STOCHASTIC MODELING AND COST ANALYSIS OF SOME INDUSTRIAL SYSTEMS

A Thesis

Submitted in fulfilment of the

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by

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CANDIDATE DECLARATION

I hereby certify that the work being presented in the thesis entitled ***"Stochastic Modeling and Cost Analysis of Some Industrial Systems"*** *in the fulfilment of requirements for the award of degree of Doctor of Philosophy, submitted in School of Mathematics of Thapar Institute of Engineering and Technology (Deemed to be University), Patiala is my own work conducted under the supervision of Dr. A.K.Lal, Dr. S.S. Bhatia and Dr. R.K.Tuteja.*

I also declare that the thesis does not contain any part of work which has been submitted for award of any other degree.

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Date: *Rachna*

This thesis is dedicated

to

my daughter

Dhanasvi

ABSTRACT

The study of operational research started during the second world war and afterwards. With the development of operational research, the study of reliability theory emerged as by product in context of defence studies. The words reliable and reliability are in use from ancient time. In fact these occur frequently in social, political, economical and practical fields to indicate the efficiency of a person or mechanical equipment. A mathematical shape to the word reliability was given later in 1950 with its scientific use for defence purpose. Realizing its importance, the study of reliability theory was developed in western world.

The present thesis entitled "Stochastic Modeling and Cost Analysis of Some Industrial Systems" is an attempt to develop the reliability models of the system with varied production capacity depending upon the demand with following objectives:

(i) Understanding the process of industries and to create model(s) for the selected industrial system.

(ii) To analyse the reliability and availability of industrial system to be considered.

(iii) To analyse various other measures like cost-benefit analysis, busy period, mean time to system failure and expected number of visits by repairmen using Semi- Markov process and Regenerative point technique.

(iv) To compare the effectiveness of the systems on the basis of the models (to be developed) taking two at a time using Semi-Markov process and Regenerative point technique.

Data on failure / repair time and various costs has been collected on visiting Kohinoor Rice Mill. The reliabilities have been developed by considering various situations existing in the mill visited. Mean Time to system Failure and various other measures of system effectiveness have been obtained using Semi-Markov process and Regenerative Point Technique. For calculating measures of system effectiveness programming is done in MATLAB. The graphical study of these measures has been done in ORIGIN. Laplace / Laplace Stieltjes transforms and convolution are used for deriving expressions for these measures of system effectiveness. For solving system of equations Cramer's rule is used.

The present thesis embodies seven chapters:

Chapter 1 is an introductory chapter. It includes fundamental concepts and methodology related to work done. In this chapter, we briefly discuss industrial significance

of the reliability and long run availability of industries. A brief summary of the available literature has also been presented in this chapter. This chapter also contains description of the system considered in this thesis. Further, in this chapter the work of the remaining chapters has also been presented.

In **chapter 2**, we discuss the behavioural analysis of a two dissimilar unit standby system working in a rice plant by making one or both units operative depending upon the demand. System consists of following components: Paddy separator, husker, separator, destoner, polisher and colour sorter. Each unit has two types of failure-one due to failure of the component colour sorter which is referred as Type –I failure and the second due to failure of any of the other component(Paddy separator, husker, destoner, polisher) referred as Type – II failure. The processing done on each of the components other than colour sorter is transferable to component of other unit but the processing pending due to failure of colour sorter is completed only on concerned unit after it is repaired. Regenerative point technique has been used to analyse the system. Both failure and repair rates are assumed to follow exponential distribution. Effect of failure rates on availability and MTSF has been carried out. Effect of revenue and cost of busy period on profit has also been discussed.

In **chapter 3,** a standby system is studied which comprises two dissimilar units out of which one is of 8 ton capacity and other is of 4 ton capacity. In this chapter, we have considered the different stages where production is less than demand or at least equal to demand. In case of production less than demand, we have analysed the system with two types of failures and variation in demand by considering the loss of goodwill. Failure time and repair time both are assumed to follow exponential distribution. Repair facility is always available for every failed unit i.e. no unit waits for repair. It is assumed that, in case of production greater than or equal to demand and if one unit is under repair and other is operative, then no other event can take place except decrease in production. Effect of revenue of different capacity on profit has also been discussed along with effect of failure rate on availability of different capacities in both the cases when production is less than demand and when production is at least equal to demand.

Chapter 4 is devoted to a stochastic model for a three-unit standby system wherein one, two or all the units may be made operative depending upon the load/demand. The system under consideration is assumed to have two shifts of working. The system to be considered comprises of three units of paddy to rice converters having different capacities. One is of 8 ton capacity and another two are of 4 ton capacity. Priority of operation is given to line of capacity 8 ton instead of 4 ton. Effect of revenue of different capacity on profit, effect of failure rate on MTSF and availability of different capacities has been discussed in this chapter. Graphical study is also carried out.

Chapter 5 describes a two similar unit (eight ton capacity) standby system which is analysed with variation of demand and two types of failures. We have considered a single unit of eight ton capacity in place of two similar units of four ton capacity each as in the preceding chapter. Keeping in mind the installation cost of one big unit in place of two small units here, we have analysed a two similar unit standby system with varying demand and two types of failures. We have considered arbitrary distribution for repair rates. Effect of failure rates on availability and MTSF has been carried out. Effect of revenue and cost of busy period on profit has also been discussed.

In chapter 6, the comparative study of effectiveness of models discussed in the chapters 3, 4 and 5 with respect to profit evaluation has been discussed. In this chapter, we also analysed the suitability of models discussed in previous chapters in different situations.

Chapter 7 is concluding chapter of thesis. The industrial significance along with the limitations and scope of the present work has also been briefly discussed in the summary and conclusion.

LIST OF PUBLICATIONS

List of Publications in Journal:

1. Analysis of a Two-Unit Standby Industrial System with Varying Demand in **International Journal of Research in Advent Technology**, Vol 3, No. 6, June 2015 (Non SCI).
   [Based on chapter 5]
2. Reliability and Economic Analysis of a System Consisting of Two Dissimilar units with variation in production in **Ciência e Técnica Vitivinícola Journal – A Science and Technology Journal,** Vol. 32, No. 8, 384-399 (2017) – SCI indexed.
   [Based on chapter 2]

List of Publications in Conference Proceedings:

1. Availability Analysis of Polytube Industry in Proceedings of National Conference on Mathematical Modelling and Simulation(MMSim10) organized by **Department of Mathematics, Guru Jambheshwar University of Science and Technology Hisar,** held on March 20-21, 2010.
2. A Stochastic Model to Study the Cost Benefit Analysis of A Two Unit System Working in a Rice plant in proceedings of **International Conference on Advances in Modeling, Optimization and Computing (AMOC-2011) organized by Department of Mathematics, IIT Roorkee**, held on December 5-7, 2011.
3. Analysis of a Three Unit Standby System Working in a Rice plant with Varying Demand with no Condition on Number of Simultaneous Repairmen in proceedings of **International Conference on History of Development of Mathematical Sciences and Symposium on Non-Linear Analysis (ICHDMS-2012) organized by Department of Mathematics, M.D.U. Rohtak**, held on November 21-24, 2012.
4. Stochastic Model to Study Two Unit Standby System Working with Varying Demand in proceedings of **International Conference on Emerging Trends in Computational and Applied Mathematics (ICCAM) organized by ITM University**, Gurgaon held on June 2-4, 272-279, 2014.

5. Reliability Analysis of A Two Unit Standby System Working In A Rice Plant With Varying Demand in proceedings of **International Conference on Innovations & Trends to Support Make in India (ICITSMI – 2017) organized by N.C. College of Engineering**, Israna, Panipat held on March 25-26, 2017.

List of Communicated papers:

1. Cost Analysis of A Two Dissimilar Unit System With Goodwill Loss When Demand Is More Than Production in **"International Journal of Reliability and Safety"**.

2. Analysis of A Three Unit System With Demand Dependent Operability of Units in **"Advances in Mechanical Engineering"**.

NOMENCLATURE

$q_{ij}(t), Q_{ij}(t)$	Probability density function (p.d.f.), cumulative distribution function (c.d.f.) of first passage time from a regenerative i^{th} state to a regenerative j^{th} state or failed j^{th} state without viving any other regenerative state in (0, t].
$q_{ij}^{(k)}(t), Q_{ij}^{(k)}(t)$	Probability density function (p.d.f.), cumulative distribution function (c.d.f.) of first passage time from a regenerative i^{th} state to a regenerative j^{th} state or failed j^{th} state visiting k^{th} state in (0, t].
$\phi_i(t)$	c.d.f. of first passage time from regenerating state to failed state
$p_{ij}(t)$	Transition probability from regenerative state to regenerative state without visiting any other state.
$p_{ij}^{(k)}(t)$	Transition probability from regenerative state to regenerative state without visiting any other state.
$m_{ij}(t)$	Contribution to mean sojourn time in regenerative i^{th} state before transiting to regenerative j^{th} state without visiting any other state.
$m_{ij}^{(k)}(t)$	Contribution to mean sojourn time in regenerative i^{th} state before .transiting to regenerative j^{th} state visiting k^{th} state once.
μ_i	Mean sojourn time in regenerative state 'i' before transiting to another state 'j'
A_i	Probability that the system is in upstate at any instant 't' with given that system entered regenerative state 'i' at t = 0.
$M_i(t)$	Probability that system is in upstate initially in regenerative state i and is in upstate at time 't' without passing through any other regenerative state or returning to itself through one or more non regenerative states.
$B_i(t)$	Probability that repair man is busy for repair at instant t, given that the system entered regenerative state 'i' at t = 0.

$W_i(t)$	Probability that repair man is busy in i^{th} regenerative state at time 't' without passing through any other state or returning to itself through one or more non regenerative states.
$E_i(t)$	Probability that the system is in down state, given that the system entered regenerative state 'i' at t = 0.
$*$	Symbol for Laplace Transform.
$**$	Symbol for Laplace-Stieltjes Transform.
(c)	Symbol for Laplace Convolution.
(s)	Symbol for Laplace-Stieltjes Convolution
L.T.	Laplace Transform.
L.S.T.	Laplace-Stieltjes Transform.

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CHAPTER-1

INTRODUCTION

In today's scientific world, the society has become fully dependent on science and technology for all its daily needs. In order to achieve maximum benefit of advancement in science and technology, trustworthy and reliable systems are in great demand. In addition, analysis of systems in terms of reliability, helps to formulate the policies of a company to provide cost effective, efficient and user friendly systems. Therefore, reliability has been become a matter of great concern for all in today's world. In fact, uninterrupted service and failure free operation is an essential requirement of large complex systems such as electric power generation and distribution plants or communication network systems. In these cases, a sudden failure of even a single component, assembly or system results in health hazard, accident or interruption in continuity of service. Similarly, sudden failure of a car brake system while it is running may cause serious accident. Although, the failure of such products may cause inconvenience on smaller scale. The problem of assuring and maintaining reliability has many responsible factors such as equipment design, quality control during manufacturing, acceptance inspection, field trials, life testing and design modifications.

The word reliable and reliability are in use from ancient time. Their occurrence in social, political, economical and practical fields is very frequent. A separate discipline, reliability analysis, has evolved for studying the reliability of such systems. Consideration of reliability is very important in planning, design and operation of the system. Reliability engineering came into existence due to complexity and automation of equipments used in world war II which resulted in several problems related to maintenance and repair. By reliability, we mean the ability of the system to function as planned. We express this ability using probabilities and expected values.

1.1 Literature Review

A mathematical shape to reliability was given in 1950 with its scientific use for defence purpose. In 1952, the US Department of Defence had established the Advisory

Group on Reliability of Electronic Equipment (AGREE). This group published its first report on reliability in 1957. Davis [18] discussed failure data and goodness of fit tests for various failure distributions. Epstein and Sobel [23] published a fundamental paper on life testing which laid the foundation of classical reliability analysis. Epstein and Sobel [24], Epstein [22] worked in the field of life testing with the assumption of exponential distribution. After these papers, the exponential failure distribution acquired a unique position in life testing and reliability analysis. Therefore, besides finding reliability of the system, investigations had been carried out to evaluate other measures. Srinivasan and Gopalan [101] concentrated on regenerative point technique to study two unit standby system. Theory of regenerative process, reliability design and stochastic process were discussed by various authors like Smith [97,98], Barlow [6], Feller [25], Polpvoko [83], Kingman [44], Chung [17], Dhillon and Singh [21] and Medhi [64]. In regenerative point technique theory of laplace transform is used which was discussed by Spiegal [99].

Singh [94] for the first time used the concepts of reliability technology to analyse the working of production system. Nakagawa [75] considered the replacement of unit at certain level of damage. Yang and Dhillon [122] calculated the availability of a robot with safety system. The need and application of reliability technology in the process industry was discussed by Michelsen [65]. Kumar et al [46, 47, 48] calculated the availability for number of systems in process industries. Kumar et al [49] analysed a two unit standby system with instructions at need. Gupta et al [34] investigated the reliability and availability analysis of serial processes of butter oil plant and behaviour analysis of the cement industry. Agnihotri et al [3] have studied the reliability analysis of boiler used in readymade garment industry. Stochastic analysis of a three unit standby system was critically analysed by Attahiru [4].

A system can fail due to common errors and human errors. Dhillon [19] studied a 4 unit redundant system with common cause failure. Gupta and Kumar [33] evaluated the availability and mean time to failure of a two-unit cold standby system with three possible states of units,i.e., good, partially failed and failed state by introducing the concept of human repair. Chaung [13, 14] extended the idea to a repairable system subjected to failure due to common cause failure and critical human error. Such failures are very common in our day to day life. Kaushik and Singh [43] performed the reliability analysis of the Naphtha fuel oil and water system under priority repair used in thermal power plant. The reliability and availability analysis for two identical unit parallel systems with common cause failure and human errors was done by Sridharan and Mohanavadivu [100].

To improve the reliability of a system, the concept of redundancy was introduced in the system. Kapoor and Kapoor [42] discussed the effect of standby redundancy on the system's reliability. Nakagava and Osaki [74] considered stochastic behaviour of a two-unit priority standby redundant system. Singh and Goel [95] investigated the availability of heating system with warm standby and imperfect switch in sugar industry. The standby redundancy allocation in series and parallel systems was discussed by Misra et al [68].

While studying the reliability analysis of industrial system through graphs, most of the researchers used hypothetical data for failure, repair and other rates i.e. the real data on these rates were not taken into consideration. Taneja [102] collected real data and discussed reliability and profit analysis of a system which consists of one main unit (used for manufacturing) and two units (used for controlling). The reliability of 2-unit parallel continuous casting plant with full installed capacity was analysed by Mathew et al [62]. Kuo-Hsiung et al [52] emphasized on the four different systems with warm standby components and standby switching failures based on their reliability and availability.

It has also been observed that environmental conditions may affect the system's reliability considerably. Such systems are very much peculiar in nature. Reliability of mechanical component under environment stress was discussed by Dhillon [20]. Singh [91] discussed the reliability analysis under fluctuating environment. Aggarwal and Kumar [1] studied the stochastic behaviour of repairable system working under fluctuating weather conditions. The comparison between two cold and warm standby outdoor electric power systems in changing weather was done by Mokadies et al [70]. Further, Goel [26] also examined a two unit standby system under different weather conditions.

The reliability models with different constraints of operation and repair on standby units were analyzed by Bhatia [8], Taneja et al [105], Taneja and Nanda [103] and Tuteja et al [115]. The reliability and profit analysis of two-unit standby system was discussed by Tuteja and Taneja [110, 111, 112], Vashistha [118] and Tuteja and Malik [109]. Lie and Jian [56] considered repairable systems with repairman having multiple vacations. If the system fails and the repair man is on vacation then system will wait for repair until the repair man is available. Analysis of systems having priority and ordinary units was done by Gupta et al [35]. The reliability models with different repair policies were discussed by Mine and Kaiwal [67], Rander et al. [86], Tuteja et al [113, 114], Murari and Goyal [72], Guo [38], Taneja and Naveen [104], Tuteja et al [106], Chandershekhar et al [12] and Yadavalli [121]. Various reliability models with varying working conditions were analysed by Goel [28], Gopalan [29], Gupta [31], Narang et al [78] and Kumar [50, 51]. Analysis of various systems with different inspection policies was done by researchers like Nakagawa [76], Agnihotri and Satangi [2], Singh and Taneja [93, 94]. Concept of deterioration was discussed by Jacob [39] and Jaggi [40].

k-out-of-n structure is also a very popular type of redundancy and is applied in industrial and military systems. Reliability and availability of such systems have been analyzed by various researchers including Chiang and Niu [16], Chaung [15], Li and Chen [55], Tuteja and Minocha [107, 108]. Pham [82], Mc. Grady [63] and Ksir and Boushaba [45] evaluated the modeling of a shared load k-out of-n: g system. k out of n cold standby systems were also discussed by Sarhan [87, 89]. Sarhan [88, 90] also extended generalized exponential distribution and modified weibull distribution. Vanderperre [117] discussed the reliability analysis of a renewable multiple cold standby system. Goyal [30] analysed reliability and profit for a 2-out-of-3 unit system.

Cost is the most important factor to increase the availability of the process industries. Prabhuswami [84] studied the reliability based optimization of manufacturing systems. Gupta et al [36] discussed with profit analysis of a two unit priority standby system subject to degradation and random shocks. Profit analysis of two unit cold standby system was discussed by Siwach et al [96]. Chander and Bansal [11] discussed the profit analysis of single unit reliability models at different failure modes. The cost analysis of two dissimilar units was discussed by Mokadies and Matta [69]. Profit analysis of standby systems was analysed by many more researchers like Mokkadies [71], Minocha et al. [66] , Singh [92] and Malik et al. [61]. Comparative study of profit analysis of two models was later reviewed by Gupta [32], Parashar [81] and Yusuf [123].

Maintainability and availability are two main aspects for any industry, which are closely related to reliability. In a reliable system, breakdowns are less frequent and hence availability is high (i.e. system functions well and is available for use). Availability and cost analysis of preventively maintained units was discussed by Vaurio [119]. Lal et al [54] analysed availability of serial process of plastic pipe manufacturing plant. Maintenance analysis helps in determining how often the system and its components should be maintained for reliable performance. Over the last two decades, many methods/techniques have been developed/presented in number of research papers to determine the optimal maintenance schedule. Preventive maintenance models were discussed by many researchers like Barlow and Hunter [5], Goel [27], Lie et al. [57], Chan [10] and Nakagawa [77] developed a model for imperfect preventive maintenance in which the effective age of the system is reduced by 'x' time the units at each preventive maintenance. Nakagawa [73] also developed optimum preventive maintenance policy for repairable system. Ntuen [80] proposed generalized models for determining minimum cost preventive maintenance. Jayabalan and Chaudhary [41] presented a model for cost optimization of maintenance scheduling for a system with assured reliability. Ramakrishna and Bawa [85] have discussed optimization of machine design criteria for higher reliability and maintainability in food processing industry. The reliability optimization of complex systems through-SOMGA was studied by Kusumdeep and Dipti [53]. A multi objective optimization of imperfective preventive maintenance policy with hidden failure rates was calculated by Wang and Pham [120].

Several systems for reliability modeling using various aspects like type of failures, types of policies for repair / replacement, maintenances, degradation and inspections etc. has been analysed by various mathematician/statisticians. Zhao [124] discussed the availability for repairable components and series systems. The dependability modeling using petri net based model was discussed by Malhotra and Trivedi [58]. Vanderperre [116] calculated the long run availability of a two unit standby system subjected to a priority rule. System with varying demand was investigated by Mallhotra and Taneja [59, 60]. Concept of optimization was figured out by Bector [7] and Dass et al [9, 37]. Data analysis for performance study industrial systems was done by using soft computing by Nguyen et al. [79]. Few of them analysed the systems by collecting real data and considering real life practical situations. Work concerning with real life situations has not been sufficiently done so far. Also, most of the study is devoted to reliability and availability analysis of the system but cost-benefit analysis has not so far been undertaken in most of the complex practical situations. The same situation also arises while evaluating the various measures like reliability, availability, mean time to system failure, busy period of repairmen and expected down time as these measures are necessary to analyse the system completely. However the work on varying demand and many other different situations have not been sufficiently studied so far and is needed to be analyzed by collecting real data. Keeping this in mind, we have considered a system undergoing all such type of situations. One of such system is Rice Plant-Paddy to rice converter system.

Keeping these in view we have attempted to study the following objectives on cost analysis of some industrial systems in the present thesis.

1.2 Objectives of the Study are listed as:

- Understanding the process of industries and to create model(s) for the selected industrial system.
- To analyse the reliability and availability of industrial system to be considered.
- To analyse various other measures like cost-benefit analysis, busy period, mean time to system failure and expected number of visits by repairmen using Semi- Markov process and Regenerative point technique.
- To compare the effectiveness of the systems on the basis of the models (to be developed) taking two at a time using semi-Markov processes and regenerative point technique.

Some fundamental concepts related to reliability, availability and the performance measures of the systems are presented below:

1.3 Basic Definitions:

1.3.1 Reliability

Reliability is associated with the civilization of mankind to compare one object/person with another. Reliability cannot be exactly measured with respect to human behaviour but can give idea how to compare a particular person in with other. The knowledge about long-term properties of material and other devices helps in framing reliable products. Reliability is be said to be a measure of performance. A person who completes his work in time is said to be more reliable than the other who does not. Now it is easy to state, that the concept of reliability is not only associated with human behaviour or activity but can also be applied to other inventions of the mankind, by directly measuring their performance or by knowing the failure rates of the equipment/systems. The growing awareness of reliability arises from the fact that there is a need for efficient, economic and continuous running of equipment/system in any organization for achieving the targeted production at a minimum cost to face the present competitive world.

Reliability engineering has developed, and advanced mainly due to high risk and complex systems. Reliability is a measure to ensure operational efficiency. Reliability of a system/device is the probability of the system/device performing its anticipated purpose adequately for the intended period of time under the given operating conditions.

The reliability can be interpreted in various different ways some of these are as:

(i) Reliability is the quality of being trustworthy or of performing consistently well.

(ii) Reliability is the degree to which the result of a measurement, calculation, or specification can be depended on to be accurate.

(iii) Reliability is the degree to which an assessment tool produces stable and consistent results.

Quantitatively, reliability of a device in time‘t’ is the probability that it will not fail in a given environment before time t. If T is a random variable representing the time till the failure of the device starting with an initial operable condition at t = 0, then reliability R (t) of device is given by

$$R(t) = P[T > t] = 1 - P[T \le t] = 1 - F(t) \tag{1.1}$$

Thus, reliability is always a function of time. It also depends on environmental conditions which may or may not vary with time. Following assumptions are made:-

(i) R (0) = 1 since the device is assumed to be operable at t = 0.

(ii) R (∞) = 0 since no device can work forever without failure.

(iii) R (t) is non-increasing function between limits 0 and 1.

1.3.2 SCOPE OF RELIABILITY

The scope of reliability can be imagined by the following facts with respect to any equipment or system:

(i) The working environment of the equipment/system.

(ii) The need of safety aspects for goods/material and men.

(iii) Degree of uncertainty about the success of operation and its improvements in system/ equipment performance.

(iv) Need for effectual, commercial and uninterrupted running of equipment/system without disturbances.

(v) Improvement in the confidence of the personal working particularly in the failure area because of safety reasons.

Any equipment or system is framed with certain objectives so that it can meet its goals in terms of production/service.

1.3.3 Objectives of Reliability

In, Design phase of any system, it is required that the said system should maintain its performance standards within the defined constraints such as cost of equipment or product, environmental conditions and availability of material or parts etc. A system or equipment normally comprises of number of units or components which make a system complex and therefore, system is dependent on complexity of the functioning of the units. It becomes more difficult to achieve satisfactory performance from such system or equipment. Following are the objectives of reliability:

(i) Uninterrupted running of system or equipment.

(ii) The adequate performance for a stated period of time.

(iii) The equipment or system should work under the specified environmental conditions.

(iv) Minimization of time for which system is down.

(v) Maintainability of device or components.

(vi) Quantify and demonstrate the life of the product.

1.3.4 Failure

"A failure is a result of joint action of many unpredictable, random processes going on inside the operative system as well as in the environment in which the system is operating". Functioning is therefore seriously impeded or completely stopped at a certain moment in time. All failures are stochastic in nature. In some cases the time of failure is easily observed but if a unit deteriorate continuously, determination of the moment of failure is not an easy task. Failure of a system is called a disappoint or death and failures results in the system being in down state.

In practice even the best design, manufacturing and maintenance efforts do not prevent the system failure. In reliability theory there are four types of failures.

(i) Early failure

(ii) Random failure

(iii) Wear out failure

(iv) Out of tolerance failure

1.3.5 Instantaneous Hazard Rate (or Failure Rate)

It is stated as the conditional probability that the system failing during the time interval (t, t + δt] given that it was operating during (0, t]. Let $r(t)\delta t$ = probability that the device has life time between t and t + δt, given that it has functioned up to time t.

$$r(t)\delta t = \Pr[t < T \le t + \delta t | T > t]$$

$$= \frac{P[t < T \le t + \delta t]}{P[T > t]} = \frac{P[T < t + \delta t] - P[T < t]}{P[T > t]}$$

$$= \frac{[1 - R(t + \delta t)] - [1 - R(t)]}{R(t)} = -\frac{R(t + \delta t) - R(t)}{R(t)} \tag{1.2}$$

Now, the instantaneous failure rate or hazard rate r(t) at time t is defined as

$$r(t) = \lim_{\delta t \to 0} -\frac{R(t + \delta t) - R(t)}{R(t)\delta t} = -\frac{R'(t)}{R(t)} = \frac{f(t)}{R(t)} \tag{1.3}$$

where f(t) is the p.d.f. of the device life time. It can be seen that

$$F(t) = \int_t^{\infty} f(u)\, du = R(t) = \exp\left[-\int_0^t r(u)\, du\right] \tag{1.4}$$

$$f(t) = r(t) \exp\left[-\int_0^t r(u)\, du\right] \tag{1.5}$$

1.3.6 Repairable Systems:-

Failed units of systems may be replaced by new ones, but this may prove to be expensive. To replace the failed unit is usually more cost effective option and failed unit are sent to a repair facility. A repairable system can be described as one when the system found is down as the result of failure. A repair facility is available where the system can be made operable again. If no repair facility is free, failed unit queue up for repair. The life times of a unit while online, while in standby as well as the repair times are all independent random variables.

Repairable systems have been the subject of intensive investigation for a long time. Different random variables can form the basis of research such as

(i) Availability and reliability

(ii) Time necessary for repair

(iii) No of repair that can be handled

(iv) Switch over time to and from the repair facilities

1.3.7 Stochastic Process

A stochastic process is a family of random variables indexed by a parameter set realising values on another set known as the state space. Both the parametric set and the state space can be either discrete or continuous.

In a stochastic process {X(t), t ε T}, where X(t), t and T respectively are the state space, parameter (generally taken to be time) and the index set. If T is countable set such as T = {0, 1, 2, 3,…}, then the stochastic process is said to be a discrete parameter process and if T = {t : −∞ < t < ∞} or T = {t : t ≥ 0}, the stochastic process is said to be continuous parametric process. The state space is classified as discrete if it is countable and continuous if it consists of an interval on the real line. In the present study, we have only dealt with discrete state space continuous time parameter stochastic process.

1.3.8 Markov Process

A stochastic process is said to be Markov Process if the future development is completely determined by the present state and is independent of the way in which the present state has developed. If $\{X(t),\ t \in T\}$ is a stochastic process such that, given the value of X(s), the value of X(t), t > s do not depend on the values of X(u), u < s, i.e. for t > s, i ∈ s

$$\Pr[X(t) = i \mid X(u),\ 0 \le u \le s] = \Pr\left[X(t) = i \mid X(s)\right] \tag{1.6}$$

Then the process $\{X(t),\ t \in T\}$ is a Markov process.

Stochastic Processes which do not possess the Markovian property are said to be non Markovian.

1.3.9 Markov Chain

A Markov Process with discrete state space is said to be a Markov Chain. Mathematically, a stochastic process $\{X_n;\ n = 0,\ 1, 2, \ldots\}$ is called a Markov Chain if, for

$j,\ k,\ j_1,\ j_2 \ldots\ldots\ldots j_{n-1} \in N.$

$$\Pr[X_n = k \mid X_{n-1} = j,\ X_{n-2} = j_1, \ldots\ldots X_0 = j_{n-1}]$$
$$= \Pr[X_n = k \mid X_{n-1} = j] = p_{jk}\,(say) \quad (1.7)$$

If the transition probabilities p_{ij} are independent of n, then the Markov chain is said to be homogeneous and if it is dependent on n the chain is said to be non-homogeneous.

1.3.10 Renewal Process

Suppose we have repairable system which starts operation at t = 0. If X_1 denotes the time to first failure and Y_1 denotes the time from first failure to next system operation (after repair) then $t_1 = X_1 + Y_1$ denotes the time of first renewal. Similarly, if X_2 denotes the time to first renewal to second failure and Y_2 denotes the time from second failure to second renewal then $t_2 = X_2 + Y_2$ and the time of second renewal is $t_1 + t_2$. In general, $t_i = X_i + Y_i$ (inter-arrival) time between the $(i-1)^{th}$ and i^{th} renewal) for i = 1, 2, 3… If we define

$S_0 = 0,\ S_n = t_1 + t_2 + \ldots t_n$ = epoch of nth renewal,

and N(t) = number of renewals during (0, t] then the process {N(t), t > 0} is called renewal process.

1.3.11 Markov Renewal Process

Let the states of a process be denoted by the set E = {0, 1, 2 …}, and let the transition of the process occur at epochs If $t_0\,(= 0),\ t_1,\ t_2, \ldots, t_n\,(t_n < t_{n+1})$.

$$\Pr\{X_{n+1} = k,\ t_{n+1} - t_n £ t \mid X_0 = i_0, \ldots, X_n = i_n : t_0, t_1, \ldots t_n\}$$
$$= Pr(X_{n+1} = k,\ t_{n+1} - t_n \le t \mid X_n = i_n\} \quad (1.8)$$

then $\{X_n, t_n\},\ n = 0, 1, 2, \ldots.,$ constitutes a Markov Renewal Process with state space E.

1.3.12 Semi-Markov Process

In the above, if we assume that the process is time homogeneous, i.e.

$$Pr\{X_{n+1} = j,\ t_{n+1} - t_n{}^3 \mid X_n = i\} = Q_{ij}(t),\ i, j \in s \quad (1.9)$$

is independent of n, then there exist limiting transition probabilities

$$P_{ij} = Q_{ij}(t) = Pr(X_{n+1} = j \mid X_n = i\} \quad (1.10)$$

Then $\{X_n,\ n = 0, 1, 2, \ldots\}$ constitutes a Markov chain with state space E and transition probability matrix (t.p.m) is given by

$$P = \left[p_{ij} \right] \tag{1.11}$$

The continuous parameter stochastic process $Y(t)$ with state space E defined by

$$Y(t) = X_n,\ t_n < t < t_{n+1} \tag{1.12}$$

is called a semi-Markov process.

In other words, we define the semi-Markov process is a process in which transition from one state to another is governed by the transition probabilities of a Markov process but the time spent in each state before a transition occurs is random variable depending upon the last transition made. Thus at transition instants the semi-Markov behaves just like a Markov process. However, the times at which transitions occur are governed by a different probability mechanism.

1.3.13 Regenerative Process

Regenerative stochastic process was defined by Smith (1955) and has been crucial in the analysis of complex systems. In this, we take a time point at which the system history prior to the time point is irrelevant to the system conditions. These points are called regeneration points. Let $X(t)$ be the state of the system at epoch t. If $t = t_1, t_2 \ldots$ are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process $\{X(t),\ t = t_1, t_2 \ldots\}$ is called regenerative process.

1.3.14 First Passage Time

Suppose that a system starts with the state j, then time taken to reach a given state k for the first time from state j is called first passage time. In general, first passage time is a measure of how long it takes to reach a given state from another state.

1.3.15 Mean Sojourn Time in a State

The expected time taken by the system in a particular state before transiting to any other state is known as mean sojourn time or mean survival time in that state. If T_i be the sojourn time in state i, then mean sojourn time in state i is

$$\mu_i = \int_0^\infty P(T_i > t)\, dt \qquad (1.13)$$

1.3.16 Mean Time to System Failure (MTSF)

The average duration between successive system failures, i.e. MTSF is defined as the expected time for which the system is in operation before it completely fails.

Suppose the reliability function for a system is given by $R(t) = 1 - F(t)$, where $F(t)$ is the failure time distribution function and $f(t) = dF(t)/(dt)$ is the failure time density function. The mean time to system failure is given by

$$\text{MTSF} = \int_0^\infty t\, f(t)\, dt = -\int_0^\infty t\left(\frac{dR(t)}{dt}\right)dt = [-tR(t)]_0^\infty + \int_0^\infty R(t)\, dt$$

$$= \int_0^\infty R(t)\, dt = \lim_{s\to 0} R^*(s) \qquad (1.14)$$

Let $\phi_0(t)$ be the cumulative distribution function of the first passage time from initial state to a failed state, then

$$R^*(s) = \frac{1-\phi_0^{**}(s)}{s}$$

Thus, we have

$$\text{MTSF} = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} \qquad (1.15)$$

1.4 Type of systems

A system means an arbitrary device consisting of various components or units or subsystems with assumption that their reliability is known for predicting reliability of whole system.

On the basis of arrangement of components in the system, the systems can be classified as:

1.4.1 Series System:

This type of system generally consists of a large number of components connected in series. If anyone of these components fails, the system fails. This is also one of the most commonly used structures.

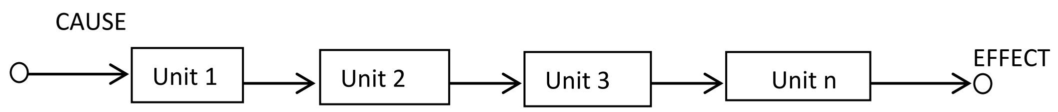

Fig. 1.1 - Series configuration

Let $R_i(t)$ be the reliability of ith components, then the system reliability is given by

$$R(t) = Pr\ (T > t] = Pr\left[min\ (T_1,\ T_2,\ T_3,\ldots,\ T_n) > t\right]\ \ [T_i > t] \tag{1.16}$$

where T_i is the life time of the i^{th} unit of the system. The system hazard rate, therefore, is

$$r(t) = \sum_{i=1}^{i=n} r_i(t) \tag{1.17}$$

where $r_i(t)$ is the instantaneous failure rate of the i^{th} unit.

1.4.2 Parallel System

This type of system generally consists of a large number of components connected in parallel. A parallel system is fail only when it's all components are failing. Parallel configuration is often referred to as redundancy. Redundancy is the technique which involves more than one component to achieve a higher reliability. Four engine aircraft which is still able to fly with only two engines working is a good example of this type of configuration. Block diagram representing a parallel configuration is shown in Fig. 1.2.

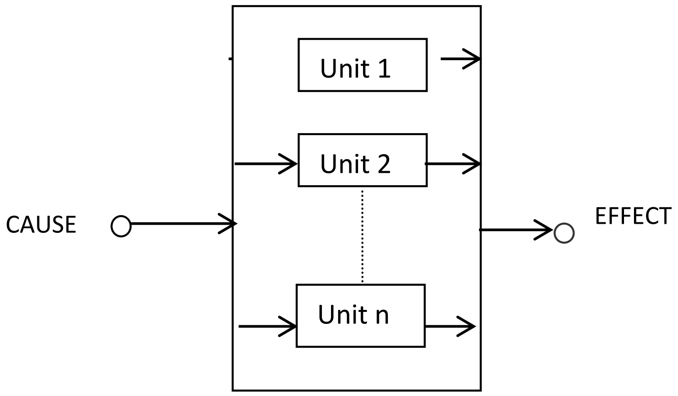

Fig. 1.2 - Parallel configuration

Suppose $R_i(t)$ and T_i be the reliability of ith component and the life time of the ith unit in time t respectively, then the system reliability is given by

$$\begin{aligned} R(t) &= Pr(T > t) = Pr\left[\ max\ (T_1,\ T_2,\ T_3,\ldots,\ T_n) > t\right] \\ &= 1 - P\ (T_1 \le t,\ T_2 \le t,\ T_3 \le t,\ldots,\ T_n \le t] \end{aligned} \tag{1.18}$$

If the units function independently, then

$$\begin{aligned} R(t) &= 1 - [1 - R_1(t)][1 - R_2(t)][1 - R_3(t)]\ldots\ldots\ldots[1 - R_n(t)] \\ &= 1 - \prod_{i=1}^{i=n}[1 - R_i(t)] \end{aligned} \tag{1.19}$$

1.4.3 Series-Parallel System:

This system consists of stage 1, stage 2... stage k connected in series.

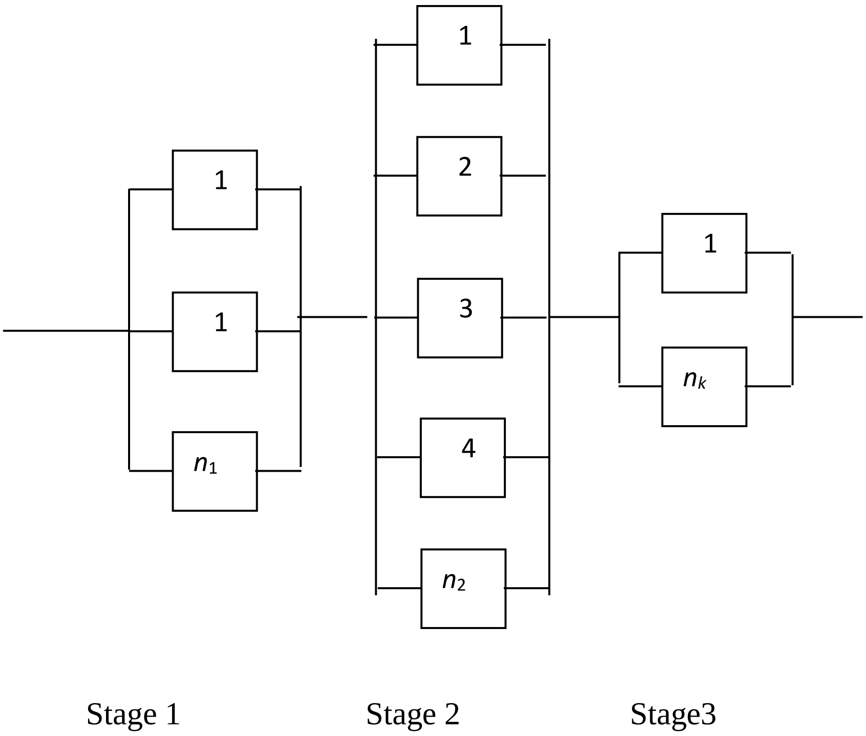

Fig. 1.3 - Series-Parallel combination of components

Each stage contains a number of redundant elements, stage 1 consisting of n_i redundant elements connected in parallel. The reliability of such a system is the product of the reliabilities of each stage. Stage i with n_i elements will have the reliability given by

$$R_i = 1 - [1 - P(X_{i1})][1 - P(X_{i2})] \ldots [1 - P(X_{in})] = 1 - \sum_{j=1}^{n_i}\left[1 - P(X_{ij})\right]$$

$$R(s) = R_1 R_2 R_3 \ldots R_K = \sum_{i=1}^{k}\left\{1 - \sum_{j=1}^{n_i}\left[1 - P(X_{ij})\right]\right\} \qquad (1.20)$$

1.4.4 Redundancy

If the state of the art is such that either it is not possible to produce highly reliable components or the cost of producing such components is very high, then we can improve the reliability of the system by introducing the technique of redundancies. This involves the deliberate creation of new parallel paths in a system to increase the reliability of the system Some of the methods of introducing redundancies in a system are mentioned below:

1.4.4.1 Element Redundancy

In element redundancy two elements are connected in parallel. In this, all the channels or paths are active from the beginning of the system till its failure. In this case the reliability of the system is higher than individual reliabilities.

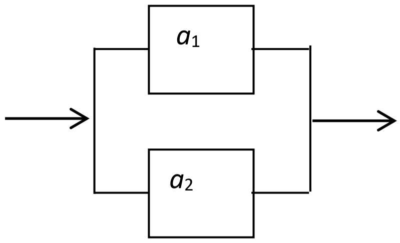

Fig. 1.4 - Element Redundancy

1.4.4.2 Standby Redundant Configuration

Redundancy is a device to improve reliability of a system. In a redundant system, more units are made available than which are necessary. There are two types of redundancy Gnedenko classified the redundancy on the basis how the units are loaded in the standby units in two ways

(i) Active Redundancy

In this case of redundancy, the system has a positive probability of failure even when it is not in operation. This may happen due to the effect of temperature, environment condition etc. Active redundancy can further be classified as hot redundancy and warm redundancy:-

(i) If the off-line unit can fail and is loaded in exactly the same way as the operating unit, it is called hot standby unit.

(ii) If the off-line unit can fail and can diminish the load, it is called warm standby unit. The probability of failure for a warm standby is less than that of failure for operative unit.

(ii) Passive or Cold Standby Redundancy

This is that form of redundancy in which the off-line unit is completely inactive and it cannot fail until it is put in place of primary unit.

Reliability R (t) of an n-unit standby system at any time instant t is given by

$$R(t) = P[\sum_{i=1}^{n} T_i > t] \qquad (1.21)$$

where T_i is the life time of i^{th} unit and all the n-units are independent.

A standby system functions as long as one of the units is available for the task on hand. A block diagram of such system is shown as in Fig. 1.2.

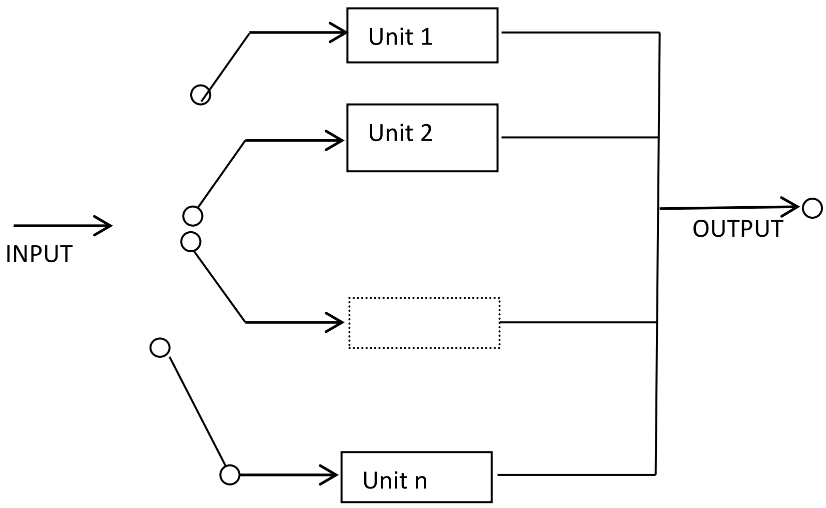

Fig.1.5 - Standby redundant configuration

1.4.5 k-out-of-n configuration

In many problems the system operates if at least k-out-of-n units function, e.g., a bridge supported by n-cables, k of which are necessary to support the maximum load. If each of n-units is identical with the same reliability then the system reliability becomes

$$R(t) = \sum_{i=k}^{n} {}^{n}C_i \, e^{-\lambda i t} (1-e^{-\lambda t})^{n-i} \qquad (1.22)$$

There exist many other configurations such as series-parallel, parallel-series, mixed parallel, etc. which are used by the industries.

1.5 Availability

When a system is often unavailable due to break downs the concerning department becomes interested to put it back into operation after each break down with proper repairs. In fact, it is concerned with availability equally as it does with reliability because of additional costs and inconvenience incurred when the system is not available. The differences between the measures reliability and availability are as follows:

(i) The reliability is an interval function while the availability is a point function describing the behaviour of the system at a specified epoch.

(ii) The reliability function precludes the failure of the system during the interval under consideration, while availability function does not impose any such restriction on the behaviour of the system.

We may categorize availability as:

1.5.1 Instantaneous (Point wise) Availability

This is the probability that the system will be able to operate within the tolerances at a given instant of time.

Let X(t) = 1, if the system is operable at time t, and X(t) = 0, when it is not operable. The availability A(t) of the system at time t is given by

$$A(t) = P\left[X(t) = 1 \mid X(0) = 1\right] \tag{1.23}$$

1.5.2 Average (Interval) Availability

It is the expected fraction of a given interval of time that the system will be able to operate within tolerances. Suppose the given interval of time is (0, T]. Then interval availability $H(0,\ T] = A(T)$ for this interval is given by

$$A(T) = \frac{1}{T}\int_0^T A(t)\, dt \tag{1.24}$$

1.5.3 Steady State (Limiting Interval) Availability

It is the expected fraction of time in the long run that the system operates satisfactory. To obtain steady state availability we simply compute

$$\lim_{T\to\infty} H(0,\ T) = \lim_{T\to\infty} A(T) \tag{1.25}$$

1.5.4 Operational Availability (A_0)

Operational availability is a measure of availability, which includes all exoerienced sources of downtime. The equation for operational availability is:

$$A_0 = \text{Uptime/ (Operation Cycle),}$$

where the operation cycle is the overall time period of operation being investigated, and uptime is the total time for which the system was functioning during the operating cycle. Thus, operational availability is the availability actually experienced by the customer.

1.6 Maintainability

Maintainability v/s Reliability: Reliability and maintainability jointly affect the availability of the equipment. Highly reliable equipment or a system may fail rarely, but, if its maintainability is poor, then it takes very long time to repair and decommission once it fails. Thus, the availability of highly reliable equipment may reduce considerably, if the maintenance is poor. Similarly equipment may have very good maintainability, but if it has poor reliability then it would fail frequently and in turn availability would get reduced. Maintainability may be given less importance in some applications like missiles and rocket propulsion etc. but, for general industrial equipments and components, maintainability has to be given more considerations.

Maintenance: Maintenance is defined as any action that restores failed units to an operational condition or retains non-failed units in an operational condition. In earlier days very few terms were used in maintenance management like repair, overhauling, preventive maintenance etc. With the involvement of experts in maintenance management several new terms were invented such as planned, scheduled, routine, periodic, breakdown, corrective and predictive maintenance etc. Maintenance actions can be classified on the basis of planning and criticality/essentiality of the jobs.

1.6.1 Corrective Maintenance

Corrective maintenance is the process to restore a failed part or system to its operational status. It is usually done by replacing or repairing a part or subsystem that is causing the system failure. Such maintenance is done at unpredictable intervals, as a component failure time is not known. The objective of corrective maintenance is to restore the system to satisfactory operative state within the shortest possible time. Corrective maintenance basically involves three steps:

I: Diagnosis of the Problem: The maintenance technician locates the failed parts or assess the cause of the system failure.

II: Repair and Replacement of Faulty Components: Once the cause of system failure is known, action is taken by replacing or repairing the components which causes the system failure. Repair action is also taken in case the component is working below acceptable reduced capacity.

III: Verification of the Repair Action: Once the components have been repaired or replaced, the maintenance technician must verify that the system is operating successfully.

1.6.2 Preventive Maintenance (PM)

Preventive maintenance is the execution of replacing components or subsystems before they fail in order to foster continuous system operation. Preventive maintenance can also be stated as the formulation for maintenance of components or equipment in order to prevent or minimize the fragmentation and devalued rates. Preventive Maintenance covers extensive areas due to which some people get misled about its coverage by thinking that it is just a routine inspection or doing minor repairs/jobs on equipment. And on the other hand some think that it includes only major jobs like overhauling and reconditioning etc. After preventive maintenance repairs the equipment's health, it is restored back nearly to the original condition. The schedule for preventive maintenance is based on observation of past behaviour of the system. The objective of preventive maintenance is to run the equipments in good conditions for a long period of time. Cost is also a factor for the scheduling of preventive maintenance. In many circumstances, it is financially more sensible to replace parts or components that have not failed at predetermined intervals rather than to wait for a system failure which may result in costly disruption in operations.

1.7 Busy Period

Let $B(t)$ be the probability that a repairman is busy with the system in the interval $(0,t]$

Then fraction of time for which system is down is given as:

$$B = \lim_{t\to\infty} B(t) = \lim_{s\to 0} s\ B^*(s) \tag{1.26}$$

1.8 Down Time

Let $D(t)$ be the probability that a is in down state in the interval $(0,t]$. Then fraction of time for which repairman is busy is given as:

$$D = \lim_{t\to\infty} D(t) = \lim_{s\to 0} s\,D^{*}(s) \tag{1.27}$$

1.9 Profit Analysis

Availability of the system leads to revenue whereas the busy period of the repairman, expected number of visits by the repairman, expected number of replacements, etc. lead to the cost of maintenance and spares. The revenue and cost function lead to the profit function of a firm, as the profit is excess of revenue over the cost of production. The profit function takes the form

P (t) = Expected revenue in (0, t] – expected total cost in (0, t]

In general, the optimal policies can more easily be derived for an infinite time span as compared to a finite span. The profit per unit time is expressed as $\lim_{t\to\infty}\frac{P(t)}{t}$ i.e.

Profit per unit time = total revenue per unit time – total cost per unit time.

For example, the profit equation may be given as

$$P_i = C_0A_0 - C_1B_0 - C_2D_0\ , \tag{1.28}$$

where

P_i = profit per unit up time of the model in the i^{th} chapter

C_0 = revenue per unit up time of the system

A_0 = the total fraction of time for which the system is up

C_1 = cost per unit time for which the repairman is busy in repairing the failed unit

B_0 = the total fraction of time for which the repairman is busy

C_2 = cost per unit time for which the system is in down state.

D_0 = the total fraction of time for which the system is in down state

1.10 Transforms and Convolutions

In Markov Model formulation we deal with set of first order differential equations. In simple models these differential equations can be easily solved but in case of complex models where we get differential equations coupled it is very difficult to solve. In such cases these differential equations are solve by using Laplace transforms. Here are some transforms and convolution which we have been used in our research work.

1.10.1 Laplace Transform

Let f(t) be a function of a positive real variable t. Then the Laplace transform (L.T.) of f(t) is defined as

$$L\left[f\left(t\right)\right] = f^*\left(s\right) = \int_0^\infty e^{-st} f\left(t\right)\, dt \tag{1.29}$$

for the range of value of s for which the integral exists. Here f (t) is called an inverse Laplace transform of f*(s) and we write

f (t) = L^{-1} {f*(s)}. The following are some important properties of Laplace transform:

(i) $L\left[\sum_{i=1}^{n} c_i f_i(t)\right] = \sum_{i=1}^{n} c_i f_i^*(s)$ (1.30)

(ii) $L\left[t^n f(t)\right] = (-1)^n \dfrac{d^n f^*(s)}{ds^n}$ (1.31)

(iii) $L\left[\int_0^t f(u)du\right] = L[F(t)] = \dfrac{f^*(s)}{s}$ (1.32)

(iv) $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sf^*(s)$ (initial value theorem) (1.33)

(v) $\lim_{t\to\infty} F(t) = \lim_{s\to 0} s\, f^*(s)$ (final value problem) (1.34)

(vi) $\lim_{s\to 0} f^*(s) = 1$ if f*(s) is L.T. of a p.d.f. (1.35)

1.10.2 Laplace Stieltjes Transform

Let X be a non-negative random variable with distribution function

$$F(x) = Pr[X \le x] \tag{1.36}$$

then Laplace Stieltjes transform (L.S.T.) of F(x) is defined, for s > 0 by

$$F^{**}(s) = \int_0^\infty e^{-sx}\, d\, F(x) \tag{1.37}$$

Therefore, we have

$$F^{**}(s) = \int_0^\infty e^{-sx} f(x)\, dx = f^*(s), \tag{1.38}$$

where f(x) = $\frac{dF(x)}{dx}$

1.10.3 Convolution

Let f(t) and g(t) be two real valued non-negative continuous functions of t, then the integral

$$\int_0^t f(t-u)g(u)du = \int_0^t g(t-u)f(u)du = f(t)\,(c)\,g(t) = L^{-1}\left[f^*(s).g^*(s)\right] \tag{1.39}$$

is called Laplace convolution of the functions f(t) and g(t).

If F(t) and G(t) be two real valued distribution functions defined for $t \geq 0$ the resulting convolution is again a distribution function and the integral

$$\int_0^t f(t-u)g(u)du = \int_0^t g(t-u)f(u)du = f(t)\,(s)\,g(t) \tag{1.40}$$

is known as Stieltjes convolution of F(t) and G(t).

1.11 Distribution Used

The family of exponential distribution is the best known and most thoroughly explored, largely through the work of Epstein (1958) and his associates. Exponential distribution plays an important role in reliability studies. Besides a number of desirable mathematical properties, it has a very important memory less property i.e. if the life length T of a structure has the exponential distribution, previous use does not affect its future life length. For example, an electric fuse whose future life distribution is practically unchanged as long as it has not yet failed. In the present work, the failure time/repair time is assumed to follow an exponential distribution.

Exponential distribution is defined as follows:

A continuous random variable having the range $0 \leq t < \infty$ is said to have an exponential distribution if it has the probability density function of the form

$$f(t) = \begin{cases} \lambda e^{-\lambda t}, & 0 \leq t < \infty \\ 0, & t < 0 \end{cases} \tag{1.41}$$

where λ is a positive constant. The corresponding distribution function is

$$F(t) = \begin{cases} 1-e^{-\lambda t}, & 0 \le t < \infty \\ 0, & t < 0 \end{cases} \qquad (1.42)$$

The hazard rate 'λ' is constant. The laplace transform of p.d.f. of exponential distribution is $\frac{\lambda}{\lambda+s}$.

In order to meet the objectives of the thesis, reliability models are developed using regenerative point technique keeping in view, the functioning of rice manufacturing plant. Therefore, it is important to discuss the working behaviour of Kohinoor Rice Plant situated at Ganaur, Sonipat, Haryana. Actual data on failure rates, repair rates and various other parameters is collected from rice plant.

1.12 Kohinoor Rice Plant – Paddy to Rice Converter:

Husk and brown rive are main components of paddy or rice grain. Brown rice contains bran which consists of outer layer and the edible portion. Rice milling is removal of husk and bran to provide the portion of rice which is edible. The process has to be done with care to prevent unnecessary breakage of the kernel and increase recovery of paddy or rice. The by-products come out in mixed or separated form as per the different rice mills. The process is usually done in case of dry paddy. Rice is considered to be staple food diet in most parts of India including the North-East states. Paddy is the most important agricultural commodity in Andhra Pradesh. The main objective of the Rice Mill is to remove husk, bran, clean and polish to obtain edible portion. One can easily achieve maximum reliability of the system by using standby component. For this purpose, one should have the knowledge which component of the system is more sensitive, depending on that one can mend the system or that particular component.

Paddy to rice converters consists of following components:

1. Elevator

Elevator (or lift) is vertical transport equipment that efficiently moves people or goods between floors (levels, decks of a building, vessel or other structures). Elevators are generally powered by electric motors that either drive traction cables and counterweight systems like a hoist, or pump hydraulic fluid to raise a cylindrical piston like a jack.

2. Cleaner

Paddy cleaner is suitable for cleaning and classifying the grain. It removes big or small impurities and stones from raw grain, rice, wheat, corn, soybean, sesame, etc. It is the ideal machine for grain processing plant or flour mill. There are three kinds of paddy cleaner according to its usage, Combined Cleaner**,** Rotary Paddy Cleaner, Vibration Cleaner, among this three, combine paddy de-cleaner is the most popular one.

3. Husker

Rice husker is an agricultural machine used to automate the process of removing the chaff (the outer husks) of grains of rice.

4. Paddy separator

It is feasible for separating brown rice from paddy efficiently under reciprocating movement of sieve plate according to different bulk densities, specific gravities and surface friction coefficients between paddy and brown rice. It is particularly suitable for separating mixed or low uniformity grain to improve the purity degree of brown rice.

5. Destoner

A destoner is a machine that removes stones and clods from soil ridges and moves them to the furrow so that the ridges are free from stones. This also helps when harvesting in wet conditions as the harvester can drive on a row of stones which helps improve traction.

6. Whitener

The function of Whitening unit is to remove all or part of the bran layer and germs from brown rice.

7. Polisher

This subsystem improves the appearance of milled rice by removing the remaining bran particles and by polishing the exterior of the milled kernel.

8. Grader

(i) **Thickness grader:** The machine used to separate grains of different thickness is known as Grader. The material is passed through cylindrical screens, which revolves for separating oversized and undersized grains. This machine is highly efficient for separating the mixture of oversized or undersized grains.

(ii) **Length Grader:** The machine used to separate the different length grainsis knows as length grader which is highly efficient for separating the mixture of different length grains. This can also be used to separate immature grains.

If the failure exists due to all the above eight subparts then we have standby i.e. we can connect the system to line running in parallel.

9. Colour sorter

A colour sorter machine separates rice grains according to differences in colour of rice grains. For colour sorting, the rice mixture will traveled through an elevator belt into a hopper on top of the machine, from which it will flow down along chutes in the colour sorter, streamlining their flow to so that they may be scanned by CCD sensors. The moment the camera detects any defect in colour, it instructs ejectors fitted in the machine to open the nozzle. The nozzle is connected to valves containing compressed air. This air is used to shoot out the defected material. If the failure exists due to this machine then we have no standby i.e. we cannot connect the system to line running in parallel.

In our research, we have considered 2 unit and 3 unit standby systems of Paddy to rice. Both similar and dissimilar units in the system have been considered. The system under consideration is assumed to have two shifts of working and the whole system undergoes for scheduled preventive/corrective maintenance before starting the second shift. Each unit has two types of failure-one due to failure of the component colour sorter having no standby which is referred as Type –I failure and the second due to failure of any of the other component- elevator, destoner, husker, paddy separator, polisher and grader having its standby referred as Type – II failure. The processing done on each of the components of the unit before the component with no standby is transferable to the corresponding component of the other unit. But the processing in pending due to failure I is completed only on the concerned unit after that component is repaired. One, two or all three units are made operative depending on the need. Otherwise after repair unit is made standby.

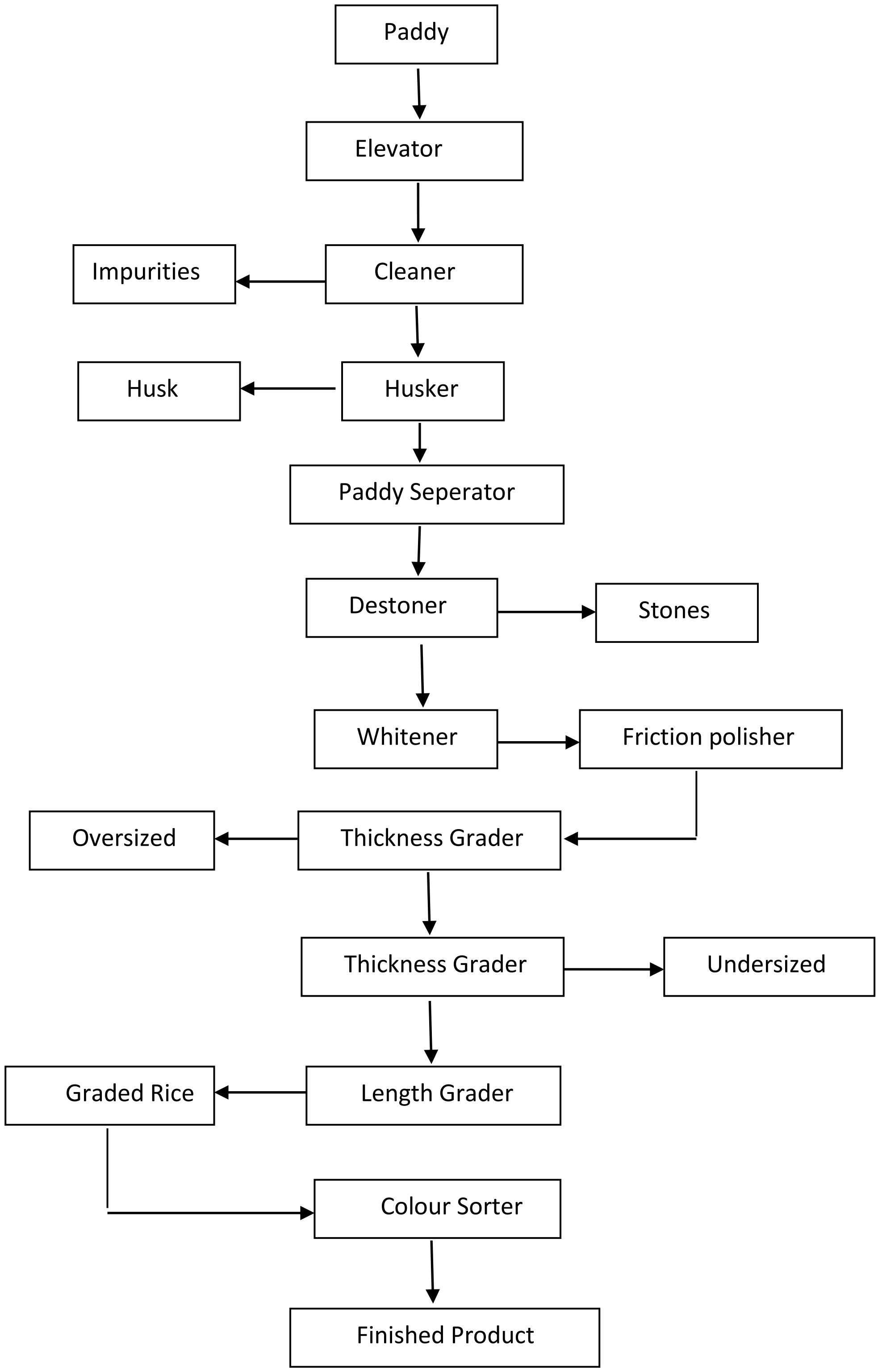

Fig 1.6 – Flow Chart of Rice Plant

1.13 Present Work

The objective of the present work is to bridge the existing gap between reliability theory and practical applications in the field of reliability engineering. The present work deals with the reliability analysis of some complex industrial systems. We, in the present thesis, collected data on failure/repair times and on some other measures of a 2 and 3 unit paddy to rice converter system – Kohinoor Rice Mill Sonipat and analyse it by doing reliability modeling for practically existing situation in the plant and also for some other relevant situations/assumptions. Comparative study among these different situations has also been carried out to arrive at very important/useful conclusions. The thesis has been divided into six chapters The attempt has been made to solve the real life problems experienced by the industries using regenerative point technique. An effort has been made in the present work to improve the profit of the industries. A brief summary of work presented in succeeding chapters is as:

Chapter 2

Reliability and Economic Analysis of a System Consisting of Two Dissimilar Units With Variation in Production

In this chapter we discuss the behavioural analysis of a two dissimilar unit standby system working in a rice plant by making one or both units operative depending upon the demand. The system is discussed with two types of failures. The processing done on each of the components other than colour sorter is transferable to component of other unit but the processing pending due to failure of colour sorter is completed only on concerned unit after it is repaired. Regenerative point technique has been used to analyse the system.

Chapter 3

Cost Analysis of a Two Dissimilar Unit System With Goodwill Loss When Demand is More Than Production

It describes a standby system which comprises of two dissimilar units one is of eight ton capacity and another is of four ton capacity. In this chapter, we have considered the different stages where production is less than demand or at least equal to demand and system is analysed. Failure time and repair time both assumes exponential distribution. Repair facility is always available for every failed unit i.e. no unit waits for repair. It is assumed that in case of production greater than or equal to demand, if one unit is under repair and other is operative then no other event can take place except decrease in production. Effect of revenue of different capacity on profit is also discussed along with effect of failure rate on availability of different capacities.

Chapter 4

Analysis of a Three Unit System with Demand Dependent Operability of Units

This chapter is devoted to a stochastic model for a three-unit standby system wherein one, two or all the units may be made operative depending upon the load/demand. The system to be considered comprises of three units of paddy to rice converters having different capacities. One is of eight ton capacity and another two are of four ton capacity. Priority of operation is given to line of capacity eight ton instead of four ton. Rest processing of the system and assumptions are same as in the preceding chapters. Effect of revenue of different capacity on profit, effect of failure rate on MTSF and availability of different capacities has been discussed in this chapter. Graphical study has also been carried out.

Chapter 5

Cost Analysis of a Two-Unit Standby Industrial System with Varying Demand

In this chapter a two similar unit (eight ton capacity) standby system is analysed with variation of demand and two types of failures. In this chapter, we have considered a single unit of eight ton capacity in place of two similar units of four ton capacity each as in the preceding chapter. Keeping in mind the installation cost of one big unit in place of two small units here, we have analysed a two similar unit standby system with varying demand and two types of failures. We have considered arbitrary distribution for repair rate. Effect of failure rates on availability and MTSF has been carried out. Effect of revenue and cost of busy period on profit has also been discussed.

Chapter 6

Comparative Analysis of the Models Discussed Under Different Situations

In this chapter, the comparative study of effectiveness of models discussed in the chapters 3, 4 and 5 with respect to profit evaluation has been discussed. In this chapter, we have also analysed the suitability of models discussed in previous chapters in different situations.

Chapter 7

Summary and Conclusion

This chapter is concluding chapter of thesis. The industrial significance along with the limitations and future scope of the present work has also been briefly discussed in the conclusion.

CHAPTER 2

Reliability And Economic Analysis Of A System Consisting Of Two Dissimilar Units With Variation In Production

The Reliability of a system is an important parameter which measures the quality of its consistence performance over its expected life span. With the development of modern technology and the world economy, the reliability aspect has been attracting imperative attentions. There is a lot of contribution in the field of reliability by various researchers including Nakagawa [76], Tuteja and Taneja [111], Gupta et al [36], Attahiru and Zhao [4], Taneja et al [105], Gupta et al [34], Minocha [66], Sarhan [90], Mathew et al (2011), Pham [82], Wang and Pham [120], Li and Jian [56], Parashar and Bhardwaj [81] and Yusuf [123] etc. Various situations have been taken into consideration by these researchers like different repair policies, k out of n systems and degradation. The concept of varying demand is important to be studied. This concept of work with varying demand has been done by Malhotra and Taneja [59, 60]. However, sufficient work has not been carried out for varied production in accordance with demand. Moreover many other aspects need to be taken into account along with fluctuating demand like considering systems with different types of failures, different shifts of working and concept of scheduled maintenance after each shift etc. As such situations may be observed in many systems, a paddy to rice converter is one of such systems. Thus incorporating the concepts of two types of failures along with variation in demand, we in the present chapter analyse reliability and cost benefit for the collected data from a rice manufacturing plant. In this chapter we develop a stochastic model for two unit standby system of a Rice Plant by making one or both the units operative depending upon the load/demand.

This chapter has been organised as follows: Section 2.1 is about description of a rice manufacturing system and assumptions of two-unit standby system are also discussed in this section. The various notations are presented in section 2.2. The mathematical formulation for stochastic model determining, transition probabilities and mean sojourn times, are developed in Section 2.3. Section 2.4 – 2.7 deals with the formulation of Mean

Time to System Failure, availability, busy period analysis and expected down time of the system. Cost Benefit analysis is done in section 2.8. Conclusions based on the present study are finally drawn in Section 2.9.

2.1 System Description and Assumptions

The system comprises of two dissimilar paddy to rice converters with different capacities i.e., of eight ton and four ton. The system under consideration is assumed to have two shifts of working and the whole system undergoes for scheduled preventive/corrective maintenance before starting the second shift. Mathematical formulation of the problem determining the transition probabilities of various states are developed considering two types of failure (Type -I which has no standby and the Type - II which has standby) for each unit. In case one unit is under repair and the other is working then after the repair, the unit is kept in standby mode or made operative according as the demand is less or more than the capacity of production by the working unit. Various reliability metrics such as MTSF, availability, busy period and profit have been discussed for measuring the system effectiveness by using semi-Markov process and regenerative point technique. Initially the system is in the working condition with both units operative. The failed unit is undertaken for repair immediately.

The following assumptions have been considered for the model:

Assumptions:

(i) After every repair the unit behaves like a completely new one.

(ii) After getting repaired the unit can be made operative or standby according to the need.

(iii) Repair facility is always available for every failed unit i.e. no unit waits for repair

(iv) Both Failure times as well as repair times follow exponential distribution.

The system is observed at suitable regenerative epochs by using regenerative point technique and the following reliability characteristics have been obtained:

(i) Mean time to system failure (MTSF)

(ii) Availability with full capacity

(iii) Availability with reduced capacity

(iv) Expected busy period of repairman in (0,t]

(v) Expected downtime

(vi) Expected profit incurred in (0,t]

2.2 Notations

Following notations have been used through out this chapter:

B_s	:	Unit with eight ton capacity in standby mode
B_o	:	Unit with eight ton capacity in operative mode
S_o	:	Unit with four ton capacity in operative mode
S_s	:	Unit with four ton capacity in standby mode
S_{op}	:	Unit with four ton capacity in pending operation state.
B_{op}	:	Unit with eight ton capacity in pending operation state.
B_r, S_r	:	System is at rest
F_{r_1}	:	Unit having Type –I failure is under repair
F_{r_2}	:	Unit having Type –II failure is under repair
λ_1	:	Rate of Type -I failure for big unit i.e. of eight ton capacity
λ_2	:	Rate of Type -II failure for big unit i.e. of eight ton capacity
β_1	:	Rate of Type -I failure for small unit i.e. of four ton capacity
β_2	:	Rate of Type -II failure for small unit i.e. of four ton capacity
γ_1	:	Rate with which the system is made operative from rest
γ_2	:	Rate with which the system goes to rest from operative state
a_1	:	Rate of Type – I repair for big unit i.e. of eight ton capacity
a_2	:	Rate of Type – II repair for big unit i.e. of eight ton capacity
b_1	:	Rate of Type – I repair for small unit i.e. of four ton capacity
b_2	:	Rate of Type – II repair for small unit i.e. of four ton capacity
$i(t)$	:	p.d.f of time to complete pending process of material at colour sorter for big unit

$I(t)$	:	c.d.f of time to complete pending process of material at colour sorter for big unit
$m(t)$	:	p.d.f of time to complete pending process of material at colour sorter for small unit
$M(t)$	:	c.d.f of time to complete pending process of material at colour sorter for small unit
p	:	Probability that the unit is not made operative after repair depending upon demand.
q	:	Probability that the unit is made operative after repair depending upon demand.
$H(t)$	:	c.d.f. of time to make operative state to standby or standby state to operative (as per demand)
$h(t)$	:	c.d.f. of time to make operative state to standby or standby state to operative (as per demand)
p_1	:	Probability that system is made operative from capacity of twelve ton to eight ton
p_2	:	Probability that system is made operative from capacity of twelve ton to four ton
p_3	:	Probability that system is made operative from capacity of 4 ton to twelve ton
p_4	:	Probability that system is made operative from capacity of 4 ton to eight ton
p_5	:	Probability that system is made operative from capacity of 8 ton to twelve ton
p_6	:	Probability that system is made operative from capacity of 8 ton to 4ton

The system of converting paddy into rice of a rice manufacturing plant has the following states:

(i) Regenerative states:

$$\left\{\begin{array}{l} S_0(B_o,S_o), S_1(B_o,S_s), S_2(B_r,S_r), S_3(B_s,S_o),\ S_4\left(B_{Fr_1},S_o\right), S_5\left(B_{Fr_1},S_o\right),\ S_6\left(B_o, S_{Fr_1}\right), \\ S_7\left(B_o, S_{Fr_2}\right),\ S_8\left(B_{op}, S_o\right),\ S_{13}\left(B_o, S_{op}\right) \end{array}\right\}$$

(ii) Failed states:

$$\left\{S_9\left(B_{Fr_1}, S_{Fr_1}\right), S_{10}\left(B_{Fr_1}, S_{Fr_2}\right), S_{11}\left(B_{Fr_2}, S_{Fr_1}\right), S_{12}\left(B_{Fr_2}, S_{Fr_2}\right)\right\}.$$

Due to complexity of system and large no. of states transition diagram of two dissimilar standby unit system is shown in the form of table.

Table 2.1: Possible states of transition

State (S_i) ***i = 0 to 13***	***Status***	***Possible transition to***	***With failure / repair rates / transition probabilities / p.d.f respectively***
0	*Bo, So*	1, 2, 3, 4, 5, 6 , 7	$p_2\, h(t), p_1\, h(t), \gamma_1, \lambda_1, \lambda_2, \beta_1, \beta_2$
1	*Bo, Ss*	0, 2, 6 , 7	$p_2\, h(t), p_1\, h(t), \beta_1, \beta_2$
2	*Br, Sr*	0, 1, 4 , 5	$p_2\, h(t), p_1\, h(t)\ , \lambda_1, \lambda_2$
3	*Bs, So*	0	γ_2
4	Bfr_1, *So*	0, 8, 9 , 10	$q\,a_1, p\,a_1, \beta_1, \beta_2$
5	Bfr_2, *So*	0, 1, 11 , 12	$q\,a_2,\ p\,a_2\ , \lambda_1, \lambda_2$
6	*Bo*, Sfr_2	0, 2, 10 , 12	$q\,b_2\ , p\,b_2, \lambda_1, \lambda_2$
7	*Bo*, Sfr_1	0, 9, 11 , 13	$q\,b_1\ , \lambda_1, \lambda_2,\ p\,b_1$
8	*Bop, So*	1	$i(t)$

9	Bfr_1, Sfr_1	4 , 7	b_1 , a_1
10	Bfr_1, Sfr_2	4 , 6	b_2 , a_1
11	Bfr_2, Sfr_1	5 , 7	b_1 , a_2
12	Bfr_2, Sfr_2	5 , 6	b_2 , a_2
13	Bo, Sop	2	$m(t)$

2.3 Transition Probabilities

The possible states of transition are exhibited in table 2.1. The epochs of entry into states 0, 1, 2, 3, 4, 5, 6, 7, 8 and 13 are regeneration points and hence these states are regenerative states. The transition probabilities p_{ij} can be obtained as:

$$p_{ij} = \lim_{s\to 0} q_{ij}^{*}(s) = \lim_{s\to 0} \int_0^\infty e^{-st} q_{ij}(t)\, dt \tag{2.1}$$

The assumptions discussed in preceding section and using equation (2.1), transition probabilities can be obtained as follows.

$$\left.\begin{array}{l}
q_{01} = p_2 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)t} h(t)\,,\ q_{02} = p_1 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)t} h(t) \\
q_{03} = \gamma_2 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)t}\, \bar{H}(t)\,,\ q_{04} = \lambda_1 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)t}\, \bar{H}(t) \\
q_{05} = \lambda_2 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)t}\, \bar{H}(t)\,,\ q_{06} = \beta_2 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)t}\, \bar{H}(t) \\
q_{07} = \beta_1 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)t}\, \bar{H}(t)\,,\ q_{10} = p_2 e^{-(\beta_1+\beta_2)t}\, h(t) \\
q_{12} = p_1 e^{-(\beta_1+\beta_2)t}\, h(t)\,,\ q_{16} = \beta_2 e^{-(\beta_1+\beta_2)t}\, \bar{H}(t)\,,\ q_{17} = \beta_1 e^{-(\beta_1+\beta_2)t}\, \bar{H}(t) \\
q_{20} = p_2 e^{-(\lambda_1+\lambda_2)t} h(t)\,,\ q_{21} = p_1 e^{-(\lambda_1+\lambda_2)t}\, h(t)\,,\ q_{30} = \gamma_1 e^{-\gamma_1 t} \\
q_{40} = q e^{-(\beta_1+\beta_2+a_1)t}\,,\ q_{48} = p e^{-(\beta_1+\beta_2+a_1)t}\,,\ q_{49} = \beta_2 e^{-(\beta_1+\beta_2+a_1)t} \\
q_{4,10} = \beta_1 e^{-(\beta_1+\beta_2+a_1)t}\,,\ q_{50} = q e^{-(\beta_1+\beta_2)t} g_2(\mathrm{t})\,,\ q_{51} = p e^{-(\beta_1+\beta_2+a_2)t}
\end{array}\right\}$$

$$
\left.\begin{aligned}
&q_{5,12}=\beta_2 e^{-(\beta_1+\beta_2+a_2)t}\ ,\ q_{5,11}=\beta_1 e^{-(\beta_1+\beta_2+a_2)t}\ ,\ q_{60}=\ q b_2 e^{-(\lambda_1+\lambda_2+b_2)t}\\
&q_{62}=p b_2\ e^{-(\lambda_1+\lambda_2+b_2)t}\ ,\ q_{6,12}=\lambda_2 e^{-(\lambda_1+\lambda_2+b_2)t}\ ,\ q_{6,10}=\lambda_1 e^{-(\lambda_1+\lambda_2+b_2)t}\\
&q_{70}=\ q b_1 e^{-(\lambda_1+\lambda_2+b_1)t}\ ,\ q_{79}=p b_1 e^{-(\lambda_1+\lambda_2+b_1)t}\ ,\ q_{7,13}=\lambda_2 e^{-(\lambda_1+\lambda_2+b_1)t}\\
&q_{7,11}=\lambda_1 e^{-(\lambda_1+\lambda_2+b_1)t}\ ,\ q_{81}\ =\ i(t)\ ,\ q_{13,2}\ =\ m(t)
\end{aligned}\right\}\qquad (2.2)
$$

The non zero elements p_{ij} are obtained as:

$$
\left.\begin{aligned}
&p_{01}=p_2h^*(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)\ ,\ p_{02}=p_1h^*(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)\\
&p_{03}=\gamma_2\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)\right),\ p_{04}=\lambda_1\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)\right)\\
&p_{05}=\lambda_2\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)\right),\ p_{06}=\beta_2\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)\right)\\
&p_{07}=\beta_1\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2)\right),\ p_{10}=p_2h^*(\beta_1+\beta_2)\ ,\ p_{12}=p_1h^*(\beta_1+\beta_2)\\
&p_{16}=\beta_2\left(1-h^*(\beta_1+\beta_2)\right)\ ,\ p_{17}=\left(1-h^*(\beta_1+\beta_2)\right),\ p_{20}=p_2h^*(\lambda_1+\lambda_2)\\
&p_{21}=p_1h^*(\lambda_1+\lambda_2)\ ,\ p_{24}=\lambda_2(1-h^*(\lambda_1+\lambda_2))\ ,\ p_{25}=\lambda_1(1-h^*(\lambda_1+\lambda_2))\\
&p_{30}=1\ ,\ p_{40}=\frac{q\,a_1}{a_1+\beta_1+\beta_2}\ ,\ p_{48}=\frac{p\,a_1}{a_1+\beta_1+\beta_2}\ ,\ p_{49}=\frac{\beta_2}{a_1+\beta_1+\beta_2}\\
&p_{4,10}=\frac{\beta_1}{a_1+\beta_1+\beta_2}\ ,\ p_{50}=\frac{q\,a_2}{a_1+\beta_1+\beta_2}\ ,\ p_{51}=\frac{p\,a_2}{a_1+\beta_1+\beta_2}\\
&p_{5,12}=\frac{\beta_2}{a_2+\beta_1+\beta_2}\ ,\ p_{5,11}=\frac{\beta_1}{a_2+\beta_1+\beta_2}\ ,\ p_{60}=\frac{q\,b_2}{b_2+\lambda_1+\lambda_2}\\
&p_{62}=\frac{p\,b_2}{b_2+\lambda_1+\lambda_2}\ ,\ p_{6,12}=\frac{\lambda_1}{b_2+\lambda_1+\lambda_2}\ ,\ p_{6,10}=\frac{\lambda_2}{b_2+\lambda_1+\lambda_2}\\
&p_{70}=\frac{q\,b_1}{b_1+\lambda_1+\lambda_2}\ ,\ p_{79}=\frac{p\,b_1}{b_1+\lambda_1+\lambda_2}\ ,\ p_{7,11}=\frac{\lambda_1}{b_1+\lambda_1+\lambda_2}\\
&p_{7,13}=\frac{\lambda_2}{b_1+\lambda_1+\lambda_2}\ ,\ p_{81}=1\ ,\ p_{13,2}=1
\end{aligned}\right\}\qquad (2.3)
$$

The mean sojourn time μ_i in the i th regenerative state is

$$
\mu_i\ =E(t)=P_r(T>t)=\int_0^\infty d\,(Q_{i\,j}(t))\qquad (2.4)
$$

$$\left.\begin{aligned}
&\mu_0 = \frac{1}{\gamma_1+\lambda_1+\lambda_2+\beta_1+\beta_2}\left(1-h^*(\gamma_1+\lambda_1+\lambda_2+\beta_1+\beta_2)\right) \\
&\mu_1 = \frac{1}{\beta_1+\beta_2}\left(1-h^*(\beta_1+\beta_2)\right) \ , \quad \mu_2 = \frac{1}{\lambda_1+\lambda_2}\left(1-h^*(\lambda_1+\lambda_2)\right) \\
&\mu_3 = \frac{1}{\gamma_2} \quad , \quad \mu_6 = \frac{1}{\lambda_1+\lambda_2+b_2} \ , \ \mu_4 = \frac{1}{\beta_1+\beta_2+a_1} \\
&\mu_5 = \frac{1}{\beta_1+\beta_2+a_2} \ , \ \mu_7 = \frac{1}{\lambda_1+\lambda_2+b_1} \ , \ \mu_8 = -i^{*\prime}(0) = 1 \\
&\mu_{13} = -m^{*\prime}(0) = 1
\end{aligned}\right\} \tag{2.5}$$

The unconditional mean time taken by the system is mathematically stated as

$$m_{ij} = \int_0^\infty t\, dQ_{ij}(t) = \int_0^\infty t\ q_{ij}(t)\, dt = -q_{ij}{}^{*}(0) \tag{2.6}$$

Thus, we get

$$\left.\begin{aligned}
&m_{01}+m_{02}+m_{03}+m_{04}+m_{05}+m_{06}+m_{07} = \mu_0 \\
&m_{10}+m_{12}+m_{16}+m_{17} = \mu_1 \\
&m_{20}+m_{21}+m_{24}+m_{25} = \mu_2 \ , \ m_{30} = \mu_3 \\
&m_{40}+m_{410}+m_{48}+m_{49} = \mu_4 \ , \ m_{50}+m_{51}+m_{5,11}+m_{5,12} = \mu_5 \\
&m_{60}+m_{62}+m_{6,10}+m_{6,12} = \mu_6 \ , \ m_{70}+m_{79}+m_{7,11}+m_{7,13} = \mu_7 \\
&m_{81} = \mu_8 \ , \ m_{13,2} = \mu_{13}
\end{aligned}\right\} \tag{2.7}$$

2.4 Mean Time to System Failure (MTSF)

In order to determine the mean time to system failure (MTSF) of the system, the failed state are considered as absorbing states, The following recursive relation for $\phi_i(t)$ have been obtained as:

$$\left.\begin{aligned}
\phi_0(t) &= Q_{01}(t)(s)\phi_1(t)+Q_{02}(t)(s)\phi_2(t)+Q_{03}(t)(s)\phi_3(t) \\
&\quad +Q_{04}(t)(s)\phi_4(t)+Q_{05}(t)(s)\phi_5(t)+Q_{06}(t)(s)\phi_6(t)+Q_{07}(t)(s)\phi_7(t) \\
\phi_1(t) &= Q_{10}(t)(s)\phi_0(t)+Q_{12}(t)(s)\phi_2(t)+Q_{16}(t)(s)\phi_6(t) \\
&\quad +Q_{17}(t)(s)\phi_7(t) \\
\phi_2(t) &= Q_{20}(t)(s)\phi_0(t)+Q_{21}(t)(s)\phi_1(t)+Q_{24}(t)(s)\phi_4(t) \\
&\quad +Q_{25}(t)(s)\phi_5(t) \\
\phi_3(t) &= Q_{30}(t)(s)\phi_0(t) \\
\phi_4(t) &= Q_{40}(t)(s)\phi_0(t)+Q_{48}(t)(s)\phi_8(t)+Q_{49}(t) + Q_{4,10}(t) \\
\phi_5(t) &= Q_{50}(t)(s)\phi_0(t)+Q_{51}(t)(s)\phi_1(t)+Q_{5,11}(t)+Q_{5,12}(t) \\
\phi_6(t) &= Q_{60}(t)(s)\phi_0(t)+Q_{62}(t)(s)\phi_2(t)+ Q_{69}(t)+Q_{6,12}(t) \\
\phi_7(t) &= Q_{70}(t)(s)\phi_0(t)+Q_{7,13}(t)(s)\phi_{13}(t)+Q_{7,10}(t) + Q_{7,11}(t) \\
\phi_8(t) &= Q_{81}(t)(s)\phi_1(t) \\
\phi_{13}(t) &= Q_{13,2}(t)(s)\phi_2(t)
\end{aligned}\right\} \tag{2.8}$$

In order to obtain MTSF, we first take Laplace Steltjes Transforms of set of equations (2.8) and then solve them for $\phi_0^{**}(s)$ as follows:

Suppose the reliability function for a system is given by R(t) = 1 –F(t), where F(t) is the failure time distribution function and f(t) = dF(t)/(dt) is the failure time density function. The mean time to system failure is given by

$$\text{MTSF} = \int_0^\infty t\, f(t)\, dt = -\int_0^\infty t\left(\frac{dR(t)}{dt}\right)dt = [-tR(t)]_0^\infty + \int_0^\infty R(t)\, dt$$

$$= \int_0^\infty R(t)\, dt = \lim_{s\to 0} R^*(s)$$

Let $\phi_0(t)$ be the cumulative distribution function of the first passage time from initial state to a failed state, then

$$R^*(s) = \frac{1-\phi_0^{**}(s)}{s}$$

Thus, we have $\text{MTSF} = \lim\limits_{s\to 0} \dfrac{1-\phi_0^{**}(s)}{s}$

$$MTSF = \lim_{s\to 0}\frac{1-\varphi_0{}^{**}(s)}{s}=\frac{N}{D}=\frac{D'(0)-N'(0)}{D'(0)}, \tag{2.9}$$

where

$$D'\left(0\right) = D_1 + D_2 + D_3$$

$$N'\left(0\right) = N_1 + N_2 + N_3$$

$$D_1 = \begin{vmatrix} \begin{array}{l} m_{03}p_{30}+m_{04}p_{40}+m_{05}p_{50}+ \\ m_{06}p_{60}+m_{07}p_{70}+m_{30}p_{03} \\ +m_{40}p_{04}+m_{50}p_{05}+m_{60}p_{06} \\ +m_{70}p_{07} \end{array} & \begin{array}{l} m_{01}+p_{05}m_{51}+ \\ m_{05}p_{51}+p_{04}p_{48}m_{81} \\ +p_{04}\mathrm{m}_{48}p_{81} \\ +\mathrm{m}_{04}p_{48}p_{81} \end{array} & \begin{array}{l} m_{02}+p_{06}\mathrm{m}_{62}+\mathrm{m}_{06}p_{62} \\ +p_{07}p_{713}m_{13,2} \\ +p_{07}m_{713}p_{13,2}+ \\ m_{07}p_{713}p_{13,2} \end{array} \\ -p_{10}-p_{16}p_{60}-p_{17}p_{70} & 1 & \begin{array}{l} -p_{12}-p_{16}p_{62} \\ -p_{17}p_{713}p_{132} \end{array} \\ -p_{20}-p_{25}p_{50}-p_{24}p_{40} & \begin{array}{l} -p_{21}-p_{25}p_{51} \\ -p_{24}p_{48}p_{81} \end{array} & 1 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} \begin{array}{l} -p_{03}p_{30}-p_{04}p_{40}-p_{05}p_{50} \\ -p_{06}p_{60}-p_{07}p_{70} \end{array} & \begin{array}{l} -p_{01}-p_{05}p_{51} \\ -p_{04}p_{48}p_{81} \end{array} & \begin{array}{l} -p_{02}-p_{06}p_{62} \\ -p_{07}p_{7,13}p_{13,2} \end{array} \\ \begin{array}{l} m_{10}+m_{16}p_{60}+m_{17}p_{70} \\ +p_{16}m_{60}+p_{17}m_{70} \end{array} & 0 & \begin{array}{l} m_{12}+m_{16}p_{62} \\ +m_{17}p_{7,13}p_{13,2}+p_{16}m_{62} \\ +p_{17}m_{7,13}p_{13,2}+p_{17}p_{7,13}m_{13,2} \end{array} \\ -p_{20}-p_{25}p_{50}-p_{24}p_{40} & \begin{array}{l} -p_{21}-p_{25}p_{51} \\ -p_{24}p_{48}p_{81} \end{array} & 1 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} \begin{array}{l} -p_{03}p_{30}-p_{04}p_{40}-p_{05}p_{50} \\ -p_{06}p_{60}-p_{07}p_{70} \end{array} & \begin{array}{l} -p_{01}-p_{05}p_{51} \\ -p_{04}p_{48}p_{81} \end{array} & \begin{array}{l} -p_{02}-p_{06}p_{62} \\ -p_{07}p_{7,13}p_{13,2} \end{array} \\ -p_{10}-p_{16}p_{60}-p_{17}p_{70} & 1 & \begin{array}{l} -p_{12}-p_{16}p_{62} \\ -p_{17}p_{7,13}p_{13,2} \end{array} \\ m_{20}+m_{25}p_{50}+m_{24}p_{40} & \begin{array}{l} m_{21}+m_{25}p_{51} \\ +m_{24}p_{48}p_{81}+p_{25}m_{51} \\ +p_{24}m_{48}p_{81}+p_{24}p_{48}m_{81} \end{array} & 0 \end{vmatrix}$$

$$
N_1 = \begin{vmatrix}
\begin{array}{l} p_{05}(p_{512}+p_{511})+p_{04}(p_{410}+p_{49}) \\ +p_{06}(p_{69}+p_{6,12})+p_{07}(p_{710}+p_{7,13}) \end{array} & \begin{array}{l} -p_{01}-p_{05}p_{51} \\ -p_{04}p_{48}p_{81} \end{array} & \begin{array}{l} -p_{02}-p_{06}p_{62} \\ -p_{07}p_{7,13}p_{13,2} \end{array} \\
p_{16}(p_{69}+p_{612})+p_{17}(p_{710}+p_{713}) & 1 & \begin{array}{l} -p_{12}-p_{16}p_{62} \\ -p_{17}p_{713}p_{132} \end{array} \\
p_{25}(p_{512}+p_{511})+p_{24}(p_{410}+p_{49}) & \begin{array}{l} -p_{21}-p_{25}p_{51} \\ -p_{24}p_{48}p_{81} \end{array} & 1
\end{vmatrix}
$$

$$
N_2 = \begin{vmatrix}
\begin{array}{l} p_{05}(p_{5,12}+p_{5,11})+p_{04}(p_{4,10}+p_{49}) \\ +p_{06}(p_{69}+p_{6,12})+p_{07}(p_{7,10}+p_{7,13}) \end{array} & \begin{array}{l} -p_{01}-p_{05}p_{51} \\ -p_{04}p_{48}p_{81} \end{array} & \begin{array}{l} -p_{02}-p_{06}p_{62} \\ -p_{07}p_{7,13}p_{13,2} \end{array} \\
\begin{array}{l} -m_{16}(p_{69}+p_{6,12})-m_{17}(p_{7,10}+p_{7,13}) \\ -p_{16}(m_{69}+m_{6,12})+p_{17}(m_{7,10}+m_{7,13}) \end{array} & 1 & \begin{array}{l} m_{12}+m_{16}p_{62} \\ +m_{17}p_{7,13}p_{13,2} \\ +p_{16}m_{62}+p_{17}m_{7,13}p_{13,2} \\ +p_{17}p_{7,13}m_{13,2} \end{array} \\
p_{25}(p_{5,12}+p_{5,11})+p_{24}(p_{4,10}+p_{49}) & \begin{array}{l} -p_{21}-p_{25}p_{51} \\ -p_{24}p_{48}p_{81} \end{array} & 1
\end{vmatrix}
$$

$$
N_3 = \begin{vmatrix}
\begin{array}{l} p_{05}(p_{5,12}+p_{5,11})+p_{04}(p_{4,10}+p_{49}) \\ +p_{06}(p_{69}+p_{6,12})+p_{07}(p_{7,10}+p_{7,13}) \end{array} & \begin{array}{l} -p_{01}-p_{05}p_{51} \\ -p_{04}p_{48}p_{81} \end{array} & \begin{array}{l} -p_{02}-p_{06}p_{62} \\ -p_{07}p_{7,13}p_{13,2} \end{array} \\
p_{16}(p_{69}+p_{6,12})+p_{17}(p_{7,10}+p_{7,13}) & 1 & \begin{array}{l} -p_{12}-p_{16}p_{62} \\ -p_{17}p_{7,13}p_{13,2} \end{array} \\
\begin{array}{l} -m_{25}(p_{5,12}+p_{5,11})-m_{24}(p_{4,10}+p_{49}) \\ -p_{25}(m_{5,12}+m_{5,11})-p_{24}(m_{4,10}+m_{49}) \end{array} & \begin{array}{l} m_{21}+m_{25}p_{51} \\ +m_{24}p_{48}p_{81}+p_{25}m_{51} \\ +p_{24}m_{48}p_{81} \\ +p_{24}p_{48}m_{81} \end{array} & 0
\end{vmatrix}
$$

2.5 Availability Analysis

2.5.1 Availability of twelve ton capacity

Let $A_i(t)$ be the probability that the system of twelve ton capacity is in upstate at instant t given that the system entered regenerative state i at $t=0$. The availability $A_i(t)$ is expressed as the following recursive relations by applying arguments of theory of regeneration process.

$$\left.\begin{array}{l}
A_0(t) = M_0(t) + Q_{01}(t)(c)A_1(t) + Q_{02}(t)(c)A_2(t) + Q_{03}(t)(c)A_3(t) + \\
\qquad Q_{04}(t)(c)A_4(t) + Q_{05}(t)(c)A_5(t) + Q_{06}(t)(c)A_6(t) + Q_{07}(t)(c)A_7(t) \\
A_1(t) = Q_{10}(t)(c)A_0(t) + Q_{12}(t)(c)A_2(t) + Q_{16}(t)(c)A_6(t) + Q_{17}(t)(c)A_7(t) \\
A_2(t) = Q_{20}(t)(c)A_0(t) + Q_{21}(t)(c)A_1(t) + Q_{24}(t)(c)A_4(t) + Q_{25}(t)(c)A_5(t) \\
A_3(t) = Q_{30}(t)(c)A_0(t) \\
A_4(t) = Q_{40}(t)(c)A_0(t) + Q_{48}(t)(c)A_8(t) + Q_{49}(t)(c)A_9(t) + Q_{4,10}(t)(c)A_{10}(t) \\
A_5(t) = Q_{50}(t)(c)A_0(t) + Q_{51}(t)(c)A_1(t) + Q_{5,11}(t)(c)A_{11}(t) + Q_{5,12}(t)(c)A_{12}(t) \\
A_6(t) = Q_{60}(t)(c)A_0(t) + Q_{62}(t)(c)A_2(t) + Q_{69}(t)(c)A_9(t) + Q_{6,12}(t)(c)A_{12}(t) \\
A_7(t) = Q_{70}(t)(c)A_0(t) + Q_{7,13}(t)(c)A_{13}(t) + Q_{7,10}(t)(c)A_{10}(t) + Q_{7,11}(t)(c)A_{11}(t) \\
A_8(t) = M_8(t) + Q_{81}(t)(c)A_1(t) \\
A_{13}(t) = M_{13}(t) + Q_{13,2}(t)(c)A_2(t)
\end{array}\right\} , \quad (2.10)$$

where

$$M_0(t) = e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_1)t}\,\bar{H}(t) \quad , \quad M_8(t) = \bar{I}(t) \; ; \; M_{13}(t) = \bar{M}(t)$$

$$\bar{H}(t) = 1 - H(t) \quad , \quad I(t) = 1 - I(t) \quad , \quad \bar{M}(t) = 1 - M(t)$$

2.5.2 Availability of four ton capacity

Similarly, if $A^1{}_i(t)$ be the probability that the system of four ton capacity is in upstate at instant t given that the system entered regenerative state i at $t=0$ then recursive relations for availability $A^1{}_i(t)$ have been obtained as :

$$
\left.
\begin{aligned}
A^1{}_0(t) &= Q_{01}(t)\text{(c)}A_1(t)+Q_{02}(t)\text{(c)}A_2(t)+Q_{03}(t)\text{(c)}A_3(t)+Q_{04}(t)\text{(c)}A_4(t) \\
&\quad +Q_{05}(t)\text{(c)}A_5(t)+Q_{06}(t)\text{(c)}A_6(t)+Q_{07}(t)\text{(c)}A_7(t) \\
A^1{}_1(t) &= Q_{10}(t)\text{(c)}A_0(t)+Q_{12}(t)\text{(c)}A_2(t)+Q_{16}(t)\text{(c)}A_6(t)+Q_{17}(t)\text{(c)}A_7(t) \\
A^1{}_2(t) &= Q_{20}(t)\text{(c)}A_0(t)+Q_{21}(t)\text{(c)}A_1(t)+Q_{24}(t)\text{(c)}A_4(t)+Q_{25}(t)\text{(c)}A_5(t) \\
A^1{}_3(t) &= Q_{30}(t)\text{(c)}A_0(t) \\
A^1{}_4(t) &= M_4(t) + Q_{40}(t)\text{(c)}A_0(t)+Q_{48}(t)\text{(c)}A_8(t)+Q_{49}(t)\text{(c)}A_9(t) \\
&\quad + Q_{4,10}(t)\text{(c)}A_{10}(t) \\
A^1{}_5(t) &= M_5(t) + Q_{50}(t)\text{(c)}A_0(t)+Q_{51}(t)\text{(c)}A_1(t)+Q_{5,11}(t)\text{(c)}A_{11}(t)+Q_{5,12}(t)\text{(c)}A_{12}(t) \\
A^1{}_6(t) &= Q_{60}(t)\text{(c)}A_0(t)+Q_{62}(t)\text{(c)}A_2(t)+Q_{69}(t)\text{(c)}A_9(t)+Q_{6,12}(t)\text{(c)}A_{12}(t) \\
A^1{}_7(t) &= Q_{70}(t)\text{(c)}A_0(t)+Q_{7,13}(t)\text{(c)}A_{13}(t)+Q_{7,10}(t)\text{(c)}A_{10}(t)+Q_{7,11}(t)\text{(c)}A_{11}(t) \\
A^1{}_8(t) &= Q_{81}(t)\text{(c)}A_1(t) \\
A^1{}_{13}(t) &= Q_{13,2}(t)\text{(c)}A_2(t)
\end{aligned}
\right\}, \tag{2.11}
$$

where

$$M_4(t) = e^{-(\beta_1+\beta_2+a_1)t} \quad , \quad M_5(t) = e^{-(\beta_1+\beta_2+a_2)t}$$

2.5.3 Availability of eight ton capacity

Similarly, if $A^2{}_i(t)$ be the probability that the system of eight ton capacity is in upstate at instant t given that the system entered regenerative state i at $t=0$ then recursive relations for availability $A^2{}_i(t)$ have been obtained as:

$$
\left.
\begin{aligned}
A^2{}_0(t) &= Q_{01}(t)\text{(c)}A_1(t)+Q_{02}(t)\text{(c)}A_2(t)+Q_{03}(t)\text{(c)}A_3(t)+Q_{04}(t)\text{(c)}A_4(t) \\
&\quad +Q_{05}(t)\text{(c)}A_5(t)+Q_{06}(t)\text{(c)}A_6(t)+Q_{07}(t)\text{(c)}A_7(t) \\
A^2{}_1(t) &= Q_{10}(t)\text{(c)}A_0(t)+Q_{12}(t)\text{(c)}A_2(t)+Q_{16}(t)\text{(c)}A_6(t)+Q_{17}(t)\text{(c)}A_7(t) \\
A^2{}_2(t) &= M_2(t) + Q_{20}(t)\text{(c)}A_0(t)+Q_{21}(t)\text{(c)}A_1(t)+Q_{24}(t)\text{(c)}A_4(t) \\
&\quad +Q_{25}(t)\text{(c)}A_5(t) \\
A^2{}_3(t) &= Q_{30}(t)\text{(c)}A_0(t) \\
A^2{}_4(t) &= Q_{40}(t)\text{(c)}A_0(t)+Q_{48}(t)\text{(c)}A_8(t)+Q_{49}(t)\text{(c)}A_9(t)+Q_{4,10}(t)\text{(c)}A_{10}(t)
\end{aligned}
\right\}
$$

$$\left.\begin{aligned}
A^2{}_5(t) &= Q_{50}(t)(c)A_0(t) + Q_{51}(t)(c)A_1(t) + Q_{5,11}(t)(c)A_{11}(t) + Q_{5,12}(t)(c)A_{12}(t) \\
A^2{}_6(t) &= M_6(t) + Q_{60}(t)(c)A_0(t) + Q_{62}(t)(c)A_2(t) + Q_{69}(t)(c)A_9(t) \\
&\quad + Q_{6,12}(t)(c)A_{12}(t) \\
A^2{}_7(t) &= M_7(t) + Q_{70}(t)(c)A_0(t) + Q_{7,13}(t)(c)A_{13}(t) + Q_{7,10}(t)(c)A_{10}(t) \\
&\quad + Q_{7,11}(t)(c)A_{11}(t) \\
A^2{}_8(t) &= Q_{81}(t)(c)A_1(t) \\
A^2{}_{13}(t) &= Q_{13,2}(t)(c)A_2(t)
\end{aligned}\right\} \quad , \qquad (2.12)$$

where

$$M_2(t) = e^{-(\gamma_1+\gamma_2+a_1)t}, \; M_6(t) = e^{-(\lambda_1+\lambda_2+b_2)t}, \; M_7(t) = e^{-(\lambda_1+\lambda_2++b_1)t}$$

Availabilities of different capacities can be found out by taking Laplace transform of set equations (2.8), (2.9), (2.10) and solving them for $A_0^*(s),\ A^1{}_0{}^*(s)$ *and* $A^2{}_0{}^*(s)$.

$$A_0^*(s) = \text{L.T.}(A_0(\text{t})) \;,\; A^1{}_0{}^*(s) = \text{L.T.}(A^1{}_0(\text{t})) \;,\; A^2{}_0{}^*(s) = \text{L.T.}(A^2{}_0(\text{t}))$$

$$A_0 = \lim_{s\to 0} s\, A_0{}^*(s) = \frac{N_{11}}{\text{derivative}(D_{11})} \qquad (2.13)$$

$$A^1{}_0 = \lim_{s\to 0} s\, A^1{}_0{}^*(s) = \frac{N_{12}}{\text{derivative}(D_{11})} \qquad (2.14)$$

$$A^2{}_0 = \lim_{s\to 0} s\, A^2{}_0{}^*(s) = \frac{N_{13}}{\text{derivative}(D_{11})} \quad , \qquad (2.15)$$

where

$$N_{11}=\begin{vmatrix} m_0 & -p_{01} & -p_{02} & -p_{04} & -p_{05} & -p_{06} & -p_{07} \\ 0 & 1 & -p_{12} & 0 & 0 & -p_{16} & -p_{17} \\ 0 & -p_{21} & 1 & -p_{24} & -p_{25} & 0 & 0 \\ m_8\, p_{48} & -p_{48}\, p_{81} & 0 & 1-p_{4,10}\, p_{10,4} & 0 & -p_{49}\, p_{96} & -p_{4,10}\, p_{10,7} \\ 0 & -p_{51} & 0 & 0 & \begin{matrix} 1-p_{5,11}\, p_{11,5} \\ -p_{5,12}\, p_{12,5} \end{matrix} & -p_{5,12}\, p_{12,6} & -p_{5,11}\, p_{11,7} \\ 0 & 0 & -p_{62} & -p_{94}\, p_{69} & -p_{6,12}\, p_{12,5} & \begin{matrix} 1-p_{6,12}\, p_{12,6} \\ -p_{69}\, p_{96} \end{matrix} & 0 \\ m_{13}\, p_{7,13} & 0 & -p_{7,13}\, p_{13,2} & -p_{7,10}\, p_{10,4} & -p_{7,11}\, p_{11,5} & 0 & 0 \end{vmatrix}$$

$$N_{12}=\begin{vmatrix} 0 & -p_{01} & -p_{02} & -p_{04} & -p_{05} & -p_{06} & -p_{07} \\ M_1 & 1 & -p_{12} & 0 & 0 & -p_{16} & -p_{17} \\ 0 & -p_{21} & 1 & -p_{24} & -p_{25} & 0 & 0 \\ M_4 & -p_{48}\, p_{81} & 0 & 1-p_{4,10}\, p_{10,4} & 0 & -p_{49}\, p_{96} & -p_{4,10}\, p_{10,7} \\ M_5 & -p_{51} & 0 & 0 & \begin{matrix} 1-p_{5,11}\, p_{11,5} \\ -p_{5,12}\, p_{12,5} \end{matrix} & -p_{5,12}\, p_{12,6} & -p_{5,11}\, p_{11,7} \\ 0 & 0 & -p_{62} & -p_{94}\, p_{69} & -p_{6,12}\, p_{12,5} & \begin{matrix} 1-p_{6,12}\, p_{12,6} \\ -p_{69}\, p_{96} \end{matrix} & 0 \\ 0 & 0 & -p_{7,13}\, p_{13,2} & -p_{7,10}\, p_{10,4} & -p_{7,11}\, p_{11,5} & 0 & 0 \end{vmatrix}$$

$$N_{13}=\begin{vmatrix} 0 & -p_{01} & -p_{02} & -p_{04} & -p_{05} & -p_{06} & -p_{07} \\ M_2 & 1 & -p_{12} & 0 & 0 & -p_{16} & -p_{17} \\ 0 & -p_{21} & 1 & -p_{24} & -p_{25} & 0 & 0 \\ 0 & -p_{48}\, p_{81} & 0 & 1-p_{4,10}\, p_{10,4} & 0 & -p_{49}\, p_{96} & -p_{4,10}\, p_{10,7} \\ 0 & -p_{51} & 0 & 0 & \begin{matrix} 1-p_{5,11}\, p_{11,5} \\ -p_{5,12}\, p_{12,5} \end{matrix} & -p_{5,12}\, p_{12,6} & -p_{5,11}\, p_{11,7} \\ M_6 & 0 & -p_{62} & -p_{94}\, p_{69} & -p_{6,12}\, p_{12,5} & \begin{matrix} 1-p_{6,12}\, p_{12,6} \\ -p_{69}\, p_{96} \end{matrix} & 0 \\ M_7 & 0 & -p_{7,13}\, p_{13,2} & -p_{7,10}\, p_{10,4} & -p_{7,11}\, p_{11,5} & 0 & 0 \end{vmatrix}$$

$$D_{11}=\begin{vmatrix} 1-q_{03}\,q_{30}-q_{04}\,q_{40} & -q_{01}-q_{04}\,q_{48}\,q_{81} & -q_{02} & -q_{05} & -q_{06} & -q_{07} & 0 & 0 \\ -q_{10} & 1 & -q_{12} & 0 & -q_{16} & -q_{17} & 0 & 0 \\ -q_{20}-q_{24}\,q_{40} & -q_{21}-q_{24}\,q_{48}\,q_{81} & 1 & -q_{25} & 0 & 0 & -q_{24}\,q_{49} & -q_{24}\,q_{4,10} \\ -q_{50} & -q_{51} & 0 & 1-q_{5,11}\,q_{11,5}-q_{5,12}\,q_{12,5} & -q_{5,12}\,q_{12,6} & -q_{5,11}\,q_{11,7} & 0 & 0 \\ -q_{60} & 0 & -q_{62} & -q_{6,12}\,q_{12,5} & 1-q_{6,12}\,q_{12,6} & 0 & -q_{69} & 0 \\ -q_{70} & 0 & -q_{7,13}\,q_{13,2} & -q_{7,11}\,q_{11,5} & 0 & 1-q_{7,11}\,q_{11,7} & 0 & -q_{7,10} \\ -q_{94}\,q_{40} & -q_{94}\,q_{48}\,q_{81} & 0 & 0 & -q_{96} & 0 & 1-q_{94}\,q_{49} & -q_{94}\,q_{4,10} \\ -q_{10,4}\,q_{40} & -q_{10,4}\,q_{48}\,q_{81} & 0 & 0 & 0 & -q_{10,7} & -q_{4,10}\,q_{49} & 1-q_{10,4}\,q_{4,10} \end{vmatrix}$$

2.6 Busy period of repairman

Following the method used in case of MTSF and Availability, recursive relations for busy period have been obtained as:

$$\left.\begin{aligned} B_0(t) &= Q_{01}(t)(c)B_1(t)+Q_{02}(t)(c)B_2(t)+Q_{03}(t)(c)B_3(t)+Q_{04}(t)(c)B_4(t) \\ &\quad +Q_{05}(t)(c)B_5(t)+Q_{06}(t)(c)B_6(t)+Q_{07}(t)(c)B_7(t) \\ B_1(t) &= Q_{10}(t)(c)B_0(t)+Q_{12}(t)(c)B_2(t)+Q_{16}(t)(c)B_6(t)+Q_{17}(t)(c)B_7(t) \\ B_2(t) &= Q_{20}(t)(c)B_0(t)+Q_{21}(t)(c)B_1(t)+Q_{24}(t)(c)B_4(t)+Q_{25}(t)(c)B_5(t) \\ B_3(t) &= Q_{30}(t)(c)B_0(t) \\ B_4(t) &= W_4(t)+Q_{40}(t)(c)B_0(t)+Q_{48}(t)(c)B_8(t)+Q_{49}(t)(c)B_9(t) \\ &\quad +Q_{4,10}(t)(c)B_{10}(t) \end{aligned}\right\}$$

$$\left.\begin{aligned}
B_5(t) &= W_5(t) + Q_{50}(t)(c)B_0(t) + Q_{51}(t)(c)B_1(t) + Q_{5,11}(t)(c)B_{11}(t) \\
&\quad + Q_{5,12}(t)(c)B_{12}(t) \\
B_6(t) &= W_6(t) + Q_{60}(t)(c)B_0(t) + Q_{62}(t)(c)B_2(t) + Q_{69}(t)(c)B_9(t) \\
&\quad + Q_{6,12}(t)(c)B_{12}(t) \\
B_7(t) &= W_7(t) + Q_{70}(t)(c)B_0(t) + Q_{7,13}(t)(c)B_{13}(t) + Q_{7,10}(t)(c)B_{10}(t) \\
&\quad + Q_{7,11}(t)(c)B_{11}(t) \\
B_8(t) &= Q_{81}(t)(c)B_1(t) \\
B_{13}(t) &= Q_{13,2}(t)(c)B_2(t)
\end{aligned}\right\}, \tag{2.16}$$

where

$$W_4(t) = e^{-\left(\beta_1+\beta_2+a_1\right)t}\,\bar{G}_1(t)\ ,\ W_5(t) = e^{-\left(\beta_1+\beta_2+a_2\right)t}$$
$$W_6(t) = e^{-\left(\lambda_1+\lambda_2+b_2\right)t}\ ,\ W_7(t) = e^{-\left(\lambda_1+\lambda_2+b_1\right)t}$$

Busy period have been obtained by taking Laplace Transform of set of equations (2.16) and solving them for $B_0^*(s)$, we get

$$B_o = \lim_{s\to 0} B_o^*(s) = \frac{N_{21}}{D_{11}}, \tag{2.17}$$

where

$$N_{21} = \begin{vmatrix}
0 & -p_{01} & -p_{02} & -p_{04} & -p_{05} & -p_{06} & -p_{07} \\
0 & 1 & -p_{12} & 0 & 0 & -p_{16} & -p_{17} \\
0 & -p_{21} & 1 & -p_{24} & -p_{25} & 0 & 0 \\
W_4 & -p_{48}p_{81} & 0 & \begin{matrix}1-p_{4,10}p_{10,4}\\ -p_{49}p_{94}\end{matrix} & 0 & -p_{49}p_{96} & -p_{4,10}p_{10,7} \\
W_5 & -p_{51} & 0 & 0 & \begin{matrix}1-p_{5,11}p_{11,5}\\ -p_{5,12}p_{12,5}\end{matrix} & -p_{5,12}p_{12,6} & -p_{5,11}p_{11,7} \\
W_6 & 0 & -p_{62} & -p_{94}p_{69} & -p_{6,12}p_{12,5} & \begin{matrix}1-p_{6,12}p_{12,6}\\ -p_{69}p_{96}\end{matrix} & 0 \\
W_7 & 0 & -p_{7,13}p_{13,2} & -p_{7,10}p_{10,4} & -p_{7,11}p_{11,5} & 0 & 0
\end{vmatrix}$$

D_{11} has been specified.

2.7 Expected Down Time

Let $E_i(t)$ be the expected down time. The recursive relations for expected down time have been obtained as:

$$\left.\begin{aligned}
E_0(t) &= Q_{01}(t)(c)E_1(t)+Q_{02}(t)(c)E_2(t)+Q_{03}(t)(c)E_3(t)+Q_{04}(t)(c)E_4(t) \\
&\quad +Q_{05}(t)(c)E_5(t)+Q_{06}(t)(c)E_6(t)+Q_{07}(t)(c)E_7(t) \\
E_1(t) &= Q_{10}(t)(c)E_0(t)+Q_{12}(t)(c)E_2(t)+Q_{16}(t)(c)E_6(t)+Q_{17}(t)(c)E_7(t) \\
E_2(t) &= Q_{20}(t)(c)E_0(t)+Q_{21}(t)(c)E_1(t)+Q_{24}(t)(s)E_4(t)+Q_{25}(t)(c)E_5(t) \\
E_3(t) &= W_3(t)+Q_{30}(t)(c)E_0(t) \\
E_4(t) &= Q_{40}(t)(c)E_0(t)+Q_{48}(t)(c)E_8(t)+Q_{49}(t)(s)\mathrm{E}_9(t)+Q_{4,10}(t)(c)\mathrm{E}_{10}(t) \\
E_5(t) &= Q_{50}(t)(c)E_0(t)+Q_{51}(t)(c)E_1(t)+Q_{5,11}(t)(s)\mathrm{E}_{11}(t)+Q_{5,12}(t)(c)\mathrm{E}_{12}(t) \\
E_6(t) &= Q_{60}(t)(c)E_0(t)+Q_{62}(t)(c)E_2(t)+Q_{69}(t)(s)\mathrm{E}_9(t)+Q_{6,12}(t)(c)\mathrm{E}_{12}(t) \\
E_7(t) &= Q_{70}(t)(c)E_0(t)+Q_{7,13}(t)(c)E_{13}(t)+Q_{7,10}(t)(c)\mathrm{E}_{10}(t) \\
&\quad +Q_{7,11}(t)(c)\mathrm{E}_{11}(t) \\
E_8(t) &= Q_{81}(t)(c)E_1(t) \\
E_{13}(t) &= Q_{13,2}(t)(c)E_2(t)
\end{aligned}\right\} , \qquad (2.18)$$

where

$$W_3(t) = e^{-\gamma_1 t}$$

Next, taking Laplace transform of set equations (2.18) and solving them for $E_0^*(s)$, we get expected down time as

$$E_o = \lim_{s\to 0} E_o^*(s) = \frac{N_{31}}{D_{11}} , \qquad (2.19)$$

where

$$N_{31} = \begin{vmatrix} p_{03}W_3 & -p_{01} & -p_{02} & -p_{04} & -p_{05} & -p_{06} & -p_{07} \\ 0 & 1 & -p_{12} & 0 & 0 & -p_{16} & -p_{17} \\ 0 & -p_{21} & 1 & -p_{24} & -p_{25} & 0 & 0 \\ 0 & -p_{48}p_{81} & 0 & \begin{matrix} 1-p_{4,10}p_{10,4} \\ -p_{49}p_{94} \end{matrix} & 0 & -p_{49}p_{96} & -p_{4,10}p_{10,7} \\ 0 & -p_{51} & 0 & 0 & \begin{matrix} 1-p_{5,11}p_{11,5} \\ -p_{5,12}p_{12,5} \end{matrix} & -p_{5,12}p_{12,6} & -p_{5,11}p_{11,7} \\ 0 & 0 & -p_{62} & -p_{94}p_{69} & -p_{6,12}p_{12,5} & \begin{matrix} 1-p_{6,12}p_{12,6} \\ -p_{69}p_{96} \end{matrix} & 0 \\ 0 & 0 & -p_{7,13}p_{13,2} & -p_{7,10}p_{10,4} & -p_{7,11}p_{11,5} & 0 & 0 \end{vmatrix}$$

D_{11} has been specified.

2.8 Cost-Benefit Analysis

The revenue and cost functions lead to the profit function of a firm. As the profit is excess of revenue over the cost of production, the profit takes the form

$P =$ Expected revenue in $(0,t]$ - Expected total cost in $(0,t]$

Using equations (2.13), (2.14), (2.15), (2.17) and (2.19) the expected profit per unit time incurred to the system is given by

$$P = C_{01}A_0 + C_{02}A^1{}_0 + C_{03}A^2{}_0 - C_1B_0 - C_2E_0, \qquad (2.20)$$

where

C_{01} = Total revenue per unit time for availability of 12 ton
C_{02} = Total revenue per unit time for availability of 4 ton
C_{03} = Total revenue per unit time for availability of 8 ton
C_1 = cost of busy period of repairman.

In order to make system profitable, the lower and upper bounds for revenue/costs are as follows:

TABLE 2.2: BOUNDS FOR REVENUE/COSTS

Revenue/Cost	Bound (Lower/Upper)	Value
C_{01}	Lower	$\left(C_1B_0 + C_2E_0 - C_{02}A^1{}_0 - C_{03}A^2{}_0\right)/A_0$
C_{02}	Lower	$\left(C_1B_0 + C_2E_0 - C_{01}A_0 - C_{03}A^2{}_0\right)/A^1{}_0$
C_{03}	Lower	$\left(C_1B_0 + C_2E_0 - C_{01}A_0 - C_{02}A^1{}_0\right)/A^2{}_0$
C_1	Upper	$\left(C_{01}A_0 + C_{02}A^1{}_0 + C_{03}A^2{}_0 - C_2E_0\right)/B_0$
C_2	Upper	$\left(C_{01}A_0 + C_{02}A^1{}_0 + C_{03}A^2{}_0 - C_1B_0\right)/E_0$

2.9 Analysis and Discussion

In this study, data for all Types of failures and repairs of the rice plant was collected in the units of per hour. Repair times are also assumed to follow exponential distribution. On the basis of this data, we have computed the failure and repair rates. . In particular case $h_1(t)=\alpha_1 e^{-\alpha_1 t}$ The values of various measures calculated and assumed are: a_1 = 0.3144 , a_2 = 0.2500, b_1=0.2346, b_2=0.1734 , γ_1 = 0.01736 , γ_2 = 0.0987 , α_1=0.01, q= 0.8 , $p_2=0.065$, $p_4=0.85$, $p_6=0.072$, $p_1=1-p_2$, $p_3=1-p_4$, $p_5=1-p_6$

Using the above measures, we have computed the following results of important reliability indices using the software 'MATLAB'. By varying $\lambda_2\,(0.015-0.045)$ for distinct values of λ_1, the values of mean time to system failure and availability are computed. Similarly, the profit is also computed by varying $C_0\,(510-525INR)$ for different choices of C_1 and results are presented. Their behaviour is exhibited in figures. Fig. 2.1 shows the behaviour of MTSF with respect to Type-II failure rate (λ_2) for distinct values of Type-I failure rate (λ_1). The graph shows that MTSF decreases with increase in the Type - II failure rate (λ_2) keeping Type - I failure rate (λ_1) constant and has greater values for lesser values of Type - II failure rate (λ_2).

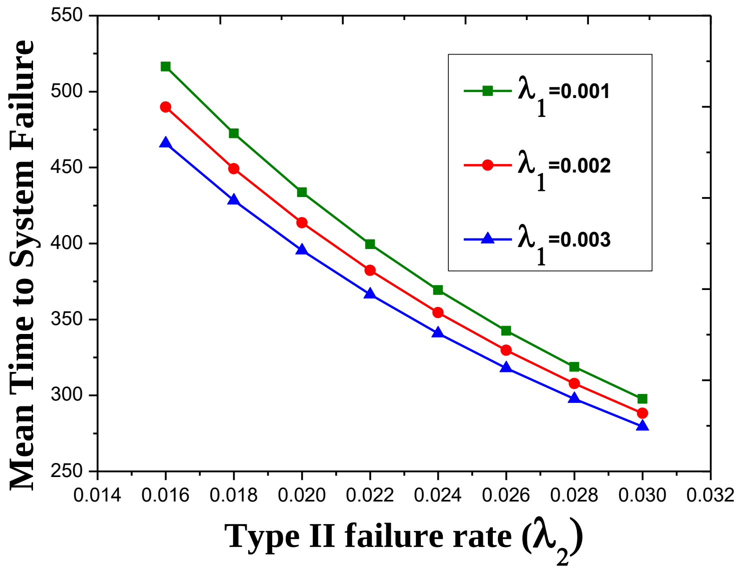

Fig. 2.1 - Effect of Type-II failure rate on Mean time to system failure for distinct values of Type-I failure rate.

Behaviour of total availability and availability of different capacity with respect to $\lambda_2\,(0.015-0.045)$ for distinct values of λ_1 has been shown in Fig 2.2, Fig 2.3, Fig 2.4, and Fig 2.5. These graphs indicate that total availability, availability of eight ton capacity and availability of twelve ton capacity of the system decreases with increase in the λ_2 and has greater values for lesser values of λ_1. Availability of four ton capacity of the system increases with increase in the λ_2 and has greater values for lesser values of λ_1.

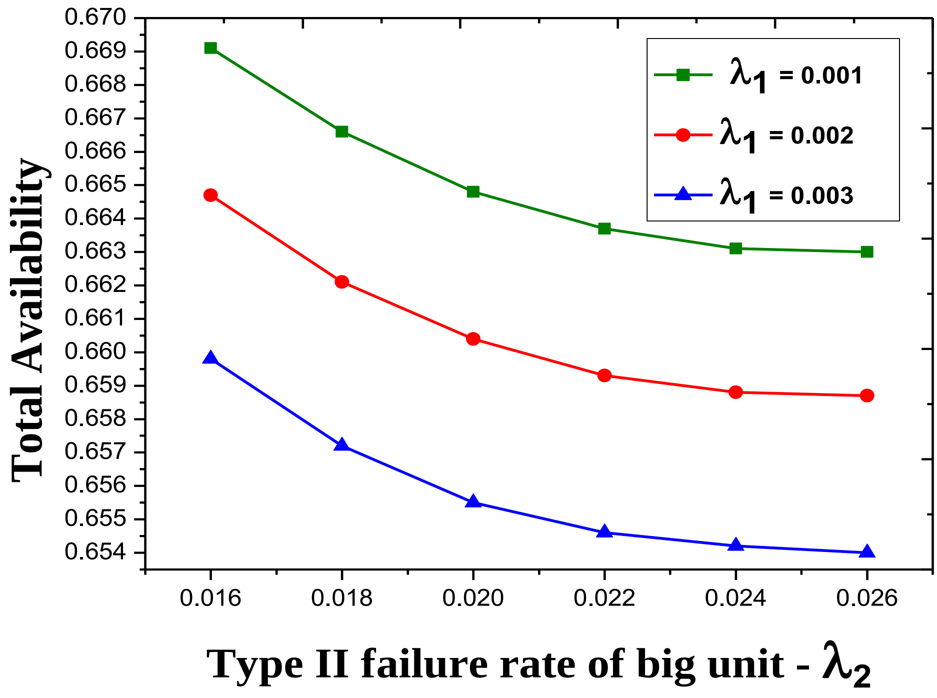

Fig. 2.2 - Effect of Type-II failure rate on total availability for different values of Type-I failure rate.

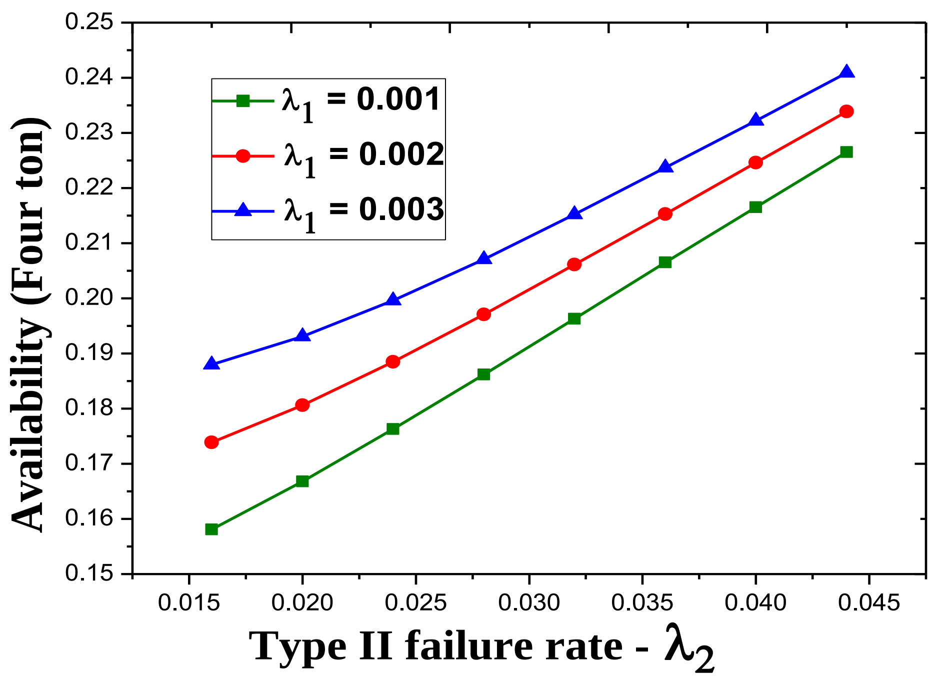

Fig. 2.3 - Effect of Type-II failure rate on Availability of four ton for different values of Type-I failure rate

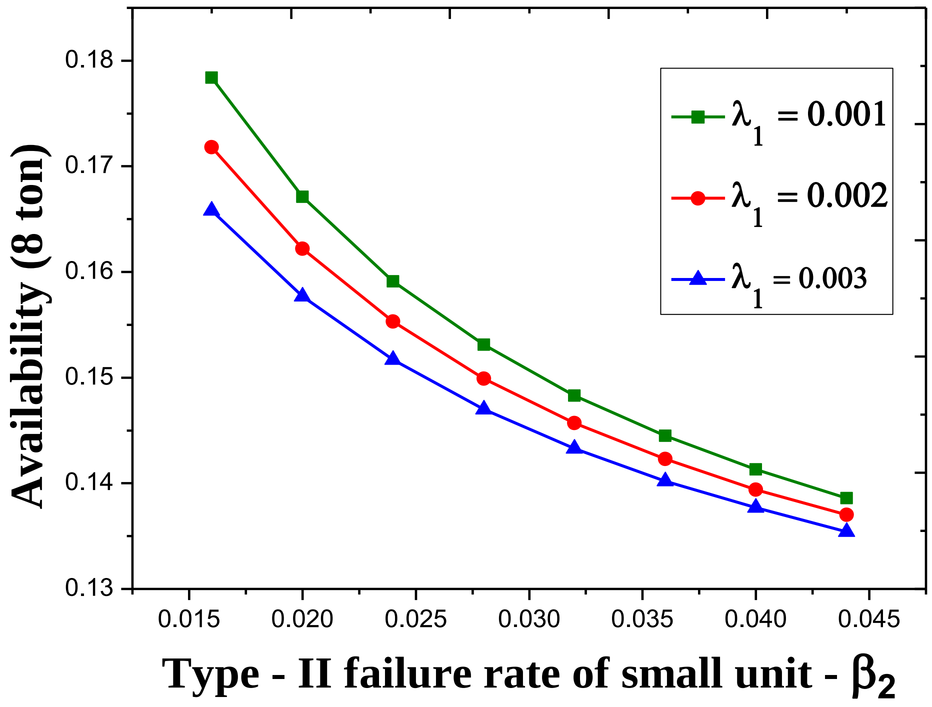

Fig. 2.4 - Effect of Type-II failure rate on Availability of eight ton for different values of Type-I failure rate

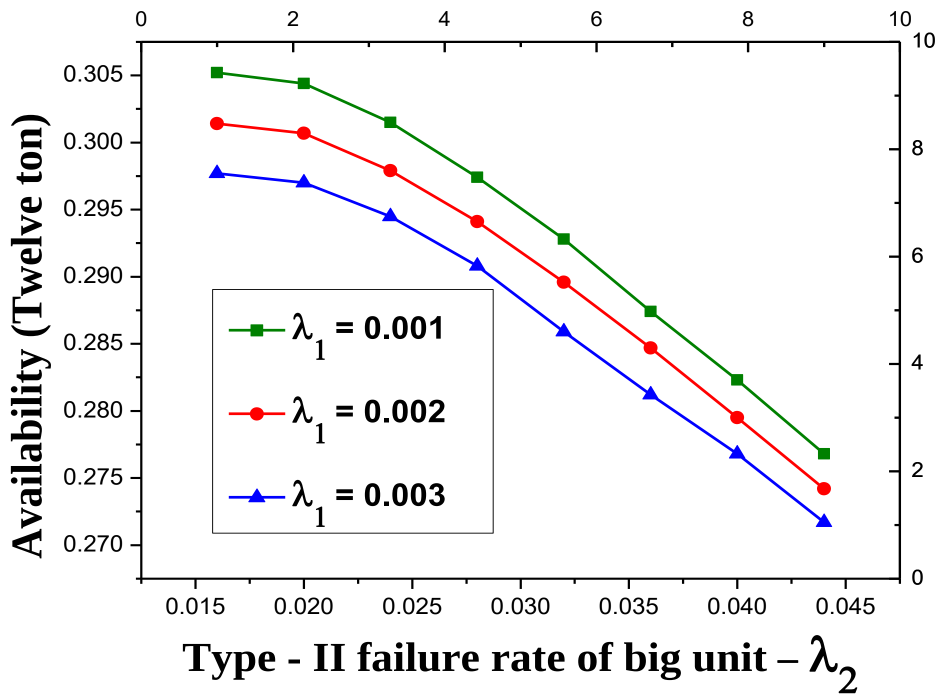

Fig. 2.5 - Effect of Type-II failure rate on Availability of twelve ton for different values of Type-I failure rate

The behaviour of profit with respect to cost of repairman (C_1) for distinct values of Type - I failure rate is shown in Fig. 2.6. It is observed from this graph that

(i) Profit decreases with the increase in both cost of busy period of repairman C_1 and Type - I failure rate.

(ii) For λ_1 = 0.05, the profit is negative or zero or positive according as $C_1 \leq$ or $\geq$ 6200. Hence, cost of busy period of repairman should be fixed greater than 6200

(iii) For λ_1 = 0.07, the profit is positive or zero or negative according as $C_1 \leq$ or $\geq$ 5250. Hence, cost of busy period of repairman should be fixed greater than 5250 (iii) for λ_1 = 0.09, the profit is positive or zero or negative according as $C_1 \leq$ or $\geq$ 4250. Hence, cost of busy period of repairman should be fixed greater than 4250.

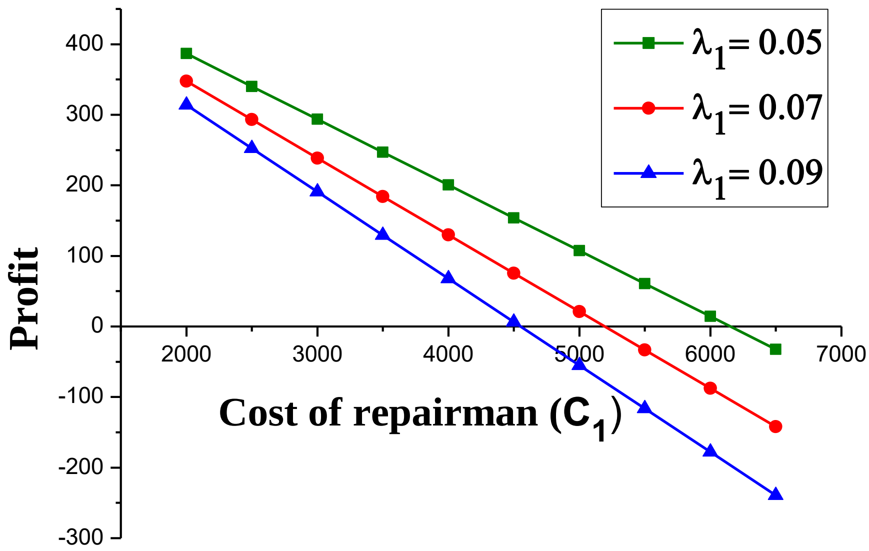

Fig. 2.6 - Effect of cost of busy period of repairman on profit achieved by the system for distinct values of Type - I failure rate .

Fig 2.7 shows the behaviour of profit achieved with respect to revenue of four ton capacity for distinct values of probability of switching to eight ton capacity from twelve ton as per the requirement. This shows that profit increases as revenue for four ton increases and decreases as the probability of switching to eight ton capacity from twelve ton increases. On comparing graphs, it reveals that

(i) Profit decreases for increasing value of revenue and has lesser value for greater value of probability of making system from twelve ton to eight ton capacity.

(ii) For p_1 = 0.1, the profit is negative or zero or positive according as $C_{02} \leq$ or $\geq$ 514.3. Hence, revenue per unit time should be fixed greater than 514.3 INR.

(ii) For p_1 = 0.9, the profit is positive or zero or negative according as C_{02} $\leq$ or $\geq$ 514.6. Hence, revenue per unit time for four ton capacity should be fixed greater than 514.60 INR.

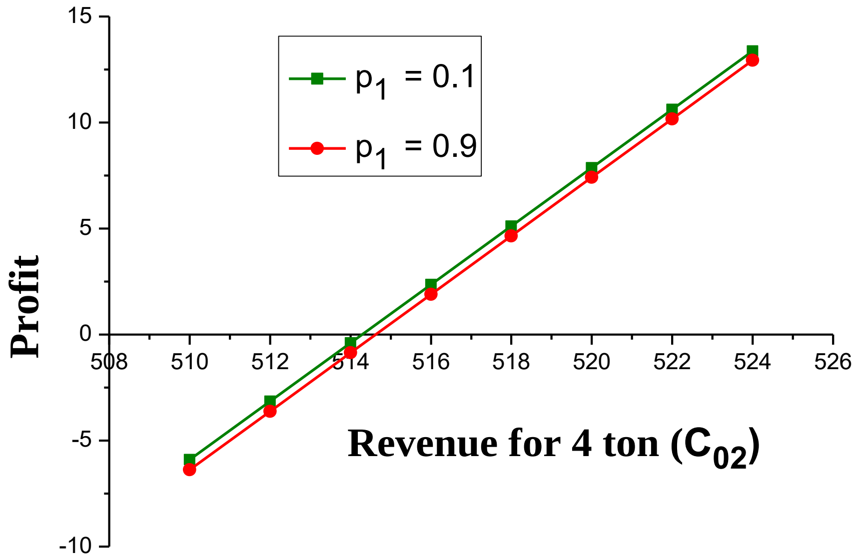

Fig. 2.7: Effect of revenue of four ton on profit achieved by the system for different values of probability of making the system at eight ton capacity from twelve ton capacity.

The behaviour of profit achieved with respect to revenue of four ton capacity for distinct values of probability of switching to twelve ton capacity from eight ton as per the requirement is shown in Fig 2.8. This shows that profit increases as revenue for four ton increases and increases as the probability of switching to twelve ton capacity from eight ton increases. On comparing graphs, it reveals that

(i) Profit decreases for increasing value of revenue and has lesser value for greater value of probability of making system at twelve ton from four ton capacity.

(ii) For p_3 = 0.1, the profit is negative or zero or positive according as $C_{02} \leq$ or $\geq$ 514.3 hence, revenue per unit time should be fixed greater than 514.3 INR.

(iii) For p_3 = 0.9, the profit is positive or zero or negative according as $C_{02} \leq$ or $\geq$ 514.25. Hence, revenue per unit time for four ton capacity should be fixed greater than 514.25 INR.

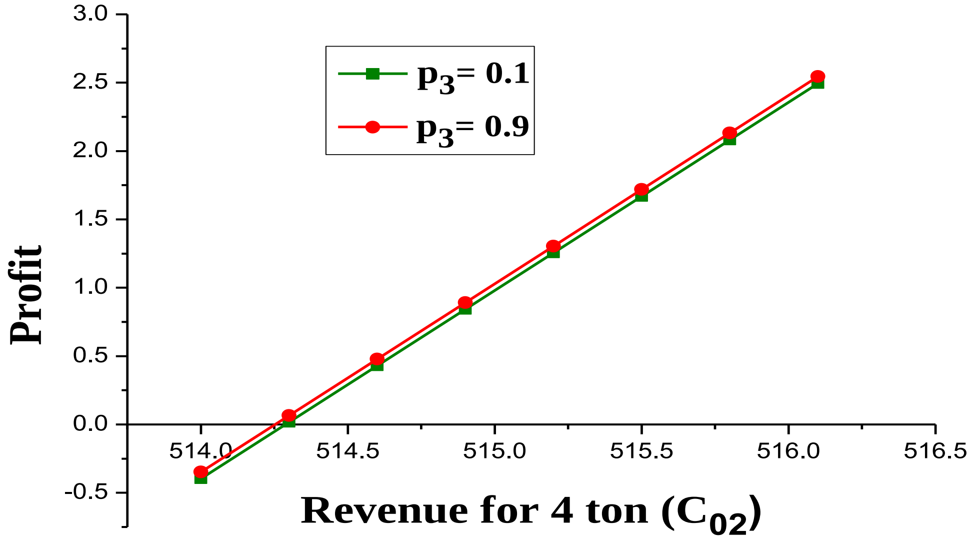

Fig. 2.8 - Effect of revenue of four ton on profit achieved by the system for different values of probability of making the system at twelve ton capacity from four ton capacity.

CHAPTER 3

Cost Analysis Of A Two Dissimilar Unit System With Goodwill Loss When Demand Is More Than Production

In the preceding chapter, cost benefit has been carried out for a system having two dissimilar units of eight ton and four ton capacities. We have analysed a two dissimilar unit standby system with variation in production and two types of failures by considering that demand can never be more than production in the preceding chapter. Thus in this chapter, we have analysed reliability and cost benefit for the system comprising of two dissimilar unit standby units by incorporating following concepts:

(i) Dissimilar standby units
(ii) Two types of failures
(iii) Variation in demand – production less than demand/production greater than or equal to demand.

This chapter has been organised as follows: Section 3.1 is about description of a rice manufacturing system and assumptions of two-unit standby system have also been discussed in this section. In section 3.2 the various notations are presented. The mathematical formulation for stochastic model determining, transition probabilities and mean sojourn times, have been developed in Section 3.3. Formulation of Mean Time to System Failure, availability ($p \geq d$ and $p < d$), busy period analysis and expected down time of the system are presented in section 3.4 – 3.7. Section 3.8 deals with the analysis of Cost Benefit of the system. Conclusions based on the present study have been finally drawn in Section 3.9.

3.1 System Description and Assumptions

This system comprises of two dissimilar paddy to rice converters with capacities of eight ton and four ton. One or both the units are made operative depending on the demand. Mathematical formulation of the problem determining the transition probabilities of various states has been developed by considering two types of failure (Type -I which has no standby and the Type - II which has standby) for each unit as in the preceding chapter. Availability analysis have been carried out by taking both cases into consideration

(i) When production is less than demand

(ii) When production is at least equal to demand.

In case if production is greater than or equal to demand and if one unit is under repair and other is operative, then no other event can take place except decrease in production. Other assumptions are same as in the preceding chapter.

The system is observed at suitable regenerative epochs by using regenerative point technique and the following reliability characteristics have been obtained:

(i) Mean time to system failure (MTSF)

(ii) Availability with full capacity ($p \geq d$)

(iii) Availability with reduced capacity ($p \geq d$)

(iv) Availability with full capacity ($p < d$)

(v) Availability with reduced capacity ($p < d$)

(vi) Expected busy period of repairman in $(0,t]$

(vii) Expected downtime

(viii) Expected profit incurred in $(0,t]$

3.2 Notations

Following notations have been used through the chapter:

B_s	:	Unit with eight ton capacity in standby mode
B_o	:	Unit with eight ton capacity in operative mode
S_o	:	Unit with four ton capacity in operative mode
S_s	:	Unit with four ton capacity in standby mode
B_r, S_r	:	System is at rest
F_{r_1}	:	Unit having Type – I failure is under repair
F_{r_2}	:	Unit having Type – II failure is under repair
λ_1	:	Rate of Type - I failure of big unit i.e. of eight ton capacity
λ_2	:	Rate of Type - II failure of big unit i.e. of eight ton capacity

β_1	:	Rate of Type - I failure of small unit i.e. of four ton capacity
β_2	:	Rate of Type - II failure of small unit i.e. of four ton capacity
p	:	Production
q	:	Demand
γ_1	:	Rate with which system is made operative from rest ($p < d$)
γ_2	:	Rate with which system goes to rest from operative state ($p \geq d$)
γ_3	:	Rate with which system is made operative ($p < d$) from operative $(p \geq d)$
a_1	:	Rate of Type – I repair for big unit i.e. of eight ton capacity
a_2	:	Rate of Type – II repair for big unit i.e. of eight ton capacity
b_1	:	Rate of Type – I repair for small unit i.e. of four ton capacity
b_2	:	Rate of Type – II repair for small unit i.e. of four ton capacity
$H_1(t)$	:	c.d.f. of going from $p \geq d$ to $p < d$ when big unit is under Type – I failure
$h_1(t)$	:	p.d.f. of going from $p \geq d$ to $p < d$ when big unit is under Type – I failure
$H_2(t)$	:	c.d.f. of going from $p \geq d$ to $p < d$ when big unit is under Type – II failure
$h_2(t)$	:	p.d.f. of going from $p \geq d$ to $p < d$ when big unit is under Type – II failure
$H_3(t)$	:	c.d.f. of going from $p \geq d$ to $p < d$ when small unit is under Type – I failure
$h_3(t)$	:	p.d.f. of going from $p \geq d$ to $p < d$ when small unit is under Type – I failure
$H_4(t)$	:	c.d.f. of going from $p \geq d$ to $p < d$ when small unit is

under Type – II failure

$h_4(t)$: p.d.f. of going from $p \geq d$ to $p < d$ when small unit is under Type – II failure

The system of converting paddy into rice of a rice manufacturing plant has the following states:

(i) Regenerative states:

$$\left\{\begin{array}{l} S_0(B_o, S_o : p \geq d), S_1\left(B_{Fr_1}, S_o : p \geq d\right), S_2\left(B_{Fr_2}, S_o : p \geq d\right), S_3\left(B_o, S_{Fr_1} : p \geq d\right) \\ S_4\left(B_o, S_{Fr_2} : p \geq d\right), S_5\left(B_r, S_r\right), S_6(B_o, S_o : p < d), S_7\left(B_{Fr_1}, S_o : p < d\right) \\ S_8\left(B_{Fr_2}, S_o : p < d\right), S_9\left(B_o, S_{Fr_1} : p < d\right), S_{10}\left(B_o, S_{Fr_2} : p < d\right) \end{array}\right\}$$

(ii) Failed states:

$$\left\{S_{11}\left(B_{Fr_1}, S_{Fr1}\right), S_{12}\left(B_{Fr_1}, S_{Fr_2}\right), S_{13}\left(B_{Fr_2}, S_{Fr_1}\right), S_{14}\left(B_{Fr_2}, S_{Fr_2}\right)\right\}$$

Table 3.1: Possible states of transition

State (S_i) ***i = 0 to 14***	***Status***	***Possible transition to***	***With failure / repair rates / transition probabilities / p.d.f respectively***
0	$B_o, S_o : p \geq d$	1, 2, 3, 4, 5 , 6	$\lambda_1, \lambda_2, \beta_1, \beta_2, \gamma_2, \gamma_3$
1	$B_{Fr_1}, S_o : p \geq d$	7	$h_1(t)$
2	$B_{Fr_2}, S_o : p \geq d$	8	$h_2(t)$
3	$B_o, S_{Fr_1} : p \geq d$	9	$h_3(t)$
4	$B_o, S_{Fr_2} : p \geq d$	10	$h_4(t)$
5	B_r, S_r	6	γ_1

6	$B_o, S_o : p < d$	7, 8, 9, 10	$\lambda_1, \lambda_2, \beta_1, \beta_2$
7	$B_{Fr_1}, S_o : p < d$	11 , 12	β_1, β_2
8	$B_{Fr_2}, S_o : p < d$	13 , 14	β_1, β_2
9	$B_o, S_{Fr_1} : p < d$	11 , 13	λ_1, λ_2
10	$B_o, S_{Fr_2} : p < d$	12 , 14	λ_1, λ_2
11	B_{Fr_1}, S_{Fr1}	7 , 9	b_1, a_1
12	B_{Fr_1}, S_{Fr_2}	7 , 10	b_2, a_1
13	B_{Fr_2}, S_{Fr_1}	8 , 9	b_1, a_2
14	B_{Fr_2}, S_{Fr_2}	8 , 10	b_2, a_2

3.3 Transition Probabilities and Mean Sojourn Time

The epochs of entry into states 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 are regenerative points. The transition probabilities p_{ij} have been obtained using the following formula.

$$p_{ij} = \lim_{s\to 0} q_{ij}^{*}(s) = \lim_{s\to 0} \int_0^\infty e^{-st} q_{ij}(t)\,dt \tag{3.1}$$

The transition probabilities have been obtained as follows:

$$\left.\begin{aligned}
&q_{01} = \lambda_1 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3)t}, \quad q_{02} = \lambda_2 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3)t} \\
&q_{03} = \beta_1 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3)t}, \quad q_{04} = \beta_2 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3)t} \\
&q_{05} = \gamma_2 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3)t}, \quad q_{06} = \gamma_3 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3)t} \\
&q_{17} = h_1(t), \quad q_{28} = h_2(t), \quad q_{39} = h_3(t) \\
&q_{4,10} = h_4(t), \quad q_{56} = \gamma_1 e^{-\gamma_1(t)}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
&q_{67} = \lambda_1 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_4)t}, \quad q_{68} = \lambda_2 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_4)t} \\
&q_{69} = \beta_1 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_4)t}, \quad q_{6,10} = \beta_2 e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_4)t} \\
&q_{76} = a_1 e^{-(\beta_1+\beta_2+a_1)t}, \quad q_{7,11} = \beta_1 e^{-(\beta_1+\beta_2+a_1)t}, \quad q_{7,12} = \beta_2 e^{-(\beta_1+\beta_2+a_1)t} \\
&q_{86} = a_2 e^{-(\beta_1+\beta_2+a_2)t}, \quad q_{8,14} = \beta_2 e^{-(\beta_1+\beta_2+a_2)t}, \quad q_{8,13} = \beta_1 e^{-(\beta_1+\beta_2+a_2)t} \\
&q_{96} = b_1 e^{-(\lambda_1+\lambda_2+b_1)t}, \quad q_{9,13} = \lambda_2 e^{-(\lambda_1+\lambda_2+b_1)t}, \quad q_{9,11} = \lambda_1 e^{-(\lambda_1+\lambda_2+b_1)t}
\end{aligned}\right.$$

(3.2)

The non zero elements p_{ij} have been obtained as:

$$p_{01}=\frac{\lambda_1}{\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3}\ ,\quad p_{02}=\frac{\lambda_2}{\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3}$$

$$p_{03}=\frac{\beta_1}{\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3}\ ,\quad p_{04}=\frac{\beta_2}{\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3}$$

$$p_{05}=\frac{\gamma_2}{\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3}\ ,\quad p_{06}=\frac{\gamma_3}{\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3}$$

$$p_{17}=1\ ,\ p_{28}=1\ ,\ p_{39}=1\ ,\ p_{4,10}=1\ ,\ p_{56}=1\ ,\ p_{67}=\frac{\lambda_1}{\lambda_1+\lambda_2+\beta_1+\beta_2}$$

$$p_{68}=\frac{\lambda_2}{\lambda_1+\lambda_2+\beta_1+\beta_2}\ ,\ p_{69}=\frac{\beta_1}{\lambda_1+\lambda_2+\beta_1+\beta_2}\ ,\ p_{6,10}=\frac{\beta_2}{\lambda_1+\lambda_2+\beta_1+\beta_2}$$

$$p_{76}=\frac{a_1}{a_1+\beta_1+\beta_2}\ ,\ p_{7,11}=\frac{\beta_1}{a_1+\beta_1+\beta_2}\ ,\ p_{7,12}=\frac{\beta_2}{a_1+\beta_1+\beta_2}$$

$$p_{86}=\frac{a_2}{a_1+\beta_1+\beta_2}\ ,\ p_{8,13}=\frac{\beta_1}{a_2+\beta_1+\beta_2}\ ,\ p_{8,14}=\frac{\beta_2}{a_2+\beta_1+\beta_2}$$

$$p_{96}=\frac{b_1}{b_1+\lambda_1+\lambda_2}\ ,\ p_{9,11}=\frac{\lambda_1}{b_1+\lambda_1+\lambda_2}\ ,\ p_{9,13}=\frac{\lambda_2}{b_1+\lambda_1+\lambda_2}$$

$$p_{10,6}=\frac{b_2}{b_2+\lambda_1+\lambda_2}\ ,\ p_{10,12}=\frac{\lambda_1}{b_2+\lambda_1+\lambda_2}\ ,\ p_{10,14}=\frac{\lambda_2}{b_2+\lambda_1+\lambda_2}$$

$$p_{11,7}=\frac{b_2}{a_1+b_2}\ ,\ p_{11,9}=\frac{a_1}{a_1+b_2}\ ,\ p_{12,7}=\frac{a_1}{a_1+b_1}\ ,\ p_{12,10}=\frac{b_1}{a_1+b_1}$$

$$p_{13,8}=\frac{a_2}{a_2+b_1}\ ,\ p_{13,9}=\frac{a_2}{a_2+b_1}\ ,\ p_{14,8}=\frac{a_2}{a_2+b_2}\ ,\ p_{14,10}=\frac{b_2}{a_2+b_2} \qquad (3.3)$$

The mean sojourn time μ_i in the ith regenerative state is as follows:

$$\mu_i = E(t) = P_r(T > t) = \int_0^\infty d\,(Q_{ij}(t)) \tag{3.4}$$

$$\left.\begin{aligned}
&\mu_0 = \frac{1}{\gamma_2+\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_3}\ ,\ \mu_1 = -h_1^*(0) = 1\ ,\ \mu_2 = -h_2^*(0) = 1\\
&\mu_3 = -h_3^*(0) = 1\ ,\ \mu_4 = -h_4^*(0) = 1\ ,\ \mu_5 = \frac{1}{\gamma_1}\\
&\mu_6 = \frac{1}{\lambda_1+\lambda_2+\beta_1+\beta_2}\ ,\ \mu_7 = \frac{1}{a_1+\beta_1+\beta_2}\ ,\mu_8 = \frac{1}{a_2+\beta_1+\beta_2}\\
&\mu_9 = \frac{1}{b_1+\lambda_1+\lambda_2}\ ,\ \mu_{10} = \frac{1}{b_2+\lambda_1+\lambda_2}
\end{aligned}\right\} \tag{3.5}$$

The unconditional mean time taken by the system is mathematically stated as

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = \int_0^\infty t\ q_{ij}(t)\,dt = -q_{ij}^*(0) \tag{3.6}$$

Thus, we get

$$\left.\begin{aligned}
&m_{01}+m_{02}+m_{03}+m_{04}+m_{05}+m_{06} = \mu_0\ ,\ m_{17} = \mu_1\ ,\ m_{28} = \mu_2\\
&m_{39} = \mu_3\ ,\ m_{4,10} = \mu_4\ ,\ m_{56} = \mu_5\ ,\ m_{76}+m_{7,11}+m_{7,13} = \mu_7\\
&m_{60}+m_{67}+m_{68}+m_{69}+m_{610} = \mu_6\ ,\quad m_{86}+m_{8,12}+m_{8,14} = \mu_8\\
&m_{96}+m_{9,11}+m_{9,13} = \mu_9\ ,\quad m_{10,6}+m_{10,12}+m_{10,14} = \mu_{10}
\end{aligned}\right\} \tag{3.7}$$

3.4 Mean Time to System Failure (MTSF)

In order to determine the mean time to system failure (MTSF) of the system, the failed states are considered as absorbing states, The following recursive relation for $\phi_i(t)$ have been obtained as:

$$\left.\begin{aligned}
&\phi_0(t)=Q_{01}(t)(s)\phi_1(t)+Q_{02}(t)(s)\phi_2(t)+Q_{03}(t)(s)\phi_3(t)+Q_{04}(t)(s)\phi_4(t)\\
&\qquad +Q_{05}(t)(s)\phi_5(t)+Q_{06}(t)(s)\phi_6(t)\\
&\phi_1(t)=Q_{17}(t)(s)\phi_7(t)\\
&\phi_2(t)=Q_{28}(t)(s)\phi_8(t)\\
&\phi_3(t)=Q_{39}(t)(s)\phi_9(t)\\
&\phi_4(t)=Q_{4,10}(t)(s)\phi_{10}(t)\\
&\phi_5(t)=Q_{56}(t)(s)\phi_6(t)
\end{aligned}\right\}$$

$$
\left.\begin{aligned}
&\phi_6(t)=Q_{60}(t)(s)\phi_0(t)+Q_{67}(t)(s)\phi_7(t)+Q_{68}(t)(s)\phi_8(t)\\
&\qquad +Q_{69}(t)(s)\phi_9(t)+Q_{6,10}(t)(s)\phi_{10}(t)\\
&\phi_7(t)=Q_{76}(t)(s)\phi_6(t)+Q_{7,11}(t)+Q_{7,13}(t)\\
&\phi_8(t)=Q_{86}(t)(s)\phi_6(t)+Q_{8,12}(t)+Q_{8,14}(t)\\
&\phi_9(t)=Q_{96}(t)(s)\phi_6(t)+Q_{9,11}(t)+Q_{9,13}(t)\\
&\phi_{10}(t)=Q_{10,6}(t)(s)\phi_6(t)+Q_{10,12}(t)+Q_{10,14}(t)
\end{aligned}\right\} \tag{3.8}
$$

Following the method used in section 2.4 MTSF has been obtained from set of equations (3.8) as:

$$
MTSF \;=\; \lim_{s\to 0}\frac{1-\varphi_0{}^{**}(s)}{s}=\frac{N}{D}=\frac{D'(0)-N'(0)}{D'(0)}, \tag{3.9}
$$

where

$$D'(0)=D_1+D_2$$

$$N'(0)=N_1+N_2$$

$$
D_1=\begin{vmatrix}
m_{05}\,p_{50}+m_{50}\,p_{05} & \begin{gathered} m_{60}+m_{02}\,p_{28}\,p_{86}+m_{03}\,p_{39}\,p_{96}+m_{04}\,p_{4,10}\,p_{10,6}+m_{01}\,p_{17}\,p_{76}\\ +\,p_{02}\,m_{28}\,p_{86}+p_{03}\,m_{39}\,p_{96}+p_{04}\,m_{4,10}\,p_{10,6}+p_{01}\,m_{17}\,p_{76}\\ +\,p_{02}\,p_{28}\,m_{86}+p_{03}\,p_{39}\,m_{96}+p_{04}\,p_{4,10}\,m_{10,6}+p_{01}\,p_{17}\,m_{76}\end{gathered}\\
0 & 1-p_{67}\,p_{76}-p_{68}\,p_{86}-p_{69}\,p_{96}-p_{6,10}\,p_{10,6}
\end{vmatrix}
$$

$$
D_2=\begin{vmatrix}
-p_{05}\,p_{50} & -p_{60}-p_{02}\,p_{28}\,p_{86}-p_{03}\,p_{39}\,p_{96}-p_{04}\,p_{4,10}\,p_{10,6}-p_{01}\,p_{17}\,p_{76}\\
0 & m_{67}\,p_{76}+m_{68}\,p_{86}+m_{69}\,p_{96}+m_{6,10}\,p_{10,6}+p_{67}\,m_{76}+p_{68}\,m_{86}+p_{69}\,m_{96}+p_{6,10}\,m_{10,6}
\end{vmatrix}
$$

$$
N_1=\begin{vmatrix}
\begin{gathered}
m_{01}\,p_{17}\,(p_{7,11}+p_{7,12})+m_{02}\,p_{28}\,(p_{8,13}+p_{8,14})\\
+m_{03}\,p_{39}\,(p_{9,11}+p_{9,13})+m_{04}\,p_{4,10}\,(p_{10,12}+p_{10,14})\\
p_{01}\,m_{17}\,(p_{7,11}+p_{7,12})+p_{02}\,m_{28}\,(p_{8,13}+p_{8,14})\\
+p_{03}\,m_{39}\,(p_{9,11}+p_{9,13})+p_{04}\,m_{4,10}\,(p_{10,12}+p_{10,14})\\
p_{01}\,p_{17}\,(m_{7,11}+m_{7,12})+p_{02}\,p_{28}\,(m_{8,13}+m_{8,14})\\
+p_{03}\,p_{39}\,(m_{9,11}+m_{9,13})+p_{04}\,p_{4,10}\,(m_{10,12}+m_{10,14})
\end{gathered} &
\begin{gathered}
m_{06}+m_{02}\,p_{28}\,p_{86}+m_{03}\,p_{39}\,p_{96}\\
+m_{04}\,p_{4,10}\,p_{10,6}+m_{01}\,p_{17}\,p_{76}\\
+p_{02}\,m_{28}\,p_{86}+p_{03}\,m_{39}\,p_{96}\\
+p_{04}\,m_{4,10}\,p_{10,6}+p_{01}\,m_{17}\,p_{76}\\
+p_{02}\,p_{28}\,m_{86}+p_{03}\,p_{39}\,m_{96}\\
+p_{04}\,p_{4,10}\,m_{10,6}+p_{01}\,p_{17}\,m_{76}
\end{gathered}\\
\begin{gathered}
-p_{67}\,(p_{7,11}+p_{7,12})-p_{68}\,(p_{8,13}+p_{8,14})\\
-p_{69}\,(p_{9,11}+p_{9,13})-p_{6,10}\,(p_{10,12}+p_{10,14})
\end{gathered} & 1-p_{67}\,p_{76}-p_{68}\,p_{86}-p_{69}\,p_{96}-p_{6,10}\,p_{10,6}
\end{vmatrix}
$$

$$N_2 = \begin{vmatrix} -p_{01}\,p_{17}\,(p_{7,11}+p_{7,12})-p_{02}\,p_{28}\,(p_{8,13}+p_{8,14}) & -p_{06}\;-p_{02}\,p_{28}\,p_{86}-p_{03}\,p_{39}\,p_{96} \\ -p_{03}\,p_{39}\,(p_{9,11}+p_{9,13})-p_{04}\,p_{4,10}\,(p_{10,12}+p_{10,14}) & -p_{04}\,p_{4,10}\,p_{10,6}-p_{01}\,p_{17}\,p_{76} \\ m_{67}\,(p_{7,11}+p_{7,12})+m_{68}\,(p_{8,13}+p_{8,14}) & \\ +m_{69}\,(p_{9,11}+p_{9,13})+m_{6,10}\,(p_{10,12}+p_{10,14}) & m_{67}\,p_{76}+m_{68}\,p_{86}+m_{69}\,p_{96}+m_{6,10}\,p_{10,6} \\ p_{67*}\,(\mathrm{m}_{7,11}+m_{7,12})+p_{68*}\,(\mathrm{m}_{8,13}+m_{8,14}) & p_{67}\,m_{76}+p_{68}\,m_{86}+p_{69}\,m_{96}+p_{6,10}\,m_{10,6} \\ +p_{69}\,(\mathrm{m}_{9,11}+m_{9,13})+p_{6,10}\,(\mathrm{m}_{10,12}+m_{10,14}) & \end{vmatrix}$$

3.5 Availability Analysis

Availability for different capacities with production greater than or equal to demand and production less than demand has been obtained as:

3.5.1 Availability with full capacity ($p \geq d$)

Let $A_i(t)$ be the probability that the system of twelve ton capacity is in upstate when production is greater or equal to demand at instant t . The availability $A_i(t)$ is expressed as the following recursive relations:

$$\left.\begin{aligned}
A_0(t) &= M_0(t) + Q_{01}(t)(c)A_1(t)+Q_{02}(t)(c)A_2(t)+Q_{03}(t)(c)A_3(t)+ \\
&\quad Q_{04}(t)(c)A_4(t)+Q_{05}(t)(c)A_5(t)+Q_{06}(t)(c)A_6(t) \\
A_1(t) &= Q_{17}(t)(c)A_7(t) \\
A_2(t) &= Q_{28}(t)(c)A_8(t) \\
A_3(t) &= Q_{39}(t)(c)A_9(t) \\
A_4(t) &= Q_{4,10}(t)(c)A_{10}(t) \\
A_5(t) &= Q_{56}(t)(c)A_6(t) \\
A_6(t) &= Q_{60}(t)(c)A_0(t)+Q_{67}(t)(c)A_2(t)+Q_{68}(t)(c)A_8(t)+ \\
&\quad Q_{69}(t)(c)A_9(t)+Q_{6,10}(t)(c)A_{10}(t) \\
A_7(t) &= Q_{76}(t)(c)A_6(t) + Q_{7,11}(t)(c)A_{11}(t)+Q_{7,13}(t)(c)A_{13}(t) \\
A_8(t) &= Q_{86}(t)(c)A_6(t) + Q_{8,12}(t)(c)A_{12}(t)+Q_{8,14}(t)(c)A_{14}(t) \\
A_9(t) &= Q_{96}(t)(s)A_6(t) + Q_{9,11}(t)(s)A_{11}(t)+Q_{9,13}(t)(c)A_{13}(t) \\
A_{10}(t) &= Q_{10,6}(t)(c)A_6(t) + Q_{10,12}(t)(c)A_{12}(t)+Q_{10,14}(t)(c)A_{14}(t) \\
A_{11}(t) &= Q_{11,7}(t)(c)A_7(t)+Q_{11,9}(t)(c)A_9(t) \\
A_{12}(t) &= Q_{12,7}(t)(c)A_7(t)+Q_{12,10}(t)(c)A_{10}(t) \\
A_{13}(t) &= Q_{13,8}(t)(c)A_8(t)+Q_{13,9}(t)(c)A_9(t) \\
A_{14}(t) &= Q_{14,8}(t)(c)A_8(t)+Q_{14,10}(t)(c)A_{10}(t)
\end{aligned}\right\} \quad (3.10)$$

where

$$M_0(t) = e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3)\,t}$$

3.5.2 Availability with full capacity ($p < d$)

If $A^1{}_i(t)$ be the probability that the system of twelve ton capacity is in upstate with production less than demand at instant t then recursive relations for availability $A^1{}_i(t)$ have been obtained as:

$$\left.\begin{aligned}
A^1{}_0(t) &= Q_{01}(t)\text{(c)}A^1{}_1(t)+Q_{02}(t)\text{(c)}A^1{}_2(t)+Q_{03}(t)\text{(c)}A^1{}_3(t)\\
&\quad +Q_{04}(t)\text{(c)}A^1{}_4(t)+Q_{05}(t)\text{(c)}A^1{}_5(t)+Q_{06}(t)\text{(c)}A^1{}_6(t)\\
A^1{}_1(t) &= Q_{17}(t)\text{(c)}A^1{}_7(t)\\
A^1{}_2(t) &= Q_{28}(t)\text{(c)}A^1{}_8(t)\\
A^1{}_3(t) &= Q_{39}(t)\text{(c)}A^1{}_9(t)\\
A^1{}_4(t) &= Q_{4,10}(t)\text{(c)}A^1{}_{10}(t)\\
A^1{}_5(t) &= Q_{56}(t)\text{(c)}A^1{}_6(t)\\
A^1{}_6(t) &= M_6(t)+Q_{60}(t)\text{(c)}A^1{}_0(t)+Q_{67}(t)\text{(c)}A^1{}_2(t)+\\
&\quad Q_{68}(t)\text{(c)}A^1{}_8(t)+Q_{69}(t)\text{(c)}A^1{}_9(t)+Q_{6,10}(t)\text{(c)}A^1{}_{10}(t)\\
A^1{}_7(t) &= Q_{76}(t)\text{(c)}A^1{}_6(t)+Q_{7,11}(t)\text{(c)}A^1{}_{11}(t)+Q_{7,13}(t)\text{(c)}A^1{}_{13}(t)\\
A^1{}_8(t) &= Q_{86}(t)\text{(c)}A^1{}_6(t)+Q_{8,12}(t)\text{(c)}A^1{}_{12}(t)+Q_{8,14}(t)\text{(c)}A^1{}_{14}(t)\\
A^1{}_9(t) &= Q_{96}(t)\text{(c)}A^1{}_6(t)+Q_{9,11}(t)\text{(c)}A^1{}_{11}(t)+Q_{9,13}(t)\text{(c)}A^1{}_{13}(t)\\
A^1{}_{10}(t) &= Q_{10,6}(t)\text{(c)}A^1{}_6(t)+Q_{10,12}(t)\text{(c)}A^1{}_{12}(t)+Q_{10,14}(t)\text{(c)}A^1{}_{14}(t)\\
A^1{}_{11}(t) &= Q_{11,7}(t)\text{(c)}A^1{}_7(t)+Q_{11,9}(t)\text{(c)}A^1{}_9(t)\\
A^1{}_{12}(t) &= Q_{12,7}(t)\text{(c)}A^1{}_7(t)+Q_{12,10}(t)\text{(c)}A^1{}_{10}(t)\\
A^1{}_{13}(t) &= Q_{13,8}(t)\text{(c)}A^1{}_8(t)+Q_{13,9}(t)\text{(c)}A^1{}_9(t)\\
A^1{}_{14}(t) &= Q_{14,8}(t)\text{(c)}A^1{}_8(t)+Q_{14,10}(t)\text{(c)}A^1{}_{14}(t)
\end{aligned}\right\},\qquad (3.11)$$

where

$$M_6(t) = e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_4)\,t}$$

3.5.3 Availability with reduced capacity for eight ton line operative ($p \geq d$)

Similarly, if $A^2{}_i(t)$ be the probability that the system of eight ton capacity is in upstate at instant t provided the system entered regenerative state i at $t=0$ then recursive relations for availability $A^2{}_i(t)$ have been obtained as:

$$\left.\begin{aligned}
A^2{}_0(t) &= Q_{01}(t)(c)A^2{}_1(t)+Q_{02}(t)(c)A^2{}_2(t)+Q_{03}(t)(c)A^2{}_3(t)+ \\
&\quad Q_{04}(t)(c)A^2{}_4(t)+Q_{05}(t)(c)A^2{}_5(t)+Q_{06}(t)(c)A^2{}_6(t) \\
A^2{}_1(t) &= Q_{17}(t)(c)A^2{}_7(t) \\
A^2{}_2(t) &= Q_{28}(t)(c)A^2{}_8(t) \\
A^2{}_3(t) &= M_3(t)+Q_{39}(t)(c)A^2{}_9(t) \\
A^2{}_4(t) &= M_4(t)+Q_{4,10}(t)(c)A^2{}_{10}(t) \\
A^2{}_5(t) &= Q_{56}(t)(c)A^2{}_6(t) \\
A^2{}_6(t) &= Q_{60}(t)(c)A^2{}_0(t)+Q_{67}(t)(c)A^2{}_2(t)+Q_{68}(t)(c)A^2{}_8(t) \\
&\quad +Q_{69}(t)(c)A^2{}_9(t)+Q_{6,10}(t)(c)A^2{}_{10}(t) \\
A^2{}_7(t) &= Q_{76}(t)(c)A^2{}_6(t)+Q_{7,11}(t)(c)A^2{}_{11}(t)+Q_{7,13}(t)(c)A^2{}_{13}(t) \\
A^2{}_8(t) &= Q_{86}(t)(c)A^2{}_6(t)+Q_{8,12}(t)(c)A^2{}_{12}(t)+Q_{8,14}(t)(c)A^2{}_{14}(t) \\
A^2{}_9(t) &= Q_{96}(t)(c)A^2{}_6(t)+Q_{9,11}(t)(c)A^2{}_{11}(t)+Q_{9,13}(t)(c)A^2{}_{13}(t) \\
A^2{}_{10}(t) &= Q_{10,6}(t)(c)A^2{}_6(t)+Q_{10,12}(t)(c)A^2{}_{12}(t)+Q_{10,14}(t)(c)A^2{}_{14}(t) \\
A^2{}_{11}(t) &= Q_{11,7}(t)(c)A^2{}_7(t)+Q_{11,9}(t)(c)A^2{}_9(t) \\
A^2{}_{12}(t) &= Q_{12,7}(t)(c)A^2{}_7(t)+Q_{12,10}(t)(c)A^2{}_{10}(t) \\
A^2{}_{13}(t) &= Q_{13,8}(t)(c)A^2{}_8(t)+Q_{13,9}(t)(c)A^2{}_9(t) \\
A^2{}_{14}(t) &= Q_{14,8}(t)(c)A^2{}_8(t)+Q_{14,10}(t)(c)A^2{}_{14}(t)
\end{aligned}\right\}, \qquad (3.12)$$

where

$$M_3(t) = \bar{H}_3(t) \ , \ M_4(t) = \bar{H}_4(t)$$

$$\bar{H}_3(t) = 1 - H_3(t) \ , \ \bar{H}_4(t) = 1 - H_4(t)$$

3.5.4 Availability with reduced capacity for four ton line operative ($p \geq d$)

If $A^3{}_i(t)$ be the probability that the system of four ton capacity is in upstate when production is greater or equal to demand at instant t then recursive relations for availability $A^3{}_i(t)$ have been obtained as follows:

$$
\left.\begin{aligned}
A^3{}_0(t) &= Q_{01}(t)(c)\,A^3{}_1(t) + Q_{02}(t)(c)\,A^3{}_2(t) + Q_{03}(t)(c)\,A^3{}_3(t) \\
&\quad + Q_{04}(t)(c)\,A^3{}_4(t) + Q_{05}(t)(c)\,A^3{}_5(t) + Q_{06}(t)(c)\,A^3{}_6(t) \\
A^3{}_1(t) &= M_1(t) + Q_{17}(t)(c)\,A^3{}_7(t) \\
A^3{}_2(t) &= M_2(t) + Q_{28}(t)(c)\,A^3{}_8(t) \\
A^3{}_3(t) &= Q_{39}(t)(c)\,A^3{}_9(t) \\
A^3{}_4(t) &= Q_{4,10}(t)(c)\,A^3{}_{10}(t) \\
A^3{}_5(t) &= Q_{56}(t)(c)\,A^3{}_6(t) \\
A^3{}_6(t) &= Q_{60}(t)(c)\,A^3{}_0(t) + Q_{67}(t)(c)\,A^3{}_2(t) + Q_{68}(t)(c)A^3{}_8(t) \\
&\quad + Q_{69}(t)(c)A^3{}_9(t) + Q_{6,10}(t)(c)A^3{}_{10}(t) \\
A^3{}_7(t) &= Q_{76}(t)(c)\,A^3{}_6(t) + Q_{7,11}(t)(c)A^3{}_{11}(t) + Q_{7,13}(t)(c)\,A^3{}_{13}(t) \\
A^3{}_8(t) &= Q_{86}(t)(c)\,A^3{}_6(t) + Q_{8,12}(t)(c)A^3{}_{12}(t) + Q_{8,14}(t)(c)\,A^3{}_{14}(t) \\
A^3{}_9(t) &= Q_{96}(t)(c)\,A^3{}_6(t) + Q_{9,11}(t)(c)A^3{}_{11}(t) + Q_{9,13}(t)(c)\,A^3{}_{13}(t) \\
A^3{}_{10}(t) &= Q_{10,6}(t)(c)\,A^3{}_6(t) + Q_{10,12}(t)(c)A^3{}_{12}(t) + Q_{10,14}(t)(c)\,A^3{}_{14}(t) \\
A^3{}_{11}(t) &= Q_{11,7}(t)(c)A^3{}_7(t) + Q_{11,9}(t)(c)\,A^3{}_9(t) \\
A^3{}_{12}(t) &= Q_{12,7}(t)(c)A^3{}_7(t) + Q_{12,10}(t)(c)\,A^3{}_{10}(t) \\
A^3{}_{13}(t) &= Q_{13,8}(t)(c)A^3{}_8(t) + Q_{13,9}(t)(c)\,A^3{}_9(t) \\
A^3{}_{14}(t) &= Q_{14,8}(t)(c)A^3{}_8(t) + Q_{14,10}(t)(c)\,A^3{}_{14}(t)
\end{aligned}\right\} , \qquad (3.13)
$$

where

$$M_1(t) = \bar{H}_1(t)\ ,\ M_2(t) = \bar{H}_2(t)\ ,\ \bar{H}_1(t) = 1 - H_1(t)\ ,\ \bar{H}_2(t) = 1 - H_2(t)$$

3.5.5 Availability with reduced capacity for eight ton line operative ($p < d$)

Let $A^4{}_i(t)$ be the probability that the system of eight ton capacity is in upstate when production is less than demand at instant t. The recursive relations for availability $A^4{}_i(t)$ have been obtained as:

$$\left.\begin{aligned}
A^4{}_0(t) &= Q_{01}(t)(c)A^4{}_1(t)+Q_{02}(t)(c)A^4{}_2(t)+Q_{03}(t)(c)A^4{}_3(t)\\
&\quad +Q_{04}(t)(c)A^4{}_4(t)+Q_{05}(t)(c)A^4{}_5(t)+Q_{06}(t)(c)A^4{}_6(t)\\
A^4{}_1(t) &= Q_{17}(t)(c)A^4{}_7(t)\\
A^4{}_2(t) &= Q_{28}(t)(c)A^4{}_8(t)\\
A^4{}_3(t) &= Q_{39}(t)(c)A^4{}_9(t)\\
A^4{}_4(t) &= Q_{4,10}(t)(c)A^4{}_{10}(t)\\
A^4{}_5(t) &= Q_{56}(t)(c)A^4{}_6(t)\\
A^4{}_6(t) &= Q_{60}(t)(c)A^4{}_0(t)+Q_{67}(t)(c)A^4{}_2(t)+Q_{68}(t)(c)A^4{}_8(t)\\
&\quad +Q_{69}(t)(c)A^4{}_9(t)+Q_{6,10}(t)(c)A^4{}_{10}(t)\\
A^4{}_7(t) &= Q_{76}(t)(c)A^4{}_6(t)+Q_{7,11}(t)(c)A^4{}_{11}(t)+Q_{7,13}(t)(c)A^4{}_{13}(t)\\
A^4{}_8(t) &= Q_{86}(t)(c)A^4{}_6(t)+Q_{8,12}(t)(c)A^4{}_{11}(t)+Q_{8,14}(t)(c)A^4{}_{14}(t)\\
A^4{}_9(t) &= M_9(t)+Q_{96}(t)(c)A^4{}_6(t)+Q_{9,11}(t)(c)A^4{}_{11}(t)+Q_{9,13}(t)(c)A^4{}_{13}(t)\\
A^4{}_{10}(t) &= M_{10}(t)+Q_{10,6}(t)(c)A^4{}_6(t)+Q_{10,12}(t)(c)A^4{}_{12}(t)\\
&\quad +Q_{10,14}(t)(c)A^4{}_{14}(t)\\
A^4{}_{11}(t) &= Q_{11,7}(t)(c)A^4{}_7(t)+Q_{11,9}(t)(c)A^4{}_9(t)\\
A^4{}_{12}(t) &= Q_{12,7}(t)(c)A^4{}_7(t)+Q_{12,10}(t)(c)A^4{}_{10}(t)\\
A^4{}_{13}(t) &= Q_{13,8}(t)(c)A^4{}_8(t)+Q_{13,9}(t)(c)A^4{}_9(t)\\
A^4{}_{14}(t) &= Q_{14,8}(t)(c)A^4{}_8(t)+Q_{14,10}(t)(c)A^4{}_{14}(t)
\end{aligned}\right\} , \qquad (3.14)$$

where

$$M_9(t) = e^{-(\lambda_1+\lambda_2+b_1)t} \quad , \quad M_{10}(t) = e^{-(\lambda_1+\lambda_2+b_2)t}$$

3.5.6 Availability with reduced capacity for four ton line operative ($p < d$)

Similarly if $A^5{}_i(t)$ be the probability that the system of four ton capacity is in upstate when production is less than demand at instant t. The recursive relations for availability $A^5{}_i(t)$ have been obtained as:

$$\left.\begin{aligned}
A^5{}_0(t) &= Q_{01}(t)(c)A^5{}_1(t)+Q_{02}(t)(c)A^5{}_2(t)+Q_{03}(t)(c)A^5{}_3(t)\\
&\quad +Q_{04}(t)(c)A^5{}_4(t)+Q_{05}(t)(c)A^5{}_5(t)+Q_{06}(t)(c)A^5{}_6(t)\\
A^5{}_1(t) &= Q_{17}(t)(c)A^5{}_7(t)\\
A^5{}_2(t) &= Q_{28}(t)(c)A^5{}_8(t)
\end{aligned}\right\}$$

$$
\left.\begin{aligned}
A^5{}_3(t) &= Q_{39}(t)(\text{c})A^5{}_9(t)\\
A^5{}_4(t) &= Q_{4,10}(t)(\text{c})A^5{}_{10}(t)\\
A^5{}_5(t) &= Q_{56}(t)(\text{c})A^5{}_6(t)\\
A^5{}_6(t) &= Q_{60}(t)(\text{c})A^5{}_0(t)+Q_{67}(t)(\text{c})A^5{}_2(t)+Q_{68}(t)(\text{c})A^5{}_8(t)\\
&\quad +Q_{69}(t)(\text{c})A^5{}_9(t)+Q_{6,10}(t)(\text{c})A^5{}_{10}(t)\\
A^5{}_7(t) &= M_7(t) + Q_{76}(t)(\text{c})A^5{}_6(t) + Q_{7,11}(t)(\text{c})A^5{}_{11}(t)+Q_{7,13}(t)(\text{c})A^5{}_{13}(t)\\
A^5{}_8(t) &= M_8(t) + Q_{86}(t)(\text{c})A^5{}_6(t) + Q_{8,12}(t)(\text{c})A^5{}_{11}(t)+Q_{8,14}(t)(\text{c})A^5{}_{14}(t)\\
A^5{}_9(t) &= Q_{96}(t)(\text{c})A^5{}_6(t) + Q_{9,11}(t)(\text{c})A^5{}_{11}(t)+Q_{9,13}(t)(\text{c})A^5{}_{13}(t)\\
A^5{}_{10}(t) &= Q_{10,6}(t)(\text{c})A^5{}_6(t) + Q_{10,12}(t)(\text{c})A^5{}_{12}(t)+Q_{10,14}(t)(\text{c})A^5{}_{14}(t)\\
A^5{}_{11}(t) &= Q_{11,7}(t)(\text{c})A^5{}_7(t)+Q_{11,9}(t)(\text{c})A^5{}_9(t)\\
A^5{}_{12}(t) &= Q_{12,7}(t)(\text{c})A^5{}_7(t)+Q_{12,10}(t)(\text{c})A^5{}_{10}(t)\\
A^5{}_{13}(t) &= Q_{13,8}(t)(\text{c})A^5{}_8(t)+Q_{13,9}(t)(\text{c})A^5{}_9(t)\\
A^5{}_{14}(t) &= Q_{14,8}(t)(\text{c})A^5{}_8(t)+Q_{14,10}(t)(\text{c})A^5{}_{14}(t)
\end{aligned}\right\} , \qquad (3.15)
$$

where

$$M_7(t) = e^{-(\beta_1+\beta_2+a_1)t}\ ,\ M_8(t) = e^{-(\beta_1+\beta_2+a_2)t}$$

Next, for obtaining availabilities of different capacities can be found out by taking Laplace Transform of set of equations (3.10), (3.11), (3.12), (3.13), (3.14), (3.15) and solving them for $A_0^*(s),\ A^1{}_0{}^*(s),\ A^2{}_0{}^*(s),\ A^3{}_0{}^*(s),\ A^4{}_0{}^*(s)\ and\ A^5{}_0{}^*(s)$, we get

$$A_0 = \lim_{s\to 0} s\ A_0*(s)\ =\ \frac{N_{11}}{\text{derivative}(D_{21})} \qquad (3.16)$$

$$A^1{}_0 = \lim_{s\to 0} s\ A^1{}_0*(s) = \frac{N_{21}}{\text{derivative}(D_{21})} \qquad (3.17)$$

$$A^2{}_0 = \lim_{s\to 0} s\ A^2{}_0*(s) = \frac{N_{31}}{\text{derivative}(D_{21})} \qquad (3.18)$$

$$A^3{}_0 = \lim_{s\to 0} s\ A^3{}_0*(s) = \frac{N_{41}}{\text{derivative}(D_{21})} \qquad (3.19)$$

$$A^4{}_0 = \lim_{s\to 0} s\ A^4{}_0*(s) = \frac{N_{51}}{\text{derivative}(D_{21})} \qquad (3.20)$$

$$A^5{}_0 = \lim_{s\to 0} s\, A^5{}_0{}^*(s) = \frac{N_{61}}{\text{derivative}(D_{21})}, \tag{3.21}$$

where

$$N_{11} = \begin{vmatrix} M_0 & -p_{06} - p_{05}p_{56} & -p_{01}p_{17} & -p_{02}p_{28} & -p_{03}p_{39} & -p_{04}p_{4,10} \\ 0 & 1 & -p_{67} & -p_{68} & -p_{69} & -p_{6,10} \\ 0 & -p_{76} & \begin{matrix} 1-p_{7,11}p_{11,7} \\ -p_{7,12}p_{12,7} \end{matrix} & 0 & -p_{7,11}p_{11,9} & -p_{7,12}p_{12,10} \\ 0 & -p_{86} & 0 & \begin{matrix} 1-p_{8,14}p_{14,8} \\ -p_{8,13}p_{13,8} \end{matrix} & -p_{8,13}p_{13,9} & -p_{8,14}p_{14,10} \\ 0 & -p_{96} & -p_{9,11}p_{11,7} & -p_{9,13}p_{13,8} & \begin{matrix} 1-p_{9,11}p_{11,9} \\ -p_{9,13}p_{13,9} \end{matrix} & 0 \\ 0 & -p_{10,6} & -p_{10,12}p_{12,7} & -p_{10,14}p_{14,8} & 0 & \begin{matrix} 1-p_{10,14}p_{14,10} \\ -p_{10,12}p_{12,10} \end{matrix} \end{vmatrix}$$

$$N_{21} = \begin{vmatrix} 0 & -p_{06} - p_{05}p_{56} & -p_{01}p_{17} & -p_{02}p_{28} & -p_{03}p_{39} & -p_{04}p_{4,10} \\ 0 & 1 & -p_{67} & -p_{68} & -p_{69} & -p_{6,10} \\ 0 & -p_{76} & \begin{matrix} 1-p_{7,11}p_{11,7} \\ -p_{7,12}p_{12,7} \end{matrix} & 0 & -p_{7,11}p_{11,9} & -p_{7,12}p_{12,10} \\ M_6 & -p_{86} & 0 & \begin{matrix} 1-p_{8,14}p_{14,8} \\ -p_{8,13}p_{13,8} \end{matrix} & -p_{8,13}p_{13,9} & -p_{8,14}p_{14,10} \\ 0 & -p_{96} & -p_{9,11}p_{11,7} & -p_{9,13}p_{13,8} & \begin{matrix} 1-p_{9,11}p_{11,9} \\ -p_{9,13}p_{13,9} \end{matrix} & 0 \\ 0 & -p_{10,6} & -p_{10,12}p_{12,7} & -p_{10,14}p_{14,8} & 0 & \begin{matrix} 1-p_{10,14}p_{14,10} \\ -p_{10,12}p_{12,10} \end{matrix} \end{vmatrix}$$

$$N_{31}=\begin{vmatrix} \begin{matrix} p_{03}M_3 \\ +p_{04}M_4 \end{matrix} & -p_{06}-p_{05}p_{56} & -p_{01}p_{17} & -p_{02}p_{28} & -p_{03}p_{39} & -p_{04}p_{4,10} \\ 0 & 1 & -p_{67} & -p_{68} & -p_{69} & -p_{6,10} \\ 0 & -p_{76} & \begin{matrix} 1-p_{7,11}p_{11,7} \\ -p_{7,12}p_{12,7} \end{matrix} & 0 & -p_{7,11}p_{11,9} & -p_{7,12}p_{12,10} \\ 0 & -p_{86} & 0 & \begin{matrix} 1-p_{8,14}p_{14,8} \\ -p_{8,13}p_{13,8} \end{matrix} & -p_{8,13}p_{13,9} & -p_{8,14}p_{14,10} \\ 0 & -p_{96} & -p_{9,11}p_{11,7} & -p_{9,13}p_{13,8} & \begin{matrix} 1-p_{9,11}p_{11,9} \\ -p_{9,13}p_{13,9} \end{matrix} & 0 \\ 0 & -p_{10,6} & -p_{10,12}p_{12,7} & -p_{10,14}p_{14,8} & 0 & \begin{matrix} 1-p_{10,14}p_{14,10} \\ -p_{10,12}p_{12,10} \end{matrix} \end{vmatrix}$$

$$N_{41}=\begin{vmatrix} \begin{matrix} p_{01}M_1 \\ +p_{02}M_2 \end{matrix} & -p_{06}-p_{05}p_{56} & -p_{01}p_{17} & -p_{02}p_{28} & -p_{03}p_{39} & -p_{04}p_{4,10} \\ 0 & 1 & -p_{67} & -p_{68} & -p_{69} & -p_{6,10} \\ 0 & -p_{76} & \begin{matrix} 1-p_{7,11}p_{11,7} \\ -p_{7,12}p_{12,7} \end{matrix} & 0 & -p_{7,11}p_{11,9} & -p_{7,12}p_{12,10} \\ 0 & -p_{86} & 0 & \begin{matrix} 1-p_{8,14}p_{14,8} \\ -p_{8,13}p_{13,8} \end{matrix} & -p_{8,13}p_{13,9} & -p_{8,14}p_{14,10} \\ 0 & -p_{96} & -p_{9,11}p_{11,7} & -p_{9,13}p_{13,8} & \begin{matrix} 1-p_{9,11}p_{11,9} \\ -p_{9,13}p_{13,9} \end{matrix} & 0 \\ 0 & -p_{10,6} & -p_{10,12}p_{12,7} & -p_{10,14}p_{14,8} & 0 & \begin{matrix} 1-p_{10,14}p_{14,10} \\ -p_{10,12}p_{12,10} \end{matrix} \end{vmatrix}$$

$$N_{51} = \begin{vmatrix} 0 & -p_{06}-p_{05}p_{56} & -p_{01}p_{17} & -p_{02}p_{28} & -p_{03}p_{39} & -p_{04}p_{4,10} \\ 0 & 1 & -p_{67} & -p_{68} & -p_{69} & -p_{6,10} \\ 0 & -p_{76} & \begin{array}{c} 1-p_{7,11}p_{11,7} \\ -p_{7,12}p_{12,7} \end{array} & 0 & -p_{7,11}p_{11,9} & -p_{7,12}p_{12,10} \\ 0 & -p_{86} & 0 & \begin{array}{c} 1-p_{8,14}p_{14,8} \\ -p_{8,13}p_{13,8} \end{array} & -p_{8,13}p_{13,9} & -p_{8,14}p_{14,10} \\ M_9 & -p_{96} & -p_{9,11}p_{11,7} & -p_{9,13}p_{13,8} & \begin{array}{c} 1-p_{9,11}p_{11,9} \\ -p_{9,13}p_{13,9} \end{array} & 0 \\ M_{10} & -p_{10,6} & -p_{10,12}p_{12,7} & -p_{10,14}p_{14,8} & 0 & \begin{array}{c} 1-p_{10,14}p_{14,10} \\ -p_{10,12}p_{12,10} \end{array} \end{vmatrix}$$

$$N_{61} = \begin{vmatrix} 0 & -p_{06}-p_{05}p_{56} & -p_{01}p_{17} & -p_{02}p_{28} & -p_{03}p_{39} & -p_{04}p_{4,10} \\ 0 & 1 & -p_{67} & -p_{68} & -p_{69} & -p_{6,10} \\ \mathrm{M}_7 & -p_{76} & \begin{array}{c} 1-p_{7,11}p_{11,7} \\ -p_{7,12}p_{12,7} \end{array} & 0 & -p_{7,11}p_{11,9} & -p_{7,12}p_{12,10} \\ M_8 & -p_{86} & 0 & \begin{array}{c} 1-p_{8,14}p_{14,8} \\ -p_{8,13}p_{13,8} \end{array} & -p_{8,13}p_{13,9} & -p_{8,14}p_{14,10} \\ 0 & -p_{96} & -p_{9,11}p_{11,7} & -p_{9,13}p_{13,8} & \begin{array}{c} 1-p_{9,11}p_{11,9} \\ -p_{9,13}p_{13,9} \end{array} & 0 \\ 0 & -p_{10,6} & -p_{10,12}p_{12,7} & -p_{10,14}p_{14,8} & 0 & \begin{array}{c} 1-p_{10,14}p_{14,10} \\ -p_{10,12}p_{12,10} \end{array} \end{vmatrix}$$

$$D_{21} = \begin{vmatrix} 1 & -q_{05}\,q_{56} & -q_{01}\,q_{17} & -q_{02}\,q_{28} & -q_{03}\,q_{39} & -q_{04}\,q_{4,10} \\ -q_{60} & 1 & -q_{67} & -q_{68} & -q_{69} & -q_{6,10} \\ 0 & -q_{76} & \begin{matrix} 1-q_{7,11}\,q_{11,7} \\ -q_{7,12}\,q_{12,7} \end{matrix} & 0 & -q_{7,11}\,q_{11,9} & -q_{7,12}\,q_{12,10} \\ 0 & -q_{86} & 0 & \begin{matrix} 1-q_{8,14}\,q_{14,8} \\ -q_{8,13}q_{13,8} \end{matrix} & -q_{8,13}\,q_{13,9} & -q_{8,14}q_{14,10} \\ 0 & -q_{96} & -q_{9,11}\,q_{11,7} & -q_{9,13}\,q_{13,8} & \begin{matrix} 1-q_{9,11}\,q_{11,9} \\ -q_{9,13}\,q_{13,9} \end{matrix} & 0 \\ 0 & -q_{10,6} & -q_{10,12}\,q_{12,7} & -q_{10,14}\,q_{14,8} & 0 & \begin{matrix} 1-q_{10,14}q_{14,10} \\ -q_{10,12}\,q_{12,10} \end{matrix} \end{vmatrix}$$

3.6 Busy period of repairman

The recursive relations for busy period have been obtained as:

$$\left.\begin{aligned}
B_0(t) &= Q_{01}(t)(\text{c})B_1(t)+Q_{02}(t)(\text{c})B_2(t)+Q_{03}(t)(\text{c})B_3(t)+Q_{04}(t)(\text{c})B_4(t) \\
&\quad +Q_{05}(t)(\text{c})B_5(t)+Q_{06}(t)(\text{c})B_6(t) \\
B_1(t) &= W_1(t)+Q_{17}(t)(\text{c})B_7(t) \\
B_2(t) &= W_2(t)+Q_{28}(t)(\text{c})B_8(t) \\
B_3(t) &= W_3(t)+Q_{39}(t)(\text{c})B_9(t) \\
B_4(t) &= W_4(t)+Q_{4,10}(t)(\text{c})B_{10}(t) \\
B_5(t) &= Q_{56}(t)(\text{c})B_6(t) \\
B_6(t) &= Q_{60}(t)(\text{c})B_0(t)+Q_{67}(t)(\text{c})B_2(t)+Q_{68}(t)(\text{c})\mathrm{B}_8(t)+Q_{69}(t)(\text{c})\mathrm{B}_9(t) \\
&\quad +Q_{6,10}(t)(\text{c})\mathrm{B}_{10}(t) \\
B_7(t) &= W_7(t)+Q_{76}(t)(\text{c})B_6(t)+Q_{7,11}(t)(\text{c})\mathrm{B}_{11}(t)+Q_{7,13}(t)(\text{c})B_{13}(t) \\
B_8(t) &= W_8(t)+Q_{86}(t)(\text{c})B_6(t)+Q_{8,12}(t)(\text{c})\mathrm{B}_{11}(t)+Q_{8,14}(t)(\text{c})B_{14}(t) \\
B_9(t) &= W_9(t)+Q_{96}(t)(\text{c})B_6(t)+Q_{9,11}(t)(\text{c})\mathrm{B}_{11}(t)+Q_{9,13}(t)(\text{c})B_{13}(t) \\
B_{10}(t) &= W_{10}(t)+Q_{10,6}(t)(\text{c})B_6(t)+Q_{10,12}(t)(\text{c})\mathrm{B}_{11}(t)+Q_{10,14}(t)(\text{c})B_{14}(t) \\
B_{11}(t) &= W_{11}(t)+Q_{11,7}(t)(\text{c})\mathrm{B}_7(t)+Q_{11,9}(t)(\text{c})B_9(t) \\
B_{12}(t) &= W_{12}(t)+Q_{12,7}(t)(\text{c})\mathrm{B}_7(t)+Q_{12,10}(t)(\text{c})B_{10}(t) \\
B_{13}(t) &= W_{13}(t)+Q_{13,8}(t)(\text{c})\mathrm{B}_8(t)+Q_{13,9}(t)(\text{c})B_9(t) \\
B_{14}(t) &= W_{14}(t)+Q_{14,8}(t)(\text{c})A_8(t)+Q_{14,10}(t)(\text{c})B_{10}(t)
\end{aligned}\right\}, \qquad (3.22)$$

where

$$W_1(t) = \bar{H}_1(t) \ , \ W_2(t) = \bar{H}_2(t) \ , \ W_3(t) = \bar{H}_3(t) \ , \ W_4(t) = \bar{H}_4(t)$$

$$W_7(t) = e^{-(\beta_1+\beta_2+a_1)t} \ , \ W_8(t) = e^{-(\beta_1+\beta_2+a_2)t} \ , \ W_9(t) = e^{-(\lambda_1+\lambda_2+b_1)t}$$

$$W_{10}(t) = e^{-(\lambda_1+\lambda_2+b_2)t} \ , \ W_{11}(t) = e^{-(a_1+b_1)t} \ , \ W_{12}(t) = e^{-(a_1+b_2)t}$$

$$W_{13}(t) = e^{-(a_2+b_1)t} \ , \ W_{14}(t) = e^{-(a_2+b_2)t}$$

$$\bar{H}_1(t) = 1 - H_1(t) \ , \ \bar{H}_2(t) = 1 - H_2(t)$$

$$\bar{H}_3(t) = 1 - H_3(t) \ , \ \bar{H}_4(t) = 1 - H_4(t)$$

Busy period has been obtained by taking Laplace transform of set equations (3.22) and solving them for $B_0^*(s)$, we get

$$B_o = \lim_{s\to 0} B_o^*(s) = \frac{N_{71}}{\text{derivative}(D_{21})}, \tag{3.23}$$

where

$$N_{71} = \begin{vmatrix} 0 & -p_{01} & -p_{02} & -p_{04} & -p_{05} & -p_{06} & -p_{07} \\ 0 & 1 & -p_{12} & 0 & 0 & -p_{16} & -p_{17} \\ 0 & -p_{21} & 1 & -p_{24} & -p_{25} & 0 & 0 \\ W_4 & -p_{48}p_{81} & 0 & 1-p_{4,10}p_{10,4}-p_{49}p_{94} & 0 & -p_{49}p_{96} & -p_{4,10}p_{10,7} \\ W_5 & -p_{51} & 0 & 0 & \begin{matrix} 1-p_{5,11}p_{11,5} \\ -p_{5,12}p_{12,5} \end{matrix} & -p_{5,12}p_{12,6} & -p_{5,11}p_{11,7} \\ W_6 & 0 & -p_{62} & -p_{94}p_{69} & -p_{6,12}p_{12,5} & \begin{matrix} 1-p_{6,12}p_{12,6} \\ -p_{69}p_{96} \end{matrix} & 0 \\ W_7 & 0 & -p_{7,13}p_{13,2} & -p_{7,10}p_{10,4} & -p_{7,11}p_{11,5} & 0 & 0 \end{vmatrix}$$

D_{21} has been specified.

3.7 Expected Down Time

The recursive relations for finding expected down time are given by:

$$
\left.\begin{array}{l}
E_3(t) = Q_{39}(t)(c)E_9(t) \\
E_4(t) = Q_{4,10}(t)(c)E_{10}(t) \\
E_5(t) = W_5(t) + Q_{56}(t)(c)E_6(t) \\
E_6(t) = Q_{60}(t)(c)E_0(t) + Q_{67}(t)(c)E_2(t) + Q_{68}(t)(c)\mathrm{E}_8(t) + Q_{69}(t)(c)\mathrm{E}_9(t) \\
\qquad + Q_{6,10}(t)(c)\mathrm{E}_{10}(t) \\
E_7(t) = Q_{76}(t)(c)E_6(t) + Q_{7,11}(t)(c)\mathrm{E}_{11}(t) + Q_{7,13}(t)(c)E_{13}(t) \\
E_8(t) = Q_{86}(t)(c)E_6(t) + Q_{8,12}(t)(c)\mathrm{E}_{11}(t) + Q_{8,14}(t)(c)E_{14}(t) \\
E_9(t) = Q_{96}(t)(c)E_6(t) + Q_{9,11}(t)(c)\mathrm{E}_{11}(t) + Q_{9,13}(t)(c)E_{13}(t) \\
E_{10}(t) = Q_{10,6}(t)(c)E_6(t) + Q_{10,12}(t)(c)\mathrm{E}_{11}(t) + Q_{10,14}(t)(c)E_{14}(t) \\
E_{11}(t) = Q_{11,7}(t)(c)\mathrm{E}_7(t) + Q_{11,9}(t)(c)E_9(t) \\
E_{12}(t) = Q_{12,7}(t)(c)\mathrm{E}_7(t) + Q_{12,10}(t)(c)E_{10}(t) \\
E_{13}(t) = Q_{13,8}(t)(c)\mathrm{E}_8(t) + Q_{13,9}(t)(c)E_9(t) \\
E_{14}(t) = Q_{14,8}(t)(c)\mathrm{E}_8(t) + Q_{14,10}(t)(c)E_{10}(t)
\end{array}\right\} \qquad (3.24)
$$

where

$$W_5(t) = e^{-\gamma_1 t}$$

Next, taking Laplace transform of set equations (3.24) and solving them for $E_0^*(s)$, we get expected down time as:

$$E_o = \lim_{s \to 0} E_o^*(s) = \frac{N_{81}}{D_{21}} \;, \qquad (3.25)$$

where

$$
N_{81} = \begin{vmatrix}
p_{03}W_3 & -p_{01} & -p_{02} & -p_{04} & -p_{05} & -p_{06} & -p_{07} \\
0 & 1 & -p_{12} & 0 & 0 & -p_{16} & -p_{17} \\
0 & -p_{21} & 1 & -p_{24} & -p_{25} & 0 & 0 \\
0 & -p_{48}\,p_{81} & 0 & \begin{array}{c} 1-p_{4,10}\,p_{10,4} \\ -p_{49}\,p_{94} \end{array} & 0 & -p_{49}\,p_{96} & -p_{4,10}\,p_{10,7} \\
0 & -p_{51} & 0 & 0 & \begin{array}{c} 1-p_{5,11}\,p_{11,5} \\ -p_{5,12}\,p_{12,5} \end{array} & -p_{5,12}\,p_{12,6} & -p_{5,11}\,p_{11,7} \\
0 & 0 & -p_{62} & -p_{94}\,p_{69} & -p_{6,12}\,p_{12,5} & \begin{array}{c} 1-p_{6,12}\,p_{12,6} \\ -p_{69}\,p_{96} \end{array} & 0 \\
0 & 0 & -p_{7,13}\,p_{13,2} & -p_{7,10}\,p_{10,4} & -p_{7,11}\,p_{11,5} & 0 & 0
\end{vmatrix}
$$

D_{21} has been specified.

3.8 Cost-Benefit Analysis

It is an important fact for any firm that the revenue and cost functions lead to the profit function. As the profit is excess of revenue over the cost of production, the profit takes the form

$$P = \text{ Expected revenue in } (0,t] \text{ - Expected total cost in } (0,t]$$

Using equations (3.16), (3.17), (3.18), (3.19), (3.20), (3.21), (3.23) and (3.25) the expected profit per unit time incurred to the system is given by

$$P = C_{01}A_0 + C_{02}A^1{}_0 + C_{03}A^2{}_0 + C_{04}A^2{}_0 + C_{05}A^2{}_0 + C_{06}A^2{}_0 - C_1B_0 - C_2E_0 - L \qquad (3.26)$$

where

C_{01} = Total revenue per unit time for availability of twelve ton ($p \geq d$)
C_{02} = Total revenue per unit time for availability of twelve ton ($p < d$)
C_{03} = Total revenue per unit time for availability of eight ton ($p \geq d$)
C_{04} = Total revenue per unit time for availability of four ton ($p \geq d$)
C_{05} = Total revenue per unit time for availability of eight ton ($p < d$)
C_{06} = Total revenue per unit time for availability of four ton ($p < d$)
C_1 = Cost of busy period of repairman
C_2 = Cost for expected down time
L = Goodwill Loss

Hence the bounds for revenue/costs for the system to be profitable are:

TABLE 3.2: BOUNDS FOR REVENUE/COSTS

Revenue/Cost	Bound (Lower/Upper)	Value
C_{01}	Lower	$\dfrac{\left(C_1B_0 + C_2E_0 + L - C_{02}A^1{}_0 - C_{03}A^2{}_0 - C_{04}A^3{}_0 - C_{05}A^4{}_0 - C_{06}A^5{}_0\right)}{A_0}$

C_{02}	Lower	$\dfrac{\left(C_1B_0 + C_2E_0 + L - C_{01}A_0 - C_{03}A^2{}_0 - C_{04}A^3{}_0 - C_{05}A^4{}_0 - C_{06}A^5{}_0\right)}{A^1{}_0}$
C_{03}	Lower	$\dfrac{\left(C_1B_0 + C_2E_0 - C_{01}A_0 - C_{02}A^1{}_0 - C_{04}A^3{}_0 - C_{05}A^4{}_0 - C_{06}A^5{}_0\right)}{A^2{}_0}$
C_{04}	Lower	$\dfrac{\left(C_1B_0 + C_2E_0 + L - C_{01}A_0 - C_{02}A^1{}_0 - C_{03}A^2{}_0 - C_{05}A^4{}_0 - C_{06}A^5{}_0\right)}{A^3{}_0}$
C_{05}	Lower	$\dfrac{\left(C_1B_0 + C_2E_0 + L - C_{01}A_0 - C_{02}A^1{}_0 - C_{03}A^2{}_0 - C_{04}A^3{}_0 - C_{06}A^5{}_0\right)}{A^4{}_0}$
C_{06}	Lower	$\dfrac{\left(C_1B_0 + C_2E_0 + L - C_{01}A_0 - C_{02}A^1{}_0 - C_{03}A^2{}_0 - C_{04}A^3{}_0 - C_{05}A^4{}_0\right)}{A^5{}_0}$
C_1	Upper	$\dfrac{\left(C_{01}A_0 + C_{02}A^1{}_0 + C_{03}A^2{}_0 - C_{04}A^3{}_0 - C_{05}A^4{}_0 - C_{06}A^5{}_0 - C_2E_0 - L\right)}{B_0}$
C_2	Upper	$\dfrac{\left(C_{01}A_0 + C_{02}A^1{}_0 + C_{03}A^2{}_0 - C_{04}A^3{}_0 - C_{05}A^4{}_0 - C_{06}A^5{}_0 - C_1B_0 - L\right)}{E_0}$

3.9 Analysis and Discussion

In this study, data for all types of failures and repairs of the rice plant was collected in the units of per hour. On the basis of these data, we have computed the failure and repair rates. . In particular case $h_1(t) = \alpha_1 e^{-\alpha_1 t}$, $h_2(t) = \alpha_2 e^{-\alpha_2 t}$ $h_3(t) = \alpha_3 e^{-\alpha_3 t}$, $h_4(t) = \alpha_4 e^{-\alpha_4 t}$
The values of various measures calculated and assumed are :
$a_1 = 0.3144$, $b_1 = 0.2346$ $\gamma_1 = 0.01736$ $\gamma_2 = 0.0987$, $\alpha_1 = 0.01$, $\alpha_2 = 0.001$
$\alpha_3 = 0.01$, $\alpha_4 = 0.001$

We have computed the following results of important reliability indices using the software 'MATLAB'. By varying λ_2 for distinct values of λ_1, the values of mean time to system failure (MTSF) and availability are computed. Similarly, the profit is also computed by varying C_0 for different choices of C_1 and results are presented. Their behaviours are exhibited in figures. Fig. 3.1 shows the behaviour of MTSF with respect to λ_2 for distinct values of λ_1. The graph shows that MTSF decreases with increase in the λ_2 keeping λ_1 constant and has greater values for lesser values of λ_1.

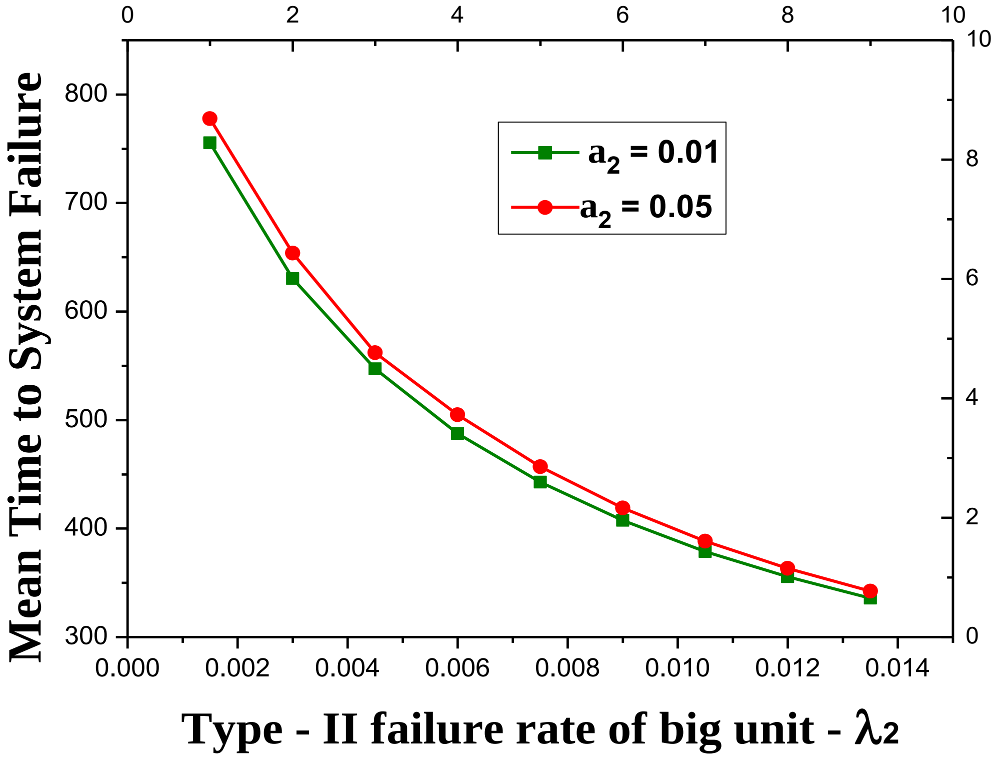

Fig. 3.1 - Effect of Type-II failure rate on Mean time to system failure for distinct values of Type-II repair rate.

Behaviour of total availability in full and reduced capacity with respect to failure rate for distinct values repair rate has been shown in Fig 3.2, Fig 3.3, Fig 3.4, Fig 3.5, Fig 3.6 and Fig 3.7. Fig. 3.2 indicates that availability of full capacity i.e. twelve ton when production is greater than or equal to demand decreases with increase in λ_2 and has greater values for higher value of a_2. Behaviour of availability in full capacity when production is less than demand is shown in Fig. 3.3 which indicates that availability in full capacity i.e. twelve ton when production is less than demand decreases with increase in λ_2 and has greater values for higher value of a_2. Fig. 3.4 indicates that availability in reduced capacity - four ton when production is less than demand increases with increase in λ_2 and has lesser values for higher value of a_2. Fig. 3.5 shows the behaviour of availability in reduced capacity - four ton when production is greater than or equal to demand which indicates that availability decreases with increase in Type II failure rate of line of four ton capacity β_2 and has lesser values for higher value of Type - II repair rate b_2. Behaviour of availability in reduced capacity – eight ton when production is greater than or equal to demand is shown in Fig. 3.6 which indicates that availability of capacity eight ton when production is greater than or equal to demand decreases with increase in λ_2 and has greater values for higher value of Type - II repair rate a_2. Fig. 3.7 indicates that availability in eight ton capacity when production is less than decreases with increase in λ_2 and has greater values for higher value of Type - II repair rate a_2.

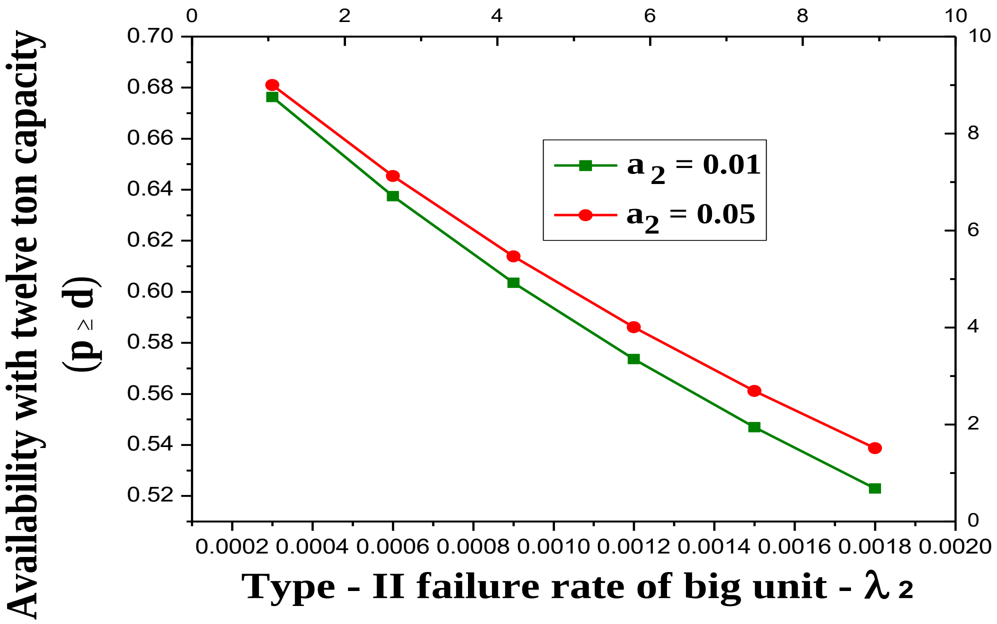

Fig. 3.2 - Effect of Type-II failure rate on Availability of twelve ton capacity (full capacity $p \geq d$) for distinct values of Type-II repair rate.

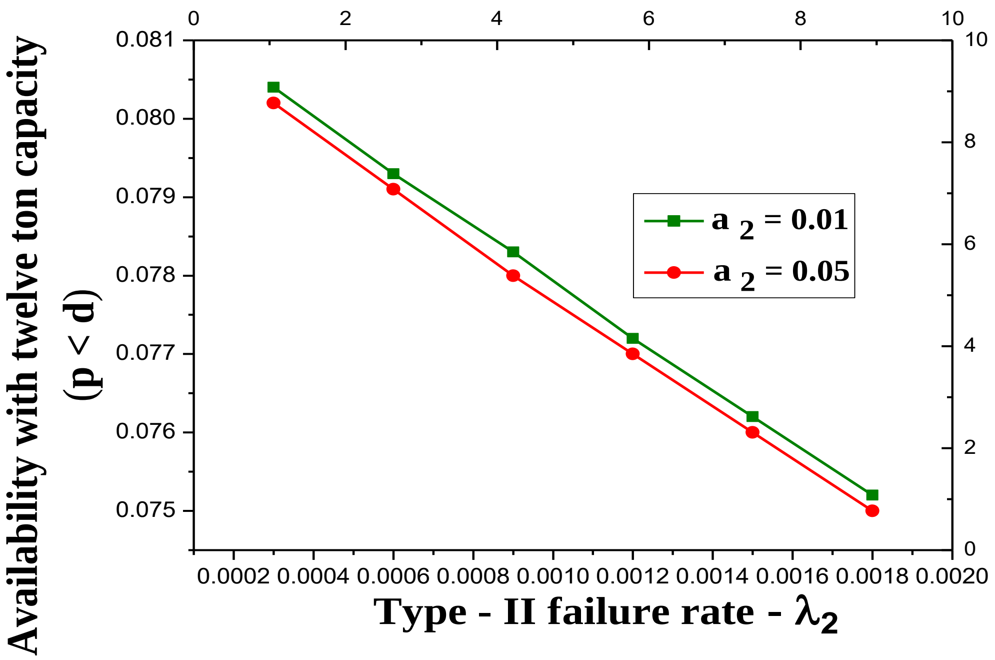

Fig. 3.3 - Effect of Type-II failure rate on Availability of twelve ton capacity (full capacity – p < d) for distinct values of Type-II repair rate.

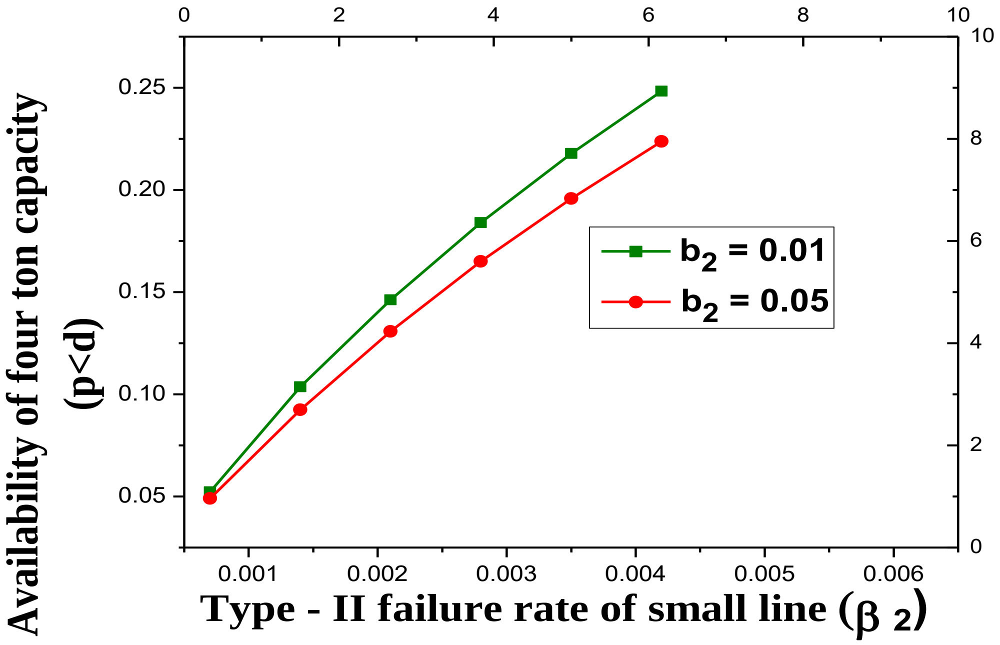

Fig. 3.4 - Effect of Type-II failure rate on Availability of four ton capacity (reduced capacity – p < d) for distinct values of Type-II repair rate.

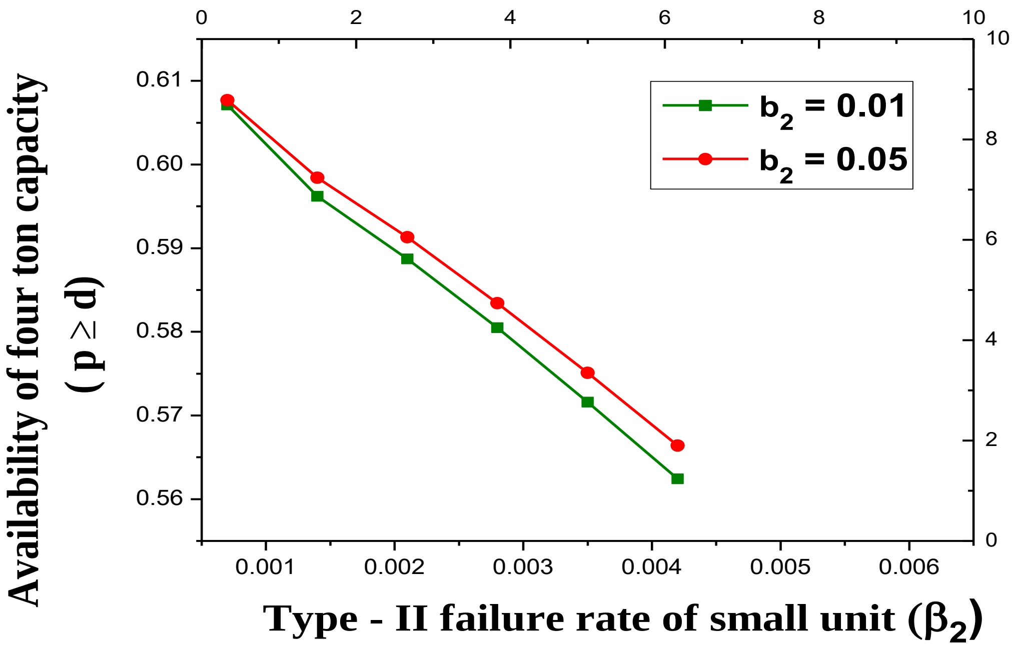

Fig. 3.5 - Effect of Type-II failure rate on Availability of four ton capacity (reduced capacity – $p \geq d$) for distinct values of Type-II repair rate.

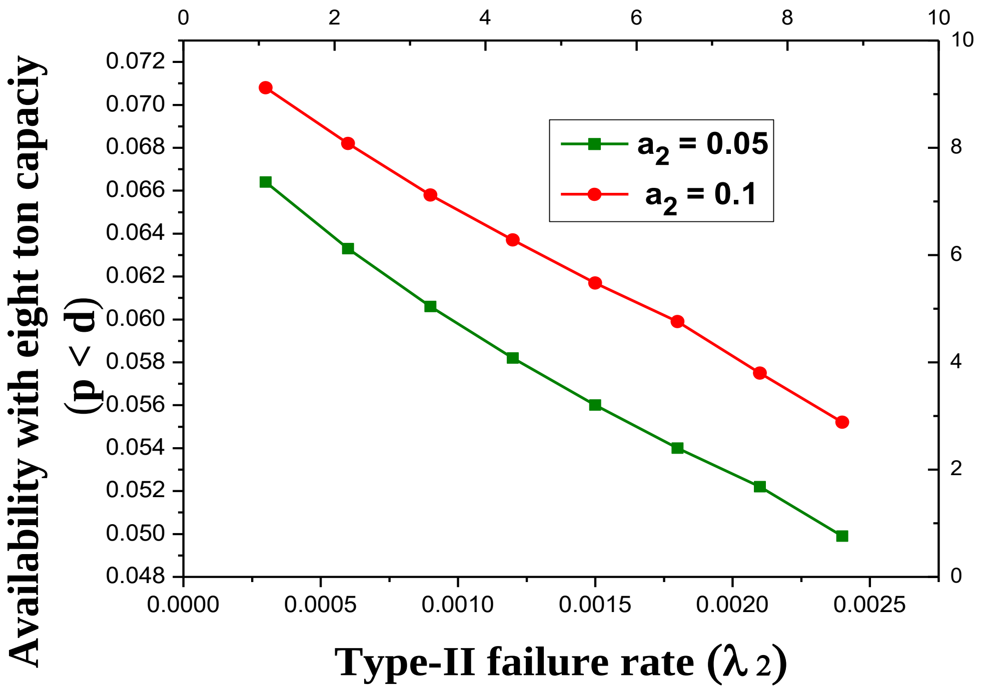

Fig. 3.6 - Effect of Type-II failure rate on Availability of eight ton capacity (reduced capacity – $p < d$) for distinct values of Type-II repair rate.

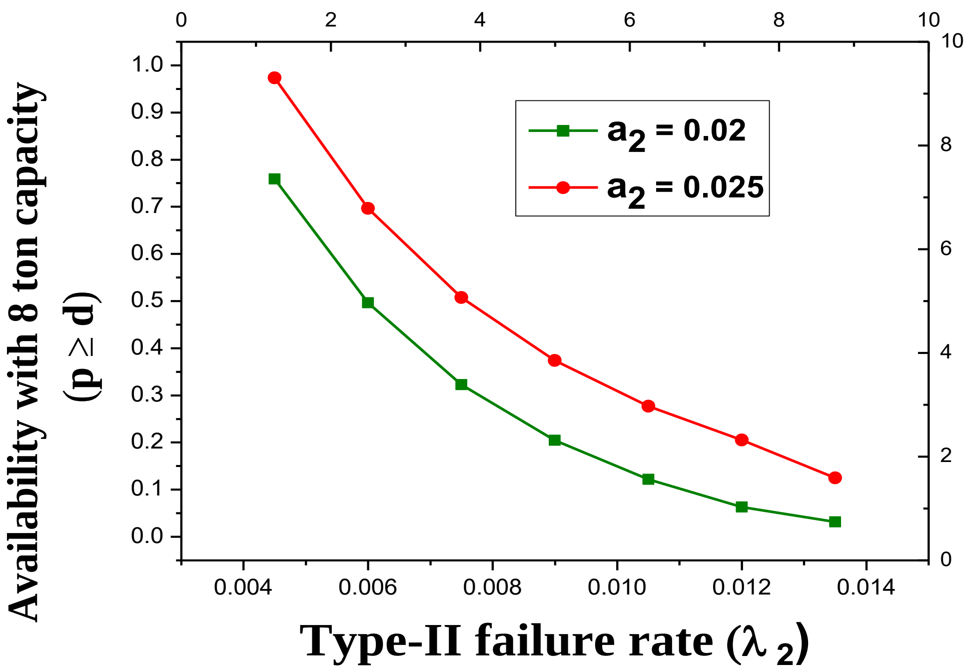

Fig. 3.7 - Effect of Type-II failure rate on Availability of eight ton capacity (reduced capacity – p ≥ d) for distinct values of Type-II repair rate.

The behaviour of profit with respect to revenue for twelve ton capacity when production is greater than or equal to demand C_{01} for distinct values of cost of repairman C_1 is shown in Fig. 3.8. Values of various measures considered are: $a_1 = 0.3144$, $b_1 = 0.2346$, $a_2 = 0.1736$, $b_2 = 0.2500$, $\lambda_1 = 0.0084$, $\lambda_2 = 0.0036$, $\beta_1 = 0.0028$ $\beta_2 = 0.0012$, $\gamma_1 = 0.01736$, $\gamma_2 = 0.0987$, $\alpha_1 = 0.01$, $\alpha_2 = 0.001$ $\alpha_3 = 0.01$, $\alpha_4 = 0.001$

$C_{02} = 1800\,INR$, $C_{05} = 1200\,INR$, $C_{06} = 600\,INR$, $C_{03} = \frac{2}{3}C_{01}$, $C_{04} = \frac{1}{3}C_{01}$

$L = 3500\,INR$, $C_2 = 1500\,INR$

It is concluded from this graph that profit increases with the increase in revenue C_{01} and decreases with increase in cost of repairman. On comparing graphs, it reveals that

(i) For $C_1 = 2000\,INR$, the profit is negative or zero or positive according as $C_{01} \leq or \geq 1252.97$. Hence, revenue for twelve ton capacity when production is greater than or equal to demand should be fixed greater than 1252.97 INR to attain the profit.

(ii) For $C_1 = 2500\,INR$, the profit is negative or zero or positive according as $C_{01} \leq or \geq 1287.03$. Hence, revenue for twelve ton capacity when production is greater than or equal to demand should be fixed greater than 1287.03 INR to attain the profit.

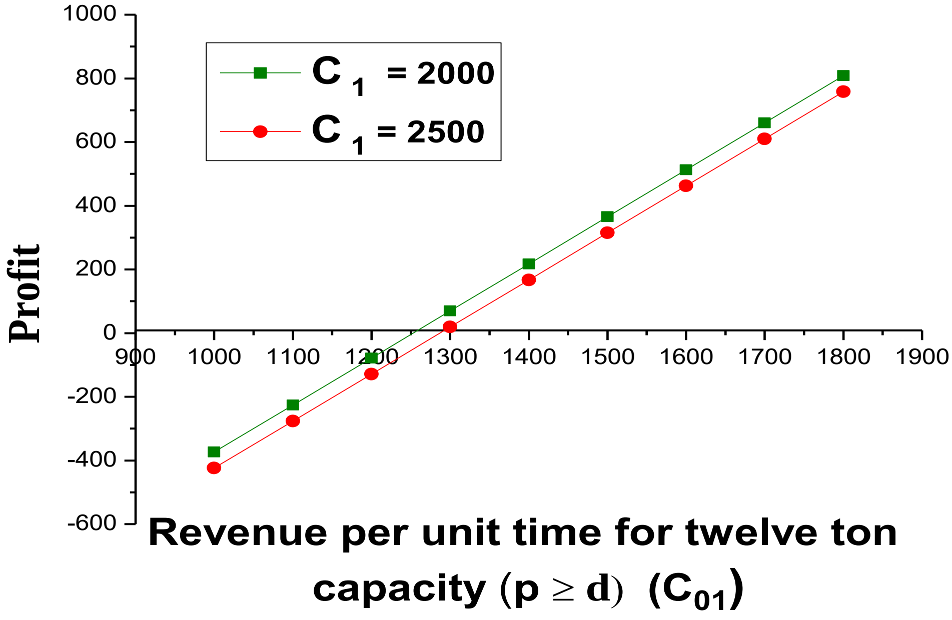

Fig. 3.8 - Effect of revenue of twelve ton capacity (p ≥ d) on profit for different values of cost of repairman.

The behaviour of profit with respect to revenue for twelve ton capacity when production is less than demand C_{02} for distinct values of cost of repairman C_1 is shown in Fig. 3.9Values of various measures considered are:

$C_{01}=1650\,INR$, $C_{03}=1100\,INR$, $C_{04}=550\,INR$, $C_{05}=\frac{2}{3}C_{02}$, $C_{06}=\frac{1}{3}C_{02}$, $C_2=1500\,INR$

It is concluded from the graph that profit increases with the increase in revenue C_{02} and decreases with increase in cost of repairman. On comparing graphs, it reveals that

(i) For $C_1 = 2000$, the profit is negative or zero or positive according as $C_{01} \leq or \geq 1062.5$. Hence, revenue for twelve ton capacity when production is greater than or equal to demand should be greater than 1062.5 INR to attain the profit.

(ii) For $C_1 = 2500$, the profit is negative or zero or positive according as $C_{01} \leq or \geq 1125.5$. Hence, revenue for twelve ton capacity when production is greater than or equal to demand should be greater than 1125.5 INR to attain the profit.

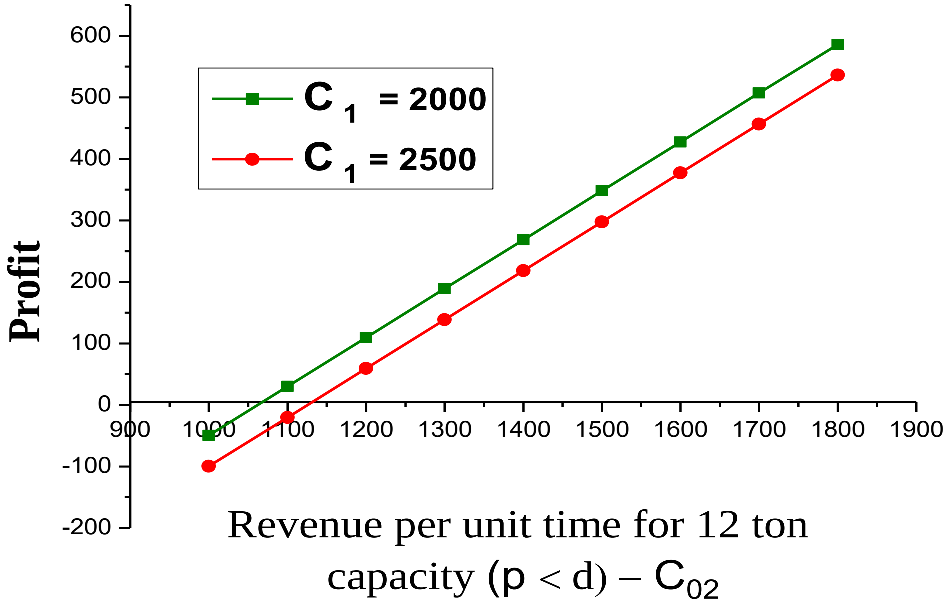

Fig. 3.9 - Effect of revenue of twelve ton capacity (p < d) on profit for different values of cost of repairman.

Fig. 3.10 shows the behaviour of profit with respect to cost of repairman for distinct values of Type II failure rate λ_2. Values of various measures are:

$\gamma_2 = 0.0987, \beta_2 = 0.0012$, $C_{01} = 1650\, INR$, $C_{03} = 1100\, INR$, $C_{04} = 550\, INR$, $C_{02} = 1800$ $C_{05} = 1200\, INR$, $C_{06} = 600\, INR$, $C_2 = 1500\, INR$

It is observed from this graph that profit decreases with the increase in cost of repairman C_1 and Type II failure rate. On comparing graphs, it reveals that

(i) For $\lambda_2 = 0.0045$, the profit is negative or zero or positive according as $C_1 \leq or \geq 4710$. Hence, repairman should not be paid more than 4710 INR to attain the profit.

(ii) For $\lambda_2 = 0.006$, the profit is negative or zero or positive according as $C_1 \leq or \geq 967$. Hence, repairman should not be paid more than 967 INR to attain the profit.

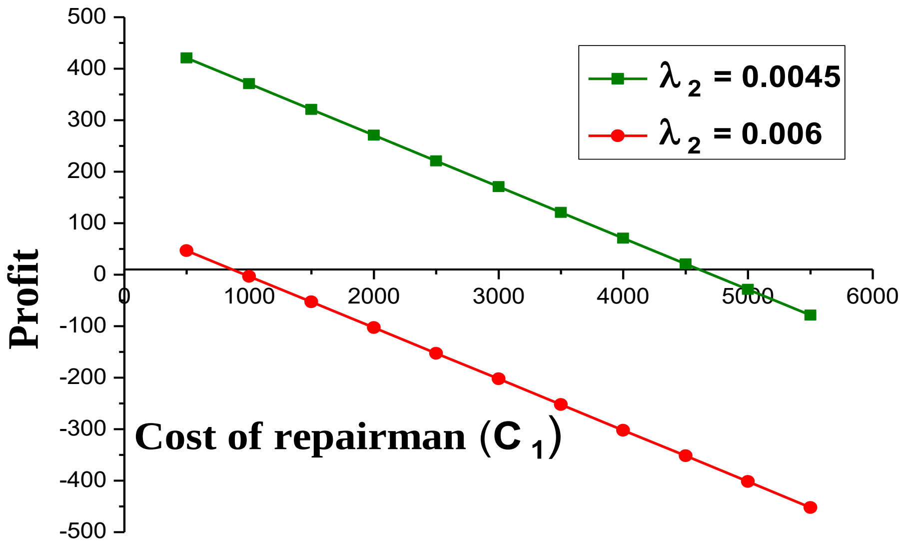

Fig. 3.10 - Effect of cost of repairman on profit for distinct values of Type II failure rate.

CHAPTER 4

ANALYSIS OF A THREE UNIT SYSTEM WITH DEMAND DEPENDENT OPERABILITY OF UNITS

In the preceding chapter we have analysed a two dissimilar unit standby system in which one unit is of 8 ton capacity and other is of four ton capacity, which were made operative depending on the demand. In this chapter profit analysis has been done by taking both the situations when production is less than demand or at least equal to demand with consideration of goodwill loss when production is less than demand. In the present chapter we have considered an additional unit of four ton capacity in the two dissimilar unit standby system to overcome the situation of goodwill loss. In the present chapter the cost benefit of of three unit standby system is obtained by considering two types of failures, variation in demand.

4.2 System Descriptions and Assumptions

System Description

This system comprises of three units one is of eight ton capacity and another two are of four ton capacity each is considered. One, two or all the three units are made operative depending on the demand. In this chapter, along with mathematical formulation of the problem determining the transition probabilities of various states by considering two types of failure for each unit, availability in full capacity and availability in reduced capacity both have been obtained by using semi –markov process and regenerative point technique. Rest processing of the system is same as in the preceding chapters. Thus incorporating the concepts of dissimilar standby units, two types of failures, variation in demand and production, we have analysed reliability and cost benefit for the system in the present chapter. Initially the system is in the working condition with both units operative. The failed unit is undertaken for repair immediately as it fails. After getting repaired the unit can be made operative or standby as per the requirement.

The following assumptions have been considered for the model:

Assumptions:

(i) After every repair the unit behaves like a totally new one.

(ii) After getting repaired the unit can be made operative or standby according to the need.

(iii) Repair facility is always available for every failed unit i.e. no unit waits for repair

(iv) Both Failure times as well as repair times are assumed to follow exponential distribution.

(v) In case of repair priority is given to line of 8 ton capacity instead of two 4 ton capacity lines when there is requirement of 8 ton.

The system has been observed at suitable regenerative epochs by using regenerative point technique and the following reliability characteristics have been obtained:

(i) Mean time to system failure (MTSF)

(ii) Availability with full capacity – sixteen ton

(iii) Availability with capacity – twelve ton

(iv) Availability with reduced capacity – eight ton

(v) Availability with reduced capacity – four ton

(vi) Expected busy period of repairman in $(0,t]$

(vii) Expected downtime

(viii) Expected profit incurred in $(0,t]$

4.3 Notations

The following notations have been used throughout this chapter:

B_s	:	Unit with 8 ton capacity in standby mode
B_o	:	Unit with 8 ton capacity in operative mode
S_o	:	Unit with 4 ton capacity in operative mode
S_s	:	Unit with 4 ton capacity in standby mode
B_r, S_r, S_r	:	System is at rest
F_{r_1}	:	Unit having Type – I failure is under repair
F_{r_2}	:	Unit having Type – II failure is under repair
λ_1	:	Rate of Type - I failure of big unit i.e. of eight ton capacity
λ_2	:	Rate of Type - II failure of big unit i.e. of eight ton capacity
β_1	:	Rate of Type - I failure of small unit i.e. of four ton capacity
β_2	:	Rate of Type - II failure of small unit i.e. of four ton capacity
$i(t)$	:	p.d.f of time to complete pending process of material at colour sorter of big unit
$I(t)$	:	c.d.f of time to complete pending process of material at colour sorter of big unit
γ_1	:	Rate with which system is made operative from rest
γ_2	:	Rate with which system goes to rest from operative state

$m(t)$	:	p.d.f of time to complete pending process of material at colour sorter of small unit
$M(t)$	:	c.d.f of time to complete pending process of material at colour sorter of small unit
p	:	Probability that after repair unit needs not to be made operative.
q	:	Probability that after repair unit is made operative .
a_1	:	Rate of Type – I repair for big unit i.e. of eight ton capacity
a_2	:	Rate of Type – II repair for big unit i.e. of eight ton capacity
b_1	:	Rate of Type – I repair for small unit i.e. of four ton capacity
b_2	:	Rate of Type – II repair for small unit i.e. of four ton capacity
p_1	:	Probability of making the system operative with capacity from sixteen to four ton as per requirement.
p_2	:	Probability of making the system operative with capacity from sixteen to eight ton as per requirement.
p_3	:	Probability of making the system operative with capacity from sixteen to twelve ton as per requirement.
p_4	:	Probability of making the system operative with capacity from four to sixteen ton as per requirement.
p_5	:	Probability of making the system operative with capacity from four to eight ton as per requirement.
p_6	:	Probability of making the system operative with capacity from four to twelve ton as per requirement.
. p_7	:	Probability of making the system operative with capacity from eight to 16 ton as per requirement.
p_8	:	Probability of making the system operative with capacity from eight to four ton as per requirement.

p_9 : Probability of making the system operative with capacity from eight to twelve ton as per requirement.

p_{10} : Probability of making the system operative with capacity from twelve to sixteen ton as per requirement.

p_{11} : Probability of making the system operative with capacity from twelve to four ton as per requirement.

p_{12} : Probability of making the system operative with capacity from twelve to eight ton as per requirement.

p_7 : Probability of making the system operative with capacity from eight to sixteen ton as per requirement.

p_8 : Probability of making the system operative with capacity from eight to four ton as per requirement.

p_9 : Probability of making the system operative with capacity from eight to twelve ton as per requirement.

p_{10} : Probability of making the system operative with capacity from twelve to sixteen ton as per requirement.

p_{11} : Probability of making the system operative with capacity from twelve to four ton as per requirement.

p_{12} : Probability of making the system operative with capacity from twelve to eight ton as per requirement.

p_{13} : Probability of making the unit of four ton capacity in operative mode from standby mode when another same unit undergoes Type-I failure.

p_{14} : Probability of making the unit of four ton capacity in operative mode from standby mode when another same unit undergoes Type-II failure.

p_{15} : Probability of not making the unit of four ton capacity in operative mode from standby mode when another same unit undergoes Type-I failure.

p_{16} : Probability of not making the unit of four ton capacity in operative mode from standby mode when another same unit undergoes Type-II failure.

p_{17} : Probability of making the unit of eight ton capacity in operative mode from standby mode when one unit of four ton capacity undergoes Type-I failure.

p_{18} : Probability of making the unit of eight ton capacity in operative mode from standby mode when one unit of four ton capacity undergoes Type-II failure.

p_{19} : Probability of making both the units of four ton capacity in operative mode from standby mode when unit of eight ton capacity undergoes Type-II failure.

p_{20} : Probability of making both the units of four ton capacity in operative mode from standby mode when unit of eight ton capacity undergoes Type-I failure.

p_{21} : Probability of making one of the units of four ton capacity in operative mode from standby mode when unit of eight ton capacity undergoes Type-I failure.

p_{22} : Probability of making one of the units of four ton capacity in operative mode from standby mode when unit of eight ton capacity undergoes Type-II failure.

p_{23} : Probability of making unit of four ton capacity in operative mode from standby mode when unit of eight ton capacity undergoes Type-I failure.

p_{24} : Probability of making unit of four ton capacity in operative mode from standby mode when unit of eight ton capacity undergoes Type-II failure.

p_{25} : Probability of making unit of four ton capacity in operative mode from standby mode when another unit of four ton capacity undergoes Type-I failure.

p_{26} : Probability of making unit of four ton capacity in operative mode from standby mode when another unit of four ton capacity undergoes Type-II failure.

p_{27} : Probability of not making unit of four ton capacity in operative mode from standby mode when unit of eight ton capacity undergoes Type-I failure.

p_{28} : Probability of not making unit of four ton capacity in operative mode from standby mode when unit of eight ton capacity undergoes Type-II failure.

p_{29} : Probability of not making unit of four ton capacity in operative mode from standby mode when another unit of four ton capacity undergoes Type-I failure.

p_{30} : Probability of not making unit of four ton capacity in operative mode from standby mode when another unit of four ton capacity undergoes Type-II failure.

The system of converting paddy into rice of a rice manufacturing plant has the following states $S_i, i = 0$ to 37,:

(i) Regenerative states

$$\left\{\begin{array}{l} S_0\left(B_o, S_o, S_o\right), S_1\left(B_s, S_s, S_o\right), S_2\left(B_o, S_s, S_s\right), S_3\left(B_o, S_o, S_s\right), S_4\left(B_r, S_r, S_r\right) \\ S_5\left(B_{Fr1}, S_o, S_o\right), S_6\left(B_{Fr2}, S_o, S_o\right), S_7\left(B_o, S_{Fr1}, S_o\right), S_8\left(B_o, S_{Fr2}, S_o\right) \\ S_9\left(B_o, S_s, S_{Fr2}\right), S_{10}\left(B_o, S_s, S_{Fr1}\right), S_{11}(B_s, S_{Fr1}, S_o), S_{12}(B_s, S_{Fr2}, S_o) \\ S_{13}(B_{Fr2}, S_o, S_s), S_{14}(B_{Fr1}, S_o, S_s), S_{15}(B_{Fr1}, S_{Fr1}, S_o), S_{16}(B_{Fr1}, S_{Fr2}, S_o) \\ S_{17}(B_{Fr2}, S_{Fr1}, S_o), S_{18}(B_{Fr2}, S_{Fr2}, S_o), S_{19}(B_o, S_{Fr1}, S_{Fr1}) \\ S_{20}(B_o, S_{Fr2}, S_{Fr1}), S_{21}(B_o, S_{op}, S_o), S_{22}(B_o, S_{Fr1}, S_{Fr2}), S_{23}(B_s, S_{op}, S_s) \\ S_{24}(B_s, S_{op}, S_o), S_{25}(B_{op}, S_o, S_o), S_{26}(B_{op}, S_o, S_{Fr1}), S_{27}(B_{Fr1}, S_{op}, S_s) \\ S_{36}(B_o, S_{op}, S_{Fr1}), S_{37}(B_o, S_{op}, S_{Fr2}), S_{30}(B_{op}, S_o, S_{Fr2}) \end{array}\right\}$$

(ii) Failed states

$$\left\{\begin{array}{l} S_4\left(B_r, S_r, S_r\right), S_{28}(B_{Fr1}, S_{Fr1}, S_{Fr1}), S_{29}(B_{Fr1}, S_{Fr2}, S_{Fr1}) \\ S_{31}(B_{Fr1}, S_{Fr2}, S_{Fr2}), S_{32}(B_{Fr2}, S_{op}, S_o), S_{33}(B_{Fr2}, S_{Fr1}, S_{Fr1}) \\ S_{34}(B_{Fr2}, S_{Fr1}, S_{Fr2}), S_{35}(B_{Fr2}, S_{Fr2}, S_{Fr2}) \end{array}\right\}.$$

In this Model we have considered the states which fulfill the requirement of four ton to sixteen ton.

Table 4.1 : Possible states of transition

State S_i i = 0 to 37	***Status***	***Possible transition to***	***With rate / probabilities respectively***

0	B_o, S_o, S_o	1, 2, 3, 4, 5, 6, 7, 8	$p_2\, h(t), p_3\, h(t)\; p_1\, h(t), \gamma_2, \lambda_1, \lambda_2, 2\beta_1, 2\beta_2$
1	B_s, S_s, S_o	0, 2, 3, 7, 8, 9, 10, 11, 12	$p_4\, h(t), p_5\, h(t)\; p_6\, h(t), p_{13}\, \beta_1, p_{14}\, \beta_2$ $p_{15}\, \beta_1, p_{16}\, \beta_2, p_{17}\, \beta_1, p_{18}\, \beta_2$
2	B_o, S_s, S_s	0, 1, 3, 5, 6, 13, 14	$p_7\, h(t), p_8\, h(t), p_9\, h(t), p_{19}\, \lambda_1, p_{20}\, \lambda_2$ $p_{21}\, \lambda_1, p_{22}\, \lambda_2$
3	B_o, S_o, S_s	0, 1, 2, 5, 6, 7, 8, 9, 10, 13, 14	$p_{10}\, h(t), p_{11}\, h(t)\; p_{12}\, h(t), p_{23}\, \lambda_1, p_{24}\, \lambda_2, p_{25}\, \beta_1$ $p_{26}\, \beta_2, p_{27}\, \lambda_1, p_{28}\, \lambda_2, p_{29}\, \beta_1, p_{30}\, \beta_2$
4	Rest	0	γ_1
5	B_{Fr1}, S_o, S_o	0, 2, 15, 16	$q a_1, p a_1, 2\beta_1, 2\beta_2$
6	B_{Fr2}, S_o, S_o	0, 2, 17, 18	$q a_2, p a_2, 2\beta_1, 2\beta_2$
7	B_o, S_{Fr1}, S_o	0, 15, 17, 19, 20, 21	$q b_1, \lambda_1, \lambda_2, \beta_1, \beta_2, p b_1$
8	B_o, S_{Fr2}, S_o	0, 3, 16, 18, 20, 22	$q b_2, p b_2, \lambda_1, \lambda_2, \beta_1, \beta_2$
9	B_o, S_s, S_{Fr2}	2, 3, 16, 18	$p b_2, q b_2, \lambda_1, \lambda_2$
10	B_o, S_s, S_{Fr1}	3, 15, 17, 23	$q b_1, \lambda_1, \lambda_2, p b_1$
11	B_s, S_{Fr1}, S_o	2, 19, 20, 24	$q b_1, \beta_1, \beta_2, p b_1$
12	B_s, S_{Fr2}, S_o	1, 2, 20, 22	$q b_2, p b_2, \beta_1, \beta_2$
13	B_{Fr2}, S_o, S_s	1, 3, 17, 18	$p a_2, q a_2, \beta_1, \beta_2$
14	B_{Fr1}, S_o, S_s	3, 15, 16, 25	$q a_1, \beta_1, \beta_2, p a_1$
15	B_{Fr1}, S_{Fr1}, S_o	5, 7, 26, 27, 28, 29	$q b_1, q a_1, p b_1, p a_1, \beta_1, \beta_2$
16	B_{Fr1}, S_{Fr2}, S_o	5, 8, 14, 29, 30, 31	$q b_2, q a_1, p b_2, \beta_1, p a_1, \beta_2$
17	B_{Fr2}, S_{Fr1}, S_o	6, 7, 11,	$q b_1, q a_2, p a_2, p b_1, \beta_2, \beta_1$

		32, 33, 34	
18	B_{Fr2}, S_{Fr2}, S_o	6, 8, 12, 13, 34, 35	$qb_2, qa_2, pa_2, pb_2, \beta_1, \beta_2$
19	B_o, S_{Fr1}, S_{Fr1}	7, 28, 33, 36	$qb_1, \lambda_1, \lambda_2, pb_1$
20	B_o, S_{Fr2}, S_{Fr1}	7, 8, 10, 29, 34, 37	$qb_2, qb_1, pb_2, \lambda_1, \lambda_2, pb_1$
21	B_o, S_{op}, S_o	3	$m(t)$
22	B_o, S_{Fr1}, S_{Fr2}	8, 9, 31, 35	$qb_2, pb_2, \lambda_1, \lambda_2$
23	B_s, S_{op}, S_s	2	$m(t)$
24	B_s, S_{op}, S_o	1	$m(t)$
25	B_{op}, S_o, S_o	1	$i(t)$
26	B_{op}, S_o, S_{Fr1}	11	$i(t)$
27	B_{Fr1}, S_{op}, S_s	14	$m(t)$
28	B_{Fr1}, S_{Fr1}, S_{Fr1}	15, 19	b_1, a_1
29	B_{Fr1}, S_{Fr2}, S_{Fr1}	15, 16, 20	b_1, b_2, a_1
30	B_{op}, S_o, S_{Fr2}	12	$i(t)$
31	B_{Fr1}, S_{Fr2}, S_{Fr2}	16, 22	b_2 , a_1
32	B_{Fr2}, S_{op}, S_o	13	$m(t)$
33	B_{Fr2}, S_{Fr1}, S_{Fr2}	17, 18, 20	b_2, b_1, a_2
34	B_{Fr2}, S_{Fr1}, S_{Fr1}	17, 19	b_1, a_2
35	B_{Fr2}, S_{Fr2}, S_{Fr2}	18, 22	b_2, a_2
36	B_o, S_{op}, S_{Fr1}	10	$m(t)$
37	B_o, S_{op}, S_{Fr2}	9	$m(t)$

4.3 Transition Probabilities

The epochs of entry into states 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 are regenerative points. The transition probabilities p_{ij} can be obtained as:

$$p_{ij} = \lim_{s\to 0} q_{ij}^{*}(s) = \lim_{s\to 0}\int_0^\infty e^{-st} q_{ij}(t)\,dt \tag{4.1}$$

The transition probabilities have been obtained as follows:

$q_{01} = p_2\, h(t)\, e^{-(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)t}$, $q_{02} = p_3\, h(t)\, e^{-(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)t}$

$q_{03} = p_1 h(t) e^{-(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)t}$, $q_{04} = \gamma_2\, e^{-(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)t}\, \bar{H}(t)$

$q_{05} = \lambda_1\, e^{-(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)t}\, \bar{H}(t)$, $q_{07} = \gamma_2\, e^{-(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)t}\, \bar{H}(t)$

$q_{08} = 2\beta_1\, e^{-(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)t}\, \bar{H}(t)$, $q_{09} = 2\beta_2\, e^{-(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)t}\, \bar{H}(t)$

$q_{10} = p_4\, h(t)\, e^{-(\beta_1+\beta_2)t}$, $q_{12} = p_5\, h(t)\, e^{-(\beta_1+\beta_2)t}$, $q_{13} = p_6\, h(t)\, e^{-(\beta_1+\beta_2)t}$

$q_{17} = p_{13}\, \beta_1\, e^{-(\beta_1+\beta_2)t}\, \bar{H}(t)$, $q_{18} = p_{14}\, \beta_2\, e^{-(\beta_1+\beta_2)t}\, \bar{H}(t)$

$q_{19} = p_{15}\, \beta_2\, e^{-(\beta_1+\beta_2)t}\, \bar{H}(t)$, $q_{1,10} = p_{16}\, \beta_1\, e^{-(\beta_1+\beta_2)t}\, \bar{H}(t)$

$q_{1,11} = p_{17}\, \beta_1\, e^{-(\beta_1+\beta_2)t}\, \bar{H}(t)$, $q_{1,12} = p_{18}\, \beta_2\, e^{-(\beta_1+\beta_2)t}\, \bar{H}(t)$

$q_{20} = p_7\, h(t)\, e^{-(\lambda_1+\lambda_2)t}$, $q_{21} = p_8\, h(t)\, e^{-(\lambda_1+\lambda_2)t}$, $q_{23} = p_9\, h(t)\, e^{-(\lambda_1+\lambda_2)t}$

$q_{25} = p_{19}\, \lambda_1\, e^{-(\lambda_1+\lambda_2)t}\, \bar{H}(t)$, $q_{26} = p_{20}\, \lambda_2\, e^{-(\lambda_1+\lambda_2)t}\, \bar{H}(t)$

$q_{2,13} = p_{21}\, \lambda_1\, e^{-(\lambda_1+\lambda_2)t}\, \bar{H}(t)$, $q_{2,14} = p_{22}\, \lambda_2\, e^{-(\lambda_1+\lambda_2)t}\, \bar{H}(t)$

$q_{32} = p_{12}\, h(t)\, e^{-(\lambda_1+\lambda_2)t}$, $q_{35} = p_{23}\, \lambda_1\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}\, \bar{H}(t)$

$q_{36} = p_{24}\, \lambda_2\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}\, \bar{H}(t)$, $q_{37} = p_{25}\, \beta_1\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}\, \bar{H}(t)$

$q_{38} = p_{26}\, \beta_2\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}\, \bar{H}(t)$, $q_{39} = p_{27}\, \beta_2\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}\, \bar{H}(t)$

$q_{3,10} = p_{28}\, \beta_1\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}\, \bar{H}(t)$, $q_{3,13} = p_{29}\, \lambda_1\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}\, \bar{H}(t)$

$q_{3,14} = p_{30}\, \lambda_2\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}\, \bar{H}(t)$, $q_{30} = p_{10}\, h(t)\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}$, $q_{40} = \gamma_1\, e^{-\gamma_1 t}$

$q_{31} = p_{11}\, h(t)\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}$, $q_{50} = q\, a_1\, e^{-(a_1+2\beta_1+2\beta_2)t}$, $q_{52} = p\, a_1\, e^{-(a_1+2\beta_1+2\beta_2)t}$

$q_{5,15} = 2\beta_1\, e^{-(a_1+2\beta_1+2\beta_2)t}$, $q_{5,16} = 2\beta_2\, e^{-(a_1+2\beta_1+2\beta_2)t}$, $q_{60} = q\, a_2\, e^{-(a_2+2\beta_1+2\beta_2)t}$

$q_{62} = p\, a_2\, e^{-(a_2+2\beta_1+2\beta_2)t}$, $q_{6,17} = 2\beta_1\, e^{-(a_2+2\beta_1+2\beta_2)t}$, $q_{6,18} = 2\beta_2\, e^{-(a_2+2\beta_1+2\beta_2)t}$

$q_{70} = q\, \mathrm{b}_1\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1)t}$, $q_{7,17} = \lambda_1\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1)t}$, $q_{7,17} = \lambda_2\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1)t}$

$q_{7,19} = \beta_1\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1)t}$, $q_{7,20} = \beta_2\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1)t}$, $q_{7,21} = q\, b_1\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1)t}$

$q_{80} = q\,b_2\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+b_2)t}$, $q_{83} = p\,b_2\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+b_2)t}$, $q_{8,16} = \lambda_1\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+b_2)t}$

$q_{8,18} = \lambda_2\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+b_2)t}$, $q_{8,20} = \beta_1\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+b_2)t}$, $q_{8,22} = \beta_2\, e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+b_2)t}$

$q_{92} = p\,b_2\, e^{-(\lambda_1+\lambda_2+b_2)t}$, $q_{93} = q\,b_2\, e^{-(\lambda_1+\lambda_2+b_2)t}$, $q_{9,16} = \lambda_1\, e^{-(\lambda_1+\lambda_2+b_2)t}$

$q_{9,18} = \lambda_2\, e^{-(\lambda_1+\lambda_2+b_2)t}$, $q_{10,3} = q\,b_1\, e^{-(\lambda_1+\lambda_2+b_1)t}$, $q_{10,15} = p\,b_1\, e^{-(\lambda_1+\lambda_2+b_1)t}$

$q_{10,17} = \lambda_2\, e^{-(\lambda_1+\lambda_2+b_1)t}$, $q_{10,23} = \lambda_1\, e^{-(\lambda_1+\lambda_2+b_1)t}$, $q_{11,19} = \beta_1\, e^{-(\beta_1+\beta_2+b_1)t}$

$q_{11,20} = \beta_2\, e^{-(\beta_1+\beta_2+b_1)t}$, $q_{11,2} = q\,b_1\, e^{-(\beta_1+\beta_2+b_1)t}$, $q_{11,24} = p\,b_1\, e^{-(\beta_1+\beta_2+b_1)t}$

$q_{12,1} = q\,b_2\, e^{-(\beta_1+\beta_2+b_2)t}$, $q_{12,2} = p\,b_2\, e^{-(\beta_1+\beta_2+b_2)t}$, $q_{12,20} = \beta_1\, e^{-(\beta_1+\beta_2+b_2)t}$

$q_{12,22} = \beta_2\, e^{-(\beta_1+\beta_2+b_2)t}$, $q_{13,1} = q\,a_2\, e^{-(\beta_1+\beta_2+a_2)t}$, $q_{13,3} = p\,a_2\, e^{-(\beta_1+\beta_2+a_2)t}$

$q_{13,17} = \beta_1\, e^{-(\beta_1+\beta_2+a_2)t}$, $q_{13,18} = \beta_2\, e^{-(\beta_1+\beta_2+a_2)t}$, $q_{14,3} = q\,a_1\, e^{-(\beta_1+\beta_2+a_1)t}$

$q_{14,15} = \beta_1\, e^{-(\beta_1+\beta_2+a_1)t}$, $q_{14,16} = \beta_2\, e^{-(\beta_1+\beta_2+a_1)t}$, $q_{14,25} = p\,a_1\, e^{-(\beta_1+\beta_2+a_1)t}$

$q_{15,5} = q\,b_1\, e^{-(\beta_1+\beta_2+b_1+a_1)t}$, $q_{15,7} = p\,b_1\, e^{-(\beta_1+\beta_2+b_1+a_1)t}$, $q_{15,26} = p\,a_1\, e^{-(\beta_1+\beta_2+b_1+a_1)t}$

$q_{15,27} = q\,a_1\, e^{-(\beta_1+\beta_2+b_1+a_1)t}$, $q_{15,28} = \beta_1\, e^{-(\beta_1+\beta_2+b_1+a_1)t}$, $q_{15,29} = \beta_2\, e^{-(\beta_1+\beta_2+b_1+a_1)t}$

$q_{16,5} = q\,b_2\, e^{-(\beta_1+\beta_2+b_2+a_1)t}$, $q_{16,8} = q\,a_1\, e^{-(\beta_1+\beta_2+b_2+a_1)t}$, $q_{16,14} = p\,b_2\, e^{-(\beta_1+\beta_2+b_2+a_1)t}$

$q_{16,29} = \beta_1\, e^{-(\beta_1+\beta_2+b_2+a_1)t}$, $q_{16,30} = p\,a_1\, e^{-(\beta_1+\beta_2+b_2+a_1)t}$, $q_{16,31} = \beta_2\, e^{-(\beta_1+\beta_2+b_2+a_1)t}$

$q_{17,6} = q\,b_1\, e^{-(\beta_1+\beta_2+b_1+a_2)t}$, $q_{17,7} = q\,a_2\, e^{-(\beta_1+\beta_2+b_1+a_2)t}$, $q_{17,11} = p\,a_2\, e^{-(\beta_1+\beta_2+b_1+a_2)t}$

$q_{17,32} = p\,b_1\, e^{-(\beta_1+\beta_2+b_1+a_2)t}$, $q_{17,33} = \beta_1\, e^{-(\beta_1+\beta_2+b_1+a_2)t}$, $q_{17,34} = \beta_2\, e^{-(\beta_1+\beta_2+b_1+a_2)t}$

$q_{18,6} = q\,b_2\, e^{-(\beta_1+\beta_2+b_2+a_2)t}$, $q_{18,8} = q\,a_2\, e^{-(\beta_1+\beta_2+b_2+a_2)t}$, $q_{18,12} = p\,a_2\, e^{-(\beta_1+\beta_2+b_2+a_2)t}$

$q_{18,13} = p\,b_2\, e^{-(\beta_1+\beta_2+b_2+a_2)t}$, $q_{18,34} = \beta_1\, e^{-(\beta_1+\beta_2+b_2+a_2)t}$, $q_{18,35} = \beta_2\, e^{-(\beta_1+\beta_2+b_2+a_2)t}$

$q_{19,7} = q\,b_1\, e^{-(\lambda_1+\lambda_2+b_1)t}$, $q_{19,36} = p\,b_1\, e^{-(\lambda_1+\lambda_2+b_1)t}$, $q_{19,28} = \lambda_1\, e^{-(\lambda_1+\lambda_2+b_1)t}$

$q_{19,33} = \lambda_2\, e^{-(\lambda_1+\lambda_2+b_1)t}$, $q_{20,7} = q\,b_2\, e^{-(\lambda_1+\lambda_2+b_2+b_1)t}$, $q_{20,8} = q\,b_1\, e^{-(\lambda_1+\lambda_2+b_2+b_1)t}$

$q_{20,10} = p\,b_2\, e^{-(\lambda_1+\lambda_2+b_2+b_1)t}$, $q_{20,29} = \lambda_1\, e^{-(\lambda_1+\lambda_2+b_2+b_1)t}$, $q_{20,34} = \lambda_2\, e^{-(\lambda_1+\lambda_2+b_2+b_1)t}$

$q_{20,37} = p\,b_1\, e^{-(\lambda_1+\lambda_2+b_2+b_1)t}$, $q_{21,3} = m(t)$, $q_{22,8} = q\,b_2\, e^{-(\lambda_1+\lambda_2+b_2)t}$, $q_{22,9} = p\,b_2\, e^{-(\lambda_1+\lambda_2+b_2)t}$

$q_{22,31} = \lambda_1\, e^{-(\lambda_1+\lambda_2+b_2)t}$, $q_{22,35} = \lambda_2\, e^{-(\lambda_1+\lambda_2+b_2)t}$, $q_{23,2} = m(t)$, $q_{24,1} = m(t)$, $q_{25,1} = i(t)$

$q_{26,11} = i(t)$, $q_{27,14} = m(t)$, $q_{28,15} = b_1\, e^{-(a_1+b_1)t}$, $q_{28,19} = a_1\, e^{-(a_1+b_1)t}$, $q_{29,15} = b_1\, e^{-(a_1+b_2+b_1)t}$

$q_{29,16} = b_2 e^{-(a_1+b_2+b_1)t}$, $q_{29,20} = a_1 e^{-(a_1+b_2+b_1)t}$, $q_{30,12} = i(t)$

$q_{31,16} = b_2 e^{-(a_1+b_2)t}$;, $q_{31,22} = a_1 e^{-(a_1+b_2)t}$, $q_{32,13} = m\ (t)$

$q_{33,17} = b_1 e^{-(a_2+b_2+b_1)t}$, $q_{33,18} = b_2 e^{-(a_2+b_2+b_1)t}$, $q_{33,20} = a_2 e^{-(a_2+b_2+b_1)t}$

$q_{34,17} = b_1 e^{-(a_2+b_1)t}$, $q_{34,19} = a_2 e^{-(a_2+b_1)t}$, $q_{35,18} = b_2 e^{-(a_2+b_2)t}$

$q_{35,22} = a_2 e^{-(a_2+b_2)t}$, $q_{36,10} = m\ (t)$, $q_{37,9} = m\ (t)$

(4.2)

The non zero elements p_{ij} have been obtained as:

$p_{01} = p_2 h^*(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)$, $p_{02} = p_3 h^*(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)$

$p_{03} = p_1 h^*(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)$, $p_{04} = \gamma_2\ h^*(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)$

$p_{05} = \lambda_1\left(1-h^*(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)\right)$, $p_{06} = \lambda_2\left(1-h^*(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)\right)$

$p_{07} = 2\beta_1\left(1-h^*(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)\right)$, $p_{08} = 2\beta_2\left(1-h^*(\lambda_1+\lambda_2+2\beta_1+2\beta_2+\gamma_2)\right)$

$p_{10} = p_4 h^*(\beta_1+\beta_2+\gamma_2)$, $p_{12} = p_5 h^*(\beta_1+\beta_2+\gamma_2)$, $p_{13} = p_6 h^*(\beta_1+\beta_2+\gamma_2)$

$p_{17} = p_{13}\beta_1\left(1-h^*(\beta_1+\beta_2)\right)$, $p_{18} = p_{14}\beta_2\left(1-h^*(\beta_1+\beta_2)\right)$, $p_{19} = p_{15}\beta_2\left(1-h^*(\beta_1+\beta_2)\right)$

$p_{1,10} = p_{16}\beta_1\left(1-h^*(\beta_1+\beta_2)\right)$, $p_{1,11} = p_{17}\beta_1\left(1-h^*(\beta_1+\beta_2)\right)$, $p_{1,12} = p_{18}\beta_2\left(1-h^*(\beta_1+\beta_2)\right)$

$p_{20} = p_7 h^*(\lambda_1+\lambda_2)$, $p_{21} = p_8 h^*(\lambda_1+\lambda_2)$, $p_{23} = p_9 h^*(\lambda_1+\lambda_2)$, $p_{25} = p_{19}\lambda_1\left(1-h^*(\lambda_1+\lambda_2)\right)$

$p_{26} = p_{20}\,\lambda_2\left(1-h^*(\lambda_1+\lambda_2)\right)$, $p_{2,13} = p_{21}\,\lambda_1\left(1-h^*(\lambda_1+\lambda_2)\right)$

$p_{2,14} = p_{22}\,\lambda_2\left(1-h^*(\lambda_1+\lambda_2)\right)$, $p_{30} = p_{10}\,h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)$

$p_{31} = p_{11}\,h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)$, $p_{32} = p_{12}\,h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)$

$p_{35} = p_{23}\,\lambda_1\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)\right)$, $p_{36} = p_{24}\,\lambda_2\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)\right)$

$p_{37} = p_{25}\,\beta_1\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)\right)$, $p_{38} = p_{26}\,\beta_2\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)\right)$

$p_{39} = p_{27}\,\beta_2\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)\right)$, $p_{3,10} = p_{28}\,\beta_1\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)\right)$

$p_{3,13} = p_{29}\,\lambda_1\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)\right)$, $p_{3,14} = p_{30}\,\lambda_2\left(1-h^*(\lambda_1+\lambda_2+\beta_1+\beta_2)\right)$

$p_{40} = 1$, $p_{50} = \dfrac{q a_1}{a_1+2\beta_1+2\beta_2}$, $p_{52} = \dfrac{p a_1}{a_1+2\beta_1+2\beta_2}$, $p_{5,15} = \dfrac{2\beta_1}{a_1+2\beta_1+2\beta_2}$

$p_{5,16} = \dfrac{2\beta_2}{a_1+2\beta_1+2\beta_2}$, $p_{60} = \dfrac{q a_2}{a_2+2\beta_1+2\beta_2}$, $p_{62} = \dfrac{p a_2}{a_2+2\beta_1+2\beta_2}$

$p_{6,17} = \dfrac{2\beta_1}{a_2+2\beta_1+2\beta_2}$, $p_{6,18} = \dfrac{2\beta_2}{a_2+2\beta_1+2\beta_2}$, $p_{70} = \dfrac{q\,\mathrm{b}_1}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1}$

$p_{7,17} = \dfrac{\lambda_1}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1}$, $p_{7,17} = \dfrac{\lambda_2}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1}$, $p_{7,19} = \dfrac{\beta_1}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1}$

$p_{7,20} = \dfrac{\beta_2}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1}$, $p_{7,21} = \dfrac{q b_1}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_1}$, $p_{80} = \dfrac{q\,\mathrm{b}_2}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_2}$

$p_{8,16} = \dfrac{\lambda_1}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_2}$, $p_{83} = \dfrac{p b_2}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_2}$, $p_{8,18} = \dfrac{\lambda_2}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_2}$

$p_{8,20} = \dfrac{\beta_1}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_2}$, $p_{8,22} = \dfrac{\beta_2}{\lambda_1+\lambda_2+\beta_1+\beta_2+\mathrm{b}_2}$, $q_{92} = \dfrac{p\,\mathrm{b}_2}{\lambda_1+\lambda_2+\mathrm{b}_2}$

$q_{93} = \dfrac{q\,\mathrm{b}_2}{\lambda_1+\lambda_2+\mathrm{b}_2}$, $p_{9,16} = \dfrac{\lambda_1}{\lambda_1+\lambda_2+\mathrm{b}_2}$, $p_{9,18} = \dfrac{\lambda_2}{\lambda_1+\lambda_2+\mathrm{b}_2}$, $p_{10,3} = \dfrac{q\,\mathrm{b}_1}{\lambda_1+\lambda_2+\mathrm{b}_1}$

$p_{10,15} = \dfrac{p\,\mathrm{b}_1}{\lambda_1+\lambda_2+\mathrm{b}_1}$, $p_{10,17} = \dfrac{\lambda_2}{\lambda_1+\lambda_2+\mathrm{b}_1}$, $p_{10,23} = \dfrac{\lambda_1}{\lambda_1+\lambda_2+\mathrm{b}_1}$, $p_{11,19} = \dfrac{\beta_1}{\beta_1+\beta_2+\mathrm{b}_1}$

$p_{11,20} = \dfrac{\beta_2}{\beta_1+\beta_2+\mathrm{b}_1}$, $p_{11,2} = \dfrac{q\,\mathrm{b}_1}{\beta_1+\beta_2+\mathrm{b}_1}$, $p_{11,24} = \dfrac{p\,\mathrm{b}_1}{\beta_1+\beta_2+\mathrm{b}_1}$, $p_{12,1} = \dfrac{q\,\mathrm{b}_2}{\beta_1+\beta_2+\mathrm{b}_2}$

$p_{12,2} = \dfrac{p\,\mathrm{b}_2}{\beta_1+\beta_2+\mathrm{b}_2}$, $p_{12,20} = \dfrac{\beta_1}{\beta_1+\beta_2+\mathrm{b}_2}$, $p_{12,22} = \dfrac{\beta_2}{\beta_1+\beta_2+\mathrm{b}_2}$, $p_{13,1} = \dfrac{q a_2}{\beta_1+\beta_2+a_2}$

$p_{13,3} = \dfrac{p a_2}{\beta_1+\beta_2+a_2}$, $p_{13,17} = \dfrac{\beta_1}{\beta_1+\beta_2+a_2}$, $p_{13,18} = \dfrac{\beta_2}{\beta_1+\beta_2+a_2}$, $p_{14,3} = \dfrac{q a_1}{\beta_1+\beta_2+a_1}$

$p_{14,15} = \dfrac{\beta_1}{\beta_1+\beta_2+a_1}$, $p_{14,16} = \dfrac{\beta_2}{\beta_1+\beta_2+a_1}$, $p_{14,25} = \dfrac{p a_1}{\beta_1+\beta_2+a_1}$, $p_{15,5} = \dfrac{q b_1}{\beta_1+\beta_2+b_1+a_1}$

$p_{15,7} = \dfrac{p b_1}{\beta_1+\beta_2+b_1+a_1}$, $p_{15,26} = \dfrac{p a_1}{\beta_1+\beta_2+b_1+a_1}$, $p_{15,27} = \dfrac{q a_1}{\beta_1+\beta_2+b_1+a_1}$

$p_{15,28} = \dfrac{\beta_1}{\beta_1+\beta_2+b_1+a_1}$, $p_{15,29} = \dfrac{\beta_2}{\beta_1+\beta_2+b_1+a_1}$, $p_{16,5} = \dfrac{q b_2}{\beta_1+\beta_2+b_2+a_1}$

$p_{16,8} = \dfrac{q a_1}{\beta_1+\beta_2+b_2+a_1}$, $p_{16,14} = \dfrac{p b_2}{\beta_1+\beta_2+b_2+a_1}$, $p_{16,29} = \dfrac{\beta_1}{\beta_1+\beta_2+b_2+a_1}$

$p_{16,30} = \dfrac{q a_1}{\beta_1+\beta_2+b_2+a_1}$, $p_{16,31} = \dfrac{\beta_2}{\beta_1+\beta_2+b_2+a_1}$, $p_{17,6} = \dfrac{q b_1}{\beta_1+\beta_2+b_1+a_2}$

$p_{17,7} = \dfrac{q a_2}{\beta_1+\beta_2+b_1+a_2}$, $p_{17,11} = \dfrac{p a_2}{\beta_1+\beta_2+b_1+a_2}$, $p_{17,32} = \dfrac{p b_1}{\beta_1+\beta_2+b_1+a_2}$

$p_{17,33} = \dfrac{\beta_1}{\beta_1+\beta_2+b_1+a_2}$, $p_{17,34} = \dfrac{\beta_2}{\beta_1+\beta_2+b_1+a_2}$, $p_{18,6} = \dfrac{q b_2}{\beta_1+\beta_2+b_2+a_2}$

$p_{18,8} = \dfrac{q a_2}{\beta_1+\beta_2+b_2+a_2}$, $p_{18,12} = \dfrac{p a_2}{\beta_1+\beta_2+b_2+a_2}$, $p_{18,13} = \dfrac{p b_2}{\beta_1+\beta_2+b_2+a_2}$

$p_{18,34} = \dfrac{\beta_1}{\beta_1+\beta_2+b_2+a_2}$, $p_{18,35} = \dfrac{\beta_2}{\beta_1+\beta_2+b_2+a_2}$, $p_{19,7} = \dfrac{q b_1}{\lambda_1+\lambda_2+b_1}$

$p_{19,36} = \dfrac{p b_1}{\lambda_1+\lambda_2+b_1}$, $p_{19,28} = \dfrac{\lambda_1}{\lambda_1+\lambda_2+b_1}$, $p_{19,33} = \dfrac{\lambda_2}{\lambda_1+\lambda_2+b_1}$

$p_{20,7} = \dfrac{q b_2}{\lambda_1+\lambda_2+b_2+b_1}$, $p_{20,8} = \dfrac{q b_1}{\lambda_1+\lambda_2+b_2+b_1}$, $p_{20,10} = \dfrac{p b_2}{\lambda_1+\lambda_2+b_2+b_1}$

$p_{20,29} = \dfrac{\lambda_1}{\lambda_1+\lambda_2+b_2+b_1}$, $p_{20,34} = \dfrac{\lambda_2}{\lambda_1+\lambda_2+b_2+b_1}$, $p_{20,37} = \dfrac{p b_1}{\lambda_1+\lambda_2+b_2+b_1}$

$p_{21,3} = 1$, $p_{22,8} = \dfrac{q b_2}{\lambda_1+\lambda_2+b_2}$, $p_{22,9} = \dfrac{p b_2}{\lambda_1+\lambda_2+b_2}$, $p_{22,31} = \dfrac{\lambda_1}{\lambda_1+\lambda_2+b_2}$

$p_{22,35} = \dfrac{\lambda_2}{\lambda_1+\lambda_2+b_2}$, $p_{23,2} = 1$, $p_{24,1} = 1$, $p_{25,1} = 1$, $p_{26,11} = 1$, $p_{27,14} = 1$

$p_{28,15} = \dfrac{b_1}{a_1+b_1}$, $p_{28,19} = \dfrac{a_1}{a_1+b_1}$, $p_{29,15} = \dfrac{b_1}{a_1+b_2+b_1}$, $p_{29,16} = \dfrac{b_2}{a_1+b_2+b_1}$

$p_{29,20} = \dfrac{a_1}{a_1+b_2+b_1}$, $p_{30,12} = 1$, $p_{31,16} = \dfrac{b_2}{a_1+b_2}$, $p_{31,22} = \dfrac{a_1}{a_1+b_2}$, $p_{32,13} = 1$

$p_{33,17} = \dfrac{b_1}{a_2+b_2+b_1}$, $q_{33,18} = \dfrac{b_2}{a_2+b_2+b_1}$, $q_{33,20} = \dfrac{a_2}{a_2+b_2+b_1}$, $p_{34,17} = \dfrac{b_1}{a_2+b_1}$

$$p_{34,19} = \frac{a_2}{a_2+b_1} \ , \ p_{35,18} = \frac{b_2}{a_2+b_2} \ , \ p_{35,22} = \frac{a_2}{a_2+b_2} \ , \ p_{36,10} = 1 \ , \ p_{37,9} = 1 \quad (4.3)$$

The mean sojourn time μ_i in the ith regenerative state is given as:

$$\mu_i = E(t) = P_r(T > t) = \int_0^\infty d\,(Q_{ij}(t)) \quad (4.4)$$

$$\mu_0 = \left(1 - h^*\left(\gamma_2 + \lambda_1 + \lambda_2 + 2\beta_1 + 2\beta_2\right)\right) \ , \ \mu_1 = \left(1 - h^*\left(\beta_1 + \beta_2\right)\right)$$

$$\mu_2 = 1 - \left(h^*\left(\lambda_1 + \lambda_2\right)\right) \ , \ \mu_4 = \frac{1}{\gamma_2} \ , \ \mu_5 = \left(1 - h^*\left(a_1 + \lambda_1 + \lambda_2 + \beta_1 + \beta_2\right)\right)$$

$$\mu_6 = \left(1 - h^*\left(a_2 + 2\beta_1 + 2\beta_2\right)\right) \ , \ \mu_7 = \left(1 - h^*\left(b_1 + \lambda_1 + \lambda_2 + \beta_1 + \beta_2\right)\right)$$

$$\mu_8 = \left(1 - h^*\left(b_2 + \lambda_1 + \lambda_2 + \beta_1 + \beta_2\right)\right) \ , \ \mu_9 = 1 - \left(h^*\left(b_2 + \lambda_1 + \lambda_2\right)\right)$$

$$\mu_{10} = 1 - \left(h^*\left(b_1 + \lambda_1 + \lambda_2\right)\right), \ \mu_{11} = \left(1 - h^*\left(b_1 + \beta_1 + \beta_2\right)\right)$$

$$\mu_{12} = \left(1 - h^*\left(b_2 + \beta_1 + \beta_2\right)\right), \ \mu_{13} = \left(1 - h^*\left(a_2 + \beta_1 + \beta_2\right)\right)$$

$$\mu_{14} = \left(1 - h^*\left(a_1 + \beta_1 + \beta_2\right)\right), \ \mu_{15} = \left(1 - h^*\left(a_1 + b_1 + \beta_1 + \beta_2\right)\right)$$

$$\mu_{16} = \left(1 - h^*\left(a_1 + b_2 + \beta_1 + \beta_2\right)\right), \ \mu_{17} = \left(1 - h^*\left(a_2 + b_1 + \beta_1 + \beta_2\right)\right)$$

$$\mu_{18} = \left(1 - h^*\left(a_2 + b_2 + \beta_1 + \beta_2\right)\right), \ \mu_{19} = 1 - \left(h^*\left(b_1 + \lambda_1 + \lambda_2\right)\right)$$

$$\mu_{20} = 1 - \left(h^*\left(b_1 + b_2 + \lambda_1 + \lambda_2\right)\right), \ \mu_{21} = -m^*(0) \ , \ \mu_{22} = 1 - \left(h^*\left(b_2 + \lambda_1 + \lambda_2\right)\right)$$

$$\mu_{23} = -m^*(0) \ , \ \mu_{24} = -m^*(0) \ , \ \mu_{25} = -i^*(0) \ , \ \mu_{26} = -i^*(0) \ , \ \mu_{27} = -m^*(0)$$

$$\mu_{28} = \frac{1}{a_1 + b_1} \ , \ \mu_{29} = \frac{1}{a_1 + b_1 + b_2} \ , \ \mu_{30} = -i^*(0) \ , \ \mu_{31} = \frac{1}{a_1 + b_2}$$

$$\mu_{32} = -i^*(0) \ , \ \mu_{33} = \frac{1}{a_1 + b_1 + b_2} \ , \ \mu_{34} = \frac{1}{a_2 + b_1} \ , \ \mu_{35} = \frac{1}{a_2 + b_2}$$

$$\mu_{36} = -m^*(0) \ , \ \mu_{37} = -m^*(0) \quad (4.5)$$

The unconditional mean time taken by the system is mathematically stated as:

$$m_{ij} = \int_0^\infty t\, dQ_{ij}(t) = \int_0^\infty t\ q_{ij}(t)\, dt = -q_{ij}^{\ *}(0) \quad (4.6)$$

Thus, we get

$$\left.\begin{array}{l}
m_{01}+m_{02}+m_{03}+m_{04}+m_{05}+m_{06}+m_{07}+m_{08} = \mu_0 \\
m_{10}+m_{12}+m_{13}+m_{17}+m_{18}+m_{19}+m_{1,10}+ m_{1,11}+m_{1,12} = \mu_1 \\
m_{20}+m_{21}+m_{23}+m_{25}+m_{26}+m_{2,13}+m_{2,14} = \mu_2 \\
m_{30}+m_{31}+m_{32}+m_{35}+m_{36}+m_{37}+m_{38}+m_{39}+m_{3,10} +m_{3,13}+m_{3,14} = \mu_3 \\
m_{40} = \mu_4 \ , m_{50}+m_{52}+m_{5,15}+m_{5,16} = \mu_5 \ , m_{60}+m_{62}+m_{6,17}+m_{6,18} = \mu_6 \\
m_{70}+m_{7,15}+m_{7,17}+m_{7,19}+m_{7,20}+m_{7,21} = \mu_7 \\
m_{80}+m_{83}+m_{8,16}+m_{8,18}+m_{8,20}+m_{8,22} = \mu_8 \\
m_{92}+m_{93}+m_{9,16}+m_{9,18} = \mu_9 \ , \ m_{10,3}+m_{10,15}+m_{10,17}+m_{10,23}= \mu_{10} \\
m_{11,2}+m_{11,19}+m_{11,20}+m_{11,24}= \mu_{11} \ , m_{12,1}+m_{12,2}+m_{12,20}+m_{12,22}= \mu_{12} \\
m_{13,1}+m_{13,3}+m_{13,17}+m_{13,18}= \mu_{13} \ , \ m_{14,3}+m_{14,15}+m_{14,16}+m_{14,25}= \mu_{14} \\
m_{15,5}+m_{15,7}+m_{15,26}+m_{15,27}+m_{15,28}+m_{15,29} = \mu_{15} \\
m_{16,5}+m_{16,8}+m_{16,14}+m_{16,29}+m_{16,30}+m_{16,31} = \mu_{16} \\
m_{17,6}+m_{17,7}+m_{17,11}+m_{17,32}+m_{17,33}+m_{17,34} = \mu_{17} \\
m_{18,6}+m_{18,8}+m_{18,12}+m_{18,13}+m_{18,34}+m_{18,35} = \mu_{18} \\
m_{19,7}+m_{19,28}+m_{19,33}+m_{19,36} = \mu_{19} \ , \ m_{23,2}= \mu_{23} \ , \ m_{24,1}= \mu_{24} \\
m_{20,7}+m_{20,8}+m_{20,10}+m_{20,29}+m_{20,34}+m_{20,37} = \mu_{20} \ , m_{21,3}= \mu_{21} \\
m_{22,8}+m_{22,9}+m_{22,31}+m_{22,35} = \mu_{22} \ , \ m_{25,1}= \mu_{25} \ , \ m_{26,11}= \mu_{26} \\
m_{27,14}= \mu_{27} \ , \ m_{30,12}= \mu_{30} \ , \ m_{36,10}= \mu_{36} \ , \ m_{37,9}= \mu_{37}
\end{array}\right\} \quad (4.7)$$

4.4 Mean Time to System Failure (MTSF)

In order to determine the mean time to system failure (MTSF) of the system, the failed states are considered as absorbing states, The following recursive relation for $\phi_i\left(t\right)$ are obtained as:

$$\begin{aligned}
\phi_0(t)=&Q_{01}(t)(s)\phi_1(t)+Q_{02}(t)(s)\phi_2(t)+Q_{03}(t)(s)\phi_3(t)+Q_{04}(t)(s)\phi_4(t)+Q_{05}(t)(s)\phi_5(t) \\
&+Q_{06}(t)(s)\phi_6(t)+Q_{07}(t)(s)\phi_7(t)+Q_{08}(t)(s)\phi_8(t) \\
\phi_1(t)=&Q_{10}(t)(s)\phi_0(t)+Q_{12}(t)(s)\phi_2(t)+Q_{13}(t)(s)\phi_3(t)+Q_{17}(t)(s)\phi_7(t)+Q_{18}(t)(s)\phi_8(t) \\
&+Q_{19}(t)(s)\phi_9(t)+Q_{1,11}(t)(s)\phi_{11}(t)+Q_{1,12}(t)(s)\phi_{12}(t) \\
\phi_2(t)=&Q_{20}(t)(s)\phi_0(t)+Q_{21}(t)(s)\phi_1(t)+Q_{23}(t)(s)\phi_3(t)+Q_{25}(t)(s)\phi_5(t)+Q_{2,13}(t)(s)\phi_{13}(t) \\
&+Q_{2,14}(t)(s)\phi_{14}(t)
\end{aligned}$$

$$\phi_3(t)=Q_{30}(t)(s)\phi_0(t) + Q_{31}(t)(s)\phi_1(t) + Q_{32}(t)(s)\phi_2(t) + Q_{35}(t)(s)\phi_5(t) + Q_{36}(t)(s)\phi_7(t) + Q_{38}(t)(s)\phi_8(t) + Q_{39}(t)(s)\phi_9(t) + Q_{3,10}(t)(s)\phi_{10}(t) + Q_{3,13}(t)(s)\phi_{3,13}(t) + Q_{3,14}(t)(s)\phi_{3,14}(t)$$

$$\phi_4(t)=Q_{40}(t)(s)\phi_0(t)$$

$$\phi_5(t)=Q_{50}(t)(s)\phi_0(t)+Q_{52}(t)(s)\phi_2(t)+Q_{5,15}(t)(s)\phi_{15}(t)+Q_{5,16}(t)(s)\phi_{16}(t)$$

$$\phi_6(t)=Q_{60}(t)(s)\phi_0(t)+Q_{62}(t)(s)\phi_2(t)+Q_{6,17}(t)(s)\phi_{17}(t)+Q_{6,18}(t)(s)\phi_{18}(t)$$

$$\phi_7(t)=Q_{70}(t)(s)\phi_0(t)+Q_{7,15}(t)(s)\phi_{15}(t)+Q_{7,19}(t)(s)\phi_{19}(t)+Q_{7,20}(t)(s)\phi_{20}(t)+Q_{7,21}(t)(s)\phi_{21}(t)$$

$$\phi_8(t)=Q_{80}(t)(s)\phi_0(t)+Q_{83}(t)(s)\phi_3(t)+Q_{8,16}(t)(s)\phi_{16}(t)+Q_{8,18}(t)(s)\phi_{18}(t)+Q_{8,20}(t)(s)\phi_{20}(t)+Q_{8,22}(t)(s)\phi_{22}(t)$$

$$\phi_9(t)=Q_{92}(t)(s)\phi_2(t)+Q_{93}(t)(s)\phi_3(t)+Q_{9,16}(t)(s)\phi_{16}(t)+Q_{9.18}(t)(s)\phi_{18}(t)$$

$$\phi_{10}(t)=Q_{10,3}(t)(s)\phi_3(t)+Q_{10,15}(t)(s)\phi_{15}(t)+Q_{10,17}(t)(s)\phi_{17}(t)+Q_{10,18}(t)(s)\phi_{18}(t)$$

$$\phi_{11}(t)=Q_{11,2}(t)(s)\phi_2(t)+Q_{11,19}(t)(s)\phi_{19}(t)+Q_{11,20}(t)(s)\phi_{20}(t)+Q_{11,24}(t)(s)\phi_{24}(t)$$

$$\phi_{12}(t)=Q_{12,1}(t)(s)\phi_1(t)+Q_{12,2}(t)(s)\phi_2(t)+Q_{12,20}(t)(s)\phi_{20}(t)+Q_{12,22}(t)(s)\phi_{22}(t)$$

$$\phi_{13}(t)=Q_{13,1}(t)(s)\phi_1(t)+Q_{13,3}(t)(s)\phi_3(t)+Q_{13,17}(t)(s)\phi_{17}(t)+Q_{13,18}(t)(s)\phi_{18}(t)$$

$$\phi_{14}(t)=Q_{14,3}(t)(s)\phi_3(t)+Q_{14,15}(t)(s)\phi_{15}(t)+Q_{14,16}(t)(s)\phi_{16}(t)+Q_{14,25}(t)(s)\phi_{25}(t)$$

$$\phi_{15}(t)=Q_{15,5}(t)(s)\phi_5(t)+Q_{15,7}(t)(s)\phi_7(t)+Q_{15,26}(t)(s)\phi_{26}(t)+Q_{15,27}(t)(s)\phi_{27}(t)+Q_{15,28}(t)+Q_{15,29}(t)$$

$$\phi_{16}(t)=Q_{16,5}(t)(s)\phi_5(t)+Q_{16,8}(t)(s)\phi_8(t)+Q_{16,14}(t)(s)\phi_{14}(t)+Q_{16,30}(t)(s)\phi_{30}(t)+Q_{16,31}(t)+Q_{16,29}(t)$$

$$\phi_{17}(t)=Q_{17,6}(t)(s)\phi_6(t)+Q_{17,7}(t)(s)\phi_7(t)+Q_{17,11}(t)(s)\phi_{11}(t)+Q_{17,32}(t)(s)\phi_{32}(t)+Q_{17,33}(t)+Q_{17,34}(t)$$

$$\phi_{18}(t)=Q_{18,6}(t)(s)\phi_6(t)+Q_{18,6}(t)(s)\phi_6(t)+Q_{18,13}(t)(s)\phi_{13}(t)+Q_{18,12}(t)(s)\phi_{12}(t)+Q_{18,34}(t)+Q_{18,35}(t)$$

$$\phi_{19}(t)=Q_{19,7}(t)(s)\phi_7(t)+Q_{19,36}(t)(s)\phi_{36}(t)+Q_{19.28}(t)+Q_{19.34}(t)$$

$$\phi_{20}(t)=Q_{20,7}(t)(s)\phi_7(t)+Q_{20,8}(t)(s)\phi_8(t)+Q_{20,10}(t)(s)\phi_{10}(t)+Q_{20,37}(t)(s)\phi_{37}(t)+Q_{20,29}(t)+Q_{20,34}(t)$$

$$\phi_{21}(t)=Q_{21,3}(t)(s)\phi_3(t)$$

$$\phi_{22}(t)=Q_{22,8}(t)(s)\phi_8(t)+Q_{22,9}(t)(s)\phi_9(t)+Q_{22,31}(t)+Q_{22,35}(t)$$

$$\phi_{23}(t)=Q_{23,1}(t)(s)\phi_2(t)$$

$$\phi_{24}(t)=Q_{24,1}(t)(s)\phi_1(t)$$

$$\phi_{25}(t)=Q_{25,1}(t)(s)\phi_1(t)$$

$$
\begin{aligned}
\phi_{26}(t) &= Q_{26,11}(t)(s)\phi_{11}(t) \\
\phi_{27}(t) &= Q_{27,14}(t)(s)\phi_{14}(t) \\
\phi_{30}(t) &= Q_{30,12}(t)(s)\phi_{12}(t) \\
\phi_{32}(t) &= Q_{32,13}(t)(s)\phi_{13}(t) \\
\phi_{30}(t) &= Q_{30,12}(t)(s)\phi_{12}(t) \\
\phi_{36}(t) &= Q_{36,10}(t)(s)\phi_{10}(t) \\
\phi_{32}(t) &= Q_{32,13}(t)(s)\phi_{13}(t) \\
\phi_{37}(t) &= Q_{37,9}(t)(s)\phi_{9}(t)
\end{aligned}
\tag{4.8}
$$

In order to obtain MTSF, we first take Laplace Steltjes Transforms of set of equations (4.8) and then solve them for $\phi_0^{**}(s)$. Thus, we get

$$MTSF \;=\; \lim_{s\to 0}\frac{1-\varphi_0{}^{**}(s)}{s} = \frac{N}{D} = \frac{D'(0)-N'(0)}{D'(0)} \tag{4.9}$$

where

$$D'(0) = \text{derivative}(D_1)$$

$$N'(0) = \text{derivative}(N_1)\ ,$$

where

$$D_1 = \begin{vmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & 0 & 0 & 0 & 0 & a_{1,11} & a_{1,12} & a_{1,13} & a_{1,14} \\
a_{21} & 1 & a_{21} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & 0 & 0 & 0 & 0 \\
a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & 0 & 0 & 0 & a_{3,11} & a_{3,12} & a_{3,13} & a_{3,14} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} & 0 & 0 & a_{4,11} & a_{4,12} & a_{4,13} & a_{4,14} \\
a_{51} & 0 & 0 & a_{54} & a_{55} & 0 & 0 & a_{58} & 0 & 0 & a_{5,11} & 0 & a_{5,13} & 0 \\
a_{61} & 0 & 0 & a_{64} & 0 & a_{66} & a_{67} & a_{68} & 0 & 0 & 0 & a_{6,12} & 0 & a_{6,14} \\
0 & 0 & a_{73} & a_{74} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & a_{7,12} & 0 & a_{7,14} \\
0 & 0 & a_{82} & a_{83} & 0 & 0 & 0 & 1 & 0 & 0 & a_{8,11} & 0 & a_{8,13} & 0 \\
0 & a_{92} & a_{93} & 0 & a_{95} & a_{96} & a_{97} & a_{98} & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{10,2} & a_{10,3} & 0 & a_{10,5} & a_{10,6} & a_{10,7} & a_{10,8} & 0 & 1 & 0 & 0 & 0 & 0 \\
a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & 0 & 0 & 0 & a_{11,8} & a_{11,9} & 0 & a_{11,11} & a_{11,12} & 0 & 0 \\
a_{12,1} & a_{12,2} & a_{12,3} & a_{12,4} & 0 & 0 & 0 & 0 & 0 & a_{12,10} & 0 & a_{12,12} & 0 & 0 \\
a_{13,1} & a_{13,2} & a_{13,3} & a_{13,4} & a_{13,5} & 0 & 0 & a_{13,8} & 0 & 0 & 0 & 0 & a_{13,13} & a_{13,14} \\
a_{14,1} & a_{14,2} & a_{14,3} & a_{14,4} & 0 & a_{14,6} & 0 & 0 & a_{14,9} & 0 & 0 & 0 & a_{14,13} & a_{14,14}
\end{vmatrix}$$

$a_{11}=1-q_{04}q_{40}-q_{05}q_{50}-q_{06}q_{60}$, $a_{12}=-q_{01}$, $a_{13}=-q_{02}-q_{05}q_{52}-q_{06}q_{62}$

$a_{14}=-q_{03}$, $a_{15}=-q_{07}$; $a_{16}=-q_{08}$, $a_{1,11}=-q_{05}q_{5,15}$, $a_{1,12}=-q_{05}q_{5,16}$

$a_{1,13}=-q_{06}q_{6,17}$, $a_{1,14}=-q_{06}q_{6,18}$, $a_{21}=-q_{10}$, $a_{23}=-q_{12}$, $a_{24}=-q_{13}$

$a_{25}=-q_{17}$; $a_{26}=-q_{18}$, $a_{27}=-q_{19}$, $a_{28}=-p_{1,10}$, $a_{29}=-q_{1,11}$, $a_{2,10}=-q_{1,12}$

$a_{31}=-q_{20}-q_{26}q_{60}-q_{25}q_{50}$, $a_{32}=-q_{21}-q_{2,13}q_{13,1}-q_{2,14}q_{14,25}q_{25,1}$

$a_{33}=1-q_{25}q_{52}-q_{26}q_{62}$, $a_{34}=-q_{23}-q_{2,13}q_{13,3}-q_{2,14}q_{14,3}$

$a_{3,11}=-q_{25}q_{5,15}-q_{2,14}q_{14,15}$, $a_{3,12}=-q_{25}q_{5,16}-q_{2,14}q_{14,16}$

$a_{3,13}=-q_{2,13}q_{13,17}-q_{26}q_{6,17}$, $a_{3,14}=-q_{2,13}q_{13,18}-q_{26}q_{6,18}$

$a_{41}=-q_{30}-q_{36}q_{60}-q_{35}q_{50}$, $a_{42}=-q_{31}-q_{3,13}q_{13,1}-q_{3,14}q_{14,25}q_{25,1}$

$a_{43}=-q_{35}q_{52}-q_{36}q_{62}$, $a_{44}=1-q_{3,13}q_{13,3}-q_{3,14}q_{14,3}$

$a_{45}=-q_{37}$, $a_{46}=-q_{38}$, $a_{47}=-q_{39}$, $a_{48}=-q_{3,10}$

$a_{4,11}=-q_{35}q_{5,15}-q_{3,14}q_{14,15}$, $a_{4,12}=-q_{35}q_{5,16}-q_{3,14}q_{14,16}$

$a_{4,13}=-q_{3,13}q_{13,17}-q_{36}q_{6,17}$, $a_{4,14}=-q_{3,13}q_{13,18}-q_{36}q_{6,18}$

$a_{41}=-q_{30}-q_{36}q_{60}-q_{35}q_{50}$, $a_{42}=-q_{31}-q_{3,13}q_{13,1}-q_{3,14}q_{14,25}q_{25,1}$

$a_{43}=-q_{35}q_{52}-q_{36}q_{62}$, $a_{51}=-q_{70}$, $a_{54}=-q_{7,21}q_{21,3}$, $a_{55}=1-q_{7,19}q_{19,7}$

$a_{58}=-q_{7,19}q_{19,36}q_{36,10}$, $a_{5,11}=-q_{7,15}$, $a_{5,13}=-q_{7,17}$, $a_{61}=-q_{80}$, $a_{64}=-q_{83}$

$a_{66}=1-q_{8,20}q_{20,8}-q_{8,22}q_{22,8}$, $a_{67}=-q_{8,20}q_{20,37}q_{37,9}-q_{8,22}q_{22,9}$, $a_{68}=-q_{8,20}q_{20,10}$

$a_{6,12}=-q_{8,16}$, $a_{6,14}=-q_{8,18}$, $a_{73}=-q_{92}$, $a_{74}=-q_{93}$, $a_{7,12}=-q_{9,16}$, $a_{7,14}=-q_{9,18}$

$a_{83}=-q_{10,23}q_{23,2}$, $a_{84}=-q_{10,3}$, $a_{8,11}=-q_{10,15}$; $a_{8,13}=-q_{10,17}$, $a_{92}=-q_{11,24}q_{24,1}$

$a_{93}=-q_{11,2}$, $a_{95}=-q_{11,19}q_{19,7}-q_{11,20}q_{20,7}$, $a_{96}=-q_{11,20}q_{20,8}$, $a_{97}=-q_{11,20}q_{20,37}q_{37,9}$

$a_{98}=-q_{11,20}q_{20,10}-q_{11,19}q_{19,36}q_{36,10}$; $a_{10,2}=-q_{12,1}$, $a_{10,3}=-q_{12,2}$

$a_{10,5}=-q_{12,20}q_{20,7}$, $a_{10,6}=-q_{12,20}q_{20,8}-q_{12,22}q_{22,8}$, $a_{10,8}=-q_{12,20}q_{20,10}$

$a_{10,7}=-q_{12,20}q_{20,37}q_{37,9}-q_{12,22}q_{22,9}$, $a_{11,1}=-q_{15,5}q_{50}$, $a_{11,2}=-q_{15,27}q_{27,14}q_{14,25}q_{25,1}$

$a_{11,3}=-q_{15,5}q_{52}$, $a_{11,4}=-q_{15,27}q_{27,14}q_{14,3}$, $a_{11,8}=-q_{15,26}q_{26,11}$

$a_{11,11}=1-q_{15,27}q_{27,14}q_{14,15}-q_{15,5}q_{5,15}$, $a_{11,12}=-q_{15,27}q_{27,14}q_{14,16}-q_{15,5}q_{5,16}$

$a_{12,1}=-q_{16,5}q_{50}$, $a_{12,2}=-q_{16,14}q_{14,25}q_{25,1}$, $a_{12,3}=-q_{16,5}q_{52}$, $a_{12,4}=-q_{16,14}q_{14,3}$

$a_{12,9}=-q_{16,30}q_{30,12}$, $a_{12,12}=1-q_{16,14}q_{14,16}-q_{16,5}q_{5,16}$, $a_{13,1}=-q_{17,6}q_{60}$

$a_{13,2}=-q_{17,32}q_{32,13}q_{13,1}$, $a_{13,3}=-q_{17,6}q_{62}$, $a_{13,4}=-q_{17,32}q_{32,13}q_{13,3}$

$a_{13,5}=-q_{17,7}$, $a_{13,8}=-q_{17,11}$, $a_{13,13}=1-q_{17,6}q_{6,17}-q_{17,32}q_{32,13}q_{13,17}$

$a_{13,14}=-q_{17,6}q_{6,18}-q_{17,32}q_{32,13}q_{13,18}$, $a_{14,1}=-q_{18,6}q_{60}$, $a_{14,2}=-q_{18,13}q_{13,1}$

$a_{14,3} = -q_{18,6}q_{62}$, $a_{14,4} = -q_{18,13}q_{13,3}$, $a_{14,6} = -q_{18,8}$, $a_{14,9} = -q_{18,12}$

$a_{14,13} = -q_{18,6}q_{6,17} - q_{18,13}q_{13,17}$, $a_{14,14} = 1 - q_{18,6}q_{6,18} - q_{18,13}q_{13,18}$

$$N_1 = \begin{vmatrix}
0 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & 0 & 0 & 0 & 0 & a_{1,11} & a_{1,12} & a_{1,13} & a_{1,14} \\
0 & 1 & a_{21} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & 0 & 0 & 0 & 0 \\
0 & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & 0 & 0 & 0 & a_{3,11} & a_{3,12} & a_{3,13} & a_{3,14} \\
0 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} & 0 & 0 & a_{4,11} & a_{4,12} & a_{4,13} & a_{4,14} \\
a_{51} & 0 & 0 & a_{54} & a_{55} & 0 & 0 & a_{58} & 0 & 0 & a_{5,11} & 0 & a_{5,13} & 0 \\
a_{61} & 0 & 0 & a_{64} & 0 & a_{66} & a_{67} & a_{68} & 0 & 0 & 0 & a_{6,12} & 0 & a_{6,14} \\
0 & 0 & a_{73} & a_{74} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & a_{7,12} & 0 & a_{7,14} \\
0 & 0 & a_{82} & a_{83} & 0 & 0 & 0 & 1 & 0 & 0 & a_{8,11} & 0 & a_{8,13} & 0 \\
a_{91} & a_{92} & a_{93} & 0 & a_{95} & a_{96} & a_{97} & a_{98} & 1 & 0 & 0 & 0 & 0 & 0 \\
a_{10,1} & a_{10,2} & a_{10,3} & 0 & a_{10,5} & a_{10,6} & a_{10,7} & a_{10,8} & 0 & 1 & 0 & 0 & 0 & 0 \\
a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & 0 & 0 & 0 & a_{11,8} & a_{11,9} & 0 & a_{11,11} & a_{11,12} & 0 & 0 \\
a_{12,1} & a_{12,2} & a_{12,3} & a_{12,4} & 0 & 0 & 0 & 0 & 0 & a_{12,10} & 0 & a_{12,12} & 0 & 0 \\
a_{13,1} & a_{13,2} & a_{13,3} & a_{13,4} & a_{13,5} & 0 & 0 & a_{13,8} & 0 & 0 & 0 & 0 & a_{13,13} & a_{13,14} \\
a_{14,1} & a_{14,2} & a_{14,3} & a_{14,4} & 0 & a_{14,6} & 0 & 0 & a_{14,9} & 0 & 0 & 0 & a_{14,13} & a_{14,14}
\end{vmatrix}$$

where

$a_{12}=-q_{01}$, $a_{13}=-q_{02}-q_{05}q_{52}-q_{06}q_{62}$, $a_{14}=-q_{03}$, $a_{15}=-q_{07}$; $a_{16}=-q_{08}$

$a_{1,11}=-q_{05}q_{5,15}$, $a_{1,12}=-q_{05}q_{5,16}$, $a_{1,13}=-q_{06}q_{6,17}$, $a_{1,14}=-q_{06}q_{6,18}$, $a_{23}=-q_{12}$

$a_{24}=-q_{13}$, $a_{25}=-q_{17}$, $a_{26}=-q_{18}$, $a_{27}=-q_{19}$, $a_{28}=-p_{1,10}$, $a_{29}=-q_{1,11}$, $a_{2,10}=-q_{1,12}$

$a_{32}=-q_{21}-q_{2,13}q_{13,1}-q_{2,14}q_{14,25}q_{25,1}$, $a_{33}=1-q_{25}q_{52}-q_{26}q_{62}$

$a_{34}=-q_{23}-q_{2,13}q_{13,3}-q_{2,14}q_{14,3}$, $a_{3,11}=-q_{25}q_{5,15}-q_{2,14}q_{14,15}$

$a_{3,12}=-q_{25}q_{5,16}-q_{2,14}q_{14,16}$, $a_{3,13}=-q_{2,13}q_{13,17}-q_{26}q_{6,17}$, $a_{3,14}=-q_{2,13}q_{13,18}-q_{26}q_{6,18}$

$a_{42}=-q_{31}-q_{3,13}q_{13,1}-q_{3,14}q_{14,25}q_{25,1}$, $a_{43}=-q_{35}q_{52}-q_{36}q_{62}$, $a_{45}=-q_{37}$, $a_{46}=-q_{38}$

$a_{44}=1-q_{3,13}q_{13,3}-q_{3,14}q_{14,3}$, $a_{47}=-q_{39}$, $a_{48}=-q_{3,10}$, $a_{4,11}=-q_{35}q_{5,15}-q_{3,14}q_{14,15}$

$a_{4,12}=-q_{35}q_{5,16}-q_{3,14}q_{14,16}$, $a_{4,13}=-q_{3,13}q_{13,17}-q_{36}q_{6,17}$, $a_{4,14}=-q_{3,13}q_{13,18}-q_{36}q_{6,18}$

$a_{42}=-q_{31}-q_{3,13}q_{13,1}-q_{3,14}q_{14,25}q_{25,1}$; $a_{43}=-q_{35}q_{52}-q_{36}q_{62}$, $a_{51}=q_{7,19}(q_{19,28}+q_{19,33})$

$a_{54}=-q_{7,21}q_{21,3}$, $a_{55}=1-q_{7,19}q_{19,7}$, $a_{58}=-q_{7,19}q_{19,36}q_{36,10}$

$a_{5,11}=-q_{7,15}$, $a_{5,13}=-q_{7,17}$, $a_{61}=-q_{80}$, $a_{64}=-q_{83}$, $a_{66}=1-q_{8,20}q_{20,8}-q_{8,22}q_{22,8}$

$a_{67}=-q_{8,20}q_{20,37}q_{37,9}-q_{8,22}q_{22,9}$, $a_{68}=-q_{8,20}q_{20,10}$, $a_{6,12}=-q_{8,16}$, $a_{6,14}=-q_{8,18}$

$a_{73}=-q_{92}$, $a_{74}=-q_{93}$, $a_{7,12}=-q_{9,16}$, $a_{7,14}=-q_{9,18}$, $a_{83}=-q_{10,23}q_{23,2}$, $a_{84}=-q_{10,3}$

$a_{8,11}=-q_{10,15}$; $a_{8,13}=-q_{10,17}$, $a_{91}=q_{11,19}(q_{19,28}+q_{19,33})+q_{11,20}(q_{20,29}+q_{20,34})$

$a_{92}=-q_{11,24}q_{24,1}$, $a_{93}=-q_{11,2}$, $a_{95}=-q_{11,19}q_{19,7}-q_{11,20}q_{20,7}$, $a_{96}=-q_{11,20}q_{20,8}$

$a_{97}=-q_{11,20}q_{20,37}q_{37,9}$, $a_{98}=-q_{11,20}q_{20,10}-q_{11,19}q_{19,36}q_{36,10}$, $a_{10,2}=-q_{12,1}$

$a_{10,1}=q_{12,20}(q_{20,29}+q_{20,34})+q_{12,22}(q_{22,31}+q_{22,35})$, $a_{10,3}=-q_{12,2}$, $a_{10,5}=-q_{12,20}q_{20,7}$

$a_{10,6}=-q_{12,20}q_{20,8}-q_{12,22}q_{22,8}$, $a_{10,7}=-q_{12,20}q_{20,37}q_{37,9}-q_{12,22}q_{22,9}$, $a_{11,1}=q_{15,28}+q_{15,29}$

$a_{10,8}=-q_{12,20}q_{20,10}$, $a_{11,2}=-q_{15,27}q_{27,14}q_{14,25}q_{25,1}$, $a_{11,3}=-q_{15,5}q_{52}$

$a_{11,4}=-q_{15,27}q_{27,14}q_{14,3}$, $a_{11,8}=-q_{15,26}q_{26,11}$, $a_{11,11}=1-q_{15,27}q_{27,14}q_{14,15}-q_{15,5}q_{5,15}$

$a_{11,12}=-q_{15,27}q_{27,14}q_{14,16}-q_{15,5}q_{5,16}$, $a_{12,1}=q_{16,29}+q_{16,31}$, $a_{12,2}=-q_{16,14}q_{14,25}q_{25,1}$

$a_{12,3}=-q_{16,5}q_{52}$, $a_{12,4}=-q_{16,14}q_{14,3}$, $a_{12,9}=-q_{16,30}q_{30,12}$

$a_{12,12}=1-q_{16,14}q_{14,16}-q_{16,5}q_{5,16}$, $a_{13,1}=q_{17,33}+q_{17,34}$, $a_{13,2}=-q_{17,32}q_{32,13}q_{13,1}$

$a_{13,3}=-q_{17,6}q_{62}$, $a_{13,4}=-q_{17,32}q_{32,13}q_{13,3}$, $a_{13,5}=-q_{17,7}$, $a_{13,8}=-q_{17,11}$

$a_{13,13}=1-q_{17,6}q_{6,17}-q_{17,32}q_{32,13}q_{13,17}$, $a_{13,14}=-q_{17,6}q_{6,18}-q_{17,32}q_{32,13}q_{13,18}$

$a_{14,1}=q_{18,34}+q_{18,35}$, $a_{14,2}=-q_{18,13}q_{13,1}$; $a_{14,3}=-q_{18,6}q_{62}$, $a_{14,4}=-q_{18,13}q_{13,3}$

$a_{14,6}=-q_{18,8}$, $a_{14,9}=-q_{18,12}$, $a_{14,13}=-q_{18,6}q_{6,17}-q_{18,13}q_{13,17}$

$a_{14,14}=1-q_{18,6}q_{6,18}-q_{18,13}q_{13,18}$

4.5 Availability Analysis

4.5.1 Availability with full capacity

Let $A^1{}_i(t)$ be the probability that the system of 16 ton capacity is in upstate when at instant t given that the system entered regenerative state i at $t=0$.The availability $A^1{}_i(t)$ is expressed as the following recursive relations:

$$A^1{}_0(t)=M_0(t)+Q_{01}(t)(c)A^1{}_1(t)+Q_{02}(t)(c)A^1{}_2(t)+Q_{03}(t)(c)A^1{}_3(t)+Q_{04}(t)(c)A^1{}_4(t)$$
$$+Q_{05}(t)(c)A^1{}_5(t)+Q_{06}(t)(c)A^1{}_6(t)+Q_{07}(t)(c)A^1{}_7(t)+Q_{08}(t)(c)A_8(t)$$
$$A^1{}_1(t)=Q_{10}(t)(c)A^1{}_0(t)+Q_{12}(t)(c)A^1{}_2(t)+Q_{13}(t)(c)A^1{}_3(t)+Q_{17}(t)(c)A^1{}_7(t)$$
$$+Q_{18}(t)(c)A^1{}_8(t)+Q_{19}(t)(c)A^1{}_9(t)+Q_{1,11}(t)(c)A^1{}_{11}(t)+Q_{1,12}(t)(c)A^1{}_{12}(t)$$
$$A^1{}_2(t)=Q_{20}(t)(c)A^1{}_0(t)+Q_{21}(t)(c)A^1{}_1(t)+Q_{23}(t)(c)A^1{}_3(t)+Q_{25}(t)(c)A^1{}_5(t)$$
$$+Q_{2,13}(t)(c)A^1{}_{13}(t)+Q_{2,14}(t)(c)A^1{}_{14}(t)$$
$$A^1{}_3(t)=Q_{30}(t)(c)A^1{}_0(t)+Q_{31}(t)(c)A^1{}_1(t)+Q_{32}(t)(c)A^1{}_2(t)+Q_{35}(t)(c)A^1{}_5(t)$$
$$+Q_{36}(t)(c)A^1{}_7(t)+Q_{38}(t)(c)A^1{}_8(t)+Q_{39}(t)(c)A^1{}_9(t)+Q_{3,10}(t)(c)A^1{}_{10}(t)$$
$$+Q_{3,13}(t)(c)A^1{}_{3,13}(t)+Q_{3,14}(t)(c)A^1{}_{3,14}(t)$$
$$A^1{}_4(t)=Q_{40}(t)(c)A^1{}_0(t)$$
$$A^1{}_5(t)=Q_{50}(t)(c)A^1{}_0(t)+Q_{52}(t)(c)A^1{}_2(t)+Q_{5,15}(t)(c)A^1{}_{15}(t)+Q_{5,16}(t)(c)A^1{}_{16}(t)$$
$$A^1{}_6(t)=Q_{60}(t)(c)A^1{}_0(t)+Q_{62}(t)(c)A^1{}_2(t)+Q_{6,17}(t)(c)A^1{}_{17}(t)+Q_{6,18}(t)(c)A^1{}_{18}(t)$$
$$A^1{}_7(t)=Q_{70}(t)(c)A^1{}_0(t)+Q_{7,15}(t)(c)A^1{}_{15}(t)+Q_{7,19}(t)(c)A^1{}_{19}(t)+Q_{7,20}(t)(c)A^1{}_{20}(t)$$
$$+Q_{7,21}(t)(c)A^1{}_{21}(t)$$
$$A^1{}_8(t)=Q_{80}(t)(c)A^1{}_0(t)+Q_{83}(t)(c)A^1{}_3(t)+Q_{8,16}(t)(c)A^1{}_{16}(t)+Q_{8,18}(t)(c)A^1{}_{18}(t)$$
$$+Q_{8,20}(t)(c)A^1{}_{20}(t)+Q_{8,22}(t)(c)A^1{}_{22}(t)$$
$$A^1{}_9(t)=Q_{92}(t)(c)A^1{}_2(t)+Q_{93}(t)(c)A^1{}_3(t)+Q_{9,16}(t)(c)A^1{}_{16}(t)+Q_{9.18}(t)(c)A^1{}_{18}(t)$$
$$A^1{}_{10}(t)=Q_{10,3}(t)(c)A^1{}_3(t)+Q_{10,15}(t)(c)A^1{}_{15}(t)+Q_{10,17}(t)(c)A^1{}_{17}(t)+Q_{10,18}(t)(c)A^1{}_{18}(t)$$
$$A^1{}_{11}(t)=Q_{11,2}(t)(c)A^1{}_2(t)+Q_{11,19}(t)(c)A^1{}_{19}(t)+Q_{11,20}(t)(c)A^1{}_{20}(t)+Q_{11,24}(t)(c)A^1{}_{24}(t)$$
$$A^1{}_{12}(t)=Q_{12,1}(t)(c)A^1{}_1(t)+Q_{12,2}(t)(c)A^1{}_2(t)+Q_{12,20}(t)(c)A^1{}_{20}(t)+Q_{12,22}(t)(c)A^1{}_{22}(t)$$
$$A^1{}_{13}(t)=Q_{13,1}(t)(c)A^1{}_1(t)+Q_{13,3}(t)(c)A^1{}_3(t)+Q_{13,17}(t)(c)A^1{}_{17}(t)+Q_{13,18}(t)(c)A^1{}_{18}(t)$$
$$A^1{}_{14}(t)=Q_{14,3}(t)(c)A^1{}_3(t)+Q_{14,15}(t)(c)A^1{}_{15}(t)+Q_{14,16}(t)(c)A^1{}_{16}(t)+Q_{14,25}(t)(c)A^1{}_{25}(t)$$

$$
\begin{aligned}
A^1_{15}(t) &= Q_{15,5}(t)(c)\,A^1_{5}(t)+Q_{15,7}(t)(c)\,A^1_{7}(t)+Q_{15,26}(t)(c)\,A^1_{26}(t)+Q_{15,27}(t)(c)\,A^1_{27}(t)\\
&\quad +Q_{15,28}(t)(c)\,A^1_{28}(t)+Q_{15,29}(t)(c)\,A^1_{29}(t)\\
A^1_{16}(t) &= Q_{16,5}(t)(c)\,A^1_{5}(t)+Q_{16,8}(t)(c)\,A^1_{8}(t)+Q_{16,14}(t)(c)\,A^1_{14}(t)+Q_{16,30}(t)(c)\,A^1_{30}(t)\\
&\quad +Q_{16,31}(t)(c)\,A^1_{31}(t)+Q_{16,29}(t)(c)\,A^1_{29}(t)\\
A^1_{17}(t) &= Q_{17,6}(t)(c)\,A^1_{6}(t)+Q_{17,7}(t)(c)\,A^1_{7}(t)+Q_{17,11}(t)(c)\,A^1_{11}(t)+Q_{17,32}(t)(c)\,A^1_{32}(t)\\
&\quad +Q_{17,33}(t)(s)\,A^1_{33}(t)+Q_{17,34}(t)(s)\,A^1_{34}(t)\\
A^1_{18}(t) &= Q_{18,6}(t)(c)\,A^1_{6}(t)+Q_{18,6}(t)(c)\,A^1_{6}(t)+Q_{18,13}(t)(c)\,A^1_{13}(t)+Q_{18,12}(t)(c)\,A^1_{12}(t)\\
&\quad +Q_{18,34}(t)(c)\,A^1_{34}(t)+Q_{18,35}(t)(c)\,A^1_{35}(t)\\
A^1_{19}(t) &= Q_{19,7}(t)(c)\,A^1_{7}(t)+Q_{19,36}(t)(c)\,A^1_{36}(t)+Q_{19,28}(t)(c)\,A^1_{28}(t)+Q_{19,34}(t)(c)\,A^1_{34}(t)\\
A^1_{20}(t) &= Q_{20,7}(t)(c)\,A^1_{7}(t)+Q_{20,8}(t)(c)\,A^1_{8}(t)+Q_{20,10}(t)(c)\,A^1_{10}(t)+Q_{20,37}(t)(c)\,A^1_{37}(t)\\
&\quad +Q_{20,29}(t)(c)\,A^1_{29}(t)+Q_{20,34}(t)(c)\,A^1_{37}(t)\\
A^1_{21}(t) &= M_{21}(t)+Q_{21,3}(t)(c)\,A^1_{3}(t)\\
A^1_{22}(t) &= Q_{22,8}(t)(c)\,A^1_{8}(t)+Q_{22,9}(t)(c)\,A^1_{9}(t)+Q_{22,31}(t)(c)\,A^1_{31}(t)+Q_{22,35}(t)(c)\,A^1_{35}(t)\\
A^1_{23}(t) &= Q_{23,1}(t)(c)\,A^1_{2}(t)\\
A^1_{24}(t) &= Q_{24,1}(t)(c)\,A^1_{1}(t)\\
A^1_{25}(t) &= M_{25}(t)+Q_{25,1}(t)(c)\,A^1_{1}(t)\\
A^1_{26}(t) &= Q_{26,11}(t)(c)\,A^1_{11}(t)\\
A^1_{27}(t) &= Q_{27,14}(t)(c)\,A^1_{14}(t)\\
A^1_{28}(t) &= Q_{28,15}(t)(c)\,A^1_{15}(t)+Q_{28,19}(t)(c)\,A^1_{19}(t)\\
A^1_{29}(t) &= Q_{28,15}(t)(c)\,A^1_{15}(t)+Q_{28,19}(t)(c)\,A^1_{19}(t)\\
A^1_{30}(t) &= Q_{30,12}(t)(c)\,A^1_{12}(t)\\
A^1_{31}(t) &= Q_{28,15}(t)(c)\,A^1_{15}(t)+Q_{28,19}(t)(c)\,A^1_{19}(t)\\
A^1_{32}(t) &= Q_{32,13}(t)(c)\,A^1_{13}(t)\\
A^1_{33}(t) &= Q_{33,17}(t)(c)\,A^1_{17}(t)+Q_{33,18}(t)(c)\,A^1_{18}(t)+Q_{33,20}(t)(c)\,A^1_{20}(t)\\
A^1_{34}(t) &= Q_{34,17}(t)(c)\,A^1_{17}(t)+Q_{34,19}(t)(c)\,A^1_{19}(t)\\
A^1_{35}(t) &= Q_{35,18}(t)(c)\,A^1_{18}(t)+Q_{35,22}(t)(c)\,A^1_{22}(t)\\
A^1_{36}(t) &= Q_{36,10}(t)(c)\,A^1_{10}(t)\\
A^1_{37}(t) &= Q_{37,9}(t)(c)\,A^1_{9}(t)
\end{aligned}
\quad , \qquad (4.10)
$$

where

$$M_0(t) = e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3)\,t} \;,\; M_{21}(t)=\bar{M}(t) \;,\; M_{25}(t)=\bar{I}(t)$$
$$\bar{M}(t)=1-M(\mathrm{t}) \;,\; \bar{I}(t)=1-I(\mathrm{t})$$

4.5.2 Availability with twelve ton capacity

Similarly if $A^2{}_i(t)$ be the probability that the system of 12 ton capacity is in upstate at instant t given that the system entered regenerative state i at $t=0$ then recursive relations for availability $A^2{}_i(t)$ have been obtained as:

$$A^2{}_0(t)=Q_{01}(t)(\mathrm{c})A^2{}_1(t)+Q_{02}(t)(\mathrm{c})A^2{}_2(t)+Q_{03}(t)(\mathrm{c})A^2{}_3(t)+Q_{04}(t)(\mathrm{c})A^2{}_4(t)+$$
$$Q_{05}(t)(\mathrm{c})A^2{}_5(t)+Q_{06}(t)(\mathrm{c})A^2{}_6(t)+Q_{07}(t)(\mathrm{c})A^2{}_7(t)+Q_{08}(t)(\mathrm{c})A^2{}_8(t)$$
$$A^2{}_1(t)=Q_{10}(t)(\mathrm{c})A^2{}_0(t)+Q_{12}(t)(\mathrm{c})A^2{}_2(t)+Q_{13}(t)(\mathrm{c})A^2{}_3(t)+Q_{17}(t)(\mathrm{c})A^2{}_7(t)+$$
$$Q_{18}(t)(\mathrm{c})A^2{}_8(t)+Q_{19}(t)(\mathrm{c})A^2{}_9(t)+Q_{1,11}(t)(\mathrm{c})A^2{}_{11}(t)+Q_{1,12}(t)(\mathrm{c})A^2{}_{12}(t)$$
$$A^2{}_2(t)=Q_{20}(t)(\mathrm{c})A^2{}_0(t)+Q_{21}(t)(\mathrm{c})A^2{}_1(t)+Q_{23}(t)(\mathrm{c})A^2{}_3(t)+Q_{25}(t)(\mathrm{c})A^2{}_5(t)$$
$$+Q_{2,13}(t)(\mathrm{c})A^2{}_{13}(t)+Q_{2,14}(t)(\mathrm{c})A^2{}_{14}(t)$$
$$A^2{}_3(t)=M_3(t)+Q_{30}(t)(\mathrm{c})A^2{}_0(t)+Q_{31}(t)(s)A^2{}_1(t)+Q_{32}(t)(\mathrm{c})A^2{}_2(t)+Q_{35}(t)(\mathrm{c})A^2{}_5(t)$$
$$+Q_{36}(t)(\mathrm{c})A^2{}_7(t)+Q_{38}(t)(\mathrm{c})A^2{}_8(t)+Q_{39}(t)(\mathrm{c})A^2{}_9(t)+Q_{3,10}(t)(\mathrm{c})A^2{}_{10}(t)$$
$$+Q_{3,13}(t)(\mathrm{c})A^1{}_{3,13}(t)+Q_{3,14}(t)(\mathrm{c})A^2{}_{3,14}(t)$$
$$A^2{}_4(t)=Q_{40}(t)(\mathrm{c})A^2{}_0(t)$$
$$A^2{}_5(t)=Q_{50}(t)(\mathrm{c})A^2{}_0(t)+Q_{52}(t)(\mathrm{c})A^2{}_2(t)+Q_{5,15}(t)(\mathrm{c})A^2{}_{15}(t)+Q_{5,16}(t)(\mathrm{c})A^2{}_{16}(t)$$
$$A^2{}_6(t)=Q_{60}(t)(\mathrm{c})A^2{}_0(t)+Q_{62}(t)(\mathrm{c})A^2{}_2(t)+Q_{6,17}(t)(\mathrm{c})A^2{}_{17}(t)+Q_{6,18}(t)(\mathrm{c})A^2{}_{18}(t)$$
$$A^2{}_7(t)=M_7(t)+Q_{70}(t)(\mathrm{c})A^2{}_0(t)+Q_{7,15}(t)(\mathrm{c})A^2{}_{15}(t)+Q_{7,19}(t)(\mathrm{c})A^2{}_{19}(t)$$
$$+Q_{7,20}(t)(\mathrm{c})A^2{}_{20}(t)+Q_{7,21}(t)(\mathrm{c})A^2{}_{21}(t)$$
$$A^2{}_8(t)=M_8(t)+Q_{80}(t)(\mathrm{c})A^2{}_0(t)+Q_{83}(t)(\mathrm{c})A^2{}_3(t)+Q_{8,16}(t)(\mathrm{c})A^2{}_{16}(t)$$
$$+Q_{8,18}(t)(\mathrm{c})A^2{}_{18}(t)+Q_{8,20}(t)(\mathrm{c})A^2{}_{20}(t)+Q_{8,22}(t)(\mathrm{c})A^2{}_{22}(t)$$
$$A^2{}_9(t)=Q_{92}(t)(\mathrm{c})A^2{}_2(t)+Q_{93}(t)(\mathrm{c})A^2{}_3(t)+Q_{9,16}(t)(\mathrm{c})A^1{}_{16}(t)+Q_{9.18}(t)(\mathrm{c})A^2{}_{18}(t)$$
$$A^2{}_{10}(t)=Q_{10,3}(t)(\mathrm{c})A^2{}_3(t)+Q_{10,15}(t)(\mathrm{c})A^2{}_{15}(t)+Q_{10,17}(t)(\mathrm{c})A^2{}_{17}(t)+Q_{10,18}(t)(\mathrm{c})A^2{}_{18}(t)$$
$$A^2{}_{11}(t)=Q_{11,2}(t)(\mathrm{c})A^2{}_2(t)+Q_{11,19}(t)(\mathrm{c})A^2{}_{19}(t)+Q_{11,20}(t)(\mathrm{c})A^2{}_{20}(t)+Q_{11,24}(t)(\mathrm{c})A^2{}_{24}(t)$$
$$A^2{}_{12}(t)=Q_{12,1}(t)(\mathrm{c})A^2{}_1(t)+Q_{12,2}(t)(\mathrm{c})A^2{}_2(t)+Q_{12,20}(t)(\mathrm{c})A^2{}_{20}(t)+Q_{12,22}(t)(\mathrm{c})A^2{}_{22}(t)$$
$$A^2{}_{13}(t)=Q_{13,1}(t)(\mathrm{c})A^2{}_1(t)+Q_{13,3}(t)(\mathrm{c})A^2{}_3(t)+Q_{13,17}(t)(\mathrm{c})A^2{}_{17}(t)+Q_{13,18}(t)(\mathrm{c})A^2{}_{18}(t)$$
$$A^2{}_{14}(t)=Q_{14,3}(t)(\mathrm{c})A^2{}_3(t)+Q_{14,15}(t)(\mathrm{c})A^2{}_{15}(t)+Q_{14,16}(t)(\mathrm{c})A^2{}_{16}(t)+Q_{14,25}(t)(\mathrm{c})A^2{}_{25}(t)$$

$$A^2_{15}(t) = Q_{15,5}(t)(c)A^2_5(t) + Q_{15,7}(t)(c)A^2_7(t) + Q_{15,26}(t)(c)A^2_{26}(t) + Q_{15,27}(t)(c)A^2_{27}(t) + Q_{15,28}(t)(c)A^2_{28}(t) + Q_{15,29}(t)(c)A^2_{29}(t)$$

$$A^2_{16}(t) = Q_{16,5}(t)(c)A^2_5(t) + Q_{16,8}(t)(c)A^2_8(t) + Q_{16,14}(t)(c)A^2_{14}(t) + Q_{16,30}(t)(c)A^2_{30}(t) + Q_{16,31}(t)(c)A^2_{31}(t) + Q_{16,29}(t)(c)A^2_{29}(t)$$

$$A^2_{17}(t) = Q_{17,6}(t)(c)A^2_6(t) + Q_{17,7}(t)(c)A^2_7(t) + Q_{17,11}(t)(c)A^2_{11}(t) + Q_{17,32}(t)(c)A^2_{32}(t) + Q_{17,33}(t)(c)A^2_{33}(t) + Q_{17,34}(t)(c)A^2_{34}(t)$$

$$A^2_{18}(t) = Q_{18,6}(t)(c)A^2_6(t) + Q_{18,6}(t)(c)A^2_6(t) + Q_{18,13}(t)(c)A^2_{13}(t) + Q_{18,12}(t)(c)A^2_{12}(t) + Q_{18,34}(t)(c)A^2_{34}(t) + Q_{18,35}(t)(c)A^2_{35}(t)$$

$$A^2_{19}(t) = Q_{19,7}(t)(c)A^2_7(t) + Q_{19,36}(t)(c)A^2_{36}(t) + Q_{19,28}(t)(c)A^2_{28}(t) + Q_{19,34}(t)(c)A^2_{34}(t)$$

$$A^2_{20}(t) = Q_{20,7}(t)(c)A^2_7(t) + Q_{20,8}(t)(c)A^2_8(t) + Q_{20,10}(t)(c)A^2_{10}(t) + Q_{20,37}(t)(c)A^2_{37}(t) + Q_{20,29}(t)(c)A^2_{29}(t) + Q_{20,34}(t)(c)A^2_{37}(t)$$

$$A^2_{21}(t) = Q_{21,3}(t)(c)A^2_3(t)$$

$$A^2_{22}(t) = Q_{22,8}(t)(c)A^2_8(t) + Q_{22,9}(t)(c)A^2_9(t) + Q_{22,31}(t)(c)A^2_{31}(t) + Q_{22,35}(t)(c)A^2_{35}(t)$$

$$A^2_{23}(t) = Q_{23,1}(t)(c)A^2_2(t)$$

$$A^2_{24}(t) = Q_{24,1}(t)(c)A^2_1(t)$$

$$A^2_{25}(t) = Q_{25,1}(t)(c)A^2_1(t)$$

$$A^2_{26}(t) = M_{26}(t) + Q_{26,11}(t)(c)A^2_{11}(t)$$

$$A^2_{27}(t) = Q_{27,14}(t)(c)A^2_{14}(t)$$

$$A^2_{28}(t) = Q_{28,15}(t)(c)A^2_{15}(t) + Q_{28,19}(t)(c)A^2_{19}(t)$$

$$A^2_{29}(t) = Q_{28,15}(t)(c)A^2_{15}(t) + Q_{28,19}(t)(c)A^2_{19}(t)$$

$$A^2_{30}(t) = M_{30}(t) + Q_{30,12}(t)(c)A^2_{12}(t)$$

$$A^2_{31}(t) = Q_{28,15}(t)(c)A^2_{15}(t) + Q_{28,19}(t)(c)A^2_{19}(t)$$

$$A^2_{32}(t) = Q_{32,13}(t)(c)A^2_{13}(t)$$

$$A^2_{33}(t) = Q_{33,17}(t)(c)A^2_{17}(t) + Q_{33,18}(t)(c)A^2_{18}(t) + Q_{33,20}(t)(c)A^2_{20}(t)$$

$$A^2_{34}(t) = Q_{34,17}(t)(c)A^2_{17}(t) + Q_{34,19}(t)(c)A^2_{19}(t) \quad , \qquad (4.11)$$

$$A^2_{35}(t) = Q_{35,18}(t)(c)A^2_{18}(t) + Q_{35,22}(t)(c)A^2_{22}(t)$$

$$A^2_{36}(t) = M_{36}(t) + Q_{36,10}(t)(c)A^2_{10}(t)$$

$$A^2_{37}(t) = M_{37}(t) + Q_{37,9}(t)(c)A^2_9(t)$$

where

$M_3(t) = e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3)t}$, $M_7(t) = e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+\gamma_2+\gamma_3)t}$, $M_8(t) = e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2+b_1+b_2)t}$

$M_{26}(t)=\bar{I}(t)$, $M_{26}(t)=\bar{I}(t)$, $M_{36}(t)=\bar{M}(t)$, $M_{37}(t)=\bar{M}(t)$

$\bar{M}(t)=1-M(\text{t})$, $\bar{I}(t)=1-I(\text{t})$

4.5.3 Availability with eight ton capacity

If $A^3{}_i(t)$ be the probability that the system of 8 ton capacity is in upstate at instant t given that the system entered regenerative state i at $t=0$ then recursive relations for availability $A^3{}_i(t)$ have been obtained as:

$$A^3{}_0(t)=Q_{01}(t)(\text{c})A^3{}_1(t)+Q_{02}(t)(\text{c})A^3{}_2(t)+Q_{03}(t)(\text{c})A^3{}_3(t)+Q_{04}(t)(\text{c})A^3{}_4(t)+$$
$$Q_{05}(t)(\text{c})A^3{}_5(t)+Q_{06}(t)(\text{c})A^3{}_6(t)+Q_{07}(t)(\text{c})A^3{}_7(t)+Q_{08}(t)(\text{c})A^3{}_8(t)$$

$$A^3{}_1(t)=Q_{10}(t)(\text{c})A^3{}_0(t)+Q_{12}(t)(\text{c})A^3{}_2(t)+Q_{13}(t)(\text{c})A^3{}_3(t)+Q_{17}(t)(\text{c})A^3{}_7(t)+$$
$$Q_{18}(t)(\text{c})A^3{}_8(t)+Q_{19}(t)(\text{c})A^3{}_9(t)+Q_{1,11}(t)(\text{c})A^3{}_{11}(t)+Q_{1,12}(t)(\text{c})A^3{}_{12}(t)$$

$$A^3{}_2(t)=M_2(t)+Q_{20}(t)(\text{c})A^3{}_0(t)+Q_{21}(t)(\text{c})A^3{}_1(t)+Q_{23}(t)(\text{c})A^3{}_3(t)+Q_{25}(t)(\text{c})A^3{}_5(t)$$
$$+Q_{2,13}(t)(\text{c})A^3{}_{13}(t)+Q_{2,14}(t)(\text{c})A^3{}_{14}(t)$$

$$A^3{}_3(t)=M_3(t)+Q_{30}(t)(\text{c})A^3{}_0(t)+Q_{31}(t)(\text{c})A^3{}_1(t)+Q_{32}(t)(\text{c})A^3{}_2(t)+Q_{35}(t)(\text{c})A^3{}_5(t)$$
$$+Q_{36}(t)(\text{c})A^3{}_7(t)+Q_{38}(t)(\text{c})A^3{}_8(t)+Q_{39}(t)(\text{c})A^3{}_9(t)+Q_{3,10}(t)(\text{c})A^3{}_{10}(t)$$
$$+Q_{3,13}(t)(\text{c})A^3{}_{3,13}(t)+Q_{3,14}(t)(s)A^3{}_{3,14}(t)$$

$$A^3{}_4(t)=Q_{40}(t)(\text{c})A^3{}_0(t)$$

$$A^3{}_5(t)=Q_{50}(t)(\text{c})A^3{}_0(t)+Q_{52}(t)(\text{c})A^3{}_2(t)+Q_{5,15}(t)(\text{c})A^3{}_{15}(t)+Q_{5,16}(t)(\text{c})A^3{}_{16}(t)$$

$$A^3{}_6(t)=Q_{60}(t)(\text{c})A^3{}_0(t)+Q_{62}(t)(\text{c})A^3{}_2(t)+Q_{6,17}(t)(\text{c})A^3{}_{17}(t)+Q_{6,18}(t)(\text{c})A^3{}_{18}(t)$$

$$A^3{}_7(t)=M_7(t)+Q_{70}(t)(\text{c})A^3{}_0(t)+Q_{7,15}(t)(\text{c})A^3{}_{15}(t)+Q_{7,19}(t)(\text{c})A^3{}_{19}(t)$$
$$+Q_{7,20}(t)(\text{c})A^3{}_{20}(t)+Q_{7,21}(t)(\text{c})A^3{}_{21}(t)$$

$$A^3{}_8(t)=M_8(t)+Q_{80}(t)(\text{c})A^3{}_0(t)+Q_{83}(t)(\text{c})A^3{}_3(t)+Q_{8,16}(t)(\text{c})A^3{}_{16}(t)$$
$$+Q_{8,18}(t)(\text{c})A^3{}_{18}(t)+Q_{8,20}(t)(\text{c})A^3{}_{20}(t)+Q_{8,22}(t)(\text{c})A^3{}_{22}(t)$$

$$A^3{}_9(t)=M_9(t)+Q_{92}(t)(\text{c})A^3{}_2(t)+Q_{93}(t)(\text{c})A^3{}_3(t)+Q_{9,16}(t)(\text{c})A^3{}_{16}(t)+Q_{9,18}(t)(\text{c})A^3{}_{18}(t)$$

$$A^3{}_{10}(t)=M_{10}(t)+Q_{10,3}(t)(\text{c})A^3{}_3(t)+Q_{10,15}(t)(\text{c})A^3{}_{15}(t)+Q_{10,17}(t)(\text{c})A^3{}_{17}(t)+Q_{10,18}(t)(\text{c})A^3{}_{18}(t)$$

$$A^3{}_{11}(t)=Q_{11,2}(t)(\text{c})A^3{}_2(t)+Q_{11,19}(t)(\text{c})A^3{}_{19}(t)+Q_{11,20}(t)(\text{c})A^3{}_{20}(t)+Q_{11,24}(t)(\text{c})A^3{}_{24}(t)$$

$$A^3_{12}(t)=Q_{12,1}(t)(c)A^3_1(t)+Q_{12,2}(t)(c)A^3_2(t)+Q_{12,20}(t)(c)A^3_{20}(t)+Q_{12,22}(t)(c)A^3_{22}(t)$$

$$A^3_{13}(t)=Q_{13,1}(t)(c)A^3_1(t)+Q_{13,3}(t)(c)A^1_3(t)+Q_{13,17}(t)(c)A^3_{17}(t)+Q_{13,18}(t)(c)A^3_{18}(t)$$

$$A^3_{14}(t)=Q_{14,3}(t)(c)A^3_3(t)+Q_{14,15}(t)(c)A^3_{15}(t)+Q_{14,16}(t)(c)A^3_{16}(t)+Q_{14,25}(t)(c)A^3_{25}(t)$$

$$A^3_{15}(t)=Q_{15,5}(t)(c)A^3_5(t)+Q_{15,7}(t)(c)A^3_7(t)+Q_{15,26}(t)(c)A^3_{26}(t)+Q_{15,27}(t)(c)A^3_{27}(t)$$
$$+Q_{15,28}(t)(c)A^3_{28}(t)+Q_{15,29}(t)(c)A^3_{29}(t)$$

$$A^3_{16}(t)=Q_{16,5}(t)(c)A^3_5(t)+Q_{16,8}(t)(c)A^3_8(t)+Q_{16,14}(t)(c)A^3_{14}(t)+Q_{16,30}(t)(c)A^3_{30}(t)$$
$$+Q_{16,31}(t)(c)A^3_{31}(t)+Q_{16,29}(t)(c)A^3_{29}(t)$$

$$A^3_{17}(t)=Q_{17,6}(t)(c)A^3_6(t)+Q_{17,7}(t)(c)A^3_7(t)+Q_{17,11}(t)(c)A^3_{11}(t)+Q_{17,32}(t)(c)A^3_{32}(t)$$
$$+Q_{17,33}(t)(c)A^3_{33}(t)+Q_{17,34}(t)(c)A^3_{34}(t)$$

$$A^3_{18}(t)=Q_{18,6}(t)(c)A^3_6(t)+Q_{18,6}(t)(c)A^3_6(t)+Q_{18,13}(t)(c)A^3_{13}(t)+Q_{18,12}(t)(c)A^3_{12}(t)$$
$$+Q_{18,34}(t)(c)A^3_{34}(t)+Q_{18,35}(t)(c)A^3_{35}(t)$$

$$A^3_{19}(t)=M_{19}(t)+Q_{19,7}(t)(c)A^3_7(t)+Q_{19,36}(t)(c)A^3_{36}(t)+Q_{19,28}(t)(c)A^3_{28}(t)$$
$$+Q_{19,34}(t)(c)A^3_{34}(t)$$

$$A^3_{20}(t)=M_{20}(t)+Q_{20,7}(t)(c)A^3_7(t)+Q_{20,8}(t)(c)A^3_8(t)+Q_{20,10}(t)(c)A^3_{10}(t)$$
$$+Q_{20,37}(t)(c)A^3_{37}(t)+Q_{20,29}(t)(c)A^3_{29}(t)+Q_{20,34}(t)(c)A^3_{37}(t)$$

$$A^3_{21}(t)=Q_{21,3}(t)(c)A^3_3(t)$$

$$A^3_{22}(t)=M_{22}(t)+Q_{22,8}(t)(c)A^3_8(t)+Q_{22,9}(t)(c)A^3_9(t)+Q_{22,31}(t)(c)A^3_{31}(t)$$
$$+Q_{22,35}(t)(c)A^3_{35}(t)$$

$$A^3_{23}(t)=Q_{23,1}(t)(c)A^3_2(t)$$

$$A^3_{24}(t)=M_{24}(t)+Q_{24,1}(t)(c)A^3_1(t)$$

$$A^3_{25}(t)=Q_{25,1}(t)(c)A^3_1(t)$$

$$A^3_{26}(t)=Q_{26,11}(t)(c)A^3_{11}(t)$$

$$A^3_{27}(t)=Q_{27,14}(t)(c)A^3_{14}(t)$$

$$A^3_{28}(t)=Q_{28,15}(t)(c)A^3_{15}(t)+Q_{28,19}(t)(c)A^3_{19}(t)$$

$$A^3_{29}(t)=Q_{28,15}(t)(c)A^3_{15}(t)+Q_{28,19}(t)(c)A^3_{19}(t)$$

$$A^3_{30}(t)=Q_{30,12}(t)(c)A^3_{12}(t)$$

$$A^3_{31}(t)=Q_{28,15}(t)(c)A^3_{15}(t)+Q_{28,19}(t)(c)A^3_{19}(t)$$

$$A^3_{32}(t)=M_{32}(t)+Q_{32,13}(t)(c)A^3_{13}(t)$$

$$A^3_{33}(t)=Q_{33,17}(t)(c)A^3_{17}(t)+Q_{33,18}(t)(c)A^3_{18}(t)+Q_{33,20}(t)(c)A^3_{20}(t)$$

$$, \qquad (4.11)$$

$$A^3{}_{34}(t) = Q_{34,17}(t)(c)\, A^3{}_{17}(t) + Q_{34,19}(t)(c)\, A^3{}_{19}(t)$$
$$A^3{}_{35}(t) = Q_{35,18}(t)(c)\, A^3{}_{18}(t) + Q_{35,22}(t)(c)\, A^3{}_{22}(t)$$
$$A^3{}_{36}(t) = Q_{36,10}(t)(c)\, A^3{}_{10}(t)$$
$$A^3{}_{37}(t) = Q_{37,9}(t)(c)\, A^3{}_{9}(t)$$

where

$$M_2(t) = e^{-(\lambda_1+\lambda_2)t}\,\bar{H}(t)\ , M_9(t) = e^{-(\lambda_1+\lambda_2+b_2)t}\ , M_{10}(t) = e^{-(\lambda_1+\lambda_2+b_1)t}$$
$$M_{20}(t) = e^{-(\lambda_1+\lambda_2+b_1+b_2)t}\ ,\ M_{22}(t) = e^{-(\lambda_1+\lambda_2+b_2)t}\ , M_{24}(t) = \bar{M}(t)\ ,\ M_{32}(t) = \bar{M}(t)$$
$$\bar{M}(t) = 1 - M(t)$$

4.5.4 Availability with capacity of four ton

If $A^4{}_i(t)$ be the probability that the system of 4 ton capacity is in upstate when at instant t given that the system entered regenerative state i at $t=0$ then recursive relations for availability $A^4{}_i(t)$ have been obtained as:

$$A^4{}_0(t) = Q_{01}(t)(s)\, A^4{}_1(t) + Q_{02}(t)(s)\, A^4{}_2(t) + Q_{03}(t)(s)\, A^4{}_3(t) + Q_{04}(t)(s)\, A^4{}_4(t) +$$
$$Q_{05}(t)(s)\, A^1{}_5(t) + Q_{06}(t)(s)\, A^1{}_6(t) + Q_{07}(t)(s)\, A^4{}_7(t) + Q_{08}(t)(s)\, A^4{}_8(t)$$
$$A^4{}_1(t) = M_1(t) + Q_{10}(t)(s)\, A^4{}_0(t) + Q_{12}(t)(s)\, A^4{}_2(t) + Q_{13}(t)(s)\, A^4{}_3(t) + Q_{17}(t)(s)\, A^4{}_7(t) +$$
$$Q_{18}(t)(s)\, A^4{}_8(t) + Q_{19}(t)(s)\, A^4{}_9(t) + Q_{1,11}(t)(s)\, A^4{}_{11}(t) + Q_{1,12}(t)(s)\, A^4{}_{12}(t)$$
$$A^4{}_2(t) = Q_{20}(t)(s)\, A^4{}_0(t) + Q_{21}(t)(s)\, A^4{}_1(t) + Q_{23}(t)(s)\, A^4{}_3(t) + Q_{25}(t)(s)\, A^4{}_5(t)$$
$$+ Q_{2,13}(t)(s)\, A^4{}_{13}(t) + Q_{2,14}(t)(s)\, A^4{}_{14}(t)$$
$$A^4{}_3(t) = Q_{30}(t)(s) A^4{}_0(t) + Q_{31}(t)(s)\, A^1{}_1(t) + Q_{32}(t)(s)\, A^4{}_2(t) + Q_{35}(t)(s)\, A^4{}_5(t)$$
$$+ Q_{36}(t)(s)\, A^4{}_7(t) + Q_{38}(t)(s)\, A^4{}_8(t) + Q_{39}(t)(s)\, A^4{}_9(t) + Q_{3,10}(t)(s)\, A^4{}_{10}(t)$$
$$+ Q_{3,13}(t)(s)\, A^4{}_{3,13}(t) + Q_{3,14}(t)(s)\, A^4{}_{3,14}(t)$$
$$A^4{}_4(t) = Q_{40}(t)(s)\, A^4{}_0(t)$$
$$A^4{}_5(t) = Q_{50}(t)(s)\, A^4{}_0(t) + Q_{52}(t)(s)\, A^4{}_2(t) + Q_{5,15}(t)(s)\, A^4{}_{15}(t) + Q_{5,16}(t)(s)\, A^4{}_{16}(t)$$
$$A^4{}_6(t) = Q_{60}(t)(s)\, A^4{}_0(t) + Q_{62}(t)(s)\, A^4{}_2(t) + Q_{6,17}(t)(s)\, A^4{}_{17}(t) + Q_{6,18}(t)(s)\, A^4{}_{18}(t)$$
$$A^4{}_7(t) = Q_{70}(t)(s)\, A^4{}_0(t) + Q_{7,15}(t)(s)\, A^4{}_{15}(t) + Q_{7,19}(t)(s)\, A^4{}_{19}(t)$$
$$+ Q_{7,20}(t)(s)\, A^4{}_{20}(t) + Q_{7,21}(t)(s)\, A^4{}_{21}(t)$$

$$A^4{}_8(t) = Q_{80}(t)(c)A^4{}_0(t) + Q_{83}(t)(c)A^4{}_3(t) + Q_{8,16}(t)(c)A^4{}_{16}(t)$$
$$+Q_{8,18}(t)(c)A^4{}_{18}(t) + Q_{8,20}(t)(c)A^4{}_{20}(t) + Q_{8,22}(t)(c)A^4{}_{22}(t)$$
$$A^4{}_9(t) = Q_{92}(t)(c)A^4{}_2(t) + Q_{93}(t)(c)A^4{}_3(t) + Q_{9,16}(t)(c)A^4{}_{16}(t) + Q_{9,18}(t)(c)A^4{}_{18}(t)$$
$$A^4{}_{10}(t) = Q_{10,3}(t)(c)A^4{}_3(t) + Q_{10,15}(t)(c)A^4{}_{15}(t) + Q_{10,17}(t)(c)A^4{}_{17}(t) + Q_{10,18}(t)(c)A^4{}_{18}(t)$$
$$A^4{}_{11}(t) = M_{11}(t) + Q_{11,2}(t)(c)A^4{}_2(t) + Q_{11,19}(t)(c)A^4{}_{19}(t) + Q_{11,20}(t)(c)A^4{}_{20}(t)$$
$$+Q_{11,24}(t)(c)A^4{}_{24}(t)$$
$$A^4{}_{12}(t) = M_{12}(t) + Q_{12,1}(t)(c)A^4{}_1(t) + Q_{12,2}(t)(c)A^4{}_2(t) + Q_{12,20}(t)(c)A^4{}_{20}(t)$$
$$+Q_{12,22}(t)(c)A^4{}_{22}(t)$$
$$A^4{}_{13}(t) = M_{13}(t) + Q_{13,1}(t)(c)A^4{}_1(t) + Q_{13,3}(t)(c)A^4{}_3(t) + Q_{13,17}(t)(c)A^4{}_{17}(t)$$
$$+Q_{13,18}(t)(c)A^4{}_{18}(t)$$
$$A^4{}_{14}(t) = M_{14}(t) + Q_{14,3}(t)(c)A^4{}_3(t) + Q_{14,15}(t)(c)A^4{}_{15}(t) + Q_{14,16}(t)(c)A^4{}_{16}(t)$$
$$+Q_{14,25}(t)(c)A^4{}_{25}(t)$$
$$A^4{}_{15}(t) = M_{15}(t) + Q_{15,5}(t)(c)A^4{}_5(t) + Q_{15,7}(t)(c)A^4{}_7(t) + Q_{15,26}(t)(c)A^4{}_{26}(t)$$
$$+Q_{15,27}(t)(c)A^4{}_{27}(t) + Q_{15,28}(t)(c)A^4{}_{28}(t) + Q_{15,29}(t)(c)A^4{}_{29}(t)$$
$$A^4{}_{16}(t) = M_{16}(t) + Q_{16,5}(t)(c)A^4{}_5(t) + Q_{16,8}(t)(c)A^4{}_8(t) + Q_{16,14}(t)(c)A^4{}_{14}(t)$$
$$+Q_{16,30}(t)(c)A^4{}_{30}(t) + Q_{16,31}(t)(c)A^4{}_{31}(t) + Q_{16,29}(t)(c)A^4{}_{29}(t)$$
$$A^4{}_{17}(t) = M_{17}(t) + Q_{17,6}(t)(c)A^4{}_6(t) + Q_{17,7}(t)(c)A^4{}_7(t) + Q_{17,11}(t)(c)A^4{}_{11}(t)$$
$$+Q_{17,32}(t)(c)A^4{}_{32}(t) + Q_{17,33}(t)(c)A^4{}_{33}(t) + Q_{17,34}(t)(c)A^4{}_{34}(t)$$
$$A^4{}_{18}(t) = M_{18}(t) + Q_{18,6}(t)(c)A^4{}_6(t) + Q_{18,6}(t)(c)A^4{}_6(t) + Q_{18,13}(t)(c)A^4{}_{13}(t)$$
$$+Q_{18,12}(t)(c)A^4{}_{12}(t) + Q_{18,34}(t)(c)A^4{}_{34}(t) + Q_{18,35}(t)(c)A^4{}_{35}(t)$$
$$A^4{}_{19}(t) = Q_{19,7}(t)(c)A^4{}_7(t) + Q_{19,36}(t)(c)A^4{}_{36}(t) + Q_{19,28}(t)(c)A^4{}_{28}(t) + Q_{19,34}(t)(c)A^4{}_{34}(t)$$
$$A^4{}_{20}(t) = Q_{20,7}(t)(c)A^4{}_7(t) + Q_{20,8}(t)(c)A^4{}_8(t) + Q_{20,10}(t)(c)A^4{}_{10}(t) + Q_{20,37}(t)(c)A^4{}_{37}(t)$$
$$+Q_{20,29}(t)(c)A^4{}_{29}(t) + Q_{20,34}(t)(c)A^4{}_{37}(t)$$
$$A^4{}_{21}(t) = Q_{21,3}(t)(c)A^4{}_3(t)$$
$$A^4{}_{22}(t) = Q_{22,8}(t)(c)A^4{}_8(t) + Q_{22,9}(t)(c)A^4{}_9(t) + Q_{22,31}(t)(c)A^4{}_{31}(t) + Q_{22,35}(t)(c)A^4{}_{35}(t)$$
$$A^4{}_{23}(t) = M_{23}(t) + Q_{23,1}(t)(c)A^4{}_2(t)$$
$$A^4{}_{24}(t) = Q_{24,1}(t)(c)A^4{}_1(t)$$
$$A^4{}_{25}(t) = Q_{25,1}(t)(c)A^4{}_1(t)$$
$$A^4{}_{26}(t) = Q_{26,11}(t)(c)A^4{}_{11}(t)$$
$$A^4{}_{27}(t) = M_{27}(t) + Q_{27,14}(t)(c)A^4{}_{14}(t)$$

$$
\begin{aligned}
&A^4{}_{28}(t)=Q_{28,15}(t)(c)\,A^4{}_{15}(t)+Q_{28,19}(t)(c)\,A^4{}_{19}(t)\\
&A^4{}_{29}(t)=Q_{28,15}(t)(c)\,A^4{}_{15}(t)+Q_{28,19}(t)(c)\,A^4{}_{19}(t)\\
&A^4{}_{30}(t)=\ Q_{30,12}(t)(c)\,A^4{}_{12}(t)\\
&A^4{}_{31}(t)\ =Q_{28,15}(t)(c)\,A^4{}_{15}(t)+Q_{28,19}(t)(c)\,A^4{}_{19}(t)\\
&A^4{}_{32}(t)=\ Q_{32,13}(t)(c)\,A^4{}_{13}(t)\\
&A^4{}_{33}(t)\ =Q_{33,17}(t)(c)\,A^4{}_{17}(t)+Q_{33,18}(t)(c)\,A^4{}_{18}(t)+Q_{33,20}(t)(c)\,A^4{}_{20}(t)\\
&A^4{}_{34}(t)\ =Q_{34,17}(t)(c)\,A^4{}_{17}(t)+Q_{34,19}(t)(c)\,A^4{}_{19}(t)\\
&A^4{}_{35}(t)\ =Q_{35,18}(t)(c)\,A^4{}_{18}(t)+Q_{35,22}(t)(c)\,A^4{}_{22}(t)\\
&A^4{}_{36}(t)=Q_{36,10}(t)(c)\,A^4{}_{10}(t)\\
&A^4{}_{37}(t)=\ Q_{37,9}(t)(c)\,A^4{}_{9}(t)
\end{aligned}
\quad , \qquad (4.13)
$$

where

$$
M_1(t)=e^{-(\beta_1+\beta_2)t}\,\bar{H}(t),\quad M_3(t)=e^{-(\lambda_1+\lambda_2+\beta_1+\beta_2)t}\,\bar{H}(t)
$$

$$
M_{11}(t)=e^{-(b_1+\beta_1+\beta_2)t},\ M_{12}(t)=\ e^{-(b_2+\beta_1+\beta_2)t},\ M_{13}(t)=e^{-(a_2+\beta_1+\beta_2)t}
$$

$$
M_{14}(t)=e^{-(a_1+\beta_1+\beta_2)t},M_{15}(t)=e^{-(a_1+b_1+\beta_1+\beta_2)t},\ M_{16}(t)=e^{-(a_1+b_2+\beta_1+\beta_2)t}
$$

$$
M_{17}(t)=e^{-(a_2+b_1+\beta_1+\beta_2)t},\ M_{18}(t)=e^{-(a_2+b_2+\beta_1+\beta_2)t}
$$

$$
M_{23}(t)=\bar{M}(t)=M_{27}(t),\ \bar{M}(t)=1-M(t)\ ,\ \bar{H}(t)=1-H(t)
$$

Next, for obtaining availabilities of different capacities can be found out by taking Laplace transform of set equations (4.10), (4.11), (4.12), (4.13), and solving them for $A^1{}_0{}^*(s),\ A^2{}_0{}^*(s),\ A^3{}_0{}^*(s),\ and\ A^4{}_0{}^*(s)$, we get

$$
A^1{}_0=\lim_{s\to 0} s\,A^1{}_0*(s)=\ \frac{N_{11}}{\text{derivative}(D_{11})} \qquad (4.14)
$$

$$
A^2{}_0=\lim_{s\to 0} s\,A^2{}_0*(s)=\ \frac{N_{21}}{\text{derivative}(D_{11})} \qquad (4.15)
$$

$$A^3{}_0 = \lim_{s \to 0} s\, A^3{}_0 * (s) = \frac{N_{31}}{\text{derivative}(D_{11})} \tag{4.16}$$

$$A^4{}_0 = \lim_{s \to 0} s\, A^4{}_0 * (s) = \frac{N_{41}}{\text{derivative}(D_{11})}\ , \tag{4.17}$$

where

$$D_{11} = \begin{vmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & 0 & 0 & a_{18} & a_{19} & a_{1,10} & a_{1,11} & 0 & 0 & 0 \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & 0 & 0 & a_{28} & a_{29} & a_{2,10} & a_{2,11} & a_{2,12} & a_{2,13} & a_{2,14} \\
a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & a_{38} & a_{39} & a_{3,10} & a_{3,11} & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{11} & a_{44} & a_{45} & 0 & a_{47} & a_{48} & a_{49} & a_{4,10} & a_{4,11} & 0 & 0 & 0 \\
a_{51} & 0 & 0 & a_{54} & 1 & 0 & 0 & a_{58} & 0 & a_{5,10} & 0 & a_{5,12} & a_{5,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{72} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & a_{7,10} & a_{7,11} & 0 & 0 & 0 \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & 0 & 0 & a_{88} & a_{89} & 0 & 0 & a_{8,12} & a_{8,13} & 0 \\
a_{91} & a_{92} & a_{93} & a_{94} & 0 & 0 & 0 & a_{98} & a_{99} & 0 & 0 & 0 & a_{9,13} & a_{9,14} \\
a_{10,1} & a_{10,2} & a_{10,3} & 0 & a_{10,5} & 0 & a_{10,7} & 0 & 0 & a_{10,10} & a_{10,11} & a_{10,12} & a_{10,13} & 0 \\
a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & 0 & 0 & a_{11,7} & 0 & a_{11,9} & a_{11,10} & a_{11,11} & a_{11,12} & a_{11,13} & a_{11,14} \\
0 & 0 & a_{11} & a_{12,4} & a_{12,5} & 0 & 0 & a_{12,8} & 0 & a_{12,10} & a_{12,11} & a_{12,12} & a_{12,13} & 0 \\
0 & 0 & a_{11} & a_{13,4} & a_{13,5} & 0 & 0 & a_{13,8} & a_{13,9} & a_{13,10} & a_{13,11} & a_{13,2} & a_{13,13} & 0 \\
0 & 0 & a_{14,1} & a_{14,2} & 0 & 0 & 0 & 0 & a_{14,9} & 0 & a_{14,11} & 0 & 0 & a_{14,14}
\end{vmatrix}$$

$a_{11} = 1 - q_{05} q_{50} - q_{08} p_{80}$, $a_{12} = -q_{01}$, $a_{13} = -q_{02} - q_{05} q_{52} - q_{06} q_{62}$

$a_{14} = -q_{03}$; $a_{15} = -q_{07}$, $a_{18} = -q_{05} q_{5,15}$, $a_{19} = -q_{05} q_{5,16}$, $a_{1,10} = -q_{06} q_{6,17}$

$a_{1,11} = -q_{06} q_{6,18} - q_{08} q_{8,18}$, $a_{21} = -q_{10} - q_{18} q_{80}$, $a_{22} = 1 - q_{1,12} q_{12,1} - q_{1,11} q_{11,24} q_{24,1}$

$a_{23} = -q_{12} - q_{1,11} q_{11,2} - q_{19} q_{92} - q_{1,12} q_{12,2} - q_{1,10} q_{10,23} q_{23,2}$, $a_{24} = -q_{13} - q_{19} q_{93} - q_{1,10} q_{10,3}$

$a_{25} = -q_{17}$, $a_{28} = -q_{1,10} q_{10,15}$, $a_{29} = -p_{18} p_{8,16} - p_{19} p_{9,16}$, $a_{2,10} = -p_{1,10} p_{10,17}$

$a_{2,11} = -q_{18} q_{8,18} - q_{19} q_{9,18}$, $a_{2,12} = -q_{1,11} q_{11,19}$, $a_{2,13} = -q_{1,11} q_{11,20} - q_{1,12} q_{12,20}$

$a_{2,14} = -q_{1,12} q_{12,22}$, $a_{31} = -q_{20} - q_{26} q_{60} - q_{25} q_{50}$, $a_{32} = -q_{21} - q_{2,14} q_{14,25} q_{25,1}$

$a_{33} = 1 - q_{25} q_{52} - q_{26} q_{62}$, $a_{34} = -q_{23} - q_{2,14} q_{14,3}$, $a_{38} = -q_{25} q_{5,15} - q_{2,14} q_{14,15}$

$a_{39} = -q_{25} q_{5,16} - q_{2,14} q_{14,16}$, $a_{3,10} = -q_{26} q_{6,17}$, $a_{3,11} = -q_{26} q_{6,18}$

$a_{42} = -q_{31} - q_{3,14} q_{14,25} q_{25,1}$, $a_{43} = -q_{32} - q_{35} q_{52} - q_{36} q_{62} - q_{39} q_{92} - q_{3,10} q_{10,23} q_{23,2}$

$a_{41} = -q_{30} - q_{36} q_{60} - q_{35} q_{50} - q_{38} q_{80}$, $a_{44} = 1 - q_{38} q_{83} - q_{39} q_{93} - q_{3,10} q_{10,3} - q_{3,14} q_{14,3}$

$a_{45} = -q_{37}$, $a_{47} = -q_{3,13}$, $a_{48} = -q_{35} q_{5,15} - q_{3,14} q_{14,15}$, $a_{49} = -q_{35} q_{5,16} - q_{39} q_{9,16} - q_{3,14} q_{14,16}$

$a_{4,10} = -q_{36} q_{6,17}$, $a_{51} = -q_{70}$, $a_{54} = -q_{7,21} q_{21,3}$, $a_{4,11} = -q_{39} q_{9,18} - q_{38} q_{8,18}$, $a_{51} = -q_{70}$

$a_{54} = -q_{7,21} q_{21,3}$, $a_{55} = 1$, $a_{58} = -q_{7,15}$, $a_{5,10} = -q_{7,17}$, $a_{5,12} = -q_{7,19}$, $q_{5,13} = -p_{7,20}$

$a_{72} = -q_{13,1}$, $a_{7,10} = -q_{13,17}$, $a_{7,11} = -q_{13,18}$, $a_{81} = -q_{15,5} q_{50}$, $a_{83} = -q_{15,5} q_{52} - p_{15,26} p_{26,11} p_{11,2}$

$a_{82} = -q_{15,27} q_{27,14} q_{14,25} q_{25,1} - q_{15,26} q_{26,11} q_{11,24} q_{24,1}$, $a_{84} = -q_{15,27} q_{27,14} q_{14,3}$, $a_{85} = -q_{15,7}$

$a_{88} = 1 - q_{15,27} q_{27,14} q_{14,15} - q_{15,5} q_{5,15} - q_{15,28} q_{28,15} - q_{15,29} q_{29,15}$

$a_{13,13}=1-q_{20,29}q_{29,20}$, $a_{14,2}=-q_{22,9}q_{92}$, $a_{14,3}=-q_{22,9}q_{93}$ $a_{14,9}=-q_{22,9}q_{9,16}-q_{22,31}q_{31,16}$

$a_{14,11}=-q_{22,9}q_{9,18}-q_{22,35}q_{35,18}$, $a_{14,14}=1-q_{22,31}q_{31,22}-q_{22,35}q_{35,22}$

$a_{89}=-q_{15,27}q_{27,14}q_{14,16}-q_{15,5}q_{5,16}-q_{15,29}q_{29,16}$, $a_{8,12}=-q_{15,28}q_{28,19}-q_{15,26}q_{26,11}q_{11,19}$

$a_{8,13}=-q_{15,29}q_{29,20}-q_{15,26}q_{26,11}q_{11,20}$, $a_{91}=-q_{16,5}q_{50}$, $a_{92}=-q_{16,14}q_{14,25}q_{25,1}-q_{16,30}q_{30,12}q_{12,1}$

$a_{93}=-q_{16,5}q_{52}-q_{16,30}q_{30,12}q_{12,2}$, $a_{94}=-q_{16,14}q_{16,3}$; $a_{98}=-q_{16,5}q_{5,15}-q_{16,14}q_{14,15}-q_{16,29}q_{29,15}$

$a_{99}=1-q_{16,5}q_{5,16}-q_{16,14}q_{14,16}-q_{16,29}q_{29,16}-q_{16,31}q_{29,16}$, $a_{9,13}=-q_{16,29}q_{29,20}-q_{16,30}q_{30,12}q_{12,20}$

$a_{9,14}=-q_{16,31}q_{31,22}-q_{16,30}q_{30,12}q_{12,22}$, $a_{10,1}=-q_{17,6}q_{60}$, $a_{10,2}=-q_{17,11}q_{11,24}q_{24,1}$

$a_{10,3}=-q_{17,6}q_{62}-q_{17,11}q_{11,2}$, $a_{10,5}=-q_{17,7}$, $a_{10,9}=-q_{17,32}q_{32,13}$

$a_{10,10}=1-q_{17,6}q_{6,17}-q_{17,33}q_{33,17}-q_{17,34}q_{34,17}$, $a_{10,12}=-q_{17,6}q_{6,18}-q_{17,33}q_{33,18}$

$a_{10,13}=-q_{17,11}q_{11,20}-q_{17,33}q_{33,20}$, $a_{11,1}=-q_{18,6}q_{60}-q_{18,8}q_{80}$, $a_{11,2}=-q_{18,12}q_{12,1}$

$a_{11,3}=-q_{18,6}q_{62}-q_{18,12}q_{12,2}$, $a_{11,4}=-q_{18,8}q_{83}$, $a_{11,7}=-q_{18,13}$, $a_{11,9}=-q_{18,8}q_{8,16}$

$a_{11,10}=-q_{18,6}q_{6,17}q_{18,34}q_{34,17}$, $a_{11,11}=1-q_{18,6}q_{6,18}-q_{18,8}q_{8,18}-q_{18,35}q_{35,18}$, $a_{11,12}=-q_{18,34}q_{34,19}$

$a_{11,13}=-q_{18,12}q_{12,20}-q_{18,34}q_{34,20}$, $a_{11,14}=-q_{18,12}q_{11,22}-q_{18,35}q_{35,22}$, $a_{12,4}=-q_{19,36}q_{36,10}q_{10,3}$

$a_{12,3}=-q_{19,36}q_{36,10}q_{10,23}q_{23,2}$, $a_{12,5}=-q_{19,7}$, $a_{12,8}=-q_{19,28}q_{28,15}-q_{19,36}q_{36,10}q_{10,15}$

$a_{12,10}=-q_{19,33}q_{33,17}-q_{19,36}q_{36,10}q_{10,17}$, $a_{12,11}=-q_{19,33}q_{33,18}$, $a_{12,12}=1-q_{28,19}q_{19,28}$

$a_{12,13}=-q_{19,33}q_{33,20}$, $a_{13,3}=-q_{20,10}q_{10,23}q_{23,2}-q_{20,37}q_{37,9}q_{92}$, $a_{13,8}=-q_{20,29}q_{29,15}-q_{20,10}q_{10,15}$

$a_{13,4}=-q_{20,10}q_{10,3}-q_{20,37}q_{37,9}q_{93}$, $a_{13,5}=-q_{20,7}$, $a_{13,9}=-q_{20,29}q_{29,16}-q_{20,37}q_{37,9}q_{9,16}$

$a_{13,10}=-q_{20,10}q_{10,17}-q_{20,34}q_{34,17}$, $a_{13,11}=-q_{20,37}q_{37,9}q_{9,18}$, $a_{13,12}=-q_{20,34}q_{34,19}$

$$N_{11}=\begin{vmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & 0 & 0 & a_{18} & a_{19} & a_{1,10} & a_{1,11} & 0 & 0 & 0\\
0 & a_{22} & a_{23} & a_{24} & a_{25} & 0 & 0 & a_{28} & a_{29} & a_{2,10} & a_{2,11} & a_{2,12} & a_{2,13} & a_{2,14}\\
a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & a_{38} & a_{39} & a_{3,10} & a_{3,11} & 0 & 0 & 0\\
a_{41} & a_{42} & a_{11} & a_{44} & a_{45} & 0 & a_{47} & a_{48} & a_{49} & a_{4,10} & a_{4,11} & 0 & 0 & 0\\
a_{51} & 0 & 0 & a_{54} & 1 & 0 & 0 & a_{58} & 0 & a_{5,10} & 0 & a_{5,12} & a_{5,13} & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & a_{72} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & a_{7,10} & a_{7,11} & 0 & 0 & 0\\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & 0 & 0 & a_{88} & a_{89} & 0 & 0 & a_{8,12} & a_{8,13} & 0\\
a_{91} & a_{92} & a_{93} & a_{94} & 0 & 0 & 0 & a_{98} & a_{99} & 0 & 0 & 0 & a_{9,13} & a_{9,14}\\
0 & a_{10,2} & a_{10,3} & 0 & a_{10,5} & 0 & a_{10,7} & 0 & 0 & a_{10,10} & a_{10,11} & a_{10,12} & a_{10,13} & 0\\
0 & a_{11,2} & a_{11,3} & a_{11,4} & 0 & 0 & a_{11,7} & 0 & a_{11,9} & a_{11,10} & a_{11,11} & a_{11,12} & a_{11,13} & a_{11,14}\\
0 & 0 & a_{11} & a_{12,4} & a_{12,5} & 0 & 0 & a_{12,8} & 0 & a_{12,10} & a_{12,11} & a_{12,12} & a_{12,13} & 0\\
0 & 0 & a_{11} & a_{13,4} & a_{13,5} & 0 & 0 & a_{13,8} & a_{13,9} & a_{13,10} & a_{13,11} & a_{13,2} & a_{13,13} & 0\\
0 & 0 & a_{14,1} & a_{14,2} & 0 & 0 & 0 & 0 & a_{14,9} & 0 & a_{14,11} & 0 & 0 & a_{14,14}
\end{vmatrix}$$

$a_{11}=M_0$, $a_{12}=-p_{01}$, $a_{13}=-p_{02}-p_{05}p_{52}-p_{06}p_{62}$, $a_{14}=-p_{03}$, $a_{15}=-p_{07}$

$a_{18}=-p_{05}p_{5,15}$, $a_{19}=-p_{05}p_{5,16}$, $a_{1,10}=-p_{06}p_{6,17}$, $a_{1,11}=-p_{06}p_{6,18}-p_{08}p_{8,18}$

$a_{21}=-p_{10}-p_{18}p_{80}$, $a_{22}=1-p_{1,12}p_{12,1}-p_{1,11}p_{11,24}p_{24,1}$, $a_{24}=-p_{13}-p_{19}p_{93}-p_{1,10}p_{10,3}$

$a_{23}=-p_{12}-p_{1,11}p_{11,2}-p_{19}p_{92}-p_{1,12}p_{12,2}-p_{1,10}p_{10,23}p_{23,2}$, $a_{25}=-p_{17}$, $a_{28}=-p_{1,10}\,p_{10,15}$

$a_{29}=-p_{18}p_{8,16}-p_{19}p_{9,16}$, $a_{2,10}=-p_{1,10}p_{10,17}$, $a_{2,11}=-p_{18}p_{8,18}-p_{19}p_{9,18}$, $a_{2,12}=-p_{1,11}p_{11,19}$

$a_{2,13}=-p_{1,11}p_{11,20}-p_{1,12}p_{12,20}$, $a_{2,14}=-p_{1,12}p_{12,22}$, $a_{31}=p_{2,14}p_{14,25}M_{25}$

$a_{32}=-p_{21}-p_{2,14}p_{14,25}p_{25,1}$, $a_{33}=1-p_{25}p_{52}-p_{26}p_{62}$, $a_{34}=-p_{23}-p_{2,14}p_{14,3}$

$a_{38}=-p_{25}p_{5,15}-p_{2,14}p_{14,15}$, $a_{39}=-p_{25}p_{5,16}-p_{2,14}p_{14,16}$, $a_{3,10}=-p_{26}p_{6,17}$

$a_{3,11}=-p_{26}p_{6,18}$, $a_{41}=p_{3,14}p_{14,25}M_{25}$, $a_{42}=-p_{31}-p_{3,14}p_{14,25}p_{25,1}$

$a_{43}=-p_{32}-p_{35}p_{52}-p_{36}p_{62}-p_{39}p_{92}-p_{3,10}p_{10,23}p_{23,2}$

$a_{45}=-p_{37}$, $a_{44}=1-p_{38}p_{83}-p_{39}p_{93}-p_{3,10}p_{10,3}-p_{3,14}p_{14,3}$, $a_{47}=-p_{3,13}$

$a_{48}=-p_{35}p_{5,15}-p_{3,14}p_{14,15}$, $a_{49}=-p_{35}p_{5,16}-p_{39}p_{9,16}-p_{3,14}p_{14,16}$, $a_{4,10}=-p_{36}p_{6,17}$

$a_{4,11}=-p_{39}p_{9,18}-p_{38}p_{8,18}$, $a_{51}=p_{7,21}M_{21}$, $a_{54}=-p_{7,21}p_{21,3}$, $a_{55}=1$, $a_{58}=-p_{7,15}$

$a_{5,10}=-p_{7,17}$, $a_{5,12}=-p_{7,19}$, $a_{5,13}=-p_{7,20}$, $a_{72}=-p_{13,1}$, $a_{7,10}=-p_{13,17}$, $a_{7,11}=-p_{13,18}$

$a_{81}=p_{15,27}p_{27,14}p_{14,25}M_{25}$, $a_{83}=-p_{15,5}p_{52}-p_{15,26}p_{26,11}p_{11,2}$

$a_{82}=-p_{15,27}p_{27,14}p_{14,25}p_{25,1}-p_{15,26}p_{26,11}p_{11,24}p_{24,1}$, $a_{84}=-p_{15,27}p_{27,14}p_{14,3}$, $a_{85}=-p_{15,7}$

$a_{88}=1-p_{15,27}p_{27,14}p_{14,15}-p_{15,5}p_{5,15}-p_{15,28}p_{28,15}-p_{15,29}p_{29,15}$

$a_{89}=-p_{15,27}p_{27,14}p_{14,16}-p_{15,5}p_{5,16}-p_{15,29}p_{29,16}$, $a_{8,12}=-p_{15,28}p_{28,19}-p_{15,26}p_{26,11}p_{11,19}$

$a_{8,13} =- p_{15,29} p_{29,20} - p_{15,26} p_{26,11} p_{11,20}$ ， $a_{91} = p_{16,14} p_{14,25} M_{25}$

$a_{92} =- p_{16,14} p_{14,25} p_{25,1} - p_{16,30} p_{30,12} p_{12,1}$, $a_{93} = - p_{16,5} p_{52} - p_{16,30} p_{30,12} p_{12,2}$, $a_{94} - p_{16,14} p_{16,3}$

$a_{98} =- p_{16,5} p_{5,15} - p_{16,14} p_{14,15} - p_{16,29} p_{29,15}$

$a_{99} = 1 - p_{16,5} p_{5,16} - p_{16,14} p_{14,16} - p_{16,29} p_{29,16} - p_{16,31} p_{29,16}$

$a_{9,13} = - p_{16,29} p_{29,20} - p_{16,30} p_{30,12} p_{12,20}$ ， $a_{9,14} = - p_{16,31} p_{31,22} - p_{16,30} p_{30,12} p_{12,22}$ ， $a_{10,1} =- p_{17,6} p_{60}$

$a_{10,2} = - p_{17,11} p_{11,24} p_{24,1}$, $a_{10,3} = - p_{17,6} p_{62} - p_{17,11} p_{11,2}$ ， $a_{10,5} =- p_{17,7}$, $a_{10,9} = - p_{17,32} p_{32,13}$

$a_{10,10} = 1 - p_{17,6} p_{6,17} - p_{17,33} p_{33,17} - p_{17,34} p_{34,17}$ ， $a_{10,12} = - p_{17,6} p_{6,18} - p_{17,33} p_{33,18}$

$a_{10,13} = - p_{17,11} p_{11,20} - p_{17,33} p_{33,20}$, $a_{11,1} = - p_{18,6} p_{60} - p_{18,8} p_{80}$ ， $a_{11,3} = - p_{18,6} p_{62} - p_{18,12} p_{12,2}$

$a_{11,4} = - p_{18,8} p_{83}$, $a_{11,7} = - p_{18,13}$ ， $a_{11,9} =- p_{18,8} p_{8,16}$, $a_{11,10} =- p_{18,6} p_{6,17} p_{18,34} p_{34,17}$

$a_{11,12} =- p_{18,34} p_{34,19}$, $a_{11,13} = - p_{18,12} p_{12,20} - p_{18,34} p_{34,20}$, $a_{11,14} =- p_{18,12} p_{11,22} - p_{18,35} p_{35,22}$

$a_{12,3} = - p_{19,36} p_{36,10} p_{10,23} p_{23,2}$ ， $a_{12,4} = - p_{19,36} p_{36,10} p_{10,3}$, $a_{12,5} = - p_{19,7}$, $a_{12,11} = - p_{19,33} p_{33,18}$

$a_{12,8} = - p_{19,28} p_{28,15} - p_{19,36} p_{36,10} p_{10,15}$ ， $a_{12,10} = - p_{19,33} p_{33,17} - p_{19,36} p_{36,10} p_{10,17}$

$a_{12,12} = 1 - p_{28,19} p_{19,28}$, $a_{12,13} = - p_{19,33} p_{33,20}$, $a_{13,3} = - p_{20,10} p_{10,23} p_{23,2} - p_{20,37} p_{37,9} p_{92}$

$a_{13,4} = - p_{20,10} p_{10,3} - p_{20,37} p_{37,9} p_{93}$, $a_{13,5} = - p_{20,7}$ ， $a_{13,8} = - p_{20,29} p_{29,15} - p_{20,10} p_{10,15}$

$a_{13,9} = - p_{20,29} p_{29,16} - p_{20,37} p_{37,9} p_{9,16}$ ， $a_{13,10} = - p_{20,10} p_{10,17} - p_{20,34} p_{34,17}$ ， $a_{14,3} = - p_{22,9} p_{93}$

$a_{13,11} = - p_{20,37} p_{37,9} p_{9,18}$, $a_{13,12} = - p_{20,34} p_{34,19}$, $a_{13,13} = 1 - p_{20,29} p_{29,20}$, $a_{14,2} = - p_{22,9} p_{92}$

$a_{14,9} = - p_{22,9} p_{9,16} - p_{22,31} p_{31,16}$ ， $a_{14,11} = - p_{22,9} p_{9,18} - p_{22,35} p_{35,18}$

$a_{14,14} = 1 - p_{22,31} p_{31,22} - p_{22,35} p_{35,22}$

$$N_{21} = \begin{vmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & 0 & 0 & a_{18} & a_{19} & a_{1,10} & a_{1,11} & 0 & 0 & 0 \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & 0 & 0 & a_{28} & a_{29} & a_{2,10} & a_{2,11} & a_{2,12} & a_{2,13} & a_{2,14} \\
0 & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & a_{38} & a_{39} & a_{3,10} & a_{3,11} & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{11} & a_{44} & a_{45} & 0 & a_{47} & a_{48} & a_{49} & a_{4,10} & a_{4,11} & 0 & 0 & 0 \\
a_{51} & 0 & 0 & a_{54} & 1 & 0 & 0 & a_{58} & 0 & a_{5,10} & 0 & a_{5,12} & a_{5,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{72} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & a_{7,10} & a_{7,11} & 0 & 0 & 0 \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & 0 & 0 & a_{88} & a_{89} & 0 & 0 & a_{8,12} & a_{8,13} & 0 \\
a_{91} & a_{92} & a_{93} & a_{94} & 0 & 0 & 0 & a_{98} & a_{99} & 0 & 0 & 0 & a_{9,13} & a_{9,14} \\
0 & a_{10,2} & a_{10,3} & 0 & a_{10,5} & 0 & a_{10,7} & 0 & 0 & a_{10,10} & a_{10,11} & a_{10,12} & a_{10,13} & 0 \\
a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & 0 & 0 & a_{11,7} & 0 & a_{11,9} & a_{11,10} & a_{11,11} & a_{11,12} & a_{11,13} & a_{11,14} \\
a_{12,1} & 0 & a_{11} & a_{12,4} & a_{12,5} & 0 & 0 & a_{12,8} & 0 & a_{12,10} & a_{12,11} & a_{12,12} & a_{12,13} & 0 \\
a_{13,1} & 0 & a_{11} & a_{13,4} & a_{13,5} & 0 & 0 & a_{13,8} & a_{13,9} & a_{13,10} & a_{13,11} & a_{13,2} & a_{13,13} & 0 \\
0 & 0 & a_{14,1} & a_{14,2} & 0 & 0 & 0 & 0 & a_{14,9} & 0 & a_{14,11} & 0 & 0 & a_{14,14}
\end{vmatrix}$$

$a_{11} = p_{08}M_8$, $a_{12} =- p_{01}$, $a_{13} =- p_{02} - p_{05}p_{52} - p_{06}p_{62}$

$a_{14} =- p_{03}$, $a_{15} =- p_{07}$, $a_{18} =- p_{05}p_{5,15}$, $a_{19} =- p_{05}p_{5,16}$, $a_{1,10} = - p_{06}p_{6,17}$

$a_{1,11} =- p_{06}p_{6,18} - p_{08}p_{8,18}$, $a_{21} = p_{18}M_8$, $a_{22} = 1 - p_{1,12}p_{12,1} - p_{1,11}p_{11,24}p_{24,1}$

$a_{23} =- p_{12} - p_{1,11}p_{11,2} - p_{19}p_{92} - p_{1,12}p_{12,2} - p_{1,10}p_{10,23}p_{23,2}$, $a_{24} =- p_{13} - p_{19}p_{93} - p_{1,10}p_{10,3}$

$a_{25} =- p_{17}$, $a_{28} =- p_{1,10}\, p_{10,15}$, $a_{29} =- p_{18}p_{8,16} - p_{19}p_{9,16}$, $a_{2,10} =- p_{1,10}p_{10,17}$

$a_{2,11} =- p_{18}p_{8,18} - p_{19}p_{9,18}$, $a_{2,12} =- p_{1,11}p_{11,19}$, $a_{2,13} =- p_{1,11}p_{11,20} - p_{1,12}p_{12,20}$

$a_{2,14} =- p_{1,12}p_{12,22}$, $a_{32} =- p_{21} - p_{2,14}p_{14,25}p_{25,1}$, $a_{33} = 1 - p_{25}p_{52} - p_{26}p_{62}$

$a_{34} =- p_{23} - p_{2,14}p_{14,3}$, $a_{38} =- p_{25}p_{5,15} - p_{2,14}p_{14,15}$, $a_{39} =- p_{25}p_{5,16} - p_{2,14}p_{14,16}$

$a_{3,10} = - p_{26}p_{6,17}$, $a_{3,11} = - p_{26}p_{6,18}$, $a_{41} = M_3 + p_{38}M_8$, $a_{42} =- p_{31} - p_{3,14}p_{14,25}p_{25,1}$

$a_{43} =- p_{32} - p_{35}p_{52} - p_{36}p_{62} - p_{39}p_{92} - p_{3,10}p_{10,23}p_{23,2}$, $a_{45} =- p_{37}$

$a_{44} = 1 - p_{38}p_{83} - p_{39}p_{93} - p_{3,10}p_{10,3} - p_{3,14}p_{14,3}$, $a_{47} = - p_{3,13}$

$a_{48} =- p_{35}p_{5,15} - p_{3,14}p_{14,15}$, $a_{49} =- p_{35}p_{5,16} - p_{39}p_{9,16} - p_{3,14}p_{14,16}$, $a_{4,10} = - p_{36}p_{6,17}$

$a_{4,11} =- p_{39}p_{9,18} - p_{38}p_{8,18}$, $a_{51} = M_7$, $a_{54} = - p_{7,21}p_{21,3}$, $a_{54} =- p_{37}$, $a_{55} = 1$

$a_{58} = - p_{7,15}$, $a_{58} =- p_{7,15}$, $a_{5,10} =- p_{7,17}$, $a_{5,12} =- p_{7,19}$, $a_{5,13} =- p_{7,20}$, $a_{72} =- p_{13,1}$

$a_{7,10} =- p_{13,17}$, $a_{7,11} =- p_{13,18}$, $a_{81} = p_{15,26}M_{26}$, $a_{83} =- p_{15,5}p_{52} - p_{15,26}p_{26,11}p_{11,2}$

$a_{82} =- p_{15,27}p_{27,14}p_{14,25}p_{25,1} - p_{15,26}p_{26,11}p_{11,24}p_{24,1}$, $a_{84} =- p_{15,27}p_{27,14}p_{14,3}$, $a_{85} = - p_{15,7}$

$a_{88} = 1 - p_{15,27}p_{27,14}p_{14,15} - p_{15,5}p_{5,15} - p_{15,28}p_{28,15} - p_{15,29}p_{29,15}$

$a_{89} =- p_{15,27}p_{27,14}p_{14,16} - p_{15,5}p_{5,16} - p_{15,29}p_{29,16}$, $a_{8,12} =- p_{15,28}p_{28,19} - p_{15,26}p_{26,11}p_{11,19}$

$a_{8,13} =- p_{15,29}p_{29,20} - p_{15,26}p_{26,11}p_{11,20}$, $a_{91} = p_{16,30}M_{30}$, $a_{92} =- p_{16,14}p_{14,25}p_{25,1} - p_{16,30}p_{30,12}p_{12,1}$

$a_{93} = - p_{16,5}p_{52} - p_{16,30}p_{30,12}p_{12,2}$, $a_{94} - p_{16,14}p_{16,3}$, $a_{98} =- p_{16,5}p_{5,15} - p_{16,14}p_{14,15} - p_{16,29}p_{29,15}$

$a_{99} = 1 - p_{16,5}p_{5,16} - p_{16,14}p_{14,16} - p_{16,29}p_{29,16} - p_{16,31}p_{29,16}$, $a_{9,13} = - p_{16,29}p_{29,20} - p_{16,30}p_{30,12}p_{12,20}$

$a_{9,14} = - p_{16,31}p_{31,22} - p_{16,30}p_{30,12}p_{12,22}$, $a_{10,2} = - p_{17,11}p_{11,24}p_{24,1}$, $a_{10,3} = - p_{17,6}p_{62} - p_{17,11}p_{11,2}$

$a_{10,5} =- p_{17,7}$, $a_{10,9} = - p_{17,32}p_{32,13}$, $a_{10,10} = 1 - p_{17,6}p_{6,17} - p_{17,33}p_{33,17} - p_{17,34}p_{34,17}$

$a_{10,12} = - p_{17,6}p_{6,18} - p_{17,33}p_{33,18}$, $a_{10,13} = - p_{17,11}p_{11,20} - p_{17,33}p_{33,20}$, $a_{11,1} = p_{18,8}M_8$

$a_{11,2} = - p_{18,12}p_{12,1}$, $a_{11,3} = - p_{18,6}p_{62} - p_{18,12}p_{12,2}$, $a_{11,4} = - p_{18,8}p_{83}$

$a_{11,7} = - p_{18,13}$, $a_{11,9} =- p_{18,8}p_{8,16}$, $a_{11,10} =- p_{18,6}p_{6,17}p_{18,34}p_{34,17}$

$a_{11,11} = 1 - p_{18,6}p_{6,18} - p_{18,8}p_{8,18} - p_{18,35}p_{35,18}$, $a_{11,12} =- p_{18,34}p_{34,19}$

$a_{11,13} = - p_{18,12}p_{12,20} - p_{18,34}p_{34,20}$, $a_{11,14} =- p_{18,12}p_{11,22} - p_{18,35}p_{35,22}$

$a_{12,1} = p_{19,36}M_{36}$, $a_{12,3} = - p_{19,36}p_{36,10}p_{10,23}p_{23,2}$, $a_{12,4} = - p_{19,36}p_{36,10}p_{10,3}$

$a_{12,5} = - p_{19,7}$, $a_{12,8} = - p_{19,28}p_{28,15} - p_{19,36}p_{36,10}p_{10,15}$, $a_{12,11} = - p_{19,33}p_{33,18}$

$a_{12,10} = - p_{19,33}p_{33,17} - p_{19,36}p_{36,10}p_{10,17}$, $a_{12,12} = 1 - p_{28,19}p_{19,28}$, $a_{12,13} = - p_{19,33}p_{33,20}$

$a_{13,1} = p_{20,37}M_{37}$, $a_{13,3} = - p_{20,10}p_{10,23}p_{23,2} - p_{20,37}p_{37,9}p_{92}$, $a_{13,5} = - p_{20,7}$

$a_{13,4} = - p_{20,10}p_{10,3} - p_{20,37}p_{37,9}p_{93}$, $a_{13,8} = - p_{20,29}p_{29,15} - p_{20,10}p_{10,15}$, $a_{13,13} = 1 - p_{20,29}p_{29,20}$

$a_{13,9} = -p_{20,29}p_{29,16} - p_{20,37}p_{37,9}p_{9,16}$, $a_{13,10} = -p_{20,10}p_{10,17} - p_{20,34}p_{34,17}$, $a_{14,2} = -p_{22,9}p_{92}$

$a_{13,11} = -p_{20,37}p_{37,9}p_{9,18}$, $a_{13,12} = -p_{20,34}p_{34,19}$, $a_{14,3} = -p_{22,9}p_{93}$, $a_{14,9} = -p_{22,9}p_{9,16} - p_{22,31}p_{31,16}$

$a_{14,11} = -p_{22,9}p_{9,18} - p_{22,35}p_{35,18}$, $a_{14,14} = 1 - p_{22,31}p_{31,22} - p_{22,35}p_{35,22}$

$$N_{31} = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & 0 & 0 & a_{18} & a_{19} & a_{1,10} & a_{1,11} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & 0 & 0 & a_{28} & a_{29} & a_{2,10} & a_{2,11} & a_{2,12} & a_{2,13} & a_{2,14} \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & a_{38} & a_{39} & a_{3,10} & a_{3,11} & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{11} & a_{44} & a_{45} & 0 & a_{47} & a_{48} & a_{49} & a_{4,10} & a_{4,11} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{54} & 1 & 0 & 0 & a_{58} & 0 & a_{5,10} & 0 & a_{5,12} & a_{5,13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{72} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & a_{7,10} & a_{7,11} & 0 & 0 & 0 \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & 0 & 0 & a_{88} & a_{89} & 0 & 0 & a_{8,12} & a_{8,13} & 0 \\ a_{91} & a_{92} & a_{93} & a_{94} & 0 & 0 & 0 & a_{98} & a_{99} & 0 & 0 & 0 & a_{9,13} & a_{9,14} \\ a_{10,1} & a_{10,2} & a_{10,3} & 0 & a_{10,5} & 0 & a_{10,7} & 0 & 0 & a_{10,10} & a_{10,11} & a_{10,12} & a_{10,13} & 0 \\ a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & 0 & 0 & a_{11,7} & 0 & a_{11,9} & a_{11,10} & a_{11,11} & a_{11,12} & a_{11,13} & a_{11,14} \\ a_{12,1} & 0 & a_{11} & a_{12,4} & a_{12,5} & 0 & 0 & a_{12,8} & 0 & a_{12,10} & a_{12,11} & a_{12,12} & a_{12,13} & 0 \\ a_{13,1} & 0 & a_{11} & a_{13,4} & a_{13,5} & 0 & 0 & a_{13,8} & a_{13,9} & a_{13,10} & a_{13,11} & a_{13,2} & a_{13,13} & 0 \\ a_{14,1} & 0 & a_{14,1} & a_{14,2} & 0 & 0 & 0 & 0 & a_{14,9} & 0 & a_{14,11} & 0 & 0 & a_{14,14} \end{vmatrix}$$

$a_{11} = p_{05}M_5 + p_{06}M_6$, $a_{12} = -p_{01}$, $a_{13} = -p_{02} - p_{05}p_{52} - p_{06}p_{62}$

$a_{14} = -p_{03}$; $a_{15} = -p_{07}$, $a_{18} = -p_{05}p_{5,15}$, $a_{19} = -p_{05}p_{5,16}$, $a_{1,10} = -p_{06}p_{6,17}$

$a_{1,11} = -p_{06}p_{6,18} - p_{08}p_{8,18}$, $a_{21} = p_{19}M_9 + p_{1,10}M_{10}p_{1,11}p_{11,24}M_{24}$

$a_{22} = 1 - p_{1,12}p_{12,1} - p_{1,11}p_{11,24}p_{24,1}$, $a_{23} = -p_{12} - p_{1,11}p_{11,2} - p_{19}p_{92} - p_{1,12}p_{12,2} - p_{1,10}p_{10,23}p_{23,2}$

$a_{24} = -p_{13} - p_{19}p_{93} - p_{1,10}p_{10,3}$, $a_{25} = -p_{17}$, $a_{28} = -p_{1,10}\,p_{10,15}$, $a_{29} = -p_{18}p_{8,16} - p_{19}p_{9,16}$

$a_{2,10} = -p_{1,10}p_{10,17}$, $a_{2,11} = -p_{18}p_{8,18} - p_{19}p_{9,18}$, $a_{2,12} = -p_{1,11}p_{11,19}$, $a_{2,13} = -p_{1,11}p_{11,20} - p_{1,12}p_{12,20}$

$a_{2,14} = -p_{1,12}p_{12,22}$, $a_{31} = M_2 + p_{25}M_5 + p_{26}M_6$, $a_{32} = -p_{21} - p_{2,14}p_{14,25}p_{25,1}$

$a_{33} = 1 - p_{25}p_{52} - p_{26}p_{62}$, $a_{34} = -p_{23} - p_{2,14}p_{14,3}$, $a_{38} = -p_{25}p_{5,15} - p_{2,14}p_{14,15}$

$a_{39} = -p_{25}p_{5,16} - p_{2,14}p_{14,16}$, $a_{3,10} = -p_{26}p_{6,17}$, $a_{3,11} = -p_{26}p_{6,18}$, $a_{45} = -p_{37}$

$a_{41} = p_{35}M_5 + p_{36}M_6p_{39}M_9 + p_{3,10}M_{10}$, $a_{42} = -p_{31} - p_{3,14}p_{14,25}p_{25,1}$

$a_{43} = -p_{32} - p_{35}p_{52} - p_{36}p_{62} - p_{39}p_{92} - p_{3,10}p_{10,23}p_{23,2}$, $a_{47} = -p_{3,13}$

$a_{44} = 1 - p_{38}p_{83} - p_{39}p_{93} - p_{3,10}p_{10,3} - p_{3,14}p_{14,3}$, $a_{48} = -p_{35}p_{5,15} - p_{3,14}p_{14,15}$

$a_{49} = -p_{35}p_{5,16} - p_{39}p_{9,16} - p_{3,14}p_{14,16}$, $a_{4,10} = -p_{36}p_{6,17}$, $a_{4,11} = -p_{39}p_{9,18} - p_{38}p_{8,18}$

$a_{54} = -p_{7,21}p_{21,3}$, $a_{55} = 1$, $a_{58} = -p_{7,15}$, $a_{5,10} = -p_{7,17}$, $a_{5,12} = -p_{7,19}$, $a_{5,13} = -p_{7,20}$

$a_{72} =- p_{13,1}$ ， $a_{7,10} =- p_{13,17}$ ， $a_{7,11} =- p_{13,18}$ ， $a_{81} = p_{15,5} M_5 + p_{15,26} p_{26,11} p_{11,24} M_{24}$

$a_{82} =- p_{15,27} p_{27,14} p_{14,25} p_{25,1} - p_{15,26} p_{26,11} p_{11,24} p_{24,1}$ ， $a_{83} =- p_{15,5} p_{52} - p_{15,26} p_{26,11} p_{11,2}$

$a_{84} =- p_{15,27} p_{27,14} p_{14,3}$ ， $a_{85} = - p_{15,7}$

$a_{88} = 1 - p_{15,27} p_{27,14} p_{14,15} - p_{15,5} p_{5,15} - p_{15,28} p_{28,15} - p_{15,29} p_{29,15}$

$a_{89} =- p_{15,27} p_{27,14} p_{14,16} - p_{15,5} p_{5,16} - p_{15,29} p_{29,16}$ ， $a_{8,12} =- p_{15,28} p_{28,19} - p_{15,26} p_{26,11} p_{11,19}$

$a_{8,13} =- p_{15,29} p_{29,20} - p_{15,26} p_{26,11} p_{11,20}$ ， $a_{91} = p_{16,5} M_5$

$a_{92} =- p_{16,14} p_{14,25} p_{25,1} - p_{16,30} p_{30,12} p_{12,1}$ ， $a_{93} = - p_{16,5} p_{52} - p_{16,30} p_{30,12} p_{12,2}$

$a_{94} - p_{16,14} p_{16,3}$ ； $a_{98} =- p_{16,5} p_{5,15} - p_{16,14} p_{14,15} - p_{16,29} p_{29,15}$

$a_{99} = 1 - p_{16,5} p_{5,16} - p_{16,14} p_{14,16} - p_{16,29} p_{29,16} - p_{16,31} p_{29,16}$

$a_{9,13} = - p_{16,29} p_{29,20} - p_{16,30} p_{30,12} p_{12,20}$

$a_{9,14} = - p_{16,31} p_{31,22} - p_{16,30} p_{30,12} p_{12,22}$ ， $a_{10,1} = p_{17,6} M_6 + p_{17,32} M_{32} p_{17,11} p_{11,24} M_{24}$

$a_{10,2} = - p_{17,11} p_{11,24} p_{24,1}$ ， $a_{10,3} = - p_{17,6} p_{62} - p_{17,11} p_{11,2}$ ， $a_{10,5} =- p_{17,7}$ ， $a_{10,9} = - p_{17,32} p_{32,13}$

$a_{10,10} = 1 - p_{17,6} p_{6,17} - p_{17,33} p_{33,17} - p_{17,34} p_{34,17}$ ， $a_{10,12} = - p_{17,6} p_{6,18} - p_{17,33} p_{33,18}$

$a_{10,13} = - p_{17,11} p_{11,20} - p_{17,33} p_{33,20}$ ， $a_{11,1} = p_{18,6} M_6$ ， $a_{11,2} = - p_{18,12} p_{12,1}$

$a_{11,3} = - p_{18,6} p_{62} - p_{18,12} p_{12,2}$ ， $a_{11,4} = - p_{18,8} p_{83}$ ， $a_{11,7} = - p_{18,13}$ ， $a_{11,9} =- p_{18,8} p_{8,16}$

$a_{11,10} =- p_{18,6} p_{6,17} p_{18,34} p_{34,17}$ ， $a_{11,11} = 1 - p_{18,6} p_{6,18} - p_{18,8} p_{8,18} - p_{18,35} p_{35,18}$

$a_{11,13} = - p_{18,12} p_{12,20} - p_{18,34} p_{34,20}$ ， $a_{11,14} =- p_{18,12} p_{11,22} - p_{18,35} p_{35,22}$

$a_{12,1} = M_{19} + p_{19,36} p_{36,10} M_{10}$ ， $a_{12,3} = - p_{19,36} p_{36,10} p_{10,23} p_{23,2}$ ， $a_{12,4} = - p_{19,36} p_{36,10} p_{10,3}$

$a_{12,5} = - p_{19,7}$ ， $a_{12,8} = - p_{19,28} p_{28,15} - p_{19,36} p_{36,10} p_{10,15}$ ， $a_{11,12} =- p_{18,34} p_{34,19}$

$a_{12,10} = - p_{19,33} p_{33,17} - p_{19,36} p_{36,10} p_{10,17}$ ， $a_{12,11} = - p_{19,33} p_{33,18}$ ， $a_{12,12} = 1 - p_{28,19} p_{19,28}$

$a_{12,13} = - p_{19,33} p_{33,20}$ ， $a_{13,1} = M_{20} + p_{20,37} p_{37,9} M_9$ ， $a_{13,3} = - p_{20,10} p_{10,23} p_{23,2} - p_{20,37} p_{37,9} p_{92}$

$a_{13,8} = - p_{20,29} p_{29,15} - p_{20,10} p_{10,15}$ ， $a_{13,4} = - p_{20,10} p_{10,3} - p_{20,37} p_{37,9} p_{93}$ ， $a_{13,5} = - p_{20,7}$

$a_{14,1} = M_{22}$ ， $a_{13,9} = - p_{20,29} p_{29,16} - p_{20,37} p_{37,9} p_{9,16}$ ， $a_{13,10} = - p_{20,10} p_{10,17} - p_{20,34} p_{34,17}$

$a_{13,11} = - p_{20,37} p_{37,9} p_{9,18}$ ， $a_{13,12} = - p_{20,34} p_{34,19}$ ， $a_{13,13} = 1 - p_{20,29} p_{29,20}$ ， $a_{14,2} = - p_{22,9} p_{92}$

$a_{14,3} = - p_{22,9} p_{93}$ ， $a_{14,9} = - p_{22,9} p_{9,16} - p_{22,31} p_{31,16}$

$a_{14,11} = - p_{22,9} p_{9,18} - p_{22,35} p_{35,18}$ ， $a_{14,14} = 1 - p_{22,31} p_{31,22} - p_{22,35} p_{35,22}$

$$N_{41}=\begin{vmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & 0 & 0 & a_{18} & a_{19} & a_{1,10} & a_{1,11} & 0 & 0 & 0\\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & 0 & 0 & a_{28} & a_{29} & a_{2,10} & a_{2,11} & a_{2,12} & a_{2,13} & a_{2,14}\\
0 & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & a_{38} & a_{39} & a_{3,10} & a_{3,11} & 0 & 0 & 0\\
a_{41} & a_{42} & a_{11} & a_{44} & a_{45} & 0 & a_{47} & a_{48} & a_{49} & a_{4,10} & a_{4,11} & 0 & 0 & 0\\
a_{51} & 0 & 0 & a_{54} & 1 & 0 & 0 & a_{58} & 0 & a_{5,10} & 0 & a_{5,12} & a_{5,13} & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & a_{72} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & a_{7,10} & a_{7,11} & 0 & 0 & 0\\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & 0 & 0 & a_{88} & a_{89} & 0 & 0 & a_{8,12} & a_{8,13} & 0\\
a_{91} & a_{92} & a_{93} & a_{94} & 0 & 0 & 0 & a_{98} & a_{99} & 0 & 0 & 0 & a_{9,13} & a_{9,14}\\
0 & a_{10,2} & a_{10,3} & 0 & a_{10,5} & 0 & a_{10,7} & 0 & 0 & a_{10,10} & a_{10,11} & a_{10,12} & a_{10,13} & 0\\
a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & 0 & 0 & a_{11,7} & 0 & a_{11,9} & a_{11,10} & a_{11,11} & a_{11,12} & a_{11,13} & a_{11,14}\\
a_{12,1} & 0 & a_{11} & a_{12,4} & a_{12,5} & 0 & 0 & a_{12,8} & 0 & a_{12,10} & a_{12,11} & a_{12,12} & a_{12,13} & 0\\
a_{13,1} & 0 & a_{11} & a_{13,4} & a_{13,5} & 0 & 0 & a_{13,8} & a_{13,9} & a_{13,10} & a_{13,11} & a_{13,2} & a_{13,13} & 0\\
0 & 0 & a_{14,1} & a_{14,2} & 0 & 0 & 0 & 0 & a_{14,9} & 0 & a_{14,11} & 0 & 0 & a_{14,14}
\end{vmatrix}$$

$a_{12}=-p_{01}$, $a_{13}=-p_{02}-p_{05}p_{52}-p_{06}p_{62}$, $a_{14}=-p_{03}$; $a_{15}=-p_{07}$, $a_{18}=-p_{05}p_{5,15}$

$a_{19}=-p_{05}p_{5,16}$, $a_{1,10}=-p_{06}p_{6,17}$, $a_{21}=p_{1,12}M_{12}+p_{1,11}M_{11}p_{1,10}p_{10,23}M_{23}+M_{1}$

$a_{1,11}=-p_{06}p_{6,18}-p_{08}p_{8,18}$, $a_{22}=1-p_{1,12}p_{12,1}-p_{1,11}p_{11,24}p_{24,1}$

$a_{23}=-p_{12}-p_{1,11}p_{11,2}-p_{19}p_{92}-p_{1,12}p_{12,2}-p_{1,10}p_{10,23}p_{23,2}$, $a_{24}=-p_{13}-p_{19}p_{93}-p_{1,10}p_{10,3}$

$a_{25}=-p_{17}$, $a_{28}=-p_{1,10}\,p_{10,15}$, $a_{29}=-p_{18}p_{8,16}-p_{19}p_{9,16}$, $a_{2,10}=-p_{1,10}p_{10,17}$

$a_{2,11}=-p_{18}p_{8,18}-p_{19}p_{9,18}$, $a_{2,12}=-p_{1,11}p_{11,19}$, $a_{2,13}=-p_{1,11}p_{11,20}-p_{1,12}p_{12,20}$

$a_{2,14}=-p_{1,12}p_{12,22}$, $a_{31}=p_{2,14}M_{14}$, $a_{32}=-p_{21}-p_{2,14}p_{14,25}p_{25,1}$, $a_{34}=-p_{23}-p_{2,14}p_{14,3}$

$a_{33}=1-p_{25}p_{52}-p_{26}p_{62}$, $a_{38}=-p_{25}p_{5,15}-p_{2,14}p_{14,15}$, $a_{39}=-p_{25}p_{5,16}-p_{2,14}p_{14,16}$

$a_{3,10}=-p_{26}p_{6,17}$, $a_{3,11}=-p_{26}p_{6,18}$, $a_{41}=p_{3,14}M_{14}+p_{3,10}p_{10,23}M_{23}$, $a_{42}=-p_{31}-p_{3,14}p_{14,25}p_{25,1}$

$a_{43}=-p_{32}-p_{35}p_{52}-p_{36}p_{62}-p_{39}p_{92}-p_{3,10}p_{10,23}p_{23,2}$, $a_{45}=-p_{37}$, $a_{47}=-p_{3,13}$

$a_{44}=1-p_{38}p_{83}-p_{39}p_{93}-p_{3,10}p_{10,3}-p_{3,14}p_{14,3}$, $a_{48}=-p_{35}p_{5,15}-p_{3,14}p_{14,15}$

$a_{49}=-p_{35}p_{5,16}-p_{39}p_{9,16}-p_{3,14}p_{14,16}$, $a_{4,10}=-p_{36}p_{6,17}$, $a_{4,11}=-p_{39}p_{9,18}-p_{38}p_{8,18}$, $a_{51}=-p_{70}$

$a_{54}=-p_{7,21}p_{21,3}$, $a_{55}=1$, $a_{58}=-p_{7,15}$, $a_{5,10}=-p_{7,17}$, $a_{5,12}=-p_{7,19}$, $a_{5,13}=-p_{7,20}$, $a_{71}=M_{13}$

$a_{72}=-p_{13,1}$, $a_{7,10}=-p_{13,17}$, $a_{7,11}=-p_{13,18}$, $a_{81}=M_{15}+p_{15,27}p_{27,14}M_{14}$

$a_{82} = -p_{15,27}p_{27,14}p_{14,25}p_{25,1} - p_{15,26}p_{26,11}p_{11,24}p_{24,1}$, $a_{83} = -p_{15,5}p_{52} - p_{15,26}p_{26,11}p_{11,2}$

$a_{84} = -p_{15,27}p_{27,14}p_{14,3}$, $a_{85} = -p_{15,7}$, $a_{9,13} = -p_{16,29}p_{29,20} - p_{16,30}p_{30,12}p_{12,20}$

$a_{88} = 1 - p_{15,27}p_{27,14}p_{14,15} - p_{15,5}p_{5,15} - p_{15,28}p_{28,15} - p_{15,29}p_{29,15}$

$a_{89} = -p_{15,27}p_{27,14}p_{14,16} - p_{15,5}p_{5,16} - p_{15,29}p_{29,16}$, $a_{8,12} = -p_{15,28}p_{28,19} - p_{15,26}p_{26,11}p_{11,19}$

$a_{8,13} = -p_{15,29}p_{29,20} - p_{15,26}p_{26,11}p_{11,20}$, $a_{92} = -p_{16,14}p_{14,25}p_{25,1} - p_{16,30}p_{30,12}p_{12,1}$

$a_{93} = -p_{16,5}p_{52} - p_{16,30}p_{30,12}p_{12,2}$, $a_{94} - p_{16,14}p_{16,3}$, $a_{98} = -p_{16,5}p_{5,15} - p_{16,14}p_{14,15} - p_{16,29}p_{29,15}$

$a_{99} = 1 - p_{16,5}p_{5,16} - p_{16,14}p_{14,16} - p_{16,29}p_{29,16} - p_{16,31}p_{29,16}$

$a_{9,14} = -p_{16,31}p_{31,22} - p_{16,30}p_{30,12}p_{12,22}$, $a_{10,1} = M_{17} + p_{17,11}M_{11}$, $a_{10,2} = -p_{17,11}p_{11,24}p_{24,1}$

$a_{10,3} = -p_{17,6}p_{62} - p_{17,11}p_{11,2}$, $a_{10,5} = -p_{17,7}$, $a_{10,9} = -p_{17,32}p_{32,13}$

$a_{10,10} = 1 - p_{17,6}p_{6,17} - p_{17,33}p_{33,17} - p_{17,34}p_{34,17}$, $a_{10,12} = -p_{17,6}p_{6,18} - p_{17,33}p_{33,18}$

$a_{10,13} = -p_{17,11}p_{11,20} - p_{17,33}p_{33,20}$, $a_{11,1} = M_{18} + p_{18,12}M_{12}$, $a_{11,2} = -p_{18,12}p_{12,1}$

$a_{11,3} = -p_{18,6}p_{62} - p_{18,12}p_{12,2}$, $a_{11,4} = -p_{18,8}p_{83}$, $a_{11,7} = -p_{18,13}$, $a_{11,9} = -p_{18,8}p_{8,16}$

$a_{11,10} = -p_{18,6}p_{6,17}p_{18,34}p_{34,17}$, $a_{11,11} = 1 - p_{18,6}p_{6,18} - p_{18,8}p_{8,18} - p_{18,35}p_{35,18}$

$a_{11,12} = -p_{18,34}p_{34,19}$, $a_{11,13} = -p_{18,12}p_{12,20} - p_{18,34}p_{34,20}$, $a_{11,14} = -p_{18,12}p_{11,22} - p_{18,35}p_{35,22}$

$a_{12,1} = M_{19} + p_{19,36}\, p_{36,10}p_{10,23}M_{10}$, $a_{12,3} = -p_{19,36}p_{36,10}p_{10,23}p_{23,2}$, $a_{12,4} = -p_{19,36}p_{36,10}p_{10,3}$

$a_{12,5} = -p_{19,7}$, $a_{12,8} = -p_{19,28}p_{28,15} - p_{19,36}p_{36,10}p_{10,15}$, $a_{12,10} = -p_{19,33}p_{33,17} - p_{19,36}p_{36,10}p_{10,17}$

$a_{12,11} = -p_{19,33}p_{33,18}$, $a_{12,12} = 1 - p_{28,19}p_{19,28}$, $a_{12,13} = -p_{19,33}p_{33,20}$

$a_{13,3} = -p_{20,10}p_{10,23}p_{23,2} - p_{20,37}p_{37,9}p_{92}$, $a_{13,4} = -p_{20,10}p_{10,3} - p_{20,37}p_{37,9}p_{93}$, $a_{13,5} = -p_{20,7}$

$a_{13,8} = -p_{20,29}p_{29,15} - p_{20,10}p_{10,15}$, $a_{13,9} = -p_{20,29}p_{29,16} - p_{20,37}p_{37,9}p_{9,16}$

$a_{13,10} = -p_{20,10}p_{10,17} - p_{20,34}p_{34,17}$, $a_{13,11} = -p_{20,37}p_{37,9}p_{9,18}$, $a_{13,12} = -p_{20,34}p_{34,19}$

$a_{13,13} = 1 - p_{20,29}p_{29,20}$, $a_{14,2} = -p_{22,9}p_{92}$, $a_{14,3} = -p_{22,9}p_{93}$, $a_{14,9} = -p_{22,9}p_{9,16} - p_{22,31}p_{31,16}$

$a_{14,11} = -p_{22,9}p_{9,18} - p_{22,35}p_{35,18}$, $a_{14,14} = 1 - p_{22,31}p_{31,22} - p_{22,35}p_{35,22}$

4.6 Busy period of repairman

The recursive relation for busy period $B_i(t)$ can be obtained as:

$$B_0(t) = Q_{01}(t)(c)B_1(t) + Q_{02}(t)(c)B_2(t) + Q_{03}(t)(c)B_3(t) + Q_{04}(t)(c)B_4(t) + Q_{05}(t)(c)B_5(t) + Q_{06}(t)(c)B_6(t) + Q_{07}(t)(c)B_7(t) + Q_{08}(t)(c)B_8(t)$$

$$B_1(t) = Q_{10}(t)(c)B_0(t) + Q_{12}(t)(c)B_2(t) + Q_{13}(t)(c)B_3(t) + Q_{17}(t)(c)B_7(t) + Q_{18}(t)(c)B_8(t) + Q_{19}(t)(c)B_9(t) + Q_{1,11}(t)(c)B_{11}(t) + Q_{1,12}(t)(s)B_{12}(t)$$

$$B_2(t) = Q_{20}(t)(c)B_0(t) + Q_{21}(t)(c)B_1(t) + Q_{23}(t)(c)B_3(t) + Q_{25}(t)(c)B_5(t) + Q_{2,13}(t)(c)B_{13}(t) + Q_{2,14}(t)(c)B_{14}(t)$$

$$B_3(t)=M_3(t)+Q_{30}(t)\text{(c)}B_0(t)+Q_{31}(t)\text{(c)}B_1(t)+Q_{32}(t)\text{(c)}B_2(t)+Q_{35}(t)\text{(c)}B_5(t)$$
$$+Q_{36}(t)\text{(c)}B_7(t)+Q_{38}(t)\text{(c)}B_8(t)+Q_{39}(t)\text{(c)}B_9(t)+Q_{3,10}(t)\text{(c)}B_{10}(t)$$
$$+Q_{3,13}(t)\text{(c)}B_{3,13}(t)+Q_{3,14}(t)\text{(c)}B_{3,14}(t)$$
$$B_4(t)=Q_{40}(t)\text{(c)}B_0(t)$$
$$B_5(t)=Q_{50}(t)\text{(c)}B_0(t)+Q_{52}(t)\text{(c)}B_2(t)+Q_{5,15}(t)\text{(c)}B_{15}(t)+Q_{5,16}(t)\text{(c)}B_{16}(t)$$
$$B_6(t)=Q_{60}(t)\text{(c)}B_0(t)+Q_{62}(t)\text{(c)}B_2(t)+Q_{6,17}(t)\text{(c)}B_{17}(t)+Q_{6,18}(t)\text{(c)}B_{18}(t)$$
$$B_7(t)=M_7(t)+Q_{70}(t)\text{(c)}B_0(t)+Q_{7,15}(t)\text{(c)}B_{15}(t)+Q_{7,19}(t)\text{(c)}B_{19}(t)$$
$$+Q_{7,20}(t)\text{(c)}B_{20}(t)+Q_{7,21}(t)\text{(c)}B_{21}(t)$$
$$B_8(t)=M_8(t)+Q_{80}(t)\text{(c)}B_0(t)+Q_{83}(t)\text{(c)}B_3(t)+Q_{8,16}(t)\text{(c)}B_{16}(t)$$
$$+Q_{8,18}(t)\text{(c)}B_{18}(t)+Q_{8,20}(t)\text{(c)}B_{20}(t)+Q_{8,22}(t)\text{(c)}B_{22}(t)$$
$$B_9(t)=Q_{92}(t)\text{(c)}B_2(t)+Q_{93}(t)\text{(c)}B_3(t)+Q_{9,16}(t)\text{(c)}B_{16}(t)+Q_{18}(t)\text{(c)}B_{18}(t)$$
$$B_{10}(t)=Q_{10,3}(t)\text{(c)}B_3(t)+Q_{10,15}(t)\text{(c)}B_{15}(t)+Q_{10,17}(t)\text{(c)}B_{17}(t)+Q_{10,18}(t)\text{(c)}B_{18}(t)$$
$$B_{11}(t)=Q_{11,2}(t)\text{(c)}B_2(t)+Q_{11,19}(t)\text{(c)}B_{19}(t)+Q_{11,20}(t)\text{(c)}B_{20}(t)+Q_{11,24}(t)\text{(c)}B_{24}(t)$$
$$B_{12}(t)=Q_{12,1}(t)\text{(c)}B_1(t)+Q_{12,2}(t)\text{(c)}B_2(t)+Q_{12,20}(t)\text{(c)}B_{20}(t)+Q_{12,22}(t)\text{(c)}B_{22}(t)$$
$$B_{13}(t)=Q_{13,1}(t)\text{(c)}B_1(t)+Q_{13,3}(t)\text{(c)}B_3(t)+Q_{13,17}(t)\text{(c)}B_{17}(t)+Q_{13,18}(t)\text{(c)}B_{18}(t)$$
$$B_{14}(t)=Q_{14,3}(t)\text{(c)}B_3(t)+Q_{14,15}(t)\text{(c)}B_{15}(t)+Q_{14,16}(t)\text{(c)}B_{16}(t)+Q_{14,25}(t)\text{(c)}B_{25}(t)$$
$$B_{15}(t)=Q_{15,5}(t)\text{(c)}B_5(t)+Q_{15,7}(t)\text{(c)}B_7(t)+Q_{15,26}(t)\text{(c)}B_{26}(t)+Q_{15,27}(t)\text{(c)}B_{27}(t)$$
$$+Q_{15,28}(t)\text{(c)}B_{28}(t)+Q_{15,29}(t)\text{(c)}B_{29}(t)$$
$$B_{16}(t)=Q_{16,5}(t)\text{(c)}B_5(t)+Q_{16,8}(t)\text{(c)}B_8(t)+Q_{16,14}(t)\text{(c)}B_{14}(t)+Q_{16,30}(t)\text{(c)}B_{30}(t)$$
$$+Q_{16,31}(t)\text{(c)}B_{31}(t)+Q_{16,29}(t)\text{(c)}B_{29}(t)$$
$$B_{17}(t)=Q_{17,6}(t)\text{(c)}B_6(t)+Q_{17,7}(t)\text{(c)}B_7(t)+Q_{17,11}(t)(s)B_{11}(t)+Q_{17,32}(t)\text{(c)}B_{32}(t)$$
$$+Q_{17,33}(t)\text{(c)}B_{33}(t)+Q_{17,34}(t)\text{(c)}B_{34}(t)$$
$$B_{18}(t)=Q_{18,6}(t)\text{(c)}B_6(t)+Q_{18,6}(t)\text{(c)}B_6(t)+Q_{18,13}(t)\text{(c)}B_{13}(t)+Q_{18,12}(t)\text{(c)}B_{12}(t)$$
$$+Q_{18,34}(t)\text{(c)}B_{34}(t)+Q_{18,35}(t)\text{(c)}B_{35}(t)$$
$$B_{19}(t)=Q_{19,7}(t)\text{(c)}B_7(t)+Q_{19,36}(t)\text{(c)}B_{36}(t)+Q_{19,28}(t)\text{(c)}B_{28}(t)+Q_{19,34}(t)\text{(c)}B_{34}(t)$$
$$B_{20}(t)=Q_{20,7}(t)\text{(c)}B_7(t)+Q_{20,8}(t)\text{(c)}B_8(t)+Q_{20,10}(t)\text{(c)}B_{10}(t)+Q_{20,37}(t)\text{(c)}B_{37}(t)$$
$$+Q_{20,29}(t)\text{(c)}B_{29}(t)+Q_{20,34}(t)\text{(c)}B_{37}(t)$$
$$B_{21}(t)=Q_{21,3}(t)\text{(c)}B_3(t)$$
$$B_{22}(t)=Q_{22,8}(t)\text{(c)}B_8(t)+Q_{22,9}(t)\text{(c)}B_9(t)+Q_{22,31}(t)\text{(c)}B_{31}(t)+Q_{22,35}(t)\text{(c)}B_{35}(t)$$
$$B_{23}(t)=Q_{23,1}(t)\text{(c)}B_2(t)$$
$$B_{24}(t)=Q_{24,1}(t)\text{(c)}B_1(t)$$
$$B_{25}(t)=Q_{25,1}(t)\text{(c)}B_1(t)$$
$$B_{26}(t)=M_{26}(t)+Q_{26,11}(t)\text{(c)}B_{11}(t)$$

$$B_{27}(t)=Q_{27,14}(t)(\text{c})B_{14}(t)$$
$$B_{28}(t)=Q_{28,15}(t)(\text{c})B_{15}(t)+Q_{28,19}(t)(\text{c})B_{19}(t)$$
$$B_{29}(t)=Q_{28,15}(t)(\text{c})B_{15}(t)+Q_{28,19}(t)(\text{c})B_{19}(t)$$
$$B_{30}(t)=M_{30}(t)+Q_{30,12}(t)(\text{c})B_{12}(t)$$
$$B_{31}(t)=Q_{28,15}(t)(\text{c})B_{15}(t)+Q_{28,19}(t)(\text{c})B_{19}(t)$$
$$B_{32}(t)=Q_{32,13}(t)(\text{c})B_{13}(t) \quad , \qquad (4.18)$$
$$B_{33}(t)=Q_{33,17}(t)(\text{c})B_{17}(t)+Q_{33,18}(t)(\text{c})B_{18}(t)+Q_{33,20}(t)(\text{c})B_{20}(t)$$
$$B_{34}(t)=Q_{34,17}(t)(\text{c})B_{17}(t)+Q_{34,19}(t)(\text{c})B_{19}(t)$$
$$B_{35}(t)=Q_{35,18}(t)(\text{c})B_{18}(t)+Q_{35,22}(t)(\text{c})B_{22}(t)$$
$$B_{36}(t)=M_{36}(t)+Q_{36,10}(t)(\text{c})B_{10}(t)$$
$$B_{37}(t)=M_{37}(t)+Q_{37,9}(t)(\text{c})B_{9}(t)$$

where

$$W_1(t)=\bar{H}_1(t)\ ;\ W_2(t)=\bar{H}_2(t)$$

$$W_3(t)=\bar{H}_3(t)\ ;\ W_4(t)=\bar{H}_4(t)$$

$$W_7(t)=e^{-(\beta_1+\beta_2)t}\,\bar{G}_1(t)\ ;\ W_8(t)=e^{-(\beta_1+\beta_2)t}\,\bar{G}_2(t)$$

$$W_9(t)=e^{-(\lambda_1+\lambda_2)t}\,\bar{R}_1(t)\ ;\ W_{10}(t)=e^{-(\lambda_1+\lambda_2)t}\,\bar{R}_2(t)$$

$$\bar{H}_1(t)=1-H_1(t)\ ;\ \bar{H}_2(t)=1-H_2(t)\ ;\ \bar{H}_3(t)=1-H_3(t)\ ;\ \bar{H}_4(t)=1-H_4(t)$$

Busy period can be obtained by taking Laplace transform of set equations (4.17) and solving them for $B_0^*(s)$, we get

$$B_o=\lim_{s\to 0}B_o^*(s)=\frac{N_{51}}{D_{11}}, \qquad (4.19)$$

where

D_{11} has been specified.

$$
N_{51} = \begin{vmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & 0 & 0 & a_{18} & a_{19} & a_{1,10} & a_{1,11} & 0 & 0 & 0 \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & 0 & 0 & a_{28} & a_{29} & a_{2,10} & a_{2,11} & a_{2,12} & a_{2,13} & a_{2,14} \\
a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & a_{38} & a_{39} & a_{3,10} & a_{3,11} & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{11} & a_{44} & a_{45} & 0 & a_{47} & a_{48} & a_{49} & a_{4,10} & a_{4,11} & 0 & 0 & 0 \\
a_{51} & 0 & 0 & a_{54} & 1 & 0 & 0 & a_{58} & 0 & a_{5,10} & 0 & a_{5,12} & a_{5,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{71} & a_{72} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & a_{7,10} & a_{7,11} & 0 & 0 & 0 \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & 0 & 0 & a_{88} & a_{89} & 0 & 0 & a_{8,12} & a_{8,13} & 0 \\
a_{91} & a_{92} & a_{93} & a_{94} & 0 & 0 & 0 & a_{98} & a_{99} & 0 & 0 & 0 & a_{9,13} & a_{9,14} \\
a_{10,1} & a_{10,2} & a_{10,3} & 0 & a_{10,5} & 0 & a_{10,7} & 0 & 0 & a_{10,10} & a_{10,11} & a_{10,12} & a_{10,13} & 0 \\
a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & 0 & 0 & a_{11,7} & 0 & a_{11,9} & a_{11,10} & a_{11,11} & a_{11,12} & a_{11,13} & a_{11,14} \\
a_{12,1} & 0 & a_{12,3} & a_{12,4} & a_{12,5} & 0 & 0 & a_{12,8} & 0 & a_{12,10} & a_{12,11} & a_{12,12} & a_{12,13} & 0 \\
a_{13,1} & 0 & a_{11} & a_{13,4} & a_{13,5} & 0 & 0 & a_{13,8} & a_{13,9} & a_{13,10} & a_{13,11} & a_{13,2} & a_{13,13} & 0 \\
a_{14,1} & 0 & a_{14,1} & a_{14,2} & 0 & 0 & 0 & 0 & a_{14,9} & 0 & a_{14,11} & 0 & 0 & a_{14,14}
\end{vmatrix}
$$

$a_{11} = p_{05} W_5 + p_{06} W_6 + p_{08} W_8$, $a_{12} =- p_{01}$, $a_{13} =- p_{02} - p_{05} p_{52} - p_{06} p_{62}$, $a_{14} =- p_{03}$

$a_{15} =- p_{07}$, $a_{18} =- p_{05} p_{5,15}$, $a_{19} =- p_{05} p_{5,16}$, $a_{1,10} = - p_{06} p_{6,17}$, $a_{1,11} =- p_{06} p_{6,18} - p_{08} p_{8,18}$

$a_{21} = p_{1,10} W_{10} + p_{19} W_9 + p_{18} W_8 p_{1,11} W_{11} + p_{1,12} W_{12}$, $a_{22} =1- p_{1,12} p_{12,1} - p_{1,11} p_{11,24} p_{24,1}$

$a_{23} =- p_{12} - p_{1,11} p_{11,2} - p_{19} p_{92} - p_{1,12} p_{12,2} - p_{1,10} p_{10,23} p_{23,2}$, $a_{24} =- p_{13} - p_{19} p_{93} - p_{1,10} p_{10,3}$

$a_{25} =- p_{17}$, $a_{28} =- p_{1,10}\, p_{10,15}$, $a_{29} =- p_{18} p_{8,16} - p_{19} p_{9,16}$, $a_{2,10} =- p_{1,10} p_{10,17}$

$a_{2,11} =- p_{18} p_{8,18} - p_{19} p_{9,18}$, $a_{2,12} =- p_{1,11} p_{11,19}$, $a_{2,13} =- p_{1,11} p_{11,20} - p_{1,12} p_{12,20}$

$a_{2,14} =- p_{1,12} p_{12,22}$, $a_{31} = p_{25} W_5 + p_{26} W_6 + p_{2,14} W_{14}$, $a_{32} =- p_{21} - p_{2,14} p_{14,25} p_{25,1}$

$a_{33} =1- p_{25} p_{52} - p_{26} p_{62}$, $a_{34} =- p_{23} - p_{2,14} p_{14,3}$, $a_{38} =- p_{25} p_{5,15} - p_{2,14} p_{14,15}$

$a_{39} =- p_{25} p_{5,16} - p_{2,14} p_{14,16}$, $a_{3,10} = - p_{26} p_{6,17}$, $a_{3,11} = - p_{26} p_{6,18}$

$a_{42} =- p_{31} - p_{3,14} p_{14,25} p_{25,1}$, $a_{41} = p_{35} W_5 + p_{36} W_6 + p_{3,14} W_{14} p_{39} W_9 + p_{38} W_8$

$a_{45} =- p_{37}$, $a_{47} = - p_{3,13}$, $a_{43} =- p_{32} - p_{35} p_{52} - p_{36} p_{62} - p_{39} p_{92} - p_{3,10} p_{10,23} p_{23,2}$

$a_{48} =- p_{35} p_{5,15} - p_{3,14} p_{14,15}$, $a_{44} =1- p_{38} p_{83} - p_{39} p_{93} - p_{3,10} p_{10,3} - p_{3,14} p_{14,3}$

$a_{49} =- p_{35} p_{5,16} - p_{39} p_{9,16} - p_{3,14} p_{14,16}$, $a_{4,10} = - p_{36} p_{6,17}$, $a_{4,11} =- p_{39} p_{9,18} - p_{38} p_{8,18}$

$a_{51} = W_7$, $a_{54} = - p_{7,21} p_{21,3}$, $a_{55} = 1$, $a_{58} =- p_{7,15}$, $a_{5,10} =- p_{7,17}$, $a_{5,12} =- p_{7,19}$

$a_{5,13} =- p_{7,20}$, $a_{71} = W_{13}$, $a_{72} =- p_{13,1}$, $a_{7,10} =- p_{13,17}$, $a_{7,11} =- p_{13,18}$

$a_{81} = W_{15} + p_{15,5} W_5 + p_{15,7} W_7 + p_{15,27} W_{27} + p_{15,28} W_{28} + p_{15,29} W_{29} + p_{15,27} p_{27,14} W_{14} + p_{15,26} p_{26,11} W_{11}$

$a_{82} =- p_{15,27} p_{27,14} p_{14,25} p_{25,1} - p_{15,26} p_{26,11} p_{11,24} p_{24,1}$, $a_{84} =- p_{15,27} p_{27,14} p_{14,3}$, $a_{85} = - p_{15,7}$

$a_{83} =- p_{15,5} p_{52} - p_{15,26} p_{26,11} p_{11,2}$, $a_{88} =1- p_{15,27} p_{27,14} p_{14,15} - p_{15,5} p_{5,15} - p_{15,28} p_{28,15} - p_{15,29} p_{29,15}$

$a_{89} =- p_{15,27} p_{27,14} p_{14,16} - p_{15,5} p_{5,16} - p_{15,29} p_{29,16}$, $a_{8,12} =- p_{15,28} p_{28,19} - p_{15,26} p_{26,11} p_{11,19}$

$a_{8,13} =- p_{15,29} p_{29,20} - p_{15,26} p_{26,11} p_{11,20}$, $a_{92} =- p_{16,14} p_{14,25} p_{25,1} - p_{16,30} p_{30,12} p_{12,1}$

$a_{91} = W_{16} + p_{16,5} W_5 + p_{16,14} W_{14} + p_{16,29} W_{29} + p_{16,30} W_{30} + p_{16,31} W_{31}$

$a_{93} = - p_{16,5} p_{52} - p_{16,30} p_{30,12} p_{12,2}$, $a_{94} - p_{16,14} p_{16,3}$; $a_{98} = - p_{16,5} p_{5,15} - p_{16,14} p_{14,15} - p_{16,29} p_{29,15}$

$a_{99} = 1 - p_{16,5} p_{5,16} - p_{16,14} p_{14,16} - p_{16,29} p_{29,16} - p_{16,31} p_{29,16}$

$a_{9,13} = - p_{16,29} p_{29,20} - p_{16,30} p_{30,12} p_{12,20}$, $a_{9,14} = - p_{16,31} p_{31,22} - p_{16,30} p_{30,12} p_{12,22}$

$a_{10,1} = W_{17} + p_{17,6} W_6 + p_{17,11} W_{11} + p_{17,32} W_{32} + p_{17,33} W_{33} + p_{17,34} W_{34}$, $a_{10,2} = - p_{17,11} p_{11,24} p_{24,1}$

$a_{10,3} = - p_{17,6} p_{62} - p_{17,11} p_{11,2}$, $a_{10,5} = - p_{17,7}$, $a_{10,9} = - p_{17,32} p_{32,13}$

$a_{10,10} = 1 - p_{17,6} p_{6,17} - p_{17,33} p_{33,17} - p_{17,34} p_{34,17}$, $a_{10,12} = - p_{17,6} p_{6,18} - p_{17,33} p_{33,18}$

$a_{10,13} = - p_{17,11} p_{11,20} - p_{17,33} p_{33,20}$, $a_{11,2} = - p_{18,12} p_{12,1}$, $a_{11,3} = - p_{18,6} p_{62} - p_{18,12} p_{12,2}$

$a_{11,1} = W_{18} + p_{18,6} W_6 + + p_{17,8} W_8 + p_{18,12} W_{12} + p_{18,34} W_{34} + p_{18,35} W_{35}$, $a_{11,4} = - p_{18,8} p_{83}$

$a_{11,7} = - p_{18,13}$, $a_{11,9} = - p_{18,8} p_{8,16}$, $a_{11,10} = - p_{18,6} p_{6,17} p_{18,34} p_{34,17}$, $a_{11,12} = - p_{18,34} p_{34,19}$

$a_{11,11} = 1 - p_{18,6} p_{6,18} - p_{18,8} p_{8,18} - p_{18,35} p_{35,18}$, $a_{11,13} = - p_{18,12} p_{12,20} - p_{18,34} p_{34,20}$

$a_{11,14} = - p_{18,12} p_{11,22} - p_{18,35} p_{35,22}$, $a_{12,1} = W_{19} + p_{19,28} W_{28} + p_{19,36} W_{36} + p_{19,33} W_{33} + p_{19,36} p_{36,10} W_{10}$

$a_{12,3} = - p_{19,36} p_{36,10} p_{10,23} p_{23,2}$, $a_{12,4} = - p_{19,36} p_{36,10} p_{10,3}$, $a_{12,5} = - p_{19,7}$

$a_{12,8} = - p_{19,28} p_{28,15} - p_{19,36} p_{36,10} p_{10,15}$, $a_{12,10} = - p_{19,33} p_{33,17} - p_{19,36} p_{36,10} p_{10,17}$

$a_{12,11} = - p_{19,33} p_{33,18}$, $a_{12,12} = 1 - p_{28,19} p_{19,28}$, $a_{12,13} = - p_{19,33} p_{33,20}$

$a_{13,1} = W_{20} + p_{20,29} W_{29} + p_{20,34} W_{34} + p_{20,37} W_{37} + p_{20,37} p_{37,9} W_9$

$a_{13,3} = - p_{20,10} p_{10,23} p_{23,2} - p_{20,37} p_{37,9} p_{92}$, $a_{13,4} = - p_{20,10} p_{10,3} - p_{20,37} p_{37,9} p_{93}$, $a_{13,5} = - p_{20,7}$

$a_{13,8} = - p_{20,29} p_{29,15} - p_{20,10} p_{10,15}$, $a_{13,9} = - p_{20,29} p_{29,16} - p_{20,37} p_{37,9} p_{9,16}$, $a_{13,12} = - p_{20,34} p_{34,19}$

$a_{13,10} = - p_{20,10} p_{10,17} - p_{20,34} p_{34,17}$, $a_{13,11} = - p_{20,37} p_{37,9} p_{9,18}$, $a_{13,13} = 1 - p_{20,29} p_{29,20}$

$a_{14,1} = W_{22} + p_{22,31} W_{31} + p_{22,35} W_{35}$, $a_{14,2} = - p_{22,9} p_{92}$, $a_{14,3} = - p_{22,9} p_{93}$, $a_{14,9} = - p_{22,9} p_{9,16} - p_{22,31} p_{31,1}$

$a_{14,11} = - p_{22,9} p_{9,18} - p_{22,35} p_{35,18}$, $a_{14,14} = 1 - p_{22,31} p_{31,22} - p_{22,35} p_{35,22}$

4.7 Expected Down Time

For expected down time $E_i(t)$ the recursive relations are given by:

$$E_0(t) = Q_{01}(t)(c)E_1(t) + Q_{02}(t)(c)E_2(t) + Q_{03}(t)(c)E_3(t) + Q_{04}(t)(c)E_4(t) + Q_{05}(t)(c)E_5(t) + Q_{06}(t)(c)E_6(t) + Q_{07}(t)(c)E_7(t) + Q_{08}(t)(c)E_8(t)$$

$$E_1(t) = Q_{10}(t)(c)E_0(t) + Q_{12}(t)(c)E_2(t) + Q_{13}(t)(c)E_3(t) + Q_{17}(t)(c)E_7(t) + Q_{18}(t)(c)E_8(t) + Q_{19}(t)(c)E_9(t) + Q_{1,11}(t)(c)E_{11}(t) + Q_{1,12}(t)(c)E_{12}(t)$$

$$E_2(t) = Q_{20}(t)(c)E_0(t) + Q_{21}(t)(c)E_1(t) + Q_{23}(t)(c)\phi_3(t) + Q_{25}(t)(c)E_5(t) + Q_{2,13}(t)(c)E_{13}(t) + Q_{2,14}(t)(c)E_{14}(t)$$

$$E_3(t) = Q_{30}(t)(c)E_0(t) + Q_{31}(t)(c)E_1(t) + Q_{32}(t)(c)E_2(t) + Q_{35}(t)(c)E_5(t) + Q_{36}(t)(c)E_7(t) + Q_{38}(t)(c)E_8(t) + Q_{39}(t)(c)E_9(t) + Q_{3,10}(t)(c)E_{10}(t) + Q_{3,13}(t)(c)E_{3,13}(t) + Q_{3,14}(t)(c)E_{3,14}(t)$$

$$E_4(t)=M_4(t)+Q_{40}(t)\text{(c)}E_0(t)$$

$$E_5(t)=Q_{50}(t)\text{(c)}E_0(t)+Q_{52}(t)\text{(c)}E_2(t)+Q_{5,15}(t)\text{(c)}E_{15}(t)+Q_{5,16}(t)\text{(c)}E_{16}(t)$$

$$E_6(t)=Q_{60}(t)\text{(c)}E_0(t)+Q_{62}(t)\text{(c)}E_2(t)+Q_{6,17}(t)\text{(c)}E_{17}(t)+Q_{6,18}(t)\text{(c)}E_{18}(t)$$

$$E_7(t)=Q_{70}(t)\text{(c)}E_0(t)+Q_{7,15}(t)\text{(c)}E_{15}(t)+Q_{7,19}(t)\text{(c)}E_{19}(t)$$
$$+Q_{7,20}(t)\text{(c)}E_{20}(t)+Q_{7,21}(t)\text{(c)}E_{21}(t)$$

$$E_8(t)=Q_{80}(t)\text{(c)}E_0(t)+Q_{83}(t)\text{(c)}E_3(t)+Q_{8,16}(t)\text{(c)}E_{16}(t)$$
$$+Q_{8,18}(t)\text{(c)}E_{18}(t)+Q_{8,20}(t)\text{(c)}E_{20}(t)+Q_{8,22}(t)\text{(c)}E_{22}(t)$$

$$E_9(t)=Q_{92}(t)\text{(c)}E_2(t)+Q_{93}(t)\text{(c)}E_3(t)+Q_{9,16}(t)\text{(c)}E_{16}(t)+Q_{9.18}(t)\text{(c)}E_{18}(t)$$

$$E_{10}(t)=Q_{10,3}(t)\text{(c)}E_3(t)+Q_{10,15}(t)\text{(c)}E_{15}(t)+Q_{10,17}(t)\text{(c)}E_{17}(t)+Q_{10,18}(t)\text{(c)}E_{18}(t)$$

$$E_{11}(t)=Q_{11,2}(t)\text{(c)}E_2(t)+Q_{11,19}(t)\text{(c)}E_{19}(t)+Q_{11,20}(t)\text{(c)}E_{20}(t)+Q_{11,24}(t)\text{(c)}E_{24}(t)$$

$$E_{12}(t)=Q_{12,1}(t)\text{(c)}E_1(t)+Q_{12,2}(t)\text{(c)}E_2(t)+Q_{12,20}(t)\text{(c)}E_{20}(t)+Q_{12,22}(t)\text{(c)}E_{22}(t)$$

$$E_{13}(t)=Q_{13,1}(t)\text{(c)}E_1(t)+Q_{13,3}(t)\text{(c)}E_3(t)+Q_{13,17}(t)\text{(c)}E_{17}(t)+Q_{13,18}(t)\text{(c)}E_{18}(t)$$

$$E_{14}(t)=Q_{14,3}(t)\text{(c)}E_3(t)+Q_{14,15}(t)\text{(c)}E_{15}(t)+Q_{14,16}(t)\text{(c)}E_{16}(t)+Q_{14,25}(t)\text{(c)}E_{25}(t)$$

$$E_{15}(t)=Q_{15,5}(t)\text{(c)}E_5(t)+Q_{15,7}(t)\text{(c)}E_7(t)+Q_{15,26}(t)\text{(c)}E_{26}(t)+Q_{15,27}(t)\text{(c)}E_{27}(t)$$
$$+Q_{15,28}(t)\text{(c)}E_{28}(t)+Q_{15,29}(t)\text{(c)}E_{29}(t)$$

$$E_{16}(t)=Q_{16,5}(t)\text{(c)}E_5(t)+Q_{16,8}(t)\text{(c)}E_8(t)+Q_{16,14}(t)\text{(c)}E_{14}(t)+Q_{16,30}(t)\text{(c)}E_{30}(t)$$
$$+Q_{16,31}(t)\text{(c)}E_{31}(t)+Q_{16,29}(t)\text{(c)}E_{29}(t)$$

$$E_{17}(t)=Q_{17,6}(t)\text{(c)}E_6(t)+Q_{17,7}(t)\text{(c)}E_7(t)+Q_{17,11}(t)\text{(c)}E_{11}(t)+Q_{17,32}(t)\text{(c)}E_{32}(t)$$
$$+Q_{17,33}(t)\text{(c)}E_{33}(t)+Q_{17,34}(t)\text{(c)}E_{34}(t)$$

$$E_{18}(t)=Q_{18,6}(t)\text{(c)}E_6(t)+Q_{18,6}(t)\text{(c)}E_6(t)+Q_{18,13}(t)\text{(c)}E_{13}(t)+Q_{18,12}(t)\text{(c)}E_{12}(t)$$
$$+Q_{18,34}(t)\text{(c)}E_{34}(t)+Q_{18,35}(t)\text{(c)}E_{35}(t)$$

$$E_{19}(t)=Q_{19,7}(t)\text{(c)}E_7(t)+Q_{19,36}(t)\text{(c)}E_{36}(t)+Q_{19,28}(t)\text{(c)}E_{28}(t)+Q_{19,34}(t)\text{(c)}E_{34}(t)$$

$$E_{20}(t)=Q_{20,7}(t)\text{(c)}E_7(t)+Q_{20,8}(t)\text{(c)}E_8(t)+Q_{20,10}(t)\text{(c)}E_{10}(t)+Q_{20,37}(t)\text{(c)}E_{37}(t)$$
$$+Q_{20,29}(t)\text{(c)}E_{29}(t)+Q_{20,34}(t)\text{(c)}E_{37}(t)$$

$$E_{21}(t)=Q_{21,3}(t)\text{(c)}E_3(t)$$

$$E_{22}(t)=Q_{22,8}(t)\text{(c)}E_8(t)+Q_{22,9}(t)\text{(c)}E_9(t)+Q_{22,31}(t)\text{(c)}E_{31}(t)+Q_{22,35}(t)\text{(c)}E_{35}(t)$$

$$E_{23}(t)=Q_{23,1}(t)\text{(c)}E_2(t)$$

$$E_{24}(t)=Q_{24,1}(t)\text{(c)}E_1(t)$$

$$E_{25}(t)=Q_{25,1}(t)\text{(c)}E_1(t)$$

$$E_{26}(t)=Q_{26,11}(t)\text{(c)}E_{11}(t)$$

$$E_{27}(t)=Q_{27,14}(t)\text{(c)}E_{14}(t)$$

$$E_{28}(t)=Q_{28,15}(t)\text{(c)}E_{15}(t)+Q_{28,19}(t)\text{(c)}E_{19}(t)$$

$$E_{29}(t) = Q_{28,15}(t)(c)E_{15}(t) + Q_{28,19}(t)(c)E_{19}(t)$$
$$E_{30}(t) = Q_{30,12}(t)(c)E_{12}(t)$$
$$E_{31}(t) = Q_{28,15}(t)(c)E_{15}(t) + Q_{28,19}(t)(c)E_{19}(t)$$
$$E_{32}(t) = Q_{32,13}(t)(c)E_{13}(t)$$
$$E_{33}(t) = Q_{33,17}(t)(c)E_{17}(t) + Q_{33,18}(t)(c)E_{18}(t) + Q_{33,20}(t)(c)E_{20}(t) \tag{4.20}$$
$$E_{34}(t) = Q_{34,17}(t)(c)E_{17}(t) + Q_{34,19}(t)(c)E_{19}(t)$$
$$E_{35}(t) = Q_{35,18}(t)(c)E_{18}(t) + Q_{35,22}(t)(c)E_{22}(t)$$
$$E_{36}(t) = Q_{36,10}(t)(c)E_{10}(t)$$
$$E_{37}(t) = Q_{37,9}(t)(c)E_{9}(t)$$

where

$$W_4(t) = e^{-\gamma_1 t}$$

Next, taking Laplace transform of set equations (4.19) and solving them for $E_0^*(s)$, we get expected down time as:

$$E_o = \lim_{s \to 0} E_o^*(s) = \frac{N_{81}}{D_{11}}, \tag{4.21}$$

where

$$N_{61} = \begin{vmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & 0 & 0 & a_{18} & a_{19} & a_{1,10} & a_{1,11} & 0 & 0 & 0 \\
0 & a_{22} & a_{23} & a_{24} & a_{25} & 0 & 0 & a_{28} & a_{29} & a_{2,10} & a_{2,11} & a_{2,12} & a_{2,13} & a_{2,14} \\
0 & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & a_{38} & a_{39} & a_{3,10} & a_{3,11} & 0 & 0 & 0 \\
0 & a_{42} & a_{11} & a_{44} & a_{45} & 0 & a_{47} & a_{48} & a_{49} & a_{4,10} & a_{4,11} & 0 & 0 & 0 \\
0 & 0 & 0 & a_{54} & 1 & 0 & 0 & a_{58} & 0 & a_{5,10} & 0 & a_{5,12} & a_{5,13} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{72} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & a_{7,10} & a_{7,11} & 0 & 0 & 0 \\
0 & a_{82} & a_{83} & a_{84} & a_{85} & 0 & 0 & a_{88} & a_{89} & 0 & 0 & a_{8,12} & a_{8,13} & 0 \\
0 & a_{92} & a_{93} & a_{94} & 0 & 0 & 0 & a_{98} & a_{99} & 0 & 0 & 0 & a_{9,13} & a_{9,14} \\
0 & a_{10,2} & a_{10,3} & 0 & a_{10,5} & 0 & a_{10,7} & 0 & 0 & a_{10,10} & a_{10,11} & a_{10,12} & a_{10,13} & 0 \\
0 & a_{11,2} & a_{11,3} & a_{11,4} & 0 & 0 & a_{11,7} & 0 & a_{11,9} & a_{11,10} & a_{11,11} & a_{11,12} & a_{11,13} & a_{11,14} \\
0 & 0 & a_{11} & a_{12,4} & a_{12,5} & 0 & 0 & a_{12,8} & 0 & a_{12,10} & a_{12,11} & a_{12,12} & a_{12,13} & 0 \\
0 & 0 & a_{11} & a_{13,4} & a_{13,5} & 0 & 0 & a_{13,8} & a_{13,9} & a_{13,10} & a_{13,11} & a_{13,2} & a_{13,13} & 0 \\
0 & 0 & a_{14,1} & a_{14,2} & 0 & 0 & 0 & 0 & a_{14,9} & 0 & a_{14,11} & 0 & 0 & a_{14,14}
\end{vmatrix}$$

D_{11} has been specified.

$a_{11}=p_{04}W_4$, $a_{12}=-p_{01}$, $a_{13}=-p_{02}-p_{05}p_{52}-p_{06}p_{62}$, $a_{14}=-p_{03}$

$a_{15}=-p_{07}$, $a_{18}=-p_{05}p_{5,15}$, $a_{19}=-p_{05}p_{5,16}$, $a_{1,10}=-p_{06}p_{6,17}$, $a_{1,11}=-p_{06}p_{6,18}-p_{08}p_{8,18}$

$a_{22}=1-p_{1,12}p_{12,1}-p_{1,11}p_{11,24}p_{24,1}$, $a_{23}=-p_{12}-p_{1,11}p_{11,2}-p_{19}p_{92}-p_{1,12}p_{12,2}-p_{1,10}p_{10,23}p_{23,2}$

$a_{24}=-p_{13}-p_{19}p_{93}-p_{1,10}p_{10,3}$, $a_{25}=-p_{17}$, $a_{28}=-p_{1,10}p_{10,15}$, $a_{29}=-p_{18}p_{8,16}-p_{19}p_{9,16}$

$a_{2,10}=-p_{1,10}p_{10,17}$, $a_{2,11}=-p_{18}p_{8,18}-p_{19}p_{9,18}$, $a_{2,12}=-p_{1,11}p_{11,19}$

$a_{2,13}=-p_{1,11}p_{11,20}-p_{1,12}p_{12,20}$, $a_{2,14}=-p_{1,12}p_{12,22}$, $a_{32}=-p_{21}-p_{2,14}p_{14,25}p_{25,1}$

$a_{33}=1-p_{25}p_{52}-p_{26}p_{62}$, $a_{34}=-p_{23}-p_{2,14}p_{14,3}$, $a_{38}=-p_{25}p_{5,15}-p_{2,14}p_{14,15}$

$a_{39}=-p_{25}p_{5,16}-p_{2,14}p_{14,16}$, $a_{3,10}=-p_{26}p_{6,17}$, $a_{3,11}=-p_{26}p_{6,18}$, $a_{42}=-p_{31}-p_{3,14}p_{14,25}p_{25,1}$

$a_{43}=-p_{32}-p_{35}p_{52}-p_{36}p_{62}-p_{39}p_{92}-p_{3,10}p_{10,23}p_{23,2}$, $a_{45}=-p_{37}$, $a_{47}=-p_{3,13}$

$a_{44}=1-p_{38}p_{83}-p_{39}p_{93}-p_{3,10}p_{10,3}-p_{3,14}p_{14,3}$

$a_{48}=-p_{35}p_{5,15}-p_{3,14}p_{14,15}$, $a_{49}=-p_{35}p_{5,16}-p_{39}p_{9,16}-p_{3,14}p_{14,16}$, $a_{4,10}=-p_{36}p_{6,17}$

$a_{4,11}=-p_{39}p_{9,18}-p_{38}p_{8,18}$, $a_{54}=-p_{7,21}p_{21,3}$, $a_{55}=1$, $a_{58}=-p_{7,15}$, $a_{5,10}=-p_{7,17}$

$a_{5,12}=-p_{7,19}$, $a_{5,13}=-p_{7,20}$, $a_{72}=-p_{13,1}$, $a_{7,10}=-p_{13,17}$, $a_{7,11}=-p_{13,18}$

$a_{82}=-p_{15,27}p_{27,14}p_{14,25}p_{25,1}-p_{15,26}p_{26,11}p_{11,24}p_{24,1}$, $a_{85}=-p_{15,7}$

$a_{83}=-p_{15,5}p_{52}-p_{15,26}p_{26,11}p_{11,2}$, $a_{84}=-p_{15,27}p_{27,14}p_{14,3}$

$a_{88}=1-p_{15,27}p_{27,14}p_{14,15}-p_{15,5}p_{5,15}-p_{15,28}p_{28,15}-p_{15,29}p_{29,15}$

$a_{89}=-p_{15,27}p_{27,14}p_{14,16}-p_{15,5}p_{5,16}-p_{15,29}p_{29,16}$, $a_{8,12}=-p_{15,28}p_{28,19}-p_{15,26}p_{26,11}p_{11,19}$

$a_{8,13}=-p_{15,29}p_{29,20}-p_{15,26}p_{26,11}p_{11,20}$, $a_{92}=-p_{16,14}p_{14,25}p_{25,1}-p_{16,30}p_{30,12}p_{12,1}$

$a_{93}=-p_{16,5}p_{52}-p_{16,30}p_{30,12}p_{12,2}$, $a_{94}-p_{16,14}p_{16,3}$; $a_{98}=-p_{16,5}p_{5,15}-p_{16,14}p_{14,15}-p_{16,29}p_{29,15}$

$a_{99}=1-p_{16,5}p_{5,16}-p_{16,14}p_{14,16}-p_{16,29}p_{29,16}-p_{16,31}p_{29,16}$, $a_{9,13}=-p_{16,29}p_{29,20}-p_{16,30}p_{30,12}p_{12,20}$

$a_{9,14}=-p_{16,31}p_{31,22}-p_{16,30}p_{30,12}p_{12,22}$, $a_{10,1}=-p_{17,6}p_{60}$, $a_{10,2}=-p_{17,11}p_{11,24}p_{24,1}$

$a_{10,3}=-p_{17,6}p_{62}-p_{17,11}p_{11,2}$, $a_{10,5}=-p_{17,7}$, $a_{10,9}=-p_{17,32}p_{32,13}$

$a_{10,10}=1-p_{17,6}p_{6,17}-p_{17,33}p_{33,17}-p_{17,34}p_{34,17}$, $a_{10,12}=-p_{17,6}p_{6,18}-p_{17,33}p_{33,18}$

$a_{10,13}=-p_{17,11}p_{11,20}-p_{17,33}p_{33,20}$, $a_{11,2}=-p_{18,12}p_{12,1}$, $a_{11,3}=-p_{18,6}p_{62}-p_{18,12}p_{12,2}$

$a_{11,4}=-p_{18,8}p_{83}$, $a_{11,7}=-p_{18,13}$, $a_{11,9}=-p_{18,8}p_{8,16}$, $a_{11,10}=-p_{18,6}p_{6,17}p_{18,34}p_{34,17}$

$a_{11,11}=1-p_{18,6}p_{6,18}-p_{18,8}p_{8,18}-p_{18,35}p_{35,18}$, $a_{11,12}=-p_{18,34}p_{34,19}$

$a_{11,13}=-p_{18,12}p_{12,20}-p_{18,34}p_{34,20}$, $a_{11,14}=-p_{18,12}p_{11,22}-p_{18,35}p_{35,22}$

$a_{12,3}=-p_{19,36}p_{36,10}p_{10,23}p_{23,2}$, $a_{12,4}=-p_{19,36}p_{36,10}p_{10,3}$, $a_{12,5}=-p_{19,7}$

$a_{12,8}=-p_{19,28}p_{28,15}-p_{19,36}p_{36,10}p_{10,15}$, $a_{12,10}=-p_{19,33}p_{33,17}-p_{19,36}p_{36,10}p_{10,17}$

$a_{12,11}=-p_{19,33}p_{33,18}$, $a_{12,12}=1-p_{28,19}p_{19,28}$, $a_{12,13}=-p_{19,33}p_{33,20}$

$a_{13,3}=-p_{20,10}p_{10,23}p_{23,2}-p_{20,37}p_{37,9}p_{92}$, $a_{13,4}=-p_{20,10}p_{10,3}-p_{20,37}p_{37,9}p_{93}$

$a_{13,5}=-p_{20,7}$, $a_{13,8}=-p_{20,29}p_{29,15}-p_{20,10}p_{10,15}$

$a_{13,9}=-p_{20,29}p_{29,16}-p_{20,37}p_{37,9}p_{9,16}$, $a_{13,10}=-p_{20,10}p_{10,17}-p_{20,34}p_{34,17}$

$a_{13,11} = -p_{20,37}p_{37,9}p_{9,18}$, $a_{13,12} = -p_{20,34}p_{34,19}$, $a_{13,13} = 1 - p_{20,29}p_{29,20}$
$a_{14,2} = -p_{22,9}p_{92}$, $a_{14,3} = -p_{22,9}p_{93}$ $a_{14,9} = -p_{22,9}p_{9,16} - p_{22,31}p_{31,16}$
$a_{14,11} = -p_{22,9}p_{9,18} - p_{22,35}p_{35,18}$, $a_{14,14} = 1 - p_{22,31}p_{31,22} - p_{22,35}p_{35,22}$

4.8 Cost-Benefit Analysis

The revenue and cost functions lead to the profit function of a firm. As the profit is excess of revenue over the cost of production, the profit takes the form

$P =$ Expected revenue in $(0,t]$ - Expected total cost in $(0,t]$

Using equations (4.13), (4.14), (4.15), (4.16), (4.18) and (4.20) the expected profit per unit time incurred to the system is given by

$$P = C_{01}A^1{}_0 + C_{02}A^2{}_0 + C_{03}A^3{}_0 + C_{04}A^4{}_0 - C_1B_0 - C_2E_0 \, , \qquad (4.22)$$

where

C_{01} = Total *r*evenue per unit time for availability of 12 ton (p $\geq$ d)
C_{02} = Total *r*evenue per unit time for availability of 12 ton ($p < d$)
C_{03} = Total *r*evenue per unit time for availability of 8 ton (p $\geq$ d)
C_{01} = Total *r*evenue per unit time for availability of 4 ton (p $\geq$ d)
C_{01} = Total *r*evenue per unit time for availability of 8 ton ($p < d$)
C_{01} = Total *r*evenue per unit time for availability of 4 ton ($p < d$)
C_1 = *C*ost of busy period of repairman
C_2 = Cost for expected down time

Hence the bounds for revenue/costs for the system to be profitable are:

TABLE 4.2: BOUNDS FOR REVENUE/COSTS

Revenue/Cost	Bound (Lower/Upper)	Value
C_{01}	Lower	$\frac{\left(C_1 B_0 + C_2 E_0 - C_{02} A^1{}_0 - C_{03} A^2{}_0 - C_{04} A^3{}_0 - C_{05} A^4{}_0 - C_{06} A^5{}_0\right)}{A_0}$
C_{02}	Lower	$\frac{\left(C_1 B_0 + C_2 E_0 - C_{01} A_0 - C_{03} A^2{}_0 - C_{04} A^3{}_0 - C_{05} A^4{}_0 - C_{06} A^5{}_0\right)}{A^1{}_0}$
C_{03}	Lower	$\frac{\left(C_1 B_0 + C_2 E_0 - C_{01} A_0 - C_{02} A^1{}_0 - C_{04} A^3{}_0 - C_{05} A^4{}_0 - C_{06} A^5{}_0\right)}{A^2{}_0}$
C_{04}	Lower	$\frac{\left(C_1 B_0 + C_2 E_0 - C_{01} A_0 - C_{02} A^1{}_0 - C_{03} A^2{}_0 - C_{05} A^4{}_0 - C_{06} A^5{}_0\right)}{A^3{}_0}$
C_{05}	Lower	$\frac{\left(C_1 B_0 + C_2 E_0 - C_{01} A_0 - C_{02} A^1{}_0 - C_{03} A^2{}_0 - C_{04} A^3{}_0 - C_{06} A^5{}_0\right)}{A^4{}_0}$
C_{06}	Lower	$\frac{\left(C_1 B_0 + C_2 E_0 - C_{01} A_0 - C_{02} A^1{}_0 - C_{03} A^2{}_0 - C_{04} A^3{}_0 - C_{05} A^4{}_0\right)}{A^5{}_0}$
C_1	Upper	$\frac{\left(C_{01} A_0 + C_{02} A^1{}_0 + C_{03} A^2{}_0 - C_{04} A^3{}_0 - C_{05} A^4{}_0 - C_{06} A^5{}_0 - C_2 E_0\right)}{B_0}$
C_2	Upper	$\frac{\left(C_{01} A_0 + C_{02} A^1{}_0 + C_{03} A^2{}_0 - C_{04} A^3{}_0 - C_{05} A^4{}_0 - C_{06} A^5{}_0 - C_1 B_0\right)}{E_0}$

4.9 Analysis and Discussion

In this study, data for all types of failures and repairs of the rice plant was collected in the units of per hour. On the basis of these data, we have computed the failure and repair rates. Some of measures have been assumed. The values of various measures calculated and assumed are:

$a_1 = 0.3144$, $b_1 = 0.2346$ $\gamma_1 = 0.01736$ $\gamma_2 = 0.0987$ $q = 0.8$; $p = 1 - q$; $\beta_2 = 0.0036$

$p_2 = 0.065$; $p_1 = 0.35$; $p_4 = 0.35$; $p_3 = 1 - p_2 - p_1$; $p_6 = 0.22$; $p_5 = 1 - p_6 - p_4$; $p_7 = 0.45$

$p_8 = 0.082$; $p_9 = 1 - p_7 - p_8$; $p_{10} = 0.025$; $p_{11} = 0.57$; $p_{12} = 0.035$; $p_{13} = 0.12$; $p_{15} = 0.42$

$p_{17} = 1 - p_{13} - p_{15}$; $p_{12} = 1 - p_{10} - p_{11}$; $p_{16} = 0.45$; $p_{14} = 1 - p_{16} - p_{18} = 0.34$; $p_{21} = 0.35$

$p_{20} = 0.065$; $p_{19} = 1 - p_{21}$; $p_{22} = 1 - p_{20}$; $p_{23} = 0.35$; $p_{24} = 0.55$; $p_{25} = 0.75$

$p_{27} = 1 - p_{23}$; $p_{26} = 0.43$; $p_{28} = 1 - p_{24}$; $p_{29} = 1 - p_{25}$; $p_{30} = 1 - p_{24}$; $\beta_1 = 0.0084$

Based on these measures we have computed the following results of important reliability indices using the software 'MATLAB'. By varying λ_1 for distinct values of a_1, the values of MTSF. Availabilities of different capacities are computed by varying λ_1 for distinct values of a_1 and by varying β_2 for distinct values of b_2 . Similarly, the profit is also computed by varying C_{04} for different choices of C_1 and results are presented. Their behaviours are exhibited in figures. Fig. 4.1 shows the behaviour of MTSF with respect to λ_2 for distinct values a_2 . The graph shows that MTSF decreases with increase in λ_2 keeping a_2 and has greater values for greater values of a_2 .

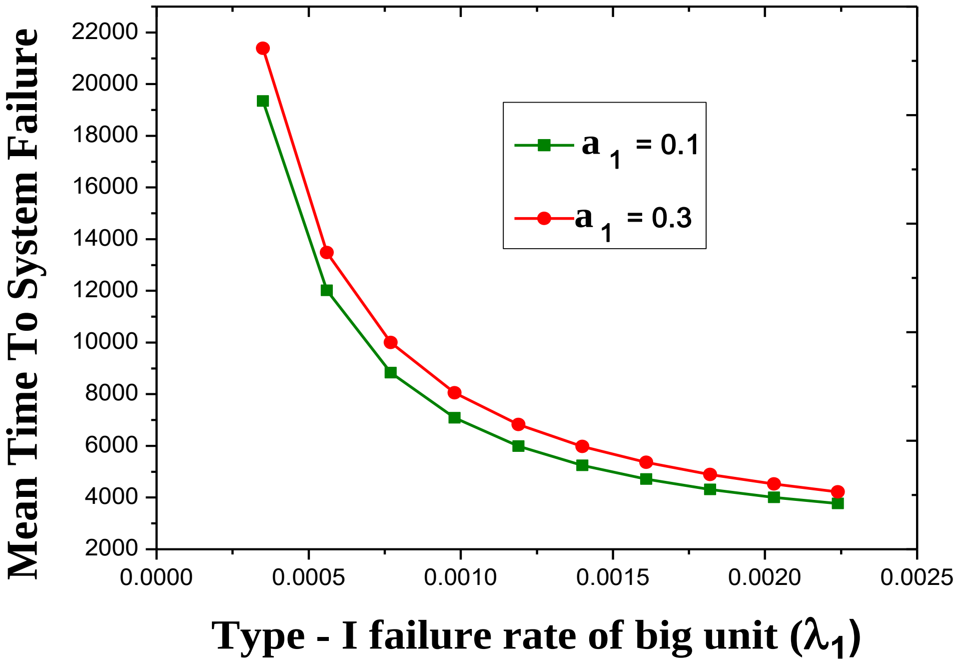

Fig. 4.1 - Effect of Type-II failure rate on Mean time to system failure for distinct values of Type-II repair rate.

Behaviour of availability in different capacities with respect to failure rate for distinct values repair rate has been shown in Fig 4.2, Fig 4.3, Fig 4.4 and Fig 4.5. Fig. 4.2 indicates that availability of full capacity i.e. sixteen ton decreases with increase in λ_1 and has greater values for higher value of a_1. Fig. 4.3 indicates that availability of twelve ton capacity decreases with increase in λ_1 and has lesser values for higher value a_1. Behaviour of availability of eight ton capacity is shown in fig. 4.4 which indicates that availability of capacity eight ton decreases with increase in λ_1 and has greater values for higher value of a_1. Fig. 4.5 shows the behaviour of availability of four ton capacity which indicates that availability decreases with increase β_2 and has lesser values for lower value of b_2.

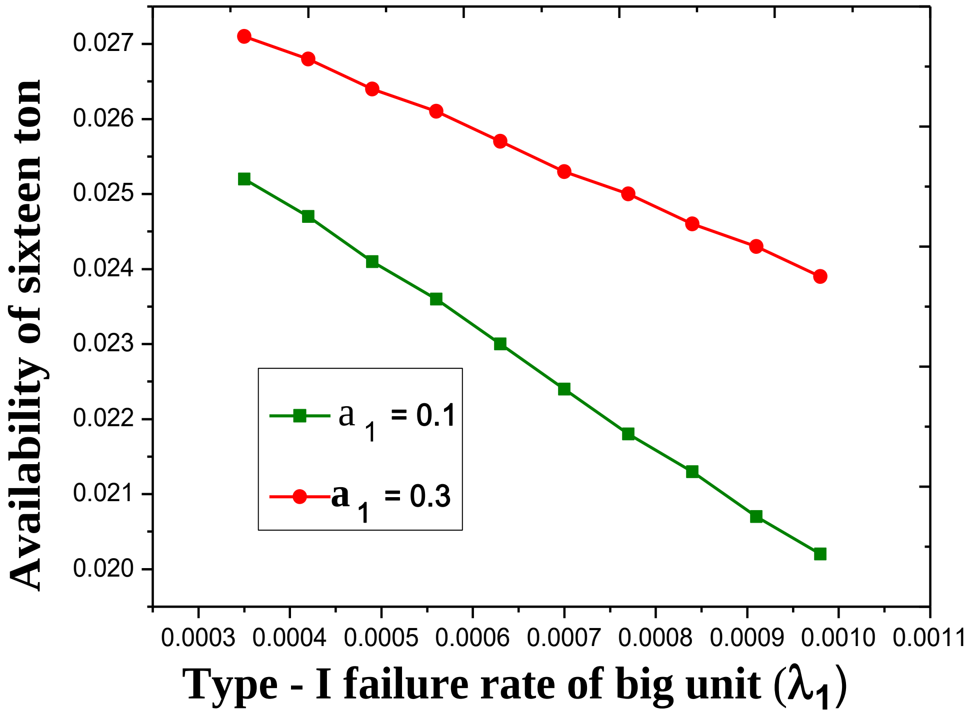

Fig. 4.2 - Effect of Type-I failure rate on Availability of sixteen ton capacity for different values of Type-I repair rate.

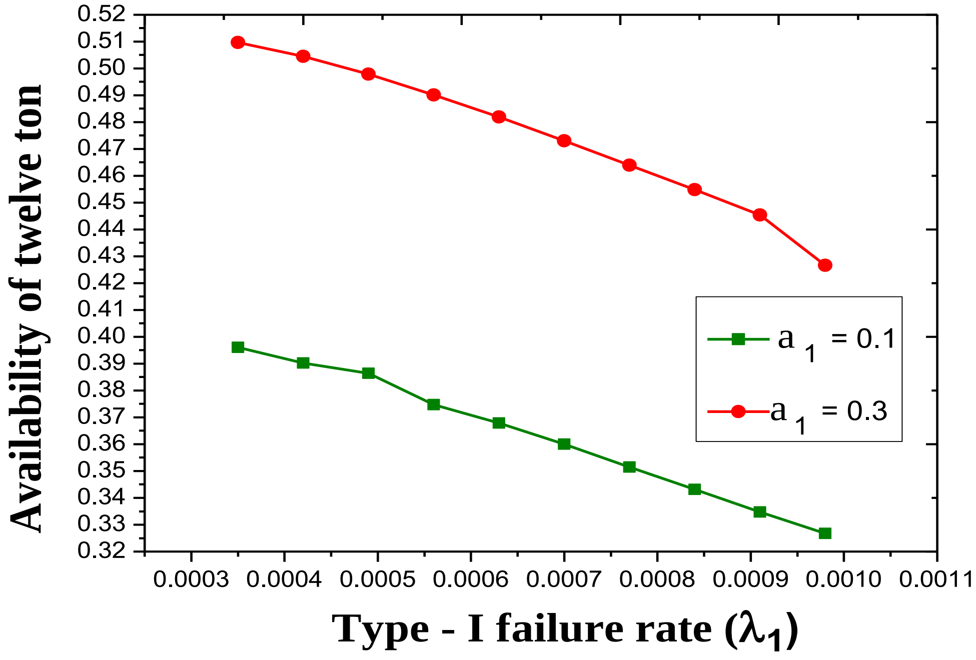

Fig. 4.3 - Effect of Type-I failure rate on Availability of twelve ton capacity for different values of Type-I repair rate.

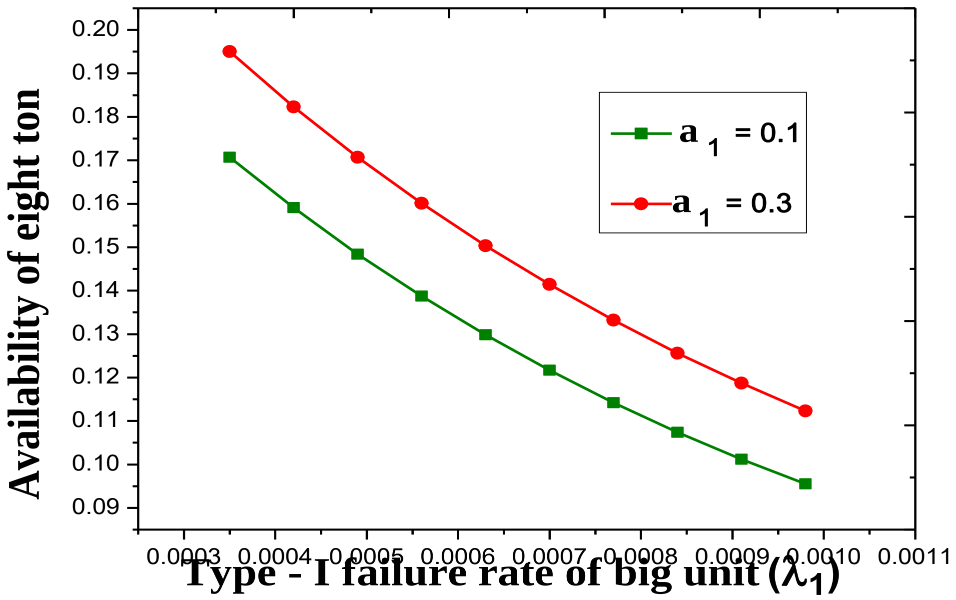

Fig. 4.4 - Effect of Type-I failure rate on Availability of eight ton capacity for distinct values of Type-I repair rate.

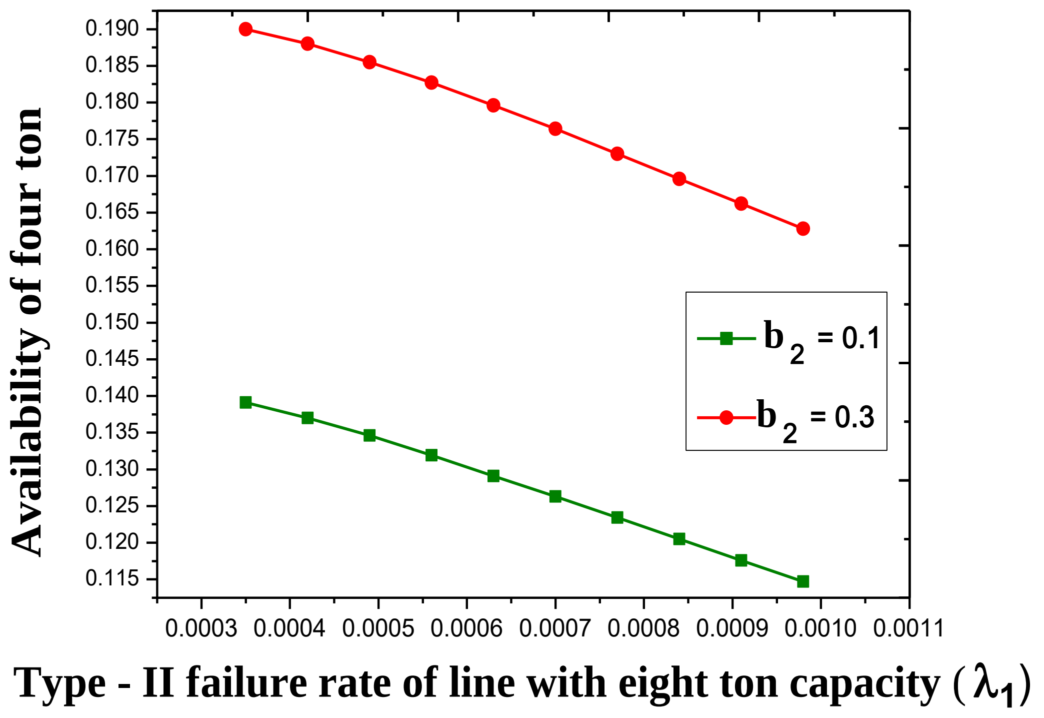

Fig. 4.5 - Effect of Type-II failure rate on Availability of 4 ton capacity for different values of type-II repair rate.

The behaviour of profit with respect to revenue for 4 ton capacity C_{04} for distinct values of cost of repairman C_1 is shown in fig. 4.6.

Values of various measures considered are:

a_1 = 0.3144 ; $b_1 = 0.2346$; a_2 = 0.1736 , $b_2 = 0.2500$; $q = 0.8$; $p = 1 - q$; γ_1 = 0.01736

$h(t) = 0.01$; γ_2 = 0.0987 $C_2 = 1200\, INR$; $C_{01} = 4C_{04}$; $C_{02} = 3C_{04}$; $C_{03} = 2C_{04}$

$p_2 = 0.065$; $p_1 = 0.35$; $p_4 = 0.35$; $p_3 = 1 - p_2 - p_1$; $p_6 = 0.22$; $p_5 = 1 - p_6 - p_4$; $p_7 = 0.45$
$p_8 = 0.082$; $p_9 = 1 - p_7 - p_8$; $p_{10} = 0.025$; $p_{11} = 0.57$; $p_{12} = 0.035$; $p_{13} = 0.12$; $p_{15} = 0.42$
$p_{17} = 1 - p_{13} - p_{15}$; $p_{12} = 1 - p_{10} - p_{11}$; $p_{16} = 0.45$; $p_{14} = 1 - p_{16} - p_{18} = 0.34$; $p_{21} = 0.35$
$p_{20} = 0.065$; $p_{19} = 1 - p_{21}$; $p_{22} = 1 - p_{20}$; $p_{23} = 0.35$; $p_{24} = 0.55$; $p_{25} = 0.75$
$p_{27} = 1 - p_{23}$; $p_{26} = 0.43$; $p_{28} = 1 - p_{24}$; $p_{29} = 1 - p_{25}$; $p_{30} = 1 - p_{24}$; $\beta_1 = 0.0084$
$\beta_2 = 0.0036$; $\lambda_1 = 0.0007$; $\lambda_2 = 0.0003$

It is observed from this graph that profit increases with the increase in revenue C_{04} and decreases with increase in cost of repairman. On comparing graphs, it reveals that

(i) For $C_1 = 3500$ INR, the profit is negative or zero or positive according as $C_{04} \le or \ge 1013.80$. Hence, revenue for 12 ton capacity when production is greater than or equal to demand should be fixed greater than 1013.80 INR to attain the profit.

(ii) For $C_1 = 4000$ INR, the profit is negative or zero or positive according as $C_{04} \le or \ge 1131.75$. Hence, revenue for 12 ton capacity when production is greater than or equal to demand should be fixed greater than 1131.75 INR to attain the profit.

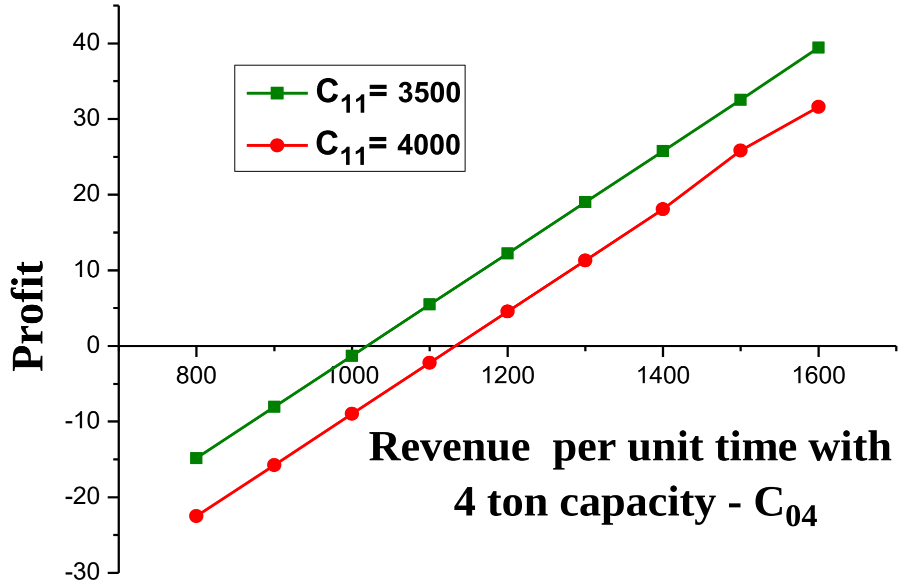

Fig. 4.6 - Effect of revenue of 4 ton capacity on profit for distinct values of cost of repairman.

Fig. 4.7 shows the behaviour of profit with respect to cost of repairman for distinct values of revenue for 4 ton C_{04}. It is observed from this graph that profit decreases with the increase in cost of repairman C_1 and increases with increase in revenue C_{04}. On comparing graphs, it reveals that

(i) For $C_{04} = 1000$ INR the profit is negative or zero or positive according as $C_1 \geq or \leq 3415.70$. Hence, repairman should not be paid more than 3415.70 INR to attain the profit.

(ii) For $C_{04} = 1500$ INR the profit is negative or zero or positive according as $C_1 \geq or \leq 4294.85$. Hence, repairman should not be paid more than 4294.85 INR to attain the profit.

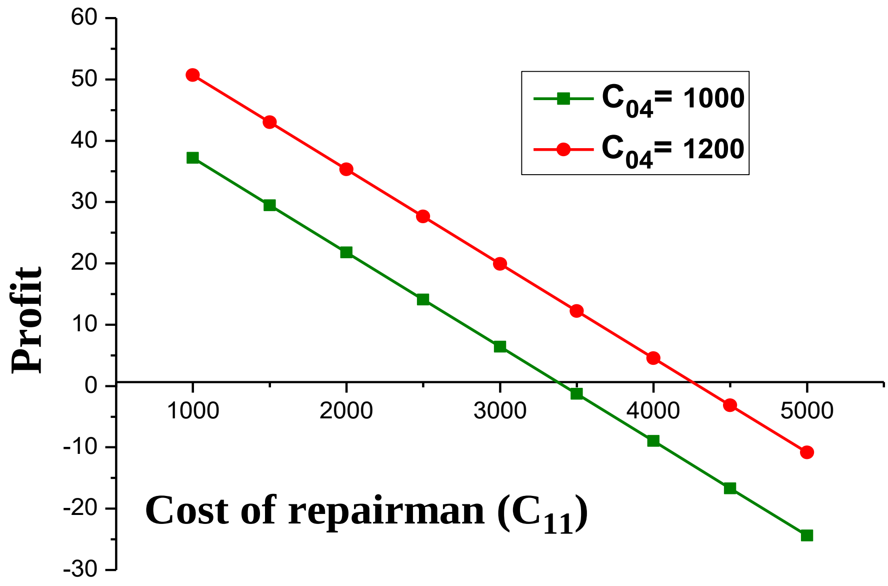

Fig. 4.7 - Effect of cost of repairman on profit for distinct values of revenue for 4 ton capacity.

CHAPTER 5

COST ANALYSIS OF A TWO-UNIT STANDBY INDUSTRIAL SYSTEM WITH VARYING DEMAND

In the preceding chapter, cost benefit has been carried out for a system having three units of capacities: eight ton, four ton and four ton with varying demand and two types of failures. Instead of considering two units of four ton capacity each, we can consider a single unit of eight ton capacity in pace of two units of four ton capacity each. Keeping in mind the installation cost of one big unit in place of two small units, we have considered a system having two similar units of eight ton capacity each. In this chapter a two similar unit standby system is analysed with variation of demand and two types of failures.

This chapter has been organised as follows: Section 5.1 describes description of a rice manufacturing system and assumptions of two-unit standby system are also discussed in this section. Section 5.2 presents various notations used in this chapter. The mathematical formulation for stochastic model determining, transition probabilities and mean sojourn times has been developed in Section 5.3. Section 5.4 – 5.6 deals with the formulation of Mean Time to System Failure, availability, busy period analysis and expected down time of the system. Cost Benefit analysis has been done in section 5.7. Conclusions based on the present study have been finally drawn in Section 5.8.

5.1 System Descriptions and Assumptions

This system has two similar paddy to rice converting units of same capacity which are made operative depending upon demand. Both the units are of eight ton capacity. The system under investigation considers the situation where the system has two shifts of working and before starting the second shift the whole system undergoes for scheduled preventive/corrective maintenance. Initially the system is in the working condition with both units operative. The failed unit is undertaken for repair immediately. After getting repaired the unit can be made operative or standby as per the requirement.

The following assumptions have been considered for the model:

(i) After repair unit behaves as totally new.

(ii) After getting repaired the unit can be made operative or standby according to the need.

(iii) There is a single repair facility.

(iv) Failure times are assumed to follow exponential distribution.

(v) Repair times are assumed to follow any arbitrary distribution.

The system has been observed at suitable regenerative epochs by using regenerative point technique and the following reliability characteristics have been obtained:

(i) Mean time to system failure (MTSF)
(ii) Availability
(iii) Expected busy period of repairman in (0,t]
(iv) Expected profit incurred in (0,t]

5.2 Notations:

B_s	:	Unit with eight ton capacity in standby mode
B_o	:	Unit with eight ton capacity in operative mode
B_{op}	:	Unit with eight ton capacity in pending operation state.
B_r, B_r	:	System is at rest
F_{r_1}	:	Unit having Type – I failure is under repair
F_{r_2}	:	Unit having Type – II failure is under repair
F_{wr_1}	:	Unit having Type – I failure is waiting for repair
F_{wr_2}	:	Unit having Type – II failure is waiting for repair
F_{R_1}	:	Repair of Type – I failure is continuing from previous state
F_{R2}	:	Repair of Type – I failure is continuing from previous state
λ_1	:	Rate of Type - I failure
λ_2	:	Rate of Type - II failure
γ_1	:	Rate with which the system is made operative from rest
γ_2	:	Rate with which the system goes to rest from operative state
$i(t)$	:	p.d.f of time to complete pending process of material at colour

		sorter
$I(t)$	:	c.d.f of time to complete pending process of material at colour sorter
p	:	Probability that the unit is not made operative after repair depending upon demand.
q	:	Probability that the unit is made operative after repair depending upon demand.
$G_1(t)$	:	c.d.f. of the repair time of for type – I failure.
$g_1(t)$	:	p.d.f. of the repair time of unit type - I failure.
$H_1(t)$	:	c.d.f. of time to make operative state standby (as per demand)
$h_1(t)$	:	p.d.f of time to make operative state standby (as per demand)
$H_2(t)$	:	c.d.f. of time to make standby state operative (as per demand)
$h_2(t)$	:	p.d.f of time to make standby state operative (as per demand)

The system of converting paddy into rice of a rice manufacturing plant has the following states:

(i) Regenerative states:

$S_0(B_0,B_0)$, $S_1(B_0,B_s)$, $S_2(B_r,B_r)$, $S_3(BF_{r_1},B_0)$, $S_4(BF_{r_2},B_0)$ and $S_5(B_{op},B_0)$

(ii) Failed and non-regenerative states are:

$S_6(BF_{R_1},BF_{WR_1})$, $S_7(BF_{R_1},BF_{Wr_1})$, $S_8(BF_{R_2},BF_{Wr_2})$ and $S_9(BF_{R_2},BF_{Wr_1})$.

Table 5.1 – Possible states of Transition

State (S_i) ***i=0 to 9***	***Status***	***Possible transition to***	***With failure / repair rates / transition probabilities / p.d.f respectively***
0	Bo, Bo	1, 2, 3 & 4	$h_1(t), \gamma_1, 2\lambda_1 \& 2\lambda_2$
1	Bo, Bs	0, 2, 3 & 4	$h_2(t), \gamma_1, \lambda_1 \& \lambda_2$
2	Br, Br	0	γ_2
3	Bfr_1, Bo	0, 5, 6 & 7	$q\,g_1(t), p\,g_1(t), \lambda_1 \& \lambda_2$
4	Bfr_2, Bo	0, 1, 8 & 9	$q\,g_2(t), p\,g_2(t), \lambda_2 \& \lambda_1$
5	B_{op}, Bo	1	$i(t)$
6	$Bfr_1, Bwfr_1$	3	$g_1(t)$
7	$Bfr_1, Bwfr_2$	4	$g_2(t)$
8	$Bfr_2, Bwfr_2$	4	$g_2(t)$
9	$Bfr_2, Bwfr_1$	3	$g_1(t)$

In the light of the system description, notations and assumptions, we present a statistical model of two unit system here.

5.3 Transition Probabilities and Mean Sojourn Time

A transition table shown in Fig.1 exhibits the various states of the system. The epochs of entry into states 0, 1, 2, 3, 4 and 5 are regenerative points. The transition probabilities p_{ij} have been obtained as:

$$p_{ij} = \lim_{s \to 0} q_{ij}^*(s) = \lim_{s \to 0} \int_0^\infty e^{-st} q_{ij}(t)\,dt \tag{5.1}$$

$$\left.\begin{aligned}
&q_{01} = h_1(t)\,e^{-(\gamma_1+2\lambda_1+2\lambda_2)t}\ ,\ q_{02} = \gamma_1\, e^{-(\gamma_1+2\lambda_1+2\lambda_2)t}\, \bar{H}_1(t)\\
&q_{03} = 2\lambda_1\, e^{-(\gamma_1+2\lambda_1+2\lambda_2)t}\, \bar{H}_1(t)\ ,\ q_{04} = 2\lambda_2\, e^{-(\gamma_1+2\lambda_1+2\lambda_2)t}\, \bar{H}_1(t)\\
&q_{10} = h_2(t)\,e^{-(\gamma_1+\lambda_1+\lambda_2)t}\ ,\ p_{12} = \gamma_1\, e^{-(\gamma_1+2\lambda_1+2\lambda_2)t}\, \bar{H}_2(t)\\
&q_{13} = \lambda_1\, e^{-(\gamma_1+2\lambda_1+2\lambda_2)t}\, \bar{H}_2(t)\ ,\ q_{14} = \lambda_2\, e^{-(\gamma_1+2\lambda_1+2\lambda_2)t}\, \bar{H}_2(t)\\
&q_{20} = 1\ ;\ q_{30} = q\,g_1(t)\,e^{-(\lambda_1+\lambda_2)t}\ ;\ q_{35} = p\,g_1(t)\,e^{-(\lambda_1+\lambda_2)t}\\
&q_{36} = \lambda_1\, e^{-(\lambda_1+\lambda_2)t}\,\bar{G}_1(t)\ ,\ q_{37} = \lambda_2\, e^{-(\lambda_1+\lambda_2)t}\,\bar{G}_1(t)\\
&q_{33}^{(6)} = e^{-\lambda_1 t}\,\bar{G}_1(t)\ ,\ q_{34}^{(7)} = e^{-\lambda_2 t}\,\bar{G}_1(t)\\
&q_{40} = q\,g_2(t)\,e^{-(\lambda_1+\lambda_2)t}\ ,\ q_{41} = p\,g_2(t)\,e^{-(\lambda_1+\lambda_2)t}\\
&q_{48} = \lambda_2\, e^{-(\lambda_1+\lambda_2)t}\,\bar{G}_2(t)\ ,\ q_{49} = \lambda_1\, e^{-(\lambda_1+\lambda_2)t}\,\bar{G}_2(t)\\
&q_{44}^{(8)} = e^{-\lambda_2 t}\,\bar{G}_2(t)\ ,\ q_{49}^{(3)} = e^{-\lambda_1 t}\,\bar{G}_2(t)\ ,\ q_{51} = i(t)
\end{aligned}\right\} \tag{5.2}$$

The non zero element p_{ij} are obtained as:

$$\left.\begin{aligned}
&p_{01} = h_1^*(\gamma_1+2\lambda_1+2\lambda_2)\ ,\ p_{02} = \frac{\gamma_1}{\gamma_1+2\lambda_1+2\lambda_2}\left(1-h_1^*(\gamma_1+2\lambda_1+2\lambda_2)\right)\\
&p_{03} = \frac{2\lambda_1}{\gamma_1+2\lambda_1+2\lambda_2}\left(1-h_1^*(\gamma_1+2\lambda_1+2\lambda_2)\right),\ p_{04} = \frac{2\lambda_2}{\gamma_1+2\lambda_1+2\lambda_2}\left(1-h_1^*(\gamma_1+2\lambda_1+2\lambda_2)\right)\\
&p_{10} = h_2^*(\gamma_1+\lambda_1+\lambda_2)\ ,\ p_{12} = \frac{\gamma_1}{\gamma_1+\lambda_1+\lambda_2}\left(1-h_2^*(\gamma_1+\lambda_1+\lambda_2)\right)\\
&p_{13} = \frac{\lambda_1}{\gamma_1+\lambda_1+\lambda_2}\left(1-h_2^*(\gamma_1+\lambda_1+\lambda_2)\right),\ p_{14} = \frac{\lambda_2}{\gamma_1+\lambda_1+\lambda_2}\left(1-h_2^*(\gamma_1+\lambda_1+\lambda_2)\right)\\
&p_{20} = 1\ ,\ p_{30} = q\,g_1^*(\lambda_1+\lambda_2)\ ,\ p_{35} = p\,g_1^*(\lambda_1+\lambda_2)\ ,\ p_{36} = \frac{\lambda_1}{\lambda_1+\lambda_2}\,g_1^*(\lambda_1+\lambda_2)\\
&p_{37} = \frac{\lambda_2}{\lambda_1+\lambda_2}\,g_1^*(\lambda_1+\lambda_2)\ ,\ p_{33}^{(6)} = 1-g_1^*(\lambda_1)\ ,\ p_{34}^{(7)} = 1-g_1^*(\lambda_2)\\
&p_{40} = q\,g_2^*(\lambda_1+\lambda_2)\ ,\ p_{41} = p\,g_2^*(\lambda_1+\lambda_2)\ ,\ p_{48} = \frac{\lambda_2}{\lambda_1+\lambda_2}\left(1-g_2^*(\lambda_1+\lambda_2)\right)\\
&p_{49} = \frac{\lambda_1}{\lambda_1+\lambda_2}\left(1-g_2^*(\lambda_1+\lambda_2)\right)\ ,\ p_{44}^{(8)} = 1-g_2^*(\lambda_2)\ ,\ p_{49}^{(3)} = 1-g_2^*(\lambda_1)\ ,\ p_{51} = 1
\end{aligned}\right\} \tag{5.3}$$

By using above equations, it can be verified that

$$\left.\begin{array}{l} p_{01}+p_{02}+p_{03}+p_{04}=1 \;\;,\;\; p_{10}+p_{12}+p_{13}+p_{14}=1 \\ p_{20}=1 \;\;;\;\; p_{30}+p_{35}+p_{36}+p_{37}=1 \\ p_{40}+p_{41}+p_{48}+p_{49}=1 \;\;,\;\; p_{51}=1 \\ p_{30}+p_{35}+p_{33}^{(6)}+p_{34}^{(7)}=1 \;\;,\;\; p_{40}+p_{41}+p_{44}^{(8)}+p_{43}^{(9)}=1 \end{array}\right\}$$

If T denotes the sojourn time in the regenerative state 'i', then mean sojourn time μ_i is obtained as:

$$\mu_i = E(t) = P_r(T>t) = \int_0^\infty d\,(Q_{ij}(t)) \tag{5.4}$$

$$\mu_0 = \frac{1}{\gamma_1+2\lambda_1+2\lambda_2}\left(1-h_1^*(\gamma_1+2\lambda_1+2\lambda_2)\right)$$

$$\mu_1 = \frac{1}{\gamma_1+\lambda_1+\lambda_2}\left(1-h_2^*(\gamma_1+\lambda_1+\lambda_2)\right) \;\;,\;\; \mu_2 = \frac{1}{\gamma_2}$$

$$\mu_3 = \frac{1}{\lambda_1}\left(1-g_1^*(\lambda_1)\right) \;\;,\;\; \mu_4 = \frac{1}{\lambda_2}\left(1-g_2^*(\lambda_2)\right)$$

$$\mu_5 = -i^{*\prime}(0) = 1$$

The unconditional mean time taken by the system is mathematically stated as

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = \int_0^\infty t\;q_{ij}(t)\,dt = -q_{ij}^*(0) \tag{5.5}$$

Thus, we get

$$m_{01}+m_{02}+m_{03}+m_{04} = \mu_0 \;\;,\;\; m_{10}+m_{12}+m_{13}+m_{14} = \mu_1$$

$$m_{20} = \mu_2 \;\;,\;\; m_{30}+m_{35}+m_{36}+m_{37} = \mu_3$$

$$m_{40}+m_{41}+m_{48}+m_{49} = \mu_4 \;\;,\;\; m_{51} = \mu_5$$

$$m_{30}+m_{35}+m_{33}^{(6)}+m_{34}^{(7)} = k_3(say) \;\;,\;\; m_{40}+m_{41}+m_{44}^{(8)}+m_{43}^{(9)} = k_4$$

5.4 Mean Time to System Failure (MTSF)

In order to determine the mean time to system failure (MTSF) of the system, the failed states are considered as absorbing states. We obtain the following recursive relation for $\phi_i(t)$:

$$\left.\begin{aligned}
&\phi_0(t)=Q_{01}(t)(s)\,\phi_1(t)+Q_{02}(t)(s)\phi_2(t)+Q_{03}(t)(s)\phi_3(t)+Q_{04}(t)(s)\phi_4(t)\\
&\phi_1(t)=Q_{10}(t)(s)\phi_1(t)+Q_{12}(t)(s)\phi_2(t)+Q_{13}(t)(s)\phi_3(t)+Q_{14}(t)(s)\phi_4(t)\\
&\phi_2(t)=Q_{20}(t)+\phi_0(t)\\
&\phi_3(t)=Q_{30}(t)(s)\phi_0(t)+Q_{35}(t)(s)\phi_5(t)+Q_{36}(t)+Q_{37}(t)\\
&\phi_4(t)=Q_{41}(t)(s)\phi_1(t)+Q_{42}(t)(s)\phi_2(t)+Q_{48}(t)+Q_{49}(t)\\
&\phi_5(t)=Q_{51}(t)(s)\phi_1(t)
\end{aligned}\right\} \tag{5.6}$$

In order to obtain MTSF using methodology of section 2.4, we first take Laplace Steltjes Transforms of equations (5.6) and then solve them for $\phi_0^{**}(s)$. Thus, we get

$$MTSF = \lim_{s\to 0}\frac{1-\varphi_0{}^{**}(s)}{s}=\frac{N}{D}, \tag{5.7}$$

where

$$\begin{aligned}
N = {}&\mu_0(-p_{14}p_{41}-p_{13}p_{35}+1)-\mu_1(-p_{01}-p_{04}p_{41}-p_{03}p_{35})\\
&-\mu_2(-p_{02}p_{12}-p_{04}p_{12}p_{41}-p_{12}p_{03}p_{35}-p_{02}+p_{02}p_{14}p_{41})\\
&-\mu_3(-p_{03}+p_{03}p_{14}p_{41}-p_{01}p_{13}-p_{04}p_{13}p_{41})\\
&-\mu_4(-p_{04}+p_{04}p_{13}p_{35}-p_{01}p_{14}-p_{03}p_{14}p_{35})\\
&-\mu_5(-p_{03}p_{35}-p_{01}p_{13}p_{35}-p_{13}p_{35}p_{04}p_{41}+p_{03}p_{35}p_{14}p_{41}
\end{aligned}$$

$$\begin{aligned}
D = 1&-p_{14}p_{41}-p_{13}p_{35}-p_{04}p_{40}-p_{02}+p_{02}p_{14}p_{41}+p_{02}p_{13}p_{35}-p_{01}p_{12}+p_{04}p_{40}p_{13}p_{35}\\
&-p_{03}p_{30}+p_{03}p_{30}p_{14}p_{41}-p_{01}p_{10}-p_{01}p_{14}p_{40}-p_{01}p_{13}p_{30}-p_{04}p_{41}p_{12}-p_{04}p_{41}p_{10}\\
&-p_{04}p_{13}p_{41}p_{30}-p_{12}p_{03}p_{35}-p_{03}p_{35}p_{10}-p_{03}p_{35}p_{14}p_{40}
\end{aligned}$$

5.5 Availability Analysis

Following the method used in preceding chapters, the availability $A_i(t)$ is expressed as the following recursive relations:

$$\left.\begin{array}{l}
A_0(t)=M_0(t)+q_{01}(t)\text{(c)}A_1(t)+q_{02}(t)\text{(c)}A_2(t)+q_{03}(t)\text{(c)}A_3(t)+q_{04}(t)\text{(c)}A_4(t)\\
A_1(t)=M_1(t)+q_{10}(t)\text{(c)}A_1(t)+q_{12}(t)\text{(c)}A_2(t)+q_{13}(t)\text{(c)}A_3(t)+q_{14}(t)\text{(c)}A_4(t)\\
A_2(t)=q_{20}(t)\text{(c)}A_0(t)\\
A_3(t)=M_3(t)+q_{30}(t)\text{(c)}A_0(t)+q_{35}(t)\text{(c)}A_5(t)+q_{33}^{(6)}(t)\text{(c)}A_3(t)\\
\qquad +q_{34}^{(7)}(t)\text{(c)}A_4(t)\\
A_4(t)=q_{41}(t)\text{(c)}A_1(t)+q_{42}(t)\text{(c)}A_2(t)+q_{44}^{(8)}(t)\text{(c)}A_4(t)+q_{43}^{(9)}(t)\text{(c)}A_3(t)\\
A_5(t)=M_5(t)+q_{51}(t)\text{(c)}A_1(t)
\end{array}\right\}, \qquad (5.8)$$

where

$$M_0(t)=e^{-(2\lambda_1+2\lambda_2)t}\,\bar{H}_1(t)\ ;\ M_1(t)=e^{-(\gamma_1+\lambda_1+\lambda_2)t}\,\bar{H}_2(t)$$

$$M_3(t)=e^{-(\lambda_1+\lambda_2)t}\,\bar{G}_2(t)\ ;\ M_4(t)=e^{-(\lambda_1+\lambda_2)t}\,\bar{G}_1(t)\ ;\ M_5(t)=\bar{I}(t)$$

$$\bar{G}_1(t)=1-G_1(t)\ ;\ \bar{H}_1(t)=1-H_1(t)\ ;\ \bar{G}_2(t)=1-G_2(t)\ ;\ \bar{H}_2(t)=1-H_2(t)$$

Next, taking Laplace transforms of equations (5.8) and solving them for $A_0^*(s)$, we get

$$A_0=\lim_{s\to 0} s\,A_0^*(s)=\frac{N_1}{D_1}, \qquad (5.9)$$

where

$$\begin{aligned}
D_1 = \mu_0\{&p_{30}\left(1-p_{44}^{(8)}\right)+p_{35}p_{40}(1-p_{13})+(p_{10}+p_{12})\\
&\left(p_{35}p_{41}+p_{35}p_{43}^{(9)}+p_{34}^{(7)}p_{41}\right)-p_{30}p_{14}p_{41}+p_{34}^{(7)}p_{40}\}\\
&+\mu_1\{p_{01}p_{30}(1-p_{44}^{(8)})+p_{04}p_{30}p_{41}+p_{35}p_{40}(p_{01}+p_{03})\\
&+p_{01}p_{40}p_{34}^{(7)}+(1-p_{02})(p_{35}p_{41}+p_{35}p_{43}^{(9)}+p_{41}p_{34}^{(7)})\}\\
&+k_3\{p_{03}p_{40}+p_{03}p_{41}-p_{03}p_{14}p_{41}+p_{01}p_{40}p_{13}+(1-p_{02}-p_{03})\\
&p_{13}p_{41}-p_{01}(p_{10}+p_{12})p_{43}^{(9)}+(1-p_{02})p_{43}^{(9)}\}+k_4\{p_{35}p_{14}\\
&(p_{01}+p_{03})-p_{01}(p_{10}+p_{12})p_{34}^{(7)}+(1-p_{02})p_{34}^{(7)}+p_{35}p_{04}\\
&(1-p_{13})+p_{30}p_{04}+p_{01}p_{14}p_{30}\}+\mu_2\{p_{01}p_{12}(p_{30}+p_{35}+p_{34}^{(7)})\\
&(p_{40}+p_{41}+p_{43}^{(9)}-p_{14}p_{41})-p_{01}p_{12}p_{34}^{(7)}p_{43}^{(9)}-p_{01}p_{12}p_{13}p_{41}p_{34}^{(7)}\\
&-p_{01}p_{12}p_{43}^{(9)}p_{14}p_{35}-p_{01}p_{12}p_{13}p_{35}(p_{40}+p_{41}+p_{43}^{(9)})
\end{aligned}$$

$$
\begin{aligned}
N_1 = {} & (M_0 + p_{01}M_1)(1 - p_{44}^{(8)} - p_{14}p_{41} - p_{33}^{(6)} + p_{33}^{(6)}p_{44}^{(8)} \\
& + p_{33}^{(6)}p_{14}p_{41} + p_{13}p_{35} + p_{13}p_{35}p_{44}^{(8)} + p_{13}p_{35}p_{14}p_{41}) \\
& + (M_4 + p_{41}M_1)(p_{03}p_{34}^{(7)} + p_{03}p_{14}p_{35} + p_{04} - p_{04}p_{33}^{(6)} \\
& - p_{13}p_{35}p_{04}) + M_3(-p_{03}p_{44}^{(8)} + p_{03} - p_{03}p_{14}p_{41} + p_{04}p_{43}^{(9)} \\
& + p_{04}p_{13}p_{41}) + p_{03}p_{35} - p_{03}p_{35}p_{44}^{(8)} + p_{03}p_{35}p_{14}p_{41} \\
& + p_{04}p_{35}p_{43}^{(9)} + p_{13}p_{35}p_{04}p_{41} + (p_{03} + p_{01}p_{13}) \\
& (p_{12}p_{35}p_{40} + p_{12}p_{35}p_{41} + p_{12}p_{35}p_{43}^{(9)} + p_{12}p_{34}^{(7)}p_{41}) \\
& + (p_{04} + p_{01}p_{14})(p_{12}p_{35}p_{43}^{(9)} + p_{12}p_{30}p_{41} + p_{12}p_{35}p_{41} \\
& + p_{12}p_{34}^{(7)}p_{41})\} + \mu_5\{(1 - p_{02} - p_{01}(p_{10} + p_{12})) \\
& (p_{14}p_{35}p_{13}p_{41} + p_{14}p_{35}p_{43}^{(9)} + p_{13}p_{35}p_{40} + p_{13}p_{35}p_{41} \\
& + p_{13}p_{35}p_{43}^{(9)}) + (p_{03} + p_{01}p_{13})((p_{10} + p_{12})p_{35}p_{40} \\
& + (p_{10} + p_{12})p_{35}p_{41} + (p_{10} + p_{12})p_{35}p_{43}^{(9)} + p_{14}p_{35}p_{40}) \\
& + (p_{04} + p_{01}p_{14})(p_{10}p_{35}p_{43}^{(9)} + p_{12}p_{35}p_{43}^{(9)} + p_{13}p_{35}p_{40})\}
\end{aligned}
$$

5.6 Busy period of repairman

Following the method used in case of MTSF and Availability recursive relations for busy period have been obtained as:

$$
\left.
\begin{aligned}
& B_0(t) = q_{01}(t)\,(c)\,B_1(t) + q_{02}(t)\,(c)\,B_2(t) + q_{03}(t)\,(c)\,B_3(t) + q_{04}(t)\,(c)\,B_4(t) \\
& B_1(t) = q_{10}(t)\,(c)\,B_1(t) + q_{12}(t)\,(c)\,B_2(t) + q_{13}(t)\,(c)\,B_3(t) + q_{14}(t)\,(c)\,B_4(t) \\
& B_2(t) = q_{20}(t)\,(c)\,B_0(t) \\
& B_3(t) = W_3(t) + q_{30}(t)\,(c)\,B_0(t) + q_{35}(t)\,(c)\,B_5(t) + q_{33}^{(6)}(t)\,(c)\,B_3(t) \\
& \qquad + q_{34}^{(7)}(t)\,(c)\,B_4(t) \\
& B_4(t) = W_4(t) + q_{41}(t)\,(c)\,B_1(t) + q_{42}(t)\,(c)\,B_2(t) + q_{44}^{(8)}(t)\,(c)\,B_4(t) \\
& \qquad + q_{43}^{(9)}(t)\,(c)\,B_3(t) \\
& B_5(t) = q_{51}(t)\,(c)\,B_1(t)
\end{aligned}
\right\} \qquad (5.10)
$$

Finally, again taking Laplace transforms of equations (5.10) and solving them for $B_0^*(s)$ we get

$$
B_0^*(s) = \frac{N_3(s)}{D_1(s)} \qquad (5.11)
$$

In steady-state, the total fraction of time for which the system is under the service of assistant repairman is given by

$$B_o = \lim_{s \to 0} B_o^*(s) = \frac{N_3}{D_1} \quad , \tag{5.12}$$

where

$$N_3 = (p_{03} + p_{04}) \begin{bmatrix} p_{48} + p_{49} + W_4 - p_{41} p_{14} (p_{48} + p_{49} + W_4) \\ - p_{13} p_{35} (p_{48} + p_{48} + W_4) \end{bmatrix}$$
$$+ (p_{03} + p_{04} + p_{13} + p_{14} + p_{04} p_{41} + p_{03} p_{35})(p_{36} + p_{37} + W_3)$$

and D_1 is already specified.

5.7 Cost Benefit Analysis

One of the objectives of reliability analysis is to optimize the profit incurred to the system. To achieve this, profit model is defined by subtracting all expected maintenance liabilities from the total revenue. Using equations (5.10) and (5.12), we get

$$P = C_0 A_0 - C_1 B_0 , \tag{5.13}$$

where C_0 is total revenue per unit time and C_1 is cost of busy period of repairman.

5.8 Conclusion and Discussions

Following particular cases are considered for analyzing the behaviour of the system.

$g_1(t) = \gamma e^{-\gamma t}$; $g_2(t) = \alpha_1 e^{-\alpha_1 t}$; $h_1(t) = \alpha e^{-\alpha t}$; $h_2(t) = \beta e^{-\beta t}$

In this study data for all types of failures and repairs of the concerned industry was collected in the units of per hour.

On the basis of these data, we have computed the following rates: $\gamma = 0.01$, $\alpha_1 = 0.0123$, $\gamma_1 = 0.01736$ and $\gamma_2 = 0.0987$. Assuming $\alpha = 0.01$, $\beta = 0.01$ and $q = 0.8$; $p = 1 - q$, we have computed the following results of important reliability indices using the software 'MATLAB'. By varying λ_2 (0.01 – 0.024) for distinct values of λ_1, the values of MTSF and availability are computed and their behaviours are exhibited in graphs

(Fig 5.2 and Fig 5.3). Similarly, the profit is also computed by varying C_0 for different choices of C_1 and results are presented in Fig 5.4.

Fig. 5.2 shows the behaviour of MTSF with respect to λ_2 for distinct values of λ_1. The graph shows that MTSF decreases with increase in the λ_2 keeping λ_1 constant and has greater values for lesser values of λ_1.

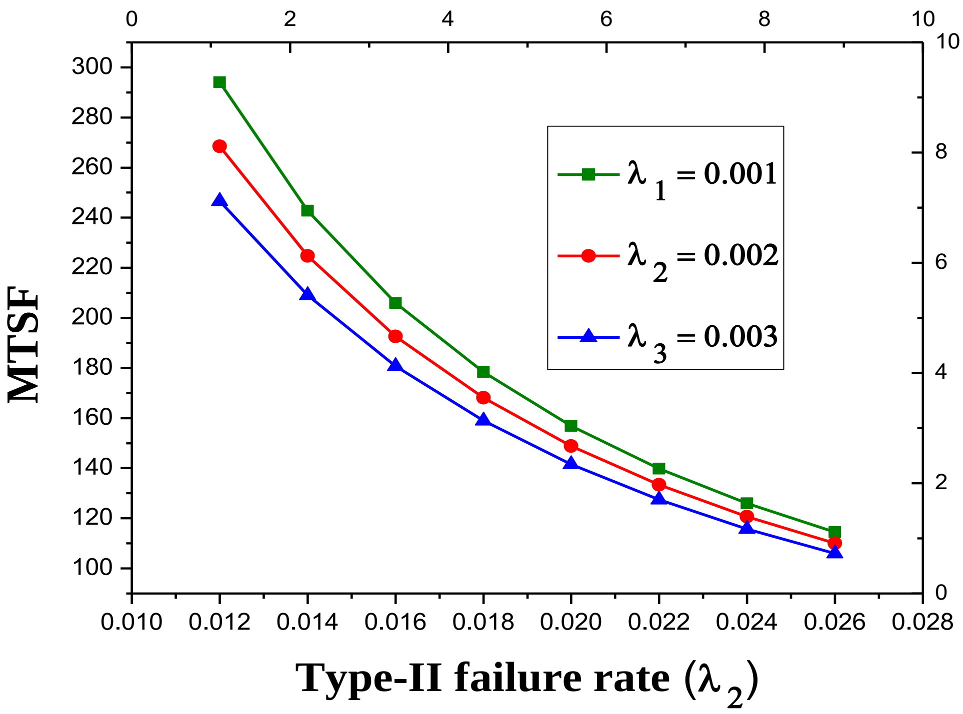

Fig. 5.1 - Effect of Type-II failure rate on Mean time to system failure for distinct values of Type-I failure rate.

Fig. 5.3 shows the behaviour of availability with respect to Type II failure rate (λ_2) for distinct values of Type - I failure rate (λ_1). This graph indicates that availability of the system decreases with increase in the Type - II failure rate (λ_2) keeping Type - I failure rate constant and has greater values for lesser values of Type - I failure rate (λ_1).

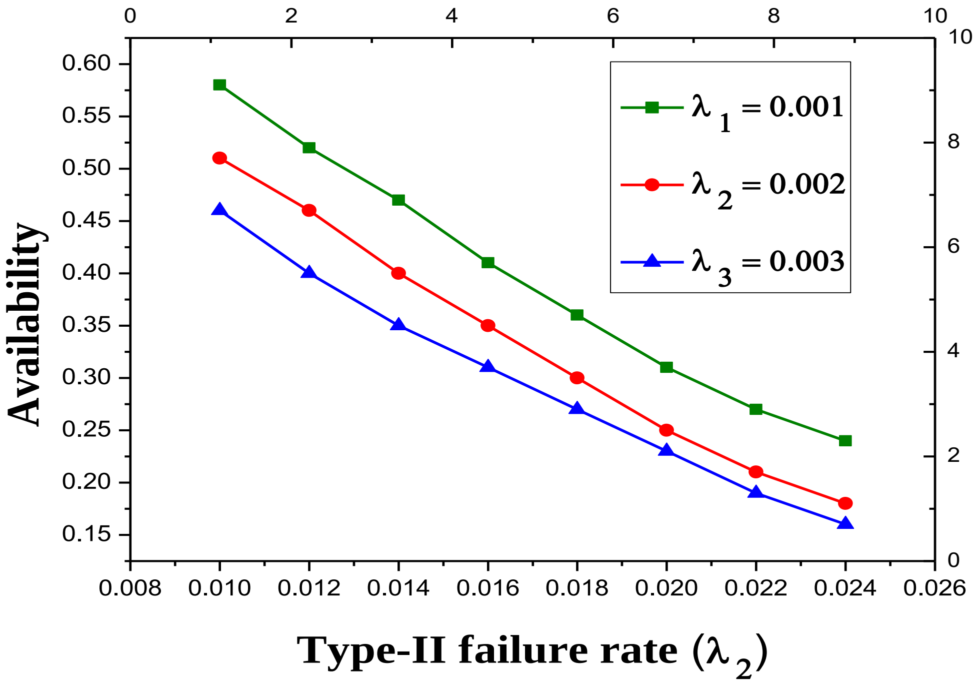

Fig. 5.3 - Effect of Type-II failure rate on availability for distinct values of Type-I failure rate

The behaviour of profit with respect to revenue C_0 for distinct values of cost of repairman C_1 is shown in Fig. 5.4. It is observed from this graph that profit decreases with the increase in revenue per unit time C_0 and has greater values for lesser values of cost of repairman C_1.

On comparing the graphs, it reveals that

(i) For C_1 = 850 INR, the profit is negative or zero or positive according as $C_0 \leq$ or $\geq$ 1449.6458.50 INR. Hence, revenue per unit time should be greater than 1449.6458 INR.

(ii) For C_1 = 900 INR, the profit is positive or zero or negative according as $C_0 \leq$ or $\geq$ 1534.9166 INR. Hence, revenue per unit time should be fixed greater than 1534.9166 INR

(iii) For C_1 = 950 INR, the profit is positive or zero or negative according as $C_0 \leq$ or $\geq$ 1620.1960 INR. Hence, revenue per unit time should be fixed greater than 1620.1960 INR.

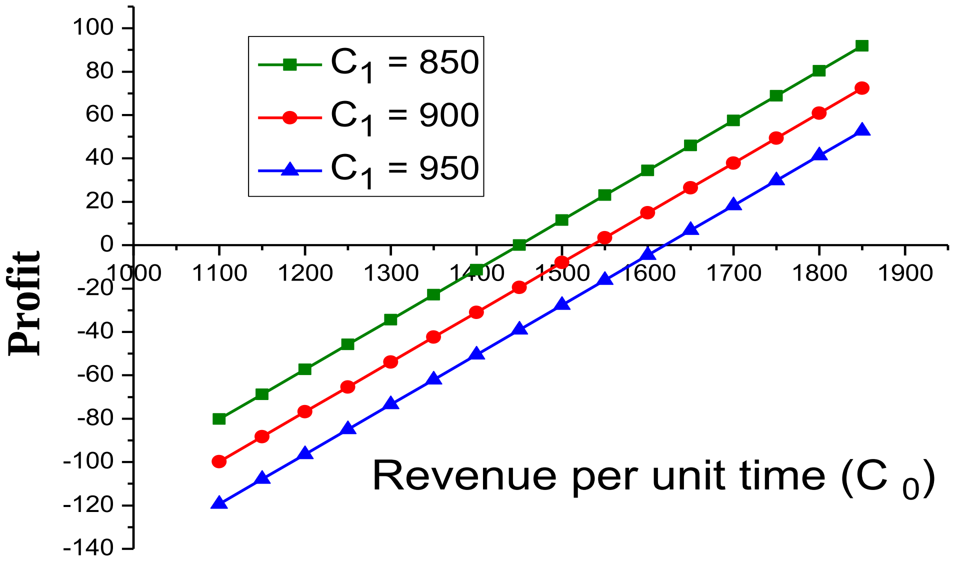

Fig. 5.4 - Effect of revenue per unit time on Profit achieved by the system for different values of cost of busy period of repairman.

CHAPTER 6

COMPARATIVE ANALYSIS OF THE MODELS DISCUSSED UNDER DIFFERENT SITUATIONS

The various reliability models have been developed and analysed for two-unit and three-unit standby systems working at “Kohinoor Foods”. These models have been analysed in terms of performance measures and cost benefit of the system considering various conditions of the system like two type of failures and variation in demand.

A system working under some specific situations cannot be considered as good as compared to other similar systems working under some different situations. A particular system may be better in one situation and may be not as good in other situation. Hence, comparison of various models studied in chapters 3, 4 and 5 by considering different situations for a system is necessary. In this chapter, we compare these models to find which model is better than the other in which particular situation.

6.1 Comparative Analysis Description and Notations

Comparative analysis of the models is done by taking two at a time. Various conclusions have been drawn observing the trend of the graphs as well as the cut-off points. For making the comparative study between two models MTSFs, Availability and Net Profit are needed.

$M_{\mathbf{i}}$	:	Model discussed in ith chapter
$MT_{\mathbf{i}}$	:	MTSF of the model discussed in ith chapter
$A_{\mathbf{i}}$	:	Availability of the model discussed in ith chapter
P_i	:	Profit of the model discussed in ith chapter
IC	:	Installation cost of additional unit
NP_3	:	P_3
NP_4	:	$P_4 - IC$
NP_5	:	$P_5 - IC$
L	:	Goodwill Loss

C_{11}	:	Cost of repairman
C_{04}	:	Revenue for four ton capacity
C_{01}	:	Revenue for twelve ton capacity

Comparative study between the models is described as follows:

6.2 Comparison Between the Models Discussed in Chapter 3 and Chapter 4

Model with two standby units and three standby units are discussed in chapter 3 and chapter 4 respectively. The model discussed in chapter 3 has two units, one is of eight ton capacity and other is of four ton capacity. In chapter 3 both cases are discussed when production is less than demand and at least equal to demand. One additional unit is taken in chapter 4 keeping in mind goodwill loss which can occur in model considered in chapter 3.

The behaviour of the MTSF with respect to Type – I failure rate of eight ton capacity unit for both models M_3 and M_4 is shown in Fig 6.1.

In order to analyse the MTSF with respect to Type – I failure rate of eight ton capacity unit for both models M_3 and M_4 the following values of various measures are considered.

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1-q$
$\gamma_1 = 0.01736/\text{hr}$ $h(t) = 0.01/\text{hr}$; $\gamma_2 = 0.0987/\text{hr}$
$p_2 = 0.065$, $p_1 = 0.35$, $p_4 = 0.35$, $p_3 = 1-p_2-p_1$, $p_6 = 0.22$; $p_5 = 1-p_6-p_4$, $p_7 = 0.45$
$p_8 = 0.082$, $p_9 = 1-p_7-p_8$; $p_{10} = 0.025$; $p_{11} = 0.57$, $p_{12} = 0.035$, $p_{13} = 0.12$, $p_{15} = 0.42$
$p_{17} = 1-p_{13}-p_{15}$, $p_{12} = 1-p_{10}-p_{11}$, $p_{16} = 0.45$, $p_{14} = 1-p_{16}-p_{18} = 0.34$, $p_{21} = 0.35$
$p_{20} = 0.065$, $p_{19} = 1-p_{21}$, $p_{22} = 1-p_{20}$, $p_{23} = 0.35$, $p_{24} = 0.55$, $p_{25} = 0.75$
$p_{27} = 1-p_{23}$, $p_{26} = 0.43$, $p_{28} = 1-p_{24}$, $p_{29} = 1-p_{25}$, $p_{30} = 1-p_{24}$, $\beta_1 = 0.0084/\text{hr}$
$\beta_2 = 0.0036/\text{hr}$

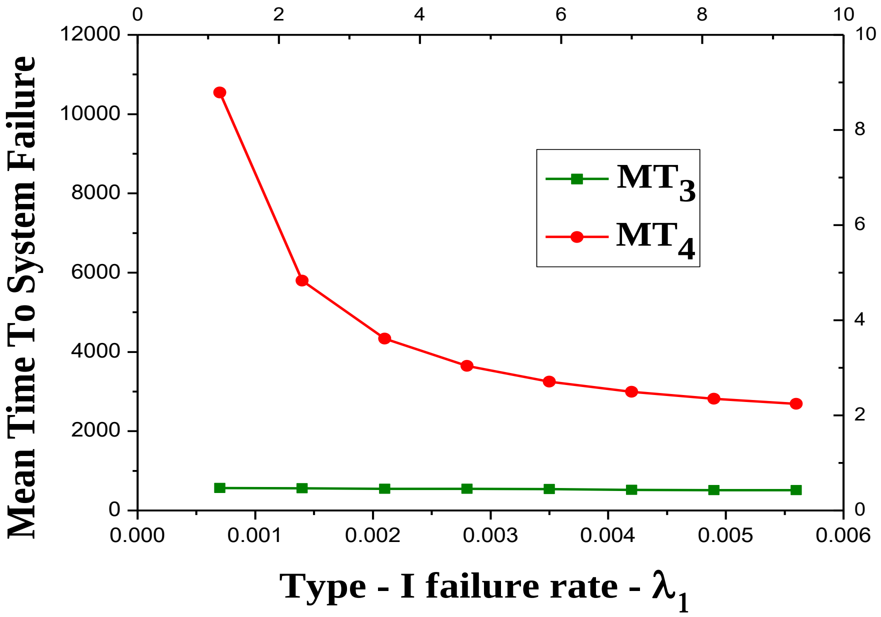

Fig. 6.1 – Effect of Type – I failure rate on MTSF of Model M_4 and Model M_3

From Fig. 6.1 it is clear that $MT_4 > MT_3$ for any value of λ_1, hence Model M_4 is better than Model M_3 for any value of Type – I failure rate.

Fig. 6.2 indicates the behaviour analysis of difference in profits NP_3 and NP_4 of the Models M_3 and M_4 with respect to cost of repairman for distinct values of revenue.

The values of various measures considered are:

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1 - q$

$\gamma_1 = 0.01736/\text{hr}$, $h(t) = 0.01/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$

$IC = 500\,INR$, $C_2 = 1500\,INR$, $L = 3500\,INR$, $C_{01} = 4C_{04}$, $C_{02} = 3C_{04}$, $C_{03} = 2C_{04}$

$p_2 = 0.065$, $p_1 = 0.35$, $p_4 = 0.35$, $p_3 = 1 - p_2 - p_1$, $p_6 = 0.22$; $p_5 = 1 - p_6 - p_4$, $p_7 = 0.45$
$p_8 = 0.082$, $p_9 = 1 - p_7 - p_8$; $p_{10} = 0.025$; $p_{11} = 0.57$, $p_{12} = 0.035$, $p_{13} = 0.12$, $p_{15} = 0.42$
$p_{17} = 1 - p_{13} - p_{15}$, $p_{12} = 1 - p_{10} - p_{11}$, $p_{16} = 0.45$, $p_{14} = 1 - p_{16} - p_{18} = 0.34$, $p_{21} = 0.35$
$p_{20} = 0.065$, $p_{19} = 1 - p_{21}$, $p_{22} = 1 - p_{20}$, $p_{23} = 0.35$, $p_{24} = 0.55$, $p_{25} = 0.75$
$p_{27} = 1 - p_{23}$, $p_{26} = 0.43$, $p_{28} = 1 - p_{24}$, $p_{29} = 1 - p_{25}$, $p_{30} = 1 - p_{24}$, $\beta_1 = 0.0084/\text{hr}$
$\beta_2 = 0.0036/\text{hr}$, $\lambda_1 = 0.0007/\text{hr}$, $\lambda_2 = 0.0003/\text{hr}$

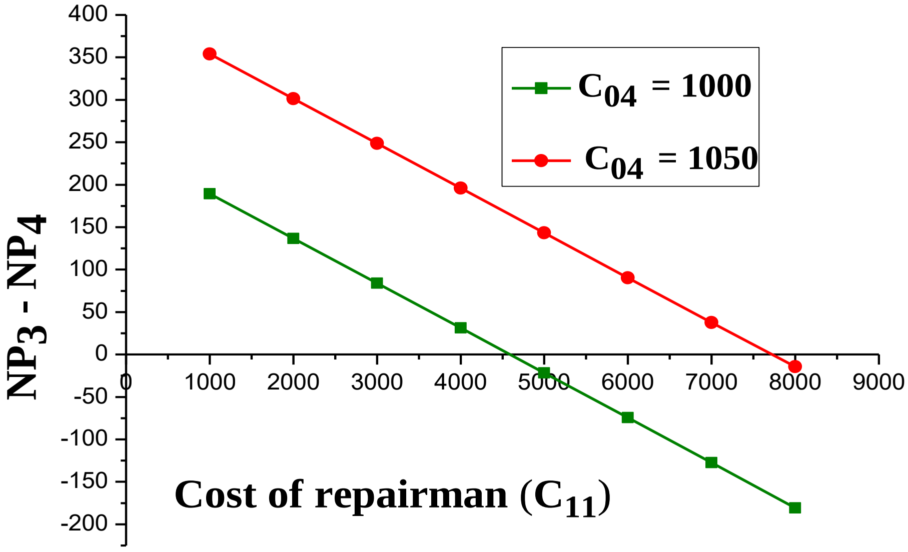

Fig. 6.2 – Difference of profits ($NP_3 - NP_4$) versus Cost of repairman for different values of Revenue.

From Fig 6.2 it has been concluded that:

(i) The difference $NP_3 - NP_4$ decreases as the cost of repairman increases. The difference becomes greater for greater values of revenue.

(ii) For $C_{04} = 1000$, $NP_3 - NP_4$ > or = or < 0 according as C_{11} < or = or > 4587.22 INR. Hence Model M_3 is better when C_{11} < 4587.22 INR and the Model M_4 is better whenever C_{11} > 4587.22 INR. Both the models are equally good if C_{11} = 4587.22 INR.

(iii) For $C_{04} = 1050$, $NP_3 - NP_4$ > or = or < 0 according as C_{11} < or = or > 7719.35 INR. Hence Model M_3 is better when C_{11} < 7719.35 INR and the Model M_4 is better whenever C_{11} > 7719.35 INR. Both the models are equally good if C_{11} = 7719.35 INR.

Fig. 6.3 indicates the behaviour analysis of difference in profits NP_3 and NP_4 of the Models M_3 and M_4 with respect to revenue for distinct values of cost of repairman.

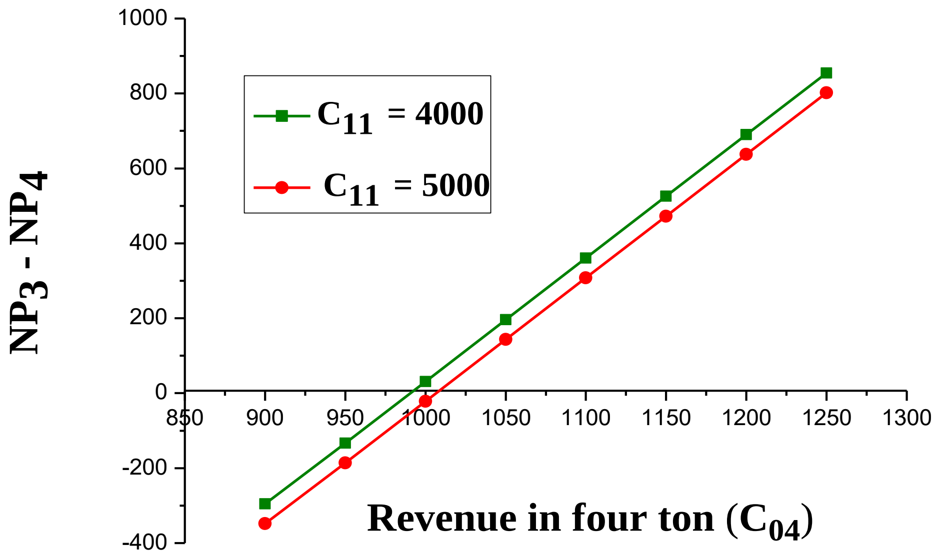

Fig. 6.3 – Difference of profits ($NP_3 - NP_4$) versus Revenue for distinct values of Cost of repairman.

From Fig 6.3 it has been concluded that:

(i) The difference $NP_3 - NP_4$ increases as the revenue of four ton increases. The difference becomes greater for lesser value of cost of repairman.

(ii) For $C_{11} = 4000$, $NP_3 - NP_4$ > or = or < 0 according as C_{04} > or = or < 990.30 INR. Hence Model M_3 is better when C_{04} > 990.30 INR and the Model M_4 is better whenever C_{04} < 990.30 INR. Both the models are equally good if C_{04} = 990.30 INR.

(iii) For $C_{11} = 5000$, $NP_3 - NP_4$ > or = or < 0 according as C_{04} > or = or < 1006.32 INR. Hence Model M_3 is better when C_{04} > 1006.32 INR and the Model M_4 is better whenever C_{04} < 1006.32 INR. Both the models are equally good if C_{04} = 1006.32 INR.

Fig. 6.4 indicates the behaviour analysis of difference in profits NP_3 and NP_4 of the Models M_3 and M_4 with respect to installation cost of addition unit for distinct values of goodwill loss.

The values of various measures considered are:

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1 - q$,

$C_{11} = 4000\, INR$, $C_2 = 1500\, INR$, $C_{04} = 1000\, INR$, $C_{01} = 4C_{04}$, $C_{02} = 3C_{04}$, $C_{03} = 2C_{04}$

$p_2 = 0.065$, $p_1 = 0.35$, $p_4 = 0.35$, $p_3 = 1 - p_2 - p_1$, $p_6 = 0.22$; $p_5 = 1 - p_6 - p_4$, $p_7 = 0.45$
$p_8 = 0.082$, $p_9 = 1 - p_7 - p_8$; $p_{10} = 0.025$; $p_{11} = 0.57$, $p_{12} = 0.035$, $p_{13} = 0.12$, $p_{15} = 0.42$
$p_{17} = 1 - p_{13} - p_{15}$, $p_{12} = 1 - p_{10} - p_{11}$, $p_{16} = 0.45$, $p_{14} = 1 - p_{16} - p_{18} = 0.34$, $p_{21} = 0.35$
$p_{20} = 0.065$, $p_{19} = 1 - p_{21}$, $p_{22} = 1 - p_{20}$, $p_{23} = 0.35$, $p_{24} = 0.55$, $p_{25} = 0.75$
$p_{27} = 1 - p_{23}$, $p_{26} = 0.43$, $p_{28} = 1 - p_{24}$, $p_{29} = 1 - p_{25}$, $p_{30} = 1 - p_{24}$, $\beta_1 = 0.0084/\text{hr}$
$\beta_2 = 0.0036/\text{hr}$, $\lambda_1 = 0.0007/\text{hr}$, $\lambda_2 = 0.0003/\text{hr}$, $\gamma_1 = 0.01736/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$

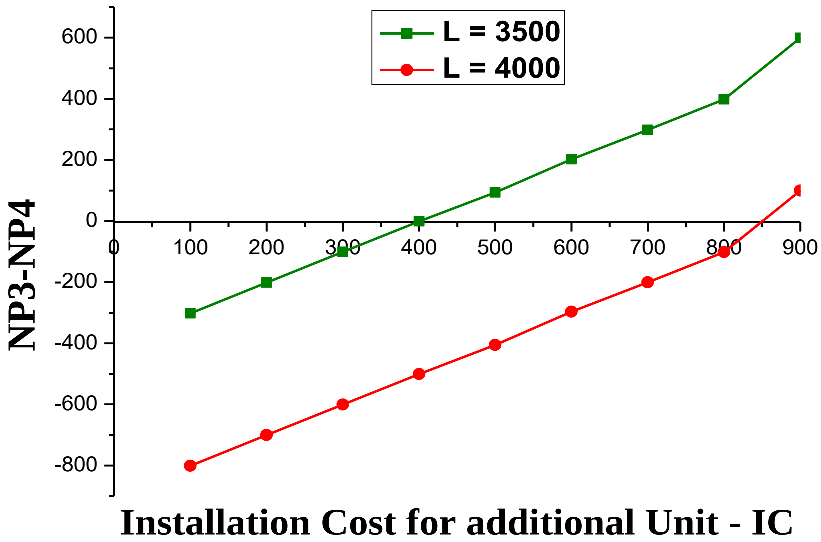

Fig. 6.4 – Difference of profits ($NP_3 - NP_4$) versus Installation Cost for different values of Goodwill Loss

From the Fig 6.4, following conclusions have been concluded:

(i) The difference $NP_3 - NP_4$ increases as the installation cost (IC) increases. The difference becomes greater for lesser values of goodwill loss (*L*).

(ii) For *L* = 3500, $NP_3 - NP_4$ > or = or < 0 according as IC > or = or < 397.440 INR. Hence Model M_3 is better whenever IC > 397.440 INR and Model M_4 is better whenever IC < 397.440 INR. Both the models are equally good if IC = 397.440 INR.

(iii) For L = 4000, $NP_3 - NP_4$ > or = or < 0 according as IC > or = or < 865.368 INR. Hence Model M_3 is better when IC > 865.368 INR and Model M_4 is better whenever IC < 865.368 INR. Both the models are equally good if IC = 865.368 INR.

Fig. 6.5 indicates the behaviour analysis of difference in profits NP_3 and NP_4 of the Models M_3 and M_4 with respect to goodwill loss for distinct values of installation cost of addition unit.

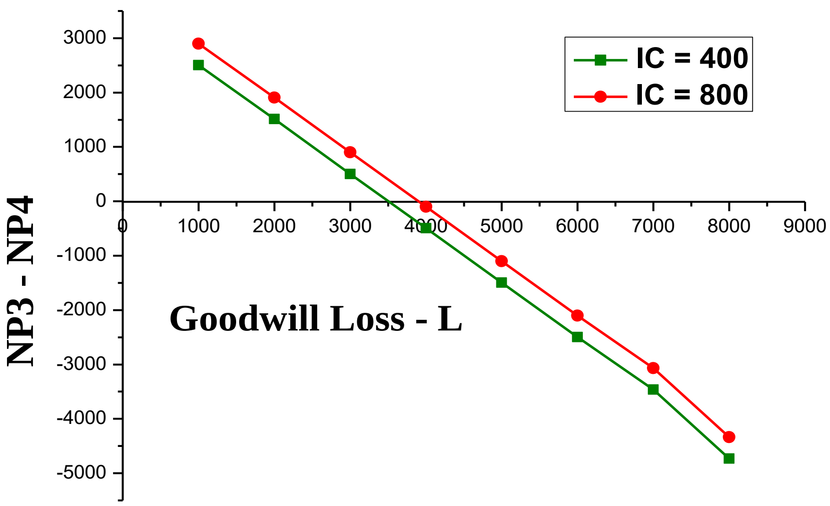

Fig. 6.5 – Difference of profits (NP_3 - NP_4) versus Goodwill Loss for different values of Installation Cost

From the Fig 6.5, it has been concluded that:

(i) The difference $NP_3 - NP_4$ decreases as the goodwill loss (L) increases. The difference becomes greater for greater values of installation cost (IC).

(ii) For IC = 400 INR, $NP_3 - NP_4$ > or = or < 0 according as L < or = or > 3497.0469 INR. Hence Model M_3 is better when L < 3497.0469 INR and the Model M_4 is better whenever L > 397.440 INR. Both the models are equally good if L = 3497.0469 INR.

(iii) For IC = 800 INR, $NP_3 - NP_4$ > or = or < 0 according as L < or = or > 3886.052 INR. Hence Model M_3 is better when L < 3886.052 INR and Model M_4 is better whenever L > 3886.052 INR. Both the models are equally good if L = 3886.052 INR.

6.3 Comparison between the Models Discussed in Chapter 3 and Chapter 5

In both Models Model M_3 and Model M_5 two standby units are discussed. The model discussed in chapter 3 i.e. M_3 has two units one is of eight ton capacity and other is of four ton capacity. In Model M_3 both cases are discussed when production is less than demand and at least equal to demand. In Model M_5 both units are of eight ton capacity. capacity of system is increased in Model M_5 keeping in mind goodwill loss which can occur in Model M_3 when production is less than demand. Installation cost is considered for increased capacity in Model M_5.

In Model M_3 both failure and repair rate are assumed to follow exponential distribution while in Model M_5 failure rate follows exponential distribution and repair rate are arbitrary. For making the comparison between the two models, various conclusions have been drawn through graphs as follows:

Fig.6.6 indicates the behaviour of the MTSF with respect to type – I failure rate of unit of eight ton capacity for both models M_3 and M_5.

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1 - q$
$\gamma_1 = 0.01736/\text{hr}$ $h(t) = 0.01/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$, $\beta_1 = 0.0084/\text{hr}$, $\beta_2 = 0.0036/\text{hr}$

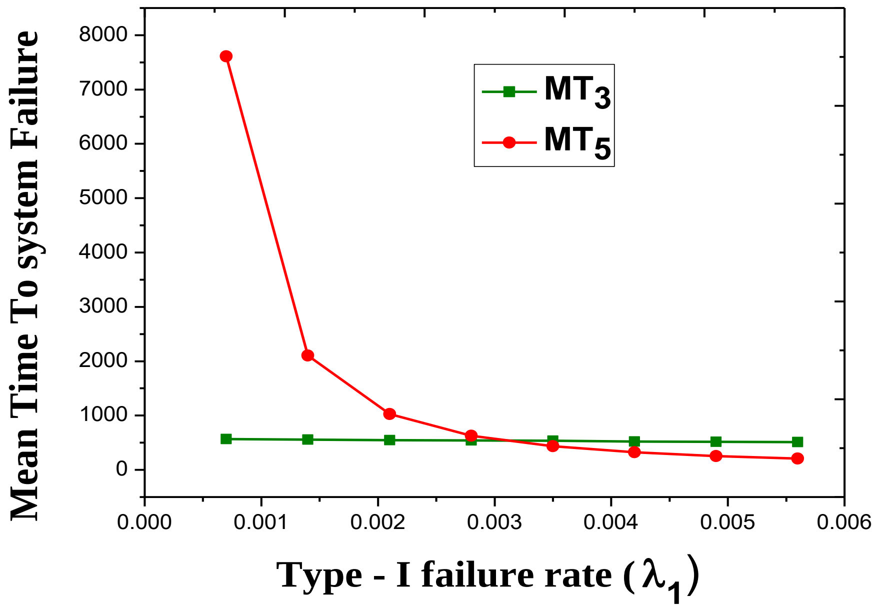

Fig. 6.6 – Effect of Type – I failure rate on MTSF of Model M_3 and Model M_5

From Fig. 6.6 following conclusions have been concluded:

(i) For $\lambda_1 > 0.003062$ $MT_5 > MT_3$ Hence Model M_5 is better than the model M_3.

(ii) For $\lambda_1 < 0.003062$ $MT_3 > MT_5$ Hence Model M_3 is better than the model M_5.

(iii) For $\lambda_1 = 0.003062$ $MT_3 = MT_5$ Hence Model M_3 and M_5 both are equally good.

Fig. 6.7 indicates the behaviour analysis of difference in profits NP_3 and NP_5 of the Models M_3 and M_5 with respect to cost of repairmen for distinct values of revenue in twelve ton capacity.

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1-q$
$\gamma_1 = 0.01736/\text{hr}$ $h(t) = 0.01/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$
$IC = 500\,INR$, $C_2 = 1500\,INR$, $L = 9000\,INR$, $C_{01} = 4C_{04}$, $C_{02} = 3C_{04}$, $C_{03} = 2C_{04}$
$\beta_1 = 0.0084/\text{hr}$, $\beta_2 = 0.0036/\text{hr}$, $\lambda_1 = 0.0007/\text{hr}$, $\lambda_2 = 0.0003/\text{hr}$

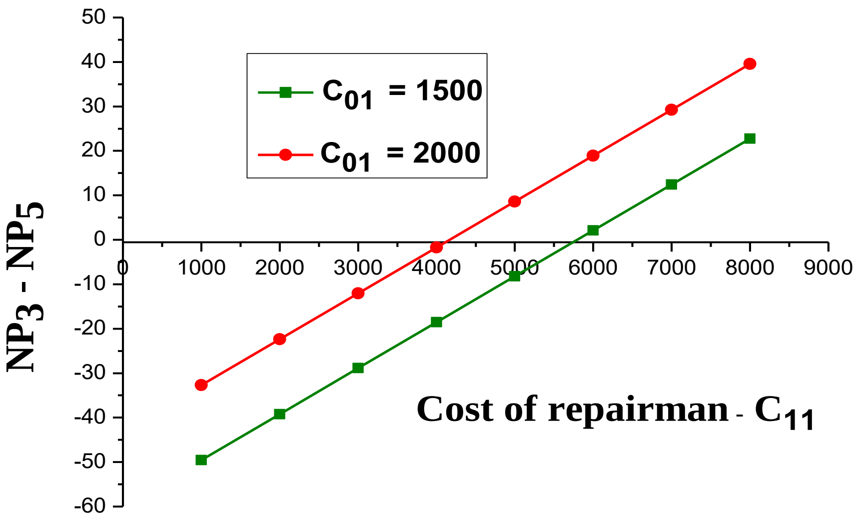

Fig. 6.7 – Difference of profits ($NP_3 - NP_5$) versus Cost of repairmen for different values of Revenue in twelve ton capacity.

From Fig 6.7 it can be concluded that:

(i) The difference $NP_3 - NP_5$ increases as the cost of repairman increases. The difference becomes greater for greater values of revenue.

(ii) For $C_{01} = 1500$, $NP_3 - NP_5$ > or = or < 0 according as C_{11} > or = or < 5795.0019 INR. Hence Model M_3 is better when C_{11} > 5795.0019 INR and the Model M_5 is better whenever C_{11} < 5795.0019 INR. Both the models are equally good if C_{11} = 5795.0019 INR.

(iii) For $C_{01} = 2000$, $NP_3 - NP_5$ > or = or < 0 according as C_{11} > or = or < 4166.2429 INR. Hence Model M_3 is better when C_{11} > 4166.2429 INR and the Model M_5 is better whenever C_{11} < 4166.2429 INR. Both the models are equally good if C_{11} = 4166.2429 INR.

Fig. 6.8 indicates the behaviour analysis of difference in profits NP_3 and NP_5 of the Models M_3 and M_5 with respect to cost of repairman for distinct values of revenue in twelve ton capacity.

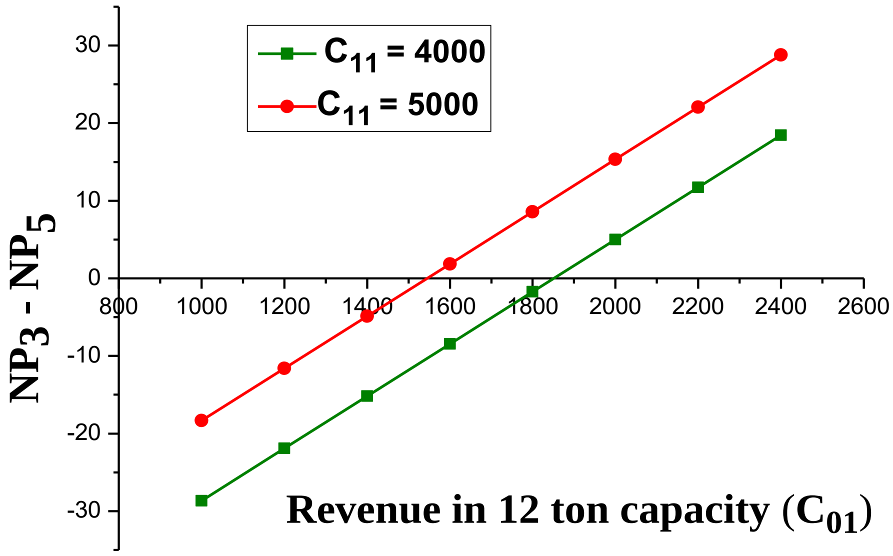

Fig. 6.8 – Difference of profits ($NP_3 - NP_5$) versus Revenue in twelve ton capacity for distinct values of Cost of repairman.

From Fig 6.8 it can be concluded that:

(iv) The difference $NP_3 - NP_5$ increases as the revenue increases. The difference becomes greater for greater values of cost of repairman.

(v) For $C_{11} = 4000$, $NP_3 - NP_5$ > or = or < 0 according as C_{02} > or = or < 1851.3717 INR. Hence Model M_3 is better when C_{02} > 1851.3717 INR and the Model M_5 is better whenever C_{02} < 1851.3717 INR. Both the models are equally good if C_{01} = 1851.3717 INR.

(vi) For $C_{11} = 5000$, $NP_3 - NP_5$ > or = or < 0 according as C_{02} > or = or < 1544.50 INR. Hence Model M_3 is better when C_{02} > 1544.50 INR and the Model M_5 is better whenever C_{02} < 1544.50 INR. Both the models are equally good if C_{02} = 1544.50 INR.

Fig. 6.9 indicates the behaviour analysis of difference in profits NP_3 and NP_5 of the Models M_3 and M_5 with respect to goodwill loss for distinct values of installation cost. The values of different measures considered are:

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1-q$

$\gamma_1 = 0.01736/\text{hr}$ $h(t) = 0.01/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$

$C_{11} = 4500\,INR$, $C_2 = 1500\,INR$, $C_{02} = 1500\,INR$, $C_{01} = 4C_{04}$, $C_{02} = 3C_{04}$, $C_{03} = 2C_{04}$

$\beta_1 = 0.0084/\text{hr}$, $\beta_2 = 0.0036/\text{hr}$, $\lambda_1 = 0.0007/\text{hr}$, $\lambda_2 = 0.0003/\text{hr}$

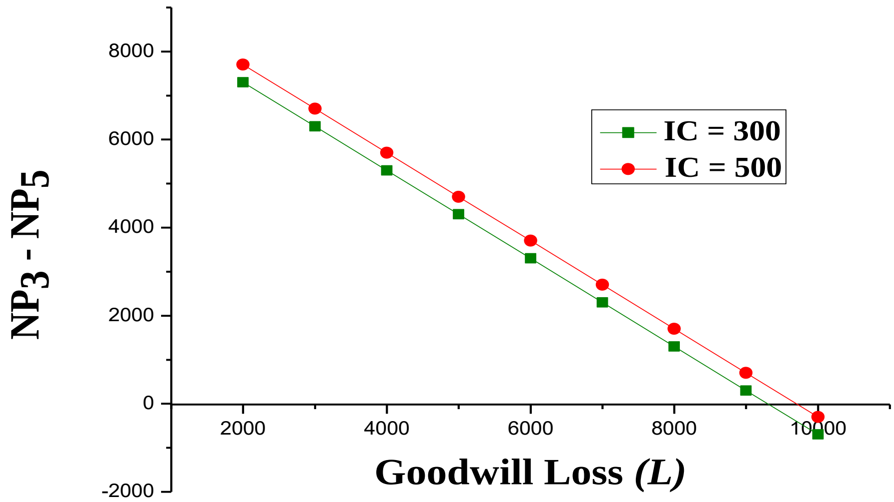

Fig. 6.9 – Difference of profits ($NP_3 - NP_5$) versus Goodwill Loss (*L*) for different values of Installation Cost (*IC*).

From Fig 6.9 it has been concluded that:

(i) The difference $NP_3 - NP_5$ decreases as the goodwill loss increases. The difference becomes greater for greater value of installation cost.

(ii) For $IC = 300$, $NP_3 - NP_5$ > or = or < 0 according as L < or = or > 9301.4069 INR. Hence Model M_5 is better when L > 9301.4069 INR and the Model M_3 is better whenever L < 9301.4069 INR. Both the models are equally good if C_{11} = 9301.4069 INR.

(iii) For $IC = 300$, $NP_3 - NP_5$ > or = or < 0 according as L < or = or > 9701.4042 INR. Hence Model M_5 is better when L > 9701.4042 INR and the Model M_3 is better whenever L < 9701.4042 INR. Both the models are equally good if L = 9701.4042 INR.

Fig. 6.10 indicates the behaviour analysis of difference in profits NP_3 and NP_5 of the Models M_3 and M_5 with respect to goodwill loss for distinct values of installation cost by considering the following values of measures.

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1 - q$
$\gamma_1 = 0.01736/\text{hr}$ $h(t) = 0.01/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$
$C_{11} = 4500\,INR$, $C_2 = 1500\,INR$, $C_{02} = 1500\,INR$, $C_{01} = 4C_{04}$, $C_{02} = 3C_{04}$, $C_{03} = 2C_{04}$
$\beta_1 = 0.0084/\text{hr}$, $\beta_2 = 0.0036/\text{hr}$, $\lambda_1 = 0.0007/\text{hr}$, $\lambda_2 = 0.0003/\text{hr}$

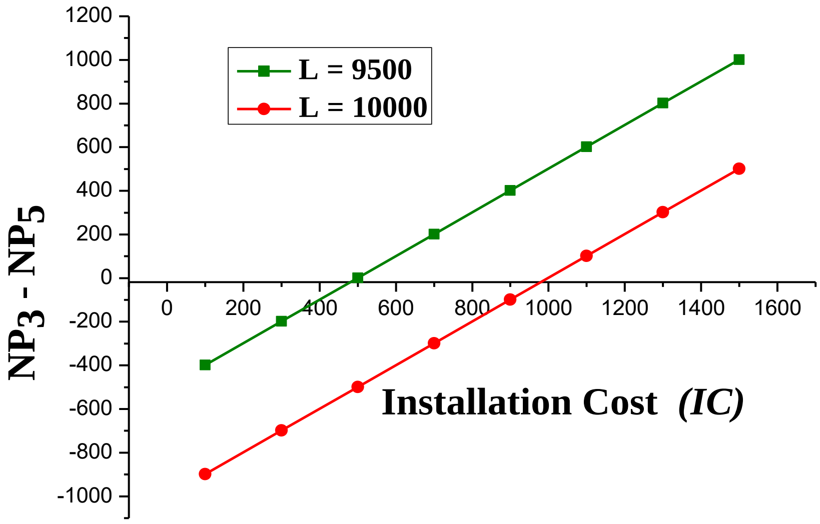

Fig. 6.10 – Difference of profits ($NP_3 - NP_5$) versus Installation Cost (*IC*) for different values of Goodwill Loss (*L*).

From Fig 6.10 it has been concluded that:

(i) The difference $NP_3 - NP_5$ increases as the installation cost increases. The difference becomes greater for lesser value of goodwill loss.

(ii) For $L = 9500$, $NP_3 - NP_5$ > or = or < 0 according as IC > or = or < 498.6146 INR. Hence Model M_3 is better when IC > 498.6146 INR and the Model M_5 is better whenever IC < 498.6146 INR. Both the models are equally good if IC = 498.6146 INR.

(iii) For $L = 10000$, $NP_3 - NP_5$ > or = or < 0 according as IC > or = or < 698.6146 INR. Hence Model M_3 is better when IC > 698.6146 INR and the Model M_5 is better whenever IC < 698.6146 INR. Both the models are equally good if IC = 698.6146 INR.

6.4 Comparison between the Models Discussed in Chapter 4 and Chapter 5

In chapter 4 model with three unit standby system is discussed while in chapter 5 two unit standby system is discussed. The model discussed in chapter 4 i.e. Model M_4 has three units one is of eight ton capacity and other two are of four ton capacity each. In chapter 5 Model M_5 has two units of eight ton capacity each. Instead of considering two units of four ton capacity each as in Model M_4, in Model M_5 we considered a single unit of double capacity in place of two units. Keeping in mind installation cost of a single unit and two units, comparison of Model M_4 and Model M_5 is done. IC_1 is taken as installation cost of single unit and IC_2 is taken as installation cost of single unit.

In Model M_4 both failure and repair rate are assumed to follow exponential distribution while in Model M_5 failure rate follows exponential distribution and repair rate are arbitrary. For making the comparison between the two models, various conclusions have been drawn through graphs as follows:

Fig. 6.11 indicates the behaviour of the MTSF with respect to Type – I failure rate of unit of eight ton capacity for both models M_4 and M_5 by considering the following values.

$a_1 = 0.15/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1 - q$
$\gamma_1 = 0.01736/\text{hr}$ $h(t) = 0.01/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$

$p_2 = 0.065$, $p_1 = 0.35$, $p_4 = 0.35$, $p_3 = 1 - p_2 - p_1$, $p_6 = 0.22$, $p_5 = 1 - p_6 - p_4$, $p_7 = 0.45$
$p_8 = 0.082$, $p_9 = 1 - p_7 - p_8$, $p_{10} = 0.025$, $p_{11} = 0.57$, $p_{12} = 0.035$, $p_{13} = 0.12$, $p_{15} = 0.42$
$p_{17} = 1 - p_{13} - p_{15}$, $p_{12} = 1 - p_{10} - p_{11}$, $p_{16} = 0.45$, $p_{14} = 1 - p_{16} - p_{18} = 0.34$, $p_{21} = 0.35$
$p_{20} = 0.065$, $p_{19} = 1 - p_{21}$, $p_{22} = 1 - p_{20}$, $p_{23} = 0.35$, $p_{24} = 0.55$, $p_{25} = 0.75$
$p_{27} = 1 - p_{23}$, $p_{26} = 0.43$, $p_{28} = 1 - p_{24}$, $p_{29} = 1 - p_{25}$, $p_{30} = 1 - p_{24}$, $\beta_1 = 0.0084\,/\,\text{hr}$
$\beta_2 = 0.0036\,/\,\text{hr}$

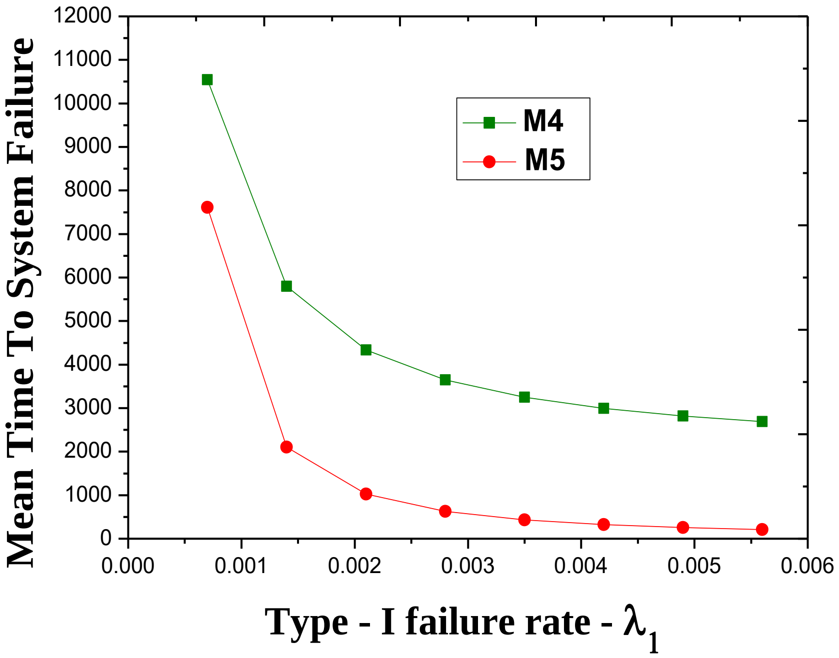

Fig. 6.11 – Effect of Type – I failure rate on MTSF of Model M_4 and Model M_5

From Fig. 6.11 it is clear that $MT_4 > MT_5$ for any value of λ_1 hence Model M_4 is better than Model M_5 for any value of Type – I failure rate.

Fig. 6.12 indicates the behaviour of the total availability with respect to Type – I failure rate of unit of eight ton capacity for both models M_4 and M_5 by considering the following values.

$a_1 = 0.3144\,/\,\text{hr}$, $b_1 = 0.2346\,/\,\text{hr}$, $a_2 = 0.1736\,/\,\text{hr}$, $b_2 = 0.2500\,/\,\text{hr}$, $q = 0.8$, $p = 1 - q$
$\gamma_1 = 0.01736\,/\,\text{hr}$ $h(t) = 0.01\,/\,\text{hr}$, $\gamma_2 = 0.0987\,/\,\text{hr}$

$p_2 = 0.065$, $p_1 = 0.35$, $p_4 = 0.35$, $p_3 = 1 - p_2 - p_1$, $p_6 = 0.22$; $p_5 = 1 - p_6 - p_4$, $p_7 = 0.45$
$p_8 = 0.082$, $p_9 = 1 - p_7 - p_8$; $p_{10} = 0.025$; $p_{11} = 0.57$, $p_{12} = 0.035$, $p_{13} = 0.12$, $p_{15} = 0.42$
$p_{17} = 1 - p_{13} - p_{15}$, $p_{12} = 1 - p_{10} - p_{11}$, $p_{16} = 0.45$, $p_{14} = 1 - p_{16} - p_{18} = 0.34$, $p_{21} = 0.35$
$p_{20} = 0.065$, $p_{19} = 1 - p_{21}$, $p_{22} = 1 - p_{20}$, $p_{23} = 0.35$, $p_{24} = 0.55$, $p_{25} = 0.75$
$p_{27} = 1 - p_{23}$, $p_{26} = 0.43$, $p_{28} = 1 - p_{24}$, $p_{29} = 1 - p_{25}$, $p_{30} = 1 - p_{24}$, $\beta_1 = 0.0084/\text{hr}$
$\beta_2 = 0.0036/\text{hr}$

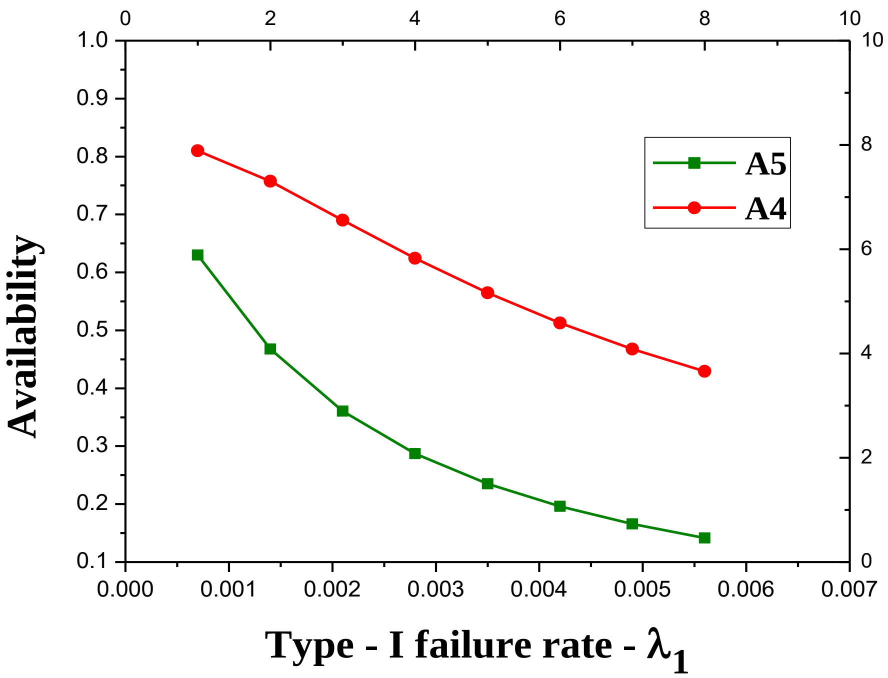

Fig. 6.12 – Effect of Type – I failure rate on MTSF of Model M_4 and Model M_5

From Fig. 6.12 it is clear that A_4 is always greater than A_5 i.e. availability is greater in case of Model M_4 whatever be the value of λ_1 hence Model M_4 is better than Model M_5 for any value of Type – I failure rate.

Fig. 6.13 indicates the behaviour analysis of difference in profits NP_4 and NP_5 of the Models M_4 and M_5 with respect to cost of repairmen for distinct values of revenue for four ton. The values of the measures considered are:

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1 - q$
$\gamma_1 = 0.01736/\text{hr}$ $h(t) = 0.01/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$ $C_{01} = 4C_{04}$, $C_{02} = 3C_{04}$, $C_{03} = 2C_{04}$

$p_2 = 0.065$, $p_1 = 0.35$, $p_4 = 0.35$, $p_3 = 1 - p_2 - p_1$, $p_6 = 0.22$; $p_5 = 1 - p_6 - p_4$, $p_7 = 0.45$
$p_8 = 0.082$, $p_9 = 1 - p_7 - p_8$, $p_{10} = 0.025$, $p_{11} = 0.57$, $p_{12} = 0.035$, $p_{13} = 0.12$, $p_{15} = 0.42$
$p_{17} = 1 - p_{13} - p_{15}$, $p_{12} = 1 - p_{10} - p_{11}$, $p_{16} = 0.45$, $p_{14} = 1 - p_{16} - p_{18} = 0.34$, $p_{21} = 0.35$
$p_{20} = 0.065$, $p_{19} = 1 - p_{21}$, $p_{22} = 1 - p_{20}$, $p_{23} = 0.35$, $p_{24} = 0.55$, $p_{25} = 0.75$, $p_{27} = 1 - p_{23}$
$p_{26} = 0.43$, $p_{28} = 1 - p_{24}$, $p_{29} = 1 - p_{25}$, $p_{30} = 1 - p_{24}$, $\beta_1 = 0.0084$ / hr , $\beta_2 = 0.0036$ / hr
$\lambda_1 = 0.0007$ / hr , $\lambda_2 = 0.0003$ / hr , $IC_1 = 1000\, INR$, $C_2 = 1500\, INR$, $IC_2 = 500\, INR$

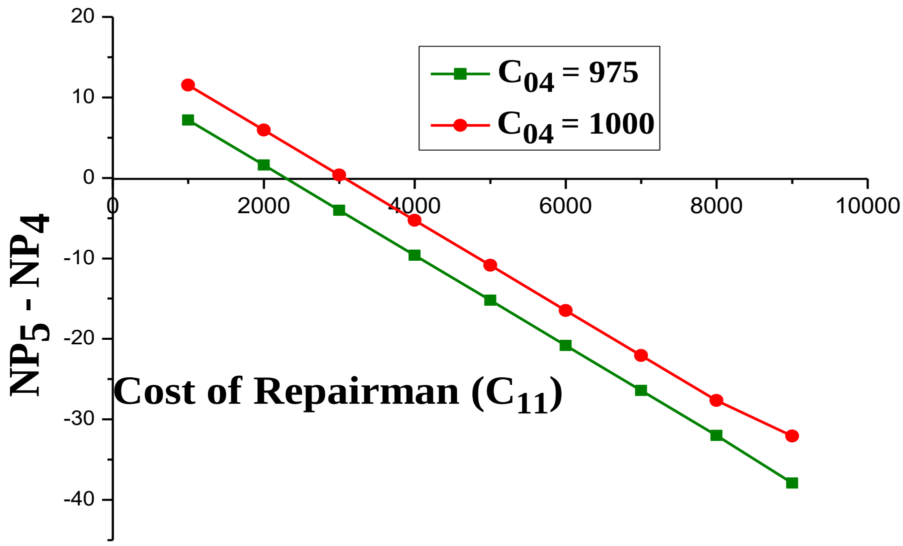

Fig. 6.13 – Effect of cost of repairmen on difference in profits ($NP_5 - NP_4$) for different values of revenue for four ton.

From Fig 6.13 it has been concluded that:

(i) The difference $NP_5 - NP_4$ decreases as the cost of repairmen increases and has higher value for higher value of revenue.

(ii) For $C_{04} = 975$, $NP_5 - NP_4$ > or = or < 0 according as C_{11} < or = or > 2286.5409 INR. Hence Model M_4 is better when C_{11} > 2286.5409 INR and the Model M_5 is better whenever C_{11} < 2286.5409 INR. Both the models are equally good if C_{11} = 5795.0019 INR.

(iii) For $C_{04} = 1000$, $NP_5 - NP_4$ > or = or < 0 according as C_{11} < or = or > 3056.6811 INR. Hence Model M_4 is better when C_{11} < 3056.6811 INR and the

Model M_5 is better whenever C_{11} > 3056.6811 INR. The models M_4 and M_5 are equally good if C_{11} = 3056.6811 INR.

Fig. 6.14 indicates the behaviour analysis of difference in profits NP_4 and NP_5 of the Models M_4 and M_5 with respect to revenue for four ton for distinct values of cost of repairman. The values of the measures considered are:

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1-q$
$\gamma_1 = 0.01736/\text{hr}$ $h(t) = 0.01/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$ $C_{01} = 4C_{04}$, $C_{02} = 3C_{04}$, $C_{03} = 2C_{04}$

$p_2 = 0.065$, $p_1 = 0.35$, $p_4 = 0.35$, $p_3 = 1 - p_2 - p_1$, $p_6 = 0.22$; $p_5 = 1 - p_6 - p_4$, $p_7 = 0.45$
$p_8 = 0.082$, $p_9 = 1 - p_7 - p_8$, $p_{10} = 0.025$, $p_{11} = 0.57$, $p_{12} = 0.035$, $p_{13} = 0.12$, $p_{15} = 0.42$
$p_{17} = 1 - p_{13} - p_{15}$, $p_{12} = 1 - p_{10} - p_{11}$, $p_{16} = 0.45$, $p_{14} = 1 - p_{16} - p_{18} = 0.34$, $p_{21} = 0.35$
$p_{20} = 0.065$, $p_{19} = 1 - p_{21}$, $p_{22} = 1 - p_{20}$, $p_{23} = 0.35$, $p_{24} = 0.55$, $p_{25} = 0.75$, $p_{27} = 1 - p_{23}$
$p_{26} = 0.43$, $p_{28} = 1 - p_{24}$, $p_{29} = 1 - p_{25}$, $p_{30} = 1 - p_{24}$, $\beta_1 = 0.0084/\text{hr}$, $\beta_2 = 0.0036/\text{hr}$
$\lambda_1 = 0.0007/\text{hr}$, $\lambda_2 = 0.0003/\text{hr}$, $IC_1 = 1000\,INR$, $C_2 = 1500\,INR$, $IC_2 = 500\,INR$

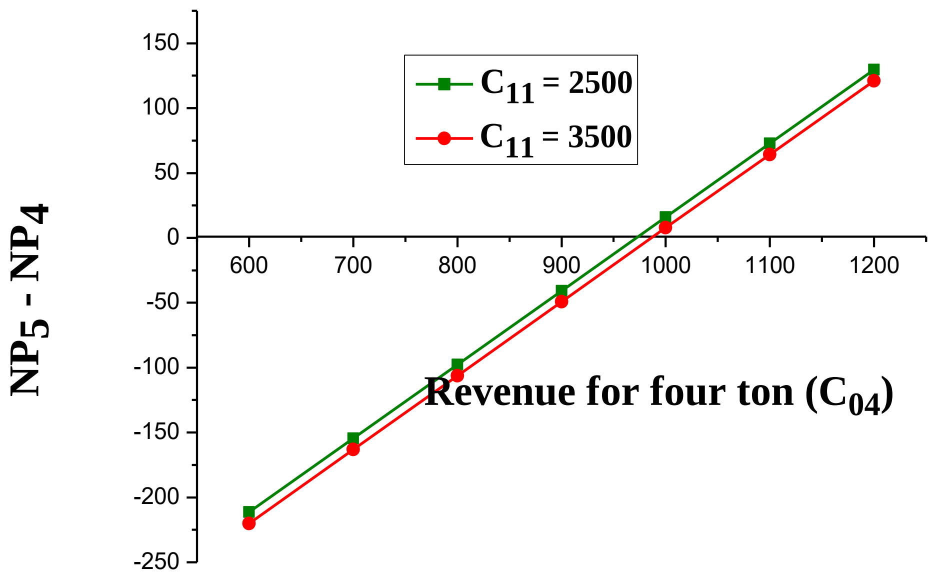

Fig 6.14 - Effect of on difference in profits (NP_5 - NP_4) for distinct values of cost of repairmen

From Fig 6.14 following conclusions have been concluded:

(i) The difference $NP_5 - NP_4$ increases as the revenue increases and has lesser value for greater value of cost of repairman.

(ii) For $C_{11} = 2500$, $NP_5 - NP_4$ > or = or < 0 according as C_{04} > or = or < 971.854 INR. Hence Model M_5 is better when C_{04} > 971.854 INR and the Model M_4 is better whenever C_{04} < 971.854 INR. Both the models are equally good if C_{11} = 971.854 INR.

(iii) For $C_{11} = 3500$, $NP_5 - NP_4$ > or = or < 0 according as C_{04} > or = or < 986.80 INR. Hence Model M_5 is better when C_{04} < 986.80INR and the Model M_4 is better whenever C_{04} > 986.80 INR. The models M_4 and M_5 are equally good if C_{04} = 986.80 INR.

Fig. 6.15 indicates the behaviour analysis of difference in profits NP_4 and NP_5 of the Models M_4 and M_5 with respect to IC_1 for distinct values of IC_2.

The values of the measures considered are:

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1 - q$
$\gamma_1 = 0.01736/\text{hr}$, $\beta_1 = 0.0084/\text{hr}$, $\beta_2 = 0.0036/\text{hr}$, $\lambda_1 = 0.0007/\text{hr}$, $\lambda_2 = 0.0003/\text{hr}$

$h(t) = 0.01/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$, $C_{04} = 1000\,INR$, $C_{01} = 4C_{04}$, $C_{02} = 3C_{04}$, $C_{03} = 2C_{04}$

$p_2 = 0.065$, $p_1 = 0.35$, $p_4 = 0.35$, $p_3 = 1 - p_2 - p_1$, $p_6 = 0.22$, $p_5 = 1 - p_6 - p_4$, $p_7 = 0.45$
$p_8 = 0.082$, $p_9 = 1 - p_7 - p_8$, $p_{10} = 0.025$, $p_{11} = 0.57$, $p_{12} = 0.035$, $p_{13} = 0.12$, $p_{15} = 0.42$
$p_{17} = 1 - p_{13} - p_{15}$, $p_{12} = 1 - p_{10} - p_{11}$, $p_{16} = 0.45$, $p_{14} = 1 - p_{16} - p_{18} = 0.34$, $p_{21} = 0.35$
$p_{20} = 0.065$, $p_{19} = 1 - p_{21}$, $p_{22} = 1 - p_{20}$, $p_{23} = 0.35$, $p_{24} = 0.55$, $p_{25} = 0.75$, $p_{27} = 1 - p_{23}$
$p_{26} = 0.43$, $p_{28} = 1 - p_{24}$, $p_{29} = 1 - p_{25}$, $p_{30} = 1 - p_{24}$, $C_1 = 3000\,INR$, $C_2 = 1500\,INR$

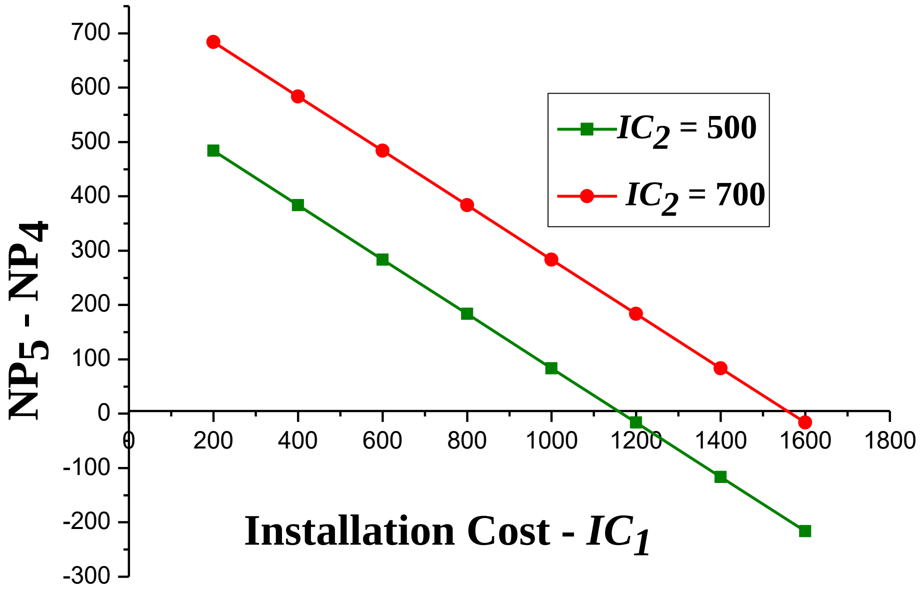

Fig. 6.15 – Difference of profits ($NP_5 - NP_4$) versus Installation Cost *(IC_1)* for different values of Installation Cost (*IC_2*).

From Fig 6.15 following conclusions have been concluded:

(i) The difference $NP_5 - NP_4$ decreases as the installation cost IC_1 increases. The difference becomes greater for greater value of installation cost IC_2.

(ii) For $IC_2 = 500$ INR, $NP_5 - NP_4$ > or = or < 0 according as IC_1 < or = or > 1165.9766 INR. Hence Model M_4 is better when IC_1 > 1165.9766 INR and the Model M_5 is better whenever IC_1 < 1165.9766 INR. Both the models are equally good if IC_1 = 1165.9766 INR.

(iii) For $IC_2 = 700$ INR, $NP_5 - NP_4$ > or = or < 0 according as IC_1 < or = or > 1565.3120 INR. Hence Model M_4 is better when IC_1 > 1565.3120 INR and the Model M_5 is better whenever IC_1 < 1165.9766 INR. The models M_4 and M_5 are equally good if IC_1 = 1565.3120 INR.

Fig. 6.16 indicates the behaviour analysis of difference in profits NP_4 and NP_5 of the Models M_4 and M_5 with respect to IC_2 for distinct values of IC_1.

The values of the measures considered are:

$a_1 = 0.3144/\text{hr}$, $b_1 = 0.2346/\text{hr}$, $a_2 = 0.1736/\text{hr}$, $b_2 = 0.2500/\text{hr}$, $q = 0.8$, $p = 1-q$
$\gamma_1 = 0.01736/\text{hr}$, $\beta_1 = 0.0084/\text{hr}$, $\beta_2 = 0.0036/\text{hr}$, $\lambda_1 = 0.0007/\text{hr}$, $\lambda_2 = 0.0003/\text{hr}$

$h(t) = 0.01/\text{hr}$, $\gamma_2 = 0.0987/\text{hr}$, $C_{04} = 1000\,INR$, $C_{01} = 4C_{04}$, $C_{02} = 3C_{04}$, $C_{03} = 2C_{04}$

$p_2 = 0.065$, $p_1 = 0.35$, $p_4 = 0.35$, $p_3 = 1 - p_2 - p_1$, $p_6 = 0.22$; $p_5 = 1 - p_6 - p_4$, $p_7 = 0.45$
$p_8 = 0.082$, $p_9 = 1 - p_7 - p_8$; $p_{10} = 0.025$; $p_{11} = 0.57$, $p_{12} = 0.035$, $p_{13} = 0.12$, $p_{15} = 0.42$
$p_{17} = 1 - p_{13} - p_{15}$, $p_{12} = 1 - p_{10} - p_{11}$, $p_{16} = 0.45$, $p_{14} = 1 - p_{16} - p_{18} = 0.34$, $p_{21} = 0.35$
$p_{20} = 0.065$, $p_{19} = 1 - p_{21}$, $p_{22} = 1 - p_{20}$, $p_{23} = 0.35$, $p_{24} = 0.55$, $p_{25} = 0.75$, $p_{27} = 1 - p_{23}$
$p_{26} = 0.43$, $p_{28} = 1 - p_{24}$, $p_{29} = 1 - p_{25}$, $p_{30} = 1 - p_{24}$, $C_1 = 3000\,INR$, $C_2 = 1500\,INR$

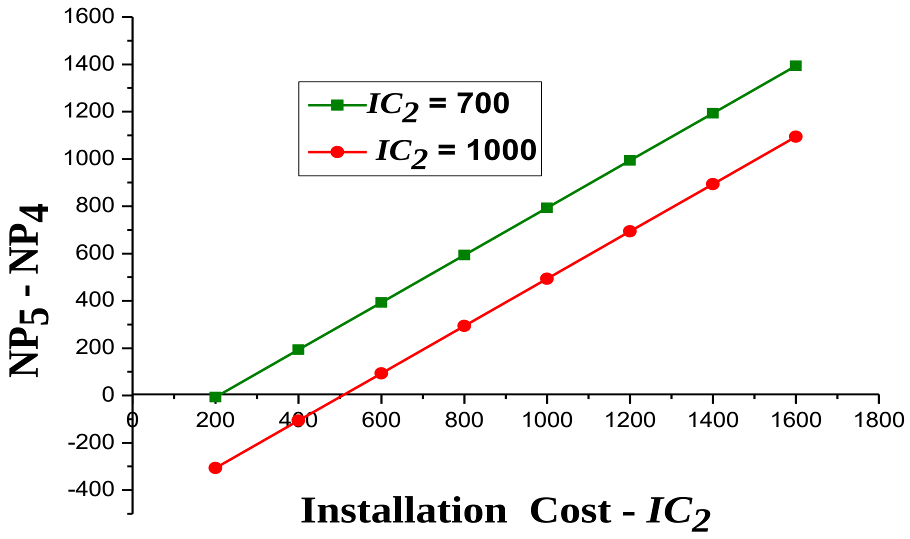

Fig. 6.16 – Difference of profits ($NP_5 - NP_4$) versus Installation Cost (IC_2) for different values of installation Cost (IC_1).

From Fig 6.16 following conclusions have been concluded:

(i) The difference $NP_5 - NP_4$ increases as the installation cost IC_2 increases. The difference becomes greater for lesser value of installation cost IC_1.

(ii) For $IC_1 = 700$, $NP_5 - NP_4$ > or = or < 0 according as IC_2 > or = or < 206.4776 INR. Hence Model M_5 is better when IC_2 > 206.4776 INR and the Model M_4 is better whenever IC_2 < 206.4776 INR. Both the models are equally good if IC_2 = 206.4776 INR.

(iii) For $IC_1 = 1000$, $NP_5 - NP_4$ > or = or < 0 according as IC_2 > or = or < 506.4776 INR. Hence Model M_5 is better when IC_2 > 506.4776 INR and the Model M_4 is better whenever IC_2 < 506.4776 INR. The models M_4 and M_5 are equally good if IC_2 = 506.4776 INR.

CHAPTER 7

SUMMARY AND CONCLUSION

7.1 Summary

In the present thesis, we have primarily developed reliability models considering different situations as variation in demand, Variation in production, production less than demand and at least equal to demand. Two unit standby and three unit standby systems are analyzed in the thesis considering these situations and two types of failures.

The present thesis entitled "Stochastic Modeling and Cost Analysis of Some Industrial Systems" contains analysis of various stochastic models for two unit and three unit rice to paddy converters. As standby units are in great demand and rice to paddy converter is one of such systems, we observed such a system working in Kohinoor foods, Sonipat. After understanding the process of paddy to rice converters, different models in different situations are developed which have been analysed as similar and dissimilar paddy to rice converters. Various measures of system effectiveness are obtained using Markov process and Regenerative point technique. Comparative study of different models is also done to identify which model is better in which situation than another model working under different situation. Measures obtained are:

1. Mean Time to system failure (MTSF)
2. Availability in different capacities
3. Availability in both case when production is less than demand and at least equal to demand
4. Expected busy period of repairman
5. Expected down time of the system
6. Profit achieved by the system
7. Graphical study is also done to understand the system behaviour in better way. For programming MAT LAB is used. Graphs are made in ORIGIN.
8. Various lower and upper bounds for failure rate, revenue per unit time and cost of repairman are obtained to analyze the behaviour of profit.

9. The lower and upper bounds are also obtained to analyze which model is better in which situation for various measures as :
    - Cost of repairman
    - Installation Cost of additional unit (IC)
    - Installation Cost of a single unit (IC_1)
    - Installation Cost of two unit (IC_2)
    - Good Will Loss (L)

In the present study we have analysed two and three unit standby systems with concept of varied production depending upon the demand. In case of production less than demand, we have analysed a model by considering goodwill loss. In the next model we considered a three unit standby system to fulfil the complete demand. In the chapter 6, we have also done the comparison of models working in different situations. The main emphasis is on cost benefit, minimization of operational cost and variation in demand. Such type of study is very useful for the industries. The work done is useful to study life times, reliability and profit analysis of standby units working in different situations.

The models discussed in the thesis are not bound to rice to paddy converters only. These models can be applied to any industry where such types of standby systems are used. As we computed all results considering the data provided by the industry, we can suggest them which model will be better in which situation. We can share the analysis done on the basis of data with them so that they can set up their system as per the requirement.

7.2 Limitations of Research Work

There are some constraints faced during the research work done which are listed as:

(i) It was a very difficult task to collect real data of industry as financial constraint due to some privacy. Kohinoor Foods provided some data for rice plant which in turn is used for finding failure/repair rates. Due to non availability of some data, some measures were to be assumed.

(ii) The models discussed in this thesis may not meet all the requirements of the industry, hence may need further modifications.

(iii) For computing the various measures of system effectiveness, we have tried to solve the system of differential equations using Laplace transform method. This method is easy when the system of differential equations is small but it becomes complex for big system of differential equation. In this thesis it is seen that a system of very large of differential equations is formed during its behavioural analysis.

7.3 Industrial Significance of the present Study

For any industry to make any system profitable, some unpleasant activities may happen in the whole process. Some equipment may undergo repair, customer may demand less or some parts may come out to defective. These kind of activities may effect the performance of the system. The study in present thesis can help to analyse some unpleasant factors which reduces the performance of the system. The present thesis provides the lower and upper bounds of minimum revenue and maximum cost of repairman which will help the industry to make the system profitable.

The present work helps the industry to analyse the various important measures of system effectiveness which in turn help to increase the profit. Our study is based on standby systems. The models developed in present study can be applied to any industry where such types of standby systems are used. Our suggestions based on this study may help these type of industries to analyse the sensitivity of the subparts of the system.

7.4 Future Scope of Study

In the present study reliability and cost benefit analysis have been carried out using regenerative point technique. The lower and upper bounds for various measures are obtained which reveal maximum possible cost of repairman, minimum revenue, minimum installation cost and minimum goodwill loss as to have a profitable system lower and upper bounds for other measures of interest may also be obtained. The models discussed in the present thesis can be applied to any industry where such type of standby systems is used. The users of such systems may fit our model to their systems may fit our model to their systems taking whatever distributions: Weibull, Gamma, Erlang distribution etc. instead of exponential distribution. Due to non availability of complete real data hypothetical values of some parameter were considered. In future we can try to collect actual data for all parameters to avoid hypothetical values.

In the present thesis, we have considered the repair and failure rates of the units as independent with each other and constant. Work on variable and dependent failure and repair rates can be done. The maintainability of the system can also be taken into consideration which is not being taken into consideration in this thesis.

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