

# ***BOTTOM MESON SPECTROSCOPY IN HQET***

Dissertation submitted for partial fulfillment of the requirement for the award of the  
degree

of

Master of Science

In

PHYSICS

Under the supervision

of

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July, 2012

## CERTIFICATE


I hereby declare that the work which is being presented in this dissertation entitled "**Bottom Meson Spectroscopy in HQET**" in partial fulfillment of requirement for the award of the degree of **Master of Science in Physics** submitted in the department of **School of Physics and Material Science, Thapar University, Patiala** is an authentic record of the initial work carried out by me under the guidance of **Dr. Alka Upadhyay, Asst. Professor, School of Physics and Material Science, Thapar University, Patiala** and refers others researcher's work which is dully listed in the references section.

The matter embodied in this dissertation has not been submitted in part or full to any other university or institute for the award of any degree.

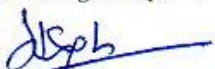
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This is to certify that the above declaration made by the student concerned is correct to the best of my knowledge and belief.

  
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## **ACKNOWLEDGEMENT**

I am highly grateful to the authorities of Thapar University, Patiala for providing this opportunity to carry out the thesis work. I acknowledge with deep sense of gratitude and humility, the inspiring help and guidance rendered by my supervisor , **Dr. Alka Upadhyay, Assistant Professor, School of Physics and Material Science, Thapar University, Patiala**, for her interest and encouragement throughout this work. Her deep insight into the problem and ability to provide constructive suggestions have been of immense value in improving the quality of my thesis work at all stages.

I would also like to give special thanks to research scholar **Mrs. Meenakshi Batra** for the help and valuable suggestions whenever I need it out for her busy schedule. Special thanks to all my friends, colleagues, and the staff at the School of Physics and Material Science for providing me a friendly atmosphere and encouraging me throughout my thesis work.

Finally I would like to express my deepest gratitude to all my beloved family members, without whom I am nothing, for providing me great opportunities, everlasting support , constant encouragement and lots of love.

*Nkaur*  
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## **ABSTRACT**

Particles and their properties can be studied through various models and theories. These models and theories are based on perturbative or non-perturbative approaches. In the present thesis, our aim is to study the heavy-light bottom mesons in the fundamental theory that involves the inclusion of two symmetries (Heavy quark symmetry and the Chiral symmetry) and combined effect of these two is called the heavy quark effective theory. One of the main aim of this thesis is to find the unknown masses of the bottom meson sector for even and odd parities with the help of the recent available data in PDG. We used the mass formulae developed by Manohar and Wise written in terms of the HQET parameters to find the unknown non strange states of B meson. Comparison have been made with the other theoretical values in literature and with the experimental result thereby verifying the validity of this theory in estimating the masses and mass splitting of the bottom mesons in the heavy-light sector.

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## UNITS IN HIGH ENERGY PHYSICS

(a)

QUANTITY	HIGH ENERGY UNIT	VALUE IN SI UNITS
Length	1 fm	$10^{-15}$ m
Energy	1 GeV= $10^9$ eV	$1.602 \times 10^{-10}$ J
Mass, $E/c^2$	1 GeV/ $c^2$	$1.78 \times 10^{-27}$ kg
$\hbar = h/(2\pi)$	$6.588 \times 10^{-25}$ GeV s	$1.055 \times 10^{-34}$ J s
$c$	$2.998 \times 10^{23}$ fm $s^{-1}$	$2.998 \times 10^8$ $ms^{-1}$
$\hbar c$	0.1975 GeV fm	$3.162 \times 10^{-26}$ J m

(b)

<b>Natural units</b>	$\hbar=c=1$
<b>Mass, <math>Mc^2/c^2</math></b>	1 GeV
<b>Length, <math>\hbar c/(Mc^2)</math></b>	1 GeV $^{-1}$ =0.1975 fm
<b>Time, <math>\hbar c/(Mc^3)</math></b>	1 GeV $^{-1}$ = $6.59 \times 10^{-25}$ s

## CONSTANTS

CONSTANT	SYMBOL	VALUE
Electronic Charge	$e$	$1.6 \times 10^{-19}$ C
Electron Mass	$m_e$	$9.1 \times 10^{-31}$ kg
Proton Mass	$m_p$	$1.67 \times 10^{-27}$ kg
Fine Structure Constant	$\alpha=e^2/(4\pi)$	1/137.06
Planck's Constant (Reduced)	$\hbar$	$6.582... \times 10^{-22}$ MeV
Strong Coupling Constant	G	1

## RELATIONS BETWEEN ENERGY UNITS

$$1 \text{ MeV} = 10^6 \text{ eV}$$

$$1 \text{ GeV} = 10^3 \text{ MeV} = 10^9 \text{ eV}$$

$$1 \text{ TeV} = 10^3 \text{ GeV} = 10^6 \text{ MeV} = 10^{12} \text{ eV}$$

## GREEK ALPHABETS

<b>GREEK LETTER NAME</b>	<b>LOWER CASE</b>	<b>UPPER CASE</b>
<b>Alpha</b>	$\alpha$	A
<b>Beta</b>	$\beta$	B
<b>Gamma</b>	$\gamma$	Γ
<b>Delta</b>	$\delta$	Δ
<b>Epsilon</b>	$\varepsilon$	E
<b>Zeta</b>	$\zeta$	Z
<b>Eta</b>	$\eta$	H
<b>Theta</b>	$\theta$	Θ
<b>Iota</b>	$\iota$	I
<b>Lambda</b>	$\lambda$	Λ
<b>Mu</b>	$\mu$	M
<b>Pi</b>	$\pi$	Π
<b>Rho</b>	$\rho$	P
<b>Sigma</b>	$\sigma$	Σ
<b>Tau</b>	$\tau$	T
<b>Phi</b>	$\phi$	Φ
<b>Chi</b>	$\chi$	X
<b>Psi</b>	$\psi$	Ψ
<b>Nu</b>	$\nu$	N
<b>Xi</b>	$\xi$	Ξ
<b>Omega</b>	$\omega$	Ω
<b>Kappa</b>	$\kappa$	K
<b>Upsilon</b>	$\upsilon$	Υ

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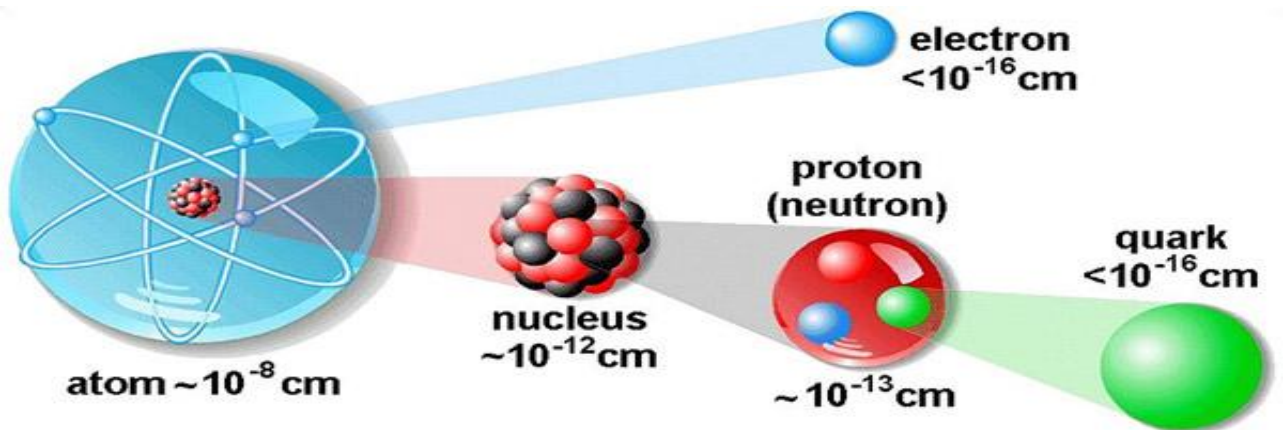
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# CHAPTER 1

## INTRODUCTION AND FUNDAMENTAL THEORY

### 1.1 ELEMENTARY PARTICLE PHYSICS

The universe is filled with matter that exists in different states, namely the solid, liquid, gas and the plasma. The question is what the matter is made of ? The answer to this question lies primarily in the realm of a particle physicist, who keeps on trying to know the ultimate constituent (or constituents) of the matter. This ultimate or the fundamental constituent, which may also be called the constituent of matter at its smallest scale of size, is also known as the elementary particle or the fundamental particle.



*Thomson(1897):Discovers electron*

*Rutherford(1909):Nuclear atom(proton)*

*Chadwick(1932):Discovers Neutron*

*SLAC(1968):Quarks in Neutrons and Protons*

*Fig. 1.1 Constituents of Matter*

At different times, each of the above mentioned particles (in figure) were considered to be fundamental. But in particle physics, an **elementary particle** or **fundamental particle** is a particle which do not have a substructure; that is, it is not known to be made up of smaller particles. If an elementary particle truly has no substructure, then it is one of the basic building blocks of the universe from which all other particles

are made. Today, we know that atoms do not represent the smallest unit of matter but the particles called quarks and leptons seem to be the fundamental building blocks. Since 1974, we consider quarks to be fundamental, because so far we have been unable to “break them apart”. As we increase the momentum of the particles in our accelerators, we are able to resolve, or see, deeper into matter. We are currently able to accelerate particles to energies of  $\sim 1 \text{ TeV}$ . High energy particles provide a way to probe, or “see” matter at the very smallest size (Recall Electron microscope example). Today, high energy accelerators produce energetic beams which allow us to probe matter at its most fundamental level. As we go to higher energy particle collisions:

- 1) Wavelength of the probe becomes smaller due to which we see finer details.
- 2) Can produce more massive objects, via  $E=mc^2$ . The physics in which all the above facts are being studied is known as elementary particle physics.

## **1.2 MOTIVATION**

The driving motivation behind particle physics experiments is the desire to determine the ultimate structure of matter and the forces of nature. Our current standard model is believed to be an effective theory, which has a deeper underlying theory reachable in the next generation of experiments. In the electroweak sector, where great successes of the past decades have predicted and verified the unification of the electromagnetic and weak nuclear forces, precision measurements at the CERN Large Electron Positron collider (LEP) and the Fermilab proton-antiproton collider (Tevatron) demand that there be either a light Higgs particle with a mass in between  $125\text{-}126 \text{ GeV}/c^2$  or a physical system mimicking its interactions. At the same time, the requirement that the theory be stable even with a Higgs, as well as the observation of cold dark matter in the universe, compellingly point to new physics at the Terascale.

In the flavour sector, a decade of increasingly precise measurements of the properties of heavy quarks has shown remarkable agreement with the standard model predictions, and we are now moving into an era of precise investigations of the neutrino sector. The demonstration by the Japanese Super-Kamiokande and Canadian SNO experiments that neutrinos flavours oscillate but that their masses are likely much smaller than those of the other elementary particles suggests that there are critical phenomena in particle physics which cannot be explained by the standard model. This physics likely had a very significant role in cosmology. There are some hints that this new physics might be accessible to upcoming experiments.

In the strong sector, the theory of quantum chromodynamics (QCD) has been successfully used to predict the behaviour of quarks and gluons at high energies observed at the HERA collider at DESY Hamburg as well as at LEP and the Tevatron, but the lower energy regime, where they are bound into particles such as protons and neutrons, remains theoretically and experimentally challenging and requires further investigation.

Supersymmetry theory, claims there is a fundamental unity, or symmetry, between matter particles and force particles. If this is true, then every quark, lepton, and gauge boson should have a heavier partner particle. Particle physicists are intently searching for these “sparticles.” Since the Standard Model does not account for gravity, high-energy physicists are developing other theories – string theory, brane theory, and theories of extra dimensions – to bring gravity into the fold. Like string theory, theories of branes and extra dimensions suggest that our universe contains many more dimensions than those we experience with our senses. This idea is already being tested in the laboratory. High-energy physicists are studying slight differences in the behaviour of matter and antimatter that might account for the way our universe evolved. The origins of the matter-antimatter asymmetry of the Universe by studying CP violation at the LHCb experiment is being investigated. Dark energy is even more puzzling. Astronomical observations suggest that some force is causing our universe to expand at an ever-increasing rate even as dark matter pulls it back together. High-energy physicists are designing experiments to find out what this energy is.

From the past two decades, a remarkable progress has been seen in the elementary particle physics which could have been possible by new generation of particle accelerators and detectors. A huge amount of experimental data has been collected from these huge machines. Before proceeding further, let us have a look over some of these huge machines.

### ***1.3 EXPERIMENTAL SETUP AT VARIOUS PLACES***

**Particle Accelerators:** Just after the Big Bang, the universe was a rapidly expanding ball of fundamental particles. As the universe expanded, it cooled and the particles decayed, changing into other fundamental particles. These particles then joined together and gradually formed the matter that we see around us today.

In particle accelerators we smash beams of particles together in head-on collisions that are energetic enough to turn the clock back to just after the Big Bang. The more energetic the collisions, the more likely we are to make fundamental particles appear again. Once we've produced fundamental particles we can study their behaviour to find out why the universe is made the way it is.

**Particle Detectors:** If we've created particles in a collision in an accelerator, we want to be able to look at them. And that's where particle detectors come in. We build these at the collision points in an accelerator and use them to identify as much of what was produced in the collision as we can.

The principle of a particle detector is simple. It will never “see” a particle directly, but shows where it has travelled, what signature tracks it leaves behind and the effect it has on the detector when it is stopped as it flies out of the collision. The data pouring out of these detectors will be analysed to answer fundamental questions about the way the universe works.....

Some of the accelerators and detectors are explained below.

### ➤ **BABAR**

BABAR is a High Energy Physics experiment located at SLAC National Accelerator Laboratory, near Stanford University, in California. The goal of the experiment is to study the violation of charge and parity (CP) symmetry in the decays of B mesons. This violation manifests itself as different behaviour between particles and anti-particles and is the first step to explain the absence of anti-particles in everyday life. To study CP violation, the BABAR experiment exploits the 9.1 GeV electron beam and the 3 GeV positron beam of the PEP-II accelerator. The two beams collide in the center of the experiment, producing  $\Upsilon(4S)$  mesons which decay into equal numbers of B and anti-B mesons. Also, BABAR performs stringent tests of new theories by precisely measuring various properties of particles and of their radioactive decays, in particular those of the bottom and charm quarks and of the tau-lepton. It also tests new ideas that predict the existence of new particles of various types, such as low-mass Higgs, dark gauge bosons, and pentaquarks.

### ➤ **TEVATRON**

The Tevatron is a circular particle accelerator in the United States, at the Fermi National Accelerator Laboratory (also known as *Fermilab*), just east of Batavia, Illinois, USA. The Tevatron is a synchrotron that accelerates protons and antiprotons in a 6.28 km ring to energies of up to 1TeV, hence its name. It has two detectors, called CDF, for Collider Detector at Fermilab, and DZero, named for its location on the Tevatron accelerator ring, containing many detection subsystems that identified the different types of particles emerging from the collisions.

The goal of the CDF experiment is to measure exceptional events out of the billions of collisions in order to:

- Look for evidence for phenomena beyond the Standard Model of particle physics,
- Measure and study the production and decay of heavy particles such as the Top and Bottom quarks, and the W and Z bosons,
- Measure and study the production of high-energy particle jets and photons.

The  $D\bar{0}$  collaboration has published results which may explain the matter-antimatter asymmetry responsible for the abundance of matter in the universe. B mesons, which oscillate between their matter and anti-matter state trillions of times each second, may take longer to decay into antimatter than matter. This would eventually lead to a slightly greater abundance of matter than antimatter, explaining why some matter remains after annihilation in the early universe.

## ➤ **BELLE**

The Belle experiment is a particle physics experiment conducted by the Belle Collaboration, an international collaboration of more than 400 physicists and engineers investigating CP-violation effects at the High Energy Accelerator Research Organisation (KEK) in Tsukuba, Japan. The Belle detector, located at the collision point of the  $e^-e^+$  asymmetric-energy collider (KEKB), is a multilayer particle detector. Extensive studies of rare decays, searches for exotic particles and precision measurements of B mesons, D mesons, and tau particles have been carried out and have resulted in almost 300 publications in physics journals. In addition, Belle has carried out special short runs at the  $\Upsilon(5S)$  resonance to study  $B_s$  mesons as well as on the  $\Upsilon(3S)$  resonance to search for evidence of Dark Matter and the Higgs Boson.

## ➤ **CLEO**

CLEO was a general purpose particle detector at the Cornell Electron Storage Ring (CESR), at the Wilson Synchrotron Laboratory at Cornell. The Cornell Electron Storage Rings produce electron-positron annihilations; the CLEO experiment detects the results of these annihilations. The objective is to study details of new quark states and couplings. The energy range of the beams is perfect for investigation of particles containing bottom quarks, and is also very convenient for finding new states containing charm quarks.

Data from Cornell is analyzed using the large parallel processing computer farm located in Florida. This has led to many discoveries concerning in particular rare decays of B mesons and new charmed baryon states<sup>[1]</sup>. Measurements such as these provide fundamental tests of the Standard Model description of strong, electromagnetic and weak decays. Practically all experimental data collected from these accelerators and detectors can be accounted for by the so-called Standard Model of particles and their interactions formulated in 1970's. It is being discussed in the next section.

## **1.4 STANDARD MODEL( $\sim 10^2$ GeV)**

At the present time, the most widely accepted theory of particle physics is the Standard Model. The discussions on the fundamental particles classification and the basic forces are based on this theory. It describes the strong, weak, and electromagnetic fundamental interactions, using mediating gauge bosons. The species of gauge bosons are the gluons,  $W^-$ ,  $W^+$ ,  $Z^0$  [2] bosons and the photons. The model also contains 24 fundamental particles (six quarks, six anti-quarks, six leptons, six anti-leptons) which are the constituents of matter. It explains seventeen parameters: masses, mixing angles, coupling constants, isospin, hypercharge, charge conjugations etc. Finally, it predicts the existence of a type of boson known as the Higgs Boson, which is yet to be discovered. The role of particle physics is to test this model in a conceivable way seeking to discover whether something more lies beyond it.

H= the missing ingredient: the Higgs Boson

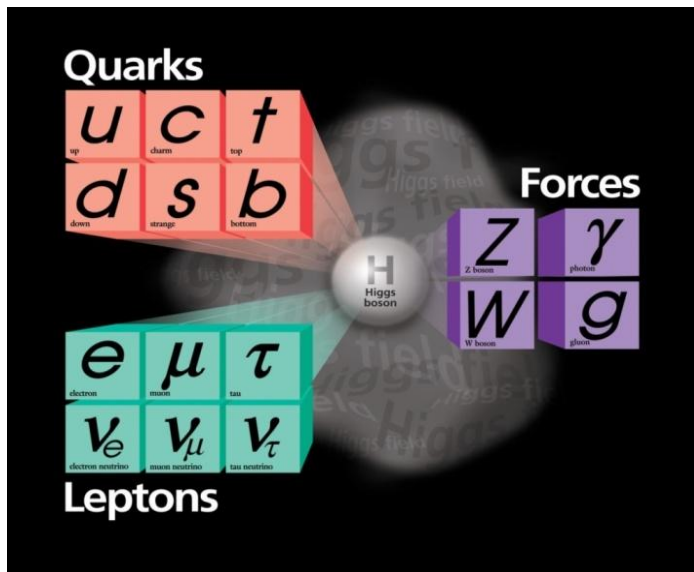


Fig.1.2 The Standard Model

### 1.4.1 QUARKS

Quarks are one type of matter particle. Most of the matter we see around us is made from protons and neutrons, which are composed of quarks. Quarks have the unusual characteristic of having a fractional electric charge, unlike the proton and neutron which have the integral charges of +1 and -1 respectively. Quarks also carry another type of charge called color charge. Each of the six “flavors” of quark can have three different “colors” namely- red , green and blue. There are six types of quarks, known as flavors: up, down , strange , charm, bottom and top . The properties which identify the quarks are listed in the table below:

GENERATION	FLAVOR & SYMBOL	ELECTRIC CHARGE	SPIN	BARYON NUMBER	S	C	B	T	MASS (GeV/c <sup>2</sup> )
1 <sup>st</sup>	Up (u)	+2/3	1/2	1/3	0	0	0	0	0.003
	Down (d)	-1/3	1/2	1/3	0	0	0	0	0.006
2 <sup>nd</sup>	Charm (c)	+2/3	1/2	1/3	0	+1	0	0	1.3
	Strange (s)	-1/3	1/2	1/3	+1	0	0	0	0.1
3 <sup>rd</sup>	Top (t)	+2/3	1/2	1/3	0	0	0	+1	175
	Bottom (b)	-1/3	1/2	1/3	0	0	+1	0	4.3

Table 1.1 Quarks and their quantum numbers(S=strangeness,C=charmness,B=bottomness,T=topness)

Because all the quarks are spin 1/2 particles so they are fermions. For each particle of matter there exists an equivalent particle with opposite quantum characteristics, called an anti-particle. For each of the six quarks, there is an antiquark ( $\bar{u}$ ,  $\bar{d}$ ,  $\bar{s}$ ,  $\bar{c}$ ,  $\bar{b}$ ,  $\bar{t}$ ) and for these antiquarks, the electric charge ( $Q$ ) and all flavor quantum numbers ( $B$ ,  $C$ ,  $S$ ,  $T$ , and Baryon number) are of opposite sign. Mass and total angular momentum ( $J$ ; equal to spin for point particles) do not change sign for the antiquarks<sup>[3]</sup>.

### 1.4.2 LEPTONS

The other type of matter particles are the leptons<sup>[4]</sup>. The best known of all leptons is the electron which governs nearly all of chemistry as it is found in atoms and is directly tied to all chemical properties. There are six types of leptons, known as flavors, forming three generations<sup>[5]</sup> as shown in the table below:

GENERATION	FLAVOUR and SYMBOL	ELECTRIC CHARGE	LEPTON NUMBER	MASSIVE RANGE
1 <sup>st</sup>	Electron ( $e^-$ )	-1	1	0.511 MeV/c <sup>2</sup>
	Electron Neutrino ( $\nu_e$ )	0	1	< 0.16 eV/c <sup>2</sup>
2 <sup>nd</sup>	Muon ( $\mu$ )	-1	1	105.68 MeV/c <sup>2</sup>
	Muon Neutrino ( $\nu_\mu$ )	0	1	< 10 eV/c <sup>2</sup>
3 <sup>rd</sup>	Tau ( $\tau$ )	-1	1	1777 MeV/c <sup>2</sup>
	Tau Neutrino ( $\nu_\tau$ )	0	1	< 18 MeV/c <sup>2</sup>

*Table 1.2 Leptons and their quantum numbers*

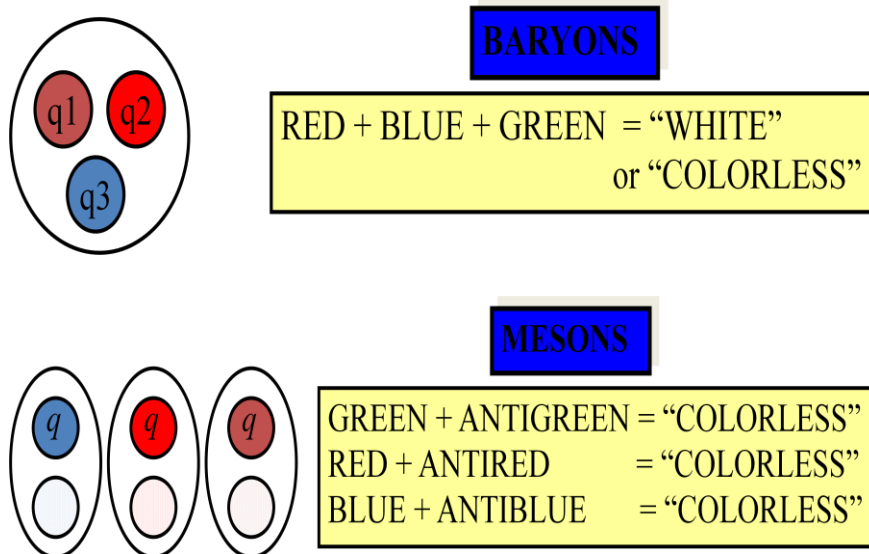
For every lepton flavor, there is a corresponding type of antiparticle, known as antilepton, that differs from the lepton in the sense that some of its properties have equal magnitude but opposite sign. All the leptons are capable of independent existence but the quarks do not.

If we allow an extra degree of freedom i.e. colors to each quark, then the total charge of  $u, c, t$  is  $(3 \cdot 3 \cdot 2/3) = 6e$ . Similarly for  $d, s, b$  is  $(3 \cdot 3 \cdot (-1/3)) = -3e$ .

The total charge of leptons is  $(3 \cdot (-1) + 3 \cdot 0) = -3e$ . Therefore, the total charge of all the fermions is zero. This is the actual condition that the standard model is free from anomalies and is a renormalizable field theory<sup>[6]</sup>.

### 1.4.3 HADRONS

Quarks are never found in free state rather they combine to form the composite particles called hadrons. Hadrons are categorized into two families: baryons (made of three quarks) and mesons (made of one quark and one antiquark). Hadrons observed in nature are colorless (but their constituents are not) as shown in the figure ahead:



*Fig. 1.3 Hadrons: Baryons and Mesons*

- **BARYONS (strongly interacting fermions):** All the baryons have half integral spins, i.e. 1/2, 3/2, 5/2 etc. and are fermions because they obey Fermi Dirac statistics. They have masses equal to or more than that of nucleon mass. Particles of this category which are heavier than nucleons are collectively known as *hyperons*. Each type of baryon has a corresponding antiparticle (antibaryon) in which the quarks are replaced by their corresponding antiquarks. Baryons are subjected to all the three types of interactions i.e. strong, weak and electromagnetic. Some of the examples of the baryons with their quantum numbers are listed in the following table :

NAME	SYMBOL	QUARK COMPOSITION	MASS (MeV/c <sup>2</sup> )	ELECTRIC CHARGE
<b>Proton</b> <sup>[7]</sup>	<i>p</i>	<i>uud</i>	938.272013±0.000023	+1
<b>Neutron</b> <sup>[7]</sup>	<i>n</i>	<i>udd</i>	939.565346±0.000023	-1
<b>Lambda</b> <sup>[7]</sup>	$\Lambda^{\circ}$	<i>uds</i>	1115.683±0.006	0
<b>Sigma</b> <sup>[7]</sup>	$\Sigma^{+}$	<i>uus</i>	1189.37±0.07	+1
<b>Sigma</b> <sup>[7]</sup>	$\Sigma^{\circ}$	<i>uds</i>	1192.642±0.024	0
<b>Sigma</b> <sup>[7]</sup>	$\Sigma^{-}$	<i>dds</i>	1197.449±0.030	-1
<b>Xi</b> <sup>[7]</sup>	$\Xi^{\circ}$	<i>uss</i>	1314.86±0.20	0
<b>Xi</b> <sup>[7]</sup>	$\Xi^{-}$	<i>dss</i>	1321.71±0.07	-1
<b>Omega</b> <sup>[7]</sup>	$\Omega^{-}$	<i>sss</i>	1672.45±0.29	-1

*Table 1.3 Baryons and their quantum numbers.*

- **MESONS (strongly interacting bosons):** All mesons have zero or integral spins i.e., 0, 1, 2, etc. and are bosons because they obey Bose Einstein statistics. Each type of meson has a corresponding antiparticle(anti-meson) in which the quarks are replaced by their corresponding antiquarks. They have masses intermediate between leptons and nucleons and are subjected to all the three types of interactions i.e., strong, weak and electromagnetic. Some of the examples of mesons are listed below:

NAME	SYMBOL	QUARK COMPOSITION	MASS (MeV/c <sup>2</sup> )	ELECTRIC CHARGE
<b>Pion</b> <sup>[8]</sup>	π <sup>+</sup>	u $\bar{d}$	139.57018±0.00035	+1
<b>Pion</b> <sup>[8]</sup>	π <sup>0</sup>	$\frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$	134.9766±0.0006	0
<b>Eta Meson</b> <sup>[18]</sup>	η	$\frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}}$	547.853±0.024	0
<b>Kaon</b> <sup>[8]</sup>	K <sup>+</sup>	u $\bar{s}$	493.677±0.016	+1
<b>Kaon</b> <sup>[8]</sup>	K <sup>0</sup>	d $\bar{s}$	497.614±0.024	0
<b>D Meson</b> <sup>[8]</sup>	D <sup>+</sup>	c $\bar{d}$	1869.60±0.16	+1
<b>B Meson</b> <sup>[8]</sup>	B <sup>+</sup>	u $\bar{b}$	5279.15±0.31	+1

*Table 1.4: Mesons and their quantum numbers*

If we take three flavors of quarks, then the quarks lie in the fundamental representation, 3 (called the triplet) of flavor SU(3). The antiquarks lie in the complex conjugate representation  $\bar{3}$ . The nine states (nonet) made out of a pair can be decomposed into the trivial representation, 1 (called the singlet), and the adjoint representation, 8 (called the octet). The notation for this decomposition is  $3 \otimes \bar{3} = 8 \oplus 1$ . If the flavor symmetry were exact, then all nine mesons would have the same mass. The physical content of the theory includes consideration of the symmetry breaking induced by the quark mass differences, and considerations of mixing between various multiplets.

The quarks of the Standard Model fall into two classes - light quarks (q → u, d, s) and heavy quarks (Q → c, b, t) and we know that the quarks and the antiquarks can be bounded together to form the mesons. These mesons can be further classified as:

- 1) **Light-Light Mesons (q $\bar{q}$ )** : Here both the quark and the antiquark are from the light quark group (q → u, d, or s and  $\bar{q} \rightarrow \bar{u}, \bar{d}, \text{ or } \bar{s}$ ). Examples of light-light mesons are : π<sup>+</sup>(u $\bar{d}$ ), K<sup>+</sup>(u $\bar{s}$ ), K<sup>0</sup>(d $\bar{s}$ ), ρ<sup>-</sup>(d $\bar{u}$ ), φ(s $\bar{s}$ ).

2) **Heavy-Light Mesons ( $q\bar{Q}$  or  $Q\bar{q}$ )** : These mesons contain light quark and heavy antiquark or vice-versa. Example of these types of mesons are :  $D^+(c\bar{d})$ ,  $D_s^+(c\bar{s})$ ,  $B^-(b\bar{u})$ ,  $B^0(b\bar{d})$ .

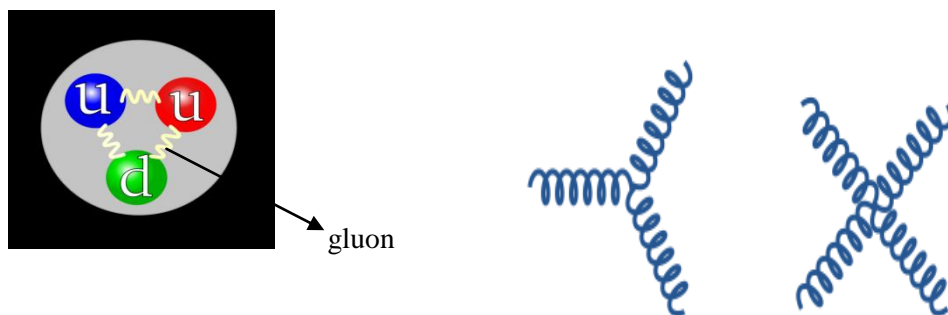
3) **Heavy-Heavy Mesons ( $Q\bar{Q}$ )** : Here both the quark and the antiquark are from the heavy quark group. Examples are:  $J/\psi(c\bar{c})$  and  $\Upsilon(b\bar{b})$ .

My work is based on the Heavy-Light Mesons. The different states of the mesons are represented by the quantum number  $J^P$ , in which J represents the total angular momentum of the system and P represents the parity of the system. Spin and orbital angular momentum of the quarks inside the meson can couple to give the total angular momentum  $J = 0, 1$ , or any other value. The mesons with total angular momentum zero and positive parity are called scalar mesons. Similarly other states are named as given below:

$J^P$	$0^-$	$0^+$	$1^-$	$1^+$	$2^+$
NAME	Pseudoscalar	Scalar	Vector	Axialvector	Tensor

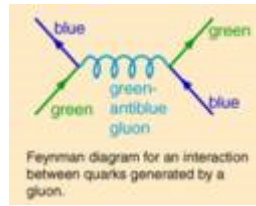
### 1.4.4 GLUONS

The objects which carry the colour force between quarks and hence glue them into hadrons are called *gluons*. Gluons carry both colour and anticolour. Unlike photons, gluons can interact with each other due to the color charge as shown in the following figure:



**Fig. 1.4 (i) Gluons binding the quarks into a proton.(ii) Three Point & Four Point interaction.**

Quarks interact by emitting and absorbing gluons, just as electrically charged particles interact through the emission and absorption of photons<sup>[9]</sup>. Since gluons carry color charge, it is fair to say that the interaction between quarks results in the exchange of color (or color charge) as shown in the figure ahead:



*Fig. 1.5 Feynman diagram for an interaction between the quarks generated by a gluon.*

Note that the gluon generates a color change for the quarks. The gluon exchange converts a blue quark into a green one and vice-versa.

### **1.4.5 HIGGS BOSON**

The Higgs boson or Higgs particle is a proposed elementary particle in the Standard Model of particle physics. The Higgs boson is named after Peter Higgs who, along with two other teams, proposed the mechanism that suggested such a particle in 1964<sup>[10]</sup>. The existence of a Higgs field and its associated Higgs boson would be the simplest of several methods to explain why some other elementary particles have mass. According to this theory, certain elementary particles obtain mass by interacting with the Higgs field which has non-zero strength everywhere, even in otherwise empty space. The Higgs boson - the smallest possible excitation of this field—is predicted to exist by the same theory, and as this would be detectable, it has been the target of a long search in particle physics.

One of the primary goals of the Large Hadron Collider (LHC) at CERN in Geneva, Switzerland - the most powerful particle accelerator and one of the most complicated scientific instruments ever built - was to test the existence of the Higgs boson and measure its properties which would allow physicists to confirm this cornerstone of modern theory.

According to the Standard Model, the Higgs particle is a boson, a type of particle that allows multiple identical particles to exist in the same place in the same quantum state. It has no intrinsic spin, no electric charge, and no colour charge. It is also very unstable, decaying into other particles almost immediately.

On 4 July 2012, the CMS and the ATLAS experimental teams at the Large Hadron Collider independently announced that they each confirmed the formal discovery of a previously unknown boson of mass between 125–127 GeV/c<sup>2</sup>, whose behaviour so far was "consistent with" a Higgs boson, while adding a cautious note that further data and analysis were needed before positively identifying the new particle as being a Higgs boson of some type.

## 1.4.6 FUNDAMENTAL INTERACTIONS

There are four different types of interactions that can occur between bodies. These interactions can take place even when the bodies are not in physical contact and together they account for all the observed forces that occur in the universe. According to the quantum field theory, all the interactions rely on mechanism of exchange of quanta. Each force field has its own quanta-analogous to photons-which act as carriers of electromagnetic interaction. So all the forces are transmitted from one particle to another by successive process of emission, propagation and absorption of such carriers. The following table summarizes the basic properties of these carriers:

FORCE CARRIER	ELECTRIC CHARGE	COLOR CHARGE	SPIN	MASS (GeV/c <sup>2</sup> )	INTERACTION MEDIATED
Photon( $\gamma$ )	0	No	1	0	Electromagnetic
Gluon(g)	0	8	1	0	Strong
W <sup>±</sup>	±1	No	1	80.4	Weak
Z <sup>0</sup>	0	No	1	91.2	Weak
Graviton(not in SM)	0	No	2	0	Gravitational

*Table 1.5 Gauge Bosons and their properties*

The four fundamental interactions are explained below:

### 1) GRAVITATIONAL INTERACTION

Carriers of gravitational interactions are gravitons, the quanta of gravitational field. However this interaction is the weakest interaction. It is 10<sup>39</sup> times weaker than the strong interaction. Detection of individual graviton is impossible because of non-availability of such a sensitive detector. However large cooperative action of gravitons can be detected.

The gravitational force between two particles with masses m<sub>1</sub> and m<sub>2</sub> is given as

$$F_{gravity} = \frac{gm_1m_2}{r^2}$$

Where r is the distance between the masses.

#### Characteristics:

- (i) It acts along the line joining the masses.

- (ii) It obeys the inverse square law.
- (iii) Its characteristics time is of the order of  $10^{-16}$  seconds.
- (iv) It is always attractive.
- (v) Its range is infinite.

The hypothetical gravitational quantum, the **graviton**, is also a useful concept in some contexts. On the atomic scale, the gravitational force is negligibly weak, but on the cosmological scale, where masses are enormous, it is immensely important in holding the components of the universe together.

## 2) ***WEAK INTERACTION***

It is  $10^{15}$  times weaker than the strong interaction. It occurs between leptons and in the decay of hadrons. It is responsible for the beta decay of particles and nuclei. In the current model, the weak interaction is visualized as a force mediated by the exchange of virtual particles, called intermediate vector bosons. The weak interactions are described by electroweak theory, which unifies them with the electromagnetic interactions.

### *Characteristics :*

- (i) Short range interactions. Its range is about  $10^{-17}$  m.
- (ii) Does not obey the inverse square law.
- (iii) Characteristics time is of the order of  $10^{-10}$  seconds.

## 3) ***ELECTROMAGNETIC INTERACTION***

It is responsible for the forces that control atomic structure, chemical reactions, and all electromagnetic phenomenon. All charged particles are acted upon by electromagnetic interaction. The production of electron-positron pair from gamma ray involves this kind of interaction. Even the binding force between the orbital electron and the positive nucleus is provided by this interaction. The strength of this interaction is much greater than gravitational and weak interactions.

### *Characteristics :*

- (i) Long range interaction(it extends upto infinity).
- (ii) It can be attractive as well as repulsive.
- (iii) Obeys inverse square law.
- (iv) Acts along the line joining the charged particle.
- (v) Its characteristic time is  $10^{-20}$  seconds.

The quantum theory of electromagnetic interactions is described by quantum electrodynamics, which is a simple form of gauge theory. It is given as:

$$F_{em} = \frac{ke^2}{r^2}$$

A dimensionless constant which characterizes the electromagnetic force is  $\alpha = \frac{2\pi ke^2}{hc} = \frac{1}{137}$ . This coupling constant is also called the “fine structure constant”.

#### 4) ***STRONG INTERACTION***

It holds the quarks and gluons together to form protons, neutrons and other particles. The word “strong” is used since the strong interaction is the strongest of the four fundamental forces. It is responsible for the force between nucleons that gives the atomic nucleus its greater stability. The strong force is mediated by gluons, acting upon quarks, antiquarks, and gluons themselves.

##### **Characteristics :**

- (i) Short range interactions. Its range is about  $10^{-15}$ m.
- (ii) Charge independent.
- (iii) Does not obey the inverse square law.
- (iv) Independent of the relative orientation of the nucleons.
- (v) Characteristic time is  $10^{-23}$  seconds.

The strong interactions are described by a gauge theory called quantum chromo dynamics (QCD). Analysis of the coupling constant with QCD gives an expression for the diminishing coupling constant. The coupling decreases logarithmically, a phenomenon known as asymptotic freedom. The coupling decreases approximately as

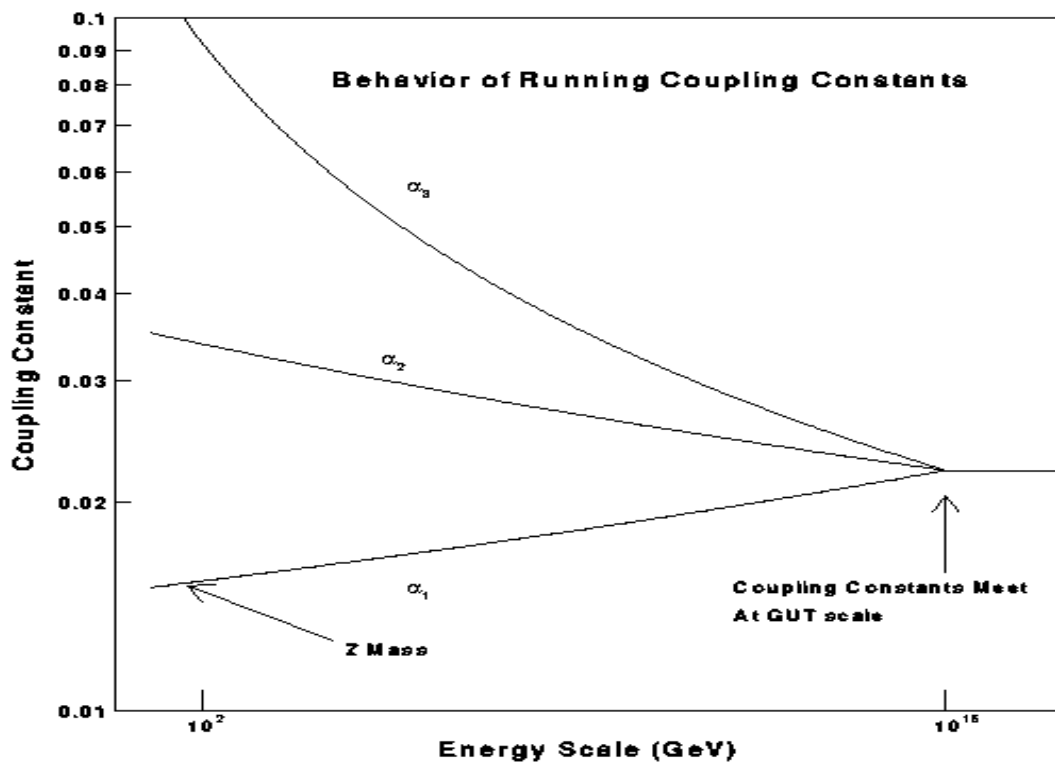
$$\alpha_s(k^2) = \frac{g_s^2(k^2)}{4\pi} = \frac{1}{\beta_0 \ln(k^2/\Lambda^2)}$$

where  $\beta_0$  is a constant. In non-abelian gauge theories, the  $\beta$  function can be negative. Conversely, the coupling increases with decreasing energy. This means that the coupling becomes large at low energies.

#### **1.4.7 UNIFICATION**

The standard model describes the strong, weak, and electromagnetic interactions in the energy range  $\leq 10^2$  GeV with the three different coupling constants:  $\alpha_3$ ,  $\alpha_2$ , and  $\alpha_1$  for the gauge groups SU(3), SU(2), and U(1) respectively. Due to renormalization, the coupling constants of each of these symmetries vary with the en-

ergy at which they are measured. Around  $10^{15}$  GeV, these couplings become approximately equal. This has led to speculation that above this energy the three gauge symmetries of the standard model are unified in one single gauge symmetry with a simple group gauge group, and just one coupling constant. Below this energy the symmetry is spontaneously broken to the standard model symmetries. Popular choices for the unifying group are the special unitary group in five dimensions  $SU(5)$  and the special orthogonal group in ten dimensions  $SO(10)$ <sup>[11]</sup>. Theories that unify the standard model symmetries in this way are called Grand Unified Theories (or GUTs), and the energy scale at which the unified symmetry is broken is called the GUT scale. Generically, grand unified theories predict the creation of magnetic monopoles in the early universe,<sup>[12]</sup> and instability of the proton<sup>[13]</sup>. Neither of which have been observed, and this absence of observation puts limits on the possible GUTs.



*Fig.1.6 Coupling Constant Unification.*

### **1.4.8 BEYOND THE STANDARD MODEL**

Today, the standard model of the strong and electroweak interactions provides a most successful description of the physics currently accessible with particle accelerators. In spite of its success, however, many open questions remain<sup>[14]</sup>- it does not incorporate the physics of general relativity such as gravitation and dark matter. It does not account for neutrino oscillations which is only possible if neutrino's have mass but

this is not explained by standard model. Standard model is theoretically self consistent but it gives rise to puzzle like CP violation and hierarchy problem. Also, there is no answer to the question as to why there are only three generations of quarks and leptons and not more than that. The model is somewhat inelegant as it contains only seventeen parameters and the basis for choosing these seventeen parameters is not yet clear. Physics Beyond the Standard Model refers to the theoretical developments needed to explain these deficiencies of the Standard Model<sup>[15]</sup>. After 1TeV, there is a concept of super-symmetry i.e. at  $10^{12}$  eV, there is fermion-boson symmetry which means each fermion will have a boson partner. It is fairly possible that standard model form an integral and important part of a more complicated theory of particles in future.

## 1.5 SYMMETRIES

There are objects in nature which exhibit various symmetries. According to Weyl , “a thing is symmetrical if there is something we can do to it so that after we have done it, it looks the same as it did before” e.g. a sphere is invariant when rotated around an axis passing through its centre. Similarly an equation possesses symmetry when its form remains unchanged following a given transformation. From this, it is apparent that the concept of symmetry has a mathematical origin but it has profound consequences in all fields of physics and there exist symmetries even in the basic physical laws themselves.

According to German mathematician E.Noether: “Every conservation principle corresponds to a symmetry in nature”. For example, the conservation of energy follows from the time-invariance of the physical systems, and the fact that physical systems behave the same regardless of how they are oriented in space gives rise to the conservation of angular momentum.

### *SU(2) SYMMETRY*

Isospin arises because the nucleon may be viewed as having an internal degree of freedom with two allowed states , the proton and the neutron, which the nuclear interaction does not distinguish. We therefore have an SU(2) symmetry in which the (n,p) form the fundamental representation. Thus the group structure of isospin symmetry is very similar to that of the usual spin. The isospin generators  $T_i$  satisfy the Lie algebra of SU(2)

$$[T_i, T_j] = i\epsilon_{ijk} T_k$$

where the indices range from 1 to 3.

The proton and neutron forms a doublet  $\begin{pmatrix} p \\ n \end{pmatrix}$  means that

$$T_3|p\rangle = \frac{1}{2} |p\rangle \quad , \quad T_3|n\rangle = -\frac{1}{2} |n\rangle$$

And  $T_+|n\rangle = |p\rangle$  ,  $T_-|n\rangle = |p\rangle$  .

The strong interaction does not distinguish n from p means that the strong interaction Hamiltonian  $H_s$  has the property

$$[T_i, H_s] = 0 \quad ; \quad i=1,2,3.$$

The concept of isospin can be extended to other hadrons. For example,  $(\pi^+, \pi^0, \pi^-)$ ,  $(\Sigma^+, \Sigma^0, \Sigma^-)$ ,  $(\rho^+, \rho^0, \rho^-)$  are (T=1) isotriplets ;  $(K^+, K^0)$ ,  $(\bar{K}^0, K^-)$ , and  $(\Xi^0, \Xi^-)$  are (T=1/2) doublets;  $\eta$ ,  $\omega$ ,  $\phi$ , and  $\Lambda$  are (T=0) isosinglets.

The strong interactions are invariant under a transformation which interchanges proton (p) and neutron (n). Since different members of the isospin multiplet have different electric charges, the electromagnetic interaction clearly does not respect the isospin symmetry.

If the symmetry is an exact one, we have

$$[T_i, H] = 0$$

for the total Hamiltonian of the system; all members of an isomultiplet would be strictly degenerate in mass. Thus the mass differences within an isomultiplet are a good measure of the symmetry breaking.

### ***SU(3) SYMMETRY***

The set of unitary 3X3 matrices with  $\det U=1$  form the group SU(3). The generators may be taken to be any  $3^2-1=8$  linearly independent traceless hermitian 3X3 matrices. Since it is possible to have only two of these traceless matrices diagonal , this is the maximum number of mutually commuting generators. This number is called the rank of the group, so that SU(3) has rank 2 and SU(2) has rank 1.

The fundamental representation of SU(3) is a triplet. The three color charges of a quark R, G, B form the fundamental representation of an SU(3) symmetry group. In this representation, the generators are 3X3 matrices. They are traditionally denoted by  $\lambda_i$  with  $i=1, \dots, 8$  and the diagonal matrices are taken to be

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad , \quad \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

with simultaneous eigenvectors

$$R = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

With this numbering of the  $\lambda_i$  matrices,  $\lambda_1, \lambda_2, \lambda_3$  correspond to the three Pauli matrices and thus they exhibit explicitly one SU(2) subgroup of SU(3). The  $\lambda_i$  are known as the Gell-Mann matrices.

## ***1.6 SYMMETRY BREAKINGS***

Symmetry can be exact, approximate or broken<sup>[16-19]</sup>. Exact means unconditionally valid; approximate means valid under certain conditions; broken can mean different things, depending on the object considered and its context. Generally the breaking of certain symmetry does not imply that no symmetry is present, but rather that the situation where this symmetry is broken is characterized by a lower symmetry than the situation where this symmetry is not broken.

Symmetry breakings are classified as:

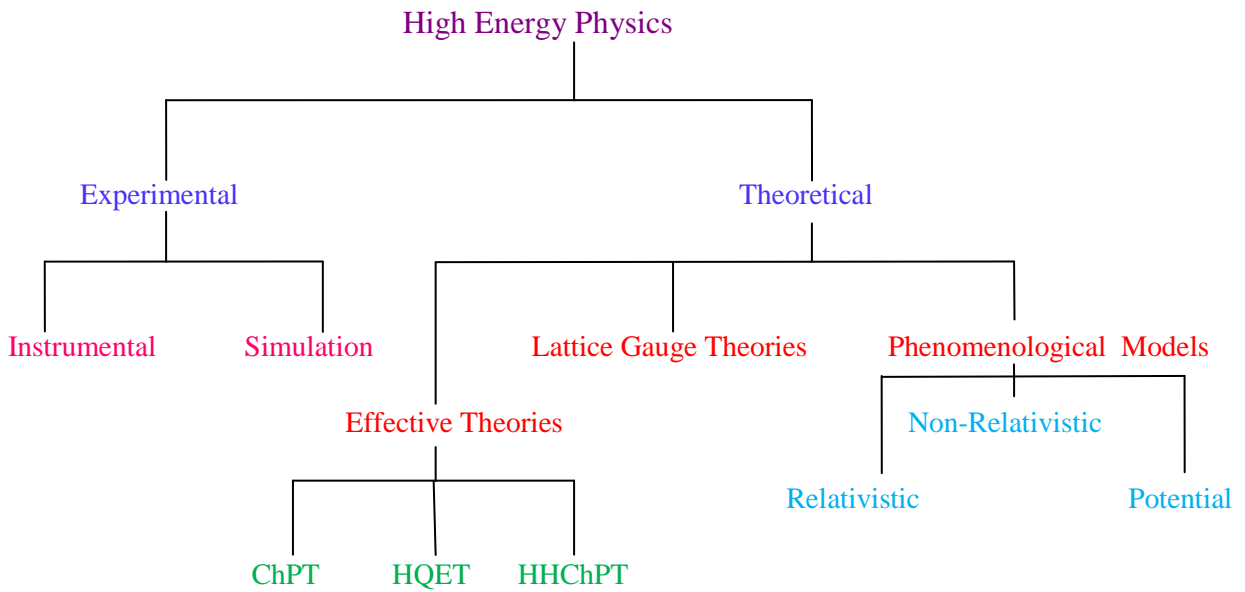
### 1.6.1 Explicit Symmetry Breaking

### 1.6.2 Spontaneous Symmetry Breaking

## CHAPTER 2

### HEAVY QUARK EFFECTIVE THEORY

There are many ways to know the masses of various states of bottom mesons which are listed in the following classification table. My thesis work focusses mainly on effective theories. These theories provide theoretical predictions that can be confronted with experimental observations.



In the following sections , these effective theories have been discussed.

#### ***2.1 EFFECTIVE FIELD THEORIES***

In physics, an effective field theory is, as any effective theory, an approximate theory, (usually a quantum field theory) that includes appropriate degrees of freedom to describe physical phenomena occurring at a chosen length scale, while ignoring the degrees of freedom at shorter distances (or, equivalently, at higher energies). Presently, in particle physics, effective field theories are written for many situations. The effective field theory of QCD called chiral perturbation theory has had better success. This theory deals with the interactions of hadrons with pions or kaons, which are the Goldstone bosons of spontaneous chiral symmetry breaking. The expansion parameter is the pion energy/momentum. For hadrons containing two heavy quarks, an effective field theory which expands in powers of the relative velocity of the heavy quarks, called non-relativistic QCD (NRQCD), has been found useful, especially when used in conjunctions with lattice QCD. For hadrons containing one heavy quark (such as the bottom or charm), an effective field

theory which expands in powers of the quark mass, called the heavy-quark effective theory (HQET), has been found useful. In this case, the large scale is the mass  $m_Q$  of the heavy quark, and the leading term of this expansion is the static limit. Heavy Quark Effective Theory is a powerful tool, which allows for numerous purely QCD based calculations. Before discussing the Effective Theories, let us understand chiral symmetry and heavy-quark symmetry first.

### 2.2.1 CHIRAL SYMMETRY

In quantum field theory, chiral symmetry is a possible symmetry of the Lagrangian under which the left-handed and right-handed parts of Dirac fields transform independently. The chiral symmetry transformation can be divided into a component that treats the left-handed and the right-handed parts equally, known as vector symmetry, and a component that actually treats them differently, known as axial symmetry.

The u, d, s quark masses are small compared with the scale  $\Lambda_{\text{QCD}}$  of non-perturbative strong interaction physics, and so it is useful to consider an approximation to QCD in which the masses of these light quarks are set to zero, and to do perturbation theory in  $m_q$  about this limit. The  $m_q \rightarrow 0$ , is known as the chiral limit because the light quark Lagrangian is

$$L = \bar{q}_L i \mathcal{D} q_L + \bar{q}_R i \mathcal{D} q_R + L_{\text{gluons}}$$

In terms of left-handed and right-handed spinors it becomes

$$L = \bar{u}_L i \mathcal{D} u_L + \bar{u}_R i \mathcal{D} u_R + \bar{d}_L i \mathcal{D} d_L + \bar{d}_R i \mathcal{D} d_R + L_{\text{gluons}}$$

(Hereby  $i$  is the imaginary unit and  $\mathcal{D}$  the well-known Dirac operator.)

The Lagrangian is unchanged under a rotation of  $q_L$  by any  $2 \times 2$  unitary matrix  $L$ , and  $q_R$  by any  $2 \times 2$  unitary matrix  $R$ . This symmetry of the Lagrangian is called flavor symmetry or chiral symmetry, and denoted as  $U(2)_L \times U(2)_R$ . It can be decomposed into

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

The vector symmetry  $U(1)_V$  acts as

$$q_L \rightarrow e^{i\theta} q_L \quad , \quad q_R \rightarrow e^{i\theta} q_R$$

and corresponds to baryon number conservation.

The axial symmetry  $U(1)_A$  acts as

$$q_L \rightarrow e^{i\theta} q_L \quad , \quad q_R \rightarrow e^{-i\theta} q_R$$

and it does not correspond to a conserved quantity because it is violated due to quantum anomaly.

The remaining chiral symmetry  $SU(2)_L \times SU(2)_R$  turns out to be spontaneously broken into the vector subgroup  $SU(2)_V$ , known as isospin. The Goldstone bosons corresponding to the three broken generators are the pions. In real world, because of the differing masses of the quarks,  $SU(2)_L \times SU(2)_R$  is only an approximate symmetry to begin with, and therefore the pions are not massless, but have small masses: they are pseudo-Goldstone bosons<sup>[20]</sup>.

### 2.2.2 HEAVY QUARK SYMMETRY

For quarks with masses  $m_Q$  ( $Q \rightarrow c, b, t$  are the heavy quarks) that are large compared with the scale of non-perturbative strong dynamics, it is a good approximation to take the  $m_Q \rightarrow \infty$  limit of QCD. In this limit, QCD has spin-flavor heavy quark symmetry, which has important implications for the properties of hadrons containing single heavy quark. There are several reasons why the strong interactions of systems containing heavy quarks are easier to understand than those of systems containing only light quarks. The first is asymptotic freedom, the fact that the effective coupling constant of QCD becomes weak in processes with large momentum transfer, corresponding to interactions at short distance scales<sup>[21,22]</sup>. The effective coupling constant

$$\alpha_s(Q^2) = \frac{g_{eff}^2}{4\pi} = \frac{12\pi}{(33 - 2N_f) \ln(Q^2 / \Lambda_{QCD}^2)}$$

decreases at large  $Q^2$ , i.e., the strong interactions become weak at short distances. At large distances, (small  $Q^2$ ), on the other hand, the coupling becomes strong, leading to non-perturbative phenomena such as the confinement of quarks and gluons on a length scale  $R_{had} \sim 1/\Lambda_{QCD} \sim 1$  fm, which determines the typical size of hadrons. Roughly speaking,  $\Lambda_{QCD} \sim 0.2$  GeV is the energy scale that separates the regions of large and small coupling constant.

When the mass of quark  $Q$  is much larger than this scale, i.e. when  $m_Q \gg \Lambda_{QCD}$ , then the effective coupling constant  $\alpha_s(m_Q)$  is small, implying that on length scales comparable to the Compton wavelength  $\lambda_Q \sim 1/m_Q$  the strong interactions are perturbative and much like the electromagnetic interactions. In fact, the quarkonium systems ( $Q\bar{Q}$ ), which have a size of order  $\lambda_Q/\alpha_s(m_Q) \ll R_{had}$ , are very much hydrogen-like. This is the reason why charmonium and bottomonium type of systems are easier to handle than the other mesons.

For systems composed of a heavy quark and other light constituents, things are most complicated. The size of such systems is determined by  $R_{\text{had}}$ , and the typical momenta exchanged between the heavy and light constituents are of order  $\Lambda_{\text{QCD}}$ . The heavy quark is surrounded by a most complicated strongly interacting cloud of light quarks, antiquarks, and gluons. This cloud is sometimes referred to as the “brown muck”.

In this case, it is the fact that  $m_Q \gg \Lambda_{\text{QCD}}$ , so,  $\lambda_Q \ll R_{\text{had}}$ , the Compton wavelength of the heavy quarks much smaller than the size of the hadron, which leads to simplifications. To resolve the quantum numbers of the heavy quark would require a hard probe with  $Q^2 \geq m_Q^2$ . The soft gluons which couple to the “brown muck” can only resolve distances much larger than  $\lambda_Q$ . Therefore, the light degrees of freedom are blind to the flavor(mass) and spin orientation of the heavy quark; they only experience its color field, which extends over large distances because of confinement.

Hence, the dynamics is unchanged under the exchange of heavy quark flavors. Only the triplet color source sitting at the centre is visible in both the cases. This gives rise to  $U(N_f)$  symmetry, where  $N_f$  is the number of heavy quark flavors. In the  $m_Q \rightarrow \infty$  limit, the static heavy quark can only interact with gluons via its chromoelectric charge. The interaction is spin independent. This leads to  $SU(2)$  spin symmetry: the dynamics is unchanged under arbitrary transformations on the spin of the heavy quark. The spin dependent interactions are proportional to the chromomagnetic moment of the quark, and so are of the order of  $1/m_Q$ . The combination of above two symmetries leads to  $U(2N_f)$  spin-flavor symmetry in the  $m_Q \rightarrow \infty$  limit. It follows that, in the  $m_Q \rightarrow \infty$  limit, hadronic systems which differ only in the flavor or spin quantum numbers of the heavy quark have the same configuration of light degrees of freedom.

The heavy-quark symmetry is an approximate symmetry, and corrections arise since the quark masses are not infinite. These corrections are of the order  $\Lambda_{\text{QCD}}/m_Q$ . The condition  $m_Q \gg \Lambda_{\text{QCD}}$  is necessary and sufficient for a system containing a heavy quark to be close to the symmetry limit. In many respects, heavy-quark symmetry is complementary to chiral symmetry, which arises in the opposite limit of small quark masses,  $m_q \ll \Lambda_{\text{QCD}}$ . There is an important distinction, however. Whereas chiral symmetry is a symmetry of the QCD Lagrangian in the limit of vanishing quark masses, heavy-quark symmetry is not a symmetry of the Lagrangian (not even an approximate one), but rather a symmetry of an effective theory which is a good approximation to QCD in a certain kinematic region. It is realized only in systems in which a heavy quark interacts predominantly by the exchange of soft gluons. In such systems the heavy quark is almost on-shell; its momentum fluctuates around the mass shell by an amount of order  $\Lambda_{\text{QCD}}$ . The corresponding changes in the velocity of the heavy quark as  $\Lambda_{\text{QCD}}/m_Q \rightarrow 0$ . The velocity becomes a conserved quantity and is no longer a dynamical degree of freedom.

Results derived on the basis of heavy-quark symmetry are model-independent consequences of QCD in a well defined limit. The symmetry breaking corrections can, at least in principle, be studied in a systematic way.

## ***2.3 CHIRAL PERTURBATION THEORY***

**Chiral perturbation theory (ChPT)** is an effective field theory constructed with a Lagrangian consistent with the (approximate) chiral symmetry of quantum chromodynamics (QCD), as well as the other symmetries of parity and charge conjugation. ChPT is a theory which allows one to study the low-energy dynamics of QCD. The effective Lagrangian is expressed in terms of those hadronic degrees of freedom which, at low energies, show up as observable asymptotic states<sup>[23]</sup>. At low energies, it implies that the  $SU(3)_L \times SU(3)_R \times U(1)_V$  of the Lagrangian is spontaneously broken down to  $SU(3)_V \times U(1)_V$  at a scale  $\Lambda_{\text{QCD}} \sim 1$  GeV, with the generation of eight Goldstone bosons (pions, kaons and  $\eta$ ). The interactions of these Goldstone bosons at energies much smaller than  $\Lambda_{\text{QCD}}$  can be calculated using an effective Lagrangian. The light quarks u, d, and s can be treated as approximately mass-less and having masses very small as compared to  $\Lambda_{\text{QCD}}$ , whereas the heavy quarks c, b and t are those whose masses are large as compared to this energy scale. The heavy quarks are integrated out of the low energy effective theory<sup>[35]</sup>. The non-vanishing masses of the light pseudo-scalars in the “real” world are related to the explicit symmetry breaking in QCD due to the light quark masses<sup>[23]</sup>.

ChPT is based on a theorem that states that apart from causality and unitarity, the contents of a quantum field theory are dictated by the symmetries it possesses. So, one has to replace the quarks and gluons of QCD by the pseudo-scalar mesons and write down the most general Lagrangian involving these particles that have the same symmetries as the QCD Lagrangian. The ChPT generating functional admits an expansion in powers of external momenta and quark masses. Although ChPT is not a renormalizable theory the results can be rendered by finite order in the expansion. The consequence is that new terms (with unknown constants) have to be included in the Lagrangian at each order in the expansion<sup>[24]</sup>.

## ***2.4 HEAVY QUARK EFFECTIVE THEORY***

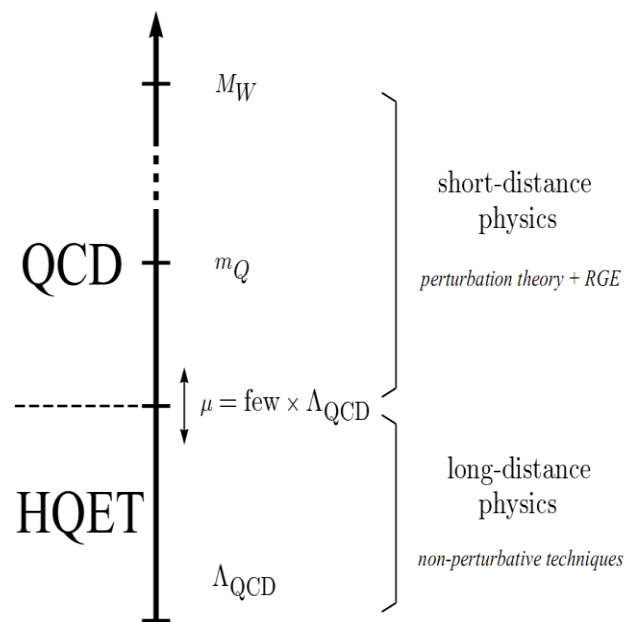
**Heavy Quark Effective Theory (HQET)** is a new approach to QCD problems involving a heavy quark. The effects of a very heavy particle often become irrelevant at low energies. So it is useful to construct a low-energy effective theory, in which this heavy particle no longer appears. Eventually, this effective theory will be easier to deal with than the full theory. The process of removing the degrees of freedom of a heavy particle involves the following steps<sup>[25]-[27]</sup>:

Firstly identify the heavy-particle fields and “integrate them out” in the generating functional of the Green functions of the theory. This is possible because at low energies the heavy particle does not appear as an external state.

However, although the action of the full theory is usually a local one, what results after this first step is a non-local effective action. The non-locality is related to the fact that in the full theory the heavy particle with mass  $M$  can appear in virtual processes and propagate over a short but finite distance  $\Delta x \sim 1/M$ .

Thus, a second step is required to obtain a local effective Lagrangian: the non-local effective action is rewritten as an infinite series of local terms in an Operator Product Expansion (OPE)<sup>[28],[29]</sup>. Roughly speaking, this corresponds to an expansion in powers of  $1/M$ . Short- and long-distance physics is separated in this step. The long-distance physics corresponds to interactions at low energies and is the same in the full and the effective theory. But short-distance effects arising from quantum corrections involving large virtual momenta (of order  $M$ ) are not reproduced in the effective theory, once the heavy particle has been integrated out.

In a third step, they have to be added in a perturbative way using renormalization-group techniques. These short-distance effects lead to a renormalization of the coefficients of the local operators in the effective Lagrangian. The heavy-quark effective theory (HQET) is constructed to provide a simplified description of processes where a heavy quark interacts with light degrees of freedom predominantly by the exchange of soft gluons<sup>[30]-[40]</sup>. Clearly,  $m_Q$  is the high-energy scale in this case, and  $\Lambda_{\text{QCD}}$  is the scale of the hadronic physics we are interested in. The situation is illustrated in the following figure:



**FIG.2.1 : Philosophy of the heavy-quark effective theory.**

At short distances, i.e. for energy scales larger than the heavy-quark mass, the physics is perturbative and described by ordinary QCD. For mass scales much below the heavy-quark mass, the physics is complicated and non-perturbative because of confinement. Our goal is to obtain a simplified description in this region using an effective field theory. To separate short- and long-distance effects, we introduce a separation scale  $\mu$  such that  $\Lambda_{\text{QCD}} \ll \mu \ll m_Q$ . The HQET will be constructed in such a way that it is identical to QCD in the long-distance region, i.e. for scales below  $\mu$ . In the short-distance region, the effective theory is incom-

plete, however, since some high-momentum modes have been integrated out from the full theory. The fact that the physics must be independent of the arbitrary scale  $\mu$  allows us to derive renormalization-group equations, which we shall employ to deal with the short-distance effects in an efficient way. Compared with most effective theories, in which the degrees of freedom of a heavy particle are removed completely from the low-energy theory, the HQET is special in that its purpose is to describe the properties and decays of hadrons which do contain a heavy quark. Hence, it is not possible to remove the heavy quark completely from the effective theory. What is possible is to integrate out the “small components” in the full heavy-quark spinor, which describe the fluctuations around the mass shell.

It is observed that<sup>[38],[39]</sup>

$$L_{\text{eff}} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \bar{h}_v (iD_\perp)^2 h_v + \frac{g_s}{4m_Q} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v + O(1/m_Q^2) \quad (2.1)$$

In the limit  $m_Q \rightarrow \infty$ , only the first terms remains:

$$L_\infty = \bar{h}_v i v \cdot D h_v \quad (2.2)$$

This is the effective Lagrangian of the HQET but it does not have the spin-flavor symmetry when  $m_Q \rightarrow \infty$  (because it does not contain the Dirac matrices and the mass term). But if the Lagrangian in Eq. (2.1) is considered, we find that it has two additional terms which breaks this symmetry. The first term in the Lagrangian which arises from the off-shell motion of the heavy quark breaks the flavour symmetry, while the second term which arises from the chromo-magnetic interaction of heavy quark spin with gluon fields breaks the spin symmetry. If the hard gluon exchange is included, then the coefficient in the Lagrangian changes and it looks like<sup>[41]</sup>:

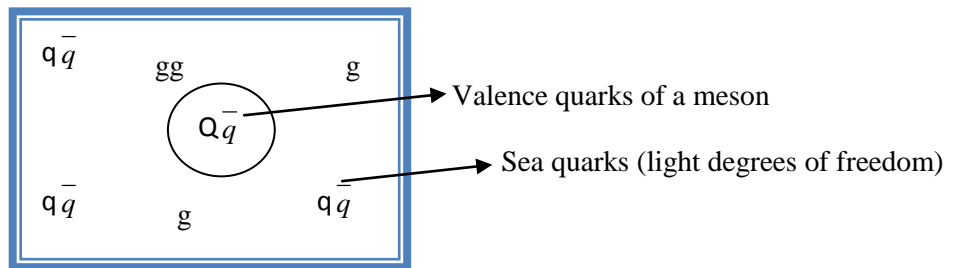
$$L = \bar{h}_v (i v \cdot D) h_v + \frac{a_1}{2m_Q} \bar{h}_v (iD)^2 h_v + \frac{g_\alpha a_2}{4m_Q} h_v G^{\alpha\beta} h_v \quad (2.3)$$

## CHAPTER 3

### FRAMEWORK

#### 3.1 QUANTUM NUMBERS FOR HEAVY-LIGHT MESONS

Heavy hadrons contain a heavy quark as well as light quarks and/or antiquarks and gluons. All the degrees of freedom other than the heavy quark are referred to as the light degrees of freedom. For example, a heavy  $Q\bar{q}$  meson has an antiquark  $\bar{q}$ , gluons, and  $q\bar{q}$  pairs as the light degrees of freedom.



*Fig. 3.1 Sea and its Structure.*

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fields form a  $\bar{3}$  representation of the light quark flavor group  $SU(3)_V$ . The  $SU(3)_V$  diagram for the  $\bar{3}$  mesons is shown below :

$$\begin{array}{ccc}
 \bar{B}_s^0, \bar{B}_s^{*0} & & \\
 \bar{b}\bar{s} & & \\
 \\
 \bar{B}^-, \bar{B}^{*-} & & \bar{B}^0, \bar{B}^{*0} \\
 \bar{b}\bar{u} & & \bar{b}\bar{d}
 \end{array}$$

**Fig .3.2  $SU(3)$  weight diagram for the spin-0 pseudoscalar and spin-1 vector  $b\bar{q}$  mesons.**

In the heavy quark limit the coupling of the heavy quark spin to the light degrees of freedom vanishes, in such a case the angular momentum and parity of the light degrees of freedom  $j^P$  can be used to classify the heavy meson states. The spectrum consists of the degenerate heavy meson doublets with definite  $j^P$ . The  $J^P = 0^-$  and  $1^-$  heavy mesons are members of the  $j^P = (1/2)^-$  ground state doublet. The lowest lying excited states,  $J^P = 0^+$  and  $1^+$  heavy mesons are the members of the  $j^P = (1/2)^+$  doublet. There is also an excited doublet of heavy mesons with  $j^P = 3/2^+$  whose members have  $J^P = 1^+$  and  $2^+$ . The  $j^P = 3/2^+$  mesons decay to the ground state by D-wave pion emission typically have width  $\Gamma \sim 20$  MeV, and therefore have well measured masses. The hyperfine splitting of all these heavy quark doublets are suppressed by  $1/m_Q$ , where  $m_Q$  is the heavy quark mass. The experimental values of the masses of B mesons with possible uncertainties in their masses is shown in the table below:

$S_l=(1/2)$	$S_l=(1/2)^-$ Ground state		$S_l=(1/2)^+$ LL excited state	
$J^P$	$0^-$	$1^-$	$0^+$	$1^+$
General states	$B$	$B^*$	$B_0$	$B_1$
$b\bar{u}$	$B^- = 5279.25 \pm 0.17$ $(m_{H_1})$	$B^{*-} = 5325.2 \pm 0.4$ $(m_{H_1}^*)$	$B_0^- = ?$ $(m_{S_1})$	$B_1^- = ?$ $(m_{S_1}^*)$
$b\bar{d}$	$B^0 = 5279.58 \pm 0.17$ $(m_{H_2})$	$B^{*0} = 5325.1 \pm 0.5$ $(m_{H_2}^*)$	$B_0^0 = ?$ $(m_{S_2})$	$B_1^0 = 5723.5 \pm 2.0$ $(m_{S_2}^*)$
$b\bar{s}$	$B_s^0 = 5366.77 \pm 0.24$ $(m_{H_3})$	$B_s^{*0} = 5415.4 \pm 1.4$ $(m_{H_3}^*)$	$B_{0s}^0 = 5839.7 \pm 0.6$ $(m_{S_3})$	$B_{1s}^0 = 5829.4 \pm 0.7$ $(m_{S_3}^*)$
	Pseudoscalars	Vector	Scalars	Axial vector

**Table 3.1 Experimental masses of B-Mesons (in  $MeV/c^2$ ).**

As we see from the table 3.1 that some of the masses of B mesons are not known. The relations between hadron masses developed in HQET by which these masses can be find out are discussed in the next sections.

### 3.2 RELATION BETWEEN MASSES

Heavy quark symmetry can be used to obtain relations between hadron masses. The hadron mass in the effective theory is  $m_H - m_Q$ , since the heavy quark mass  $m_Q$  has been subtracted from all energies. At order  $m_Q$ , all heavy hadrons containing Q are degenerate, and have the same mass  $m_Q$ . At the order of unity, the hadron masses get the contribution

$$\frac{1}{2} \langle H^{(\mathcal{Q})} | H_0 | H^{(\mathcal{Q})} \rangle \equiv \bar{\Lambda} , \quad (3.1)$$

Where  $H_0$  is the order  $1/m_Q^0$  terms in the HQET Hamiltonian obtained from the Lagrangian term  $\bar{Q}_v (i v \cdot D) Q_v$ , as well as the terms involving light quarks and gluons. In this section the hadron states  $| H^{(\mathcal{Q})} \rangle$  are in the effective theory with  $v=v_f = (1,0)$ . Here  $\bar{\Lambda}$  is a parameter of HQET and has the same value for all particles in a spin-flavor multiplet. The values will be denoted by  $\bar{\Lambda}$  for B, B\*, D, and D\* states,  $\bar{\Lambda}_\Lambda$  for the  $\Lambda_b$  and  $\Lambda_c$ , and  $\bar{\Lambda}_\Sigma$  for the  $\Sigma_b$ ,  $\Sigma_b^*$ ,  $\Sigma_c$ ,  $\Sigma_c^*$ . In the SU(3) limit,  $\bar{\Lambda}$  doesnot depend on the light quark flavor. If SU(3) breaking is included,  $\bar{\Lambda}$  is different for the  $B_{u,d}$  and  $B_s$  mesons, and will be denoted by  $\bar{\Lambda}_{u,d}$  and  $\bar{\Lambda}_s$  respectively.

At order  $1/m_Q$ , there is an additional contribution to the hadron masses given by the expectation value of the  $1/m_Q$  correction to the Hamiltonian:

$$H_1 = -L_1 = \bar{Q}_v \frac{D_\perp^2}{2m_Q} Q_v + a(\mu) g \bar{Q}_v \frac{\sigma_{\alpha\beta} G^{\alpha\beta}}{4m_Q} Q_v \quad (3.2)$$

The matrix elements of the two terms in above equation define two nonperturbative parameters,  $\lambda_1$  and  $\lambda_2$  :

$$2\lambda_1 = -\langle H^{(\mathcal{Q})} | \bar{Q}_{vr} D_\perp^2 Q_{vr} | H^{(\mathcal{Q})} \rangle, \quad (3.3)$$

$$16(S_Q \cdot S_l) \lambda_2(m_Q) = a(\mu) \langle H^{(\mathcal{Q})} | \bar{Q}_{vr} g \sigma_{\alpha\beta} G^{\alpha\beta} Q_{vr} | H^{(\mathcal{Q})} \rangle \quad (3.4)$$

Here  $\lambda_1$  is independent of  $m_Q$ , and  $\lambda_2$  depends on  $m_Q$  through the logarithmic  $m_Q$  dependence of  $a(\mu)$  as

$$a(\mu) = \left[ \frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right]^{9/(33-2N_f)} ; \quad (3.5)$$

$\lambda_{1,2}$  have the same value for all states in a given spin-flavor multiplet and are expected to be of the order of  $\Lambda_{QCD}^2$ .

The naïve expectation that the heavy quark kinetic energy is positive suggests that  $\lambda_1$  should be negative. The  $\lambda_2$  matrix element transforms like  $S_Q \cdot S_l$  under the spin symmetry, since that is the transformation property of  $\overline{Q}_{vr} \sigma_{\alpha\beta} G^{\alpha\beta} Q_{vr}$ . Only the two upper components of  $Q_{vr}$  are nonzero, since  $\gamma^0 Q_{vr} = Q_{vr}$ , and  $\overline{Q}_{vr} \sigma_{\alpha\beta} G^{\alpha\beta} Q_{vr}$  reduces to the matrix element of  $\overline{Q}_{vr} \sigma \cdot B Q_{vr}$ , where B is chromo-magnetic field. The operator  $\overline{Q}_{vr} \sigma Q_{vr}$  is the heavy quark spin, and the matrix element of B in the hadron must be proportional to the spin of the light degrees of freedom, by rotational invariance and time-reversal invariance, so that the chromo-magnetic operator contribution is proportional to  $S_Q \cdot S_l$ . Using  $S_Q \cdot S_l = (J^2 - S_Q^2 - S_l^2)/2$ , one finds that

$$\begin{aligned} m_B &= m_b + \overline{\Lambda} - \frac{\lambda_1}{2m_b} - \frac{3\lambda_2(m_b)}{2m_b}, \\ m_B^* &= m_b + \overline{\Lambda} - \frac{\lambda_1}{2m_b} + \frac{\lambda_2(m_b)}{2m_b}, \\ m_D &= m_c + \overline{\Lambda} - \frac{\lambda_1}{2m_c} - \frac{3\lambda_2(m_c)}{2m_c}, \\ m_D^* &= m_c + \overline{\Lambda} - \frac{\lambda_1}{2m_c} + \frac{\lambda_2(m_c)}{2m_c} \end{aligned}$$

The average mass of a heavy quark spin symmetry multiplet, e.g.,  $(3m_{p^*} + m_p)/4$  for the meson multiplet, does not depend on  $\lambda_2$ . The magnetic interaction  $\lambda_2$  is responsible for the  $B^* - B$  and  $D^* - D$  splitting. The observed value of the  $B^* - B$  mass difference gives  $\lambda_2(m_b) \sim 0.12 \text{ GeV}^2$ .

The above equations gives the mass relation as :

$$0.49 \text{ GeV}^2 \approx m_{B^*}^2 - m_B^2 \approx 4\lambda_2 \approx m_{D^*}^2 - m_D^2 \approx 0.55 \text{ GeV}^2$$

Upto corrections of order  $1/m_{b,c}$  and ignoring the weak  $m_Q$  dependence of  $\lambda_2$ . Similarly, one finds that

$$90 \pm 3 \text{ MeV} = m_{B_s} - m_{B_d} = \overline{\Lambda}_s - \overline{\Lambda}_{u,d} = m_{D_s} - m_{D_d} = 99 \pm 1 \text{ MeV}$$

The parameters  $\lambda_1$  and  $\lambda_2$  are nonperturbative parameters of QCD and have not been computed from first principles. It might appear that very little has been gained by using the above equations for the hadron masses in terms of  $\overline{\Lambda}$ ,  $\lambda_1$  and  $\lambda_2$ . However the same hadronic matrix elements also occur in other quantities, such as form factors and decay rates. One can then use the values of  $\overline{\Lambda}$ ,  $\lambda_1$  and  $\lambda_2$  obtained by fitting to

the hadron masses to compute the form factors and decay rates, without making any model dependent assumptions.

### 3.3 HQET FORMULA FOR CALCULATING B-MESON MASSES

We use the  $O(1/m_Q)$  HQET formulas for the mass of a heavy hadron  $X$  which contains a heavy quark  $Q$ <sup>[46]</sup>:

$$m_X^{(\mathcal{Q})} = m_Q + \bar{\Lambda}^X - \frac{\lambda_1^X}{2m_Q} + n_J \frac{\lambda_2^X}{2m_Q} \quad (3.6)$$

where  $\lambda_1^X$  and  $\lambda_2^X$  are hadronic matrix elements of the HQET operators  $\bar{h}(iD)^2 h$  and  $g_s \bar{h} \sigma^{\mu\nu} G_{\mu\nu} h/2$ , respectively, and  $n_J = +1$  for  $J=1$  states and  $n_J = -3$  for  $J=0$  states. The first  $1/m_Q$  correction,  $-\lambda_1^X/(2m_Q)$ , is the kinetic energy of the heavy quark. The second  $1/m_Q$  correction contributes to hyperfine splitting.

In general, for B and D mesons, the mass formulas for the spin state (0,1) can be written as

$$\begin{aligned} m_B &= m_b + \bar{\Lambda} - \frac{\lambda_1^B}{2m_Q} - \frac{3\lambda_2^B}{m_Q}, & m_{B^*} &= m_b + \bar{\Lambda} - \frac{\lambda_1^B}{2m_b} + \frac{\lambda_2^B}{m_b} \\ m_D &= m_c + \bar{\Lambda} - \frac{\lambda_1^D}{2m_c} - \frac{3\lambda_2^D}{m_c}, & m_{D^*} &= m_c + \bar{\Lambda} - \frac{\lambda_1^D}{2m_c} + \frac{\lambda_2^D}{m_c} \end{aligned}$$

where  $\bar{m}_H^{(\mathcal{Q})} = (3m_{H^*}^{(\mathcal{Q})} + m_H^{(\mathcal{Q})})/4$  and  $\bar{m}_S^{(\mathcal{Q})} = (3m_{S^*}^{(\mathcal{Q})} + m_S^{(\mathcal{Q})})/4$ .

The difference between the spin averaged masses of the  $j^P = \frac{1}{2}^-$  and  $j^P = \frac{1}{2}^+$  mesons, respectively, is given by

$$\bar{m}_S^{(\mathcal{Q})} - \bar{m}_H^{(\mathcal{Q})} = \bar{\Lambda}^S - \bar{\Lambda}^H - \frac{\lambda_1^S}{2m_Q} + \frac{\lambda_1^H}{2m_Q}, \quad (3.7)$$

which leads to the following formulas for the splitting of the even and odd parity states in the bottom sector:

$$\bar{m}_S^b - \bar{m}_H^b = \bar{m}_S^c - \bar{m}_H^c + (\lambda_1^S - \lambda_1^H) \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right). \quad (3.8)$$

A global fit to B- decays yields  $\lambda_1^H = -0.20 \pm 0.06 GeV^2$ <sup>[47]</sup> but the parameter  $\lambda_1^S$  is unknown. From the

spectroscopy of excited  $j^P = \frac{3}{2}^+$  D and B mesons, Ref. [48] extracts  $\lambda_1^{3/2} - \lambda_1^H = -0.23 GeV^2$ , where  $\lambda_1^{3/2}$

is the  $\lambda_1$  matrix element of the  $j^p = \frac{3^+}{2}$  doublet. The negative sign indicates that the kinetic energy of the heavy quark in the excited heavy meson is larger than that in the ground state, which agrees with intuition.

The kinetic energy of the heavy quark in the  $j^p = \frac{1^+}{2}$  states is expected to be comparable to that of  $j^p = \frac{3^+}{2}$  states.

To estimate  $\overline{m}_S^{(b)}$  with conservative errors, we take  $\lambda_1^S - \lambda_1^H = -0.2 \pm 0.1 \text{ GeV}^2$ ,  $m_c = 1.4 \text{ GeV}$  and  $m_b = 4.8 \text{ GeV}$  to find

$$\overline{m}_S^{(b)} - \overline{m}_H^{(b)} = \overline{m}_S^{(c)} - \overline{m}_H^{(c)} - 50 \pm 25 \text{ MeV} \quad (3.9)$$

We estimate the masses of excited bottom meson states ( $0^+$  and  $1^+$ ) with the help of the parameters given for the ground state and low-lying excited state using the ratio

$$\frac{m_{H^*}^{(b)} - m_H^{(b)}}{m_{H^*}^{(c)} - m_H^{(c)}} = \frac{m_{S^*}^{(b)} - m_S^{(b)}}{m_{S^*}^{(c)} - m_S^{(c)}} = \frac{m_c}{m_b}. \quad (3.10)$$

upto the  $O(1/m_Q)$  corrections. Thus, all the hyperfine splittings in the bottom sector are related to the charm sector by a universal factor.

Using the fitted values for the masses of the ground state and excited state D mesons, their spin averaged

masses for the  $j^p = \frac{1^-}{2}$  and  $j^p = \frac{1^+}{2}$  have been calculated as follows:

$$\begin{aligned} \overline{m}_{H_1}^{(c)} &= \left(3m_{H_1}^{(c)} + m_{H_1}^{(c)}\right)/4 = [3(44.4608) - 105.971]/4 = 6.85285 \\ \overline{m}_{H_3}^{(c)} &= \left(3m_{H_3}^{(c)} + m_{H_3}^{(c)}\right)/4 = [3(136.485) - 5.27028]/4 = 101.04618 \\ \overline{m}_{S_1}^{(c)} &= \left(3m_{S_1}^{(c)} + m_{S_1}^{(c)}\right)/4 = [3(460.509) + 329.722]/4 = 427.81225 \\ \overline{m}_{S_3}^{(c)} &= \left(3m_{S_3}^{(c)} + m_{S_3}^{(c)}\right)/4 = [3(483.281) + 341.077]/4 = 447.73 \end{aligned} \quad (3.11)$$

The hyperfine splittings of the B mesons have been calculated using the splittings given for the D mesons

and the relation (3.10). Since  $m_c = 1.4 \text{ GeV}$  and  $m_b = 4.8 \text{ GeV}$ , this gives  $\frac{m_c}{m_b} = 0.2916$ .

$$m_{H_1^*}^{(b)} - m_{H_1}^{(b)} = \left(\frac{m_c}{m_b}\right) \left(m_{H_1^*}^{(c)} - m_{H_1}^{(c)}\right) = (0.2916)(150.1997) = 43.798$$

$$m_{H_3^*}^{(b)} - m_{H_3}^{(b)} = \left(\frac{m_c}{m_b}\right) \left(m_{H_3^*}^{(c)} - m_{H_3}^{(c)}\right) = (0.2916)(141.756) = 41.336$$

$$\begin{aligned}
m_{S_1^*}^{(b)} - m_{S_1}^{(b)} &= \left( \frac{m_c}{m_b} \right) (m_{S_1^*}^{(c)} - m_{S_1}^{(c)}) = (0.2916)(119.432) = 34.826 \\
m_{S_3^*}^{(b)} - m_{S_3}^{(b)} &= \left( \frac{m_c}{m_b} \right) (m_{S_3^*}^{(c)} - m_{S_3}^{(c)}) = (0.2916)(141.740) = 41.331
\end{aligned} \tag{3.12}$$

So, the value for  $m_{H_1^*}^{(b)}$  and  $m_{H_3^*}^{(b)}$  will be

$$m_{H_1^*}^{(b)} = 43.798 + m_{H_1}^{(b)} = 43.798 + 5271.1 = 5314.898 \text{ MeV}$$

$$m_{H_3^*}^{(b)} = 41.336 + m_{H_3}^{(b)} = 41.336 + 5258.6 = 5299.936 \text{ MeV}$$

Using the theoretical values of bottom non-strange sector,  $m_{H_1}^{(b)} = 5271.1 \text{ MeV}$  and

$m_{H_1^*}^{(b)} = 5314.898 \text{ MeV}$ ,  $\bar{m}_{H_1} = 5303.9485 \text{ MeV}$ ,  $m_{H_3}^{(b)} = 5258.6 \text{ MeV}$  and the relation (3.9), the spin averaged masses of the B mesons have been calculated as follows:

$$\begin{aligned}
\bar{m}_{S_1}^{-b} - \bar{m}_{H_1}^{-b} &= \bar{m}_{S_1}^{-c} - \bar{m}_{H_1}^{-c} - 50 \pm 25 \text{ MeV} \\
\bar{m}_{S_1}^{-b} &= \bar{m}_{H_1}^{-b} + \bar{m}_{S_1}^{-c} - \bar{m}_{H_1}^{-c} - 50 \pm 25 \text{ MeV} \\
\bar{m}_{S_1}^{-b} &= 5303.9485 + 427.81225 - 6.85285 - 50 \pm 25 \text{ MeV} \\
\bar{m}_{S_1}^{-b} &= 5674.9079 \pm 25 \text{ MeV}
\end{aligned} \tag{3.13}$$

Similarly,

$$\begin{aligned}
\bar{m}_{S_3}^{-b} - \bar{m}_{H_3}^{-b} &= \bar{m}_{S_3}^{-c} - \bar{m}_{H_3}^{-c} - 50 \pm 25 \text{ MeV} \\
\bar{m}_{S_3}^{-b} &= \bar{m}_{H_3}^{-b} + \bar{m}_{S_3}^{-c} - \bar{m}_{H_3}^{-c} - 50 \pm 25 \text{ MeV} \\
\bar{m}_{S_3}^{-b} &= 5289.602 + 447.73 - 101.04618 - 50 \pm 25 \text{ MeV} \\
\bar{m}_{S_3}^{-b} &= 5586.285 \pm 25 \text{ MeV}
\end{aligned} \tag{3.14}$$

Thus the spin averaged masses of the B mesons are found to be

$$\begin{aligned}
\bar{m}_{H_1}^{(b)} &= \left( 3m_{H_1^*}^{(b)} + m_{H_1}^{(b)} \right) / 4 = 5303.9485 \text{ MeV} \\
\bar{m}_{H_3}^{(b)} &= \left( 3m_{H_3^*}^{(b)} + m_{H_3}^{(b)} \right) / 4 = 5289.602 \text{ MeV} \\
\bar{m}_{S_1}^{(b)} &= \left( 3m_{S_1^*}^{(b)} + m_{S_1}^{(b)} \right) / 4 = 5674.9079 \pm 25 \text{ MeV} \\
\bar{m}_{S_3}^{(b)} &= \left( 3m_{S_3^*}^{(b)} + m_{S_3}^{(b)} \right) / 4 = 5586.285 \pm 25 \text{ MeV}
\end{aligned} \tag{3.15}$$

The masses of the low-lying excited B mesons found out by solving the equations (3.12) and (3.15) are given below:

$$m_{S_1}^{(b)} = 5631.3754 \pm 25 \text{ MeV}, \quad m_{S_1^*}^{(b)} = 5666.2014 \pm 25 \text{ MeV}, \quad m_{S_3}^{(b)} = 5718.669 \text{ MeV},$$

The calculated mass splittings are:

$$B_{os}^0 - B_0^-(0^+ - 0^+) = 62.2936 \text{ MeV}$$

$$B_{1s}^0 - B_1^-(1^+ - 1^+) = 68.7986 \text{ MeV}$$

The calculated iso-spin splittings are:

$$B_s^{*0} - B_s^0 = 41.336 \text{ MeV}$$

$$B_1^- - B_0^- = 34.826 \text{ MeV}$$

Now, using the experimental values of known B and D mesons masses, the calculations are:

$$m_{H_1^*}^{(b)} - m_{H_1}^{(b)} = \left( \frac{m_c}{m_b} \right) (m_{H_1^*}^{(c)} - m_{H_1}^{(c)}) = (0.2916)(141.39) = 41.229$$

$$m_{H_3^*}^{(b)} - m_{H_3}^{(b)} = \left( \frac{m_c}{m_b} \right) (m_{H_3^*}^{(c)} - m_{H_3}^{(c)}) = (0.2916)(143.63 \pm 0.33) = 41.882 \pm 0.096$$

$$m_{S_1^*}^{(b)} - m_{S_1}^{(b)} = \left( \frac{m_c}{m_b} \right) (m_{S_1^*}^{(c)} - m_{S_1}^{(c)}) = (0.2916)(120 \pm 2) = 34.992 \pm 0.58$$

$$m_{S_3^*}^{(b)} - m_{S_3}^{(b)} = \left( \frac{m_c}{m_b} \right) (m_{S_3^*}^{(c)} - m_{S_3}^{(c)}) = (0.2916)(141.7) = 41.319 \quad (3.16)$$

The value for  $m_{H_1^*}^b$  and  $m_{H_3^*}^b$  will be

$$m_{H_1^*}^{(b)} = 41.229 + m_{H_1}^{(b)} = 41.229 + 5279.25 \pm 0.17 = 5320.479 \pm 0.17 \text{ MeV}$$

$$m_{H_3^*}^{(b)} = 41.882 \pm 0.096 + m_{H_3}^{(b)} = 41.882 \pm 0.096 + 5366.77 \pm 0.24 = 5408.652 \pm 0.336 \text{ MeV}$$

Using the experimental values of bottom non-strange sector,  $m_{H_1}^{(b)} = 5279.25 \pm 0.1 \text{ MeV}$  and

$m_{H_1^*}^{(b)} = 5325.2 \pm 0.4 \text{ MeV}$ ,  $m_{H_1}^{\bar{b}} = 5313.7125 \pm 0.325 \text{ MeV}$ ,  $m_{H_3}^{(b)} = 5366.77 \pm 0.24 \text{ MeV}$  given in Particle Data Group and the relation (3.9), the spin averaged masses of the B mesons have been calculated as follows:

$$\overline{m}_{S_1}^{\bar{b}} - \overline{m}_{H_1}^{\bar{b}} = \overline{m}_{S_1}^{\bar{c}} - \overline{m}_{H_1}^{\bar{c}} - 50 \pm 25 \text{ MeV}$$

$$\overline{m}_{S_1}^{\bar{b}} = \overline{m}_{H_1}^{\bar{b}} + \overline{m}_{S_1}^{\bar{c}} - \overline{m}_{H_1}^{\bar{c}} - 50 \pm 25 \text{ MeV}$$

$$\begin{aligned}\overline{m}_{S_1}^{-b} &= 5313.7125 \pm 0.325 + 2408 \pm 30.5 - 1973.2525 - 50 \pm 25 \text{MeV} \\ \overline{m}_{S_1}^{-b} &= 5698.46 \pm 55.825 \text{MeV}\end{aligned}\quad (3.17)$$

Similarly,

$$\begin{aligned}\overline{m}_{S_3}^{-b} - \overline{m}_{H_3}^{-b} &= \overline{m}_{S_3}^{-c} - \overline{m}_{H_3}^{-c} - 50 \pm 25 \text{MeV} \\ \overline{m}_{S_3}^{-b} &= \overline{m}_{H_3}^{-b} + \overline{m}_{S_3}^{-c} - \overline{m}_{H_3}^{-c} - 50 \pm 25 \text{MeV} \\ \overline{m}_{S_3}^{-b} &= 5329.85 \pm 1.15 + 2424.075 \pm 0.3 - 2076.1925 \pm 0.0825 - 50 \pm 25 \text{MeV} \\ \overline{m}_{S_3}^{-b} &= 5690.7325 \pm 26.5325 \text{MeV}\end{aligned}\quad (3.18)$$

Thus the spin averaged masses of the B mesons are found to be

$$\begin{aligned}\overline{m}_{H_1}^{(b)} &= \left(3m_{H_1^*}^{(b)} + m_{H_1}^{(b)}\right)/4 = 5313.7125 \pm 0.325 \text{MeV} \\ \overline{m}_{H_3}^{(b)} &= \left(3m_{H_3^*}^{(b)} + m_{H_3}^{(b)}\right)/4 = 5329.85 \pm 1.15 \text{MeV} \\ \overline{m}_{S_1}^{(b)} &= \left(3m_{S_1^*}^{(b)} + m_{S_1}^{(b)}\right)/4 = 5698.46 \pm 55.825 \text{MeV} \\ \overline{m}_{S_3}^{(b)} &= \left(3m_{S_3^*}^{(b)} + m_{S_3}^{(b)}\right)/4 = 5690.7325 \pm 26.5325 \text{MeV}\end{aligned}\quad (3.19)$$

The masses of the excited B mesons found out by solving the equations (3.16) and (3.19) are given below:

$$m_{S_1^*}^{(b)} = 5672.216 \pm 55.39 \text{MeV}, \quad m_{S_1^*}^{(b)} = 5707.208 \pm 55.97 \text{MeV}, \quad m_{S_3^*}^{(b)} = 5788.081 \pm 0.7 \text{MeV}$$

## **CHAPTER 4**

### **RESULTS and DISCUSSIONS**

The  $Q\bar{q}$  mesons are the QCD analog of the hydrogen atom. The ground-state (S-wave) meson has  $J^P = (1/2)^-$ , the excited P-wave mesons have  $J^P = (1/2)^+$  and  $(3/2)^+$ . When we switch the heavy quark spin on, each of these mesons becomes a degenerate doublet. Its components are transformed into each other by the heavy quark spin symmetry operations. We have the ground-state doublet  $0^-$ ,  $1^-$  and the excited P-wave doublets  $0^+$ ,  $1^+$  and  $1^+$ ,  $2^+$ . Splittings in these doublets (hyperfine splittings) are due to the heavy quark chromomagnetic moment interaction violating the spin symmetry, and are proportional to  $1/m_Q$ .

Heavy quark effective theory is useful theory in studying heavy-light mesons and baryons. In the present thesis, the masses of bottom mesons and their mass splittings have been calculated using a mass formula generated and applied with symmetries (Chiral symmetry and Heavy quark symmetry). Here the mass formulae depends on the HQET parameters and hence their values have been taken from the literature. The heavy quark mass limit and low quark mass limit have applications in HQET and they have been extensively studied in the present thesis. Due to the heavy quark spin symmetry, hadrons may be classified according to the light fields' angular momentum and parity  $J^P$  which are conserved quantum numbers. In other words, we can switch off the heavy quark spin using the superflavour symmetry, and then the hadron's momentum and parity will be  $J^P$ .

We studied the heavy quark effective theory with the aim to find the bottom mesons masses for ground state  $J^P = 0^-$  and  $1^-$  and low-lying excited state  $J^P = 0^+$  and  $1^+$ . We also reanalysed the new masses found here based on the new status report confirmed by the experiments. Bottom mesons studied in a framework where two symmetries have been exploited to formulate a theory that works quite well to understand these states of bottom mesons. This is known as heavy quark effective theory where the two symmetries have been combined i.e chiral symmetry and the heavy quark symmetry. Using this theory, masses for various bottom mesons have been predicted and their isospin and mass splittings have also been determined. These values are quite close to the other theoretical predictions and matching well with values predicted in other theories. The masses of the bottom mesons calculated using theoretical and experimental masses of known B and D mesons are listed here. Also, the theoretical values from other models and known experimental values of B mesons are mentioned in the table ahead:

Sr.No.	State	Calculated mass (MeV/c <sup>2</sup> ) Set I	Calculated mass (MeV/c <sup>2</sup> ) Set II	Theoretical Masses from other models (MeV/c <sup>2</sup> )	Experimental Masses (MeV/c <sup>2</sup> )
1.	$B^{*-}$	5314.898	5320.479±0.17	5321.5	5325.2±0.4
2.	$B_s^{*0}$	5299.936	5408.652±0.336	5440	5415.4±1.4
3.	$B_s^0$	5398.664	5373.518±1.304	5258.6	5366.77±0.24
4.	$B_0^-$	5631.3754±25	5672.216±55.39	5697	-
5.	$B_{0s}^0$	5718.669	5788.081±0.7	5716	5839.7±0.6
6.	$B_1^-$	5666.2014±25	5707.208±55.97	5780	-

**Table 4.1 Masses of B mesons.**

The masses of the states  $B^{*-}$ ,  $B_s^{*0}$ ,  $B_s^0$ ,  $B_0^-$ ,  $B_{0s}^0$ ,  $B_1^-$  calculated here vary from the other theoretical values by 0.12%, 2.57%, 2.66%, 0.71%, 0.04%, 1.5% respectively.

Also, the masses of the states  $B^{*-}$ ,  $B_s^{*0}$ ,  $B_s^0$ ,  $B_{0s}^0$  vary from the experimental values by 0.2%, 2.23%, 0.58%, 2.08% respectively.

The calculated mass splittings are:

$$B_{0s}^0 - B_0^-(0^+ - 0^+) = 62.2936MeV$$

$$B_{1s}^0 - B_1^-(1^+ - 1^+) = 68.7986MeV$$

The calculated iso-spin splittings are:

$$B_s^{*0} - B_s^0 = 41.336MeV$$

$$B_1^- - B_0^- = 34.826MeV$$

Therefore, these values are in close approximation to various theoretical values calculated from other models and with experimental values. So, from the above calculations, the HQET is justified to be an approximate theory which give the results matching with the experimental values to the better accuracy. Here, this theory can also be applied to calculate the other higher excited states masses and their decay widths. Recently, lots of experimental values are coming from the ongoing experiments for bottom and charm sector and hence our theory can be checked for its validity.

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