

**GENERATION AND DYNAMICS OF DISSIPATIVE OPTICAL CAVITY
SOLITONS**

A thesis submitted in the partial fulfillment of the requirements for the award of

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Master of Science

in

Physics

Submitted by

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*I dedicate my thesis to my loving family, friends and my sister Baldeep Kaur
who have always been supportive toward me.*

CERTIFICATE

I hereby certify that the work which has been presented in this thesis entitled, “**Generation and Dynamics of Dissipative Optical cavity Solitons,**” submitted in partial fulfillment of the requirements for the award of degree of **Master of Science in Physics** at **Thapar University, Patiala**, is an authentic record of my own work carried out under the supervision of **Dr. Soumendu Jana**, Assistant Professor, School of Physics and Materials Science and refers other researcher’s work which are duly listed in reference section. The matter embodied in this thesis has not been submitted for the award of any other degree of this or any other university.

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Abstract

Optical dissipative solitons, namely, cavity solitons have been investigated in different dissipative media, particularly, in vertical cavity surface emitting laser (VCSEL) cavity with different feedback configuration. For first time super Gaussian and Cosh-Gaussian type cavity solitons have been achieved. Models based on Cubic Ginzburg-Landau equations have been solved by separation method with Super Gaussian profile with and without feedback. Cosh-Gaussian cavity soliton has also been found in VCSEL with saturable absorber and frequency selective feedback. The range of parameters that promise cavity solitons has been identified. As a special case, a large continuous family of Cosh- Gaussian cavity solitons has been found that apparently resembles with conservative solitons. Cosh- Gaussian cavity solitons thus found are of versatile profile and will be of fundamental and applied research interest. The cavity solitons achieved by this investigation have huge potential applications in all optical devices, like all-optical erasable memory device, all-optical data storage, delay lines, all-optical push broom and multi-level logic systems.

CHAPTER-1

1.1 Introduction to Soliton

Soliton is a localized structure like vibration, wave, beam or pulse that retains its shape and size even after collision with other soliton. It was firstly observed by *John Scott Russell* in 1834. The soliton has been flourished to a robust multidisciplinary research topic having application in numerous fields of Physics, Chemistry, engineering and even in Biology. The concept of soliton is developed mostly in conservative systems, i.e., loss less environment. With advent of new techniques the legacy of soliton has been extended to many practical systems those are lossy in nature. Dissipative solitons are localized structures which are generated in lossy nonlinear system. In the large family of dissipative solitons, the optical dissipative solitons (ODS) attracted considerable attention for both theoretical and experimental research. During last three decades optical dissipative solitons has been grown to one of the most sought after topic of research interest.

Optical solitons are self trapped beams or pulses that don't change their shape during propagation. They originate through the interplay between two counteracting phenomena: self-diffraction/ group velocity dispersion induced broadening and nonlinearity-induced contraction (i.e., self-focusing/self-phase modulation). Thus the formation of soliton keenly requires nonlinearity of the medium.

1.1.1 Nonlinear Optics: Soliton

At the basic level the origin of soliton is related to nonlinear optics. The branch of optics that deals with propagation of highly intense light in solids, liquids, gases and plasmas or the branch that describes the light behaviour in non linear media. In this media, the matter respond in a non linear manner to the incident field i.e. the dielectric polarization P has nonlinear dependence on the electric field E of the incident light. The non linear response results in new phenomena, e.g., creation of radiation field of new frequency or modulation in phase and pulse or beam shape. These new phenomena generate because strong light change the properties of

system (refractive index and absorption coefficient), while normal light is too weak to change those properties. As of today nonlinear optics has grown to a significant size and standard. If only number is considered the nonlinear optical phenomena are greater than the linear optical phenomena.

1.1.2 History of Non linear Optics and Soliton:

In pre-laser era the basic characteristics of a light wave that determine the nature of its interaction with matter was directly dependent on polarization. The index of refraction n , the absorption coefficient, and the effective scattering cross section of the light was not dependent on the intensity of light. Only a few works may be noted in which attempts were made to study the effect of light intensity on optical phenomena. In 1923, *S.I. Vavilov and V.L. Levshin* detected a reduction in the absorption of light by uranium glass with increasing light intensity, and they reasoned that in a strong electromagnetic field a larger portion of the atoms (or molecules) are in an excited state and cannot absorb light. On the assumption that this is only one of a set of possible nonlinear effects in optics, *Vavilov* introduced the term “nonlinear optics.”

The creation of lasers opened up extensive possibilities for the study of nonlinear optical phenomena. In 1961, *P. Franken* and his co-workers (USA) discovered the frequency-doubling effect for light in crystals (the generation of the second harmonic of the light). Frequency tripling (third harmonic generation) was observed in 1962. Fundamental results in the theory of nonlinear optical phenomena were obtained in the USSR and the USA from 1961 to 1963, laying the theoretical foundation for nonlinear optics. In 1962–63 the phenomenon of stimulated Raman scattering of light was discovered and explained. This gave the impetus for the study of other types of stimulated scattering, such as stimulated Brillouin and stimulated Rayleigh scattering.

The phenomenon of self-focusing of light beams was discovered in 1965. The phenomenon of self-focusing of electromagnetic waves was predicted in a general form by *G.A. Askar'ian* (USSR) in 1962. Optical experimentation was stimulated by the theoretical work of *C. Townes* and his co-workers (USA, 1964). The work of *A.M. Prokhorov* and his co-workers made a large contribution to understanding the nature of the phenomenon.

One of the most significant and revolutionary contribution of nonlinearity is the soliton formation. The story of nonlinearity induced soliton has been started long back. In 1834, Scottish engineer, *Scott Russell* has given the concept of solitary waves. He described a translation wave that was propagating in a narrow barge canal for a miles distance [1]. He gave the idea that the water wave equations would have the solitary wave solutions. In 1847, *Stokes* has given a solution of periodic nonlinear wave train in deep water [2]. In 1965, four scientist *Los Alamos*, *Enrico Fermi*, *John Pasta*, and *Stan Ulam* were studying the energy flow in one dimensional lattice in which equal masses were attached by nonlinear springs. They expected that the energy which is given input to a long wavelength mode will be shared among all the modes of the system but the system exhibited the energy sharing among the few lowest modes only [3]. In 1965, *Norman Zabusky* and *Martin Kruskal* investigated this system and studied this system numerically by KdV equation (Korteweg and de Vries equation). They established that stable pulse like waves exist in the system that do not change their shape while travelling and preserve their shape after collision with other similar wave. They named it as ‘Soliton’ [4]. After this, *Gardner*, *Greene*, *Kruskal* and *Miura* solved the initial value problem for the KdV equation to develop a mathematical method [5]. Then in 1971, *Zakharov and Shabat* proved that this method worked for other nonlinear equations also [6]. In 1974, *Ablowitz*, *Kaup*, *Newell* and *Segur* gave the techniques to solve the nonlinear evolution equations known as ‘inverse scattering transform’. In 1985, *Airy*, *Boussinesq*, *Korteweg* and *de Vries* (KdV) described a nonlinear wave equation known as ‘a model for the evolution of long waves in a shallow one dimensional water channel’ [7]. During the last decades, mathematicians and physicist make the Soliton concept so developed in modern mathematical physics that it is now being claimed that any wave may give rise to a soiton. The existence of solitons depends on the range of physical situations. Solitons can grow both in conservative or non-conservative systems. In conservative system, there is a counter balance between nonlinearity and diffraction/dispersion of the material while in non-conservative system, in addition to the aforesaid balance, a continuous energy flow is required to keep it alive. The solitons in non-conservative system are called dissipative solitons.

The present thesis is on one such dissipative solitons. *N. Akhmediev* and *A. Ankiewicz* investigated the solitons which can be obtained from one dimensional Ginzburg-Landau and Swift-Hohenburg equations. They explained the energy flow across a Soliton and Soliton explosion. Dissipative magneto-solitons have been obtained through the Basic Cubic-Quintic

Complex Ginzburg-Landau equation. Spatial dissipative optical solitons in semiconductor optical amplifier have also been investigated. In fact, in recent years it attracted huge attention. *Ackemann* and *Firth* gave the phenomenon of cavity solitons and feedback solitons. Cavity solitons are the dissipative solitons in an optical cavity possessing nonlinear medium. In feedback solitons, a feedback mirror is aligned in front of a nonlinear optical medium. *N. N. Resonav* has given a review of dissipative solitons in laser with saturable absorption. The properties and applications of dissipative solitons in time domain were discussed by *Peschel*, *Bakonyi* and *Lederer*. Dissipative solitons are also studied in mode-locked laser system. Dissipative solitons are found in various systems such as in discrete lattice and Bose Einstein condensates [8]. Apart from dissipative optical solitons, there are optical dark solitons, i.e., which are localized nonlinear waves obtainable on a stable continuous wave background. Other new classes of solitons exist such as snake, creeping, pulsating, erupting and chaotic solitons that are not confined in time domain rather stable in spatial domain. So the concept of Soliton has become most advantageous topic of research in modern times.

1.1.3. Applications of Nonlinear Optics and Soliton:

Nonlinear optics arises due to the interaction of light having high intensity with matter. The study of nonlinear effects is very old and has a long history while the field of nonlinear optics was born after the discovery of laser, so is quite young. So the study of nonlinear effects is an active area of research and is developing in material science and source technology. Nonlinear optics has a broad range of applications. It can be used for studying the physical properties of the materials by spectroscopy or can be used as a basic for optical devices, i.e., used in optical communication, optical switching, optical computing, optical signal processing and in frequency generation. In frequency generation, we generate new frequencies by nonlinear optics effects such as harmonic generation, frequency mixing effects or Raman shifting etc. As we know, cavity Soliton is bistable, i.e., it can be set and reset by an optical control pulse. It acts as a bit, so is used for optical memory devices. It is used for erasable memory devices because it can be set or reset. It is useful to explore the mobility of cavity solitons for applications. Because stationary cavity solitons acts as 'stopped light' while drifting cavity solitons live in the cavity forever. Nonlinear optics is becoming popular in the field of computing and networking in which all optical circuitry is used which is very fast as compared to electronic circuitry. The concept of

nonlinear optics is advancing in other fields also. For example, optical properties of carbon nanotubes were studied. It was observed that by changing the diameter of carbon nanotube, we can change the absorption and emission properties of the material. These materials are used to make optical as well as thin-film electronic devices which has low cost. With the help of these devices, fully integrated electro-photonic circuits will be manufactured in future. Nonlinear optics has a huge advantage in optical communication. For long haul propagation losses should be balanced by the gain, which can be achieved by using optical amplifiers. Thus signal can propagate uninterrupted for extremely long distance over hundreds of kilometers. To design the light wave system having better capacity, nonlinear optics is used in present time. In optical fibers, there is lower distortion of signals as compared to copper wires, so it is used in transmission of multimedia such as television or telephone signals over large distances. At early age, fiber-optic system was complex and expensive, so used for long-distance transmission only, not in cities due to complex infrastructure. But now a day, it is used everywhere. The other advantage of fiber-optic communication is that it is cost effective as compared to copper based network. The prices of fiber-optic communication have been dropping since last few decades.

Introduction of soliton has revolutionized many fields of application of nonlinear optics. Soliton research has a diverge fields of applications as quantum mechanics, particle physics, molecular biology, geology, astrophysics etc. Soliton has huge technological advantage in optical fibers. In optical fibers solitons carry the information in 'bits'. Optical communication and Soliton research are two separate techniques but have so united that these are developed as a system the nonlinear optical transmission line. With the help of soliton transmission, data can be transmitted several times faster than the optical transmission.

Cavity Soliton, a kind of bistable soliton in semiconductor cavity,(and our topic of interest) can be set and reset by an optical control pulse, can be produced at any point by a appropriate address pulse. It acts as a bit, so is used for optical memory devices, data capture, storage and processing. It is used for erasable memory devices because it can be set or reset. It is useful to explore the mobility of cavity solitons for applications. Because stationary cavity solitons acts as 'stopped light' while drifting cavity solitons live in the cavity forever. Feedback solitons are used in optical buffer registers which are used in optical telecommunication for temporary storage of data and to convert serial data to parallel data. For high speed switching of

the data, optical delay lines are used. Cavity soliton has shown optical push broom effects. Optical push broom scanners are used to obtain images using spectroscopic sensors.

1.2. Elements of Soliton theory

When a pulse propagates through a non-linear medium, a number of phenomena occur in the medium. These phenomena occur due to frequency dependent refractive index. Some of these phenomena are chromatic dispersion, optical loss, self-phase modulation, cross phase modulation etc. Due to non-linear effects there is spectral and temporal change in the pulse. Group velocity dispersion broadens the pulse shape. Self phase modulation arises because refractive index is intensity dependent. The effect of self phase modulation (SPM) is that it tends to broaden the spectrum of optical pulse. When SPM & group velocity dispersion (GVD) both acts together, they affect the pulse shape in different way. There is interplay between dispersion or diffraction with non-linearity gives rise to solitons, which can be temporal or spatial in conservative system. While in non-conservative system, different structures called dissipative solitons exist which have a wide use in non-linear optics. Cross phase modulation (XPM) effect also occur in non-linear media. It occurs when at least two optical fields propagate at the same time and affect each other by the intensity dependence refractive index. Some basic linear and nonlinear phenomena, commonly occurred in nonlinear optics and soliton theory are discussed.

1.2.1 Dispersion:

When we apply the intense light to the medium, due to different speeds of the frequency components, there is delay of each component or one component reach earlier in comparison to other, leads to separation of frequency components, called dispersion. Since different frequency components move at different speeds so there is broadening of pulse, called group velocity dispersion. There are two types of dispersive media namely normal & anomalous. In normal dispersive media, higher frequency component travel slower than lower frequency components and reverse in case of anomalous dispersion.

1.2.2 Optical Non-linearity:

When we incident the intense light to a material system, the optical properties of the system gets modified. The study of phenomenon that occurs due to this modification of optical properties is called non-linear optics. Only laser light is sufficient intense to give the optical phenomenon. The phenomenon is called non-linear because response of system to applied field that depends non-linearly on the strength of applied field.

The phenomenon of nonlinearity occurs because optical properties of the materials like refractive index or absorption coefficient depend on the intensity of the light. The phenomenon of nonlinearity is not observed in free space rather it is observed in the medium. As the light passes through the nonlinear medium, its frequency or wavelength can be changed or we can say the colour of light may change. In such a media, if optical field is applied, it modifies the medium properties due to which another optical field is generated or the original field is modified.

Polarization of a material depends on the strength of applied field in case of linear optics as:

$$P(t) = \chi^{(1)}E(t) , \quad (1.1)$$

where, $\chi^{(1)}$ is linear susceptibility.

In case of non-linear optics, the polarization depends on applied field as:

$$P(t) = \chi^{(1)}.E(t) + \chi^{(2)}.E^2(t) + \chi^{(3)}.E^3(t) + \dots , \quad (1.2)$$

where, $\chi^{(2)}$, $\chi^{(3)}$ are 2nd and 3rd order non-linear optical susceptibilities, respectively.

The nonlinearity arises due to inharmonic motion of the bound electrons under the control of intense light [9]. In eq. (1.2), the first term including $\chi^{(1)}$ gives dominant contribution to polarization. The medium having the centre of symmetry have zero 2nd order susceptibility ($\chi^{(2)}$). So in such media only 1st term of eq. (1.2) contributes. When applied electric field is small, then the relation between P and E is linear as given in eq. (1.1) but for higher value of electric field ,i.e., $E \sim 10^5 - 10^8 V/m$, the polarization P has nonlinear dependence on E as shown in eq. (1.2).

There are different types of nonlinearities. These are described below:

1. Cubic or Kerr nonlinearity:

It is simplest form of nonlinearity which is related to optical Kerr effect. Kerr effect states that the refractive index of the material changes when external field is applied to it. This change in refractive index is directly proportional to the square of the applied field rather than varying linear with it. In optical Kerr effect, the electric field is due to light itself. There is a change in the refractive index which is relative to the local irradiance of light. Nonlinear optical effects such as self-focusing or self phase modulation occur due to this variation in refractive index as a result of optical Kerr effect. For this effect to occur, the light should be high intense. The expression for the refractive index is given by as [10]

$$n = n_0 + n_2|E_0|^2, \quad (1.3)$$

where, $n_2 = \frac{3\chi^{(3)}}{8n_0}$. Here, n_0 is linear refractive index, n_2 is nonlinear refractive index which depends on the third order nonlinear susceptibility $\chi^{(3)}$. The contribution of n_2 is usually called optical Kerr effect. Some of the materials which show Kerr nonlinearity are glass, *NaCl*, *CS₂*, *Si*, *GaAs* etc.

2. Competing nonlinearity: Cubic Quintic nonlinearity

In competing nonlinearity, there is a competition between cubic and quintic nonlinearity, i.e., between focusing and defocusing. It can be expressed as [11]

$$n(I) = n_p I^p + n_{2p} I^{2p}, \quad (1.4)$$

here, n_p and n_{2p} are nonlinear coefficients and are of opposite signs, p is a positive coefficient. The competing nonlinearity is said to be cubic quintic for $p = 1$. It is expressed as

$$n(I) = n_0 + n_2 I + n_4 I^2, \quad (1.5)$$

where, n_2 is 3rd order and n_4 is 5th order nonlinear coefficient and is given by:

$n_4 = \frac{5}{16n_0} \chi^{(5)}$, where $\chi^{(5)}$ is fifth order nonlinear susceptibility. Cubic Quintic nonlinearity is found in materials such as para toluene sulphonate (PTS), semiconductor doped glass, organic polymer etc.

3. Power Law nonlinearity:

Various materials including semiconductors exhibit power law nonlinearity. The power law is given by the equation: $F(s) = s^p$, in this case $0 < p < 2$ to avoid wave collapse and $p \neq 2$ to avoid self focusing singularity. The case where $p = \frac{1}{2}$ is considered in the context of Soliton turbulence.

4. Saturable nonlinearity:

In most materials, the saturation of optical nonlinearity occurs at a certain high value of optical intensity. In that case nonlinearity is described by a law called saturating law [12], given as:

$$n = \frac{n_0}{1 + \frac{I}{I_s}}, \quad (1.6)$$

where, I is intensity of incident beam and I_s is saturation intensity. In semiconductor doped fibers, soliton propagation is modeled using saturable nonlinearity not with Kerr nonlinearity because in semiconductor doped fibers and some composite materials, saturable nonlinearity occurs at not too high intensity.

There are other types of nonlinearities such as dual power law nonlinearity, exponential law, nonlinearity, log law nonlinearity. Cubic Quintic nonlinearity is also called parabolic nonlinearity [13].

1.2.3 Self Phase Modulation:

Under the influence of intense light, susceptibility varies non-linearly with electric field. The refractive index depends on the intensity of light due to which phase of light changes and frequency of light differ from central frequency. As a result of this, new frequency components generate and there is broadening of pulse which is called self phase modulation. In this case

amplitude does not change along the fiber length i.e. pulse shape remains unchanged, there is only phase shifting and this nonlinear phase shifting is given as:

$$\phi_{NL}(L, T) = |U(0, T)|^2 \left(\frac{L_{eff}}{L_{NL}} \right), \quad (1.7)$$

where, $U(0, T)$ is field amplitude at $z = 0$. This equation shows that non-linear phase shift ϕ_{NL} depend on fiber length and it increases with fiber length L . Due to fiber losses, L_{eff} is smaller than L , so L_{eff} also affects ϕ_{NL} . Also ϕ_{NL} depends on time T , that give rise to SPM. Due to SPM, instantaneous optical frequency differ from central frequency ω_0 , the difference $\delta\omega$ is called chirping and given by:

$$\delta\omega(T) = \left(\frac{L_{eff}}{L_{NL}} \right) \frac{\partial |U(0, T)|^2}{\partial T}, \quad (1.8)$$

This equation signifies that, as the pulse propagates down the fiber, new frequency components are generated and spectrum is broadened. When $\delta\omega$ is negative, frequency spectrum gets red shift and for positive $\delta\omega$, it gets blue shift.

1.2.4 Soliton: Optical Soliton

Solitons are the localized state of waves, which don't change their shape while travelling through a medium. Even after the collision solitons preserve their shape. Optical soliton is localized state of pulses or beams obtained as a result of counter balance between non-linearity & dispersion (pulse) or diffraction (beam). These localized solutions can be obtained in both conservative (with no loss) and non-conservative systems (with loss). Solitons in non-conservative systems are called dissipative solitons that depend on the external energy source to keep them alive. There is compensation between gain and loss of the system. While in conservative system, solitons exist due to compensation between non-linearity and diffraction/dispersion. Due to this balanced, localized structure is formed.

Optical soliton can be spatial, temporal and spatiotemporal. In spatial soliton, two effects self-diffraction & self-focusing balance each other so there is no change in spatial dimensions [14]. Temporal soliton are pulses that don't change their shape in both spectral & temporal domain. Here, group velocity dispersion counter balances the self phase modulation (in non-linear medium).

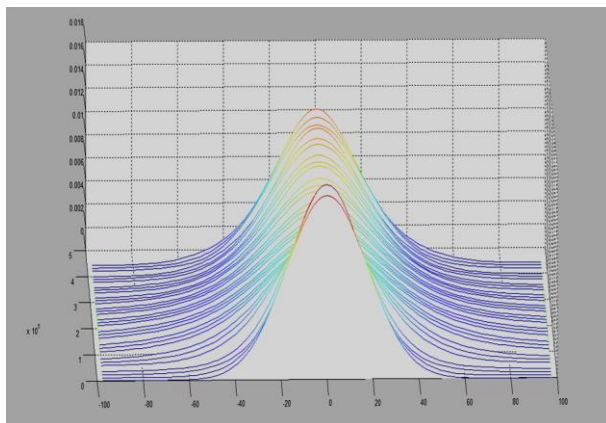


Fig. 1.1: Fundamental soliton

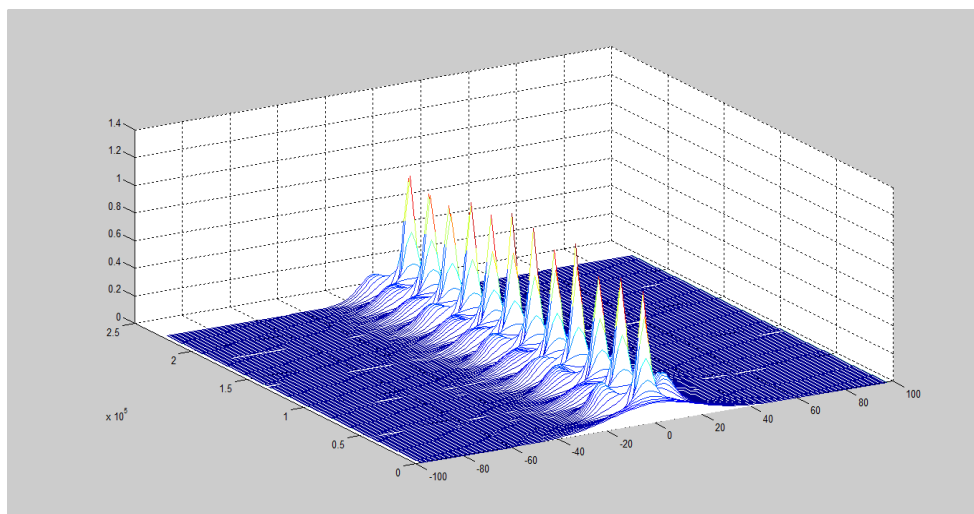


Fig. 1.2: Higher order soliton

1.2.4.1 Generation of Temporal soliton:

Temporal solitons are light pulses that don't change their shapes in both temporal and spectral domain [9,11,5-20]. They neither broaden nor contracts. To understand this mechanism, consider an optical pulse propagating in self-focusing, anomalously dispersive Kerr non-linear media. If group velocity dispersion (GVD) dominates, then trailing edge of pulse gets red shifted and leading edge is blue shifted due to which the pulse gets broadened. On the other hand, if non-linearity dominates, a chirp is produced due to SPM which is opposite to GVD that may lead to contraction. This mechanism is shown in fig.1.3. A perfect counter balance between these two

chirps leads to formation of temporal soliton keeping spectral and temporal width constant. Temporal soliton is (1+1) dimensional. The generation of temporal soliton is shown in fig. 1.3.

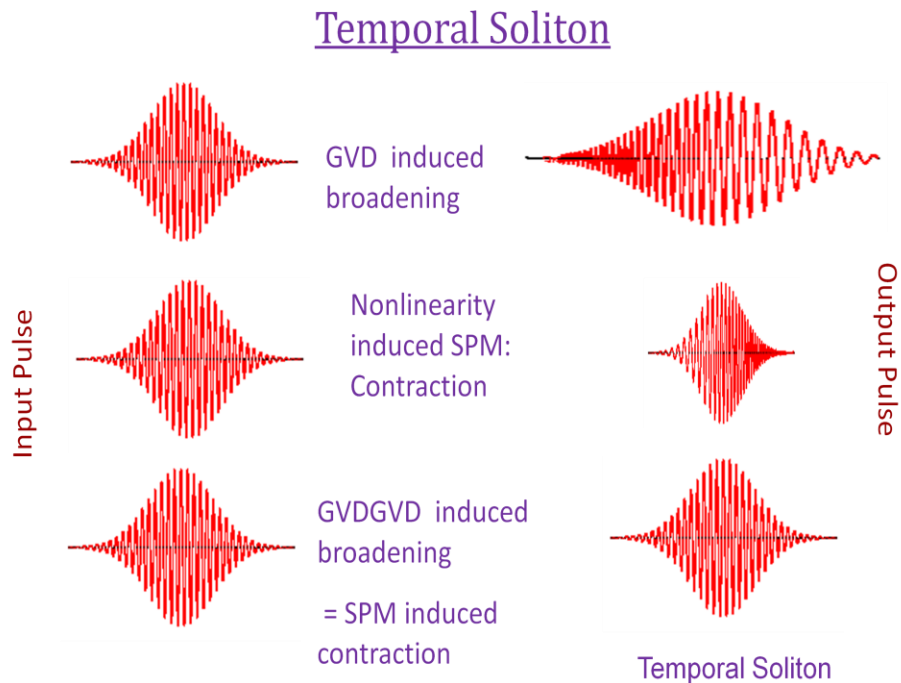


Fig. 1.3: Generation of an optical temporal soliton

1.2.4.2 Generation of spatial soliton:

An optical beam gets broadens due to self-diffraction. When this beam is propagating in non-linear media, non-linearity tends to defocus the beam. When these two effects counter balance each other [21], there is no change in spatial dimension of the beam i.e. the beam propagates as a self trapped beam leads to formation of spatial soliton as shown in fig.1.4. Consider an example of a beam with Gaussian intensity distribution is propagating in self-focusing Kerr nonlinear medium. In this medium, phase velocity is minimum at the centre region because refractive index is highest at peak region while refractive index decreases along the edges leads to maximum phase velocity at both edges. As the beam propagates, its curvature goes on increasing due to which the medium behave as a convex lens and it leads to self-focusing of beam. The beam gets broadened due to self-diffraction. These two effects compensate each other's effect & gives spatial soliton. The generation of spatial Soliton is shown in fig. 1.4.

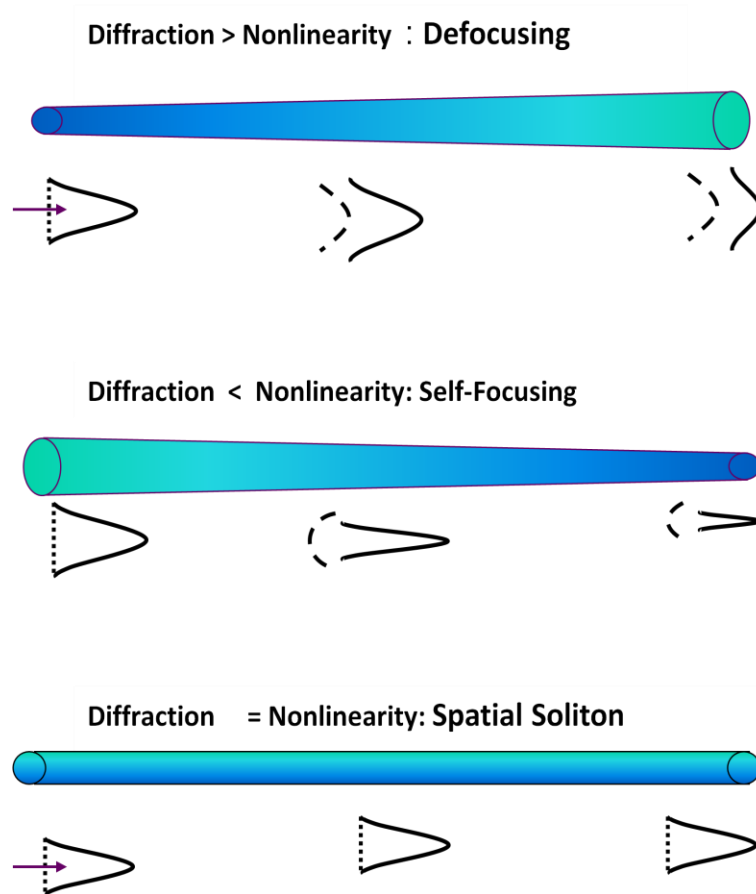


Fig. 1.4: Generation of an optical spatial soliton

1.2.4.3 Generation of spatiotemporal soliton:

When in both space and time domain, a beam remains self-localized while propagating through a non-linear media is known as spatiotemporal Soliton (STS). The complete localized spatiotemporal solitons are called ‘light bullets’ [22-23]. In temporal soliton GVD balances SPM while in spatial soliton self-diffraction balances self-focusing. The simultaneous effect of these both phenomenon gives rise to spatiotemporal soliton which is constrained in all three spatial dimensions as well as in time domain i.e. in (3+1)D. When a beam/pulse propagates through a bulk nonlinear media, diffraction in space domain, dispersion in time domain and nonlinearity acts simultaneously that gives rise to spatiotemporal soliton.

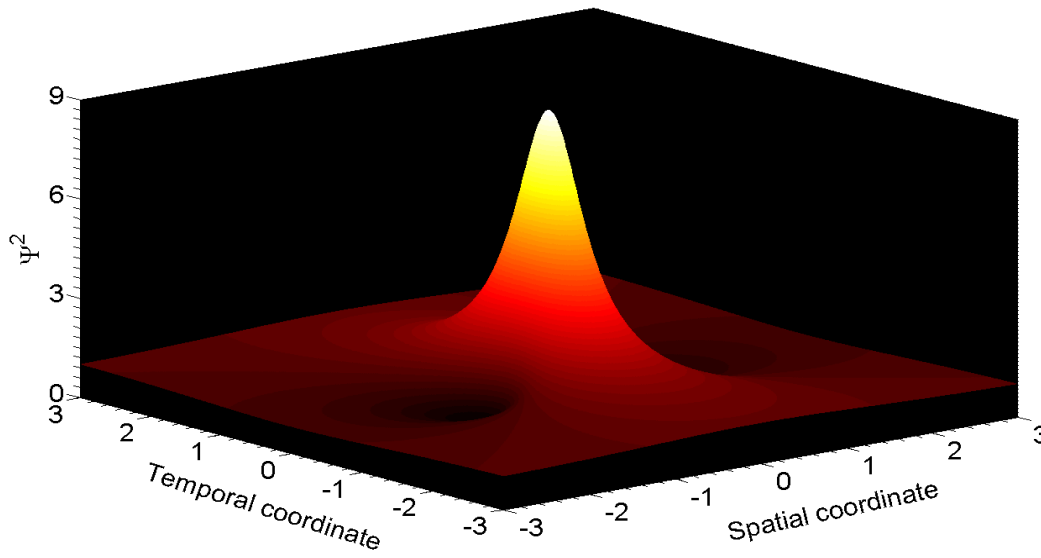


Fig. 1.5: Generation of spatiotemporal soliton

(Image Courtesy: upload.wikimedia.org/Wikipedia/commons/0/01/Soliton_de_Peregrine.png)

1.2.5 Dissipative Optical Soliton: Cavity Soliton

Dissipative optical Soliton (DOS) is a type of soliton in which there is some gain or loss of energy. This is a localized structure and exists as soon as some external energy is supplied to the system and it disappears when energy supply is discontinued. This energy is re-distributed between the parts of soliton, so soliton change its shape periodically or non-periodically. These structures depend on the processes like gain, loss, dispersion, diffraction, non-linearity etc. When overall gain and loss are compensated then we get stationary solitons. The dissipative structure can be profile of light, temperature, electric field or magnetic field etc. In case of dissipative Soliton (DS), different non-linearity interactions give rise to different structures. These solitons are fixed. They depend on the parameters of the system and have shape, amplitude and velocity fixed. There is internal energy exchange mechanism due to which energy is distributed and exchanged between various parts and soliton structure goes on changing. DS are similar to biological entities due to internal energy flow in system. The generation of conservative and dissipative soliton is shown in fig.1.6. Cavity solitons (CS) are DS which are obtained in an optical cavity and can be present or absent under the same condition. CS has freedom to move in localized direction. CS formation needs nonlinearity and diffraction that can be separate in

feedback configuration or can occur together in an optical cavity. An address beam is used to obtain a soliton and a reverse address beam is used to erase the soliton. CS are used in all optical lines or buffer registers. Stationary CSs are unstable while moving CSs are stable.

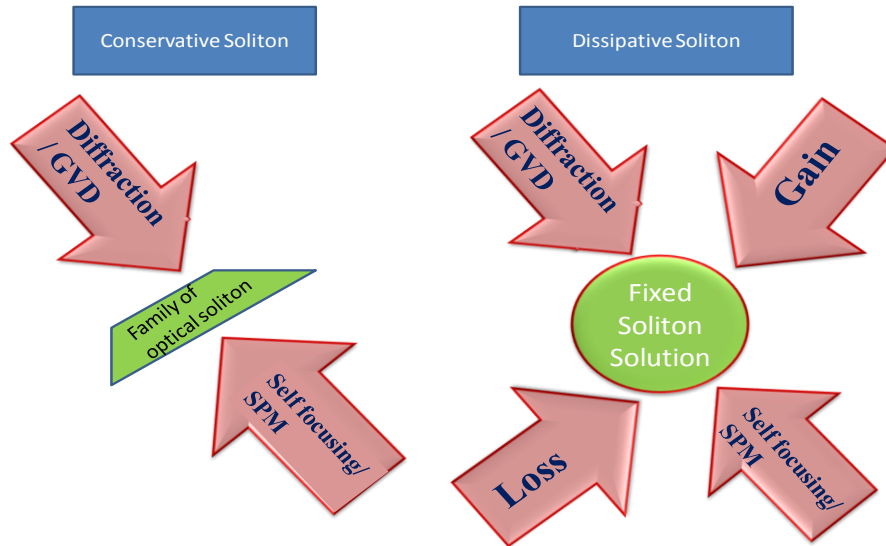


Fig. 1.6: Generation of conservative and dissipative Soliton

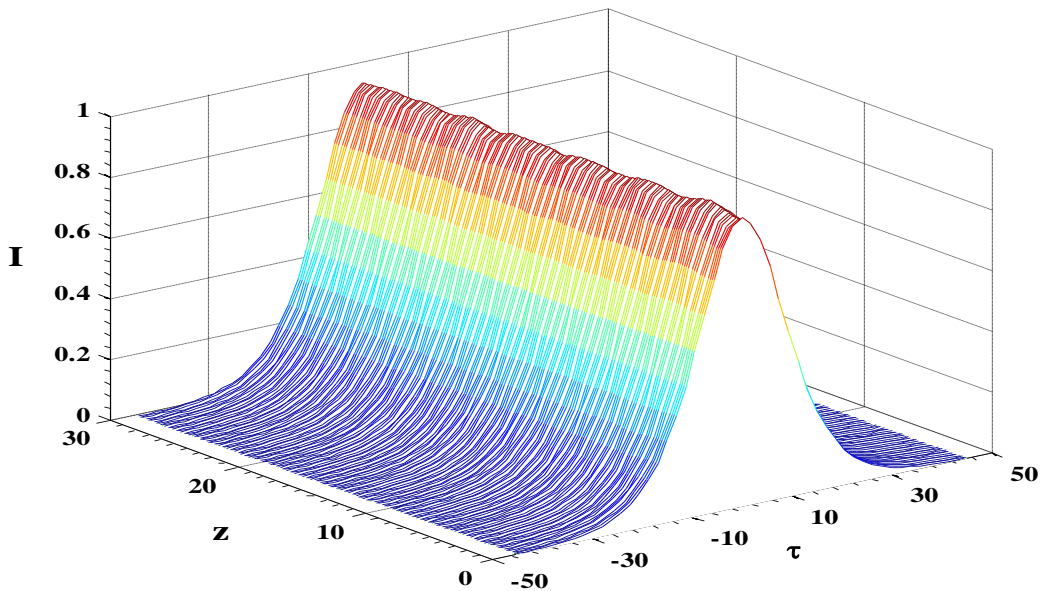


Fig.1.7: Formation of Dissipative optical soliton

1.2.6 VCSEL (Vertical Cavity Surface Emitting Laser):

Most of the cavity solitons are formed in the VCSEL.



Fig.1.8: Typical VCSEL

(Image courtesy: http://www.lasercomponents.com/fileadmin/user_upload/home/Images/_Presse/Pictures/Laser/Flat%20Window%20TO%20Can%20Package.jpg)

VCSEL are recent type of semiconductor laser invented in mid 1980's. Due to its lower manufacturing cost and reliability, it became the technology for local area networks and displacing the edge emitting laser. VCSEL has semiconductor layers grown on top of each other on a substrate called epi. Metal organic chemical vapor deposition (MOCVD) or molecular beam epitaxy (MBE) is used for growth of crystal. In VCSEL, the active layer is sandwiched between two reflected mirrors called DBR (dubbed distributed Bragg reflectors). These mirrors are highly reflective nearly 99.5~99.9%. These Bragg reflected mirrors have alternating low and high refractive index layers due to which the oscillation of light occurs in perpendicular to the layers and escapes through the top or bottom of the device. Several methods have been developed to achieve current confinement to the active area. Some of those are etching of the top mirror [24, 25], ion implantation to get highly resistive semiconductor regions [26,27] or selective lateral oxidation of an aluminum-rich layer, i.e., selective oxidation of some 10nm thick layer of semiconductor with high aluminum content like $Al_{0.98}Ga_{0.02}As$ or with $AlAs$ [28]. Proton implantation method is used to fabricate commercial VCSEL's having high productivity and reliability [29, 30]. Fig. shows a mounted bottom-emitting selectively oxidized structure of VCSEL.

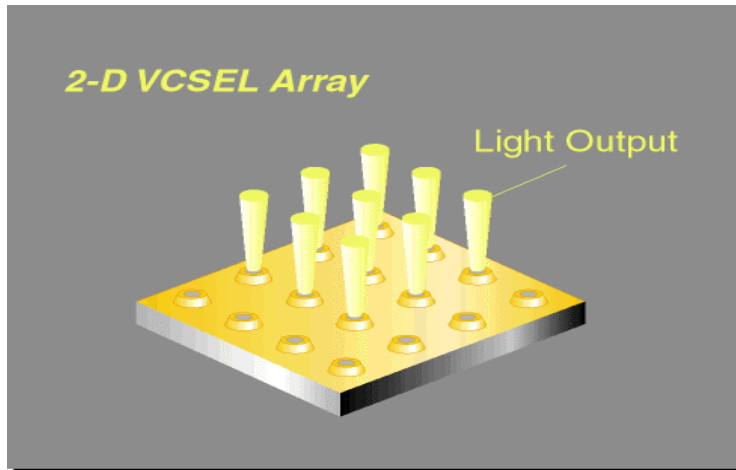


Fig.1.9: Array of VCSEL
 (Image courtesy: <http://vcsel-www.pi.titech.ac.jp/coe/image/vcsel.gif>)

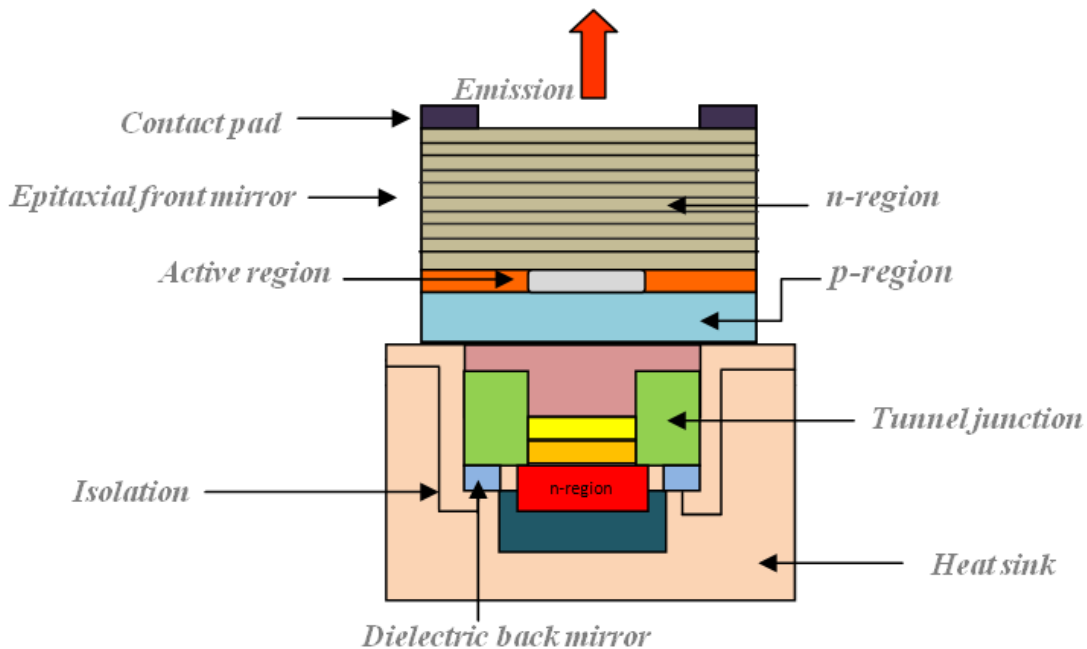


Fig. 1.8: The structure of VCSEL

Advantages of VCSEL:

VCSEL has many advantages. One of the important advantages of VCSEL over edge emitter is the longer operating lifetime and higher output power. Moreover, it has low threshold currents nearly a few μA and low noise operation. It has very stable and uniform lasing wavelength which is fixed by the cavity of short wavelength. This cavity has very least

temperature dependent, due to which the wavelength of emitted light in VCSEL has a insignificant dependence on temperature as compared to edge-emitter laser. Since VCSELs work reliably at temperature upto $80^{\circ}C$, so they are easily operated without refrigeration. These have higher power per unit area nearly $1200W/cm^2$ and are much reliable and easy to make than edge-emitter lasers.

1.3 Review on Cavity Soliton:

The concept of dissipative soliton was known properly since the last few decades. The concept of soliton comes in 1960 from the development of classical soliton theory named “non-linear modes” of integrable systems. In 1965, mathematical concept was given by *Zabusky & Kruskal* but till then there was no experimental verification that was later carried out and it was concluded that solitons are present everywhere. Dissipative soliton concept has three component parts. First is Standard Soliton theory which states that there is balance between non-linearity & dispersion or diffraction due to which stationary localized solitons exist. 2nd is non-linear dynamics theory given by *Poincare* that gives the idea of soliton bifurcations. 3rd is *Prigogine's* ideas of systems far from equilibrium that concludes that solitons are self-organized structures which exist when energy is continuously supplied. DS can be a profile of distribution of light, temperature, pressure, electric field, magnetic field, biological entity like a cell or organ etc. DS have different shapes depending on the energy supply to the system. DS are not decaying solutions. In dissipative soliton system external energy is provided and there is transfer of energy & compensation between loss and gain of system. DS occur in a system far from equilibrium and depend on the external energy source.

To study DOS optical cavities with different configuration and media are widely used. Such solitons are referred as CS too. Inside a fiber laser cavity, certain types of collisions involving soliton pairs were studied [31]. Both numerically & experimentally collisions of dissipative solitons are observed & found that two pulses form a bound state with group velocity different from single soliton. Two models of dissipative system supporting solitons: continuous CGLE and CGLE with parameter management [32] were used. With the collision of bound state soliton with single soliton, different scenarios were found ,i.e., after collision, bound state can be

destroyed or another bound state can be formed which moves with same velocity leaving previous pair at rest or Soliton triplet can be formed.

It was studied that, for normal & anomalous chromatic dispersion, there exist optical light bullets in non linear dissipative medium [21, 33]. This observation was derived from the numerical simulation of complex CGLE in space & time (3+1) dimension. The regions were observed where stable bullets [34, 35] exist and unstable bullets transformed into rockets (bullets elongated in temporal domain). These bell-shaped optical bullets were numerically demonstrated using CGLE model in anomalous dispersion and a number of interactions between stable optical bullets using temporal & spatial interaction planes were studied.

One of the most important and very unique types of spatial dissipative soliton class is CS. The CSs are stable & self localized structures in a dissipative medium like a non-linear optics cavity. The fundamental spatial dissipative soliton (SDS) is a bright spot of light on low amplitude background. It is localized in two spatial dimensions which is perpendicular to the cavity axis which is main propagation axis of travelling beam. In the longitudinal direction, the solitons are self-localized by cavity mirrors. These solitons can be found in various systems including chemistry, biology, fluid dynamics, gas discharges, granular media, optics etc. In optical solitons, generally non-linearity balances the dispersion or diffraction (in conservative system) and most of these limited to one-dimensional system like a system with Kerr-nonlinearity in which refractive index change with the intensity of incident light. But if we include dissipation, driving or feedback, we get the solitons with new properties. Among these, localized bright spots were observed in optical cavities which have a great use in semiconductors micro-resonators and potential applications in information processing. Apart from spatial cavity soliton, temporal CS has also been reported.

Most of the CSs are formed when a pattern coexist with a stable pattern. If we found such a phenomenon in a driven optical cavity with a non-linear medium, called cavity soliton and same phenomenon occur when a driven non-linear medium is placed in front of single feedback mirror, called feedback soliton. In an optical soliton, non-linearity counter balance the diffraction/dispersion, while we can also get stable soliton if the effect of feedback and dissipation is included. The DS when observed in a cavity, the localized bright spots were located, which have some properties similar to those of stable soliton & some new properties,

they were called CS. If some feedback medium is provided, like a single mirror feedback, the solitons observed are called feedback solitons.

The concept of cavity soliton began with seminal theoretical papers of Moloney & his co-workers [36, 37]. In their model, a Gaussian beam is passed through a ring cavity which contains self focusing Kerr medium & optical field passes in z -direction around the cavity, when the input field exceeds the threshold of optical bistability switch-up, a switching wave is found with most of the beam is switching up. An example of cavity system having spatial dissipative soliton is vertical cavity surface-emitting laser (VCSEL) given by *Wilmsen* in 1999. These are semiconductor microcavities closed by Bragg reflectors which have high reflectivity. To provide gain to the system, quantum well structures are used as active medium. At a certain biasing condition of holding beam, the cavity solitons are observed.

In inhomogeneous medium some properties of the system are different in different directions. Inhomogeneity of the medium has significant role in the propagation of pulse or beam through the media and thus on cavity soliton too. The dynamics of CSs were studied under the influence of spatial inhomogeneities & generalized equations of motion were derived [38]. The ability of CSs to react upon external perturbation or variations in parameters of the system were studied or it was observed that how CSs response to spatially dependent perturbations of the system parameters. It was observed that if there is large perturbation as compared to size of soliton, then modulus of velocity of soliton proportionally depend on gradient of that perturbation. If there is short scale perturbation, these solitons gets trapped at the extreme of inhomogeneities. Perturbation theory describes the shape & stability of trapped solitons. Cavity solitons are used in storage of optical information. The inhomogeneities such as system parameter variation can be used to control the position of cavity solitons [39, 40] or to steer them to the point of operation and used to make all optical full adders [41] and shift registers [42]. For e.g. smooth inhomogeneities change the direction of cavity Soliton in transverse direction and velocity of Soliton depends on the local gradient of the corresponding inhomogeneity [39, 43, 44, 20].

1.4 Motivation/ Gap:

Although dissipative optical solitons, particularly, cavity solitons have been widely investigated in lossy/ dissipative systems there is still enough scope of more comprehensive investigation considering combined influence of different kind of losses and gain mechanism. Investigations on cavity solitons mostly use *sech* type or similar bell shaped profile. To the best of our knowledge no investigation has been done with flat top profile or other not so common profile, while those may provide us with better controlling to the cavity soliton dynamics. Thus in the current investigation we study the influence of flat top profile, namely, *Super-Gaussian* profile on the generation of cavity solitons in different systems. We also explore the possibility of formation of *cosh-Gaussian* type cavity soliton. This will be helpful for realization of more versatile CS. Moreover, it is anticipated to have more control on the generated CS.

1.5. Objectives:

Thus we propose to investigate

- Generation of cavity solitons in different dissipative nonlinear medium with super-Gaussian profile.
- Cavity soliton formation with Cosh-Gaussian profile and its dynamics.

1.6 Methodology:

The theoretical work on DOS basically involves the derivation, solution and analysis of the pulse / beam propagation equations. The basic equations which govern the nonlinear dynamics of a dissipative optical soliton are: Complex-Ginzburg Landau equation (CGLE), Swift Hohenberg equation (SHE), Lugiato-Lefever equation. Also, non linear Schrodinger equation (NLSE) in conjugation with some additional dissipative term can be used for such investigation. One of the most sought after equation that describes beam/pulse dynamics in dissipative system is the complex Ginzburg Landau equation. A CGLE can be considered as a perturbed nonlinear Schrödinger equation NLSE [45]. Following are examples of CGLE [46] representing the dissipative systems:

$$\frac{\partial A}{\partial z} = (1 + i\alpha) \frac{\partial^2 A}{\partial x^2} + A - (1 - i\beta)|A|^2 A, \quad (1.9)$$

or,

$$\frac{\partial A}{\partial t} = (1 + i\alpha) \frac{\partial^2 A}{\partial x^2} + A - (1 - i\beta)|A|^2 A, \quad (1.10)$$

Note the complex coefficients of the diffraction terms (2nd terms of both equations) and the nonlinear terms (3rd terms of both equations). Imaginary terms are responsible for the loss/gain of the system.

The theoretical work on DOS primarily aims to solve the governing equations presented above. Both analytical as well as numerical methods have been used. However, the theoretical work is dominated by numerical investigation. It's mainly due to either non integrability of the governing equations and/or the complicated nature of the system. Numerical methods are basically Crank-Nicolson or Newton's method have been used. However, in principle, Split Step Fourier method (SSFM), with some modification can be used for such investigation [9]. Analytical methods are of two categories: Exact methods, like inverse scattering method [47], AKNS method [48] etc and the approximate analytical method such as Variation method [49], Moment method and Collective Variable method [50]. Since, the governing equation is non-integrable or too complicated to integrate we will avoid exact methods to find the exact solution of the system.

In our work, we will use the Separation method, which is analytical method and Split Step Fourier method, which is numerical method. These methods are described as follows.

1.6.1. Analytical Method: (Separation method)

For our analytical investigation, we will use a separation method proposed by Firth *et al.* [51], which is an analytical method, relies on graphical solution and shooting method. In separation method, the actual problem is divided into two Eigen value problems: one linear or 'spectral problem' and another nonlinear or 'soliton problem'. The spectral problem describes the cavity and the 'soliton problem' describes the beam/ pulse dynamics in the cavity. Both the

equations are solved graphically. The intersection point(s) of the linear and spectral curves represent the solutions. Thus, indicate the final solution(s) of the main problem.

This is very efficient approach as exact points of solution can be obtained from it that can be achieved by rigorous numerical simulations. This is more flexible analytical method. It can give the parametric region where solution can be found. This method can also provide some ideas regarding the stability of the CSs with high accuracy. In fact, this method looks simple but smart analytical method is much better than the existing numerical and other analytical methods to study CS. The method is explained by the following example:

Consider the CGL3 equation in one dimension as:

$$(\partial_t + 1)E = (d + iD)\partial_{xx}E + (g_0 - g_2|E|^2), \quad (1.11)$$

Separating the eq. (1.11) into two parts, i.e., in linear and nonlinear parts. By separating the linear and nonlinear parts, we define two operators as: $\hat{L} = (\partial_t + 1)$ is linear operator that describes the cavity and there is nonlinear operator given as, $\hat{N} = (d + iD)\partial_{xx} + (g_0 - g_2|E|^2)$, where g_0 is linear gain/loss, g_2 is nonlinear gain-loss, d is diffusion & D is diffraction coefficient and the term '1' in linear operator represents the cavity loss. The term $\partial_t = \frac{\partial}{\partial t}$ is linear operator and $\partial_{xx} = \frac{\partial^2}{\partial x^2}$ is nonlinear operator. The term $g_2|E|^2E$ represents cubic nonlinearity.

Solving eq. (1.11) by splitting it into two complex eigen value problems as given:

$$\hat{L}E(x, t) = LE(x, t), \quad (1.12)$$

$$\hat{N}E(x, t) = NE(x, t). \quad (1.13)$$

Assuming the trial function and substituting it in eq. (1.12) and (1.13), the linear and nonlinear eigen value, say L and N , are obtained respectively. By equating these eigen values i.e. $L = N = X - iY$ gives the solution of problem (1.11). By plotting these equations of linear and nonlinear eigen value in X, Y plane, we get different solutions for different value of parameter of trial function.

1.6.2. Numerical Method: Split Step Fourier Transformation Method:

To understand the nonlinear effects in optical fibres, numerical approach is used. Most of the numerical methods are classified into two broad categories: Finite-difference method & Pseudo-spectral method. Split Step Fourier Method (SSFM) is used to solve pulse-propagation problem in non-linear dissipative medium. Its methodology is described as:

Consider the Non-linear Schrodinger equation (NLSE) as:

$$\frac{\partial A}{\partial z} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - i\gamma |A|^2 A = 0, \quad (1.14)$$

$$\frac{\partial A}{\partial z} = \left[-i \frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} + i\gamma |A|^2 \right] A. \quad (1.15)$$

Above equation (1.15) can be written as:

$$\frac{\partial A}{\partial z} = (\widehat{D} + \widehat{N})A, \quad (1.16)$$

$$\text{where, } \widehat{D} = -i \frac{\beta_2}{2} \frac{\partial^2}{\partial t^2}, \widehat{N} = i\gamma |A|^2. \quad (1.17)$$

\widehat{D} is the differential operator that gives dispersion term and losses in a linear medium and \widehat{N} is non linear operator that accounts the effect of fibre nonlinearities on pulse propagation. In general case, Dispersion & non linearity operate together along the fibre. But in SSFM, we obtain an approximate solution. We assume that in propagation over a small distance h , dispersion and non-linearity act independently; so propagation is done from z to $z+h$ in two steps, where h is step size.

Step 1: Non-linearity act alone i.e. $\widehat{D} = 0$.

$$\text{Equation (1.16) becomes } \frac{\partial A}{\partial z} = \widehat{N}A. \quad (1.18)$$

By integration from limit $A(z, t)$ to $A(z + h, t)$:

$$\int_{A(z,t)}^{A(z+h,t)} \frac{\partial A}{A} = \widehat{N} \int_z^{z+h} \partial z, \quad (1.19)$$

$$A(z + h, t) = A(z, t) \exp(h\widehat{N}). \quad (1.20)$$

Step 2: Dispersion act alone i.e. $\hat{N} = 0$.

$$\text{Equation (1.16) becomes } \frac{\partial A}{\partial z} = \hat{D}A . \quad (1.21)$$

By integration from limit $A(z, t)$ to $A(z + h, t)$:

$$\int_{A(z,t)}^{A(z+h,t)} \frac{\partial A}{A} = \hat{D} \int_z^{z+h} \partial z, \quad (1.22)$$

$$A(z + h, t) = A(z, t) \exp(h\hat{D}) . \quad (1.23)$$

By combining step 1 & 2, solution becomes

$$A(z + h, t) \approx \exp(h\hat{D}) \exp(h\hat{N}) A(z, t). \quad (1.24)$$

Operator $\exp(h\hat{D})$ can be calculated by fourier theorem and by replacing $\frac{\partial}{\partial t}$ by $-i\omega$ i.e. time domain has to be converted into frequency domain by using fourier transformation. So operator \hat{D} can be expressed as:

$$\hat{D} = i \frac{\beta_2}{2} \omega^2, \quad (1.25)$$

$$\exp(h\hat{D}) B(z, t) = F_T^{-1} \exp(h\hat{D}(-i\omega)) F_T B(z, t). \quad (1.26)$$

$$\text{The exact solution of eq. (1.14) is given as } A(z + h, t) = \exp(h(\hat{D} + \hat{N}))A(z, t). \quad (1.27)$$

Baker-Hausdorff formula for two non-commuting operators \hat{a} & \hat{b} is given as

$$\exp(\hat{a})\exp(\hat{b}) = \exp(\hat{a} + \hat{b} + \frac{1}{2} [\hat{a}, \hat{b}] + \frac{1}{12} [\hat{a} - \hat{b}, [\hat{a}, \hat{b}]] + \dots), \quad (1.28)$$

where $[\hat{a}, \hat{b}] = \hat{a}\hat{b} - \hat{b}\hat{a}$. Using eq.(1.26) with $\hat{a} = h\hat{D}$ & $\hat{b} = h\hat{N}$, the dominant error term is found to produced from factor $\frac{1}{2}h^2 [\hat{D}, \hat{N}]$. Hence SSFM method is accurate to 2nd order of h .

The accuracy of SSFM can be improved by using a different scheme. In this procedure, eq. (1.23) can be expressed as:

$$A(z + h, t) \approx \exp(\frac{h}{2}\hat{D}) \exp(\int_z^{z+h} \hat{N}(\zeta)d\zeta) \exp(\frac{h}{2}\hat{D}) A(z, t). \quad (1.29)$$

This scheme is called symmetrized SSFM because in eq. (1.29), *exponential* operator is symmetric. In above eq. (1.29), middle term shows that the operator \hat{N} depends on z . In this procedure, non-linearity is not acting at boundary of segment; rather it is acting on the middle of segment. If there is very small value of step size h , then the middle term is approximated to $\exp(h\hat{N})$, then eq. (1.29) & (1.23) become similar. In this scheme error term depends on third order of h ; so this method is better than the previous one. The accuracy can be further improved by integrating the mid-integral in eq. (1.29) by using trapezoidal rule as shown:

$$\int_z^{z+h} \hat{N}(z) dz \approx \frac{h}{2} [\hat{N}(z) + \hat{N}(z+h)], \quad (1.30)$$

here, $\hat{N}(z+h)$ term is unknown at $z+\frac{h}{2}$, so iterative procedure is used to find the term $\hat{N}(z+h)$. For implementation of SSFM, first of all fiber length is divided into large number of segments, they may be equally spaced or not. Using eq. (1.29), optical pulse is propagated from one segment to another. Firstly field $A(z,t)$ is propagated for distance $h/2$, for this distance only dispersion term acts using eq. (1.26). When the field reached to mid of segment i.e. at $z+h/2$, non-linear term acts and field is multiplied by non-linear term. For remaining distance $h/2$ from the mid, field is propagated with dispersion and we get $A(z+h,t)$.

Chapter 2

Cavity Soliton formation using Super Gaussian profile:

We considered different models for the generation of Cavity Soliton using Super Gaussian profile.

2.1. Cubic Ginzburg-Landau model:

To study the nonlinear dynamics of a dissipative optical Soliton, the equations such as: Complex Ginzburg Landau equation (CGLE), Swift Hohenberg equation, Lugiato-Lefever equation. Also, non linear Schrodinger equation (NLSE) in conjugation with some additional dissipative term can be used for such investigation.

We consider the simplest model for cavity Soliton laser (CSL) i.e. CGLE with cubic nonlinearity [52]. We have taken chirped Super Gaussian beam to find spectral and Soliton solution for this problem.

Consider following form of CGL3 equation in one dimension as:

$$(\partial_t + 1)E = (d + iD)\partial_{xx}E + (g_0 - g_2|E|^2)E. \quad (2.1)$$

In eq. (2.1) $\hat{L} = (\partial_t + 1)$ is linear operator that describes the cavity. In eq. (2.1), the term '1' represents the cavity loss. On right hand side of eq. (2.1), there is nonlinear operator given as, $\hat{N} = (d + iD)\partial_{xx} + (g_0 - g_2|E|^2)$, where g_0 is linear gain/loss, g_2 is nonlinear gain-loss, d is diffusion & D is diffraction coefficient. The term $\partial_t = \frac{\partial}{\partial t}$ is linear operator and $\partial_{xx} = \frac{\partial^2}{\partial x^2}$ is nonlinear operator. The term $g_2|E|^2E$ represents cubic nonlinearity.

Solving eq. (2.1) by splitting it into two complex eigen value problems as given:

$$\hat{L}E(x, t) = LE(x, t), \quad (2.2)$$

$$\hat{N}E(x, t) = NE(x, t). \quad (2.3)$$

Equating these eigen values i.e. $L = N = X - iY$ gives the solution of problem (2.1). For solving the problem (2.1), consider the solution as:

$$E(x, t) = S(x)e^{-i\Omega t}, \quad (2.4)$$

By operating the linear operator on eq. (2.4), we get

$$\hat{L}E(x, t) = (\partial_t + 1)E(x, t) = \frac{\partial E(x, t)}{\partial t} + E(x, t), \quad (2.5)$$

$$\hat{L}E(x, t) = \frac{\partial(S(x)e^{-i\Omega t})}{\partial t} + S(x)e^{-i\Omega t}, \quad (2.6)$$

$$\hat{L}E(x, t) = (1 - i\Omega)E(x, t), \quad (2.7)$$

The eq. (2.7) gives the eigen value $L = 1 - i\Omega = X - iY$, which gives the family of complex eigen values parameterized by Ω . We find straight vertical line when this family is plotted in X, Y plane corresponds to $X = 1, Y = \Omega$. This straight line is representing ‘spectral curve’.

For stable background and for self localized solution, we impose the boundary condition $|E| \rightarrow 0$ and $|x| \rightarrow \infty$ on the nonlinear side of eq. (2.1) and setting $E = \delta E e^{ikx}$ (δE is small), we find:

$$\hat{N}E(x, t) = (d + iD) \frac{\partial^2(\delta E e^{ikx})}{\partial x^2} + (g_0 - g_2|E|^2)(\delta E e^{ikx}), \quad (2.8)$$

$$\hat{N}E(x, t) = -(d + iD)\delta E k^2 e^{ikx} + (g_0 - g_2|\delta E|^2)(\delta E e^{ikx}). \quad (2.9)$$

Since δE is small so neglecting second power of δE in eq. (2.9), we get

$$\hat{N}E(x, t) = -(d + iD)k^2 + g_0) \delta E e^{ikx}, \quad (2.10)$$

$$\hat{N}E(x, t) = -(d + iD)k^2 + g_0)E(x, t). \quad (2.11)$$

The eq. (2.11) implies that the nonlinear eigenvalue is given by

$$N = N_l = -(d + iD)k^2 + g_0. \quad (2.12)$$

When we plot eq. (2.12) in X, Y plane, it corresponds to half line having no intersection with spectral curve giving stable background. To find the soliton solution of eq. (2.1), for chirped Super Gaussian type, we take $E = S(x)e^{-i\Omega t}$ and trial function as,

$$S(x) = \exp\left(-\frac{1+ic}{2} \left(\frac{x}{x_0}\right)^{2m}\right). \quad (2.13)$$

Where, x_0 is the inverse width, m is Super Gaussian parameter which decides the edge sharpness. $m = 1$ gives chirped Gaussian profile while for higher values of m , profile becomes square shaped having sharper leading and trailing edges, c is chirp parameter. Super Gaussian profile is shown in fig 2.1.

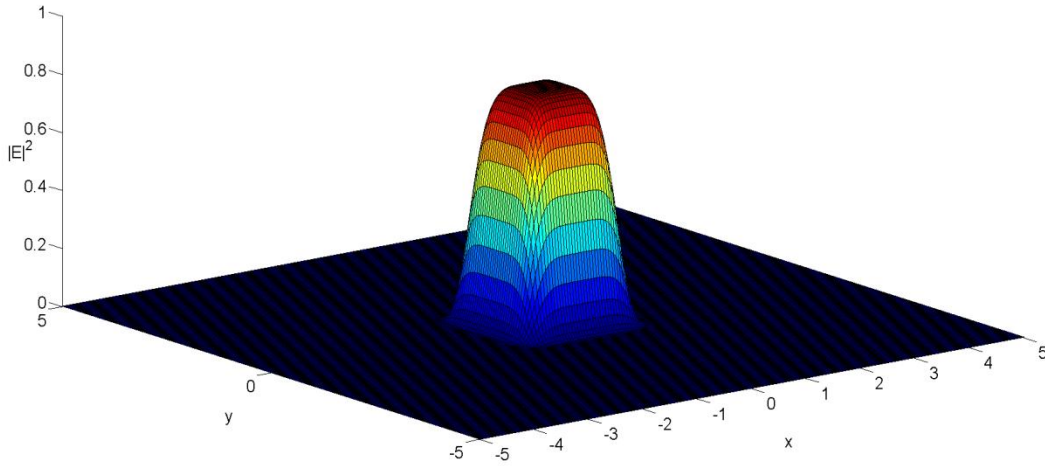


Fig. 2.1: Super Gaussian Profile for $m=3$

Applying the nonlinear operator on $E(x, t) = S(x)e^{-i\Omega t}$, where $S(x)$ is given by eq. (2.13).

$$\hat{N}E(x, t) = (d + iD) \frac{\partial^2}{\partial x^2} \exp\left(-\frac{1+ic}{2} \left(\frac{x}{x_0}\right)^{2m}\right) e^{-i\Omega t} + (g_0 - g_2 |E|^2) \exp\left(-\frac{1+ic}{2} \left(\frac{x}{x_0}\right)^{2m}\right) e^{-i\Omega t} \quad (2.14)$$

$$\begin{aligned} \hat{N}E(x, t) = & \left[(d + iD) \left(-\frac{1+ic}{2}\right) \left\{ 4m^2 \frac{x^{2m-2}}{x_0^{2m}} - 2m \frac{x^{2m-2}}{x_0^{2m}} + 4m^2 \left(-\frac{1+ic}{2}\right) \frac{x^{4m-2}}{x_0^{4m}} \right\} \right] \exp\left(-\frac{1+ic}{2} \left(\frac{x}{x_0}\right)^{2m}\right) e^{-i\Omega t} + \\ & \left(g_0 - g_2 \exp\left(-\frac{x^{2m}}{x_0^{2m}}\right) \right) \exp\left(-\frac{1+ic}{2} \left(\frac{x}{x_0}\right)^{2m}\right) e^{-i\Omega t} \end{aligned} \quad (2.15)$$

$$\hat{N}E(x, t) = \left[(d + iD) \left(-\frac{1+ic}{2} \right) \left\{ (4m^2 - 2m) \frac{x^{2m-2}}{x_0^{2m}} + 4m^2 \left(-\frac{1+ic}{2} \right) \frac{x^{4m-2}}{x_0^{4m}} \right\} + \left(g_0 - g_2 \exp \left(-\left(\frac{x}{x_0} \right)^{2m} \right) \right) \right] E(x, t), \quad (2.16)$$

$$\hat{N}E(x, t) = \left[(d + iD) \left(-\frac{1+ic}{2} \right) \left\{ (4m^2 - 2m) \frac{x^{2m-2}}{x_0^{2m}} + 4m^2 \left(-\frac{1+ic}{2} \right) \frac{x^{4m-2}}{x_0^{4m}} \right\} + \left(g_0 - g_2 \left(1 - \left(\frac{x}{x_0} \right)^{2m} \right) \right) \right] E(x, t), \quad (2.17)$$

In eq. (2.17), by putting the coefficients of x to zero we get the nonlinear eigenvalue given as:

$$N = g_0 - g_2 = X - iY, \quad (2.18)$$

From eq. (2.18), we get equations of X and Y as:

$$X = g_0 - g_2, \quad (2.19)$$

$$Y = 0. \quad (2.20)$$

Eq. (2.19) and (2.20) gives the soliton curve when plotted in X, Y plane. The spectral and Soliton curves for problem as described in eq. (2.1) are plotted as shown in fig.2.2.

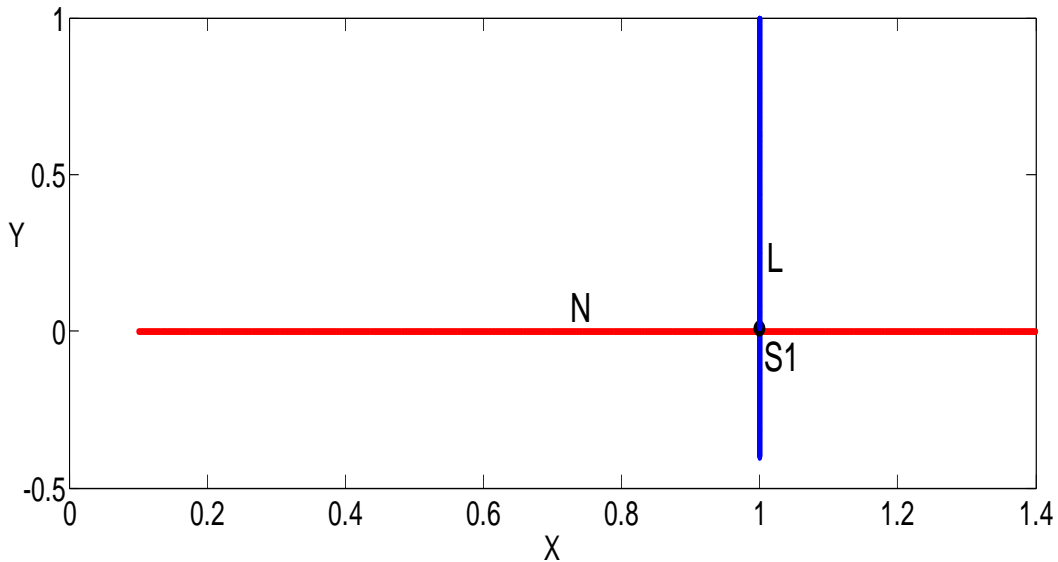


Fig.2.2: Spectral (L) and Soliton (N) curve which has one solution only

In fig.2.2, straight blue vertical line is spectral curve while red line is Soliton curve. It is evident that the two lines have only one intersection giving rise to one solution only. Moreover, the soliton line as well as soliton solution point are the same for all values of super-Gaussian parameter, e.g., $m=1, 2, 3, 4$ etc . Therefore, we get a family of super-Gaussian cavity solitons.

2.2. CGL3 Model with frequency selective feedback:

Consider a simple model of VSCSEL with frequency selective feedback. We can establish FSF in a simple way by coupling a second field F to E in \hat{L} as:

$$(\partial_t + 1)E(x, t) - \sigma^{\frac{1}{2}}F(x, t) = \hat{L}E(x, t), \quad (2.21)$$

$$\partial_t F + (\Gamma_0 + i\Omega_0)F = \sigma^{\frac{1}{2}}E. \quad (2.22)$$

here, σ is a coupling constant, Ω_0 and Γ_0 are the resonant frequency & linewidth parameter of the F field respectively. Consider the single frequency solution as taken in eq. (2.4): $E(x, t) = S(x)e^{-i\Omega t}$. The feedback field is taken as:

$$F(x, t) = S_f(x)e^{-i\Omega t}. \quad (2.23)$$

Using eq. (2.23) in eq. (2.21) gives

$$\sigma^{\frac{1}{2}}E(x, t) = \frac{\partial}{\partial t} (S_f(x)e^{-i\Omega t}) + (\Gamma_0 + i\Omega_0)S_f(x)e^{-i\Omega t}, \quad (2.24)$$

$$\sigma^{\frac{1}{2}}E(x, t) = (\Gamma_0 + i(\Omega_0 - \Omega))F(x, t). \quad (2.25)$$

From eq. (2.25), we get feedback: $F(x, t) = \frac{\sigma^{\frac{1}{2}}E(x, t)}{\Gamma_0 + i(\Omega_0 - \Omega)}$. (2.26)

Substituting eq. (2.4) and (2.26) in eq. (2.21),

$$\hat{L}E(x, t) = \frac{\partial}{\partial t} (S(x)e^{-i\Omega t}) + S(x)e^{-i\Omega t} - \frac{\sigma E(x, t)}{\Gamma_0 + i(\Omega_0 - \Omega)}, \quad (2.27)$$

$$\hat{L}E(x, t) = (1 - i\Omega)E(x, t) - \frac{\sigma E(x, t)}{\Gamma_0 + i(\Omega_0 - \Omega)}. \quad (2.28)$$

Hence, from eq. (2.28), we get the linear eigen value as:

$$L = (1 - i\Omega) - \frac{\sigma}{\Gamma_0 + i(\Omega_0 - \Omega)} = X - iY. \quad (2.29)$$

On comparing the real and imaginary parts of eq. (2.33), we get the equation of X and Y .

$$X = 1 - \frac{\text{Re}(\sigma)\Gamma_0}{\Gamma_0^2 + (\Omega_0 - \Omega)^2}, \quad (2.30)$$

$$Y = \Omega - \frac{\text{Im}(\sigma)(\Omega_0 - \Omega)}{\Gamma_0^2 + (\Omega_0 - \Omega)^2}. \quad (2.31)$$

The eq. (2.30) and (2.31) when plotted in X, Y plane, gives spectral curve as shown in fig. 2.3. The spectral curve obtained in eq. (2.31) is an asymptote to the spectral curve which is obtained in previous case of CGL3 model.

To deal with Soliton solution in this case, consider the CGL3 expression as:

$$\hat{N}E(x, t) = i\partial_{xx}E(x, t) + \mu(1 - i\alpha)(1 - |E|^2)E(x, t). \quad (2.32)$$

Eq. (2.32) represents the basic version of a standard VSCEL model. Here, μ is proportional to the injected current and represents gain in the VSCEL. The parameter $\mu > 0$, if there is gain in the system while $\mu < 1$ represents a stable off state. The parameter α is line-width enhancement or phase amplitude coupling factor and generally taken to be positive and its value is nearly 5 in VSCELS [53, 54].

Since we have taken $E(x, t) = S(x)e^{-i\Omega t}$ as in eq. (2.4), where $S(x)$ is given as by eq. (2.13) as:

$$S(x) = \exp\left[-\frac{1 + ic}{2} \left(\frac{x}{x_0}\right)^{2m}\right]$$

Using these equations in eq. (2.36), we get

$$\hat{N}E(x, t) = i\frac{\partial^2}{\partial x^2} \left[\exp\left(-\frac{1+ic}{2} \left(\frac{x}{x_0}\right)^{2m}\right) e^{-i\Omega t} \right] + \mu(1 - i\alpha)(1 - |S(x)|^2) \exp\left(-\frac{1+ic}{2} \left(\frac{x}{x_0}\right)^{2m}\right) e^{-i\Omega t}, \quad (2.33)$$

$$\begin{aligned} \hat{N}E(x, t) = & i \left[\left(-\frac{1+ic}{2} \right) \left\{ 4m^2 \frac{x^{2m-2}}{x_0^{2m}} - 2m \frac{x^{2m-2}}{x_0^{2m}} + 4m^2 \left(-\frac{1+ic}{2} \right) \frac{x^{4m-2}}{x_0^{4m}} \right\} \right] \exp \left(-\frac{1+ic}{2} \left(\frac{x}{x_0} \right)^{2m} \right) e^{-i\Omega t} + \\ & \mathbb{Q}(1 - i\alpha) \exp \left(-\frac{1+ic}{2} \frac{x}{x_0} \right) \exp \left(-\frac{1+ic}{2} \left(\frac{x}{x_0} \right)^{2m} \right) e^{-i\Omega t}, \end{aligned} \quad (2.34)$$

$$\hat{N}E(x, t) = \left[i \left(-\frac{1+ic}{2} \right) \left\{ (4m^2 - 2m) \frac{x^{2m-2}}{x_0^{2m}} + 4m^2 \left(-\frac{1+ic}{2} \right) \frac{x^{4m-2}}{x_0^{4m}} \right\} + \mathbb{Q}(1 - i\alpha) \exp \left(-\frac{1+ic}{2} \frac{x}{x_0} \right) \right] E(x, t), \quad (2.35)$$

$$\hat{N}E(x, t) = \left[i \left(-\frac{1+ic}{2} \right) \left\{ (4m^2 - 2m) \frac{x^{2m-2}}{x_0^{2m}} + 4m^2 \left(-\frac{1+ic}{2} \right) \frac{x^{4m-2}}{x_0^{4m}} \right\} + \mathbb{Q}(1 - i\alpha) \left(1 - \frac{1+ic}{2} \left(\frac{x}{x_0} \right)^{2m} \right) \right] E(x, t). \quad (2.36)$$

By equating the coefficients of x to zero, we get nonlinear eigen value as:

$$N = \mu(1 - i\alpha) = X - iY. \quad (2.37)$$

Eq. (2.37) gives the equations of X and Y as:

$$X = \mu, \quad (2.38)$$

$$Y = \mu\alpha. \quad (2.39)$$

Eq. (2.38) and (2.39) gives the Soliton curve in X, Y plane, which is plotted with eq. (2.30) and (2.31) as shown in fig. (2.3), (2.4) and (2.5).

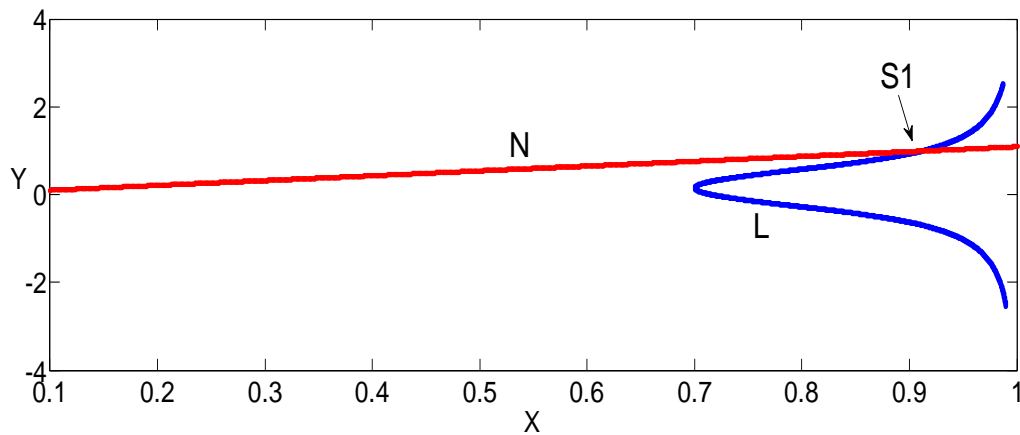


Fig.2.3: Spectral (L) and Soliton (N) lines in the complex eigen value plane for CGL3 with frequency selective feedback ($\Omega_0 = 0.16, \Gamma_0 = 0.5$)

Spectral and Soliton curves intersect at one point only. That point of intersection indicated the cavity soliton solution. Varying FSF parameters a family of solution has been obtained.

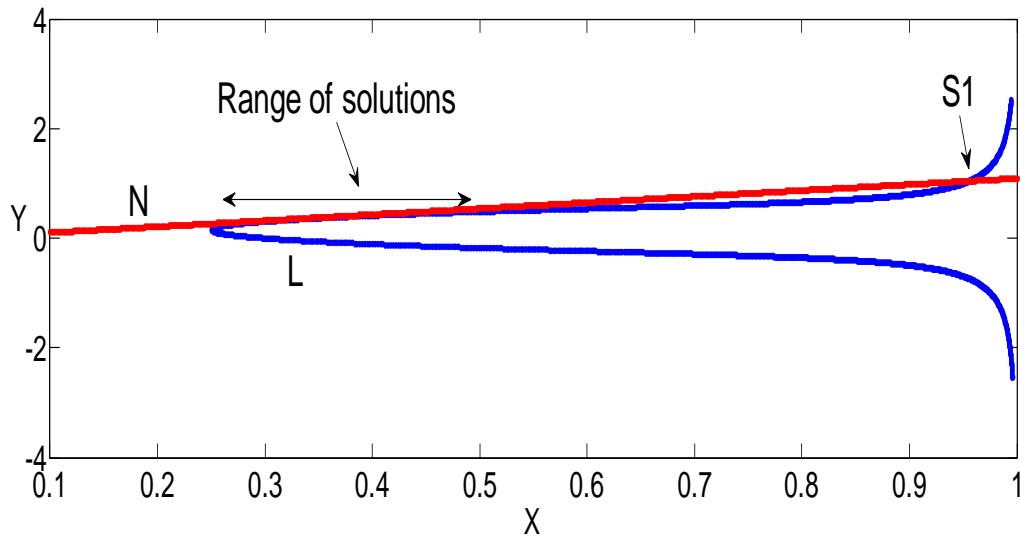


Fig. 2.4: Spectral (L) and soliton (N) curves in the complex eigen value plane for feedback parameter $\Omega_0 = 0.16, \Gamma_0 = 0.2$

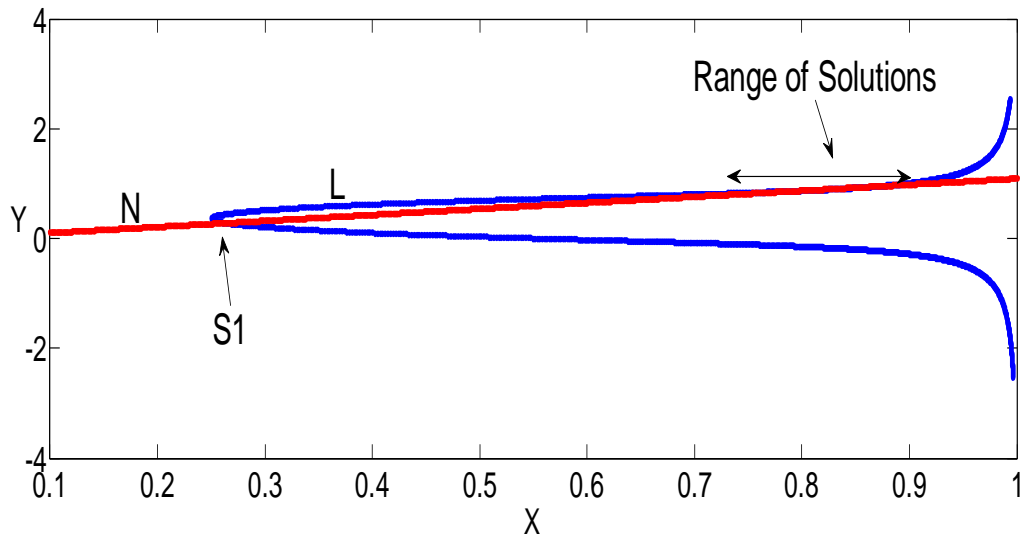


Fig. 2.5: Spectral (L) and soliton (N) curves in the complex eigen value plane for feedback parameter $\Omega_0 = 0.37, \Gamma_0 = 0.2$

Hence, for different values of feedback parameters, we get different Soliton families. In fig. (2.3) and (2.4), one Soliton is formed while in fig. (2.5), we get a range of solutions for a particular value of feedback parameters.

We get identical solution region for or all Super-Gaussian parameters, namely $m=1, 2, 3$ and 4 .

2.3. Saturable gain models and multi-dimensional dissipative solitons:

In this model, to find the cavity soliton, we use the rate equations for E and for the feedback field F as follows [55]

$$\frac{\partial E}{\partial t} = -\kappa E + \frac{\kappa E}{1+|E|^2} + \frac{i\alpha\kappa E}{1+|E|^2} + i\Delta_{\perp} E + F + i\omega_s - i\alpha\kappa E. \quad (2.40)$$

The feedback equation is given as:

$$\frac{\partial F}{\partial t} = -\lambda F + \sigma\lambda E(t - \tau), \quad (2.41)$$

here, κ is decay rate of the field in the cavity, α is the line-width enhancement factor describing phase-amplitude coupling, μ is the pump current normalized to be 1.0 at the threshold of the solitary laser, $\Delta_{\perp} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the transverse Laplacian that describes diffraction, ω_s is the detuning of the threshold frequency, σ is the feedback strength, τ is the delay time in the feedback loop and λ is the bandwidth of the filter reflection. The terms on the right side of eq. (2.41) describe losses due to which our system is dissipative. For steady state, $\frac{\partial F}{\partial t} = 0$, substitute in eq. (2.41) gives:

$$F = \sigma E(t - \tau) \quad (2.42)$$

Here, we are dealing with two transverse dimensions so we take the solution of eq. (2.40) in two dimensions. The solution is given by:

$$E(x, y, t) = S(x, y)e^{-i\Omega t} \quad (2.43)$$

$$\text{where, } S(x, y) = \exp\left(-\frac{1+ic}{2}\left\{\left(\frac{x}{x_0}\right)^{2m} + \left(\frac{y}{y_0}\right)^{2m}\right\}\right) \quad (2.44)$$

Where, x_0 is inverse width in x axis and y_0 is inverse width in y axis, c is chirp parameter and m is Super Gaussian parameter.

By rearranging and separating the linear and nonlinear terms of eq. (2.40), we get

$$\begin{aligned} -i\Omega E + \kappa E - \kappa \square E - i\alpha\kappa \square E - \sigma E e^{i\Omega\tau} - i\omega_s E + i\alpha\kappa E = & -\kappa \square |E|^2 E - i\alpha\kappa \square |E|^2 E + \\ & i\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E \end{aligned} \quad (2.45)$$

On the left side of eq. (2.45), we have linear terms while on the right side we have nonlinear terms. So left side of eq. (2.45) gives the linear operator as:

$$\hat{L}E(x, y, t) = -i\Omega E + \kappa E - \kappa \square E - i\alpha\kappa \square E - \sigma E e^{i\Omega\tau} - i\omega_s E + i\alpha\kappa E. \quad (2.46)$$

Eq. (2.46) gives the linear eigenvalue as:

$$L = -i\Omega + \kappa - \kappa \square - i\alpha\kappa \square - \sigma e^{i\Omega\tau} - i\omega_s + i\alpha\kappa = X - iY. \quad (2.47)$$

On comparing the real and imaginary parts of eq. (2.47), we get:

$$X = \kappa - \kappa \square - \sigma \cos \Omega\tau, \quad (2.48)$$

$$Y = \Omega + \alpha\kappa \square + \omega_s - \alpha\kappa + \sigma \sin \Omega\tau \quad (2.49)$$

The eq. (2.48) and (2.49) are plotted in X, Y plane, give a number of spectral curves as shown in fig. (2.6).

The nonlinear operator is given from eq. (2.45) as:

$$\hat{N}E(x, y, t) = -\kappa \square |E|^2 E - i\alpha\kappa \square |E|^2 E + i\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E. \quad (2.50)$$

By putting the solution given by eq.(2.43) and (2.44) in eq.(2.50)

$$\hat{N}E(x, y, t) = \left[(-k_0 - i\alpha k_0) \exp\left(-\left\{\left(\frac{x}{x_0}\right)^{2m} + \left(\frac{y}{y_0}\right)^{2m}\right\}\right) - i\left(-\frac{1+ic}{2}\right)(4m^2 - 2m)\left(\frac{x^{2m-2}}{x_0^{2m}} + \frac{y^{2m-2}}{y_0^{2m}}\right) - 4im^2 - 1 + ic \right] E(x, y, t). \quad (2.51)$$

By equating the coefficients of x and y to zero of eq. (2.51), we get the nonlinear eigen value as:

$$N = (-k_0 - i\alpha k_0) = X - iY, \quad (2.52)$$

$$X = -k_0, \quad (2.53)$$

$$Y = \alpha k_0. \quad (2.54)$$

Eq. (2.53) and (2.54) is plotted in X, Y plane with spectral curve given by eq. (2.48) and (2.49) in fig. 2.6. To find the soliton curve, we consider the model with local gain saturation not global which usually taken in the spatial domain. The nonlinear operator based on [56] is given as:

$$\hat{N}E = i\nabla^2 E + \Sigma(1 - i\alpha)(n - 1)E. \quad (2.55)$$

The dynamics of n is given as:

$$\partial_t n = -\gamma[n - J + |E|^2(n - 1) + d_e \nabla^2 n]. \quad (2.56)$$

Here, Σ is field-carrier coupling constant, n is electron-hole population, γ is relaxation rate, J is current density, d_e is diffusion strength, there will be stimulated absorption for $n < 1$ and stimulated emission for $n > 1$. Eq. (2.55) is taken in two dimensions. We take the approximation to eq. (2.56) that the diffusion term is neglected and for steady state $\frac{\partial n}{\partial t} = 0$, so we get equation of n as:

$$n = \frac{J-1}{1+|E|^2} \quad (2.57)$$

Using eq. (2.57) in eq. (2.55), we get the nonlinear operator as:

$$\hat{N}E = i\nabla^2 E + \Sigma(1 - i\alpha) \frac{J-1}{1+|E|^2} E \quad (2.58)$$

Using eq. (2.43) and (2.44) in eq. (2.58):

$$\hat{N}E = i \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \exp \left(- \frac{1+ic}{2} \left\{ \left(\frac{x}{x_0} \right)^{2m} + \left(\frac{y}{y_0} \right)^{2m} \right\} \right) e^{-i\Omega t} + \Sigma(1 - i\alpha) \frac{J-1}{1+|S(x,y)|^2} S(x,y) e^{-i\Omega t} \quad (2.59)$$

$$\begin{aligned} \hat{N}E = i \left(- \frac{1+ic}{2} \right) \left\{ (4m^2 - 2m) \left(\frac{x^{2m-2}}{x_0^{2m}} + \frac{y^{2m-2}}{y_0^{2m}} \right) + 4m^2 \left(- \frac{1+ic}{2} \right) \left(\frac{x^{4m-2}}{x_0^{4m}} + \frac{y^{4m-2}}{y_0^{4m}} \right) \right\} S(x,y) e^{-i\Omega t} + \\ \Sigma(1 - i\alpha)(J - 1) \left(1 - \exp \left(- \left\{ \left(\frac{x}{x_0} \right)^{2m} + \left(\frac{y}{y_0} \right)^{2m} \right\} \right) \right) S(x,y) e^{-i\Omega t}. \end{aligned} \quad (2.60)$$

$$N = i \left(- \frac{1+ic}{2} \right) \left\{ (4m^2 - 2m) \left(\frac{x^{2m-2}}{x_0^{2m}} + \frac{y^{2m-2}}{y_0^{2m}} \right) + 4m^2 \left(- \frac{1+ic}{2} \right) \left(\frac{x^{4m-2}}{x_0^{4m}} + \frac{y^{4m-2}}{y_0^{4m}} \right) \right\} + \Sigma(1 - i\alpha)(J - 1) \left(1 - 1 + \right. \\ \left. xx02m+yy02m=X-iY. \right) \quad (2.61)$$

By equating the coefficients of x and y to zero of eq. (2.61), we get the nonlinear eigen value as:

$$N = 0 = X - iY \quad (2.62)$$

Since nonlinear eigenvalue is zero, therefore we do not get any Soliton curve from eq. (2.55) for Super Gaussian wave function.

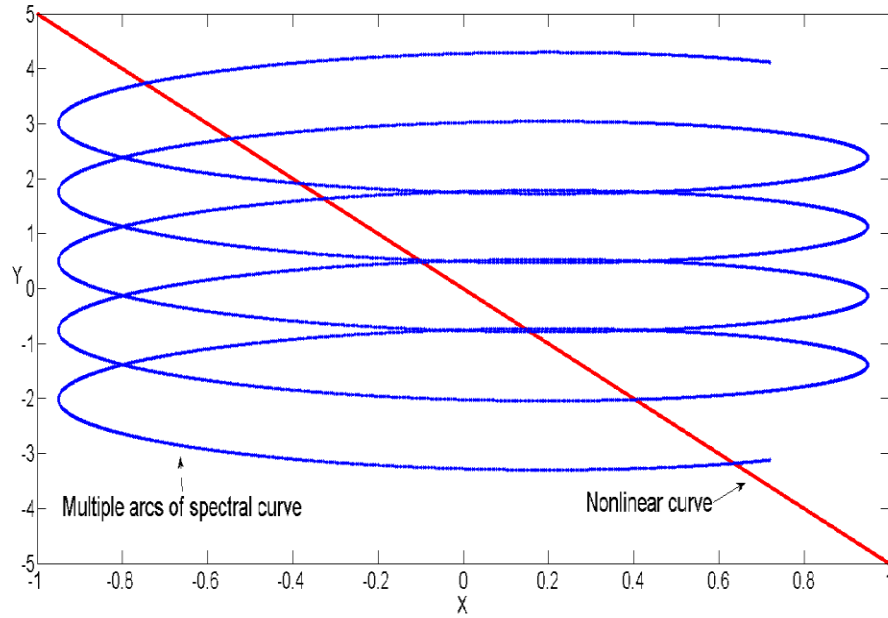


Fig. 2.6: Multiple arcs of spectral curve with 2D Soliton curve

Fig. 2.6 shows that we get multiple arcs of the spectral curve for some particular value of feedback parameters and these arcs of spectral curve intersect the 2D Soliton curve at different points giving rise to solutions. In this case too varying feedback parameter a family of solutions can be obtained.

2.4. CGLE Model for saturable absorber and frequency selective feedback:

We consider an optical cavity in VCSEL with saturable absorber and frequency selective feedback. Rate equation of the slowly varying amplitude (E) of the laser field is given as

$$\partial_t E = \left[-(1 - i\theta) + \frac{\mu(1-i\alpha)}{1+|E|^2} - \frac{\gamma(1-i\beta)}{1+s|E|^2} + i\nabla^2 \right] E + F. \quad (2.63)$$

The feedback is defined by equation:

$$\partial_t F = -(\Gamma_0 + i(\Omega_0 - \eta))F + \sigma^{1/2}E. \quad (2.64)$$

here, μ and γ are pump parameters, θ is detuning due to the medium, α and β are line width enhancement factors for active and passive materials respectively, F is the feedback field, σ is

feedback strength, λ is band-width of the filter reflection. For steady state: $\frac{\partial F}{\partial t} = 0$, using this condition in eq. (2.64), we get

$$-(\Gamma_0 + i(\Omega_0 - \eta))F + \sigma^{1/2}E = 0. \quad (2.65)$$

$$\text{This equation gives the feedback as: } F = \frac{\sigma^{1/2}E}{\Gamma_0 + i(\Omega_0 - \eta)}. \quad (2.66)$$

Using eq. (2.65) in eq. (2.63):

$$\partial_t E = \left[-(1 - i\theta) + \frac{\Re(1 - i\alpha)}{1 + |E|^2} - \frac{\gamma(1 - i\beta)}{1 + s|E|^2} + i\nabla^2 \right] E + \frac{\sigma^{1/2}E}{\Gamma_0 + i(\Omega_0 - \eta)}, \quad (2.67)$$

$$\partial_t E + (1 - i\theta)E - \frac{\sigma^{1/2}E}{\Gamma_0 + i(\Omega_0 - \eta)} = \frac{\Re(1 - i\alpha)E}{1 + |E|^2} - \frac{\gamma(1 - i\beta)E}{1 + s|E|^2} + i\nabla^2 E. \quad (2.68)$$

We consider the solution of problem as:

$$E(x, t) = S(x)e^{-i\eta t}, \text{ where } S(x) = \exp\left(-\frac{1+ic}{2} \left(\frac{x}{x_0}\right)^{2m}\right). \quad (2.69)$$

Separate the eq. (2.68) in linear and nonlinear parts, we get linear and nonlinear operator as:

$$\hat{L}E(x, t) = \partial_t E + (1 - i\theta)E - \frac{\sigma^{1/2}E}{\Gamma_0 + i(\Omega_0 - \eta)} - \mu(1 - i\alpha)E + \gamma(1 - i\beta)E, \quad (2.70)$$

$$\hat{N}E(x, t) = -\mu(1 - i\alpha)|E|^2 E + \gamma(1 - i\beta)s|E|^2 E + i\frac{\partial^2}{\partial x^2} E. \quad (2.71)$$

Using the solution $E(x, t) = S(x)e^{-i\eta t}$ in eq. (2.70)

$$\hat{L}E(x, t) = -i\eta S(x)e^{-i\eta t} + (1 - i\theta)S(x)e^{-i\eta t} - \frac{\sigma^{1/2}S(x)e^{-i\eta t}}{\Gamma_0 + i(\Omega_0 - \eta)} - \Re(1 - i\alpha)S(x)e^{-i\eta t} + \gamma(1 - i\beta)S(x)e^{-i\eta t}. \quad (2.72)$$

On simplifying the eq. (2.72), we get

$$\hat{L}E(x, t) = \left[-i\eta + (1 - i\theta) - \frac{\sigma^{1/2}}{\Gamma_0 + i(\Omega_0 - \eta)} - \Re(1 - i\alpha) + \gamma(1 - i\beta) \right] S(x)e^{-i\eta t} \quad (2.73)$$

From eq. (2.73), we find the linear eigen value given as:

$$L = -i\eta + (1 - i\theta) - \frac{\sigma^{1/2}}{\Gamma_0 + i(\Omega_0 - \eta)} - \mathbb{Q}(1 - i\alpha) + \gamma(1 - i\beta). \quad (2.74)$$

By rearranging the terms of eq. (2.74) and separating the real and imaginary parts, we get

$$L = 1 - \frac{\sigma^{1/2}\Gamma_0}{\Gamma_0^2 + (\Omega_0 - \eta)^2} - \mathbb{Q} + \gamma - i \left(\eta + \theta - \frac{\sigma^{1/2}(\Omega_0 - \eta)}{\Gamma_0^2 + (\Omega_0 - \eta)^2} - \mathbb{Q}\alpha + \gamma\beta \right) = X - iY. \quad (2.75)$$

The eq. (2.75) is plotted in X, Y plane gives spectral curve as shown in fig.2.7 and 2.8. for different values of system parameter.

So we get the real and imaginary parts of eq. (2.75) as:

$$X = 1 - \frac{\sigma^{1/2}\Gamma_0}{\Gamma_0^2 + (\Omega_0 - \eta)^2} - \mathbb{Q} + \gamma, \quad (2.76)$$

$$Y = \eta + \theta - \frac{\sigma^{1/2}(\Omega_0 - \eta)}{\Gamma_0^2 + (\Omega_0 - \eta)^2} - \mathbb{Q}\alpha + \gamma\beta. \quad (2.77)$$

By using the solution given by eq. (2.69) in eq. (2.71), we get

The nonlinear part is given as:

$$\hat{N}E(x, t) = -\mu(1 - i\alpha)|S(x)|^2S(x)e^{-i\eta t} + \gamma(1 - i\beta)s|S(x)|^2S(x)e^{-i\eta t} + i \frac{\partial^2}{\partial x^2} \exp\left(-\frac{1+ic}{2} \left(\frac{x}{x_0}\right)^{2m}\right), \quad (2.78)$$

$$\begin{aligned} \hat{N}E(x, t) = & -\mu(1 - i\alpha) \exp\left(-\left(\frac{x}{x_0}\right)^{2m}\right) S(x)e^{-i\Omega t} + \gamma(1 - i\beta)s \exp\left(-\left(\frac{x}{x_0}\right)^{2m}\right) S(x)e^{-i\Omega t} + \\ & i \left(-\frac{1+ic}{2}\right) \left\{ (4m^2 - 2m) \frac{x^{2m-2}}{x_0^{2m}} + 4m^2 \left(-\frac{1+ic}{2}\right) \frac{x^{4m-2}}{x_0^{4m}} \right\} S(x)e^{-i\Omega t}, \end{aligned} \quad (2.79)$$

$$\begin{aligned} \hat{N}E(x, t) = & \left[-\mu(1 - i\alpha) \exp\left(-\left(\frac{x}{x_0}\right)^{2m}\right) + \gamma(1 - i\beta)s \exp\left(-\left(\frac{x}{x_0}\right)^{2m}\right) + i \left(-\frac{1+ic}{2}\right) \left\{ (4m^2 - \right. \right. \\ & \left. \left. 2mx^{2m-2} - 2x^{4m-2} + 4m^2 - 1 + ic2x^{4m-2} - 2x^{4m} \right\} S(x)e^{-i\Omega t}, \end{aligned} \quad (2.80)$$

$$\hat{N}E(x, t) = \left[-\mu(1 - i\alpha) \exp\left(-\left(\frac{x}{x_0}\right)^{2m}\right) + \gamma(1 - i\beta)s \exp\left(-\left(\frac{x}{x_0}\right)^{2m}\right) + i\left(-\frac{1+i\epsilon}{2}\right) \left\{ (4m^2 - 2m)x^{2m-2} + 2x^{2m-1} + i\epsilon(2x^{4m-2} - 2x^{4m-1}) \right\} E(x, t), \right. \\ \left. (2.81) \right]$$

$$\hat{N}E(x, t) = \left[(-\mu(1 - i\alpha) + \gamma(1 - i\beta)s) \left(1 - \left(\frac{x}{x_0}\right)^{2m}\right) + i\left(-\frac{1+i\epsilon}{2}\right) \left\{ (4m^2 - 2m) \frac{x^{2m-2}}{x_0^{2m}} + 4m^2 - 1 + i\epsilon(2x^{4m-2} - 2x^{4m-1}) \right\} E(x, t), \right. \\ \left. (2.82) \right]$$

Equating the coefficient of x to zero, we get the nonlinear eigen value as:

$$N = (-\mu(1 - i\alpha) + \gamma(1 - i\beta)s) = X - iY, \quad (2.83)$$

$$X = -\mu + \gamma s, \quad (2.84)$$

$$Y = -\mu\alpha + \gamma\beta s. \quad (2.85)$$

Eq. (2.84) and (2.85) is plotted in X, Y plane along with eq. (2.76) and (2.77) as shown in fig. (2.7) and (2.8) for two different values of Γ_0 .

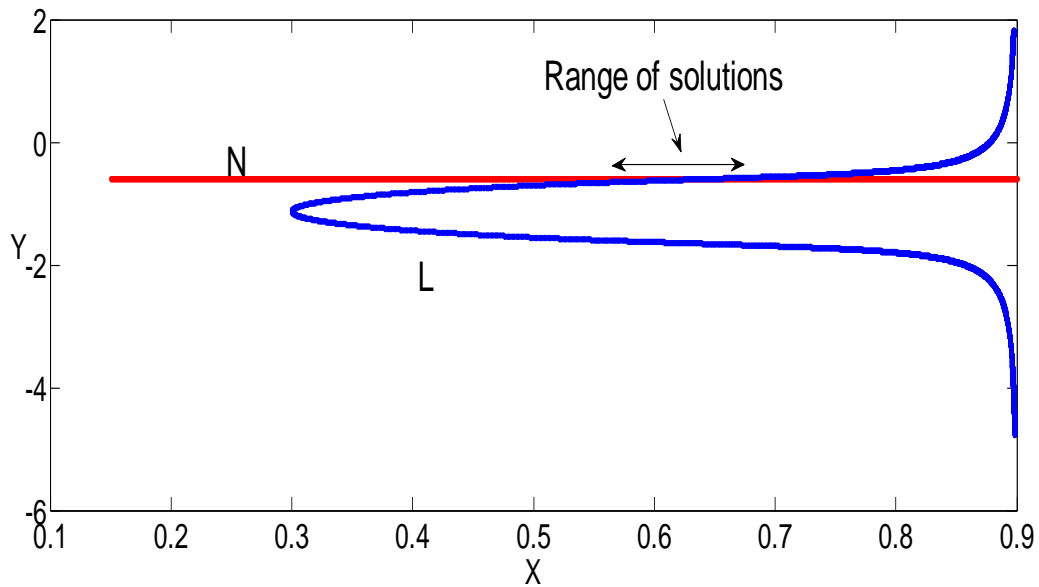


Fig.2.7: Spectral (L) and Soliton (N) lines in the complex eigen value plane for CGL3 with saturable absorber and frequency selective feedback (for $\Gamma_0 = 0.2$)

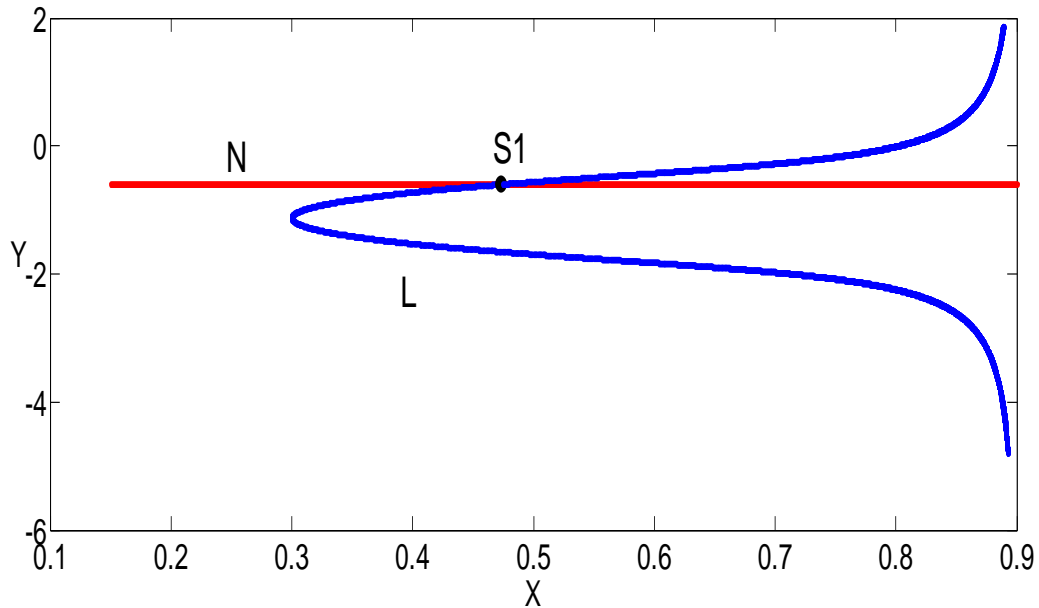


Fig.2.8: Spectral (L) and Soliton (N) lines in the complex eigen value plane for CGL3 with saturable absorber and frequency selective feedback (for $\Gamma_0 = 0.4$)

Figs. (2.7) and (2.8) shows the Soliton and spectral curves for different values of feedback parameter Γ_0 , i.e., for $\Gamma_0 = 0.2, 0.4$ respectively. We get one solution point. A typical example is given for $\Gamma_0 = 0.4$. Exciting result is obtained for $\Gamma_0 = 0.2$; wherein spectral and Soliton curve intersect over a wide range giving rise to a range of solutions. Commonly, the solution points of cavity solitons are discrete due to the two dimensional balance, i.e., broadening–compressing and loss-gain. But in this case unusually a long line of solution is obtained, which quite resembles with conservative soliton case. This finding will be very helpful for the further experimental investigation as it gives wider option to the experimentalist.

Chapter 3

3.1 Cavity Soliton with CGLE Model for cosh-Gaussian profile:

We now explore the possibility of cavity soliton formation with cosh-Gaussian profile. Two decentered Gaussian profiles combined together can give rise to a cosh-Gaussian profile. This is a bell shaped profile with a central deep. With increasing cosh-Gaussian parameter the central deep of the profile increases. Thus this profile promises a very versatile family of cavity solitons. A variety of cosh-Gaussian profile is given below in fig. (3.1).

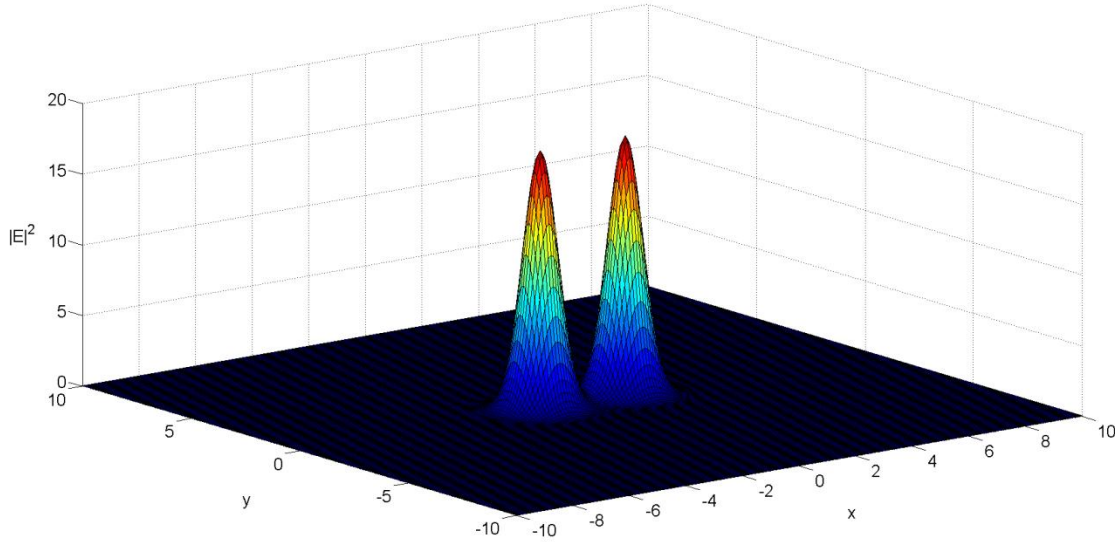


Fig. 3.1: Typical cosh- Gaussian profile in 3D

We consider an optical cavity in VCSEL with saturable absorber and frequency selective feedback. Rate equation of the slowly varying amplitude (E) of the laser field is given by eq.(29) as:

$$\partial_t E = \left[-(1 - i\theta) + \frac{\gamma(1-i\alpha)}{1+|E|^2} - \frac{\gamma(1-i\beta)}{1+s|E|^2} + i\nabla^2 \right] \quad (3.1)$$

The feedback is defined by equation:

$$\partial_t F = -(\Gamma_0 + i(\Omega_0 - \eta))F + \sigma^{1/2}E, \quad (3.2)$$

where, all parameters are defined as in chapter 2.

For steady state $\frac{\partial F}{\partial t} = 0$, using in eq. (3.0), we get

$$-(\Gamma_0 + i(\Omega_0 - \eta))F + \sigma^{1/2}E = 0. \quad (3.3)$$

$$\text{This equation gives the feedback as } F = \frac{\sigma^{1/2}E}{\Gamma_0 + i(\Omega_0 - \eta)}. \quad (3.4)$$

Using eq. (3.4) in eq. (3.1):

$$\partial_t E = \left[-(1 - i\theta) + \frac{\mu(1 - i\alpha)}{1 + |E|^2} - \frac{\gamma(1 - i\beta)}{1 + s|E|^2} + i\nabla^2 \right] E + \frac{\sigma^{1/2}E}{\Gamma_0 + i(\Omega_0 - \eta)}, \quad (3.5)$$

$$\partial_t E + (1 - i\theta)E - \frac{\sigma^{1/2}E}{\Gamma_0 + i(\Omega_0 - \eta)} = \frac{\mu(1 - i\alpha)E}{1 + |E|^2} - \frac{\gamma(1 - i\beta)E}{1 + s|E|^2} + i\nabla^2 E. \quad (3.6)$$

We consider the solution of problem as described in eq. (3.1) as:

$$E(x, t) = S(x)e^{-i\eta t}, \text{ where, } S(x) = A(\cosh(\Omega x))^{1+ic} \exp\left(-\frac{x^2}{2x_0^2}\right), \quad (3.7)$$

Where, x_0 is inverse width, Ω is cos-hyperbolic parameter and c is chirp parameter.

Separating the eq. (3.6) in linear and nonlinear parts, we get linear and nonlinear operator as:

$$\hat{L}E(x, t) = \partial_t E + (1 - i\theta)E - \frac{\sigma^{1/2}E}{\Gamma_0 + i(\Omega_0 - \eta)} - \mu(1 - i\alpha)E + \gamma(1 - i\beta)E, \quad (3.8)$$

$$\hat{N}E(x, t) = -\mu(1 - i\alpha)|E|^2E + \gamma(1 - i\beta)s|E|^2E + i\frac{\partial^2}{\partial x^2}E. \quad (3.9)$$

Using the solution $E(x, t) = S(x)e^{-i\eta t}$ in eq. (3.8)

$$\hat{L}E(x, t) = -i\eta S(x)e^{-i\eta t} + (1 - i\theta)S(x)e^{-i\eta t} - \frac{\sigma^{1/2}S(x)e^{-i\eta t}}{\Gamma_0 + i(\Omega_0 - \eta)} - \mu(1 - i\alpha)S(x)e^{-i\eta t} + \gamma(1 - i\beta)S(x)e^{-i\eta t}. \quad (3.10)$$

On simplifying the eq. (3.10), we get:

$$\hat{L}E(x, t) = \left[-i\eta + (1 - i\theta) - \frac{\sigma^{1/2}}{\Gamma_0 + i(\Omega_0 - \eta)} - \mathbb{Q}(1 - i\alpha) + \gamma(1 - i\beta) \right] S(x) e^{-i\eta t}. \quad (3.11)$$

From eq. (3.11), we find the linear eigen value given as:

$$L = -i\eta + (1 - i\theta) - \frac{\sigma^{1/2}}{\Gamma_0 + i(\Omega_0 - \eta)} - \mathbb{Q}(1 - i\alpha) + \gamma(1 - i\beta). \quad (3.12)$$

By rearranging the terms of eq. (3.12) and separating the real and imaginary parts, we get

$$L = 1 - \frac{\sigma^{1/2}\Gamma_0}{\Gamma_0^2 + (\Omega_0 - \eta)^2} - \mathbb{Q} + \gamma - i \left(\eta + \theta - \frac{\sigma^{1/2}(\Omega_0 - \eta)}{\Gamma_0^2 + (\Omega_0 - \eta)^2} - \mathbb{Q}\alpha + \gamma\beta \right) = X - iY. \quad (3.13)$$

The eq. (3.14) is plotted in X, Y plane gives spectral curve as shown in fig.3.2.

So we get the real and imaginary parts of eq. (3.13) as:

$$X = 1 - \frac{\sigma^{1/2}\Gamma_0}{\Gamma_0^2 + (\Omega_0 - \eta)^2} - \mathbb{Q} + \gamma, \quad (3.14)$$

$$Y = \eta + \theta - \frac{\sigma^{1/2}(\Omega_0 - \eta)}{\Gamma_0^2 + (\Omega_0 - \eta)^2} - \mathbb{Q}\alpha + \gamma\beta. \quad (3.15)$$

By using the solution given by eq. (3.7) in eq. (3.9), we get

The nonlinear part is given as:

$$\begin{aligned} \hat{N}E(x, t) = & -\mu(1 - i\alpha)|S(x)|^2 S(x) e^{-i\eta t} + \gamma(1 - i\beta)s|S(x)|^2 S(x) e^{-i\eta t} + i \frac{\partial^2}{\partial x^2} A(\cosh(\mathbb{Q}\Omega x))^{1+ic} \exp\left(-\frac{x^2}{2x_0^2}\right) \\ & (3.16) \end{aligned}$$

$$\begin{aligned} \hat{N}E(x, t) = & -\mu(1 - i\alpha) A^2 \cosh^2(\Omega x) \exp\left(-\frac{x^2}{x_0^2}\right) S(x) e^{-i\eta t} + \gamma(1 - i\beta) s A^2 \cosh^2(\Omega x) \exp\left(-\frac{x^2}{x_0^2}\right) e^{-i\eta t} + \\ & i \left[A(1 + ic)^2 \Omega^2 (\tanh(\Omega x))^2 + A(1 + ic) \Omega^2 - A(1 + ic) \Omega^2 (\tanh(\Omega x))^2 - 2A(1 + \right. \\ & \left. ic) \Omega x \frac{\tanh(\mathbb{Q}\Omega x)}{x_0^2} - \frac{A}{x_0^2} + A \frac{x^2}{x_0^4} \right] S(x) e^{-i\eta t}, \quad (3.17) \end{aligned}$$

$$\hat{N}E(x, t) = \left[(-\mu(1 - i\alpha) + \gamma(1 - i\beta)s)A^2 \cosh^2(\Omega x) \exp\left(-\frac{x^2}{x_0^2}\right) + i \left(A(1 + ic)^2 \Omega^2 (\tanh(\Omega x))^2 + A(1 + ic)\Omega^2 - A(1 + ic)\Omega^2 (\tanh(\Omega x))^2 - 2A(1 + ic)\Omega x \tanh(\Omega x) \right) \right] E(x, t), \quad (3.18)$$

$$\hat{N}E(x, t) = \left[(-\mu(1 - i\alpha) + \gamma(1 - i\beta)s)A^2 \cosh^2(\Omega x) \exp\left(-\frac{x^2}{x_0^2}\right) + i \left(A(1 + ic)^2 \Omega^2 (1 + \operatorname{sech}^2(\Omega x)) + A(1 + ic)\Omega^2 - A(1 + ic)\Omega^2 (1 + \operatorname{sech}^2(\Omega x)) - 2A(1 + ic)\Omega x \tanh(\Omega x) \right) \right] E(x, t). \quad (3.19)$$

In eq. (3.19), by equating the coefficients of x to zero, the remaining terms give us nonlinear eigen value as:

$$N = i \left(A(1 + ic)^2 \Omega^2 + A(1 + ic)\Omega^2 - A(1 + ic)\Omega^2 - \frac{A}{x_0^2} \right), \quad (3.20)$$

$$N = i \left(A(1 + ic)^2 \Omega^2 - \frac{A}{x_0^2} \right) = X - iY. \quad (3.21)$$

On comparing the real and imaginary parts of eq. (3.21), we get

$$X = -2Ac\Omega^2, \quad (3.22)$$

$$Y = (-1 + c^2)A\Omega^2 + \frac{A}{x_0^2}. \quad (3.23)$$

On plotting the equations (3.22) and (3.23) in X, Y plane, we get the soliton curve which intersects the spectral curve given by eq. (3.14) and (3.15) at different points for different value of parameter Ω of cosh-Gaussian profile as shown in fig. (3.2).

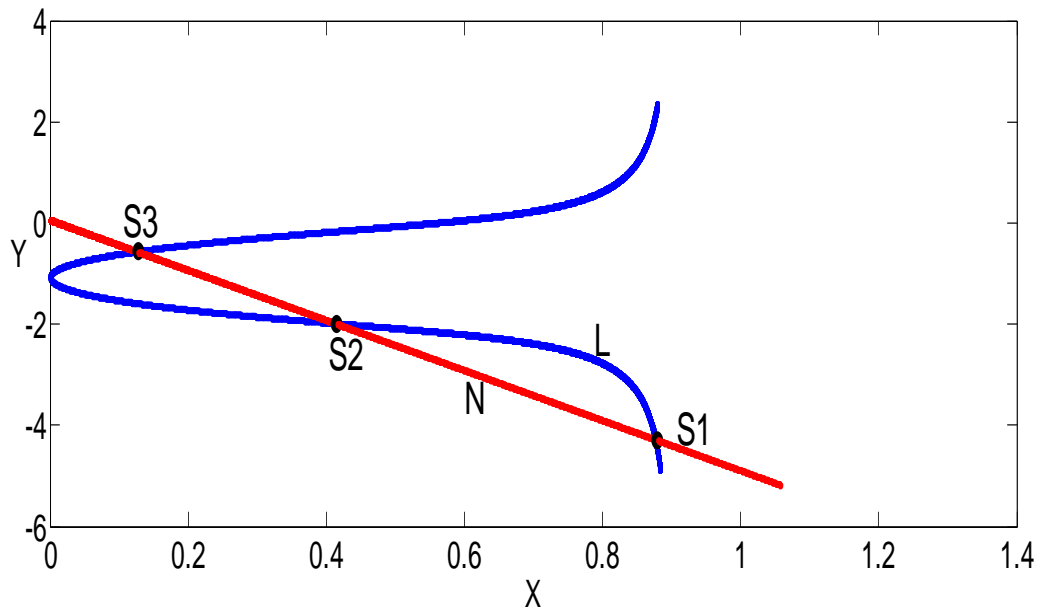


Fig.3.2: Spectral (L) and Soliton (N) curves in the complex eigenvalue plane with frequency selective feedback and saturable absorber for $\Omega = 0$ to 2.3

In fig. (3.2), spectral and Soliton curve intersects at three points giving rise to three solutions S1, S2, S3 for a particular range of Ω . Varying the system parameters a family of solutions has been obtained.

Dynamics of Cavity Soliton:

As described in above model 3.1, we get different linear and nonlinear curves which intersect at different points giving rise to different solutions. Now we have to check whether these solutions are stable or unstable through plotting and analyzing snaking curve. Fig. (3.3) shows snaking curve of above model (3.1) for three solutions S1, S2 and S3. By changing the value of parameter σ , different set of solutions are obtained for different range of η . By plotting σ vs. η , we get the curve as shown in fig. (3.3). The ziz-zag nature of plot gives rise to the name snaking.

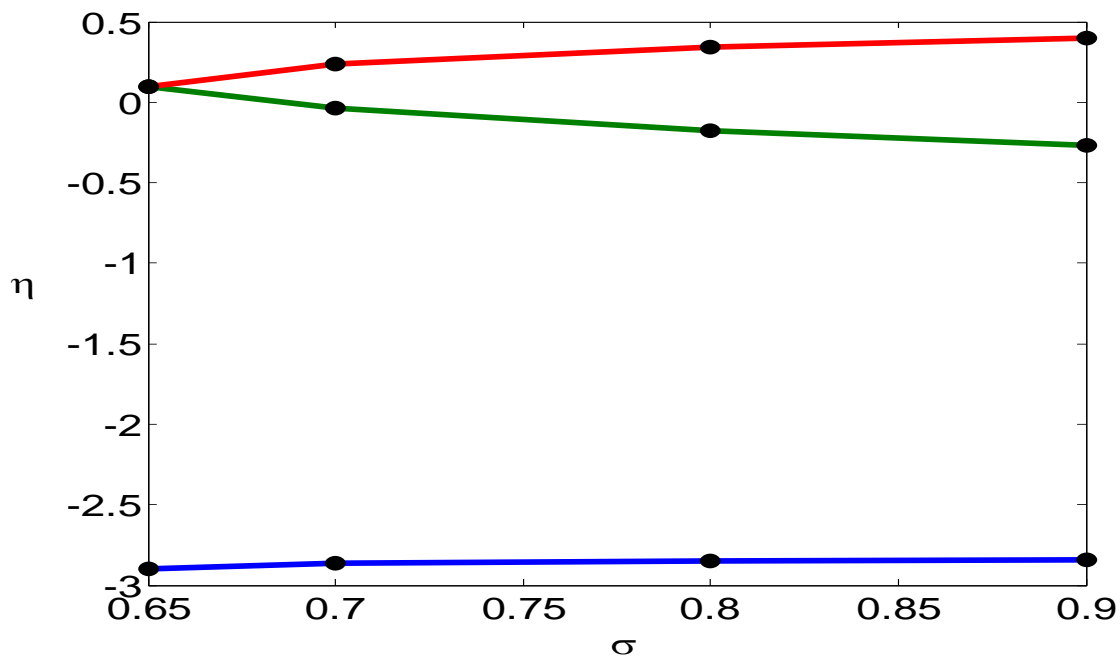


Fig. 3.3: Snaking of the curve obtained in above model (σ vs η)

From the snaking curve it is evident that, for a particular value of σ multiple values (here three) of η is possible. That means multi stable solitons are there in the system. Numerical simulation confirms the stability of the solutions.

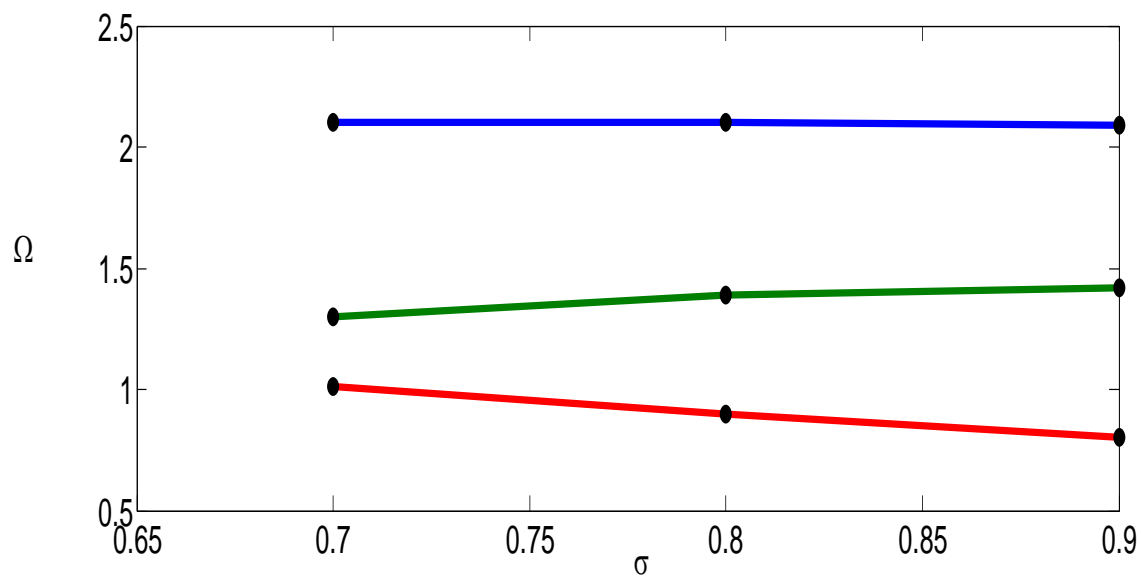


Fig. 3.4: Snaking of the curve obtained in above model (σ vs Ω)

Fig. (3.3) shows snaking of above model (3.1) for three solutions S1, S2 and S3. By changing the value of parameter σ , different set of solutions are obtained for different range of Ω . By plotting σ vs. Ω , we get the snaking curve as shown in fig. (3.3). This plot too shows the multistable nature of the cavity soliton.

Conclusions:

We have investigated Cubic Ginzburg-Landau laser model with and without feedback for Super Gaussian profile. Then we take this model for saturable absorber and frequency selective feedback for Super Gaussian and cosh-Gaussian trial function. We have solved all these models by separation method as discussed in chapter 1. By solving these models, we get different equations of linear and nonlinear parts. We have plotted these equations in X, Y eigen value plane to find different sets of solutions for different parameters of trial function. We get a number of results which are briefly discussed as:

1. For CGL model without feedback and with frequency selective feedback, we get one solution family of Super Gaussian profile.
2. For saturable gain and multidimensional model we get a large family of multidimensional (2D) cavity soliton of Super-Gaussian profile.
3. For CGLE model with saturable absorber and frequency selective feedback, a family of Super-Gaussian cavity soliton has been obtained.
4. For CGLE model that describes VCSEL with saturable absorber and frequency selective feedback, we get cavity soliton of cosh-Gaussian profile. These solitons are versatile in nature (profile) and multistable.

The cavity Soliton thus achieved has potential applications in all-optical erasable memory device, all-optical data storage, delay lines, all-optical push broom and multi-level logic systems.

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