“A Study on Transportation Problems with Imprecise Parameters”

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Submitted By

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CERTIFICATE

I hereby certify that the work which is being presented in the thesis entitled, "**A Study on Transportation Problems with Imprecise Parameters**", in partial fulfillment of the requirements for the award of degree of Master of Technology in Computer Science and Applications submitted in Computer Science and Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of Dr. Amit Kumar and refers other researcher's work which are duly listed in the reference section.

The matter presented in the thesis has not been submitted for award of any other degree of this or any other University.

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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.

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ABSTRACT

The classical transportation problem is one of the many well-structured problems in Operations Research that has been extensively studied in literature. The transportation problem is one of the subclass of the linear programming problems for which simple and practical computational procedures have been developed that take the advantage of the special structure of the problem. The transportation problem is amongst the most important special linear programming problem – in terms of the frequency with it appears in the applications and also in the simplicity of the procedure developed for its solutions.

In solving real life transportation problem we often face the state of uncertainty as well as hesitation due to various uncontrollable factors. To deal with uncertainty and hesitation many authors have suggested the fuzzy/intuitionistic fuzzy representation for the data.

In this thesis, it is shown that it is not necessary to use the arithmetic operations of fuzzy/intuitionistic fuzzy numbers to find the fuzzy/intuitionistic fuzzy optimal solution of fuzzy/intuitionistic fuzzy transportation problems. The same can also be obtained by using the arithmetic operations of real numbers. Also, as it is much easy to apply arithmetic operations of real numbers as compared to arithmetic operations of fuzzy/intuitionistic fuzzy numbers so it is better to find the use method based on arithmetic operations of real numbers as compared to method based on arithmetic operations of fuzzy numbers.

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Chapter 1

Introduction

1.1. Classical transportation problem

In today's competitive market, the pressure on organizations to find better ways to create and deliver values to customers becomes stronger. How and when to send the products to the customers in the quantities, they want in a cost-effective manner, become more challenging. Transportation model provides a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods.

The basic transportation problem was originally developed by Hitchcock [15]. The transportation can be modeled as a standard linear programming problem, which can then we solved by simplex method. Charnes and Cooper [8] developed stepping stone method which provides an alternative way of the simplex method information.

Dantzig and Thapa [10] used simplex method to the transportation problem as the primal simplex transportation method. An initial basic feasible solution for transportation problem can be obtained by using north-west corner rule, column minimum, row minimum, matrix minima, or the Vogel approximation method. The modified distribution method is useful for finding the optimal solution for the transportation problem.

1.1.1 Tabular representation

A balanced transportation problem (total availability equal to total demand) having m sources ($O_i, i = 1, 2, ..., m$) and n destinations ($D_j,\ j = 1, 2, ..., n$) can be represented as shown in Table 1.1 [10].

Table 1.1: Tabular representation of a transportation problem

Destinations Origins	D_1	D_2	$\cdots$	D_n	Availability (a_i)
O_1	c_{11}	c_{12}	$\cdots$	c_{1n}	a_1
O_2	c_{21}	c_{22}	$\cdots$	c_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	
O_m	c_{m1}	c_{m2}	$\cdots$	c_{mn}	a_m
Demand b_j	b_1	b_2	$\cdots$	b_n	$\sum_i a_i = \sum_j b_j$

Where,

a_i : quantity of sources of material available at origin $O_i, i = 1, 2, ..., m$.

b_j : quantity of sources of material required at destinations at D_j, $j = 1, 2, ..., n$.

c_{ij} : unit cost of transportation from source O_i to destination D_j.

1.1.2 Linear programming formulation

A balanced transportation problem, represented by Table 1.1, can also be formulated into the following linear programming problem [10]:

$$Minimize\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$Subject\ to$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2, \dots, m$$

$$\sum_{j=1}^{n} x_{ij} = b_j, \qquad j = 1,2, \dots, n$$

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j.$$

where,

x_{ij}: amount transported from the i^{th} origin to the j^{th} destination.

1.2 Fuzzy transportation problem

In conventional transportation models, it is assumed that a decision maker is sure about the precise values of transportation cost, availability and demand of the product. However, in real world applications, all the parameters may not be known precisely due to uncontrollable factors.

In real life problems the following cases may arise:

i. No one has the knowledge of transportation cost while the product is to be transported for first time to a destination. Therefore, there exists uncertainty about the transportation cost.

ii. If a new product is launched in the market then there always exists uncertainty about the demand of that particular product.

iii. Sometimes if there is an increase in Demand of the product in the market, supplier either meets this demand or less than the demand. Here uncertainty prevails in the supply.

iv. Due to natural disasters uncertainty exists in both supply and demand.

To deal quantitatively with imprecise information, the concepts and techniques of probability could be employed. However, probability distributions require either a priori predictable regularity or a posterior frequency distribution to construct. Moreover, the premise that imprecision can be equated with randomness is still questionable. As an alternative, uncertain values can be represented by membership functions of the fuzzy set theory [27]. The main advantages of methodologies based on fuzzy theory are that they do not require prior predictable regularities or posterior frequency distributions, and they can deal with imprecise input information containing feelings and emotions quantified based on the decision-makers subjective judgment.

Due to the same reason several authors have represented some or all the parameters of transportation problems by fuzzy numbers and proposed different methods for solving these problems.

1.2.1 Basic definitions

In this section, some basic definitions of fuzzy set, fuzzy number, triangular fuzzy number and trapezoidal fuzzy number are reviewed [12].

Definition 1.1 Let X be a non-empty set. A fuzzy set $\tilde{A}$ of X is defined as $\tilde{A} = \left\{ \left\langle x, \mu_{\tilde{A}}(x) \right\rangle / x \in X \right\}$ where $\mu_{\tilde{A}}(x)$ is called membership function which maps each element of X to a value between 0 and 1.

Definition 1.2 A fuzzy number $\tilde{A}$ is a convex normalized set on the real line R such that:

- $\tilde{A}$ is normal. It means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$.
- $\tilde{A}$ is convex. It means that for every $x_1, x_2 \in R$

  $\mu_{\tilde{A}}(x)(\lambda x_1 + (1-\lambda)x_2) \geq \min\left\{ \mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2) \right\}, \lambda \in [0,1]$.
- $\mu_{\tilde{A}}(x)$ is upper semi-continuous.

- $\sup(\tilde{A})$ is bounded in R .

Definition 1.3 A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be triangular fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & x = a_2 \\ \dfrac{x - a_3}{a_2 - a_3}, & a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases}$$

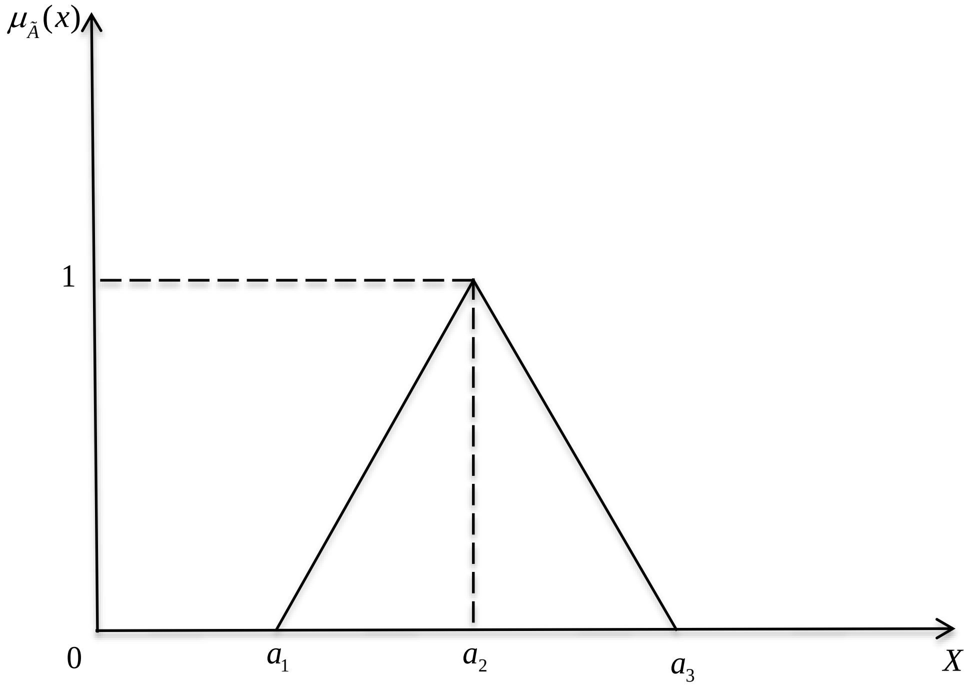

Figure 1.1: Triangular fuzzy number

Definition 1.4: A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is said to be trapezoidal fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, a_1 \le x < a_2\\ 1, \qquad a_2 \le x \le a_3\\ \dfrac{x-a_4}{a_3-a_4}, a_3 < x \le a_4\\ 0, \qquad \text{otherwise}\end{cases}$$

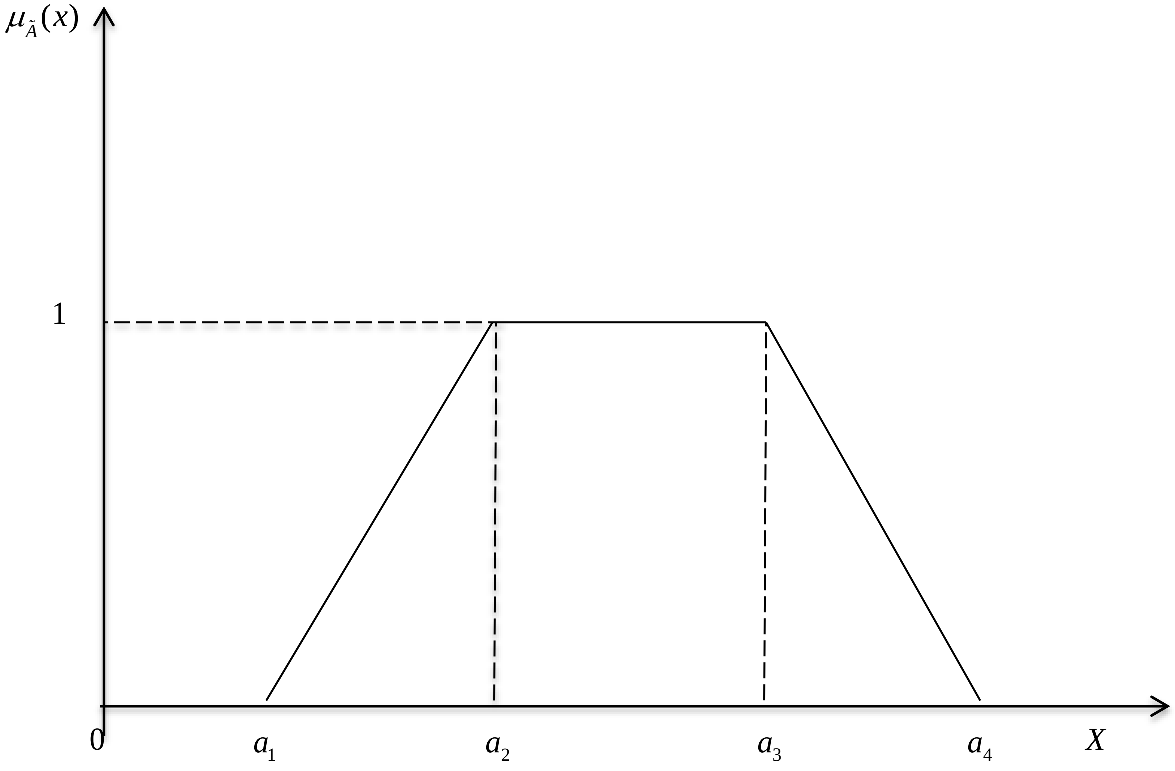

Figure 1.2 Trapezoidal fuzzy number

1.2.2 Arithmetic operations

In this section arithmetic operations of triangular and trapezoidal fuzzy numbers are presented [12].

1.2.2.1 Arithmetic operations of triangular fuzzy numbers

In this section arithmetic operations of triangular fuzzy numbers are presented.

Let $\tilde{A}=(a_1,a_2,a_3)$ and $\tilde{B}=(b_1,b_2,b_3)$ be two triangular fuzzy numbers. Then,

i. $\tilde{A}+\tilde{B}=(a_1+b_1, a_2+b_2, a_3+b_3)$.

ii. $\tilde{A}-\tilde{B}=(a_1-b_3, a_2-b_2, a_3-b_1)$.

iii. $\tilde{A}\times\tilde{B}=(\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$.

iv. $\lambda\tilde{A}=\begin{cases}(\lambda a_1, \lambda a_2, \lambda a_3) & \lambda \geq 0\\ (\lambda b_3, \lambda b_2, \lambda b_1) & \lambda < 0\end{cases}$

1.2.2.2 Arithmetic operations of trapezoidal fuzzy numbers

In this section arithmetic operations of trapezoidal fuzzy numbers are presented.

Let $\tilde{A}=(a_1, a_2, a_3, a_4)$ and $\tilde{B}=(b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. Then,

i. $\tilde{A}+\tilde{B}=(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$.

ii. $\tilde{A}-\tilde{B}=(a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$.

iii. $\tilde{A}\times\tilde{B}=(a,b,c,d)$,

Where

$$a=\min(a_1b_1, a_1b_4, a_4b_1, a_4b_4), b=\min(a_2b_2, a_2b_3, a_3b_2, a_3b_3),$$
$$c=\max(a_2b_2, a_2b_3, a_3b_2, a_3b_3), d=\max(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$$

iv. $\lambda\tilde{A}=\begin{cases}(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4) & \lambda \geq 0\\ (\lambda b_4, \lambda b_3, \lambda b_2, \lambda b_1) & \lambda < 0\end{cases}$

1.2.3 Comparison of fuzzy numbers

Comparison of fuzzy numbers, play an important role in decision making problems. Fuzzy numbers must be compared before an action is taken by decision maker. Jain [17] proposed the concept of ranking function for comparing normal fuzzy numbers. Chen [9] pointed out that in many cases it is not to possible restricting the membership function to the normal and the proposed concept of generalized fuzzy numbers. Since then, tremendous efforts are spent and significant advances are made on the development of numerous methodologies. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function, $\Re : F(\mathbb{R}) \rightarrow \mathbb{R}$, where $F(\mathbb{R})$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists.

Let $\tilde{A} = (a_1, b_1, c_1, d_1), \tilde{B} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then,

(i) $\tilde{A} >_{\Re} \tilde{B} \, iff \; \Re(\tilde{A}) > \Re(\tilde{B})$

(ii) $\tilde{A} <_{\Re} \tilde{B} \, iff \; \Re(\tilde{A}) < \Re(\tilde{B})$

(iii) $\tilde{A} =_{\Re} \tilde{B} \, iff \; \Re(\tilde{A}) = \Re(\tilde{B})$

where,

$\Re(\tilde{A}) = (a_1 + b_1 + c_1 + d_1)/4,$ and $\Re(\tilde{B}) = (a_2 + b_2 + c_2 + d_2)/4$.

1.2.4 Tabular representation

A balanced fuzzy transportation problem (total fuzzy availability equal to total fuzzy demand) having *m* sources ($O_i, i = 1, 2, ..., m$) and *n* destinations ($D_j,\ j = 1, 2, ..., n$) can be represented as shown in Table 1.2 [1].

Table 1.2: Tabular representation of a fuzzy transportation problem

Destinations Origins	D_1	D_2	$\ldots$	D_n	Fuzzy Availability $(\tilde{a}_i)$
O_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\ldots$	$\tilde{c}_{1n}$	$\tilde{a}_1$
O_2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\ldots$	$\tilde{c}_{2n}$	$\tilde{a}_2$

$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	
O_m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	$\cdots$	$\tilde{c}_{mn}$	$\tilde{a}_m$
Demand $\tilde{b}_j$	$\tilde{b}_1$	$\tilde{b}_2$	$\cdots$	$\tilde{b}_n$	$\sum_i \tilde{a}_i = \sum_j \tilde{b}_j$

1.2.5 Linear programming formulation

A balanced fuzzy transportation problem, represented by Table 1.2, can also be formulated into the following fuzzy linear programming problem [1]:

$$Minimize\ \tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$$

Subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i \qquad i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j \qquad j = 1,2,\dots,m$$

$$\tilde{x}_{ij} \geq 0\ \forall\ i,j.$$

$$\sum_{j=1}^{n} \tilde{a}_i = \sum_{i=1}^{m} \tilde{b}_j$$

1.3 Intuitionistic fuzzy transportation problem

In fuzzy sets the degree of acceptance is considered only but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than1. Intuitionistic fuzzy set (IFS) is one of the generalizations of fuzzy sets theory [3]. Out of several higher-order fuzzy sets, IFS first introduced by Attanassov [2] have been found to be compatible to deal with vagueness. The concept of IFS can be viewed as an appropriate/alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set.

Presently intuitionistic fuzzy sets are being studied and used in different fields of science. Usually, it is assumed that transportation costs are exactly known. However, these costs depend, in reality, on many factors, e.g., on the travelling time, which depends also on weather, on the present traffic, on the current situation of the road (traffic jams, road works etc.), Weight of the load etc. This means that in many situations such costs cannot be exactly known, but they are estimated [26]. Thus in estimating the transportation cost the decision maker is not very much sure. He may hesitate in predicting the transportation cost. Thus it is better to consider the cost as intuitionistic fuzzy number to deal efficiently with the uncertainty as well as hesitation arising in prediction of transportation cost.

1.3.1 Basic definitions

In this section, some basic definitions of intuitionistic fuzzy set, intuitionistic fuzzy number, triangular intuitionistic fuzzy number and trapezoidal intuitionistic fuzzy number are reviewed [26].

Definition 1.5 Let X be non-empty set. An intuitionistic fuzzy set $\tilde{A}^I$ of X is defined as $\tilde{A}^I = \left\{ \left\langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) \right\rangle / x \in X \right\}$ where $\mu_{\tilde{A}^I}(x)$ and $\nu_{\tilde{A}^I}(x)$ are membership and non-membership functions such that $\mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) : X \to [0,1]$ and $0 \le \mu_{\tilde{A}^I}(x) \le \nu_{\tilde{A}^I}(x) \le 1$ for all $x \in X$.

Definition 1.6 An intuitionistic fuzzy subset $\tilde{A}^I = \left\{ \left\langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) \right\rangle / x \in X \right\}$ of real line X is called an intuitionistic Fuzzy Number if the following conditions hold:

(i) There exists a real number *m* such that $\mu_{\tilde{A}^I}(m) = 1$ and $\nu_{\tilde{A}^I}(m) = 0$

(ii) $\mu_{\tilde{A}^I} : \mathbb{R} \to [0,1]$ is a continuous function such that $0 \le \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1$ for all $x \in X$.

(iii) The membership and non-membership functions $\tilde{A}^I$ are the following form:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0, & -\infty < x \le a_1 \\ f(x), & a_1 \le x \le a_2 \\ 1, & x = a_2 \\ g(x), & a_2 \le x \le a_3 \\ 0, & a_2 \le x < \infty \end{cases} \qquad \nu_{\tilde{A}^I}(x) = \begin{cases} 1, & -\infty < x \le a_1^{'} \\ f^{'}(x), & a_1^{'} \le x \le a_2 \\ 0, & x = a_2 \\ g^{'}(x), & a_2 \le x \le a_3^{'} \\ 1, & a_3 \le x < \infty \end{cases}$$

Where $f, f^{'}, g, g^{'}$ are functions from $\mathbb{R} \to [0,1]$, f and $g^{'}$ are strictly increasing functions and g and $f^{'}$ are strictly decreasing functions with the conditions $0 \le f(x) + f^{'}(x) \le 1$ and $0 \le g(x) + g^{'}(x) \le 1$.

Definition 1.7 A triangular intuitionistic fuzzy number $\tilde{A}^I$ is denoted by $\tilde{A}^I = (a_1, a_2, a_3)(a_1^{'}, a_2, a_3^{'})$ where $a_1^{'} \le a_1 \le a_2 \le a_3 \le a_3^{'}$ with the following membership $\mu_{\tilde{A}^I}(x)$ and non-membership function $\nu_{\tilde{A}^I}(x)$.

$$\mu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & \text{for } a_1\le x\le a_2\\ 1, & \text{for } x=a_2\\ \dfrac{x-a_3}{a_2-a_3}, & \text{for } a_2\le x\le a_3\\ 0, & \text{otherwise}\end{cases}$$

$$\nu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{x-a_2}{a_1'-a_2}, & \text{for } a_1'\le x\le a_2\\ 0, & \text{for } x=a_2\\ \dfrac{x-a_2}{a_3'-a_2}, & \text{for } a_2\le x\le a_3'\\ 1, & \text{otherwise}\end{cases}$$

Definition 1.8 A trapezoidal intuitionistic fuzzy number is denoted by $\tilde{A}^I=(a_1,a_2,a_3,a_4)(a_1',a_2,a_3,a_4')$ where $a_1'\le a_1\le a_2\le a_3\le a_4\le a_4'$ with the following membership $\mu_{\tilde{A}^I}(x)$ and non-membership function $\nu_{\tilde{A}^I}(x)$.

$$\mu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & \text{for } a_1\le x\le a_2\\ 1, & \text{for } a_2\le x\le a_3\\ \dfrac{x-a_4}{a_3-a_4}, & \text{for } a_3\le x\le a_4\\ 0, & \text{otherwise}\end{cases}$$

$$\nu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{x-a_2}{a_1'-a_2}, & \text{for } a_1'\le x\le a_2\\ 0, & \text{for } a_2\le x\le a_3\\ \dfrac{x-a_3}{a_4'-a_3}, & \text{for } a_3\le x\le a_4'\\ 1, & \text{otherwise}\end{cases}$$

1.3.2 Arithmetic operations

In this section, arithmetic operations of triangular and trapezoidal intuitionistic fuzzy numbers are presented [26].

1.3.2.1 Arithmetic operations of triangular intuitionistic fuzzy numbers

In this section arithmetic operations of triangular intuitionistic fuzzy numbers are presented.

Let $\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$ and $\tilde{B}^I = (b_1, b_2, b_3)(b_1', b_2, b_3')$ be any two triangular intuitionistic fuzzy numbers. Then,

I. $\tilde{A}^I + \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3)(a_1' + b_1', a_2 + b_2, a_3' + b_3')$

II. $\tilde{A}^I - \tilde{B}^I = (a_1 - b_3, a_2 - b_2, a_3 - b_1)(a_1' - b_3', a_2 - b_2, a_3' - b_1')$

III. $\tilde{A}^I \times \tilde{B}^I = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)(a_1' \times b_1', a_2 \times b_2, a_3' \times b_3')$ where $\tilde{A}^I, \tilde{B}^I$ are non-negative triangular intuitionistic fuzzy numbers.

1.3.2.2 Arithmetic operations of trapezoidal intuitionistic fuzzy numbers

In this section arithmetic operations of trapezoidal intuitionistic fuzzy numbers are presented.

Let $\tilde{A}^I = (a_1, a_2, a_3, a_4)(a_1', a_2, a_3, a_4')$ and $\tilde{B}^I = (b_1, b_2, b_3, b_4)(b_1', b_2, b_3, b_4')$ be any two trapezoidal intuitionistic fuzzy numbers. Then,

I. $\tilde{A}^I - \tilde{B}^I = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)(a_1' - b_4', a_2 - b_3, a_3 - b_2, a_4' - b_1')$

II. $\tilde{A}^I + \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)(a_1' + b_1', a_2 + b_2, a_3 + b_3, a_4' + b_4')$

III. $\tilde{A}^I \times \tilde{B}^I = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4; a_1' b_1', a_2 b_2, a_3 b_3, a_4' b_4')$ Where $\tilde{A}^I, \tilde{B}^I$ are non-negative trapezoidal intuitionistic fuzzy numbers.

1.3.3 Comparison of intuitionistic fuzzy numbers [26]

Let $\tilde{A}^I = (a_1, a_2, a_3,)(a_1', a_2, a_3')$ and $\tilde{B}^I = (b_1, b_2, b_3)(b_1', b_2, b_3')$ be two triangular intuitionistic fuzzy numbers. Then,

(i) $\Re(\tilde{A}^I) > \Re(\tilde{B}^I) \, iff \; \tilde{A}^I > \tilde{B}^I$

(ii) $\Re(\tilde{A}^I) < \Re(\tilde{B}^I) \, iff \; \tilde{A}^I < \tilde{B}^I$

(iii) $\Re(\tilde{A}^I) = \Re(\tilde{B}^I) \, iff \; \tilde{A}^I = \tilde{B}^I$

Where,

$$\Re(\tilde{A}^I) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

$$\Re(\tilde{B}^I) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

1.3.4 Tabular representation

A balanced intuitionistic fuzzy transportation problem (total intuitionistic fuzzy availability equal to total intuitionistic fuzzy demand) having *m* sources ($O_i, i = 1, 2, ..., m$) and *n* destinations ($D_j,\ j = 1, 2, ..., n$) can be represented as shown in Table 1.3 [26].

Table 1.3: Tabular representation of an intuitionistic fuzzy transportation problem

Destinations Origins	D_1	D_2	$\dots$	D_n	Intuitionistic fuzzy Availability (a_i)
O_1	$\tilde{c}_{11}{}^I$	$\tilde{c}_{12}{}^I$	$\dots$	$\tilde{c}_{1n}{}^I$	$\tilde{a}_1{}^I$
O_2	$\tilde{c}_{21}{}^I$	$\tilde{c}_{22}{}^I$	$\dots$	$\tilde{c}_{2n}{}^I$	$\tilde{a}_2{}^I$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	

O_m	$\tilde{c}_{m1}{}^{I}$	$\tilde{c}_{m2}{}^{I}$	$\dots$	$\tilde{c}_{mn}{}^{I}$	$\tilde{a}_m{}^{I}$
Demand $\tilde{b}_j{}^{I}$	$\tilde{b}_1{}^{I}$	$\tilde{b}_2{}^{I}$	$\dots$	$\tilde{b}_n{}^{I}$	$\sum_i \tilde{a}_i{}^{I} = \sum_j \tilde{b}_j{}^{I}$

1.3.5 Linear programming formulation

A balanced intuitionistic fuzzy transportation problem, represented by Table 1.3, can also be formulated into the following fuzzy linear programming problem [26]:

$$
\begin{aligned}
&\text{Minimize}\,\tilde{Z}^{I} = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}{}^{I}\tilde{x}_{ij}{}^{I}\\
&\text{Subject to}\\
&\sum_{j=1}^{n}\tilde{x}_{ij}{}^{I} = \tilde{a}_{ij}{}^{I}, \quad i = 1, 2, \ldots, m\\
&\sum_{i=1}^{m}\tilde{x}_{ij}{}^{I} = \tilde{b}_{ij}{}^{I}, \quad j = 1, 2, \ldots, n\\
&\textit{and}\\
&\tilde{x}_{ij}{}^{I} \geq 0, \quad \forall i, j
\end{aligned}
$$

1.4 Organization of thesis

The chapter wise summary of the thesis is as follows:

In Chapter 1 a brief introduction about transportation problem, fuzzy transportation problem and intuitionistic fuzzy transportation problem is presented..

There are different methods for solving various type of fuzzy transportation problems and intuitionistic fuzzy transportation problems. In Chapter 2, an existing method for finding paradox of a particular type of fully fuzzy transportation problem as well as an existing method to find the intuitionistic fuzzy optimal solution of a particular type of intuitionistic fuzzy transportation problem is presented.

In Chapter 3 the objectives of the thesis are discussed.

In Chapter 4 the objectives, mentioned in Chapter 3, are achieved.

Chapter 5 draws the conclusions and recommendations for future work.

Chapter 2

Literature Survey

There are a lot of papers in the area of fuzzy transportation and intuitionistic transportation problems. After reviewing some recently published papers, it is concluded that there is an alternative and easy way to solve the fuzzy transportation problem and intuitionistic fuzzy transportation problem considered by [1] and [26] respectively. Keeping the same in mind, the same is proposed in this thesis and hence, in this chapter only the existing methods [1,26] are presented in detailed manner.

2.1 Intuitionistic fuzzy transportation problem

Thamaraiselvi and Santhi [26] proposed the following method to obtain an initial basic feasible solution and optimal solution of such intuitionistic fuzzy transportation problems in which availability and demand are represented by triangular (hexagonal) intuitionistic fuzzy numbers while the per unit transportation costs are represented by real numbers.

2.1.1 Intuitionistic fuzzy initial basic feasible solution (IFIBFS)

Thamaraiselvi and Santhi [26] proposed the intuitionistic fuzzy Vogel’s approximation method (IFVAM) to obtain the intuitionistic fuzzy basic feasible solution.

The method proceeds as follows.

Step 1: Calculate the magnitude of difference between the minimum and next to minimum transportation cost in each row and column and write it as “Diff.” along the side of the table against the corresponding row/column.

Step 2: In the row /column corresponding to maximum “Diff.”, make the maximum allotment at the box having minimum transportation cost in that row/ column.

Step 3: If the maximum “Diff.” corresponding to two or more rows or columns are equal, select the top most row and the extreme left column. Repeat the above procedure until all the IF supplies are fully used and IF demands are fully received.

2.1.2 Intuitionistic fuzzy optimal solution (IFOS)

Thamaraiselvi and Santhi [26] proposed the following method to obtain the intuitionistic fuzzy optimal solution of the intuitionistic fuzzy transportation problem.

Step 1: Write the intuitionistic fuzzy transportation problem in tabular form.

Step 2: Subtract each row entries of the table from the row minimum or row reduced.

Step 3: In the table, obtained in Step 1, subtract each column entries from the column minimum. We get at least one zero in each row and each column in the resultant table.

Step 4: In the resultant table, obtained in Step 3, for every zero cost cell, count the total number of zeros in the corresponding row and column. Suppose $(i, j)^{th}$ zero cost cell is selected, count the total number of zeros in the i^{th} row and j^{th} column.

Step 5: Now select a cell of zero cost for which the number of zeros counted in Step 4 is minimum and allocate the maximum possible hexagonal intuitionistic fuzzy quantity to that cell. If tie occurs for some zeros in Step 4 then find the sum of all the elements in the corresponding row and column. Now choose the zero with maximum sum and allocate the maximum possible quantity to that cell.

Step 6: After every allocation, delete the row or column for which the demand fulfilled and the supply is depleted.

Step 7: Repeat Step 3 to Step 6 until all the demands are satisfied and all the supplies are exhausted.

2.1.3 Numerical example

Thamaraiselvi and Santhi [26] solved the intuitionistic fuzzy transportation problem, represented by Table 2.1, to illustrate their proposed method.

Table 2.1: Intuitionistic fuzzy transportation problem

Destinations Origins	D_1	D_2	D_3	D_4	IF Supply
O_1	5	6	12	9	(7,9,11,13,16, 20) (5,7,11,13,19, 20)
O_2	3	2	8	4	(6,8,11,14,19, 21) (4,7,11,14,21, 27)
O_3	7	11	20	9	(9,11,13,15, 18, 20) (8,10,13,15, 19, 22)
IF Demand	(3,4,5, 6,8,10) (2,4,5, 6,10,12)	(3,5,7, 9,12,16) (2,4,7, 9,13,17)	(6,7,9, 11,13,24) (5,6,9, 11,16,18)	(10,12,14, 16,20, 24) (8,10,14,16,20, 25)	

2.1.3.1 Intuitionistic fuzzy initial basic feasible solution (IFIBFS)

Using the intuitionistic fuzzy VAM method, proposed by Thamaraiselvi and Santhi [26], an intuitionistic fuzzy IBFS of the IFTP, represented by Table 2.1, can be obtained as follows:

Step 1: The magnitude of difference "Diff" between the minimum and next to minimum transportation cost in each row and column is shown in Table 2.2.

Table 2.2: Intuitionistic fuzzy transportation problem with "Diff"

Destinations Origins	D_1	D_2	D_3	D_4	IF Supply	Diff
O_1	5	6	12	9	(7,9,11,13,16,20) (5,7,11,13,19,20)	1
O_2	3	2	8	4	(6,8,11,14,19,21) (4,7,11,14,21,27)	1
O_3	7	11	20	9	(9,11,13,15,18,20) (8,10,13,15,19,22)	2
IF Demand	(3,4,5,6,8,10) (2,4,5,6,10,12)	(3,5,7,9,12,16) (2,4,7,9,13,17)	(6,7,9,11,13,24) (5,6,9,11,16,18)	(10,12,14,16,20,24) (8,10,14,16,20,25)		
Diff	2	4	4	5 ↑		

Step 2: Using the Step 2 identify the row/column corresponding to the highest value of "Diff". In this case it occurs at column 4. In this column minimum cost cell is $(2,4)$ and the corresponding demand and supply are $(10, 12, 14, 16, 20, 24)$ $(8, 10, 14, 16, 20, 25)$ and $(6, 8, 11, 14, 19, 25)$ $(4, 7, 11, 14, 21, 27)$ respectively. Now allocate the (minimum of the above demand and supply) maximum possible units $(6, 8, 11, 14, 19, 25)$ $(4, 7, 11, 14, 21, 27)$ to the minimum cost position $(2,4)$ and write the remaining in column 4. After removing the second row repeats the Step 1, we obtained new Table 2.3

Table 2.3: First Allocation

Destinations Origins	D_1	D_2	D_3	D_4	IF Supply	Diff
O_1	5	6	12	9	$(7,9,11,13,16,20)$ $(5,7,11,13,19,20)$	1
O_2	-	-	-	$(6,8,11,14,19,25)$ $(4,7,11,14,21,27)$	-	-
O_3	7	11	20	9	$(9,11,13,15,18,220)$ $(8,10,13,15,19,22)$	2
IF Demand	$(3,4,5,6,8,10)$ $(2,4,5,6,10,12)$	$(3,5,7,9,12,16)$ $(2,4,7,9,13,17)$	$(6,7,9,11,13,24)$ $(5,6,9,11,16,18)$	$(-15,-7,0,5,12,18)$ $(-19,-1,0,5,13,21)$		
Diff	2	5	8 ↑	0		

Step 3: In Table 2.3 the maximum value of "Diff" occurs at the third column. So allocate the maximum possible units $(6, 7, 9, 11, 13, 16)\ (5, 6, 9, 11, 16, 18)$ to the minimum cost position $(1, 3)$ and write the remaining in first row. After removing the third column repeat the steps 1 to 3. Now highest value of "Diff" occurs at second column. Now allocate the maximum possible units $(-9, -4,\ 0,\ 4,\ 9,\ 14)\ (-13,\ \text{-}9,\ 0,\ 4,\ 13,\ 18)$ to the minimum cost position $(1, 2)$. After writing the remaining in column 2, removing the first row and repeating the Step1, Table 2.4 is obtained.

Table 2.4: The next allocations

Destinations Origins	D_1	D_2	D_3	D_4	IF Supply	Diff
O_1	-	$(-9,-4,0,$ $4,9,1)$ $(13,-9,0,$ $4,13,18)$	(6,7,9, 11,13,16) (5,6,9, 11,16,18)	-	-	-
O_2	-	-	-	(6,8,11, 14,19,25) (4,7,11, 14,21,27)	-	-
O_3	7	11	-	9	(9,11,13, 15,18, 20) (8,10,13, 15,19, 22)	2

IF Demand	(3,4,5,6,8,10) (2,4,5,6,10,12)	$(-11,-4,3,9,16,24)$ $(-19,-9,3,9,22,30)$	-	$(-15,-7,0,5,12,18)$ $(-19,-11,0,5,13,21)$		
Diff	2	-	-	-		

Step 4: Repeating the same procedure, the optimal solution shown in Table 2.5, is obtained.

Table 2.5: The results

Destinations Origins	D_1	D_2	D_3	D_4	IF Supply	Diff
O_1	-	$(-9,-4,0,4,9,1)$ $(-13,-9,0,4,13,18)$	(6,7,9,11,13,16) (5,6,9,11,16,18)	-	-	-
O_2	-	-	-	(6,8,11,14,19,25) (4,7,11,14,21,27)	-	-
O_3	(3,4,5,6,8,10) (2,4,5,6,10,12)	$(-11,-4,3,9,16,24)$ $(-19,-9,3,9,22,30)$	-	$(-15,-7,0,5,12,18)$ $(-19,-11,0,5,13,21)$	-	-
IF Demand	-	-	-	-		
Diff	-	-	-	-		

It is obvious from Table 2.5 that the intuitionistic fuzzy basic feasible solution is,

$$\tilde{x}_{12} = (-9,-4,0,4,9,14)(-13,-9,0,4,13,18),$$
$$\tilde{x}_{13} = (6,7,9,11,13,16)(5,6,9,11,16,18)$$
$$\tilde{x}_{24} = (6,8,11,14,19,25)(4,7,11,14,21,27)$$
$$\tilde{x}_{31} = (3,4,5,6,8,10)(2,4,5,6,10,12)$$
$$\tilde{x}_{32} = (-11,-4,3,9,16,24)(-19,-11,0,5,13,21)$$
$$\tilde{x}_{33} = (-15,-7,0,5,12,18)(-19,-11,0,5,13,21)$$

and the minimum total intuitionistic fuzzy transportation cost is

$$\begin{aligned}\tilde{Z}^I &= 6(-9,-4,0,4,9,14)(-13,-9,0,4,13,18)+12(6,7,9,11,13,16)(5,6,9,11,16,18)\\ &+4(6,8,11,14,19,25)(4,7,11,14,21,27)+7(3,4,5,6,8,10)(2,4,5,6,10,12)\\ &+11(-11,-4,3,9,16,24)(-19,-11,0,5,13,21)+9(-15,-7,0,5,12,18)(-19,-11,0,5,13,21)\\ &=(-193,13,220,398,626,872)(-335,-124,220,398,783,1035)\end{aligned}$$

2.1.3.2 Intuitionistic fuzzy optimal solution (IFOS)

Using the method, proposed by Thamaraiselvi and Santhi [26], an intuitionistic fuzzy optimal solution of the IFTP, represented by Table 2.1, can be obtained as follows:

Step 1: Using Step 2 and Step 3 of the method, proposed by Thamaraiselvi and Santhi [26], Table 2.6 is obtained.

Table 2.6: Row and Column contain at least 1 zero

Destinations Origins	D_1	D_2	D_3	D_4	IF Supply
O_1	0	1	0	2	(7,9,11,13,16,20) (5,7,11,13,19,20)
O_2	1	0	1	0	(6,8,11,14,19,21) (4,7,11,14,21,27)
O_3	0	2	8	0	(9,11,13,15,18,20) (8,10,13,15,19,22)
IF Demand	(3,4,5,	(3,5,7,	(6,7,9,	(10,12,14,	

	6,8,10) (2,4,5,6,10,12)	9,12,16) (2,4,7,9,13,17)	11,13,24) (5,6,9,11,16,18)	16,20,24) (8,10,14,16,20,25)	

Step 2: Applying Step 4 and Step 5 for the Table 2.6. For every 0, count the total number of 0's in the corresponding row and column. Since, the zero in the cell $(1,3)$ has minimum number (2) of zeros with maximum sum (12) of elements in three first row and third column. Now allocate the maximum possible units $(6,7,9,11,13,16)(5,6,9,11,16,18)$ to the position $(1,3)$ and write the remaining in row 1. After removing the third column we obtain the following table.

Table 2.7: First allocation

Destinations Origins	D_1	D_2	D_3	D_4	IF Supply
O_1	0	1	(6,7,9,11,13,24) (5,6,9,11,16,18)	2	$(-9,-4,0,4,9,14)$ $(-13,-9,0,4,13,18)$
O_2	1	0	-	0	(6,8,11,14,19,21) (4,7,11,14,21,27)
O_3	0	4	-	0	(9,11,13,15,18,20) (8,10,13,15,19,22)
IF Demand	(3,4,5,6,8,10) (2,4,5,6,10,12)	(3,5,7,9,12,15) (2,4,7,9,13,17)	-	(10,12,14,16,20,24) (8,10,14,16,20,25)	

Step 3: Applying the Step 4 and Step 5 and allocate the maximum possible units $(3,5,7,9,12,15)(2,4,7,9,13,17)$ to the position $(2,2)$ and write the remaining in second

row. After removing the second column again apply the Step 4 to Step 6, we obtain the following table.

Table 2.8: Second allocation

Destinations Origins	D_1	D_2	D_3	D_4	IF Supply
O_1	$(-9,-4,0,4,9,14)$ $(-13,-9,0,4,13,18)$	-	(6,7,9,11,13,24) (5,6,9,11,16,18)	2	-
O_2	1	(3,5,7,9,12,16) (2,4,7,9,13,17)	-	0	$(-9,-4,2,7,14,22)$ $(-13,-6,2,7,17,25)$
O_3	0	-	-	0	(9,11,13,15,18,220) (8,10,13,15,19,22)
IF Demand	$(-11,-5,1,6,12,19)$ $(-16,-9,1,6,19,25)$	-	-	(10,12,14,16,20,24) (8,10,14,16,20,25)	

Step 4: Apply the same steps to fulfill the demand and supply and the allocation table is:

Table 2.9: The optimal solution

Destinations Origins	D_1	D_2	D_3	D_4	IF Supply
O_1	$(-9,-4,0,4,9,14)$ $(-13,-9,0,4,13,18)$	-	$(6,7,9,11,13,24)$ $(5,6,9,11,16,18)$	2	-
O_2	1	$(3,5,7,9,12,16)$ $(2,4,7,9,13,17)$	-	$(-9,-4,2,7,14,22)$ $(-13,-6,2,7,17,25)$	-
O_3	$(-11,-5,1,6,12,19)$ $(-16,-9,1,6,19,25)$	-	-	$(-10,-1,7,14,23,31)$ $(-17,-9,7,14,28,38)$	-
IF Demand	-	-	-	-	

Thus, the intuitionistic fuzzy optimal solution in terms of hexagonal intuitionistic fuzzy numbers (HIFN) is,

$$\tilde{x}_{11} = (-9,-4,0,4,9,14)(-13,-9,0,4,13,18), \tilde{x}_{13} = (6,7,9,11,13,16)(5,6,9,11,16,18),$$

$$\tilde{x}_{22} = (3,5,7,9,12,15)(2,4,7,9,13,17), \tilde{x}_{24} = (-9,-4,2,7,14,22)(-13,-6,2,7,17,25),$$

$$\tilde{x}_{31} = (-11,-5,1,6,12,19)(-16,-9,1,6,199,25), \tilde{x}_{34} = (-10,-1,7,14,23,31)(-17,-9,7,14,28,38)$$

And the total minimum intuitionistic fuzzy transportation cost is

$$\begin{aligned}
\tilde{Z}^{I} &= 5(-9,-4,0,4,9,14)(-13,-9,0,4,13,18) + 12(6,7,9,11,13,24)(5,6,9,11,16,18) \\
&+ 2(3,5,7,9,12,16)(2,4,7,9,13,17) + 4(-9,-4,2,7,14,22)(-13,-6,2,7,17,25) \\
&+ 7(-11,-5,1,6,12,19)(-16,-9,1,6,19,25) + 9(-10,-1,7,14,23,31)(-17,-9,7,14,28,38) \\
&= (-170,-6,200,366,572,792)(-318,-133,200,366,736,957)
\end{aligned}$$

2.2 Paradox in transportation problems under fuzzy environments

In some cases of the transportation problem, an increase in the supplies and demands or in other words, increase in the flow results a decrease in the optimum transportation cost. This type of behavior which means paradoxical is called transportation paradox. Acharya et al. [1] proposed a sufficient condition for the existence of paradox in a transportation problem under fuzzy environment.

2.2.1 Basic definitions

In this section some basic definitions are presented [1].

Definition 2.1 In a fuzzy transportation problem if we can obtain more flow $(\tilde{F}_1)$ with lesser transportation cost $(\tilde{Z}_1)$ than the optimum flow $(\tilde{F}_0)$ corresponding to the optimum transportation cost $(\tilde{Z}_0)$ i.e. $\tilde{F}_1 > \tilde{F}_0$ and $\tilde{Z}_1 > \tilde{Z}_0$, then we say that a paradox occurs in a fuzzy transportation problem.

Definition 2.2 If the value of the objective function is $\tilde{Z}_i$ and the flow is $\tilde{F}_i$ corresponding to the feasible solution $\tilde{x}_i$ of a fuzzy transportation problem, then the pair $(\tilde{Z}_i \ \tilde{F}_i)$ is called the fuzzy cost-flow pair corresponding to the feasible solution $\tilde{x}_i$.

Definition 2.3 A fuzzy cost-flow pair $(\tilde{Z}, \tilde{F})$ of an objective function is called fuzzy paradoxical pair if $\tilde{Z} < \tilde{Z}_0$ and $\tilde{F} > \tilde{F}_0$ where $\tilde{Z}_0$ is the optimum transportation cost and $\tilde{F}_0$ is the optimum flow of the fuzzy transportation problem.

Definition 2.4 The fuzzy paradoxical pair $(\tilde{Z}^*, \tilde{F}^*)$ is called the best fuzzy paradoxical pair of a fuzzy transportation problem if for all fuzzy paradoxical pair $(\tilde{Z}, \tilde{F})$, either $\tilde{Z}^* < \tilde{Z}$ or $\tilde{Z}^* = \tilde{Z}$ but $\tilde{F}^* > \tilde{F}$.

Definition 2.5 If $\tilde{F}_0$ is the optimum flow and $\tilde{F}_*$ be the flow corresponding to the best fuzzy paradoxical pair of a fuzzy transportation problem then $[\tilde{F}_0, \tilde{F}_*]$ is called fuzzy paradoxical range of flow.

Theorem 2.1 [1] The sufficient condition for the existence of paradoxical solution of (P) that in the optimum table of (P), $\exists$ at least one cell $(r,s) \notin B$ where we have $(\tilde{u}_r + \tilde{v}_s) < 0$, if $\tilde{a}_r$ and $\tilde{b}_s$ are replaced by $\tilde{a}_r + \tilde{l}$ and $\tilde{b}_s + \tilde{l}\,(\tilde{l} > 0)$ respectively.

$P : \tilde{Z}_0 = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$

Subject to

$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i \,;\forall\, i \in I = (1,2,...,m)$

$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{b}_i \,;\forall\, j \in J = (1,2,...,n)$

$\tilde{x}_{ij} \geq 0, \forall (i,j) \in IxJ$

Proof: Let $\tilde{Z}_0$ be the value of the objective function and $\tilde{F}_0$ be the optimum flow corresponding to the optimum solution $(\tilde{x}_0)$ of the problem (P). The dual variables $\tilde{u}_i$ *and* $\tilde{v}_j$ are given by $\tilde{u}_i + \tilde{v}_j = \tilde{c}_{ij} \forall (i,j) \in B$, Then the value of the objective function in terms of the dual variables is given by

$$\tilde{Z}_0 = \sum_i \sum_j \tilde{c}_{ij} \tilde{x}_{ij}{}^0$$

$$= \sum_i \sum_j (\tilde{u}_i + \tilde{v}_j) \tilde{x}_{ij}{}^0$$

$$= \sum_i (\sum_j \tilde{x}_{ij}{}^0)\tilde{u}_i + \sum_j (\sum_i \tilde{x}_{ij}{}^0)\tilde{v}_j$$

$$= \sum_i \tilde{a}_i \tilde{u}_i + \sum_j \tilde{b}_j \tilde{v}_j$$

Now let $\exists$ at least one cell $(r,s) \notin B,$ where if we replace $\tilde{a}_r$ and $\tilde{b}_s$ by $\tilde{a}_r + \tilde{l}$ and $\tilde{b}_s + \tilde{l}$ respectively $(\tilde{l} > 0),$ in such a way that the optimum basis remains same, then the value of the objective function $\overset{\frown}{\tilde{Z}}$ is given by

$$\overset{\frown}{\tilde{Z}} = \left[\sum_{i, i \neq r} \tilde{a}_i \tilde{u}_i + \sum_{j, j \neq s} \tilde{b}_j \tilde{v}_j + \tilde{u}_r (\tilde{a}_r + \tilde{l}) + \tilde{v}_s (\tilde{b}_s + \tilde{l}) \right]$$

$$= \left[\tilde{Z}^0 + \tilde{l}(\tilde{u}_r + \tilde{v}_s) \right]$$

The new flow $\overset{\frown}{\tilde{F}}$ is given by

$$\overset{\frown}{\tilde{F}} = \sum_i \tilde{a}_i + \tilde{l} = \sum_j \tilde{b}_j + \tilde{l} = \tilde{F}^0 + \tilde{l}$$

$$\tilde{\tilde{F}} - \tilde{F}^0 = \tilde{l} > \tilde{0}$$

Therefore the existence of paradox we must have $\tilde{\tilde{Z}} - \tilde{Z}^0 < \tilde{0}$ since $\tilde{\tilde{F}} - \tilde{F}^0 > \tilde{0}$ i.e. $\tilde{u}_r + \tilde{v}_s < \tilde{0}$ because $\tilde{l} > \tilde{0}$.

Hence the theorem is proved.

2.2.2 Existing algorithm

Acharya et al. [1] proposed the following algorithm to find all the paradoxical pairs of the problem (P).

Step 1: $i = 0$.

Step 2: Find the cost flow pair $(\tilde{Z}_0, \tilde{F}_0)$ for the optimum solution $\tilde{x}_0$.

Step 3: Find all cells $(r,s) \notin B$ such that $(\tilde{u}_r + \tilde{v}_s) < \tilde{0}$ if it exists, otherwise go to Step 8.

Step 4: Find min flow for $\tilde{l} = (1,0,0,0), \tilde{l} = (0,1,0,0), \tilde{l} = (0,0,1,0),\ \tilde{l} \approx (0,0,0,1)$ or $\tilde{l} = (1,1,1,1),$ which enters into existing basis whose corresponding cost is minimum. Let $(\tilde{Z}_i, \tilde{F}_i)$ be the new cost flow pair corresponding to the optimum solution X_i.

Step 5: $i = i + 1.$

Step 6: Write $(\tilde{Z}_i, \tilde{F}_i).$

Step 7: Find all cells $(r,s) \notin B$ such that $(\tilde{u}_r + \tilde{v}_s) < \tilde{0}$ if it exists go to Step 4, otherwise go to Step 9.

Step 8: Write paradox does not exist and go to Step 10.

Step 9: Write paradox exists and the best paradox pair $(\tilde{Z}_*, \tilde{F}_*) = (\tilde{Z}_i, \tilde{F}_i)$ for the optimum solution $\tilde{x}_* = \tilde{x}_i$.

Step 10: End.

2.2.3 Numerical example

Acharya et al. [1] considered a numerical example which consists of three origins and four destinations, the uncertain number of supply, demand and cost per unit are tabulated in Table 2.10.

Table 2.10: Fuzzy transportation problem

Destination → Origin ↓	D_1	D_2	D_3	D_4	$\tilde{a}_i$
O_1	(0,1,3,4)	(5,6,8,9)	(1,2,4,5)	(6,7,9,10)	(38,39,41,42)

O_2	(4,5,7,8)	(0,1,1,2)	(7,8,10,11)	(2,3,5,6)	(48,49,51,52)
O_3	(1,2,4,5)	(6,7,9,10)	(0,1,3,4)	(8,9,11,12)	(38,39,41,42)
	(18,19,21,22)	(23,24,26,27)	(48,49,51,52)	(33,34,36,37)	

Using fuzzy Vogel's approximation methods, takes the first row and choose its smallest entry and subtract this from the next smallest entry, and write in front of the row on the right. This is the fuzzy penalty for first row. Calculate the penalties for other rows. Similarly compute fuzzy penalties for all the columns and write them in the bottom of the Fuzzy transportation table below corresponding columns. Select the highest fuzzy penalty and observe the row or column for which this corresponds. Determine the smallest fuzzy cost in the selected row or column. Let it be $\tilde{c}_{ij}$. Allocate $\tilde{x}_{ij} = \min(\tilde{a}_i, \tilde{b}_j)$ in the $(i, j)^{th}$ cell of the given fuzzy transportation table.

So, we can find first allocation on $(\tilde{x}_{11}) = (18,19,21,23)$ and give allocation that then fuzzy availability is changed to $\tilde{a}_1 = (16,18,22,24)$ and the next allocation on the position $(\tilde{x}_{22}) = (23,24,26,27)$ and that containing new availability is $\tilde{a}_2 = (21,23,27,29)$.

The resulting matrix is shown in Table 2.11

Table 2.11: The first and second allocation

Destination → Origin ↓	D_1	D_2	D_3	D_4	$\tilde{a}_i$
O_1	(0,1,3,4) (18,19,21,22)	(5,6,8,9)	(1,2,4,5)	(6,7,9,10)	~~(38,39,41,42)~~ (16,18,22,

					24)
O_2	(4,5,7,8)	(0,1,1,2) (23,24,26,27)	(7,8,10,11)	(2,3,5,6)	~~(48,49,51,52)~~ (21,23,27,29)
O_3	(1,2,4,5)	(6,7,9,10)	(0,1,3,4)	(8,9,11,12)	(38,39,41,42)
	~~(18,19,21,22)~~	~~(23,24,26,27)~~	(48,49,51,52)	(33,34,36,37)	

And the next allocations are shown in the Table 2.12

Table 2.12: Vogel's next allocation

Destination → Origin ↓	D_1	D_2	D_3	D_4	$\tilde{a}_i$
O_1	(0,1,3,4) (18,19,21,22)	(5,6,8,9)	(1,2,4,5)	(6,7,9,10)	~~(38,39,41,42)~~ (16,18,22,24)
O_2	(4,5,7,8)	(0,1,1,2) (23,24,26,27)	(7,8,10,11)	(2,3,5,6)	~~(48,49,51,52)~~ (21,23,27,29)
O_3	(1,2,4,5)	(6,7,9,10)	(0,1,3,4) (38,39,41,42)	(8,9,11,12)	~~(38,39,41,42)~~
	~~(18,19,21,22)~~	~~(23,24,26,27)~~	~~(48,49,51,52)~~ (6,8,12,14)	(33,34,36,37)	

The obtained initial fuzzy basic feasible solution and initial fuzzy transportation cost shown in the Table 2.13.

Table 2.13: **The IFBF solution**

Destination → Origin ↓	D_1	D_2	D_3	D_4	$\tilde{a}_i$
O_1	(0,1,3,4) (18,19,21,22)	(5,6,8,9)	(1,2,4,5) (6,8,12,14)	(6,7,9,10) (4,7,13,16)	(38,39,41,42) (0,0,0,0)
O_2	(4,5,7,8)	(0,1,1,2) (23,24,26,27)	(7,8,10,11)	(2,3,5,6) (21,23,27,29)	(48,49,51,52) $(-8,-6,-2,0)$
O_3	(1,2,4,5)	(6,7,9,10)	(0,1,3,4) (38,39,41,42)	(8,9,11,12)	(38,39,41,42) $(-5,-3,1,3)$
	(18,19,21,22)	(23,24,26,27)	(48,49,51,52)	(33,34,36,37)	

Table 2.13 gives the initial basic feasible solution $\tilde{x}_0 = \{\tilde{x}_{11} = (18,19,21,22)$, $\tilde{x}_{13} = (6,8,12,14)$, $\tilde{x}_{22} = (23,24,26,27)$, $\tilde{x}_{24} = (21,23,27,29)$, $\tilde{x}_{33} = (38,39,41,42)\}$ and the cost flow pair is $(\tilde{Z}_0, \tilde{F}_0) = ((72,216,512,714),(124,127,133,136))$.

From Table 2.13, we can easily see that the cells $(1,2)$ $(2,1)$ $(2,3)$ $(3,1)$ $(3,2)$ $(3,4)$ are non-basic cells. Now, we calculate the value of fuzzy dual variables corresponding to these cells.

$\tilde{u}_1 + \tilde{v}_1 = (0,1,3,4)$, $\tilde{u}_1 + \tilde{v}_3 = (1,2,4,5)$, $\tilde{u}_1 + \tilde{v}_4 = (6,7,9,10)$, $\tilde{u}_2 + \tilde{v}_2 = (0,1,1,2)$,

$\tilde{u}_2 + \tilde{v}_4 = (2,3,5,6)$

and $\tilde{u}_3 + \tilde{v}_3 = (0,1,3,4)$.

Let $\tilde{u}_1 = (0,0,0,0)$ then, $\tilde{v}_1 = (0,1,3,4)$, $\tilde{u}_2 = (-8,-6,-2,0)$, $\tilde{v}_2 = (0,3,7,10)$, $\tilde{u}_3 = (-5,-3,1,3)$ $\tilde{v}_3 = (1,2,4,5)$ and $\tilde{v}_4 = (6,7,9,10)$.

Cost for non-basic cells for the positions $(i, j)^{th}$ are as follows:

$$(1,2)^{th} : (5,6,8,9) - [(0,0,0,0) + (0,3,7,10)]$$
$$= (-5,-1,5,9)(+ve)$$

$$(2,1)^{th} : (4,5,7,8) - [(-8,-6,-2,0) + (0,1,3,4)]$$
$$= (0,4,12,16)(+ve)$$

$$(2,3)^{th} : (7,8,10,11) - [(-8,-6,-2,0) + (1,2,4,5)]$$
$$= (2,6,14,18)(+ve)$$

$$(3,1)^{th} : (1,2,4,5) - [(-5,-3,1,3) + (0,1,3,4)]$$
$$= (-6,-2,6,10)(+ve)$$

$$(3,2)^{th} : (6,7,9,10) - [(-5,-3,1,3) + (0,3,7,10)]$$
$$= (-7,-1,9,15)(+ve)$$

$$(3,4)^{th} : (8,9,11,12) - [(-5,-3,1,3) + (6,7,9,10)]$$
$$= (-5,-1,7,11)(+ve)$$

The ranks of cost of each non-basic cell is non-negative i.e. $\tilde{d}_{ij} \geq 0$ for all non-basic $(i, j)^{th}$. Therefore, the obtained IFBFS is our optimal solution is shown in Table 2.14.

Table 2.14: The optimum solution

Destinations → Origin ↓	D_1	D_2	D_3	D_4	$\tilde{a}_i$
O_1	(0,1,3,4) (18,19,21,22)	(5,6,8,9)	(1,2,4,5) (6,8,12,14)	(6,7,9,10) (4,7,13,16)	(38,39,41,42) (0,0,0,0)
O_2	(4,5,7,8)	(0,1,1,2) (23,24,26,27)	(7,8,10,11)	(2,3,5,6) (21,23,27,29)	(48,49,51,52) $(-8,-6,-2,0)$
O_3	(1,2,4,5)	(6,7,9,10)	(0,1,3,4) (38,39,41,42)	(8,9,11,12)	(38,39,41,42) $(-5,-3,1,3)$
	(18,19,21,22)	(23,24,26,27)	(48,49,51,52)	(33,34,36,37)	

For the cell $\tilde{u}_2 + \tilde{v}_1$ and $\tilde{u}_2 + \tilde{v}_3$ both contains negative cost. So using section 2.2.4 Step 3 and further. We can easily see that $\tilde{u}_i + \tilde{v}_j < 0$ for cell $(2,1)$ and $(2,3)$. Now we have $\tilde{u}_2 + \tilde{v}_1 < 0$ and $\tilde{u}_2 + \tilde{v}_3 < 0$ where the cells $(2,1)$ and $(2,3)$ are not in the basis, so paradox exists. We take $\tilde{l} = (1,1,1,1)$ and we have for the cell $(2,1)$ the optimum cost-flow pair is $(\tilde{Z}, \tilde{F}) = ((68, 213, 511, 714), (123, 125, 135, 139))$ given in Table 2.15,

Table 2.15: The optimum cost-flow pair corresponding to the cell $(2,1)$

Destination → Origin ↓	D_1	D_2	D_3	D_4	$\tilde{a}_i$
O_1	(0,1,3,4) (19,20,22,23)	(5,6,8,9)	(1,2,4,5) (6,8,12,14)	(6,7,9,10) (3,6,12,15)	(38,39,41,42)

O_2	(4,5,7,8)	(0,1,1,2) (23,24,26,27)	(7,8,10,11)	(2,3,5,6) (22,24,28,30)	(49,50,52, 53)
O_3	(1,2,4,5)	(6,7,9,10)	(0,1,3,4) (38,39,41,42)	(8,9,11,12)	(38,39,41, 42)
	(19,20,22,23)	(23,24,26,27)	(48,49,51,52)	(33,34,36,37)	

Table 2.16: The optimum cost-flow pair corresponding to the cell $(2,1)$

Destination → Origin ↓	D_1	D_2	D_3	D_4	$\tilde{a}_i$
O_1	(0,1,3,4) (18,19,21,22)	(5,6,8,9)	(1,2,4,5) (7,9,13,15)	(6,7,9,10) (3,6,12,15)	(38,39,41, 42)
O_2	(4,5,7,8)	(0,1,1,2) (23,24,26,27)	(7,8,10,11)	(2,3,5,6) (22,24,28,30)	(48,49,51, 52)
O_3	(1,2,4,5)	(6,7,9,10)	(0,1,3,4) (38,39,41,42)	(8,9,11,12)	(38,39,41, 42)
	(18,19,21,22)	(23,24,26,27)	(48,49,51,52)	(33,34,36,37)	

For the cell $(2,3)$ the optimum cost flow pair is $(\tilde{Z},\tilde{F})=((69,214,512,715),(123,125,135,139))$ given in Table 2.14, the paradoxical cost flow pair is $(\tilde{Z},\tilde{F})=((68,213,511,714),(125,128,134,137))$ given in Table 2.16.

Chapter 3

Problem Statement

It is obvious from Chapter 2 that for solving the fuzzy transportation problem and intuitionistic fuzzy transportation problems by using the existing method [1] and the existing method [26], there is need to apply the arithmetic operations of fuzzy numbers and intuitionistic fuzzy numbers respectively.

It is very difficult to apply the arithmetic operations of fuzzy numbers and intuitionistic fuzzy numbers as well as to implement the existing methods [1, 26] with arithmetic operations of fuzzy numbers/intuitionistic fuzzy numbers in a programming language.

However, it is much easy to apply the arithmetic operations of real numbers as compared to fuzzy numbers and intuitionistic fuzzy numbers as well as to implement a method with arithmetic operations of real numbers in a programming language as compared a method with arithmetic operations of fuzzy numbers and intuitionistic fuzzy numbers.

Keeping, the same in mind, the aim of the thesis is to show that using the ranking function, the fuzzy transportation problem [1] and intuitionistic fuzzy transportation problems [26] can be transformed into equivalent crisp transportation problems as well as the solution of the fuzzy transportation problem [1] and intuitionistic fuzzy transportation problems [26] can be obtained using the solution of the transformed crisp transportation problem.

Chapter 4

Contributions

In this chapter, it is shown that using the ranking function, the fuzzy transportation problem [1] and intuitionistic fuzzy transportation problems [26] can be transformed into equivalent crisp transportation problems as well as the solution of the fuzzy transportation problem [1] and intuitionistic fuzzy transportation problems [26] can be obtained using the solution of the transformed crisp transportation problem.

4.1 Alternative algorithm to find more-for-less paradox in a transportation problem under fuzzy environment

The more-for-less paradox in transportation problem under fuzzy environment [1] can also be obtained as follows:

Step 1: Using the ranking function, transform the fuzzy transportation problem, represented by Table 1.2, into the crisp transportation problem, represented by Table 4.1.

Table 4.1: Crisp transportation problem

	D_1	D_2	D_3	D_4	Availability (a_i)
O_1	c_{11}	c_{12}	c_{13}	c_{14}	a_1
O_2	c_{21}	c_{22}	c_{23}	c_{24}	a_2
O_3	c_{31}	c_{32}	c_{33}	c_{34}	a_3
Demand (b_j)	b_1	b_2	b_3	b_4	$\sum_i a_i = \sum_j b_j$

Step 2: Find the optimal cost (Z) of the of the crisp transportation problem represented by Table 4.2.

Step 3: Check, Does there exist non-basic cells, say (p,q), in the optimal table such that $(u_p + v_q) < 0$? If exists then go to Step 4, otherwise, (optimal cost, optimum flow), i.e., (Z, F) is the only paradoxical pair.

Step 4: Add 1 unit quantity at p^{th} origin and at q^{th} destination for which $(u_p + v_q) < 0$ and then find the optimal cost.

Step 5: The cell, for which optimal cost is minimum, will enter into the optimum basis.

Step 6: Repeat Step 3 to Step 5 until $Z_i < Z_{i+1}$.

4.2 Numerical example

Acharya et al. [1] solved the transportation problem, represented by Table 2.11, to illustrate their proposed algorithm. Using the algorithm, described in Section 4.2, the same problem can be solved as follows:

Step 1: The fuzzy transportation problem, represented by Table 2.11, can be transformed into the equivalent crisp transportation problem, represented by Table 4.2.

Table 4.2: Equivalent crisp transportation problem

	D_1	D_2	D_3	D_4	Availability (a_i)
O_1	2	7	3	8	40

O_2	6	1	9	4	50
O_3	3	8	2	10	40
Demand (b_j)	20	25	50	35	130

Step 2: On solving the crisp transportation problem, represented by Table 2.11, the optimal solution, shown in Table 4.3, is obtained.

Table 4.3: Optimal table

	D_1	D_2	D_3	D_4	Availability (a_i)
O_1	2 (20)	7	3 (10)	8 (10)	40
O_2	6	1 (25)	9	4 (25)	50
O_3	3	8	2 (40)	10	40
Demand (b_j)	20	25	50	35	130

The total cost is 355 and the first paradoxical pair is (355, 130).

Step 3: It is obvious from Table 4.4 that $u_i + v_j < 0$ for non-basic cell (2, 1) and (2, 3) so there may also exist other paradoxical pairs for this problem.

Table 4.4: Optimal table with dual variables

	D_1	D_2	D_3	D_4	Availability (a_i)	u_i
O_1	2 (20)	7	3 (10)	8 (10)	40	0

O_2	6	1 (25)	9	4 (25)	50	–4
O_3	3	8	2 (40)	10	40	–1
Demand (b_j)	20	25	50	35	130	
v_j	2	5	3	8		

Step 4: Applying Step 4 of the proposed algorithm, Table 4.5 is formed for cell (2, 1) and Table 4.6 is formed for cell (2, 3).

Table 4.5: Optimal table for cell (2, 1)

	D_1	D_2	D_3	D_4	Availability (a_i)	u_i
O_1	2 (21)	7	3 (10)	8 (9)	40	0
O_2	6	1 (25)	9	4 (26)	51	–4
O_3	3	8	2 (40)	10	40	–1
Demand (b_j)	21	25	50	35	131	
v_j	2	5	3	8		

The total cost is 353 and the paradoxical pair is (353, 131).

Table 4.6: Cost flow for cell (2, 3)

	D_1	D_2	D_3	D_4	Availability (a_i)	u_i
O_1	2 (20)	7	3 (11)	8 (9)	40	0
O_2	6	1 (25)	9	4 (26)	51	–4
O_3	3	8	2 (40)	10	40	–1
Demand	20	25	51	35	131	

(b_j)					
v_j	2	5	3	8	

The total cost is 354 and $\min\{353, 354\} = 353$, therefore, we will proceed with cell (2, 1) corresponding a transportation problem, represented by Table 4.6

Now, repeating the steps of proposed algorithm, the optimal solution is given in Table 4.7.

Table 4.7: The optimal table

	D_1	D_2	D_3	D_4	Availability (a_i)	u_i
O_1	2 (29)	7	3 (11)	8 (0)	40	0
O_2	6	1 (25)	9	4 (35)	60	–4
O_3	3	8	2 (40)	10	40	–1
Demand (b_j)	29	25	51	35	131	
v_j	2	5	3	8		

The optimal cost is 336 and the optimum paradoxical pair is (336, 140).

4.3 Alternative algorithm for solving intuitionistic fuzzy transportation problem

The intuitionistic fuzzy optimal solution of the transportation problem [26] can also be obtained as follows:

Step 1: The given fuzzy transportation problem is transformed into the tabular form.

Step 2: After applying the ranking function change this table into crisp table.

Step 3: First reduce the obtained crisp matrix in row reduce form by subtracting the entry which has the least entry from each entries of its corresponding row of crisp transportation problem then reduce it in column reduction form for those column that have no zero entry .

Step 4: Calculate the cost penalties of each destination (corresponding to those cells which contain zero cost) by taking the difference of minimum costs of corresponding to row and column.

Step 5: Select the cell for which the penalty is largest and allocate b_j to this cell. In case of tie at both these aspects, it is an arbitrary choice of the decision maker.

Step 6: Check whether the resultant matrix is in row and column reduce form else repeats Step 2.

Step 7: Repeat Step 3 to Step 6 until all the allocations are made and we get the fuzzy optimal solution of fuzzy transportation problem.

4.4 Numerical example

Thamaraiselvi and Santhi [26] solved the intuitionistic fuzzy transportation problem, represented by Table 4.8, to illustrate their proposed algorithm. Using the algorithm, described in Section 4.2, the same problem can be solved as follows:

Table 4.8: Intuitionistic fuzzy transportation problem

Destinations Origins	D_1	D_2	D_3	D_4	IF Supply

O_1	5	6	12	9	(7,9,11,13,16, 20) (5,7,11,13,19, 20)
O_2	3	2	8	4	(6,8,11,14,19, 21) (4,7,11,14,21, 27)
O_3	7	11	20	9	(9,11,13,15, 18, 20) (8,10,13,15, 19, 22)
IF Demand	(3,4,5,6,8, 10) (2,4,5,6,10, 12)	(3,5,7,9,12, 16) (2,4,7,9,13, 17)	(6,7,9,11,13, 24) (5,6,9,11,16, 18)	(10,12,14,16,20, 24) (8,10,14,16,20,25)	

Replacing each intuitionistic fuzzy number of Table 4.8 by its rank, it can be transformed into Table 4.9.

Table 4.9: Equivalent crisp transportation problem

Destinations Origins	D_1	D_2	D_3	D_4	Supply
O_1	5	6	12	9	12.67 13.33
O_2	3	2	8	4	13.83 14
O_3	7	11	20	9	14.33 14.5
Demand	6	8.5	10.33	16	

	6.5	8.67	10.83	15.5	

Using the alternative algorithm, presented in Section 4.3, Table 4.9 can be transformed into Table 4.10.

Table 4.10: Row-reduced form of equivalent crisp transportation problem

Destinations Origins	D_1	D_2	D_3	D_4	Supply
O_1	0	1	1	2	12.67 13.33
O_2	1	0	0	0	13.83 14
O_3	0	4	7	0	14.33 14.5
Demand	6 6.5	8.5 8.67	10.33 10.83	16 15.5	

Using the alternative algorithm, presented in Section 4.3, Table 4.10 can be transformed into Table 4.11.

Table 4.11: The first allocation

Destinations Origins	D_1	D_2	D_3	D_4	Supply
O_1	**0** (6) (6.5)	1	1	2	6.67 6.83

O_2	-	0	0	0	13.83 14
O_3	-	4	7	0	14.33 14.5
Demand	-	8.5 8.67	10.33 10.83	16 15.5	

Using the alternative algorithm, presented in Section 4.3, Table 4.11 can be transformed into Table 4.12.

Table 4.12: The second allocation

Destinations Origins	D_1	D_2	D_3	D_4	Supply
O_1	**0** (6) (6.5)	0	0	1	6.67 6.83
O_2	-	0	0	0	13.83 14
O_3	-	-	-	0 (14.33) (14.5)	-
Demand	-	8.5 8.67	10.33 10.83	1.67 1	

Similarly, repeating the alternative algorithm, presented in Section 4.3, the optimal table is

Table 4.13: Optimal table

Destinations Origins	D_1	D_2	D_3	D_4	Supply
O_1	0 (6) (6.5)	0 (6.67) (6.83)	0	1	-
O_2	-	0 (1.83) (2.17)	0 (10.33) (10.83)	0 (1.67) (1)	-
O_3	-	-	-	0 (14.33) (14.5)	-
Demand	-	-	-	-	

The optimal solution (X, Y) of the above problem is (292.15, 298.95)

$$X = 5\times6+6\times6.67+2\times1.83+8\times10.33+4\times1.67+9\times14.33$$
$$=30+40.2+3.66+82.64+6.68+128.97$$
$$=292.15$$

$$Y=5\times6.5+6\times6.83+2\times2.17+8\times10.83+4\times1+9\times14.5$$
$$=32.5+40.98+4.34+86.64+4+130.5$$
$$=298.95$$

Chapter 5

Conclusions

In this thesis, an algorithm is presented to find the for-more-less paradox of such a transportation problem in which all the parameters are represented by fuzzy numbers. Also, an algorithm is presented to find the intuitionistic fuzzy optimal solution of such a transportation problem in which transportation costs for supplying one unit quantity of the product from sources to destinations are represented by triangular intuitionistic fuzzy numbers while the remaining parameters are represented by real numbers. Further, it is discussed that it is much easy to apply these alternative algorithms as compared to the existing algorithms [1 26].

In future, it may be tried to propose an algorithm for obtaining for-more-less paradox of such a transportation problem in which all the parameters are represented by triangular intuitionistic fuzzy numbers.

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