

**EFFECTS OF MAGNETIC FIELD ON ROTATIONALLY AND
TIDALLY DISTORTED WHITE DWARF MODELS OF STARS**

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CERTIFICATE

This is to certify that the thesis entitled “**EFFECTS OF MAGNETIC FIELD ON ROTATIONALLY & TIDALLY DISTORTED WHITE DWARF MODELS OF STARS**”, being presented in partial fulfilment of requirements for the award of the degree of Master of Science in Mathematics to School of Mathematics (SOM),Thapar Institute of Engineering & Technology, Patiala, is an authentic record of my own work carried out under the supervision of Dr. Arvind Kumar Lal.

The matter presented in this thesis has not been submitted for the award of any other degree from this or any other institute.



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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.



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ABSTRACT

In the present thesis, we have used the approach of Mohan, Saxena & Aggarwal along with the approach of Kopal to study the effects of magnetic field on Rotationally and tidally distorted White Dwarf Models of Stars.

In Theoretical Models, Stars are often regarded as self-gravitating spheres composed mainly of gases. Theoretical Analysis is the determination of Equilibrium Structure (ES) of Stars. This involves studying the distribution of mass, pressure, temperature and density throughout a star's interior. By accurately modelling these factors, scientists can gain insights into the physical processes that sustain a star's stability and prevent its collapse. These studies have been beneficial to the understanding of the stellar structure, its evolution and its behaviour. Moreover, they are used to predict the properties of stars in different stages of their lifecycle.

A binary star consists of two components that are gravitationally bonded to and orbit each other. These stars rotate about their own axis besides revolving around each other. Complex analytical studies have been used to understand the effect of rotation and tidal distortion stellar structure and evolution. While obtaining Roche equipotential surfaces effect of magnetic forces always have been neglected due to its minor effect as compared to centrifugal and gravitational forces. However, recent studies have suggested that magnetic forces should not be overlooked, as they can play a significant role in determining the shape of Roche equipotential surface of stars.

By understanding the effects of magnetic field on rotationally and tidally distorted stars, this research has made a significant effort to both theoretically and computationally analyze the significant impacts of magnetic distortion on the ES of rotationally and tidally distorted gaseous spheres, further research is needed to gain a greater insight into stellar structure.

This thesis consists of 3 Chapters. The First Chapter serves as an introduction to the topic. It consists of the brief explanation on how these binary stellar models can be used to study the effects of rotation, tidal forces and magnetic fields on the evolution of stars.

Chapter two has a brief description about the binary stars thus reinforces the concept of Roche equipotential surfaces (EQS). Here we study the effect of magnetic field on mass transfer that takes place in a binary system and how it leads to significant decrease in the total stellar mass over time. It gives us a glimpse into the methodology used by Kippenhahn and Thomas to gain a deeper understanding of the stellar environment. In order to highlight the importance of inclusion of magnetic distortion on stellar grounds, where it was not thought to be to a significant level to be included in previous stellar studies, the phenomena of magnetic braking have been described.

Chapter three is the last chapter of my thesis. It involves the numerical computations, approaches and procedures used to calculate the stellar parameters. On the basis of the current investigation, several conclusions have been drawn. The chapter comes to a close with a note on the importance of the current study, a discussion of the limitations of our methodology.

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Chapter One

1.1 INTRODUCTION

This Thesis deals with the problem of studying ES of rotationally, tidally and magnetically distorted (RTMD) white dwarf models of stars. This problem has astrophysical significance to understand the inner structure of stars. Theoretically, a Star is assumed to as a gaseous sphere both in hydrostatic and thermal equilibrium. Observations indicate that some of the stars are rotating about their axes. So, EQ of such stars get distorted by rotational forces. Some of the stars are observed as members of binary system. In this case, the ES of such stars get distorted by tidal force if it is not rotating and rotating and tidal both of its rotating. It is also observed that in binary stars magnetic field arises which results in different dynamical phenomena such as solar flare, X-ray emission etc. Obviously, the ES of this type of stars will be RTMD if it is rotating and a part of binary system. In the present thesis, we have considered the possibilities using Mohan, Saxena and Aggarwal approach along with the result earlier obtained by Vatsala (2016) [39] to determine the ES of RTMD white dwarf stars.

1.2 Literature Review

The several authors extensively studied ES of star and stars in binary system assuming to be undistorted. Since then, a great deal of research has been carried out on the issue of determining the ES of rotationally and tidally distorted stellar models.

Chandrashekhar [2] originally suggested a first order analysis to look into the rotational, tidal, force on binary stars series of problems. The effects of EQS of the stars are estimated by Roche equipotential and Authors [12,13] have suggested an analytical method of evaluating the impact of rotation and tidal deformations on ES of the stars or a system of stars. Lal et al. [16] have taken into account both rotational and tidally distortion on the ES.

In order to use this concept of rotating stars in binary systems, Kopal [14] developed a coordinate system known as the Roche coordinates. Mohan & Saxena [23] determine the cumulative effects of rotation and tidal distortions on the ES and oscillations of the white dwarf model of stars.

The literature has covered intricate analytical efforts to determine the ES of rotationally and tidally distorted stars. To study such issues, Kopal introduced the idea of Roche equipotential in 1972. Since then, this issue has been addressed by other authors, including Mohan and Singh [23], [14], [16], [17], and [21]. This method accounts for the effects of gravity, Coriolis, and centrifugal forces but ignores the impact of magnetic force. Due to the magnetic force's predicted small influence in comparison to gravitational and centrifugal forces, it has typically been disregarded when deriving Roche equipotential statistics of stars.

But the magnetic field is also one of the things that makes stars spherical. Numerous attempts have been made in the literature to factor magnetic fields impact and the resulting Lorentz force into the equation describing stellar structure. The effects of magnetic fields on the stars EQS have been examined by authors [4,5,8]. Due to the non-radial

Lorentz Force's presence, it was challenging to analyse the scenario. Djurasevic [4] examined the critical surfaces in a closed binary system by taking into account the radial component of the magnetic field, or magnetic pressure, in contrast to other writers who only considered the toroidal component of the magnetic field while formulating this problem.

Vatsala (2016) [39] studied ES of RTMD polytropic models of stars. However, effects of RTM force on White Dwarf has not been satisfactorily carried out. The study of such a magnetic field's impact on the forms and ES of rotationally and tidally distorted stars could be a fruitful search direction.

In the current work, an effort has been made to determine the influence of magnetic pressure on the ES of rotationally, tidally and distorted white dwarf stars, using the assumptions of [4] and the methodology of [16].

1.3 Averaging Methods of Kippenhahn and Thomas

Kippenhahn and Thomas [13] define the model of the topologically equivalent spherical surface, consistent with actual equipotential surfaces of rotationally and tidally distorted stellar models, to examine the effect of RTD on the ES of the star.

They define quantities such as \bar{f}, \bar{g} etc. on the actual EQS, which are certain average of the quantities \bar{f}, \bar{g} respectively.

Suppose φ represents the total potential on any random point P (x, y, z). Then $\varphi(x, y, z) = \text{constant}$, is an EQS. Let V_φ and S_φ be the volume and surface area, respectively of this equipotential surface.

They define the mean value of any function $f(x, y, z)$ over EQS

$\varphi = \text{constant}$ by its relation

$$\bar{f} = \frac{1}{S_\varphi} \int f d\sigma \quad (1.1)$$

where $d\sigma$ denotes a minor surface component of φ . Evidently \bar{f} calculated by φ , determines for all φ . Let V_ψ be the volume enclosed and S_ψ be the surface area of the EQS $\varphi = \text{constant}$. Then similar to the sphere, Kippenhahn and Thomas define a variable r_ψ by the relation:

$$V_\psi = \frac{4}{3} \pi r_\psi^3 \quad (1.2)$$

However, the surface area is given by

$$S_\psi = \int d\sigma \quad (1.3)$$

Kippenhahn and Thomas defined the function $g(x, y, z)$ by incorporating the gravitational force acting on a star.

$$g = \frac{d\varphi}{dn} \quad (1.4)$$

Here function g represents the stellar gravity. The distance dn between two neighbouring surfaces φ and $\varphi + d\varphi (= \text{constant})$, is not constant in general. it varies at different locations on the surface. They used an equation (1.14) to obtain \bar{g} and $\overline{g^{-1}}$, by using the formula:

$$\bar{g} = \frac{1}{S_\varphi} \int \frac{d\varphi}{dn} d\sigma \quad (1.5)$$

$$\overline{g^{-1}} = \frac{1}{S_\varphi} \int \left(\frac{d\varphi}{dn}\right)^{-1} d\sigma \quad (1.6)$$

Both \bar{g} and $\overline{g^{-1}}$ represent the values of g and g^{-1} over the topologically equivalent spherical surface. Between the surface φ and $\varphi + d\varphi$ equals volume:

$$dV_\varphi = \int dn d\sigma = \int \left(\frac{d\varphi}{dn}\right)^{-1} dn = S_\varphi \overline{g^{-1}} d\varphi \quad (1.7)$$

The following non-dimensional parameters are defined for this approach:

$$\begin{aligned} u &= \frac{S_\varphi}{4\pi r_\varphi^2} \\ v &= \frac{\bar{g} r_\varphi^2}{GM_\varphi} \\ w &= \frac{\overline{g^{-1}} GM_\varphi}{r_\varphi^2} \end{aligned} \quad (1.8)$$

M_φ represents the mass enclosed by ES $\varphi = \text{constant}$.

Equations (1.3) and (1.6) are pure mathematical logic assertions that were applied to obtain rotationally and tidally distorted gravity fields of stars. EQS become equi-pressure and equi-density surfaces when hydrostatic equilibrium is reached. Therefore, the pressure P_φ and density ρ_φ are constant for an equipotential surface.

Equation (1.1) and these concepts serve to explain the mass of the equipotential surface φ and $\varphi + d\varphi$.

$$dM_\varphi = dV_\varphi \rho_\varphi = 4\pi r_\varphi^2 \rho_\varphi dr_\varphi \quad (1.9)$$

We get,

$$\frac{dM_\varphi}{dr_\varphi} = 4\pi r_\varphi^2 \rho_\varphi \quad (1.10)$$

Equations (1.8) and (1.9) yields

$$\begin{aligned} d\varphi &= \frac{d\varphi}{dV_\varphi} dV_\varphi = \left(\frac{dV_\varphi}{d\varphi} \right)^{-1} \frac{dM_\varphi}{\rho_\varphi} \\ &= dM_\varphi / S_\varphi \bar{g}^{-1} \rho_\varphi \end{aligned} \quad (1.11)$$

From (1.8), we get

$$d\varphi = GM_\varphi \frac{dM_\varphi}{4\pi r_\varphi^4 \rho_\varphi u w} \quad (1.12)$$

The equation for hydrostatic equilibrium is

$$\frac{dP_\varphi}{d\varphi} = -\rho_\varphi \cdot$$

On simplification, we get

$$\frac{dP_\varphi}{dM_\varphi} = -\frac{GM_\varphi}{4\pi r_\varphi^4} f_p \quad (1.13)$$

Here ,

$$f_p = \frac{1}{u w} = \frac{4\pi r_\varphi^4}{GM_\varphi} \frac{1}{\bar{g}^{-1} S_\varphi}$$

f_p is once again determined by the value of φ . Thus, if φ is recognised, we can establish the equipotential surface, and from the geometry of the equipotentials, we can derive the values of $S_\varphi, r_\varphi, \bar{g}$ and \bar{g}^{-1} for individual equipotential surfaces. The mass M_φ is determined by the density profile ρ_φ and can be calculated by integrating equation (1.12). Kippenhahn and Thomas obtained comparable structural equations by include the influence of rotational and tidal distortions on the ES of star systems.

The rate of creation of nuclear energy (ε) in chemically homogeneous spherical domains depends on two parameters, namely density (ρ_φ) and temperature T_φ and is thus constant on an equipotential surface. Assuming L_φ is the energy emitted per unit second across a certain equipotential surface $\varphi=(\text{constant})$, then

$$\frac{dL_\varphi}{dM_\varphi} = \varepsilon \quad (1.14)$$

We may see from equation (12) that

$$\frac{dL_\varphi}{dM_\varphi} = 4\pi r_\varphi^4 \rho_\varphi \varepsilon \quad (1.15)$$

$$\frac{dT_\varphi}{dM_\varphi} = -\frac{3KL_\varphi}{64\pi^2 acT_\varphi^3 r_\varphi^4} f_T \quad (1.16)$$

From (1.9), we can write the above equation as

$$\frac{dT_\varphi}{dr_\varphi} = -\frac{3K\rho_\varphi L_\varphi}{16\pi^2 acT_\varphi^3 r_\varphi^2} f_T \quad (1.17)$$

$$\text{Where } f_T = \frac{1}{u^2 v w} \quad (1.18)$$

Thus equations (1.10), (1.13), (1.14), and (1.16), respectively, represent the fundamental stellar equations of RTD stars.

These can be expressed as follows:

$$1. \quad \frac{dM_\varphi}{dr_\varphi} = 4\pi r_\varphi^2 \rho_\varphi \quad (1.19)$$

$$2. \quad \frac{dP_\varphi}{dM_\varphi} = -\frac{GM_\varphi}{4\pi r_\varphi^4} f_P \quad (1.20)$$

$$3. \quad \frac{dL_\varphi}{dM_\varphi} = \varepsilon \quad (1.21)$$

$$4. \quad \frac{dT_\varphi}{dM_\varphi} = -\frac{3KL_\varphi}{64\pi^2 acT_\varphi^3 r_\varphi^4} f_T \quad (1.22)$$

Here $f_P = \frac{1}{uw}$ and $f_T = \frac{1}{vwu^2}$

By setting each of the distortion parameters $u = v = w = 1$, above equations can be reduced to the original fundamental equations of the ES.

The following are the boundary conditions required to satisfy the aforementioned equations:

$$M_\varphi = 0, L_\varphi = 0 \quad (1.23)$$

At centre $r_\varphi = 0, M_\varphi = M_0, L_\varphi = L_{\varphi S}, P_\varphi = 0, T_\varphi = 0$ or $P_\varphi = P_{\varphi S}$ and $T_\varphi = T_{\varphi S}$

At the free surface $r_\varphi = R_\varphi \quad (1.24)$

M_0 is the total mass of the binary stellar system, while $L_{\varphi S}, P_{\varphi S}, T_{\varphi S}$ are the values of $L_\varphi, T_\varphi, P_\varphi$ at the outermost equipotential surface.

1.4 Modified Roche Equipotential Surface

Let M_0 and M_1 be the masses of primary and secondary components of binary system assumed to be gaseous sphere

Following Kopal's approach, the masses of major and minor components are believed to be M_0 and M_1 , with M_0 being greater than M_1 i. e. $M_0 \gg M_1$. The internal arrangement of a large star is approximated by the Roche Model, which assumes that the total stellar mass is focused at its centre and that this point mass is enveloped in a transient envelope with a density profile that is inversely proportional to the squared distance from the centre. The influence of a magnetic field on the critical surfaces of rotationally and tidally distorted stars (RTD) has been in the present address. Djurašević [4] formulated the problem by taking into account the effects of radiation from both components of the binary star system while ignoring the magnetic field. In contrast, we simply considered the magnetic field in this investigation and ignored any radiation impacts. As a result, only the influence of the magnetic field attributable to the primary component is considered when calculating the potential equation. Because it is presumed to be a point mass star, the magnetic effect caused by the secondary component could not be evaluated.

Let D be the distance between the centres of primary M_0 and secondary component M_1 of binary system. Further suppose that the position of the two components of binary system is referred to rectangular system whose origin is at the centre of gravity of M_0

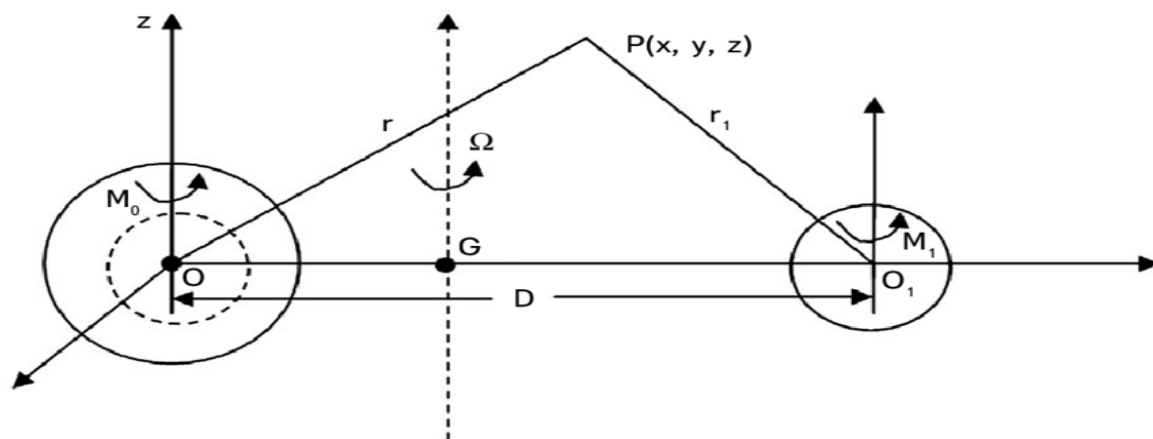


Fig 1: Axis of reference for a binary system

where

$$r_1 = \sqrt{x^2 + y^2 + z^2}, r_2 = \sqrt{(D - x)^2 + y^2 + z^2}$$

$$\text{and } r = \sqrt{(x - d_1)^2 + y^2 + z^2}.$$

These represent the separation of a point P (x, y, z) from the centre of gravity of the major star (point O), the mid-point of gravity of the minor star (point O_1) and the mid-point of gravity of the entire system (point C).

Following the approach as discussed by Vatsala (2016) [39] and earlier used by Djurašević [4], the total potential at a point P (x,y,z) that experiences magnetic forces in addition to gravitational and centrifugal forces is given by

$$\begin{aligned} \varphi = & \frac{GM_0}{r_1} + \frac{GM_1}{r_2} + \frac{1}{2}(\vec{\Omega} \times \vec{r}) \cdot (\vec{\Omega} \times \vec{r}) - \\ & \left(\frac{1}{2}\right)^{\frac{1}{4}} \frac{kT_1}{2\mu m_H} \left[1 \sqrt{1 - \left(\frac{R_1}{r_1}\right)^2}\right]^{\frac{1}{4}} \left(5 + 4 \frac{P_m}{P_g}\right) \end{aligned} \quad (1.25)$$

where the first two expressions correspond to gravitational potential, the third component is due to centrifugal force, and the final term is related to magnetic pressure.

The symbols are the same as Djurašević [4]. The third term is the cross product of the angular velocity along the -z-axis and \vec{r} .

On replacing by using binomial expansion, we get

$$\frac{kT_1}{2\sqrt{2}\mu m_H} \left(5 + 4 \frac{P_m}{P_g}\right) \sqrt{\frac{R_1}{r_1}} \quad (1.26)$$

If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{\Omega} = \Omega\hat{k}$, so the term becomes

$$\vec{\Omega} \times \vec{r} = \Omega^2[(x - d)^2 + y^2] \quad (1.27)$$

Thus, the modified expression of potential now becomes:

$$\varphi = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} + \frac{1}{2}\Omega^2[(x-d)^2 + y^2] - \frac{kT_1}{2\sqrt{2}\mu m_H} \sqrt{\frac{R_1}{r_1}} \left(5 + 4\frac{P_m}{P_g}\right) \quad (1.28)$$

Following Kopal (1972), equation (1.26) can be expressed as:

$$\varphi^* = \frac{1}{r^*} + q \left(\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^* \right) + n(1-v^2)r^{*2} - \frac{\alpha}{\sqrt{r^*}} \quad (1.29)$$

where $\varphi^* = \frac{D\varphi}{GM_0} - \frac{M_1^2}{2M_0(M_0+M_1)}$ is the non-dimensional form of φ ,

and $r^* = \left(\frac{r_0}{D}\right)$ is non-dimensional form of r_0 .

$q = \frac{M_1}{M_0}$ is the mass ratio, called as tidal parameter

For synchronous rotation we have,

$$n = \frac{(q+1)}{2}$$

Here n is called rotational parameter and magnetic parameter

α is given by:

$$\alpha = \left(\frac{D}{GM_0}\right) \frac{kT_1}{2\sqrt{2}\mu m_H} \left(5 + 4\frac{P_m}{P_g}\right) \sqrt{\frac{R_1}{r_1}} \quad (1.30)$$

Also $\lambda = \sin \theta \cos \phi$, $\mu = \sin \theta \sin \phi$, $\nu = \cos \theta$; r, θ, ϕ being the spherical polar coordinates of P. For binary systems in synchronous rotation the angular velocity Ω is identified with the Keplerian angular velocity so that

$$\Omega^2 = \frac{G(M_0+M_1)}{d^3} \quad (1.31)$$

This relation expressed in terms of the non-dimensional variables of equation (1.22h) becomes

$$n = \frac{q+1}{2} \quad (1.32)$$

Assuming the star's composition is dominated by ionised hydrogen, $\mu = \frac{1}{2}$ and substituting for the value of the Boltzmann constant $k = 1.38 \times 10^{-23} m^2 kgs^{-2} K^{-1}$, the value of α ranges from 0 and 1.

α , like tidal and rotational parameters, is a non-dimensional parameter. The surface generated by substitution $\psi^* = constant$ is called Roche Equipotential Surface.

On substituting $\alpha = 0$ in (1.29) it reduces to a Roche equipotential of rotationally and tidally distorted stars.

From a computational standpoint, equation (1.29) can be represented as follows using Legendre Polynomials $P_j(\lambda)$ as:

$$\varphi^* = \frac{1}{r^*} + q + q \sum_{j=2}^{\infty} r^{*j} P_j(\lambda) + n(1 - v^2)r^{*2} - \frac{\alpha}{\sqrt{r^*}} \quad (1.33)$$

Following Vatsala (2016) [39], r^* the shape of Roche equipotential, Volume V_ψ , Surface Area S_ψ and gravity \bar{g} and \bar{g}^{-1} of equipotential surface are given as:

$$\begin{aligned} r^* = r_0 & \left[1 - \alpha r_0^{\frac{1}{2}} + \frac{\alpha}{2} r_0 + \{qP_2 + n(1 - v^2)\} r_0^3 + \left\{ -\frac{7}{2} \alpha (qP_2 + \right. \right. \\ & \left. \left. n(1 - v^2)) \right\} r_0^{\frac{7}{2}} + qP_3 r_0^4 + \left\{ -\frac{9}{2} \alpha q P_3 \right\} r_0^{\frac{9}{2}} + qP_4 r_0^5 + \right. \\ & \left. \left\{ -\frac{11}{2} \alpha q P_4 \right\} r_0^{\frac{11}{2}} + \{qP_5 + 3P_2^2 q^2 + 6qn(1 - v^2)P_2 + 3n^2(1 - \right. \\ & \left. v^2)^2\} r_0^6 + \left\{ -\frac{13}{2} \alpha q P_5 \right\} r_0^{\frac{13}{2}} + \{qP_6 + 7P_3 P_2 q^2 + 7qn(1 - \right. \\ & \left. v^2)P_3\} r_0^7 + \left\{ -\frac{15}{2} \alpha q P_6 \right\} r_0^{\frac{15}{2}} + \{qP_7 + 8P_4 P_2 q^2 + 4P_3^2 q^2 + 8qn(1 - \right. \end{aligned}$$

$$\begin{aligned}
&v^2)P_4\}r_0^8 + \left\{-\frac{17}{2}\alpha qP_7\right\}r_0^{\frac{17}{2}} + \{qP_8 + 9P_5P_2q^2 + 9P_4P_3q^2 + \\
&9qn(1-v^2)P_5\}r_0^9 + \left\{-\frac{19}{2}\alpha qP_8\right\}r_0^{\frac{19}{2}} + \{qP_9 + 10P_6P_2q^2 + \\
&10P_5P_3q^2 + 5P_4^2q^2 + 10qn(1-v^2)P_6\}r_0^{10} + \dots]
\end{aligned} \tag{1.34}$$

$$\begin{aligned}
V_\varphi = \frac{4}{3}\pi r_0^3 D^3 \left[1 - 3\alpha\sqrt{r_0} + \frac{5}{2}\alpha^2 r_0 + 2nr_0^3 - 11\alpha nr_0^{3.5} \right. \\
\left. + \left\{\frac{12}{5}q^2 + \frac{8}{5}qn + \frac{32}{5}n^2\right\}r_0^6 + \frac{15}{7}q^2 r_0^8 + 2q^2 r_0^{10} + \dots \right]
\end{aligned} \tag{1.35}$$

$$\begin{aligned}
r_\varphi = r_0 D \left[1 - \alpha r_0^{\frac{1}{2}} + \frac{1}{2}\alpha^2 r_0 + \frac{2}{3}nr_0^3 - \frac{7}{3}\alpha nr_0^{\frac{7}{2}} + \left\{\frac{4}{5}q^2 + \frac{8}{15}qn + \right. \right. \\
\left. \left. \frac{76}{45}n^2\right\}r_0^6 + \frac{5}{7}q^2 r_0^8 + \frac{2}{3}q^2 r_0^{10} + \dots \right]
\end{aligned} \tag{1.36}$$

$$\begin{aligned}
S_\varphi = 4\pi r_0^2 D^2 \left[1 - 2\alpha r_0^{\frac{1}{2}} + 2\alpha^2 r_0 + \frac{4}{3}nr_0^3 - 6\alpha nr_0^{\frac{7}{2}} + \left\{\frac{7}{5}q^2 + \right. \right. \\
\left. \left. \frac{14}{15}qn + \frac{56}{15}n^2\right\}r_0^6 + \frac{9}{7}q^2 r_0^8 + \frac{11}{9}q^2 r_0^{10} + \dots \right]
\end{aligned} \tag{1.37}$$

$$\begin{aligned}
\bar{g} = \frac{GM_\psi}{r_0^2 D^2} \left[1 + 2\alpha r_0^{\frac{1}{2}} - 2\alpha^2 r_0 - \frac{32}{15}nr_0^3 + 2\alpha nr_0^{\frac{7}{2}} + \frac{157}{5}\alpha^2 nr_0^4 + \right. \\
\left\{-\frac{7}{5}q^2 - \frac{14}{15}qn - \frac{86}{15}n^2\right\}r_0^6 + \left\{-\frac{28}{5}\alpha q^2 - \frac{56}{15}\alpha qn - \frac{2588}{63}\alpha n^2\right\}r_0^{\frac{13}{2}} + \\
\left\{\frac{28}{5}\alpha^2 q^2 + \frac{56}{15}\alpha^2 qn + \frac{920}{7}\alpha^2 n^2\right\}r_0^7 - \frac{9}{7}q^2 r_0^8 - \frac{36}{7}\alpha q^2 r_0^{\frac{17}{2}} + \\
\left\{\frac{36}{7}\alpha^2 q^2 + \frac{728}{225}qn^2 + \frac{364}{75}nq^2\right\}r_0^9 + \left\{-\frac{1232}{75}\alpha nq^2 - \frac{224}{45}\alpha qn^2\right\}r_0^{\frac{19}{2}} + \\
\left\{\frac{11}{9}q^2 + \frac{154}{75}\alpha^2 qn^2 + \frac{77}{25}\alpha^2 nq^2\right\}r_0^{10}
\end{aligned} \tag{1.38}$$

$$\begin{aligned}
\bar{g}^{-1} = & \frac{r_0^2 D^2}{GM_\psi} \left[1 - 2\alpha r_0^{\frac{1}{2}} + 2\alpha^2 r_0 + \frac{32}{15} n r_0^3 - \frac{158}{15} \alpha n r_0^{\frac{7}{2}} + \frac{827}{30} \alpha^2 n r_0^4 + \right. \\
& \left. \left\{ -\frac{7}{5} q^2 - \frac{15}{15} q n + \frac{4066}{315} n^2 \right\} r_0^6 + \left\{ -\frac{112}{15} \alpha q n + \frac{3116}{63} \alpha n^2 \right\} r_0^{\frac{13}{2}} + \right. \\
& \left. \left\{ \frac{84}{5} \alpha^2 q^2 + \frac{616}{15} \alpha^2 q n + \frac{23426}{105} \alpha^2 n^2 \right\} r_0^7 - \frac{9}{7} q^2 r_0^8 + \frac{72}{7} \alpha q^2 r_0^{\frac{17}{2}} + \right. \\
& \left. \left\{ -\frac{36}{7} \alpha^2 q^2 - \frac{144}{7} \alpha q^2 - \frac{16}{3} \alpha^2 n^2 \right\} r_0^9 + \left\{ -\frac{1372}{75} \alpha q^2 n - \right. \right. \\
& \left. \left. \frac{952}{25} \alpha q n^2 \right\} r_0^{\frac{19}{2}} + \left\{ -\frac{11}{9} q^2 + \frac{1757}{10} \alpha^2 q^2 n + \frac{65779}{225} \alpha^2 q n^2 \right\} r_0^{10} \quad (1.39)
\end{aligned}$$

Similarly, the distortion parameters u, v, w, f_p, f_T are obtained by Vatsala (2016) [39] are given as:

$$\begin{aligned}
u = & 1 + \frac{8}{3} n r_0^3 + \frac{8}{3} \alpha^2 n r_0^4 + \left\{ -\frac{1}{5} q^2 - \frac{2}{15} q n - \frac{4}{45} n^2 \right\} r_0^6 + \left\{ \frac{6}{5} \alpha q^2 + \right. \\
& \left. \frac{4}{5} \alpha q n - \frac{676}{45} \alpha n^2 \right\} r_0^{\frac{13}{2}} + \left\{ \frac{58}{5} \alpha^2 q^2 + \frac{116}{15} \alpha^2 q n + \frac{6167}{45} \alpha^2 n^2 \right\} r_0^7 - \\
& \frac{1}{7} q^2 r_0^8 + \frac{8}{7} \alpha q^2 r_0^{\frac{17}{2}} + \left\{ \frac{13}{7} \alpha^2 q^2 - \frac{4}{5} n q^2 - \frac{8}{15} n^2 q \right\} r_0^9 + \left\{ -\frac{202}{15} \alpha n q^2 - \right. \\
& \left. \frac{404}{45} \alpha n^2 q \right\} r_0^{\frac{19}{2}} + \left\{ -\frac{1}{9} q^2 + \frac{416}{5} \alpha^2 n q^2 + \frac{832}{15} \alpha^2 n^2 q \right\} r_0^{10} \quad (1.40)
\end{aligned}$$

$$\text{where } r_0 = \frac{1}{\psi^* - q}$$

$$\begin{aligned}
v = & 1 - \frac{4}{5} n r_0^3 + \frac{44}{15} \alpha n r_0^{\frac{7}{2}} + \frac{287}{15} \alpha^2 n r_0^4 + \left\{ \frac{1}{5} q^2 + \frac{2}{15} q n - \right. \\
& \left. \frac{214}{45} n^2 \right\} r_0^6 + \left\{ \frac{22}{5} \alpha q^2 + \frac{44}{15} \alpha q n + \frac{1264}{45} \alpha n^2 \right\} r_0^{\frac{13}{2}} + \left\{ -2\alpha^2 q^2 - \right. \\
& \left. \frac{4}{3} \alpha^2 q n + \frac{397}{45} \alpha^2 n^2 \right\} r_0^7 + \frac{1}{7} q^2 r_0^8 - \frac{8}{7} \alpha q^2 r_0^{\frac{17}{2}} + \left\{ \frac{282}{25} \alpha q^2 n + \right. \\
& \left. \frac{2012}{225} \alpha q n^2 \right\} r_0^{\frac{19}{2}} + \left\{ \frac{4}{3} q^2 + \frac{488}{15} \alpha^2 q^2 n + \frac{976}{45} \alpha^2 q n^2 \right\} r_0^{10} \quad (1.41)
\end{aligned}$$

$$\begin{aligned}
w = & 1 + \frac{4}{5}nr_0^3 - \frac{44}{15}\alpha nr_0^{\frac{7}{2}} + \frac{683}{30}\alpha^2 nr_0^4 + \left\{-3q^2 - 2qn + \right. \\
& \left.\frac{842}{105}n^2\right\}r_0^6 + \left\{-\frac{22}{5}\alpha q^2 - \frac{44}{15}\alpha qn + \frac{544}{21}\alpha n^2\right\}r_0^{\frac{13}{2}} + \\
& \left\{18\alpha^2 q^2 + \frac{676}{15}\alpha^2 qn + \frac{93089}{315}\alpha^2 n^2\right\}r_0^7 - \frac{19}{7}q^2 r_0^8 + \\
& \left\{-\frac{298}{25}\alpha nq^2 - \frac{596}{75}\alpha qn^2\right\}r_0^{\frac{19}{2}} + \left\{-\frac{4}{3}q^2 + \frac{148}{75}\alpha^2 nq^2 - \right. \\
& \left.\frac{3208}{75}\alpha^2 qn^2\right\}r_0^{10} \tag{1.42}
\end{aligned}$$

$$\begin{aligned}
f_P = & 1 - \frac{52}{15}nr_0^3 + \frac{44}{15}\alpha nr_0^{\frac{7}{2}} - \frac{687}{30}\alpha^2 nr_0^4 + \left\{-\frac{14}{5}q^2 + \frac{32}{15}qn - \right. \\
& \left.\frac{4994}{525}n^2\right\}r_0^6 + \left\{\frac{16}{5}\alpha q^2 + \frac{32}{15}\alpha qn - \frac{12212}{1575}\alpha n^2\right\}r_0^{\frac{13}{2}} + \\
& \left\{-\frac{148}{5}\alpha^2 q^2 - \frac{264}{5}\alpha^2 qn - \frac{141802}{315}\alpha^2 n^2\right\}r_0^7 + \frac{20}{7}q^2 r_0^8 - \\
& \frac{52}{7}\alpha q^2 r_0^{\frac{17}{2}} + \left\{\frac{76}{225}\alpha q^2 n + \frac{1384}{45}\alpha qn^2\right\}r_0^{\frac{19}{2}} + \left\{\frac{13}{9}q^2 - \right. \\
& \left.\frac{46}{15}\alpha^2 q^2 n - \frac{21022}{225}\alpha^2 qn^2\right\}r_0^{10} \tag{1.43}
\end{aligned}$$

$$\begin{aligned}
f_T = & 1 - \frac{1417}{30}\alpha^2 nr_0^4 + \left\{\frac{16}{5}q^2 + \frac{32}{15}qn - \frac{15052}{1575}n^2\right\}r_0^6 + \\
& \left\{-\frac{12}{5}\alpha q^2 - \frac{8}{5}\alpha qn - \frac{45112}{1575}\alpha n^2\right\}r_0^{\frac{13}{2}} + \left\{-\frac{138}{5}\alpha^2 q^2 - \right. \\
& \left.\frac{772}{15}\alpha^2 qn - \frac{139891}{315}\alpha^2 n^2\right\}r_0^7 + \frac{20}{7}q^2 r_0^8 - \frac{52}{7}\alpha q^2 r_0^{\frac{17}{2}} + \\
& \left\{-\frac{253}{7}\alpha^2 q^2 - \frac{4}{5}nq^2 + \frac{56}{45}qn^2 + \frac{128}{15}\alpha^2 n^2\right\}r_0^9 + \left\{\frac{754}{75}\alpha q^2 n + \right. \\
& \left.\frac{132}{25}\alpha qn^2\right\}r_0^{\frac{19}{2}} + \left\{-\frac{53167}{150}\alpha^2 q^2 n + \frac{2683}{75}\alpha^2 qn^2\right\}r_0^{10} \tag{1.44}
\end{aligned}$$

Where the non-dimensional form of r_ψ is $r_\psi^* = \frac{r_\psi}{D}$ and terms

up to the second order of smallness in n, q and α terms up to r_0^{10} in r_ϕ are retained.

The modified structure equations determining the equilibrium structure of RTM distorted stars are as follows:

$$\frac{dM_\phi}{dr_\phi} = 4\pi D^3 r_0^3 \rho_\phi f_1 \quad (1.45)$$

$$\frac{dP_\phi}{dM_\phi} = -\frac{GM_\phi}{Dr_0^2} \rho_\phi f_2 \quad (1.46)$$

$$\frac{dL_\phi}{dM_\phi} = 4\pi \epsilon D^3 \rho_\phi r_0^2 f_1 \quad (1.47)$$

$$\frac{dT_\phi}{dM_\phi} = -\frac{3\kappa L_\phi \rho_\phi}{16\pi D \alpha c T_\phi^3 r_0^2} f_3 \quad (1.48)$$

where f_1, f_2, f_3 are distortion parameters that incorporate the effect of magnetic forces, as well rotational and tidal forces, on the equilibrium structure of stellar model.

$$\begin{aligned} f_1 = & 1 - \frac{7}{2} \alpha r_0^{\frac{1}{2}} + 6\alpha^2 r_0 + 4nr_0^3 - \frac{143}{6} \alpha nr_0^{\frac{7}{2}} + 42\alpha^2 nr_0^4 + \left\{ \frac{36}{5} q^2 + \right. \\ & \left. \frac{24}{5} qn + \frac{96}{5} n^2 \right\} r_0^6 + \left\{ -\frac{76}{5} \alpha q^2 - \frac{152}{15} \alpha qn - \frac{988}{15} \alpha n^2 \right\} r_0^{\frac{13}{2}} + \\ & \left\{ 16\alpha^2 q^2 + \frac{32}{3} \alpha^2 qn + \frac{92258}{765} \alpha^2 n^2 \right\} r_0^7 + \frac{55}{7} q^2 r_0^8 - \frac{115}{7} \alpha q^2 r_0^{\frac{17}{2}} + \\ & \left\{ \frac{120}{7} \alpha^2 q^2 + \frac{64}{5} nq^2 + \frac{128}{15} qn^2 \right\} r_0^9 + \left\{ -60\alpha q^2 n - 40\alpha qn^2 \right\} r_0^{\frac{19}{2}} + \\ & \left\{ \frac{4}{3} q^2 + \frac{832}{15} \alpha^2 q^2 n + \frac{1664}{45} \alpha^2 qn^2 \right\} r_0^{10} \end{aligned} \quad (1.49)$$

$$\begin{aligned} f_2 = & 1 + 2\alpha r_0^{\frac{1}{2}} + 2\alpha^2 r_0 - \frac{24}{5} nr_0^3 - \frac{10}{3} \alpha nr_0^{\frac{7}{2}} - \frac{239}{30} \alpha^2 nr_0^4 + \\ & \left\{ -\frac{22}{5} q^2 + \frac{16}{15} qn - \frac{10922}{1575} n^2 \right\} r_0^6 + \left\{ -\frac{36}{5} \alpha q^2 + \frac{16}{5} \alpha qn - \right. \\ & \left. \frac{82636}{1575} \alpha n^2 \right\} r_0^{\frac{13}{2}} + \left\{ -\frac{132}{5} \alpha^2 q^2 - \frac{128}{3} \alpha^2 qn - \frac{1531766}{3150} \alpha^2 n^2 \right\} r_0^7 + \\ & \frac{10}{7} q^2 r_0^8 - 6\alpha q^2 r_0^{\frac{17}{2}} + \left\{ -\frac{243}{7} \alpha^2 q^2 + \frac{1844}{75} q^2 n + \frac{1048}{225} qn^2 + \right. \end{aligned}$$

$$\frac{16}{3} \alpha^2 n^2 \} r_0^9 + \left\{ \frac{4312}{225} \alpha q^2 n + \frac{2488}{75} \alpha q n^2 \right\} r_0^{\frac{19}{2}} + \left\{ \frac{1}{9} q^2 + \frac{9266}{225} \alpha^2 q^2 n + \frac{20794}{225} \alpha^2 q n^2 \right\} r_0^{10} \quad (1.50)$$

Explicit expressions for $f_1 = \frac{dr_\varphi}{dr_0} \frac{r_\varphi^2}{D^3}$, $f_2 = \frac{f_P}{r_\varphi^2} \frac{dr_\varphi}{dr_0} D$, $f_3 = \frac{f_1}{r_\varphi^2} \frac{dr_\varphi}{dr_0} D$

Where terms up to the second order of exactness in n, q and α up to r_0^{10} in r_0 are retained.

The boundary condition are given as:

At the centre $r_0 = 0, M_\varphi = 0$

At the surface, $r_0 = r_{0s}, M_\varphi = M_0, P_\varphi = 0$ or $P_{\varphi s}$

Where M_0 is the entire mass of stellar model, $P_{\varphi s}$ denotes the outermost pressure on the EQS $\psi = constant$.

CHAPTER TWO

EQUILIBRIUM STRUCTURE OF ROTATIONALLY,
TIDALLY AND MAGNETICALLY DISTORTED
WHITE DWARF MODEL OF STARS

2.1 WHITE DWARF STARS

This section is introductory in nature. The broad problem of determining the configuration of the equilibrium structure of stars affected by RTM is of enormous importance in the field of astronomy. Challenging issues such as this promote a better understanding of the nature and environment of the internal configuration of rotating, tidally and magnetically distorted stars. There is currently no satisfactory method available for determining the combined effects of rotation, tidal, and magnetic disturbances on the equilibrium structures of white dwarf stars. Many attempts have been made in to study some specific parts of this subject in rough ways. As a result, the challenge of determining these effects on the structure and stability of a binary star system remains unsolved. Keeping its astronomical relevance in mind, there is still a lot of scope and need for future research on this topic.

As a result, we endeavoured to explore and present specific facets of this subject. Determining the combined effects of rotational, tidal, and magnetic distortions on star structure is extremely difficult and time consuming. As a result, an attempt was made to investigate the problem using reliable and appropriate simplifying assumptions.

An attempt has been made in this thesis to determine an equilibrium structure of rotationally, tidally and magnetically distorted (RTMD) white dwarf stars.

White dwarf models have been widely used in literature as representative models of low mass stars which are in their last stage of evolution. If we consider a completely degenerate white dwarf models, then following Chandrashekar [2] the equation of state can be written as:

$$P = Af(x), \rho = Bx^3$$

$$\text{Where } A = 6.01 \times 10^{22}, B = 9.82 \times 10^5 \mu_e \quad (1.50)$$

$$f(x) = x(2x^3 - 3)(x^2 + 1)^{\frac{1}{2}} + 3 \sin h^{-1} x \quad (1.51)$$

$x = \frac{P_0}{m_c}$ is a relativistic constant

μ_e represents mean molecular weight per electron

The non-linear differential equation structure is given by:

$$\frac{1}{\eta^2} \frac{d}{d\eta} \left(\eta^2 \frac{d\phi}{d\eta} \right) = - \left(\phi^2 - \frac{1}{\phi_0^2} \right)^{\frac{3}{2}} \quad (1.52)$$

Under the boundary conditions:

$$\text{At the centre, } \phi = 1, \frac{d\phi}{d\eta} = 0, \eta = 0 \quad (1.53)$$

$$\text{At the surface, } \phi = \frac{1}{\phi_0}, \eta = \eta_1$$

2.2 Equilibrium Structures of RTMD White dwarf models

Following approach of Lal (1993), the distorted equipotential surface due to rotational, tidal and magnetic force may be approximated by the appropriate Roche equipotential. Let P_φ and ρ_φ represent the pressure and density respectively on the equipotential surface $\psi = \text{constant}$ of a RTMD white dwarf model. Again, assuming that the distorted models is also a completely degenerate white dwarf model, P_φ and ρ_φ of such configuration will be connected through the relation of the form

$$P_\varphi = Af(x) \text{ and } \rho_\varphi = B(x)$$

Where A, B and f(x) are defined as in (1.50) and (1.51)

The equation which governs the hydrostatic equilibrium structure of RTMD star model can be combined together to yield

$$\frac{1}{r_\psi^2} \frac{d}{dr_\psi} \left[\frac{r_\psi^2}{\rho_\psi} u w \frac{dP_\psi}{dr_\psi} \right] = -4\pi G \rho_\psi$$

On using the value of P_φ and ρ_φ from the above equation and substituting $(x^2 + 1) = \phi_0^2 \phi_\psi^2$, it reduces to

$$\frac{\alpha^2}{r_\psi^2} \frac{d}{dr_\psi} \left[r_\psi^2 u w \frac{d\phi_\psi}{dr_\psi} \right] = - \left(\phi_\psi^2 - \frac{1}{\phi_0^2} \right)^{\frac{3}{2}}$$

Where $r_\psi = l\eta_\mu$, $l^2 = \frac{2A}{\pi G B^2 \phi_0^2}$. So, at the outer surface, $\frac{D}{l} = \frac{D}{l} \frac{\eta_u}{\eta_u} = \frac{D}{R_\psi} \eta_u = \frac{1}{K} \eta_u$, where η_u is the value radius of the undistorted stars.

The differential equation determining the equilibrium structure of RTMD white dwarf stars is given by:

$$\frac{d}{dr_0} [A(n, q, r_0, \alpha)] = - \frac{\eta_u^2}{K^2} r_0^2 B(n, q, r_0, \alpha) \left(\phi_\psi^2 - \frac{1}{\phi_0^2} \right)^{\frac{3}{2}} \quad (1.54)$$

$A(r_0, n, q, \alpha)$ and $B(r_0, n, q, \alpha)$ are obtained as follows:

$$\begin{aligned}
A(r_0, n, q, \alpha) = & \frac{r_0^2}{f_2} = 1 - 2\alpha r_0^{\frac{1}{2}} + 2\alpha^2 r_0 + \frac{24}{5} n r_0^3 - \frac{238}{15} \alpha n r_0^{\frac{7}{2}} - \\
& \frac{737}{30} \alpha^2 n r_0^4 + \left\{ \frac{22}{5} q^2 - \frac{16}{15} q n + \frac{9442}{315} n^2 \right\} r_0^6 + \left\{ -\frac{52}{5} \alpha q^2 + \frac{16}{15} \alpha q n + \right. \\
& \left. \frac{12764}{225} \alpha n^2 \right\} r_0^{\frac{13}{2}} + \left\{ -20\alpha^2 q^2 + \frac{896}{15} \alpha^2 q n + \frac{1059214}{3150} \alpha^2 n^2 \right\} r_0^7 - \\
& \frac{10}{7} q^2 r_0^8 + \frac{82}{7} \alpha q^2 r_0^{\frac{17}{2}} + \left\{ \frac{115}{7} \alpha^2 q^2 + \frac{1324}{75} q^2 n - \frac{3352}{225} q n^2 + \right. \\
& \left. \frac{496}{15} \alpha^2 n^2 \right\} r_0^9 + \left\{ \frac{39968}{225} \alpha q^2 n - \frac{3928}{75} \alpha q n^2 \right\} r_0^{\frac{19}{2}} + \left\{ -\frac{1}{9} q^2 + \right. \\
& \left. \frac{102908}{225} \alpha^2 q^2 n + \frac{20318}{45} \alpha^2 q n^2 \right\} r_0^{10}
\end{aligned}$$

and

$$\begin{aligned}
B(r_0, n, q, \alpha) = & f_1 = 1 - \frac{7}{2} \alpha r_0^{\frac{1}{2}} + 6\alpha^2 r_0 + 4n r_0^3 - \frac{143}{6} \alpha n r_0^{\frac{7}{2}} + \\
& 42\alpha^2 n r_0^4 + \left\{ \frac{36}{5} q^2 + \frac{24}{5} q n + \frac{96}{5} n^2 \right\} r_0^6 + \left\{ -\frac{76}{5} \alpha q^2 - \frac{152}{15} \alpha q n - \right. \\
& \left. \frac{988}{15} \alpha n^2 \right\} r_0^{\frac{13}{2}} + \left\{ 16\alpha^2 q^2 + \frac{32}{3} \alpha^2 q n + \frac{92258}{765} \alpha^2 n^2 \right\} r_0^7 + \frac{55}{7} q^2 r_0^8 - \\
& \frac{115}{7} \alpha q^2 r_0^{\frac{17}{2}} + \left\{ \frac{120}{7} \alpha^2 q^2 + \frac{64}{5} n q^2 + \frac{128}{15} q n^2 \right\} r_0^9 + \left\{ -60\alpha q^2 n - \right. \\
& \left. 40\alpha q n^2 \right\} r_0^{\frac{19}{2}} + \left\{ \frac{4}{3} q^2 + \frac{832}{15} \alpha^2 q^2 n + \frac{1664}{45} \alpha^2 q n^2 \right\} r_0^{10}
\end{aligned}$$

Where $r_0 = \frac{1}{\psi - q}$.

In both the above expressions terms up to the second order of smallness in n , q & α and up to r_0^{10} in r_0 are retained. Now the boundary conditions become:

$$\text{At the centre: } r_0 = 0, \phi_\psi = 1, \frac{d\phi_\psi}{dr_0} = 0 \quad (1.55)$$

$$\text{At the surface: } r_0 = r_{os}, \phi_\psi = \frac{1}{\phi_0}$$

r_{os} denotes the value of r_0 at the surface, and both of these values are non-dimensional.

2.3 Calculation of Surface Area, Volume and other Physical Parameters of RTMD White Dwarf Model of Stars

The value of r_ψ , volume V_ψ and the surface area S_ψ of a RTMD white dwarf models are obtained as

$$r_\psi = r_{os} \left(\frac{l\varepsilon_\mu}{K} \right) \left[1 - \alpha r_{os}^{\frac{1}{2}} + \frac{1}{2} \alpha^2 r_{os} + \frac{2}{3} n r_{os}^3 - \frac{7}{3} \alpha n r_{os}^{\frac{7}{2}} + \left\{ \frac{4}{5} q^2 + \frac{8}{15} qn + \frac{76}{45} n^2 \right\} r_{os}^6 + \frac{5}{7} q^2 r_{os}^8 + \frac{2}{3} q^2 r_{os}^{10} + \dots \right] \quad (1.56)$$

$$V_\psi = \frac{4\pi r_{os}^3}{3} \left(\frac{l\varepsilon_\mu}{K} \right)^3 \left[1 - 3\alpha r_{os}^{\frac{1}{2}} - 11\alpha n r_{os}^{\frac{7}{2}} + \frac{5}{2} \alpha^2 r_{os} + 2n r_{os}^3 + \left\{ \frac{12}{5} q^2 + \frac{8}{5} qn + \frac{32}{5} n^2 \right\} r_{os}^6 + \frac{15}{7} q^2 r_{os}^8 + 2q^2 r_{os}^{10} + \dots \right] \quad (1.57)$$

$$S_\psi = 4\pi r_{os}^2 \left(\frac{l\varepsilon_\mu}{K} \right)^2 \left[1 - 2\alpha r_{os}^{\frac{1}{2}} - 6\alpha n r_{os}^{\frac{7}{2}} + 2\alpha^2 r_{os} + \frac{4}{3} n r_{os}^3 + \left\{ \frac{7}{5} q^2 + \frac{14}{15} nq + \frac{56}{15} n^2 \right\} r_{os}^6 + \frac{9}{7} q^2 r_{os}^8 + \frac{11}{9} q^2 r_{os}^{10} \right] \quad (1.58)$$

When the effects of magnetic force are not explicitly considered, the expressions $A(r_0, n, q, \alpha)$ and $B(r_0, n, q, \alpha)$ reduce to their corresponding expressions as obtained by Mohan and Saxena (1983). The relationships (1.57) and (1.58) indicate the volume and surface area of RTMD stars that have been warped by rotation, tidal, and magnetic forces. To compute the volume and surface areas of the distorted white dwarf model, we must replace r_{os} for r_0 in the above expressions for r_ψ , V_ψ and S_ψ respectively.

CHAPTER THREE

Concluding Observations

3.1 MATHEMATICAL CALCULATIONS

For the supplied input parameter values, equation (1.54) has been numerically integrated using the 4th order Runge - Kutta method to produce the numerical solutions. To begin the integration at places near the centre, a series solution comparable to one variable for distorted white dwarf models [2] has been created. The following is the series solution:

$$\phi_{\psi} = 1 - \frac{1}{6} \frac{\varepsilon_u^2}{K^2} K^{*\frac{3}{2}} r_0^2 + \frac{1}{40} \frac{\varepsilon_u^4}{K^4} K^{*2} r_0^4 - \frac{\alpha K^{*5} (5K^{*2} + 14)}{5040} \varepsilon_{\mu}^{\frac{5}{2}} r_0^6 + \dots$$

where $K^* = \left(1 - \frac{1}{\phi_0^2}\right)$ (3.11)

Using fourth order Runge-Kutta method of fourth, we have solved the numerical integration. Using step size $h = 0.005$ and equation (3.11), numerical integration is carried out until ϕ_{ψ} becomes $\frac{1}{\phi_0}$. Once the outermost radius r_{0s} is determined, the corresponding volume, surface area, and shape of distorted white dwarf models can be calculated. The numerical results obtained from various parameters, the volume and surface area are shown in the Table (I, II, III and IV), with the values of different white dwarf indices $\frac{1}{\phi_0^2} = 0.2, 0.4, 0.6, 0.8$.

Table I: Volume & Surface Areas of rotationally, Tidally & Magnetically Distorted White Dwarf Model of Stars with white dwarf index $\frac{1}{\phi_0^2}=0.2, \eta_u = 3.7271$

Undistorted Case

Model No.	n	q	α	K	r_{os}	Volume $\times 10^{-2}$	Surface $\times 10^{-2}$	r_{si}
1.	0	0	0	1	1.000000	6.50445	1.745327	0.372613

Magnetic Effects

2.	0	0	0.1	0.5	0.511182	5.55010	1.477782	0.092706
3.	0	0	0.2	0.5	0.542199	5.08083	1.248044	0.102607

Rotational & Magnetic Effects

4.	0.1	0	0.1	1	0.932974	4.50110	0.741094	0.289072
5.	0.5	0	0.1	1	0.749533	3.46099	1.048678	0.171048
6.	0.6	0	0.1	1	0.727653	3.38326	1.067055	0.160352

Tidal & Magnetic Effects

7.	0	0.05	0.1	0.5	0.511182	5.55106	1.477930	0.092710
8.	0	0.1	0.1	0.5	0.511182	5.55394	1.478374	0.092724
9.	0	0.2	0.1	0.5	0.511182	5.56546	1.480151	0.092779

Rotational, Tidal & Magnetic Effects

10.	0.05	0.1	0.1	0.5	0.507001	5.48036	1.474093	0.090821
11.	0.1	0.05	0.1	0.5	0.503052	5.41093	1.469729	0.089042
12.	0.2	0.2	0.1	0.5	0.495765	5.31215	1.465523	0.085946

Table II: Volume & Surface Areas of rotationally, Tidally and Magnetically Distorted White Dwarf model of stars with White Dwarf Index $\frac{1}{\phi_0^2}=0.4$, $\eta_u = 3.5245$

Undistorted Case

Model No.	n	q	α	K	r_{os}	Volume $\times 10^{-2}$	Surface $\times 10^{-2}$	r_{si}
1.	0	0	0	1	1.0000	5.50062	1.560790	0.352377

Magnetic Effects

2.	0	0	0.1	0.5	0.515347	4.80434	1.339544	0.089800
3.	0	0	0.2	0.5	0.552460	4.52182	1.136765	0.102503

Rotational & Magnetic Effects

4.	0.1	0	0.1	1	0.937115	3.86360	0.660952	0.277172
5.	0.5	0	0.1	1	0.751803	2.96871	0.946141	0.163535
6.	0.6	0	0.1	1	0.729951	2.90566	0.963837	0.153427

Tidal & Magnetic Effects

7.	0	0.05	0.1	0.5	0.51534 7	4.80522	1.339686	0.089805
8.	0	0.1	0.1	0.5	0.51534 7	4.80785	1.340112	0.089819
9.	0	0.2	0.1	0.5	0.51534 7	4.81838	1.341815	0.089875

Rotational, Tidal & Magnetic Effects

10.	0.05	0.1	0.1	0.5	0.510861	4.73803	1.335511	0.087845
11.	0.1	0.05	0.1	0.5	0.506647	4.67263	1.330890	0.086012
12.	0.2	0.2	0.1	0.5	0.498917	4.57932	1.326058	0.082842

Table III: Volume & Surface Areas of rotationally, Tidally & Magnetically Distorted White Dwarf Model of Stars with white dwarf index $\frac{1}{\phi_0^2}=0.6$, $\eta_u = 3.6038$

Undistorted Case

Model No.	n	q	α	K	r_{os}	Volume $\times 10^{-2}$	Surface $\times 10^{-2}$	r_{si}
1.	0	0	0	1	1.000000	5.88043	1.631834	0.360311

Magnetic Effects

2.	0	0	0.1	0.5	0.517467	5.19708	1.410114	0.092945
3.	0	0	0.2	0.5	0.557888	4.96442	1.199247	0.107852

Rotational & Magnetic Effects

4.	0.1	0	0.1	1	0.939113	4.16017	0.690072	0.285305
5.	0.5	0	0.1	1	0.752988	3.19737	0.993805	0.168175
6.	0.6	0	0.1	1	0.731162	3.13174	1.013058	0.157853

Tidal & Magnetic Effects

7.	0	0.05	0.1	0.5	0.517467	5.19806	1.410268	0.092950
8.	0	0.1	0.1	0.5	0.517467	5.20098	1.410729	0.092965
9.	0	0.2	0.1	0.5	0.517467	5.21269	1.412574	0.093025

Rotational, Tidal & Magnetic Effects

10.	0.05	0.1	0.1	0.5	0.512815	5.12183	1.405463	0.090848
11.	0.1	0.05	0.1	0.5	0.508456	5.04793	1.400209	0.888888
12.	0.2	0.2	0.1	0.5	0.500490	4.94245	1.394535	0.085514

Table IV: Volume & Surface Areas of rotationally, Tidally & Magnetically Distorted White Dwarf Model of Stars with white dwarf index $\frac{1}{\phi_0^2}=0.8$, $\eta_u = 4.0446$

Undistorted Case

Model No.	n	q	α	K	r_{os}	Volume $\times 10^{-2}$	Surface $\times 10^{-2}$	r_{si}
1.	0	0	0	1	1.000000	8.31317	2.055486	0.404395

Magnetic Effects

2.	0	0	0.1	0.5	0.518832	7.40287	1.783966	0.105131
3.	0	0	0.2	0.5	0.561458	7.14094	1.519115	0.123324

Rotational & Magnetic Effects

4.	0.1	0	0.1	1	0.940361	5.90757	0.868428	0.321536
5.	0.5	0	0.1	1	0.753765	4.54212	1.255611	0.189455
6.	0.6	0	0.1	1	0.731960	4.45111	1.280507	0.177884

Tidal & Magnetic Effects

7.	0	0.05	0.1	0.5	0.518832	7.40429	1.784164	0.105137
8.	0	0.1	0.1	0.5	0.518832	7.40853	1.784758	0.105154
9.	0	0.2	0.1	0.5	0.518832	7.42550	1.787134	0.105223

Rotational, Tidal & Magnetic Effects

10.	0.05	0.1	0.1	0.5	0.514068	7.29228	1.777732	0.102704
11.	0.1	0.05	0.1	0.5	0.509614	7.18404	1.770754	0.100441
12.	0.2	0.2	0.1	0.5	0.501491	7.02948	1.763074	0.096554

3.2 CONCLUSION AND DISCUSSION OF RESULTS

While coming to the conclusion, we have observed that as soon as the white dwarf index increases, the volume, surface area and shape of the white dwarf star first increases, then decreases and then it again increases.

Whereas when compared with the undistorted model,

For white dwarf index $\frac{1}{\phi_0^2} = 0.2$,

For no rotational, tidal effect i.e., only magnetic effect is there, then the corresponding volume, surface area slightly decreases.

When there is an effect of magnetic field with rotation, volume and surface area decreases.

When there is magnetic effect with tidal effect and rotational, tidal and magnetic effect, there is a marginal change in the volume and surface area.

For white dwarf index $\frac{1}{\phi_0^2} = 0.4$,

For no rotational, tidal effect i.e., only magnetic effect is there, then there is a marginal change in the corresponding volume, surface area.

When there is an effect of magnetic field with rotation, volume and surface area slightly decreases.

When there is magnetic effect with tidal effect and rotational, tidal and magnetic effect, there is again a marginal change in the volume and surface area.

For white dwarf index $\frac{1}{\phi_0^2} = 0.6$,

For no rotational, tidal effect i.e., only magnetic effect is there, then there is a marginal change in the corresponding volume, surface area.

When there is an effect of magnetic field with rotation, volume and surface area slightly decreases.

When there is magnetic effect with tidal effect and rotational, tidal and magnetic effect, there is again a marginal change in the volume and surface area.

For white dwarf index $\frac{1}{\phi_0^2} = 0.8$,

For no rotational, tidal effect i.e., only magnetic effect is there, then there is a marginal change in the corresponding volume, surface area.

When there is an effect of magnetic field with rotation, volume and surface area slightly decreases.

When there is magnetic effect with tidal effect and rotational, tidal and magnetic effect, there is again a marginal change in the volume and surface area.

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