Image Texture Enhancement Through an Improved Grunwald-Letnikov Fractional Differential Mask

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Submitted By:-

VISHWADEEP GARG

Roll no. 800961022

Under the guidance of:-

Dr. KULBIR SINGH

Associate Professor, ECED

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING,

THAPAR UNIVERSITY

PATIALA -147001, PUNJAB, INDIA

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CERTIFICATE

I, Vishwadeep Garg hereby certify that the work which is being presented in this thesis entitled "**Image Texture Enhancement Through an Improved Grunwald-Letnikov Fractional Differential Mask**" by me in partial fulfilment of the requirement of the award of degree of Master of Engineering in Electronics and Communication Engineering from Thapar University, Patiala is authentic record of my own work carried under the supervision of Dr. Kulbir Singh and referred other researcher's work which are duly listed in reference section

The matter presented in this report has not been submitted in any University/Institute for the award of Masters of Engineering

(Vishwadeep Garg)

Signature of Student

Date:- 29/12/2011

This is certified that the above statement made by the candidate is correct to the best of my knowledge.

(Dr Kulbir Singh)

(Supervisor)

Date:- 29/12/2011

(Dr Rajesh Khanna)

Head of Department

ECED, T.U. Patiala

(Dr S.K Mohapatra)

Dean of Academic Affairs

T.U. Patiala

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ABSTRACT

The texture of an image is used to discriminate between segmented regions or to classify them but during image acquisition these textural features loses its contrast and gets blurred due to some natural or artificial effects. Image enhancement techniques therefore are required to emphasize and sharpen these textural features of image. The types of techniques that can sharpen texture features include point operations, where each pixel is modified according to particular equation; mask operation where each pixel is modified according to value of pixel's neighbours. The mask operations used till date are based on integral order derivatives, these operators can enhance the high frequency information but the low frequency information like texture do not get preserved using these techniques. In order to overcome this problem concept of Fractional Differential is given. Recently, there have been lot of interest in employing Fractional Differential in various image enhancement applications like remote sensing and navigation, for segmentation or texture enhancement. Based on classical definition of Fractional Calculus many types of Fractional Differential Filter masks are developed which can improve the image texture.

In this thesis, an improved Fractional Differential filter mask is designed which provides better feature enhancement. The fractional differential mask proposed in the work is derived from the definition of Grunwald-Letnikov Fractional Differential concept. The capability of this fractional differential mask is analyzed, by varying the different parameters of the fractional differential mask like intensity factor, fractional differential order and size of mask and processing the different types of images with this Fractional Differential mask and the effect of these processed images on Information Entropy and Average Gradient of image is presented.

The proposed filter is then compared with other Fractional Differential filter mask by implementing other Fractional Differential masks to five images and their effects on image parameters are analyzed. It is shown that the proposed mask gives better performance in terms of Information Entropy by 0.5 than Grunwald-Letnikov and Riemann-Liouville Fractional Differential Mask.

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CHAPTER 1: INTRODUCTION

1.1 Preamble

Images provide visual representation of the content that is to be examined and allow the users to reflect on them later. They are a powerful data collection medium that is stored easily and used indefinitely. With the advent of digital imaging, a whole new set of possibilities have opened up for professional and amateur users. The amateur users can now easily snap, store, edit and share images while researchers and professional users rely on them to identify areas of interest, scrutinize details and present their findings effectively.

Image Enhancement transforms images to provide better representation of the subtle details. It is an indispensable tool for researchers in a wide variety of fields including (but not limited to) medical imaging, art studies, forensics and atmospheric sciences. It is application specific: a technique suitable for one problem might be inadequate for another. For example forensic images/videos employ techniques that resolve the problem of low resolution and motion blur while medical imaging employ techniques for the texture enhancement.

1.2 Image Texture

Texture is one of the important characteristic used in identifying objects or region of interest in an image, whether the image is photomicrograph, an aerial photograph or satellite image. Texture is a combination of repeated patterns with a regular frequency. Texture is an innate property of virtually all surfaces- grain of wood, the weave of a fabric, the pattern of crop in a field etc [1]. Image textures are complex visual patterns composed of entities or regions with sub-patterns with the characteristics of brightness, colour, shape, size, etc. An image region has a constant texture if a set of its characteristics are constant, slowly changing or approximately periodic. The texture of images refers to the appearance, structure and arrangement of the parts of an object within the image [2]. Texture can be evaluated as being fine, coarse or smooth; rippled, molled, irregulated or lineated. It can be seen in all images from multispectral scanner images obtained from aircraft or satellite platforms (which the remote sensing community analyzes) to microscopic images of cell cultures or tissue samples (which the biomedical community analyzes) [3]. It contains important information about structural arrangement of surfaces and their relationship to surrounding elements.

1.3 Image Texture Enhancement

The textural properties of an image appears to carry useful information for discrimination purpose, so it is important to extract textural features of images and discuss the usefulness of these features for discriminating between different kinds of image data. And inorder to efficiently extract the textural features of image, image enhancement techniques are required which can improve or enhance the textural features of image. Texture enhancement is used in many areas such as medical image processing, pattern recognition, image restoration, robotics, interpretation of image data, remote sensing and so on. It aims at improving the visual effect of image through purposefully emphasizing local or whole characteristics features of image and impairing the characteristics which are not interested in. So the quality of image would be improved and the useful information would be enriched.

Despite a family of techniques has been greatly developed over the last couple of decades, there are only a few reliable methods for texture-enhancing are presented. Multiresolution techniques [4]-[6] seem to be attractive approaches for many applications. Texture enhancement is implemented by image sharpening, which enhanced edges of image or by contrast enhancement. The image sharpening methods are used to enhance fine detail (texture) or to enhance details that have been blurred. Common image sharpening techniques deployed are

- Unsharp masking
- High boost filtering
- Derivative filters: Two types of derivative operators used are
  - First order derivative filters: First order derivative filters are Roberts, Sobel, Prewitts.
  - Second order derivative filter: Second order derivative filter is Laplacian and Laplacian of Gaussian filter

Two common types of contrast enhancement techniques are discussed in [7] these are

- Linear Contrast Enhancement: It includes Min-Max Linear Contrast Stretch, Percentage Linear Contrast Stretch, Piecewise Linear Contrast Stretch
- Non-Linear Contrast Enhancement: It involves Histogram Equalization, Adaptive Histogram Equalization, Homomorphic Filtering

In 1986 John Canny proposed an improved edge detection operator [8] which is now known as Canny operator. Another non linear edge detector for feature extraction is Kirsch operator.

1.4 Fractional Calculus

Fractional Calculus is a field of mathematic study that grows out of the traditional definitions of the calculus integral and derivative operators in much the same way fractional exponents is an outgrowth of exponents with integer value. The Fractional Calculus (FC) is a generalisation of the traditional calculus that leads to similar concepts and tools, but with a much wider applicability. In a letter dated September 30th, 1695 L'Hopital wrote to Leibniz asking him about a particular notation he had used in his publications for the nth-derivative of the linear function $f(x) = x$, $D^n f(x)/D^n x$. Leibniz's response: "An apparent paradox, from which one day useful consequences will be drawn" [9]. In these words fractional calculus was born.

Fourier, Euler, Laplace are among the many that dabbled with fractional calculus and the mathematical consequences [10]. Many found, using their own notation and methodology, definitions that fit the concept of a non-integer order integral or derivative. In different definitions of fractional calculus are considered and fractional differential of some special kind of functions are given. The most famous of these definitions that have been popularized in the world of fractional calculus are the Riemann-Liouville (R-L) and Grunwald-Letnikov (G-L) definition. Caputo reformulated the more classic definition of the Riemann-Liouville fractional derivative in order to use integer order initial conditions to solve his fractional order differential equations. As recently as 1996, Kolowankar reformulated again, the Riemann-Liouville fractional derivative in order to differentiate no-where differentiable fractal functions
Some of the reasons why fractional calculus is catching on are [11]

- There is evidence that most of the biological signals have spectra that do not increase or decrease by multiples of 20 dB/decade. This happens, for example, with ECG, speech, music, etc. The electric power line is a channel with such characteristics.
- The long-range processes like (1/f noise sources) the fractional Brownian motion (fBm) is the most famous have attracted attention because of their importance in many practical systems. Although there are several methods for the analysis and synthesis of such signals, for example, using wavelets, modelling done with fractional derivatives has proven most efficient.

- There is the famous Curie law stating that the current in an insulator increases proportionally to a negative power of the time. This leads to the known ‘‘supercapacitors’’ that have impedance of the form $1/\omega^{\alpha}$, with $0 < a < 1$. Electrochemists have used the Constant Phase Elements (CPE) description for over 60 years. The new terminology is ‘‘fractance’’ to indicate an impedance with fractional order response. As these devices become available commercially, many of the rules for design of filters and controllers are written.

There is a long and growing list of practical applications for the increased power of the fractional calculus. These include [11]

- Thermal engineering,
- Acoustics
- Electromagnetism
- Control
- Robotics
- Viscoelasticity
- Edge detection

1.5 Objective of Thesis

1. To discuss an improved G-L Fractional Differential Mask, derived from basic G-L fractional differential definition, using Lagrange 3-point interpolation.
2. To enhance the textural features of image using this G-L Fractional Differential Mask and analyze its texture enhancing capability using parameters viz. information entropy and average gradient
3. Comparative analysis of the proposed texture enhancement technique with other fractional Differential Filter techniques

1.6 Organization of Thesis

The thesis consist of six chapters organized as below

Chapter 1: Introduction, it consist of introduction to texture, image texture enhancement and fractional calculus

Chapter 2: Literature review, study of research papers of related field in sequence has been discussed.

Chapter 3: Image Texture Enhancement, in this chapter various kinds of image enhancement techniques are discussed.

Chapter 4: The Fractional Differentiation, in this G-L based fractional differentiation is discussed, then a physical and geometric meaning of fractional differentiation is studied and its texture enhancing capability is discussed.

Chapter 5: Image Texture Enhancement Using an Improved Fractional Differential, in this an improved Fractional Differential equation is discussed and using this equation a Fractional Differential Mask is designed.

Chapter 6: Results and Discussions, in this chapter all the results simulated in MATLAB-2007. The proposed Fractional Differential is applied on different image and enhanced images are analyzed using various parameters.

Chapter 7: Conclusion and future scope, in this chapter whole work has been concluded on basis of results obtained and future scope has been given.

CHAPTER 2: LITERATURE REVIEW

A collection of numerical algorithms based on fractional calculus is given by K. Diethelm et.al [12]. Fractional-Integral based on Riemann-Liouville definition is considered and a Caputo type Fractional Differential Equations and algorithm for deriving fractional derivatives is discussed. It is shown that the analytic solution to fractional differential equations is expressed in terms of Mittag-Leffler function and algorithm to find this function is considered.

D. Cafagna [13] describes fractional calculus by considering three definitions based on G-L, Riemaan-Liouville and Caputo and giving there expression for fractional derivative and fractional integral. Fractional calculus application in field of biomedical and control engineering is discussed. Two main classes of methods for solving fractional differential equations: the frequency-domain methods and the time-domain methods are presented.

The relation showing the equality of G-L and generalised Cauchy derivatives is presented by M.D Ortigueira [14] . This establishes a bridge between two different formulations and simultaneously between the classic integer order derivatives and the fractional ones. Starting from the generalised Cauchy derivative formula, new relations are obtained. From the regularised derivative, new formulations are deduced and specialised first for the real functions and afterwards for functions with Laplace transforms obtaining the definitions proposed by Liouville. With these tools suitable definitions of fractional linear systems are obtained.

I. Poldubny [15] suggests a matrix form representation of discrete analogues of various forms of fractional differentiation and fractional integration. The numerical differentiation of integer order and the *n*-fold integration is unified using the so-called triangular strip matrices. Applied to numerical solution of differential equations, it also unifies the solution of ordinary integer- and fractional-order differential equations, and of fractional integral equations. The suggested approach leads to significant simplification of the numerical solution of fractional integral and differential equations.

Using fractional calculus a novel approach for speech signal modelling is presented by Khaled Assaleh et.al [16]. Speech signal modelling using Linear Predictive Coding is discussed. By using a few integrals of fractional order as basis function, the speech signal can be modelled accurately. It is seen that it requires less model parameters and hence is preferred over LPC

J. Lu et.al [17] proposed a new method of iris localization using fractional calculus phenomena. Image is binarized to get pupil area and finding its coarse center. Then intensity derivative is used to find coarse outer boundary and then obtain four edge points through four directions. Finally, four edge points are compared to find out outer boundary.

J. A Tanereiro Machado [18] introduces the fundamental aspects of the theory of fractional calculus and discusses its application in systems engineering. Based on different definitions fractional order derivatives and integrals are given. Two methods for implementing fractional-order derivatives, namely the frequency-based and the discrete-time approaches is presented using electric recursive circuits.

The masking operator based on basic definition of Fractional order calculus is presented by Chung-Li Fan et.al [19] and applied to fingerprint images. This masking operator can enhance the image quality according to the requirements. It is seen that this method has a better filtering capability without destroying the useful information contained in the edges. At the same time, it makes up for the shortcomings of the traditional method which cannot change the treatment effects continuously. In general, this method is a simple and effective way in image enhancement.

Yocef Ferdi [20] describes the application of fractional calculus to biomedical signal processing for enhancing useful information. Three types of digital filters are considered, namely, lowpass differentiation filter, smoothing filter, and $1/f^{\beta}$ noise generation filter

Based on the perceptual approach a texture enhancement algorithm is given by Shen-Chuan Tai et.al [21]. To improve the overshoot and undershoot problem, the Texture Detail Enhancement Algorithm (TOEA) consists of three major groups: (1) Clipped Median Filter (2) Sobel Filter and (3) Edge-weighted Contrast Enhancement i.e. the image to be enhanced passes through these stages and it is observed that the algorithm not only enhance the texture detail of original image but also preserve the whole intensity of original image and it not only reserves the brighter information of original image but also enhanced the texture detail in formation which is not easy conscious by human eye.

Defect detection in digital texture images using segmentation is given by K.N. Sivabalan et.al. [22] The given image is subjected to feature extraction using various parameters such as minimum, maximum, median. After extracting features high frequency components are eliminated using median value. The extracted image and median value of each row of image is then used for identifying defected area.

A new edge detection operator based on fractional differentiation and integration is given by Yang Haibo et.al [23]. The proposed operator features a compound derivative and based on this an edge detection algorithm is given which is implemented both in 1-D and 2-D. Comparative analysis of the algorithm with Canny and CRONE detector in presence of noise and without noise shows that the proposed algorithm is better.

Yifie Pu [24] proposed a Fractional Calculus approach to texture of digital image. Using G-L definition fractional differential mask is obtained in various directions. This fractional differential mask is then implemented on an image to improve the texture features and comparison is done with integral order derivative.

Huading Jia et.al [25] put forward and discusses multiform covering templates and algorithms of fractional derivative for digital image. The computer experiment shows the textural detail enhancing capability of fractional derivative-based texture operator is much better than integral derivative-based one for rich grained digital image.

Digital Watermarking through implementation of fractional calculus is presented by Huading Jia et.al [26]. By the sampling deviation of fractional calculus sinusoidal signal implementation of fractional calculus pseudo-random sequence is considered. A watermark embedding algorithm using fractional calculus is discussed which provides security based on fractional order. The robustness of the algorithm is shown by implementing algorithm on digital image and observing that watermark extraction sequence completely depends on knowledge of fractional order and initial phrase.

Yi-Fei Pu [27] et.al proposed a texture segmentation approach which is based on fractional differential. Using the G-L definition of fractional differential a fractional differential mask is obtained and its parameters and structures are presented in eight directions. The G-L Fractional Differential is applied on various signals to show better performance of fractional differential than integral differential. This G-L based Fractional Differential Mask is then applied on texture-enriched images for values of fractional order and texture-segmentation performance using this mask is compared with other classical algorithms for texture segmentation. And it is observed that multiscale-texture segmentation of texture-enriched images using fractional differential mask is more efficient than the classical texture segmentation algorithms.

A gradient operator based on fractional differential is constructed by Zhuzhong YANG et.al [28]. First and second order integral gradient operators are discussed and using integral order differentiation filter the fractional order differential filter is deduced. Using this fractional differential filter, the fractional differential equation for 1-D signals is obtained. For digital images based on this fractional differential equation a fractional differential gradient formula is obtained in different directions and accordingly the fractional differential gradient mask is obtained in each direction. On combining these masks a Tiansi fractional differential gradient mask is constructed for x-direction and y-direction. This Tiansi gradient mask is then applied on noise and noise-free images and its is seen that with the decreasing order of fractional differential order noise is somewhat reduced and also the peak signal to noise ratio for fractional differential gradient mask is more than that of integral differential gradient mask.

Jia Changyun et.al [29] proposed an improved fractional differential algorithm which can be used to enhance images affected with poisson noise efficiently. With the help of Taylor expansion a difference expression on a one-dimensional signal is obtained and difference coefficient of fractional order differential are derived from the expression. A fractional differential template on x and y directions are obtained using fractional differential coefficients. The fractional differential template is implemented on image affected with poisson noise. It is observed that under influence of noise integral differential operator missed some edge information but the proposed algorithm performed better in edge detection and can suppress noise.

Yi ZHANG et.al [30] uses the Riemann-Liouville equation of fractional calculus for one-dimensional signal. This equation is then extended for digital images and a fractional differential mask with fractional differential order is implemented on eight symmetric directions to obtain eight fractional differential masks. For gray scale digital image filter is then convoluted respectively on eight directions using eight fractional differential masks and for coloured image fractional differential on each component is done individually and then combined. It is observed that grayscale image processed with integral differential operator obtain edges clearly but the texture features are abandoned but image processed with fractional differential mask have clear edges as well as the texture features of image are also enhanced.

Yi-Fei Pu et.al [31] implements a class of fractional differential mask for multiscale texture enhancement. Using the G-L and Riemann-Liouville definition different types of fractional differential masks are constructed for different directions. With relative error analysis best fractional differential mask is selected. Then the capability of non-linearly texture enhancement of fractional differential is analyzed. The selected fractional differential mask is then implemented on image for various values of fractional order and to describe comprehensive information of texture details Gray Level Cooccurence matrix is obtained for different values of fractional order for different angles. The proposed mask is compared with other texture enhancement algorithms and it is observed that non-linear enhancement of texture details in an image by fractional-differential based approach is more efficient than other texture-enhancement algorithms.

Yawei Liu [32] proposed a method based on fractional differential for enhancing the remote sensed images. First, the properties of fractional differential are discussed, and some important conclusions are drawn. Then, the numerical algorithm of fractional differential is studied, and an operator is constructed, by which the operation of fractional differential can be implemented through image convolution. Lastly, a strategy based on the principle of maximum entropy is given, by which the differential order can selected automatically in the process of the image enhancement. The experimental results indicate that the proposed method is effective, and has better result for remote sensing image compared with other methods.

Zhifeng Gan et.al [33] defined a fractional-differential two-dimensional discrete gradient operator based on definition of R-L fractional calculus to enhance image texture, extract more subtle texture information and overcome lack of classical gradient operator. Using the definition of Riemann-Lioville based fractional differential equation and binomially expanding the equation the i^{th} fractional differential coefficient is obtained which is value of i-layer of mask. For texture enhancement sharpening edges must add value to original pixel and a parameter called intensity factor is implied with it. Fractional differential of image is implemented by filtering it with the non-linear filter mask and the enhancement degree is controlled by intensity factor. It is shown that with increasing order of intensity factor image gets distorted. As compared to other texture-enhancement algorithms it is seen that the operator can extract more subtle texture information and sharpen edges more efficiently than the previous classical integral differential operators.

Wang Zheng et.al [34] gives an overview of fractional-order control of background and basic knowledge of mathematics. Different definitions of fractional calculus along with their numerical algorithms are discussed. Based on this definition image signal processing analysis of fractional-order characteristics are described. A mathematical model of fractional-order systems which used for analyzing and researching processing of fractional-order systems is established.

CHAPTER 3: IMAGE TEXTURE ENHANCEMENT

3.1 Introduction

Texture variation plays an important role in identifying surface anomalies in a wide variety of objects, such as bruised flesh, and in tracking motion in slowly deforming plastic surfaces, such as ocean swells. Texture also provides a visual context for the understanding of lines. The textural properties of an image appears to carry useful information for discrimination purpose, so it is important to extract textural features of images and discuss the usefulness of these features for discriminating between different kinds of image data. Due to some natural and artificial artifacts the textural features of an image gets degraded. So image processing techniques must be employed which can improve or enhance the textural features of image so that these features can be extracted for required application. Enhancement of the image texture to extract more subtle information is implemented by changing the contrast or by employing image sharpening methods.

3.2 Contrast Enhancement Techniques

Contrast enhancement is frequently referred to as one of the most important issues in image processing. Contrast is created by the difference in luminance reflected from two adjacent surfaces [7]. In visual perception, contrast is determined by the difference in the color and brightness of an object with other objects. If the contrast of an image is highly concentrated on a specific range, the information may be lost in those areas which are excessively and uniformly concentrated. The problem is to optimize the contrast of an image in order to represent all. Sometimes during image acquisition low contrast may be result due to one of the following reasons: poor illumination, lack of dynamic range in the image sensor and wrong setting of the lens aperture. The idea behind contrast stretching is to increase the dynamic range of gray levels in the image being processed. Various contrast enhancement techniques are discussed below

3.2.1 Histogram Equalization

Histogram equalization is one of the most useful forms of nonlinear contrast enhancement. The histogram of a digital image with gray levels in the range [0, L-1] is a discrete function $h(r_k) = n_k$, where r_k is the kth gray level and n_k is the number of pixels in the image having gray level r_k

[35]. It is common practice to normalize a histogram by dividing each of its values by the total number of pixels in the image, denoted by n. Thus, a normalized histogram is given by $p\{r_k) = n_k/n$, for k = 0,1,..., L-1. Thus $p\{r_k)$ gives an estimate of the probability of occurrence of gray level r_k. When an image's histogram is equalized, all pixel values of the image are redistributed so there are approximately an equal number of pixels to each of the user-specified output gray-scale classes (e.g., 32, 64, and 256). Contrast is increased at the most populated range of brightness values of the histogram (or "peaks"). It automatically reduces the contrast in very light or dark parts of the image associated with the tails of a normally distributed histogram. Figure 3.1 shows how histogram equalization of image enhances the contrast of image.

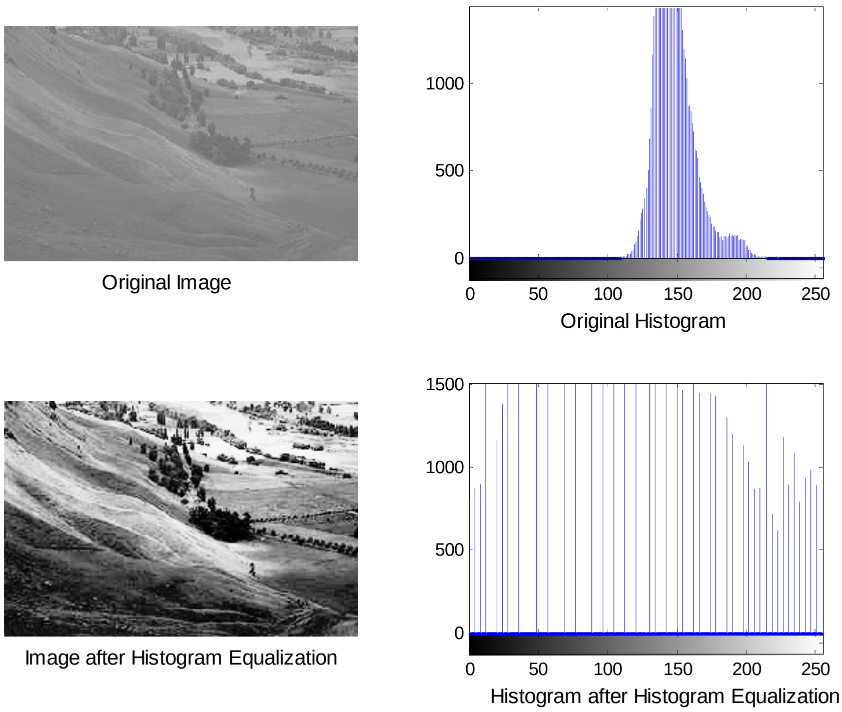


Figure 3.1: Effect of Histogram Equalization on low contrast image

Thus it is observed that histogram equalization uniformly distribute the intensity values over the full gray scale. This technique is used in image comparison processes (because it is effective in detail enhancement) and in the correction of non-linear effects.

3.2.2 Adaptive Histogram Equalization

In Adaptive Histogram Equalization the image is divided into several rectangular domains, compute an equalizing histogram and modify levels so that they match across boundaries. Adaptive Histogram Equalization (AHE) computes the histogram of a local window centered at a given pixel to determine the mapping for that pixel, which provides a local contrast enhancement [36]. Therefore regions occupying different gray scale ranges can be enhanced simultaneously. Figure 3.2 shows the effect of applying adaptive histogram equalization on a low contrast image. It is observed that adaptive histogram equalization is more suitable to bring out more detail

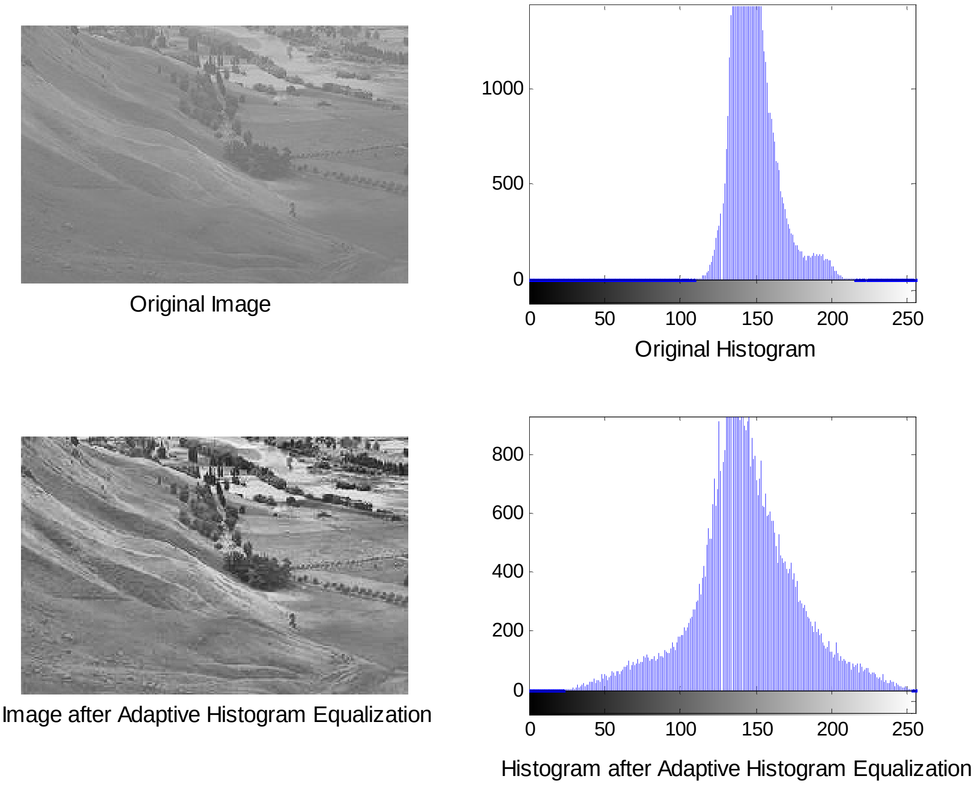


Figure 3.2: Effect of Adaptive Histogram Equalization

3.2.3 Unsharp Filtering

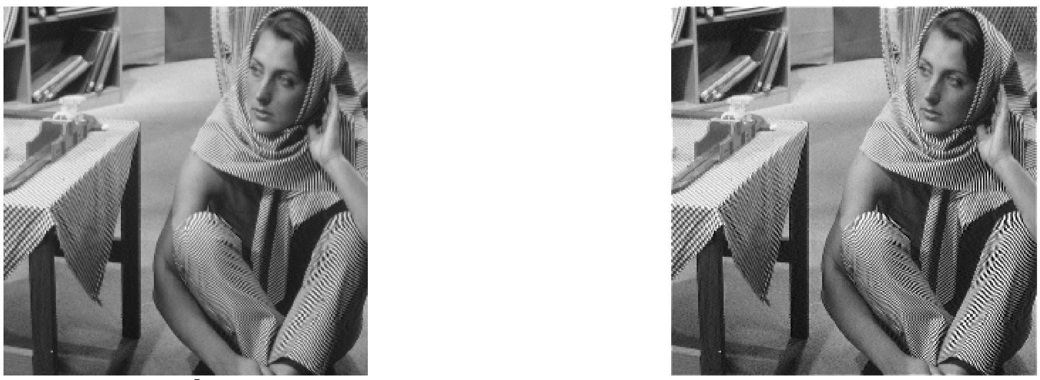
The unsharp filter is a simple sharpening operator which derives its name from the fact that it enhances edges (and other high frequency components in an image) via a procedure which subtracts an unsharp, or smoothed, version of an image from the original image. In the unsharp filtering technique, a highpass filtered, scaled version of an image is added to the image itself [37]. The prototype of unsharp filtering is defined as

$$g(x,y) = f(x,y) - f_{smooth}(x,y) \tag{3.1}$$

Where $g(x,y)$, an edge image is produced from an input image $f(x,y)$ and $f(x,y)$ is smoothed version of input image. Because all low frequency components are subtracted from the original image (*i.e.*, highpass filtered the image is obtained) only high frequency edge descriptions are obtained. Normally, it is required that a sharpening operator give us back our original image with the high frequency components enhanced. In order to achieve this effect some proportion of this gradient image is added back onto our original image. Let scaling constant factor be k hence the final enhanced image is given as

$$h(x,y) = f(x,y) + k * g(x,y) \tag{3.2}$$

Figure 3.3 shows unsharp filtering of an image and it is seen that high frequency components are enhanced



Original Image　　　　Image after Unsharp Filtering

Figure 3.3: Effect of Unsharp Filtering of an image

3.3 Image Sharpening Techniques

The principal objective of sharpening is to highlight fine detail in an image or to enhance detail that has been blurred, either in error or as a natural effect of a particular method of image acquisition. Sharpening increases the contrast around edges of objects to increase object definition. Sharpening an image is to blur it slightly. Next, the original image and the blurred version are compared one pixel at a time. If a pixel is brighter than the blurred version it is lightened further; if a pixel is darker than the blurred version, it is darkened. The result is to increase the contrast between each pixel and its neighbours. Uses of image sharpening vary and include applications ranging from electronic printing and medical imaging to industrial inspection and autonomous guidance in military systems. Types of image sharpening filters are discussed below.

3.3.1 First-Order Derivative Filter

First order derivative edge gradient is obtained by forming running difference of pixels along rows and columns of the image [38]. The gradient of a function f (x, y) at coordinates (x, y) is defined as two dimensional column vector given as

$$\nabla f \equiv grad(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \tag{3.3}$$

This vector points in the direction of the greatest rate of changes of f at location (x,y) . The magnitude (length) of vector ∇f denoted as $M(x, y)$, where

$$M(x,y) = mag(\nabla f) = \sqrt{{g_x}^2 + {g_y}^2} \tag{3.4}$$

$M(x,y)$ is referred to as the gradient image. It has the same size as the original, created when x and y are allowed to vary over all pixel locations in f. Types of first order derivative filters are

3.3.1.1 Roberts Cross Gradient Operator

It was one of the first edge detectors and was initially proposed by Lawrence Roberts in 1963 as a differential operator, the idea behind the Robert's Cross operator is to obtain Diagonal edge gradients by forming running differences of diagonal pairs of pixels [38]. The Roberts operator performs a simple, quick to compute, 2-D spatial gradient measurement on an image. It thus highlights regions of high spatial gradient which often correspond to edges. In its most common usage, the input to the operator is a grayscale image, as is the output. Pixel values at each point in the output represent the estimated absolute magnitude of the spatial gradient of the input image at that point. In theory, the operator consists of a pair of 2×2 convolution masks as shown [39]. One mask is simply the other rotated by 90°

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{bmatrix} \qquad \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{bmatrix}$$

G_x G_y

These masks are designed to respond maximally to edges running at 45° to the pixel grid, one mask for each of the two perpendicular orientations. The masks can be applied separately to the input image, to produce separate measurements of the gradient component in each orientation (call these *Gx* and *Gy*). These can then be combined together to find the absolute magnitude of the gradient at each point and the orientation of that gradient. Figure 4.5 shows the effect of applying Roberts Cross Gradient operator on an image

Original Image

Roberts Cross Gradient Operator

Figure 3.4: Image Enhancement by Roberts Cross Gradient Operator

3.3.1.2 Prewitt Operator

The Prewitt operator is used in image processing, particularly within edge detection algorithms. Technically, it is a discrete differentiation operator, computing an approximation of the gradient of the image intensity function. At each point in the image, the result of the Prewitt operator is either the corresponding gradient vector or the norm of this vector. The operator calculates the gradient of the image intensity at each point, giving the direction of the largest possible increase from light to dark and the rate of change in that direction. The result therefore shows how "abruptly" or "smoothly" the image changes at that point and therefore how likely it is that that part of the image represents an edge, as well as how that edge is likely to be oriented. The operator uses two 3 X 3 kernels which are convolved with the original image to calculate approximations of the derivatives one for horizontal changes, and one for vertical. The Prewitt 3 X 3 kernel used is given as [35]

$$\begin{bmatrix} \mathbf{-1} & \mathbf{-1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix} \qquad \begin{bmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

G_x G_y

Applying the Prewitt operator on an image will provide an edge enhanced image as shown in Figure 3.5. It is seen that contrast of edges enhanced is more than that by Roberts Operator

Original Image Prewitt Operator

Figure 3.5 Edge enhancement using Prewitt Operator

3.3.1.3 Sobel Operator

In a Sobel Operator slight variation from Prewitt operator in weight of central coefficient is done i.e. weight of 2 is used in centre coefficient. The weight value of 2 is used to achieve some smoothing by giving more importance to central point [35]. The 3×3 Sobel mask used for x and y directions are [39]

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

G_x G_y

It is seen that summation of all coefficients of the mask is zero hence in areas of constant gray level they give zero response. The effect of Sobel operator on an image is as shown in Figure 3.6.

Original Image Sobel Operator

Figure 3.6 Effect of Sobel Operator on an image

3.3.2 Second Order Derivative Operator

Second order derivatives are employed when only edge magnitudes are of interest and without regard to their orientations. There are two types of operators which hold this kind of operators these are discussed below

3.3.2.1 Laplacian Filter

Laplacian have same properties in all directions and therefore is invariant to rotation in an image. The Laplace operator is a very popular operator approximating the second derivative. The 3×3 masks for the four and eight neighbourhoods used are [35]

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Effect of applying a Laplacian filter to the image is shown in Figure 3.7

Original Image Laplacian Filtering

Figure 3.7 Effect of applying Laplacian to an image

3.3.2.2 Laplacian of Gaussian (LoG) Filter

Inorder to remove noise that occur in the Laplacian filtering, Laplacian of Gaussian filter is used. Marr-Hildreth have proposed the Laplacian of Gaussian Edge Detector [38]. Following steps are involved in LoG filtering.

- Smooth the image using Gaussian filter
- Enhance the edges using Laplacian operator
- Zero crossings denote the edge locations
- Use linear interpolation to determine sub-pixel location of edge

It is defined as

$$LoG(x,y) = -\frac{1}{\pi\sigma^4}\left[1 - \frac{x^2+y^2}{2\sigma^2}\right]e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (3.5)$$

Greater the value of σ broader is the Gaussian filter, more is the smoothing. But too much smoothing will make the detection of edges difficult. Figure 3.8 shows the effect of LoG filtering on an image

Original Image LoG Filtered

Figure 3.8 Effect of LoG filtering on an image

It can be observed that LoG filtering is less susceptible to noise unlike Laplacian so image sharpening is better using LoG filtering

CHAPTER 4: FRACTIONAL DIFFERENTIATION

The past few decades have witnessed an increasing interest in fractional derivatives, mainly due to many applications. Fractional processes defined by using fractional calculus are convenient for describing a number of problems appearing very often in applications, especially in physics, meteorology, climatology, hydrology, geophysics, economy. Fractional Differentiation is the branch of calculus that generalizes the derivative of a function to non-integer order, allowing calculations such as deriving a function to 1/2 order. The integral differentiation is considered as the special case of fractional differentiation for when the fractional order value changes to integer order values. The fractional derivative at a point *x* is a local property only when fractional order is an integer; in non-integer cases we cannot say that the fractional derivative at *x* of a function *f* depends only on the graph of *f* very near *x*, in the way that integer-power derivatives certainly do. Many definitions of fractional derivatives are given in past. The most common definition which is obtained from the integer order derivative is discussed below.

4.1 Definition

Considering the basic definition of integer order differentiation first order derivative of signal *x*(t) is given as [10]

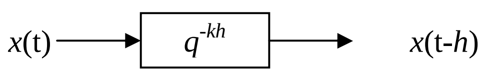
$$x'(t) = \lim_{h\to 0} \frac{x(t)-x(t-h)}{h} \tag{4.1}$$

Similarly second and third order derivative of signal *x*(t) is given by

$$x''(t) = \lim_{h\to 0} \frac{x'(t)-x'(t-h)}{h} \tag{4.2}$$

$$x''(t) = \lim_{h\to 0} \frac{x(t)-2x(t-h)+x(t-2h)}{h^2} \tag{4.3}$$

Defining a delay operator



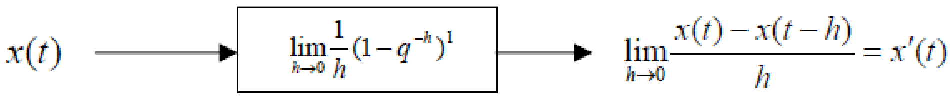
Then from the equalities

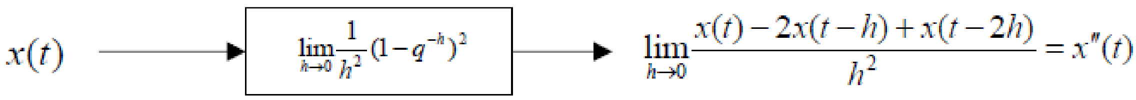
$$(1-q^{-h})^1 = 1-q^{-h} \tag{4.4}$$

$$(1-q^{-h})^2 = 1-2q^{-h}+q^{-2h} \tag{4.5}$$

$$(1-q^{-h})^m = \sum_{k=0}^{m}\binom{m}{k}(-q^{-h})^k = \sum_{k=0}^{m}(-1)^k\binom{m}{k}q^{-kh} \tag{4.6}$$

Thus we have





$$x^m(t) = \lim_{h\to 0}\frac{(1-q^{-h})^m}{h^m}x(t) \tag{4.7}$$

Changing integer *m* to fraction v from (4.7) we get

$$x^{(v)}(t) = \lim_{h\to 0}\frac{(1-q^{-h})^v}{h^v}x(t) \tag{4.8}$$

Using equation (4.7) and (4.8) the fractional derivative of signal is given as

$$x^{(v)}(t) = \lim_{h\to 0}\frac{1}{h^v}\left(\sum_{k=0}^{m}(-1)^k\binom{v}{k}q^{-kh}\right)x(t) \tag{4.9}$$

$$= \lim_{h\to 0}\sum_{k=0}^{m}h^{-v}(-1)^k\binom{v}{k}x(t-kh) \tag{4.10}$$

Equation (4.10) is a G-L derivative. Assume for all $v \in R$ (R represents the real set and $[v]$ is its integral part), the signal F (t) $\in [a,t]$, $a < t$, $a \in R$, $t \in R$, has m($m \in Z$, Z represents integer set) order continuous differentiation. When $v > 0$, m is no less than $[v]$, so v -order differentiation could be expressed as [40]

$$D^v = \lim_{h\to 0}F^{(v)}(t) = \lim_{h\to 0}\frac{h^{(-v)}}{\Gamma(-v)}\sum_{m=0}^{n-1}(-1)^m\frac{\Gamma(m-v)}{\Gamma(m+1)}F(t-mh) \tag{4.11}$$

where $\Gamma(x) = (x-1)!$ is gamma function of x

As for any quadratic integrability energy type signal that $F(t)$, the v-order fractional differential Fourier transform is

$$D^v F(t) = D_v F(t) = \frac{d^v F(t)}{dt^v} \overset{FT}{\Leftrightarrow} \left(\widehat{D}_v F\right)(\omega) = (j\omega)^v . \widehat{F}(\omega) = \hat{d}_v(\omega)\widehat{F}(\omega) \quad (4.12)$$

where v order differential operator that $\boldsymbol{D_v} = \boldsymbol{D}$ is v order differential multipliable operator of function $\hat{d}_v(\omega) = (j\omega)^v$. The fractional differential filter function can be expressed as [40]

$$\begin{cases} \hat{d}_v(\omega) = (j\omega)^v = \hat{a}_v(\omega). \exp\left(j\theta_v(\omega)\right) = \hat{a}_v(\omega). p_v(\omega), \\ \hat{a}_v(\omega) = |\omega|^v, \theta_v(\omega) = \frac{v\pi}{2}\, sgn(\omega) \end{cases} \quad (4.13)$$

4.2 Properties of Fractional Derivative

The concept of derivative is traditionally associated to an integer; given a function, we can derive it one, two, three times and so on. It can be have an interest to investigate the possibility to derive a real number of times a function. The main idea is to examine the properties of the ordinary derivative and see where and how it is possible to generalize the concepts. As often happen there is not only a way to do that; we are going to use the most intuitive and, in a certain sense, less rigorous way. Let us consider the general properties of the derivative D_t^n for $n \in N$, where n is an integer. This operator is, in fact, defined to have the following properties, all of which is applicable to the fractional derivative

(1) Associative Law: According to this property for a function f(t) and constant C

$$D_t^v[Cf(t)] = CD_t^v[f(t)]$$

(2) Distributive Law: For the function f(t) and g(t)

$$D_t^v[f(t) \pm g(t)] = D_t^v[f(t)] \pm D_t^v[g(t)]$$

(3) The operator obeys Leibniz rule for taking the derivative of product of two functions i.e.

$$D_t^v[f(t)g(t)] = \sum_{k=0}^{v} D_t^{v-k}[f(t)]D_t^k[g(t)]$$

$$= \sum_{k=0}^{v} D_t^{v-k}[f(t)]D_t^k[g(t)]$$

4.3 Physical Meaning of Fractional Differential

From (4.12), it is seen that the physical meaning of a signal's fractional differential is generalized amplitude-and-phase modulation from the viewpoint of information theory, that is to say, it is fractional phase-and-frequency modulation [40]. The results show that the amplitude of original signal is changing with its frequency as fractional power exponent, while the phase is generalized Hilbert transform of frequency.

From equation (4.11), we know that v order difference of signal is expressed as

$$\Delta^v F = \frac{1}{\Gamma(-v)} \sum_{m=0}^{n} \frac{\Gamma(m-v)}{\Gamma(m+1)} F(t-mh) \tag{4.14}$$

So, v order difference is the common difference or generalized difference. Integral differential expression can be put into (4.14), since integral difference is a special example of fractional difference. The geometric meaning of fractional derivative can be seen as generalized slope of its function curve that is called fractional slope. Fractional integral is the generalized Euclidean measurement for normal geometric image that is called fractional Euclidean measurement [40]. From kinetic viewpoints, fractional derivative is generalized derivative and the mathematic generalization for generalized flow and speed. Thus, it can be deduced that the physical meaning of fractional derivative is fractional flow or fractional speed. Generalized acceleration is included in generalized speed. Low order (when the order is in [0, 1]) fractional speed is generalized speed; it is the continuous interpolation of fractional time between zero order displacement vector and first order speed vector; it is the fractional continuous measurement for distance variation speed and direction; it is also the low order measurement for temporal balance state (when order is in [0, 1]). Low order speed shows that it has the fractional time continuous interpolation between zero order displacement and first order speed, and the difference between them is not absolute. There are not only two movement models of physical movement being discrete and continuous; it also has the third universal model that is between discrete and continuous. Gradient vector does not always orientate the maximum descending way. This is the change from linear time-space view to curve time-space view. Above first order fractional speed is generalized acceleration and the fractional time continuous interpolation between m and m+1 acceleration vector. It is the fractional continuous measurement for the speed change and its direction and also the high order measurement (when order is >1) for temporal balance state. Thus, it is said that traditional speed and acceleration is the special case for factional speed and acceleration.

Taking Gauss signal φ (t) in Figure 4.1 [40] as an example to discuss the numerical implement of any order derivative, is not losing generality. Here v = 0.00 means the original signal that is not differential or integral. The fractional differential operator has a limit, that is, $|\boldsymbol{D}^{v} s(t)| = |s^{v}(t)| < \infty$. Moreover, it is continuous, that is $\lim_{1\to 2} \boldsymbol{D}^{v1} s(t) = \boldsymbol{D}^{v2} s(t)$, $v1, v2 \in$ R. In other words, fractional differential is the continuous interpolation of neighbourhood order. It is a real number, that is, $\boldsymbol{D}^{v} s(t) \in$ R. For any v fractional differential, it has $\boldsymbol{D}^{v}$ [0] $\equiv$ 0, where [0] shows special unit leaping signal whose amplitude is always zero. As for non-zero amplitude unit leaping signal, its fractional differential is not zero. As discussed above, fractional derivative $\varphi^{v}(t)$ is fractional gradient vector. When v is integer, it orientates the maximum descending direction. Fractional derivative $\varphi^{v}(t)$ is the fractional measurement for the temporal equilibrium of the points in curve $\varphi^{v}(t)$. $\varphi^{1}(t) = 0$ is a stationary point of curve $\varphi(t)$ and the first order temporal equilibrium point, which may be a stable equilibrium point or an unstable one. It is observed that the first order derivative is not the measurement for temporal stable state of a certain point of curve, but that for its temporal equilibrium state. The temporal stable state of the point of the curve depends on the concavity or convexity of the curve or curve surface and on the co-effect of the characteristic of concavity & convexity and temporal balance state. In a broad way, the generalized stationary point corresponding to $\varphi^{v}(t) = 0$ is v order temporal fractional balanced point.

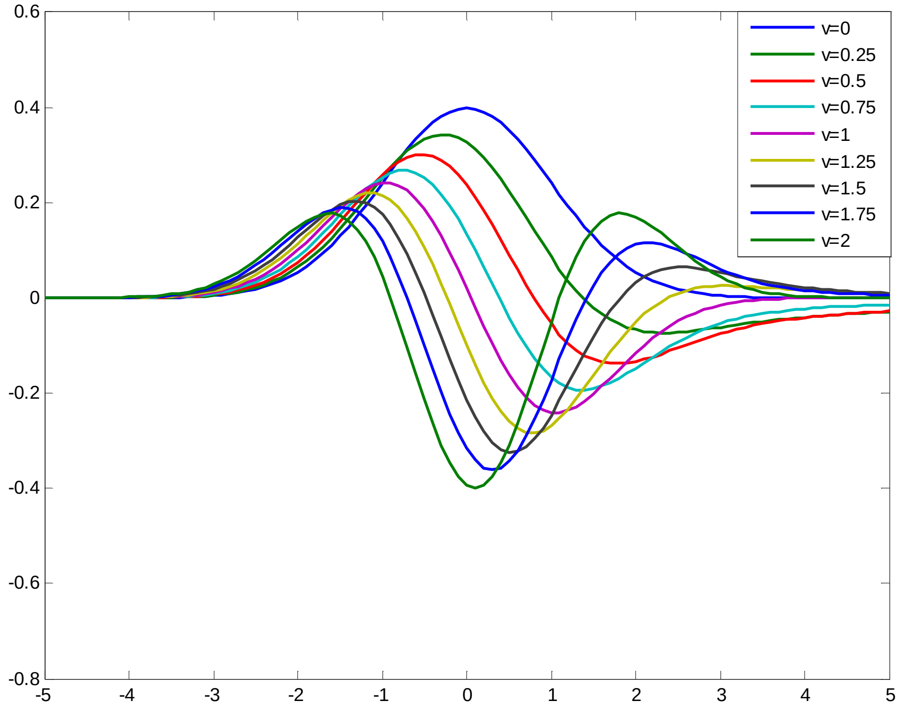


Figure 4.1: Different fractional order fractional differential of normal Gaussian signal [40]

From Figure 4.1, it is seen that the fractional differential of first order stationary point is not zero in general, and the fractional stationary points and first order stationary points are non identity. Similarly, the first order differential of the first order differential stationary point in two-dimensional curve surface is zero, but its fractional differential is not zero. So, if time t changes as -∞→0→∞, the order v of $\varphi^{v}(t) = 0$ varies as 0→1→∞ accordingly, since fractional differential is continuous. Fractional derivative indicates the high order (when order is between (1, ∞)) fractional stationary point of signal on the ascending section of signal $\varphi(t)$ when t ∈ (-∞, 0); while it indicates low order (when order is between [0, 1]) one on descending section when t ∈ (0, ∞).

In general, fractional derivative (not including integral derivative) of even symmetric signal is not symmetric and cannot keep its symmetry unchanged before and after fractional differential [40]. Thus, the stationary points of higher order fractional differential are non symmetrical. In other words, the corresponding temporal balance state of the even points on even symmetric curve is not the same, which is quite different with integral differential. At the same time, the curve of lower order fractional derivative is single apex, abrupt descending, and with long negative tail. It crosses zero line for only one time and has only one lower order (when order is between [0, 1]) stationary point. The lower fractional derivative curve, the higher one and the zero line of vertical axis may intersect at the same point. The stationary point of the higher order (when order is >1) and the lower order (when order is between [0, 1]) may be the same point. So, the fractional order v is fractional description for temporal balance state, and it is called fractional equilibrium coefficient [40].

4.4 Fractional Differential Detection for Image Texture Detail Feature

The filter function of fractional differential filter is $\hat{d}_v(\omega)$ =$(j\omega)^v$= $|\omega|^v$=$\exp\big(j\theta_v(\omega)\big)$. The amplitude characteristic is even function and phase characteristic is odd function. Thus analyzing the characteristics of fractional differential filter for $\omega > 0$. The frequency response of fractional differential filter is given in Figure 4.2.

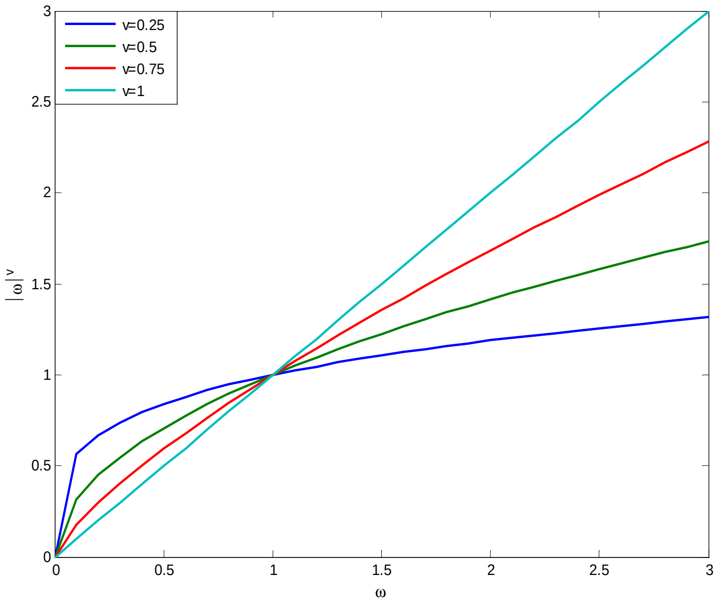
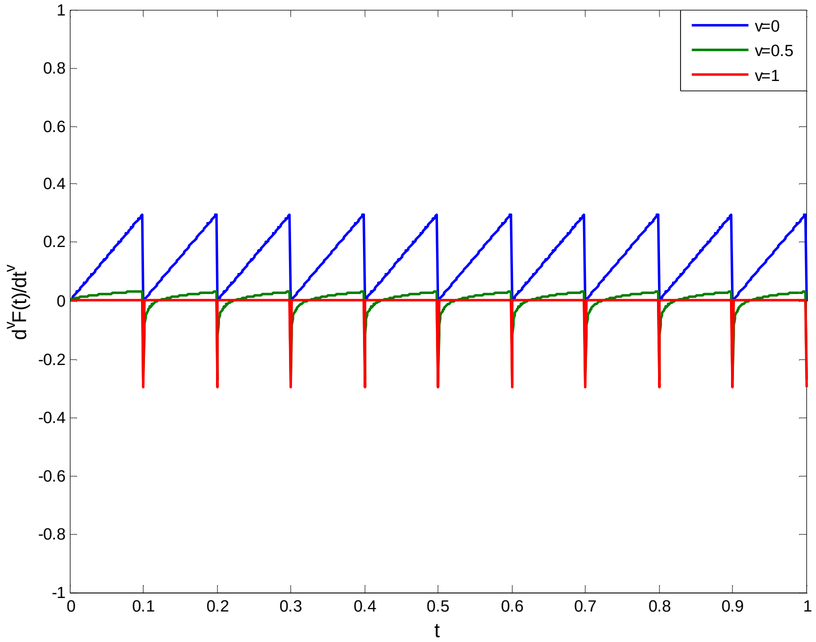


Figure 4.2: Frequency Response of Fractional Differential Filter [31]

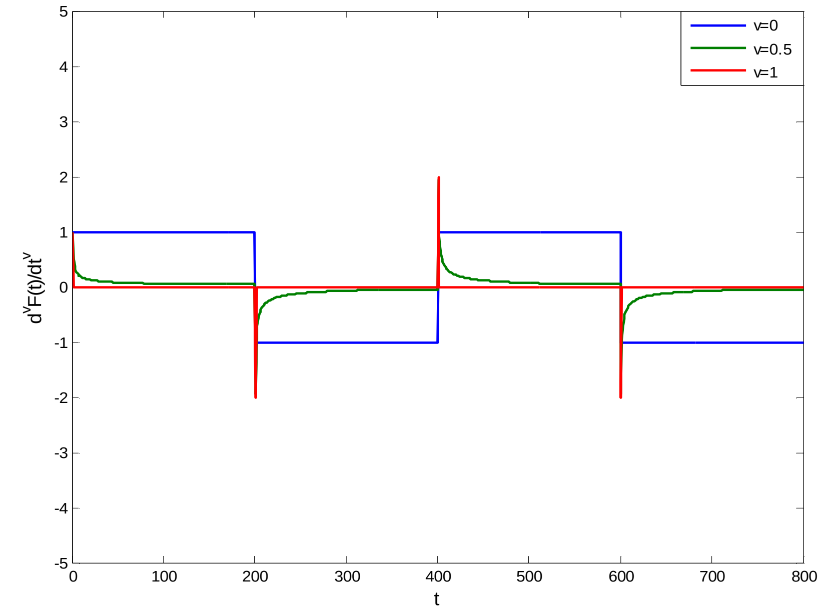
It is observed that, in viewpoints of signal processing, the frequency response of fractional differential is actually a nonlinear filter when v = 0, v-order fractional differential is all-pass filter, and its frequency response is $\hat{d}_v(\omega) = 0$ => d_v (t) = δ(t) [31]. When v < 0 it is a fractional integrator and is singular low-pass integral filter. When $v > 0$, it is fractional derivative operator, and its frequency response is $\lim_{|\omega|\to\infty}\left|\hat{d}_v(\omega)\right| \to \infty$. Here $\hat{d}_v(\omega)$, is singular high-pass differential filter. Note that, while v is increasing, the transmission bands of $\hat{d}_v(\omega)$ become narrower and high-pass characteristic is stronger. In other words, for $v > 0$, $\hat{d}_v(\omega)$ nonlinearly enhances high-frequency components of signal and nonlinearly inhibits its low-frequency components. When $0 < v < 1$, in the section of $\omega > 1$, the enhancement of high-frequency components by fractional differential is less than integral one, and the enhancement of high-frequency edge components by fractional differential is inferior to that of first-order one. However, in extremely low-frequency section of $0 < \omega < 1$, the preservation magnitude of low-frequency contour by fractional differential is superior to that by first-order one. So, fractional differential is a nonlinear attenuator while first-order differential is linear one. That is, in the extremely low-frequency section of $0 < \omega < 1$, when v becomes smaller, the attenuation of low frequency components is also less. Similarly, when $v \to 0$ it is an all-pass filter almost keeping signal unchanged.

From above-mentioned discussion, it can be said that the image's smooth area whose gray scale does not change intensively, the texture features in smooth area may be greatly attenuating and its differential result is nearly zero (the integral differential of constant is zero), when it is filtered by first-order differential-based operator such as Sobel operator, or second-order-based one such as Gauss-Laplace operator [31]. For this reason, integral differential linearly attenuates texture features and could not well hold them in such area. On the contrary, fractional differential-based operator could nonlinearly preserve textural feature in smooth area to the biggest degree. Thus, as for enhancing the texture in smooth area, fractional differential-based operator is superior to integral differential-based one.

The characteristic of fractional differential and fractional differential mask is analyzed by doing fractional differential for rectangle wave signal and saw-tooth wave signal. The results are shown in Figure 4.5



(a)



(b)

Figure 4.5: Characteristics Analyzing for Fractional Differential (a) First order and fractional differential of a Rectangular Wave (b) First order and fractional differential of a Sawtooth Wave [31]

In the Figure 4.5(a), fractional differential is from the biggest value in the singular leaping point to zero in smooth area. Note that, by default, any integral differential in smooth area approximately equals to zero (the integral differential of constant is zero), which is the remarkable difference between fractional differential and integral one. In the initial point of gray scale gradient or slope is nonzero, which nonlinearly enhances high-frequency singular information. As we know, when $v = 1$, integral differential of high-frequency singular signal is Dirac signal. When $0 < v < 1$, the enhancement of high-frequency singular information is smaller than that when $v = 1$. Therefore, integral differential is the special case of fractional differential.

In the Figure 4.5(b), the fractional differential of the slope is not zero or constant, it is a nonlinear curve. However, the integral differential of the slope is constant. From the above discussion, we can see that fractional differential could nonlinearly preserve the low-frequency contour feature in the smooth area to the furthest degree, and as well as, nonlinearly enhance high-frequency marginal information in those areas where gray scale changes frequently, and nonlinearly enhance texture details in those areas where gray scale does not change evidently.

Therefore, in brief, fractional differential could nonlinearly enhance the comprehensive texture details. When the image is processed, it requires keeping the original information, improving image quality, enhancing details and texture characteristics, and keeping the marginal details and energy as well. The requirements are difficult to obtain by traditional integral differential-based texture-enhancing algorithms, while they are easy to obtain by fractional differential-based algorithm.

CHAPTER 5: FRACTIONAL DIFFERENTIAL FILTER

Fractional differential is more efficient in image texture enhancement than the integral differential techniques in the sense that it not only enhances the high frequency components of an image but also preserve the low frequency components [31]. There are various types of fractional differential equations available, but the most common fractional differential used are G-L and R-L equations. Based on these equations fractional differential filters or masks are developed and are applied on image for texture enhancement. An improved G-L fractional differential filter is discussed below

5.1 An Improved G-L Fractional Differential Filter

From equation (4.11) i.e. G-L definition the *v*-order fractional differential of signal *F*(t) is given by

$$D^{v} = \lim_{h\to 0} F^{(v)}(t) = \lim_{h\to 0} \frac{h^{-v}}{\Gamma(-v)} \sum_{m=0}^{n-1} \frac{\Gamma(m-v)}{\Gamma(m+1)} F(t-mh) \tag{5.1}$$

To make fractional differential operator more precise we can rewrite equation (5.1) as

$$\frac{\partial^{v}}{\partial t} F(t) = \frac{h^{-v}}{\Gamma(-v)} \sum_{m=0}^{n-1} \frac{\Gamma(m-v)}{\Gamma(m+1)} F\left(t + \frac{vh}{2} - mh\right) \tag{5.2}$$

Comparing (5.1) and (5.2), (5.2) has introduced signal values of F (t) on non nodes besides supposing v = 0, ±2, ±4,..... , thus considering the three nodes $F(t+h-mh)$, $F(t-mh\,)$ and $F(t-h-mh)$ and using 3-point Lagrange interpolation expression we get

$$\begin{aligned} F(\xi) = {} & \frac{(\xi-t+mh)(\xi-t+h+mh)}{2h^2} F(t+h-mh) \\ & - \frac{(\xi-t-h+mh)(\xi-t+h+mh)}{h^2} F(t-mh) \\ & + \frac{(\xi-t-h+mh)(\xi-t+mh)}{2h^2} F(t-h-mh) \end{aligned} \tag{5.3}$$

Assuming $\xi = t + (vh/2) - mh$ and doing fractional interpolation we get

$$F\left(t+\frac{vh}{2}-mh\right) \cong \left(\frac{v}{4}+\frac{v^2}{8}\right) F(t+h-mh)$$
$$+\left(1-\frac{v^2}{4}\right) F(t-mh)$$
$$+\left(\frac{v^2}{8}-\frac{v}{4}\right) F(t-h-mh) \tag{5.4}$$

From (5.2) and (5.4) we get

$$\frac{\partial^v}{\partial t}F(t) = \frac{h^{-v}}{\Gamma(-v)}\sum_{m=0}^{n-1}\frac{\Gamma(m-v)}{\Gamma(m+1)} \times \begin{bmatrix} F_m + \frac{v}{4}(F_{m-1}-F_{m+1}) \\ +\frac{v^2}{8}(F_{m-1}-2F_m+F_{m+1}) \end{bmatrix} \tag{5.5}$$

where $F_m = F(t-mh), F_{m-1} = F(t+h-mh), F_{m+1} = F(t-h-mh)$

This is an expression of an improved G-L fractional differential

5.1.1 Design of an improved G-L Fractional Differential Filter

In general, if the image is needed to process with the nonlinear filter, the values of pixels of image is convolved with a *s*×*t* size mask. It is in the following form [32]

$$g(x,y) = \sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)f(x+s,y+t) \tag{5.6}$$

where $f(x,y)$ is a value of pixel and $w(s,t)$ is a value of mask. Considering the gradient direction, the mask is designed into an *m*×*m*-size matrix ***T*** which has *m* layers (*m* is odd natural number). There are 8 directions of ***T***, which are 0, π/8, π/4 ,3π/8, π/2,5π/8,3π/4 and 7π/8, respectively.

From (5.5) it is concluded that for $v \in (0, 1)$

$$T_i = \frac{1}{\Gamma(-v)}\left[\frac{\Gamma(i-v+1)}{(i+1)!}\left(\frac{v}{4}+\frac{v^2}{8}\right)+\frac{\Gamma(i-v)}{i!}\left(1-\frac{v^2}{4}\right)+\frac{\Gamma(i-v-1)}{(i-1)!}\left(-\frac{v}{4}+\frac{v^2}{8}\right)\right] \tag{5.7}$$

Where T_i is the value of i^{th} layer of mask, $v \in R^+$ Especially, inorder to make the sum of ***T*** equal to 0, it has

$$T_0 = -1 * \sum_{i=1}^{n} 8 * i * T_i \tag{5.8}$$

Clearly, the result of convoluting with $\boldsymbol{T}$ is the sharpening edges of the image. For the purpose of texture enhancement, the sharpening edges must add to the value of original pixel [32].

So we must change $\boldsymbol{T}$ into $\boldsymbol{R}$. It has

$$\begin{cases} R_i = \gamma T_i & ;(i > 0) \\ R_0 = (1+\gamma)T_0 & ;(i = 0) \end{cases} \tag{5.9}$$

where γ is the intensity factor. When $\gamma > 0$ and $i > 0$, R_i is negative and increased with decreasing of i, when $\gamma > 0$ and $i > 5$, R_i is almost equal to 0; when $i = 0$, R_i is positive. It means that the relativity between the central pixel and others declines when their distinct increases. It is consistent with the reality. When $v \in (0, 1)$ from (5.7) and (5.8)

$$\begin{cases} R_i = \gamma T_i & ;(i > 0) \\ R_0 = 1 - \sum_{i=1}^{n} 8 * i * R_i & ;(i = 0) \end{cases} \tag{5.10}$$

The dimension of R is an odd natural number. The mask of an improved G-L fractional differential is shown in Figure 5.1

R1	R1	R1
R1	R0	R1
R1	R1	R1

(a)

R2	R2	R2	R2	R2
R2	R1	R1	R1	R2
R1	R1	R0	R1	R1
R2	R1	R1	R1	R2
R2	R2	R2	R2	R2

(b)

Figure 5.1: The Mask of an improved G-L Fractional Differential. (a) 3 X 3 size mask (b) 5 X 5 size mask [32]

For instance, if v = 0.5 and intensity factor (γ) = 1 then from (5.7) and (5.10) a 3 X 3 size filter mask is

-0.2474	-0.2474	-0.2474
-0.2474	2.9792	-0.2474
-0.2474	-0.2474	-0.2474

Figure 5.2: The Mask of an improved G-L Fractional Differential with fractional order v = 0.5 and intensity factor (γ) = 1

It is seen from Figure 5.2 that sum of coefficients of mask is not equal to zero which is a prominent difference between fractional and integral differential.

5.1.2 Criteria for Selecting Fractional Differential Order

The fractional differential operator has been given in Fig. 5.1, and it can be used for image enhancement. The operator can enhance edges and contours, as well as reserving the texture details. But selecting the differential order v is also a difficult problem. Based on definition of image entropy, a new method is discussed in [33], by which the differential order can be selected automatically.

The entropy of a gray image I (i, j) is defined as

$$H(I) = \sum_{k=0,p(i)\neq 0}^{255} p(i) log \frac{1}{p(i)} \tag{5.11}$$

where $p(i)$,i = 0,1,...,255 is the probability distribution function of image intensity. Image entropy is a measure of the amount of information, and the larger the value of $H(I)$ is, the greater amount of information carried by image.

Mutual entropy between two images I and J can be defined as

$$H(I,J) = H(I) + H(J) - \sum_{i=0}^{255} \sum_{j=0}^{255} P(I,J) log \frac{1}{P(I,J)} \tag{5.12}$$

where $P(i, j)$ is the two-dimensional joint distribution function of I and J . The mutual entropy reflects the similarity of two images. So a reasonable selection criterion of differential order v can also be established as

$$v^* = \max_{v\in(0,1)}(H\big(I, I^{(v)}\big) \tag{5.13}$$

where $I^{(v)}$ is the enhanced image by v -order differential

CHAPTER 6: RESULTS AND DISCUSSIONS

In the thesis texture enhancing capability of an improved G-L Fractional Differential is analyzed by qualitatively comparing it with other types of texture enhancement techniques. While enhancing the texture of an image using any enhancement technique the information in the enhanced image might get decreased or increased. So the capability of any texture enhancement technique can be analyzed using two criterions

- Information Entropy: The entropy denotes measure of amount of image information. If an image has no texture its entropy is close to zero. Otherwise if image have great texture details, its entropy is greater. Entropy of gray image is given by (5.11)
- Average Gradient: It reflects the ability of contrast expression of small details, used to evaluate image clarity. More the average gradient more clearer the image is. Average Gradient of an image $F(x, y)$ is given as [41]

$$ag = \frac{1}{PQ} \times \sum_{x=1}^{P} \sum_{y=1}^{Q} \sqrt{\frac{\left(\frac{\partial F(x,y)}{\partial x}\right)^2 + \left(\frac{\partial F(x,y)}{\partial y}\right)^2}{2}} \tag{6.1}$$

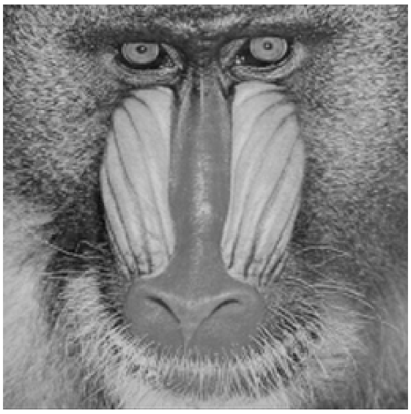
6.1 Texture Enhancement Analysis

Image texture enhancement experiment is conducted on four texture enriched images. First one is a Baboon image with high texture. The second, third, fourth and fifth are images of Bridge, Surface of Moon, Barbara,

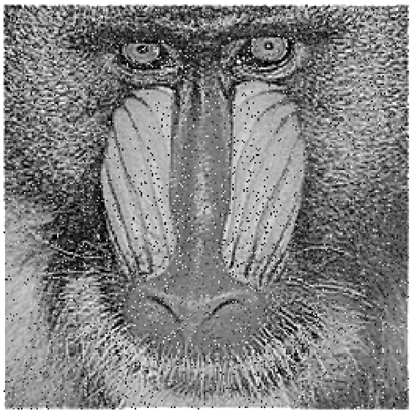
6.1.1 Effect of Varying Intensity Factor (γ)

In this texture of different images is enhanced by implementing the proposed Fractional Differential Mask of size 3 X 3 on the images and analysis is done, by varying the intensity factor of Fractional Differential Mask and keeping the fractional differential order constant and variation of information entropy and average gradient of the image is observed with variation in intensity factor.

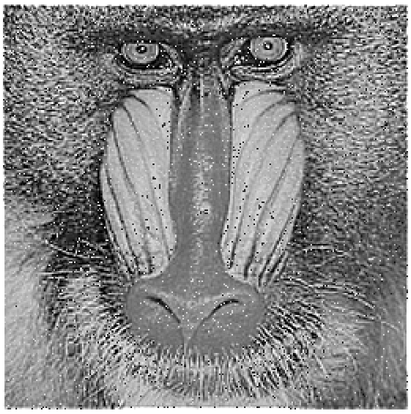
Baboon Image



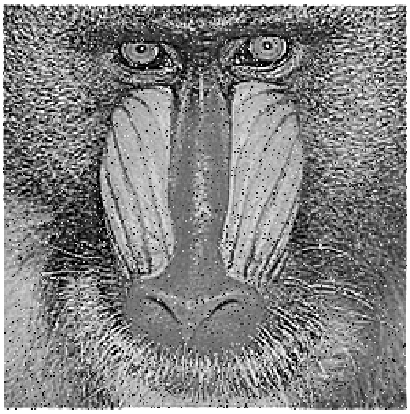
(a)



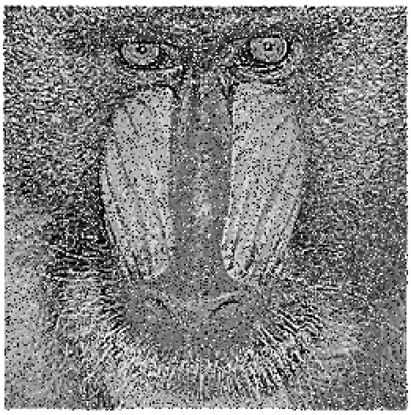
(b)



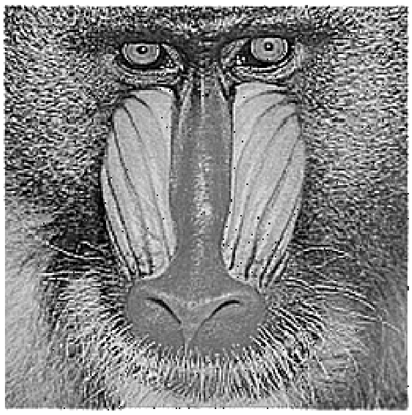
(c)



(d)



(e)



(f)

Figure 6.1: Texture enhancement of Baboon Image for fractional order ν = 0.2 for different values of intensity factor (γ). (a) Original Image, (b) γ = 1, (c) γ = 1.2, (d) γ = 1.5, (e) γ = 1.7, (f) γ = 1.9

Table 6.1: Information Entropy and Average Gradient of images in Figure 6.1

Figure	Information Entropy	Average Gradient
(a)	7.1686	9.1536
(b)	7.5715	21.2197
(c)	7.6287	23.7531
(d)	7.6815	27.5754
(e)	7.6969	30.1341
(f)	7.6966	32.6990

From Figure 6.1 and Table 6.1 it can be observed that

- With increase in the value of intensity factor (γ) the texture of an image gets enhanced.
- For the intensity factor $\gamma < 1.7$ the degree of enhancement is not deep. But at $\gamma = 1.7$, texture is more enhanced.
- With increase in intensity factor the average gradient of an image increases i.e. image gets more clearer for $\gamma <= 1.7$ but for $\gamma > 1.7$, average gradient doesn't increase much also the information entropy of the image starts decreasing i.e. with increase in intensity factor image starts losing some texture information and gets distorted
- Hence there is a trade-off between average gradient and image quality.

Image of Bridge

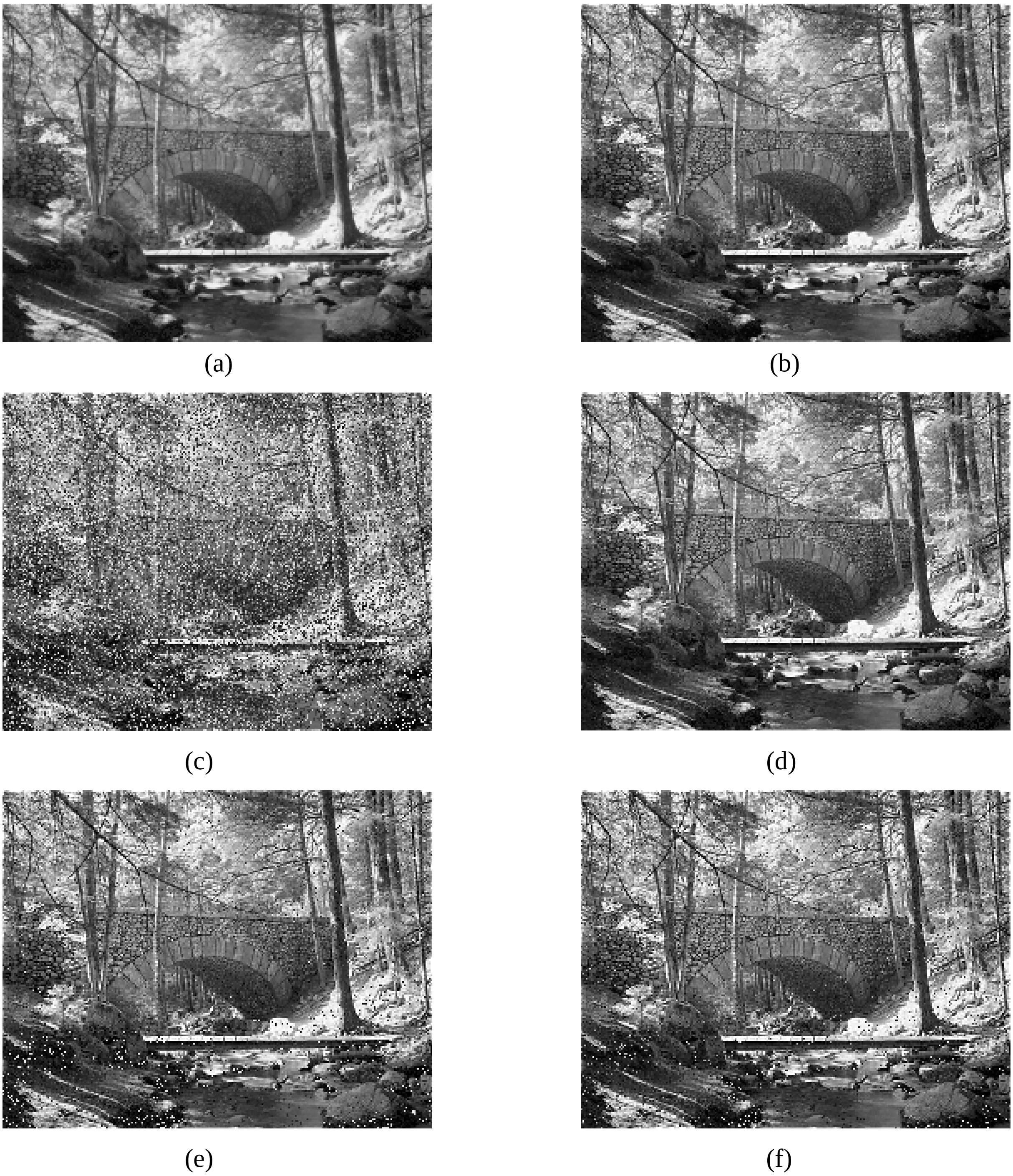


Figure 6.2: Texture enhancement of Image of Bridge for fractional order v = 0.2 for different values of intensity factor (γ). (a) Original Image, (b) γ = 1, (c) γ = 1.2, (d) γ = 1.5, (e) γ = 1.7, (f) γ = 1.9

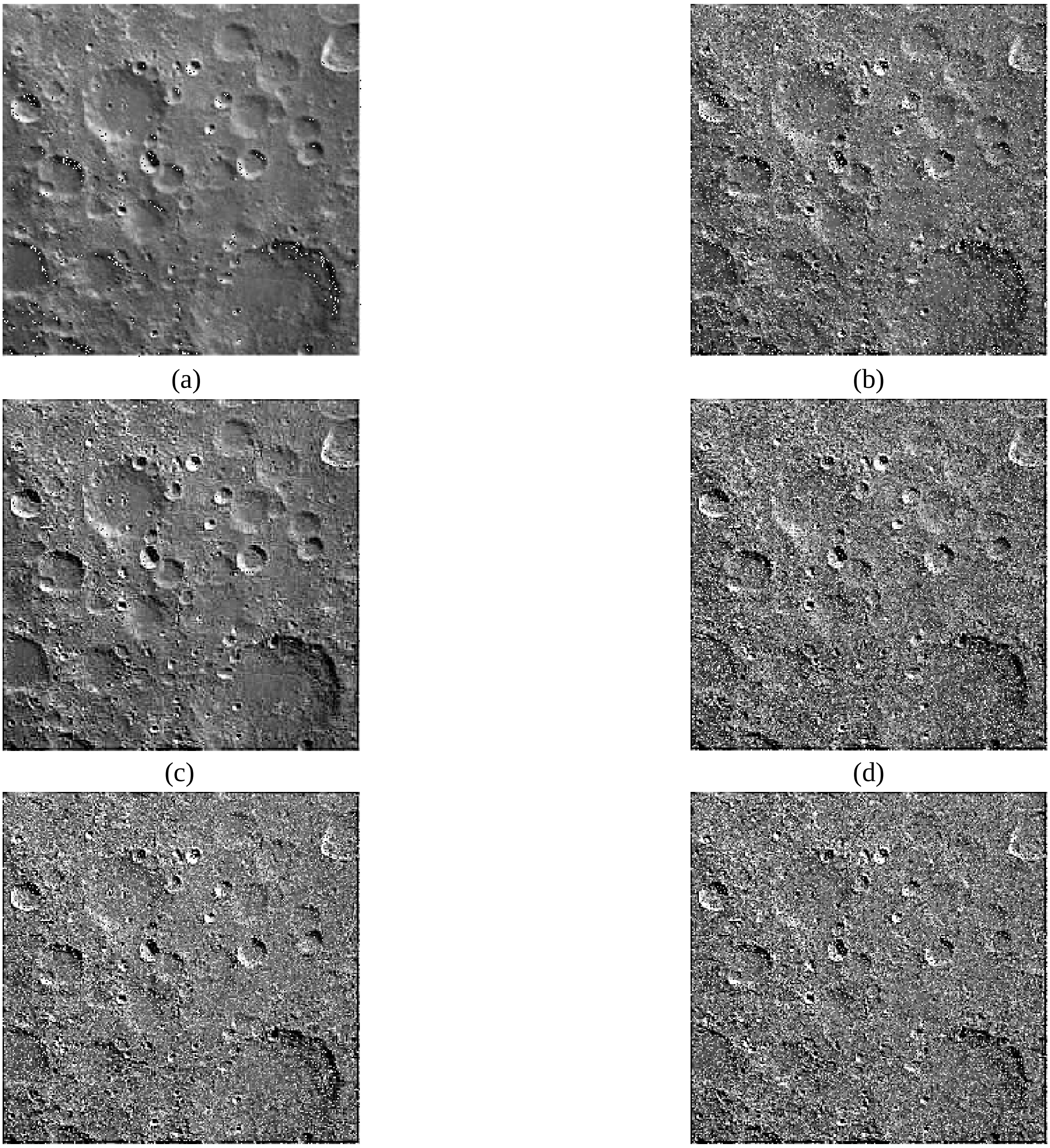
Table 6.2: Information Entropy and Average Gradient of images in Figure 6.2

Figure	Information Entropy	Average Gradient
(a)	7.7284	8.3963
(b)	7.8136	13.4902
(c)	7.8244	14.6016
(d)	7.8340	16.2984
(e)	7.8350	17.4450
(f)	7.8331	18.6010

From Figure 6.2 and Table 6.2 following conclusions can be drawn

- Texture channel becomes deeper and texture details gets clear with increase in intensity factor (γ).
- For the intensity factor $\gamma < 1.7$ the texture channel is not deep. But at $\gamma = 1.7$, texture details gets clearer.
- The average gradient of the image increases with increase in intensity factor i.e. contrast of small details gets improved but the information entropy of image gets reduced if we increase the intensity factor beyond 1.7 and results in losing texture details.

Image of Moon Surface



(a) (b) (c) (d) (e) (f)

Figure 6.3: Texture enhancement of Mars Surface with fractional order ν = 0.2 for different values of intensity factor (γ). (a) Original Image, (b) γ = 1, (c) γ = 1.2, (d) γ = 1.5, (e) γ = 1.7, (f) γ = 1.9

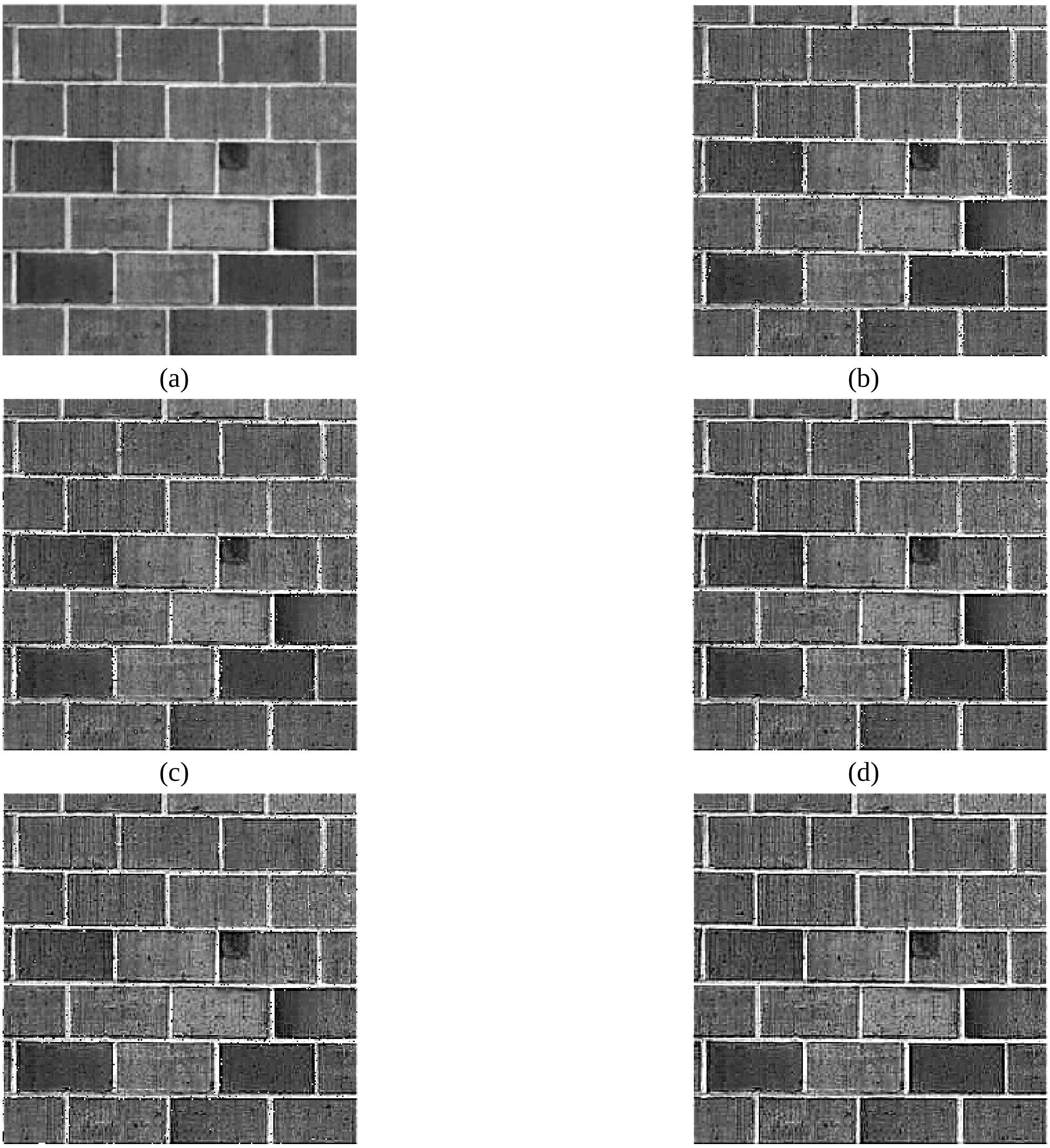
Table 6.3: Information Entropy and Average Gradient of images in Figure 6.3

Figure	Information Entropy	Average Gradient
(a)	7.0332	10.3750
(b)	7.3198	22.2058
(c)	7.3635	24.6720
(d)	7.4121	28.3960
(e)	7.4296	30.8907
(f)	7.4450	33.3926

It can be seen from Figure 6.3 and Table 6.3

- The image texture gets enhanced hence textural information increases with increase in intensity factor (γ).
- The information entropy of an image increases by a large value through fractional differential filter with an intensity factor γ = 1.9 resulting in enhancement of textural details and with increase in intensity factor information entropy gets increased.
- If intensity factor (γ) is exceeded above 1.9 the information entropy of image gets decreased hence losing lot of texture details and decreasing texture contrast

Image of Brick Wall



(a) (b) (c) (d) (e) (f)

Figure 6.4: Texture enhancement of Brick Wall with fractional order v = 0.2 for different values of intensity factor (γ). (a) Original Image, (b) γ = 1, (c) γ =1.2, (d) γ = 1.5, (e) γ = 1.7, (f) γ = 1.9

Table 6.4: Information Entropy and Average Gradient of images in Figure 6.4

Figure	Information Entropy	Average Gradient
(a)	6.9253	8.7595
(b)	7.1709	16.8510
(c)	7.1942	18.5608
(d)	7.2115	21.1490
(e)	7.2162	22.8859
(f)	7.2160	24.6298

Figure 6.4 and Table 6.4 shows that

- The abundant complex textural detail information is becoming more clear-cut after implementing fractional differential.
- This texture information increases with increase in intensity factor (γ) thus enhancing the contrast of texture.
- Since increase in intensity factor increases the textural information hence the information entropy of an image increases with intensity factor.
- The entropy reaches its maximum value for an intensity factor (γ) = 1.7 but for intensity factor (γ) > 1.7 the information entropy gets decreased resulting in loss of textural information and texture contrast.
- The average gradient of image increases with intensity factor but image gets more distorted with increase in intensity factor.

Barbara Image

Figure 6.5: Texture enhancement of Barbara Image with fractional order v = 0.2 for different values of intensity factor (γ). (a) Original Image, (b) γ = 1, (c) γ = 1.2, (d) γ = 1.5, (e) γ =1. 7, (f) γ = 1.9

Table 6.5: Information Entropy and Average Gradient of images in Figure 6.5

Figure	Information Entropy	Average Gradient
(a)	7.6321	9.2177
(b)	7.7721	22.7428
(c)	7.7333	25.5345
(d)	7.6691	29.7420
(e)	7.6204	32.5567
(f)	7.5736	35.3772

It is observed from Figure 6.5 and Table 6.5 that

- The texture details of the image get enhanced with increase in intensity factor (γ).
- It is seen that texture channel becomes deeper with increase in intensity factor but after some value of intensity factor image gets distorted and gets more susceptible to noise.
- With increase in intensity factor the information entropy of image increases and attains its maximum value at $\gamma = 1.2$ and the entropy decreases for value of $\gamma > 1.2$ hence the texture details of image becomes weak.
- Although the average gradient of image increases with intensity factor thus enhancing the contrast but image gets distorted after some value of fractional order.

6.1.2 Effect of Varying Fractional Differential Order (v)

In this texture enhancement of different images is analyzed by keeping the intensity factor (γ) constant and varying the fractional differential order (v)

Baboon Image

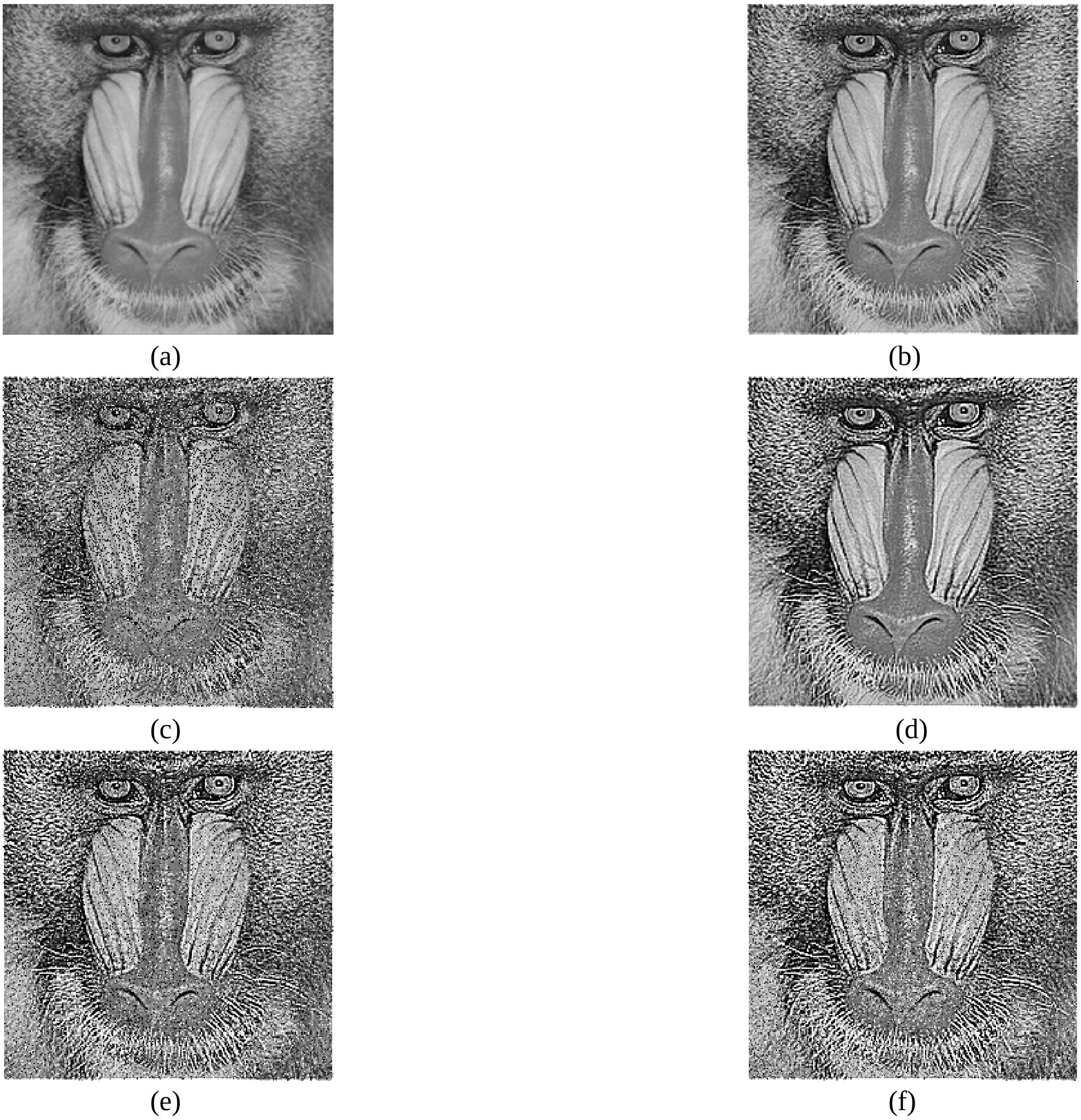


Figure 6.6: Texture Enhancement of Baboon Image with an intensity factor γ = 1 and varying fractional differential order (v), (a) v = 0, (b) v = 0.1, (c) v = 0.2, (d) v = 0.3, (e) v = 0.4, (f) v = 0.5

Table 6.6: Information Entropy and Average Gradient of Baboon image for varying fractional order

Fractional Order	Information Entropy	Average Gradient
0	7.1686	9.1537
0.1	7.3789	15.0309
0.2	7.5716	21.2197
0.3	7.6789	27.3150
0.4	7.6948	33.1318
0.5	7.6559	38.5195
0.6	7.5883	43.3331
0.7	7.5065	47.4249
0.8	7.4439	50.6422
0.9	7.3966	52.8258
1.0	7.3754	53.8095

It is observed from Figure 6.6 and Table 6.6 that

- Since fractional differential of constant is not zero so for $0 < v < 1$, the contrast of complex texture details is enhanced with increasing v, and image becomes clearer.
- With increase in fractional order (v), information entropy of image increases until v = 0.4, if fractional order is further increased above 0.4 the information entropy starts decreasing hence image loses some of its texture features.
- The average gradient of image increases with fractional differential order (v) thus enhancing the contrast of image

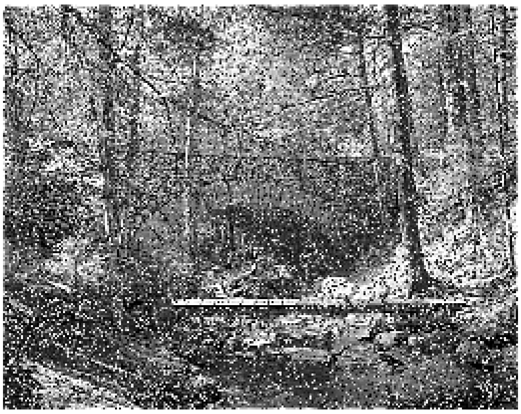
Image of Bridge

(a)

(b)

(c)

(d)



(e)

(f)

Figure 6.7: Texture Enhancement of an Image of Bridge with an intensity factor γ = 1 and varying fractional differential order (v), (a) v = 0, (b) v = 0.1, (c) v = 0.2, (d) v = 0.3, (e) v = 0.4, (f) v = 0.5

Table 6.7: Information Entropy and Average Gradient of an Image of Bridge for varying fractional order

Fractional Order	Information Entropy	Average Gradient
0	7.7284	8.3962
0.1	7.7744	10.8351
0.2	7.8136	13.4902
0.3	7.8335	16.1821
0.4	7.8324	18.7966
0.5	7.8146	21.2432
0.6	7.7877	23.4429
0.7	7.7558	25.3203
0.8	7.7260	26.8003
0.9	7.7035	27.8065
1.0	7.6940	28.2602

Following conclusions can be drawn from Figure 6.7 and Table 6.7

- For $0 < v < 0.1$ texture features of image gets enhanced than original, but texture channels are not much deeper.
- When $0.1 < v < 0.3$ the textural features enhancement increases with increase in fractional differential order
- The information entropy of image increases with increase in fractional differential order and attains a maximum value for order $v = 0.3$.
- Increasing the fractional differential order beyond 0.3 results in decrease in information entropy, since the result of fractional differential becomes close to integral differential and resulting in loss of more texture information

Image of Moon Surface

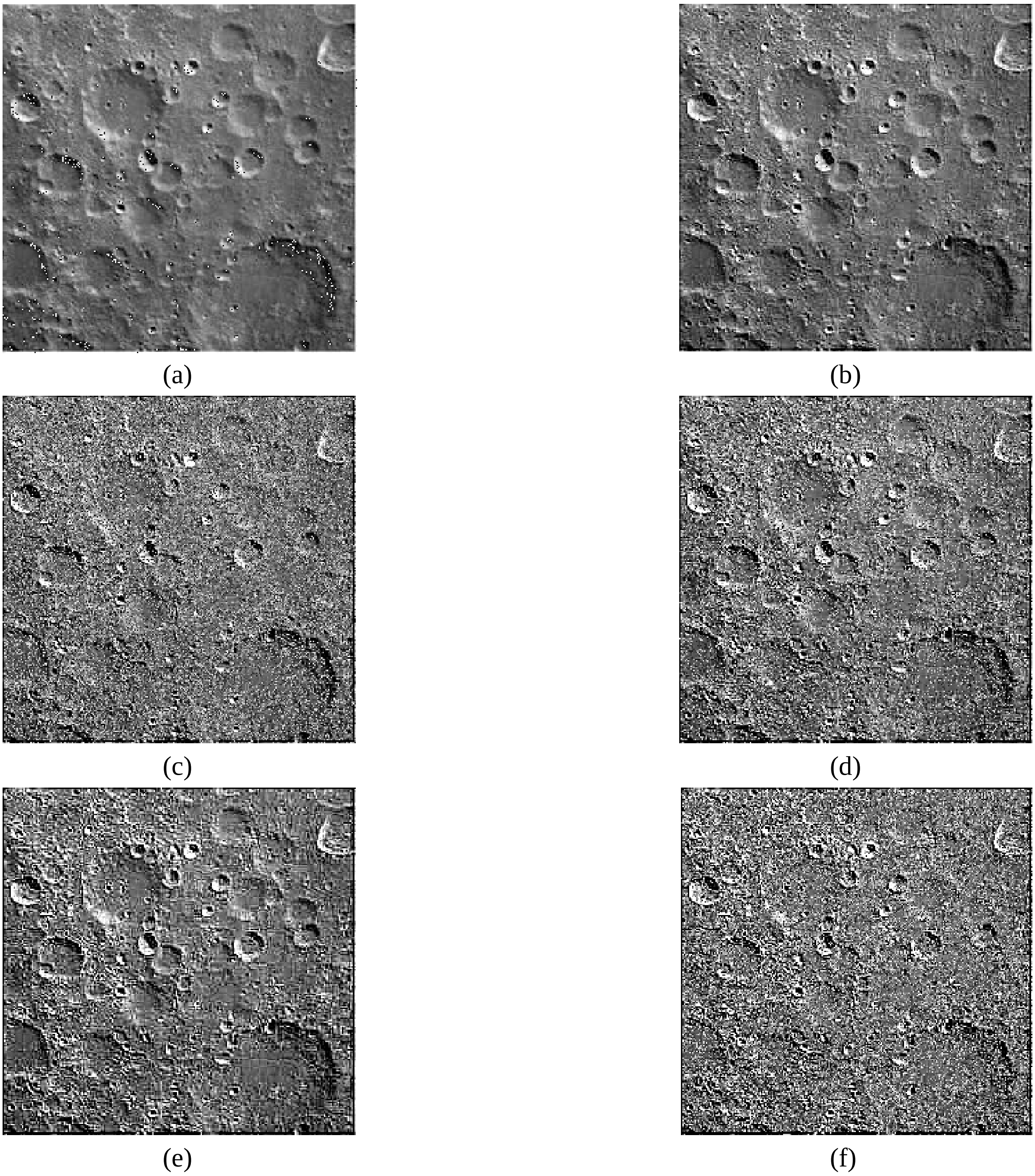


Figure 6.8: Texture Enhancement of an Image of Mars Surface with an intensity factor $\gamma = 1$ and varying fractional differential order (v), (a) $v = 0$, (b) $v = 0.1$, (c) $v = 0.2$, (d) $v = 0.3$, (e) $v = 0.4$, (f) $v = 0.5$

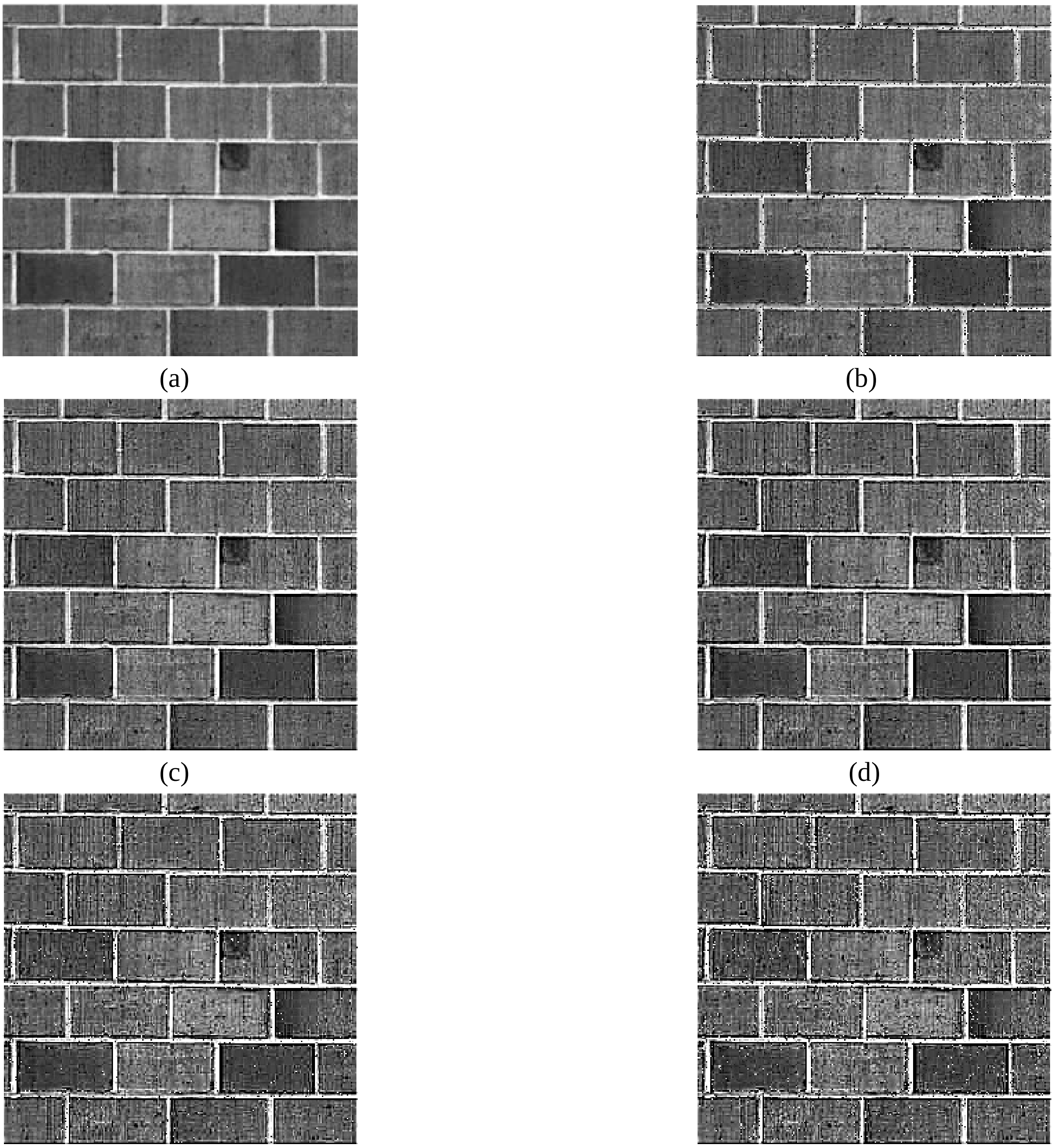
Table 6.8: Information Entropy and Average Gradient of an Image of Moon Surface for varying fractional order

Fractional Order	Information Entropy	Average Gradient
0	7.0331	10.3749
0.1	7.1698	16.1845
0.2	7.3197	22.2057
0.3	7.4085	28.1421
0.4	7.4456	33.8149
0.5	7.4495	39.0734
0.6	7.4325	43.7740
0.7	7.4058	47.7712
0.8	7.3773	50.9149
0.9	7.3540	53.0489
1.0	7.3430	54.0104

It can be shown from Figure 6.8 and Table 6.8 that

- The degree of texture enhancement is less for $0 < v < 0.5$ but as v increases beyond 0.5 the textural features of image gets enhanced and its textural contrast increases
- The information entropy of image increases for $0 < v < 0.5$ and attains maximum entropy at fractional order $v = 0.5$, but for $0.6 < v < 1$ with increase of fractional order information entropy decreases and hence textural information decreases.
- The average gradient of image increases with increase in fractional differential order.

Image of Brick Wall



(a) (b) (c) (d) (e) (f)

Figure 6.9: Texture Enhancement of an Image of Brick Wall with an intensity factor γ = 1 and varying fractional differential order (v), (a) v = 0, (b) v = 0.1, (c) v = 0.2, (d) v = 0.3, (e) v = 0.4, (f) v = 0.5

Table 6.9: Information Entropy and Average Gradient of an Image of Brick Wall for varying fractional order

Fractional Order	**Information Entropy**	**Average Gradient**
0	6.9253	8.7594
0.1	7.0623	12.6967
0.2	7.1708	16.8510
0.3	7.2104	20.9724
0.4	7.2158	24.9242
0.5	7.2128	28.5951
0.6	7.2115	31.8808
0.7	7.2073	34.6771
0.8	7.2020	36.8776
0.9	7.1973	38.3719
1.0	7.1936	39.0453

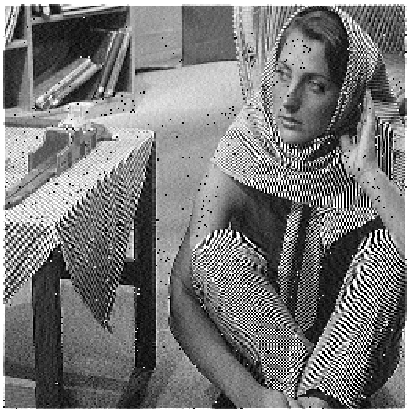
It can be concluded from Figure 6.9 and Table 6.9 that

- The texture of Brick Wall increases with increase in fractional differential order.
- It is easily seen that fractional differential not only maintain the most energy of image on the low frequency, but also nonlinearly enhance its energy over intermediate and high frequency, which leads to a richer texture details.
- With increase in fractional order (v), information entropy of image increases until v = 0.4, if fractional order is further increased above 0.4 the information entropy starts decreasing hence image loses some of its texture features.
- The average gradient of image increases with fractional differential order (v) thus enhancing the contrast of image

Barbara Image

(a)

(b)



(c)

(d)

(e)

(f)

Figure 6.10: Texture Enhancement of Barbara Image with an intensity factor γ = 1 and varying fractional differential order (v), (a) v = 0, (b) v = 0.1, (c) v = 0.2, (d) v = 0.3, (e) v = 0.4, (f) v = 0.5

Table 6.10: Information Entropy and Average Gradient of Barbara Image for varying fractional order

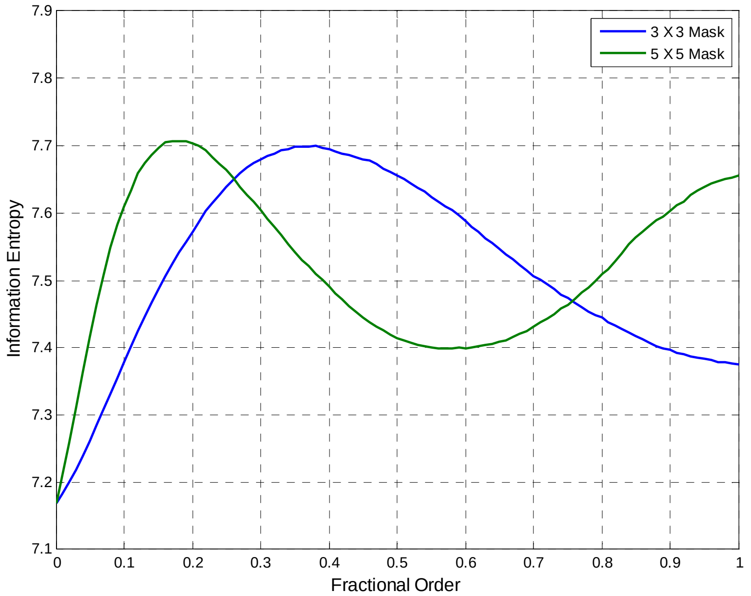
Fractional Order	Information Entropy	Average Gradient
0	7.6321	9.2177
0.1	7.7896	15.8982
0.2	7.7721	22.7428
0.3	7.6737	29.4554
0.4	7.5659	35.8530
0.5	7.4621	41.7753
0.6	7.3682	47.0651
0.7	7.2906	51.5611
0.8	7.2285	55.0960
0.9	7.1877	57.4950
1.0	7.1710	58.5758

Following conclusions can be drawn from Figure 6.10 and Table 6.10

- When $0 < v < 0.1$ the textural features enhancement increases with increase in fractional differential order
- The information entropy of image increases with increase in fractional differential order and attains a maximum value for order $v = 0.1$.
- Increasing the fractional differential order beyond 0.1 results in decrease in information entropy, since the result of fractional differential becomes close to integral differential and resulting in loss of more texture information

6.1.3 Effect of varying size of mask: Fractional Differential is implemented on images with 3 X 3 and 5 X 5 size fractional differential mask

Baboon Image



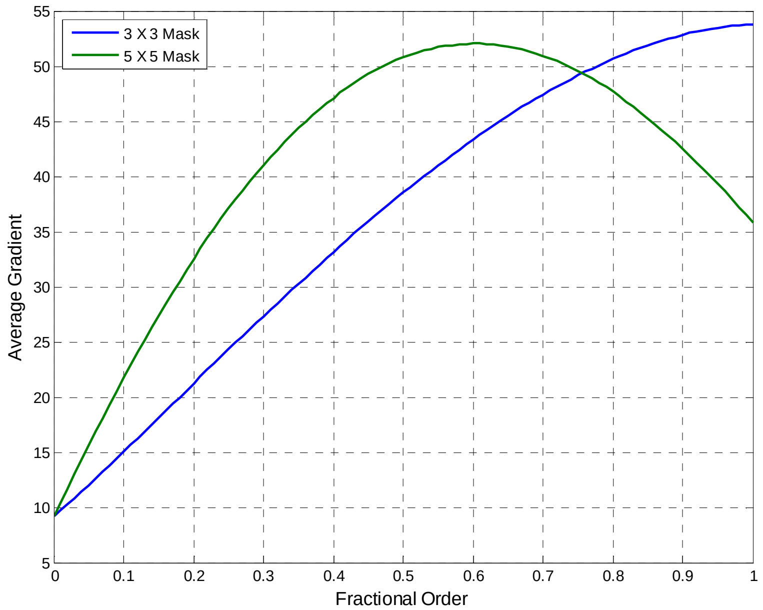


Figure 6.11: Information Entropy and Average Gradient of Baboon Image for varying fractional order through different size of mask

Table 6.11: Information Entropy and Average Gradient of Baboon Image for varying fractional order through different size of mask

Size of Fractional Differential Mask / Fractional Order	3 X 3 Size Mask		5 X 5 Size Mask	
	Information Entropy	**Average Gradient**	**Information Entropy**	**Average Gradient**
0	7.1686	9.1537	7.1686	9.1537
0.1	7.3789	15.0309	7.6105	21.7119
0.2	7.5716	21.2197	7.7031	32.5087
0.3	7.6789	27.3150	7.6048	41.0208
0.4	7.6948	33.1318	7.4906	47.1275
0.5	7.6559	38.5195	7.4147	50.7903
0.6	7.5883	43.3331	7.3994	52.0320
0.7	7.5065	47.4249	7.4311	50.9395
0.8	7.4439	50.6422	7.5092	47.6756
0.9	7.3966	52.8258	7.6021	42.4946
1.0	7.3754	53.8095	7.6563	35.7811

It is observed that for a 3 X 3 size mask the information entropy increases with increase in value of fractional order for the range $0 < v < 0.4$ thus enhancing the texture of image but after that the information entropy starts decreasing losing the texture information. On other hand if we increase the size of mask to 5 X 5 the information entropy increases for $0 < v < 0.3$ and then starts decreasing for $0.4 < v < 0.6$ but with further increment in fractional order entropy again increases. Thus entropy with 5 X 5 size mask is more than with 3 X 3 size mask at most of points.

Image of Bridge

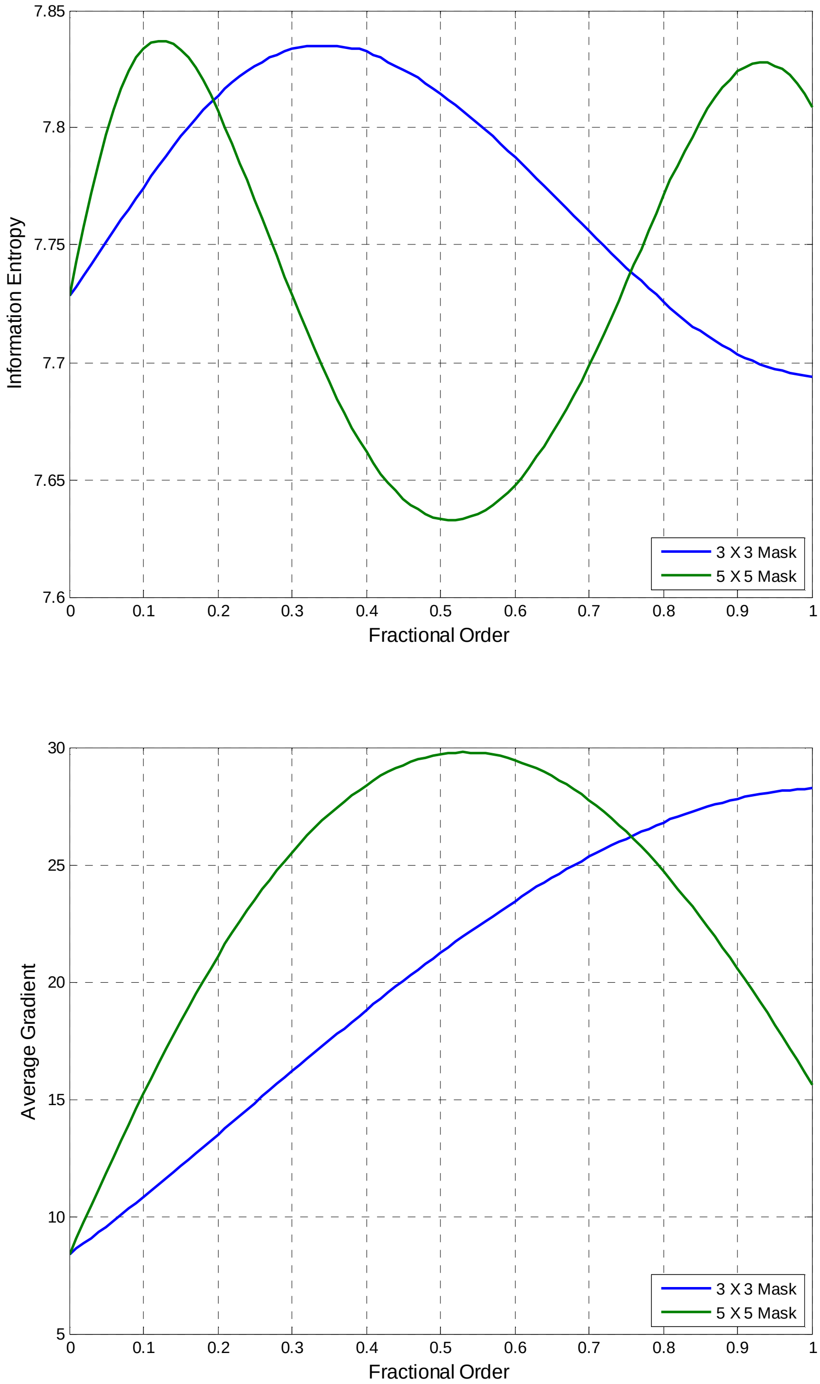


Figure 6.12: Information Entropy and Average Gradient of an Image of Bridge for varying fractional order through different size of mask

Table 6.12: Information Entropy and Average Gradient of an Image of Bridge for varying fractional order through different size of mask

Size of Fractional Differential Mask / Fractional Order	3 X 3 Size Mask		5 X 5 Size Mask	
	Information Entropy	**Average Gradient**	**Information Entropy**	**Average Gradient**
0	7.7285	8.3963	7.7285	8.3963
0.1	7.7744	10.8359	7.8335	15.2316
0.2	7.8136	13.4902	7.8070	21.1011
0.3	7.8336	16.1822	7.7289	25.5270
0.4	7.8325	18.7967	7.6617	28.4004
0.5	7.8147	21.2433	7.6331	29.7009
0.6	7.7878	23.4429	7.6475	29.4608
0.7	7.7559	25.3203	7.6985	27.7605
0.8	7.7260	26.8004	7.7713	24.7331
0.9	7.7036	27.8065	7.8239	20.5808
1.0	7.6940	28.2602	7.8088	15.6250

From Table 6.12 it can be concluded that with 5 X 5 size mask the information entropy is more than it is with 3 X 3 size mask. However after some value of fractional differential order both information entropy and average gradient starts decreasing and reaches its minimum value and after that it again starts increasing with increase in fractional order. The average gradient with 5 X 5 size mask is also more than it is for 3 X 3 size mask.

Image of Moon Surface

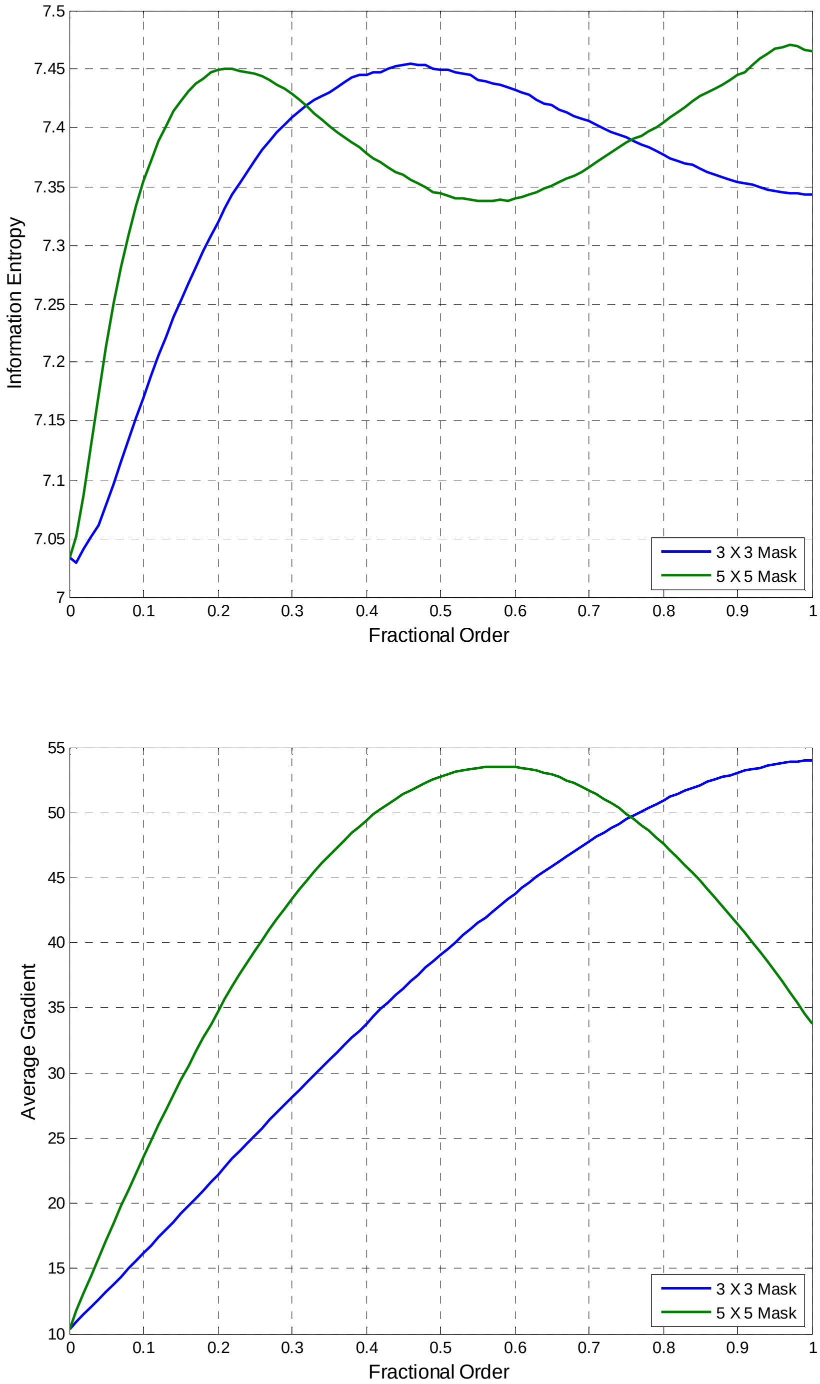


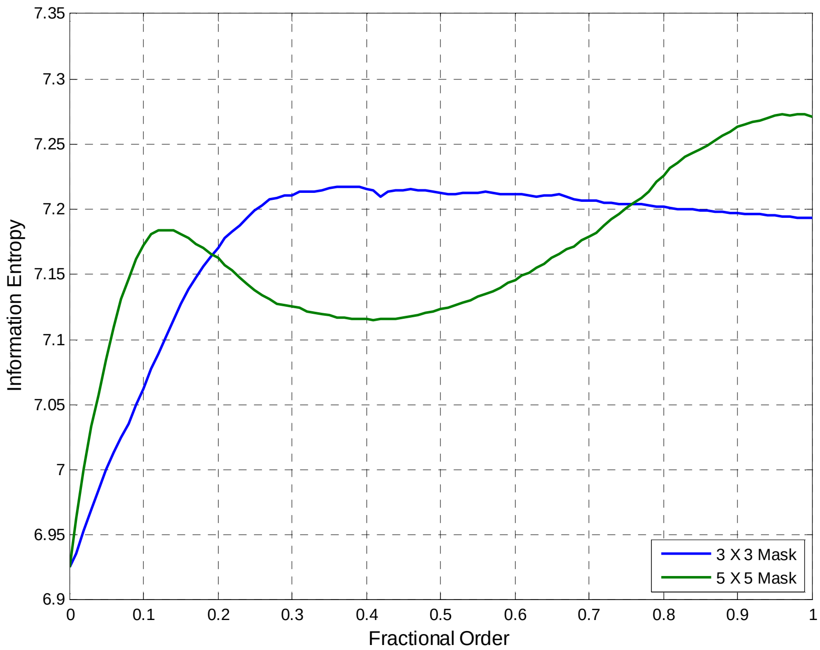
Figure 6.13: Information Entropy and Average Gradient of an Image of Moon Surface for varying fractional order through different size of mask

Table 6.13: Information Entropy and Average Gradient of an Image of Moon Surface for varying fractional order through different size of mask

Size of Fractional Differential Mask / Fractional Order	3 X 3 Size Mask		5 X 5 Size Mask	
	Information Entropy	**Average Gradient**	**Information Entropy**	**Average Gradient**
0	7.0332	10.3750	7.0332	10.3750
0.1	7.1698	16.1845	7.3547	23.5807
0.2	7.3198	22.2058	7.4491	34.7313
0.3	7.4086	28.1421	7.4291	43.3751
0.4	7.4457	33.8149	7.3787	49.3913
0.5	7.4496	39.0735	7.3438	52.7481
0.6	7.4325	43.7741	7.3396	53.4823
0.7	7.4059	47.7713	7.3660	51.7005
0.8	7.3774	50.9150	7.4053	47.5928
0.9	7.3540	53.0489	7.4447	41.4681
1.0	7.3431	54.0104	7.4651	33.7790

In Figure 6.13 the information entropy and average gradient is plotted against fractional differential order it can be seen that with 5 X 5 size mask both the information entropy and average gradient first increases with fractional differential order and after attaining maximum value starts decreasing. But after some value the information entropy again starts increasing. Here the information entropy in case of 5 X 5 size mask is greater than that for 3 X 3 size mask.

Image of Brick Wall



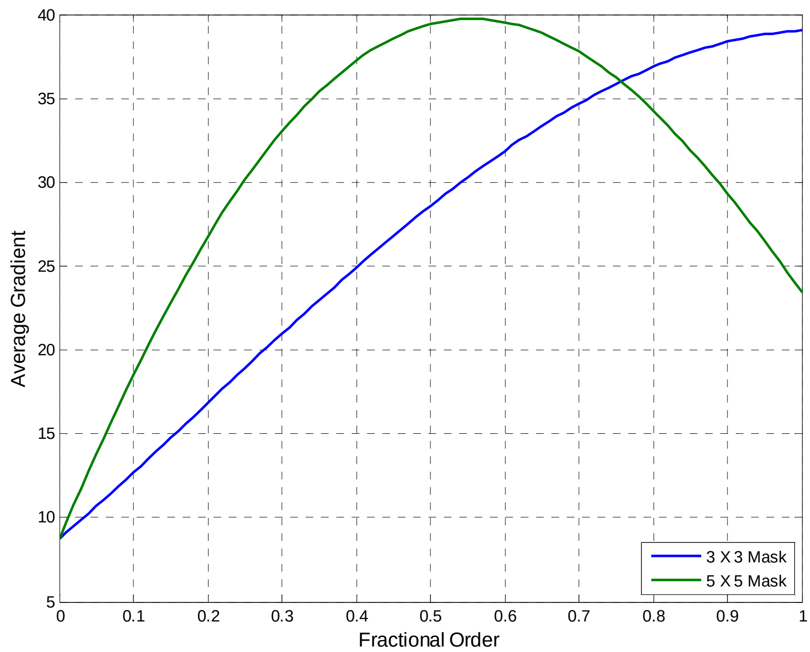


Figure 6.14: Information Entropy and Average Gradient of an Image of Brick Wall for varying fractional order through different size of mask

Table 6.14: Information Entropy and Average Gradient of an Image of Brick Wall for varying fractional order through different size of mask

Size of Fractional Differential Mask / Fractional Order	3 X 3 Size Mask		5 X 5 Size Mask	
	Information Entropy	**Average Gradient**	**Information Entropy**	**Average Gradient**
0	6.9253	8.7595	6.9253	8.7595
0.1	7.0624	12.6967	7.1721	18.5145
0.2	7.1709	16.8510	7.1626	26.7464
0.3	7.2104	20.9724	7.1256	33.0217
0.4	7.2158	24.9243	7.1156	37.2462
0.5	7.2129	28.5952	7.1234	39.4074
0.6	7.2115	31.8808	7.1460	39.5494
0.7	7.2073	34.6772	7.1790	37.7756
0.8	7.2020	36.8777	7.2264	34.2656
0.9	7.1974	38.3719	7.2632	29.3292
1.0	7.1936	39.0453	7.2708	23.4247

From Figure 6.14 it can be seen that the information entropy for 5 X 5 size mask increases for $0 < v < 0.1$ but exceeding the value beyond 0.1 results in decrease in the information entropy. After attaining minimum value at $v = 0.4$ the information entropy again increases with increase in fractional differential order. Similarly the average gradient first increases with increase in fractional order and then decreases.

Barbara Image

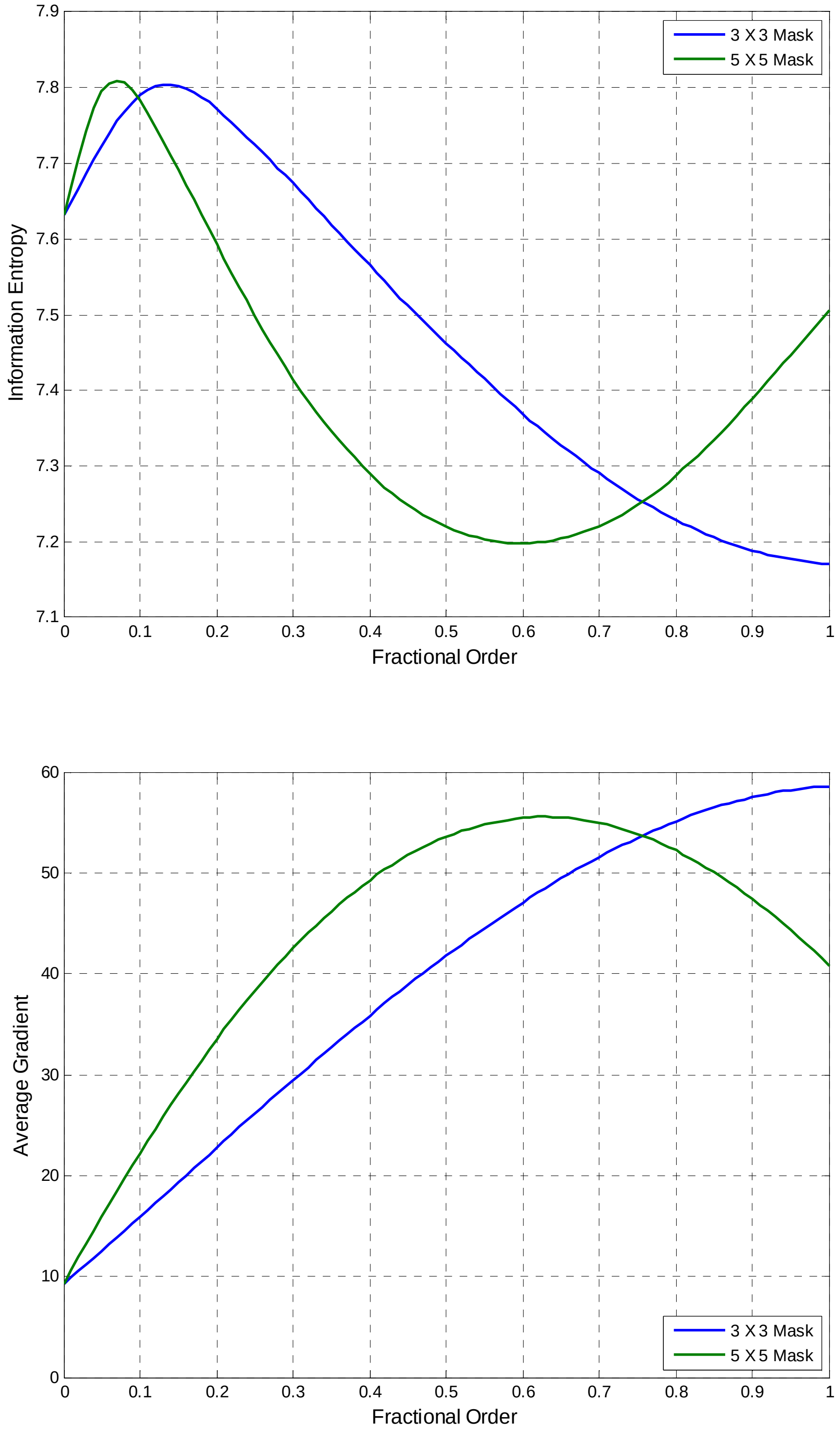


Figure 6.15: Information Entropy and Average Gradient of Barbara Image for varying fractional order through different size of mask

Table 6.15: Information Entropy and Average Gradient of Barbara Image for varying fractional order through different size of mask

Type of Fractional Differential Mask / Fractional Order	3 X 3 Size Mask		5 X 5 Size Mask	
	Information Entropy	Average Gradient	Information Entropy	Average Gradient
0	7.6321	9.2177	7.6321	9.2177
0.1	7.7897	15.8982	7.7833	22.2032
0.2	7.7721	22.7428	7.5927	33.4729
0.3	7.6737	29.4554	7.4146	42.5399
0.4	7.5660	35.8530	7.2904	49.2581
0.5	7.4621	41.7753	7.2198	53.5625
0.6	7.3683	47.0651	7.1980	55.4457
0.7	7.2907	51.5611	7.2198	54.9579
0.8	7.2286	55.0960	7.2874	52.2184
0.9	7.1878	57.4950	7.3890	47.4162
1.0	7.1710	58.5758	7.5060	40.8151

In Figure 6.15 the information entropy and average gradient is plotted against fractional differential order it can be seen that for 3 X 3 size mask both information entropy and average gradient increases with fractional order. With 5 X 5 size mask both the information entropy and average gradient first increases with fractional differential order and after attaining maximum value starts decreasing. But after some value the information entropy again starts increasing. Here the information entropy in case of 5 X 5 size mask is greater than that for 3 X 3 size mask.

6.2 Comparison to other Fractional Differential Filters

Baboon Image

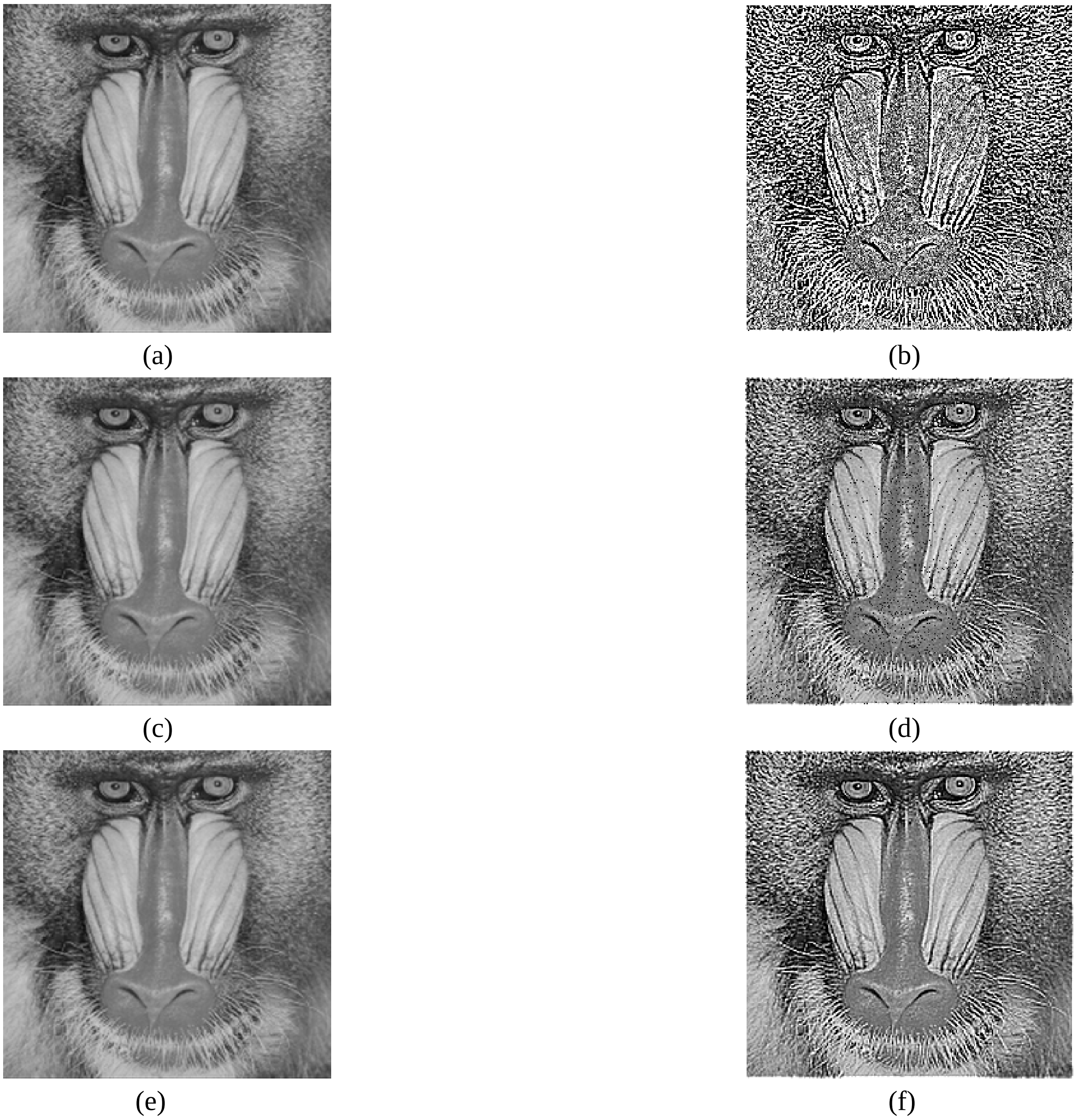


Figure 6.16: Texture enhancement capability comparison of Baboon Image, (a) Original Image (b) 0.5 order G-L fractional differential (c) Original Image (d) 0.5 order R-L fractional differential (e) Original Image (f) 0.5 order improved G-L fractional differential

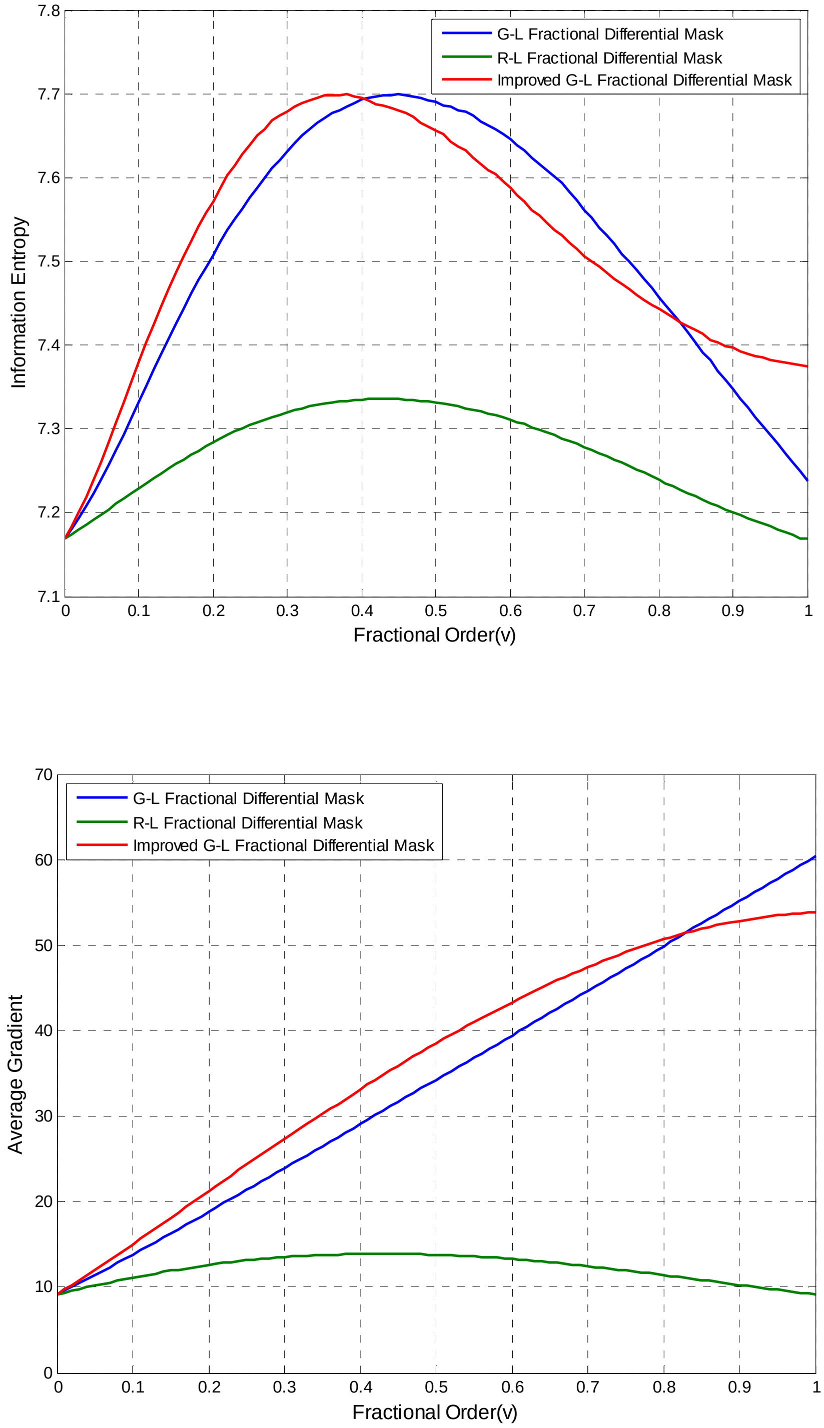


Figure 6.17: Information Entropy and Average Gradient of Baboon image for varying fractional order through different types of Fractional Differential Masks

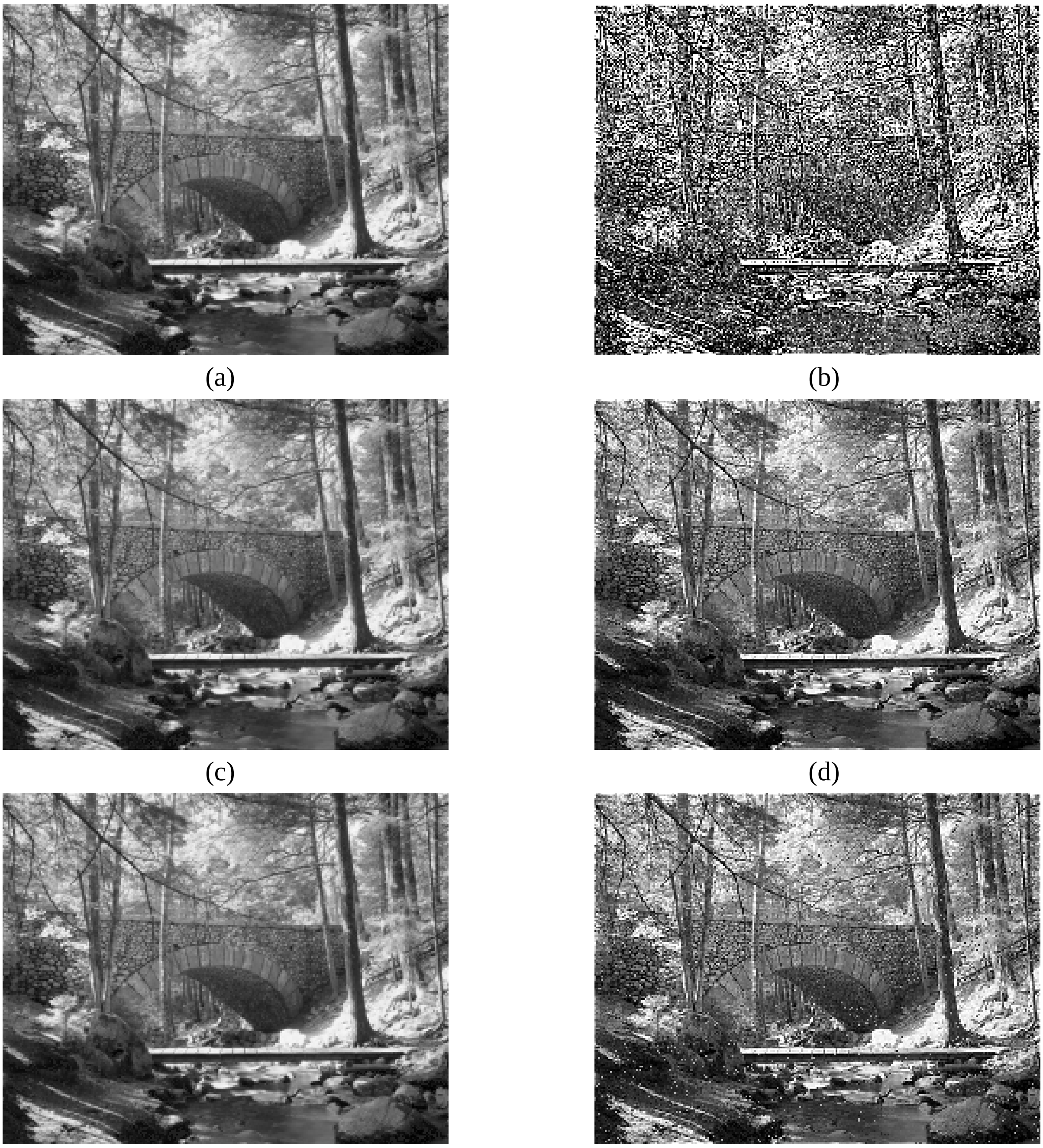
Table 6.16: Information Entropy and Average Gradient of Baboon image for varying fractional order through different types of Fractional Differential Masks

Type of Fractional Differential Mask / Fractional Order	G-L Fractional Differential Mask		R-L Fractional Differential Mask		Improved G-L Fractional Differential Mask	
	Information Entropy	**Average Gradient**	**Information Entropy**	**Average Gradient**	**Information Entropy**	**Average Gradient**
0	7.1686	9.1536	7.1686	9.1536	7.1686	9.1536
0.1	7.3320	13.8149	7.2285	11.1102	7.3788	15.0309
0.2	7.5074	18.8088	7.2837	12.5891	7.5715	21.2196
0.3	7.6310	23.9133	7.3200	13.5149	7.6789	27.3150
0.4	7.6932	29.0690	7.3350	13.9022	7.6947	33.1318
0.5	7.6899	34.2533	7.3309	13.8020	7.6559	38.5195
0.6	7.6455	39.4552	7.3110	13.2892	7.5882	43.3330
0.7	7.5612	44.6685	7.2785	12.4565	7.5064	47.4248
0.8	7.4573	49.8898	7.2399	11.4099	7.4438	50.6421
0.9	7.3482	55.1167	7.2008	10.2666	7.3965	52.8257
1.0	7.2379	60.3477	7.1686	9.1537	7.3754	53.8095

From Figure 6.16 and Table 6.16 it is observed that

- For G-L Fractional Differential Mask texture of an image increases with increase in fractional differential order (v). But after some value of fractional order the image gets distorted and gets susceptible to noise. It is seen that for $0 < v \leq 0.4$ the information entropy increases but for $v > 0.4$ the information entropy decreases resulting in loss of textural information.
- In case of R-L Fractional Differential Mask $0 < v \leq 0.4$ information entropy increases with increase in fractional differential order thus enhancing texture and is maximum at 0.4 and after that it decreases with order thereby result in loss of texture. Also the average gradient of image decreases for $v > 0.4$ thereby decreases the contrast of image
- For improved G-L Fractional Differential Mask both information entropy increases for $0 < v \leq 0.4$ hence improving texture of image and with increase in fractional differential order average gradient increases thereby increasing contrast of image. Also the entropy is greater that both G-L Fractional Differential Mask and R-L Fractional Differential Mask at some points.

Image of Bridge



(a) (b) (c) (d) (e) (f)

Figure 6.18: Texture enhancement capability comparison of an Image of Bridge (a) 0 order G-L fractional differential (b) 0.5 order G-L fractional differential (c) 0 order R-L fractional differential (d) 0.5 order R-L fractional differential (e) 0 order improved G-L fractional differential (f) 0.5 order improved G-L fractional differential

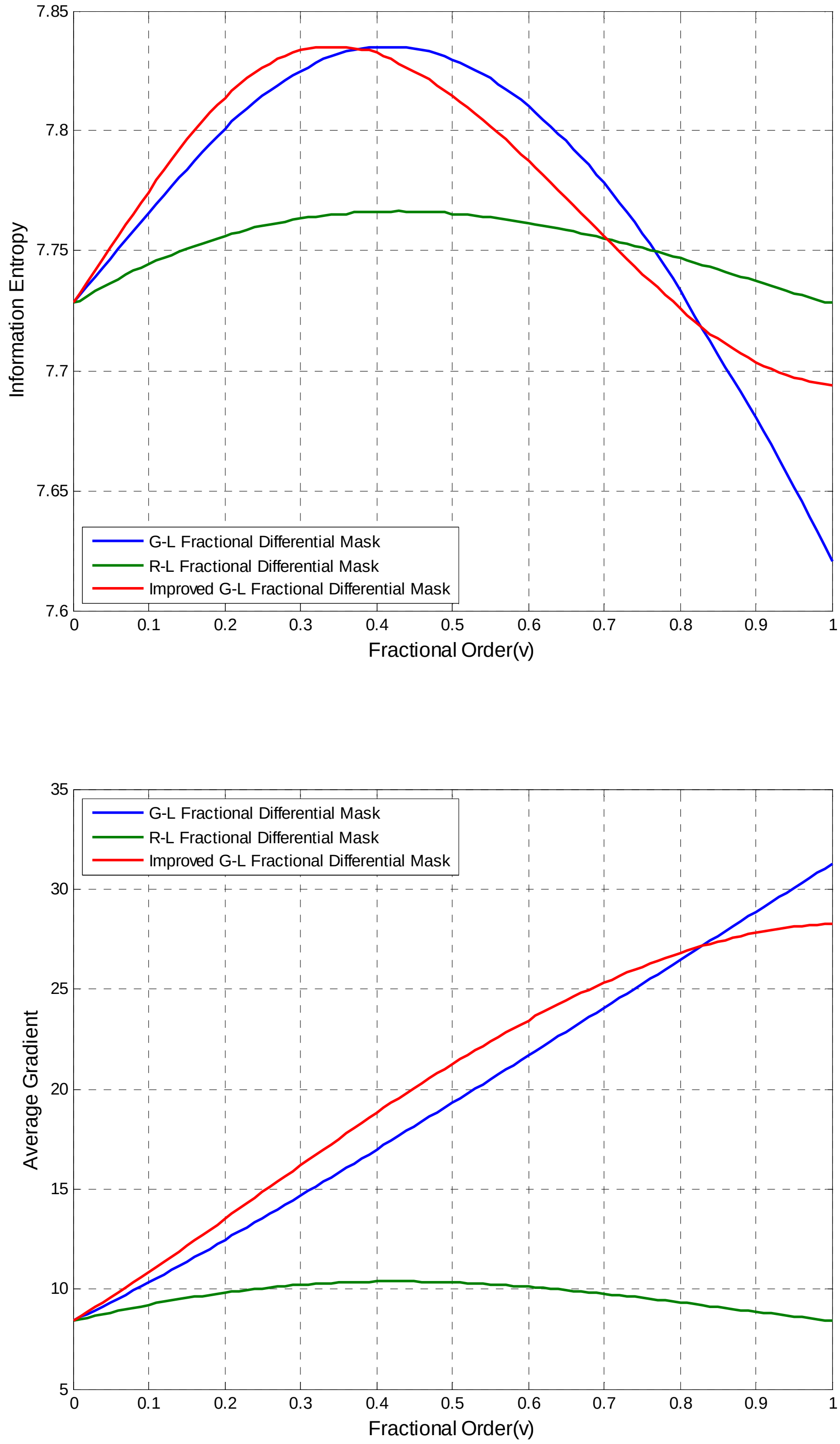


Figure 6.19: Information Entropy and Average Gradient of an Image of Bridge for varying fractional order through different types of Fractional Differential Masks

Table 6.17: Information Entropy and Average Gradient of an Image of Bridge for varying fractional order through different types of Fractional Differential Masks

Type of Fractional Differential Mask / Fractional Order	G-L Fractional Differential Mask		R-L Fractional Differential Mask		Improved G-L Fractional Differential Mask	
	Information Entropy	**Average Gradient**	**Information Entropy**	**Average Gradient**	**Information Entropy**	**Average Gradient**
0	7.7285	8.3962	7.7285	8.3962	7.7284	8.3962
0.1	7.7657	10.3266	7.7443	9.2084	7.7744	10.8352
0.2	7.8007	12.4448	7.7560	9.8176	7.8136	13.4902
0.3	7.8248	14.6722	7.7634	10.2014	7.8335	16.1821
0.4	7.8351	16.9668	7.7663	10.3630	7.8324	18.7966
0.5	7.8297	19.3043	7.7652	10.3212	7.8146	21.2432
0.6	7.8104	21.6699	7.7615	10.1077	7.7877	23.4429
0.7	7.7783	24.0549	7.7552	9.7627	7.7558	25.3203
0.8	7.7335	26.4539	7.7469	9.3316	7.7260	26.8003
0.9	7.6807	28.8634	7.7374	8.8612	7.7035	27.8065
1.0	7.6206	31.2808	7.7284	8.3962	7.6940	28.2602

It can be concluded that

- With increase in fractional differential order the texture details are enhanced through G-L Fractional Differential Mask, but the enhancement is for the range $0 < \upsilon \leq 0.4$. As the order is exceeded beyond 0.4 the texture details loses its contrast and image gets distorted. This effect can also be seen from Table 6.17 that with increase in fractional order the information entropy of an image reduces, hence with increase in fractional order there is loss in textural information.
- For R-L Fractional Differential Mask with increasing value of fractional order the texture is enhanced. But as fractional order is increased beyond some value the texture of the image do not enhance. This effect can be seen from Table 6.17 the information entropy first increases with increase in fractional then further decreases with increase in fractional order. Also the average gradient decreases after some fractional order hence decreasing contrast.
- With improved G-L Fractional Differential Mask the degree of texture enhancement increases with fractional order attains maximum, improving textural details and then decreases.

Image of Moon Surface

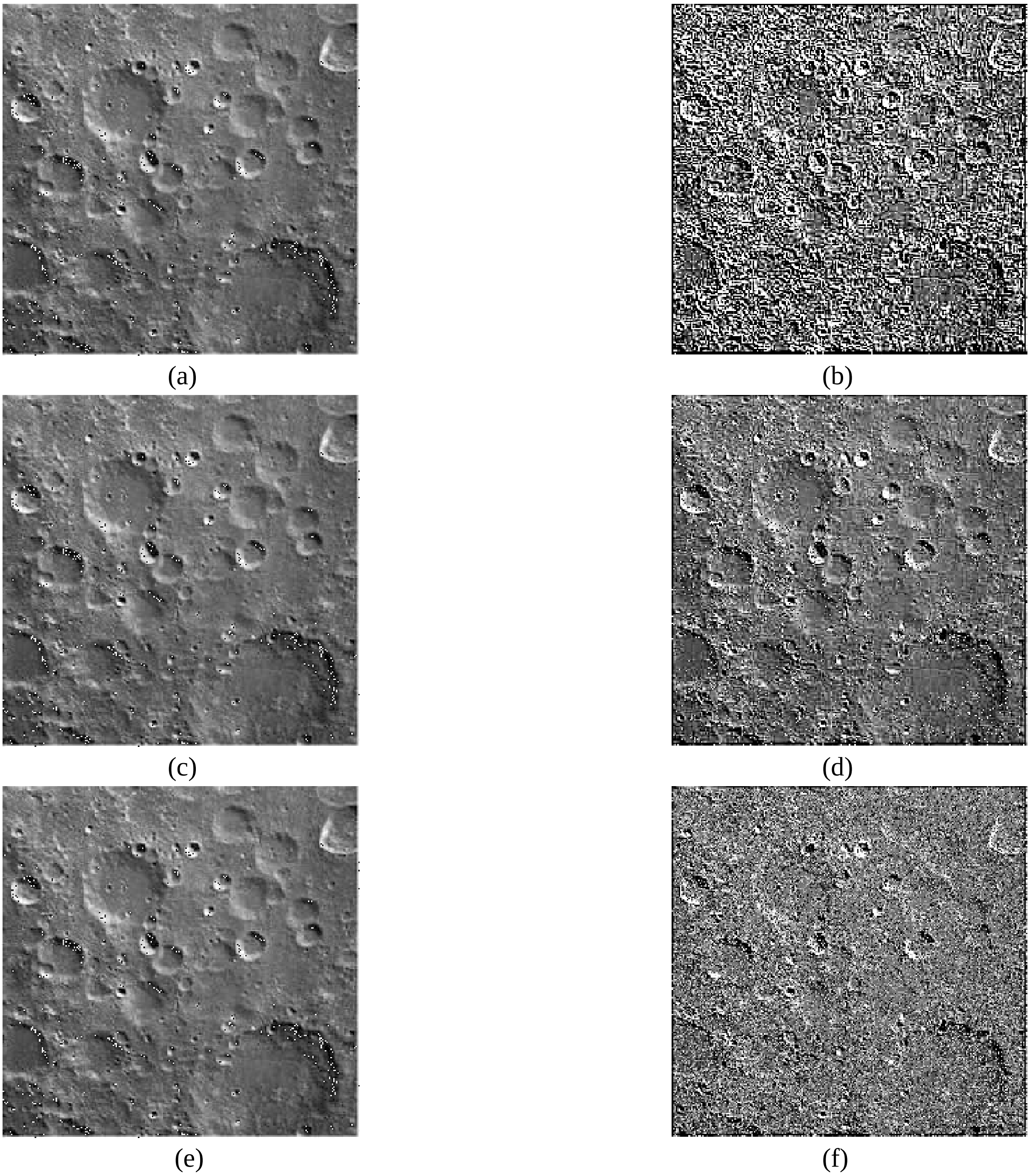


Figure 6.20: Texture enhancement capability comparison of an Image of Moon Surface (a) 0 order G-L fractional differential (b) 0.5 order G-L fractional differential (c) 0 order R-L fractional differential (d) 0.5 order R-L fractional differential (e) 0 order improved G-L fractional differential (f) 0.5 order improved G-L fractional differential

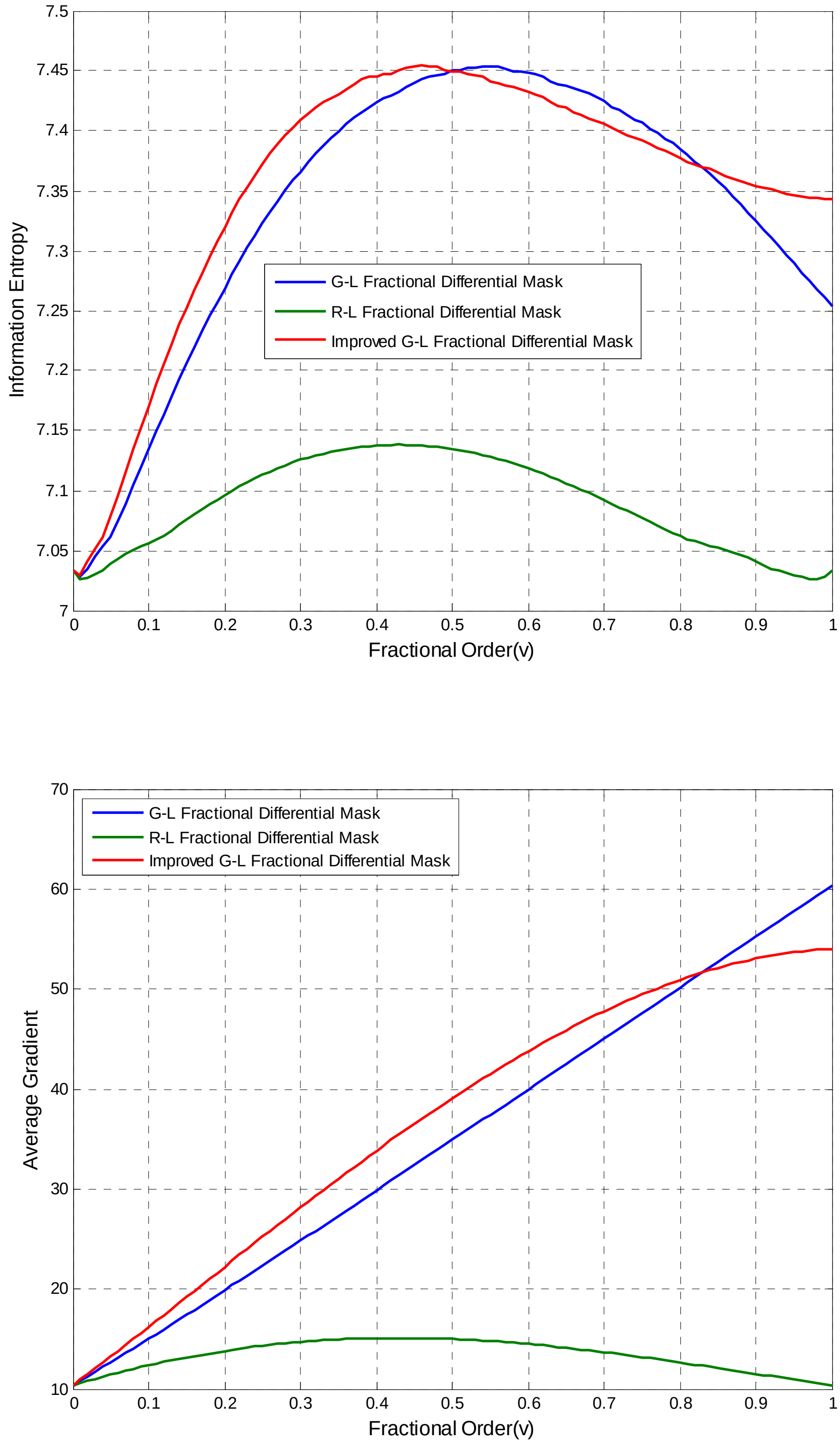


Figure 6.21: Information Entropy and Average Gradient of an Image of Moon Surface for varying fractional order through different types of Fractional Differential Masks

Table 6.18: Information Entropy and Average Gradient of an Image of Moon Surface for varying fractional order through different types of Fractional Differential Masks

Fractional Order \ Type of Fractional Differential Mask	G-L Fractional Differential Mask		R-L Fractional Differential Mask		Improved G-L Fractional Differential Mask	
	Information Entropy	**Average Gradient**	**Information Entropy**	**Average Gradient**	**Information Entropy**	**Average Gradient**
0	7.0331	10.3749	7.0331	10.3749	7.0331	10.3749
0.1	7.1349	14.9990	7.0562	12.3481	7.1698	16.1845
0.2	7.2687	19.8604	7.0965	13.8012	7.3197	22.2057
0.3	7.3660	24.8279	7.1258	14.7061	7.4085	28.1421
0.4	7.4240	29.8521	7.1378	15.0842	7.4456	33.8149
0.5	7.4505	34.9093	7.1341	14.9864	7.4495	39.0734
0.6	7.4483	39.9870	7.1182	14.4857	7.4325	43.7740
0.7	7.4247	45.0785	7.0918	13.6713	7.4058	47.7712
0.8	7.3845	50.1797	7.0618	12.6437	7.3773	50.9149
0.9	7.3248	55.2880	7.0411	11.5095	7.3540	53.0489
1.0	7.2536	60.4017	7.0331	10.3749	7.3430	54.0104

It can be concluded that

- In case of G-L Fractional Differential Mask texture information gets enhanced with increase in fractional differential order but after some value of fractional order the texture information gets loss resulting in distortion of image. It can be seen that information entropy increases for $0 < v \leq 0.5$ with increase in value v. But exceeding v beyond 0.5 result in decrease in information entropy and hence texture.
- For R-L Fractional Differential Mask the texture of an image gets enhanced with increase in value of fractional order. However, after some order the texture information couldn't preserve. Both information entropy and average gradient increases for $0 < v \leq 0.4$ and then decreases thereby decreasing contrast.
- With an improved G-L Fractional Differential Mask the texture channel becomes deeper with increase in value of fractional differential order hence increasing texture information. The information entropy increases with increase in fractional differential order for $0 < v \leq 0.4$ and the average gradient increases with increase in fractional differential order for $0 < v < 1.0$

Image of Brick wall

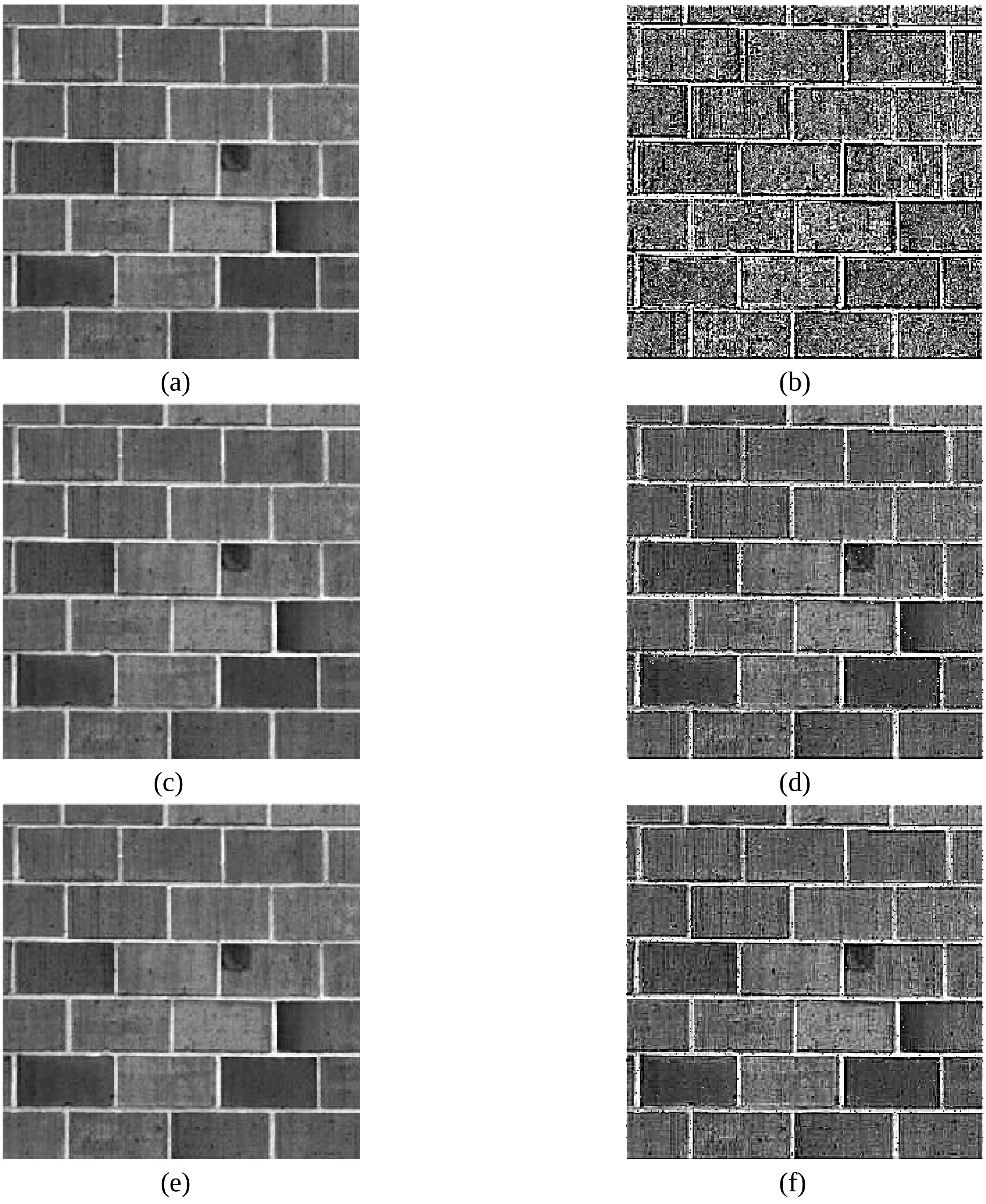


Figure 6.22: Texture enhancement capability comparison of an Image of Brick Wall (a) 0 order G-L fractional differential (b) 0.5 order G-L fractional differential (c) 0 order R-L fractional differential (d) 0.5 order R-L fractional differential (e) 0 order improved G-L fractional differential (f) 0.5 order improved G-L fractional differential

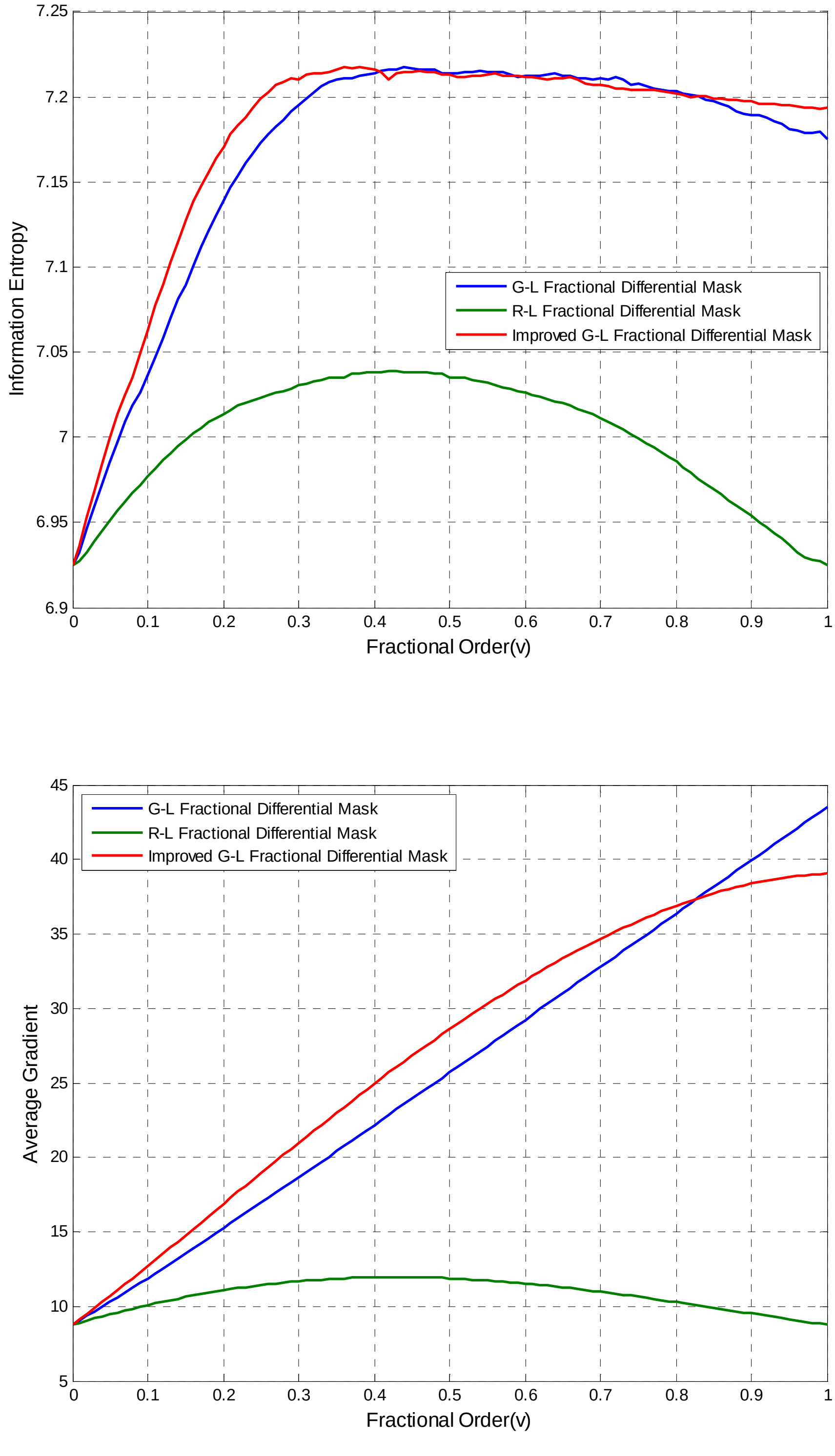


Figure 6.23: Information Entropy and Average Gradient of an Image of Brick Wall for varying fractional order through different types of Fractional Differential Masks

Table 6.19: Information Entropy and Average Gradient of an Image of Brick Wall for varying fractional order through different types of Fractional Differential Masks

Type of Fractional Differential Mask / Fractional Order	G-L Fractional Differential Mask		R-L Fractional Differential Mask		Improved G-L Fractional Differential Mask	
	Information Entropy	Average Gradient	Information Entropy	Average Gradient	Information Entropy	Average Gradient
0	6.9253	8.7595	6.9253	8.7595	6.9253	8.7595
0.1	7.0363	11.8840	6.9770	10.0790	7.0624	12.6967
0.2	7.1392	15.2290	7.0138	11.0656	7.1709	16.8510
0.3	7.1950	18.6691	7.0305	11.6836	7.2104	20.9724
0.4	7.2141	22.1626	7.0381	11.9423	7.2158	24.9243
0.5	7.2138	25.6877	7.0355	11.8753	7.2129	28.5952
0.6	7.2122	29.2335	7.0259	11.5329	7.2115	31.8808
0.7	7.2107	32.7932	7.0116	10.9771	7.2073	34.6772
0.8	7.2035	36.3629	6.9858	10.2791	7.2020	36.8777
0.9	7.1890	39.9403	6.9538	9.5145	7.1974	38.3719
1.0	7.1755	43.5233	6.9253	8.7595	7.1936	39.0453

From Figure 6.22 and Table 6.19 it is observed that

- For G-L Fractional Differential Mask texture of an image increases with increase in fractional differential order (v). But after some value of fractional order the image gets distorted and gets susceptible to noise. It is seen that for $0 < v \leq 0.4$ the information entropy increases but for $v > 0.4$ the information entropy decreases resulting in loss of textural information.
- In case of R-L Fractional Differential Mask $0 < v \leq 0.4$ information entropy increases with increase in fractional differential order thus enhancing texture and is maximum at 0.4 and after that it decreases with order thereby result in loss of texture. Also the average gradient of image decreases for $v < 0.4$ thereby decreases the contrast of image
- For improved G-L Fractional Differential Mask both information entropy increases for $0 < v \leq 0.4$ hence improving texture of image and with increase in fractional differential order average gradient increases thereby increasing contrast of image. Also the entropy is greater that both G-L Fractional Differential Mask and R-L Fractional Differential Mask at some points.

Barbara Image

(a) (b)

(c) (d)

(e) (f)

Figure 6.24: Texture enhancement capability comparison of Barbara Image (a) Original image, (b) 0.5 order G-L fractional differential (c) Original Image (d) 0.5 order R-L fractional differential (e) Original image (f) 0.5 order improved G-L Fractional Differential

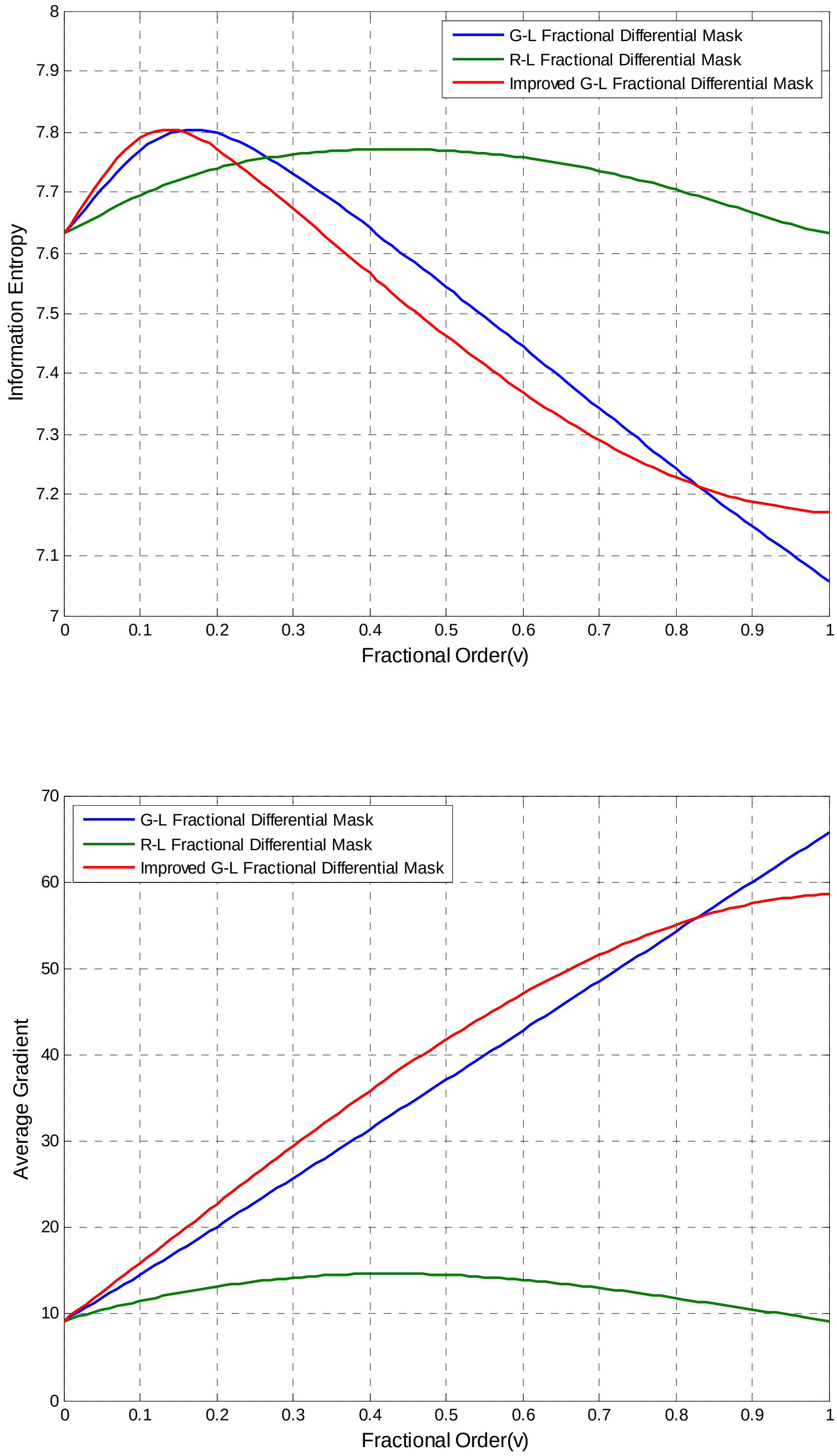


Figure 6.25: Information Entropy and Average Gradient of Barbara Image for varying fractional order through different types of Fractional Differential Masks

Table 6.20: Information Entropy and Average Gradient of Barbara Image for varying fractional order through different types of Fractional Differential Masks

Type of Fractional Differential Mask / Fractional Order	G-L Fractional Differential Mask		R-L Fractional Differential Mask		Improved G-L Fractional Differential Mask	
	Information Entropy	Average Gradient	Information Entropy	Average Gradient	Information Entropy	Average Gradient
0	7.6321	9.2177	7.6321	9.2177	7.6321	9.2177
0.1	7.7693	14.5433	7.6953	11.4976	7.7897	15.8982
0.2	7.7983	20.0826	7.7398	13.1703	7.7721	22.7428
0.3	7.7306	25.7109	7.7629	14.2080	7.6737	29.4554
0.4	7.6414	31.3852	7.7711	14.6408	7.5660	35.8530
0.5	7.5447	37.0861	7.7687	14.5289	7.4621	41.7753
0.6	7.4447	42.8037	7.7575	13.9555	7.3683	47.0651
0.7	7.3437	48.5326	7.7361	13.0211	7.2907	51.5611
0.8	7.2429	54.2693	7.7052	11.8386	7.2286	55.0960
0.9	7.1477	60.0119	7.6668	10.5285	7.1878	57.4950
1.0	7.0574	65.7588	7.6321	9.2177	7.1710	58.5758

It can be concluded that

- With increase in fractional differential order the texture details are enhanced through G-L Fractional Differential Mask, but the enhancement is for the range $0 < v \leq 0.2$. As the order is exceeded beyond 0.4 the texture details loses its contrast and image gets distorted. This effect can also be seen from Table 6.20 that with increase in fractional order the information entropy of an image reduces, hence with increase in fractional order there is loss in textural information.
- For R-L Fractional Differential Mask with increasing value of fractional order the texture is enhanced. But as fractional order is increased beyond some value the texture of the image do not enhance. This effect can be seen from Table 6.20 the information entropy first increases with increase in fractional then further decreases with increase in fractional order. Also the average gradient decreases after some fractional order hence decreasing contrast.
- With improved G-L Fractional Differential Mask the degree of texture enhancement increases with fractional order. The information entropy increases with fractional order attains maximum at $v = 0.1$, improving textural details but exceeding v beyond 0.1 results in loss of texture information.

CHAPTER 7: CONCLUSION AND FUTURE SCOPE

7.1 Conclusion

In the present work, Image Texture Enhancement is carried out by using an improved G-L Fractional Differential filter. The classical texture enhancement techniques suffers from the various drawbacks like they are susceptible to noise and gets distorted in attempt of enhancing the texture features of image. Different images are analyzed by implementing the improved G-L Fractional Differential filter and varying the filter or mask parameters and it is seen that the effect is different for different images. However there are fractional differential filters like G-L Fractional Differential filter and R-L Fractional Differential mask which can enhance the textural features of image more efficiently than the classical enhancement techniques. But it is seen that in comparison with the improved G-L Fractional Differential these enhancement techniques also suffers from various drawbacks like image distortion and less degree of enhancement. So an improved G-L based fractional differential filter mask is proposed for the texture enhancement.

When the proposed mask is implemented on different images by increasing the intensity factor or fractional order of filter mask the information entropy gets improved approximately by factor 0.5 thus enhancing textural features of image. Also for large size filter mask information entropy is more as compared to the case when filter mask of small size is implemented.

From the results it is concluded that the information entropy with improved Fractional Differential Mask is more than both G-L Fractional Differential Mask and R-L Fractional Differential Mask, hence providing more texture enhancement.

Thus, from the spectrum of the improved G-L Fractional Differential, both the texture and lightness of image are enhanced by the improved G-L Fractional Differential algorithm. And the improved Fractional Differential Mask presented in this thesis work can control the degree of texture enhancement of the image with the filter mask parameters viz. fractional order and the intensity factor.

7.2 Future Scope

The presented work can be further extended to design a Fractional Differential Filter of Digital Image using the proposed Fractional Differential Mask, and this Fractional Differential Filter can be implemented on FPGA. Also this Fractional Differential Mask can be implemented on fingerprint images, collected from the crime scene, so that there texture can be enhanced.

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