

A Thesis report on

**KINEMATIC STUDY AND TRAJECTORY TRACKING OF A
FIVE-DOF SPATIAL MANIPULATOR**

Submitted in the partial fulfillment of requirement for the award of degree of

**MASTER OF ENGINEERING
IN
CAD/CAM ENGINEERING**

Submitted by

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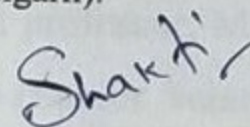
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July, 2015

DECLARATION

I hereby declare that the seminar report entitled "KINEMATIC STUDY AND TRAJECTORY TRACKING OF A FIVE-DOF SPATIAL MANIPULATOR" is an authentic record of my study carried out as requirement for the award of degree of **Master of Engineering in CAD/CAM Engineering** during 4th semester (Jan 2015 - July 2015) at Thapar University, Patiala and CSIR-CSIO, Chandigarh, under the joint supervision of **Dr. Ashish Singla** (Assistant Professor, Mechanical Engineering Department) and **Mr. Sanjeev Soni** (Manager, Bio Medical Instrumentation Department, CSIR-CSIO, Chandigarh).

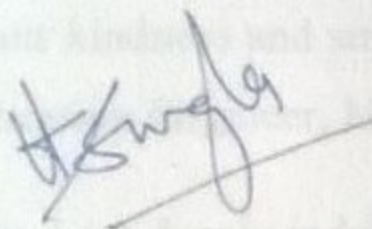
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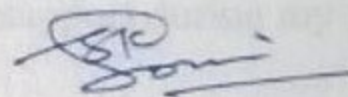
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The word 'Robot' has always been very exciting to me, like every other guy, as the world of robotics is full of possibilities. My interest towards robots increased when I watched a movie named 'Robot'. During second semester of my M.E., I got to study a subject on Robots, which acted like an introduction to the world of robots. It was back in Aug. 2014, when I got a chance to work under in the field of robotics, under the guidance of **Dr. Ashish Singla**, a person with extremely well conceptual knowledge, innovative and creative teaching style, a great speaker and motivator.

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ABSTRACT

With the development in technology, more and more automation systems and robotic manipulators are used in place of the conventional machines. These systems and robots are having complex shapes and structures. In the past many methods like D-H parameter method, S-U notation method etc. have been used for the kinematic analysis of such robotic manipulators. The scope of this thesis is limited to kinematic study and trajectory tracking of the spatial manipulators. It has been found in the literature that the pioneer work done by Denavit and Hartenberg to develop the kinematic model of a serial manipulator, has its own limitations. A couple of ambiguities are reported by different researchers in the past. In this thesis, an ambiguity of D-H parameter is observed. For complex and spatial structure, conventional methods like D-H are not able to develop the kinematic models. A spatial robot having five degree of freedom is discussed and ambiguities of D-H parameter method are highlighted. Improvements have been made to the existing D-H method, by using concept of *Dummy Frames*. These dummy frames remove the positional and orientational inconsistency in the kinematic study of Spatial Manipulator. Trajectory tracking of the spatial robotic manipulator is done using a software named *SimWise 4D*.

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LIST OF SYMBOLS

- a_i = link length parameter
 α_i = link twist parameter
 d_i = joint offset parameter
 θ_i = joint angle parameter
 x_i = unit vector along x - axis
 y_i = unit vector along y - axis
 z_i = unit vector along joint axis (z - axis)
 i = joint axis
 O_i = origin of i^{th} link-frame
 u, v, w = unit vectors of frame , at proximal end of a link
 x, y, z = unit vectors of frame , at distal end of a link
dof = degree-of-freedom
 $\alpha_i, \beta_i, \theta_i$ = joint variables
 ${}_{i-1}^i T$ = mapping of $(i - 1)^{th}$ frame to i^{th} frame
 R_x = rotation about x - axis
 R_y = rotation about y - axis
 R_z = rotation about z - axis
 q_i = joint variable
 \mathbb{R}^3 = cartesian space
 \mathbb{R}^n = joint space
 p = position vector
 a, R = fundamental rotation matrix
 A_1 = homogeneous coordinate transformation matrix
 x_1, y_1 = point vectors in \mathbb{R}^3
 p_1, p_2 and p_3 = components of position vector p

η^T = perspective vector

σ = non-zero scale factor

t_i = amount of translation along i^{th} unit vector of frame {a}

t_j = amount of translation along j^{th} unit vector of frame {a}

t_k = amount of translation along k^{th} unit vector of frame {a}

c_i = $\cos \theta_i$; $i = 1$ to n .

s_i = $\sin \theta_i$; $i = 1$ to n .

c_{ij} = $\cos (\theta_i + \theta_j)$

c_{ijk} = $\cos (\theta_i + \theta_j + \theta_k)$

s_{ij} = $\sin (\theta_i + \theta_j)$

s_{ijk} = $\sin (\theta_i + \theta_j + \theta_k)$

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Chapter 1

Introduction

1.1 What is a Robot?

Robot is a software controllable mechanical device, having multiple degrees of freedom and can be used for performing special tasks. A robot can also be defined as a mechanical or virtual artificial agent, usually an electro-mechanical machine that is guided by a computer program. A robot is an automatically operated machine that replaces human effort, though it may not resemble human beings in appearance or perform functions in a humanlike manner.

Robot can also be described as a "machine that looks like a human being and performs various complex acts (as walking or talking) of a human being", or a "device that automatically performs complicated often repetitive tasks", or a "mechanism guided by automatic controls".

So, overall it can be said that robots can perform a variety of tasks easily. They just need to be programmed according to our need. So, robots can be a very good helping hand in our lives, if used properly.

1.2 Applications of Robots

Earlier, there were times when the robots were not so advanced. In those days, robots were limited to some specific tasks only. But now a day, with the advancement of technology, robots have found their application in almost every field. Now days, robots are used in industry, medical field, military services etc.

On application basis, robots can be classified in following categories.

1.2.1 Industrial Robots

Industrial robots are robots used in an industrial manufacturing environment. Usually these are articulated arms specifically developed for such applications as welding, material handling, painting and others. If judged purely by application, this type could also include some automated guided vehicles and other robots.

As shown in Fig. 1.1, the robot is used for pick and place operation in industry. It has a gripper attached at the end effector, which is used to hold the object. In industry, there are many such tasks which are not easy for humans, as there is some risk to life. Such



Fig. 1.1: Industrial Robot used for pick and place operation [F1].

kind of tasks are easily done with the help of a robot. Another reason for using the robots in the industry is the accuracy and capacity to work for prolonged hours. The use of robots for the industrial applications is increasing day by day due to its advantages and growing need of good quality as well as large quantity of a product.

1.2.2 Domestic or Household Robots

Domestic robots are used at home for households. This type of robots includes many quite different devices such as robotic vacuum cleaners, robotic pool cleaners, sweepers, gutter cleaners and other robots that can do different chores. Also, some surveillance robots could be regarded as household robots if used in that environment.

1.2.3 Medical Robots

This is one of the most emerging sector where robots are playing a huge role. Earlier, there were times when surgeries were performed manually. The patient had to undergo lot of pain and it used to take around 15-20 days in coming back to normal life for the patient. But now doctors are doing the same surgery with the help of a robotic system. This robotic system consist of a surgeon side manipulator, a patient side manipulator, a camera and display screen. Doctor just control the patient side manipulator using surgeon side manipulator or some control system provided. The camera shows the movement of the patient side manipulator to the doctor in the display screen. Camera used is of very small size and can capture very high quality

photos and videos, thus enabling the doctor to zoom at microscopic levels to clearly see and analyze the patient side. Such surgeries are also called minimal invasive surgeries, as shown in Fig. 1.2. It takes around 2-5 days for the patient to come back to his/her normal life. Also, the pain during the surgery is also very less as compared to the classical method of surgery. Minimal Invasive surgery is costlier than a simple surgery but due to its benefits, it has become quite popular now days.



Fig. 1.2: Medical robotic system [F2].

Medical robots now days have become more popular, due to their increased accuracy and ease of use. Medical robots have made the complex surgeries made look easy and very accurate. Minimally invasive surgeries are also possible due to the medical robots. In minimally invasive surgery, surgeons use a variety of techniques to operate with less injury to the body of patient than with open surgery. In general, it is safer than open surgery and allows you to recover faster and heal with less pain and scarring during and after surgery. Minimally invasive surgery is usually done on an outpatient basis or requires only a short hospital stay. So, medical robots help a lot in a minimal invasive surgery and reduce the time taken to being normal.

Apart from that, medical robots help in performing very complex surgeries with very high accuracy. In such surgeries, the operations performed on the patient are controlled by a control panel which is controlled by the doctor. The movement of the robot is shown in the display panel. It leads to a clear understanding of operations and the motion of the robot is controlled very easily. Also, the degree of freedom of the robot is more than a human doctor. So, it

becomes easy for the robot to move in all the directions than a human doctor or the conventional method of surgery.

Now days, multiple surgeries are also performed with the help of the medical robots. A doctor sitting thousands of kilometers away from the patient can perform surgery on the patient with the help of advance technologies in the field of medical robotics.

1.2.4 Military Robots

Robots have find wide applications in military. In military, robots are used in bomb disposal, different transportation activities, reconnaissance drones. They can also be used in law enforcement, search and rescue and other related fields.



Fig. 1.3: DRDO Daksh [F3].

Indian army has a Remote Operated Vehicle named 'DAKSH', which is developed by DRDO, India (Defense Research and Development Organization), as shown in Fig. 1.3.

1.2.5 Space Robots

Space robots are the robots sent in to the space to gather some data, images etc. which can help the scientists to carry out further research. It includes robots used on the International Space Station. With such robots, scientists can do their work sitting in their labs and controlling the

robots directly. Robots can go in any kind of environment and collect data. Normally, power supply is given to the robots with the help of solar panels fitted on them, but now days, radioisotope power generators are also used. Example – Mars Curiosity Rover.

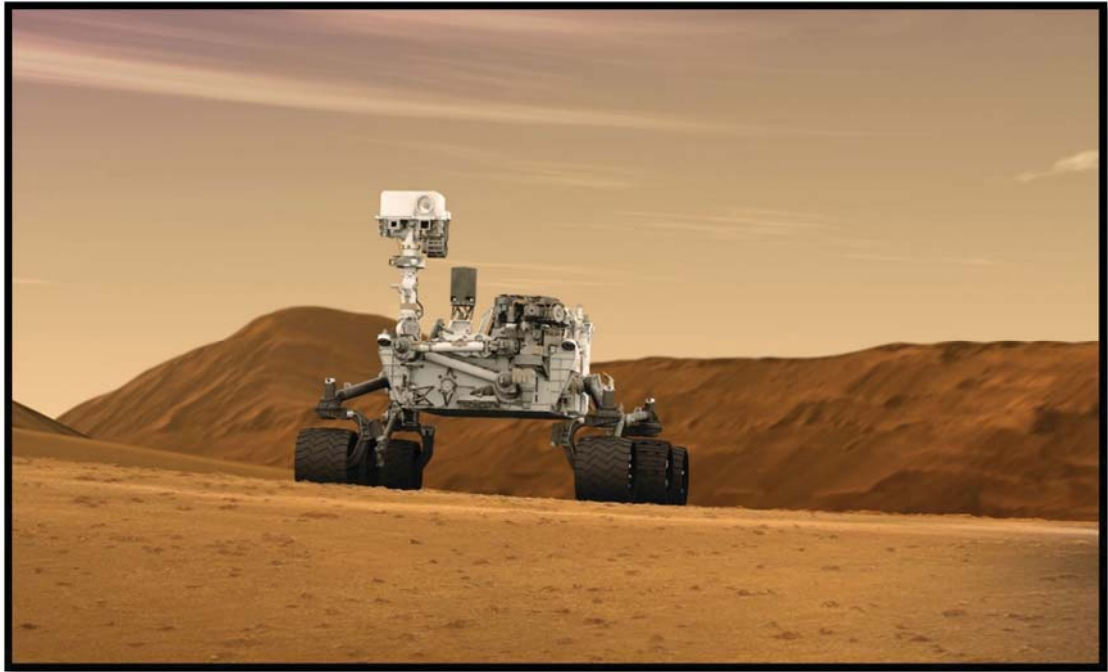


Fig. 1.4: Mars Curiosity Rover [F4].

Mars Curiosity Rover is bigger than a small car, equipped "standard" with six-wheel rocker-bogie suspension and multiple camera systems, and its power supply doesn't rely on solar panels. It uses a radioisotope power generator so that it can roam longer and farther. It can travel to more interesting places than previous missions. It consists of an expansive suite of science instruments named Sample Analysis at Mars, designed to analyze samples of material collected and delivered by the rover's arm.

1.2.6 Hobby and Competition Robots

It includes the robots that are created for hobby or the competition purposes. Line followers, sumo-bots, robots made just for fun and robots made for competition. Such kinds of robots are very popular now days. It also includes the robot football and some other games also. In the line follower robot, a robot follows the pre-defined trajectory.

1.3 Classification of Robots

There are number of ways on the basis of which robots can be classified in to different categories. Here, some important classifications are discussed one by one.

The axes of first three joints are termed as *major axis* and axes of all other joints are termed as the *minor axes*. It is basically the major axes that determine the position of the end effector. The minor axes determine the orientation of the tool or the end effector. Robots can also be classified according to their work envelope geometry.

1.3.1 Cartesian Coordinate Robots

All the major axes are prismatic or have linear motion along an axis.

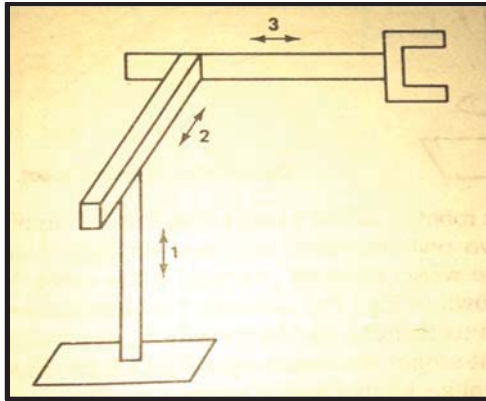


Fig. 1.5: Cartesian coordinate robots [7]

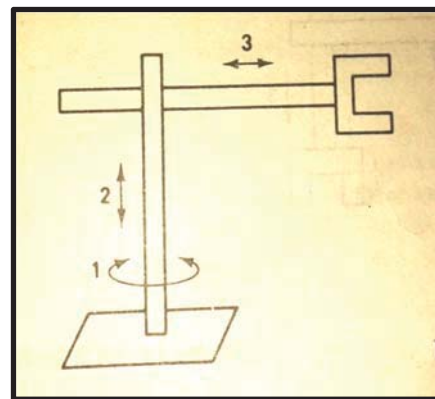


Fig. 1.6: Cylindrical coordinate robots[7]

As shown in the Fig. 1.5, all the first three links have linear motion about the major axis. These joints having such linear motion are known as prismatic joints and such a robot is known as Cartesian robot.

1.3.2 Cylindrical Coordinate Robots

As shown in Fig. 1.6, cylindrical robots have first joint as revolute joint and all other joints are prismatic joints. Such a robot is called cylindrical coordinate robot.

1.3.3 Spherical Coordinate Robots

If second joint of the cylindrical coordinate robot is replaced with revolute joint, then the robot is known Spherical Coordinate Robot. As shown in Fig. 1.7, spherical coordinate robot has first two joints as revolute joints and the third joint is prismatic joint.

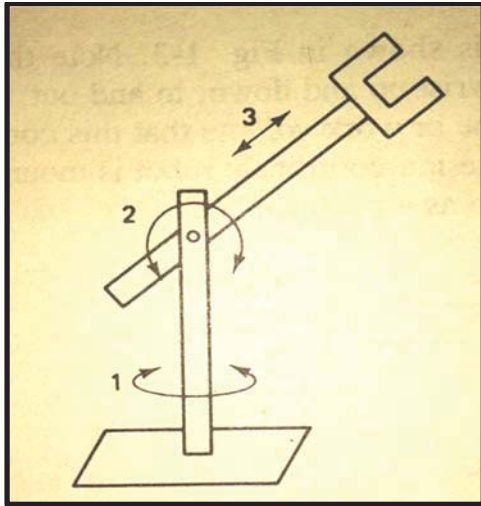


Fig. 1.7: Spherical coordinate robot [7]

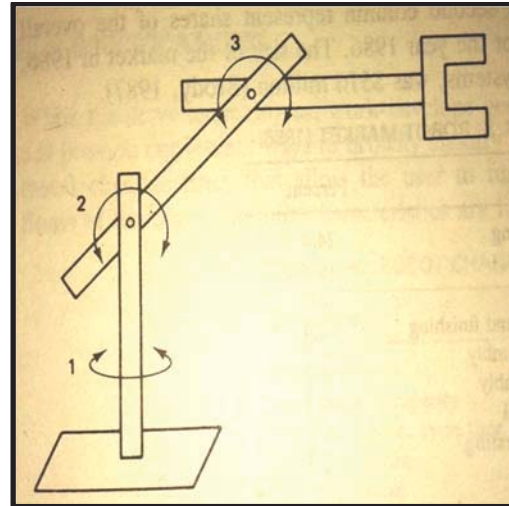


Fig. 1.8: Articulated coordinate robot [7]

1.3.4 Articulated Coordinate Robots

In the articulated robot, all the three joints are revolute joints and they are free to rotate about the major axes. An example of the articulated robot is shown in Fig. 1.8. Here, the first revolute joint swings the robot back and forth about a vertical base axis. The second joint pitches the arm up and down about a horizontal shoulder axis. The third joint pitches the forearm up and down about a horizontal elbow axis. These motions create a complex work envelope. Articulated robots are also known as *revolute robots*.

1.4 Kinematics of Robot

Kinematics of robot includes the study of position and orientation of the different joints of the robot. Different links and joints are modelled in to geometry as per their dimensions. And then suitable methods to calculate the desired positions, orientations and the link angles will be used. Kinematics of robots can be done in 2 ways, Forward and Inverse kinematics. Let us discuss both kind of kinematics in detail one by one.

1.4.1 Forward Kinematics

In the forward kinematics of the robots, calculation of positions and orientations of the robot using the joint angles, joint lengths, link angles and link lengths is done. First of all, robot is represented in to a geometrical model. Then, coordinate frames are assigned at each joint. Normally distal and proximal D-H methods are used to assign the coordinate frames.

1.4.2 Inverse Kinematics

Inverse kinematics of a robot involves the calculation of joint angles and joint distance of a robot at a particular position and orientation. In the Inverse kinematics, position and orientation of the end effector is already known.

This is very helpful in path tracking or trajectory tracking of the robot. In trajectory tracking, joint parameters at different points of the desired trajectory are calculated, normally with respect to time. Then, Joint and Link parameters are given as input to make the robot follow the desired trajectory.

1.5 Scope of Thesis

In this thesis, the kinematic study of variety of robotic manipulators has been discussed. Simple structures have been discussed first, thereafter moving towards some complex structures. Later on, it has been concluded that the dealing with spatial and complex structures, with conventional methods is not feasible. It can be concluded that kinematic study of a spatial and complex robot is a challenging task, and this is our main focus in this thesis. A software named *SimWise 4D* has been used to study the trajectory tracking of a Spatial Manipulator.

1.6 Organization of the Thesis

The organization of the thesis is as given below:

In **Chapter 2**, different methods discovered or used in the past for studying the kinematics of robotic manipulators have been discussed. It includes the study of structures of different manipulators, procedure of different methods used and their final results and comments. All these methods have been presented in an order as per their development in the past.

In **Chapter 3**, the forward kinematic study of some general manipulators is presented. The pioneer work, referred as Denavit and Hartenberg (D-H) parameter method, is used to describe the structure of some general robotic structures. Thereafter, suitable examples for two variants of D-H parameter method, i.e. distal and proximal have been presented to get the basic idea and procedure of kinematic study and model formulation.

In **Chapter 4**, Forward kinematic study of a robotic manipulator present at CSIR-CSIO has been presented using the D-H parameter method. Ambiguities of the D-H method have been highlighted, in case of a spatial robot. The problems faced with this method has been solved by making improvements to the method. Concept of *dummy frames* has been introduced.

In **Chapter 5**, Trajectory tracking of spatial manipulator has been done using *SimWise 4D*. The spatial manipulator is made to follow a straight line. All this is done by using the software *SimWise 4D*.

In **Chapter 6**, the major observations and work of this thesis are summarized. Future directions of the present work are also discussed.

Chapter-2

Literature Review

2.1 Introduction

Robotics has been a topic of discussion and research since a long time. Lot of research has been carried out in the past on robot's kinematics, dynamics, control, human robots etc. In this section, a brief discussion on the work done in the field of kinematics of robot has been done. Robots now days are involving in day to day life. Apart from that, robots have find a place in industries, medical surgeries and also in many other areas due to their accuracy, ability to work consistently without any fatigue. So, lot of researchers have worked on the methods to solve the kinematics of the robots.

Earlier, the methodologies developed for a simple configuration like a serial robot. Then, with the time passing on, researchers carried out vast research on different complex arrangement and developed more new and modified variants also. Also, some new algebraic methods have also been developed to carry out the forward and inverse kinematics of the robots. Depending upon the complexity and kind of robot, one can easily choose among these notations and methods. Also, each and every method has its own advantages and disadvantages. So, it is needed to study carefully all the methods to choose among these methods. In this chapter, all the methods which the researchers have developed over the time, mainly related to the kinematics of the robots has been studied.

2.2 Different Methods for Robot Kinematics

Many methods are available now days to study the geometry of robots with open-chain mechanism. The most common used is the D-H method. It was back in 1955 when two scientists named Jacques Denavit and Richard Hartenberg gave a notation for kinematic study of the robotic manipulators. They named it as D-H notation. After D-H notations many other methods and notations were discovered. These new theories or notations were made to remove away the difficulties or the shortcomings of the previous notations. Following are some of the methods used for the kinematic study of the robotic manipulators.

- Original D-H Parameter Method
- Distal D-H Variant
- Proximal D-H Variant
- Sheth and Uicker Notation Method
- Homogenous Matrix Transformation Method
- Lie Algebra Method
- Screw Theory Method
- Unified Notation Method for Serial & Parallel Robots

Different methods or different kind of notations for kinematic solution of a robotic manipulator are discussed below one by one.

2.2.1 Original D-H Parameter Method

The very first notation given for the kinematic analysis of robotic manipulators, known as D-H notations, was given by two scientists, named Jacques Denavit and Richard Hartenberg in 1955 [1]. They proposed a systematic notation for assigning right handed orthonormal coordinate frames for every link in an open kinematic chain of links. Using these orthonormal coordinate frames, final position and orientation of the links and end effector were calculated.

So, while using the D-H parameter method, first of all, assign orthonormal coordinate frames to all the links. Once these coordinate frames are assigned, transformations between adjacent coordinate frames can be calculated. These transformations are represented by a single standard 4×4 homogenous coordinate transformation matrix.

The Original D-H parameter method is easy to apply and has a generalized solution for all kind of the serial robots. Also, the calculations involved are not so complex, but it can be used only in the case of serial robots. In case of serial robots, it gives us an overall transform matrix, which maps the coordinates of the tool with respect to the base of the robot. This is known as the arm equation. The arm equation is obtained by multiplying different transformation matrices, obtained using the formula given by D-H notation.

In this method four parameters, popularly known as *D-H parameters*, are defined to provide the geometric description to serial mechanisms. Out of the four, two are known as *link*

parameters. They describe the relative location of two attached axes in space. Remaining two are known as *joint parameters*. They describe the connection of any link to its neighboring link

Link length (a_i) is measured from z_i to z_{i+1} along x_{i+1} . *Link twist* (α_i) is measured from z_i to z_{i+1} about x_{i+1} . *Joint offset* (d_i) is measured from x_i to x_{i+1} along z_i . *Joint angle* (θ_i) is measured from x_i to x_{i+1} about z_i [3].

A robot is composed of $(n+1)$ links, link 0 is the fixed base, link n is the terminal link, and joint i connects $(i - 1)$. A coordinate frame R_i is assigned with respect to link i . The axis of joint i is supposed along z_{i-1} , while the X_i axis is defined as the common perpendicular to z_{i-1} and z_i . Once the notations are given properly to the different frames and the axes, then joint parameters and the link parameters can be calculated easily and written in a tabular form. As shown in Fig. 2.1, notations are given to different frames and axis, using the original D-H parameter method.

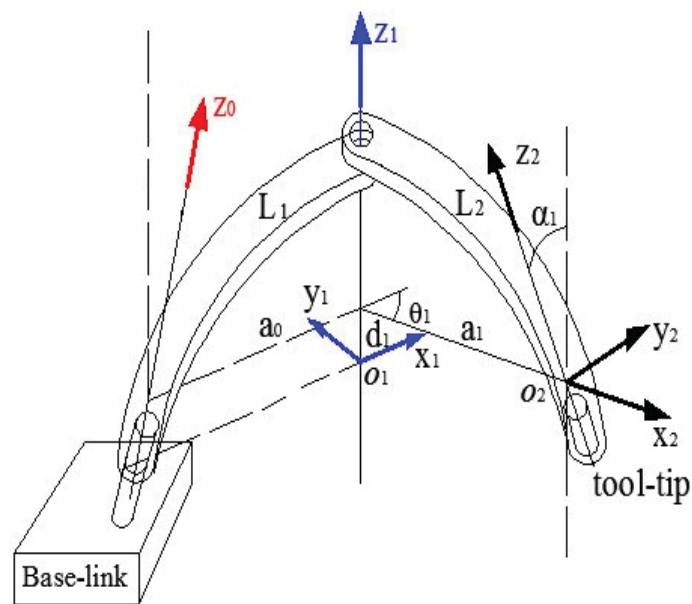


Fig. 2.1: Denavit-Hartenberg Notation [3]

The 4×4 transformation matrix which defines frame i with respect to frame $i - 1$ is obtained as a function of joint and link parameters $(\theta_i, r_i, d_i, \alpha_i)$.

Four D-H parameters can be expressed using vector forms which takes in to account the sign of displacements,

$$\cos \alpha_1 = z_1 \cdot z_2, \quad (2.1)$$

$$\sin \alpha_1 = z_1 \times z_2 \cdot x_2, \quad (2.2)$$

$$\cos \theta_1 = x_1 \cdot x_2, \quad (2.3)$$

$$\sin \theta_1 = x_1 \times x_2 \cdot z_1, \quad (2.4)$$

$$a_1 = \text{Vec}(O_1 O_2) \cdot x_2, \quad (2.5)$$

$$d_1 = \text{Vec}(O_1 O_2) \cdot z_1, \quad (2.6)$$

where $\text{Vec}(O_1 O_2)$ represents a vector from O_1 to O_2 .

Two screw transformations are involved in mapping of frame 1 to frame 2 [3]. First, about the axis z_1 by distance d_1 and angle θ_1 followed by second, about axis x_2 by distance a_1 and an angle of α_1 . Both these screw transformations finally give a single transformation matrix after multiplication.

$$T_{12} = \text{screw}(\theta_1, d_1, z_1) \cdot \text{screw}(\alpha_1, a_1, x_2), \quad (2.7)$$

$$T_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_1 & -\sin \theta_1 & 0 \\ 0 & \sin \theta_1 & \cos \theta_1 & 0 \\ d_1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_1 & 1 & 0 & 0 \\ 0 & 0 & \cos \alpha_1 & -\sin \alpha_1 \\ 0 & 0 & \sin \alpha_1 & \cos \alpha_1 \end{bmatrix} \quad (2.8)$$

$$T_{12} \triangleq \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_1 \cos \theta_1 & \cos \theta_1 & -\sin \theta_1 \cos \alpha_1 & \sin \theta_1 \sin \alpha_1 \\ a_1 \sin \theta_1 & \sin \theta_1 & \cos \theta_1 \cos \alpha_1 & -\cos \theta_1 \sin \alpha_1 \\ d_1 & 0 & 0 & \cos \alpha_1 \end{bmatrix} \quad (2.9)$$

where T_{12} is 4×4 homogeneous transformation matrix which gives position of frame 2 with respect to frame 1.

If joint i is rotational, then the joint variable q_i is equal to θ_i , while $q_i = r_i$, in case of a prismatic joint. The geometric model of a serial robot can thus be obtained by the successive multiplications of the transformation matrices.

$$T_0^n = T_0^1 \times T_1^2 \times \dots \times T_{n-1}^n \quad (2.10)$$

It is to be noted that the frame n can be always defined such that D-H parameters of frame n are always equal to zero.

This is the procedure to calculate the transformation matrices. These transformation matrices are further used to calculate the arm equation of the robot. The arm equation as shown above is nothing but simply the systematic multiplication of different matrices calculated using the formula for the transformation matrix. So, it can be said that this method is overall good for serial robots because it can easily give a generalized solution for a serial robot. Further, the calculation involves are not so complicated in this notation. So, this notation can be used easily for serial robots.

That's how, the D-H notations are used to find out the D-H parameters or the values of the joint and the link variables. Once the value for the joint and link variables is obtained, transformation matrices can be calculated easily, which ultimately gives the arm matrix or the arm equation upon multiplying all the obtained transformation matrices.

2.2.2 Distal D-H Parameter Method

The distal variant is currently the most popular form of the Denavit-Hartenberg notation found in the literature. The earlier references to the present form that are given by Kahn Roth [4]. As shown in Fig. 2.2, the original parameters $(a_1, \alpha_1, \theta_1, d_1)$ have been replaced by $(a_2, \alpha_2, \theta_2, d_2)$. But the coordinate frame indices and the link indices remain the same. It can be seen that displacement about the second joint is θ_2 or d_2 . Similarly, displacements along the first joint axis are θ_1 and d_1 . The four Denavit-Hartenberg parameters $(a_2, \alpha_2, \theta_2, d_2)$ can be expressed using vector forms. When used in vector forms, signs of the displacements are also taken in to the account.

This modification was made to make the parameter identification simpler [6] and is used world widely for kinematic description of serial robotic manipulators, as reported by Lipkin [3]. The term distal refers to the location of the link-frame relative to manipulator base and according to this variant, a frame is attached to farther end point of link, when travelling from manipulator base towards end effector.

Fig. 2.2, describes the schematic representation of distal variant. The coordinate frame having components x_1, y_1 and z_1 is attached at father end point of first moving link L_1 . Later on, this variant was popularized by Paul [6] and Shilling [7] and has been applied to different kinds of kinematic problems. There are some differences in the original version of D-H parameter and the distal variant of same. In original D-H method, link parameters, a_i and α_i , are measured

from z_i to z_{i+1} about x_{i+1} , whereas in this variant, link parameters are measured from z_{i-1} to z_i about x_i .

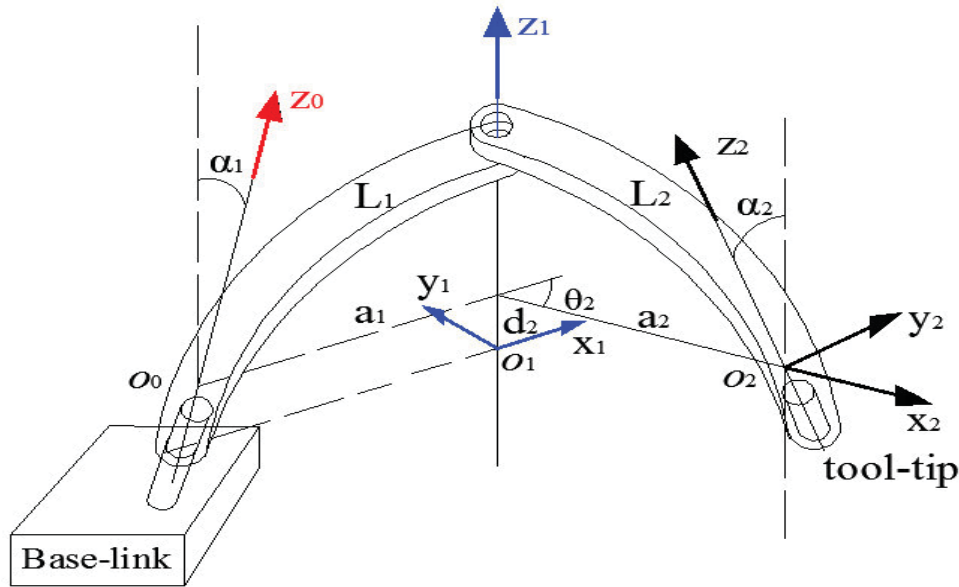


Fig. 2.2: Distal D-H notations [3].

Also, joint parameters θ_i and d_i are measured from x_{i-1} to x_i about z_{i-1} rather than measuring from x_i to x_{i+1} about z_i as in original D-H parameters method. From Fig. 2, it can be seen that a_2 and α_2 were measured about x_2 , whereas d_2 and θ_2 were measured about z_1 .

➤ **Frame Assignment Using Distal Variant of D-H Parameter Method [7]:**

- I. First of all, diagram of a given kinematic chain showing the links and joints axes is drawn. It may be termed as line diagram. Joint numbers are marked from 1 to n , starting with base and proceeding towards end-effector. Then, z_i - axis to $(i + 1)^{th}$ joint axis are assigned about which motion is taking place, for $i = 1$ to $(n - 1)$. The base frame z -axis, z_0 -axis is always aligned with 1st joint axis.
- II. A point at the intersection of axes z_{i-1} and z_i is located. Origin (O_i) is marked of i^{th} frame. A common normal is drawn to z_{i-1} and z_i from the point of intersection. Assign x -axis from O_i along common normal. If z_{i-1} and z_i don't intersect, as in case when two consecutive joint axes are parallel to each other, origin (O_i) is considered at the point of intersection between z_i and common normal. The x -axis, is assigned in the same way as before however, away from z_{i-1} . In this way, z and x axis can be assigned at each joint.
- III. Then, y -axis can be assigned according to right hand thumb rule.

IV. At the end, it can be seen that all the coordinates have been assigned. Now, corresponding D-H parameters can be calculated.

Once, D-H parameters are calculated, transformation matrices can be calculated easily. In Fig. 2.2, frame 1 can be mapped to frame 2 by a screwing motion about z_2 by distance d_2 and angle θ_2 , which is further followed by a screwing motion about x_2 by distance a_2 and angle α_2 .

$$B_{12} = \text{Screw}(\theta_2, d_2, z_2) \cdot \text{screw}(\alpha_2, a_2, x_2) \quad (2.11)$$

$$B_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 & 0 \\ 0 & \sin\theta_2 & \cos\theta_2 & 0 \\ d_2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & \cos\alpha_2 & -\sin\alpha_2 & 0 \\ 0 & \sin\alpha_2 & \cos\alpha_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.12)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_2 \cos\theta_2 & \cos\theta_2 & -\sin\theta_2 \cos\alpha_2 & \sin\theta_2 \sin\alpha_2 \\ a_2 \sin\theta_2 & \sin\theta_2 & \cos\theta_2 \cos\alpha_2 & -\cos\theta_2 \sin\alpha_2 \\ d_2 & 0 & 0 & 1 \end{bmatrix} \quad (2.13)$$

This is the transformation matrix obtained from the distal D-H parameters. All the transformation matrices can be calculated using the same formula. Then all the matrices can be multiplied in a systematic way in order to obtain an arm matrix, which ultimately maps the tool coordinates to the base frame.

Later on, velocity, total work envelope etc. can be calculated using the arm equation. These are the different techniques to solve for the forward and inverse kinematics of the robotic manipulator.

2.2.3 Proximal D-H Parameter Method

Second modified method is known as *proximal* variant of D-H parameter method. Again, reason behind this modification was same as in case of distal variant i.e. to make the parameters identification simpler. In this method, the term proximal refers location of frame, being rigidly attached to link, relative to manipulator base. According to this variant, coordinate frame is placed at that end of the link which is closer to manipulator base or which comes first, when travelling from base towards end effector. It differs from original D-H parameter method in the sense that link parameters a_i and α_i are measured from z_i to z_{i+1} about x_i rather than

measuring from z_i to z_{i+1} about x_{i+1} as in case of original method. Also, joint parameters θ_i and d_i are measured from x_{i-1} to x_i about z_i rather than measuring from x_i to x_{i+1} about z_i [2, 3].

With reference to Fig. 2.3, it can be noticed that link coordinate frame having components x_1, y_1 and z_1 is attached at closer end point of first moving link L_1 . The link parameters a_2 and α_2 are measured about x_2 , whereas d_2 and θ_2 are measured about z_2 .

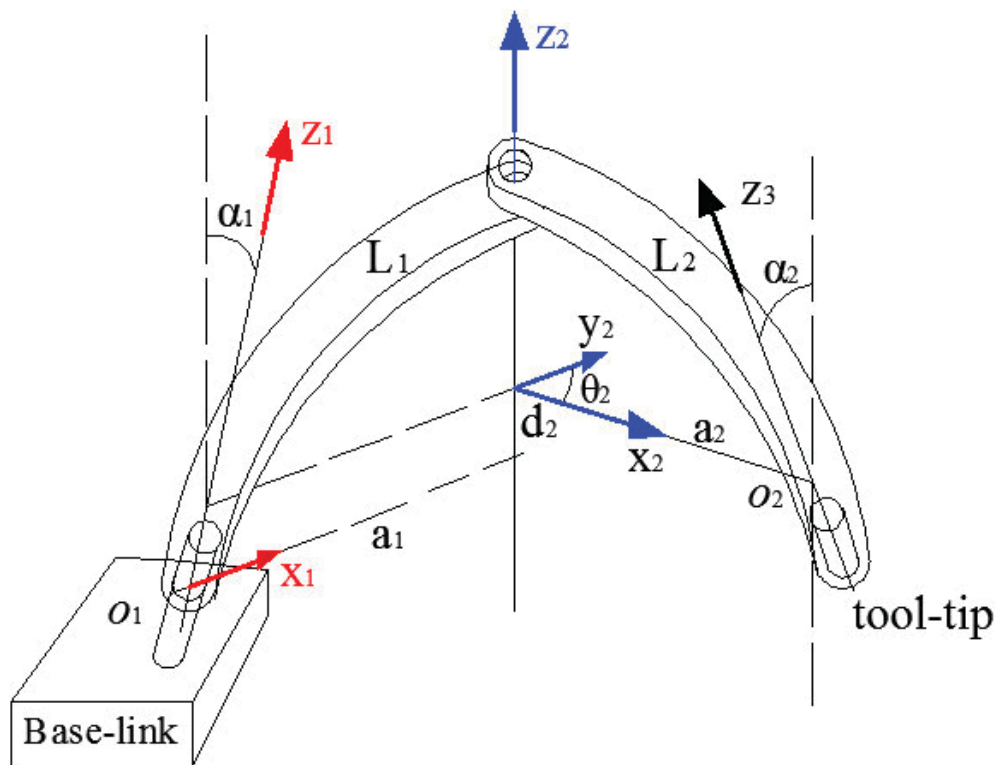


Fig. 2.3: schematic representation of proximal variant of D-H parameters method [3].

The proximal variant is less widely found in present literature than the distal variant but occurs more frequently than the original notation. In the proximal variant of the D-H parameter method, it can be noticed that,

- I. Frame i is rigidly attached to link i .
- II. Displacements θ_i and d_i are measured about z_i .
- III. Displacements α_i and a_i are measured about x_i .
- IV. The first joint displacement is θ_i or d_i .
- V. The ground is link 0.

The i_{th} joint in the chain is along z_i . Frame i is attached to the proximal end of link i to satisfy criterion 1. Since the displacements θ_i and d_i are measured about z_i criterion 2 is satisfied. Similarly, displacements α_i and a_i are measured along x_i so criterion 3 is satisfied. Finally the first joint displacement is θ_1 or d_1 so criterion 4 is satisfied and the base is link 0 so criterion 5 is satisfied. In proximal variant, all five criteria are satisfied, whereas in Original D-H, only two criteria are satisfied and in Proximal D-H, four criteria are satisfied.

➤ **Frame Assignment Using Proximal Variant [2]:**

- I. A schematic line diagram of given kinematic chain is drawn at first. Now, the next step is to recognize the joint axes. Joint axes are marked as $z_i - axis$, for $i = 1$ to $i = n$. The z -axes, z_0 and z_{n+1} , of base and end-effector frames can be chosen arbitrarily. It can be aligned with 1st joint axis for base frame and along the approach vector for end-effector frame. In this way, z -axes are assigned. Next task is to assign the $x - axes$.
- II. There is a common normal between i^{th} and $(i + 1)^{th}$ joint axes or there is a point of intersection of same axes. If there is a point of intersection, mark this point as origin of i^{th} frame. If these joint axes does not intersect, choose origin arbitrarily at the intersection of common normal and i^{th} joint axis, so that joint distance become zero. Assign x -axis along common normal from origin.
- III. Right hand thumb rule can be used to complete the link frame definition and thus y -axis can be assigned in this way.
- IV. At last, corresponding D-H parameters can be calculated.

$$B_{12} = \text{Screw}(\alpha_1, a_1, x_1).screw((\theta_2, d_2, z_2) \quad (2.14)$$

$$B_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_1 & 1 & 0 & 0 \\ 0 & 0 & \cos\alpha_1 & -\sin\alpha_1 \\ 0 & 0 & \sin\alpha_1 & \cos\alpha_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 & 0 \\ 0 & \sin\theta_2 & \cos\theta_2 & 0 \\ d_2 & 0 & 0 & 1 \end{bmatrix} \quad (2.15)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_1 & \cos\theta_2 & -\sin\theta_2 & 0 \\ -d_2\sin\alpha_1 & \cos\alpha_1 \sin\theta_2 & \cos\alpha_1 \cos\theta_2 & -\sin\alpha_1 \\ d_2\cos\alpha_1 & \sin\alpha_1 \sin\theta_2 & \sin\alpha_1 \cos\theta_2 & \cos\alpha_1 \end{bmatrix} \quad (2.16)$$

This is the transformation matrix in case of the Proximal D-H parameter.

➤ **Comparison: Original D-H, Distal and Proximal variant:**

From the above discussion, it can be concluded that proximal method is easiest to use, as it is easy to identify the parameters in case of Proximal D-H parameter method. In Proximal variant, the subscript index of axes (x_i and z_i) are same. These are the axes about which link and joint parameters are being measured, as shown in Table 2.1.

Table 2.1: Comparison of original D-H parameter method and its variants in terms of parameter representation.

Parameter (s)	Original D-H method	Distal variant	Proximal variant
$\cos \alpha_1$	$z_1 \cdot z_2$	$z_0 \cdot z_1$	$z_1 \cdot z_2$
$\sin \alpha_1$	$z_1 \times z_2 \cdot x_2$	$z_0 \times z_1 \cdot x_1$	$z_1 \times z_2 \cdot x_1$
$\cos \theta_1$	$x_1 \cdot x_2$	$x_0 \cdot x_1$	$x_0 \cdot x_1$
$\sin \theta_1$	$x_1 \times x_2 \cdot z_1$	$x_0 \times x_1 \cdot z_0$	$x_0 \times x_1 \cdot z_1$
a_1	$vec(O_1 O_2) \cdot x_2$	$vec(O_0 O_1) \cdot x_1$	$vec(O_1 O_2) \cdot x_1$
d_1	$vec(O_1 O_2) \cdot z_1$	$vec(O_0 O_1) \cdot z_0$	$vec(O_1 O_2) \cdot z_1$

But in case of original and its distal variant, the situation is different. So, it is not an easy task to remember the definition of these parameter measurement. As observed from above discussion, distal variant is more widely used as compared to original and proximal D-H method. Table 2.1 shows the summary of comparison made above.

➤ **Limitations of D-H parameter method:**

1. It is not an easy task to evaluate D-H parameters for some spatial linkages. The reason behind is that common perpendicular is required between two skew lines in space [10].
2. Kinematic analysis of only serial mechanisms can be done using D-H parameter method and its variants, as pointed by Thomas et al. [9].
3. Only z -axis is defined by D-H parameter method base coordinate frame. No unique representation of x -axis is provided. In case of tool-tip frame, i.e. n^{th} in case of distal

variant and $(n + 1)^{th}$ in case of proximal variant, there is no definite representation of x and z -axes. They are assigned arbitrarily as reported by Siciliano et al. [21].

4. Describing parallel and hybrid manipulators, which contain joints like spherical or cardan joints, i.e. joints with more than one degree of freedom, by using D-H parameter method does not give unique results as reported by Thomas et al. [9].
5. D-H parameter is based on Homogeneous link coordinate transformation matrix. This is a 4×4 matrix. It uses twelve parameters to represent only six degree of freedom. So, it highly redundant as pointed out by Sahu et al. [20].

2.2.4 Sheth and Uicker Notation

Another notation was given by Sheth and Uicker in 1971. This convention partitions displacements in joint and link displacements and decomposes these displacements into three axial screw displacements. Because of this systematic construction, the convention features several practical and theoretical preferences. Conventions for kinematic modeling have to deal with a 'trade-off' between compactness on the one hand, and generality and intuitiveness on the other hand. It was shown how the two-axial-twist description of Denavit–Hartenberg is extended to a three-axial-twist description by the Sheth–Uicker convention such that a mechanism can be described by a table that reflects the geometry of adjacent joint axes and that has a structure that reflects the topology of the mechanism type.

After the brief study, it can be concluded that this method is very complicated for the serial robot, so better option is to use the D-H method or the D-H notations. As it has been observed that, S-U notation was mainly given for the complicated robot structures like the tree robot. Due to the complex calculations involved, this method is not used for the serial robots, but this notation is very helpful for calculations in a tree structure robot.

The S-U method has successfully removed the problem of multi-parameters sets, as reported in limitations of D-H method. Each link of a serial chain is assigned with two coordinate frames in this method. The frame which is at proximal end has components u , v and w , whereas the frame at the distal end has components as x , y and z . Fig. 2.4 shows the coordinate systems assigned by using S-U Method.

Procedure to apply S-U method to a spatial mechanism:

- I. First of all, joint axes of infinite length are drawn at the points where links form the connection.

- II. Thereafter, an axis w_i is assigned to proximal end of the link and z_j to the distal end of the link.
- III. Assign w_j - axis for next link, which should be at proximal end of the link, along the z_j - axis of previous link. Then, z_k -axis is assigned at distal end of the link along the joint axis, as shown in Fig. 2.4.
- IV. Now, for the remaining two coordinate frames, u, v for proximal frame can be randomly chosen. Similarly, x, y for distal frame can also be chosen arbitrarily.
- V. A common normal is drawn from w_j to z_k , i.e. t_{jk} , where z_k is the axis assigned to later link w.r.t pair being considered.

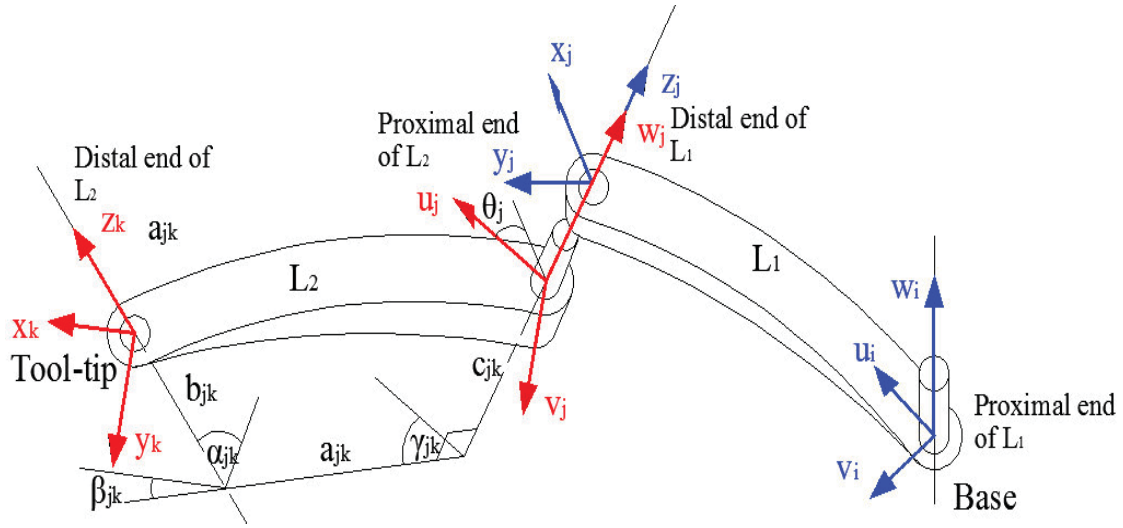


Fig. 2.4: Sheth and Uicker Notation. [3]

It is important to first understand the S-U method for first two links. If done so, it is easy to assign S-U parameters for rest of links. First link frame of the structure has components u_i, v_i, w_i at proximal end of the structure. Whereas, at its distal end, second frame has x_j, y_j, z_j components. So, if this is understood to the user, it is very easy to assign the frames and then further calculations can be done. As shown in the Fig. 2.4, second link has two frames u_j, v_j, w_j and x_k, y_k, z_k respectively at proximal and distal ends.

The S-U parameters can be defined as follows:

a_{jk} = distance measured from w_j to z_k along t_{jk} .

α_{jk} = angle measured from w_j to z_k about t_{jk} .

b_{jk} = distance measured from t_{jk} to x_k along z_k .

β_{jk} = angle measured from t_{jk} to x_k about z_k .

c_{jk} = distance measured from u_j to t_{jk} along w_j .

γ_{jk} = angle measured from u_j to t_{jk} about w_j .

θ_j = angle measured from u_j to x_j about common joint axis z_j, w_j .

The next step in S-U method is to perform coordinate mapping or transformation. Existing matrix method of coordinate transformation can be used. A numerical schemes have been presented by the authors, in order to reduce the task of data collection.

In this scheme, each transformation matrix consists of two parts, which are as follows:

- I. **Shape matrix:** The shape of the link is specified using the shape matrix. It is a constant part which specifies the shape of a link by using six parameters. It uses relative orientation of a coordinate frame, at the beginning of link and another coordinate frame present at the end of same link. A common perpendicular is found between two axes, in order to determine the constant shape parameters for the link.
- II. **Joint matrix:** It is a homogeneous link coordinate transformation matrix which represents the joint motion. It is a function of joint variable.

➤ **Advantages of S-U method:**

Following are the advantages of S-U method over the D-H parameter based methods:

- I. S-U method is applicable to all rigid link mechanisms like open loop, closed loop, hybrid structures and tree like structures.
- II. S-U method is more flexible than D-H methods. Here, in S-U method, two frames one for each link, is provided at the point of meeting of two links. So, frames can be chosen independently, which provides more flexibility. But, in case of D-H parameter method and its counter parts, there is no such flexibility.

➤ **Limitations of S-U method:**

There are some limitations also, associated with the S-U method, discussed below.

- I. It uses seven parameters as compared to four parameters by D-H, to describe joint motion and the link shape.

- II. The complexity level is very high in case of S-U method. So, normally some other method, which is simple is preferred over the S-U method. It is mostly suitable for only closed-loop chains only.
- III. Number of coordinate frames is very high. So, level of complicacy is also very high, which ultimately makes the method computationally expensive.

2.3 Summary

With reference to above discussion, methods like D-H parameter method and its variants, S-U method, Khalil and Kleinfinger method. A unified method, Quaternion theory based method, screw based displacement methods etc. has been found in literature. From this review, it has been observed that for the kinematic description of serial manipulators, the distal variant of D-H parameter method is used most widely, as compared to original D-H parameter method and its proximal variant. However, it leads to ambiguities when applied to parallel, closed-loop or tree like structured robots. Later on, these ambiguities has been removed by method proposed by Sheth and Uicker (S-U), which is the most generalized method, because of its wide range of applicability, for dealing with kinematic problem of any rigid link mechanism. Still, because of its complexity (two number of frames per link for serial chains and non-minimal parameter representation), it is applied to closed-loop robots only. Thereafter, a new geometric method has been proposed by Khalil and Kleinfinger which is also applicable to open, closed-loop or tree like structured robots and simple to use as compared to S-U method. In literature review performed in this thesis, it has been observed that this method has a wide range of applications. Another method, coined as a Unified method for serial, parallel, hybrid manipulators, has been proposed by Thomas et al. However, its application to kinematic problems is very limited as compared to other methods.

But as per the application of our work and ease of use, D-H parameter method is the method which is easy to apply, computationally not expensive and easy to modify as per our requirements.

Chapter-3

Kinematic Study of Conventional Serial Manipulators

3.1 Introduction

Robot kinematics studies the relationship between the dimensions and connectivity of kinematic chains and the position, velocity and acceleration of each of the links in the robotic system, in order to plan and control movement and to compute actuator forces and torques.

Robot Kinematics can be studied in two ways-

- Forward Kinematics
- Inverse Kinematics

Forward Kinematics- In forward kinematics, joint variables and link variables are known, and final position and orientation of the robot end effector are calculated. First of all have a look at the joint variables and link variables. With the help of geometric transformations, mapping of the coordinates of end effector of robot with respect to the base of the robot is done. Then generation of the kinematic equations establishing the relationship between the base and the end effector of the robot is done. In total there are four variables, i.e. link angle, link distance, joint angle and joint distance. Then the values of the joint variables and link variables are inserted in the kinematic equations produced. This ultimately gives the position and orientation of the tool or the end effector of the robot.

Inverse Kinematics- In case of Inverse Kinematics, the position and orientation of the end effector are known and the joint and link variables of the robot are to be calculated. Here also, development of some algorithm or a systematic procedure has been done in order to assign the link coordinates to a robotic manipulator. Successive transformations between adjacent coordinate frames, starting at tool tip and working back to the base of the robot, then led to the arm matrix. The arm matrix represents the position and orientation of the tool in the base frame as a function of joint variables. From the arm matrix, the tool configuration matrix can be obtained easily. The tool configuration matrix represents the position and orientation of the

robot end effector or tool in terms of joint variables. So, the tool configuration matrix can be further used to find out the joint variables. In this way, the inverse kinematic problem is solved.

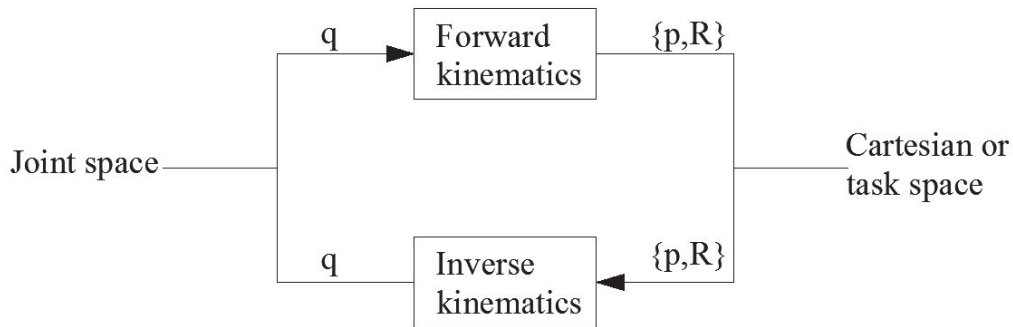


Fig. 3.1: General description of kinematics of robotic manipulators.

Above line diagram defines the Forward and Inverse Kinematics in a very simple and easy to understand way. From this line diagram it can be said that, if the Joint variables (\mathbf{q}), are known, then using the forward kinematics, position (\mathbf{p}) and orientation (\mathbf{R}) of tool-tip of end-effector can be found. And similarly, if the position and orientation of the end effector of the tool tip are known, then using the inverse kinematics operation, the joint variables of the robot links can be calculated.

Summing up the above discussion, it can be said that the Forward kinematics transforms the Joint space coordinates in to Cartesian or task space coordinates. On the other hand, Inverse kinematics transforms the Cartesian or task space coordinates in to the Joint space coordinates.

3.2 Link Coordinate Transformations

Orthonormal frames are attached to each and every link on our manipulator. Then, it is required to map the frame attached to an arbitrary point on rigid-link, into base coordinate frame to know about the relative position and orientation of different links of a manipulator in \mathbb{R}^3 . This is known as link coordinate transformation.

3.2.1 Homogeneous Transformations

Homogenous transformations include both homogenous translations and rotations. During rotations, the orientation changes, while during the translation, changes the position of the object. The orientation of coordinate frame attached rigidly to a body can be mapped to another coordinate frame with the help of pure rotation matrix (3×3) in \mathbb{R}^3 . But the position of the same, cannot be defined in \mathbb{R}^3 by a matrix of order 3×3 . After rotation of frame attached to body the position of origin remains same, as it is before rotation, but it is not true for the case

of displacement of body, resulting into change in position of origin. In simpler words, it can be said that a matrix of order 3×3 can take in to account only the rotation, but not the position of the body. So, in order to define the position of body-attached frame with respect to some other coordinate frame there is a need for another type of transformation, which is known as *translation* and is defined in four dimensional space (\mathbb{R}^4), by means of homogeneous coordinates. Homogeneous coordinates of a point (say) \mathbf{p} in \mathbb{R}^3 , w.r.t an orthonormal coordinate frame $\{A\}$ in \mathbb{R}^3 , are denoted as $[\mathbf{p}]^A$ and are given as [7]:

$$[\mathbf{p}]^A = \sigma[p_1, p_2, p_3, 1]^T \quad (3.1)$$

where p_1, p_2 and p_3 are components of point \mathbf{p} .

Thus, homogeneous coordinates of a point \mathbf{p} in \mathbb{R}^3 can be represented by a vector in \mathbb{R}^4 . The fourth component σ is a non-zero scale factor and it can have any value, which results in generation of same physical space. Hence, homogeneous coordinates are not unique, as there is a non- zero scale factor. These transformations are the *point* transformations. Now, to transform a physical point, defined in its homogeneous coordinates in \mathbb{R}^3 , from one coordinate frame to other coordinate frame a 4×4 homogeneous coordinate transformation matrix is used and is defined as:

$$T \triangleq \begin{bmatrix} \mathbf{R} & \mathbf{p} \\ \boldsymbol{\eta}^T & \sigma \end{bmatrix} \quad (3.2)$$

where

\mathbf{R} is a 3×3 matrix which represents the orientation of frame to be rotated.

\mathbf{P} is a 3×1 column vector representing position of frame to be translated.

$\boldsymbol{\eta}^T$ is perspective vector which specify a point of perspective for visual sensing with camera. It has non-zero value in case of overhead camera transformations.

3.2.2 Homogeneous Rotation and Translation

An elementary rotation or translation can be regarded as a special case of 4×4 homogeneous transformation matrix. Consider $\{A\}$, $\{B\}$ are two orthonormal coordinate frames, initially coincident, now:

- I. If rotation of frame {B} is done by an angle θ about i^{th} unit vector of frame {A}, the same can be represented in terms of homogeneous coordinates by 4×4 matrix denoted by $\text{Rot}(\theta, i)$ and is pronounced as i^{th} elementary homogeneous rotation matrix:

$${}^A_B[\mathbf{Rot}(\theta, i)] \triangleq \begin{bmatrix} & & & | & 0 \\ & [\mathbf{R}_i(\theta)]_{3 \times 3} & & | & 0 \\ - & - & - & | & 0 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad 1 \leq i \leq 3; \quad (3.3)$$

Where $[\mathbf{R}_i(\theta)]_{3 \times 3}$ is the elementary rotation matrix about i^{th} unit vector.

- II. For elementary translation of origin of frame {B} relative to the origin of frame {A}, say by amount t_i along i^{th} unit vector of frame {A} following 4×4 homogeneous transformation matrix is used:

$${}^A_B[\mathbf{Trans}(q)] \triangleq \begin{bmatrix} 1 & 0 & 0 & | & t_i \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ - & - & - & | & - \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (3.4)$$

- III. Simultaneous rotation and translation homogeneous transformation matrix can be written as:

$${}^A_B[\mathbf{T}] \triangleq \begin{bmatrix} & & & | & t_i \\ & [\mathbf{R}_i(\theta)]_{3 \times 3} & & | & t_j \\ - & - & - & | & t_k \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (3.5)$$

- IV. A point \mathbf{p} defined in local coordinate frame {A}, can be defined in global coordinate frame using homogeneous transformation as:

$${}^B\mathbf{p} = {}^B_A\mathbf{T} \quad {}^A\mathbf{p} \quad (3.6)$$

3.3 Forward Kinematics: Using Distal Variant

First of all, study of the forward kinematics of some simpler structures of robotic manipulators is done. Then, a spatial robotic configuration, which is a more complicated structure having five degree of freedom has been discussed. So, in this section, study of forward kinematics of SCARA robot, 5-axis robot and a six axis robot has been done, using the distal variant of D-H parameters.

3.3.1 SCARA Robot

A SCARA robot is a four axis robot having three revolute and one prismatic joint. SCARA stands for Selective Compliance Assembly Robot Arm. The first two joints are revolute joints, followed by a prismatic and a revolute joint.

The acronym SCARA stands for Selective Compliance Assembly Robot Arm [8]. It consists of one fixed link and four moving links, providing four independent motions in Cartesian space (R^3). The kinematic configuration of this robot has been described in Fig. 3.4.

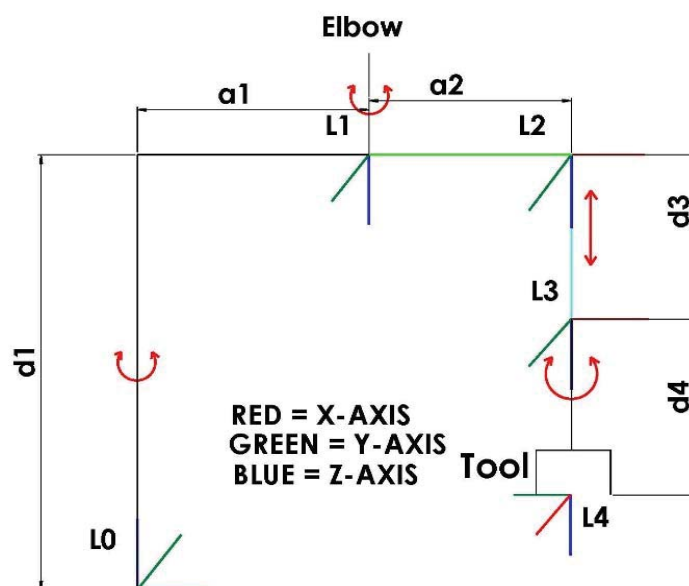


Fig. 3.2: Description of 4-axis SCARA robot according to distal variant.

There are two types of pairs between these links— i) revolute, between links (L_0, L_1) , (L_1, L_2) and (L_3, L_4) ; ii) a prismatic pair, between links L_2, L_3 . Link-frames are assigned using distal variant of D-H parameter method. The corresponding link parameters and joint parameters are as shown in Fig. 3.4 and are given in Table 3.1. Major applications of this robot is pick and place operations.

Table 3.1: Distal variant D-H parameters of SCARA robot.

S. No.	Link Length (a_i) (mm)	Link Twist (α_i) (deg.)	Joint offset (d_i) (mm)	Joint angle (θ_i) (deg.)	Home position
1.	a_1	$\alpha_1 = 180;$	d_1	θ_1	0
2.	a_2	$\alpha_2 = 0;$	$d_2 = 0;$	θ_2	0
3.	$a_3 = 0;$	$\alpha_3 = 0;$	d_3	$\theta_3 = \text{fixed}$	d
4.	$a_4 = 0;$	$\alpha_4 = 0;$	d_4	θ_4	0

To solve the forward kinematics, first of all coordinate frames to all the links are assigned. Once coordinate frames are assigned, then the transformation matrices are used to calculate the position and orientation of the respective links and finally the position and orientation of the tool tip or the end effector of the robotic manipulator. While solving a transformation matrix, input is needed in form of joint angle and joint distance, which are already available. At first, a table has been made enlisting down all the joint and link parameters. Then, values of these parameters are inserted in to transformation matrix and solution to our problem can be calculated. In this way, the forward kinematics of the robotic manipulator can be calculated. Final transformation matrix for a SCARA robot transforming the joint space coordinates in to the Cartesian coordinates has been calculated. In this way, the position and orientation of the tool tip or the end effector of the robot can be obtained.

The transformation matrix (${}^{\text{base}}_{\text{tool}}\mathbf{T}$) for a SCARA robot, transforming from base of the robot to the tool of the robot is,

$${}^{\text{base}}_{\text{tool}}\mathbf{T} = \begin{bmatrix} c_{1-2-4} & s_{1-2-4} & 0 & | & a_1c_1 + a_2c_{1-2} \\ s_{1-2-4} & -c_{1-2-4} & 0 & | & a_1s_1 + a_2s_{1-2} \\ 0 & 0 & -1 & | & d_1 - \frac{q_3}{3} - d_4 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (3.7)$$

Here, $c_{1-2-4} = \cos(q_1 - q_2 - q_4)$.

3.3.2 Forward Kinematics of a 5-Axis Robot

A manipulator whose kinematic configuration resembles very closely to the human arm anatomy, is 5-axis articulated robot. It consists of one fixed (marked as L_0) and four moving links (L_1, L_2, L_3 and L_4). All joints are of revolute type. At last link, usually known as end-

effector, two motions (pitch and roll) can be observed. Two examples of this robot can be reported, these are— i) Microbot Alpha II; ii) Rhino XR-3 [8]. Both of these manipulators has same kinematic configuration. It has been observed while calculating D-H parameters, that these parameters are same for both and are given in Table 3.2.

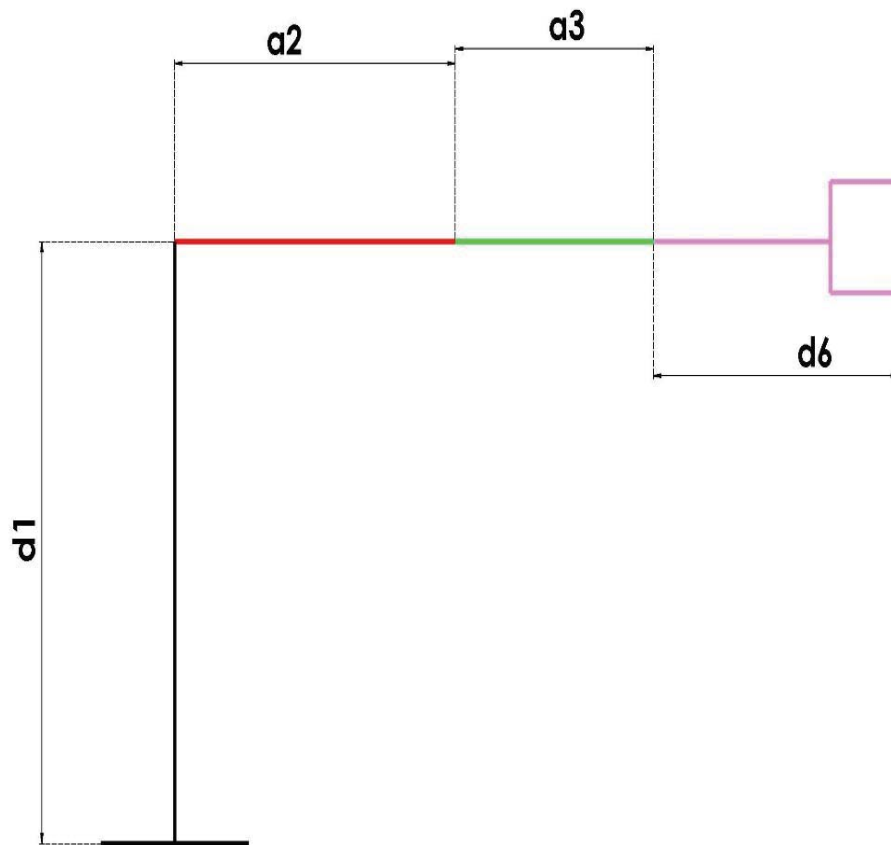


Fig. 3.3: Microbot Alpha-II- structural details [8]

The Fig. 3.3 indicates the structural details of the Microbot Alpha –II robot. This is a five axis robot. Fig. 3.4 shows coordinate frames assigned to all the links, in order to calculate the D-H parameters

In Fig. 3.4, coordinate frames are assigned to the robotic structure, using the distal variant if the D-H parameter.

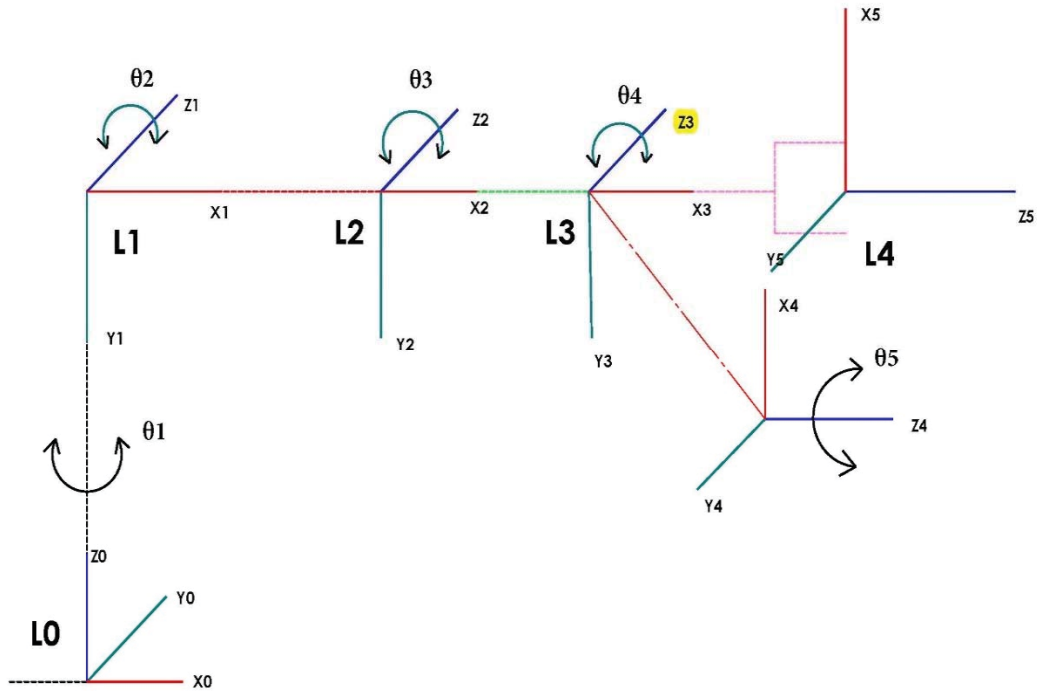


Fig. 3.4: Microbot Alpha-II – Frame assignment [8]

Now, the frames have been assigned. Next task is to calculate the D-H parameters using the distal variant.

Table 3.2: Distal variant parameters of 5-axis articulated arm.

S. No.	Link Length (a_i) (mm)	Link Twist (α_i) (deg.)	Joint offset (d_i) (mm)	Joint angle (θ_i) (deg.)	Home position
1.	$a_1 = 0;$	$\alpha_1 = -90;$	d_1	θ_1	0
2.	a_2	$\alpha_2 = 0;$	$d_2 = 0;$	θ_2	0
3.	a_3	$\alpha_3 = 0;$	$d_3 = 0;$	θ_3	0
4.	$a_4 = 0;$	$\alpha_4 = -90;$	$d_4 = 0;$	θ_4	90
5.	$a_5 = 0;$	$\alpha_5 = 0;$	d_5	θ_5	0

From these D-H parameters, the position and orientation of the robot can be calculated by simply putting the values in to the transformation matrix.

3.3.3 Forward Kinematics of a Six-Axis Robot

Now, let us perform a kinematic analysis of a six-axis articulated industrial robot, the Intelledex 660 manipulator [8]. This robot is high-precision light-assembly industrial robot, having six degree of freedom.

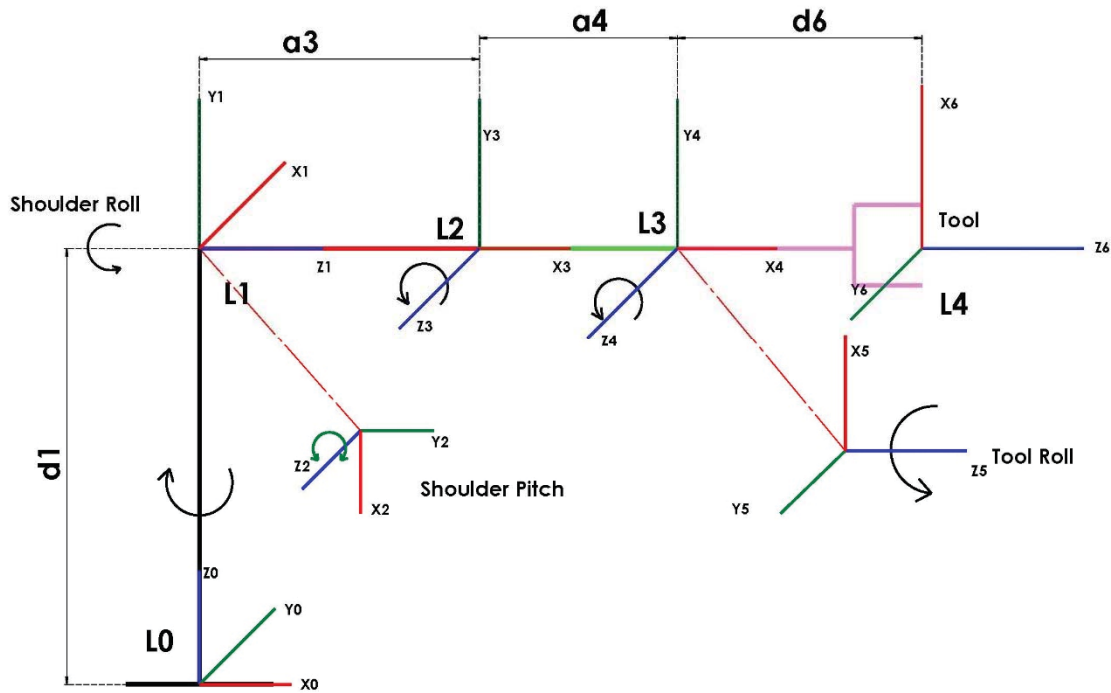


Fig. 3.5: Intelledex 660 six-axis articulated robot

The Fig. 3.5 shows the structure of the robot, its joints, coordinate frames assigned to various links. Now, let us find the D-H parameters of the same using the distal approach.

Table 3.3: Distal variant parameters of 6-axis articulated arm.

S. No.	Link Length (a_i)(mm)	Link Twist (α_i) (deg.)	Joint offset (d_i)(mm)	Joint angle (θ_i)(deg.)	Home position
1.	$a_1 = 0;$	$\alpha_1 = 90;$	d_1	q_1	90
2.	$a_2=0;$	$\alpha_2 = 90;$	$d_2 = 0;$	q_2	-90
3.	a_3	$\alpha_3 = 0;$	$d_3 = 0;$	q_3	90
4.	a_4	$\alpha_4 = 0;$	$d_4 = 0;$	q_4	0
5.	$a_5 = 0;$	$\alpha_5 = 90;$	$d_5 = 0;$	q_5	90
6	$a_6 = 0;$	$\alpha_6 = 0;$	d_6	q_6	0

Further kinematic study can be done by inserting these values in to the transformation matrix obtained by multiplying all the transformation matrices. This is the way to calculate the position and orientation of the tool tip or the end effector with respect to the base of the robot.

3.4 Forward Kinematics: using Proximal Variant

3.4.1 Forward Kinematics of a SCARA Robot

The kinematic configuration of this robot has been described in the previous section, in applications of distal variant. However, the link-frame assignment and D-H parameters calculation has been carried out in this example according to proximal variant, as shown in Fig. 3.6. The calculated D-H parameters are given in Table 3.4.

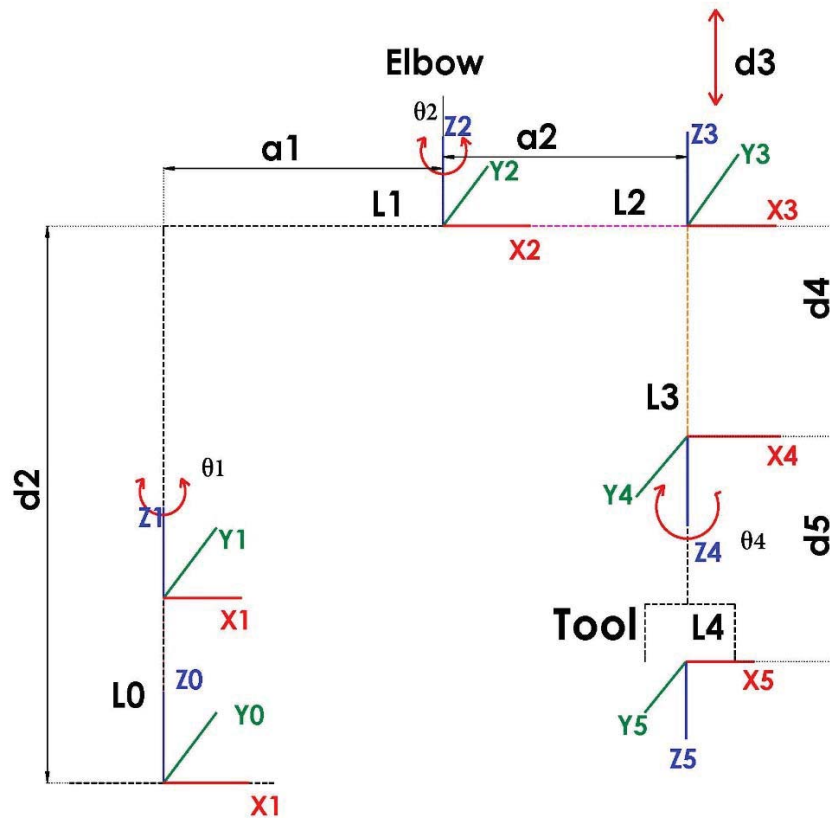


Fig. 3.6: Description of 4-axis SCARA robot according to proximal variant.

Once coordinate frames are assigned, then by using the transformation matrices, position and orientation of the respective links and finally the position and orientation of the tool tip or the end effector of the robotic manipulator can be calculated. While solving a transformation matrix, the input is joint angle and joint distance. Table below indicates the joint and link parameters.

Table 3.4: Proximal variant D-H parameters of SCARA robot.

S. No.	Link Length (a_{i-1}) (mm)	Link Twist (α_{i-1}) (deg.)	Joint offset (d_i) (mm)	Joint angle (θ_i) (deg.)	Home position
1.	$a_0 = 0;$	$\alpha_0 = 0;$	d_1	θ_1	0
2.	a_1	$\alpha_1 = 0;$	d_2	θ_2	0
3.	a_2	$\alpha_2 = 0;$	$d_3 = \text{variable}$	$\theta_3 = \text{fixed}$	0
4.	$a_3 = 0;$	$\alpha_3 = 180;$	d_4	θ_4	0
5.	$a_4 = 0;$	$\alpha_4 = 0;$	d_5	—	—

3.4.2 Forward Kinematics of a 5-Axis Robot

D-H parameters of a five axis robot is calculated using the proximal variant of the D-H parameter. Fig. 3.7 shows the structural details of a 5-axes robotic Manipulator.

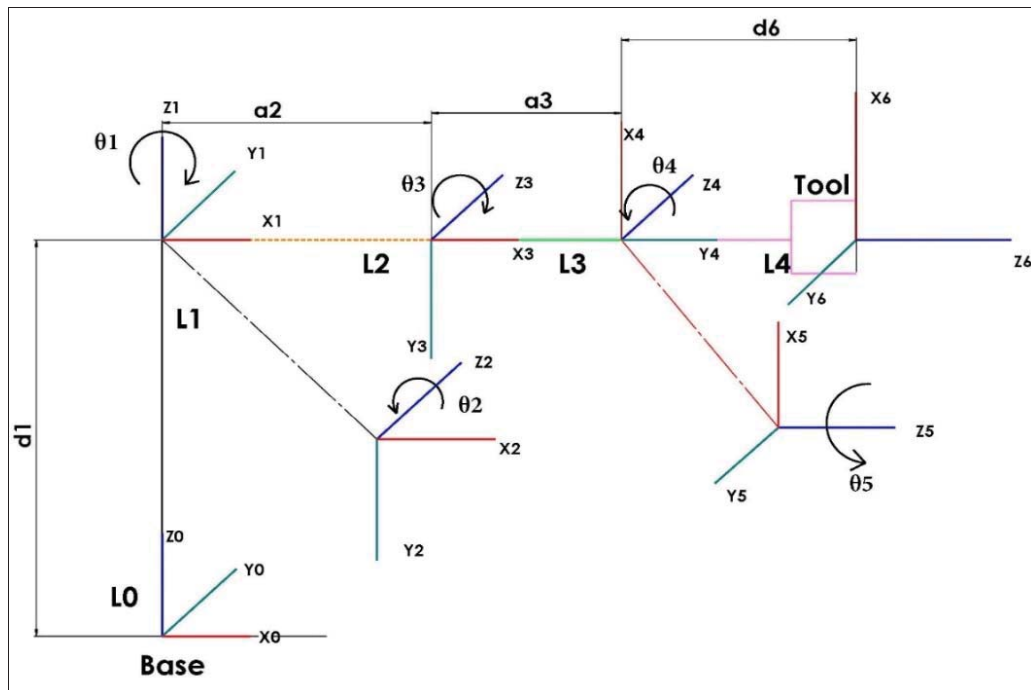


Fig. 3.7 Structural details of a 5-axis robot

Table 3.5: Proximal variant D-H parameters of articulated arm.

S. No.	Link Length (a_{i-1}) (deg.)	Link Twist (α_{i-1}) (deg.)	Joint offset (d_i) (mm)	Joint angle (θ_i) (deg.)	Home position
1.	$a_0 = 0;$	$\alpha_0 = 0;$	d_1	θ_1	0
2.	$a_1 = 0;$	$\alpha_1 = -90;$	$d_2 = 0;$	θ_2	0
3.	a_2	$\alpha_2 = 0;$	$d_3 = 0;$	θ_3	0
4.	a_3	$\alpha_3 = 0;$	$d_4 = 0;$	θ_4	+ 90
5.	$a_4 = 0;$	$\alpha_4 = -90;$	$d_5 = 0;$	θ_5	0
6.	$a_5 = 0;$	$\alpha_5 = 0;$	$d_6;$	—	—

3.4.3 Forward Kinematics of a Six-Axis Robot

The Unimation PUMA 560 is a six DOF robot [2]. It consists of one fixed (L_0) and four moving links (L_1, L_2, L_3 and L_4). All joints are of revolute type.

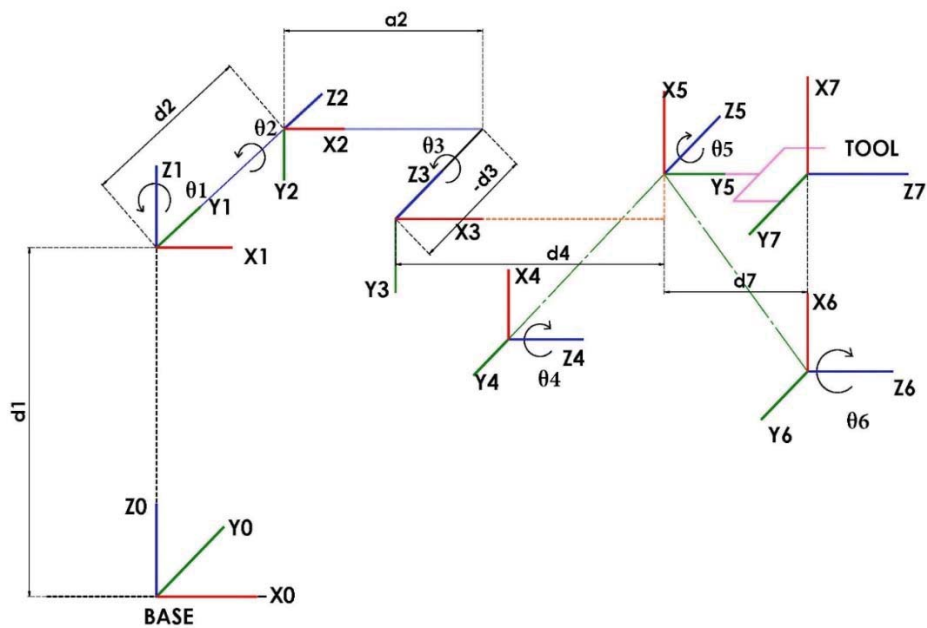


Fig. 3.8: Description of 6-axis Puma 560 robot according to proximal variant.

The end-effector of this robot has Roll-Pitch-Roll motion. The axes about which these motions takes place intersect at a point and three frames one for each, are assigned at this point. Fig. 3.8, describes link-frame assignment and marked D-H parameters (refer Table 3.6) according to proximal variant. This is the structure of the Unimation PUMA 560 robot. No, let us find out the D-H parameters of the same. Here, coordinate frames according to the proximal variant of the D-H parameter have been assigned. To calculate the transformation matrices, D-H parameters are required. Table 3.6 shows the D-H parameters for the above structure.

Table 3.6: Proximal variant D-H parameters of PUMA 560 robot.

S. No.	Link Length (a_{i-1}) (mm)	Link Twist (α_{i-1}) (deg.)	Joint offset (d_i) (mm)	Joint angle (θ_i) (deg.)	Home position
1.	$a_0 = 0;$	$\alpha_0 = 0;$	d_1	θ_1	0
2.	$a_1 = 0;$	$\alpha_1 = -90;$	d_2	θ_2	0
3.	a_2	$\alpha_2 = +90;$	$d_3 = 0;$	θ_3	-90
4.	a_3	$\alpha_3 = -90;$	d_4	θ_4	0
5.	$a_4 = 0;$	$\alpha_4 = +90;$	$d_5 = 0;$	θ_5	0
6.	$a_5 = 0;$	$\alpha_5 = -90;$	$d_6 = 0;$	θ_6	0
7.	$a_6 = 0$	$\alpha_6 = 0$	d_7	-	-

$${}^0_6T = \begin{bmatrix} r_{11}^0 & r_{12}^0 & r_{13}^0 & p_x^0 \\ r_{21}^0 & r_{22}^0 & r_{23}^0 & p_y^0 \\ r_{31}^0 & r_{32}^0 & r_{33}^0 & p_z^0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.8)$$

$$r_{11} = C_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] + S_1(S_4C_5C_6 + C_4S_6) \quad (3.9)$$

$$r_{21} = S_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] - C_1(S_4C_5C_6 + C_4S_6) \quad (3.10)$$

$$r_{31} = -S_{23}(C_4C_5C_6 - S_4S_6) - C_{23}S_5C_6 \quad (3.11)$$

$$r_{12} = C_1[C_{23}(-C_4C_5S_6 - S_4S_6) + S_{23}S_5S_6] + S_1(-S_4C_5S_6 + C_4C_6) \quad (3.12)$$

$$r_{22} = S_1[C_{23}(-C_4C_5S_6 - S_4C_6) + S_{23}S_5S_6] - C_1(-S_4C_5S_6 + C_4C_6) \quad (3.13)$$

$$r_{32} = -S_{23}(-C_4C_5S_6 - S_4C_6) + C_{23}S_5S_6 \quad (3.14)$$

$$r_{13} = -C_1(C_{23}C_4S_5) + S_{23}C_5 - S_1S_4S_5 \quad (3.15)$$

$$r_{23} = -S_1(C_{23}C_4S_5 + S_{23}C_5) - C_1S_4S_5 \quad (3.16)$$

$$r_{33} = S_{23}C_4S_5 - C_{23}C_5 \quad (3.17)$$

$$p_x = C_1[a_2C_2 + a_3C_{23} - d_4S_{23}] - d_3S_1 \quad (3.18)$$

$$p_y = S_1[a_2C_2 + a_3C_{23} - d_4S_{23}] + d_3C_1 \quad (3.19)$$

$$p_z = -a_3S_{23} - a_2S_2 - d_4C_{23} \quad (3.20)$$

The orientation and position of the tool tip can be calculated at any time by simply putting the value in the above equations, i.e. equation 3.23-3.28.

3.5 Summary

In this chapter, forward kinematics of different robotic structures has been studied. Both distal and proximal approaches of the D-H Parameters have been used. Forward kinematic model for some robots has been developed using both distal and proximal variant of the D-H parameter. In the next chapter, forward kinematics of a robotic manipulator, located at CSIR-CSIO, Chandigarh, has been discussed. Forward kinematic model the five DOF robot has been developed, using the proximal variant of the D-H parameter. Some shortcomings of the D-H parameters has been discussed and efforts have been made to improve the D-H parameter method, by doing certain modifications.

Chapter-4

Forward Kinematics Using Dummy Frame

Method

4.1 Introduction

In the previous chapter, forward kinematics of four axis, five axis and six axis robots has been studied using both distal and proximal variants of D-H parameter. The configurations used were very simple and had straight links. So, the calculations were simple. But, in this chapter robot is spatial and complex, for which the kinematic study is not easy. Certain modifications are made in the existing D-H method in order to get accurate results.

The robotic manipulator used here for the study is a robot at CSIR-CSIO, Chandigarh. This robot is a five DOF robot. It has spatial links. Forward kinematic study for the spatial and complex robotic structure is done here. Some shortcomings of the D-H method are discussed. Then, some improvements has been made to remove those shortcomings.

4.2 Case Study: A 5-DOF Spatial Robotic Manipulator

In this section, discussion is focused on kinematic study of a spatial robot at CSIR-CSIO, Chandigarh. The kinematic study here is done using *dummy frame* method, which is an extension of Proximal variant of the D-H.

4.2.1 Structural Details

First of all, have a look at the configuration and structural details of the manipulator, present at CSIR – CSIO, Chandigarh. It is a robot having 5-DOF, which can be used either in a medical surgery or any other industrial purpose. The material used in making the links is mainly cast iron. At some places, steel alloys are also used. For movement in the links, electric actuators are used, as they provide better control accuracy. The robot is slightly heavy. So, the actuators are designed in such a way that the manipulator will move slowly, but accurately.



Fig. 4.1(a): Practical model of the spatial manipulator at CSIR-CSIO, Chandigarh.

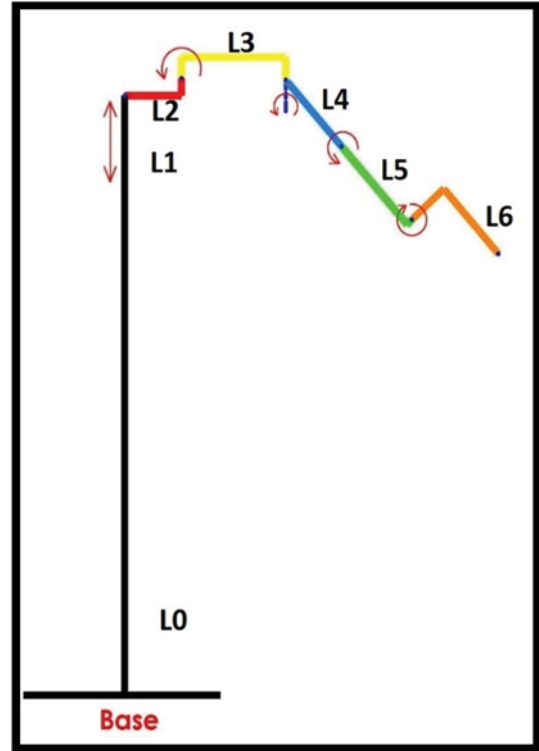


Fig. 4.1(b): Basic Kinematic structure of the spatial manipulator.

So, if used in such a surgery, accuracy will be our prime concern than speed. Fig. 4.1(a) shows the practical model of the spatial manipulator, while a line diagram is drawn in Fig. 4.1(b). From Fig. 4.1(b), it can be observed that the robot has five degree of freedom. The first joint, i.e. the link between L1 and L2 is a prismatic joint. Link L2 can move up to 206 mm downwards from the position as shown in the Fig. 4.1 right now. The next joint, between L2 and L3 is a revolute joint. Link L3 can move around L2 easily, but some constraint is provided so that it does not touch L1. Coming to the next joint between L3 and L4. It is also a revolute joint. Link L4 is mounted at some angle to link L3. The angle at which L4 is mounted to L3 is 42 degrees. Next joint, between L4 and L5, is also a revolute joint. Link 5 is basically a straight link, but the axis of rotation of next joint does not lie at the center of the axis passing through L5. So, a 20 mm portion has been drawn separately. Next, there a revolute joint between L5 and L6. The axis of rotation of this link is perpendicular to plane of link L5. So, due to this some problem may be faced while formulating the kinematic model of this manipulator. Later on, study will made on these difficulties and how to remove the error. Now, have a look at the forward kinematics of this manipulator.

4.2.2 Forward Kinematic Study

In forward kinematics, the position and orientation of the end effector or the last link of the manipulator will be calculated, using the D-H parameter method. At first, arm equation needs to be formulated. For that, D-H parameters are needed. Proximal variant of the D-H parameter method is used here. Arm equation is formed by multiplying different transformation matrices obtained.

The first step towards the kinematic study of the robotic manipulator is the assignment of orthonormal coordinate frames. After assigning the coordinate frames, kinematic model can be developed by using any suitable method for the robotic kinematic study. Most widely used methods include D-H parameter method, S-U method etc. Here, proximal variant of the D-H parameter method will be used. So, after solving the kinematics of the robot, validation of the results will be done.

For validation part Solidworks 2014 has been used. Physical model measurements are also used. In Solidworks, a 3D model of the same robot having same dimensions is made, in order to calculate the position of the end effector for the any set of joint and link variables. Fig. 4.2 shows the dimensions of the spatial manipulator to be studied.

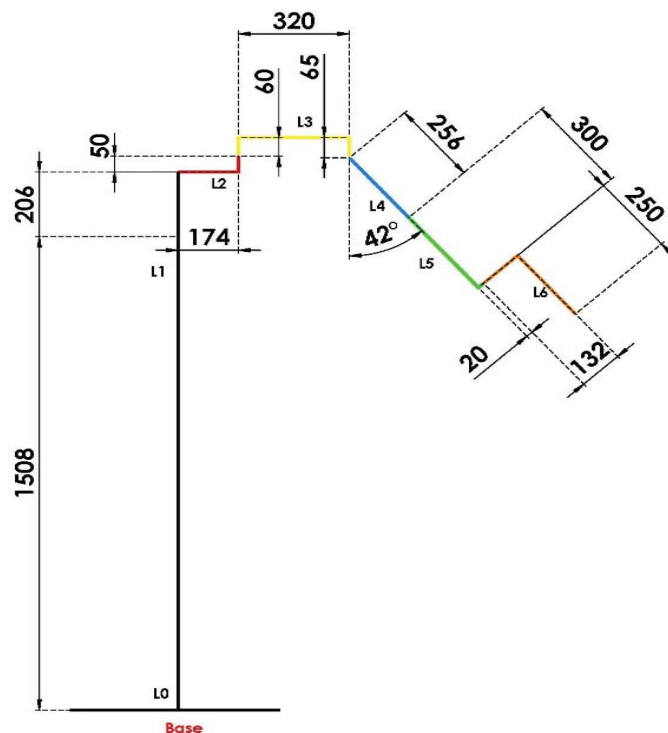


Fig. 4.2 – Line diagram of the manipulator at CSIR-CSIO, Chandigarh.

Fig. 4.2 is detailed diagram of the manipulator. It shows the dimensions of each link and kind of joints between various links. Now, next step is to assign coordinate frames to each and every link in order to start the kinematic study of the robot. Proximal variant of the D-H parameter method is used here to assign the coordinate frames. Coordinates frames in Fig. 4.3 are assigned using the proximal variant of D-H parameter method.

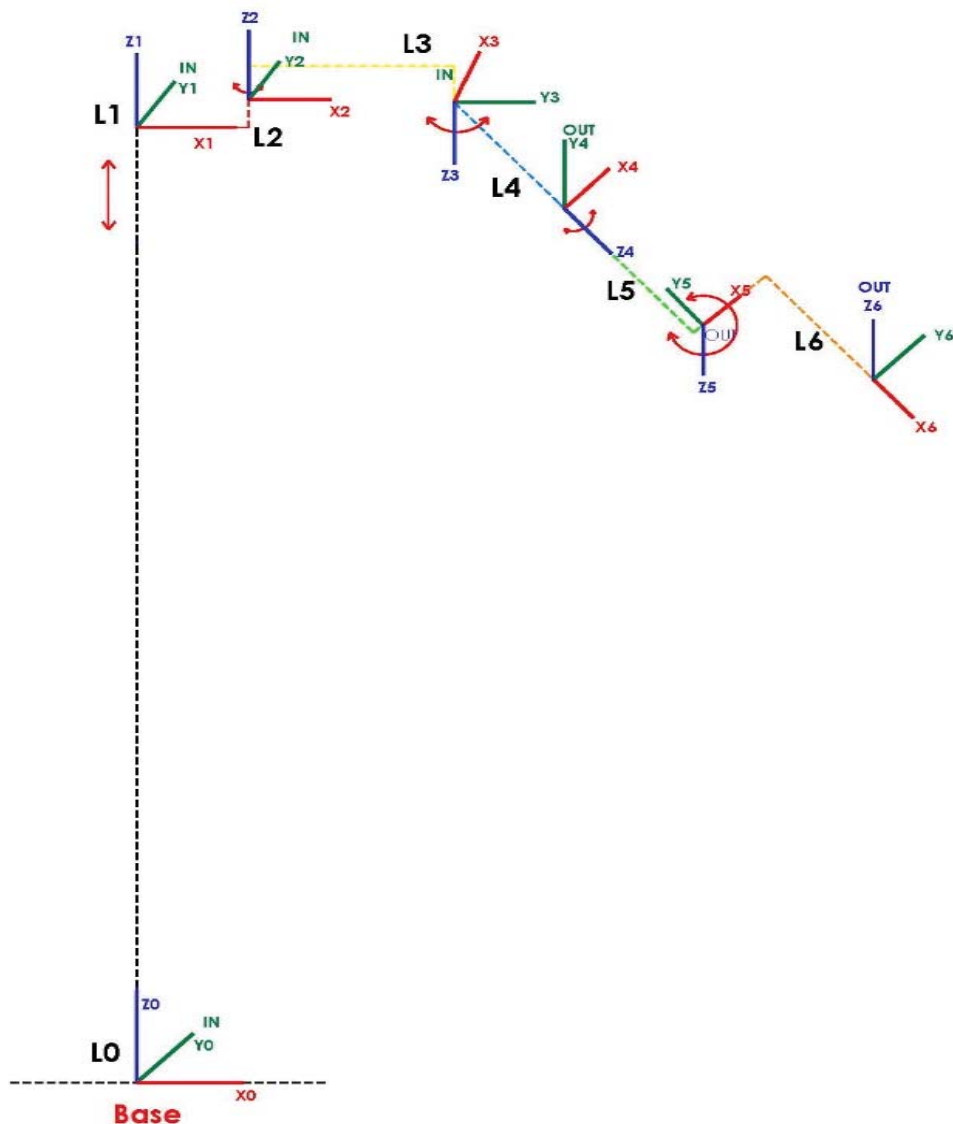


Fig. 4.3 Coordinate frame assignment of robotic manipulator

First assign the orthonormal coordinate frames for all the links. Then, next step is to calculate the D-H parameters. Then, calculate the transformation matrices for all the links. Once, the transformation matrices are calculated, arm matrix can be calculated easily by multiplying all the transformation matrices. It gives the position and orientation of the end effector. Coordinate frames are assigned to all the links of the robot. These are orthonormal coordinate frames. One

thing that can be noticed is that the z-axes are taken as axes of movement at that particular joint. These axes have been shown using different colors for each axis. RGB concept is used here. Red color has been used to represent the X- axis, Green color to represent the Y-axis and blue color for the Z-axis.

In and out notations in the Fig. 4.3 represent either is the axis is going inside the plane or coming out of the plane. Table 4.1 displays all the D-H parameter calculated using the above Fig. 4.3.

Table 4.1: D-H parameters of spatial manipulator

S. No.	Link Length (a_{i-1}) (mm)	Joint Distance (d_i) (mm)	Link Twist (α_{i-1}) (deg.)	Joint angle (θ_i) (deg.)	Home Position
1	$a_0 = 0$	d_1	$\alpha_0 = 0$	$\theta_1 = 0$	1714
2	$a_1 = 174$	$d_2 = 50$	$\alpha_1 = 0$	θ_2	0
3	$a_2 = 320$	$d_3 = 5$	$\alpha_2 = -90$	θ_3	-90
4	$a_3 = 0$	$d_4 = 256$	$\alpha_3 = -42$	θ_4	-90
5	$a_4 = 20$	$d_5 = 0$	$\alpha_4 = -90$	θ_5	0
6	$a_5 = 132$	$d_6 = 0$	$\alpha_5 = 0$	θ_6	-90

4.2.3 Development of Kinematic Model of Robotic Manipulator

To calculate the transformation matrix from i^{th} joint to $(i + 1)^{th}$ joint, using the proximal variant of the D-H parameter, following formula can be used.

$${}^{i-1}_i\mathbf{A} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & | & a_{i-1} \\ s_{\theta_i}c_{\alpha_{i-1}} & c_{\theta_i}c_{\alpha_{i-1}} & -s_{\alpha_{i-1}} & | & -s_{\alpha_{i-1}}d_i \\ s_{\theta_i}s_{\alpha_{i-1}} & c_{\theta_i}s_{\alpha_{i-1}} & c_{\alpha_{i-1}} & | & c_{\alpha_{i-1}}d_i \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.1)$$

By using Eqn. (4.1), the tool-tip of our manipulator can be transformed to its base, as given below:

$${}^0\mathbf{A} = {}^0\mathbf{A} {}^1\mathbf{A} {}^2\mathbf{A} {}^3\mathbf{A} {}^4\mathbf{A} {}^5\mathbf{A}. \quad (4.2)$$

where

$${}^0\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 1714 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.3)$$

$${}^1\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & | & 174 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 50 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.4)$$

$${}^2\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & | & 320 \\ 1 & 0 & 0 & | & 0 \\ 0 & 0 & -1 & | & -5 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.4)$$

$${}^3\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & | & 0 \\ 0.74 & 0 & 0.66 & | & 171.2 \\ -0.66 & 0 & 0.74 & | & 190.2 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.5)$$

$${}^4\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & | & 20 \\ 0 & 0 & 1 & | & 0 \\ 0 & -1 & 0 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.6)$$

$${}^5\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & | & 132 \\ -1 & 0 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.7)$$

Now, final position and orientation of the different links of the robotic manipulator can be obtained using the above equations.

$${}^0_1\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 1714 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.8)$$

$${}^0_2\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & | & 174 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 1764 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.9)$$

$${}^0_3\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & | & 494 \\ 1 & 0 & 0 & | & 0 \\ 0 & 0 & -1 & | & 1759 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.10)$$

$${}^0_4\mathbf{A} = \begin{bmatrix} 0.743 & 0 & 0.669 & | & 665.2 \\ 0 & -1 & 0 & | & 0 \\ 0.669 & 0 & -0.743 & | & 1568.8 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.11)$$

$${}^0_5\mathbf{A} = \begin{bmatrix} 0.743 & -0.669 & 0 & | & 680.16 \\ 0 & 1 & -1 & | & 0 \\ 0.669 & 0.743 & 0 & | & 1582.1 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.12)$$

$${}^0_6\mathbf{A} = \begin{bmatrix} 0.669 & 0.743 & 0 & | & 778.2 \\ 0 & 1 & -1 & | & 0 \\ 0.743 & 0.669 & 0 & | & 1670.5 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.13)$$

These are the transformation matrices obtained after multiplying individual transformation matrices. They give the position and orientation of the respective link. So, to check the accuracy and in order to validate our results, a 3D model has been developed in Solidworks and the real positions of all the links with the same set of the joint and link parameters are calculated. Comparison of both the results is done, in order to check the accuracy of this method. First, it is needed to study the Solidworks 2014 model, in order to find out the positions of the different joints, at home position.

4.3 Development of 3D Model of Robotic Manipulator

For the validation part of the forward kinematics, a 3D model of the robotic manipulator in Solidworks 2014 environment has been developed. All the dimensions of the model are kept similar to the practical version of the robotic manipulator. Some complex parts and motors in our model are not imported to the 3D model, as they do not play any considerable role in the forward kinematic study of the robot.

The 3D model, as shown in Fig. 4.1(b) is modelled in Solidworks 2014 environment. The main purpose behind the 3D model is to verify the positions of different joints. The position at any point can be measured using the measure tool in Solidworks.

At start, the UCS in Solidworks is randomly placed. So, to make sure all the observations are right, the UCS is located at the base of the robot, as all our calculations are done with respect to the base.

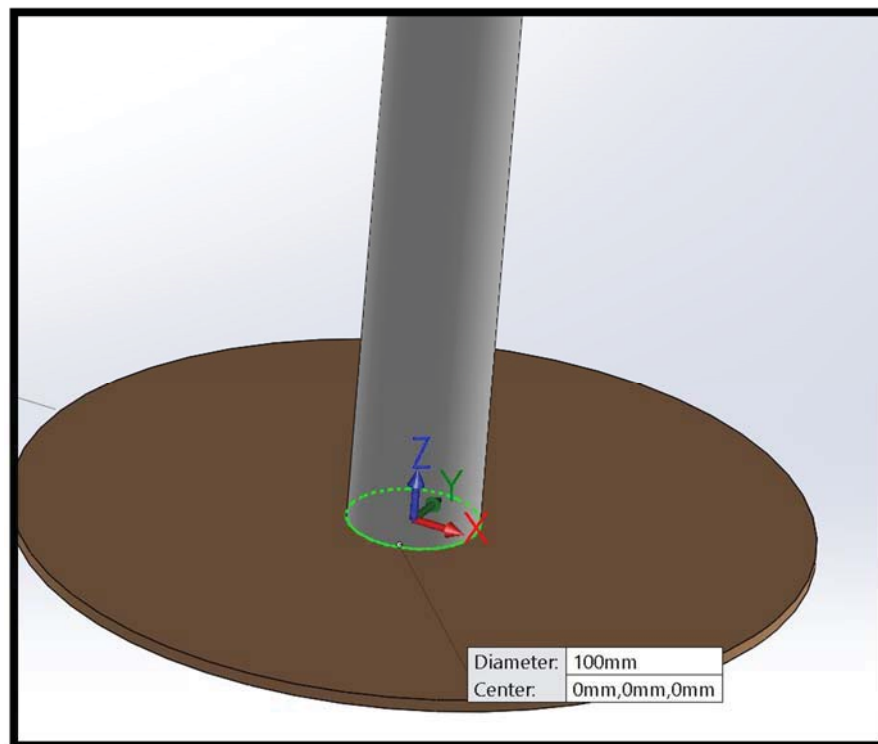


Fig. 4.4: Coordinate system placed at the base of robot

As shown in Fig. 4.4, the location of base is measured using the measure tool in Solidworks.

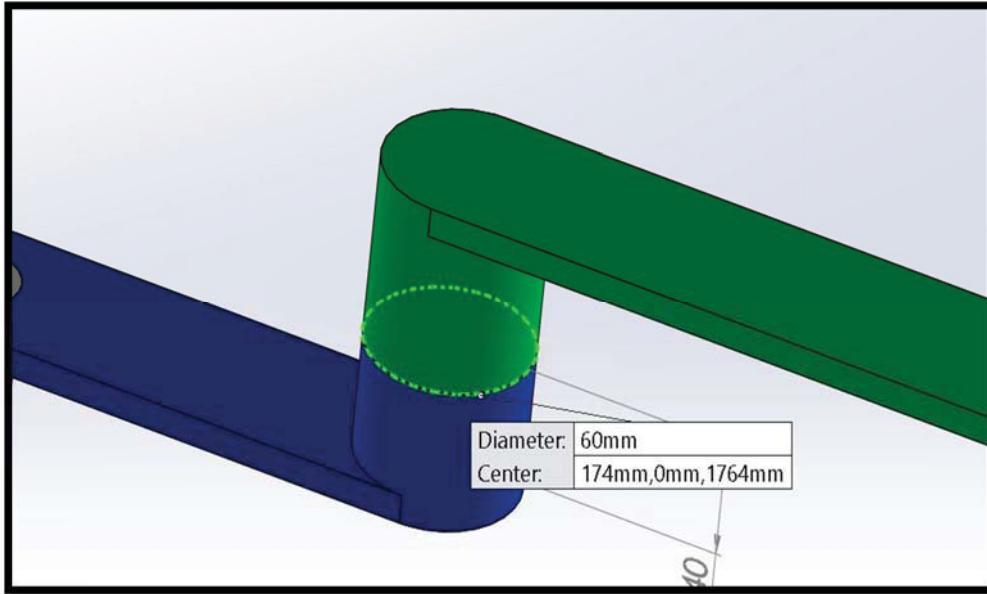


Fig. 4.5: Position of Joint 2 in the space, using measure tool in Solidworks 2014

It can be noticed that the measure tool can easily measure the position of the joint 2. According to measure tool, (174, 0, 1764) is the location of the joint 2. It means the position or location of joint 2 from base is 174 mm away towards X-axis and 1764 mm away in direction of Z-axis. Similarly, location of all the joints is measured.

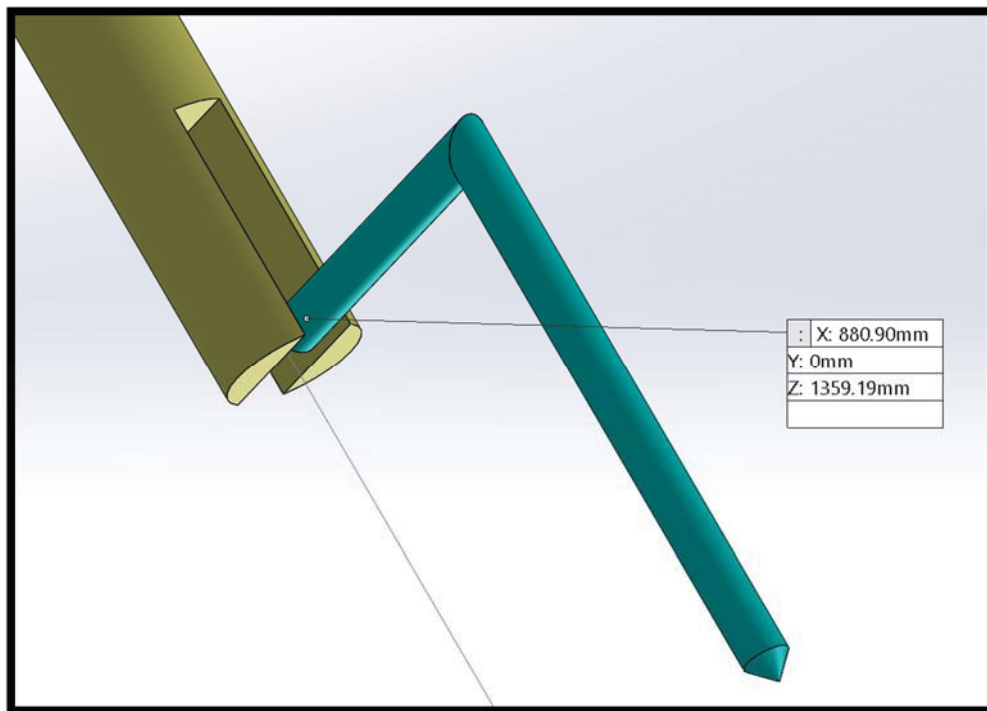


Fig. 4.6: Position of Joint 5 (Revolute Joint) in the space, using measure tool in Solidworks 2014

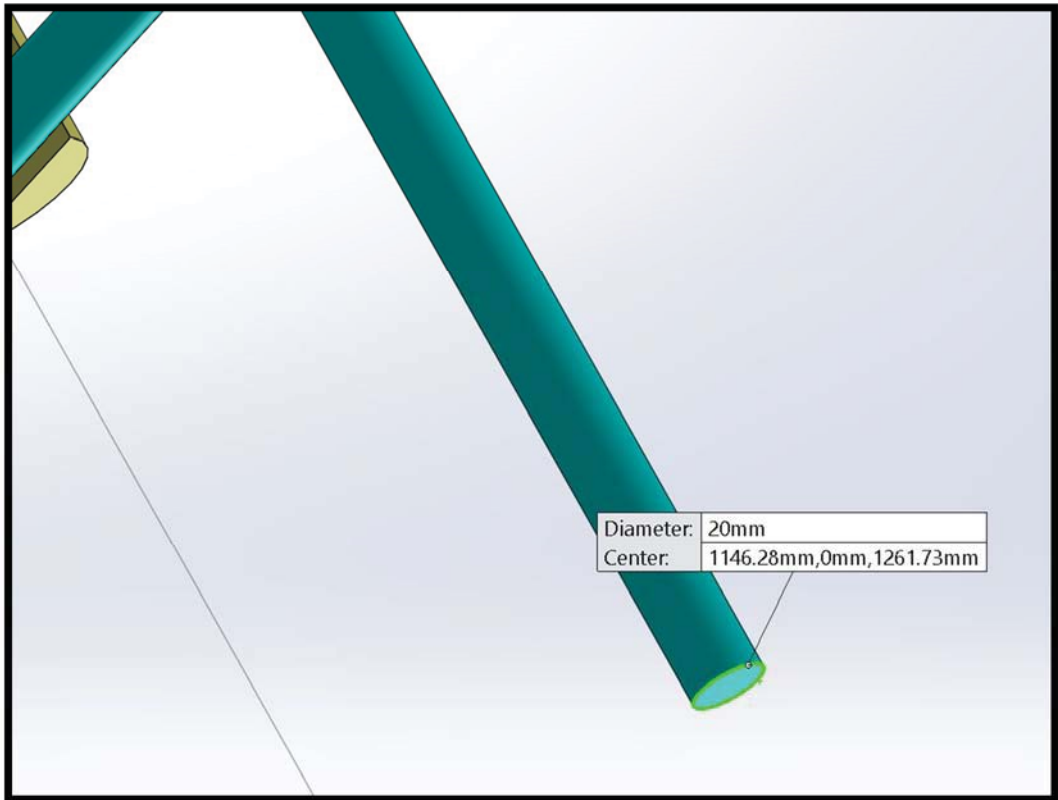


Fig. 4.7: Position of tool tip in the space, using measure tool in Solidworks 2014

So, the positions of all the joints in space in Cartesian coordinates is available now. Positions obtained from D-H method are also available. Now, comparison of both the results is done, in order to check the accuracy of the D-H parameter method.

4.3.1 Result Validation (D-H parameter method)

Comparison of the results obtained by two methods is done in order to check the accuracy of the D-H parameter method. One method is D-H parameter method, and another one is Solidworks 3D model. Positions of all the joints using both methods have been calculated.

This table compares the positions of each link, obtained from D-H parameter method to the real positions. From observations, it can be seen that the first four joints are getting the right positions, while the last two positions are not matching with the real positions. The reason behind the mismatch is that the proximal D-H parameters did not take in to account all the link lengths.

Table 4.2: Comparison sheet of positions of different joints of spatial manipulator

S. No.	Proximal D-H Parameter method			Solidworks (or real position)		
	X (mm)	Y(mm)	Z(mm)	X(mm)	Y(mm)	Z(mm)
1	0	0	1714	0	0	1714
2	174	0	1764	174	0	1764
3	494	0	1759	494	0	1759
4	665.2	0	1568.8	665.2	0	1568.8
5	680.16	0	1582.1	880.9	0	1359.2
6	778.2	0	1670.5	1146.3	0	1260.7

This is the ambiguity noticed in the proximal variant of D-H parameter method. So, in order to remove this error or the discrepancy, some modifications are to be made to the proximal variant of the D-H parameter method. D-H parameter and its variants can take in to account all the dimensions of a planar link easily and a serial robot. But here the links are spatial and complex. So, it is not possible for the D-H parameter method to solve problem accurately. It is needed to make modifications in the proximal variant of the D-H parameter method.

4.4 Ambiguities in D-H parameter method

From our observation, it can be noticed that the results of D-H parameters are not exactly matching with what are the real positions of the manipulator. Up to joint 4, positions are matching, but after that there is difference between the results obtained by two methods.

The D-H method was not able to consider the lengths of 300 and 250 in link 5 and link 6 respectively. The reason behind this is that the joint of link 5 (L5) and link 6 (L6) is such that the axis of rotation lies in a plane perpendicular to plane of both the links. That's why, D-H parameter is not able to give the correct results to us in case of spatial linkages.

4.5 Improvement in D-H parameters

As seen above, the proximal variant of the D-H parameter was not able to give us the correct result in case of our robotic manipulator. So, certain modifications are made to the proximal variant of the D-H parameter in order to solve the kinematic analysis of the spatial manipulator.

4.5.1 Concept of Dummy Frames

According to the concept of *dummy frames*, for a spatial structure, an extra frame can be used at a point, where there is no relative motion, in order to take in to account all the segments of the structure [21].

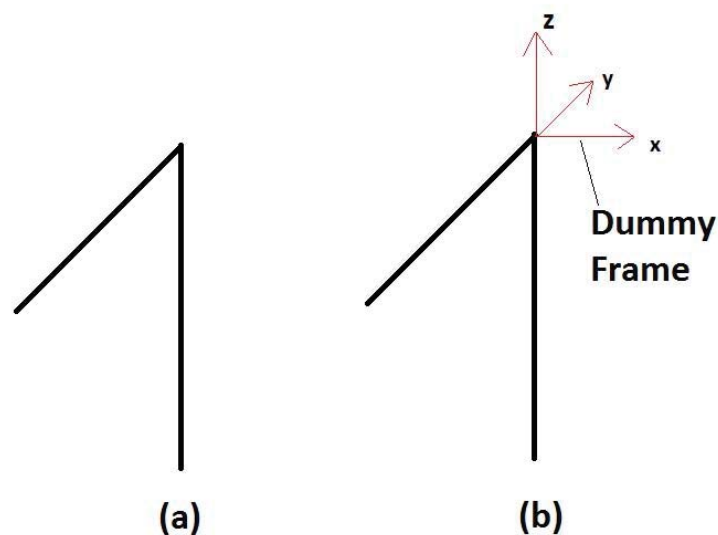


Fig. 4.8-(a): A Spatial link without dummy frame. Fig. 4.8-(b) A spatial link with dummy frame.

When this concept is applied to a spatial link, as shown in Fig. 4.8-(b), it eliminates the deficiency of D-H notation. Dummy frame is different from D-H frame. It is placed at a point where there is no motion, whereas a D-H frame is placed at a point where motion is taking place. Absence of motion at dummy frame does not imply that joint angle corresponding to dummy frame vanishes rather it becomes a constant, whose value can be determined (as angle from x_{i-1} to x_i about z_i) according to D-H algorithm [21]. According to this concept, dummy frames are introduced at two links, L5 and L6.

When this concept is applied to a spatial link, as shown in Fig. 4.9, it accounts for all the segments of spatial link into corresponding D-H parameters and eliminates the deficiency of D-H notation. Dummy frame is different from D-H frame in the sense that it is placed at a point where there is no motion, whereas a D-H frame is placed at a point where motion is taking

place. Absence of motion at dummy frame does not mean that joint angle corresponding to dummy frame vanishes rather it becomes a constant, whose value can be determined according to D-H algorithm.

➤ **Rules to Assign Dummy Frames [21]:**

1. First of all, Z-axis is assigned in the appropriate direction.
2. Next, assign X-axis according to D-H notation.
3. Use right hand thumb rule to assign Y-axis.

In the present work, two dummy frames are attached to spatial Manipulator. As there is no motion taking place about any of the axis of assigned dummy frame, that's why, they are known as dummy frames.

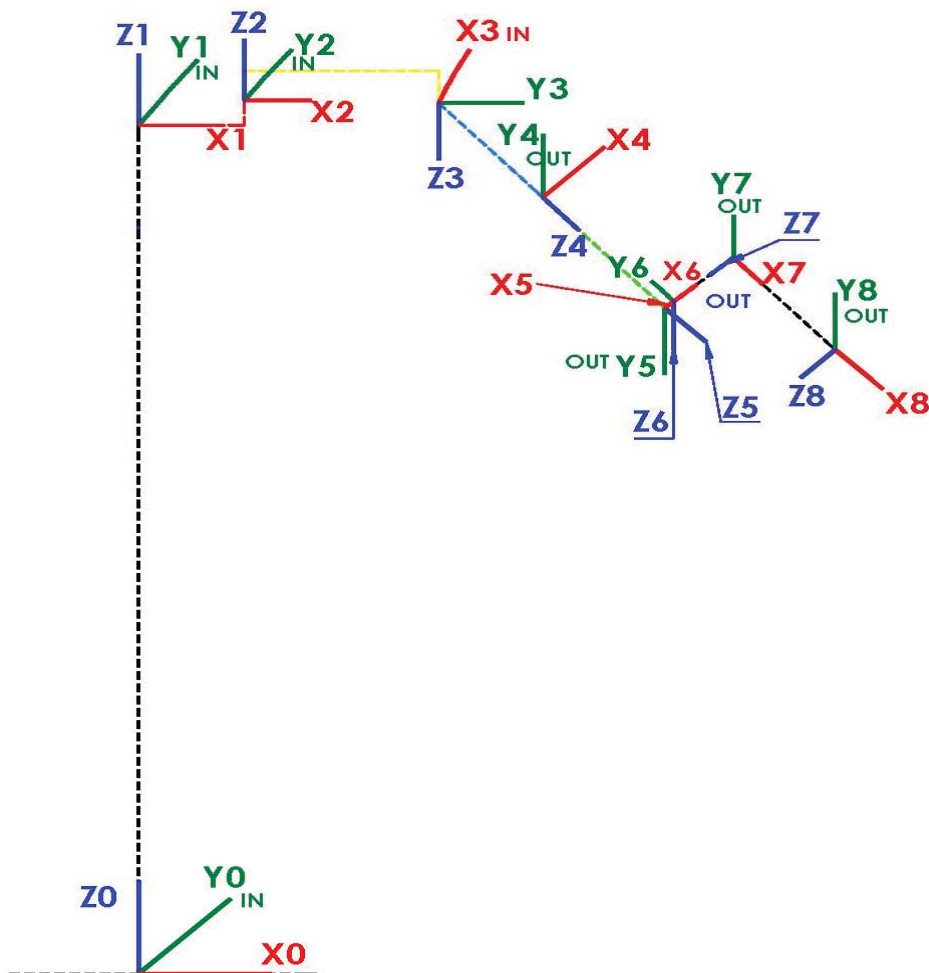


Fig. 4.9: Coordinates frames assigned along with dummy frames to the manipulator.

Fig. 4.9 and Fig. 4.10 shows the assignment of orthonormal coordinate frames to the spatial manipulator.

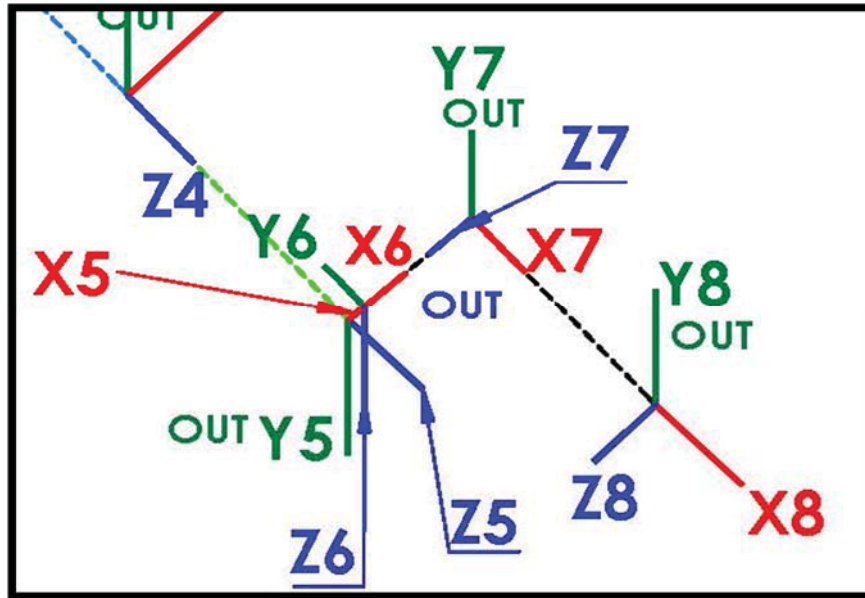


Fig. 4.10: Zoom in snap of fig.4.12 showing dummy frames assigned to the manipulator.

Fig. 4.10 shows the two dummy frames assigned, frame no. 5(X5, Y5, Z5) and frame no. 7(X7, Y7, Z7). Table 4.3 contains the D-H parameters for the manipulator.

Table 4.3: D-H parameter of robotic manipulator at CSIR-CSIO Lab

S. No.	Link Length (a_{i-1}) (mm)	Joint Distance (d_i) (mm)	Link Twist (α_{i-1}) (deg.)	Joint angle (θ_i) (deg.)	Home Position
1	$a_0 = 0$	d_1	$\alpha_0 = 0$	$\theta_1 = 0$	1714
2	$a_1 = 174$	$d_2 = 50$	$\alpha_1 = 0$	θ_2	0
3	$a_2 = 320$	$d_3 = 5$	$\alpha_2 = 90$	θ_3	-90
4	$a_3 = 0$	$d_4 = 256$	$\alpha_3 = -42$	θ_4	-90
5	$a_4 = 0$	$d_5 = 300$	$\alpha_4 = 0$	$\theta_5 = 0$	-
6	$a_5 = 20$	$d_6 = 0$	$\alpha_5 = -90$	θ_6	0
7	$a_6 = 132$	$d_7 = 0$	$\alpha_6 = 0$	$\theta_7 = -90$	-
8	$a_6 = 250$	$d_8 = 0$	$\alpha_6 = 0$	θ_8	0

To calculate the transformation matrix from i^{th} joint to $(i + 1)^{th}$ joint, this formula is used.

$${}^{i-1}_i\mathbf{A} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & | & a_{i-1} \\ s_{\theta_i}c_{\alpha_{i-1}} & c_{\theta_i}c_{\alpha_{i-1}} & -s_{\alpha_{i-1}} & | & -s_{\alpha_{i-1}}d_i \\ s_{\theta_i}s_{\alpha_{i-1}} & c_{\theta_i}s_{\alpha_{i-1}} & c_{\alpha_{i-1}} & | & c_{\alpha_{i-1}}d_i \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.14)$$

By using Eqn. (4.1), the tool-tip of our manipulator can be transformed to its base, as given below:

$${}^0_8\mathbf{A} = {}^0_1\mathbf{A} {}^1_2\mathbf{A} {}^2_3\mathbf{A} {}^3_4\mathbf{A} {}^4_5\mathbf{A} {}^5_6\mathbf{A} {}^6_7\mathbf{A} {}^7_8\mathbf{A}. \quad (4.15)$$

where

$${}^0_1\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 1714 \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.16)$$

$${}^1_2\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & | & 174 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 50 \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.17)$$

$${}^2_3\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & | & 320 \\ 1 & 0 & 0 & | & 0 \\ 0 & 0 & -1 & | & -5 \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.18)$$

$${}^3_4\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & | & 0 \\ 0.74 & 0 & 0.66 & | & 171.2 \\ -0.66 & 0 & 0.74 & | & 190.2 \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.19)$$

$${}^4_5\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 300 \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.20)$$

$${}^5_6\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & | & 20 \\ 0 & 0 & 1 & | & 0 \\ 0 & -1 & 0 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.21)$$

$${}^6_7\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & | & 132 \\ -1 & 0 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.22)$$

$${}^7_8\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & | & 250 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.23)$$

Now, final position and orientation of the different links of the robotic manipulator is:

$${}^0_1\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 1714 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.24)$$

$${}^0_2\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & | & 174 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 1764 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.25)$$

$${}^0_3\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & | & 494 \\ 1 & 0 & 0 & | & 0 \\ 0 & 0 & -1 & | & 1759 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.26)$$

$${}^0_4\mathbf{A} = \begin{bmatrix} 0.743 & 0 & 0.669 & | & 665.2 \\ 0 & -1 & 0 & | & 0 \\ 0.669 & 0 & -0.743 & | & 1568.8 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.27)$$

$${}^0_5\mathbf{A} = \begin{bmatrix} 0.743 & 0.669 & 0 & | & 866.036 \\ 0 & 1 & -1 & | & 0 \\ 0.669 & -0.743 & 0 & | & 1345.8 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.28)$$

$${}^0_6\mathbf{A} = \begin{bmatrix} 0.7431 & -0.669 & 0 & | & 880.89 \\ 0 & 1 & -1 & | & 0 \\ 0.669 & 0.743 & 0 & | & 1359.2 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.29)$$

$${}^0_7\mathbf{A} = \begin{bmatrix} 0.7431 & 0.743 & 0 & | & 978.9 \\ 0 & 1 & -1 & | & 0 \\ -0.743 & 0.669 & 0 & | & 1447.5 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.30)$$

$${}^0_8\mathbf{A} = \begin{bmatrix} 0.669 & 0.743 & 0 & | & 1146.3 \\ 0 & 1 & -1 & | & 0 \\ -0.743 & 0.669 & 0 & | & 1261.7 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad (4.31)$$

These transformation matrices give the position and orientation of the respective links with respect to the base of the robot.

4.5.2 Result Validation

Let us compare the results obtained by adding dummy frames to the manipulator with the results obtained in the Solidworks model. If results obtained by both the methods match, it means our approach of using the dummy frames is right.

Table 4.4: Comparison sheet of positions of different links

S. No.	Proximal D-H Parameter method			Solidworks (or real position)		
	X (mm)	Y(mm)	Z(mm)	X(mm)	Y(mm)	Z(mm)
1	0	0	1714	0	0	1714
2	174	0	1764	174	0	1764
3	494	0	1759	494	0	1759
4	665.2	0	1568.8	665.2	0	1568.8
5	866	0	1345.8	866	0	1345.8
6	880.9	0	1359.2	880.9	0	1359.2
7	978.99	0	1447.5	978.99	0	1447.5
8	1146.3	0	1260.7	1146.3	0	1260.7

4.6 Summary

After the detailed study, it can be concluded that forward kinematic study of a robotic manipulator having spatial links is possible only with the help of dummy frames. Without using the dummy frames, results were not matching with the physical prototype. But the concept of dummy frames changed the results and the results are in accordance with the physical prototype. So, a new concept of *dummy frames* has been introduced in this chapter.

Chapter-5

Trajectory Tracking of the 5-DOF Spatial Manipulator

5.1 Introduction

Kinematic study of spatial manipulator is done in the previous chapter. Proximal variant of the D-H parameter method is used to solve the forward kinematics of the spatial manipulator. Trajectory tracking is one of the most important tasks for a robotic manipulator. As it is not an easy task to control the robot to follow a certain path. In this Chapter, main focus has been put on the trajectory tracking. The spatial manipulator is controlled in such a way that it follows the straight line as defined by user. For this purpose, a software names *SimWise 4D* has been used.

5.2 Trajectory Tracking

To make the robot follow a specified path is called trajectory tracking. In literature, methods like Joint Space Scheme has been used for trajectory tracking of the robots. In this chapter, trajectory tracking is done with the help of a software known as *SimWise 4D*. The spatial manipulator is made to follow a straight line in this chapter. At first, basic concepts of *SimWise 4D* are discussed and then trajectory tracking is done later on. The process starts with importing the 3D model in to the *SimWise 4D*.

5.2.1 Importing the 3D Model

The process starts with importing the 3D model in to the *SimWise 4D* environment. There are options in *SimWise 4D* to import a 3D model directly from Solidworks, PTC Creo and Inventor. From other software, 3D model can be imported through IGES file. One thing that need to be taken care is that the assembly and parts should be present in the same folder of the computer, otherwise *SimWise 4D* will not show the parts in its environment. There is one more option in *SimWise 4D*, the most convenient one, using this option the whole assembly can be built in *SimWise 4D* itself. This option gives most accurate results and is very simple to use.

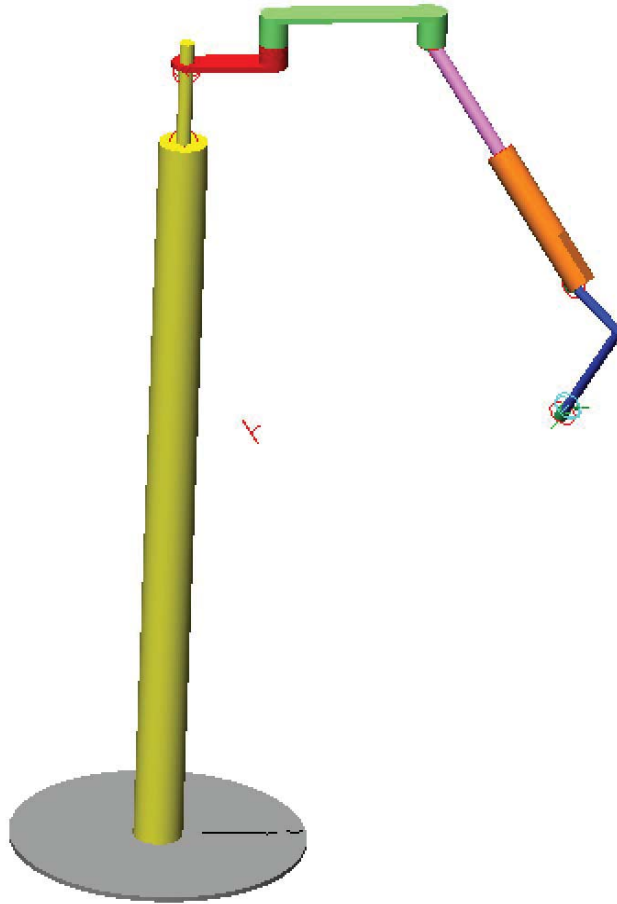


Fig. 5.1: Spatial Manipulator in *SimWise 4D* environment.

Fig. 5.1 shows the 3D model of the spatial manipulator, assembled in *SimWise 4D*. All the links of the spatial manipulator are clearly visible in the Fig. 5.1. This is the same manipulator which was discussed in the previous chapter for kinematic analysis. All the parts are imported separately in the *SimWise 4D* environment and assembled in the *SimWise 4D* itself. There are some options present in the *SimWise 4D* to give different kind of joints between the links. Required joint types are selected and assembled to form the spatial manipulator.

5.2.2 Assigning the Coordinate Frames

After importing the 3D model to the *SimWise 4D*, next step is to assign the coordinate frames to all links. The coordinate frames are assigned in order to assign meters (measuring tool in *SimWise 4D*) at required points. On the spatial manipulator, meters are placed at all the links.

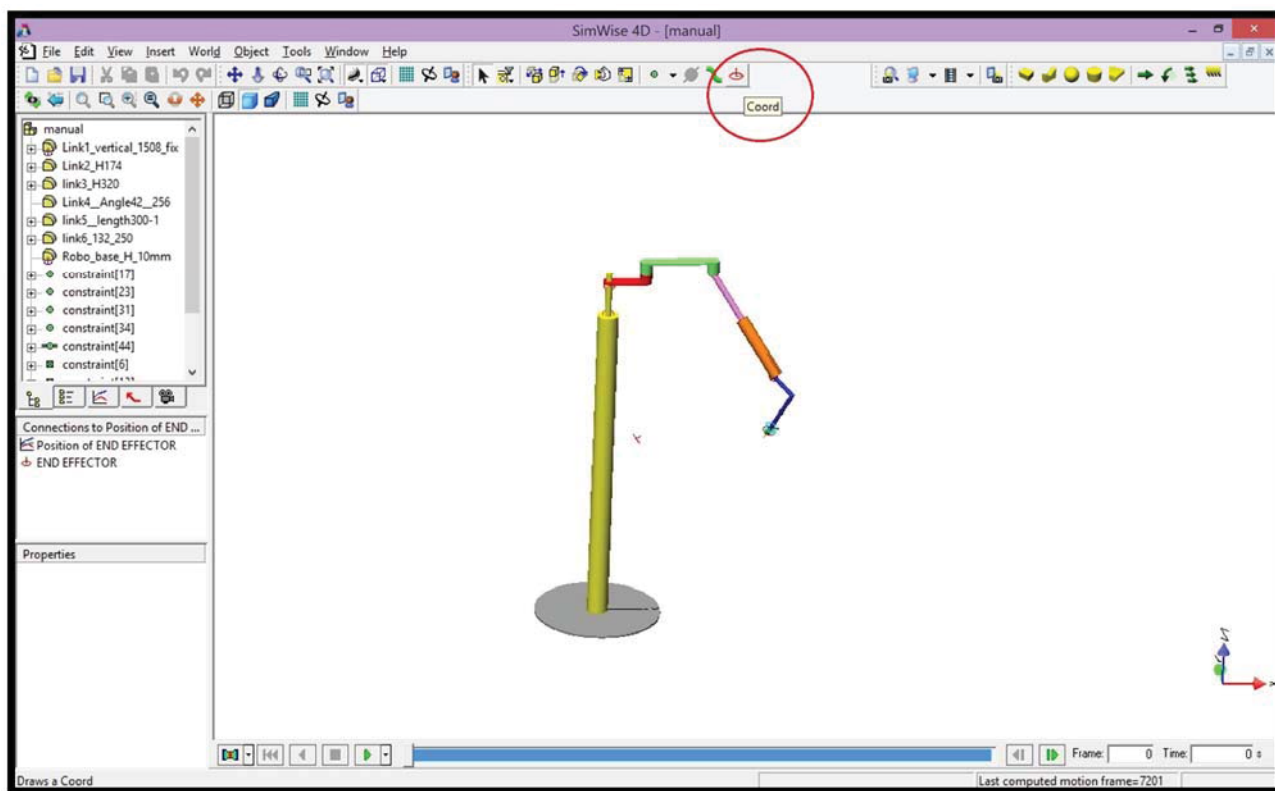


Fig. 5.2: Frame assignment to Manipulator in *SimWise 4D* environment.

Fig. 5.2 shows the screenshot of the *SimWise 4D* environment. An option to insert frame is given in the taskbar at the top. Using this option, coordinate frames are inserted at every link of the spatial manipulator.

5.2.3 Inserting Meter

In *SimWise 4D*, meters are inserted at required points for different kind of measurements. An option of meter is given in the Insert menu. Using this tool, a meter can be inserted at any point. A meter tool in *SimWise 4D* can be used to measure position, velocity, acceleration, linear momentum, orientation, angular velocity, angular acceleration, displacement, force, torque etc. Here, meter tool is used to measure the position and orientation of all links. Meter tool gives the output in form of either a graph or digital form. This data can be copied to an excel sheet by using simple copy paste commands. The meters located at all the links will give data regarding the positions of the all the links at any particular time. This data is then used to make a MATLAB code with the help of which, spatial manipulator will trace the required trajectory. Two meters are assigned at every link, one for the position and another for the orientation of the spatial manipulator.

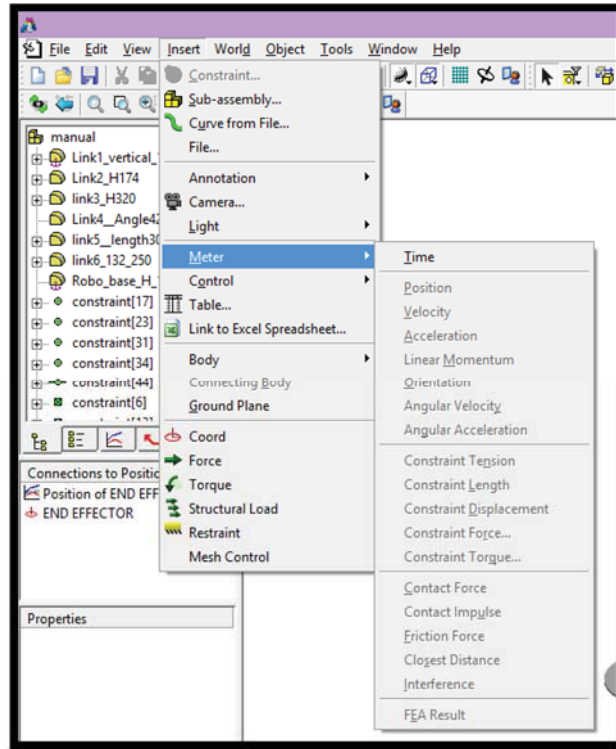


Fig. 5.3: *SimWise 4D*: Inserting meter.

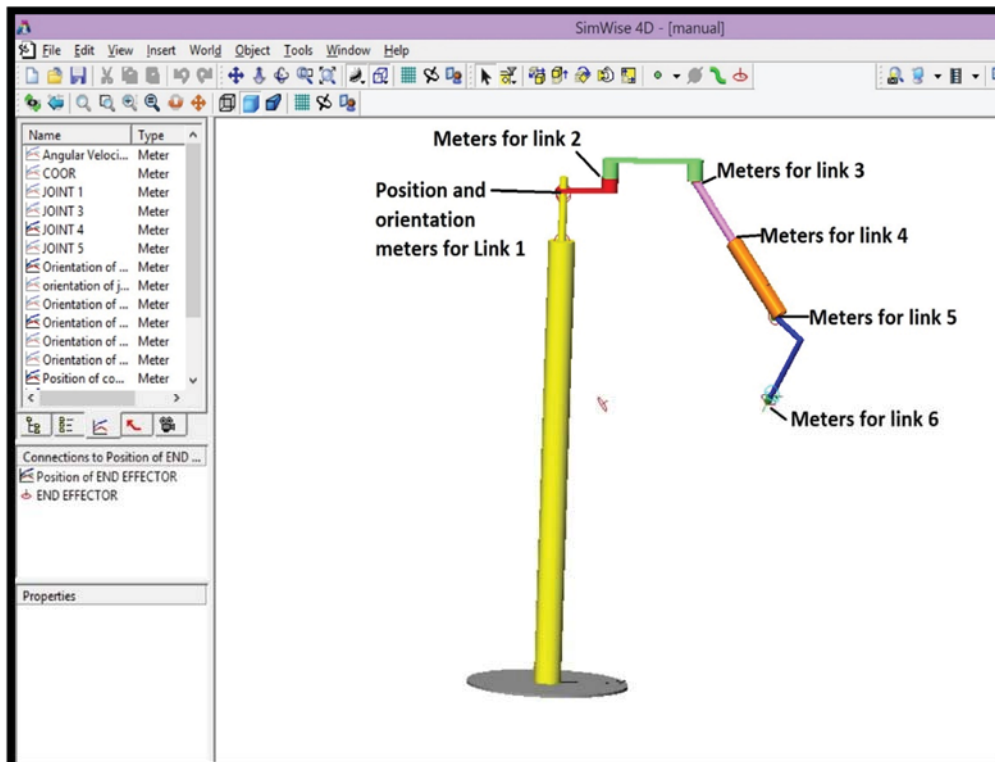


Fig. 5.4: *SimWise 4D*: Meter Locations.

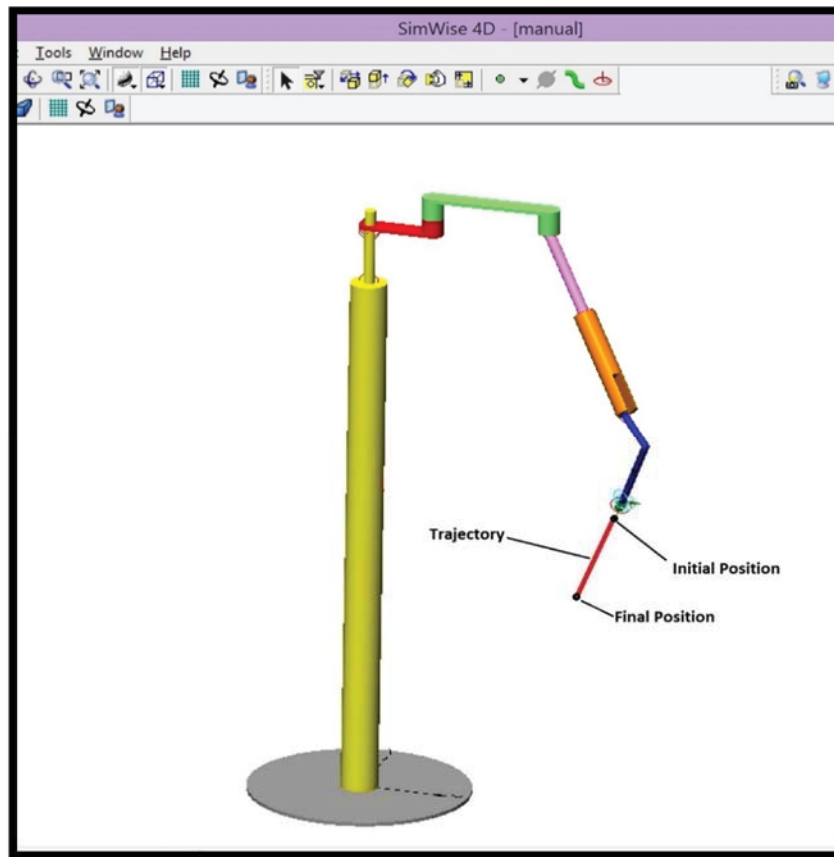


Fig. 5.5: *SimWise 4D*: Manipulator end effector at Initial Position.

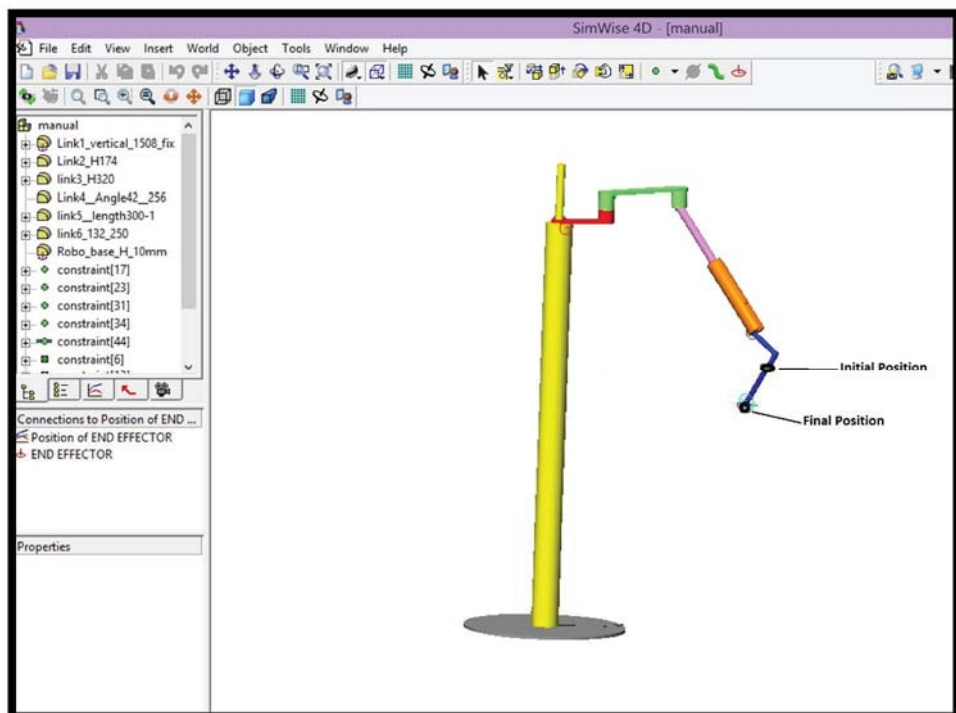


Fig. 5.6: *SimWise 4D*: Manipulator Final Position.

In Fig. 5.4, all the meters inserted in the model are shown. At each position, meters are inserted to measure the position and orientation of the links. In Fig. 5.5 and 5.6, Initial and Final positions of the end effector is shown.

5.2.4 Trajectory Following

As shown in Fig. 5.5, a straight line is drawn in Red color. This is the required trajectory for the spatial manipulator to follow. First the Manipulator is moved on the desired trajectory, by using *SimWise 4D*. Using the *SimWise 4D*, position records of every link at particular period of time has been created, with frequency of 1 second. Table 5.1 shows the position record of meter attached at link 1.

Table 5.1: Position record of Meter 1

Time (sec)	X(mm)	Y(mm)	Z(mm)
1	0.00	0.00	1682.89
2	0.00	0.00	1682.25
3	0.00	0.00	1680.34
4	0.00	0.00	1677.15
5	0.00	0.00	1672.66
6	0.00	0.00	1666.86
7	0.00	0.00	1659.71
8	0.00	0.00	1651.21
9	0.00	0.00	1641.33
10	0.00	0.00	1630.04
11	0.00	0.00	1617.34
12	0.00	0.00	1603.21
13	0.00	0.00	1587.64
14	0.00	0.00	1570.61
15	0.00	0.00	1552.11
16	0.00	0.00	1532.15
17	0.00	0.00	1520.53
18	0.00	0.00	1520.50
19	0.00	0.00	1520.46
20	0.00	0.00	1520.42
21	0.00	0.00	1520.39
22	0.00	0.00	1520.35
23	0.00	0.00	1520.32
24	0.00	0.00	1520.28
25	0.00	0.00	1520.25
26	0.00	0.00	1520.21
27	0.00	0.00	1520.17
28	0.00	0.00	1520.14
29	0.00	0.00	1520.10
30	0.00	0.00	1520.07
31	0.00	0.00	1520.03
32	0.00	0.00	1520.00

33	0.00	0.00	1519.96
34	0.00	0.00	1519.92
35	0.00	0.00	1519.89
36	0.00	0.00	1519.85
37	0.00	0.00	1519.82
38	0.00	0.00	1519.78

It can be observed that the reading of positions in X and Y remain constant, but in direction of Z, it decreases with time. While the orientation which is in degrees, remain constant for all 38 units of time.

Table 5.2: Path and Orientation record of Meter 2

Time (sec)	X(mm)	Y(mm)	Z(mm)
1	174.00	0.00	1722.89
2	174.00	0.00	1722.25
3	174.00	0.00	1720.34
4	174.00	0.00	1717.15
5	174.00	0.00	1712.66
6	174.00	0.00	1706.86
7	174.00	0.00	1699.71
8	174.00	0.00	1691.21
9	174.00	0.00	1681.33
10	174.00	0.00	1670.04
11	174.00	0.00	1657.34
12	174.00	0.00	1643.21
13	174.00	0.00	1627.64
14	174.00	0.00	1610.61
15	174.00	0.00	1592.11
16	174.00	0.00	1572.15
17	174.00	0.00	1560.53
18	174.00	0.00	1560.50
19	174.00	0.00	1560.46
20	174.00	0.00	1560.42
21	174.00	0.00	1560.39
22	174.00	0.00	1560.35
23	174.00	0.00	1560.32
24	174.00	0.00	1560.28
25	174.00	0.00	1560.25
26	174.00	0.00	1560.21
27	174.00	0.00	1560.17
28	174.00	0.00	1560.14
29	174.00	0.00	1560.10
30	174.00	0.00	1560.07
31	174.00	0.00	1560.03
32	174.00	0.00	1560.00
33	174.00	0.00	1559.96
34	174.00	0.00	1559.92
35	174.00	0.00	1559.89
36	174.00	0.00	1559.85

37	174.00	0.00	1559.82
38	174.00	0.00	1559.78

These position records are saved for all the links. Table 5.3 shows the meter readings at joint between links L2 and L3.

Table 5.3: Path and Orientation record of Meter 3

Time (sec)	X(mm)	Y(mm)	Z(mm)
1	493.85	9.93	1717.89
2	493.83	10.54	1717.25
3	493.76	12.37	1715.34
4	493.63	15.39	1712.15
5	493.40	19.56	1707.66
6	493.04	24.81	1701.86
7	492.48	31.11	1694.71
8	491.69	38.36	1686.21
9	490.60	46.51	1676.33
10	489.15	55.49	1665.04
11	487.28	65.22	1652.34
12	484.93	75.63	1638.21
13	482.04	86.68	1622.64
14	478.53	98.29	1605.61
15	474.35	110.41	1587.11
16	469.43	122.98	1567.15
17	466.37	130.08	1555.53
18	466.36	130.10	1555.50
19	466.35	130.12	1555.46
20	466.34	130.14	1555.42
21	466.33	130.16	1555.39
22	466.32	130.18	1555.35
23	466.31	130.20	1555.32
24	466.30	130.23	1555.28
25	466.29	130.25	1555.25
26	466.28	130.27	1555.21
27	466.27	130.29	1555.17
28	466.26	130.31	1555.14
29	466.26	130.33	1555.10
30	466.25	130.36	1555.07
31	466.24	130.38	1555.03
32	466.23	130.40	1555.00
33	466.22	130.42	1554.96
34	466.21	130.44	1554.92
35	466.20	130.46	1554.89
36	466.19	130.48	1554.85
37	466.18	130.51	1554.82
38	466.17	130.53	1554.78

Table 5.4 shows the meter readings at joint between L3 and L4. These readings indicate the movement of the links. Similarly, positions for all the links are stored in tabular form. Once these positions are stored, can be used to program the Manipulator to follow the same path.

Table 5.4: Path and Orientation record of Meter 4

Time (sec)	X(mm)	Y(mm)	Z(mm)
1	629.99	-94.58	1527.27
2	629.88	-94.09	1526.63
3	629.53	-92.63	1524.72
4	628.93	-90.22	1521.53
5	628.06	-86.87	1517.04
6	626.88	-82.63	1511.23
7	625.38	-77.51	1504.09
8	623.50	-71.57	1495.59
9	621.21	-64.84	1485.71
10	618.47	-57.37	1474.42
11	615.23	-49.19	1461.72
12	611.46	-40.34	1447.59
13	607.11	-30.86	1432.02
14	602.14	-20.79	1414.99
15	596.51	-10.16	1396.49
16	590.17	1.00	1376.53
17	586.36	7.35	1364.91
18	586.35	7.37	1364.87
19	586.34	7.39	1364.84
20	586.32	7.41	1364.80
21	586.31	7.43	1364.77
22	586.30	7.45	1364.73
23	586.29	7.47	1364.70
24	586.28	7.49	1364.66
25	586.27	7.51	1364.62
26	586.25	7.52	1364.59
27	586.24	7.54	1364.55
28	586.23	7.56	1364.52
29	586.22	7.58	1364.48
30	586.21	7.60	1364.45
31	586.19	7.62	1364.41
32	586.18	7.64	1364.37
33	586.17	7.66	1364.34
34	586.16	7.68	1364.30
35	586.15	7.70	1364.27
36	586.14	7.72	1364.23
37	586.12	7.74	1364.20
38	586.11	7.76	1364.16

Table 5.5: Path and Orientation record of Meter 5

Time (sec)	X(mm)	Y(mm)	Z(mm)
1	777.77	-207.32	1290.93
2	777.54	-206.97	1290.30
3	776.86	-205.94	1288.39
4	775.73	-204.22	1285.19
5	774.12	-201.81	1280.70
6	772.03	-198.73	1274.90
7	769.43	-194.96	1267.75
8	766.30	-190.53	1259.25
9	762.64	-185.43	1249.37
10	758.41	-179.68	1238.08
11	753.61	-173.27	1225.39
12	748.20	-166.22	1211.26
13	742.18	-158.54	1195.68
14	735.53	-150.23	1178.65
15	728.23	-141.30	1160.16
16	720.27	-131.77	1140.21
17	715.59	-126.26	1128.59
18	715.58	-126.24	1128.55
19	715.56	-126.23	1128.52
20	715.55	-126.21	1128.48
21	715.53	-126.19	1128.45
22	715.52	-126.18	1128.41
23	715.50	-126.16	1128.37
24	715.49	-126.14	1128.34
25	715.48	-126.12	1128.30
26	715.46	-126.11	1128.27
27	715.45	-126.09	1128.23
28	715.43	-126.07	1128.20
29	715.42	-126.06	1128.16
30	715.40	-126.04	1128.12
31	715.39	-126.02	1128.09
32	715.38	-126.01	1128.05
33	715.36	-125.99	1128.02
34	715.35	-125.97	1127.98
35	715.33	-125.96	1127.95
36	715.32	-125.94	1127.91
37	715.30	-125.92	1127.87
38	715.29	-125.91	1127.84

Table 5.5 shows the meter readings at joint between L4 and L5. Table 5.6 shows the meter readings at joint between L5 and L6. All the dimensions and readings are in mm.

Table 5.6: Path and Orientation record of Meter 6

Time (sec)	X(mm)	Y(mm)	Z(mm)
1	768.11	-180.36	999.70
2	767.83	-180.08	999.06
3	766.99	-179.23	997.13
4	765.58	-177.82	993.92
5	763.61	-175.83	989.41
6	761.06	-173.27	983.57
7	757.92	-170.11	976.40
8	754.19	-166.36	967.86
9	749.86	-161.99	957.94
10	744.91	-157.02	946.63
11	739.35	-151.42	933.91
12	733.17	-145.20	919.76
13	726.35	-138.34	904.17
14	718.91	-130.84	887.13
15	710.82	-122.71	868.63
16	702.10	-113.93	848.68
17	697.02	-108.82	837.06
18	697.00	-108.80	837.02
19	696.99	-108.79	836.99
20	696.97	-108.77	836.95
21	696.96	-108.76	836.91
22	696.94	-108.74	836.88
23	696.93	-108.73	836.84
24	696.91	-108.71	836.81
25	696.90	-108.69	836.77
26	696.88	-108.68	836.74
27	696.86	-108.66	836.70
28	696.85	-108.65	836.66
29	696.83	-108.63	836.63
30	696.82	-108.62	836.59
31	696.80	-108.60	836.56
32	696.79	-108.58	836.52
33	696.77	-108.57	836.49
34	696.76	-108.55	836.45
35	696.74	-108.54	836.41
36	696.72	-108.52	836.38
37	696.71	-108.51	836.34
38	696.69	-108.49	836.31

Using the above data obtained from *SimWise 4D*, a MATLAB code was prepared to check the validity of these points. The code prepared in the MATLAB gives the output, which is visually similar to the motion planned in *SimWise 4D*. Fig. 5.7 shows the motion of the spatial manipulator in MATLAB. Here, position of the robot is demonstrated at some time intervals. Main objective is to check whether the positions obtained by *SimWise 4D* are correct or not.

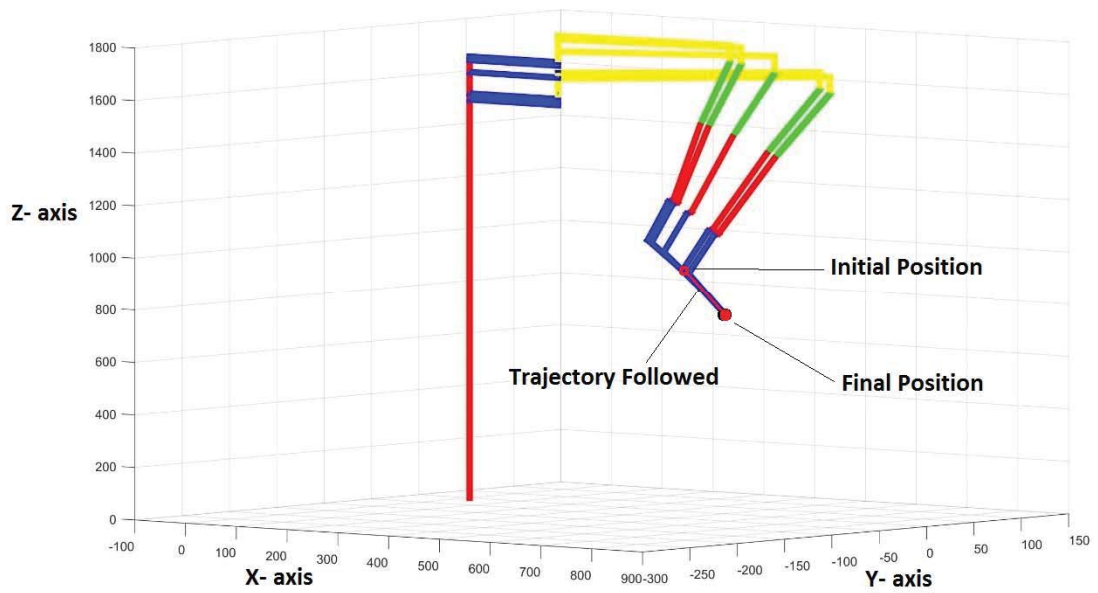


Fig. 5.7: Trajectory followed by Spatial Manipulator in MATLAB.

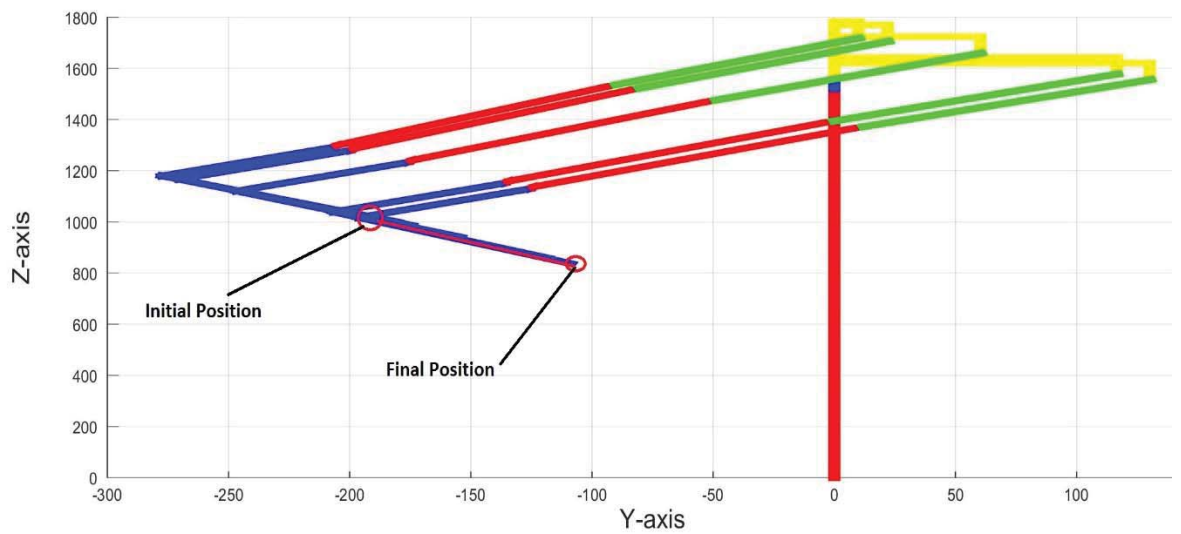


Fig. 5.8: Trajectory followed by Spatial Manipulator in MATLAB: Y-Z plane view.

As shown in Fig. 5.8, the same trajectory is followed by the MATLAB code. In this way, any trajectory, if possible can be planned using the *SimWise 4D*.

5.4 Summary

This chapter shows how the position of different links of spatial manipulator is calculated, using *SimWise 4D*, which later on used to plan a straight line trajectory for the end effector. By observing the above tabular data and the MATLAB output, it can be said that the positions

given by *SimWise 4D* are correct. So, *SimWise 4D* has been successfully used for trajectory tracking for spatial manipulator.

It can further be used to avoid any obstacle in the way of the trajectory. This property of *SimWise 4D* can be very useful in automation industries, medical surgeries etc.

Chapter 6

Conclusions and Future Directions

6.1 Conclusions

From the current work, the following conclusions can be drawn:

- From literature review, it can be concluded that D-H parameter, along with its variants, distal and proximal is the most widely used method to study the kinematic analysis of a robot. However, it is applicable to open-loop serial manipulators only. Also, D-H Parameter method is computationally much better than all other methods discussed by various authors. Among the original D-H, distal D-H and proximal D-H, distal D-H Parameter method is the most popular one.
- For a spatial manipulator, an ambiguity has been discussed in D-H parameter method, which was later on removed by doing an improvement in the proximal variant of D-H Parameter Method. The improvement is made by using the concept of *dummy frames*. The introduced concept removes the inconsistency in D-H parameter method, observed during the kinematic study of spatial manipulator.
- Trajectory tracking of a spatial manipulator is done with the help of *SimWise 4D*.

6.2 Future Directions

Now days, robotics is involving itself in almost all the fields of human interest. So, apart from what has been studied in this thesis, lot of future work is possible in this direction Let us discuss some of the possible work, which can be done in near future.

- The work present in this thesis is limited to the kinematic study only, which can be extended for the dynamic study of robotic manipulators.
- The work proposed here was limited up to only trajectory tracking. It can further be extended to trajectory tracking with obstacle avoidance. A control system can also be designed to control the manipulator by using *SimWise 4D*. Acceleration, velocity etc. of the manipulator can also be controlled, which can increase the degree of automation in existing manual systems.

- Trajectory tracking with obstacle avoidance has lot of applications in industries like automation systems, civil industries, medical surgeries etc. Any system can be designed using *SimWise 4D*, which can be very helpful in above mentioned industries.

REFERENCES

- [1] Denavit, J. and Hartenberg, R. S., “*A kinematic notation for lower pair mechanisms based on matrices*”, ASME Journal of Applied Mechanics, vol. 6, pp. 215–221, (1955).
- [2] John. J. Craig, “*Introduction to Robotics: Mechanics and Control*”, Reading, MA: Addison-Wesley, (1986).
- [3] Harvey Lipkin, “*A note on Denavit–Hartenberg notation in robotics*”, Proceedings of ASME International Design Engineering Technical Conferences & Computers and Information in Engineering Conference (IDETC/CIE), pp. 921–926, Long Beach, California USA (2005).
- [4] Kahn, M. E. and Roth, B., “*The near minimum-time control of open-loop articulated kinematic chains*”, ASME Journal of Dynamic Systems, Measurement, and Control, vol. 6, pp. 215–221, (1971).
- [5] C. R. Rocha, C. P. Tonetto, A. Dias, “*A comparison between the Denavit–Hartenberg and the screw-based methods used in kinematic modeling of robot manipulators*”, Journal of Robotics and Computer-Integrated Manufacturing vol. 27, pp. 723–728, (2011).
- [6] R.P, Paul, “*Robot Manipulators Mathematics, Programming and Control*”, MIT Press, Cambridge, MA 1981.
- [7] Robert J. Shilling, “*Fundamentals of Robotics Analysis and Control*”, Prentice Hall, Third Impression (2008).
- [8] Nicolas A. Aspragathos and John K. Dimitros, “*A comparative study on three methods for robot kinematics*”, IEEE Transaction on Man and Cybernetics Systems, vol. 28, pp. 135–145, (1998).
- [9] U. Thomas, I. Maciuszek, F.M Wahl, “*A Unified Notation for Serial, parallel and hybrid kinematic structures*”, Proceedings of IEEE, International Conference on Robotics and Automation, pp. 2868–2873, (2002).

- [10] P. N. Sheth, J. J. Uicker, “*A generalized symbolic notation for mechanisms*”, Journal of Engineering for Industry Transactions of the ASME, vol.93, pp. 102–112, (1971).

- [11] B. Roth, “Overview on Advanced Robotics: Manipulation”, Proceedings of IEEE, International Conference on Robotics and Automation (ICRA-1985), pp. 559–570, Tokyo, Japan, (1985).

- [12] W. Khalil, J. F. Kleinfinger, “*A new geometric notation for open and closed-loop robots*,” Proceedings of IEEE International Conference on Robotics and Automation, vol. 3, pp. 1174–1179, (1986).

- [13] Bruno Siciliano, Lorenzo Sciavicco, Luigi Villani, Giuseppe Oriolo, “*Robotics, Modelling, Planning and Control*”, Advanced Textbook in Control and Signal Processing 2009.

- [14] W. Khalil, G. Garcia and J. F. Delagrade, “*Calibration of geometric parameters of robots without external sensors*”, Proceedings of IEEE, International Conference on Robotics and Automation, vol. 3, pp. 3039–3044, (1995).

- [15] W. Khalil, “*Dynamic modelling of robots using recursive Newton-Euler techniques*”, International Conference on Informatics in Control, Automation and Robotics, (2010).

- [16] Peter I. Corke, “*A simple and systematic approach to assigning D-H parameters*”, IEEE Transactions on Robotics, vol. 23, pp. 590–594, (2007).

- [17] J. Ronney, “*A comparison of representation of general spatial screw displacement*”, Environ. Planning (England), Series- B, vol. 5, pp. 89–99, (1978).

- [18] Janez Funda, Russell H. Taylor and Richard P. Paul, “*On homogeneous transformations, quaternion and computational efficiency*”, Transactions of IEEE, on Robotics and Automation, vol. 6, pp. 382–388, (1990).
- [19] S. S. Rattan, “*Theory of Machines*”, Tata McGraw Hills Education Private Limited, (2009).
- [20] S. Sahu, B. B. Biswal, Bidyadhar Subudhi, “*A novel method for representing robot kinematics using quaternion theory*”, Proceedings of IEEE, International Conference on Computational Intelligence, Control And Computer Vision In Robotics and Automation, pp.76–82, (2008).
- [21] Amanpreet Singh, Ashish Singla and Sanjeev Soni, “*D-H Parameters Augmented with Dummy Frames for Serial Manipulators containing Spatial Links*”, The 3rd IEEE International Symposium on Robot and Human Interactive Communication, pp. 975-980,(2014).

ONLINE REFERENCES

- [F1] http://www.kaynakrobotu.com/images/Robot_Assemblaggio_6_jpg61039527-fc64-423c-9cd0-c6f04a5de1e2Large.jpg
- [F2] http://www.robotsurgery.ie/assets/images/animation/da_Vinci_S_HD_System.jpg
- [F3] http://en.wikipedia.org/wiki/File:DRDO_Daksh_ROV.jpg
- [F4] http://mars.jpl.nasa.gov/msl/images/msl20110526_MSL_Artist_Concept_PIA14165-full.jpg