ROCHE COORDINATES FOR ROTATIONALLY AND MAGNETICALLY DISTORTED BINARY STARS

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By

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CERTIFICATE

Certified that the thesis entitled **'ROCHE COORDINATES FOR ROTATIONALLY AND MAGNETICALLY DISTORTED BINARY STARS'** submitted by Ms. **Palak Singla** (301504024) in partial fulfilment of the requirement for the award of the degree of **Master in physics,** submitted in the School of Physics and Material Science(SPMS), Thapar University, Patiala, Punjab is the record of candidate's own independent and original research work carried out by her under the supervision and guidance of **Dr. A.K. Lal** .

The matter embodied in this thesis has not been submitted in part or full to any other university or institute for award of any degree.

Date: 17 JULY 2017

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It is certified that the above statement made by the student is correct to the best of my/our knowledge and belief.

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Contents

ABSTRACT

Apart from gravitational and centrifugal forces, magnetic field also plays an important role while studying the equilibrium structures and also period of oscillations of rotating stars as well as stars in binary system. A new system of co-ordinates was introduced by Kopal[19] which is known as Roche system of co-ordinates and he also emphasized that how this system of co-ordinates can be used to study the problems of vibrations of rotationally and tidally distorted stellar models.

Mohan and Singh [24], Singh [18], Singh and Gupta [19], have discussed the system of Roche co-ordinates for certain types of distorted stars.

However after the formulation of equipotential surfaces of rotationally, tidally and magnetically distorted stars the Roche coordinates for these stars have not been taken into account.

So the present thesis is an attempt to develop expressions for Roche coordinates and related parameters for Roche equipotential surfaces for stars distorted by rotation and magnetic field.

Chapter wise summary of work done in this thesis is as follows:

Chapter 1 is just introductory in nature. We study the significance in terms of astronomy of deducing the equilibrium structure of tidally, rotationally and magnetically deformed binary Stars. Then there is a brief literature review. Then we proceed by introducing some mathematical methods. And at last we define in detail a system of Roche equipotential including the effects of magnetic field.

In chapter 2 we will discuss in detail a system of binary star and two types of mass transfer in such systems. Further we will proceed to study effects of magnetic fields on the mass transmission and reduction along with generations of poloidal and toroidal magnetic fields. Next we will explain the concept of magnetic braking and how it affects the binary system. At last we will compile our chapter by evaluating the effect of magnetic field on the shapes of the binary system.

Chapter 3 is the final chapter that comprises an introduction to curvilinear coordinate system which has been included to explain the basis of Roche Coordinates. Finally we will derive expressions of Roche coordinates for modified roche equipotential surface for rotationally and/or magnetically distorted system of stars and their metric coefficients.

Chapter 1

INTRODUCTION

This chapter is just introductory in nature. In section 1.1 we study the significance in terms of astronomy of deducing the equilibrium structure of tidally, rotationally and magnetically deformed binary Stars. In section 1.2 there is a brief literature review.Then we proceed by introducing some mathematical methods in section 1.2.1. And at last in section 1.3 we define in detail a system of Roche equipotential including the effects of magnetic field.

1.1 Importance Of Studying Tidally, Rotationally and Magnetically Deformed Gaseous Spheres In Astrophysics

A star is a subtle balance between the outward pressure force (expanding it outwards) and gravitational force (pulling the star inwards). During other physical experiments a theoretical idea can be immediately assessed but the research of a star or system of stars takes into view of a collection of coincident theories, estimation techniques and computational mastery in order to get strongly valid interpretations about the celestial environment. This quandary emerges as the outcome of the convoluted features of the system established at such a high energy scale due to nuclear collisions and also due to unexplored and unmapped landscapes of the celestial environment that we study.

Theoretically, a star is examined as an elementary self-gravitating (because of inward pull) globule of highly crowded molecular clouds, prevailing in both hydrostatic and thermal equilibrium.

Stars being the most commonly and widely observed celestial objects, represent the basic building blocks of the galaxies. Through celestial studies astrophysicists are successful in determining the age, composition and also the distribution of these gaseous globules in the known galaxies. By the analysis of this collected data, it helps us in uncover the very origination, stellar evolution and stellar dynamics of that specific galaxy.

The stellar evolution is basically responsible for the creation and universal distribution of the heavy elements present in the periodic table, like C, N, O and also their characteristics are linked to the one‘s of the planetary systems which may combine with them. As a result of this, the analysis of the celestial world becomes fundamental to the subject of astronomy.

Greater number of the discovered stars spins about their particular axis. The distortion of equilibrium structure of such stars is because of tides, provided the star is not rotating. Non-uniformities on the exterior surface of the stars give us an opportunity of studying the stellar magnetic fields and stellar structures for various types of stars. Surface non-uniformities are developed differently because they depend upon the velocity of the rotating star. The magnetic fields also play an important role at various stages involved for the stellar formation and hence lead to evolution. Magnetic fields are the main reason for the loss in angular momentum of the stars that are new. Magnetic fields are also a powerful energy source for a large number of dynamic phenomenons (for example solar flares, star spots, X-ray emission) which usually occur at the layers of a star. The equilibrium model of a star appearing in a binary system or a multiple system will be tidally distorted gaseous sphere if it is not rotating; rotationally and tidally distorted if the star is rotating and rotationally, tidally and magnetically distorted if the star is rotating as well as producing magnetic field.

1.2 Literature Review

In past many researchers have done a detailed study on the equilibrium structures of the stars, by considering that the star is undistorted. Numerous authors have dealt with the problems in order to seek the equilibrium structures of tidally and rotationally distorted model of a stars. A first order analysis for the investigation of the tidal,rotational and also binary stars problem was proposed by Chandrashekhar **[4]** . Along with that authors in **[14,15]** have provided a rational approach for evaluating the effect of rotation as well as tidal deformations on the stellar equilibrium structures and/or a system of stars by computing the real equipotential stellar surfaces by the help of Roche Equipotentials . Lal et al **[20]** have managed to considered the equilibrium structures of tidally as well as rotationally distorted primary components of the binary stars by taking into account the impact of mass variation within the internal part of the star. A perturbation theory to calculate the effect of differential rotation upon the adiabatic non-radial models of stellar oscillations has been developed earlier. Sensible evaluations of results and the impacts of Coriolis force along with the ellipticity, in parallel, by the means of this perturbation technique has been performed.

Hamiltonian operator was created upto the 2^{nd} order in eigen-frequencies and upto 1^{st} order in eigen-functions.

Kopal **[17]** laid a foundation of a coordinate system, known as the Roche coordinates, for investigating the problems of rotating stars in a binary star systems. Mohan and Saxena **[26]** employed the averaging technique along with Kopals results on Roche equipotentials in order to find the cumulative effects of rotation and tidal distortions on the equilibrium structures as well as oscillations of the polytropic model of stars.

Complex analytical studies have been already discussed in the literature in order to find the equilibrium structure of tidally and rotationally distorted stars. Kopal in 1972, introduced the concept of Roche equipotentials for analyzing such problems. Soon after that many authors **[17]**, Mohan and Singh **[26]**, **[20,21,24]**, have addressed themselves to this problem. Roche approximation was employed in order to formulate the potential of a rotating star and also a star in a binary system. But this technique takes into account just the centrifugal, Coriolis and gravitational forces and it does not takes the effect of magnetic force. In general, the magnetic force has been neglected while obtaining the Roche equipotential surface of stars as its impact is assumed to be very small in comparison to those of centrifugal and gravitational forces.

However,it has been observed that magnetic field is also one of the factors causing a distortion in sphericity of stars. Much efforts have been made in literature to include the effect of magnetic fields into the equation of stellar structure. Authors **[2,5,10]** have studied the effects of magnetic field on the equipotential surfaces of the stars. It was really difficult to examine the situation owing to the presence of the non-radial component of Lorentz Force. While numerous authors have formulated this problem by considering only the toroidal component of the magnetic field, Djuraevi **[2]** investigated the critical surfaces in closed binary system by also taking into consideration the radial component of the magnetic field magnetic pressure.

However after the formulation of equipotential surfaces of rotaionally, tidally and magnetically distorted stars the Roche coordinates for these stars have not been successfully tackled in the literature. In the present thesis an attempt have been made to obtain explicit expressions for Roche coordinates of Roche equipotential surfaces experiencing gravitation,centrifugal and magnetic forces..

1.3 Elementary equations that determine the equilibrium

Not all the stars in the galaxy are of the same mass, age and composition. Because of this each one of them has uniquely different internal structure. Many of the stellar structure models has been developed to display the stellar interior structure of a star in detail, thus, helps the astronomers in predicting the luminosity, colour, energy production and also the future of the star or system of stars. From the information collected through the study of the stars in different regions of electromagnetic spectrum and specifically the precise observations of the pulsating sun modes and also neutrinos provide the information required to fabricate the models of interior of stellar. Hence astronomers found a way to learn about the interior of a star without physically studying it.

1.3.1 Mathematical Models

A mathematical model of the internal structure of a star is created by employing the information acquired from its stellar surface and by understanding the behavior of gases in different conditions. The mathematical model in a basic sense is the set of equations that describe the way in which certain mechanisms takes place layer by layer inside a star. Fortunately enough the stellar interior is absolutely (downright) gaseous all the way to the centre. Hence, equations are less heavy and thus relatively simple to work with.

(a) Temperature
It is the measure of the unsystematic motion (average kinetic energy)of the gaseous particles. At higher temperature, kinetic energy is also higher.

(b) Pressure
Pressure is the magnitude of the force per unit area of contact. Therefore, hot gas expands to exert pressure on its surroundings.

(c) Mass Density
It is the measure of total amount of mass per unit volume. The gaseous materials can be easily compacted to greater densities and lower volumes.

The synergy of these 3 parameters is to explain the material that is under study and is expressed by equation of state.Thus, it relates pressure, temperature and density.The primary equations of equilibrium structure of the star in thermal and hydrostatic equilibrium are discovered in related work. They relate it to the so-called challenge to find the equilibrium structure of the stellar models. Let ρ and P denote the density and pressure at a point, correspondingly at a distance r from the centre of sphere.

Conservation of Mass

Consider M(r) be the mass that is enclosed within the radius r. Thus the mass confined within the hollow sphere of radius r to r + dr is expressed as

$$M(r + dr) - M(r) = \rho(r)dV \tag{1.1}$$

Here,
r is the star‘s radial coordinate, M(r) is stellar mass enclosed within a star of radius (r) and P(r) is the star‘s density.
$dV = 4\pi r^2 dr$,is the volume of shell of star. LHS of equation (1.1) can be written as

$$\frac{dM}{dr} = \rho(r)4\pi r^2 \tag{1.2}$$

Momentum Conservation (Hydrostatic Equilibrium)

For a star to be in hydrostatic equilibrium, gravitational force at any point on the star must be equal to pressure at that point. To explain this, consider a shell of the radius r. Also, let the mass per unit area of a particular shell, ρdr. And weight per unit area of shell is $g\rho dr$. This gravitational force must be balanced by the equal amount of pressure force, experienced between the both sides of the shell. For the shell element with radius r to r + dr, the area element will be dA.

Thus a coordinate system is established in which the radial component grows larger towards the outer side. Consequently, the pressure force acting inwards on a star with radius r is positive whereas the force of pressure on the outer side with radius r + dr is negative.

Therefore,

$$P(r)dA - P(r+dr)dA = [P(r) - P(r+dr)]dA = -\frac{dP}{dr}drdA \tag{1.3}$$

The gravitational force is negative owing to spherical symmetry pointing towards center of star.

$$-\frac{GM_r dm}{r^2} \tag{1.4}$$

where dm is mass of small element taken into consideration.
Thus, we know

$$dm = \rho(r)dV = \rho(r)dAdr \tag{1.5}$$

Putting the value of dm in the equation (1.4), we obtain following

$$-\frac{G\rho(r)dAdrM_r}{r^2} \tag{1.6}$$

Using Newtonss 2nd law , we get the following expression

$$\rho(r)dA\frac{d^2r}{dt^2} = -\frac{dP}{dr}dA - \frac{G\rho(r)dAM_r}{r^2} \tag{1.7}$$

after cancelling the factors dr and dA we obtain

$$\rho(r)\frac{d^2r}{dt^2} = -\frac{dP}{dr} - \frac{G\rho(r)M_r}{r^2} \tag{1.8}$$

As the stellar system is in the hydrostatic equilibrium, the cumulative force must disappear, i.e.,

$$\frac{dP}{dr} - \frac{G\rho(r)M_r}{r^2} = 0 \tag{1.9}$$

$$\frac{dP}{dr} = \frac{G\rho(r)M_r}{r^2} \tag{1.10}$$

Conservation of Energy

Star emits energy through radiation. Thus this reduction should be cancelled out by energy emitted from the interior nuclear reactions happening at the core of the star.

Now, let L(r) be the energy flow over the gaseous sphere of radius r. The total loss of energy from the shell with radius r to r + dr is

$$L(r+dr) - L(r) = \frac{dL(r)}{dr}dr \tag{1.11}$$

Let us suppose is amount of energy produced per kg. Then the total energy produced in this shell is given as

$$dE = \varepsilon\rho(r)4\pi r^2 dr \tag{1.12}$$

Thermal equilibrium can be kept only when the loss of radiation is same as gain of energy (from the nuclear fusion). Thus

$$\frac{dL(r)}{dr}dr = \varepsilon\rho(r)4\pi r^2 dr \tag{1.13}$$

$$or \tag{1.14}$$

$$\frac{dL(r)}{dr} = \varepsilon\rho(r)4\pi r^2 \tag{1.15}$$

In astrophysical drawback, where the thermal feature of the model are neither required nor investigated, comes in the role of equilibrium composition of the stars, which is estimated through use of the equations (1.2 and 1.10) with the set of equations and their boundary conditions.

At the center r = 0 , M(r) = 0
At the surface r = 0 , M(r) = 0 ; P = 0 or P_s ; ρ= 0 or ρ_s

Numerous studies (theoretical as well as numerical) about the equilibrium structures of stars are mainly found in the **[4,21,12,31,37]** .

1.4 Modified Roche Equipotential surface including the effects of magnetic field

To introduce the Roche equipotential concept we assume that the binary system consists of two components i.e. primary and secondary stars, out of these the primary component is assumed exceptionally larger than the secondary component, which is in fact considered as point mass. The secondary is believed to cause tidal effects in the primary component. Also, both the components of binary system primary and secondary rotate around their own axis and revolve about their axis passing through the common center of mass or gravity.

Following Kopals approach , let us suppose M_0 and M_1 be the masses of two components , such that $M_0 >> M_1$. Let they be separated by distance D. The configuration of the internal part of the larger star is estimated by Roche Model, that takes into consideration the fact that the sum total stellar mass is fully concentrated at the center point and also this point mass is enclosed by a mobile envelope where its density profile is in inverse proportion to the distance square from the center.

Normally, it has been observed that magnetic field imparts a negligible affect upon the structure and shape of equipotential surfaces. Apart from this there were several challenges that resulted presence of the non-radial component of Lorentz force. Hence, the use of magnetic pressure that is the radial component of Lorentz force is made in the present work to obtain the equation of the potential. Here effect of magnetic field on the critical surfaces of RTD stars following the assumptions of **[2]** in which the author studied the problem of determination of equipotential surfaces. The magnetic field was neglected by Djuraevi **[2]** when he formulated the problem that incorporated the effects of radiation because of both the components in the binary system. On the other hand contradicting this, we have just considered the effects due to magnetic field and totally neglected any of the radiation effects in our study. So only the effect of magnetic field owing to the primary component is considered while forming potential equation. The effect due to secondary one isnt considered as it is already presumed to be a point mass star.

Let us consider a rectangular system of Cartesian coordinates to describe the position of stars where the center of gravity of the primary star is taken as origin and the center of gravity of that of the secondary is taken at D distance from primary, so that the center of gravity C of the binary system considered, lies on x-axis with coordinates (d1, 0, 0) where

$$d_1 = \frac{M_1 D}{M_1 + M_0} \tag{1.16}$$

D is separation distance between the two stars (primary M0 and secondary M1).

A perpendicular axis i.e. Z-axis is considered orthogonal to the plane of orbit of the 2 stars. Let Ω symbolize angular velocity for rotation of a system about the axis which is perpendicular to xy plane and that passes through the gravitational center C of the system.

In a binary star system, the primary star rotates about its axis OZ possessing angular velocity Ω_1, and also it rotates about axis parallel to z-axis passing from shared centre

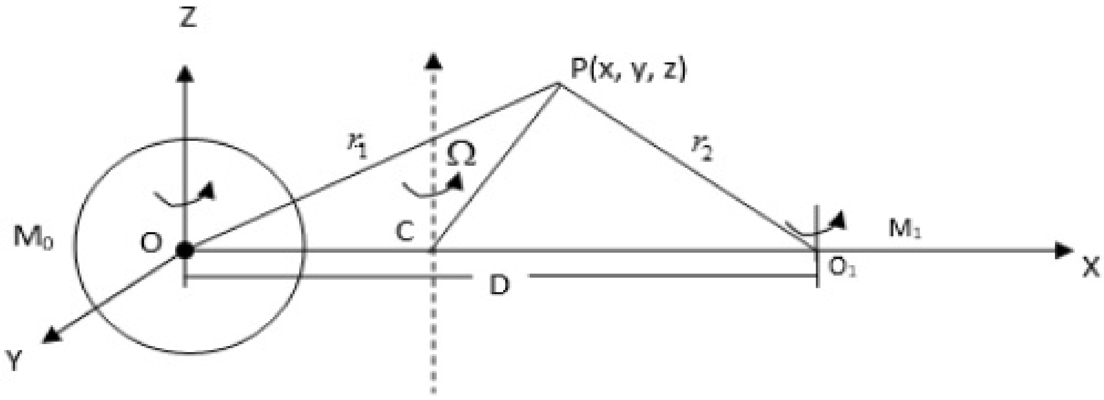

FIGURE 1.1: Coordinate Axis for reference for a binary system

of mass denoted by C with angular velocity Ω. Point P will experience three kinds of forces they are gravitational forces, centrifugal forces and magnetic pull or force.

If it lies in the vicinity of the primary component of double star system

$$r_1 = \sqrt{x^2 + y^2 + z^2}, \quad r_2 = \sqrt{(D - x)^2 + y^2 + z^2} \quad and \quad r = \sqrt{(x - d_1)^2 + y^2 + z^2} \tag{1.17}$$

The above equations represent net distance of a point P (x, y, z) from that of centre of gravity of primary star (point O) and secondary star (point O_1) and the centre of mass of this two body system (point C) respectively.

Further following the assumptions of Djuraevi as well as making use of classical dynamics, the potential obtained at a point P (x, y, z) that experience effects of the magnetic forces along with the gravitational and centrifugal force, is written as

$$\Psi = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} + \frac{1}{2}(\vec{\Omega} * \vec{r}).(\vec{\Omega} * \vec{r}) - \left(\frac{1}{2}\right)^{\frac{1}{4}} \frac{kT_1}{2\mu m_H} \left[1 - \sqrt{1 - \left(\frac{R_1}{r_1}\right)^2}\right]^{\frac{1}{4}} \left(5 + 4\frac{P_m}{P_g}\right) \tag{1.18}$$

where the first term from right is due to gravitational potential of mass M_0, the second one is due to gravitational potential of mass M_1, 3^{rd} one is for centrifugal force and so the last term was included because of the magnetic pressure.

As stated by Djuraevi **[2]**, μ is for mean molecular weight, T_1 stands for effective temperature of the primary component, , m_H represents atomic mass unit, k is the Boltzmann constant, P_m denotes magnetic pressure, P_g denotes the gas pressure and R_1 is for the radius of primary star. If we expand the last term of the equation (1.18) by Binomial approximation, we get

$$\frac{kT_1}{2\sqrt{2}\mu m_H}\left(5+4\frac{P_m}{P_g}\right)\sqrt{\frac{R_1}{r_1}} \tag{1.19}$$

The third term contains the cross product of the angular velocity of the stars thats along z- axis and vector $\overrightarrow{r}$. We know, if $\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\overrightarrow{\Omega} = \Omega k$ as a result the term becomes

$$\overrightarrow{\Omega} * \overrightarrow{r} = \Omega^2[(x-d)^2 + y^2] \tag{1.20}$$

Now the new expression for potential is as follows

$$\Psi = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} + \frac{1}{2}\Omega^2[(x-d)^2+y^2] - \frac{kT_1}{2\sqrt{2}\mu m_H}\left(5+4\frac{P_m}{P_g}\right)\sqrt{\frac{R_1}{r_1}} \tag{1.21}$$

Following the approach of Kopal (1972), equation (1.18) can be non-dimensionalized as:

$$\overset{*}{\Psi} = \frac{1}{\overset{*}{r}} + q\left(\frac{1}{\sqrt{1-2\lambda\overset{*}{r}+\overset{*}{r}^2}} - \lambda\overset{*}{r}\right) + n(1-v^2)\overset{*}{r}^2 - \frac{\alpha}{\sqrt{\overset{*}{r}}} \tag{1.22}$$

where $\overset{*}{\Psi} = \frac{D\Psi}{GM_0} - \frac{M_1^2}{2M_0(M_0+M_1)}$ is the way to obtain non-dimensional form of potential Ψ, $\overset{*}{r} = (r_1/D)$ is the non-dimensionalized form of r_1 , $q = \frac{M_1}{M_0}$ is the mass ratio and is called tidal parameter n in the above equation is known as rotational parameter.

If $n = \frac{(q+1)}{2}$ then it is synchronous rotation and $n \neq \frac{(q+1)}{2}$ then it is asynchronous rotation. $\alpha = \left(\frac{D}{GM_0}\right)\frac{kT_1}{2\sqrt{2}\mu m_H}\left(5+4\frac{P_m}{P_g}\right)\sqrt{\frac{R_1}{r_1}}$ stands for the non dimensional magnetic parameter.

The use of spherical polar coordinates is made in the above equation where $\lambda = \sin\theta\cos\varphi$, $\mu = \sin\theta\sin\varphi$ and $\nu = \cos\theta$. And $\frac{P_q}{P_m}$ represents the plasma parameter.

If we assume that the internal and external composition of the star is dominated by ionized hydrogen, $\mu = \frac{1}{2}$, and inserting the value of k (Boltzmann constant)=$1.38 \times 10^{-23}\ m^2\ kg\ s^{-2}\ K^{-1}$, the value of ,,, obtained from its expression above varies between zero and one. ,,, is a non-dimensional parameter just like tidal and rotational parameters. The different effects that magnetic field has on the binary system can be closely examined by varying the values of this parameter. For the condition $\frac{P_g}{P_m} = 0$, the effect caused due to magnetic pressure completely vanishes and so now the equipotential surfaces are defined just by gas pressures gradient, gravitational forces and centrifugal forces. Apart from this, the changes in the Roche model due to gas pressure are very low and almost negligible hence they can be neglected. If we consider $\alpha = 0$, then the resulting expression takes the form as

$$\overset{*}{\Psi}= \frac{1}{\overset{*}{r}} + q\left(\frac{1}{\sqrt{1-2\lambda\,\overset{*}{r}+\overset{*}{r^2}}} - \lambda\,\overset{*}{r}\right) + n(1-v^2)\,\overset{*}{r^2} \tag{1.23}$$

Equation (1.23) is similar to the one obtained in **[17]**. Setting q = 0, (1.22) takes the form of magnetically rotating stellar model without considering tidal effects.

On setting $\alpha = 0$ and n = 0, the equation (1.22) becomes the special case of pure tidal distortion.

The equation (1.22) can also be written by making the use of Legendre Polynomials $P_j(\lambda)$ as:

$$\overset{*}{\Psi}= \frac{1}{\overset{*}{r}} + q + q\sum_{j=2}^{\infty}\overset{*}{r^j}\,P_j(\lambda) + n(1-v^2)\,\overset{*}{r^2} - \frac{\alpha}{\sqrt{\overset{*}{r}}} \tag{1.24}$$

Chapter 2

SHAPE OF ROCHE EQUIPOTENTIAL SURFACE OF TIDALLY, ROTATIONALLY AND MAGNETICALLY DISTORTED BINARY STARS

In this chapter we will discuss in detail a system of binary star and two types of mass transfer in such systems. Further we will proceed to study effects of magnetic fields on the mass transmission and reduction along with generations of poloidal and toroidal magnetic fields. Next we will explain the concept of magnetic braking and how it effects the binary system. At last we will compile our chapter by evaluating the effect of magnetic field on the shapes of the binary system.

2.1 Binary Stars

A binary star system includes 2 stars of non-identical masses orbiting around their shared center of mass. These play a significant role in the modern astrophysics as a result of this when we compute their orbits, the probability to determine the stellar masses of their component gaseous spheres, comes high. Consequently various other key parameters of stars like their radius, density profile etc. often can be calculated, though indirectly. Their research has successfully been able to conclude an empirical mass-luminosity relationship (written as MLR) that successfully estimates the mass of single stars of our galaxy.

The size of the stars (that appear within the main sequence chain) increases till a certain point with their evolution. And beyond that certain point, when a star exceeds its Roche lobe, the stellar material flow to the binary companion, usually via accretion discs. Roche lobe is hence defined as a distinctively formed (tear drop) region that outlines a binary star system, that is bound to the stars gravitational pull. Any material lying beyond Roche lobe of the star either escapes the binary star system entirely, orbit both of the stars or descend to the binary component, as a concern to what were its former energy, position and momentum. To visualize it easily, let us consider a close binary system where one of the component stars has expanded (i.e. increased in size).

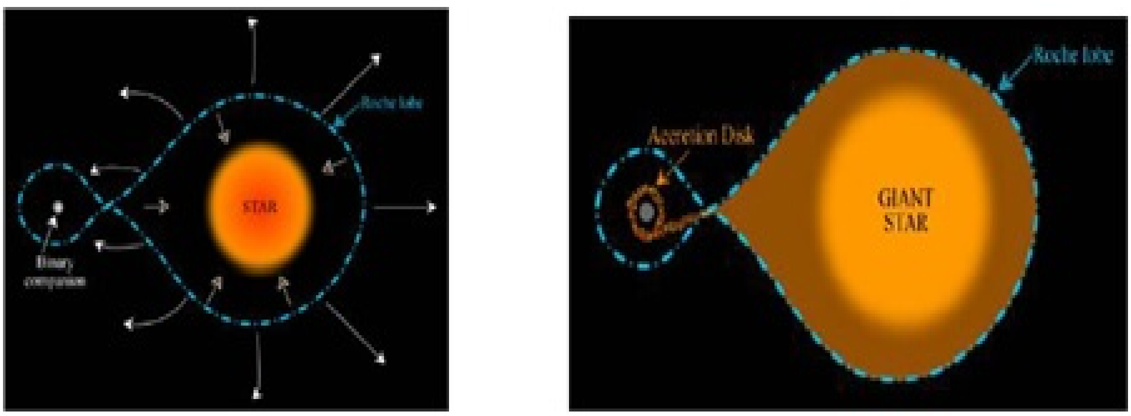

Figure 2.1

The above diagram represents that once a star exceeds its limit of (size) Roche lobe, its stellar material floods towards its binary star component. This phenomenon has been termed as Roche-lobe overflow. It takes place through a mathematical point known as internal Lagrangian point, which creates the possibility of mass transfer. It can be defined as a physical point at which the gravitational forces of the two coupled stars cancel each other. It accounts for a number of astronomical phenomenons like X-ray binary system, novae, etc.

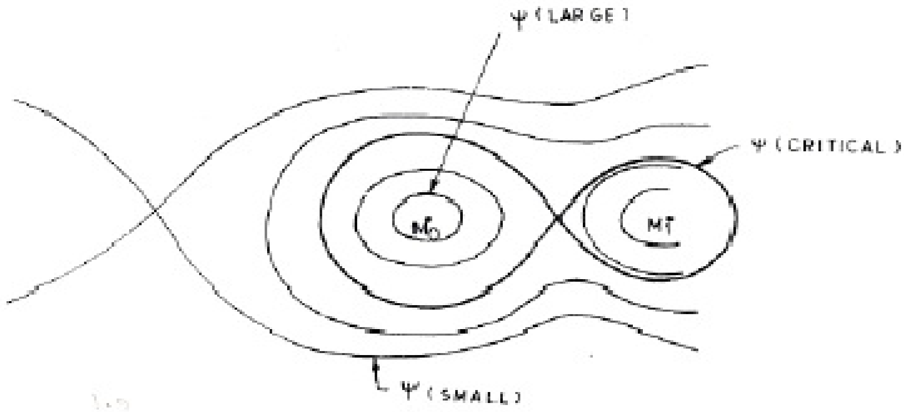

FIGURE 2.2: Roche equipotential surface in two dimension

Mass transfer is of two types.

1. **Conservative Mass Transfe**r: This happens when none of substance been expelled leaves a binary system. During this type of mass transmission, the systems orbital element may change due to transfer of angular momentum between the star and its binary counterpart. Now, let us assume a stellar environment comprising total mass $M = M_1 + M_2$ (here M_1 is the mass of the primary component and M_2 is that of the secondary component). Eccentricity e of the system along with its total angular momentum $J = M_1 M_2 \sqrt{\frac{G\alpha(1-e^2)}{M_1 M_2}}$, will be conserved. Here G represents universal gravitational constant. So J=0, M=0 and also $M_1 = -M_2$.

Hence if M_1 is the mass of the star that is shredding its mass, then $\dot{M}_1$ is clearly negative. Thus we conclude that when the mass exhausting star is more massive then the stellar path can suffer a definite shrink and hence period decreases. On the other hand if we consider that the star is not as big as its significant binary component, the stellar trajectory will expand with rise in its period.

2. **Non-Conservative Mass Transfer**: In this angular momentum as well as the stellar mass is discharged from the system. Orbital Angular Momentum of a two body system is defined as

$$J = \left(\frac{M_1 M_2}{M_1 + M_2}\right) \alpha^2 \Omega = \left(\frac{q}{(1+q)^2}\right) M \alpha^2 \Omega \tag{2.1}$$

where, $\left(\frac{M_1 M_2}{M_1 = M_2}\right) \alpha^2 = I$ is the Moment of inertia. $\Omega = \frac{2\pi}{P}$ will be the angular speed and P is the orbital period.

2.2 Effects of Magnetic Fields on The Mass Transmission and Reduction

Now a days it has convinced astrophysicists that close binaries having a relatively cool F-K type star show an enhanced magnetic activities. Further the close binary systems with orbital periods smaller than 5 to 6 days possess certain types of characteristics due to their rapid rotations. Various authors [**22,25,28**] have studied the primary as well as secondary components in closed binaries to display various magnetic properties which are time dependent, and that gives rise to numerous variations that are been observed in the brightness of their light curves, cyclic variations in the orbital period of binary systems, radiations of UV, X-rays and Infrared.

2.3 Generations of Poloidal and Toroidal Magnetic Fields

We are well aware that the generator or dynamo mechanism is most probable and obvious source of huge scale production of magnetic field in the stars with convective layers. The differential rotation that has been observed between the radiative core together with the convective envelop, leads to the deformation of the poloidal field by winding up the field and this further produces an extra toroidal field component, which generates a Lorentz force that works against the shear or deformation caused by the poloidal field. The very effect of magnetic fields on the mass as well as angular momentum transmission or/and loss of mass of both binary companions are really convincing.

In close binary systems, the orbital angular momenta and spin angular momentum are coupled very strongly, so it happens that the stellar spindown cause a lowering of the orbital period of binary system even when there is no mass transfer. Here magnetic field is a significant factor on which the value of coupling constant totally depends and may become notable if the field is very strong. There exists an additional effect of magnetic field that is it alters the rotation by providing a torque to the star through mass overflow.

2.4 Magnetic Braking

The result of exchange of mass is the mass transfer, generally from a primary component to a secondary one during a whole contact phase. Along with this magnetic braking is also a usual occurrence for all closed binary stars system. The loss of angular momentum has a very important role in birth and future evolution of closed binary stars. The magnetized stellar winds at first are directed towards the outer surface of the active star however these stellar winds are twisted (distorted) because of rapid star rotation. The magnetic field lines drag away the charged particles (trapped in the star wind) present within star. As a result angular momentum is transfered from a star to the charged particles via magnetic field. As the star wind leaves the stars surface, the charged particles are pulled away by the strong magnetic fields. This further effects the rotation of the star and slows it down. As an e.g., in close binary systems where synchronization is expected of the rotational and orbital periods, the reduction of rotational angular momentum usually takes place at the cost of orbital angular momentum. Because of this the period drops that is constituents spin up which is followed by their approaching movements towards each other to generate a single rapidly rotating star.

It has been observed that closed binary systems are very active magnetically and also it is accepted that these systems lose their mass as well as angular momentum by the magnetized gust. In addition to this even the constituents are however not considerably separated . Hence it is expected that the magnetic field interactions within the 2 constituents to be extremely high, so their impact upon the AM (angular momentum) loss is really significant the reason being the formation of magnetic loops from the arising magnetic fields at its surface of either both or one binary components.

Next the magnetic torque created from the magnetic field in the wind is further dependent upon the extent of magnetic field. No doubt, the in depth knowledge of this notion as well as experimental verifications of data and quantitive formulations will remain one of the major challenges of astrophysicists for future.

There is large no. of charged particles contained in the stellar wind which show a departure from the star radially. Also at the external surface of the star the rotational surface velocity components and tangential components are equal. It is expected that the speed of the same particles reduce to quite lower rate, at really large distances in space. But it has been observed that in Suns case, the speed of the particles measured is of the order of about 1-10 km/s. Which is ((102 103)) times ahead to the expected ones. This is because charged particles do not only flow radially outwards instead also travel through the twisted open field lines.

Hence it is important that the magnetic fields energy / volume which needs to be greater than the particles own kinetic energy, so that the particles trajectory is not dominated by gravitational field but rather by magnetic field lines direction.

When some of the magnetic field lines of the star are open as well as reconnect to that of the companion star, then these particles can either hit (collide) the particles which are governed by the companion star along with the magnetic field lines in the similar way as that of the primary star or it can fall into the very own atmosphere of the secondary companion star (fig 2.1 and 2.2). The angular momentum of the stars is carried by these particles.

As already explained that due to rapid rotation of the star, the bending down of magnetic field lines happens. Now this bending of field lines form a curvature that further is reason for production of a force acting on the stellar plasma.

The magnetic braking is small when we assume that the stellar rotational poles coincide with magnetic poles because of which the dissipation of angular momentum of the double star system reduces. Though when the field is concerned either near or at the equatorial plane itself, then the magnetic braking is at its peak and so maximum amount of angular momentum is been removed. Through observations made by astrophysicists it has been inferred that magnetic braking is prominent in late F-K type of stars.

In binary star system the component gaseous spheres (10-100 times) rotate at a much faster rate in comparison to rotation of that of a single star because of the presence of tidal (or gravitational) interactions among the components of binaries. Hence we expect that the generation of lines of magnetic field at the stars surface of the binary system to be very intense in comparison to that of a single star.

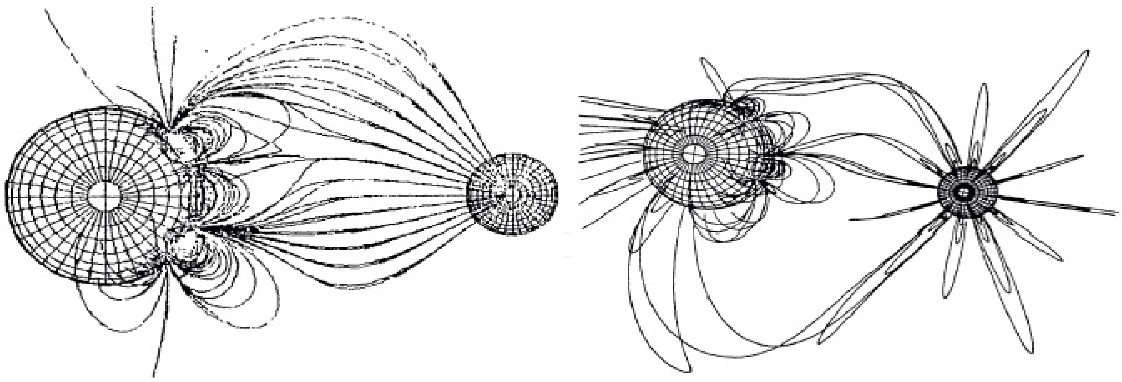

FIGURE 2.3: The formation of magnetic loops in a binary system at a large scale

2.5 Effect of magnetic field on the shapes of the binary system

For depicting the shape of Roche equipotential surfaces, we need to set $\overset{*}{\Psi}=$ constant at the left hand side of (1.22) (or (1.24)). It has been observed that for bigger values of $\overset{*}{\Psi}$, Roche equipotential surface consists two separate ovals (whose shape differ a little from spheres) across the two mass center points of the binary system. As we reduce the value of $\overset{*}{\Psi}$, the surface starts to extend towards the center of mass of the system unless a certain critical value of say $\overset{*}{\Psi}_1$, is reached, at this the ovals now form a dumb bell like configuration across a point on the axis that joins the centers of mass. The value of $\overset{*}{\Psi}$ at this point is a feature of mass ratio of each and hence it is called Roche limit. For smaller values, the surfaces collapses in a single dumb bell like surface that surrounds both of the stars.

In accordance with the approach discussed by Kopal [**19**] and Lal et al. [**20**], the Roche limits and along with that the position of the point of contract of the dumb bell shape was been computed. To calculate the Roche limits, the initial step is to compute the critical point P_1 at which $\overset{*}{\Psi} = \overset{*}{\Psi}_1$ that is the a common point of contact for the Roche lobes. The point is situated on the x-axis, the axis which joins the centers of gravity of the two masses. P_1 is featured by the vanishing of the total potential force. That is, at P_1

$$\overset{*}{\Psi}_x = \overset{*}{\Psi}_y = 0 \tag{2.2}$$

the x coordinate of P_1 (say x_1) is given by the vanishing value of $\overset{*}{\Psi}_x$ that is a root of the equation written below

$$(q+1)x^5 + \frac{\alpha}{2}x^{9/2} - (3q+2)x^4 - \alpha x^{7/2} + (3q+2)x^3 + \frac{\alpha}{2}x^{5/2} - x^2 + 2x - 1 = 0 \tag{2.3}$$

The equation (2.2) has in general five roots and the root lying between 0 and 1 is the our desired Roche limit. On obtaining the accurate value of x_1 , the next step is finding the corresponding value of $\overset{*}{\Psi}_x$ which will be $\overset{*}{\Psi}_1$.

$\overset{*}{\Psi}_1 = \overset{*}{\Psi}_x \ (x_1, 0, 0)$

which on simplification gives

$$\overset{*}{\Psi}_1 = \frac{1}{x_1} + q\left[\frac{1}{1-x_1} - x\right] + nx_1^2 - \frac{\alpha}{\sqrt{x_1}} \tag{2.4}$$

Further it is important to calculate $P_{4,5}$ in the x-y plane (see Fig. 2.4). These points are featured by the vanishing of $\frac{dy}{dx}$ at x = x_1. Their coordinates $x_{4,5}$ and $y_{4,5}$ are been obtained on solving these 2 equations simultaneously,

$$\overset{*}{\Psi}\ (x, y, 0) = \overset{*}{\Psi}_1 \tag{2.5}$$

$$\overset{*}{\Psi}_x\ (x, y, 0) = 0 \tag{2.6}$$

On simplifying, $\overset{*}{\Psi}\ (x, y, 0) = \overset{*}{\Psi}_1$ becomes

$$\overset{*}{\Psi}_1 = \frac{1}{\sqrt{x^2+y^2}} + q\left[\frac{1}{\sqrt{(1-x)^2+y^2}} - x\right] + n(x^2+y^2) - \frac{\alpha}{\sqrt{x^2+y^2}} \tag{2.7}$$

and $\overset{*}{\Psi}_x\ (x,y,0) = 0$finally takes the form as

$$-x + q(x^2+y^2)^{3/2}\left[\frac{1-x}{((1-x)^2+y^2)^{3/2}} - 1\right] + 2n(x^2+y^2)^{3/2} - \frac{\alpha}{2}(x^2+y^2)^{1/4} = 0 \tag{2.8}$$

Once the values of $x_{4,5}$ are calculated, the values of corresponding coordinates in x-z plane, $z_{6,7}$ are the roots of the equation

$$\overset{*}{\Psi}\ (x_{4,5},0,z) = \overset{*}{\Psi}_1 \tag{2.9}$$

Hence the values of $\overset{*}{\Psi}_1, x_1, x_{4,5}, y_{4,5} and z_{6,7}$ are calculated for different values of q, n and using expressions (2.1) - (2.8).

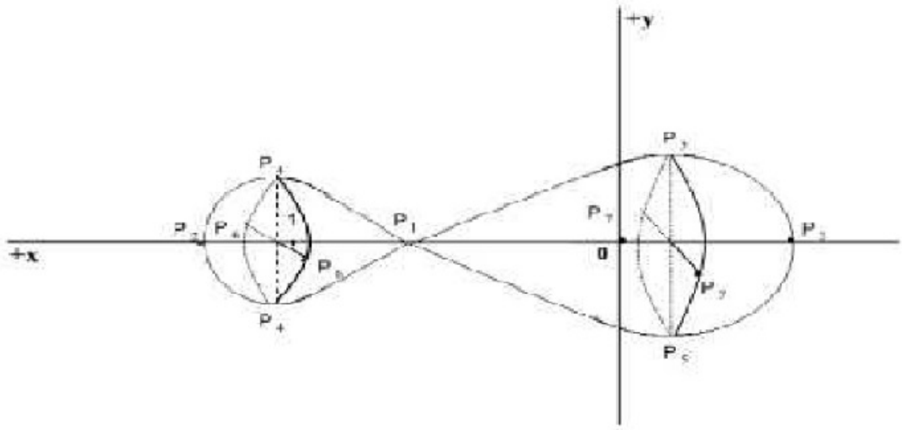

FIGURE 2.4: Schematic view of contact binary at roche limit

The shapes of the Roche equipotential surfaces depicted in Fig 2.4 for certain values of q, n and are compared with the shapes of those systems where effects of magnetic field is neglected. It is observed that the shapes do not differ much when the value of α is small.

Chapter 3

ROCHE COORDINATES AND METRIC COEFFICIENTS FOR ROTATIONALLY AND MAGNETICALLY DISTORTED BINARY STARS

This chapter is the final chapter that comprises an introduction to curvilinear coordinate system which has been included to explain the basis of Roche Coordinates. Finally we will derive expressions of Roche coordinates for modified Roche equipotential surface for rotationally and/or magnetically distorted system of stars and their metric coefficients.

3.1 An introduction to Curvilinear coordinate system

Though cartesian system of orthogonal coordinates is very easy to use, it is sometimes more convenient to resolve our work with other systems coordinate. The ability to change all of the expression and variables in a particular given problem, when a different set of coordinate is been chosen is one of those skills required in a physicist. We will further introduced and explained a general method of how to express any variable along with expression in an arbitrary system curvilinear coordinate.

The concept of orthogonal curvilinear coordinates.

The cartesian system of coordinate is quite easy to handle. Once we fix the origin in space and three perpendicular axis are attached to this origin, any point in space can now be uniquely determined just by set of three real numbers, its called cartesian coordinates. Now starting from this cartesian one as our basis, a curvilinear coordinate system could be defined. If x, y, z are the cartesian coordinates and u, v, w are curvilinear ones these can be expressed as functions of x, y, z, accordingly as:

$$u = u(x, y, z) \tag{3.1}$$

$$v = v(x, y, z) \tag{3.2}$$

$$w = w(x, y, z) \tag{3.3}$$

The inversion of these functions show x, y, z-dependency on u, v, w:

$$x = x(u, v, w) \tag{3.4}$$

$$y = y(u, v, w) \tag{3.5}$$

$$z = z(u, v, w) \tag{3.6}$$

There are infinite number of curvilinear systems that can be defined by using equations (3.1 - 3.3) and (3.4 - 3.6). Our main interest is orthogonal curvilinear coordinate systems, as defined as follows. By the definition of point we know that any point in space can be determined by intersection of three planes:
$u = const, \qquad v = const, \qquad w = const$

Let us call these curved surfaces as coordinate surfaces. The three curves, called coordinate curves, can now said to be formed by the intersection of any pair of these surfaces. Three straight lines can be drawn as tangents to each coordinate curve at the point in space. As a result in an orthogonal curved system the above drawn three tangent lines will be perpendicular for all the points in space (see Figure 3.1).

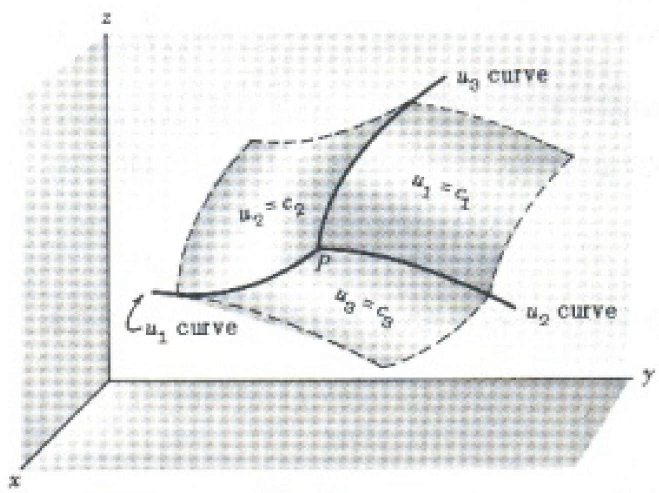

Fig. 4

FIGURE 3.1

Let us now express differential operators, like the divergence or the gradient, in curvilinear coordinates. In order to do this it is convenient to initiate from the infinitesimally small increment in Cartesian system of coordinates,

$$dr = \frac{\partial r}{\partial u}du + \frac{\partial r}{\partial v}dv + \frac{\partial r}{\partial w}dw \tag{3.7}$$

where $\partial r/\partial u, \partial r/\partial v \quad and \quad \partial r/\partial w$ are tangent vectors, respectively, to the coordinate curves along u, v and w, at P. These three vectors are mutually perpendicular, as we are working on orthogonal system of curvilinear coordinates. Let eu, ev and ew, denote unit-lenght vectors along $\partial r/\partial u, \partial r/\partial v and \partial r/\partial w$, respectively. If we further define by hu, hv and hw as:

$$h_u \equiv \left|\frac{\partial r}{\partial u}\right| \qquad h_v \equiv \left|\frac{\partial r}{\partial v}\right| \qquad h_w \equiv \left|\frac{\partial r}{\partial w}\right| \tag{3.8}$$

then the infinitesimally small increment (3) can be modified as:

$$dr = h_u du e_u + h_v dv e_v + h_w dw e_w \tag{3.9}$$

Equation (3.9), and its associated definitions (3.8), are key tool in the deriving many fundamental quantities used and applied in differential calculus, when we shift from a cartesian system to curvilinear coordinate system. We now consider an example of polar coordinates, (r, θ), in the plane. x and y that are functions of r and θ according to:

$$x = r\cos(\theta) \qquad y = r\sin(\theta) \tag{3.10}$$

To derive a correct expression for dr (dx, dy) we need to first compute hr and h. From (3.8) we get:

$$h_r = \left| \left(\frac{\partial x}{\partial r}, \frac{\partial y}{\partial r} \right) \right| = \sqrt{\cos^2(\theta) + \sin^2(\theta)} = 1 \tag{3.11}$$

$$\tag{3.12}$$

$$h_\theta = \left| \left(\frac{\partial x}{\partial \theta}, \frac{\partial y}{\partial \theta} \right) \right| = \sqrt{[-r\sin(\theta)]^2 + [r\cos(\theta)]^2} = r \tag{3.13}$$

Thus, dr is written as:

$$dr = dre_r + rd\theta e_\theta \tag{3.14}$$

With the above result we can now derive the form of several quantities in terms of polar coordinates. For example, the case of line element is given by:

$$dl \equiv \sqrt{dr.dr} = \sqrt{(dr)^2 + r^2(d\theta)^2} \tag{3.15}$$

and area element is written as:

$$dS = h_r h_\theta dr d\theta = r dr d\theta \tag{3.16}$$

For the three dimensional case in general, the line element is given by:

$$dl \equiv \sqrt{dr.dr} = (h_u du)^2 + (h_v dv)^2 + (h_w dw)^2 \tag{3.17}$$

and in addition to this the volume element is given by:

$$dV \equiv [(e_u.dr)e_u].\{[(e_v.dr)e_v] \times [(e_w.dr)e_w]\} = h_u h_v h_w du dv dw \tag{3.18}$$

To compute curl it is really important to have resolved expressions for the surface elements orthogonal to each curve coordinate. These elements can be simply given by:

$$dS_u = h_v h_w dv dw \qquad dS_v = h_u h_w du dw \qquad dS_w = h_u h_v du dv \tag{3.19}$$

3.2 A system of Roche Coordinates

To further study the problems related to close binary star systems, Kopal and Kitamura **[40]** put forward a new system of coordinates known as Roche coordinates. As our chapter will proceed we will begin to discuss certain types of theoretical aspects of the system of Roche coordinates. We have derived the equation for a system of Roche coordinates in section, the direct equations have been obtained by incorporating the effects of magnetic field along with rotational effects.

If we consider a real star then most of its mass is concentrated at the centre or very close to it. Hence the structure is comparable to Roche model where whole of the mass of the star is assumed to be taken at the centre point and this point is encircled by a cover in which density is supposed to depend inversely to some positive power of separation distance from the centre.

Through some serious calculations, Chandrasekhar **[4]** has proved that stars with central density to mean density ratio equal to 100 or above that, then for such systems the Roche model for rotating configurations will represent the real type of equipotential surfaces of this rotating star having an error $< 1\%$. So, our assumption of equipotential surfaces as defined by equation is in close estimate to equipotential surfaces of mostly all rotating stars in case of closed binary systems.

Some of the analysis of mathematical properties was performed by Kopal along with his coworkers for the system of Roche coordinates. In order to make a presentation of a real model of stars which is under the effect of tidal, rotational and magnetic deformations the Roche coordinates obtained by equipotential surfaces given by Roche model are used.

The important characteristics of the system of Roche coordinates will be presented in the proceeding sections. The rotational, tidal and magnetic forces cause distortion in the equipotential surface of a real star. By choosing these equipotential surfaces as first coordinate, the choice of the other two is made by keeping in mind the condition that they make a triple orthogonal system. Let us take the ξ coordinate as:

$$\xi = \frac{1}{r} + q[\frac{1}{\sqrt{1 - 2\lambda r + r^2}} - \lambda r] + nr^2(1 - v^2) = constant \tag{3.20}$$

and choicing of the other two coordinates is made such that they satisfy the condition of orthogonality:

$$\eta = \cos^{-1}\lambda - \frac{q}{\sqrt{1-\lambda^2}} \sum_{j=2}^{4} \frac{r^{j+1}}{j+1} \acute{P_j} \lambda \tag{3.21}$$

$$\zeta = cos^{-1} \frac{v}{\sqrt{1-\lambda^2}} \tag{3.22}$$

Let us recall that for rotationally,tidally and magnetically distorted stars the expression for potential is:

$$\Psi = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} + \frac{1}{2}\Omega^2[(x-d)^2 + y^2] - \frac{kT_1}{2\sqrt{2}\mu m_H}\left(5 + 4\frac{P_m}{P_g}\right)\sqrt{\frac{R_1}{r_1}} \tag{3.23}$$

it can be non-dimensionalized as:

$$\overset{*}{\Psi} = \frac{1}{\overset{*}{r}} + q\left(\frac{1}{\sqrt{1 - 2\lambda \overset{*}{r} + \overset{*}{r^2}}} - \lambda \overset{*}{r}\right) + n(1 - v^2)\overset{*}{r^2} - \frac{\alpha}{\sqrt{\overset{*}{r}}} \tag{3.24}$$

where $\overset{*}{\Psi}= \frac{D\Psi}{GM_0} - \frac{M_1^2}{2M_0(M_0+M_1)}$ is the way to obtain non-dimensional form of potential Ψ, $\overset{*}{r}= (r_1/D)$ is the non-dimensionalized form of r_1 , $q = \frac{M_1}{M_0}$ is the mass ratio and is called tidal parameter n in the above equation is known as rotational parameter. The equation can also be written by making the use of Legendre Polynomials $P_j(\lambda)$ as:

$$\overset{*}{\Psi}= \frac{1}{\overset{*}{r}} + q + q\sum_{j=2}^{\infty} \overset{*}{r}^{j}\ P_j(\lambda) + n(1-v^2)\ \overset{*}{r}^{2} - \frac{\alpha}{\sqrt{\overset{*}{r}}} \tag{3.25}$$

there is a need to find $\overset{*}{r}$, for a mentioned value of $\overset{*}{\Psi}$ and given values of θ and ϕ, cannot be computed explicitly.Hence Kopal derived a series expansion for $\overset{*}{r}$ in the terms of other parameters. Following **[17]** and **[26]** and considering non-dimensional variable our first approximation towards the distance of equipotential surface to that from the center, several iterations has been carried out to calculate the value of $\overset{*}{r}$ in terms of r_0.

$$\overset{*}{r}= r_0[1 - \alpha r_{0^{1/2}} + \frac{\alpha}{2}r_0 + \{qP_2 + n(1-\nu^2)\}r_0^3 + \left\{\frac{-7}{2}\alpha(qP_2 + n(1-\nu^2))\right\} r_0^{7/2} + qP_3r_0^4 + \left\{\frac{-9}{2}\alpha qP_3\right\} r_0^{9/2} + qP_4r_0^5 + \left\{\frac{-11}{2}\alpha qP_4\right\} r_0^{11/2} + qP_5 + 3P_2^2q^2 + 6qn(1-\nu^2)P_2 + 3n^2(1-\nu^2)^2r_0^6 \tag{3.26}$$

3.3 Expressions of Roche coordinates for modified Roche equipotential surface for magnetically distorted stars

A curvilinear system of coordinates called Roche coordinates have been introduced, here ξ has been already defined by Roche equipotential surface along with η and ζ as orthogonal coordinates to ξ as well as amoung themselves. The orthogonal coordinates must satisfy the below given conditions as mentioned by Kopal **[19]**.

$$\xi_x\eta_x + \xi_y\eta_y + \xi_z\eta_z = 0 \tag{3.27}$$

$$\xi_x\zeta_x + \xi_y\zeta_y + \xi_z\zeta_z = 0 \tag{3.28}$$

$$\eta_x\zeta_x + \eta_y\zeta_y + \eta_z\zeta_z = 0 \tag{3.29}$$

Let ξ be also wriiten as $\overset{*}{\Psi}$ such that the potential expression obtained under the effect of rotation,tidal and magnetic field is

$$\overset{*}{\Psi}= \frac{1}{\overset{*}{r}} + q + q\sum_{j=2}^{\infty} \overset{*}{r}^{j}\ P_j(\lambda) + n(1-\nu^2)\ \overset{*}{r}^{2} - \frac{\alpha}{\sqrt{\overset{*}{r}}} \tag{3.30}$$

To obtain the coordinates under the effect of only magnetic field we ignore the rotation and tidal effects and for that purpose we insert the values q=n=0.Hence the expression obtained is:

$$\overset{*}{\Psi} = \frac{1}{\overset{*}{r}} - \frac{\alpha}{\sqrt{\overset{*}{r}}} \tag{3.31}$$

By partially differentiating equation (3.31) with respect to r,λ and ν, we get:

$$\overset{*}{\Psi}_{\overset{*}{r}} = \frac{-1}{\overset{*}{r}^2} + \frac{\alpha}{2\overset{*}{r}^{3/2}} \tag{3.32}$$

$$\overset{*}{\Psi}_{\lambda} = 0 \tag{3.33}$$

$$\overset{*}{\Psi}_{\nu} = 0 \tag{3.34}$$

The transformation of coordinates follows that

$$\overset{*}{\Psi}_x = \lambda \overset{*}{\Psi}_{\overset{*}{r}} - \frac{(1-\lambda^2)}{\overset{*}{r}} \overset{*}{\Psi}_{\lambda} - \frac{\lambda\nu}{\overset{*}{r}} \overset{*}{\Psi}_{\nu} \tag{3.35}$$

$$\overset{*}{\Psi}_y = \mu \overset{*}{\Psi}_{\overset{*}{r}} - \frac{(\lambda\mu}{\overset{*}{r}} \overset{*}{\Psi}_{\lambda} - \frac{\lambda\mu}{\overset{*}{r}} \overset{*}{\Psi}_{\nu} \tag{3.36}$$

$$\overset{*}{\Psi}_z = \nu \overset{*}{\Psi}_{\overset{*}{r}} - \frac{(\lambda\nu}{\overset{*}{r}} \overset{*}{\Psi}_{\lambda} - \frac{(1-\nu^2)}{\overset{*}{r}} \overset{*}{\Psi}_{\nu} \tag{3.37}$$

using the above expression we get

$$\overset{*}{\Psi}_x = \lambda \left[\frac{-1}{\overset{*}{r}^2} + \frac{\alpha}{2\overset{*}{r}^{3/2}}\right] \tag{3.38}$$

$$\overset{*}{\Psi}_y = \mu \left[\frac{-1}{\overset{*}{r}^2} + \frac{\alpha}{2\overset{*}{r}^{3/2}}\right] \tag{3.39}$$

$$\overset{*}{\Psi}_z = \nu \left[\frac{-1}{\overset{*}{r}^2} + \frac{\alpha}{2\overset{*}{r}^{3/2}}\right] \tag{3.40}$$

Hence we obtain the value of

$$(\overset{*}{\Psi}_x^2 + \overset{*}{\Psi}_y^2 + \overset{*}{\Psi}_z^2)^{-1/2} = \left[\frac{-1}{\overset{*}{r}^2} + \frac{\alpha}{2\overset{*}{r}^{3/2}}\right]^{-1} \tag{3.41}$$

The diffrential equations that generate equipotential surface according to Kopal[**19**] are

$$\overset{*}{\Psi}_x\, dx + \overset{*}{\Psi}_y\, dy + \overset{*}{\Psi}_z\, dz = 0 \tag{3.42}$$

the equations that generate orthogonal lines to equipotential surface are

$$\frac{dx}{\overset{*}{\Psi}_x} = \frac{dy}{\overset{*}{\Psi}_y} = \frac{dz}{\overset{*}{\Psi}_z} \tag{3.43}$$

the integration constants of above equation generate the Roche coordinates η and ζ that correspond to angular coordinates of spherical polars. Further to transform the equation (3.43) to form a suitable integrand, we devide it by normal area element ds, we know

$$\frac{dy/ds}{dx/ds} = \frac{\overset{*}{\Psi}_y}{\overset{*}{\Psi}_x} \; and \; \frac{dz/ds}{dx/ds} = \frac{\overset{*}{\Psi}_z}{\overset{*}{\Psi}_x} \tag{3.44}$$

the direction cosines of normal vector must obey the condition

$$(dx/ds)^2 + (dy/ds)^2 + (dz/ds)^2 = 1 \tag{3.45}$$

the equation (3.38 - 3.44) results

$$\frac{dx}{ds} = \frac{-\overset{*}{\Psi}_x}{(\overset{*}{\Psi}{}_x^2 + \overset{*}{\Psi}{}_y^2 + \overset{*}{\Psi}{}_z^2)^{-1/2}} \tag{3.46}$$

$$\frac{dy}{ds} = \frac{-\overset{*}{\Psi}_y}{(\overset{*}{\Psi}{}_x^2 + \overset{*}{\Psi}{}_y^2 + \overset{*}{\Psi}{}_z^2)^{-1/2}} \tag{3.47}$$

$$\frac{dz}{ds} = \frac{-\overset{*}{\Psi}_z}{(\overset{*}{\Psi}{}_x^2 + \overset{*}{\Psi}{}_y^2 + \overset{*}{\Psi}{}_z^2)^{-1/2}} \tag{3.48}$$

as $\xi(x, y, z)$ is a diminishing function of s, as a result we take the negative value of the square root. Employing the spherical polar coordinates

$x = r \sin\theta \cos\phi \quad y = r \sin\theta \sin\phi \quad z = r \cos\theta$

we obtain

$$\frac{dx}{d\overset{*}{r}} = \overset{*}{r}\frac{d\lambda}{\overset{*}{r}} + \lambda \tag{3.49}$$

$$\frac{dy}{d\overset{*}{r}} = \overset{*}{r}\frac{d\mu}{\overset{*}{r}} + \mu \tag{3.50}$$

$$\frac{dz}{d\overset{*}{r}} = \overset{*}{r}\frac{d\nu}{\overset{*}{r}} + \nu \tag{3.51}$$

as the element ds is taken very small, so we can replace it by line element dr accordingly apply first order approximation

$$\frac{dx}{d\overset{*}{r}} = -\lambda \tag{3.52}$$

$$\frac{dy}{d\overset{*}{r}} = -\mu \tag{3.53}$$

$$\frac{dz}{d\overset{*}{r}} = -\nu \tag{3.54}$$

by comparing (3.49 - 3.51) and (3.52 - 3.54) we obtain

$$\overset{*}{r}\frac{d\lambda}{d\overset{*}{r}} + \lambda = -\lambda \tag{3.55}$$

$$\overset{*}{r}\frac{d\mu}{d\overset{*}{r}} + \mu = -\mu \tag{3.56}$$

$$\overset{*}{r}\frac{d\nu}{d\overset{*}{r}} + \nu = -\nu \tag{3.57}$$

integrating the above gives

$$\lambda_1 = \frac{C_1}{\overset{*}{r}^2} \tag{3.58}$$

$$\mu_1 = \frac{C_2}{\overset{*}{r}^2} \tag{3.59}$$

$$\nu_1 = \frac{C_3}{\overset{*}{r}^2} \tag{3.60}$$

here $C_1, C_2 \quad and \quad C_3$ are integration constants that can be obtain by the condition

$$\lambda_1^2 + \mu_1^2 + \nu_1^2 = 1 \tag{3.61}$$

followed by this we obtain $C_1 = \lambda_0$, $C_2 = \mu_0$, $C_3 = \nu_0$ where zero subscript denotes zeroth order approximation. $\lambda_0, \mu_0 \; and \; \nu_0$ in terms of spherical coordinates are written as

$$\lambda_0 = \cos\theta \sin\phi \qquad \mu_0 = \sin\theta \sin\phi \qquad \nu_0 = \cos\theta \tag{3.62}$$

with the condition $\lambda_0^2 + \mu_0^2 + \nu_0^2 = 1$ for approximation of zeroth order.By using the dependence of $\lambda_0, \mu_0 \; and \; \nu_0 \; on \; \eta \; and \; \zeta$, we suggest to make the use of following substitutions for the case of rotational distortions as

$$\lambda_1 = \cos\eta \sin\zeta \qquad \mu_1 = \sin\eta \sin\zeta \qquad \nu_1 = \cos\zeta \tag{3.63}$$

using equation(3.63), we get

$$\cos\eta = \frac{\lambda_1}{\sqrt{1 - \nu_1^2}} \tag{3.64}$$

and also

$$cos\zeta = \frac{\nu_0}{\overset{*}{r}^2} \tag{3.65}$$

the expression for η in equation (3.65) is exact, the expression for ζ is correct up to 2^{nd} order term in n and q.

3.4 The metric coefficients for magnetically distorted stars

In our chosen system of coordinates (ξ, η, ζ) if we assume ζ= constant and then choose η and ζ in such a manner that they satisfy condition of mutual orthogonality w.r.t. ζ and among each other.

The general equations of orthogonality are same as in equation(3.27 - 3.29)

Also the value of r^* for the case of magnetic distortion is

$$r^* = r_0[1 - \alpha r_0^{1/2} + \frac{\alpha}{2} r_0] \tag{3.66}$$

If we assume

$$x = x(\xi, \eta, \zeta); \qquad y = y(\xi, \eta, \zeta); \qquad z = z(\xi, \eta, \zeta) \tag{3.67}$$

In these the surface ξ=constant, η= constant and ζ=constant form orthogonal set of coordinates. In 3D transformation the metric coefficients can be written as

$$(dx)^2 + (dy)^2 + (dz)^2 = h_1^2(d\xi)^2 + h_2^2(d\eta)^2 + h_3^2(d\zeta)^2 \tag{3.68}$$

Now in this part we will be dealing with explicit expression of metric coefficients i.e. h_1, h_2 *and* h_3 that are associated with the system of Roche coordinates (ξ, η, ζ) for the Roche model of stars distorted by rotation,tidal and magnetic field.The metric coefficients are defined by:

$$h_1 = (\xi_x^2 + \xi_y^2 + \xi_z^2)^{-1/2} \tag{3.69}$$
$$h_2 = (\eta_x^2 + \eta_y^2 + \eta_z^2)^{-1/2} \tag{3.70}$$
$$h_3 = (\zeta_x^2 + \zeta_y^2 + \zeta_z^2)^{-1/2} \tag{3.71}$$

The value of h_1 obtained by substituting the value of r in terms of r_0 is

$$h_1 = -r_0^2 \left[1 - \frac{3}{2}\alpha r_0^{1/2} - \left(1 - \frac{5}{4}\alpha\right) \alpha r_0 + \frac{5}{4}\alpha^2 r_0^{3/2}\right] \tag{3.72}$$

Further it can be shown that

$$\eta_x = -\frac{\mu}{r(1-\nu^2)}, \quad \eta_y = -\frac{\lambda}{r(1-\nu^2)} \; and \; \eta_z = 0 \tag{3.73}$$

from equations (3.70) and (3.73)it can be shown

$$h_2 = r\sqrt{(1-\nu^2)} \tag{3.74}$$

By putting the values of r_0

$$h_2 = r_0 \left[1 - \alpha r_0^{1/2} + \frac{\alpha}{2} r_0 - \cos^2 \zeta \left(\frac{1}{2} r_0^4 + \frac{\alpha}{2} r_0^{9/2} + \frac{\alpha}{4} r_{0^5} \right) \right] \tag{3.75}$$

And now the expression for h_3 can be obtained by the expression

$$h_3 = (\zeta_x^2 + \zeta_y^2 + \zeta_z^2)^{-1/2} \tag{3.76}$$

and the obtained expression is

$$h_3 = \frac{r^3(1-\nu^2)^{1/2}}{3\nu} \left[1 - \frac{r^4}{18v^2} - \frac{r^2}{3} \right] \tag{3.77}$$

3.5 Expressions of Roche coordinates for modified Roche equipotential surface for rotationally and magnetically distorted stars

As already discussed a curvilinear system of coordinates called Roche coordinates have been introduced, here ξ has been already defined by Roche equipotential srface along with η and ζ as orthogonal coordinates to ξ as well as amoung themselves. The orthogonal coordinates must satisfy the below given conditions as mentioned by Kopal[**19**].

$$\xi_x \eta_x + \xi_y \eta_y + \xi_z \eta_z = 0 \tag{3.78}$$

$$\xi_x \zeta_x + \xi_y \zeta_y + \xi_z \zeta_z = 0 \tag{3.79}$$

$$\eta_x \zeta_x + \eta_y \zeta_y + \eta_z \zeta_z = 0 \tag{3.80}$$

Let ξ be also wriiten as $\overset{*}{\Psi}$ such that the potential expression obtained under the effect of rotation,tidal and magnetic field is

$$\overset{*}{\Psi} = \frac{1}{\overset{*}{r}} + q + q \sum_{j=2}^{\infty} \overset{*}{r}^{j} P_j(\lambda) + n(1-\nu^2)\, \overset{*}{r}^{2} - \frac{\alpha}{\sqrt{\overset{*}{r}}} \tag{3.81}$$

To obtain the coordinates under the effect of only rotation and magnetic field we ignore the tidal effects and for that purpose we insert the values q=0.Hence the expression obtained is:

$$\overset{*}{\Psi} = \frac{1}{\overset{*}{r}} + n(1-\nu^2)\, \overset{*}{r}^{2} - \frac{\alpha}{\sqrt{\overset{*}{r}}} \tag{3.82}$$

By partially differentiating equation (3.82) with respect to r,λ and ν, we get:

$$\overset{*}{\Psi}_{\overset{*}{r}} = \frac{-1}{\overset{*}{r}^{2}} + 2n\,\overset{*}{r}\,(1-\nu^2) + \frac{\alpha}{2\overset{*}{r}^{3/2}} \tag{3.83}$$

$$\overset{*}{\Psi}_{\lambda} = 0 \tag{3.84}$$

$$\overset{*}{\Psi}_{\nu} = -2n\,\overset{*}{r}^{2}\,\nu \tag{3.85}$$

The transformation of coordinates follows that

$$\overset{*}{\Psi}_{x} = \lambda\,\overset{*}{\Psi}_{\overset{*}{r}} - \frac{(1-\lambda^2)}{\overset{*}{r}}\,\overset{*}{\Psi}_{\lambda} - \frac{\lambda\nu}{\overset{*}{r}}\,\overset{*}{\Psi}_{\nu} \tag{3.86}$$

$$\overset{*}{\Psi}_{y} = \mu\,\overset{*}{\Psi}_{\overset{*}{r}} - \frac{(\lambda\mu}{\overset{*}{r}}\,\overset{*}{\Psi}_{\lambda} - \frac{\lambda\mu}{\overset{*}{r}}\,\overset{*}{\Psi}_{\nu} \tag{3.87}$$

$$\overset{*}{\Psi}_{z} = \nu\,\overset{*}{\Psi}_{\overset{*}{r}} - \frac{(\lambda\nu}{\overset{*}{r}}\,\overset{*}{\Psi}_{\lambda} - \frac{(1-\nu^2)}{\overset{*}{r}}\,\overset{*}{\Psi}_{\nu} \tag{3.88}$$

using the above expression we get

$$\overset{*}{\Psi}_{x} = \lambda\left[\frac{-1}{\overset{*}{r}^{2}} + 2rn + \frac{\alpha}{2\overset{*}{r}^{3/2}}\right] \tag{3.89}$$

$$\overset{*}{\Psi}_{y} = \mu\left[\frac{-1}{\overset{*}{r}^{2}} + 2rn + \frac{\alpha}{2\overset{*}{r}^{3/2}}\right] \tag{3.90}$$

$$\overset{*}{\Psi}_{z} = \nu\left[\frac{-1}{\overset{*}{r}^{2}} + \frac{\alpha}{2\overset{*}{r}^{3/2}}\right] \tag{3.91}$$

Hence we obtain the value of

$$(\overset{*}{\Psi}{}^{2}_{x} + \overset{*}{\Psi}{}^{2}_{y} + \overset{*}{\Psi}{}^{2}_{z})^{-1/2} = \overset{*}{r}^{2}\left[1 + \frac{\alpha\sqrt{\overset{*}{r}}}{2} - \frac{\alpha^2\,\overset{*}{r}}{8} + 2n(1-\nu^2)\,\overset{*}{r}^{3} - n\alpha(1-\nu^2)\,\overset{*}{r}^{7/2} - 2\,\overset{*}{r}^{2}\,(1-\nu^2)\,\overset{*}{r}^{6}\right] \tag{3.92}$$

The diffrential equations that generate equipotential surface according to Kopal[19] are

$$\overset{*}{\Psi}_{x}\,dx + \overset{*}{\Psi}_{y}\,dy + \overset{*}{\Psi}_{z}\,dz = 0 \tag{3.93}$$

the equations that generate orthogonal lines to equipotential surface are

$$\frac{dx}{\overset{*}{\Psi}_{x}} = \frac{dy}{\overset{*}{\Psi}_{y}} = \frac{dz}{\overset{*}{\Psi}_{z}} \tag{3.94}$$

the integration constants of above equation generate the Roche coordinates η and ζ that correspond to angular coordinates of spherical polars.Further to transform the equation

(3.94) to form a suitable integrand, we devide it by normal area element ds, we know

$$\frac{dy/ds}{dx/ds} = \frac{\overset{*}{\Psi}_y}{\overset{*}{\Psi}_x} \, and \, \frac{dz/ds}{dx/ds} = \frac{\overset{*}{\Psi}_z}{\overset{*}{\Psi}_x} \tag{3.95}$$

the direction cosines of normal vector must obey the condition

$$(dx/ds)^2 + (dy/ds)^2 + (dz/ds)^2 = 1 \tag{3.96}$$

the equation (3.89 - 3.95) results

$$\frac{dx}{ds} = \frac{-\overset{*}{\Psi}_x}{(\overset{*}{\Psi}{}_x^2 + \overset{*}{\Psi}{}_y^2 + \overset{*}{\Psi}{}_z^2)^{-1/2}} \tag{3.97}$$

$$\frac{dy}{ds} = \frac{-\overset{*}{\Psi}_y}{(\overset{*}{\Psi}{}_x^2 + \overset{*}{\Psi}{}_y^2 + \overset{*}{\Psi}{}_z^2)^{-1/2}} \tag{3.98}$$

$$\frac{dz}{ds} = \frac{-\overset{*}{\Psi}_z}{(\overset{*}{\Psi}{}_x^2 + \overset{*}{\Psi}{}_y^2 + \overset{*}{\Psi}{}_z^2)^{-1/2}} \tag{3.99}$$

as $\xi(x, y, z)$ is a diminishing function of s, as a result we take the negative value of the square root. Employing the spherical polar coordinates

$x = r \sin\theta \cos\phi \quad y = r \sin\theta \sin\phi \quad z = r \cos\theta$

we obtain

$$\frac{dx}{d\overset{*}{r}} = \overset{*}{r}\frac{d\lambda}{\overset{*}{r}} + \lambda \tag{3.100}$$

$$\frac{dy}{d\overset{*}{r}} = \overset{*}{r}\frac{d\mu}{\overset{*}{r}} + \mu \tag{3.101}$$

$$\frac{dz}{d\overset{*}{r}} = \overset{*}{r}\frac{d\nu}{\overset{*}{r}} + \nu \tag{3.102}$$

$$\tag{3.103}$$

as the element ds is taken very small, so we can replace it by line element dr accordingly apply first order approximation

$$\frac{dx}{d\overset{*}{r}} = \lambda M \tag{3.104}$$

$$\frac{dy}{d\overset{*}{r}} = \mu M \tag{3.105}$$

$$\frac{dz}{d\overset{*}{r}} = \nu N \tag{3.106}$$

here

$$M = [1+\frac{n\alpha^2}{4}-\frac{2n}{\overset{*}{r}}-\frac{\alpha n}{\sqrt{\overset{*}{r}}}-\frac{3}{8}\alpha^2\overset{*}{r}+\frac{\alpha^2}{16}\overset{*}{r}^{3/2}-4n^2(1-\nu^2)\overset{*}{r}^{2}+2n^2\alpha(1-\nu^2)\overset{*}{r}^{5/2}+2n(1-\nu^2)\overset{*}{r}^{3}-$$
$$2n\alpha(1-\nu^2)\overset{*}{r}^{7/2}+\frac{n\alpha^2}{2}(1-\nu^2)\overset{*}{r}^{4}+4n^3(1-\nu^2)\overset{*}{r}^{5}-2n^2(1-\nu^2)\overset{*}{r}^{6}+n^2\alpha(1-\nu^2)\overset{*}{r}^{13/2}] \tag{3.107}$$

$$N = [1-\frac{3}{8}\alpha^2\overset{*}{r}+\frac{\alpha^2}{16}\overset{*}{r}^{3/2}+2n(1-\nu^2)\overset{*}{r}^{3}-2n\alpha(1-\nu^2)\overset{*}{r}^{7/2}+\frac{n\alpha^2}{2}(1-\nu^2)\overset{*}{r}^{4}-$$
$$2n^2(1-\nu^2)\overset{*}{r}^{6}+n^2\alpha(1-\nu^2)\overset{*}{r}^{13/2}] \tag{3.108}$$

by comparing (3.100 - 3.103) and (3.104 - 3.106) we obtain

$$\overset{*}{r}\frac{d\lambda}{\overset{*}{r}}+\lambda=\lambda M \tag{3.109}$$
$$\overset{*}{r}\frac{d\mu}{\overset{*}{r}}+\mu=\mu M \tag{3.110}$$
$$\overset{*}{r}\frac{d\nu}{\overset{*}{r}}+\nu=\nu N \tag{3.111}$$

integrating the above gives

$$\lambda_1 = C_1^{'}\exp(M_1) \tag{3.112}$$
$$\mu_1 = C_2^{'}\exp(M_1) \tag{3.113}$$
$$\nu_1 = C_3^{'}\exp(N_1) \tag{3.114}$$

here

$$M_1 = [\frac{n\alpha^2}{4}\log\overset{*}{r}+\frac{2n}{\overset{*}{r}}+\frac{\alpha n}{\sqrt{\overset{*}{r}}}-\frac{3}{8}\alpha^2\overset{*}{r}+\frac{3\alpha^2}{8}\overset{*}{r}^{3/2}-2n^2(1-\nu^2)\overset{*}{r}^{2}+\frac{4n^2\alpha}{5}(1-\nu^2)\overset{*}{r}^{5/2}+\frac{2n}{3}(1-\nu^2)\overset{*}{r}^{3}$$
$$-\frac{4n\alpha}{7}(1-\nu^2)\overset{*}{r}^{7/2}+\frac{n\alpha^2}{8}(1-\nu^2)\overset{*}{r}^{4}+\frac{4}{5}n^3(1-\nu^2)\overset{*}{r}^{5}-\frac{n^2}{3}(1-\nu^2)\overset{*}{r}^{6}+\frac{2}{13}n^2\alpha(1-\nu^2)\overset{*}{r}^{13/2}] \tag{3.115}$$

$$N_1 = [-\frac{3}{8}\alpha^2\overset{*}{r}+\frac{3\alpha^2}{8}\overset{*}{r}^{3/2}+\frac{2n}{3}(1-\nu^2)\overset{*}{r}^{3}-\frac{4n}{7}\alpha(1-\nu^2)\overset{*}{r}^{7/2}+\frac{n\alpha^2}{8}(1-\nu^2)\overset{*}{r}^{4}-$$
$$\frac{n^2}{3}(1-\nu^2)\overset{*}{r}^{6}+\frac{2}{13}n^2\alpha(1-\nu^2)\overset{*}{r}^{13/2}] \tag{3.116}$$

here $C_1, C_2 \quad and \quad C_3$ are integration constants that can be obtain by the condition

$$\lambda_1^2+\mu_1^2+\nu_1^2=1 \tag{3.117}$$

followed by this we obtain $C_1 = \lambda_0$, $C_2 = \mu_0$, $C_3 = \nu_0$ where zero subscript denotes zeroth order approximation. λ_0, μ_0 *and* ν_0 in terms of spherical coordinates are written as

$$\lambda_0 = \cos\theta \sin\phi \qquad \mu_0 = \sin\theta \sin\phi \qquad \nu_0 = \cos\theta \tag{3.118}$$

with the condition $\lambda_0^2+\mu_0^2+\nu_0^2 = 1$ for approximation of zeroth order.By using the dependence of λ_0, μ_0 *and* ν_0 *on* η *and* ζ, we suggest to make the use of following substitutions for the case of rotational distortions as

$$\lambda_1 = \cos\eta \sin\zeta \qquad \mu_1 = \sin\eta \sin\zeta \qquad \nu_1 = \cos\zeta \tag{3.119}$$

using equation(3.119), we get

$$\cos\eta = \frac{\lambda_1}{\sqrt{1-\nu_1^2}} \tag{3.120}$$

and also

$$cos\zeta = \nu_0 \exp(N_1) \tag{3.121}$$

the expression for η in equation 3.65 is exact, the expression for ζ is correct up to 2^{nd} order term in n and q.

3.6 The metric coefficients for rotationally and magnetically distorted stars

In our chosen system of coordinates (ξ, η, ζ) if we assume ζ= constant and then choose η and ζ in such a manner that they satisfy condition of mutual orthogonality w.r.t. ζ and among each other.

The general equations of orthogonality are same as in equation(3.78 - 3.80)

Also the value of r^* for the case of rotational and magnetic distortion is

$$r^* = r_\circ\{1 - \alpha r_\circ^{\frac{1}{2}} + \frac{\alpha}{2} r_\circ + n(1-v^2) r_\circ^3 - \frac{7}{2}\alpha n(1-v^2) x_\circ^{\frac{7}{2}} + 3n^2(1-v^2)^2 x_\circ^6\} \tag{3.122}$$

If we assume

$$x = x(\xi, \eta, \zeta); \qquad y = y(\xi, \eta, \zeta); \qquad z = z(\xi, \eta, \zeta) \tag{3.123}$$

In these the surface ξ=constant, η= constant and ζ=constant form orthogonal set of coordinates. In 3D transformation the metric coefficients can be written as

$$(dx)^2 + (dy)^2 + (dz)^2 = h_1^2(d\xi)^2 + h_2^2(d\eta)^2 + h_3^2(d\zeta)^2 \tag{3.124}$$

Now in this part we will be dealing with explicit expression of metric coefficients i.e. h_1, h_2 *and* h_3 that are associated with the system of Roche coordinates (ξ, η, ζ) for the Roche model of stars distorted by rotation,tidal and magnetic field.The metric coefficients are defined by:

$$h_1 = (\xi_x^2 + \xi_y^2 + \xi_z^2)^{-1/2} \tag{3.125}$$
$$h_2 = (\eta_x^2 + \eta_y^2 + \eta_z^2)^{-1/2} \tag{3.126}$$
$$h_3 = (\zeta_x^2 + \zeta_y^2 + \zeta_z^2)^{-1/2} \tag{3.127}$$

The value of h_1 obtained by substituting the value of r in terms of r_0 is

$$h_1 = \left(1 - \frac{cos^2\zeta \exp(-2N)}{2}\right)(r_\circ\{1-\alpha r_\circ^{\frac{1}{2}} + \frac{\alpha}{2}r_\circ + n(1-v^2)r_\circ^3 - \frac{7}{2}\alpha n(1-v^2)x_\circ^{\frac{7}{2}} + 3n^2(1-v^2)^2 x_\circ^6\}) \tag{3.128}$$

Further it can be shown that

$$\eta_x = -\frac{\mu}{r(1-v^2)}, \quad \eta_y = -\frac{\lambda}{r(1-v^2)} \; and \; \eta_z = 0 \tag{3.129}$$

from equation (3.126) and (3.129) it can be shown

$$h_2 = r\sqrt{(1-v^2)} \tag{3.130}$$

Similarly by putting the values of r_0 and ζ, h_2 *and* h_3 can be obtained as

$$h_2 = r\sqrt{(1-v^2)} \qquad\qquad h_3 = (\zeta_x^2 + \zeta_y^2 +_z^2)^{-1/2} \tag{3.131}$$

3.7 Conclusion

In the present work we have developed some explicit expressions for a modified Roche equipotential for a system of rotationally, tidally and magnetically distorted star. We developed a system of Roche coordinates i.e. $\xi, \eta \ and \ \zeta$ for rotationally and/or magnetically distorted stars.The system of these coordinates has been introduced by Kopal and it was claimed that these coordinates can be used to study various problems of distorted stars. In the absence of magnetic field i.e. for α=0 these expressions reduce to form for equipotential surface of rotationally and tidally distorted star.

The expressions developed in this thesis for Roche coordinates of rotationally and/or magnetically distorted star can further be used to study the problems of vibrational stability of rotationally and magnetically distorted models of star under the influence of radial and non radial modes of oscillator.

It must be noted that in the present work we have taken terms with 2^{nd} order smallness in n,q and α, the work can be extended to higher orders by explicitly forming some new expressions.

Moreover a similar kind of approach to binary stars can be made with the help of Clairauts coordinates.

It should be noticed that we have taken into consideration the Roche equipotential surface of the binary system according to Roche model of the stars.This present work can be extended to more realistic type of stars instead of using Roche model of stars to study their internal structure.

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