ALGORITHMIC APPROACH TO STUDY EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS

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Hamit Kumar

Roll no:-300903004

Under the guidance of

Dr. A.K. Lal

Associate Professor

School of Mathematics and Computer Applications
Thapar University, Patiala-147004(PUNJAB), INDIA

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Dedicated to My Divine Brothers, Parents and God

CERTIFICATE

This is to certify that the thesis "**ALGORITHMIC APPROACH TO STUDY EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS**" submitted by Mr. Hamit Kumar of M.Sc (Mathematics and Computing), Thapar University, Patiala, was carried out by me under supervisions of Dr. A.K. Lal. He has not submitted this material for credit towards any other degree at Thapar University Patiala or any other University.

Hamit Kumar
(Hamit Kumar)

This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

Dr. A.K. Lal
Supervisor
School of Mathematics and Computer Applications
Thapar University, Patiala

Countersigned by:

Dr. S.S Bhatia
Professor and Head 15/7/11
School of Mathematics and Computer Application
Thapar University, Patiala

Dr. S. K. Mohapatra
Dean of Academic Affairs
Thapar University, Patiala

ABSTRACT

In the present thesis, we have proposed a algorithmic approach for Mohan,Saxena and Aggarwal method to determine the effect of rotation and tidal distortion in the equilibrium structure of polytropic models of stars. Although efforts have been made to develop analytic expression in series form whose closed form solutions have not possible. Even these series solutions do not provide any analytic expression which could provide some result of practical significance. It may, therefore, be of interest to see if instead of developing detailed series expansion of distortional parameters r_ψ, u, v, w, f_p, and f_T etc. required in determining the equilibrium structure and periods of oscillations of rotationally and tidally distorted stars. In fact, this does not seem to be difficult proposition in view of the availability of fast computing machines. Thus, there is need of developing an efficient computational method which directly evaluates these distortional parameters without using their explicit expressions in the form of series available in the literature..

The thesis consists of three chapters. Chapter-I is introductory in nature. We briefly discuss the astrophysical significance of the problem of determining the equilibrium structures of rotationally and tidally distorted stellar models. A brief survey of the literature available on this subject and summary of the work presented in succeeding chapters of this thesis also appears in this chapter.

In Chapter-II we first present suitable Gauss quadrature formula for evaluating double integral having singularity at the end of the intervals and how it has been used to evaluate double integral appearing in most of the mathematical expressions for distortional and structural parameters of rotationally and tidally distorted stellar models. An algorithm to use Mohan, Saxena and Agarwal (62,63) approach has been formalized in this chapter to determine the effects of rotation and tidal forces on the equilibrium structure without using the explicit expressions for the distortional parameters available in the literature.

The algorithm developed in Chapter-II has next been used in Chapter-III to determine the equilibrium structure of rotationally and tidally distorted polytropic models of stars. The equilibrium structure and other physical parameters of various rotationally and tidally distorted polytropic models with polytropic indices 1.5, 3.0 and 4.0 have been computed. The results thus obtained have been compared with the results earlier obtained by Mohan and Saxena (57,58)

for the same models using the explicit expressions for the distortional parameters r_ψ, u, v, w, f_p, and f_T. Certain conclusions based on this study have also been drawn. The astrophysical importance of the present work as well as the limitation of the scope of the present work is finally discussed in this chapter.

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Dated: 15-7-11

Hamit kumar
(Hamit Kumar)

CONTENTS

CHAPTER - I

INTRODUCTION

This chapter is introductory in nature. In section 1.1 we first explain in brief the astrophysical significance of the theoretical study of the problem of determining the effects of rotation and/or tidal distortions on the equilibrium structures of gaseous spheres. A brief survey of the literature available on the subject is presented in Section 1.2. In section1.3 we present how Kippenhahn and Thomas (30) averaging technique to derive system of differential equations which determine the equilibrium structures of rotationally and tidally distorted gaseous spheres. The concepts of Roche equipotentials and Roche limits are then introduced in section 1.4. In section 1.5 we present how Mohan et al (62, 63) used Kippenhahn and Thomas (30) approach in conjunction with certain result on Roche equipotentials to obtain the system of differential equation governing the equilibrium structures of rotationally and tidally distorted stellar models .A brief summary of the work presented in the succeeding chapters of this thesis is finally presented in section 1.6.

1.1 SIGNIFICANCE OF DETERMINING THE EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED GASEOUS SPHERES

The theoretical model of a star is essentially a self gravitating gaseous sphere in hydrostatic and thermal equilibrium. Theoretical studies of the problems of the equilibrium structure of gaseous spheres have often been carried out to understand the nature of the internal structures, responsible for various observed phenomena of the stars. Whereas some of the stars are observed as single stars others are observed in groups of two or more stars. Observations also indicate that some of the stars are rotating about their axes of rotation. Many of the stars in binary and multiple systems are also known to be rotating about their axes as well as revolving around each other. Thus, if we assume the equilibrium model of a single nonrotating star as a

gaseous sphere, the equilibrium model of a rotating star will be rotationally distorted gaseous sphere. Similarly, the equilibrium model of a star appearing in a binary or a multiple system, then such a model will be a tidally distorted gaseous sphere if it is not rotating and a rotationally and tidally distorted gaseous sphere if the star is rotating as well.

Analytic studies of the problems of rotating stars and stars in binary systems have engaged the attention of astrophysicist since long time with a view to analyze and understand the observational behavior of such stars. In a binary system of stars, the two stars normally rotate about their own axis as well as revolve about their common center of mass. In a majority of binary stars, one star called primary, is generally more massive compared to its companion star. Contact binaries also been observed in which outermost surfaces of the two stars just touch each other

Keeping this in view an attempt has been made in the present thesis to investigate certain aspects of the problems of equilibrium structures of rotationally and tidally distorted gaseous spheres which still need further investigations.

1.2 BRIEF SURVEY OF THE LITERATURE

Most of the theoretical studies about the equilibrium structures of the stars have been carried out in literature by assuming the star to be an undistorted spherical gaseous sphere. Extensive literature is now available on this subject (see for instance Chandrasekhar (9), Eddington (19), Mentzel et al. (52), Cox and Giuli (14), Kippenhahn and Weigert (31), Clement (11), Kopal (33-35), Tassoul (89), Cox (13), Bohm-Vitense et al (3).

The theoretical investigation related to the problems of determining the equilibrium structures and stability of rotating and self gravitating objects, possibly begun with the work of Newton. He was the first to realize the importance of the law of gravitation for explaining the figures of celestial bodies. Later on Maclaurin, Clairaut, Laplace, Legendre, Jacobi, Poincare etc. contributed ideas, necessary for the development of the general theory of rotating bodies. Maclaurin, Jacobi, Kelvin and Jeans investigated in detail the problem of structure and stability of rotating liquid masses assuming uniform rotation.

In the year 1923, Edward Arthur Milne developed a technique for constructing the first detailed model for a slowly rotating star in pure radiative equilibrium. Later on in the year 1933, the technique of Milne was generalized and applied to slightly distorted polytropes by Chandrasekhar. The effect of uniform rotation on slow rotating Cowling star obeying simple

Kramer's opacity has been studied by Sweet and Roy (88). Much of the work on the effect of rotation on stellar interiors is summarized in the review article of Authors such as Kruszewski (38), Limber (45), Roberts (74), James (29), Hurley and Roberts (27), Roxburgh and Durney (77), Martin (50), Sackmann and Anand (79), Linnell (46,47), Endal and Sofia (22), Smith (85), Lubow (49), Kopal (34), Durney (18), , Deupree (16), Einsel and Spurzem (21), Maeder and Zahn (53), Reyniers and Smeyers (73) and Sood and Singh (86) have also investigated the problems of equilibrium structures of rotating stars. Meynet and Meader (53) studied the effects of rotation on the equilibrium structure and evolution of massive stars. Mender et al (51) investigated the theoretical models of low mass pre- main sequence rotating stars.

Equilibrium structures of stars which appear in binary and multiple systems are likely to be affected by both the rotational as well as the tidal effects of the companion stars. Attempts have been made in literature to determine the effects of rotation and tidal distortions on the equilibrium structure the stars in binary and multiple systems. In a series of papers Chandrasekhar (6,7,8) developed a first order analysis which he applied to the study of the rotational problem, the tidal problem and the binary star problem. The method, however, was found unsuitable when the separation between the components is only a few times the undisturbed radius of the primary. Monaghan (64) modified it to get more accurate results near the surface.

The method of Monaghan and Roxburgh (66) to study the structure of the primary component of a synchronous close binary was further extended by Naylor and Anand (67). Kippenhahn and Thomas (30) suggested a practical way of analyzing the effects of rotation and tidal distortions on the equilibrium structures of stars by approximating the actual equipotentials surfaces of the star by Roche equipotentials.

Chan and Chau (4) developed a method which allows an efficient and accurate investigation of the structure and evolution of a rotationally and tidally distorted star in close binary systems. Tassoul and Tassoul (91) considered the meridional circulation in rotating stars and mean steady motions in rotationally and tidally distorted stars. Later, Tassoul and Tassoul (90) extended the earlier work to study the reflection effects in close binaries when there is meridional circulation in rotating stars. Nelson et al (68) have discussed the evolution of rotationally and tidally distorted low-mass close binary systems. . Lopezorti et al (48) analyzed the equilibrium configurations of close binary systems by expanding the auto gravitational, centrifugal and tidal potentials in Clairaut coordinates. Lal et al (41) have discussed the

equilibrium structures of rotationally and tidally distorted primary component of binary stars taking into account the effect of mass variation inside the star.

Mohan and Singh (60) have used the Kippenhahn and Thomas (30) averaging technique in conjunction with certain results of Kopal (32) on Roche equipotential to study the effect of rotation and tidal distortions on the small adiabatic oscillations of stars in binary system. Sepenisky et al (82) investigated the existence and properties of equipotential surfaces and Lagrangian points in non-synchronous, eccentric binary star and planetary systems under the assumption of quasi-static equilibrium. Roche (75) studied effect of slow uniform rotation on the tidal effects in close binary system. Deupree and Karkas (17) studied the structure and evolution of close binary stars using the two-dimensional stellar structure algorithm. They have computed a series of solar composition for stellar evolution sequences of binary.

Orlov (69) have generalized the Roche model as is applied in the case of double star. In this model the point nuclei of the Roche model has been substituted by polytropic gas nuclei of finite dimensions. Plavec (71) presented tables of Roche model for the use of investigators in close binary systems. Avni and Schiller (2) studied the Roche potential systems where the stellar rotation axis is not aligned with the orbital revolution axis. Eggleton (20) computed the effective radii of Roche lobes and compared the results with the earlier results available in literature. Mochnacki (54) accurately integrated Roche model for close binary system in synchronous rotation to give volume, radii, surface area, mean gravities and mean inverse gravities in normalized form. Seidov (81) derived the exact analytical formula for the potential and mass ratio as a function of Lagrangian point's position in the classical Roche model of the close binary stars. Csataryova and Skopal (15) derive approximate analytical formulas for the basic parameters of the Roche lobe, its radius and the position of the L_1 point for asynchronously rotating component in a binary system. Lal et al (42) studied the validity of series expression being used for determining the position of a point on a Roche equipotential in case of rotating stars and stars in binary systems.

Most of the authors have studied the equilibrium structures of stars having solid body rotation. The influence of uniform rotation on the global structure of the white dwarf models has been considered by Chandrasekhar (10), Suda (87) and Lal et al. (40). The most detailed models of uniformly rotating white dwarf are due to Anand (1), Roxburgh (76),Monaghan et al (54). Some of the authors such as Ostriker and Tassoul (70),Shapiro and Teukolsky (82) have noted the stability analysis of uniformly rotating white dwarf stars. Ostriker and Bodenheimer (69),

Smart and Monaghan (84), and Blinnikov (5) extensively analysed the models of zero-temperature white dwarf in non-uniform rotation, Hachisu et al. (25) studied the fate of merging double white dwarfs and presented a numerical method, Lal el at (40) presented a method for computing equilibrium structure of differentially rotating and tidally distorted white dwarf models of stars.

Kopal (32) introduced a system of coordinates, which he called Roche coordinates, to study the problems of rotating stars and stars in binary system. Mohan and Singh (59) considered the use of Roche coordinates in solving the problems of small adiabatic oscillations of rotationally and tidally distorted stellar models. Mohan and Saxena (57,58) used the Kippenhahn and Thomas (48) averaging technique in conjunction with Kopal's results on Roche equipotentials to determine the combined effects of rotation and tidal distortions on the equilibrium structures and oscillations of the polytropic models of the stars. This approach is presented in detail by Saxena (80). Later this approach was also used by Mohan and Agarwal (62,63) to study the effects of rotation and tidal distortions on the structure of composite models of stars. The technique was subsequently formalized by Mohan et al (57,58) and used to study the problems of equilibrium structures and oscillations of rotationally and tidally distorted main sequence stars. Lal (40) studied in detail the equilibrium structures and periods of oscillations of differentially rotating stellar models. Later on Singh and Sharma (83) also studied the oscillations of differentially rotating stars in binary system.

The approximation of exact equipotential surface of rotationally and tidally distorted stars by corresponding Roche equipotential used in this method may not be very much justified for the white dwarfs stars. However still, it will be of interest to see how the equilibrium structures of white dwarfs stars are affected by rotation and tidally distorted with the present approach. In fact, Lal et al () applied this technique to study the equilibrium structures of differentially rotating and tidally distorted white dwarf models of stars using this approach.

1.3 AVERAGING TECHNIQUE OF KIPPENHAHN AND THOMAS

In order to study the effects of rotation and tidal distortions on the equilibrium structure of gaseous spheres, Kippenhahn and Thomas (30) developed the concept of topologically equivalent spherical surfaces corresponding to actual equipotential surfaces of a rotationally and tidally distorted model. They define on these equivalent spherical surfaces, quantities such as $\overline{f}, \overline{g}$ etc. which denote certain averages of the quantities f, g, respectively on the actual equipotential surfaces. If ψ denotes the total potential due to gravitation, rotation and tidal

forces of a rotationally and tidally distorted model at an arbitrary point $P(x,y,z)$ then $\psi(x,y,z)$ = constant, is an equipotential surface. Let V_ψ be the volume enclosed by the equipotential surface. ψ = constant and S_ψ the surface area of this equipotential surface. For any function $f(x,y,z)$ they define $\bar{f}$ as its mean value over the equipotential surfaces

Ψ=constant by the relation

$$\bar{f} = \frac{1}{S_\psi} \int_{\psi=const} f \, d\sigma \tag{1.1}$$

where $d\sigma$ denotes the surface element of the equipotential surface ψ = constant. Clearly $\bar{f}$ is a function of equipotential surface ψ =constant only and can be obtained as equation (1.1) for each equipotential surface ψ =constant. Kippenhahn and Thomas (1970) also define a variable r_ψ in analogy with the radius of sphere by the relation

$$V_\psi = \frac{4}{3}\pi r_\psi^3 \tag{1.2}$$

Also by definition

$$S_\psi = \int_{\psi=const} d\sigma \tag{1.3}$$

Obviously, in general, S_ψ is not equal to$4\pi r_\psi^2$. Kippenhahn and Thomas (58) define a function $g(x,y,z)$ by the relation

$$g = \frac{d\psi}{dn} \tag{1.4}$$

This g corresponds to the force of gravity of a sphere. The distance dn between two neighboring surfaces ψ = constant and $\psi + d\psi$ = constant is, in general, not constant (i.e. not same at all points of the surface). They used (1.4) to compute the mean values $\bar{g}$ and $\overline{g^{-1}}$ with the help of relations

$$\bar{g} = \frac{1}{S_\psi} \int_{\psi=const} \frac{d\psi}{dn} d\sigma$$

$$\overline{g^{-1}} = \frac{1}{S_\psi} \int_{\psi=const} \left(\frac{d\psi}{dn}\right)^{-1} d\sigma \tag{1.5}$$

Both $\bar{g}\ and\ \overline{g^{-1}}$ are functions of ψ alone and represent the value of g and g^{-1} respectively over the topologically equivalent spherical surface. The volume dV_ψ between the surface $\boldsymbol{\psi}$ = constant and $\psi + d\,\psi$ = constant is given by

$$\boldsymbol{dV_\psi = \int_{\psi=const} dnd\sigma = \int_{\psi=const} \left(\frac{d\psi}{dn}\right)^{-1} dn = S_\psi \overline{g^{-1}} d\psi} \qquad (\mathbf{1.6})$$

Kippenhahn and Thomas (1970) also defined nondimensional parameters u, v and w as

$$u = \frac{S_\psi}{4\pi r_\psi^2}, v = \frac{\bar{g} r_\psi^2}{GM_\psi}, w = \frac{\overline{g^{-1}} GM_\psi}{r_\psi^2} \qquad (1.7)$$

where M_ψ is the mass enclosed by equipotential surface ψ =constant.

We may thus regard the equipotential surface ψ = constant to be topologically equivalent to a sphere of radius r_ψ for which various functions are defined by the above relations. It may be noticed that if ψ is the gravitational potential of a sphere then the surface ψ = constant is spherical surface with $r_\psi = r$ for which u =1 and $g = GM_\psi/r_\psi^2$ is constant on these spheres and therefore v and w are constants and equal to 1.

Equations (1.1) to (1.7) are purely mathematical definitions, which have been applied by Kippenhahn and Thomas (30) to gravitational fields of gaseous spheres distorted by rotational and tidal forces. In hydrostatic equilibrium the equipotential surfaces are also surface of equipressure and equidensity. Therefore, on an equipotential surface the pressure P_ψ and the density ρ_ψ are also constant. Using these concepts, Kippenhahn and Thomas (30) obtained the equations governing the equilibrium structure of a rotationally and tidally distorted stellar model in the following manner

From equation (1.2) the mass dM_ψ between the equipotential surface ψ = constant and $\psi + d\psi$ = constant is given by

$$dM_\psi = dV_\psi \rho_\psi = 4\pi r_\psi^2 \rho_\psi\, dr_\psi \qquad (1.8)$$

Thus, we get

$$\frac{dM_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \qquad (1.9)$$

From equation (1.6) and (1.8) we have

$$d\psi = \frac{d\psi}{dV_\psi} dV_\psi = \left(\frac{dV_\psi}{d\psi}\right)^{-1} \frac{dM_\psi}{\rho_\psi} = \frac{dM_\psi}{S_\psi \overline{g^{-1}} \rho_\psi} \tag{1.10}$$

Using relations (1.7), we get

$$d\psi = \frac{GM_\psi dM_\psi}{4\pi r_\psi^4 \rho_\psi uw} \tag{1.11}$$

The conditions for hydrostatic equilibrium, $dP_\psi / d\psi = -\rho_\psi$, can now be written with equation (1.7) in the form

$$\frac{dP_\psi}{dM_\psi} = -\frac{GM_\psi}{4\pi r_\psi^4} f_p \tag{1.12}$$

Where

$$f_p = \frac{1}{uw} = \frac{4\pi r_\psi^4}{GM_\psi} \frac{1}{S_\psi \overline{g^{-1}}}$$

The factor $\boldsymbol{f_p}$ is a function of $\boldsymbol{\psi}$ only. If $\boldsymbol{\psi}$ is known the equipotential surface can be determined, and then consequently values of $\boldsymbol{S_\psi = r_\psi = \overline{g}}$ and $\boldsymbol{\overline{g^{-1}}}$ for each equipotential surface can be obtained simply from the geometry of the equipotentials. The mass $\boldsymbol{M_\psi}$ which depends on the density distribution $\boldsymbol{\rho_\psi}$ can be determined by integrating the equation (1.9). Similarly the other structure equations derived by Kippenhahn and Thomas (48), which includes the effects of rotation and tidal distortions on the equilibrium structure of gaseous sphere are as follows.

For chemically homogenous spheres, nuclear energy generation rate ε depends only upon density ρ_ψ and the temperature T_ψ and are, therefore, constant on equipotential surface. Thus, if L_ψ is the energy which passes per second through the equipotential surface $\psi =$ constant, then

$$\frac{dL_\psi}{dM_\psi} = \varepsilon \tag{1.13}$$

Using equation (1.8), it can be written as

$$\frac{dL_\psi}{dr_\psi} = 4\pi r_\psi^2 \rho_\psi \varepsilon \tag{1.14}$$

If the energy is transported by radiation, then the energy transport equation is

$$F_\psi = -\frac{4acT_\psi^3}{3\kappa}\frac{d\psi}{dn}\frac{dT_\psi}{dM_\psi}\frac{4\pi r_\psi^4 uw}{GM_\psi} \tag{1.15}$$

where F_ψ is the radioactive flux on the equipotential surface $\psi =$ constant. By integrating F_ψ over the equipotential surface ψ = constant, we get

$$\begin{aligned} L_\psi &= \int_{\psi=const} F_\psi d\sigma \\ &= -\frac{4acT_\psi{}^3}{3\kappa}\frac{dT_\psi}{dM_\psi}uw\frac{4\pi r_\psi^4}{GM_\psi}\int_{\psi=const}\left(\frac{d\psi}{dn}\right)d\sigma \\ &= \frac{64\pi^2 acT_\psi^3 r_\psi^4}{3\kappa}u^2 vw\frac{dT_\psi}{dM_\psi} \end{aligned} \tag{1.16}$$

So that

$$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa L_\psi}{64\pi^2 acT_\psi^3 r_\psi^4}f_T \tag{1.17}$$

Using equation (1.8), this equation can be expressed as

$$\frac{dT_\psi}{dr_\psi} = -\frac{3\kappa L_\psi \rho_\psi}{16\pi acT_\psi^3 r_\psi^2}f_T \tag{1.18}$$

where

$$f_T = \frac{1}{u^2 vw}$$

Equations (1.9), (1.12), (1.13) and (1.17) which are the four basic equations governing the equilibrium structure of a gaseous sphere distorted by rotational and tidal forces.

These reduce to the normal equations used for determining the equilibrium structures of spherical models of stars when distortion parameters u, v, w are set one each. The boundary conditions which the above equations have to satisfy are:

$$M_\psi = 0, L_\psi = 0, \text{ at the centre } r_\psi = 0 \tag{1.19a}$$

$$M_\psi = M_0, L_\psi = L_{\psi S}, P_\psi = 0, T_\psi = 0$$

or

$$P_\psi = P_{\psi S}, \; T_\psi = T_{\psi S} \text{ at the free surface } \; r_\psi = R_\psi \tag{1.19b}$$

Where M_0 is the total mass of the model and $L_{\psi S}$, $P_{\psi S}, T_{\psi S}$ are the values of L_ψ, P_ψ, T_ψ respectively, on the outermost equipotential surface.

1.4 ROCHE EQUIPOTENTIAL OF DISTORTED STARS

Roche equipotentials have often been used to analyze the problems of rotationally and tidally distorted models of stars. In order to introduce the concept of Roche equipotential, we assume two components of a close binary system known as primary and secondary star. The primary star is supposed to be much more massive than the secondary which is assumed to be a point mass causing tidal effects on the more massive primary component. Both the components of binary system are assumed to be rotating about their axes as well as revolving about their common center of mass. Following Kopal (32), Mohan and Singh (56), and Mohan et al (57,58) certain results on Roche equipotential which are of practical interest to the present study, are summarized below:

Suppose that M_0 and M_1 are the masses of the two components of a close binary system separated by distance *D*. Further suppose that the primary component of this system of mass M_0 is much larger than its companion star of mass $M_1 = (M_0 \geq M_1)$ which can be regarded as a point mass. Next suppose that the position of the two components is referred to a rectangular system of cartesian coordinates with origin at the center of gravity of mass M_0, the $x-$ axis along the line joining the mass centers of two components and $z-$axis perpendicular to the plane of the orbit of the two components (Fig. 1.1). Thus the total potential ψ due to the gravitational and disturbing force acting at an arbitrary point $P(x, y, z)$, which is not inside any of these two gaseous spheres is given by:

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_1} + \frac{\Omega^2}{2}\left[\left(x - \frac{M_1 D}{M_0 + M_1}\right)^2 + y^2\right] \tag{1.20}$$

where $r^2 = x^2 + y^2 + z^2$ and $r_2^2 = (D - x)^2 + y^2 + z^2$ represent the squares of the distances of *P* from the center of gravity of the two components, Ω denotes the angular velocity of rotation of the system about an axis perpendicular to the xy-plane and passing through the center of gravity of the system and *G* the constant of gravitation.

The first, second and third term on the right hand side of equation (1.20) respectively represent the potential which arises due to the mass M_0 of the primary component, the disturbing potential of its companion of mass M_1 and the potential arising from the centrifugal force respectively. Equation (1.20) strictly holds at points which are outside the components of binary system.

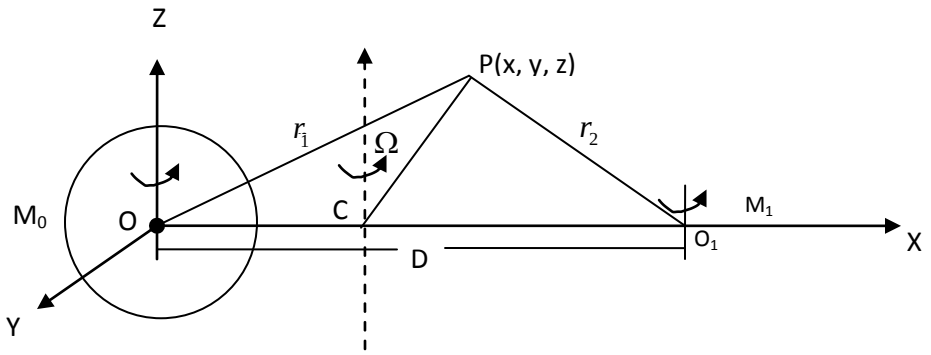

Fig 1.1: Axis of reference for a binary system

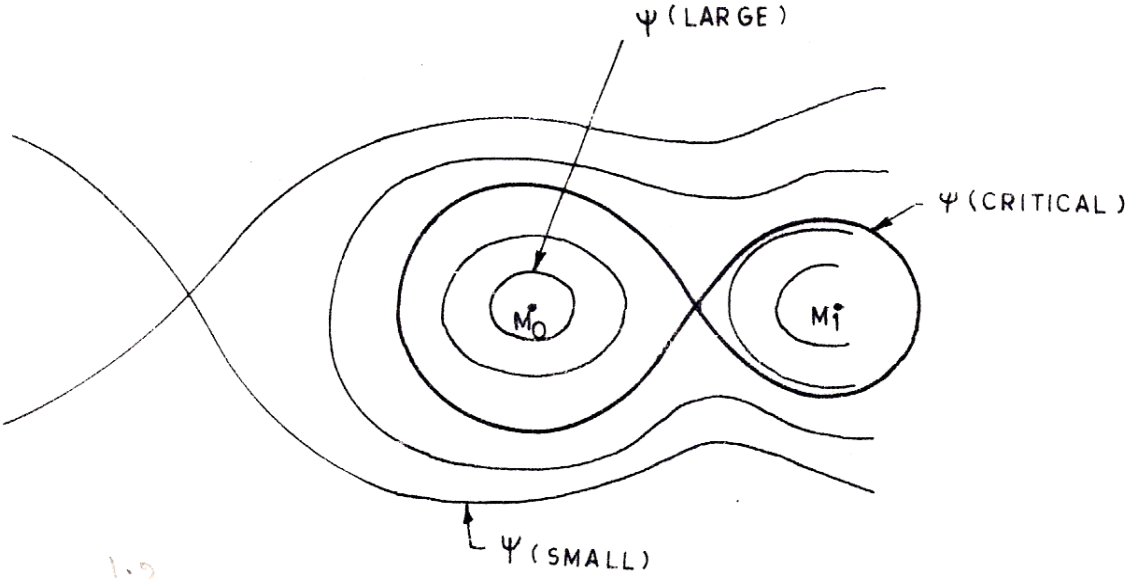

Fig 1.2: Roche equipotential surfaces (two dimensional)

In case, we assume Roche model for the primary and a point mass for the secondary component, equation (1.19) holds everywhere. In Roche model, it is assumed that the total mass of a star is concentrated at its center and this point mass is surrounded by an evanescent envelope in which density varies inversely as the square of the distance from its center.

Equation (1.20) can be expressed in nondimensional form as

$$\psi^* = \frac{1}{r^*} + q\left[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^*\right] + nr^{*2}(1-\nu^2) \tag{1.21}$$

where

$$\psi^* = \frac{D\psi}{GM_0} - \frac{M_1^2}{2M_0(M_0+M_1)}$$

is the nondimensional form of total potential ψ and $r^* = r/D$ is nondimensional form of r . Also$\lambda = \sin\theta\cos\emptyset$, $\mu = \sin\theta\sin\emptyset$ and $\nu = \cos\theta$ ($r, \theta, \emptyset$ being the polar spherical coordinate of the point$P$). Moreover,

$$q = \frac{M_1}{M_0} \tag{1.22}$$

is a nondimensional parameter representing the ratio of mass of the secondary over primary and $2n$ represent the square of the normalized angular velocity Ω .The equation (1.20) reduces to the potential of a purely rotating spherical model if $q = 0$. For $n = 0$ it reduces to the potential of a non-rotating spherical model distorted by the tidal effects of the companion alone.

For a binary system in synchronous rotation, the angular velocity Ω is identical with Keplerian angular velocity, that is,

$$\Omega^2 = G\frac{(M_0+M_1)}{D^3} \tag{1.23}$$

then we get a relation of the type

$$n = \frac{q+1}{2} \tag{1.24}$$

The surfaces generated by setting ψ = constant on the left hand side of equation (1.21) are referred to as Roche equipotentials. Roche equipotentials in nondimensional form may be represented by ψ^*= constant where ψ^* is same as defined in equation (1.24). The form of Roche-equipotential depends entirely upon the values of ψ. If ψ is large the corresponding equipotentials consist of two separate ovals, closed around each of the two mass points (Fig. 1.2). For specified values of M_0,M_1, Ω and D the right hand side of equation (1.21) can be large only if r and r_2 becomes small. Therefore, large values of ψ correspond to equipotentials which

differ little from spheres surrounding each of the two mass centers. With decreasing values of ψ , these spherical equipotential surfaces become oval shaped and get elongated in the direction of the center of gravity of the system until for a certain critical value of ψ, which is characteristic of each mass ratio, both oval shaped surfaces unite at a single point on the x-axis to form a dumbbell like configuration. These limiting values of ψ are called Roche limits. For certain mass ratios Kopal (32) computed the numerical values of Roche limits in the case of synchronous binary stars for values of q ranging from zero to one. Defining a non dimensional variable r_0 by the relation

$$r_o = \frac{1}{\psi^* - q} \tag{1.25}$$

Kopal (32) has also shown that on the surface of Roche equipotentials, $(r, \theta, \emptyset)$ are connected through the relation

$$r^* = r_0[1 + C_3 r_0^3 + C_4 r_0^4 + C_5 r_0^5 + C_6 r_0^6 + C_7 r_0^7 + C_8 r_0^8 + C_9 r_0^9 + \ldots] \tag{1.26}$$

where

$$C_3 = qP_2 + n(1 - \nu^2), C_4 = qP_3, C_5 = qP_4$$

$$C_6 = qP_5 + 3{C_3}^2, C_7 = qP_6 + 7q{C_3}^2 P_3$$

$$C_8 = qP_5 + 8qC_3C_4 + 4q^2{P_3}^2$$

$$C_9 = qP_8 + 9qC_3P_5 + 9q^2P_3P_4$$

Here, $P_J = P_J(\lambda)$ are the Legendre polynomials and terms up to second order of smallness in n and q have been retained in equation (1.27). This relation helps to obtain the shape of a Roche equipotentials ψ^* = constant.

The volume V_ψ enclosed by the equipotential surface ψ^* = constant is given by

$$V_\psi = \frac{2}{3} \int_{-1}^{1} \int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^3 d\lambda d\nu}{\mu} \tag{1.27}$$

Kopal (32) has shown that the explicit expression of V_ψ in terms of r_0 defined by equation (1.26), can be represented as

$$V_\psi = \frac{4}{3} D^3 r_0^3 [1 + 2nr_0^3 + (\frac{12}{5} q^2 + \frac{8}{5} nq + \frac{32}{5} n^2) r_0^6 + \frac{15}{7} q^2 r_0^8 + 2q^2 r_0^{10} + \ldots] \tag{1.28}$$

where terms up to second order of smallness in n and q are retained.

Following the approach of Kopal (32), Mohan and Singh (59) obtained the explicit expressions for the surface area S_ψ and the values of averages or parameters r_ψ, g and $\overline{g^{-1}}$on the Roche equipotential ψ^*= constant. These are

$$S_\psi = 2\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^2 d\lambda d\nu}{\mu}$$

$$= 4\pi D^2 {r_0}^2\left[1+\frac{4}{3}{r_0}^3+\left(\frac{7}{5}q^2+\frac{14}{15}nq+\frac{56}{15}n^2\right){r_0}^6+\frac{9}{7}{r_0}^8q^2+\frac{11}{9}{r_0}^{10}q^2+\cdots\right] \quad (1.29)$$

$$r_\psi = \left(\frac{3V_\psi}{4\pi}\right)^{1/3}$$

$$= Dr_0\left[1+\frac{2n}{3}r_0^3\left(+\frac{4}{5}q^2+\frac{8}{15}nq+\frac{76}{45}n^2\right)r_0^6+\frac{5}{7}r_0^8q^2r_0^8+\frac{2}{3}r_0^{10}q^2+\cdots\right] \quad (1.30)$$

$$\bar{g} = \frac{2}{S_\psi}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}}\left(\frac{d\psi}{dn}\right)\frac{r^2 d\lambda d\nu}{\mu}$$

$$= \frac{GM_\psi}{D^2}\frac{1}{r_0^2}\left[1-\frac{8n}{3}r_0^3-\left(3q^2+2nq+\frac{40}{9}n^2\right)r_0^6-\frac{51}{14}r_0^8q^2-\frac{13}{3}r_0^{10}q^2+.\ldots\right] \quad (1.31)$$

$$\overline{g^{-1}} = \frac{2}{S_\psi}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}}\left(\frac{d\psi}{dn}\right)^{-1}\frac{r^2 d\lambda d\nu}{\mu}$$

$$= \frac{D^2}{GM_\psi}r_0^2\left[\ 1+\frac{8n}{3}r_0^3+\left(\frac{31}{5}q^2+\frac{62}{15}nq+\frac{584}{45}n^2\right)r_0^6+\frac{101}{14}r_0^8q^2+\frac{75}{9}r_0^{10}q^2\ldots\right] \quad (1.32)$$

where

$$\frac{d\psi}{dn} = \left(\psi_r^2+\frac{1-\lambda^2}{r^2}\psi_\lambda^2+\frac{x}{r^2}\psi_\nu^2-\frac{2\lambda(1-x)^{\frac{1}{2}}}{r^2}\psi_\lambda\psi_\nu\right)^{\frac{1}{2}} \quad and\ x = 1-\nu^2$$

inverting the relation (1.39) they also obtain

$$r_0 = r_\psi^*\left[1-\frac{2n}{3}r_\psi^{*2}-\left(\frac{4}{5}q^2+\frac{8}{15}nq-\frac{4}{45}n^2\right)r_\psi^{*6}-\frac{5}{7}q^2r_\psi^{*8}-\frac{5}{7}q^2r_\psi^{*10}\ldots\right] \quad (1.33)$$

Where $r_\psi^* = r_\psi/D$ being the nondimensional form . In all the above expressions terms up to second order of smallness in n and q have been retained.

1.5 MOHAN, SAXENA AND AGARWAL'S APPROACH FOR COMPUTING THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED GASEOUS SPHERES

Mohan, Saxena and Agarwal (62, 63) used the concept of Roche equipotentials proposed by Kopal in conjunction with Kippenhahn and Thomas's averaging approach to explicitly obtain equations governing the equilibrium structures and periods of radial and nonradial oscillations of rotationally and/or tidally distorted stars and applied these to analyze the problems of rotating stars and stars in binary systems. In this section we briefly review their approach.

In order to determine the inner structure of a rotationally and tidally distorted gaseous sphere, the system of equations (1.19) has to be integrated numerically subject to the boundary conditions (1.20) specified therein. Therefore, the evaluation of the actual equipotential surface of a rotationally and tidally distorted gaseous sphere is complicated. Kippenhahn and Thomas (30) proposed that for evaluation of the distortion parameters u, v, w, f_p, f_T etc., the actual equipotential surface may be replaced by Roche equipotential surface (It may be noted that this approximation is reasonably valid for most of the models of the actual stars. In fact as far back as 1933, Chandrasekhar had shown that for stars whose central density bears to the mean density a ratio of 100 or more, the Roche model of a rotating configuration will represent the actual equipotential surfaces of the star within an error of less than one percent).

Once the equipotential surfaces of a rotationally and tidally distorted star are approximated by the Roche equipotentials, the results obtained by Kopal (32) and Mohan and Singh (59) may be used to evaluate explicitly the values of the distortion parameters u, v, w, f_p and f_T appearing in stellar structure equations (1.12) and (1.18). Using equations (1.7), (1.12), (1.1.18) and (1.29 – 1.34) the explicit expressions of the distortions parameters u, v, w, f_P and f_T on the equipotential surface as obtained by Mohan et al () are

$$u = \frac{S_\psi}{4\pi r_\psi^2}$$

$$= [1 - \left(\frac{q^2}{5} + \frac{2}{15}nq + \frac{4}{45}n^2\right) r_\psi^{*6} - \frac{1}{7}q^2 r_\psi^{*8} - \frac{1}{9}q^2 r_\psi^{*10} + \ldots] \qquad (1.34a)$$

$$v = \frac{\bar{g} r_\psi^2}{GM_\psi}$$

$$= [1 - \frac{4n}{3} r_\psi^{*3} - \left(\frac{7}{5}q^2 + \frac{14}{15}nq + \frac{68}{45}n^2\right) r_\psi^{*6} - \frac{31}{14}q^2 r_\psi^{*8} - 3q^2 r_\psi^{*10} + \ldots] \qquad (1.34b)$$

$$w = \frac{\overline{g^{-1}}}{r_\psi^2} GM_\psi$$

$$= [1 + \frac{4n}{3} r_\psi^{*3} + \left(\frac{23}{5}q^2 + \frac{46}{15}nq + \frac{212}{45}n^2\right) r_\psi^{*6} + \frac{81}{14}q^2 r_\psi^{*8} + 7q^2 r_\psi^{*10} + \ldots] \qquad (1.34c)$$

$$f_p = \frac{1}{uw}$$

$$= [1 - \tfrac{4n}{3} r_\psi^{*3} - \left(\tfrac{22}{5} q^2 + \tfrac{44}{15} nq + \tfrac{128}{45} n^2\right) r_\psi^{*6} - \tfrac{79}{14} q^2 r_\psi^{*8} - \tfrac{62}{9} q^2 r_\psi^{*10} - ..] \qquad (1.34d)$$

and

$$f_T = \frac{1}{u^2 vw}$$

$$= [1 - \left(\tfrac{14}{5} q^2 + \tfrac{28}{15} nq + \tfrac{56}{45} n^2\right) r_\psi^{*6} - \tfrac{46}{14} q^2 r_\psi^{*8} - \tfrac{34}{9} q^2 r_\psi^{*10} - \ldots \qquad (1.34e)$$

where $r_\psi^* = \frac{r_\psi}{D}$ is the nondimensional form of r_ψ and terms up to second order of smallness in n and q are retained.

The values of M_ψ, P_ψ, L_ψ etc. on the various equipotential surfaces of a rotationally and tidally distorted gaseous spheres may now be obtained by solving the system of differential equations (1.19) with boundary conditions (1.20) and using the values of distortion parameters f_p and f_T as given in (1.35).

It may be noted that while approximating the equipotential surfaces of a rotationally and tidally distorted model by Roche equipotentials, the structure of the star is not approximated by the structure of a Roche model. In the case of no distortion (*n=q=0),* equation (1.35) gives $u = v = w = f_p = f_T = 1$ and the system of differential equations (1.19) reduce to the equations governing the equilibrium structure of the original undistorted star and not of the Roche model. Usual methods for stellar structure equations such as Henyey et al (48) method can be now used to integrate the system of differential equation (1.19) governing the equilibrium structure of a rotationally and tidally distorted gaseous sphere. At every step, the values of the distortion parameters u, v, w, f_p and f_T have to be computed using (1.35). In case the thermal properties are not considered important and only hydrostatic equilibrium of a rotationally and tidally distorted gaseous spheres is to be investigated then we need only to integrate equation (1.9) and (1.12) subject to the boundary conditions

At the center $\mathrm{r}_\psi = 0$ $\mathrm{M}_\psi = 0$ (1.35a)

and at the free surface $\mathrm{r}_\psi = \mathrm{R}_\psi$

$\mathrm{M}_\psi = \mathrm{M}_0$, $\mathrm{P}_\psi = 0$, $\rho_\psi = 0$ or $\mathrm{P}_\psi = \mathrm{P}_{\psi S}$, $\rho_\psi = \rho_{\psi S}$ (1.35b)

In case the star is being distorted by rotational forces alone (or tidal forces alone) we may set $q=0$ ($n=0$) in (1.43) and still use the above approach to determine the equilibrium structure of corresponding purely rotationally distorted or purely tidally distorted model. For obtaining the structure of the primary component of a synchronous binary system we may set $n=(q+1)/2$.

Mohan and Saxena (57, 58) found it more convenient to work with r_0 in place of M_ψ or r_ψ as independent variable by using (1.26) which is connected with variable r_ψ through relation (1.34). Saxena (80) expressed the system of differential equations governing the equilibrium structure of a rotationally and tidally distorted stellar model as

$$\frac{dM_\psi}{dr_0} = 4\pi D^3 \rho_\psi r_0^2 f_1 \qquad (1.36a)$$

$$\frac{dP_\psi}{dr_0} = -\frac{GM_\psi}{Dr_0^2}\rho_\psi f_2 \qquad (1.36b)$$

$$\frac{dL_\psi}{dr_0} = 4\pi\varepsilon D^3 \rho_\psi r_0^2 f_1 \qquad (1.36c)$$

$$\frac{dT_\psi}{dr_0} = \frac{3\kappa l_\psi}{16\pi DacT_\psi^3}\frac{\rho_\psi}{r_0^2} f_3 \qquad (1.36d)$$

Here f_1, f_2 and f_3 are functions of n, q and r_0 incorporating the effects of rotation and tidal distortions on the equilibrium structure equations of a stellar model. The explicit expressions for these distortion parameters as given by Saxena (80) are

$$f_1 = \left[1 + 4nr_0^3 + \left(\frac{36}{5}q^2 + \frac{24}{5}nq + \frac{96}{5}n^2\right)r_0^6 + \frac{55}{7}q^2 r_0^8 + \frac{26}{3}q^2 r_0^{10} + \ldots\right] \qquad (1.37)$$

$$f_2 = [1 - \left(\frac{2}{5}q^2 + \frac{4}{15}nq + \frac{16}{15}n^2\right)r_0^6 - \frac{9}{14}q^2 r_0^8 - \frac{8}{9}q^2 r_0^{10} + \ldots \qquad (1.38)$$

And

$$f_3 = [1 + \frac{4nr_0^3}{3} + \left(\frac{6}{5}q^2 + \frac{4}{5}nq + \frac{224}{45}n^2\right)r_0^6 + \frac{24}{14}q^2 r_0^8 + \frac{20}{9}q^2 r_0^{10} + \ldots] \qquad (1.39)$$

In these above expressions terms up to second order of smallness in n, q and up to r_0^{10} in r_0 are retained. The boundary conditions governing the system of differential equations (1.45) are:

At the center $r_o = 0, M_\psi = 0, L_\psi = 0$ (1.40a)

and at the free surface, r_o=r_{os}

$$M_\psi = M_0,\ L_\psi = L_{\psi S}$$

$$P_\psi = 0, \rho_\psi = 0, T_\psi = 0 \ or \ \ P_\psi = P_{\psi S}, \rho_\psi = \rho_{\psi S}, T_\psi = T_{\psi S} \tag{1.40b}$$

Where r_{os} being the value of r_o at the free surfaces. In fact

$$r_{os} = \frac{1}{\psi_S^* - q} \tag{1.41}$$

where ψ_S^* is the nondimensional form of the total potential ψ on the outermost equipotential surface of a rotationally and tidally distorted stellar model. In the case of no distortion $f_p = f_T = 1$ and the above equations reduce to the usual equations governing the equilibrium structure of an undistorted gaseous sphere.

1.6 THE PRESENT WORK

The problem of determining the structure of equilibrium structures of stars distorted by rotation tidal forces has a great importance in astrophysics. Such a problems will help in better understanding the nature of inner structure of rotating stars and stars in binary /or multiple systems. Thus, there is need for in depth investigation of determining the equilibrium structure of gaseous sphere.

Analytic study of determining the equilibrium structure of rotationally and tidally distorted stellar models is quite complex. Therefore, investigation attempted to solve such a problems in some approximate way. In one such attempt Mohan,Saxena and Agarwal (62,63) used Kippenhahnanl Thomas averaging technique together with the results of Kopal (32) on Roche equipotential ,to determine the effects of rotation and tidal forces on the equilibrium structures and eigenfrequencies of small barotropic radial and non radial modes of oscillations of the theoretic models of the stars. They also implemented this approach in the case of the polytropic models of the stars as well as certain realistic main sequence stars. Several authors such as Lal (40),Sharma (83), Saini(79) ,Pathenia (72) used this method to study some astrophysical problems.

In the present thesis, we have proposed an algorithmic approach for Mohan, Saxena and Aggarwal method to determine the effect of rotation and tidal distortion in the equilibrium structure of polytropic models of stars. Although efforts have been made to develop analytic expression in series form whose closed form solutions have not possible. Even these series

solutions do not provide any analytic expression which could provide some result of practical significance. It may, therefore, be of interest to see if instead of developing detailed series expansion of distortional parameters r_ψ, u, v, w, f_p, and f_T etc. required in determining the equilibrium structure and periods of oscillations of rotationally and tidally distorted stars. In fact, this does not seem to be difficult proposition in view of the availability of fast computing machines. Thus, there is need of developing an efficient computational method which directly evaluates these distortional parameters without using their explicit expressions in the form of series available in the literature. The computation of these parameters plays an important role in determining the various structural parameters of distorted stars.

The thesis consists of three chapters. Chapter I is introductory in nature. We briefly discuss the astrophysical significance of the problem of determining the equilibrium structures of rotationally and tidally distorted stellar models. A brief survey of the literature available on this subject and summary of the work presented in succeeding chapters of this thesis also appears in this chapter.

In Chapter-II we first present suitable Gauss quadrature formula for evaluating double integral having singularity at the end of the intervals and how it has been used to evaluate double integral appearing in most of the mathematical expressions for distortional and structural parameters of rotationally and tidally distorted stellar models. An algorithm to use Mohan, Saxena and Agarwal (62, 63) approach has been formalized in this chapter to determine the effects of rotation and tidal forces on the equilibrium structure without using the explicit expressions for the distortional parameters available in the literature.

The algorithm developed in Chapter II has next been used in Chapter III to determine the equilibrium structure of rotationally and tidally distorted polytropic models of stars. The equilibrium structure and other physical parameters of various rotationally and tidally distorted polytropic models with polytropic indices 1.5, 3.0 and 4.0 have been computed. The results thus obtained have been compared with the results earlier obtained by Mohan and Saxena (57,58) for the same models using the explicit expressions for the distortional parametersr_ψ, u, v, w, f_p, and f_T. Certain conclusions based on this study have also been drawn. The astrophysical importance of the present work as well as the limitation of the scope of the present work is finally discussed in this chapter.

CHAPTER-II

DEVELOPMENT OF ALGORITHM TO COMPUTE DISTORTIONAL AND STRUCTURAL PARAMETRERS FOR ROTATIONALLY AND TIDALLY DISTORTED STARS

Mohan, Saxena and Aggarwal (62,63) used the averaging technique of Kippenhahn and Thomas (30) and utilizes the results of Roche equipotential obtained by Kopal (32) to determine the equilibrium structure and periods of oscillation of rotationally and tidally distorted stellar models. They demonstrated this approach to study the equilibrium structure of rotationally and tidally distorted polytropic stars. This method applied by Lal (41), Saini (79), Pathnia (72) to determine the equilibrium structure of rotationally and tidally distorted various stellar structures.

Once the equipotential surface of a rotationally and tidally distorted models are approximated by the Roche equipotential, the results obtained by Kopal (32), Mohan and Singh (59) have been used to evaluate explicitly the values of the distortional parameters$r_\psi, S_\psi, \overline{g}, \overline{g^{-1}}, u, v, w, f_p, and\ f_T$ appearing in stellar structure equations. In fact, the problems of determining equilibrium structure and periods of oscillation of rotationally and tidally stellar models have been carried forward by computing their parameter only. While formulating the explicit expression for these parameters, it was realized that the aforesaid method is very complicative and time consuming process to formulate the explicit expression for distortional parameters. Thus, there is a need of some alternate ways to use these parameters efficiently without developing these explicit expressions.

In the present chapter we have proposed an algorithm to use Mohan, Saxena and Agarwal (62, 63) approach to determine the effect of rotational and tidal forces on to equilibrium structure of gaseous sphere which does not use the explicit expressions for those distorted parameters available in literature. These algorithms may be helpful not only to study the equilibrium structure but also periods of oscillation of rotationally and tidally distorted stars by direct integrating their respective equations. The computation of the distortional parameters starts from the evaluation of the volume V_ψ enclosed by the equipotential surface $\psi =$ constant which is given as:

$$V_\psi = \frac{2}{3}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^3 d\lambda dv}{\sqrt{1-\lambda^2-v^2}} \tag{2.1}$$

The explicit expression of this integral determines by Kopal (32) in the form of infinite series is a very complicated mathematical procedure. The problem becomes more complex when this infinite series will be further used to develop the distortional parameters. Therefore, we propose numerical method to evaluate this double integral directly and use this result whenever required in the computation of distortional and structure parameter.

This section is organized as follows: While evaluating the inner integral of the given double integral, a singularity occurred at the end of the interval. In section 2.1 we apply suitable Gauss quadrature formula handling the singularities at the end of the interval A methodology of five point Gauss- Chebyshev quadrature formula required for inner integration has been developed in section 2.2. This section also discusses the five -point Gauss- Legendre quadrature formula for evaluating outer integration of the double integration. The methodology developed in section 2.2 has been subsequently used in section 2.3 to formalize in algorithm to compute the volume and other distortional parameters required to obtain the equilibrium structure of rotationally and tidally distorted stellar models.

2.1 Development of suitable Gauss quadrature formula for evaluating double integral having singularity

Before discussing suitable numerical integration method handing singularity at the end of interval, we first illustrate with example the limitation and suitability of a particular numerical method. Consider the following integral which is similar to the integral generally used in computing the structural parameters of the rotationally and tidally distorted stars.

$$I = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{f(x,y)dxdy}{\sqrt{1-x^2-y^2}} \qquad (2.2)$$

Analytically, this integral comes out to be 2π exactly for $f(x,y) = 1$. In order to solve double integral (2.2) with $f(x,y) = 1$ numerically by Gaussian quadrature formula, we first translated the interval $[-\sqrt{1-x^2}, \sqrt{1-x^2}]$ to $[-1,1]$ and then applied Gauss quadrature formula. This results in the formula

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{dxdy}{\sqrt{1-x^2-y^2}} = \int_{-1}^{1} \sqrt{1-x^2} \sum_{j=1}^{n} c_{n,j} \frac{1}{\sqrt{1-x^2-(\sqrt{1-x^2})r_{n,j}}} \qquad (2.3)$$

where $c_{n,j}$ and $r_{n,j}$ are the weights and the nodes respectively for n point Gauss Legendre quadrature formula. The Gaussian quadrature is further applied to approximate the integral on the right hand side of equation (2.3). On using five-point Gauss-Legendre quadrature formula$(n = 5)$, it gives the approximation 5.6508201576688, which widely deviates from the actual value 2π. This deviation might be one of the reason that the function $\frac{1}{\sqrt{1-x^2-y^2}}$ has algebraic singularity at $y = \pm\sqrt{1-x^2}$.

In order to avoid the singularity, we have further, solved the same double integral using Gauss-Chebyshev five point quadrature formula for inner part and five point Gauss -Legendre formula for outer part of the double integral. Interestingly, the computed result is very accurate to 2π. This numerical experiment motivates us to use a mixed quadrature formula to solve such a singular integral.

We now discuss in brief Gaussian type quadrature formulas, especially useful in dealing with singular integrals. In order to evaluate integral

$$\int_{a}^{b} g(x)\, dx \qquad (2.4)$$

where $g(x)$ has an algebraic singularity at a and /or b, one transforms the integral into

$$\int_{-1}^{1} f(x)w(x)dx \qquad (2.5)$$

where $g(x) = f(x)w(x)$ and $w(x) = (1-x)^{\alpha}(1+x)^{\beta}, \alpha > -1, \beta > -1$ for appropriate exponents α and β. In the special case $\alpha = \beta = -\frac{1}{2}$, these are just the Chebyshev polynomials.

For the special case, one gets the very attractive rule of $(k+1)$ -point's quadrature formula to evaluate (2.5) having singularity at $x = \pm 1$.That is,

$$\int_{-1}^{1} \frac{f(x)}{(1-x^2)^{1/2}} dx \approx \frac{\pi}{k+1} \sum_{i=0}^{k} f(\xi_i) \tag{2.6}$$

For which all the weights coincide with $\frac{\pi}{k+1}, k = 0,1,2,\ldots$ and the nodes $\xi_i's$ are the Chebyshev points given by

$$\xi_i = \cos\left(\frac{2i+1}{k+1}\frac{\pi}{2}\right), i = 0,1,\ldots k \tag{2.7}$$

Keeping the results of integral (2.6) in view, we have extended it for the general integral having singularity at $x = \pm a$.That is to compute the following integral

$$I = \int_{-a}^{a} \frac{f(x)}{\sqrt{a^2 - x^2}} dx \tag{2.8}$$

Thus, the two- point Gauss quadrature formula of (2.8) can be written as:

$$\int_{-a}^{a} \frac{f(x)}{\sqrt{a^2 - x^2}} dx = \lambda_0 f(x_0) + \lambda_1 f(x_1) \tag{2.9}$$

where λ_0, λ_1 are weights and x_0 , x_1 are the nodes respectively. As these are four unknowns, this method will be exact for $f(x) = 1, x, x^2, x^3$

when $f(x) = 1, \int_{-a}^{a} \frac{dx}{\sqrt{a^2 - x^2}} = \lambda_0 + \lambda_1$

$$\lambda_0 + \lambda_1 = \pi \tag{2.10}$$

$$f(x) = x, \int_{-a}^{a} \frac{xdx}{\sqrt{a^2 - x^2}} = \lambda_0 x_0 + \lambda_1 x_1$$

$$\lambda_0 x_0 + \lambda_1 x_1 = 0 \tag{2.11}$$

$$f(x) = x^2, \int_{-a}^{a} \frac{x^2 dx}{\sqrt{a^2 - x^2}} = \lambda_0 {x_0}^2 + \lambda_1 {x_1}^2$$

$$\lambda_0 {x_0}^2 + \lambda_1 {x_1}^2 = \frac{a^2\pi}{2} \tag{2.12}$$

and $f(x) = x^3$, $\int_{-a}^{a} \frac{x^3 dx}{\sqrt{a^2 - x^2}} = \lambda_0 {x_0}^3 + \lambda_1 {x_1}^3$

$$\lambda_0 {x_0}^3 + \lambda_1 {x_1}^3 = 0 \tag{2.13}$$

Eliminating λ_0 from (2.11) and (2.13), we get

$$\lambda_1 x_1 (x_1 - x_0)(x_1 + x_0) = 0 \tag{2.14}$$

Since $\lambda_1 \neq 0$ *and* $x_1 \neq x_0$, from equation (2.14), we get either $x_1 + x_0 = 0$ or $x_1 = 0$.

If x_1=0, then from equation (2.11), we get $x_0 = 0$ which contradicts the equation (2.12)

and hence $x_1 \neq 0$ .Thus

$$x_0 = -x_1 \tag{2.15}$$

On substitute (2.15) in (2.11), we get

$$\lambda_0 = \lambda_1 \tag{2.16}$$

On substitute (2.16) in (2.10), we get

$$\lambda_0 = \lambda_1 = \frac{\pi}{2} \tag{2.17}$$

Further using (2.15) and (2.17) in (2.12), we get

$$2x_0^2 = a^2$$

Therefore,

$$x_0 = \pm \frac{a}{\sqrt{2}} \; and \; x_1 = \mp \frac{a}{\sqrt{2}} \tag{2,18}$$

Thus, The quadrature formula on substituting the values of $\lambda_0, \lambda_1, x_0$ and x_1 in (2.9) finally becomes:

$$\int_{-a}^{a} \frac{f(x)}{\sqrt{a^2 - x^2}} dx = \frac{\pi}{2}\left[f\left(-\frac{a}{\sqrt{2}}\right) + f\left(\frac{a}{\sqrt{2}}\right)\right] \tag{2.19}$$

Clearly equation (2.6) is a special case of (2.19) for $k = 1$ and $a = 1$.

In order to write the five-point Gauss-Chebyshev quadrature formula for evaluating the integral (2.8), we first present some important properties of Chebyshev polynomials. By definition, the Chebyshev polynomial $T_k(x)$ of degree k is given ($on\ -1 \leq x \leq 1$) by the rule

$$T_k(cos\ \theta) = \cos(k\theta) \tag{2.20}$$

where $x = \cos(\theta)$. Thus,

$$T_0(x) = 1, T_1(x) = x \tag{2.21}$$

and all the higher degree Chebyshev polynomials can be written using following recurrence relation

$$T_{k+1} = 2xT_k(x) - T_{k-1}(x), \quad k = 1,2, \ldots \tag{2.22}$$

For getting the better result we want to extend the previous formula (2.19) to five-point Gauss-Chebyshev quadrature for integrating (2.8) . This can be written as

$$\int_{-a}^{a} \frac{f(x)}{\sqrt{a^2 - x^2}} dx = \lambda_0 f(x_0) + \lambda_1 f(x_1) + \lambda_2 f(x_2) + \lambda_3 f(x_3) + \lambda_4 f(x_4) \tag{2.23}$$

where λ_i and x_i are the weights and nodes of the quadrature formula. As we know that the nodes of the five-point Gauss-Chebyshev quadrature formula are the root of the five degree Chebyshev polynomial, the zeros of the five degree Chebyshev polynomial

$$T_5(x) = 16x^5 - 20x^3 + 5x \tag{2.24}$$

are obtained as:

$$x = 0, \pm\sqrt{\frac{5 + \sqrt{5}}{8}}, \pm\sqrt{\frac{5 - \sqrt{5}}{8}} \tag{2.25}$$

Also, we have seen in equation (2.6) that the weights of the quadrature formula coincide with $\frac{\pi}{k+1}$. The weights in this case for $k = 4$ become $\frac{\pi}{5}$. Thus, the five Point Gauss-Chebyshev quadrature formula finally becomes

$$\int_{-a}^{a} \frac{f(x)}{\sqrt{a^2 - x^2}} dx = \frac{\pi}{5}[f(0) + f(a\sqrt{\frac{5 + \sqrt{5}}{8}}) + f(-a\sqrt{\frac{5 + \sqrt{5}}{8}}) + f(-a\sqrt{\frac{5 + \sqrt{5}}{8}}) + f(-a\sqrt{\frac{5 + \sqrt{5}}{8}}) + f(-a\sqrt{\frac{5 + \sqrt{5}}{8}})\} \tag{2.26}$$

In order to evaluate (2.2), we first use the result of five-point Gauss-Chebyshev quadrature formula (2.26) for inner part of (2.2) and we get

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{f(x,y)dxdy}{\sqrt{1-x^2-y^2}} \approx \frac{\pi}{5}[\int_{-1}^{1} f(x,0)dx + \int_{-1}^{1} f(x,\left(\sqrt{1-x^2}\right)\sqrt{\frac{5+\sqrt{5}}{8}})dx$$

$$+ \int_{-1}^{1} f(x,\left(-\sqrt{1-x^2}\right)\sqrt{\frac{5+\sqrt{5}}{8}})dx + \int_{-1}^{1} f(x,\left(\sqrt{1-x^2}\right)\sqrt{\frac{5-\sqrt{5}}{8}})dx$$

$$+ \int_{-1}^{1} f(x,(-\sqrt{1-x^2})\sqrt{\frac{5-\sqrt{5}}{8}})dx\ ]$$

where $g(x,y) = \frac{f(x,y)}{\sqrt{1-x^2-y^2}}, w(x,y) = (1-x^2-y^2)^{-1/2};$ (2.27)

We next apply five point Gauss-Legendre quadrature formula to each integral of (2.27) for which weights λ_i's and nodes x_i's are given below as:

$$\lambda_1 = 0.2369268850, \lambda_2 = 0.4786286705, \lambda_3 = 0.5688888889$$

$$\lambda_4 = 0.4786286705, \lambda_5 = 0.2369268850$$

$$x_1 = 0.9061798459, x_2 = 0.5384693101, x_3 = 0$$

$$x_4 = -0.5384693101, x_5 = -0.9061798459$$

Thus, we have obtained a quadrature formula to evaluate the double integral (2.2) having singularities $y = \pm\sqrt{1-x^2}$ at the end of the interval, which can be written as

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{f(x,y)\,dxdy}{\sqrt{1-x^2-y^2}} \approx \frac{\pi}{5}[\sum_{j=1}^{5}\lambda_j[f(x_{j,}0) + f(x_{j,}(\sqrt{1-x^2})\sqrt{\frac{5+\sqrt{5}}{8}})$$

$$+ f(x_{j,}(-\sqrt{1-x^2})\sqrt{\frac{5+\sqrt{5}}{8}}) + f(x_{j,}\left(\sqrt{1-x^2}\right)\sqrt{\frac{5-\sqrt{5}}{8}})$$

$$+ f(x_{j,}\left(-\sqrt{1-x^2}\right)\sqrt{\frac{5-\sqrt{5}}{8}})] \quad (2.28)$$

2.2 Algorithmic approach to compute the structural parameters of rotationally and tidally distorted models

The methodology developed in section (2.1) can be used to compute the structural parameters of rotationally and tidally distorted stars as these parameters are expressed

mathematically in the form of the double integrals. Further, this approach can be used to compute the distortional parameters r_ψ, S_ψ, $\overline{g}$, $\overline{g^{-1}}$, u, v, w, f_p, and f_T.

In this section we develop an algorithm to compute the volume, the surface area and the shape of rotationally and tidally distorted stellar models directly integrating their respective double integrals instead of using their explicit expressions available in the literature.

The volume V_ψ enclosed by the equipotential surface ψ =constant given in (2.1) can also be written as

$$V_\psi = \frac{2}{3}D^3 \int_{-1}^{1} \int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^{*3}\, d\lambda dv}{\sqrt{1-\lambda^2-v^2}} \tag{2.29}$$

Here $f(\lambda, v) = r^{*3}$ and $w(\lambda, v) = \frac{1}{\sqrt{1-\lambda^2-v^2}}$. The radius vector r^*, represents the shape of a Roche equipotential surface can be determined by solving the nondimensional form of a total potential equation (1.21). That is,

$$\psi^* = \frac{1}{r^*} + q\left[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^*\right] + n r^{*2}(1-v^2) \tag{2.30}$$

In order to find r^*, we apply fixed-point iteration method with specified error tolerance. An equation of the form

$$r^* = F(r^*) \tag{2.31}$$

can be derived from equation (2.30) so that any solution of (2.31) is the solution of (2.30), the iteration function $F(r^*)$ for solving (2.30) can be chosen as

$$F(r^*) = [\psi^* - \mathrm{q}[(1-2\lambda r^* + r^{*^2})^{-\frac{1}{2}} - \lambda r^*] - \mathrm{n} r^{*^2}(1-v^2)]^{-1} \tag{2.32}$$

The iterative sequence generated by the recursion formula

$$r^*_{i+1} = F(r_i^*), \quad i = 0,1,2,\ldots \tag{2.33}$$

will converge to a point r^* for which equation (2.30) is satisfied. The value of r^* thus computed with desired accuracy is written as r^*_{exp} for avoiding the confusion between the values of r^*.

The fixed-point iteration technique proposed in this section may be used to determine the value of r^* which gives us the shape of outermost equipotential surfaces of rotationally and tidally distorted gaseous spheres. Numerically computation for (2.33) has been performed with initial approximation $r_o = \frac{1}{\psi^* - q}$, for some specified values of ψ^*, n, q. This value of ψ^* and q were selected in such a manner that the distorted models are well within the

respective Roche lobes. The value of $r_{exp}{}^{*}$ is computed with desired accuracy of 0.000005

Once the radius vector r^{*} is computed, we are now ready to evaluate volume V_{ψ} of an equipotential surface ψ =constant given by (2.29) and then distortional parameter r_{ψ} (the radius of the topologically equivalent sphere). In the subsequent section we formalize the algorithm to compute the structural parameters such as volume V_{ψ} and surface area S_{ψ} of the equipotential surface ψ =constant.

2.2.1 Algorithm for computing structural parameters

Step 1: Initialize $n, q. \psi^{*}, D$, sum = 0 and compute $r_0 = \frac{1}{\psi^{*}-q}$.

Step 2: Use five-point Gauss-Chebyshev quadrature rule (2.26) to evaluate inner integral ranges $-\sqrt{1-\lambda^2}$ to $\sqrt{1-\lambda^2}$, having singularity at the end of the interval and then five-point Gauss - Legendre quadrature formula to the outer integral ranges -1 to 1 for the double integral (2.29).

Step 3: For j=1, 2… 5 compute:

$$sum = sum + \frac{\lambda_j \pi}{5}[f(x_j, 0) + f(x_{j,}\left(\sqrt{1-x^2}\right)\sqrt{\frac{5+\sqrt{5}}{8}}) \quad + f(x_{j,}\left(-\sqrt{1-x^2}\right)\sqrt{\frac{5+\sqrt{5}}{8}})$$

$$+ \ f(x_{j,}\left(\sqrt{1-x^2}\right)\sqrt{\frac{5-\sqrt{5}}{8}}) + f(x_{j,}\left(-\sqrt{1-x^2}\right)\sqrt{\frac{5-\sqrt{5}}{8}})]$$

where x_j and λ_j are the nodes and weights of the five-point Gauss Legendre quadrature formula, respectively. While computing r^{*} using fixed point iteration formula (2.33), assigned x_j to λ and $\pm(\sqrt{1-x_j^2}\sqrt{\frac{5-\sqrt{5}}{8}}), \pm(\sqrt{1-x_j^2}\sqrt{\frac{5+\sqrt{5}}{8}})$ and 0 to ν. Making use of desired value of $f(\lambda, \nu)$, the double integral of (2.29) has finally been evaluated.

Step 4: The volume V_{ψ} enclosed by the equilipotential surface ψ =constant is next computed by

$$V_{\psi} = \frac{2}{3} D^3 * sum$$

Step 5: The radius r_{ψ} of a topologically equivalent spherical surface is next computed by

$$r_\psi = (\frac{3V_\psi}{4\pi})^{1/3} \tag{2.34}$$

Step 6: The surface area S_ψ enclosed by the equipotential surface ψ =constant given by

$$S_\psi = 2\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^2 d\lambda dv}{\sqrt{1-\lambda^2-v^2}} = 2D^2\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^{*2} d\lambda dv}{\sqrt{1-\lambda^2-v^2}} \tag{2.35}$$

is next computed following the step 1 to 3, taking $f(\lambda, v) = r^{*2}$and obtained by the relation

$$S_\psi = 2D^2 * sum$$

In order to compute the average of gravity, we have to evaluate the following integrals

$$\overline{g} = \frac{2D^2}{S_\psi}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} (\frac{d\psi}{dn}) \frac{r^{*2} d\lambda dv}{\sqrt{1-\lambda^2-v^2}} \tag{2.36}$$

$$\overline{g^{-1}} = \frac{2D^2}{S_\psi}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} (\frac{d\psi}{dn})^{-1} \frac{r^{*2} d\lambda dv}{\sqrt{1-\lambda^2-v^2}} \tag{2.37}$$

where

$$\left(\frac{d\psi}{dn}\right) = (\psi_{r^*}^2 + \frac{1-\lambda^2}{r^{*2}}\psi_\lambda^2 + \frac{1-v^2}{r^{*2}}\psi_v^2 - \frac{2\lambda v}{r^{*2}}\psi_\lambda\psi_v)^{\frac{1}{2}}$$

$$\psi_r = 2nr^*(1-v^2) + q\left(1-2\lambda r^* + r^{*^2}\right)^{-\frac{3}{2}}(\lambda - r^*) - q\lambda - \frac{1}{r^{*2}}$$

$$\psi_\lambda = q(1-2\lambda r^* + r^{*^2})^{-\frac{3}{2}}r^* - qr^*$$

$$\psi_v = -2vnr^{*^2}$$

$$f(\lambda, v) = r^{*^2}\left(\frac{d\psi}{dn}\right) \; and \;\; w(\lambda, v) = \frac{1}{\sqrt{1-\lambda^2-v^2}}$$

The algorithm to compute (2.36) is given below:

Step 7: First compute $\left(\frac{d\psi}{dn}\right)$ by using the value of r^* computed from step 3 in which we assign x_j to λ and $\pm(\sqrt{1-x_j^2}\sqrt{\frac{5-\sqrt{5}}{8}}), \pm(\sqrt{1-x_j^2}\sqrt{\frac{5+\sqrt{5}}{8}})$ and 0 to v.

Step 8: Making use of $f(\lambda,\nu)=r^{*^2}\left(\frac{d\psi}{dn}\right)$ computed the double integral associated with the expression (2.36).

Step 9: $\bar{g}$ is finally computed by

$$\bar{g}=\frac{2D^2}{S_\psi}*sum$$

Step 10: t=the double integral for $\overline{g^{-1}}$ has been computed by using $f(\lambda,\nu)=r^{*^2}(\frac{d\psi}{dn})^{-1}$,

Step: 11: The inverse of gravity $\overline{g^{-1}}$ is finally computed by

$$\overline{g^{-1}}=\frac{2D^2}{S_\psi}*sum$$

Step 12: The structural parameters $V_\psi, S_\psi, r_\psi, \bar{g}, \overline{g^{-1}}$ thus computed using algorithmic approach is next computed by their corresponding explicit expressions earlier obtained by Mohan, Saxena and Agarwal (62,63) given by equations (1.27), (1.28), (1.29), (1.30) and (1.31), respectively.

Step 13: The results of these structural parameters obtained by two different approaches have finally been compared.

2.3Algorithmic approach to compute the distortional parameters of rotationally and tidally distorted models

Following the averaging approach of Kippenhahn and Thomas (30), distortional parameters $u, v\ and\ w$ have been computed directly using the following algorithms discussed below.

Step 14: Following the Steps (5) and (6), computed u by the relation

$$u=\frac{S_\psi}{4\pi r_\psi^2} \tag{2.38}$$

Step 15: Following the Steps (5), (8) and (9), computed v using the expression

$$v=\frac{\bar{g}r_\psi^2}{GM_\psi} \tag{2.39}$$

Step 16:.Following the Steps (5), (10) and (11), computed w using the expression

$$w = \frac{\overline{g^{-1}}}{r_\psi^2} GM_\psi \tag{2.40}$$

Step 17: Once the value of u and w are known, using the Steps (14) and (16) the value of f_p is next computed by the following relation.

$$f_p = \frac{1}{uw} \tag{2.41}$$

. **Step 18:** Following the Steps (14), (15) and (16), computed f_T.using the relation

$$f_T = \frac{1}{u^2 vw} \tag{2.42}$$

Aw we know that the parameters f_1, f_2 and f_3 are certain function of n, q and r_0 incorporating the effect of rotation and tidal forces to the distorted models, can be computed using the following algorithm:

Step 19: In order to compute f_1 by the following relation

$$f_1 = \frac{r_\psi^2 \frac{dr_\psi}{dr_0}}{D^3 r_0^2} \tag{2.43}$$

we first compute r_ψ for different values of r_0 using step (5).

Algorithm to compute f_1 ,f_2 and f_3

Step 20: Next, compute $\frac{dr_\psi}{dr_0}$ using forward difference for initial point, central difference for middle points and backward difference for the end point.

Step 21: Finally computed f_1 using the steps (5) and (20).

$$f_2 = \frac{Dr_0^2 \frac{dr_\psi}{dr_0}}{r_\psi^2} f_p \tag{2.44}$$

Step 21: Following the steps (19) and (20) computed f_2.

Step 22: Apply steps (5), (19), and (20) to compute f_3 which is given by

$$f_3 = \frac{Dr_0^2 \frac{dr_\psi}{dr_0}}{r_\psi^2} f_T \qquad (2.45)$$

In the subsequent chapter we shall apply this algorithm to determine the equilibrium structures of rotationally and tidally distorted polytropic models of the stars. In fact, this algorithm can be further used to study the periods of oscillations of various types of rotationally and tidally distorted stellar model.

CHAPTER-II1

ALGORITHMIC APPROACH TO DETERMINE THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

Polytropic models have frequently been used in literature to understand the inner structures of realistic stars. Chandrasekhar developed the theory of distorted polytropes in the year 1933 to study the problems of rotationally and or tidally distorted polytropes. Since then several investigators such as Roberts (74), James (29), Hurley and Roberts (26), Monaghan and Roxburgh (66), Monaghan (65), Kovetz (37), Smith (85), Linnell (46, 47) .have discussed the structures of uniformly rotating polytropes. Naylor and Anand (67) studied the structure of rotating polytropes assuming the primary component being a synchronous close binary system. However not much attention has been paid to the problem of determining the effects of tidal distortions in the presence of rotation on the equilibrium structures of polytropic models.

In the present chapter we consider the feasibility of using the algorithm developed in section 2.2 of Chapter – II to determine the effects of rotation and tidal distortions on the equilibrium structures of polytropic models. In Sections 3.1 we first set up boundary value problem which determines the equilibrium structure of a rotationally and tidally distorted polytropic model. Numerical results for the inner structures, volumes, surface areas and shapes of certain rotationally and or tidally distorted polytropic models are obtained in Section 3.2 using the algorithm formalized in Chapter-II. Conclusions based on the present study are finally drawn in section 3.3.

3.1 Mathematical Model Determining Equilibrium Structure of a Rotationally and Tidally Distorted Polytropic Models

In this section we use the algorithm developed in Chapter – II to obtain the equilibrium structures of rotationally and tidally distorted polytropic models. If we suppose a polytropic model is subjected to rotation and tidal distortions, then its structure will become a rotationally and tidally distorted polytropic model.

Following the approach of Section 1.2 we shall approximate the equipotential surfaces of this distorted model by Roche equipotentials. Let P_ψ denote the pressure and ρ_ψ the density on the equipotential surface ψ = constant of the distorted model. Then the values of the density and the pressure on the equivalent equipotential surface of the corresponding spherical model will also be ρ_ψ and P_ψ respectively. We shall assume that the distorted model also behaves like a polytropic model so that P_ψ and ρ_ψ are also connected through the polytropic type of relations

$$P_\psi = P_{c\psi}\theta_\psi^{N+1} \text{ and } \rho_\psi = \rho_{c\psi}\theta_\psi^N \tag{3.1}$$

Where $P_{c\psi}$ and $\rho_{c\psi}$ are values of P_ψ and ρ_ψ at the centre and θ_ψ is some average of θ on the equipotential surface ψ = constant. The index N used in equation (3.1) is called the polytropic index of the model which lies between 0 to 5 for the practical interest to the problem of stellar structure. In fact, N represents the central condensation of the model. Equations (1.36a) and (1.36b) which govern the hydrostatic equilibrium of a rotationally and tidally distorted gaseous sphere can be combined to yield

$$\frac{1}{r_0^2}\frac{d}{dr_0}\left[\frac{r_0^2}{\rho_\psi f_2}\frac{dP_\psi}{dr_0}\right] = -4\pi G D^2 P_\psi f_1 \tag{3.2}$$

On substituting the values of P_ψ and ρ_ψ from relation (3.1), this equation can be expressed in the nondimensional form as

$$\frac{1}{r_0^2}\frac{d}{dr_0}\left[\frac{r_0^2}{f_2}\frac{d\theta_\psi}{dr_0}\right] = -\frac{D^2}{\alpha^2}\theta_\psi^N f_1 \tag{3.3}$$

where

$$\alpha^2 = \frac{(N+1)P_{c\psi}}{4\pi G\rho_{c\psi}^2}$$

and f_1 *and* f_2 are the functions of distortion parameters n, q and r_0 are same as defined in equation (2.42 -2.43).

As regards the boundary conditions, since P_ψ and ρ_ψ must be maximum at the centre and zero at the free surface, we should have θ_ψ maximum at the centre and zero at the free surface. These obviously lead to the conditions: θ_ψ=1 and $\frac{d\theta_\psi}{dr_0} = 0$ at the centre and θ_ψ=0 at the free surface. Thus the boundary conditions which equation (3.3) must satisfy are:

at the centre: r_0=0, θ_ψ=1, $\frac{d\theta_\psi}{dr_0}$=0,

and at the surface: $r_0 = r_{0s}, \theta_\psi$=0, (3.4)

r_{0s} being the value of r_0 on the free surface.

It may be noted that the approximation of the equipotential surfaces by Roche equipotentials has not basically altered the structures of polytropic model because in the absence of any distortions ($f_1 = f_2 = 1$) equation (3.3) reduces to the usual Lane - Emden equation governing the equilibrium structure of an undistorted polytropic model of index N in nondimensional form and not to the equation governing the equilibrium structure of an undistorted Roche model.

The quantity α as defined in equation (3.3) is of the dimension of length. If we set $r_\psi = \alpha\xi$, then ξ will be a nondimensional variable defined for the equivalent spherical model .it corresponds to the usual Emden variable ξ of the Lane-Emden equation for an undistorted spherical polytropic model. In the case of a rotationally and tidally distorted polytropic model which is the primary component of a binary system, let k denotes the ratio between the undistorted radius of the primary and D the distance between the centers of the two components of the binary system, we get

$$\frac{D}{\alpha} = \frac{D\xi_u}{\alpha\xi_u} = \frac{D}{R_u}\xi_u = \frac{1}{k}\xi_u \tag{3.5}$$

Where ξ_u is the value of ξ at the outer surface of the undistorted polytropic model. For a polytropic model distorted by rotational forces alone we should take $k = 1$. In the case of the polytropic model being the primary component of a binary system the value of k must be such that the outermost surface of the primary component lies well within the Roche lobe otherwise the two stars will coalesce. Using the substitution (3.5), equation (3.3) can be written as

$$\frac{d}{dr_0}\left(\frac{r_0^2}{f_2}\frac{d\theta_\psi}{dr_0}\right) = -\frac{\xi_u^2}{k^2} r_0^2 \theta_\psi^N f_1 \tag{3.6}$$

and further simplification gives

$$\frac{d^2\theta_\psi}{dr_0^2} + \frac{d\theta_\psi}{dr_0}\left(\frac{2r_0 f_2 - r_0^2 f_2'}{r_0^2 f_2}\right) = -\frac{\xi_u^2}{k^2}\ \theta_\psi^N f_1 f_2 \tag{3.7}$$

where

$$f_1 = \frac{r_\psi^2 \frac{dr_\psi}{dr_0}}{D^3 r_0^2} \quad , \qquad f_2 = \frac{{Dr_0}^2 \frac{dr_\psi}{dx}}{r_\psi^2} \quad \text{and} \qquad D = \frac{\alpha\, \xi_u}{k}$$

Equation (3.7) subject to the boundary conditions (3.4) determines the equilibrium structure of a rotationally and tidally distorted polytropic model. On setting $q = 0$ the equation can be used to determine equilibrium structure of a polytropic model distorted by rotation alone. If we set $n = 0$ then the equation can be used to determine the equilibrium structure of polytropic model distorted by tidal forces alone. Also, by setting n= (q+1)/2 this equation can be used to determine the equilibrium structure of the primary component of a synchronous rotating binary system.

3.2 Algorithmic approach to determine equilibrium structure of rotationally and tidally distorted polytropic model

In order to determine the numerical solution of the second –order nonlinear differential equation (3.7) subject to the boundary conditions (3.4), we can start integration from the centre ($r_0 = 0$) using θ_ψ =1 and $\frac{d\theta_\psi}{dr_0}$=0 as the initial conditions for certain specified values of N, ξ_u, n, q and k, which denotes respectively the polytropic index, the radius of the undistorted polytropic model, the nondimensional measure of angular velocity of rotation, the ratio of the mass of the companion and the ratio of the undistorted radius of the primary to the distance between the centers of the primary and secondary. As there is a singularity at the point near centre, a series solution similar to the one available for undistorted polytropic models (see Chandrasekhar (9), page 97) was developed to start the integration. The series solution is given by

$$\theta_\psi = 1 - \frac{k^2}{6} r_0^2 + \frac{Nk^4}{120} r_0^4 - \frac{2nk^2}{15} r_0^5 - \frac{N(8N-5)k^6}{3 \times 5040} r_0^6 + \frac{Nnk^4}{70} r_0^7 + \left[\frac{N(122N^2 - 183N - 70)k^8}{9 \times 362880} - \frac{(3q^2 + 2nq + 8n^2)k^2}{36} \right] r_0^8 + \cdots \qquad (3.8)$$

The integration is to be continued till θ_ψ first become zero. The value of r_0 (i.e, r_{0s}) when θ_ψ first becomes zero determines the outermost free surface of the model. Once the solution of equation (3.7) is obtained, we know the values of θ_ψ for various values of the nondimensional independent variable r_0 varying from zero to r_{0s}. The pressure P_ψ and the density ρ_ψ on various equipotentials surface of the distorted model may now be obtained through the relations in the same manner as is done for undistorted polytropic models. The other physical parameters of rotationally and tidally distorted polytropic stars are then also compared.

The procedure just discussed has been formalized in following algorithm.

Step 1: Initialize n, q, k, N, ξ_u, accuracy, step size $h = 0.01$ and starting value of $r_0 = 0.005$

Step 2: Compute D using $\frac{\alpha\xi_u}{k}$, taking $\alpha = 1$ and the value of the parameter k has been taken as one for rotationally distorted model and 0.5 for tidally distorted or rotationally and tidally distorted models.

Step 3: Following the Step 5 of Chapter – II, Compute r_ψ using the relation (2.34).

Step 4: To obtain various distortional parameters, compute derivative of r_ψ with respect to r_0 using forward difference for initial point, central difference for middle points and backward difference method for the end points.

Step 5: Next compute distortional parameters f_1 and f_2 which are required in the differential equation (3.7). These can be computed using the formulas given in equation (3.7)

Step 6: Following the same procedure discussed in Step 4, the derivative of f_2 with respect to r_0 also can be computed.

Step 7: To start the integration using Runge-Kutta fourth order method, take the initial value of θ_ψ and $\frac{d\theta_\psi}{dx}$ from series solution given in equation (3.8).

Step 8: While using Runge-Kutta fourth order method to solve second order non linear differential equation (3.7) we need to compute two functions per iteration which are given as:

$$f(r_0, \theta_\psi, z_\psi) = z_\psi \tag{3.9}$$

$$g(r_0, \theta_\psi, z_\psi) = \left(\frac{1}{f_2}\frac{df_2}{dx} - \frac{2}{r_0}\right) z_\psi - \frac{\xi_u^2}{k^2}\theta_\psi^N f_1 f_2 \tag{3.10}$$

Step 9: Computation is continued till $\theta_\psi \geq 0$ and stop the process as soon as θ_ψ becomes negative.

Step 10: Finally used linear interpolation to get the desired value of r_{0s} corresponding to the value of θ_ψ very close to zero.

Step 11: Once the value of r_{0s} for the rotationally and tidally distorted polytropic models for the specified parameters is known, the equation (3.7) has been normalized using $x = \frac{r_0}{r_{0s}}$. The expressions for f, g, f_1 and f_2 are next changed to the following form

$$f(x, \theta_\psi, z_\psi) = z_\psi \tag{3.11}$$

$$g(x, \theta_\psi, z_\psi) = \left(\frac{1}{f_2}\frac{df_2}{dx} - \frac{2}{x}\right) z_\psi - \frac{\xi_u^2}{k^2}\theta_\psi^N f_1 f_2 \, r_{0s}^2 \tag{3.12}$$

$$f_1 = \frac{r_\psi^2 \frac{dr_\psi}{dx}}{D^3 r_{0s}^2 x^2} \tag{3.13}$$

$$f_2 = \frac{Dx^2 r_{0s} \frac{dr_\psi}{dx}}{r_\psi^2} \tag{3.14}$$

Step 12: Following the Steps 2 to 11 and initializing the value n, q, k, N, $\xi_u, x = 0.005, h = 0.01$ and accuracy, computed the inner structure of a rotationally and tidally distorted polytropic models.

Step 13: Once the value of r_{0s} is known, the surface area, the shape and the volume enclosed by an equipotential surface located in the interior of a rotationally and tidally distorted polytropic model may be determined following the Steps 4 to 6 of Chapter – II.

In order to compute the inner structure and some observable physical parameters of rotationally and tidally distorted polytropic models, we have developed a computer code based on the present algorithm and presented in the appendix.

3.3: Concluding Observations

In table 3.1 we present the values of r_{0s} for various types of rotationally and tidally distorted polytropic models with polytropic indexes $N = 1.5, 3.0, 4.0$, respectively. The value of θ_ψ for various typed of distorted polytropic models with indices $N = 1.5, 3.0 \; and \; 4.0$ are shown in the Tables 3.2-3.4. The values of the volumes and the surface areas obtained for each of the distorted model are next obtained by assuming $\alpha = 1$ and are presented in Table 3.5.

The results of the Table 3.2 are computed for the same parameters of n and q as earlier used by Mohan and Saxena (57,58). The result presented in Tables 3.2-3.4 for the values of θ_ψ exhibit the similar behavior of the variability of θ_ψ with the inclusion of distortional effects as earlier reported by Mohan and Saxena (57,58) and the other authors. In fact the results computed by the presented algorithm approach agree with the corresponding results of Mohan and Saxena (57,58) up to two to three significant digits.

The results for the volumes and the surface areas given in Table 3.5 show that for all the polytropic models of indices $N = 1.5, 3.0, 4.0$, the volumes and the surface area of distorted models are larger compared to their corresponding values for the undistorted models. It is noticed that the effect of rotation on the volume and surface area are larger in comparison to the tidal effect. Similar trends in the variability of the results were also observed by Mohan and Saxena (57,58). The results shown in parenthesis in Table 3.5 are the results of the corresponding model earlier computed by Mohan and Saxena (57,58). Our results, clearly agree with the Mohan and Saxena (57,58) result upto three significant digits.

In the present thesis, an effort has been made to develop an algorithm for Mohan, Saxena, Agarwal (57, 58) approach with which equilibrium structures of rotationally and tidally distorted gas sphere could be computed very efficiently and accurately without developing detailed series expansions of distortion parameters u, v, w, f_p and f_T. The proposed algorithm directly evaluates these distortional parameters while computing the structural parameters of distorted stars. This algorithm can further be used to study the periods of oscillations of rotationally and tidally distorted stars. The present study could be possible in view of the fast computing machines.

The results of structural parameters obtained by the present algorithmic approach agree with their corresponding results of Mohan and Saxena (57, 58) correct upto two to three decimal places only. A more accurate result can be achieved by modifying the present algorithm with the inclusion of some other numerical techniques to solve the double integrals whose efficient and accurate computation plays important role in the present study.

It would be worthwhile to develop techniques which account for exact potential instead of Roche equipotential. It should be possible in view of the availability of the efficient computational software for this purpose.

From the astrophysical view point, it will be worthwhile to incorporate the present algorithm into certain available computer codes for stellar structure and stellar pulsation and

apply it to determine the equilibrium models and trace the evolutionary tracks of certain realistic models of rotating stars and stars in binary system.

Table 3.1: Values of r_{os} for various types of rotationally and tidally distorted polytropic models with polytropic indices N

Model No.	N	Q	K	N=1.5	N=3.0	N=4.0
Undistorted Model						
1.	0.000	0.00	1.0	1.000000	1.000000	1.000000
Tidally Distorted Models						
2.	0.000	0.10	0.5	0.499952	0.498846	0.491012
3.	0.000	0.50	0.5	0.499063	0.498190	0.490510
Rotationally Distorted Models						
4.	0.020	0.00	1.0	0.993712	0.996009	0.993476
5.	0.050	0.00	1.0	0.984160	0.990804	0.991373
6.	0.100	0.00	1.0	0.967738	0.981691	0.987454
Rotationally and tidally distorted models						
7.	0.020	0.20	0.5	0.499472	0.498574	0.490887
8.	0.100	0.20	0.5	0.497872	0.497704	0.490552
9..	0.150	0.01	0.5	0.496861	0.497154	0.490339
Synchronous Binary System						
10..	0.525	0.05	0.5	0.489494	0.493141	0.488789

11.	0.550	0.10	0.5	0.488910	0.492813	0.488633

Table 3.2: Values of θ_ψ for rotationally and tidally distorted polytropic models for N=1.5

$x = \frac{r_0}{r_{0s}}$	Undistorted k=1.0,n=0, q=0.0	Rotationally Distorted k=1.0,n=0.1 q=0.0	Tidally distorted k=0.5, n=0, q=0.1	Rotationally and tidally distorted k=0.5,n=0.1 q=0.2	Synchronous Binary System k=0.5,n=0.55, q=0.1
0.0000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1050	0.975873	0.977387	0.975886	0.976084	0.976927
0.2050	0.910439	0.915854	0.910463	0.911169	0.914184
0.3050	0.811152	0.821804	0.811192	0.812574	0.818497
0.4050	0.688456	0.704199	0.688512	0.690540	0.699272
0.5050	0.553898	0.573091	0.553968	0.556416	0.567025
0.6050	0.418280	0.438207	0.418358	0.420870	0.431831
0.7050	0.290283	0.307880	0.290361	0.292553	0.302170
0.8050	0.175765	0.188423	0.175835	0.177398	0.184259
0.9050	0.077666	0.083886	0.077719	0.078491	0.081828
1.0000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 3.3: Values of θ_ψ for rotationally and tidally distorted polytropic models for N=3.0

$x = \frac{r_0}{r_{0s}}$	Undistorted k=1.0,n=0, q=0.0	Rotationally Distorted k=1.0,n=0.1 q=0.0	Tidally distorted k=0.5, n=0, q=0.1	Rotationally and tidally distorted k=0.5,n=0.1 q=0.2	Synchronous Binary System k=0.5,n=0.55, q=0.1
0.0000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1050	0.919388	0.922102	0.919740	0.920080	0.921530
0.2050	0.743817	0.750918	0.744732	0.745618	0.749408
0.3050	0.556166	0.565503	0.557404	0.558562	0.563535
0.4050	0.398871	0.408008	0.400164	0.401290	0.406137
0.5050	0.278689	0.286341	0.279895	0.280833	0.284865
0.6050	0.189127	0.194926	0.190203	0.190910	0.193933
0.7050	0.121989	0.126021	0.122936	0.123425	0.125489
0.8050	0.070689	0.073192	0.071524	0.071824	0.073063
0.9050	0.030546	0.031767	0.031286	0.031427	0.031980
1.0000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 3.4: Values of θ_ψ for rotationally and tidally distorted polytropic models for N=4.0

$x = \frac{r_0}{r_{0s}}$	Undistorted k=1.0,n=0, q=0.0	Rotationally Distorted k=1.0,n=0.1 q=0.0	Tidally distorted k=0.5, n=0, q=0.1	Rotationally and tidally distorted k=0.5,n=0.1 q=0.2	Synchronous Binary System k=0.5,n=0.55, q=0.1
0.0000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1050	0.722455	0.727218	0.729309	0.729659	0.731116
0.2050	0.429444	0.434963	0.437549	0.437955	0.439646
0.3050	0.267112	0.271466	0.273762	0.274075	0.275377
0.4050	0.174621	0.177903	0.179930	0.180157	0.181091
0.5050	0.116793	0.117285	0.121132	0.121294	0.121953
0.6050	0.077675	0.079591	0.081320	0.081433	0.081889
0.7050	0.049571	0.051057	0.052706	0.052783	0.053086
0.8050	0.028435	0.029586	0.031183	0.031231	0.031414
0.9050	0.011967	0.012843	0.014413	0.014436	0.014521
1.0000	0.000000	0.000000	0.000000	0.000000	0.000000

Model No.	n	q	N=1.5		N=3.0		N=4.0	
			Volume $\times 10^{-2}$	Surface $\times 10^{-2}$	Volume $\times 10^{-3}$	Surface $\times 10^{-2}$	Volume $\times 10^{-3}$	Surface $\times 10^{-3}$
Undistorted Models								
1.	0.00	0.00	2.0432 (2.0432)	1.6776 (1.6776)	1.37474 (1.37474)	5.9774 (5.9774)	14.0569 (14.0625)	2.81672 (2.81785)
Tidally Distorted Models								
2.	0.00	0.10	2.04346 (2.0449)	1.67769 (1.6787)	1.36526 (1.3754)	5.95128 (5.9816)	13.3175 (14.0799)	2.71697 (2.81993)
3.	0.00	0.50	2.05901 (2.0664)	1.68424 (1.6890)	1.37735 (1.39232)	5.97952 (6.0251)	13.4298 (14.2581)	2.72943 (2.84128)
Rotationally Distorted Models								
4.	0.02	0.00	2.08892 (2.0903)	1.70248 (1.7035)	1.41513 (1.41815)	6.09534 (6.1061)	14.3609 (14.6047)	2.85709 (2.88977)
5.	0.05	0.00	2.16833 (2.1672)	1.74495 (1.7452)	1.49180 (1.49352)	6.31201 (6.3199)	15.2601 (15.5466)	2.97440 (3.01302)

6.	0.1	0.00	2.34453 (2.3083)	1.83614 (1.8219)	1.67001 (1.64074)	6.79615 (6.7362)	15.2897 (17.4648)	2.97824 (3.56101)
Rotationally and tidally distorted models								
7.	0.02	0.20	2.05099 (2.0533)	1.68159 (1.6831)	1.37194 (1.38287)	5.96989 (6.0025)	13.3894 (14.1665)	2.72641 (2.83128)
8.	0.1	0.20	2.07434 (2.0770)	1.69422 (1.6960)	1.39368 (1.40507)	6.03241 (6.0664)	13.6324 (14.4394)	2.75914 (2.86744)
9.	0.2	0.01	2.08954 (2.1031)	1.70239 (1.7104)	1.40796 (1.43057)	6.07323 (6.1405)	13.7923 (14.7611)	2.78053 (2.91036)
Synchronous Binary Systems								
10..	0.525	0.05	2.21768 (2.2032)	1.77076 (1.7646)	1.53119 (1.53024)	6.42047 (6.4240)	15.1802 (16.0220)	2.96312 (3.07460)
11.	0.55	0.10	2.23042 (2.2214)	1.77719 (1.7744)	1.54347 (1.54860)	6.45338 (6.4754)	15.3141 (16.2558)	2.979930 (3.10452) .

Table 3.5: Volumes and Surface areas of rotationally and tidally distorted polytropes

PROGRAM: TO COMPUTE THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODEL

```
#include<stdio.h>

#include<conio.h>

#include<math.h>

# define max  230

double f11(double,double);

double f12(double,double);

double f(double,double);

double volume();

double theta(double);

double dtheta(double);
```

```
double gbarinverse(double);

double surface();

double g1();

double fun(double,double,double);

double gun(double,double,double,int);

double n1, q, r, k, N, shi, zu1, D;

double  rr[6],c[6],a,b,t1,t2;

double  i,xz[max],drz[max],h=0.005,rs,f2[max],f1[max],df2[max],r0[max],u[max],w[max],

a1, a2, a3, a4, a5;

void main()

{

double value2,check,value1,value3,value4,value5,value6;

double y0,u0,t=0.005,k1,k2,k3,k4,m1,m2,m3,m4,h1=.01,xx[max],th[max],y;

int count=0,j=0,m;

primtf("enter the value of  n1, q, r, k, N, zu1");

scanf("%lf %lf %lf %lf %lf", &n1, &q, &r, &k, &N, &zu1);

D=zu1/k;
```

THESE ARE THE POINTS OF GAUSS-LEGENDRE AND CHEBYSHEV FORMULA

```
c[1]=0.2369268850;

c[2]=0.4786286705;

c[3]=0.5688888889;

c[4]=0.4786286705;

c[5]=0.2369268850;
```

```
rr[1]=0.9061798459;

rr[2]=0.5384693101;

rr[3]=0;

rr[4]=-0.5384693101;

rr[5]=-0.9061798459;

t1=(5+sqrt(5))/8.0;

t2=(5-sqrt(5))/8.0;
```

THIS PART COMPUTE DISTORTED PARAMETER RSI,U,W BY DIRECT COMPUTING THE FORMULA FOR VOLUME,SURFACE AREA AND AVERAGE OF GRAVITY

```
for(i=0.005;i<=1+h1;i=i+.005)

{

r=i;

zu=(1.0/r)+q;

value1=volume();

value2=(3*value1)/(16*atan(1));

value3=surface();

rs=pow(value2,1.0/3);

value4=16*atan(1)*rs*rs;

u[j]=value3/value4;

value5=gbarinverse(value3);

value6=value5/(pow(rs,2));

w[j]=value6;

r0[j]=i;
```

```
xz[j++]=rs;

count++;

}
```

THIS PART COMPUTE DERIVATIVE OF RSI W.R.T.X

```
drz[0]=(xz[1]-xz[0])/h;

for(m=1;m<count-1;m++)

drz[m]=(xz[m+1]-xz[m-1])/(2*h);

drz[count-1]=(xz[count-1]-xz[count-2])/h;
```

THIS PART COMPUTE F1 AND F2 (DISTORTED PARAMETERS

```
for(m=0;m<count;m++)

{

f1[m]=pow(xz[m],2)*drz[m]/(pow(D,3)*pow(r0[m],2));

f2[m]=D*drz[m]*pow(r0[m],2)/(pow(xz[m],2)*u[m]*w[m]);

}
```

THIS PART COMPUTE DERIVATIVE OF F2

```
df2[0]=(f2[1]-f2[0])/h;

for(m=1;m<count-1;m++)

df2[m]=(f2[m+1]-f2[m-1])/(2*h);

df2[count-1]=(f2[count-1]-f2[count-2])/h;
```

TO INITIAL THE THSI AND ZSI

```
m=0;

j=0;

count=0;
```

```
printf("the w is %lf",w[count-2]);

y0=theta(.005);

u0=dtheta(.005);

th[0]=y0;

xx[0]=u0;
```

USE RUNGE- KUTTA FOURTH ORDER METHOD TO COMPUTE THSI AND ZSI

```
while((y0)>=0.0)

{

k1=h1*fun(t,y0,u0);

m1=h1*gun(t,y0,u0,m);

k2=h1*fun(t+h1/2,y0+k1/2,u0+m1/2);

m2=h1*gun(t+h1/2,y0+k1/2,u0+m1/2,++m);

k3=h1*fun(t+h1/2,y0+k2/2,u0+m2/2);

m3=h1*gun(t+h1/2,y0+k2/2,u0+m2/2,m);

k4=h1*fun(t+h1,y0+k3,u0+m3);

m4=h1*gun(t+h1,y0+k3,u0+m3,++m);

y0=y0+(k1+2*k2+2*k3+k4)/6;

u0=u0+(m1+2*m2+2*m3+m4)/6;

t=t+h1;

j++;

th[j]=y0;

xx[j]=t;

count++;
```

```
if(count%10==0)

printf("y0=%lf  u0=%lf t=%lf\n",y0,u0,t);

}
```

USE LINEAR INTERPOTATIONAL TO GET THSI CORRECT TO FIVE DECIMAL PLACES

```
y=.0000005;
t=(y-th[j])*xx[j-1]/(th[j-1]-th[j])+(y-th[j-1])*xx[j]/(th[j]-th[j-1]);

printf("the value is %lf",t);

}

double fun(double a,double b,double c)

{

return(0*a+0*b+c);

}

double gun(double a,double b,double c,int m)

{

float z,store;

z=(pow(zu1,2)*f2[m]*pow(fabs(b),N)*f1[m])/pow(k,2);

store=(df2[m]/f2[m])-(2./a);

store=store*c;

z=store-z;

return(z);

}
```

THIS PART EVALUATE THE VOLUME OF THE STAR

```
double  volume()
```

```
{
int j;
double ans1=0.0;
for(j=1;j<=5;j++)
{
a1=f(rr[j],-sqrt(1.0-rr[j]*rr[j])*sqrt(t1));
a2=f(rr[j],sqrt(1.0-rr[j]*rr[j])*sqrt(t1));
a3=f(rr[j],0.0);
a4=f(rr[j],-sqrt(1.0-rr[j]*rr[j])*sqrt(t2));
a5=f(rr[j],sqrt(1.0-rr[j]*rr[j])*sqrt(t2));
ans1=ans1+c[j]*(a1+a2+a3+a4+a5);
}
ans1=pow(D,3)*ans1*8*atan(1)/15.0;
return(ans1);
}
double f(double l,double v)
{
double a,b,r1;
r1=r;
step1:
a=sqrt(1-2*l*r1+r1*r1);
a=1.0/a;
b=shi-q*(a-l*r1)-n1*r1*r1*(1.0-v*v);
```

```
b=1.0/b;

if(fabs(b-r1)>.0000005)

{

r1=b;

goto step1;

}

return(r1*r1*r1);

}

double  theta(double x)

{

float  store1,store2,store;

store1=-(pow(k,2)*pow(x,2))/6+(N*pow(k,4)*pow(x,4))/120-(2*n1*pow(k,2)*pow(x,5))/15-
pow(k,6)*N*(8*N-5)*pow(x,6)/(3*50
40)+(pow(k,4)*N*n1*pow(x,7))/70;

store2=(pow(k,8)*N*(122*N*N-183*N-70))/(9*362880)-
(pow(k,2)*(3*q*q+2*n1*q+8*n1*n1))/36;

store2=store2*pow(x,8);

store=1.0+store1+store2;

return(store);

}

double  dtheta(double x)

{

float  store1,store2,store;
```

```
store1=-(pow(k,2)*pow(x,1))/3+(N*pow(k,4)*pow(x,3))/30-(2*n1*pow(k,2)*pow(x,4))/3-
(pow(k,6)*N*(8*N-5)*pow(x,5))/(2520)+(pow(k,4)*N*n1*pow(x,6))/10;

store2=(pow(k,8)*N*(122*N*N-183*N-70))/(9*362880)-
(pow(k,2)*(3*q*q+2*n1*q+8*n1*n1))/36;

store2=8*store2*pow(x,7);

store=store1+store2;

return(store);

}
```

THIS PART EVALUATE THE SURFACE AREA OF THE STAR

```
double  surface()

{

int j;

double ans1=0.0;

for(j=1;j<=5;j++)

{

a1=f11(rr[j],-sqrt(1.0-rr[j]*rr[j])*sqrt(t1));

a2=f11(rr[j],sqrt(1.0-rr[j]*rr[j])*sqrt(t1));

a3=f11(rr[j],0.0);

a4=f11(rr[j],-sqrt(1.0-rr[j]*rr[j])*sqrt(t2));

a5=f11(rr[j],sqrt(1.0-rr[j]*rr[j])*sqrt(t2));

ans1=ans1+c[j]*(a1+a2+a3+a4+a5);

}

ans1=pow(D,2)*ans1*8*atan(1)/5.0;

return(ans1);
```

```
}

double f11(double l,double v)

{

double a,b,r1;

r1=r;

step1:

a=sqrt(1-2*l*r1+r1*r1);

a=1.0/a;

b=shi-q*(a-l*r1)-n1*r1*r1*(1.0-v*v);

b=1.0/b;

if(fabs(b-r1)>=.0000005)

{

r1=b;

goto step1;

}

return(r1*r1);

}
```

THIS PART EVALUATE THE INVERSE OF GRAVITY OF THE STAR

```
double  gbarinverse(double value7)
{

int j;

double ans1=0.0;

for(j=1;j<=5;j++)

{
```

```
a1=f12(rr[j],-sqrt(1.0-rr[j]*rr[j])*sqrt(t1));

a2=f12(rr[j],sqrt(1.0-rr[j]*rr[j])*sqrt(t1));

a3=f12(rr[j],0.0);

a4=f12(rr[j],-sqrt(1.0-rr[j]*rr[j])*sqrt(t2));

a5=f12(rr[j],sqrt(1.0-rr[j]*rr[j])*sqrt(t2));

ans1=ans1+c[j]*(a1+a2+a3+a4+a5);

}

ans1=pow(D,2)*ans1*8*atan(1)/(5.0*value7);

return(ans1);

}

double f12(double l,double v)

{

double a,b,r1,r2,wl,wv,wr,wn,store;

r1=r;

step1:

a=sqrt(1-2*l*r1+r1*r1);

a=1.0/a;

b=shi-q*(a-l*r1)-n1*r1*r1*(1.0-v*v);

b=1.0/b;

if(fabs(b-r1)>=.0000005)

{

r1=b;

goto step1;}
```

```
store=(1-2*l*r1+r1*r1);

store=pow(store,1.5);

store=1.0/store;

r2=1-v*v;

wr=2*n1*r1*r2+q*(l-r1)*store-q*l-(1.0/(r1*r1));

wl=q*store*r1-q*r1;

wv=-2*v*n1*r1*r1;

wn=pow(wr,2)+pow(wl,2)*(1-l*l)/(r1*r1)+(r2*pow(wv,2))/(r1*r1)-(2*l*v*wl*wv)/(r1*r1);

wn=sqrt(wn);

wn=(1.0/wn)*D*D;

return(r1*r1*wn);}
```

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