

Modification for the Generalized Quasi–Orthogonal Space Time Block Codes

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DECLARATION

I, hereby declare that the work, which is being presented in the dissertation, entitled "**Modification for the Generalized Quasi-Orthogonal Space Time Block Codes**" in partial fulfilment of the requirements for the award of degree of Master of Engineering in Electronics and Communication Engineering submitted at Electronics and Communication Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the guidance of **Dr. Sanjay Sharma (Professor and Head)**, Electronics and Communication Department and refers other research's work which are duly listed in reference section. The matter presented in this dissertation has not been submitted in any other University/Institute for the award of degree.

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ABSTRACT

In the recent years, demand for mobile communication systems with high data rates has increased significantly. To satisfy this huge communications demand search for new methods has been going that exploiting the limited resources such as bandwidth and power as efficient as possible. MIMO (Multiple input and Multiple Output) systems is one of the techniques with multiple antenna elements at both link ends that is an efficient solution for future wireless communications systems as they provide high data rates by exploiting the spatial domain under the constraints of limited bandwidth and transmit power. Space-Time Block Coding (STBC) is a MIMO transmit strategy which exploits transmit diversity. STBCs can be divided into two main classes, namely, Orthogonal Space-Time Block Codes (OSTBCs) and Non-Orthogonal Space-Time Block Codes (NO-STBCs). The Quasi-Orthogonal Space-Time Block Codes (QO-STBCs) belong to class of NO-STBCs and have been an intensive area of research. The O-STBCs achieve full diversity with low decoding complexity, but at the cost of some loss in data rate. Full data rate is achievable in connection with full diversity only in the case of two transmit antennas. For more than two transmit antennas full data rate can be achieved with QSTBCs with a small loss of the diversity gain.

The main aim of this work is to provide a unified theory of QSTBCs for four transmit antennas and one receive antennas. The dissertation consists of two main parts: In the first part detailed analyses of the QSTBCs transmission without any channel knowledge at the transmitter is given and in the second part transmission with QSTBCs assuming partial channel state (CSI) information at the transmitter is illustrated.

In the first part of this dissertation a definition of QSTBC for four transmit antennas is given. It is shown that different QSTBCs are obtained by linear transformations and that already known codes can be transformed into each other. The (4×1) MIMO channel in the case of applying quasi-orthogonal codes can be transformed into an equivalent highly structured virtual (4×4) MIMO channel matrix. The structure of the equivalent channel is of key importance for the performance of the QSTBCs. The off-diagonal elements of the virtual channel matrix are responsible for some signal self-interference at the receiver. The closer these off-diagonal elements of the virtual channel matrix are to zero, the closer is the code to an orthogonal code. Based on the six different self-interference parameter, performance of corresponding six QOSTBC types has been analysed. After that, effect of decoding method on the performance of the code is illustrated. Finally, new QOSTBC is

proposed whose decoding complexity is very less compared to conventional QOSTBC without losing any performance.

In the second part of the dissertation, transmit antenna shuffling (TAS) method is demonstrated to improve the QSTBC transmission when partial CSI is available at the transmitter. It is shown that QSTBCs can achieve full diversity and nearly exact orthogonality with a small amount of feedback bits returned from the receiver back to the transmitter. Later on, TAS method is demonstrated on the new QOSTBC code to improve its transmission strategy when some feedback is available at the transmitter. Lastly, performance of both conventional and proposed QOSTBC is compared.

CONTENTS

DECLARATION	<i>i</i>
ACKNOWLEDGEMENT	<i>ii</i>
ABSTRACT	<i>iii</i>
CONTENTS	<i>v</i>
LIST OF FIGURES AND TABLES	<i>viii</i>
ABBREVIATIONS	<i>ix</i>
1 INTRODUCTION	1
1.1 Advancement in Wireless Communication	1
1.2 Wireless Channel	2
1.3 Large-Scale Fading	3
1.4 Small-Scale Fading	4
1.5 Rayleigh Flat Fading Channel	4
1.6 Diversity	5
1.7 Signal Model for MIMO System	6
1.8 Capacity of MIMO System	7
1.9 Organization of Dissertation	9
2 LITERATURE REVIEW	11
3 SPACE TIME CODING AND QOSTBCs	16
3.1 Space Time Coding	16
3.2 Space Time Code Design Criteria	17
3.3 Space Time Block Codes	22
3.3.1 Alamouti Scheme	22
3.3.2 Decoding of Alamouti Scheme	24
3.3.3 Generalization of Space Time Block Codes	25
3.4 Structure of QO-STBC	26

3.5	Pairwise Decoding of QO-STBC	27
3.6	Rotated QO-STBC	29
3.7	Optimum Rotation & Performance of QO-STBC	32
4	DESIGN OF NEW QO-STBCs FROM EXISTING ONES	35
4.1	Popular QO-STBCs	35
4.1.1	Jafarkhani Code	35
4.1.2	ABBA Code	35
4.1.3	Papadias and Foschini (PF) Code	35
4.2	Other QO-STBCs Obtained by Linear Transformation	36
4.3	Equivalent Virtual Channel matrix (EVCN)	37
4.4	Common Properties of the EVCN	39
4.5	EVCN for Known QO-STBCs	40
4.5.1	EVCN for Jafarkhani Code	40
4.5.2	EVCN for ABBA Code	40
4.5.3	EVCN for Papadias and Foschini (PF) Code	40
4.6	Useful QO-STBC Types	41
4.7	Grammian Matrix G	43
4.8	Statistical Properties of the Channel dependent Self Interference Parameter X	43
4.9	Receiver Algorithm for QO-STBCs	45
4.9.1	Maximum Ratio Combining	45
4.9.2	Maximum Likelihood Decoding	46
4.9.3	Linear Receiver	47
4.10	Proposed QO-STBC	48
4.10.1	Construction Method	48
5	QO-STBC WITH PARTIAL FEEDBACK INFORMATION	52
5.1	Introduction	52

5.2	Transmit Antenna Shuffling for Conventional QO-STBC	52
5.3	Transmit Antenna Shuffling for Proposed QO-STBC Scheme	57
5.4	Comparison of Conventional QO-STBC Scheme and Proposed QO-STBC Scheme using TAS	61
6	CONCLUSION AND FUTURE SCOPE	63
6.1	Conclusion	63
6.2	Future Scope	64
	Appendix A	65
	References	69
	List of Publications	74

LIST OF FIGURES AND TABLES

List of Figures

- Fig. 1.1** : An example of different paths in wireless channels.
- Fig. 1.2** : Rayleigh distribution with $\alpha = 0.5$.
- Fig. 1.3** : Multiple antennas communication system.
- Fig. 1.4** : MIMO capacity for different transmit and receive antennas.
- Fig. 3.1** : Representation of Alamouti scheme.
- Fig. 3.2** : Bit error rate versus SNR for Alamouti scheme.
- Fig. 3.3** : Performance of QO-STBC.
- Fig. 3.4** : Constellation rotation for BPSK constellation set ($d_{\min} = 2$).
- Fig. 3.5** : Comparison of bit error rate for rotated and non-rotated QOSTBC.
- Fig. 3.6** : Minimum CGD for different rotation; QO-STBC using BPSK.
- Fig. 3.7** : Constellation diagram for QPSK & 8-PSK.
- Fig. 3.8** : Minimum CGD using QPSK.
- Fig. 3.9** : Minimum CGD using 8-PSK.
- Fig. 3.10**: BER curve for QO-STBC for different rotations using QPSK modulation.
- Fig. 4.1** : Comparison of performance of different QO-STBCs.
- Fig. 4.2** : BER performance of different QO-STBC formed after linear transformation.
- Fig. 4.3** : Distribution of $|X|$.
- Fig. 4.4** : Comparison of QO-STBC with different decoding techniques.
- Fig. 4.5** : Performance of proposed QOSTBC scheme.
- Fig. 5.1** : Closed loop QO-STBC scheme.
- Fig. 5.2** : Performance of conventional QO-STBC using TAS.
- Fig. 5.3** : (a), (b), (c), (d), (e) and (f) shows the performance of different shuffled proposed codes under optimum channel conditions.
- Fig. 5.4** : Performance of proposed QO-STBC using TAS.
- Fig. 6.5** : Comparison of performance of proposed QO-STBC and conventional QO-STBC using TAS.

List of Tables

- Table 4.1** : Six QO-STBCs with different interference parameter β .
- Table 5.1** : Six different shuffling having different value of b .

ABBREVIATIONS

MIMO	Multiple Input Multiple Output
STC	Space-Time Coding
STBC	Space-Time Block Code
O-STBC	Orthogonal Space-Time Block Code
QO-STBC	Quasi- Orthogonal Space-Time Block Code
LOS	Line of Sight
QPSK	Quadrature Phase Shift Keying
MRRC	Maximum Ratio Receiver Combining
EVC	Equivalent Virtual MIMO Channel Matrix
CR	Constellation Rotation
ML	Maximum Likelihood
CGD	Coding Gain Distance
SVD	Singular Value Decomposition
TAS	Transmit Antenna Shuffling
CSI	Channel State Information

1.1 Advancement in Wireless Communication

Today communication technologies have become a very important part of human life. Wireless communication systems have opened new magnitude in communications. Wireless communication gives promise of portability, mobility, and accessibility. Although wired communication brings more stability, better performance, and higher reliability, it comes with the necessity of being restricted to a certain location or a bounded environment. As a result, customers use wireless systems more often. With the advent of Internet and multimedia applications in next generation wireless communications, the demand for wide-band high data rate communication services is growing. As the available radio spectrum is limited, higher data rates can be achieved only by designing more promising signaling techniques. One of the most promising future technologies in mobile radio communications is multi antenna elements at the transmitter and at the receiver.

MIMO stands for multiple-input multiple-output and means multiple antennas at both link ends of a communication system, i.e. at the transmitter and at the receiver side. The idea behind MIMO is that the transmit antennas at one end and the receive antennas at the other end are “connected and combined” in such a way that the quality (bit error rate (BER), or the data rate) for each user is improved [8, 38].

Several transmission schemes have been proposed that utilize the MIMO channel in different ways, e.g., spatial multiplexing, space-time coding or beamforming. Space-time coding (STC), introduced first by Tarokh *at el.* [4], is a promising method where the number of the transmitted code symbols per time slot are equal to the number of transmit antennas. These code symbols are generated by the space-time encoder in such a way that diversity gain, coding gain, as well as high spectral efficiency are achieved.

Space-time coding finds its application in cellular communications as well as in wireless local area networks. There are various coding methods as space-time trellis codes (STTC), space-time block codes (STBC), space-time turbo trellis codes and layered space-time (LST) codes [4, 14]. A main issue in all these schemes is the exploitation of redundancy to achieve high reliability, high spectral efficiency and high performance gain. The design of STC amounts to find code matrices that satisfy certain optimality criteria. In particular, STC

schemes optimize a trade-off between the three conflicting goals of maintaining a simple decoding algorithm, obtaining low error probability, and maximizing the information rate. In the last few years, researchers have made an enormous effort to understand space-time codes, their performance and their limits. The purpose of this work is to explain the concept of space-time block coding in a systematic way. This thesis provides an overview of STBC design principles and performance. The main focus is devoted to so-called quasi-orthogonal space-time block codes (QO-STBCs) [16]. Our goal is to provide a unified theory of QO-STBCs for four transmit antennas and one receive antenna and to analyze their performance on Rayleigh fading channels, with and without channel state information (CSI) available at the transmitter.

1.2 Wireless Channel

Wireless channel is logical connection over a medium such as air that connects data source to data sink. There are many different paths between the transmitter and the receiver results in receiving different versions of the transmitted signal at the receiver. These separate versions experience different path loss and phases. To depict such channel model, different possible paths for the received signals are needed to study.

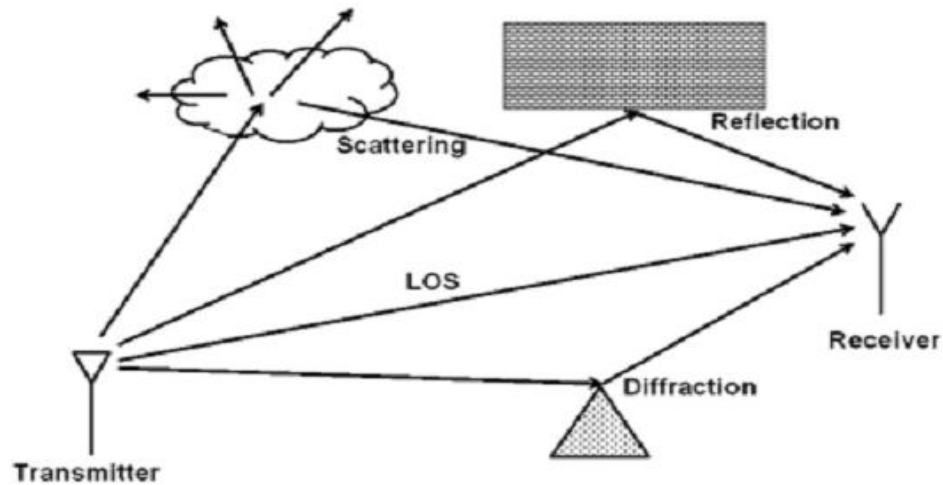


Fig.1.1: An example of different paths in wireless channels [10].

If there is a direct path between the transmitter and the receiver, it is called the line of sight (LOS). A LOS is not the only path that an electromagnetic wave can take from a transmitter to a receiver. An electromagnetic wave may reflect when it meets an object that is much

larger than the wavelength. Through reflection from many surfaces, there are different paths resulting in power strengths and phases other than those of the LOS path. Another way that electromagnetic waves propagate is diffraction. Diffraction occurs when the electromagnetic wave hits a surface with irregularities like sharp edges. Finally, scattering also occurs where there are a large number of objects smaller than the wavelength between the transmitter and the receiver. This results in scattering of wave and many copies of the wave propagate in many different directions. There are also other phenomena's that affect the propagation of electromagnetic waves like absorption and refraction.

Power of the received signal reduces in different ways because of these propagation mechanisms. There are two general aspects of such a power reduction that require separate treatments. One aspect is the large-scale effect which corresponds to the characterization of the signal power over large distances or the time-average behaviors of the signal. This is called attenuation or path loss and sometimes large-scale fading. The other aspect is the small-scale fading, or just fading which corresponds to the characterization of the signal over short distances or short time intervals. In the next sections, explanation of model of behavior of large-scale and small-scale fading is illustrated.

1.3 Large-Scale Fading

Large-scale fading or attenuation is caused by many factors including propagation losses, antenna losses, and filter losses. The average received signal, or the large-scale fading factor, decreases logarithmically with distance. The logarithm factor, or the path gain exponent, depends on the propagation medium and the environment between the transmitter and the receiver. For example, for a free space environment, like that of satellite communications, the exponent is two. In other words, the average received power P_R is proportional to d^{-2} , where d is the distance between the transmitter and the receiver. For other propagation environments, like urban areas, the path loss exponent is usually greater than 2. In other words, if the average transmitted power is P_T , average received power given by [10]:

$$P_R = \beta d^{-\nu} P_T \quad (1.1)$$

Here ν is the path loss exponent and β is a parameter that depends on the frequency and other factors. This is sometimes also called the log-distance path loss model as the path loss and the distance have a logarithmic relationship.

1.4 Small-Scale Fading

Small-scale fading or equivalently fading is caused by interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times. These signals, called multipath waves, combine at the receiver. This resulting signal can vary widely in amplitude and phase over a short period of time, equivalently a short travel distance, is such that the large-scale path loss effects may be ignored. The randomness of multipath effects and fading results in the use of different statistical arguments to model the wireless channel. Therefore, statistical models are needed to investigate the behavior of the amplitude and power of the received signal.

1.5 Rayleigh flat fading channel

In this work, narrowband systems are considered, in which the bandwidth of the transmitted signal is smaller than the channel's coherence bandwidth, which is defined as the frequency range over which channel fading process is correlated. This type of fading is called as flat fading or frequency non-selective fading. For this Rayleigh distribution is commonly used to describe the statistically time-varying nature of the received envelope of a flat fading channel. For a typical mobile wireless channel in indoor and urban areas, it is assumed that the direct line of sight is obstructed and the receiver obtains only reflected waves from the surrounding objects. In a multipath channel with I multiple paths, transmitting a signal over the carrier frequency f_c results in receiving the sum of I components from different paths plus a Gaussian noise as follows:

$$r(t) = \sum_{i=1}^I a_i \cos(2\pi f_c t + \phi_i) + n(t) \quad (1.2)$$

where a_i and ϕ_i are the amplitude and phase of the i -th component respectively and $n(t)$ is Gaussian noise. Expanding the cosine terms in above equation results in:

$$r(t) = \cos(2\pi f_c t) \sum_{i=1}^I a_i \cos(\phi_i) - \sin(2\pi f_c t) \sum_{i=1}^I a_i \sin(\phi_i) + n(t) \quad (1.3)$$

Let $A = \sum_{i=1}^I a_i \cos(\phi_i)$ and $B = \sum_{i=1}^I a_i \sin(\phi_i)$, where A and B are summation of I random variables since object in environment is randomly located. When the number of reflected waves is large, according to central limit theory, two components A and B are independent

identically distributed Gaussian random variable. The envelope of the received signal is now given by

$$R = \sqrt{A^2 + B^2} \quad (1.4)$$

Since A and B are i.i.d.(independent & identically distributed) zero mean Gaussian random variable, the envelope follows a Rayleigh distribution with variance σ^2 . The probability density function of the Rayleigh distribution is given by [10].

$$P(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} & r \geq 0 \\ 0 & r < 0 \end{cases} \quad (1.5)$$

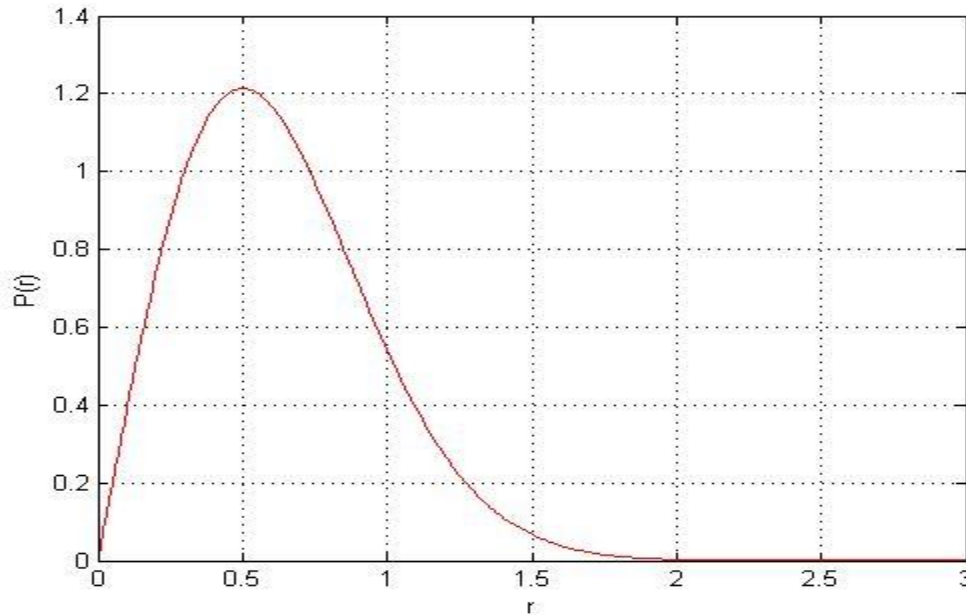


Fig.1.2: Rayleigh distribution with $\sigma = 0.5$.

1.6 Diversity

Fading makes it extremely difficult for the receiver to recover the transmitted signal unless the receiver is provided with some form of diversity, i.e. replicas of the same transmitted signal with uncorrelated attenuation. In fact, diversity combining technology has been one of the most important contributors to reliable wireless communications. Ways to achieve diversity include:

Temporal Diversity: In this scheme, channel coding in conjunction with time interleaving is

used. Thus replicas of the transmitted signal are provided to the receiver in the form of redundancy in the temporal domain. However, in slow fading channels, temporal diversity is not an option for delay-sensitive applications.

Frequency Diversity: In this scheme, the fact that signals that are transmitted on different frequencies tend to experience different fading effects is exploited. Thus replicas of the transmitted signal are provided to the receiver in the form of redundancy in the frequency domain. However, this scheme is not bandwidth-efficient.

Spatial Diversity: In this scheme, spatially separated antennas are used to provide diversity in the spatial domain. Diversity combining technique is then used to select or combine the signals that have been transmitted or received on different antennas.

Spatial diversity is attractive as diversity can be obtained with no penalty in bandwidth efficiency. It can be implemented by deploying multiple antennas at the transmitter and/or the receiver. Depending on the location of the antennas, wireless communication system employing spatial diversity can be classified into the following three configurations:

Single Input Multiple Output (SIMO): When there are single transmit antenna but multiple receive antennas, i.e. receive diversity.

Multiple Input Single Output (MISO): When there are multiple transmit antennas but one receive antenna, i.e. transmit diversity.

Multiple Input Multiple Output (MIMO): When there are multiple transmit antennas and multiple receive antennas, i.e. both transmit and receive diversity are used. Besides providing spatial diversity, it has been shown that the capacity of a wireless channel grows linearly with the number of transmit and receive antennas, hence a MIMO system can be used to boost the capacity of wireless channel too.

1.7 Signal Model for MIMO System

Consider a wireless communication system [8] with two users. One is the transmitter and the other is the receiver. The transmitter has T transmit antennas and the receiver has R receive antennas as illustrated in Figure 1.3. There exists a wireless channel between each pair of transmit and receive antennas. The channel between the t -th transmit antenna and the r -th receive antenna can be represented by the random propagation coefficient h_{tr} , which are Rayleigh distributed.

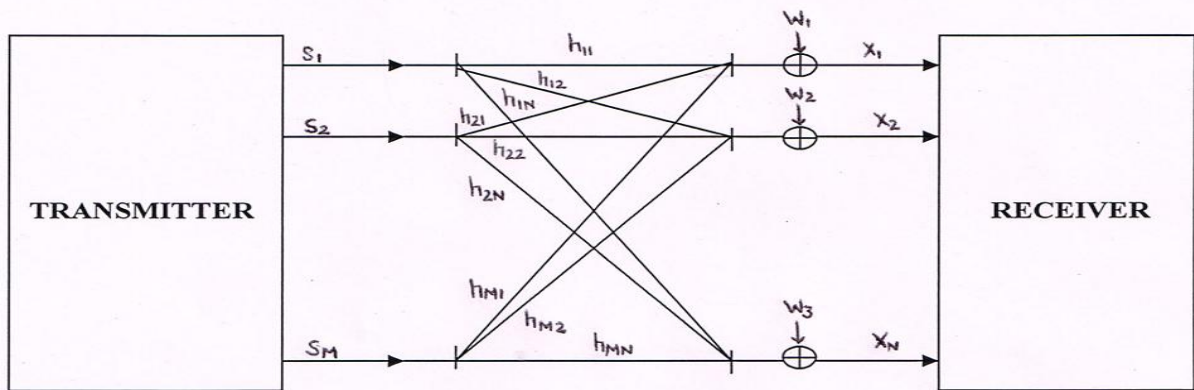


Fig.1.3: Multiple antennas communication system [8].

To send information to the receiver, at every transmission time, the transmitter feeds signals s_1, s_2, \dots, s_T to its T transmit antennas respectively. The antennas then send the signals simultaneously to the receiver. Every receive antenna at the receiver obtains a signal that is a superposition of the signals from every transmit antenna through the fading coefficient. The received signal is also corrupted by noise. If the noise at the r -th receive antenna is denoted by w_r , the received signal at the r -th receive antenna is

$$x_r = \sum_{t=1}^T h_{tr} s_t + w_r \quad (1.6)$$

In vector form, above equation is represented as

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_R \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,R} \\ h_{2,1} & h_{2,2} & \dots & h_{2,R} \\ \vdots & \vdots & \dots & \vdots \\ h_{T,1} & h_{T,2} & \dots & h_{T,R} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_M \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_R \end{bmatrix} \quad (1.7)$$

So, system equation can be written as, $\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{w}$

1.8 Capacity of MIMO System

The maximum error-free data rate that a channel can support is called the channel capacity. The channel capacity for SISO AWGN channels was derived by Claude Shannon. In contrast to AWGN channels, multiple antenna channels combat fading and cover a spatial dimension. The capacity of a deterministic SISO channel with an input-output relation $r = \mathbf{H}\mathbf{s} + \mathbf{n}$ is given by:

$$C = \log_2(1 + \rho|\mathbf{H}|^2) \quad (1.8)$$

Here, average SNR at each receiver branch is $\rho = P/\sigma_n^2$ and P is average power at the output of each receive antennas. $|H|^2$ is the normalized channel power transfer characteristics. For MIMO channel, channel capacity [1, 2] is given by:

$$C = \log_2 [\det (I_{n_r} + \frac{\rho}{n_t} HH^H)] \quad (1.9)$$

where n_t and n_r are numbers of transmit and receive antenna

For fixed value of n_r , if $n_t \rightarrow \infty$, $\lim_{n_t \rightarrow \infty} \frac{1}{n_t} H^H H = I_{n_r}$

$$C = n_r \log(1 + \rho) \quad (1.10)$$

which grows linearly with increase in number of receive antenna.

For fixed value of n_t , if $n_r \rightarrow \infty$, $\lim_{n_r \rightarrow \infty} \frac{1}{n_r} H H^H = I_{n_t}$

$$C = n_t \log \left(1 + \frac{\rho n_r}{n_t} \right) \quad (1.11)$$

which grows almost linearly with increase in number of transmit antennas. Therefore, comparing with the single antenna capacity, the capacity of multiple antenna systems increases linearly with $\min\{n_t n_r\}$. So, multiple antennas systems then give significant capacity improvement than single antenna systems.

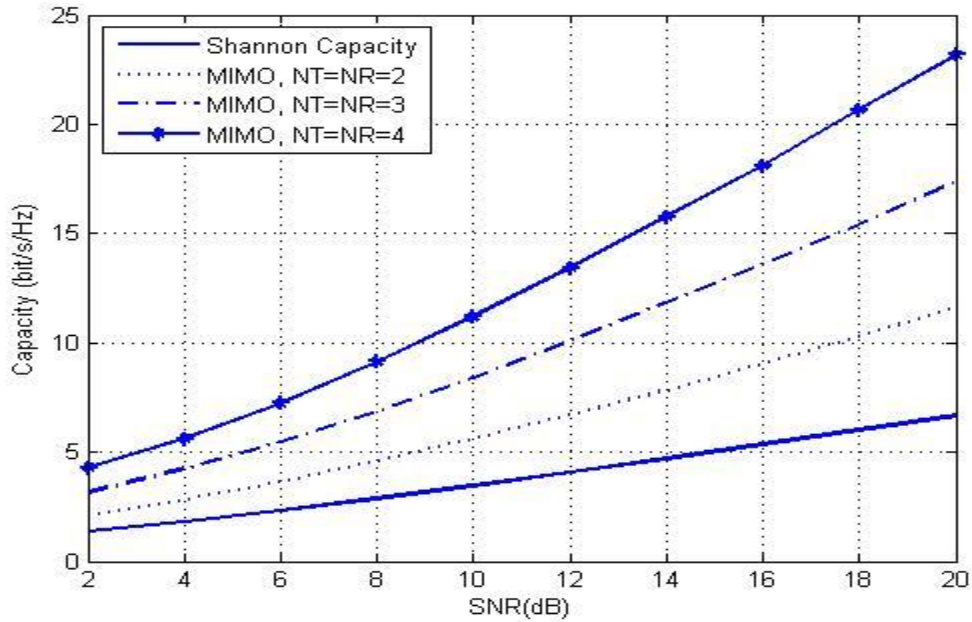


Fig.1.4: MIMO capacity for different transmit and receive antennas.

1.9 Organization of Dissertation

This dissertation consists of six chapters including introduction as follows:

Chapter 2 deals with literature review in which study on existing space time block codes and their decoding technique is discussed.

Chapter 3 provides a systematic discussion of STCs and a more systematic discussion of space-time block coding (STBC). Firstly, Alamouti STBC is explained that provides a transmit diversity of two. Orthogonal designs are presented and their performance is evaluated by simulations. After this, it defines Quasi-Orthogonal Space time Block Codes. It gives the detailed structure of QO-STBC and presents pairwise decoding. Later on, impacts of rotation of constellation set on QO-STBC code is demonstrated and study the optimum choice of the rotation for different constellation and evaluate performance of QO-STBC on this optimum rotation.

Chapter 4 is devoted to the analysis of quasi-orthogonal STBCs in open-loop transmission systems. For QO-STBCs only examples have been reported in the literature without systematic analysis and precise definition. The matrices of known QO-STBCs have been analyzed and new versions of QO-STBCs have been presented. Also, the design of the receiver structure for the QO-STBC proposed has been studied. The primary goal of this chapter is to provide a unified theory of QO-STBCs for four transmit antennas and one receive antenna. It introduces a concept of extending O-STBCs to QO-STBCs and show how families of codes with essentially identical code properties but different transmission properties can be generated. It analyzes the structuring property of the equivalent virtual MIMO channel matrix (EVCN) resulting from the QO-STBCs after which the transmission problem in an equivalent form much more suitable for the system performance analysis. After this, it discusses the performance of various receivers under QO-STBC transmission. Finally, this chapter presents a new QO-STBC scheme.

Chapter 5 provides very simple method to improve the QO-STBC transmission strategy when the transmitter knows the channel. In this receiver returns a small amount of the CSI that enables the transmitter to minimize the interference parameter resulting from the non-orthogonality of all QO-STBCs. Here, it employs closed loop QO-STBC with transmit

antenna shuffling. In this way full diversity and nearly full-orthogonality can be achieved. Lastly, it uses transmit antenna shuffling in both conventional and proposed QO-STBC scheme and compares the results of both the scheme.

Chapter 6 highlights the conclusion of the dissertation and tells about future prospects.

CHAPTER 2 LITERATURE REVIEW

The literature review summarizes, interprets, and evaluates existing "literature" (or published material) in order to establish current knowledge of a subject. The purpose for doing so relates to ongoing research to develop that knowledge. The literature review may resolve a controversy, establish the need for additional research, and define a topic of inquiry.

V. Tarokh *et al.* [13] introduced space-time block coding, a new paradigm for communication over Rayleigh fading channels using multiple transmit antennas. Data is encoded using a space-time block code and the encoded data is split into N streams which are simultaneously transmitted using N transmit antennas. Maximum likelihood decoding is achieved in a simple way through decoupling of the signals transmitted from different antennas rather than joint detection. This uses the orthogonal structure of the space-time block code. The classical mathematical framework of orthogonal designs is applied to construct space-time block codes. A generalization of orthogonal designs is shown to provide space-time block codes for both real and complex constellations for any number of transmit antennas.

W. Su *et al.* [33] introduced signal constellation rotation to achieve full diversity in QO-STBC. With the quasi-orthogonal structure, the quasi-orthogonal STBCs still have a fast ML decoding, but do not have the full diversity. The performance of these codes is better than that of the codes from orthogonal designs at low signal-to-noise ratio (SNR), but worse at high SNR. This is due to the fact that the slope of the performance curve depends on the diversity. It is desired to have the quasi-orthogonal STBCs with full diversity to ensure good performance at high SNR. In this paper, to achieve this goal the signal constellations are chosen properly. Specifically, it is proposed that half of the symbols in a quasi-orthogonal design are chosen from a signal constellation set (A) and the other half of them is chosen from a rotated constellation ($e^{j\theta} A$). The resulting STBCs can guarantee both full diversity and fast ML decoding. Moreover, the optimum selections of the rotation angles for some commonly used signal constellations are obtained.

U. park *et al.* [28] proposed a new quasi-orthogonal space time block coding (QO-STBC) scheme which uses simple linear decoding. A conventional QO-STBC scheme can achieve the full rate, but at the cost of decoding complexity and diversity gain. These disadvantages of the conventional QO-STBC scheme are mainly a result of interference terms in the detection matrix. This proposed QO-STBC scheme eliminates interference terms. The proposed method achieves improved diversity gain with respect to the conventional QO-STBC scheme, as well as a great reduction in decoding complexity.

S. Kundu *et al.* [23] presented a maximum likelihood (ML) decoder with the lowest computational complexity known to-date for full-diversity where arbitrary size Quasi-Orthogonal Space-Time Block Codes (QO-STBCs) with symbols from square or rectangular quadrature amplitude modulation (QAM) constellations is used. It is started with the formulation of an explicit joint two-complex symbol decoder for general QO-STBCs with arbitrary complex symbols and then derives the proposed ML decoder for QO-STBCs with QAM symbols. The complexity savings are made possible by a simplified quadratic ML decoding statistic that utilizes algebraically the structure of the signal points of the QAM constellation.

H. Jafarkhani [16] firstly proposed QO-STBC. It has been shown that a complex orthogonal design that provides full diversity and full transmission rate for a space-time block code is not possible for more than two antennas. Previous attempts have been concentrated in generalizing orthogonal designs which provide space-time block codes with full diversity and a high transmission rate. In this work, rate one codes which are quasi-orthogonal is designed which provide partial diversity. The decoder of the proposed codes works with pairs of transmitted symbols instead of single symbols.

S. M. Alamouti [11] presented a simple two branch transmit diversity scheme. Using two transmit antennas and one receive antenna the scheme provides the same diversity order as maximal-ratio receiver combining (MRRC) with one transmit antenna, and two receive antennas. It is also shown that the scheme may easily be generalized to two transmit antennas and M receive antennas to provide a diversity order of $2M$. The new scheme does not require any bandwidth expansion any feedback from the receiver to the transmitter and its computation complexity is similar to MRRC.

C. Yuen *et al.* [30] considered quasi-orthogonal (QO) space-time block code (STBC) with minimum decoding complexity (MDC-QO-STBC). Later on, formulate its algebraic structure and propose a systematic method for its construction. It is shown that a maximum-likelihood (ML) decoder for this MDC-QO-STBC, for any number of transmit antennas, only requires the joint detection of two real symbols. Assuming the use of a square or rectangular quadratic-amplitude modulation (QAM) or multiple phase-shift keying (MPSK) modulation for this MDC-QO-STBC, the optimum constellation rotation angle is also obtained, in order to achieve full diversity and optimum coding gain. It is also shown that the maximum achievable code rate of this MDC-QO-STBC is 1 for three and four antennas and $3/4$ for five to eight antennas. The proposed MDC-QO-STBC has several desirable properties, such as a more even power distribution among antennas and better scalability in adjusting the number of transmit antennas, compared with the coordinate interleaved orthogonal design (CIOD) and asymmetric CIOD (ACIOD) codes. For the case of an odd number of transmit antennas, MDC-QO-STBC also has better decoding performance than CIOD.

Z. Chen *et al.* [32] proposed a novel method of extending any QO-STBC constructed for 4 transmit antennas to a closed-loop scheme based on circulant matrix. It is shown that with the aid of multiplying the entries of QO-STBC code words by the appropriate phase factors which depend on the channel information, the proposed scheme can improve its transmit diversity with one bit feedback. The performances of the proposed scenario extended from Jafarkhani's QO-STBC as well as its optimal constellation rotated scheme are analyzed. The simulation results suggest that there is a significant SNR advantage in the proposed scheme which is able to be designed easily.

Y. Li *et al.* [29] investigated the performance of QO-STBC for dual-polarized MIMO channels. It is shown that when QO-STBC is used with dual-polarized antennas, there is a need to connect the power amplifiers (PAs) to the differently polarized antennas carefully, in order to avoid more than 1dB performance loss at high cross-polar discrimination (XPD). Also, it is shown that, if the QO-STBC with constellation rotations, which achieves full diversity in uni-polarized MIMO channels, is used, the performance loss due to the bad PA-to-antenna connections is mitigated. However, when the XPD is high, the QO-STBC without full diversity, if connected properly, can achieve better performance than the QO-STBC with

full diversity when connected badly, which shows the importance of a proper connection at high XPD. So, to guarantee robust performance of QO-STBC at any XPD, besides designing codes to have full diversity and good diversity product, connecting the PAs and the antennas properly is also important.

H. K. Shah *et al.* [40] analyzed the performance of QO-STBC over correlated channels. As, Space Time Block Code (STBC) schemes are widely used in MIMO systems for link performance improvement. For more than two transmit antenna systems, Quasi Orthogonal STBC (QO-STBC) and Constellation Rotation QOSTBC (CR-QO-STBC) is found to be appropriate. Various proposed STBC schemes are evaluated over Rayleigh fading channels, where there is no consideration for correlation between antennas. Inadequate distance between antennas creates correlated channels. Over this channel the performance of various schemes must be evaluated. The performance of QO-STBC and CR-QO-STBC systems over correlated wideband channels is analyzed. The effect of correlation over CRQOSTBC and QO-STBC systems is presented by varying antenna spacing parameter in kronecker channel model. The comparison between CR-QO-STBC and QO-STBC over correlated and uncorrelated channels is also discussed in this paper on the basis of simulations.

C. Yuen *et al.* [26] searched for high rate QO-STBC. Quasi-Orthogonal Space-Time Block Code (QO-STBC) is attractive because it achieves higher code rate than orthogonal STBC and lower decoding complexity than non-orthogonal STBC. In this paper, first the algebraic structure of QO-STBC is derived, and then a novel graph-based search algorithm is applied to find high-rate QO-STBCs with code rates greater than 1. From the four-antenna codes found using this approach, it is found that the maximum code rate is limited to $5/4$ with symbol wise diversity level of four, and 4 with symbol wise diversity level of two. The maximum likelihood decoding of these high-rate QO-STBCs can be performed on two separate sub-groups of symbols. The rate- $5/4$ codes are the first known QO-STBCs with code rate greater 1 that has full symbol wise diversity level.

C. Toker *et al.* [20] proposed two feedback methods to achieve full diversity for Quasi-orthogonal space-time block codes (QO-STBCs). In the first method, signals radiated from various antennas are rotated by phasors according to feedback from the receiver, whereas the second method is based upon antenna weighting selection. For high to moderate feedback

error rates, it is demonstrated that the proposed methods outperform the quantized transmit beamformer. The performance improvement is also investigated for these closed-loop methods when the transmitted signal is error control coded.

Y. Yu *et al.* [36] proposed a transmit antenna shuffling scheme for quasi-orthogonal space-time block codes (QO-STBCs). It is shown that by adaptively mapping the space-time sequences of the QO-STBC to the appropriate transmit antennas depending on the channel condition, the proposed scheme can improve its transmit diversity with limited feedback information. The performance of the scheme with various numbers of shuffling patterns is analyzed. The bit error probability of the schemes is evaluated by simulations. It is demonstrated that with the linear zero-forcing (ZF) and the minimum mean squared error (MMSE) receivers, the closed-loop QO-STBC using two feedback bits can achieve almost the same performance as the ideal 4-path diversity and it is about 4-5 dB better than the open loop schemes.

Z. Q. Taha *et al.* [22] proposed modification to the conventional coding/decoding scheme. Space-time coding can achieve transmit diversity and power gain over spatially uncoded systems without sacrificing bandwidth. There are various approaches in coding structures, including space-time block codes. A class of space-time block code namely quasi-orthogonal space-time block codes can achieve the full rate, but the conventional decoders of these codes experience interference terms resulting from neighboring signals during signal detection. The presence of the interference terms causes an increase in the decoder complexity and a decrease in the performance gain. So, modified codes will improve performance, reduce decoding complexity, and improve robustness against channel estimation errors as well.

D. Wang *et al.* [21] presented optimal rotations for quasi-orthogonal space-time block codes (STBC) with MPSK symbols, where the optimality means that the optimal diversity product is achieved. The optimal diversity products of quasi-orthogonal STBC with rotated MPSK constellations are given. The sensitivity of the diversity product to the rotation angle offset is studied.

3.1 Space Time Coding

As, mobile receivers are typically required to be small and cost-effective, it may not be practical to deploy receive diversity at the mobile terminal. This motivates many researchers to consider transmit diversity by deploying multiple antennas at the base station. By employing multiple transmit antennas to provide transmit diversity, the BER can be significantly reduced, such that the BER (bit error rate) curve decays faster with SNR (signal to noise ratio). This is due to the multiple transmit antennas providing higher spatial diversity level. However, unlike receive diversity that can be achieved by simply performing the diversity combining at the receiver side, transmit diversity requires some form of signal processing, generally known as space time coding, on the transmitted signals in order to achieve signal enhancement at the receiver. Space-Time Coding (STC) [4] is a technique that combines coding, modulation and signal processing to achieve transmit diversity. The first STC proposed in the literature is Space-Time Trellis Code (STTC) [14], which has a good decoding performance but its decoding complexity increases exponentially with the transmission rate. In addressing the issue of decoding complexity of STTC, Space-Time Block Code (STBC) was subsequently proposed. Alamouti [11] discovered a remarkable STBC scheme for two transmit antennas. This scheme supports linear decoding complexity for maximum-likelihood (ML) decoding, which is much simpler than the decoding of STTC. It can achieve the same diversity gain as a corresponding STTC for two transmit antennas, though with a shortfall in coding gain. Despite the lower coding gain, Alamouti's scheme is very appealing in terms of implementation simplicity. Hence it motivates a search for similar schemes for more than two transmit antennas, to achieve diversity level higher than two. As a result, Orthogonal Space-Time Block Code (O-STBC) was introduced by Tarokh *et al.* [13] O-STBC is a generalization of the Alamouti's scheme to an arbitrary number of transmit antenna. It retains the property of having linear maximum-likelihood decoding with full transmit diversity.

Although O-STBC can provide full diversity at low computational cost, but its code rate is less than one for more than two antennas. As rate-1 O-STBC with complex constellation is not possible for more than two transmit antennas, O-STBC design for more than two transmit

antennas will always suffers capacity loss. To address the issue of capacity loss, various non-orthogonal STBC designs have been proposed. An interesting one among them is the Quasi-Orthogonal STBC (QO-STBC) [16], which is designed to achieve a higher code rate than O-STBC by partially (instead of fully, as in the case of other non-orthogonal STBCs) relaxing the orthogonality of an O-STBC, so that the ML decoding of the full-rate QO-STBC for four transmit antennas can be achieved by jointly detecting two out of four complex symbols in the codeword, and separately doing the same for the remaining two complex symbols. Due to these advantage of QO-STBC, as well as its ability to achieve full transmit diversity by using constellation rotation, complete study on QO-STBC is provided in this monograph and to seek further improvements in its design.

3.2 Space Time Code Design Criteria

To design good codes, defining a design criterion is important parameter. For example consider linear block code transmitted over binary symmetric channel. In this bit error rate depends on the hamming distance between the codeword pairs. So, design criterion for such a code is to maximize the minimum hamming distance among all codeword pairs. Similarly for an additive white Gaussian noise (AWGN) channel, a good design criterion is to maximize the minimum Euclidean distance among all possible codeword pairs. In this section, design criteria that guarantee the maximum possible diversity gain and coding gain is derived for space time codes. In space time code [10], a codeword is a $T \times N$ matrix. Let us assume that a codeword C^l is transmitted.

$$C^1 = \begin{bmatrix} C_{1,1}^1 & C_{1,2}^1 & \dots & C_{1,N}^1 \\ C_{2,1}^1 & C_{2,2}^1 & \dots & C_{2,N}^1 \\ \vdots & \vdots & \vdots & \vdots \\ C_{T,1}^1 & C_{T,2}^1 & \dots & C_{T,N}^1 \end{bmatrix} \quad (3.1)$$

An error occurs if the decoder mistakenly decides that another codeword C^2 has been transmitted.

$$C^2 = \begin{bmatrix} C_{1,1}^2 & C_{1,2}^2 & \dots & C_{1,N}^2 \\ C_{2,1}^2 & C_{2,2}^2 & \dots & C_{2,N}^2 \\ \vdots & \vdots & \vdots & \vdots \\ C_{T,1}^2 & C_{T,2}^2 & \dots & C_{T,N}^2 \end{bmatrix} \quad (3.2)$$

So, pairwise error probability of transmitting C^l and detecting as C^2 , is expressed by:

$$P(C^1 \rightarrow C^2) \quad (3.3)$$

which is an important measure of performance for the code. A good design criterion is to minimize the maximum pairwise error probability. In general, when codebook contains I codewords, the probability of error when C^l is transmitted, is upper bounded by

$$P(\text{error}|C^l \text{ is sent}) \leq \sum_2^I P(C^1 \rightarrow C^2) \quad (3.4)$$

For calculating pairwise error probability, calculate the average error by computing the expected value over the distribution of the H (assuming fixed known channel matrix H). As explained earlier, input-output relationship between channels is given as

$$R = CH + \bar{N} \quad (3.5)$$

where, R = received matrix ($T \times M$) ; H = Channel Matrix($N \times M$) ;

C = codeword matrix ($T \times N$) ; \bar{N} = noise matrix ($T \times M$) ;

and N , M , T are no. of transmit antenna , no. of receiver antenna and no. of time slots respectively. Assume that variance of noise sample is $E [|n_{T,M}|]^2 = N_0 = 1/\gamma$. Consider distribution of received signal for the known codeword C and channel matrix H is $f(R|C, H)$. As it has assumed that Gaussian noise \bar{N} with independent components, for fixed C and H , so received vector R is also a multivariate, multidimensional Gaussian random variable. Therefore,

$$\begin{aligned} f(R|C, H) &= \frac{1}{(\pi N_0)^{\frac{M \times M}{2}}} \exp \left\{ \frac{-Tr[(R-CH)(R-CH)^H]}{N_0} \right\} \quad (3.6) \\ &= \left(\frac{\gamma}{\pi} \right)^{M \times M / 2} \exp \{ -\gamma Tr[(R - CH)(R - CH)^H] \} \\ &= \left(\frac{\gamma}{\pi} \right)^{M \times M / 2} \exp \{ -\gamma \|R - CH\|_F^2 \} \end{aligned}$$

Using property of Frobenius norm for matrix $(R - CH)$ which is defined as $\|A\|_F = \sqrt{Tr(AA^H)}$ where A is any matrix and Tr stands for trace of matrix. Maximum likelihood decoding decides in favour of a codeword that maximizes $f(R|C, H)$. Equivalently, ML decoding finds the codeword \hat{C} that solves the following minimization problem:

$$\hat{C} = \arg_C \min \{ Tr[(R - CH)(R - CH)^H] \} \quad (3.7)$$

Let assume that codeword C^l is transmitted. The pairwise error probability if by chance C^2 is detected, is calculated by

$$\begin{aligned} P(C^1 \rightarrow C^2 | H) &= P([(\|R^1 - C^1 H\|_F)^2 - (\|R^1 - C^2 H\|_F)^2] > 0 | H) \\ &= P(\text{Tr}[(R^1 - C^1 H)^H (R^1 - C^1 H) - (R^1 - C^2 H)^H (R^1 - C^2 H)] > 0 | H) \end{aligned} \quad (3.8)$$

By putting value of $R^1 = C^1 H + \bar{N}^1$, which is the received vector corresponding to C^l , rewrite above equation as

$$P(C^1 \rightarrow C^2 | H) = P(\text{Tr}[(C^1 - C^2)H + \bar{N}^1]^H (C^1 - C^2)H + \bar{N}^1) - \bar{N}^{1H} \bar{N}^1 < 0 | H) \quad (3.9)$$

By simple mathematical arrangement above equation is written as

$$P(C^1 \rightarrow C^2 | H) = P(X > (\|(C^2 - C^1)H\|_F^2 | H)) \quad (3.10)$$

where $X = \text{Tr}[\bar{N}^{1H} \cdot (C^2 - C^1) \cdot H + H^H \cdot (C^2 - C^1) \cdot \bar{N}^1]$ is a mean Gaussian random variable with variance $2N_0 \|(C^2 - C^1) \cdot H\|_F^2 = (2/\gamma) \cdot \|(C^2 - C^1) \cdot H\|_F^2$.

Any Gaussian probability density function may be given by

$$\begin{aligned} P(x \geq x_0) &= P(y > (x_0 - m)/\sigma) \\ &= Q((x_0 - m)/\sigma), \end{aligned} \quad (3.11)$$

where m and σ are mean and standard deviation of variable x respectively. Therefore, using Q function for calculating pairwise error probability is given by

$$P(C^1 \rightarrow C^2 | H) = Q\left(\frac{(\|(C^2 - C^1)H\|_F^2)}{\sqrt{(2/\gamma)(\|(C^2 - C^1)H\|_F^2)}}\right) = Q\left(\sqrt{(\gamma/2)(\|(C^2 - C^1)H\|_F^2)}\right) \quad (3.12)$$

where $Q(x) = \frac{1}{2\pi} \int_x^\infty e^{-\frac{y^2}{2}} dy$. Now, calculate $(\|(C^2 - C^1) \cdot H\|_F^2)$ to find conditional pairwise error probability. So,

$$\begin{aligned} (\|(C^2 - C^1) \cdot H\|_F^2) &= \text{Tr}[(C^2 - C^1)H]^H (C^2 - C^1)H \\ &= \text{Tr}[(C^2 - C^1)^H H^H (C^2 - C^1)H] \end{aligned} \quad (3.13)$$

Let $A(C^1, C^2) = D^H D = (C^2 - C^1)^H (C^2 - C^1)$, where A is code distance matrix and D is code difference matrix. So, above equation becomes as

$$(\|(C^2 - C^1) \cdot H\|_F^2) = \text{Tr}[H^H \cdot A(C^1, C^2) \cdot H] \quad (3.14)$$

Using the singular value decomposition, $A(C^1, C^2)$ which is real symmetric matrix can be written as :

$$A(C^1, C^2) = E^H V E \quad (3.15)$$

where V is diagonal matrix with $V = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N)$.

Therefore, $(\|(C^2 - C^1)H\|_F^2 = \text{Tr}[H^H \cdot E^H \cdot V \cdot E \cdot H])$. And (N, M) th element of $V \cdot H$ denoted by $\beta_{N,M}$. In matrix form, $(\|(C^2 - C^1)H\|_F^2$ is given by:

$$\begin{aligned} & (\|(C^2 - C^1) \cdot H\|_F^2 \\ &= \text{Tr} \left\{ \begin{bmatrix} \beta_{1,1}^* & \beta_{2,1}^* & \dots & \beta_{N,1}^* \\ \beta_{1,2}^* & \beta_{2,2}^* & \dots & \beta_{N,2}^* \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1,M}^* & \beta_{2,M}^* & \dots & \beta_{N,M}^* \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_N \end{bmatrix} \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \dots & \beta_{1,M} \\ \beta_{2,1} & \beta_{2,2} & \dots & \beta_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{N,1} & \beta_{N,2} & \dots & \beta_{N,M} \end{bmatrix} \right\} \end{aligned} \quad (3.16)$$

or,

$$(\|(C^2 - C^1) \cdot H\|_F^2 = \text{Tr} \left\{ \begin{array}{ccc} \sum_{n=1}^N \lambda_n |\beta_{n,1}|^2 & & \dots \\ & \sum_{n=1}^N \lambda_n |\beta_{n,2}|^2 & \dots \\ & & \dots & \sum_{n=1}^N \lambda_n |\beta_{n,M}|^2 \end{array} \right\} \quad (3.17)$$

Calculating trace in last equation, finally

$$(\|(C^2 - C^1) \cdot H\|_F^2 = \sum_{m=1}^M \sum_{n=1}^N \lambda_n |\beta_{n,m}|^2 \quad (3.18)$$

Putting this in equation to find pairwise error probability as

$$P(C^1 \rightarrow C^2 | H) = Q \left(\sqrt{\binom{Y}{2} \sum_{m=1}^M \sum_{n=1}^N \lambda_n |\beta_{n,m}|^2} \right) \quad (3.19)$$

As $Q(x)$ can be approximated as $Q(x) = \frac{1}{2} e^{-x^2/2}$. So, pair wise error probability is given with upper bound as:

$$P(C^1 \rightarrow C^2 | H) = \frac{1}{2} \exp(-(\gamma/4) \sum_{m=1}^M \sum_{n=1}^N \lambda_n |\beta_{n,m}|^2) \quad (3.20)$$

Since $\beta_{n,m}$ are gaussian, their magnitude is Rayleigh with probability density function

$$f(|\beta_{n,m}|) = 2 |\beta_{n,m}| \exp(-|\beta_{n,m}|^2) \quad (3.21)$$

Using the distribution of $|\beta_{n,m}|$, expected value of the pairwise error probability is calculated as:

$$P(C^1 \rightarrow C^2) = E[P(C^1 \rightarrow C^2 | H)] \leq \frac{1}{\prod_{n=1}^N \left[1 + \left(\frac{\gamma \lambda_n}{4}\right)\right]^M} \quad (3.22)$$

Here, it is assumed that $A(C^1, C^2)$ is full rank, which means none of its eigen value is zero. On the other hand, if some eigen value is zero, $\lambda_r = \lambda_{r+1} = \dots = \lambda_N = 0$ when $N > r$, or rank is $r < N$. So, under high SNR (neglecting one in denominator), above equation can be rewritten [10] as:

$$P(C^1 \rightarrow C^2) = \frac{1}{(\prod_{n=1}^N \lambda_n)^M (0.25\gamma)^{rM}} \quad (3.23)$$

The pairwise probability indicates that the probability of error decreases as $(0.25\gamma)^{rM}$ increases or rM factor increases, which is diversity gain of space time code. The maximum diversity gain possible through combining M transmit and N receive antenna is NM . Thus, to obtain maximum diversity gain, the space time code must be design such that $A(C^1, C^2)$ is full rank matrix or $M \times N$ difference matrix D between any two codewords has full rank equal to N . This design criterion is called *Rank Criterion*.

The coding gain associated with the pairwise error probability depends on the $(\prod_{n=1}^N \lambda_n)^M$ or equivalently the determinant of matrix $A(C^1, C^2)$. Coding gain distance (CGD) between codewords C^1 and C^2 is defined as $CGD(C^1, C^2) = \det[A(C^1, C^2)]$. Thus a high coding gain is achieved by maximizing the minimum of the determinant of $A(C^1, C^2)$ over all codeword pairs. This design criterion is called *Determinant Criterion*.

So, for any two codewords C^i and C^j where $i \neq j$, the rank criterion suggests that error matrix $D = (C^i - C^j)$ has to be full rank for all combinations in order to obtain full diversity NM . The

determinant criterion says that the minimum determinant of $A(C^1, C^2)$ among all combinations has to be large enough to obtain high coding gain. Using these criterions, least probability of error is achieved.

3.3 Space Time Block Codes

In this section, design of space-time block codes (STBCs) to transmit information over a multiple antenna wireless communication system is illustrated. It is assumed that fading is quasi-static and flat and also consider a wireless communication system where the transmitter contains N transmit antennas and the receiver contains M receive antennas. These notations are used for the input–output relation of the MIMO channel. The goal of space-time coding is to achieve the maximum diversity of NM , the maximum coding gain, and the highest possible throughput. In addition, the decoding complexity is very important. In a typical wireless communication system the mobile transceiver has a limited available power through a battery and should be a small physical device. To improve the battery life, low complexity encoding and decoding is very crucial. On the other hand, the base station is not as restricted in terms of power and physical size. One can put multiple independent antennas in a base station. Therefore, in many practical situations, a very low complexity system with multiple transmit antennas is desirable. Space-time block coding is a scheme to provide these properties. Despite the name, a STBC can be considered as a modulation scheme for multiple transmit antennas that provide full diversity and very low complexity encoding and decoding. Firstly, Alamouti code is introduced, which is a simple two-branch transmit diversity scheme. The key feature of the scheme is that it achieves a full diversity gain with a simple maximum-likelihood decoding algorithm. Later on, space-time block codes for more than two antennas are presented with different rates.

3.3.1 Alamouti Scheme

The Alamouti scheme is historically the first space-time block code to provide full transmit diversity for systems with two transmit antennas. A simple example for two transmits antennas:

$$G = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \quad (3.24)$$

This is simpler type of STBC which is called as orthogonal space time block code (O-STBC) because its columns are orthogonal to each other.

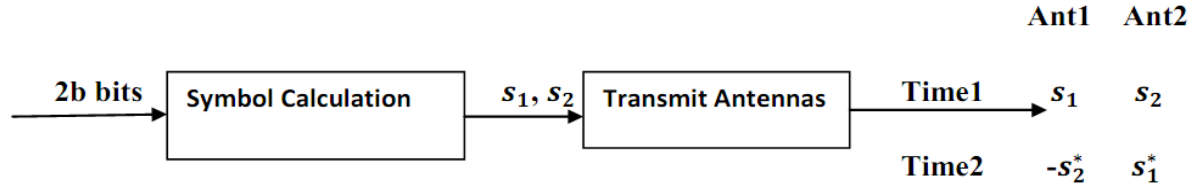


Fig.3.1: Representation of Alamouti scheme.

Consider any two distinct code sequences C and C' are generated by the inputs (s_1, s_2) and (s'_1, s'_2) respectively, where (s_1, s_2) is not equal to (s'_1, s'_2) .

$$C = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \quad C' = \begin{bmatrix} s'_1 & s'_2 \\ -s_2'^* & s_1'^* \end{bmatrix}$$

The codeword difference matrix is given by:

$$D(C, C') = \begin{bmatrix} s_1 - s'_1 & s_2 - s'_2 \\ s_2'^* - s_2^* & s_1^* - s_1'^* \end{bmatrix} \quad (3.25)$$

Since the columns of the code matrix are orthogonal, the columns of codeword difference matrix are orthogonal as well as the code matrix. The codeword distance matrix is given by:

$$A(C, C') = D(C, C') \cdot D^H(C, C') \quad (3.26)$$

$$= \begin{bmatrix} |s_1 - s'_1|^2 + |s_2 - s'_2|^2 & 0 \\ 0 & |s_1 - s'_1|^2 + |s_2 - s'_2|^2 \end{bmatrix}$$

It is clear that the distance matrices of any two distinct codewords have a full rank of two. In other words, this scheme can achieve a full transmit diversity of two. Here codeword distance matrix has two identical eigen values. The minimum eigen value is equal to the minimum squared Euclidean distance between any two transmitted code sequences remains the same as in the uncoded system. Therefore, this scheme does not provide any coding gain relative to the uncoded modulation scheme.

3.3.2 Decoding of Alamouti Scheme

Let r_1 and r_2 be the received signal at the instant t_1 and t_2 respectively.

$$\begin{cases} r_1 = \alpha_1 s_1 + \alpha_2 s_2 + n_1 \\ r_2 = -\alpha_1 s_2^* + \alpha_2 s_1^* + n_2 \end{cases} \quad (3.27)$$

The maximum-likelihood detection amounts to minimizing the decision metric

$$|r_1 - \alpha_1 s_1 - \alpha_2 s_2|^2 + |r_2 + \alpha_1 s_2^* - \alpha_2 s_1^*|^2 \quad (3.28)$$

over all possible values of s_1 and s_2 , where α is path gain.

Rewrite (3.27) in a matrix form as

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2^* \\ \alpha_2 & -\alpha_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \quad (3.29)$$

and channel matrix is given by $H = \begin{bmatrix} \alpha_1 & \alpha_2^* \\ \alpha_2 & -\alpha_1^* \end{bmatrix}$

Multiply both sides of (3.29) by H^H

$$\begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{bmatrix} = H^H \begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = (|\alpha_1|^2 + |\alpha_2|^2) \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \quad (3.30)$$

It consists of two separate equations for decoding the two transmitted symbols. This separation is the main reason for the simple ML decoding, which is only possible by using orthogonal STBC (O-STBC). It shows that the following equation is the main reason for simple ML decoding and maximum diversity so that the code satisfies the rank criterion and therefore provides the maximum possible diversity:

$$H^H H = (|\alpha_1|^2 + |\alpha_2|^2) I_2 \quad (3.31)$$

which is a result of the orthogonality of the columns of G . If the number of the transmit antennas is larger than two, the code design goal is to construct high-rate complex transmission matrices G with low decoding complexity that achieve the full diversity.

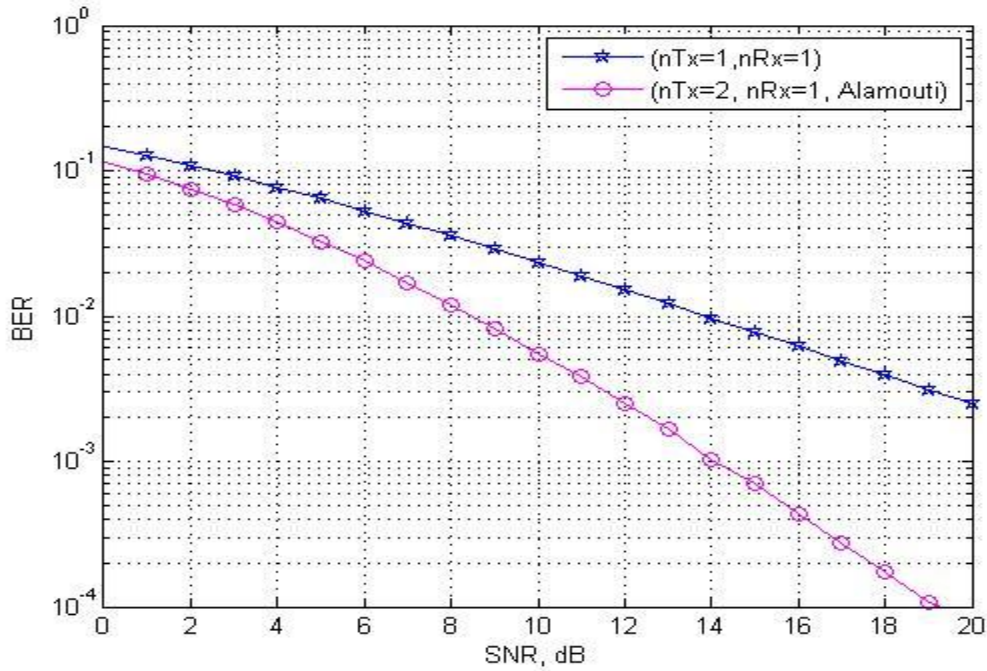


Fig.3.2: Bit error rate versus SNR for Alamouti scheme.

3.3.3 Generalization of Space Time Block Codes

In general, a complex space–time block code is given by a $M \times N$ transmission matrix G where, M represents the number of time slots for transmitting one block of symbols and N represents the number of transmit antennas. The elements of the matrix are linear combinations of indeterminate $x_1, x_2, x_3, \dots, x_k$ and their conjugates [13]. It is required that

$$G^H G = (|x_1|^2 + \dots + |x_k|^2) I \quad (3.32)$$

where G^H the Hermitian of G and I is the identity matrix. The rate of G is defined to be

$$R = K / T \quad (3.33)$$

This is due to the fact that the code transmits K constellation symbols in T time slots. Rate of a O-STBC for more than two antennas is always less than one or $R \leq 1$. So, emphasis is given to design codes which provide full diversity by generalizing the theory of orthogonal designs. Here, STBCs which achieved rate of $1/2$ and $3/4$ for three transmit antennas are presented. The codes are designed using orthogonal designs theory, which are transmission matrices with orthogonal columns so that simple decoding which can separately recover transmit symbols is possible.

O-STBC with rate 1/2 symbol per time slot: Rate 1/2 space-time block code with three antennas over complex signal constellation is given by [13]:

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & s_3 \\ -s_2 & s_1 & -s_4 \\ -s_3 & s_4 & s_1 \\ -s_4 & -s_3 & s_2 \\ s_1^* & s_2^* & s_3^* \\ -s_2^* & s_1^* & -s_4^* \\ -s_3^* & s_4^* & s_1^* \\ -s_4^* & -s_3^* & s_2^* \end{bmatrix}$$

In given code matrix \mathbf{S} , four complex symbols are taken at a time and transmitted via three transmit antennas in eight time slots. Thus, the symbol rate is 1/2.

O-STBC with rate 3/4 symbol per time slot: Rate 3/4 space-time block code with three antennas over complex signal constellation is given by [13]

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & s_3/\sqrt{2} \\ -s_2^* & s_1^* & s_3/\sqrt{2} \\ s_3^*/\sqrt{2} & s_3^*/\sqrt{2} & (-s_1 - s_1^* + s_2 - s_2^*)/2 \\ s_3^*/\sqrt{2} & -s_3^*/\sqrt{2} & (s_2 + s_2^* + s_1 - s_1^*)/2 \end{bmatrix}$$

In code matrix \mathbf{S} , three complex symbols are taken at a time and transmitted via three transmit antennas in four time slots. Thus, the symbol rate is 3/4.

3.4 Structure of QO-STBC

It is impossible to construct full rate orthogonal designs with complex elements in transmission matrix for more than two antennas. For two antennas, Alamouti scheme is the only example of a full-rate full-diversity complex space time block codes. The main property of an orthogonal design is simple separate decoding. To design full rate codes for more than two antennas, the simple separate decoding property is relaxed and consider codes for which decoding pairs of symbols independently is possible. These codes are called Quasi-orthogonal space time block codes (QO-STBC). Consider following QO-STBC [10]

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \quad (3.34)$$

Now, denote the i^{th} column of S by V_i where $i=1 \rightarrow 4$. For any indeterminate variable s_i ,

$$\langle V_1, V_2 \rangle = \langle V_1, V_3 \rangle = \langle V_2, V_4 \rangle = \langle V_3, V_4 \rangle = 0 \quad (3.35)$$

where $\langle V_i, V_j \rangle$ is the inner product of vectors V_i and V_j . Therefore subspace created by V_1 and V_4 is orthogonal to subspace created by V_2 and V_3 . That why these codes are called Quasi-Orthogonal. With regard to matrix S , four symbols (s_1, s_2, s_3, s_4) from a complex constellation are taken at a time and transmitted via four transmit antennas (no. of columns) in four symbol periods (no. of rows) so that code rate is one. Assuming a flat-fading channel over four time slots and a single receive antenna, the received signals are expressed by:

$$r = Sh + n \quad (3.36)$$

where r denotes the vector of four temporally successive signal samples at the receive antenna. The channel are denoted by $h = [h_1, h_2, h_3, h_4]^T$ and $n = [n_1, n_2, n_3, n_4]^T$ is the noise vector.

3.5 Pairwise Decoding of QO-STBC

The orthogonality of the subspaces of the matrix S results in the possibility of decoding pairs of symbols independently. In matrix form, equation (3.36) can be rewritten as:

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} \quad (3.37)$$

Maximum Likelihood (ML) decoding amounts to find the codeword S that maximizes the density function $f(r|S, h)$. Equivalently, ML decoding finds the codeword S that solves the following minimization problem by equation (3.7) as:

$$arg_G \min \{ \text{Tr} [(r - Sh)(r - Sh)^H] \} \quad (3.38)$$

Simple algebraic manipulation shows that the ML decoding amounts to minimizing the following sum, where f_{14} and f_{23} are functions of (s_1, s_4) and (s_2, s_3) respectively.

$$f_{14}(s_1, s_4) + f_{23}(s_2, s_3) \quad (3.39)$$

where,

$$\begin{aligned}
f_{14}(s_1, s_4) = & \sum_{m=1}^M \left[(|s_1|^2 + |s_4|^2) \left(\sum_{n=1}^4 |h_{n,m}|^2 \right) \right. \\
& + 2\text{Real}\{(-h_{1,m}r_{1,m}^* - h_{2,m}^*r_{2,m} - h_{3,m}^*r_{3,m} - h_{4,m}r_{4,m}^*)s_1 \\
& + (-h_{4,m}r_{1,m}^* - h_{3,m}^*r_{2,m} - h_{2,m}^*r_{3,m} - h_{1,m}r_{4,m}^*)s_4\} \\
& \left. + 4\text{Real}\{h_{1,m}h_{4,m}^* - h_{2,m}^*h_{3,m}\}\text{Real}\{s_1s_4^*\} \right] \\
f_{23}(s_2, s_3) = & \sum_{m=1}^M \left[(|s_2|^2 + |s_3|^2) \left(\sum_{n=1}^4 |h_{n,m}|^2 \right) \right. \\
& + 2\text{Real}\{(-h_{2,m}r_{1,m}^* + h_{1,m}^*r_{2,m} - h_{4,m}^*r_{3,m} + h_{3,m}r_{4,m}^*)s_2 \\
& + (-h_{3,m}r_{1,m}^* - h_{4,m}^*r_{2,m} + h_{1,m}^*r_{3,m} - h_{2,m}r_{4,m}^*)s_3\} \\
& \left. + 4\text{Real}\{h_{2,m}h_{3,m}^* - h_{1,m}^*h_{4,m}\}\text{Real}\{s_2s_3^*\} \right]
\end{aligned}$$

where M is the no of possible combination between two constellation symbols. Since $f_{14}(s_1, s_4)$ is independent of (s_2, s_3) and $f_{23}(s_2, s_3)$ is independent of (s_1, s_4) , the pairs (s_1, s_4) and (s_2, s_3) can be decoded separately. Therefore, the ML decoding results in minimizing $f_{14}(s_1, s_4)$ over all possible values of s_1 and s_4 and minimizing $f_{23}(s_2, s_3)$ over all possible values of s_2 and s_3 .

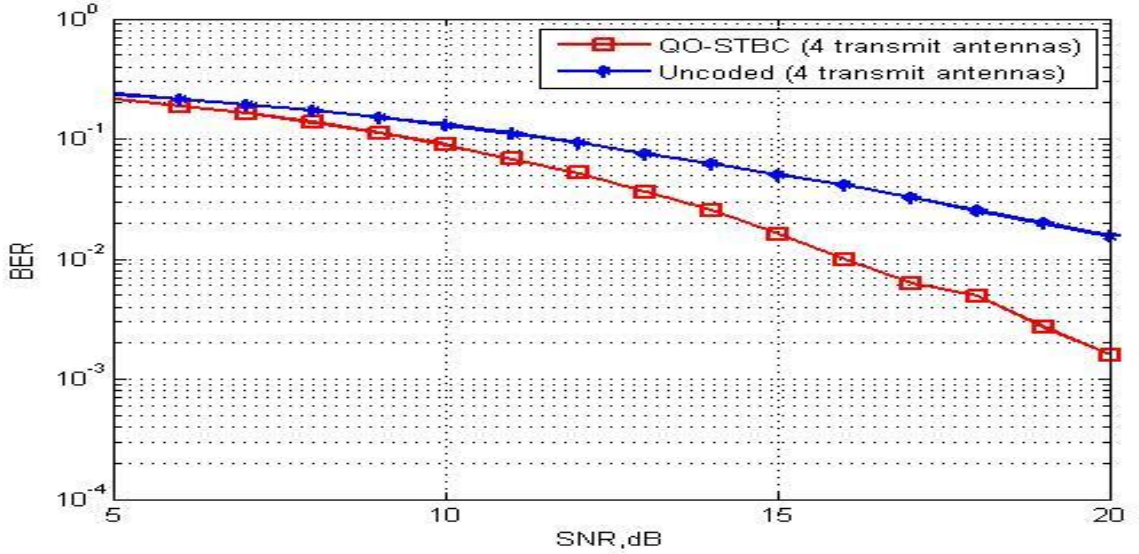


Fig.3.3: Performance of QO-STBC.

3.6 Rotated QO-STBC

Although QO-STBC can achieve higher code rates than O-STBC, it generally does not provide full transmit diversity directly. It is easy to show that minimum rank of the difference matrix $D(C^i, C^j)$ is two for QO-STBC explained in this section for regular symmetric constellations like BPSK, QPSK, 8-PSK etc. The maximum diversity of four is impossible in this case if all the symbols are taken from same constellation. To provide full diversity, different constellations for different transmitted symbols are used. Let in above explained QO-STBC, \tilde{s}_3 and \tilde{s}_4 are rotated version of s_3 and s_4 respectively, so replace s_3 and s_4 by \tilde{s}_3 and \tilde{s}_4 respectively in the QO-STBC code. The resulting code is very robust since it provides full diversity; rate one and pairwise decoding with good performance. Let the desired QO-STBC code is C which is of Jafarkhani type, is given by

$$C = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2^* & c_1^* & -c_4^* & c_3^* \\ -c_3^* & -c_4^* & c_1^* & c_2^* \\ c_4 & -c_3 & -c_2 & c_1 \end{bmatrix} \quad (3.40)$$

An error occurs if the decoder mistakenly decides that transmitted code is C^2 in place of C^1 , where C^1 and C^2 is given as

$$C^1 = \begin{bmatrix} c_1^1 & c_2^1 & c_3^1 & c_4^1 \\ -c_2^{1*} & c_1^{1*} & -c_4^{1*} & c_3^{1*} \\ -c_3^{1*} & -c_4^{1*} & c_1^{1*} & c_2^{1*} \\ c_4^1 & -c_3^1 & -c_2^1 & c_1^1 \end{bmatrix} \quad C^2 = \begin{bmatrix} c_2^2 & c_1^2 & c_3^2 & c_4^2 \\ -c_1^{2*} & c_2^{2*} & -c_3^{2*} & c_4^{2*} \\ -c_4^{2*} & -c_3^{2*} & c_2^{2*} & c_1^{2*} \\ c_3^2 & -c_4^2 & -c_1^2 & c_2^2 \end{bmatrix}$$

For examining the decoding error probability of C , A is important parameter which is expressed as $A(C^1, C^2) = (C^2 - C^1)^H (C^2 - C^1)$. The expression of A given by [12]:

$$A = \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix} \quad (3.41)$$

where $\alpha = \sum_{i=1}^4 |\Delta_i|^2$, $\beta = 2 \times \text{Re}(\Delta_1 \times \Delta_4^* - \Delta_2 \times \Delta_3^*)$, and $\Delta_i = c_i^1 - c_i^2$, represents the possible code symbol errors, where c^1 and c^2 are symbols of C^1 and C^2 codeword respectively. The determinant of the codeword distance matrix is

$$\text{Det}(A) = (\alpha + \beta)^2 \times (\alpha - \beta)^2 \quad (3.42)$$

Simple algebraic manipulation by putting value of α and β , shows that

$$\text{Det}(A) = [(|\Delta_1 + \Delta_4|^2 + |\Delta_2 - \Delta_3|^2) \times (|\Delta_1 - \Delta_4|^2 + |\Delta_2 + \Delta_3|^2)]^2 \quad (3.43)$$

Minimum value of $\text{Det}(A)$ occurs when half of the symbols have error, i.e. either $\Delta_1 = \Delta_4 = 0$ or $\Delta_2 = \Delta_3 = 0$. Hence when considering minimum determinant value (assuming that $\Delta_2 = \Delta_3 = 0$), above equation can be simplified as:

$$\text{Det}(A) = [(|\Delta_1 + \Delta_4|^2) \times (|\Delta_1 - \Delta_4|^2)]^2 \quad (3.44)$$

It is easy to see from equation (3.44), when $\Delta_1 = \pm \Delta_4$, the determinant value of the codeword distance matrix A would become zero, which implies that the codeword distance matrix A would not have full rank and so, code C cannot achieve full diversity. Such a situation can easily happen due to the symmetry of conventional constellation sets such as phase shift keying (PSK) and quadrature amplitude modulation (QAM). To achieve non-zero determinant value and hence full diversity, $\Delta_1 \neq \Delta_4$. This can be achieved by using the constellation rotation (CR) technique.

For example, consider the binary phase shift keying (BPSK) constellation set $[-1, 1]$ for all symbols, the possible values of Δ_i is $[0, 2, -2]$ for all values of i , hence $|\Delta_1 \pm \Delta_4|$ can be easily be zero. But if first symbol c_1 uses a non-rotated constellation set as shown in Fig.3.4(a), Δ_1 takes values $[0, 2, -2]$, while fourth symbol c_4 uses a rotated constellation set as shown in Fig.3.4(b), Δ_4 takes values $(0, 2\exp(j\theta), -2\exp(j\theta))$, so a non-zero value for $|\Delta_1 \pm \Delta_4|$ can always be obtained. Similarly BPSK constellation rotation can be applied to c_2 and c_3 under assumption $\Delta_1 = \Delta_4 = 0$. Therefore, by using rotated version of third symbol c_3 and fourth symbol c_4 , a simple and effective means to achieve full diversity in QO-STBC is obtained.

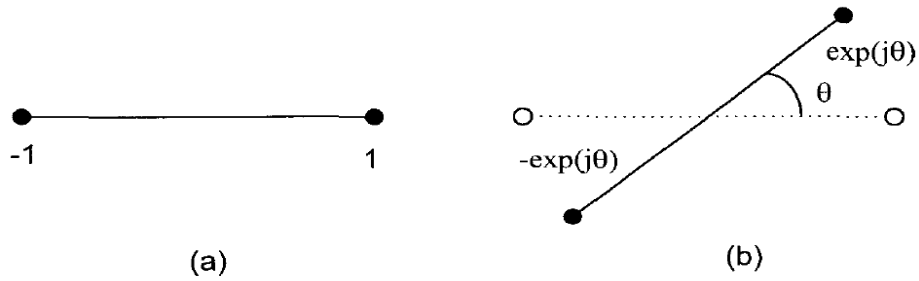


Fig.3.4: Constellation rotation for BPSK constellation set ($d_{min} = 2$) [12].

Besides giving full diversity, constellation rotation also provide an extra degree of freedom to maximize the minimum determinant value to achieve maximum coding gain for the QO-STBC. The C code after appropriate CR , denoted as C_{CR} , is able to achieve full diversity, which is given by [12]

$$C_{CR} = \begin{bmatrix} c_1 & c_2 & c_3 e^{j\theta} & c_4 e^{j\theta} \\ -c_2^* & c_1^* & -(c_4 e^{j\theta})^* & (c_3 e^{j\theta})^* \\ -(c_3 e^{j\theta})^* & -(c_4 e^{j\theta})^* & c_1^* & c_2^* \\ c_4 e^{j\theta} & -c_3 e^{j\theta} & -c_2 & c_1 \end{bmatrix} \quad (3.45)$$

where the factor $e^{j\theta}$ denotes constellation rotation angle for the symbols c_3 and c_4 . It can be seen that both C and C_{CR} perform same at low SNR region. However, as the SNR increases, the performance of C loses out, as it is not able to achieve full transmit diversity and hence its BER slope is smaller than the rotated version. On the other hand, C_{CR} , which employs constellation rotation, performs consistently better than C because C_{CR} achieves full transmit diversity. Because of the use of constellation rotation the decoding complexity of the QO-STBC increases.

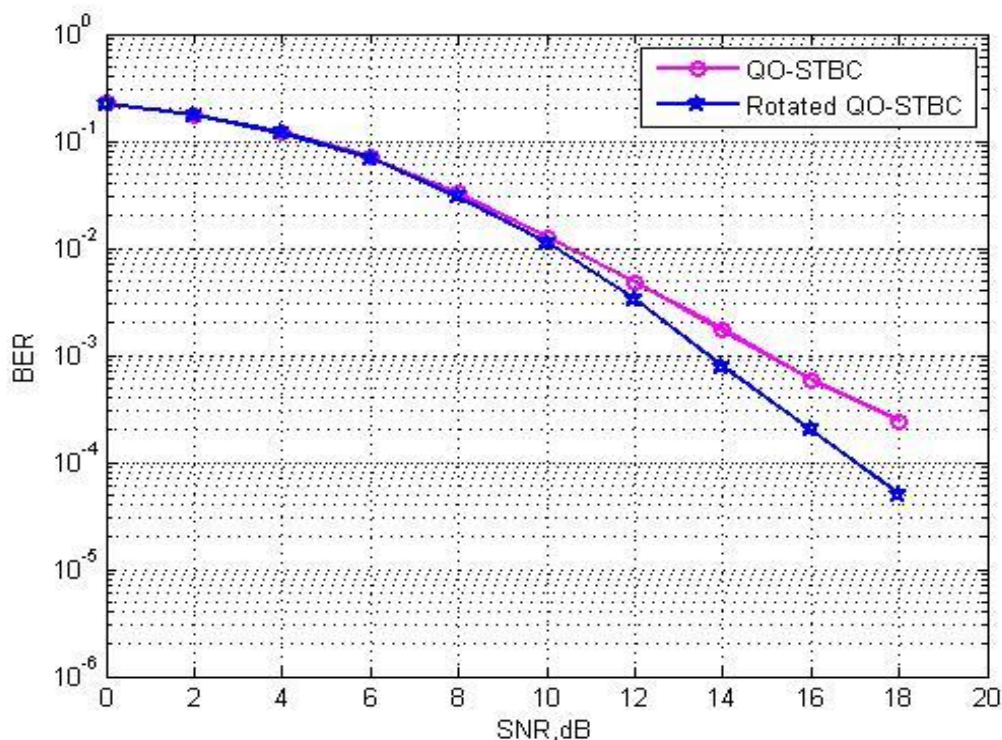


Fig.3.5: Comparison of bit error rate for rotated and non-rotated QO-STBC.

3.7 Optimum Rotation and Performance of QO-STBC

To find optimum angle of constellation rotation to attain full diversity for a given constellation, the minimum determinant of code distance matrix A should be maximized. From equation (3.44) determinant of code distance matrix is given by:

$$\begin{aligned} \text{Det}(A) &= [(|\Delta_1 + \Delta_4|^2) \times (|\Delta_1 - \Delta_4|^2)]^2 \\ &= |\Delta_1^2 - \Delta_4^2|^4 \end{aligned} \quad (3.46)$$

Let $\Delta_1 = c_1^1 - c_1^2$ and $\Delta_4 = \tilde{c}_4^1 - \tilde{c}_4^2$. Putting these in equation (3.46), rewrite above equation as:

$$\text{Det}(A) = |(c_1^1 - c_1^2)^2 - (\tilde{c}_4^1 - \tilde{c}_4^2)^2|^4 \quad (3.47)$$

where c_1^1 and c_1^2 are symbols from the original constellation and \tilde{c}_4^1 and \tilde{c}_4^2 are symbols from the rotated constellation. Let d_{min} the minimum Euclidean distance between the constellation points in fixed signal constellation, which is same for both rotated and original constellation, which is expressed as $d_{min} = |c_i^1 - c_i^2|$, where $i = 1 \rightarrow 4$. In terms of Euclidean distance, determinant of A in equation (3.47) can also be written as [10]:

$$\begin{aligned} \text{Det}(A) &= |(c_1^1 - c_1^2)^2 - (c_4^1 e^{j\theta} - c_4^2 e^{j\theta})^2|^4 \\ &= |(c_1^1 - c_1^2)^2 - e^{2j\theta} (c_4^1 - c_4^2)^2|^4 \\ &= |d_{min}^2 - e^{2j\theta} d_{min}^2|^4 \\ &= |1 - e^{2j\theta}|^4 d_{min}^8 \\ &= |2\sin\theta|^4 d_{min}^8 \end{aligned} \quad (3.48)$$

As $\text{Det}(A)$ also called code gain distance (CGD), so minimum possible CGD between codewords C^1 and C^2 is upper bounded by:

$$(\text{CGD})_{\min} \leq |2\sin\theta|^4 d_{min}^8 \quad (3.49)$$

Also, if in equation (3.47), $\tilde{c}_4^1 = \tilde{c}_4^2$, $(\text{CGD})_{\min}$ equal to the power of eight of minimum Euclidean distance d_{min} , given by:

$$(\text{CGD})_{\min} \leq d_{min}^8 \quad (3.50)$$

So, to find optimum rotation that maximizes CGD, two upper bound on CGD in (3.49) and (3.50) have considered. By analyzing (3.49) and (3.50), it is deduced that (3.49) is the tighter upper bound than (3.50) for $\theta < \pi/6$. This means that the upper bound (3.50) is not achievable for an L-PSK where $L > 6$. For BPSK constellation, $\theta = \pi/2$ achieves the upper bound (3.50). In this case, the original constellation consists of $\{1, -1\}$ and the rotated constellation consists of $\{-j, j\}$. For optimum rotation $\theta = \pi/2$, $\text{CGD}(\pi/2) = 256$ which is maximum possible CGD_{\min} . Figure 3.6 shows the minimum value of CGD for different rotations using BPSK. CGD_{\min} is a non increasing function of the rotation $0 \leq \theta \leq \pi/2$.

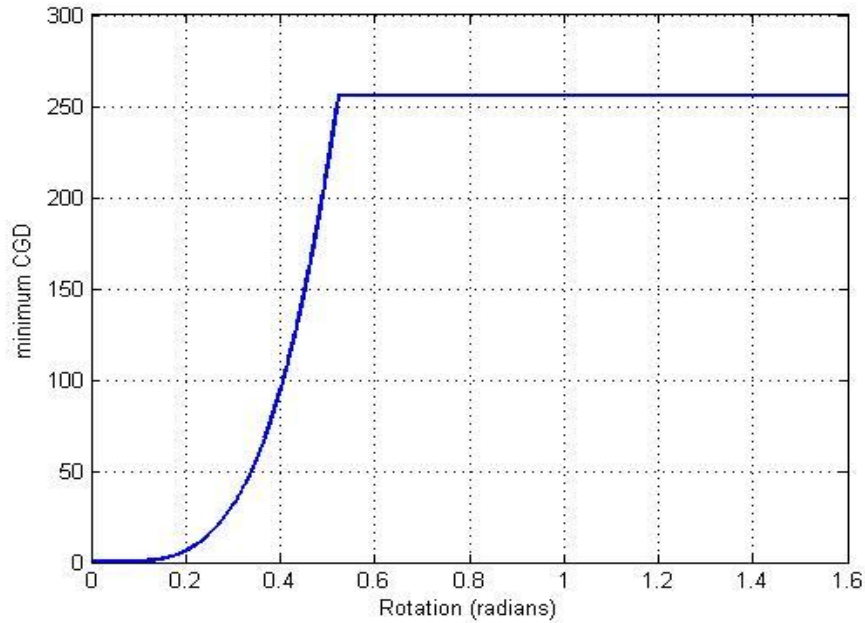
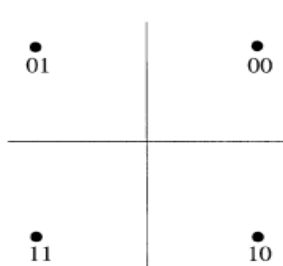
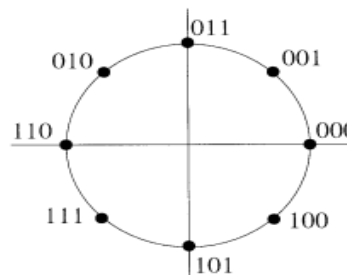


Fig.3.6: Minimum CGD for different rotation; QO-STBC using BPSK.

For QPSK constellation, $\theta = \pi/4$ is optimum rotation which achieve the upper bound (3.49) with maximum possible $\text{CGD}_{\min} = 16$. Figure 3.8 shows the minimum value of CGD for different rotation using QPSK and Figure 3.10 shows performance of QO-STBC using QPSK with different rotations. And for 8-PSK, $\theta = \pi/6$ is optimum rotation which achieve the upper bound (3.49). Figure 3.9 shows the minimum value of CGD for different rotation using 8-PSK.



(QPSK constellation $d_{min} = \sqrt{2}$)



(8-PSK constellation $d_{min} = 0.7654$)

Fig.3.7: Constellation diagram for QPSK and 8-PSK [33].

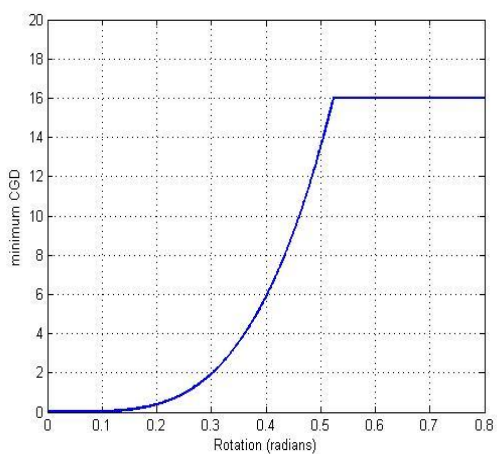


Fig.3.8: Minimum CGD using QPSK.

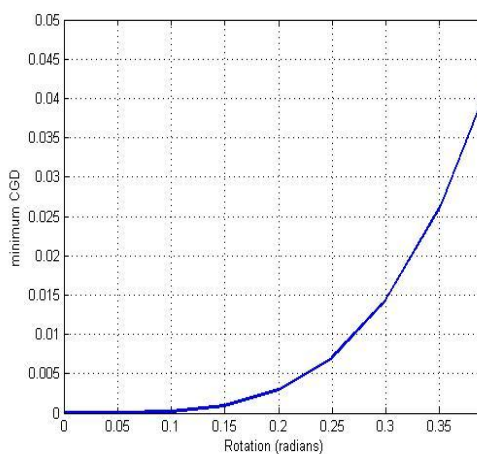


Fig.3.9: Minimum CGD using 8-PSK.

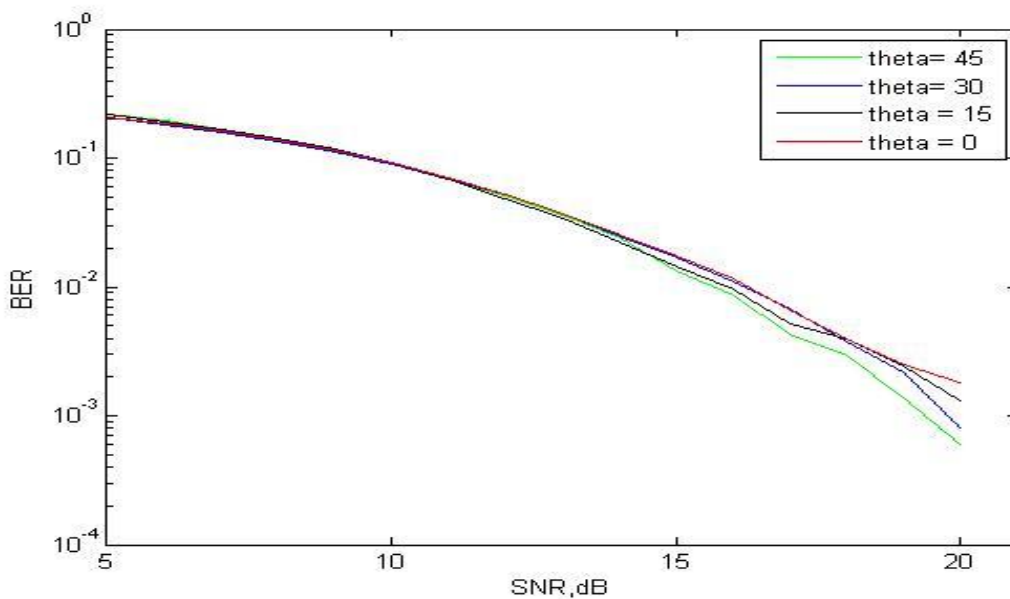


Fig.3.10: BER curve for QO-STBC for different rotations using QPSK modulation.

4.1 Popular QO-STBCs

4.1.1 Jafarkhani Code

Jafarkhani [16] proposed STBCs from quasi-orthogonal designs. For four antennas, a QOSTBC was constructed from the Alamouti scheme so it is called Extended Alamouti code since it straightforwardly extends the concept of alamouti:

$$\mathbf{S}_1 = \begin{bmatrix} A_{12} & A_{34} \\ -A_{34}^* & A_{12}^* \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \quad (4.1)$$

where A_{12} and A_{34} are the two (2×2) matrices based on Alamouti scheme of two transmit antennas, represented as

$$A_{12} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \quad \text{and} \quad A_{34} = \begin{bmatrix} s_3 & s_4 \\ -s_4^* & s_3^* \end{bmatrix}$$

4.1.2 ABBA code

ABBA code [27] proposed by Tirkkonen et al. as follows:

$$\mathbf{S}_2 = \begin{bmatrix} A_{12} & A_{34} \\ A_{34} & A_{12} \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ s_4^* & -s_3^* & s_2^* & -s_1^* \end{bmatrix} \quad (4.2)$$

4.1.3 Papadias and Foschini (PF) Code

Finally code proposed by Papadias and Foschini [22] that cannot be described by sub- blocks like the first two codes, as follows:

$$\mathbf{S}_3 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & -s_1 & s_2 \\ s_4^* & s_3^* & -s_2^* & -s_1^* \end{bmatrix} \quad (4.3)$$

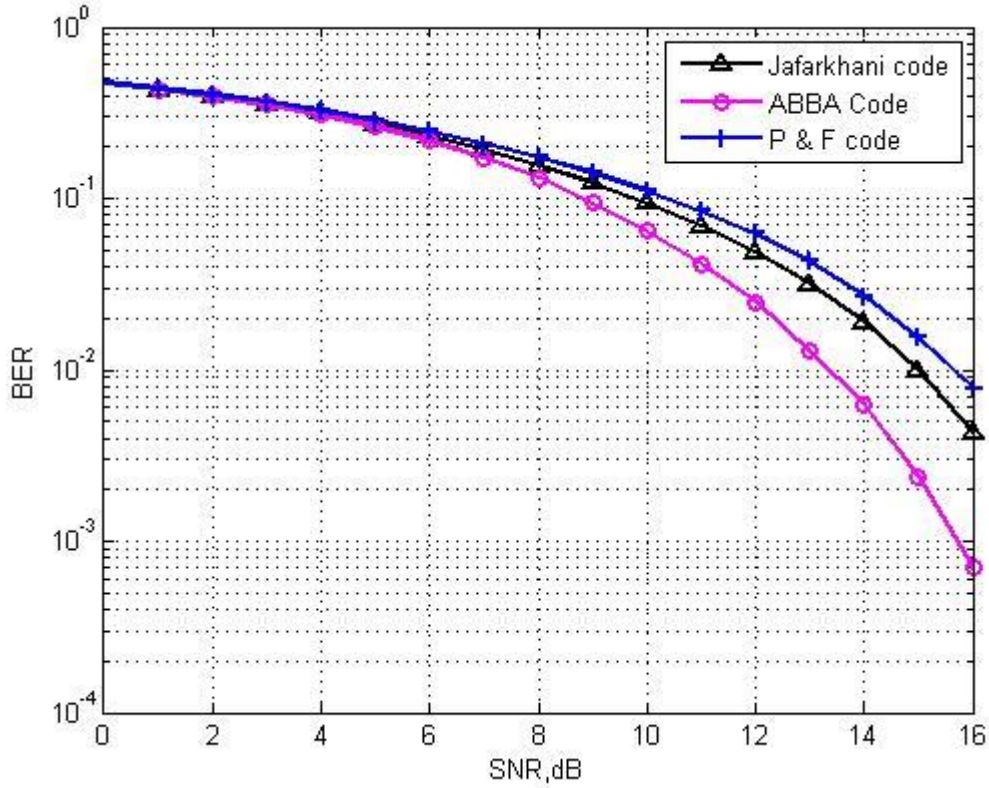


Fig.4.1: Comparison of performance of different QO-STBCs.

4.2 Other QO-STBCS Obtained by Linear Transformation

Although similar in appearance, the codes S_1 , S_2 , and S_3 show quite different behavior on particular channels. This is due to the fact that various QO-STBCs can be translated into each other by linear transformations [37]. Consider the following linear transformations on a given QO-STBC:

i) The first transformation of a given code matrix permutes rows and columns in a code matrix. This is performed by the application of an elementary permutation matrix. Let the matrix be

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (4.4)$$

is a permutation matrix that changes third and fourth row or the third and the fourth column of an 4×4 matrix S , if P multiplies S from the left or from the right. So, $S_{\text{new}} = PS$ whose third and fourth rows are switch and $S_{\text{new}} = SP$ whose third and fourth columns are switch.

ii) A second class of linear transformations changes the sign of either a column or row of the original matrix S . This can be performed by multiplying S with diagonal matrix \bar{I} , which has only +1 values at its diagonal entries except at i -th diagonal position it has a -1. Depending on left or right multiplication of the diagonal matrix to the code matrix S , such a matrix either changes sign of i -th column or sign of i -th row.

iii) Similarly, an operator I^* can be applied to change the entries of the column no. i or the entries of row no. j to their conjugate complex values. Note that all this operations can easily perform without essential hardware or software complexity. Applying one or more such operations on a given codeword matrix, new codes are generated. For example, by applying first linear transformation on code matrix S_I , new code matrix is obtained, which is given by:

$$S_{\text{new}} = PS_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_4 & -s_3 & -s_2 & s_1 \\ -s_3^* & -s_4^* & s_1^* & s_2^* \end{bmatrix} \quad (4.5)$$

4.3 Equivalent Virtual Channel Matrix (EVCN)

An important characteristic of QO-STBCs is their unique equivalent, highly structured, virtual MIMO channel matrix H [37]. The EVCN has a similar structure as the code matrix S of the underlying QO-STBC. Let us consider a QO-STBC denoted by S and a (4×1) frequency flat MISO channel h . Then received vector is given by:

$$r = Sh + n \quad (4.6)$$

where r denotes the vector of four temporally successive signal samples at the receive antenna. The channel are denoted by $h = [h_1, h_2, h_3, h_4]^T$ and $n = [n_1, n_2, n_3, n_4]^T$ is the noise vector. Assuming (4×1) MISO system (which means four transmit antenna and one receive antenna) and take code matrix S as S_I , then equation (4.6) is given by:

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} \quad (4.7)$$

By expanding above equation, received signals within four successive time slots are given as:

$$r_1 = s_1 h_1 + s_2 h_2 + s_3 h_3 + s_4 h_4 \quad (4.8)$$

$$r_2 = s_2^* h_1 - s_1^* h_2 + s_4^* h_3 - s_3^* h_4 \quad (4.9)$$

$$r_3 = s_3^* h_1 + s_4^* h_2 - s_1^* h_3 - s_2^* h_4 \quad (4.10)$$

$$r_4 = s_4 h_1 - s_3 h_2 - s_2 h_3 + s_1 h_4 \quad (4.11)$$

By complex conjugating the received signal in second and third time slots, modified received signals within four successive time slots can be written as:

$$r_1 = s_1 h_1 + s_2 h_2 + s_3 h_3 + s_4 h_4 \quad (4.12)$$

$$r_2^* = s_2 h_1^* - s_1 h_2^* + s_4 h_3^* - s_3 h_4^* \quad (4.13)$$

$$r_3^* = s_3 h_1^* + s_4 h_2^* - s_1 h_3^* - s_2 h_4^* \quad (4.14)$$

$$r_4 = s_4 h_1 - s_3 h_2 - s_2 h_3 + s_1 h_4 \quad (4.15)$$

In matrix notation, this can be written as:

$$\begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \\ n_3^* \\ n_4 \end{bmatrix} \quad (4.16)$$

So, modified received signal vector \mathbf{y} can be written as:

$$\mathbf{y} = \begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \mathbf{H}\mathbf{s} + \bar{\mathbf{n}} \quad (4.17)$$

where $\mathbf{s} = [s_1, s_2, s_3, s_4]^T$, $\bar{\mathbf{n}} = [n_1, n_2^*, n_3^*, n_4]^T$ and \mathbf{H} is called equivalent virtual (4×4) channel matrix which is given by

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \quad (4.18)$$

which can be interpreted as an equivalent, highly structured, virtual(4×4) MIMO channel matrix(EVCM), replacing the (4×1) channel vector h . In this way, the QO-STBC pretends a virtual, specifically structured (4×4) MIMO channel with four transmit and four virtual receive antennas. This EVCM simplifies the analysis of the QO-STBCs. Similarly, EVCM for other QO-STBCs can be derived.

4.4 Common Properties of the EVCM

i) The virtual channel matrix H has a block structure very similar to the corresponding non-orthogonal matrix S .

ii) Matrix obtained from multiplication of H and its hermitian, called as Grammian matrix or detection matrix, which is given by

$$G = H^H H \quad (4.19)$$

For, H defined for Jafarkhani code, G is given by:

$$G = \begin{bmatrix} h_1^* & h_2 & h_3 & h_4^* \\ h_2^* & -h_1 & h_4 & -h_3^* \\ h_3^* & h_4 & -h_1 & -h_2^* \\ h_4^* & -h_3 & -h_2 & h_1^* \end{bmatrix} \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \quad (4.20)$$

$$= \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix}$$

where $\alpha = \sum_{i=1}^4 |h_i|^2$ and $\beta = 2\text{Re}(h_1 h_4^* - h_2 h_3^*)$

iii) G is a sparse matrix with real-valued gain factor α at its main diagonal and a self interference factor β some off-diagonal positions.

iv) The self interference factor β can be real or purely imaginary depending on which QO-STBCs we used.

v) If any of the linear transformations is worked out then the quasi-orthogonal structure of G is not changed, only the value of β changes.

vi) The interference parameter β is responsible for the non-orthogonality of the code. The closer β is to zero, the closer is the code to an orthogonal code with $G = \alpha I$.

vii) Since β is the only term that changes its value with the correct linear transformations and the value of β depends on the particular channel realization influences directly performance of code and leads to complex and computationally expensive decoding.

4.5 EVCM for Known QO-STBCs

4.5.1 EVCM for Jafarkhani Code

By complex conjugating the second and third receive vector of Jafarkhani code S_1 , EVCM

for Jafarkhani code can be derived as:
$$\mathbf{H}_1 = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix}$$

The corresponding Grammian matrix G_1 results in:

$$\begin{aligned} G_1 &= \mathbf{H}_1^H \mathbf{H}_1 \\ &= \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix} \end{aligned} \quad (4.21)$$

where $\alpha = \sum_{i=1}^4 |h_i|^2$ and $\beta = 2\text{Re}(h_1 h_4^* - h_2 h_3^*)$. Since the terms α and β are real, G_1 is a real symmetric matrix ($G_1^T = G_1$)

4.5.2 EVCM for ABBA Code

By complex conjugating the second and fourth receive vector of ABBA code S_2 , EVCM for

ABBA code can be derived as:
$$\mathbf{H}_2 = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ -h_4^* & h_3^* & -h_2^* & h_1^* \end{bmatrix}$$

The corresponding Grammian matrix G_2 results in:

$$\begin{aligned} G_2 &= \mathbf{H}_2^H \mathbf{H}_2 \\ &= \begin{bmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & \beta \\ \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{bmatrix} \end{aligned} \quad (4.22)$$

where $\alpha = \sum_{i=1}^4 |h_i|^2$ and $\beta = 2\text{Re}(h_1 h_3^* + h_2 h_4^*)$. Since the terms α and β are real, G_2 is also a real symmetric matrix ($G_2^T = G_2$)

4.5.3 EVCM for Papadias and Foschini (PF) Code

By complex conjugating the second and fourth receive vector of Papadias and Foschini code S_3 ,

EVCM for Papadias and Foschini code can be derived as: $\mathbf{H}_3 = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3 & h_4 & h_1 & -h_2 \\ -h_4^* & -h_3^* & h_2^* & h_1^* \end{bmatrix}$

The corresponding Grammian matrix G_3 results in:

$$\begin{aligned} G_3 &= \mathbf{H}_3^H \mathbf{H}_3 \\ &= \begin{bmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & -\beta \\ -\beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{bmatrix} \end{aligned} \quad (4.23)$$

where $\alpha = \sum_{i=1}^4 |h_i|^2$ and $\beta = 2\text{Im}(h_1 h_3^* + h_2 h_4^*)$. Since the terms α and β are real and imaginary respectively, G_3 is a Hermitian matrix ($G_3^H = G_3$)

4.6 Useful QO-STBC Types

Using linear transformation of the QO-STBC, the sparse structure of the grammian G matrix is preserved and only the value of interference parameter β may be changed. The parameter β is always a real or imaginary part of the sum or difference of two complex terms which are the product of two channel coefficients in case of four transmit antenna. The six distinct β values that can occur with linear transformation of Jafarkhani QO-STBCs are listed below.

$$\begin{aligned} \beta_1 &= 2\text{real}(h_1 h_4^* - h_2 h_3^*) & \beta_4 &= -2\text{real}(h_1 h_4^* + h_2 h_3^*) \\ \beta_2 &= 2\text{real}(h_1 h_2^* - h_3 h_4^*) & \beta_5 &= -2\text{real}(h_1 h_2^* + h_3 h_4^*) \\ \beta_3 &= 2\text{real}(h_1 h_3^* - h_2 h_4^*) & \beta_6 &= -2\text{real}(h_1 h_3^* + h_2 h_4^*) \end{aligned}$$

Six different QO-STBC S_1, S_2, S_3, S_4, S_5 and S_6 corresponding to real-valued self interference parameter $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ and β_6 respectively, are given in table I. The corresponding QO-STBCs can be found by starting with an arbitrarily QO-STBC and then applying appropriate linear transformation as discussed earlier in section 4.2. In figure 4.2, performance of these six QO-STBCs has been evaluated using QPSK modulation in quasi-static Rayleigh fading channel.

Table 4.1: Six QO-STBCs with different interference parameter β

β	Jafarkhani type QO-STBC Codes	β	Jafarkhani type QO-STBC Codes
β_1	$S1 = \begin{bmatrix} S_1 & S_2 & S_3 & S_4 \\ -S_2^* & S_1^* & -S_4^* & S_3^* \\ -S_3^* & -S_4^* & S_1^* & S_2^* \\ S_4 & -S_3 & -S_2 & S_1 \end{bmatrix}$	β_2	$S2 = \begin{bmatrix} S_1 & S_2 & S_4 & S_3 \\ -S_2^* & S_1^* & S_3^* & -S_4^* \\ -S_3^* & -S_4^* & S_2^* & S_1^* \\ S_4 & -S_3 & S_1 & -S_2 \end{bmatrix}$
β_3	$S3 = \begin{bmatrix} S_1 & S_4 & S_3 & S_2 \\ -S_2^* & S_3^* & -S_4^* & S_1^* \\ -S_3^* & S_2^* & S_1^* & -S_4^* \\ S_4 & S_1 & -S_2 & -S_3 \end{bmatrix}$	β_4	$S4 = \begin{bmatrix} -S_1 & S_2 & S_3 & S_4 \\ S_2^* & S_1^* & -S_4^* & S_3^* \\ S_3^* & -S_4^* & S_1^* & S_2^* \\ -S_4 & -S_3 & -S_2 & S_1 \end{bmatrix}$
β_5	$S5 = \begin{bmatrix} -S_1 & S_2 & S_4 & S_3 \\ S_2^* & S_1^* & S_3^* & -S_4^* \\ S_3^* & -S_4^* & S_2^* & S_1^* \\ -S_4 & -S_3 & S_1 & -S_2 \end{bmatrix}$	β_6	$S6 = \begin{bmatrix} -S_1 & S_4 & S_3 & S_2 \\ S_2^* & S_3^* & -S_4^* & S_1^* \\ S_3^* & S_2^* & S_1^* & -S_4^* \\ -S_4 & S_1 & -S_2 & -S_3 \end{bmatrix}$

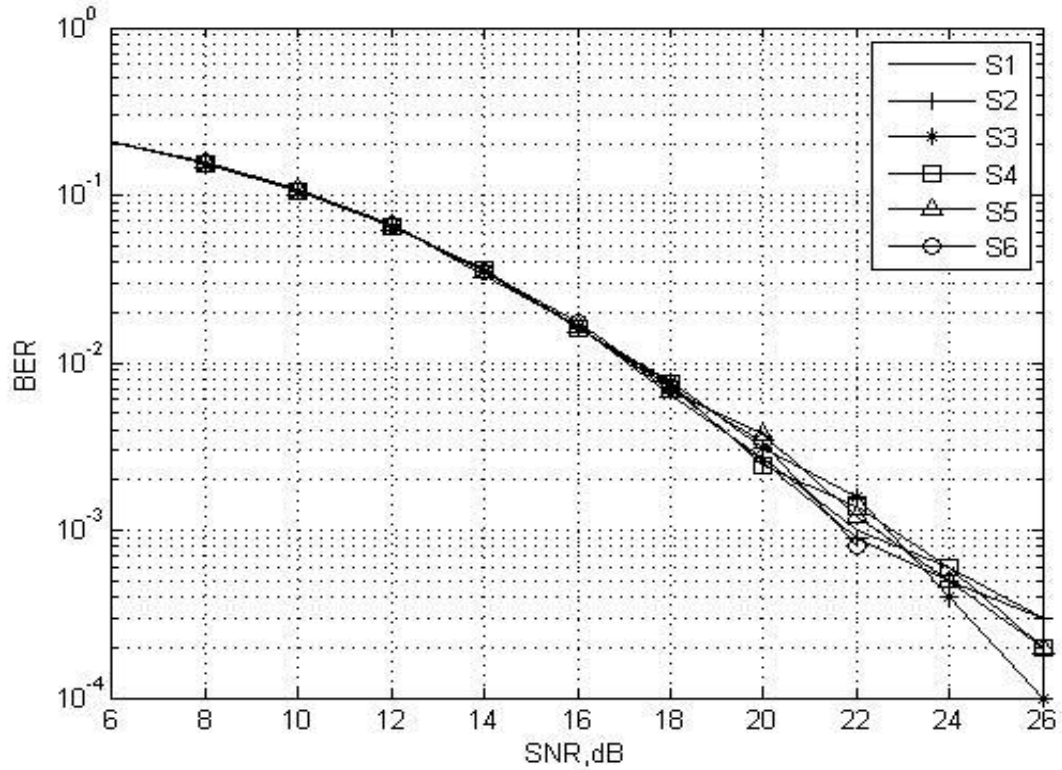


Fig.4.2: BER performance of different QO-STBC formed after linear transformation.

4.7 Grammian Matrix G

Grammian matrix takes essential part in decoding of QO-STBC code. So, it is also called as detection matrix. For Jafarkhani code, grammian matrix is given by:

$$G = \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix} \quad (4.24)$$

This grammian matrix can also be written as:

$$G = \alpha \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & -X & 0 \\ 0 & -X & 1 & 0 \\ X & 0 & 0 & 1 \end{bmatrix} \quad (4.25)$$

where X is self-interference parameter,

$$X = \frac{\beta}{\alpha} = \frac{2\text{Re}(h_1 h_4^* - h_2 h_3^*)}{\sum_{i=1}^4 |h_i|^2}$$

In general way, Grammian matrix is expressed as:

$$G = \alpha \begin{bmatrix} I_2 & X W_k \\ X W_k & I_2 \end{bmatrix} ; k = 1, 2 \quad (4.26)$$

where $W_1 = \begin{bmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{bmatrix}$ or $W_2 = \begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix}$, W_1 results in case of Jafarkhani type QO-STBC and W_2 results in case of ABBA type or PF type QO-STBC. Grammian matrix G should approximate a scaled identity matrix as far as possible to achieve full diversity and optimum BER performance. If G is a scaled identity matrix, the above matrix is changed to orthogonal STBC. So, a simple linear matrix multiplication of the modified received vector y by H^H to decouple the vector component of s perfectly can be used which result in full diversity of order four. Otherwise, X leads to partial interference between symbol pairs. So, X should be small as possible.

4.8 Statistical Properties of the Channel Dependent Self Interference Parameter X

Consider a Rayleigh fading channel. If the channel coefficients h_i are complex i.i.d. Gaussian distributed random variables, then the probability density function (pdf) of self interference

parameter X can be easily derived. It is known that self interference parameter X for Jafarkhani code is given by [37]:

$$X = \frac{\beta}{\alpha} = \frac{2\text{Re}(h_1 h_4^* - h_2 h_3^*)}{\sum_{i=1}^4 |h_i|^2} \quad (4.27)$$

$$= \frac{h_1 h_4^* - h_2 h_3^* + h_1^* h_4 - h_2^* h_3}{|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2}$$

Now, calculate pdf of $(X + 1)$ instead of X , so $(X + 1)$ is given by after rearranging as:

$$(X + 1) = \frac{|h_1 + h_4|^2 + |h_1 - h_4|^2}{|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2} \quad (4.28)$$

Since h_i are i.i.d complex valued Gaussian distributed variables, the variables $(h_1 + h_4)$ and $(h_2 - h_3)$ are also i.i.d complex Gaussian distributed variables. Assume some transformations in above equation as:

$$m = (h_1 + h_4) / \sqrt{2} \quad n = (h_2 - h_3) / \sqrt{2}$$

$$\bar{m} = (h_1 - h_4) / \sqrt{2} \quad \bar{n} = (h_2 + h_3) / \sqrt{2}$$

Now Equation (4.28) can be rewritten as:

$$\frac{X + 1}{2} = \frac{|m|^2 + |n|^2}{|m|^2 + |n|^2 + |\bar{m}|^2 + |\bar{n}|^2} = \frac{|\chi_1|^2}{|\chi_1|^2 + |\chi_2|^2} \quad (4.29)$$

where χ_1^2 and χ_2^2 are statistically independent random variable having chi-square distribution with degree of freedom $v_1 = 4$ and $v_2 = 4$ respectively. But whole equation (4.29) is beta(p, q) distributed with $p = v_1/2 = 2$ and $q = v_2/2 = 2$ degree of freedom. So, resultant pdf of (4.29) is given by:

$$f(r) = \frac{1}{B(p,q)} r^{p-1} (1-r)^{q-1} \quad (4.30)$$

where $B(p,q) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}$. By putting the values of p and q in above equation, the pdf for

$\left(\frac{X+1}{2}\right)$ is given by:

$$f(r) = 6r(1-r) \quad (4.31)$$

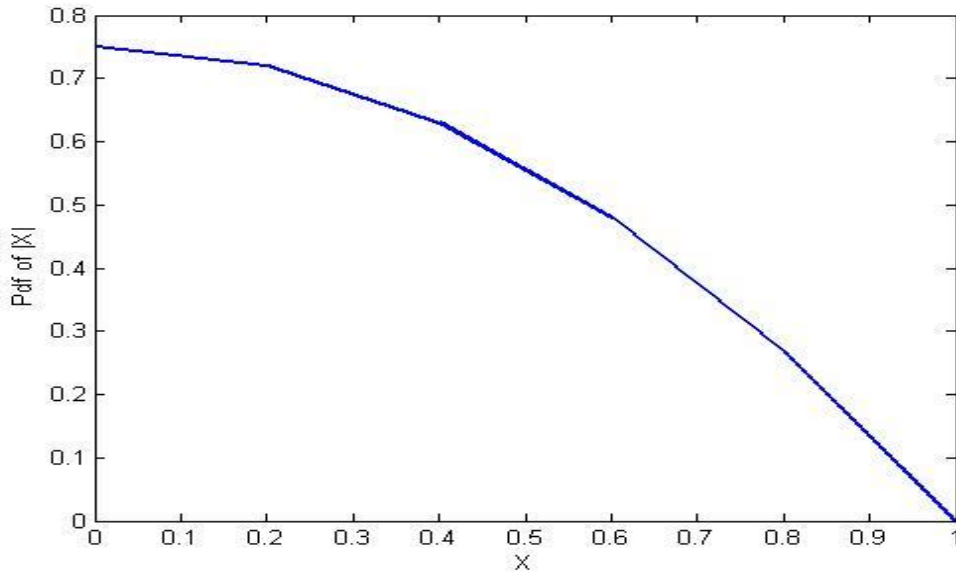


Fig.4.3: Distribution of $|X|$.

As the pdf of $\left(\frac{X+1}{2}\right)$ is known, the pdf of X can be easily find by using transformation formula, which is given by [15]

$$f_X(x) = \frac{3}{4}(1-x^2) \quad ; |x| < 1 \quad \text{otherwise } f_X(x) = 0 \quad (4.32)$$

The function $f_X(x)$ is represented in figure 4.3. It is shown that $|X|$ is highly distributed around zero. Probability density function of all the interference parameters (which is defined as the sum or the difference of two terms which are the product of two i.i.d complex-valued channel coefficients that are i.i.d complex-valued Gaussian distributed) is given by equation (4.32).

4.9 Receiver Algorithm for QO-STBCs

4.9.1 Maximum Ratio Combining

It is the simplest way to decode a QO-STBC. This can be done in a very simple way by multiplying modified received signal vector y with H^H . Then new decision vector z is given by:

$$\begin{aligned} z &= H^H y = H^H (Hs + \bar{n}) = H^H Hs + H^H \bar{n} \\ &= Gs + H^H \bar{n} \end{aligned} \quad (4.33)$$

where G is grammain matrix, $G = H^H H$. For Jafarkhani type QO-STBC, G has the following form which is given by:

$$G = \alpha \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & -X & 0 \\ 0 & -X & 1 & 0 \\ X & 0 & 0 & 1 \end{bmatrix} \quad (4.34)$$

where X is self-interference parameter, $X = \beta / \alpha$. Corresponding to MRC (maximum ratio combining) receiver operating as above, it is deduced that decoupling can be performed not only by one symbol but splitting up the decision vector z into subsets of z_1 and z_4 , z_2 and z_3 . So, due to symmetry in G matrix, after MRC, four input-four output systems can be perfectly decoupled into sets of two input-two output systems. Mathematically it is explained as:

$$z = \alpha \begin{bmatrix} s_1 + Xs_4 \\ s_2 - Xs_3 \\ s_3 - Xs_2 \\ s_4 + Xs_1 \end{bmatrix} + H^H \bar{n} \quad (4.35)$$

And by grouping z into two pairs, we have

$$\begin{bmatrix} z_1 \\ z_4 \end{bmatrix} = \alpha \begin{bmatrix} 1 & X \\ X & 1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_4 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_4 \end{bmatrix} \quad (4.36)$$

$$\begin{bmatrix} z_2 \\ z_3 \end{bmatrix} = \alpha \begin{bmatrix} 1 & X \\ X & 1 \end{bmatrix} \begin{bmatrix} s_2 \\ s_3 \end{bmatrix} + \begin{bmatrix} \tilde{n}_2 \\ \tilde{n}_3 \end{bmatrix} \quad (4.37)$$

where \tilde{n}_i $i=1, \dots, 4$ are the receive noise terms after MRC. After MRC this additional decoding step is required to retrieve the signal terms.

4.9.2 Maximum Likelihood Decoding

For minimum error probability, maximum likelihood decoding [37] is optimal choice. But this optimality is obtained at the cost of a decoding complexity which increases as number of transmit antenna and symbol constellation size increases. Generally, ML decoder selects the signal vector s that minimizes the distance D between the received vector and all possible output vector $H^H s_i$ (which are free from noise) where s_i stands for all possible transmit vectors. For a specific transmit vector s , distance D is given by

$$D(s) = \| y - H^H s \|^2 \quad (4.38)$$

Using QPSK modulation, $4^4 = 256$ combination of s vector is considered. This means that firstly 256 distance metrics is calculated and from all of these, ML algorithm searches for minimum value. Corresponding to this minimum distance metric, best possible transmitted code s is selected.

4.9.3 Linear Receiver

There are two types of linear receiver which are used frequently: (1) Zero forcing receivers (2) Minimum mean square error (MMSE) receivers. These receivers can reduce decoding complexity but they typically suffer from noise enhancement. Linear detection can be described by:

$$\hat{s} = (H^H H + \mu I)^{-1} z \quad (4.39)$$

where $\mu = 0$ for zero forcing receiver and $\mu = \sigma_n^2$ for the MMSE receiver. And σ_n^2 is noise variance which leads to small improvement compared to zero forcing receiver. In practice it can be difficult to find correct value of noise variance parameter σ_n^2 . Therefore MMSE is not used in practice and so, it is not discuss in later sections. Zero forcing receiver separates the received signal into its components by [37]:

$$\hat{s} = (H^H H)^{-1} z = s + (H^H H)^{-1} H^H \bar{n} = s + \tilde{n} \quad (4.40)$$

The zero forcing receivers decouple the channel matrix into parallel scalar channel with additive noise. Then, zero forcing receivers decode each streams independently ignoring noise correlation. No doubt ZF receiver reduces decoding complexity but it can leads to significant performance degradation.

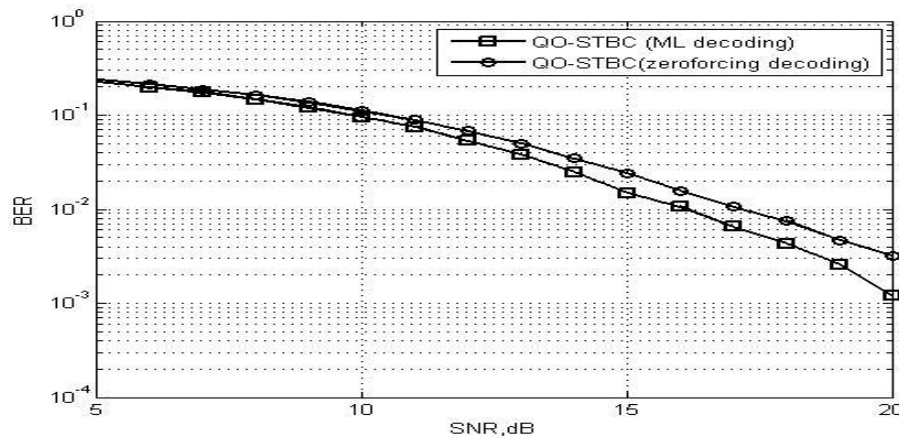


Fig.4.4: Comparison of QO-STBC with different decoding techniques.

4.10 Proposed QO-STBC

Proposed QO-STBC for four transmit and one receive antenna is given by:

$$\begin{bmatrix} (s_1 + s_2 + s_3 - s_4)/2 & (s_1 - s_2 + s_3 + s_4)/2 & (-s_1 + s_2 + s_3 + s_4)/2 & (s_1 + s_2 - s_3 + s_4)/2 \\ (-s_1^* + s_2^* - s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (-s_1^* + s_2^* + s_3^* + s_4^*)/2 \\ (s_1^* - s_2^* - s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (s_1^* - s_2^* + s_3^* + s_4^*)/2 \\ (s_1 + s_2 - s_3 + s_4)/2 & (s_1 - s_2 - s_3 - s_4)/2 & (-s_1 + s_2 - s_3 - s_4)/2 & (s_1 + s_2 + s_3 - s_4)/2 \end{bmatrix}$$

4.10.1 Construction Method

As, discussed earlier, Jafarkhani code for four transmit antenna and one receive antenna, is given by [16]:

$$S = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \quad (4.41)$$

where s_i is input modulated symbols for the QO-STBC encoder. Corresponding equivalent virtual channel matrix (EVCN) of this code is given by

$$H = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \quad (4.42)$$

where h_i is channel coefficient for each transmit-receive antenna pair. So, received signal vector \mathbf{y} can be written as:

$$\mathbf{y} = H\mathbf{s} + \bar{\mathbf{n}} \quad (4.43)$$

where $\mathbf{s} = [s_1, s_2, s_3, s_4]^T$, $\bar{\mathbf{n}} = [n_1, n_2^*, n_3^*, n_4]^T$ and n_i is the complex white Gaussian noise added in the i -th time slot. In case of orthogonal scheme, the received signals are decoded using a Gramian matrix or detection matrix D defined as $H^H H$ where H^H is Hermitian of H . For an O-STBC scheme, the detection matrix is always a diagonal matrix and this enables simple linear decoding. However for QO-STBC scheme this cannot be applied because detection matrix is not diagonal. For example, the detection matrix for the aforementioned four transmit antenna QO-STBC scheme is expressed by:

$$D = H^H H = \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} \quad (4.44)$$

where, $a = \sum_{i=1}^4 |h_i|^2$ represents channel gain for the four transmit antennas,

$b = 2\text{Re}(h_1 h_4^* - h_2 h_3^*)$ represents interference terms.

For decode this, more complex decoding method to detect the estimate \hat{C} was introduced which has the high decoding complexity due to process of the inverse of the matrix. This method is called as zero forcing decoding, and given by [37]:

$$\hat{C} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{R}_n \quad (4.45)$$

Due to the presence of interference terms in detection matrix D , described above, complex decoding is required. So, elimination of interference terms is necessary for using simple linear decoding at receiver. As detection matrix D is symmetric, which means $D^T = D$ and so, by using property of symmetric matrix, D can be expressed as

$$D = Q D_n Q^T \quad (4.46)$$

where Q is orthogonal matrix and D_n is diagonal matrix whose diagonal elements are eigen value of D . Equation (4.46) can also be written as

$$Q^T D Q = D_n \quad (4.47)$$

By equation (4.47) it is clear that by pre and post multiplying the detection matrix D with Q^T and Q , diagonal matrix D_n which is interference free is obtained. In Z.Q.Taha *et al.* [22], they used the same concept of symmetry of detection matrix but value of Q was taken as general unitary matrix for constructing their code. But in the proposed scheme singular value decomposition is used to derive value of Q (Appendix A). So Q is given by:

$$Q = \begin{bmatrix} 0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix} \quad (4.48)$$

In matrix form, D_n can be written as:

$$D_n = \begin{bmatrix} 0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix} \quad (4.49)$$

New interference free detection matrix D_n is given by

$$D_n = \begin{bmatrix} a+b & 0 & 0 & 0 \\ 0 & a+b & 0 & 0 \\ 0 & 0 & a-b & 0 \\ 0 & 0 & 0 & a-b \end{bmatrix} \quad (4.50)$$

Then, new channel matrix is derived by D_n as:

$$\begin{aligned} D_n &= Q^T D Q = Q^H D Q = Q^H H^H H Q \\ &= (HQ)^H \cdot (HQ) \end{aligned} \quad (4.51)$$

So, new channel matrix is defined as $H_n = HQ$, which can be expressed as following matrix:

$$H_n = \begin{bmatrix} (h_1 + h_2 - h_3 + h_4)/2 & (h_1 - h_2 + h_3 + h_4)/2 & (h_1 + h_2 + h_3 - h_4)/2 & (-h_1 + h_2 + h_2 + h_4)/2 \\ (h_2^* - h_1^* - h_4^* - h_3^*)/2 & (h_2^* + h_1^* + h_4^* - h_3^*)/2 & (h_2^* - h_1^* + h_4^* + h_3^*)/2 & (-h_2^* - h_1^* + h_4^* - h_3^*)/2 \\ (h_3^* + h_4^* + h_1^* - h_2^*)/2 & (h_3^* - h_4^* - h_1^* - h_2^*)/2 & (h_3^* + h_4^* - h_1^* + h_2^*)/2 & (-h_3^* + h_4^* - h_1^* - h_2^*)/2 \\ (h_4 - h_3 + h_2 + h_1)/2 & (h_4 + h_3 - h_2 + h_1)/2 & (h_4 - h_3 - h_2 - h_1)/2 & (-h_4 - h_3 - h_2 + h_1)/2 \end{bmatrix} \quad (4.52)$$

Now, corresponding to H_n , new encoding matrix which is expressed as:

$$S_n = \begin{bmatrix} (s_1 + s_2 + s_3 - s_4)/2 & (s_1 - s_2 + s_3 + s_4)/2 & (-s_1 + s_2 + s_3 + s_4)/2 & (s_1 + s_2 - s_3 + s_4)/2 \\ (-s_1^* + s_2^* - s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (-s_1^* + s_2^* + s_3^* + s_4^*)/2 \\ (s_1^* - s_2^* - s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (s_1^* - s_2^* + s_3^* + s_4^*)/2 \\ (s_1 + s_2 - s_3 + s_4)/2 & (s_1 - s_2 - s_3 - s_4)/2 & (-s_1 + s_2 - s_3 - s_4)/2 & (s_1 + s_2 + s_3 - s_4)/2 \end{bmatrix} \quad (4.53)$$

The new encoding matrix S_n is Quasi-orthogonal, but its channel matrix H_n is orthogonal matrix, so simple linear decoding can be used to reduce decoding complexity at the receiver without losing performance in terms of bit error rate. The decoding matrix is given by:

$$\hat{C} = H_n^H H_n C_n + H_n^H \bar{n} \quad (4.54)$$

where $C_n = [s_1 \ s_2 \ s_3 \ s_4]^T$, $\bar{n} = [n_1 \ n_2^* \ n_3^* \ n_4]^T$

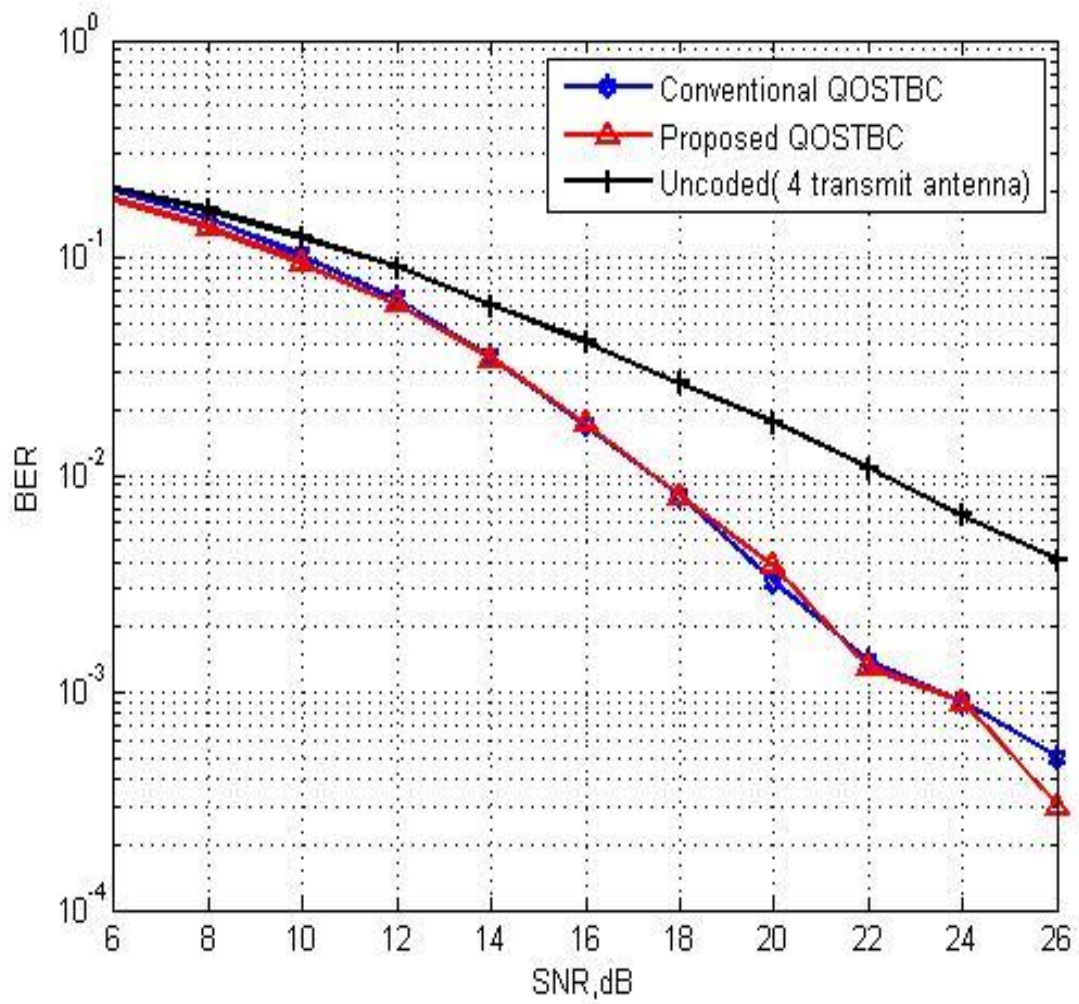


Fig.4.5: Performance of proposed QO-STBC scheme.

CHAPTER 5

QO-STBC WITH PARTIAL FEEDBACK INFORMATION

5.1 Introduction

Transmit diversity can be used as:

Feedback Scheme: This involves the feedback of channel state information (CSI, typically including channel gain and phase information) from the receiver to the transmitter in order to adapt the transmitter to the channel during the next transmission epochs. It is also commonly known as the “closed-loop” system.

Feedforward Scheme: This involves the receiver making use of feedforward information sent by the transmitter, such as pilot symbols, to estimate the channel, but no channel feedback information is sent back to the transmitter. It is also commonly known as the “open-loop” or “coherent” system.

In previous chapters various space-time coding schemes have been presented. In the analysis of these codes it is usually assumed that the channel state information (CSI) is known perfectly at the receiver but not at the transmitter. On the other hand, CSI can be made available at the transmitter. In fact, the transmitter should exploit any channel information available. This knowledge, whether partial or complete, can be advantageously exploited to adapt the transmission strategy in order to optimize the system performance. Full channel knowledge at the transmitter implies that the instantaneous channel transfer matrix H is known at the transmitter.

5.2 Transmit Antenna Shuffling for Conventional QO-STBC

Transmit antenna shuffling [36] is one of the feedback scheme which is used to improve performance of code. In this scheme, adaptively mapping of space time sequences of the code to appropriate transmit antennas depending on channel condition takes place, so that transmit diversity can improved with limited feedback information. Here, transmit antenna shuffling is applied in Jafarkhani code for four transmit antennas so that optimum antenna shuffling pattern can be selected to improve the transmit diversity. Detection matrix aforementioned four transmit antenna QO-STBC scheme is given by:

$$D = H^H H = \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} \quad (5.1)$$

where $a = \sum_{i=1}^4 |h_i|^2$ represents channel gain for the four transmit antennas, $b = 2\text{Re}(h_1 h_4^* - h_2 h_3^*)$ represents interference terms. Received signal is given by:

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} \quad (5.2)$$

After applying a complex conjugate operation to the second and the third elements of the received signal, modified received signal is then given by:

$$\begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \\ n_3^* \\ n_4 \end{bmatrix} \quad (5.3)$$

or, in matrix notation, rewrite above equation as

$$y = \begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = Hs + \bar{n} \quad (5.4)$$

For decoding this signal, above equation is multiplied by H^H to get estimated transmitted signal \tilde{s}

$$\tilde{s} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \end{bmatrix} = H^H \begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + H^H \begin{bmatrix} n_1 \\ n_2^* \\ n_3^* \\ n_4 \end{bmatrix} \quad (5.5)$$

$$\tilde{s} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \end{bmatrix} = \begin{bmatrix} as_1 + bs_4 \\ as_2 - bs_3 \\ as_3 - bs_2 \\ as_4 + bs_1 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{n}_3 \\ \tilde{n}_4 \end{bmatrix} \quad (5.6)$$

where \tilde{n}_i is noise component after multiplication by H^H . For error free decoding, these interference term b must be zero or small so that effect of this parameter in decoding is minimum. The parameter b is strongly dependent on channel coefficients. Also it is known

that by using shuffling in transmit antenna, six possible different shuffled codes are obtained for which detection matrices are same but having different values of b as given in Table 5.1. So, at the transmitter end that shuffled code is used which has minimum $|b|$. The block diagram of the proposed QO-STBC scheme using transmit antenna shuffling scheme (TAS) scheme with four transmit antennas and one receive antenna is given in Fig.5.1. Here, an antenna shuffling structure [1B 4 3 2] between the QO-STBC encoder and four transmit antennas is implemented.

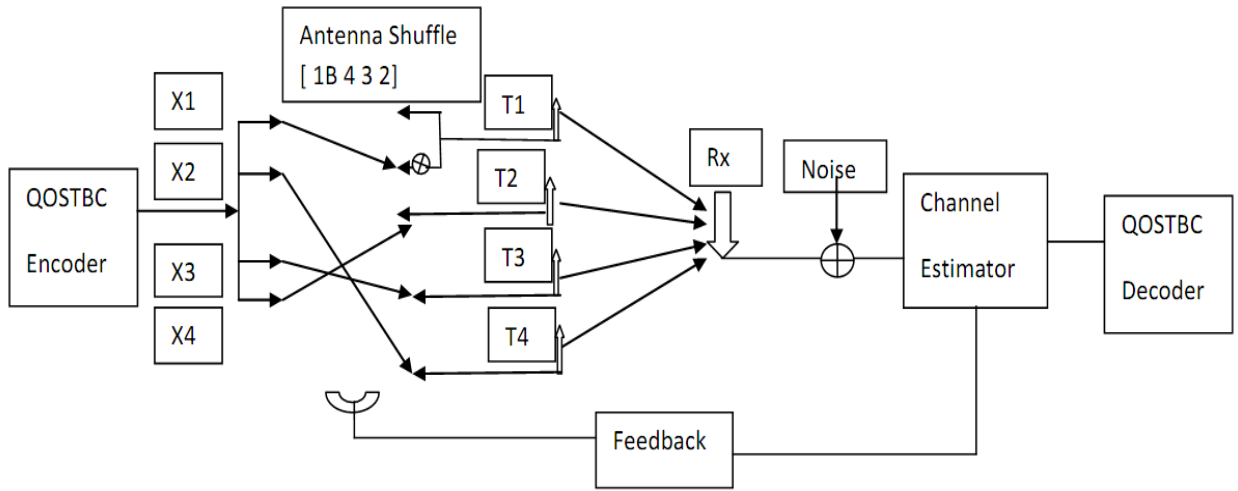


Fig.5.1: Closed loop QO-STBC scheme.

For having different value of b , possible number of antenna shuffling patterns for the proposed QO-STBC code is six, which are shown in Table 5.1.

Table 5.1: Six different shuffling having different value of b

Index	Six Different Shuffling	Value of b
1	[1A,2,3,4]	$b_1 = 2\text{real}(h_1h_4^* - h_2h_3^*)$
2	[1A,2,4,3]	$b_2 = 2\text{real}(h_1h_3^* - h_2h_4^*)$
3	[1A,4,3,2]	$b_3 = 2\text{real}(h_1h_2^* - h_3h_4^*)$
4	[1B,2,3,4]	$b_4 = -2\text{real}(h_1h_4^* + h_2h_3^*)$
5	[1B,2,4,3]	$b_5 = -2\text{real}(h_1h_3^* + h_2h_4^*)$
6	[1B,4,3,2]	$b_6 = -2\text{real}(h_1h_2^* + h_3h_4^*)$

From Table 5.1 pattern [1A,2,4,3] means the four rows of the QO-STBC will be transmitted from antenna 1,2,4 and 3 respectively and pattern [1B,3,4,2] representing that the four rows of the QO-STBC are transmitted from antenna 1(with 180° phase shift before transmission), 3, 4 and 2 respectively. Six different codes corresponding to six different types of transmit antenna shuffling {codes from [i] to [vi] corresponding to different shuffle having index (1) to (6) respectively} is given by:

$$\begin{aligned}
 \text{[i]} & \begin{bmatrix} S_1 & S_2 & S_3 & S_4 \\ -S_2^* & S_1^* & -S_4^* & S_3^* \\ -S_3^* & -S_4^* & S_1^* & S_2^* \\ S_4 & -S_3 & -S_2 & S_1 \end{bmatrix} & \text{[iv]} & \begin{bmatrix} -S_1 & S_2 & S_3 & S_4 \\ S_2^* & S_1^* & -S_4^* & S_3^* \\ S_3^* & -S_4^* & S_1^* & S_2^* \\ -S_4 & -S_3 & -S_2 & S_1 \end{bmatrix} \\
 \text{[ii]} & \begin{bmatrix} S_1 & S_2 & S_4 & S_3 \\ -S_2^* & S_1^* & S_3^* & -S_4^* \\ -S_3^* & -S_4^* & S_2^* & S_1^* \\ S_4 & -S_3 & S_1 & -S_2 \end{bmatrix} & \text{[v]} & \begin{bmatrix} -S_1 & S_2 & S_4 & S_3 \\ S_2^* & S_1^* & S_3^* & -S_4^* \\ S_3^* & -S_4^* & S_2^* & S_1^* \\ -S_4 & -S_3 & S_1 & -S_2 \end{bmatrix} \\
 \text{[iii]} & \begin{bmatrix} S_1 & S_4 & S_3 & S_2 \\ -S_2^* & S_3^* & -S_4^* & S_1^* \\ -S_3^* & S_2^* & S_1^* & -S_4^* \\ S_4 & S_1 & -S_2 & -S_3 \end{bmatrix} & \text{[vi]} & \begin{bmatrix} -S_1 & S_4 & S_3 & S_2 \\ S_2^* & S_3^* & -S_4^* & S_1^* \\ S_3^* & S_2^* & S_1^* & -S_4^* \\ -S_4 & S_1 & -S_2 & -S_3 \end{bmatrix}
 \end{aligned}$$

Adaptive antenna shuffling algorithm

It is assumed that the CSI information is perfectly known at the receiver and both transmitter & receiver has knowledge of different possible shuffling of the transmit antennas at different channel conditions. Each antenna shuffling is associated with a particular value of channel dependent parameter e.g. b . The association of the channel dependent parameter and antenna shuffling is given in Table 5.1. The steps of the algorithm are:

- Calculate the values of the channel dependent parameter b at current channel condition using the channel coefficients. The values of b are calculated according to the groupings of channel coefficients as given in Table 5.1.
- Select the index of the grouping providing $\min |b|$ from all calculated $|b|$ values for transmission to the transmitter as feedback.
- The transmitter selects the antenna shuffling associated with the minimum value of the b

- The transmitter shuffles the antennas according to the selected antenna shuffling based on minimum value of $|b|$.
- At receiver, multiply the received signal by the hermitian of channel matrix of transmitted shuffled code so that we decode symbol using simple linear decoding.

Since, six different type of shuffling is used, so three bit feedback is used to tell transmitter about which shuffling undergoes during transmission having minimum value of $|b|$. This in turns increase the performance of conventional QO-STBC code.

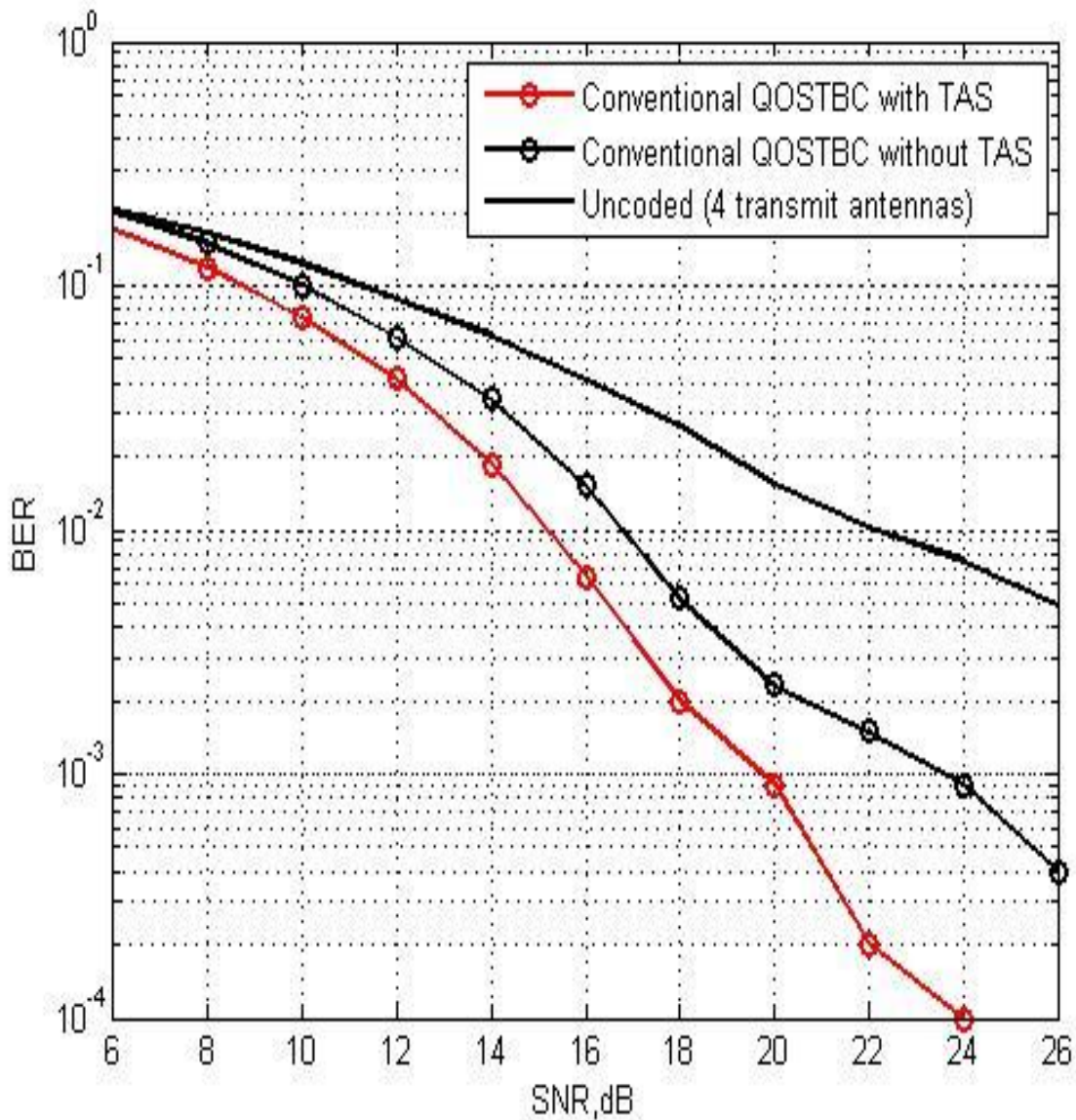


Fig.5.2: Performance of the conventional QO-STBC using TAS.

5.3 Transmit Antenna Shuffling for Proposed QO-STBC Scheme

In this section, transmit antenna shuffling is applied in proposed QO-STBC code for four transmit antennas so that optimum antenna shuffling pattern can be selected to improve the transmit diversity. Detection matrix for proposed QO-STBC scheme for four transmit antenna and one receive antenna, is given by:

$$D = H^H H = \begin{bmatrix} a+b & 0 & 0 & 0 \\ 0 & a+b & 0 & 0 \\ 0 & 0 & a-b & 0 \\ 0 & 0 & 0 & a-b \end{bmatrix} \quad (5.7)$$

where $(a+b)$, $(a+b)$, $(a-b)$ and $(a-b)$ are total channel gains for four transmit antenna respectively. Received signal is given by:

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} (s_1 + s_2 + s_3 - s_4)/2 & (s_1 - s_2 + s_3 + s_4)/2 & (-s_1 + s_2 + s_3 + s_4)/2 & (s_1 + s_2 - s_3 + s_4)/2 \\ (-s_1^* + s_2^* - s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (-s_1^* + s_2^* + s_3^* + s_4^*)/2 \\ (s_1^* - s_2^* - s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (s_1^* - s_2^* + s_3^* + s_4^*)/2 \\ (s_1 + s_2 - s_3 + s_4)/2 & (s_1 - s_2 - s_3 - s_4)/2 & (-s_1 + s_2 - s_3 - s_4)/2 & (s_1 + s_2 + s_3 - s_4)/2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} \quad (5.8)$$

After applying a complex conjugate operation to the second and the third elements of the received signal, modified received signal is then given by:

$$\begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \begin{bmatrix} (h_1 + h_2 - h_3 + h_4)/2 & (h_1 - h_2 + h_3 + h_4)/2 & (h_1 + h_2 + h_3 - h_4)/2 & (-h_1 + h_2 + h_3 + h_4)/2 \\ (h_2^* - h_1^* - h_4^* - h_3^*)/2 & (h_2^* + h_1^* + h_4^* - h_3^*)/2 & (h_2^* - h_1^* + h_4^* + h_3^*)/2 & (-h_2^* - h_1^* + h_4^* - h_3^*)/2 \\ (h_3^* + h_4^* + h_1^* - h_2^*)/2 & (h_3^* - h_4^* - h_1^* - h_2^*)/2 & (h_3^* + h_4^* - h_1^* + h_2^*)/2 & (-h_3^* + h_4^* - h_1^* - h_2^*)/2 \\ (h_4 - h_3 + h_2 + h_1)/2 & (h_4 + h_3 - h_2 + h_1)/2 & (h_4 - h_3 - h_2 - h_1)/2 & (-h_4 - h_3 - h_2 + h_1)/2 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \\ n_3^* \\ n_4 \end{bmatrix} \quad (5.9)$$

or, in matrix notation, rewrite above equation as: $y = \begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = Hs + \bar{n}$.

For decoding this signal, above equation is multiplied by H^H to get estimated transmitted signal \tilde{s} which is given by:

$$\tilde{s} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \end{bmatrix} = H^H \begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \begin{bmatrix} a+b & 0 & 0 & 0 \\ 0 & a+b & 0 & 0 \\ 0 & 0 & a-b & 0 \\ 0 & 0 & 0 & a-b \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + H^H \begin{bmatrix} n_1 \\ n_2^* \\ n_3^* \\ n_4 \end{bmatrix} \quad (5.10)$$

$$\tilde{s} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \end{bmatrix} = \begin{bmatrix} (a+b)s_1 \\ (a+b)s_2 \\ (a-b)s_3 \\ (a-b)s_4 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{n}_3 \\ \tilde{n}_4 \end{bmatrix} \quad (5.11)$$

where \tilde{n}_i is noise component after multiplication by H^H . For error free decoding, these channel gains $(a+b)$ and $(a-b)$ must be more positive which are strongly dependent on channel coefficients. As channel gain is described by two factors a and b , where a is always positive quantity but b can be positive large or negative large quantity depending on the channel condition, which in turn decrease the positivity of half of the channel gains. So, for all channel gains, to have consistency in its positivity, value of b should be minimized. It is known that by using shuffling in transmit antenna, six different shuffled codes is possible for which detection matrices are same but having different values of channel gain because of different value of b which are same as given in Table 5.1. So, at the transmitter end, that shuffled code is used for which $|b|$ has minimum value. For this, similar antenna shuffling algorithm is used which is discussed earlier in case of transmit antenna shuffling for conventional QO-STBC. Here, also six different type of shuffling is used, so three bit feedback is used to tell transmitter about which shuffling undergoes during transmission having minimum value of $|b|$. This in turns increase the performance of proposed QO-STBC scheme. Six different codes corresponding to six different types of transmit antenna shuffling {codes from [i] to [vi] corresponding to different shuffle having index (1) to (6) respectively} is given by:

[i]

$$\begin{bmatrix} (s_1 + s_2 + s_3 - s_4)/2 & (s_1 - s_2 + s_3 + s_4)/2 & (-s_1 + s_2 + s_3 + s_4)/2 & (s_1 + s_2 - s_3 + s_4)/2 \\ (-s_1^* + s_2^* - s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (-s_1^* + s_2^* + s_3^* + s_4^*)/2 \\ (s_1^* - s_2^* - s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (s_1^* - s_2^* + s_3^* + s_4^*)/2 \\ (s_1 + s_2 - s_3 + s_4)/2 & (s_1 - s_2 - s_3 - s_4)/2 & (-s_1 + s_2 - s_3 - s_4)/2 & (s_1 + s_2 + s_3 - s_4)/2 \end{bmatrix}$$

[ii]

$$\begin{bmatrix} (s_1 + s_2 + s_3 - s_4)/2 & (s_1 - s_2 + s_3 + s_4)/2 & (s_1 + s_2 - s_3 + s_4)/2 & (-s_1 + s_2 + s_3 + s_4)/2 \\ (-s_1^* + s_2^* - s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (-s_1^* + s_2^* + s_3^* + s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 \\ (s_1^* - s_2^* - s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (s_1^* - s_2^* + s_3^* + s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 \\ (s_1 + s_2 - s_3 + s_4)/2 & (s_1 - s_2 - s_3 - s_4)/2 & (s_1 + s_2 + s_3 - s_4)/2 & (-s_1 + s_2 - s_3 - s_4)/2 \end{bmatrix}$$

[iii]

$$\begin{bmatrix} (s_1 + s_2 + s_3 - s_4)/2 & (s_1 + s_2 - s_3 + s_4)/2 & (-s_1 + s_2 + s_3 + s_4)/2 & (s_1 - s_2 + s_3 + s_4)/2 \\ (-s_1^* + s_2^* - s_3^* - s_4^*)/2 & (-s_1^* + s_2^* + s_3^* + s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 \\ (s_1^* - s_2^* - s_3^* - s_4^*)/2 & (s_1^* - s_2^* + s_3^* + s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 \\ (s_1 + s_2 - s_3 + s_4)/2 & (s_1 + s_2 + s_3 - s_4)/2 & (-s_1 + s_2 - s_3 - s_4)/2 & (s_1 - s_2 - s_3 - s_4)/2 \end{bmatrix}$$

[iv]

$$\begin{bmatrix} -(s_1 + s_2 + s_3 - s_4)/2 & (s_1 - s_2 + s_3 + s_4)/2 & (-s_1 + s_2 + s_3 + s_4)/2 & (s_1 + s_2 - s_3 + s_4)/2 \\ -(-s_1^* + s_2^* - s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (-s_1^* + s_2^* + s_3^* + s_4^*)/2 \\ -(s_1^* - s_2^* - s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (s_1^* - s_2^* + s_3^* + s_4^*)/2 \\ -(s_1 + s_2 - s_3 + s_4)/2 & (s_1 - s_2 - s_3 - s_4)/2 & (-s_1 + s_2 - s_3 - s_4)/2 & (s_1 + s_2 + s_3 - s_4)/2 \end{bmatrix}$$

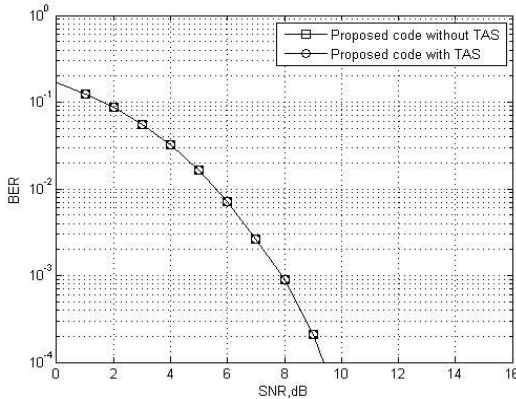
[v]

$$\begin{bmatrix} -(s_1 + s_2 + s_3 - s_4)/2 & (s_1 - s_2 + s_3 + s_4)/2 & (s_1 + s_2 - s_3 + s_4)/2 & (-s_1 + s_2 + s_3 + s_4)/2 \\ -(-s_1^* + s_2^* - s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (-s_1^* + s_2^* + s_3^* + s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 \\ -(s_1^* - s_2^* - s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (s_1^* - s_2^* + s_3^* + s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 \\ -(s_1 + s_2 - s_3 + s_4)/2 & (s_1 - s_2 - s_3 - s_4)/2 & (s_1 + s_2 + s_3 - s_4)/2 & (-s_1 + s_2 - s_3 - s_4)/2 \end{bmatrix}$$

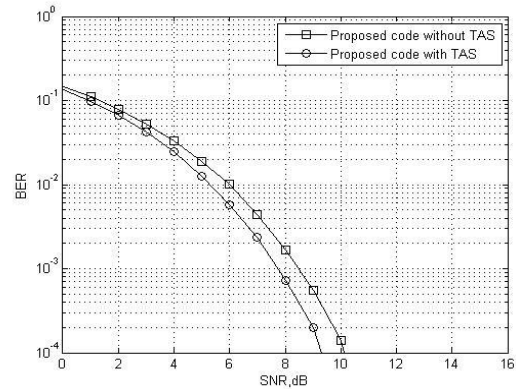
[vi]

$$\begin{bmatrix} -(s_1 + s_2 + s_3 - s_4)/2 & (s_1 + s_2 - s_3 + s_4)/2 & (-s_1 + s_2 + s_3 + s_4)/2 & (s_1 - s_2 + s_3 + s_4)/2 \\ -(-s_1^* + s_2^* - s_3^* - s_4^*)/2 & (-s_1^* + s_2^* + s_3^* + s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 \\ -(s_1^* - s_2^* - s_3^* - s_4^*)/2 & (s_1^* - s_2^* + s_3^* + s_4^*)/2 & (s_1^* + s_2^* + s_3^* - s_4^*)/2 & (-s_1^* - s_2^* + s_3^* - s_4^*)/2 \\ -(s_1 + s_2 - s_3 + s_4)/2 & (s_1 + s_2 + s_3 - s_4)/2 & (-s_1 + s_2 - s_3 - s_4)/2 & (s_1 - s_2 - s_3 - s_4)/2 \end{bmatrix}$$

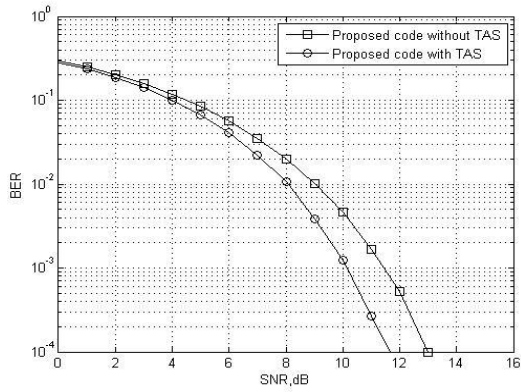
Figure 5.3 shows the bit error rate performance of different QO-STBC under their optimum channel condition. In this case, it is assumed channel is static which means that it does not change during the whole data transmission and QPSK modulation is used. As it is closed loop system, three bit feedback is used to tell the transmitter which index of b has minimum value. Corresponding to the index, the shuffled code is transmitted. For example, when $|b_2|$ is minimum, shuffled code [ii] is selected; when $|b_3|$ is minimum, shuffled code [iii] is selected and similarly for others. So, in Fig.5.3, figure (a) is for code [i] when $|b_1|$ is minimum; figure (b) is for code [ii] when $|b_2|$ is minimum and similarly for others.



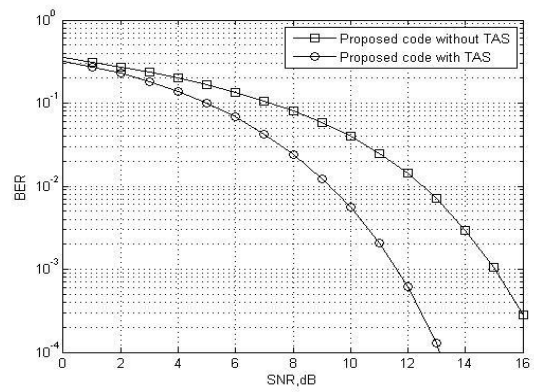
(a) For code [i], when b_1 is minimum.



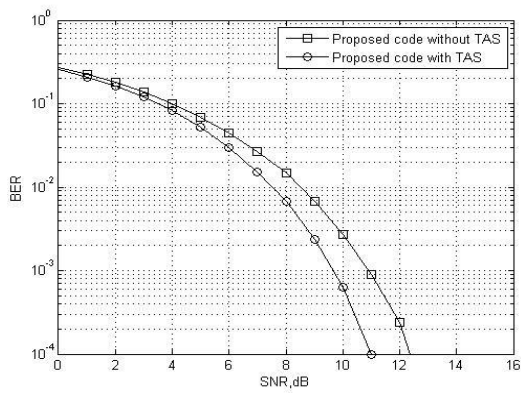
(b) For code [ii], when b_2 is minimum.



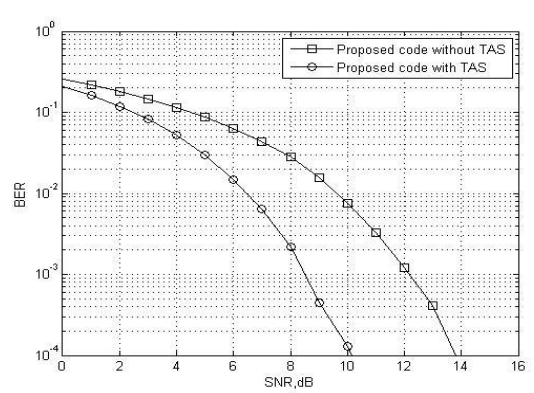
(c) For code [iii], when b_3 is minimum.



(d) For code [iv], when b_4 is minimum.



(e) For code [v], when b_5 is minimum.



(f) For code [vi], when b_6 is minimum.

Fig.5.3: (a), (b), (c), (d), (e) and (f) shows the performance of different shuffled proposed codes under optimum channel conditions respectively.

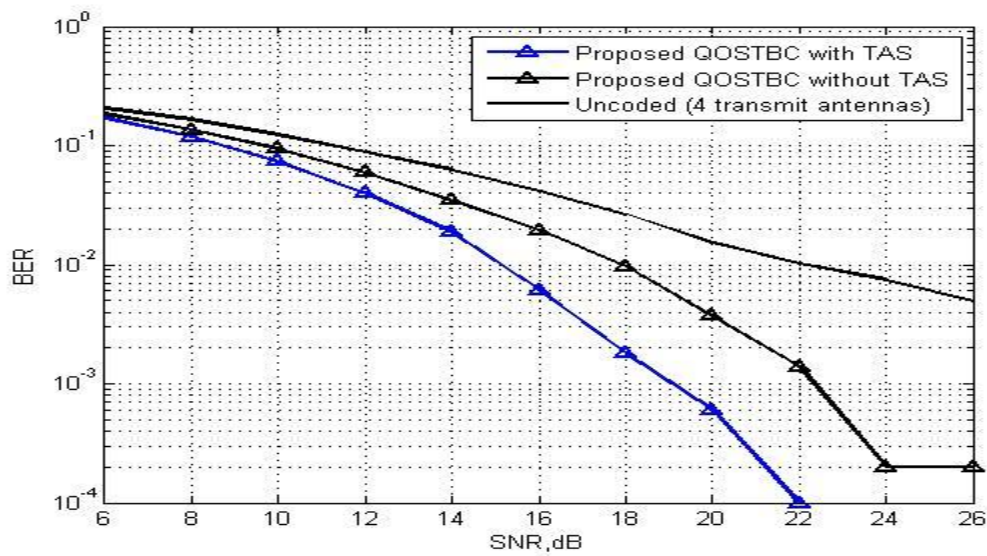


Fig.5.4: Performance of proposed QO-STBC using TAS.

5.4 Comparison of Conventional QO-STBC Scheme and Proposed QO-STBC Scheme using TAS

Estimated transmitted signal for conventional QO-STBC scheme is given by:

$$\begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \end{bmatrix} = \begin{bmatrix} as_1 + bs_4 \\ as_2 - bs_3 \\ as_3 - bs_2 \\ as_4 + bs_1 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{n}_3 \\ \tilde{n}_4 \end{bmatrix} \quad (5.12)$$

Estimated transmitted signal for proposed QO-STBC scheme is given by:

$$\begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \end{bmatrix} = \begin{bmatrix} (a+b)s_1 \\ (a+b)s_2 \\ (a-b)s_3 \\ (a-b)s_4 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \tilde{n}_3 \\ \tilde{n}_4 \end{bmatrix} \quad (5.13)$$

where \tilde{s}_i is estimated symbol ; s_i is transmitted symbol and \tilde{n}_i is noise component

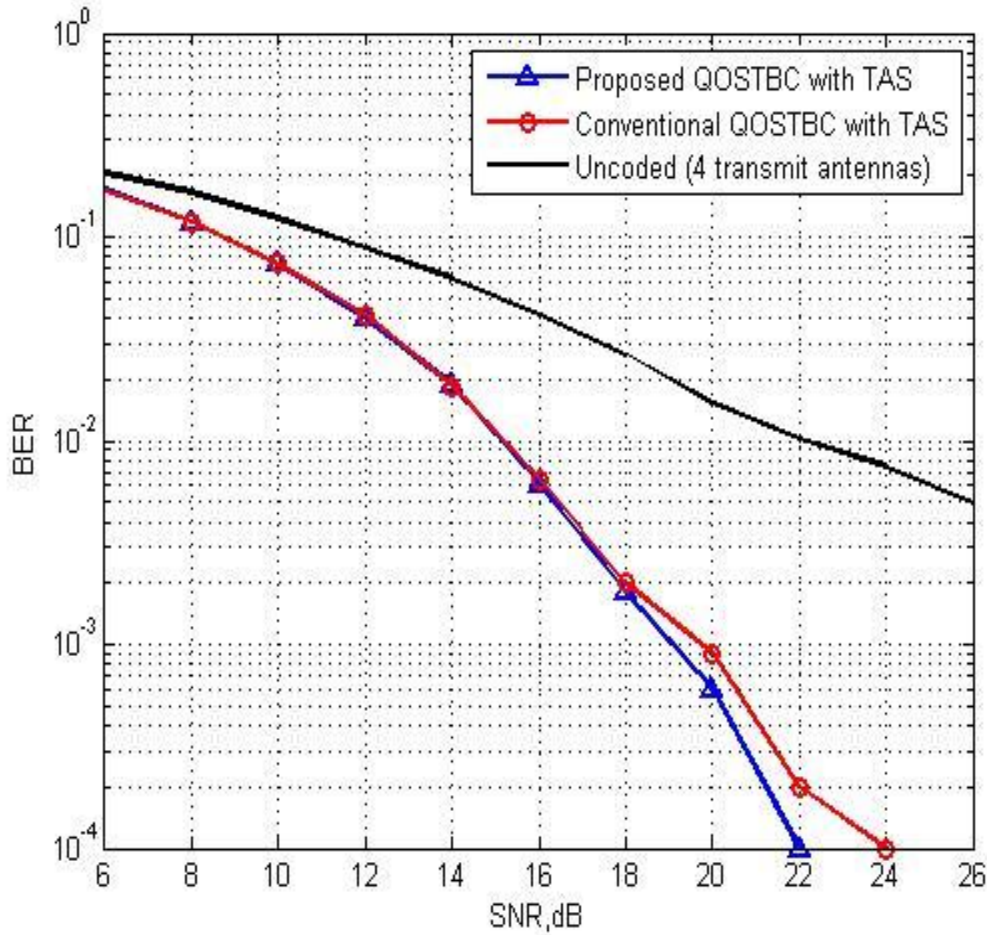


Fig.5.5: Comparison of performance of proposed QO-STBC and conventional QO-STBC using TAS.

By using transmit antenna shuffling scheme, minimum value of parameter b is obtained in both the cases, which is the main reason behind the increase in performance of both schemes. So, the value of parameter b should be equal to zero. But in practical, value of b is not always equal or nearly equal to zero. In conventional QO-STBC, for example, if this condition occurs, $(as_1 + bs_4)$ is not nearly equal to s_1 , which means simple linear decoding is not possible and degradation in the performance takes place. But on the other hand, in proposed QO-STBC code, for $(a + b)s_1$, increase or decrease in value of b , is added to s_1 itself so that simple linear decoding can be used which increase the performance of proposed code.

6.1 Conclusion

It is well-known that QO-STBC is developed from quasi-orthogonal designs, where the orthogonality is relaxed to provide higher code rate. QO-STBC allows a trade-off between higher rate and decoding complexity unlike in STBC where simple linear decoding is used. Basically two types of decoder are used for decoding symbol in QO-STBC, one is maximum likelihood decoding and other is zero forcing decoding. In maximum likelihood decoding, decoder of QO-STBC processes pairs of transmitted symbols instead of a single symbol. Two maximum likelihood detectors are used in parallel to decode pairs of transmitted symbols in QO-STBC. The decision metric of QO-STBC is the sum of two terms; thus minimizing the decision metric is equivalent to minimizing two terms independently. Two maximum likelihood detectors are used either in sequence or in parallel. Therefore, decoding pairs for QO-STBC is more complex than decoding single symbols for space–time block codes. This results in higher complexity decoding at the receiver. Specifically, the decoding complexity increases with the modulation level. In order to minimize the decision metric using maximum likelihood method, the receiver computes the decision metric over all possible symbols of a constellation or modulation level and decides in favor of the constellation symbols that minimize the decision metric. As the size of the constellation increases, the receiver must minimize the decision metric over large number of symbols. This will, subsequently, increase the transmission delay when high modulation schemes or more antennas are employed. For low modulation level, maximum likelihood decoding is good choice but practically we have to use high modulation for achieving high data rate. On the other hand, by using zero forcing decoding there is no problem of transmission delay with increase in modulation level, so it is used more often. But due to the process of inverse of the matrix use in this type of decoding, the technique has the high decoding complexity. So, there is need to search new code for which decoding complexity is minimized with least transmission delay.

In this work, new QO-STBC is proposed whose decoding complexity is very much lower than decoding complexity of above mentioned two decoding schemes. New QO-STBC is developed by using the symmetry property of the detection matrix of Jafarkhani code. The new encoding matrix S_n is Quasi-Orthogonal but its equivalent virtual channel matrix H_n is

Orthogonal matrix, so that simple linear decoding is possible for this code which in turns reduces transmission delay. Hence, decoding complexity of proposed code is much lower than conventional QO-STBC without losing BER performance. Further, transmission strategy can be improved when CSI must be known to the transmitter. For this closed loop QO-STBC is employed using transmit antenna shuffling (TAS) for both proposed QO-STBC and conventional QO-STBC. It is clear that at bit error probability of 10^{-4} , the proposed QO-STBC with TAS provides about 2dB power gain over the conventional QO-STBC. So, by using transmit antenna shuffling technique, proposed code has minimum decoding complexity and can be achieved better BER performance than any other QO-STBC schemes.

6.2 Future Scope

The new QO-STBC has been proposed which can be a good candidate for future wireless communication systems. The proposed QO-STBCs provide high transmission rate with simple decoding algorithms and minimum transmission delay. The application of this new code could well be used in future Long term evolution (LTE) systems so that burden at the receiver reduces. Further improvements in code rate of QO-STBC, is another parameter for research by optimizing structure of QO-STBC. There is also chance to explore structure of QO-STBC using Clifford algebra, which is also an area of further research.

Appendix A

Singular value decomposition is based on linear algebra which says that a rectangular matrix D can be broken down into the product of three matrices, orthogonal matrix U , a diagonal matrix S and the transpose of an orthogonal matrix W . The theorem is usually presented like this:

$$D=USW^T \quad (\text{A.1})$$

where $UU^T=I$, $WW^T=I$; the columns of U are orthogonal eigenvectors of DD^T , the columns of W are orthogonal eigen vectors of D^TD , and S is a diagonal matrix containing the square roots of eigen values from U or W in descending order. Start with the matrix

$$D=\begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} \quad (\text{A.2})$$

In order to find U , start with DD^T

$$\begin{aligned} DD^T &= \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} \\ &= \begin{bmatrix} a^2 + b^2 & 0 & 0 & 2ab \\ 0 & a^2 + b^2 & -2ab & 0 \\ 0 & -2ab & a^2 + b^2 & 0 \\ 2ab & 0 & 0 & a^2 + b^2 \end{bmatrix} \end{aligned} \quad (\text{A.3})$$

Now, find the eigen values and corresponding eigenvectors of DD^T . It is known that eigen vectors are defined by the equation $A\vec{v}=\lambda\vec{v}$, applying this to DD^T gives us:

$$\begin{bmatrix} a^2 + b^2 & 0 & 0 & 2ab \\ 0 & a^2 + b^2 & -2ab & 0 \\ 0 & -2ab & a^2 + b^2 & 0 \\ 2ab & 0 & 0 & a^2 + b^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (\text{A.4})$$

Rewrite above matrix as the set of equations

$$(a^2 + b^2 - \lambda) x_1 + (2ab) x_4 = 0 \quad (\text{A.5})$$

$$(a^2 + b^2 - \lambda) x_2 + (-2ab) x_3 = 0 \quad (\text{A.6})$$

$$(-2ab) x_2 + (a^2 + b^2 - \lambda) x_3 = 0 \quad (\text{A.7})$$

$$(2ab) x_1 + (a^2 + b^2 - \lambda) x_4 = 0 \quad (\text{A.8})$$

which are solved by setting determinant of coefficient matrix to zero,

$$\begin{bmatrix} a^2 + b^2 - \lambda & 0 & 0 & 2ab \\ 0 & a^2 + b^2 - \lambda & -2ab & 0 \\ 0 & -2ab & a^2 + b^2 - \lambda & 0 \\ 2ab & 0 & 0 & a^2 + b^2 - \lambda \end{bmatrix} = 0 \quad (\text{A.9})$$

which works out as

$$\Rightarrow (a^2 + b^2 - \lambda) [(a^2 + b^2 - \lambda)^3 - 4(a^2 + b^2 - \lambda)a^2b^2 - 2ab [2ab (a^2 + b^2 - \lambda)^2 - 2ab(4a^2b^2)]] = 0 \quad (\text{A.10})$$

$$\Rightarrow (a^2 + b^2 - \lambda)^4 - 8(a^2 + b^2 - \lambda)^2 a^2b^2 + 16 a^4b^4 = 0 \quad (\text{A.11})$$

gives us eigen values $\lambda = (a+b)^2$; $\lambda = (a+b)^2$; $\lambda = (a-b)^2$ and $\lambda = (a-b)^2$. Plugging λ back to the original equations gives us eigen vectors.

For $\lambda = (a+b)^2$,

$$(a^2 + b^2 - (a+b)^2) x_1 + (2ab) x_4 = 0 \quad \text{or} \quad (-2ab) x_1 + (2ab) x_4 = 0 \quad (\text{A.12})$$

$$(a^2 + b^2 - (a+b)^2) x_2 + (-2ab) x_3 = 0 \quad \text{or} \quad (-2ab) x_2 + (-2ab) x_3 = 0 \quad (\text{A.13})$$

$$(-2ab) x_2 + (a^2 + b^2 - (a+b)^2) x_3 = 0 \quad \text{or} \quad (-2ab) x_2 + (-2ab) x_3 = 0 \quad (\text{A.14})$$

$$(2ab) x_1 + (a^2 + b^2 - (a+b)^2) x_4 = 0 \quad \text{or} \quad (2ab) x_1 + (-2ab) x_4 = 0 \quad (\text{A.15})$$

Solving these equation we get, $x_1 = x_4$ and $x_2 = -x_3$. Thus, corresponding to eigen value $\lambda = (a+b)^2$, eigen vectors are $[1 \ 1 \ -1 \ 1]$ and $[1 \ -1 \ 1 \ 1]$.

For $\lambda = (a-b)^2$,

$$(a^2 + b^2 - (a-b)^2) x_1 + (2ab) x_4 = 0 \quad \text{or} \quad (2ab) x_1 + (2ab) x_4 = 0 \quad (\text{A.16})$$

$$(a^2 + b^2 - (a-b)^2) x_2 + (-2ab) x_3 = 0 \quad \text{or} \quad (2ab) x_2 + (-2ab) x_3 = 0 \quad (\text{A.17})$$

$$(-2ab) x_2 + (a^2 + b^2 - (a-b)^2) x_3 = 0 \quad \text{or} \quad (-2ab) x_2 + (2ab) x_3 = 0 \quad (\text{A.18})$$

$$(2ab) x_1 + (a^2 + b^2 - (a-b)^2) x_4 = 0 \quad \text{or} \quad (2ab) x_1 + (2ab) x_4 = 0 \quad (\text{A.19})$$

Solving these equation we get, $x_1 = -x_4$ and $x_2 = x_3$. Thus, corresponding to eigen value $\lambda = (a-b)^2$, eigen vectors are $[1 \ 1 \ 1 \ -1]$ and $[-1 \ 1 \ 1 \ 1]$.

These eigen vectors become column vectors in a matrix ordered by the size of the corresponding eigen value. In the matrix below, the eigen vectors for $\lambda = (a+b)^2$ are in column one and two, and , the eigen vectors for $\lambda = (a-b)^2$ are in column three and four.

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{bmatrix} \quad (\text{A.20})$$

Finally, by applying Gram-Schmidt orthonormalization process to the column vector, above matrix is converted into an orthogonal matrix. Begin by normalizing

For $\vec{v}_1 = [1 \ 1 \ -1 \ 1]$

$$\begin{aligned} \vec{u}_1 &= \vec{v}_1 / |\vec{v}_1| = [1 \ 1 \ -1 \ 1] / \sqrt{1^2 + 1^2 + (-1)^2 + 1^2} = [1 \ 1 \ -1 \ 1] / 2 \\ &= [0.5 \ 0.5 \ -0.5 \ 0.5] \end{aligned} \quad (\text{A.21})$$

For $\vec{v}_2 = [1 \ -1 \ 1 \ 1]$

$$\begin{aligned} \vec{w}_2 &= \vec{v}_2 - \vec{u}_1 \cdot \vec{v}_2 * \vec{u}_1 \\ \vec{u}_2 &= \vec{w}_2 / |\vec{w}_2| = [1 \ -1 \ 1 \ 1] / \sqrt{1^2 + (-1)^2 + 1^2 + 1^2} = [1 \ -1 \ 1 \ 1] / 2 \\ &= [0.5 \ -0.5 \ 0.5 \ 0.5] \end{aligned} \quad (\text{A.22})$$

For $\vec{v}_3 = [1 \ 1 \ 1 \ -1]$

$$\begin{aligned} \vec{w}_3 &= \vec{v}_3 - \vec{u}_1 \cdot \vec{v}_3 * \vec{u}_1 - \vec{u}_2 \cdot \vec{v}_3 * \vec{u}_2 \\ \vec{u}_3 &= \vec{w}_3 / |\vec{w}_3| = [1 \ 1 \ 1 \ -1] / \sqrt{1^2 + 1^2 + 1^2 + (-1)^2} = [1 \ 1 \ 1 \ -1] / 2 \\ &= [0.5 \ 0.5 \ 0.5 \ -0.5] \end{aligned} \quad (\text{A.23})$$

For $\vec{v}_4 = [-1 \ 1 \ 1 \ 1]$

$$\begin{aligned} \vec{w}_4 &= \vec{v}_4 - \vec{u}_1 \cdot \vec{v}_4 * \vec{u}_1 - \vec{u}_2 \cdot \vec{v}_4 * \vec{u}_2 - \vec{u}_3 \cdot \vec{v}_4 * \vec{u}_3 \\ \vec{u}_4 &= \vec{w}_4 / |\vec{w}_4| = [-1 \ 1 \ 1 \ 1] / \sqrt{(-1)^2 + 1^2 + 1^2 + 1^2} = [-1 \ 1 \ 1 \ 1] / 2 \\ &= [-0.5 \ 0.5 \ 0.5 \ 0.5] \end{aligned} \quad (\text{A.24})$$

All these \vec{u} 's to give an orthogonal matrix U which is given by

$$\begin{bmatrix} 0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix} \quad (\text{A.25})$$

In order to find W , starts with $D^T D$. As D is the symmetric matrix, so $D^T D = DD^T$. This means that value of W will be equal to the value of U which is derived by DD^T . So, W^T is given by

$$\begin{bmatrix} 0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix} \quad (\text{A.26})$$

For finding S , take square roots of the non-zero eigen values and populate the diagonal with them, putting largest first. So, S is given by:

$$\begin{bmatrix} a+b & 0 & 0 & 0 \\ 0 & a+b & 0 & 0 \\ 0 & 0 & a-b & 0 \\ 0 & 0 & 0 & a-b \end{bmatrix} \quad (\text{A.27})$$

So, $D = USW^T$ is given by:

$D =$

$$\begin{bmatrix} 0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} a+b & 0 & 0 & 0 \\ 0 & a+b & 0 & 0 \\ 0 & 0 & a-b & 0 \\ 0 & 0 & 0 & a-b \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix} \quad (\text{A.28})$$

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