Conservation plan for Spanish Imperial Eagle via mathematical modeling

Thesis submitted in partial fulfillment of the requirement for the award of the degree of

Masters of Science

In

Mathematics and Computing

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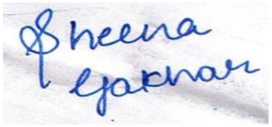
CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled "Conservation plan for Spanish Imperial Eagle via mathematical modeling" in partial fulfilment of the requirements for the award of degree of Master of Science, School of Mathematics, Thapar Institute of Engineering and Technology, Patiala is an authentic record of my own work carried out under the supervision of Dr. Parimita Roy.

The material submitted in this thesis has not been submitted for the award of any other degree of this or any other university.



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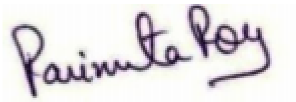

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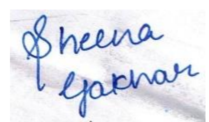

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ABSTRACT

Our planet is now in the midst of its sixth mass extinction of plants and animals. Despite evidence that diseases can increase extinction risk in wild animals (Woodroffe 1999; Daszak et al. 1999), few researchers have investigated the factors associated with disease-mediated declines or extinctions. Inspite of availability of tools and data no analytical framework is currently available to do this and predictive models are not validated against historical evidence of the interaction between risk predictors and risk change. Therefore, we urgently need to adopt strategies that limits disease mediated destruction if we want to avoid an acceleration of global extinctions. In this work we mainly focus on the dissemination of rabbit hemorrhagic disease in the rabbit population and its subsequences on Imperial Eagle population extinction. We designed a new eco-epidemiological model with simple law of mass action and Holling type-2 functional response. We perform stability analysis for both non-spatial and spatial model around the equilibrium equilibrium points. We will also gives conservation measures depending on our simulation result of the designed model system. Numerical simulation results confirm the analytical finding and generate beautiful patterns.

CONTENTS

Chapter-1

Introduction and Literature Review

1.1 Introduction

On Earth life is expected to die at a fast rate and it is also likely expected that soon the sixth biological extinction will take place (Dirzo et al. 2014). Naturally the Extinction of species take place at rate of one to five species per year but the rate is increasing from 1000 to 10,000 time (Chivian and Bernstein 2008). In the recent decades, there has a increase rate of extinction and decrease in bio-diversity. This leads to the concern towards conservation (Tittensor et al. 2014). Then increase in the disappearing of the species many more things can be affected. Human resources, economic growth, health and security are the things that area going to be most affected. There is an urgent need to know the major cause of extension of these species. Four main causes of extinction as suggested by Diamond (1989) are extinction-habitat loss, species invasion, over killing and co-extinction. But disease is not suggested as possible to species extinction or endangerment. Habitat loss and over exploitation are also most contributing factors in extinction (IUCN 2004). Hence we can roughly estimate that infectious disease causes no harm or lesser harm in isolation. Disease can also drive many species to extinction directly or indirectly. This doesn't seem surprising, but until today it hadn't been proven. Infectious diseases causes consequently population declines in wildlife, for example canine distemper virus in Serengeti lions (Roelke-Parker et al. 1996), Ebola outbreaks in African apes (Leroy et al. 2004), and multiple bacteria that affect amphibian populations (Daszak et al. 1999; Pounds et al. 2006, Schloegel et al. 2006).

The Spanish Imperial Eagle (*Aquilla Heliaca)* is rarest bird in this world. The size of reproductive population is so small in the Iberian Peninsula and due to unreliable factors there is a huge risk of extinction. The estimated population is only 250 brace (pairs), and hence the most threatened bird in Europe. Almost the entire reproductive population in the world can be found in Spain, however a pair successfully nested in Morocco in 1995 and another in 2006 in Portugal (González, 1996). The Spanish imperial eagle is now classified as 'endangered bird' in the Red Book of nesting birds of Spain and it is protected by Action Plan of the European Commission.

This bird mainly predates on rabbit population. In the places where substitute prey existed, such as Donana, the eagles ate the aquatic birds instead of the rabbits. In this thesis, conservation status of this bird is analyzed using mathematical model, using its current situation and highlighting the measures needed for its total recovery. The main threats reported for its mortality are electrocution, habitat fragmentation, poisoning, shooting and the decline of main prey, i.e. rabbit due to viral of haemorrhagic disease. RHD or Myxomatosis is a viral disease that caused big fall in the rabbit population and had catastrophic consequences for Spanish Imperial Eagle. It thought that many brace (pairs) stopped breeding when they suddenly broke of their main food while not having access to any abundant substitute prey.

1.2. Biological preliminaries

Susceptible population:

In this susceptible population, the units are free from infections i.e. this population have an active potential threat of infection as the units are healthy.

Infected population:

In this infected population, the units that are infected and also those units which can transmit the infectious disease. And this transmitted disease having adequate contacts with the susceptible class of the populations.

1.3. Tools of analysis

Stability:

The mathematical model or the mathematical equations which describe the physical phenomenon are in most of the cases of ordinary differential equations.

Local Stability:

Local stability of an equilibrium point means that if you put the system somewhere nearby the point then it will move itself to the equilibrium point in some time.

Lozinki measure:

Let A be a square matrix and $\|.\|$ be an induced matrix norm. The associated logarithmic norm μ of A is defined as

$$\mu(A) = \lim_{h \to 0^+} \frac{\| I + hA \| - 1}{h}$$

Here I is the identity matrix of the same dimension as A, and h is real, positive number. The limit $h \to 0^-$ equals $-\mu(-A)$ and is in general different from the logarithmic norm $\mu(A)$, as $-\mu(-A) \leq \mu(A)$ for all matrices.

Hurwitz Theorem:

The necessary and sufficient conditions for the real parts of all roots of the polynomial with the real coefficients

$$L(\lambda) = \lambda^n + a_1\lambda^{n-1} + ... + a_n$$

With real coefficients this is the positivity for all the principal diagonals

$$H_n = \begin{bmatrix} a_1 & 1 & 0 & 0 & 0 & .. & 0 \\ a_3 & a_2 & a_1 & 1 & 1 & .. & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 & .. & 0 \\ : & : & : & : & : & .. & 0 \\ 0 & 0 & 0 & 0 & 0 & .. & a_n \end{bmatrix}$$

It should be noted that the principal diagonal of the matrix H_n exhibits the coefficients of the polynomial $L(\lambda)$ in the order of their numbers from a_1 to a_n. The principal diagonal minors of the matrix by

$$D_1 = |a_1|, \quad D_2 = \begin{vmatrix} a_1 & 1 \\ a_3 & a_2 \end{vmatrix}, \ldots\ldots\ldots\ldots\ldots\ldots, D_n = |Hn|$$

This theorem is not suitable for large 'n'. To observe this suitability, lets apply it to polynomials of the second degree, third degree and fourth degree:

1. $\lambda^2 + a_1\lambda + a_2$

   The Hurwitz condition reduce to $a_1 > 0$ and $a_2 > 0$

2. $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$

The Hurwitz conditions reduce to $a_1 > 0$, $a_2 > 0$, $a_3 > 0$ and $a_1 a_2 - a_3 > 0$

3. $\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4$

The Hurwitz conditions reduce to $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, $a_4 > 0$ and $a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 > 0$.

From the Hurwitz condition it follows that all the $a_i > 0$, $i = 1, 2, \ldots n$; however, the positivity of all the coefficients is not enough for the real part of all the roots of L(λ) to be negative.

1.4. Motivation/state of the art

Particularly, we wish to increase the population in Portugal and Spain. And also wish to develop the overall ecosystem. This work is motivated by

(i) The quality of breeding Eagle to be improved for existing and also for new pairs of breeding.

(ii) The maintenance to be promoted and also to promote the conservation of main prey 'rabbit' that enhance the species 'natural recolonisation'.

(iii) To promote regeneration of the forest cover with native species.

1.5. Problem description and solution strategies

In Europe, The Spanish imperial Eagle is the most threatened bird. The ecological and eco-epidemiological model is not sufficient to understand the spatial spread of the disease. In the present study we will investigate the drivers of extinction risk using past changes to inform future extinction risk predictions by building spatial mathematical models and will give its application to conservation practice. Mathematical models which are both structurally and dynamically complex will be designed and the following will be answered

1. Which variables contribute to species decline and extinction?
2. What effect disease have on extinction?
3. When to vaccinate the species so that the process is cost effective?
4. How past changes to the drivers of extinction risk can be used to inform future extinction risk predictions?
5. This project will also make a step towards understanding the co extinction. It depends on the extinction of species. In this thesis the conservation actions should include the spatial planning, in areas of special conservation interest.

Chapter-2

Protecting and Conserving Spanish Imperial Eagle: A Mathematical Modeling Analysis

Imperial Eagle (Aquila *Heliaca*) is the rarest bird in this world and this is now classified as 'endangered(threatened) bird' in the Red Book of nesting birds of Spain and it is protected by Action Plan of the European Commission. Currently, the Spanish imperial eagle is confined to Portugal and also in Spain. The size of reproductive population is small in the Iberian Peninsula and due to the unreliable environmental factors there is a huge risk of extinction. Its main threats are mortality and it is caused by poisoning, habitat fragmentation, shooting and also the decline of main prey i.e. the European rabbit due to viral of *Haemorrhagic* disease. RHD (rabbit haemorrhagic disease) is that virus in which that infects only rabbits, and this virus has been used in some countries to control the population of rabbit. RHD is immensely hard to establish in the wild. With the viral of disease there is 75% of rabbits that will die in their burrows underground. Due to the difficulty of this, the morbidity and mortality estimates for haemmorrhagic disease that have a broad range. In the wild, the explosions in the rabbits vary depending on season, breeding cycles and geographical locations. Some areas see a high morbidity and mortality among the population of rabbit which is followed by calmer periods. Those Countries which are uninfected by the haemorrhagic disease may assign restrictions on importation from the endemic countries. In this work, we wish to amplify(increase) in the population of the Spanish Imperial eagle in Portugal and Spain, and also improve the overall Iberian population. We offer a new eco-epidemiological model that investigating a haemorrhagic disease (RHD) disease in the rabbit prey population, that used by a predator population (Eagle) and also discuss the rate of transmission in the context of the eco-epidemiology. By the relationship of bio-geographical relationship between the species of conservation concern, the Spanish imperial eagle and European rabbit, from this relation we explore the value for predator conservation.

2.1 Basic assumptions and mathematical modeling

In the present work, we have studied the impact of rabbit hemorrhagic disease on the dynamics of Spanish Imperial Eagle population. Its model is to primarily identifies the mathematical viable conditions. These conditions determine the predator population (Eagle) that can be able to either survive or becomes extinct. The model considers the following fact:

I. Eagle depends only on the Rabbits for survival as rabbit is the only food for eagle.

II. The species of the predator consumes the species of prey because predators cannot distinguish between the population of infected and susceptible.

III. There is an exponentially decreased in the predator population whenever there is absence of prey.

Considering these assumptions, the model can be constructed using non-linear differential equations.

$$\frac{dS}{dt} = r\,S\,(1-(S+I)/k) - bSI - w_1 SP/(S+A), \tag{1.1}$$

$$\frac{dI}{dt} = bSI - w_2 IP/(I+A) - d_1 I, \tag{1.2}$$

$$\frac{dP}{dt} = w_3 SP/(S+a) + w_4 IP/(I+A) - d_2 P. \tag{1.3}$$

Table2.1.1. Parameter values of model and their biological meanings.

Parameters	**Description**
R	Intrinsic growth
K	Carrying capacity
B	Infection rate
w_1	Predation rate
w_2	Predation rate
w_3	Conversion rate
w_4	Conversion rate
d_1	Death rate
d_2	Death rate
A	Environment protection rate

2.2. Analysis of non-spatial model

These following results show boundedness of system. The ecological point of view for the boundedness system is important.

2.2.1. Boundedness of the system

Theorem1: The solution of a system which initiate in the R_+^3 are bounded if $w_2 w_3 > w_1 w_4$.this condition holds.

Proof. We define $W = S + I + \frac{w_1 P}{w_3}$. (1.4)

On differentiating equation (1.4) and using eqn (1.1)-(1.3), we get

$$\frac{dW}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{w_1}{w_3}\frac{dP}{dt}$$

$$\frac{dW}{dt} + \mu W = rS(1-(S+I)/k) - bSI - w_1 SP/(S+A) + bSI - w_2 IP/(I+A) - d_1 I$$
$$+ w_3 SP/(S+A) + w_4 IP/(I+A) - d_2 P + \mu(S + I + w_1 P/w_3)$$

$$\frac{dW}{dt} + \mu W = rS(1-(S+I)/k) + (w_1 w_4/(w_3) - w_2) IP/(I+a) - d_1 I - w_1/w_3 d_2 P +$$
$$\mu S + \mu I + \mu \frac{w_1}{w_3} P$$

$$\frac{dW}{dt} \le \frac{k}{4r}(\mu + r)^2 - \frac{w_1}{w_3}(d_2 - \mu)P - (d_1 - \mu)I$$

Now, if we assume the value of $\mu < \min(d_1, d_2)$ then the right side of the above inequality is bounded. And from this boundednes we find w >0 such that $(dW/\ dt) + \mu W = \phi$

From these above equations, we have

$$W(t) \le e^{-\mu t} W(0) + \frac{\phi}{\mu}(1 - e^{-\mu t}),$$

Moreover, we have

$$\limsup_{t\to\infty} W(t) \le \frac{\phi}{\mu} = L_0,$$

it is independent of initial condition..

This result implies the interacting species that will not grow abruptly or exponentially for long-time interval. Due to limited resourceabundance each species is bounded.

2.2.2. Existence and stability analysis

The Jacobian matrix of a system is given by

J = $\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$, where

$$b_{11} = -bI - \frac{r}{k} + \frac{PSw_1}{(A+S)^2} - \frac{Pw_1}{A+S}, \quad b_{12} = -\frac{r}{k} - bS, \quad b_{13} = -\frac{Sw_1}{A+S}$$

$$b_{21} = bI, \quad b_{22} = -d_1 + bS + \frac{IPw_2}{(A+I)^2} - \frac{Pw_2}{A+I}, \quad b_{23} = -\frac{Iw_2}{A+I}$$

$$b_{31} = -\frac{PSw_3}{(A+S)^2} + \frac{Pw_3}{A+S}, \quad b_{32} = -\frac{IPw_4}{(A+I)^2} + \frac{Pw_4}{A+I}, \quad b_{33} = -d_2 + \frac{Sw_3}{A+S} + \frac{Iw_4}{A+I}$$

The system has the equilibrium points which are as follows:

1. The planar equilibrium point is $E_1(d_1/b, ((-d_1+bk)r)/(b(d_1k+r), 0)$ always exist if $d_1 > bk$.
2. The planar equilibrium point is $E_3(\frac{Ad_2}{-d_3+w_3}, \frac{rw_3(d_2(A+k)-kw_3)}{d_2kw_1(d_2-w3)}, 0)$ always exist if $d_3 > w_3$.

Theorem2: The planar equilibrium point $E_1(d_1/b, ((-d_1+bk)r)/(b(d_1k+r), 0)$ is asymptotically stable if $w_3 \le \frac{(Ab+d_1)(d_1-bk)rw_4}{d_1(-d_1r+bkr+Ab(d_1k+r))}, b < \frac{d_1}{k}, \frac{d_1}{A+k} \le b$.

Proof. The eigen values of $E_1(d_1/b, ((-d_1+bk)r)/(b(d_1k+r), 0)$ are

$$-\frac{bk^2r + r^2 + \sqrt{r(4d_1^3k^3 + 4d_1kr(-2bk^2+r) + r(-bk^2+r)^2 + d_1^2(-4bk^4+8k^2r))}}{2k(d_1k+r)},$$

$$-d_2 + \frac{d_1w_3}{Ab+d_1} + \frac{(-d_1+bk)rw_4}{-d_1r+bkr+Ab(d_1k+r)}$$

So E_1 will be asymptotically stable if $w_3 \le \dfrac{(Ab+d_1)(d_1-bk)rw_4}{d_1(-d_1r+bkr+Ab(d_1k+r))}, b<\dfrac{d_1}{k}, \dfrac{d_1}{A+k} \le b.$

Theorem 3: The equilibrium point $E_2(k,0,0)$ is locally asymptotically stable if $d_2 > \dfrac{kw_4}{A+k}$.

Proof. The eigenvalues about $E_2(k,0,0)$ are

$$\frac{-d_1k-bk^2-r-\sqrt{(d_1k+bk^2+r)^2-4k(bd_1k^2+d_1r+bkr)}}{2k},$$

$$\frac{-d_1k-bk^2-r+\sqrt{(d_1k+bk^2+r)^2-4k(bd_1k^2+d_1r+bkr)}}{2k},$$

$$-d_2+\frac{kw_4}{A+k}$$

So E_2 will be asymptotically stable if $d_2 > \dfrac{kw_4}{A+k}$.

Theorem 4: The planar equilibrium points $E_3(\dfrac{Ad_2}{-d_3+w_3}, \dfrac{rw_3(d_2(A+k)-kw_3)}{d_2kw_1(d_2-w3)}, 0)$ is asymptotically stable if $a_1>0, a_2>0, a_3>0$ and $a_1a_2-a_3>0$.

Proof. Characteristic equation for the equilibrium point $E_3(\dfrac{Ad_2}{-d_3+w_3}, \dfrac{rw_3(d_2(A+k)-kw_3)}{d_2kw_1(d_2-w3)}, 0)$

is $\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0.$

where

$$a_1=\frac{d_2{}^3(A+k)rw_1+d_2{}^2(A(Ab+d_1)k-(A+3k)r)w_1w_3+d_2(k(-Ad_1+3r)w_1+(A+k)rw_2)w_3{}^2-kr(w_1+w_2)w_3{}^3}{Ad_2kw_1(d_2-w_3)w_3}$$

$$a_2=\frac{1}{A^2d_2{}^2k^2w_1(d_2-w_3)w_3}r(Ad_2{}^4(Ab+d_1+d_2)k(A+k)w_1+d_2{}^3(-Ak(3(d_1+d_2)k+A(d_1+2(d_2+bk)))w_1+(A+k$$
$$+d_2{}^2k(A(3(d_1+d_2)k+A(d_2+bk))w_1-3(A+k)rw_2)w_3{}^2+d_2k(-A(d_1+d_2)kw_1+(A+3k)rw_2)w_3{}^3-k^2rw_2w_3{}^4)$$

$$a_3=\frac{r(d_2(A+k)-kw_3)(A(Ab+d_1)d_2{}^2kw_1+d_2-Ad_1kw_1(A+k)rw_2)w_3-krw_2w_3{}^2)}{A^2d_2k^2w_1w_3}.$$

From Routh-Hurwitz rule if $a_1>0, a_2>0, a_3>0$ and $a_1a_2-a_3>0$ then E_3 will be asymptotically stable.

Theorem 5: For thee equilibrium point $E^*(S^*, I^*, P^*)$ will be locally asymptotically stable if the roots of the equation $\lambda^3 + B_1\lambda^2 + B_2\lambda + B_3 = 0$ of the Jacobian matrix $J(E^*)$ satisfy Routh-Hurwitz rule i.e. $B_1 > 0, B_1B_2 > B_3$ where

$$J = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

and

$$B_1 = -a_{11} - a_{22} - a_{33},$$

$$B_2 = a_{11}\, a_{22} + a_{22}a_{33} + a_{11}\, a_{33} - a_{12}a_{21} - a_{13}a_{31} - a_{23}\, a_{32},$$

$$B_3 = a_{11}\, a_{22}\, a_{32} + a_{12}\, a_{21}\, a_{33} + a_{13}\, a_{22}\, a_{31} - a_{11}\, a_{22}\, a_{33} - a_{12}\, a_{13}a_{23} - a_{13}a_{21}\, a_{32},$$

also ,

$$a_{11} = -bI^* - \frac{r}{k} + \frac{P^*S^*w_1}{(A+S^*)^2} - \frac{P^*w_1}{A+S^*},\ a_{12} = -\frac{r}{k} - bS^*, a_{13} = -\frac{S^*w_1}{A+S^*}\ a_{21} = bI^*,$$

$$a_{22} = d_1 + bS^* + \frac{I^*P^*w_2}{(A+I^*)^2} - \frac{P^*w_2}{A+I^*},\ a_{23} = -\frac{I^*w_2}{A+I^*},\ a_{31} = -\frac{P^*S^*w_3}{(A+S^*)^2} + \frac{P^*w_3}{A+S^*},$$

$$a_{32} = -\frac{I^*P^*w_4}{(A+I^*)^2} + \frac{P^*w_4}{A+I^*} \quad , \quad a_{33} = -d_2 + \frac{S^*w_3}{A+S^*} + \frac{I^*w_4}{A+I^*}.$$

2.3. Numerical simulations

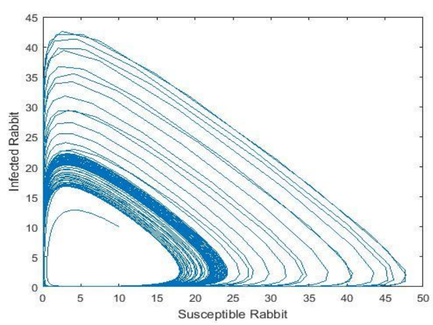
The system is investigated to numerically of global dynamic behavior. The aim to discover the model system for the existence of complex dynamics along with deterministic chaos and extinction dynamics. The Runge-Kutta method is used for the system investigation with the different parameter values and initial conditions. The values of parameters are selected on a basis as reported in Jorgensen (1979) and other related works that are in eco-epidemiology research and these parameter values are ecologically realistic. The arrangement of the parameter values which are fixed for simulation experiment are as follows:

r =8.85, k = 100, b = 2.681, w_1 = 0.25, w_2 = 4.1, w_3 =0.201, w_4 = 1.8, d_1 = 7.45 ,

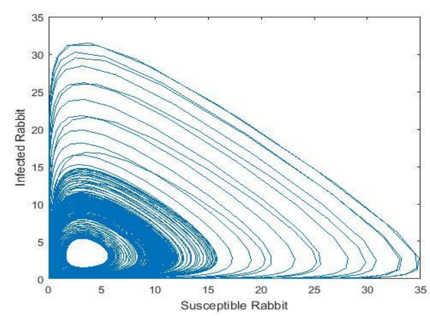
A=10 (1.5)

The estimated values which are estimated that are hypothetical and these values are to be chosen carefully that do not break the real scenario.

We present the simulation experiments and also establish the stability of the global of a model system (1.1-1.3) about the two equilibrium point E^* and E_3. We observe all species coexists when b is small. In this, there is a predator-free equilibrium point E_3, which is most undesirable and can be seen in figure 1.2(b). From this, we determine that we increase the parameter value b, and the extinction of the predator is permanently after the value of threshold. Consequently, it is advised to control the value of parameter b, for saving the eagle population from complete extinction and control the value of parameter b by depopulation, disinfection, surveillance and quarantines to effectively eradicate the infected population.



(a)



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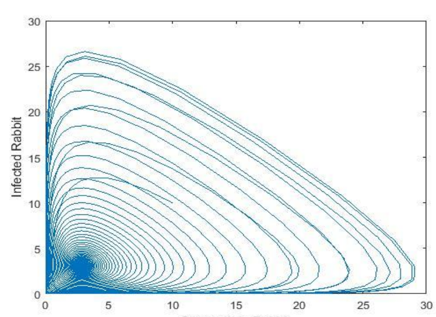
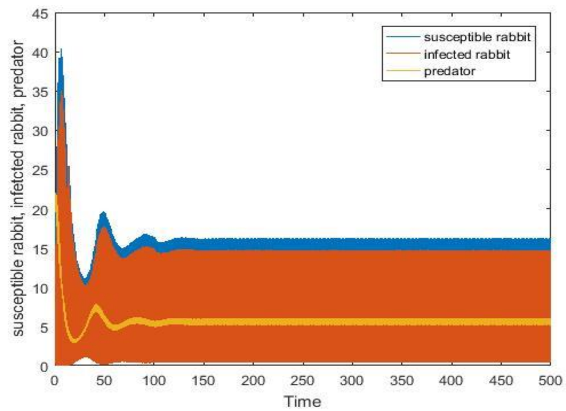
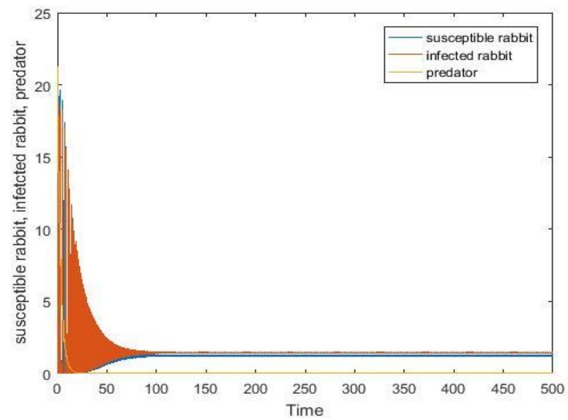


(c)

Figure 1.1. Attractors for different value of d_2 and keeping the others parameters be fixed as given in Eq. (12) (a) Chaos for $d_2 = 0.2298$, (b) Limit cycle for $d_2 = 0.398$, (c) Stable focus for $d_2 = 0.4698$.



(a)



(b)

Figure 1.2. (a) Co-existence of all three species for $b = 1.92$, (b) infected prey stability with healthy prey and extinction of the predator for $b = 5.992$ .other parameters are same as (a)

2.4. Conclusions and Discussion

The serious conservation has been raised for the extension of RHD. The extension of Haemorrhagic disease (RHD) had direct influence on Eagle population. In the thesis, we investigated a eco-epidemiological system of diffusive that conforms the attributes of the significant measures of the susceptible rabbits and infected rabbits-Imperial Eagle food chain. We achieved the solution of the conditions for the boundedness. And also for the non- diffusive model there is an existence and stability of the system. From simulations, it very is interesting to know that if we change the values of rate of the death of the population of predator d_2 the dynamics of the system changes rapidly ranging from chaos to stable focus. This gives us an idea that d_2 can be bifurcation parameter. It is also interesting to note that at the higher value of the disease of the transmission coefficient b, the predator can also at verge extinction. This suggests that recruitment failure owing to low rabbit numbers can be a primary cause of the population of eagle is decreased.

Chapter-3

Impact of diffusion in the distribution of species

Incorporating Brownian random movement that provides the sound biologically for the approximation of the path of movement of animal that is based on the distinct location data, and it is the powerful method to quantify the distributions. This chapter is an extension of the model discussed in chapter 2 incorporating local Brownian random motion in two-dimensional space. We suppose that population of prey and population of predator that perform energetic movements in *x* and *y* directions. These movements has ignored and also it relevant biologically. For the better opportunity and better food, the various requirements are occurred in random action of animals. On the other locations of spatial locations, the animals are immigrated on the demand of food availability and living conditions demand. In the model given in chapter 2, we include the diffusion term for animal movements. The movements are distributed randomly and uniformly in all the directions. The model with diffusion is written as

$$\frac{\partial S}{\partial t} = r(1-(S+I)/k) - bSI - w_1 SP/(S+A) + D_1 \nabla^2 S \qquad (2.1)$$

$$\frac{\partial I}{\partial t} = bSI - w_2 IP/(I+A) - d_1 I + D_2 \nabla^2 I \qquad (2.2)$$

$$\frac{\partial P}{\partial t} = w_3 SP/(S+A) + w_4 IP/(I+A) - d_2 P + D_3 \nabla^2 P \qquad (2.3)$$

the model system (2.1)-(2.3) is analyzed under zero flux boundary condition

$\frac{\partial S}{\partial n} = \frac{\partial I}{\partial n} = \frac{\partial P}{\partial n} = 0,\ z \in \partial\Omega, t > 0,$ and under the initial conditions given by

$S(z,0) = S_0 > 0, I(z,0) = I_0 > 0, P(z,0) = P_0 > 0$ **where** $z = (x, y) \in \Omega = [0, L] \times [0, L]$.

D_1, D_2, D_3 are diffusion coefficients of the susceptible prey, the infected prey and the predator. In above, *n* is outward unit normal vector of the boundary $\partial\Omega$ and the homogeneous Neumann boundary condition is being considered. The Laplacian operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, is used to describe the local Brownian random motion in two-dimensional space.

3.1. Analysis of spatial model

For the model system, we establish the instability of diffusion driven. Now firstly we deliver some basic results.

Let M_n(R) be the linear space of $n \times n$ matrices with entries in R, and $J^* = (b_{ij})_{nxn} \in M_n(R)$. For $1 \le k \le n$, let I_k denote the set $I_k = \{ (i_1, i_2, ..., i_k \mid 1 \le i_1 < i_2 .... < i_k \le n) \}$. Set $I_0 = \varnothing$. For any J= $(i_1, i_2, ..., i_k) \in I_k$. Let $P_j(J^*)$ denote the $k \times k$ principal sub matrix of J^*

$$\begin{pmatrix} b_{i_1 i_1} & b_{i_1 i_2} \cdots & b_{i_1 i_k} \\ b_{i_2 i_1} & b_{i_2 i_2} \cdots & b_{i_2 i_k} \\ b_{i_k i_1} & b_{i_k i_2} \cdots & b_{i_k i_k} \end{pmatrix}$$

And we define $P_{\varnothing}(J^*)$ =1. If D= $(D_1, D_2, ..., D_n)$ is a diagonal matrix with non- negative entries, let $D_j = \prod_{j \in J} D_j$ for $J \in I_k$ and $D_{\varnothing}$ =1. Let $J' = \{1, 2, ..., n\} / J$ for $J \in I_k$. The determinant $(J^* - D)$ be stated as a polynomial in D_j as

$$\det(J^* - D) = \sum_{k=0}^{n} (-1)^k \sum_{J \in I_k} \det(P_{J'}(J^*)) D_J$$

$$= \det(J^*) + \sum_{k=0}^{n} (-1)^k \sum_{J \in I_k} \det(P_{J'}(J^*)) D_J + (-1) \prod_{j=1}^{n} D_J$$

If D= λI_{nxn}, then above equation is the characteristics polynomials of J^*.

Definition 3.1. The matrix J^* is said to be assure the minor conditions if $(-1)^k \det(P_j(J^*)) \ge 0$ for all $J \in I_k$ and $1 \le k \le n$. The minor condition is said to be strict if the inequalities of minor conditions are strict.

Theorem 1. If $J^* = (b_{ij})_{3x3}$ is stable and also assures minor conditions then $(J^* - D)$ is also stable for all $D \geq 0$. The positive steady state $E^*(S^*, I^*, P^*)$ of the spatiotemporal system is asymptotically stable for $D > 0$.

Proof. Let $J^* = (b_{ij})_{3x3}$ be stable. By Routh-Hurwitz rule, J^* is stable iff $tr(J^*) < 0$, $\det(J^*) < 0$, and $tr(J^*)a_2 < \det(J^*)$,

Where a_2 is sum of all the 2 x 2 principle minors of J^*. Now, assume that J^* is satisfying the minor condition. We wish to show that $(J^* - D)$ also assures Routh-Hurwitz rule for an arbitrary $D \geq 0$. First, $tr(J^* - D) = tr(J^*) - tr(D) < 0$, since $tr(J^*) < 0$ and $D \geq 0$. It follows from the above equation and from the minor condition in which that $\det(J^* - D) < 0$. It verify the condition that $tr(J^* - D)a_2 < \det(J^* - D)$, where a_2 is sum of all 2x2 principal minors of $(J^* - D)$. We have,

$$(J^* - D) = \begin{pmatrix} b_{11} - D_1 & b_{12} & b_{13} \\ b_{21} & b_{22} - D_2 & b_{23} \\ b_{31} & b_{32} & b_{33} - D_3 \end{pmatrix}, \text{ and}$$

$$a_2 = \begin{pmatrix} b_{11} - D_1 & b_{12} \\ b_{21} & b_{22} - D_2 \end{pmatrix} + \begin{pmatrix} b_{11} - D_1 & b_{13} \\ b_{31} & b_{33} - D_3 \end{pmatrix} + \begin{pmatrix} b_{22} - D_2 & b_{23} \\ b_{32} & b_{33} - D_3 \end{pmatrix},$$

$$=$$

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} - b_{11}D_2 - b_{22}D_1 + D_1D_2 + \begin{pmatrix} b_{11} & b_{13} \\ b_{31} & b_{33} \end{pmatrix} - b_{11}D_3 - b_{33}D_1 + D_1D_3 + \begin{pmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{pmatrix} - b_{22}D_3 - b_{33}D_2 + D_2D_3$$

Thus,

$$tr(J^* - D)a_2 = (b_{11} + b_{22} + b_{33} - D_1 - D_2 - D_3)a_2$$

$$= (tr(A) - D_1 - D_2 - D_3)(a_2 - b_{11}D_2 - b_{22}D_1 + D_1D_2 - b_{11}D_3 - b_{33}D_1 + D_1D_3 - b_{22}D_3 - b_{33}D_2 + D_2D_3)$$

And

$$\det(J^* - D) = \det(J^*) - (\begin{pmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{pmatrix} D_1 + \begin{pmatrix} b_{11} & b_{13} \\ b_{31} & b_{33} \end{pmatrix} D_2 + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} D_3) + D_1(b_{33}D_2 + b_{22}D_3 - D_2D_3) + b_{11}D_2D_3$$

3.2. Diffusive instability

Diffusive Instability is the constant state which is asymptotically stable for the model system of non-spatial but it is not stable with respect to the solutions of model system of spatial. The constant state for homogeneous of E^* is stable asymptotically iff

1. $(-1)^n \det(J^*) > 0$

2. $\mu(J^{[2]}) < 0$

(Where μ is the lozinki measure that used in proving the stability of spatial model.)

holds. It will be unstable state by diffusion of any of the minor conditions $(-1)^k \det(P_J(J^*)) > 0$ fails to hold.

Theorem2. Assume that J is stable and it satisfies $(-1)^k \det(P_J(J^*)) < 0$ for some $1 \le k \le 3$ and $J \in I_k$ .then, there exists $D \ge 0$ such that $(J^* - D)$ is unstable.

Proof It is sufficient to show the existence of $D \ge 0$ such that $(-1)^3 \det(J^* - D) < 0$.

We desire D such that D_i =0 for $i \in J$ and $D_i = D > 0$ for $i \in J'$.

$$(-1)^3 \det(J^* - D) = (-1)^3 \det(J^*) + (-1)^k \det(P_J(J^*)) d^{3-k}$$

Since $(-1)^k \det(P_J(J^*)) < 0, (-1)^3 \det(J^* - D) < 0,$ if D is sufficiently large.

Examples: The parameter values are r = 8.85, k =100, b= 2.681, w_1 = 0.25, w_2 =4.1, w_3 = 0.201, w_4 = 1.8, d_1 7.45, d_2 =0.4698, and the jacobian matrix J^* for the homogeneous constant state is stable. Now for diffusion instability to be occur, we show that J^* satisfies $(-1)^k \det(P_J(J^*)) < 0$, for some $1 \le k \le 3$ and $J \in I_k$. For showing above result, we calculate all principle minors.

Solution: 1. The first order principal minor axis is

$$(-1)^3 \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = -51.05225818626699 < 0$$

2. The second order minor principal axis is

$$(-1)^2 \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = -51.023 < 0,$$

$$(-1)^2 \begin{pmatrix} b_{11} & b_{13} \\ b_{31} & b_{33} \end{pmatrix} = 0.04289326478963609 > 0,$$

$$(-1)^2 \begin{pmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{pmatrix} = 7.187854329681826 > 0.$$

3. The third order minor principal axis is

$$(-1)\left[b_{11}\right] = 7.175812975384314 > 0,$$

$$(-1)\left[b_{22}\right] = -4.99081133072875 < 0,$$

$$(-1)\left[b_{33}\right] = 0.$$

Hence, we detect that the condition $(-1)^k \det(P_J(J^*)) < 0$, is satisfied for some $1 \le k \le 3$ and $J \in I_k$. Then, there exists $\mathrm{D} \ge 0$ such that $(J^* - D)$ is unstable .

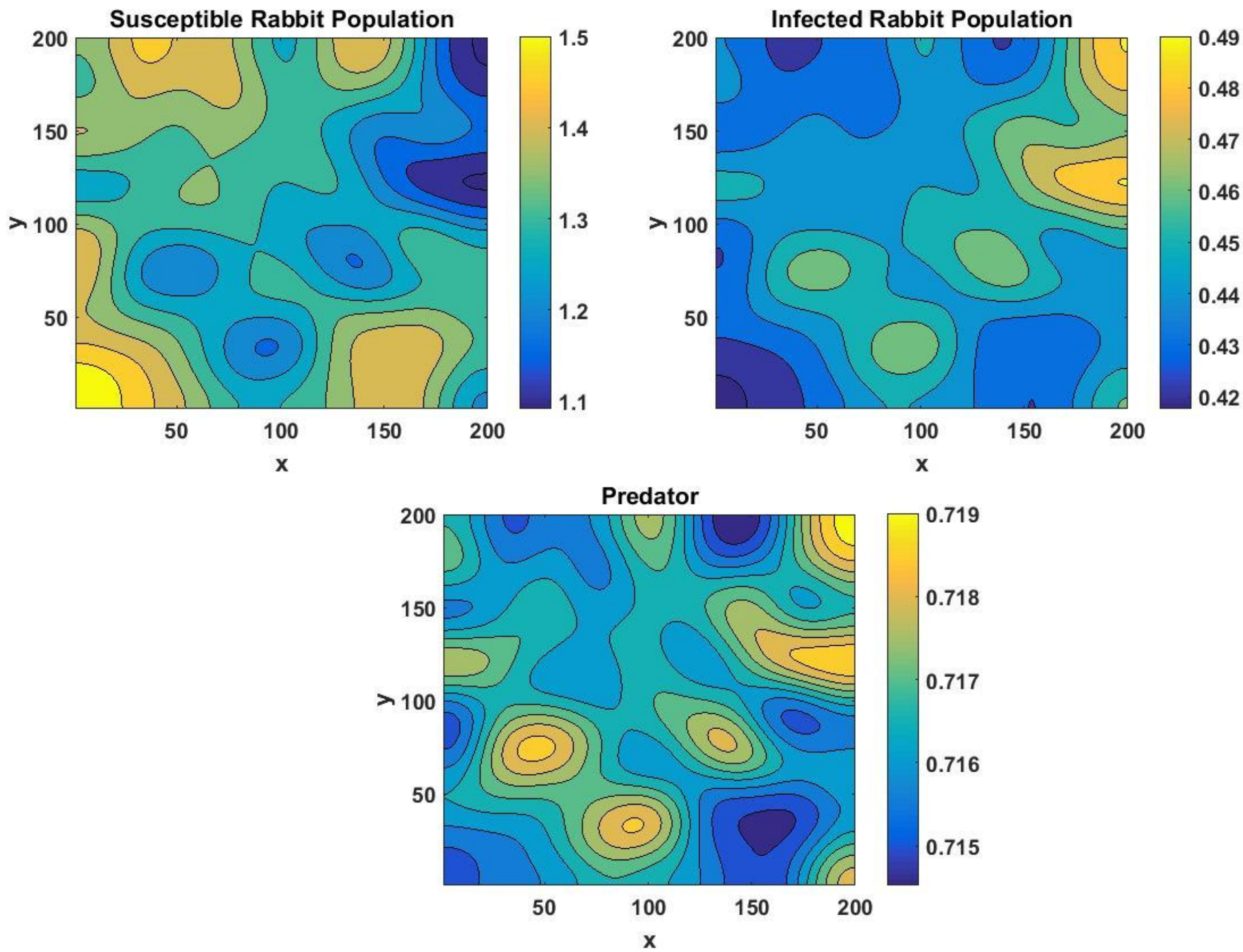
3.3. Numerical simulation

In this, we present the simulations of the diffusion model system in two-dimensional diffusion. To examine the dynamics of spatiotemporal of a model system, we present an extensive of the numerical simulations which extended the model (2.1)-(2.3) spatially in two-dimensional spaces. All the simulations uses non-zero initial condition and boundary conditions for the zero-flux with size 200×200 (discretized through $(x_0, x_1, x_2 \ldots x_N)$ and $(y_0, y_1, y_2 \ldots y_N)$, with N = 300. The lattice constant Δh defines the lattice point between the spacing. For the $\Delta h \to 0$, the difference approaches the derivatives. In the present study, we set $\Delta t = 0.001$; $\Delta h = 0.25$. We employ explicit and standard five-point approximation for the 2D Laplacian with the Neumann (zero-flux) boundary conditions. The plot gives spaces vs. population densities. The dynamics of the infected and susceptible prey is simulated for the following set of values of the parameters are r = 8.85, k = 100, b = 2.681, $w_1 = 0.25$, $w_2 = 4.1$, $w_3 = 0.201$, $w_4 = 1.8$, d_1 =7.45, $d_2 = 0.4698$. (2.4)

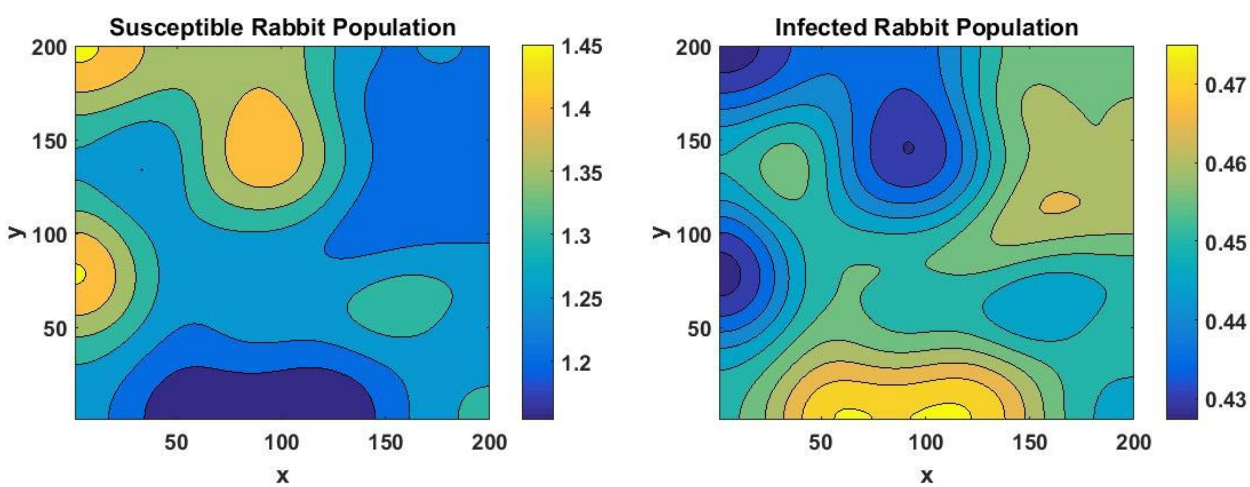
These are the parameter values which are same for non- diffusion model.

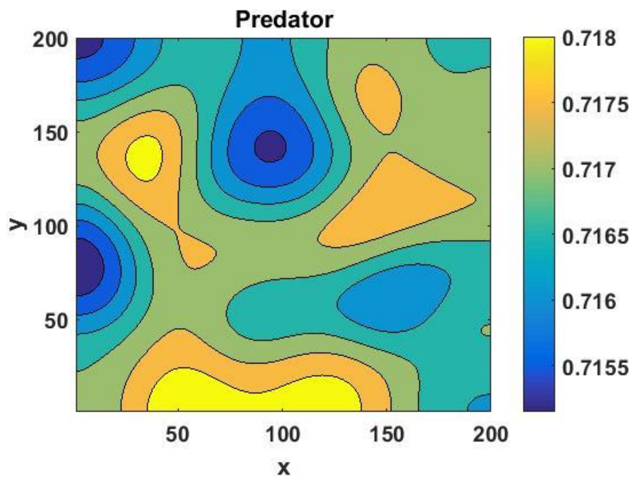
3.3.1. Effect of diffusion coefficient

In this, we consider the dynamics of spatiotemporal model system (2.1)-(2.3) with the fixed values of parameters as given in Eq. (2.5) and varying the diffusion coefficients D_1, D_2, D_3. The values of the parameter are to be selected from the Turing space. These patterns are shown in Fig. 2.1, 2.2 and 2.3. From Fig. 2.1 we observe that when D_1 increases, infection tends to increase and predator population decreases. When D_2 is increased, the density of predator decreases and gets concentrated in a small circular clustered pattern (c.f. Fig 2.2). Finally, when we increase D_3 from 1 to 10 we observe that infected and predator shows similar pattern distribution. We also find that the number of predator population increases on increasing D_3. Hence, we can say that the population of predator have to actively diffuse and also have to perform the movement of active random in the search for favorite food i.e. rabbit population.


Susceptible Rabbit Population
Infected Rabbit Population
Predator
x
y
0.719
0.718
0.717
0.716
0.715

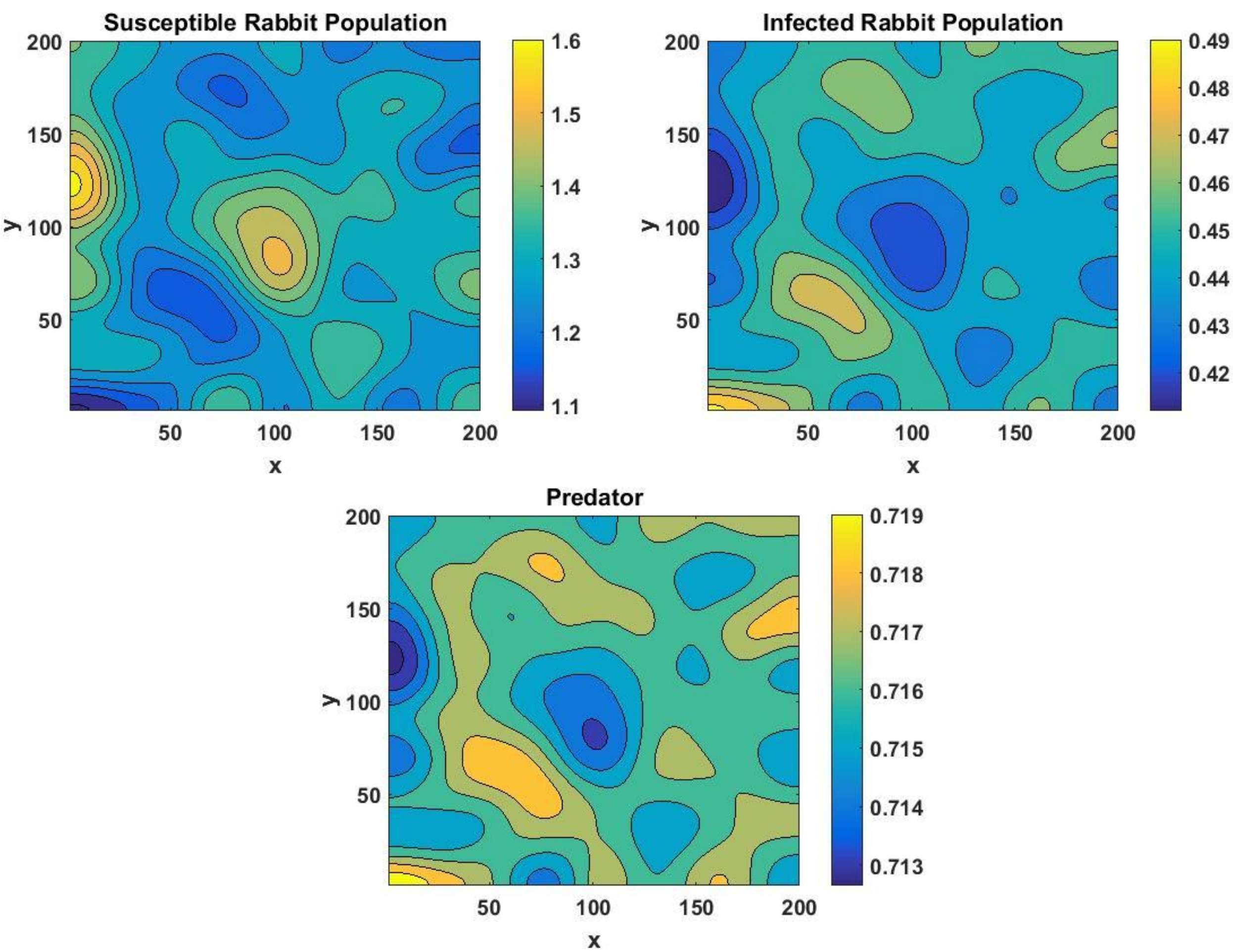
(a)


Susceptible Rabbit Population
Infected Rabbit Population
x
y

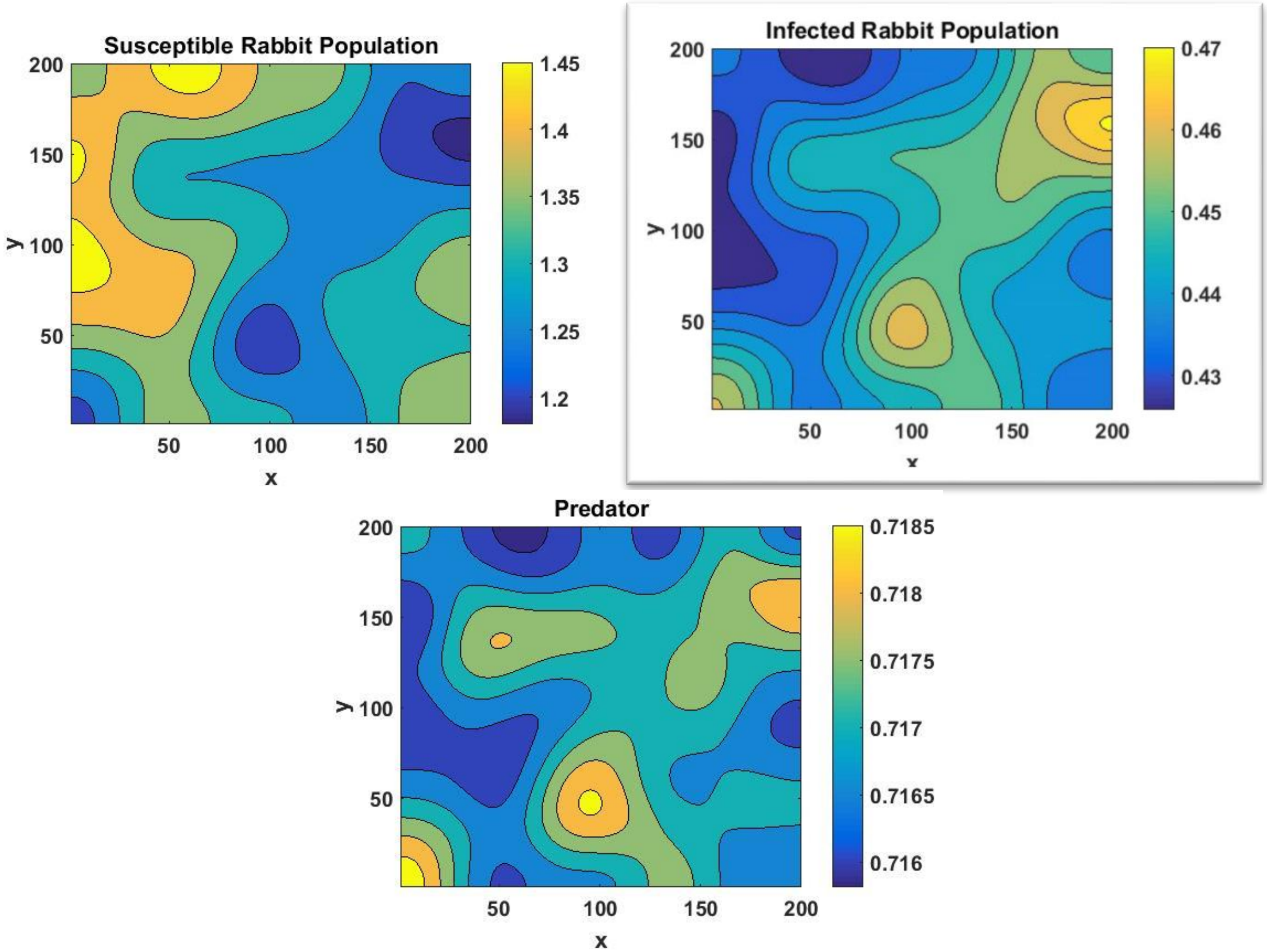


(b)

Figure 2.1 Turing patterns are produced through the simulating model system (2) varying D_1 at t = 30 days with (a) $D_1 = 0.5$, (b) $D_1 = 1$.

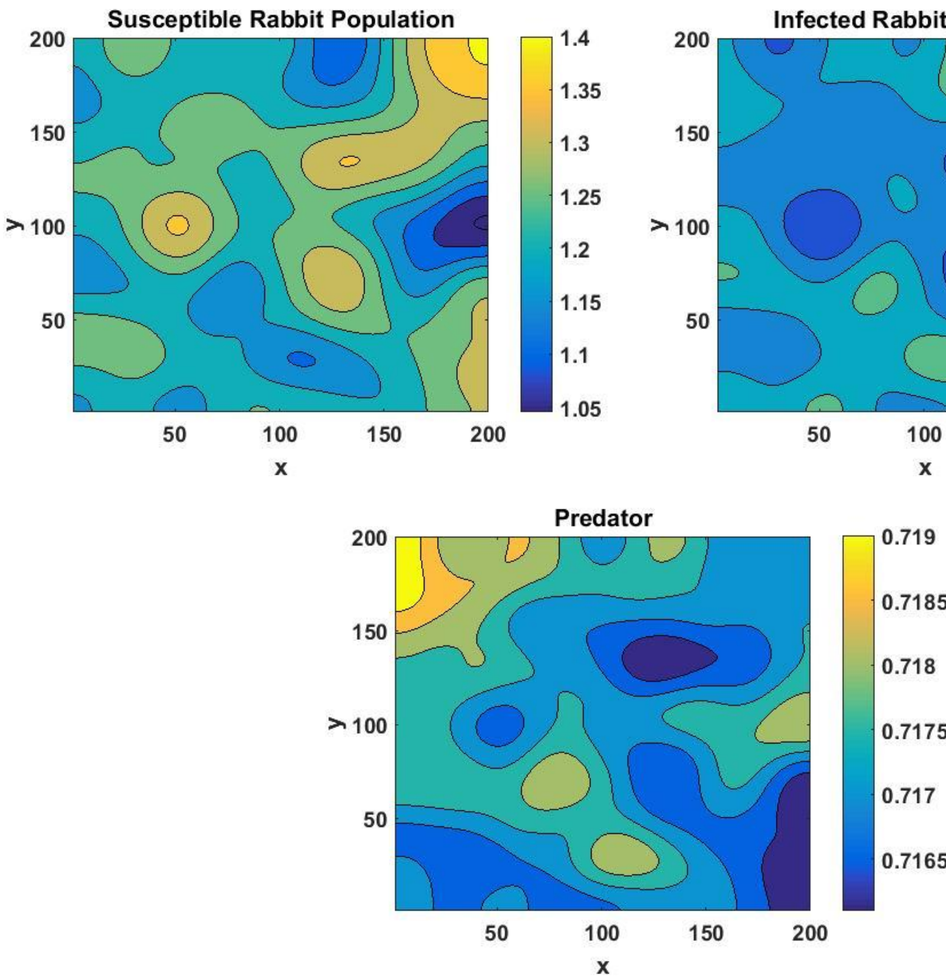


(a)

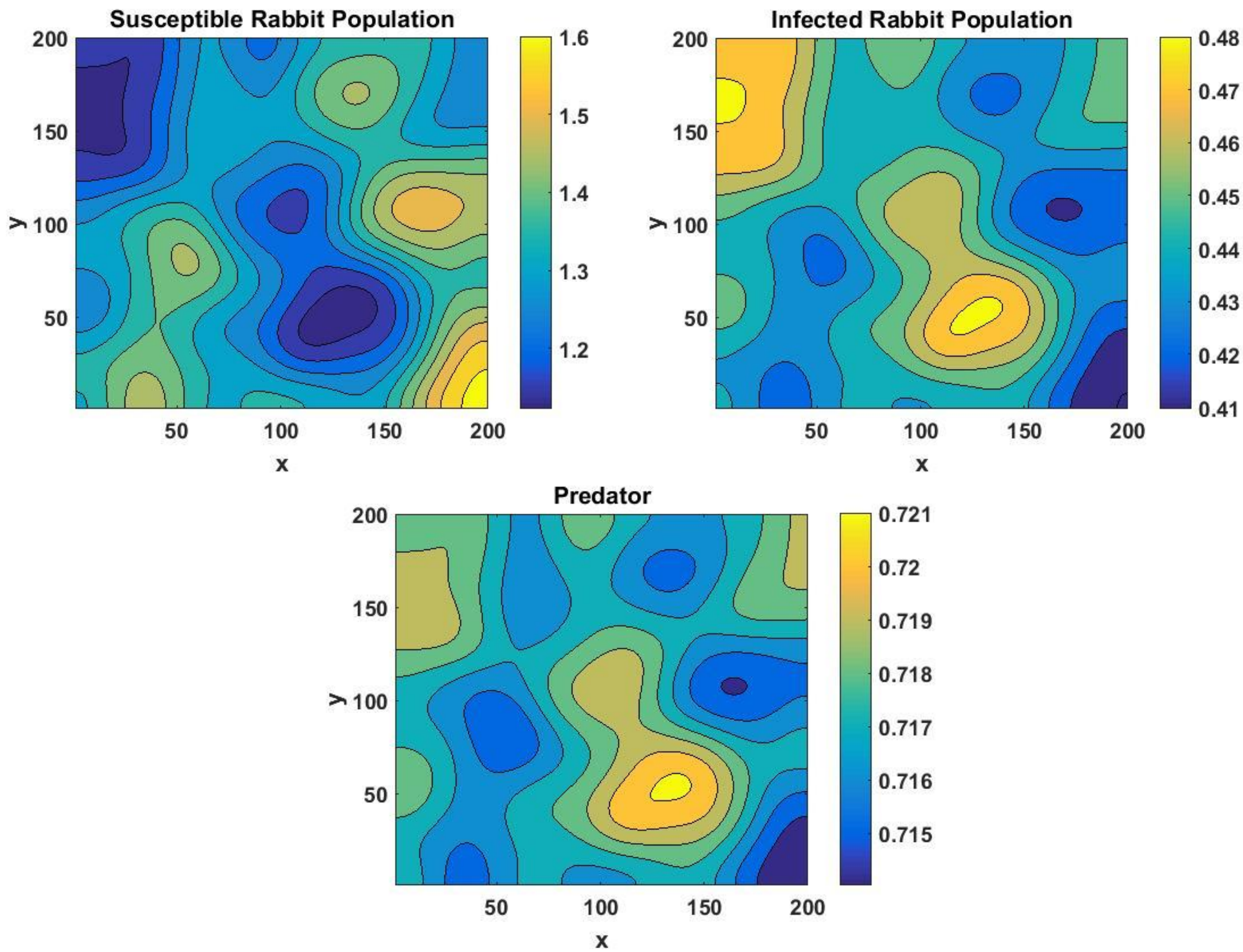


(b)

Fig. 2.2 Turing patterns are produced through the simulating model system (2.1)-(2.3) varying D_2 at $t = 30$ days with (a) $D_2 = 0.01$, (b) $D_2 = 0.5$.


Susceptible Rabbit Population
Infected Rabbit Population
Predator
x
y
200
150
100
50
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
0.5
0.49
0.48
0.47
0.46
0.45
0.44
0.719
0.7185
0.718
0.7175
0.717
0.7165

(a)

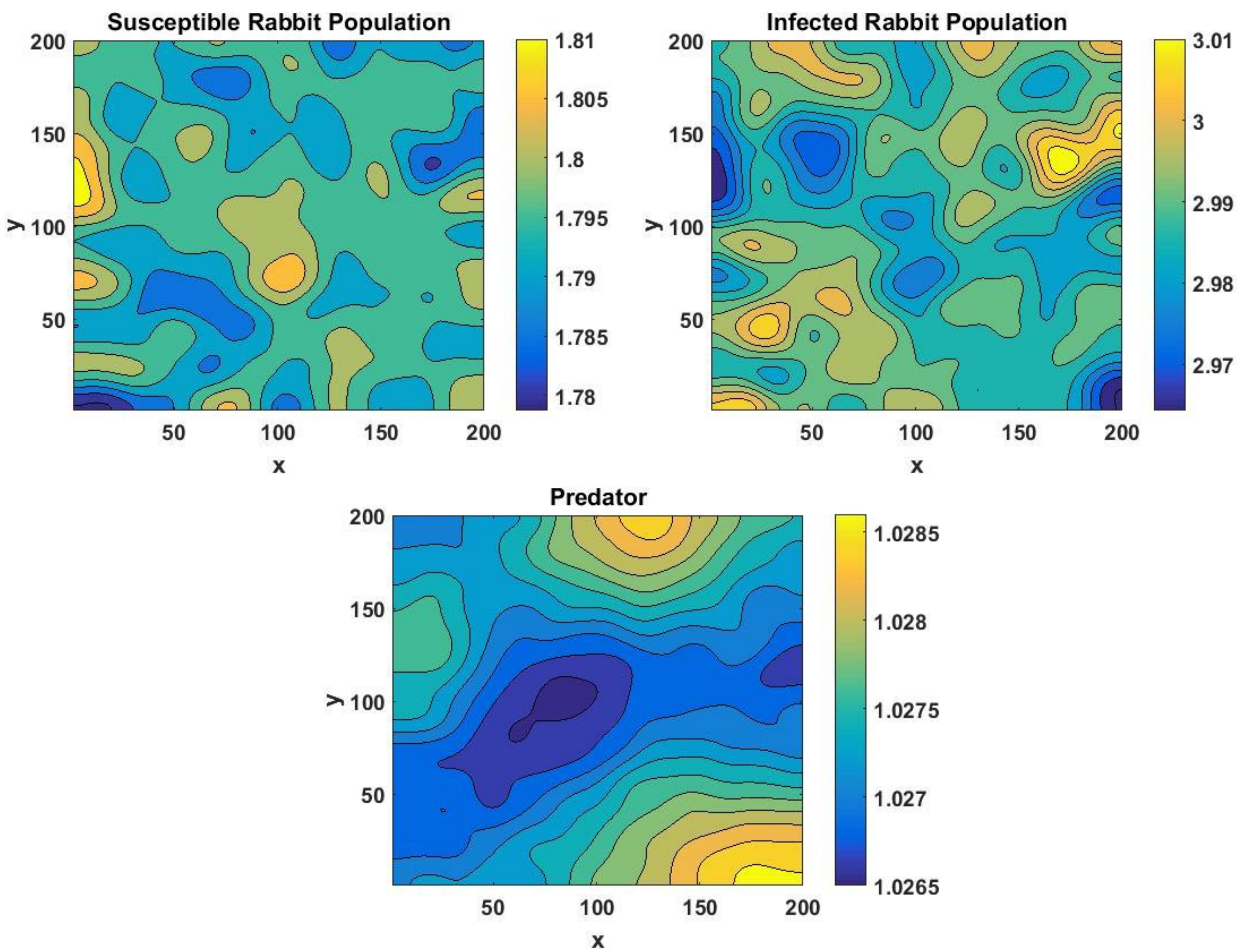


(b)

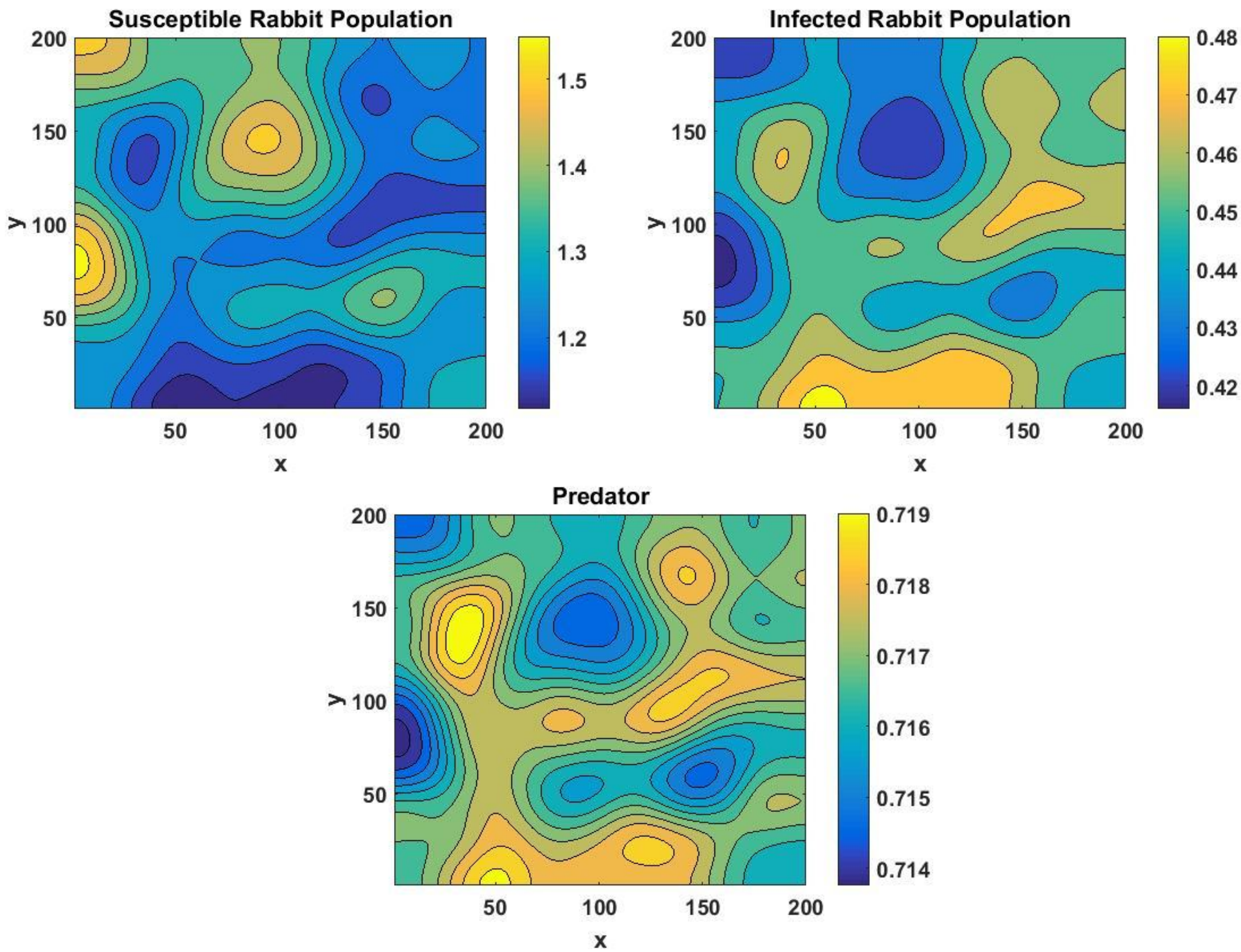
Figure 2.3 Turing patterns are produced through the simulating model system (2.1)-(2.3) varying D_3 at *t* = 30 days with (a) $D_3 = 1$, (b) $D_3 = 10$.

3.3.2. The evolutionary process of turing pattern formation

The processes of evolutionary of Turing pattern formations are studied. The parameters of a system were fixed as *r* = 8.85, k = 100, b = 2.681, $w_1 = 0.25$, $w_2 = 4.1$, $w_3 = 0.201$, $w_4 = 1.8$, d_1 =7.45, $d_2 = 0.4698$. At different time levels for the prey and predator population of infected, the different spatial patterns were appeared. We considered 100 iterations as one day. The formation in spots and circular patterns leads to the random perturbation with the homogeneous state. In this , we observe initially in about 10 days the population of predator is present and the density predator is maximum at the corner of the domain. As increasing the time level, we observe that there is an increment in disease and decrement in the predator population.

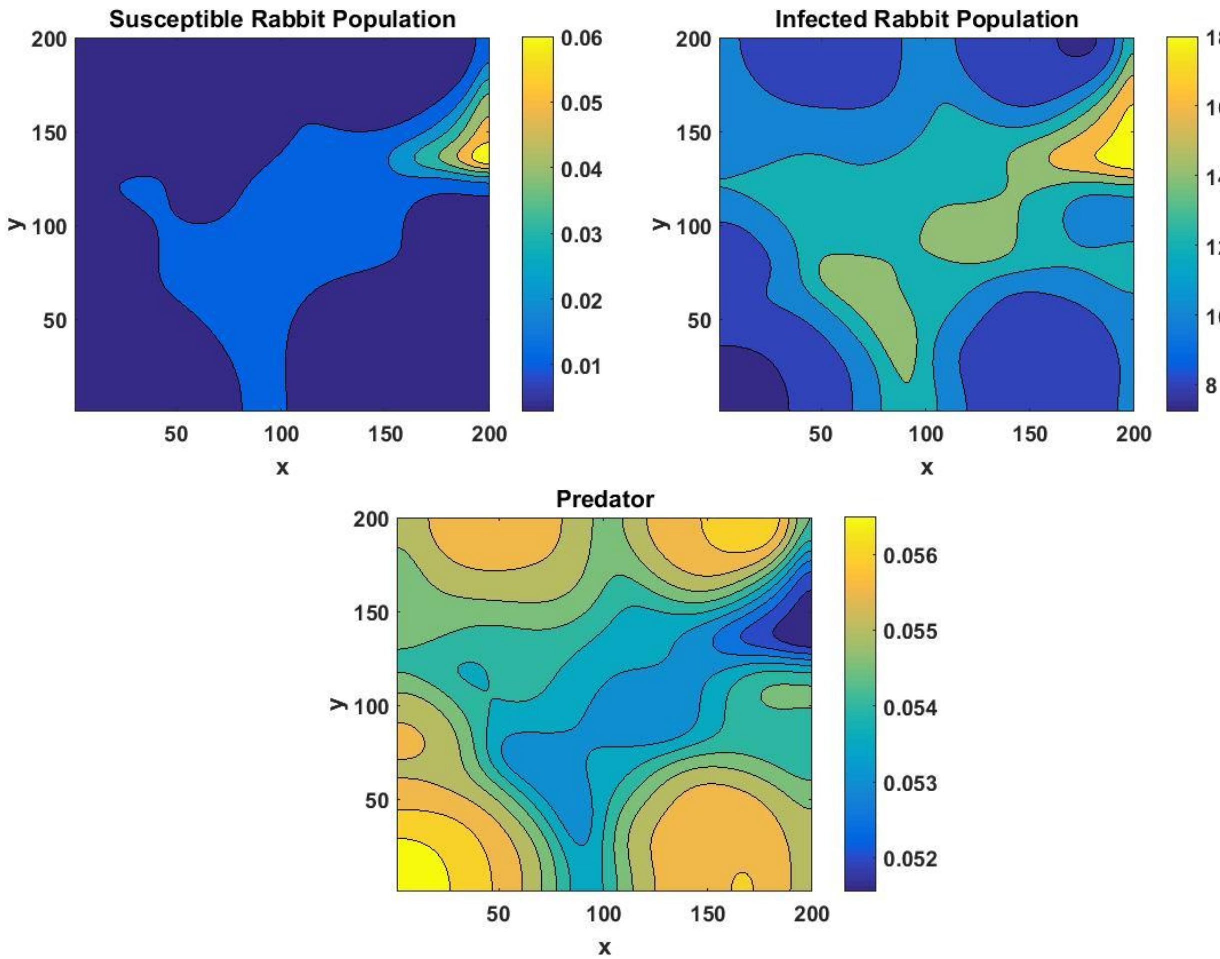

Susceptible Rabbit Population
Infected Rabbit Population
Predator
x
y
200
150
100
50
1.81
1.805
1.8
1.795
1.79
1.785
1.78
3.01
3
2.99
2.98
2.97
1.0285
1.028
1.0275
1.027
1.0265

(a)



Susceptible Rabbit Population
Infected Rabbit Population
Predator
x
y

(b)



(c)

Fig. 2.4. Snapshots of patterns formation for the time evolution of susceptible, infected prey and predator population at time (a) t = 10 days, (b) t = 30 days, (c) t = 50 days, respectively with d_1 =0.5, d_2 =0.1, d_3 =5.

3.4. Conclusions and discussion

The serious conservation has been raised for the extension of RHD. The extension of haemorrhagic disease (RHD) had direct impact on Eagle population. In this paper, we investigated a eco-epidemiological of diffusive system. The attributes of the susceptible rabbits and infected rabbits-Spanish Eagle food chain coincide the quantitative measures. We achieved the solution of the system conditions for the boundedness. For the non-diffusive model, there is an existence and stability of the system. With the reaction diffusion dynamics system the eco-epidemiological model help us to infer the distribution of species. In the time and space the disease is spread. We investigated the spread dynamics by varying diffusion coefficients D_1, D_2 and D_3 for the susceptible and the predator respectively. We found that movement of predator population is beneficial for their survival. Beautiful Turing patterns are generated when we increase the time to observe the species distribution.

Chapter-4

Summary and Conclusions

The thesis consists of four chapters in which second and third chapter consists of eco-epidemiological modeling for deciphering the dynamics between rabbit suffering with RHD and imperial eagle aiming at designing policy plans control measure. Numerical experiments presented the proposed system taking the values of paramters biologically relevant which support our analytical findings.

Chapter 2 explains the mathematical modeling of non- spatial and its stability. Further it consists of numerical simulation that shows that the small change in death rate of predator population effects the dynamics of the system giving rise to limit cycle, chaos and stable focus. We also emphasize on reducing the value of b for saving predator population.

Chapter 3 explains the mathematical modeling of spatial model and its stability. . The analysis of the formation of Turing patterns in detail are selected by the reaction-diffusion system under the zero flux boundary conditions are presented Further it consists of numerical simulation that shows the effect of diffusion rates with respect to time and space.

One aspect overlooked in both chapters is that the changes in infection rate can be a function of the factors that are extrinsic to the infection, such as host behavioral change or seasonal change in climate. Seasonal change can further elaborate patterns of incidence. . The analysis of the formation of Turing patterns in detail are selected by the reaction-diffusion system under the zero flux boundary conditions are presented. Accurate quantitative prediction of parameters is practically impossible, particularly given the possible complexities of dynamic wildlife host population as mentioned above.

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