

# **DESIGN AND ANALYSIS OF SHARPENED CIC FILTERS**

*Dissertation submitted in the partial fulfilment of requirements for the award of degree of*

**Master of Engineering  
in  
Wireless Communication Engineering**

Submitted by

**Anirudh Singhal**

**Roll No. 801363003**

Under guidance of

**Dr. Sanjay Sharma**

**Professor & HOD (ECED)**



**ELECTRONICS AND COMMUNICATION ENGINEERING DEPARTMENT**

**THAPAR UNIVERSITY**

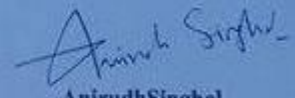
**(Established under the section 3 of UGC Act, 1956)**

**PATIALA-147004 (PUNJAB)**

## DECLARATION

I hereby declare that the work, which is being presented in the dissertation, entitled "Design and Analysis of Sharpened CIC Filters" is an authentic record of my own work carried out as requirement for the award of Master of Engineering in Wireless Communication Engineering at Thapar University, Patiala under the guidance of Dr. Sanjay Sharma, Professor & Head (ECED) during June to November, 2014.

Date: 06 July, 2015

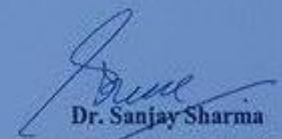


Anirudh Singh

Roll No. 801363003

It is certified that the above statement is correct to the best of my knowledge and belief.

Date: 6/7/2015

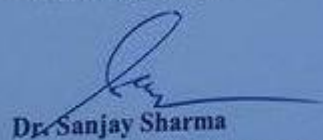


Dr. Sanjay Sharma

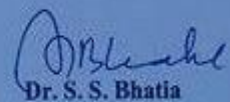
Professor & Head (ECED)

Thapar University, Patiala

Countersigned by:



Dr. Sanjay Sharma  
Professor and Head (ECED)  
Thapar University, Patiala



Dr. S. S. Bhatia

Dean, Academic Affairs  
Thapar University, Patiala

## ACKNOWLEDGEMENT

I would like to express my gratitude to my mentor **Dr. Sanjay Sharma (Professor and Head)** Electronics and Communication Engineering Department, Thapar University, Patiala, for his advice, kind assistance, and invaluable guidance. It has been a great honour to work under him.

I am also thankful to **Dr. Amit Kumar Kholi, P.G. Coordinator**, Electronics and communication Engineering Department, for the motivation and inspiration that triggered me for this work.

I am greatly indebted to all of my friends who constantly encouraged me and also would like to thank all the faculty members of Electronics and Communication Engineering Department for the full support of my work. I am also thankful to the authors whose work have been consulted and quoted in this work.

Finally, I would like to thank my parents for allowing me to realize my own potential. All the support they have provided me over the years was the greatest gift anyone has ever given me.

AnirudhSinghal

## ABSTRACT

Cascaded-integrator-comb (CIC) filter is the simplest decimation filter. However, its magnitude response has a high pass-band droop, which is not tolerable in various application. CIC compensator is a technique used for the reduction of the droop. CIC filter is an FIR structure, which consists of cascaded integrator stages working at the higher sampling rate and the same number of comb stages working at the low sampling rate. A number of cascaded integrator comb pairs are chosen to meet the design requirements for aliasing or imaging errors. Although, the CIC filters can implement decimation and interpolation efficiently in the hardware for a wide range of rate change factors, yet CIC filter response is lacking in a flat pass-band response and better transition bandwidth. To circumvent these problems, a compensation FIR filter can be employed in cascade with the CIC filter to provide frequency correction as well as spectrum shaping.

The main aim of this work is to improve the magnitude response of CIC filters and also solve the passband droop problem. For improving the passband and the transition band features of the CIC filter and for improving the performance of CIC filter there are many techniques such as compensation filter cascaded with CIC filter, sharpening technique, polyphase decimation FIR filter to achieve wide broadband compensation of the CIC filter and maximally-flat based compensator filter. The droop can be reduced by modifying the original CIC structure or by connecting an additional filter called CIC compensator in cascade with the CIC decimator. The former approach is based on a technique called sharpening. In recent years, several methods for the design of CIC compensators have been developed and now compensation is combined with a sharpening technique.

In this dissertation, we implemented the CIC compensation filter that is based on maximally flat criterion. Also study the maximally flat criterion and different technique to improve CIC filter response. The method is based on the minimization of error function in the least-square sense. The main goal of this CIC filter's design is to explain the wideband compensation as well as to efficiently implement the compensator filters. Additionally, the multiplierless implementation is proposed. In this method Leibniz rule is used but when solved the set of linear equation to get the

filter's coefficients for high order of FIR filter order, complexity increase in solving the equations.

After that we proposed a new design of CIC compensator filter that is different from the existing one. This design is also based on maximally flat criterion but will use different technique to get filter coefficients and improve the passband droop problem for same order of FIR filter. The compensator's coefficients are obtained by solving a linear system of equations. In this method Bernoulli numbers and Riemann zeta function are used and to get filter's coefficients linear system of equation in matrix form is used. Due to this complexity is reducing to solve the equations. Later, we compared the results of new design with different orders with the previous design of CIC compensator. Then we found that results of new proposed design are far better than the existing one, which makes it less complex and more efficient. The proposed method gives the better magnitude response of compensated CIC filter.

# CONTENTS

<b><i>DECLARATION</i></b>	<i>i</i>
<b><i>ACKNOWLEDGEMENT</i></b>	<i>ii</i>
<b><i>ABSTRACT</i></b>	<i>iii</i>
<b><i>CONTENTS</i></b>	<i>v</i>
<b><i>LIST OF FIGURES</i></b>	<i>vii</i>
<b><i>ABBREVIATIONS</i></b>	<i>viii</i>
<b>1. INTRODUCTION</b>	1
1.1 Integrator	2
1.2 Comb Filter	3
1.3 Architecture of CIC Filter	4
1.4 CIC Filters in Decimation and Interpolation	7
1.5 Frequency Characteristics of CIC Filter	7
1.6 Bit Growth in CIC Filter	8
1.7 Motivation of Thesis	9
1.8 Problem Formation	9
1.9 Objectives	10
1.10 Improved Methods of CIC Filters	10
1.11 Advantages and Disadvantages of CIC Filter	11
1.12 Applications of CIC Filter	12
1.13 Organization of Dissertation	12
<b>2. LITERATURE REVIEW</b>	13
<b>3. MAXIMALLY FLAT CIC COMPENSATOR FILTER</b>	23
3.1 Maximally Flat Compensators	24
3.2 CIC Compensator Response for Different Order of FIR Filter	26
3.3 Compare the CIC Compensator Response for Different order of FIR Filter	33

<b>4. DESIGN OF COMPENSATOR FILTER FROM EXISTING ONES</b>	35
4.1 Ideal CIC Compensator	35
4.2 Maximally Flat Compensators	36
4.3 CIC Compensator Response for Different Order of FIR Filter	39
4.4 Compare the CIC Compensator Response for Different order of FIR Filter	44
<b>5. RESULTS AND DISCUSSION</b>	47
<b>6. CONCLUSION AND FUTURE SCOPE</b>	52
6.1 Conclusion	52
6.2 Future Scope	53
<b>REFERENCES</b>	54
<b>LIST OF PUBLICATIONS</b>	57

## LIST OF FIGURES

- Figure 1.1:** Basic Integrator
- Figure 1.2:** Basic Comb Filter
- Figure 1.3:** CIC Decimator Filter
- Figure 1.4:** CIC Interpolator Filter
- Figure 1.5:** Multi-stage CIC filter gain response
- Figure 1.6:** Expanded view of the multi-stage CIC filter gain response
- Figure 3.1:** CIC Compensator for FIR filter order  $L=2$
- Figure 3.2:** Magnitude Response of Compensated CIC filter for  $L=2$
- Figure 3.3:** CIC Compensator for FIR filter order  $L=4$
- Figure 3.4:** Magnitude Response of Compensated CIC filter for  $L=4$
- Figure 3.5:** CIC Compensator for FIR filter order  $L=6$
- Figure 3.6:** Magnitude Response of Compensated CIC filter for  $L=6$
- Figure 3.7:** CIC Compensator for FIR filter order  $L=8$
- Figure 3.8:** Magnitude Response of Compensated CIC filter for  $L=8$
- Figure 3.9:** Compensated Filter Response for Different Order of FIR Filter
- Figure 3.10:** Magnitude Response of Compensated CIC Filter
- Figure 3.11:** Full Magnitude Response of CIC Filter for Different order of FIR Filter
- Figure 4.1:** CIC Compensator for FIR filter order  $L=2$
- Figure 4.2:** Magnitude Response of Compensated CIC filter for  $L=2$
- Figure 4.3:** CIC Compensator for FIR filter order  $L=4$
- Figure 4.4:** Magnitude Response of Compensated CIC filter for  $L=4$
- Figure 4.5:** CIC Compensator for FIR filter order  $L=6$
- Figure 4.6:** Magnitude Response of Compensated CIC filter for  $L=6$
- Figure 4.7:** CIC Compensator for FIR filter order  $L=8$
- Figure 4.8:** Magnitude Response of Compensated CIC filter for  $L=8$
- Figure 4.9:** Compensated Filter Response for Different Order of FIR Filter
- Figure 4.10:** Magnitude Response of Compensated CIC Filter
- Figure 4.11:** Full Magnitude Response of CIC Filter for Different order of FIR Filter
- Figure 5.1:** Magnitude Response of Compensator Filter for different method  $L=6$
- Figure 5.2:** Magnitude Response of Compensated CIC filter for different method  $L=6$
- Figure 5.3:** Magnitude Response of Compensator Filter for different method  $L=8$
- Figure 5.4:** Magnitude Response of Compensated CIC filter for different method  $L=8$

## ABBREVIATIONS

<b>CIC</b>	Cascaded Integrator Comb
<b>FIR</b>	Finite Impulse Response
<b>IIR</b>	Infinite Impulse Response
<b>DSP</b>	Digital Signal Processing
<b>SDR</b>	Software Defined Radio
<b>GCF</b>	Generalized Comb Filter
<b>SCIC</b>	Sharpened Cascaded Integrator Comb
<b>SNR</b>	Signal to Noise Ratio
<b>SRC</b>	Sample Rate Conversion

# CHAPTER 1

## INTRODUCTION

---

In the present scenario data converters are being used in almost all the application of narrow band extraction from wideband sources and narrow band construction of wideband signals is becoming more important. These functions require two basic signal processing procedures: decimation and interpolation. If the required sample-rate is an integer multiple of the existing one, it is adequate to oversample the input signal with the help of an interpolation filter. Similarly, if the existing sample-rate is an integer multiple of the one that is desired, then the process of decimation or down-sampling can be commissioned. Decimation is used to reduce the sampling rate by passing a signal through low pass or band pass filter. Interpolation is used to increase the sampling rate [1].

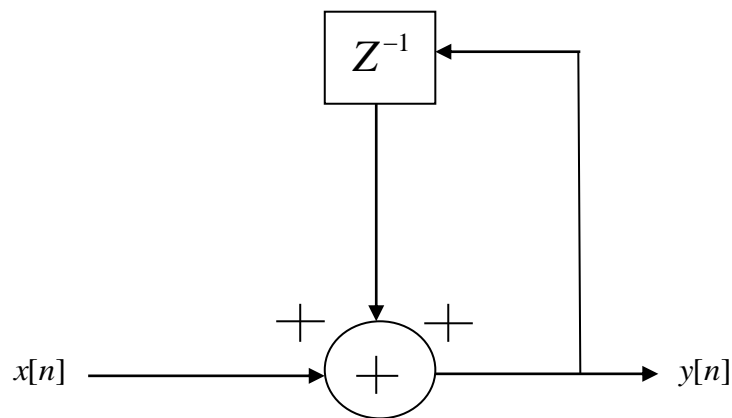
Hogenauer[2] proposed a class of digital linear phase finite impulse response (FIR) filters for the decimation and interpolation, which require no multipliers (use limited storage), making them a viable substitute in the conventional implementation for certain applications. The CIC architecture can be used for both interpolation and decimation. In the CIC filter design, the pass-band response and stop-band rejection characteristics are managed by three-integer parameters, which are number of stages, the differential delay and the number of bits in input/output registers. For example, higher the number of stages, higher will be the existing stop-band attenuation. CIC filter use in Software Defined Radio (SDR) systems [3].

The CIC filters are multiplication free filters with limited storage requirements, which make them ideal for the high speed data converters. Hogenauer[2] presented an FIR structure, which consists of cascaded integrator stages working at the higher sampling rate and the same number of comb stages working at the low sampling rate. A number of cascaded integrator comb pairs are chosen to meet the design requirements for aliasing or imaging errors. Although, the CIC filters can implement decimation and interpolation efficiently in the hardware for a wide range of rate change factors, yet CIC filter response is lacking in a flat pass-band response and better transition bandwidth [4]. To circumvent these problems, a compensation FIR filter can be employed in cascade with the CIC filter to provide frequency correction as well as spectrum shaping.

The CIC filters are an unparalleled class of the digital filters that present a computationally effective manner of implementing the narrowband low-pass filter for anti-aliasing. The CIC filter uses only delays and summation units, and it does not require multiplication operations as in an FIR filter [1]. Occasionally, these are referred to as filters without multipliers. The filter can be implemented by cascading either the integrator with the comb filter or vice-versa. The low-pass frequency response can be improved by cascading N number of CIC filter stages. These filters also have a linear phase response like the FIR filters. The FIR filters can be employed for the interpolation as well as decimation. The two basic building of a CIC filter are an integrator and a comb [5].

### 1.1 INTEGRATOR:-

An integrator is simply a single-pole IIR filter with a unity feedback coefficient, and it may be shown in figure 1.1 [6].



**Figure 1.1: Basic Integrator**

It has the low-pass characteristics and obviously no multipliers are required. The time-domain output may be shown as

$$y[n] = y[n-1] + x[n] \quad (1.1)$$

And, the output in the z-domain may be as shown as

$$Y[z] = Y[z^{-1}] + X[z] \quad (1.2)$$

It may also written as

$$Y[z](1 - z^{-1}) = X[z] \quad (1.3)$$

The transfer function for a single integrator on the z-plane is

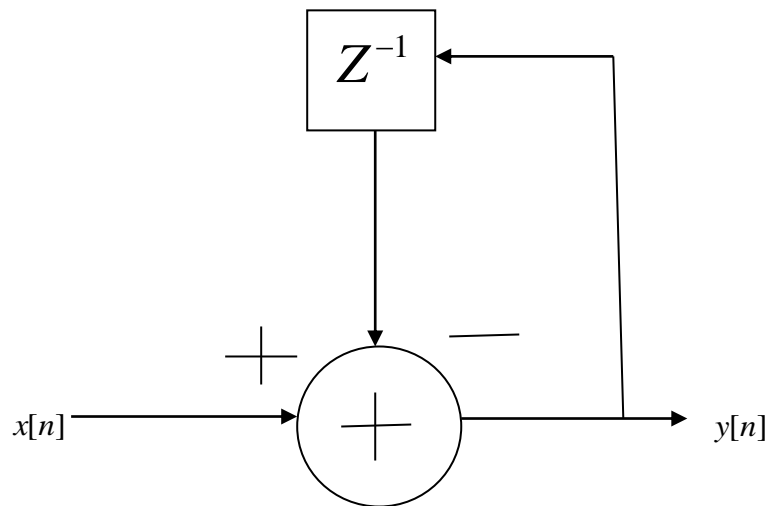
$$H_I(z) = \frac{1}{1 - z^{-1}} \quad (1.4)$$

This system is also known as an accumulator. The integrator section of CIC filters consists of N digital integrator stages operating at high sampling rate,  $f_s$ . Each stage is implemented as a one pole filter with a unity feedback coefficient.

### 1.2 COMB FILTER:-

A comb filter is used to add delay version of a signal itself. The frequency response of a comb filter consists of a series of regularly spaced spikes, giving the appearance of a comb [6].

The general structure of comb filters as shown in figure 1.2.



**Figure 1.2: Basic Comb Filter**

Comb filter section operates at the low sampling rate,  $f_s / R$ , where R is the integer rate change factor. This section consists of N comb stages with a differential delay of M samples per stage. The differential delay is a filter design parameter used to control the filter's frequency response.

The time-domain output may be shown as

$$y[n] = x[n] - x[n - RM] \quad (1.4)$$

And, the output in the z-domain may be as shown as

$$Y[z] = X[z] - Z^{-RM} X[z] \quad (1.5)$$

It may also be written as

$$Y[z] = X[z](1 - Z^{-RM}) \quad (1.6)$$

The system function for a single comb stage referenced to the high sampling rate is

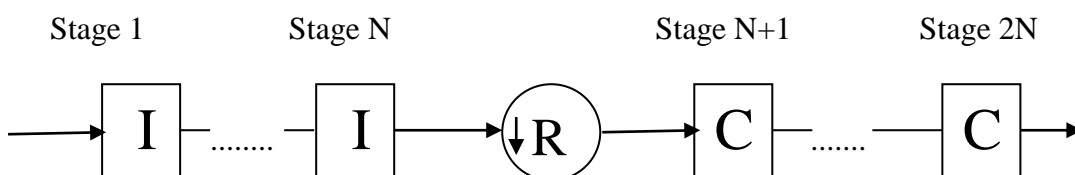
$$H_C(z) = 1 - z^{-RM} \quad (1.7)$$

Comb filters find applications in a wide range of practical systems such as in the rejection of power line harmonics and in the suppression of clutter from fixed objects in moving-target-indicator (MTI) radars.

### 1.3 ARCHITECTURE OF CIC FILTER:-

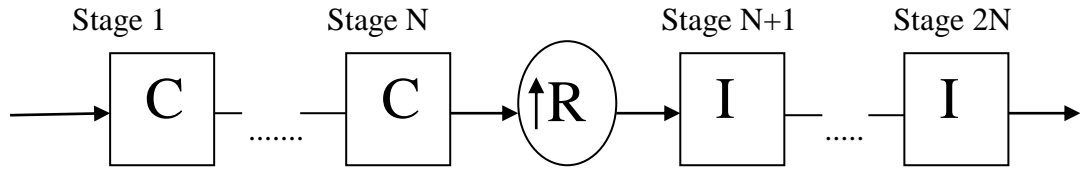
The CIC filter is a class of hardware-efficient linear phase FIR digital filters. The CIC filters realize sampling rate decrease (decimation) and sampling rate increase (interpolation) without using multipliers. A CIC filter consists of an equal number of stages of the ideal integrator filters and the comb filters. Its frequency response may be tuned by selecting the suitable number of cascaded integrator and comb filter pairs. The extremely symmetric structure of the CIC filter authorizes effective implementation in the hardware [7]. However the disadvantage of a CIC filter is that, its pass-band is not flat, which is disagreeable in many applications. Fortunately, this problem can be alleviated by using a compensation filter. The CIC filter can also be implemented very efficiently in hardware due to its symmetric structure. The CIC decimator would have  $N$  cascaded integrator stages clocked at  $f_s$ , followed by a change in the rate by a factor  $R$ , followed by  $N$  cascaded comb stages consecutively running at  $f_s/R$ .

Between the two filters sections is a rate change switch. CIC decimation would have  $N$  cascaded integrator stages clocked at  $f_s$ , followed by a rate change by a factor  $R$ , followed by  $N$  cascaded comb stages running at  $f_s / R$  as shown in figure 1.3 [2].



**Figure 1.3: CIC Decimator Filter**

CIC interpolator would be N cascaded comb stages running at  $f_s / R$ , followed by a zero-suffer, followed by N cascaded integrator stages running at  $f_s$  as shown in figure 1.4 [2].



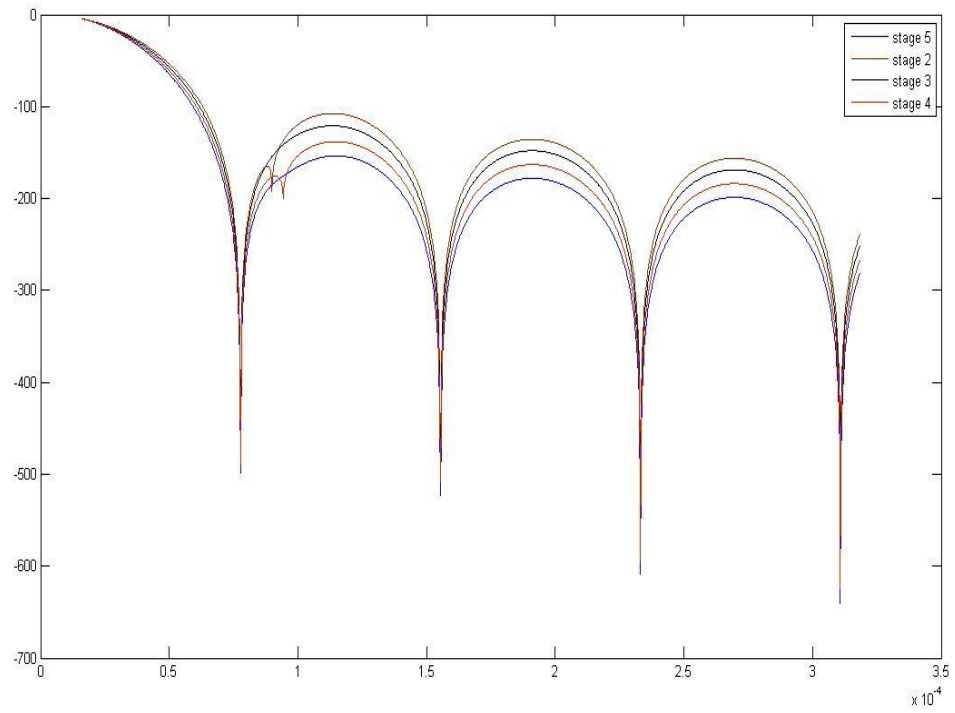
**Figure 1.4: CIC Interpolator Filter**

The transfer function for a CIC filter at  $f_s$  is [2]

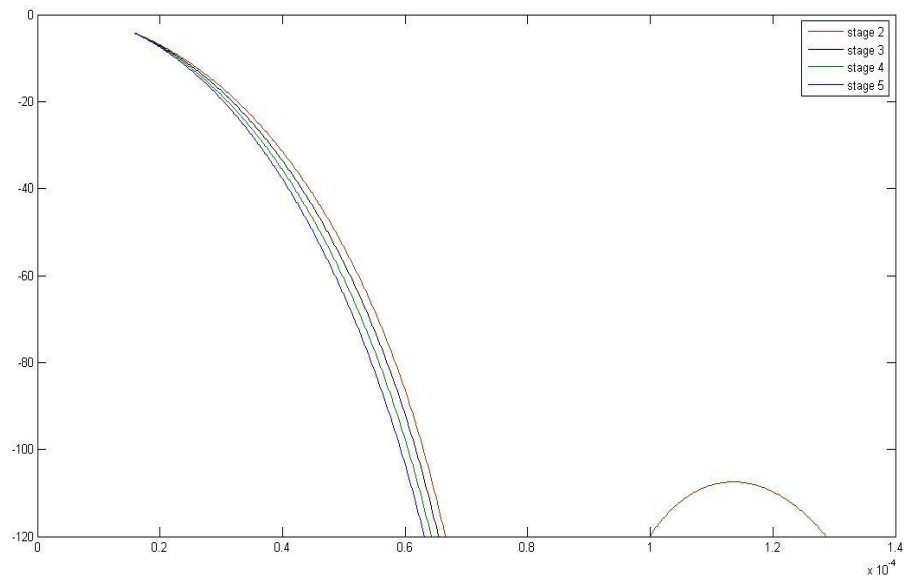
$$H_{CIC}(z) = H_I^N(z) H_C^N = \frac{(1 - z^{-RM})^N}{(1 - z^{-1})^N} = \left[ \sum_{k=0}^{RM-1} z^{-k} \right]^N \quad (1.8)$$

This equation shows that even though a CIC has integrators in it, which by them has an infinite impulse response, a CIC filter is equivalent to N FIR filters, each having a rectangular impulse response. Since all of the coefficients of these FIR filters are unity, and therefore symmetric, a CIC filter has a linear phase response and a constant group delay. The numerator represents the transfer function of a differentiator and the denominator indicates the transfer function of an integrator. A very poor magnitude response of the comb filter is improved by cascading the several identical comb filters.

The figure 1.5 exemplifies that how the multi-stage realization improves the selectivity and the stop-band attenuation of the overall filter. As the number of FIR filter order increase passband droop problem improve in magnitude response of compensated CIC filter. The selectivity and stop-band attenuation are augmented with the increasing number of comb filter sections. The filter has multiple nulls with multiplicity equal to the number of sections. Consequently, the stop-band attenuation in the null intervals is very high. The figure 1.6 illustrates a monotonic decrease of the magnitude response in the pass-band, called the pass-band droop. When FIR filter orders increases, get the different filter coefficients and get the improve magnitude response in compensated CIC filter.



**Figure 1.5: Multi-stage CIC filter gain response**



**Figure 1.6: Expanded view of the multi-stage CIC filter gain response**

#### 1.4 CIC FILTERS IN DECIMATION AND INTERPOLATION:-

The CIC filters are utilized in the multi-rate systems for constructing the efficient decimators and interpolators. The comb filter's capacity to perform filtering without multiplications is highly admirable to be applied to the exalted rate signals [8]. Moreover, the CIC filters are suitable for the large conversion factors because the low-pass bandwidth is very small. In the multi-stage decimators with the large conversion factor, the comb filter sounds to be the best solution for the first decimation stage. However in the interpolators, the comb filter is convenient for the last interpolation stage. The multi-rate application of the comb filters was first proposed by Hogenauer and since that time, the well known Hogenauer filters have influenced many researchers and practicing engineers.

#### 1.5 FREQUENCY CHARACTERICS OF CIC FILTER:-

CIC filters have a low pass frequency characteristic. The frequency response is given equation (1.8) evaluated by [1]

$$z = e^{j\frac{2\pi f}{R}} \quad 0 \leq f \leq R/2 \quad (1.9)$$

Where  $f$  is the frequency relative to the low sampling rate,  $f_s / R$ . The cut off frequency for this filter is  $f_c$  again relative to the low sample rate. The magnitude response at the output of the filter can be shown to be [1]

$$|H_{CIC}(f)| = \left| \frac{\sin \pi Mf}{\sin \frac{\pi f}{R}} \right|^{2N} \quad (1.10)$$

By using the relation  $\sin x \approx x$  for small value of  $x$  and some algebra, we can approximate this function for large value of  $R$  as [1]

$$|H_{CIC}(f)| \approx \left| RM \frac{\sin \pi Mf}{\pi Mf} \right|^{2N} \quad 0 \leq f < \frac{1}{M} \quad (1.8)$$

We can notice from the magnitude response that nulls exist at multiples of  $f = \frac{1}{M}$ .

Thus the difference delay  $M$  can be used as a design parameter to control the placement of nulls. For CIC decimation filters, the region around every  $M^{th}$  null is

folded into the passband causing aliasing errors; and for CIC interpolation filters, imaging occurs in the region around these same nulls.

For practical design problems, the aliasing/imaging errors can be characterized by the maximum error over all aliasing/imaging bands. For a large class of filter design problems where  $f_c \leq \frac{1}{2M}$ , lower edge of the first aliasing/imaging band at [2]

$$f_{AI} = 1 - f_c \quad (1.11)$$

Another thing we can notice is that the passband attenuation is a function of the number of stages. As a result, while increasing the number of stages improves the imaging/aliasing rejection, it also increases the passband droop. We can also notice that the DC gain of the filter is a function of the rate change

### 1.6 BIT GROWTH IN CIC FILTER:-

For CIC decimators, the gain  $G$  at the output of the final comb section is [1]

$$G = (RM)^N \quad (1.12)$$

Assuming two's complement arithmetic, we can use this result to calculate the number of bits required for the last comb due to bit growth. If  $B_{in}$  is the of input bits, then the number of output bits,  $B_{out}$ , is [1]

$$B_{out} = \lceil N \log_2 RM + B_{in} \rceil \quad (1.13)$$

It also turn out that  $B_{out}$  bits are needed for each integrator and comb stage. The input needs to be sign extended to  $B_{out}$  bits, but LSB's can either be truncate or rounded at later stages.

For a CIC interpolator, the gain  $G$  at the  $i^{th}$  stage [1]

$$G_i = \begin{cases} 2^i & i = 1, 2, \dots, N \\ \frac{2^{2N-i} (RM)^{i-N}}{R}, & i = N + 1, \dots, 2N \end{cases} \quad (1.14)$$

As a result the register width,  $W_i$ , at  $i^{th}$  stage is [1]

$$W_i = \lceil B_{in} + \log_2 G_i \rceil \quad (1.15)$$

And

$$W_N = B_{in} + N - 1 \quad (1.16)$$

If  $M=1$ . Rounding or truncation cannot be used in CIC interpolator, except for the result, because the small errors introduced by rounding or truncation can grow without bound in the integrator section. For decimators, integrator overflow is not a problem as long as two's complement math is used.

### **1.7 MOTIVATION OF THE THESIS:-**

In many communication and signal processing systems, it is highly desirable to implement an efficient narrow-band filter that decimate or interpolate the incoming signals. Fast sampling rate offers several benefits, including the ability to digitize wideband signals, reduced complexity of anti-alias filters, and lower noise power spectral density. The decimation filter is a sampling rate conversion system, performs low-pass filtering as well as down-sampling operation and hence widely used in speech processing and communication systems application.

Cascaded-integrator-comb (CIC) filter is the simplest decimation filter. A number of cascaded integrator comb pairs are chosen to meet the design requirements for aliasing or imaging errors. Although, the CIC filters can implement decimation and interpolation efficiently in the hardware for a wide range of rate change factors, yet CIC filter response is lacking in a flat pass-band response and better transition bandwidth.

The CIC filters are an unparalleled class of the digital filters that present a computationally effective manner of implementing the narrowband low-pass filter for anti-aliasing. The CIC filter uses only delays and summation units, and it does not require multiplication operations as in an FIR filter. Occasionally, these are referred to as filters without multipliers. The filter can be implemented by cascading either the integrator with the comb filter or vice-versa. The low-pass frequency response can be improved by cascading  $N$  number of CIC filter stages. These filters also have a linear phase response like the FIR filters. The FIR filters can be employed for the interpolation as well as decimation.

### **1.8 PROBLEM FORMALATION:-**

The simplest multiplier less filter is CIC filter. The CIC filters are multiplication free filters with limited storage requirements, which make them ideal for the high speed data converters. Hogenauer[2] presented an FIR structure, which consists of cascaded

integrator stages working at the higher sampling rate and the same number of comb stages working at the low sampling rate. CIC filter consists of an equal number of stages of the ideal integrator filters and the comb filters. A number of cascaded integrator comb pairs are chosen to meet the design requirements for aliasing or imaging errors.

The disadvantage of a CIC filter is that, its pass-band is not flat, which is disagreeable in many applications and the frequency response of CIC filter is fully determined by only three integer parameters (R, M and N) resulting in a limited range of filter characteristics. The magnitude response of this filter has a high passband droop. The main aims of this work to improve the magnitude response of CIC filters and also solve the passband droop problem. The droop can be reduced by modifying the original CIC structure or by connecting an additional filter called CIC compensator in cascade with the CIC decimator. The former approach is based on a technique called sharpening. In recent years, several methods for the design of CIC compensators have been developed and now compensation is combined with a sharpening technique.

### **1.9 OBJECTIVES:-**

In most applications it is required to have a flat passband, otherwise the original signal may be destroyed. Unfortunately, the CIC filter alone suffers with a passband droop, which in many cases, cannot be accepted. The research objectives of this thesis are as follows

- ✓ Implement a compensator that is based on maximally flat criterion and solve this CIC compensator for higher order of CIC filter.
- ✓ Propose a new CIC compensator filter to improve the magnitude response of CIC filter.
- ✓ Comparison the results of magnitude response.

### **1.10 IMPROVED METHOD OF CIC FILTERS:-**

The simplest multiplier less filter is CIC filter. However, the magnitude response of this filter has a high passband droop, which is not tolerable in many applications. For improving the passband and the transition band features of the CIC filter and improving the performance of CIC filter there are many techniques, such as compensation filter cascaded with CIC filter, sharpening technique, polyphase

decimation FIR filter to achieve wide broadband compensation of the CIC filter and maximally-flat based compensator filter.

The droop can be reduced by modifying the original CIC structure or by connecting an additional filter called CIC compensator in cascade with the CIC decimator. The former approach is based on a technique called sharpening [9]. The sharpening of CIC filters is described in [10] and [11]. Filter sharpening can be used to improve the response of a CIC filter. In most applications it is required to have a flat passband, otherwise the original signal may be destroyed. Unfortunately, the CIC filter alone suffers with a passband droop, which in many cases, cannot be accepted. The big droop is due the sinc-like characteristic of the filter. Hence, it is of a great interest to get a flat passband using a compensation filter. The compensation filter will take the form of the inverse of the CIC filter frequency response in the passband, and attenuate as much as possible in the stopband. This is the one way to improve the response of CIC filter [12]-[13].

In the latter approach, CIC compensators are designed to approximate the inverse amplitude response of the CIC filter. In recent years, several methods for the design of CIC compensators have been developed and now compensation is combined with a sharpening technique [14].

### **1.11 ADVANTAGES & DISADVANTAGES OF CIC FILTER:-**

There are some advantages of the CIC filters [2]:-

- ✓ No multipliers are required
- ✓ No storage is required for filter coefficients
- ✓ Intermediate storage is reduced by integrating at high sampling rate and comb filtering at low sampling rate
- ✓ Little external control or complicated local timing is required
- ✓ The same filter design can be used for a wide range of rate change factor  $R$  with the addition of a scaling circuit and minimal change to the filter timing.

Some problems encountered with the CIC filters include the following [2]:-

- ✓ Register widths can become large for large rate change factors
- ✓ The frequency response is fully determined by only three integer parameters ( $R$ ,  $M$  and  $N$ ) resulting in a limited range of filter characteristics.

### **1.12 APPLICATION OF CIC FILTER:-**

The application for CIC filters seems to be in areas where high sampling rates make multipliers an uneconomical choice and areas where large rate change factors would require large amount of coefficient storage or fast impulse response generation[2].

### **1.13 ORGANIZATION OF DESSERTATION:-**

This dissertation consists of six chapters including introduction as follows:

*Chapter 2* deals with literature review in which study on existing improve method of CIC filter technique is discussed.

*Chapter 3* discuss about the CIC compensation filter which is based on maximally flat criterion. Also discuss about the maximally flat criterion and different technique to improve CIC filter response.

*Chapter 4* provides the new design of CIC compensator filter which is also based on maximally flat criterion but design is done by different techniques.

*Chapter 5* deals with the results of new design of CIC compensator filter and compare with another method.

*Chapter 6* highlights the conclusion of the dissertation and tells about future prospects.

## CHAPTER 2

### LITERATURE REVIEW

---

The literature review summarizes, interprets, and evaluates existing "literature" (or published material) in order to establish current knowledge of a subject. The purpose for doing so relates to ongoing research to develop that knowledge. The literature review may resolve a controversy, establish the need for additional research, and define a topic of inquiry.

**Zhang *et al.*** [15] present a paper to improve the performance of CIC filter. In this paper they used the compensation filter which has the inverse magnitude response to CIC filter was proposed, which improved the passband and transition band features of the CIC filter and improved the performance of CIC filter.

CIC filter is equivalent to a number of rectangular windows cascaded recursive filter form, and therefore has a more significant performance limitations. CIC filter has low attenuation and a droop in the desired passband that is dependent upon the decimation factor  $R$  and the number  $N$  of section in cascade.

In this paper, the magnitude response of CIC filter has been improved by introducing a compensation filter. To improve the magnitude drop, some compensation filters that have a magnitude response such as the sine compensator and cosine filters have been proposed.

We concluded that with the help of inverse sinc compensation filter improved and became flat passband, transition zone of it's became sharp, as a result the performance of the filter has been improved.

**Dolecek *et al.*** [16] present a paper new two-stage CIC based decimation filter for input signals occupying  $\frac{3}{4}$  of the digital band. They assumed that the decimation factor  $M$  to be an even number. The decimation factor of the first stage is  $M/2$ , whereas, that of the second stage is 2. They introduced a sine based compensator filter to decrease the passband droop of the CIC filter and a cosine filter to improve the overall stopband characteristics. As a result the signal-to-noise (SNR) is improved. They used the polyphase decomposition method of the comb filter. The polyphase components of the comb filters are moved to the lower rate which is  $M/2$  less than the input rate. Consequently there is no filtering at the high input rate. The proposed filter

performs decimation efficiently using only additions/subtractions making it attractive for software radio applications. They introduced sine-based compensator to improve the passband characteristics of the CIC filter. The proposed filter performs decimation using only additions/subtractions and exhibits higher SNR compared to that of the corresponding CIC filter, as the expense of slight increase of complexity.

**Mitral *et al.*** [17] present a paper on “Efficient Sharpening of CIC Decimation Filter”. In this paper, they proposed an efficient sharpening of a CIC decimation filter for an even decimation factor. The proposed structure consists of two main sections: a section composed of a cascade of the first order moving average filters, and a sharpening filter section. The proposed decomposition scheme allows a sharpening section to operate at half of the input rate. In addition, the sharpened CIC filter is of length that is half of that of the original CIC filter. With the aid of the polyphase decomposition, the polyphase sub filters of the first section can also be operated at the half of the input rate.

Frequency response of the CIC filter exhibits a linear phase, lowpass  $\sin Mx / \sin x$  characteristics with a droop in the desired frequency range in the passband that is dependent upon the decimation factor  $M$ . The signal components aliased into the passband, which is caused by the down-sampling, appear on both sides of the nulls. The worst case aliasing occurs in the passband near the first null at  $1/M$ .

The resulting sharpened filter has a significantly improved frequency response, i.e. the reduced passband droop and improved alias rejection. The main drawback of this structure is that the filtering is performed at the high input rate and for a given decimation factor  $M$ , the complexity increases with the number of the stages  $K$ .

To overcome these problems they implemented the filter in two stages and allowing filtering at a rate lower than the high input rate while reducing the length of the sharpened CIC filter to one half of that of the original CIC filter.

Using the polyphase decomposition the first section can also be moved to a lower rate and implemented efficiently by using dedicated shift and add multipliers and eliminating redundant operations using the data broadcast structure.

**Li *et al.*** [18] present a paper on “Compensation Method for CIC Filter in Digital Down Converter”. In this paper, a method with the structure of polyphase decimation FIR filter is proposed to achieve wide broadband compensation of the CIC filter and sampling rate conversion. The paper focused on the usage of FIR filters to

compensate CIC filters with the coefficients obtained by simulation, which extend the bandwidth and improves the flexibility of the compensation filters.

There are many methods to compensate CIC filter, i.e. based on sharpening technique, the interpolation binomial compensation method and inverse sinc function compensation. But these methods cannot guarantee the flatness of passband when the bandwidth is wide. Because of advantage of FIR filters, i.e. adaptability, flexibility, easily adjusted coefficients and the structure of polyphase filters can be used to FIR filters, most of the commercial down-conversion chip still uses the FIR filters to compensate CIC filters.

It can be seen that CIC filter is a rectangular window. The highly symmetric structure allows efficient implementation in hardware. The disadvantage of CIC filter is that its passband is not flat, which is undesirable in wide application. Single stage CIC filter is difficult to meet the practical requirements for its poor stopband droop. We can use multistage CIC filter to increase the stopband attenuation to meet practical demands. However, with the number of the cascaded CIC filter increasing, CIC filter's passband performance will significantly become worse. Generally, the order is limited to 5 orders. The method, using the inverse of the magnitude response of CIC filter, is proposed to design a compensation filter.

They used the mean and square error can be used to measure the performance of the filter after compensation. If the order of the filter is fixed, in order to get the best compensation effect, the square error should be minimum. They observed that if the order of the compensation filter is at the range of 12 to 24, the compensation effect is better. After getting the FIR coefficient with the best compensation effect, in order to reduce the computational speed and save hardware resources, they used polyphase decimation FIR filter to complete data decimation. They observed that after FIR compensation filter significantly reduces passband drop and improves stop band rejection.

From this paper it concluded that sharpening techniques and the interpolation binomial compensation method cannot meet broadband application. FIR filter, based on polyphase structure, is used to compensate CIC filter in view of small decimation. It enlarges the compensation bandwidth and improves the flexibility of the compensation. Filter coefficient is properly quantified, it can ensure that the flatness of the passband and stopband attenuation both obtain high computational efficiency.

**Stephen et al.** [19] present a paper on “High Speed Sharpening of Decimation CIC Filter”. In this paper they present a modification of an existing technique which compensates for passband droop in the frequency response of CIC filter for power of two decimation factor. The new techniques delivers passband droop correction comparable to existing techniques, while allowing faster operation than for existing structures by moving integrator away from the filter input.

They used a sharpened technique developed by Kaiser and Hamming .This technique allows the programmable FIR filter to be replaced with a constant coefficient FIR filter, which, despite resulting in increased hardware consumption by the CIC portion of the circuit, leads to a more hardware efficient solution overall.

The hardware implementations of both CIC and sharpened filters result in a series of integrators running at the input (high) sample rate. As these sections are recursive, they cannot be pipelined, thus limiting the critical path to one adder operation. Non-recursive structures can be pipelined (at bit-level for increased speed) thus reducing the critical path to a fraction of an arithmetic operation and allowing the circuit to be clocked at a faster rate.

**Harris et al.** [20] present a paper on “On Design of Two Stage CIC Compensation Filter”. This paper presents the compensation filter design for the two-stage CIC decimation filter. The goal is twofold: to avoid the integrator section at high input rate and obtain a low wideband passband droop of the overall filter. To this end the decimation is split into two stages with the cascaded less order RRS filters at each stage. The first stage can be implemented either in non recursive form or using the polyphase decomposition. The simple compensation filter and the sharpening are applied to the second section where RRS filter is implemented as a CIC filter. The resulting structure is a multiplier less and with no integrators at high input rate. Additionally, the structure exhibits a low passband droop and high stopband attenuation.

This paper presents a new multiplierless two-stage decimation filter. Unlike to CIC filter the proposed structure does not have the integrator section at the high input rate. Using the polyphase decomposition of the filter at the first stage all filtering is moved to the lower rate which is  $M/2$  times less than the input rate. Additionally the structure exhibits a low wideband passband droop and a high attenuation. The price is the slightly increased complexity.

**Karnati et al.** [21] present a paper “A Power-Efficient Polyphase Sharpened CIC Filter for Sigma-Delta ADCs”. This paper presents a power-efficient poly-phase sharpened CIC filter for sigma-delta ADCs. In this scheme, by using a cascade of CIC filter and SCIC filter with proper optimization of the two-stage decimation structure, the power consumption can be considerably reduced compared to conventional SCIC filter architectures. Furthermore, poly-phase implementations for both the CIC and SCIC filter sections are used to further reduce the power. The proposed filter is designed with CMOS 111m technology, where the lowest power consumption is achieved among similar filter architectures.

In this paper, a power efficient poly-phase sharpened CIC filter for sigma-delta ADCs has been proposed. The cascaded filter architecture consists of the first stage CIC filter and the second stage SCIC filter. The proposed scheme is finalized by finding the optimum decimation ratio of each stage that gives minimum power consumption, whereas maintaining the passband and stopband characteristics. In addition, the power consumption of the proposed filters was further reduced by poly-phase implementation of each stage. As a result, the proposed design shows the lowest power consumption among similar filter architecture with equivalent performance. Although a second order cascaded CIC and SCIC filter with decimation ratio of 64 is used for the design, the proposed scheme can be applied to filters with arbitrary order and decimation ratio.

**Mitra et al.** [22] present a paper on “A New Two Stage Sharpened Comb Decimator”. This paper presents a new sharpened comb decimator structure consisting of a cascade of a comb-filter based decimator and a sharpened comb decimator. The proposed realization scheme allows the sharpened section to operate at a lower rate that depends on the decimation factor of the first section. Using a polyphase decomposition, the sub filters of the first section can also be operated at this lower rate.

A new efficient structure for a sharpened comb factor-of  $M$  decimation filter is proposed. The structure consists of two main sections: a cascade of comb filters followed by down-sampling with a factor  $M_1$ , and a sharpened comb filter followed by downsampling with a factor  $M_2$ , where  $M=M_1M_2$ . This arrangement allows the sharpening operation to move to a lower rate which is  $M_1$  times less than the high input rate. Applying a polyphase decomposition to the first section filter, the subfilters of the first section can also be operated at the  $M_1$  times lower rate. The alias rejection

of the proposed filter does not depend on the values for a given but the passband droop is increased with the increase of  $M1$ . The proposed sharpened filter has much better alias rejection than the corresponding comb filter and the original sharpened comb filter. In order to obtain further increase in the alias rejection, the RS filter can be cascaded with the sharpened filter in the second section. The passband droop of the proposed filter is similar to the corresponding original sharpened filter and much smaller than the corresponding comb filter.

**Dolecek** [23] presents a paper “On Design of CIC Decimator Filter with Improved Response”. A simple decimation filter based on the CIC filter and a sine compensator is presented. The design parameters are the number  $K$  of cascaded CIC filters and the parameter  $b$  defining the coefficient of the compensator filter. The proposed filter performs decimation efficiently using only additions/subtractions making it attractive for software radio (SWR) applications.

A simple method of designing a multiplier-free CIC-based decimation filter is proposed. The designed filter keeps the simplicity of the CIC filter while significantly improving the gain response in the passband of interest. The number  $K$  of cascaded CIC filters controls the alias rejection while the parameter  $b$  of the compensation filter controls the droop in the passband of interest. For a wider passband of interest the value of  $b$  is generally smaller. Comparison with some known methods shows that the similar results are obtained using the proposed filter but requiring lesser computational efforts.

**Vazquez et al.** [24] present a paper on “Maximally Flat CIC Compensation Filter: Design and Multiplier less Implementation”. This brief introduces a design and implementation of maximally flat cascaded integrator comb compensation filters. In particular, we consider second- and fourth-order linear phase filters for narrow-band and wideband compensation. Closed-form equations for the computation of the filter coefficients are given. The multiplier less implementation is also considered. The number of adders is a function of the decimation factor  $D$  and the number of stages  $N$ . The implementation complexity is discussed, and comparisons with some methods reported in the literature are provided.

This brief introduces a design of maximally flat CIC passband compensator filters. The filter coefficients are obtained by solving a set of linear equations. We described in detail two special cases, i.e., second- and fourth-order filters for the narrow-band and wideband compensations, respectively. The corresponding filter coefficients have

closed-form equations. The multiplier less implementation complexity of the proposed filters depends on the decimation factor  $D$  and the number of stages  $N$ . However, there is a restriction on the values of the decimation factor: for the narrow-band design, decimation factor  $D$  must be a power of two, whereas for the wideband design.

**Laddomade *et al.*** [25] present a paper on “An Economical Class of Droop-Compensated Generalized Comb Filters: Analysis and Design”. In this brief, we address the design of economical recursive generalized comb filters (GCFs) by proposing an efficient technique to quantize the multipliers in the  $z$ -transfer function employing power-of-2 (PO2) terms. GCFs are efficient anti-aliasing decimation filters with improved selectivity and quantization noise rejection performance around the so-called folding bands with respect to classical comb filters. The proposed quantization technique guarantees perfect pole-zero cancelation in the rational  $z$ -transfer function of the GCFs, thus totally avoiding instability problems. Moreover, we propose the use of a simple droop compensator for the sake of recovering the passband droop distorting the useful digital signal in the baseband. A design example is proposed with the aim of showing the application of the proposed technique, and a practical architecture of a sample third-order GCF is discussed.

This brief has focused on the design of recursive multiplier less GCFs by proposing an effective technique to jointly quantize the coefficients in the  $z$ -transfer function in such a way as to meet perfect pole-zero cancelation. We have also proposed the use of a very efficient second order droop compensator to recover the passband distortion introduced by this class of filters and briefly addressed the design of multiplier less higher order GCFs.

**Mitra *et al.*** [26] present a paper on “Simple Method for Compensation of CIC Decimation Filter”. A simple second-order sine-based CIC (cascaded-integrator-comb) compensator is presented. The design parameter is the integer  $b$ , which depends on the number  $K$  of the cascaded CIC filters. The proposed filter performs compensation efficiently using only three additions/subtractions.

A simple multiplier free sine-based compensator with only two adders is proposed. The number  $K$  of the cascaded CIC filters defines the parameter  $b$  depending on whether the compensation is in the narrowband or wideband. The proposed filter is convenient to the passband CIC compensation in the band less than  $3/5$  of the total

band after decimation. Comparison with some known methods show that the proposed method requires less computational efforts and is generally less complex.

**Gao *et al.*** [27] present a paper on “An Improved Architecture and Implementation of CIC Filter”. In this paper an improved version of the non-recursive carry-save-adder-based structure for CIC decimation filters is proposed for high speed applications. By employing parallel processing techniques, the improved structure can further increase the sampling rate of CIC filters. Low-complexity implementation of the parallel stages is also discussed.

By employing parallel processing techniques, the improved structure can further increase the sampling rate of the non-recursive structure for CIC decimation filters. Power consumption and hardware (area) are significantly saved by the following means: 1) Parallel processing is only applied to the critical stages which dominate the sampling rate of the non-recursive structure 2) Reduced-complexity implementation of the parallel stages.

**Singh *et al.*** [28] present a paper on “Design and Implementation of CIC Based Decimation Filter for Improved Frequency Response”. The over sampled output of a sigma delta modulator is decimated to Nyquist sampling rate by Decimation filters. The decimation filters work two fold, they decimate the sampling rate by a factor of OSR [over sampling rate] in doing so they remove the out band quantization noise resulting in an increase in resolution. The speed, area and power consumption of oversampled converter are governed largely by decimation filters in sigma-delta A/D converters. The paper presents a design and implementation of a sigma-delta digital decimation filter. The decimation filter structure is based on cascaded-integrated Comb (CIC) filter. A second decimation filter using CIC for large rate change and cascaded FIR filters, for small rate changes, to improve the frequency response. The proposed structure is even more hardware efficient.

In this paper the CIC based decimation filter design was discussed and its implementation was presented. The implementation is presented by describing each block and the additional coder circuit required. The use for large rate change was highlighted. They are very economic decimation structure compared to FIR or IIR for large rate change due to lack of multipliers. The frequency limitations of the CIC filter was discussed. The multistage implementation where CIC is used for large rate change and FIR cascading at lower rates was found to have a very sharp and precise response and it was even more hardware efficient. The FIR structures used must be

hardware efficient. One such structure is use of half band FIR where the numbers of taps are reduced by half.

**Saud *et al.*** [29] proposed a simple modification to CIC filter that enhances its performance at expense of requiring few extra computations per output sample. He said that although Cascaded-integrator-comb (CIC) filters perform sample rate conversion (SRC) efficiently using only additions/subtractions. However, the limited number of tuning parameters may make conventional CIC filters unsuitable for SRC in software radio (SWR) systems. So he proposed a simple modification to the CIC filter that enhances its SRC performance at the expense of requiring a few extra computations per output sample is proposed. Simulation results show that the modified CIC filter outperforms the conventional CIC filter for the purpose of SRC in SWR.

The modified CIC filter provides higher SNRs and better image attenuation than the conventional CIC filter by adjusting the zeros of the filter to target high-power image components. SWR systems can take advantage of this flexibility when the wideband input contains narrowband channels with a high dynamic range. An SWR receiver can measure the power of different channels and correspondingly adjust the delays of the CIC filter to minimize aliasing caused by high-power narrowband channels. The modified CIC filter gains this improved performance over the conventional CIC filter at the expense of a small increase in the number of computations.

**Dolecek *et al.*** [30] presents new optimal sharpening techniques. In this new work, it is shown that, for stringent magnitude specifications, sharpening compensated comb filters requires a lower-degree sharpening polynomial compared to sharpening comb filters without compensation, resulting in a solution with lower computational complexity. Using a simple three-addition compensator and an optimization-based derivation of sharpening polynomials, we introduce an effective low-complexity filtering scheme. The goal of the optimization problem was to minimize the min-max error over the frequency bands of interest of the sharpened filter. The sharpening coefficients are guaranteed to be integers scaled by power-of-2 terms, thus resulting in low-complexity structures. Moreover, it was shown that the use of compensated comb filters, instead of combs only as basic building blocks in the sharpened filter, results in lower complexity structures for the same magnitude characteristics. Finally, it was shown that the proposed method provides better magnitude characteristic than other

sharpening-based approaches for two-stage comb-based structures since it is able to correct the passband droop introduced by the first stage comb filter.

**Yoshida *et al.*** [31] present a paper on “FIR band-pass digital differentiators with flat passband and equiripple stopband characteristics”. Maximally flat digital differentiators are widely used as narrow-band digital differentiators because of their high accuracy around their center frequency of flat property. To obtain highly accurate differentiation over narrow-band, it is important to avoid the undesirable amplification of noise. In this paper, they introduce a design method of linear phase FIR band-pass differentiators with flat passband and equiripple stopband characteristics. The center frequency at the passband of the designed differentiators can be adjusted arbitrarily. Moreover, the proposed transfer function consists of two functions, i.e. the passband function and the stopband one. The weighting coefficients of the passband function are derived using a closed-form formula based on Jacobi Polynomial. The weighting coefficients of the stopband function are achieved using Remez algorithm.

**Dolecek *et al.*** [32] introduce a design and implementation of maximally flat cascaded integrator comb compensation filters. In particular, they consider second- and fourth-order linear phase filters for narrow-band and wideband compensation. Closed-form equations for the computation of the filter coefficients are given. The multiplierless implementation is also considered. The number of adders is a function of the decimation factor  $D$  and the number of stages  $N$ . The implementation complexity is discussed, and comparisons with some methods reported in the literature are provided. The filter coefficients are obtained by solving a set of linear equations. They described in detail two special cases, i.e., second- and fourth-order filters for the narrow-band and wideband compensations, respectively. The corresponding filter coefficients have closed-form equations. The multiplierless implementation complexity of the proposed filters depends on the decimation factor  $D$  and the number of stages  $N$ .

### MAXIMALLY FLAT CIC COMPENSATOR FILTER

---

The simplest multiplier less filter is CIC filter. However, the magnitude response of this filter has a high passband droop, which is not tolerable in many applications. For improving the passband and the transition band features of the CIC filter and improving the performance of CIC filter there are many techniques, such as compensation filter cascaded with CIC filter, Sharpening technique, polyphase decimation FIR filter to achieve wide broadband compensation of the CIC filter and maximally-flat based compensator filter.

The droop can be reduced by modifying the original CIC structure or by connecting an additional filter called CIC compensator in cascade with the CIC decimator. The former approach is based on a technique called sharpening. Filter sharpening can be used to improve the response of a CIC filter. In most applications it is required to have a flat passband, otherwise the original signal may be destroyed. Unfortunately, the CIC filter alone suffers with a passband droop, which in many cases, cannot be accepted. The big droop is due the sinc-like characteristic of the filter. Hence, it is of a great interest to get a flat passband using a compensation filter. The compensation filter will take the form of the inverse of the CIC filter frequency response in the passband, and attenuate as much as possible in the stopband. This is the one way to improve the response of CIC filter.

In the latter approach, CIC compensators are designed to approximate the inverse amplitude response of the CIC filter. In recent years, several methods for the design of CIC compensators have been developed and now compensation is combined with a sharpening technique.

This chapter will discuss about the CIC compensation filter that is based on maximally flat criterion. The method is based on the minimization of error function in the least-square sense. The main goal of this brief is to explain the wideband compensation as well as to efficiently implement the compensator filters. Additionally, the multiplierless implementation is proposed.

### 3.1 MAXIMALLY FLAT COMPENSATORS:-

This section discuss about the design of linear phase FIR compensation filter, with the maximally flat magnitude response. The proposed compensated CIC filter is expressed as [32]

$$G(z) = H(z)P(z^D) \quad (3.1)$$

Where  $H(z)$  is the transfer function of CIC filter [2]

$$H(z) = \left( \frac{1}{D} \frac{1-z^{-D}}{1-z^{-1}} \right)^N \quad (3.2)$$

Where  $D$  is the decimation factor and  $N$  is the number of CIC stages.  $P(z)$  is the linear phase filter. In general, there are four stages of linear phase filters. However, the type 2-4 FIR filter imply at least one zero on unit circle, which is inconvenient for the expanded filter design.

The CIC decimation filter is followed by  $2^{\text{nd}}$  decimation stage. The decimation factor  $\nu$  of the second stage determines the passband edge frequency  $w_p$ , where the worst passband droop occurs,  $w_p = \pi / D\nu$ . Depending on the value of decimation factor we design the CIC compensator.

From equation (3.2), the frequency response of CIC filter is given by

$$H(e^{jw}) = e^{-j(D-1)Nw/2} H_R(w) \quad (3.3)$$

$H_R(w)$  is a real valued function given by

$$H_R(w) = \left( \frac{1}{D} \frac{\sin(Dw/2)}{\sin(w/2)} \right)^N \quad (3.4)$$

Type 1 linear phase FIR compensation filter  $P(z)$  as

$$P(z) = \sum_{n=0}^L a_n z^{-n} \quad (3.5)$$

Where  $L$  is an even integer and is the order of  $P(z)$ ,  $a_n$ ,  $n=0, \dots, L$  are the filter coefficients, and  $a_n$  satisfies  $a_n = a_{L-n}$ . The corresponding frequency response is [33]

$$P(e^{iw}) = e^{-jwL/2} P_R(w) \quad (3.6)$$

Where the real valued function  $P_R(w)$  is expressed as [33]

$$P_R(w) = a_{L/2} + 2 \sum_{n=0}^{L/2-1} a_n \cos(w(L/2 - n)) \quad (3.7)$$

The corresponding frequency response of  $G(z)$  can be written as

$$G(e^{jw}) = e^{-jw((D-1)N+LD)/2} H_R(w) P_R(Dw) \quad (3.8)$$

From the above equation it is clear that the overall filter  $G(e^{jw})$  has a linear phase, which avoids phase distortion of the input signal in the passband. Now impose the maximally flat condition onto the magnitude response.

In order to design the compensation filter  $P(z)$ , error function define as [34]

$$E(w) = P_R(Dw) H_R(w) - 1 \quad (3.9)$$

The condition that the error function  $E(\omega)$  is maximally flat at  $\omega = 0$  is that it has as many derivatives as possible that are vanishing at  $\omega = 0$  [35]. Since the error function is an even function of  $\omega$ , its odd indexed derivatives evaluated at  $\omega = 0$  are automatically zero [35]. Therefore, the maximally flat conditions are

$$E(0) = 0 \quad (3.10)$$

$$\frac{d^p E(w)}{dw^p} = 0 \quad (3.11)$$

Where  $p$  is even and positive integer, i.e.  $p=2q$  for  $q=1, \dots, L/2$ .

Using (3.9), (3.10) implies

$$P_R(0) = 1 \quad (3.12)$$

Now, substituting (3.9) into (3.11) and using the general Leibniz rule for the  $p$ th derivative of a product [32]

$$\frac{d^p H_R(w)}{dw^p} + \sum_{l=1}^p \binom{p}{l} \left[ \frac{d^l P_R(Dw)}{dw^l} \frac{d^{p-l} H_R(w)}{dw^{p-l}} \right]_{w=0} = 0 \quad (3.13)$$

Where the binomial coefficient is given by

$$\binom{p}{l} = \frac{p!}{l!(p-l)!} \quad (3.14)$$

The odd indexed derivatives of  $P_R(Dw)$  evaluated at  $w=0$  are zero. Therefore, from (3.7) [32]

$$\left[ \frac{d^l P_R(Dw)}{dw^l} \right]_{w=0} = \begin{cases} 2(-1)^{l/2} D^l \sum_{n=0}^{L/2-1} (L/2 - n)^l a_n & l = \text{even} \\ 0 & l = \text{odd} \end{cases} \quad (3.15)$$

Substituting (3.15) into (3.13), (3.11) can be written as [32]

$$2 \sum_{l=1}^q \binom{2q}{2l} (-1)^l D^{2l} \times \sum_{n=0}^{L/2-1} \binom{L/2-n}{2l} a_n \left[ \frac{d^{2(q-l)} H_R(w)}{dw^{2(q-l)}} \right]_{w=0} = - \frac{d^{2q} H_R(w)}{dw^{2q}} \quad (3.16)$$

For  $q = 1, \dots, L/2$ . The coefficients of the linear phase maximally flat compensation filter  $P(z)$  of the order  $L$  are obtained by solving the set of linear equations (3.16).

### 3.2 CIC COMPENSATOR RESPONSE FOR DIFFERENT ORDER OF FIR FILTER:-

This section focuses on the design of the CIC compensator filter based on linear phase filter. The resulting magnitude characteristic of the compensated CIC filter exhibits a maximally flat characteristic at  $w = 0$ . In this section compare the four order of FIR filter order and in the last compare the results.

For FIR linear phase filter order  $L=2$ , the transfer function of  $P(z)$  is given by

$$P(z) = a_0 + a_1 z^{-1} + a_0 z^{-2} \quad (3.17)$$

Accordingly, the maximally flat conditions given by (3.12) and (3.16) are equivalent to

$$a_1 + 2a_0 = 1 \quad (3.17)$$

$$a_0 = \frac{1}{2D^2} \frac{d^2 H_R(w)}{dw^2} \quad (3.18)$$

Where

$$\frac{d^2 H_R(w)}{dw^2} = - \frac{N(D^2 - 1)}{12} \quad (3.19)$$

Solving the set of linear equations (3.17) and (3.18)

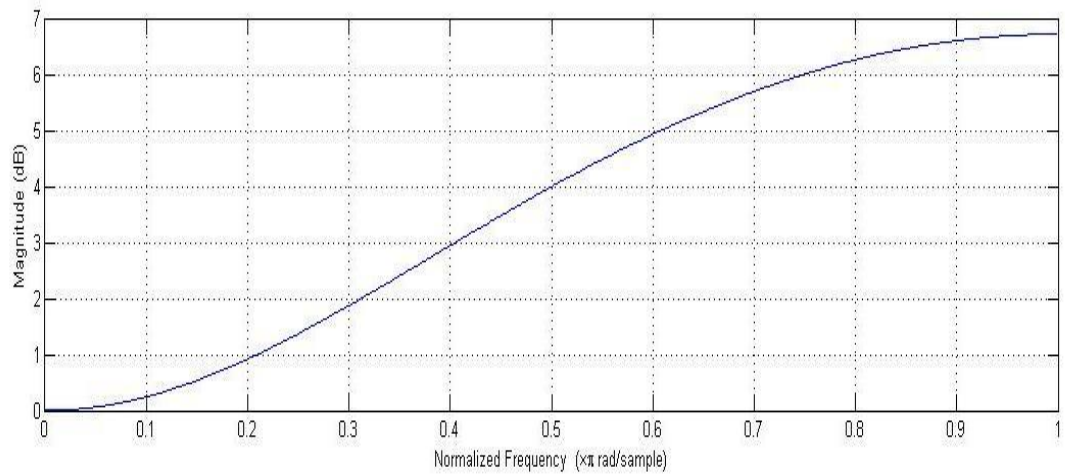
$$a_0 = - \frac{N}{32} \frac{1 - D^{-2}}{1 - 2^{-2}}, \quad a_1 = 1 - 2a_0 \quad (3.20)$$

Solving the above equations and get the filter coefficients value of FIR linear phase filter order  $L=2$ . Now consider the CIC filter stage  $N=7$  and decimation factor  $D$  is equal to 2048. For these following design parameters filter coefficients values are

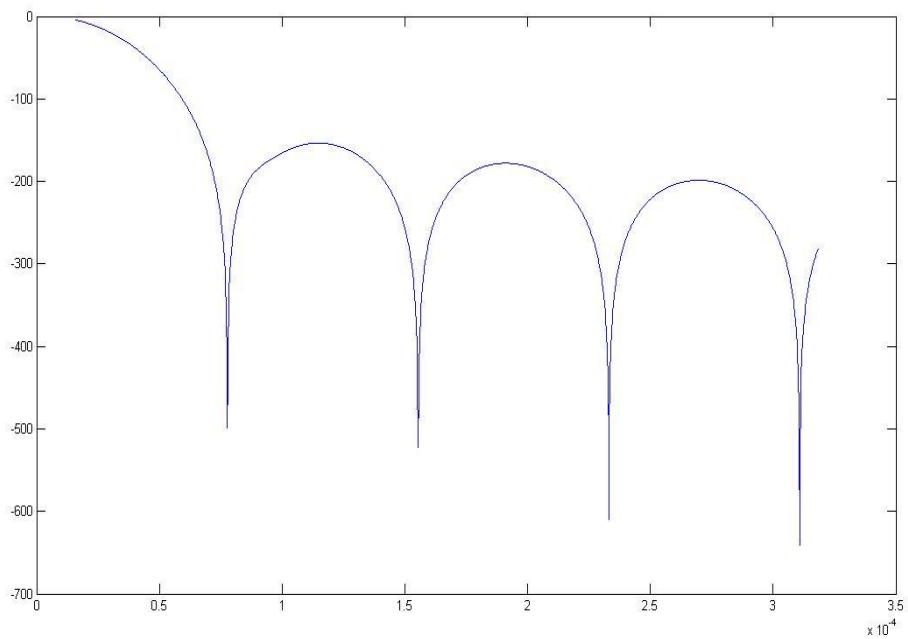
$$a_0 = -0.2917$$

$$a_1 = 1.5833$$

For these filter coefficients value compensated CIC filter and magnitude response of compensated CIC filter shown in figure 3.1 and 3.2 respectively.



**Figure 3.1: CIC Compensator for FIR filter order L=2**



**Figure 3.2: Magnitude Response of Compensated CIC filter for L=2**

For the following design parameters figure 3.1 shows the CIC compensator response and figure 3.2 shows the magnitude response of the compensated CIC filter. The compensated CIC filter has same attenuation in the alias band as the CIC filter.

For FIR linear phase filter order L=4, the transfer function of  $P(z)$  is given by

$$P(z) = a_0 + a_1z^{-1} + a_2 + a_1z^{-3} + a_0z^{-4} \quad (3.21)$$

The maximally flat conditions (3.12) and (3.16) are rewritten as

$$2a_0 + 2a_1 + a_2 = 1 \quad (3.22)$$

$$4a_0 + a_1 = \frac{1}{2D^2} \left. \frac{d^2 H_R(w)}{dw^2} \right|_{w=0} \quad (3.23)$$

$$16a_0 + a_1 = \frac{1}{2D^4} \left( 6 \left[ \left. \frac{d^2 H_R(w)}{dw^2} \right]_{w=0}^2 - \left. \frac{d^4 H_R(w)}{dw^4} \right|_{w=0} \right) \quad (3.24)$$

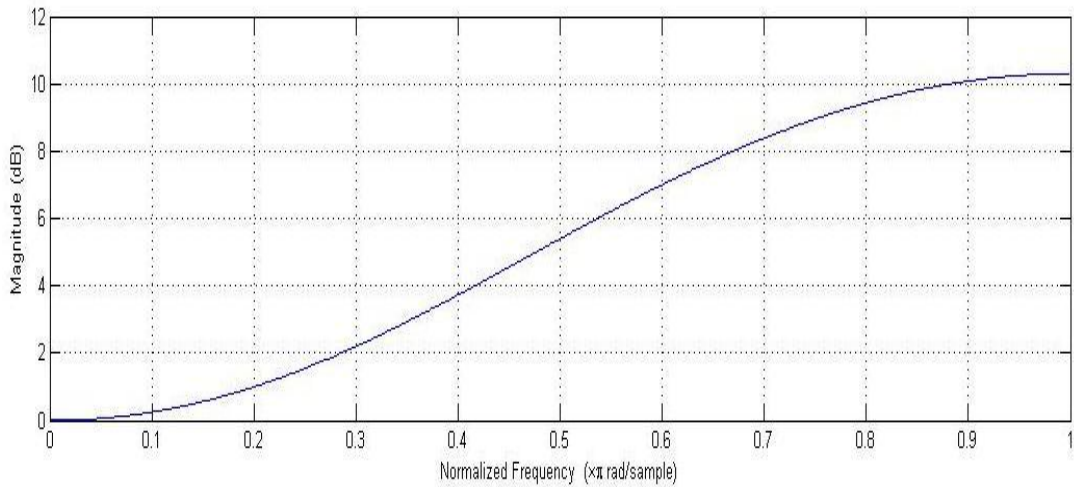
Where

$$\left. \frac{d^4 H_R(w)}{dw^4} \right|_{w=0} = \frac{N(D^2 - 1)(D^2(5N - 2) - 5N - 2)}{240} \quad (3.25)$$

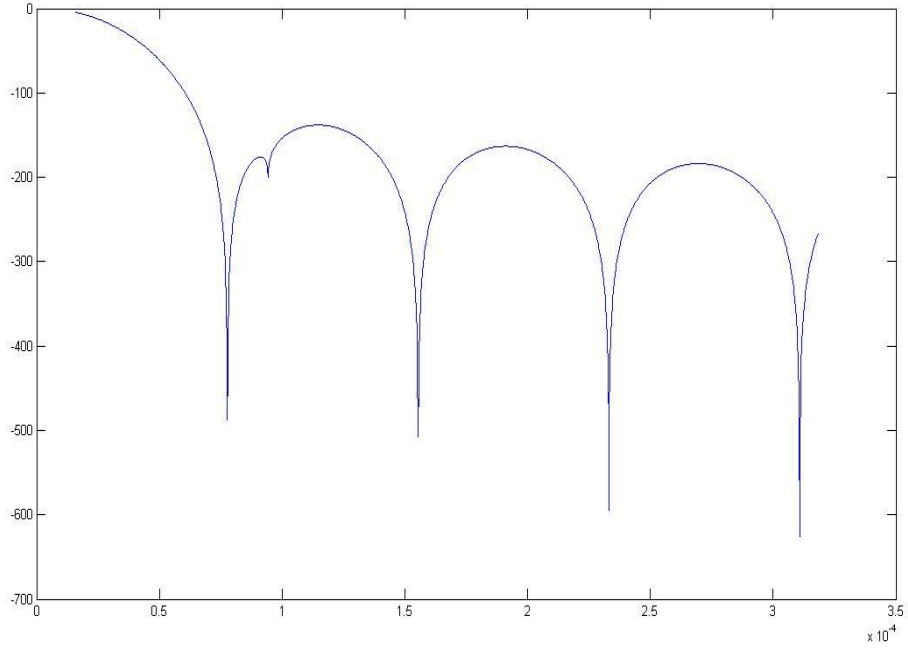
Solving the above equations and get the filter coefficients value of FIR linear phase filter order L=4. Now consider the CIC filter stage N=7 and decimation factor D is equal to 2048. For these following design parameters filter coefficients values are

$$a_0 = 0.0693, \quad a_1 = -0.5687, \quad a_2 = 1.990$$

For these filter coefficients value compensated CIC filter and magnitude response of compensated CIC filter shown in figure 3.3 and 3.4 respectively.



**Figure 3.3: CIC Compensator for FIR filter order L=4**



**Figure 3.4: Magnitude Response of Compensated CIC filter for L=4**

For the following design parameters figure 3.3 shows the CIC compensator response and figure 3.4 shows the magnitude response of the compensated CIC filter. The compensated CIC filter has same attenuation in the alias band as the CIC filter. For L=4 FIR filter order CIC compensator magnitude response is better than the FIR filter L=2. In L=4 passband droop is less than the L=2.

For FIR linear phase filter order L=6, the transfer function of  $P(z)$  is given by

$$P(z) = a_0 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_2z^{-4} + a_1z^{-5} + a_0z^{-6} \quad (3.26)$$

The maximally flat conditions (3.12) and (3.16) are rewritten as

$$a_3 = 1 - 2a_0 - 2a_1 - 2a_2 \quad (3.27)$$

$$9a_0 + 4a_1 + a_2 = \frac{1}{2D^2} \left. \frac{d^2 H_R(w)}{dw^2} \right|_{w=0} \quad (3.28)$$

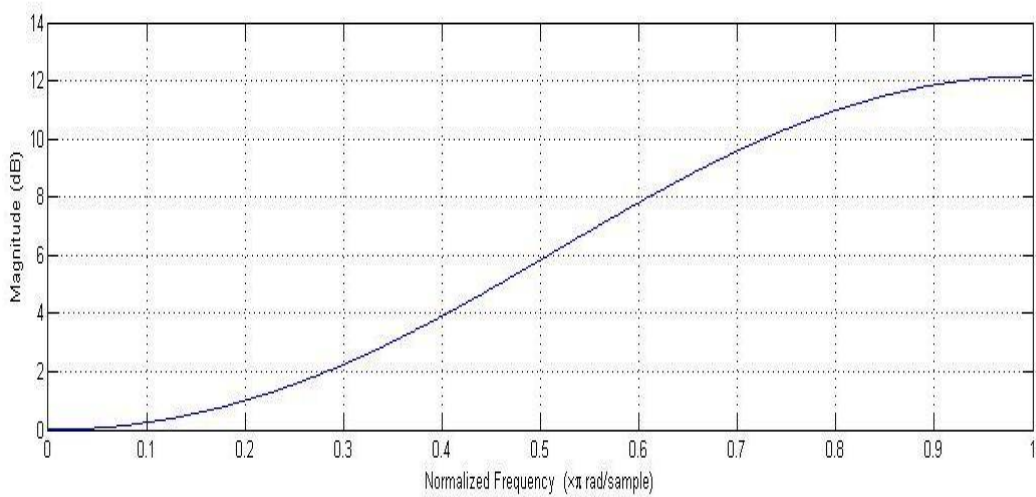
$$81a_0 + 16a_1 + a_2 = \frac{1}{2D^4} \left[ 6 \left[ \left. \frac{d^2 H_R(w)}{dw^2} \right]_{w=0}^2 - \left. \frac{d^4 H_R(w)}{dw^4} \right|_{w=0} \right] \quad (3.29)$$

$$729a_0 + 64a_1 + a_2 = -\frac{1}{2D^6} \left[ 30 \frac{d^2 H_R(w)}{dw^2} \frac{d^4 H_R(w)}{dw^4} - 90 \left[ \left. \frac{d^2 H_R(w)}{dw^4} \right]^3 - \left. \frac{d^6 H_R(w)}{dw^6} \right|_{w=0} \right] \quad (3.30)$$

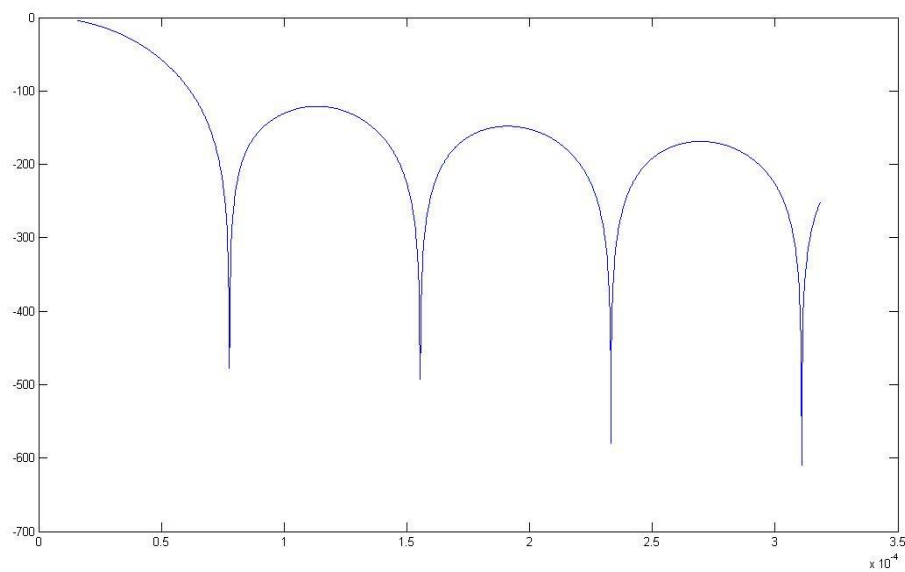
Solving the above equations and get the filter coefficients value of FIR linear phase filter order  $L=6$ . Now consider the CIC filter stage  $N=7$  and decimation factor  $D$  is equal to 2048. For these following design parameters filter coefficients values are

$$a_0 = -0.0122, \quad a_1 = 0.1423, \quad a_2 = -0.7512, \quad a_3 = 2.2423$$

For these filter coefficients value compensated CIC filter and magnitude response of compensated CIC filter shown in figure 3.5 and 3.6 respectively.



**Figure 3.5: CIC Compensator for FIR filter order  $L=6$**



**Figure 3.6: Magnitude Response of Compensated CIC filter for  $L=6$**

For the following design parameters figure 3.5 shows the CIC compensator response and figure 3.6 shows the magnitude response of the compensated CIC filter. The compensated CIC filter has same attenuation in the alias band as the CIC filter. For L=6 FIR filter order CIC compensator magnitude response is better than the FIR filter L=2 and L=4. In L=6 passband droop is less than the L=2 and L=4.

For FIR linear phase filter order L=8, the transfer function of  $P(z)$  is given by

$$P(z) = a_0 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_3z^{-5} + a_2z^{-6} + a_1z^{-7} + a_0z^{-8} \quad (3.31)$$

The maximally flat conditions (3.12) and (3.16) are rewritten as

$$a_4 = 1 - 2a_3 - 2a_2 - 2a_1 - 2a_0 \quad (3.32)$$

$$16a_0 + 9a_1 + 4a_2 + a_3 = \frac{1}{2D^2} \left. \frac{d^2 H_R(w)}{dw^2} \right|_{w=0} \quad (3.33)$$

$$256a_0 + 81a_1 + 16a_2 + a_3 = \frac{1}{2D^3} \left[ 6 \left[ \left. \frac{d^2 H_R(w)}{dw^2} \right|_{w=0} \right]^2 - \left. \frac{d^4 H_R(w)}{dw^4} \right|_{w=0} \right] \quad (3.34)$$

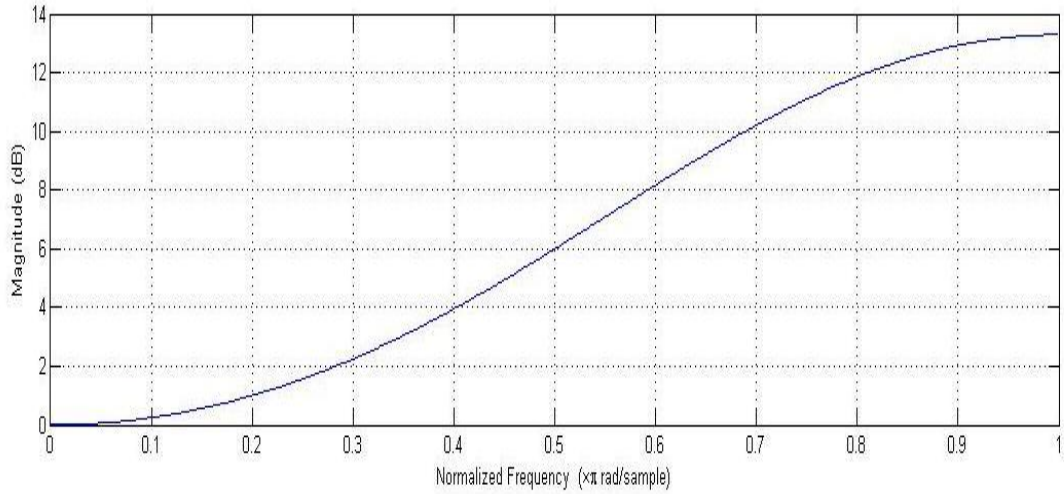
$$4096a_0 + 729a_1 + 64a_2 + a_3 = -\frac{1}{2D^6} \left[ 30 \frac{d^2 H_R(w)}{dw^2} \frac{d^4 H_R(w)}{dw^4} - 90 \left[ \left. \frac{d^2 H_R(w)}{dw^2} \right|_{w=0} \right]^3 - \left. \frac{d^6 H_R(w)}{dw^6} \right|_{w=0} \right] \quad (3.35)$$

$$65536a_0 + 6561a_1 + 256a_2 + a_3 = \frac{1}{2D^8} \left[ 56 \frac{d^2 H_R(w)}{dw^2} \frac{d^6 H_R(w)}{dw^6} - 1260 \left[ \left. \frac{d^2 H_R(w)}{dw^2} \right|_{w=0} \right]^2 \frac{d^4 H_R(w)}{dw^4} + 70 \left[ \left. \frac{d^4 H_R(w)}{dw^4} \right|_{w=0} \right]^2 + 2520 \left[ \left. \frac{d^2 H_R(w)}{dw^2} \right|_{w=0} \right]^3 \frac{d^2 H_R(w)}{dw^2} - \left. \frac{d^8 H_R(w)}{dw^8} \right|_{w=0} \right] \quad (3.36)$$

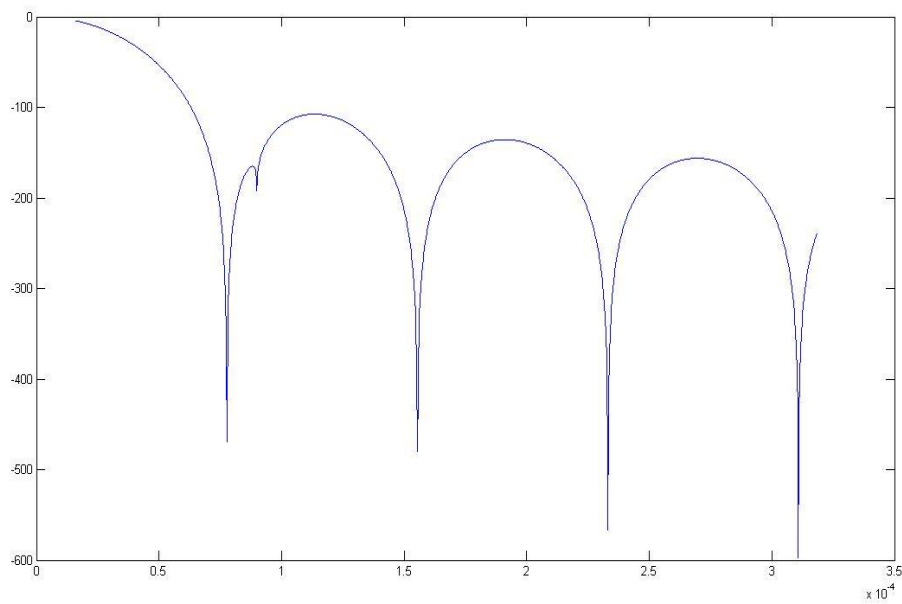
Solving the above equations and get the filter coefficients value of FIR linear phase filter order L=8. Now consider the CIC filter stage N=7 and decimation factor D is equal to 2048. For these following design parameters filter coefficients values are

$$a_0 = 0.0022, \quad a_1 = -0.0299, \quad a_2 = 0.2042, \quad a_3 = -0.8751, \quad a_4 = 2.3971$$

For these filter coefficients value compensated CIC filter and magnitude response of compensated CIC filter shown in figure 3.7 and 3.8 respectively.



**Figure 3.7: CIC Compensator for FIR filter order L=8**

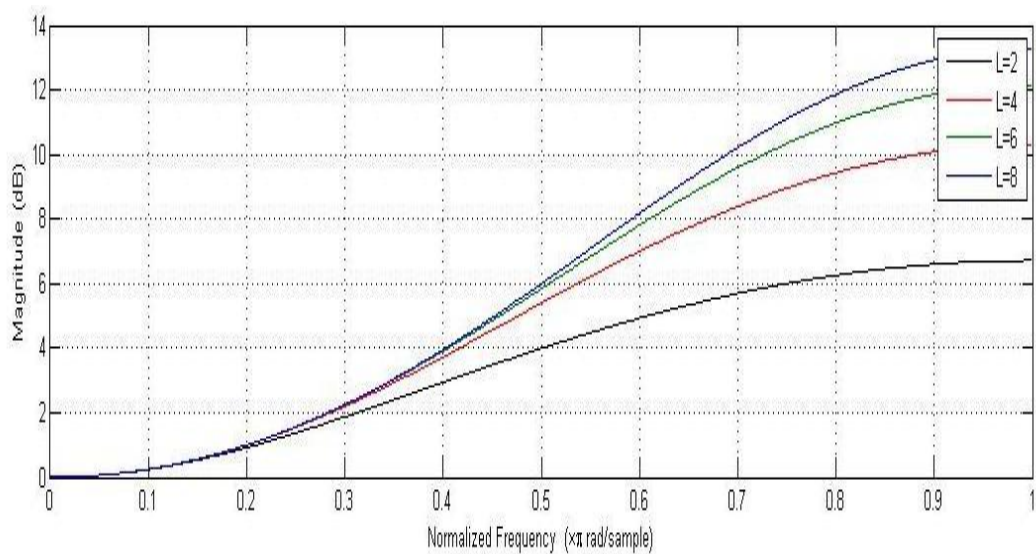


**Figure 3.8: Magnitude Response of Compensated CIC filter for L=8**

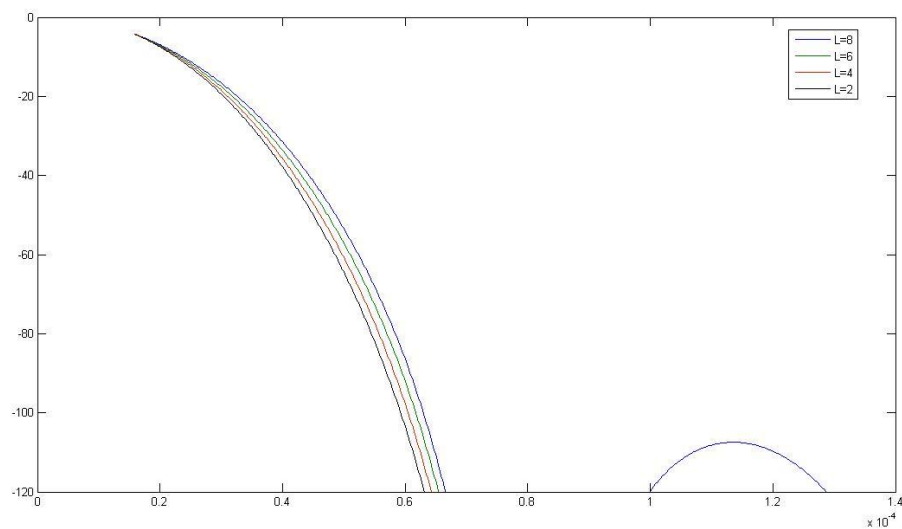
For the following design parameters figure 3.7 shows the CIC compensator response and figure 3.8 shows the magnitude response of the compensated CIC filter. The compensated CIC filter has same attenuation in the alias band as the CIC filter. For L=8 FIR filter order CIC compensator magnitude response is better than the FIR filter L=2, L=4 and L=6. In L=8 passband droop is less than the L=2, L=4 and L=6.

### 3.3 COMPARE THE CIC COMPENSATOR RESPONSE FOR DIFFERENT ORDER OF FIR FILTER:-

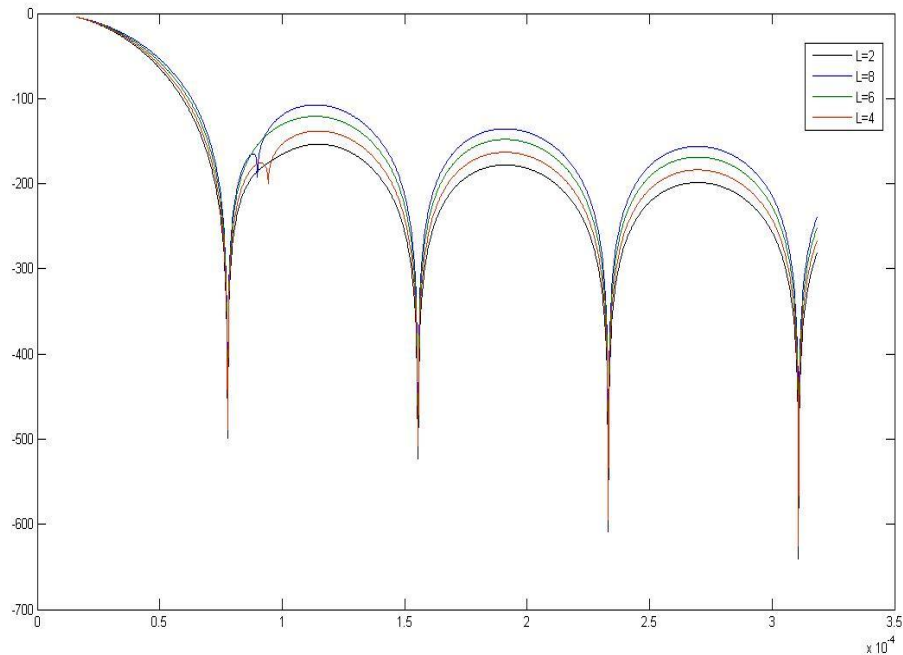
This chapter discuss about the brief design and implementation of maximally flat CIC filter. Consider the FIR filter order  $L=2,4,6,8$ . Now compare the compensated filter and compensated CIC filter magnitude response for the CIC filter stage  $N=7$  and decimation factor  $D$  is equal to 2048.as shown in figure 3.9, 3.10 and 3.11 respectively.



**Figure 3.9: Compensated Filter Response for Different Order of FIR Filter**



**Figure 3.10: Magnitude Response of Compensated CIC Filter**



**Figure 3.11: Full Magnitude Response of CIC Filter for Different order of FIR Filter**

From the above figure and the given design parameter value it concluded that as the number of FIR filter order increase passband droop problem improve in magnitude response of compensated CIC filter. The filter coefficients are obtained by solving a set of linear equations. The corresponding filter coefficients have closed form equations. The multiplier-less implementation complexity of the compensated CIC filter depends on the decimation factor and number of stages. So with the help of maximally flat criterion and FIR filter order improve the passband droop in compensated CIC filter.

**DESIGN OF COMPENSATOR FILTER FROM EXISTING ONES**

---

This chapter will discuss the new design of compensated CIC filter that is also based on maximally flat criterion but we will use different technique to get filter coefficients and improve the passband droop problem for same order of FIR filter. CIC filter is the simplest decimation filter but passband droop occurs in CIC filter magnitude response. For solving this problem, another compensation technique is used which is based on maximally flat error criterion. In this technique the compensator's coefficients are obtained by solving a linear system of equations, which is formed by using a straight forward procedure.

This section considers the design of CIC compensators, which is based on maximally flat approximation. In this method the ideal CIC compensator is discussed, later about closed form method for the design of compensators with a high number of coefficients is discussed.

**4.1 IDEAL CIC COMPENSATOR:-**

As from earlier discussion, the transfer function of CIC filter is given by [2]

$$H_{CIC}(z) = \left( \frac{1}{D} \frac{1-z^{-D}}{1-z^{-1}} \right)^N \quad (4.1)$$

Where D is the decimation factor of CIC filter and N is the number of CIC filter stages. From (4.1), the frequency response of the CIC filter is given by

$$H(e^{jw}) = e^{-j(D-1)Nw/2} H_R(w) \quad (4.2)$$

Where  $H_R(w)$  is real valued function given by

$$H_R(w) = \left( \frac{1}{D} \frac{\sin(Dw/2)}{\sin(w/2)} \right)^N \quad (4.3)$$

The CIC compensator is connected to the output of the CIC decimator. Therefore, the ideal amplitude response of the compensator is obtained as the inverse amplitude response of the decimator filter relative to the low sampling rate, as in

$$H_c(w) = \frac{1}{H_{CIC}\left(\frac{w}{D}\right)} \quad (4.4)$$

Put (4.4) into (4.3), the ideal compensator's response is obtained as

$$H_c(w) = \left[ D \frac{\sin\left(\frac{w}{2D}\right)}{\sin\left(\frac{w}{2}\right)} \right]^N \quad (4.5)$$

#### 4.2 MAXIMALLY FLAT COMPENSATORS:-

This section discuss the about the linear phase FIR filter with an arbitrarily high number of coefficients that approximates the response of ideal CIC compensator in a maximally flat response. The linear phase FIR compensation filter  $P(z)$  as

$$P(z) = \sum_{n=0}^L a_n z^{-n} \quad (4.6)$$

Where  $L$  is an even integer and is the order of  $P(z)$ ,  $a_n$ ,  $n=0, \dots, L$  are the filter coefficients, and  $a_n$  satisfies  $a_n = a_{L-n}$ . The corresponding frequency response is

$$P(e^{jw}) = e^{-jwL/2} P_R(w) \quad (4.7)$$

Where the real valued function  $P_R(w)$  is expressed as [33]

$$P_R(w) = a_{L/2} + 2 \sum_{n=0}^{L/2-1} a_n \cos(w(L/2 - n)) \quad (4.8)$$

To obtain the impulse response of the CIC compensator, error function is [34]

$$E(w) = P_R(w) - H_c(w) \quad (4.9)$$

To satisfy the maximally flat criterion at  $w=0$ , the first  $k$  derivative at  $w=0$  of the error function are set to 0, as in

$$\frac{d^k E(w)}{d\omega^k} = 0 \quad (4.10)$$

Now put the value of (4.9) into (4.10)

$$\left. \frac{d^k P_R(w)}{d\omega^k} \right|_{w=0} = \left. \frac{d^k H_c(w)}{d\omega^k} \right|_{w=0} \quad (4.11)$$

The response of  $H_c(w)$  and  $P_R(w)$  are even functions of  $w$ . Therefore, their odd derivative evaluated at  $w=0$  are zero.

The even order derivatives of  $H_c(\omega)$  when  $\omega \rightarrow 0$  can be expressed as

$$\frac{d^k H_c(\omega)}{d\omega^k} = \sum_{q=1}^{k/2} (2q-1) \left[ \frac{N}{2} \frac{d^{k-2q+1} F(\omega)}{d\omega^{k-2q+1}} \right]^q \quad (4.12)$$

Where

$$F(\omega) = \frac{1}{D} \cot\left(\frac{\omega}{2D}\right) - \cot\left(\frac{\omega}{2}\right) \quad (4.13)$$

It is clear that from (4.12) finding the derivatives of  $H_c(\omega)$  is reduced to finding the derivatives of  $F(\omega)$ . To obtain a closed form expression for the derivatives of  $F(\omega)$ , we use the power series expansion of the cotangent function. It is given by

$$\cot(x) = \frac{1}{x} - \sum_{m=1}^{\infty} \frac{2^{2m} |B_{2m}|}{2m!} x^{2m-1} \text{ for } |x| < \pi \quad (4.14)$$

Where  $B_{2m}$  are the Bernoulli numbers. The Bernoulli numbers are given in an explicit form, as in [36]

$$B_{2m} = \frac{(-1)^{m-1} 2(2m)!}{(2\pi)^{2m}} \zeta(2m) \quad (4.15)$$

Where  $\zeta(s)$  is the Riemann zeta function defined as [36]

$$\zeta(s) = \sum_{r=1}^{\infty} \frac{1}{r^s} \quad (4.16)$$

Now substituting the value of (4.14) into (4.13)

$$F(\omega) = \sum_{m=1}^{\infty} \frac{2^{2m} |B_{2m}|}{(2m)!} \left[ \left(\frac{\omega}{2}\right)^{2m-1} - \left(\frac{\omega}{2D}\right)^{2m-1} \right] \quad (4.17)$$

The derivatives of  $F(\omega)$  can be obtained as [37]

$$\frac{d^p F(\omega)}{d\omega^p} = \sum_{m=\lceil \frac{p+1}{2} \rceil}^{\infty} \frac{2^{2m-p-1} |B_{2m}|}{m(2m-p-1)!} \left[ \left(\frac{\omega}{2}\right)^{2m-p-1} - \frac{1}{D^{p+1}} \left(\frac{\omega}{2D}\right)^{2m-p-1} \right] \quad (4.18)$$

Where  $\lceil \cdot \rceil$  denotes the next higher integer value.

In (4.12), the order of derivative of  $F(\omega)$  is  $p = k - 2q + 1$ . Since  $k$  is even, it follows that  $p$  is odd. For an odd  $p$ ,  $m$  in (4.18) starts with  $(p+1)/2$  rather than with  $\lceil (p+1)/2 \rceil$ . Moreover, if  $\omega \rightarrow 0$ , only the term with  $m = (p+1)/2$  remains. Consequently, the odd order derivatives of  $F(\omega)$  when  $\omega \rightarrow 0$  are given by

$$\frac{d^p F(\omega)}{d\omega^p} = 2 \frac{|B_{p+1}|}{p+1} \left( 1 - \frac{1}{D^{p+1}} \right) \quad (4.19)$$

Finally, by substituting (4.19) into (4.12), we obtain

$$\frac{d^k H(\omega)}{d\omega^k} = \sum_{q=1}^{k/2} (2q-1) \left[ \frac{N |B_{k-2q+2}|}{k-2q+2} \left( 1 - \frac{1}{D^{k-2q+2}} \right) \right]^q \quad (4.20)$$

The odd indexed derivatives of  $P_R(w)$  evaluated at  $w=0$  are zero. Therefore, from (4.8) [32]

$$\left[ \frac{d^k P_R(w)}{dw^k} \right]_{w=0} = \begin{cases} 2(-1)^{k/2} D^k \sum_{n=0}^{L/2-1} (L/2 - n)^k a_n & k = \text{even} \\ 0 & k = \text{odd} \end{cases} \quad (4.21)$$

There are three equations. Put the value of (4.20) and (4.21) into the (4.11) and a linear system of equations can be formed to obtain the CIC compensator coefficients. A linear system of equations has a unique solution if the number of variables and the number of equations are equal. Using (4.11), the system is obtained in the form

$$AX = B \quad (4.22)$$

Where matrix A represents the equation (4.21), matrix X represents the  $a_n$  in the (4.21) and matrix B represents the equation (4.20). B is a column matrix.

$$A = [A_{u,v}], A_{u,v} = 2(-1)^u D^{2u} \left( v - \frac{1}{2} \right)^{2u} \quad (4.23)$$

Where

$$u = 1, 2, \dots, (L/2 - 1), \quad v = 1, 2, \dots, (L/2 - 1) \quad (4.24)$$

Column matrix B is

$$B = [B_u] \quad (4.25)$$

Where

$$B_u = \frac{d^{2u} H_c(\omega)}{d\omega^{2u}}, \quad u = 1, 2, \dots, (L/2 - 1) \quad (4.26)$$

The solution of system in (4.22) is found as

$$X = A^{-1}B \quad (4.27)$$

Note that X contains compensator's coefficient  $a_1, a_2, \dots, a_{(L/2-1)}$ , whereas coefficient  $a_0$  is still unknown. It is determine by setting  $E(\omega) = 0$  as  $\omega \rightarrow 0$ . Using (4.9), it follows that

$$P_R(0) = H_c(\omega) \quad (4.29)$$

By substituting (4.5) and (4.8) into (4.29), the value of coefficient  $a_0$  is obtained as

$$a_0 = 1 - 2 \sum_{n=1}^{(L/2-1)} a_n \quad (4.30)$$

Finally, the impulse response of the CIC compensator is formed as

$$h = [a_{(L/2-1)}, \dots, a_1, a_0, a_1, \dots, a_{(L/2-1)}] \quad (4.31)$$

### 4.3 CIC COMPENSATOR RESPONSE FOR DIFFERENT ORDER OF FIR FILTER:-

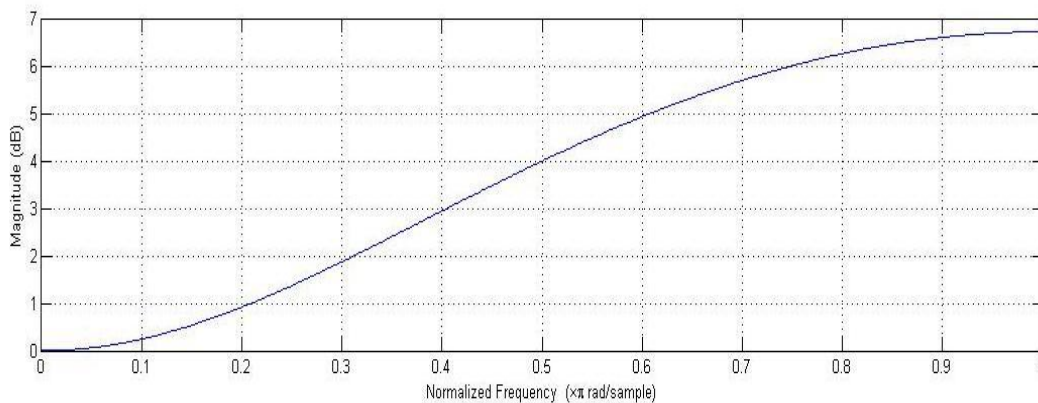
This section focuses on the design of the CIC compensator filter based on linear phase filter. The resulting magnitude characteristic of the compensated CIC filter exhibits a maximally flat characteristic at  $w=0$ . In this section compare the four order of FIR filter order and in the last compare the results.

For FIR linear phase filter order  $L=2$ , consider the CIC filter stage  $N=7$  and decimation factor  $D$  is equal to 2048. For these following design parameters filter coefficients values are

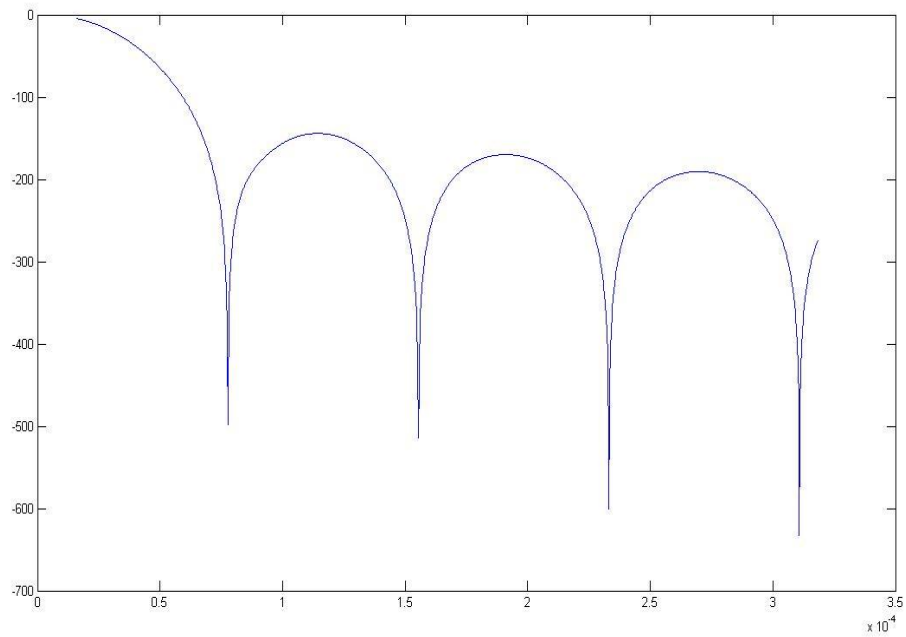
$$a_0 = -0.2917$$

$$a_1 = 1.5833$$

For these filter coefficients value compensated CIC filter and magnitude response of compensated CIC filter shown in figure 4.1 and 4.2 respectively.



**Figure 4.1: CIC Compensator for FIR filter order L=2**



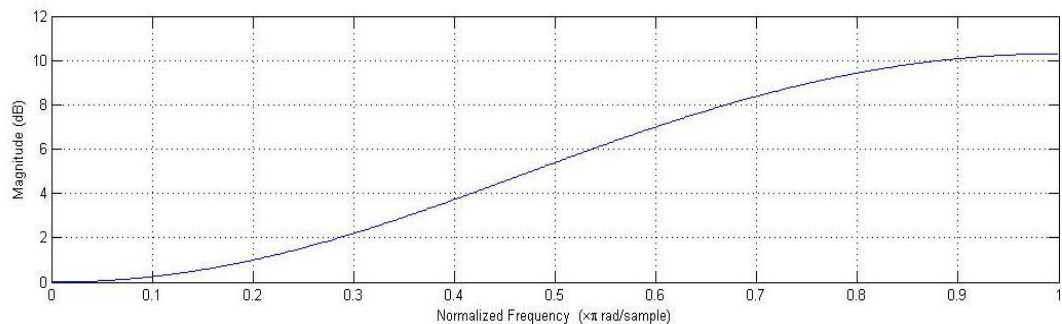
**Figure 4.2: Magnitude Response of Compensated CIC filter for L=2**

For the following design parameters figure 4.1 shows the CIC compensator response and figure 4.2 shows the magnitude response of the compensated CIC filter. The compensated CIC filter has same attenuation in the alias band as the CIC filter.

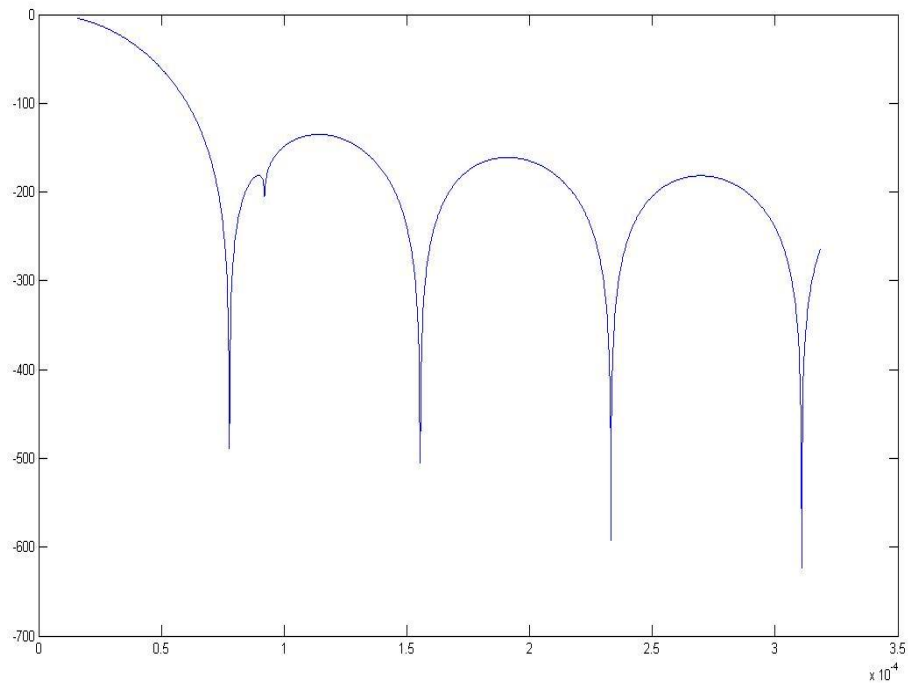
For FIR linear phase filter order L=4, consider the CIC filter stage N=7 and decimation factor D is equal to 2048. For these following design parameters filter coefficients values are

$$a_0 = 0.0693, \quad a_1 = -0.5687, \quad a_2 = 1.990$$

For these filter coefficients value compensated CIC filter and magnitude response of compensated CIC filter shown in figure 4.3 and 4.4 respectively.



**Figure 4.3: CIC Compensator for FIR filter order L=4**



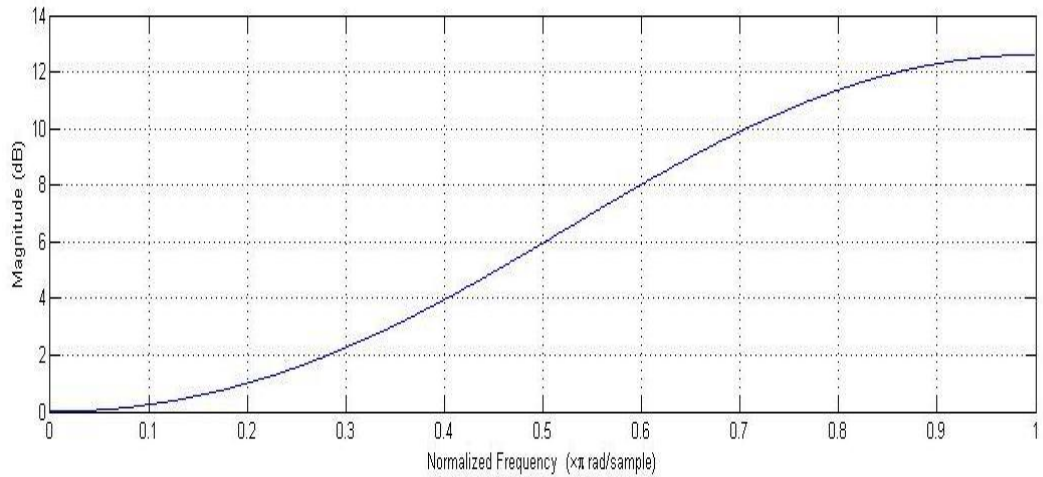
**Figure 4.4: Magnitude Response of Compensated CIC filter for L=4**

For the following design parameters figure 4.3 shows the CIC compensator response and figure 4.4 shows the magnitude response of the compensated CIC filter. The compensated CIC filter has same attenuation in the alias band as the CIC filter. For L=4 FIR filter order CIC compensator magnitude response is better than the FIR filter L=2. In L=4 passband droop is less than the L=2.

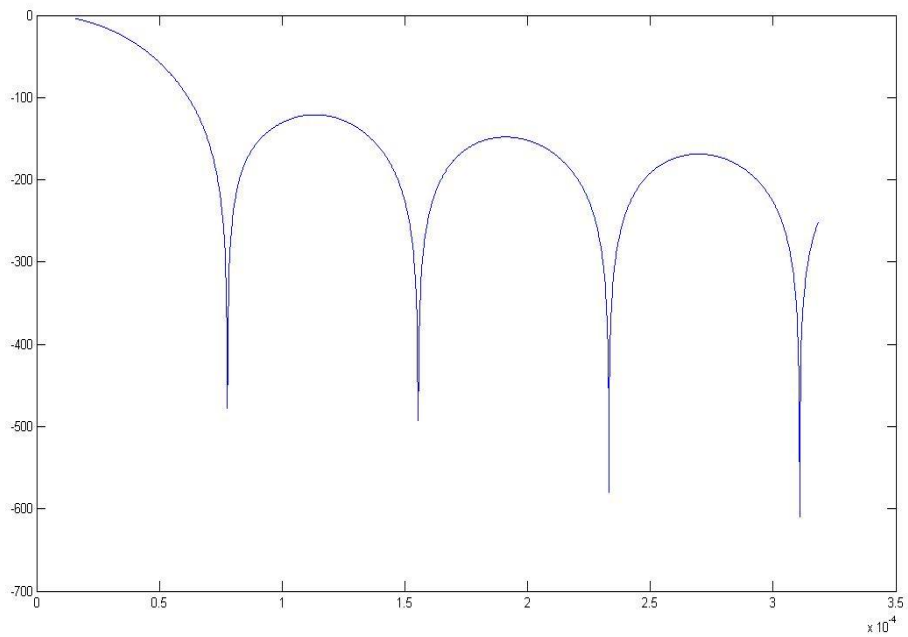
For FIR linear phase filter order L=6, consider the CIC filter stage N=7 and decimation factor D is equal to 2048. For these following design parameters filter coefficients values are

$$a_0 = -0.01555, \quad a_1 = 0.16256, \quad a_2 = -0.80196, \quad a_3 = 2.3099$$

For these filter coefficients value compensated CIC filter and magnitude response of compensated CIC filter shown in figure 4.5 and 4.6 respectively.



**Figure 4.5: CIC Compensator for FIR filter order L=6**



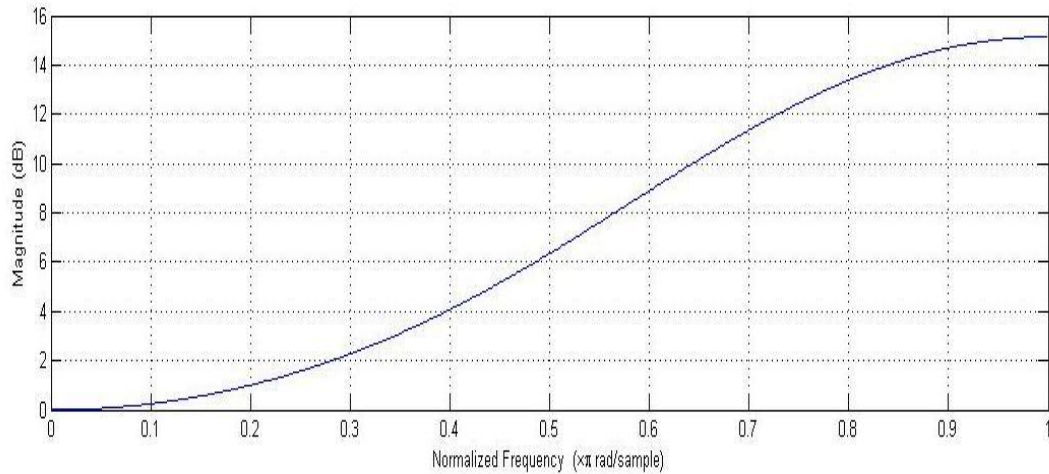
**Figure 4.6: Magnitude Response of Compensated CIC filter for L=6**

For the following design parameters figure 4.5 shows the CIC compensator response and figure 4.6 shows the magnitude response of the compensated CIC filter. The compensated CIC filter has same attenuation in the alias band as the CIC filter. For L=6 FIR filter order CIC compensator magnitude response is better than the FIR filter L=2 and L=4. In L=6 passband droop is less than the L=2 and L=4.

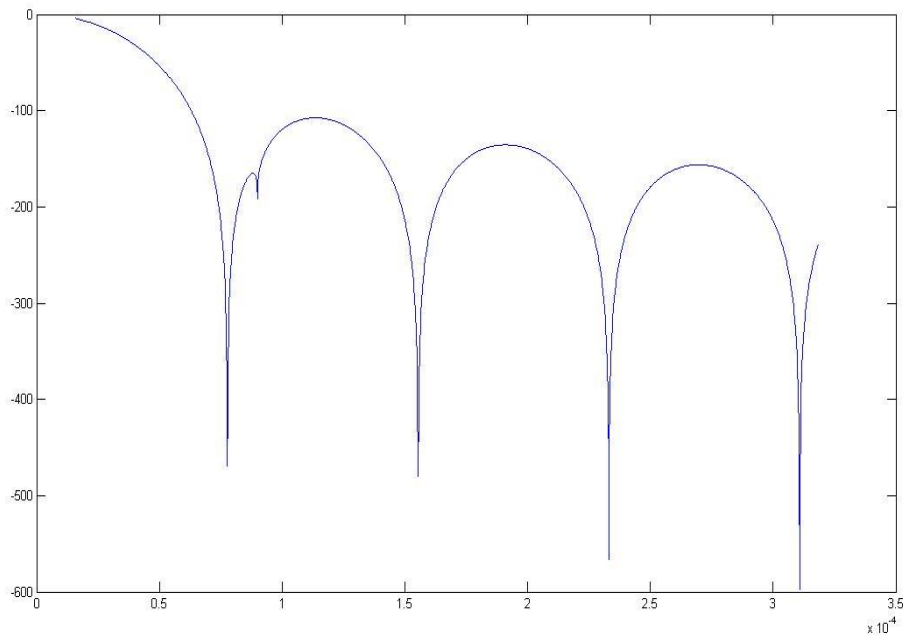
For FIR linear phase filter order  $L=8$ , consider the CIC filter stage  $N=7$  and decimation factor  $D$  is equal to 2048. For these following design parameters filter coefficients values are

$$a_0 = 0.0056, \quad a_1 = -0.0607, \quad a_2 = 0.3207, \quad a_3 = -1.1183, \quad a_4 = 2.7054$$

For these filter coefficients value compensated CIC filter and magnitude response of compensated CIC filter shown in figure 4.7 and 4.8 respectively.



**Figure 4.7: CIC Compensator for FIR filter order  $L=8$**

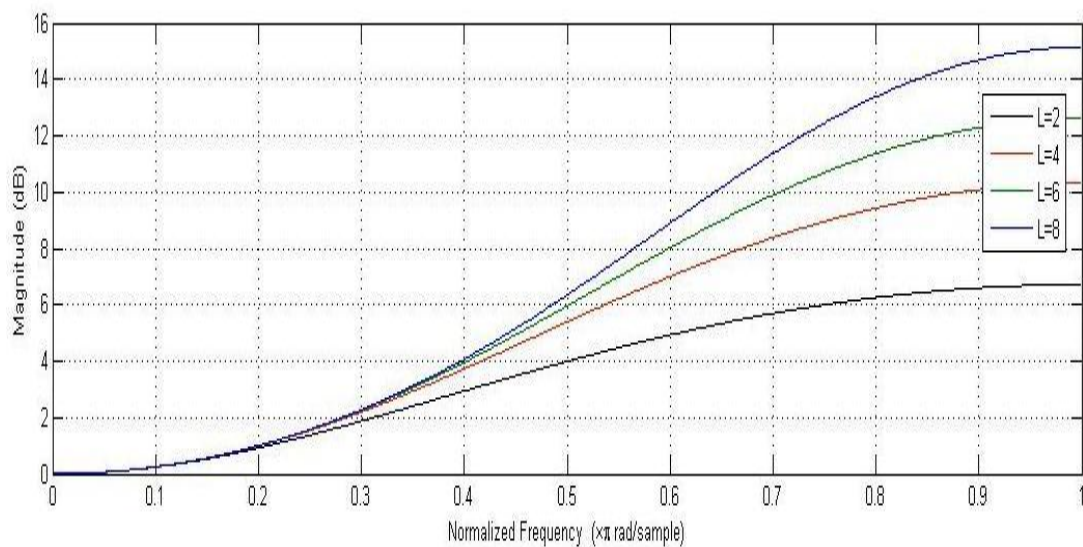


**Figure 4.8: Magnitude Response of Compensated CIC filter for  $L=8$**

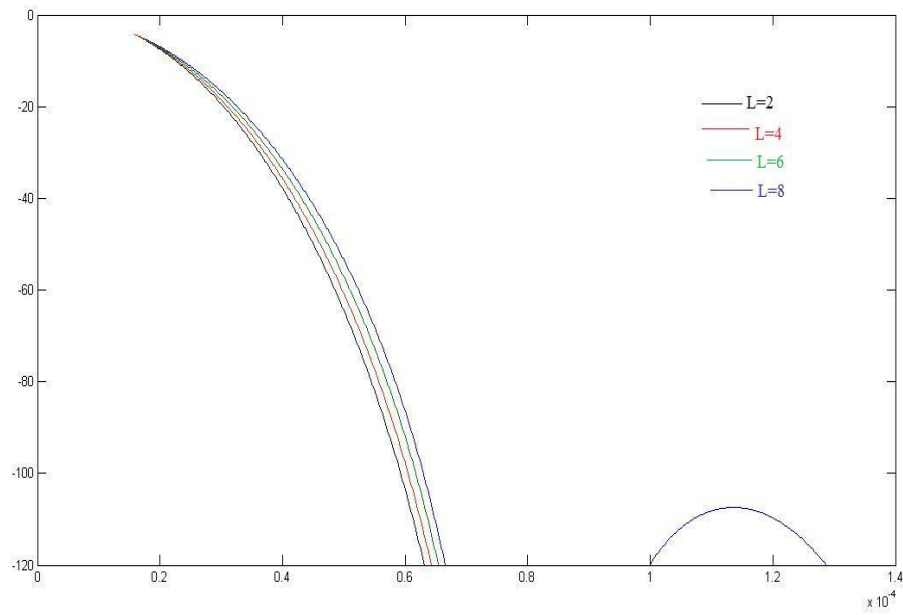
For the following design parameters figure 4.7 shows the CIC compensator response and figure 4.8 shows the magnitude response of the compensated CIC filter. The compensated CIC filter has same attenuation in the alias band as the CIC filter. For L=8 FIR filter order CIC compensator magnitude response is better than the FIR filter L=2, L=4 and L=6. In L=8 passband droop is less than the L=2, L=4 and L=6.

#### 4.4 COMPARE THE CIC COMPENSATOR RESPONSE FOR DIFFERENT ORDER OF FIR FILTER:-

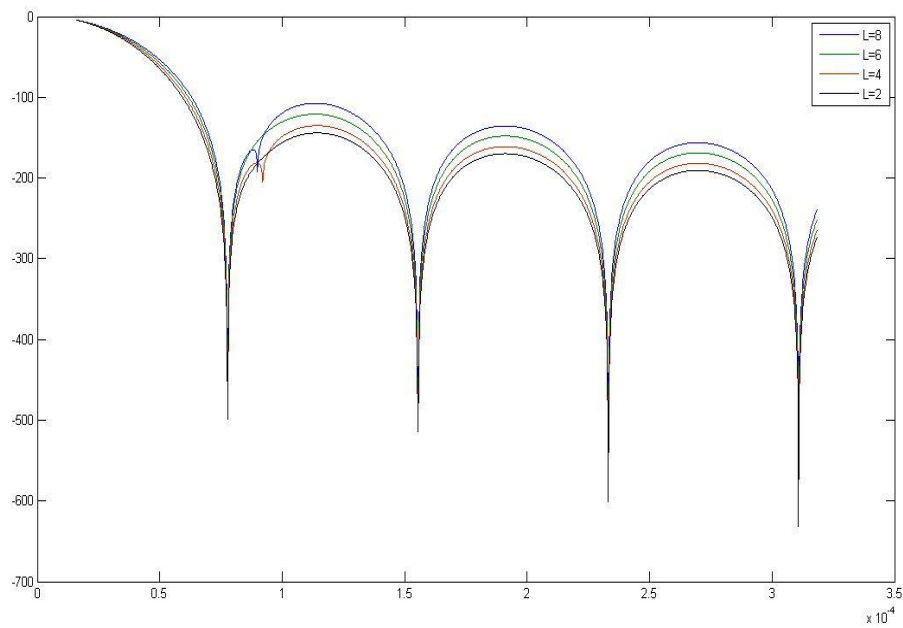
This chapter discussed about the brief design and implementation of maximally flat CIC filter. Consider the FIR filter order L=2,4,6,8. Now compare the compensated filter and compensated CIC filter magnitude response for the CIC filter stage N=7 and decimation factor D is equal to 2048.as shown in figure 4.9, 4.10 and 4.11 respectively.



**Figure 4.9: Compensated Filter Response for Different Order of FIR Filter**



**Figure 4.10: Magnitude Response of Compensated CIC Filter**



**Figure 4.11: Full Magnitude Response of CIC Filter for Different order of FIR Filter**

From the above figure and the given design parameter value it concluded that as the number of FIR filter order increase passband droop problem improve in magnitude response of compensated CIC filter. The filter coefficients are obtained by solving a

set of linear system of equations. The corresponding filter coefficients have closed form equations. The multiplier-less implementation complexity of the compensated CIC filter depends on the decimation factor and number of stages. So with the help of maximally flat criterion and FIR filter order improve the passband droop in compensated CIC filter.

Figure 4.9 shows the magnitude response of compensator obtained for the CIC filter. It is clear that the compensators with high number of coefficients approximate the ideal response better. Figure 4.10 and 4.11 show the magnitude response of compensated CIC filter. It is clear that method is suitable for both narrow band and wide band compensator.

The present method is based on closed form expression resulting in a straight forward design. The Bernoulli numbers are calculated using the zeta function. This expression can be easily implemented since the zeta function is a part of common tools for signal processing.

## CHAPTER 5

### RESULTS AND DISCUSSION

---

In this chapter the results of proposed design of CIC compensator are compared with the existing design. Method 1 is based on maximally flat criterion [32] that is the existing one design. In this method Leibniz rule is used but when solved the set of linear equation to get the filter's coefficients for high order of FIR filter order, complexity increase in solving the equations. Method 2 is also based on maximally flat criterion that is proposed design. In this method Bernoulli numbers and Riemann zeta function are used and to get filter's coefficients linear system of equation in matrix form is used. Due to this complexity is reducing to solve the equations.

Method 2 is better than method 1. In method 2 we get improved results. For FIR filter order  $L=2$  and  $L=4$  we get same filter coefficients. In FIR filter order  $L=2$  and  $L=4$  there are no improvement in compensated CIC filter passband results. But for FIR filter order  $L=6$  and  $L=8$  we get the different filter coefficients and get the improve magnitude response in compensated CIC filter.

For FIR linear phase filter order  $L=2$ , consider the CIC filter stage  $N=7$  and decimation factor  $D$  is equal to 2048. For both methods we get the same filter's coefficients which are

$$a_0 = -0.2917, \quad a_1 = 1.5833$$

There are no improvements in magnitude response of compensator and compensated CIC filter. For FIR linear phase filter order  $L=4$ , consider the CIC filter stage  $N=7$  and decimation factor  $D$  is equal to 2048. For both methods we get the same filter's coefficients which are

$$a_0 = 0.0693, \quad a_1 = -0.5687, \quad a_2 = 1.990$$

For these same filter coefficients value compensated CIC filter and magnitude response of compensated CIC filter are same. There are no improvements in magnitude response of compensated CIC filter.

As we discussed in the brief that for low FIR filter order  $L=2$  and  $L=4$ , we get same filter coefficients. In FIR filter order  $L=2$  and  $L=4$  there are no improvement in compensated CIC filter passband results. When FIR filter orders increase, we get the different filter coefficients and get the improve magnitude responses in compensated CIC filter.

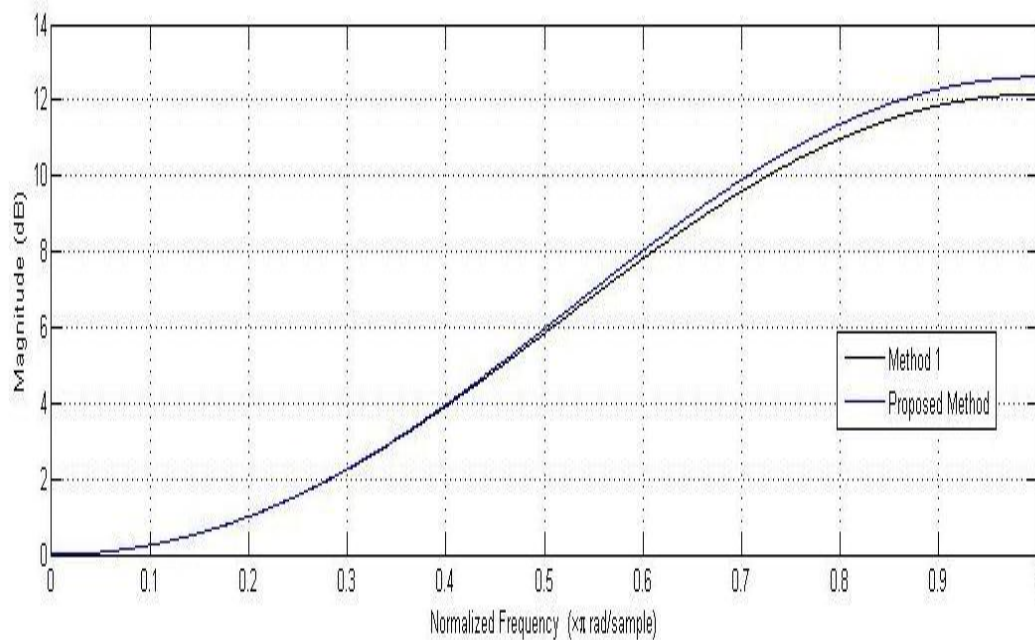
Now we will discuss the high order of FIR filter order. For FIR linear phase filter order  $L=6$ , consider the CIC filter stage  $N=7$  and decimation factor  $D$  is equal to 2048. For these following design parameters filter coefficients values of method 1 are

$$a_0 = -0.0122, \quad a_1 = 0.1423, \quad a_2 = -0.7512, \quad a_3 = 2.2423$$

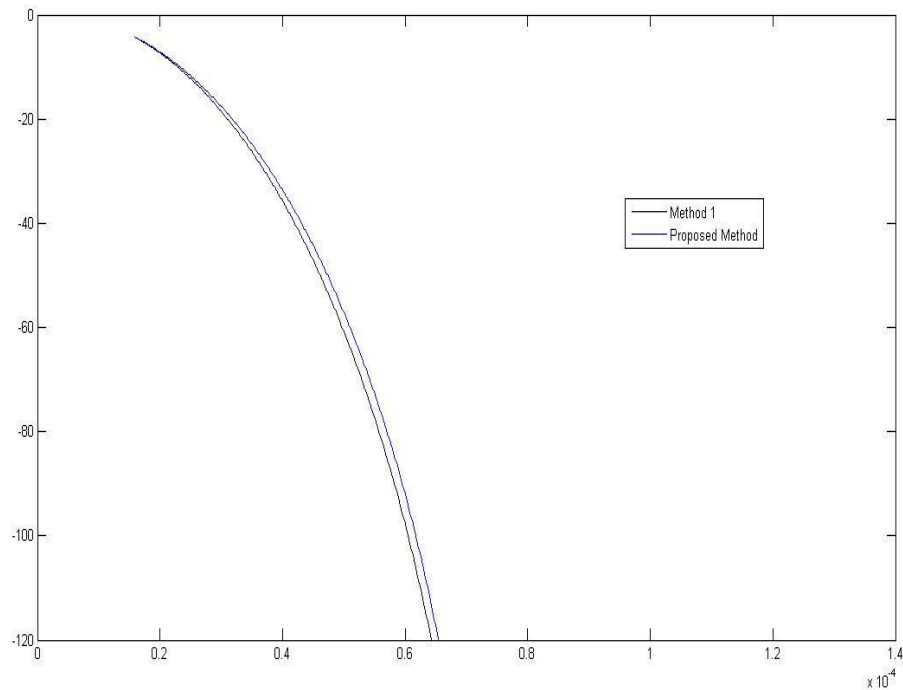
And for improved method 2 filter's coefficients are

$$a_0 = -0.01555, \quad a_1 = 0.16256, \quad a_2 = -0.80196, \quad a_3 = 2.3099$$

The filter's coefficients values of method 2 are better than method 1. For these filter coefficients values compare the magnitude response of compensated CIC filter and compensated CIC filter shown in figure 5.1 and 5.2 respectively.



**Figure 5.1: Magnitude Response of Compensator Filter for different method (L=6)**



**Figure 5.2: Magnitude Response of Compensated CIC filter for different method (L=6)**

For the following design parameters figure 5.1 shows the CIC compensator response and figure 5.2 shows the magnitude response of the compensated CIC filter. From the above figures it is clear that the magnitude response of compensator CIC filter is better in method 2. Passband droop is also seen less in method 2 comparing with method 1.

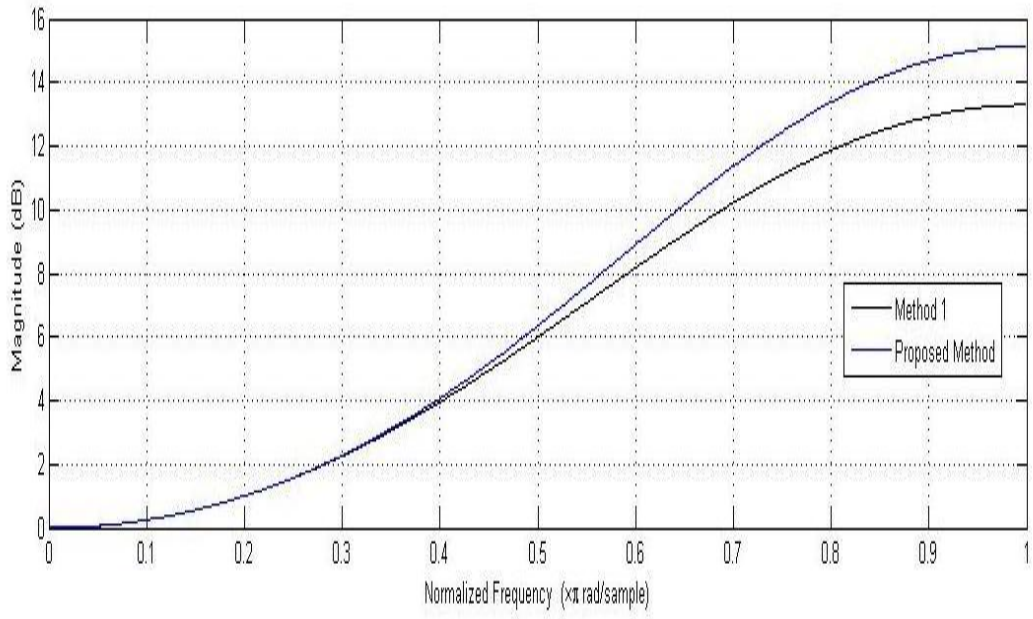
For FIR linear phase filter order  $L=8$ , consider the CIC filter stage  $N=7$  and decimation factor  $D$  is equal to 2048. For these following design parameters filter coefficients values of method 1 are

$$a_0 = 0.0022, \quad a_1 = -0.0299, \quad a_2 = 0.2042, \quad a_3 = -0.8751, \quad a_4 = 2.3971$$

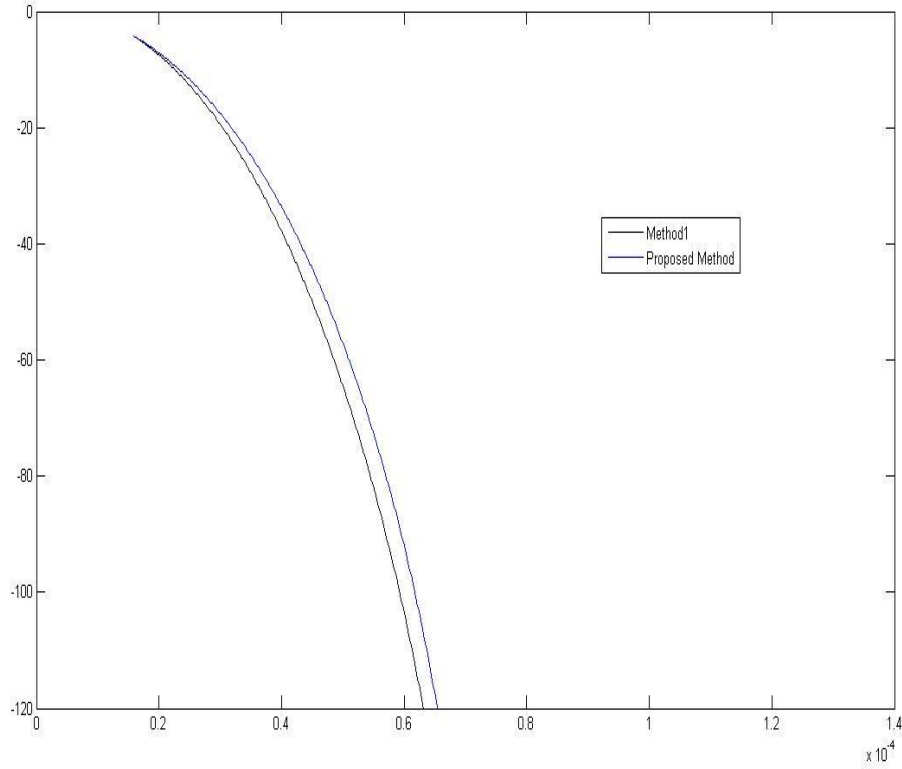
And for improved method 2 filter's coefficients are

$$a_0 = 0.0056, \quad a_1 = -0.0607, \quad a_2 = 0.3207, \quad a_3 = -1.1183, \quad a_4 = 2.7054$$

The filter's coefficients values of method 2 are better than method 1. For these filter coefficients values compare the magnitude response of compensated CIC filter and compensated CIC filter shown in figure 5.3 and 5.4 respectively.



**Figure 5.3: Magnitude Response of Compensator Filter for different method (L=8)**



**Figure 5.4: Magnitude Response of Compensated CIC filter for different method (L=8)**

For the following design parameters figure 5.3 shows the CIC compensator response and figure 5.4 shows the magnitude response of the compensated CIC filter. From the above figures it is clear that the magnitude response of compensator CIC filter is better in method 2. Passband droop is also seen less in method 2 comparing with method 1.

From the above discussion and the given design parameter value we conclude that as the number of FIR filter order increases, magnitude response of the CIC filter improves in respect of passband droop problem. Method 2 gives the better magnitude response of compensated CIC filter. The filter coefficients are obtained by solving a set of linear system of equations. The corresponding filter coefficients have closed form equations. The multiplier-less implementation complexity of the compensated CIC filter depends on the decimation factor and number of stages. So with the help of maximally flat criterion and FIR filter order improve the passband droop in compensated CIC filter. Method 2 is based on closed form expression resulting in a straight forward design. The Bernoulli numbers are calculated using the zeta function. This expression can be easily implemented since the zeta function is a part of common tools for signal processing.

#### 6.1:- CONCLUSION:-

CIC filter is the simplest decimation filter but its magnitude response has a high passband droop. For improving the passband and the transition band features of the CIC filter and improving the performance of CIC filter we can use many techniques, such as compensation filter cascaded with CIC filter, Sharpening technique, polyphase decimation FIR filter to achieve wide broadband compensation of the CIC filter and maximally-flat based compensator filter.

The droop can be reduced by modifying the original CIC structure or by connecting an additional filter called CIC compensator in cascade with the CIC decimator. The former approach is based on a technique called sharpening. Filter sharpening can be used to improve the response of a CIC filter. In most applications it is required to have a flat passband, otherwise the original signal may be destroyed. Unfortunately, the CIC filter alone suffers with a passband droop, which in many cases, cannot be accepted. The big droop is due the sinc-like characteristic of the filter. Hence, it is of a great interest to get a flat passband using a compensation filter. The compensation filter will take the form of the inverse of the CIC filter frequency response in the passband, and attenuate as much as possible in the stopband. This is the one way to improve the response of CIC filter.

In the latter approach, CIC compensators are designed to approximate the inverse amplitude response of the CIC filter. In recent years, several methods for the design of CIC compensators have been developed and now compensation is combined with a sharpening technique.

This brief compares the results of proposed design of CIC compensator with the existing design and gives the result on the basis of comparison. Method 1 is based on maximally flat criterion, which is discussed in chapter-3. In this method Leibniz rule is used but after solving the set of linear equation to get the filter's coefficients for high order of FIR filter order, complexity increase in solving the equations. Method 2 is also based on maximally flat criterion, which is discussed in chapter-4. In this method Bernoulli numbers and Riemann zeta function are used and to get filter's

coefficients linear system of equation in matrix form is used. Due to this complexity is reducing to solve the equations.

Method 2 is better than method 1. In method 2 we get improved results regarding passband droop. For FIR filter order  $L=2$  and  $L=4$ , we get same filter coefficients. In FIR filter order  $L=2$  and  $L=4$  there are no improvement in compensated CIC filter passband results. But for FIR filter order  $L=6$  and  $L=8$ , we get the different filter coefficients and get the improve magnitude response in compensated CIC filter. For higher order of FIR filter's coefficients method 2 give the better improvement results in magnitude response compare to method 1.

From the above discussion we conclude that as the number of FIR filter order increases, passband droop problem improve in magnitude response of compensated CIC filter. Method 2 gives the better magnitude response of compensated CIC filter. The filter coefficients are obtained by solving a set of linear system of equations. The corresponding filter coefficients have closed form equations. The multiplier-less implementation complexity of the compensated CIC filter depends on the decimation factor and number of stages. So with the help of maximally flat criterion and FIR filter order improve the passband droop in compensated CIC filter. Method 2 is based on closed form expression resulting in a straight forward design. The Bernoulli numbers are calculated using the zeta function. This expression can be easily implemented since the zeta function is a part of common tools for signal processing.

## **6.2 FUTURE SCOPE:-**

This research work have proposed and simulated the CIC filters with compensation for interpolation and decimation in the evolving field of communication systems for the different wireless standards and engineering applications. The proposed design of CIC compensator which is based on maximally flat criterion provides improved magnitude response of CIC filter. The application for CIC filters seems to be in areas where high sampling rates make multipliers an uneconomical choice and areas where large rate change factors would require large amounts of coefficient storage or fast impulse response generation. Further improvement in the compensator using another technique such as optimal sharpening, is also chance to get better magnitude response of CIC filter.

## REFERENCES

---

- [1] M. P. Donadio, "CIC Filter Introduction," *lowegian Publication*, 2000  
(<http://www.mikrocontroller.net/attachment/51932/cic2.pdf>)
- [2] E. B. Hogenaur, "An economical class of digital filters for decimation and interpolation," California : *IEEE Trans. on Acoustic, Speech and Signal Processing*, Vols. ASSP-29, NO.2, pp. 271-274, April 1981.
- [3] F. J. A. d. Aquino, C. A. F. d. Rocha and L. S. Resende, "Design of CIC Filters for Software Radio System," *IEEE*, pp. 225-228, 2006.
- [4] T. Hentschel and G. Fettweis, "Sample rate conversion for software radio," *IEEE Commun. Mag.*, vol. 38, no. 8, pp. 142-150, August 2000.
- [5] A. N. 455, "Understanding CIC Compensation Filters," *ALTERA*, April 2007
- [6] Proakis, John G. and Manolakis, Dimitris G. "Digital Signal Processing". s.l. : *Pearson Education*, 2007.
- [7] U. Meyer-Baese, "Digital Signal Processing with FPGA," *Springer*, May 2001.
- [8] R. E. Crochiere and L. R. Rabiner, "Multirate Digital Signal Processing", *Englewood Cliffs, New Jersey: Prentice-Hall*, 1983.
- [9] ] J. Kaiser and R. Hamming, "Sharpening the response of a symmetric nonrecursive filter by multiple use of the same filter," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-25, no. 5, pp. 415–422, Oct. 1977.
- [10] A. Y. Kwentus, Z. Jiang, and A. N. Willson, Jr., "Application of filter sharpening to cascaded integrator-comb decimation filters," *IEEE Trans. Signal Process.*, vol. 45, no. 2, pp. 457–467, Feb. 1997.
- [11] G. Jovanovic Dolecek and S. K. Mitra, "A new two-stage sharpened comb decimator," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 52, no. 7, pp. 1414–1420, Jul. 2005.
- [12] G. Jovanovic Dolecek and S. K. Mitra, "Simple method for compensation of CIC decimation filter," *Electron. Lett.*, vol. 44, no. 19, pp. 1162–1163, Sep. 2008.
- [13] G. Jovanovic Dolecek, "Simple wideband CIC compensator," *Electron. Lett.*, vol. 45, no. 24, pp. 1270–1272, Nov. 2009.
- [14] G. Jovanovic Dolecek and F. Harris, "Design of wideband CIC compensator filter for a digital IF receiver," *Digit. Signal Process*, vol. 19, no. 5, pp. 1204–1213, Sep. 2000.

- [15] S. Zhang, J. Qi and J. Bao, "The improvement of design for CIC Compensation Filter," *International Conference on Electronics, Communications and Control*, pp. 1712-1715, 2011.
- [16] G. J. Dolecek and S. K. Mitra, "A New Two-Stage CIC-Based Decimation Filter," *International Conference on Image and Signal Processing and Analysis*, vol. 55, pp. 218-223, sept 2007.
- [17] G. Jovanovic-Dolecek and S. K. Mitra, "Efficient Sharpening of CIC Decimation Filter," *International Conference on Acoustics, Speech, and Signal Processing*, pp. 385-388, 2003.
- [18] M. Li, S. Zhang and X. Diao, "Compensatin Method for the CIC Filter in Digital Down Converter," *International Conference on Electrical and Control Engineering*, pp. 1024-1027, 2011.
- [19] G. Stephen and R. Stewart, "High-speed sharpening of decimating CIC Filter," *International Electronics Letter*, vol. 40, no. 21, july 2004.
- [20] G. J. Dolecek and F. Harris, "On Design of CIC Decimation Filter with Improved Response," *International Conference on Communications, Control and Signal Processing*, pp. 903-908, 2008.
- [21] N. R. Karnati, K.-S. Lee, J. Carletta and R. Veillette, "A Power-Efficient Polyphase Sharpened CIC Filter for Sigma-Delta ADCs," *International Midwest Symposium on Circuits and System*, 2011.
- [22] G. Jovanovic-Dolecek and S. K. Mitra, "A New Two Stage Sharpened Comb Decimator," *IEEE Transactions on Circuits and Systems Part I*, vol. 52, no. 7, pp. 1414-1420, July 2005.
- [23] G. J. Dolecek and S. K. Mitra, "On Design of CIC Decimator Filter with Improved Response," *ISCCSP*, pp. 1072-1076, March 2008.
- [24] A. Fernandez-Vazquez and G. J. Dolecek, "Maximally Flat CIC Compensation Filter: Design and Multiplier less Implementation," *IEEE Trans on Circuits and Systems—II*, vol. 59, no. 2, pp. 275-279, Feb 2012.
- [25] G. J. Dolecek and M. Laddomada, "An Economical Class of Droop-Compensated Generalized Comb Filters: Analysis and Design," *IEEE Trans. on Circuits and Systems—II*, vol. 57, no. 4, pp. 113-117, April 2010.
- [26] G. J. Dolecek and S. Mitra, "Simple Method for Compensation of CIC Decimation Filter," *International Electronics Letter*, vol. 44, no. 19, September 2008.

- [27] Y. Gao, L. Jia and H. Tenhunen, Hannu, "An Improved Architecture and Implementation of Cascaded Infigrator-Comb Decimation Filters," *IEEE Pacific Rim Conference on Communications, Computers and Signal Processing*, pp. 317-320, 1999.
- [28] R.K.Singh, Richa and A. Katiyar, Amit, "Design and Implementation of CIC Based Decimation filter For Improved Frequency Response." *International Conference on Emerging Trends in Electronic and Photonic Devices & Systems*, pp. 236-240, 2009.
- [29] W. A. Abu-Al-Saud and G. L. Stuber, "Modified CIC Filter for Sample Rate Conversion in Software Radio Systems," *IEEE Signal Process. Letters*, vol. 10, pp. 152–154, May 2003
- [30] David Ernesto Troncoso Romero, Massimiliano Laddomada, and Gordana Jovanovic Dolecek, "Optimal Sharpening of Compensated Comb Decimation Filters: Analysis and Design" in *Hindawi Publishing Corporation Scientific World Journal* Volume 2014, Article ID 950860
- [31] T. Yoshida, Y. Sugiura and N. Aikawa, "FIR Band-Pass Differentiators with Flat Passband and Equiripple Stopband Characteristic" in *Signal Processing Conference (EUSIPCO)*, pp. 701-705, 2014.
- [32] G. J. Dolecek and A. Fernandez-Vazquez, "Maximally Flat CIC Compensation Filter: Design and Multiplier less Implementation," *IEEE Trans on Circuits and Systems—II*, vol. 59, no. 2, pp. 275-279, Feb 2012.
- [33] S. K. Mitra, "Digital Signal Processing: A Computer Based Approach", 3rd ed. *New York: McGraw-Hill*, 2006
- [34] A. Fernandez-Vazquez and G. Jovanovic-Dolecek, "A general method to design GCF compensation filter," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 56, no. 5, pp. 409–413, May 2009.
- [35] I. W. Selesnick, "Low-pass filter realizable as all-pass sums: Design via a new flat delay filter," *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.*, vol. 46, no. 1, pp. 40–50, Jan. 1999.
- [36] I. S. Gradshteyn and I. M. Ryzhik, "Table of Integrals, Series, and Products. Amsterdam", *the Netherlands: Elsevier*, 2007.
- [37] G. Molnar and M. Vucic, "Closed-Form Design of CIC Compensators Based on Maximally Flat Error Criterion", *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 58, no. 12, pp. 409–413, December 2011.

## LIST OF PUBLICATIONS

---

- [1] A. Singhal and S. Sharma, "Sharpening of CIC Filter Based on Maximally Flat Error Criterion," communicated in Elsevier, May 2015