Reliability, Availability and Maintainability (RAM) Analysis of an Industrial Process

A THESIS

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MATHEMATICS

By

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Candidate's Declaration

I, hereby certify that the work being presented in the thesis entitled **Reliability, Availability and Maintainability (RAM) Analysis of an Industrial Process,** in fulfillment of the requirements for the award of the degree of Doctor of Philosophy, submitted in the School of Mathematics and Computer Applications (SMCA) of Thapar University, Patiala, is an authentic record of my own work carried out during a period from June, 2008 to December 2013 under the supervision of Dr. A. K. Lal, Dr. S. S. Bhatia and Dr. A.S. Reddy.

The matter embodied in this thesis has not been submitted by me for the award of any other degree.

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ABSTRACT

The contribution made in this thesis is helpful to manufacturing industries for studying the performance of industrial process. The design of manufacturing process, its maintenance and satisfactory performance play vital role in order to produce reliable product. In order to analyse the performance of industry, a methodical scheme based on theory of Boolean algebra and Markov model has been developed in the present work. In addition, this thesis introduces a new numerical approach for evaluating transient computation of Markovian system of equations that are obtained by using supplementary variable technique. This approach incorporates different numerical methods including Finite Difference Methods, Lagrange's interpolation method and Simpson's one-third method. Further, an optimal maintenance-scheduling model has been developed by giving constraints on availability and cost of maintenance to find the optimal maintenance rates of industrial manufacturing machines or system when they underwent through preventive and corrective maintenance action.

The proposed methodology is demonstrated on a fabric industry. From this application, it is concluded that mixer and slasher modules are sensitive towards the performance of industry. Further, systems of these modules have been analyzed by using the maintenance model in order to study maintenance rates of units of these systems.

This research work started with following three research objectives for studying the performance of the manufacturing process:

1. To conceptualize a RAM model for the selected industrial process/system
2. To transform the conceptualized RAM model into a quantitative and simulation ready model
3. To assess the influence of maintenance on the industrial process/system's reliability and availability through simulation

Keeping these objectives in view, the research work has been carried out. The whole research work is divided into seven chapters. The chapter wise summary of the thesis is as follows:

Chapter 1

Chapter 1 is introductory. In this chapter, apart from preliminary concepts to be used in the sequel, the literature on performance analysis of the systems along with a brief plan of the results presented in the subsequent chapters has been discussed.

Chapter 2

In this chapter, a methodical scheme is proposed to compute performance metric of the industrial process. This scheme utilizes the concepts of Boolean algebra and stochastic process, and is referred as Assiduity Progression Diagnosis (APD). The proposed scheme overcomes the limitation of stochastic models in which state space grows beyond certain limits while implementing on industrial process. Due to which number of equations in the system of differential equations, governing the performance of the industry, increases abruptly and it becomes a difficult task to compute performance parameters. This scheme helps the manufacturing industries to predict Reliability, Availability and Maintainability (RAM) of industrial process. The model thus developed helps to determine reliability, three kinds of availability (steady state, time dependent and inherent availability), maintainability, average production and other related metrics of the industrial process by using state probabilities efficiently. The proposed model has finally been implemented to the fabric industry for its performance analysis and has later been discussed in Chapter 5. The contents of this chapter meet our first objective.

Chapter 3

This chapter is in concerned with performance analysis of an industrial system where stochastic models are developed by using the supplementary variable technique (SVT) for a system having constant and variable transient rates. This technique can be applied to the RAM model (APD, as discussed in Chapter 2) when failure and repair rates of modules of an industry are variable. However, in this case the performance metrics such as reliability, time dependent availability and mean time to failure of process will not be computed from resulting mathematical model. In fact, the analytical methods such as Laplace Transform method, Lagrange's method and separation of variable method have been used to solve the resultant system of simultaneous linear partial differential equations but analytical solution is so intricate that the industry persons will not be able to use them conveniently. In order to overcome this situation, a numerical technique is developed in this chapter to solve such a complex mathematical problem. The proposed numerical approach consists of three numerical methods (finite difference schemes, Simpson's one-third method and Lagrange's interpolation) which iteratively compute the transient solution of the Markovian model taken under consideration. This hybrid method is named as Lagrange Finite Difference Simpson Method (LFDSM). The contents of this chapter meet the second research objective.

Chapter 4

Three important criteria: (i) minimum replacement cost rate (ii) maximum availability and (iii) lower bound on the mission reliability are frequently used in literature for scheduling maintenance of an industrial system. By selecting anyone of these criteria, the other two criteria are generally ignored in the case of system maintenance. This ignorance may not provide flexibility for the decision makers to finalize the maintenance scheduling of the system. In this chapter, the problem of maintenance scheduling of the industrial system in terms of optimal failure and repair rates is considered. An optimization model is presented to maximize the total profit from the output of five units working system by imposing constraints on availability and maintenance cost of the system. The reliability criteria can also be used in this model under some conditions. This model is developed by taking into account the concept of stochastic process based on supplementary variable technique and optimization theory. A decision strategy, using the maintenance cost constraint, has next been proposed for a decision maker. The application of the proposed model is finally demonstrated on two industrial systems and has been discussed in Chapter 6. The contents of this chapter fulfil the third research objective.

Chapter 5

This chapter deals with the application of the APD model developed in Chapter 2 on a fabric industry to analyze its performance analysis. Certain parameters such as reliability, time dependent availability, steady state availability, maintainability, and average production metrics are used as performance measures for overall behavior of fabric industry to achieve desired production targets. The behavior analysis of APD model reveals that the mixer and slasher system of fabric industry are the most sensitive and affect more to the overall performance of the industry. A comparative study has also been presented in this Chapter between the APD model and the traditional approach for evaluating the performance of industry by using stochastic models.

Chapter 6

In this chapter, the optimal results for failure and maintenance rates for only mixer and slasher systems have been obtained by using the optimal maintenance scheduling methodology as discussed in Chapter 4. Further, in this chapter we have also discussed that how the steady state behavioral analysis can be used instead of transient analysis for studying optimal maintenance scheduling of these systems.

Chapter 7

In this concluding chapter, limitation and scope of the methodologies developed in this thesis to study RAM analysis of selected industrial process has been discussed. The industrial significance of the results as well as a scope for further work on this topic are also presented in the concluding chapter.

RESEARCH PUBLICATIONS

Papers published in journals:

1. Manwinder Kaur, Arvind Kumar Lal, S. S. Bhatia and A.S. Reddy, "Numerical solution of stochastic partial differential difference equation arising in reliability engineering" *Applied Mathematics and Computation*, 219(14) : 7645:7652, 2013. *(SCI-Impact factor 1.349)*
2. Manwinder Kaur, Arvind Kumar Lal and S. S. Bhatia, "Reliability distribution of an industrial process" *International Journal of Research in Advent Technology,* 1(5): 587-592, 2013.
3. Manwinder Kaur, Arvind Kumar Lal, S. S. Bhatia and A.S. Reddy, "On use of corrective maintenance data for performance analysis of preventively maintained textile industry" *Journal of Reliability and Statistical Studies*, 6(2):51-63, 2013.
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1. Manwinder Kaur, Arvind Kumar Lal, S. S. Bhatia and A.S. Reddy, "Optimal maintenance scheduling of textile weaving Process", DST sponsored National Meet of Research Scholars in Mathematical Sciences held at IIT Roorkee, Dec 19-23, 2009. (This was awarded as best poster presentation)
2. Manwinder Kaur, Arvind Kumar Lal, S. S. Bhatia and A.S. Reddy, "Reliability analysis of preventively maintained system using finite element method", International Congress of Mathematicians-2010 held at Hyderabad, Aug 19-27, 2010, pp 213.
3. Manwinder Kaur, Arvind Kumar Lal, S. S. Bhatia and A.S. Reddy, "Reliability, availability and maintainability (RAM) model for industrial process", 14th Punjab Science congress held at SLIET, Longowal Feb. 7-9, 2011, pp 166.
4. Manwinder Kaur, Arvind Kumar Lal, S. S. Bhatia and A.S. Reddy, "Performance analysis of industrial System under corrective and preventive maintenance", International Conference of Quality and Reliability Engineering held at Bangalore, Dec 20-22, 2011, pp. 77.

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2. Manwinder Kaur, Arvind Kumar Lal, S. S. Bhatia and A.S. Reddy, "Optimal maintenance rates top maximize profit of three unit system" communicated with a refereed journal.

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NOTATIONS

a	:	acceptable value of availability for optimal maintenance model
b	:	constant probability that preventive maintenance is carried out unsatisfactorily (i.e. $b=1$) and this leads to system to failed state immediately. On the other hand, if the preventive maintenance carried satisfactorily then $b=0$
c	:	z % of total profit obtained from the system working output. Here, z can be any arbitrary number, which is set by industry for covering maintenance cost of the system
m	:	total number of modules
g	:	good state of the unit for idle system
o	:	operating state
cm	:	corrective maintenance condition of unit
pm	:	preventive maintenance condition of unit
Ω	:	state space of the stochastic process
$N(t)$	:	state of the system or process at time t.
$R(t)$	:	reliability of the process or system
$A(t)$	:	time dependent availability of the process or system at time t
$M(t)$	:	maintainability of the process
$X(t)$	:	supplementary variable, for time already spent in maintenance service by time t of a unit receiving service.
$Y(t)$	:	supplementary variable, for time already spent in operating state by time t for a unit without failing the system
$P_i(t)$	:	probability of being in i^{th} state of the transition diagram at time t
A_∞	:	long-run availability or steady state availability of the process
A_i	:	inherent availability of the process
C_r	:	revenue cost of the system
C_i	:	preventive maintenance cost (per unit time) of j^{th} state, for $j = 1,2,3$ and corrective maintenance cost (per unit time) of j^{th} state, for $j = 4,5, 6$

λ_i	:	failure rate of the i^{th} module
T_i	:	the time needed for i^{th} module repairs
u_{ij}	:	the i^{th} unit of the system in j^{th} state, j can be operating (o) or failed (f) state
Z_1	:	Boolean variable represents the state of the k-out-of-n systems working within a module. If at least k systems are working then it is assumed as true (1) otherwise false (0)
Z_2	:	Boolean variable represents the production of required items based on availability of stored items for a module. This factor represents false value when a module fails to produce required items through raw material availability
α	:	constant failure rate of one unit system
α_i	:	constant failure rate of i^{th} unit of the system, $i = 1, 2, 3$
$\beta(x)$	:	variable repair rate of one unit system
$\beta_i(x)$	:	transition (downtime) rate for i^{th} preventive maintenance state of the system with elapsed repair time x, $i = 1, 2, 3, 4, 5$
$\eta_i(x)$	:	corrective maintenance repair (failure replacement) rate of the i^{th} state of the system having elapsed repair time x, $i = 1, 2, 3, 4, 5$
$\gamma_i(y)$	:	the first order probability that the unit transit from normal to failed state in the interval $(y, y+\Delta)$, conditioned that it has not failed up to time y, $i = 4, 5, 6, 7$
L_i, U_i	:	lower and upper limit of $\frac{\alpha_i}{\beta_i(x)}$,respectively
l_i, u_i	:	lower and upper limit of $\frac{\gamma_i(y)}{\eta_i(x)}$, respectively
$P_0(t)$	:	probability that system is working with full capacity represented by state zero
$P_1(x,t)$	:	probability that the system is in state one, which signifies repair state of the system at time 't' with elapsed repair time x
$P_{i+3}(y,x,t)$	:	probability that system is in state i at time t such that system has elapsed failure time y and elapsed repair time x due to unit Z where $i = 1, 2, 3$ *and* $Z = A_1, A_2, A_3$
$\lim_{t\to\infty} P_i(t)$	:	P_i

ABBREVIATIONS

APD	:	Assiduity Progression Diagnosis
BDM	:	Backward Difference Method
CTMC	:	Continuous Time Markov Chain
CSSF	:	Cost Spent on System Functioning (Maintenance cost)
DTMC	:	Discrete Time Markov Chain
EP	:	Estimated Production
FMEA	:	Failure Mode Effect Analysis
FTA	:	Fault Tree Analysis
fixed cost	:	Indirect and direct cost involve in maintenance
GAA	:	Geometric Average Approach
LFDSM	:	Lagrange Finite Difference Simpson Method
LRA	:	Long Run Availability
MTTF	:	Mean time to (process) failure
MTTR	:	Mean time to (process) repair
RAM	:	Reliability, Availability, and Maintainability
RBD	:	Reliability Block Diagram
SVT	:	Supplementary Variable Technique
TDA	:	Time Dependent Availability
TP	:	Total profit per unit of time of three-unit system operating

Chapter 1
INTRODUCTION

The main aim of the manufacturing industries is to produce reliable products that work without failure under stated conditions. The scheduled commitments of producing goods can only be accomplished if the manufacturing processes are efficient and, machinery and facilities are maintained for continued availability. Performance study of manufacturing industries is the need of the time to improve production process. Billions of bucks, which are lost due to negligence of maintenance, can be saved by analyzing performance study well in time. Negligence can serve as costly affair such as loss of space shuttle Columbia, the chemical spills at Bhopal (India) and the recent electricity outage in North America. Three behavioral metrics: reliability, availability and maintainability are used in general for studying the performance of production systems. The same metrics can also be used to analyze the performance of the process as well. These metrics frequently appear in the field of reliability engineering.

1.1. Scope of Present Work

The research in the field of reliability engineering for performance analysis of systems has abruptly increased [1-15, 17-30, 32-132, 134-136, 138-143 and 146-154] since 1980. The main reason of this development is a high concern of the manufactures towards company's reputation in the market as customer demand good quality and long life usability product. But the quality of product cannot be improved just by improving the raw material of manufactured items but it also depends on various factors. Some of these factor are, design of manufacturing process and its improvement afterwards with advent technology, the availability of machines to meet the demand, quantity of reliable products produced in a given time interval and the maintenance of the manufacturing machine for better performance. A good quality product of any industry improves the standard of business and helps the nation in general to make a future business deals and thus in the interest of economic growth of a country the importance of performance analysis of industrial process has increased significantly. This study is contributed to develop a reliability, availability and maintainability model for industrial process.

1.2. Literature review

Reliability engineering is a collaborative field of physical sciences and various disciplines of engineering. The research work within each engineering disciplines (computer science [89], mechanical and electronic system engineering [131] and related engineering [102]) varies from branch to branch.

Reliability was considered as a vital concept during the world war second. In this direction, the initial steps were taken by establishing Joint Navy and army groups of USA in June 1943. In 1950, an adhoc committee was constituted to improve the reliability of general equipments by the Department of Defense, USA. And later in 1952, this department constituted a permanent group called as Advisory Group on the Reliability of Electronic Equipment (AGREE). In 1957, AGREE published a document on the reliability of electronic equipments used by military. The recommendations of this report of AGREE on reliability were adopted by National Aeronautics and Space Administration (NASA) and number of other high profile supplier's contractors and purchasers of equipment involving high technology. Further, the department of defense in 1965 issued reliability programs for systems and equipment, which were revised in 1980. After that, the need of reliability in many products and systems has been emphasized and a number of areas have been identified. From initial research in reliability studies, it has been found that the size of the system, intricacy of the specified functions, the length of the useful interval of life variable and the degree of hostility of system's environment, influence the efficient performance of system [2]. Many mathematical models have been developed to help the manufacturer to make efficient systems by developing reliable components and parts.

Keeping in view the outcome of this initial research of this field, reliability engineering can be divided into following categories:

1. Testing procedures [2]
2. System improvement techniques [38, 76, 148, 152, 153]
3. Theoretical maintenance models [90]
4. Failure analysis [18]
5. Risk and safety analysis of system [4]
6. Performance studies of systems [43, 45, 51, 52, 62, 68, 82, 86, 91, 124]
7. Life models of system [110]

Some of these categories, which have mostly been observed and used for system engineering, are discussed as follows:

The study of failure data of the system plays a major role in the reliability engineering. Various statistical techniques have been applied on the failure data to provide logic based conclusions. In 1952, Davis [17] summarized the statistical techniques for analyzing failure data obtained from operations performed by machines and people. His initiatives later appeared as failure distribution of systems and were presented in the work Lie and Chun [79]. Mendenhall and Harder [84] later discussed the role of failure time distributions. The failure data then was classified into sudden and delayed failure times as discussed in Kao [76]. Various failure distributions widely used in this field are: exponential distribution [79], Weibull mixture distribution [13] and Rayleigh distribution [111]. The concepts for calculating frequency of system failure are discussed by Singh and Patton [125].

The studies of failure data analysis are further extended with the evaluation of performance parameters using the failure data of systems. Various studies are available in the literature which make use of reliability, availability and different measures to evaluate the system efficiency. The evaluations of system performance are mostly studied by using the reliability or availability parameters. In 1961, Barlow and Hunter [8] analyzed the reliability of one unified system. Then, Bellmore and Jensen [58], Osaki [93], Nakagawa [90], Aggarwal *et al.* [2], Dhillon and Rayapati [24] presented different approaches for analyzing system reliability. Page and Perry [96] extended work on reliability by presenting failure causes while evaluating system performance.

The above studies of performance parameters have been further extended by evaluating reliability or availability for different kinds of systems such as for series and parallel network systems. Kodama [61] did the probabilistic analysis of a multicomponent series-parallel system. Flehinger [32] used redundancy technique for improving system reliability. Zacks [153] in his book presented different probability and statistical methods for estimating and predicting reliability and availability. In 1975, Barlow and Proschan [9] introduced an excellent book, which deals with probabilistic models connecting lifetime of complex systems for reliability studies. Proschan and Esary [30] showed the relationship between a system and component failure rates on performance with mathematical concept. Later, Gaver [42] discussed the redundant repairable systems by using exponential failure and repair times. Effect of stand by redundancy on the system has been discussed by Kapur and Kapoor [59] using markovian approaches. Osaki [94] and Singh [126] explained redundancy for switch over devices to improve the performance. Further, time redundancy for system improvement has been discussed by Krishana and Singh [63]. Azaron *et al.* [6] presented stand by redundancy for time dependent systems. Here, redundancy for a system means that if a system fails during the operating conditions then a stand by system starts working to keep the system in operating condition.

Maintenance models play an important role in reliability engineering in order to improve the performance of system. In this regard, various authors have presented different types of maintenance policies. Flehinger [33] proposed a general model for maintained systems while Osaki and Asakurae [95] discussed the preventive maintenance policy. Lie and Chun [79] developed new method for preventive maintenance policy. Wortman *et al.* [149] presented maintenance strategy for deteriorating systems. Smith [128] presented impact of preventive maintenance on plant availability. Smith and Dekker [129] studied preventive maintenance policy in 1-out-of-*n* unit system under the uptime and downtime costs. Sandoh *et al.* [110] proposed a new discrete preventive maintenance policy method. Most of these studies considered cost parameters as a major measure for maintenance theory [23, 33, 40, 67, 79, 90, 95, 98, 128, 129, 134, 135, 139-143, 146-152].

Besides these factors as discussed above, risk and safety are the other factors, which come under the performance studies for improving systems to provide safe environment during system use. Some techniques that are found in relation to these factors are Fault Tree Analysis (FTA) [66, 68, 86, 124], Failure Mode and Effect Analysis (FMEA) [18, 28, 82]. These techniques help in studying the failures, their causes, corresponding effects on environment and people around, and for preventing the failures in future before occurrence by giving remedial measures. These also help the manufacturer to assign warning signs during product usage, which in turn helps its client to be aware of the dangers from any hazardous affects. Other than these, many testing procedures that have come into existence for improving the product performances during design phase [2].

It is apparent from the Table 1.1 given below that the stochastic models have high degree of acceptance for studying performance analysis of system [1, 3, 5, 11, 21, 22, 26, 27, 29, 33-41, 43-56, 60, 61, 64-67, 69-75, 78, 91-93, 98-101, 103-108, 112-121, 127, 130, 131, 133-135, 138-152, 154]. In order to carry out the performance analysis by using this approach, a mathematical model undergoes transitions from one state to another state and then the Chapman-Kolmogorov differential equations associated with them are formulated. There are two main limitations associated with this approach. One is the combinatorial explosion of the number of states [123] and other one is associated with transient state analysis of the resulting equations [91]. Besides having these limitations, performance studies of some manufacturing systems by using this method are found in [43, 49, 50, 78, 91, 98 and 105].

Table 1.1: Methods for analysing process industries and their systems.

Performance analysis techniques	References
Stochastic models with markovian property, also known as Markov models	[1, 3, 5, 11, 21, 22, 26, 27, 29, 33-41, 43-56, 60, 61, 64-67, 69-75, 78, 91-93, 98-101, 103-108, 112-121, 127, 130, 131, 133-135, 138-152, 154]
Fuzzy Based techniques	[34, 36, 62, 68, 86, 120, 124]
FMEA	[18, 28, 82]
FTA	[66,68,86,124]
Bayesian Technique	[111,123]

Some of the complex manufacturing systems such as plastic manufacturing plant [51], urea fertilizer plant [35, 67, 69-74], butter oil manufacturing plant [50, 68], chemical plant [47], powder production [52], paper plant [106-108], piston manufacturing plant [78], and power plant [52-54, 99] have been studied and analyzed on the basis of stochastic models. All of these studies were carried out to find the performance analysis of systems by studying their operating and reduced working conditions. Further, a critical review of performance metrics has shown that availability and reliability have significantly been used in the existing case studies and are referred in Table 1.2. Though, very few have considered Reliability, Availability and Maintainability (RAM) analysis. In 1995, Salehfar *et al.* [109] did a RAM analysis of electro-hydraulic steering system for improving its reliability. Herder *et al.* [57] studied performance analysis of the Lexan plant at GE industrial, plastics. They developed and implemented RAM model by using Reliability Block Diagram (RBD). In order to analyze these parameters they have used the Monte Carlo simulation. However, Sachdeva *et al.* [106-108] have used stochastic petri net method for the performance analysis of various industrial systems through steady state availability. The results for performance analysis were analyzed by using Monte Carlo simulation method.

Techniques such as Fault Tree Analysis (FTA) [66, 68, 86, 124], Failure Mode and Effect Analysis (FMEA) [18, 28, 82], Bayesian technique [123] and Boolean function [7] are also used for performance studies. However, these techniques have been used by very few authors to study the performance of industrial systems in comparison to stochastic models.

Table 1.2: Classification of performance metrics

Performance parameters	References
Reliability	[1, 5-8, 13-15, 21, 26, 32-34, 43, 48, 50-53, 57, 58, 63, 66-68, 80, 89, 93, 96-99, 105, 107, 114, 123, 127, 130, 135 , 136, 139, 150, 151, 154]
Availability	[1, 3, 26, 39-42, 45, 47-51, 54, 68, 69, 71, 73-75, 78, 81, 91, 100, 106, 108, 112, 113, 117, 119-121, 128, 130, 132, 136, 138, 146]
RAM	[57, 62, 117]
Cost based performance analysis	[129, 135, 142, 147, 148]

The above critical analysis motivated us to use the stochastic models for studying the RAM analysis of manufacturing process due to its wide applicability in reliability engineering. Further, we study these models with respect to our research objectives keeping in mind the gaps as discussed in next section.

1.3. Thesis Motivation

The field of reliability engineering has attracted several researchers for its potential contribution to uncertainty problems of system operation, failure mechanism, behavioral analysis, performance improvements, and their safety measures. A number of research contributions in this field have been made on performance analysis of

- (i) equipments [30, 70, 73, 94, 95, 121 and 130],
- (ii) systems [32-34, 36, 40-46, 48, 52, 61-65, 72-75, 85, 96, 98, 100, 101, 105, 107, 108, 110, 117-120, 125, 126, 132 and 138] and
- (iii) industries [35, 39, 47, 49-51, 53-57, 66-70, 78, 99, 106, 112-116, 122, 127].

In case of these performance evaluation studies, stochastic models with markovian property are widely accepted methodology for studying behavioral analysis of system performance. However, this methodology has some limitations. In the recent studies, researchers have turned their attention towards approaches such as Fault Tree Analysis (FTA) [66, 68, 86, 124], Reliability Block Diagram (RBD) [57], and Failure Mode Effects Analysis (FMEA) [18, 28, 82] for studying performance of manufacturing plants. The aforesaid methodologies are probabilistic and useful to study uncertain behavior of the system or an industry.

Although extensive research has been carried out on the performance analysis of systems [29, 34-56, 64, 65, 67-75, 78, 88, 105-108, 116-121, 130, 134-136, 138-143] and process [48, 50, 51, 69, 112-115] through stochastic models but still there is a need to improve the existing studies for performance analysis of the industry. In the present studies, the following gaps are found while implementing stochastic models to an industrial process.

1. The purpose of manufacturing performance analysis is to estimate the performance of machines during operating hours in order to achieve the required production target. Thus, there must be a way to analyze the average number of items to be produced based on ongoing performance analysis of the process industry. The performance studies available in the literature [112-115] only help in evaluation of performance of the industrial process through the analysis of steady state availability without considering the quantity of production. However, very few authors discuss the performance analysis based on reliability and maintainability metrics. Reliability and availability of industrial process / systems are discussed in [26, 49-53, 67, 70-75, 78, 101, 105, 106-130] without considering maintenance and related costs. Some profit based performance studies for an industrial system are found in [64, 65, 97, 105, 121, 134, 135, 139-143, 146-148, 150-152] by using stochastic model. The available performance studies [26, 49-53, 67, 70-75, 78, 101, 105, 106-130] do not contribute much in estimating the number of machines required to meet the increasing demand of consumers.
2. In the recent studies of behavioral analysis [27, 29, 37, 38, 47, 48, 49, 51, 52, 112-114, 147, 150], performance of various industrial systems having variable failure and repair rates is evaluated by using a supplementary variable technique. In fact, integral – differential equations are obtained using supplementary variable technique [16] for time dependent performance analysis of the system. The existing analytical methods to solve such equations are not efficient for this kind of performance studies.

This research study is an effort to model a performance of a selected industrial process under the following research objectives keeping in view the gaps found in earlier studies.

1.4. Objectives

The following three research objectives were set to find a suitable model for studying the performance of manufacturing process:

1. To conceptualize a RAM model for the selected industrial process/system
2. To transform the conceptualized RAM model into a quantitative and simulation ready model
3. To assess the influence of maintenance on the industrial process/system's reliability and availability through simulation

Keeping these objectives in view, the research work has been carried out on the following work elements.

1. Understanding the selected industrial process/system.
2. Review of literature for understanding RAM modeling of the system/process.
3. Conceptualization of RAM model.
4. Collection of relevant primary and secondary data on the system/process in question.
5. Review of models and equations commonly used in the RAM modeling
6. Transforming the conceptual model into quantitative model.
7. Simulation studies on the RAM model.

The approach followed for the completion of the above mentioned work elements is briefly described below:

Understanding the selected industrial process/system: Under this element of work, the selected industrial process will be surveyed with the objective of understanding the following:

- The system, and its sub-systems and components
- Functioning of the system, and the parameters and factors that influence the system's reliability and availability
- Maintenance and service practices of the industry in relation to the system in question
- Either a power loom or a knitting machine of textile industry will be chosen for the study. The survey will include physically observing the system and its functioning, and interviewing of the personnel associated with the system's operation and maintenance including servicing.

Review of literature for understanding RAM modeling of the system/process: Review of literature on how others have tried to model repairable/maintainable systems will be focused on. This element of the work is expected to help in RAM modeling the system in question.

Conceptualization of RAM model: Conceptualization of the RAM model will be based on the understanding obtained about the system through the first work element. The second work element is supposed to provide guidance in the conceptualization.

Collection of relevant primary and secondary data on the system/process in question: Quantitative information specially on the following will be collected from secondary data sources:

- Information on failures
- Repair, maintenance and servicing of the system
- System availability and operational availability information
- For filling the gaps in information/data and for additional targeted data primary data collection will be relayed on. Efforts will be to collect data for multitude of standardized systems for a specified period of time.

Review of models and equations commonly used in the RAM modeling: Models and equations that have been used by others in RAM modeling will be reviewed here in order to equip with the alternative models and equations that can be used in the construction of the quantitative model for the system in question.

Transforming the conceptual model into quantitative model: Information collected under the fourth work element will be used in this transformation process. The transformation will be done in the light of the reviewed models and equations and concentrate on constructing a simulation model by which one can assess the reliability and availability of the system under different repair, maintenance and servicing regimes. Depending on the need, the constructed model or any portion thereof, will be calibrated and validated on the basis of the data/information collected, under the forth work element.

Simulation studies on the RAM model: This element of work is about the simulation studies on the RAM model constructed. Purpose of these studies is to assess the impact of different repair, maintenance and servicing policies and scenarios on the reliability and availability of the system. Further, it is supposed to help in prioritizing the repair, and maintenance activities for the industry for maximizing the reliability and availability functions.

1.5. Preliminaries: Basic concepts and definitions

Basic concepts and definitions required for the proposed study are discussed as follows:

1.5.1. Unit (U)

The least subdivision of a system or an item that cannot be dissembled without being destroyed is known as part or unit or element of the system.

1.5.2. System (S)

It can be characterized as a group of sub systems or units, especially integrated to perform a specific operational function or functions.

1.5.3. Modules (M)

It can be characterized as a group of similar systems, integrated to perform a specific operation.

1.5.4. Industry (I)

It is a big complex system consisting of large and small similar as well as different system(s) to manufacture a product.

In this work, we consider it as $U \subset S \subseteq M \subset I$.

1.5.5. Reliability

The Electronics Industries Association, U.S.A. (formally known as Radio Electronics Television Manufactures Association (RETMA)) states reliability as, "the probability of performing its purpose adequately for the period of time intended under the operating and environmental conditions encountered." This is considered to be standard definition as it stresses on four important factors: probability, adequate performance, time, and operating conditions.

According to British Standards Institution "reliability as the characteristic of an item expressed by the probability that it will perform a required function under stated conditions for a stated period of time."

Probabilistically, reliability, $R(t)$ can be interpreted as,

$$R(t) = P[T > t], \; t \geq 0 \tag{1.1}$$

where, T is the random variable representing failure time or time to failure. From this, the unreliability or the failure probability distribution function $F(t)$ can be defined as

$$F(t) = 1 - R(t) = P[T \le t] \tag{1.2}$$

Further, reliability can be written in terms of failure density function $f(t)$ as

$$R(t) = \int_t^{\infty} f(t)dt \tag{1.3}$$

where,

$$f(t) = \frac{dF(t)}{dt}$$

Alternatively, if N is considered as the total number of components to be tested for time t, $N_s(t)$ as the number of the components surviving in the test after time t and $N_f(t)$ as the number of failure components in the test after time t, then

$$N = N_f(t) + N_s(t) \tag{1.4}$$

Here, reliability can be interpreted as the probability of surviving components. By classical definition of probability, estimate of reliability is equal to the ratio of number of components surviving a test to the total number of components present at the beginning of the test. Thus,

$$\hat{R}(t) = \text{Probability of success} = \frac{N_s(t)}{N} \tag{1.5}$$

Similarly, unreliability is equal to the probability of failure

$$F(t) = \frac{N_f(t)}{N} \tag{1.6}$$

At any time t,

$$R(t) + F(t) = 1 \tag{1.7}$$

The event of survival components is complementary to the event of failure components. That is, a component can either survive or fail.

Using Eq. (1.4) in Eq. (1.5), we have

$$R(t) = \frac{N - N_f(t)}{N} = 1 - \frac{N_f(t)}{N} \tag{1.8}$$

Differentiating both sides, we get

$$\frac{dR(t)}{dt} = -\frac{1}{N}\frac{dN_f(t)}{dt} \tag{1.9}$$

On rearranging,

$$\frac{dN_f(t)}{d\,t} = -N\frac{dR(t)}{dt} \tag{1.10}$$

Thus, $\frac{dN_f(t)}{d\,t}$ is the number of failure components in the time interval dt, between times t and $t+dt$. In other words, we can say that it is the rate of change of component failure at time t which is still in the testing phase. Dividing Eq. (1.10) by $N_s(t)$, we get the rate of failure or instantaneous probability of failure per component. This is also called failure or hazard rate and, denoted by *h(t)*,

$$h(t) = \frac{1}{N_S(t)}\frac{dN_f(t)}{dt} = -\frac{N}{N_S(t)}\frac{dR(t)}{dt} \tag{1.11}$$

$$h(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt} = -\frac{d\left(\ln R(t)\right)}{dt}$$

On integrating, we have,

$$-\int_0^t h(t)dt = \ln R(t) \tag{1.12}$$

Thus, reliability $R(t)$ is given by

$$R(t) = e^{-\int_0^t h(t)dt} = e^{-H(t)} \tag{1.13}$$

where, $H(t) = \int_0^t h(t)dt$ is called cumulative hazard function.

Eq. (1.13) can be considered as a generic expression for failure as it is applicable to both exponential and non-exponential failure distributions.

Hazard rate can also be expressed as:

$$h(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt} \tag{1.14}$$

$$= -\frac{1}{R(t)}\frac{d\left[1-F_T(t)\right]}{dt} = -\frac{1}{R(t)}\left[-\frac{dF_T(t)}{dt}\right] \tag{1.15}$$

By using failure probability density function, we get

$$h(t) = \frac{f(t)}{R(t)} \tag{1.16}$$

1.5.6. Availability

It is the probability of system being in operating condition at any time t, given that it was in operating condition at $t = 0$.

Based on time interval, availability is classified as:

Instantaneous availability: It is defined as probability that the system is in operating state at any time t. Symbolically,

$$A(t) = E[z(t)] \tag{1.17}$$

where $z(t)$ is an indicator function with value one for in failed state at time t, otherwise zero. Here, E stands for expectations.

Average uptime availability: Proportion of time during which system is available for use in a specified interval (0, T) is known as average uptime availability. It is given as:

$$A(T) = \frac{1}{T}\int_0^T A(t)dt \tag{1.18}$$

Steady state availability: It defined as the probability that the system is operating for time interval which is considered to be large.

$$A(\infty) = \underset{T\to\infty}{Lt}\, A(T) = \underset{T\to\infty}{Lt}\, \frac{1}{T}\int_0^T A(t)dt \tag{1.19}$$

On the basis of down time, availability defined in three ways as:

Inherent availability: Considering corrective maintenance only, availability of the system is defined as the proportion of time during which system is operating, without considering preventive maintenance down time supply time and waiting time.

$$A_i = \frac{MTBF}{MTBF + MTTR} \tag{1.20}$$

where, Mean Time to Repair (MTTR) is defined as the average expected time to repair a system after failure.

Achieved availability: Considering corrective maintenance and preventive maintenance down time only, availability of the system is defined as the proportion of time during which system is operating without considering supply time and waiting time.

$$A_a = \frac{MTBF}{MTBF + M} \tag{1.21}$$

where, M=Mean maintenance downtime due to breakdown and preventive maintenances actions.

Operational availability: Considering supply time and waiting time along with corrective maintenance and preventive maintenance down time only, availability of the system is defined as the proportion of time during which system is operating.

$$A_O = \frac{\text{MTBF} + \text{Ready Time}}{\text{MTBF} + \text{Ready Time} + \text{MDF}} \tag{1.22}$$

where, Ready Time $= (\text{operational cycle - MTBF - MDT})$,
MDT = Mean Down Time = M + delay time due to supply and administrative factors.

1.5.7. Maintainability

It is defined as the probability that the system will be restored to a fully operational condition within a specified period of time. Alternatively, using complementary function, it can be defined as

$$M(t) = 1 - P[\text{ System will not be repairable during (0, t)}] \tag{1.23}$$

Basically, there are two types of Maintenance:

Preventive maintenance: When system is periodically inspected, some components are replaced and required adjustments are made before system fails. It is helpful to increase the reliability and thus to prolong life of the systems by overcoming the effects of aging, fatigue and wear. Serviceability of the system is also considered to be part of preventive maintenance.

Corrective maintenance: It is carried out once the system has failed. Its aim is to bring the system from failed state to the operating state as soon as possible to increase its availability.

1.5.8. Mean Time Between Failures (MTBF)

It is the average expected time between failures. By definition, this includes the time required to restore a system after failure. It is denoted by

$$MTBF = \frac{1}{n}\sum_{i=1}^{n} t_i \tag{1.24}$$

where, $t_1, t_2, t_3, ..., t_n$ the observed failure times of an item.

1.5.9. Mean Time to Failures (MTTF)

It is the average expected time to failures. By definition does not include the time required to restore a system after failure. It is denoted by

$$MTTF = \int_0^{\infty} R(t)dt \tag{1.25}$$

1.5.10. Reliability Networks

For measuring reliability of a complex and big systems, we break it down into small subsystem network. These reliability networks can be explained as given below:

Let $\mathrm{E_i}$ and $\mathrm{E'_i}$ are the events of satisfactory and unsatisfactory operation of the i^{th} component.

Series systems: Failure of any one of the $n-$components in this configuration results in the failure of entire system. So, reliability as

$$\begin{aligned} \mathrm{R_{ss}(t)} &= \mathrm{P[E_1 \cap E_2 \cap E_3 \cap E_4 \cap ... \cap E_n]} \\ &= \mathrm{P[E_1].P[E_2/E_1].P[E_3/E_1E_2]...} = \prod_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{P_i(t)} \end{aligned} \tag{1.26}$$

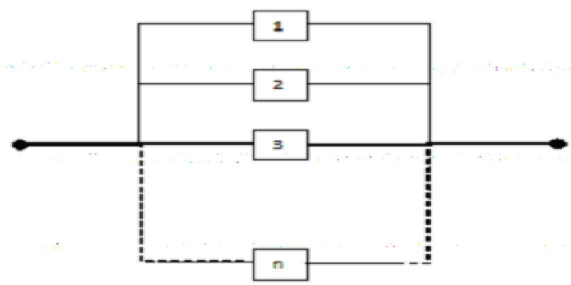
For example: Railway lines physically are parallel to each other but reliability wise they are in series. Failure of any line fails the track system of railway.

Figure 1.1: Series system network

Parallel systems: Unlike series systems, parallel system fails only when all of its components fail. Thus, reliability calculated as

$$R_{ps}(t) = P\left[\bigcup_{i=1}^{n} E_i\right] = 1 - P\left[\bigcap_{i=1}^{n} E'_i\right] = 1 - F_{ps}(t) \qquad (1.27)$$

Figure 1.2: Parallel system network



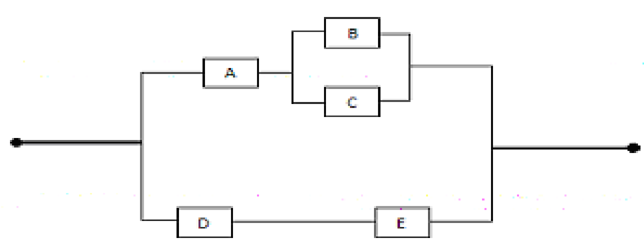
Series-Parallel Systems: Such system networks are combination of both series and parallel system networks. Reliability of Fig. 1.3 calculated as

$$R_{sps} = (1 - R_{DE})(1 - R_{ABC}) \qquad (1.28)$$

where, $R_{BC} = R_B + R_C - R_B R_C,\ \ R_{ABC} = R_A.R_{BC},\ \ R_{DE} = R_D.R_E$

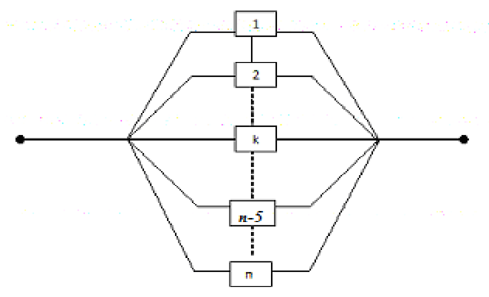
For example: Cycle, two tires are in series and paddles in series with back tire but brakes are in parallel with each other.

Figure 1.3: Series parallel system Network



k*-out-of-*n*-systems:* Such systems operate till minimum *k*-out - of - *n* components survive, system shows failure whenever *(n-(k+1))* component failure occurs. Series and parallel are special cases of this system as: For *k=n*, *k*-out-of-*n* reduced to series system, and for *k=1*, it reduced to parallel system.

Figure 1.4: *k*-out-of-*n system network*



Reliability as,

$$R_s = \sum_{i=k}^{m} {}^{m}C_i.[1-F(t)]^i F(t)^{m-i} \quad (1.29)$$

This implies that system successfully operate when *k*, *k+1*, *k+2* or *n* components survives and reliability can be obtained by using Binomial distribution with failure probability F(t) of a component. The failure probability distribution function of the system can be calculated as

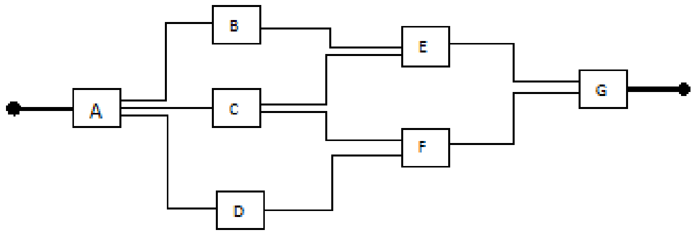
$$F_s = \sum_{i=0}^{k-1} {}^{m}C_i.[1-F(t)]^i F(t)^{m-i} \quad (1.30)$$

Example: 1. Consider a power plant, where two of its four generators are required to meet the customers demand, is *2*-out-of-*4* systems

2. In 6-cyclinder automobile, at least *4*-cylinders are required for firing but less than four fail to fire, automobile cannot be driven. So, automobile is *4*-out-of-*6* systems.

Complex systems or non-series parallel system: These systems are most difficult to recognize, as these systems are tough to break down into either series or parallel for reliability calculations.

Figure 1.5: Complex series system network



In Fig. 1.5, component C has two paths leading away from it whereas B and D have only one. So, for doing reliability calculations of such system, different types of methods are used like decomposition methods, event space method and path tracing methods.

1.5.11. Boolean algebra

Let $(B, \neg, \wedge, \vee, 0, 1)$ be a Boolean algebra. A function $f : B^n \to B$ associated with a Boolean expression in n variables is called Boolean function [137]. The basic operations/expressions of Boolean algebra on Boolean variables (as x, y) are the following ones:

Conjunction (AND): It is denoted by '$\wedge$'. If $(x, y) \in B$ then $x \wedge y = 1$ if $x = y = 1$ and $x \wedge y = 0$ otherwise.

Disjunction (OR): It is denoted by '$\vee$'. If $(x, y) \in B$ then $x \vee y = 0$ if $x = y = 0$ and $x \vee y = 1$ otherwise.

Negation (NOT): It is denoted by '$\neg$'. If $(x, y) \in B$ then $\neg x = 0$ if $x = 1$ and $\neg x = 1$ if $x = 0$.

These relation are shown in truth table format as follows:

Table 1.3: Truth Table

x	y	$x \wedge y$	$x \vee y$
0	0	0	0
1	0	0	1
0	1	0	1
1	1	1	1

1.5.12. Stochastic process

Let t be a parameter assuming values in a set T, and $N(t)$ represents a random variable or stochastic variable for every $t \in T$. The family or collection of random variables $\{N(t), t \in T\}$ is called stochastic process [83]. The parameter or index t is taken as time and $N(t)$ random variable as state of the process at time t. The set of all possible values that the random variable $N(t)$ can assume is called the state space of the process.

If $N(t) = i$, then the process is said to be in state i at time t, whenever the process is in state i, there is a fixed probability. The system will be in next state j, given that it is in state i, with transition probability P_{ij}. Let, we suppose that

$$P\{N(t+1) = j \mid N(t) = i, N(t-1) = i_{t-1}, ..., N(t-1) = i_1, N(0) = i_0\} = P_{ij} \tag{1.31}$$

for all states $i_0, i_1, ..., i_{t-1}, i, j$ and all $t \geq 0$. Such a process is known as Markov Chain for discrete state space. The Eq. (1.31) can be interpreted as, the conditional distribution of any future state $N(t+1)$ given past states $N(0), N(1), ..., N(t-1)$ and present states $t \geq 0$, is independent of the past states and depends only on the present states. This is called the markovian property[83].

Markov chains are further classified into discrete and continuous markov chains depending on the index set. If the index set T is countable, we have discrete time markov chain and if T an interval of the real line, we have a continuous-time markov chain. For more details in respect to resulting state transition differential difference system of linear equations, please consider the readings [9, 29, 83]. In this thesis, we use stochastic process, stochastic model or Markov models as synonyms.

1.6. Thesis Outline

Keeping the objectives as discussed in Section 1.4, the research work has been carried out. The overall research work is divided into seven chapters of this thesis. The chapter wise summary of the thesis is as follows:

Chapter 1

Chapter 1 is introductory. In this chapter, apart from preliminary concepts to be used in the sequel, the literature on performance analysis of the systems along with a brief plan of the results presented in the subsequent chapters has been discussed.

Chapter 2

In this chapter, a methodical scheme is proposed to compute performance metric of the industrial process. This scheme utilizes the concepts of Boolean algebra and stochastic process, and is referred as Assiduity Progression Diagnosis (APD). The proposed scheme overcomes a limitation of stochastic models in which state space grows beyond certain limits while implementing on industrial process. Due to which number of equations in the system of differential equations, governing the performance of the industry, increases abruptly and it becomes a difficult task to compute performance parameters. This scheme helps the manufacturing industries to predict Reliability, Availability and Maintainability (RAM) of industrial process. The model thus developed helps to determine reliability, three kinds of availability (steady state, time dependent and inherent availability), maintainability, average production and other related metrics of the industrial process using state probabilities efficiently. The proposed model has finally been implemented to the fabric industry for its performance analysis and later been discussed in Chapter 5. The contents of this chapter meet our first objective.

Chapter 3

This chapter is concerned with some recent studies [39-41, 47, 48, 51, 112-114, 130, 151, 152] of performance analysis of an industrial system where stochastic models are developed by using the supplementary variable technique (SVT) [16] for a system having constant and variable transient rates. This technique can be applied to the RAM model (APD, as discussed in Chapter 2) when failure and repair rates of modules of an industry are variable. However, in this case the performance metrics such as reliability, time dependent availability and mean time to failure of process will not be computed from resulting mathematical model. In fact, the analytical methods such as Laplace Transform method [112, 114, 130, 142, 146, 147, 150, 152], Lagrange's method [40, 48, 52, 112, 115, 152] and separation of variable method [29] have been used to solve the resultant system of simultaneous linear partial differential equations but analytical solution is so intricate that the industry persons will not be able to use them conveniently. In order to overcome this situation, a numerical technique is developed in this chapter to solve such a complex mathematical problem. The proposed numerical approach consists of three numerical methods (finite difference schemes, Simpson's one-third method and Lagrange's interpolation) which iteratively compute the transient solution of the Markovian model taken under consideration. This hybrid method is named as Lagrange Finite Difference Simpson Method (LFDSM). The contents of this chapter meet the second research objective.

Chapter 4

Three important criteria: (i) minimum replacement cost rate (ii) maximum availability and (iii) lower bound on the mission reliability are frequently used in literature for scheduling maintenance of an industrial system. By selecting anyone of these criteria, the other two criteria generally ignored in the case of system maintenance. This ignorance may not provide flexibility for the decision makers to finalize the maintenance scheduling of the system. In this chapter, the problem of maintenance scheduling of the industrial system in terms of optimal failure and repair rates is considered. An optimization model is presented to maximize the total profit from the output of five units working system by imposing constraints on availability and maintenance cost of the system. The reliability criteria can also be used in this proposed model under some conditions. This model is developed by taking into account the concept of stochastic process based on supplementary variable technique and optimization theory. A decision strategy, using the maintenance cost constraint, has next been proposed for a decision maker. The application of the proposed model is finally demonstrated on two industrial systems and has been discussed in Chapter 6. The contents of this chapter fulfil the third research objective.

Chapter 5

This chapter deals with the application of the APD model developed in Chapter 2 on a fabric industry to analyze its performance analysis. Certain parameters such as reliability, time dependent availability, steady state availability, maintainability, and average production metrics are used as performance measures for overall behavior of fabric industry to achieve desired production targets. The behavior analysis of APD model reveals that the mixer and slasher system of fabric industry are the most sensitive and affect more to the overall performance of the industry. A comparative study has also been presented between the APD model and the traditional approach [24] of evaluating the performance of industry using stochastic models.

Chapter 6

In this chapter, the optimal results for failure and maintenance rates for only mixer and slasher systems have been obtained by using the optimal maintenance scheduling methodology as discussed in Chapter 4. Further, in this chapter we have also discussed that how the steady state behavioral analysis can be used instead of transient analysis for studying optimal maintenance scheduling of these systems.

Chapter 7

In this concluding chapter, limitation and scope of the methodologies developed in this thesis to study RAM analysis of selected industrial process has been discussed. The industrial significance of the results as well as a scope for further work on this topic are also presented in the concluding chapter.

Chapter 2
RELIABILITY, AVAILABILITY AND MAINTAINABILITY (RAM) MODEL OF AN INDUSTRIAL PROCESS

In this chapter, a methodical scheme to compute performance metric of the industrial process is proposed. This scheme utilizes the concepts of Boolean algebra and stochastic process, and is referred as Assiduity Progression Diagnosis (APD) . The proposed scheme overcomes a limitation of stochastic models in which state space grows beyond certain limits while implementing on industrial process. This causes an abrupt increase in number of equations in the system of differential equations, governing the performance of the industry and consequently it becomes a difficult task to compute performance parameters. This scheme helps the manufacturing industries to predict Reliability, Availability and Maintainability (RAM) of industrial process. The model thus developed helps to determine reliability, three kinds of availability (steady state, time dependent and inherent), maintainability, average production and other related metrics of the industrial process by using the state probabilities efficiently. The proposed model has finally been implemented to the fabric industry for its performance analysis and has been discussed in Chapter 5.

The following sections are included in this chapter. Section 2.1 deals with background of performance studies of industrial systems based on stochastic models. In Section 2.2, scheme is proposed to model the performance of an industrial process. This scheme is finally presented as an algorithm in Section 2.3.

2.1 Background of the problem

The behavior analysis using stochastic process, in general, provides an efficient probabilistic tool to study failure analysis of the system. However, the only reason to avoid the stochastic model by practitioners is its growing state space. To understand this phenomenon of state space more clearly we discuss it in details as follows:

Suppose there are two possible states to define operational conditions of a unit system. The system can be either in operating state or in failed state. Then, a stochastic process $\{N(t),\, t\}$ defined on state space is $\Omega = \{u_{1o}, u_{1f}\}$. The total number of possible states to model the system's performance is 2^1. In case of a two-unit system having series network, the stochastic process has the

The contents of this chapter have been communicated with a refereed Journal.

state space, $\Omega = \{(u_{1o}, u_{2o}), (u_{1f}, u_{2o}), (u_{1o}, u_{2f}), (u_{1f}, u_{2f})\}$ and thus total number of possible states for this case is 2^2.

Now consider a three-unit system. Each having two states then, the possible state space of the stochastic process will have 2^3 states and is given by $\Omega = \{(u_{1o}, u_{2o}, u_{3o}), (u_{1f}, u_{2o}, u_{3o}),$ $(u_{1f}, u_{2o}, u_{3f}), (u_{1f}, u_{2f}, u_{3o}), (u_{1o}, u_{2o}, u_{3f}), (u_{1o}, u_{2f}, u_{3o}), (u_{1o}, u_{2f}, u_{3f}), (u_{1f}, u_{2f}, u_{3f})\}$

Similarly, for a n unit system, each unit having two possible states then total number of states will be 2^n. In case of a $n-$unit system, each unit having three states, then the possible number of states will be 3^n. Thus, for a system having n units and each unit having s possible states then the possible number of states of the state space will be s^n. Now, suppose a number of states for each unit of system varies. If, s_i state represents the state of i^{th} unit of $n-$unit system, then the total number of possible states in a state space of stochastic model will be $\prod_{i=1}^{n} s_i$. Therefore, the state space of the stochastic model grows with increase in number of components/ units or subsystems. Generally, industries are large complex systems and have different number of similar- dissimilar machines for producing the items. The performance analysis of industry (using stochastic process) becomes difficult because these machines induce a combinatorial explosion of the number of states for stochastic model. Consequently, this increases the size of transition matrix for performance evaluation and thus the computations will becomes very difficult to estimate probabilities of each state.

In addition to above, availability of raw material and full operational conditions of machines also plays a significant role to evaluate performance of process industry and can not be ignored. However, available literature [49-53, 56, 67, 78, 106, 108] on the performance analysis discusses the performance evaluation of production systems by using operational conditions of machines without giving much consideration to availability of raw material. Besides this, the behavioral analysis of system in existing performance analysis gives weightage to any one of three metrics: reliability, availability, or maintainability. These performance metrics are important and have their own advantage. For example, reliability helps to estimate the life of the system while the availability helps to study the performance of system at that instant of time. The parameter, availability, helps to evaluate system being in operational conditions for period of time. Therefore, it is worth to improve the existing methodology for studying the performance analysis of industrial process. Keeping these in view, we propose a methodical scheme in this chapter to improve the industrial performance by analyzing the RAM model of an industrial process. In the next section, methodical scheme, referred as Assiduity Progression Diagnosis (APD) is presented. This scheme utilizes the concept of Boolean algebra, stochastic model and Geometric Average Approach (GAA). The main aim of this scheme is to implement the stochastic models for evaluating performance of industry besides its limitation of growing state space.

2.2 Assiduity Progression Diagnosis (APD)

In this section, we first present a detailed description of proposed approach for an industrial process and finally conclude it in the form of algorithm. The proposed approach is as follows:

In order to reduce exponential growth of state space of stochastic model for evaluating performance of manufacturing process, the industrial process machines/systems are first partitioned into several modules of the selected industrial process. Each module has a fixed number (say n) of similar machines/systems and is assumed as a k-out-of-n type network (see Section 1.1.5).

2.2.1 Transition diagram and Boolean function

Based on process partition, a diagrammatic form of state space is next developed under the assumption that each module has only two states- the failed and operating states. This diagrammatic form is known as transition diagram of the process. We now defined the operational and failed state of an industrial process and its modules by using the theory of Boolean algebra as under:

Let us consider a set $B = \{0,1\}$. A Boolean function $f_{M_i} : B^2 \to B$, representing the state of the i^{th} module is defined as,

$$f_{M_i}(Z_1, Z_2) = Z_1 \wedge Z_2, \text{ for } i = 1, 2, ..., m \tag{2.1}$$

where the Boolean variable Z_1 represents the state of the k-out-of-n systems working within a module. If at least k systems are working then Z_1 is assumed as true (1) otherwise false (0). Other Boolean variable, Z_2 represents the production of required items based on availability of stored items for a module. This factor represents false value when a module fails to produce required items through availability of raw material otherwise true.

A Boolean function $f_P : B^m \to B$, representing the state of the process is defined as

$$f_P(f_{M_1}, f_{M_2}, ..., f_{M_m}) = \bigwedge_{i=1}^{m} f_{M_i}(Z_1, Z_2) \tag{2.2}$$

Using these Boolean functions, a performance sheet is proposed to collect the required data and to figure out the state of the module using Eq. (2.1). This sheet is given in Table 2.1**.** This proposed sheet is also helpful for computing the failure and repair rates of this model.

2.2.2 Mathematical formulation of performance metrics

First, a transition diagram is to be developed by using the methodology discussed in Section 2.2.1. Using this transition diagram, the Chapman-Kolmogorov differential equations is next to be developed for process industry following the mnemonic rule of [131] to formulate performance parameters. The resulting Chapman-Kolmogorov differential equations are then solved either by using method of Laplace transforms [29] or by using Runge-Kutta fourth order method [78] to estimate the probability of each state of the state space. A detailed study of computational procedure is presented in Chapter 3. The following transient probabilities, $p_{0i}(t)$ of stochastic model are sufficient to approximate the performance parameters [29].

$$p_{0i}(t) = P[N(t) = i \mid N(0) = 0] \tag{2.3}$$

using initial distribution, $P_0(0) = 1, P_i(0) = 0, \textit{for}\ i = 1,2,3,...,m$

By using these probabilities of stochastic model, following performance parameters of industry (having *m* modules) are next computed:

Reliability of the process industry:

$$R(t) = 1 - \sum_{i=1}^{m} P_i(t) \tag{2.4}$$

Time dependent availability:

$$A(t) = 1 - \sum_{i=1}^{m} P_i(t) \tag{2.5}$$

Long run availability:

$$A_\infty = \underset{t\to\infty}{Lt}\ A(t) = 1 - \sum_{i=1}^{m} P_i \tag{2.6}$$

The Mean Time To (process) Failures (MTTF) of the process:

$$MTTF = \int_0^t R(t)dt \quad (2.7)$$

The Mean Time To Repair (MTTR) of the process:

$$MTTR = \frac{\sum_{i=1}^{m} \lambda_i T_i}{\sum_{i}^{m} \lambda_i} \quad (2.8)$$

The inherent availability, A_i of the process:

$$A_i = \frac{MTTF}{MTTF + MTTR} \quad (2.9)$$

Maintainability of the process:

$$M(t) = 1 - e^{(\frac{-t}{MTTR})} \quad (2.10)$$

Next, we propose an evaluation parameter for estimating the average production quantity produced by the industry using the probability value of states of the transition diagram. The parameter, estimated production (EP) is proposed as:

$$\text{EP} = \sum_{i}^{m} X_i P_i \quad (2.11)$$

where, X_i represents the number of items or goods that are produced by industry per unit of time when the system is in the i^{th} state of the transition diagram, and m represents the total number of modules.

The performance metrics are assumed to help in making decision on target fulfillment with the following conditions:

1. If the reliability or availability of the process is near one for a given period under specified operating and maintenance conditions then the industry will produce the estimated production,

2. Otherwise, an appropriate decision must be taken by industries to achieve the desired target, either by improving the maintenance conditions within the module or by improving the modules that affect the performance of an industrial process.

Thus, reliability helps us to ensure to what probabilistic extent estimated production target of industry can be achieved with ongoing maintenance and operating conditions. Also the availability ensures the longevity of the estimated target under the current maintenance conditions when production fails and is fixed.

2.2.3 Simulating Mathematical Formulation

The results for performance metrics (reliability and time dependent availability) can be obtained by solving the Chapman-Kolmogorov differential equations for various choices of failure (or repair) rates of a module by keeping values of failure-repair rates fixed for other modules. The computed results help us to access the effect of a module's failure (or repair) rate on performance metric of an industrial process. In case of time dependent availability, if the results show marginal change (i.e. for λ_1=0.5, $R(t)$ is 0.9998 and for λ_1=0.6, $R(t)$ is 0.9997) then average out this effect by implementing Geometric Average Approach (GAA) [87] on failure and repair rates of a module and estimate new values of failure-repair rates (i.e. new $\lambda_1 = \sqrt{(0.5 \text{ x } 0.6)} = 0.54$). With the new values of failure-repair rates, solve the Chapman-Kolmogorov differential equations again only for steady state (by imposing condition $t \to \infty$). The new estimated results now help to find the optimal values of failure and repair rates required for long-run availability. This can also be referred from flow chart shown in Fig. 2.1

2.3 Algorithm of Assiduity Progression Diagnosis (APD)

The approach discussed above has finally been formalized in the following algorithmic form:

1. First, divide the production process into several modules so that each module has a fixed number (say n) of similar systems and is a k-out-of-n type [43].
2. Construct the state transition diagram of the process based on relation between each module to define the whole production process and determine state of each module using conjunction relation described in Eq. (2.1).

3. Develop Chapman-Kolmogorov differential equations using the transition diagram of a stochastic process, and solve them to determine performance metrics such as reliability, time dependent availability, and average production.
    a. Check whether the reliability or availability is near to one.
        i. If so, proceed with present maintenance plan.
        ii. Else, consult the maintenance management for changes within module either to increase the number of systems, or to improve the failure and repair rates values for achieving required availability of industry, proceed to Step 4.
4. To find the optimal failure and repair rates (Fig 2.1):
    a. Compute results of performance metrics under different variations of failure and repair rates of modules.
    b. If the percentage change is marginal in the simulated results of performance metrics for given variations of failure and repair rates then implement geometric averaging approach otherwise proceed to next step.
    c. Compute and compare the results for steady state availability of process industry for combined variation of failure and repair rates of sensitive modules in order to find the optimal results for maximum availability.
5. Compute maintainability using Eq. (2.10) in order to access the chances of industry to restore back to its operational state after failure.

Figure 2.1: Flow chart for performance evaluation through stochastic models

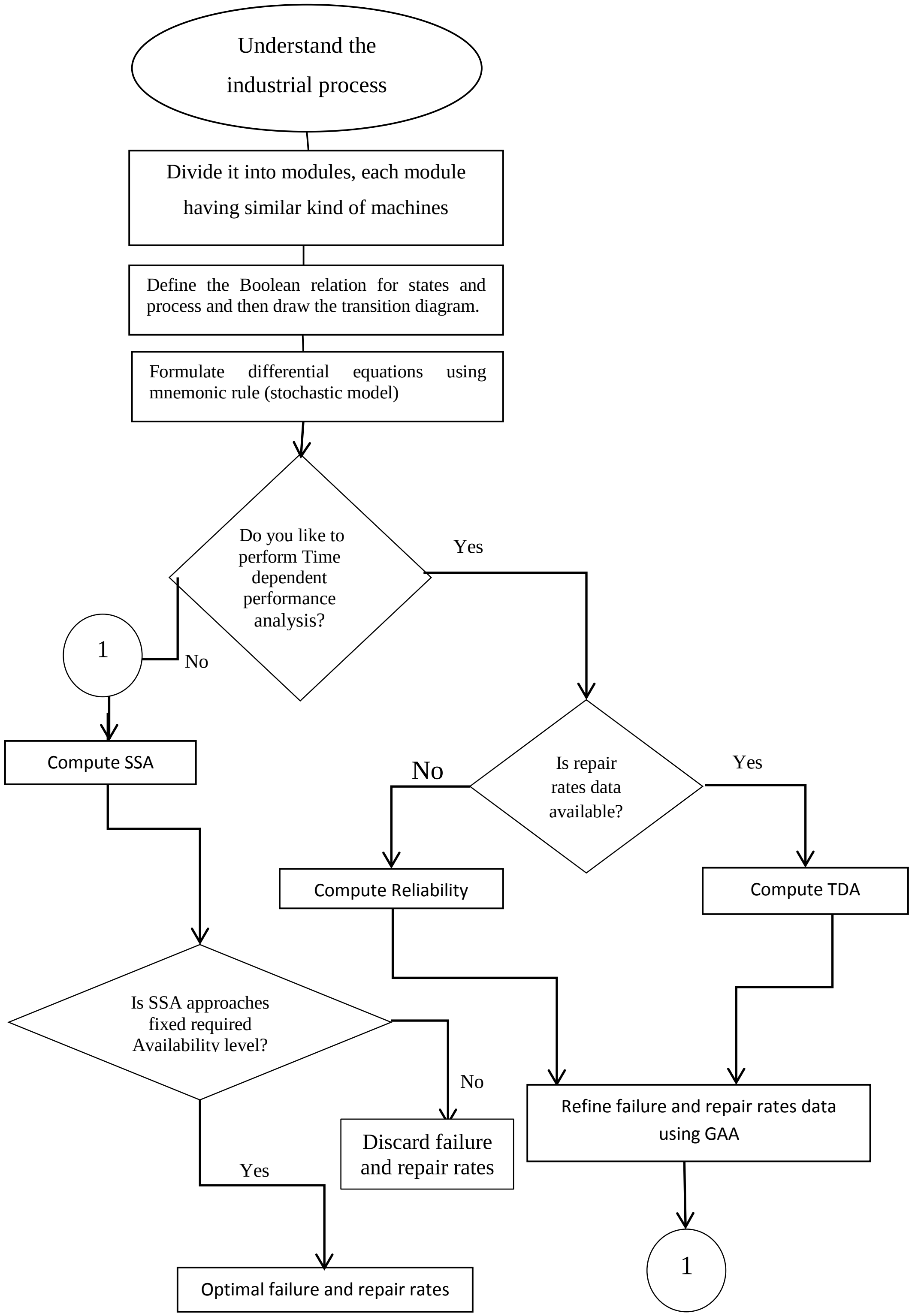


Table 2.1: Data record sheet for APD model

1	Data record on		100 day
2	Data record for module		X
3	Remark, if any		None
4	Total machines/system, n		3
5	Module type		2 out of 3
6	No. of items produced per hour by this module		3
7	Average required items per hour from this module output		2
8	Storage	a Available today	zero
		b Minimum	None
		c Maximum allowed	7
9	Average machine failure rate of this module		once in 90 days
10	Average repair time for a system		one hour

11 Collect the following information to estimate Boolean variables (Z_1, Z_2). Use the following table to input values in next step.

S. no.	**Entry time**	**Required no of items to be produced per hour**	**Actual no. of items produced per hour**	**Cumulative storage available after adjusting with requirements**	Z_1	Z_2	**Remark if any**
1	10:00 AM	2	3	1	True	True	
2	11:00 AM	2	3	2	True	True	
3	12:00 PM	2	2	2	True	True	M_1 system down, repairmen absent
4	1:00 PM	3	2	2	True	True	
5	2:00 PM	3	2	1	True	True	
6	3:00 PM	3	2	0	True	False	
7	4:00 PM	3	1	0	False	False	M_2 system also down, repairmen absent
8	5:00 PM	3	1	0	False	False	

where

Z_1 : whether the k number of system are working

Z_2 : check whether required number is produced with present production and storage availability, if so supply 1 else 0

12. State of the module is interpreted as below using conjunction relation (Eq. 2.1),

S.No.	Z_1	Z_2	Module state
1	True	True	True (operating)
2	True	True	True (operating)
3	True	True	True (operating)
4	True	True	True (operating)
5	True	True	True (operating)
6	True	False	False (failed)
7	False	False	False (failed)
8	False	False	False (failed)

13. Using the information of Step 12, Compute module failure rate as $\frac{\text{no.of hours module fail}}{\text{Total no of hours module works}}$.

For example, for the given data in table, module failure rate $=\frac{3}{8}$ = 0.375 per hour =0.00625 per minute.

14. Use the below Table for computations of module repair rate. It is equal to $\frac{\text{No. of machines required}}{\text{Total time taken to repair}}$

For example, for given data, repair rate $=\frac{2}{135}$ = 0.014815 per minute

Day	100 and 101th				
S.no	**Systems failed at time**	**Repair time starts at**	**Repair time ends at**	**System back in operation**	**Total time**
System I					
1	12:00am, day:100	repair unavailable	-	-	-
2	failed yesterday, day:101	10:00am	10:45am	11:00am	45 mins
System II					
1	4:00pm	repair unavailable	-	-	-
2	failed yesterday, day:101	11:00am	12:30pm	01:00pm	90 mins
System III					
1	-	-	-	-	-
2	-	-	-	-	-
Total					**135 mins**
Total systems					**2**

Chapter 3
COMPUTATIONAL APPROACH FOR RAM MODEL

Gaver [42] studied the performance of a system with non-markovian model by converting it into markovian model using supplementary variable technique [16]. Markovian system of equations obtained by using supplementary variable technique is frequently used in reliability engineering for analyzing performance of the system having variable transition rates [1, 3, 5, 27, 39-41, 47-49, 51, 112-114, 146, 147, 150-152]. Gaver's model [42] is accepted as basic and the most simplest model for studying the performance of the system using supplementary variable technique. However, in his work, Gaver discussed only the steady state analysis of the resulting system of equations. In this chapter, a numerical method is proposed to solve the markovian system of equations obtained by using supplementary variable technique in order to study the performance of the system having constant failure and variable repair rate through transient state analysis.

This chapter is organized as follows: Section 3.1 is introductory which presents background of the problem along with brief survey of solutions techniques available in literature for analyzing markovian models. Section 3.2 deals with the mathematical model of system developed by Gaver [42]. In Section 3.3, we have introduced the proposed numerical method to obtain the solution of this model. The results obtained by this method are finally presented in Section 3.4. Further, the effectiveness of this method is also discussed in this Section.

3.1. Background of the problem

Markovian models are analyzed in two ways namely, steady state and transient state analysis. Steady-state analysis has been the focus of most of the performance studies in the area of manufacturing systems. The transient-state analysis helps in studying issues like accumulated performance rewards over finite intervals, first passage times, sensitivity analysis, settling time computation, and deriving the behavior of queuing models as they approach equilibrium, and is hence necessary for time dependent analysis [6]. Transient-state analysis, in reliability theory, plays an important role, as it is helpful in studying the time dependent availability and reliability of the system.

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For system performance, several authors [1, 3, 5, 44, 50, 78, 81, 91, 103, 104, 133] have obtained transient solution of Markovian models by assuming constant repair and failure rates. Narahari and Viswanadham [91] have discussed Lapalace transform inversion and matrix exponential to compute the transient solution of Markovian models by taking into account the methodology of [30, 104, 133]. They found lot of difficulties while computating the large Markovian systems. Tombuyses and Devooght [136] investigated four explicit and six implicit Runge - Kutta methods for obtaining transient solution of such models by emphasizing on stability, amount of numerical work and accuracy to solve. Amiri and Tari [3] developed the Markov models for analyzing the transient availability and survivability of the system by using eigen vectors and eigenvalues. Lindermann *et al.* [80] discussed numerical methods for reliability of Markov closed fault – tolerant system.

In order to study the performance of system Gaver [42], Arora [5], Sridharan and Mohanavadivu1 [130] discussed the Markov models by considering maintenance states. This type of work can also be found in the recent research [39-41, 47, 48, 51, 52, 112-114, 130, 151, 152]. These authors developed a system of complex mathematical equations consisting of simultaneous linear ordinary and partial differential equations that determine the reliability and availability of their respective industrial systems. They have used Laplace Transform [112, 114, 130,142, 146, 147, 150, 151], Lagrange's methods [40, 48, 52, 112, 115, 152] and separation of variable method [27] in their respective work to solve the resultant system of simultaneous differential difference equations. The analytical solutions obtained by these authors are so intricate that the industry persons may not use these results conveniently in their problems for performance analysis. Thus, there is need to apply some numerical methods which can be used to solve industrial problem of such kind. In this chapter, an efficient numerical method is proposed to fill this gap. The proposed numerical method is combinational numerical approach consisting of three numerical methods (finite difference schemes, Simpson's one-third method and Lagrange's interpolation), which iteratively compute the transient solution of the Markov model taken under study and is named as Lagrange Finite Difference Simpson Method (LFDSM).This simple numerical approach helps to approximate the solution of Markovian models having variable repair and failure rates.

3.2. Gaver's Model

For studying the performance of a system, Gaver [42] in 1963 discussed the following differential equations assuming variable repair rate and constant failure rate of the subsystems.

$$\frac{dP_0(t)}{dt} = -\alpha P_0(t) + \int_0^x \beta(x) P_1(x,t)dx \tag{3.1}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right) P_1(x,t) + \beta(x) P_1(x,t) = \alpha P_0(t) \tag{3.2}$$

Also, $$P_1(t) = \int_0^\infty P_1(x,t)\, dx \tag{3.3}$$

This system is subjected to the boundary condition

$$P_1(0,t) = -\alpha P_0(t) \tag{3.4}$$

and initial conditions

$$P_0(0) = 1,\; P_1(x,0) = 0 \tag{3.5}$$

The presence of the term $\beta(x)$ in the Eq. (3.2) renders to the markovian property of the process. To restore the Markovian property of the model a supplementary variable $'x'$ is introduced which represents elapsed repair time as discussed in [16, 42].

3.3. Proposed Numerical Method

In order to solve system of differentio-integral equations (3.1-3.3) together with boundary condition (3.4) and initial condition (3.5), the following notations are used to make the problem in computable form

$$P_0(t) = A(t) \tag{3.6}$$

$$P_1(t) = B(t) \tag{3.7}$$

$$P_1(x,t) = B(x,t) \tag{3.8}$$

and

$$\int_0^x \beta(x)\, B(x,t)dx = Z(t) \tag{3.9}$$

where, $Z(t) \neq B(t)$

Thus, by using the equations (3.6-3.9), the equations (3.1-3.2) can be written as:

$$\frac{dA(t)}{dt} = -\alpha\, A(t) + Z(t) \tag{3.10}$$

$$\frac{\partial B(x,t)}{\partial x} + \frac{\partial B(x,t)}{\partial t} + \beta(x)\, B(x,t) = \alpha\, A(t) \tag{3.11}$$

The boundary and initial conditions take the form

$$A(0) = 1, \;\; B(x,0) = 0 \tag{3.12}$$

and $$B(0,x) = -\alpha\, A(0) \tag{3.13}$$

In order to solve this problem, we obtain finite difference analogous of equations (3.10) and (3.11) by replacing the derivatives with their corresponding approximations in a rectangular region; $R = \{(x,t) : 0 \le x \le c, a \le t \le b\}$ by partitioning it into a grid consisting of M and N rectangles with sides $\Delta x = g$ and $\Delta t = h$ as shown in Fig. 3.1.We shall use Backward Difference Method (BDM) to compute approximations $\left\{ B_{m,n} : m = 0,1,\cdots,M \right\}$ in successive rows for $n = 1,2,\cdots, N$. The grid spacing is uniform with $x_{i+1} = x_i + g, i = 0,1,\cdots,N$ in every row and with $t_{j+1} = t_j + h,\; j = 0,1,\cdots,M$ in every column. The backward difference formulae for $B_x(x,t)$ and $B_t(x,t)$ are given as

$$B_x(x,t) = \frac{B(x-g,t) - B(x,t)}{g} \tag{3.14}$$

$$B_t(x,t) = \frac{B(x,t-h) - B(x,t)}{h} \tag{3.15}$$

The BDM will be used in Eq. (3.11) together with boundary condition (3.12) to obtain $B(x,t)$ for the specified values of repair rates $\beta(x)$. Using the approximation $B_{m,n}$ for $B(x_m,t_n)$, $B_{m-1,n}$ for $B(x_m - g, t_n)$, $B_{m,n-1}$ for $B(x_m, t_n - h)$ in equation (3.14) and (3.15) and further substituting into Eq. (3.11) with β_m for $\beta(x)$, we get

$$\frac{B_{m-1,n} - B_{m,n}}{h} + \frac{B_{m,n-1} - B_{m,n}}{g} = -\beta_m\, B_{m.n} + \alpha A_n \tag{3.16}$$

On simplification, we obtain the three - point formula as

$$B_{m,n} = \frac{\alpha\, g\, hA_n + g\, B_{m-1,n} + h\, B_{m,n-1}}{gh\beta_m + h + g} \tag{3.17}$$

and

$$Z_m = \int_0^x \beta_m B_{m,n}\, dt, \tag{3.18}$$

where $m = 1, 2, \cdots, M,\ n = 1, 2, \cdots, N$.

Boundary condition (3.12) can be replaced by finite difference analogue

$$B_{0,n} = -\alpha\, A_0\,,\ n = 1, 2, \cdots, N \tag{3.19}$$

On solving the Eq. (3.11), the values of the $B(x,t)$ will be used in Eq. (3.9) to compute the integral by making use of Simpson's one-third method. As industries do not provide sufficient data for repair rates of subsystems so the missing values of those repair rates, wherever required in computation, have been estimated by using Lagrange's interpolation method. Since all the parameters on the right hand side of Eq. (3.10) are known so we are now ready to solve Eq. (3.10) using forward difference formula, given by

$$\frac{dA(t)}{dt} = \frac{A(t+h) - A(h)}{h} + O(h) \tag{3.20}$$

Eq. (3.10), then becomes

$$A(t+h) - (1-\alpha\, h)\, A(t) = h\, Z\,(t) \tag{3.21}$$

and after discretization, Eq. (3.21) takes the form

$$A_{n+1} = (1-\alpha h)\, A_n + h\, Z_n\,,\ \ n = 0, 1, 2, \cdots, N \tag{3.22}$$

The initial condition finally becomes

$$A_0 = 1,\ B_{m,0} = 0,\ m = 1, 2, \cdots, M \tag{3.23}$$

Using equation (3.18) and (3.23), A_1 can be computed from Eq. (3.22) and is given by

$$A_1 = (1-\alpha h) A_0 + hZ_1 \tag{3.24}$$

where, $Z_1 = \int_0^1 \beta_m \, B_{m,0} \, dx$

Similarly, A_2 can be computed by using Eq. (3.22) as:

$$A_2 = (1 - \alpha h) A_1 + h Z_2 \tag{3.25}$$

where,

$$Z_2 = \int_0^2 \beta_m \, B_{m,1} \, dx \tag{3.26}$$

In order to calculate Z_2, we first compute $B_{m,1}$ by using Eq. (3.17) for $n = 1$, we get

$$B_{m,1} = \frac{a\, g\, h\, A_1 + gB_{m-1,1} + hB_{m,0}}{g + h + gh\beta_m} \tag{3.27}$$

Since $B_{m,0}$ is known from initial condition and other parameters $B_{m-1,1}$, A_1, α and β_m are also known, we can compute $B_{m,1}$ for all $m = 1, 2, \cdots, M$. Once $B_{m,1}$ is obtained, Z_2 can be computed using Simpson's one-third method for point $m = 1, 2, \cdots, M$.

3.3.1. Algorithm

The step-by-step calculation is now formalized in the following algorithm known as Lagrange Finite Difference Simpson Method (LFDSM):

Step 1: Input the known parameters of the equations, that is, failure rate α, repair rate $\beta(x)$, elapsed time x and number of subdivisions N and M along x and $t-$axes.

Step 2: Set the initial conditions.

Step 3: Using Lagrange's interpolation method computed the missing value of repair rates at different nodal point from the available values of repair rate.

Step 4: For $j = 1, 2, \cdots, n$ do:

a. Calculate $P_1(0, j) = -\alpha P_0(j)$
b. For $i = 1 \; to \, m$, do:
    i. Compute $P_1(x,t)$ using backward difference scheme in Eq. (3.2).
    ii. Compute $Z(t) = \int_0^\infty \beta(x) P_1(x,t) dx$ using Simpson's one-third method
c. Compute $P_0(t)$ using forward difference scheme in Eq. (3.1)

Step 5: Calculate $P_1(t)$ using Simpson's one-third method in Eq. (3.3).

Step 6: Check sum of both probabilities and noted down outputs: $P_1(x,t)$, $P_0(t)$, $P_1(t)$, Sum.

3.4. Results and discussion

For the computation purposes, we take the following input parameters: failure rate α =0.05 per unit time, elapsed repair time $x = 1$ and the values of repair rate $\beta(x)$ can be taken from Table 3.1. We also provide the number of subdivisions M and N for each run of the algorithm so that a uniform mesh along the t and x axis can be generated using step sizes $h = g = 0.1$, 0.05, 0.025, 0.167.

Following the algorithm given in Section 3.3 and the above-mentioned data, computation has been carried out to obtain $P_1(x,t)$. The values of $P_1(x,t)$ have been plotted against elapsed repair time x and time t, and are shown in Fig (3.2-3.5). It is observed from the figures that the concentration profiles of probability state remain unchanged with the increase in number of subdivisions as 10, 20, 30 and 60.

The results corresponding to time dependent probabilistic states have been presented in Table 3.2 and this table exhibits the numerical solution of the probabilistic states values obtained by the proposed numerical method. From the tabulated values, it is observed that the sum of probability values for each markovian state comes out one with the small percentage error (0.02% to 0.05%). This validates the correction of the computed results using the proposed method.

Validation of the results implies appropriateness of LFDSM for studying the transient state behavior of system when a failure and repair rate shows variations. Subsequently, proposed methodology can be used to solve any complex models for determining the transient behavior of the system as discussed in the works of [48, 112].

Figure 3.1: Rectangular mesh for $B(x,t)$

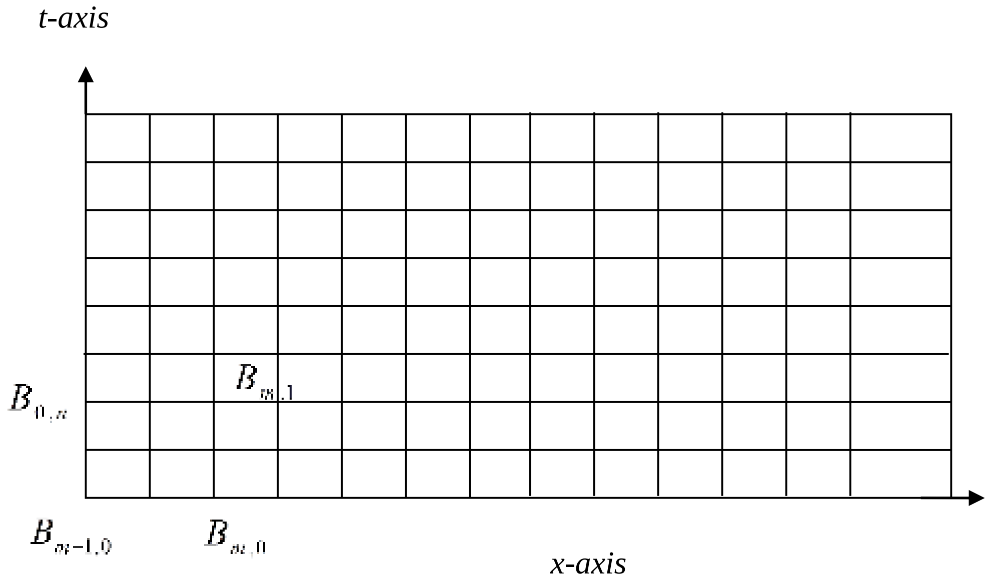

Figure 3.2: Concentration profile of $P_1(x,t)$ for 10 subdivisions with elapsed repair time

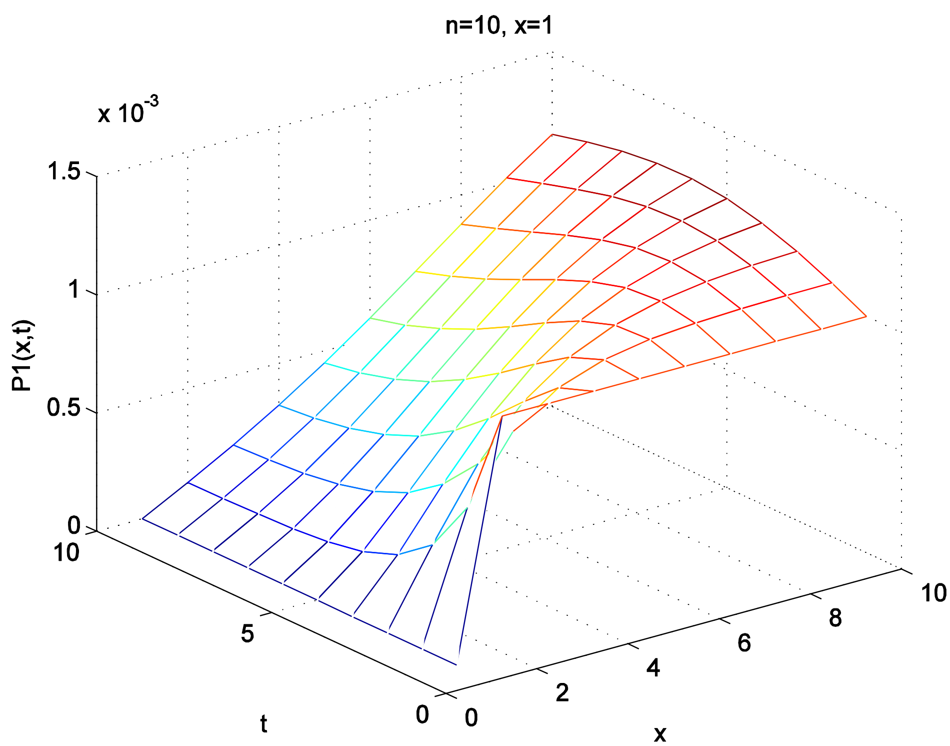

Figure 3.3: Concentration profile of $P_1(x,t)$ for 20 subdivisions with elapsed repair time

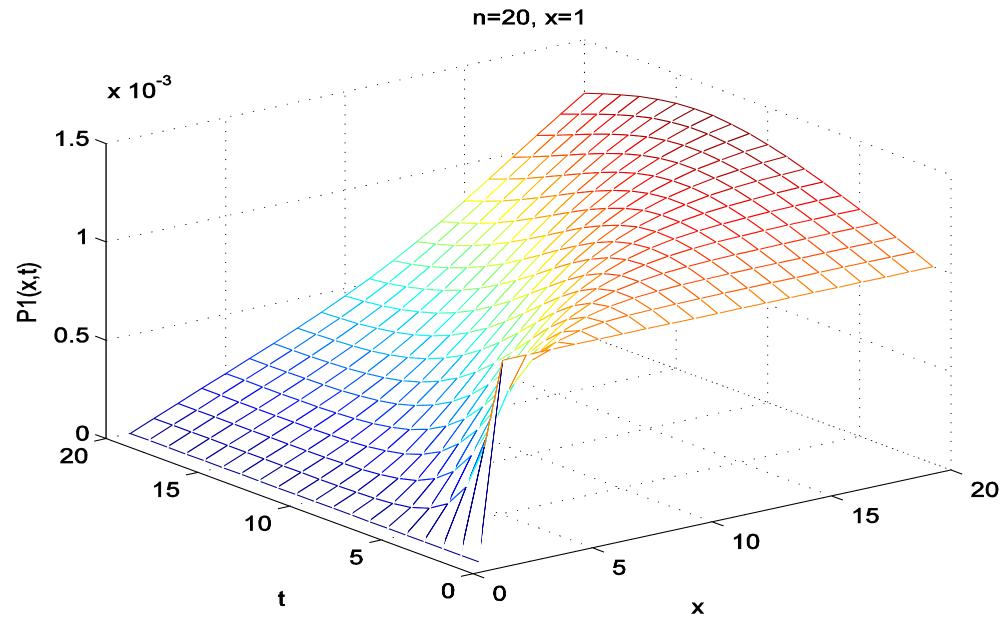

Figure 3.4 : Concentration profile of $P_1(x,t)$ for 40 subdivisions with elapsed repair time

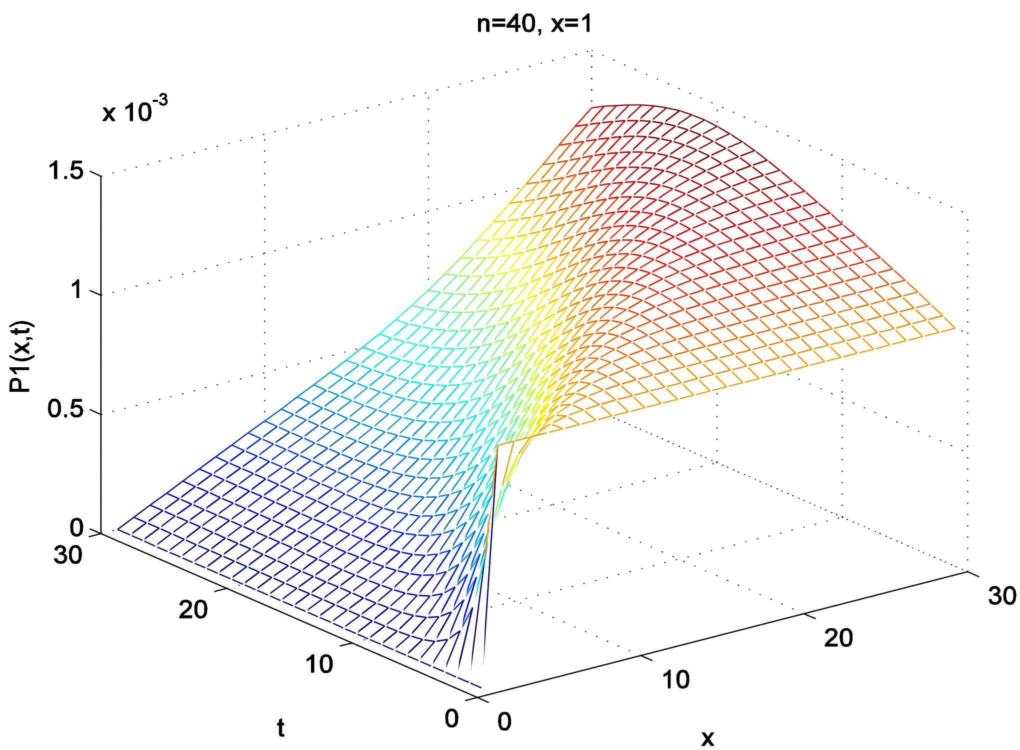

Figure 3.5: Concentration profile of $P_1(x,t)$ for 60 subdivisions with elapsed repair time

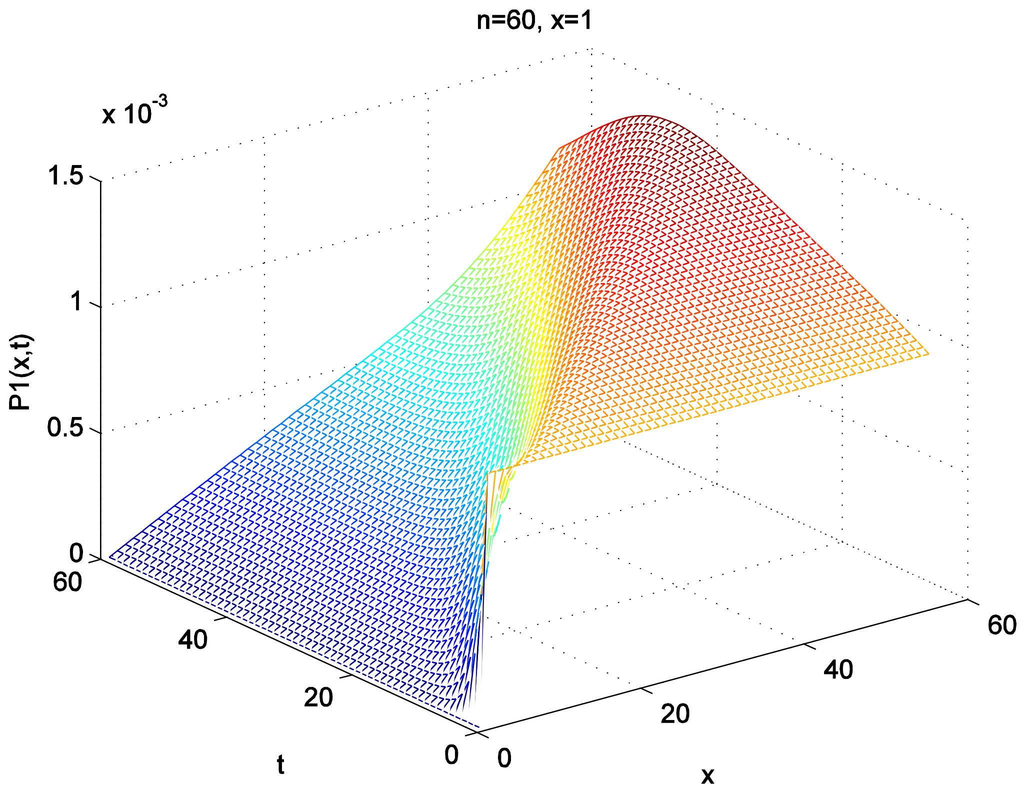

Table 3.1: Variable repair rate data for the system at given elapsed times

x	0	1	2	3	4	5	11	8	6	7
$\beta(x)$	0.04733	0.03215	0.01745	0.00396	0.00034	0.08644	0.0382	0.01269	0.03338	0.03917

Table 3.2: Numerical solution of probabilistic states using the LFDSM

TIME	*h=0.1*			*h=0.05*			*h=0.025*			*h=0.0167*		
	$P_0(t)$	$P_1(t)$	*Sum*	$P_0(t)$	$P_1(t)$	*Sum*	$P_0(t)$	$P_1(t)$	*Sum*	$P_0(t)$	$P_1(t)$	Sum
0	1	0	1	1	0	1	1	0	1	1	0	1
0.1	0.9999	0.0003	1.0002	0.9999	0.0003	1.0002	0.9999	0.0003	1.0002	0.9999	0.0003	1.0002
0.2	0.9998	0.0005	1.0003	0.9998	0.0004	1.0002	0.9998	0.0005	1.0003	0.9998	0.0005	1.0003
0.3	0.9997	0.0006	1.0003	0.9997	0.0006	1.0003	0.9997	0.0006	1.0003	0.9997	0.0006	1.0003
0.4	0.9996	0.0008	1.0004	0.9996	0.0007	1.0003	0.9996	0.0008	1.0004	0.9996	0.0008	1.0004
0.5	0.9995	0.0009	1.0004	0.9995	0.0009	1.0004	0.9995	0.0009	1.0004	0.9995	0.0009	1.0004
0.6	0.9994	0.001	1.0004	0.9994	0.0011	1.0005	0.9995	0.001	1.0005	0.9995	0.001	1.0005
0.7	0.9994	0.001	1.0004	0.9994	0.0011	1.0005	0.9994	0.0011	1.0005	0.9994	0.0011	1.0005
0.8	0.9993	0.0011	1.0004	0.9993	0.0012	1.0005	0.9993	0.0012	1.0005	0.9993	0.0012	1.0005
0.9	0.9992	0.0012	1.0004	0.9992	0.0012	1.0004	0.9992	0.0013	1.0005	0.9992	0.0013	1.0005

Chapter 4
OPTIMAL MAINTENANCE-SCHEDULING MODEL

It has been observed from literature [7, 34-36, 39-41, 47, 48, 69, 112] that stochastic model provides an efficient tool to analyze the performance of any industrial system. However, this model does not contribute much while evaluating optimal maintenance times under the constraints of availability and maintenance cost. In this chapter, an optimal maintenance-scheduling model is presented to find the maintenance times corresponding to different maintenance states of the system to maximize the profit of the system by giving constraints on availability and maintenance cost. The application of this model is demonstrated on two industrial systems, and has been discussed in Chapter 6.

This chapter is divided into two sections. In Section 4.1 the research background of the problem is discussed. The optimal maintenance-scheduling model is presented in Section 4.2.

4.1 Background of the Problem

For scheduling the maintenance of industrial system, three important criteria are frequently used in literature: (i) minimum replacement cost rate, (ii) maximum availability, (iii) lower bound on the mission reliability. By selecting anyone of these criteria, the other two criteria are generally ignored in the case of system maintenance. According to Kuo [76], this ignorance may not provide flexibility for the decision makers to finalize the maintenance scheduling of the system. All the three criteria are equally important when an optimal maintenance scheduling of the system is taken into consideration. In order to deal with this problem, the following literature has been reviewed for maintenance scheduling of the system:

Various optimization theories are available in reliability engineering for the improvement of the system performance during production and burn-in phase. This has been extensively discussed in the book authored by Kuo [76]. Recently, Billionnet [12] presented an alternative formulation to redundancy allocation problem for maximizing reliability of a series parallel system. On the other hand, behavioral studies of manufacturing systems [27, 40, 48-51, 55, 67, 73, 105, 108, 112-114, 119, 130, 139, 142, 146] based on stochastic model does not provide significant work in order to

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The contents of this chapter have been published in Quality and Reliability Engineering: Recent trends and future directions, Chapter 29: 342-352, of Allied Publishers, Bangalore, 2013. The modified form of the model presented in this chapter is communicated with a refereed Journal.

evaluate optimal maintenance times for given constraints on availability and maintenance cost. However, some of the authors [47, 52, 112-114] have taken maintenance states of a unit of the system into account to study the overall behavior of the system. To study this behavior, these authors have used preventive action and failure replacement times for modelling. Aforesaid study fails to provide an optimal maintenance times corresponding to different maintenance states of the system to maximize the profit of the system by giving constraints on availability and maintenance cost. In the following section, we have proposed an optimal maintenance-scheduling model to tackle with this problem for a five-unit system.

4.2 Optimal Maintenance-Scheduling Model

The proposed model is divided into three parts: (i) Stochastic model, (ii) Optimal model and (iii) Decision strategy. These models have been discussed as under:

4.2.1 Stochastic model

First, a stochastic model for a five-unit system has been developed under two maintenance policies − preventive and corrective maintenance. The preventive maintenance is a repair cum inspection strategy carried out to avoid the early failures by evaluating the system and its units after a fixed interval of time whereas the corrective maintenance is carried out when system shows sudden failure and requires replacement of unit to restore it back to its operating state.

The state space of stochastic process for this five-unit system with these maintenance policies is described through transition diagram (Fig. 4.1). The following assumptions are considered to develop the stochastic model:

i. All of the five units are initially in operating state.
ii. Four out of five units have four states − operating, good, preventive maintenance, and corrective maintenance. Each unit is good as new after each repair. The fifth unit undergoes through replacement either upon failure or after a fixed interval of time.
iii. Each unit has independent repair and maintenance facilities to handle preventive and corrective maintenance actions.

Following Shakuntala [112], the system of differential equations (4.2, 4.3, 4.5 and 4.6) are developed by using the transition diagram (Fig. 4.1). The probability consideration of operating and other maintenance states associated with transition diagram, as shown in Fig. 4.1, gives the following equations:

$$P_0(t+\Delta)-P_o(t)+\left(\sum_{i=1}^{5}\alpha_i-\sum_{i=1}^{4}\gamma_i(y)\right)P_o(t)\Delta=\beta_5P_5(t)\Delta+\sum_{i=1}^{4}\int(1-b)\beta_i(x)P_i(x,t)dx\Delta$$

$$+\sum_{i=1}^{4}\int(\eta_i(x)P_{5+i}(y,x,t)dxdy\,\Delta+o(\Delta), \qquad (4.1)$$

As $\Delta\to 0$, the above equation takes the form

$$P_0'(t)+\left(\sum_{i=1}^{5}\alpha_i+\sum_{i=1}^{4}\gamma_i(y)\right)P_o(t)=\sum_{i=1}^{4}\int(1-b)\beta_i(x)P_i(x,t)dx+\beta_5(x)P_5(t)$$

$$+\sum_{i=1}^{4}\int(\eta_i(x)P_{i+5}(y,x,t)dx\,dy \qquad (4.2)$$

Similarly, equation for fifth state of the transition diagram is obtained as:

$$P_5'(t)=\alpha_5P_o(t)-\beta_5P_5(t) \qquad (4.3)$$

The difference equations relating to the probability that the system is in i^{th} state (for $i=1,2,3,4$) at time $t+\Delta$ with probability of being in various states at time t is given as.

$$P_i(x+\Delta,t+\Delta)-P_i(x,t)[1-\beta(x)\Delta]=\alpha_iP_0(t)\Delta+o(\Delta) \qquad (4.4)$$

Adding and subtracting $P_i(x+\Delta,t)$ to Eq. (4.4),

$$\left[P_i(x+\Delta,t)-P_i(x,t)\right]+\left[P_i(x+\Delta,t+\Delta)-P_i(x+\Delta,t)\right]+P_i(x,t)\beta(x)\Delta=\alpha_iP_0(t)\Delta+o(\Delta) \qquad (4.5)$$

on dividing Eq. 4.5 by Δ and taking limit $\Delta\to 0$, we get

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\beta_i(x)\right)P_i(x,t)=\alpha_iP_o(t) \qquad (4.6)$$

Similarly, we obtain following equation relating the probability that system in $(5+i)^{th}$ state (for $i=1,2,3,4$).

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\eta_i(x)\right)P_{5+i}(y,x,t)=\gamma_i(y)P_o(t)+b\beta_i(x)P_i(x,t) \qquad (4.7)$$

with boundary conditions:

$$\left.\begin{aligned} &P_{5+i}(y,0,t)=\gamma_i(y)P_o(t) \\ &P_{5+i}(0,x,t)=b\beta_i(x)P_o(t) \\ &P_i(0,t)=\alpha_i P_o(t), \qquad\qquad i=1,\ldots,4 \end{aligned}\right\} \tag{4.8}$$

and initial conditions:

$$\left.\begin{aligned} &P_0(0)=1, \\ &P_i(x,0)=0, \qquad\qquad i=1,\ldots,5 \\ &P_{i+5}(x,y,0)=0, \qquad i=1,\ldots,4 \end{aligned}\right\} \tag{4.9}$$

Also,

$$P_i(t)=\int_0^\infty P_i(x,y,t)dx, \qquad i=1,\ldots,4 \tag{4.10}$$

$$P_{5+i}(t)=\int_0^\infty\int_0^\infty P_{i+5}(x,y,t)dxdy, \quad i=1,\ldots,4 \tag{4.11}$$

The solution of equations (4.2, 4.3, 4.6, and 4.7-4.11) has been used to evaluate the following performance metrics:

(i) Time dependent availability of the system is the probability of being in operating state at that instant of time and is given as:

$$A(t)=1-\sum_{i=1}^{5}P_i(t)-\sum_{i=1}^{4}P_{5+i}(t). \tag{4.12}$$

This above equation also leads to computation of system reliability when all repair rates are zero.

(ii) Long-run availability of the system

$$A_\infty=\lim_{t\to\infty}A(t)=1-\lim_{t\to\infty}\sum_{i=1}^{5}P_i(t)-\lim_{t\to\infty}\sum_{i=1}^{4}P_{5+i}(t). \tag{4.13}$$

The availability metric so obtained has been used in the optimization model as mentioned in next subsection.

4.2.2 Optimization model

In this subsection, an optimization model is proposed to maximise the total profit obtained from the output of the working five-unit system by giving constraints on maintenance cost and availability of the system. This model determines the optimal ratio of failure and maintenance repair rates of the system that maximizes total profit (T_p) by using the results of stochastic model presented in Subsection 4.2.1. The corresponding optimization model is defined as follows:

$$Max\ T_p = C_r\, A(t) - CSSF \tag{4.14}$$

subject to

$$\mathrm{L_i} \le \frac{\alpha_i}{\beta_i(x)} \le U_i, \quad l_\mathrm{i} \le \frac{\gamma_i(y)}{\eta_i(x)} \le u_i, \quad \text{for } i = 1, ..., 5. \tag{4.15}$$

when the selected values of variables satisfy the following two constraints:

i) Maintenance cost or Cost Spent on System Functioning ($CSSF$) constraint:

$$CSSF \le c, \tag{4.16}$$

where,

$CSSF = \sum_{\mathrm{i=0}}^{9} C_i P_i(t) - \text{fixed cost}$, and c is z % of the total profit obtained from the system working output. Here, z can be any arbitrary number, which is set by industry for covering maintenance cost of the system. The cost parameters (C_i) can either be assigned fixed values or can be obtained by using the maintenance cost models as discussed by Nakagawa [90]. The value of c in Eq. (4.5) is decided by the industries themselves.

ii) Availability ($A(t)$) constraint:

$$A(t) \ge a, \quad \sum_{i=0}^{9} P_i(t) = 1. \tag{4.17}$$

where, a is acceptable value of availability of system for optimal maintenance model.

The availability $A(t)$ is considered to be equivalent to $R(t)$ in the absence of maintenance or repair rates as mentioned earlier.

In the next subsection, a decision strategy is proposed by using the maintenance cost constraint of aforesaid optimization model.

4.2.3 Decision strategy

The three-step procedure of this strategy for a five-unit system is described as follows:

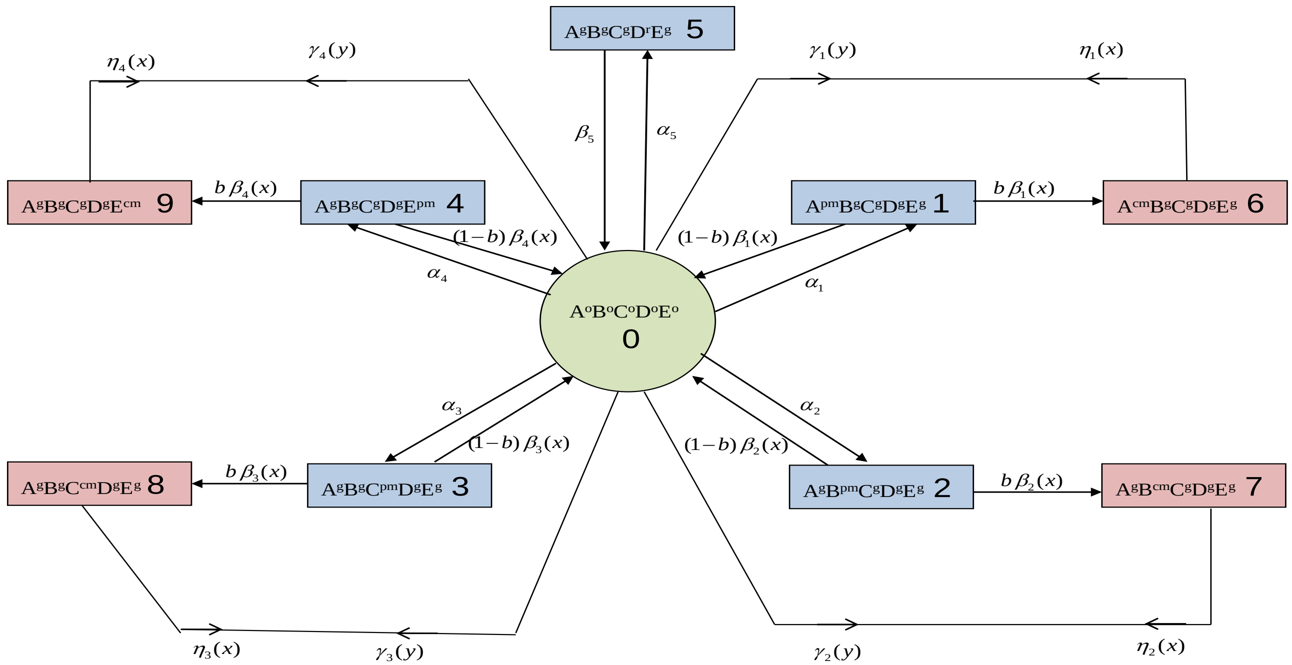
1. If CSSF is less than or equal to c , then use the ongoing maintenance schedule.
2. If not, reschedule the maintenance action by using the optimal solution of equation (4.14-4.17). That is, the ratio of the failure to maintenance rates are converted back into mean time to failure (MTTF) to mean time to maintenance of the system units. The respective time of maintenance so obtained are assigned to each unit of the system.
3. In addition, if *CSSF* cost exceeds fifty percent of overall profit of the system then replace the system.

This decision strategy helps the management of the concerned industry the following ways:

(i) for rescheduling mean time to maintenance of units of the system, and

(ii) for deciding the replacement of the system with a new system.

Figure 4.1: Transition diagram of system



In this figure, the blue states represent preventive maintenance state of units of the system while the red states show the corrective maintenance state of units for a system. The operating state of the system is represented by a green circle. The fifth unit (D) of the system is replaced after a fixed interval of time and replacement carried within fixed interval of time too. This state of the system is represented in blue color.

Chapter 5
RELIABILITY, AVAILABILITY AND MAINTAINABILITY ANALYSIS OF A FABRIC INDUSTRY

This chapter deals with the application of the APD model developed in Chapter 2 on a fabric industry to analyze its performance analysis. Further, a comparative study has also been presented between the application of proposed APD model and the traditional approach [24, 50-51, 69, 78] of evaluating the performance of industry using stochastic models.

This chapter is divided into five Sections. Section 5.1 discusses the manufacturing process of fabric in industry. The application of APD model on fabric industry for performance evaluation has been discussed in Section 5.2. In Section 5.3 a mathematical formulation of Chapman-Kolmogorov differential difference equations determining various performance parameters has also been developed and analyzed using stochastic process in traditional approach for fabric industrial production system. A comparative discussion is finally summarized in Section 5.4.

5.1 Description of Fabric Manufacturing Process

In a fabric industry, raw production material (that is, threads) is stored in a moisture free storage room to maintain the elasticity and strength of threads for consistent production. The fabric base is then prepared by winding a definite number of thread ends (in a precise order of given length) over a cylindrical beam with a warper system. Meanwhile, a mixture of polyvinyl alcohol, starch, binder, and organic softener is prepared in another system, called a mixer. This mixture is used to coat the thread on the cylindrical beam with a slasher system. The slasher wets the threads with water before passing them through the prepared mixture. Further, the threads are dried without disturbing their order. This procedure of coating is called sizing. Then, the loom system uses two cylindrical beams of coated (dry) threads to interlace the series of vertical parallel threads on the first cylindrical beam with the horizontal parallel threads on the other cylindrical beam. In this way, the final product is created (i.e., the fabric). At the end, a fabric-inspecting system is used to check the quality of the fabric before it is sent for packaging.

5.2 Performance Evaluation using APD model

The step-by-step procedure of APD algorithm (as described in Chapter 2) is described in the following subsections:

The contents of Section 5.2 have been communicated with a refereed Journal.

5.2.1 Transition diagram of the process using Boolean function

The fabric industrial process discussed in Section 5.1 has been divided into six modules, namely A, B, C, D, E, and F. The detail of these modules is given in Table 5.1. Following the approach discussed in Chapter 2, the transition diagram of the fabric industry has been constructed (as shown in Fig 5.1) by using all the possible states of the modules and the process. The state $f_{M_i}(Z_1, Z_2)$ of the i^{th} module M_i and the state $f_P(f_{M_1}, f_{M_2}, ..., f_{M_6})$ of the process are respectively defined as**,**

$$f_{M_i}(Z_1, Z_2) = Z_1 \wedge Z_2, \text{ for } i = 1, 2, ..., 6 \tag{5.1}$$

$$f_P(f_{M_1}, f_{M_2}, ..., f_{M_6}) = \bigwedge_{i=1}^{6} f_{M_i}(Z_1, Z_2) \tag{5.2}$$

where, Z_1, Z_2 are Boolean variables.

In addition, only those failure states of the industry has been considered that lead to failure of production within given time of interval for constructing the transition diagram. These states are ${}^{16}f_{M_p}$, ${}^{24}f_{M_p}$, ${}^{28}f_{M_p}$, ${}^{30}f_{M_p}$, ${}^{31}f_{M_p}$, ${}^{32}f_{M_p}$ and have been shown with bold face in Table 5.2.

5.2.2 Mathematical formulation for Assiduity Progression Diagnosis (APD)

In this subsection, the Chapman-Kolmogorov differential difference equations of the fabric industry have been developed by assuming constant failure and repair rates of each module. Considering probability of each state in the transition diagram (Fig. 5.1) and by using the mnemonic rule [131], the following system of differential equations are obtained:

$$\frac{dP_0(t)}{dt} = \sum_{i=1}^{5} \mu_i P_i(t) - \sum_{i=1}^{5} \lambda_i P_0(t) \tag{5.3}$$

$$\frac{dP_i(t)}{dt} = \lambda_i P_0(t) - \mu_i P_i(t), \text{ where } i = 1, ..., 5. \tag{5.4}$$

with initial conditions

$$P_0(0) = 1, \quad P_i(0) = 0, \text{ where } i = 1, ..., 5. \tag{5.5}$$

The solution of these equations (5.3-5.5) thus obtained are used to compute the reliability and Time Dependent Availability (TDA) of the fabric industry by using the algorithm presented in Subsection 5.2.3.

The Long-Run Availability (LRA) is computed by imposing condition $t \to \infty$ in Eq. (5.4) and this leads to following equation:

$$P_i = \delta_i\, P_0, \quad \text{where } \delta_i = \frac{\lambda_i}{\mu_i} \text{ for i = 1,2,...5} \tag{5.6}$$

The Eq. (5.6), for $i = 1,2,...,5$ is solved recursively with the normalizing condition:

$$\sum_{i=0}^{5} P_i = 1 \tag{5.7}$$

The analytic solution of long run availability (LRA) of the process thus obtained is

$$A_\infty = \frac{1}{1+\sum_{i=1}^{5}\delta_i} \tag{5.8}$$

The probabilities $P_1,\ldots,P_5$ can also be computed by using equations (5.6) and (5.8). The parameter, estimated production (EP) is thus computed as:

$$\text{E P} = \sum_{i=0}^{6} X_i P_i \tag{5.9}$$

where, X_i represents the number of fabric rolls produced by industry per hour when the system was in the i^{th} state of the transition diagram, as shown in Fig. 5.1.

Moreover, maintainability, and other parameters (like MTTR and MTBF) can also be evaluated in the similar manner as discussed in Chapter 2.

5.2.3 Computational approach for reliability, time dependent availability and target production of fabric industry

The algorithm for solving system of differential equations (5.3-5.4) with initial conditions (Eq. 5.5) is as follows:

Step 1: Input the initial time, step size, number of products manufactured per hour for each state and transition rates (i.e., failure and repair rates) of various modules of industry as shown in transition diagram (Fig. 5.1).

Step 2: Set initial distribution as $P_0(0) = 1,\ P_i(0) = 0,\ \ for\ i = 1,...,5.$

Step 3: Apply the Runge-Kutta fourth-order method on system of differential equations (5.3)-(5.4) to obtain transition probabilities $p_{00}(t),\ p_{01}(t),...,\ p_{05}(t).$

Step 4: Determine the probability of each state and check if the sum of probabilities is one. If the sum is one, then proceed to Step 5. Otherwise, check the correction of program implemented in Step 3.

Step 5: Check, if the repair rates are zero for all (or any of the states) then compute the reliability using Eq. (2.4), otherwise compute the time dependent availability using Eq. (2.5).

Step 6: Compute the estimated production of the industry by using Eq. (5.9) at time, *t=60* minutes only.

Step 7: Finally, print the output values of reliability or availability and the estimated production target of the concerned industry.

The other transition probabilities ($p_{ji}(t) = P[N(t) = i \mid N(0) = j]$) of the transition probability matrix can be obtained by replacing the step 2 with initial distribution, $P_j(0) = 1,\ P_i(0) = 0,$ $for\, i \neq j;\ i, j = 0,1,2,...,5$. Then the computations of other rows of transition probability matrix is carried out in the similar way.

Following this algorithm, performance parameters such as time dependent availability, reliability have been computed by performing 72000 iterations assuming the step size of iteration $h = 0.005$, as one minute. The results thus computed for reliability and time dependent are presented, and are shown in Fig. 5.2 and in Fig. 5.3-5.4, respectively. The results for time dependent availability have also been presented in Tables 5.6-5.10 of process industry.

Further, Geometric Average Approach (GAA) has been next implemented on failure and repair rates. For example, for module B (warping): when the failure rate of this module increased from 0.056- 1.00 % then time dependent availability (Table 5.6) of the process decreased by 0.11- 0.46 %. Therefore, by using GAA, the new values of failure rate of this module is redefined as $\lambda_{1a} = \sqrt[6]{\prod_{j=1}^{6} \lambda_{1j}}$. By observing similar kind of observations on other failure and repair rates, GAA is implemented for failure and repair rates of other modules. The detailed mathematical formulation of new failure and repair rates using GAA is shown in Table 5.3 and the corresponding values thus obtained are used and can be referred from Table 5.4. This approach is used to avoid those results of performance parameter, which reveals that the failure and repair rates variations shows insignificant change in the time dependent availability. The new values of failure and repair rate values are next used in computations of long run availability (or steady state availability) from solution of equations (5.6 -5.8), and results are shown in Table 5.4.

5.2.4 Behavioral analysis of fabric industry by using APD model

The process performance is analyzed to study the effect of module failure on reliability of the fabric industry (Fig. 5.2).

It is observed from the graph that the failure of the mixer (state 2, Fig. 5.2) and slasher module (state 3, Fig. 5.2) affects the reliability more in comparison to other remaining modules. It also indicates that the reliability of the process decreases by 0.4 -15.46 % as time varies from 30 to 1080 minutes under different absorbed states (states having repair rate equal to zero). In case, if all of the modules of the process industry fail simultaneously (a rare event, when all states are absorbed) then the reliability of the process comes out to be 0.9952 during the first thirty minutes of its operations.

Now, the MTTF is next estimated by using Eq. (2.7) where the reliability values are integrated over time 30 to 360 minutes with Simpson's one-third method. The computed result comes out to be 310 operating minutes. This indicates that the process industry continues to operate up to approximately five hours without process failure.

Time dependent availability of the process has next been computed for various choices of failure and repair rates of modules. These results are shown in Fig. 5.3-5.4 and are also presented in Tables 5.6-5.10. The computed results show that mixer module of the fabric industry affects the performance of this industry more than the other remaining modules while the fabric inspection module has less affect (Fig. 5.3). Similarly, the affects of on-going maintenance on the performance of this industry have again been computed by varying repair rates of each modules and are shown in Fig. 5.4. These results indicate that performance parameter is more affected by the mixer and slasher modules of the process industry than the remaining modules.

It has been found that there is a marginal change upto two decimal places in the computed results of the time dependent availability of the process industry with varying failure and repair rates of the modules. In order to average out this marginal change, the new values of failure and repair rates are evaluated by using Geometric Average Approach (GAA), formulations for which are presented in Table 5.3. The computations of long run availability have been carried out with these new transition rates and the results are shown in Table 5.4. It is observed that LRA of the process decreases from 0.729% to 16.3 % with respect to change in failure and repair rates of the mixer and sizing modules. Results further reveal that the mixer module affects the LRA of industry more than the sizing module.

The MTTR of the process is next computed by using Eq. (2.8) and it comes out 0.43 per hour. This result implies that the average repair rate of the process industry is approximately two hours and twenty minutes when any of the module fails. By using this parameter, the maintainability of the process has also been computed with Eq. (2.10). The computed results as shown in Table 5.5, reveal that there is a 99 % chance of successful repair of the process.

The process industry produces 180 fabric rolls per hour with different production lines. Each roll contains 5 meters of cloth. Here, it is mentioned that industry also imports some fabric rolls from different production units to produce 180 fabric rolls. As per the rough estimates provided by the management of the concerned fabric industry, we assign $X_0 = 180$, $X_1 = 170$, $X_2 = 178$, $X_3 = 157$, $X_4 = 164$ and $X_5 = 173$. The expected production target of the process industry is approximated to be 179 rolls per working hour based on Eq. (5.9). Therefore, we conclude that an industry can produce approximately 179 fabric rolls per hour of operation with 99% reliable production under the current maintenance conditions.

The next section deals with performance analysis of the same fabric industry by using a stochastic model in traditional approach.

5.3 Performance Modelling using Stochastic Model in Traditional Approach

In fact, APD model discussed in Section 5.2 consists of 212 numbers of systems and are further divided into six modules. For each system three kinds of working states: Operating, failure and availability of raw material have been assumed. If a stochastic model for performance analysis of this industry is to be developed (considering all factors as in APD model), then stochastic model will have all the 3^{212} states in its state space. Consequently, the order of the matrix associated with a system of linear differential equation (Chapman-Kolomogorov differential equation) will be $3^{212} \times 3^{212}$ and thus making it very difficult to solve such a complex mathematical problem.

In order to simplify this complex model, it has been shown in this section that how the stochastic model is used to study the performance of fabric industry in a traditional approach. In view of the traditional approach, a production line has been used as a system and this system has further been divided into subsystem. As per the assumption on used by Gupta *et al.* [50, 51], Lal *et al.*

The contents of Section 5.3 have been published in Journal of Reliability and Statistical Studies, 6(2):51-163, 2013.

[78], Shakuntla [112], we have also assumed that some of the system can have three states namely operating, failed and reduced states. However, availability of raw material state has not been considered in this case (as assumed in the case of APD model).

The fabric production system is divided into six subsystems namely A, B, C, D, E, and F. The brief description of these subsystems is as follows:

1. Subsystem A is a storage room that kept the raw material of the fabric for favorable production.
2. Subsystem B is a machining operation for preparation of base for a woven fabric.
3. Subsystem C is mixer system used to prepare a mixture. The mixture is utilized to coat the threads of the base prepared to give further strength to its threads.
4. Subsystem D is a machining process of coating threads of the base of the woven fabric. This process is known as sizing.
5. Subsystem E is machining process for weaving fabric to interlacing a series of vertical parallel threads of a base with horizontal parallel threads of other base placed horizontally to the earlier one.
6. Subsystem F is machining process of inspecting fabric to check the quality of the fabric before packaging.

For each subsystem, two kinds of failure i.e. major and minor have been considered. Minor failure are those which can be repaired during the working conditions (reduced states) while the major failure are those in which system shows complete breakdown. The subsystem C, E and F are subject to major failure while subsystems B and D are subject to both major and minor failures. The subsystem, A is considered to have no failure.

5.3.1 Transition Diagram

The transition diagram for this system is shown in Fig. 5.6. The following notations are considered for the transition diagram.

A, B, C, D, E, F : Represent that systems are working in good states.

$\overline{B}, \overline{D}$: Subsystems B and D are working with reduced capacity

λ_i (i=1,$\cdots$,7) : Failure rates of the subsystems $C, E, F, B, D, \bar{B}, \bar{D}$ respectively

μ_i (i=1,$\cdots$, 7) : Repair rates of subsystems $C, E, F, B, D, \bar{B}, \bar{D}$ respectively

b, c, d, e, f : The failed state of subsystems B, C, D, E, and F, respectively.

The following assumptions are considered in order to carry out the performance analysis for maintenance scheduling of fabric production system through stochastic model:

i. Repairs and failures are statistically independent with each other.
ii. Units of repair and failure rates are taken as per day.
iii. There is no simultaneous failure among the sub systems.
iv. Subsystems B and D can fail completely only through reduce states. Repair of these subsystems is allowed in reduced state only up to a certain limit.
v. Repair is carried out when the system is in the reduced or failed state.
vi. Component based preventive maintenance for each subsystem is carried to avoid frequent failures within.
vii. Repaired components are treated as new components.

5.3.2 Mathematical Formulation

In order to obtain transition probabilities for estimating time dependent availability of fabric production system, Chapman-Kolmogorov differential difference equations have then been formulated by using mnemonic rule on the transition diagram shown in Fig. 5.6. The mathematical formulation has been carried out in both transient and steady states as discussed in the next subsection.

5.3.3 Transient state for time dependent availability analysis

The following system of linear differential equation is obtained at time $t+\Delta t$ using the mnemonic rule with probability consideration of various states shown in the transition diagram as shown in Fig. 5.6. The differential equation for the state one is obtained as:

$$P_1(t+\Delta t) = [1-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)]\Delta t\, P_1(t) + \mu_1 \Delta t\, P_5(t) + \mu_2 \Delta t\, P_6(t) +$$
$$\mu_3 \Delta t\, P_7(t) + \mu_4 \Delta t\, P_2(t) + \mu_5\, \Delta t\, P_3(t) + \mu_6 \Delta t\, P_{14}(t) + \mu_7\, \Delta t\, P_{15}(t) \quad (5.10)$$

Dividing (5.10) both sides by Δt and taking limit as $\Delta t \to 0$, we get

$$\frac{d\,P_1(t)}{dt} + \alpha_1 P_1(t) = \mu_1 P_5(t) + \mu_2 P_6(t) + \mu_3 P_7(t) + \mu_4 P_2(t) + \mu_5 P_3(t) + \mu_6 P_{14}(t) + \mu_7 P_{15}(t) \quad (5.11)$$

Similarly, system of differential equations for other states is given as:

$$\frac{d\,P_2(t)}{dt}+\alpha_2 P_2(t)=\mu_1 P_8(t)+\mu_2 P_9(t)+\mu_3 P_{10}(t)+\lambda_4 P_1(t)+\mu_7 P_{17}(t) \tag{5.12}$$

$$\frac{d\,P_3(t)}{dt}+\alpha_3 P_3(t)=\mu_1 P_{11}(t)+\mu_2 P_{12}(t)+\mu_3 P_{13}(t)+\lambda_5 P_1(t)+\mu_6 P_{16}(t) \tag{5.13}$$

$$\frac{d\,P_4(t)}{dt}+\alpha_4 P_4(t)=\lambda_5 P_2(t)+\mu_3 P_{20}(t)+\mu_1 P_{18}(t)+\mu_2 P_{19}(t)+\lambda_4 P_3(t) \tag{5.14}$$

$$\frac{d\,P_{4+i}(t)}{dt}+\mu_i P_{4+i}(t)=\lambda_i P_1(t), \qquad i=1,2,3 \tag{5.15}$$

$$\frac{d\,P_{7+i}(t)}{dt}+\mu_i P_{7+i}(t)=\lambda_i P_2(t), \qquad i=1,2,3 \tag{5.16}$$

$$\frac{d\,P_{10+i}(t)}{dt}+\mu_i P_{10+i}(t)=\lambda_i P_3(t), \qquad i=1,2,3 \tag{5.17}$$

$$\frac{d\,P_{14}(t)}{dt}+\mu_6 P_{14}(t)=\lambda_6 P_2(t) \tag{5.18}$$

$$\frac{d\,P_{15}(t)}{dt}+\mu_7 P_{15}(t)=\lambda_7 P_3(t) \tag{5.19}$$

$$\frac{d\,P_{16}(t)}{dt}+\mu_6 P_{16}(t)=\lambda_6 P_4(t) \tag{5.20}$$

$$\frac{d\,P_{17}(t)}{dt}+\mu_7 P_{17}(t)=\lambda_7 P_4(t) \tag{5.21}$$

$$\frac{d\,P_{17+i}(t)}{dt}+\mu_i P_{17+i}(t)=\lambda_i P_4(t), \qquad i=1,2,3 \tag{5.22}$$

where,

$$\alpha_1=\sum_{i=1}^{5}\lambda_i;\quad \alpha_2=\sum_{\substack{i=1,\\ i\neq 4}}^{6}\lambda_i+\mu_4;\quad \alpha_3=\sum_{i=1,}^{4}\lambda_i+\mu_5+\lambda_7;\quad \alpha_4=\lambda_1+\lambda_2+\lambda_3+\lambda_7+\lambda_6$$

with the initial conditions:

$$P_i(0)=\begin{cases}1, & \text{if } i=1\\ 0, & \text{otherwise}\end{cases} \tag{5.23}$$

The system of differential equations (5.11-5.22) is called Chapman-Kolmogorov differential difference equations. Time dependent availability $A(t)$ of the system in the transient state can be computed by using the relation

$$A(t) = \sum_{i=1}^{4} P_i(t) \tag{5.24}$$

5.3.4 Steady state for long run availability analysis

Steady state probability of the subsystems can be obtained by imposing the restriction $t \to \infty$ on the system of linear differential equations (5.11-5.22). By doing so, the system of linear differential equations (5.11-5.22) reduces to the following system of homogeneous linear equations:

$$\alpha_1 P_1 = \mu_1 P_5 + \mu_2 P_6 + \mu_3 P_7 + \mu_4 P_2 + \mu_5 P_3 + \mu_6 P_{14} + \mu_7 P_{15} \tag{5.25}$$

$$\alpha_2 P_2 = \mu_1 P_8 + \mu_2 P_9 + \mu_3 P_{10} + \lambda_4 P_1 + \mu_7 P_{17} \tag{5.26}$$

$$\alpha_3 P_3 = \mu_1 P_{11} + \mu_2 P_{12} + \mu_3 P_{13} + \lambda_5 P_1 + \mu_6 P_{16} \tag{5.27}$$

$$\alpha_4 P_4 = \lambda_5 P_2 + \mu_3 P_{20} + \mu_1 P_{18} + \mu_2 P_{19} + \lambda_4 P_3 \tag{5.28}$$

$$\mu_i P_{4+i} = \lambda_i P_1, \quad i = 1, 2, 3 \tag{5.29}$$

$$\mu_i P_{7+i} = \lambda_i P_2, \quad i = 1, 2, 3 \tag{5.30}$$

$$\mu_i P_{10+i} = \lambda_i P_3, \quad i = 1, 2, 3 \tag{5.31}$$

$$\mu_6 P_{14} = \lambda_6 P_2 \tag{5.32}$$

$$\mu_7 P_{15} = \lambda_7 P_3 \tag{5.33}$$

$$\mu_6 P_{16} = \lambda_6 P_4 \tag{5.34}$$

$$\mu_7 P_{17} = \lambda_7 P_4 \tag{5.35}$$

$$\mu_i P_{17+i} = \lambda_i P_4, \quad i = 1, 2, 3 \tag{5.36}$$

The solution of the systems of homogenous linear equations (5.25-5.36) are finally used to study the long run availability (A_∞) in the steady state for fabric production system by computing the following relation

$$A_\infty = \sum_{i=1}^{4} P_i \qquad (5.37)$$

In order to obtain time dependent availability of the fabric production system, the results are approximated numerically using Runge-Kutta fourth order method following the work of [50, 51 78] for the system of linear differential difference equations (5.11-5.22) with initial conditions (Eq. 5.23), assuming the step size of iteration as, $h = 0.005$, as one day. This leads to computations of time dependent availability (Eq. 5.24) from $t = 30$ to $t = 360$ days for various choices of failure rates of the subsystems. The computed results are shown in Fig. 5.7. The data used for failure and repair rate parameters is presented in Table 5.11. In order to find the optimal choices for failure and repair rates of this sensitive system, the behavior analysis (in terms of long run availability) is presented below:

Next, the system of homogeneous linear equations (5.25-5.36) has been solved numerically by using Gauss elimination method with partial pivoting method. In order to obtain probability values of each state using this method one of the equations of the system (5.25-5.36) is replaced with normalizing condition to avoid the trivial solution. The normalizing condition is given as:

$$\sum_{i=1}^{20} P_i = 1 \qquad (5.38)$$

Once unknown $P_i(t),\ i = 1,...,20$ are computed, the long run availability or steady state availability A_∞ is obtained by using Eq. (5.37) for various combination of failure and repair rates of the subsystems.

5.3.5 Behavioral Analysis of fabric production system using traditional approach

By varying the failure rate λ_1 of subsystem C and keeping other parameters values fixed (as shown in Table 5.11), time dependent availability for one year has been computed with the interval of 30 days. The results of this subsystem are shown in Fig. 5.6*a*. Similarly, by varying the failure rate $\lambda_i,\ i = 2,...,7$ of other subsystems separately and taking other parameters fixed (as given in Table 5.11), the time dependent availability of the fabric production system have also been computed. The results thus obtained are shown in Fig. 5.6 (*b*-*g*). The results exhibited in figures (5.6*a*-5.6*g*) reveal that the warping and mixer are the only sensitive subsystems in comparison to other remaining subsystems.

The results has been computed for long run availability of the production line by varying parameters of both the sensitive subsystems and are presented in Table 5.12-5.14 for B and C, respectively. From Table 5.12, it has been observed that the optimal choices of pair wise failure and repair rate of subsystem B (i.e., λ_4, μ_4) are [(0.005536, 1.69999), (0.005557, 2.9), (0.005794, 1.69999), (0.005794, 2.9)] corresponding to required long-run availability of the system [0.98960, 0.98960, 0.98960, and 0.98960]. From Table 5.13, it is noticed that for maintaining maximum availability of the system, the optimal choices for pair wise failure and repair rates of subsystem B (i.e., λ_6, μ_6) are [(0.002092, 2.73), (0.00185, 2.73), (0.002092, 1.533), (0.00185, 1.53)] corresponding to the required long run availability [0.98965, 0.98967, 0.98962, 0.98963].

Similarly, for subsystem C, optimal choices for pair-wise combination of failure and repair rate, i.e. (λ_1, μ_1) [(0.001329, 0.5), (0.00139, 0.50024), (0.00139, 0.512, 0.524)] are obtained from Table 5.14 corresponding to required long run availability are [0.98960, 0.98960, 0.98966, 0.98972].

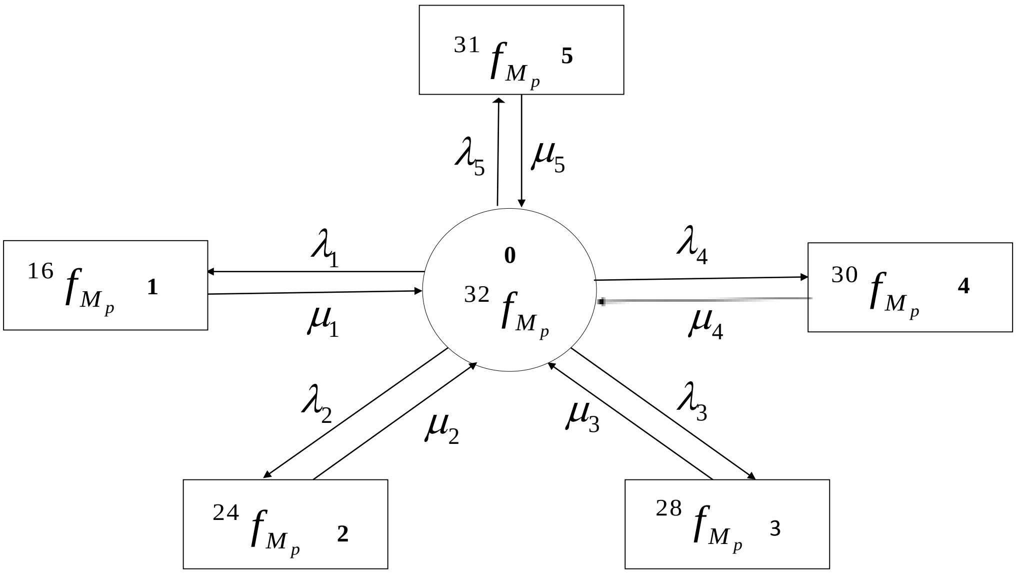
5.4 Comparative Discussion

In this section, a comparative study between the two methodologies discussed in Section 5.2 and 5.3 have been critically analyzed.

S.no	APD Model	Traditional Approach
1	This method is based on the performance of all the working systems in industry.	In this approach, performance of only those systems is considered which are working in a production line.
2	This method leads to a simple model for performance analysis in spite of high level of complexity (as discussed in Section 5.3).	Level of complexity is reduced in this method but it gives complex mathematical model for the performance analysis.
3	The number of equations required to analyze the performance of the system is six only.	This approach requires twenty equations even after reducing its complexity.
4	The solution of the system of equations obtained by this method can easily be obtained either analytically or by using any of the numerical method.	The analytical method for solving the system of equations, obtained by using traditional method, is not so simple. Even numerical method is time consuming in this case.

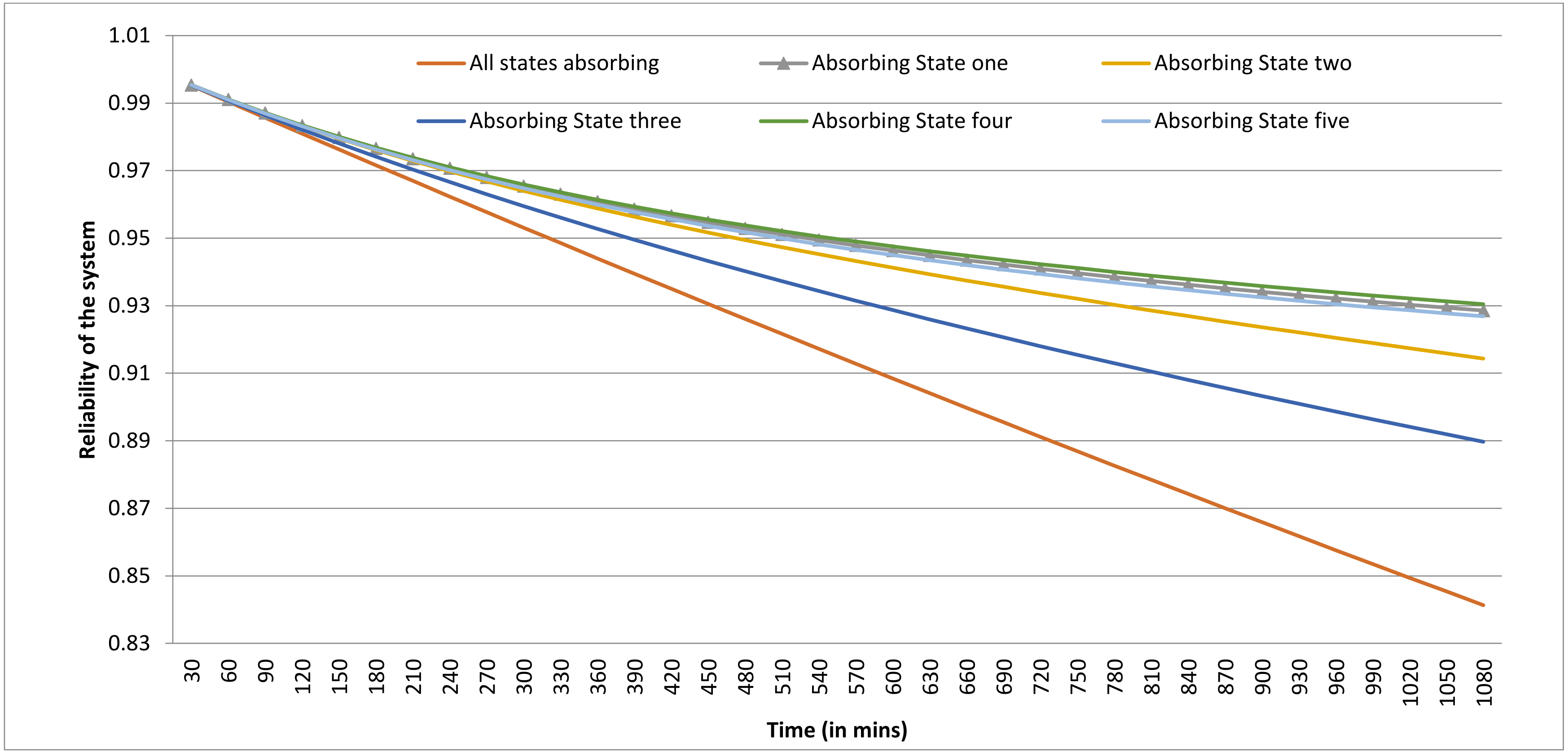
5	The availability of raw material is considered at each module level for achieving the desired production target.	This method completely ignores the availability of raw material.
6	The advantage of using this approach is to analyze various performance parameter of industry such as MTTF, MTTR, reliability, three kinds of availability and maintainability	This approach can also be used to analyze various performance parameters but most of the authors have used the availability parameters for performance analysis of the system. However, some authors have also considered MTTR and reliability as performance parameters.
7	This model provides target estimation parameter to estimate average production within existing maintenance condition	The earlier studies of the performance analysis in this approach do not considered the target estimation parameter.

Figure 5.1: State transition diagram of the fabric industry based on Table 5.1-5.2



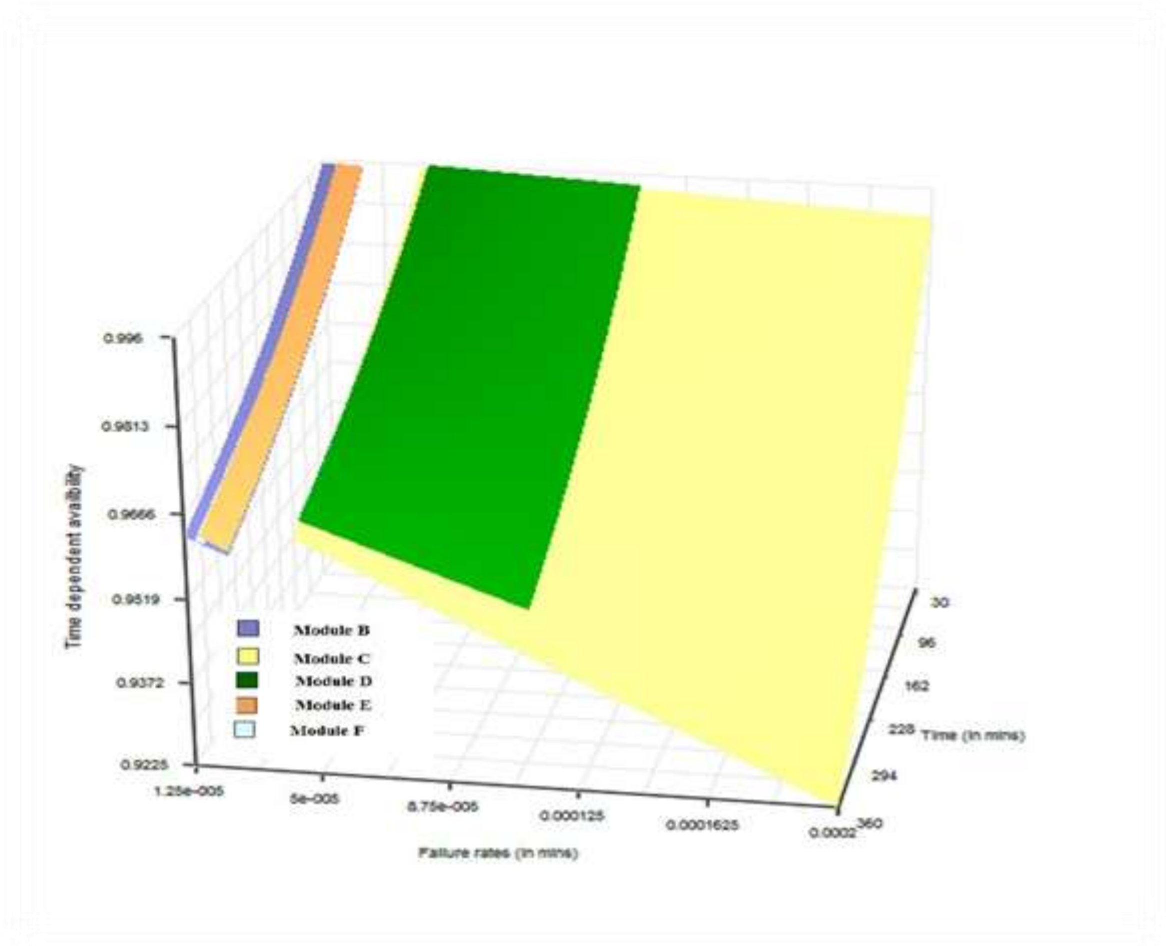
The failed states of the process are represented by rectangles and are numbered from one to five. The operating state of the process (when all modules are working satisfactorily) is denoted by a zero and symbolized by a circle. The constant failure and repair rates of the modules are represented by λ_i and μ_i $(i = 1,...,5)$, respectively, for modules *B*, *C*, *D*, *E*, and *F*

Figure 5.2: Effects of absorbing states (repair rate is zero) on the reliability of the process



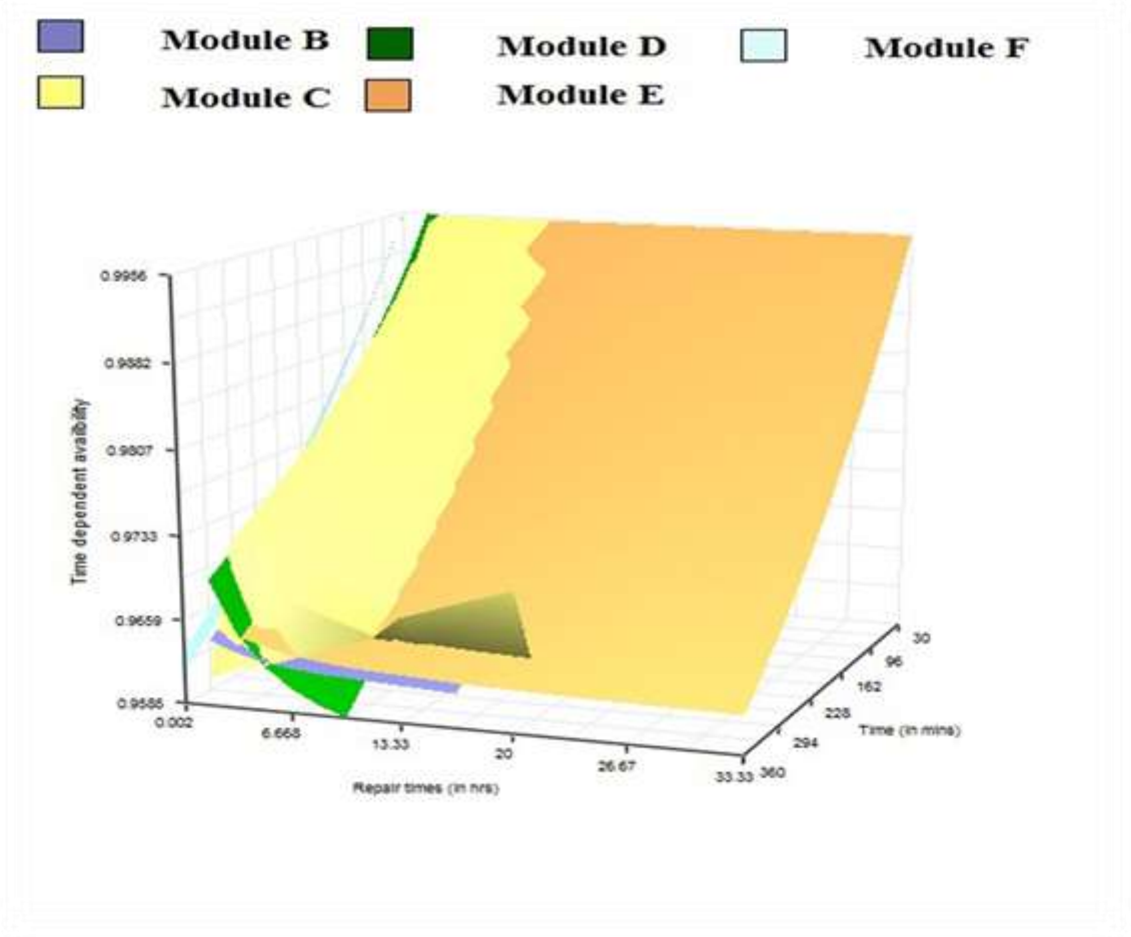
In this figure, the i^{th} absorbing state represents that process transits from operating state to i^{th} state of the transition diagram and it fails to return in operating state after entering in the i^{th} state. The value of *i* can be 1, 2,3,4,5. All states are absorbing when modules have zero repair rates.

Figure 5.3: Effect of varying modules failure rates on the time dependent availability of the process



The colour, which covers the maximum surface area, represents the greater effects of module failure on the TDA. The detailed computational values given in Table 5.6-5.10 for failure rates variations represented by λ_{ij} in the each table.

Figure 5.4: Effect of varying modules repair times on the time dependent availability of the process



The colour, which covers the maximum surface area, represents the greater effects of module failure on the TDA. The detailed computational values given in Table 5.6 - 5.10 for failure rates variations represented by μ_{ij} in the each table.

Figure 5.5 : Transition diagram of fabric production system

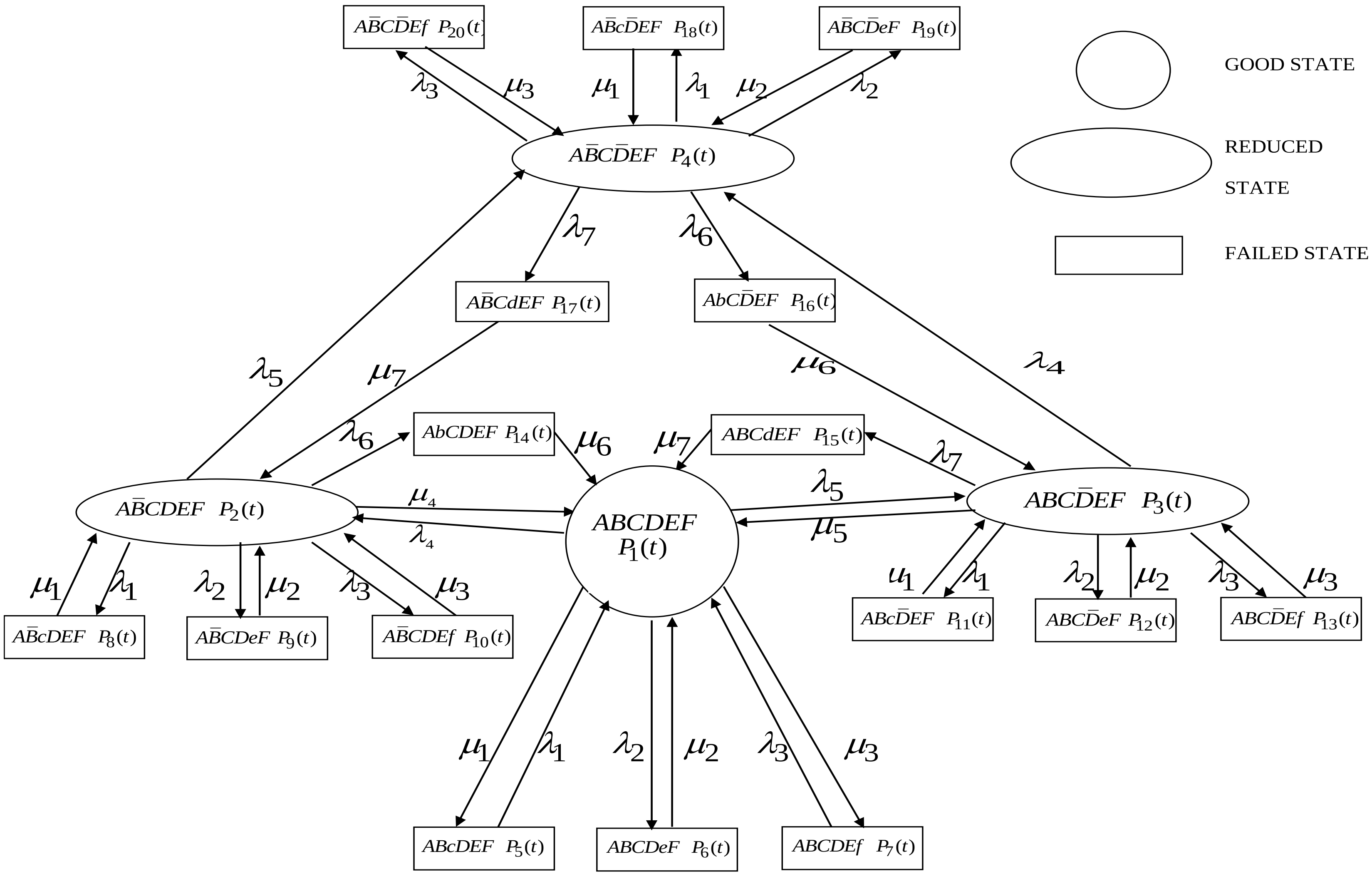


Figure 5.6 Effect of failure rates on time dependent availability of fabric production system

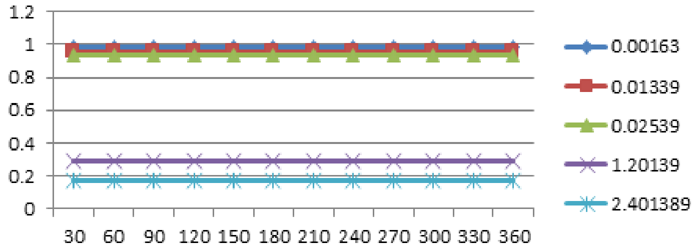


Figure *a*: Failure rate (λ_1)

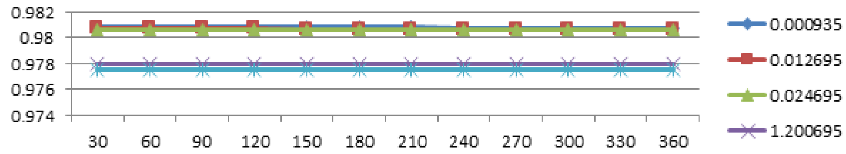


Figure *b*: Failure rate (λ_2)

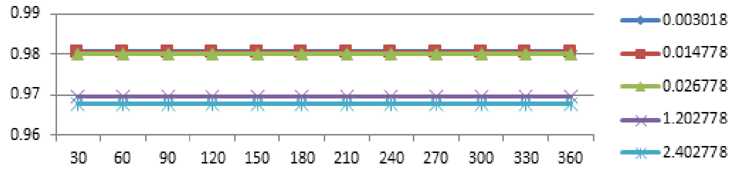


Figure *c*: Failure rate (λ_3)

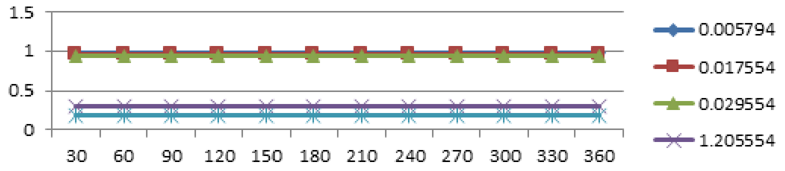


Figure *d*: Failure rate (λ_4)

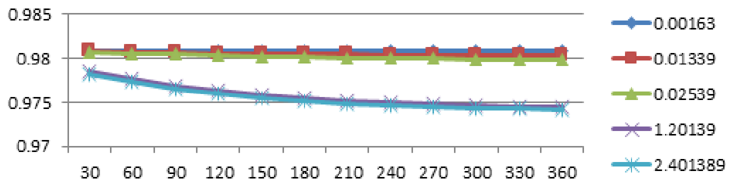


Figure *e*: Failure rate (λ_5)

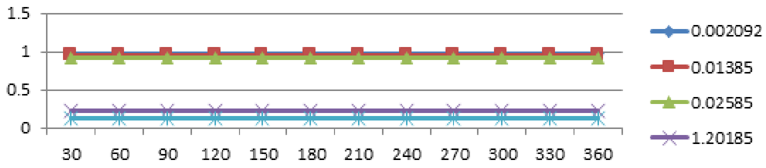


Figure *f*: Failure rate (λ_6)

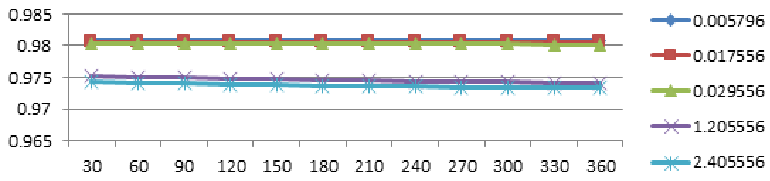


Figure *g*: Failure rate (λ_7)

Table 5.1: Details of modulewise process description with assumptions used to construct transition diagram (Figure 5.1)

Modules and their Notations	**Functioning of the Modules**	**No. of systems in each Module**	**Operational Assumptions ***	**Failure Causes for each Module**
Storage Room (A)	Stores the raw material	3	Never fails	Fire and Moisture effecting strength of thread
Warper (B)	Prepare base for fabric	3	Failed when two warper systems fails out of three	Component based failure like stop motion, scissors and electric circuit break-up within system
Mixture (C)	IT prepares mixtures to strengthen the thread	1	Failed when any failure cause occur	Failure of cylinder, cooker, motor, pressure gauge, safety valves
Slasher (D)	It is process of coating the thread with mixture and then drying the threads	3	Two out of three system module	Failure of dryers, automatic control systems, press rollers, belts, beams, motors, hydraulic system
Loom(E)	It interlaces threads to woven fabric as final product.	200	180 out of 200 systems module	Thread breakage due to excess speed, any jam due to fibers, any internal circuit damage
Fabric Inspection System (F)	Check the final product quality	2	Failed when both systems failed together	Glass breakage, sensitivity failure of sensors, other internal failures within system
*General Assumptions: Some other assumptions in relation to above that repair and failures are statistically independent with each other. There are no simultaneous failures among the modules and each module has skilled maintenance personals. Plant considered to be failed with failure of any module.				

Table 5.2: The Boolean table for possible states for process state

Possible states, j=	f_{M_1}	f_{M_2}	f_{M_3}	$\cdot f_{M_4} \cdot$	f_{M_5}	f_{M_6}	$^{j} f_{Mp}$
1	1	0	0	0	0	0	0
2	1	0	0	0	0	1	0
3	1	0	0	0	1	0	0
4	1	0	0	0	1	1	0
5	1	0	0	1	0	0	0
6	1	0	0	1	0	1	0
7	1	0	0	1	1	0	0
8	1	0	0	1	1	1	0
9	1	0	1	0	0	0	0
10	1	0	1	0	0	1	0
11	1	0	1	0	1	0	0
12	1	0	1	0	1	1	0
13	1	0	1	1	0	0	0
14	1	0	1	1	0	1	0
15	1	0	1	1	1	0	0
16	**1**	**0**	**1**	**1**	**1**	**1**	**0**
17	1	1	0	0	0	0	0
18	1	1	0	0	0	1	0
19	1	1	0	0	1	0	0
20	1	1	0	0	1	1	0
21	1	1	0	1	0	0	0
22	1	1	0	1	0	1	0
23	1	1	0	1	1	0	0
24	**1**	**1**	**0**	**1**	**1**	**1**	**0**
25	1	1	1	0	0	0	0
26	1	1	1	0	0	1	0
27	1	1	1	0	1	0	0
28	**1**	**1**	**1**	**0**	**1**	**1**	**0**
29	1	1	1	1	0	0	0
30	**1**	**1**	**1**	**1**	**0**	**1**	**0**
31	**1**	**1**	**1**	**1**	**1**	**0**	**0**
32	**1**	**1**	**1**	**1**	**1**	**1**	**1**

Table 5.3: The failure and repair rates using geometric average approach (GAA)

Module	Failure rates		Repair rates	
	Formulae using Geometric average Approach	Variations in TDA	Formulae using Geometric average Approach	Variations in TDA
B	$\lambda_{1a} = \sqrt[6]{\prod_{j=1}^{6} \lambda_{1j}}$	0.002-0.036 %	$\mu_{1a} = \sqrt[6]{\prod_{j=1}^{6} \mu_{1j}}$	0.003-0.005%
C	$\lambda_{2a} = \sqrt[3]{\prod_{j=1}^{3} \lambda_{2j}}$, $\lambda_{2b} = \sqrt[3]{\prod_{j=4}^{6} \lambda_{2j}}$, $\lambda_{2c} = \sqrt[6]{\prod_{j=1}^{6} \lambda_{2j}}$	0.012-0.035 %, 0.110-0.458% and 0.012-0.458 %	$\mu_{2a} = \sqrt[3]{\prod_{j=1}^{3} \mu_{2j}}$, $\mu_{2b} = \sqrt[3]{\prod_{j=4}^{6} \mu_{2j}}$, $\mu_{2c} = \sqrt[6]{\prod_{j=1}^{6} \mu_{2j}}$	0.009-0.012%, 0.016-0.017%, and 0.009-0.017%.
D	$\lambda_{3a} = \sqrt[4]{\prod_{j=1}^{4} \lambda_{3j}}$, $\lambda_{3b} = \sqrt[2]{\prod_{j=5}^{6} \lambda_{3j}}$	0.013-0.028% and 0.06-0.186 %	$\mu_{3a} = \sqrt[3]{\prod_{j=1}^{3} \mu_{3j}}$, $\mu_{3b} = \sqrt[3]{\prod_{j=4}^{6} \mu_{3j}}$, $\mu_{3c} = \sqrt[6]{\prod_{j=1}^{6} \mu_{3j}}$	0.013-0.018%, 0.02-0.023 % and 0.013-0.023 %
E	$\lambda_{4a} = \sqrt[6]{\prod_{j=1}^{6} \lambda_{4j}}$	0.001-0.024%	$\mu_{4c} = \sqrt[6]{\prod_{j=1}^{6} \mu_{4j}}$	0.002-0.003%
F	$\lambda_{5a} = \sqrt[6]{\prod_{j=1}^{6} \lambda_{5j}}$	0.002-0.014%	$\mu_{5c} = \sqrt[6]{\prod_{j=1}^{6} \mu_{5j}}$	0.002-0.005%

The change in TDA up to two decimal places considered as marginal percentage change to explain the application of GAA. In addition, in this table, the constant failure and repair rate of i^{th} state of transition diagram for j^{th} value is represented by λ_{ij} and μ_{ij}.

Table 5.4: Long-run availability analysis using GAA failure and repair rates computations (shown in Table 5.3)

		λ_{1a}=1.634E-05, μ_{1a}=0.00307, λ_{5a}=1.49E-05, μ_{4a}=1.973E-05, μ_{5a}=0.00421					
		λ_{3a}=5.39E-05			λ_{3b}=9.62E-05		
		λ_{2a} =4.86E-05	λ_{2b}=1.28E-04	λ_{2c}=7.88E-05	λ_{2a}=4.86E-05	λ_{2b}=1.28E-04	λ_{2c}=7.88E-05
μ_{2a}=5.50E-03	μ_{3a}=5.50E-03	0.9599	0.9468	0.9549	0.9529	0.9400	0.9479
	μ_{3b}=2.03E-03	0.9447	0.9321	0.9398	0.9264	0.9143	0.9218
	μ_{3c}=3.34E-03	0.9541	0.9412	0.9491	0.9427	0.9301	0.9379
μ_{2b}=2.50E-03	μ_{3a}=5.50E-03	0.9297	0.8734	0.9074	0.9231	0.8676	0.9011
	μ_{3b}=2.03E-03	0.9155	0.8608	0.8938	0.8983	0.8456	0.8775
	μ_{3c}=3.34E-03	0.9243	0.8686	0.9022	0.9136	0.8592	0.8920
μ_{2c}=1.14E-03	μ_{3a}=5.50E-03	0.9503	0.9256	0.9395	0.9434	0.9161	0.9011
	μ_{3b}=2.03E-03	0.9354	0.9115	0.9249	0.9174	0.8916	0.9074
	μ_{3c}=3.34E-03	0.9352	0.9541	0.9599	0.9285	0.9472	0.9529

Table 5.5: Maintainability of the process

Time in hrs.	1	2	3	4	5	6	7	8	9	10
Maintainability	0.419	0.662	0.804	0.886	0.934	0.961	0.978	0.987	0.992	0.996

Table 5.6: Effects of failure and repair rate variations of module B on time dependent availability of process

	λ_{1i}						μ_{1i}					
$i=$	1	2	3	4	5	6	1	2	3	4	5	6
Time (in mins)	1.25E-05	1.32E-05	1.39E-05	1.67E-05	0.00002	0.000025	0.01000	0.00500	0.00333	0.00250	0.00200	0.00100
30	0.99543	0.99541	0.99539	0.99531	0.99521	0.99507	0.99543	0.99540	0.99540	0.99539	0.99539	0.99538
60	0.99121	0.99117	0.99114	0.99098	0.99080	0.99052	0.99128	0.99119	0.99116	0.99114	0.99113	0.99110
90	0.98732	0.98726	0.98721	0.98699	0.98672	0.98632	0.98750	0.98732	0.98725	0.98721	0.98718	0.98713
120	0.98371	0.98364	0.98357	0.98329	0.98295	0.98244	0.98404	0.98376	0.98364	0.98357	0.98353	0.98344
150	0.98038	0.98029	0.98021	0.97987	0.97946	0.97885	0.98085	0.98047	0.98030	0.98021	0.98014	0.98001
180	0.97728	0.97718	0.97708	0.97669	0.97622	0.97552	0.97792	0.97744	0.97721	0.97708	0.97700	0.97681
210	0.97441	0.97429	0.97419	0.97374	0.97322	0.97243	0.97521	0.97464	0.97435	0.97419	0.97408	0.97383
240	0.97173	0.97161	0.97149	0.97101	0.97043	0.96956	0.97270	0.97204	0.97170	0.97149	0.97135	0.97104
270	0.96925	0.96911	0.96898	0.96846	0.96783	0.96689	0.97037	0.96964	0.96923	0.96898	0.96882	0.96844
300	0.96693	0.96679	0.96665	0.96609	0.96541	0.96440	0.96821	0.96740	0.96694	0.96665	0.96645	0.96600
330	0.96477	0.96462	0.96447	0.96387	0.96316	0.96209	0.96619	0.96532	0.96480	0.96447	0.96424	0.96371
360	0.96275	0.96259	0.96244	0.96181	0.96106	0.95993	0.96431	0.96338	0.96281	0.96244	0.96218	0.96156
Other parameters	$\lambda_1=\lambda_{1i}$, λ_2=0.0000476, λ_3=0.0000667, λ_4=0.0000185, λ_5=0.0000133, μ_1=0.0025, μ_2=0.00143, μ_3=0.0033, μ_4=0.001, μ_5=0.005.						λ_1=0.0000139, λ_2=0.0000476, λ_3=0.0000667, λ_4=0.0000185, λ_5=0.0000133, $\mu_1=\mu_{1i}$, μ_2=0.00143, μ_3=0.0033, μ_4=0.001, μ_5=0.005.					

Table 5.7: Effects of failure and repair rate variations of module C on time dependent availability of process

	λ_{2i}						μ_{2i}					
$i=$	1	2	3	4	5	6	1	2	3	4	5	6
Time (In mins.)	4.35E-05	4.76E-05	5.56E-05	8.33E-05	0.000125	0.0002	0.01000	0.00500	0.00333	0.00143	0.00125	0.00083
30	0.99551	0.99539	0.99516	0.99435	0.99313	0.99095	0.99555	0.99546	0.99543	0.99539	0.99539	0.99538
60	0.99137	0.99114	0.99068	0.98910	0.98674	0.98249	0.99172	0.99140	0.99128	0.99114	0.99112	0.99109
90	0.98755	0.98721	0.98654	0.98424	0.98078	0.97460	0.98839	0.98777	0.98752	0.98721	0.98718	0.98710
120	0.98402	0.98357	0.98271	0.97971	0.97523	0.96721	0.98547	0.98451	0.98411	0.98357	0.98352	0.98339
150	0.98075	0.98021	0.97915	0.97550	0.97004	0.96030	0.98289	0.98159	0.98100	0.98021	0.98012	0.97992
180	0.97772	0.97708	0.97585	0.97158	0.96519	0.95382	0.98060	0.97895	0.97818	0.97708	0.97697	0.97669
210	0.97491	0.97419	0.97278	0.96792	0.96066	0.94775	0.97855	0.97658	0.97560	0.97419	0.97403	0.97366
240	0.97230	0.97149	0.96992	0.96450	0.95641	0.94205	0.97670	0.97444	0.97326	0.97149	0.97130	0.97082
270	0.96987	0.96898	0.96726	0.96131	0.95244	0.93671	0.97504	0.97250	0.97112	0.96898	0.96875	0.96816
300	0.96761	0.96665	0.96478	0.95832	0.94871	0.93168	0.97353	0.97074	0.96916	0.96665	0.96636	0.96566
330	0.96550	0.96447	0.96246	0.95553	0.94520	0.92696	0.97215	0.96915	0.96737	0.96447	0.96413	0.96330
360	0.96354	0.96244	0.96029	0.95291	0.94191	0.92252	0.96107	0.96205	0.96244	0.96574	0.96770	0.97089
Other parameters	λ_1=0.0000139, $\lambda_2=\lambda_{2i}$, λ_3=0.0000667, λ_4=0.0000185, λ_5=0.0000133, μ_1=0.0025, μ_2=0.00143, μ_3=0.0033, μ_4=0.001, μ_5=0.005.						λ_1=0.0000139, λ_2=0.0000476, λ_3=0.0000667, λ_4=0.0000185, λ_5=0.0000133, μ_1=0.0025, $\mu_2=\mu_{2i}$, μ_3=0.0033, μ_4=0.001, μ_5=0.005.					

Table 5.8: Effects of failure and repair rate variations of module D on time dependent availability of process

	λ_{3i}						μ_{3i}					
$i=$	1	2	3	4	5	6	1	2	3	4	5	6
Time (In mins.)	4.55E-05	0.00005	5.56E-05	6.67E-05	8.33E-05	0.000111	0.01000	0.00500	0.00333	0.00250	0.00200	0.00167
30	0.99599	0.99586	0.99571	0.99539	0.99492	0.99413	0.99557	0.99544	0.99539	0.99537	0.99535	0.99534
60	0.99228	0.99204	0.99173	0.99114	0.99024	0.98875	0.99175	0.99131	0.99114	0.99105	0.99099	0.99096
90	0.98883	0.98849	0.98806	0.98721	0.98593	0.98382	0.98842	0.98756	0.98721	0.98702	0.98690	0.98682
120	0.98563	0.98519	0.98465	0.98357	0.98196	0.97928	0.98548	0.98414	0.98357	0.98326	0.98306	0.98292
150	0.98265	0.98213	0.98149	0.98021	0.97829	0.97511	0.98285	0.98103	0.98021	0.97974	0.97945	0.97924
180	0.97988	0.97929	0.97855	0.97708	0.97490	0.97127	0.98048	0.97817	0.97708	0.97646	0.97605	0.97576
210	0.97729	0.97663	0.97581	0.97419	0.97176	0.96773	0.97831	0.97556	0.97419	0.97338	0.97285	0.97248
240	0.97488	0.97416	0.97326	0.97149	0.96885	0.96446	0.97632	0.97315	0.97149	0.97050	0.96984	0.96937
270	0.97262	0.97185	0.97089	0.96898	0.96615	0.96144	0.97448	0.97092	0.96898	0.96780	0.96700	0.96643
300	0.97051	0.96969	0.96867	0.96665	0.96364	0.95865	0.97276	0.96886	0.96665	0.96526	0.96432	0.96365
330	0.96853	0.96767	0.96659	0.96447	0.96131	0.95606	0.97116	0.96696	0.96447	0.96289	0.96180	0.96101
360	0.96667	0.96577	0.96465	0.96244	0.95914	0.95367	0.96965	0.96518	0.96244	0.96065	0.95942	0.95851
Other parameters	λ_1=0.0000139, λ_2=0.0000476, $\lambda_3=\lambda_{3i}$, λ_4=0.0000185, λ_5=0.0000133, μ_1=0.0025, μ_2=0.00143, μ_3=0.0033, μ_4=0.001, μ_5=0.005.						λ_1=0.0000139, λ_2=0.0000476, λ_3=0.0000667, λ_4 =0.0000185, λ_5=0.0000133, μ_1=0.0025, μ_2=0.00143, $\mu_3=\mu_{3i}$, μ_4=0.001, μ_5=0.005.					

Table 5.9: Effects of failure and repair rate variations of module E on time dependent availability of process

	λ_{4i}						μ_{4i}					
$i =$	1	2	3	4	5	6	1	2	3	4	5	6
Time (mins.)	1.67E-05	1.72E-05	1.85E-05	0.00002	2.22E-05	0.000025	0.00500	0.00250	0.00167	0.00100	0.00067	0.00050
30	0.99544	0.99543	0.99539	0.99535	0.99528	0.99520	0.99542	0.99540	0.99540	0.99539	0.99539	0.99539
60	0.99124	0.99121	0.99114	0.99105	0.99092	0.99076	0.99125	0.99118	0.99116	0.99114	0.99113	0.99112
90	0.98736	0.98732	0.98721	0.98708	0.98689	0.98666	0.98746	0.98731	0.98725	0.98721	0.98718	0.98717
120	0.98378	0.98372	0.98357	0.98341	0.98316	0.98285	0.98399	0.98374	0.98365	0.98357	0.98353	0.98351
150	0.98046	0.98038	0.98021	0.98000	0.97970	0.97932	0.98082	0.98046	0.98033	0.98021	0.98014	0.98011
180	0.97738	0.97729	0.97708	0.97684	0.97649	0.97604	0.97792	0.97744	0.97725	0.97708	0.97699	0.97695
210	0.97452	0.97442	0.97419	0.97391	0.97350	0.97299	0.97526	0.97466	0.97441	0.97419	0.97407	0.97401
240	0.97187	0.97176	0.97149	0.97118	0.97073	0.97015	0.97282	0.97209	0.97177	0.97149	0.97134	0.97126
270	0.96940	0.96928	0.96898	0.96864	0.96814	0.96750	0.97058	0.96971	0.96933	0.96898	0.96880	0.96870
300	0.96711	0.96697	0.96665	0.96628	0.96573	0.96503	0.96851	0.96751	0.96706	0.96665	0.96642	0.96630
330	0.96496	0.96482	0.96447	0.96407	0.96347	0.96272	0.96661	0.96548	0.96496	0.96447	0.96420	0.96406
360	0.96297	0.96281	0.96244	0.96200	0.96137	0.96056	0.96485	0.96360	0.96300	0.96244	0.96212	0.96195
Other parameters	λ_1=0.0000139, λ_2=0.0000476, λ_3=0.0000667, $\lambda_4 = \lambda_{4i}$, λ_5=0.0000133, μ_1=0.0025, μ_2=0.00143, μ_3=0.0033, μ_4=0.001, μ_5=0.005.						λ_1=0.0000139, λ_2=0.0000476, λ_3=0.0000667, λ_4 =0.0000185, λ_5=0.0000133, μ_1=0.0025, μ_2=0.00143, μ_3 =0.0033, $\mu_4 = \mu_{4i}$, μ_5=0.005.					

Table 5.10: Effects of failure and repair rate variations of module F on time dependent availability of process

	λ_{5i}						μ_{5i}					
$i =$	1	2	3	4	5	6	1	2	3	4	5	6
Time (in mins.)	1.25E-05	1.33E-05	1.39E-05	1.54E-05	1.67E-05	1.82E-05	0.01000	0.00667	0.00500	0.00333	0.00250	0.00200
30	0.99541	0.99539	0.99537	0.99533	0.99530	0.99525	0.99542	0.99540	0.99539	0.99538	0.99538	0.99537
60	0.99118	0.99114	0.99111	0.99103	0.99096	0.99089	0.99123	0.99117	0.99114	0.99110	0.99109	0.99107
90	0.98726	0.98721	0.98716	0.98706	0.98696	0.98686	0.98738	0.98727	0.98721	0.98714	0.98710	0.98708
120	0.98364	0.98357	0.98352	0.98339	0.98327	0.98314	0.98384	0.98367	0.98357	0.98346	0.98340	0.98336
150	0.98029	0.98021	0.98014	0.97999	0.97985	0.97970	0.98057	0.98035	0.98021	0.98004	0.97995	0.97989
180	0.97718	0.97708	0.97701	0.97684	0.97669	0.97652	0.97754	0.97726	0.97708	0.97687	0.97674	0.97666
210	0.97429	0.97419	0.97411	0.97392	0.97376	0.97357	0.97473	0.97441	0.97419	0.97391	0.97375	0.97365
240	0.97160	0.97149	0.97141	0.97121	0.97103	0.97083	0.97212	0.97175	0.97149	0.97116	0.97096	0.97083
270	0.96910	0.96898	0.96890	0.96869	0.96850	0.96829	0.96969	0.96928	0.96898	0.96860	0.96836	0.96820
300	0.96677	0.96665	0.96656	0.96634	0.96614	0.96592	0.96742	0.96698	0.96665	0.96621	0.96593	0.96574
330	0.96459	0.96447	0.96438	0.96415	0.96395	0.96372	0.96530	0.96483	0.96447	0.96398	0.96366	0.96344
360	0.96256	0.96244	0.96234	0.96210	0.96190	0.96166	0.96332	0.96283	0.96244	0.96189	0.96153	0.96129
Other parameters	λ_1=0.0000139, λ_2=0.0000476, λ_3=0.0000667, λ_4=0.0000185, $\lambda_5 = \lambda_{5i}$, μ_1=0.0025, μ_2=0.00143, μ_3=0.0033, μ_4=0.001, μ_5=0.005.						λ_1=0.0000139, λ_2=0.0000476, λ_3=0.0000667, λ_4=0.0000185, λ_5=0.0000133, μ_1=0.0025, μ_2=0.00143, μ_3=0.0033, μ_4=0.001, $\mu_5 = \mu_{5i}$					

Table 5.11: Failure and repair rates data

C	λ_1	0.0014	μ_1	0.5
E	λ_2	0.0007	μ_2	0.33
F	λ_3	0.00028	μ_3	0.5
B	λ_4	0.0056	μ_4	0.5
D	λ_5	0.0014	μ_5	0.33
$\overline{B}$	λ_6	0.0019	μ_6	0.33
$\overline{D}$	λ_7	0.0056	μ_7	0.33

Table 5.12: Systems availability results to detect optimized value of failure (λ_4) and repair rate (μ_4) of warping

λ_4 \ μ_4	0.5	0.50024	0.512	0.524	1.7	2.9
0.005557	<u>0.98960</u>	<u>0.98960</u>	<u>0.98960</u>	<u>0.98960</u>	<u>0.98960</u>	<u>0.98960</u>
0.005794	0.98959	0.98959	0.98959	0.98959	0.98960	0.98960
0.017554	0.98950	0.98950	0.98950	0.98950	0.98951	0.98951
0.029554	0.98941	0.98941	0.98941	0.98941	0.98942	0.98942
1.205554	0.98854	0.98854	0.98852	0.98851	0.98770	0.98738
2.405554	0.98885	0.98885	0.98883	0.98882	0.98793	0.98747
Underlined values represent optimal choices of long-run availability corresponding to the pair wise failure and repair rates.						

Table 5.13: Systems availability results to detect optimized value of failure (λ_6) and repair rate (μ_6) of warping

λ_6 \ μ_6	0. 33	0. 334	0. 345	0. 357	1. 53	2. 73
0.00185	0.98960	0.98960	0.98960	0.98960	<u>0.98962</u>	<u>0.98965</u>
0.00209	0.98960	0.98960	0.98960	0.98961	<u>0.98963</u>	<u>0.98967</u>
0.01385	0.98986	0.98986	0.98986	0.98987	0.98993	0.99002
0.02585	0.99012	0.99012	0.99013	0.99014	0.99020	0.99031

Underlined values represent optimal choices of long-run availability corresponding to the pair wise failure and repair rates.

Table 5.14: Systems availability results to detect optimized value of failure (λ_1) and repair rate (μ_1) of mixer

λ_1 \ μ_1	0. 5	0.50024	0. 512	0. 524	1. 7	2. 9
0.00139	<u>0.98960</u>	<u>0.98960</u>	<u>0.98966</u>	<u>0.98972</u>	0.99152	0.99185
0.00163	0.98913	0.98913	0.98920	0.98927	0.99138	0.99177
0.01339	0.96664	0.96665	0.96723	0.96779	0.98463	0.98780
0.02539	0.94472	0.94474	0.94578	0.94680	0.97783	0.98378
1.20139	0.29321	0.29331	0.29813	0.30299	0.58328	0.70323
2.4019	0.17210	0.17217	0.17550	0.17887	0.41317	0.54472

Underlined values represent optimal choices of long-run availability corresponding to the pair wise failure and repair rates.

Chapter 6
BEHAVIOR ANALYSIS OF INDUSTRIAL SYSTEMS

It is observed from the analysis of APD model on fabric industry (as discussed in Chapter 5) that the mixer and slasher modules are more sensitive towards the overall performance of industrial process**.** In this chapter, the feasibility of optimal maintenance-scheduling model, developed in Chapter 4, has been investigated by using the systems of these modules.

This chapter is divided into three main sections. Section 6.1 describes the functioning of the systems of mixer and slasher modules, and their role in fabric industry. In addition, information on data collection for parameters of mathematical models discussed in Chapter 4 has been presented. The mathematical formulation for the systems of these modules has been carried out in Section 6.2. Finally, behavior analysis of these systems is presented in Section 6.3.

6.1. Description of Industrial Modules

The role and description of systems of mixer and slasher modules of industry is discussed as under:

6.1.1. Mixer module

Mixer module has one system, called as mixer. It is used to prepare a mixture. The mixture thus prepared is used to coat the threads of the woven fabric. Further, the mixer is divided into five main units: cooker gasket, pressure gauge, safety valve, mixer filter, and motor and are respectively represented by A, B, C, D and E. These units work under three types of maintenance actions: preventive maintenance, corrective maintenance, and replacement. The replacement is given to filter unit only, while the preventive and corrective maintenance actions are carried on the remaining units to maintain the performance of this system.

6.1.2. Slasher module

Slasher module has three identical systems, and these systems are generally known as slasher or sizing systems. These identical systems are used to coat the threads of the woven fabric with the gelatinous mixture in order to improve smoothness and elasticity of the yarn. Each of these system is further divided into three main units: sizing, drying and separating unit, and are respectively represented A_1, A_2, A_3. In order to maintain the performance levels of these systems, preventive and corrective maintenance are carried on the units of these systems.

The contents of sections (6.1.1, 6.2.1, 6.3.1) have been published in Quality and Reliability Engineering: Recent trends and future directions, Chapter 29: 342-352, published by Allied Publishers, Bangalore, 2013 while the content of sections (6.1.2, 6.2.2, 6.3.2) is communicated with a refereed Journal.

6.1.3. Data collection

The data for various parameters of the slasher and mixer modules has been collected from a fabric industry. The number of failures, repair carried, operating hours, and time for each repair for units of systems of both the modules have been recorded. The maximum and minimum values of constant failure- repair rates are next obtained to define upper (u_i, U_i) and lower (l_i, L_i) limits for each unit of both systems. The maintenance sheet given in Table 6.1 can also be used for recording the data. The hypothetical values for cost parameters (i.e. C_r, C_i, c) have been taken for both the systems. The collected data has been used in the further analysis of both the mathematical models of mixer and slasher industrial system.

6.2. Mathematical Model for Industrial Systems

In the following section, the implementation of model discussed in Chapter 4 has been carried out on systems of both the modules. The results of these models have been evaluated by using long run availability (Eq. 4.12) instead of time-dependent availability in equations (4.12, 4.15 and 4.17) due to unavailability of complete maintenance data for transition rates.

6.2.1. Mixer system

The same set of mathematical formulation as discussed in Chapter 4 has been used to model the behavior of this system. A mathematical model for mixer system has been developed by assuming constant rate of transitions by considering the transition diagram as shown in Fig. 4.1. The equations (4.1-4.4) reduce to the following equations by taking limit $t \to \infty$

$$\left(\sum_{i=1}^{5}\alpha_i + \sum_{i=1}^{4}\gamma_i\right)P_o = \sum_{i=1}^{5}\beta_i P_i + \sum_{i=1}^{4}\eta_i P_{i+5} \tag{6.1}$$

and

$$\beta_i P_i = \alpha_i P_o, \qquad \text{where} \quad i = 1,..,5 \tag{6.2}$$

$$\eta_i P_{5+i} = \gamma_i P_o + b\beta_i P_i, \quad \text{where} \quad i = 1,..,4 \tag{6.3}$$

Solving these equations recursively with normalizing condition, that is, sum of all probabilities is one, the following expression for long run availability (Eq. 4.13) is obtained

$$A_\infty = \frac{1}{1 + \sum_{i=1}^{5}\frac{\alpha_i}{\beta_i} + \sum_{i=1}^{4}\frac{\gamma_i + b\alpha_i}{\eta_i}}. \tag{6.4}$$

Thus, the optimal model discussed in Chapter 4 takes the following form by making use of collected data:

$$Max\ T_p = 100\,A_\infty - CSSF$$

subject to

$$0.1 \le \frac{\alpha_i}{\beta_i} \le 1;\ ,0.1 \le \frac{\gamma_i}{\eta_i} \le 1, \quad \text{where } i = 1,...,4 \tag{6.5}$$

satisfying following constraints

$$\left.\begin{array}{c} 10\,P_0 + 1.2\,P_1 + 1.8\,P_2 + 1.5\,P_3 + P_4 + 5\,P_5 + 2\,P_6 + 3.8\,P_7 + 2\,P_8 + 3.5\,P_9 \le\ 24 \\ \mathrm{A}_\infty \ge\ 0.7;\ \sum_{j=0}^{9} P_j = 1. \end{array}\right\} \tag{6.6}$$

6.2.2. Slasher system

The following system of differential equations for three-unit slasher system has been developed by using the transition diagram (Fig. 6.1). This system of equations has been obtained by assuming constant and variable transition rates.

$$P_0'(t) + \left(\sum_{i=1}^{3}\alpha_i + \sum_{i=1}^{3}\gamma_i(y)\right)P_o(t) =$$

$$\sum_{i=1}^{3}\int(1-b)\beta_i(x)P_i(x,t)dx + \sum_{i=1}^{3}\int(\eta_i(x)P_{i+3}(y,x,t)dx\,, \tag{6.7}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \beta_i(x)\right)P_i(x,t) = \alpha_i P_o(t), \ \ i = 1,2,3 \tag{6.8}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial t} + \eta_i(x)\right)P_{i+3}(y,x,t) = \gamma_i(y)P_o(t) + b\beta_i(x)\,P_i(x,t), \ \ i = 1,2,3 \tag{6.9}$$

with boundary conditions:

$$\left.\begin{array}{l} P_{i+3}(y,0,t) = \gamma_i(y)P_o(t), \\ P_{i+3}(0,x,t) = b\beta_i(x)P_o(t), \\ P_i(0,t) = \alpha_i P_o(t)\,, \qquad i = 1,2,3 \end{array}\right\} \tag{6.10}$$

and initial conditions:

$$P_0(0) = 1,\ \ P_i(x,0) = 0, \quad P_{i+3}(y,x,0) = 0, \quad i = 1,2,3. \tag{6.11}$$

Also,

$$P_i(t)=\int_0^\infty P_i(x,y,t)dx, \qquad i=1,...,3 \tag{6.12}$$

$$P_{i+3}(t)=\int_0^\infty\int_0^\infty P_{i+3}(x,y,t)dxdy, \quad i=1,...,3. \tag{6.13}$$

Assuming constant rate of transition, the set of equations (6.7-6.9) reduce to the following equation by taking limit $t \to \infty$

$$\left(\sum_{i=1}^{3}\alpha_i+\sum_{i=1}^{3}\gamma_i\right)P_o=\sum_{i=1}^{3}(1-b)\beta_i P_i+\sum_{i=1}^{3}\eta_i P_{i+3}, \tag{6.14}$$

$$\beta_i P_i=\alpha_i P_o, \tag{6.15}$$

$$\eta_i P_{i+3}=\gamma_i P_o+b\beta_i P_i, \quad \textit{for} \;\; i=1,2,3. \tag{6.16}$$

The resulting equations (6.14-6.16) are then solved recursively with normalizing condition, that is, the sum of all probabilities is one. This leads to computations of state probabilities of each state of stochastic process. The Eq. (4.13) of Chapter 4 takes the following expression for long run availability:

$$A_\infty=\lim_{t\to\infty}A(t)=1-\lim_{t\to\infty}\sum_{i=1}^{3}P_i(t)-\lim_{t\to\infty}\sum_{i=1}^{3}P_{3+i}(t)=\frac{1}{1+\sum_{i=1}^{3}\frac{\alpha_i}{\beta_i}+\sum_{i=1}^{3}\frac{\gamma_i+b\alpha_i}{\eta_i}}. \tag{6.17}$$

Other state probabilities $(i=1,2,3)$ are obtained as,

$$P_i=\frac{\alpha_i}{\beta_i}P_o, \tag{6.18}$$

$$P_{i+3}=\left(\frac{\gamma_i+b\alpha_i}{\eta_i}\right)P_o(t). \tag{6.19}$$

Thus, the optimal maintenance-scheduling model takes the following form:

$$Max\ T_p = 100A_\infty - CSSF$$

subject to

$$0.002667\le\frac{\alpha_1}{\beta_1}\le 2,\ 0.05\le\frac{\alpha_2}{\beta_2}\le 2,\ 0.0125\le\frac{\alpha_3}{\beta_3}\le 3$$

$$0.0014\le\frac{\gamma_1}{\eta_1}\le 4,\ 0.00025\le\frac{\gamma_2}{\eta_2}\le 5,\ 0.000625\le\frac{\gamma_3}{\eta_3}\le 6 \tag{6.20}$$

satisfying the constraints

$$0.8P_0+1.2P_1+1.5P_2+10P_3+2P_4+20P_5+2P_6-2\le\ 16, \tag{6.21}$$

$$A_\infty \geq 0.7\ ,\ \sum_{i=0}^{6} P_i = 1. \tag{6.22}$$

6.3. Behavior Analysis of the Industrial Systems

In the following sections, the behavior analysis of mixer and slasher system is presented:

6.3.1. Mixer system

The results for optimal maintenance scheduling of mixer system has been obtained manually by preparing a spreadsheet with the empirical data, and then these results have been compared for given constraints. The solution that satisfies the given constraints with respect to objective functions is given as:

$$\frac{\alpha_1}{\beta_1}=0.300,\ \frac{\alpha_2}{\beta_2}=0.300,\ \frac{\alpha_3}{\beta_3}=0.299,\ \frac{\alpha_4}{\beta_4}=0.303,\ \frac{\alpha_5}{\beta_5}=0.300,\ \frac{\gamma_1}{\eta_1}=0.000695,\ \frac{\gamma_2}{\eta_2}=0.000693,$$

$$\frac{\gamma_3}{\eta_3}=0.000278,\ \frac{\gamma_4}{\eta_4}=0.000834.$$

The behavioral analysis of each unit of this system for its availability (Fig. 6.2) and profit (Fig. 6.3) has been carried out for perfect maintenance conditions (b=0). It has been found that maintenance of motor (unit E) has greater impact on profit of the system and filter unit (D) of the system has least impact. In case of availability, motor of the mixer system has shown the least affect whereas the filter has affected the most.

6.3.2. Slasher system

The following optimal results have been obtained by solving the equations (6.15-6.18) by using the optimization toolbox of MATLAB, under the imperfect maintenance condition (b=1):

$$\frac{\alpha_1}{\beta_1}=0.002667,\ \frac{\alpha_2}{\beta_2}=0.05,\ \frac{\alpha_3}{\beta_3}=0.012,\frac{\gamma_1}{\eta_1}=0.0014,\frac{\gamma_2}{\eta_2}=0.00025,\frac{\gamma_3}{\eta_3}=0.000625.$$

The behavioral analysis of each unit of this system for its availability and profit has been carried out for imperfect maintenance conditions (b=1) and are shown in Fig 6.4 and Fig. 6.5, respectively. From these figures, it has been found that maintenance of sizing unit (A_1) has a greater effect on availability and profit of the system. However, drying unit (A_2) of the system has shown the least affect in case of availability, whereas separating unit has shown the least affect in case of profit.

Table 6.1: Maintenance sheet for industry persons

Industry name and MS number:

Maintenance manager Name:

Name of repairman, handling this complete system :

Representations of units of the system

A	Cooker Gasket	C	Safety valve	E	Motor
B	Pressure gauge	D	Mixture filter		

Date and time when system start operating for first time:

System wear out time:

Suggested figure(s) for maintenance

Unit	**PM carried after 's' hours**	**Time taken by repair man to carry out PM of unit (in hours)**	**System shows sudden breakdown after 't' hours due to**	**Time taken by system to repair the system back in operation after correcting sudden fault/failure**
A	150	0.5	7200	5 hours
B	10	0.08	4800	3.5 hours
C	20	0.1667	2400	0.666 hours
D	100	0.25	-	-
E	40	1	600	5 hours

For Maintenance record

Preventive Maintenance record, maintained by repairman under supervision of maintenance managers for each round

Round	For 'A' unit	For 'B' unit	For 'C' unit	For 'D' unit	For 'E' unit
1	145 / 0.5*****	10/0.5	100/20	100/0.25	40/1 (**#1)**
On Date:	9 Oct 2011*	3 Oct 2011	6 Oct 2011	6 Oct 2011	3 Oct 2011
2					
On date					
3					
On date					
4					
…					

Corrective Maintenance record, maintained by repairman under supervision of maintenance managers for each round

Round	Unit A	Unit B	Unit C	Unit D	Unit E
1	7200 hrs. /3 hrs.	None	None	None	None
On Date:	6 March 2011	--	--	--	--
2.					
On date					
3.					
…					
Remarks**					

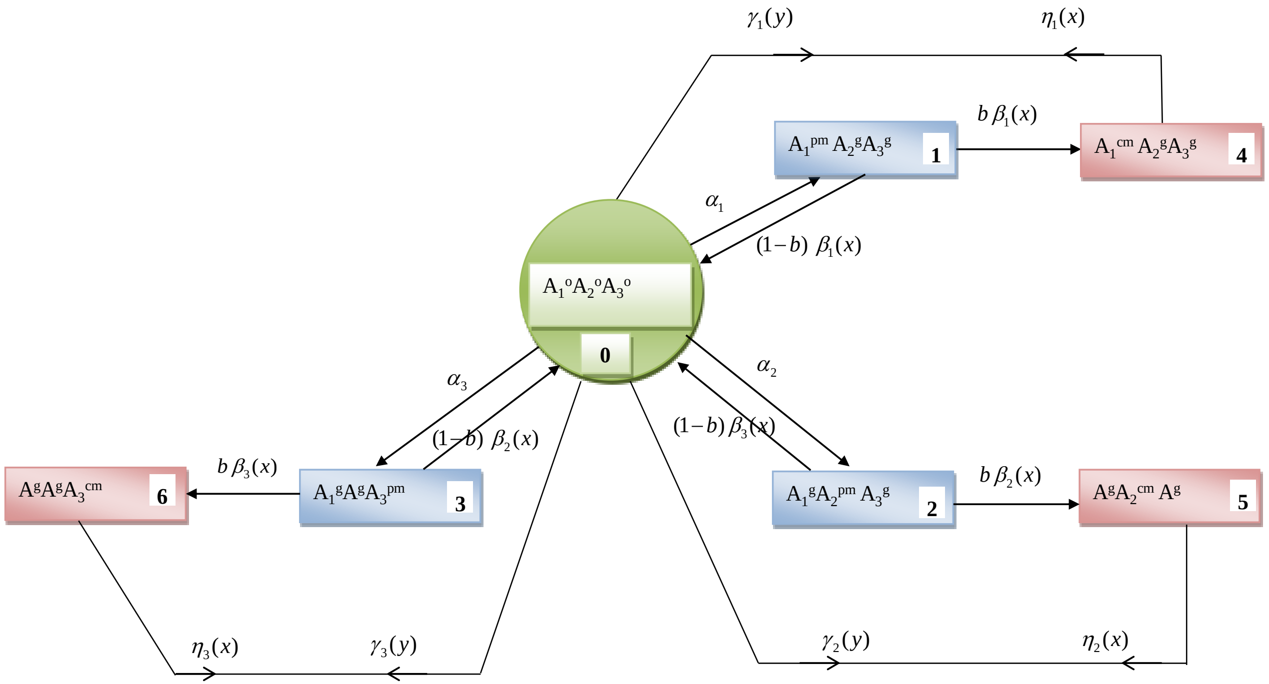
#1	Motor shows early signs of failure than expected failure time, keep on check after every 5 to 10 hrs. of system operation

*After working of x hours / PM carried by repairman within t hours on some date given

** Mark "#" to corresponding failure to time ratio (for example see Unit E data for PM tasks) to give any remark for suspecting something which can damage or any information to keep the system running.

After complete damage of system, like wear out time : Main authorities are responsible to submit it dully signed copies to industrial main unit

Figure 6.1: State transition diagram of three unit system having preventive and corrective maintenance policies



In this diagram, blue states represent preventive maintenance state of system units while the red states show the corrective maintenance state of system units. The operating state of the system is represented by a green circle.

Figure 6.2: Availability of the mixer system

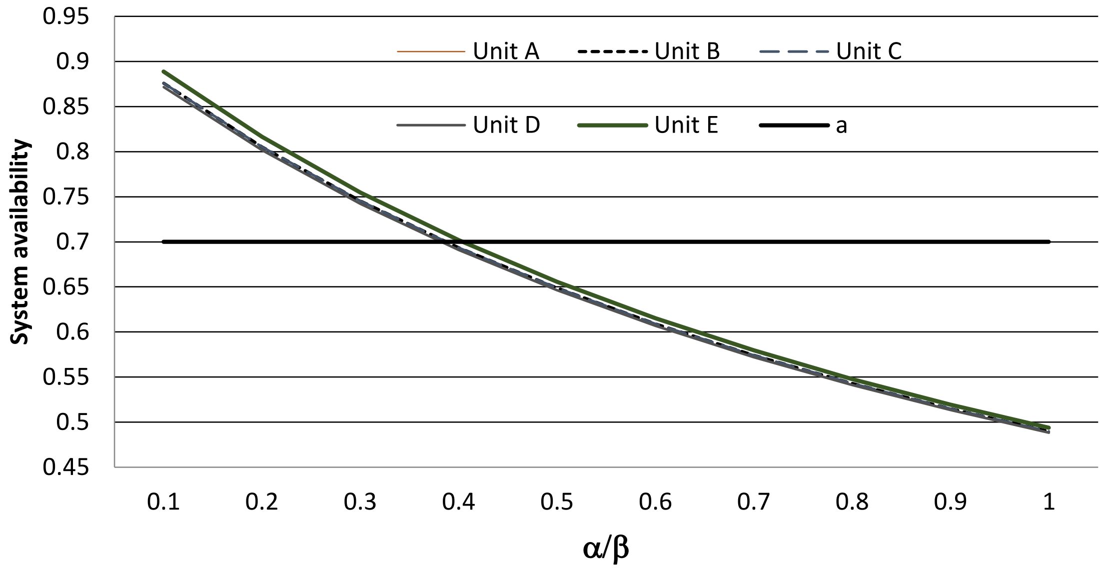


Figure 6.3: Profit analysis of the mixer system

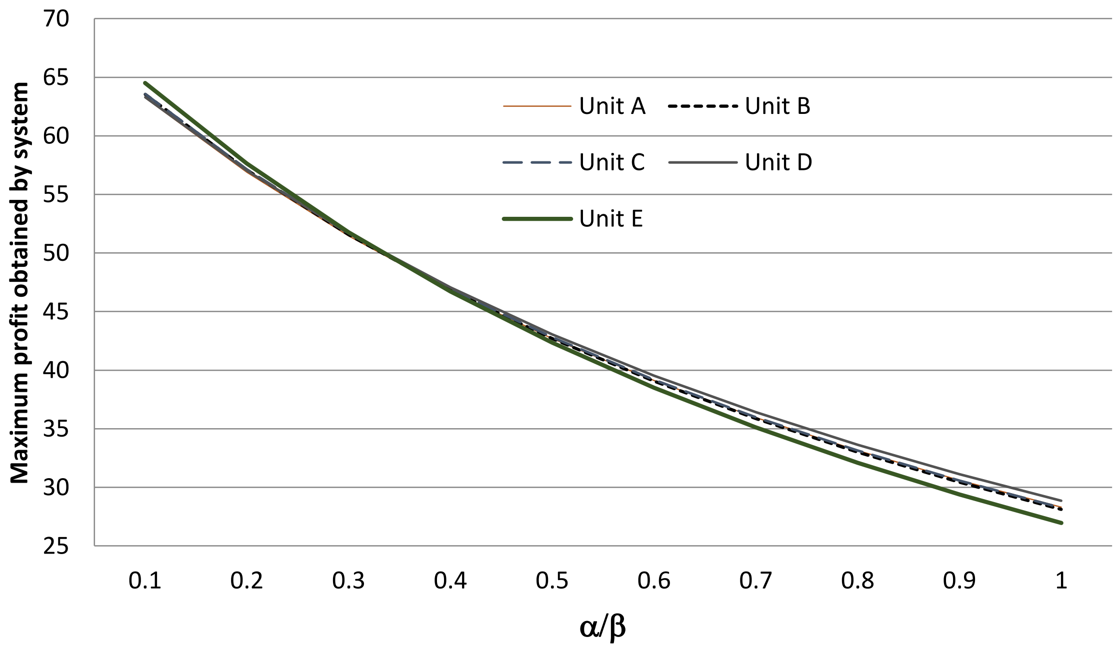


Figure 6.4: Slasher system long-run availability

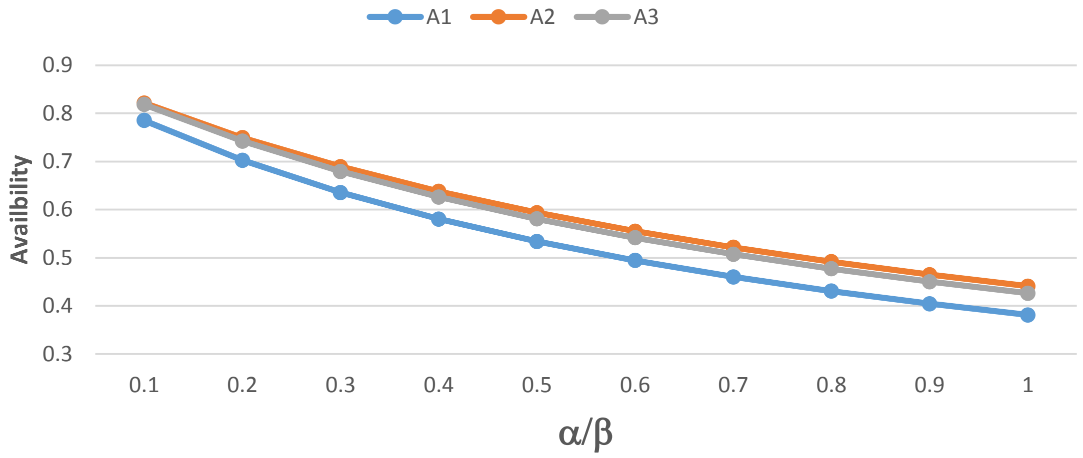
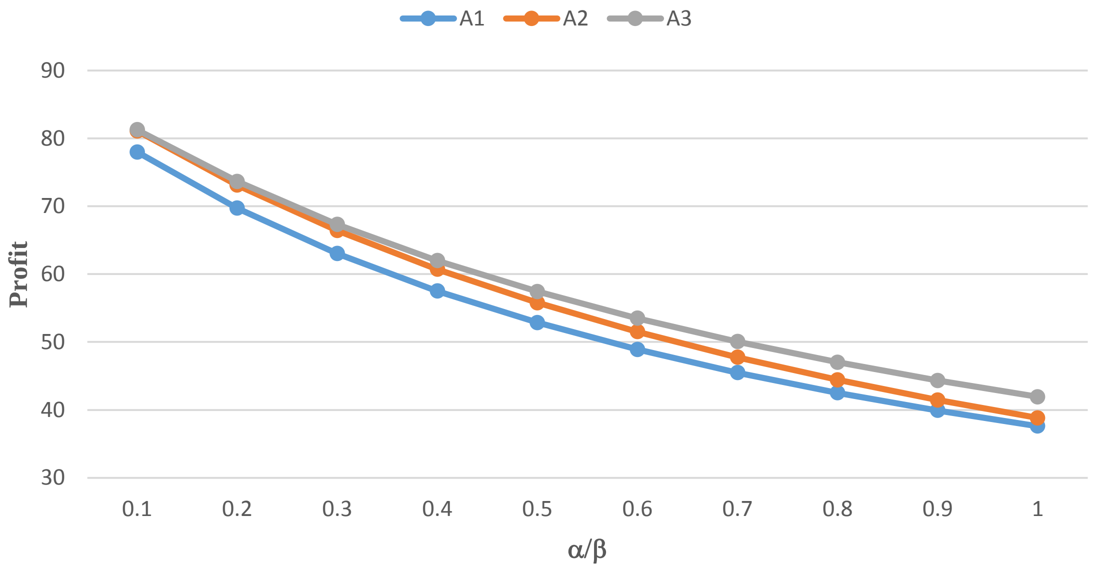


Figure 6.5: Profit analysis of slasher system



Chapter 7
CONCLUDING OBSERVATIONS

The outcome of the work against each of the objective and the conclusion drawn are presented in this concluding chapter. Further, the limitation and the scope of the methodologies developed in the present work to study RAM analysis of selected industrial process has been discussed. Lastly, the industrial significance of the results as well as a scope for further work on this topic has been presented.

7.1 Research Outcome

7.1.1 Objective 1: To conceptualize a RAM model for the selected industrial process/ system

A RAM model has been conceptualized for the selected fabric industry. This model is referred as Assiduity Progression Diagnosis (APD). The model so developed has been found very helpful in determining- reliability, time dependent availability, long-run availability, inherent availability, maintainability, average production and other related metrics (such as MTTR, MTBF) of the process by using the values of state probabilities. This model is developed and discussed in Chapter 2 whereas application of this model is presented in Chapter 5. A comparative study between the two methodologies (APD model and traditional approach) have been critically analyzed in Chapter 5, from which it is concluded that APD model is better alternate to study performance analysis of industrial process.

7.1.2 Objective 2: To transform the conceptualized RAM model into a quantitative and simulation ready model

A numerical simulation procedure which is a combination of different numerical approaches, such as, finite difference schemes, Simpson's one-third method and Lagrange interpolation method and which iteratively compute the transient solution of the stochastic model taken under consideration. This hybrid method, named as Lagrange Finite Difference Simpson Method (LFDSM) has the main advantage of computing the solution very efficiently.

7.1.3 Objective 3: To assess the influence of maintenance on the industrial process/ system's reliability and availability through simulation

An optimization model has been presented in Chapter 4 to maximize the total profit obtained from the output of system wherein constraints on the availability and maintenance cost of the system are imposed. The reliability criteria can also be used in this proposed model under some condition. This model has been developed by taking into account the concept of stochastic process based on supplementary variable technique and optimization theory. It has been found that a decision strategy of this model helps the management of the concerned industry in rescheduling mean time to maintenance of units of the system, and in deciding the replacement of the system with a new system. The application of this model has been demonstrated on two industrial systems in Chapter 6.

7.2 Practical Implications

7.2.1 Concluding remarks of performance analysis of Industrial Process

It is observed from the overall performance analysis that the fabric industry is 99% reliable to produce 179 fabric rolls per hour. The industry is 99% available for the manufacturing production. It requires one hour and fifty minutes per day of average repair time to restore it back to 99% maintainability. From the simulated results, we have recommended to the maintenance management of the industry to pay close attention to the critical modules- mixer and slasher, since these modules shows higher impact on industrial performance in comparison to other modules.

7.2.2 Concluding remarks based on behavior analysis of Industrial system

The two systems of industrial modules, mixer and slasher have been studied in Chapter 6 to assess the feasibility of optimal maintenance-scheduling model. The optimal ratios of the transition rates have been obtained in this chapter from application of optimal maintenance-scheduling model, which can be used to assign the maintenance times to each unit of both the systems. Further, from the behavior analysis of both systems of industrial modules, we found motor and filter as sensitive units of the system while in case of slasher, sizing unit is more effective than the other remaining units.

7.3 Significance of Present Work

In this section, we outline some of the advantages of the work discussed in thesis over the existing performance analysis available in literature, and are as follows:

1. The APD model developed in Chapter 2 reduces the size some of the mathematical model for studying the performance analysis as well as it reduces the size of transition diagram, which saves time and computational efforts
2. The APD model considers storage factor (availability of raw material), which is ignored in some existing performance analysis. This helps to analyze the performance of industry with respect to production of items more effectively.
3. The APD model uses mathematical conjunction relation to define states of the modules and process industry. This methodology helps us to how estimate production quantity by using the probabilities of the states of the transition diagram.

4. The present research work discusses the LFSDM which computes the numerical results of integral differential equations obtained more efficiently.
5. The optimal maintenance model thus developed in Chapter 4 helps us to find the optimal maintenance rates.
6. The maintenance strategy is an added advantage of the optimal maintenance model.
7. The present study helpful for industry to get the reliable production process system with maximum availability at minimum maintenance cost.

7.4 Limitations of Present Work

The limitations of the present work are as follows:

1. The APD model discussed in thesis is fundamental in nature for analyzing performance of process industry. However, this model does not meet all the industrial requirements for better production and hence needs further modifications.
2. This research contribution (APD model) does not consider the issue of safety analysis during the production process. This is one of the major factor for industries besides performance analysis.
3. Maintenance strategy can be explored in a more better way by considering other factors, such as faults, failure causes of systems.
4. The proposed approach, discussed in the Chapter 2, uses the mean (Eq. (2.11)) for estimating production target. A better alternative could be found which considers fluctuating demand and other factors, which influence the production quantity.

7.5 Future Scope of the Present Work

In the present analysis, the system of integral differential equations have been solved for a one-unit system by using the proposed LFSDM in Chapter 3. It would be interesting to see its application for an n-unit system ($n > 1$). In the present analysis of fabric industry using APD model, we have considered the constant failure and repair rates (Chapter 2). However, this model has great scope for analyzing the performance of an industry with variable transition rates. The present work can further be improved by studying other factors that contributes more in improving the module's operational reliability based on individual machine failures. In the present study, an optimal maintenance model has been discussed for a fixed minimum maintenance cost for each state of stochastic process (Chapter 4). However, in reality the maintenance cost varies, thus it would be interesting to see how the use of cost models [90] contributes in improving this model.

In the theory of reliability engineering, generally it is considered that system will follow exponential distribution. By assuming exponential distribution, we develop Chapman-Kolmogorov equations for markovian models. It is our interest to check whether the distribution of reliability of the process follows exponential distribution? A MINITAB software is used in order to infer this hypothesis from the reliability computations of fabric industry from APD analysis when all states are absorbed. By using this software four probability plots are obtained to check fitting of exponential, Weibull, normal and gamma distribution to the reliability computations. The probability plots are shown in Fig. 7.1 for these distributions.

The results shown in Fig. 7.1 suggests that gamma distribution is the best fit while the normal and Weibull shows equally good fit in comparison to exponential distribution. Therefore, it is concluded that in order to study reliability of an industry, the gamma distribution* can be beneficiary than the other three distributions for further research studies.

7.6 Industrial Significance of the Thesis Work

To achieve the mission of the industrial process, unexpected events always happen: customers may change their orders, machines may break down, workers may be absent, raw material may not arrive on time, processed parts may be defective, etc. Such randomness affects the performance of the industry and complicates decision-making process. Due to these unexpected disturbances a significant amount of time of manufacturing managers is lost. The present study provides an important model (APD) in analyzing some of these factors which reduce the performance of industry. Such performance studies require only maintenance and failure data in order to maintain the operational levels of manufacturing process.

The APD model for an industrial process helps the manufactures to analyze various important performance metrics. So, this model can be applicable to any other industrial processes by dividing the process into different modules. Then the solution of Chapman-Kolmogorov differential equations governing the performance of industrial process provides guidance towards sensitive modules of the industry. The successful implementation of this model for the fabric industry supports its convenient application and can be taken as example for studying other industrial process performance metrics. In the nutshell, the present model help in evaluating performance metrics like reliability, three different kinds of availability, maintainability, MTBF, MTTR of the process industry using the probability values of each states. As saying, a healthy man can work better and so is the reliable manufacturing process can manufacture quality products, which meets the demand of the clients and efficiently stand against its competitors.

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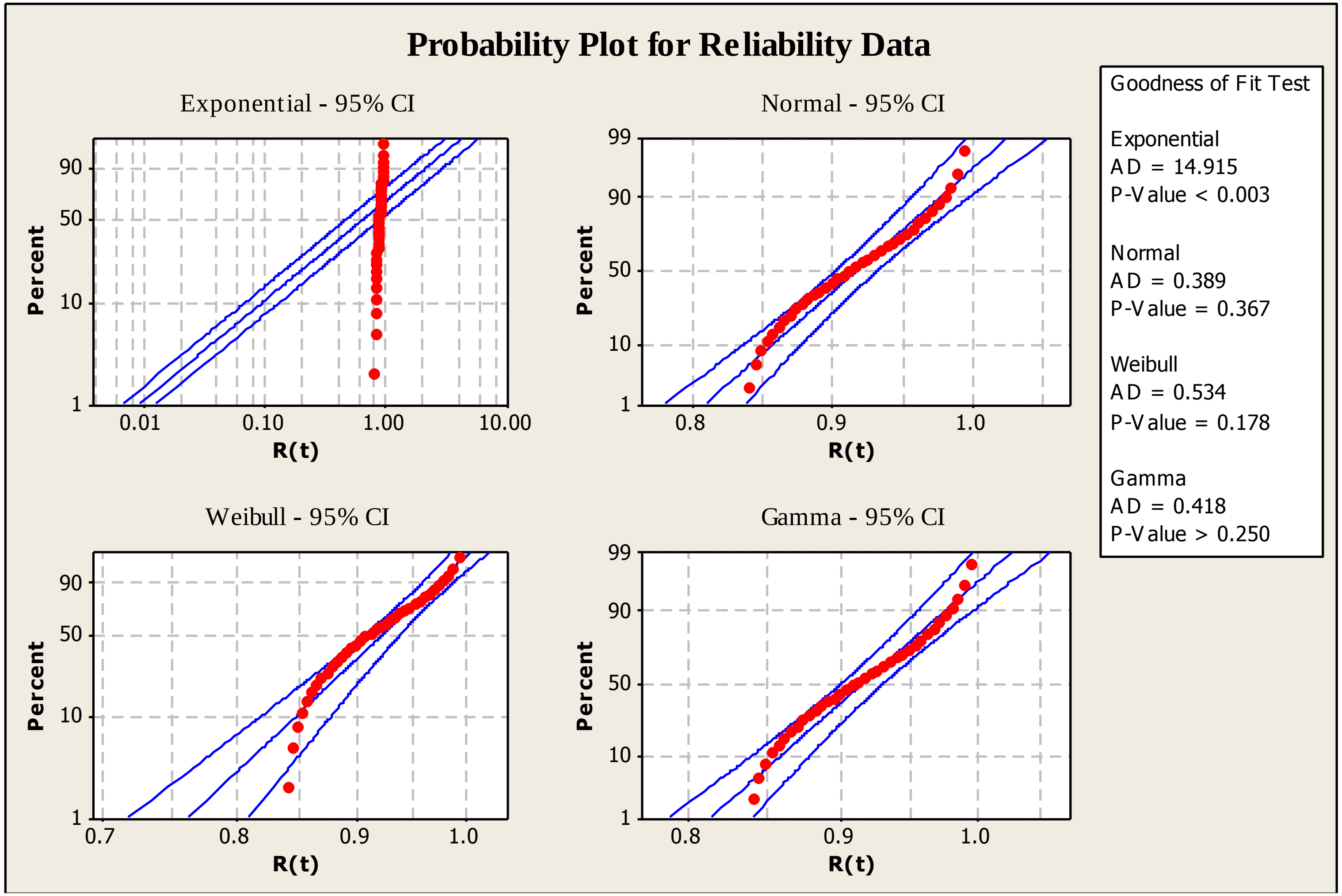
* The contents of this section have been published in International Journal of Research in Advent Technology, 1(5):587-592, 2013.

Figure 7.1 : Probability plots fitting on reliability data for Exponential, Weibull, Normal and Gamma distributions



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