

FRACTIONAL ORDER APPROACH FOR EDGE DETECTION OF LOW CONTRAST IMAGES

A Dissertation submitted in fulfillment of the requirements for the Degree

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Submitted by

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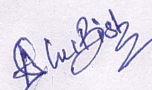
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DECLARATION

I hereby declare that the work which is being presented in the dissertation entitled, "**Fractional order approach for edge detection of low contrast images**", in partial fulfilment of requirements for the award of degree of **Master of Engineering in Electronic Instrumentation and Control**, submitted in the Department of Electrical and Instrumentation Engineering at Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Swati Sondhi, Assistant Professor, EIED. It refers others research work which is appropriately recorded in reference section. The matter contained in this dissertation has not been submitted, neither partially nor in full, in any other University/Institute for the award of any other degree.

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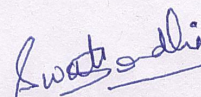


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It is certified that the above statement made by the student is correct to the best of my knowledge and belief.

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LIST OF ABBREVIATIONS

Abbreviations	Details
RGB	Red Green Blue
FOD	Fractional Order Derivative/Differentiation
G-L	Grunwald-Letnikov
R-L	Riemann-Liouville
MSE	Mean Square Error
SNR	Signal to Noise Ratio
PSNR	Peak Signal to Noise Ratio
MATLAB	Matrix Laboratory
HE	Histogram Equalization
SSR	Single-scale Retinex
GLCM	Gray Level Co-occurrence Matrix
QMF	Quadrature Mirror Filter
LPF	Low Pass Filter
HPF	High Pass Filter
IFD	Improved Fractional Differentiation
S-G	Savitzky Golay
FD	Fractional Differential
MRI	Magnetic Resonance Imaging
FIM	Fractional Integral Mask
VFOD	Variable Fractional Order Differentiator
IQM	Image Quality Measurements
ROI	Region of Interest
HVS	Human Visual System
WMSE	Weighted Mean Square Error
IOD	Integer Order Derivative
IQA	Image Quality Assessment
FPGA	Field-Programmable Gate Array

ABSTRACT

In digital image processing, image enhancement techniques are considered important in many applications where the subjective quality of images is important for human interpretation. It serves as a fundamental task to improve interpretability and appearance of an image and is applicable in every field where images are to be understood and analyzed. Contrast enhancement is one of the important enhancement operations in any subjective evaluation of image quality. Many images like underwater images, satellite images, medical images as well as various real time images may suffer from poor contrast due to various factors affecting the surrounding environment during image capturing. Underwater images usually suffer from degraded visibility. Light attenuates and scatters in water resulting in low contrast and haziness in the scenes. Therefore, the main problems to be dealt with in underwater environment are poor contrast, non-uniform lighting, haziness and blurring. Hence, in order to study underwater images, it becomes utmost important to extract the invisible or unclear edges. Since edge detection is widely used in high level processing fields like computer vision, feature extraction, image segmentation etc., various mathematical tools have been developed which aim at identifying these edges in an image. It was indicated that the edge detection methods operationally are a mixture of image smoothing and image differentiation. These integer order differential operators suffer from poor accuracy and noise immunity. In the presented work, a method for edge detection by using fractional order differentiation based approach has been realized. Considering the G-L based fractional differential operator's basic definition and implementation, a filter is devised and its applicability for texture enhancement is analyzed. Various different underwater images have been used for experimentation using both conventional as well as fractional order differential operators. Further, the results are compared with another approach based on Riemann Liouville (R-L) fractional differential operator. The analysis of tests proves that the proposed method displays better results for detecting edges of low contrast underwater images with high accuracy, good sharpness and reveals more information than conventional methods as well as R-L definition based method.

CHAPTER 1

INTRODUCTION

1.1 BASIC IMAGE ENTITIES

1.1.1 Description of an image

An image is nothing but a 2-D signal. It is a rectangular grid having a definite height and width measured in terms of pixels. Mathematically, it is defined as a function $f(x, y)$ where x and y represents the horizontal and vertical coordinates respectively and the amplitude of f at any pair of coordinate (x, y) is called the intensity or gray level of the image at that point. When the values of the coordinates (x, y) and the amplitude f are all finite and discrete, we call it a digital image [1]. They are basically sampled from an analog signal at particular set points and further mapped together as a grid of pixels or dots. The image acquisition process has been illustrated in Figure 1.1.

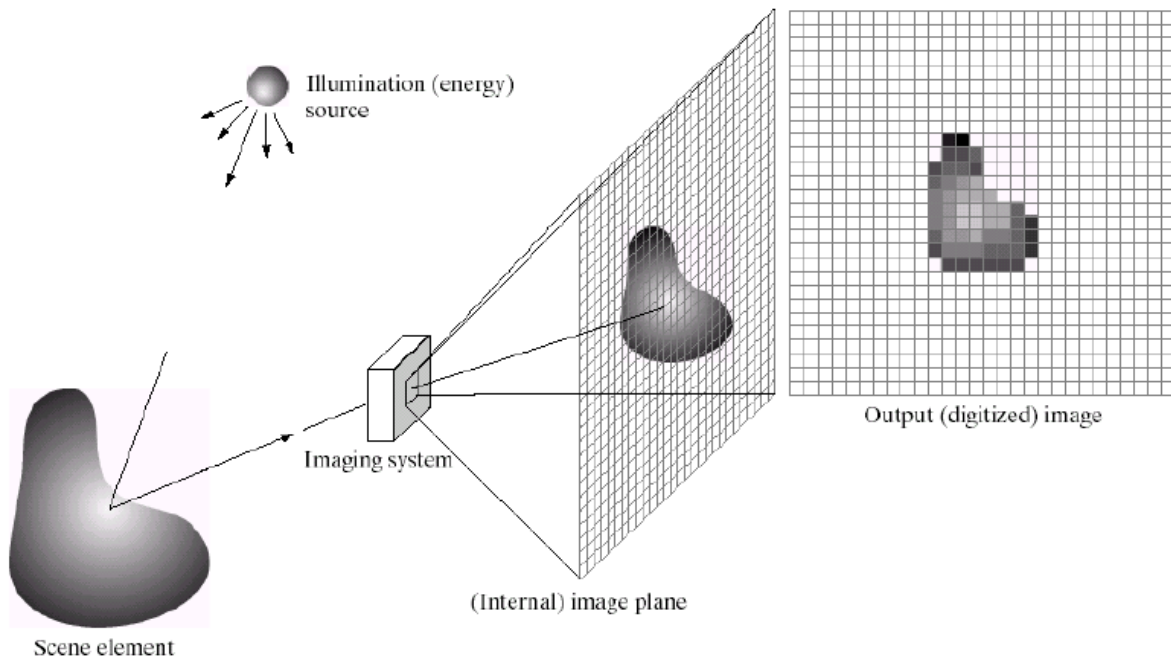


Figure 1.1 Image acquisition process

Most of the images are generated by the combination of an illumination source and the reflection or absorption of energy from that source by the elements of the scene being images [1].

Let the illumination component be $i(x, y)$ and reflectance component be $r(x, y)$, then the image function $f(x, y)$ can be written as:

$$f(x, y) = i(x, y)r(x, y) \quad (1.1)$$

	0	1	2	3	4	:	N-1
0	f(0,0)	f(1,0)	f(2,0)	:	:	:	f(N-1,0)
1	f(0,1)	f(1,1)	f(2,1)	:	:	:	f(N-1,1)
2	f(0,2)	f(1,2)	f(2,2)	:	:	:	f(N-1,2)
3	f(0,3)	f(1,3)	f(2,3)	:	:	:	f(N-1,3)
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
M-1	f(0,M-1)	f(1,M-1)	f(2,M-1)	:	:	:	f(N-1,M-1)

Figure 1.2 General form of image matrix

Figure 1.2 illustrates the general image in matrix form where each block shows picture element of the image. Spatial resolution is defined as the smallest distinct detail in an image, denoted as pixel per unit distance and lines per unit distance [2]. It determines how closely the lines are placed in an image. Intensity resolution is defined as the smallest change in the intensity levels. It is given as:

$$L = 2^k \quad (1.2)$$

where L is number of intensity levels and k determines the bit of image.

For instance, if an image has 256 intensity levels, then it will be called an 8-bit image.

1.1.2 Pixels

The word *pixel* is a combination of two words ‘*pix*’ and ‘*el*’ which denotes picture and elements respectively. A digital image, represented as 2-dimensional image, is composed of a finite number of elements, each of which has a particular location and values of these elements are referred to as picture elements, image elements, pels or pixels. Therefore, each sample of the

original image represents a pixel as illustrated in Figure 1.3. Pixels are usually represented by using dots or squares. Pixel values typically represent gray levels, colors, heights, opacities etc. [3]. Pixels where the intensity of image changes suddenly are called edge pixels. Edges are formed by joining these edge pixels. Edge pixels are detected by using edge detectors.

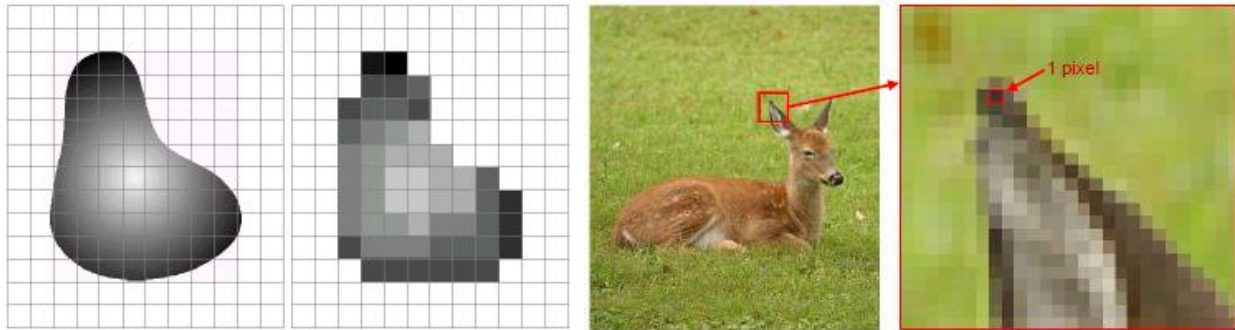


Figure 1.3 Representation of a pixel in an image. [3]

A pixel p has four horizontal and vertical neighbors at coordinate (x, y) . These are given as:

$(x+1, y), (x-1, y), (x, y+1), (x, y-1)$

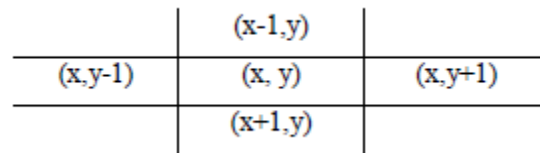


Figure 1.4 Horizontal and vertical neighbors $N_4(p)$ of pixel, P

These pixels are denoted as $N_4(p)$ and are known as 4-neighbors of P as illustrated in Figure 1.4.

If (x, y) is at the border of the image then some of its neighboring pixels will lie outside the digital image.

An image also has four of diagonal neighbors of P which are at coordinates:

$(x-1, y-1), (x-1, y+1), (x+1, y-1), (x+1, y+1)$

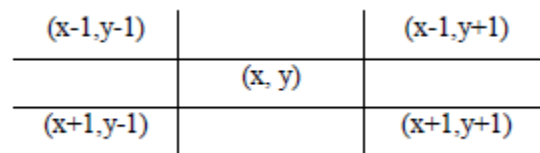


Figure 1.5 Diagonal neighbors $N_d(p)$ of pixel, P

These pixels are denoted by $N_D(p)$ as illustrated in Figure 1.5. The combination of $N_4(p)$ and $N_D(p)$ is called 8-neighbors of P . It is denoted by $N_8(p)$. The illustration of 8-neighbors is shown in Figure 1.6.

$(x-1,y-1)$	$(x-1,y)$	$(x-1,y+1)$
$(x,y-1)$	(x,y)	$(x,y+1)$
$(x+1,y-1)$	$(x+1,y)$	$(x+1,y+1)$

Figure 1.6 Eight neighbors $N_8(p)$ of pixel, P

An image is a visual representation of anything, be it an object, a place or a thing. It is an optical counterpart of the object of reference which is produced by lens, mirror or through luminous rays. Generally, there are three preferably used formats for an image, which are described as [3]:

- a. 1 sample per pixel (Grayscale image)
- b. 3 samples per pixel (RGB image)
- c. 4 samples per pixel (RGB and “Alpha” i.e. Opacity)

These image formats are illustrated with a reference image in Figure 1.7.



Figure 1.7 Image formats: a) 1 sample per pixel, b) 3 samples per pixel, c) 4 samples per pixel.

1.1.3 Edges

Edges are defined as abrupt discontinuities in intensity of image, the point where brightness changes sharply [4]. Edges represent the difference of colors, patterns and outline of shape or texture. They are used to find corresponding points from number of images which are of same scene. Edges provide the information about scene object due to the geometrical variations. Edges are formed due to the variations in coordination, depth of scene, surface color, illumination and

reflectance [4] as shown in Figure 1.8. As intensity of an image is proportional to the luminance of the picture, therefore edges are entitled by variations in intensities of the image.

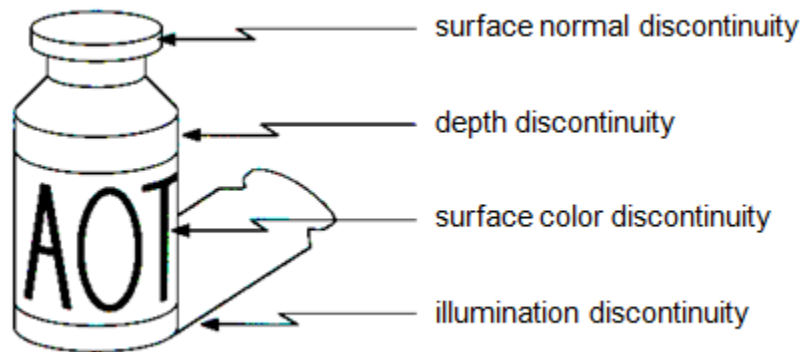


Figure 1.8 Factors affecting the origin of edges [5]

Edge detection is defined as mathematical method which is used to identify the points at which brightness of image changes abruptly. The edge profiles of step signal, smoothed step signal, and differentiated step signal with noise [4] is illustrated in Figure 1.9.

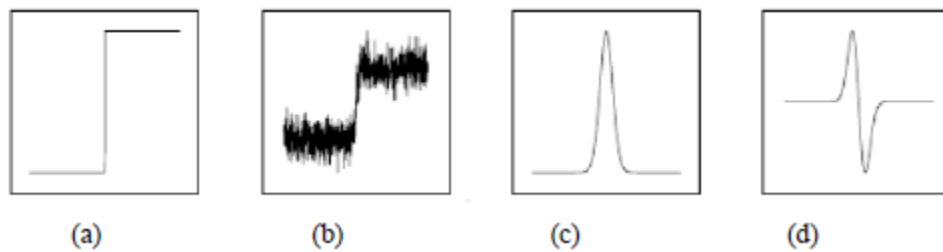


Figure 1.9 Profile of a) Ideal step edge, b) Smooth step edge with noise, c) FOD of smoothed edge, d) Second derivative of smoothed edge.

1.2 EDGE DETECTION

Edge detection is a terminology in image processing that refers to the algorithms which aim at identifying edges in an image. Edge detection is a fundamental asset in image analysis because it reduces unnecessary information in an image while conserving its structure by locating the areas with strong intensity contrasts. Further, edge detection depicts the ability to measure the gray level transitions in a meaningful manner [6] as illustrated in Figure 1.10. It is generalized in the areas of feature selection, segmentation and extraction in Computer Vision. An edge detector inputs a digital image and produces an edge outlet as output.

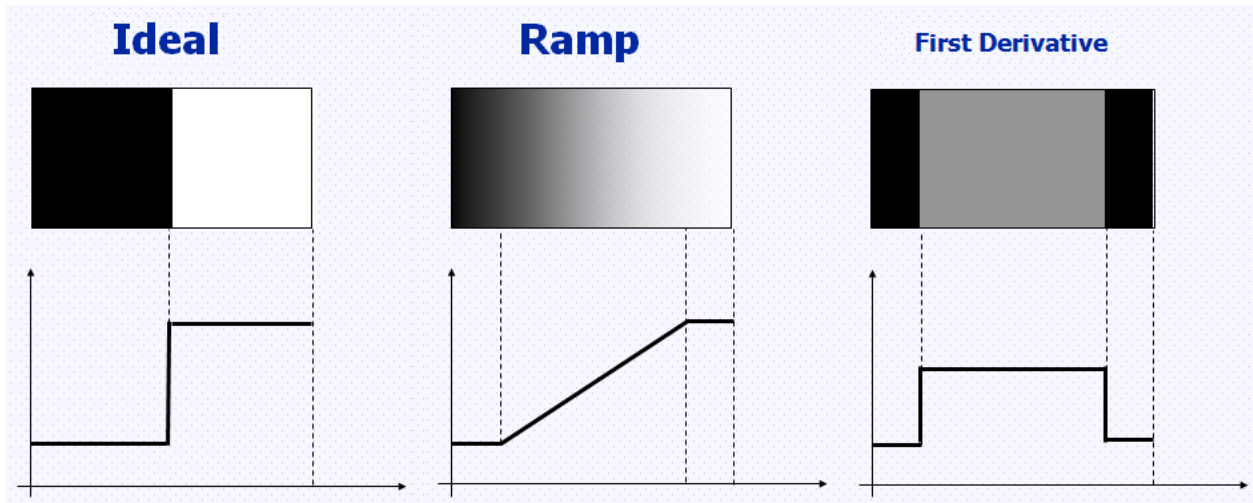


Figure 1.10 Gray level transitions in case of ideal edges, ramp edges and first derivative of the ramp edges. [6]

The edge detection methods are categorized into two groups: *gradient based* and *zero-crossing based*. In gradient based methods edges are detected first by quantifying the gradient magnitude of image intensity function. After that it searches its local maxima and minima in the direction of first order derivative in image. The zero-crossing methods locate zeroes in second order derivative of an image to compute edges. It is also termed as Laplacian of an image. The edge detection methods include three operations: differentiation, labeling and smoothing. Differentiation implies to extracting the desired derivatives of an image. Smoothing is used to reduce noise and normalizing numerical differentiation. Labeling is used to increase signal to noise ratio (SNR) and localizing the edges. It increases the SNR by suppressing the false edges.

1.3 STANDARD EDGE DETECTORS

Edge detection is a key tool in image analysis. Various integer order edge detectors are often used in preprocessing of image, especially gradient based operators.

First order gradient is obtained by taking the difference between subsequent pixels along rows and columns in an image. The gradient of a function $f(a,b)$ at any point (a,b) is computed as:

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} G_a \\ G_b \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial a} \\ \frac{\partial f}{\partial b} \end{bmatrix} \quad (1.3)$$

The magnitude of gradient ∇f is denoted as:

$$G(a,b) = \sqrt{G_a^2 + G_b^2} \approx |G_a| + |G_b| \quad (1.4)$$

$G(a,b)$ is referred to as *gradient image*. It has the same size as that of the original image [7].

Gradient based edge detection is best used for abrupt discontinuities. It shows superior performance for less noisy image.

Four major standard gradient based detectors are Roberts edge detector, Prewitt edge detector, Sobel edge detector and Canny edge detector. A brief review of each detector is discussed.

1.3.1 Roberts Operator

Lawrence Roberts proposed the first ever edge detector in 1963 [8]. The author proposed 2*2 convolution mask pairs which are simple and easy to compute. The approach is based on obtaining the diagonal edge gradients by computing the differences of the diagonal pixel pairs.

The pixel differences are computed at interpolated points $\left(i + \frac{1}{2}, j + \frac{1}{2}\right)$ instead of (i, j) .

$$\begin{array}{cc} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \\ G_x & G_y \end{array}$$

Figure 1.11 Roberts gradient along x and y directions [8]

Figure 1.11 depicts the Roberts mask operators along x and y directions respectively. Both masks are oriented perpendicular to each other and respond to edges at 45° to the pixel grid.

1.3.2 Prewitt Operator

S. Prewitt proposed 3*3 convolution mask pairs [8]. The differences are calculated at the centre pixel of the mask. Technically, Prewitt operator is useful in computation of first order derivative of the image intensity at each point. It provides a path from darker to brighter values, depicting the direction of maximum rate of change of intensities. Thus, it shows how smoothly the image converts at that point and how likely is that part of image represents an edge. Mathematically, an

image being a two variable function, at each image point the gradient will be computed in both horizontal and vertical directions as depicted in Figure 1.12.

$$\begin{matrix} \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} & \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \\ G_x & G_y \end{matrix}$$

Figure 1.12 Prewitt gradient along x and y directions [8]

1.3.3 Sobel Operator

Irwin Sobel proposed the idea of a 3*3 gradient operator at a talk in SAIL in 1968 [8]. Technically, Sobel operator is a modified version of Prewitt operator, where the weight on the central pixel coefficient is doubled as illustrated in Figure 1.13. Usually, the value of central pixel is kept variable. This increment in value helps in improved smoothing. Though the representation of Sobel operator is an inaccurate approximation, but it is satisfactory for some applications with quite good accuracy. [8]

$$\begin{matrix} \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} & \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \\ G_x & G_y \end{matrix}$$

Figure 1.13 Sobel gradient in x and y directions [8]

1.3.4 Canny Operator

John F. Canny developed the Canny Operator in 1986. It is a multi stage algorithm defined for edge detection. Technically, the process consists of 5 main steps [8].

- a) Noise smoothening by using Gaussian filter.
- b) Finding the image gradient using Sobel or Prewitt operator.
- c) Determine edge direction using the gradients and resolve the edge directions adaptively.
- d) Non maxima Suppression: Keep only local maxima values in the gradient. It gives a thin line for the edge.
- e) Apply double thresholding to broadcast only potential edges.

However, the edge detection algorithm is simple and quite accurate, but with new challenges and robustness requirement in detection, every step of Canny's method needs advanced changes and improvement for better use.

1.4 FRACTIONAL CALCULUS

The term fractional calculus is more than 300 years old. It is the generalization of the ordinary integral differentiation and integration to non-integer arbitrary order [9]. Nowadays, the fractional calculus, particularly fractional derivatives, is widely discussed and attracts many engineers and scientists. In the past few decades, fractional calculus became a popular and powerful tool for enhanced modeling and better control of processes in various fields of engineering and technology.

1.4.1 Historical origin of fractional calculus

G. W. Leibniz (1646 - 1716) and Issac Newton (1642 – 1727) have independently discovered calculus in the late 17th century. The discovery of calculus became so remarkable that in its recognition J. V. Neumann (1903 – 1957) quoted: *'The calculus was the first achievement of modern mathematics and it is highly difficult to overestimate its importance'*. [10] Leibniz, in his

discovery of calculus, was the first one to introduce the symbol $\frac{d^n y}{dx^n} = D^n y$ for the n^{th} derivative, where n is a non-negative integer. Fractional Calculus was introduced on September 30, 1695 when Leibniz, in a letter to L'Hospital (1661 - 1704), raised a question about the possibility of generalizing the meaning of derivatives with integer order to derivatives with non integer orders [11]. In curiosity, L'Hospital cross-questioned Leibniz about wanting to know the result for the derivative of order $n = 1/2$, i.e.

$$\text{for } \frac{d^n y}{dx^n} \text{ what if } n = \frac{1}{2} ?$$

to which Leibniz replied: *"It will lead to a paradox from which one day useful consequences will be drawn"* [11, 12] and in fact, his vision became a reality. That question raised by Leibniz is still an ongoing research topic. Several mathematicians like Liouville, Riemann, Weyl, Fourier, Abel, Grunwald and Letnikov have contributed a major part to the theory of fractional calculus.

There are several applications of fractional calculus in physics, mechanics, signal processing, image processing and automatic control theory [13, 14]. However, it is still difficult to broaden the numerical methods for fractional calculus due to its complex definitions. Many researchers have tried to solve the fractional integrals, fractional derivatives and fractional differential equations theoretically and later in Matlab.

1.4.1.1 Lacroix Definition

Lacroix, in 1819, in his 800 paged book, developed the formula for the n^{th} derivative of $y = x^k$, where k being a positive integer, as:

$$d^n x^k = \frac{k!}{(k-n)!} x^k \quad (1.5)$$

1.4.1.2 Leibnitz Definition

After having two years of research on fractional calculus, Leibniz, in a letter to J. Bernoulli, in 1677, mentioned a proposed possible approach to fractional order differentiation in that sense that for non-integer values of n the definition could be the following:

$$\frac{d^n e^{kx}}{dx^n} = k^n e^{kx} \quad (1.6)$$

1.4.1.3 Euler's Definition

L. Euler, in 1730, proposed an approach towards solving fractional differentiation. He suggested his approach is also applicable for negative and rational order values. The relationship is as following:

$$\frac{d^\alpha x^\beta}{dx^\alpha} = \beta(\beta-1)\dots(\beta-\alpha+1)x^{\beta-\alpha} \quad (1.7)$$

$$\Gamma(\beta+1) = \beta(\beta-1)\dots(\beta-\alpha+1)\Gamma(\beta-\alpha+1) \quad (1.8)$$

$$\frac{d^\alpha x^\beta}{dx^\alpha} = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} x^{\beta-\alpha} \quad (1.9)$$

Taking $\beta = 1$ and $\alpha = \frac{1}{2}$, Euler obtained:

$$\frac{d^{0.5}x}{dx^{0.5}} = \sqrt{\frac{4x}{\pi}} \quad \left(= \frac{2}{\sqrt{\pi}} x^{1/2} \right) \quad (1.10)$$

1.4.1.4 Fourier Definition

However, the first step to generalization of the notion of differentiation for arbitrary functions was done by J. B. J. Fourier in 1822. After introducing his formula:

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(z) dz \int_{-\infty}^{\infty} \cos(px - pz) dp, \quad (1.11)$$

Fourier made a remark that

$$\frac{d^n f(x)}{dx^n} = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(z) dz \int_{-\infty}^{\infty} \cos\left(px - pz + n\frac{\pi}{2}\right) dp, \quad (1.12)$$

and this relationship could serve as a definition of the n^{th} order derivative for non-integer n .

As seen above, the Euler, Fourier and Leibnitz are among the many that introduced an approach with fractional calculus. Many others also initiated a trend by using their own methodology and notation, definitions that suitable for the concept of non-integer order integral or derivative [15].

1.4.2 Definitions related to fractional order calculus

Fractional calculus is considered in different definitions and fractional differential of some special kind of function is given. The most famous definitions in the world of fractional calculus are Grunwald Letnikov (G-L), Riemann-Liouville (R-L) and Caputo definitions [16]. The most classic definition by Caputo is a modification of Riemann-Liouville definition in order to solve his fractional order differential equations by using integer order initial conditions [17].

1.4.2.1 Grunwald Letnikov (G-L) Definition

The Grunwald–Letnikov fractional order derivative can be expressed as [9]:

$${}_a D_t^\alpha f(x) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{j=0}^{\left[\frac{x-a}{h} \right]} (-1)^j \binom{\alpha}{j} f(x-jh) \quad (1.13)$$

Where $\binom{\alpha}{j}$ is the binomial coefficient and a is the initial value. By contrast, the G-L derivative starts with the derivative instead of the integral. Being an adept method to compute fractional order derivative, it gives good approximation to fractional derivative for sufficiently lower value of h . Also the precision of this method is high.

1.4.2.2 Riemann-Liouville (R-L) Definition

The Riemann–Liouville integral is defined by [9, 17]:

$${}_a J_t^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-\tau)^{\alpha-1} f(\tau) d\tau \quad (1.14)$$

Where $0 < \alpha < 1$ and a is the initial value, usually set to be 0.

1.4.2.3 Caputo Definition

Caputo's definition can be expressed as [17]:

$${}_a D_x^\alpha f(x) = \frac{1}{\Gamma(1-\gamma)} \int_a^x (x-t)^{-\gamma} f^{m+1}(t) dt \quad (1.15)$$

Where $\alpha = m + \gamma$, and $0 < \gamma \leq 1$, m is an integer

1.5 OBJECTIVES OF THESIS

- i) To devise an analytical model of fractional order differentiator based filter using G-L definition of fractional calculus.
- ii) To develop an algorithm for the edge detection operation utilizing the proposed fractional differentiator based filter and compare the results obtained with the conventional filters.
- iii) To compare the performance of proposed filter as well as other similar approaches and conventional operators based on various performance metric parameters like MSE and PSNR.

1.6 ORGANIZATION OF THESIS

The dissertation comprises of the following chapters which are summarized below as an overview of the compiled work.

Chapter 1: Introduction

This chapter gives a brief review of basic terms and definitions related to the theory on which the thesis work is based. It consists of theory on basics of an image, edge detection operators and its techniques, fundamentals of fractional calculus as well as the main objectives of the dissertation work.

Chapter 2: Literature Survey

In this chapter, a brief knowledge about the various related researches in the field of edge detection and fractional order filter based approaches are discussed in sequence. It gives a brief information regarding the various methodologies adopted by the researchers for improvement in image enhancement approach, especially in edge detection. This chapter presents the previous work done, their achievements and limitations.

Chapter 3: Proposed Methodology

This chapter comprises of the objectives and the methodology proposed in this dissertation work. It gives the sequence wise objectives to be achieved in the methodology as well as the steps involved in achieving those objectives. The methodology consists of two proposed algorithms; one is defined for filter designing by deriving window coefficient matrix while the other is defined for detection of edges using this proposed filter.

Chapter 4: Results and Discussions

This chapter consists of three parts. Part 1 is experimentation i.e. a set of images on which the experiment is performed. Part 2 displays the results obtained after applying the proposed methodology on the set of images being taken. All results are obtained in MATLAB R2014a. Further, there is comparison of results with the other related approaches as well as the conventional approaches. Part 3 gives information about the image quality assessment where the MSE and PSNR is discussed, analyzed and evaluated on the proposed approach.

Chapter 5: Conclusion and Future Scope

This chapter confers the conclusion about the work based on the obtained results. Further, the future scope of implementation of the proposed methodology is described.

CHAPTER 2

LITERATURE SURVEY

In recent years, various authors have proposed different methodologies for image enhancement as well as for fractional differentiation based approach in image processing. However some have shown a qualitative improvement in the conventional approaches while some others have discovered their own methodologies as per their perspective. The related work is discussed in this chapter.

Q. Chen *et. al.* [18] developed a non linear filter algorithm in which a 2-D fractional differential mask is constructed. The author presented the mask structure and parameters in eight directions through which the orientation of contrast enhancement of the test image is being controlled. The developed fractional order mask is made on R-L FOD approach. In the designing of filter, concentration is being put on discrete expression of FOD mask. The author has revised and simplified the numerical form of fractional differential to multiplication and addition. He called this algorithm as CRL operator. Further, after developing the fractional mask for all eight directions, they have calculated the sum of coefficients of filter mask and proved that as the sum is non-zero, the response of FOD filter is non-zero in the regions of constant gray level values of pixels. The author suggested that this algorithm is applicable for remote sensing images, infrared images as well as medical images.

The experimentation and analysis of the algorithm shows that this method has a good feedback for contrast enhancement of dark images. It is enhancing edges somewhat effectively and also with the higher values of fractional order, ν , clearer and detailed features are obtained. Further, the author has compared this methodology to HE and SSR methods and analyzed using the Gray Level Co-occurrence Matrix (GLCM) that the proposed method produce bigger contrast values than HE and SSR method. However, the computation complexity of algorithm is high, which causes an increase in computation time.

K. Singh *et. al.* [19] modeled a discrete time FOD using Legendre's polynomials. The author used it for the estimation of FOD of any given signal. The proposed filter is based on Savitzky-Golay approach where the filter is designed for data smoothening and the technique is based on fitting of the least square polynomial. The author has fitted the given signal using the Legendre polynomials and then computed the filter coefficients using least square method. The Riemann-Liouville (R-L) FOD definition is used to calculate the weight matrix. Further, the calculated weight matrix is convolved with the signal in order to compute the required FOD of the given signal.

The analysis of the proposed method shows that it is applicable for calculating the FOD of both noise free as well as noisy signals. However, the accuracy of this method is not high, but its usefulness cannot be overshadowed. This approach is further implemented by the author for low contrast and hazy images.

K. Singh *et. al.* [20] proposed a newly designed edge detection algorithm for low contrast images. The author has modeled a fractional order differentiator in the previous discussed paper in the literature. Here, the work is based on implementing a new model using the same approach, however, this time the author is using Chebyshev polynomial based FOD for approximation of the filtering operation on the low contrast images. Using Quadrature Mirror Filter (QMF) concept, HPF and LPF are implemented separately. Then, a pre processing is performed using the fractional order differentiator in which the low pass filtered image is enhanced using relaxation coefficient. At last, by using Sobel edge detection method on the final image, observations are being made by varying the relaxation coefficient.

The author implemented the approach on low contrast satellite images. It was found that the conventional operators were not able to detect edges in case of highly complexed satellite images due to its smoothness and hazy nature. However, extracting edges in these cases is not easy; the newly developed masking based edge detection algorithm is playing a major role and is performing well for lower values of relaxation coefficient.

C. Chi *et. al.* [21] has provided new numerical methodologies, based on Taylor and Fourier series techniques, for computation and simulation of fractional order derivatives by using Matlab software. The author reviewed some basic FOD definitions like the G-L definition, the R-L definition and the Caputo definition and remarked their similarities and outcomes. Further, the author computed the n^{th} derivative of various basic functions like $\sin(x)$ and $\cos(x)$ by transforming their integer order Fourier series into fractional order, and implemented it on MATLAB to see their results. Similarly, the fractional order derivative of a function is computed using its fractional order Taylor series.

The author concluded using this approach that the higher the order of approximation of the function using Fourier or Taylor series is, the higher will be the accuracy. Also the G-L method is more efficient in calculating the FOD, with high accuracy.

M. Xu *et. al.* [22] developed a new non linear filter algorithm, for image quality enhancement, based on variable-order fractional operators. The author has divided the process of construction of variable order fractional order derivative mask into two parts. In the first step, filter coefficients are obtained using high order fractional derivative discretization formula. Once the discrete coefficients of fractional derivatives are obtained, a FOD mask is constructed. In the second step, a gradient based method is used for optimizing the mask values and the fractional order adaptively. The R-L definition is being used in this process.

The author experimented and analyzed the proposed algorithm on heart based images. The images are taken from medical data. The images were low contrast images having noise immunity. A discrete method based on higher order is showing improved performance as compared to the traditional first order discrete methods for enhancement of edges. The results shows higher average gradient and contrast values for the proposed method. In addition, a higher PSNR value is also obtained. Also, due to optimization, the noise can be easily suppressed and smoothed data can be obtained effectively.

V. Garg *et. al.* [23] proposed a non-linear 2-D FOD operator for texture enhancement. This operator is an enhanced version of G-L derivative implied fractional differentiator. This 2-D

isotropic gradient operator mask is executed on different digital images having texture rich information. The author has performed two algorithms. In the first algorithm, the intensity factor is kept constant while the fractional order is varied from 0 to 1. It exhibits that the consistency of the image, using enhanced G-L fractional derivative mask, is directly proportional to the fractional order, unveiling a significant increase with increase in the order. But for higher values of fractional differential order, it results in image distortion. However, the distortion is less than the traditional G-L fractional derivative mask. In the second part, the intensity factor is varied while keeping the order fixed. The degree of enhancement of image feature is controlled by intensity factor.

The author has also calculated the average gradient and information entropy in the digital images. From the quantitative analysis, it is distinguished that the information entropy is directly proportional to the intensity factor. But for higher values of intensity factor, there will be loss of textural information.

Y. Pu *et. al.* [24] implemented a new mathematical approach which intends to the derivation of high précised fractional differential masks. Six fractional differential masks are derived using the approach constructed on G-L and R-L definitions. The author then demonstrated the most efficient mask by both theoretical and experimental analysis. The structured mask with parameters on the direction of positive x-coordinates comes out to be the most efficient. Further, the experimentation is accomplished on real data sets. The non linearity of the algorithm for texture enhancement and the capability of multi scale FOD masks are discussed in the experimentation.

The author studied the texture segmentation and texture enhancement of multi scale FOD masks and analyzed that the proffered algorithm is superior and more effective than the traditional integral differential-based approaches.

C. Gao *et. al.* [25] aspired to introduce an improved fractional differentiation (IFD) operator, which is concerned on enhancement of image. After the evolution of the FOD and its applications in the advanced signal processing, the author tried to refine the numerical

calculations by utilizing the IFD method established on piecewise quadratic interpolation equation. In this work, eight IFD masks are put into practice in all eight directions. The masks so enacted are symmetric and directional independent. For that, a newly built operator called IFD operator is proposed which would be applied for enhancement of image. For the digital images with enriched texture information, experimentation is being done to check the capability of non linear fractional differentiator for enhancement of comprehensive texture details using IFD. In this approach, the variation of derivative order is less as it can select values up to 3 only. This is done for expanding the transition width of the operator. This also helps in selecting the required data only as it favors enhancement preciseness.

Further, quantitative analysis is done by calculating the entropy and average gradient. The average gradient is sensitive to contrast of lesser details. It will help in evaluation of clarity in the image. The entropy is calculated to obtain the information regarding the consistency in the image. On observation, the consistency details become clearer and the texture channels become deeper by using IFD operator.

X. Chen *et. al* [26] proposed a new methodology on improving the limitations of Roberts edge detection algorithm. The author has discussed mainly on how to use the advantages of fractional differential method to overcome the Roberts method shortcomings. This paper presents fractional differential operator characteristics for enhancing detailed texture features in an image, which will help in improving the standard Roberts edge detection operator. In the newly described mathematical approach, a 3*3 fractional order differential mask is structured on the eight central symmetric directions. Further, a combination of newly constructed mask with traditional Roberts operator is put forward.

The experimentation and results showcase that the new algorithm can detect thin and delicate edges besides doing edge enhancement with higher accuracy, more detailing and fine distinctions. This method can be used as a first step or as a reference towards pre processing of any application comprising of rich textured images.

H. Jalab *et. al.* [27] worked on designing an image consistency augmentation technique using Savitzky Golay (S-G) differentiator. S-G filter is used for smoothening of images and enhancing the textural information. By abstracting the S-G FOD operator, the author calculated the FOD of the given image by utilizing the moving weighted matrix window. It is remarked that the abstracted fractional differential is highly effective because of its delicateness to the sharp fluctuations of the pixel intensities.

In this paper, the author has used an abstracted FOD rooted on the Savitzky Golay (S-G) approach. In the abstracted method, Srivastava-Owa fractional operators are used for texture enhancement of an image. The S-G filter is introduced for computing the numerical results. Since S-G filter is used for smoothening of images, it is also called as a smoothening filter. This method is utilized to retain higher frequency in the data, thus minimizing the deformation of important features. As per the quantitative analysis, by evaluating the interpretation of the suggested algorithm by differentiating the four statistical measures, it is perceived that this new algorithm can manage the orientation of image consistency. Also, the fractional order of the parameters can also be controlled.

H. Jalab *et. al.* [28] instigated a texture magnification technique for thoracic aorta images. The author manipulated fractional differential (FD) masks rooted on Srivastava-Owa maneuvered operators. Further, the FD operator has been propagated and a 2-D isotropic gradient mask is constructed. The discrete numeric of each of the FD masks is acquired by moving the mask window. The output of each image portion is 8 valued. It represents the information on consistency of each image. The author analyzed the consistency enhancing capability of the FD masks rooted on the visual interpretation. Further, by utilizing Sobel edge filter with GLCM matrix, the relationship between fractional power parameters and texture enhancing details can be computed.

The consistency of the image is strengthened by the algorithm established on FD masks. Further, the proffered algorithm controls the orientation of consistency improvement by utilizing the fractional order parameters of the multi fractional derivative model.

C. Li *et. al.* [29] proposed an adaptive FOD image amplification algorithm based on image complexity. The principle of the algorithm is based on calculating the value of fractal image differential dimension in terms of differential box. Further, the differential dimension box calculation is based on the image complexity. Next, a mathematical relationship is formed between fractional differential value and standard FOD algorithm. At last, based upon the fractal dimension of image complexity, the optimal fractional order will be determined. Making use of the fractional differential algorithm with the determined value, an adaptive fractional differential enhancement of the target image is acquired.

The experimentation and study of the proffered algorithm manifests that the target image will preserve the low frequency information, and in the mean time it will also strengthen the high frequency information. This enhanced optimal effect will improve the resultant image effectively by avoiding the false edges.

A. Sparavigna *et. al.* [30] discussed on the possibilities and use of fractional differential calculus as a tool for the processing of astronomical images. Generally, the content matter in the astronomical is very vast and a little information loss can cause a huge limitation to its application. The main content matters contained in any astronomical image are faint object galactic matter, nebulosity, stars and planetary surfaces. The objective of the proposed fractional differentiator will be to enhance the astronomical objects and obtain as much detail as possible on the targeted cosmic image. The author has proposed various situations and examples to show how effectively one can achieve the results from the image processing of astronomical images through fractional differentiation. For that, long exposure times are required, but it will also enhance various types of noise in the astronomical images such as atmospheric moisture, pollution, dead pixels, stray light, amp glow etc. which will appear in the final exposed image. Further, removal of such type of noise will distort the image details and as a result the objects will look fainter and weaker in contrast.

Therefore, the proposed fractional differentiator will be able to scan and examine an image to detect the faint, weaker edges and enhances them suitably so not much information loss can occur. This can become an interesting source for further scientific purpose, for instance to discover supernovae, comets and very faint galaxies.

B. Mathieu *et. al.* [31] demonstrated that how the introduction of an edge detection algorithm established on FOD can improve the principle of detection of thin and weaker edges, selectivity and noise immunity. The objective is to analyze the fractional derivative applied to the parabolic step-type transition, and to compare its maximum abscissa value to that of the inflexion point. Further, main focus is put on improving the selectivity of inflection point detection. For that improvement, an operator is designed at the point where the parabolic step response shows a cusp on the inflection point abscissa. This can be achieved by the synthesis of a newly propounded detector, being named as CRONE detector. It is acquired through the propagation of the order of fractional derivative.

The author evaluated the distinctiveness of the CRONE detector. It is remarked that when the derivative order is between 1 and 2, the detector favors selectivity, while when the derivative order is between -1 and 1, it favors noise immunity. The evaluation also presented a comparison of CRONE performance with that of the standard Prewitt's gradient, which showed that the CRONE detector provide boosted noise immunity.

S. Hemalatha *et. al.* [32] proposed a modified model of the G-L definition specified FD operator. Considering the basic definition of the G-L specified FD operator, a filter is devised and its applicability for texture enhancement of images is analyzed. For boosting the consistency of the gray scale and colored images, the traditional G-L operator is improvised based on the fact that the gray level of a pixel is dependent on its neighboring pixels. This dependency is determined by the spatial autocorrelation function. For that, the filter coefficients having zero value in the standard G-L filter are made non-zero by distributing the autocorrelation feature across the pixels in the neighborhood.

The author experimented the approach on various standard texture enriched images. It is analyzed that the textural features of the image are enhanced nonlinearly and also, the contrast is enhanced by using the modified filter. Further, based on the quantitative analysis using the GLCM measures, it is proved that the proposed modified G-L fractional order filter is performing better than the histogram equalization approach, conventional G-L fractional order filter and adaptive FD filter.

Z. Wang *et. al.* [33] proffered a new algorithm based on the wavelet de-noising principle. The newly devised algorithm is rooted for edge detection of images. When image is processed with white noise, initially a threshold value is set and image is filtered based on that value. Next, a fractional mask is being proffered. For that, the differential equation is constructed by rooting the standard G-L FOD definition. The experimentation of the mask versus the conventional operator is done for the edge detection in images. It is demonstrated that the fractional order can extract edges better than the integer order differential. Then, using the threshold wavelet algorithm, the image edges are extracted by combination of various orders of the fractional mask.

The experimentation of the proposed method is implemented on medical images as well as remote sensing images. The author compared the proposed algorithm to the traditional algorithm and concluded that the proposed method is working effectively in extracting the edge information in an image. Moreover, based on the quantitative analysis, it is also proved by the author that the FD mask operator can restrict the noise well, since it has a good PSNR ratio, and good positioning performance.

D. Tian *et. al.* [34] introduced a new FOD operator for feature extraction principle. The application of this operator is realized for medical images. The proffered operator is a popularized Sobel operator rooted on G-L FOD definition. This FOD operator, being a global operator, is based on the fact that it extracts more information from its neighboring pixels. Therefore, the author generalized the conventional Sobel operator into fractional order and utilized its frequency characteristic for extraction of structural features in the image. Further, by calculating the average gradient, a threshold point is set. By utilizing the threshold value, the pixels below a minimum threshold is neglected the required structural features can be extracted from a less crowded image.

The author experimented the novelty of the operator on MRI images as well as ultrasound images. The experimentation shows that for larger fractional order, superior features are preserved. The only disadvantage is less noise immunity. However, the proffered FOD operator fetches good visual results and better feature extraction effects.

M. Mekideche *et. al.* [35] investigated the conventional Canny operator and, keeping in mind its limitations, proposed a new method called as modified Canny operator. The main aim was to reduce the computational time by skipping the smoothing step. For that, the author introduced a fractional integral mask (FIM). It is shown that instead of smoothing operation on the image by using the traditional integer gradient masks, which are sensitive to noise, it should be just convolved with the newly investigated FIM. The coefficients of the FIM are computed by utilizing the fitting method defined by the Grunwald-Letnikov FOD. It is remembered that FIM is directly related to the fractional order α . This relation determines the how much a pixel is contributing in an image. This fact determines the potential of the FIM to unveil the faded and thinner edges by adjusting the value of α .

The author interpreted FIM efficiency in terms of computation time, noise removal and capability of diagnosis of weaker edges. The experimental study and examination exhibits that the weaker edges can be strengthened by fine tuning the parameter α . The author also described FIM as a prominent application tool for low contrast satellite images.

H. Yang *et. al.* [36] proffered an edge detection method using FOD approach. In this paper, the author obtained a new edge detection order by using fractional order differentiation and integration. In the conventional integer order filters, smoothing is a necessary step for improving noise immunity, but somehow the accuracy suffers as then the weaker edges can't be predicted easily. Therefore, the author introduced Ye and Yang fractional order operator, which eliminates the preprocessing smoothing step and provide a new process towards compromising the tuning between noise immunity and accuracy. In this approach, the gradient masks are calculated utilizing the derivative while suppressing all the points other than maxima. A double threshold is applied to bridge a link between the edge points.

The author collated the interpretation of the proffered operator with the traditional Canny operator as well as with CRONE operator in terms of noise immunity and accuracy, and analyzed that the proposed method is superior and effective.

F. Dong *et. al.* [37] proposed a noise removal method constructed on the combination of different FOD regularizations. By combination of different FOD in different textures, the newly proposed edge-texture detector function can conserve the structural features and also circumvents the staircase effect during noise removal effectively. In the smoother regions, the fractional order greater than 2 is regularized for noise removal. In the region of edges, the fractional order should be regularized in (0,1] to conserve the edges.

The experimentation and results demonstrates that the proposed model have superior interpretation than the conventional second derivative model. Further, using the quantitative analysis, it is clear that the PSNR and SNR value of the proposed model is higher than the latter models. Thus, the main advantage of this framework includes better texture and repetitive structures while superiorly eliminating the staircase effect.

C. Tseng *et. al.* [38] investigated the design problem and application of the variable fractional order differentiator (VFOD). Initially, the author calculated the FD of a function and defined its value utilizing the propagation of the Cauchy integral formula. Then, the least square method using suitable weight implementation technique has been manifested to design the variable FOD based on FIR. Finally, the designed VFOD is put into application to combine the effect of inverse frequency $1/f$ noises.

The author minimized the error using the proposed VFOD filter. The experimental results indicate that the proffered method can combine and produce the $1/f$ noises more accurately and effectively than any of the traditional FOD methods.

J. Zhang *et. al.* [39] proposed a noise removal method constructed on FOD method instead of the traditional integer order gradient method of the image. In the de noising process, it becomes utmost important to conserve the finest scaling features from distortion. Further, the author computed the numerical analysis utilizing the fractional difference in discrete form. For analyzing the filtering characteristics of the FOD operators, Fourier transformation pair for fractional order is studied.

The experimentation and numerical results indicates that the proffered model is not only performing superior for conserving fine textures but also giving a better de noising by improving the signal to noise ratio value of the noisy image.

X. Pan *et. al.* [40] deduced a novel complex fractional differential mask which is derived by the implementation of fractional calculus. Further, a new compound derivative is proposed by the amalgamation of fractional differentiation and fractional integration, in which the FOD mask is employed. The forward differentiation is realized by mask convolution while the reverse integration is obtained by filter convolution. The feasibility of the mask can be determined. It exhibits effectiveness in introduction of complex calculation in signal detection. The differentiator together with the complex mask forms as the newly embedded operator for detection of edges.

From the experimentation and results, it can be seen that the length of the mask should be set at smaller values. Similarly the width of the filter function is to be kept low. This implies that the author has reduced the complexity of the function. Further, the 1-D and 2-D experimental results exhibit that without the noise, the newly proffered detector can determine the edges accurately, while in presence of the noise factor, the mask is suppressing the noises effectively. Finally the quantitative study demonstrates that the proffered complex mask compound derivative operator is outperforming the Canny operator.

Y. Liu *et. al.* [41] proposed a new approach for augmenting the texture details of remote sensing images based on fractional differential theory. Initially, the author draws some important conclusion based on studying the properties of fractional order differentiation. As the texture details are a dominant factor for remote sensing images, and the standard conventional approaches are not much effective, a new operator is proffered by discussing the algorithm of fractional differential. In the presented method, convolution is performed between the fractional differential and the given image. At last, selectivity of the differential order is automatically controlled based on the entropy principle.

The analysis of the results exhibits the effectiveness of the proffered method. It also demonstrates that the proffered method is performing superiorly than the traditional methods utilizing the Histogram Equalization or Laplacian operator.

H. Mingliang *et. al.* [42] presented an information extraction algorithm based on fractional order differentiation as per its characteristics in signal processing. The algorithm is carried out for Poisson noise images. In the fractional differentiation in image processing, as the pattern of fractional differential operators must be changed into difference form, the author derived the fractional difference formula using the G-L FOD definition. The gray scale value of the pixels is incremented by employing the repeated differentiators. Several iterations are being run on the image and the pixel with maximum grayscale value is extracted.

As compared to the integer order differential algorithm, not only the high frequency information i.e. edge information can be extracted to larger extent but also the texture information is extracted from the smooth region that they can't locate. Further, the amount of texture information can be controlled by adjusting the iterations count. Also this algorithm provides noise immunity effectively.

Z. Gan *et. al.* [43] determined a new mask, for boosting the consistency of images. This mask is employed on the Riemann-Liouville (R-L) FOD definition. From the spectrum of 1-D R-L fractional differential, the consistency and fadedness of the image is strengthened by this mask. A 2-D isotropic gradient mask is approximately constructed based on the R-L fractional differential definition. A non linear filter is convoluted with the mask to execute the image. Further, the orientation of the consistency of the images is controlled by the variational fractional order and the intensity factor.

The experimental analysis demonstrates that the proposed operator can extract the information precisely and can upgrade the edges more prominently. Also, in contrast to the traditional Laplacian operator or the Histogram Equalization technique, this proposed method shows outstanding result and outperforms the latter operators in terms of texture enhancement. The disadvantage of this method is that as the R-L fractional derivative is sensitive to noise, the noise

will be expanded. Therefore suitable practices should be administered for minimizing the effects of noise.

M. Hou *et. al.* [44] studied the integer order differential operators and draw a conclusion that the texture and edge information can't be easily identified using integer order operators in image processing of natural images. So, in order to overcome this limitation, the author prospered a FOD algorithm which not only extracts textural information but simultaneously the marginal information can be analyzed. On the basis of the drift problem in the marginal detection of the fractional differential operator, the author also developed a fractional order template in this study.

The experimental analysis of the results exhibits that the marginal information and the image consistency were extracted simultaneously by using the proposed approach. This dual working principle is useful for image recognition. Further, the consistency in the information of a composite image can be differentiated easily by using the proposed method.

A. Yusra *et. al.* [45] studied various types of image quality measurements (IQM). Image quality assessment is approached for inspecting the image quality. Further, the various measures are correlated to draw an analysis. Since different techniques are available for quality measurement but each has its own limitations and therefore none is considered to be perfect for quality measurement. The author studied various IQM techniques based on pixel difference measures like MSE, SNR and PSNR, neural networks, region of interest (ROI), edge detection and human visual system (HVS).

A good IQM should be precise in measurement and steady in approach. It should be uniform in speculating the image quality. After analyzing the limitations of each studied approach, the author concluded that despite the subjective measures are expensive and a time consuming process, but it still outperforms objective measures. In addition, the objective measures are simpler and an open source but it needs a lot of improvement to cooperate with the subjective measures.

K. Zhang *et. al.* [46] presented a new method for assessing the quality of digital images. The author used the traditionally used mean square error technique and applied learning to it. Thus due to the applied learning, the new method will vary the iterations based on the number of inputs given and rotate each iterations the number of times such that an adaptive solution is obtained. This new approach is constructed on correlation between the pixels and weighted mean square error (WMSE). Thus the proffered technique is dependent on two factors, i.e. the mean square error and the pixel correlation in the specified neighboring area. The problem involved with the traditional MSE and PSNR measures is that it does not take visual analysis of image counterparts into account. The visual approach is sensitive and can vary from person to person. It is a self perspective approach and different results will be obtained for different visions.

After experimentation and analysis on the standard Lena image, the author observed that WMSE satisfies conformity and visual judgment. Also, although WMSE technique requires more computation time, but since it is a modification of the MSE technique, it is mathematically tractable and is monotonous with different degrees of image distortion. However, only by selecting a suitable mask size and an ordered function β , an adaptive satisfactory performance can be obtained.

D. Poobathy *et. al.* [47] compared various traditional edge recognition operators and drawn an analysis. The comparison is made on the basis of MSE and PSNR values as applied on an image after detecting the edges using the conventional operators like Sobel, Prewitt, Canny, Laplace of Gaussian (LoG) and Roberts. The performance of each image is evaluated with combined results of MATLAB and Java. The edge detection algorithm is applied on MATLAB while the PSNR and MSE values are obtained using Java programming.

The author experimented all the traditional approaches on a group of four images. A comparative analysis is drawn by validating the PSNR and MSE values. The interpretation of the suitable algorithms is evaluated based on the quantitative analysis. After the experimentation and results, the author finds Canny operator as superlative among all the traditional operators in terms of accuracy in edge detection. Then, LoG, Prewitt, Sobel, and Robert's algorithm are graded in the respective order.

CHAPTER 3

PROPOSED METHODOLOGY

Underwater imaging raises new difficulties and intrudes considerable challenges due to absorption and scattering effects of light. They suffer from low contrast and degraded vision of farther objects due to the debilitation of the propagated light. Thus, when such images are captured, quality of the image derogates [48]. Therefore, the image is captured via camera from a small distance in order to maintain a suitable quality. Hence it becomes necessary to pre process these images before applying any high level image processing.

These images have texture edges and smoothness as the major dominant factors present in them, which allows us to apply the low pass filtering operations which extricates their texture edges. Various popular edge preserving filtering techniques have been applied for underwater preprocessing however it is found out that edge extraction in low contrast underwater images having smoothness property is strenuous [49,50]. Also, the inbuilt conventional algorithms for edge detection fail to provide satisfactory results in these types of images. These traditional edge detection techniques have some major drawbacks. For instance these integral order filters are susceptible to noise, making the edges gets distorted.

By visual interpretation, it is noticed that texture features are sharp details in images and it is inferred that differential operators may be considered for highlighting the textural information in images. Basically, first and second order differential operators such as gradient and Laplacian operators are used to highlight the edges and boundaries in images. It was also proved in the recent literature that fractional differential operators are found to be more appropriate for image textural features than integral differential operators. Thus, fractional differential operators are considered for texture enhancement in images.

The aim of this chapter is to discuss the objectives and methodology of the curriculated work in a detailed manner.

3.1 OBJECTIVES OF THE METHODOLOGY

The objectives to be fulfilled in the methodology of the dissertation are as follows:

- (i) To propose an analytical fractional order model.

As texture plays an important role in the low level image analysis, therefore texture based image enhancement is done. In order to attain texture enhancement in images, a fractional order derivative (FOD) model is proposed.

- (ii) Algorithm to design the filter based on the proposed fractional order model.

Considering the G-L based fractional differential operator's basic definition and implementation, a filter is devised and its applicability for texture enhancement is analyzed. Legendre polynomials based FOD has been used for design of filter.

- (iii) Algorithm to apply the edge detection operation.

Utilizing the proposed FOD filter in combination with any conventional edge detector (we have used Sobel edge detector), an algorithm for effective edge detection is designed.

- (iv) To compare the results obtained using the proposed approach with the conventional edge detection operators using integer order operation from the point of view of various performance metric parameters.

3.2 PROPOSED METHODOLOGY

3.2.1 Selection of Grunwald–Letnikov (G-L) FOD model

The Grunwald–Letnikov (G-L) fractional order derivative can be obtained as:

$${}_i D_i^\alpha f(x) = \lim_{k \rightarrow 0} \frac{1}{k^\alpha} \sum_{j=0}^{\lfloor \frac{x-i}{k} \rfloor} (-1)^j \binom{\alpha}{j} f(x - jk) \quad (3.1)$$

Where $\binom{\alpha}{j}$ is the coefficient in binomial form and i is the initial value.

By contrast, the G-L derivative starts with the derivative instead of the integral. Being an adept method to compute fractional order derivative, it fits better for lower value of k . Also it can be verified that precision of this method is high.

It is examined that, for majority of testing functions, G-L and Caputo derivatives are identical. A little variation occurs with the constant part.

Further, Caputo's derivative can be obtained as:

$${}_i D_x^\alpha f(x) = \frac{1}{\Gamma(1-k)} \int_i^x (x-t)^{-k} f^{m+1}(t) dt \quad (3.2)$$

Where $\alpha = m+k$ and $0 < k \leq 1$, m is an integer.

3.2.2 Design of Filter

We are designing a fractional order filter based on Legendre Polynomials as the filter coefficients. A brief theory on Legendre Polynomial and the concept of designing of filter using the same is discussed in this section.

3.2.2.1 Legendre Polynomials

Legendre polynomials (or Legendre functions) are a particular class of functions which are distinctly convenient for approximating other functions. It plays a key role in many problems of numerical integration, numerical differentiation, numerical solution of ordinary polynomials, uniform approximation, partial approximation and least square approximation of continuous functions and implied calculation of such approximation.

In mathematics, Legendre functions are solved from the following Legendre's differential equation:

$$\frac{d}{dx} \left[(1-x^2) \frac{d}{dx} P_n(x) \right] + n(n+1) P_n(x) = 0 \quad (3.3)$$

or

$$(1-x^2)P'' - 2xP' + n(n+1)P = 0 \quad (3.4)$$

where $n \in R$.

The equation (3.4) described above is called a legendre equation of order n .

The Legendre polynomials $P_n(x)$ for $n=1,2,3,\dots$ can be determined using Rodrigue's formula as:

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n \quad (3.5)$$

Therefore, for individual values of n , Legendre polynomials of distinct degrees are obtained using Rodrigue's formula.

$$\begin{aligned} P_0(x) &= 1 \\ P_1(x) &= x \\ P_2(x) &= \frac{1}{2}(3x^2 - 1) \end{aligned} \quad (3.6)$$

and so on.

The graph of these polynomials (upto $n = 5$) is shown in Figure 3.1.

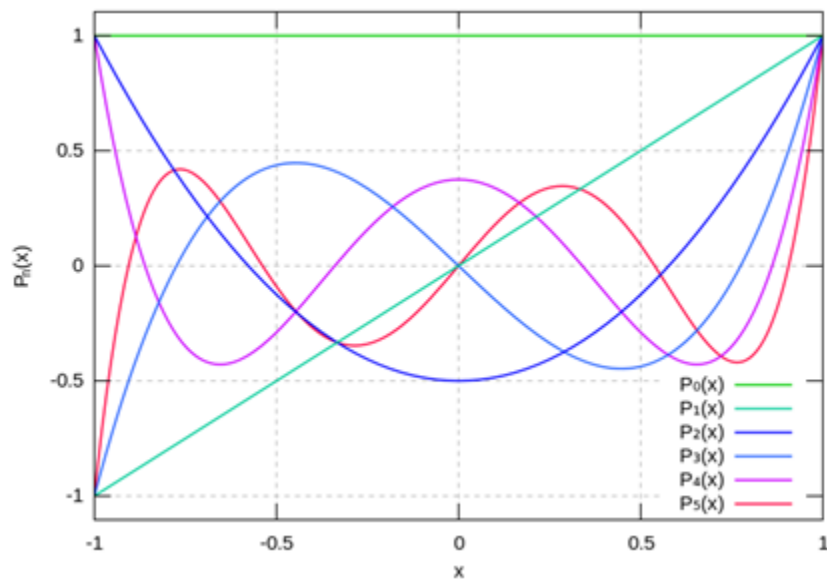


Figure 3.1 Graph of legendre's polynomials

Legendre's polynomials of distinct orders can be used to fit any given function. This property of Legendre polynomials will be used to define the proposed filter coefficients as shown in the next section.

3.2.2.2 Legendre polynomials based algorithm

Let's consider two higher order functions as $X(t)$ and $\tilde{X}(t)$, which are differentiable in R .

$X(t)$ is the measured function while $\tilde{X}(t)$ is the given function.

Thus, the relationship between the measured function and the given function can be obtained as:

$$X(t) = \tilde{X}(t) + E(t) \quad (3.7)$$

where $E(t)$ is the approximate error.

The current work comprehends smoothening of the measured function by estimating the dn^{th} derivative, t point window filter and a n -degree polynomial. Using the Savitzky-Golay approach, any least square polynomial can be expressed as:

$$X(t) = \sum_{k=0}^n z_k Q_k(t) \quad (3.8)$$

where $X(t)$ is a n -degree polynomial function, which is used to approximate the given signal, $t = 1, 2, 3, \dots, L$ is the region of t^{th} point in the filtering window and z_k is the k^{th} coefficient of the polynomial. The coefficients z_k can be obtained by using least square method. Q_k denotes equivalent value of Legendre polynomial respective to t^{th} point and k^{th} coefficient.

Equation (3.8) is further expanded as:

$$\begin{aligned} Q_0(1)z_0 + Q_1(1)z_1 + Q_2(1)z_2 + \dots + Q_n(1)z_n &= x_1 \\ Q_0(2)z_0 + Q_1(2)z_1 + Q_2(2)z_2 + \dots + Q_n(2)z_n &= x_2 \\ Q_0(3)z_0 + Q_1(3)z_1 + Q_2(3)z_2 + \dots + Q_n(3)z_n &= x_3 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots & \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots & \\ Q_0(L)z_0 + Q_1(L)z_1 + Q_2(L)z_2 + \dots + Q_n(L)z_n &= x_L \end{aligned} \quad (3.9)$$

For better understanding, we will use matrix notation further, and (3.9) can be re-written as

$$X = QZ + e \quad (3.10)$$

where $X = [x_1, x_2, \dots, x_L]^T$ denotes the observed function points in the filtering window matrix, $Z = [z_1, z_2, \dots, z_L]^T$ denotes the coefficient matrix and e is the estimated error.

Q is a Legendre polynomial matrix of order $L \times (n+1)$ and can be defined as

$$Q = \begin{bmatrix} Q_0(1) & Q_1(1) & Q_2(1) & \dots & Q_n(1) \\ Q_0(2) & Q_1(2) & Q_2(2) & \dots & Q_n(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Q_0(L) & Q_1(L) & Q_2(L) & \dots & Q_n(L) \end{bmatrix} \quad (3.11)$$

The values of components of Q can be computed by using equation (3.5). Thus the components of the matrix Q are the values of Legendre polynomial at specific point. Vector Z , storing the coefficients of the best-approximated polynomial, is calculated as:

$$Z = (Q^T Q)^{-1} Q^T X \quad (3.12)$$

Further, equation (3.10) can be solved using equations (3.5) and (3.12). The estimation of given function is computed as:

$$\tilde{X} = QZ = Q(Q^T Q)^{-1} Q^T X = WX \quad (3.13)$$

where W symbolize the filtering window's coefficient matrix. It can be employed for smoothening of the given function. Different window coefficient matrix can be utilized for different smoothing.

The FOD, relative to the filtering window coefficient matrix W , can be obtained by equation (3.1).

By applying various properties of FOD on equation (3.12), we obtain,

$$\tilde{X}_i^\alpha = Q_i^\alpha Z = W_i^\alpha X = z(Q^T Q)^{-1} Q^T X \quad (3.14)$$

Where \tilde{X}_t^α indicates the α^{th} derivative of the t^{th} point in the window filter and W_t^α expresses the α^{th} derivative coefficient of the t^{th} point in the window filter.

3.2.2.3 Overview of filter designing algorithm by deriving window coefficient matrix

In the basic approach for the designing of FOD filter, we need to provide the length of the differentiator filter, L , and the order of the polynomial, n , manually. Further, the order of derivative α can be chosen based on the fact that at which value of the derivative order we are getting the maximum amount of signal data. Thus, after confirming these input arguments, we need to concentrate on deriving the Legendre coefficients. Next, the constant value, z , is computed for the fractional order n^{th} derivative of the signal x^n .

At last, combining the results using the equation (3.14), we procure the window matrix for the fractional order, α . This matrix is derived to be used as a low pass filter. Further, a high pass filter can be prevailed using some relation.

Algorithm Overview:

Input arguments: L, n, α

Output arguments: W, h_0

L : Differentiator Filter Length

n : Polynomial Order

α : Derivative Order

Q : Legendre polynomial matrix

z : Constant value

W : Window Matrix

Γ : Gamma Function

start

for $a \leftarrow 1$ to L

for $b \leftarrow 0$ to n

Enumerate matrix Q_{ab}

$$Q_{ab}(a) = 2aQ_j(a) - Q_{b-1}(a)$$

{ Here $Q_0(a) = 1$, $Q_1(b) = b$ }

end

end

for $a \leftarrow 1$ to L

for $b \leftarrow 0$ to n

Compute $z = \frac{\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+1-\alpha)} a^{n-\alpha}$

$$W_t^\alpha = z(Q^T Q)^{-1} Q^T$$

$$G_0(a) \leftarrow W_t^\alpha$$

$$h_0(a) = (-1)^L G_0(a)$$

end

end

end

3.2.3 Algorithm for detection of edges

We have derived the low pass filter and high pass filter using the window coefficient filter matrix in the previous section. Now, we need to define an approach for edge detection by combining the proposed filtering operation on any standard edge detection algorithm. We are using Sobel edge detector for our approach.

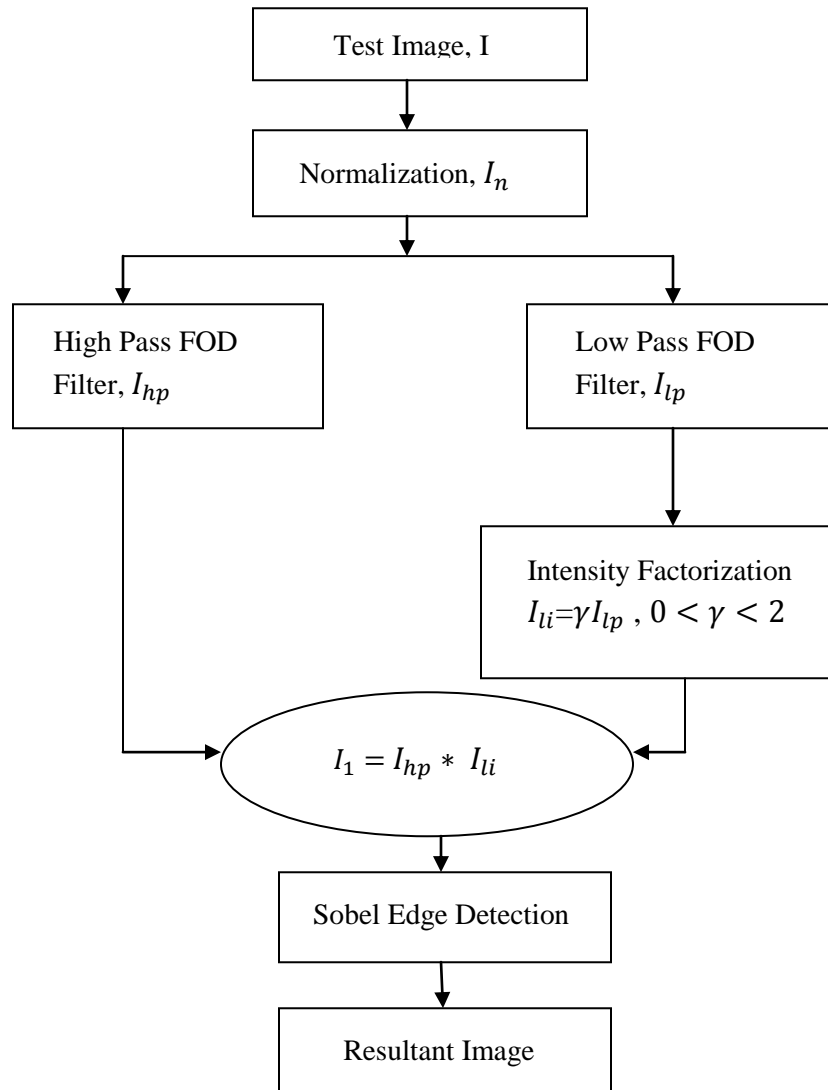


Figure 3.2 Flow chart of proposed edge detection algorithm

Figure 3.2 demonstrates the flow chart of the proposed edge detection algorithm method. The test image, I , has to be normalized initially. The normalized image, I_n , is then processed with the designed FOD filter, created using Legendre polynomial. The LPF and HPF filtering operations are performed on the normalized image separately. I_{li} symbolizes the observed image of the LPF filtering operation followed by the intensity factor. The intensity factor varies from 0 to 2. The role of intensity factor is to enhance the weaker and smaller edges and brighten the contrast of the observed image prevailed using the low pass filtering operation. I_{hp} signifies the output image of the HPF filtering operation. Multiplication operation has been performed between I_{li}

and I_{hp} . At last, for detecting the edges, the traditional sobel edge detection operation is implied on this resultant image.

Algorithm Overview

An overview of the basic steps of the proposed algorithm:

I : Test Image

γ : Intensity Factor

I_n : Normalized (or equalized) image

I_{lp} : Extracted image with LPF

I_{hp} : Extracted image with HPF

I_1 : Unprocessed Image

I_E : Resultant Pre-processed Image

$h_0()$: High pass filtering operation

$G_0()$: Low pass filtering operation

$Sobel()$: Conventional Sobel edge detection operation

$Norm()$: Image normalization

start

$$I_n = Norm(I)$$

$$I_{hp} = h_0(I_n)$$

$$I_{lp} = G_0(I_n)$$

$$I_{li} = \gamma I_{lp}$$

$$I_1 = I_{hp} * I_{li}$$

$$I_E = Sobel(I_1)$$

end

CHAPTER 4

RESULTS AND DISCUSSIONS

In this chapter, the capability of our proposed fractional order differentiator is validated. The edge detection of noiseless/noisy image is analyzed using the traditional techniques as well as using our newly improved fractional order filter technique as described in the previous chapter. Further, a qualitative comparison is made based on the increment or decrement of the information in the observed image.

4.1 EXPERIMENTATION

The edge detection analysis is conducted on six test images. All test cases taken into consideration are underwater images having smoothness properties as well as low contrast. The suggested algorithm is tested over a wide range of image dimensions from 330x287 pixels to 1200x900 pixels as shown in Figure 4.1.

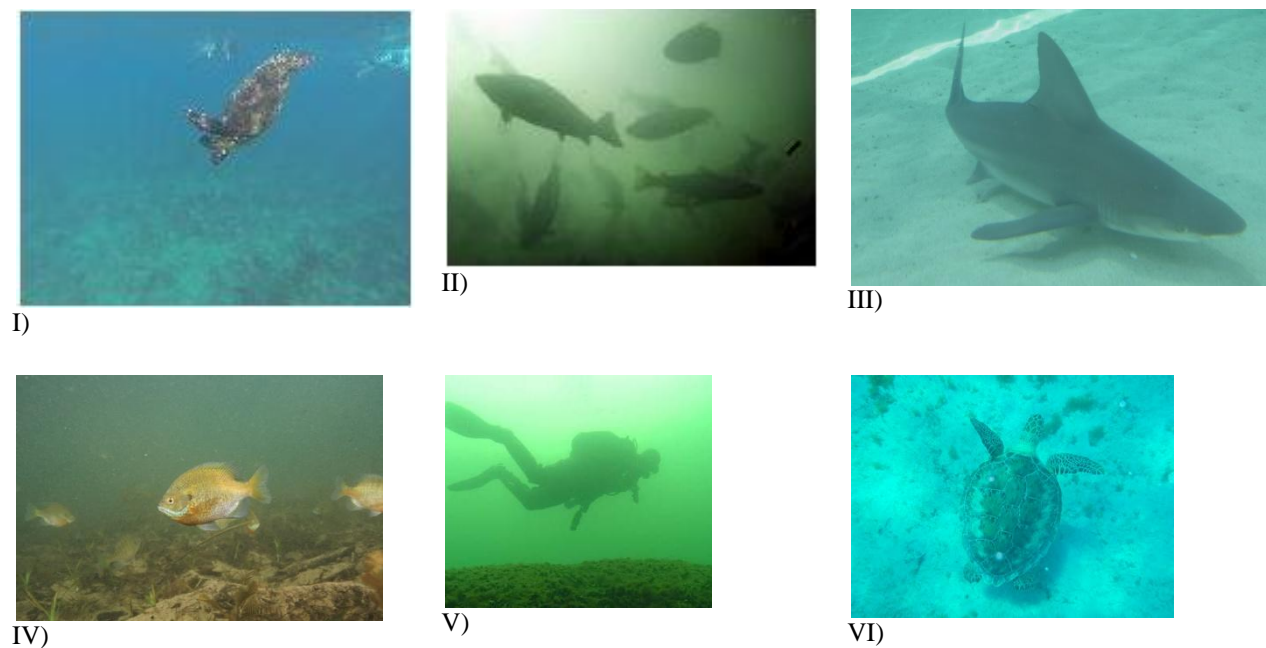


Figure 4.1 Experimental Images: I) Test Image 1 (1200x900), II) Test Image 2 (600x420), III) Test Image 3 (900x600), IV) Test Image 4 (550x367), V) Test Image 5 (330x287), VI) Test Image 6 (590x442)

4.2 RESULTS AND DISCUSSIONS

A lesser known characteristic is that the integer order derivative (IOD) is a local operator while the fractional order derivative (FOD) is a global operator. Thus the FOD operator can be considered to detect more information from the neighboring elements as compared to IOD. Keeping this fact in mind, we combine our FOD differentiator to the conventional 1st order Sobel detector for low level analysis of image and extracting its texture features.

4.2.1 Edge detection by using proposed fractional order differentiator algorithm

The newly built algorithm has been experimented and validated on the test images as shown in the Figure 4.1. The results for the images used with the proposed algorithm followed by the Sobel operator in a noise-free environment are discussed below.

For experimentation purpose, we obtained the values as $\alpha = 0.3$ and $L = 9$ where α is the order of differentiation and L is the filter length. For the value of order of differentiation, we need to perform Image Quality Assessment (IQA) on the results attained at various fractional values of α . We have assessed the results for 9 values of the fractional order which are $\alpha = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$ and 0.9 respectively. From the IQA using Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR), we concluded that for $\alpha = 0.3$, the highest range of PSNR is prevailed. This implies that the observed signal will be closer to the original signal and a minimum error will be acquired.

Further, as we kept the variation of intensity factor between 0 and 2, we are performing the algorithm for six different values of intensity factor (γ). The experimentation values of factor coefficients being taken are $\gamma = 0.3, 0.6, 0.9, 1.2, 1.5$ and 1.8 respectively.

As procured from the results of Test Image 1 in Figure 4.2, we remarked that more texture edges are enhanced for lower value of intensity factor. Though, it is also noticed that the noise is also increasing in some cases, we are working on improving the noise immunity.

Similarly for Test Image 2 to Test Image 6, the results of the proffered approach are shown in Figure 4.3, Figure 4.4, Figure 4.5, Figure 4.6 and Figure 4.7 respectively.

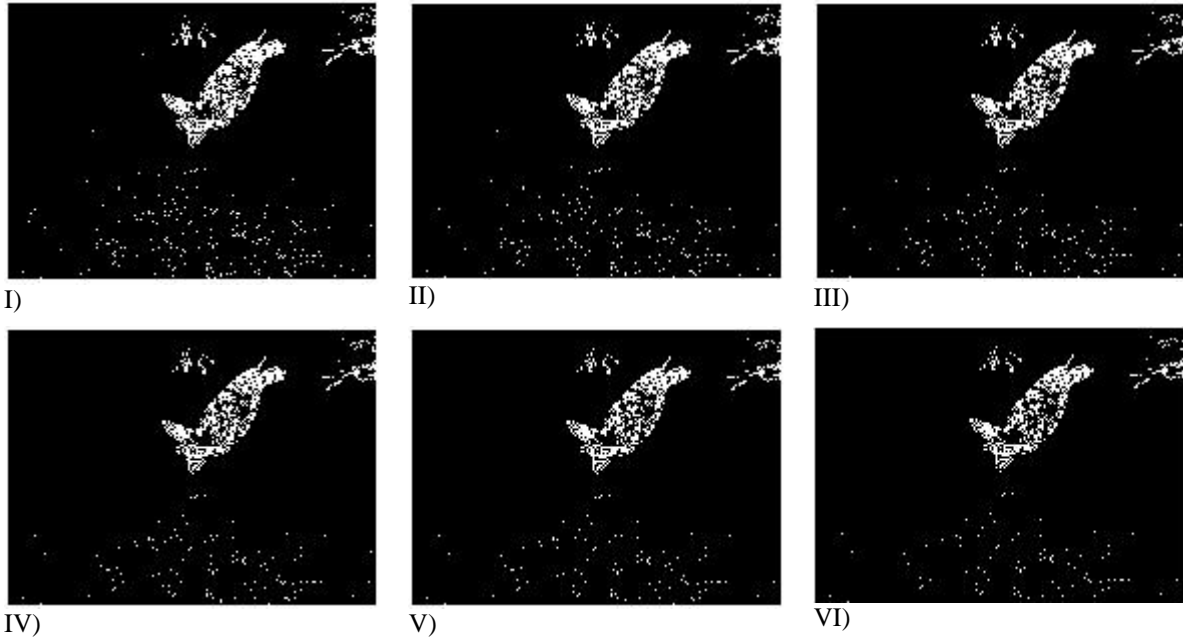


Figure 4.2 Detected edges of Test Image 1 by the proffered approach at I) $\gamma = 0.3$, II) $\gamma = 0.6$, III) $\gamma = 0.9$, IV) $\gamma = 1.2$, V) $\gamma = 1.5$, VI) $\gamma = 1.8$

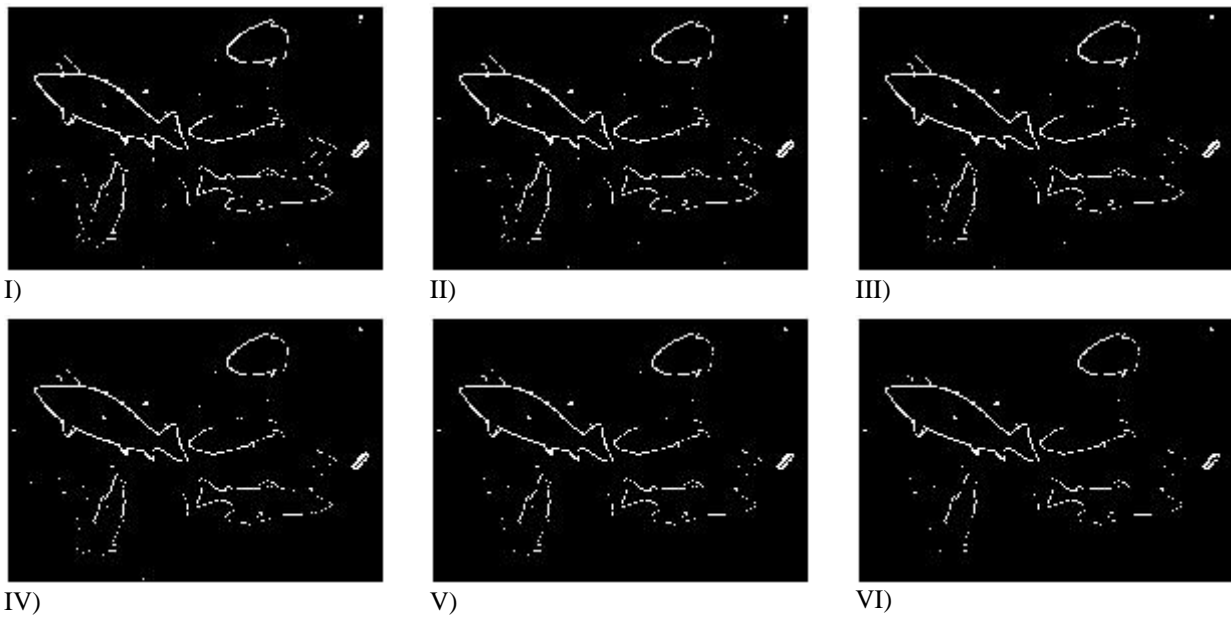


Figure 4.3 Detected edges of Test Image 2 by the proffered approach at I) $\gamma = 0.3$, II) $\gamma = 0.6$, III) $\gamma = 0.9$, IV) $\gamma = 1.2$, V) $\gamma = 1.5$, VI) $\gamma = 1.8$

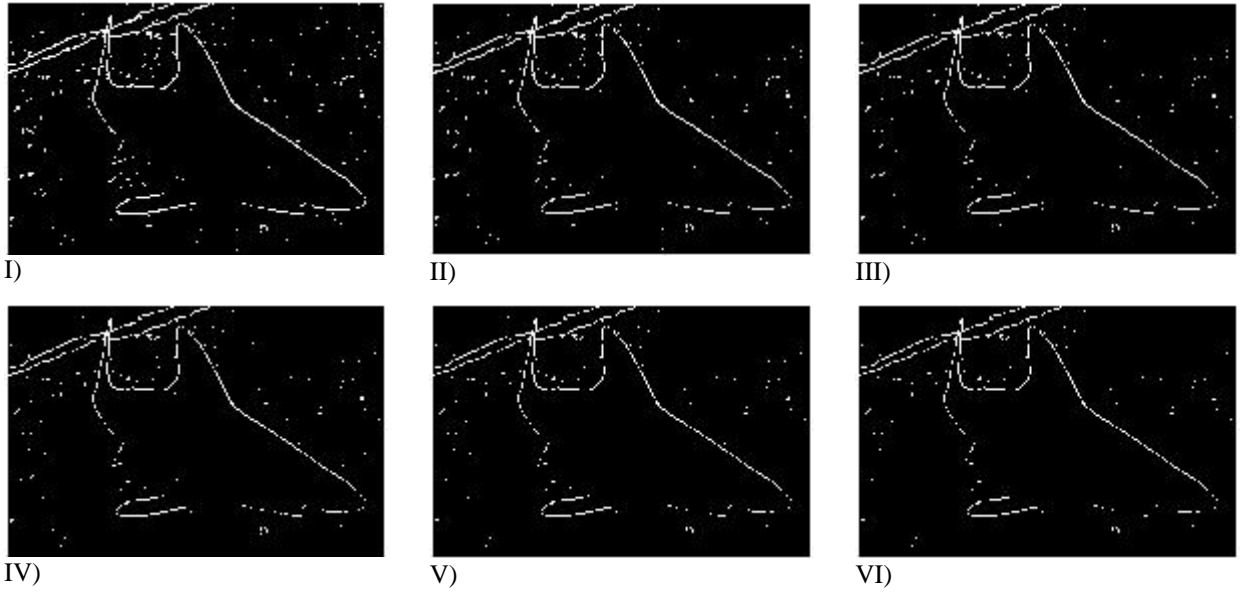


Figure 4.4 Detected edges of Test Image 3 by the proffered approach at I) $\gamma = 0.3$, II) $\gamma = 0.6$, III) $\gamma = 0.9$, IV) $\gamma = 1.2$, V) $\gamma = 1.5$, VI) $\gamma = 1.8$

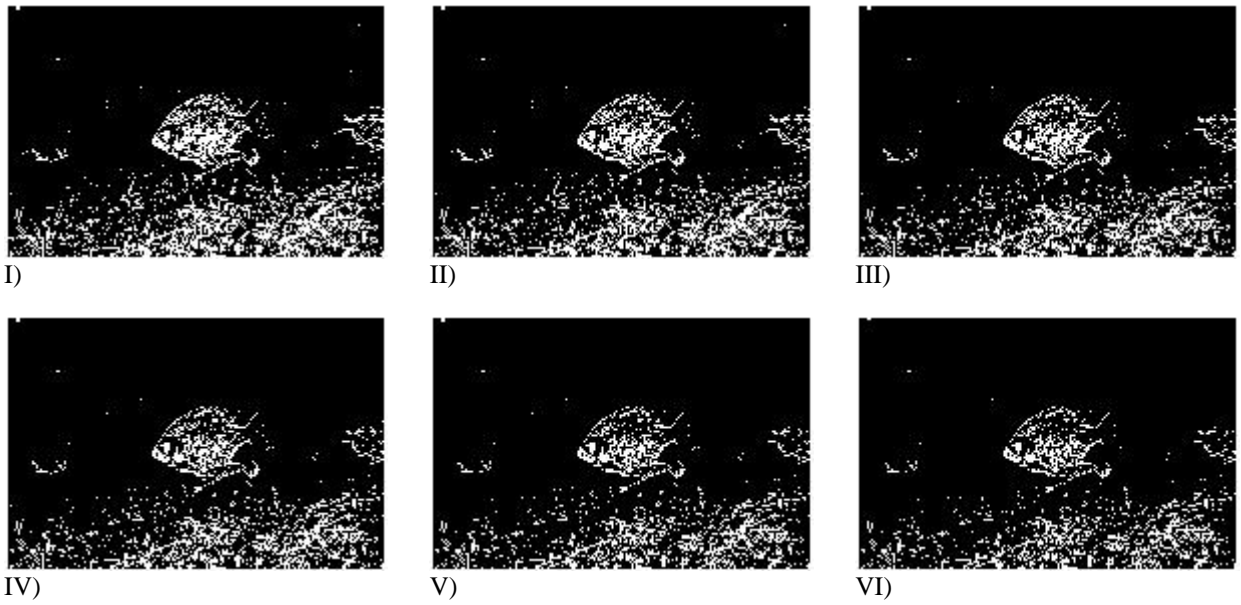


Figure 4.5 Detected edges of Test Image 4 by the proffered approach at I) $\gamma = 0.3$, II) $\gamma = 0.6$, III) $\gamma = 0.9$, IV) $\gamma = 1.2$, V) $\gamma = 1.5$, VI) $\gamma = 1.8$

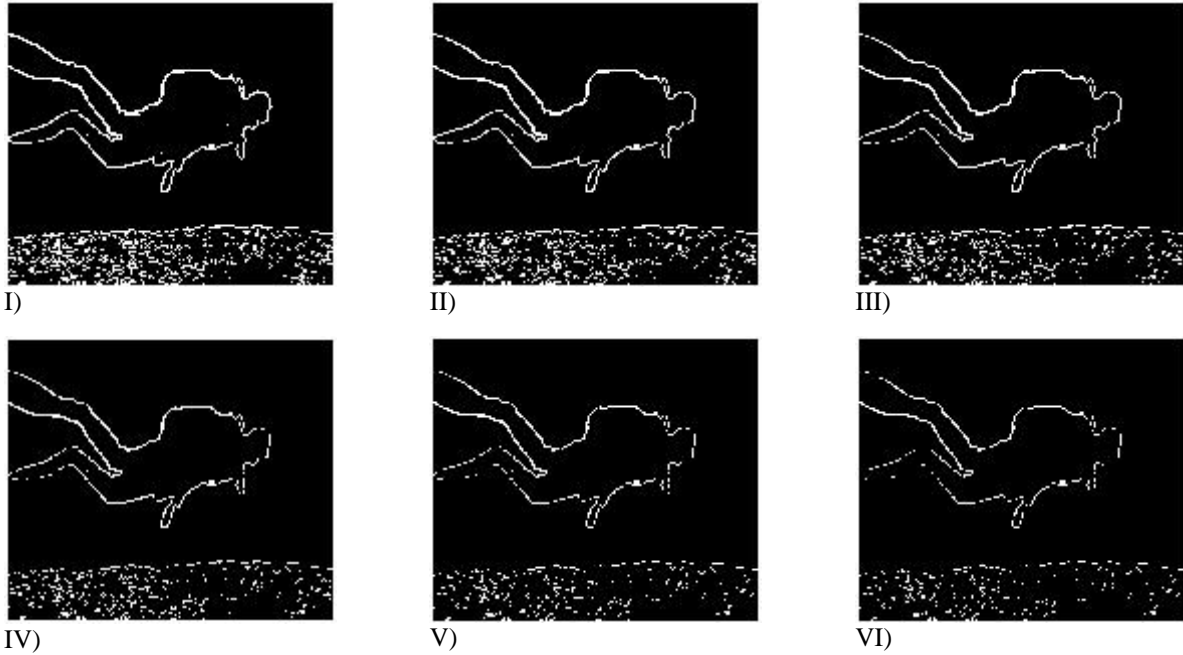


Figure 4.6 Detected edges of Test Image 5 by the proffered approach at I) $\gamma = 0.3$, II) $\gamma = 0.6$, III) $\gamma = 0.9$, IV) $\gamma = 1.2$, V) $\gamma = 1.5$, VI) $\gamma = 1.8$

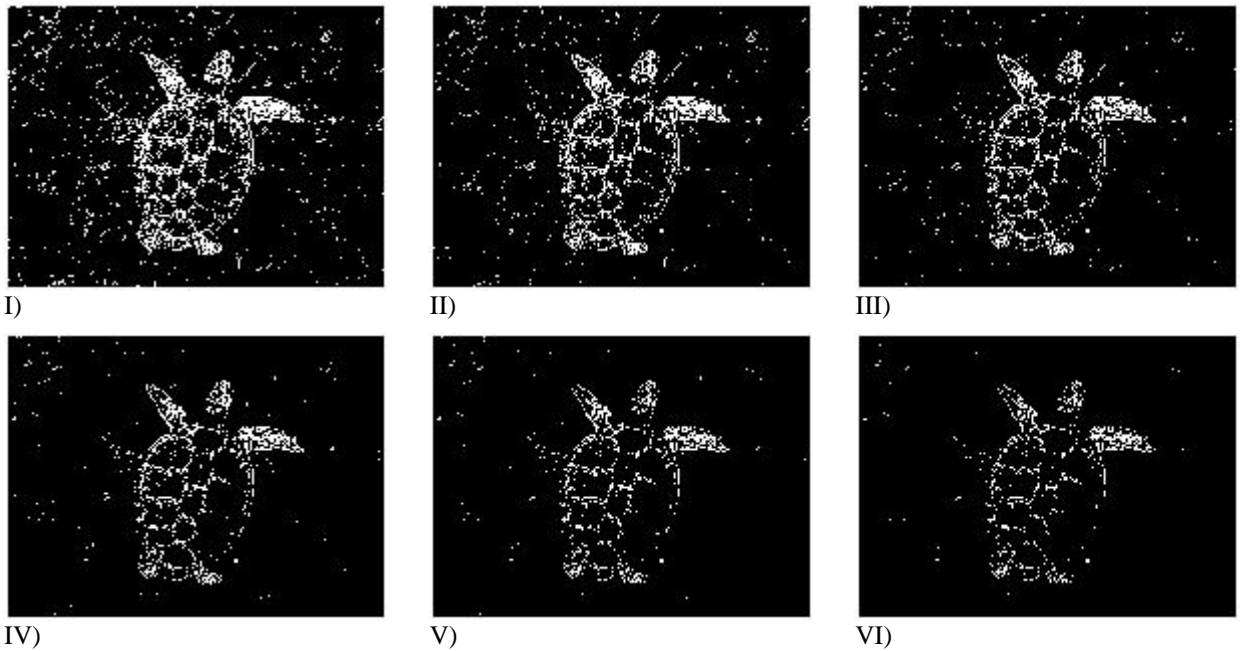


Figure 4.7 Detected edges of Test Image 6 by the proffered approach at I) $\gamma = 0.3$, II) $\gamma = 0.6$, III) $\gamma = 0.9$, IV) $\gamma = 1.2$, V) $\gamma = 1.5$, VI) $\gamma = 1.8$

4.2.2 Comparison of proposed algorithm with conventional methods

Figure 4.2 to Figure 4.7 displays the results obtained from the proposed fractional order edge detection algorithm for various values of intensity factor (γ). Now, we will compare the outcast results from our proposed algorithm with that of the customary algorithms.

Figure 4.8 exhibits the results attained for Test image 1 by prevailing edge detection methods. Figure 4.8 (I), (II), (III), (IV), (V) and (VI) shows edge detected images using Sobel operator, Prewitt operator, Canny operator, Roberts operator, Histogram Equalization (HE) Method and R-L Method respectively. It is distinguished that the customary methods as well as HE and R-L methods results does not provide satisfactory results for Test image 1.

Similarly, Figure. 4.9 to Figure 4.13 displays the results procured from the occurring edge detection algorithms for Test image 2 to Test image 6 respectively. The results of these methods also show the inefficiency in detecting the edges.

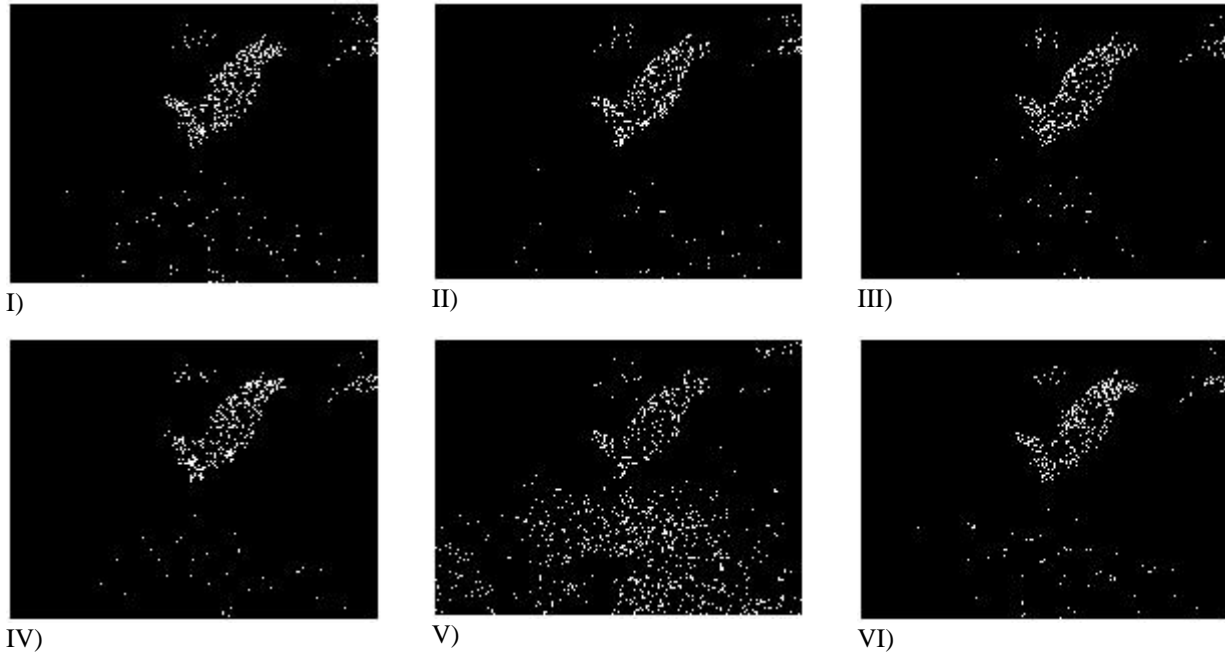


Figure 4.8 Edges of Test Image 1 by I) Prewitt method, II) Sobel method, III) Roberts method, IV) Canny method, V) Histogram Equalization Method, VI) R-L Method

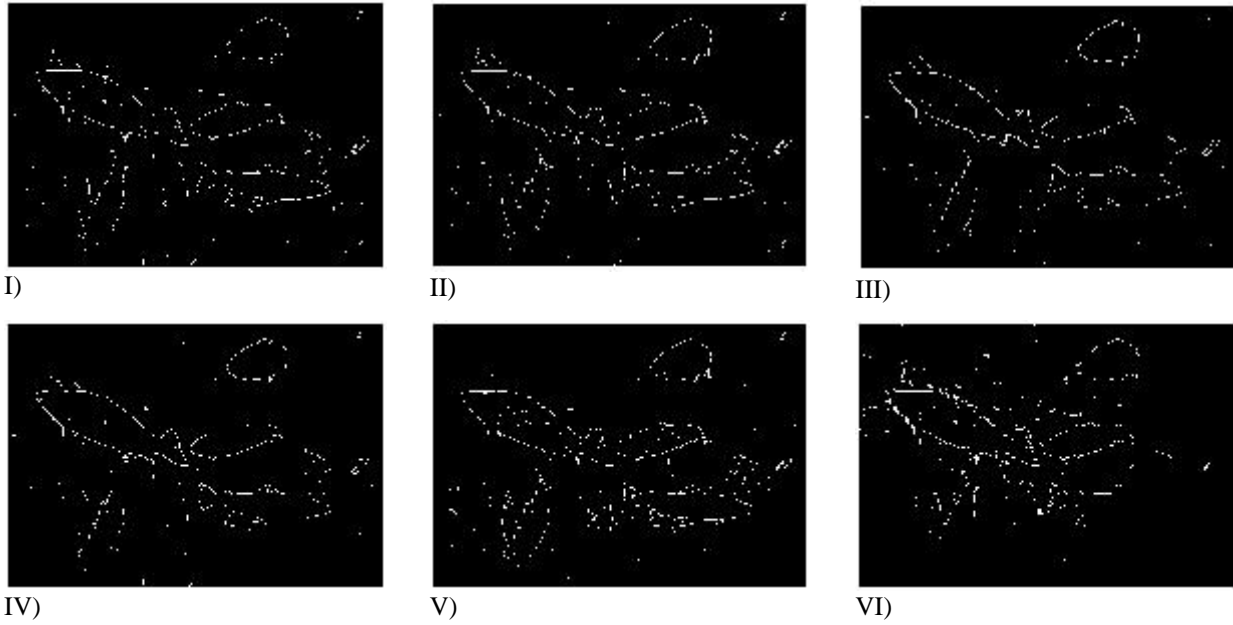


Figure 4.9 Edges of Test Image 2 by I) Prewitt method, II) Sobel method, III) Roberts method, IV) Canny method, V) Histogram Equalization Method, VI) R-L Method.

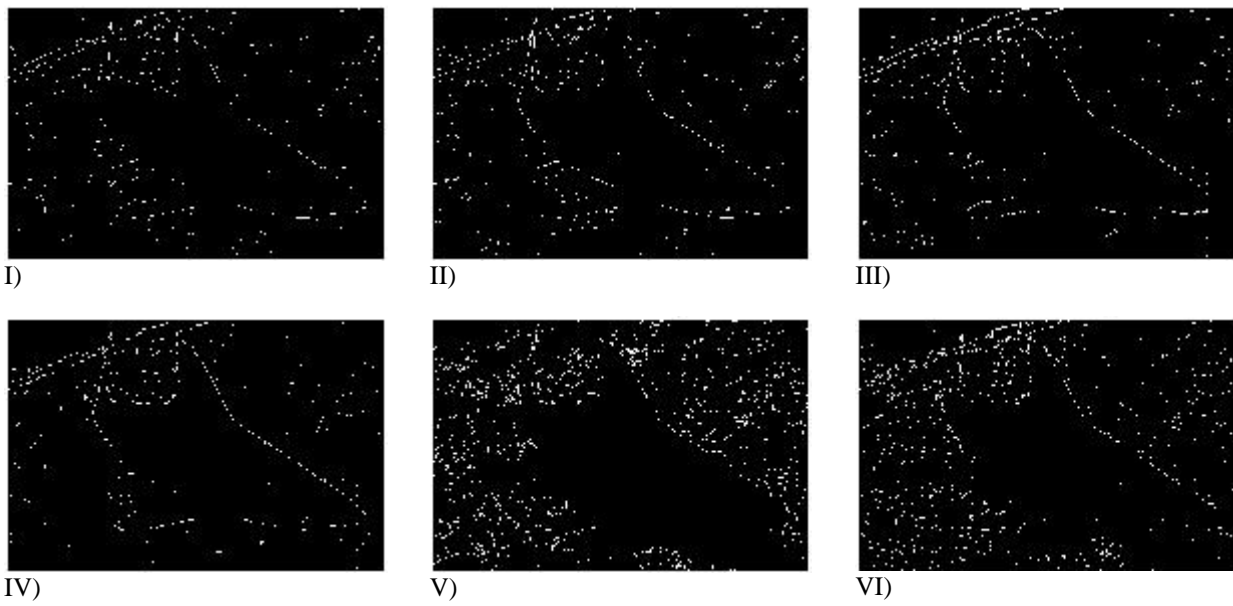


Figure 4.10 Edges of Test Image 3 by I) Prewitt method, II) Sobel method, III) Roberts method, IV) Canny method, V) Histogram Equalization Method, VI) R-L Method

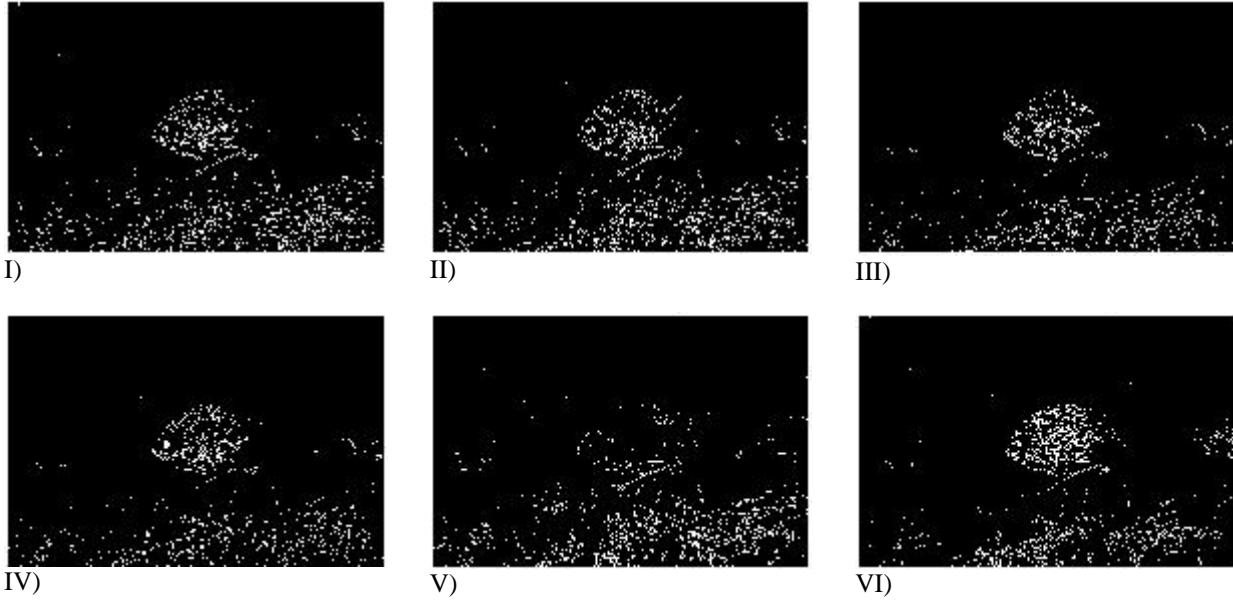


Figure 4.11 Edges of Test Image 4 by I) Prewitt method, II) Sobel method, III) Roberts method, IV) Canny method, V) Histogram Equalization Method, VI) R-L Method

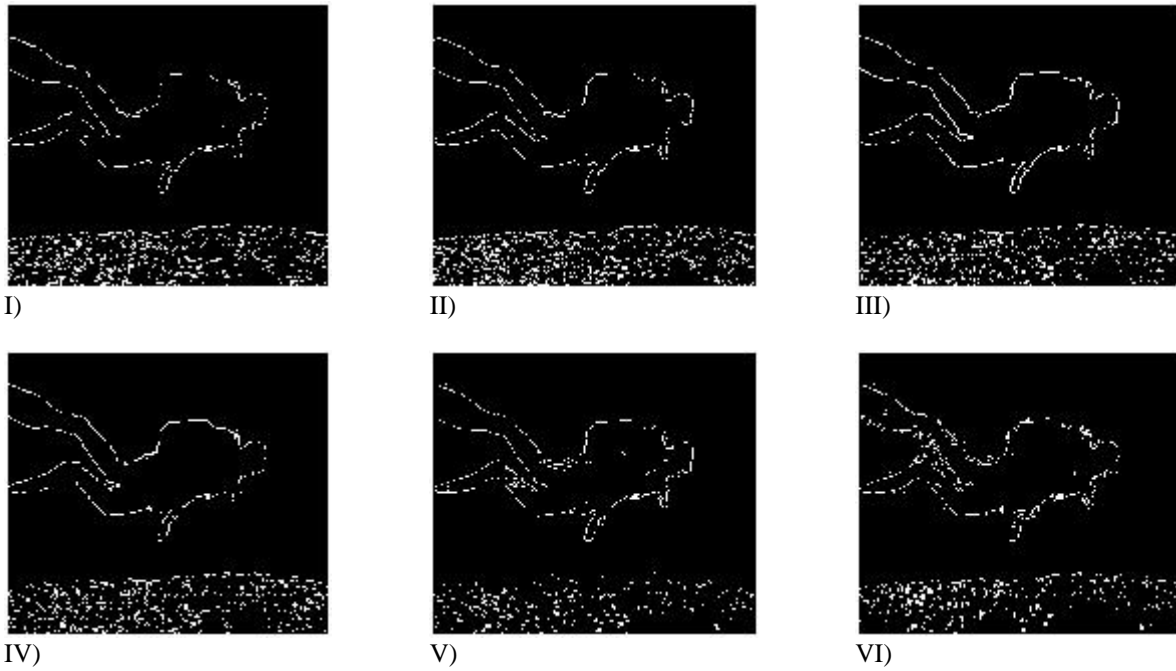


Figure 4.12 Edges of Test Image 5 by I) Prewitt method, II) Sobel method, III) Roberts method, IV) Canny method, V) Histogram Equalization Method, VI) R-L Method

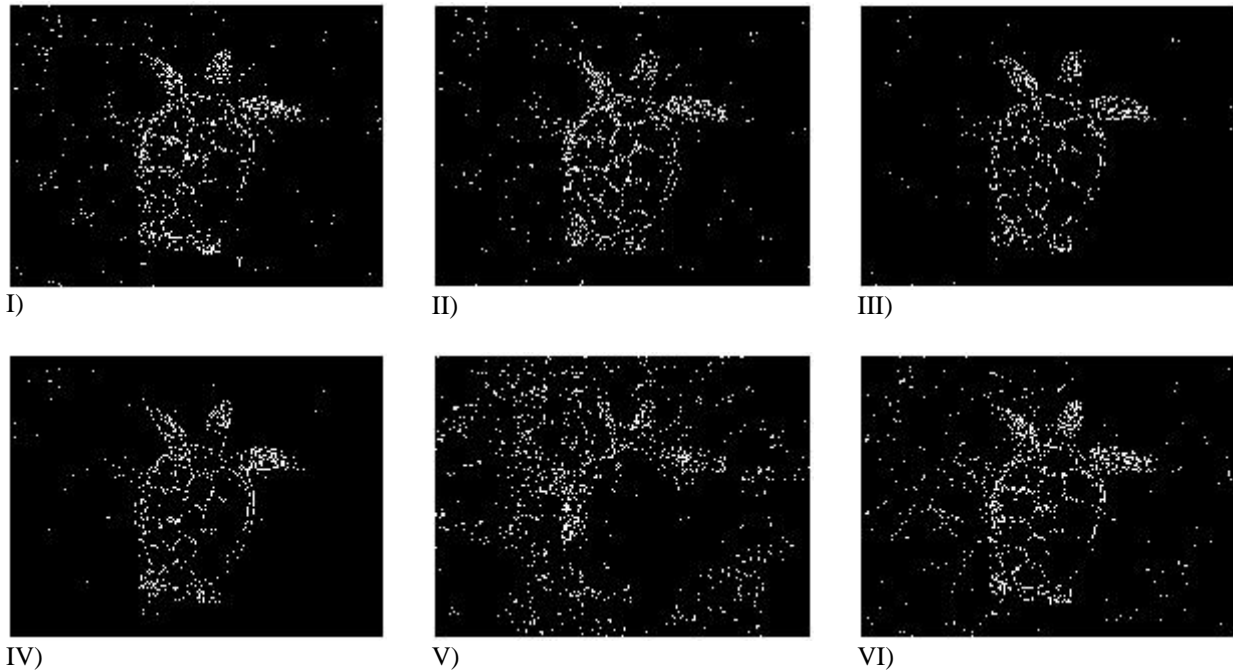


Figure 4.13 Edges of Test Image 6 by I) Prewitt method, II) Sobel method, III) Roberts method, IV) Canny method, V) Histogram Equalization Method, VI) R-L Method

4.3 IMAGE QUALITY ASSESSMENT

The quality maintenance of an image is a very sensitive task. The quality of an image can degrade from the image acquisition step to its visual appearance to an observer. A digital image is subject to many kinds of distortions during the storing, acquisition, processing, compressing, reproducing and transmitting stages. These distortions will lead to quality degradation in the image. Therefore, the Image Quality Assessment (IQA) is being done to assess the quality and amount of degradation in an image.

There are two standard methods for evaluating the quality of an image, the subjective method and the objective method. In practice, the subjective method evaluation is considered too inconvenient, expensive and time consuming, since we got to choose a number of observers, unveil them to various images and catechize them to rate each image depending on their judgments. However, a human analysis can differ depending upon the place of reference, psychological differences as well as difference in outlook and viewpoint. This method is inappropriate most of the times. Hence, measuring an image quality by subjective method is a hard and complicated process. On the other hand, an objective metric evaluation plays a variety

of role in the image processing applications. Firstly, it uses dynamic approach to assess and monitor the image quality without human interference. Secondly, it is used for optimization of algorithms by adjusting different parameters in an image. Thirdly, it can be used as a standard benchmark method in image processing systems.

The objective image quality matrices are classified into various groups depending on the availability of the original undistorted image. The recent approaches are Full Reference, Reduced Reference and No Reference Quality Assessment. In *Full Reference* approach, a reference image is to be known. When the reference image is partially present, in form of separate sets, or some extracted features are present. In that case *Reduced Reference* Quality Assessment is approached. When the reference image is not given or is not approachable, a doubted blind assessment is desired. This is termed as *No Reference* Quality Assessment. Our work is based on Full Reference Quality Assessment of the image.

Full Reference image quality measures is grouped into six sections of objective image assessment, which are pixel difference-based measures, context-based measures, correlation-based measures, spectral distance-based measures, edge-based measures and Human Visual System-based measures. The prediction of a good IQA must be accurate and consistent.

We are assessing our images in terms of pixel difference based measures as this measure is widely used, have clear physical meaning, and mathematically simpler and easier approach for the evaluation as compared to other measures. It comprises of the Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR).

4.3.1 Mean Square Error (MSE):

The MSE quantifies the normal squared contrast between the estimator and the parameter. It is computed by averaging the squared intensity differences between the distorted and original (undistorted) signal. MSE specifies the average difference of the pixels between the original reference image and the edge detected image. The higher the value of MSE, the larger will be the difference between the distorted and original image. [46]

$$MSE = \frac{1}{ij} \sum_0^{i-1} \sum_0^{j-1} \|I_1(x, y) - I_2(x, y)\|^2 \quad (4.1)$$

Where, I_1 is the original undistorted image; I_2 is the processed image; i and j represents the number of rows and columns in the input image respectively.

In case of image restoration and image compression, MSE should be as minimum as possible for better enhancement. But, for edge detection, the MSE could also procure higher values. This indicates that more edge points and weaker edges are traceable,

4.3.2 Peak Signal to Noise Ratio (PSNR):

Peak signal-to-noise ratio, is a ratio between the maximum possible power of a signal and the power of corrupting noise. The PSNR is generally expressed in terms of the decibel (dB) scale. PSNR gives a roughly measure of quality of reconstruction relative to human perception. A greater value of PSNR indicates that the reconstruction of lossy or lossless compression is of higher quality. Here, the noise obtained due to compression will be the main source of error. However, sometimes in edge detection, the value of PSNR should be minimum in order to acquire the required results. [45, 47]

The PSNR is calculated in terms of MSE as:

$$PSNR = abs\left(10\log_{10}\left(\frac{s^2}{MSE}\right)\right) \quad (4.2)$$

Where s is the maximum variation in the input image data. The value of $s = 511$ for a 9-bit image while $s = 1023$ for a 10-bit image.

PSNR is basically the signal to noise ratio (SNR) when the value of every pixel reaches to the maximum value i.e. s .

4.3.3 Analysis of MSE and PSNR results

In computer vision, various edge detection algorithms are available for the pre-processing stage of image enhancement but Sobel, Prewitt, Robert's and Canny are the universally applied conventional algorithms. In this section, a comparison is made between these operators by checking their respective MSE and PSNR values on the observed images prevailed by using these algorithms on the test images.

Table 4.1: Comparison of conventional operators

OPERATOR	MSE	PSNR
Test Image 1		
Sobel	75.5521	29.0811
Prewitt	93.1402	25.8234
Roberts	103.3192	23.3221
Canny	61.6132	31.6132
Test Image 2		
Sobel	103.1864	23.4629
Prewitt	124.4589	19.2823
Roberts	133.8964	17.4073
Canny	94.3496	25.3467
Test Image 3		
Sobel	95.2231	24.7859
Prewitt	123.8443	20.5768
Roberts	132.7651	18.5756
Canny	91.3566	26.4789
Test Image 4		
Sobel	75.5743	29.3428
Prewitt	93.5472	25.4532
Roberts	103.6198	23.1253
Canny	62.3845	31.1156

OPERATOR	MSE	PSNR
Test Image 5		
Sobel	62.4593	31.4123
Prewitt	74.5435	29.6556
Roberts	89.6543	26.5844
Canny	50.6954	34.6888
Test Image 6		
Sobel	60.9978	32.4534
Prewitt	73.5445	30.6793
Roberts	90.1124	26.9987
Canny	49.0846	35.2423

Table 4.1 describes the numerical analysis of the MSE and PSNR values on the resultant images obtained by using the suitable conventional operator on the respective test image. It is observed that the canny operator shows more efficiency with lower Mean Square Error values, while the Robert's algorithm shows the least efficiency by having higher range of MSE values. Thus, it is concluded that Canny edge detection is performing better than other operators in terms of detection of weaker edges. Further, Sobel, Prewitt and Robert's edge detection algorithm are ranked in order.

Table 4.2 Sobel operation using fractional order operator

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Image 1									
MSE	74.7765	73.8648	72.2873	73.9965	74.8763	75.6090	76.1843	78.0956	81.0121
PSNR	29.0314	29.2432	29.2650	29.1911	29.0678	28.6744	28.0775	26.9976	21.7654
Image 2									
MSE	106.5434	105.6567	104.0995	105.8103	106.7690	107.4998	107.9965	109.7121	113.9870
PSNR	23.5383	23.6299	23.6654	23.5984	23.4002	22.7640	22.0921	19.2251	14.6209

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Image 3									
MSE	99.4112	98.3667	97.9943	98.8421	99.7612	100.4321	100.9323	102.6643	107.6588
PSNR	24.7432	24.9565	25.0435	24.9114	24.6865	23.7765	23.0765	20.3421	15.4304
Image 4									
MSE	73.4574	72.4388	71.8945	72.7633	73.6007	74.4098	74.9054	76.8934	81.7865
PSNR	29.5501	29.6376	29.6520	29.6224	29.4987	28.8532	28.1654	24.9114	18.6534
Image 5									
MSE	67.4586	66.3884	65.9976	66.8654	67.7897	68.4856	68.9776	70.7003	75.8779
PSNR	31.8224	31.9543	31.9744	31.9234	31.8076	31.0664	30.3410	26.8956	21.3320
Image 6									
MSE	65.4332	64.4005	63.8216	64.5765	65.5776	66.3891	66.8754	68.7077	73.7474
PSNR	32.7312	32.8645	32.8861	32.8309	32.7112	31.9996	31.4562	27.9943	22.4405

Table 4.2 represents the use of fractional order filter with Sobel algorithm. Further using Sobel Operator with the fractional order, it is analyzed that the lowest values of Mean Square error are obtained at order 0.3. This means it gives the highest range of Peak signal to noise ratio values at the same order, since MSE and PSNR are inversely proportional to each other. Thus, when the order is near 0.3, we will obtain a minimum error between the original and observed image, i.e. the obtained observed image will be more close to the original image. Also, when the differential order is smaller, the enhancement is more and in case of edge detection, the edges being obtained are more crisp and clear.

CHAPTER 5

CONCLUSION AND FUTURE SCOPE

5.1 CONCLUSION

Although it is not easy to compute fractional derivatives and integrals due to lack of numerical methodologies, still fractional order calculus is playing an important role in many engineering and research fields. However, based on the proposed methodology, it is found out that edge extraction in low contrast underwater images having smoothness property is strenuous. The inbuilt standard algorithms for edge detection fail to provide satisfactory results in these types of images. In the presented work, we applied a newly proposed mathematical approach in which edge detection is carried out by using fractional order derivative filter. The newly designed filter, based on G-L definition, plays a major role in the designed algorithm. The pre processed image using FOD based approach followed by Sobel edge detector performed better than all the traditional algorithms. Though the computation time is increased due to heavier algorithms, but this newly designed filter can detect edges with higher accuracy, better sharpness and with more detailing.

When the proposed filter is implemented on low contrast images by varying the intensity factor and keeping the fractional order same, we observe that with the decreasing value of intensity factor, lower noise and better edges is produced. Thus, it can be said that the algorithm is performing better for low value of intensity factor. It enhances the high frequency components and preserves the low frequency ones.

From the image quality assessment using MSE and PSNR values, it is concluded that MSE and PSNR are inversely proportional. Further, the higher the value of PSNR, the better the edge information and good noise immunity.

Further, it has also been observed that the proposed algorithm based on Grunwald-Letnikov (G–L) definition also displayed better results than the other fractional order techniques using Histogram Equalization (HE) and Riemann-Liouville (R-L) definition methods.

5.2 FUTURE SCOPE

The present work can be further implemented using artificial intelligence techniques like genetic algorithm for applications like enhanced fingerprint scanning, biometric scanning etc. or using fuzzy logic for more optimized results. Also, the integrated machine learning and image recognition can be implemented using this method in a brilliant way having precise edge detection with a minimum error detection possibility.

Further, dynamic de noising can be applied in case of non stationary or real time images. For that, the proposed methodology can be implemented on FPGA for real time applications.

The proposed methodology can also be extended for three dimensional images.

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