous binary star and planetary systems under the assumption of quasi-static equilibrium. Roche(61) studied the effect of slow uniform rotation on the tidal effects in close binary system.

Orlov(55) have generalized the Roche model as is applied in the case of double star. In this model the point nuclei of the Roche model has been substituted by polytropic gas nuclei of finite dimensions. Plavec(57) presented tables of Roche model for the use of investigators in close binary systems. Eggleton (20) computed the effective radii of Roche lobes and compared the results with the earlier results available in literature. Mochnack i(51) accurately integrated Roche model for close binary system in synchronous rotations give volume, radii, surface area, mean gravities and mean inverse gravities in normalized form. Seidov (64) derived the exact analytical formula for the potential and mass ratio as a function of Langrangian point’s position in the classical Roche model of the close binary stars. Csataryova and Skopal(15) derive approximate analytical formulas for the basic parameters of the Roche lobe, its radius and the position of the L_1 point for asynchronously rotating component in a binary system. Lal et al(40) studied the validity of series expression being used for determining the position of a point on a Roche equipotential in case of rotating stars and stars in binary systems.

Most of the authors have studied the equilibrium structures of stars having solid body rotation. The influence of uniform rotation on the structure of the white dwarf models has been considered by Chandrasekhar(10), Suda(68) and Lal(38) et al. The most detailed models of uniformly rotating white dwarf are due Anand(1), Roxburgh(62), Monaghan et al(51). Some of the authors such as Ostriker and Tassoul(56), Shapiro and Teukolsky(65) have noted the stability analysis of uniformly rotating white dwarf stars. Ostriker and Bodenheimer(55) Smart and Monaghan(66), and Blinnikov(5) extensively analyzed the models of zero temperature white dwarf in non-uniform rotation, Hachisu et al(25). studied the fate of merging double white dwarfs and presented a numerical method, Lal el at(38) presented a method for computing equilibrium structure of differentially rotating and tidally distorted white dwarf models of stars.

1.3 BASIC EQUATIONS DETERMINIG THE EQUILIBRIUM STRUCTURE OF GASEOUS SPHERE

The system of basic equations of the equilibrium structure of gaseous sphere in hydrostatic and thermal equilibrium, are well established in literature. These equations pertain to the problem of the equilibrium structure of stellar models. Let P and ρ denote the pressure and the density at a point, respectively, distant r from the center of the sphere.

1.3.1 Mass conservation

Let $M(r)$ be the mass contained within radius r, then the mass contained with the shell from r to $r+dr$ is then

$$M(r+dr)-M(r)=\rho(r)\,dV \tag{1.1a}$$

where $dV=4\pi r^2 dr$ is the volume of the shell. Now the left-hand-side can be rewritten as,

$$\frac{dM(r)}{dr}dr=\rho(r)4\pi r^2 dr \tag{1.1b}$$

Cancelling out the factor dr, we obtain

$$\frac{dM(r)}{dr}=\rho(r)4\pi r^2 \tag{1.1c}$$

1.3.2 Hydrostatic equilibrium

For a star in hydrostatic equilibrium, gravity is balanced by pressure. Now, for a small element in a shell from r to $r+dr$. The area of this element is dA.

A coordinate system, where the radial component increases outwards, establishes.

Then the pressure force acting on the inner side of radius r is positive, while the pressure force acting on the outer side of radius $r+dr$ is negative.

The total pressure force is then

$$P(r)dA-P(r+dr)dA=[P(r)-P(r+dr)]dA=-\frac{dP}{dr}dr\,dA \tag{1.2a}$$

The gravity force, due to spherical symmetry, points toward the centre and therefore has a negative sign i.e.

$$-\frac{GM_r\,dm}{r^2} \tag{1.2b}$$

where dm is the mass of the element. Obviously

$$dm = \rho(r)\,dV = \rho(r)\,dA\,dr \tag{1.2c}$$

Therefore the gravity is given by

$$-\frac{GM_r\rho(r)dADr}{r^2} \tag{1.2d}$$

So from Newton's second law, we have

$$\rho(r)dAdr\frac{d^2r}{dt^2} = -\frac{dP}{dr}dr\,dA - \frac{GM_r\rho(r)dAdr}{r^2} \tag{1.2e}$$

We can cancel out the factors dr and dA, and arrive at:

$$\rho(r)\frac{d^2r}{dt^2} = -\frac{dP}{dr} - \frac{GM_r\rho(r)}{r^2} \tag{1.2f}$$

If the star is in hydrostatic equilibrium, then the sum of all forces must vanish, i.e.,

$$-\frac{dP}{dr} - \frac{GM_r\rho(r)}{r^2} = 0 \tag{1.2g}$$

Moving the second term to the right hand side, we have

$$\frac{dP}{dr} = -\frac{GM_r\rho(r)}{r^2} \tag{1.2h}$$

1.3.3 Energy conservation

Stars lose energy via radiation. The radiation loss must be balanced by energy generated by nuclear reactions. The energy conservation equation express this in mathematical terms as:

Let $L(r)$ be the energy flow across the sphere with radius r, in units of W, then the net energy loss in the shell from r to $r+dr$ is

$$L(r+dr) - L(r) = \frac{dL(r)}{dr}dr \tag{1.3a}$$

If ε is the energy generation per kg, then the total energy generated in the shell is

$$dE = \varepsilon\rho(r)4\pi r^2 dr \tag{1.3b}$$

For the gas to be in thermal equilibrium, the radiation loss must be equal to the energy gain from nuclear burning. Therefore we have

$$\frac{dL(r)}{dr}dr = \varepsilon\rho(r)4\pi r^2 dr \tag{1.3c}$$

dr Cancels out, so we have

$$\frac{dL(r)}{dr} = \varepsilon\rho(r)4\pi r^2 \tag{1.3d}$$

In problems, where the thermal properties of the model are either not to be investigated or are not important, the equilibrium structure of the gaseous sphere may be determined by solving equations (1.1c and 1.2h) using some suitable equations of state together with boundary conditions

At the centre $r = 0,\, M(r) = 0$

At the surface $r = 0,\, M(r) = 0, P = 0, or\, P_s, \rho = 0 or\, \rho_s$

A number of the theoretical as well as numerical studies regarding the equilibrium structure of gaseous spheres, particularly those which have particular reference to the problems of the equilibrium structures of the stars are available in literature (Chandrasekhar (8), Eddington (19), Meznal et al. (49), Cox and Giuli (13), Kippenhahn and Weigert (30)).

1.4 AVERAGING TECHNIQUE OF KIPPENHAHN AND THOMAS

In order to study the effects of rotation and tidal distortion on the equilibrium structure of gaseous spheres, Kippenhahn and Thomas (29) developed the concept of topologically equivalent spherical surfaces corresponding to actual equipotential surfaces of a rotationally and tidally distorted model. They define on these equivalent spherical surfaces, quantities such as $\overline{f}, \overline{g}$ etc. which denote certain averages of the quantities f, g, respectively on the actual equipotential surfaces. If ψ denotes the total potential due to gravitation, rotation and tidal forces of a rotationally and tidally distorted model at an arbitrary point P (x, y, z) then $\psi\,(x, y, z)$ = constant, is an equipotential surface. Let V_ψ be the volume enclosed by the equipotential surface. ψ = constant and S_ψ the surface area of equipotential surface. For any function $f\,(x, y, z)$ they define $\overline{f}$ as its mean value over the equipotential surfaces

ψ = constant by the relation

$$\overline{f} = \frac{1}{S_\psi} \int_{\psi = const} f\, d\sigma \tag{1.4.1}$$

Where $d\sigma$ denotes the surface element of the equipotential surface ψ =constant. Clearly $\overline{f}$ is a function of equipotential surface ψ = constant only and can be obtained for each equipotential surface ψ =constant. Kippenhahn and Thomas (1970) define a new variable r_ψ in analogy with the sphere by the relation

$$V_\psi = \frac{4}{3}\pi r_\psi{}^3 \tag{1.4.2}$$

Also by definition

$$S_\psi = \int_{\psi = const} d\sigma \tag{1.4.3}$$

Obviously, in general, $S_\psi \neq 4\pi r_\psi^2$. Kippenhahn and Thomas define a function g(x, y, z) by the relation as

$$g = \frac{d\psi}{dn} \tag{1.4.4}$$

This g corresponds to the force of gravity of a sphere. dn is the distance between two neighboring surfaces ψ = constant and $\psi + d\psi$ = constant, is not constant (i.e. not same at all points of the surface). They used the equation (1.4.4) to compute the mean values $\overline{g}$ and $\overline{g^{-1}}$ using the relations

$$\overline{g} = \frac{1}{S_\psi} \int_{\psi = const} \frac{d\psi}{dn} d\sigma \tag{1.4.5}$$

$$\overline{g^{-1}} = \frac{1}{S_\psi} \int_{\psi = const} \left(\frac{d\psi}{dn} \right)^{-1} d\sigma \tag{1.4.6}$$

Both $\overline{g}$ and $\overline{g^{-1}}$ are functions of ψ alone and represent the value of $\overline{g}$ and $\overline{g^{-1}}$ respectively over the topologically equivalent spherical surface. The volume dV_ψ between the surface $\psi =$ constant and $\psi + d\psi =$ constant is given as

$$dV_\psi = \int_{\psi = const} dn d\sigma = \int_{\psi = const} \left(\frac{d\psi}{dn} \right)^{-1} dn = S_\psi \overline{g^{-1}} d\psi \tag{1.4.7}$$

Kippenhahn and Thomas also defined non-dimensionless parameters *u*, *v* and *w* as

$$u = \frac{S_\psi}{4\pi r_\psi{}^2},\ v = \frac{\overline{g} r_\psi{}^2}{GM_\psi},\ w = \frac{\overline{g^{-1}} GM_\psi}{r_\psi{}^2} \tag{1.4.8}$$

where M_ψ is the mass enclosed by equipotential surface ψ = constant.

We may thus regard the equipotential surface ψ = constant to be topologically equivalent to a sphere of radius r_ψ. If ψ be the gravitational potential of sphere then the surface ψ = constant is spherical surface with $r_\psi = r$ for which $u = 1$ and $g = GM_\psi / r_\psi^2$ is constant on these spheres and therefore v and w are constants and equal to 1

The above defined equations are purely mathematical definitions, which have been applied by Kippenhahn and Thomas to gravitational fields of gaseous spheres distorted by rotational and tidal forces. In hydrostatic equilibrium the equipotential surfaces are also surface of equipressure and equidensity. Therefore, on an equipotential surface the pressure P_ψ and the density ρ_ψ are also constant. Using these concepts Kippenhahn and Thomas obtained the equations governing the equilibrium structure of rotationally and tidally distorted stellar model in the following manner.

From equation (1.4.2) the mass dM_ψ between the equipotential surface ψ = constant and $\psi + d\psi =$ constant is given by

$$dM_\psi = dV_\psi \rho_\psi = 4\pi r_\psi^2 \rho_\psi dr_\psi \tag{1.4.9}$$

Thus, we get

$$\frac{dM_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \tag{1.4.10}$$

From equation (1.4.7) and (1.4.9) we have

$$d\psi = \frac{d\psi}{dV_\psi} dV_\psi = \left(\frac{dV_\psi}{d\psi}\right)^{-1} \frac{dM_\psi}{\rho_\psi} = \frac{dM_\psi}{S_\psi \overline{g^{-1}} \rho_\psi} \tag{1.4.11}$$

Using relation (1.4.8), we get

$$d\psi = \frac{GM_\psi dM_\psi}{4\pi r_\psi^4 \rho_\psi uw} \tag{1.4.12}$$

The conditions for hydrostatic equilibrium, $dP_\psi / d\psi = -\rho_\psi$, can now be written with equation (1.4.8) in the form

$$\frac{dP_\psi}{dM_\psi} = -\frac{GM_\psi}{4\pi r_\psi^4} f_p \tag{1.4.13}$$

where

$$f_p = \frac{1}{uw} = \frac{4\pi r_\psi^4}{GM_\psi} \frac{1}{S_\psi \overline{g^{-1}}}$$

The factor f_p is a function of ψ only. If ψ is known, then the equipotential surface can be determined and then consequently values of $S_\psi, r_\psi, \bar{g}$ and $\overline{g^{-1}}$ for each equipotential surface can be obtained simply from the geometry of the equipotentials. The mass M_ψ which depends on the density distribution ρ_ψ can be determined by integrating the equation (1.4.10). Similarly the other structure equations derived by the Kippenhahn and Thomas, which includes the effects of rotation and tidal distortions on the equilibrium structure of gaseous sphere as follows.

For chemically homogeneous spheres, nuclear energy generation rate ε depends only upon the density ρ_ψ and the temperature T_ψ and are, therefore, constant on equipotential surface. Thus, if L_ψ is the energy which passes per second through the equipotential surface ψ = constant, then

$$\frac{dL_\psi}{dM_\psi} = \varepsilon \qquad (1.4.14)$$

Using equation (1.4.9), it can be written as

$$\frac{dL_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \varepsilon \qquad (1.4.15)$$

If the energy is transported by the radiation, then the energy transport equation is

$$F_\psi = -\frac{4acT_\psi^3}{3\kappa} \frac{d\psi}{dn} \frac{dT_\psi}{dM_\psi} \frac{4\pi r_\psi^4 uw}{GM_\psi} \qquad (1.4.16)$$

where F_ψ is the radioactive flux on the equipotential surface ψ = constant. By integrating F_ψ over the equipotential surface ψ = constant, we get

$$L_\psi = \int_{\psi=const} F_\psi d\sigma$$

$$= -\frac{4acT_\psi^3}{3\kappa} \frac{d\psi}{dn} \frac{dT_\psi}{dM_\psi} uw \frac{4\pi r_\psi^4}{GM_\psi} \int_{\psi=const} \left(\frac{d\psi}{dn}\right) d\sigma$$

$$= \frac{64\pi^2 acT_\psi^3 r_\psi^4}{3\kappa} u^2 vw \frac{dT_\psi}{dM_\psi} \tag{1.4.17}$$

So that

$$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi}{64\pi^2 acT_\psi^3 r_\psi^4} f_T \tag{1.4.18}$$

Using equation (1.4.9), this equation can be expressed as

$$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi \rho_\psi}{16\pi acT_\psi^3 r_\psi^2} f_T \tag{1.4.19}$$

where

$$f_T = \frac{1}{u^2 vw}$$

Equations (1.4.10), (1.4.13), (1.4.14), and (1.4.18) which are the four basic equations governing the equilibrium structure of a gaseous sphere distorted by rotation and tidal forces.

These reduced to the normal equation used for determining the equilibrium structure of spherical models of stars when distortion parameters u, v, w are set one each. The boundary conditions which the above equations have to satisfy are:

$M_\psi = 0, L_\psi = 0$, at the centre $r_\psi = 0$ (1.4.19a)

$M_\psi = M_0$, $L_\psi = L_{\psi s}$, $P_\psi = 0$, $T_\psi = 0$

Or $P_\psi = P_{\psi s}$, $T_\psi = T_{\psi s}$ at the free surface $r_\psi = R_\psi$ (1.4.19b)

where M_0 is the total mass of model and $L_{\psi s}, P_{\psi s}, T_{\psi s}$ are the values of the L_ψ, P_ψ, T_ψ respectively, on the outermost equivalent surface.

1.5 THE PRESENT WORK

The problem to study the equilibrium structure of stars distorted by rotational and tidal effect has great importance in astrophysics. Such a problem will help better to understand inner structure of binary system. Therefore, there is need to study the equilibrium structure of gaseous sphere.

Analytic study of determining the equilibrium structure of rotationally and tidally distorted stellar models is quite complex. Therefore, investigators attempted to solve such problems in some approximate way. In one such attempt Mohan, Saxena and Aggarwal used Kippenhahn and Thomas averaging technique together with the results of Kopal on Roche equipotential, to determine the effects of rotation and tidal distortion on the equilibrium structure of binary stars. However, the problems of determining the equilibrium structure of aligned and non synchronously rotating stars have not been satisfactory tackled so far.

In the present thesis an attempt has been made to study the effect of aligned and non synchronous rotation on the equilibrium structure of rotation and tidal distortion on the binary system.

The thesis consists of three chapters and chapterwise summary of the work is presented below. Chapter one is introductory in nature where we discussed the astrophysical significance of studying the equilibrium structure of rotationally and tidally distorted stellar models. Chapter two deals with the concept of modified Roche equipotential of distorted stars which accounts for the aligned, non synchronous rotation of the binary system.The explicit expression for distortional parameters u, v, w, f_P and f_T are obtained using Kippenhahn and Thomas averaging technique along with the results of Roche equipotential. In this chapter we have also discussed how to find the equilibrium structure of rotationally and tidally distorted stellar models in presence of aligned and non synchronous rotation.

The methodology developed in Chapter-II is next used in chapter III to determine the equilibrium structure of non synchronous rotating and tidally distorted polytropic models of stars. Computation is carried out to determine the inner structure as well as various physical parameters of the rotationally and tidally distorted polytropic models with various values of degree of non synchronous parameter *f*. Conclusion based on the study has finally been discussed.

CHAPTER - II

MODIFIED ROCHE EQUIPOTENTIAL OF DISTORTED STARS

The section 2.1 involves the concept of modified equipotential of stars distorted by aligned and non synchronous rotation. The explicit expressions for volume, surface area, gravitational acceleration and distortional parameters have also developed in this section. In section 2.2 we have presented how Mohan et al. used Kippenhahn and Thomas approach in conjunction with certain result on Roche equipotentials to determine the system of differential equation for rotationally and tidally distorted binary components in the influence of aligned and non synchronous rotation.

2.1 MODIFIED ROCHE EQUIPOTENTIAL OF DISTORTED STARS

Roche equipotential have been used to analyze the problems of rotationally and tidally distorted system of stars. In order to introduce the concept of Roche equipotential, we assume two components of a close binary system known as primary star and secondary star. The primary star is supposed to be much more massive as compare to secondary star which is supposed to be point mass causing tidal effects on the primary star. Both the components of binary system are assumed to be rotating about their axis as well as revolving about their centre of mass. Following Kopal, Mohan and Singh, and Mohan et al approach, certain results are summarized below:

Suppose $M_o \; and \; M_1$ be the masses of the two components of a close binary system separated by the distance D. Suppose that the primary component of this system having mass M_o is much larger than secondary star of mass M_1 which can be regarded as a point mass i.e. $\left(M_o \geq M_1\right)$.

Following Bisikalo et al.(2), the shape of the mass losing components in a semi-detached binary system can be determined, assuming (i) the circular orbits, (ii) the rotation of the star is synchronized with the orbital rotation $\Omega_* = \Omega$, and (iii) the stars are strongly concentrated, so that gravitational field would be considered as point mass. We assume the Cartesian coordinate system where X axis is the line connecting centre of stars, the Z-axis is perpendicular to the plane of the orbit with the rotation axis and Y-axis forms the right handed coordinate system whose origin is at the centre of mass of the donor star. Then the potential due to gravitational and disturbing forces acting at an arbitrary point P(x,y,z) is given as

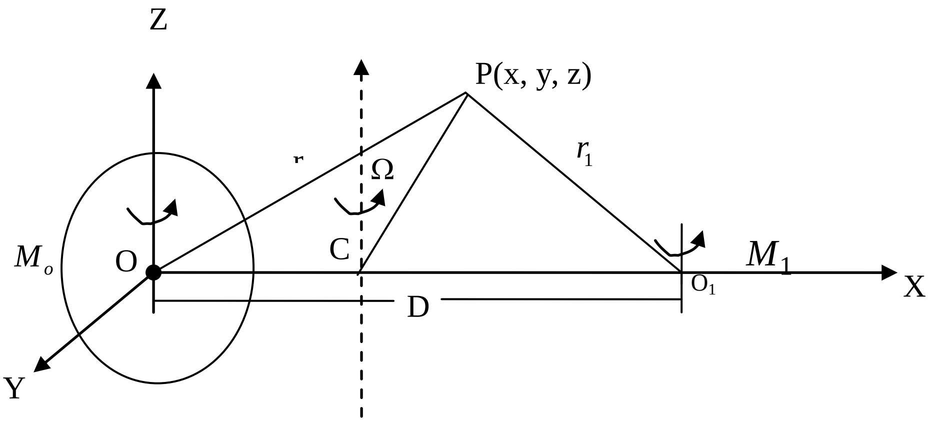

Fig 1.1: Axis of for a binary system

$$\psi = \frac{GM_o}{r} + \frac{GM_1}{r_1} + \frac{1}{2}\Omega^2\left[(x-x_c)^2 + y^2\right] \tag{1.20}$$

Where $r^2 = x^2 + y^2 + z^2$ *and* $r_1^2 = (D-x)^2 + y^2 + z^2$ represent the square of distances of P from centre of gravity of the two components of the binary star, Ω be the orbital angular velocity about an axis perpendicular to xy- plane and passing through centre of mass. Also $x_c = \frac{M_1 D}{M_o + M_1}$. The first term, second and third term of (1.20) are the potential arises due to mass M_o , M_1 and the potential fom centrifugal force respectively.

The situation changes when the shape of potential differs from (1.20), in that case Roche approximation is not applied. We now analyze the situation when rotation is misaligned and non-synchronous. Following Bisikalo et al.(2) the total potential for the aligned, non synchronous rotation including all the forces in rotating frame of reference, except Corolis force and the pressure gradient is given as

$$\psi = \frac{GM_o}{r} + \frac{GM_1}{r_1} + \frac{1}{2}\left[\left(x-x_c\right)^2 + y^2\right] + \frac{1}{2}(\Omega_*{}^2 - \Omega^2)(x^2 + y^2) \tag{1.21}$$

$$\psi = \frac{GM_o}{r} + \frac{GM_1}{r_1} + \frac{1}{2}\left[(x-x_c)^2 + y^2\right] + \frac{1}{2}(f^2-1)(x^2+y^2) \tag{1.21}$$

where $f = \frac{\Omega_*}{\Omega}$.

When $f=1$, that is, $\Omega_* = \Omega$, the potential coincides with (1. 20) which is for aligned and synchronous rotation.

Equation (1.21) in non dimensionless form is given as

$$\psi^* = \frac{1}{r^*} + q\left[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^*\right] + nf(1-\nu^2)r^* \tag{1.22}$$

where

$\psi^* = \frac{D\psi}{GM_o} - \frac{M_1^{\ 2}}{2M_o(M_o+M_1)}$ is non-dimensionless form of potential

$q = \frac{M_1}{M_0}$ be mass ratio. $r^* = \frac{r}{D}$ is non-dimensional form of r.

$\lambda = S\text{in}\theta Cos\phi,\ \mu = Sin\theta Sin\phi,\ \nu = Cos\theta$ be the spherical polar coordinates.

Equation (1.22) reduces to potential of a purely nonsynchronously rotating spherical model if q=0. For n = 0 it reduces to the potential of non-rotating spherical model distorted by tidal effects of secondary component of binary star alone.

Defining a non-dimensional variable r_o by the relation

$$r_o = \frac{1}{\psi^* - q} \tag{1.23}$$

Kopal has also shown that on the surface of Roche equipotentials (r,θ,ϕ) are connected by the relation

$$r^* = r_o\left[1 + C_3 r_o^3 + C_4 r_o^4 + C_5 r_o^5 + C_6 r_o^6 + C_7 r_o^7 + C_8 r_o^8 + C_9 r_o^9 + C_{10} r_o^{10} \ldots\right] \tag{1.24}$$

where

$$
\begin{aligned}
&C_3 = qP_2 + nf(1-\nu^2),\ C_4 = qP_3,\ C_5 = qP_4 \\
&C_6 = qP_5 + 3C_3{}^2,\ C_7 = qP_6 + 7qC_3P_3 \\
&C_8 = qP_7 + 8qC_3P_4 + 4q^2P_3^2 \\
&C_9 = qP_8 + 9qC_3P_5 + 9q^2P_3P_4 \\
&C_{10} = qP_9 + 10q^2P_3P_5 + 10qC_3P_6 + 5q^2P_4^2
\end{aligned}
$$

This $P_j = P_j(\lambda)$ is Legendre polynomial and terms up to second order of smallness in n, q and f have been retained in equation (1.24). This relation will help in obtaining the shape of Roche equipotentials $\psi^* =$ constant.

The volume V_ψ enclosed by the equipotential surface $\psi^* =$ constant is given by

$$V_\psi = \frac{2}{3}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^3}{\mu}\, d\lambda d\nu \qquad (1.25)$$

Kopal has given the explicit expression of V_ψ in terms of r_o that can be defined by (1.25), can be represented as

$$V_\psi = \frac{4}{3}\pi r_o{}^3 D^3\left[1 + 2nfr_o{}^3 + \left(\frac{12}{5}q^2 + \frac{8}{5}nfq + \frac{32}{5}n^2 f^2\right)r_o{}^6 + \frac{15}{7}q^2 r_o{}^8 + 2q^2 r_o{}^{10}\ldots\right] \qquad (1.26)$$

Following then approach of Kopal (31), Mohan and Singh (52), the explicit expression for the surface area S_ψ and the values of parameters $r_\psi, \overline{g}, \overline{g^{-1}}$ on the Roche equipotential ψ^* = constant has been calculated. These are

$$S_\psi = 2\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^2}{\mu}\, d\lambda d\nu$$

$$S_\psi = 4\pi r_o{}^2 D^2\left[1 + \frac{4}{3}nfr_o{}^3 + \left(\frac{7}{5}q^2 + \frac{14}{15}qnf + \frac{56}{15}n^2 f^2\right)r_o{}^6 + \frac{9}{7}q^2 r_o{}^8 + \frac{11}{9}q^2 r_o{}^{10}\ldots\right] \qquad (1.27)$$

$$r_\psi = \left[\frac{3}{4\pi}V_\psi\right]^{1/3}$$

$$= r_o D\left[1+\frac{2}{3}nfr_o^{\,3}+\left(\frac{4}{5}q^2+\frac{8}{15}qnf+\frac{76}{45}n^2f^2\right)r_o^{\,6}+\frac{5}{7}q^2r_o^{\,8}+\frac{2}{3}q^2r_o^{\,10}\ldots\right] \tag{1.28}$$

$$\begin{aligned}\bar{g} &= \frac{2}{S_\psi}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}}\left(\frac{d\psi}{dn}\right)\frac{r^2}{\mu}\,d\lambda\, d\nu \\ &= \frac{GM_\psi}{r_o^{\,2}D^2}\left[1-\frac{8}{3}nfr_o^{\,3}-\left(2q^2+\frac{4}{3}qnf+\frac{28}{9}n^2f^2\right)r_o^{\,6}-\frac{15}{7}q^2r_o^{\,8}-\frac{7}{3}q^2r_o^{\,10}\ldots\right]\end{aligned} \tag{1.29}$$

$$\begin{aligned}\overline{g^{-1}} &= \frac{2}{S_\psi}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}}\left(\frac{d\psi}{dn}\right)^{-1}\frac{r^2}{\mu}\,d\lambda\, d\nu \\ &= \frac{r_0^{\,2}D^2}{GM_\psi}\left[1+\frac{8}{3}nfr_o^{\,3}+\left(\frac{26}{5}q^2+\frac{52}{15}qnf+\frac{524}{45}n^2f^2\right)r_o^{\,6}+\frac{40}{7}q^2r_o^{\,8}-\frac{11}{9}q^2r_o^{\,10}\ldots\right]\end{aligned} \tag{1.30}$$

where $r_\psi^{\,*} = r_\psi / D,\ r_\psi^{\,*}$ being the non dimensional form r_ψ. In all the above expressions terms upto second order of smallness in n and q are retained.

2.2 MOHAN, SAXENA and AGARWAL's APPROACH FOR COMPUTING THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED GASEOUS SPHERES

Mohan, Saxena and Agarwal used the concept of Roche equipotentials proposed by Kopal in conjunction with Kippenhahn and Thomas's averaging approach to explicitly obtain equations governing the equilibrium structures of rotationally and tidally distorted stars. In this section we briefly review their approach.

In order to determine the inner structure of rotationally and tidally binary components, the system of equations (1.19a), (1.19b) has to be integrated numerically subject to the boundary conditions (1.20) specified therein. Therefore, the evaluation of the actual equipotential surface of a rotationally and tidally distorted gaseous sphere is complicated. Kippehahn ad Thomas proposed that for the evaluation of the distorted parameters u, v, w, f_P, f_T etc., the actual equipotential surface may be replaced by Roche equipotetial surface.

Once the equipotential surfaces of a rotationally and tidally distorted star are approximated by the Roche equipotentials, the results obtained by Kopal and Mohan and Singh may be used to evaluate explicitly the values of the distortion parameters $u, v, w, f_P \; and \; f_T$ appearing in the stellar structure equations (1.13) and (1.19). Using the explicit expressions of the distorted parameters, their values are calculated below

$$u = \frac{S_\psi}{4\pi r_\psi^{\,2}}$$

$$= 1 - \left(\frac{1}{5}q^2 + \frac{2}{15}qnf + \frac{4}{45}n^2 f^2\right) r_o^{\,6} - \frac{1}{7}q^2 r_o^{\,8} - \frac{1}{9}q^2 r_o^{\,10} + \ldots \qquad (1.31a)$$

$$v = \frac{\overline{g} r_\psi^{\,2}}{GM_\psi}$$

$$= 1 - \frac{4}{3}nfr_o^{\,3} - \left(\frac{2}{5}q^2 + \frac{4}{15}nfq + \frac{128}{45}n^2 f^2\right) r_o^{\,6} - \frac{5}{7}q^2 r_o^{\,8} - q^2 r_o^{\,10} + \ldots \qquad (1.31b)$$

$$w = \frac{\overline{g^{-1}} GM_\psi}{r_\psi^{\,2}}$$

$$= 1 + \frac{4}{3}nfr_o^{\,3} + \left(\frac{18}{5}q^2 + \frac{12}{5}qnf + \frac{272}{45}n^2 f^2\right) r_o^{\,6} + \frac{30}{7}q^2 r_o^{\,8} - \frac{23}{9}q^2 r_o^{\,10} + \ldots \qquad (1.31c)$$

$$f_p = \frac{1}{uw}$$

$$= 1 - \frac{4}{3}nfr_o^{\,3} - \left(\frac{17}{5}q^2 + \frac{34}{15}qnf + \frac{188}{45}n^2 f^2\right) r_o^{\,6} - \frac{29}{7}q^2 r_o^{\,8} + \frac{8}{3}q^2 r_o^{\,10} \ldots \qquad (1.31d)$$

and

$$f_T = \frac{1}{u^2 vw}$$

$$=1-\left(\frac{14}{5}q^2+\frac{28}{15}qnf+\frac{56}{45}n^2f^2\right)r_o{}^6-\frac{23}{7}q^2r_o{}^8+\frac{34}{9}q^2r_o{}^{10}\ldots \qquad (1.31e)$$

where $r_\psi{}^*=\frac{r_\psi}{D}$ is the non-dimensional form of r_ψ ad terms upto second order of smallness in n and q are retained.

The values of M_ψ, P_ψ, L_ψ etc. on the various equipotential surfaces of a rotationally and tidally distorted gaseous spheres may now be obtained by solving the system of differential equations (1.19a), (1.19b) with boundary conditions (1.20) and using the values of distortion parameters f_P and f_T as given in (1.35).

It may be noted that while approximating the equipotential surfaces of a rotationally and tidally distorted model by Roche equipotentials, the structure of the star is not approximated by the structure of a Roche model. In case of no distortion (n=q=0 ; f=1), equation (1.35) gives $u=v=w=f_P=f_T=1$ and the system of differential equation (1.20) governing the equilibrium structure of rotationally and tidally distorted gaseous sphere. At every step, the values of the distortion parameters u, v, v, f_p and f_T have to be computed using (1.35). In case the thermal properties are not considered important and only hydrostatic equilibrium of a rotationally and tidally distorted gaseous spheres is to be investigated then we need only to integrate equation (1.9) and (1.12) subject to boundary conditions.

At the centre $r_\psi=0 \quad M_\psi=0$

and at the free surface $r_\psi=R_\psi,\ M_\psi=M_0, P_\psi=0, \rho_\psi=0$ or $P_\psi=P_{\psi s}, \rho_\psi=\rho_{\psi s}$

In case the star is being distorted by rotational forces alone (or tidal forces alone) we may set $q=0\,(n=0)$ in (1.43) and still use the above approach to determine the equilibrium structure of corresponding purely rotationally distorted or purely tidally distorted model. For obtaining the structure of primary component of synchronous binary system we may set $n=(q+1)/2$

Mohan and Saxena (57, 58) found it more convenient to work with r_0 in place of M_ψ or r_ψ through the relation (1.34). Saxena (80) expressed the system differential equation governing the equilibrium structure of rotationally and tidally distorted stellar model as

$$\frac{dM_\psi}{dr_0} = 4\pi D^3 \rho_\psi r_0{}^2 f_1$$

$$\frac{dP_\psi}{dr_0} = \frac{GM_\psi}{Dr_0^2} \rho_\psi f_2 \qquad (1.\ 32)$$

$$\frac{dL_\psi}{dr_0} = 4\pi\varepsilon D^3 \rho_\psi r_0{}^2 f_1$$

$$\frac{dT_\psi}{dr_0} = \frac{3\kappa l_\psi}{16\pi DacT_\psi^3} \frac{\rho_\psi}{r_0^2} f_3$$

wher f_1, f_2 and f_3 are functions of n, q, f and r_0 incorporating the effects of nonsynchronous rotation and tidal distortions on the equilibrium structure of a stellar model. The explicit expressions for these parameters are given as

$$f_1 = \frac{r_\psi^2}{r_o^2 D^3} \frac{dr_\psi}{dr_o}$$

$$= 1 + 4nfr_o{}^3 + \left(\frac{36}{5} q^2 + \frac{24}{5} qnf + \frac{96}{5} n^2 f^2 \right) r_o{}^6 + \frac{55}{7} q^2 r_o{}^8 + \frac{26}{3} q^2 r_o{}^{10} \ldots \quad (1.33)$$

$$f_2 = \frac{f_P r_o^2}{r_\psi^2} \frac{dr_\psi}{dr_o} D$$

$$= 1 - \left(\frac{3}{5} q^2 + \frac{2}{5} qnf + \frac{4}{15} n^2 f^2 \right) r_o{}^6 + \frac{6}{7} q^2 r_o{}^8 + \frac{26}{3} q^2 r_o{}^{10} \ldots \qquad (1.34)$$

$$f_3 = \frac{r_o^2 f_T}{r_\psi{}^2} \frac{dr_\psi}{dr_o} D$$

$$= 1 + \frac{4}{3} nfr_o{}^3 + \left(\frac{6}{5} q^2 + \frac{4}{5} qnf + \frac{224}{45} n^2 f^2 \right) r_o{}^6 + \frac{12}{7} q^2 r_o{}^8 + \frac{88}{9} q^2 r_o{}^{10} \ldots \quad (1.35)$$

In these above expressions terms up to second order of smallness in n, q and up to $r_o^{10}\, in\, r_o$ are retained. The boundary conditions for the system of differential equation (1.45) are

At the centre $r_o = 0, M_\psi = 0, L_\psi = 0$

At the surface,

$$r_o = r_{os}$$
$$M_\psi = M_o\,, L_\psi = L_{\psi s}$$

$$P_\psi = 0, \rho_\psi = 0, T_\psi = 0 \;\; or \;\; P_\psi = P_{\psi s}\,, \rho_\psi = \rho_{\psi s}\,, T_\psi = T_{\psi s}$$

where r_{os} being the value at the free surfaces.

In fact,

$$r_{os} = \frac{1}{\psi_s^* - q}$$

where ψ_s^* is the non-dimensional form of the total potential ψ on the outermost equipotential surface of rotationally and tidally distorted stellar model. For $f_P = f_T = 1$, the equations reduces for undistorted gaseous sphere.

CHAPTER – III

EQUILIBRIUM STRUCTURE OF ALIGNED AND NON-SYNCHRONOUSLY ROTATING POLYTROPIC MODEL OF STARS

Polytropic models have been used in literature to understand the inner structures of realistic stars. Chandrasekhar( ) developed the theory of distorted polytropes in order to study the effect of aligned and non synchronous rotation of distorted polytropes. Then several investigators such as Roberts (74), James(29), Hurley and Roberts, Monaghan and Roxburgh( ) , Monaghan( ), Kovetz( ), Smith( ), Linnell ( ) have discussed the structures of uniformly rotating polytropes. Naylor and Anand( ) studied the structure of rotating polytropes assuming the primary component being a synchronous close binary system.

In section 3.1, the boundary value problem has been developed to study the equilibrium structure of polytropic binary system. In section 3.2

3.1 INTRODUCTION TO POLYTROPIC MODELS OF STARS

In a polytropic mode, l the pressure P and the density ρ are related with

$$P = P_c\theta^{N+1} \, and \, \rho = \rho_c\theta^N \tag{3.1}$$

where P_c and ρ_c are the pressure and density at the centre of the model and θ is a parameter depending upon the distance of the arbitrary point from the centre. The term N is polytropic index lies between 0 and 1. Index N is a measure of central concentration of the model. A polytropic model with N=0 has a homogenous structure in which density is uniform whereas if N=5 means highly centrally condensed model whose radius extends to infinity.

The second order nonlinear differential equation determining the equilibrium structure of a polytropic model of index N is given by the solution of non linear differential equation

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^N \tag{3.2}$$

subject to the boundary conditions

$$\begin{aligned}&\theta = 1, \ \frac{d\theta}{d\xi} = 0 \ \text{at the centre} \ \xi = 0\\ &and\\ &\theta = 0 \ \text{at the surface} \ \xi = \xi_u\end{aligned} \tag{3.3}$$

Equation (3.2) is known as Lane-Emden equation. Analytic solution of this equation are possible for N=0, 1 and 5. The solution of Lane-Emden equation satisfies the conditions (3.3) and thus helps to compute parameters of the star having inner structure as a polytrope of index N

3.2 MATHEMATICAL MODEL FOR DETERMINING THE EQUILIBRIUM STRUCTURE OF ALIGNED AND NON SYNCHRONOUS ROTATING AND TIDALLY DISTORTED STAR

Let P_ψ *and* ρ_ψ be the pressure and density on the equipotential surface ψ = constant of the distorted model. Then the values of the density and the pressure on the equivalent equipotential surface of the corresponding spherical model will be ρ_ψ *and* P_ψ respectively. It is assumed that the distorted model behave as polytropic model so that P_ψ *and* ρ_ψ are connected through the polytropic type of relations

$$P_\psi = P_{c\psi}\theta_\psi^{N+1} \text{ and } \rho_\psi = \rho_{c\psi}\theta_\psi^{N} \tag{3.4}$$

Where $P_{c\psi}$ and $\rho_{c\psi}$ are values of P_ψ and ρ_ψ at the centre and θ_ψ is some average of θ on the equipotential surface $\psi =$ constant. The index N used in equation (3.4) is called the polytropic index of the model which lies between 0 to 5 for the practical interest to the problem of stellar structure. The fact, N represents the central condensation of the model. Equation (1.36a) and (1.36b) which govern the hydrostatic equilibrium rotationally and tidally distorted gaseous sphere can be combined to yield

$$\frac{1}{r_0^{\,2}}\frac{d}{dr_0}\left[\frac{r_0^2}{\rho_\psi f_2}\frac{dP_\psi}{dr_0}\right] = -4\pi G D^2 P_\psi f_1 \tag{3.5}$$

On substituting the values of P_ψ and ρ_ψ from relation (3.4), this equation can be expressed in the non dimensional form as

$$\frac{1}{r_o^{\,2}}\frac{d}{dr_0}\left[\frac{r_0^2}{\rho_\psi f_2}\frac{d\theta_\psi}{dr_0}\right] = -\frac{D^2}{\alpha^2}\theta_\psi^N f_1 \tag{3.6}$$

where

$$\alpha^2 = \frac{(N+1)P_{c\psi}}{4\pi G\rho_{c\psi}^2}$$

and f_1 and f_2 are the functions of distortion parameters n, q, f and r_0. The boundary conditions are specified as follows: P_ψ and ρ_ψ must be maximum at the centre and zero at free surface, we should have θ_ψ maximum at the centre and zero at the free surface.

These lead to the condition that $\theta_\psi =1$ and $\frac{d\theta_\psi}{dr_0}=0$ at the centre and $\theta_\psi =0$ at the free surface. Thus the boundary conditions satisfied by (3.5) are

$$\begin{aligned}&\text{At the centre}: r_o=0,\ \theta_\psi=1,\ \frac{d\theta_\psi}{dr_o}=0\\&\text{At the surface}: r_o=r_{os},\ \theta_\psi=0\end{aligned}\tag{3.7}$$

where r_{os} is the value of r_o at surface.

In the absence of any distortion $(f_1=f_2=1)$, equation (3.6) becomes Lane-Emden equation for an undistorted polytropic model of index N.

The quantity α defined as defined in (3.6) is of the dimension of length. The term ξ will become non dimensional defined for equivalent spherical model if $r_\psi=\alpha\xi$. It coincides with Emden variable ξ of the Lane-Emden equation for an undistorted spherical polytropic model. The dimensionless k is the ratio of the undistorted radius R_ψ of primary and D the distance between the centres of the two components.

$$\frac{D}{\alpha}=\frac{D\xi_u}{\alpha\xi_u}=\frac{D}{R_\psi\xi_u}=\frac{1}{k}\xi_u \tag{3.8}$$

where ξ_u is the value of ξ at the outermost surface of the undistorted polytropic model.

For k=1, only rotational forces will distort polytropic model. In case the polytropic model being the primary component then K must be such that the outermost surface of the primary component lies within the Roche lobe. From (3.6) and (3.8)

$$\frac{d}{dr_o}\left(\frac{r_o^2 d\theta_\psi}{f_2 dr_o}\right)=-\frac{\xi_u^2}{k^2}r_o^2\theta_\psi^N f_1 \tag{3.8a}$$

or
$$\frac{d}{dr_o}\left(A(r_o,n,q)\frac{d\theta_\psi}{dr_o}\right)=-\frac{\xi_u^2}{k^2}r_o^2\theta_\psi^N B\left(r_o,n,q\right) \tag{3.8b}$$

where

$$A\left(r_o,n,q\right)=\frac{r_o^2}{f_2}=r_o^2\left[1-\left(\frac{3}{5}q^2+\frac{2}{5}qnf+\frac{4}{15}n^2f^2\right)r_o^6-\frac{6}{7}q^2r_o^8-\frac{26}{3}q^2r_o^{10}...\right]$$

$$B(r_0,n,q)=f_1=\left[1+4nfr_o^3+\left(\frac{36}{5}q^2+\frac{24}{5}qnf+\frac{96}{5}n^2f^2\right)r_o^6+\frac{55}{7}q^2r_o^8+\frac{26}{3}q^2r_o^{10}...\right]$$

Equation (3.8) subject to boundary conditions (3.7) determines the equilibrium structure of aligned and non synchronously rotationally and tidally distorted polytropic model. For q=0 , the can only be used to determine the effect of rotational forces. For n=0, then it allows to study the equilibrium structure of polytropic model distorted by tidal forces alone.

In this section we present explicit expressions for computing the volume, surface area and the shape of a rotationally and tidally distorted polytropic model. Following the approach of Mohan and Saxena the total volume enclosed by rotationally and tidally distorted polytropic model is given by

$$V_\Psi = \frac{4\pi}{3}\left(\frac{l\xi_u}{K}\right)^3 r_0^3\left[1+2nr_{0S}^3+\left(\frac{12}{5}q^2+\frac{8}{5}nqf+\frac{32}{5}n^2f^2\right)r_{0S}^6+\frac{15}{7}q^2r_{0S}^8+2q^2r_{0S}^{10}+\dots\right]$$

$$S_\Psi = 4\pi r_{0S}^2\left(\frac{l\xi_u}{K}\right)^2\left[1+\frac{4}{3}nr_{0S}^3+\left(\frac{7}{5}q^2+\frac{14}{15}nqf+\frac{56}{15}n^2f^2\right)r_{0S}^6+\frac{9}{7}q^2r_{0S}^8+\frac{11}{7}q^2r_{0S}^{10}+\dots\right]$$

3.3 NUMERICAL COMPUTATION

In order to determine the numerical solution of the (3.8), we start integrate from the centre r_o=0, using $\theta_\psi = 1$ *and* $d\theta_\psi / dr_o$ as the initial conditions. N, ξ_u, n, q and K respectively denotes the polytropic index ,the radius of undistorted polytropic model, angular velocity of rotation , the ratio of the mass of companion and the ratio of the undistorted radius of the primary to the distance D between primary and secondary component.

Therefore, for obtaining the numerical solution, (3.8) has been integrated using fourth order Runge-Kutta method. The series solution is

$$\theta_\psi = 1-\frac{k^2}{6}r_o^6+\frac{Nk^4}{120}r_o^4-\frac{2nk^2}{15}r_o^5-\frac{N(8N-5)k^6}{15120}r_o^6+\frac{Nnk^4}{70}r_o^7\dots \qquad (3.9)$$

This series solution coincides with the one obtained for undistorted polytropic model obtained by Chandrasekhar. The value of r_o when θ_ψ becomes zero determines the outermost free surface of model. Further pressure P_ψ *and density* ρ_ψ on various equipotentials surface of the distorted model can be obtained and the other physical parameters of distorted polytropic stars are also computed.

Table 3.1: Values of $\theta_\psi, P_\psi, \rho_\psi$ for undistorted, tidally distorted and rotationally & tidally distorted

N=1.5, ξ_u=3.653750

Undistorted Models n=0.0, q=0.0, k=1.0, f=0.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6949	0.4250	0.1811	0.0000
P_Ψ	1.0000	0.7997	0.4025	0.1177	0.1396	0.0000
ρ_Ψ	1.0000	0.8745	0.5792	0.2770	0.0771	0.0000
n=0.0, q=0.0, k=1.0, f=0.05						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6949	0.4250	0.1811	0.0000
P_Ψ	1.0000	0.7997	0.4025	0.1177	0.0140	0.0000
ρ_Ψ	1.0000	0.8745	0.5792	0.2770	0.0771	0.0000
n=0.0, q=0.0, k=1.0, f=0.2						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6949	0.4250	0.1811	0.0000
P_Ψ	1.0000	0.7997	0.4025	0.1177	0.1396	0.0000
ρ_Ψ	1.0000	0.8745	0.5792	0.2770	0.0771	0.0000
n=0.0, q=0.0, k=1.0, f=0.8						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6949	0.4250	0.1811	0.0000
P_Ψ	1.0000	0.7997	0.4025	0.1177	0.1396	0.0000
ρ_Ψ	1.0000	0.8745	0.5792	0.2770	0.0771	0.0000
n=0.0, q=0.0, k=1.0, f=0.9						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6949	0.4250	0.1811	0.0000
P_Ψ	1.0000	0.7997	0.4025	0.1177	0.1396	0.0000
ρ_Ψ	1.0000	0.8745	0.5792	0.2770	0.0771	0.0000
n=0.0, q=0.0, k=1.0, f=1.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6949	0.4250	0.1811	0.0000
P_Ψ	1.0000	0.7997	0.4025	0.1177	0.1396	0.0000
ρ_Ψ	1.0000	0.8745	0.5792	0.2770	0.0771	0.0000
Tidal distortion models N=0.0, q=0.1, k=0.5, f=0.0						
x	0.0	0.2	0.4	0.6	0.8	1.0

θ_ψ	1.0000	0.9145	0.6950	0.4250	0.1812	0.0000
P_Ψ	1.0000	0.7998	0.4026	0.1177	0.0140	0.0000
ρ_Ψ	1.0000	0.8746	0.5793	0.2770	0.0771	0.0000
N=0.0, q=0.1, k=0.5, f=0.05						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6950	0.4250	0.1812	0.0000
P_Ψ	1.0000	0.7998	0.4026	0.1177	0.140	0.0000
ρ_Ψ	1.0000	0.8746	0.5793	0.2770	0.0771	0.0000
n=0.0, q=0.1, k=0.5, f=0.2						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6950	0.4250	0.1812	0.0000
P_Ψ	1.0000	0.7998	0.4026	0.1177	0.0140	0.0000
ρ_Ψ	1.0000	0.8746	0.5793	0.2770	0.0771	0.0000
n=0.0, q=0.1, k=0.5, f=0.8						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6950	0.4250	0.1812	0.0000
P_Ψ	1.0000	0.7998	0.4026	0.1177	0.0140	0.0000
ρ_Ψ	1.0000	0.8746	0.5793	0.2770	0.0771	0.0000
n=0.0, q=0.1, k=0.5, f=0.9						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6950	0.4250	0.1812	0.0000
P_Ψ	1.0000	0.7998	0.4026	0.1177	0.0140	0.0000
ρ_Ψ	1.0000	0.8746	0.5793	0.2770	0.0771	0.0000
n=0.0, q=0.1, k=0.5, f=1.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6950	0.4250	0.1812	0.0000
P_Ψ	1.0000	0.7998	0.4026	0.1177	0.0140	0.0000
ρ_Ψ	1.0000	0.8746	0.5793	0.2770	0.0771	0.0000
Rotational and tidal distortion n=0.1, q=0.1, k=0.5, f=0.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9145	0.6949	0.4249	0.1811	0.0000
P_Ψ	1.0000	0.7997	0.4025	0.1177	0.0140	0.0000
ρ_Ψ	1.0000	0.8745	0.5792	0.2770	0.0771	0.0000
n=0.1, q=0.1, k=0.5, f=0.05						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.9146	0.6950	0.4251	0.1812	0.0000
P_Ψ	1.0000	0.7999	0.4027	0.1178	0.0140	0.0000

ρ_{Ψ}	1.0000	0.8746	0.5795	0.2771	0.0772	0.0000
n=0.1, q=0.1, k=0.5, f=0.2						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_{ψ}	1.0000	0.9146	0.6953	0.4254	0.1814	0.0000
P_{Ψ}	1.0000	0.8001	0.4032	0.1181	0.0140	0.0000
ρ_{Ψ}	1.0000	0.8747	0.5798	0.2775	0.0773	0.0000
n=0.1, q=0.1, k=0.5, f=0.8						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_{ψ}	1.0000	0.9150	0.6965	0.4268	0.1823	0.0000
P_{Ψ}	1.0000	0.8009	0.4048	0.1190	0.0142	0.0000
ρ_{Ψ}	1.0000	0.8753	0.5812	0.2789	0.0778	0.0000
n=0.1, q=0.1, k=0.5, f=0.9						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_{ψ}	1.0000	0.9151	0.6966	0.4271	0.1825	0.0000
P_{Ψ}	1.0000	0.8011	0.4051	0.1192	0.0142	0.0000
ρ_{Ψ}	1.0000	0.8753	0.5814	0.2791	0.0780	0.0000
n=0.1, q=0.1, k=0.5, f=1.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_{ψ}	1.0000	0.9152	0.6968	0.4273	0.1826	0.0000
P_{Ψ}	1.0000	0.8012	0.4053	0.1194	0.0142	0.0000
ρ_{Ψ}	1.0000	0.8755	0.5817	0.2793	0.0780	0.0000

For N=3.0, ξ_u =6.896850

Undistorted Models n=0.0, q=0.0, k=1.0, f=0.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7532	0.4059	0.1932	0.0731	0.0000
P_Ψ	1.0000	0.3219	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4273	0.0669	0.0072	0.0004	0.0000
n=0.0, q=0.0, k=1.0, f=0.05						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7532	0.4059	0.1932	0.0731	0.0000
P_Ψ	1.0000	0.3219	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4273	0.0669	0.0072	0.0004	0.0000
n=0.0, q=0.0, k=1.0, f=0.2						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7532	0.4059	0.1932	0.0731	0.0000
P_Ψ	1.0000	0.3219	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4273	0.0669	0.0072	0.0004	0.0000
n=0.0, q=0.0, k=1.0, f=0.8						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7532	0.4059	0.1932	0.0731	0.0000
P_Ψ	1.0000	0.3219	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4273	0.0669	0.0072	0.0004	0.0000
n=0.0, q=0.0, k=1.0, f=0.9						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7532	0.4059	0.1932	0.0731	0.0000
P_Ψ	1.0000	0.3219	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4273	0.0669	0.0072	0.0004	0.0000
n=0.0, q=0.0, k=1.0, f=1.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7532	0.4059	0.1932	0.0731	0.0000
P_Ψ	1.0000	0.3219	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4273	0.0669	0.0072	0.0004	0.0000
Tidal distortion models n=0.0, q=0.1, k=0.5, f=0.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7533	0.4060	0.1931	0.0731	0.0000
P_Ψ	1.0000	0.3220	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4274	0.0669	0.0072	0.0004	0.0000

N=0.0, q=0.1, k=0.5, f=0.05						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7533	0.4060	0.1931	0.0731	0.0000
P_Ψ	1.0000	0.3220	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4274	0.0669	0.0072	0.0004	0.0000
n=0.0, q=0.1, k=0.5, f=0.2						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7533	0.4060	0.1931	0.0731	0.0000
P_Ψ	1.0000	0.3220	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4274	0.0669	0.0072	0.0004	0.0000
n=0.0, q=0.1, k=0.5, f=0.8						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7533	0.4060	0.1931	0.0731	0.0000
P_Ψ	1.0000	0.3220	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4274	0.0669	0.0072	0.0004	0.0000
n=0.0, q=0.1, k=0.5, f=0.9						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7533	0.4060	0.1931	0.0731	0.0000
P_Ψ	1.0000	0.3220	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4274	0.0669	0.0072	0.0004	0.0000
n=0.0, q=0.1, k=0.5, f=1.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7533	0.4060	0.1931	0.0731	0.0000
P_Ψ	1.0000	0.3220	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4274	0.0669	0.0072	0.0004	0.0000
Rotational and tidal distortion n=0.1, q=0.1, k=0.5, f=0.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7533	0.4060	0.1931	0.0731	0.0000
P_Ψ	1.0000	0.3220	0.0271	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4275	0.0669	0.0072	0.0004	0.0000
n=0.1, q=0.1, k=0.5, f=0.05						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7534	0.4060	0.1932	0.0731	0.0000
P_Ψ	1.0000	0.3222	0.2713	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4276	0.0669	0.0072	0.0004	0.0000
n=0.1, q=0.1, k=0.5, f=0.2						
x	0.0	0.2	0.4	0.6	0.8	1.0

θ_ψ	1.0000	0.7536	0.4063	0.1934	0.0732	0.0000
P_Ψ	1.0000	0.3226	0.0273	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4280	0.0671	0.0072	0.0004	0.0000
n=0.1, q=0.1, k=0.5, f=0.8						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7545	0.4075	0.1943	0.0738	0.0000
P_Ψ	1.0000	0.3241	0.0276	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4295	0.0677	0.0073	0.0004	0.0000
n=0.1, q=0.1, k=0.5, f=0.9						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7547	0.4078	0.1945	0.0739	0.0000
P_Ψ	1.0000	0.3243	0.0276	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4298	0.0678	0.0074	0.0004	0.0000
n=0.1, q=0.1, k=0.5, f=1.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.7548	0.4080	0.1946	0.0740	0.0000
P_Ψ	1.0000	0.3246	0.0277	0.0014	0.0000	0.0000
ρ_Ψ	1.0000	0.4300	0.0679	0.0074	0.0004	0.0000

FOR N=4.0 and $\xi_u = 14.97155$

Undistorted Models n=0.0, q=0.0, k=1.0, f=0.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4409	0.1789	0.0800	0.0300	0.0000
P_Ψ	1.0000	0.0167	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0379	0.0010	0.0000	0.0000	0.0000
n=0.0, q=0.0, k=1.0, f=0.05						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4409	0.1789	0.0800	0.0300	0.0000
P_Ψ	1.0000	0.0167	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0379	0.0010	0.0000	0.0000	0.0000
n=0.0, q=0.0, k=1.0, f=0.2						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4409	0.1789	0.0800	0.0300	0.0000
P_Ψ	1.0000	0.0167	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0379	0.0010	0.0000	0.0000	0.0000
n=0.0, q=0.0, k=1.0, f=0.8						

x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4409	0.1789	0.0800	0.0300	0.0000
P_Ψ	1.0000	0.0167	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0379	0.0010	0.0000	0.0000	0.0000
n=0.0, q=0.0, k=1.0, f=0.9						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4409	0.1789	0.0800	0.0300	0.0000
P_Ψ	1.0000	0.0167	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0379	0.0010	0.0000	0.0000	0.0000
n=0.0, q=0.0, k=1.0, f=1.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4409	0.1789	0.0800	0.0300	0.0000
P_Ψ	1.0000	0.0167	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0379	0.0010	0.0000	0.0000	0.0000
Tidal distortion models n=0.0, q=0.1, k=0.5, f=0.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4407	0.1787	0.0798	0.0298	0.0000
P_Ψ	1.0000	0.0166	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.03773	0.0010	0.0000	0.0000	0.0000
N=0.0, q=0.1, k=0.5, f=0.05						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4407	0.1787	0.0798	0.0298	0.0000
P_Ψ	1.0000	0.0166	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.03773	0.0010	0.0000	0.0000	0.0000
n=0.0, q=0.1, k=0.5, f=0.2						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4407	0.1787	0.0798	0.0298	0.0000
P_Ψ	1.0000	0.0166	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.03773	0.0010	0.0000	0.0000	0.0000
n=0.0, q=0.1, k=0.5, f=0.8						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4407	0.1787	0.0798	0.0298	0.0000
P_Ψ	1.0000	0.0166	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.03773	0.0010	0.0000	0.0000	0.0000
n=0.0, q=0.1, k=0.5, f=0.9						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4407	0.1787	0.0798	0.0298	0.0000

P_Ψ	1.0000	0.0166	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.03773	0.0010	0.0000	0.0000	0.0000
n=0.0, q=0.1, k=0.5, f=1.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4407	0.1787	0.0798	0.0298	0.0000
P_Ψ	1.0000	0.0166	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.03773	0.0010	0.0000	0.0000	0.0000
Rotational and tidal distortion n=0.1, q=0.1, k=0.5, f=0.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4407	0.1787	0.0798	0.0298	0.0000
P_Ψ	1.0000	0.0166	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0377	0.0010	0.0000	0.0000	0.0000
n=0.1, q=0.1, k=0.5, f=0.05						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4408	0.1788	0.0798	0.0298	0.0000
P_Ψ	1.0000	0.0166	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0378	0.0010	0.0000	0.0000	0.0000
n=0.1, q=0.1, k=0.5, f=0.2						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4411	0.1789	0.0799	0.0299	0.0000
P_Ψ	1.0000	0.0167	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0379	0.0010	0.0000	0.0000	0.0000
n=0.1, q=0.1, k=0.5, f=0.8						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4421	0.1796	0.0804	0.0302	0.0000
P_Ψ	1.0000	0.0169	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0382	0.0010	0.0000	0.0000	0.0000
n=0.1, q=0.1, k=0.5, f=0.9						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4423	0.1797	0.0804	0.0303	0.0000
P_Ψ	1.0000	0.0169	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.03827	0.0010	0.0000	0.0000	0.0000
n=0.1, q=0.1, k=0.5, f=1.0						
x	0.0	0.2	0.4	0.6	0.8	1.0
θ_ψ	1.0000	0.4425	0.1798	0.0805	0.0303	0.0000
P_Ψ	1.0000	0.0170	0.0002	0.0000	0.0000	0.0000
ρ_Ψ	1.0000	0.0383	0.0010	0.0000	0.0000	0.0000

UNDISTORTED MODEL		
Polytropic index N=1.5, n=0.0, q=0.0, k=1.0		
f	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$
0.0	2.0432	1.6776
0.05	2.0432	1.6776
0.2	2.0432	1.6776
0.8	2.0432	1.6776
0.9	2.0432	1.6776
1.0	2.0432	1.6776
UNIFORMLY ROTATION		
Polytropic index N=1.5, n=0.1, q=0.0 ,k=1.0		
f	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$
0.0	2.0432	1.6776
0.05	2.0530	1.6830
0.2	2.0848	1.7003
0.8	2.2361	1.7824
0.9	2.2640	1.7977
1.0	2.2925	1.8133
TIDAL DISTORTION		
Polytropic index, N=1.5 n=0.0, q=0.1, k=0.5		
f	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$
0.0	2.0438	1.6779
0.05	2.0438	1.6779
0.2	2.0438	1.6779
0.8	2.0438	1.6779
0.9	2.0438	1.6779
1.0	2.0438	1.6779
ROTATIONALLY AND TIDALLY DISTORTED		
Polytropic index, N=1.5 n=0.1, q=0.1, k=0.5		
f	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$
0.0	2.0438	1.6779
0.05	2.0451	1.6786
0.2	2.0493	1.6809
0.8	2.0663	1.6901
0.9	2.0692	1.6917
1.0	2.0721	1.6933
NON SYNCHRONOUS DISTORTED MODEL		
Polytropic index, N=1.5 n=0.2, q=0.2, k=0.5		
f	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$
0.0	2.0456	1.6787

0.05	2.0483	1.6802
0.2	2.0566	1.6847
0.8	2.0917	1.7037
0.9	2.0978	1.7070
1.0	2.1040	1.7104

3.4 Conclusion: -

Results presented in table 3.1 are the values of $\theta_\psi, \rho_\psi \text{and } \mathrm{P}_\psi$ for the respective undistorted, tidally distorted, rotationally and tidally distorted polytropic model for N=1.5, 3.0 and 4.0.

The results for the volume and surface area given in table 3.2 show that for all the polytropic models of indices 1.5, 3.0, 4.0, the volumes and surface areas of distorted models are larger compared to there corresponding values for the undistorted models. It has also been seen that compared to their tidal effects, the effects produced by rotational distortion, the volume and surface area of the distorted models are larger. Our results agrees with all the results earlier obtained by Mohan and Saxena( ) for f =1. As soon as f decreases from 1 to 0, the respectives values decrease with respect to the corresponding values earlier obtained by Mohan and Saxena( ).

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