EFFECT OF THERMAL GRADIENT AND REINFORCEMENT GRADIENT ON THE STEADY STATE CREEP IN THE FUNCTIONALLY GRADED CYLINDER

A dissertation submitted
in partial fulfilment of the requirements for the degree of

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in

CAD/CAM Engineering

by

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CERTIFICATE

This to certify that the thesis entitled "***Effect of Thermal Gradient and Reinforcement Gradient on the Steady State Creep in the Functionally Graded Cylinder***" being submitted by **Gurdeep Singh** for the award of the degree of **Master of Engineering** in **CAD/CAM Engineering** at **Thapar University**, is a record of bonafide research work carried out by him under my supervision and guidance. He has worked for six month on the above problem at the Department of Mechanical Engineering, Thapar University, Patiala and this has reached the standard fulfilling the requirements and the regulation relating to the degree. The contents of this thesis, in full or part, have not been submitted to any other university or institution for the award of any degree.

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This is to certify that above statement made by the student is correct to the best of my knowledge and belief.

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(Gurdeep Singh)

Abstract

The present work investigates the steady state creep in an FGM cylinder with varying thermal gradient .The creep behaviour of the FGM cylinder is described by the threshold stress based creep law. In this work the reinforcement is distributed non- linearly in the FGM cylinder and its effect on the stresses and strain rates is investigated. Firstly the influence of reinforcement particle content on strain rates is investigated at constant temperature condition there after investigate is carried out to determine the effect of thermal gradient. The strain rates reduce significantly by increasing the particle content at the inner surface of the FGM cylinder than at the outer surface of the cylinder at constant temperature. There is no larger variation observed in the radial and axial stress but the tangential stress vary significantly due to thermal gradient in the FGM cylinder. The effect of linear and non- linear variation in temperature profile on the strain rates is also investigated. The strain rates decrease throughout the FGM cylinder, when the temperature is minimum at the inner side and maximum at the outer surface of the FGM cylinder. Thus, The FGM cylinder with non – linear temperature profile, maximum particle content and minimum temperature at the inner surface than the outer surface, undergoes the minimum distortion.

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Nomenclature

V(r) = Particle content at any radius

V_b = Particle content at the outer radius

a = Inner Radius

b = Outer radius

m = Reinforcement gradation index

V (avg) = Average particle content

T(r) = Temperature at any radius

T_a = Temperature at the inner radius

T_b = Temperature at the outer radius

M(r) = Creep parameter

R = Gas constant

E = Young's Modulus

Q = True activation energy

n = True stress exponent

A' = Structure dependent parameter

σ_e = Effective stress

$\dot{\varepsilon_e}$ = Effective creep rate

P = Particle size

$\dot{\varepsilon_r}$ = Radial strain rate

$\dot{\varepsilon_\theta}$ = Tangential strain rate

$\dot{\varepsilon_z}$ = Axial strain rate

$\dot{u_r}$ = Radial deformation

p = Internal pressure in the cylinder

q = External pressure in the cylinder

r = Radius

σ_θ = Tangential stress

σ_r = Radial stress

σ_z = Axial stress

Chapter 1

1.1 Composite

The idea of composite invention drive from the natural composite available like sea shell, bones, wood, teeth etc [30]. These all are naturally available which having good thermal and mechanical properties but human had done some variation for making a man made composite by changing the size and orientation of the reinforcement. Everything that surrounds us is matter. The matter consists of material which the human being uses for making the different parts or elements for structural applications. From the start of human life different types of materials were developed to achieve the required design needs. From the start only single phase or one constituent of material were present for making the different parts which were used for structural applications. But with the advancement of technology and development of different types of approaches, different types of materials were developed to meet the requirements of engineering design. Now with the development of new materials by the combination of two or more materials which are known as alloys were developed. In this case the constituents' materials are melted first then they are mixed in the liquid form and form a new material with different properties than the parent material. Alloys are not macroscopically heterogeneous and their physical properties do not alter.

Similarly in the plastic they consist of another filler material for making the plastic but by the use of this filler material their physical property does not alter significantly.

On the other hand when the constituent material not melted up to the liquid state consist of two or more physical and chemically distinct phase of material are combined together is known as composite material. This is the new class of smart materials which are used mostly for large number of structural applications in spite of their larger cost. These materials are used where cost was outweigh by their benefits.

Composite are used by different fields as given below:

a. The Automotive Industry
b. Aerospace Industry
c. Transportation and Infrastructure Industry
d. Sports
e. Biomedical

f. Computer
g. Satellite & Missile
h. Rocket
i. Agriculture Industry

The composite are used because of the following properties light weight, better corrosion resistance, higher toughness, better aesthetics, wear resistance , higher damping property, better behaviour at higher temperature and thermal conductivity etc

The composite material generally consists of two phase of material, one is the uniform throughout and is called the matrix while the other is distributed phase, is called the reinforcement. The distribution of reinforced material may be uniform throughout the material or not depend on the requirement of design or service condition. The man made composite consists of concrete, plywood, fibreglass and fibrous composite. Composite materials are familiar because of their higher strength application.

The characteristics of the composite material depend on the following:

1 The property of the matrix phase material.

2 The property of the reinforced material.

3 The interaction between the matrix and the reinforcement phase in the composite.

1.2 Parameter for Selection of Composite

Different factor for selection of composite for particular application are given below:

- Strength
- Weight
- Toughness
- Corrosion
- Damping property
- Behaviour at higher temperature

1.3 Types of Composite

Composite can be classified on the basis of types of matrix and types of reinforcement used [27].

The different types of matrix materials consist of:

1) Polymer Matrix Material
2) Metal Matrix Material
3) Ceramic Metal Material
4) Carbon and Graphite Matrix Material

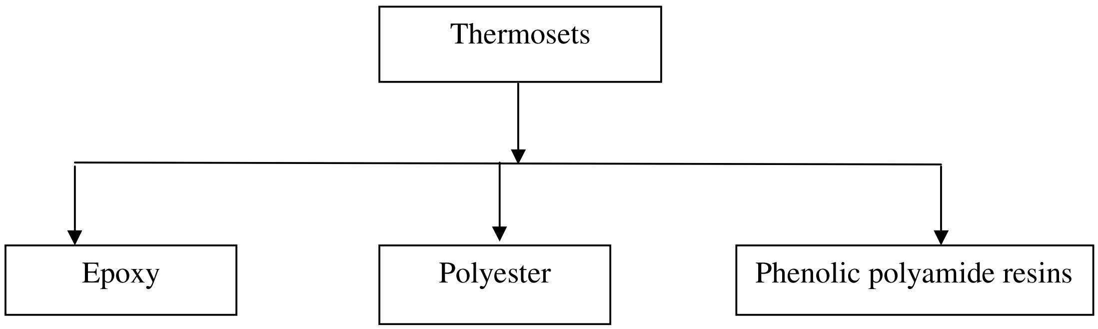
Various matrix materials and their operating temperature [26]

	Matrix Material	Temperature Range
i.	Polymers	<260 ℃
ii.	Metal matrix	260-750 ℃
iii.	Glass	750-1150 ℃
iv.	Carbon and graphite	1150-1400 ℃

Matrix materials are used for the following function in the composite:

- Act as a binder in the composite.
- Shifting stress and load within the composite.
- Provide the supports to the whole structure.
- Most of the matrix material materials are relatively cheaper.

1.3.1 Polymer Matrix Material: - These materials can be processed easily and having a light weight, desirable mechanical property can be achieved during the manufacturing of the polymer matrix material. It is further classified as thermosets and thermoplastic. Thermoset has a well bounded arrangement of molecular structure in a 3-dimensional structure after curing. They are very flexible and they can hold their partially cured structure for a long period of time. On the other hand the thermoplastic has one or two dimensional molecular structure and it is used at a high temperature condition. The composite made from thermoplastic material has come out with the new class of composite. This improves the basic property of the material to a great extent. The advantages of using thermoplastic over thermo set material are that there is no chemical reaction involved.



Epoxy material matrix is used for printed circuit board in computer application. Polyester resins are also used in the large range of applications. In liquid form it is stored for the long period of time at room temperature and by the use of catalyst it can be cured in very short duration of time. The different polymer matrix materials are epoxy, nylon, PPS, polyester.

1.3.2 Metal Matrix Material:- The metal matrix is used in the large number of applications because of the following reasons:

i. High strength.
ii. High toughness.
iii. High impact strength.
iv. High modulus of elasticity.
v. With stand a temperature more than polymer.

The mostly used metals for matrix material are Aluminium, Titanium, and having a low specific gravity. Magnesium can also be used as a matrix material has a low specific gravity but it react with oxygen and causes corrosion. Mostly Al matrix material is used. The strength to weight ratio of the composite more than the alloy materials.

1.3.3 Ceramic Matrix Material:- These materials are used for application where the component has to be operated at more than 1000 ^{o}C temperature. It's used for nose cone of missiles, turbine blades etc. Glass ceramic matrices are made from CAS (calcium alumina silicate) and LAS (lithium alumina silicate). Ceramics consists of solid materials which were bonded by strong ionic bonding in general and in some case covalent bonding. It has good corrosion resistance, high melting point, stability at elevated temperature and good compressive strength proved that ceramic material is a good choice for high temperature range of application. Ceramic has high modulus of elasticity and low strain rate. Due to this, there is insufficient elongation of matrix and this cause the insufficient transfer of load to the reinforcement and the ceramic composite fail whether the enough volume content of the reinforcement. The matrix is reinforcement to employ the higher tensile strength of the fibre to develop a matrix of the high load carrying capacity.

1.3.4 Carbon and Graphite Matrix: - It is special type of matrix materials which is used for high temperature with higher strength and resistivity which is not affected due to higher temperature. This type of matrix made by the use of CVD (chemical vapour deposition) technique. The material used for these type of matrix consist of PAN (polyacrylenitrile), Rayon etc.

1.4 Introduction to Reinforcement Material

Reinforcement consists of material which is embedded in the matrix. Reinforcement consists of small size particle, short fibre, whisker, flakes etc [30]. The distribution of this reinforcement uniform may be uniform or varied in the structural element. Particle is considered to be spherical in shape, so no proper orientation is to be considered, fibre has larger length along the an axis as compared to other two dimension. Whiskers have proper orientation but are small in size as compared to the fibre.

These different types of reinforcement materials have their own characteristic according to which they are used for different engineering applications.

1.5 Classification of Composite on the Basis of Reinforcement [30]:

1) Particle reinforced composites
2) Short fibre or whisker reinforcement composite
3) Continuous fibre or sheet reinforcement
4) Laminate composite.

1.6 Functionally Graded Materials [FGMs]

Alloys consists of those material in which the two or more material are combined together after the melting of these materials, but in composite materials the elements are not melt. They are in powder form of small size claded into the uniformity of another material known as matrix. The combination of these constituent forms a new material known as composite. Now with the advancement of technology and future demand, a new type of material has been developed to meet the future challenge with the extraordinary property like thermo-mechanical and electronic property.

Functionally graded material [29] is new class of material having composional gradient from one structural element to other. Some property of it is similar to composite material, in which the gradient is completely eliminated. The reinforcement is uniform throughout the structure and involves a compromise with some property of the material but the term compromise completely eliminate in the functionally graded material.

Functionally graded material characterised by the structural and compositional variation over the volume of the element, outcome of this corresponding variation in the property. It is the new generation of material which take vast interest during the current period of time. These material acts as a thermal barrier for aerospace structural application and for other structural element which operated under high temperature condition. For example functional graded materials which are reinforced with the ceramic material in the metal matrix are able to operate at high temperature. The ceramic enhance the property of thermal barrier in the structural element and the metal matrixes provide the mechanical strength to the structure and reduce the chance of fracture.

FGM is a one type of the composite in which the content of the reinforcement variation in the any direction corresponding to this variation, the property of the structure also varies along that direction. Due to this gradient property these material finds larger area of application in different field of engineering.

1.7 Types of FGM

The FGM materials are classified or grouped on the basis of nature of gradient used given below:

1. Property of individual constituents.
2. Relative amount of constituents.
3. Size and shape of constituents.
4. Degree of bonding between constituents.
5. Orientation of various constituents.
6. Fraction of gradient type

Ceramic metal matrix functional graded composite show melioration in property like light weight, thermal barrier, higher strength, dimensional stability, higher strength to density ratio, higher stiffness to density ratio, good fatigue and wear resistance, able to withstand high temperature environment etc.

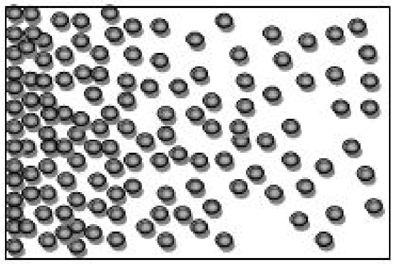


Fig.1.1a. Content variation

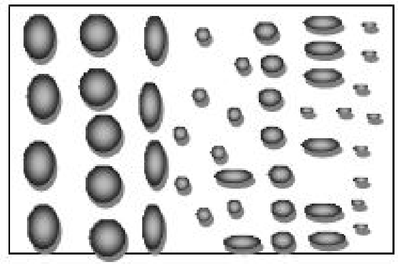


Fig.1.1b. Size variation

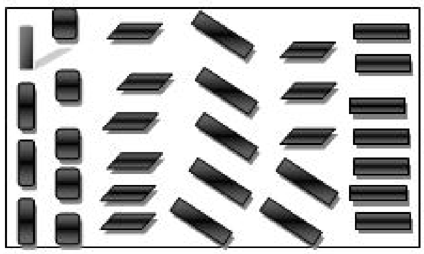


Fig.1.1c. Orientation variation

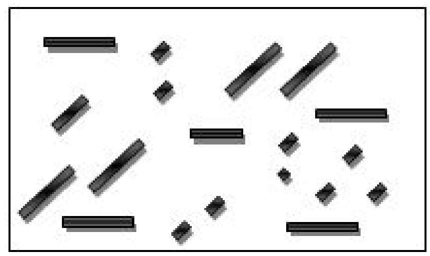


Fig.1.1d. Whisker variation

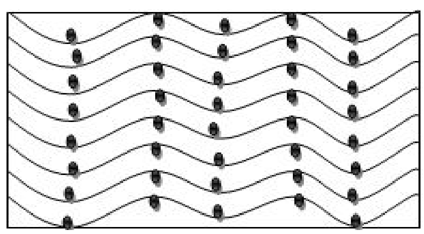


Fig.1.1e. Woven type

Length: – Fibre can be short or long in dimension (length), long continuous fibre are easy to orientate in particular direction than the short fibre but on the other hand the cost of processing of short fibre is less than the long fibre.

Orientation: – The anisotropy or orientation variations of the fibre in the composite have great impact on the mechanical properties of the composite. The composite has different strength and toughness property along the direction of fibre than the perpendicular to the axis of the fibre.

Shape: – Most of the shapes of the ingredients are considered to be circular but it may also have different shape such as square or hexagonal etc. The atomic packing factor in structure also depends on the arrangement of ingredient in the structure.

Material: – There is different type of composite available for different application. So the mechanical performance is directly influenced by the material of the composite or the material of matrix phase and material of the reinforcement.

1.8 Application of the FG Materials According to Implementation in Different Field [28]

1. Industrial and structural application: pressure vessels, eye glass frame, fuel tanks, turbine blade, musical instrument, helmets, laptops cases, X-Ray tables etc.
2. Automobile and aerospace equipment application:

	FGM Composite Type	Structural Application
I.	SiC- SiC	Combustion Chamber
II.	SiC_w /Al- Alloys	Diesel engine piston
III.	Al-SiC	Engine cylinder liners
IV.	Al-C	Drive Shaft
V.	Al-SiC	Flywheels, Racing car brake
VI.	SiC_p/Al-Alloys	Shocks Absorber
VII.	E-glass/Epoxy	Leaf Spring
VIII.	TiAl-SiC fibres	Rocket Nozzle, Heat Exchanger
IX.	TiAl-SiC fibres	Solar Panels, Turbine wheels, Spacecraft,
X.	TiAl-SiC fibres	leading edge of missiles and space shuttle
XI.	C and glass fibres	Propulsion Shaft
XII.	Graphite /Epoxy	Cylinder pressure hull, Composite piping system
XIII.	Al/SiC	Plunging Cylinder

There are different types of fabrication processing technique available for producing different types of FG material. The main two classifications of FG materials are given below:

- Thin FGM
- Bulk FGM

Thin FGMs are those materials in which a section is coated with small layer of thin FG material. But in bulk as the name suggests the volume content of the FGM is more. Surface Coating or thin section is produced by the physical or chemical vapour deposition, Plasma Spraying, Self- Propagating High temperature Synthesis (SHS) etc.

Bulk FGM is produced using powder metallurgy technique, centrifugal casting method, solid free form etc. Functionally Graded Materials find their application in aerospace, automobile, medicine, sport, energy, sensors, optoelectronic etc.

As the fabrication process is improved, cost of powder is reduced and the overall process cost is reduced, hence expanding the application of FGM in different field. Owing to the importance of FGM there are lots of research efforts are going on at improving the material processing, fabrication and properties of the FGM.

1.9 Creep

Investigation show that the strength of metal decreases with the increase in temperature. It mainly occurs due to the movement of atoms at high temperature. To control this movement by using those material which have good thermal barrier and mechanical properties. Metallurgical stability of metal and alloys also get disturbed at high temperature exposure for a long period of time another affect of high temperature environment is the intergranular penetration of oxide and catastrophic oxidation must be eliminated. One effect of creep is for example the creep in the turbine blade will cause the blade to touch the casing and a result the failure of the blade. So the creep is topic of concern for engineers for that component which operates under high temperature and stress.

Due to this adverse effect the monolithic materials may not perform well at elevated temperatures then the alloys developed for production a number of material with efficient high temperature properties but now increase demand for extraordinary material which have strength and able to withstand high temperature for the longer period of time. There is a larger number of equipment which is operated at elevated temperature for longer period of time like a steam power plant, boiler, steam turbine etc. The high temperature strength of material always consider with respect to time scale. At normal temperature the properties are independent of time but at high temperature it behaves like a viscoelastic material.

A material subjected to a constant tensile load at an elevated temperature will creep [26]. The creep test is used to measure the dimensional change that occur at high temperature and stress rupture test is used to measure the effect of long term load sustaining capacity. These tests provide the information for better understanding of the mechanism of deformation and engineering significance of the test data to interpret it.

1.10 Stages of Creep

The creep response of the material divided into the following three stages:

1. Primary creep (transiting creep)
2. Secondary creep (steady state creep)
3. Tertiary creep

The different stages of the creep plotted in Fig.1.2. When the strain vs time response of the material is plotted. From the three stage of the creep, steady state creep is more significant than the other two stages.

1.11 Creep Curve

The time dependent deformation of any structure when it is subjected to constant loading condition called creep. The creep of any component is determined by the tensile testing at constant temperature and loading condition. When the specimen is subjected to these conditions the change in dimension (strain) observed in the specimen after the passage of time. Now the curve is drawn between the strain and time scale. Curve A is general shape of creep curve and slope of this curve is creep rate. As the specimen is loaded initially, an abrupt change in the dimension occur is known as the primary creep. After that the strain rate reaches a steady value. Last the strain rate increase rapidly with respect to time and the fracture occurs in the structure. These different stage of creep depended on the applied load and temperature of surroundings. During the creep experiment the load is kept constant but when the load acts for longer period of time cause the change in length and corresponding reduction in the cross sectional area occurs and the stress value increases. The technique to carry out the test under constant stress by compensation for the change in the dimension was developed. When this method for compensation was used the creep curve in stage iii (tertiary creep) get delay as shown by the dotted line but in engineering case load remain constant not stress condition.

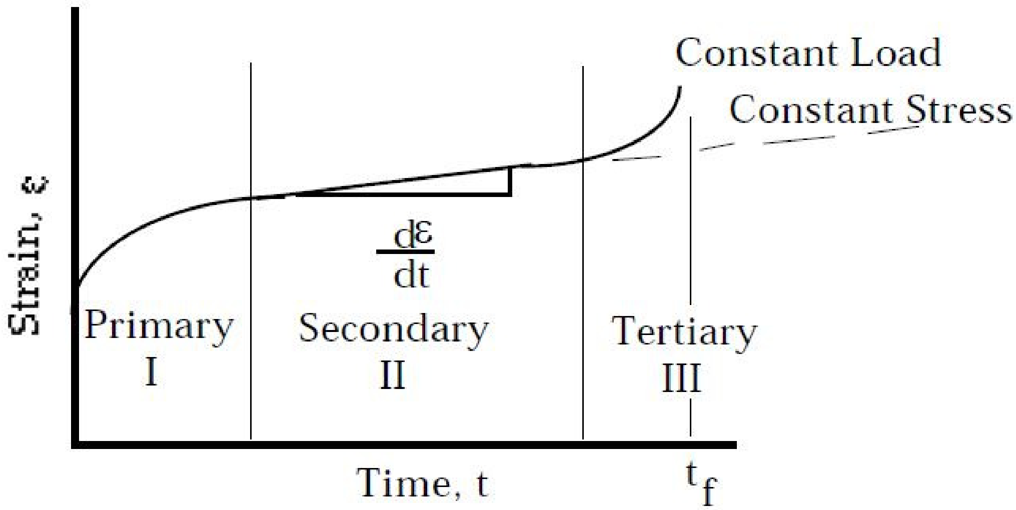
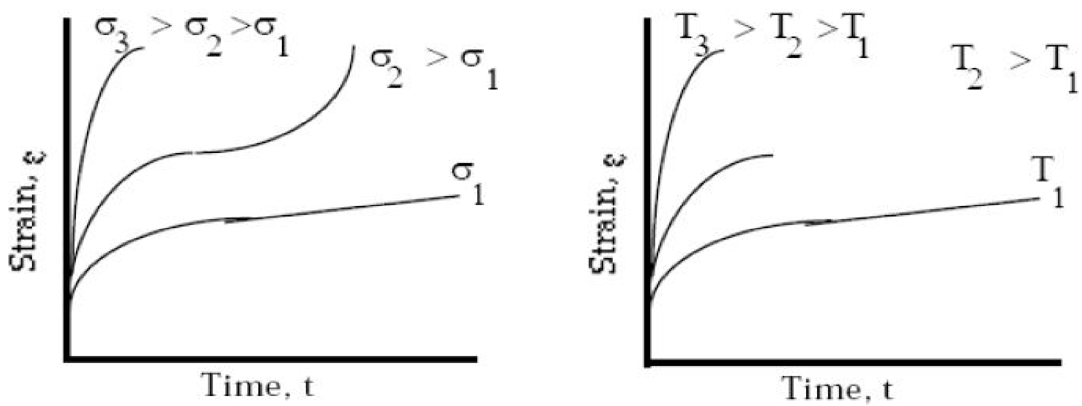


Fig.1.2 (Strain time curve for creep test) [26]

The strain vs time plot as given below show the variation in strain due to change in stress and temperature such as the strain increse in short time duration as the stress increeses similar case happened in high temperature case as the temperature increse the strain rate increse .The rate of this deformation is function of time of exposure at elevated temperature (service condition), material property and the magnitude of applied load.



(a) **(b)**

Fig.1.3 (a, b) (Effect of stress and temperature on strain time creep curves) [26]

At high temperature the strength becomes very much dependent on strain rate. A strong dependence of strength for different materials becomes important. High temperature of one material may not be high for another material. Creep can also be defined as the deformation at absolute temperature is greater than one half of the absolute melting temperature. The relative temperature is known as the homologous temperature. Creep is relative phenomenon which may occur at temperature not normally consider high. Some example related to this point given below:

- Ice melt at 0^c=273K and known to creep at -50^c =223K. Then the homologous temperature is 223/273=0.82 which is more than 0.5. So this is coherent with the definition of creep.
- Tin solder melts at 200 ℃ =473K and solder joint is known to creep at room temperature 20 ℃ =293K. The homologous temperature is 0.60. This is more than the 0.5. So this is also coherent with the definition of creep.

Chapter 2

2.1 Cylinder

The Cylinder is regarded as thin Cylinder when the thickness (t) is less than the 1/20 of the diameter of the Cylinder if its thickness is more than this then this Cylinder is called the Thick Cylinder. It's axis symmetric component and finds large amount of application in the different field of Engineering.

Axis-symmetric component such as a cylindrical vessel is more often used as the basic process component in various structural and engineering applications such as chemical, nuclear industry, petroleum and military industry (pressure vessels, pipes, accumulator shells, boiler Pressure Vessels or Cylinder, Hydraulic Cylinders, Fuel Tanks ,Gun Barrels, Pipes, Boilers, Engine Cylinder and Air Compressor unit, Cylinders for Aerospace industries, Nuclear Reactors and Military applications, Industrial Gases) or a Media Transportation of high-pressurized fluids [2,10]. In usually these applications, the Cylinder has to operate under severe Mechanical and Thermal Load whether it's varies or constant. Due to this, cylinder was subjected to significant deformation and thus reducing its service life [2, 20, and 15]. Therefore, the analysis of long term steady state creep deformations is very important in these applications of cylinder. The operated temperature and pressures of the cylinder may be constant or varying.

2.2 Mode of failure of Cylinder

In general two failure modes are considered:

1. Longitudinal failure
2. Circumferential failure

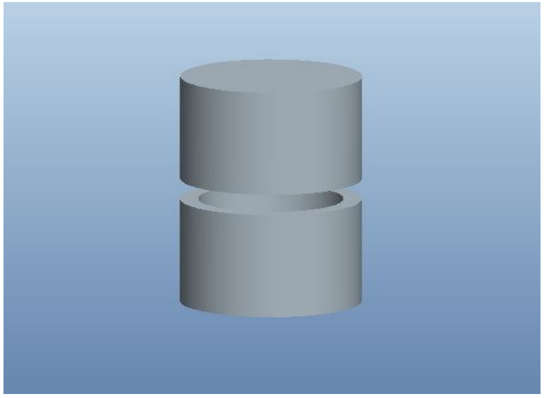


Fig.2.1(Longitudinal Failure of Cylinder)

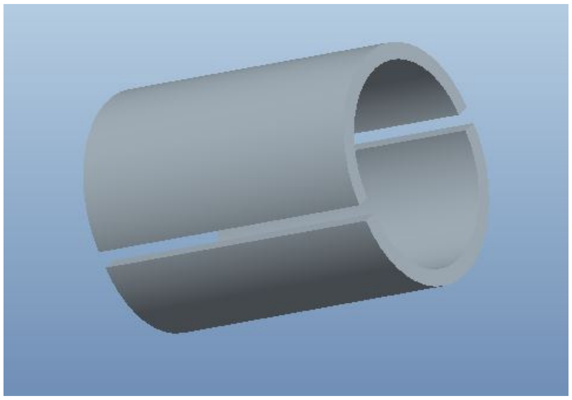


Fig.2.2 (Circumferential Failure of Cylinder)

2.3 Literature Review

The cylinders are used in the large number of engineering applications. Several investigators have carried out creep analysis of cylinder is discussed here under:

Rimrott (1959) carried out the analytical solution of a hollow thick cylinder subjected to internal pressure by the implementation of general assumption for the isotropic cylinder. By the use of finite strain theory, the creep rate of thick cylinder increase by the increase internal pressure. The creep rate of same material element in tension under constant stress may have different value. It was that small strain rate assumption was not valid in all the condition of working of cylinder. It was conclude that the assumption of small deformation in the cylinder may not true in all the case of working. The possibility of the larger strain may not be over looked for this reason, used the finite strain theory under the longer strain condition

Taira and Ohtani (1968) carried out analysis of creep and rupture for a cylinder subjected to internal pressure, made of low carbon steel and 5/2 Cr-IMo steel, experimentally and Theoretical analysis of stress was derived under the multi axial creep stress theory and finite strain theory. In this work the theoretical and experimental result data was compared under the internal pressure condition of the thick cylinder, For experimental testing two condition were taken, i.e. with internal pressure and without internal pressure.

Zhu *et al*. (2000) investigated creep behaviour of aluminium strengthened by fine aluminium carbide particles and reinforced by silicon carbide particulates. For comparison, the results for two dispersion strengthened (DS) AlC alloys denoted DS AlC1 and DS AlC2 were considered. (C1 and C2 meaning the contents of carbon in wt% which define the volume fractions of Al4C3 particles in DS AlC alloys as well as in SiC: AlC composite matrices). The volume fraction of SiC particulates in composites, denoted SiC: AlC1, SiC: AlC2 and SiC: AlC3, was fixed to 10 vol%. The creep in both DS AlC alloys as well as in SiC: AlC composites were investigated.

Gupta and Pathak (2001) carried out the analysis of the thick walled cylinder by the condition of compressibility and incompressibility of the material. In this work the Seth transition theory was used. The cylinder was considered to be made of compressible and incompressible material. Thermal stress component along the radius of the cylinder was considered. Incompressible material cylinder had higher value at the outer radius than the compressible material. Introduction of thermal effect cut down the stress at the outer surface.

It was conclude that the cylinder of compressible material under pressure and temperature had larger strain than the incompressible material cylinder.

Singh (2003) described the creep analysis in an isotropic FGM rotating disc of Al–SiC composite. Creep response in an isotropic FGM rotating disc of Al–SiC particulate composites with composition gradient along the radial direction using Norton's law. Results were compared with those in an isotropic disc having the same average particle gradient at the same temperature.

Gupta *et at.* (2004a) investigated steady state creep and material parameters in a rotating disc of Al–SiC_P composite. The steady state creep rates were calculated for various combinations of material parameters (like particle size and particle content) and temperatures. The optimum content of reinforcement was used for the minimum creep rates in disc.

Gupta *et al.* (2004b) described creep behavior of a rotating functionally graded composite disc operating under thermal gradient. The disc was made of Al-SiC_p reinforcement having a thermal gradient in the radial direction and linear particle distribution in the radial direction.

Calhoglu *et al*. (2006) described the elastic–plastic stress analysis of an orthotropic rotating disc. The analysis was done analytically at different angular velocity. The disc was manufactured from curvilinear steel fibers-reinforced aluminium metal–matrix by means of compression moulding method. The analytical solution gave the radial displacement component at each point for both the elastic and plastic solutions at the different angular velocities.

Bayat *et al.* (2007) presented the thermo elastic analysis of a functionally graded rotating disk with small and large deflections. In this work, axis-symmetric bending and thermal loading was discussed, when the material of reinforcement graded in the radial direction of the disc by the power law function and the result were compared for the homogeneous and the FG disc.

You *et al*. (2007) analyzed creep deformations and stresses in thick-walled cylindrical vessels of functionally graded materials subjected to internal pressure. In this work thick wall FGM cylindrical vessel subjected to internal pressure then by the use of Norton law of creep analysis was used for the different material parameter. Stress and strain rate were calculated under the plane stress condition along the radial direction of the cylinder.

Singh (2008) described one parameter model for creep in a whisker reinforced anisotropic rotating disc of Al–SiC_w composite. The influence of anisotropy was investigated in terms of a single parameter. Stress distribution and creep rate were determined in the metal matrix composite made of Al and containing 20 vol% of SiC whiskers.

Abrinia *et al.* (2008) investigated the stress in the FGM thick cylinder under the effect of pressure and temperature. Modulus of elasticity (E) and coefficient of thermal expansion was varied from inner radius to outer radius as a function of power law relationship. The Non-Homogeneity was created by the use of non-dimensional parameter (β) which was assigned any arbitrary value for the analysis. This value caused the change in the value of Young's Modulus and coefficient of thermal expansion along the radial direction of the cylinder and the corresponding values of stress and strain rates were examined. Individually effect of pressure, temperature and combination of both under different value of β was analyzed. The effect of different loading condition on the deformation of the FGM cylinder was analysed.

Singh *et al.* (2009) analysed the creep in a functionally graded composite thick cylinder subjected to only internal pressure the cylinder was made up of Al-SiC particle in which the content of the SiC-particle varied linearly from inner radius to outer radius and the effect of liner gradient on creep stress and creep rate in the composite cylinder was analyzed. The strain rates in the cylinder decreases significantly over the entire radius by varying the distribution of reinforced SiC particle in the composite cylinder but average content of reinforcement remain same in the composite cylinder. The radial stress in the cylinder gets reduced by increase in the gradient of reinforcement. Tangential, axial and effective stresses were higher at inner radius and decrease toward the outer radius due to increase in particle gradient in the cylinder.

Sharma (2009) carried out the creep analysis of rotating cylinder by using the transition theory. He studies the less compressible material at the inner side and more compressible material at the outer surface was considered. The result for the different value of angular velocity and temperature was calculated analytically. By the development of different differential equations the creep stresses were calculated for the homogenous and non homogenous thick cylinder were estimated under the different condition of working. The stress and strain rates in the cylinder and concluded that the cylinder with less compressibility at the inner surface than the outer surface is on the safe side for the design than the vice-versa.

Singh and Gupta (2009) described the effect of material parameter on the creep rate in the thick composite cylinder under internal pressure .The cylinder was made of Al-SiC particle, the creep law used for effective creep rate in the cylinder was threshold stress based law with stress exponent 5. In the study the volume content of SiC particle remains same or homogenous throughout the entire radius (10%, 20%, and 30%) and stress distribution and strain rates in the cylinder were analysed for different volume content, particle size, temperature. In this isotropic composite cylinder the strain rate reduce significantly by using the small size and more particle content of SiC.

Singh and Gupta (2010) investigated the creep in the thick composite cylinder subjected to internal and external pressures. Cylinder was made of Al-SiC particle. The effect of varying size and content of SiC particle on the stresses developed in the cylinder and on the strain rates in the cylinder were investigated. The creep behaviour was described by the threshold stress based law and stress exponent was taken as 5.The results obtained were compared with the Al alloys (RR59) cylinder which was used by Johnson *et al* .[39] under the operating temperature of 350 ^{o}C. It was observed that there was a very small variation in the stress distribution in the cylinder but strain rate reduced significantly by varying size and content of the SiC particle in the thick composite cylinder.

Kursun *et al.* (2011) observed the stress analysis of functionally graded disc under thermal and mechanical loads. In this case the author used Mat lab and analyzed the creep rate in the disc when the disc was subjected to internal pressure and temperature, linearly varying from the inner radius to outer radius.

Singh and Gupta (2011) described effect of anisotropy on steady state creep in functionally graded cylinder. in this case the cylinder was subjected to internal pressure and the effect of orientation of the reinforcement on the strain rate was investigated.

Callioglu (2011) carried out the analytical solution for a disc made of functionally graded material by using the theory of elasticity. The gradation of material was done by the use of power law distribution of reinforcement along the radius of the disk. He investigated the influence the internal and external pressure, grading, centrifugal and thermal loading on the stress and strain in the FG hollow disc and the equation from the theory of elasticity under equilibrium was developed. Disc operated under the internal and external pressure investigated by taking the different graded index value (0.5, 0.25, 0,-0.25,-0.5) under the steady state temperature and constant angular velocity. The property of the disc such as density, Young's modulus, co-efficient of thermal expansion and temperature varied along the radial direction. It was conclude that by increase the value of grading index parameter (n) radial and tangential stresses and radial deformation increases in the disc due to increases in centrifugal force. Tangential stress had higher value at inner radius than at the outer radius of the disc

Singh and Gupta (2012) investigated the steady state creep in the FG composite cylinder subjected to internal pressure under the thermal gradient. The authors analyzed the effect of particle and thermal gradient on the stress distribution and strain rate in the thick cylinder. The content of the reinforced particle varied linearly from inner radius to outer radius of the cylinder. The study revealed that the radial stress decrease along the radial direction from the inner radius to outer radius but tangential ,axial, effective stress increase near the inner radius but then show a significant decrease toward the outer radius due to particle and thermal gradient alone in the composite cylinder. By keeping the same average content the strain rate decrease to a larger extent by keeping the particle gradient and thermal gradient alone or together in the composite cylinder. The radial / tangential strain rate in the composite cylinder with uniform distribution of reinforcement but the thermal gradient in the radial direction lower than the similar cylinder having the same particle content and operated under the constant temperature.

Garg and Salaria (2013) investigated the effect of reinforcement gradient on steady state creep in a variable thickness rotating disc made of non-linear FGM. The creep behaviour of the disc was described by a threshold stress-based law. Creep rate was obtained and compared it with the uniform FGM material distribution in the disc.

Garg *et al.* (2013) described the effect of thermal gradient on steady state creep in a rotating FGM disc having a linear varying thickness and linear thermal gradient. The content of SiCp in the disc decreased non-linearly from inner radius to outer radius. The results were also estimated for the similar disc but operated at constant temp and the creep rate was compared in both cases.

Loghman *et al.* (2013) investigated the effect of particle content, size and temperature on magneto-thermo-mechanical creep behaviour of composite cylinders. In this case steady-state creep behaviour of thick-walled rotating cylinders made of Al-SiC composites was investigated. Under the uniform magnetic field in the axial direction and steady state heat conduction in the radial direction and inertia force due to rotation was considered. It was observed that the composite is to be developed or designed the particle content and size play a major role and the magnetic field also lower the stress and effective strain rate.

Thakur (2014) investigated the strain rates and thermal stress in a rotating thick wall cylinder under steady state temperature condition by the using the Seth is transition theory. According to this theory no assumption was taken like the material is incompressible, the deformation was assumed to be small etc. The strain rates at different radii ratio were estimated and compared the results by considering the compressibility and incompressibility of the material during the analysis under the steady state thermal condition. The stress distribution in an isotropic cylinder with reference to different radii ratio was also analysed. It was concluded that cylinder having lesser angular velocity and radii ratio with increase temperature become plastic as compare to cylinder having larger radii ratio. Incompressible material has higher circumferential stress at inner surface than the compressible material in the cylinder.

Khanna *et al.* (2015) investigated the steady state creep in a Al-SiC_p rotating disc having a varying thickness profiles and reinforcement gradient. Yielding of disc material was according to the Tresca Criteria and creep by the threshold stress based law. In this study the FGM disc of non- linear distribution of SiC particle along the radial direction. Under different volume content of reinforcement the corresponding stress and strain rates were calculated. First the authors considered the different thickness profiles of the disc and the profile design which showed the minimum strain rate was taken for the effect of variation of particle content (reinforcement gradient) analysis and the corresponding particle content, Stresses and strain rates were examined in the radial and tangential direction of the disc.

Chapter 3

3.1 Mathematical Modelling

In the present study Al matrix composite reinforced by SiC particle has variation in properties in the radial direction of the Cylinder due to the variation in the content of reinforcement. The content of the reinforcement is to decrease non- linearly from the inner radius (a) to the outer radius (b). So the density of the Cylinder varies in the radial direction of the Cylinder.

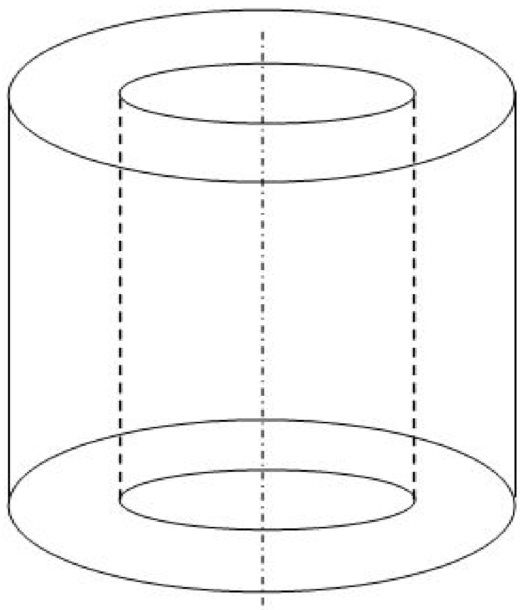


Fig.3.1 Thick cylinder

The content of reinforcement at any radius (r), as taken by the earlier study [25], is given by:

$$ - \qquad (1) $$

Where

m = Reinforcement gradation index

V_b = Particle content at the outer radius

The average particle content in the Cylinder is given as

$$ \frac{\quad}{\quad} \qquad (2) $$

Substituting $V(r)$ from Eq. (1) into Eq. (2) and integrating, one obtains:

$$v_b = \frac{(b^2-a^2)(V_{avg})(m+2)(b^2)}{2(b^{m+2}-a^{m+2})} \tag{3}$$

For linear temperature distribution

$$T(r) = T_a + (r-a/b-a)\,[T_b - T_a] \tag{3a}$$

and

For non-linear temperature distribution

$$T(r) = T_a + (r-a\,/\,b-a)^2\,[T_b - T_a] \tag{3b}$$

Where T_a, T_b, and $T_{(r)}$ are

T_a = Internal temperature of the cylinder

T_b = External temperature of the cylinder

T(r) = Temperature at any radius in the cylinder

Creep law based on threshold stress (σ_0) given by

$$\dot{\varepsilon}_e = A'\left(\frac{\sigma_e-\sigma_O}{E}\right)^n exp^{\left(\frac{-Q}{RT}\right)} \tag{4}$$

Where

A'=structure dependent parameter, σ_e = Effective Stress, n =True Stress Exponent, Q=True Activation Energy, E=Temperature Dependent Young's Modulus, R= Gas Constant,

T = Operating Temperature

Alternatively,

$$\dot{\varepsilon}_e = M(r)\,[\{\sigma_e - \sigma_0\,(r)\}]^n \tag{5}$$

Where $M(r) = \frac{1}{E}\left(A'\,exp\frac{-Q}{RT}\right)^{\frac{1}{n}}$ is a creep parameter

In the composite material the value of the parameter M(r) and $\sigma_o(r)$ depends on the particle size (P=1.7μm), particle content V(r) and the operating temperature T(r) according to the following relations as derived in the earlier work [21].

$$M(r) = 0.02876 - \frac{0.00879}{P} - \frac{14.0266}{T(r)} + \frac{0.03224}{V(r)} \quad (8)$$

$$\sigma_0(r) = -0.084P - 0.0232\,T(r) + 1.1853\,V(r) + 22.207 \quad (9)$$

There are some assumptions for the present analysis of the cylinder given below:

I. Cylinder is made up of incompressible material.
II. Pressure is applied gradually and then held constant.
III. Steady state condition of stress is assumed
IV. Elastic deformations are small and neglected as compared to creep deformation.

The strain rate is mutually related to the displacement rate given by the equation.

$$\dot{\varepsilon_r} = \frac{d\dot{u_r}}{dr} \quad (10)$$

$$\dot{\varepsilon_\theta} = \frac{\dot{u_r}}{r} \quad (11)$$

Where $\dot{\varepsilon_r}, \dot{\varepsilon_\theta}$ the radial and tangential strains rates in the cylinder and $\dot{u}_r$ is the radial displacement rate.

Differentiation of Eq. (11) w. r. t. ‘r’, we get,

$$\frac{d\dot{\varepsilon_\theta}}{dr} = -\frac{\dot{u}_r}{r^2} + \frac{1}{r}\frac{d\dot{u}_r}{dr}$$

$$\frac{d\dot{\varepsilon_\theta}}{dr} = \frac{1}{r}\left[-\frac{\dot{u}_r}{r} + \frac{d\dot{u}_r}{dr}\right]$$

From Eq. (10) and (11) by eliminating $\dot{u}_r$ we get

$$r\frac{d\dot{\varepsilon_\theta}}{dr} = \dot{\varepsilon_r} - \dot{\varepsilon_\theta} \quad (12)$$

The cylinder is subjected to the following boundary condition,

$$\sigma_r = -p \quad \text{at} \quad r = a \quad (13)$$

$$\sigma_r = -q \quad \text{at} \quad r = b \quad (14)$$

Where, p and q are internal and external pressure in the cylinder.

Where, the negative sign implies that the load acting compressive in nature. Now we consider the small element of the Cylinder subjected to internal and external pressure.

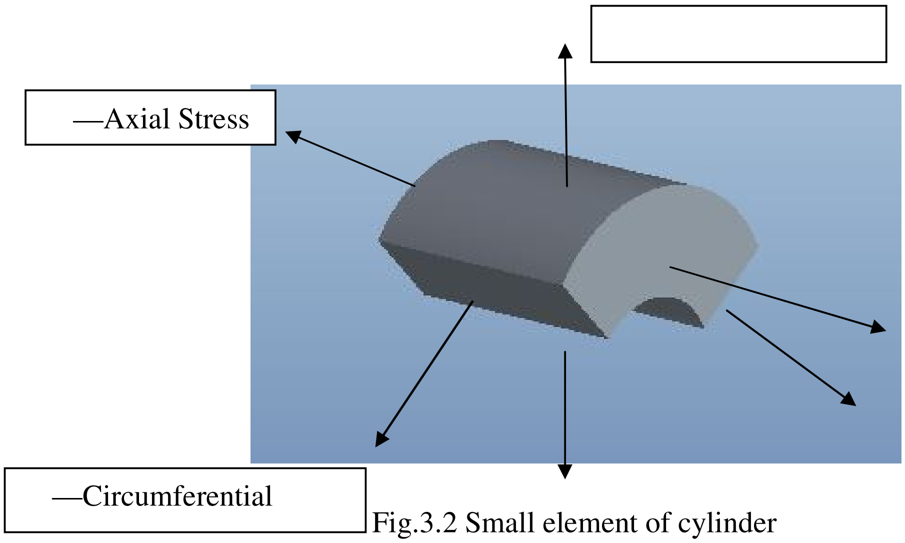


Fig.3.2 Small element of cylinder

Considering the equilibrium of forces acting on the element of the cylinder in the radial direction, one obtains:

(15)

Assuming the incompressibility condition for creep strain in the cylinder, we get:

(16)

The equation for multi axial creep in the FGM material, when the principal axes are the axes of reference, are written as [11]

(17)

(18)

(19)

According to Von-Mises Yield criteria, the effective stress is given by:

$$\sigma_e = [\frac{1}{\sqrt{2}}\{((\sigma_\theta - \sigma_z)^2) + ((\sigma_z - \sigma_r)^2) + ((\sigma_r - \sigma_\theta)^2)\}]^{1/2} \tag{20}$$

For the close end cylinder made of an incompressible material, the plane strain condition exists i.e. the axial strain is zero .So the Eqs. (10), (11) and (16) are simplified to:

$$\dot{u}_r = \frac{C}{r} \tag{21}$$

On differentiating of Eq. (21) w. r. t. 'r', we get:

$$\frac{d\dot{u}_r}{dr} = -\frac{C}{r^2}$$

Where, C is the constant of integration

Now using Eq. (21) in Eqs. (10) and (12) we get:

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = -\frac{C}{r^2} \tag{22}$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} = \frac{C}{r^2} \tag{23}$$

The plane strain condition ($\dot{e}_z = 0$) eq. (19) i.e.

$$\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_e}{2\sigma_e}[2\sigma_z - \sigma_r - \sigma_\theta]$$

$[2\sigma_z - \sigma_r - \sigma_\theta] = 0$ can be written as:

Or

$$\sigma_z = \frac{\sigma_r + \sigma_\theta}{2} \tag{24}$$

Now putting the value σ_z from Eq. (24) into Eq. (20), we get:

$$\sigma_e = \sqrt{\left[\frac{3}{2}\right]}(\sigma_\theta - \sigma_r) \tag{25}$$

Now putting $\dot{\varepsilon}_r$ and $\dot{\sigma}_z$ from Eqs (22) and (24) into Eq. (17), we get:

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{2\sigma_e}[2\sigma_r - \sigma_\theta - \sigma_z]$$

$$-\frac{C}{r^2} = \frac{\dot{\varepsilon}_e}{2\sigma_e}[2\sigma_r - \sigma_\theta - \frac{\sigma_r+\sigma_\theta}{2}]$$

After solving it, we get

$$\sigma_\theta - \sigma_r = \frac{4}{3}\frac{\sigma_e}{\dot{\epsilon}_e}\frac{C}{r^2} \tag{26}$$

$$\sigma_\theta - \sigma_r = \frac{4}{3}\frac{C}{r^2}\frac{\sigma_e}{\dot{\epsilon}_e}$$

Now put the value σ_e *and* $\dot{\epsilon}_e$ from Eqs. (25) and (5)

$$\sigma_\theta - \sigma_r = \frac{4}{3}\frac{C}{r^2}\frac{(\sigma_\theta-\sigma_r)\sqrt{\left[\frac{3}{2}\right]}}{M(r)[\{\sigma_e - \sigma_0(r)\}]^n}$$

$$\sigma_\theta - \sigma_r = \frac{4}{3}\frac{C}{r^2}\frac{(\sigma_\theta-\sigma_r)/\sqrt{\frac{3}{2}}}{M_{(r)}\left[\left\{\sqrt{\frac{3}{2}}(\sigma_\theta-\sigma_r)-\sigma_0(r)\right\}\right]^n}$$

$$(\sigma_\theta - \sigma_r) = \frac{C^{\frac{1}{n}}\left[\frac{2}{\sqrt{3}}\right]^{\frac{n+1}{n}}}{M(r)\, r^{\frac{2}{n}}} + \frac{2}{\sqrt{3}}\sigma_o$$

$$\sigma_\theta - \sigma_r = \frac{I_1}{r^{\frac{2}{n}}} + I_2 \tag{27}$$

Where $\quad I_1 = \frac{C^{\frac{1}{n}}\left[\frac{2}{\sqrt{3}}\right]^{\frac{n+1}{n}}}{M(r)}$, and (28)

$$I_2 = \frac{2}{\sqrt{3}}\sigma_o(r) \tag{29}$$

Eq. (15) written as

$$r\frac{d\sigma_r}{dr} = \frac{I_1}{r^{\frac{2}{n}}} + I_2$$

Integrate the above Eq., we get

$$\int_a^r d\sigma_r = \int_a^r \frac{I_1(r)}{r^{\frac{2+n}{n}}} + \int_a^r \frac{I_{2}(r)}{r}\, dr \tag{30}$$

Let $X_3 = \int_a^r \frac{I_1(r)}{r^{\frac{2+n}{n}}}$, $\quad X_4 = \int_a^r \frac{I_2(r)}{r}\, dr$

Now the Radial stress is given by,

$$\sigma_r = X_3 + X_4 - p \tag{31}$$

Substitute σ_r from Eq. (31) into Eq. (17), we get:

$$\sigma_\theta = X_3 + X_4 + \frac{I_1}{r^{\frac{2}{n}}} + I_2 - p \tag{32}$$

To calculate the valve of constant C, substitute the value of boundary condition from Eqs. (13) and (14)

σ_r = - p   at   r = a

σ_r = - q   at   r = b

Integrate the Eq. (30) under the limit ‘a’ and ‘b’, written as:

$$-q = \int_a^b \frac{I_1}{r^{\frac{2+n}{n}}} + \int_a^b \frac{I_2}{r}\, dr \;\; - p$$

$$p - q = \int_a^b \frac{I_1}{r^{\frac{2+n}{n}}} + \int_a^b \frac{I_2}{r}\, dr$$

Put the value of I_1 and I_2 in the above equation, we get:

$$p - q = \int_a^b \frac{\frac{C^{\frac{1}{n}} \frac{2}{\sqrt{3}}^{\frac{n+1}{n}}}{M(r)}}{r^{\frac{2+n}{n}}} + \int_a^b \frac{\frac{2}{\sqrt{3}}\sigma_o}{r}\, dr = C^{\frac{1}{n}} \int_a^b \frac{\frac{\frac{2}{\sqrt{3}}^{\frac{n+1}{n}}}{M(r)}}{r^{\frac{2+n}{n}}} + \int_a^b \frac{\sigma_o \frac{2}{\sqrt{3}}}{r}\, dr$$

$$\text{let } X_2 = \int_a^b \frac{\frac{\frac{2}{\sqrt{3}}^{\frac{n+1}{n}}}{M(r)}}{r^{\frac{2+n}{n}}}\;,\quad X_1 = \int_a^b \frac{\sigma_o \frac{2}{\sqrt{3}}}{r}\, dr$$

$$p - q = C^{\frac{1}{n}} X_3 + X_4$$

$$C = \left[\frac{p - q - X_1}{X_2}\right]^n \tag{33}$$

Now putting the value from equation (24) ,(25) in equation 17 ,we get

$$\dot{\varepsilon_r} = \frac{\dot{\varepsilon_e} * 2}{(2)(\sigma_\theta - \sigma_r)\sqrt{3}} \left[\left\{2\sigma_r - \sigma_\theta - \frac{\sigma_r + \sigma_\theta}{2}\right\}\right]$$

After solving it, we get:

$$-\dot{\varepsilon}_r = \frac{\dot{\varepsilon_e}\sqrt{3}}{2} \quad (34)$$

Now putting the value from Eqs (24) and (25) in Eq. (18), we get:

$$\dot{\varepsilon_\theta} = \frac{\dot{\varepsilon_e}}{(2)\sigma_e}[\{2\sigma_\theta - \sigma_z - \sigma_r\}]$$

$$\dot{\varepsilon_\theta} = \frac{\dot{\varepsilon_e}*2}{(2)(\sigma_\theta-\sigma_r)\sqrt{3}}[\left\{2\sigma_\theta - \frac{\sigma_r+\sigma_\theta}{2} - \sigma_r\right\}]$$

On solving it, we get:

$$\dot{\varepsilon_\theta} = \frac{\dot{\varepsilon_e}\sqrt{3}}{2} \quad (35)$$

Now equating the equation (34) and (35), we get

$$-\dot{\varepsilon}_r = \dot{\varepsilon}_\theta = \frac{\dot{\varepsilon_e}\sqrt{3}}{2}$$

Now by considering the isotropic FGM Cylinder the strain rate in the radial and tangential direction is 87% of the effective strain rate.

3.2 Numerical Scheme of Computation

For computation of stress and strain rates in the FGM cylinder the systematic procedure is given as under:

- Calculate the value of X_1 and X_2 by solving the integral.
- Then obtain the value of constant (C)
- Thereafter compute the $I_1(r)$ and $I_2(r)$
- Now compute the numerical integration for X_3 and X_4
- After this calculate the radial stress and tangential stress from Eqs (30) and (32).
- Calculate the Effective stress from Eq. (20).
- Now calculate the effective creep rate from Eq. (5).
- After all this the radial and tangential strain rate from Eqs (17) and (18) can be estimated.

Chapter 4

4.1 Problem Formulation

The literature consulted so far reveals that large amount of work has been done on a cylinder made of uniform composite. In most of these works the cylinder is considered a rotating element made of isotropic composite material. Some work is also seen on functionally graded cylinder subjected to internal pressure and external pressure. But no work is reported for a cylinder made of functionally graded composite with non linear distribution of reinforcement (SiC_p) in the Al matrix subjected to thermal loads. So it is proposed to investigate creep in a functionally graded cylinder under the thermal gradient subjected to internal and external pressure. The temperature is varied linearly and non-linearly in the FGM cylinder. Mathematical analysis shall be carried out for the above mentioned cylinder under the imposed boundary conditions and creep stresses and strain rates will be estimated by writing a computer code in Matlab. The results obtained will be validated for their correctness and further analysis will be carried out for the different conditions.

In light of the above said it is proposed to investigate the effect of particle content and thermal gradient on the creep behaviour of FGM thick cylinder subjected to internal and external pressure and made of Al- SiC particle.

4.2 Dimensions of the Cylinder and Distribution of Reinforcement

The dimensions of the cylinder, inside diameter, d_i (=*50.8mm*), outside diameter, d_o(=*101.6mm*), internal pressure, p(=*85.25MPa*) and external pressure, q(=*42.62MPa*) are taken similar to the earlier work [16] for the FGM cylinder.

Chapter 5

5.1 Validation

Before discussing the result from the present work, it becomes necessary to validate the computer code which has been developed on the basis of mathematical model. The radial, tangential and axial stress and corresponding strain rate computed for the same dimension cylinder under the similar boundary condition. A good agreement observed between the results obtained from the present analysis and the result available in the literature [16].

Table 1. Data for validation

Creep parameter, M =1.7723x$10^{-4}$$s^{-1/5}$/MPa,   = 24.38MPa
Internal pressure (*p*) = 281.23MPa,
External pressure (*q*) = 0
Cylinder inner radius (*a*) = 20mm
Cylinder outer radius (b) = 100mm

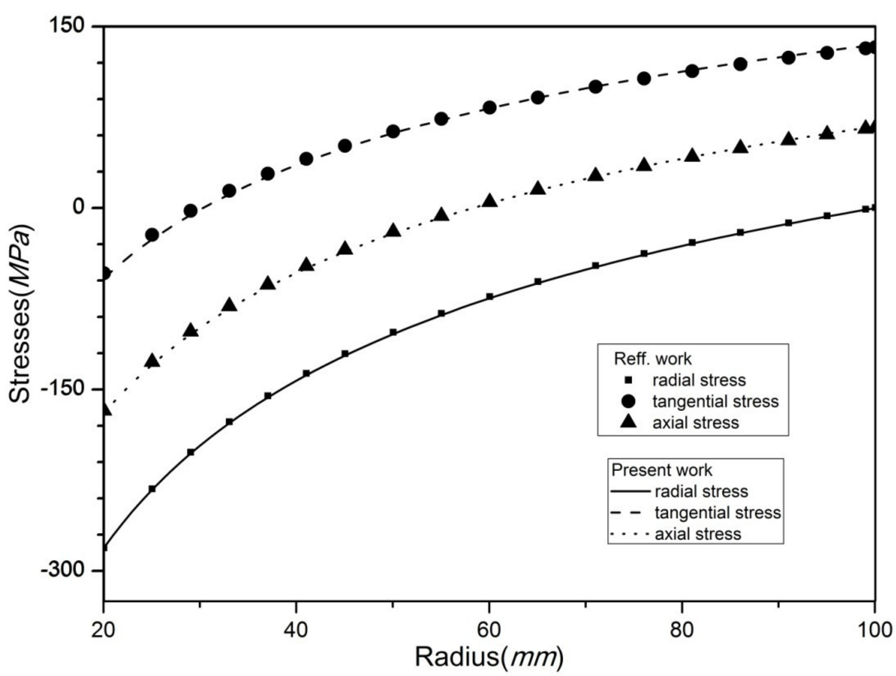


Fig.5.1 Comparison of stresses (present analysis and literature)

5.2 Results and Discussions

The creep stresses and strain rates have been estimated for the different reinforcement particle distribution under different thermal gradient. The effect of particle gradient and thermal gradient is discussed in the following section:

5.3 Effect of Particle Content

The result are obtained for the uniform composite cylinder (m=0) and for the FGM cylinder in which the particle content varies from the inner to outer radius of the cylinder. The variation of reinforcement at any radius is given by the equation (1). In present work, for two cases the particle content is more at inner face of the cylinder and decreases non – linearly toward the outer face of the cylinder, similarly for other two cases for the particle content is less at the inner face and more at the outer face [19]. The value of the gradation index (m) is taken as negative (m= -0.25,-0.50), which decreases the particle content from the inner face to outer face and for the reverses distribution increasing the particle content from inner face to outer face the gradation index is taken as positive (m=0.25,0.50). For composite cylinder the value of gradation index zero (m=0), which give the uniform distribution of particle content. The purpose of this work is to evaluate the creep in the cylinder at different values of gradation index, and to find which particle distribution gives the minimum creep rates.

Table 2 Distribution of particle content (V_{avg}=20%)

Cylinder Notation	Gradation Index (m)	Particle Content (V %)		Particle Gradient
		V_i	V_o	PG
C1	0	20	20	0
C2	0.25	17.96	21.37	3.41
C3	0.50	16.10	22.82	6.72
C4	-0.25	22.21	18.60	3.61
C5	-0.50	24.61	17.96	6.65

The Table (2) depicted the particle content at the inner surface (V_i) and at the outer surface (V_o) for the different gradation index(m). The distribution of reinforcements (SiC_p) along the radial direction is depicted in the Fig.5.2. The distribution is followed by the power law distribution but the average content of the SiC_p remains same in the five cases (refer Table 2).

The effect of varying particle content on the stresses and strain rates in the cylinder as depicted in Figs.5.3 to 5.7.

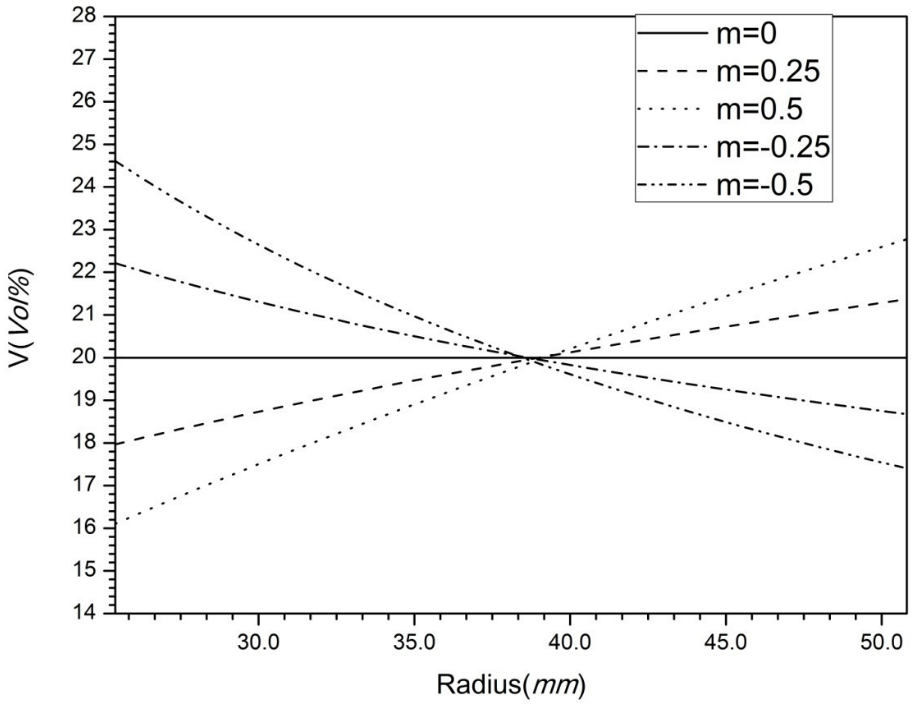


Fig.5.2 Variation of particle content in the cylinder

5.3.1 Effect on Stresses

The radial stress remains compressive in nature throughout the cylinder due to the imposed boundary condition, as given in equation (13) and (14). The magnitude of the radial stress goes on decreasing from the inner surface to the outer surface of the cylinder along the radial direction of the thick cylinder (refer Fig.5.3a). When the particle gradation index (m=-0.5), particle content more at the inner surface than at the outer surface of the cylinder. The magnitude of the radial stress is lower in the middle region of the FGM cylinder(C5) than the uniform composite cylinder (C1)and the magnitude of radial stress of the FGM cylinder (C5) decreases by -1.186 MPa (2%) than the uniform composite cylinder(C1) .when the particle gradation index(m=0.5), then particle content at the inner surface of the FGM cylinder(C3) is lesser than the uniform composite cylinder(C1) then the magnitude of the radial stress goes on increasing by the 2.38 MPa (4.02%) in the middle region than the uniform composite cylinder(C1). So due to the variation in the particle content does not have a significant effect on the radial stress except for some change in the middle region. Radial stress does not show any larger variation at the inner and outer surface of the cylinder due to boundary condition.

Tangential stress remains compressive in nature near the inner surface and change to tensile toward the outer surface of the cylinder and it also become zero somewhere in the middle region (refer Fig.5.3b). As the particle gradation increases (m=0.25, 0.50), particle content decreases near the inner surface and keeps on increases toward the outer surface of the cylinder. With this variation of particle content magnitude of tangential stress (compressive) increases at the inner surface and keeps on decreasing up to middle radius and become zero somewhere in the middle region and becomes tensile after the middle region and increases toward the outer surface of the FGM cylinder. With the increases in particle content near the inner surface tangential stress decreases and continues to decreases toward the outer surface and reaches zero at the middle region and again increase (tensile) toward the outer surface but its magnitude remain low in comparison to uniform composite cylinder (C1). When the particle content in the FGM cylinder (C3) at the inner surface is less (m=0.5) than the uniform composite cylinder (C1) the magnitude of the tangential stress increase by the 7.15MPa (35.6%, compressive) but as the particle content in the FGM cylinder (C5) increase (m=-0.5) at the inner surface than the uniform composite cylinder (C1) the magnitude of tangential stress decrease by 7.76MPa (38.6%, compressive). At the outer surface of the FGM cylinder (C3), particle content more (m=0.5) than the uniform composite cylinder (C1) it causes the increases in the tangential stress by 7.24MPa (46.3%, tensile) than the uniform composite cylinder (C1) and due to reduction in particle content in the FGM cylinder (C5) than uniform cylinder (C1) cause the reduction in the tangential stress by 6.67MPa (43.4%, tensile).

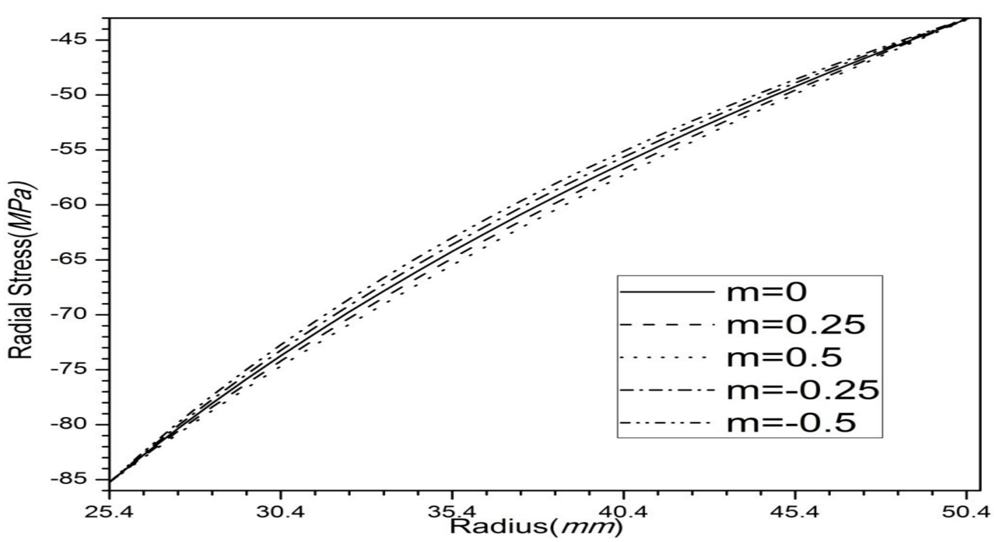


Fig.5.3a Variation in radial stress in cylinder with varying particle content

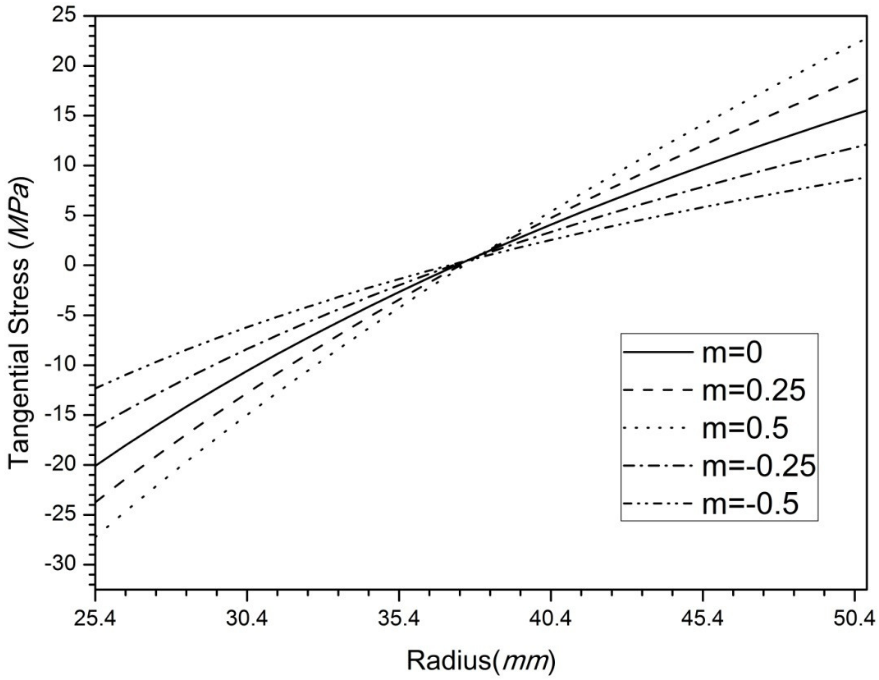


Fig.5.3b Variation in tangential stress in cylinder with varying particle content

Axial stress, as depicted in the Fig. 5.3c, remains compressive in nature through the entire radius of the cylinder. The magnitude of the axial stress is more at the inner surface and decreases toward the outer surface. With the increases in the value of particle gradation index (m), particle content at the inner surface decreases as compared to the uniform composite cylinder (C1) and the magnitude of the axial stress increases at the inner surface and becomes equal with the uniform composite cylinder at radius of 40.4mm and after that it starts decreasing towards the outer surface. As the particle content increases (C5) at the inner surface of the FGM cylinder the magnitude of the axial stress is decreased than the uniform composite cylinder (C1) and goes on decreasing till a radius value of 40.4 mm and after that its magnitude goes on increasing as moving towards the outer surface. As the particle content, decreases (C3) at the inner surface of the FGM cylinder, the magnitude of the axial stress, increases by 3.57MPa i.e. (6.78%) in comparison to the uniform composite cylinder (C1). It means the magnitude of the axial stress is inversely proportional to the particle content. With the increases in particle content (C5) at the inner surface cause the reduction in the magnitude of axial stress is observed by 3.88MP (7.33%) than in the uniform composite cylinder (C1).

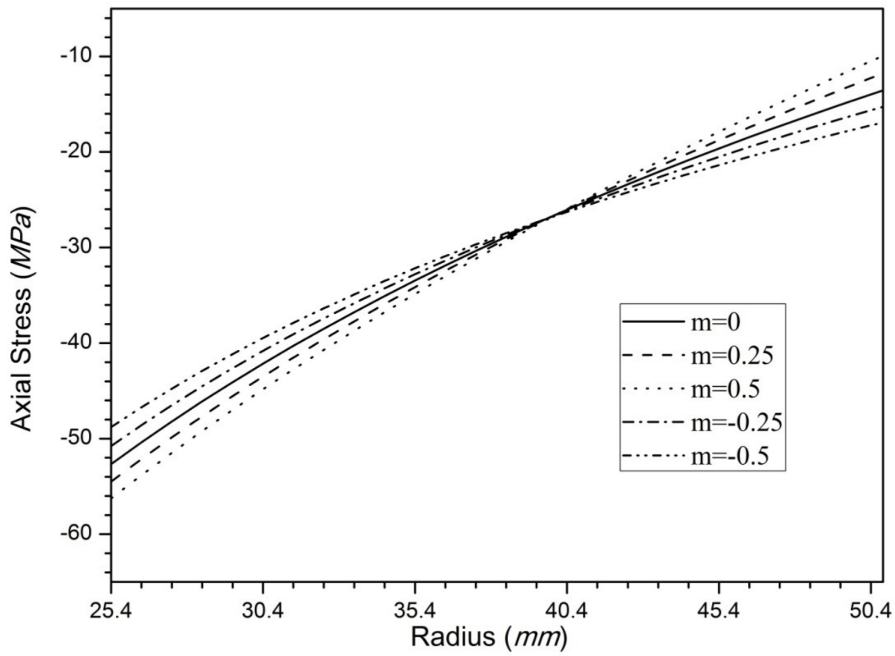


Fig. 5.3c Variation in axial stress with varying particle content

5.3.2 Effect on Strain Rates

The strain rates in the cylinder depend on the effective strain rate which further depends on threshold (σ_0) and effective stress (σ_e). The threshold stress depends on the particle content (V), particle size (P) and operating temperature (T) and the effective stress depends on the radial, tangential and axial stresses in the cylinder. The Threshold stress (σ_0) increases as the particle content at the inner surface increases and vice-versa and decrease at the outer surface due to reduction in the particle content. Average particle content in all the cylinders is kept same i.e. V (avg)=20%.

Creep parameter (M) depends on the particle size (P), particle content (V), and operating temperature (T) as in equation (8). As the particle content (C5) at the inner surface of the FGM cylinder increases the value of the creep parameter decrease (refer Fig.5.4). As the particle content decreases at the inner surface of the FGM cylinder (C3) the creep parameter (M) increases but goes on decreasing towards the radial direction. The decreases in the value of stress difference, as shown in the Fig.5.5. The stress difference (σ_e-σ_o) decrease toward the radial direction of the cylinder. It is also clear that the reduction in the stress difference (σ_e–

σ_o) is more in case when the gradation index is m=-0.5 in the FGM cylinder (C5) than the uniform composite cylinder (C1).

It is important to mention here that the value of stress difference decreases throughout the cylinder. When the particle gradation index (m=0.5), particle content at the inner surface is minimum in comparison to the uniform composite cylinder (C1) and the stress difference is also lower in the FGM cylinder (C3) than the uniform composite cylinder (C1) and becomes equal to the uniform composite cylinder (C1) at radius 31.4mm after that its value remains higher than the uniform composite cylinder. With the increases in particle content in the FGM cylinder (C5) at the inner surface the stress difference is more than the uniform composite cylinder (C1) and equal to uniform composite cylinder at radius 31.4mm after that the stress difference is lower than the uniform composite cylinder(C1).

So the reduction in stress difference is more at gradation index (m= -0.5) then the uniform composite cylinder (C1) by 1.61MPa (6.4%)at the inner surface when the particle content(C5) is more at the inner surface cause the increase in stress difference, by 1.29MPa (5.11%) than the uniform composite cylinder (C1). As the particle content at the inner surface increase (C5) it causes the decrease in the creep parameter and increases in stress difference and is maximum at inner surface after that stress difference goes on decreasing and matches with the uniform composite cylinder at radius 31.4 after that it again decreases. Due to this, overall reduction in the creep rate in the FGM cylinder (C5) when the particle content more at inner surface and reduction in the creep rate by $4.21 x 10^8$ (29.27%) than the uniform composite cylinder (C1). But as the particle content low at inner surface (C3) cause the increase in creep rate by $6.02*10^8$ (41.8%) than the uniform composite cylinder(C1). The variation in the creep rate is more in case of the composite and FGM cylinder (m=0, m=0.25, 0.5) than the creep rate in the FGM cylinder in which the gradation index (m=-0.25,-0.5). The deduction in creep rate is observed due to increases in the threshold stress (σ_o) and decrease in the creep parameter (M) as the particle content of SiC_p increases at the inner side of the FGM cylinder. In the present case the minimum creep rate estimated in FGM cylinder (C5) at constant temperature condition (refer Fig.5.6).

The radial and tangential strain in the cylinder also depends on the effective creep rate as given in the equation (17), (18). The axial strain rate is taken as zero under the plane strain condition ($\dot{e_z} = 0$).

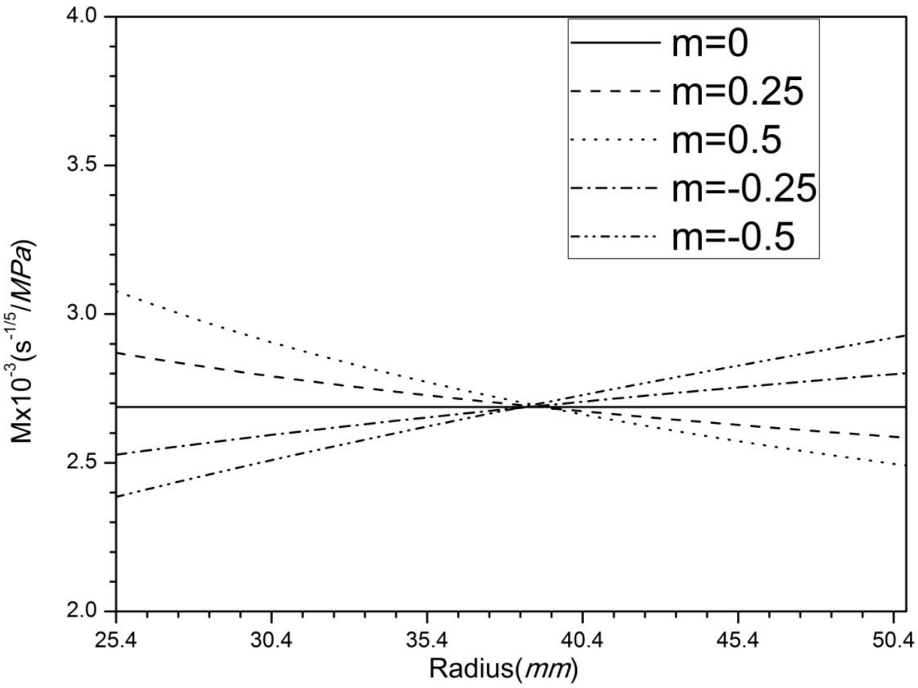


Fig.5.4 Variation in creep parameter in cylinder with varying particle content

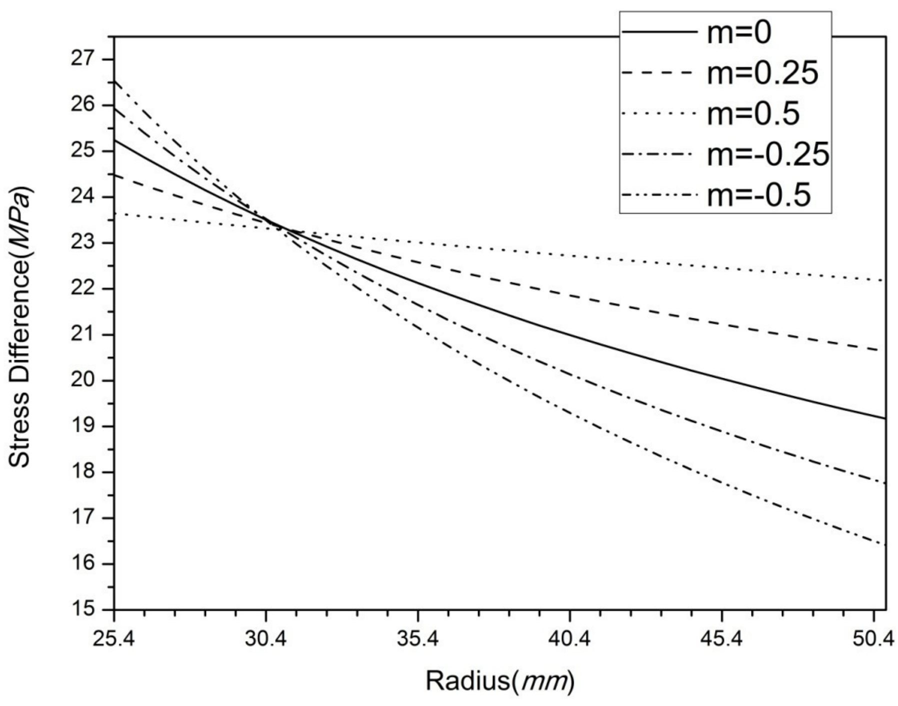


Fig.5.5 Variation in stress difference (σ_e-σ_o) with different particle content

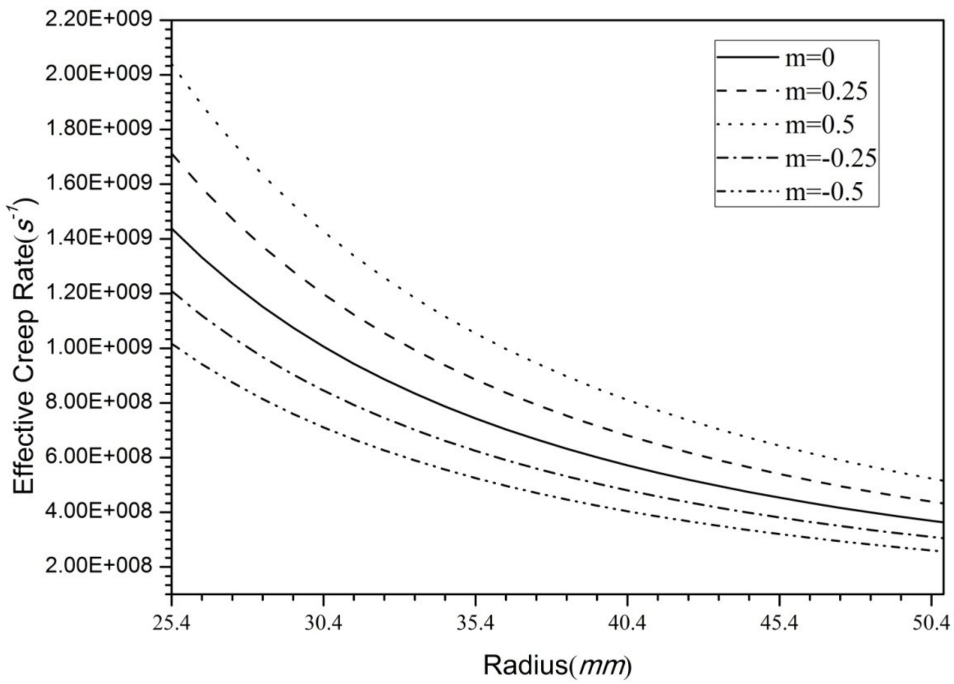


Fig.5.6 Variation in effective creep rate with different particle content

The radial and tangential strain rate in the cylinder under the constant temperature condition as given in the Figure (5.7a) and (5.7b).From the Fig.5.6, it attributed that increases in the particle content at the inner surface of the FGM cylinder (C5) cause the reduction in the creep rate of the cylinder. The influence of particle content on the tangential and radial strain rates is similar to those observed for the effective creep rate. The Increase in the particle content of SiC_p in the cylinder causes the decreases inter- particle spacing. This causes the increases in the threshold stress [20] and decrease in the creep parameter. Both these factor impart to reduction in the strain rates in the cylinder. The tangential and radial strain rates in the cylinder are same in magnitude but inverse in nature due to incompressibility condition given by the equation (16). Refer Fig. (5.7a) and (5.7b). As the particle content more (C5) inner side of the cylinder reduction in the magnitude of radial and tangential strain rates by 3.63×10^8(29.3%) than the uniform composite cylinder(C1) but as the particle content decrease (C3) at the inner side increase the strain rate by 5×10^8 (40.32%) than the uniform composite cylinder (C1). In the decreases trends of particle content in the FGM cylinder (C5) from inner radius to outer radius the magnitude of the strain rates decreases at the outer radius by 0.89×10^8 (28.8%) than the uniform composite cylinder (C1). If the particle content (C3) is more at the outer surface the magnitude of the strain rates increases by 1.33×10^8 (43%) than the uniform composite cylinder (C1). the variation in strain rates observed to minimum and the chance of distortion reduce in case when gradation index m=-0.5 than the other cases (m=0, 0.25, 0.5, -0.25) at constant temperature and use this work for thermal gradient analysis. In all this discussion the cylinder is operated under the constant temperature condition.

Now by taking the gradation index (m) =-0.5, the influence of temperature gradient (TG) on the strain rate discuss in detail.

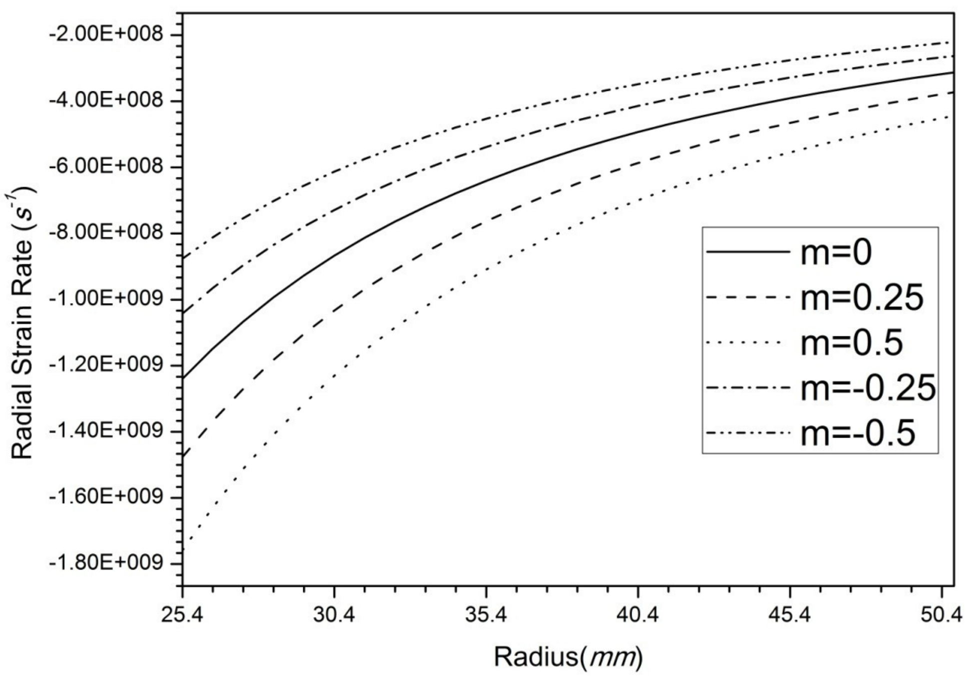


Fig.5.7a Variation in radial strain rate in cylinder with varying particle content

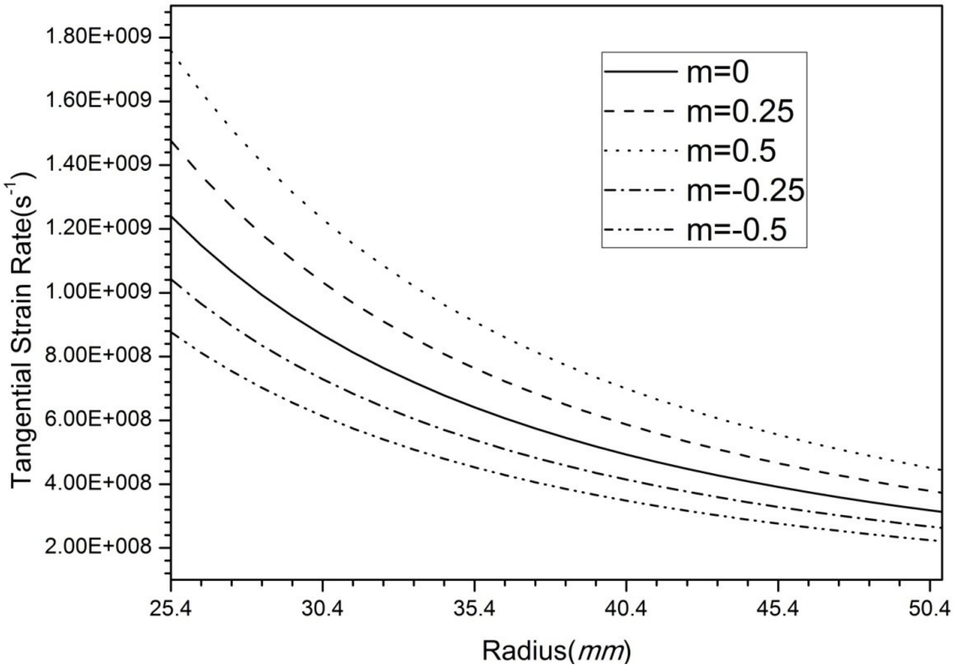


Fig.5.7b Variation in tangential strain rate in cylinder with varying particle content

5.4 Effect of Thermal Gradient

The creep stresses and strain rates have also been estimated for an FGM cylinder with gradation index (m=-0.5) under different thermal gradients as given in the Tables 3 and 4. The results obtained for the FGM cylinder under thermal gradient are compared with the same FGM cylinder subjected to constant temperature.

The temperature has been varied from the inner surface to the outer surface of the cylinder. Broadly two temperatures distributions have been considered as given below:

- Temperature varying linearly from inner to outer surface of the cylinder.
- Temperature varying non-linearly from inner to outer surface of the cylinder.

Further, five different thermal gradients (TG), as given in the Tables.3 and 4, for each of the temperature distribution profile have been considered to investigate their effect on the stresses and strain rates in the FGM cylinder

Table 3 Description of thermal gradient in the FGM cylinder (Linear temperature profile)

Notations	Temperature		Thermal Gradient (TG)
	T_a	T_b	$TG = (T_b - T_a)$
TG=LD(80)	663	583	80
TG=LD(40)	643	603	40
TG=0	623	623	0
TG=LI(40)	603	643	40
TG=LI(80)	583	663	80
TG=LD (50)	648	598	50

For both the temperature profiles, the stresses and strain rates are compared under same thermal gradient. The variation in temperature for both the profiles i.e. linear and non-linear distribution is shown in the Fig.5.8a and 5.8b.

Table 4 Description of thermal gradient in the FGM cylinder (Non-linear temperature profile)

Notations	Temperature		Thermal Gradient (TG)
	T_a	T_b	$TG = (T_b - T_a)$
TG=NLD(80)	663	583	80
TG=NLD(40)	643	603	40
TG=0	623	623	0
TG=NLI(40)	603	643	40
TG=NLI(80)	583	663	80
TG=NLD (50)	648	598	50

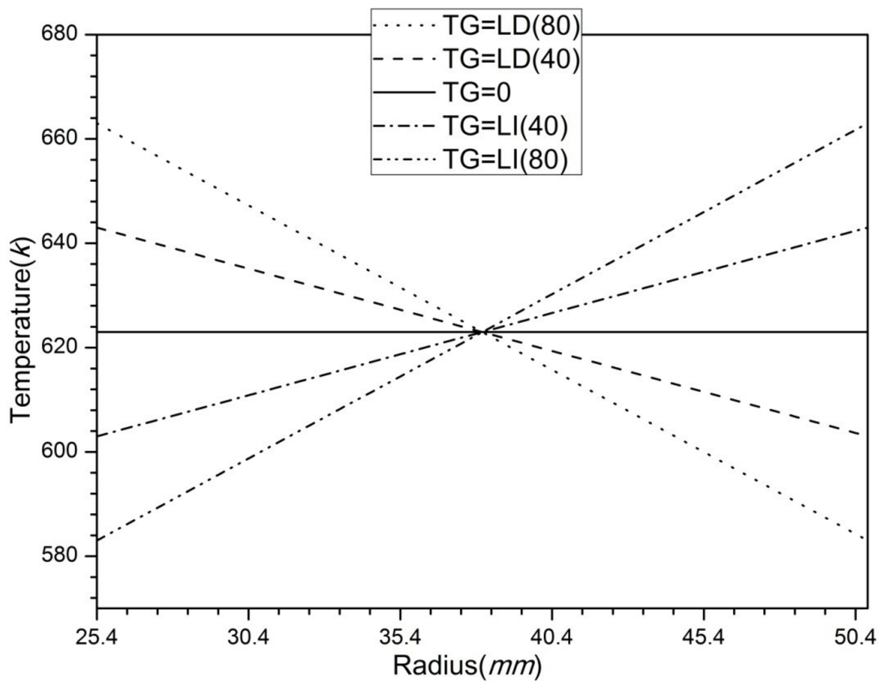


Fig .5.8a Variation in temperature (Linear)

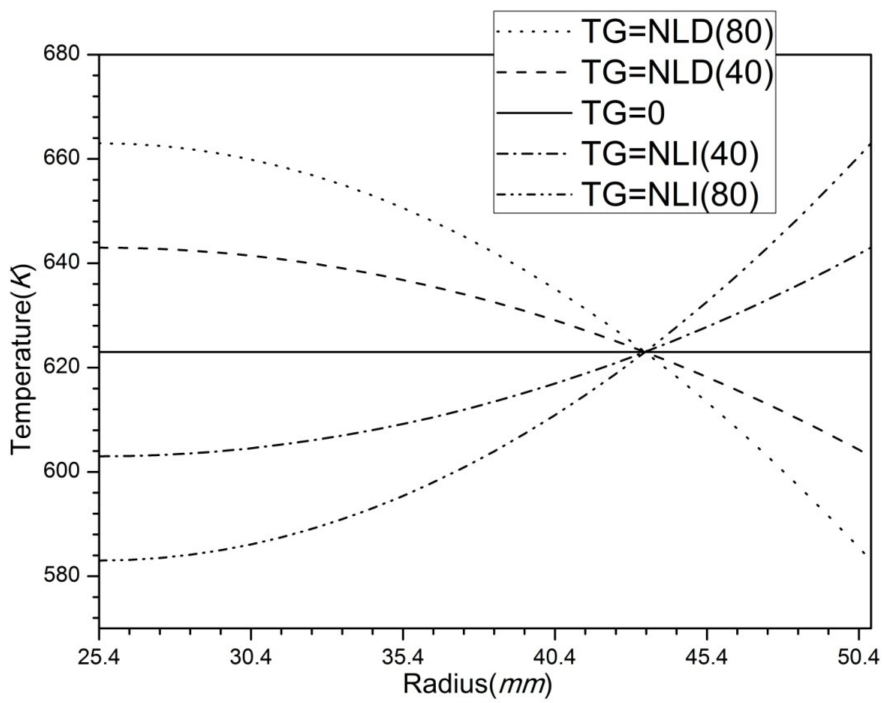


Fig.5.8b Variation in temperature (Non- linear)

As depicted in the Figs. 5.8a, 5.8b, the temperature is assumed to remain uniform in one of the FGM cylinders and in other two cases temperature is kept maximum at the inner surface and goes on decreasing (linearly and non linearly) to minimum towards the radial direction of the FGM cylinder and in the other two cases, temperature is minimum at the inner surface and goes on increasing (linearly and non linearly) to maximum towards the outer surface of the cylinder.

5.4.1 Effect on Stresses

This section, discusses the effect of thermal gradient (linearly and non- linearly) on the radial stress in the FGM cylinder (refer Fig.5.9a, 5.9b). The radial stress remains compressive in nature throughout the cylinder and imposed thermal gradient does not have a significant effect on this. But a slight variation in the radial stress is observed at the middle region of the FGM cylinder. The magnitude of radial stress is maximum at the middle region for the FGM cylinder in which the temperature decreases [TG=LD(80), TG=NLD(80)], for both temperature profiles, from the inner surface to the outer surface of the cylinder . The magnitude of the radial stress increases by 2.53MPa (4.24%) in the FGM cylinder subjected to thermal gradient as compared to the FGM cylinder subjected to uniform temperature. As the thermal gradient decreases [TG=LD(40), TG=NLD(40)] the variation in magnitude of the radial stress at the middle region decreases than the uniform temperature FGM cylinder. When the temperature is minimum at the inner surface than the uniform temperature FGM cylinder (refer Fig.5.8a, 5.8b) and the temperature increasing from inner surface to the outer surface [TG=LI(80),TG=NLI(80)] of the FGM cylinder the magnitude of the radial stress decrease by 2.61MPa (4.4%) in the middle region of the FGM cylinder than the uniform temperature (TG=0) FGM cylinder.

Tangential stress remains compressive in nature at the inner surface, as shown in the Figs. 5.10a, 5.10b, of the FGM cylinder for most of the cases considered except the cylinder TG=LI(80). In the cylinder TG=LI(80), tangential stress is tensile at the inner surface. The magnitude of the tangential stress observed is minimum in case of non–linear distribution of temperature [TG=NLD(80)] at inner surface and changes to maximum tensile at the outer surface. As the temperature decreases from the inner surface to the outer surface [TG=LD(80),TG=NLD(80)] of the FGM cylinder the magnitude of the tangential stress at the inner surface increases by 11.81MPa (95.7%,compressive) for the linear temperature profile and increase by 8.05MPa (65.3%,compressive) for the non-linear temperature profile in comparison to the FGM cylinder having uniform temperature distribution. At the outer surface of the FGM cylinders [TG=LD(80),TG=NLD(80)] temperature is lower than the uniform temperature cylinder in both cases, the tangential stress increases by 23MPa (258.4%,tensile) for linear temperature variation and 30.94MPa (349.2%,tensile) for non-linear temperature profile temperature FGM cylinder than the uniform temperature cylinder. In the other temperature distribution, when the temperature is minimum [TG=LI(80),TG=NLI(80)] at the inner surface than the uniform temperature FGM cylinder(TG=0) than the magnitude of the tangential stress increase by 9.06MPa (73.5%,tensile) in linear temperature and 6.49MPa (52.63%,tensile) in non linear temperature profile than the constant temperature FGM cylinder. At the outer surface, the temperature is more [TG=LI(80), TG=NLI(80)] than the uniform temperature FGM cylinder in both cases and the tangential stress is compressive in nature (refer Fig.5.10a, 5.10b) and the magnitude decreases by 7.35MPa (82.6%) for linear and 4.96MPa (55.9%) for the non-linear temperature profile than the uniform temperature FGM cylinder. When the temperature is low [TG=LI(80),TG=NLI(80)] at the inner surface of the FGM cylinder, the magnitude of the tangential stress is maximum in case of linear temperature profile than the non linear temperature profile and tensile in nature at the inner surface of the cylinder than the uniform temperature cylinder. More the thermal gradient (linear or non linear) more the variation observed in the tangential stress in the radial direction throughout the FGM cylinder. In both case (linear and non linear) the variation along the radial direction is maximum in cases [TG=LD(80), TG=NLD(80)].

Axial stress remains compressive in nature throughout the FGM cylinder (both for linear and non linear temperature variation) as depicted in Fig. 5.11a, 5.11b. The magnitude of the axial stress is more at the inner surface of the FGM cylinder and decreasing toward the outer surface. When the temperature is more at the inner surface [TG=LD(80), TG=NLD(80)] of the FGM cylinder, the magnitude of the axial stress increases by 5.9MPa (12%) for linear variation of temperature and increases by 4.03MPa (8.26%) for the non linear variation of the temperature than the uniform temperature FGM cylinder. At the outer surface of the cylinder [TG=LD(80),TG=NLD(80)] temperature is lower than the uniform temperature cylinder and the magnitude of the axial stress decreases by 10.80MPa (64.3%), for linear and 14.80MPa(88%)**,** for non linear temperature variation than the uniform temperature FGM cylinder. Overall variation in the axial stress is more due to the decreasing (linear and non- linear) temperature [TG=LD(80), TG=NLD(80)] than the increasing [TG=LI(80),TG=NLI(80), TG] temperature trends. As the temperature is minimum at the inner surface [TG=LI(80),TG=NLI(80)] than the uniform temperature FGM cylinder, the magnitude of the axial stress decreases by 16.86MPa (34.56%) at inner surface and increases by 5.2MPa (31%) at the outer surface (temperature is more than the uniform temperature FGM cylinder) in linear variation temperature profile and for non linear temperature variation , the magnitude decreases by 9.09MPa (18.6%) at the inner surface and increase by 6.4MPa (38.1%) at the outer surface (temperature is more than the uniform temperature) of FGM cylinder.

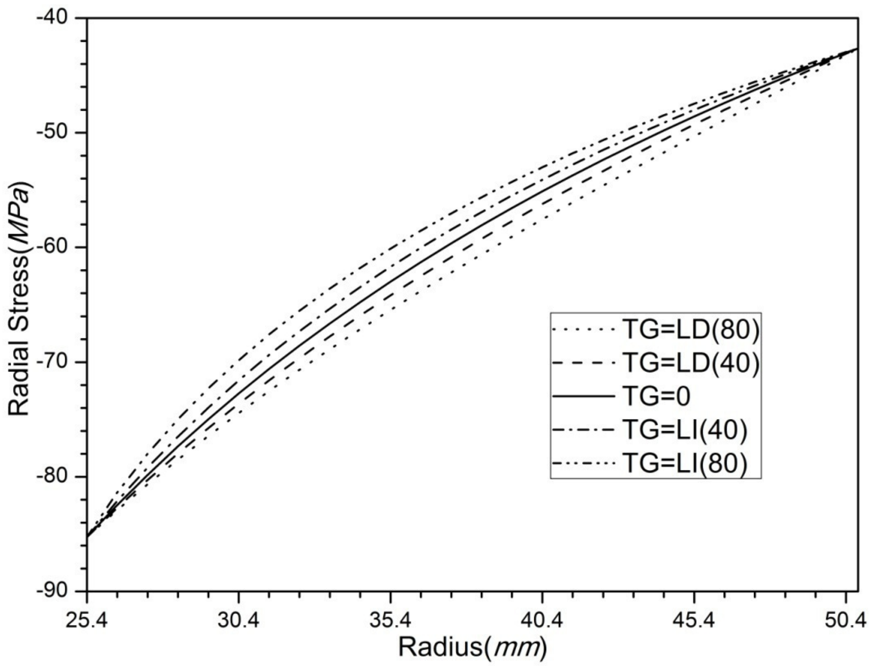


Fig.5.9a Variation in radial stress with thermal gradient (Linear temperature profile)

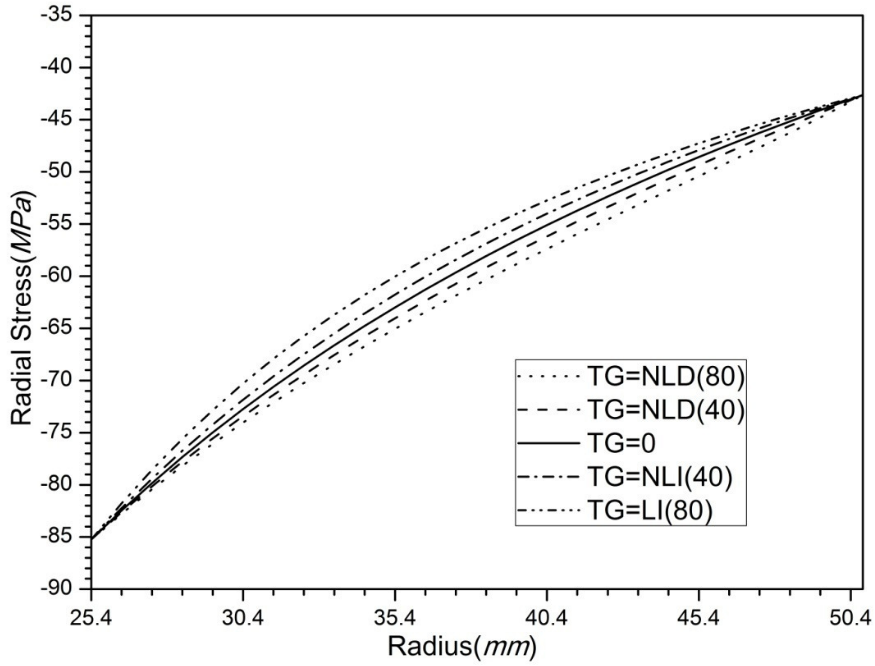


Fig.5.9b Variation in radial stress with thermal gradient (Non-linear temperature profile)

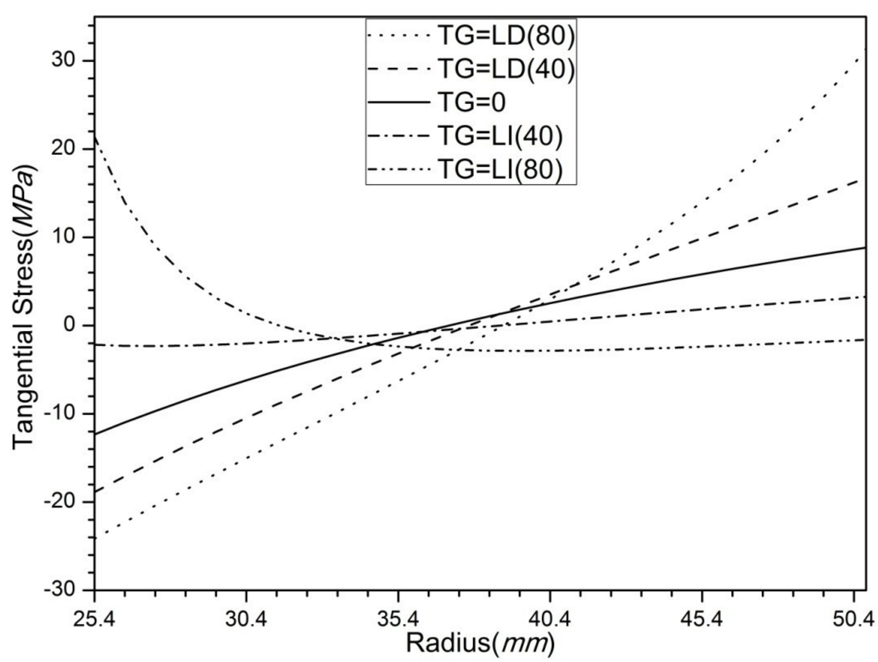


Fig.5.10a Variation in tangential stress with thermal gradient (Linear temperature profile)

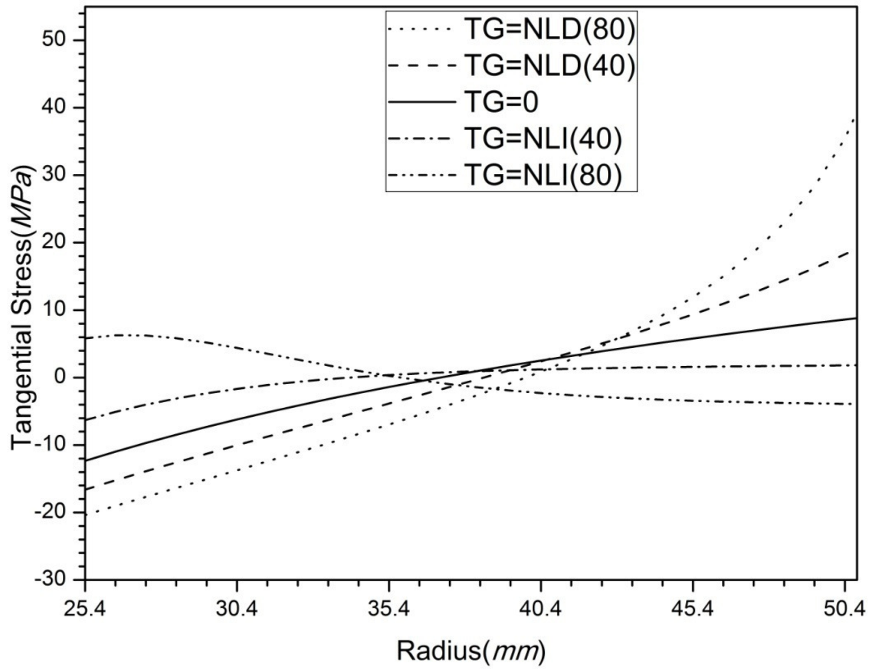


Fig.5.10b: Variation in tangential stress with thermal gradient (Non-linear temperature profile)

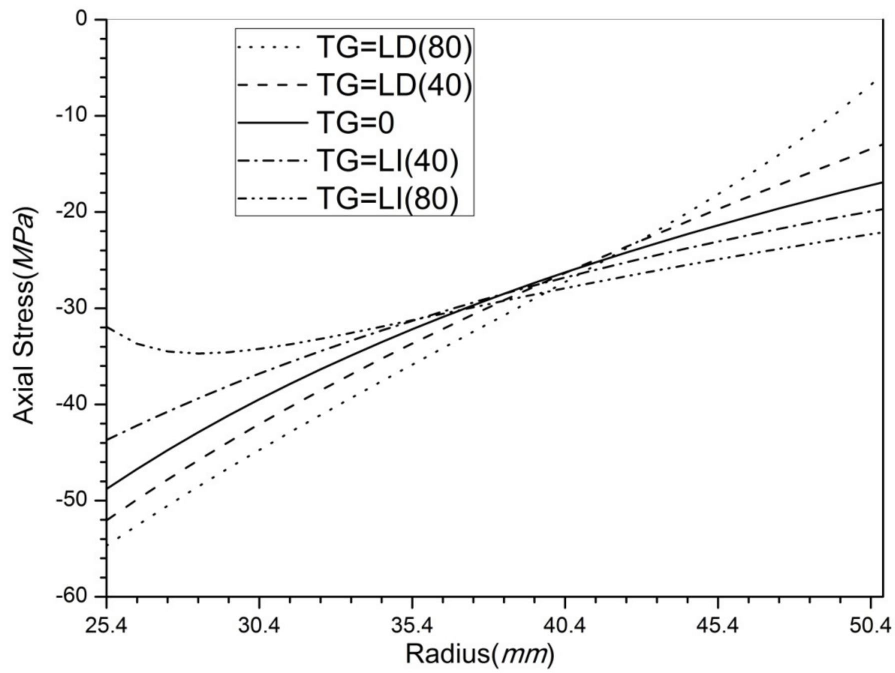


Fig.5.11a Variation in axial stress with thermal gradient (Linear temperature profile)

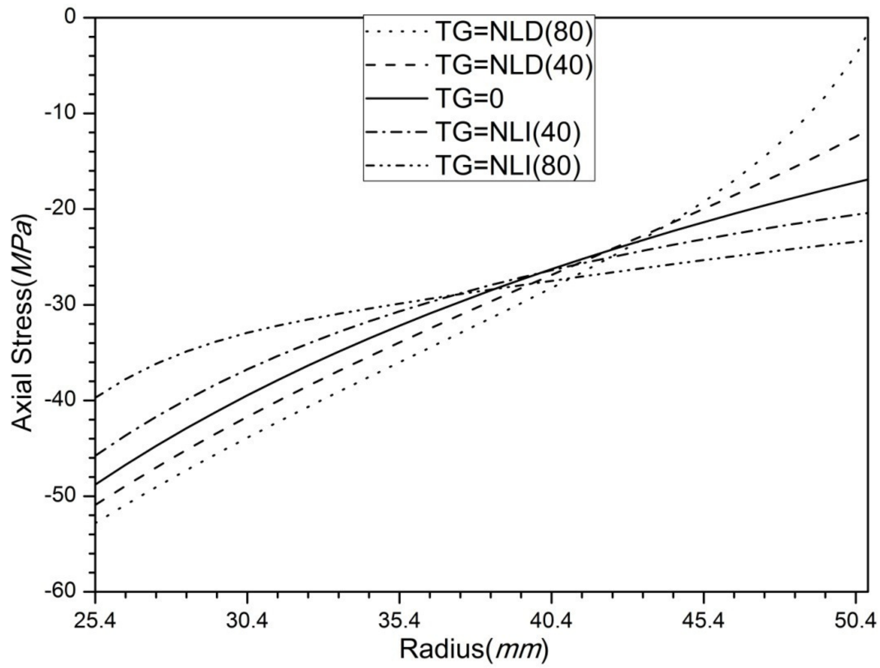


Fig.5.11b Variation in axial stress with thermal gradient (Non-linear temperature profile)

5.4.2 Effect on Strain Rates

To investigate the effect of thermal gradient (TG) on the strain rates in the FGM cylinder under the six different cases of thermal gradient as given in the Table 3, 4. The strain rates depend on the effective creep rate and the effective creep rate ($\dot{\varepsilon}_e$) depends on the creep parameter (M) and the stress difference (σ_e-σ_o). The creep parameter (M) also depends on the particle size (P), particle content (V) and the operating temperature (T). The variation of creep parameter (M) as depicted in Figs.5.12a, 5.12b for the linear and non–linear temperature variation profile. The value of creep parameter is minimum at minimum temperature and maximum at the maximum temperature in both the temperature profile.

The stress difference (σ_e-σ_o), as shown in the Figs. 5.13a, 5.13b, plays a very significant role for the reduction in creep rate. The effective creep rate depends on the product of creep parameter (M) and the stress difference. As the temperature is minimum at the inner surface the magnitude of the stress difference is maximum but creep parameter is minimum as the temperature in minimum and it causes the overall reduction in the effective creep rate in the cylinder (refer Fig.5.14a, 5.14b). For the increasing temperature profile [TG=LI(80), TG=NLI(80)], the stress difference is maximum at the inner surface for the linear variation temperature profile than the non–linear temperature profile in the FGM cylinder and it is decreasing significantly towards the outer surface. Due to this, the effective creep rate decreases, the reduction in creep rate is more in case of non–linear temperature profile cylinder and is more significant than the linear temperature profile cylinder under the similar thermal gradient[TG=LI(80)]. As the temperature is maximum at the inner surface of the [TG=LD(80), TG=NLD(80)] than the stress difference decreases by 9.35MPa (35.24%) for linear and by 5.8MPa (22%) for the non linear temperature distribution of temperature than the uniform temperature cylinder and at the outer surface with same thermal gradient, temperature is lower than the constant temperature FGM cylinder increases the stress difference by 18.65MPa (114.1%) for linear and increases by 26.47MPa (161.8%) for the non linear temperature variation. The increase in stress difference is more in case of TG=NLD(80) than the TG=LD(80) at the outer surface of the cylinder cause the more creep rate in the FGM cylinder.

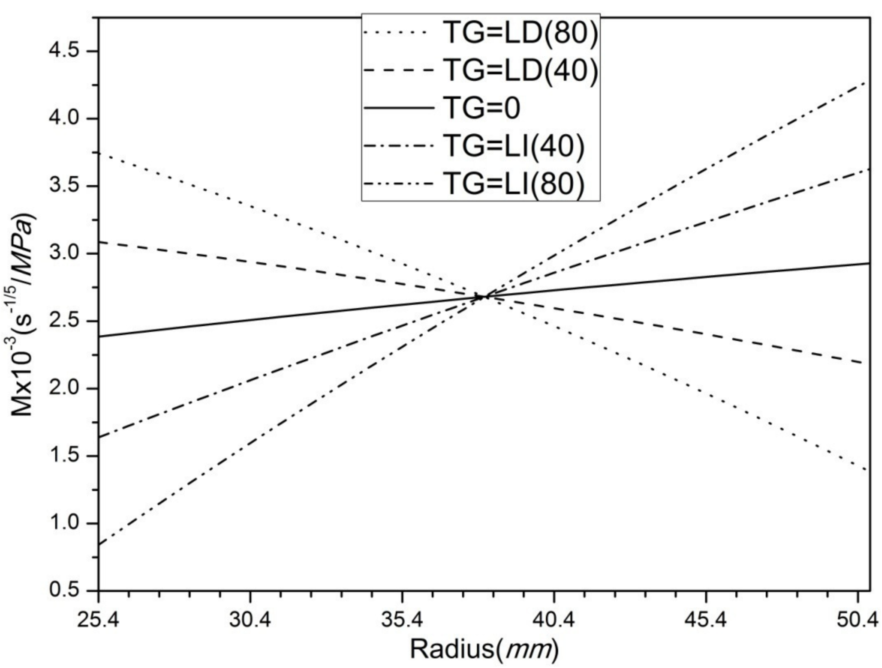


Fig.5.12a Variation in creep parameter with thermal gradient
(Linear temperature profile)

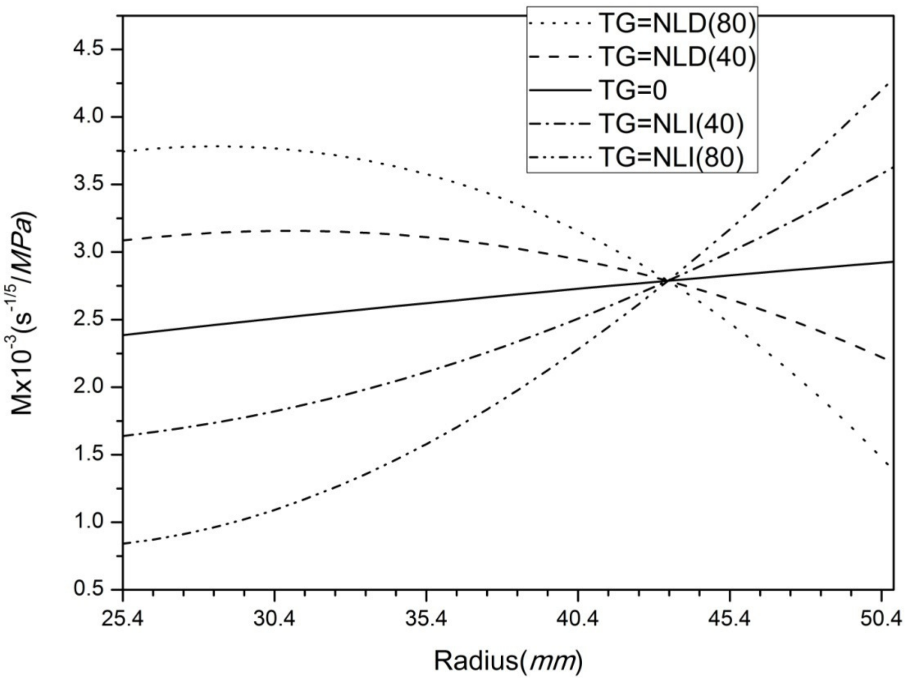


Fig.5.12b Variation in creep parameter with thermal gradient
(Non- linear temperature profile)

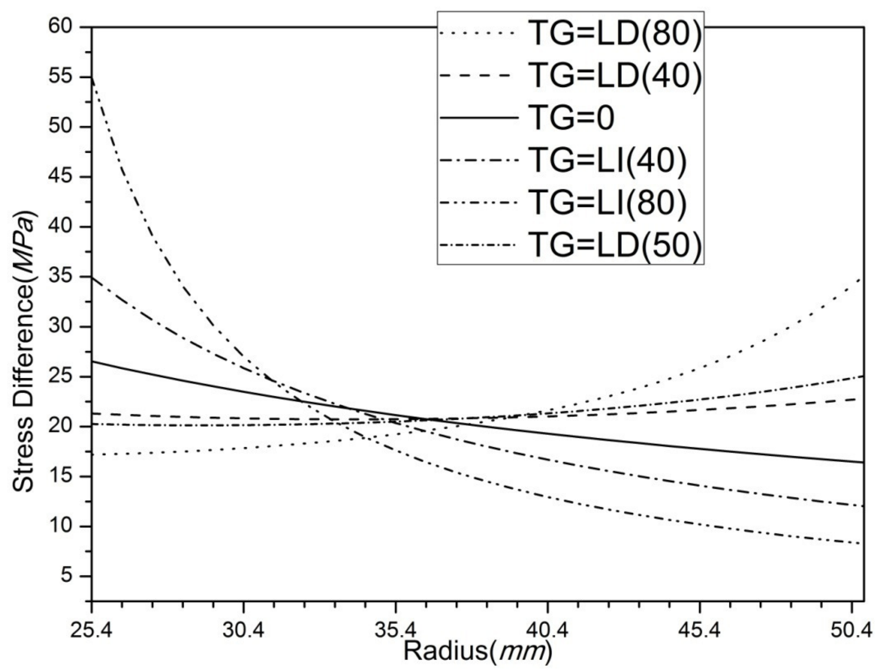


Fig.5.13a Variation in stress difference (σ_e-σ_o) with thermal gradient
(Linear temperature profile)

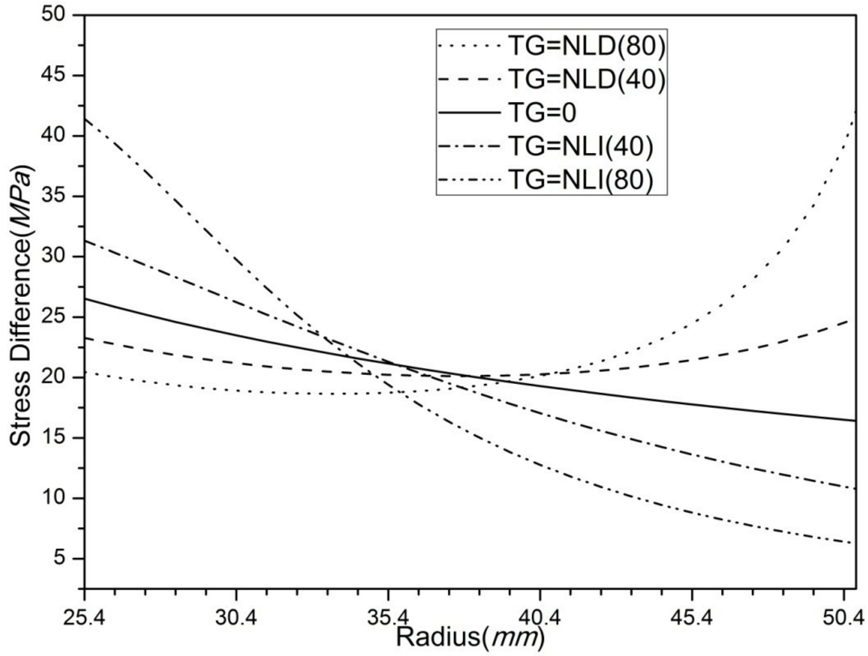


Fig.5.13b Variation in stress difference (σ_e-σ_o) with thermal gradient
(Non-linear temperature profile)

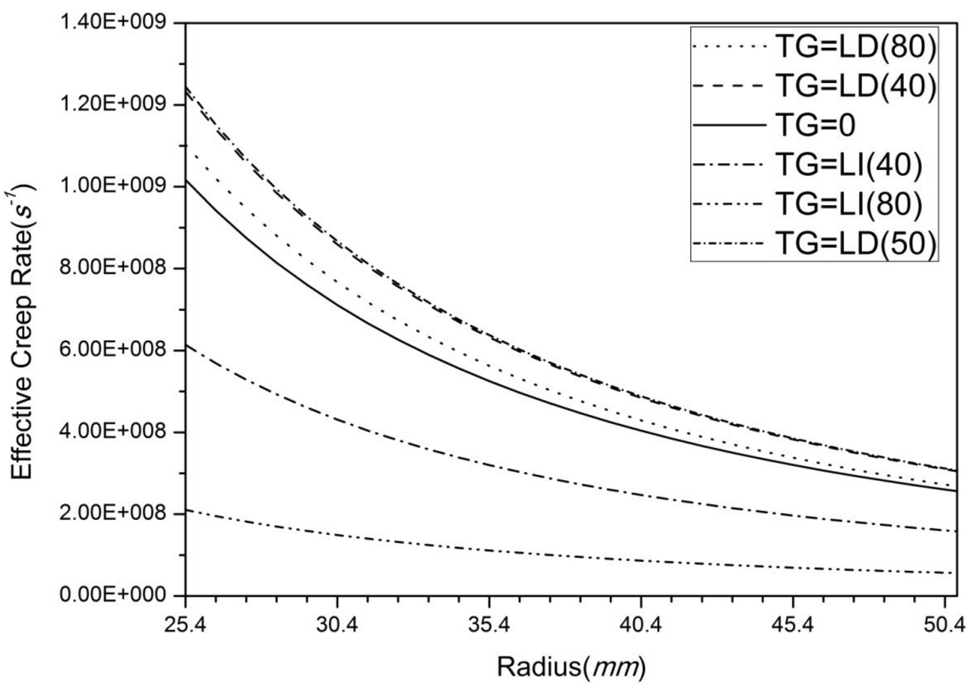


Fig.5.14a Variation in effective creep rate with thermal gradient
(Linear temperature profile)

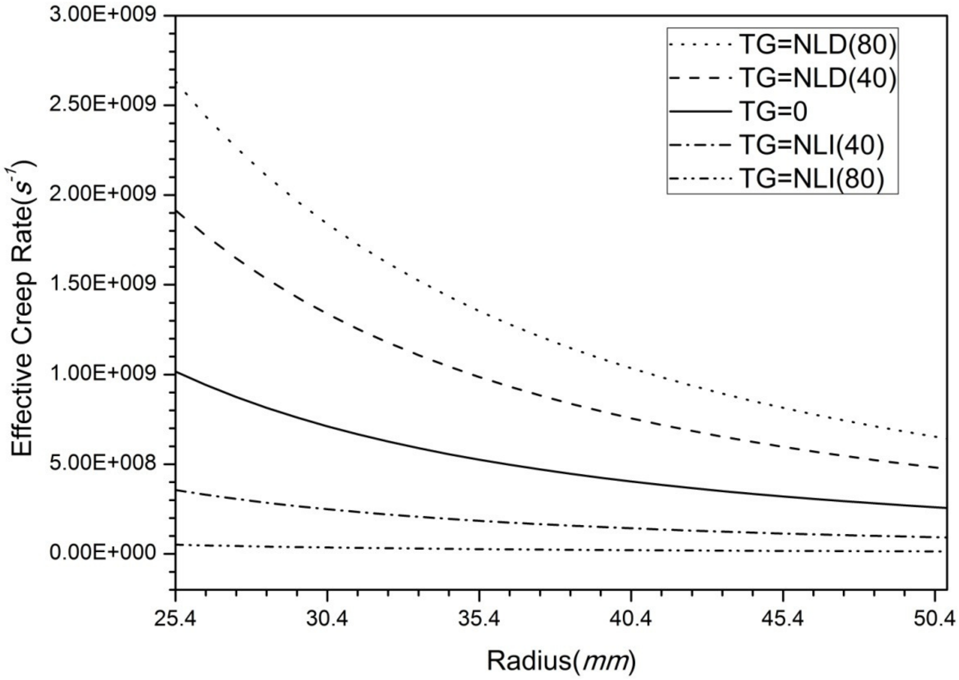


Fig. 5.14b Variation in effective creep rate with thermal gradient
(Non-linear temperature profile)

Radial and tangential strain rates under the effect of thermal gradient are depicted in the Figs. (5.15 a, b and 5.16 a, b). It has been observed that the radial strain remains compressive throughout and decreases towards the outer surface, however the rate of decrease is more significant near the inner surface. The radial stain rate is minimum when the temperature profile has increasing trend. , Further it has also been observed that for linearly decreasing temperature profile, the maximum strain rate observed up to the thermal gradient of 50, after that the radial strain rate start decreasing. The tangential strain also varies in the same manner as the radial strain rate. In the FGM cylinder having decreasing temperature trend [TG=LD(80), TG=NLD(80)], it has been observed that the magnitude of the radial and tangential strain rates are lower for the linear temperature profile at the inner surface in comparison to the non-linear temperature distribution profile. For the other temperature distribution i.e. when the cylinder is having increasing temperature distribution from inner to the outer surface [TG=LI (80), TG=NLI(80)], both the strain rates are much more uniform than the cylinder having [TG=0] uniform temperature distribution. For FGM cylinder [TG=NLI(80)], strain rates are minimum and uniform throughout in the radial direction.

In the FGM cylinder having decreasing temperature trend [TG=LD (50)], the magnitude of the radial strain rate is increases by 10.73×10^8 (22.3%) at the inner surface and by 0.42×10^8 (19%) at the outer surface than the uniform temperature FGM cylinder. For the other temperature distribution i.e. when the cylinder is having increasing temperature distribution from inner to the outer surface [TG=LI (80)] than the uniform temperature cylinder [TG=0], the magnitude of strain rates decreases by 6.95×10^8 (79.29%) at the inner surface and by 1.72×10^8 (78.18%) at the outer surface. But in non-linear temperature profile [TG=NLI(80)], the magnitude of strain rates decrease by 8.33×10^8 (94.98%) at the inner surface and 2.08×10^8 (94.6%) at the outer surface than the uniform temperature FGM cylinder.

Form the above discussion, it can be concluded that the cylinder having increasing temperature distribution from inner to the outer surface under same thermal gradient, the magnitude of the strain rates minimum for non-linear temperature profile in comparison to the linear temperature profile, for both the strain rates. But the variation in magnitude of the tangential strain rate is opposite in nature from the radial strain rate.

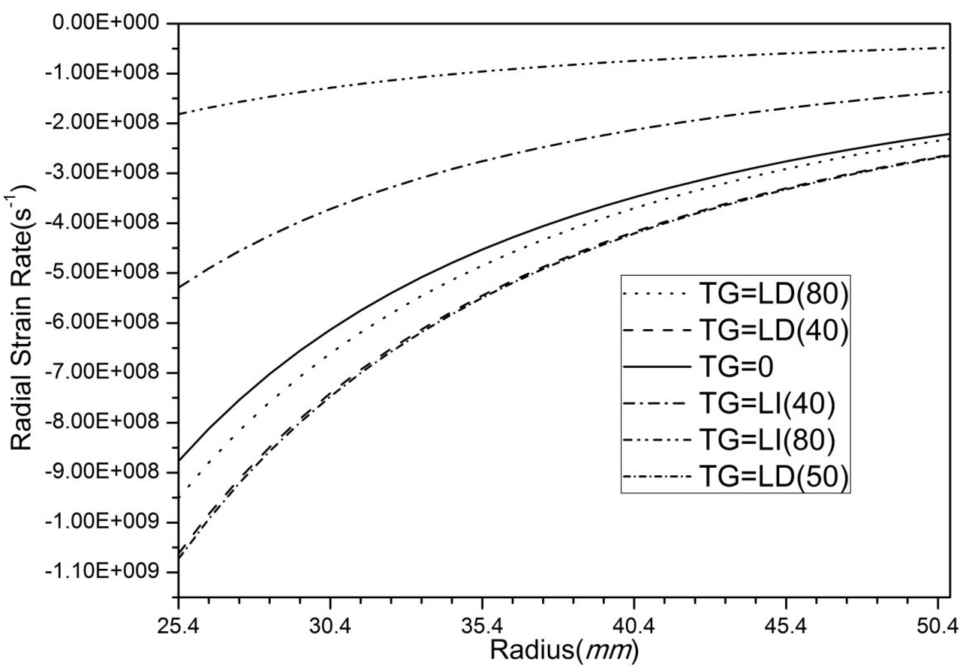


Fig. 5.15a Variation in radial strain rate with thermal gradient
(Linear temperature profile)

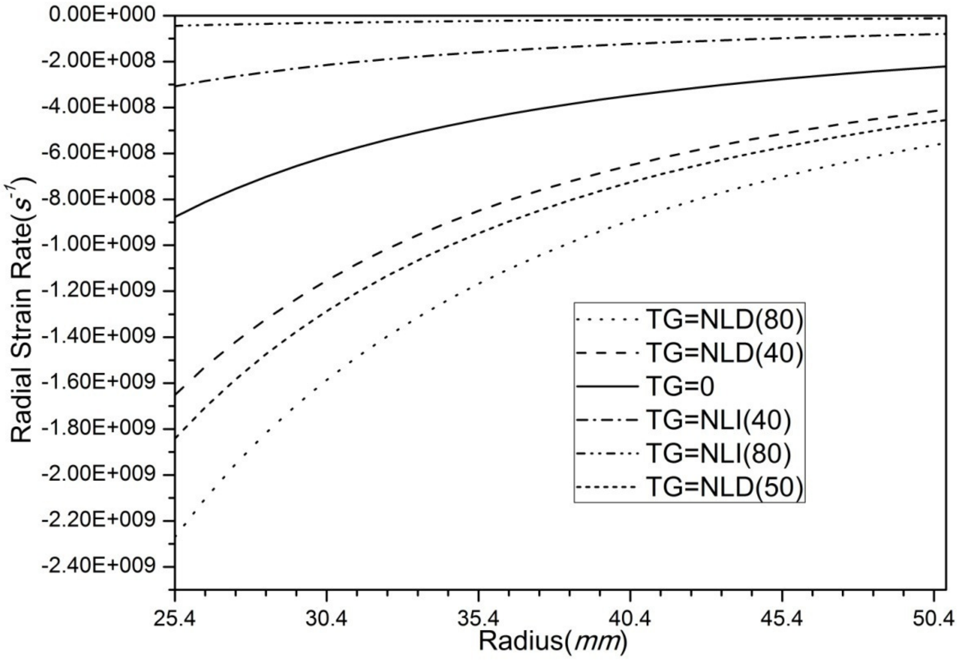


Fig. 5.15b Variation of radial strain rate with thermal gradient
(Non-linear temperature profile)

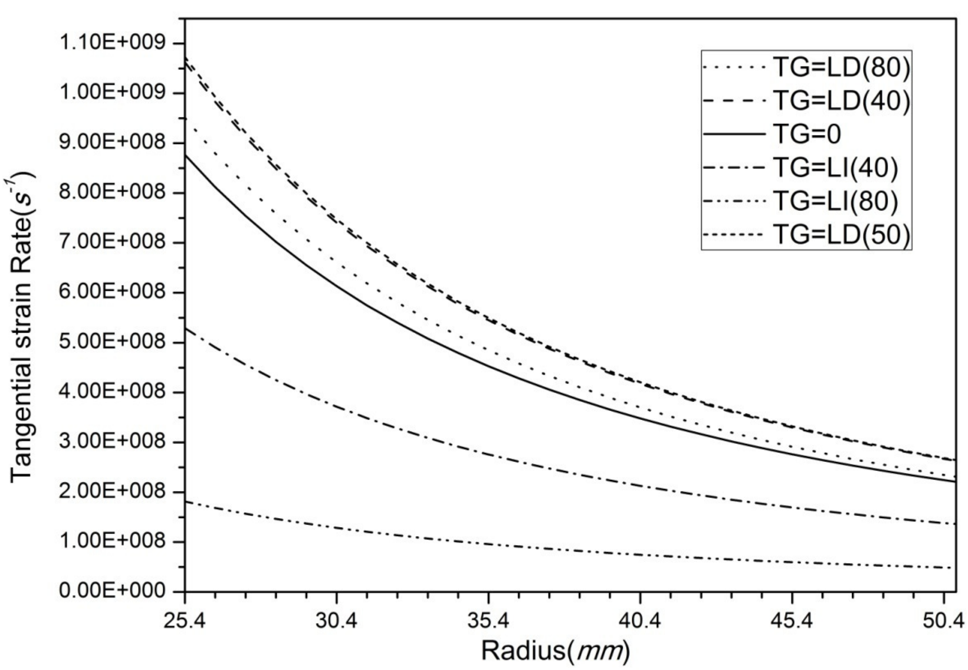


Fig.5.16a Variation of tangential strain rate with thermal gradient
(Linear temperature profile)

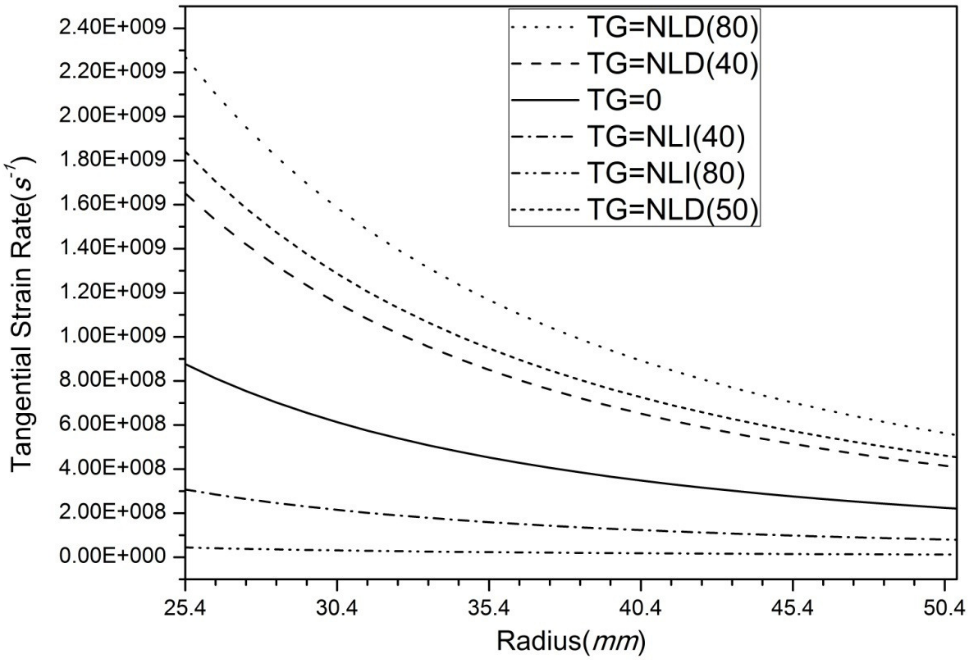


Fig. 5.16b Variation of tangential strain rate with thermal gradient (Non-linear temperature profile)

Chapter 6

6.1 Conclusions

From the previous discussion following conclusion can be drawn:

a) The magnitude of the tangential and axial stresses in the FGM cylinder decreases as the content of SiC_p decreases along the radial direction.

b) The effective creep rate decrease to great extent by increase in the reinforcement content.

c) The radial and tangential strain rates decreases by the decrease in particle content along the radial direction.

d) The nature of tangential stress changes when temperature distribution profile changes from increasing to decreasing from inner surface to the outer surface.

e) The reduction in the effective creep rate is more significant in the non-linear temperature profile with increasing trend of temperature from inner to the outer surface.

f) The strain rates increase by the increase in the thermal gradient up to a certain limit and start decreasing beyond that for linearly decreasing temperature profile.

6.2 Future Scope of Work

The present work may be further extended to:

- The creep analysis of cylinder which made of some other composite materials.
- The effect of anisotropy on the creep behaviour of the FGM cylinder can be investigated.
- The analysis can be carried out for the rotating cylinder.

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