OPTIMIZATION AND PERFORMANCE ANALYSIS OF LINEAR DISPERSIVE STBC CODES WITH DIFFERENT ESTIMATORS.

Dissertation Submitted towards the partial fulfilment of requirement for the award of degree of

Masters of Engineering

In

Electronics and Communication Engineering

Submitted by

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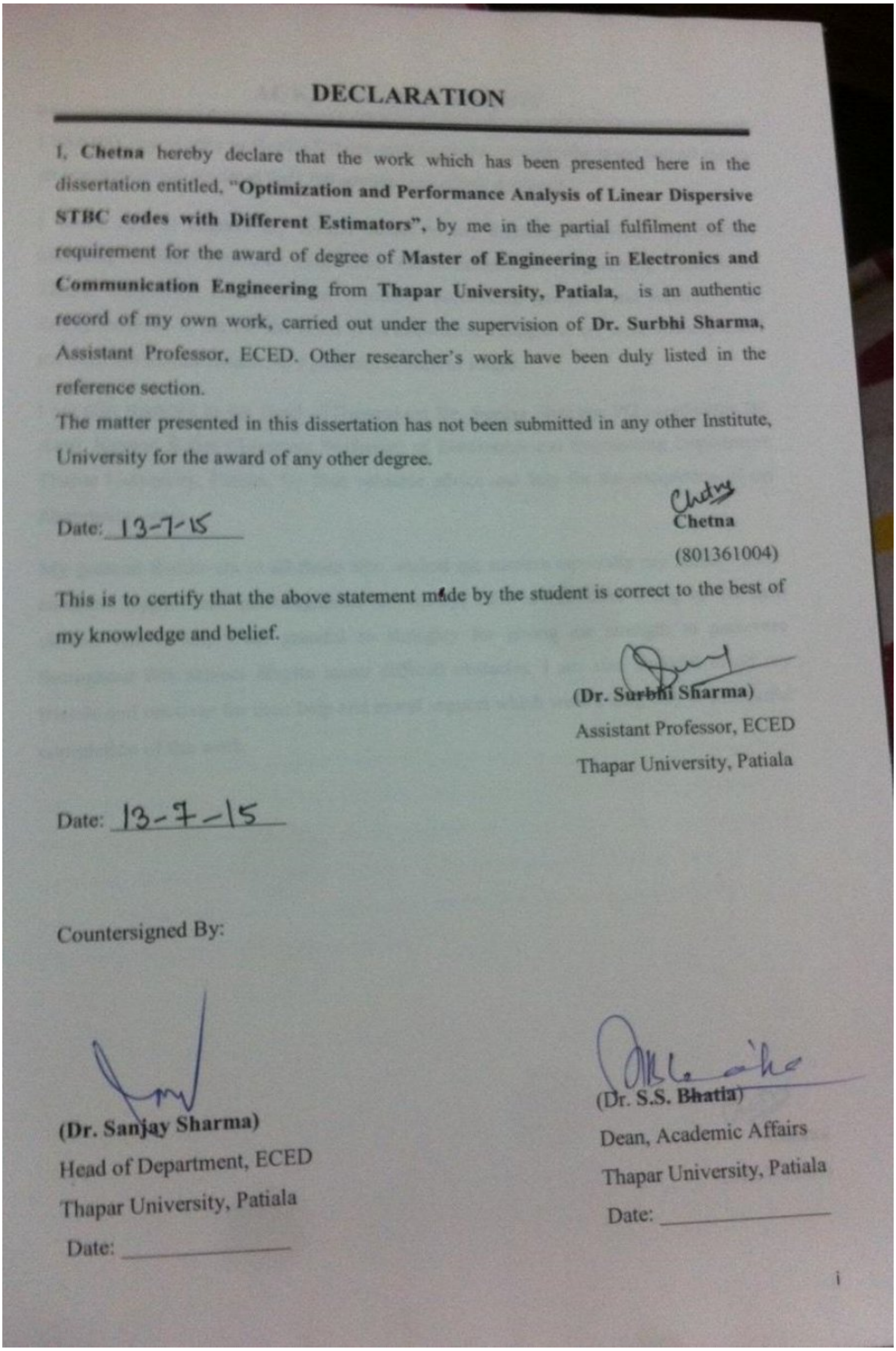
THAPAR UNIVERSITY

(Established under the section 3 of UGC Act, 1956)

PATIALA – 147004 (PUNJAB)

June, 2015



DECLARATION

I, **Chetna** hereby declare that the work which has been presented here in the dissertation entitled, **"Optimization and Performance Analysis of Linear Dispersive STBC codes with Different Estimators"**, by me in the partial fulfilment of the requirement for the award of degree of **Master of Engineering** in **Electronics and Communication Engineering** from **Thapar University, Patiala**, is an authentic record of my own work, carried out under the supervision of **Dr. Surbhi Sharma**, Assistant Professor, ECED. Other researcher's work have been duly listed in the reference section.

The matter presented in this dissertation has not been submitted in any other Institute, University for the award of any other degree.

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This is to certify that the above statement made by the student is correct to the best of my knowledge and belief.

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ACKNOWLEDGEMENT

I take this opportunity to thank all those magnanimous persons who stood behind me as an inspiration and rendered their full service throughout my thesis.

I would like to express my special thanks to my dissertation advisor, **Dr. Surbhi Sharma,** Assistant Professor, Electronics and communication Engineering Department, Thapar University, Patiala for her continous unrelenting guidance and support. I am deeply indebted to her for she stood as an inspiration throughout my work and explained me even the minute details very patiently at various stages.

I am also thankful to the Head of Department **Dr. Sanjay Sharma**, PG coordinator **Dr. Amit Kumar Kohli** (Associate Professor) of Electronics and Engineering Department, Thapar University, Patiala, for their valuable advice and help for the completion of my dissertation work.

My greatest thanks are to all those who wished me success especially my family. Their constant support and encouragement has always showed me silver lining in the dark clouds. Above all, I am grateful to almighty for giving me strength to persevere throughout this project despite many difficult obstacles. I am also thankful to all my friends and relatives for their help and moral support which went a long way in successful completion of the work.

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ABSTRACT

The Linear Dispersion Codes, a type of STBC, have been of great importance nowadays, as they subsume the important characteristics of the STBC codes and have decoding simplicity of VBLAST codes. These codes perform well in case of both Ergodic capacity and the error probability. The salient advantage of the code is its ability to get designed for any configuration of transmit and receive antennas, codeword lengths and rate. It is for this reason that we extend our work on the optimization of these codes in the thesis work.

We considered a flat fading, open loop MIMO communication scheme where the transmitter having multiple antennas encodes the desired message signal into codeword matrix using an Optimized Linear Dispersion encoder. The designing and optimization of the encoder has been inspired from the different design methods. Non vanishing determinant property has been employ-yed to check whether it obtains an optimal diversity trade-off. The codes designed have been verified to satisfy NVD property. Further, Rank and determinant criteria has been obtained for the Integer Forcing receiver and used to optimize the encoder matrix.

The performance of the optimized Linear dispersive Encoder has been evaluated using the Sphere Decoder and has been compared with different estimators such as Zero Forcing, MMSE receivers. A new decoding scheme, Integer forcing equalization scheme has been combined with the Encoder to efficiently estimate the codeword matrix. Monte Carlo simulations were used to compare its performance with other linear estimators and decoders, in terms of bit error probability. It has been observed that the designed model achieves better performance as compared to conventional linear receivers and have lower complexity than that of the Sphere Decoder.

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LIST OF ABBREVIATIONS

APSK	Amplitude Phase Shift Keying
BER	Bit Error Rate
BLAST	Bell Labs Layered Space Time Architecture
BLER	Block Error rate
CSI	Channel State Information
DMT	Diversity Mulitplexing Tradeoff
DoT	Department of Telecommunications
FP	Fincke Pohst
GLST	Group Layered Space Time
GSM	Global System for Mobile Communication
IF	Integer Forcing
IF-SIC	Integer Forcing- Successive Interference Cancellation
ITU	International Tele communication Unit
LDC	Linear dispersion Codes
LST	Layered Space Time
LTE	Long Term Evolution
MIMO	Multiple-Input Multiple-Output
ML	Maximum likelihood
MMSE	Maximum Mean Square error
MMSE-DFE	MMSE- Decision Feedback Equalizers
MRRC	Maximal Ratio Receiver Combining
NVD	Non Vanishing Determinant
OFDM	Orthogonal Frequency Division Multiplexing
OSTBC	Orthogonal Space Time Block Codes
PAPR	Peak To Average Power Ratio
PEP	Pairwise Error Probability
PSK	Phase Shift Keying
QAM	Quadrature Amplitude Modulation
QoGST	Quasi orthogonal Group Space Time
QOSTBC	Quasi-orthogonal Space Time Block Codes

SD	Sphere Decoding
SISO	Single Input Single output
SM-OSTBC	Spatially Modulated- Orthogonal Space Time Block Codes
SNR	Signal to Noise Ratio
SVD	Singular Value Decomposition
STBC	Space Time Block codes
STC	Space Time Coding
STTC	Space Time Trellis Codes
TC	Turbo Codes
ZF	Zero Forcing
ZF-DFE	Zero–Forcing Decision Feedback Equalizers

NOTATIONS USED

Throughout the thesis, lower case letter a, boldface lower case letter **a,** boldface uppercase letter **A** represent scalar, vector in column form and a matrix respectively.$|\ |$, $\|\square\|$,$\det(\square)$ represent absolute value of a, Euclidean norm of **a,** and determinant of **A** respectively. The $(\cdot)^*$,$(\cdot)^{\square}$,$(\cdot)^{\square}$ represents the complex conjugate, trace and hermition respectively while $(\cdot)^{\square\square}$represents the inverse of the matrix. The bit error probability of an event $\square$ is denoted by $\square_{\square}(\square)$ while the pairwise error probability of a event is represented by $P(\square)$ and $\square[\cdot]$ represent the expectation and ***I*** represents the identity matrix. $\mathbb{R}, \mathbb{C}$ and $\mathbb{Z}$ represents the set of real numbers, complex numbers and Integers respectively. $\square\square\square(\cdot)$ is an operator used to form vector from the successive columns from the matrix. $\square$ represents the type of QAM modulation used and C represents the capacity. The $\sum$represents the submission while the $\|\cdot\|$ represents the Frobenius norm.

CHAPTER 1

INTRODUCTION

From the time, when the very first long-distance transatlantic radio communication was uncovered by Italian physicist Guglielmo Marconi, wireless technology has been changing at a striking rate since 113 years that is from transmission of single-bit Morse code to the 5th generation cellular system supporting the data rates of 1Gb/sec. The dominance of laptop, smart phones and tablet reveals that wireless technology has deep effect on our daily lives. In 2015, there are almost billion mobile-cellular users and 2 billion smartphone users in the world as provided by the International Telecommunication Unit-[ITU] report [1]. These numbers show the milestones achieved since 2000 (738 million users) when global leaders came together for a UN Summit to establish the UN Millennium Development Goals (MDGs) to make the world connected on a large scale. The first generation of mobile system using analog communication was commenced in the early 1980s. In the next decade, more sophisticated digital transmission technology was integrated in the second generation of cellular communication system. The commercial applications of 2G were limited to speech only with a theoretical data transfer speed of 9.6 kilobits per second (kb/s). With the supporting multimedia and other high bit-rate services in mind, the third generation system and its descendant, 4G Long Term Evolution (LTE) aimed at a peak data rate of several megabits per second (Mb/s) [2]. With the increasing number of users enjoying the freedom from being physically connected, the demand on bandwidth and spectral efficiency is growing at a striking rate.

1.1. Motivation

In order to meet the current demand of wireless market, information throughput required is of several orders of magnitude higher than the data rates, made available by state-of-the-art technology. On the other hand, the available Bandwidth is limited and a huge portion of bandwidth is dedicated to satellites, military applications, radio broadcasting and TV. Further, the government has also reserved a large block of spectrum. As a result, radio spectrum resource is very limited and hence, extremely expensive. A few MHz or even KHz of bandwidth can cost thousands of crores for a mobile telecommunication company to acquire.

The main motive of telecommunications is to reliably transmit information/data between distanced locations via wireless medium with an adequate quality and at a certain rate. Practically the wireless channel would severely damage the transmitted signals, causing the received power level to fluctuate rapidly. Such an undesirable interference caused to the channel is generally defined as channel fading. Thermal noise generated by the electronic circuits in receivers also affects the reliable signal detection and recovery [3]. Therefore, design of modern wireless communication system primarily has focus on improving the data/information rate within the limited spectrum and reducing the fading effects due to the time varying nature of the wireless channel and other real-world limitations and thus achieving the quality of wire-line communication. Modern wireless communication structure is under the constraint of two major resources, i.e. total transmitted power and spectral bandwidth available. Transmitting the information from the transmitter to the receiver at a certain rate requires certain spectral bandwidth and power, both of which are under strict control of government organizations, like Department of Telecommunications (DoT) in India. In different cases, one of the available resources is more precious than the other. Hence, the communication systems are therefore, classified as power-limited and bandwidth-limited systems. For bandwidth-limited systems, one should make the best use of bandwidth in the system, at the expense of power. Similarly, for the power limited systems, the primary focus is on the judiciously utilising the total power.

1.2. MIMO and Diversity schemes

The issue of maximizing the data rate has been addressed by Shannon’s work in the field of information theory [4]. He stated that the maximum possible transmission rate is bounded by the channel capacity at which information can be transmitted over the wireless channel with small probability of error, despite the presence of unwanted signals, called noise. Because of the limitations on the available Bandwidth of radio spectrum, data rate as well as spectral efficiency can be maximized primarily using the more efficient signalling techniques like Turbo codes and trellis codes. Using the error control codes, such as turbo codes (TC) [5] and low density parity check codes (LDPC) [6] perform very close to the Shannon capacity limit in systems with single-input single-output (SISO) systems. Capacity Gain can also be increased by increasing the number of antennas at either or both ends of the wireless communication link. Systems having multiple transmit and receive antennas are termed as multiple-input multiple-utput (MIMO) systems as shown in Fig. 1.1., where $h_{m,n}$represents the channel fading coefficients from n^{th} transmit antenna to m^{th} receive antenna.

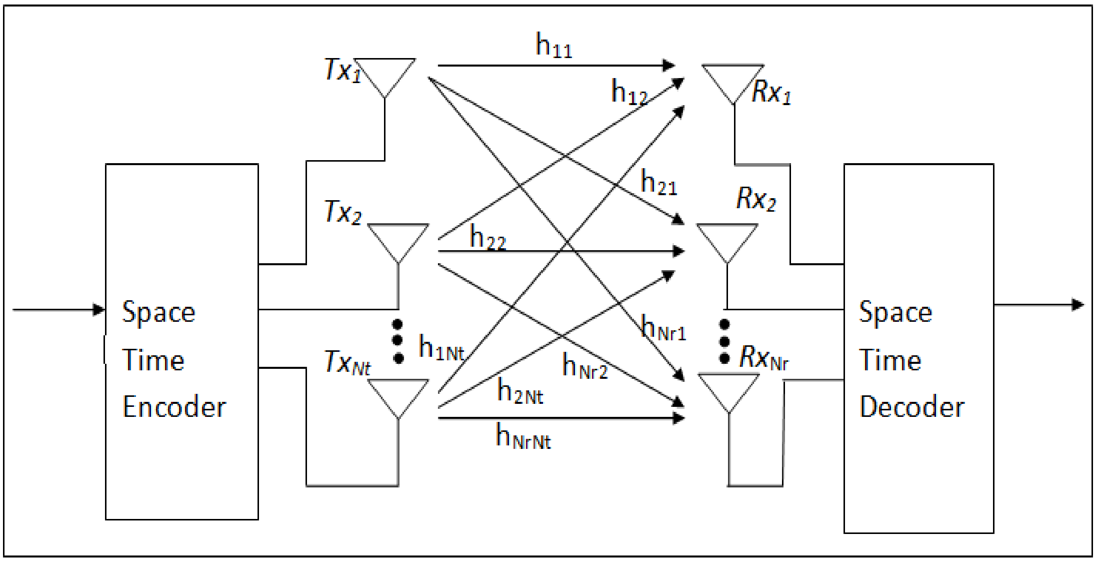

Fig. 1.1. Block diagram of Multiple Input Multiple Output (MIMO) system

Researchers have proven from the information-theoretical viewpoint that enormous capacity gains can be achieved for MIMO channels because of the diversity modulation obtained from the signal paths, corresponding to different antennas [7 - 10]. It has been pointed out that the capacity limit of MIMO channels scaled almost linearly with the number of antennas. The basic concept behind diversity is that the surrounding environment generally varies rapidly with time. The radio signals emitting from different transmit antennas propagate through several independent paths and hence face different environments. The combined received signals received at the receiver end, are more likely to have sufficient power strength to support reliable detection at the receiver. Hence it has been concluded that the diversity intends to mitigate the deleterious effects of fading. For single-user MIMO systems, maximum diversity order is limited by $N_T \times N_R$, where N_T is the number of transmit antennas and N_R is the number of receive antennas [11]. Mathematically, the relationship between the received SNR, γ and the probability of error P_b (e) is asymptotically formulated as $P_b(e) = \gamma^{-d}$. Graphically, 'diversity' or 'diversity order' can be observed from the slope of the error probability vs signal-to-noise ratio (SNR) curve. There are various types of diversity:

- **Temporal Diversity**: When the identical messages are transmitted in different time slots, then temporal diversity is achieved. Thus uncorrelated signal are received at the receiver at different time slots. The minimum time difference required between different time slots is the coherence time of the channel. This scheme is useful in the fast fading environments where the coherence time is small. Whereas in slow fading channels, coherence time is large and hence, lead to larger delays. Further, because of the redundancy introduced in the time domain, Time diversity resulted in bandwidth inefficiency.
- **Frequency Diversity**: In this technique, the same message is transmitted using different frequencies. The sole criterion behind the different frequencies is that the messages must deal with independent fading i.e. all the frequencies must be uncorrelated. That is, the frequency separation must be greater than the coherence bandwidth. Frequency diversity finds its use in the mobile communication where replicas of the message signal are sent to the receiver in the form of redundancy in frequency domain using the spread spectrum techniques such as Direct sequence spread spectrum, frequency hopping. This technique is useful only when the coherence bandwidth of the channel is smaller than the spreading bandwidth. Like the transmit diversity, this scheme also introduces loss in bandwidth efficiency as redundancy is introduced in the frequency domain.
- **Antenna/Spatial Diversity:** In this technique, multiple antennas or antenna arrays are employed at the transmitter and/or the receiver side. The different antennas are placed separately so that the individual signals are uncorrelated. The separation of a few wavelength $(3 - 10\lambda)$ is provided to obtain uncorrelation among the signals. The benefit of this technique is that it does not induce any loss in Bandwidth efficiency like that of Temporal and frequency diversity. This property is useful in high data rate wireless applications [12]. Depending on the position of multiple antennas, this scheme is of two types:

Receive Diversity: When multiple antennas are used at the receiver side, then it is termed as Receive diversity. The duplicate copies of the transmitted signals are combined to increase the overall SNR and mitigate the effects of fading. Receive diversity is extensively used at the base stations. A base station in the GSM systems generally contains two receiving antennas [13]. Such a configuration provides significant improvement in data rate and spectral efficiency for the wireless link from mobile station (end user) to base station.

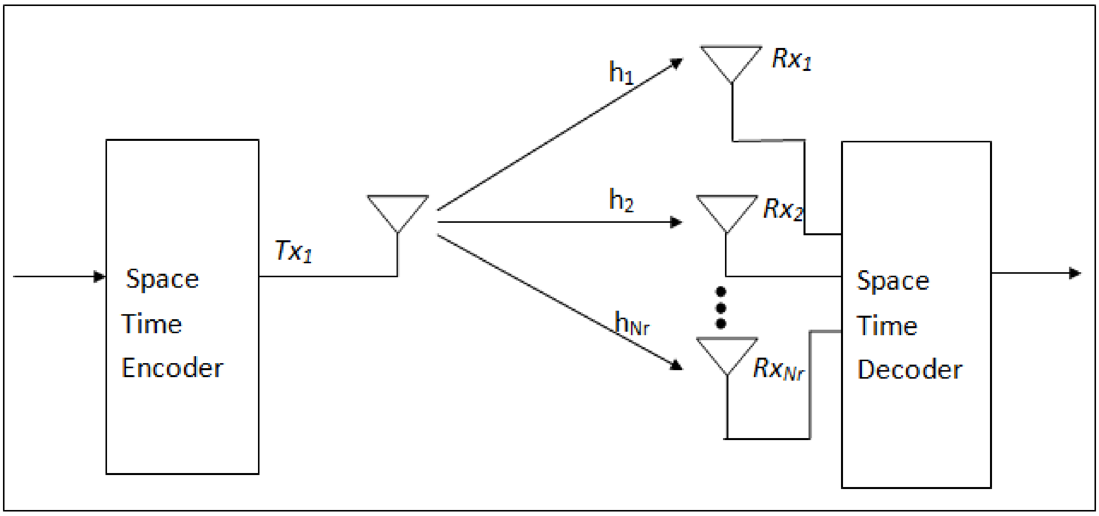

Fig. 1.2. Block Diagram of Receive Diversity

Transmit Diversity: When the multiple antennas are used at the transmitter side, it is termed as Transmit diversity. In practice, the strength of received signals might attenuate deeply during transmission because of time-variant fading from the wireless channels and interference from other users, which could lead to unreliable recovery of transmitted information at the receiver. Using Transmit diversity as shown in Fig. 1.3., the affects of attenuation are reduced. Transmit diversity is used for the downlink (from base station to end user) and it offers potential increase in performance.

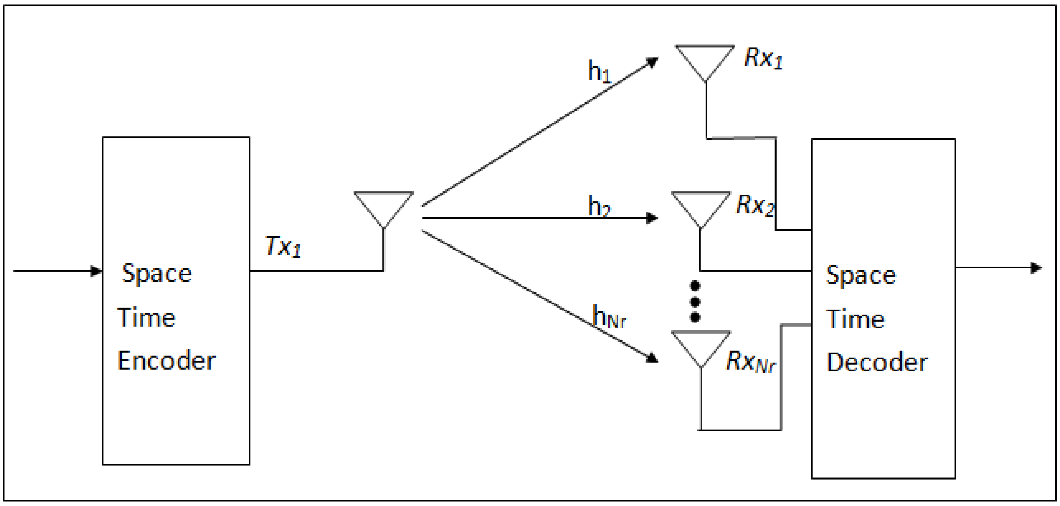

Fig. 1.3. Block Diagram of Transmit Diversity

1.3. Space Time Coding

During the early 90's, various delay transmit modulation schemes (Delay diversity) were proposed for operation in the base stations. By activating two or more transmit antennas interchangeably, same diversity gain can be achieved by Delay diversity as that acquired with receive diversity but the former gain was at a cost of less efficiency in terms of required SNR for a same bit error probability. Space-time coding (STC) is a real-world application of diversity technique, capable of providing substantial capacity gain promised by the MIMO channels. The family of STC consists of space-time block coding (STBC) [14–16] and layered space-time (LST) [17] coding, space-time trellis coding (STTC) [18] schemes. These distinct coding techniques are designed to achieve diversity gains and/or coding gains, as well as high spectral efficiency with the introduction of encoding operations across both space and time domains, thus utilizing the spatial and time diversity. The original scheme of STC was based on trellis codes until an easier version using block code was proposed in 1998 [19]. Both STTC and STBC focused on enhancing the error performance and mitigating the effects of channel-induced fading by transmitting replica of information sent, through different paths. Hence, provides the full diversity gain. On the other hand, LST is designed for full multiplexing gain and higher attainable transmission rate. It operates by sending independent information in parallel across different channels. However the requirement posed by the LST, that the number of receive antennas must be greater than transmit antennas, limited its use. Whereas, STBCs are primarily constructed from orthogonal designs, in which the data streams to be transmitted are encoded in blocks and are distributed among spaced antennas across time. Code orthogonality in STBCs facilitated a low-complexity decoding via linear processing at the receiver side but has have an adverse effect on the coding gain. On the other hand, STTC possessed both the diversity advantage as well as enormous coding gain, but they exhibit high complexity while decoding due to the Joint Maximum Likelihood sequence estimation. Hence, the STBCs are more likeable in practice due to implementation simplicity at the transmitter side and simple as well as optimal decoding at the receiver side. The Alamouti code [14] is chronologically the first STBC scheme that provides simultaneously full transmit diversity and full transmission rate. In other words, it exploited full diversity gain without sacrificing the transmission rate. Originally, it was proposed for two transmit antennas, later

Tarokh et al. [15] extended this concept of STBC for higher number of antennas, using theory of orthogonal designs. As a result, the Alamouti code is deemed as a special case of the orthogonal STBC (OSTBC) [16].Using the merits of both, the LST and the orthogonal codes, Linear Dispersion STBC were proposed by Hassibi *et.al* [20] The benefit of Linear Dispersion codes is that they can be designed for any number of transmit and receive antennas, simple decoding and satisfy the information theoretic optimality criteria. This technique involves breaking of data into sub-streams and then using the dispersion matrices to disperse the sub-streams into space and time.

1.4. Detection and Estimation Methods

Multiple antenna technologies play an essential role in the design of most of the successful wireless communication systems due to their potential to increase the spectral efficiency and the transmission data rates. Several wireless standards have incorporated MIMO communications to take the advantage of the diversity brought by multiple antennas and enhance their system performance. But the major challenge face in such systems is the designing of efficient and low-complexity receivers. Various strategies have been proposed for the decoding of the sent codewords at the receiver side. Maximum likelihood (ML) decoding, Sphere decoding [21, 22], and Successive Cancelling and Nulling [23, 24], Zero Forcing, MMSE linear receivers are discussed here.

1.4.1. Maximum Likelihood Detection

The Maximum Likelihood detection is based on finding the most likely transmitted block. Consider the linear MIMO diagram shown in Fig. 1.4. The sent information is corrupted by the non ideal Channel matrix characteristics and additive white Gaussian noise $\boldsymbol{W}$ as shown.

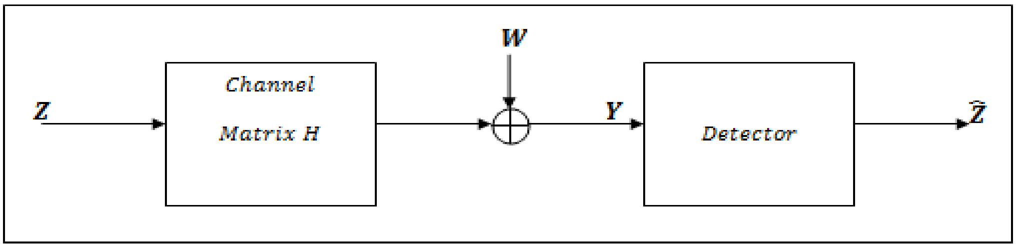

Fig. 1.4. A simplified MIMO communication system diagram showing transmitted symbol,$\boldsymbol{s} \in \mathbb{C}^{N_T}$, channel matrix $\boldsymbol{H} \in \mathbb{R}^{N_R\times N_T}$, additive white Gaussian noise, $\boldsymbol{W} \in \mathbb{R}^{N_R}$, received vector $\boldsymbol{Y} = \mathbb{R}^{N_R}$and the detected symbol vector, $\hat{\boldsymbol{s}} \in \mathfrak{R}^{N_T}$.

The ML decoder searches over all the possible symbol combinations and finds the symbol that solves the following minimization problem,

$$\hat{s} = \underset{s}{\arg\min}\{Tr(y - Hs)^H \cdot (y - Hs)\} \tag{1}$$

Expanding the cost function and considering $y^H \cdot y$ is independent of the transmitted codeword and the choice of the codeword.

$$\begin{aligned} \hat{s} &= \underset{s}{\arg\min}\{Tr[y^H \cdot y + H^H \cdot s^s \cdot s \cdot H - H^H \cdot s^s \cdot y - y^H \cdot s \cdot H]\} \\ &= \underset{s}{\arg\min}\{Tr[H^H \cdot s^s \cdot s \cdot H - 2\Re\,(H^H \cdot s^s \cdot y)]\} \end{aligned} \tag{2}$$

It has been proved as an accurate method and possesses very low error rate, but suffers from high computational complexity of searching over all possible codewords. It’s complexity is of the exponential order. This high order of complexity limits its use for higher configurations.

1.4.2. Sphere Decoding Detection

In order to decrease the complexity, the second algorithm known as Sphere Decoding has been proposed [21, 22]. It reduced the number of possible codewords to be evaluated. Hence, in Sphere decoding only those codewords which are located within a sphere of certain radius r, centred around the received signal, are considered for maximum likelihood detection as shown in Fig. 1.5.

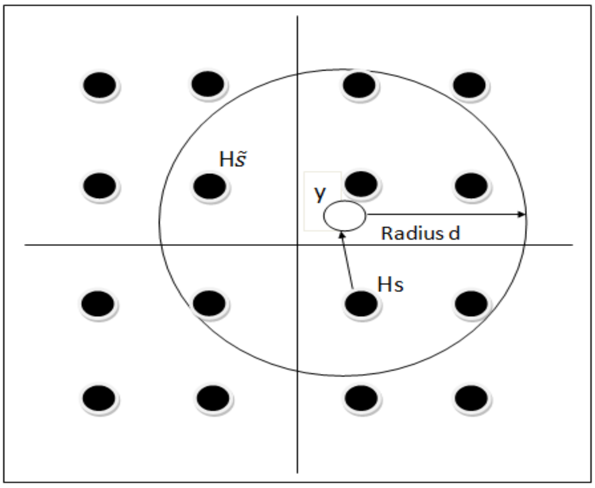

Fig. 1.5. Illustration of Sphere Decoding Technique. Here, s is transmitted and y is received signal. The points which lie in the sphere of radius d are evaluated and the node having the least distance is selected.

The remaining codewords are directly rejected. This technique exhibits polynomial order complexity. Hence it is more favourable than ML brute force decoding technique and hence, classified as Simplified ML decoding. The sphere decoding is further divided into two types, The ***Finke Pohst*** enumeration and the ***Schnorr Euchner***[22]. In the former case, the search radius is specified and the search for the node having minimum weight starts with depth first algorithm shown in Fig. 1.6.

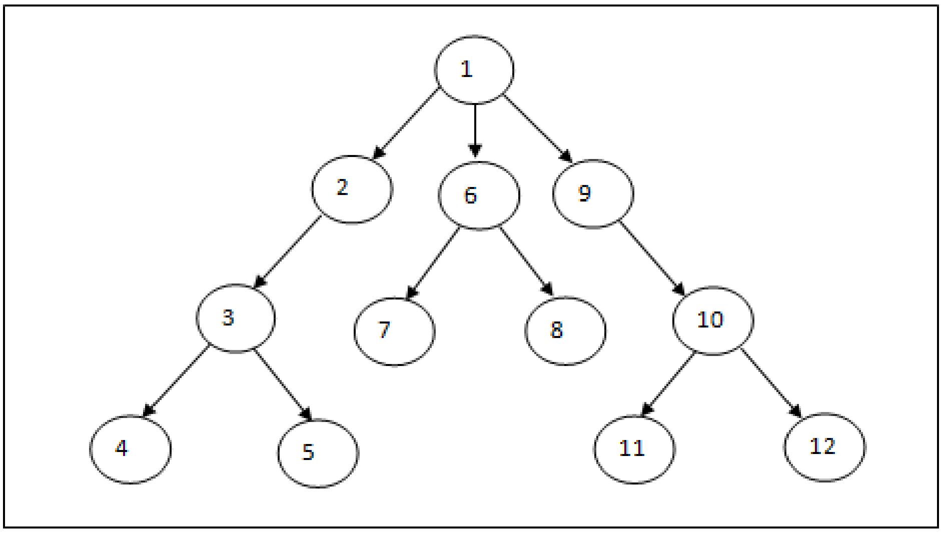

Fig. 1.6. Depth First Algorithm

At each stage, the node under consideration would be expanded if its weight has been found to be lesser than the squared of the radius defined. Otherwise, if the weight of the node under consideration is found to be larger than this distance threshold, then the current search path would be terminated because it cannot possibly lead to a closest lattice point. Upon path termination, the next node to be considered would be the next sibling of its parent. Following this, the search process would continue until all the nodes having weights less than defined radius has been found. The drawback of Fincke Pohst (FP) method was its sensitivity of performance to the choice of already specified radius. If the radius is too large, more number of nodes would be computed and hence the complexity would be high. However if the search radius is very small, no node could be detected and for the next time, decoder would start with a much larger radius. The Schnorr Euchner enumeration adds small modification to the FP method. Here, the radius is not already specified. Initially it is equal to infinity. Afterwards it is assigned the weight of the first detected node. Hence, after detecting the first node, the radius would be updated and the new nodes would be searched within this radius till a leaf node with a lower radius is not achieved. Here, the radius would update again. This adaptive way of setting the radius makes it better than FP decoding method.

1.4.3. Estimation using Zero Forcing

The Zero Forcing (ZF) receiver forces the channel matrix into an identity matrix using the pseudo-inverse of the channel matrix. This helps in reducing the inter-symbol interference to zero in a noise-free channel. Hence, it is beneficial when ISI is more prominent as compared to the noise. The ZF receiver is usually employed with MIMO systems.

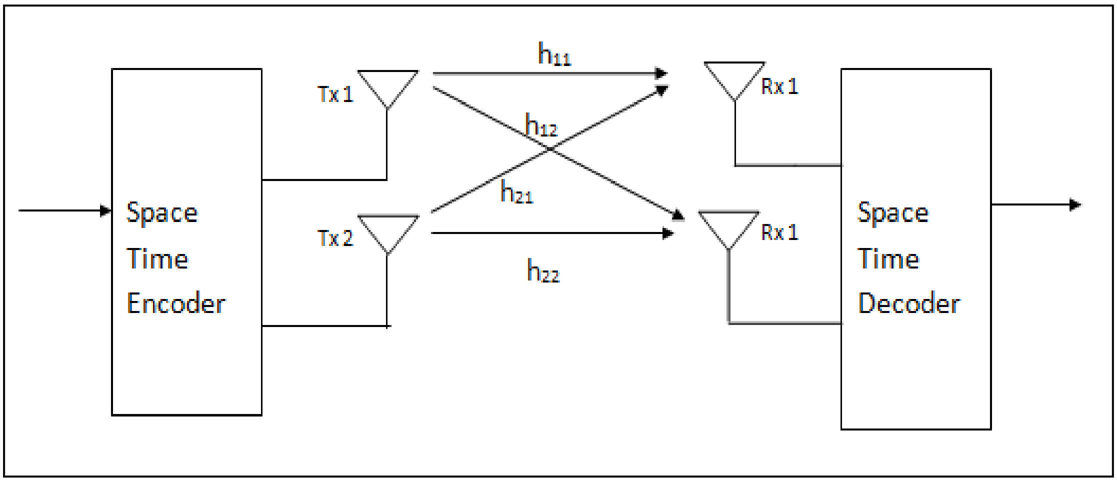

Fig. 1.7. Block Diagram of 2 × 2 MIMO channel scheme

For a 2 × 2 MIMO channel shown in Fig. 1.7., the received symbol on the first receiver is:

$$y_1 = h_{1,1}s_1 + h_{1,2}s_2 + w_1 = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + w_1 \qquad (3)$$

and the received symbol on the second receiver is:

$$y_2 = h_{2,1}s_1 + h_{2,2}s_2 + w_2 = \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + w_2 \qquad (4)$$

where y_1 and y_2 are the received symbols on first and second receiver antenna respectively, $h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna, $h_{2,1}$ is the channel from 1st transmit antenna to 2nd receive antenna, $h_{1,2}$ is the channel from 2nd transmit antenna to 1st receive antenna, $h_{2,2}$ is the channel from 2nd transmit antenna 2nd to receive antenna, s_1 and s_2 are the two transmitted symbols and w_1 and w_2 are the noise components on 1st and 2nd receive antennas.

Thus, $$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Equivalently, we can say that,

$$\boldsymbol{Y} = \boldsymbol{HS} + \boldsymbol{W} \tag{5}$$

To restore the sent symbols, an estimator needs to have a matrix $\overline{\boldsymbol{Y}}_{\boldsymbol{ZF}}$such that $\overline{Y}_{ZF}Y = I$. A pseudo-inverse of channel matrix **H,** $\overline{\boldsymbol{Y}}_{\boldsymbol{ZF}}$ has been proposed in the ZF technique as shown below,

$$\overline{\boldsymbol{Y}}_{\boldsymbol{ZF}} = (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-\boldsymbol{1}}\boldsymbol{H}^{\boldsymbol{T}} \tag{6}$$

The matrix $\overline{Y}_{ZF}$, is also termed as Pseudo-inverse of the channel matrix, H.
Solving the matrix multiplication for $\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H}$,

$$\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H} = \begin{bmatrix} {h_{1,1}}^* & {h_{2,1}}^* \\ {h_{1,2}}^* & {h_{2,2}}^* \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} = \begin{bmatrix} \left|h_{1,1}\right|^2 + \left|h_{1,2}\right|^2 & {h_{1,1}}^* h_{1,2} + {h_{2,1}}^* h_{2,2} \\ {h_{1,2}}^* h_{1,1} + {h_{2,2}}^* h_{2,1} & \left|h_{2,1}\right|^2 + \left|h_{2,2}\right|^2 \end{bmatrix}$$

These off-diagonal terms tries to nullify the interfering terms when performing the equalization. While doing so, amplification of noise occurs. Hence, in the presence of more prominent noise, Zero forcing estimator is not beneficial.

1.4.4. Estimation using MMSE receiver

The limitation of noise amplification of ZF receiver is however reduced with the help of MMSE receiver at low SNRs. The MMSE receiver tries to minimize the mean square error (MSE), which is a common measure of estimator quality. It does not completely remove the ISI, but tries to reduce the total power of noise and ISI at the receiver side.

To minimize the mean square error, MMSE estimator tries to find out the coefficient vector $\overline{\boldsymbol{Y}}_{\boldsymbol{MMSE}}$**,** so as to minimize the MMSE criterion for a known variable **Y,** and an unknown variable **Z**

$$\mathrm{E}\{[\overline{\boldsymbol{Y}}_{\boldsymbol{MMSE}}\boldsymbol{Y} - \boldsymbol{Z}][\overline{\boldsymbol{Y}}_{\boldsymbol{MMSE}}\boldsymbol{Y} - \boldsymbol{Z}]^{\boldsymbol{H}}\}$$

The solution to this criterion for MMSE follows,

$$\overline{\boldsymbol{Y}}_{\boldsymbol{MMSE}} = (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H} + \tfrac{1}{P}\boldsymbol{I}_{\mathbf{N_R}})^{-1}\boldsymbol{H}^{\boldsymbol{T}} \tag{7}$$

Hence, the estimated codeword will be,

$$\widetilde{\boldsymbol{Z}} = \overline{\boldsymbol{Y}}_{\boldsymbol{MMSE}} * \boldsymbol{Y} = (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H} + \frac{1}{P}\boldsymbol{I}_{\mathbf{N_R}})^{-1}\boldsymbol{H}^{\boldsymbol{T}} * \boldsymbol{Y}$$

From (9), It has been inferred that for a high SNR regime, the performance of MMSE receiver is comparable to that of Zero Forcing receiver.

1.4.5. Estimation using Successive Nulling and Cancellation

Successive Nulling and Cancellation is a non-linear detection scheme, employed usually with the BLAST receivers, for estimation of the transmitted symbols in a sequential order. It shows improvement in the performance with the increase in complexity. In this method, the received symbols are detected one by one, starting from the signal having the strongest SNR to the one having the weakest SNR. During the detection of each symbol, the effects of previously detected symbols are reduced from the received signals before the detection of the new symbol, known as cancelling and the effect of remaining undetected symbols would be considered as interference and is nullified. The Nulling stage can be realized using different equalization perceptions such as zero-forcing (ZF) or minimum mean square error (MMSE) equalization. However if the cancelling stage is ignored, then this method is termed as equalization. ZF and MMSE are the most commonly used estimators.

1.5. Thesis Objective

For High data rate communication, we need error-free reliable communication supporting high rates. Space time coding technique helps in achieving diversity gains along with the high spectral efficiency with the help of Spatial and time diversity. The decoding scheme chosen at the receiver side also affects the error performance. In this thesis, we have worked on designing and optimizing of Linear dispersive STBCs so as to achieve better error performance at higher rates. The different decoding schemes have been analysed by comparing their BER at different SNRs. The main objectives of the thesis work are:

- Designing and optimizing the Linear Dispersive Space time Block codes.
- Simulation of Linear Dispersive STBCs with Sphere Decoding technique.
- Simulation of Linear Dispersive STBCs with Integer Forcing technique.
- Comparison of error performance of Linear Dispersive STBCs with various decoding techniques, such as Sphere Decoding, Zero Forcing, MMSE, Integer forcing.

1.6. Organization of Thesis

This thesis included five chapters. An outline of each chapter is given below.

Chapter 1 gives us introduction to the wireless technology, Multiple Input Multiple output systems, Space Time Coding and various detection techniques such as ML decoding, Sphere Decoding, Successive Nulling and Cancellation, Zero forcing and MMSE.

Chapter 2 dealt with Literature survey. The research work relevant to the thesis work has been discussed here in detail.

Chapter 3 presents the detailed study of Linear Dispersion Block codes with various designing procedures, Non Vanishing Determinant property and the Integer Forcing (IF) receivers. The designing and performance of the IF receivers have been studied mathematically analysed with the Rank and determinant criteria.

Chapter 4 included the proposed design for the designing and optimization of the Frame based Linear Dispersion STBCs in conjunction with Sphere Decoder and Integer forcing receivers.

Chapter 5 included the results, supported by the Monte Carlo simulations. In this chapter an analysis of error performance of Optimized Linear Dispersion Codes along with the Sphere Decoder has been done. The SNR vs BER performance of Integer forcing decoder is compared and analysed with the Zero forcing and MMSE receivers.

Chapter 6 concluded the thesis by summarising the results and suggesting future work.

The end section lists the important references that have been referred throughout the thesis and without which the thesis work could not have been accomplished.

CHAPTER 2

LITERATURE SURVEY

A SIMPLE TRANSMIT DIVERSITY TECHINIQUE FOR WIRELESS COMMUNICATION [14]

Siavash M. Alamouti presented a new scheme which utilised the transmit diversity. This scheme design was particularly for two transmit antennas and one receive antenna. It was required that there should be simultaneous transmission of symbols at each interval of time. In particular, there would be two signals transmission from the two antennas at each instant of time. During the first interval of time, the signal transmitted from first antenna was denoted by s_1 and from second antenna was denoted by s_2. During the next symbol period signal $(-s_2{}^*)$ was transmitted from first antenna and $s_1{}^*$ signal was transmitted from second antenna. These signals are mixed at the receiver side and are detected by the maximum likelihood detector. No feedback is needed from the receiver to the transmitter in this scheme. Hence that system is said to have CSI only at the receiver. It was observed that, using this configuration of two transmit antennas and one receive antenna, the diversity order of the scheme is equal to that of MRRC with one transmit and two receive antennas was achieved. For the same value of total radiated power, this scheme has disadvantage of 3-dB due to the simultaneous transmission of two distinct symbols from two transmitters. Otherwise, its performance is similar to MRRC for the case when total power has been doubled.

SPACE TIME CODES FOR HIGH DATA RATE WIRELESS COMMUNICATION: PERFORMANCE CRITERIA AND CODE CONSTRUCTION (1999) [16]

Vahid Tarokh *et al.* proposed a performance criterion for the block codes and the trellis codes known as Rank and Determinant criteria. With the help of this criteria, they stated that we can achieve the best trade off between data rate, trellis complexity and diversity. Performance of the codes designed was evaluated from the rank and determinant of the matrices of the distinct pairs of code sequences. It is designed assuming that the fading at each link between transmitter and receiver is Rayleigh or rician. For quasi static Rayleigh fading channel, the diversity gain is deliberated from the rank of the certain matrices and coding gain from the determinant .For fast fading channel, diversity gain is deliberated from the generalised Hamming distance of certain matrices and coding gain from the generalised Product distance. Rank criteria stated that the diversity advantage of m*n is achieved if the codeword difference matrix B is full rank .whereas the Determinant criteria states that the minimum of the determinant of A where A is $B^H * B$, taken over all combinations must be capitalized on. In Fast fading channel, the Distance criteria states that the difference between any two codewords for a same transmitter must be non zero for transmitter number varying from 1 to m, while the Product criteria stated that the minimum of the product of A must be capitalized.

HIGH-RATE CODES THAT ARE LINEAR IN SPACE AND TIME (2002) [20]

Babak Hassibi *et al.* proposed codes that surpass both the V-BLAST and orthogonal codes and called them Linear Dispersive Space Time Block Codes .It was stated that VBLAST has a big disadvantage that it can't work with fewer receive antenna with reference to transmit antennas and contains no built-in technique for the prevention against deep fades. Also, the orthogonal codes have poor performance with large number of antennas at elevated data rates. So they offered a transmission scheme in which the codes broke the data-streams into sub-streams of data that are dispersed in time and space. These codes are designed on the criteria of maximizing the mutual information and can be utilized for any configuration of transmit and receive antennas. These codes can be easily encoded and decoded. They argued that the mutual information criterion is the requirement to attain capacity .the capacity achieved herein is advances to the Shannon capacity. They also revealed that information-theoretic optimality has a theoretical association with low pair-wise error probability but if diversity criterion alone is used to design space–time codes at high spectral efficiencies, it may not lead to code designs with higher capacity.

A FASTEST DECODABLE CODE STRUCTURE FOR LINEAR DISPERSION CODES (2009)[21]

X. G. Dai *et al.* proposed a new design for fast-decodable, flexible-rate, full-rank linear dispersion codes (LDCs) with arbitrary numbers of transmitters and receivers. The code words formed by the LDCs encoder represented a linear combination of certain basis matrices. It was also stated that the LDCs would consists of as many orthogonal rows as possible in a dispersion matrices. The decoding is done using the

Simplified Sphere Decoding mechanism. It was observed that with this proposed code, the number of levels in the tree search has been substantially reduced. The reduction in the number of levels is due to the modulation scheme used and the total number of transmitted symbols. Hence the complexity of the sphere decoder (SD) at-the receiver is low down. Monte Carlo computer simulation performed on this architecture has revealed that the LDCs with and without the orthogonal structure have al-most identical bit-error-rate (BER) performances. However, the complexity at the decoding side is substantially reduced using the proposed family of LDCs with Orthogonal row structure and it is more significant for the LDCs with high level of modulation.

A FULL RATE FULL DIVERSITY SPACE TIME BLOCK CODE WITH NON VANISHING DETERMINANTS AND SIMPLIFIED MAXIMUM LIKELIHOOD DECODING (2008) [22]

Javier M. Paredes *et al.* proposed a new 2*2 full-diversity, full-rate linear dispersion space-time block code (STBC) design by augmenting the generator of the lattice of the Alamouti's orthogonal STBC and then optimizing it, so as to satisfy the criterion of the maximal worst codeword difference determinant. The proposed STBC has been proved to satisfy the Non- Vanishing Determinant Property (NVD) and hence achieved the optimal diversity-multiplexing gain (DMG) trade-off. A simplified maximum likelihood decoding is also proposed for decoding which offered reduced computational complexity as compared to other decoding schemes. The proposed scheme offered as simplified way to detect the symbols as compared to the standard Maximum Likelihood and Sphere decoders. Thus, the decoding complexity of the proposed design of STBC lied in between that of the standard sphere decoder and the symbol-by-symbol decoder. This eventually reduces the cost of overall decoding procedure. Monte Carlo simulations were observed for the error performance of the designed architecture with that of Golden codes, algebraic codes, full rate-full diversity codes. Its performance has been shown to be comparable to that of the best full-rate STBCs known so far.

INTEGER-FORCING LINEAR RECEIVERS (2014) [25]

Jiening Khan *et.al*. proposed a new receiver architecture scheme for multiple input multiple output channels that acts as a link between optimal Joint ML decoder and conventional linear receivers. The receiver is designed on the criterion of maximum achievable rate. The decoder converts the received data streams into integer combination of independent data streams sent by the different transmitters. Inspired from the compute and Forward mechanism, the decoder at the receiver end, would easily recover the linear combinations of data streams. Then those streams of data are solved for the original data streams. The designing technique for the Integer forcing coefficient matrix has been proposed on the criteria of maximizing the achievable rate. It has been shown that the added complexity in Integer forcing receiver only lies in the search for the Integer forcing coefficient matrix, because the rest of the procedure is similar to Zero forcing Receiver. If this matrix is properly chosen then it can operate quite close to the MIMO outage capacity. The performance of the proposed architecture has been compared with that of conventional receivers. It has been shown that Integer forcing techniques outperforms the V- Blast, Zero Forcing, MMSE receivers and approaches joint ML decoding at High SNRs. This arrangement also provides optimal diversity multiplexing trade-off for the Integer forcing architecture which was sub-optimal in case of Zero forcing and MMSE receiver.

INTEGER-FORCING MIMO LINEAR RECEIVERS BASED ON LATTICE REDUCTION (2013) [26]

Amin Saksad *et al.* proposed a method based on Hermite Korkine Zolotareff and Minkowski lattice based reduction algorithm to design the Integer forcing coefficient matrix for the IF receiver. The criteria behind the designing of integer forcing coefficient matrix was uni-modular and inverted matrix along with the full rank property. The advantages of IF linear receiver over the lattice reduction-aided MIMO detector have been assessed. The designed receiver has achieved receive diversity, a lower bound on Ergodic Rate and outperformed lattice reduction-aided detectors in error performance. The complexity of the designing method has been very less when compared to the exhaustive research method. Lenstra-Lenstra-Lovasz (LLL) lattice reduction algorithm (CLLL) is also investigated to solve the problem. But it has been bound that the CLLL algorithm do not provide full diversity irrespective of the fact that it provided lower complexity as compared to Hermite Korkine Zolotareff and Minkowski lattice based reduction. Comparison of the coded- block error rate and bit error rate of the proposed approach for the 2 × 2 and 4 × 4 MIMO channels with that of other linear receivers has been performed. Simulation results demonstrated that the proposed approach outperformed the lattice reduction-aided MIMO detectors, minimum mean square error (MMSE) receiver and the zero-forcing (ZF) receiver.

LINEAR DISPERSION CODES FOR MIMO SYSTEMS BASED ON FRAME THEORY (2002) [27]

Robert W. Heath *et al.* proposed codes based on frame theory for MIMO Rayleigh fading channels. Their design criterion was based on generating random orthogonal matrices and then applying different criteria to select the best code. The different criteria used were tight frame criteria, standard power criteria, average power criterion. Then Rank and Determinant criteria is used to select the matrix having the maximum diversity and coding gain. Upon generating the random orthogonal matrices either by QR decomposition method or House holder decomposition method, rearrangement of the basis matrices was done to perform the tight frame relationship check. It was observed that the tight frame structure provides us the capacity optimal codes .Then basis matrices are checked for the standard power criteria and average power criterion. It was stated that there were still a large number of frame based codes satisfying theses constraints. That is, they had similar Ergodic capacity and hence further optimization of the codes is done on the basis of error performance constraint. Rank and determinant criteria was used to ensure good error rate performance. First, the rank of the difference matrices is checked and based upon that, the matrices having the maximum ranks are selected for the diversity check. The matrix having the largest value of the determinant is the selected code. This is the method used for the designing of codes .They have also given an algorithm to realize this optimization.

DIVERSITY MULTIPLEXING TRADE-OFF PERFORMANCE OF LINEAR DISPERSIVE CODES (2008) [28]

H. Yang *et.al.* studied the DMG trade-off function and examined it for Linear dispersion codes as well as some other codes such as VBLAST, QoGST, GLST, LSTBC over both block and fast fading channels. They unearthed that the VBLAST scheme have high multiplexing gain but low diversity gain. GoSTC, GLST, LSTBC have better trade-off as compared to VBLAST because those schemes utilized group transmission and reception. But their multiplexing gain is lower because their transmission rate is low. Hence, they stated that the LD codes have the highest diversity multiplexing trade-off performance among the above mentioned codes in case of block fading as well as fast fading channel. They further stated that there is also an increase in delay and decoding complexity because of T.

LINEAR DISPERSION CODES FOR ASYNCHRONOUS COOPERATIVE MIMO SYSTEMS (2011) [29]

Nan Wu *et al.* offered a joint optimization scheme which is based on jointly optimizing the capacity and symbol error probability. The method consists of generation of random codes each time and checking for their minimum determinant and capacity (only if the calculated value of determinant is greater than the previous one). The sent codewords are detected with the help of least square method. The coherent detection method is used for decoding. The codes are also designed to transmit low data rate information. Simulations revealed that the code performs better than the codes designed earlier. They have improved diversity gain, low outage probability, easy design and low functioning complexity.

STOCHASTIC GRADIENT ALGORITHMS FOR DESIGN OF MINIMUM ERROR-RATE LINEAR DISPERSION CODES IN MIMO WIRELESS SYSTEMS (2006) [30]

Xiaodong Wang *et al*. proposed a design method for the Linear dispersion STBCs. They stated that the traditionally designed LD codes were created using the maximization of average mutual information but it does not guarantee good error performance. Hence, they offered a design scheme for Linear Dispersion codes that tend to minimize the block error rate (BLER) in MIMO channels considering arbitrary fading channels and different detectors. As the linear dispersion codes considered here have been derived from the Frame Theory. In this paper, it has been stated that their error rate does not acknowledge an explicit form. Therefore, a deterministic optimization method could not be used in the designing of the minimum-error-rate LD codes. Hence, they proposed a simulation-based optimization methodology for the design using stochastic approximation along with simulation-based gradient estimation. The gradient estimation has been done using the score function method. The proposed method could be used to design the minimum-error-rate LD codes for a number of detectors including the maximum-likelihood (ML) detector and several suboptimal detectors. It can be used to design optimal codes for arbitrary fading channels. It accumulates the knowledge of spatial fading correlation at the transmitter and the receiver side. Simulation results proved that codes generated by the proposed new design scheme outperformed the codes designed based on algebraic number theory.

SPACE-TIME LINEAR DISPERSION CODES BASED ON OPTIMAL ALGORITHMS (2009) [31]

Yan Liang *et al.* proposed a designing method for the construction of Linear Dispersion STBC. They used two more design constraints in addition to the maximizing the mutual information. They utilized the Pairwise error probability and Block error rate. They optimized the code to improve the channel capacity and minimizing the BLER. The proposed scheme can be used as a common scheme to optimize the code. Simulation results showed that there is increase in the achievable capacity. Their method was to iteratively update the matrices exploiting the gradient computation method. They further analysed that combined method (maximizing the mutual information and minimizing the BLER) is better than using the mutual information or average BLER alone.

ON THE OPTIMUM DESIGN OF SPACE-TIME LINEAR DISPERSION CODES (2005) [32]

Jibing Wang *et al.* proposed a designing method for the Linear dispersive STBC minimizing the union bound based on the exact value of PEP while considering the transmit and receive correlation. In the beginning, they have discovered that the union bound on PEP converges to block error probability at medium to high SNR. Then they utilized the gradient descent method to fine out the optimized code. In the later part, they considered that the system consisted of the spatial fading correlation. Then they derived the exact PEP for the Space time code for the spatially correlated channels. They also found out that receive correlation always degraded the BER performance. While the transmit correlation, can either improve or degrade the performance. Numerical optimization showed that the designed codes performed well as compared to already designed codes on frame theory or algebraic theory.

LDC CONSTRUCTION WITH A DEFINED STRUCTURE (2003) [33]

A.Agustín *et al*. proposed a new construction for Linear dispersive code, called Quasi Orthogonal Design. In this paper, they provided a 2 step approach to generate matrices that can achieve a high data rate and orthogonal properties. At first, they made a selection of K orthogonal matrices and then, they employed scrambling matrices in the generation of Linear Dispersion codes. It has been shown that these codes attain the upper bound on diversity with a low value of error probability. We can generate any configuration of matrices with the help of these codes. The designing criterion has 2 degrees of freedom, firstly at the selection of basis matrices and secondly, the complex period of scrambling matrices. With the use of MMSE receiver, the results are shown to be better than the Bell codes and the codes designed by Hassibi.

DESIGN OF LINEAR DISPERSION CODES FOR PRACTICAL MIMO-OFDM SYSTEMS (2007) [34]

Xiaodong Wang *et.al.* have considered the earlier designed dispersive codes and proposed a designing method for the Linear dispersive codes applied to the wide band MIMO systems. They go for a 2 step procedure, i.e., minimizing the average un-coded(without outer codes) block error rate and maximizing the ergodic mutual information. A stochastic gradient descent algorithm considering either maximum likelihood or linear zero-forcing decoding can be used for the numerical computation. The results are shown to be adjustable to any number of OFDM symbol intervals, any number of transmit/receive antennas and for any statistical fading channel model. Different code design examples are presented in practical next-generation communication systems for application purposes.

AUTOMATIC DESIGN OF ORTHOGONAL OR NEAR- ORTHOGONAL LINEAR DISPERSIVE SPACE TIME BLOCK CODES (2009) [35]

George Jongren presented a procedure to design the weighting matrices of the LD codes. The upper bound on the union bound of the codeword error probability is taken as performance gauger. The deigning problem corresponded to minimize the performance gauger while satisfying a standard constraint on the average output power. A criterion function in closed form was obtained by means of upper bounding the true codeword error probability with the use of the union bound principle, along with a well-known upper bound on the average pairwise error probability. A simple Gradient Search technique was used to solve the design problem numerically and a large number of codes were constructed. The weighting matrices were updated at every iterations, first taking a leap in the gradient side and then normalizing it to satisfy the power constraint. The designed codes have been studied and they turned out to be either OSTB codes, or so-called near-orthogonal codes. For transmit antenna-array sizes and code rates, it has been proved in this paper that OSTBC are optimal among all the LD codes. The near-orthogonal property of the designed codes played an important role in the making of low-complexity linear MMSE decoding, significantly better than earlier case. Simulation results have demonstrated that the designed codes performed excellent, both in terms of codeword error probability and information theoretic capacity of a definite effective channel.

SPATIALLY MODULATED ORTHOGONAL SPACE TIME BLOCK CODES WITH NON-VANISHING DETERMINANTS (2014) [36]

Marco Di Renzo *et al.* proposed a multiple-input multiple output transmission scheme for Spatially Modulated Orthogonal Space-Time Block Codes (SM–OSTBC). In this scheme, transmitted codeword matrices were generated by multiplying Spatial Constellation codeword matrices with codewords constructed from Orthogonal Space-time Block Codes (O–STBCs). The Spatially Constellated codeword matrices provides a way of carrying information bits along with the O–STBC codeword and thus, second-order transmit diversity is achieved by the SM–OSTBC scheme when they satisfy the non-vanishing determinant property. They have presented a systematic method to design the Spatial constellated codewords for an even number of transmit antennas having number greater than 3. A single-stream maximum likelihood (ML) decoder, having a low computational complexity requirement and a sphere decoder with further diminished signal processing complexity were developed using the structure of the SM–OSTBC codewords and the orthogonality of the O–STBCs. The bit error rate (BER) performance of the projected scheme is evaluated using both computer simulations and theoretical union bound analysis. Simulation results have confirmed that the proposed SM–OSTBC schemes have outperformed many existing MIMO arrangements, including the Alamouti STBC, Sezginer STBC, Spatially Modulated STBCs, STBC–Spatially modulated and V– BLAST schemes.

OPTIMIZATION OF FAST-DECODABLE FULL-RATE STBC WITH NON-VANISHING DETERMINANTS (2011) [37]

Tian Peng Ren *et al*. stated that Full-rate space-time block codes with non-vanishing determinants achieve optimal diversity-multiplexing trade-off value but they also incur elevated decoding complexity. For fast decoding, They proposed a STBC structure having special QR decomposition feature. A simplified form of fast-decodable code structure with fast ML decoding capability has been adopted and optimized of the code is done analytically. It has been shown that the constellation topology of the symbols such as QAM, APSK, or PSK, made a critical impact on the presence of non-vanishing determinants for the full-rate STBC. They demonstrated that, for APSK-STBC scheme to achieve non-vanishing determinant value, the topology need to be an APSK constellation topology with constellation points lying on square grid. It can not be a conventional topology. They presented a methodology to analytically optimize the full-rate STBCs with maximum value of vanishing determinants to achieve maximum coding gain at specific constellation dimensions. Simulation on the BER , coding gain and code PAPR (peak to average power ratio) have shown that the proposed APSK topology leads to lower PAPR than QAM, and provided better BER at high SNR.

A LOW COMPLEXITY DECODER FOR QUASI-ORTHOGONAL SPACE TIME BLOCK CODES (2011) [38]

Samer J. Alabed *et al.* proposed a low-complexity, suboptimal decoder for coherent and non-coherent, quasi-orthogonal space time block codes (QOSTBC) with three and four transmit antennas. Using Mapping techniques along CSI known at the transmitter, Decoder has been proposed for coherent and non coherent QSTBCs. They stated that the designed decoder enjoys near to linear complexity and roughly the same performance as the optimal maximum-likelihood (ML) decoder. Performance and complexity of the proposed decoder is evaluated with several schemes such as such as MMSE detector, Full search decoder. There has been improvement in the BER vs SNR curve at High SNRs. The average number of operations per symbol vs SNR curve for the coherent QO-STBC using 16-QAM modulation and different decoders has been evaluated and Comparison have shown that the proposed QOSTBC decoder have a substantially reduced complexity and Hence, an improved performance to-complexity trade-off has been achieved.

IMPROVED PERFECT SPACE-TIME BLOCK CODES (2013) [39]

K. Pavan Srinath *et al.* proposed improved Perfect space-time block codes (STBCs) from the known Perfect space-time block codes. Earlier the design was based on given four criteria i.e. the full-rateness, non-vanishing determinant property, uniform average transmitted energy per antenna per time slot and constellation cubic shaping for the Perfect space-time block codes. Transmission at uniform average energy per antenna per time slot was an important perspective for energy efficiency of STBCs while the Constellation Cubic Shaping criterion demanded that the generator matrix of the lattice from which each layer of the perfect STBC is sliced must be unitary. In this paper, it has been shown that unitariness is not a necessity for energy efficiency for space-time coding with finite input constellations. Hence, an alternative criterion is provided by them, that helped to obtain full-rate STBCs with higher normalized non vanishing minimum determinants than perfect STBCs. Using this, two STBCs, one for 4 transmit antennas and second for 6 transmit antennas have been presented and they are verified for larger normalized minimum determinants than the comparable perfect STBCs which till that date, had the best-known normalized minimum determinants. The proposed shaping criterion could be used to check if better STBCs, in terms of coding gain, can be attained for arbitrary number of transmit antennas.

INTEGER-FORCING LINEAR RECEIVER DESIGN OVER MIMO CHANNELS (2012) [40]

Lili Wei and Wen Chen presented an algorithm to design Integer forcing coefficient matrix according to the channel conditions. The criterion is to make the IF coefficient matrix full rank and maximizing the total achievable rate. At first, a viable searching set is generated with the help of Fincke-Pohst method, instead of generating the whole integer search space Then a group of integer vectors are picked within the searching set to construct the full rank IF coefficient matrix. The performance of the proposed receiver architecture is then compared with that of standard linear detection methods known as Zero Forcing and MMSE receivers. Average rate has been evaluated for different schemes. It has been found that the average rate of proposed Integer Forcing Receiver design is greater than the Zero forcing and MMSE receiver and closer to the channel capacity.

COMBINING SPACE-TIME BLOCK MODULATION WITH INTEGER FORCING RECEIVERS (2012) [41]

Uri Erez *et al.* proposed an Integer Forcing receiver structure in combination with linear transmit diversity methods. It was shown that for quasi static flat independent Rayleigh fading channel using linear pre- and post-processing and standard scalar coding and decoding techniques, this combination approaches closely the outage capacity of the channel. Open loop mode is considered where the transmitter does not have any information about the channel. The outage probability of different schemes like IF-Golden and IF-Golden-SIC have been analysed. It has been found that the basic IF code when used with independent schemes outperforms both , IF-Golden and IF- Golden-SIC. The performance has been found close to the ML decoding technique in case of outage probability.

PERFORMANCE OF PRECODED INTEGER-FORCING FOR CLOSED-LOOP MIMO MULTICAST (2014) [42]

Elad Domanovitz *et al.* considered the problem of multicasting over multiple-input multiple output channels to a reserved number of users, and with space-only linear pre-coding. They presented a practical transceiver model that approached the MIMO multicast system capacity for a reasonable number of users using linear space optimized pre-coding in combination with the Integer-Forcing Receiver architecture with Successive Interference cancellation, known as IF-SIC equalization. The Channel is assumed to be closed loop i.e. the CSI is available to the transmitter, thus allowing it to optimize the pre-coding matrix so as to achieve the maximize transmission rate. Numerical optimization is done to find an optimal pre-coding matrix for the case of two or three users. It has been observed that in the case of two users, Pre-coded IF-SIC architecture suffered negligible loss with respect to Capacity. While for three-user arrangement, the gap is more prominent at small values of SNR. Nevertheless, for an outage probability of 0.001 or 0.1%, the maximum loss in the SNR regime is less than 10% of capacity. Hence, in both the cases, pre-coded IF-SIC significantly outperformed the other referred schemes such as, Golden-IF-SIC and Alamouti. It was also experimentally found that the loss with respect to the multicast capacity increases with the increase in the number of users. However, it the 0.1% outage efficiency of multicast system is much higher than the performance achievable via open-loop pre-coding for large number of users .Hence, IF-SIC, employing a one dimensional modulo operation, outperforms the performance of the open loop pre-coding.

PRECODED INTEGER-FORCING UNIVERSALLY ACHIEVES MIMO CAPACITY TO WITHIN A CONSTANT GAP (2015)[43]

Or Ordentlich et al. proposed a method of pre-coding the data streams using the Linear dispersion encoder. An integer-forcing linear receiver has been used at the receiver side. After this, the SISO decoding is done to recover the original data streams. The analytical results for the integer-forcing linear receivers have been developed to support its behaviour which were earlier restricted to transmitting independent streams of data from different transmitters. They have derived lower bounds on the effective SNR seen by the IF receiver in terms of the minimum distance between the symbols transmitted using QAM modulation. The analytical model provided that the performance of the IF receiver only at the high SNR regime. Hence, they proved that the pre-coded IF achieved rates within a constant gap from the compound MIMO capacity. The capacity only depends upon the number of transmit antennas present and independent on the number of receive antennas. Hence the scheme is insensitive to degrees-of-freedom mismatch. They have further stated that the scheme that achieved a constant gap from capacity would be DMT optimal under any fading characteristics.

LINEAR RECEIVERS DESIGN WITH SLOWEST DESCENT METHOD(2015)[44]

In this paper, Lili Wei, Et al. considered the problem of IF linear receiver's design with respect to the channel conditions. Practical and efficient algorithms have been proposed to design the integer-forcing full rank coefficient matrix, based on the Slowest Descent Method such that the total achievable rate is maximized. This design is having reduced complexity as it does not follow the exhaustive research. Initially searching set is generated with integer vector search near the continuous- valued lines of least metric which is increased from the continuous minimize/maximize in the Euclidean vector space. Then integer vectors are selected within searching set to construct the full rank IF coefficient matrix using the Greedy Search algorithm. The performance of IF matrix is first evaluated with the number of slowest ascent lines that are chosen for generating candidate search set. It was observed that when the number of slowest ascent lines are too less, then there may be some chance that the full rank IF coefficient matrix may not form .Hence the successful construction probability will increase with that number of lines. However, at some point, the successful construction probability becomes one and after that it does not changes with the increase in the number of lines. The result of the proposed methods also approaches the output of exhaustive search method.

INTEGER-FORCING ARCHITECTURES FOR MIMO: DISTRIBUTED IMPLEMENTATION AND SIC (2013)[45]

Jiening Zhan et al proposed a distributed architecture for the Integer-Forcing receiver where the receiver does not perform at front-end linear projection. Thus, this model represents a system where processing of outputs is performed at distributed parts and their processed signals are collected by a central unit. A number of decoders are used.They attempt to recover two linear equations of the messages. The recovered equations of sent messages are collected by the central unit and an inverse operation is performed to solve for the original equations as in the non distributed architecture. This reduces the signal processing complexity at the decoder side and allows for distribution in the MIMO system. For simplicity, a symmetric MIMO channel is considered in this paper, where M = N. They stated that although distributedness comes at a penalty in performance, the Integer-Forcing architecture still achieved both rate and diversity gains over conventional linear receivers. From the outage performance, it has been observed that the Integer-Forcing linear receiver achieves closer to Optimal performance at high SNR. Although the enforcement of a distributed architecture of the Integer-Forcing linear receiver results in a small rate penalty, the slope of the receiver with distributed decoders, decays faster than traditional linear architectures percentage.

ON COMPLEX LLL ALGORITHM FOR INTEGER FORCING LINEAR RECEIVERS (2013)[46]

Sakzad et al. proposed a low-complexity method for choosing the Integer Forcing coefficient matrix. It was a combination of three low-complexity methods .The first two were based on CLLL algorithm and the last one was based on SVD decomposition. As all the three algorithms had low computational complexity, the combining algorithm is said to have lower computational complexity than the exhaustive search. They proposed a selective combining technique to choose the rows of A from the rows of candidate search set generated. Performance evaluation of the proposed low-complexity solution, with other known linear receivers for the 2 x 2 MIMO channel, is done in terms of ergodic rate and un-coded probability of error . It was observed that combined CLLL-SVD solution performs almost the same as IF receiver with exhaustive search and IF receiver based on Minkowski lattice reduction at low and moderate SNRs. This architecture also helped in achieving a lower bound on the ergodic rate of the IF receiver. Thus, it outperformed the ZF and MMSE receivers in terms of probability of error, and traded off error performance for computational complexity in comparison with the lattice reduction methods including HKZ and Minkowski method and the exhaustive search method.

DESIGN OF LINEAR DISPERSION CODES: ASYMPTOTIC GUIDELINES AND THEIR IMPLEMENTATION (2005)[49]

Ramy H. Gohary and Timothy N. Davidson have shown that the set of linear-dispersion (LD) codes subsumes a diverse class of STBC codes, the OSTBC and BLAST schemes. The codes proposed in this paper guarantees minimum output mean-square error (MSE) without any information loss. Additionally, these codes have been shown to minimize the high-SNR average pairwise error probability (PEP). In this paper, a systematic and an efficient constellation-independent designing method has been presented for finding unitary matrices. The designing procedure was based on minimizing the lower bound on the mean square error at the output of a linear pre-processor at the receiver, maximization of an upper bound on the mutual information along with minimizing the lower bound on the high-SNR average PEP. The properties of a unitary coded system facilitate them to propose a row interleaving scheme for LD codes that significantly improved the system performance at high SNR by transmitting row matrix over different channel realizations, without any increase in the complexity of the detector.

LINEAR ORTHOGONAL DISPERSIVE CODES FOR MIMO SYSTEM (2004) [50]

Daiming Yang *et al.* proposed Linear orthogonal dispersive codes LODC. They employed the a group of complete orthogonal vector basis of vector space in the construction of these codes. LODC has been used to transmit low rate information sequence along with better power efficiency. Optimal design of LODC has been proposed to search for a space time code with high joint optimized capacity and low symbol error probability. The method has simple design concept, good diversity gain and multiplexing gain and reasonably low implementation complexity. It has been observed that LODC has enhanced diversity gain and outage probability performance.

ON FULL DIVERSITY LINEAR DISPERSION CODES WITH PARTIAL INTERFERENCE CANCELLATION GROUP DECODING (2008)[51]

Xiaoyong Guo *et.al.* proposed a partial interference cancellation group decoding for LDC codes. A design criterion has been proposed using the PIC decoding at the receiver end.The decoding helped the codes to achieve full diversity. A PIC group decoder decodes the symbols entrenched in an STBC by separating them into several groups and decoding of each group is done individually after a linear PIC operation. It can be observed as an intermediary decoding method between the (ML) receiver and a zero-forcing (ZF) receiver. The partial interference cancellation group decoding provides an outline to regulate complexity performance tradeoff by choosing the dimensions of the symbols.

WIDELY LINEAR MMSE RECEIVERS FOR LINEAR DISPERSION SPACE TIME BLOCK CODES (2010) [52]

Konstantinos N. Plataniotis *et al.* projected a new receiver construction for the linear-dispersion (LD) codes, subsuming the V-BLAST and the quasi orthogonal codes. They suggested the use of widely-linear minimum-mean-squared-error (WL-MMSE) estimates of transmitted symbols in place of maximum likelihood (ML) detection of the symbols. Assuming that the channel is a closed loop system such that an error and delay-free feedback channel would be available through which the bits can be sent to the transmitter. The linear receiver determined LDC from a set of LDCs yielded minimum instantaneous error probability. It then sends the bit index through the feedback channel signifying the best selected Linear Dispersion code matrix to the transmitter. Simulation results show that the given suboptimal receiver performed near to the optimal one, in addition to reducing the receiver’s complexity.

CHAPTER 3

LINEAR DISPERSION SPACE TIME BLOCK CODES

For high data rates and a large number of transmit antennas, OSTBCs and STTCs has suffered from complexity order and/or performance shortcomings. In STTCs, the number of trellis states varies exponentially with the increase in number of transmit antennas [20] and finding the high data rate OSTBCs (>1/2) for a large number of transmit antennas has been a tough task. Hence, these codes generally exhibited poor performance at high rate and/or with many transmitting antennas. Hassibi *et al.* [20] presented a new type of codes that are non-orthogonal and linear. They are referred as linear dispersion codes (LDCs), since they operate by dividing the data into sub-streams that are dispersed in linear combinations over space and time. The LDCs provide coding gain as well as diversity gain while keeping decoding simple for an arbitrary number of transmit and receive antennas.

3.1. Encoding of Linear Dispersion Codes

Linear dispersion Encoders are encoders in which the codewords are expressed as linear combination of certain basis matrices with the weights defined by the transmitted symbols. For a generalised MIMO channel system having MIMO channel scheme having N_T transmit antennas, N_R number of receive antennas, K modulated symbols sent each time from constellation $\mathbb{r}$ in T symbol periods and channel is assumed to be frequency flat channel with channel state information (CSI) known only at the reception, the generalized LDC structure is given by $\boldsymbol{Z}$,

$$\boldsymbol{Z} = \sum_{k=1}^{K} s_k \boldsymbol{C}_k + s_k{}^{*} \boldsymbol{D}_k \tag{8}$$

where the symbols to be transmitted, $\boldsymbol{s}$ are divided into K substreams, $\boldsymbol{s} = \{s_1, s_2, s_3, \dots\dots s_K\}$ and $\boldsymbol{C}_k$ and $\boldsymbol{D}_k$ are the k^{th} dispersion matrices corresponding to the s_K symbol, having the dimensions $T \times N_T$. The code is completely established by the dispersion matrices but the individual codeword depend upon the choice of s_k symbol. Decomposing $s_k = \alpha_k + j\beta_k$, where α_k corresponds to the real part of s_k and $\beta_k$corresponds to the imaginary part of s_k .The LD code can also be written as:

$$\boldsymbol{Z} = \sum_{k=1}^{K} \alpha_k \boldsymbol{A}_k + j\beta_k \boldsymbol{B}_k \tag{9}$$

where $\boldsymbol{A}_k = \boldsymbol{C}_k + \boldsymbol{D}_k$ and $\boldsymbol{B}_k = \boldsymbol{C}_k - \boldsymbol{D}_k$.

It has been assumed that the variance of $\{\alpha_1, \alpha_2, \alpha_3, \dots \dots \alpha_K\}$ and $\{\beta_1, \beta_2, \beta_3, \dots \dots \beta_K\}$ tend to have value equal to $\frac{1}{2}$. That is the transmitted signal **s** has been normalised as $\sum \boldsymbol{s}\boldsymbol{s}^* = T \times N_T$.

3.2. Decoding of Linear Dispersion Codes

The special property of the LD codes is the linearity in the variables $\{\alpha_k, \beta_k\}$ that led to efficient V-BLAST like coding schemes. The system equation would be:

$$\boldsymbol{Y} = \sqrt{\frac{SNR}{N_T}}\boldsymbol{H} \cdot \boldsymbol{Z} + \boldsymbol{W} \tag{10}$$

$$\boldsymbol{Y} = \sqrt{\frac{SNR}{N_T}}\boldsymbol{H} \cdot \left(\textstyle\sum_{k=1}^{K} \alpha_k \boldsymbol{A}_k + j\beta_k \boldsymbol{B}_{\boldsymbol{k}}\right) + \boldsymbol{W} \tag{11}$$

Decomposing the matrices into their real and imaginary parts, we obtain

$$\boldsymbol{Y}_{\Re} + \boldsymbol{Y}_{\Im} = \sqrt{\frac{SNR}{N_T}}(\boldsymbol{H}_{\Re} + j\boldsymbol{H}_{\Im}) \cdot \left(\sum_{k=1}^{K} \alpha_k(\boldsymbol{A}_{\Re,k} + j\boldsymbol{A}_{\Im,k}) + j\beta_k(\boldsymbol{B}_{\Re,k} + j\boldsymbol{B}_{\Im,k})\right) + (\boldsymbol{W}_{\Re} + j\boldsymbol{W}_{\Im})$$

where $\boldsymbol{H}_{\Re}$ is the real part of $\boldsymbol{H}$ and $\boldsymbol{H}_{\Im}$ corresponds to the imaginary part of $\boldsymbol{H}$.

Solving the equation,

$$\boldsymbol{Y}_{\Re} = \sqrt{\frac{SNR}{N_T}}\sum_{k=1}^{K}[\alpha_k(\boldsymbol{A}_{\Re,k}\boldsymbol{Y}_{\Im} - \boldsymbol{A}_{\Im,k}\boldsymbol{H}_{\Im}) + \beta_k(-\boldsymbol{B}_{\Im,k}\boldsymbol{H}_{\Re} - \boldsymbol{B}_{\Re,k}\boldsymbol{H}_{\Im})] + \boldsymbol{W}_{\Re}$$

$$\boldsymbol{Y}_{\Im} = \sqrt{\frac{SNR}{N_T}}\sum_{k=1}^{K}[\alpha_k(\boldsymbol{A}_{\Im,k}\boldsymbol{H}_{\Re} - \boldsymbol{A}_{\Re,k}\boldsymbol{H}_{\Im}) + \beta_k(\boldsymbol{B}_{\Re,k}\boldsymbol{H}_{\Re} - \boldsymbol{B}_{\Im,k}\boldsymbol{H}_{\Im})] + \boldsymbol{W}_{\Im}$$

Substituting the columns of $\boldsymbol{Y}_{\Re}$,$\boldsymbol{Y}_{\Im}$, $\boldsymbol{H}_{\Re}$, $\boldsymbol{H}_{\Im}$, $\boldsymbol{W}_{\Re}$ and $\boldsymbol{W}_{\Im}$ as $y_{\Re,n}, y_{\Im,n}, h_{\Re,n}, h_{\Im,n}, w_{\Re,n}$, $w_{\Im,n}$ and defining

$$\boldsymbol{A}_k = \begin{bmatrix} \boldsymbol{A}_{\Re,k} & -\boldsymbol{A}_{\Im,k} \\ \boldsymbol{A}_{\Im,k} & \boldsymbol{A}_{\Re,k} \end{bmatrix}, \boldsymbol{B}_k = \begin{bmatrix} -\boldsymbol{B}_{\Im,k} & -\boldsymbol{B}_{\Re,k} \\ \boldsymbol{B}_{\Re,k} & -\boldsymbol{B}_{\Im,k} \end{bmatrix} \text{ and } h_n = \begin{bmatrix} h_{\Re,n} \\ h_{\Im,n} \end{bmatrix}$$

where $\mathrm{n} = 1, \dots \dots, N_R$

Assembling the equations in the $\boldsymbol{Y}_{\Re}$ and $\boldsymbol{Y}_{\Im}$ to form single real system of equations, we get:

$$\underbrace{\begin{bmatrix} y_{\Re,1} \\ y_{\Im,1} \\ y_{\Re,2} \\ y_{\Im,2} \\ \vdots \\ y_{\Re,N_R} \\ y_{\Im,N_R} \end{bmatrix}}_{\triangleq\, \boldsymbol{y}} = \sqrt{\frac{SNR}{N_T}} \cdot \boldsymbol{\mathcal{H}} \underbrace{\begin{bmatrix} \alpha_1 \\ \beta_1 \\ \alpha_2 \\ \beta_2 \\ \vdots \\ \alpha_K \\ \beta_K \end{bmatrix}}_{\triangleq\, \boldsymbol{s}} + \underbrace{\begin{bmatrix} w_{\Re,1} \\ w_{\Im,1} \\ w_{\Re,2} \\ w_{\Im,2} \\ \vdots \\ w_{\Re,N_R} \\ w_{\Im,N_R} \end{bmatrix}}_{\triangleq\, \boldsymbol{w}} \tag{12}$$

hence, we can write the linear equation,

$$\boldsymbol{y} = \sqrt{\frac{SNR}{N_T}} \cdot \boldsymbol{\mathcal{H}}\boldsymbol{s} + \boldsymbol{w} \tag{13}$$

where $\boldsymbol{\mathcal{H}}$ is the equivalent channel and defined as:

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \boldsymbol{A}_1 h_1 & \boldsymbol{B}_1 h_1 & \cdots & \boldsymbol{A}_K h_1 & \boldsymbol{B}_K h_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{A}_1 h_K & \boldsymbol{B}_1 h_K & \cdots & \boldsymbol{A}_K h_K & \boldsymbol{B}_K h_K \end{bmatrix}$$

Since the original channel $\boldsymbol{H}$ and the dispersion matrices were known to the receiver, we can say that $\boldsymbol{\mathcal{H}}$ is also known to the receiver. Using the equation directly, the receiver can perform the decoding procedure.

3.3. Design Procedures for Linear Dispersion Codes

Researchers proposed different methods for improving the performance of these codes, based on designing, optimization and employing different schemes at the receiver side, for efficient detection. Some of them have been discussed below:

- ***Based on criteria of maximizing the ergodic capacity***: Hassibi *et al* [20] proposed a designing method for the Linear dispersion encoder. It has been stated that the designed codes approximated to the upper bound on the Capacity. The design criteria to achieve maximum ergodic capacity, proposed by them was as follows:

1) Choose the number of symbols to be sent, $K \leq N_R \cdot T$.
2) Choose the basis matrices $\{\boldsymbol{A}_k, \boldsymbol{B}_k\}$ to solve the optimization problem:

$$C_{LD} = \max_{\boldsymbol{A}_k, \boldsymbol{B}_k, k=1,\ldots,K} \frac{1}{2T} E \log\det\left(\mathrm{I}_{2\mathrm{KT}} + \frac{SNR}{N_T}\, \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^*\right)$$

Along with the above problem was solved for the dispersion matrices $\{\boldsymbol{A}_k, \boldsymbol{B}_k\}$, following constraints were also considered :

a. $\sum_{k=1}^{K}(Tr\boldsymbol{A}_k{}^*\boldsymbol{A}_k + Tr\boldsymbol{B}_k{}^*\boldsymbol{B}_k) = 2T \cdot N_T$

b. $Tr\boldsymbol{A}_k{}^*\boldsymbol{A}_k = Tr\boldsymbol{B}_k{}^*\boldsymbol{B}_k = \frac{T \cdot N_T}{K}$

c. $\boldsymbol{A}_k{}^*\boldsymbol{A}_k = \boldsymbol{B}_k{}^*\boldsymbol{B}_k = \frac{T}{K}\boldsymbol{I}_{N_T}$

- In another paper [27], ***average error probability*** has been used as a performance metric. Average error probability for falsely detecting $\boldsymbol{Z^j}$ matrix when codeword $\boldsymbol{Z^i}$ was sent, can be defined as:

$$\mathrm{P}(\boldsymbol{Z^i} \to \boldsymbol{Z^j}) = \frac{1}{\left|I_{N_R N_T} + \frac{SNR}{4N_0}\boldsymbol{R}_z \otimes \boldsymbol{I}_{N_T}\right|}$$

At high SNR,

$$\mathrm{P}(\boldsymbol{Z^i} \to \boldsymbol{Z^j}) \le \frac{1}{\left(\frac{SNR}{4N_0}\right)^{\mathrm{rank}(\boldsymbol{R}_z)\cdot N_T} \prod_{n=1}^{\mathrm{rank}(\boldsymbol{R}_z)} \lambda_n{}^{N_R}}$$

where $\boldsymbol{R}_z = (\boldsymbol{Z^i} - \boldsymbol{Z^j})^H(\boldsymbol{Z^i} - \boldsymbol{Z^j})$.

The diversity advantage of the code has been determined by the smallest product, rank $(\boldsymbol{R}_z) \cdot N_T$ while the coding advantage has been defined as $\prod_{n=1}^{\mathrm{rank}(\boldsymbol{R}_z)} \lambda_n{}^{N_R}$. Hence, they proposed a method in which dispersion matrices are optimized with respect to the Rank and Determinant criteria [15]. They also made a proposition that a fully diverse code has full rank codeword matrices [27].

- An additional constraint of ***block error rate*** has been considered in [31] along with the already existing Pairwise error probability and the maximization of ergodic capacity schemes to optimize the LD codes. They iteratively updated the matrices while exploiting the gradient computation method. For the average Pairwise error probability, determinant value, det $(\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^*)^{\frac{-1}{2}}$ has been verified since minimizing the determinant value would minimize the average value of PEP. It was further analysed that combined method (maximizing the mutual information and minimizing the BLER along with minimizing the error probability) has been better than using the mutual information or average BLER method only. To maximize the upper bound on the capacity, LD codes were designed to follow the following constraint:

$$\sum_{k=1}^{K}(Tr\boldsymbol{A}_k{}^*\boldsymbol{A}_k + Tr\boldsymbol{B}_k{}^*\boldsymbol{B}_k) = 2N_T \cdot \boldsymbol{I}_{2T}$$

- In another technique, ***Quasi Orthogonal codes*** were used for the generation of Linear Dispersion codes. At first, K orthogonal matrices were generated and then, scrambling matrices were employed in the generation of Linear Dispersion codes. It has been shown that employing this method, codes attain the upper bound on diversity with a lower value of error probability. Further, any configuration of matrices could be generated with the help of this method. When used with the ***MMSE receiver***, the BER results have been shown to be better than that of Hassibi [20].
- In another method [22], LD codes designing used the ***algebraic model*** of design and used the criteria of maximal worst codeword difference matrix determinant for optimization, the optimized codes were then evaluated with the ***Non vanishing determinant (NVD)*** property and employed ***Sphere decoding*** at the receiver end. It was stated that Sphere decoding helped in reducing the overall decoding cost.

Non Vanishing Determinants

A Space Time Block Code scheme is said to have a non vanishing determinant if the determinant of the codeword sent and the difference matrix obtained is non-zero. It has been analytically supported by Rajan *et al.* [39], that the lower bound on DMG trade-off is proportional on the rank of the difference matrix. That is, the maximum diversity gain, $d(r)$ achievable by an STBC scheme at a particular rate r, has been lower bounded by the rank of the difference matrix,

$$d(r) \leq n\mu\left(1 - \frac{r}{\gamma}\right) \tag{14}$$

where μ is defined as the rank of difference matrix $\left(Z - \breve{Z}\right)$, γ be the rate (average number of symbols transmitted per channel use) of the codeword set, $\acute{\boldsymbol{Z}}$ with a constellation, $\mathbb{r}$, i.e. $\gamma = \frac{1}{n}\log_{\mathbb{r}}|\boldsymbol{Z}|$ with the assumption that the number of transmit antennas are equal to the receive antennas, i.e. $n = N_T = N_R$.

For a sent codeword Z, from the codeword set $\acute{Z}$,

$$\delta = \left\|\det\left(\boldsymbol{Z} - \breve{\boldsymbol{Z}}\right)\right\|^2 \neq 0 \qquad \forall\, \boldsymbol{Z} \neq \breve{\boldsymbol{Z}}\ ,\boldsymbol{Z},\breve{\boldsymbol{Z}} \in \acute{\boldsymbol{Z}} \tag{15}$$

Hence, NVD property ensures that the determinant of any such matrix does not becomes arbitrarily small with the increase in SNR and thus, prevents the nullification of the coding gain. Further, this property has been cited as a sufficient condition to achieve an optimal Diversity-multiplexing trade-off for a MIMO channel scheme, provided that the number of receivers is greater than that of transmitters [22]. Another advantage of a NVD satisfied code is that the coding gain also remains constant even if there is increase in the constellation size.

- In another scheme [42], a practical transceiver model has been proposed using linear space optimized pre-coding technique in combination with the ***Integer-Forcing Receiver***. Assuming the channel to be closed loop, optimization of the LD matrix is done to achieve the maximum transmission rate. It was stated that using this combination, the maximum loss in the SNR regime is less than 10% of capacity for an outage probability of 0.001 or 0.1%. It was also observed that, pre-coded IF-SIC significantly outperformed the Golden-IF-SIC and Alamouti code. Inspired by the achievable gains of this receiver, we extend its use with the LDC encoder. A brief description of which is given below.

Integer Forcing Technique

Practical deployments in wireless systems are limited by processing and computational capabilities. Hence, suboptimal low-complexity linear (ZF, MMSE) and non-linear (ZE-DFE, MMSE-DFE) receivers are commonly used. Although the hardware complexity of these receivers is low, they offer poor performance in terms of error probability and Diversity multiplexing Trade-off. In order to improve the performance, a new architecture of MIMO receivers, termed as Integer Forcing receiver (IF) has been introduced by Zhan *et al.* [25, 26]. Here, each encoder uses the same linear code and the receiver exploits the code-level linearity to recover equations of the transmitted messages. Instead of inverting the channel as in the case of conventional linear receivers, this scheme uses linear projection matrix to force the effective channel to a ***full-rank integer matrix* A**.

As in the case of traditional linear receivers, each element of the effective output is then sent to a separate decoder. However, since each encoder uses the same linear code, each decoder can recover an integer linear combination of the codewords. The integer-forcing receiver is free to choose the set of equation coefficients **A** to be any full-rank integer matrix. The resulting integer combinations of codewords can be mapped back to a set of full-rank messages over a finite field. Finally, the individual messages vectors are recovered from the set of full-rank equations of message vectors. The details of the architecture are provided given in Fig. 3.1.

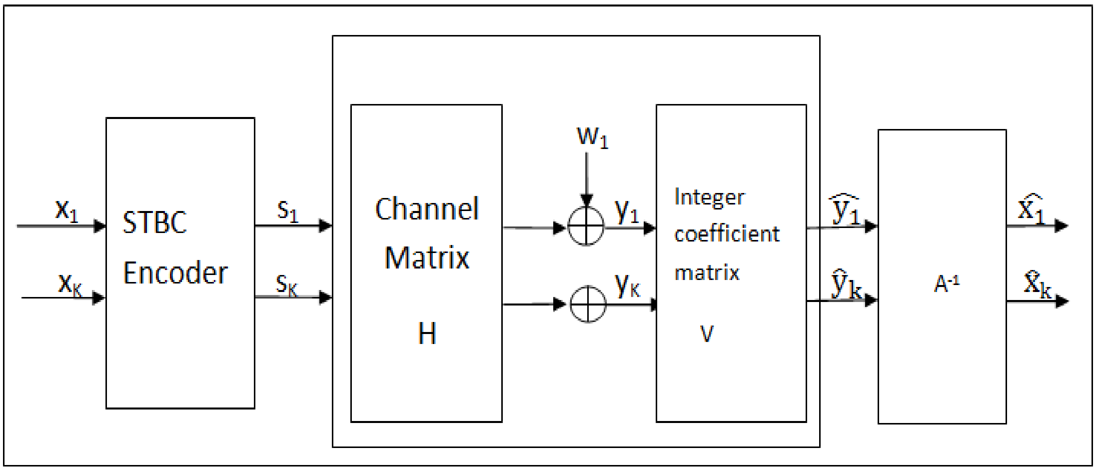

Fig. 3.1. Block diagram of Integer Forcing receiver

Prior to decoding of the information, the receiver projects the channel output **Y**, with Integer Forcing coefficient matrix **V**, to get an effective Integer Matrix.

$$\widehat{\boldsymbol{Y}} = \boldsymbol{V} * \boldsymbol{Y}$$

$$= \boldsymbol{V} * (\boldsymbol{HS} + \boldsymbol{W}) = \boldsymbol{A} * \boldsymbol{S} + \widetilde{\boldsymbol{W}} \qquad (16)$$

where **A** is the Integer matrix, $\boldsymbol{S}$ contains the sent symbols. Since **A** is a full- rank matrix, transmitted codewords are recovered by a simple matrix inversion,

$$\widehat{\boldsymbol{S}} = \boldsymbol{A}^{-1} * \widehat{\boldsymbol{Y}} \qquad (17)$$

***Achievable rates*:**

In integer forcing (IF) receiver, the receiver tries to design an equalization matrix **V** , where $\mathbf{V} \in \Re^{\mathrm{L}\times\mathrm{K}}$, such that after the projection, the resulting IF matrix $\boldsymbol{A}$ satisfies that $\mathbf{A} \in \mathbb{Z}^{\mathrm{L}\times\mathrm{L}}$ and the achievable rate R_m is maximized, where $R_m$is defined as:

$$R_m = \frac{1}{2}\log\left(\frac{P}{\|v_m{}^2\|+P\left\|\boldsymbol{H}^T v_m - a_m\right\|^2}\right) \qquad (18)$$

where $\boldsymbol{V} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots\ldots \boldsymbol{v}_L]^T$ and $\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots\ldots \boldsymbol{a}_L]^T$and P is the signal to noise ratio. It has been stated that for a fixed Integer Forcing Coefficient matrix, the computation rate is maximized by choosing,

$$\boldsymbol{v}_{\boldsymbol{m}}^{\boldsymbol{T}} = \boldsymbol{a}_{\boldsymbol{m}}^{\boldsymbol{T}} * \left(\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H} + \frac{1}{P}\boldsymbol{I}_{\mathrm{N_R}}\right)^{-1} \boldsymbol{H}^T \qquad (19)$$

Using the optimal value of $\boldsymbol{v}_{\boldsymbol{m}}$ in R_m, we get

$$R_m = \frac{1}{2}\log\left(\frac{1}{a_m^T \breve{Q} a_m}\right) \qquad (20)$$

where $\breve{\boldsymbol{Q}} = \boldsymbol{I}_{N_t} - \boldsymbol{H}^{\boldsymbol{T}}\left(\boldsymbol{H}\boldsymbol{H}^{\boldsymbol{T}} + \frac{1}{P}\boldsymbol{I}_{N_r}\right)^{-1}\boldsymbol{H}$. Thus the total achievable rate is,

$$R_{total} \triangleq \underset{|\boldsymbol{A}| \neq 0}{max} \; \underset{\mathrm{m}}{\min} \; L\, R_m$$

$$R_{total} \triangleq \underset{|\boldsymbol{A}| \neq 0}{max} \; \underset{\mathrm{m}}{\min} \; L \log\left(\frac{1}{a_m^T \breve{Q} a_m}\right) \tag{21}$$

Hence, the design criteria for the optimum A matrix would be,

$$\boldsymbol{A} = \arg \underset{|\boldsymbol{A}| \neq 0}{max} \; \underset{\mathrm{m}}{\min} \frac{L}{2} \log\left(\frac{1}{\boldsymbol{a_m^T \breve{Q} a_m}}\right)$$

$$= \arg \underset{|\boldsymbol{A}| \neq 0}{min} \; \underset{\mathrm{m}}{\max} \; \boldsymbol{a_m^T \breve{Q} a_m} \tag{22}$$

Solving this optimization has been a critical work. Several contributions for the designing of Integer forcing receivers, based on the achievable rate came subsequently into light in [40, 41]. Lili Wei [40] proposed a design method for the IF receivers using ***Fincke Pohst Candidate set searching traditional method*** [47]. The combination of IF receiver with STBC modulation has been studied in [41]. It has been shown that IF could be applicable to perfect Space Time codes. In addition, authors in [42] proposed a design criterion for codes that achieve full diversity under IF decoding based on satisfying a non-vanishing singular value property. Those perfect codes are shown to attain full diversity using IF decoding. It was further proved that this receiver architecture, when coupled with space time linear precoding, is able to achieve the capacity of an open loop multiple-input multiple-output channel, up to a constant gap that depends only on the number of transmit antennas. The gap, however, is quite large and thus provides performance guarantees that are useful only for high values of capacity, hence bridges the rate gap between traditional linear receivers and the joint ML receiver at the cost of some additional signal processing [43].

Rank and Determinant Criteria for the Integer Forcing Receiver

Vahid Tarokh[16] proposed a performance criterion for the block codes and the trellis codes known as Rank and Determinant criteria. In this paper, we expanded the Rank and Determinant criteria with Integer Forcing for the LD codes. To minimize the codeword error probability and to optimize the linear dispersion matrix, an additional Integer coefficient matrix V, is incorporated in the distribution of received signal having known value of **r, H**, defined as, $f(\frac{r}{\boldsymbol{Z}}, \mathbf{H})$

$$f\left(\frac{r}{\boldsymbol{Z}}, \mathbf{H}\right) = ß \cdot exp\left\{-Tr\left(\frac{1}{(\pi \mathrm{N_o})^{Nr*Nr/2}} \frac{((r-\boldsymbol{V}*\boldsymbol{H}*\boldsymbol{Z})^H*(r-\boldsymbol{V}*\boldsymbol{H}*\boldsymbol{Z}))}{\mathrm{N_o}}\right\}\right. \tag{23}$$

where r is the received signal, ß is equal to $\frac{1}{\pi \mathrm{N_o}^{N_r * N_r/2}}$ [12]. Assuming that all the codewords, **Z** are equally likely to be transmitted, the probability of pairwise error probability can be approximated as,

$$\mathrm{P}(\boldsymbol{Z^i} \rightarrow \boldsymbol{Z^j}|\boldsymbol{H}) = \mathrm{Q}(\frac{\|(\boldsymbol{Z^i}-\boldsymbol{Z^j})*\boldsymbol{A}\|_F^2}{\sqrt{2*\mathrm{N_o}*\|(\boldsymbol{Z^i}-\boldsymbol{Z^j})*\boldsymbol{A}\|_F^2}} \tag{24}$$

where **A**= **V·H**, represents the Integer matrix, $\|\boldsymbol{D}\|_F$ represents the Frobenius norm of **D**, and is defined as,

$$\|\mathbf{D}\|_\mathrm{F} = \sqrt{\mathrm{Tr}(\mathbf{D} * \mathbf{D^H})} = \sqrt{\mathrm{Tr}(\mathbf{D^H} * \mathbf{D})}$$

Hence, $\left\|(\boldsymbol{Z^i} - \boldsymbol{Z^j}) * \boldsymbol{A}\right\|_F$ can be written as,

$$\begin{aligned} \left\|(\mathbf{Z^i} - \mathbf{Z^j}) * \mathbf{A}\right\|_F &= \sqrt{\mathrm{Tr}((\mathbf{Z^i} - \mathbf{Z^j}) * \mathbf{A})^{\mathbf{H}}(\mathbf{Z^i} - \mathbf{Z^j}) * \mathbf{A})} \\ &= \sqrt{\mathrm{Tr}(\mathbf{A^H} * (\mathbf{Z^i} - \mathbf{Z^j})^{\mathbf{H}} * (\mathbf{Z^i} - \mathbf{Z^j}) * \mathbf{A})} \end{aligned} \tag{25}$$

Solving (24),

$$\mathrm{P}(\boldsymbol{Z^i} \rightarrow \boldsymbol{Z^j}|\boldsymbol{H}) = Q(\sqrt{\frac{1}{2\mathrm{N_o}}} \cdot \left\|(\boldsymbol{Z^i} - \boldsymbol{Z^j}) * \boldsymbol{A}\right\|_F) \tag{26}$$

Employing the combination of House Holder representation and QR decomposition, $\left\|(\boldsymbol{Z^i} - \boldsymbol{Z^j})\right\|_F$ can be written in terms of eigen values of matrix **G** as,

$$\left\|(\mathbf{Z^i} - \mathbf{Z^j}) * \mathbf{A}\right\|_\mathrm{F} = \mathbf{G} = \mathbf{NUN^H}$$

where $\mathbf{U} = diag\left[u_1, u_{2,} u_3, \dots\right]$ contains the eigen values of the matrix **G** and columns of N are eigenvectors of $\left\|(\boldsymbol{Z^i} - \boldsymbol{Z^j}) * \boldsymbol{A}\right\|_F$**.** Hence (26) reduces to

$$\left\|(\boldsymbol{Z^i} - \boldsymbol{Z^j}) * \boldsymbol{A}\right\|_F = Tr[\boldsymbol{A^H} * \boldsymbol{N^H} \cdot \boldsymbol{U} * \boldsymbol{N} * \boldsymbol{A}] \tag{27}$$

Since elements of integer matrix A, are statistically found to exhibit gaussian behavior. The elements of $N \cdot A$ are also gaussian.We denote the $(m,n)^{th}$ elements of $N \cdot \boldsymbol{A}$ as $\alpha_{m,n}$.

Therefore,
$$\left\|(\boldsymbol{Z^i} - \boldsymbol{Z^j}) * \boldsymbol{A}\right\|_F = \sum_{n=1}^{N_r} \sum_{m=1}^{N_t} u_m \left|\alpha_{m,n}\right|^2 \tag{28}$$

Using the value of $\left\|(\boldsymbol{Z^i} - \boldsymbol{Z^j}) * \boldsymbol{A}\right\|_F$ from (28) in (26)

$$\mathrm{P}(\boldsymbol{Z^i} \rightarrow \boldsymbol{Z^j}|\boldsymbol{H}) = Q(\sqrt{\frac{1}{2\mathrm{N_o}}} \cdot \sum_{n=1}^{N_r} \sum_{m=1}^{N_t} u_m \left|\alpha_{m,n}\right|^2) \tag{29}$$

Using the Upper bound on Q function,$Q(x) \leq \frac{1}{2} e^{\frac{-x^2}{2}}$, the upper bound on pairwise error probability is given as,

$$\mathrm{P}(\boldsymbol{Z^i} \rightarrow \boldsymbol{Z^j}|\boldsymbol{H}) \leq \frac{1}{2} exp(\frac{-1}{4\mathrm{N_o}} \cdot (\sum_{n=1}^{N_r} \sum_{m=1}^{N_t} u_m \left|\alpha_{m,n}\right|^2)) \tag{30}$$

Since elements $\alpha_{m,n}$ are Gaussian, hence using Rayleigh distribution probability density function for $|\alpha_{m,n}|$ as,

$$f(|\alpha_{m,n}|) = 2|\alpha_{m,n}|\exp(-|\alpha_{m,n}|^2)$$

The expected value of pairwise error probability would be,

$$\mathrm{P}(\boldsymbol{Z^i} \rightarrow \boldsymbol{Z^j}) = \mathrm{E}\left[\mathrm{P}(\boldsymbol{Z^i} \rightarrow \boldsymbol{Z^j}|\boldsymbol{H})\right] \leq \frac{1}{\prod_{n=1}^{p}[1+\frac{u_n}{4N_O}]^{N_r}} \tag{31}$$

Here, p = rank of $((\boldsymbol{Z^i} - \boldsymbol{Z^j})^H * (\boldsymbol{Z^i} - \boldsymbol{Z^j}))$ matrix. If the codeword difference matrix, $|(\boldsymbol{Z^i} - \boldsymbol{Z^j})|$ has full rank, all the eigen values will be non zero. Hence, the coding gain would be maximum. For high SNR, the one in the denominator can be neglected. Hence the upper bound on pairwise error probability will be,

$$\mathrm{P}(\boldsymbol{Z^i} \rightarrow \boldsymbol{Z^j}) \leq \frac{1}{\prod_{n=1}^{p}[\frac{u}{4N_O}]^{N_r}} \leq \frac{(4N_O)^{p*N_r}}{\prod_{n=1}^{p} u_n{}^{N_r}} \tag{32}$$

Hence for lower bit error rate, the factor in the denominator, $\prod_{n=1}^{p} u_n{}^{N_r}$should be maximized which implies that p and $\prod_{n=1}^{p} u_n$should be maximum for an optimized code. Maximizing the value of former will ensure the diversity gain and latter will ensure the coding gain.

CHAPTER 4

PROPOSED WORK

In this thesis work, we have designed and optimized the Linear Dispersion codes using different criteria and employed different estimators to evaluate its performance. Initially it has been evaluated with Sphere Decoder, Zero Forcing and MMSE receiver. Then considering the complexity of the Sphere Decoder and low performance of the conventional estimators, Integer Forcing has been suggested and designed with the LD codes.

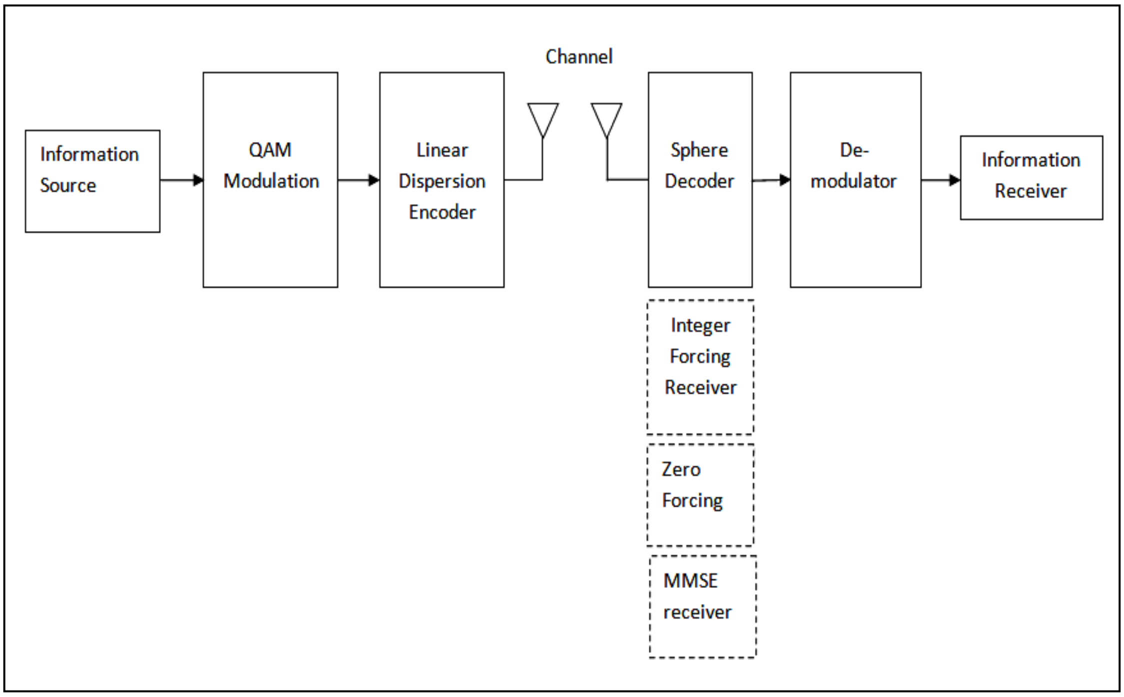

Fig. 4.1. System model of proposed work

4.1. Designing and Optimization of LD-STBC

In this thesis, we have considered a MIMO channel scheme where N_T corresponded to the number of transmit antennas, N_R corresponded to the number of receive antennas, K were the number of modulated symbols sent each time from constellation $\mathbb{r}$ in T symbol periods and channel is assumed to be frequency flat channel with channel state information (CSI) known only at the reception. Let $\{\mathbf{s_k}\}|_{k=0}^{K-1}$ represents the K modulated symbols to be sent. Using a combination of earlier proposed methods, a design criterion has been suggested. Inspired by the frame theory [27], the encoder matrix does not follow a defined pattern rather it is a pseudo random matrix.

For the designing of LD codes, initially a number of random orthogonal matrices $£_q$, of dimension (N_T × T) × K, have been generated using QR decomposition as the by-product of QR decomposition is an orthogonal matrix. On the fetched orthogonal matrices, a set of criteria and constraints mentioned in this section are applied. From generated q random matrices, a number of matrices that satisfied the Tight frame criteria have been selected,

Tight frame criteria $$\{£ * £^{\boldsymbol{H}}\} = \frac{T}{K}\boldsymbol{I}_K \tag{33}$$

This criterion transforms the dispersion matrix £ a unitary matrix with weighing value, T/K. The matrices which satisfied the above criterion have been evaluated with the standard power constraint and equal power constraint.

Standard Power Constraint $$tr\left\{\sum_{k=0}^{K-1} \boldsymbol{M_k} * \boldsymbol{M_k^H}\right\} = T \tag{34}$$

Standard Power Constraint is used to limit the total power to be transmitted. Then the selected matrices have been scaled to satisfy the equal power constraint criteria mentioned in (35). This criterion suggests that each dispersion matrix should be distributed with the equal average power in order to transmit each symbol with same overall power from N_T antennas during *T* channel intervals.

$$tr\left\{\boldsymbol{M_k} * \boldsymbol{M_k^H}\right\} = \frac{T}{K} \qquad \mathrm{k} = 0, 1 \ldots\ldots, \mathrm{K} - 1 \tag{35}$$

The matrices satisfying the above criteria are then evaluated with the diversity constraint. This constraint has been very restrictive, for the reason that it forces the symbols to be dispersed with equal energy in all dimensions and checks whether the codeword matrix is a full rank matrix. A full rank codeword matrices has been proved to exhibit full diversity property [48]. This constraint helps the code to demonstrate full diversity.

Diversity Criteria, $$\boldsymbol{M_k} * \boldsymbol{M_k^H} = \frac{1}{K}\boldsymbol{I_T} \qquad \mathrm{k} = 0, 1\ldots\ldots, \mathrm{K} - 1 \tag{36}$$

Thus the short listed codes are said to be diversity optimal. The codes satisfying the above mentioned codes are then verified with the Non Vanishing determinant property (NVD). The NVD property helps in achieving an optimal diversity-multiplexity trade off and has been considered as a sufficient condition for the optimality for any arbitrary fading channel, provided the number of receive antenna must be greater than or equal to the transmit antenna[37]. The basic condition for NVD is that δ should not approach to zero value for any pair of sent, $\boldsymbol{Z}$ and erroneous received signal, $\breve{\boldsymbol{Z}}$.

Non Vanishing Criteria,

$$\boldsymbol{\delta} = \left\|\mathbf{det}\left(\boldsymbol{Z}-\breve{\boldsymbol{Z}}\right)\right\|^{2} \neq \mathbf{0} \qquad\qquad \forall\, \boldsymbol{Z} \neq \breve{\boldsymbol{Z}} \tag{37}$$

At the final stage, the newly designed Rank and determinant criterion for Integer forcing, mentioned in the section 3.3, has been used to select the most effective dispersion matrix. This criterion evaluated the codes on the basis of their diversity and coding gain. Diversity gain is estimated from the rank of the codeword difference matrix. Therefore the matrix has to be Full rank. Among the matrices having full rank, the matrix having the maximum value of minimum determinant of the codeword difference matrices would be chosen. As the matrix with the maximum value of minimum determinant of the codeword difference matrices will provide the highest coding gain.

4.2. LDC with Sphere Decoder

Sphere Decoding has been used in the thesis work to evaluate the performance of Optimized Linear Dispersive Space time block codes. Schnorr Euchner method has been practised in which the channel matrix H having the dimensions of $N_R \times N_T$where $N_R \geq N_T$, has been factorised into

$$\boldsymbol{H} = \boldsymbol{Q}\breve{\boldsymbol{R}} = \boldsymbol{Q}\begin{bmatrix}\boldsymbol{R}\\ \mathbf{0}\end{bmatrix}$$

where Q is an orthogonal matrix having dimensions $N_R \times N_R$, R is an upper triangular , invertible matrix with $N_T \times N_T$ dimensions and 0 is a zero matrix of the $(N_R{-}N_T) \times N_T$ dimensions. The decoder's target is to find out $\mathrm{l_{opt}}$ from the codebook $\mathbb{C}^{N_T}$ that results in minimal norm of the difference with received vector Y.

$$\mathrm{l_{opt}} = \underset{\boldsymbol{Z}\,\in\,\mathbb{C}^{\mathrm{T}\times N_T}}{\arg min} |\boldsymbol{Y}-\boldsymbol{H}\boldsymbol{Z}|^2 = \underset{\boldsymbol{Z}\,\in\,\mathbb{C}^{\mathrm{T}\times N_T}}{\arg min} \left|\boldsymbol{Q}^{\boldsymbol{T}}\boldsymbol{Y}-\breve{\boldsymbol{R}}\boldsymbol{Z}\right|^2 \tag{38}$$

where $\boldsymbol{Z}$ is the sent Linear Dispersive modulated codeword. Because of the zero matrix of the $N_R{-}N_T \times N_T$ dimensions, the search here, is not affected by the last $(N_R{-}N_T)$ rows of vector $\left|\boldsymbol{Q}^{\boldsymbol{T}}\boldsymbol{Y}-\breve{\boldsymbol{R}}\boldsymbol{Z}\right|^2$. Hence, equation (38) has be rewritten as:

$$\mathrm{l_{opt}} = \underset{\boldsymbol{S}\,\in\,\mathbb{C}^{\mathrm{T}\times N_T}}{\arg min} |\breve{\boldsymbol{Y}}-\boldsymbol{R}\boldsymbol{Z}|^2 \tag{39}$$

where $\widetilde{\boldsymbol{Y}} = [\boldsymbol{Q}^{\boldsymbol{T}}\,\boldsymbol{Y}]_1^{N_T}$extracted the first N_T elements of the orthogonally transformed Y.Due to QR decomposition, matrix **R** can be written as,

$$\boldsymbol{R} = \begin{bmatrix} \psi \boldsymbol{I}_{N_T} & \acute{\boldsymbol{A}} \\ 0 & \acute{\boldsymbol{B}} \end{bmatrix}.$$

where $\acute{\boldsymbol{A}}$ is a general matrix of dimensions $\frac{N_T}{2} \times \frac{N_T}{2}$ and $\acute{\boldsymbol{B}}$ is a $\frac{N_T}{2} \times \frac{N_T}{2}$ upper triangular matrix and $\psi$is a constant. Hence, splitting the $\boldsymbol{Z}$ matrix into two parts,

$$\boldsymbol{Z} = [\boldsymbol{Z_1}^{\boldsymbol{T}} \quad \boldsymbol{Z_2}^{\boldsymbol{T}}]^T$$

we get,

$$\mathrm{l_{opt}} = \underset{\boldsymbol{Z}\ \in\ \mathbb{C}^{\mathrm{T}\times N_T}}{\arg min} \left\| \widetilde{\boldsymbol{Y}} - \begin{matrix} \psi \boldsymbol{Z_1} + \acute{\boldsymbol{A}}\boldsymbol{Z} \\ \boldsymbol{B}\acute{\boldsymbol{Z}}_{\boldsymbol{2}} \end{matrix} \right\|^2 \qquad (40)$$

Hence, the sphere decoder searched for the possible $\frac{N_T}{2}$ symbols and after estimating these symbols, symbol by symbol decoding is done to determine the left $\frac{N_T}{2}$ symbols.

4.3. LDC with Integer Forcing Receiver

Integer forcing has been employed for the received LD encoded message symbols. The designing technique [40] included the generation of a viable searching set using the Fincke Pohst (FP) Candidate set searching traditional method [47]. From the generated searching set, an optimum Integer forcing coefficient matrix $\boldsymbol{A}$ has been constructed using the full rank criteria [40]. System Model of Linear Dispersive STBC encoder with an Integer Forcing Linear Receiver has been shown in Fig. 4.1.

Let the search set matrix found using the FP method is $\boldsymbol{\vartheta}$ and $\boldsymbol{t_l}$ represents $\boldsymbol{l^{th}}$ column of the matrix $\boldsymbol{\vartheta}$. Define a function f($\boldsymbol{t_l}$) for each column $\boldsymbol{t_l}$,

$$\mathrm{f}(\boldsymbol{t_l}) = \boldsymbol{t_l}^{\boldsymbol{T}}\boldsymbol{S}\boldsymbol{t_l} \qquad (41)$$

where **S** is the matrix obtained by

$$\boldsymbol{S} = \boldsymbol{I}_{N_t} - \boldsymbol{H}^{\boldsymbol{T}}\left(\boldsymbol{H}\boldsymbol{H}^{\boldsymbol{T}} + \tfrac{1}{P}\boldsymbol{I}_{N_r}\right)^{\boldsymbol{-1}}\boldsymbol{H} \qquad (42)$$

The column of the searching set $\boldsymbol{\vartheta}$ were sorted, in conformity with the increasing value of f($\boldsymbol{t_l}$) such that

$$\boldsymbol{\vartheta} = \{\boldsymbol{t_1}, \boldsymbol{t_2}, \ldots, \boldsymbol{t_l} : f(\boldsymbol{t_1}) \leq\ f(\boldsymbol{t_2}).,\leq \cdots .f(\boldsymbol{t_l})\} \qquad (43)$$

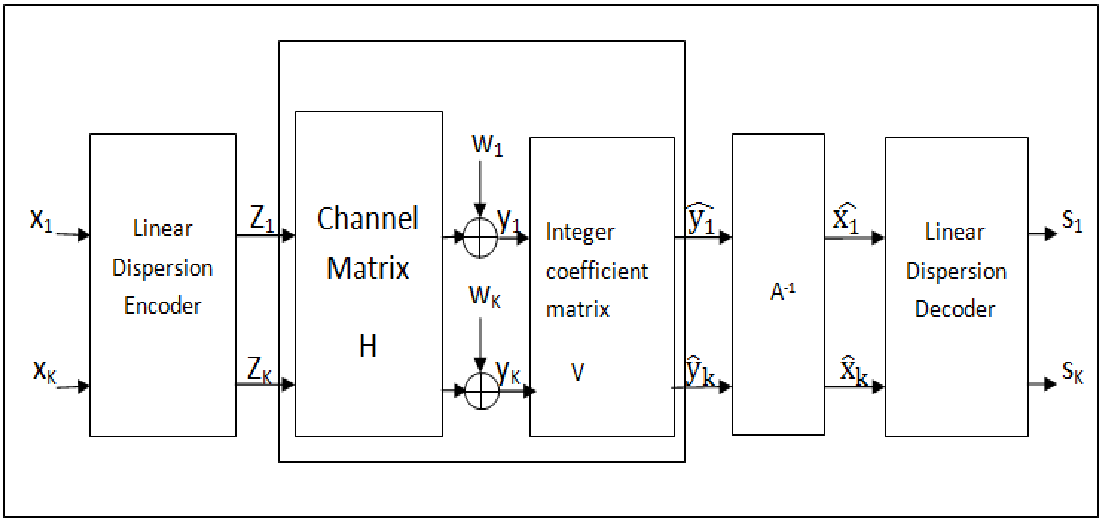

Fig. 4.2. System Model of Linear Dispersive STBC encoder with an Integer Forcing Linear Receiver.

The number of columns in the search set $\boldsymbol{\vartheta}$**,** must be greater than the value of N_T for the successful generation of IF coefficient matrix. For each column $\boldsymbol{l}$, a set of L columns, $\boldsymbol{t_l}$ to $\boldsymbol{t_{l+L-1}}$ has been considered from the set, $\boldsymbol{\vartheta}$ and checked for the full rank. If that set exhibits full rank, then the matrix of L columns would be termed as optimal Integer forcing matrix A. The process persisted until the non singular matrix has not been achieved. The Integer forcing coefficient matrix V can be derived from IF matrix A by:

$$\boldsymbol{v_m^T} = \boldsymbol{a_m^T} * \left(\boldsymbol{H^T H} + \tfrac{1}{P}\boldsymbol{I}_{\mathrm{N_R}}\right)^{-1} \boldsymbol{H^T} \tag{44}$$

where $\boldsymbol{a_m}$ represents the columns of Integer Matrix and $\mathbf{v_m}$ represents the columns of integer forcing coefficient matrix **V.** Hence, using the integer forcing coefficient matrix **V**, $\boldsymbol{\hat{Y}}$matrix has been received**,**

$$\boldsymbol{\hat{Y}} = \boldsymbol{V} * \boldsymbol{Y} = \boldsymbol{V} * (\boldsymbol{HZ} + \boldsymbol{W}) = \boldsymbol{A} * \boldsymbol{Z} + \boldsymbol{\widetilde{W}} \tag{45}$$

where $\mathbf{\widetilde{W}} = \mathbf{V*W}$. The original message has been decoded using the $\boldsymbol{A^{-1}}$ matrix and LD decoder.

$$\boldsymbol{\hat{Z}} = \boldsymbol{A^{-1}} * \boldsymbol{\hat{Y}}$$

$$\boldsymbol{vec(X)} = \boldsymbol{£^{-1}} * \boldsymbol{\hat{Z}} \tag{46}$$

where $\mathbf{£^{-1}}$ is the inverse of LD code.

CHAPTER 5

RESULTS AND DISCUSSIONS

The LDC codes have been designed and optimized using the method proposed in the section 4.1. To evaluate its performance, different detection schemes have been employed. First of all, the Sphere Decoding has been used to evaluate the performance of the proposed codes. Afterwards, Zero Forcing, MMSE and Integer Forcing have been compared on the basis of complexity and performance.

5.1. Performance Evaluation of Optimized LD codes with Sphere Decoding Technique

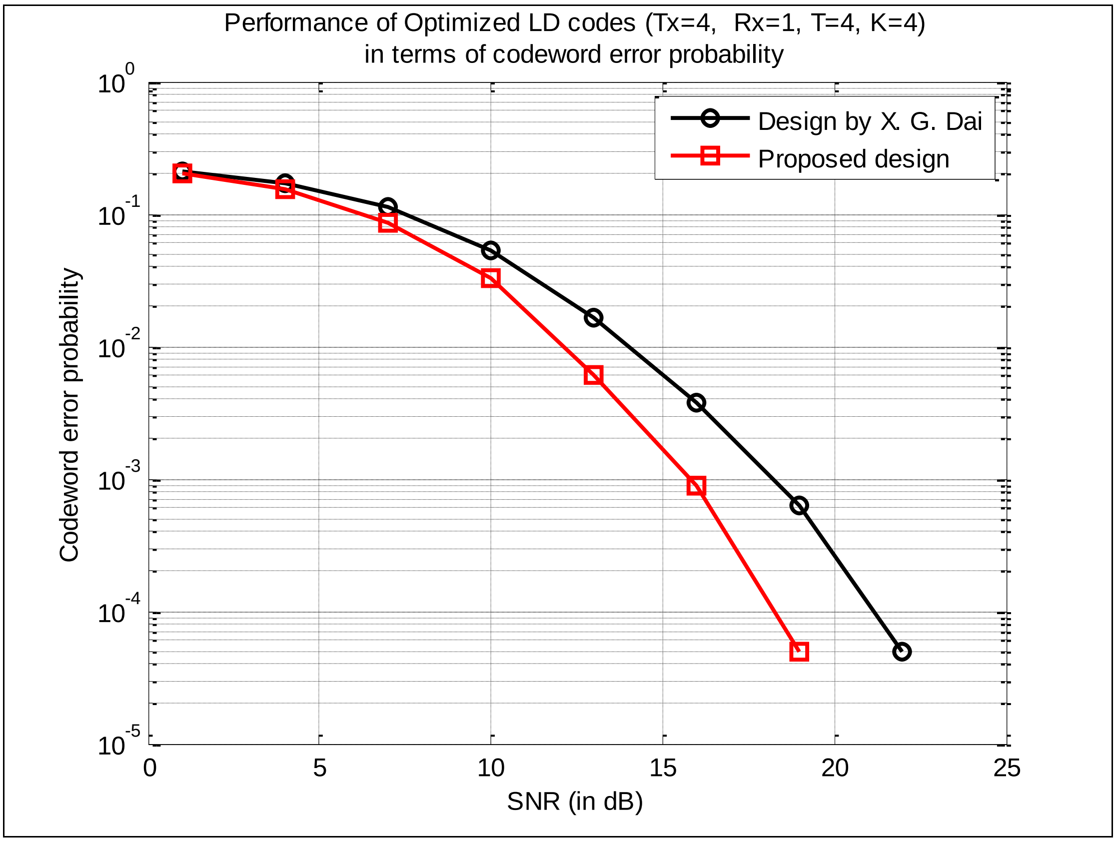

Fig. 5.1. BER performance of optimized Linear Dispersive Codes in comparison with earlier designed codes with Sphere Decoding.

In the thesis work, we compared the performance of the optimized LD code having the configuration of (Tx=4, Rx=1, T=4,K=4) with the LD codes given by X. G. Dai[21]. Sphere Decoding has been used at the receiver side. From the given Fig. 5.1., It has been observed that the optimized Linear dispersive codes have shown better performance as compared to the earlier designed codes. At BER of 10^{-4}, the SNR improvement of 5dB have been observed when compared with the X. G. Dai. codes.

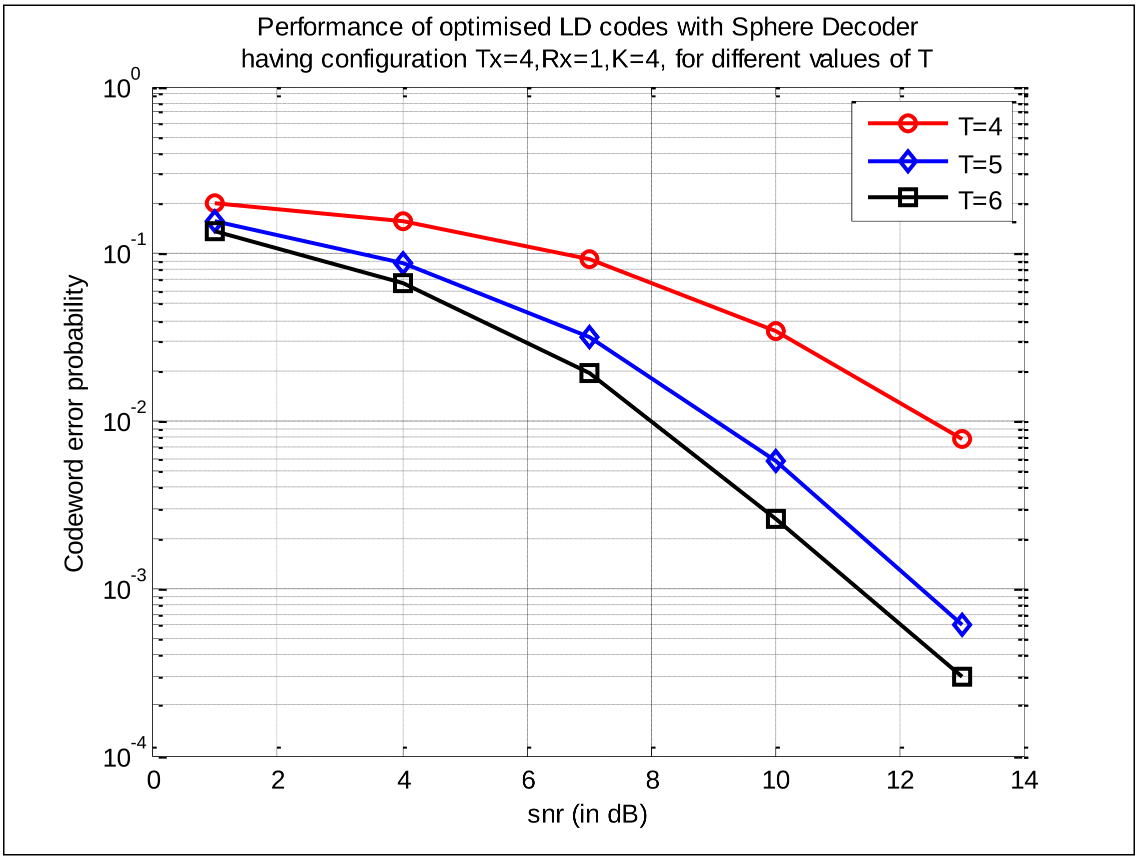

Fig. 5.2. BER performance of optimized Linear Dispersive Codes with Sphere Decoder for increasing values of Time interval *T*.

In Fig. 5.2., we have examined the effect of increasing the value of *T*, keeping the number of symbols, *K* fixed. The error rate was estimated using 25000 Monte Carlo simulations. It has been observed that the error rate has been reduced with the increase in time intervals, *T*. The increase in the value of *T* causes an increase in the size of the error difference matrix. It allows for the increase in the rank. Thus, better performance is achieved.

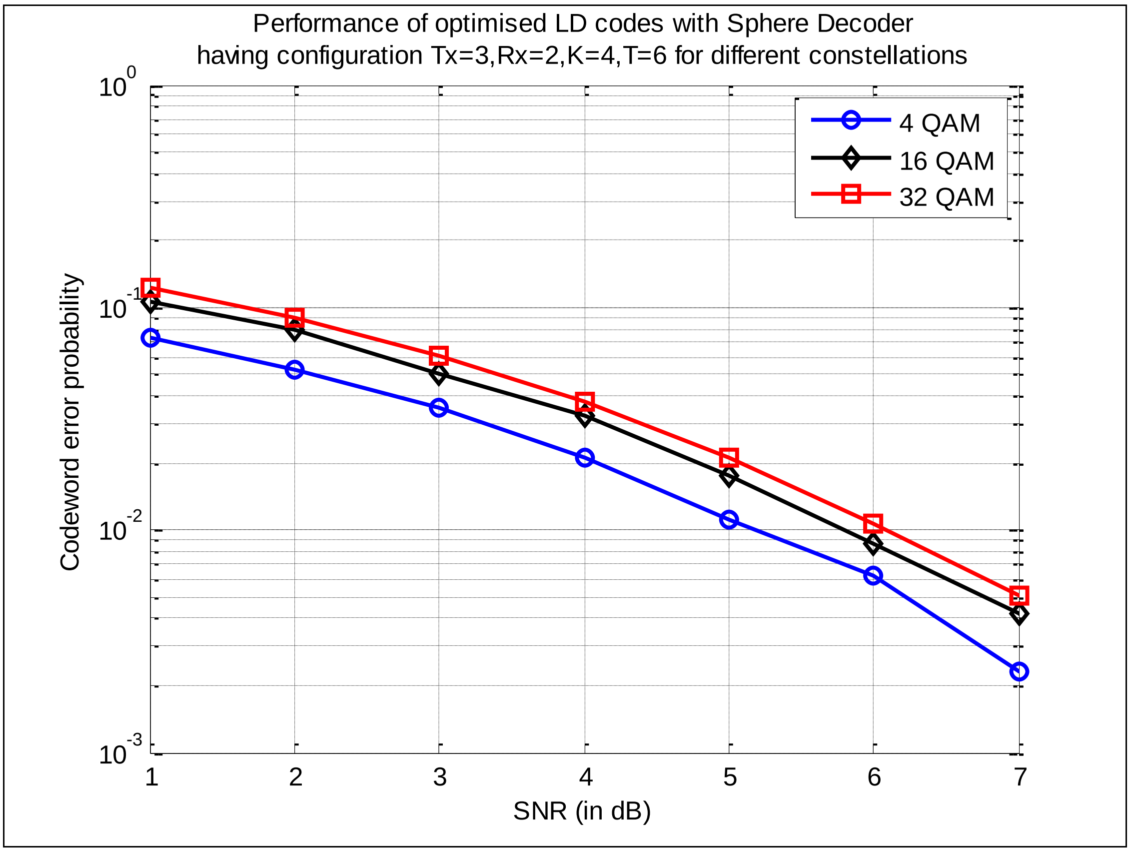

Fig. 5.3. BER performance of optimized Linear Dispersive Codes with Sphere Decoder for different constellations.

In Fig. 5.3., the optimized LD codes' performances have been evaluated for different constellations, namely 4QAM, 16QAM and 32 QAM. Schnorr Euchner Method of sphere decoding has been employed. It has been observed that the error rate increases with the increase in the constellation size for the same SNR, because of the increase in constellation size leads to a reduction in Euclidean distance.

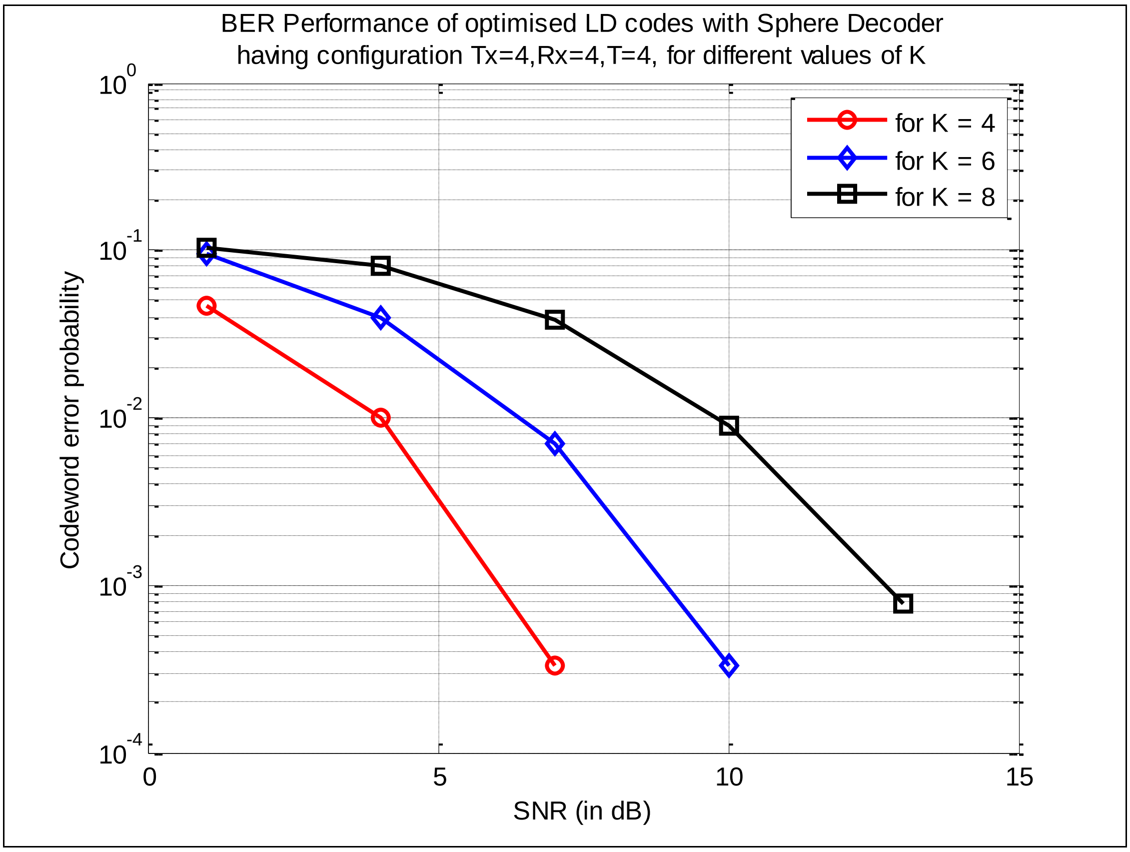

Fig. 5.4. BER performance of optimized Linear Dispersive Codes with Sphere Decoder for different numbers of symbols, K sent.

In Fig. 5.4., the number of symbols to be sent has been increased considerably and the effect of this change has been evaluated. The configuration of Tx=4, Rx=4, *T*=4 has been used, along with the different number of symbols, *K*= 4, 6, 8. It has been observed that the performance degrades with the increase in the number of symbols.

5.2. Performance Evaluation with Integer Forcing Receiver

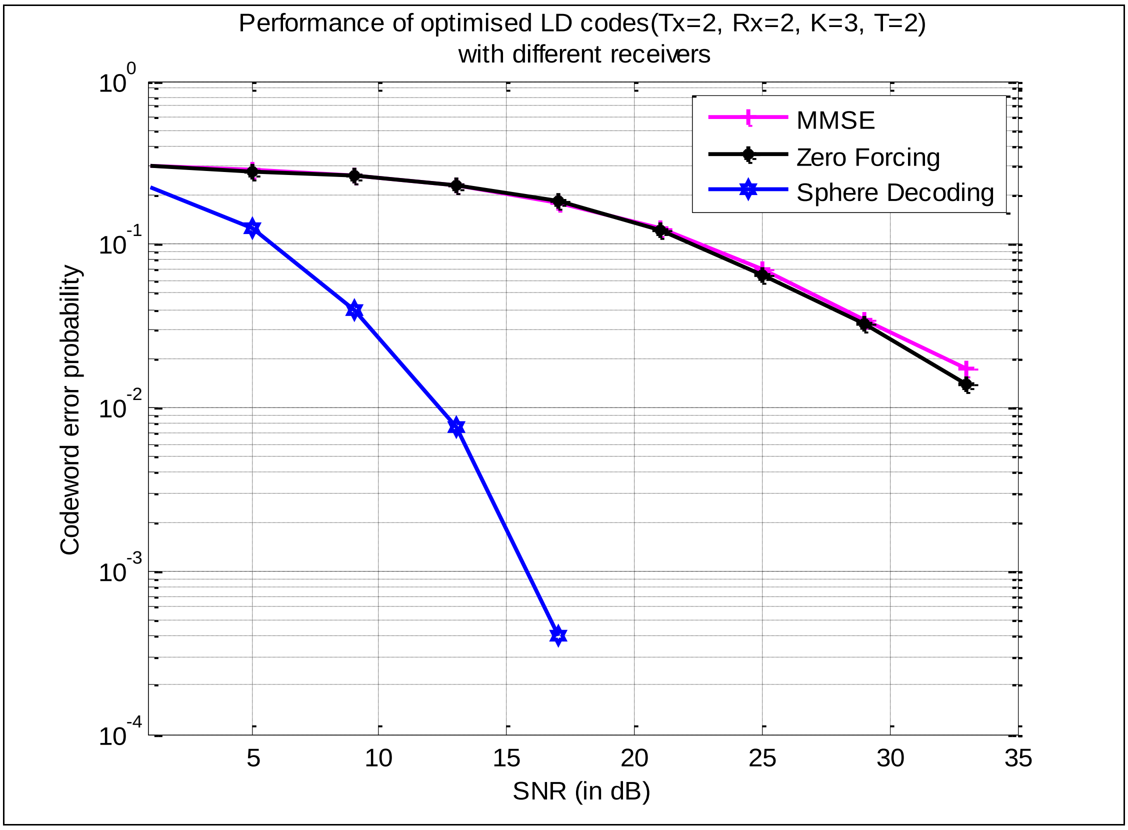

Fig. 5.5(a). Performance evaluation of optimized LD codes with Sphere decoding in comparison to the ZF and MMSE receivers

In Fig. 5.5.(a)., the performance of optimized LD code has been evaluated using different receivers and estimators. Zero Forcing receivers as well as MMSE receivers has been used and compared for the configuration of Tx=2, Rx=2, *K*=3, *T*=2. It has been verified that the complex receivers i.e. Sphere Decoder gives better performance as compared to the linear conventional estimators i.e. Zero Forcing and MMSE receiver which shows low complexity in decoding. At an error rate of 10^{-2}, there has been 19 dB loss in SNR when Low complexity receivers are used as the SNR requirement in case of Sphere decoder has been 13 dB and in case of ZF, MMSE receiver has been 32 dB.

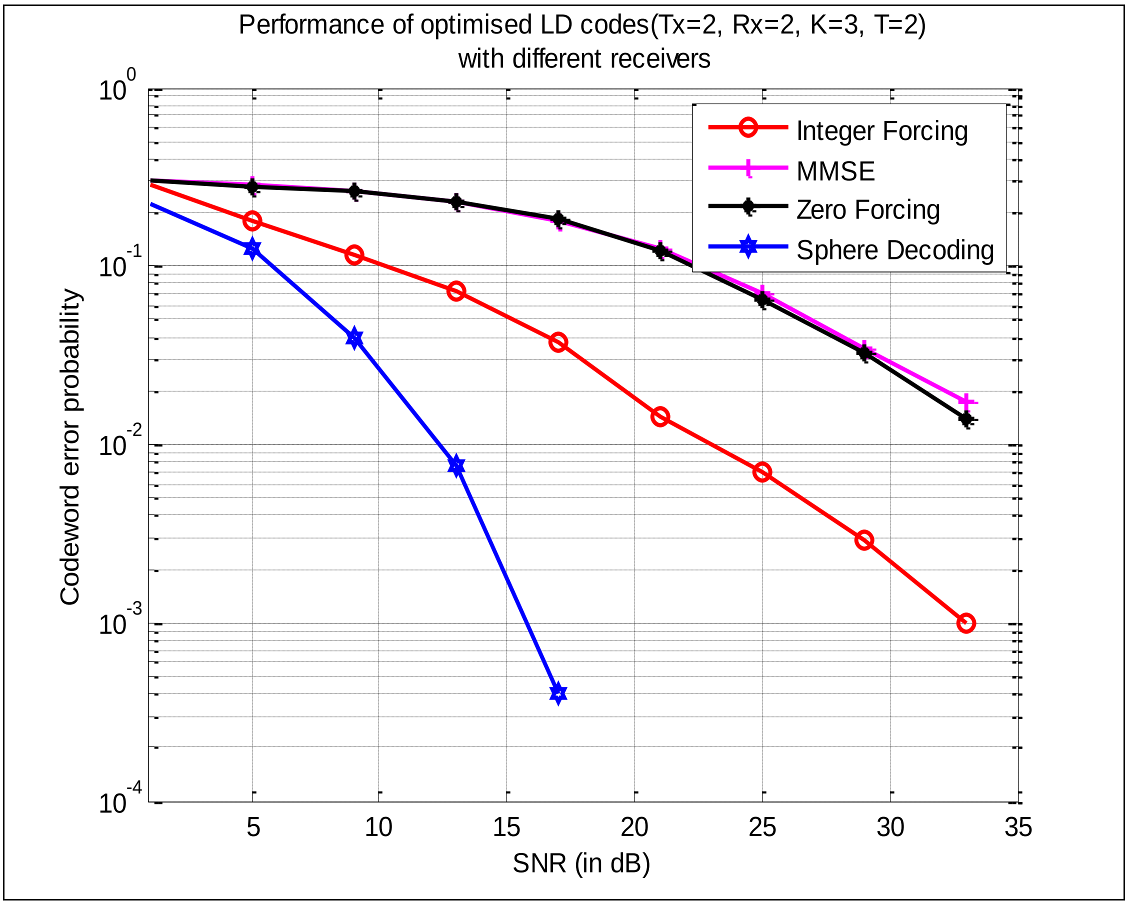

Fig. 5.5(b). Performance evaluation of optimized LD codes with Sphere decoding in comparison to the Integer forcing, ZF and MMSE receivers

Hence, a new estimator has been studied and combined with the optimized LD codes. From Fig. 5.5.(b)., It has been observed that the Integer Forcing receiver shows a significant improvement in performance over the conventional linear receivers(ZF and MMSE). It has reduced SNR requirement for given BER as compared to the linear receivers while incurring a small increse in complexity. Th performance of Integer forcing receiver is however, lower than the sphere Decoding but the high performance is at the cost of too high complexity**.**

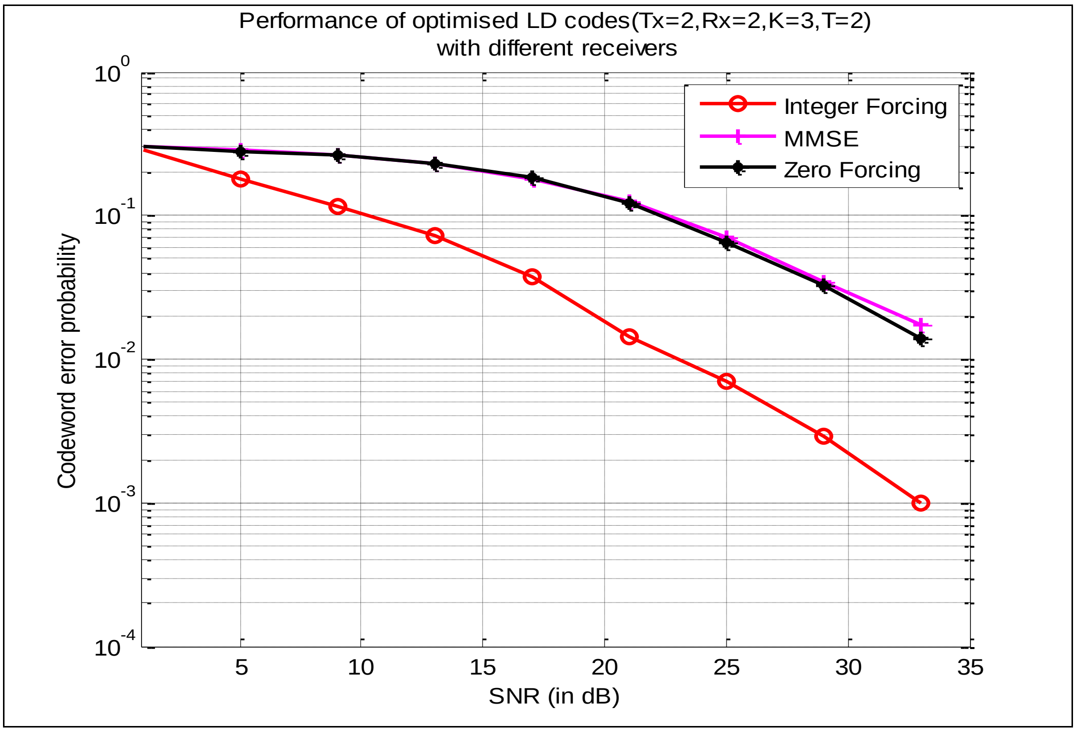

Fig. 5.6.(a). Performance evaluation of optimized LD codes Integer forcing, ZF and MMSE receivers for the configuration of Tx=2, Rx=2, K=3, T=2.

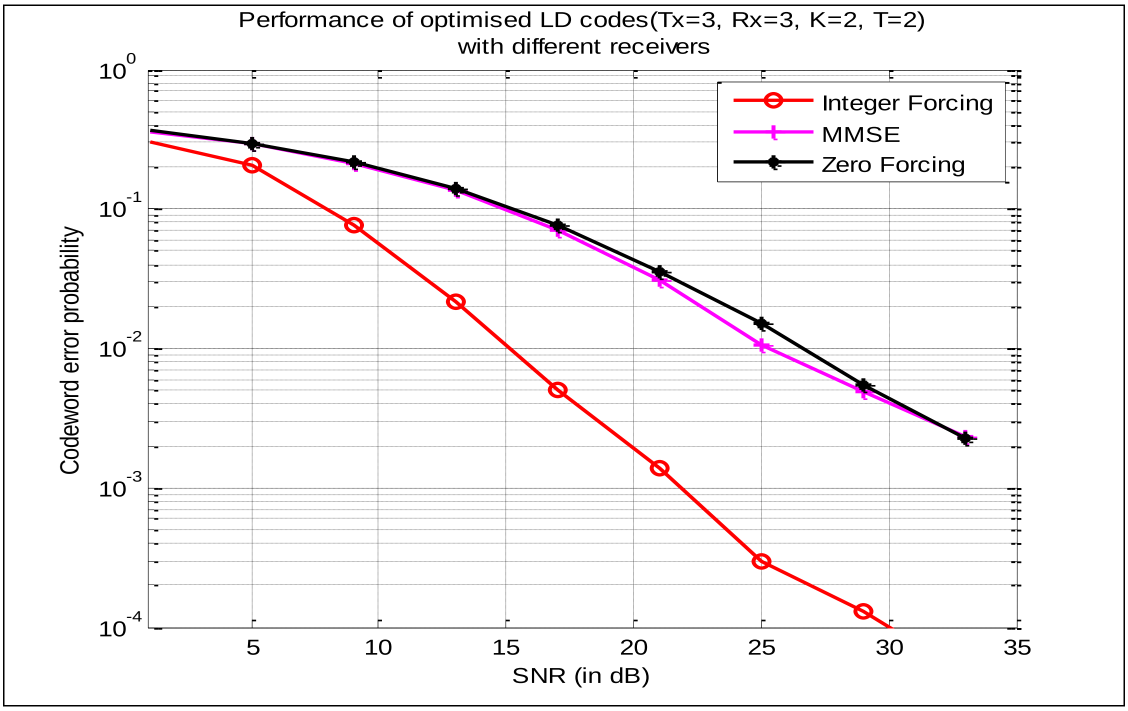

Fig. 5.6.(b). Performance evaluation of optimized LD codes Integer forcing, ZF and MMSE receivers for the configuration of Tx=3, Rx=3, *K*=2, *T*=2.

The performance of the optimized code with Integer Forcing receiver has been compared with the Zero Forcing and MMSE receivers with different configurations, Tx=2, Rx=2, T=3, K=2 and Tx=3, Rx=3, T=2, K=2. From both the graphs, it has been observed that the Integer Forcing receiver performed better than the conventional linear receivers. The performance has been evaluated and shown in the Table 5.1.

Table 5.1. SNR improvement of the proposed codes for various receivers (in dB)

Receiver	**Expression for the Equalization matrix for different schemes**	**SNR required at 10^{-2} bit error rate for(2Tx, 2Rx,3K, 2T)**	**Improvement of IF over ZF and MMSE (in dB)**	**SNR required at 10^{-3} bit error rate for(3Tx, 3Rx,2K, 3T)**	**Improvement of IF over ZF and MMSE (in dB)**
Zero forcing	$(\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\boldsymbol{T}}$	33.5	12.5	36.5	14.5
MMSE	$(\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H} + \frac{1}{P}\boldsymbol{I}_{\mathrm{N_R}})^{-1}\boldsymbol{H}^{\boldsymbol{T}}$	33	12	36.5	14.5
Integer forcing	$\boldsymbol{v}_{\boldsymbol{m}}^{\boldsymbol{T}} = \boldsymbol{a}_{\boldsymbol{m}}^{\boldsymbol{T}} * (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H} + \frac{1}{P}\boldsymbol{I}_{\mathrm{N_R}})^{-1}\boldsymbol{H}^{T}$	21	-	22	-

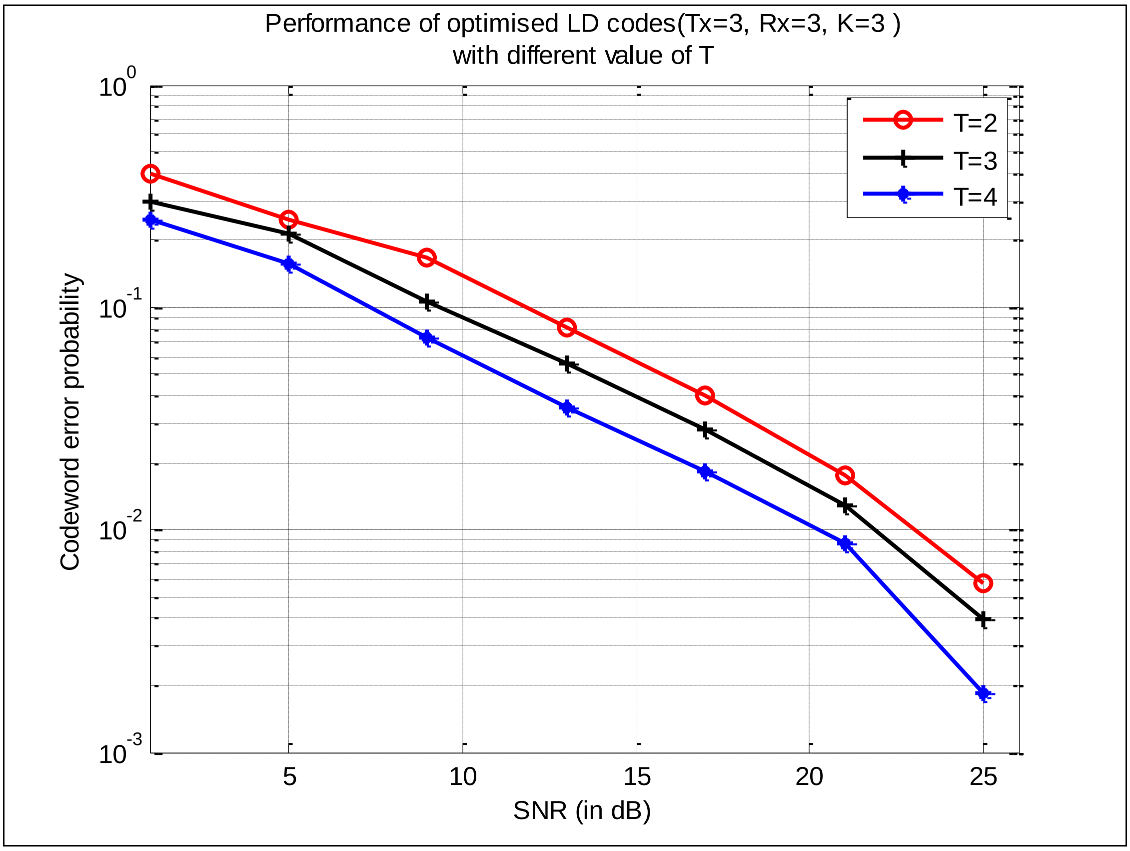

Fig. 5.7. BER performance of optimized LD codes with Integer Forcing receiver for increasing values of T.

In Fig. 5.7., the performance of the optimized LD codes has been studied for different values of *T*. The effect of increasing the value of *T*, keeping the number of symbols, *K* fixed has been evaluated. It has been verified that the there has been reduction in the error rate for the same BER, with the increase in timing intervals, *T* because of the increase in the size of the error difference matrix.

CHAPTER 6

CONCLUSION AND FUTURE SCOPE

The Linear dispersive codes have been designed and optimized using different criteria and their performance has been evaluated with different decoders. In this section, brief overview of the thesis work and future scope have been mentioned.

6.1 Conclusion

The proposed method for the designing and optimization of Linear Dispersive Space Time Block Codes has been proved to be better than the earlier designed codes. There has been 5 dB improvement in the SNR at 10^{-4} bit error rate when Sphere Decoding has been employed with the optimized Linear Dispersion codes. The combination of Tight frame criteria, Standard Power Constraint, Equal Power Constraint, Diversity criterion facilitated us in the designing of the matrices while the NVD property and the Rank and Determinant Criterion offers optimization and helped selecting the best matrix for the encoding. The selected matrix has achieved full rate and an optimal Diversity-Multiplexing Gain tradeoff. The performance evaluation of LD codes with different receivers, namely Zero Forcing, MMSE receiver and Integer Forcing receiver has proved that the Integer Forcing receiver has been a better estimator as compared to the ZF, MMSE in terms of the BER performance while incurring a small increase in complexity. The improvement of 14.5 dB has been achieved using Integer forcing over ZF and MMSE receiver at an error rate of 10^{-3} using the Monte Carlo simulations.

6.2 Future Scope

- Proposed work can be combined with different cooperative diversity schemes.
- Proposed work can be extended with other designing techniques proposed for the Integer Forcing coefficient matrix such as Lattice reduction technique, etc.
- Proposed work can be explored with cognitive radios.

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